Transient SH-Wave Interaction with a Cohesive Interface

THESIS

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Abstract

Characterization of material damage at the interface between two bodies using non-destructive evaluation (NDE) techniques is a field of study that is important from the point of view of both, research and application. In order to provide useful design engineering tools to the practicing engineer in the field of NDE it is necessary to build robust models that can be easily implemented. In the present thesis SH body waves are chosen as a representative candidate to assess material damage and degradation at an interface. In addition a well-established material damage law in the form of cohesive zones from fracture mechanics is considered. The combined effects of transient wave motion and the cohesive boundary condition at the interface are studied in order to develop a methodology for potential use in an NDE application scenario. Parameter variation studies offer promising results to the practicing engineer. Specifically, the trends of displacement and traction along the interface are evaluated between the bonded and damaged material conditions. Parameter variations allow for insight into how altering the material parameters and wave setup affect macro-behavior of the interface.
Dla pogoń za wiedzą, i do mojej rodziny.

Dedication
Acknowledgments

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Chapter 1: Introduction

1.1) Background

The iterative design process in any engineering discipline serves many purposes; these include meeting design criteria to satisfy the customer’s needs, ensuring ease of manufacturing and maintenance as well as designing components and systems against overuse, misuse and catastrophic failure. The latter are of utmost importance in disciplines where devices and systems support human life. Systems commonly subjected to more rigorous design analyses against failure include aircraft components, structural components such as bridge supports and skyscraper columns and girders, as well as vehicle components. This is evident in the works of Tilly [1] who explored fatigue loading and its effect on concrete reinforcement bars of various types. Catastrophic failure was shown to occur even with fatigue loading of small cracks, a reality many structures enduring cyclic loading show. Failure loads could also be caused by wave propagation through the components or systems as will be explored further in the course of this work.

The focus of the work being presented in this thesis contains important aspects of wave propagation through solids as previously stated. Previous research such as what has been studied in geophysical research has been important in determining the mechanisms of seismological behavior as early as the 1950s and 1960s by Toksöz and Anderson [2]. Working off of Love’s generalized plane stress equations for surface waves, stress-strain...
matrices were related for a flat plate with a central wave source. Such models were used to predict the behavior of large geological structures like ice sheets or tectonic plates. Such research predictions have been especially important in seismic wave propagation through soil as indicated by Pascoe et al. [3]. The presented research reduced dimensionality to one space dimension and time with the elastodynamics equations for coupled P-SV motion. The finite-difference technique was used with an algorithm based on the Alekseev-Milhailenko method for this type of wave motion which showed a high degree of accuracy to the exact wave solution for an inhomogeneous half space representing a layer of soil. Their work among others has been crucial in understanding wave dynamics and their impact on stress-strain values for different types of setups.

As was alluded to, many types of wave motion could be coupled wave motions where more than one type of wave travel is occurring simultaneously. Represented in Figure 1.1 are three of the basic types of body waves. These include P-waves and S-waves (shear waves) of two types of which the latter will be of study in this thesis. Waves studied in civil engineering are typically surface waves of which include Love, Stoneley and Rayleigh waves. The SV wave is polarized horizontally from SH waves which are polarized vertically. The presented methods however can be extended to any of the three body wave types in most cases.

Combining the various types of waves to be studied with the various applications or setups possible provides for a large variety of research study capabilities. It should be noted however as was the case Toksöz and Anderson [2] and Pascoe et al. [3] that a half space infinite media must be used to apply known solutions to the problem. Many problems
have been approximated with this setup such as geographical regions of soil which at large are large flat plans that are susceptible to wave impulses traveling through. However, other problems exist and have been studied with layered half spaces as shown by the work of Doong [4], Thapar [5] and Mandal et al. [6]. The work of Doong was primarily with unperturbed boundaries as was the work of Mandal et al. while Thapar explored the effect of boundary perturbations on Rayleigh wave amplitudes. Doong focused on interface problems including those with frictional interactions of an SH wave pulse. It should be noted that the work of Doong was a primary source for boundary element codes and reference in this thesis. This interface type of study also allows for material characterization.

Figure 1.1: P-Wave and S-Wave Representation for Travel Direction

1 http://alabamaquake.com/education.html
The interface to be used for damage detection and material characterization in the study that follows will be used with two sets of boundary conditions. These boundary conditions will allow for comparisons between perfectly bonded materials for calibration against damaged materials. The boundary condition selection for the damaged interface case is critical in evaluating the viability of this damage detection method. A classic candidate for this damage model is the “cohesive zone” model used in fracture mechanics. Cohesive zones are used to approximate the non-linear behavior at a crack tip and will be used as the primary source of damage modeling in this method.

Nearly all parts or structures have flaws or imperfections within their material structure. Flaws can range in scale from nano to micro-level flaws to macro-level flaws, the latter two of which include cracks. Fracture mechanics as the study of these material flaws and their behavior aims at calculating exact or approximate stress values for various setups, many of which are idealized or physically impossible such as the infinite body setup shown. Zehnder’s book [7] on fracture mechanics describes many idealized cases for stress field analysis and such a sample case of an infinite body with a center crack in uniform tension is shown in Figure 1.2 [7].

![Figure 1.2: Crack Setup in Infinite Body Under Tension](image)
Many analytical equations exist for such setups described. However, finite element softwares such as ANSYS and Abaqus can calculate values used to evaluate cracks such as $K_I$ and the J-integral easily and quickly. More complex geometries and loading types are available for use in such programs. For more complex problems and investigations into crack influence on macro-structure behavior, computational solutions to crack problems have been studied extensively for unique and realistic geometries as shown by the works of Brenner [8], Oliver et al. [9], and Sánchez et al [10]. Brenner for example presented a multi-grid method for calculating the singularity crack tip solutions and stress intensity factor. Such modifications to the finite element equation are improvements to hand-written codes that exist.

In all that has been studied with fracture mechanics, one popular and continuously evolving area of research is that of the area of cohesive zones. This is where the damage model to be used in the wave propagation study presented is derived. Singularities at the crack tip present problems to analytical equation-based methods and are still challenging for finite element solutions to accurately compute displacement and stress action at crack tips. Cohesive zones were initially theorized and developed by Barenblatt [11], Dugdale [12], and Bilby et al. [13] around 1960. Cohesive zone study was initiated to completely describe crack opening tractions using displacements of each crack face. It should be noted that one of the first applications by Dugdale in [12] was for steel sheets containing slits, a similar setup to what was shown in Zehnder [7].

Initial motivation for cohesive zone models was their accuracy in simulating the de-bonding of the atomic structures inside a propagating crack tip. Use of cohesive zones
simulating this de-bonding for dynamic fracture models was extensively studied by Falk et al. [14]. The idea behind this model was to simulate the hardening and softening process of material at the tips of cracks. Singularities were removed from crack tips mathematically using this approximation with continuous traction displacement laws being proposed of various types. Such methods have been used as shown by Remmers et al. [15] to approximate opening displacements for calculating crack growth. The referenced study suggested the exponential nature of the complete cohesive zone law which can be approximated but must be approached from linear laws in discrete element codes that use linear systems for solution. The suggested exponential cohesive zone law describing crack opening displacement to crack opening tractions is shown in Figure 1.3 [11].

![Figure 1.3: Exponential Cohesive Zone Law](image)

Two portions of the plot in describe two separate behaviors of the cohesive zone. The maximum traction value is named as the “cohesive strength” or “$t_0$” labeled $T_c$ in the plot. The variable “$t_0$” will be used in this thesis. The maximum displacement jump, which will be further explained later, is the largest displacement jump on this plot indicating ultimate failure of the tip of the crack. This displacement jump is the difference between displacement of each crack face and is named “crack tip opening displacement (CTOD)”
denoted by \( \ddot{\delta}_0 \) labeled as such in the plot. The portion of this exponential law prior to reaching \( t_0 \) is known as the “hardening” portion of the law which is followed by the “softening” portion. This will be explained further during equation formulation.

Such exponential cohesive zone laws are useful and accurate in predicting the true behavior of damaged material at the tip of a crack, however implementation in a boundary element code is not possible based on the linear system of equations used in such codes. Approximations to this exponential model must linearize each portion of the cohesive zone law. Chalivendra et al [16] used cohesive zone models by first obtaining experimental data on cohesive separation from a projectile fired into a plate. Data in this study confirmed the nonlinear nature of the law through experimentation and implementation into finite element codes was possible with special cohesive elements. However for boundary element problems, linear approximations to these laws are a requirement. Shown in Figure 1.4 are examples of linear approximations to the softening portion of the exponential law. These and other approximations were further described by Park and Paulino [17].
Various forms of the cohesive zone model have been proposed and as described in Chandra [18], many researchers have suggested that the cohesive strength and CTOD values are most critical in determining accuracy or validity of various cohesive zone models. However, Chandra investigates and demonstrates the importance of the shape of the cohesive zone laws for predicting mechanical macro-response of materials. In support of this work was research conducted by Falk et al. [14] which more specifically shows that cohesive law shapes are critical factors in crack branching problems. The presented results showed that the scale of branching depends on whether or not the relation is initially linear or rigid. Both sets of work would suggest a model similar to that of d) in Figure 1.4. However, work since this research has shown contrary data supporting other parameters’ importance in accuracy of the law.
Tvergaard and Hutchinson [19] indicated that for their ductile fracture specimen models the results for crack growth indicated a more significant dependence on $t_0$ and $\delta_0$ than the shape of the cohesive zone curves. This result implies that the more important facture in crack growth is the fracture energy of the crack, calculated by finding the area under these cohesive zone curves. Both results previously discussed have validity in different cases but de Borst et al. [20] supported that the shape of the curve drives fracture in brittle materials more readily than ductile materials. This is an important finding for the work to be presented in this thesis as Aluminum was selected a base material of choice for simulation. Its high ductility tends towards the models being sensitive to the parameters selected for the cohesive strength and CTOD.

To implement the ideas discussed of wave propagation for damage detection using the cohesive zone as a damage model, boundary element methods are especially useful for such problems especially those with material interface setups. Mendelsohn and Doong [21] investigated frictional interface interaction using a boundary element method with two layered semi-infinite half spaces to study behavior of an existing crack at a material interface. Similar methods of boundary truncation used by Mendelsohn and Doong were used by Halpern [22] to demonstrate how to apply boundary conditions for discretized one-dimensional wave solutions. Such methods again must be used in boundary element computational codes to narrow the window of solution in space dimensions. Mokashi [23] in his dissertation used a direct boundary element formulation to model a linear softening cohesive zone ahead of a crack tip in a simply supported beam with an edge crack.
Discretization took place purely along the boundary and J-integral calculations along the crack interface were conducted for use in non-linear vibrations.

Combining the ideas of cohesive zones (Mokashi [23]) with wave propagation and boundary element methods (Doong [4]) provides the framework to investigate wave propagation through a cohesive interface for damage detection. The background of this non-destructive evaluation technique will now be given to identify components of prior work to be used and important implications and applications to this and future wave propagation research.
1.2) Focus of Thesis

The objective of the work presented in this thesis is to investigate material characterization of an interface between two dissimilar materials. Material characterization occurs nondestructively by sending an SH wave pulse through two semi-infinite bodies separated by an infinite interface. The infinite interface is used to simulate two primary material behavior cases; first a completely bonded interface is used between the two materials and a cohesive zone is then implemented along the boundary. Truncation of the semi-infinite bodies occurs in an area of approximately (60 m x 1000 m) to create a finite mesh size along the boundary. An out of plane line source wave was also used to send the wave pulse.

The goal of sending an SH wave pulse through the interface is to qualitatively and quantitatively understand the displacement-traction relationship behavior at the interface for wave arrival. The bonded interface case is primarily used for code calibration and comparison to work done by Doong [21]. Following verification of the bonded case is the implementation of the cohesive zone laws to implement a damage model. Equation modification is required for the linear system of equations to accurately capture this material behavior. Use of a cohesive zone for wave propagation has not been completed previously after a thorough literature review of other relevant wave propagation and cohesive zone work.

Parameter variation occurs for material mismatch at the interface as well as various cohesive zone parameters. A straight line softening law is used to approximate the exponential softening portion of the cohesive zone curve previously described. Hardening
is therefore considered to have taken place with interface tractions being equal to the cohesive strength initially during simulation.

To implement the above models a direct boundary element formulation was implemented as discussed. The boundary element formulation was implemented with a linear system of equations in MATLAB and solution was obtained transiently by stepping in time. A linear system of equations was formed to solve for traction and displacement along the bonded interface and displacements of both sides were solved for in the case of the cohesive zone law. Boundary conditions to reduce the number of unknowns along the boundary will be explained for each case.

Future research for this work includes simulating rapid vibrational wave pulses being sent from the line source of high frequency. A linear model was used for the softening portion of the cohesive zone but a bi-linear model could be used to increase the accuracy of the simulation. Further changes must be made to the code to track wave reflection and transmission in each body. Current 3D maps of displacement show general wave propagation but do not track wave reflection and transmission from one body to another. A further significant addition to the code would be alteration of the out of plane line source wave to represent a plane wave pulse at a specified distance from the interface. This along with further parameter variation summarizes future research efforts that must be made with these models and codes.
1.3) **Significance of Research**

As previously stated, cohesive zone research has been popular in fracture mechanics because of the simplicity of the model for crack openings and its potential accuracy in predicting total fracture energy. Much work has been done including that most recently by Dean [24] in implementing cohesive zone models with boundary element methods, specifically for a center cracked beam under cyclic axial loading. What is lacking in literature related to cohesive zones is wave propagation results for any material with such damage laws. The presented research aims to shed light on this phenomena.

Wave propagation through cohesive zones has implications for nondestructive evaluation of materials. Materials with an unknown amount of damage could be evaluated by comparing bulk or macro-response to some known documented baseline. For example, if a material exhibited a certain percentage larger displacement or vibrational response at an interface, this could indicate the presence of micro or macro-cracks based on the level of increased compliance. With cohesive zones as the basis for this research, a fair amount of accuracy could be predicted for this material behavior and comparisons could be made.

Ultimately, this research code could be taken and modified to simulate a variety of situations. This may include different types of waves as well since some materials or applications may require a P-wave instead of shear waves. Many industrial applications include designing against shock failure from wave propagation. These are especially pertinent in applications such as the aerospace industry where parts like turbine blades or wingspan joints must retain rigidity and strength even with damage from repeated cycling caused by takeoff and landings.
1.4) Overview of Thesis

The presented thesis contains five chapters of work. Chapter 1 is an introduction to wave propagation, cohesive zone and its relevance in fracture mechanics and the boundary element method used in this research. The second chapter is formulation for the boundary element method and the pertinent bonded and cohesive zone boundary condition adjustments to the equations. The third chapter presents normalized results versus time for displacement and traction for the bonded interface case and displacement, displacement jump and traction for the cohesive zone case. Results are presented for parameter variation with material mismatch, source distance and source strength for both boundary conditions as well as constant cohesive energy, and altering cohesive energies through $t_0$ and $\delta_0$ separately. Chapter 4 presents preliminary unloading results for one set of material parameters. Chapter 5 concludes the thesis and presents opportunities for future work.
Chapter 2: Formulation

The problem considered in the present work is that of an infinite interface between two isotropic, homogeneous elastic bodies with different material moduli. Three types of body waves can propagate through the combined system. They are in-plane, longitudinal (P) and shear (SV) waves, and anti-plane shear waves (SH). These three waves can exist independently of each other. Surface waves such as Stoneley, Love, Lamb and Rayleigh waves are not considered. The governing equations for in-plane motion are coupled, while SH motion is independent of the other two. Therefore, SH waves are good candidates for dynamic material characterization of the interface. The resulting integral formulation is presented next.

2.1) Governing Equations

The starting point in the formulation is a review of equations of motion and stress-strain equations for a homogeneous isotropic body in Cartesian coordinates. These equations from Doong’s work [4] are shown as follows.

\[ \sigma_{ij,j} + \rho f_i = \rho \ddot{u}_i \]  

(2.1)
\[\sigma_{ij} = \lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij}\]  
(2.2)

\[\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})\]  
(2.3)

\[\omega_{ij} = \frac{1}{2} (u_{i,j} - u_{j,i})\]  
(2.4)

Displacement of a material point is represented by \(u_i\), and from continuum mechanics, the stress tensor is represented by \(\sigma_{ij}\). The mass density and body force per unit volume density are represented by \(\rho\) and \(f_i\) respectively. The variables \(\epsilon_{ij}\) and \(\omega_{ij}\) are the strain and rotation tensors also taken from continuum mechanics. In the presented notation, \(\lambda\) and \(\mu\) are Lame’s constants the latter of which is the same as shear modulus \(G\) which will be used moving forward. For a perfectly isotropic material \(\lambda = \frac{2Gv}{1-2v}\). It should be noted in equation formulation that a comma represents spatial differentiation and dots represent time differentiation. Repeated indices from continuum mechanics imply the summation convention and \(\delta_{ij}\) is the Kronecker delta.

Initial conditions for the dynamic problem to be presented are based on displacement at the initial time of 0 seconds. Displacements and velocities at any material point are considered to be functions of position only as shown by the following two equations.

\[u_i(x, 0) = u_0(x)\]  
(2.5)

\[\dot{u}_i(x, 0) = v_0(x)\]  
(2.6)
The general mixed boundary conditions along the boundary $\partial B$ of body B are later specified in this chapter. These boundary conditions must be satisfied for all of time for each material characterization set of results to be discussed.

Following the work of Doong [4] by substituting the strain relationship into the equation of motion, Navier’s equation is obtained as shown.

$$G u_{i,jj} + \frac{G}{1-2\nu} u_{j,ji} + b_i = 0, i = 1,2,3.$$ (2.7)

In reality, (2.7) represents three equations to account for equilibrium in the three principal directions where $\nu$ is Poisson’s ratio. The term $b_i$ represents a body force per unit volume in the $i^{th}$ direction. It should be noted that in comparison to the work of Doong [4], the body force term denoted by $b_i$ is equal to the force and acceleration terms as shown.

$$b_i = \rho (f_i - \ddot{u}_i)$$ (2.8)

Assuming a unit load in the $x_3$ direction, Green’s functions for fundamental displacement and traction tensors can be found and are considered known. This is used to transform (2.7) into an integral equation in space and time for a two-dimensional elastic domain $\Omega$ over a boundary $\Gamma$.

2.2) Direct Boundary Element Formulation

The basis for boundary element method codes is given by Betti’s Reciprocal Work Theorem as shown.

$$\int_{\Omega} \sigma_{ij}^* \varepsilon_{ij} d\Omega = \int_{\Omega} \sigma_{ij} \varepsilon^*_{ij} d\Omega$$ (2.9)
As previously mentioned, the formulation discussed is for a two state solid which has boundaries $\Gamma$ and $\Gamma^*$ enclosing regions $\Omega$ and $\Omega^*$ respectively. Betti’s Reciprocal Work Theorem is a conservation of work statement, whereas the work done by the strains in $\Omega$ on the stresses in $\Omega^*$ is equal to the work done by the strains in $\Omega^*$ on the stresses in $\Omega^*$. The setup for these two fields which are self-contained by $\Gamma^*$ are shown in Figure 2.1.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Two State Fields for Boundary Element Formulation}
\end{figure}

After applying divergence theorem to (2.9) and using equilibrium on a volume element which gives $\sigma_{ij,j} + b_i = 0$ and $t_i = \sigma_{ij}n_j$ as a definition for tractions, (2.10) is derived from Brebbia and Mansur [25].

$$\int_{\Omega} b^*_i u_i dA + \int_{\Gamma} t^*_i u_i ds = \int_{\Omega} b_i u^*_i dA + \int_{\Gamma} t_i u^*_i ds$$  \hspace{1cm} (2.10)

Integration is done on each side of the equation over the bodies and boundaries for displacements and tractions of each region from Figure 2.1. This integration is also done with respect to time and the final form of this elastodynamics equation is shown.

$$\int^t_0 \int_{\Omega} b^*_i u_i dAdt + \int^t_0 \int_{\Gamma} t^*_i u_i dsdt = \int^t_0 \int_{\Omega} b_i u^*_i dAdt + \int^t_0 \int_{\Gamma} t_i u^*_i dsdt$$  \hspace{1cm} (2.11)
These are again represented by the “starred” and “un-starred” fields. To further define the body force terms in the equation, the Dirac delta function is implemented to find an impulse response to the equation. Letting \( \delta(\xi - x) \) represent the Dirac delta function to form the following equations

\[
\int_{\Omega} \delta(\xi - x)f(x)d\Omega(x) = 0, \quad \xi \notin \Omega \tag{2.12}
\]

\[
\int_{\Omega} \delta(\xi - x)f(x)d\Omega(x) = f(\xi), \quad \xi \in \Omega \tag{2.13}
\]

From previous discussion, \( b^*_i \) represents a body force per unit volume in the 3-direction. The resulting body force load is described by (2.14) as such.

\[
b^*_3 = \delta(\xi - x) \tag{2.14}
\]

Substituting this body force equation into the reciprocal representation in (2.11) results in the following.

\[
u_j(\xi) = \int^t_0 \int_{\Gamma} U^*_i(\xi, x)t_j(x)ds(x)dt - \int^t_0 \int_{\Gamma} T^*_i(\xi, x)u_j(x)ds(x)dt + \int^t_0 \int_{\Omega} U^*_i(\xi, x)b_j(x)dA(x)dt, \quad i = 3 \tag{2.15}
\]

This statement is a representation of displacements at any point \( \xi \) in the domain \( \Omega \).

From Mokashi [23], taking the limit of Eqn. (2.15) when \( \xi \) approaches the boundary \( \Gamma \) of \( \Omega \) produces the same statement except the term on the left is multiplied by 0.5. With this knowledge, the following two statements are used in tandem.

\[
c_{ij}u_j(\xi) = \int^t_0 \int_{\Gamma} U^*_i(\xi, x)t_j(x)ds(x)dt - \int^t_0 \int_{\Gamma} T^*_i(\xi, x)u_j(x)ds(x)dt + \int^t_0 \int_{\Omega} U^*_i(\xi, x)b_j(x)dA(x)dt, \quad i = 3 \tag{2.16}
\]
\[ c_{ij} = \begin{cases} 1, & \xi \in B \\ \frac{1}{2}, & \in \partial B \\ 0, & otherwise \end{cases} \]

The tensorial quantities \( U_{ij}^* \) and \( T_{ij}^* \) represent displacement and traction Green’s tensors and were given in Doong [4] as shown for an out of plane shear wave source in one of the two regions discussed.

\[
U_{ij} = \frac{1}{2\pi\rho} \left[ \left( \frac{2t^2-r^2}{c_1^2} \right) H(t-r/c_1) - \left( \frac{2t^2-r^2}{c_2^2} \right) H(t-r/c_2) \right] \frac{r_i r_j}{r^4} - \left( H \left( t - \frac{r}{c_1} \right) \sqrt{\frac{t^2 - r^2}{c_1^2}} - H \left( t - \frac{r}{c_2} \right) \sqrt{\frac{t^2 - r^2}{c_2^2}} \right) \delta_{ij} \right]
\]

\[ T_{ij} = \sigma_{ikj} n_k \text{ where } \sigma_{ikj} = -\rho \left( c_1^2 - 2c_2^2 \right) \frac{\partial u_{ip}}{\partial x_p} \delta_{ik} - \rho c_2^2 \left( \frac{\partial u_{ij}}{\partial x_k} + \frac{\partial u_{kj}}{\partial x_i} \right) \]

Wave speed is denoted by \( c_1 \) and \( c_2 \) for bodies 1 and 2 in the formulation. Rayleigh wave speed is calculated by \( c = \sqrt{\frac{\mu}{\rho}} \) which varies based on the shear modulus of each body’s material. \( H(...) \) is defined as the Heaviside function, \( t \) is the time and the variable \( r \) is based on the distance between the impulse source point and each element’s position.

The described boundary element method is used in conjunction with a discretized boundary which in the case of interface problems is just along the infinite interface. If taking \( M \) line elements along the interface where \( \Gamma_k = 1,2,\ldots,M \) and assuming the tractions and displacements at each element to be constant, the resulting equation is simplified with the two expressions shown.
\[ t_i(x) = t^k_i = \text{constant}; \; x \in \Gamma^k \]  
(2.18)

\[ u_i(x) = u^k_i = \text{constant}; \; x \in \Gamma^k \]  
(2.19)

\[ i = 3 \; \text{for SH wave motion} \]

Using Eqn. (2.18) and Eqn. (2.19) into Eqn. (2.16) along the interface results into the two summation equations shown for SH wave motion.

\[ \frac{1}{2} u^l_1 = \sum_{k=1}^{M} \left[ 1 U_{33}^l 1 \tau_3^k + 1 T_{33}^l 1 u_3^k - 1 b_3^k \right] \]  
(2.20)

\[ \frac{1}{2} u^l_2 = \sum_{k=1}^{M} \left[ 2 U_{33}^l 2 \tau_3^k + 2 T_{33}^l 2 u_3^k \right] \]  
(2.21)

\[ l = 1, 2, \ldots, M \]

The extra term in Eqn. (2.20) is the body force wave source input to the system of equations. The equations can be re-written to be set equal to known constants as shown in Eqn. (2.22) and Eqn. (2.23).

\[ \sum_{k=1}^{M} \left[ 1 U_{33}^l 1 \tau_3^k + \left( 1 T_{33}^l - \frac{1}{2} \delta_{lk} \right) 1 u_3^k \right] = 1 b_3^k \]  
(2.22)

\[ \sum_{k=1}^{M} \left[ 2 U_{33}^l 2 \tau_3^k + \left( 2 T_{33}^l - \frac{1}{2} \delta_{lk} \right) 2 u_3^k \right] = 0 \]  
(2.23)

Matrix representations of the two previous equations are shown in their final basic form in Eqn.’s (2.24) and (2.25).

\[ \begin{bmatrix} 1 U_{33} \end{bmatrix} \begin{bmatrix} 1 \tau_3 \end{bmatrix} + \begin{bmatrix} 1 T_{33} \end{bmatrix} \begin{bmatrix} 1 u_3 \end{bmatrix} = \begin{bmatrix} 1 b_3 \end{bmatrix} \]  
(2.24)

\[ \begin{bmatrix} 2 U_{33} \end{bmatrix} \begin{bmatrix} 2 \tau_3 \end{bmatrix} + \begin{bmatrix} 2 T_{33} \end{bmatrix} \begin{bmatrix} 2 u_3 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \]  
(2.25)

A complete matrix representation of the system of equations is shown. Note that the system for SH wave motion through an interface between two dissimilar solids is in terms of two equations for every node with four unknowns per node before applying boundary
conditions. For simplicity and clarity the directional indices are dropped considering SH motion involves $u_3$ only.

\[
\begin{bmatrix}
U_1 & \cdots & T_1 \\
\vdots & \ddots & \vdots \\
U_2 & \cdots & T_2
\end{bmatrix}
\begin{bmatrix}
\tau_1 \\
\vdots \\
u_1
\end{bmatrix}
= 
\begin{bmatrix}
b_1 \\
\vdots \\
0
\end{bmatrix}
\]

The discussed model represents a two body formulation for wave propagation through an infinite interface. The ellipses represent the number of equations required based on the number of elements $M$ along the interface. The two sets of boundary conditions used in modeling material response are discussed in the following sections.

2.3) Bonded Material Interface Problem

For calibration purposes, a bonded interface case was studied to compare to the work of Doong [4]. Two semi-infinite layered solids separated by an infinite interface were setup in MATLAB and fictitious boundaries of approximately $40m \times 1000m$ were setup for simulation. As shown in Figure 2.2, an out of plane line source was placed at a distance $d$ from the interface and at a vertical distance symmetric from the truncation boundaries.
In accordance with Eqn.’s (2.24) and (2.25), displacements and tractions at every element along the interface are assumed to be unknown and constant. $B_1$ and $B_2$ represent the two dissimilar materials with shear modulus mismatch as discussed further in the results section.

For a bonded interface setup using Figure 2.2, displacements of each body at the interface are coupled ($u_1 = u_2$), and tractions are set equal and opposite for bodies 1 and 2 ($t_1 = -t_2$). Using these boundary conditions with the original matrix representations from Eqn. (2.24) and (2.25) results in Eqn. (2.26) and (2.27). It is a choice to solve for tractions and displacements only along body 1 or body 2 at the interface. By choice, body 1 was selected to find unknown tractions and displacements.

\[
\begin{bmatrix}
_1U_{33}
\end{bmatrix}
\begin{bmatrix}
_1\tau_3
\end{bmatrix}
+ \begin{bmatrix}
_1T_{33}
\end{bmatrix}
\begin{bmatrix}
_1u_3
\end{bmatrix} = \begin{bmatrix}
_1b_3
\end{bmatrix}
\] 
(2.26)

\[
-\begin{bmatrix}
_2U_{33}
\end{bmatrix}
\begin{bmatrix}
_1\tau_3
\end{bmatrix}
+ \begin{bmatrix}
_2T_{33}
\end{bmatrix}
\begin{bmatrix}
_1u_3
\end{bmatrix} = [0]
\] 
(2.27)
Combining the two discussed equations into one matrix representation results in the final linear system of equations for the bonded material interface simulations.

\[
\begin{bmatrix}
\begin{bmatrix}
U_1 \\
\vdots \\
-2U_1 \\
\end{bmatrix} & \cdots & \begin{bmatrix}
T_1 \\
\vdots \\
-2T_1 \\
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
\tau_1 \\
\vdots \\
\end{bmatrix} \\
\begin{bmatrix}
u_1 \\
\vdots \\
\end{bmatrix}
\end{bmatrix} =
\begin{bmatrix}
\begin{bmatrix} b_1 \\
\vdots \\
0 \end{bmatrix}
\end{bmatrix}
\]

The terms \[\begin{bmatrix} U_1 \end{bmatrix}\] and \[\begin{bmatrix} T_1 \end{bmatrix}\] represent displacement and traction green’s matrices that determine when elements along the interface are excited by the source wave. It should be noted that in practice, these are implemented as row vectors within the larger global matrix which is \(2M \times 2M\) in size.

Modifications to the original equations were also done to accommodate the cohesive zone law representing material damage. The formulations to follow identify the new equations to accommodate damage.

2.4) Cohesive Zone Material Interface Problem

\textit{Loading}

As shown in Figure 1.3 the cohesive zone law has been hypothesized to be exponential in nature, however for boundary element methods, linear approximations of any governing material laws must be implemented. One key assumption in the presented research is that hardening has occurred within the cohesive zone along the interface. This means that only the softening portion of the law is to be approximated. For loading where the wave pulse has a strength that is at least constant or increasing in time, the following discussion presents the formulation for the equations of the cohesive zone model.
As discussed, many approximations to this softening portion have been used, however one of the simpler models was implemented in this research. The linear softening cohesive zone law being used is shown in Figure 2.3.

![Figure 2.3: Linear Softening Cohesive Zone Approximation](image)

The equation for the linear softening relationship is shown in Eqn. (2.28).

$$t = t_0 \left( 1 - \frac{|\delta|}{\delta_0} \right)$$  \hspace{1cm} (2.28)

Two important parameters identified in this relationship to be used throughout the presented research are *cohesive strength*, shown as $t_0$ and *crack tip opening displacement (CTOD)* denoted by $\delta_0$. It should also be noted that displacement jump denoted by $\delta$ is the difference between displacements of body 1 and 2 at the interface. With this modification to Eqn. (2.28) the final version of the linear softening cohesive zone law is shown in Eqn. (2.29).

$$\tau = t_0 \left( 1 - \frac{|1u_3 - 2u_3|}{\delta_0} \right)$$  \hspace{1cm} (2.29)

Equation (2.29) can be used to define tractions along $B_1$ or $B_2$ depending on the direction of interface shift at each time step, but for simplicity it will be used to define
tractions along $B_1$. This in conjunction with Eqn. (2.24) results in the modified cohesive zone equation for $B_1$ as shown in Eqn. (2.30).

$$\left[ U_{33}^1 \right] t_0 \left( 1 - \frac{|u_3^1 - u_3^2|}{\delta_0} \right) + \left[ T_{33}^1 \right] [u_3] = \left[ b_3^1 \right]$$

(2.30)

The absolute value function was removed and applied after solution in the MATLAB code. After simplifying, the final version of the general cohesive zone representation for $B_1$ is shown in Eqn. (2.31).

$$\left[ U_{33}^1 \right] t_0 - \left[ U_{33}^1 \right] t_0 \frac{|u_3^1 - u_3^2|}{\delta_0} + \left[ T_{33}^1 \right] [u_3] = \left[ b_3^1 \right]$$

$$\therefore \left[ T_{33}^1 \right] - \frac{\left[ U_{33}^1 \right] t_0}{\delta_0} \right) [u_3] - \frac{\left[ U_{33}^1 \right] t_0}{\delta_0} [2u_3] = \left[ b_3^1 \right] - \left[ U_{33}^1 \right] t_0$$

(2.31)

The second boundary condition used where tractions are equal and opposite from $B_1$ to $B_2$ is still implemented in the cohesive zone formulation. Using this boundary condition, Eqn. (2.32) describes the cohesive zone relationship to implement for $B_2$.

$$-\tau_1 = \tau_2 \quad \therefore \tau_2 = -t_0 \left( 1 - \frac{|u_3^1 - u_3^2|}{\delta_0} \right)$$

(2.32)

Using Eqn. (2.32) with (2.25) results in the similar formulation for $B_2$.

$$-\left[ U_{33}^2 \right] t_0 \left( 1 - \frac{|u_3^1 - u_3^2|}{\delta_0} \right) + \left[ T_{33}^2 \right] [u_3] = [0]$$

$$\therefore \left[ U_{33}^2 \right] t_0 [u_3] + \left[ T_{33}^2 \right] - \frac{\left[ U_{33}^2 \right] t_0}{\delta_0} \right) [u_3] = \left[ U_{33}^2 \right] t_0$$

(2.33)

Combining Eqn.’s (2.32) and (2.33) into one matrix representation for the final linear system of equations for the cohesive zone material interface simulations.
During loading, the matrix representation of the system of equations applies until the input wave source has a force value decreasing versus time. At this time step, unloading commences and a new formulation is required based on element unloading towards the origin of the cohesive zone plot, explained in the unloading section.

**Unloading**

Solutions in the code presented tracked body force values versus time, and unloading was initiated when body force source quantities began to decrease versus time. Figure 2.4 demonstrates the unloading line assumed during unloading towards the origin.

![Cohesive Zone Element During Unloading](image)

Figure 2.4: Cohesive Zone Law Representing Unloading of Element
Each element during the unloading initiation time step is tracked based on its position on the plot represented by \((\alpha, \beta)\). The dotted line in Figure 2.4 represents the unloading line which is used in the new formulation. This line is defined as shown in Eqn. (2.34)

\[
\tau_1 = \frac{\beta}{\alpha} \delta
\]  

(2.34)

Using the definition of displacement jump, the equation is modified to its final form for cohesive zone unloading. This equation is applied for every element along the interface for their specific \((\alpha, \beta)\) at that time step.

\[
\tau_1 = \frac{\beta}{\alpha} |1_u - 2_u|
\]  

(2.35)

Using equation (2.24) the formulation for \(B_1\) is again modified for unloading to the origin for the cohesive zone. The resulting equation is shown in (2.36).

\[
\begin{bmatrix}
  |1U_{33}\frac{\beta}{\alpha}|
\end{bmatrix}1_u - 2_u + \begin{bmatrix}
  1T_{33}1_u \end{bmatrix} = \begin{bmatrix}
  1b_3 \end{bmatrix}
\]  

(2.36)

The absolute value function was again removed and applied after solution in the MATLAB code. After simplifying, the final version of the general cohesive zone unloading representation for \(B_1\) is shown in Eqn. (2.37).

\[
\begin{bmatrix}
  |1U_{33}\frac{\beta}{\alpha}|
\end{bmatrix}1_u - 2_u + \begin{bmatrix}
  1T_{33}1_u \end{bmatrix} = \begin{bmatrix}
  1b_3 \end{bmatrix}
\]

(2.37)

\[
\left(\begin{bmatrix}
  1T_{33} + \frac{\beta}{\alpha}1U_{33} \end{bmatrix}1_u - \frac{\beta}{\alpha}1U_{33}2_u \right) = \begin{bmatrix}
  1b_3 \end{bmatrix}
\]

The second boundary condition is the same as before where tractions are equal and opposite from \(B_1\) to \(B_2\). This condition is implemented again in the unloading cohesive zone formulation. Using this boundary condition, Eqn. (2.38) describes the unloading cohesive zone relationship to implement for \(B_2\).

\[
-\tau_1 = \tau_2 \therefore \tau_2 = -\frac{\beta}{\alpha} |1_u - 2_u|
\]  

(2.38)
Using Eqn. (2.38) with (2.25) results in the similar formulation for $B_2$:

$$\left[ 2 U_{33} \right] \frac{\beta}{\alpha} u_3 \left| _{1} \right. - 2u_3 \left| _{2} \right. + \left[ 2 T_{33} \right] [2u_3] = [0]$$

$$\frac{\beta}{\alpha} u_3 \left| _{1} \right. - \left( \left[ 2 T_{33} \right] + \frac{\beta}{\alpha} [2 U_{33}] \right) [2u_3] = [0]$$

Combining Eqn.’s (2.37) and (2.39) into one matrix representation for the final linear system of equations for the cohesive zone material interface simulations.

$$\begin{bmatrix} \left[ \begin{bmatrix} T_1 \\ \vdots \\ -\frac{\beta}{\alpha} U_2 \end{bmatrix} \right] + \frac{\beta}{\alpha} \left[ \begin{bmatrix} U_1 \end{bmatrix} \right] & \cdots & \left[ \begin{bmatrix} T_1 \\ \vdots \\ -\frac{\beta}{\alpha} U_2 \end{bmatrix} \right] \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ 0 \end{bmatrix}$$

### 2.5 Solution Scheme

A boundary element method solution scheme was originally developed by Doong [21] using FORTRAN code to solve interface wave propagation problems. Two semi-infinite bodies were layered with an infinite interface between. The interface was used for several kinds of boundary conditions. Frictional sliding was studied between the two bodies for stick-slip conditions in the original code. The code was modified to study bonded conditions at the interface with wave propagation in this research. An interface of approximately 400 m in length was originally studied but modifications in the presented research were made to lengthen the interface to gather more data.

The original code was written in FORTRAN and was converted to MATLAB for simulation. Subroutines in FORTRAN were converted to separate MATLAB script files instead of functions for simplicity. In general, the code had three main sections to it. The first section was variable definition and mesh generation. The last two portions of the code
calculated values for just the first time step and then looped to calculate results for the remaining time steps respectively. Mesh generation included defining elements and nodes along the interface and for the source region in body 1. Three node quadratic elements were used with undistorted shape functions. Outward normal of the elements were defined to assign positive and negative values for tractions during solution.

The general solution scheme includes defining the current time, body forces and calculating the displacement and traction Green’s matrices for each time step. An initial “stiffness” matrix is assembled during the second portion of the code before the first time step solution is calculated. This consisted of the discussed Green’s matrices which are initially set for the first time step and used for the stiffness matrix. A similar green’s function calculation occurs for the source point contribution for each element based on position and current time. This multiplied with the body force applied to each element creates the vector used in matrix algebra to find the solution of displacement and traction at each element (for the bonded boundary condition). This is done in MATLAB using Gauss elimination $A^{-1}b = x$.

For the remaining time steps during solution, the green’s function matrices are recalculated and added to the column vector containing the body force contributions. This avoids making modifications to the global stiffness matrix every time step. Body forces are re-defined for the new time step and the functions are recalculated based on wave progression through the solid (in this case Body 1). During unloading for the cohesive zone model previously discussed, this portion of the code re-adjusts the global stiffness matrix
to account for element unloading towards the origin. For the bonded interface case, there is no adjustment so the values are continuously re-calculated.

The general solution scheme is shown in Figure 2.5. It should be noted that the described solution algorithm is repeated for four material mismatch cases. This outer loop is visible in the scheme shown.
Figure 2.5: Wave Propagation Research Code Flowchart
To run the MATLAB code for this research, various material parameters had to be defined such as shear modulus and density as well as other parameters like interface length, number of elements, time step increment and similar variables. 150 quadratic elements connected end to end were used for an interface of length 1000 m along the vertical direction of Figure 2.2. The distance from source point to interface was kept constant except when making this variation the parameter of study. Typical distances ranged from 1 m to 5 m from the interface. Typical cohesive zone values used for the baseline simulations were taken from the work of Dean [24] where $t_0 = 50$ MPa and $\delta_0 = 0.8$ mm.

Chapter three presents results for various material parameter variations based on the described model and algorithm. Specific material parameter values are defined in this chapter.
Chapter 3: Results

As described in chapter 2, the formulation as a linear system was programmed into MATLAB to solve several transient wave propagation problems. This chapter presents first bonded interface results for a shear wave propagating from a single element. Cohesive zone results are then compared for several parameter variations. Material mismatch, source distance, source strength and cohesive zone variation are all parameter variations included in the discussion. The common material parameters used throughout all simulations include shear moduli representing aluminum and aluminum like materials. In the simulations, $G_1 = 1.163 \text{ GPa}$, $\rho_1 = \rho_2 = 2,002.31 \text{ kg/m}^3$. The shear modulus of body 2 is varied in one set of simulations as a parameter variation, however the remaining simulations use $G_2 = 4G_1$. Time stepping was used where $\Delta t = 0.0015625 \text{ s}$.

3.1) Bonded Interface Results

3.1.1) Bonded Interface Varying Material Mismatch

A perfectly bonded condition was first implemented for calibration in MATLAB using the boundary conditions described and shown in Eqn’s (2.17) and (2.18). Displacement and traction values for the 150 elements comprising the interface were
solved at each time step. Four simulations were run with shear moduli of $n*G_2$ where “$n$” was set to 1-4. The results for displacement were plotted versus time and are shown first for element 75 (center element). This center element will be the primary study of the research as inaccuracy in the data increases as the wave reaches the fictitious boundaries created in MATLAB. It should be noted that most of the figures shown are of normalized displacement and traction. Displacement was normalized over the length of the interface and traction was normalized against the nominal pulse strength.

![Figure 3.1: Element 75 Normalized Displacement vs. Time for Increasing Material Mismatch](image_url)
Figure 3.1 indicates that increasing material mismatch decreases the maximum displacement of the interface. As expected, displacements along the interface died out as time progressed. It should be noted inversely that decreasing material mismatch has a larger effect on the maximum displacement at the interface as the mismatch approaches a material match between shear moduli. Figure 3.2 shows the traction plots versus time for the same simulation varying material mismatch.

Figure 3.2: Element 75 Normalized Traction vs. Time for Increasing Material Mismatch

A similar effect is shown from the displacement plot previously discussed, however in the plot of traction versus time for the center element along the interface, the maximum traction value occurs for a material mismatch of $4G_1 = G_4$. Change in peak traction values
decreased with increasing material mismatch. The traction values however die out as time increases in the same way displacements react to the wave pulse.

To demonstrate wave interaction as it proceeds along the interface in the vertical direction, elements’ displacement and traction values were plotted versus time towards the truncation boundary. The effect of the wave losing strength is visible as seen from the displacement plot in Figure 3.3 for displacement versus time of element 60.

![Figure 3.3: Element 60 Normalized Displacement vs. Time for Increasing Material Mismatch](image)

From Figure 3.3 it’s shown that there is a finite time before the wave arrival occurs as shown by the delay in peak displacement. Smaller wave reactions are shown for the three
material mismatch cases. This is caused by the increased Rayleigh wave speed for a larger shear modulus in Body 2. The wave in Body 2 in each case provides an additional displacement reaction for those three cases that happens earlier because of the increased wave speed. The maximum displacement in element 60 is 5.6 times smaller (on average) than that of element 75 for each case shown. The displacements similarly die out as time increases after the wave arrival from Body 1. The increase in increased displacement for decreasing material mismatch is again shown.

Traction values for element 60 were plotted in Figure 3.4. Similar effects from element 75 are again shown with a time delay and decrease in wave strength evident.

Figure 3.4: Element 60 Normalized Traction vs. Time for Increasing Material Mismatch
Similarly to displacement for element 60, there was a factor of 5.2 on average in decreased peak traction upon wave arrival from Body 1. Smaller peaks were again seen from the wave propagation happening in Body 2 at an increased wave speed for the material mismatch cases.

3.1.2) Bonded Interface Varying Source Position

The second set of parameter variation conducted with the bonded interface code was varying the SH wave source position. Seven positions ranging from a distance of 1 m to 30 m were used to investigate the effect of source position on the maximum displacement and traction values. A consistent material mismatch was used of \(4G_1 = G_2\). The same interface length of 1000 m and source strength of 50 MPa was used as previously stated. The center element’s displacement values were plotted versus time for the various source positions and the results are shown in Figure 3.5.
The first observation from Figure 3.5 was that initial wave arrival takes an additional 0.00625 seconds for every 5 meters of additional source distance. This is a validation of the calibration of the bonded interface code and model. The second observation was that peak displacements decreased steadily as the source distance increased. This result was expected as the source wave lost strength traveling through the medium before reaching the interface. Similarly to the differing material mismatch cases, the displacement at the center element shows fast decay to a steady state value of zero.

Next, tractions were plotted for the same element versus time for increasing source distance. These results are shown in Figure 3.6
The traction plots for each increasing source distance case show similar results, in that the wave arrival clearly occurs an additional time from the previous case based on similar distance spacing’s chosen. The peak traction values also show a decrease in wave strength for increasing source distance as expected. It should be noted that the tractions were normalized against the source strength, and therefore by increasing the source distance by a factor of 30 reduced the peak traction value by 27.27% of the value for $D = 1$ m.

Element 60 which was located 100 m from the center of the model was also used to study the effect of increasing source distance placement. The same values of
displacement and traction were studied for this element versus time. Figure 3.7 presents the displacement versus time results for element 60.

![Graph](image)

Figure 3.7: Element 60 Normalized Displacement vs. Time for Increasing Source Distance

In the data for element 60 which was offset from the center element, the effect of wave arrival time delay was less pronounced. Most peak displacements occurred in a closer time range for all increasing source distances. It’s expected that this result is an effect stemming from the wave contributions after transmission to Body 2 with a faster wave speed. This is seen in the smaller initial peak for the first case of \( D = 1 \) m. A second observation from this data was that peak displacements increased from the \( D = 1 \) m and \( D = 5 \) m cases before decreasing slightly in the \( D = 25 \) m and \( D = 30 \) m cases. There is not a
strong correlation from this point on the interface of decreasing peak displacement for increasing source wave distance. The values however do reach a steady state value of zero after some time period.

Similarly traction values for the same element were plotted versus time and the results are shown in Figure 3.8.

![Figure 3.8: Element 60 Normalized Traction vs. Time for Increasing Source Distance](image)

It was also difficult in the plot of traction to draw a correlation between peak traction values and increasing source distance. It’s again expected that this result was related to the additional wave contributions from transmission to Body 2 where wave speed was larger. The time delay for wave arrival was also less pronounced for this element and
subsequently other elements farther from the center of the interface. It should be noted that the peak traction values were 6% on average of the initial wave strength, a factor of 4.5 from the center element data.

3.1.3) Bonded Interface Varying Source Strength

The third set of parameter variation conducted with the bonded interface code was varying the SH wave source strength. Six source strengths ranging from a strength of 50 MPa to 300 MPa were used to investigate the effect of source strength on the maximum displacement and traction values. A consistent material mismatch was used of $4G_1 = G_2$ and a source distance of $D = 5$ m was also used. Interface length and element number did not change during this parameter variation. The center element’s normalized displacement values were plotted versus time for the various source positions and the results are shown in Figure 3.9.
The first thing observed from Figure 3.9 was that peak displacements increased linearly at the same time step with increasing source strength. Decay also occurred at a similar rate between the different source strengths studied. Peak displacement at $P_6 = 300$ MPa was observed to be significantly larger than those for the material mismatch and source distance variations at a factor of exactly 6. Material mismatch variation only increased peak displacement to less than 50% of the shown effect in increasing the source strength. It was clear from the three parameter variations that peak displacements were most directly affected by source strength, followed by decreasing material mismatch.

Figure 3.9: Element 75 Normalized Displacement vs. Time for Increasing Source Strength
However, further investigation into fractional material mismatch may lead to larger peak displacements based on the trends shown in Figure 3.1.

The normalized tractions for element 75 with increasing source strength were plotted and shown in Figure 3.10.

Figure 3.10: Element 75 Normalized Traction vs. Time for Increasing Source Strength

The trends in Figure 3.10 of normalized traction are similar to those in the normalized displacement plot previously discussed for increasing source strength. The increase in peak traction was similarly observed for increasing source strength and the increase between each case of source strength was the same showing a linear relationship. Decay occurred
at similar rates for the increasing source strengths as shown previously as tractions decayed to a steady state value of zero for all cases.

In element 60 at a distance of 100 m from the center of the interface, normalized displacements and tractions were also plotted to compare the effect of increasing source strength. The normalized displacement values for element 60 were plotted in Figure 3.11.

![Figure 3.11: Element 60 Normalized Displacement vs. Time for Increasing Source Strength](image)

There was a noticeable delay in action at the plotted element’s data caused by the delay in wave arrival to that element as shown previously. A similar effect of linearly increasing displacements was shown in the data at the wave arrival time step. Decay of
displacements to a value of zero was also seen in the data, however the peak displacement values was again decreased from the center element. The peak displacement values were 4.89 times smaller at this element as shown previously in other displacement plots at element 60. The primary effect of increasing source strength was on the increase in displacement.

To complete the bonded interface data on source strength, traction values for the same element versus time was plotted. The results are shown in Figure 3.12.

![Figure 3.12: Element 60 Normalized Traction vs. Time for Increasing Source Strength](image)

Similarly for normalized traction plot versus time at element 60, a delay was shown in wave arrival for this element. Peak normalized traction values were again 4.89 times
smaller than those at element 75. The results for the bonded interface correlate well between element 75 and element 60 for material mismatch, varying source distance and source strength, however varying source strength appears to only have altered the maximum displacement values and tractions shown. The percent decreases from element 75 to 60 are the same as shown between displacements and tractions as verified. Decay was again observed to a steady state value of zero.

A sample of spatial data was also plotted at several time steps to show out of plane displacement along the interface. This data is shown in Figure 3.13.

Figure 3.13: Spatial Representations of Bonded Interface Displacements
One problem that was encountered during simulations was that an interface “heaving” was occurring as the faster transmitted wave in Body 2 reached the truncation boundary. It was suspected that additional sources of error inherent in BEM when reaching truncation boundaries was the cause of errors adding to the total interface displacement, causing the additional displacement of the interface at these time steps. This was noticed in the bottom left picture of Figure 3.13 as the interface displacement reached a value of over 15 mm. It should be noted that the difference between peak displacements and non-excited interface elements was about 1-2 mm after the first few time steps where wave strength was the strongest.

After studying the effects of varying the discussed parameters on the bonded material interface, the cohesive zone law equations discussed in Chapter 2 of this thesis were implemented in the MATLAB code. The same interface length and single SH wave source element were used to study effects of both changing the cohesive zone law and also changing the same parameters discussed for the bonded interface case. The goal of the cohesive zone study was to compare differences in interface behavior so that damaged material characterization could be achieved.

3.2) Cohesive Zone Interface Results

3.2.1) Cohesive Zone Varying Laws with Constant Cohesive Energy

The cohesive zone condition was implemented in MATLAB using the boundary conditions described and shown in Eqn’s (2.22) and (2.24). A primary difference in the solution scheme between the bonded and damaged interface boundary conditions was that
with the cohesive zone equations, displacements of Body 1 and 2 were solved and tractions were calculated based on the pertinent cohesive zone law at hand. Displacement values for the 150 elements comprising the interface were solved at each time step. The first parameter variation done with the linear softening cohesive zone model was studying various laws representing the same cohesive energy. Cohesive energy was defined as area under the linear softening law or \( E_{\text{cohesive}} = \frac{1}{2} \delta_0 t_0 \). Using \( t_0 = 50 \text{ MPa} \) and \( \delta_0 = 0.8 \text{ mm} \) for the baseline cohesive zone model gives a baseline cohesive energy of 20 kN/m, This value was used to vary the cohesive zone law while keeping the area under the softening law constant. The four set of cohesive zone laws used in this section of parameter variation is shown in Figure 3.14.
The results for displacement were plotted versus time and are shown first for element 75 (center element). This center element will continue to be the primary study of the research as inaccuracy in the data increases as the wave reaches the fictitious boundaries created in MATLAB. It should be noted again that most of the figures shown are of normalized displacement and traction. Displacement was normalized over the length of the interface and traction was normalized against the nominal pulse strength of 50 MPa and a source distance of $D = 5$ m and a material mismatch of $4G_1 = G_2$. The displacement values of element 75 were plotted versus time for each of the cohesive zone laws and the results are shown in Figure 3.15.
Figure 3.15: Element 75 Normalized Displacement vs. Time for Constant Cohesive Energies

An initial observation from the data for varying cohesive energies was that the peak displacements in comparison to those during source distance variation from the bonded case remained nearly unchanged when implementing the cohesive zone along the interface. The percent difference in peak displacement across the constant cohesive zone cases shown was 4.50%. The variation with $t_0$ being minimized exhibited a slightly larger displacement, most likely due to a larger corresponding $\delta_0$. All four variations on the cohesive zone resulted in very similar displacements and steady state decays to a displacement of zero.
A more interesting plot was that of displacement jump versus time for the elements. Since the displacement coupling does not occur in the cohesive zone, the opening displacement is where the elements should exhibit the most action. A plot of displacement jump versus time for the center element was created and is shown in Figure 3.16.

![Figure 3.16: Element 75 Normalized Displacement Jump vs. Time for Constant Cohesive Energies](image)

Immediately upon inspection of the plot of displacement jump, it’s noticeable that varying the laws while maintaining a constant cohesive energy had a large effect on the peak displacement jump calculated. Each law of decreasing maximum cohesive traction $t_0$ had an increased maximum displacement jump. This is a direct result of the subsequent
increases in the maximum crack tip opening displacements $\delta_0$ (CTOD). For every increase in $\delta_0$, the maximum displacement jump values are approximately double the increase in $\delta_0$. For example, increasing $\delta_0$ from 0.8 mm to 1.6 mm resulted in maximum (normalized) displacement jumps of $0.2\times10^{-6}$ and $0.85\times10^{-6}$ respectively. The trend is similar for the other differences in $\delta_0$. The decay to a steady state displacement jump of zero is also consistent between the different laws which tends towards the conclusion that keeping cohesive energy constant does not quicken or slow the reaction of the interface to the wave source.

For completeness, traction values were calculated from the displacement jumps at each element according to the law in Eqn. (2.19). Signs of traction were assigned based on which direction Body 1 and 2 were moving at each time step. The results for element 75 for traction at the interface were plotted in Figure 3.17.
Figure 3.17: Element 75 Normalized Traction vs. Time for Constant Cohesive Energies

The first observation from Figure 3.17 was that regardless of cohesive zone law considering the constant energy cases, the interface was excited 30.84% of the pulse strength of 50 MPa. It should be noted that the elements in the cohesive zone law reached a minimum cohesive zone traction during exciting because of the nature of the linear softening law, so tractions decreased when elements were excited. Each case of the cohesive zone law resulted in the same rate of decay to the steady state tractions and each element along the interface reached its respective value of $t_0$. Another interesting observation is that the amount of traction decrease for each case was approximately the same as the maximum traction value excited during the simulation demonstrated in Figure
3.2. This may indicate that the traction increase or decrease for the bonded and damaged interfaces are the same when cohesive energies studied are kept constant.

To study the effect of changing cohesive zone laws for a constant cohesive energy on the decay of wave strength along the interface, the same values as previously discussed were plotted for element 60, 100 m away from the center element. The first plot of displacement versus time is shown in Figure 3.18.

![Figure 3.18: Element 60 Normalized Displacement vs. Time for Constant Cohesive Energies](image)

Similarly from the plot for the center element, changing cohesive zones for a constant cohesive energy did not have a major effect on the maximum displacement upon
wave arrival. The difference in maximum displacements from element 75 to 60 was a factor of 4.89 on average. This is similar to what was shown for the bonded interface case even though there was an increased compliance along the interface caused by implementing the damage model. Decay to a value of zero occurred as expected again and the delay of wave arrival was unaffected by the damage model as expected and proven with comparison to any of the bonded interface figures.

Displacement jump at the off center position was plotted and is shown in Figure 3.19.

![Figure 3.19: Element 60 Normalized Displacement Jump vs. Time for Constant Cohesive Energies](image-url)
As shown with the center element, varying the laws even while maintaining a constant cohesive energy had a large effect on the maximum displacement jump calculated. The increase in displacement jump was non-linear with increasing the maximum opening displacement $\delta_0$. The decrease in displacement jumps from the center element were by a factor of 4.89. Displacement jump decay and wave arrival times were similar to the simulations run for the bonded and damaged law interfaces being unchanged.

The cohesive tractions at the off center element were also plotted for comparison. This data is shown in Figure 3.20.

![Figure 3.20: Element 60 Normalized Traction vs. Time for Constant Cohesive Energies](image)
The first observation from Figure 3.20 was that regardless of cohesive zone law considering the constant energy cases, the interface was excited 5.6% of the nominal pulse strength of 50 MPa. This is decreased from the center element’s data since the wave loses strength traveling through the solid and interface. It should be again noted that the elements in the cohesive zone law reached a minimum cohesive zone traction during exciting because of the nature of the linear softening law, so tractions decrease when elements were excited. The amount of traction decrease for each case was the same as the maximum traction value excited during the simulation demonstrated in the bonded interface case. This can be seen from Figure 3.8. This supports the notion that traction increase or decrease for the bonded and damaged interfaces are the same when cohesive energies studied are kept constant. One important takeaway from the off center element is that the elements are not excited as much down the cohesive zone plots as those during initial wave arrival at the center of the interface. This suggests that focus should continue to be on the point of wave arrival as the wave loses strength.

The second set of parameter variation was keeping $t_0$ constant and varying $\delta_0$. This represents various rates of linear softening and the data was accordingly different with the changing cohesive energy driving different levels of compliance in the interface.

3.2.2) **Cohesive Zone Varying Laws with Constant $t_0$ and Varying $\delta_0$**

The same cohesive zone condition implemented in MATLAB as before was used with Eqn’s (2.22) and (2.24). The displacements for each body at the interface were again solved transiently in time and tractions were calculated from displacement jumps
The results for displacement were plotted versus time and are shown first for element 75 (center element). Displacement was normalized over the length of the interface and traction was normalized against the nominal pulse strength of 50 MPa and a source
distance of $D = 5$ m and a material mismatch of $4G_1 = G_2$. The displacement values of element 75 were plotted versus time for each of the cohesive zone laws and the results are shown in Figure 3.22.

![Normalized Displacement vs. Time for $t_0 = 50$ MPa Varying $\delta_0$](image)

Figure 3.22: Element 75 Normalized Displacement vs. Time for $t_0 = 50$ MPa Varying $\delta_0$

Displacement plots for varying $\delta_0$ values were nearly identical on a macro-scale in magnitude and time variance. The difference between the peak displacement values from the minimum to maximum value of $\delta_0$ was 0.93%. The maximum displacement values for each variance were similar to those of the constant cohesive energy case and rate of decay remained unchanged while varying $\delta_0$. The displacement jump for varying $\delta_0$ values was
also calculated and plotted to demonstrate the effect of various softening rates on that parameter. This plot is shown in Figure 3.23.

![Figure 3.23: Element 75 Normalized Displacement Jump vs. Time for Constant Cohesive Energies](image)

From the displacement jump plot for the center of the interface, it should be noted principally that increasing $\delta_0$ by the same amount resulted in a constant increase in displacement jump. This result matches the conclusions made about keeping the cohesive energy constant in Figure 3.16, which were that the amount of increase in $\delta_0$ directly correlated to the increase in peak displacement jump. The rate of decays from Figure 3.23 towards a steady state displacement jump of zero were also similar between the variation
of $\delta_0$. The main effect of varying $\delta_0$ for a constant $t_0$ is the effect of altering the magnitude of displacement jump.

The tractions at the center element were plotted versus time to explore the effect that the displacement jump results had. This plot is shown in Figure 3.24.

![Figure 3.24: Element 75 Normalized Traction vs. Time for $t_0 = 50$ MPa Varying $\delta_0$](image)

It is clear from the plot of normalized traction versus time at the center of the interface that varying the maximum opening displacement values had no effect on peak traction values of the excited element. The difference in values across the variation was less than 1%. The rate of decay of this peak traction was also unaffected. The peak tractions
were all 73.22% on average of the source strength and $t_0$ which were both 50 MPa. The effect of source strength on traction will be discussed in a later section.

To study the same properties as wave strength decreases, element 60 was again used to compare displacement, displacement jump and traction versus time. The first plot of displacement for this offset element is shown in Figure 3.25.

![Normalized Displacement vs. Time for $t_0 = 50$ MPa Varying $\delta_0$](image)

**Figure 3.25:** Element 60 Normalized Displacement vs. Time for $t_0 = 50$ MPa Varying $\delta_0$

A time delay for wave arrival is indicated in the plot of normalized displacement versus time for the offset element. This time delay was unaffected by altering $\delta_0$ and the rate of decay was also unaffected as typical of the previous simulations. There was again very little difference in peak displacements between the various $\delta_0$ and the peak
displacement was the same as peak displacement when varying the cohesive zone laws while keeping the cohesive energy constant. The difference across the peaks for varying $\delta_0$ was less than 1%.

The displacement jump values versus time for the same offset element were plotted and are shown in Figure 3.26.

![Figure 3.26: Element 60 Normalized Displacement Jump vs. Time for Constant Cohesive Energies](image)

Similarly to the results for the center of the interface, increasing the maximum opening displacement resulted in a proportional increase in maximum displacement jump. This increase however was not as large as with the center element due to the loss in wave
strength during travel along the interface. The increase between each peak displacement jump decreased by a factor of 4.90 in the 100 m travel of the wave. There may be some correlation to loss of displacement jump peak value versus distance from initial wave impact, however this was not part of the presented study. Decay and wave arrival times again remained unchanged.

Traction values were plotted versus time as well for the offset element to compare to the center of the interface and the results are found in Figure 3.27.

![Figure 3.27: Element 60 Normalized Traction vs. Time for $t_0 = 50$ MPa Varying $\delta_0$](image)

A similar effect was shown for traction at the offset element. The peak traction values were only 5.47% of the initial wave strength and of $t_0$ and the change in peak traction...
from the center of the interface was the same as that noted in the constant cohesive energy case. Decay and wave arrival time were unaffected again compared to the center element results. The primary effect of altering $\delta_0$ was in increasing or decreasing the peak displacement jump values by that proportional amount. This is important in understanding what parameters of the cohesive zone law effect the overall behavior of the interface.

The third set of parameter variation was keeping $\delta_0$ constant and varying $t_0$. This represents various rates of linear softening as well but by varying $t_0$ instead, the displacement and displacement jump increases are meant to be compared to altering $\delta_0$.

### 3.2.3) Cohesive Zone Varying Laws with Constant $\delta_0$ and Varying $t_0$

The same cohesive zone condition implemented in MATLAB as before was used with Eqn’s (2.22) and (2.24). The displacements for each body at the interface were again solved transiently in time and tractions were calculated from displacement jumps calculated. The third parameter variation done with the linear softening cohesive zone model was studying various laws representing the same maximum opening displacement $\delta_0$ while varying $t_0$. Using $t_0 = 50 \text{ MPa}$ and $\delta_0 = 0.8 \text{ mm}$ for the baseline cohesive zone model gives a baseline cohesive energy of 20 kN/m. The other $t_0$ values used were 25 MPa, 50 MPa, 75 MPa, 100 MPa. Figure 3.28 shows the sample cohesive zone laws used for these parameter variation simulations.
The results for displacement were plotted versus time and are shown first for element 75 (center element). Displacement was normalized over the length of the interface and traction was normalized against the nominal pulse strength of 50 MPa and a source distance of $D = 5$ m and a material mismatch of $4G_1 = G_2$. The displacement values of element 75 were plotted versus time for each of the cohesive zone laws and the results are shown in Figure 3.29.
The plot of displacement versus time indicates very little effect on the overall macro-level response of the interface at the center. It was noted that the values for peak displacement increased non-linearly for decreasing $t_0$, however the change is very small with the largest difference of 1.2% being between $t_0 = 50$ MPa and $t_0 = 25$ MPa. The displacement values for each $t_0$ value in the cohesive zone laws were nearly identical and the decay and steady state value tending towards zero indicate that this parameter variation does have a large effect. The maximum displacement values were comparable to those of the simulations varying $\delta_0$ as well as the simulations varying the cohesive energy. To study
the effect this parameter variation had on displacement jump, the displacement jump values were plotted versus time. These results are shown in Figure 3.30.

![Figure 3.30: Element 75 Normalized Displacement Jump vs. Time for $\delta_0 = 0.8$ mm Varying $t_0$](image)

The main impact of varying maximum opening displacement on displacement jump at the center of the interface was that decreasing this maximum traction increased the maximum displacement jump nonlinearly. Decreasing $t_0$ while holding the opening displacement fixed decreases the cohesive energy of the bond. This decrease in cohesive energy results in an increased compliance of the interface and larger displacement jump values. The rate of decay and decay to a value of zero again remained unchanged. This
result is important in showing that changing $t_0$ unlike opening displacement has a nonlinear impact on peak displacement jump. The overall peak displacement jumps here were smaller than those seen in varying the cohesive laws while keeping the cohesive energies constant. It’s suspected that varying both $\delta_0$ and $t_0$ had the combined effect on increasing the maximum displacement jump seen in those simulations.

The tractions were also plotted for the center of the interface versus time and these results are shown in Figure 3.31.

![Figure 3.31: Element 75 Normalized Traction vs. Time for $\delta_0 = 0.80$ mm Varying $t_0$](image)

From Figure 3.31 the effect of varying $t_0$ was only seen in the steady state traction values since the center of the interface approached each respective $t_0$ as time progressed.
The decrease in traction value was approximately the same for each variation of $t_0$ at 26.82% of the nominal pulse strength. As shown in keeping the cohesive energy constant, the amount of traction decrease for each case was approximately the same as the maximum traction value excited during the simulation demonstrated in Figure 3.17. The rates of decay were unchanged, and therefore changing $t_0$ only shifted the steady state value of traction.

For comparison to other parameter variation, the off-center element 60 data was collected and displacement, displacement plot and traction were plotted versus time. The first plot of displacement versus time is shown in Figure 3.32.
The maximum peak displacement decreased by a factor of 4.89 from the center of the interface. This is the same decrease seen in comparison to varying $\delta_0$ and the constant cohesive energy variation. Other than the decrease in maximum displacement, varying $t_0$ did not distinguish differences between the $\delta$ for peak values, wave arrival delay, or rate of decay. A plot of displacement jump was used to further investigate the effects of altering $t_0$ on the cohesive zone plots. This data is shown in Figure 3.33.

![Figure 3.33: Element 60 Normalized Displacement Jump vs. Time for $\delta_0 = 0.8$ mm Varying $t_0$](image)

The same effect seen with the central element was seen in the offset element as far as increasing displacement jump with decreasing $t_0$. The decrease in $t_0$ displayed a
nonlinear response in increasing displacement jump as shown before, and the decrease in displacement jump between peaks was 4.89 times smaller by moving 100 m from the center of the interface. Displacement jump approaching zero remained unchanged as usual. The final value plotted was traction at the offset element. The traction values versus time were plotted and this is shown in Figure 3.34.

Figure 3.34: Element 60 Normalized Traction vs. Time for $\delta_0 = 0.80$ mm Varying $t_0$

The traction plots look similar to Figure 3.20 in that the wave arrival was delayed to the same time step and the element studied was excited less than the central element. The increase in minimum traction value was 5.44% of the nominal pulse strength, supporting the results from the constant cohesive energy case where traction values were
excited by 5.6%. The decay rates and steady state value of \( t_0 \) where consistent with previous cohesive zone results, suggesting that the effect of changing \( t_0 \) on maximum displacement jump is non-linear with similar effects as altering other parameters for other variables.

3.2.4) **Cohesive Zone with Constant \( \delta_0 \) and \( t_0 \) Varying Material Mismatch**

After varying the cohesive zone laws in three ways, the fourth set of parameter variation was keeping a constant baseline cohesive zone law of \( t_0 = 50 \) MPa and \( \delta_0 = 0.8 \) mm while varying material mismatch. Comparisons to the bonded interface data were made based on the figures to be presented for differing material mismatch. The parameters used in these simulations were the same as before with the exception of the baseline cohesive zone used for all four material mismatches, as well as the source point being moved to a distance of \( D = 1 \) m to match the bonded interface case’s constants. The results for the central element will first be used again and then comparison to an offset element will be presented. The first plots presented are displacement, displacement jump and traction for varying material mismatch. The plot of displacement versus time for the central element is in Figure 3.35.
In comparison to Figure 3.1, the displacement at the center of the interface was actually smaller by introducing the cohesive zone when focusing on varying material mismatch. The displacements in Figure 3.35 were 4.5% smaller on average for each material mismatch. This difference was most accentuated for $G_2 = 4G_1$. As before, decreasing the material mismatch increased the displacement of the interface nonlinearly. Rate of decay from the peak values was approximately the same with the steady state displacement values being zero. Displacement jump values were also plotted versus time for the same element. This data is found in Figure 3.36.
Figure 3.36: Element 75 Normalized Displacement Jump vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm Varying Material Mismatch

The plot of displacement jump versus time for the center element varying material mismatch is similar to the plot displacement but interestingly enough, the relationship between increasing material mismatch and displacement jump is proportional. However, for each increase in material match, it appears that there is some maximum displacement jump value reached, as the difference between peaks decreased for increasing material mismatch. This is slightly different from the displacement case previously shown where displacement decreased with increasing displacement jump. The difference in
displacement followed the opposite pattern. Further studies could be conducted to determine if a minimum displacement and maximum displacement jump could be reached by continuing to increase material mismatch. Decay of displacement jump towards a value of zero was the same as previously shown. Traction values for the center element discussed were also plotted versus time and these values are shown in Figure 3.37.

Figure 3.37: Element 75 Normalized Traction vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm

Varying Material Mismatch

The plot of traction versus time for the center region of the interface was similar to the trends shown in plotting displacement jump. Increasing material mismatch further excited the element with the same wave pulse, however it appears there is some maximum
traction value being approached as material mismatch was increased. Further study should be conducted on where a maximum value may lie. It should be noted in comparison to Figure 3.2 that during the bonded interface simulation, the center interface was excited to 27% of the wave strength of 50 MPa. In this set of simulations the element was excited to 28.88% of the maximum opening traction. This increase is most likely related to the increased compliance in the interface.

For comparison to the center element, the same off-center element 60 was used at a distance of 100 m to plot the same values versus time. Displacement values were plotted versus time and the results for this off-center element are shown in Figure 3.38.
Figure 3.38: Element 60 Normalized Displacement vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm

Varying Material Mismatch

Similarly to Figure 3.35, increasing material mismatch decreased maximum displacement, however the difference between peaks decreased, again suggesting some minimum value may exist for further material mismatch cases. The actual values for the cohesive zone case were 3.5% smaller on average to those from the bonded interface case in Figure 3.3, suggesting adding the cohesive zone law to the interface did have a small effect on wave strength decay during travel along the interface. One thing also noted from the material mismatch plots for the cohesive zone case, was that there was no smaller peaks before wave arrival. These were estimated to be caused by the increased wave speed in...
Body 2 and inaccuracy present when that wave reaches the truncation boundary. In the cohesive zone simulations, it is unknown why these peaks were not present as the size of the interface and other parameters were the same.

The displacement jump values were also plotted versus time for element 60 and these values are shown in Figure 3.39.

Figure 3.39: Element 60 Normalized Displacement Jump vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm Varying Material Mismatch

The maximum displacement jumps behaved the same way as with element 75 where increasing material mismatch increased displacement jump while the difference...
between the peaks was decreasing towards a peak value. The decrease in displacement jump from element 75 to element 60 was at a factor of about 5. This is consistent with the decrease from other simulations between the two elements of a factor of 4.8-5 on average. As always, the decrease in value to zero and rate of decay remained the same. The traction values were also plotted after calculating the displacement jump values and these were plotted versus time in Figure 3.40.

![Figure 3.40: Element 60 Normalized Traction vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm](image)

Varying Material Mismatch

The traction plot at the off center element in comparison to the center element is similar in trend. Increasing material mismatch increased the traction peak (an actual
traction minima), and the difference between peaks decreased as usual. The actual peak values were very similar to those found in Figure 3.4. The primary difference caused by increasing material mismatch was in decreasing displacement but increasing displacement jump and tractions nonlinearly. Decays and peak values were similar to what was found in the bonded interface case.

3.2.5) Cohesive Zone with Constant $\delta_0$ and $t_0$ Varying Source Distance

After varying material mismatch with a constant cohesive zone law, the fifth set of parameter variation was keeping a constant baseline cohesive zone law of $t_0 = 50$ MPa and $\delta_0 = 0.8$ mm while varying source distance. Comparisons to the bonded interface data were made based on the figures to be presented for differing source distance. The parameters used in these simulations were the same as before with the exception of the material mismatch now being a constant $4G_1 = G_2$. The results for the central element will first be used again and then comparison to an offset element will be presented. The first plots presented are displacement, displacement jump and traction for varying material mismatch. The plot of displacement versus time for the central element is in Figure 3.41.
Varying Source Distance

As seen in the plot of displacement versus time for various source distances, the peak displacement value decreased as the source distance was increased. This is due to the source weakening before it arrives at the interface and supports the similar results seen for the bonded interface case in Figure 3.1. The values and shapes of the curves are nearly identical to what was shown for the bonded interface. The difference between the cohesive zone and bonded interface case was less than 1% on average. The increase in time delay varied linearly with increasing source distance as expected. The displacement jump values for the same element were plotted to further study what effects changing source distance...
may have with the cohesive zone element. These results were plotted and shown in Figure 3.42.

Figure 3.42: Element 75 Normalized Displacement Jump vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm Varying Source Distance

The first observation from the displacement jump curve was that the shape and spacing of the displacement jump curves was very similar to that of displacement. The displacement jump values were orders of approximately 50 smaller than the displacement values while still following the same trend over time. It should be noted that by increasing the source distance by a factor of 30, the displacement jump peak value decreased by about
30%. This decrease in displacement jump is expected to have affected traction values which are shown in Figure 3.43.

![Figure 3.43: Element 75 Normalized Traction vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm Varying Source Distance](image)

As seen before in the plots of displacement and displacement jump versus time, the plot of traction shows decreasing maximum values as source distance increased. The shapes of the curves match the first two plots in this section since tractions have been calculated from displacement jumps. It should be noted that the elements were excited to a minimum of 71.11% of $t_0$ for a source distance of $D = 1$ m while moving the source to a distance of $D = 30$ m increased the peak traction value to 79.57% of $t_0$. Since the cohesive
zone is in a state of linear softening, this means that for the 30% decrease in displacement jump, an 8% decrease in traction values between the two extremes was shown. This is related to the effect of displacement in Body 2 being also affected by the increase in source distance.

To again compare the differences in displacement, displacement jump and traction when increasing the source distance, the values for the offset element used in previous simulations were plotted versus time. The displacement plot is shown in Figure 3.44.

![Figure 3.44: Element 60 Normalized Traction vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm Varying Source Distance](image)

Varying Source Distance
As noticed in the same plot for the bonded interface, wave action occurred more closely together for varied source distances. This was again caused by the second wave traveling in body 2 at a different speed and arriving to make its contribution at a different time in each simulation. The interesting observation about Figure 3.44 besides the fact that the values and shapes of displacement versus time are nearly the same as the bonded interface case, is that the smaller initial peak caused by the wave transmission in Body 2 is gone. This is in comparison to what was shown in Figure 3.3. Other than this difference, the plots are nearly identical. For better comparison, displacement jump was plotted versus time and shown in Figure 3.45.
Figure 3.45: Element 60 Normalized Displacement Jump vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm Varying Source Distance

The plot of displacement jump shows the similar trend as seen with displacement where peak displacement jumps were lumped closer together even with the increasing source distance. The drop in displacement jump from the center of the element was on the order of about 3. This is smaller than other parameter variations but consistent with the other results for material mismatch. This suggests that relatively between the center of the interface and 100 m away, increasing the source distance did not affect the wave strength loss between those two points. It also did not affect the magnitude of displacement jump.
peak in comparison to varying other parameters since these results matched the bonded interface results.

Traction values were calculated from the displacement jump values versus time. These were plotted in Figure 3.46.

![Figure 3.46: Element 60 Normalized Traction vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm](image)

Varying Source Distance

As previously shown in several traction results, there was a strong correlation between the traction percentage excited with the bonded interface, and the difference from $t_0$ during excitation for the cohesive zone case. In the bonded interface case shown in Figure 3.4, the peak traction excited was roughly 6% of 50 MPa or the wave strength. Here in the
cohesive zone case, this was the difference from $t_0$ which equaled the wave strength during exciting. There is more potential work to be done on whether or not altering $t_0$ with the alternate source distances would change these values, however a combination of the two sets of results presented for each of those parameter variations would be expected.

### 3.2.6) Cohesive Zone with Constant $\delta_0$ and $t_0$ Varying Source Strength

After varying source distance with a constant cohesive zone law, the sixth and final set of parameter variation was keeping a constant baseline cohesive zone law of $t_0 = 50$ MPa and $\delta_0 = 0.8$ mm while varying source strength. Comparisons to the bonded interface data were made based on the figures to be presented for differing source strengths. The parameters used in these simulations were the same as before with the exception of the source distance now being a constant $D = 5$ m. The results for the central element will first be used again and then comparison to an offset element will be presented. The first plots presented are displacement, displacement jump and traction for varying material mismatch. The plot of displacement versus time for the central element is in Figure 3.47.
Increasing the source strength as shown in Figure 3.47 increased the peak displacement at the center of the interface linearly with linear increases in source strength. In comparison with the bonded interface displacement results shown in Figure 3.1, there was a slight increase in displacement of 1.3% for addition of the cohesive zone, however the difference between peaks, rate of decay and steady state values of zero displacement were similar (within 1-2%). It’s expected that the slight increase in displacement was caused by the additional compliance in the interface from the linear softening damage.
model. The displacement jump values were also plotted versus time for the center of the interface as shown in Figure 3.48.

![Figure 3.48: Element 75 Normalized Displacement vs. Time for \( t_0 = 50 \text{ MPa}, \delta_0 = 0.80 \text{ mm} \)](image)

Varying Source Strength

Similarly for displacement jump, the difference in peak displacement jumps were evenly spaced apart, indicating a linear relationship between increasing source strength and peak displacement jump. Increasing the source strength by a factor of 6 increased peak displacement jump by a factor of approximately 5.7. As shown with the bonded interface, the primary effect of increasing source strength was in the increasing of peak displacement.
The tractions for the center of the interface were also plotted versus time as done before and this plot is shown in Figure 3.49.

![Figure 3.49: Element 75 Normalized Displacement vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm](image)

Varying Source Strength

As shown in the plot of displacement jump versus time, the traction values increased linearly with increase in pulse strength. The difference between maximum and minimum tractions in comparison to the bonded interface case in Figure 3.2 was the same. In the plot of traction for increasing source strength shown, the peak traction occurred at a difference of 1.61 normalized traction from the minimum value. This matches the figure
shown for the bonded interface case. The traction values tended towards $t_0$ value with similar decay rates as usual.

For comparison to the center of the interface, displacement, displacement jump and traction values were plotted versus time for the offset element number 60. This element at a location of 100 m away was used again for comparison to show the decay in wave strength. The values for displacement at this point were plotted versus time in Figure 3.50.
times were the same as shown in the bonded interface case. The bonded interface data was shown in Figure 3.3. As shown in other displacement plots for the offset element, the time delay for wave arrival was unchanged and the rate of decay to a displacement of zero was unchanged from increasing the pulse strength.

Displacement jump versus time was also plotted for this offset element and the data is shown in Figure 3.51.

![Normalized Displacement Jump vs. Time](image)

Figure 3.51: Element 60 Normalized Displacement Jump vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm Varying Source Strength

Displacement jump at the interface showed the same linear increase in displacement jump for linear increases in pulse strength as shown in the plot for the center of the
interface. The displacement jumps decreased by a factor of approximately 5 from the center of the interface to the shown element’s data 100 m away. This factor is consistent in both the bonded interface data and cohesive zone data as previously stated. The displacement jump values are also very small in comparison to the displacement values, being on the order of 50-60 times smaller than the displacement values.

Traction values were plotted lastly for this element versus time and this plot is shown in Figure 3.52.

Figure 3.52: Element 60 Normalized Traction vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm

Varying Source Strength
Traction values peaked at the same time step as previous simulations for the offset element studied and the peak value was about 65% of the maximum traction $t_0$. This decrease in traction of 35% matches the increase in traction shown in Figure 3.4 for the bonded interface case. These results matched continuously for the cohesive zone results and bonded interface results suggesting a strong relationship between the two. The increase in traction was again linear with the linear increases in source strength. The rates of decays were similar and consistent with the simulations prior.

For understanding of where along the $t-\delta$ law the elements were located for the baseline cohesive model, the values were plotted for four separate time steps. These plots are shown in Figure 3.53.
The model’s shown data for traction and displacement jump along the cohesive zone plot represents data close to $t_0$ and displacement jumps of zero for a small pulse strength of $P = 50$ MPa. The same data was plotted for a model with a much stronger strength of $P = 300$ MPa to compare how far down the linear softening law the elements were driven. This data was plotted in Figure 3.54.
It’s shown that by increasing the pulse strength, elements were driven nearly halfway down the cohesive zone linear softening law. Increasing the pulse further would theoretically drive separation of the interface whenever displacement jump at any element is greater than CTOD.

The data in chapter 3 presented was for a square wave pulse without unloading for the cohesive zone model. To study the effects of unloading in conjunction with the theory on unloading in the cohesive zone model equations, several simulations were run with
various trapezoidal pulses with steep unloading slopes. The data for unloading with the cohesive zone interface model is presented in chapter 4.
Chapter 4: Unloading Results

Chapter 2 gave an introduction to the unloading formulation when loading of the interface was removed. According to equations (2.37) and (2.39) the solution scheme is in terms of interface displacements for Body 1 and 2. The key to solution using unloading is tracking the time history of the body force terms. At onset of unloading, the point of last traction and displacement jump was tracked for every element denoted by \((\alpha, \beta)\). These principles were used to induce one set of unloading data for comparison to the pure loading cases in Chapter 3. In an attempt to avoid abrupt jumps in displacement or traction, unloading was done using a “trapezoidal” rule where the unloading occurred over three time steps initially. Figure 4.1 shows the body force versus time plot to demonstrate the unloading influence of the body forces.
Figure 4.1: Cohesive Zone Data on Law for Baseline Model P = 300 MPa

As shown from the body force profile the decrease in body force is very rapid in comparison to the total time of simulation. Using this input to the code, one simulation was run using a source distance of \( D = 1 \) m, \( G_2 = G_1 \), \( t_0 = 50 \) MPa, \( \delta_0 = 0.8 \) mm, source strength “\( P'\) = 300 MPa to excite the cohesive zone sufficiently and the same time step and dimension parameters as before. Results for displacement, displacement jump and traction for the central element were stored and the first chart for displacement versus time is shown in Figure 4.2. Results were normalized versus total interface length and nominal source strength of 50 MPa as before.
Figure 4.2: Element 75 Normalized Displacement vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm

Unloading Body Forces

The main difference as shown in Figure 4.2 between the loading and unloading data was that there was a rapid decrease to zero displacement during unloading. For the bonded and cohesive zone loading cases, the displacements continued to decrease to zero approaching them asymptotically. To further study the effect of unloading on the cohesive zone, the displacement jump data was plotted versus time. This data is the driving factor in where along the cohesive zone the elements fall during unloading. This plot is included in Figure 4.3.
Figure 4.3: Element 75 Normalized Displacement Jump vs. Time for $t_0 = 50$ MPa, $\delta_0 = 0.80$ mm Unloading Body Forces

The plot for normalized displacement jump is similar in trend to the plot shown for displacement during unloading of the interface. There is a slight ridge upon zooming closer to the cohesive unloading time steps most likely caused by the exact time step for unloading selected. The displacement jumps quickly died to zero upon unloading showing the main effect of introducing the trapezoidal unloading. The plot of traction versus time was also plotted for the center of the interface and is shown in Figure 4.4.
Figure 4.4: Element 75 Normalized Traction vs. Time for \( t_0 = 50 \) MPa, \( \delta_0 = 0.80 \) mm

Unloading Body Forces

The plot of traction versus time shows an interesting trend during unloading. The two bodies slip past each other as shown by the quick change in sign of the traction around \(.24 \) s. Note that the traction actually changes sign in one time step. Elements approaching a traction of zero would actually be representing open separation of the interface. The tractions approach the origin as described in Chapter 2 along the line defining the history of traction and displacement jump for this element prior to unloading. The plot in Figure 4.5 shows the history of this element along the cohesive zone envelope during unloading.
As seen from this data, at the time of unloading, the central element is located near $t_0$ on the cohesive zone plot on either the positive or negative side of the interface slip. The yellow marker indicates the last time step when unloading is mostly occurring and shows the element approaching zero from the previous point on the cohesive zone envelope. A plot showing this data zoomed in closer to the region where the element exists on the cohesive zone envelope was made and is shown in Figure 4.6.
Unloading Body Forces for Various Time Steps

A black line was plotted from the time step as unloading occurred to show that the central element tended towards the origin shown by the yellow data marker. As will be discussed in Chapter 5 for future work, there is future work to be done on adjusting the loading and unloading body force profile to maximize the distance along the cohesive zone envelope that the elements are excited. Introducing the unloading equations however showed the viability for such a simulation and potential for also introducing reloading to be discussed.
Chapter 5: Summary and Conclusions

This final chapter of the thesis sheds light on the significance of the scientific contribution presented and summarizes the results from Chapters 3 and 4. It also outlines the future work to be potentially completed and concludes the thesis.

5.1) Significance of Research

Significant work has been completed to study the effect of cohesive zones on material response to cracks as outlined by the works of Dean [24], and Mokashi [23]. Further work has been done on wave propagation through interfaces with various frictional properties governed by stick or stick-slip phenomena as shown by the works of Doong [4] and Mendelsohn with Doong [21]. These two ideas however have never combined to study the effect of wave propagation through a cohesive zone. This was therefore the first known attempt of studying this especially computationally using the boundary element method.

5.2) Conclusions

Data was collected for wave propagation through the interface in MATLAB for both a bonded and cohesive zone interface. It was shown in Chapter 3 that material mismatch
variations had non-linear effects on peak displacements and traction versus time. The effect was theorized to be caused by wave speed depending on the square root of shear modulus. Varying source distance decreased peak tractions and displacements non-linearly for linear increases in source distance. The primary linear effect in parameter variation was seen when linearly increasing source strength. Peak tractions and displacements were seen to increase exactly linearly with this increase in source strength as expected.

Introducing the cohesive zone allowed for the described parameter variations as well as variations with the cohesive zone law. Keeping the cohesive energy constant was shown to still have a non-linear effect on increasing peak displacement and traction values as well as displacement jump. The effect was shown to be most prominent in the plot of displacement jump. Further studying variations of $t_0$ and $\delta_0$ separately showed that increasing $\delta_0$ had a linear effect on peak displacement and traction values while increasing or decreasing $t_0$ induced the nonlinear change. The combined effect for keeping cohesive energy constant was seen from these two phenomena.

5.3) Future Work

With a base code built for wave propagation through a cohesive zone, there are several components to be added to make the code more robust. The first bit of future work to be completed is to further investigate unloading response of the interface based on the work done in Chapter 4 and the equations from the end of Chapter 2. The exact profile of unloading should be studied further to excite the elements along the cohesive zone for unloading to the origin from a farther point along the envelope. Work should also be done
to implement re-loading into the code for a cyclic MHz level pulse through the interface simulating rapid vibrations through the solid.

Other work that could be done would be to alter the type of body wave sent through the interface. SV and P waves are two other polarization types of waves that could be used to excite the interface. The tractions studied would no longer be in the out of plane direction however and extensive work to modify the equations would have to be accomplished with the Green’s functions. The wave source could also be switched from an out of plane line source to a plan wave source with some incident angle.

5.4) Summary

The purpose of this thesis is to study wave propagation through a cohesive zone transiently using the boundary element method. A background was given on the history of cohesive zones and wave propagation research as well as how they could be tied together in work such as what was presented. Relevant history was also given to the boundary element method and reasons for selecting such a method as the computational means for obtaining the presented results. Formulation was shown for how the boundary element method was obtained as well as relevant equations and boundary conditions to implement both the bonded and cohesive zone interfaces. This was done for both loading only and loading with unloading along the interface.

Results were presented for material parameter variation for the bonded interface case for material mismatch for shear moduli, source pulse distance as well as source pulse strength. Results were presented for the central element along the interface as well as an
offset element 100 m away. The cohesive zone material parameter variation involved first changing cohesive zone law parameters. Cohesive zone energy was kept constant while varying \( t_0 \) and \( \delta_0 \) followed by keeping either \( t_0 \) or \( \delta_0 \) constant while varying the other. Displacement, displacement jump and tractions were presented versus time for the same elements. This was followed with the data for the original parameter variations done for the bonded interface using the cohesive zone code. Preliminary results for one trapezoidal unloading case were also presented in Chapter 4.

All of the data presented could be used in a non-destructive evaluation mindset for comparing interface behavior for bonded and damaged joints. Knowledge of some baseline data such as was shown for the bonded interface could be used to classify a bond or interface between two materials at a certain damage level based on increased displacements or different behaviors for testing with different source waves. These tools could continue to prove useful for testing components in service more quickly and efficiently than is possible in a lab setting.
References


[24] M. C. Dean, "The Effect of Implementing a Boundary Element Cohesive Zone Model with Unloading-Reloading Hysteresis on Bulk Material Response," The Ohio State University, Columbus, 2014.

