Radar Sensing Based on Wavelets

Dissertation

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Abstract

Radar waveform design is an active research area for decades. With the advent of advanced digital signal processing, high speed computing, high frequency electronics, and solid state power amplifiers, emerging radar systems (such as UWB radar, multiple-input and multiple-output (MIMO) radar, cognitive radar, etc.) are expecting more from their waveforms. Taking advantage of the new techniques, scientists and engineers are able to implement new waveforms to achieve significantly better performance for conventional radar systems, namely target detection including range, speed, and shape. The objective of this dissertation is to exploit a practical way to build flexible waveforms for the modern radar.

On the other hand, conventional radar systems detect targets or pixels of an area individually. Each target or pixel generates a set of data in real-time, which must be recorded for off-line processing. When the number of elements is increased, phased array radar is able to generate narrow beams, which can detect more targets or cover larger areas for data collection in high definition. The disadvantage is the increased time in sensing since narrow beams need more time to cover the same area than wider beams. To address this issue, the sensing mechanism needs to be studied. The objective of this dissertation is to exploit a new sensing mechanism, named transform sensing, to cover wider areas, tracking more moving objects, and providing high resolution of the target area with limited times of sensing.
Because the waveform design and transform sensing in this dissertation are all based on wavelets, the dissertation introduces the wavelet basics. Then the wavelet based waveform is presented. This waveform is generated by concatenating wavelet packets, and can suppress range sidelobes more effectively than the traditional Linear Frequency Modulated (LFM) waveform. In addition, the wavelet based waveform can de-couple its envelope and carrier for range and velocity estimations, respectively, because of which the speed detection and range detection using the proposed waveform is more stable for high speed targets than the LFM waveform. Consequently, the wavelet based waveform produces higher accuracy (and resolution) in range or velocity detection. The wavelet based waveform can be applied to the Synthetic Aperture Radar (SAR) for improved performance. The range and velocity detection is directly associated with range and azimuth detection for the stripmap SAR, we further compare the wavelet based waveform with the traditional LFM in the stripmap SAR simulation.

On the other hand, this dissertation discusses the feasibility of transform sensing using wavelets. First, the way in which different sensing patterns can be formed and achieve a coarse-to-detail spatial resolution is discussed. Then, realization of different sensing patterns by the phased array is discussed in detail. Because the transform sensing is feasible to be realized by the state of the art technique, a simulation generating wavelets for different sensing patterns is shown. A preliminary experiment is further implemented.
This is dedicated to my parents and my wife Xiang Zhang.
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Chapter 1: Introduction

The radar waveform design is for improving radar performance, including maximum signal-to-noise-ratio, and optimal extraction of target information, such as range, speed, shape, and recognition [1, 2]. Because of the potential improvement on radar performance, radar waveform has been a key topic of research for long time and is still active today.

In this chapter, we first review the progress in the radar waveform designs in recent years. In fact, the waveform design has a long history research and is still under active investigation for existing radar systems. With the advent of ultra-wide bandwidth (UWB) electronics, advanced digital signal processing, high speed computing, and high frequency electronics, and solid state power amplifier, emerging radar systems (such as UWB radar, multiple-input and multiple-output (MIMO) radar, cognitive radar, etc.) are expecting more from their waveforms. These new technologies enable the scientists and engineers to implement new waveforms, which were not possible when these new technologies were not available, to achieve significantly better performance for conventional radar systems, namely target detection including its range, speed, and shape.

The study on the waveform design is also pushed by more applications of radar in our daily life. Traditionally radar has been primarily used in military for target
detection or for image construction. Today radar sensing is being used in a wide scope of applications including, for example, automobile radar. It is perceived that automobile radar will be ubiquitous in the future, installed on every vehicle for safety consideration, even for fully automated vehicles. In addition, microwave radars are also being used for human detection on mobile robots, and for probing of concealed objects, etc. Different applications naturally demand for different waveforms in order to push the performance to the limit, for any particular purpose of applications. Researchers are therefore continuously studying the radar waveform design, from the fundamental pulse compression to more advanced waveforms for future radar systems.

Since the method for radar sensing in this dissertation is based on wavelets, basics of wavelets are also introduced in this chapter.

1.1 Fundamentals

1.1.1 Radar Waveform Fundamentals

Similar to wireless communication systems, radar typically transmits a baseband signal carried by a high frequency sinusoid signal, i.e.:

\[ x(t) = s(t) \exp(j2\pi f_c t) \]  (1.1)

where \( s(t) \) is the baseband waveform and \( f_c \) the carrier frequency. Because of the properties of radiated electromagnetic energy, the ideal received signal reflected by a single point target will be:

\[ y(t) = A x(t - \tau) \exp(j2\pi (f_c + f_d)t - j2\pi f_c t \tau) \]  (1.2)

In the above equation, the time delay \( \tau \) is caused by the range of the target, the Doppler frequency \( f_d \) is determined by the radical velocity of the target, and the
amplitude $A$ is affected by the cross-section of the target when the target range, and transmitter and receiver gains are given. Apparently, how the three parameters $A$, $f_d$ and $\tau$ are altered can reveal the information associated with a target.

The range resolution is the ability of a radar system to resolve targets along the range direction. Take the rectangular signal rect($\frac{t}{T}$) as the baseband waveform for example, where $T$ is the duration of the radar pulse (radar pulse width), and the rectangular function is defined as below:

$$\text{rect}(x) = \begin{cases} 
1, & -\frac{1}{2} \leq t \leq \frac{1}{2} \\
0, & \text{else}
\end{cases} \quad (1.3)$$

The received signal reflected from a single static target is:

$$\text{rect}\left(\frac{t - \frac{2R}{c}}{T}\right) \exp\left(j2\pi fc\left(t - \frac{2R}{c}\right)\right), \quad (1.4)$$

where $R$ is the range of the static target and $c$ the speed of light. The term $\frac{2R}{c}$ is equivalent to the delay $\tau$ in Eq.1.2. This is because the electromagnetic energy typically travels through the space in a straight line at the constant speed $c$. Thus delay $\tau$ can be used to measure the range of the static target. When the rectangular waveform is used to detect two static targets, the received signal is:

$$A_1\text{rect}\left(\frac{t - \frac{2R_1}{c}}{T}\right) \exp\left(j2\pi fc\left(t - \frac{2R_1}{c}\right)\right) + A_2\text{rect}\left(\frac{t - \frac{2R_2}{c}}{T}\right) \exp\left(j2\pi fc\left(t - \frac{2R_2}{c}\right)\right), \quad (1.5)$$

where $R_1$ and $R_2$ correspond to the positions of the two targets, respectively. If the two targets are very close to each other, the two reflected signals generated by the two different targets mix together (Fig.1.1), and are difficult to resolve due to the radar pulse width (RPW).
The range resolution using the rectangular waveform can be defined as using the bandwidth of the waveform $B$:

$$S_r = \frac{c}{2B} \quad (1.6)$$

According to Eq.1.6, radar can improve its range resolution via different waveforms so long as the bandwidth is increased. The challenge is how to increase the range resolution, while transmitting enough energy for long range target detection. According to Eq.1.6, one can increase the range resolution via a high bandwidth waveform, while the pulse width can be maintained large. Therefore, a high time-bandwidth product waveform has the potential to increase the range resolution without sacrificing the transmission power. Scientists call this kind of waveform pulse compression waveform.

One of the most widely used pulse compression waveforms is the linear frequency modulated (LFM) waveform [3] or the chirp waveform. The frequency of LFM linearly
increases (up-chirp) or decreases (down-chirp) with time. Therefore, the waveform is feasible to be achieved by the analog circuits. The mathematical equation for LFM is:

\[
\text{rect} \left( \frac{t}{T} \right) \exp(j \pi kt^2) \exp(j 2\pi f_c t),
\]

(1.7)

where \( k \) is the rate of frequency increase or the chirp rate. The instantaneous frequency of the baseband waveform is a function of time: \( f(t) = k \text{rect} \left( \frac{t}{T} \right) \). In the beginning of the pulse, \( f \left( -\frac{T}{2} \right) = -\frac{kT}{2} \). At the end of the pulse, \( f \left( \frac{T}{2} \right) = \frac{kT}{2} \). As a result, the bandwidth of LFM is \( kT \). The time-bandwidth product of LFM is \( kT^2 \).

According to Eq.1.6, the LFM waveform can simply increase its chirp rate \( k \) in order to obtain the high range resolution and keep a wide pulse width at the same time.

The ambiguity function is used to evaluate how the returned pulse is distorted due to the receiver matched filter:

\[
\chi(\tau, f_d) = \int_{-\infty}^{\infty} s(t)s^*(t - \tau) \exp(j 2\pi f_d t) dt
\]

(1.8)

When the Doppler frequency is not considered, the ambiguity function reduces to the autocorrelation of \( s(t) \), i.e.

\[
\chi(\tau, 0) = \int_{-\infty}^{\infty} s(t)s^*(t - \tau) dt
\]

(1.9)

Fig.1.2 shows a LFM waveform with a 3\( \mu \)s pulse width and 64MHz bandwidth. The autocorrelation of the waveform produces a sharp mainlobe with serious sidelobes. The peak to sidelobe ratio (PSLR) of LFM is \(-13.2dB\). These sidelobes are highly responsible for blocking nearby weak reflections from small targets or for blurring SAR images.
One traditional way to suppress the sidelobes is using windowing. Harris [4] provides an extensive list of windows and their properties. Coming along with the sidelobe suppression, the windowing procedure brings a side effect of increased main-lobe width, which also degrades the range resolution in target detection. Comparing with LFM, the stepped frequency waveform [5] increases (or decreases) its frequency discretely by a fixed frequency increment (or decrement) each time. The frequency jump burst (FJB) [6] divides LFM into multiple sub-pulses in the time domain, and transmits each separately. Consequently, these discrete versions of LFM perform similarly as the original LFM waveform in range detection.
Aiming at suppressing range sidelobes, some researchers have selected nonlinear frequency modulated waveforms. Griffiths [7] generates nonlinear frequency waveform using equation:

$$\hat{s}(t) = \hat{s}_1(t) + \hat{s}_2(t)$$  \hspace{1cm} (1.10)

A central linear FM component $\hat{s}_1(t)$ and the additional higher FM rate portion $\hat{s}_2(t)$ are plotted in Fig.1.3. The PSLR of his proposed waveform is $-50.4dB$, which is much lower than the traditional LFM waveform. Instead of the piecewise nonlinear FM waveform, Witte [8] further studies a continuous nonlinear FM waveform to suppress the sidelobes to below $-70dB$. However, the nonlinear FM waveforms increase the width of the mainlobe comparing with LFM, and thus to lower the range resolution. In addition, the nonlinear FM waveforms are sensitive to the Doppler effect. That is, the range of the target is sensitive to the speed of the target. Different speeds can result in different range data for an identical target. Consequently the range
resolution becomes lower when the target is in motion. The details of this issue will be further discussed in the next section.

1.1.2 Wavelets Basics

Daubechies established orthonormal bases of compactly supported wavelets in [9, 10]. The basic recursion equations that generate the scaling function \( \varphi(t) \) and the wavelet function \( \psi(t) \) are shown below:

\[
\varphi(t) = \sum_{n} h(n) \sqrt{2} \varphi(2t - n)
\]  
(1.11)

\[
\psi(t) = \sum_{n} g(n) \sqrt{2} \varphi(2t - n),
\]  
(1.12)

where

\[ g(n) = (-1)^n h(L - n - 1), \quad n = 0, ..., L - 1. \]  
(1.13)

In Eq. (1.13), \( L \) is the length of the coefficients \( h(n) \).

Different coefficients \( h(n) \) generate different wavelets and scaling functions, so all information about the wavelets and scaling function is contained in \( h(n) \). However, the coefficients cannot be arbitrary chosen; they need to satisfy several constraints [11]:

a. \[ \sum_{n} h(n) = \sqrt{2} \]  
(1.14)

b. \[ \sum_{n} h(2n) = \sum_{n} h(2n + 1) \]  
(1.15)

c. if \[ \int \varphi(t) \varphi(t - k) dt = \delta(k) \]  
(1.16)

then \[ \sum_{n} h(n)h(n - 2k) = \delta(k). \]  
(1.17)
The wavelet coefficients not only satisfy the necessary conditions in Eqs. (1.14), (1.15),
and (1.17), but they also give the order of vanishing moments. For example, with Daubechies wavelets, the order of vanishing moments equals half the number of coefficients. The order of vanishing moments is defined as $M$ in Eq. (1.18):

$$\int_{-\infty}^{\infty} \psi(x)x^m dx = 0, \quad m = 0, \ldots, M - 1. \quad (1.18)$$

According to [11], the orthogonality condition of the wavelet coefficients gives not only Eq. (1.17), but also the following equations:

$$\int \psi(t)\psi(t-k)dt = \delta(k) \quad (1.19)$$
$$\int \varphi(t)\psi(t-k)dt = 0. \quad (1.20)$$

To analyze $h(n)$ and $g(n)$ in the frequency domain, we denote the spectra of $h(n)$ and $g(n)$ as $H(w)$ and $G(w)$, respectively. $H(w)$ and $G(w)$ can be used to set necessary conditions for the coefficients as follows:

a. $\sum_{n} h(n) = \sqrt{2} \iff H(0) = \sqrt{2}$ \quad (1.21)
b. $\sum_{n} h(2n) = \sum_{n} h(2n+1) \iff H(\pi) = 0$ \quad (1.22)
c. $\sum_{n} h(n)h(n-2k) = E\delta(k)$

$$\iff |H(w)|^2 + |H(w+\pi)|^2 = 2. \quad (1.23)$$

Then a cascade algorithm (by recursion of Eqs. (1.11) and (1.12)) calculates the approximate scaling function and wavelet function as follows:

$$\Phi(w) = \left[ \prod_{k=0}^{\infty} \frac{1}{\sqrt{2}} H\left(\frac{w}{2^k}\right) \right] \Phi(0) \quad (1.24)$$
$$\Psi(w) = \left[ \frac{1}{\sqrt{2}} G(w) \prod_{k=1}^{\infty} \frac{1}{\sqrt{2}} H\left(\frac{w}{2^k}\right) \right] \Psi(0), \quad (1.25)$$
where $\Phi(w)$ and $\Psi(w)$ are the spectra of the scaling function and the wavelet function, respectively. It is impossible to generate a scaling function (or wavelets) with full resolution because the algorithm has an infinite cascade. However, in reality, the iteration can only be performed by a limited number of times. Every iteration will result in a wavelet of higher resolution until a desired one is generated (Fig. 1.4).

1.2 Related Works

1.2.1 Phase Waveform

One approach for designing new radar waveforms is through phase coding. The phase coding approach introduces a set of pulse compression waveforms. This approach divides the radar pulse into sub-pulses, and then phase coding is performed on each sub-pulse. A well-known phase coding approach is Barker coding [12], which codes each sub-pulse with a phase shift of 0 or 180 degrees (i.e. bi-phase coding) accordingly as shown in Fig. 1.5.
By employing the phase coding, the sidelobes of the autocorrelation of the Barker code have an equal magnitude of \( \frac{1}{N} \) as shown in Fig.1.6, where \( N = 7 \) is the length of the code. However, Barker codes cannot be of arbitrary length. The maximum possible length is \( N = 13 \), resulting in limited sidelobe suppression. A set of concatenated Barker codes can increase the length of the waveform, but the sidelobe suppression is still limited by \( N = 13 \). Therefore, the minimum PSLR of the Barker code is \(-22.28dB\).

Another phase coding approach is the Golay complementary sequences, which can be used to construct bi-phase waveforms as well. The difference is that the Golay code consists of a pair of bi-phase sequences. The summation of the autocorrelations of the two sequences produces a delta function. As a result, transmitting the pair of bi-phase sequences of waveforms separately in time can totally remove the sidelobes in range.
However, the bi-phased code is highly sensitive to the Doppler shift. To overcome this problem, Pezeshki et al., propose using the Prouhet-Thue-Morse (PTM) sequence to construct Doppler-resilient sequences of Golay pairs [13, 14]. However, the PTM sequence requires transmitting a long sequence of pulses. Galati [15] proposes to nest an up-chirp and down-chirp waveform into complementary codes, which produces range sidelobes similar to that of LFM.

In addition to bi-phase coding, polyphase codes provide more flexibility than bi-phase codes. The Frank code [16] or its modified versions - P1, P2, and Px codes [17] - are polyphase codes that approximate the LFM waveform. Some other LFM-like polyphase codes are P3 and P4 codes [18] as well as Golomb Polyphase codes [19].

There are intensive studies on finding constant-amplitude zero-autocorrelation (CAZAC) waveforms. Early works use different names for this kind of waveform [20] including: polyphase codes with good periodic [21] or optimum correlation properties [22], perfect autocorrelation [23] or root-of-unity sequences [24], bi-unimodular sequences [25], and bent functions [26]. To identify an appropriate CAZAC waveform for the radar is still an interest in the radar community [27]. Benedetto and Donatelli [20] propose the periodic CAZAC code, which can produce the mainlobe without sidelobes. In [23], a polyphase code with the length of 75 is shown as an example. When unimodular is not a limitation, Soltanalian [28] found virtually perfect phase-quantized sequences with extremely small sidelobes (PSLR is less than $-250\, dB$). Unfortunately, the magnitude of the sequence is not constant, and the dynamic range of the transmitted waveform is not as good as other poly-phased waveforms. Chen [29] utilizes a cyclic algorithm to synthesize probing sequences with good aperiodic autocorrelation properties, and to suppress the range sidelobes to a
lower than $-30 dB$ PSLR via a sequence of length 400. Stringer [30] applies multi-objective evolutionary algorithms (MOEAs) to the phase coded waveform design, and has achieved a lower than $-30 dB$ PSLR via a sequence of length 64. Jakabosky [31] applies the Marginal Fisher’s information algorithm to suppress the sidelobes of a continuous phased waveform to $-30 dB$ via a sequence of length 64.

When SNR is not a critical issue for the receiver, bi-phase or polyphase codes using mismatched filter is a selection. Baden [32] shows an orthogonal filtering approach for the orthogonal code pairs, and generates filters for mismatched pulse compression with low sidelobes near the mainlobe. The distribution of the sidelobes’ energy can be adjusted by a weight function. Griep [33] introduces mismatched poly-phase codes with low PSLR for a pulse compression system which has specific temporal and frequency characteristics. Polyphase codes’ limitation is the sensitivity to the Doppler effect [34], and the complexity of waveform generation for various purposes of applications.

To decrease the complexity of radar systems, some of today’s radar designs use pseudo-random-noise (PRN) code approaches [35, 36]. The PRN codes are easy to generate, and can achieve relatively good sidelobe suppression [37]. In recent studies, Axelsson [38] studies random bi-phase modulated waveform for improving the range resolution, but the waveform introduces high sidelobes. Pralon [39] studies random phase/frequency modulated waveform for effective sidelobe suppression. Another recent study on PRN shows that complex PNR waveforms are capable of reducing the near-sidelobes significantly [40, 41].

In recent years, Ultra-wideband (UWB) radar is studied for the high range resolution due to the use of wide bandwidth. Comparing with traditional sinusoid carrier
based waveforms, UWB radar transmits very short pulses with extremely wide bandwidth. In [42], Gaussian monocycle shapes are used for UWB. The pulse compression of the UWB radar usually uses a train of properly weighted and shifted Gaussian pulses based on selected codes [43]. Therefore, the waveform design is equivalent to finding a good range ambiguity bi-phase code. Nijsure [44] studies a bi-phase code based on Walsh-Hadamard code matrix for the UWB cognitive radar networks.

A different approach related to coding waveform is the frequency coding waveform. Similar to the phase coded waveform, the Costas waveform divides a relatively long pulse into N equal-duration sub-pulses with each sub-pulse centered at a different frequency. Generally, the frequencies are spaced equally. When the frequencies increase linearly with the sub-pulses, a stepped frequency waveform will generate. As we discussed in the last subsection, the stepped frequency waveform performs similarly as LFM, whose ambiguity function (Eq. 1.8) produces sidelobes in both the range and Doppler frequency directions. The Costas waveform [45] transmits a sequence of hopping-frequency waveforms.

Fig. 1.7 shows the comparison between the stepped frequency waveform and the Costas waveform with 10 sub-pulses. The row represents the time sequences, and the column denotes the frequency. A '1' specifies the frequency associated with the indicated sub-pulse. If the ambiguity function of Costas waveform is discretely calculated according to the sub-pulse duration and unit hopping-frequency, the ambiguity function will have an ideal range and the Doppler sidelobe behavior, which provide unambiguous Doppler and range information. However, the continuous ambiguity function still shows $-13.2dB$ sidelobes in the range and Doppler frequency directions. In a recent study, Bell [46] attempts to further push away the annoying sidelobes
Figure 1.7: The stepped frequency waveform (top) and the Costas waveform (bottom) using a hopping-frequency waveform. However, the near-sidelobes still exist due to the property of the autocorrelation of the hopping frequency waveform.

### 1.2.2 MIMO Radar Waveform

Because of the development in the area of field-programmable gate array (FPGA) and digital signal processor (DSP) in recent years, pulse compression in the digital domain becomes possible. Fig.1.8 shows a digital system for in-phase and quadrature signals in a single receiver. After demodulation and filtering, the received analog signals are converted into digital signals. The complex baseband signals are then cross-correlate with the reference waveform in the frequency domain via Fourier transform to increase the digital processing speed. The results then go through an inverse Fourier transform channel to obtain the cross-correlation in the digital domain.
Taking the advantage of digitalization in radar applications, MIMO radar systems have been studied intensively in recent years. Different from the phased array radar which transmits an identical waveform in all the antenna elements to generate a high gain in a specific direction, the MIMO radar transmits different waveforms on all the antenna elements involved. In [47], Li introduces the MIMO radar waveform in details.

According to [47], the technology of MIMO can be briefly introduced here. Assume that there are $M_t$ transmitting elements, $M_r$ receiving elements, and $\tau_i(\theta)$ is the time delay for the signal transmitted from the transmitting element number $i$ to the point target, or transmitted from the point target to the receiving element number $i$. $x(n) = [x_1(n), x_2(n), ..., x_{M_t}(n)]^T$ is the transmitted signal for each transmitting element, $a(\theta) = [e^{j2\pi f_c \tau_1(\theta)}, e^{j2\pi f_c \tau_2(\theta)}, e^{j2\pi f_c \tau_3(\theta)}, ..., e^{j2\pi f_c \tau_{M_t}(\theta)}]^T$ is the phase shift of the baseband waveform from the transmitting element to the point target. Similarly, $y(n) = [y_1(n), y_2(n), ..., y_{M_r}(n)]^T$ is the received signal for each received element,
\[ \mathbf{b}(\theta) = \begin{bmatrix} e^{j2\pi f_c \tilde{\tau}_1(\theta)} & e^{j2\pi f_c \tilde{\tau}_2(\theta)} & e^{j2\pi f_c \tilde{\tau}_3(\theta)} & \ldots & e^{j2\pi f_c \tilde{\tau}_{M_t}(\theta)} \end{bmatrix}^T \]

is the phase shift of the target reflected signals to the receiving element. When there are targets reflecting signals back to the radar receivers, the received signal will be:

\[ \mathbf{y}(n) = \sum_{k=1}^{K} \beta_k \mathbf{b}^c(\theta_k) \mathbf{a}^* (\theta_k) \mathbf{x} (n) + \varepsilon (n) \quad (1.26) \]

where \( \beta_i \) is the complex radar cross section (RCS) of the target \( i \), \( \varepsilon(n) \) is the interference-noise, \( (\bullet)^c \) denotes the complex conjugate, and \( (\bullet)^* \) denotes the conjugate transpose. The MIMO radar targets can be detected by resolving \( \{\beta_k\}_{k=1}^{K} \) and \( \{\theta_k\}_{k=1}^{K} \) via \( \{\mathbf{y}(n)\}_{n=1}^{N} : \tilde{\beta}_k = \beta_k \), \( \tilde{\theta}_k = \theta_k \).

The estimation of the targets parameter can be expressed as below:

\[ \sum_{k=1}^{K} \tilde{\beta}_k \mathbf{b}^c(\theta_k) \mathbf{a}^* (\theta_k) = \sum_{k=1}^{K} \beta_k \mathbf{b}^c(\theta_k) \mathbf{a}^* (\theta_k) \quad (1.27) \]

or

\[ \mathbf{\hat{B}} \mathbf{\hat{\beta}} = \mathbf{B} \mathbf{\beta} \quad (1.28) \]

where

\[ \mathbf{B} = \begin{bmatrix} \mathbf{a}^c(\theta_1) \otimes \mathbf{b}^c(\theta_1) & \ldots & \mathbf{a}^c(\theta_K) \otimes \mathbf{b}^c(\theta_K) \end{bmatrix} \]

\[ \beta = [\beta_1 \ldots \beta_K] \]

\( \mathbf{\hat{B}} \) and \( \mathbf{\hat{\beta}} \) are the estimation of \( \mathbf{B} \) and \( \beta \), respectively, and \( \otimes \) denotes the Kronecker product operator. Li [47] considers the Cramer-Rao bound of \( \{\theta_k\} \), and uses the Slepian-Bangs formula for estimating the parameters.

When the transmit array is the same as the receive array, the sample covariance matrix of the target reflected waveforms can be expressed as

\[ \mathbf{\hat{A}}^* \mathbf{\hat{R}}_{xx} \mathbf{\hat{A}} \quad (1.29) \]

where

\[ \mathbf{\hat{A}} = [\beta_1^* \mathbf{a}(\theta_1), \beta_2^* \mathbf{a}(\theta_2), \beta_3^* \mathbf{a}(\theta_3), \ldots, \beta_{M_t}^* \mathbf{a}(\theta_{M_t})] ; \]

\[ \mathbf{\hat{R}}_{xx} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{x}(n) \mathbf{x}^*(n). \]
When orthogonal waveforms are implemented in the transmitter and $N > M$, $\hat{\mathbf{R}}_{xx}$ becomes a scaled identity matrix. If the number of targets $K$ is less than $M_t$, $\tilde{\mathbf{A}}^*\hat{\mathbf{R}}_{xx}\tilde{\mathbf{A}}$ has full rank, and all the parameters can be resolved.

From the above analysis, one can see that constructing such a set of orthogonal waveforms in time domain is a preliminary step for the MIMO radar. Various approaches have been developed to design new radar waveforms such that the advantages of the MIMO radar can be fully taken. Kemkemian [48] obtains the orthogonality between the transmitted waveforms via Doppler Division Multiple Access (DDMA). Rabideau [49] discusses the code division multiple access (CDMA), time division multiple access (TDMA), and frequency division multiple access (FDMA) techniques, and further uses these conventional techniques to create sets of displaced waveforms for the MIMO radar. Cai [50] has designed orthogonal binary code waveforms for the MIMO radar. Takayama [51] further verifies the gold code based MIMO radar in the radar platform. Galati [52] proposes to use Costas codes and the phase noise signals to generate orthogonal waveforms for the MIMO radar. Han [53] and Chen [54] use frequency hopping codes. Majumder [55] presents a set of waveforms generated by direct sequence spread spectrum (DSSS) coding on LFM. Guang [56] utilizes Costas array coding (CAC) and quadratic congruence coding (QCC) in the MIMO radar. Sharma [57] uses phased coded waveforms based on the Kumar sequences. Yang [58, 59] produces chaotic phased coded waveforms for the MIMO radar.

Given the same amount of array elements, one advantage of the MIMO radar is its ability to resolve more targets in different directions comparing with the traditional phased array approach. Consider the situation that the transmitter array and the receive array are different in number, i.e. $M_t \neq M_r$. If the receive array is the
subset of the transmit array, $a^c(\theta) \otimes b^c(\theta)$ has only $M_t + M_r - 1$ distinct elements. However, when there are no sharing elements between the transmit and receive arrays, $a^c(\theta) \otimes b^c(\theta)$ has $M_t M_r$ distinct elements. Therefore, the number of maximum possible distinct objects to be resolved by the MIMO [45] is $K_{\text{max}} \in \max\left(\frac{M_t + M_r - 2}{2}, \frac{M_t M_r}{2}\right)$, which is up to $M_t$ times that of its phased array counterpart.

Another advantage of the MIMO radar is its capability to adjust its beamforming via the waveform design. Because the power in a direction $\theta$ is given by:

$$P(\theta) = a^*(\theta) R_{xx} a(\theta)$$

$R_{xx}$ can be optimized to control the spatial power distribution via the transmitted waveform. Li [45] considers different spatial beam patterns, such as the uniform elemental power for target searching, and multiple beams with minimized sidelobes for multiple targets tracking.

For applications in reality, the target response should be considered as a finite impulse response (FIR), which can be a Gaussian random vector with zero mean. Based on the information theory, Tang [60] proposes the MIMO radar waveform design in colored noise to maximize mutual information and relative entropy. Yang [61] designs waveforms that maximize the conditional mutual information between the random target impulse response and the reflected waveforms, and develop transmitted waveforms that minimize the mean-square error (MSE) in estimating the target impulse response. Naghibi [62] designs the MIMO radar in the presence of clutters.

Waveform design for MIMO is for improving the performance of the new radar system to meet the orthogonal waveform requirement as transmitted by individual radar elements in MIMO. Research to discover new radar waveforms for improving the performance in both conventional and new structures still continues. In the next
chapter, this dissertation introduce an emerge effort in recent years which is called wavelet-based waveform.

1.3 Organization of this Dissertation

The rest of this dissertation is organized as follows.

Chapter 2 will introduce the method to build wavelet based waveform. First, Daubechies order-4 scaling function is compared with low rate LFM signal, since the scaling function is one key component to build the wavelet based waveform. Then the method to build the wavelet based waveform is introduced. Though the wavelet based waveform suppress near-sidelobes in the range detection ideally, it generates far-sidelobes. The methods to reduce the far-sidelobes is also introduced. One can adjust the signs and sequences of the wavelet packets to generate noise like far-sidelobes, or can push the far-sidelobes out of the detection bounds.

Chapter 3 describes the advantages of the wavelet-based waveform, which is its high resolution ability for both range and speed detection when the detected targets are still or move. First, the simulation and experiment of the wavelet-based waveform for still targets detection are described. Then, how Doppler effect is used for speed detection, and how different waveforms affect the speed detection is studied. A further study of how Doppler effect influences range detection by different waveforms is also conducted. Then, how different waveforms affect the resolution of the synthetic aperture radar is discussed. Simulations results of SAR using the wavelet based waveform are presented, which illustrate the advantage of the wavelet based waveform in both range and velocity detections, particularly in the strip-map SAR.
Chapter 4 summarizes an alternative way for the phased array to increase its sensing efficiency, named *transform sensing*. First, the transform sensing is compared with the traditional sequential scanning for their efficiency. Then, we introduce how different sensing patterns can be formed and achieve a coarse-to-detail spatial resolution. Realization of different sensing patterns by the phased array is discussed in detail. A simulation generating wavelets for different sensing patterns is shown, and a preliminary experiment is implemented.

Chapter 5 summarizes this dissertation, and suggests future research directions to extend the work to a broader scope of visual tracking problems.
Chapter 2: Wavelet Based Waveform

2.1 Introduction

In this dissertation, we introduce a completely new approach that uses a set of discrete wavelets to design new radar waveforms. Wavelets and wavelet transforms have been used for voice, image, and video processing in recent years, including the JPEG2000 image compression standards [63]. However, there are only limited studies using wavelets to construct radar waveforms. Mohseni [64] generated ultra-wideband (UWB) radar waveforms via wavelet packet based on orthogonal frequency-division multiplexing (OFDM) radar signals. Lester [65] studied periodic continuous wavelets as radar waveforms. Fenlian [66] discussed the relationship between wide-band ambiguity functions and wavelet transforms. Wang [67] proposed transmitting single wavelet functions and performing wavelet decomposition at the receiver instead of using matched filters. Bonneau [68] uses wavelet packets as filter banks to decompose the chirp waveform according to the target spectral energy. The purpose is to improve radar detection probability. While the SAR platform employs the chirp waveform to generate the sinc function in the both range and azimuth directions, Xu [69] uses wavelet transform to improve moving target detections. However, none of these works consider using the discrete scaling function and wavelets to construct
radar waveforms (except an early work by the current authors [70]). We consider that features of multi-resolution representations of signals in both the time and frequency domains empower designing radar waveforms in the form of multiple wavelets, which are equivalent to multiple and separable subbands. Such waveforms are adaptive to the environment as well as detection purposes by adjusting the magnitude and phase of one or multiple subbands (wavelets).

For that purpose, we propose a new adaptive radar pulse based on wavelets that differs from the conventional LFM signal as the frequency in the signal is not linearly modulated to increase the bandwidth for the purpose of pulse compression. Instead, the signal increases or decreases its bandwidth via subbands. The bandwidth of each wavelet subband is adaptive, i.e., it is not necessarily equal in the time duration of the signal. Such a waveform can meet the needs of cognitive radar [71]. In this chapter, we show how such a radar waveform can be constructed by using wavelets and their corresponding scaling function. Our theory have verified the advantage of sidelobe suppression in autocorrelation using the wavelet-based waveform, which is a surprising discovery during the course of our study. However, far-sidelobes emerge as a side effect of the near-sidelobe suppression. We introduce an optimization method to suppress the far-sidelobes. After optimization, they can be suppressed to the noise level and, thus, can be ignored as noise always exists. Since the Costas waveform is another hopping frequency waveform built by windowed sinsoid waveforms as components, we compare the wavelet based waveform with the Costas waveform. We also discussed another method to deal with the far-sidelobes of the wavelet based waveform, which is inserting time interval between the adjacent wavelet packets so that the far-sidelobes will be pushed outside the detection bounds.
This chapter is organized as follows. Section 2.2 presents the basic components to build wavelet based waveform. We compared the scaling function with low rate LFM signal, and introduced the property of wavelet packets, which is fundamental of the wavelet based waveform. Section 2.3 shows the wavelet based waveform, and analyze the range and doppler ambiguity of this waveform. Further, we discussed the way to suppress or push away far-sidelobes and compare the wavelet based waveform with the Costas waveform. Section 2.4 concludes the chapter.

2.2 Components to build wavelet based waveform

2.2.1 Daubechies Order-4 Scaling Function Compares with Low Rate LFM Signal

LFM is a conventional radar signal, which is defined as follows:

\[ f(t) = \sin \left( (f_0 + \beta t) t + \theta \right), \quad (2.1) \]

where \( f_0 \) is the carrier frequency, \( \beta \) is the chirp rate, and \( \theta \) is the phase angle. For convenience, we choose Daubechies-4 (D-4) wavelets in which the order of the vanishing moment \( M = 4 \). To compare with a D-4 scaling function, we set the carrier frequency to 0 Hz, the signal duration to 3 s, and \( \beta \) to 0.59 rad/s, which is a low-rate LFM. In this way, the D-4 scaling function and the LFM signal have the same 3 dB bandwidth and signal duration such that we can fairly compare the two waveforms.

Fig. 2.1 shows the low rate LFM signal as well as the scaling function, both lasting for 3 s. In Fig. 2.2, the spectra of the 3 s scaling function and the low rate LFM signal are shown. In Fig. 2.3, the autocorrelations of the scaling function and the LFM are shown. The LFM signal has a narrower mainlobe and two serious near-sidelobes. The autocorrelation of the scaling function appears as a spike with little
sidelobes, which is ideal for radar, since the resolution and distinction between two close objects are improved. In Fig. 2.3, the autocorrelation of the scaling function has a peak (mainlobe) to sidelobe ratio of 22.69 dB, whereas that of the LFM is 13.2 dB.

![Figure 2.1: D-4 scaling function and low-rate LFM signal](image)

To determine why the two waveforms perform differently, the following mathematical analysis is in order. The autocorrelation of LFM has been discussed by many publications [72]. Assume the LFM function is

$$s(t) = \begin{cases} 
e^{i\pi \beta t^2}, & \text{if } -\frac{T}{2} \leq t \leq \frac{T}{2}, \\ 0, & \text{otherwise}, \end{cases}$$

(2.2)

where $T$ is the duration of the radar pulse.

The autocorrelation of $s(t)$ is a sinc function:

$$\phi_s(t) = T\Lambda \left( \frac{t}{T} \right) \text{sinc} \left( \pi \beta T t \Lambda \left( \frac{t}{T} \right) \right),$$

(2.3)
where

\[
\Lambda(t) = \begin{cases} 
1 + t, & \text{for } -1 \leq t \leq 0, \\
1 - t, & \text{for } 0 \leq t \leq 1, \\
0, & \text{otherwise.}
\end{cases} 
\]  

(2.4)

The sinc function determines LFM’s peak to near-sidelobe ratio is 13.2 dB.

On the other hand, the autocorrelation of a discrete scaling function can be iteratively obtained as follows. Let \( c(k) \) be the coefficients of the autocorrelation:

\[
c(k) = 2 \sum_{l=0}^{L-l-k} h(l)h(l + k). 
\]  

(2.5)

Then the autocorrelation function can be iteratively generated as below [73]:

\[
\phi(k/2) = \phi(k) + \frac{1}{2} \sum_{l \in N} c(2l - 1)(\phi(k - 2l + 1) \\
+ \phi(k + 2l - 1)). 
\]  

(2.6)

Every iteration results in an autocorrelation with higher resolution but lower peak to near-sidelobe ratio. For example, the autocorrelation of the Daubechies order-4 (D-4) scaling function generated by one iteration (based on the roughest scaling function)
Figure 2.3: Autocorrelation of D-4 scaling function and low-rate LFM signal has a 24.08 dB peak to near-sidelobe ratio. When the iteration time is greater than 8, the peak to near-sidelobe ratio converges to 22.69 dB, which is significantly greater than 13.2 dB. The autocorrelations generated via different iteration times are shown in Fig. 2.4.

Figure 2.4: Autocorrelation of D-4 scaling function generated by 1–8 iterations
Differences in the two waveforms’ spectra can also provide insight into the different autocorrelations between the D-4 scaling function and the LFM. It is known that the sinc function in the time domain has a rectangular spectrum in the frequency domain. Due to time-frequency duality, a rectangular function in the time domain has a sinc spectrum in the frequency domain. Therefore, if we aim to achieve a rectangular autocorrelation, the signal should have a sinc spectrum. The D-4 scaling function does have a sinc like spectrum (see Fig. 2.2), while the LFM signal has a smoothly decaying spectrum. As a result, the LFM signal has a sinc autocorrelation in the time domain, but the scaling function has a spike-shaped autocorrelation.

However, a single scaling function has a very low compression rate due to the narrow bandwidth. Achieving a high pulse-compression ratio entails increasing the bandwidth of the scaling function. One way to do so is to compress the scaling function and pad zeros after it. In this way, a new scaling function with a broader bandwidth is generated (Fig. 2.5). The compressed scaling signal has a wider bandwidth and maintains the spike-shaped autocorrelation. Unfortunately, the effective waveform is shorter than the original scaling function, and the padding zeros will waste transmitted energy as nothing is carried by the radar signal. Consequently, increasing the bandwidth of a radar signal using wavelets to maintain the time duration of the pulse becomes the new approach. In the next section, the form of the wavelet-based waveform is formally introduced.

2.2.2 The basics of wavelet-based waveform

Beylkin and Saito studied the autocorrelation of wavelets in a theoretical work [73]. They show that the summation of the autocorrelation of the scaling function and the
autocorrelation of the wavelet is the autocorrelation of the narrowed-by-2 scaling function (Fig. 2.6). One way to explain the above result uses the spectra of the scaling function and the wavelet as shown in Fig. 2.7. The Fourier transform of the wavelet $G(w)$ is the complement of the scaling function $H(w)$. Thus, the summation of the spectrum of the scaling function and that of the wavelet is the spectrum of the compressed scaling function. In other words, the spectrum of a compressed scaling function can be divided into the spectrum of a scaling function plus that of a wavelet. The formal mathematical proof is shown below [73].

By Eq. (1.13), one has

$$G(w) = H(w + \pi).$$

Let the spectra of the autocorrelations of $\varphi(t)$ and $\psi(t)$ be $S_\varphi(w)$ and $S_\psi(w)$, respectively. According to Eqs. (1.24) and (1.25), we have the following:

$$S_\varphi(w) = \varphi(w) \cdot \varphi(w)^*$$

$$= \left| \frac{1}{\sqrt{2}} H(w) \right|^2 \prod_{k=1}^{\infty} \frac{1}{\sqrt{2}} H \left( \frac{w}{2^k} \right) \Phi(0) \right|^2$$

Figure 2.5: Comparison of compressed scaling function (blue) with LFM signal having same bandwidth (red)
Figure 2.6: Summation of autocorrelations of a D-4 scaling function and a wavelet equals autocorrelation of a narrowed-by-2 scaling function

\[ S_\psi(w) = \psi(w) \cdot \psi(w)^* = \left| \frac{1}{\sqrt{2}} G(w) \right|^2 \prod_{k=1}^{\infty} \left| \frac{1}{\sqrt{2}} H \left( \frac{w}{2^k} \right) \Phi(0) \right|^2. \]  

(2.9)
Then, from Eqs. (1.23) and (2.7), the summation of $S_\varphi(w)$ and $S_\psi(w)$ is:

\[
S_\varphi(w) + S_\psi(w) = \left( \frac{1}{2} |H(w)|^2 + \frac{1}{2} |G(w)|^2 \right) \cdot \left[ \prod_{k=1}^{\infty} \frac{1}{\sqrt{2}} H \left( \frac{w}{2^k} \right) \Phi(0) \right] \cdot \Phi(0) \cdot \left[ \prod_{k=1}^{\infty} \frac{1}{\sqrt{2}} H \left( \frac{w}{2^k} \right) \Phi(0) \right] \cdot \Phi(0) = S_\varphi \left( \frac{w}{2} \right),
\]

where $S_\varphi \left( \frac{w}{2} \right)$ is the spectrum of the autocorrelation of the narrowed-by-2 scaling function.

Based on the above approach, the concept of wavelet packets can be applied here to further divide a waveform into a number of constructing signals that are called wavelet packets [11]. Fundamentally, wavelet packets are formed by iterating the
original scaling function and the wavelet into waveforms with narrower bands. The process includes interpolating, splitting, and filtering the scaling function such that it becomes a new scaling function plus a wavelet with half the bandwidth of the original scaling function. The same can be applied to the wavelet (or the new scaling function) to form a lower half band and a higher half band. This iteration continues generating a number of wavelet packets, each of which takes a subband of the entire waveform bandwidth. An example of eight packets (subbands) is shown in Fig. 2.8, in which each packet spans the same bandwidth. In Fig. 2.9, these eight wavelet packets are plotted in the time domain. The iteration to generate four wavelet packets $W_0(w)$,
$W_1(w)$, $W_2(w)$, $W_3(w)$ is shown below:

$$W_0(w) = \left[ \frac{1}{\sqrt{2}} H(w) \frac{1}{\sqrt{2}} H\left(\frac{w}{2}\right) \prod_{k=2}^{\infty} \frac{1}{\sqrt{2}} H\left(\frac{w}{2^k}\right) \right] \Phi(0)$$

$$W_1(w) = \left[ \frac{1}{\sqrt{2}} H(w) \frac{1}{\sqrt{2}} G\left(\frac{w}{2}\right) \prod_{k=2}^{\infty} \frac{1}{\sqrt{2}} H\left(\frac{w}{2^k}\right) \right] \Phi(0)$$

$$W_2(w) = \left[ \frac{1}{\sqrt{2}} G(w) \frac{1}{\sqrt{2}} H\left(\frac{w}{2}\right) \prod_{k=2}^{\infty} \frac{1}{\sqrt{2}} H\left(\frac{w}{2^k}\right) \right] \Phi(0)$$

$$W_3(w) = \left[ \frac{1}{\sqrt{2}} G(w) \frac{1}{\sqrt{2}} G\left(\frac{w}{2}\right) \prod_{k=2}^{\infty} \frac{1}{\sqrt{2}} H\left(\frac{w}{2^k}\right) \right] \Phi(0).$$

The summation of autocorrelations of all the wavelet packets is:

$$S_W = S_{W_0} + S_{W_1} + S_{W_2} + S_{W_3}$$

$$= \left( \left| \frac{1}{\sqrt{2}} H(w) \frac{1}{\sqrt{2}} H\left(\frac{w}{2}\right) \right|^2 + \left| \frac{1}{\sqrt{2}} H(w) \frac{1}{\sqrt{2}} G\left(\frac{w}{2}\right) \right|^2 \right. \right.$$

$$+ \left. \left| \frac{1}{\sqrt{2}} G(w) \frac{1}{\sqrt{2}} H\left(\frac{w}{2}\right) \right|^2 + \left| \frac{1}{\sqrt{2}} G(w) \frac{1}{\sqrt{2}} G\left(\frac{w}{2}\right) \right|^2 \right)$$

$$\cdot \left| \prod_{k=2}^{\infty} \frac{1}{\sqrt{2}} H\left(\frac{w}{2^k}\Phi(0)\right) \right|^2$$

$$= \left( \left| \frac{1}{\sqrt{2}} H\left(\frac{w}{2}\right) \right|^2 + \left| \frac{1}{\sqrt{2}} G\left(\frac{w}{2}\right) \right|^2 \right)$$

$$\cdot \left| \prod_{k=2}^{\infty} \frac{1}{\sqrt{2}} H\left(\frac{w}{2^k}\Phi(0)\right) \right|^2$$

$$= \left| \prod_{k=2}^{\infty} \frac{1}{\sqrt{2}} H\left(\frac{w}{2^k}\Phi(0)\right) \right|^2$$

$$= S_\varphi\left(\frac{w}{4}\right),$$

where $S_\varphi\left(\frac{w}{4}\right)$ is the spectrum of the autocorrelation of the narrowed-by-4 scaling function.

The advantage of cascading the wavelet packets is increased bandwidth of a radar waveform with the same time duration. For example, if the pulse width is 1 $\mu$s, one scaling function riding the radar pulse will have a bandwidth of 1.5 MHz. If we embed eight wavelet packets in the same pulse width, the bandwidth can be increased to 96
MHz. In Fig. 2.10, the summation of the autocorrelations of eight wavelet packets is shown in the foreground layer, and the autocorrelations of the eight wavelet packets are shown in the background layer. One can see that for each individual wavelet packet, the autocorrelation is not ideal (with many sidelobes), but the summation of the autocorrelations produces a spike-shaped autocorrelation with little sidelobes.
To compare the wavelet-based waveform with the conventional LFM waveform of the same bandwidth, the time-frequency diagrams of the LFM and the proposed waveform are shown in Fig. 2.11. Both waveforms have the same bandwidth, $\Delta F = 64\Delta f$. The LFM waveform increases the frequency linearly and continuously, while the proposed waveform increases the frequency discretely at subband granularity according to the packets used in the waveform. One can see the time-frequency blocks in the wavelet-based waveform instead of a single line. In each time duration $\Delta t$, there is a wavelet packet that spans a subband of the spectrum. Within $\Delta t$, one cannot resolve a particular frequency for each point of time, only a subband spanned by the corresponding wavelet. The wavelet-based waveform is more flexible for adjusting the magnitude and phase for specific frequency subbands as mentioned earlier, since the adjustments can be focused on a particular wavelet packet. In reality, a wavelet based waveform is generated by a chosen sequence of wavelet packets. One example is shown in Fig. 2.12, which has eight wavelet packets. Each of the eight packets is shown individually in Fig. 2.9. When they are cascaded sequentially, a radar waveform is formed.

Fig. 2.13 compares the autocorrelation functions of an LFM and of the wavelet-based waveform with the same duration and bandwidth. One can see that the autocorrelation of the wavelet based waveform does not have significant near-sidelobes, which are nearly adjacent to the mainlobe, whereas the LFM generates two significant near-sidelobes. The wavelet-based waveform has a peak to near-sidelobe ratio of 22.69 dB in comparison to 13.2 dB for the LFM waveform with the same bandwidth. Another advantage of the proposed waveform is that each subband of the wavelet packets can have a different bandwidth. One can use the adaptive wavelet packets
Figure 2.11: Time-frequency diagram of LFM waveform (left) and wavelet based waveform (right)

approach to adjust each packet’s bandwidth, resulting in different bandwidths for each packet. In Fig. 2.14, the time-frequency diagram of a proposed waveform generated by adaptive wavelet packets is shown. Unlike in Fig. 2.11, the blocks in the time-frequency diagram have various sizes. The user can adjust the sizes according to target response and application purpose. In Fig. 2.15, the radar waveform generated by the adaptive wavelet packets of Fig. 2.14 is shown along with the autocorrelation of the new radar waveform. The peak to near-sidelobe ratio of this new proposed waveform is still 22.69 dB since the overall bandwidth remains the same.

One may notice that the wavelet-based waveform generates significant far-sidelobes, which are far away from the mainlobe. The far-sidelobes do not block nearby targets but they may block targets whose distances are far away from the mainlobe. The latter is not desirable either. Fortunately, the far-sidelobes of the wavelet-based waveform can be reduced to become noise like signals without influencing near-sidelobe suppression. This can be done by selecting different signs and sequences of the wavelet packets. The details of this selection process will be introduced later.
2.3 Performance Analysis of Wavelet-Based Waveforms

2.3.1 Range and Doppler ambiguity

In the following, we formalize the waveform detection problem using wavelets. Suppose the sequence of ordered wavelets is:

\[ W_0(t), W_1(t), W_2(t), W_3(t), \ldots, W_{N-1}(t), \]  \hspace{1cm} (2.13)
where, for any \( j \), \( W_j(t) \) is a wavelet packet when \( 0 \leq t \leq \Delta t \); otherwise, \( W_j(t) = 0 \).

Note \( W_0(t) \) is the scaling function. Then the wavelet-based waveform is be given by:

\[ \mu(t) = \sum_{n=0}^{N-1} W_n(t - n\Delta t). \]  

(2.14)

The delay-Doppler ambiguity function is defined as

\[ \chi(\tau, v) = \int_{-\infty}^{\infty} \mu^*(\lambda)\mu(\lambda - \tau)e^{j2\pi v\lambda}d\lambda. \]  

(2.15)

To understand the physical principles behind using wavelet based waveforms as detection signals, define the cross-correlation function:

\[ \phi_{nm}(\tau, v) = \int_{-\infty}^{\infty} W_n^*(\lambda)W_m(\lambda - \tau)e^{j2\pi v\lambda}d\lambda. \]  

(2.16)

When \( m = n \), Eq. (2.16) becomes the autocorrelation function \( \phi_{nn} \).

Suppose only two wavelet packets, i.e., the scaling function and a wavelet, are used. The spectra of the wavelet packets are \( \Phi(w) \) and \( \Psi(w) \) as shown in Eqs. (1.24)
Figure 2.15: Proposed waveform (left) generated by using adaptive wavelet packets and its autocorrelation (right)

and (1.25). As an example, consider the frequency domain of the cross-correlation function, Eq. (2.16), between the wavelet packets:

\[
\phi_{01}(w, v) = \Phi(w - v)\Psi^*(w)
\]

\[
= \left[ \frac{1}{\sqrt{2}} H(w - v) \prod_{k=1}^{\infty} H \left( \frac{w - v}{2^k} \right) \right] \Phi(0)
\]

\[
\cdot \left[ \frac{1}{\sqrt{2}} G^*(w) \prod_{k=1}^{\infty} \frac{1}{\sqrt{2}} H^* \left( \frac{w}{2^k} \right) \right] \Phi^*(0).
\]

(2.17)

When \( v = 0 \), one has:

\[
\phi_{01}(w, 0) = \frac{1}{\sqrt{2}} H(w) \frac{1}{\sqrt{2}} G^*(w) \left[ \prod_{k=1}^{\infty} \frac{1}{\sqrt{2}} H \left( \frac{w}{2^k} \right) \Phi(0) \right]^2.
\]

(2.18)

Transforming \( \phi_{01}(w, 0) \) to the time domain, we obtain

\[
\phi_{01}(\tau, 0) = \sum_n [h(n) * g(-n)] R_\varphi(2\tau - n),
\]

(2.19)

where * is the convolution operator and \( R_\varphi(2t) \) is the autocorrelation of the narrowed-by-2 scaling function. Similarly, for \( 2^n \) wavelet packets, the cross-correlation between


any two wavelet packets will be equal to the convolution of the autocorrelation of the narrowed-by-$2^n$ scaling function and the wavelet coefficients (or up-sampled wavelet coefficients).

From Eqs. (2.14) and (2.15), we obtain

$$
\chi(\tau, v) = \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} W_n^*(\lambda - n\Delta t) \times \sum_{m=0}^{N-1} W_m(\lambda - \tau - m\Delta t)e^{j2\pi v \lambda}d\lambda.
$$

By using Eq. (2.16), we can obtain

$$
\chi(\tau, v) = \sum_{n=0}^{N-1} e^{j2\pi nv \Delta t} \times [\phi_{nn}(\tau, v) + \sum_{m=0, m \neq n}^{N-1} \phi_{nm}(\tau - (n - m)\Delta t, v)].
$$

Note that $\phi_{nm}(\tau, v) = 0$ when $\tau \geq \Delta t$, since the two wavelets are fully separated.

First, we consider the case when $v = 0$. As it is shown in Eq. (2.21), the $\phi_{nm}$ term is zero when $|\tau| \geq \Delta t$. So

$$
\chi(\tau, 0) = \sum_{n=0}^{N-1} \phi_{nn}(\tau, 0).
$$

As we discussed in Section 2.2.2, the summation of the autocorrelations of the wavelet packets equals the autocorrelation of the narrowed scaling function. Regardless of the sequence of the wavelet packets, the autocorrelation of the wavelet sequence has a resolution of $\Delta t/N$. However, when $|\tau| > \Delta t$ the autocorrelation of the waveform is completely determined by $\phi_{nm}(\tau, 0)$, where $n \neq m$.

Now consider the case when $\tau = 0$. Because the wavelet-based waveform is generated by multiplying a carrier with the wavelets, the waveform is amplitude modulated. As the wavelets' bandwidth is at the lower end of the spectrum in comparison with the
frequency of the carrier, the Doppler effect only applies to the carrier. Consequently, one may obtain

\[ \chi(0, v) = \int_{-NT}^{NT} e^{j2\pi v\lambda} d\lambda \]  
\[ |\chi(0, v)| = \left| \frac{\sin(\pi vNT)}{\pi vNT} \right|. \]  

The \( \tau = 0 \) axis shows that the velocity ambiguity function is a sinc function related to the duration of the transmitted waveform.

In the \( v = 0 \) axis, the wavelet-based waveform can effectively suppress near-sidelobes. However, the far-sidelobes that are generated by the term \( \phi_{nm} \) (where \( n \neq m \)) will influence the radar resolution. A figure describes the autocorrelation of the waveform in a straightforward manner. If the wavelet packets are transmitted in an ascending sequence, i.e., \([W_0, W_1, W_2, W_3, W_4, W_5, W_6, W_7]\), the received signal will be received in the same sequence. The cross-correlation between the received and the reference signals is shown in Fig. 2.16.

![Packet correlation operations. Autocorrelation (top); Cross-correlation when waveforms are shifted by one packet (bottom)](image)

Figure 2.16: Packet correlation operations. Autocorrelation (top); Cross-correlation when waveforms are shifted by one packet (bottom)
The center of the autocorrelation (plotted at the top of Fig. 2.16) is the summation of the autocorrelations of each individual packet. Thus the magnitude of the mainlobe of the proposed waveform’s autocorrelation is eight times the peak of a single scaling function. Similarly, the bottom of Fig. 2.16 reveals cross-correlation among wavelet packets such as \( W_0 \ast W_1 + W_1 \ast W_2 + W_2 \ast W_3 + W_3 \ast W_4 + W_4 \ast W_5 + W_5 \ast W_6 + W_6 \ast W_7 \).

Unfortunately, the cross-correlation between two wavelet packets, i.e., \( \phi_{nm} \) (where \( n \neq m \)), is non-zero, which causes the far-sidelobes.

The cross-correlation between the wavelet packets, i.e., \( \phi_{nm} \) (where \( n \neq m \)), can be more conveniently studied in the frequency domain. In Fig. 2.8, it is shown that the spectra of the D4-wavelet packets overlap. That means that the wavelet packets are not completely orthogonal. The spectrum overlap between two packets causes ripples in the cross-correlation, and the summation of all the overlaps generates the far-sidelobes. Fig. 2.17 shows how wavelet packet 1 cross-correlates with the other seven packets in the wavelet-based waveform of eight wavelet packets.

![Cross-correlation between wavelet packet 1 and other wavelet packets](image)

Figure 2.17: Cross-correlation between wavelet packet 1 and other wavelet packets
2.3.2 Reduction of the Far-Sidelobes

Adjust Signs and Sequences of the Wavelet Packets

If the wavelet packets are transmitted in a different sequence with different signs, the cross-correlation between the received and reference waveforms could be as shown in Fig. 2.18. A new waveform is generated by concatenating the packets in this specific sequence with these particular signs. The new waveform has the same duration and bandwidth as the original one as they are constructed using the same set of wavelet packets. The centers of the autocorrelations of the two waveforms as plotted at the top of Figs. 2.16 and 2.18, respectively, are the same, which equals the summation of all individual autocorrelations as is proved in Eq. (2.22). The far-sidelobes of the two waveforms are different due to the different sequences and signs of the wavelet packets (see the bottom of Figs. 2.16 and 2.18). As a result, the new waveform that is generated by different sequences and signs of the same set of wavelet packets does not affect the near-sidelobes and the mainlobe, but reduces the far-sidelobes. The approach here is similar to the Costas waveform, which uses frequency hopping to suppress sidelobes.

Select Signs and Sequences of the Wavelet Packets

In practice, the selection process searches all possible signs and sequences of the wavelet packets to identify the “best” ones for far-sidelobe minimization. It is mathematically impossible, let alone justifiable, to develop a formulation for this purpose. This is due to the myriad variety of wavelets and the order thereof that one can choose to form the radar waveform. On the other hand, a complete search may
Figure 2.18: Packet correlation operations: autocorrelation (top); cross-correlation when waveforms are shifted by one packet (bottom)

produce the best sequence and signs. Once the search is completed, no more computation is needed to form the waveform. Thus, search is a one-time process that can be performed off-line.

Formally, the possible signs and sequences of the wavelet packets are enumerated to generate different waveforms. Then the peaks of the far-sidelobes in each waveform is compared. The waveform with the minimum peak is chosen as “the best” in terms of far-sidelobe suppression.

For the wavelet-based waveform of eight wavelet packets, there are $10,321,920 = 2^8 \times 8!$ possibilities. It takes 2 h to complete the computation using a laptop with an Intel Core i5-2520 processor and 8 GB RAM. To increase computational efficiency, we fix the first wavelet packet as the scaling function with zero phase. This reduces the total number of iterations to $645,120 = 2^7 \times 7!$, which can be completed in 7 min 30 s by the same laptop. We can fix the first wavelet packet as changing its sign is
equivalent to changing the signs of all the other packets simultaneously, which does not affect the magnitude of the autocorrelation. Also, changing the position of the first packet only alters the relative positions of the far-sidelobes, not the magnitude. Consequently, the computation efficiency can be significantly improved.

An optimal solution for the wavelet-based waveform constructed by eight wavelet packets is shown in Fig. 2.19. The peak to far-sidelobe ratio of the autocorrelation of the original waveforms is 10.75 dB (shown in Fig. 2.13). The peak to far-sidelobe ratio of the autocorrelation of the optimized waveform is 20.63 dB (shown in Fig. 2.20). We have also compared the autocorrelation of this optimized waveform with that of an LFM waveform with the same bandwidth and same duration as shown in Fig. 2.20. It shows that the optimized waveform can effectively suppress the near-sidelobes as well as the far-sidelobes to the noise level. For the wavelet-based waveform, the peak to largest sidelobe energy ratio is 18.0 dB. For LFM, this energy ratio is 14.3 dB. As discussed in the earlier subsection, if a large number of wavelet packets are used,
the far-sidelobes can be suppressed to an even lower level. In the meantime, the mainlobe’s peak energy remains the same.

Figure 2.20: Autocorrelation comparison between LFM (red) and optimized waveform (blue). Full view of the autocorrelation is on left; close-up of center of mainlobe is on right

For the adaptive wavelet packets shown in Fig. 2.14, we also construct the optimized waveform. Using an ascending sequence of wavelet packets, the peak to far-sidelobe ratio of the proposed waveform is 11.90 dB (shown in Fig. 2.15), and that with the optimized sequence and signs is 19.66 dB (shown in Fig. 2.21).

From the discussion above, one can see that the wavelet-based waveform can suppress far-sidelobes by optimizing the signs and sequences of the wavelet packets. It should be noted that due to the unit volume of the ambiguity function, the suppressed sidelobes may appear somewhere else. The optimization process lowers the peak value of the sidelobes such that they appear as noise.
2.3.3 Comparison with the Costas waveform

Costas proposed a waveform based on frequency hopping [45]. For each item in the Welch code, a single frequency window function is transmitted. In each window, the waveform proposed in this paper transmits a frequency band instead (Fig. 2.22). Because of the wavelets’ time-frequency localized property, the cross-correlation of the wavelet packets will only affect the nearby frequency packets. The single frequency window function, which is the sinc function in the frequency domain, spreads in all frequency bands, thus impacting all other frequency window functions.

![Figure 2.22: Comparison between Costas and wavelet-based waveforms](image)

Figure 2.22: Comparison between Costas and wavelet-based waveforms
The range ambiguity function of the Costas waveform is proved to have resolution $\Delta t/N$ (where $N$ is the length of the Costas code, and $\Delta t$ is the duration of the window function for each code). This resolution is the same as that of the waveform proposed in this paper. For sidelobes, the Costas waveform is proved to be zero for every $\Delta t$ shift and a sinc function at the center of the autocorrelation. In contrast, as shown in Fig. 2.3, the center of the autocorrelation of the wavelet-based waveform is the autocorrelation of the narrowed scaling function, which will suppress the sidelobes much more effectively than the sinc function. Further, by Eq. (1.20), the cross-correlation among wavelet packets is zero for every $\Delta t/3$ step of Daubechies order-4 wavelets.

As proved for the Costas waveform [45], a longer code sequence decreases the far-sidelobes relative to the increase of the center peak of the autocorrelation. The waveform proposed in this paper will also suppress the far-sidelobes more effectively as more wavelet packets are used. When the sequence of the Costas code increases, a sinc function caused by the middle of the autocorrelation will increase range ambiguity. Similarly, the wavelet-based waveform will also be limited by the autocorrelation of the scaling function when the wavelet packet sequence increases. However, the autocorrelation of the scaling function has much smaller sidelobes than that of the sinc function, which helps improve range resolution.

The velocity ambiguity function of the Costas waveform is proved to be the same as the waveform proposed in this paper. However, both waveforms assume that the Doppler effect results in a single frequency shift due to the high ratio between the carrier frequency and the baseband signal bandwidth. For wide-band waveforms, the
Costas waveform seriously degrades when the signal shrinks or expands. The wavelet-based waveform will not degrade via shrinking or expanding as a constant amplitude amplifier will be able to filter out the baseband waveform, focusing only on the carrier. The carrier will only shift a single frequency when shrinking or expanding.

To its advantage, the Costas waveform can fully use the amplifier’s dynamic range, but the wavelet-based waveform cannot do so, since it is an amplitude-modulated signal. Consequently, the power amplifier’s range is not fully utilized. This is the trade-off between the utilization of the amplifier and the resolution of range/Doppler using the proposed approach.

### 2.3.4 Push the far sidelobes out of the detection bounds

The wavelet-based waveform as shown in Fig. 2.23 is a sequence of sub-pulses, each being a wavelet packet. Since between any two packets, there exist overlaps in the frequency domain, we can also introduce time gaps between two neighboring packets. The purpose is to perform exact autocorrelations for each packet, eliminating the effects of its cross-correlation with any other packet. This can help eliminate the far-sidelobes caused by the continuous cross-correlation between wavelet packets. Meanwhile, the near-sidelobes are determined by the autocorrelation of scaling function, and are thus much smaller than the LFM and other traditional waveforms. Some traditional waveforms using windowed sinusoid waveforms as sub-pulses (such as stepped frequency waveform [74] and Costas waveform [45]) generate sinc function like near-sidelobes, and cannot suppress these sidelobes effectively. The phase coded waveforms (such as Barker code [12], Frank code [16], and P1, P2, Px codes [17, 18]) cannot remove sidelobes totally, and its performance degrades seriously as the Doppler
effect increases. For the proposed new waveform, an example signal consisting of eight D-4 wavelet packets is shown in Fig. 2.23.

Figure 2.23: The wavelet-based waveform consisting of eight D-4 wavelet packets

The expression of the wavelet-based waveform can be written as:

\[
\mu(t) = \sum_{n=0}^{N-1} P_n(t - n\Delta t).
\]  

(2.25)

where \( P_n(t) \) is the sub-pulse of the waveform, and the sub-pulse duration is \( \Delta t = T_1 + T_2 \), where \( T_1 \) is the duration of each wavelet packet, and \( T_2 \) is the delay separating two consequent wavelet packets;

\[
P_n(t) = \begin{cases} 
W_n(t), & 0 \leq t \leq T_1 \\
0, & T_1 < t \leq T_2
\end{cases}
\]  

(2.26)
where \( W_n(t) \) is the wavelet packet, \( n \) refers to the number of the packet, and '0' between \( T_1 \) and \( T_2 \) in Eq. (2.26) means the silent interval between the wavelet packets.

The cross-correlation between the sub-pulse is defined as:

\[
\phi(\tau) = \int P_n(t)P^*_m(t-\tau)dt,
\]

\[
= \begin{cases} 
  \int W_n(t)W^*_m(t-\tau)dt, & |\tau| \leq T_1, \\
  0, & T_1 \leq |\tau| \leq T_1 + T_2.
\end{cases}
\]

(2.27)

The autocorrelation of \( \mu(t) \) is:

\[
\chi(\tau) = \int \sum_{n=0}^{N-1} P_n(t-n\Delta t)\sum_{m=0}^{N-1} P_m(t-m\Delta t-\tau).
\]

(2.28)

Note that \( \phi_{nm}(\tau) = 0 \), when \( \tau \geq \Delta t \), since the two sub-pulses are fully separated. Eq. (2.28) implies the following two cases.

a) When \( |\tau| \leq \Delta t \), the \( \phi_{nm} \) term equals zero, and \( \chi(\tau) \) is totally determined by the summation of the autocorrelations of the packets, i.e. the \( \phi_{nn} \) term. We obtain

\[
\chi(\tau) = \phi_{00}(\tau) + \phi_{11}(\tau) + ... + \phi_{NN}(\tau).
\]

(2.29)

Recall in Eq. (2.10), the summation of the autocorrelation of \( N \) wavelet packets is equal to narrowed-by-N scaling function (i.e. wavelet packet number 0). Thus,

\[
\chi(\tau) = \begin{cases} 
  \phi_{00}(N\tau), & |\tau| \leq \frac{T_1+T_2}{N}, \\
  0, & \frac{T_1+T_2}{N} \leq |\tau| \leq T_1 + T_2,
\end{cases}
\]

\[
= \begin{cases} 
  \int W_0(t)W^*_0(t-N\tau)dt, & |\tau| \leq \frac{T_1}{N}, \\
  0, & \frac{T_1}{N} \leq |\tau| \leq T_1 + T_2.
\end{cases}
\]

(2.30)

Eq. (2.30) shows that the summation of the autocorrelations of the packets achieve effective range suppression, when \( |\tau| \leq T_1 + T_2 \).

b) When \( |\tau| \geq \Delta t \), the \( \Phi_{nn} \) terms will all equal zero, and \( \chi(\tau) \) is totally determined by the \( \Phi_{nm} \) term. Because the wavelet packets are not totally orthogonal with each
other, the $\Phi_{nm}$ term will not always be zeros. Consequently, the cross-correlation between wavelet packets generates far-sidelobes [70], which should be avoided.

Fig. 2.3 shows that the autocorrelation of the D-4 scaling function has nearly spike like main-lobe with very small near-sidelobes, which leads to effective sidelobe suppression. However, the D-4 wavelet packets are only orthogonal to each other at discrete points. Transmitting them in sequence leads to far-sidelobes [70]. The near-sidelobes affect the detection of nearby targets. For example, when a fighter launches a missile, the missile might be immersed by the near-sidelobes of the fighter itself. The far-sidelobes may also be harmful to the detection when targets are dense. For example, when several fighters are flying inside a same detection zone, each of the fighter generates a main-lobe with far-sidelobes. One of the main-lobes might be buried by far-sidelobes caused by other targets. Therefore, both the near-sidelobes and the far-sidelobes can be harmful to resolve targets. In our study, we assume that the detected targets are far from the sensing system, and are all within the range $R_0 + c\Delta t$, where $R_0$ refers to the range of the target closest to the sensing system. This assumption is reasonable, since the sensing system is always far from the illuminated targets (Fig. 2.24). In this case, the cross-correlation between the transmitted waveform and the received signal will only be determined by the summation of $\phi_{nm}$ terms, which leads to a spike like main-lobe with very small near-sidelobes, and the $\phi_{nm}$ terms will not influence the detection of the targets.

We compare the wavelet-based waveform with the same non-zero duration and bandwidth LFM waveform in Fig. 2.25. Here non-zero duration means that the durations of all the sub-pulses have no intervals of $T_2$. The duration of both the two waveforms are $0.36\mu s$. More specifically, $T_1$ of the wavelet-based waveform is $0.045\mu s$,
and $T_2$ is 0.09\,$\mu$s. The wavelet-based waveform in Fig. 2.25 uses D-4 wavelet packets as sub-pulses. Note that each sub-pulse of the wavelet-based waveform is one narrow band of the whole waveform bandwidth. Thus, the radar does not have to transmit the waveforms in full-bandwidth. Instead, the radar can transmit each sub-pulse using a narrow band transmitter. This property cannot be achieved by the phase coded waveforms or other Costas waveform. The autocorrelation of the waveforms scaled in decibel is shown in Fig. 2.26. The wavelet-based waveform has only small near-sidelobes of -22.07 dB, and eliminates the far-sidelobes totally. In contrast, the near-sidelobes of LFM is -13.2 db, and the sidelobes decrease slowly.

Fig. 2.27 shows the transmitted waveform and received scatters of two detected targets. If the two detected targets are too far from each other, the far-sidelobes
of the two targets interfere with each other. We propose to adjust $T_2$ in order to avoid interference of the far-sidelobes among targets. The sub-pulses interval $T_2$ determines the maximum detect domain, which equals the maximum measuring range $R_{\text{max}}$ minus the minimum measuring range $R_{\text{min}}$. When $T_2$ increases, the detectable domain will increase. The equation below shows the lower bound of $T_2$,

$$
T_2 \geq \frac{(R_{\text{max}} - R_{\text{min}})}{2c} - T_1. 
$$

(2.31)

Figure 2.25: The LFM waveform (top) and the proposed D-4 wavelet packets waveform (bottom) with the same bandwidth and non-zero duration

However, when the radar is using duplexer to switch between transmitting and receiving operations, $T_2$ cannot be arbitrary long, because $T_2$ also determines the
Figure 2.26: The autocorrelation of the LFM waveform (top), and the autocorrelation of the proposed D-4 wavelet packets waveform with the same bandwidth and duration (bottom).

length of the proposed waveform. The length of the wavelet-based waveform will increase as $T_2$ increases, and thus it will enlarge the minimum measuring range $R_{min}$ ("blind range") [75], which is the minimum distance to the sensing targets from the radar. Consequently, it is necessary to limit $T_2$ such that the transmitting pulse leaves
Figure 2.27: $T_2$ is adjusted to increase detected domain. Fig. 2.27(a) shows the two targets to be within the detect domain. Fig. 2.27(b) shows the two targets to be out of the detect domain. Fig. 2.27(c) shows the two targets to be within the detect domain after adjusting $T_2$. 

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the antenna completely and the radar unit can switch to the receiving operation. We obtain:

\[ R_{\text{min}} = \frac{c(N\Delta t + t_{\text{recovery}})}{2}, \]  

(2.32)

where \( c \) is the speed of light, \( N \) the number of wavelet packets used by the wavelet-based waveform, \( \Delta t \) the duration of each sub-pulse, and \( t_{\text{recovery}} \) the time gap between finishing transmission and starting receiving [75]. Therefore, the upper bound of \( T_2 \) should be:

\[ T_2 \leq \frac{2R_{\text{min}}}{c} - \frac{t_{\text{recovery}}}{N} - T_1. \]  

(2.33)

The upper and lower bounds of \( T_2 \) imply that the wavelet-based waveform can be applied when the radar is always far from the detection zone, and the domain of the detection zone is smaller than the distance of the radar. Synthetic Aperture Radar (SAR) can easily satisfy this condition.

2.4 Conclusion

In this chapter, we have proposed a new radar waveform based on the scaling function and wavelets in the wavelet transform domain. One significant advantage of the proposed waveform is effective sidelobe suppression. The near-sidelobes of the proposed signal can be suppressed by more than 2 times in comparison to LFM. This effective sidelobe suppression can be used to improve radar range resolution and thus performance at detecting small targets. Further study on the far-sidelobes has suggested that selecting the best signs and sequences of the wavelet packets via search can effectively suppress the far-sidelobes to the noise level. Another advantage of the proposed signal is that the new radar waveform’s adaptability to targets and environments as the magnitudes and phases of specific frequency subbands can be
adjusted by modifying the corresponding wavelet packets. Thus the proposed signal is more suitable for cognitive radar than LFM.

The proposed wavelet-based waveform is rooted in two areas. The first is the Costas waveform that divides the pulse duration into equal sub-periods; in each sub-period, a different frequency is chosen for optimization of range and Doppler resolution. The proposed wavelet-based waveform has a sub-band in each sub-period for reducing sequence lengths in the waveform and improving sidelobe suppression performance. The second area is filter bank theory. In fact, wavelets are generated by a special set of filter banks via iteration of a scaling function via interpolating, splitting, and filtering [11]. The current work leverages the well-developed theory and practice of wavelets and wavelet transforms instead of developing a generic filter bank from scratch. Future studies show that the far-sidelobes of the new waveform can be pushed outside the detection bounds by inserting time intervals between the adjacent packets.
Chapter 3: Advanced Applications of Wavelet Based Waveforms

3.1 Introduction

In the previous Chapter, a wavelet-based waveform (WBW) is proposed for effective sidelobe suppression to improve range resolution for static targets. The wavelet based waveform is further implemented via simulation and experiments for range detections of static targets. On the other hand, the envelop and the carrier of the received waveform can be decoupled for range and velocity estimations, respectively. The decoupling process maintains the stability of the wavelet based waveform in range detection for moving targets. At the same time, the wavelet based waveform can eliminate the Doppler spread term, thus achieving high velocity resolution. The latter is due to the fact that the Doppler frequency of the carrier is generated by the velocity of the object only and not the spread term.

The wavelet based waveform can particularly improve the performance of Synthetic Aperture Radar (SAR) [76], which uses the relative motion between the antenna and the target zone to plot long range 2D target zone image (even 3D target zone image). The strip-map SAR [77] uses an antenna which is fixed on the radar platform, and integrates multiple pulse responses of the target zone from different
platform positions. The 2D image requires SAR to obtain the target zone information in both range and azimuth directions [78, 79]. The range direction information can be obtained by cross-correlation between the transmitted waveform and the received signals. The azimuth information [80] can be extracted by integrating multiple pulses according to the Doppler frequency caused by the relative movement between the platform and the target zone. Due to the advantage as cited earlier, the wavelet based waveform can improve the resolution of SAR image, which will be proved by both mathematical analysis and simulation results in this paper.

This chapter is organized as follows. In Section 3.2, we show experiments and simulations to verify the wavelet based waveform in range detection. In Section 3.3, we study how Doppler effect is used for speed detection, and how different waveforms affect the speed detection. A further study of how Doppler effect influences range detection by different waveforms is also conducted in this section. In Section 3.4, a study of how different waveforms affect the resolution of the synthetic aperture radar is discussed. In Section 3.5, simulations results using the proposed waveform are presented, which illustrate the advantage of the proposed waveform in both range and velocity detections, particularly in the strip-map SAR. Section 3.6 concludes the chapter.

3.2 Wavelet Based Waveform for Range Detection

3.2.1 Wavelet-based waveform experiment

A new software-defined S-band radar (SDSR) had been developed by the Air Force Research Laboratory (AFRL) [81]. The S-band radar can generate arbitrary waveforms up to four billion samples per second with two horn antennas operating from
2.6–3.95 GHz the wavelet-based waveform of eight Daubechies order-4 (D-4) wavelet packets (Fig. 2.19) and the LFM waveform with the same bandwidth and duration are both tested using this platform to detect a static flat-plate object at distances of 1.5 m, 3.1 m, and 4.6 m, respectively. Because the same amplifier is used, the LFM signal will better utilize the amplifier’s dynamic range, thus transmitting more power than the wavelet-based waveform. The cross-correlation between the wavelet-based waveform and the background scatter noise is shown in Fig. 3.1. The background noise shows that during the experiments, there are two small objects at ranges 0.3 m and 8.7 m hidden in the background.

![Figure 3.1: Background noise of experiment; the right is a close-up of the left](image)

For the 1.5 m target, Fig. 3.2 shows the matched filter output for the proposed (upper) and the LFM (bottom) waveforms, respectively. As described in Fig. 3.2, LFM has nearly two times the near-sidelobes of the wavelet-based waveform. These near-sidelobes can influence close objects and degrade radar resolution.
Figure 3.2: Matched filter output for the 1.5 m flat-plate object (upper: wavelet based waveform, bottom: LFM); the right is a close-up of the left.

For the 3.1 m target, Fig. 3.3 shows a similar comparison between the wavelet-based waveform and the LFM waveform. The near-sidelobes are effectively suppressed by the wavelet-based waveform.

For the 4.6 m target, the two background small objects at ranges 0.3 m and 8.7 m can be distinguished by the wavelet-based waveform (inside the green circle in Fig. 3.4), since the amplitudes of the two objects are smaller than the sidelobes of the wavelet-based waveform. However, for LFM, the strong near-sidelobes exceed the reflected signal and overshadow the two objects. This 4.6 m experiment proves that the proposed signal can effectively suppress sidelobes for detecting small objects. Further, the experimental results illustrate that the wavelet-based waveform can achieve higher sidelobe compression than LFM and hence better range resolution for object detection.
3.2.2 Simulation on two targets

To further verify the advantages of the wavelet-based waveform, we study more complicated cases using simulation. In the simulation, LFM and the wavelet-based waveforms are compared to resolve two targets. Both the LFM and wavelet-based waveforms are 1 µs in duration with the same bandwidth (3 dB) of 96 MHz. The SNR in the experiment is set to be 30 dB for both waveforms.

The first simulation is performed when there are two close targets; a small target is placed at 34.4 m and a big target is placed at 35.6 m. The small target is 1/5 the size of the large one. The result is shown in Fig. 3.5. The small target inside the green circle can be spotted when the wavelet-based waveform is transmitted. For LFM, the small target, which should be inside the green circle, is obscured by the large sidelobe near the large target and cannot be clearly detected.
The second simulation is performed where two targets are far to each other; a small target is placed at 10 m and a big target is placed at 35.6 m. The small target is 1/5 the size of the large one. The result is shown in Fig. 3.6. The small target inside the green circle can be spotted in both waveforms.

The simulation result has shown that the wavelet-based waveform can resolve two close targets more clearly than LFM because the near-sidelobe of the wavelet-based waveform is much smaller than that of LFM. When two targets are far apart from each other, the wavelet-based waveform has similar performance to LFM.

### 3.3 Wavelet Based Waveform for Speed Detection

#### 3.3.1 Doppler effect for speed detection

The Doppler effect is the shrink or spread of waveforms when a source is moving toward or away from the receiver. This change of waveform shape is due to the relative motion between the source and the receiver. To understand the Doppler effect, first
Figure 3.5: Two targets placed 1.2 m apart. The mainlobe is for the large target; the circle should contain the small target. Wavelet-based waveform at top; LFM waveform at bottom.

assume that the length of a transmitted waveform is $T$. If both the source and the receiver of the sensing system is stationary, the receiver will receive the waveform with a duration $T$. This is because the receiver is receiving the waveform at a speed that the source is producing. Now, if either the source or the receiver moves toward the other, the receiver will receive the complete waveform in a time shorter than $T$. This is because the receiver is receiving the waveform at a speed that the source is producing plus a relative speed between the receiver and the source. Conversely, if the source and the receiver are moving apart, the receiver will receive the complete waveform in a time longer than $T$.

In radar systems, however, the speed of waveform transmission is the speed of light, which is much faster than the relative speed between the sensing system and the target. Shrink or spread of the waveform is very slight, and thus difficult to measure. On the other hand, the carrier frequency is sensitive to the speed of light.
Figure 3.6: Two targets placed 25.6 m apart. The mainlobe is for the large target; the circle should contain the small target. Wavelet-based waveform at top; LFM waveform at bottom.

Therefore, speed detection uses the variation of carrier frequency to measure the speed between the sensing system and the target. Assume the transmitted carrier frequency to be $f_c$. The received carrier frequency $f_r$ is:

$$f_r = f_c \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}, \quad (3.1)$$

where $v$ is the relative speed of the sensing system and the target, and $c$ the speed of light. The Doppler frequency (or "beat frequency") [82] is:

$$f_d = f_r - f_c \approx \frac{2vf_c}{c}. \quad (3.2)$$

However, the transmitted signal is always a wide bandwidth waveform. Hence, the Doppler frequency is applied to not only the carrier frequency, but also to the other frequency of the transmitted waveform via spread (or shrink) of the transmitted waveform. This spread (or shrink) causes errors when using the Doppler frequency to
measure the speed of an interested target. How the Doppler spread term is generated in frequency modulated waveform will be discussed in the following subsection.

3.3.2 Doppler spread term on LFM

For frequency modulated waveforms, the Doppler spread term [74] always accompanies with the Doppler frequency. For the LFM waveform, the Doppler spread term can be understood through the following analysis.

Let LFM be expressed as:

$$s_{LFM} = exp(j2\pi f_c t + j\pi kt^2),$$  \hspace{1cm} (3.3)

where \( f_c \) is the transmitted carrier frequency, and \( k \) the chirp rate. When the detected target is at range \( R_0 \) with velocity \( v \) moving towards the sensing system, the received signal is:

$$r_{LFM} = exp\left(j2\pi f_c \left(t - \frac{2(R_0 - vt)}{c}\right) + j\pi k \left(t - \frac{2(R_0 - vt)}{c}\right)^2\right) \hspace{1cm} (3.4)$$

$$= exp\left(j2\pi f_c \left((1 + \frac{2v}{c}) t - \frac{2R_0}{c}\right) + j\pi k \left((1 + \frac{2v}{c}) t - \frac{2R_0}{c}\right)^2\right). \hspace{1cm} (3.5)$$

The phase of \( r_{LFM}(t) \) can be calculated as:

$$\phi = 2\pi f_c \left((1 + \frac{2v}{c}) t - \frac{2R_0}{c}\right) + \pi k \left((1 + \frac{2v}{c}) t - \frac{2R_0}{c}\right)^2 \hspace{1cm} (3.6)$$

The derivative of the phase equals the frequency of the returned waveform:

$$\frac{d\phi}{2\pi dt} = f_c + 2\frac{vf_c}{c} + k \left(1 + \frac{2v}{c}\right)^2 t - \frac{2kR_0}{c} \left(1 + \frac{2v}{c}\right). \hspace{1cm} (3.7)$$

After the stretch process, \( f_c \) and \( kt \) are removed. The remaining frequency components are:

$$\frac{d\phi}{2\pi dt} - f_c - kt = 2\frac{vf_c}{c} + \frac{4vk}{c} t + \frac{4v^2}{c^2} t - \frac{2kR_0}{c} - \frac{4kvR_0}{c^2}. \hspace{1cm} (3.8)$$
The terms involving $c^2$ are relatively small, since $c^2$ is much greater than $4v^2t$ and $4kvR_0$, and thus ignored. The term $-\frac{2kR_0}{c}$ is used to detect the range, and is removed due to the focusing on the Doppler frequency. The remaining part of the frequency component of the de-ramped signal is:

$$\frac{2vf_c}{c} + \frac{4vk}{c}t,$$

where $\frac{2vf_c}{c}$ is the Doppler frequency used to detect the velocity of the target, and $\frac{4vk}{c}t$ is the spread term of the Doppler frequency. The latter, which is caused by frequency modulation of LFM, will degrade the accuracy of velocity estimation. For SAR, which uses the Doppler frequency to locate targets along the azimuth direction, the spread term degrades the image resolution in that direction. There are a few studies tackling this issue. Tao [83] analyzed the Doppler spread term, and reduced the spread by introducing a scaling processing. However, the mitigation relies on the estimation of the target speed, and further post processing. Similarly, Liu [84] needs the coarse estimation of the target radial velocity, and post processing to reduce the Doppler effects. Therefore, we consider using a different waveform without the Doppler spread term.

### 3.3.3 Advantage of the wavelet-based waveform in speed detection

For the wavelet-based waveform, the Doppler spread term can be totally ignored since the carrier can be extracted to measure the Doppler frequency. Assuming the transmitted baseband signal to be $w(t)$, i.e., a wavelet carried by a carry frequency of $f_c$, the transmission waveform is:

$$s_{wp}(t) = w(t)exp(j2\pi f_c t).$$

(3.10)
When the detected target is at range \( R_0 \) with velocity \( v \) moving towards the sensing system, the received signal is:

\[
    r_{wp}(t) = w\left(t - \frac{2(R_0 - vt)}{c}\right) \exp\left(j2\pi f_c \left(t - \frac{2(R_0 - vt)}{c}\right)\right).
\]

(3.11)

A simple constant amplitude amplifier can be applied to the received signal to produce the carrier without the wavelet envelope, and only the carrier signal is maintained:

\[
    \exp\left(j2\pi f_c \left(t - \frac{2(R_0 - vt)}{c}\right)\right).
\]

(3.12)

Let \( \phi = 2\pi f_c \left(t - \frac{2(R_0 - vt)}{c}\right) \). Then the frequency of this carrier signal is:

\[
    \frac{d\phi}{2\pi dt} = f_c - \frac{2vf_c}{c}.
\]

(3.13)

After removing the carrier frequency, the Doppler frequency is retained without the spread term:

\[
    \frac{2vf_c}{c}.
\]

(3.14)

Comparing with LFM as shown in Eq. (3.9), the wavelet-based waveform does not have the spread term. Thus the velocity of the target can be accurately detected. The reason for the Doppler spread term is that the frequency modulated waveform cannot isolate the baseband signal with the carrier, because the modulation mode blends the two frequencies together. On the other hand, the wavelet-based waveform guarantees that the baseband signal can be removed by a constant amplitude amplifier at the receiver end. In this way, it can be exactly identified how the carrier frequency is changed. Consequently, the wavelet-based waveform can totally eliminate the Doppler spread term, which is highly responsible for degrading speed detection.
3.3.4 Doppler effect on range detection

For the LFM waveform is used to detect the ranges of moving targets, the Doppler frequency and the frequency spread term accompanying the range frequency is shown in Eq. (3.8). Typically, the stretch process (or matched filter) will be applied to extract the range frequency in order to measure the range of the target. When the target is moving, the combination of the other frequency components (shown in Eq. (3.8)) caused by the Doppler effect results in range ambiguity. The wavelet-based waveform, however, can remove the Doppler frequency and the frequency spread term by envelop detector, which can recover a shrink (or spread) baseband waveform. Because most moving targets are moving at a speed much lower than the speed of light, the shrink (or spread) is relatively small to the baseband waveform, and does not impact the range detection.

3.4 Synthetic Aperture Radar Using the wavelet-based waveform

Synthetic Aperture Radar (SAR) is a form of radar that uses its relative motion between an antenna and its target zone to obtain a fine spatial resolution image. The strip-map SAR is always mounted on the side of its carrier aircraft. When the aircraft is flying at a constant speed, SAR senses its illuminated area by multiple pulses. Each pulse can resolve the detected targets along the range direction (i.e. the direction perpendicular to the flying direction). Integrating multiple pulses resolves the detected targets along the azimuth direction (i.e. the flying direction). In the previous section the performance of different waveforms in range and speed detections
has been studied. This section focuses SAR solution particularly in the azimuth direction.

### 3.4.1 SAR basics

A strip-map SAR uses the frequency shift among different pulses for resolving the target along the azimuth direction. Fig. 3.7 shows the geometry of SAR resolving targets along the azimuth direction. Assume that a rectangle pulse with carrier frequency $f_c$ is transmitted to sense targets. The pulse width is $T_{\text{pulse}}$, the interval between pulses is a slow time unit $s$, and $N$ is the total number of pulses that are used to resolve the targets along the azimuth direction. The $N$ pulses can be written in the formula below:

$$\cos(2\pi f_c t) g\left(\frac{t - ns}{T_{\text{pulse}}}\right),$$

(3.15)

where $g(t)$ is the gate function with gate width $T_{\text{pulse}}$, and $n$ the pulse number.

![Figure 3.7: Geometry of SAR to resolve the targets along the azimuth direction](image)

The relative speed between the aircraft and targets is (Fig. 3.7):

$$v_d = v \cos \alpha,$$

(3.16)
where $v$ is the constant speed of the aircraft, and $\alpha$ the angle between the flight path and the direction vector from the radar to the target. According to the geometrical relationship in Fig. 3.7,

$$\cos \alpha = \frac{L - vns}{\sqrt{R^2 + (L - vns)^2}},$$

(3.17)

where $L$ is the azimuth distance between the aircraft and a target. When $R \gg L - vns$,

$$\cos \alpha \approx \frac{L - vns}{R}.$$  

(3.18)

This relative speed between the aircraft and the targets causes the Doppler frequency $f_d$,

$$f_d = \frac{2v_d f_c}{c} = \frac{2f_c v(L - vns)}{cR}.$$  

(3.19)

This Doppler frequency is added to the returned signal. Thus the returned signal is:

$$r_L(t) = \cos(2\pi f_c t + 2\pi f_d t) g\left(\frac{t - ns}{T_{\text{pulse}}}\right)$$

(3.20)

$$= \cos(2\pi f_c t + 2\pi \frac{2f_c v(L - vns)}{cR} t) g\left(\frac{t - ns}{T_{\text{pulse}}}\right).$$

(3.21)

In SAR applications, every pulse is sampled for synthesizing the aperture. Thus we may have $t = ns$ and obtain:

$$r_L(ns) = \cos \left(2\pi f_c \left(1 + \frac{2vL}{cR}\right) ns - \pi \frac{4f_c v^2}{cR} (ns)^2\right).$$

(3.22)

The equation above shows that the received pulses can be integrated to obtain a slow time LFM signal, where $f_c$ is the carrier frequency, $\frac{2vL f_c}{cR}$ the frequency caused by the different azimuth position of the target, and $\frac{4f_c v^2}{cR}$ the chirp rate of the slow time LFM. After a de-ramp process, the carrier frequency and the slow time LFM is removed, and the frequency caused by the different azimuth positions of the targets is reserved. SAR can use this frequency to locate the azimuth position of the target.
Fig. 3.8 shows how the two targets, separated by $\Delta l$ along the azimuth direction, are resolved by the de-ramp process.

Figure 3.8: De-ramp process to resolve the targets along the azimuth direction. Frequencies for the two objects with the reference signal (top) and when the reference signal is removed following a de-ramp process (bottom)

However, the rectangle pulse waveform cannot generate enough power for long range detection with a desired resolution. Compression waveforms are used as pulses
instead of using a narrow pulse with high power. How different compression waveforms affect the azimuth resolution is discussed in the remainder of this section.

3.4.2 Disadvantages using LFM

When LFM is used as sensing waveform, the Doppler frequency is always accompanied by the spread term as shown in Eq. (3.9). The received pulses of Eq. (3.20) will be changed to:

\[
r_L(t) = \cos \left( 2\pi f_c t + 2\pi \left( \frac{2v_d f_c}{c} + \frac{4v_d k}{c} \left( \frac{\sqrt{R^2 + (L - v ns)^2} - R}{c} \right) \right) t \right) g \left( \frac{t - ns}{T_{pulse}} \right). \tag{3.23}
\]

Using Eqs. (3.16) and (3.18), one obtains:

\[
r_L(t) = \cos \left( 2\pi f_c t + 2\pi \left( \frac{2v f_c L - v ns}{c R} + \frac{4vk (L - v ns)^3}{c^2 R^2} \right) t \right) g \left( \frac{t - ns}{T_{pulse}} \right). \tag{3.24}
\]

After the synthesis process, one obtains:

\[
r_L(ns) = \cos \left( 2\pi f_c \left( 1 + \frac{2v L}{c R} \right) ns - \pi \frac{4f_c v^2}{c} (ns)^2 + 2\pi \frac{4vk(L - v ns)^3}{c^2 R^2} \right). \tag{3.25}
\]

\(\phi\) is used to represent the phase of the return slow time signal. The frequency of the return slow time signal is the derivative of \(\phi\):

\[
\frac{d\phi}{d(ns)} = 2\pi f_c \left( 1 + \frac{2v L}{c R} \right) - 2\pi \frac{4f_c v^2}{c} n s - 2\pi \frac{4vk(L - v ns)^3}{c^2 R^2} \left( L^3 - 6L^2(v ns) + 9L(v ns)^2 - 4(v ns)^3 \right). \tag{3.26}
\]

Here, we consider \(L \ll v N s\) for simplification (it means that the azimuth position of the target is close to the center of the synthetic aperture). Thus the frequency can be approximated as:

\[
r_L(ns) \approx 2\pi f_c \left( 1 + \frac{2v L}{c R} \right) - 2\pi \frac{4f_c v^2}{c} n s - 2\pi \frac{16vk(v ns)^3}{c^2 R^2} \tag{3.27}
\]
The frequency of the received slow time signal by LFM is different from that of
the rectangle pulse by the spread term $-2\pi \frac{16vk(ns)^3}{c^2 R^2}$. This spread term always accompa-
nies with the azimuth frequency term $\frac{4\pi f_c vL}{cR}$, after the stretch process. When the
strip-map SAR uses that frequency to locate the azimuth position of the target, this
spread term degrades the azimuth resolution.

3.4.3 Advantages using the wavelet-based waveform

When the wavelet-based waveform is used to resolve the azimuth direction, the
Doppler frequency can be recovered without the spread term by extracting the carrier
frequency. Therefore, after a constant amplitude amplifier, the returned pulses will
have the following slow time signal:

$$r_{L}(ns) = \cos \left( 2\pi f_c \left( 1 + \frac{2vL}{cR} \right) ns - \pi \frac{4f_c v^2}{c} (ns)^2 \right). \quad (3.28)$$

Assuming the phase of this signal to be $\phi$, the frequency is the derivative of the phase:

$$\frac{d\phi}{d(ns)} = 2\pi f_c \left( 1 + \frac{2vL}{cR} \right) - 2\pi \frac{4f_c v^2}{c} ns. \quad (3.29)$$

The spread term in Eq. (3.27) is removed, and a unique frequency caused by
the azimuth position of the target can be obtained. Fig. 3.9 shows the de-ramp
outputs of LFM and the wavelet-based waveform. It demonstrates that when the
LFM waveform is used as radar pulse, the spread term causes the non-linearity of the
azimuth frequency, which degrades the resolution along the azimuth direction. Using
the wavelet-based waveform, the azimuth frequency is linear and produces no overlap
after the de-ramp process. Therefore, the azimuth direction is no long affected by the
non-linearity.
Figure 3.9: The results of the slow time de-ramp process for LFM (top) and the wavelet-based waveform (bottom)

### 3.5 Simulation Result

In this section we present two simulation results. One shows how the wavelet-based waveform performs in range detection under a serious Doppler effect, and the result is compared with LFM. The other is on a strip-map SAR.
Table 3.1: First Simulation Parameters

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Detected Target Speed</th>
<th>Carrier Frequency</th>
<th>Pulse Duration</th>
<th>Bandwidth Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment 1</td>
<td>250m/s</td>
<td>10GHz</td>
<td>1µs</td>
<td>533MHz</td>
</tr>
<tr>
<td>Experiment 2</td>
<td>1000m/s</td>
<td>77GHz</td>
<td>1µs</td>
<td>533MHz</td>
</tr>
</tbody>
</table>

In the simulation below, the baseband signal of the wavelet-based waveform has eight wavelet packets, each lasting for 0.045µs, and the interval between two neighboring packets is 0.09µs. Therefore, the duration of the wavelet-based waveform is 1µs, and its bandwidth is 533MHz. To compare with the same non-zero duration and bandwidth LFM, the baseband LFM lasts for 0.036µs (= 8 × 0.045µs). Fig. 2.25 shows the two sensing waveforms in the time domain.

In this simulation, the sensing platform is not moving. The LFM waveform is compared with the same bandwidth and duration proposed waveform to detect the ranges of moving targets at two speeds, which are 250m/s and 1000m/s respectively. The first speed is the typical cruising speed of a jet aircraft, while the maximum second is about the speed record of a jet aircraft (SR-71 Blackbird maximum speed). The parameters are shown in Table 3.1. For LFM, the received signal has to first cross-correlate with the transmitted waveform (i.e. modulated waveform), and then pass the envelop detector. For the wavelet-based waveform, the received signal first passes an envelope detector to remove the carrier, and then cross-correlates with the transmitted baseband waveform.
In the first experiment, Fig. 3.10 shows a comparison of the matched filter outputs using the two waveforms. Both are not much influenced by the speed of the targets. It shows that the wavelet-based waveform can suppress the sidelobes more effectively than the LFM waveform. In the second experiment, Fig. 3.11 shows the matched filter outputs, when the speed of the detected target is increased to 1000m/s, and the carrier frequency to 77GHz. It is shown that the Doppler frequency increases, and thus the accuracy of range by LFM is affected. The wavelet-based waveform can still locate the targets accurately. Meanwhile, the wavelet-based waveform can still maintain the main-lobe, and suppress the sidelobes to a small level (below -20dB). The simulation results here have demonstrated that the wavelet-based waveform can resist the Doppler effect in range detection, but not LFM. Consequently, the wavelet-based waveform is superior to LFM for range detection regardless of the status of the target, in motion or stationary.

In the second simulation, we compare LFM with the wavelet-based waveform on a strip-map SAR system. We assume the strip-map SAR using carrier frequency 10GHz, pulse width 1µs, minimum measuring range 2850m, maximum measuring range 3150m, aircraft speed 100m/s, integrate-length 250m (i.e. fly for 2.5 second), and pulse bandwidth 192MHz. The two pulse waveforms in the time domain are shown in Fig. 3.12. The pulse width of LFM is 1µs, and the total pulse width of the wavelet-based waveform is also 1µs. The wavelet packet based waveform has periodical gaps by inserting silent intervals between the packets. The interval is determined by Eq. (2.31). Consequently, the wavelet-based waveform has eight packets, each lasts for 0.125µs (= 1µs/8), and the delay between every two packets is 1.4µs.
Figure 3.10: The results of range detection using the LFM (top) and the wavelet-based waveform (bottom) when a detected target is moving in a speed of 250m/s

In the range direction (Fig. 3.13), the impulse response width (IRW) of LFM is 0.69m, and the wavelet-based waveform is 0.51m. The peak-to-sidelobe ratio (PSLR) of LFM is -13.47dB, and the wavelet-based waveform is -22.52dB. The integrated side-lobe ratio (ISLR) of LFM is -10.29dB, and the wavelet-based waveform is -21.82dB. Therefore, the wavelet-based waveform has a narrow mainlobe with very small sidelobes, and the sidelobes decay fast. However, LFM has serious sidelobes, and the sidelobes decay slowly.
Figure 3.11: The results of range detection using the LFM (top), and the wavelet-based waveform (bottom) when a detected target is moving in a speed of 1000m/s.

In the azimuth direction (Fig. 3.14), the IRW for the LFM waveform is 0.38m, and the wavelet-based waveform is 0.35m. The PSLR for the LFM waveform is -14.58dB, and the wavelet-based waveform is -13.28dB. The ISLR for LFM is -12.84dB, and the wavelet-based waveform is -9.91dB. Due to the Doppler effect, the wavelet-based waveform changes the carrier frequencies only corresponding to the azimuth positions of targets, while the LFM waveform introduces the Doppler spread term (The third term in Eq. (3.27)). the wavelet-based waveform can generate a better azimuth resolution than the LFM waveform.
The point spread function consisting of both range and azimuth responses are shown in Fig. 3.15. The wavelet-based waveform has a focused point response with small ripples. The LFM waveform generates spread point response in the azimuth direction, and many ripples in the range direction.
Figure 3.13: Matched filter output in range for the LFM (blue) and proposed waveform (red), Fig. 3.13(b) is the zoomed-in version of the center portion of Fig. 3.13(a).

The third simulation further applies the two waveforms to a strip-map SAR platform. The pulse repetition frequency (PRF) is 500 Hz, and the antenna length is 2m. A reflectivity map is obtained online [85], and the raw data is generated based on
Figure 3.14: A single target response in the azimuth direction by LFM (blue) and the wavelet-based waveform (red), Fig. 3.14(b) is generated by normalizing Fig. 3.14(a)
Figure 3.15: The point spread function for the LFM (top) and the wavelet-based waveform (bottom)

the reflectivity map and the moving SAR platform. Then, we process the SAR raw data using the Range-Doppler algorithm [76]. The simulation program is based on Schlutz’s work [86] with additional modifications as follows. Since the Doppler spread term of LFM is not considered in the generation of the raw data in his work, therefore, we introduce the last term in Eq. (3.27)) to his program, and adapt his program to different waveforms. Figs. 3.16 and 3.17 show the results of the strip-map SAR using
the two waveforms, respectively. In the range direction, the IRW for LFM is 0.69m, and the wavelet-based waveform is 0.55m. The PSLR for LFM is -13.46dB, and the wavelet-based waveform is -22.52dB. The ISLR for LFM is -10.31dB, and the wavelet-based waveform is -21.82dB. Therefore, the wavelet-based waveform can improve the range detection as compared to LFM via sharp mainlobe and well suppressed side-lobes. In the azimuth direction, the IRW for LFM is 0.71m, and the wavelet-based waveform is 0.70m. The PSLR for LFM is -20.25dB, and the wavelet-based waveform is -20.29dB. The ISLR for LFM is -28.27dB, and the wavelet-based waveform is -28.29dB. Because the platform is not generating an ideal LFM signal along the azimuth direction, both the waveforms generate a wide mainlobe with small sidelobes as compared to the sinc function. According to the last term in Eq. (3.27) when the ratio of bandwidth to carrier frequency and (or) the platform speed further increases, the Doppler spread term of LFM will further influence the azimuth resolution.

3.6 Conclusion

In this chapter, the wavelet packet based waveform is discussed for improving radar performance, including the strip-map synthetic aperture radar. The proposed waveform takes the advantage of auto-correlations and cross-correlation features of wavelet packets. Because of the separation of wavelet packets in both time and frequency domains, the autocorrelation of the proposed waveform has sharp-rising main-lobe but small sidelobes. This study also has found that the proposed waveform can separate the carrier and the baseband waveform effectively. The retrieved carrier frequency can determine the speed of moving target without the Doppler spread term, which causes velocity ambiguity in the LFM waveform. Meanwhile, the recovered
baseband waveform can resist the Doppler effect on range estimation, due to the removal of the Doppler frequency and the Doppler spread term. Consequently, the proposed waveform has the ability to resolve the range and speed of moving targets with higher resolution than the traditional LFM waveform.

The advantage of the proposed waveform can be directly shown when it is applied to strip-map synthetic aperture radar, which uses both range and velocity to
Figure 3.17: Strip-map SAR image using the wavelet-based waveform

resolve targets along the range and azimuth directions, respectively. Because the proposed waveform can remove the Doppler spread term, it increases the SAR image resolution along the azimuth direction. Since the proposed waveform almost eliminates all the sidelobes in the range detection, it enhances the image resolution in the range direction as well. Both mathematical analysis and simulation have shown that
the proposed waveform can improve radar performance including SAR image quality significantly.
Chapter 4: Transform Sensing

4.1 Introduction

For the purpose of better efficiency when employing phased array radar with a large number of elements, we propose a new idea, called *transform sensing*. By this new approach, the array will form beams in parallel to generate various desired patterns rather than a sharp beam in one particular direction. The desired patterns constitute a transformation, thus the space is sensed in the transformed domain. In this way, sensing can cover a wider area at a high speed, and critical areas can still be closely observed in a selected resolution. Statistically, the new approach can reduce the volume of data to be collected by one order of magnitude, while the high resolution is still gained by the increased number of elements. Consequently covering a wide area and collecting data on multiple targets, both become possible concurrently. The proposed transform sensing of phased array radar is based on the digital beam-forming (DBF) technique. We consider that the digital beam-forming technique already exists and will be widely used in the near future.

The structure of this chapter is as follows. In Section 4.2, we introduce how different sensing patterns can be formed and achieve a coarse-to-detail spatial resolution.
In Section 4.3, realization of different sensing patterns by the phased array is discussed in detail. In Section 4.4, a simulation generating wavelets for different sensing patterns is shown. In Section 4.5, a preliminary experiment is implemented. Finally, the paper is concluded in Section 4.6.

4.2 Sensing Pattern

4.2.1 Traditional sensing pattern

Traditional phased array directs its beam via changing the phase between elements linearly. Fig. 4.1 shows that when the phase difference between the elements is the delay $\tau$, a narrow beam in a specific angle $\theta$ will be generated.

With the development of electronic technique, more and more array elements can be integrated into a phased array system, resulting in a narrower beam (Fig. 4.2). The advantage of the narrow beam pattern is the high angle resolution. However, the deficiency of the traditional phased array also emerges. The narrow beam pattern takes a long time for the phased array to scan the scene. Consider an S-band (2 GHz carrier frequency) phased array radar as an example. Assume the pulse width to be $1\mu s$, maximum range 60nm, and PRF $1,250 Hz$. To keep the matter easy, only the azimuth angle is considered, and the phased array having 100 elements in the azimuth direction. The beam-width varies from 1.02° to 1.44° when $-45^\circ$ to $45^\circ$ needs to be covered. To improve the signal-to-noise ratio (SNR), 22 pulses are assumed to integrate per beam-width. Consequently, around 1.3 second is needed to finish the 90° scanning. When the spatial resolution needs to be increased by six times, the number of array elements has to be increased to 600, and the scanning time increases to 7.8 seconds. That is too long a time for a dynamic situation. In recent years, video
Figure 4.1: Traditional Phased Array

functionality is being pushed into radar systems such as the video synthetic aperture radar (ViSAR). Visual sensing generates multiple frames in a second. That imposes even a greater challenge to the sequential scanning approach.

4.2.2 Transform sensing pattern

Without loss of generality, consider only a one-dimensional area using the discrete cosine transform (DCT). The following equation represents the transformation:

\[ X(k) = \sum_{n=0}^{N-1} x(n) \cos\left(\frac{\pi}{N} \left( n + \frac{1}{2} \right) \right), k = 0, 1, ..., N - 1, \quad (4.1) \]

where \( x(n) \) represents the reflected signals from the \( n \)th target object, \( nth \) is the coefficient in the frequency domain, and \( X(k) \) is the number of the coefficients in the frequency domain, reflecting the resolution of the sensor. Therefore, we are proposing to generate beam pattern in consistent with the transform basis, i.e. \( \cos\left(\frac{\pi}{N} \left( n + \frac{1}{2} \right) \right) \) for DCT. If we are transmitting this sinusoid signal in a specific frequency to the space as the beam pattern and receive the integration of all the weighted reflections
of the targets, the sensing result will be the transform coefficients for the Fourier frequency. For convenience, the delay of the signal due to the range is not considered here. Note that $X(k)$ is the integration of all the weighted reflections of the targets. Many of the $X(k)s$ may be null, and can be discarded in real-time without affecting the resolution.

Alternatively one can use the discrete wavelet transform (DWT) [3] by the following two equations:

$$Y_l(k) = \sum_{n=0}^{N-1} x(n) h[2k - n], k = 0, 1, ..., N - 1,$$  \hfill (4.2)

$$Y_h(k) = \sum_{n=0}^{N-1} x(n) g[2k - n], k = 0, 1, ..., N - 1,$$  \hfill (4.3)

Because the wavelets basis can adapt its width (i.e. the so called time-frequency localization) according to the different resolution requirements, wavelets are more appropriate for the transform sensing. The varying width of wavelets as transform
Figure 4.3: The sensing pattern needs to be generated by the phased array basis has its potential to adapt to the signal-to-noise ratio, we will further discuss this later. Consequently, we will focus on the wavelet transform based transform sensing. Therefore, every sensing pattern is a wavelet (or wavelet packet), and each received sensing result will be a wavelet coefficient.

Because of the redundant nature of the transformed coefficients, one can reduce the number of sensing in real-time. Starting from using the lowest frequency band, one can sense the entire space using different frequency band each time. If in the previous sensing, it is judged that a subspace has no targets or is sparse, one can eliminate the higher frequency sensing in that particular subspace. To understand the transformed sensing idea better, consider the forming of a simple wavelet transform using Haar wavelets. The scaling function of the wavelet is shown in Fig. 4.3(A) which represents a low-pass filter, and its corresponding wavelet is shown in Fig. 4.3(B), Fig. 4.3(C) and Fig. 4.3(D) demonstrate two wavelets, which are one-level higher than that in Fig. 4.3(B) to represent higher frequency sensing.

To further compare the sensing speed, we show the scanning sequence of the traditional phased array sensing pattern and of the proposed sensing pattern for single
target detection in Fig. 4.4 and Fig. 4.5, respectively. Assume both the phased arrays can resolve $N$ angles in the azimuth direction. Since the target is possible to appear in any angle, the average searching time for the traditional sequential scanning is $\frac{N^2}{2}$. The proposed sensing pattern chooses sensing pattern according to the last sensing result. The first sensing can tell whether a target existed in the scene, the second sensing can further tell if the target is in the left part or right part of the scene. If the target is in the left, a third sensing can further decompose the left part into two parts, and tell a more precise position of the target. So for so on, the exact location of the target can be resolved. The average searching time decreases to $\log_2 N$.

In Fig. 4.6, the elements of the phased array work together to form sensing patterns for the entire space in parallel. Assume there are eight elements. Using our approach, we will be able to generate sensing patterns, corresponding to (A), (B), and (C) respectively. Fig. 4.2(D) shows a ”close observation” case, in which the eight elements
form a very sharp beam to fulfill the wavelet transform locally, taking the multi-resolution-analysis advantage of the wavelets. The rest of the space can be ignored since no targets have been revealed by earlier transformed sensing at lower frequency. Recall that the total number of coefficients in the transformed domain is the same as that of pixels in the spatial domain. When a large number of the coefficients is null, which could be 90% or even higher of the total coefficients, the transformed sensing can reduce the sensing time by one order of magnitude without sacrificing the resolution, and in some cases even to two or higher.

We further study the potential of transform sensing in the 2D scanning for dense targets. Fig. 4.7(a) shows the traditional sequential sensing result using 100% sensing pattern, i.e. the traditional phased array will scan pixel by pixel to draw the image. Fig. 4.7(b) shows the transform sensing result using only 20% sensing pattern. The transform sensing will choose the appropriate sensing pattern based on the previous
sensing results, thus it will not use all the sensing patterns to recover the image. The two sensing results are comparable; however, the transform sensing is 5 times faster than the tradition phased array to finish the scanning.

Using transformation to reduce the volume of data in representation is well known in image processing. The innovation is that the transformed sensing is performed online, without recording the raw data first and processing it later. Another approach for reducing the sensing times is compressive sensing. However, the compressive sensing is more practical to be applied in the scene, where its samples are random and sparse in the representative space. As the reflectivity map in the frequency domain is tend to be concentrated in the low frequency components, the advantage of compressive sensing may be difficult to show. Meanwhile, the compressive sensing cannot be solved stably, and its calculation cost is very high. We propose this novel approach, by which all the elements in the phased array radar collectively constitute a transformation to cover the entire space, but not to scan the space in sequence. The approach designs phase delays in a complicated yet efficient way, rather than a
Figure 4.7: The sensing results comparison between the traditional phased array using 100% sensing time and the transform sensing using 20% sensing time
fixed phase-delay between the elements. The details of realizing such a transformed beam pattern are stated in the next section.

4.3 Generating Transformed Sensing Pattern

Creating a transformed sensing pattern in a space entails alternating the sensing power when the radar surveys a space using the phased array radar. The amplitude is alternated in such a way that it generates a desired transformed sensing pattern in the space. We propose to develop a simultaneous beam forming technique, which can generate the desired transformed sensing in parallel.

4.3.1 Generating transform sensing pattern

Fig. 4.8 illustrates how the technique works. By conventional beamforming in the phased array radar, if each array element transmits a sinusoidal wave with a user-defined phase in proportion to the position of the element, one beam in a specific direction will be generated (Fig. 4.1). In the new approach each element is to transmit multiple sinusoidal waves each with its own phase delay according to different beam directions, multiple beams can be generated simultaneously. Furthermore the amplitude of each wave is modulated according to the transform basis. As a result, a transformed sensing pattern is formed in parallel.

Mathematically, the approach can be explained as follows. Assume we have a set of discrete beam pattern in the azimuth direction \([A_1, A_2, A_3, ..., A_m]\) as a transform basis. We need to assign different angles as \(A_1 \rightarrow \theta_{A1}, A_2 \rightarrow \theta_{A2}, A_3 \rightarrow \theta_{A3}, ..., A_m \rightarrow \theta_{Am}\), corresponding to the different beams. The waveform of each array
Figure 4.8: The phased array radar form beams in multiple directions simultaneously.

The element equation should be

\[
\text{element}_1 = A_1 \cos(w_c t + 1 \times \Phi_{A1}) + A_2 \cos(w_c t + 1 \times \Phi_{A2}) + \ldots + A_m \cos(w_c t + 1 \times \Phi_{Am}),
\]

\[
\text{element}_2 = A_1 \cos(w_c t + 2 \times \Phi_{A1}) + A_2 \cos(w_c t + 2 \times \Phi_{A2}) + \ldots + A_m \cos(w_c t + 2 \times \Phi_{Am}),
\]

\[
\text{element}_n = A_1 \cos(w_c t + n \times \Phi_{A1}) + A_2 \cos(w_c t + n \times \Phi_{A2}) + \ldots + A_m \cos(w_c t + n \times \Phi_{Am}),
\]

where \( w_c \) is the carrier frequency, and \( \Phi_{Ai}, i = 1, 2, \ldots, m \) is the phase delay between elements to generate beam in angle \( \theta_{Ai} \). Since the amplitudes vary in terms of the azimuth beams, the desired transformed sensing pattern is generated when the phased array elements collaborate by using different \( \Phi_{in} \) for each coefficient.

However, according to the above equations, each element should use multiple delay lines and amplifiers since multiple sinusoidal waves need to be transmitted simultaneously, which cannot be realized in state of the art. Our study discovers that the element equation can be simplified. Consider element 1 as an example. We can
see that
\begin{align*}
element_1 &= A_1 \cos (w_c t + 1 \Phi_{A1}) + A_2 \cos (w_c t + 1 \Phi_{A2}) + \ldots + A_m \cos (w_c t + 1 \Phi_{Am}) \\ &= \sum_{i=1}^{m} (A_i \cos(w_c t) \cos(\Phi_{Ai}) - A_i \sin(w_c t) \sin(\Phi_{Ai})) \\ &= \cos(w_c t) \sum_{i=1}^{m} (A_i \cos(\Phi_{Ai})) + \sin(w_c t) \sum_{i=1}^{m} (-A_i \sin(\Phi_{Ai}))
\end{align*}

Let \( x = \sum_{i=1}^{m} (A_i \cos(\Phi_{Ai})) \), and \( y = \sum_{i=1}^{m} (-A_i \sin(\Phi_{Ai})) \). Then one has
\begin{equation}
element_1 = x \cos(w_c t) + y \sin(w_c t) \tag{4.4}
\end{equation}

Assume \( a_1 = \sqrt{x^2 + y^2} \), and \( \Phi_1 = \arctan\left(-\frac{y}{x}\right) \). Then we can obtain
\begin{equation}
element_1 = a_1 \cos(w_c t + \Phi_1) \tag{4.5}
\end{equation}

After simplification, an element can transmit a cosine wave with amplitude \( a_i \) and a specific phase \( \Phi_i \). Similarly, each element can transmit a different cosine wave with its own amplitude and phase, i.e. \( \element_n = a_n \cos(w_c t + \Phi_n) \). In this way, a user-defined beam pattern can be realized spatially.

Eq. 4.5 states that the elements in the phased array no longer coordinate in a conventional way using a fixed phase delay between elements, but have each a special phase and amplitude calculated according to the desired sensing pattern. That means first the cost to implement the new approach will not be higher than the conventional phased array radar. Secondly it opens the door to flexible operations of the phased array radar, forming a sensing pattern or forming a sharp beam. The phased array can still form a single beam for high-resolution sensing, or form a small sensing pattern for a reduced field of view. In both cases, the power of the radar beams will increase to benefit the signal to noise ratio (SNR). Consequently, the phased array radar becomes adaptive to the resolution, the field of view, or the desired transformation.
4.3.2 Beam coverage and SNR

One interesting problem in transformed sensing is related to the signal-to-noise ratio (SNR). With wide coverage, less power is distributed to each coefficient. In contrast, if only one narrow beam is used to generate the coefficient one by one, each coefficient could have higher power. However, the narrow beam will need to scan for a long time to achieve wide coverage. This is not desirable as stated earlier. In reality, there is a trade-off between the coverage and SNR. In our study, we propose to establish the relationship between the trade-off and the DWT transform theorems.

Consider the wavelet transform as an example. Assume that the wavelet coefficients are $h(n)$ and $g(n)$, which are the low-pass and high-pass filter coefficients, respectively. Further, $w_1(n) = h(n)$ and $w_2(n) = g(n)$ can be used as the narrow beam coefficients to scan the space. When a wide area is required to survey simultaneously, one can decompose $w_1(n)$ or $w_2(n)$, or both to generate wavelet packets with more discrete points for a wider field of view. Each packet is equivalent to a frequency component in the DCT. If all the packets are used, a phased radar can cover the entire space of sensing in parallel without scanning.

Wavelet packets can be generated by simple up-sampling and convolution calculations:

$$w_{11}(n) = w_1 \left( \frac{n}{2} \right) \otimes h(n),$$  \hspace{1cm} (4.6)

$$w_{12}(n) = w_1 \left( \frac{n}{2} \right) \otimes g(n),$$  \hspace{1cm} (4.7)

$$w_{21}(n) = w_2 \left( \frac{n}{2} \right) \otimes h(n),$$  \hspace{1cm} (4.8)

$$w_{22}(n) = w_2 \left( \frac{n}{2} \right) \otimes g(n),$$  \hspace{1cm} (4.9)
where $\otimes$ is the convolution operator, $w_{11}(n)$ and $w_{12}(n)$ are the wavelet packets generated by decomposing $w_1(n)$, and $w_{21}(n)$ and $w_{22}(n)$ are the wavelet packets generated by decomposing $w_2(n)$.

According to different SNR requirements, beam shapes based on wavelets can vary. When the required SNR is low, a wider beam generated by $w_{11}(n)$ can be used to quickly sense the entire space and determine whether interesting targets exist. When the required SNR is high, a narrow beam $w_1(n)$ can be used to scan each pixel individually and ensure sufficient power concentration during each scan.

In summary, we can generate a narrow beam (Fig. 4.9) for sequential scanning. Or we can decompose the narrow beam into multiple wide beams (Fig. 4.9). Each
wide beam will cover a wide area, but few of them may be able to obtain a good sensing result (Fig. 4.7). Therefore, sensing efficiency increases.

4.3.3 System Implementation

We need to consider how the new algorithm can be implemented by a phased array radar using available hardware and software technologies. As discussed in subsection 4.3.1, each array element should transmit a cosine wave with unique magnitude and phase in order to realize the DWT beam pattern in space. The block diagram of such a phased array is shown in Fig. 4.10.

First, consider the phase delay requirement. From Eq. 4.5, one can see that each element will have its own phase and delay. Then the number of discrete points forming the beam pattern in the transformation is related to the number of different delays directly. When the number of discrete points increases, we may need more precise control of the phase and magnitude for each array element. Possible numbers of discrete points, on the other hand, are related to the number of elements of a phased array radar. That is, more elements can implement more detailed beam pattern of a transform. On the future study, we will investigate what the maximum number of elements in a phased array radar in the current state-of-the-art.

Secondly, different transform coefficients affect the detection and data compression result. A number of popular DWT (such as D2, D4, and D8 wavelets [9]) should be considered according to the nature of targets and applications. Practically the sidelobe of beam of the radar elements may also affect the performance, especially when a transform needs sharp contrast between neighboring patterns.
Finally we need to consider the element antenna design. Omni-directional antennas as equivalent to Fig. 10 are needed. The reason for omni-direction is to transmit the power as wide a field of view as possible to take advantage of the transformed sensing approach. To take the advantage of digital beam forming, the magnitude and phase adjustment can both be done in digital signal processor. Then each of the omni-directional antenna element is controlled by a single processor, and the elements are separated in space (usually in distance $\lambda/2$). In this way, a desired waveform can be generated in the space according to Eq. 4.5.

Significantly fewer omni-directional antennas need to be involved on the receiver side since the radar is to receive the coefficients, which are integration of the individual echo signals. When transmitter shapes scaling beam pattern, the receiver will receive the scaling transform coefficients. When transmitter shapes other beam pattern, the receiver will receive the other transform coefficients, so forth and so on. After the receiver summing up the received signals by all the elements, the coefficients are generated.

4.4 Simulation

To prove the feasibility of manipulating different transformed sensing, we have performed simulation study as described below. In the simulation, thirty two elements are utilized to generate different waveforms. The waveform for each element is generated as shown in Eq. 4.5. We attempt to generate 4 different wavelet packets in the spatial domain using 4 discrete points, $A_1$, $A_2$, $A_3$, and $A_4$, i.e. wavelet packet No. 1 $[1,1,1,1]$, wavelet packet No. 2 $[1,1,-1,-1]$, wavelet packet No. 3 $[1,-1,-1,1]$, and wavelet packet No. 4 $[1,-1,1,-1]$. We further set the element distance to $\lambda/2$ (one
half of the carrier wavelength) and the carrier frequency to 2GHz. Eventually, we obtained four different beam patterns in the azimuth direction as shown in Fig. 4.11.

One can see that wavelet packet No. 1 can provide rough detection in the space since the pattern is uniform in the direction. The remaining 3 wavelet packets can provide more detailed information in the space, and resolve the targets in better resolutions since the waveform generates alternately different patterns (1 and -1). The one at the right bottom corner of Fig. 4.11 generates the highest resolution.

Fig. 4.12 shows the results of the resolved four spatial directions by addition or subtraction of the four wide beam wavelet packets as shown in Fig. 4.11. The result is equivalent to a single beam, scanning the space.
Figure 4.11: Beam patterns generated by the wavelet packets of [1,1,1,1], [1,1, -1,-1], [1,-1,-1,1] and [1,-1,1,-1], respectively.

Figure 4.12: Resolved beams using beam patterns in Fig. 4.11
4.5 Experiment

To further verify the idea of transform sensing, an experiment using one transmitter element and two receiver elements is implemented. Note that the transform sensing idea should typically be implemented in the transmitter side. Since this preliminary experiment is going to show that the integration of sensing is practical in the state-of-art, and we consider that the transmitter and receiver elements is symmetric for the phased array, the receiver will be the focus in this experiment.

The antennas used in this experiment is K-MC4 radar transceiver. Fig. 4.13 shows the transmitter and receivers on the radar transceiver. Since the two receivers are placed separately in the azimuth direction, the two working together can differentiate targets along azimuth direction.

Transform sensing will first generate a wide beam to see whether there is a target existed in the scene, and then find out where is the exact location of the target.
According to the Eq. 4.5, if a two times wider beam is going to be generated, half of the element will transmit sinusoid wave close to magnitude zero. It is consistence with the traditional phased array theorem. Therefore, in the experiment we first use a single receiver element to find out whether a target is in the scene. Then, the phase information between the two elements will be used to find out the azimuth direction of the interested target.

In the experiment, the transceiver is working in 24GHz, and only Doppler frequency is detected to find out the speed of the target. A metal ball is rolling from the left of the transceiver to the right of it. A single receiver element generates Fig. 4.14, which shows that a target is appeared in the scene. Then, we focused on the target reflected area (i.e. red area in Fig. 4.14), and further find out the phase difference between the two elements (Fig. 4.15). It showed that the interested target is moving from the left to the right (i.e. the color of the interested target in Fig. 4.15 gradually goes from blue to red.)

4.6 Conclusion

In this chapter, we have proposed a new beamforming technique called transform sensing. The new beamforming technique can generate different sensing patterns for the space corresponding to different transformations. In this way, individual target echo signals become raw data while the received data are coefficients of certain transform such as DWT. The proposed transform sensing can reduce the radar scanning time, and compress radar raw data. Meanwhile it enables adaptive sensing, which uses multi-resolution analysis to select sampling for higher resolution. Both theoretical analysis and simulation verified it is feasible to generate transform sensing pattern.
Figure 4.14: The reflectivity of target detected by the magnitude of a single receiver

Figure 4.15: The phase difference between two receiver elements based on the target detected in Fig. 4.14
using the phased array radar. A preliminary experiment also shows the potential of the integration between array elements in the state of art.

The proposed transform sensing of phased array should be further verified by experiments on phased array systems. The sidelobe of the beam by the radar elements may impose a challenge to the proposed approach, especially when a selected transform needs sharp contrast between neighboring beam patterns. How the sidelobe affect the performance of the proposed approach will be an interesting problem to study in theory as well.
Chapter 5: Contributions and Future Work

We have discussed variety radar waveform designs in this dissertation. Since radar waveform design can improve the radar overall sensitivity and performance as well as extract more information from interested targets such as RCS, studies on radar waveform design have continued in the entire history of radar since it was invented almost seventy-five years ago. With the advent of high speed digital processor and solid state power amplification, it is becoming more feasible to design waveforms for extremely high resolution in both range and speed detections and for adaptation to features of targets and environments.

In this dissertation, a wavelet-based radar waveform is designed based on the scaling function and wavelets in the wavelet transform domain. One significant advantage of the wavelet-based waveform is effective sidelobe suppression. The near-sidelobes of the proposed signal can be suppressed by more than 2 times in comparison to LFM. This effective sidelobe suppression can be used to improve radar range resolution and thus performance at detecting small targets. Further study on the far-sidelobes has suggested that selecting the best signs and sequences of the wavelet packets via search can effectively suppress the far-sidelobes to the noise level, or one can push the far-sidelobes out of the detection bounds via add time intervals between the adjacent wavelet packets.
Furthermore, the wavelet-based waveform can separate the carrier and the baseband waveform effectively. The retrieved carrier frequency can determine the speed of moving target without the Doppler spread term, which causes velocity ambiguity in the LFM waveform. Meanwhile, the recovered baseband waveform can resist the Doppler effect on range estimation, due to the removed of the Doppler frequency and the Doppler spread term. Consequently, the wavelet-based waveform has the ability to resolve the range and speed of moving targets with higher resolution than the traditional LFM waveform. The advantage of the wavelet-based waveform can be directly shown when it is applied to strip-map synthetic aperture radar, which uses both range and velocity to resolve targets along the range and azimuth directions, respectively. Because the wavelet-based waveform can remove the Doppler spread term, it increases the SAR image resolution along the azimuth direction. Since the wavelet-based waveform almost eliminates all the sidelobes in the range detection, it enhances the image resolution in the range direction as well. Both mathematical analysis and simulation have shown that the wavelet-based waveform can improve radar performance including SAR image quality significantly. Another advantage of the proposed signal is that the new radar waveform’s adaptability to targets and environments as the magnitudes and phases of specific frequency subbands can be adjusted by modifying the corresponding wavelet packets. Thus the waveform is more suitable for cognitive radar than LFM.

In the remaining of the dissertation, we discussed a new beamforming technique called transform sensing. The new mechanism of the phased array can generate different sensing patterns for the space corresponding to different transformations. In this way, individual target echo signals become raw data while the received data are
coefficients of certain transform such as DWT. The transform sensing can reduce the radar scanning time, and compress radar raw data. Meanwhile it enables adaptive sensing, which uses multi-resolution analysis to select sampling for higher resolution. Both theoretical analysis and simulation verified it is feasible to generate transform sensing pattern using the phased array radar. A preliminary experiment also shows the potential of the integration between array elements in the state of art. Transform sensing of phased array should be further verified by experiments on phased array systems. The sidelobe of the beam by the radar elements may impose a challenge to the approach, especially when a selected transform needs sharp contrast between neighboring beam patterns. How the sidelobes affect the performance of the approach will be an interesting problem to study in theory as well.
Bibliography


