UNIFORM DAMPING NODE CONTROL THEORY UTILIZING SHAPED
PIEZOELECTRIC FILM

A Thesis

Presented in Partial Fulfillment of the Requirements for
the Degree Master of Science in the
Graduate School of The Ohio State University

By

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* * * * *

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ABSTRACT

There have been many active vibration control systems implemented using smart materials. These vibration suppression systems usually require separate sensors and actuators, which are placed according to criteria established based on collocation issues and actuation authority. Uniform Damping Control (UDC) is a method of vibration suppression with characteristics that make it desirable for many applications. It is a method of control that dampens motion over an entire structure by damping to every mode of vibration at the same decay rate. It is advantageous because the natural modes and the natural frequencies of the structure are not altered, allowing for energy-efficient vibration control. In addition to UDC, Node Control Theory (NCT) provides a means for actuator and sensor placement. NCT states that one can control the lowest N modes participating in a response by placing discrete sensor/actuator pairs at the nodes of the (N+1) mode. Utilizing NCT leads to a controlled system with that has frequency and modal invariance and uniform damping.

In this thesis, UDC and NCT were used together to implement Uniform Damping Node Control (UDNC) to control the vibrations of a cantilevered beam. The control algorithm was implemented using shaped polyvinylidene fluoride (PVDF) film, which provided point forces at the locations designated by the UDNC theory. The experiment
was initially run with external point displacement sensors used for feedback control. The results obtained were in agreement with the properties of UDNC.

A secondary objective of this thesis was to eliminate the external point displacement sensors. Self-sensing piezoelectric actuators were investigated, but a self-sensing PVDF actuator was never successful. Next, utilizing shaped PVDF sensors was considered. An analytical study was performed in Matlab, and it demonstrated that pennant shaped sensors could be utilized to simulate point sensors. An experiment was run to test these analytical predictions, and the results validated that the pennant shaped PVDF sensor simulates a point sensor. Finally, UDNC was implemented utilizing pennant shaped sensors and actuators.
Dedicated to Amy,

for her patience, love, and support.
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CHAPTER 1

LITERATURE REVIEW

The primary objective of this thesis is to implement a near-optimal vibration control algorithm, Uniform Damping Node Control (UDNC), using piezoelectric film in a novel manner. The piezoelectric film, a distributed actuator, will be spatially shaded in a manner that will produce discrete point forces at specific locations. Finally, any external sensors necessary to the system will be eliminated and replaced with piezoelectric components.

The purpose of any vibration control is to dampen the system response to unwanted excitations. While traditional vibration control methods are usually adequate, active vibration control holds potential for great improvements. Innovations in piezoelectric film modeling and technology as well as the rapid advancements in computers have made active vibration control systems more cost efficient and effective than previously possible. Active vibration control systems hold many advantages over passive systems. Active systems have a high bandwidth, are lightweight, and are very effective at low frequencies. Piezoelectric components are highly efficient and can be imbedded into systems to act as both sensors and actuators.
1.1 Uniform Damping Node Control Theory

UDNC is a method of vibration suppression with characteristics that make it desirable for many applications. Uniform Damping Node Control (UDNC) theory originated from the theory of natural control, which was developed by Meirovitch and Silverberg in 1983 [1]. Natural control is named such because it is a method that seeks to preserve the natural properties of the plant. Thus, natural control is a control method in which the natural coordinates (modal coordinates) for the open-loop system match the natural coordinates for the closed-loop system, and the open loop and closed loop eigenfunctions are identical. One difficulty with natural control is that ideally it requires distributed sensors and actuators, while discrete controls are usually the desired method of implementation. Meirovitch and Silverberg examined using discrete controls to approximate distributed controls and found that it could be done reasonably well. Silverberg further explored natural control in 1989, demonstrating that the control fuel consumption is near minimal when the control system preserves the natural properties of the structure, as in natural control [2]. This paper examined natural control in more detail, and developed a relationship between the physical properties of the plant and the control parameters of the closed loop system. This study revealed that the controls were most efficient when all of the modal damping rates are identical.

Silverberg developed uniform damping control, a natural control algorithm (for flexible space structures) in which all modes of the structure are damped at a uniform exponential rate [3]. Uniform damping control is a distributed control algorithm with three important characteristics: the damped natural frequencies of the open and closed loop systems were identical, the modal shapes of the open and closed loop systems were
identical, and all modes were damped at the same decay rate. Silverberg determined that the best dynamic performance was achieved when all modes of a structure were dampened at the same exponential decay rate. In addition, uniform damping control requires that the vibration must be suppressed over the entire structure at the same decay rate, which implies distributed controls. Although UDNC has many desirable characteristics, it is difficult to implement it due to its distributed nature. A method of implementation is discussed at the end of Silverberg's paper, but it requires large numbers of control forces, and the placement of the control forces is arbitrary.

Washington and Silverberg [4] added bias or steady-state calibration to this work using distributed actuators, in which the control forces were assumed to have a proportional, derivative, and integral component. Rosetti and Sun [5] extended UDNC to rings.

Weaver and Silverberg introduced node control, which directed where the discrete control forces and sensors should be placed in order to implement a natural control theory (such as uniform damping control) [6]. Node control theory (NCT) states that one can control the lowest N modes participating in a response by placing discrete sensor/actuator pairs at the nodes of the (N+1) mode. Utilizing NCT leads to a controlled system with that has frequency and modal invariance and uniform damping. Unlike many discrete control algorithms, node control is spillover resistant, due to the placement of the sensors and actuators. A general mathematical proof of NCT was developed by Silverberg with the help of Hendry and the Gauss-Jacobi quadrature theory [7,8]. Utilizing the Gauss-Jacobi quadrature theory, one is able to calculate the control gains for each control force. The union of uniform damping theory and NCT will be called Uniform Damping Node Control theory, or UDNC for the rest of this thesis.
UDNC combines the best aspects of uniform damping control with the ease of implementation of node control. With UDNC we have a discrete control algorithm that is near-optimal, energy efficient, and we have methods to compute every control gain necessary to implement the system.

1.2 Piezoelectric Film

Piezoelectric materials can directly transfer mechanical energy to electrical energy, and vice versa. Due to this unique ability that is known as the piezoelectric effect, piezoelectric materials have often been used as sensors and actuators in smart structures in literature. Usually, piezoelectric ceramics, or zirconate titanate (PZT) are utilized as an actuator, due to their ability to create relatively large actuation forces. Piezoelectric polymers such as polyvinylidene fluoride (PVDF) are usually utilized as a sensor due to their flexibility and high $g_{31}$ constant. However, in certain applications, the high flexibility of PVDF makes it preferable as an actuator. Bailey and Hubbard utilized PVDF as a distributed actuator to perform vibration control on a cantilevered beam utilizing a Lyapunov based control scheme [9]. Hubbard and Burke shaped piezoelectric film in order to create unique actuation forces for controlling beams with various boundary conditions [10, 11]. Burke and Hubbard demonstrated that piezoelectric film can be utilized to create moments and point forces, depending on the spatial distribution of the film. A pennant shaped spatial distribution applied to PVDF will be utilized to create a point force actuator in this thesis. Piezoelectric sensors can also be shaped in order to perform signal processing in the spatial domain [12]. Lee and Moon utilized shaped piezoelectric materials to implement modal sensors and actuators.
Shaped PVDF sensors will be employed in this thesis, and it will be demonstrated that a pennant shaped PVDF sensor can be utilized in place of a discrete sensor.

1.3 Piezoelectric Sensoriactuator

A piezoelectric self-sensing actuator simultaneously actuates and senses in a closed loop system. Self-sensing actuators have many possible applications, due to their desirable characteristic of perfect sensing and actuating collocation. In this thesis self-sensing actuators were investigated with the intention of implementing UDNC on the cantilevered beam without the external point sensors. Self-sensing piezoelectric actuators were developed almost concurrently by Dosch, Inman, and Garcia [13, 14] and Anderson, Hagood, and Goodliffe [15]. Dosch, Inman, and Garcia developed a self-sensing actuator circuit based on a capacitance-matching principle. This circuit met with some success, but was limited by the difficulty of perfectly matching the capacitance of the piezoelectric element.

Anderson, Hagood, and Goodliffe developed a piezoelectric self-sensing actuator, and performed experiments using the design of Dosch et al. Anderson explored the sensitivity of the self-sensing actuator to small errors in the reference capacitance, which proved to be a major issue. The authors of the paper found it extremely difficult to match the reference capacitance to the hysteretic piezoelectric capacitance. In order to achieve the best results, Anderson employed a hybrid reference capacitor consisting of part PZT ceramic and part passive ceramic capacitor [14].

Cole and Clark [16] developed an adaptive method of compensation for piezoelectric sensoriactuators. This method was in contrast to the analog methods
developed earlier in that it did not depend on matching the piezoelectric element’s capacitance accurately. Cole and Clark stated that capacitance matching introduced many difficulties, including changes in the piezoelectric due to environmental fluctuations. Further error can occur due to different hysteresis effects in the piezoelectric element at different frequencies. The digital compensation technique developed by Cole and Clark eliminated some of the difficulties encountered with the previous analog designs, but the electronics became more complicated in the process. In addition to the complications that arise when coupling digital and analog circuitry, this method required either an LMS or RLS filter to be programmed into a DSP. Vipperman and Clark [17] extended the work of Cole and Clark for practical implementation of the self-sensing actuator. Their experimental results showed that the adaptive self-sensing actuator was very accurate over a large bandwidth. In addition, they discovered a method by which the blocked capacitance of the sensoriaactuator could be estimated dynamically.

Fannin and Saunders [18] developed an analog adaptive piezoelectric sensoriaactuator that utilized an analog LMS filter. This method was deemed superior to the design of Cole and Clark because it allowed the amplitude and phase, rather than just the magnitude, of the signal to be adjusted, making it more accurate. In addition, the analog components were cheaper than their digital counterparts. Fannin and Saunders replaced the reference capacitor with an adaptive high pass filter in their design. However, the electronics employed by Fannin and Saunders to implement their design were far from simple, and they required hand-tuning potentiometers each time the device was used.
The goal of this thesis is to implement uniform damping control on a cantilevered beam utilizing the NCT for sensor and actuator placement. The control gains will be calculated using the Gauss-Jacobe Quadrature theory. Pennant-shaped PVDF film that simulates point forces will be used as the actuator for the system. Initially, external point sensors will be utilized, but these will eventually be eliminated from the system and replaced with a piezoelectric sensor.
CHAPTER 2

BEAM MODELING AND UDNC THEORY

The main objective of this thesis was to implement Uniform Damping Node Control Theory on a cantilevered beam utilizing spatially shaded piezoelectric film actuators. In this section the model of the plant (the cantilevered beam) will be developed, and then the mathematics governing Uniform Damping Node Control Theory will be developed.

2.1 Beam Modeling

A beam is a structure with a small cross-section compared to its length. In this discussion, a constant cross-section will be assumed throughout the length. The displacement characteristics of the beam will be considered for transverse motion only, where the displacement is \( u = u(x, t) \). In this section a derivation of the model of the cantilevered beam used in the experiments will be presented [18]. A differential beam element of length \( dx \) (see Figure 2.1) is displaced such that its centerline \( dx \) is unstretched. A distributed force \( p(x, t) \) is also assumed to be acting on the beam.

Summing forces normal to the centerline yields

\[
V - \left( V + \frac{\partial V}{\partial x} \ dx \right) + p \ dx = (\rho A dx) \frac{\partial^2 u}{\partial t^2} \tag{2.1}
\]

where \( \rho \) is the mass per unit volume.
Figure 2.1: A Differential Beam Element of Length \( dx \) in Bending

Canceling terms and dividing equation (2.1) by \( ds \) yields

\[
- \frac{\partial V}{\partial x} + p = (\rho A) \frac{\partial^2 u}{\partial t^2}.
\]  

(2.2)

Now sum moments about the center of the differential element:

\[
-M + (M + \frac{\partial M}{\partial x} \, dx) - Vdx - \frac{\partial V}{\partial x} \, dx = 0.
\]  

(2.3)

We have assumed that there is no rotary inertia of the element by equating the sum of the moments to be zero. However, we can eliminate \( \frac{\partial V}{\partial x} \, dx \) as a higher order term, and we can cancel other terms to get

\[
V = \frac{\partial M}{\partial x}.
\]  

(2.4)

From classical Euler-Bernoulli theory, the curvature of a beam at any location is proportional to the bending moment at that location:
\[ M = EI \frac{\partial^3 u}{\partial x^2}. \]  

(2.5)

\( E \) is Young’s modulus of elasticity, \( I \) is the area moment of inertia of the cross-sectional area, and \( \frac{\partial^3 u}{\partial x^2} \) is defined as the bending moment.

Plugging (2.4) and (2.5) into (2.2) yields

\[ -\frac{\partial^2}{\partial x^2} (EI \frac{\partial^3 u}{\partial x^2}) + p = (\rho A) \frac{\partial^2 u}{\partial t^2}. \]  

(2.6)

Assuming that the beam has a constant mass density, equation (2.6) simplifies to

\[ p = \rho A \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4}. \]  

(2.7)

For free vibration, \( p=0 \), and equation (2.7) simplifies to

\[ \rho A \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4} = 0. \]  

(2.8)

We assume that the solution to (2.8) may be separated into variables of space and time:

\[ u(x,t) = \phi(x) \Psi(t). \]  

(2.9)

Substituting equation (2.9) into (2.8) yields the following:

\[ \rho A \phi \Psi'' + EI \phi'' \Psi = 0. \]  

(2.10)

Dividing equation (2.10) through by \( \phi \Psi \) we get

\[ \frac{\phi''}{\phi} = -\left( \frac{\rho A}{EI} \right) \frac{\Psi''}{\Psi}. \]  

(2.11)
Since the spatial and temporal variables are now separated, each side of (2.11) must be equal to a constant, \( \alpha^4 \) where \( \alpha^4 = \frac{\omega^2 \rho A}{EI} \) and \( \omega \) is the natural frequency of any mode. Separating equation (2.11) into its space and time components yields

\[
\phi^{IV} - \alpha^4 \phi = 0 \tag{2.12}
\]

\[
\Psi''' + \frac{\alpha^4 EI}{\rho A} \Psi = 0. \tag{2.13}
\]

First the spatial solution (equation (2.12)) will be solved. The solution of equation (2.12) can be written in the form

\[
\phi = C_1 \sin \alpha x + C_2 \cos \alpha x + C_3 \sinh \alpha x + C_4 \cosh \alpha x. \tag{2.14}
\]

Thus, equation (2.14) represents the solution of the beam's lateral position in space with respect to length, or the modal shape. Now, consider a cantilevered (clamped-free) beam with the following boundary conditions:

at \( x = \ell \)

\[
\frac{\partial^2 u}{\partial x^2} = 0 \quad \text{(moment = 0)} \tag{2.15}
\]

at \( x = \ell \)

\[
\frac{\partial^3 u}{\partial x^3} = 0 \quad \text{(shear = 0)} \tag{2.16}
\]

at \( x = 0 \)

\[
u = 0 \quad \text{(deflection = 0)} \tag{2.17}
\]

at \( x = 0 \)

\[
\frac{\partial u}{\partial x} = 0 \quad \text{(slope = 0)}. \tag{2.18}
\]

From equation (2.17) we deduce the following relationship:

\[
C_2 + C_4 = 0
\]

\[
\therefore C_2 = -C_4. \tag{2.19}
\]
Differentiating (2.14) and applying (2.18) we can deduce

\[ C_1 + C_3 = 0 \]

\[ \therefore C_3 = -C_1. \] (2.20)

Now we have

\[ \phi(x) = C_1 \sin \alpha x + C_2 \cos \alpha x - C_1 \sinh \alpha x - C_2 \cosh \alpha x. \] (2.21)

From equation (2.15) we get

\[ \frac{\partial^2 \phi(\ell)}{\partial x^2} = 0 = -\alpha^2 C_1 \sin \alpha \ell - \alpha^2 C_2 \cos \alpha \ell - \alpha^2 C_1 \sinh \alpha \ell - \alpha^2 C_2 \cosh \alpha \ell \] (2.22)

and from equation (2.16) we get

\[ \frac{\partial^3 \phi(\ell)}{\partial x^3} = 0 = -\alpha^3 C_1 \cos \alpha \ell + \alpha^3 C_2 \sin \alpha \ell - \alpha^3 C_1 \cosh \alpha \ell - \alpha^3 C_2 \sinh \alpha \ell. \] (2.23)

Solve equation (2.23) for \( C_2 \) in terms of \( C_1 \)

\[ C_2 = C_1 \frac{-\alpha^3 \cos \alpha \ell \sin \alpha \ell - \alpha^3 \cosh \alpha \ell}{\alpha^3 \sinh \alpha \ell - \alpha^3 \sin \alpha \ell} \] (2.24)

Substitute this into equations (2.21) and (2.22)

\[ \phi(x) = C_1 (\sin \alpha x - \sinh \alpha x) + C_1 \frac{-\alpha^3 \cos \alpha \ell \sin \alpha \ell - \alpha^3 \cosh \alpha \ell}{\alpha^3 \sinh \alpha \ell - \alpha^3 \sin \alpha \ell} (\cos \alpha x - \cosh \alpha x) \] (2.25)

\[ \frac{\partial^3 \phi(\ell)}{\partial x^3} = 0 = -\alpha^2 C_1 (\sin \alpha \ell + \sinh \alpha \ell) - C_1 \frac{-\alpha^3 \cos \alpha \ell - \alpha^3 \cosh \alpha \ell}{\alpha \sinh \alpha \ell - \alpha \sin \alpha \ell} (\cos \alpha \ell + \cosh \alpha \ell) \] (2.26)

Equation (2.26) can be rewritten as

\[ \frac{\partial^3 \phi(\ell)}{\partial x^3} = 0 = -\alpha^2 C_1 (\sin \alpha \ell + \sinh \alpha \ell)(\sinh \alpha \ell - \sin \alpha \ell) + C_1 \alpha^3 (\cos \alpha \ell + \cosh \alpha \ell)^2 \] (2.27)

We know that \( C_1 = 0 \) is a trivial solution, so the inside expression in equation (2.27) must be zero. Using the identities
\[(\sin x)^2 + (\cos x)^2 = 1, \quad (\cosh x)^2 - (\sinh x)^2 = 1\]

We can simplify equation (2.27) to get

\[\cos(\alpha \ell) \cdot \cosh(\alpha \ell) = -1\]  \hspace{1cm} (2.28)

Therefore the eigenvalues for a cantilevered beam are the solutions to (2.28). The first four solutions are

\[\left(\alpha \ell\right) = 1.875104069, 4.694091133, 7.854757438, 10.99554073.\]

As one can see the solution has an infinite number of terms, with each solution representing a different mode. Dividing equation (2.21) by \(C_2\) yields

\[\phi(x) = \gamma \sin \alpha x + \cos \alpha x - \gamma \sinh \alpha x - \cosh \alpha x\]  \hspace{1cm} (2.29)

where \(\gamma = C_1/C_2\).

Solving (2.24) for the first four values of \(\gamma\) from the first four eigenvalues yields

\[\gamma = -0.7340955138, -1.018467319, -0.9992244962, -1.000033553.\]

Plotting equation (2.21) against a normalized length \(L=0\) to 1 for each of the first four eigenvalues gives the first four modal shapes shown in Figure 2.2.

To estimate the first natural frequency for the beam, the first eigenvalue

\[\left(\alpha \ell\right) = 1.875104069\]  \hspace{1cm} is plugged into the equation \(\alpha^4 = \frac{\omega^2 \rho A}{EI}\). Rearranging the equation we can solve for \(\omega\):

\[\omega = \frac{\alpha^2}{\ell^2} \sqrt[4]{\frac{EI}{\rho A}}.\]  \hspace{1cm} (2.30)

Now the temporal solution (equation (2.13)) will be solved. We can rewrite equation (2.13) as
\[ \Psi'' + \omega^2 \Psi = 0. \tag{2.31} \]

Figure 2.2: First Four Modes of Vibration of a Cantilevered Beam

It is now helpful to consider a mass on a spring (Figure 2.3). If we model the system using Newton's law of motion

\[ m \ddot{x} = \sum \text{forces}. \tag{2.32} \]
We get

\[ m\ddot{x} + kx = 0. \quad (2.33) \]

![Mass Spring System](image)

**Figure 2.3: Mass Spring System**

Note that equation (2.31) is very similar to equation (2.33). If we define \( k/m \) as \( \omega^2 \), then the two equations are exactly the same. If we model the beam as a mass with a spring and damper in free vibration (Figure 2.4) using Newton's law of motion we get

\[ m\ddot{x} + c\dot{x} + kx = 0. \quad (2.34) \]

![Mass Spring Damper System](image)

**Figure 2.4: Mass – Spring – Damper System**
To solve this differential equation we assume a solution to equation (2.34) of the form
\[ x = Qe^{nt} \]
we get
\[ (mr^2 + cr + k)Qe^{nt} = 0. \]  
(2.35)

We know that \( Qe^{nt} \) cannot be zero for all values of \( t \) (this would be a trivial solution), so equation (2.35) reduces to
\[ mr^2 + cr + k = 0. \]  
(2.36)
The roots of equation (2.36) are
\[ r_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}. \]  
(2.37)

To simplify terms and conform to normal notation, equation (2.34) can be rewritten as
\[ \ddot{x} + \frac{c}{m} \dot{x} + \frac{k}{m} x. \]  
(2.38)

We define \( \omega_n \) as \( \sqrt{\frac{k}{m}} \) (as stated previously) and \( \zeta \) as \( \frac{c}{2\sqrt{km}} \). We can rewrite equation (2.38) as
\[ \ddot{x} + 2\zeta \omega_n \dot{x} + \omega_n^2 x = 0. \]  
(2.39)

In equation (2.39) \( \zeta \) is the damping ratio and \( \omega_n \) is the natural frequency. This is the standard differential equation used to model a vibrating element, and in this thesis it is the equation used to model the cantilevered beam. It is important to note that equation (2.39) only models one mode of vibration, so the cantilevered beam is actually modeled by an infinite number of these equations.

2.2 Development of UDNC Theory
Now that the equations of motion for the cantilevered beam (the plant in the experiments performed for this thesis) the theory of Uniform Damping Node Control (UDNC) theory will be developed. The Uniform Damping Control theory will be developed first, leading to the distributed control law. The Node Control theory will be developed next, and, using the Gauss-Jacobi Quadrature theory, the discrete control law will be derived.

UDNC is a near-optimal, energy efficient "natural" control theory [1, 2]. Meirovitch and Silverberg developed natural control, which preserves the natural properties of the structure, including the damped natural frequency and the modal shapes. These characteristics have been shown to be energy-efficient. Silverberg developed Uniform Damping Control theory, which is the manifestation of natural control [3]. In the following section, the mathematics of Uniform Damping Control will be developed.

The equation of motion for a beam with a constant cross-section and external control force \( f(x,t) \) is found in equation (2.10). We can rewrite equation (2.10) in the more conventional form

\[
m \frac{\partial^3 u(x,t)}{\partial t^2} + EI \frac{\partial^4 u(x,t)}{\partial x^4} = f(x,t).
\] (2.40)

where \( u \) is the lateral displacement and \( m \) is the mass density per unit length (assumed constant throughout the structure). To simplify the analysis the motion of the structure is split up into an infinite sum \( u_\epsilon(x,t) \):

\[
  u(x,t) = u_1(x,t) + u_2(x,t) + u_3(x,t) + \ldots.
\] (2.41)
The displacement $u_r(x,t)$ is split into space and time variables (the natural decomposition) as shown:

$$ u_r(x,t) = \phi_r(x)u_r(t) . \quad (2.42) $$

The spatial variable $\phi_r(x)$ is called the natural mode of vibration or modal shape, and the temporal (or amplitude) variable, $u_r(t)$, is called the modal displacement.

The control force $f(x,t)$ can also be expressed as an infinite sum of control forces $f_r(x,t)$

$$ f(x,t) = f_1(x,t) + f_2(x,t) + f_3(x,t) + \ldots . \quad (2.43) $$

The control force can be broken down into its natural decomposition, the spatial dependence $\rho(x)\phi_r(x)$ called the natural mode of control, and the time dependence $f_r(t)$ called the modal control force:

$$ f_r(x,t) = \rho(x)\phi_r(x)f_r(t) . \quad (2.44) $$

The natural modes of vibration must satisfy the orthonormality conditions

$$ \int_D \rho(x)\phi_r^T(x)\phi_s(x)dD = \delta_{rs} . \quad (2.45) $$

$$ \int_D \phi_r(x)L\phi_s(x)dD = \omega_r^2\delta_{rs} . \quad (2.46) $$

where $\delta_{rs}$ is the Kronecker-delta and $\omega_r$ is the natural frequency of the $r^{th}$ mode.
The natural decompositions (equations (2.42) and (2.44)) are substituted into equation (2.40) and the orthonormality conditions (equations (2.45) and (2.46)) are applied to obtain the natural equations of motion for the structure:

\[ \ddot{u}_r(t) = -\omega_r^2 u_r(t) + f_r(t). \] (2.47)

The theory of uniform damping control consists of designing a set of control forces \( f(x,t) \) to achieve "natural" vibration suppression. The control forces are functions of the plant's motion, implying feedback control. A proportional-derivative control scheme will be utilized, in which the control forces are a linear function of the displacement and the velocity of the structure, expressed as

\[ f(x,t) = -Gu(x,t) - H\dot{u}(x,t). \] (2.48)

\( G \) and \( H \) are the proportional and derivative linear control gain matrix operators designed for specific applications. The control forces can be distributed or discrete depending on the operators \( G \) and \( H \), and the measurements of the displacements and velocities of the structure (\( u(x,t) \) and \( \dot{u}(x,t) \)) can also be distributed or discrete. The control law equation (2.48) is plugged into equation (2.40) to obtain the equations of motion for the controlled structure

\[ m\ddot{u}(x,t) + H\dot{u}(x,t) + (EI \frac{\partial^2}{\partial x^2} + G)u(x,t). \] (2.49)
Now the displacement of the structure will be examined in more detail. It can be expressed as a linear combination of the complex closed loop modes of the structure \( \psi_i(x) \)

\[
u(x,t) = \psi_1(x)C_1 e^{\lambda_1 t} + \psi_2(x)C_2 e^{\lambda_2 t} + \ldots \tag{2.50}
\]

in which \( C_i \) are constants that depend on the initial conditions and the closed loop eigenvalues are \( \lambda_i = -\alpha_i + i\beta_i \). The real parts of the closed loop eigenvalues, \( \alpha_i \), are exponential decay rates, and the imaginary parts, \( \beta_i \), represent oscillation frequencies of the structure. At this point it is necessary to explicitly define vibration suppression. The vibrations of the structure will be controlled in such a manner that the motion at any point \( x^r \) on the structure is bounded by the following:

\[
\|\nu(x^r, t)\| \leq U_0 e^{-\mu t} \tag{2.51}
\]

In equation (2.51) \( U_0 \) is the initial condition defined by

\[
U_0 = |\psi_1(x^r)C_1| + |\psi_2(x^r)C_2| + \ldots \tag{2.52}
\]

If equation (2.51) is compared to equation (2.52) it becomes apparent that the only way to achieve the desired control is for all modes of vibration to decay at an exponential rate defined by

\[
\alpha_i \geq \alpha \tag{2.53}
\]

Equation (2.53) characterizes Uniform Damping Theory as a "natural" control theory. The first ramification of equation (2.53) is that satisfactory vibration control will
arise from dampening the modes at an exponential rate equal to $\alpha$ – any further dampening will exceed the required performance and waste energy. Thus, each and every mode of vibration must be dampened at the same exponential rate. Because equation (2.53) must be satisfied for the entire structure, all points in the structure must be dampened at the same exponential rate $\alpha$. It has been shown that efficiency drops when the structure’s closed-loop frequencies vary from the natural frequencies, and also when the closed-loop modes differ from the open-loop modes [20-22]. Thus, the best uniform damping control is obtained when both the closed-loop modes of vibration and their frequencies are close to their open-loop counterparts, and all modes are damped at the same exponential decay rate, $\alpha$. In the interest of energy efficient vibration control, neither the natural modes nor the natural frequencies of the closed-loop system will be altered from their open-loop counterparts.

Now we shall consider the natural equations of motion for the structure (equation (2.47)) again but substitute $f_r(t) = -g_r u_r(t) - h_r u_r(t)$ as described in equation (2.48).

$$\ddot{u}_r(t) = -\omega_r^2 u_r(t) - g_r u_r(t) - h_r u_r(t)$$  \hspace{1cm} (2.54)

Rearrange equation (2.54) into the form

$$\ddot{u}_r(t) + (\omega_r^2 + g_r) u_r(t) + h_r u_r(t) = 0.$$  \hspace{1cm} (2.55)

Consider the Laplace transform of equation (2.55)

$$\left(s^2 + h_r s + (\omega_r^2 + g_r)\right)\mathcal{L}\{u_r(t)\} = 0.$$  \hspace{1cm} (2.56)

It is desired to solve for the control gain operators $g_r$ and $h_r$ in terms of the exponential decay rate, $\alpha$. A second order closed-loop system will be considered
\[ s^2 + 2\zeta \omega_n s + \omega_n^2 = 0. \quad (2.57) \]

As stated by the stipulations of uniform damping control, every mode must decay by the exponential rate \( \alpha \) and every mode must oscillate at or near its undamped natural frequency. Thus, the second order system (equation (2.57)) of this description will be described by the equation

\[ (s - \lambda_r)(s + \lambda_r) = 0 \quad (2.58) \]

where \( \lambda_r = -\alpha + i\beta_r \) (where \( \alpha \) is the constant exponential decay rate for all modes, and \( \beta_r \) represent oscillation frequencies of the structure). Expanding equation (2.58) yields

\[ s^2 + 2\alpha s + \alpha^2 + \beta_r^2 = 0 \quad (2.59) \]

Now if we compare equation (2.59) with equation (2.56) the following two conclusions can be made:

\[ \omega_r^2 + g_r = \alpha^2 + \beta_r^2 \quad (2.60) \]

\[ h_r = 2\alpha \quad (2.61) \]

Applying the fact that the closed-loop frequencies \( (\beta_r) \) must be the same as the open-loop frequencies \( \omega_r \) we conclude that

\[ g_r = \alpha^2 \quad (2.62) \]

Substitute equations (2.61) and (2.62) into equation (2.48) for the temporal solution of the control force:

\[ f_r(t) = -\alpha^2 u_r(t) - 2\alpha \dot{u}_r(t). \quad (2.63) \]
and the distributed control forces can be described by inserting equation (2.63) into equation (2.44)

\[ f_r(x,t) = \rho(x)\phi_r(x)\left(-\alpha^2 u_r(t) - 2\alpha \dot{u}_r(t)\right). \]  

(2.64)

The decay rate \( \alpha \) can be selected to damp \( C \) percent of a structure’s motion in \( t \) seconds using the following equation [4]:

\[ -\alpha = \frac{\ln(1-C)}{t} \]  

(2.65)

2.3 Node Control Theory

The distributed control forces derived from uniform damping control are applied in a discrete spatial manner as prescribed by the NCT. NCT states that the control forces for a structure with \( M \) modes participating in the dynamics should be placed at the nodes of the \( M+1^{th} \) mode [6]. Note that a cantilevered beam has \( M \) nodes in the \( M+1^{th} \) mode. Thus, NCT dictates the placement of the discrete control forces. Considering the modal displacement and the control forces in terms of their modal decompositions gives

\[ u(x,t) = \sum_{r=1}^{M} u_r(x,t) = \sum_{r=1}^{M} \phi_r(x)u_r(t). \]  

(2.66)

\[ f(x,t) = \sum_{r=1}^{M} f_r(x,t) = \sum_{r=1}^{M} \rho(x)\phi_r(x)f_r(t). \]  

(2.67)

If we multiply both sides of equation (2.66) by \( \phi_s \) and integrate we obtain

\[ \int \sum_{r=1}^{M} u_r(x,t)\phi_s(x)dx = \int \sum_{r=1}^{M} \phi_r(x)\phi_s(x)u_r(t)dx. \]  

(2.68)
Considering the orthonormality condition on the right hand side, equation (2.68) simplifies to

\[ \int u(x,t) \phi_r(x) \, dx = u_r(t). \]  

(2.69)

Similarly, the control forces can be shown to be

\[ f_r(t) = \int \phi_r(x) f(x,t) \, dx. \]  

(2.70)

Now if we consider the control forces to be spatially discrete, with one control force applied to each node of the \( M+j^th \) mode (as prescribed by NCT) we have

\[ f(x,t) = \sum_{i=1}^{M} f_i(t) \delta(x-x_i). \]  

(2.71)

Plugging equation (2.71) into equation (2.70) yields

\[ f_r(t) = \int \phi_r(x) \sum_{i=1}^{M} f_i(t) \delta(x-x_i) \, dx. \]  

(2.72)

Now consider the discrete control forces in terms of the position and its derivative measured at a discrete point \( x_i \):

\[ f_r(t) = -G_i u(x_i,t) - H_i \dot{u}(x_i,t). \]  

(2.73)

Plugging equation (2.73) into equation (2.72) yields

\[ f_r(t) = \int \phi_r(x) \sum_{i=1}^{M} \left[ -G_i u(x_i,t) - H_i \dot{u}(x_i,t) \right] \delta(x-x_i) \, dx. \]  

(2.74)
Considering equation (2.63), we can write the displacement and velocity of a point as the summation of functions of space and time:

\[ u_i(x_i, t) = \sum_{p=1}^{M} \phi_p(x_i) u_p(t). \]  \hspace{1cm} (2.75)

\[ \dot{u}_i(x_i, t) = \sum_{p=1}^{M} \phi_p(x_i) \dot{u}_p(t). \]  \hspace{1cm} (2.76)

Plug equations (2.75) and (2.76) into equation (2.74) to obtain

\[ f_r(i) = \int \phi_r(x) \sum_{i=1}^{M} \left[ -G_i \sum_{p=1}^{M} \phi_p(x_i) u_p(t) - H_i \sum_{p=1}^{M} \phi_p(x_i) \dot{u}_p(t) \right] \delta(x - x_i) dx. \]  \hspace{1cm} (2.77)

We can rewrite equation (2.74) into a more convenient form

\[ f_r(t) = \int \phi_r(x) \sum_{i=1}^{M} \sum_{p=1}^{M} \left[ -G_i \phi_p(x_i) u_p(t) - H_i \phi_p(x_i) \dot{u}_p(t) \right] \delta(x - x_i) dx \]  \hspace{1cm} (2.78)

Evaluating the integral, equation (2.78) simplifies to

\[ f_r(t) = \sum_{i=1}^{M} \sum_{p=1}^{M} \phi_r(x_i) \phi_p(x_i) \left[ -G_i u_p(t) - H_i \dot{u}_p(t) \right] \]  \hspace{1cm} (2.79)

Thus, equation (2.79) defines the control law for the discrete case. It is desirable to compare the discrete control law to the distributed control law (equation (2.64)) in order to solve for the discrete gain values. In order to compare the distributed control law to the discrete control law, the Gauss-Jacobi Quadrature (GJQ) theory will be employed. The GJQ theory is stated as follows [7, 8]:

25
Let \( x_1 < x_2 < \ldots < x_m \) denote the zeros of an \( m \) degree orthogonal polynomial \( \phi_m(x) \) on interval \( (a, b) \), and \( \Psi(x) \) is an arbitrary combination of the polynomial \( \phi_m(x) \) in \( \pi_{2m-1} \). The function \( w(x) \) is a weighting function, and the Christoffel numbers \( \Lambda_k \) are uniquely determined as

\[
\Lambda_i^{-1} = \sum_{r=1}^{m} \phi_r^2(x_i). \tag{2.80}
\]

One can then solve for the integral of \( \Psi(x) \):

\[
\int_a^b w(x) \Psi(x) \, dx = \sum_{i=1}^{m} \Lambda_i \Psi(x_i). \tag{2.81}
\]

We can extend the GJQ theory to include orthogonal sinusoids, and choose

\[
\Psi(x) = \phi_r(x)\phi_p(x). \tag{2.82}
\]

\[
w(x) = \rho(x). \tag{2.83}
\]

Plugging equations (2.82) and (2.83) into equation (2.81) yields

\[
\int_0^L \rho(x) \phi_r(x)\phi_p(x) \, dx = \sum_{i=1}^{m} \Lambda_i \phi_r(x_i)\phi_p(x_i). \tag{2.84}
\]

Applying the orthogonality condition

\[
\int \rho(x)\phi_r(x)\phi_p(x) = \delta_{rp}. \tag{2.85}
\]

If equation (2.85) is plugged into equation (2.84) the result

26
\[
\delta_{rp} = \sum_{i=1}^{m} \Lambda_i \phi_r(x_i) \phi_p(x_i) \rightarrow \frac{\delta_{rp}}{\sum_{i=1}^{M} \Lambda_i} = \sum_{i=1}^{m} \phi_r(x_i) \phi_p(x_i). \tag{2.86}
\]

Plugging equation (2.86) into equation (2.79) yields

\[
f_r(t) = \sum_{i=1}^{M} \sum_{p=1}^{M} \frac{\delta_{rp}}{\Lambda_i} \left[-G_i u_r(t) - H_i \dot{u}_r(t) \right]. \tag{2.87}
\]

Evaluating the Kronecker delta function simplifies equation (2.87) to

\[
f_r(t) = \sum_{i=1}^{M} \frac{1}{\Lambda_i} \left[-G_i u_r(t) - H_i \dot{u}_r(t) \right]. \tag{2.88}
\]

Recall equation (2.44) to get

\[
f_r(x,t) = \rho(x) \phi_r(x) \sum_{i=1}^{M} \frac{1}{\Lambda_i} \left[-G_i u_r(t) - H_i \dot{u}_r(t) \right]. \tag{2.89}
\]

Compare equation (2.89) (the discrete control law) to equation (2.64) (the distributed control law) to yield

\[
f_r(x,t) = \rho(x) \phi_r(x) \left[-\alpha^2 u_r(t) - 2\alpha \dot{u}_r(t) \right] = \rho(x) \phi_r(x) \sum_{i=1}^{M} \frac{1}{\Lambda_i} \left[-G_i u_r(t) - H_i \dot{u}_r(t) \right]. \tag{2.90}
\]

Simplifying terms in equation (2.90) yields

\[
G_i = \alpha^2 \Lambda_i, \tag{2.91}
\]

\[
H_i = 2\alpha \Lambda_i. \tag{2.92}
\]
Thus, we have determined the control gains for each discrete actuator, which is placed according to the node control theory. Note that the control forces are placed at the nodes of the N+1\textsuperscript{th} mode, which will therefore be theoretically impossible to excite. As higher modes require much more energy to excite, UDNC theory is a discrete control algorithm that is spillover resistant. In addition, the discrete sensors placed at the nodes of the N+1\textsuperscript{th} mode will not measure any of the N+1\textsuperscript{th} mode in the response of the structure.
CHAPTER 3

PIEZOELECTRIC FILM MODELING

3.1 Piezoelectric Constitutive Equations

In this chapter, the constitutive equations for piezoelectric materials will be reviewed and the equations for piezoelectric actuators will be developed. Based on the work done by Hubbard, Burke, Lee, and Moon [10-12], the theory for a discrete piezoelectric actuator will be developed.

Piezoelectric materials can directly transfer mechanical energy to electrical energy, and vice versa. More specifically, piezoelectric materials generate a charge in proportion to a mechanical stress (direct piezoelectric effect), and conversely they produce a strain when they are subjected to an electric field (the converse piezoelectric effect). Thus, piezoelectric materials can be utilized as both actuators and sensors due to their electro-mechanical coupling characteristics.

The Curie brothers discovered the piezoelectric effect (the ability to transfer mechanical energy to electrical energy exhibited by piezoelectric materials) in 1880. Common piezoelectric materials include the piezoceramic lead zirconate titanate (PZT), known for its relatively large actuation authority, and piezoelectric polymers such as polyvinylidene fluoride (PVDF). Piezoelectric film is a flexible, lightweight, tough polymer film with low density and excellent sensitivity. The piezoelectric film used in
this thesis has a silver ink electrode deposited on both of its faces to conduct voltage. Piezoelectric film has a pyroelectric (temperature) effect that enables it to convert thermal energy to electrical energy. Piezoelectric film does not have the actuation authority that piezoelectric ceramics possess, but piezoelectric film is capable of producing a higher voltage than piezoelectric ceramics for an identical applied mechanical stress. This makes piezoelectric film an excellent sensor, and it is utilized in the sensor industry for many applications. However, in this project the piezoelectric film is utilized as an actuator and sensor on a lightly damped cantilevered beam. Figure 3.1 shows the coordinate system for a piezoelectric material.

![Coordinate System Piezoelectric Element](image)

**Figure 3.1: Coordinate System Piezoelectric Element**

The constitutive relationships for piezoelectric film as developed by the Institute of Electrical and Electronics Engineers [23] are given in tensor form by

\[
S_p = s^E_{pq} T_q + d_{ip} E_i \quad \text{Converse Effect} \tag{3.1}
\]

\[
D_i = d_{iq} T_q + e^E_{ij} E_j \quad \text{Direct Effect} \tag{3.2}
\]

where

30
T – stress vector (N/m²)
S – strain vector (m/m)
E – vector of applied electric field (V/m)
D – vector of electric displacement (C/m²)
s – vector of compliance coefficients (m²/N)
d – vector of piezoelectric constants (m/m/(V/m))
ε – vector of permittivity (C/m²/(V/m))

and superscript T indicates a constant stress, and superscript E indicates a constant electric field.

The pyroelectric effect in PVDF modifies the constitutive equations as seen in equations (3.3) and (3.4):

\[ S_p = s_{pq}T_q + d_{ip}E_i + \alpha_p^{\varepsilon} \tau \text{ Converse Effect} \]  (3.3)

\[ D_i = d_{iq}T_q + \varepsilon_q^{\tau} E_j + \alpha_i^{\tau} \text{ Direct Effect.} \]  (3.4)

However, the pyroelectric effect in the PVDF will be neglected in this thesis for simplicity, so the constitutive relationships will be defined by equations (3.1) and (3.2).

In matrix form, equations (3.1) and (3.2) become

\[
\begin{bmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6
\end{bmatrix}
=
\begin{bmatrix}
s_{11} & s_{12} & s_{13} \\
s_{12} & s_{11} & s_{13} \\
s_{13} & s_{13} & s_{33} \\
s_{45} & & \\
s_{55} & & \\
s_{66}
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6
\end{bmatrix}
+
\begin{bmatrix}
d_{31} \\
d_{32} \\
d_{33} \\
d_{45} \\
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\]  (3.5)
\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix}
= \begin{bmatrix}
d_{31} \\
d_{32} \\
d_{33}
\end{bmatrix}^T \begin{bmatrix}
T_1 \\
T_2 \\
T_3
\end{bmatrix} + \begin{bmatrix}
e_{11} \\
e_{13}
\end{bmatrix} \begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}.
\] (3.6)

The Converse Effect (equation (3.1)) is used for actuation and the Direct Effect (equation (3.2)) is used for sensing. It should be noted that the piezoelectric constant \(d_{ij}\) is the electro-mechanical coupling term that relates the strain in the \(j\)-direction to the electric field in the \(i\)-direction. Unlike PZT, PVDF is not transversely isotropic from an electrical perspective, resulting in a \(d_{31}\) constant that is not identical to the \(d_{32}\) constant. Thus, for a material with no applied stress and an electric field applied in the \(3\) direction, the strain in the \(1\)-direction is calculated by multiplying the electric field by the \(d_{31}\) constant. This is illustrated in equation (3.7):

\[
S_1 = +d_{31}E_3
\] (3.7)

3.2 Uniform Strain Model for Actuation

In order to develop the model of the discrete force actuator, it is necessary to go through some basic models of piezoelectric actuators. In this section a model of the beam and piezoelectric actuator will be developed based on the uniform strain model developed by Crawley and De Luis [24, 25]. There are currently three methods commonly used to model the effects of piezoelectric actuators on beams: the block-force method, the uniform strain method, and the Bernoulli-Euler method. The Bernoulli-Euler model is the most accurate model, but it is also the most complex. The uniform strain model is simple and accurate when the structure is much thicker than the
actuator. In this thesis, the beam is 12.5 mil (317.5μm) thick, and the actuator is 52μm thick, giving a beam-to-actuator thickness ratio of 6:1. Figure 3.2 shows how the uniform strain model converges to the Bernoulli-Euler model as the thickness ratio increases. The uniform strain model conforms very closely to the Bernoulli-Euler model for a thickness ratio of 6:1.

![Normalized Curvature vs. Thickness Ratio for Uniform Strain and Bernoulli-Euler Models](image)

**Figure 3.2: Comparison of Uniform Strain and Bernoulli-Euler Models**

For the uniform strain model, the piezoelectric actuator is assumed to have a uniform strain distribution across its thickness, meaning that the actuator only extends or contracts, never bending. The forces created by the strain in the actuator are transmitted to the bonding layer (the epoxy) through a shear stress. In the ideal case, a perfect bond
at the actuator permits forces to be transmitted to the structure at concentrated points at the end of the actuator.

In the experiments conducted for this thesis, a piezoelectric film actuator was bonded to the side of a cantilevered beam. This configuration was a combination of extension and bending actuation, so both cases will be derived and combined. The total strain of the beam is

$$
\varepsilon_b^t = \varepsilon_b^e + \varepsilon_b^b
$$

(3.8)

where $\varepsilon_b^e$ is the strain in the beam due to extension and $\varepsilon_b^b$ is the strain in the beam due to bending.

In the pure bending case, the forces at the pins cause a moment to be applied to the structure:

$$
M = F t_b
$$

(3.9)

The stress in the beam is

$$
\sigma_b^b(z) = \frac{Mz}{I}.
$$

(3.10)

A symmetric beam with thickness $t_b$ and moment of inertia $I_b$ has a stress at the top surface (due to bending) of

$$
\sigma_b^b = \frac{Mt_b}{2I_b} = \frac{Ft_b^2}{2I_b}.
$$

(3.11)

The stress-strain relationships for the beam is given by

$$
\varepsilon_b^b = \frac{\sigma_b^b}{E_b}.
$$

(3.12)

The stress in the beam due to extension is given by the expression
\[
\sigma_b^e = \frac{F}{A_b}.
\]

(3.13)

The strain in the beam due to extension is given by

\[
\varepsilon_b^e = \frac{F}{b_b t_b E_b}.
\]

(3.14)

Recall that \( I_b = \frac{bh^3}{12} = \frac{b_b t_b^3}{12} \). Substituting equations (3.12) and (3.14) into equation (3.8) yields an overall strain in the beam:

\[
\varepsilon_b^e = \frac{F \left( \frac{t_b}{2} \right)^2}{b_b t_b E_b} + \frac{F}{b_b t_b E_b} = \frac{4F}{E_b t_b b_b}.
\]

(3.15)

The strain in the actuator is given by the expression

\[
\varepsilon_c = \frac{d_{31} V}{t_c} - \frac{F}{b_c t_c E_c}
\]

(3.16)

where \( \frac{d_{31} V}{t_c} \) is the strain the piezoelectric actuator creates due to the piezoelectric effect and \( \frac{F}{b_c t_c E_c} \) is the surface-restraint from the beam, which reduces the overall strain.

The displacement of the beam (equation 3.15) and actuator (equation 3.16) must be equal at the beam/actuator interface, so the two values for strain are equated:

\[
\frac{4F}{E_b t_b b_b} = \frac{d_{31} V}{t_c} - \frac{F}{b_c t_c E_c}.
\]

(3.17)

Solving equation (3.17) for \( F \) yields
\[ F = \frac{d_{31}V}{4} \left( \frac{t_c}{E_b b_s t_b} + \frac{1}{E_c b_c t_c} \right) \]  

(3.18)

which can be reduced to

\[ F = \frac{12d_{31}V}{t_c t_b^2} \left( \frac{E_b I_b E_c I_c}{3E_b I_b + 4E_c I_c} \right). \]  

(3.19)

In order to develop the model for the discrete force actuator, it is necessary to have the equation for the moment generate by the piezoelectric actuator. From equation (3.19), this moment is given by

\[ M = \frac{F t_b}{2} = \frac{6d_{31}V}{t_c t_b} \left( \frac{E_b I_b E_c I_c}{3E_b I_b + 4E_c I_c} \right). \]  

(3.20)

3.3 Shaped PVDF Film

PVDF sensors and actuators can be shaped to produce an integrating effect that can be utilized for various applications. Burke and Hubbard implemented shaped PVDF on beams with various boundary conditions [10, 11]. Piezoelectric materials are dielectric, so an electric charge generated in the film will only be collected if there is a surface electrode present. Thus, a piezoelectric sensor only reports a voltage proportional to the strain of the material covered by electrode material. Similarly, when an electric field is applied for actuation, only the area of the piezoelectric film that is covered with active electrode material will be affected by the electric field. Shaped piezoelectric sensors and actuators can be produced by etching or removing portions of the electrode, cutting the film to the desired shape, or polarizing the material in a varying manner. This method is called spatial aperture shading. The spatial aperture shading of
the electrode provides an integrating effect that enables the distributed actuator to produce a number of simulated effects like forces and moments. An example will be worked with a cantilevered beam to demonstrate.

Consider the equation of motion of a beam (equation (2.7)). Now consider a strain-inducing actuator such as PVDF film, which generates a moment (equation 3.20). If the PVDF actuator is bonded to one side of the beam, the equation of motion can be expressed as a combination of these expressions [9]:

\[
\frac{\partial^2}{\partial x^2} \left[ EI \frac{\partial^2 w(x,t)}{\partial x^2} - kV(x,t) \right] + \rho_b A_b \frac{\partial^2 w(x,t)}{\partial t^2} = 0. \tag{3.21}
\]

where \( k \) is a constant that encompasses the electro-mechanical coupling relationship and the material and geometric properties of the film and beam:

\[
k = \frac{6d_{31}}{t_s t_b} \left( \frac{E_b I_b E_c I_c}{3E_b I_b + 4E_c I_c} \right). \tag{3.22}
\]

Each term in equation (3.21) has the units of a moment. Note that the expression \( kV(x,t) \) is identical to equation (3.20). The actuator contributes to the displacement of the beam through the second term in equation (3.21).

\[
k \frac{\partial^2}{\partial x^2} [V(x,t)]. \tag{3.23}
\]

Note that the \( k \) term in equation (3.23) is only a constant that encompasses the electro-mechanical coupling relationship and the material and geometric properties of
the film and beam and is not dependent on $x$, so it is taken out of the spatial derivative operator. The voltage across the film can be separated into a function of time and space:

$$V(x,t) = V_{\text{max}} \Lambda(x) \rho(t).$$  

(3.24)

where $V_{\text{max}}$ is the maximum amplitude, and $\Lambda(x)$ and $\rho(t)$ are both normalized:

$$-1 \leq \Lambda(x) \leq 1$$ \hspace{1cm} (3.25)

$$-1 \leq \rho(t) \leq 1.$$ \hspace{1cm} (3.26)

With the voltage defined by equation (3.24), the two spatial derivatives will be applied to $\Lambda(x)$, the spatial function:

$$\frac{\partial^2 V(x,t)}{\partial x^2} = V_{\text{max}} \Lambda^{\prime\prime}(x) \rho(t).$$ \hspace{1cm} (3.27)

We will describe the spatial distribution of the electrode $\Lambda(x)$ with singularity functions.

![Figure 3.3: Spatially Uniform Actuator](image)

Figure 3.3: Spatially Uniform Actuator
Figure 3.3 shows the spatial distribution $\Lambda(x)$ for a spatially uniform actuator (a rectangular actuator). The spatial distribution is described by two step functions, $h(x)$ (which starts at $x=0$) and $-h(x-1)$ (which starts at $x=1$). Taking two derivatives of $\Lambda(x)$ with respect to space yields two point moments, one at $x=0$ and one at $x=1$:

$$\Lambda''(x) = \delta'(x) - \delta'(x-1)$$

Where $\delta'$ is a point moment

Next a triangular actuator will be analyzed. Figure 3.5 shows a triangular actuator and its spatial distribution $\Lambda(x)$. Note that the spatial distribution $\Lambda(x)$ is described by a ramp which consists of two step functions $h(x)$ (which starts at $x=0$) and $-h(x-1)$ (which starts at $x=1$) multiplied by $(1-x)$.

Taking two derivatives of $\Lambda(x)$ with respect to space yields two point forces, one at $x=0$ and one at $x=1$, and one point moment at $x=0$, as shown in Figure 3.6:
\[ \Lambda(x) = (1-x)[h(x)-h(x-1)] \]

Figure 3.5: Triangular Actuator

\[ \Lambda''(x) = \delta'(x) - \delta(x) - \delta(x-1) \]

Where \( \delta \) is defined as a point force

Figure 3.6: Forces Resulting from a Triangular Actuator
The two previous examples demonstrate that discontinuities in the amplitude of the actuator will result in point moments, and discontinuities in the slope of the actuator will result in point forces. Note that if the triangular actuator in Figure 3 was applied to a cantilevered beam, the point force and point moment created at \( x=0 \) would have no effect (being at the clamped end). This means the only force that would affect the beam would be the point force at \( (x=1) \). Thus, utilizing spatial shading, the distributed actuator can be made to simulate discrete forces. These discrete forces will be utilized in the UDNC algorithm for vibration control of the cantilevered beam.

3.4 Piezoelectric Film Sensors

In the effort to eliminate the external sensors from the system (the eddy current sensor and the laser sensor), two options were considered: the self-sensing piezoelectric actuator (see Chapter 4), and the extra PVDF film on the beam not utilized as an actuator. This section will examine utilizing the extra film as strain sensors.

PVDF film has been utilized as a strain sensor quite often in literature [12, 26-29]. Lee and Moon [12] came up with the equation relating the charge or voltage to the (two dimensional) strain in a plate and sensor shape:

\[
q_k(t) = -z_k \int_s F P_o \left[ e_{31}^0 \frac{\partial^2 u}{\partial x^2} + e_{32}^0 \frac{\partial^2 u}{\partial y^2} + 2e_{36}^0 \frac{\partial^2 u}{\partial x \partial y} \right] dxdy
\]  

(3.28)

where \( z_k \) is the thickness profile of the structure, \( F(x,y) \) is the effective surface electrode shape, \( P_o(x,y) \) is the polarization profile of the piezoelectric film, \( e_{31}^0 \) is the
directional charge constant in the x-direction, $e_3^0$ is the directional charge constant in the y-direction, and $e_3^0$ is the directional charge constant in the cross-axis.

However, in this thesis, only one-dimensional structures are considered, so we can simplify equation (3.28) for a cantilever beam:

$$q_k(t) = -z_k e_{31}^0 \int_0^L \mathcal{H}(x) \frac{\partial^2 u}{\partial x^2} dx$$  \hspace{1cm} (3.29)

where $\mathcal{H}(x)$ represents the one-dimensional polarization profile and the effective surface electrode shape of the piezoelectric film. It is important to note that the sensor voltage can be found by dividing equation (3.29) by $C_p$, the effective PVDF capacitance. Thus, from equation (3.29), the PVDF film produces a voltage proportional to the integral of the beam curvature and the spatial profile of the film (where electrode is present) [12].

![Graphs of First Mode Vibration: Deflection, Slope, Curvature, and Shear/EI](image)

**Figure 3.7: First Mode Deflection, Slope, Curvature, and Shear/EI**
It is advantageous to now consider the theory concerning beam strain. Consider the quantities of deflection, slope, moment, and shear, as commonly defined in a mechanics of materials book:

Deflection: \[ \delta = u \]  
(3.30)

Slope \[ \theta = \frac{du}{dx} \]  
(3.31)

Moment \[ M = \frac{d\theta}{dx} EI = \frac{d^2u}{dx^2} EI \]  
(3.32)

Shear \[ V = \frac{dM}{dx} EI = \frac{d^3u}{dx^3} EI \]  
(3.33)

The deflection and slope of a cantilevered beam is illustrated in Figure 3.8.

![Figure 3.8: Cantilevered Beam Deflection and Slope](image)

The modal shapes of the beam have already been calculated (Section 2.1), so it is possible to find the slope, moment, and shear forces along the beam for any mode (Figure 3.7). Furthermore, we can relate beam strain to the curvature of the beam utilizing Figure 3.9, which shows a beam element in bending. If we consider the element undergoing positive bending, the elongation \( \Delta \) of section AB is
\[ \Delta = -z \left( \frac{d\theta}{dx} \right) dx \]  

(3.34)

where \( z \) is the distance from the centroidal axis and \( \theta \) is the slope of the element at \( x \).

We can write the strain along section AB as

\[ \varepsilon_x(z) = \frac{\Delta}{dx} = -z \frac{d\theta}{dx}. \]  

(3.35)

Thus, we can calculate the strain as a function of the curvature of the beam \( \left( \frac{d\theta}{dx} \right) \) and the distance from the centroidal axis \( z \) [30]. The implication of equations (3.29) and (3.35) can be summed up in the following: a piezoelectric sensor only reports a voltage proportional to the strain of the material covered by electrode material.

Figure 3.9: Beam Element in Bending
The PVDF film sensor will return a voltage proportional to the integral of the beam strain where electrode material is present to gather the charge that the film produces. Consider the PVDF sensor/actuator bonded to a cantilevered beam shown in Figures 3.10 and 3.11, with the clamped end of the beam on the left side and the free end of the beam on the right side. The PVDF is chemically etched to separate the actuator (the black pennant shape) and the sensor (the surrounding white area). This sensor electrode shape will pick up some modes of vibration of the cantilevered beam better than others. For example, consider the first mode of the cantilevered beam (Figure 3.7) compared to the second mode (Figure 3.12). The sensor shown in Figure 3.10 will produce a much stronger signal for the second mode of the cantilevered beam than the first mode. The curvature of the beam in the first mode is highest near the clamped end of the beam, where there is only a small portion of the sensor electrode present, implying that a small amount of charge will be collected near the clamped end. The curvature of the beam in the second mode is high around the center of the beam, and when the sensor surface electrode shape of Figure 3.10 is applied to equation (3.29) for the second mode of the beam, a relatively high charge or voltage will result. In fact, calculations performed in Matlab revealed that the second mode of vibration of the beam would produce signals approximately ten times greater than the first mode of vibration for the sensors pictured in Figures 3.10 and 3.11 (see Appendix).

Figure 3.10: Shaped PVDF Sensor and Actuator (Long Pennant)
To compensate for the disproportionate sensor response to the higher modes of vibration, some signal conditioning was required. Due to mismatch of the shape of the sensor and the curvature of the beam in the first mode, the sensor signals needed to be filtered in order to amplify the first mode of vibration and attenuate the higher modes. Figure 3.13 shows the analog circuitry utilized to amplify and filter the response from the PVDF film sensors opposite the pennants. The signal from the PVDF sensor is sent through a voltage follower, a low-pass RC circuit (in order to attenuate the high amplitude signals the sensor will pick up from the higher modes of vibration), and a non-inverting amplifier. Note that all op-amps utilized in circuits for this thesis were 741 op-amps. The voltage follower acts as a high impedance amplifier for the sensor (chosen to avoid impedance mismatching with the sensor), and the R1 resistor prevents the op-amp from charging the film (which has an inherent capacitance) [17].
Figure 3.12: Second Mode Deflection, Slope, Curvature, and Shear/EI

Figure 3.13: Signal Conditioning Circuitry for PVDF Sensors Opposite the Pennants
The third alternative to external sensors was to utilize a separate PVDF pennant as a point sensor. This alternative required a different etching pattern on the PVDF. Instead of one pennant centered on the face of the beam for actuation, two pennants (one sensor and one actuator) were etched on the face of each side of the beam as seen in Figures 3.14 and 3.15.

![Figure 3.14: PVDF Pennant Sensor and Actuator (Long Pennants)](image)

![Figure 3.15: PVDF Pennant Sensor and Actuator (Short Pennants)](image)

For one-dimensional structures, shaping the PVDF in a pennant shape results in a signal proportional to positioning a point sensor at the end of the pennant. Conceptually, the pennant shape can be seen as a point sensor if one considers the discussion about shaped PVDF film in Section 3.3, in which point actuators were derived from pennant shaped film utilizing singularity functions. Consider the converse nature of piezoelectric materials and the Direct Effect (equation (3.2)), and it becomes apparent that pennant shaped film will not only simulate discrete point forces, it will simulate a point sensor.
Numerical analysis in Matlab confirmed that a pennant shaped PVDF sensor would report a signal proportional to a point sensor positioned at the end of the pennant (see Appendix). However, a pennant sensor cannot be said to be a perfect replacement for a point sensor. The results of the Matlab analysis can be seen in Figure 3.16. To create Figure 3.16, equation (3.29) was evaluated (the constants $z_k$ and $e^{0}_{H}$ were set equal to 1 for simplicity) for the curvature of the first four normalized modal shapes $\left(\frac{\partial^2 u}{\partial x^2}\right)$ multiplied by an effective surface electrode shape $z(x)$ representative of the pennant shaped sensor whose length was varied across the entire length of the beam. For example, to calculate the points on Figure 3.16 at normalized length 0.5, the curvature of each mode (for modes 1-4) was multiplied by a vector $z(x)$ representing the surface electrode shape of a linearly varying pennant shape which was normalized to be 1 at the clamped end of the beam (length=0) and 0 at length 0.5, and this was integrated over the entire length of the beam. Comparing Figure 3.16 to Figure 2.2 (the first four modal shapes of a cantilevered beam), it is apparent that for each mode the predicted sensor response is very similar to the actual displacement of the beam. For example, the nodes for each mode are in the same location for the predicted sensor response as for the actual modal shape. However, there are notable differences. Near the boundaries, the slope is different for the predicted sensor response compared to the actual modal shape for each mode. Also, the predicted sensor responses for each mode seem to resemble the modal shapes multiplied by a linearly varying function (the pennant shape).
Experiments were performed to verify the analytical predictions of Matlab. A beam with a PVDF pennant sensor on each side was etched so that each pennant terminated on a node of the third mode (Figures 3.10 and 3.11). The beam was excited by an electrodynamic shaker at the first, second, third, and fourth natural frequencies. The results can be seen in Table 3.1. The Measured Ratio is the Long Pennant Signal divided by the short pennant signal, and the Predicted Ratio is the ratio of the predicted sensor response (Figure 3.16) at length 0.8765 (where the long pennant terminates)
divided by the predicted sensor response at length 0.5036 (where the short pennant terminates).

<table>
<thead>
<tr>
<th>Mode Shape</th>
<th>Long Pennant Signal (Vmax)</th>
<th>Short Pennant Signal (Vmax)</th>
<th>Measured Ratio</th>
<th>Predicted Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>421 mV</td>
<td>275 mV</td>
<td>1.53</td>
<td>1.38</td>
</tr>
<tr>
<td>Mode 2</td>
<td>182 mV</td>
<td>781 mV</td>
<td>0.233</td>
<td>0.305</td>
</tr>
<tr>
<td>Mode 3</td>
<td>60 mV</td>
<td>81 mV</td>
<td>0.75*</td>
<td>0*</td>
</tr>
<tr>
<td>Mode 4</td>
<td>70 mV</td>
<td>500 mV</td>
<td>0.14</td>
<td>0.27</td>
</tr>
</tbody>
</table>

* When the beam was excited in the third mode the signal from both sensors was not sinusoidal.

Table 3.1: Experimental Verification of PVDF Pennant Point Sensor

Table 3.1 shows some very interesting results. If the pennants were perfect point sensors, the sensor signal ratio and the modal shape ratio would be identical. It is obvious that this is not the case. However, many observations that were made reinforced the notion that the pennant shaped PVDF sensor is similar to a point sensor. First of all, the two ratios are very close for the first two mode shapes. The ratios begin to diverge at the third mode, but the ratio comparison is not the only method to evaluate the pennant sensors. One should note that if the pennants were perfect point sensors they would not have picked up any signal when the beam was excited in the third mode. While the sensors did have some response when the third mode was excited, it was relatively small. In addition, the sensor signal for the third mode was not sinusoidal, indicating that there was also an external structural resonance occurring (perhaps of the beam clamp). Finally, observing the ratios for each of the modes, we notice that the results
seem to get worse as the mode shape becomes more complex. This would explain some of the error observed, for if one of the pennants weren’t etched properly, the error would become more significant for the complex modes, which have large amounts of curvature towards the free end of the cantilevered beam.

Taking all of these things into consideration, it was determined that the pennant shaped PVDF sensor would make an acceptable point sensor. The sensors would only be utilized for the first two modes, and the experimental results showed that the pennants would make suitable sensors for the first two mode shapes.

The final step for implementing the PVDF pennant sensor is the signal conditioning. Since the pennant shape is very similar to a point sensor (the desired signal), no filtering is necessary. The only necessary conditioning is to amplify the signal from the pennant sensor, in order to match the range of the control system’s A/D converter (-10 to +10V). The circuit utilized to perform this amplification is shown in Figure 3.17.

![Circuit Diagram](image)

**Figure 3.17: Signal Conditioning for Pennant Shaped PVDF Sensor**
CHAPTER 4

PIEZOELECTRIC SELF-SENSING ACTUATOR

One of the objectives of this research was to eliminate the external sensors from the UDNC theory vibration control system (the laser sensor and the eddy current sensor). The obvious alternative to these external sensors was to utilize the PVDF as a strain sensor. One method that has recently been developed is to utilize the piezoelectric material as a self-sensing actuator.

Self-sensing piezoelectric actuators were developed almost concurrently by Dosch, Inman, and Garcia [13], and Anderson, Hagoon, and Goodliffe [15]. The self-sensing actuator is designed to take advantage of the piezoelectric material's inherent converse properties to permit a single piezoelectric element to function as both a sensor and an actuator. The common voltage model of a piezoelectric sensor is a voltage source that is proportional to the strain of the material, in series with a capacitor that represents the capacitance of the piezoelectric element (Figure 4.1).

![Figure 4.1: Equivalent Circuit of Piezoelectric Device](image-url)
The self-sensing actuator, or sensoriactuator, is based on this simple electrical model. The self-sensing actuator works in the following manner: a control signal (voltage) is applied across the piezoelectric element (the primary branch) in order to control a structure (Figure 4.2). The same control signal is simultaneously applied across a reference branch, which contains a passive capacitance element that matches the capacitance of the piezoelectric element. The electrical response of the piezoelectric material is from two sources: the applied electrical signal, and the piezoelectric effect (by which the piezoelectric material produces a charge or voltage proportional to applied stress). The purpose of the reference branch is to match the electrical response of the piezoelectric material. The signal emanating from the reference branch is subtracted from the signal emanating from the primary branch, leaving the response of the piezoelectric element due to mechanical strains (or stresses).

The self-sensing piezoelectric actuator was built according to U.S. Patent 5,347,870 [14], designed by Dosch, Mayne, and Inman, except for a simple modification: the capacitors labeled C2 and C3 in Figure 4.2 (the patent originally intended these to be AC voltage dividers) were removed, because the control voltages applied to the PZT patch utilized as the piezoelectric element in the primary branch did not exceed the input voltage limits of the operational amplifiers used in the circuit. The actual circuit built can be seen in Figure 4.3. Although the final goal was to implement a self-sensing actuator utilizing PVDF as the piezoelectric element, PZT was utilized in the initial attempt. PZT worked well as the piezoelectric element in the sensoriactuator circuit, because it has a relatively high capacitance, and PZT requires relatively low voltages for actuation purposes.
The PZT self-sensing actuator was utilized to implement positive position feedback vibration control on a cantilevered beam. The results can be seen in Figure 4.4. The closed loop system could be improved with some simple alterations to the circuitry that would allow higher voltages to be applied to the self-sensing actuator. The circuit

![Circuit Diagram]

Figure 4.2: Self Sensing Actuator Designed by Dosch et al.

used in the experiment was implemented so that the voltage applied to the self sensing actuator was limited to +/-15 V (the 741 op-amp input voltage was the limiting factor).

The circuit of Figure 4.3 was observed to function properly, but there were obvious limitations. As explained by Cole and Clark [16], the analog compensation method (employed by Dosch, Mayne, and Inman) that did not account for changes in the piezoelectric capacitance over time and changes in the environment. Additionally,
hysteresis in the piezoelectric constant and permittivity were not modeled, and therefore were not accounted for. Anderson, Hagood, and Goodliffe [15] examined the effect of errors in matching the reference capacitance Cref to Cpzt, and found that small errors could have significant effects on closed-loop system performance.

Figure 4.3: Modified Self-Sensing Actuator Implemented
In order to compensate for mismatches between the reference capacitance and the piezoelectric element's capacitance, a phase shifter was added to the circuit (Figure 4.5). The phase shifter was tuned by adjusting the resistor value R32, which altered the phase lag added to the signal. In addition to adjusting the phase, the phase shifter attenuates the signal's amplitude. The potentiometer labeled R22 is used to adjust the amplitude of the reference branch to accommodate for the phase shifter.

The circuit of Figure 4.5 was difficult to implement. The digital and analog circuits developed by Vipperman, Clark, and Cole [17] and Fannin and Saunders [18] utilized adaptive methods to alter the gains for capacitance compensation, while the
circuit of Figure 4.5 is obviously passive. The circuit of Figure 4.5 is tuned by inputting a signal into both the primary and reference branches of the circuit at V1, adjusting the phase gradually, and observing the output signal. The circuit is properly tuned when the output signal is minimum. The circuit in Figure 4.5 was tested and was observed to function properly for a narrow bandwidth. The circuit had to be retuned for any adjustment in the input signal frequency, which made it very difficult to use this modified circuit in a vibration control scheme (where multiple modes and therefore multiple frequencies would be present). Frequency response tests also demonstrated that the circuit was only viable for a narrow bandwidth. The frequency response of the phase shifter portion of the circuit can be seen in Figure 4.6. The frequency response data was taken using SigLab (see Appendix). The phase shifter circuit is obviously only viable for a relatively narrow bandwidth, because neither the phase nor the magnitude have any flat areas of response. The self-sensing actuators developed by Vipperman, Clark, and Cole [17] and Fannin and Saunders [18] were considered as alternatives, but were determined to be too complicated for the scope of this thesis.

Implementing a self-sensing actuator utilizing PVDF film as the piezoelectric element proved difficult. Both self-sensing actuator circuits described previously (Dosch’s circuit and the modified circuit) were built utilizing PVDF film as the piezoelectric element. The low capacitance of the PVDF element (~ 5-10nF, compared to 0.237µF for the PZT) was difficult to measure and match accurately (in the reference branch of the circuit). Compounding this difficulty is the fact that the electrical response of the piezoelectric element (the unwanted portion of the response that we try to cancel with the reference branch of the circuit) is a function of the capacitance of the
piezoelectric element and the control voltage applied to it. The high control voltages necessary to utilize the PVDF film as an actuator result in large electrical response in the piezoelectric element. The signal produced in the piezoelectric material due to the mechanical response is relatively small, and any error in matching the piezoelectric element’s capacitance will result in large error. The higher compliance in PVDF also gives larger shifts in capacitance when the structure moves, leading to large errors in the reference capacitance and measurement.

The high control voltages necessary for utilizing PVDF as an actuator created further difficulties, because it was necessary to step the high voltages down to a safe level for the operational amplifiers in the analog circuitry (+/-18V). In contrast, PZT works well as the piezoelectric element in the sensoractuator circuit, because it has much higher capacitance (which is easier to measure and match in the reference branch of the circuit), and PZT requires lower voltages for actuation purposes. These two advantages of PZT elements also lead to a smaller ratio between the electrical response and the mechanical response of the piezoelectric material, resulting in less error due to mismatching the reference capacitor. Thus, the only self-sensing actuator successfully implemented was the circuit of Figure 4.3 with PZT as the piezoelectric element.
Figure 4.5: Self Sensing Actuator With Phase Shifter
Figure 4.6: Phase Shifter Frequency Response

The best way to implement a piezoelectric self-sensing actuator with a PVDF film element appears to be an adaptive sensoriactuator. The adaptive piezoelectric self-sensing actuator has the ability to match the capacitance more accurately than passive techniques. The method developed by Fannin and Saunders [18] should be attempted first, as it should be able to compensate for the electrical response of the piezoelectric element most accurately. This relatively complex circuit was not attempted in this thesis due to time constraints. Thus, a self-sensing piezoelectric actuator was built utilizing a PZT element, but not a PVDF element.
CHAPTER 5:

EXPERIMENTAL METHODOLOGY AND RESULTS

5.1 UDNC Implementation Utilizing External Sensors

The dimensions and properties of the beam and PVDF used in the first two experiments are given in Table 5.1. The beam’s dimensions and properties were used to estimate the natural frequencies and modal shapes of the beam. This information was used to determine the placement of the actuators. The PVDF was bonded to the film with a thin layer of Devcon 2-Ton Epoxy (see Appendix). The leads to the PVDF were wires soldered to copper tape, which was placed on the silver ink electrode PVDF.

The first two modes of vibration were targeted to be controlled for the experiment. According to the NCT, two actuators and two sensors are necessary to control the first two modes of vibration. Thus, PVDF was bonded onto both sides of the beam, resulting in one actuator on each side. The experiment was run three different ways: with external point sensors (a laser sensor and an eddy current sensor), utilizing the extra PVDF film as a strain sensor (and filtering the resulting signal), and utilizing a second PVDF pennant as a point sensor. The external sensors used to monitor the displacement of the beam were placed at the points dictated by the NCT (the two nodes of the third mode). Utilizing the beam theory discussed in Chapter 2, the nodes of the

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third mode were calculated to be at 127.9mm and 220.4mm from the base of the beam, respectively. An eddy current sensor was placed at the first node of the third mode, and a Microtrak 7000 Laser Displacement Sensor was directed at the second node of the third mode. The laser sensor was utilized at the second node of the third mode (which was closer to the free end of the beam) because the beam displacements were larger than at the first node of the third mode (which was closer to the clamped end of the beam). The PVDF film was etched into a pennant shape, shown in Figure 5.1. This shape was chosen to maximize the point force amplitude while assuring that the force will be directed through the center of the width of the beam (to avoid twisting).

<table>
<thead>
<tr>
<th>Beam Properties</th>
<th>PVDF Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Steel</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>210 x 10^9 N/(m^2)</td>
</tr>
<tr>
<td>Length</td>
<td>2.54 x 10^-1 m</td>
</tr>
<tr>
<td>Thickness</td>
<td>3.048 x 10^-4 m</td>
</tr>
<tr>
<td>Width</td>
<td>3.81 x 10^-2 m</td>
</tr>
<tr>
<td>Density</td>
<td>7850 kg/(m^3)</td>
</tr>
</tbody>
</table>

Table 5.1: Beam and PVDF Properties

The silver ink electrode of the PVDF was chemically etched using Methyl Ethyl Ketone as the etchant. The pennants were shaped so that they terminated at the nodes of
the third mode (as prescribed by NCT). This methodology led to one pennant being 220.4mm long (Figure 5.1), and the other was 127.9mm long (Figure 5.2). The third mode of the cantilevered beam with the two pennants superimposed to show their relative positions is shown in Figure 5.3. Note that the extra film, as indicated by the white portions in Figures 5.1 and 5.2, was not utilized as a sensor initially (the laser sensor and eddy current sensors were later replaced by the extra film). Modal testing was performed on the beam, and the nodes of the third mode were observed to occur at the points predicted by beam theory.

![Figure 5.1: Shaped PVDF Sensor and Actuator (Long Pennant)](image1)

![Figure 5.2: Shaped PVDF Sensor and Actuator (Short Pennant)](image2)

A schematic of the vibration control system can be seen in Figure 5.4. Figure 5.5 shows the Simulink diagram used to implement the controller in dSpace. Note that although the controller in Figure 5.4 was implemented digitally in dSpace, it could have been implemented using analog electronics. Each sensor and actuator pair was utilized in a UDNC control system with gains chosen according to equations (2.91) and (2.92).
An exponential decay rate of 0.5 was chosen for $\alpha$, and Matlab was used to calculate the values for $\Lambda_1$ and $\Lambda_2$. Note that the low pass filters labeled Transfer Fcn1 and Transfer Fcn2 are necessary to counter the tendency of a PD controller to amplify high frequencies. The output from dSpace was amplified up to 450V maximum using a 100X Kepco model BOP 1000M amplifier (there were two amplifiers; one for each actuator). The Kepco amplifier could have amplified the signal up to a maximum of 1000V, but arcing across the face of the PVDF electrode was observed when 600V was applied to the film. To avoid the potentially damaging electrical arcing, the voltage was limited to 450V for all experiments performed for this thesis. The amplified signal was then applied to the appropriate PVDF actuator through a lead soldered to conductive copper tape that was placed on the silver ink electrode.

Figure 5.3: Two Pennants Superimposed on Third Mode of Cantilevered Beam
Due to its position on the beam, the laser sensor observed a high first-mode content. The second node of the third mode (Figure 5.3) (where the laser sensor observed the beam motion) is close to the node of the second mode (so second mode response was low), and it is near the anti-node of the first mode (so first mode response was high). Thus, because the longer pennant (which terminated at a length of 220.4mm) terminated at a point fairly close to the node of the second mode and close to the anti-node of the first mode, it was implicitly more effective for suppressing the first mode of vibration. Similarly, for the closed-loop system it was observed that the short pennant primarily suppressed the second mode. In addition, the eddy current sensor observed a high second mode content due to its proximity to the anti-node of the second mode (so the second mode response was high), and since it was far from the anti-node of the first...
mode, the first mode response was low. Both sensors picked up virtually no third mode response, because they were positioned at the nodes of the third mode. Errors in sensor placement are responsible for any third mode response that was observed (for the external sensors).

The beam displacement with respect to time (data from the laser sensor) can be seen in Figure 5.6. In the first portion (labeled A), both the first and second modes are present in the response, and neither pennant is active (open loop). Between 1 and 3 seconds, the long pennant is active, but the short pennant is off. Section B of Figure 5.6 shows that the long pennant is very effective for damping the first mode, but not the second mode. Between time 3 and 5.5 seconds, both pennants are active (closed loop). In section C of Figure 5.6, one can see that the response of the beam is quite low.

![Simulink Diagram](image)

Figure 5.5: Simulink Diagram For UDNC Theory Implemented with External Sensors
Figure 5.6: Beam Displacement vs. Time

Figure 5.7 shows a frequency analysis of the beam’s response before and after UDNC theory was applied (data from the laser sensor). Both the first and the second mode were dampened significantly. It is important to note that the open-loop frequencies of the first and second modes are nearly identical to the closed-loop frequencies, matching the properties of UDNC. It was also observed that there was no visible change in the modal shape when control was activated, further validating UDNC. Figure 5.8 shows a frequency analysis of the beam’s response before and after UDNC theory was applied (data form the eddy current sensor). The high magnitude of the
second mode of vibration is due to two causes. The cantilevered beam was given a high second mode input, and the eddy current sensor (from which this data was taken) was positioned near an anti-node of the second mode, where second mode response is high.

Control for the third mode was unsuccessful, because the sensors and actuators were placed directly on the nodes of the third mode, and the system is uncontrollable with respect to that mode.

Figure 5.7: Frequency Analysis of Beam Response (from laser sensor)
Figure 5.8: Frequency Analysis of Beam Response (data from Eddy Current Sensor)

5.2 Vibration Control Utilizing PVDF Sensor

After UDNC was successfully implemented utilizing the laser and eddy current sensors, an effort was made to eliminate the external sensors from the system, to simplify and reduce the cost of the system. Self-sensing actuators were investigated, but building a functioning self-sensing actuator utilizing PVDF as the piezoelectric element was never accomplished (see Chapter 4).

An obvious alternative to external sensors was to utilize the extra film (that was not utilized as an actuator) as a sensor on either side of the beam (see Figures 5.1 and 5.2). The extra PVDF film opposite the short pennant (Figure 5.2) replaced the laser
sensor, and the extra PVDF film opposite the long pennant (Figure 5.1) replaced the eddy current sensor (see Film Sensor section of Chapter 3 for more detail). UDNC calls for point sensors, however, and the extra film utilized as a sensor is not equivalent to a point sensor. The temporal information will be correct, but the relative modal magnitudes will be incorrect. In order to compensate for the distorted signal of the PVDF opposite the pennant, filters were utilized in an attempt to employ the extra film as a sensor in a UDNC scheme. This met some success, but it was never possible to utilize the PVDF opposite the pennant as a point sensor, which is necessary for true UDNC.

The design of the filters in the signal conditioning for the film sensors were based on the discussion about the extra film being utilized as a sensor in the Piezoelectric Film Sensor section of Chapter 3. As the laser sensor had been primarily a first-mode sensor, the low pass filter in the signal conditioning circuitry for the sensor opposite the short pennant had a cutoff frequency just above the first natural frequency of the beam. Similarly, the eddy current sensor had been primarily a second mode sensor, so the low pass filter in the signal conditioning for the sensor opposite the long pennant had a cutoff frequency just above the second natural frequency of the beam. Figure 5.9 shows a schematic of the vibration control system without the external sensors. The controller was implemented digitally using dSpace, and the Kepco amplifier was used. The signal conditioning was performed by simple analog electronics described in the Film Sensor section. The Simulink diagram utilized to implement the controller in dSpace is shown in Figure 5.10.
Although the system utilized the PVDF sensors, the data analyzed was taken using the eddy current sensor positioned at the first node of the third mode (positioned at the same place it was in the previous experiments). The data from the PVDF sensors would have been difficult to compare to the data from the previous experiments, because of the filters used in the signal conditioning. Furthermore, because the extra film opposite the pennant is not a point sensor, this experiment was not true UDNC implementation.

The beam displacement with respect to time (data from the eddy current sensor) can be seen in Figure 5.11. It is important to note that the input to the shaker was different in this experiment than in the previous test. The input to the shaker had a
higher first mode content and a lower second mode content than in the previous experiment. In the first six seconds (labeled A) of Figure 5.11, the response contains both the first and

Figure 5.10: Simulink Diagram For Vibration Control Implemented with PVDF Film Sensors
second mode, and neither pennant is active (open loop). Between 6 and 14.5 seconds, the long pennant is active, but the short pennant is off. Section B of Figure 5.11 shows that the long pennant is very effective for damping the first mode. Between 15.5 and 20 seconds, both pennants are active (closed loop). In Section C of Figure 5.11, one can see that the response of the beam is quite low. A frequency analysis of the beam's open and closed loop responses can be seen in Figure 5.12. The closed loop beam response in the first mode was reduced by a factor of more than 14, and the closed loop beam response in the second mode was reduced by more than a factor of 10. Again, the open-loop frequencies of the first and second modes were observed to be nearly identical to the closed-loop frequencies, matching the properties of UDNC. It was also observed that there was no visible change in the modal shape when control was activated.

Figure 5.11: Beam Displacement vs. Time of System with PVDF Film Sensors
5.3 UDNC Implementation Utilizing PVDF Pennant Sensors

Although utilizing the extra PVDF as a sensor was successful, it was not true UDNC, because the extra PVDF was not a point sensor. To utilize PVDF as a point sensor in a UDNC system, pennant shaped PVDF sensors were employed. A new beam was made, and 110 μm thick PVDF was bonded to both sides of the beam (52 μm thick PVDF was utilized on the previous beam). The beam itself was made from the same steel and had the same dimensions as the previous beam. The silver ink electrode of the beam was chemically etched onto either side of the beam according to the patterns shown in Figures 5.13 and 5.14. The actuator pennant was chosen to be twice as large as the sensor pennant in order to maximize the actuation force produced. The size of the
sensor was not very important, because the sensor signal could be amplified quite easily (while the actuation voltages were already quite high, and any further amplification risks arcing across the electrode).

It was necessary to amplify the sensor signal because the voltages produced by the narrow pennant used as a sensor were relatively low (on the order of 200-400mV), and the 12-bit A/D converter used had a range of −10V to +10V. This works out to a resolution of 5mV, which isn’t very good when the signal being measured is only 200 mV. Thus, the sensor signal was amplified by the circuit shown in Figure 3.17. The circuit is simply a voltage follower followed by a non-inverting amplifier, and the resistor labeled RF in the circuit was varied until the gain of the circuit was about 22, so that the sensor signal was much larger than the 5mV resolution of the A/D converter. The Simulink diagram utilized to implement the controller in dSpace is shown in Figure 5.15.

![Figure 5.13: Long Pennant Sensor and Actuator](image1)

![Figure 5.14: Short Pennant Sensor and Actuator](image2)
Figure 5.15: Simulink Diagram For UDNC Theory Implemented with PVDF Film Pennant Sensors

The time response of the beam can be seen in Figure 5.16. In the first three seconds (labeled A) of Figure 5.16, both the first and second modes are present in the response, and neither pennant is active (open loop). Between 3 and 6.5 seconds, the long pennant is active, but the short pennant is off. Section B of Figure 5.16 shows that the long pennant is very effective for damping the first mode. Between 6.5 and 8.5 seconds, both pennants are active (closed loop). In Section C of Figure 5.16, one can see that the response of the beam is quite low. A frequency analysis of the beam’s open and closed loop responses can be seen in Figure 5.17. The closed loop beam response in
the first mode was reduced by a factor of more than 8, and the closed loop beam response in the second mode was reduced by more than half. Again, the open-loop frequencies of the first and second modes were observed to be nearly identical to the closed-loop frequencies, matching with the stipulations of UDNC. It was also observed that there was no visible change in the modal shape when control was activated.

Figure 5.16: Beam Displacement vs. Time of System with PVDF Film Pennant Sensors

The results from the system utilizing the PVDF pennant sensors should be compared to the results obtained from the system utilizing external sensors (the laser sensor and eddy current sensor). The data presented in Figure 5.8 should not be compared to the data presented in Figure 5.17, however, because the PVDF actuators
were very different for the systems used to obtain the data. The actuators were different sizes, and the PVDF film used for each of the two actuators had a different thickness. Because of these differences, the beam used to implement UDNC with PVDF pennant sensors was used to implement UDNC using the external sensors. The results can be seen in Figure 5.18. Comparing Figure 5.18 with Figure 5.17, one will observe that the results of the two systems are nearly identical. The system with the PVDF pennant sensors (Figure 5.17) attenuated the first mode a little better than the system with the external sensors (Figure 5.18), but the second mode was attenuated better by the system with the external sensors.

![Open and Closed Loop Frequency Components of System with PVDF Film Pennant Sensors](image.png)

**Figure 5.17:** Open and Closed Loop Frequency Components of System with PVDF Film Pennant Sensors
Utilizing the PVDF pennant sensors introduces two different possible sources of error. One possible source of error associated with utilizing the PVDF sensors is that any twisting of the beam will distort the PVDF sensor signal. Of course, theoretically, a beam will not twist, but some degree of twisting always occurs experimentally. In the experiments performed for this thesis, the cantilevered beams were excited utilizing an electrodynamic shaker attached to the top of the beam with a flexible threaded nylon stinger (see Figure 5.19). This asymmetric excitation definitely contributed to beam twisting. Alternative methods of exciting the beam were considered, but this method was utilized due to its simplicity. Beam twisting was never observed with the laser.
displacement sensor or the eddy current sensor. However, twisting was observed to significantly affect the PVDF sensor output.

![Electrodynamic Shaker](image)

**Figure 5.19: Experimental Setup**

The other possible source of error introduced by utilizing PVDF pennant sensors is caused by not positioning/etching the pennant properly on the beam. Even if the pennant is etched perfectly on the beam, the beam must also be positioned in the clamp very precisely. The error is most significant if the pennant is not positioned correctly at the base of the beam. Since a cantilevered beam has the highest levels of strain right at the clamped end, it is very important that the pennant sensor be positioned correctly, so that the base of the pennant starts at the edge of the clamp (Figure 5.20). If the sensor is positioned so that the base of the pennant is clamped (the beam is too short) (Figure 81
5.21), the sensor will be positioned incorrectly with respect to the modal shape of the 
N+1\textsuperscript{th} mode of the beam – the pennant will not terminate exactly on the node of the 
N+1\textsuperscript{th} mode. If the sensor is positioned so that the base of the pennant is too far from the 
clamp (the beam is too long) (Figure 5.22), two problems occur. As before, the sensor 
will be positioned incorrectly with respect to the modal shape of the N+1\textsuperscript{th} mode of the 
beam – the pennant will not terminate exactly on the node of the N+1\textsuperscript{th} mode. In 
addition, if the sensor is not positioned right at the edge of the clamp, the sensor will not 
observe the high levels of strain present at the base of the beam, and a large error will 
result.

![Figure 5.20: Pennant/Beam Positioned Properly](image)

![Figure 5.21: Beam too Short/Pennant Base Under Clamp](image)

![Figure 5.22: Beam too Long/Pennant Too Far from Clamp](image)
An experiment was performed to test the effect of incorrect beam placement in the clamp. The beam was positioned in three different ways; the beam was too short (Figure 5.21), the beam was the correct length (Figure 5.20), and the beam was too long (Figure 5.22). The results can be seen in Table 5.2.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Long Pennant Maximum Signal</th>
<th>Short Pennant Maximum Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Too Short – Length = 0.2485 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode 1</td>
<td>522 mV</td>
<td>325 mV</td>
</tr>
<tr>
<td>Mode 2</td>
<td>189 mV</td>
<td>875 mV</td>
</tr>
<tr>
<td>Mode 3</td>
<td>105 mV</td>
<td>175 mV</td>
</tr>
<tr>
<td>Mode 4</td>
<td>75 mV</td>
<td>205 mV</td>
</tr>
<tr>
<td>Beam Correct Length – Length = 0.2540 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode 1</td>
<td>421 mV</td>
<td>275 mV</td>
</tr>
<tr>
<td>Mode 2</td>
<td>182 mV</td>
<td>781 mV</td>
</tr>
<tr>
<td>Mode 3</td>
<td>60 mV</td>
<td>81 mV</td>
</tr>
<tr>
<td>Mode 4</td>
<td>70 mV</td>
<td>500 mV</td>
</tr>
<tr>
<td>Beam Too Long – Length = 0.2590 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode 1</td>
<td>230 mV</td>
<td>150 mV</td>
</tr>
<tr>
<td>Mode 2</td>
<td>218 mV</td>
<td>310 mV</td>
</tr>
<tr>
<td>Mode 3</td>
<td>309 mV</td>
<td>756 mV</td>
</tr>
<tr>
<td>Mode 4</td>
<td>60 mV</td>
<td>185 mV</td>
</tr>
</tbody>
</table>

Table 5.2: Experimental Results for Various Beam Lengths

The results from Table 5.2 demonstrate that the most significant error results from the pennant being positioned too far from the beam clamp (the beam is too long,
Figure 5.22). This is most evident in the large response observed for the third mode for both pennant sensors. Comparing the longer beam to the beam that was the correct length, third mode response increased by a factor of 5X for the long pennant sensor, and it increased by a factor of 7X for the short pennant sensor. This was expected, because the pennant sensors will not observe the large levels of strain present at the base of the beam. When the beam was too short, the error in the third mode is not as significant. This was also expected, because now the main source of error is due to the pennant not terminating right on the node of the third mode of the beam. Keep in mind that the 5mm increase (or decrease) in length only represents a 2% increase (or decrease) in the length of the beam, which is rather insignificant. It is obviously very important that the pennant is etched so that it is the correct length, but it is even more important that the beam be positioned properly in the clamp, so the pennant begins right at the edge of the clamp.
CHAPTER 6:

CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

In an effort to experimentally verify UDNC, the first and second modes of vibration were targeted for control in a cantilevered steel beam. UDNC was utilized in conjunction with NCT in order to suppress the vibration of the beam. Spatially etched PVDF provided discrete actuation forces at the nodes of the third mode. UDNC was verified experimentally for the first time when control proved successful, and further verification occurred when the third mode (not targeted) proved uncontrollable. Thus, discrete sensors (laser sensor and eddy-current sensor) and actuators were utilized to successfully control a distributed system.

In an effort to eliminate the external sensors, three alternatives were considered. Self-sensing piezoelectric actuators were investigated first, with the ultimate goal being to utilize the pennant actuators as self-sensing actuators. Self-sensing actuators were implemented using PZT patches, but a PVDF self-sensing actuator was never successfully implemented. The second alternative considered was to utilize the extra PVDF film (not utilized as an actuator) as a sensor. This proved to be a viable alternative, and successful vibration control was implemented. However, the extra PVDF film was not a point sensor, so the vibration control system implemented was not
truly UDNC. The third alternative to external sensors considered was to utilize a separate PVDF pennant as a point sensor. For one-dimensional structures such as a cantilevered beam, shaping the PVDF in a pennant shape results in a signal proportional to positioning a point sensor at the end of the pennant. UDNC was successfully implemented utilizing PVDF pennant shaped sensors and actuators, and the results were comparable to the system implemented with external sensors. While utilizing PVDF pennant shaped sensors and actuators, it was observed that the placement of the PVDF was vital to obtaining accurate results.

6.2 Future Work

The original objective of this thesis was to implement the UDNC experimentally utilizing PVDF film, and this was achieved. However, during the research and experimentation leading up to the successful implementation of UDNC, certain subjects emerged that deem further exploration. Self-sensing piezoelectric actuators were never successfully implemented on a PVDF element. If the pennants utilized as actuators with the external sensors were utilized as self-sensing actuators, it is possible that better results could be obtained. An effort should be made to implement the adaptive self-sensing actuators developed by Fannin [18] and Clark [17] on a PVDF film element. In addition, the analytical investigations performed to determine the nature of the pennant shaped PVDF sensor need to be continued in further depth, and tested experimentally. Proof that the pennant sensor is a point sensor was never obtained, but much of the experimental observations and analytical investigations lead to this conclusion. UDC theory should be implemented on a cantilevered beam utilizing modal sensors and
actuators, and the results should be compared to the results obtained in this thesis.

Finally, experimental implementation of UDNC on a two dimensional structure should be performed utilizing PVDF elements.
REFERENCES


APPENDIX A

METHODOLOGY FOR ETCHING PVDF FILM, MAPLE AND MATLAB SCRIPTS, AND SIGLAB TUTORIAL

The Appendix includes a detailed explanation of the methodology for shaping the piezoelectric film and adhering it to the steel beam, the Maple calculations used to calculate the eigenvalues and natural modes of the cantilevered beam, and the Matlab code used to predict the performance of the shaped PVDF sensor.

A.1 Etching and Cutting Piezoelectric Film and Adhering it to a Steel Beam

A sheet of 8” X 11”, 52μm thick piezoelectric film (part # 2-1004346-0) was purchased from Measurement Specialties, Inc. (610-650-1500, www.msiusa.com). This film had a silver ink electrode that covered 7.5” X 10.5” on both sides.

First the film was cut so that it would cover the entire face of one side of the beam (about 1.5” X 11”). The film was applied to a clean cutting board and taped at all four corners using a small amount of Scotch™ tape, keeping the film secure and taut. The line on which the cut was to be made was measured and marked using an ultra fine Sharpie™ marker. An X-acto™ knife was used to cut the film.

Next the new corners of the film were taped to the cutting board. It was necessary to etch the electrode away from the edge of the film, to prevent the film from electrical shorting (to the steel beam or to the other side of the film). A line was drawn
about 1.5mm away from the cut using the marker, and one long piece of scotch tape was used to mask the electrode on both sides to protect the areas where etching was not desired. The tape was pressed down firmly so that no etchant could seep under the edge. Rubber gloves should be worn during the following procedure, as methyl ethyl ketone should not touch the skin. A cotton swab was dipped in methyl ethyl ketone (the etchant) and rubbed along the electrode that was to be removed. The excess ketone was wiped away and cleaned using alcohol, and the process was repeated on the other side of the film.

The etching process described above can also be utilized to etch various patterns on the film for shaped sensors and actuators. Always be careful to mask the electrode with Scotch™ tape carefully and remove excess ferric chloride with alcohol.

Next the film was attached to the beam. First the beam was scratched with sandpaper to create a rough surface, improving the film’s adherence to the beam. The beam and back of the film were cleaned with alcohol to remove dirt and dust particles. A small amount of conductive silver epoxy (BIPAX® TRA-DUCT BA-2902 manufactured by TRA-CON) was used to ground the film to the beam. The silver epoxy was mixed, and a small portion placed in the middle of the beam. 2-Ton® Clear Epoxy manufactured by ITWDevcon was used to adhere the film to the beam. The epoxy was mixed and applied to small brayer (a rubber roller used to apply ink prints), and a very thin uniform layer of epoxy was applied to the entire beam around the dot of conductive silver epoxy. The two types of epoxy were not mixed, as this could reduce the conductivity of the silver epoxy. The film was carefully placed on the beam, and
another soft rubber brayer was used to press the film down and remove air bubbles. The brayer was applied until the air bubbles were nearly all gone. A piece of wax paper was then placed on top of the film (in case there is epoxy on the front of the film), and weights were placed on the beam and film for the duration of the curing time for the epoxy.

A.2 Maple Calculations to find Eigenvalues and Natural Modes of the Cantilevered Beam

> restart;
> Digits:=32;

\[
\text{Digits := 32}
\]

> eq1 := \cos(\beta) \cosh(\beta) + 1; \quad \text{eq 1 is eq 2.28}

\[
eq 1 := \cos(\beta) \cosh(\beta) + 1
\]

> plot(eq1, beta = 0..12, -10..10);
There are an infinite number of solutions to this equation, so we have limited the range to the first four solutions.

![Figure A.1: Eigenvalues of Cantilevered Beam](image-url)
> b1 := fsolve(eq1, beta=2);
b1 := 1.8751040687119611664453082410782

> b2 := fsolve(eq1, beta=5);
b2 := 4.6940911329741745764363917780198

> b3 := fsolve(eq1, beta=8);
b3 := 7.8547574382376125648610085827646

> b4 := fsolve(eq1, beta=11);
b4 := 10.995540734875466990667349107855

> L := 1; We normalize the length of the beam to 1 for simplicity
L := 1

We will now solve for the constant gamma, which is defined as C1/C2, from equation
2.24
> gamma1 := (sinh(b1*L) - sin(b1*L)) / (-cos(b1*L) - cosh(b1*L));
γ1 := -0.73409551375891275582878278897694

> gamma2 := (sinh(b2*L) - sin(b2*L)) / (-cos(b2*L) - cosh(b2*L));
γ2 := -1.0184673187592194050327411920450

> gamma3 := (sinh(b3*L) - sin(b3*L)) / (-cos(b3*L) - cosh(b3*L));
γ3 := -0.99922449651742828721635759805425

> gamma4 := (sinh(b4*L) - sin(b4*L)) / (-cos(b4*L) - cosh(b4*L));
γ4 := -1.0000335532517133983990162491446

> phi1 := gamma1*sin(b1*x) + cos(b1*x) - gamma1*sinh(b1*x) -
cosh(b1*x);
φ1 := -0.73409551375891275582878278897694
    sin(1.8751040687119611664453082410782 x) +
    cos(1.8751040687119611664453082410782 x) +
    0.73409551375891275582878278897694
    sinh(1.8751040687119611664453082410782 x) -
    cosh(1.8751040687119611664453082410782 x)

> plot(phi1, x=0..1); This is the first modal shape
Figure A.2: First Modal Shape of Cantilevered Beam

\[
\phi_2 := \gamma_2 \sin(b_2 x) + \cos(b_2 x) - \gamma_2 \sinh(b_2 x) - \cosh(b_2 x)
\]
\[
\phi_2 := -1.0184673187592194050327411920450 \\
\sin(4.6940911329741745764363917780198 x) \\
+ \cos(4.6940911329741745764363917780198 x) + \\
1.0184673187592194050327411920450 \\
\sinh(4.6940911329741745764363917780198 x) \\
- \cosh(4.6940911329741745764363917780198 x)
\]

> plot(\phi_2, x=0..1); This is the second modal shape
Figure A.3: Second Modal Shape of Cantilevered Beam

\[
\phi_3 := \gamma_3 \sin(b_3 x) + \cos(b_3 x) - \gamma_3 \sinh(b_3 x) - \cosh(b_3 x);
\]

\[
\phi_3 := -0.999224496517428282721635759805425
\]

\[
\sin(7.8547574382376125648610085827646 x) + \cos(7.8547574382376125648610085827646 x) +
0.999224496517428282721635759805425
\]

\[
\sinh(7.8547574382376125648610085827646 x) - \cosh(7.8547574382376125648610085827646 x)
\]

> plot(\(\phi_3, x=0..1\)); This is the third modal shape

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Figure A.4: Third Modal Shape of Cantilevered Beam

\[
\phi_4 := \text{gamma4} \cdot \sin(b4 \cdot x) + \cos(b4 \cdot x) - \text{gamma4} \cdot \sinh(b4 \cdot x) - \cosh(b4 \cdot x);
\]
\[
\phi_4 := -1.0000335532517133983990162491446
\sin(10.995540734875466990667349107855 \ x) +
\cos(10.995540734875466990667349107855 \ x) +
1.0000335532517133983990162491446
\sinh(10.995540734875466990667349107855 \ x) -
\cosh(10.995540734875466990667349107855 \ x)
\]

\[
> \text{plot}(\phi_4, x=0..1);
\]

This is the fourth modal shape
Figure A.5: Fourth Modal Shape of Cantilevered Beam

A.3 Matlab Code Used to Predict the Performance of the Shaped PVDF Sensor

```
%beam_vib7
%8-24-01
%Bruce Isler
%This Matlab file will calculate the first four modal shapes of a
cantilevered beam
%Plots for Beam Vibration Analysis

close all
clear all
format long;
xi=0:0.0001:1;

beta1=3.516^.5; %beta is equal to alpha in equation 2.29, and is
calculated in Maple using
%equation 2.28
```
\texttt{\textcolor{red}{\texttt{gammal=-0.7341; \; %gamma is defined as C1/C2, from equation 2.29. Gamma values are calculated in Maple}}

\texttt{\textcolor{red}{\texttt{x=(cos(betal*xi)-cosh(betal*xi))+gammal*(sin(betal*xi)-sinh(betal*xi)); \% x is the first modal shape, from equation 2.29}}

\texttt{\textcolor{red}{\texttt{beta2=22.034^\cdot.5; \\textcolor{red}{\texttt{gamma2=-1.01847;}}}}

\texttt{\textcolor{red}{\texttt{x2=(cos(beta2*xi)-cosh(beta2*xi))+gamma2*(sin(beta2*xi)-sinh(beta2*xi));}}

\texttt{\textcolor{red}{\texttt{beta3=61.697^\cdot.5; \\textcolor{red}{\texttt{gamma3=-9.9922;}}}}

\texttt{\textcolor{red}{\texttt{x3=(cos(beta3*xi)-cosh(beta3*xi))+gamma3*(sin(beta3*xi)-sinh(beta3*xi));}}

\texttt{\textcolor{red}{\texttt{beta4=120.902^\cdot.5; \\textcolor{red}{\texttt{gamma4=-1.00003;}}}}

\texttt{\textcolor{red}{\texttt{x4=(cos(beta4*xi)-cosh(beta4*xi))+gamma4*(sin(beta4*xi)-sinh(beta4*xi));}}

\texttt{\textcolor{red}{\texttt{figure}}

\texttt{\textcolor{red}{\texttt{SUBPLOT(4,1,1), plot(xi,x/2); title('Vibration Modes 1-4: Deflection vs. Normalized Length');ylabel('First Mode');grid;}}

\texttt{\textcolor{red}{\texttt{subplot(4,1,2), plot(xi,x2/2);ylabel('Second Mode');grid;}}

\texttt{\textcolor{red}{\texttt{subplot(4,1,3), plot(xi,x3/2);ylabel('Third Mode');grid;}}

\texttt{\textcolor{red}{\texttt{subplot(4,1,4), plot(xi,x4/2);ylabel('Fourth Mode');grid;}}

\texttt{xlabel('Normalized Length')}}

\texttt{\textcolor{red}{\texttt{figure}}

\texttt{\textcolor{red}{\texttt{SUBPLOT(4,1,1), plot(xi(1:10000),1e4*diff(x)); ylabel('First Mode');grid; title('Vibration Modes 1-4: Slope vs. Normalized Length');}}

\texttt{\textcolor{red}{\texttt{SUBPLOT(4,1,2), plot(xi(1:10000),1e4*diff(x2)); ylabel('Second Mode');grid;}}

\texttt{\textcolor{red}{\texttt{SUBPLOT(4,1,3), plot(xi(1:10000),1e4*diff(x3)); ylabel('Third Mode');grid;}}

\texttt{\textcolor{red}{\texttt{SUBPLOT(4,1,4), plot(xi(1:10000),1e4*diff(x4)); ylabel('Fourth Mode');grid; xlabel('Normalized Length')}}

\texttt{\%Added 7-12-01}

\texttt{\%strain=diff(x)}

\texttt{\%PVDF sensor is strain sensor, but it picks up integration of strain. We'll start with short sensor,}

\texttt{\% which is located at x=0.5035}

\texttt{\% long pennant occurs at x=0.8765}

\texttt{\textcolor{red}{\texttt{strain1=(.006*.0254+.000052/2)*diff(diff(x))*1e8; \; %beam thickness is}}

\texttt{\textcolor{red}{\texttt{.012".}}}

\texttt{\textcolor{red}{\texttt{strain2=(.006*.0254+.000052/2)*diff(diff(x2))*1e8;}}

\texttt{\textcolor{red}{\texttt{strain3=(.006*.0254+.000052/2)*diff(diff(x3))*1e8;}}

\texttt{\textcolor{red}{\texttt{strain4=(.006*.0254+.000052/2)*diff(diff(x4))*1e8;}}}

\texttt{\%\% ***** it is necessary to multiply the derivative by a factor of 1e4 because Matlab

\texttt{99}
%% takes a numerical derivative, which is really just dx, and we need dx/dxi, and since there are 1e4 points in xi, we must compensate %% by multiplying by multiplying by 1e8 since there are two derivatives taken.

% to integrate the strain, we sum up each element of the strain vector. This integration of strain % approximates what the pvdf sensor sees. Note that this is not an actual estimate of the voltage the % PVDF sensor would output - we would need a better estimate of z, the distance from the centroidal axis, and the % d31 constant. However, these estimates are all proportional to the actual voltages, so they will do.

shortpen1=sum(.5036^-1*.0001*(.5036-xi(1:5036)).*strain1(1:5036))
shortpen2=sum(.5036^-1*.0001*(.5036-xi(1:5036)).*strain2(1:5036))
shortpen3=sum(.5036^-1*.0001*(.5036-xi(1:5036)).*strain3(1:5036))
shortpen4=sum(.5036^-1*.0001*(.5036-xi(1:5036)).*strain4(1:5036))

longpen1=sum(.8675^-1*.0001*(.8675-xi(1:8675)).*strain1(1:8675))
longpen2=sum(.8675^-1*.0001*(.8675-xi(1:8675)).*strain2(1:8675))
longpen3=sum(.8675^-1*.0001*(.8675-xi(1:8675)).*strain3(1:8675))
longpen4=sum(.8675^-1*.0001*(.8675-xi(1:8675)).*strain4(1:8675))

for jj=1:9999
   ii(jj)=jj*.0001;
   shortpen1s(jj)=sum(ii(jj)^-1*(ii(jj)-xi(1:jj)).*strain1(1:jj)*.0001); % we must multiply our pennant shape ii(jj)^-1*(ii(jj)-xi(1:jj)) by the strain vector, and we multiply this by dx, or .0001
   shortpen2s(jj)=sum(ii(jj)^-1*(ii(jj)-xi(1:jj)).*strain2(1:jj)*.0001);
   shortpen3s(jj)=sum(ii(jj)^-1*(ii(jj)-xi(1:jj)).*strain3(1:jj)*.0001);
   shortpen4s(jj)=sum(ii(jj)^-1*(ii(jj)-xi(1:jj)).*strain4(1:jj)*.0001);
   shortpen1ss(jj)=sum(strain1(1:jj));
   shortpen2ss(jj)=sum(strain2(1:jj));
   shortpen3ss(jj)=sum(strain3(1:jj));
   shortpen4ss(jj)=sum(strain4(1:jj));
end

figure
SUBPLOT(4,1,1), plot(ii,1e4*shortpen1s); title('Vibration Modes 1-4: Predicted Pennant Sensor Response vs. Normalized Length');xlabel('First Mode');grid;
subplot(4,1,2), plot(ii,1e4*shortpen2s);xlabel('Second Mode ');grid;
subplot(4,1,3), plot(ii,1e4*shortpen3s);xlabel('Third Mode ');grid;
subplot(4,1,4), plot(ii,1e4*shortpen4s);xlabel('Fourth Mode ');grid;
xlabel('Normalized Length')

figure
SUBPLOT(4,1,1), plot(ii,shortpen1ss); title('Vibration Modes 1-4:Rectangular Sensor Response vs. Normalized Length');ylabel('First Mode');grid;
subplot(4,1,2), plot(ii,shortpen2ss);ylabel('Second Mode ');grid;
subplot(4,1,3), plot(ii,shortpen3ss);ylabel('Third Mode ');grid;
subplot(4,1,4), plot(ii,shortpen4ss);ylabel('Fourth Mode ');grid;
figure
plot(ii,shortpen2s./shortpen1s)
hold on; plot(xix2./x,'g--')

plot(ii,shortpen3s./shortpen1s,'r')
hold on; plot(xix3./x,'c--')
grid on

plot(ii,shortpen4s./shortpen1s,'m')
hold on; plot(xix4./x,'k--')
grid on
title('Agreement Between Modal Shape Ratios and Point Sensor Response Ratios')
xlabel('Normalized Length')
ylabel('Modal Shape Ratio')
legend('shortpen2s./shortpen1s','x2./x','shortpen3s./shortpen1s','x3./x','shortpen4s./shortpen1s','x4./x')

%The following calculations are to predict the behavior of the PVDF film opposite the pennant that is centered
% on the beam.
oppshortsens1=sum(.5036^-1*(xi(1:5036)).*strain1(1:5036))*.0001+sum(strain1(5038:end))*0.001
oppshortsens2=sum(.5036^-1*(xi(1:5036)).*strain2(1:5036))*.0001+sum(strain2(5038:end))*0.001
oppshortsens3=sum(.5036^-1*(xi(1:5036)).*strain3(1:5036))*.0001+sum(strain3(5038:end))*0.001
oppshortsens4=sum(.5036^-1*(xi(1:5036)).*strain4(1:5036))*.0001+sum(strain4(5038:end))*0.001

opplongsens1=sum(.8675^-1*(xi(1:8675)).*strain1(1:8675))*.0001+sum(strain1(8675:end))*0.001
opplongsens2=sum(.8675^-1*(xi(1:8675)).*strain2(1:8675))*.0001+sum(strain2(8675:end))*0.001
opplongsens3=sum(.8675^-1*(xi(1:8675)).*strain3(1:8675))*.0001+sum(strain3(8675:end))*0.001
opplongsens4=sum(.8675^-1*(xi(1:8675)).*strain4(1:8675))*.0001+sum(strain4(8675:end))*0.001

%%%%%%%%%%%%%%%%%Results
%oppshortsens1 = -2.474835726048171e-004
%oppshortsens2 = 0.00220968793002
%oppshortsens3 = -0.0028089066262
%oppshortsens4 = 0.00321823235393
%
%opplongsens1 = -1.547356844639282e-004
%opplongsens2 = 0.00155281766713
%opplongsens3 = -0.00280910132248
%opplongsens4 = 0.00385590178627
A.4 SigLab Tutorial

SigLab is a hardware/software package that works with Matlab to perform frequency response analysis. It is simple to use and enables an engineer to obtain a transfer function of a system quickly and accurately. There are many different frequency response tools available in SigLab; for this thesis the Sine Sweep method was utilized.

Set up the BNC cables to the front of the SigLab box for testing as follows: attach a BNC T adapter to Output 1, and attach one of the leads of the BNC T adapter to Input 1. Attach the other lead from Output 1 to the input of the system. The output of the system should be attached to Input 2. The Output 2 channel should not be attached to anything.

To start SigLab, simply start Matlab on a computer with the SigLab hardware and software installed. Type vss at the command prompt. Press the set up button at the top of the screen. This will bring up the SigLab interface shown in Figure A.6.

The first two lines of the set up screen help to define the frequency intervals over which the system will be tested. To add a frequency span, click the mouse on ADD SPAN, and to delete a span, click the mouse on DEL SPAN. Below each span is a column of variables and settings that specify how each span will be tested. The first is
Sweep Type, which should usually be set to linear. Tracking Bandwidth should be set to a bandwidth that is lower than the lowest frequency in the span that is being tested.

![Figure A.6: SigLab Set Up Screen](image)

The lower the tracking bandwidth is set, the longer the testing will take. To monitor the estimated time each span will take to test, view the row labeled Acquisition time. The number of averages is the number of times each span will be tested and averaged. Increase this for better accuracy, but keep in mind that increasing the number of averages will also increase the acquisition time. The Number of steps is the number of
steps that will be tested within each frequency range. Increase the number of steps for increased resolution within a frequency range. **Inter step delay** is the amount of time that SigLab will pause after each data point is tested; if the system being tested has a long settling time, this delay should be increased. It is recommended that this be set to 50ms. The Acquisition time is the total time it will take to test each frequency range with the settings selected. This is a number that should not be altered directly – instead, adjust other settings such as **Number of steps** or **Tracking Bandwidth** if the acquisition time needs to be decreased. **Level control channel** should be set to **Out1** if the cables are set up as described above. **Control level** is the maximum voltage that SigLab will send out of **Output 1** into the system for testing. This setting will vary depending on the system, but the maximum is 10V. **Ch 1 AC** sets the range of the A/D converter for **Input 1**. This setting should match the **Control level** setting. **Ch 2 AC** sets the range of the A/D converter for **Input 2**. This should be set to the maximum voltage expected out of the system.