A STUDY OF DIFFERENCE TONES
AS RELATED TO
CHORAL SINGING

A Thesis
Presented in Partial Fulfillment of the Requirements
for the Degree Master of Arts

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By
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INTRODUCTION

The title of this thesis will not give pause to an individual who happily possesses a blended knowledge of acoustical physics, music, and psychology. For the average reader, however, it may be wise to rely on Helmholtz, the fountainhead of acoustics, and Abe Pepinsky, a modern musicologist, for a definition of our subject.

In the foreword of his classic work on acoustics, Helmholtz brings us to the boundaries of physical and physiological acoustics, of musical science and esthetics, and of physics, philosophy, and art.¹ Mr. Pepinsky sketches the whole picture of musical knowledge from Pythagoras to the present, then directs our attention to the phase of systematic musicology called acoustics. Acoustics, he defines as the "descriptive analysis of music in its physical nature."

Under the heading of musical acoustics he states:

(1) The musician leaves acoustics of rooms and auditoriums to physicists and architects;
(2) The musician-composer stimulates the design of instruments, but artisans do the work;
(3) Transmission by radio, sound films, and phonographic reproduction are left to electrical engineers.

He finally delimits our field by listing problems of musical performance as

intonation, temperament, principles of orchestration; and those related to the actual performance

of music (playing and singing) as venting woodwinds, lipping brasses, use of vibrato, striking keys of claviature type, and diction.\textsuperscript{2}

Several acousticians have urged the study of difference tones, which will be defined in the next chapter. Redfield writes:

Quality is much influenced by subjective beat-tones. Composers of the future will not only employ new harmonic systems, they will also learn to utilize much more effectively the harmonic materials now employed. At present, if a composer writes he generally expects to hear just these two notes, but what the ear really has represented to it is

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As the advertisers of a few years back had it; "something new has been added." Wood says that "difference tones should not be ignored as part of the (tonal) complex by composers."\textsuperscript{4} Dayton C. Miller urges:

While beat-tones are purely subjective, they affect the ear as do real tones, and they have great influence on the tone quality on many instrumental and vocal sounds as perceived by the ear, a fact which has not been adequately investigated in the study of tone color.\textsuperscript{5}

Incidently, Pepinsky says that the masking effect of instruments has been investigated mostly with pure tones, and that complex tones should be investigated.\textsuperscript{6}

\textsuperscript{5}Dayton C. Miller, \textit{The Science of Musical Sounds}
\textsuperscript{6}Pepinsky, loc. cit.
lack exists in the field of difference tones, simply because complex tones, such as those produced by the voice, are much more difficult to handle than pure ones. There is no lack of data on laboratory instruments (pure tones), but that on the behavior of complex tones is very meager.

Other sensitive persons have wondered at the peculiar attraction of the tonal complex when listening to a string quartet, or to an a capella choir. They wonder what can be the cause of the difference in effect of closed and open harmony.

It is, indeed, realized that there is some vague difference between "open harmony" and "closed harmony"; but the precise laws governing the distribution of harmonic parts is as yet (1930) terra incognita. The principles controlling harmonic matrices and the effective use of them are matters for the future harmonist to discover. When these principles are known, the harmonic materials employed in present or future systems of harmony can be utilized to much greater advantage than is at present possible. The composer of the future will employ his harmonic matrices with a nice discrimination, marshalling them at will to serve his purposes. 7

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7 Redfield, op. cit., p. 209.
Chapter I

HISTORICAL REVIEW

Many writers have credited the discovery of difference tones to Sorge and Tartini. A.T. Jones, by going to the actual sources for his information, has conclusively proved that Giuseppe Tartini, celebrated Italian violinist, and Jean Baptiste Romieu, a French musician, independently discovered difference tones in 1714 and 1742, respectively. Georg Andreas Sorge, German organist, 1745, claims no discovery.1

According to Sir William Bragg, in a lecture at Royal Institution:

The subject of combination tones in music had formerly given rise to a great deal of argument, but the discussion had died down some 50 or 60 years ago (1878-88) and had almost been forgotten. At one time, the question whether the combination tone had an actual existence or were merely produced in the ear of the listener had been hotly debated, Helmholtz holding the former view and Koenig the latter... More recently, however, combination tones have been applied to the explanation of certain phenomena in broadcasting and in atomic science. The modulated carrier wave radiates the product of the acoustic and radio frequencies, and the sum and difference side bands correspond to the sum and difference combination tones.2

Joseph Peterson wrote his doctor’s dissertation at the

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2 W. Bragg, "Combination Tones in Sound and Light," Engineering, CXLVI (Nov. 18, 1938) p. 593.
University of Chicago, 1908. This is the only known work devoted solely to combination tones, which Peterson classifies as secondary auditory phenomena. By touching the high spots in his very thorough history of the subject, we find that many last-ditch battles were fought over this will-o’-the-wisp by men of science.

Briefly, Peterson recounts that Young’s Theory - "all beats, beyond the point of fusion, go over into combination tones" - was widely accepted, because the first difference tone has the same frequency as that of the beats of the primaries. But physical theory supported by Ohm’s Law is opposed to the beat theory.\(^3\) Lord Raleigh has expressed Ohm’s law in this way:

It is found by experiment that, whenever according to theory a simple (pendular) vibration is present, the corresponding tone can be heard, but whenever the simple vibration is absent, then the tone cannot be heard. We are, therefore, justified in asserting that simple tones and vibrations of a circular type are indissolubly connected.\(^4\)

Peterson reminds us of Helmholtz’s experiment with the piano which essentially illustrates his (Helmholtz’s) view of tonal analysis in the ear. "Raise the dampers and sing a vowel at the sounding board: the sympathetic resonance of the strings directly re-echoes the same vowel." The ear,

\(^3\) Joseph Peterson, *Combination Tones and Other Related Auditory Phenomena*, pp. 1-4.

like the piano, by means of the basilar membrane fibers,\textsuperscript{5} takes up the various constituent vibrations of any complex wave. From this resonance theory, we understand why Helmholtz could not accept the beat-tone theory of Thomas Young.

As the two primary tones diverge, the sections of the basilar membrane affected by these tones also separate farther and farther and finally cease altogether to overlap. The beats, dependent on this overlapping of vibrating sections, must consequently gradually diminish in intensity as they increase in frequency, and at a certain point must 'run out.'\textsuperscript{6} Beats, then according to Helmholtz's view can occur only when their generating tones lie near together in pitch....It is important to note that on Helmholtz's theory the place of stimulation in the cochlea, rather than the manner (frequency), is directly the determining condition of the pitch of the experienced tone.\textsuperscript{7}

As with the beat theory versus the asymmetry theory of combination tone propagation, so it was and is with the objective - subjective controversy. Helmholtz lead the objective side of the debate, although he admitted tones of a subjective character too. Koenig, his chief opponent, found no objective evidence of combination tones or beat-tones. Peterson concludes:

It may safely be inferred that if combination tones from primaries generated independently exist objectively at all, they are extremely weak....If they could all be shown to exist objectively the question as to their cause might be left entirely for physicists to an-

\textsuperscript{5} See ear chart, Appendix D, p. 78.
\textsuperscript{6} Actually, this does not happen. See the account of the slide whistle demonstration, p. 28, footnote 41.
\textsuperscript{7} Peterson, op. cit., p. 15.
swer. It is an appreciation of this fact that has caused several acousticians in recent years to investigate rather carefully the question of objectivity with reference to various so-called resultant tones. 8

We shall see that investigators since Peterson's time seem to agree that combination tones are mainly or altogether subjective in character. Objective tones, when they exist at all, are weak.

8 Ibid, p. 65.
Chapter II

A DISCUSSION OF THEORIES OF DIFFERENCE TONES
AND RELATED PHENOMENA

In order to explain the phenomenon of difference tones, it will be necessary to discuss the physical theories by which difference tones are accounted for. Also, it will be necessary to describe the phenomena which are closely related to difference tones, in order to avoid any confusion as to what difference tones are and what they are not. Finally, we shall undertake a close critical scrutiny of difference tones per se.

The once-upon-a-time scene opens upon Fourier, French physicist, 1768-1830. He formulated a theorem which was destined to prove invaluable to all study of vibration; light and sound. FOURIER'S THEOREM stated: "for each periodic curve there is just one definite set of simple harmonic curves that will combine to produce it."1 This theorem is used to obtain accurate measurements of the harmonic structure of a sound wave.

In 1843 we arrive at the contribution to sound by Georg Ohm, who is much better known in the field of electricity. We turn to Raleigh for OHM'S LAW OF ACOUSTICS:

1 A.T. Jones, Sound, p. 259.
This, in effect, is what is stated by Ohm's law, namely, that the ear can hear a particular tone in a sound wave which reaches it, only if a pendular vibration corresponding to that tone actually exists in the air. ²

For further enlightenment, Miller quotes Helmholtz:

"Any complex sound can be analyzed into a sum of simple pendular vibrations, and to each simple vibration corresponds a simple tone which the ear may hear." Helmholtz amended the law as follows: "Quality of tone depends solely on the number and relative strength of its partials, and in no respect of their difference of phase."³

Bartholomew describes the principle of SUPERPOSITION as follows: "Two or more different wave trains from one or many different sources, may exist in the air at the same time without destroying each other."⁴ Huygens, a Dutch philosopher, first advanced this principle for light waves in 1690, Fourier applied it to sound waves, and Young applied it to water waves. Raleigh points out that in order to apply Ohm's law to this principle, it is assumed that the vibrations concerned are infinitely small. And "one apparent exception to the law has long been known. This is the combination tone discovered by Sorge and Tartini in the last century."⁵

² Ll. S. Lloyd, Music and Sound, p. 78.
³ Miller, op. cit., p. 62. Helmholtz was supported in this "phase" statement by Miller, Lloyd, and Agnew, but was opposed by Koenig and Lindig. (Redfield, op. cit., p. 63) Fletcher's work refutes Helmholtz's statement.
⁵ Raleigh, op. cit., p. 286.
Perkins' vivid diagram and comment on superposition are reproduced below.\(^6\)

**INTERFERENCE** is a very important theory in dealing with the interaction of sound waves, and may be demonstrated with the Herschel-Quincke tube.\(^7\) It may also be experienced by rotating a vibrating tuning fork near one ear.

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6 When waves from two exactly similar sources combine, they produce hyperbolic loci of minimum and maximum intensity, or nodal (y) and antinodal (z) lines. This may be shown by enclosing a source of sound, such as a whistle, in a box from which the sound issues through two holes, the common source insuring the identity of frequencies, which is absolutely necessary. According to Huygens' principle, the two holes may be regarded as independent sources of sound, and as they are equidistant from the whistle W, the waves issuing from them are in the same phase. (Perkins, H.A., College Physics, p. 248).

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7 The Herschel-Quincke tube (see Fig. 2) is so constructed that one sound path may be lengthened or shortened by a slide-trombone arrangement. As the crook DBE is gradually lengthened beats and silence alternate due to constructive and destructive interference. (Lecture-demonstration, Physics 645, Maxwell, Summer Quarter, 1948).
On the subject of interference Stewart says:

If two tones of exactly the same frequency, amplitude, and phase (compressions and rarefactions coincide) are superposed, they will result in an intensity four times that of one tone alone. It the two superposed tones are exactly out-of-phase, there will be silence.8

However, such extreme theoretical cases are never actually experienced in listening to a musical performance because:

(1) Most ordinary sounds are mixed (different wave lengths) so that a point which is a maximum for some constituents of the sound may be a minimum for others and vice versa. This tends to even out the loudness of the sound. Therefore, the magnitude of the sound is not in ratio to the size of the ensemble.

(2) Then again, we have two ears, and if one of them is at a point of maximum loudness, the other is some distance from the maximum point for short waves and may even be a minimum, and this, again, tends to average the effects of superposition.9

A commonly experienced instance of superposition is that of "BEATS." Two sources of sound of nearly the same frequency result in a periodic alternation of sound and silence called beats. Beats are due to the two wave trains getting in and out of step at regular intervals.10

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8 G.W. Stewart, Introductory Acoustics, p. 35.
10 Perkins, loc. cit. Beats may be understood from Fig. 3 below, where the two dotted curves represent the vibrations of the two tones reaching the ear.
Consequently, a listening person experiences a periodic waxing and waning of the sound. To cite a concise definition from Stevens and Davis: "Beats are the periodic variations of the amplitude of the sound pressure at a point due to the interference of two sound waves of different frequencies." 11

Woods uses the resonance theory to account for beats:

Two tones act on transverse fibres of the basilar membrane. If two notes are sufficiently close in pitch for the two groups of excited fibres to overlap, then the fibres constituting the overlap will be the seat of the beating. If the tones are too widely separated in pitch for the corresponding groups of fibres to have any members in common beats cannot be heard. 12

In considering the next physical phenomenon, we assume two vibrating bodies of the same frequency. "If two tuning forks of the same vibration frequency are placed at some distance from each other and one is sounded and damped, the other will be heard to continue the note. It is said to vibrate sympathetically, or resonate the tone." This is referred to as RESONANCE, or sympathetic vibration. 13

A vibrating body may vibrate as a whole, in two equal segments, or in any number of equal segments. Also a body

can vibrate in several of these modes at the same time, thus producing a complex tone consisting of wave components corresponding to the modes of vibration.

The peculiar quality of a sound which enables us to distinguish between different sources giving the same pitch depends upon the overtones, (partials, or HARMONICS) present. Helmholtz made the first careful qualitative analysis of complex tones with the aid of resonators, each of which responded to but one frequency. More recently, D. C. Miller, an American physicist, has analyzed complex sound vibrations by means of an ingenious device which he named the phonodeik.\footnote{Perkins, op. cit., p. 242.}

Below Olson and Massa compare five methods of harmonic analysis:

(1) \textbf{Graphical}. The "Fischer-Hinnen" method involves laborious computations, and is restricted to low order harmonics and a sharp oscillogram.
(2) \textbf{Mechanical}. This machine acts as a synthesizer. Settings for the various component waves are assumed, and the stylus traces a wave that is equal to the sum of the motions imparted to it individually by several mechanical sources of different frequencies, amplitudes, and phases. The resulting plotted wave is compared with the wave being analyzed, and if it does not agree in shape, new conditions must be assumed and another comparison made. This method requires great labor, is inefficient, and needs an accurate oscillogram of a fixed size equal to the size of the curve plotted by the machine.
(3) \textbf{Electrical Resonance}. The various components are measured by means of selective circuits that can pick out the individual components at will. No disadvantages are noted.
(4) \textbf{Heterodyne}. This method uses the heterodyne principle similar to the method used in picking out radio stations in the superheterodyne receiver. The advantages of this method over the simpler resonance methods described is that the de-
tection of a particular frequency component is more positive due to the higher selectivity of the system. With this type, the harmonic component can be measured to within one or two cycles over the entire audio-frequency range, while the resonance method can only work over a limited range for a given size condenser and inductance.

(5) A sensitive Vacuum tube voltmeter with a very sharp frequency characteristic. The operation and structure are very technical.

We have been discussing objective harmonics. There is another type of harmonics called AURAL HARMONICS, which do not exist objectively, but are heard subjectively in the ear. They might be confused with difference tones, except that they occur at higher frequencies. They occupy the same frequency ranges in relation to their fundamentals as do objective harmonics, but they are created only in the ear by the asymmetrical, non-linear response of the auditory mechanism to a pure objective tone. They will be discussed in conjunction with subjective difference tones, (p. 26).

The RESONANCE THEORY was hinted at by Cotugno, 1736-1822, and Charles Bell, 1774-1842.

It is a common assumption of such theories that there is some structure in the cochlea with enough distinct parts varying in length, thickness, or tautness (like piano strings) to be capable of acting as selective resonators, vibrating analytically to the various components in a complex wave, and stimulating the underlying hair cells. Helmholtz finally settled on the basilar membrane: 'The fibres of the basilar membrane, which are transverse and in tension, act like resonators fairly heavily damped.'.... Assuming that the basilar fibers are capable of selectively vibrating to 11,000 different pitches, then the problem of analysis of complex tones and their combinations is solved, and also the capa-
city for independent experiencing of tones. It is due to physical isolation of parts. On the other hand, all phenomena demonstrating interaction of parts, as beats, combination tones, and fusion, require explanation.  

The term COMBINATION TONE is applied to tones which result from the combination of two tones; it refers to both difference and summation tones.

When two tones of different pitch are sounded together moderately loud, both are heard separately, and there may also be heard a deeper tone of a frequency equal to the difference between the frequencies of the other two; for example, if two tuning forks of 600 cycles and 1,024 cycles respectively yield a difference tone of 224 cycles. There is also another combination tone, the summational tone (discovered by Helmholtz), whose frequency is the sum of the two tones evoking it. It is much more difficult to hear, and to explain. 

Combination tones are not to be confused with upper partials (see harmonics, pp. 12-13), because they (combination tones) are not heard when only one generating tone is sounded, but appear with the second generating tone. 

Since the SUMMATION TONE is of little importance in music, we will dispose of it before taking up the real subject of our thesis, the difference tone. Summation tones are more difficult to hear because they lie in the region of frequencies already occupied by harmonics of the original sounds. 

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17 Helmholtz, op. cit., p. 153.
"The masking of high-pitch tones by low is greater than of low tones by high. This may account, in part at least, for the fact that difference tones were discovered (1714) before summation tones (1856) and are easier to hear."\textsuperscript{19} "Summation tones may form extremely inharmonic intervals with the generators. Luckily (for our enjoyment of music) they are weak."\textsuperscript{20}

\textsuperscript{19} Jones, \textit{op. cit.}, p. 259.
\textsuperscript{20} Helmholtz, \textit{op. cit.}, p. 156.
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<td><strong>Harmonium</strong></td>
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<td><strong>Pipe organ</strong></td>
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<tr>
<td><strong>Piano</strong></td>
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<td><strong>Violin</strong></td>
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<td><strong>Cello (visual)</strong></td>
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<td><strong>Trombone &amp; hum (may be sum)</strong></td>
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<td><strong>Soprano &amp; Flute</strong></td>
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<tr>
<td><strong>2 Sopranos in 3rds</strong></td>
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<td><strong>Male Voices</strong></td>
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<td>Romieu 1742</td>
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<td>Helmholtz 1858-85</td>
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<td>Sustained</td>
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<td>Pure</td>
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<td>In same air mass (adjacent)</td>
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<td>In bass staff (DT)</td>
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<td>700-2000 (DT)</td>
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<tr>
<td>3000 (DT)</td>
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<tr>
<td>High (generators)</td>
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<tr>
<td>Hi. Freq. Gen. inaud.</td>
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<tr>
<td>Almost any 5th</td>
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<tr>
<td>Small interval</td>
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<tr>
<td>Just intonation</td>
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<tr>
<td>Tempered interval &quot;horrible bass&quot; (Sound DT first, (Sound &quot;near&quot; gen., (Add 2nd gen.</td>
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<td>Non-linear) theory</td>
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<tr>
<td>Asinico-theory</td>
</tr>
</tbody>
</table>
DIFFERENCE TONES

Difference tones have been variously called terzi suoni, Tartini tones, differential tones, resultant tones, beat tones, and "advantages impurities." The tones which cause them have been called primaries, generators, score, or scored tones.

A comprehensive grasp of difference tones may be had by consulting the tables on pages 17 and 18. These tables show sources, favorable conditions, and theories of difference tones. Also the names, in chronological order, of the men who studies them. It is easily seen that there is wide disagreement between contemporaries and also between early and later authorities. Not all writers comment on all points, so there are many blank spots. The greatest divergence lies in the beat, asymmetry, and asinic theories; and in the subjective versus the objective character of difference tones.

According to the older theorists, Chaldni, Lagrange, Young, etc., the explanation of difference tones presented no particular difficulty. As the generators separated in pitch, the beats quicken and at last become too rapid for appreciation as such, passing into a difference tone, whose frequency is continuous with the frequency of the beats. This view of the matter, which has commended itself to many writers, was rejected by Helmholtz, as inconsistent with Ohm's law, and that physicist (Helmholtz) has elaborated an alternative theory, according to which the failure

21 See table 1, p. 17.
is not in Ohm's law, but in the principle of superposition.22

Lloyd lists Helmholtz's objections to Young's beat-tone theory.

(1) It does not explain summation tones:
(2) Two loud notes (of different frequencies) on the siren excite a membrane tuned to the difference tone frequency; Rücker and Edser showed the same result with forks:
(3) The ear hears motion in the air as a tone of a particular frequency only if a perfectly simple vibration of that frequency actually exists in the air (Ohm's law).

Lloyd continues:

Helmholtz concluded that asymmetry of the ear would produce the effect of combination tones, as distinguished from rapid beating. Sometimes loud asymmetrical tone sensation is louder than an objective combination tone in the air.23

Raleigh cites counter arguments:

Hermann asserts that he cannot hear summation tones nor find any one able to do so; and he regards this difficulty as a serious objection to Helmholtz's theory, according to which the summation and difference tones should be about equally strong. Koenig believes that a summation tone may in reality be a difference tone derived from the upper partials of the generators. This cannot be arithmetically disputed.

But, as Raleigh goes on to show, difference tones are more easily explicable upon the basis of Helmholtz's theory:

Upon the whole this theory seems to afford the best explanation of the facts thus far (1894) considered, but it presupposes a more ready departure from superposition of vibrations within the ear than would have been expected.24

22 Raleigh, op. cit., p. 248.
24 Raleigh, loc. cit.
John Tyndall ties up his objections to the beat tone theory with interference:

This (Young's) explanation harmonized with the fact that the number of beats, like the vibrations of the resultant tone, is equal to the difference between the two sets of vibration. But beats are louder than any continuous (resultant tone) sound. Beats can be heard after the generators cease. This depends in part upon the sense of hearing, but it also depends upon the fact that when two notes of the same intensity produce beats, the amplitude of the vibrations of the air particles is at times destroyed, and at times doubled. But by doubling the amplitude we quadruple the intensity. Hence, when two notes of the same intensity produce beats, the sound incessantly varies between silence and a tone of four times the intensity of either of the interring tones....If, therefore, the resultant tones were due to the beats of their primaries, they ought to be heard, even when the primaries are feeble. But they are not heard.25

The Encyclopaedia Britannica, 14th edition, enters this objection to the beat tone theory: "the roughness of beats and the difference tone can sometimes be heard together, which seems to indicate that they act in a different way on the cochlea." It answers the counter charges of Hermann with this statement:

It is argued that if the ear works on the principle of resonance they (beat and combination tones) ought not to be heard. This would be true if the conducting mechanism of the ear were entirely free from distortion, and if the cochlea were a perfect resonator. It is probable that the ear fulfils neither of these conditions.26

26 Encyclopaedia Britannica, loc. cit.
Lastly, we cite the keen, modern version of Stevens and Davis, seconded by Culver:

Since beats and difference tones occur at the same frequency, people have sometimes tended to confuse them, and to speak of the difference tone as though it were merely the tone perceived as a result of rapid beats. Actually, however, beats and difference tones have essentially nothing to do with one another. They are produced by two entirely different principles. If the ear were a more sharply tuned analyzer, we should hear no beats, but we should still hear difference tones. On the other hand, if the ear were a linear system, producing no distortion, we should hear no difference tones, but we should still hear beats. As it is, we hear both beats and difference tones simultaneously. 27

After considering the beat and asymmetry theories we come to still another one. Jack Cotton advances a new theory for the generation of combination tones. He calls it the ASINIC THEORY.

A group of rhythmic units forms a periodic wave which we may call an asinic wave if there is no sine wave with frequency corresponding to that of the rhythmic unit...It is assumed that periodic asinic sound waves are just as obvious to the ear as to the eye, and may arouse a sensation of pitch corresponding to the asinic wave frequency quite independently of any sine wave of corresponding frequency.

This explanation of combination tones would be untenable if the ear were known to act merely as a harmonic analyzer. However, it is generally recognized that so simple a picture of the ear's function as that of the "piano" theory, for example, is probably inadequate.

The phenomena observed may be explained just as well, perhaps, by the subjective tone theory, but such an explanation involves the participa-

27 Stevens and Davis, op. cit., p. 244.
tion to a much higher order terms in the nonlinear response equation than has been heretofore assumed.\textsuperscript{28}

Such thoroughly accredited acousticians as Helmholtz, Raleigh, Rücker and Edser, Waetzman, Barton, Redfield, Bragg, and Jeans have demonstrated the OBJECTIVE existence of difference tones. However, in examining their reports closely, we invariably find that ingenious devices, sensitive apparatus, and "certain" conditions are necessary to prove the existence of this very elusive objective phenomenon.

When consulting the "sources" table, page 17, the reader will find three items with the word visual in parenthesis. Rüker, Edser, and Waetzman "attach great importance to the fact that their existence is known independently of the mechanism of the human ear."\textsuperscript{29}

The present writer stumbled onto another example of visual evidence while double-stopping on the cello. Certain intervals bowed on the A and D strings caused sometime the G, and sometime the C string to vibrate with considerable visible amplitude. However, the writer heard no tone to correspond with the difference tone frequency.

\textsuperscript{28} J.C. Cotton, "Beats and Combination Tones at Intervals Between the Unison and the Octave," \textit{Acoustical Society of America Journal}, VII (1935) p. 44.

\textsuperscript{29} E.H. Barton, \textit{A Textbook on Sound}, p. 404.
All of which leads us to reaffirm Peterson's statement made in 1908, that objective tones are weak.

We, therefore, leave objective difference tones without regret for a more promising phenomenon: the subjective difference tone.
Chapter IV

Subjective Difference Tones

We come now to the core of this thesis: SUBJECTIVE DIFFERENCE TONES.\(^1\) As a proof that difference tones are usually subjective we have many statements to the effect that they have no objective basis and that there is no evidence of them in the wave form. Culver says:

Nerve impulses reaching the brain centers contain disturbances which have no objective genesis....The data thus far available (1941) appears to indicate that no wave motion corresponding to the combination tone frequency exists in the transmitting medium.\(^2\)

Colby arrives at his conclusion that combination tones are subjective thus:

The experimental evidence at hand leads to the conclusion that, with intense primary sounds and sensitive means of detection, combination tones produced in the air can be observed. But it is found that under favorable circumstances combination tones can be heard when the primary sounds are comparatively weak. This leads to the conclusion that combination tones which we hear are mostly generated in the ear.\(^3\)

Wood suggests a test for determining the character of combination tones:

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\(^1\) In studying the relative importance of objective and subjective difference tones, the writer collected about 450 words on objective tones and about 3800 words on subjective tones.

\(^2\) Culver, op. cit., p. 47, 51.

\(^3\) Colby, op. cit., p. 186.
Frequently the combination tone can be heard without a resonator (placed to the ear) and is not strengthened when the resonator is used. In this case the combination tone must be produced inside the ear itself.  

Subjective tones, sometimes called "pseudo" tones or non-linearity tones, are generally classified as follows:

(1) Combination, difference, and summation tones from pure primaries;
(2) Fundamental from adjacent middle partials;
(3) Upper partials from fundamental (harmonics);
(4) Upper partials from adjacent middle partials.

Class one above has been discussed throughout this paper and classes three and four are described under aural harmonics, p. 14. Class two, which may be treated as difference tones, seems to warrant most attention of seven authors.

From Fletcher's experiments (1924) with the pitch of complex tones, it appears that the fundamental (when filtered out) is supplied in the form of a subjective difference tone:

The Bell Telephone Company produced two sets of records (discs). They sound much alike, speech and music being perfectly intelligible, although all the fundamental tones are missing in one set, except in the form of difference tones created by the ear of the listener.  

Miller points out an example that should have occurred to all of us:

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4 Wood, op. cit., p. 85.
5 Wood, loc. cit.
For the lower sounds (violin G string) the fundamental is weak as indeed it must be, since these tones are lower than the fundamental resonance of the body of the violin. The ear perceives a fundamental in the lower tones of the violin, and this must result from a beat-tone produced by adjacent higher partials which are strong.6

Another application of class two has been with us for a long time: "An example...is found in the small loud speaker (6" or less) of a radio whose lowest frequency is about c-262, yet we hear some bass notes."7 On the subject of the bass supplied by small radios we have the vociferous, dissenting voice of McLachlan:

In the early days of broadcasting (before and after World War I) it was erroneously argued by some, and unfortunately put into practice, that as difference tones were created subjectively, there was no necessity to transmit the bass register at strength. The fallacy is readily discerned, for, in listening to an orchestra, the ear is under the influence not only of subjective difference tones, but of the actual objective lower tones emitted by the various instruments.8

Fig. 4

In the diagram above the overtones emitted by the

6 Miller, op. cit., p. 197.
7 Culver, op. cit., pp. 51-2.
8 N.W. McLachlan, Sound and Music, p. 10.
organ pipe are harmonics of the fundamental frequency (512, 768, 1024) and hence successive overtones differing in frequency by an amount equal to the frequency of the fundamental. This fulfills the condition for subjective difference tones. Although the objective fundamental has been filtered out, the ear supplies it subjectively.9

Wood reminds us that "the small horn of the early gramophone was a high-pass filter." To those of us who heard it, it evidently did not pass enough adjacent partials to provide a satisfactory bass.

Wood also makes the important point that the fundamental in NOT produced by INharmonic partials. "Drum music and applause don't sound natural over small radios because inharmonic partials will not combine to produce the fundamental frequency." The ear is not a rectifier in this case.10,11

9 Colby, op. cit., p. 187.
10 Wood, loc. cit.
11 This "unnatural" effect is a common experience. However, if inharmonic partials will not combine to form a fundamental, how can we account for Helmholtz's "horrible bass," of which he complains throughout his consonance-dissonance theory? Also, in my demonstration with slide whistle and fife (p.48), when the fife pitch remained fixed and the continuous pitch of the slide whistle was made to slowly descend from a unison, a CONTINUOUS difference tone was heard by many observers to commence at lowest audible pitch and gradually rise in pitch to be lost somewhere between the two primaries.

![Diagram of harmonic relationships](image-url)
As to the effect of subjective tones on QUALITY, we have only generalizations. Miller assures us that "the subjective partials (classes 3 and 4) have great influence on the tone quality on many instrumental and vocal sounds as perceived by the ear. This influence has never been fully appreciated." Redfield adds: "Quality is much influenced by subjective beat-tones." Jeans says: "The brain may hear tones of pitches which were entirely lacking in music as originally played." Culver says: "The original tone may be materially changed." And Pepinsky calls our attention to the "difference between what the composer writes and what the listener actually hears in performance."

These statements indicate that our acousticians of the immediate past have been nibbling at the idea of quality as related to subjective tones. It obviously remains for future research to clear this up.

On the question of who hears subjective tones, Culver tells us that "the ear may add partials to a pure tone, dependent upon the particular listener," thereby inferring

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12 Miller, op. cit., p. 184.
14 Jeans, op. cit., p. 231.
15 Culver, op. cit., p. 51.
16 Pepinsky, loc. cit.
the factor of individual differences.\textsuperscript{17} Jeans speculates on which subjective tones are heard: "There is no means of telling, from general principles alone, which of this vast array of frequencies represent \textit{audible} sounds, and which are unimportant."\textsuperscript{18} Perhaps modern research methods and equipment may remedy this situation. I believe this sobering thought from Jeans is quite necessary because some acousticians are "carried away" with their enthusiasm. Helmholtz and Redfield blithely score pages of theoretical difference tones, forgetting that most of them are probably experienced by few ears.

In formulating a theory of the cause of subjective tones, writers beginning with Helmholtz use the terms asymmetry and non-linearity. Sometimes they are used jointly, sometimes singly, but implying generally a similar connotation. For me, asymmetry is a physiological, and non-linearity a mathematical term.

In order to thoroughly understand asymmetry and non-linearity we will first consider the condition of SYMMETRY and LINEARITY.

If a vibratory body is displaced, it exerts a restoring force. It may be said that there is symmetry, if the magnitude of this force is independent of the direction of the displacement and de-

\textsuperscript{17} Culver, \textit{op. cit.}, p. 48.
\textsuperscript{18} Jeans, \textit{op. cit.}, p. 234.
pendent only upon the amount. The condition of symmetry is not sufficient to cause a vibrator to give only that frequency which is impressed upon it. In addition, the restoring force must be proportional to the displacement. 19

Culver writes that "the displacement of the diaphragm is strictly proportional to the excitation...and...the ear gives a linear response to low intensity." 20 Wood quotes Helmholtz as saying that there is a "linear response for free vibrating systems for small amplitudes, even when acted on by two forces." 21

Although Helmholtz, Barton, Bragg, Stewart, McLachlan, Colby, Jones, Culver, and Wood have written many pages on the subject of asymmetry and non-linearity, thoroughgoing statements by Jeans, and Trimmer and Firestone should suffice:

The ear is endowed with a property which the eye does not possess, namely, that of creating waves of entirely new frequency out of the disturbances which fall upon it. Because of this, the brain may hear tones of pitches which were entirely lacking in the music as originally played. ...A very slight knowledge of the ear-drum and its attachments tells us that it does not possess symmetry. On one side there is a complicated mechanism of bone which transmits the motion of the ear-drum to the brain. When the ear-drum moves in an outward direction, there is nothing but its own elasticity to check its motion and pull it back; when it moves inwards, its motion is further impeded by this bony structure. ...Sum and difference tones occur when the sound-curve is transmitted by any unsymmetrical structure whatever. 22

22 Jeans, op. cit., p. 231.
Non-linearity is explained by Trimmer and Firestone:

According to the theory of aural non-linearity, the phase effect and the exceptions to Ohm's law are both attributed to a single feature of the hearing organs, namely, their non-linearity. This characteristic is conveniently described on the assumption that the external sound pressure wave, which comes to the ear drum, if it is large in amplitude, is distorted as it proceeds toward the nerve endings which actually send off the auditory message to the brain. This distortion, or non-linearity, may be attributed to various elements of the middle ear or inner ear structures. It is usually assumed that the displacement at the nerve endings causes a linear response; that is, that all the distortion occurs before the wave reaches the nerve end-organs. This assumption of non-linearity gives immediate explanation of pitch perceptions contrary to Ohm's law.\(^{23}\)

Wever, Bray, and Lawrence reveal that asymmetry is not found in the ear drum and ossicles:

The results (of the experiments with normal and operated ears of quinea piga) do not support the assumptions made by Helmholtz and others that there is a significant degree of nonlinearity and asymmetry in the movements of the drum and ossicles. Distortion in these structures is not excluded but it is shown to be unimportant in relation to the distortion that appears beyond the stapes. We consider the most probable source of distortion to be the processes in the inner ear through which mechanical vibrations are transformed into electrical effects.\(^{24}\)


In another experiment they found that:

Transformation, the process by which a portion of the energy in a stimulus is thrown into overtones and combination tones, takes place almost certainly in the cochlea, and most probably in the hair cells of the organ of Corti. 25

Jeans has insisted that most transducers are non-linear at some time or other. Here Colby considers the air as such:

Combination tones exist in the air surrounding two sources of sound of different frequencies, if the air is non-linear. Air is non-linear, but whether it is sufficiently non-linear to produce audible combination tones at ordinary intensities is doubtful. Non-linearity of the air increases with pressure variation. 26


26 Colby, op. cit., p. 185.
Chapter V

Uses of Beats

The phenomenon of beats has several practical applications. One is the two-toned or double whistle. The interval between the two pitches ranges from a semi-tone to a minor third. Mine, a Boy Scout whistle, is a whole step, Bb to C.

Another non-musical application is described by Wood:

A device for the detection of gas in mines consists of two identical whistles. One is blown with pure compressed air from a cylinder, the other by air of the mine. When mine air becomes contaminated with lighter-than-air gases, the frequency of the second whistle rises, creating beats with the tone of the first whistle. The faster the beats, the greater the contamination.¹

Beats are also used in acoustical research, in what is called the "beat-beat" method of probing for aural harmonics. This method measures both the intensity and frequency of subjective tones. An auxiliary tone is introduced at a close frequency and allowed to beat with the aural harmonic. Beats are strongest when the intensity of the beating tones is equal, hence, "beat-beat." If beats are three per second, the tone we are searching for is three cycles per second from the exploring frequency.

Musical applications of beats are the organ tremolo stops, and tuning the unison, fifth etc. Two organ stops

¹ Wood, op. cit., p. 20.
using the beat principle of mistuned unisons are the "voix celest" (string) and "unda maris" (flute). Culver mentions the eliminating of beats to tune a unison. As most musicians know, the beats become slower as a unison is approached. Associated with this idea is the beat theory of dissonance. Helmholtz, with his usual thoroughness, produced a definitive piece of work on this subject. Some of the simpler applications are tuning the octave and the fifth.

Even with simple tones imperfect octaves can be distinguished from perfect ones, by the beats produced by the former. If beats are due to overtones they affect the octave tone; if beats are due to difference tones they affect the lower tone and they tend to disappear as the sound dies away. The frequency of the beats is the same for overtones and difference tones. When the sounds are much reduced, the mistuning fails to make itself apparent.

mistuned) (301
5th) (200) - imperfect 8va. 200
101

101

99 (2nd order d.t.)

beat tone - 2 c.p.s.

Fig. 6

Beats also serve to distinguish the imperfect from the justly intoned fifth, even in the case of two simple tones. (See Fig. 6 above) Beats of the disturbed fifth are more difficult to hear than those of the disturbed octave. Redfield cites the familiar use of beats by piano tuners:

2 Jeans, op. cit., p. 46.
3 Raleigh, op. cit., p. 462.
Tuners do not depend upon their sense of pitch except in the upper part of the keyboard; even the best sense of pitch is more too inaccurate for tuning purposes. For the proper tuning of the piano or organ dependence is placed upon beats. ⁴

⁴ Redfield, op. cit., p. 61.
Chapter VI

Uses of Difference Tones

We shall discuss four non-musical, and four musical uses of difference tones.

(1) Jack Quinn used the twin-oscillator to supply pure tones of precise frequency for tonal memory records. He filtered out sums and harmonics and had pure difference tones left for putting on the discs.\(^1\)

(2) Some audiometers, for the testing of hearing, make use of the same principle.\(^2\)

(3) Telephone diaphragm. A good modern telephone is designed to transmit frequencies from 300 - 2400. The main frequencies of both male and female voices lie below this range. The telephone transmits chiefly harmonics and

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\(^1\) Jack Quinn, "Experiment and Research on the Production of Tonal Memory Records."

\(^2\) The twin-oscillator of beat-frequency oscillator for auditory research consists essentially of an electrical organization composed of two circuits so designed that each generates a high frequency alternating current of the order of 100,000 cycles. One of the generators operates at a fixed frequency, while the other is variable. If and when the output frequencies of these two alternating current generators is fed into a suitable common electrical network, a third alternating current comes into being, and this third current will have a frequency which equals the difference between the two primary frequencies. Thus by varying the frequency of one of the generators, one may secure a beat note of any desired frequency between, say, 0 - 40,000 c.p.s.; and this beat tone closely approximates a pure tone. (Culver, \textit{op. cit.}, p.55)
out of these the ear-drum of the listener re-
constructs the main tones as difference tones. 3

(4) Loud speakers in radio sets. Many radios cut
out frequencies below 250 (about middle c), yet
we hear the string basses, the brass basses, and
male voices with absolute clearness. All of
these sources are rich in harmonics, out of
which our ears create the missing fundamental
tones and lower harmonics as difference tones. 4

(5) Tartini made his pupils acquainted with difference
tones to be used as a guide to correct intonation in double
stopping. 5 The writer found one violin teacher who uses
this device with pupils who are having difficulty with
double stopping. His teacher told him of it.

(6) Acoustic bass. The pipe organ sometimes uses the
second and third harmonic of the desired tone. 6 It was
first used by Vogler (see appendix, p. 73).

(7) Mixture stop in old organs. In old organs
the mixture stop often served an entirely differ-
ent purpose from that of the modern organ, which
is a synthesis of tones to create a complex tone
of greater richness. The harmonics provided by
its shrill pipes combined to produce the funda-
mental as a combination tone, much as in the
"acoustic bass" of today. By using pipes of the
pitches to which the ear is most sensitive, the
organ builders of past days got rich full organ
tone out of an instrument smaller than an upright
piano. 7

5 Lloyd, op. cit., p. 39.
6 Jeans, loc. cit.
7 Ibid.
(8) Color. Difference Tones help to color the harmonic complex presented to the ear, and thus have their part in determining whether that complex shall be pleasing or displeasing. This is explained below by Helmholtz.

(9) Helmholtz's theory of consonance and dissonance has been studied and developed by Ellis (his translator), Norden, and Jeans. We may follow Helmholtz's discussion of the effect of combination tones on consonance and the implications for tempered and just intonation below.

In the major thirds the combination tones of the first order (produced by the prime tones) are merely doubles of the tones of the triad in deeper octaves.

\[ \begin{array}{c}
\text{\textit{Fig. 7}}
\end{array} \]

For the minor triad, on the other hand, the combination tones of the first order, which are easily (?) audible, begin to disturb the harmonious effect. They are not near enough indeed to beat, but they do not belong to the harmony. This disturbing action of the combination tones on the harmoniousness of minor triads is certainly too slight to give them the character of dissonances, but they produce a sensible increase of roughness, for all cases where just intonation is employed, that is, where the mathematical ratios of the intervals are preserved.

In the ordinary tempered intonation of our keyed instruments, the roughness due to the com-

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8 Redfield, op. cit., p. 208.
Very important implications of beats and difference tones for choral singing are found in the transcribed excerpts from Norden's article on "A New Theory of Untempered Music, with Special Reference to a Capella Music." See appendix B, pp. 74-5.

Combination tones is proportionably less marked (less contrast between major and minor?), because of the much greater roughness due to the imperfection of the consonances....

Modern (1885) harmonists are unwilling to acknowledge that the minor triad is less consonant than the major. They have probably made all their experiments with tempered instruments, on which, indeed, this distinction may perhaps be allowed to be a little doubtful. But on justly intoned instruments and with a moderately piercing quality of tone, the difference is very striking and cannot be denied. The old musicians too, who composed exclusively for the voice, and were consequently not driven to enfeeble consonances by temperament, show a most decided feeling for that difference. To this feeling I attribute the chief reason for their avoidance of a minor chord at the close.

For the rest of this discussion - of intervals wider than the octave, etc., - refer to Helmholtz's *Sensations of Tone*. 
Chapter VII

The Experiment

In designing the experiment, the writer attempted to choose materials and a method that would serve to investigate vocal difference tones and still be direct and simple to administer. (In handling vocal intervals there is danger of bogging down, because of insecurity of pitch).

It was decided to concentrate on the first order difference tone, because most acousticians agree that all other combination tones—summation tones and higher order difference tones—are weak, difficult to hear, and unimportant musically.¹ We could hardly do otherwise, since our observers used only the unaided ear. No filters, amplifiers, or other electronic devices were used.

¹ (Redfield, op. cit., p. 206)

Helmholtz and many of his followers attached considerable importance to resultant notes arising from partials and from resultant notes themselves. There is no doubt that such notes exist; but it is quite certainly a mistake to attach much importance to them. It is surprising that so keen a thinker as Helmholtz should completely have lost sight of their extreme weakness. In my opinion composers cannot safely ignore the resultant tones produced by the notes in the score, but may safely disregard the extremely weak resultant notes produced by partials and by resultant notes themselves. Difference tones form a harmonic matrix in which the score is embedded.
Since this was to be a study of difference tones as related to choral singing, voices were used to produce the generating of primary tones. In order to cover the normal vocal range all types of voices were used. Soprano, alto, tenor, baritone, and bass voices were paired in various sequences of diads, and in one instance were used in an 8-part sequence. To keep the procedure from becoming unwieldy, only diatonic intervals were used. This covered the human voice range reasonably well, without creating intonation difficulties for the singers, or fatigue in the observers.

The piano was used to keep the singers on pitch. In one instance, however (see 0 group alto-tenor, p. 52), an observer suggested that we proceed without the piano, so that the piano tone would not distract him. We did this, then, without getting off pitch. Just intonation would have been much more preferable, but was impossible without special planning, devices, and training of the singers.

The best singers - from the standpoint of all-around quality, poise, experience, pitch, etc. - were always persuaded to perform the diads, to insure reasonably accurate performance. The singers in the F group were rehearsed briefly and privately in pairs, because this was the first group the writer worked with and he wasn't sure how well they would perform such an unmusical sequence of sounds (not unlike the performance of a piano tuner). The combined voices
used for the O group diads were rehearsed briefly in the D sequence. (see p. 52)

The writer was fortunate in acquiring 86 individual observers in four groups, who varied considerably in age, and in academic and musical ability.² No two groups submitted to exactly the same tests. To compensate for this diversity the results are scored as percentages along with the total number of responses. (p. 57)

The F group consisted of a mixed chorus class of 26 boys and girls, grades 7-10, in Fairview School, a small township school in central Indiana. Their only other similar experience in listening was in taking the revised Seashore Measures of Musical Talent. They were the most musical - instrumental and vocal - pupils in the school. They had recently done a very creditable job with Cadman's operetta, "Meet Arizona," as singers, dancers, and orchestra players, and also with the varied musical activities of the school year.

The R group consisted of 12 girls, grades 7 - 10, of Raleigh School, another small rural school of central Indi-

² Seashore, C.E., "Illusions of Hearing," U. of Iowa Studies

Seashore states: "In certain experiments in physics in which the recording of observations depends upon hearing, the observer must be certified first as to his capacity for the hearing of the attributes under consideration." Since the techniques and equipment of the Bell Laboratories (see p. 35 for the "best-beat" method for measuring frequency and intensity of subjective tones) were not available, we had to rely upon the unchecked subjective response of the observers. This, however, was valuable in that we got data which heretofore did not exist, whereby we could compare the answers of a number of observers.
ana. They had taken the Seashore test and had had two years experience singing two-part choruses.

The O group consisted of 24 volunteers from the Ohio State University Summer Chorus. The full chorus of 85 high school, college, and graduate students rehearsed 10 hours weekly for 5 weeks and appeared in 3 campus concerts under as many conductors, in a wide range of choral works. They classified themselves as follows: 3 choral and instrumental directors, 2 choral directors, 6 graduate students, 1 graduate student not in music, 5 college, 3 high school students, and 7 unclassified.

The K group consisted of 24 members of the King Avenue Methodist Choir, Columbus, O. Thirty minutes for the experiment were taken from the first part of a regular Thursday night rehearsal. This is a volunteer city choir capable of regularly contributing two (medium to difficult) anthems, introits, and responses, to the Sunday service 10 months a year with one weekly rehearsal. By their own classification they were: 1 research chemist, 1 X-ray technician, 3 graduate, 4 college, 2 high school students, and 8 unclassified.

The only conditions of the experiment not discussed elsewhere are probably the general attitude of the subjects and the acoustical properties of the rooms.

The attitude of the singers and observers generally was cooperative. The singers quickly learned what was wanted of
them and eagerly did their best to produce it. The observers were quiet and attentive, and seemed generally interested, though sometimes puzzled, in the problem of hearing a tone of which most of them had been unaware until the demonstration. The "descending" tenor for the K group got the giggles while doing his stint, possibly due to the unusual character of the sequence. However, we repeated the affected diads and went on with the experiment without seeming damage.

Acoustically, the four rooms used in the experiment were those in which the groups held their regular rehearsals. Generally they could all be described as on the "live" side, but not objectionably so. All had high ceilings. In detail they were as follows.

For the F group a medium sized class room was used. Almost half of one wall consisted of tall closed windows, another wall was almost covered by a glassed-in bookcase, and the remaining two consisted of varnished wood, blackboard, and plaster. It had a softwood floor and flat painted plaster ceiling on wooden lath.

The R group room was about one-third the size of the F room, and had only 3 windows, one of which was half open. Excepting the blackboard area, the walls and ceiling were of flat painted plaster.

The O group used a large rehearsal room with a barrel
plaster ceiling, and concave vaulting from ceiling to the half-walls. The walls are deeply niched for access to the many dormer windows, and have a varnished wood wainscot four feet high. The floor is of polished hardwood.

The K group used the rear corner (opposite the pulpit and choir tiers) section of the church. The partition between this rear room and the main auditorium was open. It has a hardwood floor, plaster walls up to a shallow gallery over which a plaster ceiling arches.

No modern acoustical devices such as microphone, loud speaker, amplifier, or recording, was used. Since it is hard to see just how any of these devices would have helped, and there was a possibility of loss or distortion with their use, it was decided to dispense with them. We used the "bare" voice on the "bare" ear.

In scoring the diads for the use of pianist and singers, most of the sequences began with a unison. Then one voice repeated the first tone while the other voice descended diatonically for 12 or 15 intervals. (See pp. 50, 52, 54 for scoring used with the various groups) The observers did not see any of this notation, but it would not have mattered much if they had because they were listening for unscored tones.

The response sheets (which see, pp. 51, 55) were design-
to provide a convenient method of checking responses by the observers, and questions were added to bring out additional information regarding the observers' experience with difference tones, etc.

At the actual experiment, after the response sheets and pencils were distributed, the observers were asked to read the description of difference tones at the head of the sheet. This was followed by as much verbal explanation as seemed necessary.

The purpose of the demonstration was to give the observers a first hand experience with difference tones. All the instruments used in the demonstration produced loud, and fairly pure primes. This resulted in difference tones eas-

---

3 On the subject of listening techniques we glean certain hints from Helmholtz, (mostly on his observation of upper partials, pp. 49-52).

(1) The observer must not be distracted by extraneous noise.
(2) The attention of the observer is gradually drawn to the phenomenon to be observed.
(3) Aids are used until he knows what to look for, then they may be discarded.
(4) Success depends upon mastery over attention.
(5) A musically trained ear may not hear better than an untrained one, but has the advantage of being able to imagine how the tone sought should sound.
(6) Our ear does not ordinarily analyze a tonal complex into its components because such a minute analysis would serve no useful purpose.
ily heard by most of the observers. (See D group response to the demonstration, p. 56) The F and R groups were given only the double-whistle demonstration. This was possibly a beat rather than a difference tone, but the difference between the two was not explained, to avoid confusion. The O and K groups were given four different demonstrations as listed on the response sheet (p.55) and the tabulation for K group (p.56). The difference tones generated by the four different generators may be described as a deep boom with the double-whistle; a lighter tone with the Pan-pipes, but as strong as the generators; a still lighter but more musical tone from the fifes; and from the fife and slide whistle a continuous glissando from lowest audible beat-tone up to somewhere between the generating tones. (See footnote 11, p. 28).

The observers were asked to check their own individual responses. No odium was connected with a "NO". They were instructed to believe that there existed no correct answer but their own. We wanted to know if they heard something besides the two primes or not. We wanted a true subjective response.

When the briefing was over, the first diad was sounded on the piano for the singers; they intoned it with the vowel being used, ff, while the writer signaled a duration of 4 slow beats. An interval of time was allowed between
diads for the observers to mark their responses on the blank. Once started, the sequence was continued without interruption until a change of voices was necessary.
Table 3
Intervals used in Difference Tone Experiment - Groups F & R
With Responses of Observers

Sopranos

Yes 6 27 28 28 20 21 26 20 26 24 23 27
No 28 7 6 5 14 13 8 14 8 10 11 7
Blank 1

Altos

Yes 8 11 15 13 16 16 12 10 12 15 14 15
No 14 11 7 9 6 6 9 12 10 7 8 7
Blank 1

Tenors

Yes 14 10 17 13 15 12 7 14
No 8 12 5 9 7 10 15 8

Basses

Yes 6 19 19 19 19 18 21 11 20 16 16 19
No 18 5 5 5 5 6 3 13 4 8 8 5

Group F heard all above except C 1-4 (1 tenor lacked range)
Group R heard only A series.
FAIRVIEW SCHOOL

Experiment in

Hearing of Difference Tones

A difference tone is a tone caused by two other tones.
Example: A two-toned whistle generates a third tone.
The third tone (difference tone) is always lower than
the upper generating tone.
Scientists have proved that a difference tone caused by
an instrument (e.g. organ) can be heard by the human ear.
We want to know if those caused by voices are heard.

A 1 2 3 4 5 6 7 8 9 10 11 12
Yes
No

B 1 2 3 4 5 6 7 8 9 10 11 12
Yes
No

C 1 2 3 4 5 6 7 8 9 10 11 12
Yes
No

D 1 2 3 4 5 6 7 8 9 10 11 12
Yes
No
Table 4
Intervals used in Difference Tone Experiment - Group O - Group A
With Responses of Observers

Vowel "AH"
Yes  2  10  10  10  6  7  5  7  6  7  8  8  8  8  4
No  18  9  10  10  14  13  15  12  12  13  13  13  13  13  15
Blank  1  2  1  1  1  1  1  2  3  2  2  1

Vowel "AH"
Yes  13  14  9  7  11  4  8  10  7  8  6  10  8  6
No  9  9  13  15  11  19  14  10  14  14  16  11  13  15
Blank  1  1  1  1  2  1  1  1

Piano not used

Vowel "Ah"
Yes  4  8  6  5  7  7  10  1  4  10  9  8  5  7  10  6
No  12  9  11  1  1  9  9  6  14  13  6  8  9  12  10  6  11
Blank  1  1  1  1  2  1

Bass omitted 1st time
Bass sung 2nd time
Results of Experiments - Group 0

1. Have you ever been aware of difference tones before this demonstration?

"No - Perhaps subconsciously."

"Have heard, but could not name."

"Once with individual voice with large resonance cavities, similar to harmonic overtone."

"In string music, vocal and choral groups."

"From course in Acoustics."

Not aware - 18

2. What use have you made of them?

"To color and augment volume."

"To listen more."

None - 22

General Observation:

"The two sopranos singing together seemed to produce a physical disturbance in my ear, but it didn't seem to have pitch." (This observer heard no difference tones).

Table 5

Effect of Relative Positions of Singers on Difference Tones

<table>
<thead>
<tr>
<th>Facing</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>Not sure</th>
<th>Blank</th>
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</thead>
<tbody>
<tr>
<td>Face front</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Face to face</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Backs to aud.</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Back to back</td>
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<td>3</td>
<td>4</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Table 6**

Intervals used in Difference Tone Experiment - Group K - With Responses of Observers

![Music notation]

<table>
<thead>
<tr>
<th>Vowel &quot;Ah&quot;</th>
<th>Yes</th>
<th>4</th>
<th>11</th>
<th>13</th>
<th>14</th>
<th>7</th>
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<th>5</th>
<th>4</th>
<th>5</th>
<th>2</th>
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<tr>
<td>No</td>
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<td>10</td>
<td>7</td>
<td>6</td>
<td>14</td>
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<td>14</td>
<td>15</td>
<td>15</td>
<td>18</td>
<td>17</td>
<td>18</td>
<td></td>
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<tr>
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<td>1</td>
<td>1</td>
<td>2</td>
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</tbody>
</table>

![Music notation]

<table>
<thead>
<tr>
<th>Vowel &quot;Ah&quot;</th>
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<th>5</th>
<th>5</th>
<th>11</th>
<th>12</th>
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</tbody>
</table>
DIFFERENCE TONE EXPERIMENT

A Difference Tone is a tone caused by the interference of two other tones of different pitch. The 3rd tone (Difference Tone) is always lower than the upper generating tone. Scientists have proved that Difference Tones caused by an instrument, e.g., organ, are heard by the human ear. We want to know if those caused by voices are heard and if they have any practical value in Choral Singing.

Demonstration with:
Double-whistle, Pan-pipes, Fifes, Fife with Slide Whistle

In series A to C, check your response in the yes or no box to the question: Do you hear a Difference Tone?

A 1 2 3 4 5 6 7 8 : 9 10 11 12 13 14 15 16
Yes
No
B 1 2 3 4 5 6 7 8 : 9 10 11 12 13 14 15 16
Yes
No
C 1 2 3 4 5 6 7 8 : 9 10 11 12 13 14 15 16
Yes
No
D Is bass satisfactory in either the first or second chord sequence? Which is more satisfactory?

E Rate according to strength of Difference Tone; 1 for strongest, 4 for weakest: 1st 2nd 3rd 4th position

F Compare for more prominent difference tone: AH-AYE; AH-OH.

I Have you ever been aware of difference tones before this demonstration? If so, state circumstances on reverse side.

II What use have you made of them?

III Please state school rank: High School - College - Graduate - Choral Director - other
Table 7

1. Hearing of Difference Tones in Demonstration - Group K

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Yes</th>
<th>No</th>
<th>No Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double-whistle</td>
<td>21</td>
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<td>1</td>
</tr>
<tr>
<td>Pan-pipes</td>
<td>17</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Fifes</td>
<td>18</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Fife and Slide Whistle</td>
<td>14</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

3. Were you ever aware of difference tones before the demonstration?

Yes - 4        No - 8        No response - 7

Table 8

4. Compare for more prominent difference tone with the use of different vowels.

<table>
<thead>
<tr>
<th></th>
<th>AH Compared With</th>
<th>AYE</th>
<th>AH Compared With</th>
<th>OH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Choice</td>
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</tr>
<tr>
<td>Difference Noted</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>No Difference Noted</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Didn't Hear</td>
<td></td>
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<tr>
<td>No response</td>
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</tbody>
</table>

5. What use have you made of them? None - 4 No response - 15

6. Comment: None - 16

"I have endured the harsh ones, enjoyed the others." 
"Two-toned whistle, organ diads."
"On violin"
"Organ music, but I didn't know what they were."
"Band and orchestra experience."
Table 9

Tabulation of Yes - No Responses by Intervals

<table>
<thead>
<tr>
<th></th>
<th>SS*</th>
<th>AA*</th>
<th>TT*</th>
<th>BB*</th>
<th>SA*</th>
<th>TB*</th>
<th>AT*</th>
<th>Ttl % Yes</th>
<th>Ttl % No</th>
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<tbody>
<tr>
<td>Unison</td>
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<td>13</td>
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<td>6</td>
<td>6</td>
<td>38</td>
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<td>14</td>
<td>12</td>
<td>18</td>
<td>28</td>
<td>95</td>
<td>61</td>
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<tr>
<td>2nd</td>
<td>Yes</td>
<td>49</td>
<td>11</td>
<td>5.5</td>
<td>19</td>
<td>10.5</td>
<td>128</td>
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<td>35</td>
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<tr>
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<td>61</td>
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<td>3rd</td>
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<td>4th</td>
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<td>31</td>
<td>84</td>
<td>37</td>
</tr>
</tbody>
</table>

Blanks (no response) were divided equally between Yes and No.

The major 2nd and 3rd and the minor 6th and 7th in the Alto-Tenor sequence were not entered on this sheet because the intervals recorded above are minor 2nd and 3rd and major 6th and 7th.

*S-Soprano, A-Alto, T-Tenor, B-Bass
Chapter VIII

Interpretation

When contemplating p. 57, we find some expected, and some unexpected results.

Note the highly negative results with the unison, octave, and double octave: 77, 63, and 78% respectively. This is to be expected because; in the case of the unison no difference tone is possible (400-400= 0), with the octave the difference tone is probably masked by the lower prime of the same frequency (400-200= 200), and with the double octave the difference tone would be a 5th below the upper prime and an eleventh above the lower prime (400-100= 300). This tone at 300 is very hard to hear because it has been observed that the difference tone becomes progressively weaker as it climbs higher toward the upper prime after crossing the lower prime (See slide whistle diagram, p.28).

We find a good average affirmative response (65%) on the third, and an even better one with the female voices (68%). (See table 1, p. 17, for sopranos in 3rds.). We are surprised that the fifth did not get a more positive affirmative response. However, the general conviction that the fifth produces a "honey" of a difference tone (Sorge, Vögler, and many modern observers) is borne out in the baritone responses of 19 to 5 (79%). All the baritone responses except the unison and octave are highly affirmative. It is a
pity that these results are somewhat suspect because of the small number of samples. This should be investigated further.

The "D" sequence sung for the 0 group was intended to determine the practicality of strengthening a weak bass with a proper choice of intervals in the chord above. It is hard to believe that the results prove anything conclusively. The test is inherently defective because it is impossible to compare a weak bass when singing alone with one singing with the whole choir.

Although the above test was inconclusive, we experienced a quite positive experience during a rehearsal of the Ohio State Summer Chorus. The selection "up" was Gretchaninoff's "Holy Radiant Light", arranged by Cain. The chorus came to page 5, 4th measure. (See score, p. 76). Mr. Diercks, the director, thought this would be an ideal place to try to strengthen a weak bass, because there was just one person present who could touch the scored low C#. He asked for a C# octave with the basses, then asked for a G natural from the tenors. This diminished 5th (not as scored) produced an electrifying rough reinforcement of the Sva bass C#. Later when the male voices sang the parts as scored, there was noted a reinforcement of the low bass, but it was less pronounced and smoother. This sensation as described was noticed by the direction and several members of the chorus.
The "E" test with the O group was to determine what effect the relative positions of the two singers had on the strength of the difference tone produced. Two sopranos sang the minor third e-g in the treble staff with AH in four different facing positions. They touched shoulders as in normal choir spacing and didn't vary the distance when turning. They attempted to maintain a uniform intensity throughout. Although the sampling is small, the results indicate that the face-to-face and the backs-to-audience positions produced more prominent difference tones than did the back-to-back and the front facing.

Two tests with the K group, which were not run with any other group, deserve mention. The demonstration of difference tones was checked for validity by asking the observers to check on their blanks if they heard difference tones. (The demonstration is described in detail on p. 47). The table on p. 56 shows the most positive results of any test in the entire experiment. It is interesting to note this in connection with the answers to the question directly below on the same page. Only four had previously been aware of difference tones but the majority heard them in the demonstration after briefing. It is admitted that these difference tones from loud, pure primes are hard to miss, and they are much easier to hear than those caused by complex tones, especially voices.
In the number 4 item - comparison of vowel effect-, the answers are so widely dispersed that they are hardly worth notice.
Chapter IX

Questionnaire

A questionnaire on difference tones was sent to ten choral authorities. Those who showed their interest by answering were:

Wilfred C. Bain
Dean, School of Music
Indiana University
Bloomington, Ind.

Max T. Krone
Professor of Music
University of Southern California
Los Angeles, Calif.

Robert Shaw
Director of Choral Music
Juilliard School of Music
New York, N.Y.

Thompson Stone, Conductor
The Handel and Haydn Choral Society
Boston, Mass.

A. A. Wihtol
Arranger-composer
Kama Music Publishers
Glendale 6, Calif.

Peter Wilhousky
Choral Supervisor
New York City Schools

John Finley Williamson, President
Westminster Choir College
Princeton, N.J.
I am addressing this inquiry to a small group of outstanding choral authorities in the United States. With your valuable assistance, I hope to throw some light on the neglected subject of Difference Tones as related to Choral Music.

According to many acousticians, a difference tone results when the frequencies of two strong generating tones differ 20 or more per second. We no longer hear a single, average, beating tone, but we hear the separate generating tones plus a beat (?) tone or difference tone. Its frequency is the difference between the frequencies of the generators.

Examples of difference tones of the first order (most easily heard) as scored by Helmholtz. Numerals indicate ratios of frequencies; quarter notes are difference tones.

![Fig. 8](image)

Observers report that the most favorable conditions for hearing difference tones are to have the generators loud, sustained, pure, sources adjacent and agitating the same air mass; one says the best range for audibility of difference tones is 700 - 2000 , another prefers the bass staff range.
They have reported audible difference tones generated by tones from tuning forks, siren, singing flames, double-whistle, Panpipe, fifes, violin, organ, harmonium, concerto, piano, trumpets in fanfare, two sopranos in thirds, soprano and flute, and male voices.

Two musical applications are (1) faking a 32 foot tone by sounding organ pipes of approximately 16 and 10 2/3 feet, and (2) using difference tones as a basis for judging "pure" intonation of double-stops on the violin (Tartini). Helmholtz and N.L. Norden assure us that choral works of the classical period are much more effective when sung in just intonation which is based on consideration of the difference tones produced.

Although I beg permission to mention your name in my thesis, I assure you that your replies to the following questions will remain anonymous, to avoid any unpleasant controversy.

Will you be so kind as to reply to these questions by the middle of July (the old academic imperative, you know!)?

1 Have you ever been aware of difference tones, without consciously using them as a choral device?

2 Do you use difference tones to augment a weak bass?

3 What other uses have you made of difference tones?

4 Do you feel that difference tones are too unimportant to be considered?

5 Relate other experiences with difference tones, (conditions, etc.).

6 Would you like a summary of the results of this inquiry?

Gratefully yours,

Maurice Worland

Graduate Adviser
RESULTS OF QUESTIONNAIRE TO CHORAL AUTHORITIES

1. Have you ever been aware of difference tones, without consciously using them as a choral device?

   Yes - 4

2. Do you use difference tones to augment a weak bass?

   No - 4   Yes - 1

3. What other uses have you made of difference tones?

   None - 4

4. Do you feel that difference tones are too unimportant to be considered?

   "They cannot be controlled."   "Who knows."   "No."

   "In most situations, yes. However, a study of difference tones might result in better choral recordings."

   "No, and I am more and more interested in their use in choral writing and choral performance."

5. Relate other experiences with difference tones, conditions, etc.   No response - 2

   "When by a reputable composer, I feel the effect is intentional."

   "With male voices. More likely with straight tone (no vibrato)."

   "In testing two sopranos singing in thirds before a mike, I was surprised to learn how prominent the difference tones became in the upper range. It would have been most disturbing in a broadcast. (This, I
observed from the control room."

"Obviously different rooms possessing different acoustics give varying emphasis to these tones."

Paraphrase and quotations from a letter written in answer chiefly to question 4 - the importance of difference tones.

(1) The classical composers up to Beethoven's time, wrote delicate and ornate music to be performed in a small, ("dead"), room (chamber music). It was for a small audience which was expected to be capable of "Intensive or intricate listening." This audience could appreciate "minute musical effects and secondary sounds."

The composers and performers of the Romantic period made music for a large audience in a large (reverberant) hall. They abandoned the delicate classical technique (pianists used the pedal excessively) and "proved that the public is not interested in the delicate fine shades of the classical period." We are concerned with stirring the larger emotions of the modern listener, not with his finer sensibilities.

(2) With reference to a capella singing:

Fortunately we can keep the composition in pitch, especially if the composition is written by one of the classical writers, but ... the various parts in their relation to each other are constantly going either sharp or flat. That being the case your theory of "difference tones" would be of lit-
The practical value.

(3) More than 90% of choral people cannot read the score. That is, they cannot properly "hear" the true choral effect simply by scanning the notation.

What happens is this; a composer writes a choral composition, it is printed and a copy is sent to the prospective customer, then he takes it to the piano and plays it and judges according to the sound he hears on the piano. Now we know that this sound is frequently completely erroneous, because by sympathetic vibrations, tones and harmonies are produced that will not be there when the composition is used by human voices....It is also true that the sustained tones by the voices are not maintained in similar manner by the piano and thus there is a gross misrepresentation in the reading of the music. It seems to me that your work is of value in this respect.
Chapter X

Conclusions

From a consideration of the evidence gathered from previous writers and from the findings of the present writer's experiments we may conclude that:

(1) Difference tones are mostly subjective.

(2) Individual differences in the hearing of difference tones are as widely divergent as would be expected in dealing with a subjective phenomenon.¹

(3) Most persons are unaware of difference tones.

(4) The average person can be made aware of difference tones from pure primaries. A course of ear training to hear difference tones seems feasible. It should progress from pure primes to complex.

(5) For some persons, difference tones are easier to

¹ McLachlan has an interesting point to make about individual differences in hearing: Although (as far as the author is aware) there are no published results showing the relationship between pressure and displacement of the ear drum, it is probable that as 'no two persons are alike,' no two aural characteristics are alike. Moreover, the subjective tones will vary according to the individual, which taken in conjunction with other effects means that musical or sound sensation is a purely personal matter, being different for each individual. (McLachlan, Noise, p.23)
hears than are harmonics. 2

(6) Most persons have difficulty in hearing difference tones from complex primaries.

(7) Under certain conditions (still unknown), difference tones are heard which are strong enough to change the tonal complex considerably.

2 Seashore states that "most ears do not sense upper partials." (Seashore, C.E., "Illusions of Hearing," University of Iowa Studies, Series No. 157, (June 15, 1928) p.5.)
Chapter XI

Recommendations

Further study of choral difference tones is indicated as follows:

(1) The effect of vowel changes.
(2) The position of singers in relation to each other.
(3) The effect of different types of room acoustics.
(4) Justly intoned diads. It might be possible to rig an apparatus, consisting of oscillators, stroboscope, and ear phones, that will transmit just intervals to the singers who can then relay them to the observers. A recording of the sung intervals might be checked by stroboscope for accuracy. Other methods might be to have just intervals recorded on a disc for ea-phoning to the singers, or use a piano retuned to just intonation in one key.

(5) The Bell Telephone Laboratories are carrying out a most thorough plan of research in the human voice field. It will be wise to scrutinize all their work for further development along the lines of this thesis. This development will certainly be reported in the Acoustical Society of America Journal.
APPENDIX A

(A Transcription)


Seventeen books make statements about the discovery, not including general physics texts. Tartini's Trattato di Musica was published in Padua in 1754. On pages 13 and 14 of this work Tartini tells of terzi suoni (third sounds, difference tones), and gives a number of illustrative cases. He makes no statement as to who was the discoverer of these sounds, but says simply 'There was then discovered a new harmonic phenomenon.' In most of the illustrations, as is well known, he gives the pitch of the third sound an octave too high.

In view of the error of an octave which Tartini frequently made in the pitches of difference tones, it is interesting to notice that early in the first chapter of a second publication, De Principi dell' Armonica Musicale, Padua, 1767, he points out that two tones which are an octave apart are not as easy to distinguish as tones which form other accords.

In the second chapter of De Principi (p.36) he says that since foreign authors have claimed the discovery for their nations, he feels constrained to publish the facts about the discovery.

In the year 1714, when a youth of about 22 years, he accidentally discovered this phenomenon on a violin in Ancona, where not a few who remember the fact are still living. He immediately began communicating the discovery to professors of the violin without reserve or mystery. He made it the fundamental rule of perfect tuning for the boys in the school which he opened in 1728 at Padua, and which still exists. And thus the knowledge of the phenomenon spread throughout Europe.

Sorge published the first part of his Vorgemach der musicalischen Composition, in Lobenstein in 1745. On p.13 is a brief statement dealing with difference tones, but nothing is said of the discovery.
If one has tuned a fifth, e.g., $\tilde{c}$ $\tilde{g}$ perfectly on an organ, the $c$ will also be very faintly audible; which is noticeable too when tuning the major third of the Sesquialter; which consists of a fifth and third. Indeed, even two Flutes Douce produce, when the $\tilde{c}$ and $\tilde{a}$ are blown in tune together, still the third tone, namely an $f$, which stands as a test.

The Anweisung zur Stimmung, Hamburg, 1744, is a small book in the form of a dialogue between a student and his teacher. The student asks why it is that when two notes are tuned to an interval of a fifth there can be heard along with them a third sound an octave below the lower tone of the fifth. The answer is naive and entertaining as to be worth quoting:

Nature plays her lovely game in this matter and shows that the 1 is still lacking in 2 - 3 and that she would like to have such a sound present along with the others, so that the order of 1 - 2 - 3, i.e., $c$ $\tilde{c}$ $\tilde{g}$ be complete.......And not only the Fifths do this, but also the Thirds, for if one puts a Major Third in a set of organ-works in tune, a third, deeper sound is likewise audible at the same time, for the reason that Nature wants to have 3, 2, and 1 with 5 - 4. Therefore when one has tuned 5 - 4 - 3 or $\tilde{a}$ $\tilde{g}$ $\tilde{c}$ the fourth tone announces itself, too, namely c with the 2; indeed, if one pays close attention, I even suppose the fifth tone will be heard with the 1, because Nature endures no vacuum.

I turn lastly to Romieu. His paper, 'New Discovery of Low Harmonics whose resonance is very easily discerned in the chords of wind instruments,' appears on page 77 of a volume entitled Assemblee Publique de la Societe Royale des Sciences held in the grand salon of the town hall of Montpellier, December, 1751. He describes the discovery in these words:

Wishing to tune a little organ pipe with the instrument called tone, and having stopped both of them up to make them sound together, I was surprised to hear independent of their own two sounds, a third deep and very audible sound. By several experiments repeated a long time after the observation of the deep sound made above eight or nine months ago, and which I communi-
cated to the meeting on April 29, 1751, I found that it was always the common and opposite harmonic of the two sounds which produced it; so that it had for the number of its vibration the largest common divisor terms of their agreement.¹ ²

From this statement it appears to be clear that Romieu was an independent discoverer of difference tones, and that he made the discovery about the year 1742 or 1743. On taking account of all the facts here available it appears that Tartini was probably the first to discover difference tones, and that Romieu discovered them independently some years later. In the three of Sorge's works that I have examined I have found no evidence that he thought of himself as a discoverer of difference tones, or that he regarded these tones as having anything like the importance that Tartini attached to them. Even if Sorge did make the discovery independently it is probably that he made it later than Tartini, for Sorge was born in 1703 and consequently was only seven years old when Tartini made the discovery.

Appendix

In connection with the discovery of difference tones, it is of interest to note that Romieu attributed them to the blending of beats when sufficiently rapid - a hypothesis which was later adopted by Thomas Young, Joseph Louis Lagrange and Rudolph Koenig. Romieu was doubtless the first to suggest this hypothesis.

It is also of interest to know that the modern use of a difference tone from two pedal pipes in an organ to produce a lower tone was proposed about 1800. Abt Vogler says that by this means it is possible to save 'space, tin, and above all wind.' He tells of a case in the cathedral in Schleswig where he employed a 16' pipe and its fifth to produce a 32' tone. This was done in the presence of various organists, 'and all of these skilled music-connoisseurs who had listened to (or heard of) my mixture of registers from afar, had to concede that the latter (aforementioned) experiment with my artificial 32' tone had the merit of strength and distinctness.' He gave a similar exhibition on an organ in Magdeburg.

¹ Romieu's French passage translated by courtesy of F.L. Preston, Instructor, Romance Language Dept., Ohio State University.
² I think this volume is a rare one, in the Library of the University of Montpellier.
APPENDIX B

(A Transcription)


One of the worst fallacies in present-day music is the endeavor to teach a capella music with the assistance of a piano in equal temperament. After a number of rehearsals with piano, the conductor omits the instrumental accompaniment - and the chorus is in a state approaching chaos. The singers have been taught false intervals, but are supposed to sing pure ones, for all a capella music must be sung in just intonation. As we shall show, no singer can sing a capella in any temperament.

Singers cannot sing tempered tones because in performing they do not have time to count the beats as does the piano or organ tuner. The guide for the just interval is the absence of any beats. Constant association with temperament may cause him to reach for the distorted interval without premeditation, but he can never consciously judge how sharp or flat to sing the pitch.

Singers (in the early centuries) were taught accurate intervals by means of the monochord.

While the major 3rd is correct in MEAN-TONE intonation, the 5th and minor 3rd are each 6 cents flat, so that, if this temperament were revived for teaching a capella music, the intervals would still be far from correct, excepting the major 3rds.

Now, when an interval is true, these prominent DIFFERENCE TONES are in sympathy with it, particularly in the major mode. But when the interval is distorted, all the difference tones are sharp or flat of the proper tones, and a "buzz" of conflicting tones results.

On the piano, the dissonant difference tones are not so bad because the sound (primaries) dies quickly. But with the sustained tones of the organ, or orchestra, or a capella singing it is very noticeable. Bach said equal temperament was best for the piano - but not for organ. In fact, although the modern organ is tuned in equal temperament, the mixture stops are in just intonation.

When a chorus sings in tune in just intonation, the intervals of the women's voices create difference tones that support the men's voices, and the lower overtones of the men's voices re-inforce the women's tones (with harmonics and summation tones), more, however, in the major than in the minor mode, because the true bass of a minor chord is not present in the chord (as a difference tone), whereas in the major chord it is.
We have taught intervals as though they were made of isolated tones; they are not. Every interval carries with it its satellites, which consist of difference tones and summation tones. Therefore, intervals shade off from the most unified to the most dissonant by almost imperceptible degrees. We cannot classify them.....into perfect consonances, imperfect consonances, and dissonances.

It is not difficult to tune a one-manual or a two-manual reed organ, if one wishes to teach a chorus pure intonation for a capella singing. A one-manual organ will suffice to teach all important intervals and will play in one key (in both its major and minor forms). A two-manual organ has the advantages of being able to modulate and to add "altered" chords to the diatonic ones. Regular practice with such an instrument will improve a chorus remarkably and will give it a feeling for the correct pitches. Such practice will not impair its ability to perform accompanied music, for, when an accompaniment is played, the chorus will tune to it. (But when thrown on its own resources, it will immediately sing in untempered intonation again - after sufficient association with this intonation.

About the only time many of the matters discussed in this article receive serious attention is when an announcement of a new-type keyboard appears. But music, fortunately, is not limited by the keyboard. All the problems here outlined are present all the time in all non-keyboard music. A keyboard containing the entire range of tones needed for correct intonation will very likely come in due course, but a further modification of the present equally-tempered system (by introducing quarter-tones, third-tones, etc.) will fail to solve the problems here pointed out.

It is high time that musicians and developing students should learn the facts of musical theory in such a manner that they will find them useful at all times. They should realize what these facts actually are, and what is to be done with them when they are applied to equal temperament - which is but one way, and not the best, of spacing the rungs of the musical ladder.
Appendix C
Holy Radiant Light

Octavo No. 8081
Alexandre Gretchaninoff
Ed. and arr. Noble Cain

G. Schirmer, Inc., copyright 1937
## Appendix D

### The Human Hearing Mechanism

<table>
<thead>
<tr>
<th>Part of ear</th>
<th>Structure</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer ear</td>
<td>Pinna, canal, ear drum, 3/100 inch thick</td>
<td>Gathers and transmits sound to drum. Drum vibrates to variations in air pressure.</td>
</tr>
<tr>
<td>Middle ear</td>
<td>Ossicles; hammer, anvil, stirrup (stapes)</td>
<td>Ossicles pass vibrations of drum to oval window into inner ear. Lever system decreases size of movements but increases pressure variation 30 to 60 times.</td>
</tr>
<tr>
<td>Inner ear</td>
<td>Cochlea; resembles snail shell, has spiral cavity which contains basilar and tectorial membranes, organs of Corti.</td>
<td>Hearing is specialized form of touch. 24,000 fibers, sensitive to pitch variations, stimulate hair cells (pitch and quality). Tectorial membrane detects dynamic changes (?) Organs of Corti probably responsible for distortion (subjective tones).</td>
</tr>
</tbody>
</table>
Fig. 9 - Diagrammatic section through right ear. R, pinna; G, auditory meatus; T, tympanum; P, chain of bones; O, oval window; S, cochlea; Vt, scala vestibuli; Pt, scala tympani; B, semicircular canals; E, eustachian tube; r, round window.

Fig. 10 - Cochlea in transverse section.

Fig. 11 - Cori's Organ.
23 15th Avenue,
Columbus, Ohio
June 3, 1948

Educational Department,
Bell Telephone Laboratories, Inc.,
463 West Street,
New York City.

Gentlemen:

I am a graduate student in music at Ohio State University, and am working on the thesis problem: "A Study of Difference Tones as Related to Choral Singing."

I am registered in Physics 645 here: "Acoustics for Students of Music and Speech; an elementary, non-mathematical treatment of acoustics with applications to music."

I would greatly appreciate any reports, papers, or monographs prepared by your organization on the general subject of combination tones, and especially on difference tones generated by human voices. If you know of materials on this subject from other sources, kindly list them for me. In order to be of use in my work, I would have to have this material within thirty days.

Sincerely yours,

Maurice Worland

Graduate Advisor
June 16, 1948

MR. MAURICE WORLAND

23 15th Avenue
Columbus, Ohio

Dear Mr. Worland:

In reply to your letter of June 3rd, requesting information on musical tones, we are pleased to refer you to the following articles which are probably available in your college library:


You may also find further information, bearing on the subject of your inquiry, in other issues of the Journal of the Acoustical Society of America.

Very truly yours,

F.L. HUNT
Publication Department
June 28, 1948

23 15th Avenue
Columbus 10, Ohio

Mr. F.L. Hunt
Publication Department
Murray Hill Laboratory
Murray Hill, N.J.

Dear Mr. Hunt:

In reply to your letter of June 16th, I wish to point out to you that my inquiry of June 3rd specified material published by the Bell Laboratories on DIFFERENCE TONES, not musical tones in general.

I have followed your suggestion of consulting the A.S.A. Journal, and have found valuable material, but I had hoped that you and your staff might direct me to material on the specific subject of DIFFERENCE TONES.

Sincerely yours,

Maurice Worland
BIBLIOGRAPHY

This bibliography is restricted to difference tone material and to that of related phenomena. No work is listed that does not have a bearing on this subject. The writer found The Industrial Arts Index (1913-1947), published in annual volumes by H. W. Wilson Co., N.Y. to be a most valuable reference work.


Bell Telephone Laboratories, Inc., Technical Reports, Reprints, Papers. This material is much more accessible in the ASAJ.


Bragg, W., "Combination Tones in Sound and Light," Engineering, CXLVI (Nov. 18, 1938) p. 593.


Encyclopaedia Britannica, 14th Edition.


Peterson, J., Combination Tones and Other Related Auditory Phenomena, doctor's dissertation, Department of Psychology, University of Chicago, 1908.


