Essays on Macroeconomics: Structural Analysis of Fiscal Policies and Jobless Recoveries

Dissertation

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By

Zhen Cui, M.A.

Graduate Program in Economics

The Ohio State University

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Dissertation Committee:
Paul Evans, Co-Advisor
Pok-Sang Lam, Co-Advisor
Byoung Hoon Seok
Lucia Dunn
Abstract

My research studies how fiscal policies affect an economy and uncovers the cause behind the “Jobless Recovery.”

The first essay “Consumption Response to Tax Cuts in Life-Cycle Economies” studies the aggregate consumption response to a tax cut in a life-cycle economy. I construct a general equilibrium model where households have a finite lifespan and smooth consumption using one asset. The asset is associated with a stochastic adjustment cost. The model replicates the lifetime mean net worth profile observed in the data. The implied aggregate marginal propensity to consume (MPC) out of the tax cut ranges from 14% to 43%. In contrast to the frictionless version, this model generates a substantial rise in aggregate consumption under the tax cut without needing a large aggregate MPC. The model also indicates the quantitative importance of how a tax cut is financed.

The second essay “A Model of the Consumption Response to Government Expenditures” develops a structural model to examine the effect of an increase in government spending. The model has finitely-lived households who smooth consumption using two assets. The first asset is a low return free-to-adjust asset lent by the households to the government; the second asset is a high return costly-to-adjust asset used as capital by a representative firm. Working-age households supply a fixed amount of labor to the firm. The government raises taxes to finance its spending and maintains
a balanced budget. Prices are perfectly flexible and adjust to clear all markets. The model generates an interesting result: the rise in government spending crowds out investment and drives down total output. Therefore, this paper shows that a rise in government spending can lead to a recession in a general equilibrium model with capital, flexible prices, and a fixed labor supply. The result is reversed if I endogenize labor supply by allowing households to choose their hours of work. Specifically, consumption drops substantially, causing labor supply to rise on impact. Since capital is predetermined, total output increases.

The third essay “Jobless Recoveries and Sectoral Skill-Biased Structural Change” studies the slow job market recovery in the post-1990 U.S. I document four stylized facts. (1) The U.S. job market has taken significantly longer to recover after each post-1990 recession. (2) The recovery of goods sector employment was slow after each post-1990 recession. (3) The educational attainment of service sector workers has surpassed that of goods sector workers since 1990. (4) The skill premium of workers with college-plus education has increased faster in the service sector than in the goods sector. I argue that Facts (3) and (4) indicate a post-1990 skill-biased structural change (SBSC) in the service sector. The SBSC has prevented unemployed workers in the goods sector from relocating to the service sector, causing a sluggish job market recovery at the aggregate and goods sector level. I then develop a model where SBSC is integrated via a labor adjustment cost and a reallocation shock. The simulation results show that the model successfully accounts for Facts (1) and (2).
To my family and friends
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Vita

June 2004 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Shanghai ShiXi High School

August to December 2006 . . . . . . . . . . . . . . . . . Intern,
Silicon Valley Bank - Europe

May to August 2007 . . . . . . . . . . . . . . . . . . . . . . . Summer Analyst,
Western Reserve Partners, LLP

May 2008 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . B.A. Economics & Mathematics,
Colgate University

July 2008 to June 2009 . . . . . . . . . . . . . . . . . Research Assistant,
Child Trends, Inc.

September 2009 . . . . . . . . . . . . . . . . . . . . . . . M.A. Economics,
The Ohio State University

September 2010 to present . . . . . . . . . . . . . . . Graduate Teaching Associate,
Department of Economics,
The Ohio State University

Fields of Study

Major Field: Economics
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Chapter 1: Consumption Response to Tax Cuts in Life-Cycle Economies

1.1 Introduction

Fiscal stimulus plans such as tax cuts have become a popular tool used by the U.S. government to fight recessions. For example, during the 2001 recession, the U.S. Congress passed two tax relief acts: the Economic Growth and Tax Relief Reconciliation Act (EGTRRA) of 2001 and the Jobs and Growth Tax Relief Reconciliation Act (JGTRRA) of 2003. For the Great Recession, President Obama proposed the ‘Making work pay’ tax break in 2009 and pushed back the expiration date of the payroll tax cut in 2011.

There is an abundant collection of empirical studies examining the consumption response to these stimulus plans. Shapiro and Slemrod (2003) use the Survey of Consumers and two separate follow-up surveys to see whether the 2001 EGTRRA tax rebates stimulated spending. The study concludes that about twenty percent of households said they would spend most of the tax rebates. Shapiro and Slemrod (2009) use the Survey of Consumers again and find that the aggregate propensity to spend from the 2008 rebates was about one third. Based on the Consumer Expenditure Survey (CEX), Misra and Surico (2011) discover that the consumption response to the 2001 income tax rebates is significantly associated with income, and
to a lesser extent, with liquidity and age. Parker, Souleles and McClelland (2011, hereafter PSM) also rely on CEX to measure how households’ spending responded to the mid-2008 stimulus payments. Their results show that households spent 12% of the stimulus payments on non-durables in the first three months. Older and lower-income households have a more prominent response to the stimulus payments. In summary, the empirical studies suggest: (1) the aggregate MPC out of a stimulus plan ranges from 10% to 35%; (2) the consumption response to the stimulus plan varies with age, income, and asset liquidity.

However, very few studies use dynamic structural models to analyze the consumption response to a stimulus plan. Heathcote (2005) develops a general equilibrium (GE) model where households are infinitely lived and own a costless-to-adjust asset.\(^1\) Hence, the model fails to replicate a hump-shaped lifetime wealth distribution due to the missing age dimension. Kaplan and Violante (2014, hereafter KV) propose a life-cycle model within a two-asset framework: one low-return costless-to-adjust asset (i.e. liquid) and one high-return costly-to-adjust asset (i.e. illiquid). The two-asset framework creates ‘wealthy hand-to-mouth’ households, namely households who have little liquid asset but rich in illiquid wealth. The existence of these households helps the model generate a large MPC out of the actual receipt of tax rebates and a small MPC out of the news of future tax rebates. Unfortunately, the model is a partial equilibrium (PE) exercise and absent of the labor-leisure margin. Their solution method also requires a considerable amount of computational time and resources (see their Appendix E.3.5). Huntley and Michelangeli (2011) is another study using

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\(^1\)The Appendix in Heathcote (2005) presents an over-lapping-generation(OLG) version of the model. However, the OLG version assumes a partial equilibrium setting and perfectly altruistic households.
a two-asset model: one costless-to-adjust taxable asset and one costly-to-adjust tax-deferred asset. They focus on examining the MPC out of tax rebates among young and middle-aged households. The model also has a PE setting and fails to include households’ labor supply decisions.

This paper intends to fill the literature gap between the empirical research and the theoretical studies. Motivated by the theories and empirical facts above, I set up a dynamic structural model with three main features: a built-in lifecycle, heterogeneous households, and an asset adjustment cost. In particular, households have a finite lifespan and differ between each other in labor productivity and personal wealth. They make labor supply decisions and use a single asset to smooth their consumption. The asset is associated with a stochastic adjustment cost. This frictional one-asset setting is inspired by the two-asset framework. The single-asset setting can be considered as an environment where borrowing is allowed against one’s illiquid wealth. However, the adjustment cost ensures that such borrowing is not perfect or completely frictionless as in the traditional one-asset model. It creates a layer of illiquidity to the asset. The computational time and required memory space are both lowered substantially. Lastly, the model also accounts for the effect of price changes by incorporating a GE framework. A fiscal stimulus plan undoubtedly affects both households and firms. Those effects will be reflected through the adjustment of prices, such as wage and interest rate. The price changes will in turn affect the households and firms’ behaviors again. Thus, it is important to study the aggregate consumption response to a stimulus plan under a GE setting.

I now briefly describe the main results of this paper. First, the proposed model successfully replicates the hump-shaped wealth distribution observed in the data thanks
to the built-in lifecycle and household heterogeneity. The empirical studies cited earlier have suggested that consumption responses vary among households of different ages, income, and wealth (Misra and Surico 2011; PSM 2011). Therefore, the success of reproducing a close-to-data wealth distribution is indispensable.

Second, I subject the model economy to a temporary tax cut and examine its transitional dynamics. Assuming that the government maintains a balanced budget, I let the tax cut be financed (1) via government expenditures and (2) through transfer payments. The two experiments suggest that the model can generate an aggregate MPC ranging from 14% to 43%, consistent with the empirical estimates of 10%-35%.

The experiments also indicate that the financing decision of the tax cut is quantitatively important in a dynamic life-cycle economy. When funded through transfer payments, the tax cut drags the economy into a prolonged recession. The recession disappears if the tax cut is financed using government spending.

Last, the adjustment cost enables the model to generate a much larger increase in aggregate consumption under the transfer-financed tax cut. Even though the corresponding MPC is much smaller, the rise in households’ disposable wealth is actually much bigger compared to the government-spending financing scheme. Specifically, the reduction in transfers prompts elder households to resort to their asset for consumption smoothing. Those households pay the adjustment cost and gain access to

\[ \text{The comparison must not be taken too literally. In the literature, MPC is estimated as the change in consumption divided by the change in disposable income (or the total amount of tax cut incurred). In this paper, MPC is calculated as the change in consumption divided by the change in disposable wealth. The change in disposable wealth is the sum of the change in disposable income and the change in disposable (or readily accessible) asset savings. The reason for the discrepancy is as follows. In addition to income, tax cuts in this paper affect the amount of savings households can access and convert into consumption due to the assumed adjustment cost. Specifically, the change in households’ income changes their ability and willingness to pay the adjustment cost and thus gain access to their savings. In the literature, households are assumed to always have access to their savings, indicating that tax cuts only affect income.} \]
their financial wealth. The same group of households choose not to do so when the tax cut is funded by government expenditures. This is because their income is intact in the latter case. The traditional frictionless model fails to generate this result.

The rest of the paper proceeds as follows. Section 2 lays out the model while Section 3 describes the parameterization. Section 4 first presents the steady state result. Then it conducts two experiments: one uses government spending to fund a tax cut and the other uses transfer payments. The transitional dynamics of the tax cut are compared and contrasted between the two experiments. Section 5 concludes.

1.2 Model

Demographics

The economy has a continuum of households indexed by their age \( j = 1, 2, \ldots, J \). Households retire at age \( J^ω \) and retirement lasts for \( J^r \) periods (i.e. \( J^ω + J^r = J \)). Hence, all households exit the economy at age \( J \) without uncertainty. Moreover, households who exit the economy are replaced with newborns, keeping the total population constant. Newborns enter the economy as workers with zero initial wealth.

Preferences

Households are forward-looking and have time-separable, expected logarithm utility in both consumption and leisure:

\[
\mathbb{E}_0 \left\{ \sum_{j=1}^{J^ω} \beta^{j-1} \left[ \log c_j + \psi \log(1 - n_j) \right] + \sum_{j=J^ω+1}^{J} \beta^{j-1} \log c_j \right\}, \quad \psi > 0
\]

where \( c_j \) and \( n_j \) denote the household’s non-durable consumption and hours of work at age \( j \), respectively. Households do not work after retirement (i.e. \( n_j = 0 \) for \( j = J^ω + 1, \ldots, J \)), so the leisure utility term drops out for retirees.
Earnings

At the beginning of every period, each working household draws an exogenous labor productivity, \( \epsilon \), which follows a Markov process. The household then chooses the number of hours it wants to work, \( n \), while taking wage as given. Thus, the household’s income can be represented as \((1 - \tau)w\epsilon n\), where \( w \) is the wage per efficiency unit of labor and \( \tau \) is the proportional labor income tax rate.

Each retired household receives a lump-sum tax-free government transfer, \( T_{j,\epsilon,\omega} \), at every period. Transfers grow linearly as retirees age in order to account for rising government-provided health care. Also, higher transfers are given to retirees who drew a higher labor productivity at their last working period.\(^3\)

Financial Assets

There is only one asset, denoted as \( a \), in the economy. The asset has a zero borrowing limit and earns a gross return of \( R = 1/q \). Also, the asset is associated with a disutility adjustment cost, \( \kappa \). More specifically, at the beginning of each period, every household has an i.i.d. draw of \( \kappa \) from the uniform distribution, \( U(0, \bar{\kappa}) \), where the lowerbound is 0 and the upperbound is \( \bar{\kappa} \). After seeing both of its \( \kappa \) and \( \epsilon \) draws, each household decides whether or not to adjust its asset holdings (i.e. deposit or withdraw). If the household decides to adjust the asset, it has to pay the fixed cost it has drawn. If the household decides not to adjust the asset, it consumes its current income, pays no fixed cost, and its current asset will earn a gross return of \( R \). Due to the positive adjustment cost, some households may have non-zero balances in their

\(^3\)It would have been more realistic to use \( \bar{\epsilon}_j \), which is the average \( \epsilon_j \) draws over the working lifetime. However, this approach is only feasible for a small number of \( \epsilon \) states, given the speed concern and limited computer memory. Therefore, similar to Huntley and Michelangeli (2011) and KV (2014), I approximate the transfers using \( \epsilon_j \omega \) instead.
accounts after they exit the economy. Those leftover balances will be redistributed to all the retirees in the form of government transfers.

**Household Problem**

I formulate the household problem in a recursive fashion. The state variables for a single household at each period are its current-period productivity draw, $\epsilon_j$, and its current asset holding, $a_j$. The value function of a household at age $j$ is $V_j(a_j, \epsilon_j)$ and depends on $V_j^0(a_j, \epsilon_j)$ and $V_j^1(a_j, \epsilon_j, \kappa_j)$, which are the value functions conditional on not adjusting and adjusting the asset.

During each working period, the household will choose not to adjust the asset if $V_j^0(a_j, \epsilon_j) \geq V_j^1(a_j, \epsilon_j, \kappa_j)$. In other words, it solves the following problem:

$$
V_j^0(a_j, \epsilon_j) = \max_{0 \leq n_j \leq 1} \left[ \log(c_j) + \psi \log(1 - n_j) \right] + \beta E_j V_{j+1}(a_{j+1}, \epsilon_{j+1})
$$

(1.1)

where:

$$
c_j = (1 - \tau)w\epsilon_j n_j
$$

$$
a_{j+1} = \frac{a_j}{q}
$$

The household will choose to adjust if $V_j^0(a_j, \epsilon_j) < V_j^1(a_j, \epsilon_j, \kappa_j)$. Then it solves:

$$
V_j^1(a_j, \epsilon_j, \kappa_j) = \max_{c_j, n_j, a_{j+1}} \left[ \log(c_j) + \psi \log(1 - n_j) \right] - \kappa_j + \beta E_j V_{j+1}(a_{j+1}, \epsilon_{j+1})
$$

(1.2)

subject to:

$$
c_j + qa_{j+1} \leq (1 - \tau)w\epsilon_j n_j + a_j
$$

$$
a_{j+1} \geq 0
$$

$$
c_j \geq 0
$$

$$
0 \leq n_j \leq 1.
$$
As a result, there exists a threshold adjustment cost, $\tilde{\kappa}_j$, which makes the household in each period indifferent between adjusting and not adjusting the asset. In particular, $V^0_j(a_j, \epsilon_j) = V^1_j(a_j, \epsilon_j, \tilde{\kappa}_j)$, where:

$$\tilde{\kappa}_j = \mathbb{E}_j(a_j, \epsilon_j) - V^0_j(a_j, \epsilon_j),$$

and

$$\mathbb{E}_j(a_j, \epsilon_j) = \max_{c_j, n_j, a_j+1} \left[ \log(c_j) + \psi \log(1 - n_j) \right] + \beta \mathbb{E}_j V_{j+1}(a_{j+1}, \epsilon_{j+1}).$$

The actual threshold is $\hat{\kappa}_j = \min \{ \bar{\kappa}, \max \{0, \tilde{\kappa}_j\} \}$, with $0 \leq \tilde{\kappa}_j \leq \bar{\kappa}$. Therefore, if the household draws a cost lower than $\hat{\kappa}_j$ this period, it will choose to adjust the asset. If the household draws a cost higher than $\hat{\kappa}_j$, it will choose not to adjust the asset.

Given the threshold adjustment cost, $V_j(a_j, \epsilon_j)$ can be computed as the following:

$$V_j(a_j, \epsilon_j) = \int_0^{\hat{\kappa}_j} V^1_j(a_j, \epsilon_j, \kappa) F(d\kappa) + \int_{\hat{\kappa}_j}^{\bar{\kappa}} V^0_j(a_j, \epsilon_j) F(d\kappa)$$

$$= \int_0^{\hat{\kappa}_j} [\mathbb{E}_j(a_j, \epsilon_j) - \kappa] F(d\kappa) + [1 - F(\hat{\kappa}_j)] V^0_j(a_j, \epsilon_j)$$

$$= F(\hat{\kappa}_j) \mathbb{E}_j(a_j, \epsilon_j) - \int_0^{\hat{\kappa}_j} \kappa F(d\kappa) + [1 - F(\hat{\kappa}_j)] V^0_j(a_j, \epsilon_j), \quad (1.3)$$

where $F(\kappa)$ is the c.d.f. at $\kappa$ of the uniform distribution $U(0, \bar{\kappa})$ and $F(d\kappa)$ is its p.d.f.

To solve the retiree’s problem, we set $n_j$ to zero, and replace $(1 - \tau) w c_j n_j$ with $\mathcal{T}_{j, \epsilon_j} \mu_j([a_j \times \epsilon_j])$ and $\mathbb{E}_j V_{j+1}(a_{j+1}, \epsilon_{j+1})$ with $V_{j+1}(a_{j+1}, \epsilon_{j+1})$.

**Government**

The government collects income taxes from workers and pays transfers to retirees. Government expenditure, $G$, is not valued by households. Thus, the government budget constraint can be written as:

$$\tau \sum_{j=1}^{j^w} w \epsilon_j n_j \mu_j([da_j \times d\epsilon_j]) = G + \sum_{j=J^w+1}^{j} \mathcal{T}_{j, \epsilon_j} \mu_j([da_j \times d\epsilon_j]), \quad (1.4)$$

where $\mu_j([a_j \times \epsilon_j])$ is the distribution of households of age $j$ over $a_j$ and $\epsilon_j$. 
Firm

The economy has a representative firm which hires $N$ in labor and borrows $K$ in capital from households. In return, it produces non-durable goods for households to consume. The production technology is standard Cobb-Douglas, $F(K, N) = K^\alpha N^{1-\alpha}$. Capital depreciates at rate $\delta$. The firm maximizes its profit by choosing $K$ and $N$ so that:

$$w = (1 - \alpha)(\frac{K}{N})^\alpha, \tag{1.5}$$

$$\frac{1}{q} = \alpha(\frac{K}{N})^{\alpha-1} + 1 - \delta. \tag{1.6}$$

General Equilibrium

A recursive competitive equilibrium consists of a set of functions,

$$(w, q, V^0_j, V^1_j, V_j, c_j, n_j, a_{j+1}, K, N, \Gamma),$$

that satisfies the following conditions.

1. **Individual Household Optimization:**
   
   Each household solves its own optimization problem by choosing consumption, hours of work, and next period’s asset holdings. In other words, $V^0_j, V^1_j$, and $V_j$ satisfy (2.1),(2.2), and (2.3), while $c_j, n_j$, and $a_{j+1}$ are the associated policy functions.

2. **Representative Firm Optimization:**
   
   The representative firm maximizes its profit by choosing $N$ and $K$ as shown in (2.5) and (2.6).
3. **Labor Market Clearing Condition:**

At equilibrium, the prevailing wage, \( w \), equates the amount of labor demanded by the firm to the amount of labor supplied by working-age households. Thus,

\[
N = \sum_{j=1}^{J} \int \epsilon_j n_j \mu_j (\{ da_j \times d\epsilon_j \}).
\]

4. **Capital Market Clearing Condition:**

The asset held by the households is treated as productive capital in the economy. At equilibrium, the prevailing gross return, \( R \), equates the amount of capital demanded by the firm to the amount of asset held by the households. Thus,

\[
K = q \sum_{j=1}^{J} \int a_j \mu_j (\{ da_j \times d\epsilon_j \}).
\]

5. **Goods Market Clearing Condition:**

The final good produced by the firm is used for its own capital investment and the consumption of households and government. Thus,

\[
K^\alpha N^{1-\alpha} = \sum_{j=1}^{J} \int c_j \mu_j (\{ da_j \times d\epsilon_j \}) + K' - (1-\delta)K + G
\]

where:

\[
K' = q \sum_{j=1}^{J} \int a_{j+1} \mu_j (\{ da_j \times d\epsilon_j \})
\]

\( G \) satisfies (2.4).

\(^4\)Please see Appendix A for derivation.
6. Law of Motion of the Distribution:

\( \Gamma \) is defined as follows. For all \( K \subset \mathcal{X} \),

\[
\mu_{j+1}(K, \epsilon_{j+1}) = \\
\int \left\{ \int \left[ 1 - F(\hat{\kappa}_j) \right] I_{a_{j+1} - a_{j}} d\pi(\epsilon_{j+1}|\epsilon_j) \mu_j([da_j \times d\epsilon_j]) \right\} \mu_j([da_j' \times d\epsilon_j])
\]

\[+ \int \left\{ \int \left[ F(\hat{\kappa}_j) I_{a_{j+1} = a_{j}} d\pi(\epsilon_{j+1}|\epsilon_j) \mu_j([da_j \times d\epsilon_j]) \right] \mu_j([da_j' \times d\epsilon_j]), \]

\( \forall j = 1, \ldots, J - 1. \)

Here, \( a_{j+1}^* \) solves (2.2), \( \pi(\epsilon_{j+1}|\epsilon_j) \) denotes the transitional probability from \( \epsilon_j \) to \( \epsilon_{j+1} \), and \( I \) is an indicator function which takes one if \( a_{j+1} \in K \) and zero otherwise.

Also, households who exit the economy this period are replaced with newborns.\(^5\)

Newborns enter the economy with zero initial wealth. Thus,

\[
\mu_1(0, \epsilon_1) = \int \left\{ \int d\pi(\epsilon_1|\epsilon_j) \mu_j([da_j \times d\epsilon_j]) \right\} \mu_j([da_j' \times d\epsilon_j]), \quad \text{where } j = J.
\]

1.3 Parameterization

1.3.1 Steady State

The model has an annual frequency. I assume that each household enters the economy as a worker at age 22, retires at age 60, and dies without uncertainty when it reaches age 78 (the U.S. life expectancy in 2011 is 78.6 years). Therefore, I set the

\(^5\)If the exiting household chooses to adjust its asset, it will deplete its wealth because no bequest motive is assumed.
lifespan of a household to 56 years (i.e. \( J = 56 \)). The household works for the first 37 years (i.e. \( J^\omega = 37 \)) and becomes a retiree for the last 19 years. The discount factor \( \beta \) is 0.968, targeting a four-percent annual real interest rate (i.e. \( R \equiv 1/q = 1.043 \)) at the steady state.

For working households, I approximate their idiosyncratic labor productivity process using the PSID family public data over the 1968-1993 period. In particular, I closely follow the estimation procedure outlined in Heathcote et al. (2010). I then use the Tauchen (1986) method to generate an AR(1) 5-state Markov chain based on the estimates I obtain from the PSID data (i.e. autocorrelation \( \rho_\epsilon = 0.752 \), variance of innovations \( \sigma^2_\epsilon = 0.006 \)). Moreover, the leisure preference parameter, \( \psi \), is set to 0.8, targeting an aggregate labor of 0.37 (i.e. \( N = 0.37 \)) at the steady state. The proportional income tax rate, \( \tau \), is 26% as reported in Kiefer et al. (2002, Table 5, prior-EGTRRA overall on wages).

For retired households, I adopt \( T_{j,\epsilon,j^\omega} = \chi \cdot \epsilon_{j^\omega} f(j) \) as the functional form to approximate the amount of transfers a retiree receives at age \( j \). Here, \( \chi \) is common across all retirees and set to 0.943 so that government spending is about 19% of total output (i.e. \( G/Y = 0.188 \)). To account for rising government-provided health care, \( f(j) \) is linearly increasing in age \( j \).

For technology parameters, I choose the capital production share and the depreciation rate (i.e. \( \alpha = 0.27 \) and \( \delta = 0.08 \)) to target the annual capital-to-output ratio of 2.2 and investment-to-capital ratio of 0.08, respectively.

Last but not least, the upper bound of the adjustment cost, \( \bar{\kappa} \), is set to 5.0. At this value, the total fraction of households who choose to adjust their assets is 28.6% (10% among workers and 66% among retirees).
1.3.2 Transition

To study the transitional dynamics, I assume the economy initially (i.e. date 0) was at its steady state and then a tax cut takes effect (i.e. date 1). My transitional economy is solved using the perfect foresight algorithm described in Guerrieri and Lorenzoni (2011, Appendix).

A tax cut is defined as a sequence of tax rates $\{\tilde{\tau}_t\}_{t=t^a}^{t^b}$, where $t^a$ denotes the first year of tax rate change and $t^b$ denotes the last year of the change. The cut is a relative 10% decrease in the income tax rate (i.e. $\tilde{\tau} = 23.4\%$). The length of the tax cut is one year. I consider two financing methods of the tax cut: a decrease in government spending and a reduction in transfer payments. Both experiments are exercises in balancing the government budget.

Table 1 lists all the parameter values.

1.4 Results

1.4.1 Steady State

I start my analysis by presenting two hump-shaped figures: average saving made by households over their lifetime and average wealth of households in each age group (Figures 1 and 2). The two figures show that households save more and more while working and their wealth starts to accumulate. There are three forces motivating this saving behavior. First, all households enter the economy with zero initial wealth. Thus, young households have to start saving immediately so as to smooth their future consumption in case they encounter a low productivity draw. Second, while households are working, their income is on average higher than the transfers they
will receive after they retire. Hence, young households still want to save more despite no income uncertainty during retirement. Last, the stochastic adjustment cost motivates households to save. Young households have a much longer time horizon. This indicates an uncertainty not only over future labor productivities but future adjustment costs. In the event of drawing a low productivity and a high adjustment cost, households may still have the option to withdraw money from their accounts. They can use part of their savings to pay for the cost.

Although households save more and more as a worker, they start to save less once they reach the retirement age. There are three forces motivating this dissaving behavior. First, because of the predetermined lifespan, older households realize that they are going to die soon. They no longer need to save more for the future. Second, all households in the economy are assumed to be non-altruistic, so they have no incentive to save and leave a bequest to their offspring. Last, the average income of retirees is lower than workers. Older households rely more on withdrawing from their savings to smooth consumption. Since households face the adjustment cost even in their last period of life, some of them may choose not to pay the cost and leave a positive balance behind. This is why the average wealth held by the oldest age cohort, as shown in Figure 2, is close to zero but still positive. The leftover wealth will be redistributed to retirees as transfers in the next period.

To sum up, the saving and dissaving behaviors described above are a direct result of the three important features of the model: a finite lifespan, idiosyncratic labor productivities, and an asset adjustment cost. The wealth distribution generated by the model replicates the trend shown in the data. Figure 3 compares the normalized wealth distribution in the model to the lifetime mean net worth computed from the
Survey of Consumer Finance (SCF) 2007. The model-generated average wealth tracks the data fairly well until the early retirement stage. The model assumes no bequest motive, so most households deplete their accounts before exiting the economy. In real life, people want to leave some of their wealth to the offspring. Also, real people do not know when they will die but the model assumes they do. Thus, we see a discrepancy between the wealth profile in the model and its empirical counterpart during the later years.

1.4.2 Transition

This section studies how the model economy responds to a relative ten percent drop in the income tax rate for a year before it reverts to the steady state rate. Given that the government maintains a balanced budget, the tax cut can be financed through a concurrent decrease either in government spending or retirees’ transfer payments. Both schemes are explored and their transitional dynamics are compared and contrasted.

My first focus is to compare the aggregate MPC between the two financing methods. While the transfer-financing method lowers retirees’ transfer income, the tax cut raises workers’ after-tax wage income. As a result, there is no wealth effect to a first approximation. Aggregate consumption rises because the higher wage makes working-age households work more (i.e. substitution effect). If the tax cut is funded

6I choose 2007 to avoid any large distortions to households’ wealth profile caused by the Great Recession.

7As footnoted in Introduction, MPC is computed as the change in consumption divided by the change in disposable wealth. The change in disposable wealth is the sum of the change in disposable income and the change in readily accessible asset savings. Since tax cuts in this paper affect the availability of income and savings, it is only appropriate for the calculation of MPC to account for both effects.
by government spending, retirees’ income is unaffected and workers’ income is augmented, leading to a wealth effect in addition to the substitution effect. Therefore, the government-financing method gives us an upper bound for the MPC and the transfer-financing method indicates a lower bound for the MPC.

Figures 4 and 5 show how much of the increase in total disposable wealth is spent on consumption until the economy returns to the steady state. The two figures suggest that the aggregate MPC on impact is roughly 43% in the first experiment and 14% in the second. In other words, a life-cycle one-asset model with a financial friction can generate an aggregate MPC of 14% - 43% under a reasonable parameterization.

My second focus is to compare the aggregate consumption responses between the two financing scenarios. Figure 6 presents the impulse responses of main variables under the two cases. It shows that the increase in aggregate consumption is much larger on impact under the transfer-financed tax cut. This result may seem counterintuitive, but it becomes clear once the graphic impulse response is expressed mathematically:

$$\frac{C_t - C^*}{C^*} = \frac{MPC_t \cdot (wealth_t - wealth^*)}{C^*},$$

where $C^*$ and $wealth^*$ denote their respective steady state values. As illustrated earlier, MPC is much smaller when the tax cut is funded by transfers than by government spending. Hence, the equation above implies that the change in disposable wealth should be much bigger under the transfer financing scenario. Figures 4 and 5 confirm the implication but raise the question of why.

Figure 7 compares the total consumption and adjustment of workers with retirees. We see a big jump in the percent of retirees who choose to adjust when the tax cut is financed via transfers. This is because retirees rely more on savings to smooth consumption than workers do. When transfer payments are reduced to fund the tax
cut, more retirees resort to their savings. Figure 6 also shows a large decrease in investment, indicating households are indeed withdrawing from their savings. More retirees adjust, and their disposable wealth rises when they adjust and thus gain access to savings. Meanwhile, both financing schemes affect the workers in the same way, lowering their income tax rate by ten percent. Figure 7 shows that the total consumption and adjustment of workers have identical impulse responses under the two schemes. As a result, the increase in disposable wealth is much larger under the transfer-financed tax cut.

The behavior above can be rationalized in an alternative way. Retirees know that tomorrow’s transfers will go back to its steady state level. If they adjust today instead of tomorrow to boost today’s consumption, their tomorrow’s consumption will not suffer substantially. Thus, the total retirees’ consumption slightly decreases after the tax cut ends. The initial consumption rise is short-lived. Particularly, the total consumption of retirees jumps up on impact and is then trailed by mild cuts for the next 20 periods. In a sense, the negative effect on consumption from reduced transfers is delayed. Households choose to spend the money now and spread out the inevitable cutdown of their consumption over the future.

To summarize the analysis above, this paper demonstrates that it is quantitatively possible to generate a big increase in aggregate consumption without needing a large MPC. The liquidity constraint induced by the adjustment cost is crucial to produce this result. Figure 8 compares the impulse responses under the same two financing schemes but without the adjustment cost. In other words, Figure 8 shows the transitions a standard frictionless life-cycle model would generate. The big consumption increase once observed under the transfer-financed tax cut disappears. This
is because the change in disposable wealth is no longer much different between the two cases: retirees have access to their savings every period. As a result, the aggregate consumption in both cases increase by less than 2% and the response is actually weaker under the transfer-financed tax cut.

The remainder of the section will briefly discuss the transitional dynamics of other variables under the two financing schemes. When the tax cut is funded via government spending, the extra income strengthens working-age households’ ability to save by making the adjustment cost more affordable. The increase in savings leads to an increase in investment. Also, since the tax cut is temporary, the substitution effect dominates and aggregate labor increases. Capital is predetermined on impact, so the rising labor supply drives the total output and real interest rate up and the real wage down. After the impact, capital first climbs up as a result of the rising investment and then reverts to its steady state. All variables return to their steady states within five years of the tax cut.

When the tax cut is funded via transfers, retirees resort to their savings for consumption smoothing. Thus, aggregate savings go down and investment decreases. Aggregate labor still increases on impact due to the strong substitution effect. Given the predetermined capital stock, the total output and real interest rate increase while the real wage declines. After the tax cut, aggregate labor quickly returns to its steady state. However, the initial drop in investment causes capital to fall before it gradually rises back. In stark contrast to the first scheme, the prolonged recovery of capital drags the economy into a recession after the small initial boom. It takes the entire economy twenty years to transition back to its steady state. The discrepancy of
the two transitional dynamics points out that the financing decision of a tax cut is quantitatively important.

Despite the different dynamics of capital, the real interest rate has two almost identical responses. The small magnitude of the real interest rate change corroborates Aiyagari (1994)’s claim that households’ ability to insure themselves by accumulating assets is a powerful mechanism.

1.5 Conclusion

The main results of this paper can be summarized as follows. First, the model generates a hump-shaped lifetime saving profile and replicates fairly well the net worth distribution observed in the data. Second, the aggregate MPC ranges from 14% to 43% in a life-cycle single-asset economy with a financial friction. Third, the financial friction is essential for the life-cycle single-asset model to generate a big increase in aggregate consumption in response to a tax cut. Last, the financing decision of a tax cut is quantitatively important in a dynamic life-cycle economy.
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<td>U.S. life expectancy is 78.6</td>
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<td>PSID 1968-1993</td>
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<td>common age transfer profile</td>
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Table 1.1: Parameter Values
Figure 1.1: Average Saving Made by Each Age Cohort

Figure 1.2: Average Wealth Held by Each Age Cohort

Figure 1.3: SCF2007 Mean Net Worth vs. Normalized Model Wealth
Figure 1.4: G-Financed: MPC = 43% on Impact

Figure 1.5: Transfer-Financed: MPC = 14% on Impact
Figure 1.6: Transitional Dynamics: Benchmark
Figure 1.7: Consumption & Adjustment: Workers vs. Retirees
Figure 1.8: Transitional Dynamics: No Adjustment Cost
Chapter 2: A Model of the Consumption Response to Government Expenditures

2.1 Introduction

Recent events, such as the enactment of the American Recovery and Reinvestment Act of 2009, have rekindled interest among economists in studying the impact of rising government spending on the aggregate economy. Moreover, the rise in government spending has long been an important feature in the U.S. economy (Figure 1). In the past seven decades, total government expenditures have increased from 10% of GDP in 1929 to 20% of GDP in 2012 (Figure 2). At the same time, a ten percentage points increase in total tax receipts as a fraction of GDP is observed because of the expanding government expenditures (Figure 3). In addition to the total tax increase, Figure 3 shows that the composition of tax revenue has also changed. Receipts from personal taxes have become the most important component of the total U.S. tax revenue, increasing from less than 20% of the total receipts in 1929 to around 50% of the total receipts in 2012.

There is an abundant collection of studies that develop general equilibrium models to examine the effects of government spending on an aggregate economy. Compared

\[8\text{According to the Recovery Act, the latest estimated spending for the Senate is $552 billion and $545 billion for the House.}\]
to Neoclassical models (Aiyagari, Christiano and Eichenbaum 1992; Baxter and King 1993; Burnside, Eichenbaum and Fisher 2004), increases in government expenditures typically generate more prominent increases in output in New Keynesian models (Gali, López-Salido and Vallés 2007; Monacelli and Perotti 2008; Christiano, Eichenbaum and Rebelo 2011; Woodford 2011). The majority of these models feature an infinitely lived representative household who smooths its consumption using a single asset.\footnote{Gali, López-Salido and Vallés (2007) have two types of households: rule-of-thumb and traditional Ricardian.}

I find the single asset setting and lack of heterogeneity in these models unsatisfying. First, it is through a household’s decision on saving, consumption, and hours of work that the effects of rising government spending propagate and eventually give rise to an impact on the aggregate economy. Empirical studies have shown that households’ consumption response to a fiscal policy change is significantly associated with age, income, and wealth liquidity. Specifically, households who are older or have low income and low liquid wealth are more likely to spend all the extra money after receiving fiscal stimulus payments (Johnson et al. 2006; Misra and Surico 2011; Parker et al. 2011). However, an infinitely lived representative household model not only fails to have the age dimension but generates a single income and wealth level. Therefore, the aggregate saving and consumption response resulting from such models might not be empirically plausible.

Second, a single asset setting assumes that all households in the economy hold only one type of asset. This is a strong assumption given that in reality people tend to make different saving decisions depending on the type of asset they are considering. The decision of how much to save for retirement is certainly different from the decision of how much cash to hold on hand. The type of asset households own also influences their
consumption behaviors. For example, a cash-constrained household might choose to cut down its consumption when facing a temporary tax increase, even though the household owns a house. In addition, Lusardi et al. (2011) find from their data that almost half of U.S. households probably do not have the financial capacity to come up with $2,000 within a month. This finding is hard to reconcile with the fact that the median net worth of a U.S. household was around $89,500 in 2001 (Table 1). However, the finding is consistent with the household portfolio data if we look through the lens of liquid and illiquid wealth. In particular, the median liquid wealth for a typical American household was less than $3,500 in 2001. The majority of a household’s wealth was actually held in illiquid form rather than in liquid form (Figure 4). Lastly, it is a known fact that single-asset models without financial frictions fail to generate a big enough aggregate consumption response that is in line with the data (Carroll 1997). A two-asset setting is able to resolve this issue (Kaplan and Violante 2011, hereafter KV). Therefore, it would be informative to see whether the addition of a second asset could change the response dynamics of a model.

Last, as pointed out by Heathcote et al. (2009), heterogeneity has non-trivial effects on aggregate equilibrium quantities and prices. For example, Huggett (1993) finds that the precautionary saving motive resulting from idiosyncratic uninsurable income risk reduces the equilibrium real interest rate. Heathcote (2005) shows that changes in the timing of taxes is neutral in a representative household model because of Ricardian equivalence, but have significant real effects in a heterogeneous-agent incomplete-market model.

Hence, in this paper I develop a general equilibrium model where the economy is populated with finitely lived households. These households smooth their consumption
using two assets. The first asset (i.e. liquid asset) is a low return free-to-adjust asset lent by the households to the government. The second asset (i.e. illiquid asset) is a high return costly-to-adjust asset used as capital by a representative firm. All households draw an adjustment cost from the same i.i.d. distribution at the beginning of each period before deciding whether to adjust the illiquid asset. Working-age households also face idiosyncratic labor productivity risks and supply labor to the representative firm. The government raises labor income taxes to finance its spending and thus maintains a balanced budget. Prices are perfectly flexible and adjust to clear all markets. I refer to this model as the two-asset model.

The two-asset model can be easily modified to a model which I refer to as the one-asset model. Particularly, the two assets are only differentiated by an adjustment cost. When the adjustment cost is set to zero, the economy essentially only has one asset (i.e. liquid asset). In other words, the one-asset model is an economy where there is only the liquid asset.

I subject both the two-asset and one-asset models to the same fiscal stimulus shock, namely a transient rise in government spending. As a result, my main findings can be summarized as follows. First, if I assume a fixed labor supply for working-age households, the addition of a second asset does not affect the qualitative result generated by the one-asset model. The rise in government spending crowds out investment, leading to a subsequent drop in capital which drives down total output in both economies. Therefore, this paper shows that a rise in government expenditure can lead to a recession in a general equilibrium model with capital, flexible prices, and a fixed labor supply.
Second, if I endogenize labor supply by allowing households to choose their hours of work, the addition of a second asset leads to a complete departure from the qualitative result generated by the one-asset model. Aggregate labor falls in the one-asset economy but rises in the two-asset economy thanks to a much stronger consumption response under the two-asset setting. Since capital is predetermined, total output on impact decreases in the one-asset economy but increases in the two-asset economy. After the impact date, capital is crowded out by the rise in government spending in both economies. The drop in capital overwhelms the rise in labor, and total output is mostly reduced after the impact date in the two-asset economy. Therefore, the addition of a second asset has real effects on the aggregate economy under the endogenous labor supply setting.

The rest of the paper proceeds as follows. Section 2 first lays out both the two-asset and one-asset models with labor supply fixed. Next, it gives a detailed description of my parameterization. Then it compares the transitional dynamics generated by the two models in response to a rise in government spending. The assumption of fixed labor supply is relaxed in Section 3. This section first describes how the two models are augmented with an endogenous labor supply and briefly describes the parameterization. It then highlights the difference the addition of a second asset makes by comparing the transitional dynamics generated by the two models in response to a rise in government spending. Section 4 concludes.
2.2 A Life-Cycle Model with Fixed Labor Supply

2.2.1 Model

Demographics

The economy has a continuum of households indexed by their age $j = 1, 2, \ldots, J$. Households retire at age $J^\omega$ and retirement lasts for $J^r$ periods (i.e. $J^\omega + J^r = J$). Hence, all households exit the economy at age $J$ without uncertainty. Moreover, households who exit the economy are replaced with newborns, keeping the total population constant. Newborns enter the economy as workers with zero initial wealth.

Preference

Households are forward-looking and each of them has a time-separable, expected utility:

$$E_0 \sum_{j=1}^{J} \beta^{j-1} \log c_j$$

where $c_j$ is each household’s consumption at age $j$.

Earnings

For each working household, its labor supply is exogenously given and fixed at $h$. In every period, the household is also assigned an exogenous labor productivity $\epsilon$, which follows a Markov process. Thus, the household’s income can be represented as $(1 - \tau)wh \epsilon h$, where $w$ is the wage per efficiency unit of labor and $\tau$ is the proportional labor income tax rate.

For each retired household, it receives a lump-sum tax-free government transfer $\rho(j, \epsilon_{J^\omega})$ in every period. Thus, transfers depend on the retiree's current age $j$ and his last-working-period labor productivity $\epsilon_{J^\omega}$. 

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Financial Assets

There are two assets, denoted as $a$ and $m$, in the economy. The illiquid asset $a$ is restricted to be non-negative and earns a gross return of $R^a = 1/q^a$. An adjustment cost $\kappa$ is associated with this asset. In particular, at the beginning of each period, every household has an i.i.d. draw of $\kappa$ from the same uniform distribution $U(0, \bar{\kappa})$, where the lower-bound is zero and the upper-bound is $\bar{\kappa}$. After seeing its current idiosyncratic state $\epsilon$, each household decides whether to adjust its illiquid asset holdings (i.e. deposit or withdraw). If the household decides to adjust the asset, it has to pay the fixed cost it has drawn. If the household decides not to adjust the asset, it pays no fixed cost and its current illiquid wealth earns a gross return of $R^a$. The adjustment cost is set to zero for all the households who are at their last period of life. Thus, there will be no positive illiquid wealth left behind when households exit the economy. The liquid asset $m$ has a fixed zero borrowing limit, earns a gross return of $R^m = 1/q^m$, and is costless to adjust.

Household Problem

I formulate the household problem in a recursive fashion. The state variables for a single household at each period are its age $j$, its current asset holdings of $a$ and $m$, and the current period draw of both productivity $\epsilon$ and cost $\kappa$. The value function of a household is $V(a, m, \epsilon, j, \kappa)$, which depends on $V^0(a, m, \epsilon, j)$ and $V^1(a, m, \epsilon, j, \kappa)$. $V^0(a, m, \epsilon, j)$ and $V^1(a, m, \epsilon, j, \kappa)$ are the value functions conditional on not adjusting and adjusting the illiquid asset, respectively.
During each working period, the household will choose not to adjust the asset if
\( V^0(a, m, \epsilon, j) \geq V^1(a, m, \epsilon, j, \kappa) \). In other words, it solves the following problem:

\[
V^0(a, m, \epsilon, j) = \max_{c, m'} u(c) + \beta \mathbb{E}_{\epsilon'} \left[ \int_{\kappa'} V(a', m', \epsilon', j + 1, \kappa') H(d\kappa') \right] \tag{2.1}
\]

subject to:

\[
c + q^m m' \leq (1 - \tau) w \epsilon h + m \\
a' = \frac{a}{q^a} \\
m' \geq 0.
\]

The household will choose to adjust the illiquid asset if
\( V^0(a, m, \epsilon, j) < V^1(a, m, \epsilon, j, \kappa) \). Then it solves:

\[
V^1(a, m, \epsilon, j, \kappa) = \max_{c, a', m'} u(c) + \beta \mathbb{E}_{\epsilon'} \left[ \int_{\kappa'} V(a', m', \epsilon', j + 1, \kappa') H(d\kappa') \right] \tag{2.2}
\]

subject to:

\[
c + q^a a' + q^m m' \leq (1 - \tau) w \epsilon h + a + m - \kappa \\
a' \geq 0 \\
m' \geq 0.
\]

Here, \( H(\kappa') \) is the c.d.f. at \( \kappa' \) of the uniform distribution \( U(0, \bar{\kappa}) \) and \( H(d\kappa') \) is its p.d.f.

Therefore, the ex-post value function of a household is

\[
V(a, m, \epsilon, j, \kappa) = \max\{V^0(a, m, \epsilon, j), V^1(a, m, \epsilon, j, \kappa)\}. \tag{2.3}
\]

To solve the retiree’s problem, I substitute \( (1 - \tau) w \epsilon h \) with \( \rho(j, \epsilon, j, \kappa) \) and replace

\[
\mathbb{E}_{\epsilon'} \left[ \int_{\kappa'} V(a', m', \epsilon', j + 1, \kappa') H(d\kappa') \right] \text{ with } \int_{\kappa'} V(a', m', \epsilon, j + 1, \kappa') H(d\kappa').
\]

Appendix C1 has a detailed description of the computational algorithm used to solve the household problem.
**Government**

The government collects income taxes from workers, pays transfers to retirees, and borrows money from all households by issuing a one-period bond $M$ at price $q^m$. Government expenditures $G$ are not valued by households. Let $e_j = (\epsilon_j, \kappa_j)$ be the vector of the productivity and adjustment cost drawn by a household aged $j$ in the current period, and $s_j = (a_j, m_j)$ be the vector of the household’s illiquid and liquid wealth. The government budget constraint can be written as:

$$\tau \sum_{j=1}^{J_\omega} \int w h \epsilon_j \mu_j([ds_j \times de_j]) + q^m M' = G + \sum_{j=J_\omega+1}^{J} \int \rho(j, \epsilon_{J_\omega}) \mu_j([ds_j \times de_j]) + M$$  \hspace{1cm} (2.4)

where $\mu_j([s_j \times e_j])$ is the distribution of households of age $j$ over $s_j$ and $e_j$.

**Firm**

The economy has a representative firm who hires labor and borrows capital from households. In return, it produces a homogeneous final good for households to consume. The production technology is standard Cobb-Douglas, $F(K, N) = K^\alpha N^{1-\alpha}$. Capital depreciates at rate $\delta$. The firm maximizes its profit by choosing $K$ and $N$ so that:

$$w = (1 - \alpha) \left( \frac{K}{N} \right)^\alpha,$$  \hspace{1cm} (2.5)

$$\frac{1}{q^a} = \alpha \left( \frac{K}{N} \right)^{\alpha-1} + 1 - \delta.$$  \hspace{1cm} (2.6)

**General Equilibrium**

A recursive competitive equilibrium consists of a set of functions,

$$(q^a, q^m, w, V^0, V^1, V, c, a', m', K, N, M, \Gamma),$$

that satisfies the following conditions.
1. **Individual Household Optimization:**

\[ V^0, V^1, \text{ and } V \text{ satisfy (2.1), (2.2), and (2.3), and } c, a', \text{ and } m' \text{ are the associated policy functions.} \]

2. **Representative Firm Optimization:**

The representative firm chooses \( N \) and \( K \) as shown in (2.5) and (2.6).

3. **Labor Market Clearing Condition:**

\[ N = \sum_{j=1}^{J} \int h\epsilon_j \mu_j([ds_j \times de_j]). \]

4. **Capital Market Clearing Condition:**

\[ K = q^a \sum_{j=1}^{J} \int a_j \mu_j([ds_j \times de_j]). \]

5. **Bond Market Clearing Condition:**

\[ M = \sum_{j=1}^{J} \int m_j \mu_j([ds_j \times de_j]). \]

6. **Goods Market Clearing Condition:**

\[ zK^\alpha N^{1-\alpha} = \sum_{j=1}^{J} \int c_j \mu_j([ds_j \times de_j]) + K' - (1 - \delta)K + G + \mathcal{K} \]

where:

\[ K' = q^a \sum_{j=1}^{J} \int a_{j+1} \mu_j([ds_j \times de_j]), \]

\[ \mathcal{K} = \sum_{j=1}^{J} \int \kappa_j I_{adj=1} \mu_j([ds_j \times de_j]), \]

\[ G \text{ satisfies (2.4).} \]

Here, \( I \) is an indicator function which takes one if a household adjusts the illiquid asset and zero otherwise.
7. Law of Motion of the Distribution:

\[ \Gamma \text{ is defined as follows. For all } K \subset \mathcal{K} \text{ and } M \subset \mathcal{M}, \]
\[ \mu_{j+1}(\frac{K}{q^n},M,\epsilon_{j+1},\kappa_{j+1}) = \]
\[ \int \left\{ \int I_{s_{j+1} = s_{j+1}^*} d\pi(\epsilon_{j+1}|\epsilon_j) H(d\kappa_{j+1}) \mu_j([ds_j \times de_j]) \right\} \mu_j([ds_j' \times de_j']) \]
\[ \text{for } j = 1, \ldots, J - 1. \]

\[ \mu_1(\frac{K}{q^n},M,\epsilon_1,\kappa_1) = \]
\[ \int \left\{ \int I_{s_1 = s_1^*} d\pi(\epsilon_1|\epsilon_j) H(d\kappa_1) \mu_j([ds_j \times de_j]) \right\} \mu_j([ds_j' \times de_j']). \]

Here, \( s_{j+1}^* = (a_{j+1}^*,m_{j+1}^*) \) solves (2.3), \( \pi(\epsilon_{j+1}|\epsilon_j) \) denotes the transitional probability from \( \epsilon_j \) to \( \epsilon_{j+1} \), and \( I \) is an indicator function which takes one if \( s_{j+1} \in (\frac{K}{q^n},M) \) and zero otherwise.

One-Asset Model

The two assets in the model described above are differentiated by a stochastic adjustment cost. The adjustment cost is drawn from a uniform distribution \( U[0, \bar{\kappa}] \), where zero is the lower-bound and \( \bar{\kappa} \) is the upper-bound. Therefore, if \( \bar{\kappa} \) is set to zero, the two-asset model becomes equivalent to a one-asset model. In other words, the one-asset model is an economy where there is only the liquid asset.

2.2.2 Parameterization

Steady State: Two-Asset

The model has an annual frequency. I assume that each household enters the economy as a worker at age 22, retires at age 60, and dies without uncertainty when it reaches age 78 (e.g. the U.S. life expectancy in 2011 is 78.6 years). Therefore, I set
the lifespan of a household to 56 years (i.e. $J = 56$). The household works for the first 37 years (i.e. $J^\omega = 37$) and becomes a retiree for the last 19 years. The discount factor $\beta$ is 0.956, targeting a four-percent annual real interest rate (i.e. $1/q$) at the steady state.

For working households, I approximate their idiosyncratic labor productivity process using the PSID family public data over the 1968-1993 period. In particular, I closely follow the estimation procedure outlined in Heathcote et al. (2010). Appendix B1 describes the procedure and my sample selection criteria in detail. I then use the Tauchen (1986) method to generate an AR(1) 5-state Markov chain based on the estimates I obtain from the PSID data (i.e. autocorrelation $\rho$ = 0.752, variance of innovation $\sigma^2$ = 0.006). Moreover, each household devotes one-third of its time to working (i.e. $h = 1/3$) and is subject to an income tax rate of 26% (i.e. $\tau = 0.26$) as reported in Kiefer et al. (2002, Table 5, prior-EGTRRA overall on wages).

For retired households, I adopt $\rho(j, \epsilon J^\omega) = \chi \cdot p_j \epsilon J^\omega$ as the functional form to approximate the amount of transfers a retiree receives at age $j$.\(^{10}\) Here, $\chi$ is common across all retirees, representing a deterministic age factor that differentiates retirees from workers. The value of $\chi$ is set to 5.0 so that the ratio of mean wage income to mean transfers is equal to 1.2.\(^{11}\) The second component, $p_j$, captures a deterministic age-specific effect on retirees’ earnings. Based on SCF 2001, I first compute the social security income received by each age group $j$ as a percentage of the total social

\(^{10}\)My definition is similar to that in KV(2011). KV(2011) define a retiree’s income as $y_j = \exp(\chi J^\omega + \alpha + \psi j + z J^\omega), where \chi J^\omega$ is a deterministic age profile common across all retired households, $\alpha$ is a household-specific fixed effect, $\psi$ is a household-specific age-earnings profile, and $z J^\omega$ is a stochastic idiosyncratic component following a first-order Markov process. $J^\omega$ denotes the last working period of a household.

\(^{11}\)Based on SCF 2001, the ratio of mean wage income to mean social security income among households who are aged 60 to 78 is 1.2.
security income received by all households aged from 60 to 78. Then I set each $p_j$ to its corresponding percentage value. The last component, $\epsilon_{j,\omega}$, denotes the stochastic labor productivity drawn by the household at its last working period. Please see Appendix B3 for more details.

For the bond market, the total supply of liquid assets $M$ is 0.306, targeting a 3-4% interest rate spread. The government expenditure $G$ is set to balance the government budget constraint at the steady state.

For the capital market, I choose the capital production share and the depreciation rate (i.e. $\alpha = 0.25$ and $\delta = 0.07$) to target an annual capital-to-output ratio of 2.7 and an annual investment-to-capital ratio of 0.063, respectively.

Last but not least, the upper-bound of adjustment cost is set to 1.5 (i.e. $\bar{\kappa} = 1.5$). At this value, the total fraction of households who choose to adjust their illiquid assets is 19.8%, matching an adjustment rate of 15-20% as estimated from SCF 2001 by KV(2011).

**Steady State: One-Asset**

For the one-asset model, $\bar{\kappa}$ is set to zero and the model frequency remains annual. The discount factor $\beta$ is changed to 0.965, targeting a four-percent annual real interest rate. Meanwhile, the capital production share $\alpha$ is set to 0.3 so that the annual capital-to-output ratio is 2.44. The values of all other parameters are kept unchanged.

**Transition**

To study the transitional dynamics, I assume the economy initially (i.e. date 0) was at its steady state, and then a relative three-percent rise in government spending takes effect (i.e. date 1). The length of the fiscal stimulus is set to one year. The
government raises taxes to finance its spending and thus maintains a balanced budget. My transitional economy is solved using the perfect foresight algorithm described in Guerrieri and Lorenzoni (2011, Appendix).

Tables 3 and 4 list all the parameter values.

2.2.3 A Fiscal Stimulus Experiment

I subject both the one-asset and two-asset economies to the same fiscal stimulus policy (i.e. a relative 3% increase in G). I then examine how differently the two economies respond to the policy. Figures 5 and 6 show the transitional dynamics of the major economic variables in both models.

For the one-asset economy, aggregate capital is predetermined on the impact date, and so are total output and wage since the labor supply is fixed. Meanwhile, the rise in government expenditures crowds out investment and consumption. This is because the government maintains a balanced budget by increasing taxes. The rise in the income tax rate reduces working-age households’ disposable income and limits both their abilities to save and to consume.

After the impact date, government spending reverts to its steady state level, marking the end of the fiscal stimulus. During these subsequent periods, aggregate capital first decreases as a result of the drop in investment and then gradually rises back. The movements of total output and wage exhibit the same pattern given the fixed labor supply. Moreover, the real interest rate moves in the opposite direction as capital thanks to the inverse relationship between capital and the marginal product of capital. At the same time, the transitional path of consumption mimics that of wage due to the fixed labor supply.
For the two-asset economy, the transitional dynamics are similar to those of the one-asset economy. The government expenditure multiplier is zero on impact because of the predetermined capital and fixed labor supply. Investment and consumption are crowded out by government spending. After the impact date, total output first decreases and then rises back to its steady state, indicating a recession following the temporary rise in government spending.

To sum up, if the labor supply is assumed to be fixed, the addition of a second asset does not alter the qualitative result generated by the one-asset model. Total output remains unchanged on impact because capital is predetermined and labor supply is fixed. Meanwhile, investment is crowded out, leading to a drop in capital which drives down total output subsequently in both the one-asset and two-asset economies. Therefore, in contrast to some recent New Keynesian models, the experiment here shows that the government expenditure multiplier is zero in general equilibrium models with capital, flexible prices, and a fixed labor supply. Moreover, the economy plunges into a recession afterwards.

### 2.3 A Life-Cycle Model with Endogenous Labor Supply

In this section, I relax the assumption of fixed labor supply by making hours of work a choice variable for working-age households.

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12 For example, the New Keynesian models in Woodford (2011) and Christiano, Eichenbaum and Rebelo (2011) can generate a government expenditure multiplier in excess of one when the nominal interest rate binds at zero.
2.3.1 Model

Demographics

The economy is still populated with a continuum of households indexed by their age \( j = 1, 2, \ldots, J \). Households work for the first \( J^\omega \) periods and then become retirees for the next \( J^r \) periods before exiting the economy (i.e. \( J^\omega + J^r = J \)). Hence, all households exit the economy at age \( J \) without uncertainty. Moreover, households who exit the economy are replaced with newborns, keeping the total population constant. Newborns enter the economy as workers with zero initial wealth.

Preference

Households now value both consumption and leisure. Each of them has an expected lifetime utility:

\[
E_0 \left\{ \sum_{j=1}^{J^\omega} \beta^{j-1} \left[ \log c_j + \psi \log(1 - n_j) \right] + \sum_{j=J^\omega+1}^{J} \beta^{j-1} \log c_j \right\}, \quad \psi > 0
\]

where \( c_j \) and \( n_j \) denote the household’s consumption and hours of work at age \( j \), respectively. Since households do not work after retirement (i.e. \( n_j = 0 \) for \( j = J^\omega + 1, \ldots, J \)), the leisure utility term drops out for the retirees thanks to the log functional form.

Financial Assets

The setup for financial assets remains the same as in the model with a fixed labor supply. The economy has one costless-to-adjust liquid asset \( m \) with a gross return of \( R^m = 1/q^m \), and one costly-to-adjust illiquid asset \( a \) with a gross return of \( R^a = 1/q^a \). A borrowing limit of zero is imposed on both assets. The adjustment cost \( \kappa \) is an i.i.d. draw from a uniform distribution \( U(0, \bar{\kappa}) \), where the lower-bound is zero and
the upper-bound is $\bar{\kappa}$. The adjustment cost is set to zero for the oldest households, eliminating the possibility of positive leftover illiquid wealth.

**Earnings**

For each working household, it draws an exogenous labor productivity $\epsilon$ at the beginning of every period. The productivity follows a Markov process. After seeing its $\epsilon$ and adjustment cost draw $\kappa$, the household chooses the number of hours it wants to work (i.e. $n$) while taking wage $w$ as given. Thus, the household’s income is $(1 - \tau)w\epsilon n$, where $\tau$ is labor income tax rate.

Each retired household still receives a lump-sum tax-free government transfer $\rho(j, \epsilon, \kappa)$ in every period.

**Household Problem**

The formulation of the household problem is similar to the model with a fixed labor supply. Therefore, I briefly list the three key equations below.

For each working household, it will choose not to adjust the illiquid asset if $V^0(a, m, \epsilon, j) \geq V^1(a, m, \epsilon, j, \kappa)$. Then it solves:

$$V^0(a, m, \epsilon, j) = \max_{c, m', n} u(c, n) + \beta \mathbb{E}_{\epsilon'} \left[ \int_{\kappa'} V(a', m', \epsilon', j + 1, \kappa') H(d\kappa') | \epsilon' \right] \quad (2.7)$$

subject to:

$$c + q^m m' \leq (1 - \tau)w\epsilon n + m$$

$$a' = \frac{a}{q^n}$$

$$m' \geq 0$$

$$0 \leq n \leq 1.$$
Then it solves:

\[
V^1(a, m, \epsilon, j, \kappa) = \max_{c, a', m', n} \left\{ u(c, n) + \beta \mathbb{E}_{\epsilon' \mid \kappa'} \left[ \int_{\kappa'} V(a', m', \epsilon', j, \kappa' + 1) H(d\kappa') \right] \right\}
\]

subject to:

\[
c + qa' + q^m m' \leq (1 - \tau)w\epsilon n + a + m - \kappa
\]

\[
a' \geq 0
\]

\[
m' \geq 0
\]

\[
0 \leq n \leq 1.
\]

The ex-post value function of a household is

\[
V(a, m, \epsilon, j, \kappa) = \max\{V^0(a, m, \epsilon, j), V^1(a, m, \epsilon, j, \kappa)\}.
\]

To solve the retiree's problem, I set \(n\) to zero, substitute \((1 - \tau)w\epsilon n\) with \(\rho(j, \epsilon_{j, \omega})\), and remove the expectation over \(\epsilon'_j\).

Appendix C2 has a detailed description of the computational algorithm used to solve the household problem.

**Government**

The role of the government remains the same in this economy as it is in the economy with a fixed labor supply. Its budget constraint can be written as:

\[
\tau \sum_{j=1}^{J} \int wn_j \epsilon_j \mu_j([ds_j \times de_j]) + q^m M' = G + \sum_{j=J}^{J+1} \int \rho(j, \epsilon_{j+1}) \mu_j([ds_j \times de_j]) + M
\]

where \(\mu_j([s_j \times e_j])\) is the distribution of households of age \(j\) over \(s_j = (a_j, m_j)\) and \(e_j = (\epsilon_j, \kappa_j)\).
Firm

A representative firm produces a homogeneous final good for households to consume by using the Cobb-Douglas production technology, \( F(K, N) = K^\alpha N^{1-\alpha} \). Capital depreciates at rate \( \delta \). The firm maximizes its profit by choosing \( K \) and \( N \) so that:

\[
\begin{align*}
    w &= (1 - \alpha)\left(\frac{K}{N}\right)^\alpha, \\
    \frac{1}{q^a} &= \alpha\left(\frac{K}{N}\right)^{\alpha-1} + 1 - \delta.
\end{align*}
\]  

(2.11)  

\( (2.12) \)

General Equilibrium

A recursive competitive equilibrium consists of a set of functions,

\[
(q^a, q^m, w, V^0, V^1, V, c, n, a', m', K, N, M, \Gamma),
\]

that satisfies the following conditions.

1. \textit{Individual Household Optimization}:

   \( V^0, V^1, \) and \( V \) satisfy (2.7),(2.8), and (2.9), and \( c, n, a', \) and \( m' \) are the associated policy functions.

2. \textit{Representative Firm Optimization}:

   The representative firm chooses \( N \) and \( K \) as shown in (2.11) and (2.12).

3. \textit{Labor Market Clearing Condition}:

   \[
   N = \sum_{j=1}^{J_w} \int n_j e_j \mu_j([ds_j \times de_j]).
   \]

4. \textit{Capital Market Clearing Condition}:

   \[
   K = q^a \sum_{j=1}^{J} a_j \mu_j([ds_j \times de_j]).
   \]
5. **Bond Market Clearing Condition:**

\[ M = \sum_{j=1}^{J} \int m_j \mu_j([ds_j \times de_j]). \]

6. **Goods Market Clearing Condition:**

\[ z K^\alpha N^{1-\alpha} = \sum_{j=1}^{J} \int c_j \mu_j([ds_j \times de_j]) + K' - (1 - \delta)K + G + K \]

where:

\[ K' = q^a \sum_{j=1}^{J} \int a_{j+1} \mu_j([ds_j \times de_j]), \]

\[ \mathcal{K} = \sum_{j=1}^{J} \int \kappa_j \mu_j([ds_j \times de_j]), \]

\[ G \text{ satisfies (2.10).} \]

7. **Law of Motion of the Distribution:**

\[ \Gamma \text{ is defined as follows. For all } K \subset \mathcal{K} \text{ and } M \subset \mathcal{M}, \]

\[ \mu_{j+1}(\frac{K}{q^a}, M, \epsilon_{j+1}, \kappa_{j+1}) = \]

\[ \int \left\{ \int I_{s_{j+1}=s_{j+1}^*} d\pi(\epsilon_{j+1} | \epsilon_j) H(d\kappa_{j+1}) \mu_j([ds_j \times de_j]) \right\} \mu_j([ds_j' \times de_j']). \]

for \( j = 1, ..., J - 1. \)

\[ \mu_1(\frac{K}{q^a}, M, \epsilon_1, \kappa_1) = \int \left\{ \int I_{s_1=(\frac{\kappa_1}{q^a}, M)} d\pi(\epsilon_1 | \epsilon_j) H(d\kappa_1) \mu_1([ds_j \times de_j]) \right\} \mu_1([ds_1' \times de_1']). \]

**One-Asset Model**

For the one-asset model, the economy only has the liquid asset (i.e. \( \bar{\kappa} = 0 \)). Working-age households not only choose their consumption and savings in liquid assets but the number of hours they want to work. Meanwhile, the choice variables for retired households are still consumption and liquid asset savings.
2.3.2 Parameterization

Steady State: Two-Asset

The model has an annual frequency. Except for the leisure preference parameter $\psi$, all other parameters here, together with their targets, are the same as they are in the fixed labor supply case. Therefore, this section only briefly describes the parameters whose values have changed. Please see Tables 3 and 4 for details.

First, the leisure preference parameter $\psi$ is set to 0.6, targeting an aggregate labor of 0.3-0.4 at the steady state. Second, the common age profile for transfers, $\chi$, is changed to 15 so that the ratio of mean wage income to mean transfers is about 1.33. Last, to keep an interest rate spread of 3-4%, the total supply of liquid assets $M$ is set to 0.203.

Steady State: One-Asset

For the one-asset model, $\bar{\kappa}$ is set to zero and the model frequency is annual. The discount factor $\beta$ is 0.965, targeting a four-percent annual real interest rate. The capital production share $\alpha$ is set to 0.29 so that the annual capital-to-output ratio is 2.28. The values of all other parameters are kept same as in the two-asset case.

Transition

To study the transitional dynamics, I again assume the economy initially (i.e. date 0) was at its steady state, and then a relative three-percent rise in government spending takes effect (i.e. date 1). The length of the fiscal stimulus is one year. The government raises taxes to finance its spending and thus maintains a balanced budget.
2.3.3 A Fiscal Stimulus Experiment

I subject both the one-asset and two-asset models to the same fiscal stimulus policy (i.e. a relative 3% increase in G). I then examine how differently the two economies respond to the policy. Figures 7 and 9 show the transitional dynamics of the major economic variables in both models.

One-Asset Economy

Similar to the fixed labor supply case, aggregate capital is predetermined on the impact date. Since the government uses taxes to maintain a balanced budget, the rise in its spending crowds out consumption and investment.

Unlike the fixed labor supply case, total output drops on the impact date because aggregate labor decreases. The intuition behind this decrease in labor can be explained using a representative household’s problem. For example, a representative household solves the problem below:

\[
\max_{c_t, n_t, k_{t+1}} \quad \mathbb{E}\left\{ \sum_{t=1}^{T} \beta^t [\log c_t + \psi \log(1 - n_t)] \right\}
\]

subject to:

\[
c_t + q_t k_{t+1} \leq (1 - \tau) w_t n_t + k_t.
\]

After taking first-order conditions, we can obtain the following labor-leisure condition:

\[
\frac{\psi}{1 - n_t} = (1 - \tau) \frac{w_t}{c_t}.
\]

Hence, there are two opposing effects on \( n \). The first effect is the labor income tax effect. The government raises the labor income tax rate \( \tau \) to fund its spending. As \( \tau \) increases on the right hand side, \( n \) on the left hand side has to decrease. The second effect comes from \( \frac{w}{c} \). Figure 7 shows that wage increases and consumption decreases.
on the impact date. Therefore, \( \frac{w}{c} \) goes up, putting an upward pressure on \( n \). In the end, the first effect overwhelms the second effect and aggregate labor decreases.\(^{13}\)

To further corroborate the above-described intuition, I replace the proportional labor income tax with a lump-sum tax. Figure 8 shows the transitional dynamics of the economy. On the impact date, aggregate labor increases, causing output to rise. This is because the labor income tax effect disappears. When \( \frac{w}{c} \) goes up, \( n \) increases.

The analysis above reveals an important observation. For a one-asset model with an endogenous labor supply, the government expenditure multiplier can be negative in the presence of a proportional labor income tax.

**Two-Asset Economy**

Similar to the one-asset model, aggregate capital is predetermined on the impact date. The rising government spending crowds out consumption, investment, and aggregate savings in liquid assets (i.e. \( M' \)).

In contrast to the one-asset model, total output increases on the impact date because aggregate labor jumps up. The intuition behind this increase in labor can be again explained by examining the representative household’s first-order condition:

\[
\frac{\psi}{1 - n_t} = (1 - \tau) \frac{w_t}{c_t}.
\]

\(^{13}\)The qualitative result remains intact if I change the utility functional form to \( \log(c) - \frac{\psi}{1 + \psi} (n)^{\frac{1 + \psi}{\psi}} \), where \( \psi \) is set to 0.5 targeting a steady-state aggregate labor of 0.4 (i.e. \( N = 0.4 \)). However, the change of functional form does change the impulse responses quantitatively. The responses on the impact date are roughly one order of magnitude bigger under the new functional form. Particularly, aggregate consumption decreases by one percent instead of one tenth of percent from the steady state on impact. Aggregate labor also initially decreases by more than ten percent instead of one percent from the steady state. A similar result carries over to the two-asset economy.
The relative decrease in consumption in the two-asset economy is ten times bigger than that of the one-asset economy (i.e. 1% vs. 0.1%). As a result, the effect of $\frac{w}{c}$ overwhelms the labor income tax effect and aggregate labor jumps up on impact.

The much stronger consumption response in the two-asset model merits an explanation. In both the one-asset and two-asset models, only households who are wealth constrained have to cut down their consumption considerably. The non-constrained households can use their savings for consumption smoothing.

In the one-asset economy, the empirical counterpart to the wealth in the model is net worth. Therefore, the magnitude of the aggregate consumption response to a fiscal stimulus depends on how many households are net worth constrained.

In the two-asset economy, the empirical counterparts are liquid and illiquid wealth. Appendix B2 details the definition of the two wealths. On one hand, the existence of adjustment cost prevents households from accessing their illiquid wealth. On the other hand, the illiquid asset enjoys a higher return, so the majority of a household’s wealth will be stored in its illiquid account. Therefore, the number of households who are liquid wealth constrained determines how big an aggregate consumption response the two-asset model can generate.

In the data, there are many more households who are liquid wealth constrained. Thus, under a parametrization that is in line with the data, aggregate consumption drops more in the two-asset model.

I adopt the definition of wealth-constrained households as suggested in KV (2011) and use the same dataset (i.e. SCF 2001) to measure the fraction of wealth-constrained households in both the one-asset and two-asset economies. Specifically, I assume that
households are surveyed at the midpoint of a pay period and their expenditures remain constant across two pay periods. Then households whose wealth (i.e. net worth in one-asset model and liquid wealth in two-asset model) is less than half of their earnings per pay period are considered as wealth constrained. The frequency of pay dates is assumed to be monthly. Table 5 reports my empirical estimates and their model counterparts.\textsuperscript{14}

As shown in Table 5, the number of liquid-wealth constrained households in the two-asset economy is much bigger than the number of net-worth constrained households in the one-asset economy. Even though my two-asset model underestimates the fraction of liquid-wealth constrained households,\textsuperscript{15} it still generates an aggregate consumption response big enough to overcome the labor income tax effect, which the one-asset model fails to do.

After the impact date, government spending returns to its steady state level. Aggregate capital first decreases because of the drop in investment. As a result, wage decreases, making households feel poorer and prompting a further increase in aggregate labor. However, the drop in capital outweighs the rise in labor and total output is mostly reduced after the impact date. As investment recovers, capital eventually rises back to its steady state level, and so do wage and total output.

\textsuperscript{14}KV (2011) find that 5.6\%-7.1\% of households are net-worth constrained and 30-42\% of households are liquid-wealth constrained. My empirical estimates are higher than theirs, because my definition of earnings excludes benefits, such as unemployment and disability insurances, but theirs does.

\textsuperscript{15}The underestimation might be due to two factors. First, households are not allowed to borrow in the model, but people often borrow through credit cards in real life. Second, the only channel for households in the model to partially insure themselves against future adversities (e.g. low productivity draws) is to save in one or both assets. In reality, policies, such as food stamps and unemployment benefits, often reduce one’s incentive to save. As a result, households hold more liquid wealth in my model when compared to the data.
To sum up, under an endogenous labor supply and a proportional labor income tax, the addition of a second asset can lead to a complete departure from the qualitative result generated by the one-asset model. On impact, aggregate labor drops in the one-asset economy but increases in the two-asset economy. Since capital stock is predetermined, the government expenditure multiplier is negative in the one-asset model but positive in the two-asset model. After the impact date, capital is crowded out by the rise in government spending and total output drops below its steady state in the two-asset economy.

2.4 Conclusion

In this paper, I develop a general equilibrium model with four main features: (1) a built-in lifecycle; (2) an income process estimated using the U.S. earnings data; (3) two assets differentiated by an adjustment cost; (4) idiosyncratic risks with respect to labor productivity and adjustment cost. A one-asset version of the model can be obtained by setting the adjustment cost to zero.

I subject both the two-asset and one-asset versions of the model to a rise in government spending. As a result, my main findings are as follows. First, for both versions of the model, under a fixed labor supply, the rise in government spending crowds out investment, causing capital to drop and thus driving down total output. Second, under an endogenous labor supply, aggregate labor falls in the one-asset economy but a substantial decrease in consumption causes it to rise in the two-asset economy. Capital is predetermined on impact, so total output decreases in the one-asset economy but increases in the two-asset economy. However, the increase in total output in the two-asset economy is short-lived.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J^\omega$</td>
<td>retirement age</td>
<td>60 as retirement age</td>
</tr>
<tr>
<td>$J^r$</td>
<td>retirement length</td>
<td>78.6 as U.S. life expectancy</td>
</tr>
<tr>
<td>$J$</td>
<td>total life span</td>
<td>$J = J^\omega + J^r$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>household discount factor</td>
<td>4-6% annual real interest rate</td>
</tr>
<tr>
<td>$\psi$</td>
<td>household leisure preference</td>
<td>1/3 time spent working</td>
</tr>
<tr>
<td>$h$</td>
<td>working hours</td>
<td>1/3 time spent working</td>
</tr>
<tr>
<td>$\tau$</td>
<td>labor income tax rate</td>
<td>Kiefer et al.(2011, Table 5)</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>labor productivity persistence</td>
<td>PSID 1968-1993</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>variance of innovation to labor productivity</td>
<td>PSID 1968-1993</td>
</tr>
<tr>
<td>$\chi$</td>
<td>common age effect on transfers</td>
<td>SCF 2001</td>
</tr>
<tr>
<td>$p_j$</td>
<td>age-specific effect on transfers</td>
<td>SCF 2001</td>
</tr>
<tr>
<td>$M$</td>
<td>total supply of liquid assets</td>
<td>3-4% interest spread</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>capital production share</td>
<td>annual $K/Y = 2.36$, NIPA 1954-2010</td>
</tr>
<tr>
<td>$\delta$</td>
<td>capital depreciation rate</td>
<td>annual $I/K = 0.08$, NIPA 1954-2010</td>
</tr>
<tr>
<td>$\bar{\kappa}$</td>
<td>adjustment cost upper-bound</td>
<td>15-20% adjustment, KV(2011)</td>
</tr>
</tbody>
</table>

Table 2.1: Household Portfolio Composition, SCF 2001

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median</td>
<td>Mean</td>
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<tr>
<td>Earnings plus benefits (age 22 − 59)</td>
<td>50,391</td>
<td>73,291</td>
</tr>
<tr>
<td>Net worth</td>
<td>89,451</td>
<td>289,100</td>
</tr>
<tr>
<td>Net liquid wealth</td>
<td>3,468</td>
<td>89,737</td>
</tr>
<tr>
<td>Cash, checking, saving, MM accounts</td>
<td>3,676</td>
<td>89,737</td>
</tr>
<tr>
<td>Directly held MF, stocks, bonds</td>
<td>0</td>
<td>24,179</td>
</tr>
<tr>
<td>Credit card debt</td>
<td>0</td>
<td>2,390</td>
</tr>
<tr>
<td>Net illiquid wealth</td>
<td>78,546</td>
<td>199,363</td>
</tr>
<tr>
<td>Housing net of mortgages</td>
<td>37,986</td>
<td>98,142</td>
</tr>
<tr>
<td>Vehicles net of installment loans</td>
<td>7,475</td>
<td>10,929</td>
</tr>
<tr>
<td>Retirement accounts</td>
<td>2,451</td>
<td>70,704</td>
</tr>
<tr>
<td>Life insurance</td>
<td>0</td>
<td>12,210</td>
</tr>
<tr>
<td>Certificates of deposit</td>
<td>0</td>
<td>6,043</td>
</tr>
<tr>
<td>Saving bonds</td>
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<td>1,334</td>
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Table 2.2: Parameter Description
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fixed</th>
<th>Endogenous</th>
<th>Endogenous Lump-Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J^\omega$</td>
<td>37</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>$J^\tau$</td>
<td>19</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>$J$</td>
<td>56</td>
<td>56</td>
<td>56</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.965</td>
<td>0.965</td>
<td>0.974</td>
</tr>
<tr>
<td>$\psi$</td>
<td>$-$</td>
<td>0.6</td>
<td>$(N = 0.39)$</td>
</tr>
<tr>
<td>$h$</td>
<td>$1/3$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.26</td>
<td>0.26</td>
<td>$-$</td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>0.752</td>
<td>0.752</td>
<td>0.752</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>$\chi$</td>
<td>$5.0 \ (\frac{(1-\tau)w_\bar{n}}{\rho} = 1.3)$</td>
<td>$15.0 \ (\frac{(1-\tau)w_\bar{n}}{\rho} = 1.3)$</td>
<td>$15.0 \ (\frac{(1-\tau)w_\bar{n}}{\rho} = 1.3)$</td>
</tr>
<tr>
<td>$p_j$</td>
<td>Appendix B3</td>
<td>Appendix B3</td>
<td>Appendix B3</td>
</tr>
<tr>
<td>$M$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.30</td>
<td>0.29</td>
<td>0.30 $(K/Y = 2.37)$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07 $(I/K = 0.07)$</td>
</tr>
<tr>
<td>$\bar{\kappa}$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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Table 2.3: Parameter Values: One-Asset Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fixed</th>
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<td>$J^\tau$</td>
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<td>19</td>
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<tr>
<td>$J$</td>
<td>56</td>
<td>56</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.956</td>
<td>0.956</td>
</tr>
<tr>
<td>$\psi$</td>
<td>$-$</td>
<td>0.6</td>
</tr>
<tr>
<td>$h$</td>
<td>$1/3$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>0.752</td>
<td>0.752</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>$\chi$</td>
<td>$5.0 \ (\frac{(1-\tau)w_\bar{n}}{\rho} = 1.2)$</td>
<td>$15.0 \ (\frac{(1-\tau)w_\bar{n}}{\rho} = 1.3)$</td>
</tr>
<tr>
<td>$p_j$</td>
<td>Appendix B3</td>
<td>Appendix B3</td>
</tr>
<tr>
<td>$M$</td>
<td>0.306</td>
<td>0.203</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.25</td>
<td>0.27</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>$\bar{\kappa}$</td>
<td>1.50</td>
<td>1.50</td>
</tr>
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</table>

Table 2.4: Parameter Values: Two-Asset Model
Table 2.5: Wealth-Constrained Households as Percentage, SCF2001 vs. Model
Note: monthly pay frequency and both models have an endogenous labor supply.

<table>
<thead>
<tr>
<th></th>
<th>Two-asset</th>
<th>One-asset</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Liquid-Wealth Constrained</td>
<td>Net-Worth Constrained</td>
</tr>
<tr>
<td>SCF2001</td>
<td>43.22%</td>
<td>12.06%</td>
</tr>
<tr>
<td>Model</td>
<td>33.46%</td>
<td>13.13%</td>
</tr>
</tbody>
</table>

Figure 2.1: Real U.S. Government Spending (in 2009 dollar), 1929-2012
Figure 2.2: U.S. Government Spending as Fraction of GDP, 1929-2012

Figure 2.3: U.S. Taxation, 1929-2012
Figure 2.4: Median Liquid and Illiquid Wealth by Age Cohorts, SCF 2001
Figure 2.5: Transitional Dynamics: One-Asset with Fixed Labor
Figure 2.6: Transitional Dynamics: Two-Asset with Fixed Labor
Figure 2.7: Transitional Dynamics: One-Asset with Endogenous Labor
Figure 2.8: Transitional Dynamics: One-Asset with Endogenous Labor and Lump-Sum Tax
Figure 2.9: Transitional Dynamics: Two-Asset with Endogenous Labor
Chapter 3: Jobless Recoveries and Sectoral Skill-Biased Structural Change

3.1 Introduction

The aggregate U.S. labor market has shown a puzzling business cycle phenomenon: the recovery phase of total non-farm employment after a recession has been unusually long, which was not observed in recessions before 1990. A growing body of literature has been trying to explain this jobless recovery phenomenon from a variety of angles. For example, Bachmann (2001) constructs a model that generates jobless recoveries based on the trade-off between a firm’s intensive margin (i.e. hours worked) and extensive margin (i.e. number of workers). Jaimovich and Siu (2012) argue that jobless recoveries should be attributed to job polarization. Meanwhile, Shimer (2012) demonstrates how a combination of wage rigidity and weak aggregate demand can lead to a sluggish recovery in the labor market. This paper adopts a sectoral perspective and uses both empirical evidence and a structural model to show that sectoral skill-biased structural change (SBSC) can explain the three post-1990 jobless recoveries.

Linking sectoral shifts to the cyclicality of aggregate labor market is not a novel approach (Lilien 1982; Abraham and Katz 1986). Compared to service-providing sectors, recessions tend to hit goods-producing sectors harder and result in higher unemployment. Since it takes time for an unemployed worker in one sector to find
a job in another sector, the aggregate unemployment rate can stay high for a long time. An increasing number of papers have started to explore the jobless recovery by revisiting this sectoral explanation. Garin et al. (2011) attribute the growing importance of reallocative shocks relative to aggregate shocks as the main mechanism enabling their model to generate jobless recoveries, but no empirical evidence is provided to justify such a mechanism. Pilossoph (2012) presents a search model of labor reallocation where intersectoral labor mobility frictions can only account for ten percent of observed aggregate unemployment movements. Thus, this paper makes a contribution in two aspects: (i) it empirically establishes a link between structural changes at the sectoral level and the jobless recovery in the aggregate economy; (ii) it uses a two-sector model to demonstrate that the observed structural change can well explain the jobless recovery phenomenon.

I first use the BLS establishment survey and the IPUMS-CPS March Series to establish the following four stylized facts.

1. The U.S. job market has taken significantly longer to recover after each post-1990 recession.

2. The recovery of goods sector employment was slow after each post-1990 recession.

3. The educational attainment of service sector workers has surpassed that of goods sector workers since 1990.

4. The skill premium of workers with college-plus education has increased faster in the service sector than in the goods sector.
These empirical facts suggest that SBSC in the service sector has prevented the unskilled workers who are laid off in the goods sector from relocating to the service sector. Thus, it takes longer for an unemployed worker in the goods sector to find a new job, leading to a sluggish job market recovery at both the aggregate and goods sector level.

I then develop a two-sector structural model that successfully generates a jobless recovery. The SBSC in the service sector is incorporated into the model via two components. The first component is a sector-specific labor adjustment cost. Based on Fact 3, it has become more costly for unemployed workers in the goods sector to find a job in the service sector due to the rising educational barrier. The second component is a reallocation shock. Fact 4 indicates that service sector workers, at least among the skilled, have become more productive than goods sector workers. To reflect this change, I add to the model a reallocation shock, making workers in the service sector more productive than those in the goods sector. The model with these two built-in components represents the post-1990 economy and the one without the components represents the pre-1990 economy. Both models are calibrated to fit the general characteristics of the U.S. economy during their respective time periods. A comparison of the simulation results reveals that the model is only able to reproduce a jobless recovery under the post-1990 conditions.

The rest of the paper is organized as follows. Section 2 details the four stylized facts. While Section 3 presents the structural model, Sections 4 and 5 explain its calibration procedure and simulation results. Section 6 concludes.
3.2 Stylized Facts

**Fact 1:** The U.S. job market has taken significantly longer to recover after each post-1990 recession.

An interesting phenomenon has emerged from the U.S. job market: total non-farm employment has taken significantly longer to recover to its peak level since the early 1990s (Table 1). For instance, it took non-farm employment a maximum of three quarters to return to its peak level before 1990 but ten quarters after 1990.

There are two factors that determine the employment recovery process: the number of workers flowing into the employment pool and the speed of that flow. A plot of total employment growth rate against a horizontal time-axis captures both factors (Figure 1). First, Figure 1 shows that the total U.S. employment growth is procyclical and averages around 0.4% quarterly in the post World War II era. Before 1990, the growth rate came back and rose above the mean immediately after each recession. Such quick recoveries are no longer observed for the three most recent recessions. Second, the rebound after the 1990, 2001, and 2007 recession were rather weak; the positive deviations of the growth rate from the mean never reached the pre-1990 level. Third, the employment growth has become noticeably less volatile since 1990, a phenomenon some literature associates with the Great Moderation (Stock and Watson 2002; Faberman 2012). Hence, a clear difference can be seen in the aggregate employment business cycle movements before and after 1990.

**Fact 2:** The recovery of goods sector employment was slow after each post-1990 recession.
Following the U.S. Bureau of Labor Statistics, I categorize mining, construction, and manufacturing as the goods sector. The service sector consists of transportation, utilities, trade, financial activities, and services. Figure 2 plots the difference between the sector-specific employment growth rate and the aggregate employment growth rate.\footnote{Aaronson et al. (2004) conduct a similar analysis on manufacturing durables employment.}

First, while a long-run declining trend can be observed for the goods sector (see horizontal green line), a long-run growing trend can be observed for the service sector (see horizontal red line). The quarterly employment growth rate in the goods sector was on average 0.43\% lower than the aggregate employment growth rate over the past 58 years. The quarterly employment growth rate in the service sector was on average 0.16\% higher than the aggregate employment growth rate during the same period. Therefore, jobs have been permanently moving out of the goods sector.

Second, the relative goods sector employment growth rate recovered immediately after each recession before 1990. However, the recovery was quite slow after each post-1990 recession. Before 1990, the service sector was able to absorb some of the unemployment from the goods sector. The relative employment growth rate in the service sector jumped up while the relative employment growth rate in the goods sector dropped. After 1990, the absorption of this kind significantly weakened. The relative employment growth rate in the service sector stabilized around the long-run average, while the relative employment growth rate in the goods sector continued to drop in each recession. The fact that more unemployed workers need to find a job within the sector naturally leads to a much slower employment recovery in the goods sector.
sector and thus at the aggregate level.

**Fact 3:** The educational attainment for service sector workers has surpassed that of goods sector workers since 1990.

In 1976, the IPUMS-CPS March Series started to report respondents’ years of schooling, and which industry they worked in if they were employed in the previous year. I focus on full-time employees who are male and aged 18 to 64. Following Autor et al. (2008), I define a person who has completed less than 12 years of schooling as a high school dropout, exactly 12 years as a high school graduate, 13-15 years as some college, and 16 or more years as college-plus.

Figures 3 and 4 show the weighted percentage of workers by their educational attainment in the goods and service sector, respectively. Before 1990, the majority of workers in the two sectors only had high school education. In particular, about 45% of full-time workers in the goods sector and 40% in the service sector were high school graduates. After 1990, while the educational level of most workers in the goods sector stayed unchanged, workers with college-plus education became the majority in the service sector. Although the proportion of workers who have high school diplomas has been declining in the goods sector, those workers still consist of 40% of the

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17 The result does not change if I include females. See Figures 1 and 2 in Appendix D.

18 Unfortunately, Frazis and Stewart (1996) report that the CPS changed its educational attainment questions in 1992. The previous questions were: ‘What is the highest grade or year of regular school [the respondent] has ever attended?’ and ‘Did [the respondent] complete the grade?’ In 1992, these two questions were replaced with one single question: ‘What is the highest level of school [the respondent] has completed or the highest degree has received?’ The CPS provides a recoded variable EDUC which combines the pre-1992 and post-1992 questions to bridge the break in the series. Figures 3 and 4 show that the break is still visible even with the use of the recoded variable EDUC. However, the change of questions applies to both sectors and the long-run trend is the main focus here, so the result arising from the comparison between the two sectors should still be valid.
sectoral workforce.

**Fact 4:** *The skill premium of workers with college-plus education has increased faster in the service sector than in the goods sector.*

I also take a look at the skill premium in both goods and service sectors. The sample selection is the same as described in Fact 3.\(^{19}\) The hourly wage is computed as a respondent’s total pre-tax wage and salary income received last year divided by the product of weeks worked last year and usual hours worked per week. The hourly wage is then normalized to 1999 U.S. dollars. Figure 5 depicts the difference between the skill premium in the service and goods sector for high school graduates, some college, and college-plus workers. The skill premium for high school graduates within each sector is computed as the difference between the mean log hourly wage of high school graduates and the mean log hourly wage of high school dropouts. For some college and college-plus, the calculation method is the same; I use the sectoral mean hourly wage of high school dropouts as the common denominator.\(^{20}\)

Figure 5 shows that the between-sector skill premium gap of college-plus workers has widened at a noticeable rate. It has deviated from the common trend exhibited by the skill premium gap of high school graduates and some college. Specifically, a worker with college-plus education receives a much higher hourly wage in the service sector than those in the goods sector. In other words, workers with college-plus education are more productive in the service sector than those in the goods sector.

\(^{19}\)The result does not change if I include females, though the pattern is less obvious. See Figure 3 in Appendix D.

\(^{20}\)The pattern is most noticeable with high school dropouts as the common denominator.
To sum up, Facts 3 and 4 indicate that there has been an SBSC in the service sector during the post-1990 period. This SBSC in the service sector has prevented unemployed workers in the goods sector from relocating to the service sector. As a result, we observe a sluggish recovery in both aggregate and goods sector employment in the post-1990 recessions. The next section constructs a structural model where the SBSC is built into the model in the form of a labor adjustment cost and a reallocation shock. The model is able to replicate Facts 1 and 2.\textsuperscript{21}

### 3.3 Model

#### 3.3.1 Environment

The model economy has one representative household and two sectors, each of which has a representative firm. The two sectors are denoted as $g$ for goods and $s$ for services. There are two types of labor represented by 0 as unskilled and 1 as skilled.\textsuperscript{22} Only unemployed labor can switch sectors but no labor can change its type. A quadratic cost has to be paid to move labor across sectors. All newly-moved labor will join production in the following period.

\textsuperscript{21}International trade could also be an important factor leading to the jobless recovery phenomenon. Though it is beyond this paper’s scope to study the relation between trade and jobless recoveries, I did examine Facts 3 and 4 using an alternative definition, namely the tradable and non-tradable goods sector. While the educational attainment result still holds, no noticeable trend can be seen regarding the cross-sector difference in skill premium. See Appendix D for details. Thus, the explanation offered in this paper is only appropriate for the sectoral categorization of goods and services.

\textsuperscript{22}Here, unskilled workers can be considered as those who have non-college education and skilled workers as those who have college education or more. For consistency reasons, I also examine the composition of college and non-college workers in both the goods and service sectors, as well as the cross-sector difference in skill premium for college workers. The empirical results, namely Facts 3 and 4, still hold under this broader educational attainment definition. See Figures 4, 5, and 6 in Appendix D for details.
3.3.2 Law of Motion for Labor

A number of variables need to be defined before the law of motion for labor can be specified. Let \( i \in \{0, 1\} \) denote the labor type and \( j \in \{g, s\} \) the sector. The total amount of type-\( i \) labor in the economy is indicated by \( L_i \). Since labor participation decisions are not the focus of this paper, \( L_i \) represents the labor participation force and is treated as a parameter instead of an endogenous variable. Let \( n_{ji} \) be the amount of type-\( i \) labor employed in sector \( j \). I then pool the unemployed type-\( i \) labor in both sectors and denote the sum as \( u_i \). Let \( m_{ji} \) be the fraction of unemployed type-\( i \) labor that is moved to sector \( j \) (i.e. \( j \) is the destination sector) and becomes employed without uncertainty in the next period. The representative household chooses \( m_{ji} \) by solving its optimization problem which will be detailed in Section 3.6. Lastly, each labor type is subject to a sector-specific exogenous separation rate, \( \chi_{ji} \). The law of motion for labor, where an apostrophe denotes the next period, is as follows.

\[
\begin{align*}
n'_{ji} & = (1 - \chi_{ji}) n_{ji} + m_{ji} u_i \quad (3.1) \\
u'_i & = \sum_{j \in \{g, s\}} \chi_{ji} n_{ji} + (1 - \sum_{j \in \{g, s\}} m_{ji}) u_i \quad (3.2) \\
L_i & = \sum_{j \in \{g, s\}} n_{ji} + u_i \quad (3.3) \\
1 & = L_0 + L_1 \quad (3.4)
\end{align*}
\]
3.3.3 Timing

There exist two shocks: an aggregate TFP shock and a reallocation shock. At the beginning of each period, the two shocks hit the economy. After seeing both shocks, the two representative firms make their production decisions and wage is paid to their hired labor. Then the household chooses how much of the unemployed labor to move to each sector for next period’s production (i.e. $m_{ji}$). At the end of the period, all exogenous labor separations are realized. Figure 6 graphically illustrates the timing of these events.

3.3.4 Production

Let $y_j$ denote the intermediate good produced by sector $j$ and $p_j$ the price of the good. The final good, $Y$, is produced for the household to consume using the following technology:

$$Y = \left( \alpha \frac{1}{\rho} (y_g)^{\frac{\rho-1}{\rho}} + (1 - \alpha) \frac{1}{\rho} (y_s)^{\frac{\rho-1}{\rho}} \right)^{\rho-1},$$

(3.5)

where $\alpha$ is the production share of sector-$g$’s good and $\rho$ is the elasticity of substitution between the two intermediate goods.

Normalizing the price of final good to one, I can write the optimal demand for each intermediate good as:

$$y_g = \frac{\alpha Y}{(p_g)^{\rho}},$$

(3.6)

$$y_s = \frac{(1 - \alpha) Y}{(p_s)^{\rho}}.$$  

(3.7)

3.3.5 Firm’s Problem

The representative firm in sector $j$ hires both type-0 and type-1 labor. Since employment is the main focus, I abstract capital from the production and assume labor
as the only input. More specifically, I assume a standard Cobb-Douglas production technology for both sectors:

\[ y_j = ze_j(n_{j0})^{\nu_j}(n_{j1})^{1-\nu_j}, \]  

where \( z \) is the aggregate TFP, \( \epsilon_j \) is the sector-specific productivity, and \( \nu_j \) is the production share of type-0 labor in sector \( j \).

Now let \( Z \) denote the set including all aggregate states, namely \( Z \equiv \{ z, N_{g0}, N_{s0}, N_{g1}, N_{s1} \} \), where \( N_{ji} \) is the aggregate type-\( i \) labor employed in sector \( j \). Let \( w_{ji} \) be the sector-specific wage for type-\( i \) labor. The state variables for the firm are \( \epsilon_j \) and \( Z \). I can then formulate the firm’s problem in a recursive fashion:

\[ J(\epsilon_j, Z) = \max_{\{n_{j0}, n_{j1}\}} \{p_jy_j - w_{j0}n_{j0} - w_{j1}n_{j1} + E_{\epsilon_j', Z'}d(Z, Z')J(\epsilon_j', Z')\} \]  

subject to:

\[ 0 \leq n_{ji} \leq L_i, \quad i \in \{0, 1\}. \]

Here, \( d(Z, Z') \) is the firm’s discount factor, which should be consistent with the household’s problem described in the next section.\(^{23}\)

### 3.3.6 Household’s problem

The representative household values consumption and leisure. In each period, it chooses its current consumption and next period’s labor supply while taking the prevailing wage as given. The household also pays a quadratic labor adjustment cost.

\(^{23}\)\( d(Z, Z') = \beta U'(c(Z')) / U'(c(Z)) \), where \( \beta \) is the household’s discount factor and \( U'(c) \) is its marginal utility of consumption. This simple expression of the firm’s discount factor relies on two assumptions: the logarithmic utility in consumption and the separability in utility between consumption and leisure.
I formulate the household’s problem in a recursive manner as well.

\[
V(n_{g0}, n_{s0}, n_{g1}, n_{s1}, Z) = \max_{\{c,m_{ji}, n'_{ji}\}} \{ \log(c) - \psi(n_{g0} + n_{s0} + n_{g1} + n_{s1}) + \beta \mathbb{E}_{Z'} V(n'_{g0}, n'_{g1}, n'_{s0}, n'_{s1}, Z') \}
\]

subject to:

\[
0 \leq c \leq \sum_{i,j} w_{ji}n_{ji} - \sum_{i,j} \left( \frac{\phi_{ji}m_{ji}^2}{2} \right)n_{ji}
\]

quadratic cost

\[
0 \leq m_{ji} \leq 1
\]

\[
n'_{ji} = (1 - \chi_{ji})n_{ji} + m_{ji}u_i
\]

\[
u_i = L_i - \sum_{j} n_{ji}, \text{ where } j \in \{g, s\} \text{ and } i \in \{0, 1\}.
\]

Here, \(n_{ji}\) represents the extensive margin, so this is a model with no intensive margin.

### 3.3.7 Equilibrium Prices

To close the model, the equilibrium prices of intermediate goods, \(p_j\), clear the goods market. Specifically, for the final good:

\[
Y = c + \sum_{i,j} \left( \frac{\phi_{ji}}{2} m_{ji}^2 \right)n_{ji},
\]

where \(Y\) satisfies (3.5), \(j \in \{g, s\}\), and \(i \in \{0, 1\}\). For the intermediate goods, \(y_g\) and \(y_s\) are produced using technology (3.8), and satisfy (3.6) and (3.7) respectively.

Meanwhile, the equilibrium wage, \(w_{ji}\), clears the labor market. Specifically,

\[
n^h_{ji} = n^f_{ji},
\]

where \(n^h_{ji}\) is the amount of labor the household supplies and \(n^f_{ji}\) is the amount of labor demanded by the firms.
3.4 Calibration

The model has a quarterly frequency. I choose the household’s discount rate of $\beta = 0.99$, corresponding to an annual real interest rate of four percent. The labor disutility parameter, $\psi$, is set to 0.33.

3.4.1 Labor Market

I use monthly data from CPS 1975-2010 to calibrate the labor market parameters in my model. As defined in Fact 2, I divide all industries to two categories: goods and services. I define both high school dropouts and high school graduates as type-0 labor (i.e. unskilled). If a person has some college or college-plus education, I define him as type-1 labor (i.e. skilled).

Following Shimer (2005), the monthly exogenous labor separation rate is calculated as follows.

$$
\chi_{ji,t} = \frac{u_{ji,(t+1)}}{e_{ji,t} \cdot (1 - \frac{1}{2} \sum_j f_{ji,t})},
$$

where $e_{ji,t}$ is the number of type-$i$ workers employed in sector $j$ at month $t$, $u_{ji,t}$ is the number of type-$i$ unemployed workers at month $t$ whose last job was in sector $j$, and $f_{ji,t}$ is the fraction of type-$i$ unemployed workers who found a job in sector $j$ at month $t$. I compute $f_{ji,t}$ as:

$$
f_{ji,t} = \frac{e_{ji,(t+1)} - e_{ji,t}}{\sum_j u_{ji,t}}.
$$

Since the model frequency is quarterly, I first obtain the quarterly average of the monthly separation rates and then calculate the mean of the quarterly rates for two periods, 1975-1989 and 1990-2010. Table 2 lists the values of $\chi_{ji}$ used in the model.
I adjust for frequency consistency between data and the model in the same manner wherever it is appropriate for all remaining parameters.

To calculate type-i labor force, I first compute $\tilde{L}_{i,t} = \sum_j e_{ji,t} + \sum_j u_{ji,t}$, so $\tilde{L}_{i,t}$ is the actual number of workers. Assuming that the economy has a constant population of one, I then set $L_{i,t} = \frac{\tilde{L}_{i,t}}{\sum_i \tilde{L}_{i,t}}$. Table 2 also lists the values of $L_i$ used in the model.

Based on my empirical findings, I assume no SBSC during the pre-1990 period and set all quadratic labor adjustment cost to 2.0 (i.e. $\phi_{ji} = 2.0 \quad \forall j, i$). For the post-1990 period, I assume that the SBSC in services makes it twice as costly for unemployed labor to move to services as to goods (i.e. $\phi_{gi} = 2.0, \phi_{si} = 4.0$).

### 3.4.2 Production Technology

I use industry value added in current dollar 1975-2010 from BEA, together with the CPS data, to calibrate the sectoral production technology. The income share of type-0 labor of the total output in sector $j$ determines $\nu_j$. Therefore, I first obtain sector $j$’s nominal gross output, $y_{j,t}$, from the industry value added data. Following the procedure outlined in Fact 4, I then use the CPS data to compute the mean nominal hourly wage for type-i labor in sector $j$, $w_{ji,t}$. Assuming 40 work hours per week (i.e. 1,920 hours per year), I calculate $\tilde{\nu}_{ji,t} = (1920 \cdot w_{ji,t} \cdot e_{ji,t})/y_{j,t}$. Further assuming that the production only involves the two types of labor, I set $\nu_{j,t} = \tilde{\nu}_{j0,t}/(\tilde{\nu}_{j0,t} + \tilde{\nu}_{j1,t})$. Table 3 lists the values of $\nu_j$ used in the model.

For the final goods production, I set $\rho = 2.0$ based on Broda and Weinstein (2006) that the median elasticity for three-digit sectors is about 2.2. I use the BEA data again to pin down the goods sector’s production share, $\alpha$. Specifically, I compute the
sectoral share of aggregate value added for both goods and services. Then I assume the economy only has two sectors and obtain the values of $\alpha$ as listed in Table 3.

### 3.4.3 Shocks

I use the same CPS and industry value added data to calibrate the shock process. I retrieve non-detrended sectoral Solow residuals as follows:

$$
\begin{align*}
    y_{j,t} &= Z_t \cdot \epsilon_{j,t} \cdot (e_{j0,t})^{\nu_j} (e_{j1,t})^{1-\nu_j}, \\
    \log Z_t + \log \epsilon_{j,t} &= \log y_{j,t} - \nu_j \log(e_{j0,t}) - (1 - \nu_j) \log(e_{j1,t}).
\end{align*}
$$

As illustrated above, the time trend of TFP (i.e. $Z_t$) and the time trends of two sector-specific productivities (i.e. $\epsilon_{j,t}$) are not separately identifiable. Hence, a normalization is needed. Particularly, I assume that the sector-$g$ productivity (i.e. $\epsilon_{g,t}$) is equal to one. I can then compute the aggregate TFP shock for the post-1990 period as:

$$
\log Z_t = \log y_{g,t} - \nu_g \log(e_{g0,t}) - (1 - \nu_g) \log(e_{g1,t}).
$$

The reallocation shock, $\epsilon_{t}$, can be calculated as:

$$
\log \epsilon_t = \log A_{s,t} - \log A_{g,t} = \log \epsilon_{s,t}.
$$

In other words, the reallocation shock is equivalent to the sector-$s$ shock given the normalization.

I detrend $\log Z_t$ and $\log \epsilon_t$ using a time trend and run a bivariate AR(1) regression to obtain the estimates for the two shock processes:

$$
\begin{pmatrix}
    \log Z_t \\
    \log \epsilon_t
\end{pmatrix}
= \begin{pmatrix}
    0.67 & 0.02 \\
    0.19 & 0.02 \\
    0.20 & 0.99 \\
    0.13 & 0.01
\end{pmatrix}
\begin{pmatrix}
    \log Z_{t-1} \\
    \log \epsilon_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
    u_{z,t} \\
    u_{\epsilon,t}
\end{pmatrix}
$$

76
where $\sigma^2_z = 0.002$ and $\sigma^2_\epsilon = 0.0007$ are the variances of innovations, and $\text{cov}(u_z, u_\epsilon) = -0.001$. Also, $u_{z,t}$ and $u_{\epsilon,t}$ are assumed to move in the opposite direction. The negative $\text{cov}(u_z, u_\epsilon)$ obtained from the bivariate AR(1) process appears to validate the nature of the assumed reallocation shock, which makes the productivity in the service sector higher relative to the goods sector during a recession.

For the pre-1990 period, I assume no reallocation shock. The AR(1) regression for the detrended series $\{\log Z_t\}$ indicates $\rho_z = 0.49$ and $\sigma^2_z = 0.0013$.

### 3.5 Simulation Results

Figures 7 and 8 show the impulse responses of total output (i.e. $Y$) and aggregate employment (i.e. $N = \sum_{ji} n_{ji}$) for the pre-1990 and post-1990 period, respectively. The pre-1990 figure indicates that the aggregate employment, together with the total output, recovers soon after a negative TFP shock. The post-1990 figure presents a very slow recovery in the aggregate employment, even though the total output bounces back rather quickly.

In addition, Figure 9 compares the pre-1990 impulse response of goods sector employment to its post-1990 counterpart. The figure shows a swift recovery of goods sector employment before 1990 but a prolonged recovery after 1990. Therefore, the structural model successfully generates a slow job market recovery, particularly in the goods sector, only under the post-1990 conditions. I now analyze the role each major model ingredient has played in generating such contrasting results.

Figure 7 reflects the standard effect of aggregate TFP shocks. A negative TFP shock causes the output of both sectors to fall. It also makes the marginal product
of labor (MPL) in both sectors decrease, leading to a drop in employment. When aggregate TFP starts to recover, we see a simultaneous recovery of sectoral output and employment. This is not surprising because sectoral output is determined by the sectoral employment which moves with the MPL due to the substitution effect. The aggregate employment (total output) is a (weighted) sum of the two sectoral employment (output). Thus, the total output moves in the same direction and lockstep with the aggregate employment.

Reallocation shocks allow the total output and aggregate employment to move in the opposite direction. Figure 10 presents the impulse responses of both sectoral and aggregate output and employment to a reallocation shock. The reallocation shock makes the MPL in services relatively higher than in goods. Therefore, we see an immediate increase in the service employment as well as its sectoral output. Since labor is now less productive in goods than in services, the employment in goods drops and so does its output. The increase in the services employment is overshadowed by the drop in the goods employment. Meanwhile, the rise in the services output overwhelms the decrease in the goods output. As a result, we observe an increase in total output but a decrease in aggregate employment.

To generate a jobless recovery, we also need $\rho_c > \rho_z$ so that $Y$ can decouple from $N$. More specifically, a negative TFP shock forces the total output to decrease on the impact date even in the presence of a reallocation shock. Since the TFP shock is less persistent than the reallocation shock, the downward pressure on $Y$ quickly dies out and the total output recovers. On the other hand, the lingering reallocation shock keeps the aggregate employment, specifically the goods sector employment, lower and thus generates a slow job market recovery.
Figure 8 shows an overshooting of $Y$, namely the total output rises above its steady state before slowly reverting to the steady state again. The variances of the two shocks determine the magnitude of the overshooting. The larger is $|\sigma_z|$ than $|\sigma_e|$, the less pronounced is the overshooting. This is because the total output is subject to a compound of two effects. The first effect is induced by the negative TFP shock which causes $Y$ to fall. The second effect comes from the reallocation shock which drives up $Y$ but keeps $N$ down. A larger $|\sigma_z|$ strengthens the first effect and a smaller $|\sigma_e|$ weakens the second effect. The data indeed suggest a bigger $|\sigma_z|$, but the magnitude is not big enough to prevent an overshooting of $Y$.

Last, the adjustment cost is crucial in regulating the response of sectoral employment. If we remove the adjustment cost (i.e. $\phi_{ji} = 0$), the employment in goods and services will move in the opposite direction (Figure 11). This is because when labor in services is more productive, it is also costless to move labor to services. Both factors prompt the household to move more labor to services immediately, leading to an immediate increase in the services employment despite the negative TFP shock. This result is counterfactual; labor market data suggest that both sectoral employment initially decreases during recessions. The same logic applies to the case when $\phi_{gi} > \phi_{si}$. Therefore, making the labor adjustment cost higher for services is necessary to generate an initial decrease in its sectoral employment.

3.6 Conclusion

To conclude, this paper first documents four stylized facts. Based on the four facts, it argues that the service sector has experienced an SBSC during the post-1990 era, causing the onset of the jobless recovery phenomenon. Then the paper
proposes a structural model where the SBSC is incorporated as two main features: a sector-specific labor adjustment cost and a reallocation shock raising the relative productivity in the service sector. The model successfully generates a slow recovery in both aggregate and goods sector employment despite a rather quick rebound of total output.

The results here have an important policy implication: stabilization policies aiming to increase aggregate demand as an effort to boost the job market will be ineffective. Instead, policy makers need to adopt policies that can quickly facilitate unemployed workers in the goods sector to gain necessary skills and become suitable to work in the service sector.
Figure 3.1: Total Non-Farm Quarterly Employment Growth, 1954Q2-2012Q2

Figure 3.2: Sectoral Employment Quarterly Growth, 1954Q2-2012Q2
Figure 3.3: Educational Attainment for Males in Goods Sector, 1975-2011

Figure 3.4: Educational Attainment for Males in Service Sector, 1975-2011
Figure 3.5: Difference btw Service and Goods Sector Skill Premium, 1975-2011

Figure 3.6: Timing of the Events
Figure 3.7: Pre-1990 Impulse Response of $Y$ and $N$

Figure 3.8: Post-1990 Impulse Response of $Y$ and $N$
Figure 3.9: Pre-1990 vs Post-1990 Impulse Response of Goods Employment

Figure 3.10: Post-1990 Reallocation Shock Only
Figure 3.11: Post-1990 Benchmark vs. No Adjustment Cost

<table>
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<tr>
<td>1953Q2 - 1954Q2</td>
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<td>1</td>
</tr>
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<td>1969Q4 - 1970Q4</td>
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<td>6</td>
</tr>
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<td>4</td>
</tr>
<tr>
<td>1980Q1 - 1980Q3</td>
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<td>1</td>
</tr>
<tr>
<td>1981Q3 - 1982Q4</td>
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<td>2007Q4 - 2009Q2</td>
<td>7</td>
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Data Source: NBER; BLS Establishment Survey

Table 3.1: Total Non-Farm Employment Recovery Timeline
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</tr>
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<td>$\phi_{si}$</td>
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Table 3.2: Labor Market Parameters

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<td>share of services VA</td>
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Table 3.3: Production Technology Parameters
References


Appendix A: Derivation of Aggregate Capital

We can always derive the goods market clearing condition using the budget constraint of a representative household. Below is the representative household’s budget constraint:

\[ c + qa' = (1 - \tau)wN + transfer + a. \]

Now let \( K = qa \). We can rewrite the budget constraint as:

\[ c + K' = (1 - \tau)wN + transfer + \frac{K}{q} \]

where

\[ w = (1 - \alpha)\frac{Y}{N} \]

\[ \frac{1}{q} = \alpha \frac{Y}{K} + 1 - \delta. \]
Hence,

\[ c + K' = (1 - \tau)(1 - \alpha)\frac{Y}{N} \cdot N + \text{transfer} + K \cdot (\alpha \frac{Y}{K} + 1 - \delta) \]

\[ = (1 - \alpha)Y - \tau(1 - \alpha)Y + \text{transfer} + \alpha Y + (1 - \delta)K \]

\[ = Y - \tau(1 - \alpha)Y + \text{transfer} + (1 - \delta)K \]

\[ = Y - (\tau w N - \text{transfer}) + (1 - \delta)K. \]

Thus, we have derived the goods market clearing condition:

\[ c + K' - (1 - \delta)K + G = Y. \]

This confirms that \( q \) is needed to calculate \( K \).
Appendix B: Income and Wealth Data

B.1 Labor Income Process

I use the Panel Study of Income Dynamics (PSID) over the 1968-1997 period to estimate the labor income process in my model.\textsuperscript{24} The statistical model and estimation method are adopted from Heathcote et al.(2010).

B.1.1 Sample Selection

My sample selection criteria mainly follow Heathcote et al.(2010) with the exception that I only include households whose heads have remained the same over the entire period. I use the PSID family public data but restrict my focus on heads of households.\textsuperscript{25} The following list details my sample selection criteria.

- The head of household is considered unchanged if 1) no individual moves in or out of the family, 2) there is a change in members other than head or wife only, or 3) the head is the same but the spouse left, died, or has changed.

- The head is aged between 25 and 60.\textsuperscript{26}

\textsuperscript{24}The frequency of the PSID data have changed from annual to biennial since 1997. My model has an annual frequency, so I exclude the biennial PSID data from my analysis.

\textsuperscript{25}Heathcote et al.(2010) argue that endogenous labor participation choices can contaminate the estimation of income dynamics for secondary earners.

\textsuperscript{26}In the PSID, while age of the head refers to the survey year, questions about income and the number of hours worked refer to the previous year. Following Heathcote et al.(2010), I do not adjust this timing discrepancy.
- The head’s annual number of hours working for money is bigger than 260.

- The head’s annual labor income is positive.\textsuperscript{27}

\textbf{B.1.2 Estimation Method}

First, I retrieve log hourly wage residuals from the following Mincerian regression which is run separately year by year.\textsuperscript{28}

\[
\log E_t = \beta_0 + \beta_1 \text{male} + \beta_2 \text{age} + \beta_3 (\text{age} \times \text{age}) + \beta_4 \text{college} + \beta_5 \text{white}
\]

\(E_t\): hourly wage at year \(t\)

\(\text{male}\): 1 if male; 0 otherwise

\(\text{college}\): 1 if went to college (regardless if a degree is received); 0 otherwise

\(\text{white}\): 1 if white; 0 otherwise

Second, I assume the log hourly wage residual follows a permanent-transitory statistical process. More specifically, let \(w_{i,c,t}\) be the log hourly wage residual for individual \(i\) of cohort \(c\) at year \(t\).

\[
w_{i,c,t} = z_{i,c,t} + \epsilon_{i,c,t}, \quad z_{i,c,t} = z_{i,c,t-1} + \eta_{i,c,t},
\]

where \(\epsilon_{i,c,t}\) and \(\eta_{i,c,t}\) are i.i.d. across individuals and uncorrelated with each other or over time. Therefore, \(\epsilon_{i,c,t}\) can be interpreted as a transitory income shock and \(\eta_{i,c,t}\) as a permanent income shock. I can then express the variances of the two shocks, \(\sigma_{\epsilon,t}\)

\textsuperscript{27}The labor income includes labor part of farm income and business income, bonuses, overtime, commissions, professional practice, and labor part of income from roomers and boarders or business income.

\textsuperscript{28}I define the hourly wage as the head’s annual labor income divided by the annual number of hours he worked. All wages are deflated using annual BEA price indexes for GDP (2009=100).
and $\sigma_{\eta,t}$, as follows.

$$\text{var}_c(w_{i,c,t}) - \text{cov}_c(w_{i,c,t+1}, w_{i,c,t}) = \sigma_{\epsilon,t},$$

$$\text{var}_c(w_{i,c,t}) - \text{cov}_c(w_{i,c,t}, w_{i,c,t-1}) = \sigma_{\eta,t} + \sigma_{\epsilon,t}. $$

Since neither $\sigma_{\epsilon,t}$ nor $\sigma_{\eta,t}$ depends on cohort $c$ under the true model, I can estimate variances at year $t$ by averaging across all sample cohorts.

B.1.3 Results

The table below lists the summary statistics of the shocks. Following KV(2011), I treat transitory shocks as measurement errors. Therefore, the variance of labor income shocks in my model is set to 0.006, the average value for the variances of permanent shocks from 1968 to 1993. I decide to select the average value from 1968 to 1993 for two reasons. First, the sample size drops below 500 after 1994. Second, the PSID data are released in two stages: early release and final release. The early release version is more preliminary. The 1994-onward data are still at the early release stage.

B.2 Asset Holdings

I use the Summary Extract Dataset of Survey of Consumer Finance 2001 (SCF 2001) to calculate the mean and median liquid and illiquid wealth held by households of each age group.

29 Similar to Heathcote et al.(2010), my estimates of $\sigma_{\eta,t}$ have negative values. Heathcote et al.(2010) attribute the negative values to potential misspecification, an issue they call for future research to resolve.

30 Heathcote et al.(2010) find the variance of permanent shocks to be 0.007, a value very close to mine.
<table>
<thead>
<tr>
<th>Year</th>
<th># of Obs.</th>
<th>(\text{var}(w_{i,t}))</th>
<th>(\text{corr}(w_{i,t+1}, w_{i,t}))</th>
<th>(\sigma_{\epsilon,t})</th>
<th>(\sigma_{\eta,t})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1968</td>
<td>2,931</td>
<td>0.324</td>
<td>0.712</td>
<td>0.110</td>
<td>n.a.</td>
</tr>
<tr>
<td>1969</td>
<td>2,697</td>
<td>0.280</td>
<td>0.761</td>
<td>0.063</td>
<td>0.003</td>
</tr>
<tr>
<td>1970</td>
<td>2,602</td>
<td>0.292</td>
<td>0.760</td>
<td>0.069</td>
<td>0.005</td>
</tr>
<tr>
<td>1971</td>
<td>2,510</td>
<td>0.295</td>
<td>0.784</td>
<td>0.057</td>
<td>0.015</td>
</tr>
<tr>
<td>1972</td>
<td>2,409</td>
<td>0.314</td>
<td>0.758</td>
<td>0.075</td>
<td>0.000</td>
</tr>
<tr>
<td>1973</td>
<td>2,322</td>
<td>0.314</td>
<td>0.743</td>
<td>0.086</td>
<td>−0.010</td>
</tr>
<tr>
<td>1974</td>
<td>2,205</td>
<td>0.299</td>
<td>0.733</td>
<td>0.078</td>
<td>−0.007</td>
</tr>
<tr>
<td>1975</td>
<td>2,117</td>
<td>0.305</td>
<td>0.753</td>
<td>0.075</td>
<td>0.009</td>
</tr>
<tr>
<td>1976</td>
<td>1,962</td>
<td>0.307</td>
<td>0.746</td>
<td>0.073</td>
<td>0.004</td>
</tr>
<tr>
<td>1977</td>
<td>1,840</td>
<td>0.321</td>
<td>0.780</td>
<td>0.071</td>
<td>0.016</td>
</tr>
<tr>
<td>1978</td>
<td>1,755</td>
<td>0.320</td>
<td>0.779</td>
<td>0.061</td>
<td>0.009</td>
</tr>
<tr>
<td>1979</td>
<td>1,660</td>
<td>0.346</td>
<td>0.755</td>
<td>0.091</td>
<td>−0.004</td>
</tr>
<tr>
<td>1980</td>
<td>1,567</td>
<td>0.330</td>
<td>0.726</td>
<td>0.085</td>
<td>−0.010</td>
</tr>
<tr>
<td>1981</td>
<td>1,481</td>
<td>0.345</td>
<td>0.755</td>
<td>0.070</td>
<td>0.030</td>
</tr>
<tr>
<td>1982</td>
<td>1,398</td>
<td>0.386</td>
<td>0.681</td>
<td>0.129</td>
<td>−0.019</td>
</tr>
<tr>
<td>1983</td>
<td>1,187</td>
<td>0.368</td>
<td>0.561</td>
<td>0.149</td>
<td>−0.038</td>
</tr>
<tr>
<td>1984</td>
<td>1,201</td>
<td>0.414</td>
<td>0.776</td>
<td>0.082</td>
<td>0.113</td>
</tr>
<tr>
<td>1985</td>
<td>1,118</td>
<td>0.442</td>
<td>0.772</td>
<td>0.093</td>
<td>0.017</td>
</tr>
<tr>
<td>1986</td>
<td>1,044</td>
<td>0.462</td>
<td>0.794</td>
<td>0.078</td>
<td>0.034</td>
</tr>
<tr>
<td>1987</td>
<td>980</td>
<td>0.505</td>
<td>0.808</td>
<td>0.132</td>
<td>−0.011</td>
</tr>
<tr>
<td>1988</td>
<td>884</td>
<td>0.421</td>
<td>0.833</td>
<td>0.065</td>
<td>−0.016</td>
</tr>
<tr>
<td>1989</td>
<td>808</td>
<td>0.434</td>
<td>0.805</td>
<td>0.078</td>
<td>−0.001</td>
</tr>
<tr>
<td>1990</td>
<td>739</td>
<td>0.450</td>
<td>0.747</td>
<td>0.117</td>
<td>−0.023</td>
</tr>
<tr>
<td>1991</td>
<td>657</td>
<td>0.440</td>
<td>0.752</td>
<td>0.071</td>
<td>0.037</td>
</tr>
<tr>
<td>1992</td>
<td>610</td>
<td>0.548</td>
<td>0.675</td>
<td>0.168</td>
<td>0.011</td>
</tr>
<tr>
<td>1993</td>
<td>527</td>
<td>0.579</td>
<td>0.501</td>
<td>0.221</td>
<td>−0.022</td>
</tr>
<tr>
<td>1994</td>
<td>488</td>
<td>0.879</td>
<td>0.416</td>
<td>0.520</td>
<td>0.002</td>
</tr>
<tr>
<td>1995</td>
<td>436</td>
<td>0.849</td>
<td>0.487</td>
<td>0.475</td>
<td>0.014</td>
</tr>
<tr>
<td>1996</td>
<td>400</td>
<td>0.694</td>
<td>0.311</td>
<td>0.452</td>
<td>−0.132</td>
</tr>
<tr>
<td>1997</td>
<td>271</td>
<td>0.868</td>
<td>n.a.</td>
<td>n.a.</td>
<td>0.626</td>
</tr>
</tbody>
</table>

### 1968-93 mean

<table>
<thead>
<tr>
<th>(\text{var}(w_{i,t}))</th>
<th>(\text{corr}(w_{i,t+1}, w_{i,t}))</th>
<th>(\sigma_{\epsilon,t})</th>
<th>(\sigma_{\eta,t})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.372</td>
<td>0.752</td>
<td>0.088</td>
<td>0.006</td>
</tr>
</tbody>
</table>

### 1968-97 mean

<table>
<thead>
<tr>
<th>(\text{var}(w_{i,t}))</th>
<th>(\text{corr}(w_{i,t+1}, w_{i,t}))</th>
<th>(\sigma_{\epsilon,t})</th>
<th>(\sigma_{\eta,t})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.426</td>
<td>0.705</td>
<td>0.135</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Table B.1: Summary Statistics, PSID 1968-1997

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When defining the liquid and illiquid wealth, I try to follow KV (2011, Table 2) as closely as possible. The following list describes how I define the liquid wealth.

- Variable LIQ is used as the estimate of cash, checking, saving, and MM accounts.

- I use NMMF, STOCKS, and BOND as the estimate of directly held MF, stocks, and bonds.

- Credit card debt is set to the value of CCBAL.

- Hence, the liquid wealth is defined as the sum of LIQ, NMMF, STOCKS, and BOND, minus CCBAL.

For the illiquid wealth, I adopt the following definition.

- The difference between HOUSES and MRTHEL is used as housing net of mortgage.

- Vehicles net of installment loans is the difference between VEHIC and INSTALL.

- I use RETQLIQ, CASHLI, CDS, and SAVBD as the estimate of retirement accounts, life insurance, certificates of deposit, and saving bonds.

- Hence, the illiquid wealth is defined as the sum of the above-listed variables.

A household’s net worth is defined as the sum of liquid and illiquid wealth. I construct my sample by retaining only households who are aged 22 to 78 and excluding those with net worth in the top 5%. All statistics reported in Table 1 and Figure 4 are weighted and inflation adjusted to 2010 U.S. dollars.
B.3 Transfers

I use the same SCF 2001 sample to determine the value of $p_j$ in my transfer function. $p_j$ is defined as $\frac{\text{age group } j\text{'s social security income}}{\text{total social security income of age 60-78}}$. I use SSRETINC as the estimate of social security income received by each household of age $j$. First, I compute the weighted sum of social security income received by all households of age $j$ in the data. Second, I compute the weighted sum of social security income received by all households who are 60- to 78-year-old. Last, I set $p_j$ to the ratio of the two. The figure below shows all the values of $p_j$. 

Figure B.1: Deterministic Age-specific Effect $p_j$ on Retirees’ Earnings
Appendix C: Numerical Solution Method

C.1 Household Problem with Fixed Labor Supply

The household problem within each period can be rewritten as a two subperiod problems.

C.1.1 Subperiod 2 Problem

Let $b_2$ be the within-period non-wage resources available to a particular household for its consumption and liquid asset savings. Then $b_2 = (m + a - q' a' - \kappa)$ for households who choose to adjust their illiquid assets (i.e. adjusters), and $b_2 = m$ for households who choose not to adjust their illiquid assets (i.e. non-adjusters). Also, let $j$ be the age of a household, $H(\kappa)$ the c.d.f. of adjustment cost, and $V^1$ the household’s value function during subperiod 1. Now I can formulate the subperiod 2 value function for each household as follows,

$$V^2(a', b_2, \epsilon, j) = \max_{c, m'} u(c) + \beta \sum_{l=1}^{N(\epsilon)} \pi(\epsilon'|\epsilon) \int_{\kappa'} V^1(a', m', \epsilon'|j + 1, \kappa') H(d\kappa')$$

subject to:

$$c + q^m m' \leq (1 - \tau) w \epsilon h + b_2$$

$$m' \geq 0.$$
For retirees, I substitute \((1 - \tau)w\epsilon h\) with \(\rho(j, \epsilon J\omega)\) and remove the expectation over \(\epsilon_t\).

**C.1.2 Subperiod 1 Problem**

Let \(b_1 = (m + a - \kappa)\) be the within-period post-\(\kappa\) wealth of an adjuster. The subperiod 1 value function for an adjuster is as follows,

\[
E^{A1}(b_1, \epsilon, j) = \max_{a'} V^2(a', b_1 - qa', \epsilon, j)
\]

subject to:
\[
a' \geq 0.
\]

The subperiod 1 value function for a non-adjuster is simply \(V^2(\frac{a}{\epsilon^2}, m, \epsilon, j)\).

There are three cases to consider when computing the subperiod 1 value function.

1. **Case 1**
   The household cannot afford positive consumption if it does not adjust \(a\). In other words, \(c \equiv (1 - \tau)\epsilon hw + m - q^m m \leq 0\). Then \(V^1(a, m, \epsilon, j, \kappa) = E^{A1}(b_1, \epsilon, j)\).

2. **Case 2**
   The household chooses whether to adjust or not. Then \(V^1(a, m, \epsilon, j, \kappa) = \max\{V^2(\frac{a}{\epsilon^2}, m, \epsilon, j), \ E^{A1}(b_1, \epsilon, j)\}\).

3. **Case 3**
   The household cannot afford positive consumption if it adjusts \(a\). In other words, \(c \equiv (1 - \tau)\epsilon hw + m + a - q^m m - qa - \kappa \leq 0\). Then \(V^1(a, m, \epsilon, j, \kappa) = V^2(\frac{a}{\epsilon^2}, m, \epsilon, j)\).

Here, \(m\) and \(a\) are the borrowing limit for liquid and illiquid assets, respectively. In this paper, both \(m\) and \(a\) are set to zero.

After obtaining the subperiod 1 value function at each \(\kappa(n)\), I compute
\[
\int_\kappa V^1(a, m, \epsilon, j, \kappa)H(d\kappa) = \sum_{n=1}^{N(\kappa)} V^1(a, m, \epsilon, j, \kappa(n)) \cdot wt(n). \]

Here, \(wt(n)\) is the probability weight for each \(\kappa(n)\) and \(N(\kappa)\) is the total number of grid points on the adjustment cost grid. For retirees, I substitute \((1 - \tau)w\epsilon h\) with \(\rho(j, \epsilon J\omega)\).
C.2 Household Problem with Endogenous Labor Supply

The household problem within each period can still be rewritten as a two subperiod problems as illustrated above. The difference here is that I add labor choice to subperiod 2 problem for working-age households. For retired households, the two subperiod problems remain identical to those in the fixed labor supply case. Unless stated otherwise, all the notations used in this section are consistent with the fixed labor supply case.

C.2.1 Subperiod 2 Problem for Workers

Let $c_{low}$ and $L_{low}$ denote the lower-bound for a household’s consumption and leisure, respectively. Then I can formulate the subperiod 2 problem:

$$V_2(a', b_2, \epsilon, j) = \max_{m', c, n} \left[ u(c, n) + \beta \sum_{l=1}^{N(\epsilon)} \pi(\epsilon_l' | \epsilon) \int_{\kappa'} V_1(a', m', \epsilon_l', j + 1, \kappa') \, H(d\kappa') \right]$$

subject to:

$$c + q^m m' \leq (1 - \tau)w \epsilon n + b_2$$

$$m' \geq 0$$

$$c \geq c_{low}$$

$$n \leq 1 - L_{low}.$$
For any potential $m'$ considered above, I can find a corresponding optimal $c^*(m', b_2, \epsilon)$ by solving the problem below:

$$\max_c u(c, n)$$

subject to:

$$n = \frac{c + q^m m' - b_2}{(1 - \tau)w\epsilon},$$

$$R_{low} \leq c \leq b_2 + w\epsilon(1 - L_{low}) - q^m m'.$$

Therefore, the corresponding optimal $n^* = \frac{(c^* + q^m m' - b_2)}{(1 - \tau)w\epsilon)$.

### C.2.2 Subperiod 1 Problem for Workers

The subperiod 1 problem remains the same as that in the fixed labor supply case. However, the upper- and lower-bounds for the three cases to consider have changed.

Let $R_{low} = c_{low} - w\epsilon(1 - L_{low})$ be the minimum within-period non-wage wealth that makes $c_{low}$ and $L_{low}$ possible. The three cases to consider when computing the subperiod 1 value function are as follows.

1. **Case 1**
   - The household cannot afford $c_{low}$ or $L_{low}$ if it does not adjust $a$. In other words, $m \leq R_{low} + q^m m$. Then $V^1(a, m, \epsilon, j, \kappa) = E^{A1}(b_1, \epsilon, j)$.

2. **Case 2**
   - The household chooses whether to adjust or not. Then
     $$V^1(a, m, \epsilon, j, \kappa) = \max\{V^2(\frac{a}{q^m}, m, \epsilon, j), \quad E^{A1}(b_1, \epsilon, j)\}.$$

3. **Case 3**
   - The household cannot afford $c_{low}$ or $L_{low}$ if it adjusts $a$. In other words, $a + m - \kappa \leq R_{low} + q^a a + q^m m$. Then $V^1(a, m, \epsilon, j, \kappa) = V^2(\frac{a}{q^m}, m, \epsilon, j)$.

### C.3 Algorithm for Household Problem

Below, I briefly describe the algorithm used to solve the decision problem of a household who is $j$ years old.
1. At each $\epsilon$:

i Obtain a set of bivariate Chebyshev polynomial coefficients $cpv$ over $(a, m)$ using $V(a, m, \epsilon, j + 1)$. Notice $V(a, m, \epsilon, J + 1) = 0$ for $j = J$.

ii Loop over each set of $(a, b2)$ to obtain subperiod 2 value function $V^2(a, b2, \epsilon, j)$ using $cpv$ from (i).

iii Obtain another set of bivariate Chebyshev polynomial coefficients $cpv2$ over $(a, b2)$ using $V^2(a, b2, \epsilon, j)$.

iv Loop over each set of $(a, m)$ to obtain $V^0(a, m, \epsilon, j)$.

   - At each $\kappa$, I compute subperiod 1 value function $V^1(a, m, \epsilon, j, \kappa)$. Specifically, I first use $cpv2$ from (iii) to approximate the value function for adjusters $E^{AI}(a + m - \kappa, \epsilon, j)$ and the value function for non-adjusters $V^2(a/q^#, m, \epsilon, j)$. Then I obtain $V^1$ based on the three cases listed under either the fixed labor supply problem or the endogenous labor supply problem.

   - I set $V^0(a, m, \epsilon, j) = \sum_{n=1}^{N(\kappa)} V^1(a, m, \epsilon, j, \kappa(n)) \cdot wt(n)$.

2. At each $\epsilon$:

Loop over each set of $(a, m)$ to obtain $V(a, m, \epsilon, j) = \sum_{\epsilon'} \pi(\epsilon'|\epsilon)V^0(a, m, \epsilon', j)$ for workers. For retirees, I remove the expectation calculation over $\epsilon'$.
Appendix D: Additional Figures for Educational Attainment and Skill Premium

This appendix includes additional graphs for educational attainment and skill premium. Unless stated otherwise, the definitions of sectors, educational attainment, and skill premium are the same as in Chapter 3.

Figures 1 and 2 illustrate the sectoral educational attainment for both male and female workers.

Figure 3 shows the difference between the skill premium in the goods and service sector for high school graduates, some college, and college-plus workers.

Figures 4 and 5 indicate the composition of college and non-college workers in the goods and service sector, respectively. I group together workers who are either high school dropouts or high school graduates and define them as non-college workers. The college workers are those who have either some college or college plus education.

Figure 6 shows the difference between the skill premium in the goods and service sector for college workers. The skill premium for college workers within each sector is computed as the difference between the mean log hourly wage of college workers and the mean log hourly wage of non-college workers.

Figures 7 and 8 indicate the composition of college and non-college workers in the tradable and non-tradable goods sector, respectively. The tradable goods sector
consists of mining and manufacturing, while the non-tradable sector includes construction, transportation, utilities, trade, financial activities, and services. In other words, the tradable goods sector is equivalent to the previously-defined goods sector excluding construction. The non-tradable sector combines the previously-defined service sector with construction.

Figure 9 shows the difference between the skill premium in the tradable and non-tradable sector for college workers.
Figure D.1: Educational Attainment for Workers in Goods Sector, 1975-2011

Figure D.2: Educational Attainment for Workers in Service Sector, 1975-2011
Figure D.3: Difference btw Service and Goods Sector Skill Premium, 1975-2011

Figure D.4: College vs. Non-college Workers in Goods Sector, 1975-2011
Figure D.5: College vs. Non-college Workers in Service Sector, 1975-2011

Figure D.6: Difference btw Service and Goods Sector Skill Premium for College Workers, 1975-2011
Figure D.7: College vs. Non-college Workers in Tradable Sector, 1975-2011

Figure D.8: College vs. Non-college Workers in Non-Tradable Sector, 1975-2011
Figure D.9: Difference btw Tradable and Non-Tradable Sector Skill Premium for College Workers, 1975-2011