INCREMENTAL CAPITAL COSTS
OF LOCAL TELEPHONE SERVICE

A Thesis
Presented in Partial Fulfillment of the Requirements
for the Degree Master of Science

by
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Approved by
[Signature]
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Department of Industrial
and Systems Engineering
To my parents and friends.
ACKNOWLEDGMENTS

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CHAPTER 1
INTRODUCTION

1.1 General

The costs incurred by any telephone operating company can be divided into capital and non-capital costs. Non-capital costs include expenses for maintenance, traffic, commercial and marketing, construction for station connections, general office salaries and "other." The usual utility and regulatory practice, with respect to capital costs, is to identify parts of the capital budget by subdividing it into five principal plant categories: (1) Land and Buildings (L&B), (2) Central Office Equipment (COE), (3) Outside Plant (OSP), (4) Station Equipment (SE) and Station Connections (SC), and (5) General Equipment (GE). Expensing of station connection costs, and the FCC's 1982 Second Computer Inquiry (CI II) deregulating station equipment have reduced previously capitalized expenditures to much lower levels. Expenditures for COE now constitute the major portion of the construction budget; the California Public Utilities Commission (7) concluded that COE expenditures constituted over 45 percent of the 1983 total capital spending of Pacific Telephone and Telegraph Company. Table 1.1 shows some representative COE and OSP percentage figures for the New York Telephone Company.
**TABLE 1.1**

**COE AND OSP PERCENTAGE EXPENDITURES**

<table>
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<tr>
<th>Account</th>
<th>1982</th>
<th>1983</th>
<th>1984</th>
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<td>COE</td>
<td>46.2%</td>
<td>54.4%</td>
<td>59.2%</td>
</tr>
<tr>
<td>OSP</td>
<td>19.1%</td>
<td>21.2%</td>
<td>22.8%</td>
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Source: New York Public Service Commission (10)

* 1982 and 1983 are based on actual recorded values; 1984 on forecasts.

1.2 Problem Definition

This is a cost-of-service study, which is primarily interested in determining Ohio Bell's central office capital costs. The definition of a central office that will be used throughout this study, as stated in the Bell System's Engineering Manual (3), is: "The term is sometimes used to refer to a telephone company building in which a switching system, that connects lines to lines and lines to trunks, is located and to include other equipment (such as transmission system terminals) that may be located in such a building." The types of central office equipment currently used by telephone companies are: step by step, crossbar, electronic, and digital. Digital offices constitute the latest technological development; no digital offices, however, presently exist in Ohio Bell's territory. At this time, electronic offices and particularly 1 and 1A ESS (Electronic Switching System), which will be examined in this study, are the most widely used central office categories.
The major objective of this study is to develop and test a method to determine the nature of the central office capital cost function\(^1\) for a selected ESS central office. This objective can be equivalently stated as determining the incremental capital costs of engineering measures of output of a central office. Central office capital costs can be divided in two broad categories: (1) capital costs incurred when building a new office, and (2) capital costs incurred when planning capacity expansions for existing offices. There are two major reasons for treating these two cases separately: (1) the build-up of a brand new central office exhibits significant (one-time) start-up costs (e.g. for purchasing memory modules, the generic program, etc.), while a usual capacity expansion situation has small or zero start-up cost, (2) the magnitudes of the additional engineering measures of output are significantly different. For example, the build-up of a small ESS office typically involves the installation of more than ten thousand lines, while an average capacity expansion involves the addition of a few hundred lines per year. Speaking in economic terms, such a difference in magnitude may produce significant incremental cost differences, if economies or diseconomies of scale exist. This study will examine only the most frequently occurring case, that of planning facility additions to an existing office. Different levels of growth for each planning variable will be assumed, so as to reach conclusions about the nature of the capital cost function.

\(^1\)Capital costs are determined by the Central Office Equipment Engineering Systems model.
The total cost of a capacity expansion will be expressed as
\( C(e_1, e_2, \ldots, e_n) \), where \( e_1 \) are attributes of the central office output
vector \( \mathbf{a} \). For example, \( e_1 \) could be the growth of single-party
residential main stations, \( e_2 \) the growth of PBX or direct indial
business trunks, \( e_3 \) the growth of busy-hour intraoffice usage (business
and/or residential), \( e_4 \) the growth of busy-hour interoffice local or
toll usage (business and/or residential), \( e_5 \) the number of calling
attempts by residences, and \( e_6 \) the number of calling attempts by
businesses. This study will restrict it's attention to local basic
telephone service; therefore, only local service variables will be
considered. Variables related to toll or special vertical services
will be held constant at their current levels. Detailed explanations
concerning the variable selection procedure will be given in chapter
III.

The problem definition is concluded by presenting the major metho-
diological differences of this study as compared to usual cost-of-
service studies; they are: (I) this is an incremental cost study, and
not an average cost one, (II) this is a "bottom-up" approach using data
from a specific central office, and not a "top-to-bottom" approach that
would use company-wide cost and demand data, (III) this is a designed
experiment, in which the sizes of the expansions are specified by the
experimenter, and, therefore, historic cost and demand data are not
used. Such a methodology is selected in an attempt to overcome
multicollinearity problems, which are present in most cost-of-service
studies. The above concepts will be defined in subsequent sections,
where the differences will be clearly pointed out.
1.3 Average and Marginal Cost

The following two approaches with respect to estimating capital and non-capital costs of telephone services have been developed: (1) fully distributed costs (FDC), and (2) marginal costs (MC). The FDC method is essentially based on long-run average cost pricing. Let us initially assume for simplicity that a firm produces just a single commodity, it has a total installed plant capacity equal to K, and its respective output level is equal to q. The total cost of production is equal to the sum of the capacity cost (CC(·)) and the variable cost (VC(·)). The capacity cost is an increasing function of the plant size K: \( \frac{\delta CC(K)}{\delta K} > 0 \). In the short run, the plant capacity K remains constant (CC(K)=b, where b is a positive constant). Average total cost (ATC), average variable cost (AVC), and average capacity cost (ACC) are defined as the respective total, variable, and capital costs divided by the output level:

\[
ATC = \frac{CC(K) + VC(q)}{q}, \quad AVC = \frac{VC(q)}{q}, \quad ACC = \frac{CC(K)}{q}
\]

By definition, marginal cost is the difference in total cost resulting from a unit difference in quantity. Mathematically, marginal cost (MC) is the first derivative of the total cost function with respect to output, or the slope of the total cost curve at a given
point in time. Therefore: \( MC = \frac{dTC(K, q)}{dq} \). Incremental cost (IC) is the
difference in total cost resulting from a difference in quantity of
more than one unit. Therefore:

\[
IC = \frac{TC(K_i, q+\Delta q) - TC(K_j, q)}{\Delta q}
\]

where: \( \Delta q > 1 \) and \( K_i > K_j \)

Incremental costs are used instead of marginal costs in most cost-of-
service studies. Engineering models, in particular, may not be sensi-
tive enough to account for just one additional unit of service; incre-
mental costs can provide a reasonable approximation to marginal costs
when adding more than one unit of service.

In reality, a telecommunications firm is a multi-output producer.
Fuss and Waverman (9) list several examples from the wide variety of
services that a telephone company provides, such as: residential local
switched calls, business local switched calls, residential and business
Intrastate toll switched calls, private wire services, teletypewriter
exchange service, Intrastate access to long-distance carriers, and a
variety of broad band data services. The estimation of the average
total cost of a particular product or service for a multi-product firm,
requires using accounting (something referred to as separation
procedures) to identify the portion of common or joint costs for which
this service is responsible. Once these costs are identified, the
average total cost can be estimated by dividing the total cost of that
service by its respective output quantity.
Let $q_1, q_2, \ldots, q_n$ represent the outputs of $n$ services provided by the shared capacity $K$ ($K$ is a vector). The (long-run) total cost function can be expressed as: $TC = TC(K, q_1, q_2, \ldots, q_n)$. The marginal cost of the $i$th service is:

$$MC_i = \frac{\delta TC(K, q_1, q_2, \ldots, q_i, \ldots, q_n)}{\delta q_i}$$

Therefore, the marginal cost of the $i$th service is the difference in total cost resulting from a unit difference in the output of the $i$th service, when all other output quantities remain constant. For more details on common and joint cost separation methodologies see Pollard (17).

Kahn (12) recommends the use of fully distributed costs for those portions of demand that are sufficiently inelastic. For the case of elastic demand Kahn and Bonbright (6) have identified the following defects of FDCs: (1) they do not provide a reliable measure of the change in future costs in the case that a particular service is expanded or curtailed, (2) for long-run decreasing cost conditions, FDCs consistently overestimate incurred costs. On the other hand, for long-run increasing cost conditions, FDCs are lower than the actual costs of increased service, (3) allocations of costs to different services do not accurately reflect their respective cost responsibilities, (4) peak utilization costs are not properly recognized.

The Federal Communications Commission (FCC) (8) has expressed the following conclusions with respect to MC theory: "...there is considerable merit in the basic concepts of marginal or incremental
cost pricing, and these concepts could have broad applicability in the communications field provided that certain theoretical and practical problems identified herein can be resolved, ... immediate application would appear to be in the pricing of peak/off peak usage and other services within the basic monopoly service offerings,..." However, AT&T's concept of marginal cost was rejected by the FCC for the following reasons: (1) failure to meet the basic objectives of marginal cost pricing theory, (2) uncertainty of demand and technological forecasts, (3) incapability of preventing cross-subsidization between monopoly and competitive services, and (4) incompleteness of accounting records.

1.4 Short and Long-run Marginal Cost

A marginal cost approach can be based either on short-run marginal cost (SRMC) or on long-run marginal cost (LRMC). The economic terms "short-run" and "long-run" do not have any relationship to time; most firms at any given point in time make both short-run and long-run decisions. The difference is that SRMC relates to decisions concerning output quantities under a constant plant capacity, while LRMC relates to decisions concerning the acquisition of additional fixed factors of production (e.g. new equipment, additional manpower, etc.).

Kahn defines SRMC as follows: "Short-run marginal cost is simply the change in total variable cost caused by producing one additional unit. Variable costs include any sacrifice of future value or any future realization of higher costs that are causally attributable to present production." Depreciation, cost of capital, maintenance, replacement, and various other overhead expenses belong to SRMC, as
long as they are incurred as a function of use. For example, the part of depreciation related to physical deterioration with time belongs to SRMC; on the other hand, the part of depreciation related to technological obsolescence is not a function of use and, should not be included in SRMC calculations. According to Kahn, the economic ideal would be to base cost estimation and prices on SRMC, with appropriate adjustments for the problem of second best (this problem will be discussed later in this section). He, however, admits that such an approach may permit wide price fluctuations along the SRMC function, depending on the relation of demand to capacity.

A feasible solution to the problem of estimating incurred costs should be based on LRMC, which includes: (1) the average incremental variable cost of the additional units produced and (2) the incremental capital cost per unit of additional capacity. LRMC is forward-looking, and as such must be applied to a future time period sufficiently long to insure that production factors have been adjusted to the changed levels of demand (AT&T[1]). Figure 1.1 shows the relationship between SRMC, LRMC, ATC, and AVC for the case of long-run constant returns to scale.

Pollard (18) states that under the regulatory constraint, which specifies that total revenues should be equal to total costs, it may be wrong to equate prices and LRMCs. The above, is usually referred to as the "second best pricing problem." The basic theorem known as the inverse elasticity or simple Ramsey-pricing rule indicates conditions under which prices are required to deviate from marginal costs in a
Fig. 1.1 The unit costs of a firm under long-run constant returns
Source: Kahn (12)

ATC: average total cost
AVC: average variable cost
SRMC: short-run marginal cost
LRMC: long-run marginal cost
LRAC: long-run average cost
systematic manner. Kamershen and Keenan (13) state the inverse elasticity rule as follows: "Under certain strong assumptions quasi-optimal pricing by a multiproduct monopolist operating under a profit constraint mandates that for each product the percentage deviation of quasi-optimal price from marginal costs must be inversely proportionate to its own-price elasticity of demand."

In addition to the above pricing theory, peak load pricing theory also utilizes the principles of marginal cost pricing. While several articles exist in this area, a literature review is beyond the scope of this project. The interested reader is, however, referred to the review article of Bailey and Lindeberg (2), where references to various other publications are listed.

1.5 "Top-to-Bottom" vs. "Bottom-Up" Approach

The usual regulatory approach for rate-making purposes is to use company-wide demand and cost data to determine the average total cost of providing service to different classes of customers; this approach can be described as "top-to-bottom." When attempting to determine the cost responsibilities (average or incremental costs) of various factors such as usage or lines, by using aggregate data in a regression equation, one loses track of peculiarities occurring on a central office basis. For example, the addition of the same number of lines to central offices that have a different number of already installed lines may produce widely divergent cost effects. This is the case, when the termination of the new lines require the use of certain additional expensive equipment (e.g. line link networks, trunk link networks, memory units, etc.).
The intention of this project is to examine Ohio Bell's cost and demand data on a central office basis; this approach can be described as "bottom-up." It is believed that this method can at least provide some additional insights to estimating incremental costs. It is not however, free of problems; conclusions reached from the analysis of specific central offices, or random samples may not be general in nature.

1.6 Multicollinearity

A linear statistical model relating a response $Y$ to a set of $k$ independent variables $X_1, X_2, \ldots, X_k$ is mathematically expressed as follows:

$$ Y = B_0 + B_1X_1 + B_2X_2 + \ldots + B_kX_k + \epsilon $$

where $B_0, B_1, B_2, \ldots, B_k$ represent the unknown parameters and $\epsilon$ the random error component. For example, $Y$ may represent the cost of a central office capacity expansion, while $X_1$ through $X_k$ the growth of several engineering variables such as: lines, trunks, local or long distance usage, local or toll calls, etc. The $n$ ($n \geq k$) joint observations on $(Y, X_1, X_2, \ldots, X_k)$ are presented in two data matrices (notation adapted from Mendelhall (15)):

$$ Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}, \quad X = \begin{bmatrix} 1 & X_{11} & X_{21} & \cdots & X_{k1} \\ 1 & X_{12} & X_{22} & \cdots & X_{k2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{1n} & X_{2n} & \cdots & X_{kn} \end{bmatrix} $$

The coefficients of the general linear model can be estimated by using the following matrix equation:

$$ \hat{B} = (X'X)^{-1}X'Y. $$
The power of the model, and the appropriate interpretation of the estimated coefficients depend on the fulfillment of the underlying assumptions of the linear model. One of the basic assumptions is that the data matrix $X$, which is of order $nxk$, has rank $k$; in other words, no linear dependence exists between the explanatory variables. If the rank of $X$, and hence the rank of the $X'X$ matrix, is less than $k$ the inversion of the $X'X$ matrix is impossible. In this case, the least-squares estimators cannot be calculated. Very serious problems exist when the assumption of linear independence is just satisfied, as would be indicated by a determinant of $X'X$ that is nearly equal to zero; this would for all practical purposes mean that at least some of the independent variables are highly collinear.

Johnston (11) points out the following consequences of multicollinearity: (1) the precision of the estimation drops significantly, and it becomes very difficult to obviate the relative influences of the $X_i$ variables. The estimated coefficients have very large and possibly highly correlated errors, (2) variables with coefficients not significantly different from zero are sometimes incorrectly dropped from the regression equation, (3) estimates of coefficients may be very sensitive to particular sets of data.

Aggregate historic telecommunications demand data often exhibit multicollinearity problems. For example, if the cost of a central office capacity expansion is expressed as a function of line gain, increase in local usage, and increase in toll usage, while the sum of local and toll usage per main station has remained approximately constant over the examined time period, the cost effects of increased
usage and line gains may be indistinguishable. Costs attributable to line gains are partly due to usage increases and vice-versa. We will have more to say on the topic of multicollinearity in section 2.4, where Rohlf's (19) marginal cost approach is reviewed.

A good way of overcoming multicollinearity problems is to use orthogonal experimental design models, whenever possible. If one can choose the levels of the independent variables prior to the data collection, he can design the experiment so that the columns of the X matrix will be mutually orthogonal. If this is the case, $X'X$ is a diagonal matrix and the estimators of all regression coefficients are uncorrelated. The $2^k$ factorial design is an orthogonal design for fitting the multiple linear regression model (Montgomery (16)).

Another technique for attempting to obviate the effects of multicollinear variables is ridge regression. However, ridge regression is beyond the scope of this project, since our approach attempts to remove the problems of collinearity at the experimental design stage rather than the analysis stage where ridge regression might be applicable.

1.7 Summary

The purpose of this chapter was to establish the basic framework of the problem and to clarify the issues involved. Initially, the usual capital and non-capital cost subdivisions were presented, and the importance of central office costs was intoned. We, then, stated the basic objective of this project, which is to develop a method to estimate the incremental capital costs of engineering measures of output of a central office for the case of a capacity expansion. Methodological
differences of this approach as compared to other cost-of-service methods were also pointed out. The concepts of average, short-run, and long-run marginal cost for a multiproduct firm were defined. Advantages and disadvantages of the "top-to-bottom" and the "bottom-up" approach were then presented. Finally, the problem of multicollinearity, which frequently occurs when using historic data, was examined and the reader was introduced to orthogonal experimental designs.
CHAPTER II
LITERATURE REVIEW

2.1 Introduction

The purpose of this chapter is to review three models. The first is the Central Office Equipment Engineering System (4), a computerized model that telephone operating companies use for designing new central offices, or for planning facility additions to existing offices. The second is the Service Cost Information System (5); this model's basic objective is to develop a base office equation to describe the total Engineered, Furnished and Installed Cost for building a new 1/1A ESS central office. The third is a study of marginal costs of telephone service in Washington, D.C., that was prepared by Rohlfis (19) for the Public Service Commission of the District of Columbia.

2.2 Central Office Equipment Engineering System

2.2.1 Objectives and Use

Central office equipment are telephone facilities that must be expanded in discrete increments to meet the continuously increasing demand for telephone services. The Bell Laboratories have developed a series of computer programs to assist telephone engineers in planning capacity expansions for various kinds of central offices. The Central Office Equipment Engineering System (COEES) is an interactive time-sharing computer program designed to satisfy the following objectives:

(1) Permit telephone engineers to determine the capabilities of existing equipment to handle the busy-hour traffic.
(II) Identify and evaluate all possible relief plans or alternatives for a central office in a four year period of growth.

(III) Determine the time that equipment relief should be scheduled, and the resulting economic relief period.

(IV) Estimate the quantities and costs of the necessary equipment categories.

(V) Estimate the service consequences of possible variances in demand, traffic load, and installed equipment quantities.

COEES is mainly used when planning facility additions, or the build-up of brand new 1/1A ESS local or combined local/toll central offices. Table 2.1 provides an exhaustive list of the central office configurations for which COEES can be used, and describes the type of assistance that the model can provide.

2.2.2 Model Description - Inputs

COEES consists of six input forms that cover a total of twenty-three pages and involve four hundred and eighty variables. Most of the variables require the specification of four busy-hour values, one for each of the four years considered. Busy hour inputs are usually estimated as the average of the ten day highest busy-hour values; the absolute maximum value is rarely used. Some variables are required for estimating basic service offerings, others are purely technical, while the rest represent special features offered.

The nature of each of the six input forms can, in short, be described as follows:
<table>
<thead>
<tr>
<th>CENTRAL OFFICE TYPE</th>
<th>ASSISTANCE IN:</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1/1A ESS (1)</td>
<td>facility planning, equipment ordering, engineering information</td>
</tr>
<tr>
<td>No. 2/2B ESS</td>
<td>facility planning, advanced engineering information</td>
</tr>
<tr>
<td>(No. 5 No. 5XB Crossbar)</td>
<td>facility sizing</td>
</tr>
<tr>
<td>SXS (Step by step)</td>
<td>pricing</td>
</tr>
<tr>
<td>Source: Ohio Bell</td>
<td></td>
</tr>
<tr>
<td>(1) ESS: electronic switching system</td>
<td></td>
</tr>
</tbody>
</table>
(1) Form 6403: The purpose of this form is to provide planning forecast data for main stations (MS) and access lines, together with office characteristics of local or combined local/toll central offices. Input data are subdivided in the following categories: (1) Total Office, (2) Area Transfer, (3) Plain Old Telephone Service (POTS), (4) Centrex/ESSX, (5) Remote Switching System, and (6) Loop Range Extensions.

(II) Form 6414: The purpose of this form is to provide trunk forecast data. Trunk forecasts are subdivided in the following categories: (1) Outgoing trunks, (2) Incoming trunks, (3) 2-way trunks (4) Outgoing to operators or desks, (5) Incoming from operators or desks, (6) Centrex trunks, (7) Miscellaneous trunks, (8) Data terminal lines, (9) Miscellaneous trunk data.

(III) Form 6423: The purpose of this form is to provide traffic load and call mix data. Input variables include the following: (1) Network data (e.g. line to junctor ratio, number of line link networks, peak factors, various usage measures, etc.), (2) Memory data, (3) Call capacity data (e.g. absolute busy-hour calls, Centrex calls, remote system station calls, special features, peak-call factors, etc.).

(iv) Form 6430: This form is a service circuit engineering worksheet containing receiver, transmitter, and various circuit data (e.g. ringing, tone, announcement).
(v) Form 6508: The purpose of this form is to provide generic program data for local, combined local/toll, and toll/tandem central offices.

(vi) Form 6513: The inputs of this form assist in sizing the stores of 1 and 1A ESS offices (e.g. call processing stores, translation stores, other memory characteristics).

It is worth noting at this point, that most inputs require engineering judgement; for example, poor selection of peak usage or peak call factors can cause serious service deficiencies. The uniformity and the interactive nature of the model assist in speeding up routine computations.

2.2.3 Outputs

Every run of COEES for an 1/1A ESS central office provides four major categories of output reports; the following:

(I) Office Equipment Plan Summary Report: This report identifies the present office status, and presents equipment requirements for each of the four busy seasons considered.

(II) Capacity Analysis Report: The purpose of this report is to conduct a service check of the proposed equipment installations. Post-installation central office capacities are verified to satisfy usage and main station demand requirements.

(III) Pricing Report: Equipment prices, furnished by Western Electric, are considered in order to calculate the necessary capacity expansion costs. Western Electric and Bell Labs claim that the pricing data provided have proven to be a sufficiently accurate approximation of incurred costs.
(iv) Alternative Plans Evaluation Report: Eight alternative plan combinations that can satisfy the demand for one, two, three or four future years are considered, and their respective present worth equivalents (PWE) are calculated. For comparison purposes, the PWE, the total cost, the installation, dial, and generic costs, and the number of frames used are listed for each alternative. The minimum PWE plan is not necessarily followed; the final plan selection also depends on operating costs, construction budget availabilities, and the manager's confidence on the forecasted traffic values.

2.2.4 Comments

The major concern about the COEES model is that it's cost minimization procedure does not seem to explicitly incorporate the variable operating costs of each alternative for the four year planning period. The natural vehicle for an analysis of total central office costs is a multi-input multi-product cost function. Various equipment quantities, and the man-hours necessary for engineering and installation, may be considered as the multiple inputs. The number of business and residential lines, the number of incoming, outgoing, intraoffice, and interoffice hundred call seconds (CCS), and the number of intraoffice, originating, terminating, incoming, outgoing and thru-switched calls may be considered as a potential list of output quantities.

Installing on optimum size plant involves minimizing the present worth of the total central office costs for the four year planning period. Mathematically, this can be stated as follows:
\[
\min PW = \sum_{i=0}^{3} C_i(K_i) \cdot (P/F, a\%, i) + \sum_{j=1}^{4} V_j(K_{ij}, q_{ij}) \cdot (P/F, a\%, j)
\]

where:

- \( PW \) = present worth of the four year total central office costs
- \( C_i \) = total capital costs in year \( i \)
- \( V_j \) = yearly variable operating costs (incurred at time \( j \))
- \( K_i \) = vector of capacities to be installed in year \( i \)
- \( q_{ij} \) = vector of output quantities at time \( j \)
- \( P/F \) = present worth factor
- \( a\% \) = allowed rate of return

Capital costs are incurred in the beginning of the respective year. Estimating variable operating costs, however, is not a trivial task; the output vector \( q_{ij} \) changes within the yearly periods considered. For the purpose of evaluating alternative plans, a sufficient approximation might be to consider average yearly output quantities incurred at the middle of the respective year. The capital and operating costs of each alternative during the four year planning period can be represented as in Figure 2.1.

![Diagram of PW, C1, V1, C2, V2, C3, V3, C4, V4, + Year 1, + Year 2, + Year 3, + Year 4]

Fig. 2.1 Capital and operating costs incurred during a four year period.
COEES is a Western-Electric proprietary model. Lack of documentation does not allow us to conclude whether or not variable operating costs are properly incorporated in the process of selecting the optimum alternative plan.

2.3 Service Cost Information System

2.3.1 General

The objective of the Service Cost Information System (SCIS) is to develop a base office equation to describe the total material cost or the total Engineered, Furnished, and Installed cost (E,F,&I) of a brand new No. 1/1A ESS central office. The model involves only plain old telephone service (POTS) offices having an all remreed network, miniaturized and combined trunks, zero percent touch-tone, no coin services, main distributing frame, and protector frame.

The SCIS model in connection with Long Run Incremental Analysis and standard Service Cost programs, such as CAPCOST and TESTCOST, are used by the Bell Companies for making pricing decisions for 1/1A ESS vertical services (including features).

2.3.2 Addressed Methodology

The SCIS algorithm involves four steps. In the first step, nine offices covering a fairly wide range of line sizes, and call and usage volumes are engineered and subsequently costed with COEES.

In the second step, the E,F,&I cost of each of the nine offices is partitioned into eight terms: Getting-Started Cost, Line Termination Cost, Intraoffice Usage Cost, Interoffice Usage Cost, Originating Call Cost, Terminating Call Cost, Outgoing Call Cost, and Incoming Call Cost.
The getting-started cost (GS) is defined as the part of the E,F,\&l cost necessary for establishing a new 1/1A ESS office, independent of the traffic carried or the number of lines installed. It includes generic program costs, memory costs, frame costs, and breakage allowances. The GS cost is assigned to various services in proportion to the long-run cost per unit of capacity, and the number of capacity units used by each service. The long-run cost per unit of capacity is calculated as follows:

\[
\text{Long-run cost per unit of capacity} = \frac{\text{Getting-Started Cost}}{\text{Maximum usable cycles} \times \text{Long-run average processor cycle times} \times \text{Processor utilization factor} \times \text{Adjustment factor}}
\]

The central processor's capacity is measured in cycles; after adjusting for maintenance functions, the maximum usable capacity for 1 ESS is 497,454·10^6 cycles, while for 1A ESS it is 2982,8571·10^6 cycles. Peak factor 1 is an allowance for the busy-hour of the high traffic days as compared to the busy-hour of the normal ones; it's default value is assumed equal to 1.3. Peak factor 2 is an allowance for quarter hour surges, and assumes that one hour of traffic is equal to 3.85 quarter hours. The processor utilization factor of an individual office depends on the assumed demand profile of the office over its expected lifetime, and is computed by the formula:

\[
\text{Processor Utilization} = \frac{\text{Present Worth of Processing Capacity Utilized Factor}}{\text{Present Worth of Maximum Usable Capacity}}
\]

The processor cycle adjustment factor is used only for 1A ESS offices. It reflects the reduction in the number of usable cycles due to the absence of a signal processor.
The line termination cost is equal to the sum of the working line cost and the line link network termination cost. The working line cost accounts for translation and protector frame costs. The total line link network (LLN) termination cost is given by the formula:

\[ \text{Total LLN termination cost} = \frac{\text{cost}}{\text{LLN termination} \cdot \text{terminations}} \]

The number of LLN terminations is a function of the number of working lines, the line fill factor, and the number of originating plus terminating CCS per working line during the busy-hour and at deloading.

The calculation of the intraoffice usage cost takes into account the fact that intraoffice calls, depending on the configuration of the office, can be switched in two different ways, as shown in figure 2.2. The interoffice usage cost accounts for usage generated by calls that either originate at a customer line and complete to an outgoing trunk, or come over an incoming trunk and terminate at a customer line.

The originating call cost accounts for the cost of various equipment categories, such as: receivers, announcement, tone, permanent signal and overflow circuits, disconnect and time scan junctor registers, and various hopper types. The terminating call cost mainly accounts for the cost of ringing, tone and announcement circuits. The outgoing call cost includes the cost of various transmitter types (multifrequency, dial pulse, revertive pulse), and customer dial pulse receivers. The incoming call cost includes the cost of receivers, call store words, registers, and hoppers.

In the third step, a nonlinear regression equation of the form \( Y = ax^b \) is developed for each of the usage (intraoffice, interoffice) and call (originating, terminating, outgoing, incoming) related cost
Fig. 2.2 Switching an intraoffice call
terms. In the above equation, Y represents one of the above mentioned cost terms, and X the respective measure of output of the central office, while a, b are the estimable parameters of the model. All call and usage related terms are found to exhibit declining unit capacity costs as their respective total call and usage outputs increase.

Figure 2.3 shows the total incoming call cost versus the number of incoming calls, while figure 2.4 exhibits the declining nature of the incoming cost per call as the number of incoming calls increases.

The fourth step is the development of the total capital cost equation. The E,F, & I cost is equal to the sum of the six cost values estimated in step 3, the GS cost, and the line termination cost. The final form of the E,F, & I cost equation is the following:

TOTAL E,F, & I COST = GETTING STARTED COST

\[ + 8.35 \times \text{WORKING LINES} \]
\[ + \begin{cases} 84.47 \text{ IF 2:1H} \\ 75.5 \text{ IF 3:1 H} \ast \text{LLN TERMINATIONS} \\ 59.04 \text{ IF 4:1R} \\ 54.33 \text{ IF 6:1R} \end{cases} \]
\[ + \begin{cases} 19.76 \times \text{(INTRAOFFICE CCS)}^{1.0} \\ 8.06 \times \text{(INTRAOFFICE CCS)}^{1.0} + 1.697 \times \text{(LLN)}^{1.861} \end{cases} \]
\[ + 21.80 \times \text{(OUTGOING + INCOMING CCS)}^{1.0} \]
\[ + 20.24 \times \text{(ORIGINATING CALLS)}^{0.862280} \]
\[ + 24.34 \times \text{(TERMINATING CALLS)}^{0.87829} \]
\[ + 42.84 \times \text{(OUTGOING CALLS)}^{0.6807} \]
\[ + 46.17 \times \text{(INCOMING CALLS)}^{0.71679} \]
Fig. 2.3 Total incoming call cost vs. the number of incoming calls

Fig. 2.4 Cost per incoming call vs. the number of incoming calls
In the SCIS model, Bell Labs claim that for each of the nine selected offices the total E,F,&I cost value obtained from the above equation has a small difference from the total cost estimated by COEES (2 to 5 percent).

2.3.3 Comments

The major concern about the SCIS model has to do with partitioning the total cost obtained from COEES into eight terms. Although the above is a very significant step, the documentation available does not provide any insights about the method used. Our "best guess" is that multiple runs of COEES, with one variable changing at a time while keeping the rest constant, were made for various variable levels. Differences between equipment requirements per category and the total cost figures indicated by COEES, were then used to approximate the relative influences of all equipment types on each of the eight cost terms of interest. Being unable to determine whether or not the above method was used, we have concern about the extent to which costs are the result of some allocation scheme, and as to whether the resulting partial cost levels are dependent on the expansion sizes and the allocation methods.

Technically, the eight terms included in the E,F,&I cost equation provide a reasonable representation of the services offered by a central office. The values that the explanatory variables can take cover fairly wide ranges, but significant questions are raised about whether nine sets of data points are enough for sufficiently approximating the unit costs of any possible capacity expansion. The SCIS
documentation available to us does not provide any comparisons between estimated and actual costs for central offices other than the selected ones.

The idea of examining capital costs on a central office basis is considered to be a good one. It needs to be examined, however, if the selected offices are representative on a company basis. Finally, the model does not take into account the cost of the necessary service checking equipment, which according to Ohio Bell's telephone engineers contribute to central office capital costs.

2.4 Rohlfs's Study

2.4.1 Approach

The objective of Rohlfs's study is to estimate the marginal costs of telephone service in Washington, D.C. Marginal costs are estimated through econometric analysis of actual cost and demand data for the 1960 to 1983 period and forecasts of future costs and demand that in some cases go through 1984, while in others through 1985. Demand, cost, inflation, and technology forecasts are used as provided by the C&P Telephone Company.

Rohlfs's study treats a telephone company as a multiproduct firm. Local and long distance usage, the number of business, residential, and Centrex lines, and the number of directory assistance and operator-handled calls are some of the multiple outputs considered.

Capital costs are subdivided in three categories: Central Office Costs, Outside Plant Costs, and "Other." The real annualizing factors per capital cost category are estimated as a function of the following parameters: inflation rates per capital cost category, debt equity
ratio, nominal cost of debt and equity, expected central office equipment and outside plant lives, federal and state income tax rates. Non-capital costs are subdivided in four categories: Maintenance and Service Connection Costs, Traffic Expenses, Commercial and Marketing Expenses, General Office Salaries and "Other." Non-capital costs are deflated according to reported and forecasted wage inflation rates, as experienced by the C&P Telephone Company.

The analysis of cost and demand data is done by using the multiple linear regression technique. To correct for serial correlation and obtain unbiased test statistics, Rohlf uses the Cochran-Orcutt iterative procedure (see Johnston (11)). Various demand variables are tested in each cost equation; the ones finally included are selected on the basis of having large t statistics (statistically significant at the 5 percent level).

2.4.2 Rohlf's Conclusions

Rohlf expresses Central Office (plus Land and Building) construction costs as a function of the ESS access line gain, and the number of existing Centrex lines. Specifying the growth of ESS lines as an explanatory variable is preferred to specifying the total access line gain, since future expansions will almost wholly be ESS. The central office cost and demand sample includes data from 1975 to 1985, because the relationship between ESS additions and costs did not stabilize until 1975. Rohlf claims that including a constant term in the central office cost equation makes no economic sense, and that conclusions about economies or diseconomies of scale can be reached by comparing the marginal cost of new capacity to the average cost of the
capacity in place. Other variables such as the existing level and the
growth of local or long distance usage, the number of existing
residential lines, and the number of existing business lines other than
CO Centrex were tested and found to have no significant positive
effect. Discrepancies between the actual and the fitted time-series
are explained by the fact that C&P substantially overestimated growth
during the 1980 to 1982 period. As a result of this, excess capacity
exists in major business districts of Washington, D.C., and, therefore,
the marginal costs for these areas are projected to be equal to zero.
The coefficient of the ESS access line gain in the central office cost
equation is converted to an Incremental cost figure by multiplying by
the respective annualizing factor.

Outside Plant construction costs in year (t) are expressed as
function of the outside plant costs in year (t-1), the gain in resi-
dential lines in year (t), and the number of existing residential
lines. In general, a model that relates current expenses to previous
year's costs and demand is commonly referred in econometrics as "the
Koyck distributed lag" (see Johnston (11)). Rohls explains the above
statistical finding by the fact that outside plant construction need
not occur in the same year that demand increases. The outside plant
incremental cost of residential line gain is obtained by dividing the
respective coefficient by unity minus the coefficient of the previous
year's expenses, and then multiplying by the outside plant annualizing
factor. Similar to the case of central office costs, differences
between actual and estimated cost values are due to the overestimation
of the growth rates of demand.
The primary components of the "Other" cost category are computer hardware and software, automobiles, and office equipment. The statistical analysis performed suggests that the "Other" construction is a fixed cost, not significantly related to demand variables.

Maintenance and service connection costs in year (t) are expressed as a function of the inward movement of residential lines not connected to ESS in year (t-1), the inward movement of business lines in year t, the existing number of business lines in year (t-1), and a dummy variable representing the 1983 effect of divestiture on service connection costs. The statistical finding of relating costs to values of variables lagged by one year, is explained by the fact that it takes time for the company to increase its maintenance work force in response to changes in these variables. Since a statistically significant constant term cannot be included in the equation, Rohlf concludes that maintenance and service connection costs exhibit constant returns to scale.

Traffic expenses in year (t) are expressed as a function of traffic expenses in year (t-1), the number of directory assistance calls, and the number of operator handled calls in year (t). The primary determinants of commercial and marketing expenses are the previous year's expenses, and the fraction of access lines connected to ESS. In the absence of demand variables, Rohlf forecasts zero marginal commercial and marketing costs. Finally, statistical analysis for the General Office Salaries and Other category indicates that the above expenses are not constant, but depend on the existing number of access lines.
2.4.3 Comments

From an overall point of view, the econometric study of Rohlfs is a significant contribution to the public utility literature, and a useful application of the concepts of marginal cost theory for multi-product firms. As is frequently the case, however, the analysis of unplanned data by multiple linear regression suffers from multicollinearity. Local usage, for example, was tested and found to be statistically insignificant in all cost equations. From an engineering point of view such a conclusion is highly unlikely, since local and toll usage partly determine the size of a central office capacity expansion. The incremental capital costs of the ESS line gain and the number of existing Centrex lines, which were estimated in the central office cost equation, are partly due to usage; multicollinearity problems, however, do not allow a separate estimation of these effects. Rohlfs himself is aware of this situation; he states that the large coefficient for CO Centrex in the central office cost equation is probably due to intercom usage. He further suggests that if intercom usage is costly, local and long distance usage must be even more costly. The problem that he faces is that the sum of local and long distance usage per line has remained relatively constant, while construction did not respond to minor changes in the total usage per line.

Rohlfs estimates marginal costs under the usual economic assumption of fixed proportions. In other words, the ratio of inputs to outputs remains constant. Pollard and Henderson (18) point out that the above assumption may provide fairly good marginal cost estimates only for the near future. They proceed by stating that, because of ESS
technological advancements, it is unlikely that the proportions of labor and capital, for example, have remained constant for twenty years. They finally express the opinion that labor related costs are underestimated, while capital costs are overestimated because of the relatively increased productivity of capital in recent years.

2.5 Summary

Chapter 11 contains a review of three models; the Central Office Equipment Engineering System (COEES), the Service Cost Information System (SCIS), and Rohlf's study. COEES is an engineering model designed to assist in the selection of the optimum central office capacity expansion plan for a four year period of growth. A description of input and output forms, and a comment on the incorporation of variable operating costs in the cost minimization procedure have been presented.

The objective of the SCIS model is to develop an equation to express the total cost of a new 1/1A ESS office in terms of eight engineering measures of output. The basic steps involved are: engineering and costing nine offices with COEES, partitioning the total cost into eight terms, regressing for the six call and usage related terms, and developing the total cost equation by adding the above six terms, the getting started cost, and the line termination cost. The cost partitioning scheme used is considered to be of doubtful accuracy.

Rohlf estimates the long-run marginal costs of telephone service in Washington, D.C. He uses regression analysis of historic cost and
demand data to estimate seven capital and non-capital cost functions. His study is well designed and applied; the use of undesigned data, however, causes significant multicollinearity problems.
CHAPTER III
PROBLEM APPROACH

3.1 Introduction

Chapter II has presented a review of three models, that are either
directly used in this study (COESS), or have some similarities to the
approach that will be taken (SCIS, Rohls's study.) The purpose of
this chapter is to present the method that is applied in this project
to estimate the incremental capital costs of engineering measures of
output of a central office. Sections 3.2 and 3.3 discuss the experimen-
tal plan, and the method of analysis of the experimental results.
The underlying assumptions, and a detailed discussion of each step
followed are presented in subsequent sections of chapter III.

3.2 The Experimental Plan

The intention of this project is to select a particular 1A ESS
central office, and use various "what if" expansion scenarios to esti-
mate the incremental capital costs of variables that represent the
demand for local basic telephone service. The cost of a central office
capacity expansion can be expressed as a function of the change in the
output levels of several variables related to local service, such as
the number of business and residential main stations (lines), the total
intraoffice and interoffice local usage, and the total number of origi-
nating, terminating, and thru-switched local calls. The cost of de-
signing an experiment to estimate the cost-effects of several measures
of output of a central office is prohibitive; under certain assumptions, the number of experimental design variables is reduced to four.

The possible range of values of each variable lies between a minimum and a maximum value. The minimum yearly level of growth of each variable over the four year planning horizon is set so as: Minimum output level in year \( j \) = Maximum output level in year \((j-1)\) \((j = 1,2,3,4)\). The selection of the maximum yearly levels of growth is based on the actual output increases for the 1979 to 1983 period, and Ohio Bell's forecasts for the 1984 to 1987 period. Sixty-one different capacity expansion scenarios, that include changing one, two, three, or four variables at a time are then used, and their respective costs are obtained from the Central Office Equipment Engineering System. More specifically, Ohio Bell's telephone engineers have, under certain assumptions, transformed the assumed growth levels to the required inputs of the COEES program, and have provided the yearly costs of the eight alternative plans that the model identifies.

3.3 The Experimental Results

The capacity expansion costs of interest to this project are those of the expansions that may occur at the beginning of the first year. Their respective expansion sizes are sufficient for satisfying the demand for one, two, three, or four years in the future. The above expansion sizes and costs are the only ones that can be accurately identified; the timing, the sizing, and the cost of any subsequent facility additions, as identified in the four year plan, are reconsidered whenever customer demand exceeds the available capacity.
again. In other words, a new four year plan based on updated forecasts is then developed. The new plan, however, may call for a different expansion size; if the size of an expansion has any positive or negative effect on costs, any difference in size between the second expansion under the old plan and the first expansion under the new plan, may produce significantly different incremental cost estimates. The correct incremental cost figures, however, are those obtained from the new plan, since this is what the company will actually implement.

The problem is mathematically formulated through equations that use dummy (zero-one) variables. Four equations corresponding to different expansion sizes are developed; the cost estimates obtained from each equation are then used to estimate the incremental capital costs of all variables as a function of the expansion size. Figure 3.1 shows the basic steps involved in the design and analysis of the experimental model.

3.4 Objectives

The objectives of this project are:

(1) To select a few variables that can represent the demand for local basic telephone service.

(II) To develop equations that can express the cost of a capacity expansion as a function of the growth levels of engineering measures of output of a central office.

(III) To estimate the incremental capital costs of each measure of output for different expansion sizes.
Select an 1A ESS central office

Select the experimental design variables

Determine the maximum yearly growth levels

Formulate the problem mathematically

Analyze the COEES total cost figures

Estimate the incremental capital costs

Fig. 3.1 Basic steps of the experimental design model.
3.5 Office Selection

It has already been stated that the intention of this project is to examine only one central office. Any 1 or 1A ESS office that currently requires a capacity expansion can be selected. The reader should, however, be aware of the fact that all end results may depend both on the selected central office type, and on the current output levels of the examined office.

Seventy 1 and 1A ESS local or combined local/toll offices exist in Ohio Bell's territory. A class 5 local end office transmits the incoming traffic to other local area offices (local traffic), or to toll offices (toll traffic). A class 4/5 combined local/toll office serves partly as a local and partly as a toll office, and is used in areas with low traffic loads. Since Ohio Bell has a very small number of combined local/toll offices, a class 5 local end office is selected.

A main difference between 1 and 1A ESS offices is in the memory sizes used. An 1 ESS central office uses a 32K memory module, while an 1A ESS uses a 256K memory module. The purchase of additional memory units incurs a large capital cost; a type 1 ESS central office may at some point in time require additional memory units, while the 1A ESS memory module usually lasts throughout the expected life of the office. It should be noted, that most expansion situations experienced by Ohio Bell do not require memory additions. For the purposes of this project, a class 5 1A ESS local end office with the following characteristics is selected:
Number of single lines 22,174
Number of existing direct indial (DID) business trunks 440
Total busy-hour intraoffice usage 9,678 (CCS)
Total busy-hour interoffice local usage 73,166 (CCS)

According to Ohio Bell's forecasts, the above listed levels of output constitute the December 1984 configuration of the selected office.

3.6 Problem Variables-Assumptions

There are four primary variables of interest:

(i) Growth of single lines.
(ii) Growth of direct indial (DID) business trunks.
(iii) Growth of intraoffice usage.
(iv) Growth of interoffice local usage.

Ohio Bell subdivides customer lines in the following main categories: flat rate residential (FRR), measured rate residential (MRR), measured rate business (MRB), Centrex lines, PBX trunks, direct indial (DID) business trunks, residential four element pricing lines, remote call lines, foreign exchange lines, coin lines, test lines, and two-party lines. The number of coin lines, test lines, and two-party lines is assumed to remain constant over the next four years. In addition to the above, touch-tone and dial pulse lines are added in proportion to the respective number of existing lines. Centrex systems can be thought as being composed of a number of single lines, and, therefore, the growth of Centrex lines can be included in the "single-line growth term." PBX trunks were commonly used in the past for handling both incoming and outgoing PBX traffic, while the number of PBX trunks had a
one to one correspondence to the number of terminal equipment used. Today, the growth of PBX trunks is low; they are sometimes used for handling incoming traffic, but modern traffic engineering calls for using DID trunks. According to Ohio Bell’s telephone engineers, single lines and PBX trunks have similar line termination costs. The growth of single lines can be calculated by the formula:

\[
\text{Growth of single lines} = 0 \sum_{i=1}^{n} \text{growth of single line category (I)}.
\]

The growth of DID trunks is the second independent variable that this project considers. Only recently has Ohio Bell started to forecast the DID trunk growth; this factor, however, is expected to exhibit a significant growth over the next few years.

Although this case study considers the growth of single lines as one variable, Ohio Bell’s central office planning process may require a separate specification of its components. In this case, we suggest the use of the following formula:

\[
\text{Growth of single} = \text{Yearly growth in the examined central office} \times \frac{\text{Number of existing lines for single line category (I)}}{\text{Total number of existing single lines in the examined central office}}.
\]

The growth levels per single line category are, therefore, assumed to be proportional to the respective number of existing lines in the selected central office.
Technically, the additional costs that an increase in the number of installed lines or trunks can impose are: (i) line termination costs, (ii) cost for adding line link network (LLN) capacity; depending on demand requirements, whole or half LLNs can be added, (iii) cost for adding junctors, that connect the LLNs to the trunk line networks (TLNs), and (iv) cost for adding memory units.

In addition to the two line related terms, two usage related variables, intraoffice and interoffice local usage growth, are introduced. The interoffice local traffic growth is assumed to be uniformly distributed to all trunk group connections. The additional costs that increased usage can impose are: (i) cost for adding LLNs, or connection paths between LLNs and TLNs. A predominately residential low usage end office, for example, typically has a 4:1 concentration ratio; each LLN can serve 4096 main stations, while 1024 path connections between LLNs and TLNs exist. Maintaining the same service standards under increased usage conditions may require using a 2:1 concentration ratio (2048 main stations: 1024 path connections) and, therefore, an additional LLN may be necessary, (ii) cost for additional memory units, (iii) cost for additional interoffice trunking, (iv) cost for additional service circuits.

Since the intention of this project is to restrict its attention to local service variables only, the total interoffice toll usage is held constant (at its current total level). This assumption does not imply that the growth of toll usage is not an important factor. On the contrary, toll usage may be more costly than local usage, since it generally requires the use of a slightly higher amount of equipment.
(eg. transmitters, tape readings, etc.). Additional unit cost differences may be due to the fact that local usage is transmitted through 7 digit trunks, while toll usage through 10 digit trunks.

An 1A ESS central processor has the capability to process approximately 170,000 calls during the busy-hour. Central office demand requirements exceeding this limit are unusual; for example, the central office that was selected for the purposes of this project exhibits a fifty percent call utilization rate during the busy-hour. Since the central processor is the only central office item that is engineered based on calling attempts, and it's current percentage utilization is so low, it makes no difference in cost if one assumes that the total number of calls remains constant, or if calls are increased in proportion to the respective usage growth levels. In this project, it is assumed that all the call inputs of the central office planning process remain constant at their current levels, for purposes of simplicity.

A major assumption used in this project is that all variables related to customer calling features or special services (e.g. speed-calling, call-waiting, call-forwarding, Inwats lines, Outwats lines, Centrex tie lines, other business offerings, etc.) are held constant at today's levels. This assumption enables us to estimate the incremental capital costs of variables related to local basic service, apart from offered features. Finally, the common utility practice of using up to ninety-five percent of each LLN or TLN, because of space needed for line or trunk rearrangements, and that of keeping five percent spare trunks are followed when engineering an office expansion.
3.7 Variable Growth Levels

The selection of appropriate yearly levels of growth is based on the actual output increases for the 1979 to 1983 period, and Ohio Bell's forecasts for the 1984 to 1987 period. Initially, eight 1A ESS offices of various sizes were selected, and a data request was submitted to Ohio Bell seeking for the following information:

(1) The existing configuration of the eight offices expressed in terms of the following measures of output: single lines, DID trunks, total Intraoffice usage, and total Interoffice local usage. Information was also requested on the division of single lines to subcategories.

(2) The actual levels of growth for single lines, DID trunks, Intraoffice usage, and Interoffice local usage for the 1979 to 1983 period, and future trends for the 1984 to 1987 period.

(3) The yearly levels of average Intraoffice and Interoffice local usage per main station and DID trunk.

Ohio Bell's response to the above data request has indicated, that for all four measures of output the sizes of the yearly additions vary from office to office. In this project, we do not attempt to forecast future growth; based on Ohio Bell's actual and forecasted values, reasonable upper limits of yearly growth for the selected office are established. Table 3.1 shows the upper limits of yearly growth that this project considers.
TABLE 3.1

UPPER LIMITS OF YEARLY GROWTH

<table>
<thead>
<tr>
<th>Variable</th>
<th>Busy Season</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Single line growth</td>
<td>350</td>
</tr>
<tr>
<td>DID Trunk Growth</td>
<td>90</td>
</tr>
<tr>
<td>Intraoffice usage</td>
<td>300</td>
</tr>
<tr>
<td>growth</td>
<td></td>
</tr>
<tr>
<td>Interoffice local</td>
<td>1800</td>
</tr>
<tr>
<td>usage growth</td>
<td></td>
</tr>
</tbody>
</table>

Source: Author's assumptions

3.8 Formulation of the Problem

3.8.1 Introduction to the $2^k$ Factorial Design

Factorial designs are used in multifactor experiments to study the joint effects of all factors on the response variable. A special case of a factorial design is the $2^k$ factorial; each replicate of the experiment contains $2^k$ treatment combinations (observations), and involves $k$ qualitative or quantitative variables (factors) each at only 2 levels. All $k$ variables involved, are varied independently of each other. The effect of a factor (main effect) is defined to be the change in response produced by a change in the level of a factor, and is usually denoted by a capital Latin letter. If the change in response between the levels of one factor is not the same at all levels of other factors, interactions between factors exist. The two levels of a factor are arbitrarily called "low" and "high"; the high level of a factor in a treatment combination is denoted by the presence of the
respective lower case letter, while the low level is denoted by the absence of the corresponding letter.

As example, a $2^3$ factorial design involves 3 factors (A, B, C) each at 2 levels, while each experimental replication contains 8 treatment combinations. An observation taken under the high levels of factors A and B, and the low level of factor C is represented as (ab). The observations of a $2^3$ factorial may be described by the following linear statistical model:

\[ Y_{ijkl} = \mu + a_i + b_j + c_k + (ab)_{ij} + (ac)_{ik} + (bc)_{jk} + (abc)_{ijk} + \xi_{ijkl} \]

where $\mu$ is the overall average effect, $a_i$, $b_j$, $c_k$ are the effects of the $i^{th}$ level of factor A, the $j^{th}$ level of factor B, and the $k^{th}$ level of factor C respectively, $(ab)_{ij}$, $(ac)_{ik}$, $(bc)_{jk}$ are the two factor interactions, $(abc)_{ijk}$ is the three factor interaction, and $\xi_{ijkl}$ is the random error component. The assumptions of a $2^3$ fixed factorial design are: (1) all factors are fixed, (2) the design is completely randomized, (3) the error terms are independent and normally distributed, (4) the response is approximately linear over the range of the factors chosen, (5)

\[ \sum_{i=1}^{2} a_i = \sum_{j=1}^{2} b_j = \sum_{k=1}^{2} c_k = 0 \]

(main effects are defined as deviations from the overall average effect). Interaction effects exhibit a similar property; for example the interaction between factors A and B is defined so that:
The observations of a $2^3$ factorial design may equivalently be described by the following linear statistical model:

$$Y = B_0 + B_1X_1 + B_2X_2 + B_3X_3 + B_4X_1X_2 + B_5X_1X_3 + B_6X_2X_3 + B_7X_1X_2X_3 + \varepsilon$$

where $B_0$ is the overall average response, and $X_1$, $X_2$, $X_3$ take the values +1 or -1 depending on whether the observation described is taken under the high or the low level of factors A, B, and C. In the above linear regression model, all regression coefficients $B_i$ ($i=1, 2, ..., 7$) are equal to one half the estimates of the factor effects. This is true, because a regression coefficient measures the effect of a unit change in a variable on the average response, while an effect represents the change in response for a change of two units in a variable (from -1 to +1).

Montgomery (16) states that a factorial design has the following advantages over the one factor at a time method: (1) a factorial design is more efficient, since fewer observations are needed to produce average main effects just as precise as those obtained from the one factor at a time method, (2) in the presence of interaction, conclusions reached from the single factor method may be seriously in error, (3) factorial designs allow the effects of a factor to be estimated at several levels of other factors, while the conclusions reached are valid over the experimental region.

3.8.2 The Mathematical Model

In this project, four $2^4$ factorial designs are used to estimate the incremental capital costs of four engineering measures of output of a central office for different capacity expansion sizes. The four
variables considered are: single line growth, DID trunk growth, intra-office usage growth, and interoffice local usage growth. In every design, each variable is used at two levels; the low level represents an addition large enough to satisfy the demand for (j) future years ($j=0,1,2,3$), while the high level represents a capacity expansion size that can satisfy the demand requirements for ($j+1$) future years. The low level of each variable is coded as 0, while the high level as 1. As opposed to the ($-1,+1$) formulation (see section 3.8.1), where the resulting designs are orthogonal, the (0,1) coding scheme does not produce orthogonal designs. Under the ($-1,+1$) formulation, however, the main effects are defined as deviations from the overall average effect, while the cost differences of interest to this project are those produced when the levels of the variables are changed from "low" to "high."

This section uses the following general notation:

- $a_i =$ single line growth level up to and including year $i$
- $b_i =$ DID trunk growth level up to and including year $i$
- $c_i =$ intraoffice usage growth level up to and including year $i$
- $d_i =$ interoffice local usage growth level up to and including year $i$
\(a_0, b_0, c_0, d_0\) represent zero growth for the respective demand variable. The four experimental design models used, are presented throughout the rest of this section.

(a) Design 1

In design 1, the low level of each variable is set equal to zero; the high level is set so as to satisfy the demand requirements for the next year. Table 3.2 shows the respective low and high variable settings for design 1.

**TABLE 3.2**

**DESIGN 1: LOW AND HIGH VARIABLE LEVELS**

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Variable Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
</tr>
<tr>
<td>Single line growth</td>
<td>0</td>
</tr>
<tr>
<td>DID trunk growth</td>
<td>0</td>
</tr>
<tr>
<td>Intraoffice usage growth</td>
<td>0</td>
</tr>
<tr>
<td>Interoffice local usage growth</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Author's assumptions

The 16 observations of a 2^4 factorial design are obtained from different combinations of the above variable settings, and represent different capacity expansion situations. The difference between the capital cost of any capacity expansion within the variable ranges considered, and the capital cost of all the low expansion (observation (1): 0 lines, 0 trunks, 0 intraoffice CCS, and 0 interoffice local CCS are added) can be described by the following linear model:

\[
\Delta TC_1(x_1, x_2, x_3, x_4) = B_1 x_1 + B_2 x_2 + B_3 x_3 + B_4 x_4 + B_5 x_1 x_2 + B_6 x_1 x_3 + B_7 x_1 x_4 + B_8 x_2 x_3 + B_9 x_2 x_4 + B_{10} x_3 x_4 + B_{11} x_1 x_2 x_3 + B_{12} x_1 x_2 x_4 + B_{13} x_1 x_3 x_4 + B_{14} x_2 x_3 x_4 + B_{15} x_1 x_2 x_3 x_4
\]  

(1)
where:

\[ X_1 = \begin{cases} 
0 & \text{if the growth of single lines is equal to 0} \\
1 & \text{if the growth of single lines is equal to } a_1
\end{cases} \]

\[ X_2 = \begin{cases} 
0 & \text{if the growth of DID trunks is equal to 0} \\
1 & \text{if the growth of DID trunks is equal by } b_1
\end{cases} \]

\[ X_3 = \begin{cases} 
0 & \text{if the growth of Intraoffice usage is equal to 0} \\
1 & \text{if the growth of Intraoffice usage is equal to } c_1
\end{cases} \]

\[ X_4 = \begin{cases} 
0 & \text{if the growth of Interoffice local usage is equal to 0} \\
1 & \text{if the growth of Interoffice local usage is equal to } d_1
\end{cases} \]

The coded variables \( X_1, X_2, X_3, X_4 \) can take any value between 0 and 1. As example, if the real single line growth is equal to \( z \) (\( 0 \leq z \leq a_1 \)) then:

\[ X_1 = z/a_1. \]

\( B_1 \) through \( B_{15} \) are the estimable coefficients of the model.

The reader, however, should be aware of the fact that this is a mathematical and not a statistical model. Sixteen observations are used to estimate 15 coefficients (in other words, the 16 total degrees of freedom are divided to 15 degrees of freedom for the model sum of squares, and just 1 degree of freedom for the error sum of squares. In this case, the error sum of squares is equal to zero, and no statistical evaluations of the model (e.g. t-statistics, confidence intervals) are possible, if the experimenter does not have prior information about the error variance.

The derivative of \( \Delta TC_1 \) with respect to \( X_1 \) is mathematically defined as the change in the slope of the cost surface \( \Delta TC_1(X_1, X_2, X_3, X_4) \) for a unit change in the coded variable \( X_1 \), and is given by the formula:

\[
\frac{\delta \Delta TC_1}{\delta X_1}(X_2, X_3, X_4) = B_1 + B_5X_2 + B_6X_3 + B_7X_4 + B_{11}X_2X_3 + B_{12}X_2X_4 + B_{13}X_3X_4 + B_{15}X_2X_3X_4 \tag{2}
\]
In other words, equation (2) estimates the change in the total expansion cost for a single line growth of \( x_1 \) units, and any growth levels for DID trunks, intraoffice usage, and interoffice local usage within the expansion limits considered. Therefore, the average incremental capital cost of a single line (ICC_1), for any single line growth level between 0 and \( x_1 \), is given by the formula:

\[
ICC_1 = \frac{\delta ATC_1(x_2, x_3, x_4)}{\delta x_1} / a_1 = (B_1 + B_5x_2 + B_6x_3 + B_7x_4 + B_{11}x_2x_3 + B_{12}x_2x_4 + B_{13}x_3x_4 + B_{15}x_2x_3x_4) / a_1
\]  

(3)

It should be noted, that \( x_2, x_3, \) and \( x_4 \) are substituted in both (2) and (3) in coded units.

The incremental capital cost of a DID trunk (ICC_2), an intraoffice CCS (ICC_3), and an interoffice local CCS (ICC_4) are defined and calculated in a similar manner; their respective formulas are given in table 3.3.
### TABLE 3.3

**DESIGN I: INCREMENTAL CAPITAL COST FORMULAS**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Incremental Capital Cost</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single lines</td>
<td>$\text{ICC}_1 = \frac{\delta \Delta TC_1}{\delta x_1}(x_2,x_3,x_4)/a_1$</td>
<td>$0 \leq \text{single line growth} \leq a_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 \leq x_2,x_3,x_4 \leq 1$</td>
</tr>
<tr>
<td>DID trunks</td>
<td>$\text{ICC}_2 = \frac{\delta \Delta TC_1}{\delta x_2}(x_1,x_3,x_4)/b_1$</td>
<td>$0 \leq \text{DID trunk growth} \leq b_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 \leq x_1,x_3,x_4 \leq 1$</td>
</tr>
<tr>
<td>Intraoffice usage</td>
<td>$\text{ICC}_3 = \frac{\delta \Delta TC_1}{\delta x_3}(x_1,x_2,x_4)/c_1$</td>
<td>$0 \leq \text{intraoffice usage growth} \leq c_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 \leq x_1,x_2,x_4 \leq 1$</td>
</tr>
<tr>
<td>Interoffice local usage</td>
<td>$\text{ICC}_4 = \frac{\delta \Delta TC_1}{\delta x_4}(x_1,x_2,x_3)/d_1$</td>
<td>$0 \leq \text{interoffice local usage growth} \leq d_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 \leq x_1,x_2,x_3 \leq 1$</td>
</tr>
</tbody>
</table>

Source: Author's calculations
(b) Design II

In design II, the low and high levels of all four variables are set so as to satisfy the demand requirements for one and two future years respectively. Table 3.4 shows the respective low and high variable settings.

**TABLE 3.4**

DESIGN II: LOW AND HIGH VARIABLE LEVELS

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Variable Level</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Single line growth</td>
<td>$a_1$</td>
<td>$a_2$</td>
</tr>
<tr>
<td>DiD trunk growth</td>
<td>$b_1$</td>
<td>$b_2$</td>
</tr>
<tr>
<td>Intraoffice usage growth</td>
<td>$c_1$</td>
<td>$c_2$</td>
</tr>
<tr>
<td>Interoffice local usage growth</td>
<td>$d_1$</td>
<td>$d_2$</td>
</tr>
</tbody>
</table>

Source: Author's assumptions

The difference between the capital cost of any capacity expansion, within the variable ranges considered, and the capital cost of all the low expansion (observation (1); $a_1$ lines, $b_1$ trunks, $c_1$ intra-office CCS, and $d_1$ interoffice local CCS are added) can be described by the following linear model:

$$
\Delta TC_2(x_1,x_2,x_3,x_4) = B_1x_1 + B_2x_2 + B_3x_3 + B_4x_4 + B_5x_1x_2 + B_6x_1x_3 + B_7x_1x_4 + B_8x_2x_3 + B_9x_2x_4 + B_{10}x_3x_4 + B_{11}x_1x_2x_3 + B_{12}x_1x_2x_4 + B_{13}x_1x_3x_4 + B_{14}x_2x_3x_4 + B_{15}x_1x_2x_3x_4 \quad (4)
$$

Table 3.5 shows the coding scheme used and its interpretation.
TABLE 3.5
DESIGN II: CODING METHOD

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coded Units</th>
<th>Real Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>0</td>
<td>$a_1$ single lines</td>
</tr>
<tr>
<td>$X_2$</td>
<td>1</td>
<td>$a_2$ single lines</td>
</tr>
<tr>
<td>$X_3$</td>
<td>0</td>
<td>$b_1$ DID trunks</td>
</tr>
<tr>
<td>$X_4$</td>
<td>1</td>
<td>$b_2$ DID Trunks</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_1$ Intraoffice CCS</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_2$ Intraoffice CCS</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>$d_1$ Interoffice local CCS</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>$d_2$ Interoffice local CCS</td>
</tr>
</tbody>
</table>

Source: Author's assumptions

The derivative of $\Delta TC_2$ with respect to $X_1$ represents the change in
the capital cost of an expansion of $a_1$ lines, $b_1$ trunks, $c_1$ Intraoffice
CCS, and $d_1$ Interoffice local CCS due to an instantaneous addition of
$(a_2-a_1)$ more lines, when $X_2$ trunks, $X_3$ Intraoffice CCS, and $X_4$
Interoffice local CCS are simultaneously added; It is given by the
formula:

$$\frac{\Delta TC_2}{\delta X_1}(x_2,x_3,x_4)=b_1^1+b_5^1x_2^1+b_6^1x_3^1+b_7^1x_4^1+b_1^2x_2x_3+b_1^2x_2x_4+b_1^2x_3x_4+b_1^2x_2x_3x_4$$

(5)

The average Incremental capital cost of a single line (ICC$_1$), for any
single line growth level between $a_1$ and $a_2$, is given by the formula:

$$ICC_1=(\frac{\Delta TC_1}{\delta X_1}(1,1,1)+\frac{\Delta TC_2}{\delta X_1}(x_2,x_3,x_4))/a_2=\frac{\Delta TC_1}{\delta X_1}(1,1,1)+b_1^1+b_5^1x_2^1+b_6^1x_3^1+b_7^1x_4^1+b_1^2x_2x_3+b_1^2x_2x_4+b_1^2x_3x_4+b_1^2x_2x_3x_4}{a_2}$$

(6)
The first term on the right hand side of equation (6) represents the part of the capital cost due to the addition of \( a_1 \) lines, when the existing central office configuration is expanded by \( a_1 \) lines, \( b_1 \) trunks, \( c_1 \) intraoffice CCS, and \( d_1 \) interoffice local CCS. The incremental capital cost of a single line is, therefore, equal to the change in cost due to the addition of \( a_1 \) lines, plus the change in cost due to the instantaneous addition of \((a_2-a_1)\) more lines (given by formula (5)), divided by the total number of added lines. It should be noted that \( X_2, X_3, \) and \( X_4 \) are substituted in both (5) and (6) in coded units.

The incremental capital cost of a DID trunk (ICC\(_2\)), an intra-office CCS (ICC\(_3\)), and an interoffice local CCS (ICC\(_4\)) are defined and calculated in a similar manner; their respective formulas are given in table 3.6.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Incremental Capital Cost</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single lines</td>
<td>$ICC_1 = \frac{\Delta TC_1(1,1,1) + \Delta TC_2(x_2,x_3,x_4)}{\Delta X_1}/a_2$</td>
<td>$a_1 \leq \text{single line growth} \leq a_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 \leq x_2,x_3,x_4 \leq 1$</td>
</tr>
<tr>
<td>DID trunks</td>
<td>$ICC_2 = \frac{\Delta TC_1(1,1,1) + \Delta TC_2(x_1,x_3,x_4)}{\Delta X_2}/b_2$</td>
<td>$b_1 \leq \text{DID trunk growth} \leq b_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 \leq x_1,x_3,x_4 \leq 1$</td>
</tr>
<tr>
<td>Intraoffice usage</td>
<td>$ICC_3 = \frac{\Delta TC_1(1,1,1) + \Delta TC_2(x_1,x_2,x_4)}{\Delta X_3}/c_2$</td>
<td>$c_1 \leq \text{intraoffice usage growth} \leq c_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 \leq x_1,x_2,x_4 \leq 1$</td>
</tr>
<tr>
<td>Interoffice local usage</td>
<td>$ICC_4 = \frac{\Delta TC_1(1,1,1) + \Delta TC_2(x_1,x_2,x_3)}{\Delta X_4}/d_2$</td>
<td>$d_1 \leq \text{interoffice local usage growth} \leq d_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 \leq x_1,x_2,x_3 \leq 1$</td>
</tr>
</tbody>
</table>

Source: Author's calculations
(c) Designs III & IV

In design III, the low and high variable settings are set so as to satisfy the demand requirements for two and three future years respectively. Similarly, the low and high variable settings in design IV, represent addition sizes large enough to satisfy the customer demand for three and four future years respectively. The exact same mathematical formulation is used, and the capital cost functions are estimated similarly. The only representation differences are: (i) the low variable levels for design III are \(a_2, b_2, c_2,\) and \(d_2,\) while for design IV \(a_3, b_3, c_3,\) and \(d_3,\) (ii) the high variable levels for design III are \(a_3, b_3, c_3,\) and \(d_3,\) while for design IV \(a_4, b_4, c_4,\) and \(d_4,\) (iii) the total cost change in design III is represented by \(\Delta TC_3,\) while in design IV by \(\Delta TC_4,\) (iv) the estimable coefficients in design III are represented by \(B_i,\) while in design IV by \(B_i (i=1,2,...,14,15).\)

The incremental capital cost formulas for designs III and IV are given by equations (7) and (8).

Design III: \[ ICC_i = \left( \frac{\delta \Delta TC_1(1,1,1)}{\delta X_i} + \frac{\delta \Delta TC_2(1,1,1)}{\delta X_i} + \frac{\delta \Delta TC_3(X_j, X_k, X_l)}{\delta X_i} \right)/e_3i \] (7)

Design IV: \[ ICC_i = \left( \frac{\delta \Delta TC_1(1,1,1)}{\delta X_i} + \frac{\delta \Delta TC_2(1,1,1)}{\delta X_i} + \frac{\delta \Delta TC_3(1,1,1)}{\delta X_i} + \frac{\delta \Delta TC_4(X_j, X_k, X_l)}{\delta X_i} \right)/e_4i \] (8)

where: (1) \(i\) takes the values 1, 2, 3, 4 when it represents lines, trunks, intraoffice, and interoffice local usage respectively,
(2) $X_j, X_k, X_l$ represent variables other than estimated one, and (3) $e_{31}$ and $e_{41}$ represent the high levels of the estimated variable (1) in designs III and IV respectively.

3.9 Summary

Chapter III has presented the basic steps of the method that is applied in this project to estimate the incremental capital costs of four engineering measures of output of a central office for different expansion sizes. They are as follows: (1) select one 1A ESS office, (II) select four variables that represent the demand for local basic telephone service, (III) determine the "low" and "high" variable levels, (IV) formulate the problem mathematically, by using $2^k$ factorial designs with dummy (zero-one) variables, (v) develop the incremental capital cost formulas.
CHAPTER IV
APPLICATION AND ANALYSIS OF THE RESULTS

4.1 Introduction

The purpose of this chapter is to present the application of the method that has been discussed in Chapter III, the analysis of the results, and the estimation of the incremental capital costs of the variables considered. Finally, the average incremental capital costs of each measure of output, and the incremental capital cost of an average customer are calculated.

4.2 The Application

The developed method has been applied under the assumption that the high level of all engineering measures of output considered increases linearly during the four year planning horizon. As example, figure 4.1 shows the upper limit of the total single line growth during the next four years.

The above assumption is made for the purposes of this application; the upper limits of the total growth of demand, $a_1, b_1, c_1, d_1$ ($i=1,2,3,4$), depend on demand forecasts, and can take any values that the experimenter considers appropriate. In addition to the above, it is assumed that all plant additions occur at the beginning of year 1. Table 4.1 shows the values of $a_1, b_1, c_1, d_1$ used for the purposes of this project.
Fig. 4.1 Upper limit of total single line growth
TABLE 4.1

TOTAL GROWTH OF DEMAND

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total single line growth</td>
<td>a1=350</td>
<td>a2=700</td>
<td>a3=1050</td>
<td>a4=1400</td>
</tr>
<tr>
<td>Total DID trunk growth</td>
<td>b1=90</td>
<td>b2=180</td>
<td>b3=270</td>
<td>b4=360</td>
</tr>
<tr>
<td>Total Intraoffice usage growth</td>
<td>c1=300</td>
<td>c2=600</td>
<td>c3=900</td>
<td>c4=1200</td>
</tr>
<tr>
<td>Total Interoffice local usage growth</td>
<td>d1=1800</td>
<td>d2=3600</td>
<td>d3=5400</td>
<td>d4=7200</td>
</tr>
</tbody>
</table>

Source: Author's assumptions

4.3 Data Collection

To obtain the capital costs of the sixty-one different capacity expansion situations that are included in the four designs, fifteen runs of the Central Office Equipment Engineering System (COEES) computer program are necessary. In fact, sixty-four observations (sixteen per design) are needed, but the "all high" observation of design (1) is the same as the "all low" observation of design (1+1)(Vi=1,2,3), while the cost of no expansion is equal to zero.

To minimize the number of necessary runs, each of the fifteen capacity expansion scenarios (runs) was designed so as to provide the capital cost of the same observation type for all four designs. As example, the capital costs of all type (a) observations are provided by
the same scenario. Table 4.2 shows that this is achieved by using the variable levels of observation (a) in design 1 (350 lines, 0 trunks, 0 intraoffice CCS, 0 interoffice local CCS) as the first year's inputs, and assuming that the growth levels in each of the next three years are equal to 350 lines, 90 trunks, 300 intraoffice CCS, and 1800 interoffice local CCS.

**TABLE 4.2**

**INPUT LEVELS FOR OBSERVATIONS (a)**

<table>
<thead>
<tr>
<th>Input Variables</th>
<th>Busy Season</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Growth of single lines</td>
<td>350</td>
</tr>
<tr>
<td>Growth of DID trunks</td>
<td>0</td>
</tr>
<tr>
<td>Growth of intraoffice usage</td>
<td>0</td>
</tr>
<tr>
<td>Growth of interoffice local usage</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Author's assumptions

The alternatives that the COEES model identifies can provide the capital costs that are incurred in the beginning of the first year, if the current plant configuration is expanded by increments equal to the demand requirements for one, two, three, or four future years. Table 4.3 shows the data points that COEES can provide, when the variable levels of table 4.2 are used as inputs, together with the interpretation of each data point.
TABLE 4.3
COEES DATA POINTS

<table>
<thead>
<tr>
<th>Variable</th>
<th>Single line growth</th>
<th>DID trunk growth</th>
<th>Intraoffice usage growth</th>
<th>Interoffice local usage growth</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation (a)</td>
<td>350</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>design 1</td>
</tr>
<tr>
<td>Observation (a)</td>
<td>700</td>
<td>90</td>
<td>300</td>
<td>1800</td>
<td>design 11</td>
</tr>
<tr>
<td>Observation (a)</td>
<td>1050</td>
<td>180</td>
<td>600</td>
<td>3600</td>
<td>design 111</td>
</tr>
<tr>
<td>Observation (a)</td>
<td>1400</td>
<td>270</td>
<td>900</td>
<td>5400</td>
<td>design 1111</td>
</tr>
</tbody>
</table>

Source: Author's calculations

4.4 The Results

Ohio Bell’s telephone engineers have, under certain assumptions, transformed the growth levels, that the fifteen capacity expansion scenarios use, to the required inputs of the COEES model, and have provided the yearly costs of the plant additions of interest. In addition to the assumptions listed in section 3.6 (chapter III), two additional assumptions were made: (1) Interoffice local traffic growth is assumed to be 100% multi-frequency, (II) the growth in DID trunks is assumed to be 50% dial pulse and 50% touch tone out-pulsing.

Tables 4.4, 4.5, 4.6, and 4.7 present the plant addition sizes and the total capital costs of all observations in designs I, II, III, and IV. The last column in each of the above tables is calculated as the difference between the cost of an observation and the cost of the "all low" observation (1) of the corresponding design.
### Table 4.4
**Design 1: Total Capital Costs**

<table>
<thead>
<tr>
<th>Observation</th>
<th>Single Lines</th>
<th>Trunks</th>
<th>Variable Growth Levels</th>
<th>Capital Cost (×10⁵)</th>
<th>ΔTC₁ (×10⁵)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>a</td>
<td>350</td>
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<td>0</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>b</td>
<td>0</td>
<td>90</td>
<td>0</td>
<td>0</td>
<td>74</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>76</td>
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</tr>
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<td>300</td>
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</tr>
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<td>1800</td>
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<td>90</td>
<td>300</td>
<td>1800</td>
<td>340</td>
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Source: Ohio Bell
### TABLE 4.5
**DESIGN II: TOTAL CAPITAL COSTS**

<table>
<thead>
<tr>
<th>Observation</th>
<th>Single Lines</th>
<th>DID Trunks</th>
<th>Variable Growth Levels</th>
<th>Capital Cost (×10^3)</th>
<th>ΔTC2 (×10^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Intraoffice Usage</td>
<td>Interoffice Local Usage</td>
<td></td>
</tr>
<tr>
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*Source: Ohio Bell*
<table>
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<th>Single Lines</th>
<th>DID Trunks</th>
<th>Variable Growth Levels</th>
<th>Capital Cost</th>
<th>ΔTC3</th>
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</table>

Source: Ohio Bell
### TABLE 4.7
DESIGN III: TOTAL CAPITAL COSTS

<table>
<thead>
<tr>
<th>Observation</th>
<th>Single Lines</th>
<th>DID Trunks</th>
<th>Variable Growth Levels</th>
<th>Capital Cost (#10^3)</th>
<th>ΔTC₄ (#10^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>1050</td>
<td>270</td>
<td>900</td>
<td>5400</td>
<td>566</td>
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<td>900</td>
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<td>270</td>
<td>1200</td>
<td>7200</td>
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</tr>
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<td>360</td>
<td>1200</td>
<td>7200</td>
<td>688</td>
</tr>
</tbody>
</table>

*Source: Ohio Bell*
4.5 Analysis of the Results

The original intention of this project was to develop the simplest possible set of equations that can express the total cost of an expansion as a function of the growth levels of demand; in other words, it was expected that at least some of the second, third, and fourth order interactions between lines, trunks intraoffice usage and interoffice local usage would be negligible. Two simple examples can illustrate that this is not the case. Observations (1), (a), (b) and (ab) in design 1 (see table 4.4), for example, constitute a $2^2$ factorial design where: (1) single lines and DID trunks are the two variables involved, (11) the low level of lines is 0, while the high level is 350, (111) the low level of trunks is 0, while the high level is 90, (iv) intraoffice usage, and interoffice local usage are kept equal to zero. Figure 4.2 shows that the effect of adding 350 lines at two different levels of trunk additions (0 and 90) are quite different. This means that the additive model is not appropriate, and that the two factor interaction term between lines and trunks should be included in the model. The second example from design 1, considers single lines as the first factor, and DID trunks, Intraoffice usage, and Interoffice local usage combined as the second factor. The capital cost figures of observations (1), (a), (bcd), and (abcd) in table 4.4, together with figure 4.3, can illustrate the fact that higher order interaction terms should be included in the model.

Section 4.5 uses the notation introduced in section 3.8.2.
Fig. 4.2 An example of the significance of a second order interaction term.
Fig. 4.3 An example of the significance of a higher order interaction term
The reader can reach similar conclusions for other higher order interaction terms, by examining the data of tables 4.4, 4.5, 4.6 and 4.7 in a similar manner. The interaction terms come out to be substantially different from zero, because the same quantities of some equipment types that are added when engineering an addition due to the growth of one or more factors, are able to support additions due to the growth of one or more of the remaining factors too.

4.5.1 Design $i^3$

The difference between the capital cost of any capacity expansion within the variable ranges of design $i$, and the capital cost of the "all low" expansion in design $i$, is described by the following linear model:

$$\Delta TC_i = 30X_1 + 74X_2 + 30X_3 + 270X_4 - 28X_1X_2 - 29X_1X_3 - 29X_1X_4 - 29X_2X_3 - 5X_2X_4 - 30X_3X_4 + 29X_2X_3 + 28X_1X_4 + 30X_1X_3X_4 + 29X_2X_3X_4 - 30X_1X_2X_3X_4$$

The incremental capital cost of a single line ($ICC_1$), when the number of single lines added is between 0 and 350, the number of trunks added is between 0($X_2=0$) and 90($X_2=1$), the number of Intraoffice CCS added is between 0($X_3=0$) and 300($X_3=1$), and the number of Interoffice local CCS added is between 0($X_4=0$) and 1800($X_4=1$), is a function of $X_2$, $X_3$, and $X_4$ and is given by the formula:

$$ICC_1 = (30 - 28X_2 - 29X_3 - 29X_4 + 29X_2X_3 + 28X_2X_4 + 30X_3X_4 - 30X_2X_3X_4)/350$$

$^3$Throughout the rest of section 4.5 all costs and coefficients are expressed in thousands.
The incremental capital cost of a DID trunk (ICC$_2$), an intra-office CCS (ICC$_3$), and an interoffice local CCS are, according to Table 3.3, given by the formulas:

\[
\text{ICC}_2 = (74 - 28X_1 - 29X_3 - 5X_4 + 29X_1X_3 + 28X_1X_4 + 29X_3X_4 - 30X_1X_3X_4) / 90
\]

\[
\text{ICC}_3 = (30 - 29X_1 - 29X_2 - 30X_4 + 29X_1X_2 + 30X_1X_4 + 29X_2X_4 - 30X_1X_2X_4) / 300
\]

\[
\text{ICC}_4 = (270 - 29X_1 - 5X_2 - 30X_3 + 28X_1X_2 + 30X_1X_3 + 29X_2X_3 - 30X_1X_2X_3) / 1800
\]

4.5.2 Designs II, III, & IV

Formulas similar to those of Section 4.5.1, are developed for designs II, III, and IV, according to Table 3.6 and formulas (7) and (8) of Chapter III; they are as follows:

(1) Design II

\[
\Delta T C_2 = 11 + 22X_2 + 4X_3 + 27X_4 - 3X_1X_2 - 3X_1X_3 - 3X_1X_4 - 3X_2X_3 - 4X_2X_4 + 4X_3X_4 + 3X_1X_2X_3 + 3X_1X_2X_4 + 3X_1X_3X_4 + 4X_2X_3X_4 - 3X_1X_2X_3X_4
\]

\[
\text{ICC}_1 = (12 - 3X_2 - 3X_3 - 3X_4 + 3X_2X_3 + 3X_2X_4 + 3X_3X_4 - 3X_2X_3X_4) / 700
\]

\[
\text{ICC}_2 = (90 - 3X_1 - 3X_3 - 4X_4 + 3X_1X_3 + 3X_1X_4 + 4X_3X_4 - 3X_1X_3X_4) / 180
\]

\[
\text{ICC}_3 = (4 - 3X_1 - 3X_2 - 4X_4 + 3X_1X_2 + 3X_1X_4 + 3X_2X_4 - 3X_1X_2X_4) / 600
\]

\[
\text{ICC}_4 = (290 - 3X_1 - 4X_2 - 4X_3 + 3X_1X_2 + 3X_1X_3 + 4X_2X_3 - 3X_1X_2X_3) / 3600
\]

(11) Design III

\[
\Delta T C_3 = 6X_1 + 22X_2 + 5X_3 + 31X_4 - 4X_1X_2 - 4X_1X_3 - 4X_1X_4 - 4X_2X_3 + 116X_2X_4 - 4X_3X_4 + 3X_1X_2X_3 + 4X_1X_2X_4 + 4X_1X_3X_4 + 4X_2X_3X_4 - 3X_1X_2X_3X_4
\]

\[
\text{ICC}_1 = (15 - 4X_2 - 4X_3 - 4X_4 + 3X_2X_3 + 4X_2X_4 + 4X_3X_4 - 3X_2X_3X_4) / 1050
\]

\[
\text{ICC}_2 = (109 - 4X_1 - 4X_3 + 116X_4 + 3X_1X_3 + 4X_1X_4 + 4X_3X_4 - 3X_1X_3X_4) / 270
\]

\[
\text{ICC}_3 = (6 - 4X_1 - 4X_2 - 4X_4 + 3X_1X_2 + 4X_1X_4 + 4X_2X_4 - 3X_1X_2X_4) / 900
\]

\[
\text{ICC}_4 = (317 - 4X_1 + 116X_2 - 4X_3 + 4X_1X_2 + 4X_1X_3 + 4X_2X_3 - 3X_1X_2X_3) / 5400
\]
(iii) Design IV

\[ \Delta T_{C4} = 4x_1 + 22x_2 + 4x_3 + 28x_4 + 71x_1x_2 - 3x_1x_3 - 4x_1x_4 + 71x_2x_3 + 70x_2x_4 - 4x_3x_4 - 72x_1x_2x_3 - 70x_1x_2x_4 + 4x_1x_3x_4 - 70x_2x_3x_4 + 71x_1x_2x_3x_4 \]

\[ ICC_1 = (15 + 71x_2 - 3x_3 - 4x_4 - 72x_2x_3 - 70x_2x_4 + 4x_3x_4 + 71x_2x_3x_4) / 1400 \]

\[ ICC_2 = (247 + 71x_1 + 71x_3 + 70x_4 - 72x_1x_3 - 70x_1x_4 - 70x_3x_4 + 71x_1x_3x_4) / 360 \]

\[ ICC_3 = (6 - 3x_1 + 71x_2 - 4x_4 - 72x_1x_2 + 4x_1x_4 - 70x_2x_4 + 71x_1x_2x_4) / 1200 \]

\[ ICC_4 = (462 - 4x_1 + 70x_2 - 4x_3 - 70x_1x_2 + 4x_1x_3 - 70x_2x_3 + 71x_1x_2x_3) / 7200 \]

4.6 Average Incremental Capital Costs

The formulas that were presented in sections 4.5.1 and 4.5.2 can be used to estimate the incremental capital cost of any engineering measure of output given that the growth levels of the remaining three variables are known, and lie within the expansion limits of a particular design. Table 4.8 presents example incremental capital cost calculations for design I; the results clearly indicate that the incremental costs vary significantly depending on the addition size. The above variability is mainly due to the fact that some pieces of equipment may be added to satisfy small additional demand, while they would be capable of satisfying a larger capacity expansion. The reader must always keep in mind that incremental capital costs are calculated for a future time period; the additional complication introduced due to the uncertainty of the demand forecasts, and the significant variability of the calculated incremental costs themselves, indicate the necessity of introducing an averaging scheme.
<table>
<thead>
<tr>
<th>DID trunk growth</th>
<th>Intraoffice usage growth</th>
<th>Interoffice local usage growth</th>
<th>ICC₁ ($/line)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>85.71</td>
</tr>
<tr>
<td>18</td>
<td>60</td>
<td>360</td>
<td>45.83</td>
</tr>
<tr>
<td>54</td>
<td>180</td>
<td>1080</td>
<td>9.26</td>
</tr>
<tr>
<td>90</td>
<td>300</td>
<td>1800</td>
<td>2.86</td>
</tr>
<tr>
<td>Single line growth</td>
<td>Intraoffice usage growth</td>
<td>Interoffice local usage growth</td>
<td>ICC₂ ($/trunk)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>822.22</td>
</tr>
<tr>
<td>70</td>
<td>60</td>
<td>360</td>
<td>720.00</td>
</tr>
<tr>
<td>210</td>
<td>180</td>
<td>1080</td>
<td>680.89</td>
</tr>
<tr>
<td>350</td>
<td>300</td>
<td>1800</td>
<td>755.56</td>
</tr>
<tr>
<td>Single line growth</td>
<td>DID trunk growth</td>
<td>Interoffice local usage growth</td>
<td>ICC₃ ($/intraoffice CCS)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100.00</td>
</tr>
<tr>
<td>70</td>
<td>18</td>
<td>360</td>
<td>52.27</td>
</tr>
<tr>
<td>210</td>
<td>54</td>
<td>1080</td>
<td>8.00</td>
</tr>
<tr>
<td>350</td>
<td>90</td>
<td>1800</td>
<td>0.00</td>
</tr>
<tr>
<td>Single line growth</td>
<td>DID trunk growth</td>
<td>Intraoffice usage growth</td>
<td>ICC₄ ($/interoffice local CCS)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150.00</td>
</tr>
<tr>
<td>70</td>
<td>18</td>
<td>60</td>
<td>144.69</td>
</tr>
<tr>
<td>210</td>
<td>54</td>
<td>180</td>
<td>142.47</td>
</tr>
<tr>
<td>350</td>
<td>90</td>
<td>300</td>
<td>146.11</td>
</tr>
</tbody>
</table>
The average incremental capital costs of each design are calculated as follows: (I) eleven equally spaced points are selected for each of the four variables, (II) $11^3 = 1331$ incremental capital cost data points per variable are calculated, and (III) the average incremental capital cost of a variable is approximated by the average value of all the above estimates. Table 4.9 shows the average incremental capital costs of all the measures of output for all four designs. As example, the average incremental cost of a DID trunk in design II is equal to $484.00$, and can be interpreted as the average cost of a trunk when any number of lines between 350 and 700, any number of trunks between 91 and 180, any number of Intraoffice CCS between 300 and 600, and any number of Interoffice local CCS between 1800 and 3600, are added to the present configuration of the examined central office in the beginning of the first year.

Comparisons between the total capital cost figures in tables 4.4, 4.5, 4.6, and 4.7, together with the average incremental capital costs in table 4.9 indicate that DID trunk growth and Interoffice local usage growth are the two most important factors that drive the capacity expansion costs of this office. Single line growth and Intraoffice usage growth seem to contribute less. One must always keep in mind that the above conclusions apply to only the selected office and within the capacity expansion limits considered.

4.7 The Incremental Cost of an Average Customer

The incremental cost of an average customer is calculated under the following assumptions:
TABLE 4.9  
AVERAGE INCREMENTAL CAPITAL COSTS

<table>
<thead>
<tr>
<th>Design</th>
<th>Single lines</th>
<th>DID trunks</th>
<th>Intraoffice usage</th>
<th>Interoffice local usage</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>14.28</td>
<td>674.94</td>
<td>14.16</td>
<td>142.20</td>
<td>0 ≤ single line growth ≤ 350</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0 ≤ DID trunk growth ≤ 90</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0 ≤ Intraoffice usage growth ≤ 300</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0 ≤ Interoffice local usage growth ≤ 1800</td>
</tr>
<tr>
<td>II</td>
<td>13.39</td>
<td>484.00</td>
<td>1.87</td>
<td>79.61</td>
<td>350 ≤ single line growth ≤ 700</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>90 ≤ DID trunk growth ≤ 180</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>300 ≤ Intraoffice usage growth ≤ 600</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1800 ≤ Interoffice local usage growth ≤ 3600</td>
</tr>
<tr>
<td>III</td>
<td>10.83</td>
<td>612.47</td>
<td>2.64</td>
<td>69.18</td>
<td>700 ≤ single line growth ≤ 1050</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>180 ≤ DID trunk growth ≤ 270</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>600 ≤ Intraoffice usage growth ≤ 900</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3600 ≤ Interoffice local usage growth ≤ 5400</td>
</tr>
<tr>
<td>IV</td>
<td>15.27</td>
<td>857.92</td>
<td>10.31</td>
<td>64.97</td>
<td>1050 ≤ single line growth ≤ 1400</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>270 ≤ DID trunk growth ≤ 360</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>900 ≤ Intraoffice usage growth ≤ 1200</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5400 ≤ Interoffice local usage growth ≤ 7200</td>
</tr>
</tbody>
</table>

Source: Author's calculations
(1) The average busy-hour Intraoffice usage per main station is equal to 0.44 CCS.

(ii) The average busy-hour Interoffice usage per main station is equal to 3.94 CCS.

(iii) Interoffice usage is approximately 80% local and 20% toll.

(iv) The average busy-hour total usage per DID trunk is equal to 19.72 CCS.

The above listed assumptions are based on data that Ohio Bell has provided. We furthermore assume that the average busy-hour Intraoffice usage per DID trunk is equal to 1.622 CCS, while the average busy-hour Interoffice local usage per DID trunk is equal to 7.75 CCS.

The Incremental capital cost of an average single line customer (Alcc\(_1\)) and an average DID trunk customer (Alcc\(_2\)) are given by the formulas:

\[
\begin{align*}
\text{Alcc}_1 &= e_1 + e_3 \cdot 0.44 + e_4 \cdot 3.152 \\
\text{Alcc}_2 &= e_2 + e_3 \cdot 1.622 + e_4 \cdot 7.75 
\end{align*}
\]

where:

- \(e_1\): average incremental capital cost of a single line
- \(e_2\): average incremental capital cost of a DID trunk
- \(e_3\): average incremental capital cost of an Intraoffice CCS
- \(e_4\): average incremental capital cost of an Interoffice local CCS.

Table 4.10 presents the Incremental capital costs (of local service only) for an average single line customer, and an average DID trunk customer.
TABLE 4.10
INCREMENTAL CAPITAL COST OF AN AVERAGE CUSTOMER

<table>
<thead>
<tr>
<th>Design</th>
<th>A1CC₁</th>
<th>A1CC₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>468.74</td>
<td>1799.96</td>
</tr>
<tr>
<td>II</td>
<td>265.14</td>
<td>1104.01</td>
</tr>
<tr>
<td>III</td>
<td>230.05</td>
<td>1152.90</td>
</tr>
<tr>
<td>IV</td>
<td>224.50</td>
<td>1378.16</td>
</tr>
</tbody>
</table>

Source: Author's calculations

For the selected central office and within the expansion limits considered, the incremental capital of an average single line customer decreases as the size of the capacity expansion increases, while the incremental capital cost of an average DID trunk customer varies.

4.8 Summary

In chapter IV, we have applied the method that was discussed in chapter III. The assumptions, and the variable growth levels used were presented in section 4.2. Sections 4.3 and 4.4 described the way that the COEES runs were designed, and presented the capital cost figures, as provided by Ohio Bell. The incremental capital cost formulas developed were listed in section 4.5. Section 4.6 explained the averaging scheme that was used to approximate the average incremental capital costs. Finally, section 4.8 presented the method used to calculate the incremental capital cost of an average single line or DID trunk customer.
CHAPTER V
SUMMARY AND FURTHER RESEARCH

5.1 Summary

The objectives of this cost-of-service study are: (i) to develop a method to determine the nature of the central office capital cost function, (ii) to test the method in a pilot study for a selected central office, and (iii) to estimate the average incremental capital costs of four engineering measures of output for different capacity expansions that may occur in the beginning of the planning period. The cost of expanding an existing office is expressed as a function of the growth of single lines, direct indial business trunks, intraoffice usage, and interoffice local usage. Variables related to toll or special vertical services are held constant at their current levels.

Four $2^4$ factorial designs are used to analyze the total capital cost figures. In each design, the low and high levels of each variable represent addition sizes large enough to satisfy the demand for $(j)$ and $(j+1)$ future years respectively $(j=0,1,2,3)$. The problem is mathematically formulated by using dummy (zero-one) variables, while the incremental capital cost of each variable is expressed as a function of the growth levels of the remaining three variables.

The developed method is applied under the assumption of a linear increase in demand during the four year planning horizon. The analysis of the capital cost figures, that were provided by Ohio Bell, indicates that all interaction terms in the models are substantially different.
from zero. Interoffice local usage growth, and DID trunk growth are the two most important factors that drive the capacity expansion costs of the selected office. In addition to the above, the incremental capital cost estimates are found to vary significantly depending on the addition size. In each design, the average incremental capital cost of a variable is approximated by the average value of 1331 incremental capital cost estimates. Finally, the average incremental cost estimates and the average busy-hour intraoffice, and interoffice local usage per main station and DID trunk are used to calculate the incremental cost of an average customer. The incremental capital cost of an average single line customer decreases as the size of the capacity expansion increases, while the incremental capital cost of an average DID trunk customer varies.

5.2 Further Research

The model that is presented in this study must be considered as a first step towards using factorial designs for calculating the incremental costs of providing telephone service. The model can be extended and generalized in a variety of ways; some ideas that can be further investigated are the following:

(1) A few simple modifications to the data collection procedure, and the method of analysis can lead to the calculation of the long-run average incremental capital costs of engineering measures of output of a central office for a four year period of growth. According to Kahn (12), a four year period is an appropriate time length for incremental cost calculations. Assuming that the "all low" expansion
for the four year period is zero, the "all high" expansion is 1400 lines, 360 trunks, 1200 intraoffice CCS, and 7200 interoffice local CCS, and that the high level of demand increases linearly, the inputs of the COEES model for obtaining the cost of observation (a), for example, are 350 for the single line growth, and 0 for the remaining variables for all four busy seasons. The minimum present worth of each of the 16 observations, as identified by the COEES model, can be described by the following linear model:

\[
\min PW(X_1, X_2, X_3, X_4) = B_1 X_1 + B_2 X_2 + B_3 X_3 + B_4 X_4 + B_5 X_1 X_2 + B_6 X_1 X_3 + B_7 X_1 X_4 + B_8 X_2 X_3 + B_9 X_2 X_4 + B_{10} X_3 X_4 + B_{11} X_1 X_2 X_3 + B_{12} X_1 X_2 X_4 + B_{13} X_1 X_3 X_4 + B_{14} X_2 X_3 X_4 + B_{15} X_1 X_2 X_3 X_4
\]

The low level of each variable is coded as 0, while the high level as 1. The long run incremental capital cost of a single line (LRIC\textsubscript{1}), for example, is given by the formula:

\[
LRIC_1 = \frac{PW(X_2, X_3, X_4)}{1400}
\]

The rest of the analysis is the same as that of design 1 in this study.

(I1) The total interoffice toll usage, that has been kept constant for the purposes of this project, is a factor that a future study should examine. Since for at least the examined office, intraoffice usage has been found to be relatively inexpensive, one may consider reducing the number of COEES runs required by combining intraoffice and
Interoffice local usage growth into a single factor (local usage growth). If the follow-up study uses the same variable growth levels for lines and trunks, while the local usage growth inputs are equal to the sum of the respective intraoffice and interoffice local usage growth levels used herein, one may conclude that the incremental capital cost of local usage is mainly due to interoffice local usage growth.

(iii) The same addition sizes, that have been used for this office may be assumed as the variable growth levels for one or two additional offices of different existing sizes. If the average incremental capital costs, or the long-run average incremental costs of (i), are of similar magnitude to those found for the examined office, one may conclude that the existing office configuration does not significantly affect the incremental cost estimates, which are mostly a function of the expansion sizes.

(iv) The most complicated extension of this study is to attempt to estimate the company-wide incremental capital costs. If (iii) concludes that the incremental capital costs are not a function of the existing office configuration, one may attempt to group central offices based on the growth levels of the demand variables, and then estimate the incremental costs for representative offices from each group. If, however, (iii) concludes that the incremental costs are a function of the existing office configuration, the grouping
of the offices has to be based both on the existing and the growth levels of the variables considered. Representative offices from each group can then be selected, and a similar analysis can be performed.
GLOSSARY OF TECHNICAL TERMS

A

access line: A circuit between a subscriber and a switching center.

B

busy-hour: The uninterrupted period of 60 minutes for which the average intensity is at maximum.

C

call forwarding: A service feature available in some switching systems where calls can be rerouted automatically from one line to another or to an attendant.

call waiting: Service whereby a subscriber will receive a beep tone when he is on the phone, indicating another caller is trying to reach him.

CAPCOAST: A program used by Ohio Bell for capital cost calculations.

CCS: Hundred call seconds.

central office: In telephone operations, the facility housing the switching system and related equipment that provides service for customers in the immediate geographical area.

central processor: The part of a computer which contains the logic, computation and control circuits. It controls the interpretation and execution of instructions and, sometimes, contains memory.

CENTREX: Service providing a business telephone customer with direct inward dialing to its phone extensions and direct outward dialing for them.

circuit: A pair of complementary channels, which provide bi-directional communication, with associated equipment terminating in two exchanges.
coin telephone service: A service which permits outgoing telephone calls after insertion of adequate coins or tokens and, without payment, incoming calls.

concentration ratio: The ratio of the number of incoming main stations to the number of path connections between the line and trunk link networks.

COEES: Central Office Equipment Engineering System.

crossbar switch: A switch having a plurality of vertical and horizontal paths, and electromagnetically operated mechanical means for interconnecting any one of the vertical paths with any one of the horizontal paths.

D

dial pulse: One of two methods of dialing digits from a telephone (the other being key pulse). Dial pulses are generated by alternately opening and closing a contact in the telephone through which dc current flows.

digital switch: A time switch established to interconnect nominally discontinuous electric signals that change from one state to another in discrete steps.

direct inward dialing (DID): A feature of PBXs which allows callers to dial from the public network straight to a wanted extension without intervention by an operator.

E

electronic central office: Modern telephone central office using solid state devices (not electromechanical switches).

EES: Electronic Switching System.

1, 1A ESS: Central office types.

F

FCC: Federal Communications Commission.

G

generic program: A set of instructions for a central office that is the same for all offices using that type of switching system.
GS: Getting Started Cost = Cost incurred for building a new office independent of the lines installed or the traffic carried.

Intercom telephone: Common name for systems permitting two or more extension phones to intercommunicate with each other.

Interoffice: Between two telephone switching centers or telephone offices.

Intraoffice: Within the same telephone switching center or telephone office.

Junction: The interface equipment at the end of any Interoffice circuit or Intraoffice trunk which provide circuit and signaling compatibility.

Line fill factor: The ratio of the number of working lines to the number of lines available.

Line link network (LLN): The network whose nodes are line exchanges interconnected by line circuits.

Loop: A single connection from a switching center or an individual message distribution point to the terminals of an end instrument.

Main distributing frame: A distribution frame on one part of which terminate the permanent outside lines entering the central office building and on another part of which terminate the subscriber line multiple cabling, and the trunk multiple cabling.

Multi-frequency line: Line subdivision.

Network: A combination of circuits and terminals serviced by a single switching or processing center.

Out-pulsing trunk: Trunk category.
outside plant: That portion of intrabase communication systems extending from the main distributing frame outward to the telephone instrument or the terminal connections for other technical components.

PBX trunk: A trunk used to interconnect a PBX with its servicing circuit.

Plain Old Telephone Service (POTS): Service restricted to push button dialing, nationwide and international direct dialing, and accurate and regular telephone bills.

private wire: A circuit provided for a customer's private use.

protector frame: Device on the main distributing frame of a central office to provide protection against high voltage and high currents.

price elasticity of demand: A measure of the responsiveness of demand to changes in price, assuming that all other factors remain constant.

receiver: Telephone receiver found in the handset or as a loudspeaker.

register: A device accessible to a number of input circuits, which accepts and stores information relating to a called number or service.

remote switching system: Can be located in the same building as the main switching apparatus or remote from the central office. It provides local switching with limited routing facilities.

SCIS: Service Cost Information System.

step-by-step exchange: An electromechanical exchange comprising of switching stages working one after the other independently of the state of the following stages.

store: A storage memory unit in which information may be held until needed.

switching system: Central office types, PBXs, key telephone systems, switchboards, etc.
tape readings: Magnetic or punched paper tape used for information storage.

TESTCOST: A computer program used by Ohio Bell.

toll central office: Central office whose main function is the completion of long-distance calls.

traffic: Messages sent and received.

transmitter: In telephony, a microphone.

trunk: A communication channel between two switching systems.

trunk link network (TLN): The network whose nodes are trunk exchanges interconnected by trunk circuits.
REFERENCES


