Disclosure in the Presence of Network Effects

Dissertation

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Abstract

This thesis investigates how a firm’s incentives to publicly disclose privately-held information are affected by the presence of network effects. Network effects are common phenomena arising when the value a consumer derives from a product depends on the number of other users. Consumers in markets with network effects make product adoption decisions strategically in an effort to join a large network. The analysis demonstrates that disclosures in markets with network effects lead to important interactions between consumer purchasing and firm production decisions. These interactions, termed customer feedback loops, cause a firm to worry less about divulging trade secrets and more about conveying an advantage over its rival. As a consequence, network effects encourage the firm to publicly release information it would otherwise be reluctant to disclose.
To Lisa
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# Table of Contents

Abstract ................................................................. ii
Dedication ................................................................. iii
Acknowledgments .......................................................... iv
Vita .............................................................................. v
List of Figures ............................................................... viii
1. Introduction .............................................................. 1

2. Related Literature ....................................................... 10
   2.1 Disclosure Policy .................................................. 10
   2.2 Network Effects .................................................... 12

3. Model and Results ....................................................... 15
   3.1 Model ................................................................. 15
      3.1.1 Producer Competition ...................................... 15
      3.1.2 Consumer Preferences ..................................... 15
      3.1.3 Disclosure Policy .......................................... 19
   3.2 Results ............................................................... 20
      3.2.1 Monopoly ..................................................... 20
      3.2.2 Duopoly ....................................................... 26
   3.3 Extensions and Alternative Specifications ....................... 40
      3.3.1 Both Firms Acquire Private Information ................. 40
      3.3.2 Discretionary (Ex Post) Disclosure ...................... 43
      3.3.3 Firms Pre-Commit to Quantities ......................... 53
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3.4 Price Competition</td>
<td>60</td>
</tr>
<tr>
<td>3.3.5 Negative Network Effects (Congestion)</td>
<td>66</td>
</tr>
<tr>
<td>4. Discussion and Conclusion</td>
<td>69</td>
</tr>
<tr>
<td>References</td>
<td>72</td>
</tr>
<tr>
<td>Appendix</td>
<td>76</td>
</tr>
<tr>
<td>A. Proofs</td>
<td>76</td>
</tr>
</tbody>
</table>
List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Timeline</td>
<td>19</td>
</tr>
<tr>
<td>3.2 Binary cost example of monopoly outcomes.</td>
<td>27</td>
</tr>
<tr>
<td>3.3 Binary cost example of Cournot outcomes when correlation is high ((\rho \to 1)) and firms produce incompatible ((\kappa = 0)) goods.</td>
<td>38</td>
</tr>
<tr>
<td>3.4 Binary cost example of Cournot outcomes when correlation is low ((\rho = 0)) and firms produce incompatible ((\kappa = 0)) goods.</td>
<td>39</td>
</tr>
<tr>
<td>3.5 Discretionary disclosure equilibrium in the monopoly case.</td>
<td>48</td>
</tr>
<tr>
<td>3.6 Discretionary disclosure equilibrium in the duopoly case when correlation is low ((\rho = 0)) and firms produce incompatible ((\kappa = 0)) goods.</td>
<td>54</td>
</tr>
<tr>
<td>3.7 Discretionary disclosure equilibrium in the duopoly case when correlation is high ((\rho = 1)) and firms produce compatible ((\kappa = 1)) goods.</td>
<td>55</td>
</tr>
<tr>
<td>3.8 Comparison of correlation cutoffs when firms commit and do not commit upfront to production quantities.</td>
<td>61</td>
</tr>
<tr>
<td>3.9 Binary cost example of the impact of negative network effects on Cournot outcomes when correlation is low ((\rho = 0)) and firms produce incompatible ((\kappa = 0)) goods.</td>
<td>68</td>
</tr>
</tbody>
</table>
Chapter 1: Introduction

The incentives influencing a firm’s decision to publicly disclose privately held information are complex. When establishing its disclosure policy, a firm considers the consequences of sharing information with a diverse set of interested observers including consumers, competitors, suppliers, and capital market constituents. In many cases, the desire to disclose information to (or withhold information from) one set of stakeholders may either be offset or reversed by the consequences of disclosing to another group. This thesis adds to our understanding of such incentives by examining how network effects alter the product market consequences of disclosure.

Network effects are a prevalent force arising when the benefits of using a product are influenced by the number of consumers who purchase similar or compatible goods. The telephone is a classic example of a product exhibiting network effects. A single telephone is worth little on its own but provides value through its ability to connect with other phones. As additional phones emerge and increase the number of potential connections, the values of existing phones rise. While network effects arise directly as a result of connections between consumers, they also arise indirectly in non-communication markets where having a variety of complementary goods is
valuable to consumers. If a large customer base leads to an increase in offerings of complementary products and services (spare parts, servicing, software, etc.), then a consumer’s utility will increase as more individuals purchase the product. Examples of industries exhibiting network effects include consumer electronics, telecommunications, transportation, retailing, financial services, and health care.

The analysis in this thesis demonstrates that network effects alter a firm’s disclosure incentives by reducing the cost of sharing proprietary information with a rival while increasing the benefits of disclosure-induced coordination between a firm, its rival, and their consumers. As a result, network effects encourage a firm to disclose information it would otherwise choose to conceal for competitive reasons.

To elaborate, a firm’s disclosure incentives are affected by the presence of network effects because consumers of network goods make purchasing decisions strategically, based on expectations, in an effort to join a large network. Expectations of the number of products that will be sold (or network size) are a key factor in consumer purchasing decisions because the value a consumer will derive from a product depends on the number of other consumers he/she will be able to ‘connect with’ after the purchase is complete. When forming expectations of network size (and determining their willingness to pay for a firm’s product), consumers typically rely on external information sources such as product reviews, market reports, and word of mouth. These sources are, in turn, influenced by a firm’s information releases. Accordingly, a firm striving to maximize demand for its product will design its disclosure policy with the ability to influence consumer expectations in mind. A judiciously chosen

2
policy will nudge consumer expectations in the right direction precisely at the time the firm is poised to benefit most from widespread adoption of its product.

The primary finding of this thesis is that positive network effects encourage a firm to publicly release privately held information. This finding is presented in a model that incorporates key elements of the extant literature on disclosure and adds in network effects. The model entails firms which choose production quantities in a product market with network effects. The strength of the network effects and the degree of compatibility between the firms’ products vary exogenously. Network effect strength refers to the marginal benefit a consumer gains when an additional consumer purchases a product. Compatibility refers to the ability of consumers who purchase goods from different firms to derive network benefits from each other’s adoption. The firms’ production costs are uncertain but may be correlated, and the firms obtain private information regarding their own costs. Prior to the time production decisions are made and consumers determine their willingness to pay for the firms’ products, the firms independently decide whether to disclose their private information.

The underlying effects of disclosure on competition in this setting are consistent with the effects found in more traditional settings without network effects (see Gal-Or 1985, 1986 and Darrough 1993 for early examples or Raith 1996 and Bagnoli and Watts 2011 for related surveys). Two opposing effects warrant discussion. On one hand, disclosure is costly to the disclosing firm because it provides information a rival can use to fine-tune its strategy and compete more effectively. For example, if the disclosure informs the rival that its own cost will likely be lower than previously
expected, then the rival will respond by increasing production. Disclosure is beneficial, on the other hand, because it conveys the disclosing firm’s production incentives and, consequently, improves stochastic coordination between the firms. For example, when the information suggests the disclosing firm’s cost is relatively low, the rival infers the firm will produce a relatively large quantity. The rival responds by restricting production and yielding market share.

Ex ante, the expected net cost of disclosure is positive or negative depending on the degree of correlation between the firms’ production costs. The expected cost of disclosure depends on correlation because disclosure of a single piece of news can inform market participants outside the firm in several different ways. First, a disclosure informs about the disclosing firm’s production cost. Second, a disclosure may inform about the rival’s cost. Third, a disclosure can alter expectations about the relative production efficiency of the two firms. When there is no correlation, a disclosure does not provide any information about the rival’s production cost so the expected cost of sharing information with the rival is reduced. When correlation is perfect, a disclosure cannot provide any information about relative efficiency because an efficiency advantage does not exist. Consequently, a disclosure under perfect correlation does comparatively less to promote coordination between the firms and the expected benefit of disclosure is relatively small. With intermediate levels of correlation, all three pieces of information are contained in a disclosure and impact the decisions of the firms. Accordingly, a firm establishing its disclosure policy chooses to disclose
information when correlation is below a threshold and to withhold when correlation exceeds the threshold.

While the underlying effects of disclosure remain in the current setting, the impact of disclosure on the competitive decisions of a rival and, ultimately, firm profitability is more nuanced due to the introduction of network effects. The subtlety arises because a disclosure in the presence of network effects has significantly more influence on the purchasing decisions of consumers than a disclosure absent network effects. The disclosure is useful to consumers because it provides information about the firms’ production incentives and helps them determine how large the firms’ respective networks will become. Disclosure of this information causes important interactions, termed customer feedback loops, between consumer purchasing and firm production decisions. Intuitively, a disclosure convincing consumers that a firm has incentives to produce a larger quantity than originally anticipated increases consumer expectations of the firm’s network size. This change in consumer expectations results in a boost of consumers’ willingness to pay for the firm’s product. The firm, in turn, expands production and further increases the expected size of its network. The reverse progression occurs when a disclosure reduces expected network size. In the end, a disclosure of positive news in the presence of network effects results in quantities and profits that are extremely high while outcomes with negative news are relatively much lower.

Since a disclosure provides multiple pieces of information about each of the firms to consumers, a single disclosure results in multiple customer feedback loops for each
of the firms. Specifically, feedback loops materialize in response to news about a firm’s cost and to news about the competitive position of a firm relative to its rival. The relative importance of the individual feedback loops (and the ultimate impact of disclosure on firm profits) depends on the degree of compatibility between the firms’ products. When compatibility between the firms’ products is low (e.g. Sony’s Blu-ray Disc and Toshiba’s HD DVD; Sony’s Playstation and Microsoft’s Xbox), information regarding relative production efficiency is the most useful part of a disclosure to consumers who are seeking to join the largest network. Consequently, the feedback loop associated with news about relative production efficiency is the most significant loop when compatibility is low. When compatibility is high (e.g. HP’s Pavilion and Dell’s Inspiron), consumers see the firms’ networks as one so they are not concerned about relative network size. In this case, consumers are more concerned about the size of the firms’ joint network. Consequently, the news about the absolute level of the firms’ costs results in larger feedback loops than the news about relative efficiency.

The conclusion of this thesis is that network effects encourage firms to disclose information they would otherwise withhold for competitive reasons. More precisely, the correlation threshold below which a firm is willing to disclose information increases as network effects become stronger. Network effects impact a firm’s disclosure policy because customer feedback loops alter the expected costs and benefits of disclosure. When compatibility is low, disclosure-induced feedback loops allow a firm to enhance its competitive position by conveying a production cost advantage over its rival. A
disclosure convincing customers that the disclosing firm is better positioned to dominate the market will (because of feedback loops) also convince a rival to retreat. The disclosing firm will be left to enjoy its largest possible network. As a consequence, network effects reduce the expected cost of revealing proprietary information to a rival while augmenting the expected benefit of disclosing information about relative efficiency. When compatibility is high, feedback loops cause an anticipated increase in the production quantity of one firm to increase the value of another firm’s products. Thus, the cost of sharing information that motivates a rival to ramp up production is reduced. Furthermore, feedback loops enhance the benefits of disclosure by improving coordination between the firms and customers. Coordination is beneficial because it maximizes the size of the firms’ joint network and, consequently, maximizes customers’ willingness to pay for the firms’ products precisely when the cost of production is low. As a result, network effects again reduce the cost of disclosing private information while increasing the benefits. For all levels of compatibility, network effects increase the benefits of disclosure while reducing the costs. Accordingly, network effects persuade firms to publicly disclose privately held information.

Summarizing, the key findings of this thesis are: (1) public disclosure in the presence of network effects results in a revision of consumer expectations that gives rise to customer feedback loops; (2) customer feedback loops, in turn, lead a firm to be less concerned about divulging trade secrets than about demonstrating a competitive advantage; and, consequently, (3) network effects induce firms to disclose privately held information they would otherwise choose to withhold. These results are robust to
various extensions and alternative specification of the primary model. Supplemental analyses included in this thesis consider settings in which a firm makes a disclosure decision after observing the information and firms choose prices rather than quantities. In each supplemental examination, the primary conclusions of the thesis persist, but with additional considerations.

In addition to offering a theoretical understanding of the influence network effects have on disclosure incentives, the findings of this thesis may also provide insight into the empirical research on disclosure. One immediate implication of this thesis is that it demonstrates a positive association between a firm’s propensity to disclose information and the strength of network effects in its industry. This connection may, in part, explain cross-industry variation in the degree of financial transparency as has been documented in studies such as Chen et al. 2002, Brown et al. 2006, and Ellis et al. 2011 (see Dye and Sridhar 1995 and Arya and Mittendorf 2007 for additional, complementary, explanations). The results of this thesis may also shed light on the connection between product market competition and disclosure. Though many firms assert that competitive pressures undermine disclosure incentives, the empirical evidence of this relation has thus far been mixed (see, e.g., Beyer et al. 2010 and Berger 2011 for reviews). The results herein suggest that if one were to isolate industries with relatively weak network effects, the tendency for competition to dampen disclosure may be more apparent in archival data, whereas the relationship may be muted by network effects elsewhere.
The remainder of the thesis is organized as follows: Chapter 2 reviews the related literature in more detail. Chapter 3 outlines the primary model and presents the results. Chapter 4 provides additional discussion and concludes the analysis.
Chapter 2: Related Literature

This study draws upon the streams of literature examining (1) a firm’s incentives to publicly disclose privately-held information in the presence of a rival and (2) the economic consequences of network effects. This chapter summarizes the existing related literature and theoretical results most relevant to this thesis.

2.1 Disclosure Policy

A firm’s choice of whether to publicly disclose private information is a particularly complex one due to the diversity of interested observers. While the capital markets benefits of such disclosures are widely accepted (see Verrecchia 2001, Dye 2001 and Beyer et al. 2010. for reviews), the conclusions of research examining the product market consequences of disclosure are less straightforward. An important tension a firm faces when making a disclosure decision is between the desire to hide information rivals can use to compete more effectively and the desire to achieve the benefit of quasi-Stackelberg leadership by revealing production intentions to rivals. A vast literature, beginning with Gal-Or (1985, 1986) and Vives (1984) in economics and Darrough (1993) in accounting, has examined how these incentives play out in various contexts. Gal-Or (1985) and Vives (1984) examine oligopoly models in which firms acquire
private information regarding a common demand parameter and choose whether to reveal the information to rival firms. Firms then consider the revealed information when choosing production quantities. Gal-Or (1985) and Vives (1984) both show that the equilibrium under these conditions entails no information sharing. Vives (1984) also demonstrates, however, that information sharing is a dominant strategy when firms choose prices rather than quantities. Gal-Or (1986) examines a similar model in which the uncertainty is about unknown private production costs and demonstrates that the equilibrium entails information sharing when firms choose quantities but no information sharing when firms choose prices. Darrough (1993) also examines firms’ incentives to disclose private information and confirms the findings of Gal-Or (1985, 1985) and Vives (1984). Darrough concludes that the consequences of disclosure policy depend on the specific type of competition firms are engaged in and the type of private information firms have.

Given the varied findings regarding the proprietary costs and benefits of disclosure, attempts to synthesize and generalize the results have been undertaken in Raith (1996) and Bagnoli and Watts (2011). Raith (1996) provides a general model that includes the models of previous studies as special cases. Bagnoli and Watts (2011) further generalize the information structure to allow for intermediate values cases in which the information is privately useful to the firm and partially useful to its rival. Despite these endeavors to reconcile the varied perspectives, the general conclusion of this body of research remains that the form of retail competition (price or quantity)
and the nature of the information (private-value or common-value) are critical factors in evaluating the attractiveness of disclosure.

Additional examples of work examining firms’ ex ante incentives to disclose private information in the presence of retail market competitors include, among others, papers by Maleug and Tsutsui (1996) who examine ex ante disclosure when firms receive signals about the slope of the market demand curve; Hwang and Kirby (2000) who consider how disclosure incentives are impacted when a disclosure influences the entry decision of a potential rival; Arya and Mittendorf (2007) who illustrate that firms’ ex ante incentives to withhold private information from competitors may be undercut by the desire to boost analyst following; and Bagnoli and Watts (2013) who show that when a firm has private information about a rival’s business, then the firm has ex ante incentives to disclose it.

2.2 Network Effects

Network effects arise when the value of a product to one user depends on the number of other users that adopt the product. This phenomenon was first discussed by Rohlfs (1974), but the seminal modern contributions on network effects are a series of papers by Farrell and Saloner (1985, 1986a, 1986b) and Katz and Shapiro (1985, 1986). The first set of papers analyzes the trade-off between the benefits of standardization and the benefits of variety while the latter examines the strategic implications of network effects.

The introduction of network effects into a market alters the dynamics of competition and consumption behavior. When evaluating the worth of a network good,
consumers consider how many other consumers will likely purchase a product. Consequently, if consumers in general believe a firm’s product will be widely adopted (and that the network of consumers using the firm’s products will be large), then consumers’ willingness to pay for the product will increase and the firm’s profits will rise. Conversely, if consumers believe a firm’s product will flop, then profits will suffer. In some cases, the success (or failure) of a firm is tied as much to consumer expectations of network size as to the underlying value of the firm’s products (see, for example, Katz and Shapiro 1985). A firm operating in this scenario can enhance its competitive position by working to convince consumers and rivals that its platform will ultimately become standard while rival, incompatible products will be stranded. The policies guiding firm behavior must be selected carefully because firm behavior can impact consumer expectations just enough to tip the market in (or out of) the firm’s favor (see Shapiro and Varian 1999 for related discussion).

The first example of research examining the impact of network effects on firm behavior is pioneering study by Katz and Shapiro (1985). Katz and Shapiro (1985) examine a single period oligopoly model of firms choosing whether to produce compatible products and shows there is a tendency to too little compatibility relative to the social optimum. Katz and Shapiro (1986) extend the model to multiple periods and show, however, that private firms may have socially excessive compatibility incentives because compatibility serves as a means of relaxing price competition during early stages of industry growth. Other papers addressing the impact of network effects of competition include work by Conner (1995) who finds that an innovator in a
market with strong network effects may benefit from having a clone competitor even if the innovator can foreclose such competition; Xie and Sirbu (1995) who show that an innovating firm can achieve faster diffusion of its product and gain a higher profit by having a compatible competitor enter the market at an early stage rather than by being a monopolist; and Economides (1996) who similarly finds that a monopolist in a market with strong network effects has incentive to encourage entry by competitors. Additional relevant work includes Dhebar and Oren (1985) and Xie and Sirbu (1995) which examine the impact of network effects on pricing strategy; Padmanabhan et al. (1997) which examines product upgrade strategies; and Sun et al. (2004) which considers various product strategies, such as product line extensions and licensing, in the presence of network effects.

Given that prior research has identified numerous ways in which firms alter their behavior to exploit network effects, it is plausible that firms also benefit from adjusting their disclosure practices. This thesis formally examines this supposition and demonstrates that network effects encourage firms to disclose information they would otherwise choose to withhold for competitive reasons.
3.1 Model

3.1.1 Producer Competition

A firm, denoted $F$, and its rival, denoted $R$, are (Cournot) competitors in a product market with network effects. Each firm’s (marginal) cost of production is uncertain. In particular, $F$’s cost is $c_F = \bar{c} + \delta$ where $\bar{c}$ is the expected cost and $\delta$, $\delta \in [\delta_L, \delta_H]$, is a mean zero noise term with variance $\sigma^2$. $R$’s expected cost, $E[c_R]$, is also $\bar{c}$, and the firms’ costs are jointly distributed such that $E[c_R|\delta] = \bar{c} + \rho \delta$, where $\rho$, $0 \leq \rho \leq 1$, indicates the degree of correlation in the firms’ costs. Prior to the time the firms make their production decisions, $F$ privately observes $\delta$.

3.1.2 Consumer Preferences

Following Katz and Shapiro (1985), the firms face a continuum of consumers who each purchase either zero or one unit of a good. The utility, $u$, an individual consumer of type $\alpha$ derives from purchasing a product from firm $i$, $i \in \{F,R\}$, has two components:

$$u = \alpha + \eta Q_i.$$
The first component of consumer utility, $\alpha$, denotes the basic, standalone value of the product to the consumer. This standalone value varies across the continuum of consumers and ranges over the interval $[z, a]$. The lower-bound $z$ is sufficiently small to avoid corner solutions, where all consumers purchase a product. Furthermore, $a$ is sufficiently large that the firms produce positive quantities.

The second component, $\eta Q_i$, denotes the network benefit associated with firm $i$’s product. In this linear framework, $\eta$, $0 \leq \eta < 1$, denotes the strength of the network effects while $Q_i$ denotes the size of the associated network. The network effects literature commonly assumes positive network effects ($\eta \geq 0$), but the restriction is not necessary for this analysis. The case of $\eta < 0$ is discussed in Section 3.3. The restriction $\eta < 1$ ensures the existence of an equilibrium in the quantity decision sub-game. Firm $i$’s network size is $Q_i = q_i + \kappa q_j$, where $q_i$ and $q_j$ denote firm $i$ and $j$’s production quantities, respectively, and $\kappa$, $0 \leq \kappa \leq 1$, denotes the degree of compatibility between the firms’ products. When $\kappa = 0$, the two firms’ goods are incompatible and consumers who purchase goods from different firms do not derive any benefit from each other’s adoption. If $\kappa = 1$, the firms’ goods are perfectly compatible and consumers derive the same network benefit from every good that is purchased regardless of which firm produced it. An intermediate value of $\kappa$ indicates partial compatibility. With partial compatibility, consumers purchasing different goods derive some network benefit from each other but would receive greater benefit if they were to purchase goods from the same firm.
Consumers act to maximize their individual surpluses and make purchasing decisions before actual network sizes are known. Consequently, a consumer purchases a product from the firm whose good provides the largest expected surplus, provided the expectation is positive. If expected surplus is negative for both firms’ products, then the consumer stays out of the market. The expected surplus for a consumer of type $\alpha$ who purchases a product from firm $i$ is $E[\alpha + \eta Q_i - p_i] = \alpha + \eta S_i - p_i$, where $S_i \equiv E[Q_i]$ denotes the expected size of firm $i$’s network, and $p_i$ denotes the purchase price of firm $i$’s product. A consumer of type $\alpha$ purchases a product if and only if there exists a firm $i$ such that

$$\alpha + \eta S_i - p_i \geq 0. \quad (3.1)$$

Rearranging (3.1) provides

$$\alpha \geq p_i - \eta S_i. \quad (3.2)$$

Section 3.2.1 considers the monopoly case in which only $F$ produces and sells a positive number of units. In this case, only consumers for which $\alpha \geq p_F - \eta S_F$ purchase a product. Given the distribution of $\alpha$, there are $a - (p_F - \eta S_F)$ such consumers, who each purchase a single unit. Thus, if $F$ sells a total of $q_F$ units, then $p_F$ must be set such that $a - (p_F - \eta S_F) = q_F$. Rearranging provides the monopolist’s (inverse) demand function written as

$$p_F = a - q_F + \eta S_F.$$

Section 3.2.2 considers the case in which both firms produce and sell a positive number of units. In this case (given the homogeneity of the products), prices must
be set such that all consumers are indifferent between $F$ and $R$’s products. To see this, suppose that $F$’s product is preferred by one consumer for a given set of prices. Because $\alpha + \eta S_F - p_F > \alpha + \eta S_R - p_R$ is equivalent to $\eta S_F - p_F > \eta S_R - p_R$, it follows that $F$’s product is preferred by all consumers and that demand for $R$’s product is zero. Similarly, if prices are such that $R$’s product is preferred by one of the consumers, then demand for $F$’s product is zero. It must be the case, therefore, that if the two firms produce and sell positive quantities of the good, then consumers with positive expected surpluses are indifferent between the firm’s products. Formally, the following condition must be satisfied:

$$\alpha + \eta S_F - p_F = \alpha + \eta S_R - p_R. \quad (3.3)$$

Rewriting, (3.3) yields

$$p_F - \eta S_F = p_R - \eta S_R. \quad (3.4)$$

Let $\theta$ denote the value given in (3.4). From (3.2), only consumers for whom $\alpha \geq \theta$ purchase a product. Given the distribution of $\alpha$, there are $a - \theta$ such consumers (who each purchase a single unit). Thus, if the two firms firms sell a total of $q_F + q_R$ units, then prices must be set such that

$$a - \theta = q_F + q_R. \quad (3.5)$$

Restating (3.5),

$$a - (p_i - \eta S_i) = q_F + q_R \quad \text{for} \quad i = F, R. \quad (3.6)$$

Rearranging (3.6) provides the demand function for firm $i \in \{F, R\}$ written as

$$p_i = a - q_F - q_R + \eta S_i.$$
3.1.3 Disclosure Policy

At the outset, \( F \) establishes a policy determining whether \( \delta \) will be publicly disclosed once it is acquired. If \( F \) chooses to disclose, the pronouncement is assumed to be truthful. The focus of this thesis is on the influence network effects have on \( F \)'s decision to disclose \( \delta \). Specifically, given that disclosure of its private information will impact consumer expectations and purchasing decisions as well as \( R \)'s production choice, what is \( F \)'s preferred disclosure policy and how is the policy impacted by network effects? Summarizing the model, correlation between the firms' costs increases with \( \rho \), the strength of the network effects increases with \( \eta \), and compatibility between the firms' products increases with \( \kappa \). Working backward through the game to determine outcomes under each disclosure policy, and then comparing \( F \)'s expected profits reveals the preferred policy. The analysis considers subgame perfect, rational expectations equilibria. The timing of events is summarized in Figure 3.1.
3.2 Results

3.2.1 Monopoly

The analysis in this section initially considers the impact of network effects on a firm’s production quantity, profits, and disclosure policy when the firm enjoys monopoly power (i.e. \( q_R = 0 \)). The monopoly case excludes the impact of a disclosure on the competitive decisions of a rival and focuses the analysis solely on the impact of the disclosure on consumer expectations and purchasing decisions. This special case isolates the novel aspect of this paper’s results and, in doing so, highlights the important role of customer feedback loops in leading the informed firm to prefer disclosure.

Monopoly Equilibrium Under Disclosure

Under disclosure, the monopolist and consumers are both aware of \( \delta \) and make quantity and purchasing decisions accordingly. Given the information \( \delta \) and a conjecture of consumer expectations of network size, \( \hat{S}_F(\delta) \), the firm chooses its quantity \( q_F \) to solve

\[
\max_{q_F} \left( a - q_F + \eta \hat{S}_F(\delta) \right) q_F - (\bar{c} + \delta) q_F. \tag{3.7}
\]

The first order condition of (3.7) is provided in (3.8), with superscript \( d \) reflecting disclosure.

\[
q_F^d(\delta) = \frac{a - (\bar{c} + \delta) + \eta \hat{S}_F(\delta)}{2} \tag{3.8}
\]

At the time the monopolist chooses quantity, consumers determine their willingness to pay for the firm’s product. Under disclosure, consumers’ expectation of
network size, denoted $S^d_F$, is based on $\delta$ and a conjecture of the firm’s production quantity, $\hat{q}_F(\delta)$, as stated in equation (3.9).

$$S^d_F(\delta) = \hat{q}_F(\delta)$$ (3.9)

The firm and consumers’ conjectures must be correct in equilibrium. Formally, conjectures under disclosure must satisfy equations (3.10) and (3.11).

$$\hat{S}_F(\delta) = S^d_F(\delta)$$ (3.10)

$$\hat{q}_F(\delta) = q^d_F(\delta)$$ (3.11)

Equations (3.8) - (3.11) characterize the equilibrium under disclosure. Combining equations (3.8) - (3.11) and computing profits reveals the equilibrium outcomes under disclosure as summarized in Lemma 1 (all proofs are in the appendix).

**Lemma 1.** Under disclosure, the monopoly equilibrium quantity and profits (denoted $\pi^d_F$) are:

(i). $q^d_F(\delta) = \frac{a - \bar{c}}{2 - \eta} - \frac{\delta}{2 - \eta}$

(ii). $\pi^d_F(\delta) = \left[ \frac{a - \bar{c}}{2 - \eta} - \frac{\delta}{2 - \eta} \right]^2 = [q^d_F(\delta)]^2$.

**Monopoly Equilibrium Under No Disclosure**

Under no disclosure, the firm chooses its quantity, based on $\delta$ and its conjecture of consumer expectations of network size, $\hat{S}_F$, to solve

$$\max_{q_F} \left( a - q_F + \eta\hat{S}_F \right) q_F - (\bar{c} + \delta) q_F.$$ (3.12)
The first order condition of (3.12) is provided in (3.13), with superscript \( \phi \) reflecting no disclosure.

\[
q_F^\phi(\delta) = \frac{a - (\bar{c} + \delta) + \eta \hat{S}_F}{2} \quad (3.13)
\]

Consumers form expectations of network size and make purchasing decisions without information about the firm’s cost. Consequently, consumer expectations of network size are as stated in equation (3.14).

\[
S_F^\phi = E[\hat{q}_F(\delta)] \quad (3.14)
\]

Under no disclosure, rationality requires the conjectures to satisfy equations (3.15) and (3.16).

\[
\hat{S}_F = S_F^\phi \quad (3.15)
\]

\[
\hat{q}_F(\delta) = q_F^\phi(\delta) \quad (3.16)
\]

Combining equations (3.13)-(3.16) and computing profits reveals the equilibrium outcomes under no disclosure as summarized in Lemma 2.

**Lemma 2.** Under no disclosure, the monopoly equilibrium quantity and profits \( \pi_F^\phi \) are:

(i). \[
q_F^\phi(\delta) = \frac{a - \bar{c}}{2 - \eta} - \frac{\delta}{2}
\]

(ii). \[
\pi_F^\phi(\delta) = \left[\frac{a - \bar{c}}{2 - \eta} - \frac{\delta}{2}\right]^2 = \left[q_F^\phi(\delta)\right]^2.
\]

\( F \)’s quantities under the two policies reflect some intuitive features. First, greater demand (captured by \( a \)) leads to higher quantity while higher cost \( (\bar{c} + \delta) \) leads to
lower quantity. Second, the quantity increases with the strength of the network effects ($\eta$) as well as consumer expectations of network size. An increase in the strength of the network effects or the expected network size boosts consumers’ willingness to pay for the firm’s products. The firm responds with increased production.

A comparison of $q^d_F(\delta)$ and $q^\phi_F(\delta)$ provides insight into the influence of consumer feedback loops on the firm’s equilibrium quantity, profits and disclosure policy. Under both policies, an increase in the strength of the network effects ($\eta \rightarrow 1$) leads to a larger quantity and higher profits, but the impact of the news, $\delta$, on the outcomes differs. Under disclosure, the impact of the news on quantity is $\frac{\delta}{2-\eta}$ while under no disclosure the impact is $\frac{\delta}{2}$. The difference arises because a disclosure informs consumers about the firm’s production incentives and results in a customer feedback loop. When the firm discloses its production cost is low ($\delta < 0$), consumers infer the quantity produced will be large along with the resulting network. When cost is high, consumers infer the resulting network will be relatively small. A disclosure that changes consumers’ expectation of a firm’s network size also changes consumers’ willingness to pay for the firm’s product. A change in consumers’ willingness to pay leads to an additional change in the firm’s production decision. Withholding the information prevents a feedback loop from occurring and results in relatively moderate quantities. The lack of a feedback loop is evident in the fact that the impact of the news under no disclosure ($\frac{\delta}{2}$) does not change as the network effects strengthen. The firm adjusts its quantity according to the news, but consumers remain ignorant of the change in network size that will result. When cost is low, consumers do not adjust
their willingness to pay upward. The firm, consequently, produces a lower quantity and obtains lower profits under no disclosure than it would under disclosure when cost is low. When the production cost is high, the firm produces a higher quantity and enjoys higher profits under no disclosure than it would under disclosure. Succinctly, the impact of the news on firm profitability is less significant under no disclosure.

Importantly, the intensity of the feedback loop caused by disclosure and the ultimate impact of the information under disclosure \((\frac{\delta}{2-\eta})\) become greater as the network effects strengthen. Without network effects \((\eta = 0)\), the impact of \(\delta\) under disclosure is identical to that under no disclosure \((\frac{\delta}{2})\). The disclosure has no bearing on consumer behavior because consumers do not derive any benefit from a network and are, therefore, unconcerned about network size. With network effects, however, information about the production quantity is relevant to consumers and becomes more so as the network effects become stronger. Accordingly, the impact of the news under disclosure becomes larger under disclosure as the network effects strengthen.

**Monopolist’s Optimal Disclosure Policy**

The monopolist establishes its disclosure policy based on expected profits. Lemma 3 provides the expectations, with \(\Pi_F^d\) and \(\Pi_F^\phi\) denoting expected monopoly profits under disclosure and no disclosure respectively. A direct comparison of \(\Pi_F^d\) to \(\Pi_F^\phi\) reveals the optimal disclosure policy as summarized in Proposition 1.
Lemma 3. Expected monopoly profits under disclosure and no disclosure are:

(i). \( \Pi^d_F \equiv E[\pi^d_F(\delta)] = \left( \frac{a - \bar{c}}{2 - \eta} \right)^2 + \left( \frac{\sigma}{2 - \eta} \right)^2 \)

(ii). \( \Pi^\phi_F \equiv E[\pi^\phi_F(\delta)] = \left( \frac{a - \bar{c}}{2 - \eta} \right)^2 + \sigma^2 \).

Proposition 1.

(i). The monopolist (strictly) prefers disclosure if and only if network effects exist \( (\eta > 0) \).

(ii). The net benefit of disclosure is increasing in the strength of the network effects, i.e. \( \frac{d}{d\eta}[\Pi^d_F - \Pi^\phi_F] > 0 \).

Proposition 1 shows that if network effects are present, then the firm prefers a policy of disclosure. Furthermore, the expected benefit of disclosure increases as the network effects strengthen. A policy of disclosure is appealing in the presence of network effects because disclosure when the realized production cost is relatively low \( (\delta < 0) \) causes a positive feedback loop and results in a relatively large quantity and high profits. A policy of disclosure is costly, however, because a negative feedback loop occurs when the realized cost is high. The proposition reveals that the expected benefit of positive feedback loops outweighs the expected cost of negative feedback loops. Stated differently, convexity in the monopolist’s profit function creates a preference for dispersion of production quantities. As the equilibrium quantities under disclosure are more disperse relative to those under no disclosure, the firm prefers disclosure. Intuitively, a policy of disclosure ensures consumers’ willingness to pay is
high at the same time production cost is low, which is when the firm benefits most from widespread adoption of its product. Thus, the conclusion of this subsection is that the presence of positive network effects encourages the monopolist to publicly disclose its privately held information. To demonstrate this finding, Figure 3.2 provides a binary cost illustration of monopoly outcomes both with and without network effects. Panel 3.2a shows equilibrium quantities and profits without network effects. Panel 3.2b shows outcomes with network effects. Network effects increase expected profits under both policies, but expected profits are higher under disclosure.

3.2.2 Duopoly

As demonstrated in the previous subsection, disclosure in the presence of network effects has an important impact on consumer purchasing decisions and, consequently, firm profitability. The analysis of this subsection incorporates a competitor in order to provide insight into the competitive ramifications of disclosure in the presence of network effects. The general finding of this subsection is that network effects encourage the informed firm to disclose information it would otherwise avoid disclosing for competitive reasons.

**Duopoly Equilibrium Under Disclosure**

Under disclosure, \( F, R \) and consumers each make their respective decisions based on the information \( \delta \). Given \( \delta \), a conjecture of \( R \)'s quantity, denoted \( \hat{q}_R(\delta) \), and a conjecture of consumer expectations of network size \( \hat{S}_F(\delta) \), \( F \) chooses its quantity \( q_F \).
This figure highlights the importance of customer feedback loops in leading to dispersed equilibrium outcomes. For $\eta = 0$, disclosure has no bearing on consumer purchasing decisions so outcomes are identical under disclosure and no disclosure. For $\eta > 0$, disclosure of low cost leads to higher quantities than no disclosure while disclosure of high cost leads to lower quantities. Expected profits are higher under disclosure. $F$ subscripts are suppressed while $q_H$ and $q_L$ denote the monopolist’s equilibrium quantities given high and low cost, respectively.
to solve
\[
\max_{q_F} \left( a - q_F - \hat{q}_R(\delta) + \eta \hat{S}_F(\delta) \right) q_F - (\bar{c} + \delta)q_F.
\] (3.17)

Similarly, given \( \delta \), a conjecture of \( F \)'s quantity \( \hat{q}_F(\delta) \), and a conjecture of consumer expectations of network size \( \hat{S}_R(\delta) \), \( R \) chooses its quantity \( q_R \) to solve
\[
\max_{q_R} \left( a - q_R - \hat{q}_F(\delta) + \eta \hat{S}_R(\delta) \right) q_R - (\bar{c} + \rho \delta)q_R.
\] (3.18)

The first order conditions of (3.17) and (3.18) are provided in (3.19) and (3.20), respectively.
\[
q^d_F(\delta) = \frac{a - (\bar{c} + \delta) - \hat{q}_R(\delta) + \eta \hat{S}_F(\delta)}{2}
\] (3.19)
\[
q^d_R(\delta) = \frac{a - (\bar{c} + \rho \delta) - \hat{q}_F(\delta) + \eta \hat{S}_R(\delta)}{2}
\] (3.20)

Consumers form their expectations of the firms’ network sizes and establish willingness to pay based on \( \delta \) and conjectures of the firms’ quantities as stated in equations (3.21) and (3.22).
\[
S^d_F(\delta) = \hat{q}_F(\delta) + \kappa \hat{q}_R(\delta)
\] (3.21)
\[
S^d_R(\delta) = \hat{q}_R(\delta) + \kappa \hat{q}_F(\delta)
\] (3.22)

Rationality requires the firms and consumers’ conjectures to satisfy equations (3.23) - (3.26).
\[
\hat{S}_F(\delta) = S^d_F(\delta)
\] (3.23)
\[
\hat{S}_R(\delta) = S^d_R(\delta)
\] (3.24)
\[
\hat{q}_F(\delta) = q^d_F(\delta)
\] (3.25)
\[
\hat{q}_R(\delta) = q^d_R(\delta)
\] (3.26)

28
Jointly solving equations (3.19) - (3.26) and computing profits reveals the equilibrium outcomes under disclosure as summarized in Lemma 4.

**Lemma 4.** Under disclosure, the duopoly equilibrium quantities and profits are:

(i). \[ q_F^d(\delta) = \frac{a - \bar{c}}{3 - \eta(k + 1)} - \frac{\delta}{3 - \eta(k + 1)} - \frac{\delta (1 - \rho) (1 - \eta k)}{(3 - \eta(k + 1))(1 - \eta(1 - k))} \]

and \[ \pi_F^d(\delta) = \left[q_F^d(\delta)\right]^2 \]

(ii). \[ q_R^d(\delta) = \frac{a - \bar{c}}{3 - \eta(k + 1)} - \frac{\rho \delta}{3 - \eta(k + 1)} + \frac{\delta (1 - \rho) (1 - \eta k)}{(3 - \eta(k + 1))(1 - \eta(1 - k))} \]

and \[ \pi_R^d(\delta) = \left[q_R^d(\delta)\right]^2 \].

The firms’ quantity choices under disclosure exhibit the same intuitive features as found in the monopoly case, which are reflected in the first two terms of each firm’s quantity. Competition introduces a key addition, however, which is evident in the final term of the firms’ equilibrium quantities. The additional feature is that each firm’s quantity increases with its expected cost advantage relative to the other firm. The relative cost advantage is captured by the difference \( E[c_R|\delta] - c_F = -\delta(1 - \rho) \). \( F \) holds the advantage when the difference is positive, while \( R \) enjoys the advantage otherwise. When a firm’s production cost is low relative to its rival’s, the low cost firm produces a relatively larger quantity.

The quantities presented in Lemma 4 also demonstrate that multiple feedback loops occur as a result of disclosure in the presence of a competitor and network effects. Disclosure in this situation provides several pieces of information to consumers which each result in feedback loops. First, the disclosure informs about each firm’s absolute cost of production. The impact of this information is evident in the second term...
of each firm’s quantity. The impact of the information about a firm’s absolute cost is 
\[ \frac{\delta}{3-\eta(\kappa+1)} \] 
for \( F \) and 
\[ \frac{\rho\delta}{3-\eta(\kappa+1)} \] 
for \( R \). Note that with perfect correlation between the firms’ costs \( (\rho = 1) \), the impact of the information is identical for the two firms. With no correlation, the impact on \( R \)’s quantity disappears. As compatibility between the firms’ products increases \( (\kappa \to 1) \) and the network effects strengthen \( (\eta \to 1) \), the magnitude of the impact of information about absolute cost increases for both firms.

The other piece of information in the disclosure that causes feedback loops is the news regarding the firms’ relative production efficiency. The impact of this information is evident in the third term of each firm’s quantity, which represents the adjustment made due to the cost advantage \( (-\delta(1-\rho)) \). The impact of the information on a firm’s quantity is the firm’s efficiency advantage scaled by the term 
\[ \frac{1-\eta\kappa}{(3-\eta(\kappa+1))(1-\eta(1-\kappa))} \]. With perfect correlation, a relative cost advantage does not exist so the impact disappears. For lower levels of correlation, the difference in expected efficiency is larger along with the magnitude of the adjustment. The magnitude of the response also changes as a result of changes in compatibility and network effect strength. As compatibility increases, the magnitude decreases because a relative advantage is of less concern to consumers who begin to view the firms’ networks as one. In a sense, an increase in compatibility softens competition and makes a relative advantage less important. The adjustment made to quantities in response to an efficiency advantage shrinks as a result.

The impact of \( \eta \) on the magnitude of the adjustment for relative efficiency is slightly more subtle. When compatibility is low, an increase in the strength of the
network effects leads to a larger response to relative production efficiency news for each firm. Low compatibility implies consumers derive little benefit from each other’s adoption if they purchase goods from different firms. Consequently, an increase in network effect strength causes consumers to become increasingly concerned about joining the larger network. The information regarding relative efficiency becomes increasingly influential as a result. When compatibility is high, an increase in $\eta$ leads to a smaller response to news about relative efficiency for each firm.

**Duopoly Equilibrium Under No Disclosure**

Without disclosure, $F$ continues to choose its quantity based on $\delta$, while $R$ and consumers make their respective decisions without the information. Specifically, given $\delta$ and its conjectures of $R$’s quantity, $\hat{q}_R$, and consumer expectations of network size, $\hat{S}_F$, $F$ chooses its quantity to solve

$$
\max_{q_F} \left( a - q_F - \hat{q}_R + \eta \hat{S}_F \right) q_F - (\bar{c} + \delta) q_F.
$$

(3.27)

Given its conjectures of $F$’s quantity, $\hat{q}_F(\delta)$, and consumer expectations of network size, $\hat{S}_R$, $R$ chooses its quantity to maximize expected profits. Specifically, $R$ chooses $q_R$ to solve

$$
\max_{q_R} E \left[ \left( a - q_R - \hat{q}_F(\delta) + \eta \hat{S}_R \right) q_R - (\bar{c} + \rho \delta) q_R \right].
$$

(3.28)

The first order conditions of (3.27) and (3.28) are provided in (3.29) and (3.30), respectively.

$$
\hat{q}_F^\phi(\delta) = \frac{a - (\bar{c} + \delta) - \hat{q}_R + \eta \hat{S}_F}{2}.
$$

(3.29)

$$
\hat{q}_R^\phi = \frac{a - \bar{c} - E [\hat{q}_F(\delta)] + \eta \hat{S}_R}{2}.
$$

(3.30)
Consumers form their expectations of the firms’ network sizes and establish their willingness to pay based on conjectures of the firms’ quantities as stated in equations (3.31) and (3.32).

\[
S_F^\phi = E [\hat{q}_F(\delta) + \kappa \hat{q}_R] \hspace{1cm} (3.31)
\]

\[
S_R^\phi = E [\hat{q}_R + \kappa \hat{q}_F(\delta)] \hspace{1cm} (3.32)
\]

Rationality requires conjectures to satisfy equations (3.33) - (3.36).

\[
\hat{S}_F = S_F^\phi \hspace{1cm} (3.33)
\]

\[
\hat{S}_R = S_R^\phi \hspace{1cm} (3.34)
\]

\[
\hat{q}_F(\delta) = \hat{q}_F^\phi(\delta) \hspace{1cm} (3.35)
\]

\[
\hat{q}_R = \hat{q}_R^\phi \hspace{1cm} (3.36)
\]

Jointly solving equations (3.29) - (3.36) and computing profits reveals the equilibrium outcomes under no disclosure as summarized in Lemma 5.

**Lemma 5.** Under no disclosure, the duopoly equilibrium quantities and profits are:

(i). \( q_F^\phi(\delta) = \frac{a - \bar{c}}{3 - \eta(1 + \kappa)} - \frac{\delta}{2} \) and \( \pi_F^\phi(\delta) = \left[ q_F^\phi(\delta) \right]^2 \)

(ii). \( q_R^\phi = \frac{a - \bar{c}}{3 - \eta(\kappa + 1)} \) and \( \pi_R^\phi = \left[ q_R^\phi \right]^2 \).

The quantities without disclosure are similar to those under disclosure with two important differences. First, \( R \) can no longer tailor its quantity decision to \( \delta \) and chooses the same quantity in every state. Second, there are no customer feedback loops associated with the news \( \delta \). \( F \) benefits from withholding the information by
avoiding the negative feedback loops that would result from disclosure when cost is high. The firm suffers, however, from precluding the positive loops that would result otherwise.

**Optimal Disclosure Policy**

$F$ establishes its disclosure policy based on expected profits. Lemma 6 provides the expectations, with $\Pi^d_F$ and $\Pi^\phi_F$ denoting expected profits under disclosure and no disclosure respectively. A direct comparison of $\Pi^d_F$ to $\Pi^\phi_F$ reveals the optimal disclosure policy as described in Proposition 2.

**Lemma 6.** $F$’s expected profits under disclosure and no disclosure are:

(i). $\Pi^d_F \equiv E[\pi^d_F(\delta)] = \left[ \frac{a - \bar{c}}{3 - \eta(\kappa + 1)} \right]^2 + \sigma^2 \left[ \frac{2 - \rho + \eta\kappa\rho - \eta}{(1 - \eta(1 - \kappa))(3 - \eta(\kappa + 1))} \right]^2$

(ii). $\Pi^\phi_F \equiv E[\pi^\phi_F(\delta)] = \left[ \frac{a - \bar{c}}{3 - \eta(\kappa + 1)} \right]^2 + \frac{\sigma^2}{4}$.

**Proposition 2.**

(i). The firm prefers disclosure if and only if correlation is sufficiently low, i.e. $\rho < \rho^*(\eta) \equiv \frac{1}{2} + \frac{\eta}{2} \left[ \frac{(1 - \kappa)(1 - \eta) + \eta\kappa^2}{1 - \eta\kappa} \right]$.

(ii). The cutoff $\rho^*(\eta)$ is increasing in $\eta$.

Proposition 2 provides the necessary and sufficient conditions for which $F$ prefers disclosure and shows that the firm prefers disclosure when correlation is below a cutoff $\rho^*(\eta)$. The proposition also demonstrates that the correlation threshold below which the firm is willing to disclose increases as the network effects strengthen. The
general message of Proposition 2 is that network effects encourage disclosure. Without network effects \((\eta = 0)\), the cutoff \(\rho^*\) reduces to \(\frac{1}{2}\) (see Part (i) of Corollary 1). This special case replicates the results of previous studies showing that firms prefer to disclose private-value information and withhold common-value information (See Gal-Or 1985, 1986 and Darrough 1993 as extended by Bagnoli and Watts 2011). \(F\) prefers disclosure for levels of correlation below the threshold because the disclosure tells the rival more about relative production efficiency than about its absolute cost. Informing the rival about relative efficiency is beneficial because the information improves coordination between firms: more efficient firms expand while the less efficient firms yield market share. Informing \(R\) about \(R\)'s production cost is detrimental for \(F\) because the information prompts the rival to compete more intensely when its efficiency is high. The increase in \(R\)'s intensity when cost is low prevents \(F\) from fully capitalizing on a lower than expected production cost. When correlation exceeds the threshold, \(F\) prefers withholding information in order to avoid an increase in the level of competition when its cost is low.

With network effects \((\eta > 0)\), \(F\) continues to prefer disclosure for low levels of correlation, but the threshold \(\rho^*(\eta)\) is higher and increases as the network effects strengthen \((\eta \to 1)\). The cutoff increases with \(\eta\) because coordination between firms and consumers becomes more valuable as the network effects become stronger. Consumers prefer to join a larger network and are willing to pay more for the products of the firm whose network they believe will be relatively large. Without disclosure, consumers remain uncertain about the firms’ network sizes and, thus, unwilling to
pay a higher price for products on larger networks. Disclosure informs consumers about network size and leads to customer feedback loops. Customer feedback loops magnify the effects of both efficiencies and inefficiencies, leading a comparatively efficient firm to produce a much higher quantity under disclosure than no disclosure. Additionally, consumers become increasingly concerned about network size and less about other characteristics of the products as $\eta$ increases. Consequently, information about relative network size becomes increasingly influential in production decisions (due to feedback loops) while information about the rival’s cost becomes less important. Accordingly, $F$ becomes increasingly eager to disclose information about relative production efficiency and decreasingly reluctant to inform the rival about its production cost. The cutoff increases with $\eta$ as a result.

Corollary 1 considers several special cases of Proposition 2 in order to enhance the intuition underlying the result. Part (i) of the corollary considers a situation with no network effects ($\eta = 0$) and provides the benchmark to compare with results from previous research. Parts (ii) and (iii) provide additional insight into $F$’s disclosure choice by examining the extreme cases of perfect compatibility and total incompatibility between the firms’ products.
Corollary 1.

(i). Absent network effects ($\eta = 0$), $F$ prefers disclosure for $\rho < \rho^* = \frac{1}{2}$.

(ii). If the firms’ products are perfectly compatible ($\kappa = 1$), $F$ prefers disclosure for

$$\rho < \rho^*(\eta) = \frac{1}{2} + \frac{\eta}{2(1-\eta)}.$$ 

(iii). If the firms’ products are perfectly incompatible ($\kappa = 0$), $F$ prefers disclosure

for $\rho < \rho^*(\eta) = \frac{1}{2} + \frac{\eta(2-\eta)}{2}.$

Corollary 1 Part (ii) examines a situation in which the firms’ products are perfectly compatible ($\kappa = 1$). When goods are compatible, an increase in $\eta$ softens competition. The rival is less threatening with strong network effects because an increase in the expected number of units sold by the rival increases willingness to pay for both firms’ products. Consequently, $F$ does not suffer much from disclosing information that encourages $R$ to increase production when the firms’ costs are low. $F$ benefits, however, from disclosing information that convinces consumers that the size of the firms’ joint network will be large. Since an increase in $\eta$ decreases the cost of informing $R$ about its cost and increases the benefit of informing consumers about network size, an increase in $\eta$ encourages $F$ to disclose the information.

Part (iii) of the corollary considers the opposite special case in which products are incompatible ($\kappa = 0$). With incompatible products, competition between the firms is most intense. Product incompatibility and consumers’ desire to join a large network make winner-take-all outcomes increasingly likely as the network effects strengthen. It seems natural to speculate that the intensity of competition in this situation would
cause a firm to be unwilling to publicly disclose privately held information. In this case, however, a firm can enhance its competitive position by working to convince consumers that its product will dominate the market. Consequently, a strengthening of the network effects encourages $F$ to disclose its information. More specifically, increasing $\eta$ causes the information contained in the disclosure regarding relative production efficiency to be increasingly influential in consumer purchasing decisions. When the relative advantage is large enough, feedback loops induce the well positioned firm to produce an extremely large quantity and cause the disadvantaged firm to drastically restrict production. Consequently, the well positioned firm dominates the market and enjoys relatively large profits while the disadvantaged firm’s profits are relatively small. Importantly, intensifying consumers’ desire to join a large network ($\eta \rightarrow 1$) allows smaller and smaller advantages to be large enough to tip the market in a firm’s favor. Consequently, an increase in $\eta$ increases the expected benefit of enhancing coordination by disclosing information about relative efficiency while simultaneously decreasing the expected cost of sharing information with the rival.

Figures 3.3 and 3.4 illustrate the results of Part (iii) of the corollary for high and low levels of correlation, respectively. When correlation is high ($\rho \rightarrow 1$), $F$ prefers no disclosure without network effects. With network effects ($\eta \rightarrow 1$), disclosure dominates and the preference is reversed. When correlation is low ($\rho = 0$), $F$ prefers disclosure both with and without network effects. In this case, network effects confirm the initial preference to disclose. It is important to note that if the products are perfectly incompatible ($\kappa = 0$) while correlation is perfect ($\rho = 1$), then the firm
Figure 3.3: Binary cost example of Cournot outcomes when correlation is high (ρ → 1) and firms produce incompatible (κ = 0) goods.

This figure illustrates the ability of network effects to reverse a firm’s preference to withhold information. For η = 0, disclosure allows R to compete more effectively and causes F’s equilibrium quantities to be relatively moderate. As a result, F’s expected profits under no disclosure exceed profits under disclosure. For η → 1, disclosure leads to feedback loops which result in relatively extreme outcomes. Consequently, expected profits under disclosure exceed profits under no disclosure. The network effects change F’s disclosure preference because they cause the information contained in the disclosure regarding F’s relative advantage to be extremely influential and the information about R’s cost to be unimportant. F subscripts are suppressed while $q_H$ and $q_L$ denote F’s equilibrium quantities given high and low cost, respectively.
This figure illustrates the ability of network effects to confirm a firm’s preference to disclose information. For $\eta = 0$, disclosure informs $R$ about relative efficiency and leads $F$’s equilibrium quantities to be relatively extreme. As a result, $F$’s expected profits under disclosure exceed profits under no disclosure. For $\eta > 0$, disclosure leads to feedback loops which result in even more extreme outcomes. Consequently, the network effects intensify $F$’s incentives to disclose and expected profits under disclosure further exceed profits under no disclosure. $F$ subscripts are suppressed while $q_H$ and $q_L$ denote $F$’s equilibrium quantities given high and low cost, respectively.
prefers withholding for any \( \eta \). A strengthening of the network effects does not elicit disclosure in this case because an efficiency advantage does not exist. Intuitively, if the rival perfectly mimics \( F \)’s production technology, then \( F \) will be unable to convey an advantage over the rival and will, thus, not benefit from sharing its private information.

### 3.3 Extensions and Alternative Specifications

In order to succinctly convey the key findings of this thesis and to highlight the underlying forces, some ancillary practical issues are excluded from the primary model. This section considers various extensions to and alternative specifications of the primary model in order to reflect such considerations. Analysis in this section demonstrates the robustness of the primary conclusion that positive network effects encourage a firm to disclose privately held information. Subsection 3.3.1 considers the symmetric case in which both firms acquire private information. Subsection 3.3.2 considers the impact of network effects on a firm’s discretionary disclosure decision. Subsection 3.3.3 allows firms to commit to network sizes before consumers determine their willingness to pay. Subsection 3.3.4 assumes firms choose prices and Subsection 3.3.5 discusses the case of negative network effects.

### 3.3.1 Both Firms Acquire Private Information

The primary model presumes that one of the two firms is privy to private information. This subsection extends the primary model by assuming that both firms acquire private information regarding their production costs. The analysis demonstrates that
the optimal disclosure policy derived under the assumptions of the primary model is a dominant strategy for the firms.

This extension requires the following generalization of the base setup: $F$ obtains an early read of $\delta$ while $R$ obtains an early read of an additional noise term $\lambda$. The parameters $\delta$ and $\lambda$ are mean zero with variances $\sigma_\delta^2$ and $\sigma_\lambda^2$, respectively. $F$’s expected production cost conditional on $\delta$ and $\lambda$ is $E[c_F|\delta, \lambda] = \bar{c} + \delta + \rho\lambda$ and $R$’s conditional expected cost is $E[c_R|\delta, \lambda] = \bar{c} + \rho\delta + \lambda$. The unconditional expected production cost of both firms is $\bar{c}$. The noise terms $\delta$ and $\lambda$ are independent. Note that the primary model is a special case of this setup and can be replicated with the restriction $\sigma_\lambda^2 = 0$.

The sequence of play follows as in the primary model. First, the two firms establish their disclosure policies. Second, $F$ and $R$ privately observe $\delta$ and $\lambda$, respectively, and disclose the information according to the established policies. Third, the firms choose quantities while consumers form expectations of network size and determine willingness to pay. Finally, retail demand is satisfied and profits are realized. Under this formulation, there are four possible disclosure regimes: (1) both firms withhold, (2) $F$ discloses while $R$ withholds, (3) $F$ withholds while $R$ discloses, and (4) both firms disclose. Working backward through the game to determine outcomes under each disclosure regime, and then comparing the firms’ expected profits under each regime reveals the preferred disclosure policies. Lemma 7 provides the firms’ expected profits under each disclosure regime. Proposition 3 demonstrates that the disclosure policy stated in Proposition 2 forms a dominant strategy.
Lemma 7. The firms’ expected profits under the four possible disclosure regimes are as follows:

(i). Expected profits when both firms withhold (denoted $\Pi_{\phi\phi}^F$ and $\Pi_{\phi\phi}^R$) are:

(a) $\Pi_{\phi\phi}^F = \left[ \frac{a_0 - \bar{c}}{3 - \eta(\kappa + 1)} \right]^2 + \frac{\sigma^2}{4}$

(b) $\Pi_{\phi\phi}^R = \left[ \frac{a_0 - \bar{c}}{3 - \eta(\kappa + 1)} \right]^2 + \frac{\sigma^2}{4}$.

(ii). Expected profits when $F$ discloses while $R$ withholds ($\Pi_{d\phi}^F$ and $\Pi_{d\phi}^R$) are:

(a) $\Pi_{d\phi}^F = \left[ \frac{a_0 - \bar{c}}{3 - \eta(\kappa + 1)} \right]^2 + \sigma^2 \left[ \frac{1 - 2 - \rho + \eta \rho - \eta}{(1 - \eta(1 - \kappa))(3 - \eta(\kappa + 1))} \right]^2$

(b) $\Pi_{d\phi}^R = \left[ \frac{a_0 - \bar{c}}{3 - \eta(\kappa + 1)} \right]^2 + \sigma^2 \left[ \frac{1 - \eta(\kappa - \rho) - 2 \rho}{(1 - \eta)(3 - \eta(\kappa + 1))} \right]^2 + \frac{\sigma^2}{4}$.

(iii). Expected profits when $F$ withholds while $R$ discloses ($\Pi_{\phi d}^F$ and $\Pi_{\phi d}^R$) are:

(a) $\Pi_{\phi d}^F = \left[ \frac{a_0 - \bar{c}}{3 - \eta(\kappa + 1)} \right]^2 + \sigma^2 \left[ \frac{1 - \eta(\kappa - \rho) - 2 \rho}{(1 - \eta)(3 - \eta(\kappa + 1))} \right]^2 + \sigma^2 \left[ \frac{1 - \eta(\kappa - \rho) - 2 \rho}{(1 - \eta)(3 - \eta(\kappa + 1))} \right]^2$

(b) $\Pi_{\phi d}^R = \left[ \frac{a_0 - \bar{c}}{3 - \eta(\kappa + 1)} \right]^2 + \sigma^2 \left[ \frac{1 - \eta(\kappa - \rho) - 2 \rho}{(1 - \eta)(3 - \eta(\kappa + 1))} \right]^2$.

(iv). Expected profits when both firms disclose ($\Pi_{dd}^F$ and $\Pi_{dd}^R$) are:

(a) $\Pi_{dd}^F = \left[ \frac{a_0 - \bar{c}}{3 - \eta(\kappa + 1)} \right]^2 + \sigma^2 \left[ \frac{1 - \eta(\kappa - \rho) - 2 \rho}{(1 - \eta)(3 - \eta(\kappa + 1))} \right]^2 + \sigma^2 \left[ \frac{1 - \eta(\kappa - \rho) - 2 \rho}{(1 - \eta)(3 - \eta(\kappa + 1))} \right]^2$

(b) $\Pi_{dd}^R = \left[ \frac{a_0 - \bar{c}}{3 - \eta(\kappa + 1)} \right]^2 + \sigma^2 \left[ \frac{1 - \eta(\kappa - \rho) - 2 \rho}{(1 - \eta)(3 - \eta(\kappa + 1))} \right]^2 + \sigma^2 \left[ \frac{1 - \eta(\kappa - \rho) - 2 \rho}{(1 - \eta)(3 - \eta(\kappa + 1))} \right]^2$.

Proposition 3. The disclosure policy stated in Proposition 2 is a dominant strategy.

Specifically, a firm prefers disclosure if and only if $\rho < \rho^*(\eta) \equiv \frac{1}{2} + \frac{\eta}{2} \left[ \frac{(1 - \kappa) + (1 - \eta) - \eta \kappa^2}{1 - \eta \kappa} \right]$. 

42
3.3.2 Discretionary (Ex Post) Disclosure

The analysis in this thesis is focused on the impact of network effects on the design of a firm’s disclosure policy. This subsection, however, considers the impact of network effects on a firm’s discretionary disclosure decision. The analysis identifies a unique disclosure equilibrium and demonstrates that the presence of network effects encourages a firm to disclose more good news (low cost realizations) and less bad (high cost realizations) than would be disclosed otherwise.

The major difference between a disclosure policy decision and a discretionary disclosure decision is that a discretionary disclosure decision is contingent upon the information itself. Consequently, market participants outside the firm learn something about the firm’s private knowledge even if the firm chooses not to disclose it. If a firm opts not to disclose, then outsiders consider the firm’s incentives and adjust their expectations accordingly.

Absent any additional frictions, e.g. an exogenous cost of disclosure (Verrecchia 1983) or uncertain information endowment (Dye 1985, Jung and Kwon 1988), the standard “unraveling” of discretionary disclosure (Grossman and Hart 1980, Grossman 1981, Milgrom 1981) occurs in this setting. For example if a monopolist’s cost is lower than what consumers would believe absent disclosure, then the monopolist discloses this fact in order to boost the willingness to pay of its customers. Failure to disclose conveys that the realized cost is higher than previously expected. The net result is that full revelation of the firm’s private information is the only equilibrium outcome.
In order to prevent the standard unraveling result, this subsection assumes the firm incurs an exogenous cost of disclosure. This cost, denoted \( \psi \), \( \psi > 0 \), can be interpreted as the cost of verifying and disseminating the information. The sequence of events proceeds as follows: First, \( F \) privately observes \( \delta \) and decides whether to disclose it. Next, the firms choose production quantities while consumers form expectations of network size and determine willingness to pay. Finally, retail demand is satisfied and payoffs are realized. To simplify the analysis, assume \( \delta \) is distributed uniformly over the set \([-\Delta, \Delta]\). Additionally, assume that \( R \)'s production cost is \( c_R = \bar{c} + \delta \) with probability \( \rho \) and \( c_R = \bar{c} \) with probability \( (1 - \rho) \). Furthermore, to ensure the existence of an equilibrium in the quantity decision sub-game and that the firms choose non-negative quantities, assume \( (a - \bar{c}) > 6\Delta \) and \( \eta < \frac{1}{2} \).

**Monopoly**

In order to highlight the impact of consumer feedback loops on a firm’s discretionary disclosure decision, this subsection begins with the monopoly case. Importantly, a monopolist in this setting has no reason to disclose when there are no network effects. Absent network effects, disclosure does not impact consumer purchasing decisions. Consequently, the monopolist’s profits under disclosure equal the profits under no disclosure less the exogenous cost of disclosure. With network effects, however, the monopolist benefits from informing consumers that its production cost is low. Informing consumers that production cost is low positively affects consumer expectations of network size and increases willingness to pay. Additionally, the benefit of
disclosing cost realizations that are below what consumers would expect absent disclosure increases as the network effects strengthen. Consequently, a firm discloses more information in the presence of network effects than it would otherwise.

Lemma 8 provides the firm’s profits under disclosure and no disclosure, where \( E[\delta|\phi] \) denotes consumers’ expectation of \( \delta \) given the firm has chosen not to disclose.

**Lemma 8.** When the monopolist makes a discretionary disclosure decision, the firm’s profits under disclosure and no disclosure are:

\[
\begin{align*}
\text{(i)} & \quad \pi_F^d(\delta) = \left[ \frac{a - \bar{c}}{2 - \eta} - \frac{\delta}{2 - \eta} \right]^2 - \psi \\
\text{(ii)} & \quad \pi_F^\phi(\delta) = \left[ \frac{a - \bar{c}}{2 - \eta} - \frac{\delta}{2} - \frac{\eta E[\delta|\phi]}{2(2 - \eta)} \right]^2.
\end{align*}
\]

The profits presented in Lemma 8 are similar to those presented in Lemmas 1 and 2 with two important differences. First, profits under disclosure are reduced by the amount \( \psi \). Second, since consumers’ expectations regarding \( \delta \) are impacted when the firm chooses not to disclose, a feedback loop occurs under no disclosure despite the fact that \( \delta \) is not directly revealed. Specifically, \( E[\delta|\phi] \) is not necessarily equal to \( E[\delta] = 0 \) under no disclosure. Instead, consumers consider the firm’s disclosure incentives and adjust their expectations (and willingness to pay) accordingly.

As the firm makes its disclosure decision in an effort to maximize firm profits, the firm will disclose the information, \( \delta \), when \( \pi_F^d(\delta) - \pi_F^\phi(\delta) > 0 \) and will withhold otherwise. Equivalently, the firm will disclose the information when

\[
\frac{[4(a - \bar{c} - \delta) - \eta(E[\delta|\phi] - \delta)]\eta(E[\delta|\phi] - \delta)}{4(2 - \eta)^2} > \psi. \tag{3.37}
\]
Given the restrictions on \((a - \bar{c})\) and \(\eta\), note that the left hand side of equation (3.37) is decreasing in \(\delta\) for any \(E[\delta|\phi]\). More formally, 
\[
\frac{d[\pi^d_F - \pi^\phi_F]}{d\delta} = -\eta(2(a - \bar{c} - \delta) + (2 - \eta)(E[\delta|\phi] - \delta)) \leq 0.
\]
Thus, if the firm prefers disclosure for a realization of \(\delta\), then the firm also prefers disclosure for all lower realizations of \(\delta\). Consequently, if a non degenerate disclosure equilibrium exists, then there exists a cutoff, denoted \(\delta^*(\eta) \in [-\Delta, \Delta]\), such that \(E[\delta|\phi] = \frac{\delta^*(\eta) + \Delta}{2}\).

The unique cutoff \(\delta^*(\eta)\) can be identified by recognizing that the firm is indifferent between disclosure and no disclosure when \(\delta = \delta^*(\eta)\). Specifically, \(\pi^d_F(\delta^*) = \pi^\phi_F(\delta^*)\) must be satisfied. This indifference condition can be rewritten as:

\[
\frac{\left[4(a - \bar{c} - \delta^*) - \eta \left(\frac{\delta^* + \Delta}{2} - \delta^*\right)\right] \eta \left(\frac{\delta^* + \Delta}{2} - \delta^*\right)}{4(2 - \eta)^2} = \psi. \quad (3.38)
\]

Proposition 4 identifies the cutoff \(\delta^*(\eta)\) and demonstrates that the cutoff increases as the network effects strengthen \((\eta \rightarrow \frac{1}{2})\). The cutoff increases because as the value of a large network increases for consumers, the firm has more to gain from informing consumers that its production cost is lower than what would be expected absent disclosure. Consequently, network effects encourage a monopolist making a discretionary disclosure decision to disclose more information than would be disclosed without network effects.
Proposition 4.

(i). When the monopolist makes a discretionary disclosure decision, there exists a cutoff $\delta^*(\eta)$ such that the firm discloses if and only if $\delta < \delta^*(\eta)$, where

$$\delta^*(\eta) \equiv \frac{4\eta(a - \bar{c} + \Delta) - \Delta\eta^2 - 4\sqrt{\eta(\eta(a - \bar{c} - \Delta)^2 + \psi(2 - \eta)^2(8 - \eta))}}{(8 - \eta)\eta}.$$ 

(ii). Disclosure is nontrivial, i.e. $\delta^*(\eta) > -\Delta$, when $\eta$ is sufficiently large.

(iii). The cutoff $\delta^*(\eta)$ increases as the network effects strengthen.

Figure 3.5 illustrates the discretionary disclosure equilibrium in the monopoly case. Absent network effects, disclosure does not impact consumer purchasing decisions so the firm chooses to avoid the cost of disclosure by withholding any realized $\delta$. With network effects, the monopolist discloses all realizations of $\delta$ that are below the cutoff $\delta^*(\eta)$.

Duopoly

Incorporating a competitor into the analysis complicates the inquiry somewhat due to the fact that the informed firm may face conflicting incentives to disclose. As shown in the monopoly case, the firm benefits from disclosing low realizations of $\delta$ to consumers and withholding high realizations. While the firm always benefits from convincing consumers that its production cost is low and that its network will be large, its desire to share low or high cost news with the rival depends on the degree of correlation between the firms’ costs. When correlation is low, disclosing a low $\delta$ conveys an advantage over the rival and convinces the rival to withdraw. Thus, when
This figure presents the monopolist’s profits under disclosure and no disclosure over the set $[-\Delta, \Delta]$ and illustrates the impact of network effects on a monopolist’s decision to disclose $\delta$ on a discretionary basis. The monopolist discloses when $\delta < \delta^*(\eta)$. For $\eta = 0$, disclosure does not impact consumer purchasing decisions. As a result, the monopolist’s profits under disclosure equal the profits under no disclosure less the exogenous cost of disclosure, $\psi$. Consequently, the monopolist withholds all realizations of $\delta$ when $\eta = 0$ and $\delta^*(0) = -\Delta$. For $\eta > 0$, the monopolist benefits from disclosing low realizations of $\delta$ because the disclosure positively impacts consumer expectations of network size and leads to positive feedback loops. As $\eta$ increases, the benefit of disclosing realizations of $\delta$ that are below what consumers expect absent disclosure increases. Consequently, the range of $\delta$ for which the monopolist discloses information grows larger as $\eta$ increases.
correlation is low, the incentives to disclose low cost realizations to consumers and the rival reinforce one another. As network effects strengthen, the benefit of disclosing cost realizations that are below what would be expected absent disclosure increases and the firm winds up disclosing more often. When correlation is high, however, the firm benefits from disclosing high cost news to the rival because the news informs the rival that its production cost is likely higher than previously expected and encourages the rival to scale back production. Thus, when correlation is high, the incentives to hide high cost news from consumers but disclose it to the rival oppose one another. When network effects are weak or non existent, the desire to disclose bad news to the rival dominates. When network effects are strong enough, the desire to hide bad news from consumers dominates. Summarizing, the firm discloses less bad news and more good news the presence of network effects than it would otherwise.

$F$’s duopoly profits in the discretionary disclosure case are computed in the same fashion as was done in the disclosure policy case (Section 3.2) with two important differences. First, $F$’s profits under disclosure are reduced by the exogenous cost of disclosure $\psi$. Second, the rival and consumers’ expectations of $\delta$ under no disclosure are no longer necessarily equal to the unconditional expected value of 0. Consequently, customers and the rival’s expectations and decisions are impacted by the news even if $F$ chooses not to disclose. Lemma 9 provides the firm’s profits under disclosure and no disclosure, where $E[\delta|\phi]$ denotes $R$ and consumers’ expectation of $\delta$ given $F$ has chosen not to disclose.
Lemma 9. When $F$ makes a discretionary disclosure decision, the firm’s profits under disclosure and no disclosure are:

(i). $\pi_F^d(\delta) = \left[ \frac{a - \bar{c}}{3 - \eta(k + 1)} - \frac{\delta}{3 - \eta(k + 1)} - \frac{\delta(1 - \rho)(1 - \eta \kappa)}{(3 - \eta(k + 1))(1 - \eta(1 - \kappa))} \right]^2 - \psi$

(ii). $\pi_F^\phi(\delta) = \left[ \frac{a - \bar{c}}{3 - \eta(k + 1)} - \frac{\delta}{2} - \frac{E[\delta|\phi] (1 - \eta^2 (1 - \kappa^2) + 2\eta(1 - \kappa(1 - \rho)) - 2\rho)}{2(3 + \eta^2 (1 - \kappa^2) - 2\eta(2 - \kappa))} \right]^2$

As in the monopoly case, the firm will disclose $\delta$ when $\pi_F^d(\delta) - \pi_F^\phi(\delta) > 0$ and will withhold otherwise. As noted above, $F$’s willingness to disclose a particular realization of $\delta$ depends on the degree of correlation between the firms’ costs. Three cases follow:

Case 1: $\rho = \bar{\rho}(\eta) \equiv \frac{1}{2} + \frac{\eta}{2} \left[ \frac{(1 - \kappa) + (1 - \eta) + \eta \kappa^2}{1 - \eta \kappa} \right]$

For $\rho = \bar{\rho}(\eta)$, the benefits of disclosure are 0 while the cost remains $\psi$. More formally, $\pi_F^d(\delta) - \pi_F^\phi(\delta) = -\psi < 0$. The benefit of disclosure is 0 at this specific level of correlation because the firms’ response to news about relative production efficiency is exactly offset by the response to the news about $R$’s production cost. As a result, the quantity chosen by $F$ is the same under disclosure and no disclosure. Consequently, $F$ withholds all realizations of $\delta$ when $\rho = \bar{\rho}(\eta)$.

Case 2: $\rho < \bar{\rho}(\eta) \equiv \frac{1}{2} + \frac{\eta}{2} \left[ \frac{(1 - \kappa) + (1 - \eta) + \eta \kappa^2}{1 - \eta \kappa} \right]$

For $\rho < \bar{\rho}(\eta)$, the benefits of disclosure decrease as the realized cost increases. Formally, the difference $\pi_F^d(\delta) - \pi_F^\phi(\delta)$ is decreasing in $\delta$. As a result, if a non degenerate disclosure equilibrium exists for $\rho < \bar{\rho}(\eta)$, then there exists a cutoff such that $E[\delta|\phi] = \frac{\delta^*(\eta) + \Delta}{2}$. 

50
Case 3: $\rho > \tilde{\rho}(\eta) \equiv \frac{1}{2} + \frac{\eta}{2} \left[ \frac{(1-\kappa)+(1-\eta)+\eta^2}{1-\eta\kappa} \right]$

For $\rho > \tilde{\rho}(\eta)$, the benefits of disclosure increase as the realized cost increases. Formally, the difference $\pi_F^d(\delta) - \pi_F^0(\delta)$ is increasing in $\delta$. As a result, if a non degenerate disclosure equilibrium exists for $\rho > \tilde{\rho}(\eta)$, then there exists a cutoff, such that

$$E[\delta|\phi] = \frac{-\Delta + \delta^*(\eta)}{2}.$$

For Cases 2 and 3, unique cutoffs are identified in the same manner as is done in the monopoly case. $F$ is indifferent between disclosure and no disclosure when $\delta = \delta^*(\eta)$. Specifically $\pi_F^d(\delta^*) = \pi_F^0(\delta^*)$ must be satisfied. In both cases, if an interior cutoff exists then the cutoff increases with $\eta$.

When correlation is low (Case 2), $F$ discloses when the realized $\delta$ is below the cutoff $\delta^*(\eta)$. The cutoff increases with $\eta$ because a strengthening of the the network effects enhances the benefit of disclosing realizations of $\delta$ that are below what the expectation of $\delta$ would be absent disclosure. Absent network effects, $F$ benefits from disclosing low realizations of $\delta$ because disclosure informs $R$ that $F$’s competitive position relative to $R$ is stronger than $R$ would expect absent disclosure. The disclosure persuades $R$ to reduce production quantities and, in turn, allows $F$ to expand. When network effects are present, customer feedback loops magnify the impact of the disclosure on the firms’ production decisions. Feedback loops become more powerful as network effects strengthen and, as a result, the benefits of disclosure increase. Consequently, network effects encourage a firm to disclose more information than it would disclose without network effects when correlation between the firms’ costs is low.
When correlation is high (Case 3), $F$ discloses when the realized $\delta$ is above the cutoff $\delta^*(\eta)$. The cutoff increases with $\eta$ because a strengthening of the network effects increases the cost of disclosing realizations of $\delta$ that are above consumers’ expectation of $\delta$ absent disclosure. Absent network effects, $F$ benefits from disclosing high realizations of $\delta$ because disclosure informs $R$ that its production cost is likely higher than $R$ would expect absent disclosure. The disclosure persuades $R$ to reduce its production quantity and, in turn, lessens the impact of high production costs on $F$’s production decision. When network effects are present, however, disclosing high realizations of $\delta$ is costly because the disclosure informs consumers that the firms’ networks will be smaller than would be expected absent disclosure. Disclosing the news leads to negative feedback loops, which further reduce the firms’ profitability. As network effects strengthen, the impact of the negative feedback loops increases. Consequently, network effects encourage a firm to disclose less information than would otherwise be disclosed when correlation is high. Proposition 5 summarizes the findings for all levels of correlation.

**Proposition 5.**

(i). When $F$ makes a discretionary disclosure decision and $\rho = \bar{\rho}(\eta) \equiv \frac{1}{2} + \frac{\eta}{2} \left[ \frac{(1-\kappa)+(1-\eta)+\eta \kappa^2}{1-\eta \kappa} \right]$, $F$ withholds for any realization of $\delta$.

(ii). When $F$ makes a discretionary disclosure decision and correlation is low, i.e. $\rho < \bar{\rho}(\eta)$, there exists a cutoff $\delta^*(\eta)$ such that the firm discloses if and only if $\delta < \delta^*(\eta)$. The cutoff $\delta^*(\eta)$ is increasing in $\eta$. 

52
(iii). When $F$ makes a discretionary disclosure decision and correlation is high, i.e. 
\[ \rho > \bar{\rho}(\eta), \] there exists a cutoff $\delta^*(\eta)$ such that the firm discloses if and only if
\[ \delta > \delta^*(\eta). \] The cutoff $\delta^*(\eta)$ is increasing in $\eta$.

Figures 3.6 and 3.7 illustrate the discretionary disclosure equilibrium in the duopoly case for low and high levels of correlation, respectively. When correlation is low ($\rho = 0$), $F$ discloses realizations of $\delta$ that are below the cutoff $\delta^*(\eta)$. Figure 3.6 shows that the range of $\delta$ for which $F$ discloses becomes larger as network effects strengthen. When correlation is high ($\rho = 1$), $F$ discloses realizations of $\delta$ that are above the cutoff $\delta^*(\eta)$. Figure 3.7 shows that the range of $\delta$ for which $F$ discloses becomes smaller as network effects strengthen.

3.3.3 Firms Pre-Commit to Quantities

The primary analysis presumes that firms choose quantities and consumers form their expectations of network size simultaneously. Consequently, an individual consumer’s willingness to pay for the firms’ products is not influenced by the firms’ quantity choices. Firms, thus, take consumer expectations as given when they make production decisions. This assumption can be rephrased to say that firms are unable to commit to production quantities (or network sizes) prior to the time consumers determine their willingness to pay for the products. The assumption reflects the fact that when network goods are durable, consumers must make purchasing decisions before actual network sizes are known (see Katz and Shapiro 1985 and Hagiu and Halaburda 2013 for related discussion). This subsection considers the alternative case in which firms commit to output levels before consumers determine willingness
Figure 3.6: Discretionary disclosure equilibrium in the duopoly case when correlation is low ($\rho = 0$) and firms produce incompatible ($\kappa = 0$) goods.

This figure presents $F$’s profits under disclosure and no disclosure over the set $[-\Delta, \Delta]$. The figure illustrates the impact of network effects on $F$’s decision to disclose $\delta$ on a discretionary basis. When correlation is low, $F$ discloses when $\delta < \delta^*(\eta)$ and withholds otherwise. For $\eta = 0$, $F$ benefits from disclosing low realizations of $\delta$ because the disclosure encourages $R$ to restrict production while $F$ expands. For $\eta > 0$, feedback loops enhance the benefits of the disclosure by magnifying the impact of the news on $R$ and $F$’s production decisions and profits. Consequently, the range of $\delta$ for which $F$ discloses information expands as $\eta$ increases.
Figure 3.7: Discretionary disclosure equilibrium in the duopoly case when correlation is high ($\rho = 1$) and firms produce compatible ($\kappa = 1$) goods.

This figure presents $F$’s profits under disclosure and no disclosure over the set $[-\Delta, \Delta]$. The figure illustrates the impact of network effects on $F$’s decision to disclose $\delta$ on a discretionary basis. When correlation is high, $F$ discloses when $\delta > \delta^*(\eta)$ and withholds otherwise. $F$ benefits from disclosing high realizations of $\delta$ because the disclosure encourages $R$ to restrict production and reduces the impact of high production costs on $F$’s profits. Disclosing high realizations of $\delta$ is costly, however, because the information negatively impacts consumer expectations of network size and reduces willingness to pay. For $\eta = 0$, the cost of disclosing high realizations of $\delta$ to consumers is zero. As $\eta$ increases, the cost of disclosing high realizations of $\delta$ increases. Consequently, the range of $\delta$ for which $F$ discloses information shrinks as $\eta$ increases.
to pay. Consequently, consumers consider actual quantity levels when determining the value of a product. Firms, in turn, consider the impact their production decisions will have on consumers’ willingness to pay when they choose production quantities.

This scenario entails the following sequence of events: First, $F$ establishes its disclosure policy. Second, $F$ privately observes $\delta$ and discloses according to policy. Third, $F$ and $R$ commit to production quantities. Fourth, consumers observe production quantities and determine willingness to pay. Fifth, transactions occur and payoffs are realized. Additionally, to ensure the existence of an equilibrium in the quantity decision sub-game, assume $\eta < \frac{1}{2}$.

Analysis of the monopoly case is omitted because (as the disclosure comes before the firm commits to a production quantity) disclosure has no impact on consumer purchasing decisions in this setting. Consequently, a monopolist is indifferent between disclosure and no disclosure. In the duopoly case, however, disclosure in this setting has an indirect impact on the purchasing decisions of consumers. Since a rival adjusts its production decision in response to a disclosure and consumers, in turn, base their willingness to pay on the quantity decisions of the firms, the impact of a disclosure on the firms’ profits is magnified by the presence of network effects. The analysis demonstrates that, as in the no commitment case, a firm prefers disclosure when correlation between the firms’ production costs is below a cutoff. Furthermore, the correlation threshold below which the firm is willing to disclose information increases as network effects strengthen.
Duopoly Equilibrium Under Disclosure

When firms pre-commit to quantities, consumers base their willingness to pay on actual network sizes rather than conjectures. Specifically, demand for firm $i$’s product is $p_i = a - q_i - q_j + \eta(q_i + \kappa q_j)$ for $i, j \in \{F, R\}$ and $j \neq i$. Under disclosure, $F$ and $R$ both choose quantities based on the information $\delta$. Given $\delta$ and a conjecture of $R$’s quantity, $\hat{q}_R(\delta)$, $F$ chooses its quantity $q_F$ to solve

$$\max_{q_F} \left( a - q_F - \hat{q}_R(\delta) + \eta (q_F + \kappa \hat{q}_R(\delta)) \right) q_F - (\bar{c} + \delta)q_F. \quad (3.39)$$

Similarly, given $\delta$ and a conjecture of $F$’s quantity, $\hat{q}_F(\delta)$, $R$ chooses its quantity $q_R$ to solve

$$\max_{q_R} \left( a - q_R - \hat{q}_F(\delta) + \eta (q_R + \kappa \hat{q}_F(\delta)) \right) q_R - (\bar{c} + \rho \delta)q_R. \quad (3.40)$$

The first order conditions of (3.39) and (3.40) are provided in (3.41) and (3.42), respectively.

$$q^d_F(\delta) = \frac{a - (\bar{c} + \delta) - \hat{q}_R(\delta)(1 - \eta \kappa)}{2(1 - \eta)} \quad (3.41)$$

$$q^d_R(\delta) = \frac{a - (\bar{c} + \rho \delta) - \hat{q}_F(\delta)(1 - \eta \kappa)}{2(1 - \eta)} \quad (3.42)$$

Rationality requires the firms’ conjectures to satisfy equations (3.43) and (3.44).

$$\hat{q}_F(\delta) = q^d_F(\delta) \quad (3.43)$$

$$\hat{q}_R(\delta) = q^d_R(\delta) \quad (3.44)$$

Jointly solving equations (3.41) - (3.44) and computing profits reveals the equilibrium outcomes under disclosure as summarized in Lemma 10.
Lemma 10. When firms commit to quantities, the duopoly equilibrium quantities and profits under disclosure are:

(i). \( q_d^F(\delta) = \frac{a - \bar{c}}{3 - \eta(2 + \kappa)} - \frac{\delta}{3 - \eta(2 + \kappa)} - \frac{\delta(1 - \rho)(1 - \eta \kappa)}{(3 - \eta(2 + \kappa))(1 - \eta(2 - \kappa))} \)
and \( \pi_d^F(\delta) = (1 - \eta) \left[ q_d^F(\delta) \right]^2 \)

(ii). \( q_d^R(\delta) = \frac{a - \bar{c}}{3 - \eta(2 + \kappa)} - \frac{\rho \delta}{3 - \eta(2 + \kappa)} + \frac{\delta(1 - \rho)(1 - \eta \kappa)}{(3 - \eta(2 + \kappa))(1 - \eta(2 - \kappa))} \)
and \( \pi_d^R(\delta) = (1 - \eta) \left[ q_d^R(\delta) \right]^2 \).

Duopoly Equilibrium Under No Disclosure

Without disclosure, \( F \) chooses its quantity based on \( \delta \), while \( R \) makes its decision without the information. Specifically, given \( \delta \) and its conjecture of \( R \)’s quantity, \( \hat{q}_R \), \( F \) chooses its quantity to solve

\[
\max_{q_F} \left( a - q_F - \hat{q}_R + \eta (q_F + \kappa \hat{q}_R) \right) q_F - (\bar{c} + \delta) q_F. \tag{3.45}
\]

Given its conjecture of \( F \)’s quantity, \( \hat{q}_F(\delta) \), \( R \) chooses its quantity to maximize expected profits. Specifically, \( R \) chooses \( q_R \) to solve

\[
\max_{q_R} \mathbb{E} \left[ \left( a - q_R - \hat{q}_F(\delta) + \eta(q_R + \kappa \hat{q}_F(\delta)) \right) q_R - (\bar{c} + \rho \delta) q_R \right]. \tag{3.46}
\]

The first order conditions of (3.45) and (3.46) are provided in (3.47) and (3.48), respectively.

\[
\hat{q}^F(\delta) = \frac{a - (\bar{c} + \delta) - \hat{q}_R(1 - \eta \kappa)}{2(1 - \eta)} \tag{3.47}
\]

\[
\hat{q}^R = \frac{a - \bar{c} - \mathbb{E}[\hat{q}_F(\delta)](1 - \eta \kappa)}{2(1 - \eta)} \tag{3.48}
\]
Rationality requires the firms’ conjectures to satisfy equations (3.49) and (3.50).

$$\hat{q}_F(\delta) = q^\phi_F(\delta)$$  \hspace{1cm} (3.49)

$$\hat{q}_R = q^\phi_R$$  \hspace{1cm} (3.50)

Jointly solving equations (3.47) - (3.50) and computing profits reveals the equilibrium outcomes under disclosure as summarized in Lemma 11.

**Lemma 11.** When firms commit to quantities, the duopoly equilibrium quantities and profits under no disclosure are:

(i).  \[ q^\phi_F(\delta) = \frac{a - \check{c}}{3 - \eta(2 + \kappa)} - \frac{\delta}{2(1 - \eta)} \quad \text{and} \quad \pi^\phi_F(\delta) = (1 - \eta) \left[ q^\phi_F(\delta) \right]^2 \]

(ii).  \[ q^\phi_R = \frac{a - \check{c}}{3 - \eta(2 + \kappa)} \quad \text{and} \quad \pi^\phi_R = (1 - \eta) \left[ q^\phi_R \right]^2. \]

**Optimal Disclosure Policy**  \( F \) establishes its disclosure policy based on expected profits. Lemma 12 provides the expectations. A direct comparison of \( \Pi^d_F \) to \( \Pi^\phi_F \) reveals the optimal disclosure policy as described in Proposition 6.

**Lemma 12.** \( F \)'s expected profits under disclosure and no disclosure are:

(i).  \[ \Pi^d_F \equiv E[\pi^d_F] = (1 - \eta) \left[ \left( \frac{a - \check{c}}{3 - \eta(2 + \kappa)} \right)^2 + \left( \frac{\sigma(2 - \rho - \eta(2 - \kappa) \rho)}{(3 - \eta(2 + \kappa))(1 - \eta(2 - \kappa))} \right)^2 \right], \]

(ii).  \[ \Pi^\phi_F \equiv E[\pi^\phi_F] = (1 - \eta) \left[ \left( \frac{a - \check{c}}{3 - \eta(2 + \kappa)} \right)^2 + \left( \frac{\sigma}{2(1 - \eta)} \right)^2 \right]. \]
Proposition 6.

(i). When firms pre-commit to production quantities, the informed firm prefers disclosure if and only if correlation is sufficiently low, i.e. \( \rho < \rho^* (\eta) \equiv \frac{1}{2} + \frac{\eta(1-\kappa)}{2(1-\eta)} \).

(ii). The cutoff \( \rho^* (\eta) \) is increasing in \( \eta \).

The disclosure policy described in Proposition 6 is similar to the disclosure policy described in the main section of the paper (see Proposition 2). Specifically, \( F \) prefers disclosure when correlation between the firms’ production costs is below a threshold. Furthermore, the correlation threshold below which the firm is willing to disclose information increases as network effects strengthen. Comparing the correlation cutoff given in Proposition 2 with the cutoff given in Proposition 6 reveals that the former exceeds the latter when \( \eta \) is positive but small. Specifically, the correlation cutoff given in Proposition 2 is larger for \( 0 < \eta < \frac{1}{2} \left( \frac{3-2\kappa}{1-\kappa^2} - \frac{\sqrt{5-4\kappa(3-2\kappa)}}{1-\kappa^2} \right) \). This implies that firms who cannot commit to production quantities disclose more information than firms who can commit to production quantities when network effects are somewhat weak. Figure 3.8 illustrates this finding for \( \kappa = 0 \).

3.3.4 Price Competition

This subsection considers a setting in which firms competing in a market with network effects choose prices rather than quantities. The analysis confirms the results of prior research by demonstrating that when firms choose prices and network effects are nonexistent, the well informed firm prefers withholding cost information for all levels of correlation. The analysis also demonstrates that if network effects are strong
Figure 3.8: Comparison of correlation cutoffs when firms commit and do not commit upfront to production quantities.

This figure plots the correlation cutoffs given in Proposition 2 (no commitment) and Proposition 6 (commitment) for $0 \leq \eta < \frac{1}{2}$ and $\kappa = 0$. The solid line represents the cutoff presented in the no commitment case while the dotted line represents the cutoff given in the commitment case. The figure shows that the cutoff under no commitment is larger when $\eta$ is low.
enough, then the firm prefers disclosing information when correlation between the
firms’ costs is low. Furthermore, as network effects become more powerful, the firm
is willing to disclose increasingly more information. The results in this subsection are
noteworthy in that they show that the primary finding of this thesis is not subject
to the ‘flipping’ that commonly occurs when the type of competition assumed in
the analysis (quantity or price) changes. Specifically, in both Cournot and Bertrand
settings, network effects encourage disclosure.

In order to avoid the Bertrand Paradox in which firms compete away profits,
analysis of this setting requires the introduction of a parameter $\gamma$, $0 \leq \gamma < 1$, which
represents the degree of differentiation between the firms’ products with respect char-
acteristics other than network size. Accordingly, demand for firm $i$’s product takes
the form $p_i = a - q_i - \gamma q_j + \eta S_i$. Inverting the demand functions yields
$q_i(p_i, p_j) = \frac{a}{1 + \gamma} - \frac{p_i - \gamma p_j - \eta S_i + \gamma \eta S_j}{1 - \gamma^2}$ for $i, j \in \{F, R\}$ and $j \neq i$.

To ensure interior solutions, assume $\gamma > \bar{\gamma} \equiv \frac{1}{2} \left( \sqrt{5 + 4\kappa} - 1 \right)$ and $\eta < \bar{\eta} \equiv \frac{2 - \gamma - \gamma^2}{1 - \kappa}$.
The first restriction ($\gamma > \bar{\gamma}$) eliminates the unlikely situation in which consumers of
highly differentiated goods derive network benefits from each other’s adoption. The
second restriction ($\eta < \bar{\eta}$) ensures an equilibrium exists in the price setting sub-game.

**Price Equilibrium Under Disclosure**

Given $\delta$, a conjecture of $R$’s price, denoted $\hat{p}_R(\delta)$, conjectures of consumer expec-
tations of network size, $\hat{S}_F(\delta)$ and $\hat{S}_R(\delta)$, $F$ chooses its price, $p_F$, to solve

$$
\max_{p_F} \left( p_F - (\bar{c} + \delta) \right) \left( \frac{a}{1 + \gamma} - \frac{p_F - \gamma \hat{p}_R(\delta) - \eta \hat{S}_F(\delta) + \gamma \eta \hat{S}_R(\delta)}{1 - \gamma^2} \right). \tag{3.51}
$$
Similarly, given $\delta$, a conjecture of $F$’s price, $\hat{p}_F(\delta)$, and conjectures of consumer expectations of network size, $\hat{S}_F(\delta)$ and $\hat{S}_R(\delta)$, $R$ chooses $p_R(\delta)$ to solve

$$\max_{p_R} \left( p_R - (\bar{c} + \rho \delta) \right) \left( \frac{a}{1 + \gamma} - \frac{p_R - \gamma \hat{p}_F(\delta) - \eta \hat{S}_R(\delta) + \gamma \eta \hat{S}_F(\delta)}{1 - \gamma^2} \right).$$

(3.52)

The first order conditions of (3.51) and (3.52) are provided in (3.53) and (3.54), respectively.

$$p_F(\delta) = \frac{a(1 - \gamma) + (\bar{c} + \delta) + \gamma \hat{p}_R(\delta) - \eta \gamma \hat{S}_R(\delta) + \eta \hat{S}_F(\delta)}{2}$$

(3.53)

$$p_R(\delta) = \frac{a(1 - \gamma) + (\bar{c} + \rho \delta) + \gamma \hat{p}_R(\delta) - \eta \gamma \hat{S}_F(\delta) + \eta \hat{S}_R(\delta)}{2}$$

(3.54)

Consumer expectations of the network sizes are stated in (3.55) and (3.56).

$$S_F(\delta) = q_F(\hat{p}_F(\delta), \hat{p}_R(\delta)) + \kappa q_F(\hat{p}_R(\delta), \hat{p}_F(\delta))$$

(3.55)

$$S_R(\delta) = q_R(\hat{p}_R(\delta), \hat{p}_F(\delta)) + \kappa q_R(\hat{p}_F(\delta), \hat{p}_R(\delta))$$

(3.56)

Rationality requires conjectures to satisfy equations (3.57) - (3.60).

$$\hat{S}_F(\delta) = S_F(\delta)$$

(3.57)

$$\hat{S}_R(\delta) = S_R(\delta)$$

(3.58)

$$\hat{p}_F(\delta) = p_F(\delta)$$

(3.59)

$$\hat{p}_R(\delta) = p_R(\delta)$$

(3.60)

Combining equations (3.53) - (3.60) and computing profits provides outcomes under disclosure as summarized in Lemma 13.
Lemma 13. When firms choose prices, the equilibrium prices, denoted $p_F^d(\delta)$ and $p_R^d(\delta)$, and $F$'s profits under disclosure are:

(i). $p_F^d(\delta) = \frac{a(1-\gamma^2+c+\delta(1+\gamma-\eta(1+\kappa)))}{2+\gamma(1-\gamma)-\eta(1+\kappa)} - \frac{\delta(1-\rho)(1-\gamma^2)(\gamma-\eta\kappa)}{(2+\gamma(1-\gamma)-\eta(1+\kappa))(2-\gamma(1+\gamma)-\eta(1-\kappa))}$ and

(ii). $\pi_F^d(\delta) = (1 - \gamma^2) \left( \frac{a - \bar{c} - \delta}{2+\gamma(1-\gamma)-\eta(1+\kappa)} - \frac{\delta(1-\rho)(\gamma-\eta\kappa)}{(2+\gamma(1-\gamma)-\eta(1+\kappa))(2-\gamma(1+\gamma)-\eta(1-\kappa))} \right)^2$.

Price Equilibrium Under No Disclosure

Given $\delta$, a conjecture of $R$'s price, $\hat{p}_R$, conjectures of consumer expectations of network size, $\hat{S}_F$ and $\hat{S}_R$, $F$ chooses its price $p_F$ to solve

$$\max_{p_F} \quad (p_F - (\bar{c} + \delta)) \left( \frac{a}{1+\gamma} - \frac{p_F - \gamma \hat{p}_R - \eta \hat{S}_F + \gamma \eta \hat{S}_R}{1-\gamma^2} \right).$$

(3.61)

Given a conjecture of $F$'s price, $\hat{p}_F(\delta)$, and conjectures of consumer expectations of network size, $\hat{S}_F$ and $\hat{S}_R$, $R$ chooses $p_R$ to solve

$$\max_{p_R} \quad E \left[ (p_R - (\bar{c} + \rho \delta)) \left( \frac{a}{1+\gamma} - \frac{p_R - \gamma \hat{p}_F(\delta) - \eta \hat{S}_R + \gamma \eta \hat{S}_F}{1-\gamma^2} \right) \right].$$

(3.62)

The first order conditions of (3.61) and (3.62) are provided in (3.63) and (3.64), respectively.

$$p_F(\delta) = \frac{a(1-\gamma) + (\bar{c} + \delta) + \gamma \hat{p}_R - \eta \hat{S}_R + \eta \hat{S}_F}{2}$$

(3.63)

$$p_R = \frac{a(1-\gamma) + \bar{c} + \gamma E[\hat{p}_F(\delta)] - \eta \hat{S}_F + \eta \hat{S}_R}{2}$$

(3.64)

Consumer expectations of the network sizes are stated in (3.65) and (3.66).

$$S_F = E \left[ q_F(\hat{p}_F(\delta), \hat{p}_R) + \kappa q_R(\hat{p}_R, \hat{p}_F(\delta)) \right]$$

(3.65)
\[ SR = E[q_R(\hat{p}_R, \hat{p}_F(\delta)) + \kappa q_F(\hat{p}_F(\delta), \hat{p}_R)] \] (3.66)

Rationality requires conjectures to satisfy equations (3.67) - (3.70).

\[ \hat{S}_F = S_F \] (3.67)

\[ \hat{S}_R = S_R \] (3.68)

\[ \hat{p}_F(\delta) = p_F(\delta) \] (3.69)

\[ \hat{p}_R = p_R \] (3.70)

Combining equations (3.63) - (3.70) and computing profits provides outcomes under disclosure as summarized in Lemma 14.

**Lemma 14.** When firms choose prices, the equilibrium prices, denoted \( p_F^\phi(\delta) \) and \( p_R^\phi \), and \( F \)'s profits under no disclosure are:

(i). \( p_F^\phi(\delta) = \frac{a(1-\gamma^2) + \bar{c}(1+\gamma - \eta(1+\kappa))}{2+\gamma(1-\gamma) - \eta(1+\kappa)} + \frac{\delta}{2} \) and \( p_R^\phi = \frac{a(1-\gamma^2) + \bar{c}(1+\gamma - \eta(1+\kappa))}{2+\gamma(1-\gamma) - \eta(1+\kappa)} \)

(ii). \( \pi_F^\phi(\delta) = (1 - \gamma^2) \left( \frac{a - \bar{c}}{2+\gamma(1-\gamma) - \eta(1+\kappa)} - \frac{\delta}{2(1-\gamma^2)} \right)^2 \).

**Optimal Disclosure Policy**

\( F \)'s expected profits under disclosure and no disclosure when firms choose prices are presented in Lemma 15. A direct comparison of \( \Pi_F^d \) to \( \Pi_F^\phi \) reveals the optimal disclosure policy as summarized in Proposition 7.
Lemma 15. When firms choose prices, $F$’s expected profits under disclosure and no disclosure are:

(i). \[ \Pi^d_F = (1 - \gamma^2) \left[ \frac{a - \bar{c}}{2 + \gamma(1 - \gamma) - \eta(1 + \kappa)} \right]^2 + \left( \frac{\sigma(2 - \gamma^2 - \eta - \gamma \rho + \eta \kappa \rho)}{4 + \eta^2(1 - \kappa^2) - 4\eta + \gamma^4 - (5 - 2\eta)\gamma^2 + 2\eta \kappa \gamma} \right)^2 \]

(ii). \[ \Pi^\phi_F = (1 - \gamma^2) \left[ \frac{a - \bar{c}}{2 + \gamma(1 - \gamma) - \eta(1 + \kappa)} \right]^2 + \left( \frac{\sigma(1 - \gamma^2)}{2(1 - \gamma^2)} \right)^2. \]

Proposition 7.

(i). When firms choose prices, the informed firm prefers disclosure if and only if correlation is sufficiently low, i.e. $\rho < \rho^*(\eta) = \frac{2 + \eta \kappa^2 - \eta \gamma^4 - 2\eta \kappa \gamma}{2(1 - \gamma^4)(\gamma - \eta \kappa)}$.

(ii). Disclosure is nontrivial, i.e. $\rho^*(\eta) > 0$, when $\eta$ is sufficiently large.

(iii). The cutoff $\rho^*(\eta)$ increases as the network effects strengthen.

3.3.5 Negative Network Effects (Congestion)

The analysis in this thesis has, until this point, considered only positive network effects. While positive network effects are common, a nontrivial number of products exhibiting negative network effects exist and warrant discussion here. Negative network effects arise when an individual user of a product becomes worse off as additional users join the network. For example, traffic congestion on a freeway or network congestion over limited bandwidth are both viewed as negative network effects. The primary model can be extended to include negative network effects by allowing $\eta$ to range between -1 and 1. Analysis of the model proceeds as shown in Section 3.2 and the results incorporating negative network effect are similar to those presented...
in Propositions 1 and 2. Specifically, as $\eta$ increases (decreases), a well informed firm becomes increasingly willing (less willing) to disclose its privately held information.

The additional insight gained by including negative network effects in the model is that disclosure in the presence of negative network effects leads to equilibrium quantities that are relatively moderate compared to those absent disclosure. For example, while information that cost is low prompts a firm to increase production, disclosure of the information reduces the willingness to pay of consumers who are wary of a large network. A negative feedback loop ensues and counteracts the incentive to boost production. Consequently, the equilibrium quantities and profits of a low cost firm that discloses are smaller than they would be otherwise. The opposite result occurs when the realized cost is high. As discussed in Section 3.2, convexity in the firm’s profit function leads to a preference for dispersion of production quantities. In the case of negative network effects, quantities under no disclosure are more disperse than quantities under disclosure. The firm prefers a policy of no disclosure as a result.

Figure 3.9 illustrates the impact of negative network effects on a duopolist’s desire to disclose private information. Absent network effects, the firm benefits from disclosing when correlation between the firms’ costs is low. Negative network effects, however, reverse the firm’s preference.
This figure illustrates the ability of negative network effects to reverse a firm’s preference to disclose information. For $\eta = 0$, disclosure informs $R$ about relative efficiency and leads $F$’s equilibrium quantities to be relatively dispersed. As a result, $F$’s expected profits under disclosure exceed profits under no disclosure. For $\eta < 0$, disclosure leads equilibrium quantities that are relatively moderate. Consequently, the negative network effects reverse $F$’s incentives to disclose and expected profits under no disclosure exceed profits under disclosure. $F$ subscripts are suppressed while $q_H$ and $q_L$ denote $F$’s equilibrium quantities given high and low cost, respectively.
Chapter 4: Discussion and Conclusion

This thesis examines the impact of network effects on the consequences and design of a firm’s disclosure policy. An important assumption of this thesis is that the purchasing decisions of consumers’ are affected by the disclosures firms make regarding their production incentives. More specifically, the model assumes consumers consider disclosures about firms’ production costs. The ability of these types of disclosures to influence a consumer’s purchasing decision can be verified by examining product reviews disseminated by firms such as Consumer Reports and X-bit labs. X-bit labs, in particular, maintains a popular product review website that focuses on computer technologies, PC components, information technology, and related products. The stated objective of the website is to help computer enthusiasts and information technology professionals make informed decisions. Product reviews posted on the website commonly include information regarding the manufacturing costs of products. The purpose for providing such information is to help consumers and third party developers assess the long term viability of a product or platform. One particularly appropriate example of a review from X-bit labs is about the Nintendo Wii gaming console (Shilov 2009). The article cites a Credit Suisse analyst report (which, in turn,
refers to the firm’s accounting reports) in order to provide evidence that the cost of manufacturing the Nintendo Wii has declined significantly over time. The article argues that the decline in cost will allow Nintendo to broaden its market presence and, in turn, spur production by third party game developers. One implication is that consumers who value having a wide variety of games to play on their console will find the Wii more appealing after reading the article.

Given that consumers of network goods base purchasing decisions on the information firms disclose regarding their production incentives, the analysis in this thesis demonstrates that positive network effects encourage firms to reveal information they would otherwise be hesitant to share publicly. Intuitively, a firm designing its disclosure policy looks upon disclosure favorably because positive network effects reduce the cost of sharing information with a rival while increasing the value of coordination between firms and consumers. Disclosure informs market participants which firm is better positioned to generate a large network. Consumers in turn become willing to pay more for the products of the firm with this advantage and less for others. Feedback loops ensue and the well positioned firm winds up dominating the market. In a sense, a policy favoring disclosure allows firms to alternate as monopolists in a network market depending on which firm has the advantage. As monopoly profits exceed oligopoly profits, a firm prefers stochastic sharing of monopoly profits and, thus, prefers a policy of disclosure. An additional finding of the thesis is that the benefits of disclosure increase as network effects strengthen. As consumers become more concerned about joining the largest network and relatively less concerned about
other characteristics of a product, disclosures have a larger impact on the purchasing
decisions of customers. Winner-take-all outcomes become increasingly common and
expected profits given a policy of disclosure increase as a result.

While the model presented herein entails stylized notions of disclosure, informa-
tion, and network effects, practical applications of the results are plentiful. A firm
motivated to publicly reveal its privately held information can do so in a myriad of
ways. In particular, a firm may adjust its external reporting system in order to provide
less aggregated, more timely, and more frequent disclosures. Additionally, products
exhibiting network effects are prevalent and come from a wide range of industries in-
cluding information technology, financial services, health care, energy, transportation,
and retailing. Given the prevalence of network markets and their impact on other
firm behavior, future research may examine the consequences of network effects for
other aspects of accounting including cost measurement, information flow within an
organization, and performance evaluation.
References


Appendix A: Proofs

Proof of Lemma 1

Combining equations (3.8) - (3.11) provides $q^d_F(\delta) = \frac{a-\bar{c}}{2-\eta} - \frac{\delta}{2-\eta}$ and $S^d_F(\delta) = \frac{a-\bar{c}}{2-\eta} - \frac{\delta}{2-\eta}$. The monopolist’s equilibrium profits are $\pi^d_F(\delta) = \left[a - q^d_F(\delta) + \eta S^d_F(\delta)\right] q^d_F(\delta) - (\bar{c} + \delta) q^d_F(\delta)$.

\[\square\]

Proof of Lemma 2

Combining equations (3.13)-(3.16) provides $q^\phi_F(\delta) = \frac{a-\bar{c}}{2-\eta} - \frac{\delta}{2}$ and $S^\phi_F = \frac{a-\bar{c}}{2-\eta}$. The monopolist’s equilibrium profits are $\pi^\phi_F(\delta) = \left[a - q^\phi_F(\delta) + \eta S^\phi_F q^\phi_F(\delta)\right] q^\phi_F(\delta) - (\bar{c} + \delta) q^\phi_F(\delta)$.

\[\square\]

Proof of Lemma 3

Lemma 1 and Lemma 2 provide the monopolist’s profits under disclosure and no disclosure, respectively. Taking expectations of $\pi^d_F(\delta)$ and $\pi^\phi_F(\delta)$ with respect to $\delta$, and noting $E[\delta] = 0$ and $E[\delta^2] = \sigma^2$ yields expected profits for the monopolist under each policy.

\[\square\]
Proof of Proposition 1

Lemma 3 provides expected monopoly profits under disclosure ($\Pi^d_F$) and no disclosure ($\Pi^\phi_F$).

(i). $\Pi^d_F > \Pi^\phi_F \iff [\sigma^2/(2-\eta)]^2 > [\sigma^2/2]^2 \iff \eta > 0.$

(ii). $\frac{d[\Pi^d_F - \Pi^\phi_F]}{d\eta} = \frac{2\sigma^2}{(2-\eta)^3} > 0.$ □

Proof of Lemma 4

Combining equations (3.19) - (3.26) provides $q^d_F(\delta), q^d_R(\delta), S^d_F(\delta),$ and $S^d_R(\delta)$. $F$’s equilibrium profits are

$$\pi^d_F(\delta) = (a - q^d_F(\delta) - q^d_R(\delta) + \eta S^d_F(\delta)) q^d_F(\delta) - (\bar{c} + \delta) q^d_F(\delta).$$

$R$’s profits are

$$\pi^d_R(\delta) = (a - q^d_R(\delta) - q^d_F(\delta) + \eta S^d_R(\delta)) q^d_R(\delta) - (\bar{c} + \rho \delta) q^d_R(\delta).$$ □
Proof of Lemma 5

Combining equations (3.29) - (3.36) provides \( q_F^\phi(\delta) \), \( q_R^\phi \), \( S_F^\phi \), and \( S_R^\phi \). F’s equilibrium profits are

\[
\pi_F^\phi(\delta) = \left( a - q_F^\phi(\delta) - q_R^\phi + \eta S_F^\phi \right) q_F^\phi(\delta) - (\bar{c} + \delta) q_F^\phi(\delta).
\]

R’s profits are

\[
\pi_R^\phi = E \left[ \left( a - q_R^\phi - q_F^\phi(\delta) + \eta S_R^\phi \right) q_R^\phi - (\bar{c} + \rho \delta) q_R^\phi \right].
\]

\[\Box\]

Proof of Lemma 6

Lemma 4 and Lemma 5 provide the informed firm’s profits under disclosure and no disclosure, respectively. Taking expectations of \( \pi_F^\phi(\delta) \) and \( \pi_F(\delta) \) with respect to \( \delta \), and noting \( E[\delta] = 0 \) and \( E[\delta^2] = \sigma^2 \) yields expected profits for the firm under each policy.

\[\Box\]

Proof of Proposition 2

Lemma 6 provides expected profits under disclosure (\( \Pi_F^d \)) and no disclosure (\( \Pi_F^\phi \)).

(i). \( \Pi_F^d > \Pi_F^\phi \) \( \iff \) \( \sigma^2 \left[ \frac{2-\rho+\eta \kappa - \eta}{(1-\eta(1-\kappa))(3-\eta(1-\kappa+1))} \right]^2 > \frac{\sigma^2}{4} \iff \rho < \rho^*(\eta) \equiv \frac{1}{2} + \frac{\eta}{2} \left[ \frac{(1-\kappa)+(1-\eta)+\eta \kappa^2}{1-\eta \kappa} \right]. \)

(ii). \( \frac{d}{d\eta} \rho^*(\eta) = \frac{2-\kappa-\eta(1-\kappa^2)(2-\eta \kappa)}{2(1-\eta \kappa)^2} > 0. \)

\[\Box\]
Proof of Corollary 1

Proof follows from the conditions stated in the proof of Proposition 2 after substituting the appropriate parameter restrictions. □

Proof of Lemma 7

Let $D_F \in \{\delta, \phi\}$ denote $F$’s disclosure and $D_R \in \{\lambda, \phi\}$ denote $R$’s, where $\phi$ indicates no disclosure. Given $\delta$, the disclosures of both firms, a conjecture of consumer expectations of $F$’s network size $\hat{S}_F(D_F, D_R)$, and a conjecture of $R$’s quantity $\hat{q}_R(\lambda, D_F, D_R)$, $F$ chooses its quantity $q_F$ to solve

$$\max_{q_F} E_{\lambda} \left[ \left( a - q_F - \hat{q}_R(\lambda, D_F, D_R) + \eta \hat{S}_F(D_F, D_R) \right) q_F - c_F \right]_{\delta, D_R}. \tag{A.1}$$

Similarly, given $\lambda$, the disclosures of both firms, a conjecture of consumer expectations of $R$’s network size $\hat{S}_F(D_F, D_R)$, and a conjecture of $F$’s quantity $\hat{q}_F(\delta, D_F, D_R)$, $R$ chooses $q_R$ to solve

$$\max_{q_R} E_{\delta} \left[ \left( a - q_R - \hat{q}_F(\delta, D_F, D_R) + \eta \hat{S}_R(D_F, D_R) \right) q_R - c_R \right]_{D_F, \lambda}. \tag{A.2}$$

The first order conditions of (A.1) and (A.2) are provided in (A.3) and (A.4), respectively.

$$q_F(\delta, D_F, D_R) = \frac{a - E_{\lambda}[c_F|\delta, D_R] - E_{\lambda}[\hat{q}_R(\lambda, D_F, D_R)|D_R] + \hat{S}_F(D_F, D_R)}{2} \tag{A.3}$$

$$q_R(\lambda, D_F, D_R) = \frac{a - E_{\delta}[c_R|D_F, \lambda] - E_{\delta}[\hat{q}_F(\delta, D_F, D_R)|D_F] + \hat{S}_R(D_F, D_R)}{2} \tag{A.4}$$

Consumers form their expectations of the firms’ network sizes and establish willingness to pay based on the firms’ disclosures and conjectures of the firms’ quantities as
stated in equations (A.5) and (A.6).

\[ S_F(D_F, D_R) = E_\delta \left[ \hat{q}_F(\delta, D_F, D_R) \bigg| D_F \right] + \kappa \ E_\lambda \left[ \hat{q}_R(\lambda, D_F, D_R) \bigg| D_R \right] \]  \hspace{1cm} (A.5)

\[ S_R(D_F, D_R) = E_\lambda \left[ \hat{q}_R(\lambda, D_F, D_R) \bigg| D_R \right] + \kappa \ E_\delta \left[ \hat{q}_F(\delta, D_F, D_R) \bigg| D_F \right] \]  \hspace{1cm} (A.6)

Rationality requires conjectures satisfy equations (A.7) - (A.10).

\[ \hat{S}_F(D_F, D_R) = S_F(D_F, D_R) \]  \hspace{1cm} (A.7)

\[ \hat{S}_R(D_F, D_R) = S_R(D_F, D_R) \]  \hspace{1cm} (A.8)

\[ \hat{q}_F(\delta, D_F, D_R) = q_F(\delta, D_F, D_R) \]  \hspace{1cm} (A.9)

\[ \hat{q}_R(\lambda, D_F, D_R) = q_R(\lambda, D_F, D_R) \]  \hspace{1cm} (A.10)

Combining equations (A.3) - (A.10) and substituting in the appropriate disclosure decisions \((D_F \) and \(D_R\)) provides \(F\)’s quantities and profits under each disclosure regime as follows (the superscripts denote \(F\) and \(R\)’s disclosure decisions, respectively):

\( \begin{align*}
\text{(i).} \quad & q_{\phi\phi}^F(\delta) = \frac{a - c}{3 - \eta(\kappa + 1)} - \delta \quad \text{and} \quad \pi_{\phi\phi}^F(\delta) = \left[ q_{\phi\phi}^F(\delta) \right]^2 \\
\text{(ii).} \quad & q_{\phi\phi}^F(\delta) = \frac{a - c}{3 - \eta(\kappa + 1)} - \frac{\delta(2 - \rho + \eta \kappa - \eta)}{(1 - \eta)(1 - \kappa)(3 - \eta(\kappa + 1))} \quad \text{and} \quad \pi_{\phi\phi}^F(\delta) = \left[ q_{\phi\phi}^F(\delta) \right]^2 \\
\text{(iii).} \quad & q_{\phi\phi}^d(\delta, \lambda) = \frac{a - c}{3 - \eta(\kappa + 1)} + \frac{\lambda(1 - (\kappa - \rho) - 2\rho)}{(1 - \eta)(1 - \kappa)(3 - \eta(\kappa + 1))} - \frac{\delta}{2} \quad \text{and} \quad \pi_{\phi\phi}^d(\delta, \lambda) = \left[ q_{\phi\phi}^d(\delta, \lambda) \right]^2 \\
\text{(iv).} \quad & q_{\phi\phi}^{dd}(\delta, \lambda) = \frac{a - c}{3 - \eta(\kappa + 1)} + \frac{\lambda(1 - (\kappa - \rho) - 2\rho)}{(1 - \eta)(1 - \kappa)(3 - \eta(\kappa + 1))} - \frac{\delta(2 - \rho + \eta \kappa - \eta)}{(1 - \eta)(1 - \kappa)(3 - \eta(\kappa + 1))} \quad \text{and} \quad \pi_{\phi\phi}^{dd}(\delta, \lambda) = \left[ q_{\phi\phi}^{dd}(\delta, \lambda) \right]^2.
\end{align*} \)
$R$’s outcomes are omitted as they are symmetric to $F$’s. Taking expectations with respect to $\delta$ and $\lambda$, while noting $E[\delta] = 0$, $E[\lambda] = 0$, $E[\delta\lambda] = 0$, $E[\delta^2] = \sigma_\delta^2$, and $E[\lambda^2] = \sigma_\lambda^2$, yields the expected profits presented in the lemma. □

**Proof of Proposition 3**

Taking $R$’s disclosure policy as given and comparing $F$’s expected profits under disclosure and no disclosure reveals that the disclosure policy forms a dominant strategy. Specifically, when $R$ chooses to withhold, $F$ prefers disclosure when \( \Pi_{F}^{d\phi} > \Pi_{F}^{\phi\phi} \). When $R$ chooses to disclose, $F$ prefers disclosure when \( \Pi_{F}^{dd} > \Pi_{F}^{\phi d} \). Substituting the expressions from Lemma 7 and simplifying shows that in both cases $F$ prefers disclosure if and only if \( \sigma_\delta^2 \left[ \frac{2 - \rho + \eta \rho - \eta}{(1-\eta(1-\kappa))(3-\eta(\kappa+1))} \right]^2 > \frac{\sigma_\delta^2}{4} \), which is equivalent to the condition provided in the proof of Proposition 2 part (i). Symmetric arguments can be applied to $R$’s decision. □

**Proof of Lemma 8**

(i). The equilibrium under disclosure is characterized by equations (3.8) - (3.11).

Combining equations (3.8) - (3.11) provides \( q_{F}^{d}(\delta) = \frac{a-c}{2-\eta} - \frac{\delta}{2-\eta} \) and \( S_{F}^{d}(\delta) = \frac{a-c}{2-\eta} - \frac{\delta}{2-\eta} \). The monopolist’s equilibrium profits under disclosure are \( \pi_{F}^{d}(\delta) = [a - q_{F}^{d}(\delta) + \eta S_{F}^{d}(\delta)] q_{F}^{d}(\delta) - (\bar{c} + \delta) q_{F}^{d}(\delta) - \psi \).

(ii). The equilibrium under no disclosure is characterized by equations (3.13) - (3.16).

Combining equations (3.13) - (3.16) provides \( q_{F}^{\phi}(\delta) = \frac{a-c}{2-\eta} - \frac{\delta}{2-\eta} - \frac{\eta E[\delta|\phi]}{2(2-\eta)} \) and \( S_{F}^{\phi} = \frac{a-c}{2-\eta} - \frac{E[\delta|\phi]}{2-\eta} \). The monopolist’s profits under no disclosure are \( \pi_{F}^{\phi}(\delta) = [a - q_{F}^{\phi}(\delta) + \eta S_{F}^{\phi}] q_{F}^{\phi}(\delta) - (\bar{c} + \delta) q_{F}^{\phi}(\delta) \).

□
Proof of Proposition 4

If network effects are nonexistent or weak relative to the cost of disclosure $\psi$, then the indifference condition stated in equation (3.38) cannot be satisfied. Specifically, for $\eta = 0$, the left hand side of equation (3.38) is zero while the right hand side is strictly greater than zero. Furthermore, the left hand side of the condition is strictly decreasing in $\delta^*$ and its limit as $\delta^* \to -\Delta$ is $\frac{4(a-\bar c+\Delta)-\eta\Delta\eta\Delta}{2(1-\eta)^2} > 0$. Consequently, if $\frac{4(a-\bar c+\Delta)-\eta\Delta\eta\Delta}{2(1-\eta)^2} < \psi$, then the indifference condition cannot be satisfied. Rearranging terms, if $0 \leq \eta < \eta \equiv \frac{4\Delta-4\Delta+4\Delta^2+8\psi-\sqrt{(4\Delta-4\Delta+4\Delta^2+8\psi)^2-32\psi(\Delta^2+2\psi)}}{2(\Delta^2+2\psi)}$, the indifference condition cannot be satisfied and a cutoff does not exist. In words, when network effects are weak or nonexistent, the cost of disclosure $\psi$ outweighs the benefits of disclosure for all $\delta \in [-\Delta, \Delta]$. Consequently, the monopolist withholds all realizations of $\delta$.

For $\eta \geq \eta$, however, solving for the cutoff, $\delta^*(\eta)$, yields:

$$
\delta^*(\eta) = \frac{4\eta(a-\bar c+\Delta) - \Delta\eta^2 - 4\sqrt{\eta(a-\bar c-\Delta)^2 + \psi(2-\eta)^2(8-\eta)}}{(8-\eta)\eta}.
$$

This proves parts (i) and (ii) of the proposition.

Furthermore, $\frac{d}{d \eta} \frac{d \delta^*(\eta)}{d \eta} > 0$ proves part (iii) of the proposition. \qed
Proof of Lemma 9

(i). The equilibrium under disclosure is characterized by equations (3.19) - (3.26).

Combining equations (3.19) - (3.26) provides \( q^d_F(\delta), q^d_R(\delta), S^d_F(\delta), \) and \( S^d_R(\delta). \)

\( F \)'s equilibrium profits are

\[
\pi^d_F(\delta) = (a - q^d_F(\delta) - q^d_R(\delta) + \eta S^d_F(\delta)) q^d_F(\delta) - (\bar{c} + \delta) q^d_F(\delta) - \psi.
\]

(ii). The equilibrium under disclosure is characterized by equations (3.29) - (3.36).

Combining equations (3.29) - (3.36) provides

\[
q^\phi_F(\delta) = \frac{a - \bar{c}}{3-\eta(\kappa+1)} - \frac{\delta}{2} - \frac{E[\delta] \phi((1-\eta^2(1-\kappa^2)+2\eta(1-\kappa(1-\rho))-2\rho)}{2(3+\eta^2(1-\kappa^2)-2\eta(2-\kappa))},
\]

\[
q^\phi_R = \frac{a - \bar{c}}{3-\eta(\kappa+1)} + \frac{E[\delta] \phi(1-2\rho+\eta(\rho-\kappa))}{3+\eta^2(1-\kappa^2)-2\eta(2-\kappa)},
\]

\[
S^\phi_F = \left[ \frac{a - \bar{c}}{3-\eta(\kappa+1)} + \frac{E[\delta] \phi(2-\rho-\eta(1-\kappa\rho))}{(1-\eta(1-\kappa))(3-\eta(\kappa+1))} \right] + \kappa \left[ q^\phi_R \right], \quad \text{and}
\]

\[
S^\phi_R = \left[ q^\phi_R \right] + \kappa \left[ \frac{a - \bar{c}}{3-\eta(\kappa+1)} + \frac{E[\delta] \phi(2-\rho-\eta(1-\kappa\rho))}{(1-\eta(1-\kappa))(3-\eta(\kappa+1))} \right].
\]

\( F \)'s equilibrium profits are

\[
\pi^\phi_F(\delta) = \left( a - q^\phi_F(\delta) - q^\phi_R + \eta S^\phi_F \right) q^\phi_F(\delta) - (\bar{c} + \delta) q^\phi_F(\delta).
\]

\( \square \)
Proof of Proposition 5

$F$ will disclose when $\pi^d_F(\delta) - \pi^\phi_F(\delta) > 0$ and withhold otherwise. Lemma 9 provides $F$'s profits under disclosure and no disclosure:

$$\pi^d_F(\delta) = \left[ \frac{a - \bar{c}}{3 - \eta(\kappa + 1)} - \frac{\delta}{3 - \eta(\kappa + 1)} - \frac{\delta(1 - \rho)(1 - \eta \kappa)}{(3 - \eta(\kappa + 1))(1 - \eta(1 - \kappa))} \right]^2 - \psi$$

$$\pi^\phi_F(\delta) = \left[ \frac{a - \bar{c}}{3 - \eta(\kappa + 1)} - \frac{\delta}{2} - \frac{E[\delta|\phi] (1 - \eta^2 (1 - \kappa^2) + 2\eta(1 - \kappa(1 - \rho)) - 2\rho)}{2 \left( 3 + \eta^2 (1 - \kappa^2) - 2\eta(2 - \kappa) \right)} \right]^2.$$

**Case 1:** For $\rho = \bar{\rho}(\eta) \equiv \frac{1}{2} + \frac{\eta}{2} \left[ \frac{(1 - \kappa) + (1 - \eta) + \eta \kappa^2}{1 - \eta \kappa} \right]$, $\pi^d_F(\delta) - \pi^\phi_F(\delta) = -\psi < 0$. Consequently, $F$ will not disclose when $\rho = \bar{\rho}(\eta)$. This proves (i).

**Case 2:** For $\rho < \bar{\rho}(\eta) \equiv \frac{1}{2} + \frac{\eta}{2} \left[ \frac{(1 - \kappa) + (1 - \eta) + \eta \kappa^2}{1 - \eta \kappa} \right]$, the difference $\pi^d_F(\delta) - \pi^\phi_F(\delta)$ decreases with $\delta$ and, thus, $E[\delta|\phi] = \frac{\delta^*(\eta) + \Delta}{2}$. The unique cutoff is identified by solving for the $\delta^*(\eta)$ implied by the indifference condition $\pi^d_F(\delta^*) = \pi^\phi_F(\delta^*)$. While the expression for $\delta^*(\eta)$ is not provided here, it can be shown that $\frac{d \delta^*(\eta)}{d \eta} > 0$. This proves (ii).

**Case 3:** For $\rho > \bar{\rho}(\eta) \equiv \frac{1}{2} + \frac{\eta}{2} \left[ \frac{(1 - \kappa) + (1 - \eta) + \eta \kappa^2}{1 - \eta \kappa} \right]$, the difference $\pi^d_F(\delta) - \pi^\phi_F(\delta)$ increases with $\delta$ and, thus, $E[\delta|\phi] = \frac{-\Delta + \delta^*(\eta)}{2}$. The unique cutoff is identified by solving for the $\delta^*(\eta)$ implied by the indifference condition $\pi^d_F(\delta^*) = \pi^\phi_F(\delta^*)$. While the expression for $\delta^*(\eta)$ is not provided here, it can be shown that $\frac{d \delta^*(\eta)}{d \eta} > 0$. This proves (iii).
Proof of Lemma 10

Combining equations (3.41) - (3.44) provides $q_F^d(\delta)$ and $q_R^d(\delta)$. $F$’s equilibrium profits are

$$
\pi_F^d(\delta) = (a - q_F^d(\delta) - q_R^d(\delta) + \eta (q_F^d(\delta) + \kappa q_F^d(\delta))) q_F^d(\delta) - (\bar{c} + \delta)q_F^d(\delta).
$$

$R$’s profits are

$$
\pi_R^d(\delta) = (a - q_R^d(\delta) - q_F^d(\delta) + \eta (q_R^d(\delta) + \kappa q_R^d(\delta))) q_R^d(\delta) - (\bar{c} + \rho \delta)q_R^d(\delta).
$$

\[ \square \]

Proof of Lemma 11

Combining equations (3.47) - (3.50) provides $q_F^{\phi}(\delta)$ and $q_R^{\phi}$. $F$’s equilibrium profits are

$$
\pi_F^{\phi}(\delta) = (a - q_F^{\phi}(\delta) - q_R^{\phi} + \eta (q_F^{\phi}(\delta) + \kappa q_F^{\phi}(\delta))) q_F^{\phi}(\delta) - (\bar{c} + \delta)q_F^{\phi}(\delta).
$$

$R$’s profits are

$$
\pi_R^{\phi} = E \left[ (a - q_R^{\phi} - q_F^{\phi}(\delta) + \eta (q_R^{\phi} + \kappa q_F^{\phi}(\delta))) q_R^{\phi} - (\bar{c} + \rho \delta)q_R^{\phi} \right].
$$

\[ \square \]
Proof of Lemma 12

Lemma 10 and Lemma 11 provide the informed firm’s profits under disclosure and no disclosure, respectively. Taking expectations with respect to δ, noting $E[\delta] = 0$ and $E[\delta^2] = \sigma^2$, yields $F$’s expected profits under each policy.

Proof of Proposition 6

Lemma 12 provides expected profits under disclosure and no disclosure.

(i). $\Pi^d_F > \Pi^\phi_F \iff \sigma^2 \left[ \frac{2 - \rho - \eta(2 - \kappa)}{(3 - \eta(2 + \kappa))(1 - \eta(2 - \kappa))} \right] > \frac{\sigma^2}{4(1-\eta)^2} \iff \rho < \rho^*(\eta) \equiv \frac{1}{2} + \frac{\eta(1 - \kappa)}{2(1 - \eta)}$.

(ii). $\frac{d}{d\eta}\rho^*(\eta) = \frac{1 - \kappa}{2(1 - \eta)^2} \geq 0$.

Proof of Lemma 13

Combining equations (3.53) - (3.60) provides $p^d_F(\delta)$, $p^d_R(\delta)$, $S^d_F(\delta)$, and $S^d_R(\delta)$. $F$’s equilibrium profits are

$$\pi^d_F(\delta) = (p^d_F(\delta) - (\bar{e} + \delta)) \left( \frac{a}{1 + \gamma} - \frac{p^d_F(\delta) - \gamma p^d_R(\delta) - \eta S^d_F(\delta) + \gamma \eta S^d_R(\delta)}{1 - \gamma^2} \right).$$
Proof of Lemma 14

Combining equations (3.63) - (3.70) provides $p^g_F(\delta), p^g_R, S^g_F,$ and $S^g_R$. $F$’s equilibrium profits are

$$\pi^g_F(\delta) = \left( p^g_F(\delta) - (\bar{c} + \delta) \right) \left( \frac{a}{1 + \gamma} - \frac{p^g_F(\delta) - \gamma p^g_R - \eta S^g_F + \gamma \eta S^g_R}{1 - \gamma^2} \right).$$

□

Proof of Lemma 15

Lemma 13 and Lemma 14 provide the informed firm’s profits under disclosure and no disclosure, respectively. Taking expectations with respect to $\delta$, noting $E[\delta] = 0$ and $E[\delta^2] = \sigma^2$, yields $F$’s expected profits under each policy.

□

Proof of Proposition 7

Lemma 15 provides expected profits under disclosure and no disclosure. A comparison of $F$’s expected profits under disclosure and no disclosure proves (i) and (ii). Specifically,

$$\Pi^d_F > \Pi^\phi_F \iff \begin{cases} \eta > \frac{1 - \gamma \kappa - \sqrt{(1 - \kappa^2) \gamma^4 - (1 - 2 \kappa^2) \gamma^2 - 2 \kappa \gamma + 1}}{1 - \kappa^2} \equiv \bar{\eta} \\
0 \leq \rho < \rho^*(\eta) \equiv \frac{(2 + \eta \kappa^2 - \eta) \eta + \gamma^4 - \gamma^2 - 2 \eta \kappa \gamma}{2 (1 - \gamma^2) (\gamma - \eta \kappa)}.
\end{cases}$$

Furthermore, $\frac{d}{d\eta} \left[ \frac{(2 + \eta \kappa^2 - \eta) \eta + \gamma^4 - \gamma^2 - 2 \eta \kappa \gamma}{2 (1 - \gamma^2) (\gamma - \eta \kappa)} \right] > 0$ for $\eta \geq \bar{\eta}$ proves part (iii). □