Extrasolar Planet Detection and Characterization With the KELT-North Transit Survey

DISSERTATION

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By

Thomas G. Beatty

Graduate Program in Astronomy

The Ohio State University

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Dissertation Committee:
Professor B. Scott Gaudi, Advisor
Professor Andrew P. Gould
Professor Marc H. Pinsonneault
Abstract

My dissertation focuses on the detection and characterization of new transiting extrasolar planets from the KELT-North survey, along with an examination of the processes underlying the astrophysical errors in the type of radial velocity measurements necessary to measure exoplanetary masses. Since 2006, the KELT-North transit survey has been collecting wide-angle precision photometry for 20% of the sky using a set of target selection, lightcurve processing, and candidate identification protocols I developed over the winter of 2010-2011. Since our initial set of planet candidates were generated in April 2011, KELT-North has discovered seven new transiting planets, two of which are among the five brightest transiting hot Jupiter systems discovered via a ground-based photometric survey. This highlights one of the main goals of the KELT-North survey: to discover new transiting systems orbiting bright, $V<10$, host stars. These systems offer us the best targets for the precision ground- and space-based follow-up observations necessary to measure exoplanetary atmospheres. In September 2012 I demonstrated the atmospheric science enabled by the new KELT planets by observing the secondary eclipses of the brown dwarf KELT-1b with the Spitzer Space Telescope. For the first time, these eclipse observations demonstrated that hot, transiting, brown dwarfs have
atmospheres similar to other, cold, brown dwarfs, and not to hot Jupiters. This opens up the use of the transiting brown dwarfs as objects of comparative study relative to the directly imaged cold brown dwarfs. Additionally, the strong focus on statistical repeatability I brought to the design of the KELT-North candidate selection process means that the results from the survey may be used in the future for a rigorous statistical analysis of the new, and old, transiting planets discovered by KELT-North. This will be only the fourth such analysis done using a transit survey, and, with approximately 80,000 target dwarf stars, will use the largest sample size to date. As a prelude to this project, my dissertation also provides the first \textit{a priori}, descriptive, formulation of the astrophysical sources of uncertainty in radial velocity measurements. Exoplanetary masses are typically measured using radial velocity, and a thorough understanding of the sources of error in these observations provides crucial insight into the selection biases in searches for extrasolar planets, and allows for the design of more efficient surveys in the future.
Dedication

To my mother and father.
Acknowledgments

I would like to express my special appreciation and sincere gratitude to my advisor, Scott Gaudi, for his patience, guidance, and understanding during the past five years. I first met Scott almost nine years ago in Cambridge, and it was my experience doing science with him while I was at Harvard that made me decide to pursue a PhD in astronomy. It has been a pleasure to go from undergraduate through graduate school under his tutelage, and his attention to detail has continually pushed me towards becoming a better scientist.

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Thank you as well to David Will and Michael Savage, for keeping all of the many computers necessary to run KELT-North operational, and fixing problems – sometimes before I even knew they existed.
I am especially indebted to my parents and my brothers for their support, both before and during my time in graduate school. This dissertation is, undoubtedly, the result of having watched The Dream is Alive so many times.

Finally, my special thanks to Catherine Grier for her love and company over the past four years. I look forward to the many years to come.
Vita

March 10, 1983 .................. Born – New York, NY

2006 ......................... B.A. Astrophysics, Harvard University

2009 ......................... M.S. Physics, Massachusetts Institute of Technology

2009 – 2011 .................... Distinguished University Fellow, The Ohio State University

2011 – 2012 .................... Graduate Teaching and Research Associate,
                            The Ohio State University

Publications

Research Publications


Fields of Study

Major Field: Astronomy
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Chapter 1: Introduction

1.1. History and Background

It is interesting to consider that the idea of detecting extrasolar planets via radial velocity (RV) and transits was first proposed nearly fifty years before the discovery of HD 209458 b. Struve (1952) neatly outlined the entire first ten years of the field, by suggesting the possibility of what we now call hot Jupiters, and noting that these would have relatively large RV variations and eclipse probabilities. Struve (1952) thought that a Jupiter-mass planet would be marginally detectable in both RV and photometric observations using the best equipment available at the time. If we assume this to be the case, a determined observational campaign could hypothetically have found the first planetary companion to a Sun-like star by the early 1960s.

Instead, the first clearly planetary companion to a Sun-like star was not discovered through RV measurements until 1995 (Mayor & Queloz 1995), and the first transiting planet around HD 209458 was not observed until 2000 (Charbonneau et al. 2000; Henry et al. 2000). Transiting planets have been immensely important in our study of exoplanets, by enabling measurements of planetary masses and radii. This means that in addition to resolving the \( \sin(i) \) degeneracy present for RV planets, transiting planets also allow for measurements of the density and surface gravity of planets. These measurements have immediate application towards understanding of planetary atmospheres, interiors, migration, and formation. In addition, one can
directly measure the atmospheric composition and dynamics of transiting exoplanets via transit transmission spectroscopy (Charbonneau et al. 2002), secondary eclipse observation (Charbonneau et al. 2005), and orbital phase-curve mapping (Knutson et al. 2007).

As a result of the dramatic scientific possibilities offered by transiting planets, shortly after their publication of the transit of HD 209458 b the team behind Charbonneau et al. (2000) reorganized themselves into the Transatlantic Exoplanet Survey (TrES), a dedicated search for transiting planets. They were shortly followed by over 22 other surveys (Horne 2003), including Vulcan (Borucki et al. 2001), RAPTOR (Vestrund et al. 2002), PISCES (Mochejska et al. 2002), EXPLORE (Mallén-Ornelas et al. 2003), BEST (Rauer et al. 2003), OGLE-III (Udalski et al. 2002), XO (McCullough et al. 2005), and the precursors to the HAT (Bakos et al. 2002) and SuperWASP (Kane et al. 2003) surveys. Of all of these “first-generation” transit surveys, only TrES, OGLE-III, and XO detected new transiting planets in their first years of operation, and typically only one to three new planets were announced every year.

During this time, the low detection rate meant that every new transiting planet was of individual scientific interest. For example, HD 209458 b was the first “inflated” hot Jupiter to be detected, while HD 149026 b was the first giant planet observed to have an inferred core mass of $100\,M_\oplus$ (Sato et al. 2005). These two results immediately launched a debate within the community, that is still ongoing, over the precise mechanism responsible for setting the radii of hot Jupiters (see Rauscher & Showman 2014, for an example of current theoretical grappling with the problem.). Other individual highlights from the early years of transiting planets were the longitudinal temperature map of HD 189733 b (Knutson et al. 2007), the first secondary eclipse measurement of an exoplanet (Charbonneau et al. 2005) and
the detection of the transits of the hot Neptune GJ 436 b (Gillon et al. 2007), which was the smallest transiting planet known until the Kepler mission.

In 2007, however, a phase change in the field occurred, as the rate of discovery began to steadily increase from around 2 new transiting planets per year up to 10 per year, then 20, then 50, and even 121 new transiting planets in 2011. The sharp increase in the detection rate occurred for two reasons. First, the second generation ground-based surveys – HAT and SuperWASP – began to come on line in 2007, and were then followed by the Kepler spacecraft’s first sets of discoveries in 2010 and 2011. Unlike the first generation projects, these surveys have extremely large fields of view. SuperWASP, for example, images 488 deg$^2$ per pointing, compared to 36 deg$^2$ for the TrES survey. In addition, both HAT and SuperWASP have largely solved the problems of telescope robotization and data transfer necessary for their telescopes to operate remotely and without frequent downtimes due to hardware issues. Together, this means that these two ground-based surveys are able to image more of the sky, more reliably, than earlier photometric transit surveys.

Second, by 2007 there was considerably more understanding of the sheer amount of effort necessary to run a transit survey. On the surface, detecting planetary transits seems to be simple; indeed, Charbonneau et al. (2000) first detected the transit of HD 209458b using a 10cm telescope in the parking lot on the UC Boulder campus. The numbers, too, appear to be in favor of easy detections. If we assume that roughly 1% of dwarf stars host a hot Jupiter, and those hot Jupiters have a roughly 10% transit probability, then 1 out of every 1,000 stars should show a transit. If we now design a photometric survey to take photometric observations of 10,000 stars with a precision of 1% or better, then simple binomial probability tells us that we are nearly guaranteed to discover (with a probability of 99.9%) at least 1 transiting planet, and on average we expect to find 10. On this basis, the first
generation of transit surveys were started in the first half of the 2000’s, with the expectation that over a hundred transiting planets would be discovered per year for only a moderate expenditure of time and resources.

Unfortunately, this simple and appealing calculation fails for four reasons. First, the frequency of hot Jupiters seems to be lower for transit surveys than for RV surveys. Typically, transit surveys find a hot Jupiter frequency of around 0.5%, not the 1% found by the RV searches (Wright et al. 2012, and references therein). Though this difference in the frequency of hot Jupiters is still not well understood, this immediately halves the expected number of detections.

Second, a substantial fraction of the stars within a survey field that show better than 1% photometric precision are either giants or early main-sequence stars that are too large to enable detectable transit dips from a Jupiter-sized planet (Gould & Morgan 2003; Brown 2003).

Third, robust transit detections usually require more than one transit in the data. This fact, coupled with the small fraction of the orbit a planet actually spends in transit, and the typical observing losses at single-site locations due to factors such as weather, create low window probabilities for the typical single-site transit survey in the majority of orbital period ranges of interest (von Braun et al. 2009).

Lastly, requiring better than 1% photometric precision in the data is not a sufficient condition for the successful detection of transits: identifiable transits need to surpass some kind of a detection threshold, such as a signal-to-noise ratio (S/N) threshold. The S/N of the transit signal depends on several factors in addition to the photometric precision of the data, such as the depth of the transit and the number of data points taken during the transit event. Additionally, ground-based photometry typically exhibits substantial auto-correlation in the time series data points, on the
timescales of the transits themselves. This additional red noise, which can come from a number of environmental and instrumental sources, substantially reduces the statistical power of the data (Pont et al. 2006).

Thus, nearly all of the first generation of transit surveys that began in the early 2000s were shutdown, or in the process of halting, by 2007, usually having made no new discoveries. In their place came the wide-field ground surveys like HAT and SuperWASP, who were more prepared for the large data volumes, extensive follow-up needs, and autonomous controls necessary to efficiently execute a photometric transit survey.

More importantly, as a result of the sharp increase in the number of new transiting planets detected from 2007 onwards, the field has transitioned over the last seven years from pure discovery, wherein every new transiting planet is newsworthy, to the details of planetary characterization. Individual planets, aside from the smallest ones being discovered, have become less interesting in themselves, and more interesting for what they can tell us about the statistical properties of exoplanets. Given that transiting planets are so far the only way in which we may robustly infer the radii of extrasolar planets, their exact orbital properties, and the composition and dynamics of their atmospheres, the statistical characteristics of this group of objects is one of the main ways that the areas of planetary interiors, system dynamics, migration, and formation will acquire more data (e.g., Chabrier et al. 2009; Winn et al. 2010; Cowan & Agol 2011).

That being said, it is often hard to draw statistical conclusions from the results of transit surveys. Typically, the surveys do not adopt rigorous, quantified or quantifiable detection criteria. Promising planet candidates are often followed up even if they are beyond the stated signal-to-noise (S/N) or magnitude limits of
the survey. While this is understandable from the stand point of survey operations, wherein the goal is to discover as many planets as possible, understanding and quantifying how the survey teams select candidates is vital to appropriately deriving the statistical properties of extrasolar planets. Indeed, the results of calculations in Beatty & Gaudi (2008) demonstrate that the actual statistics depend crucially on the specific magnitude and S/N threshold used.

Because the selection cut-offs used by a survey are often poorly constrained, though many hot Jupiters have been detected by transit surveys, almost none of the surveys have done detailed statistical and demographic analyses of their results. The only other surveys that have published hot Jupiter statistics are the OGLE-III (Gould et al. 2006), SuperLupus (Bayliss & Sackett 2011) and Kepler (e.g., Howard et al. 2012) surveys. The lack of statistics from transit surveys is particularly lamentable given the difference found by Wright et al. (2012) between hot Jupiter frequencies from RV surveys and the transit searches.

The primary exception to this statistical re-orientation of the field is the continuing scientific utility of transiting planets around bright, V<10, host stars. These systems continue to offer us the best targets for conducting the precise transmission spectroscopy and secondary eclipse measurements necessary to learn about the atmospheres of transiting planets (Madhusudhan et al. 2014). Indeed, eight of brightest transiting systems with V<10 have been observed repeatedly from both ground and space to investigate their atmospheres, and the results from these eight systems provide some of the best observational constraints on theoretical models of planetary atmospheres and interiors (Seager & Deming 2010). Of course, the relative paucity of bright dwarf stars with V<10 means that there are only a handful of transiting planets known around bright stars. Indeed, only 25, or 11%, of the currently known transiting planets orbit stars with V<10. This means that
finding even one more transiting planet around a V<10 will therefore generally allow for precision atmospheric follow-up observations in a completely new region of parameter space (e.g., Showman et al. 2011).

With an eye towards this lack of statistics coming from transit surveys, and in an effort to discover new planets around the bright host stars that allow for precision follow-up observations, the KELT-North transit survey began operation in 2006, and we have been actively generating and vetting planet candidates since April 2011. Like HAT and SuperWASP, the operation of KELT-North has benefited greatly from understanding and implementing the lessons taught by the first generation of photometric transit surveys.

1.2. The KELT-North Transit Survey

The Kilodegree Extremely Little Telescope (KELT) North transit survey is a single-site photometric survey designed to search for transiting hot Jupiters around bright (8<V<11) main sequence stars. The telescope is composed of a Mamiya 645 80mm f/1.9 camera lens attached to a 4096×4096 Apogee AP16E CCD. This is mounted on a Paramount ME equatorial robotic mount, and the system is located at Winer Observatory, AZ. The combination of optics and CCD give KELT-North a field of view of 26° × 26°, with a plate scale of 23” pixel⁻¹. The survey images are taken using a Kodak Wratten #8 red-pass filter, which makes KELT-North’s bandpass a widened version of the traditional Johnson-Cousins R filter. The system hardware is described in more detail in Pepper et al. (2007) and Siverd et al. (2012). Importantly for the data reduction process, since KELT-North is on an equatorial mount, the East and West pointings towards a given field need to be reduced separately, owing to the optics and detector rotating 180 degrees as the mount
crosses the meridian. The lightcurves from the East and West pointing are combined after the TFA step of the reduction process described below.

The specific magnitude range of the KELT-North target stars place the survey in a unique location of exoplanetary detection space. The magnitude range covered by KELT-North targets is largely unsurveyed by other RV or transit surveys, though planets around these bright stars afford us the best opportunities to conduct detailed and precise follow-up observations. Furthermore, while RV surveys specifically select their stars to be cooler than 6250K, and only 10% of Kepler’s targets are hotter than 6250K, nearly 50% of KELT-North targets are hotter than 6250K. Other surveys avoid these stars because they are rapidly rotating, which makes the RV detection of planets difficult. But as we have demonstrated with the detection of KELT-1b, KELT-5b, and KELT-7b, all of which have $v \sin (i) > 50 \text{ km/s}$, this does not need to be an impediment to exoplanet discoveries. KELT-North is thus surveying large sections of stellar parameter space that are mostly untouched by other surveys.

After the images are taken by the KELT-North telescope at Winer Observatory, I conduct the data reduction, initial candidate selection, and the coordination of follow-up observations. The KELT-North lightcurves are generated using a custom version of the ISIS image subtraction routines, in combination with point-spread-function fitting and source identification using the DAOPHOT photometry package. After the set of lightcurves for all of the DAOPHOT-identified point sources are generated using ISIS, I attempt to remove all of the non-main sequence stars from consideration. To do so, I match all of the stars in a given field with the Tycho-2 and 2MASS catalogs, which provides proper motions and JHK magnitudes. Using this catalog information, I construct a reduced proper motion diagram (Gould & Morgan 2003) for all of the stars, and identify as giants all stars which fail to satisfy the reduced proper motion versus J-H color cut described in Section 2.3 of Siverd et
For a typical field, this will leave somewhere between 10,000 and 30,000 putative dwarf stars.

The next stage is to run the dwarfs through the trend filtering algorithm (TFA; Kovács et al. 2005) to reduce systematic noise in the lightcurves. The version of TFA used by KELT-North takes the 150 closest stars to the star being detrended as comparisons, and then removes any correlated features in the lightcurve ensemble. After running TFA, the separate East and West lighcurves are combined into one lightcurve for the object. I then use the box-fitting least-squares algorithm (BLS; Kovács et al. 2002) to search for transit-like signals in the lightcurves. To select the initial set of candidates in each field, I use hard cuts along six of the BLS statistical outputs to pick the most likely candidates. Following this, the initial candidates are discussed by the KELT-North team, and the best are sent on for further spectroscopic and photometric follow-up.

It is important to note here that the hard statistical cut-offs I use in selecting the initial planet candidates is another distinguishing feature of the KELT-North survey: it is operated to be as statistically controlled as possible. Based on my work regarding the theory of transiting planet searches and their yields (Beatty & Gaudi 2008), I purposefully designed KELT-North’s post-processing and candidate selection process so that planet frequencies and demographics using the KELT-North data would be straightforward to calculate. This meant extensive testing of the detrending algorithm to ensure it was not reducing transit depth, and a comprehensive examination of the transit search process and its outputs, by injecting and recovering transits in real lightcurves, to determine the optimal statistical cuts to use in the initial candidate selection and to characterize the aggregate statistical nature of the KELT-North observations. My focus on statistics also meant keeping the reduction process and statistical thresholds for candidate selection fixed. There
have been very few statistics papers from other transit surveys largely because they
did not do this, since the other large surveys are largely concerned with discovering
as many planets as possible.

The final stage of the KELT-North survey is the follow-up of promising planet
candidates. In terms of the daily operation of the survey, this is often the most
important, since KELT-North generally has about a 85:1 ratio of planet candidates
to actual, new, planets. Over the last three years, KELT-North has looked at
approximately 300 planet candidates with low-precision RV observations, and 300
candidates with photometric observations – about 17 per month. Our initial RV
follow-up, which has also been precise enough to determine orbits for most of the
KELT planets, is performed using the TRES spectrograph on the 1.5m Tillinghast
reflector at the Fred Lawrence Whipple Observatory on Mount Hopkins, AZ. The
photometric follow-up of KELT-North candidates is done by a network of professional
and amateur observers across North America and Europe. These observers typically
use telescopes from 0.5m to 1.2m, and usually take lightcurves with a per point
precision of 0.2% to 1%.

For each candidate, I have reviewed the results of the observations as they
arrive, determined whether the system is still a viable candidate, and reported this
to the rest of the KELT-North team. This has involved fitting single-lined RV orbits
and photometric lightcurves to determine the potential system parameters. In the
case of particularly interesting systems, I have also organized specialized observing
resources, such as Keck HIRES RV or adaptive optics imaging time.
1.3. Scope of the Dissertation

This dissertation is organized into four subsequent chapters. In chapter 2, I investigate various astrophysical contributions to the statistical uncertainty of precision radial velocity measurements of stellar spectra, including photon noise, resolution, wavelength, effective temperature, surface gravity, metallicity, macroturbulence, and stellar rotation. This builds on my previous work on photometric transits surveys from Beatty & Gaudi (2008), and illustrates the sort of detailed understanding of measurement errors necessary for the accurate statistics of exoplanets. I derive approximate analytic expressions to these uncertainties in radial velocity measurements, and use these expressions to determine the optimal wavelength regions and dominant contributions to the statistical uncertainties in precision radial velocity measurements as a function of stellar effective temperature and mass for main sequence stars.

In Chapter 3 I report the discovery of KELT-2Ab, a hot Jupiter transiting the bright (V=8.77) primary star of the HD 42176 binary system. KELT-2A is the third brightest star with a transiting planet identified by ground-based transit surveys, and the ninth brightest star overall with a transiting planet. This planetary system is precisely the sort of system that KELT was designed to find: a hot Jupiter transiting a bright star. The host is a slightly evolved late F-star likely in the very short-lived "blue-hook" stage of evolution, with $T_{\text{eff}} = 6151 \pm 50\text{K}$, $\log g = 4.030^{+0.013}_{-0.028}$ and $[\text{Fe/H}] = -0.018 \pm 0.069$. The inferred stellar mass is $M_\star = 1.308^{+0.028}_{-0.025}M_\odot$ and the star has a relatively large radius of $R_\star = 1.828^{+0.070}_{-0.034}R_\odot$. The planet is a typical hot Jupiter with period $4.113791 \pm 0.00001\text{ days}$ and a mass of $M_P = 1.522 \pm 0.078M_J$ and radius of $R_P = 1.286^{+0.065}_{-0.047}R_J$. This is mildly inflated as compared to models of irradiated giant planets at the $\sim4\text{ Gyr}$ age of the system. I also measure the relative
motion of KELT-2A and -2B over a baseline of 38 years, robustly demonstrating for the first time that the stars are bound.

Finally, Chapter 4 describes secondary eclipse observations of the highly irradiated transiting brown dwarf KELT-1b, and is illustrative of the scientifically interesting follow-up work enabled by the bright host stars of the KELT planets. These observations represent the first constraints on the atmospheric dynamics of a highly irradiated brown dwarf, the atmospheres of irradiated giant planets at high surface gravity, and the atmospheres of brown dwarfs that are dominated by external, rather than internal, energy. Using the Spitzer Space Telescope, I measure secondary eclipse depths at 3.6 \( \mu \text{m} \) and at 4.5 \( \mu \text{m} \), and find tentative evidence for the secondary eclipse in the \( z' \) band. For the first time, I compare the IRAC colors of brown dwarfs and hot Jupiters as a function of effective temperature. Importantly, my measurements reveal that KELT-1b has a \([3.6] - [4.5]\) color of 0.07 \( \pm \) 0.11, identical to that of isolated brown dwarfs of similarly high temperature. In contrast, hot Jupiters generally show redder \([3.6] - [4.5]\) colors of \( \sim 0.4 \), with a very large range from \( \sim 0 \) to \( \sim 1 \). Evidently, despite being more similar to hot Jupiters than to isolated brown dwarfs in terms of external forcing of the atmosphere by stellar insolation, KELT-1b appears to have an atmosphere most like that of other brown dwarfs. This suggests that surface gravity is very important in controlling the atmospheric systems of substellar mass bodies, and argues that hot, transiting brown dwarfs may be usefully compared to cold, directly imaged brown dwarfs.
Chapter 2: Astrophysical Sources of Uncertainty in Precision Radial Velocities and Their Approximations

Current generation radial velocity (RV) surveys for exoplanets mostly focus on relatively bright, \( V < 8.5 \), stars using single-object spectrographs. Since they are using single-object spectrographs, the surveys target one star at a time and expose down to a desired continuum signal-to-noise (SN) ratio. The \( V \) cut-off used by the RV surveys is generally driven by the faint limit of the Hipparcos results (e.g., Marcy et al. 2005); current RV surveys deliberately pre-select all of their stars to ensure that they will be dwarfs, chromospherically quiet, and do not have known stellar companions – so that they will be good RV targets. Initially the surveys restricted their targets to single G- and K-dwarfs. This choice was driven on the one side by the faintness of M-dwarfs, which makes getting a high SN ratio in a reasonable exposure time difficult, and on the other side by the rapid rotation of stars hotter than the Kraft Break (Kraft 1970) at 6250K, which makes precision RV measurements more difficult. Over the last two decades, RV surveys have thus surveyed effectively all of the single G- and K-dwarfs brighter than \( V < 8.5 \) for Jupiter-mass planets out to periods of 5.5 years (see, for example, Cumming et al. 2008; Wright 2005).

In the last ten years, a new generation of RV surveys arose that targeted M-dwarfs (M2K and the Keck M-dwarf Survey, Apps et al. 2010; Johnson et al. 2010), fainter high-metallicity stars (N2K, Fischer et al. 2005), and former A-stars that are now sub-giants (“Retired A Stars,” Johnson et al. 2007). These newer surveys use the same observing mode as before: single-object spectrographs and
exposing to a desired SN ratio. They are nearly complete for Jupiter-mass planets out to several hundred days, though they have yet to survey all of the possible stars available.

As the field moves forward, next generation of RV surveys will target fainter stars in broader observing modes. This will be enabled by the results from the GAIA mission, which will allow for vetting of target stars down to $V < 20$ (de Bruijne 2012), and also by the exhaustion of unsurveyed bright stars. There will also be a considerable demand for RV resources to follow-up planet candidates from the upcoming TESS (Ricker et al. 2010) and PLATO (Rauer et al. 2013) missions, as well as residual planet candidates from the Kepler (Borucki et al. 2010) mission. Importantly, these photometric exoplanet surveys do not pre-select their targets in the same manner as the traditional RV surveys (if they pre-select at all). To vet the candidates from these missions will therefore necessitate precision RV measurements of stars that are potentially more active, hotter, more evolved, and rotating faster than the targets of the first-generation surveys.

In all of this, understanding the sources of velocity uncertainty in stellar RV measurements is critical. This allows us to both appropriately understand the detection sensitivities of current RV surveys, and more efficiently design and execute future precision RV searches for exoplanets. In particular, consider a multi-object precision RV survey, which would have design considerations very different from present single-star searches. Firstly, a multi-object survey will probably image a much narrower spectral region than a single-object survey at a similar resolution, on the order of $100\,\text{Å}$ for a multi-object (e.g., Fűrész et al. 2008) versus $1500\,\text{Å}$ for a single object survey (e.g., Marcy & Butler 1996). This has the immediate implication that choosing the wavelength range used by a multi-object survey will be much more important than in a single-object survey. Secondly, a multi-object RV
survey would probably not do the extensive pre-survey vetting of targets done, for example, by the N2K survey. This will give their target lists a higher dispersion in stellar mass, rotation velocities, activity, surface gravity, and metallicity than current single-object surveys. Finally, multi-object surveys will operate in an observing mode more similar to photometric transit surveys for exoplanets, as compared to traditional RV surveys. Instead of exposing to reach the same signal-to-noise ratio for each of the survey’s target stars, a multi-object survey will expose for the same time on each star. This will make the detection sensitivities of a multi-object fundamentally different from a single-object survey in terms of stellar mass, effective temperature, and metallicity. A proper understanding of how these sensitivities change is vital to the initial design of a multi-object survey, and understanding the exoplanet statistics from the survey when it has completed.

There have been several efforts to study the sources of radial velocity uncertainty since the beginnings of precision RV surveys. The first description of how to numerically calculate the expected velocity uncertainty using the properties of an observed spectrum was given by Butler et al. (1996), who were interested in comparing theoretical Poisson-limited velocity uncertainties against their actual observations. In particular, they did not discuss how the uncertainty varies with stellar properties. Bouchy et al. (2001), after their own derivation of the photon-limited RV precision, were the first to examine how velocity uncertainty changes as a function of spectral type, stellar rotation velocity, and spectroscopic resolution. Again, these results were used as part of a general discussion of the capabilities of the CORALIE (Queloz et al. 2000) and the (then future) HARPS (Mayor et al. 2003) spectrographs. Bouchy et al. (2001) thus concentrated their analysis on a limited range of spectral types, rotations, and resolutions, and drew descriptive conclusions from their results (for example, they inferred that
uncertainty is proportional to $v \sin i$ when $v \sin i$ is large). More recently, Bottom et al. (2013) considered the sources of uncertainty in RV measurements as part of an examination into how to optimize RV surveys of GKM stars. Unlike Butler et al. (1996) and Bouchy et al. (2001), Bottom et al. (2013) did not derive an equation for the photon-limited uncertainty in an RV observation, but took model spectra, added Gaussian noise, and then fit for the Doppler shift in the noisy spectra using cross-correlation techniques. The authors intent was to better replicate the true process behind the measurement of stellar RVs. Bottom et al. (2013) consider wider and more finely-spaced ranges of wavelength, temperature, and spectral resolution compared to Bouchy et al. (2001), but they do not consider in detail the effect of changing stellar rotation. Similarly to Bouchy et al. (2001), Bottom et al. (2013) restrict themselves to descriptive conclusions regarding the sources of velocity uncertainty.

In this chapter, I aim to provide a more thorough description of photon-limited stellar velocity uncertainties. Starting from a basic derivation of how to calculate the velocity uncertainty in a spectrum, my intent is to consider, in detail, the effects of effective temperature, surface gravity, metallicity, stellar rotation, spectral resolution, and macroturbulence on the uncertainty in an RV measurement. This allows me to make not only descriptive, but prescriptive conclusions regarding the sources of uncertainty. For example, I discuss precisely why stellar rotation has the effect it does on velocity uncertainties, starting from the shape of the rotation kernel itself. I am thus able to simply and numerically describe how rotation affects the velocity uncertainty across all rotation velocities.

My ultimate goal is to provide a simple and transparent description of the various sources of velocity uncertainty in RV measurements, so as to give the reader a clear picture of the interlocking forces at work. To this end, I use my results to
provide several simplifying approximations that capture the dominant sources of velocity uncertainty as function of stellar mass. This allows me to better understand why current RV surveys achieve the precision they do, and to provide guidance for the design of future surveys.

2.1. Photon-limited Radial Velocity Precision

We begin by considering an imagined one-dimensional spectrum described by an arbitrary function of intensity versus velocity. How well can we measure the velocity positions of this function’s features? Put another way, if we have a measured curve in the x-y plane with uncertainties along the y-axis, how do we transform this into uncertainties along the x-axis? This general problem has been considered before by Butler et al. (1996) and Bouchy et al. (2001), and the following derivation of the general answer is similar.

Mechanically, when one measures a velocity from an observed spectrum, the usual method is to take a template spectrum of the star (either from models or a previous observation) and cross-correlate the template against the observed spectrum to determine a velocity offset. Conceptually, the cross-correlation method is the practical implementation of maximum-likelihood estimation. The cross-correlation function output by the spectral template matching is thus the likelihood function, and the cross-correlation maximum is the maximum-likelihood velocity.

For our purposes, we will assume that the uncertainties in our imagined spectrum are purely from Poisson noise, and free of any systematics. We will also assume the uncertainties are uncorrelated and have no covariances. This allows us to calculate the standard deviation in the mean of the measured velocity using the
maximum-likelihood function with only the first order terms in the general error propagation equation,

\[ \sigma_V^2 = \sigma_1^2 \left( \frac{\partial V}{\partial V_1} \right)^2 + \sigma_2^2 \left( \frac{\partial V}{\partial V_2} \right)^2 + \ldots = \sum_i \left[ \sigma_i^2 \left( \frac{\partial V}{\partial V_i} \right)^2 \right], \quad (2.1) \]

where \( V \) is our arbitrary velocity function composed of individual points \( V_i \), each with uncertainty \( \sigma_i \). Following Butler et al. (1996), we arrive at the needed partial derivative by describing \( V \) as a weighted function of the individual values of \( V_i \),

\[ V = \frac{\sum_i w_i V_i}{\sum_i w_i}, \quad (2.2) \]

with each point weighted by \( w_i = 1/\sigma_i^2 \). The partial derivative in Equation (2.1) then becomes

\[ \frac{\partial V}{\partial V_i} = \frac{w_i}{\sum_i w_i} = \frac{1/\sigma_i^2}{\sum_i 1/\sigma_i^2}. \quad (2.3) \]

Substituting and simplifying,

\[ \sigma_V^2 = \left( \sum_i \frac{1}{\sigma_i^2} \right)^{-1}. \quad (2.4) \]

For spectra, the uncertainty \( \sigma_i \) in each velocity point is a function of the uncertainty in the measured intensity \( I \) and the local slope of the spectrum, \( dI/dV \),

\[ \sigma_i = \frac{\sigma_{I,i}}{dI/dV|_i}. \quad (2.5) \]

Assuming through Poisson statistics that \( \sigma_{I,i} = \sqrt{I_i} \), and subsisting the above into Equation (2.4)

\[ \sigma_V^2 = \left[ \sum_i \frac{(dI/dV)^2|_i}{I_i} \right]^{-1}. \quad (2.6) \]
This is generally how well we can measure the velocity positions of the features in an arbitrary function of intensity versus velocity.

2.1.1. Centroiding absorption lines

Let us first consider the case of a Gaussian absorption line. For compactness, let us define $G(V_i, V_0, \Theta_G)$ as the appropriately normalized Gaussian distribution centered at $V_0$ and with a full-width at half-max (FWHM) $\Theta_G$:

$$G(V_i, V_0, \Theta_G) = \sqrt{\frac{4 \ln 2}{\pi \Theta_G^2}} \exp \left[ -\frac{(V_i - V_0)^2}{\Theta_G^2/(4 \ln 2)} \right].$$

(2.7)

If we take a spectrum composed of points separated by a constant $\Delta V$ in velocity, and if this line absorbs $N_{\text{tot}}$ photons, then a spectrum containing only this line can be described by

$$N_{\gamma}(V_i) = (I_0 - N_{\text{tot}} G[V_i, V_0, \Theta_G]) \Delta V,$$

(2.8)

where $I_0$ is the continuum level, in units of photons per unit velocity. The factor of $\Delta V$ is the velocity span of a pixel. We can rewrite this equation in terms of the velocity equivalent width of the line, $W \equiv N_{\text{tot}}/I_0$, as

$$N_{\gamma}(V_i) = I_0 \Delta V (1 - W G) = N_{\gamma,\text{cont}} (1 - W G).$$

(2.9)

Substituting into Equation (2.6), we get

$$\sigma^2_V = \left[ \sum_i (V_i - V_0)^2 \left( \frac{2 \sqrt{2 \ln 2}}{\Theta_G} \right)^4 \frac{I_0^3 \Delta V^2 W^2 G^2}{I_0 \Delta V (1 - W G)} \right]^{-1}. $$

(2.10)

By the Euler-Maclaurin formula, we can approximate this sum with the integral

$$\sigma^2_V = \left[ \frac{1}{\Delta V} \int_{-\infty}^{\infty} (V - V_0)^2 \left( \frac{2 \sqrt{2 \ln 2}}{\Theta_G} \right)^4 \frac{I_0^3 \Delta V^2 W^2 G^2}{I_0 \Delta V (1 - W G)} dV \right]^{-1}. $$

(2.11)
Unfortunately, this integral has no analytic solution. As a limiting case, consider a shallow absorption line such that $1 - W G \approx 1$. Now we may analytically solve the above equation to get

$$\sigma_V^2 = \left[ \frac{I_0 W^2}{4 \sqrt{\pi}} \left( \frac{2 \sqrt{2 \ln 2}}{\Theta_G} \right)^3 \right]^{-1},$$

(2.12)

or,

$$\sigma_V = \left( \frac{\sqrt{\pi}}{2 (2 \ln 2)^{3/4}} \right) \frac{\Theta_G^{3/2}}{W \sqrt{I_0}} \approx 0.69 \frac{\Theta_G^{3/2}}{W \sqrt{I_0}}.$$  

(2.13)

In numerical tests (Figure 2.1), we find that this approximation is valid for lines with depths less than about 10% of the continuum level. For lines deeper than this the exponential dependence on the width of the Gaussian increases. By the time the depth of the line is 95% of the continuum level we find that the uncertainty in the centroid scales roughly as $\Theta_G^2$.

The uncertainty in the centroid of a shallow Gaussian absorption line therefore scales approximately as the FWHM of the line to the three-halves, and not linearly with $\Theta$ as is the case for a standard Gaussian. Conceptually, we can understand this by comparing a shallow Gaussian absorption line in a background continuum to a regular Gaussian with a base of zero. Let us take the simple case where the absorption line is composed of a single point displaced downwards, from a continuum of value unity, by $1/\Theta$, and the regular Gaussian is a single point displaced upwards from zero by $1/\Theta$. We will set the width of both lines to $\Theta$, so that there is a constant number of photons adsorbed or emitted for any value of $\Theta$.

If we now use Equation (2.6) to calculate the velocity uncertainty for these two lines, the key difference between these two scenarios is the value of $I$, since the
square of the slopes for both will be the same: $1/\theta^2$. For the regular Gaussian, $I$ will be a constant value, independent of $\Theta$. Thus Equation (2.6) gives $\sigma_V \propto \Theta$ for the case of the regular Gaussian. For the absorption line in a continuum, under the assumption that it is shallow $I$ will be equal to the width, $\Theta$, times the continuum level. Thus $I \propto \Theta$, and, from Equation (2.6), $\sigma_V \propto \Theta^{3/2}$.

Similarly to the Gaussian, we can work through the corresponding derivation for a Lorentzian absorption line with FWHM $\Theta_L$ to find

$$\sigma_V = \sqrt{\frac{\pi}{2}} \frac{\Theta_L^{3/2}}{W \sqrt{I_0}} \approx 1.25 \frac{\Theta_L^{3/2}}{W \sqrt{I_0}}. \quad (2.14)$$

This analytic solution is, again, under the assumption that the line depth is negligible relative to the continuum level. As can be seen in Figure 2.1, the exponent on $\Theta_L$ increases as the line depth increases.

Often, spectral lines are effectively described with Voigt profiles: the convolution of a Lorentzian and a Gaussian. Though there is no analytic description of a Voigt profile, we numerically examined how the uncertainty in the centroid scales with the Voigt width. We used Olivero & Longbothum (1977)'s approximation for the effective FWHM of a Voigt profile,

$$\Theta_{voigt} = 0.5346 \Theta_L + \sqrt{0.2166 \Theta_L^2 + \Theta_G^2}, \quad (2.15)$$

where $\Theta_L$ is the FWHM of the Lorentzian component and $\Theta_G$ is the Gaussian FWHM. We directly calculated the uncertainty in the centroid using Equation (2.6) for various values of $\Theta_L$ and $\Theta_G$. As with the pure Gaussian and pure Lorentzian, we find that the uncertainty in measuring the centroid of a Voigt profile is proportional
to $\Theta_{\text{voigt}}^{3/2}$. For the constant of proportionality relating $\sigma_V$ and $\Theta_{\text{voigt}}$, we found numerically that it varied as a function of the ratio $\Theta_L/\Theta_G$.

$$\sigma_V = 0.96 \left( \frac{\Theta_L}{\Theta_G} \right)^{1/2} \frac{\Theta_{\text{voigt}}^{3/2}}{W \sqrt{I_0}}.$$  \hspace{1cm} (2.16)

This form of the leading constant is good to 10% over the range of $1/5 < \Theta_L/\Theta_G < 5$.

In addition to spectral absorption lines, we will also need to consider the role stellar rotation plays in setting measured RV uncertainties. We begin by considering the shape of the kernel itself to determine the FWHM of the rotation kernel: $\Theta_{\text{rot}}$.

To do so, we must first assume a limb-darkening law. For simplicity, we use a simple linear limb-darkening law: $I = I_0(1 - \epsilon + \epsilon \cos \theta)$, where $I_0$ is the intensity at the center of the stellar disk, $\theta$ is the angle of the surface to our line of sight, and $\epsilon$ is the limb-darkening coefficient. Following Gray (2008), the normalized rotation kernel is then

$$G(\Delta v) = \frac{2(1 - \epsilon)\sqrt{1 - (\Delta v/v_{\text{rot}})^2} + \frac{1}{2}\pi \epsilon(1 - (\Delta v/v_{\text{rot}})^2)}{\pi v_{\text{rot}}(1 - \epsilon/3)},$$ \hspace{1cm} (2.17)

where $v_{\text{rot}}$ is the rotation speed at the limb of the star. For the case of no limb-darkening ($\epsilon = 0$) this reduces to

$$G(\Delta v) = \frac{2}{\pi v_{\text{rot}}} \sqrt{1 - (\Delta v/v_{\text{rot}})^2}.$$ \hspace{1cm} (2.18)

To find $\Theta_{\text{rot}}$ we then set $G(\Delta v) = 1/\pi v_{\text{rot}}$ (i.e., half the maximum), $\Delta v = \Theta_{\text{rot}}/2$, and solve. Thus

$$\Theta_{\text{rot}} = \sqrt{3} \ v_{\text{rot}} \ \text{ (for } \epsilon = 0).$$ \hspace{1cm} (2.19)
At the other extreme of $\epsilon = 1$, we may solve Equation (2.17) to find

$$\Theta_{\text{rot}} = \sqrt{2} \ v_{\text{rot}} \ (\text{for } \epsilon = 1).$$

(2.20)

Aside for the cases of $\epsilon = 0$ and $\epsilon = 1$, Equation (2.17) allows for no simple analytic formula for $\Theta_{\text{rot}}$ as a function of $v_{\text{rot}}$ and $\epsilon$. We therefore numerically measured the FWHM of several calculated kernels between $0 < \epsilon < 1$. We found that in between the two limb-darkening extremes the the FWHM went linearly with $\epsilon$, such that

$$\Theta_{\text{rot}} = [(\sqrt{2} - \sqrt{3}) \epsilon + \sqrt{3}] \ v_{\text{rot}}.$$

(2.21)

For reference, a Sun-like star observed at 5500Å would have $\epsilon \approx 0.75$, and $\epsilon \approx 0.4$ if observed at 10,000Å.

Now, similar to the absorption line profiles, we may use Equation (2.6) to determine how velocity uncertainty scales with the width of the rotation kernel. We first consider the case of a fully limb-darkened kernel with $\epsilon = 1$ and equivalent width $W$ subtracted from a continuum. Thus the spectrum is given by $N_{\gamma}(V_i) = N_{\gamma,\text{cont}}(1 - W G)$. This represents the ideal case of a $\delta$-function absorption line being rotationally broadened by the kernel. After making the appropriate substitutions into Equation (2.6), and again assuming that $1 - W G \approx 1$, we find that

$$\sigma_V \approx \sqrt{\frac{2/3}{2^{3/2}} \ \frac{\Theta_{\text{rot}}^{3/2}}{W \sqrt{I_0}}} \approx 0.49 \ \frac{\Theta_{\text{rot}}^{3/2}}{W \sqrt{I_0}} \ (\text{for } \epsilon = 1).$$

(2.22)

Note that since rotational broadening conserves flux, $W \equiv 1$. We will, however, include the equivalent width term as an illustration of the similarity between the rotation kernel and the absorption line profiles.
Unfortunately, $\epsilon = 1$ is the only case for which we may calculate $\sigma_V$ directly from the rotation kernel itself using Equation (2.6). For all other values of $\epsilon$ the slope of the kernel goes to infinity as $\Delta V \rightarrow v_{rot}$. We were not able to find an appropriate analytic or numeric integral to avoid this, so we instead convolved kernels for $\epsilon < 1$ with a normalized Gaussian of small fixed width ($\sigma = 0.1$) and measured the velocity uncertainties of the resulting lines for $v_{rot} > 25$ km s$^{-1}$. We found that the velocity uncertainty continued to be proportional to $\Theta_{rot}^{3/2}$, with the constant of proportionality varying with roughly linearly $\epsilon$, such that

$$\sigma_V \approx (0.347 + 0.146 \epsilon) \frac{\Theta_{rot}^{3/2}}{W \sqrt{I_0}}$$

(2.23)

This is accurate to 2% over $0 \leq \epsilon \leq 1$.

Interestingly, if one rewrites Equation (2.23) in terms of $v_{rot}$, rather than $\Theta_{rot}$, the $\epsilon$ dependence of $\sigma_V$ nearly cancels out. Put another way, velocity uncertainties are not strongly effected by the precise amount of limb-darkening in the stellar photosphere. The difference in $\sigma_V$ between $\epsilon = 0$ and $\epsilon = 0.75$ (the locations of the minimum and maximum of the proportionality coefficient) is only 5%. This is somewhat dependent on our choice of a linear limb-darkening law, but to first-order this result should hold for more complicated limb-darkening laws.

Figure 2.2 shows the directly calculated uncertainties for a rotation kernel, Gaussian, Lorentzian, and Voigt profile with the same equivalent width as a function of FWHM. The overplotted lines are what we expect for the uncertainty based on Equations (2.13), (2.14), and (2.16). For reference, Figure 2.3 shows all four profiles, each one with $\Theta = 1$ and $W = 0.2$. 

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2.2. Calculating Stellar Velocity Uncertainties

Our ultimate goal is to calculate the photon-limited uncertainty expected for RV observations of a main sequence star with a certain exposure time. To achieve this, we will examine how the velocity uncertainties of synthetic spectra – calculated directly using Equation (2.6) – change as a function of instrumental and stellar properties. From our consideration of the uncertainty in centroiding absorption lines, we anticipate that the velocity uncertainty should scale as

\[ \sigma_V \propto \bar{\Theta}^{3/2} \sqrt{I_0} \sqrt{N_{\text{lines}}} \bar{W}, \]

where \( I_0 \) is the continuum flux level, \( \bar{\Theta} \) and \( \bar{W} \) are the average FWHM and equivalent widths of the lines in the spectrum of interest, and \( N_{\text{lines}} \) is the number of lines in the spectrum. The continuum level is set by the luminosity and spectral energy distribution of the target star, the exposure time, the size of the telescope, and the throughput of the optical system. The average FWHM of the spectral lines is determined by the spectral resolution of the instrument, the observed wavelength range, as well as a host of stellar parameters such as mass, temperature, age, rotational velocity and metallicity. We chose to restrict ourselves to stars that are on the main sequence, dwarfs from 2600K to 7600K. This allows us to parameterize stellar properties in terms of one parameter (e.g., mass, or effective temperature). This leaves us with seven main parameters that will determine the average FWHM of the spectral lines: spectral resolution, wavelength range, stellar rotation, atmospheric macroturbulence, effective temperature, surface gravity, and metallicity.

For simplicity, we chose to quantify \( N_{\text{lines}} \), the number of lines present in a spectrum, and \( \bar{W} \), their average equivalent widths, using separate and purely
descriptive functions of effective temperature, surface gravity, and metallicity. Thus Equation (2.24) becomes

$$\sigma_v \propto \sqrt[3]{\bar{\Theta}^{3/2}} f(T_{\text{eff}}) f(\log g) f([\text{Fe/H}]).$$  \hspace{1cm} (2.25)

This is the approximation we will use in our fitting.

We used two different sets of synthetic spectra in our fitting: one set from the BT-Settl model spectra (Allard et al. 2012) and another calculated using Kurucz model atmospheres (Kurucz 1992). The two sets provide us with different pieces of information: the BT-Settl spectra cover the temperature range of interest, but have a (relatively) coarse wavelength spacing, while the Kurucz spectra are extremely finely spaced in wavelength but are only available for a narrower set of temperatures. We therefore used the BT-Settl spectra to examine the effect of stellar effective temperature and surface gravity, and used the Kurucz spectra to model line broadening mechanisms like spectral resolution and stellar rotation. Since one can generically think of these latter mechanisms as externalities imposed upon “perfect” stellar spectra (i.e., rotational broadening is not an intrinsic part of creating absorption lines) we expected the results we find using the Kurucz spectra to be applicable to our results using the BT-Settl spectra once we correct for the difference in wavelength spacing. As described later, we ultimately found this to be the case.

We used flux-normalized spectra in three broad bands: “optical” spectra from 4000Å to 6500Å, “red” spectra from 6500Å to 10000Å, and “near-infrared” spectra from 10000Å to 25000Å. The BT-Settl spectra used the Asplund et al. (2009) solar abundances, had solar metallicity with no alpha-enhancement, and were spaced 200K apart from 2600K to 7600K. The wavelength spacing in the spectra was
variable, with a finer spacing occurring around the absorption lines. On average the wavelength spacing was 0.05Å (R ≈ 100,000) in the optical, 0.05Å (R ≈ 165,000) in the red, and 0.2Å (R ≈ 88,000) in the NIR.

For the Kurucz spectra we used the odfnew versions of the Kurucz model atmospheres with no alpha-enhancement, and generated the spectra with v2.76 of Gray & Corbally (1994)’s spectrum code. For the optical and red spectra we used a fixed wavelength spacing of 0.001Å (R ≈ 6 × 10^6) and for the NIR spectra we used 0.005Å (R ≈ 3.5 × 10^6). In all the bands we set the microturbulent velocity to 1 km s^{-1} and left the macroturbulent velocity at zero. We considered the effect of macroturbulence separately. The Kurucz models covered effective temperatures from 4000K to 7500K with a spacing of 250K.

To investigate wavelength dependent features, we divided all our spectra into 100Å chunks. This partially isolates individual line groupings, like the Mg B triplet, so that we can test whether these groupings react to changes in the same way as the general continuum. Furthermore, splitting the spectra into 100Å chunks allowed us to mirror the actual analysis procedures of current multi-order RV surveys, and is representative of the amount of spectral information available in proposed multi-object surveys. For each chunk, we calculated the expected velocity uncertainty. After transforming each chunk from wavelength to velocity space, we normalized the chunks so that each had \( I_0 = 1 \) photon per m s^{-1}.

We now wish to see how the uncertainties of the 100Å chunks vary as a whole when the spectral resolution, stellar rotation, macroturbulence, effective temperature, and \( \log g \) change.
2.2.1. Spectral Resolution, $v \sin(i)$ and macroturbulence: Kurucz

Conceptually, the effect on a spectrum of changing spectral resolution ($R$), stellar rotation ($v \sin i$) or the macroturbulent velocity ($v_{mac}$) can be viewed as an externality imposed upon a “perfect” spectrum with $R = \infty$, $v \sin i = 0$ and $v_{mac} = 0$. Regardless of the underlying stellar parameters, the velocity uncertainty of a spectrum should vary with the same functional form for $R$, $v \sin i$ and $v_{mac}$. We therefore rewrite Equation (2.24) to

$$\sigma_V = \sigma_{V,0} \left[ \varphi_{rel}(R, v \sin i, v_{mac}) \right]^{3/2},$$  \hspace{1cm} (2.26)

where $\sigma_{V,0}$ is the velocity uncertainty of a “perfect” spectrum with $R = \infty$, and no rotation or macroturbulence. $\varphi_{rel}(R, v \sin i, v_{mac})$ is the increase in the average FWHM of the spectral lines caused by changes in $R$, $v \sin i$, and macroturbulence relative to that of the perfect spectrum. We defined $\varphi_{rel}$ such that $\varphi_{rel}(\infty, 0, 0) = 1$ and $\varphi_{rel}(0, \infty, 0) = \varphi_{rel}(0, 0, \infty) = \infty$. We assumed that the $R$, $v \sin i$ and $v_{mac}$ contributions to $\varphi_{rel}$ were separable, and we find that this is approximately true.

We first considered the $R$ dependence of $\varphi_{rel}$ using a Kurucz-based spectrum of a a 5750K, $\log(g) = 4.5$, $[\text{Fe/H}]=0.0$, Sun-like star split up into 100Å spectral chunks. We used a Kurucz-based spectrum – instead of a BT-Settl spectrum – because of the extremely fine wavelength spacing available with the Kurucz spectra. Our Kurucz spectrum had a wavelength spacing of 0.001Å, as compared to a median spacing of 0.05Å in the BT-Settl spectrum. Though the BT-Settl spacing gives a very well sampled spectrum for most applications, we will see that for our specific examination of line broadening mechanisms the 0.05Å spacing has a noticeable effect, by effectively setting a base spectral resolution of $R \approx 105,000$ (see Figure 2.5).
We assumed that the effect of instrumental spectral resolution could be approximated by convolving a spectrum with a Gaussian of FWHM equal to \( c/R \). For therefore convolved each 100Å chunk with Gaussians corresponding to a range of R-values, and calculated the velocity uncertainties. To check for any wavelength dependent effects, we normalized all of the chunks to have a relative velocity uncertainty of unity at \( R=50,000 \) and calculated the median velocity uncertainty as a function of resolution for all the chunks. Figure 2.4 shows the absolute values of the chunk velocity uncertainties for a 5750K star from \( R=10,000 \) to \( R=200,000 \) in the top row. We have separated the chunks into “optical”, “red” and “NIR”. In absolute terms there is a range in uncertainties across the chunks as a result of specific features in specific locations. However, here we are concerned with the overall behavior of the chunks as a function of R. If we normalize each chunk relative to its velocity uncertainty at \( R=50,000 \) (the middle row of Figure 2.4), the behaviors of the chunks begin to appear similar. One effect to note is that the dispersion of the chunks in the middle row panels increases as one moves to the red; that is, while the optical chunks all behave very similarly, the NIR chunks show more variation relative to each other. Specifically, many of the chunks in the NIR seem to be less affected by changing resolution than the optical chunks.

This occurs because the less affected chunks, which are mostly at longer wavelengths, have molecular bandheads as their primary spectral features. Since the lines that make up the bandheads are very closely spaced, they blend together into composite lines with a large widths even at high resolutions. This lessens the effect of increasing spectral resolution in resolving these features.

The red line in the middle row of Figure 2.4 is the median relative velocity uncertainty across all the chunks, and the bottom row of Figure 2.4 shows the fractional difference between all the chunks and this median. The fractional
difference across the entire optical and red wavelength ranges is rarely more than 20%, while the NIR chunks stay within about 30% of the calculated chunk median.

We repeated the above procedure to numerically calculate the median velocity uncertainty vs. R using all the Kurucz-derived spectra from 4000K to 7500K in steps of 250K. Across this temperature range the results for the optical, red, and NIR chunks were similar to our illustrative, 5750K, example. The velocity uncertainties of the chunks roughly stayed within 25% of the calculated median.

To determine the R dependence of $\varphi_{rel}$, we fit to the chunk median velocity uncertainties. Since we are interested in the relative change in velocity uncertainty for the chunks, we re-normalized each chunk median so that the velocity uncertainty at $R=3 \times 10^6$ was unity, such that $\varphi_{rel}(\infty, 0, 0) = 1$. To describe the average FWHM of the spectral lines in the chunks, we fit the chunk median as a Voigt profile with some inherent width $\Theta_0$, such that the relative increase in the average FWHM of the chunks increased as

$$\varphi_{rel} = \frac{0.5346\Theta_0 + \sqrt{0.2166\Theta_0^3 + \Theta_R^2}}{\Theta_0},$$

where $\Theta_R = c/R$. For each wavelength region, we fit the measured median velocity uncertainties as a function of resolution using Equations (2.26) and (2.27) and by finding the best-fit value of $\Theta_0$. Figure 2.4 shows the median chunk velocity uncertainty in the optical for a 5750K star as a function of $\Theta_R$ in black, overlaid by with our best-fit in green. The residuals to the best-fit are no more than 3% across the entire range of resolutions at 5750K.

As an illustration of the effects of the different wavelength spacing in the Kurucz spectra and the BT-Settl spectra, Figure 2.5 also shows a similarly calculated curve for the velocity uncertainty of a 5800K, $\log(g) = 4.5$ BT-Settl spectrum in
red. Note that the BT-Settl curve asymptotes to a significantly higher velocity uncertainty at high resolution (low $\Theta_R$). We interpret this as a result of the coarser wavelength spacing in the BT-Settl spectrum, which imposes a base “resolution” of $R \approx 5250\text{Å}/0.05\text{Å} = 105,000$ in the optical. Indeed, if we include a base of $R=105,000$ and recalculate how the velocity uncertainty in the BT-Settl spectrum scales with changing spectral resolution, the red BT-Settl curve in Figure 2.5 transforms to the orange line in Figure 2.5, which nearly matches the Kurucz-based results. As a further test we also generated a Kurucz spectrum with a 0.05Å wavelength spacing, and its curve is shown in blue. This coarser Kurucz spectrum nearly matches the BT-Settl results, which makes us confident that the difference between the 0.001Å Kurucz spectrum and the 0.05Å BT-Settl spectrum is primarily a result of the different wavelength spacings. Figure 2.5 also illustrates why we used the Kurucz-based spectra for our examination of line broadening mechanisms: the unbroadened BT-Settl spectra are not sampled finely enough to represent $\varphi_{\text{rel}}(\infty, 0, 0) = 1$.

We note that at low-$R$ in Figure 2.5 there is an offset between the Kurucz and BT-Settl spectra. We were not able to completely determine the cause of this offset, which is equal to about 0.15 dex at $R=3,000$ and about 0.05 dex at $R=30,000$. Partially, this difference is caused by the coarser wavelength spacing in the BT-Settl spectra, which gives them a shallower slope in Figure 2.5, since it takes a larger value of $R$ to dominate the inherent line widths and cause the velocity uncertainty to begin to scale as $\propto R^{-1.5}$.

It is also worth noting at this point that other authors’ (e.g., Hatzes & Cochran 1992; Bouchy et al. 2001; Bottom et al. 2013) numerical calculations of the dependence of how velocity uncertainty scales with spectral resolution find that at low-$R$ the uncertainty goes approximately as $\sigma_V \propto R^{-1}$ (Hatzes & Cochran 1992;
Bouchy et al. (2001) or \( \sigma_V \propto R^{-1.2} \) (Bottom et al. 2013). There are two connected points to consider here. First, as we have seen, we mathematically expect the velocity uncertainty to scale as \( \Theta^{-1.5} \), which is a combination of \( \Theta_R \) and the inherent line width \( \Theta_0 \). This means that considering only \( \Theta_R \), as these authors do, treats only a portion of the problem.

Second, as we shall see, for a Sun-like star in the optical, \( \Theta_R \) dominates the inherent line width (i.e., \( \Theta_R > 10\Theta_0 \)) only for resolutions less than 6,000. Thus Bouchy et al. (2001) and Bottom et al. (2013), who consider down to \( R=10,000 \), find \( \sigma_V \propto R^{-1} \) and \( \sigma_V \propto R^{-1.2} \), respectively, since they are largely fitting over the transition regime between \( \Theta_R \) and \( \Theta_0 \). Hatzes & Cochran (1992) directly measure the velocity uncertainty at \( R=2,500 \) and find \( \sigma_V \propto R^{-1} \), but we consider this result a poor fit to their measurements, since this line passes substantially underneath the \( R=2,500 \) point. Indeed, if we take the three points in Figure 1 of Hatzes & Cochran (1992) and fit them using our formalism, we recover \( \sigma_V \propto R^{-1.5} \).

If we plot the median relative velocity uncertainty for a 5750K, \( \log g = 4.5 \), star against the underlying Voigt width instead of spectral resolution (Figure 2.6), we can immediately see that the best fit to the calculated uncertainties goes as \( \sigma_V \propto \Theta^{3/2} \).

We did the resolution fitting over the temperature range covered by the Kurucz models – 4000K to 7500K – in the optical, red, and NIR. We found that the best-fit value of \( \Theta_0 \) decreased roughly linearly with temperature and was slightly different in each regime. Specifically, \( \Theta_0 \) goes as

\[
\Theta_0 = 8.36987 \text{ km s}^{-1} - 3.26466 \left( \frac{T_{\text{eff}}}{5800K} \right) \text{ km s}^{-1} \text{ for } 4000 \text{ to } 6500\text{Å} \\
\Theta_0 = 4.28146 \text{ km s}^{-1} + 0.54190 \left( \frac{T_{\text{eff}}}{5800K} \right) \text{ km s}^{-1} \text{ for } 6500 \text{ to } 10000\text{Å} \\
\Theta_0 = 8.18532 \text{ km s}^{-1} - 1.75910 \left( \frac{T_{\text{eff}}}{5800K} \right) \text{ km s}^{-1} \text{ for } 10000 \text{ to } 25000\text{Å}
\]
These equations are good to 3% in the optical and 5% in the red and NIR.

Figures 2.7, 2.8, and 2.9 show the residuals to our fits to the median chunk velocity uncertainties as a function of effective temperature and spectral resolution. On average, our fits are within 3% to 7% of the calculated median uncertainty. The largest differences occur in the optical at very low and very high temperatures for resolutions near 60,000. Our fits under-shoot at low temperatures and over-shoot at high temperatures, both by about 20%.

We next turned to the effect of stellar rotation on the velocity uncertainty. We presumed that the effect of changing $v_{rot}$ is similar to spectral resolution $R$, in that convolving a spectrum with a rotation kernel is similar to convolving a spectrum with a Gaussian, and that we could describe the change in the line width similarly as

$$\varphi_{rel} = \frac{0.5346\Theta_0 + \sqrt{0.2166\Theta_0^2 + \Theta_{rot}^2}}{\Theta_0}. \quad (2.29)$$

Here $\Theta_{rot}$ is the FWHM of the rotation kernel. The assumption that convolving a spectrum with the rotation kernel is similar to convolving a spectrum with a Gaussian is important to keep in mind. As is shown in Figure 2.3, the shape of the rotation kernel is definitely non-Gaussian. Therefore, we will need to approximate the broadening caused by the rotation kernel by an appropriate Gaussian for the purposes of Equation (2.29).

To do so, we set Equations (2.13) and (2.23) equal to each other and solve for $\Theta_G$. Thus,

$$\Theta_{G, equiv} \approx \left(\frac{0.347 + 0.146\epsilon}{0.69}\right)^{2/3} \Theta_{rot}. \quad (2.30)$$
As we noted in Section 2.1, the velocity uncertainty caused by the rotational broadening at a particular rotation velocity is only weakly dependent on the precise value of $\epsilon$. The difference in the proportionality constant relating $\Theta_{rot}^{3/2}$ and $\sigma_V$ varies by only 5% from minimum to maximum. We will therefore take the average value, which occurs at $\epsilon = 0.5$, for all of our results. This makes $\Theta_{G,equiv} = 0.72 \Theta_{rot}$.

Having determined $\Theta_{G,equiv}$ for rotation, we know wish to know how velocity uncertainty scales with stellar rotation velocity. In a manner similar to how we approached spectral resolution, we calculated how the velocity uncertainty in the wavelength chunks changed as $v_{rot}$ went from 0 to 25 km/s using Kurucz-based spectra. We used the AVSINI routine packaged with the SPECTRUM code to apply the rotation kernel to our spectra using with $\epsilon = 0.5$. As one can see in Figure 2.10, changing $v_{rot}$ is similar to changing spectral resolution in that it largely effects all of the wavelength chunks in the same way. The middle row of Figure 2.10 shows the relative change in the velocity uncertainty normalized to $v_{rot} = 5$ km s$^{-1}$ for a 5750K, $\log(g) = 4.5$, [Fe/H]=0.0 star in our three wavelength regimes. Similar to our approach to fitting the effect of changing spectral resolution, we also calculated median values for our entire temperature range.

We then fit the chunk averages in the same manner as for spectral resolution. In doing so, we found that rotational velocity affects the relative velocity uncertainty of a spectrum in exactly the same way as does spectral resolution. That is, when $\Theta_{G,equiv} = \Theta_R$ the velocity uncertainty is exactly the same across our entire temperature range.

In addition to rotation, we also considered the effect of macroturbulence in the stellar atmosphere. For simplicity we assumed simple isotropic macroturbulence, so that the effect of macroturbulence with velocity $v_{mac}$ is the same as convolving a
spectrum with a normalized Gaussian with standard deviation $v_{mac}/2$. Under this assumption the FWHM of the macroturbulence kernel is then simply

$$\Theta_{mac} = 2\sqrt{2\ln 2} \frac{v_{mac}}{2} \approx 1.18 v_{mac}.$$  \hspace{1cm} (2.31)

Note that in reality, the effects of rotation and macroturbulence are difficult to observationally separate when $v_{rot} \approx v_{mac}$ (e.g., Valenti & Fischer 2005). This is partially a result of the fact that the effect of macroturbulence is not isotropic, and partially because macroturbulent and rotational broadening are observed as a disk-integrated broadening profile.

Typical macroturbulent velocities for field dwarfs are on the order of a few km s$^{-1}$ (Valenti & Fischer 2005; Gray 2008; Bruntt et al. 2010), with mid F-dwarfs at about 6 km s$^{-1}$ and decreasing linearly with spectral type to about 1.5 km s$^{-1}$ for an early K-dwarf. We used the empirical relation for $v_{mac}$ as a function of temperature determined by Bruntt et al. (2010), which we list along with other stellar properties in Section 4.

We therefore will use the same results we had for spectral resolution (Voigt line profiles, temperature dependence) and apply it to rotation and macroturbulence. Putting this all together, we may rewrite Equation (2.26) as

$$\varphi_{rel} = \left( \frac{0.5346\Theta_0(T_{eff}) + \sqrt{0.2166\Theta_0^2 + \Theta_R^2 + 0.518\Theta_{rot}^2 + \Theta_{mac}^2}}{\Theta_0(T_{eff})} \right)^{3/2}. \hspace{1cm} (2.32)$$

2.2.2. Temperature: BT-Settl

Temperature affects both the width and the number of lines usable for radial velocity measurements in a spectrum, and unlike the line broadening mechanisms considered in above, stellar temperature should be considered an intrinsic part of
line generation. Without a detailed treatment of how spectral lines are created, it is therefore difficult to arrive at a physically motivated analytic expression for how the velocity uncertainty in a spectrum changes along with effective temperature. While the thermal velocity width of the lines will scale simply as the square-root of the effective temperature, the pressure of the atmospheric layer where these lines are generated will change as well. These two competing effects – temperature width and pressure width – are not easily separable (Figure 2.11). In addition, the number of lines in a spectrum depends upon a host of factors such as opacities, atomic energy levels, and ionization equilibria that also provide no simple scaling with temperature.

We therefore determined a purely numerical and descriptive scaling for how the relative velocity uncertainty in a spectrum changes with effective temperature. Due to their availability over a greater range of temperatures we used the BT-Settl models for this fitting. We again used 100Å chunks sliced out of spectra with effective temperatures of 2600K to 7600K and \( \log g = 4.5 \) in the three wavelength ranges we are considering.

The top panels of Figure 2.12 show the effect of changing temperature on the velocity uncertainty for each of the chunks. It is immediately apparent from Figure 2.12 that changing the effective temperature acts in a much less self-similar way across the chunks as compared to the external line-broadening mechanisms we considered previously. Not surprisingly, while the behavior of the optical chunks is roughly self-similar (left side of the middle row of Figure 2.12), the NIR chunks (right side of the middle row of Figure 2.12) show considerable differences. This is largely caused by the different line generation mechanisms at optical and NIR wavelengths. While the optical is mostly populated by atomic lines that change strength relatively slowly with effective temperature, the NIR chunks possess more molecular lines that have a sharp temperature dependence. For example, while a
4000K, log($g$) = 4.5, spectra from 24500Å to 24600Å is a forest of CO molecular lines, that same 100Å chunk in a 7000K, log($g$) = 4.5, spectrum has only one atomic Fe and one atomic Mg line as its major spectral features.

This also illustrates the vital importance of choosing the appropriate wavelength range in the NIR when designing an RV survey. For example, our illustrative 24500Å to 24600Å chunk is a perfect example of a wavelength region that would be a reasonable choice for an RV survey focusing on K and M stars, but it would be a bad choice for a NIR survey that would observe FGK dwarfs. We consider the choice of wavelength range in more detail in the discussion section.

To generally describe the behavior of the wavelength chunks as a function of temperature, we roughly approximated the chunk medians by taking a least squares polynomial fit to the three wavelength regions, which yielded

$$
\begin{align*}
    f(T_{\text{eff}})_{\text{Opt}} &= -2.130 + 8.493 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right) - 9.582 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right)^2 + 4.239 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right)^3 \\
    f(T_{\text{eff}})_{\text{Red}} &= -2.799 + 11.04 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right) - 12.86 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right)^2 + 5.62 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right)^3 \\
    f(T_{\text{eff}})_{\text{NIR}} &= -3.015 + 11.44 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right) - 12.43 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right)^2 + 5.018 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right)^3
\end{align*}
$$

The bottom panels of Figure 2.12 show the fractional difference between each chunk’s relative velocity uncertainties and the chunk medians. The relations in Equation (2.33) replicate the chunk medians to with 4%. Relative to the medians, while the optical chunks are relatively coherent, one can see the moving towards redder wavelengths causes the chunks to behave much more chaotically, for the reasons outlined above.

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2.2.3. Surface Gravity: BT-Settl

Surface gravity, through pressure broadening effects, can change both the depth and width of spectral lines, and thus the RV velocity uncertainty of a spectrum. The exact response of lines to changes in surface gravity is dependent upon several factors. For example, a decrease in gravity can cause a weak line to either gain or lose strength depending upon the ionization state of the atoms. We therefore approached surface gravity effects in a manner similar to effective temperature, by determining a purely numerical and descriptive scaling of velocity uncertainty.

We took BT-Settl spectra from 2600K to 7600K with surface gravities of \( \log(g) = 4.0 \) and \( \log(g) = 5.0 \) and calculated the velocity uncertainties of each relative to \( \log(g) = 4.5 \) (Figure 2.13). We again split each spectrum up into 100Å chunks and examined all three of our wavelength bands. The results in all three bands were roughly the same, with the greatest relative difference occurring around 4500K and the smallest differences happening towards the ends of the temperature range.

Since the chunk medians in all three bands are nearly symmetric about unity, we decided to describe the change in velocity error as a linear function of surface gravity relative to what the velocity uncertainty would be for \( \log g = 4.5 \). Specifically,

\[
f(\log g) = m \cdot \Delta g + 1
\]  

with \( \Delta g = \log g - 4.5 \). The slope \( m \) depends upon the effective temperature and the band observed,

\[
m_{\text{Opt}} = 0.537477 - 1.96120 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right) + 1.14867 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right)^2
\]

\[
m_{\text{Red}} = 0.745084 - 2.63396 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right) + 1.55381 \left( \frac{T_{\text{eff}}}{5800\text{K}} \right)^2
\]
\[ m_{\text{NIR}} = 1.060520 - 3.49375 \left( \frac{T_{\text{eff}}}{5800K} \right) + 1.99397 \left( \frac{T_{\text{eff}}}{5800K} \right)^2, \]

Figure 2.14 shows the measured values of \( m \) across the temperature range, overplotted by the fits from Equation (2.35). Note that these results are only accurate for surface gravities between \( 4.0 \leq \log g \leq 5.0 \).

Since the the extremes of our considered temperature range – where we expect the surface gravity to be farthest away from 4.5 – show the smallest effect due to changing gravity, we expect surface gravity to play a relatively minor role in determining the velocity error of a spectrum. Indeed, when we include how we expect surface gravity to vary with stellar mass (Equation [2.42]) we find that changing surface gravity is never more than a 10% effect, and is frequently less.

2.2.4. Metallicity: Kurucz

Finally, we consider the effect of differing metallicity, which we parameterized as [Fe/H], on velocity uncertainties. Similarly to the effect of changing effective temperature and changing surface gravity, we expected that changing metallicity would alter the velocity uncertainty in a spectrum in a way that is difficult to capture analytically from \textit{a priori} arguments. We therefore confined ourselves to a numeric, descriptive, scaling for the effect of changing [Fe/H].

To do so, we used three sets of Kurucz-model spectra at fixed effective temperatures of 5000K, 5750K, and 6500K. All three sets had a fixed surface gravity of \( \log g = 4.5 \), no alpha-enhancement, and metallicities of [Fe/H]=[-2.0,-1.5,-1.0,-0.5,0.0,0.5]. We used Kurucz based spectra, instead of BT-Settl spectra, due to the availability of Kurucz models with a wide range of [Fe/H] values. The BT-Settl spectra, while more physically motivated in terms of what metallicities are
available for a given effective temperature and surface gravity, do not provide an arbitrary range of metallicities with no alpha-enhancement. As a result, our results are confined to the optical wavelengths, from 4000Å to 6500Å, where the Kurucz line-lists are robust.

Figure 2.15 shows the results of varying the metallicity on the velocity uncertainties for a 5750K, log $g = 4.5$ star in the optical. The results for the 5000K and 6500K spectra were similar, differing by at most 15% at the low metallicity end. As one would expect, the velocity uncertainties for all of the chunks increases as $\text{[Fe/H]}$ decreases; a result of the absorption features in the spectra becoming weaker and less numerous. Some of the specific 100Å chunks are strongly effected by this, as their major lines are drastically diminished at $\text{[Fe/H]}=-2.0$.

We fit to the red chunk median in the middle panel of Figure 2.15 to find the change in velocity uncertainty relative to $\text{[Fe/H]}=0.0$. This gave

$$f(\text{[Fe/H]}) = 10^{-0.27 \text{[Fe/H]}}.$$  \hspace{1cm} (2.36)

For all three effective temperatures, this result is accurate to 15% over the range of $\text{[Fe/H]}$ values we considered, with the highest difference occurring at $\text{[Fe/H]}=-2.0$.

2.2.5. Final Expressions

Putting together all of our results from the preceding analysis, we arrive at a semi-analytic expression for the velocity uncertainty using one, or several, of the 100Å chunks as a function of $R$, $v \sin i$, $T_{\text{eff}}$, log $g$, and $\text{[Fe/H]}$:

$$\sigma_V = \left( \frac{0.5346 \Theta_0(T_{\text{eff}}) + \sqrt{0.2166 \Theta_0^2 + \Theta_R^2 + 0.518 \Theta_{\text{rot}}^2 + \Theta_{\text{mac}}^2}}{\Theta_0(T_{\text{eff}})} \right)^{3/2}$$  \hspace{1cm} (2.37)
\[
\frac{1}{\sqrt{\sum I_{0,i} / \sigma_{V,i}^2}} f(T_{\text{eff}}) f(\log g) f([\text{Fe/H}]).
\]

Where we have \(\Theta_0(T_{\text{eff}})\) per Equation (2.28), \(f(T_{\text{eff}})\) per Equation (2.33), \(f(\log g)\) per Equation (2.34) and \(f([\text{Fe/H}])\) per Equation (2.36). The leading summation term is a sum over all of the 100Å chunks observed, with \(\sigma_{V,i}\) as the velocity uncertainty of the individual chunks for \(R=\infty, v_{\text{rot}}=0, v_{\text{mac}}=0\), and for a continuum level of 1 photon per velocity element. The true continuum level in each chunk is incorporated via \(\sqrt{I_{0,i}}\). Note that we are defining the continuum here in velocity-space, and not pixel-space as is conventional in the observational literature.

Table 2.1 lists values of \(\sigma_{V,i}\), in km s\(^{-1}\), for 100Å chunks between 4000Å and 25000Å for temperatures between 2600K and 7600K and a constant \(\log g=4.5\). This information is also displayed graphically for our three wavelength regions, in Figures 2.16, 2.17, and 2.18. Table 1 was calculated using the BT-Settl spectra, which used the Asplund et al. (2009) solar abundances and had solar metallicity with no alpha-enhancement. The values in Table 1 are normalized for a continuum level of 1 photon per km s\(^{-1}\). Spectroscopic observations usually quote their S/N per pixel \((SN_{\text{pix},i})\), which can be converted into the appropriate units for \(I_{0,i}\) by taking

\[
I_{0,i} = \frac{SN_{\text{pix},i}^2 n_{\text{pix},i}}{\Delta V_{\text{chunk},i}},
\]

where \(\Delta V_{\text{chunk},i}\) is the velocity span of the wavelength chunk, and \(n_{\text{pix},i}\) is the number of pixels on the detector used to observe the chunk.

### 2.3. Stellar Properties

Having so far considered the dependence of velocity uncertainties on stellar parameters independently of what is physically reasonable, we know wish to apply
this formalism towards real stars. Our intent is to determine what is the dominant source of the velocity uncertainty for main-sequence stars.

We have chosen to focus on main sequence stars so that we may use stellar mass as a single variable to then calculate all of the stellar properties that determine velocity uncertainty. As above, the stellar properties we are interested in are rotation velocity, macroturbulence, effective temperature, and surface gravity. Additionally, we also wish to know the overall bolometric luminosities and radii of the stars. The first allows us to calculate the continuum level of the spectra, while the second will be necessary to estimate rotation velocities from the rotation periods predicted from stellar gyrochronology relations. To that end, we fit relations for effective temperature, luminosity, radius, and surface gravity from the measurements listed in Table 1 of Torres et al. (2010) for stars cooler than 7600K:

\[ T_{\text{eff}} = 5603K \left( \frac{M_*}{M_\odot} \right)^{0.41} \approx 5800K \left( \frac{M_*}{M_\odot} \right)^{0.5}, \]  

(2.39)

\[ L_* = 1.06L_\odot \left( \frac{M_*}{M_\odot} \right)^{4.48} \approx 1.0L_\odot \left( \frac{M_*}{M_\odot} \right)^{4.5}, \]  

(2.40)

\[ R_* = 1.12R_\odot \left( \frac{M_*}{M_\odot} \right)^{1.12}, \]  

(2.41)

and

\[ \log(g) = 4.96 - 0.58 \left( \frac{M_*}{M_\odot} \right) \approx 5 - 0.5 \left( \frac{M_*}{M_\odot} \right). \]  

(2.42)

For all the functions except the radius relation we have also given rough approximations, which we use when simplifying our numeric results. We note that these relations are only roughly consistent with each other, a result of our collapsing
stars of different main sequence ages and metallicities onto single relations. Unlike temperature, luminosity, radius, and surface gravity, the rotation velocity does not display a simple scaling with stellar mass, and so we consider it separately and in more detail in the section.

For the macroturbulent velocity, we used the empirical relation for $v_{mac}$ as function of effective temperature determined by Bruntt et al. (2010),

$$v_{mac} = 2.3 \text{ km s}^{-1} + 2.9 \times 10^{-3} \left( \frac{T_{\text{eff}} - 5700 \text{ K}}{\text{K/km s}^{-1}} \right)$$

$$+ 5.9 \times 10^{-7} \left( \frac{T_{\text{eff}} - 5700 \text{ K}}{\text{K/km s}^{-1}} \right)^2 .$$

Bruntt et al. (2010) make the point that this relation is only valid for stars with $\log(g) > 4.0$ and between 5000K to 6500K. We use Equation (2.43) to estimate the macroturbulent velocity for stars from 5000K to 7600K, and set a constant macroturbulent velocity of 0.51 km s$^{-1}$ (the value of Equation (2.43) at 5000K) for all stars cooler than 5000K. Above 6500K, thus should not introduce large errors into our results, because the rotational velocities of these hot stars are at least five times larger than the calculated macroturbulent velocities.

In addition to these stellar properties, there is also RV “jitter” in stars, which causes additional uncertainty in precision velocity measurements. Astrophysical jitter is generally a result of either star-spots on the stellar photosphere, or short-period solar-like asteroseismic oscillations. For the latter, many of the existing RV surveys mitigated the effect of asteroseismic jitter by integrating on a star for longer than the oscillation periods, which are typically about 5 minutes. Unfortunately, the jitter caused by star-spots has no comparable solution, other than avoiding stars with high activity indices. Typical jitter values for main-sequence stars cooler than 6300K and with average activity levels are measured to be around 3 to 4 m s$^{-1}$.
As has been noted by all of these authors, this undoubtedly includes jitter from astrophysical and instrumental sources. Indeed, Isaacson & Fischer (2010) find that their measured jitter in K-dwarfs is completely uncorrelated with stellar activity; they therefore conclude that the jitter displayed by these stars is likely a result of instrumental effects.

Since velocity jitter is caused by additional astrophysical and instrumental sources, the appropriate way to incorporate it into our formalism is to add the jitter value for a given star in quadrature to the Poisson velocity uncertainty calculated using Equation (2.37). RV jitter is therefore not a component we need to consider in determining the dominant sources of Poisson velocity uncertainty in main-sequence stars. We thus leave it aside for now, other than to note the importance of jitter in using our results to fully model an RV survey.

2.3.1. Stellar rotation

Since stellar rotation strongly broadens stellar lines, we undertook a detailed examination of the true rotational speeds, \( v_{\text{rot}} \), of stars within our mass range. In general, stars with outer convective envelopes, from \( 0.4M_\odot \) to the Kraft Break (Kraft 1970) at \( 1.1M_\odot \) will magnetically brake over the first billion years of their lives and coalesce onto a single mass-rotation-age relation. This is the basis of stellar gyrochronology. Stars less massive than \( 0.4M_\odot \) generally do not brake effectively, and so do not evolve onto a single mass-rotation-age relationship. Similarly, stars more massive than the Kraft Break mass of \( 1.1M_\odot \) have very thin outer convective envelope and retain almost all of their primordial angular momentum. These stars will slightly lengthen their rotational periods due to the gradual increase of their
radii on the main sequence, but this is change is on the order of 2%. These heavier stars also do not, therefore, evolve onto a single mass-rotation-age relationship.

For stars between $0.4M_\odot$ and $1.1M_\odot$ we used the modified Kawaler spin down model developed by Epstein & Pinsonneault (2012) to determine the rotation periods of stars at a certain mass and age. We then used the mass-radius relation in Equation (2.41) to convert the rotation periods into rotation speeds. For stars older than 0.5 Gyr this spin-down model predicts a tight mass-rotation-age relation down to $0.4M_\odot$, with more scatter as one goes to younger ages and lower mass. We linearly interpolated between the available model grid points in mass and age and took the median rotation period as the rotational period of all the stars with at that mass and age.

For stars less massive than $0.4M_\odot$, we treated $v_{\text{rot}}$ as a distribution, with velocities uniformly distributed in velocity between zero and some upper bound $v_{\text{max}}$. This roughly replicates the distribution of M-dwarf rotation velocities observed by Reiners et al. (2012). The upper bound was set equal to the Kawaler rotation velocity at the high mass end, $v_{\text{rot}}(0.4M_\odot)$, which is an age dependent quantity, and increased linearly with mass through 10 km s$^{-1}$ at $0.2M_\odot$. Thus for stars with $M_\ast < 0.4M_\odot$:

\[
v_{\text{rot}}(M_\ast) = [10 \text{ km s}^{-1} - v_{\text{rot}}(0.4M_\odot)] \frac{0.4M_\odot - M_\ast}{0.2M_\odot} + v_{\text{rot}}(0.4M_\odot).
\] (2.44)

We treated stars heavier than $1.1M_\odot$ in a similar manner. Based on the observations and discussion in Gaige (1993) and Reiners & Schmitt (2003), we treated the $v\sin i$ distribution of stars heavier than $1.1M_\odot$ as uniformly distributed in velocity between zero and a mass dependent upper bound. For the massive stars, this upper bound was set to the Kawaler rotation velocity at $1.1M_\odot$, $v_{\text{rot}}(1.1M_\odot)$,
and the bound increased linearly with mass through 100 km s\(^{-1}\) at \(1.5M_\odot\). Therefore for massive stars with \(M_\star > 1.1M_\odot\) we have

\[
v_{\text{max}}(M_\star) = [100 \text{ km s}^{-1} - \nu_{\text{rot}}(1.1M_\odot)] \frac{M_\star - 1.1M_\odot}{0.4M_\odot} + \nu_{\text{rot}}(1.1M_\odot). \tag{2.45}
\]

### 2.4. Dominant Sources of Velocity Uncertainty

We now wish to determine the dominant astrophysical sources of velocity uncertainty in an RV measurement of main sequence field stars. To do so, we calculated the change in the velocity uncertainty caused by the \(I_{0,i}, \Theta, T_{\text{eff}}, \log g,\) and [Fe/H] terms in Equation (2.37). This gives us the relative change in velocity uncertainty as a function of stellar properties, independent of the specific wavelength chunk (or chunks) chosen. The specific wavelength information is provided in Equation (2.37) by \(\sigma_{V,i}\), and serves to simply set the appropriate absolute value of the uncertainty.

We begin by calculating how the relative velocity uncertainty scales as a function of stellar mass in the wavelength ranges we consider. We used the relations in Section 4 to determine the effective temperature, surface gravity, and luminosity as a function of mass. For the luminosity, we included the effect of overall changes in the bolometric luminosity, normalized to a 5800K star, by Equation (2.40), and the effect on the observed luminosity caused by the shifting of the blackbody emission across the specific wavelength range being observed. We refer to this as the “blackbody effect.” This blackbody term means that the exact results will still depend on the specific wavelength chunk used to calculate the relative uncertainties. For each of our three bands (optical, red, and NIR), we used a 100Å chunk in the middle of the wavelength range to calculate the blackbody effect. In our tests, using chunks at the extreme of our wavelength bands changes the calculated uncertainties by 5% or less, and does not affect our ultimate conclusions.
As a fiducial example, we set \( R = 60,000 \), \([\text{Fe/H}] = 0.0\), and the stellar age to 2.0 Gyr. Recall that the age will set the rotation velocity of stars between 0.4 \( M_\odot \) and 1.1 \( M_\odot \), with younger stars rotating more rapidly. We chose 2.0 Gyr so as to be broadly representative of a typical field FGK dwarf in the Solar neighborhood (Nordström et al. 2004).

Figures 2.19, 2.20, and 2.21 show the expected uncertainty as a function of mass for the optical, red, and NIR as the solid red line. To illustrate how the overall uncertainty is determined by the underlying stellar parameters, these figures also show how the uncertainty changes if we fix all but one of the physical processes that affect the velocity uncertainty and depend on stellar mass. These parameters are changes to line strengths and numbers due to effective temperature as per Equation (2.33) (\( T \)), changes in the the overall bolometric luminosity (\( L \)), luminosity changes from the blackbody peak shifting relative to the spectral bandpass (\( BB \)), stellar rotation (\( VS \)), macroturbulence (\( VM \)), surface gravity changes (\( G \)), and spectral resolution (\( R \)).

There are two things to immediately note. First, below 0.4 \( M_\odot \) and above 1.1 \( M_\odot \) the \( VS \) curve is for the maximum observed \( v \sin i \) at each mass. Second, the changing effect of spectral resolution as a function of mass is a result of the average inherent line widths varying with effective temperature, per Equation (2.28). Larger inherent line widths (e.g., at lower temperatures in the optical) cause finite spectral resolution to have a smaller effect on the velocity uncertainties.

In general, these three figures demonstrate that there are two general regimes for the RV errors of F-M main sequence stars: luminosity and temperature dominated uncertainties below 1.1 \( M_\odot \) when stellar rotation is low, and rotation dominated uncertainties for stars above that mass. In particular, the rapid increase in the
average rotation for more massive stars means that in all three bands the maximum velocity uncertainty raises sharply in this regime, becoming an order of magnitude larger than it would be for a Sun-like star at just \( \approx 1.25M_\odot \).

Below \( 1.1M_\odot \), on the other hand, the three wavelength regions behave differently. This primarily due to the changing effect of the blackbody peak shifting relative to the observed bandpass (the BB line in all three figures). In the optical (Figure 2.19) the blackbody effect causes larger uncertainties for lower mass stars as the peak shifts into the red, particularly below \( 0.6M_\odot \). In the red (Figure 2.20), the blackbody term is nearly constant, as the peak is moving through this wavelength regime. By the time we reach the NIR (Figure 2.21), the blackbody term finally begins to reduce the velocity uncertainties of lower mass stars relative to solar-mass stars.

To illustrate this different behavior in the different bandpasses, and to understand the dominant source of velocity uncertainty in a conceptually straightforward manner, we derived simple approximations to how the velocity uncertainty of low-mass \( (M_* < 1.1M_\odot) \) stars scaled with stellar mass. In the optical, the blackbody effect nearly cancels the effect of changing effective temperature, leaving the bolometric luminosity as the dominant source of uncertainty. In the red, where the blackbody effect is nearly constant, we approximate the velocity uncertainty as a combination of luminosity and temperature effects. Finally, in the NIR, we must also include the blackbody effect into our approximation using luminosity and temperature. We find that in the NIR the BB term is well fit by a simple linear relation with stellar mass, such that \( \sigma \propto 0.4(M_*/M_\odot) + 0.6. \)
For our approximations for the relative uncertainty scaling below $1.1M_\odot$ we therefore have:

\[
\sigma_{V,\text{Opt}} \propto \frac{1}{\sqrt{L_*/L_\odot}} \quad \text{for } M_* < 1.1M_\odot
\]  

\[
\sigma_{V,\text{Red}} \propto \frac{f_{\text{Red}(T_{\text{eff}})}}{\sqrt{L_*/L_\odot}} \quad \text{for } M_* < 1.1M_\odot
\]  

\[
\sigma_{V,\text{NIR}} \propto \frac{[0.4(M_*/M_\odot) + 0.6]f_{\text{NIR}(T_{\text{eff}})}}{\sqrt{L_*/L_\odot}} \quad \text{for } M_* < 1.1M_\odot.
\]  

Above $1.1M_\odot$, where stellar rotation dominates, we would approximate the velocity error as $\sigma_V \propto \infty$, as the rapidly increasing rotation velocities widen the lines to unusability for detecting all but the most massive companions. Note that, in detail, since values of $v\sin i$ are observed to be uniformly distributed in this high-mass regime, some stars will have a $v\sin i$ low enough that rotation will not dominate the velocity uncertainties.

The red lines in Figures 15, 16, and 17 show these approximations. Note that we have normalized each to match the directly calculated velocity uncertainty at $0.5M_\odot$. This serves to account for the added uncertainty arising from our assumed value for spectral resolution, $R$, and the non-zero rotation of the low-mass stars. One can see the approximate velocity uncertainty agrees well with the calculated velocity uncertainty for lower mass stars, but the two diverge as mass increases above $M_* \approx 0.8M_\odot$. This is a result of our ignoring the effects of stellar rotation in making our approximations.
2.5. Discussion

In addition to the above approximations for the scaling of velocity uncertainty as a function of stellar mass, there are three general points regarding RV surveys that are interesting to consider.

First, velocity precision does not scale linearly with $v \sin i$ for high rotation, or as $R^{-1}$ for low resolution, as is frequently claimed in the literature. Instead, as we have shown, it goes appropriately as $(v \sin i)^{3/2}$, or $R^{-3/2}$. This arises because there is continuum emission beneath absorption lines, which causes the measured velocity uncertainty to scale as $\Theta^{3/2}$, and not as simply $\Theta$, as would be the case if there were no continuum. Additionally, it is also important to remember that the velocity uncertainty is set by the overall line width, which is a combination of the inherent line width and the effects from broadening mechanisms like $v \sin i$ and $R$. By considering the total resulting line width, we are thus able to fit for and describe the velocity precision as a function of $v \sin i$ and $R$ over the entire range of these two parameters, and not just at the high $v \sin i$ or low $R$ limits.

Second, Figure 2.4 illustrates how arbitrarily increasing the spectral resolution if an instrument does not give arbitrarily low velocity uncertainties. Specifically, there is a “knee” in the uncertainty curves around $R=100,000$, after which increased spectral resolution has a much diminished effect. This occurs because once a spectrograph reaches $R=100,000$, the instrument is able to resolve almost all the lines in a stellar spectrum; further increases to $R$ therefore do not provide substantially more information. For multi-object stellar RV surveys, Figures 2.19, 2.20, and 2.21 also show that there is a limited utility to spectral resolutions above $R=60,000$. One can see in these three figures that the effect of having $R=60,000$ (the
horizontal dashed-line labeled “R”) is the dominant source of velocity uncertainty for an extremely small range of masses.

Instead, the dominant error source in radial velocity measurements will either be caused by luminosity ($M_* \ll 0.8M_\odot$) or rotational velocity ($0.8M_\odot \ll M_*$). This means that relatively small detector arrays can be used effectively for multi-object RV surveys: 1024 pixels along the spectral dispersion axis would allow for a 100Å spectral chunk to be imaged at $R=60,000$. As an example, if we ignore systematic and instrumental uncertainties, if such a survey observed a Sun-like star from 5100-5200Å with $S/N=200$ per pixel, we predict that the Poisson velocity uncertainty on an individual observation would be $6\; m\; s^{-1}$.

More generally, we note that stellar rotation will limit the utility of increased spectral resolution – even in slowly rotating field stars. For $R=60,000$, for example, the effective line broadening caused by limited spectral resolution is equal to the amount of line broadening caused by 4.25 km s$^{-1}$ of stellar rotation. For stars rotating faster than this, further increasing $R$ will thus provide a small change in the measured velocity uncertainty. Generically, the limiting spectral resolution for a given stellar rotation velocity will be

$$\frac{R_{\text{lim}}}{60K} = \frac{4.25 \; \text{km s}^{-1}}{v_{\text{rot}}}.$$  \hspace{1cm} (2.47)

$R=60,000$ or $v_{\text{rot}} = 4.25 \; \text{km s}^{-1}$ is also the point at which $\Theta_R$ or $\Theta_{\text{rot}}$ are both nearly the same as $\Theta_0$ for a Sun-like star.

Third, Figures 2.16, 2.17, and 2.18 show the importance of choosing the appropriate wavelength for an RV survey, particularly if one cannot cover a wide range of wavelengths. To investigate this in more detail, we collapsed the uncertainty values behind Figures 2.16, 2.17, 2.18, and Table 1 along the temperature axis
to see what are the best locations for observations. We divided the temperature range into M-stars (2600K to 4000K), K-stars (4000K to 5200K), G-stars (5200K to 6000K), and F-stars (6000K to 7600K) and took the median uncertainty values across these ranges for each 100Å wavelength chunk in Table 1. Figure 2.22 shows the results, visualized in four different ways. First, the upper left panel shows the median uncertainties of each 100Å chunk for the four spectral types (labeled “Raw (linear)”). The lower left panel shows these same chunk medians, but now we have normalized them according to the fraction of the overall blackbody luminosity that each chunk occupies (labeled “BB Normalized (linear)”). In terms of Figures 2.19, 2.20, and 2.21, this factors in the “BB” line. The two right panels in Figure 2.22 are similarly “Raw” or “BB Normalized,” but we have now combined the chunks into equally wide logarithmic bins, rather than plotting them linearly as before. This is meant to replicate the true observing mode of spectrographs: at fixed resolution an instrument can image proportionally more of a spectrum at proportionally longer wavelengths. The lower right panel of Figure 2.22 therefore most directly informs the selection of a proper observing wavelength.

In the optical, spectra longwards of 5500Å provide relatively little information for F-stars, and provide an inferior amount of velocity information for later spectral types as compared to most of the shorter wavelengths. For a survey covering all spectral classes with a limited wavelength range, the best wavelengths to look at in the optical would be from 5000Å to 5200Å, which contains the Mg b triplet, and has been noted previously (Latham 1985). Shorter wavelengths than this, while advantageous for FG stars, provide little gain for either K- or M-stars.

In the “red,” between 6500Å and 10000Å, the best regions for a general exoplanet survey are in the I or i’ bands around 7000Å. Moving further out into the red, to z’, provides less velocity information at almost all effective temperatures.
Indeed, as shown in the bottom right panel of Figure 13, observing from 7000Å to 8000Å in I or i′ has the lowest base uncertainty for observations of M-dwarfs over the entire wavelength range.

To investigate this in more detail, Figure 2.23 shows four of the temperatures that make up the M-dwarfs in Figure 2.22 displayed in a similar manner. One can see that between 3800K (an M0) and 2600K (roughly an M7) the shape of the raw uncertainties as a function of wavelength are roughly self-similar, but the flux normalized uncertainties flatten out as one moves to cooler temperatures. This is, of course, a result of the peak of the stellar SED moving towards longer wavelengths as the temperature decreases. Nevertheless, although the peak of a 2600K blackbody has moved out to 11,000Å, one can clearly see in the lower panels of Figure 2.23 that the most efficient wavelength region to observe all of the M-dwarf temperatures we consider is shortwards of 10,000Å, between 6000Å and 9000Å. The M-dwarf spectra at these wavelengths have approximately 10 times the velocity information (i.e., 1/10 the uncertainty) as compared to the NIR, and thus remain the most efficient observing location even after accounting for the relative amount of stellar emission. Bottom et al. (2013) arrived at a similar conclusion for the best wavelength regime to observe M-dwarfs. This illustrates the importance of considering the velocity information available in a wavelength region, and not just the shape of the stellar SED, when designing an RV survey targeted at M-dwarfs.

If we now consider only the NIR, it is interesting to compare Figures 2.18 and 2.22 against the available transmission windows in the atmosphere. We can roughly approximate these windows by considering the NIR photometry bands: J (11000Å to 13500Å), H (15000Å to 17000Å), and K (20000Å to 23000Å). For a general RV survey of all spectral types, the optimum observing band would be in J, since the uncertainty for FGK spectral types steadily increases towards longer wavelengths.
For a survey targeting only later spectral types, the decision is less clear cut. H-band has the lowest uncertainty for observing M-dwarfs (down 10% compared to J and down 40% compared to K). At the same time, if one wished to include K-dwarfs in this survey then the optimum observing bandpass would again be J. It is interesting to note that the best wavelength regions to observe M-dwarfs are precisely in between J, H, and K. This is a result of molecular lines from water appearing in the cooler stars’ atmospheres, but this same water absorption in Earth’s atmosphere is precisely what sets the location of the NIR observing bands.

2.6. Summary

We have considered the sources of velocity uncertainty in stellar RV measurements. In doing so, we are able to describe the basic mechanisms that cause velocity uncertainties, what the dominant driver is behind stellar velocity uncertainties at various stellar masses and in various wavelength regions, and furnish several points for consideration when designing an RV survey.

At a basic level, we demonstrate that the velocity uncertainty of a weak spectral absorption line in a continuum scales as \( \Theta^{3/2} \), where \( \Theta \) is the FWHM of the line, and not linearly with \( \Theta \) as one expects when there is no continuum emission, and has been previously suggested in the literature. Using model spectra, we then calculated how the velocity uncertainty changes as a function of spectral resolution, stellar rotation, stellar effective temperature, stellar surface gravity, and stellar metallicity. By dividing our model spectra up into 100Å-wide chunks, we find that the effects of resolution, rotation, and surface gravity operate on the chunks in a largely self-similar manner – regardless of the specific wavelength or spectral features within a chunk. Effective temperature presents a more complicated picture, with different chunks behaving very differently. We numerically fit a rough relation to the
chunk medians, but the variation between chunks as a function of temperature is one illustration of the importance of carefully choosing the wavelength range used in an RV survey.

With these basic relations established, we are able to calculate how the velocity uncertainty scales as a function of stellar mass. For stars more massive than $1.1M_\odot$, we find that the rapidly increasing stellar rotation dominates the predicted uncertainties. Below $1.1M_\odot$, the velocity uncertainty is set by a combination of competing effects from changes in stellar luminosity, temperature and surface gravity. In the optical, between 4000Å and 6500Å, we find that almost all of these effects cancel, leaving the velocity uncertainty to be predominately set by the overall bolometric luminosity of the target star for a fixed distance. This is not true in the red (6500Å to 10000Å) or the NIR (10000Å to 25000Å), where one must also account for temperature (red) or temperature and the effect of the blackbody peak shifting relative to the observed wavelength range (NIR). We give simple approximations for how the velocity error scales with mass for each of these three wavelength regimes.

More generally, our consideration of velocity errors in RV surveys highlights two important points for consideration. First, after a certain point increasing spectral resolution provides diminished returns. This primary occurs because once one has resolved the lines in a spectrum increased resolution provides little more information, and because, depending on the stars being surveyed, stellar rotation will provide the dominant source of velocity uncertainty – not spectral resolution.

Second, the most efficient wavelength region to operate an RV survey for M-dwarfs is between 6000Å to 9000Å. Although the peak emission for M-dwarfs is generally longwards of these wavelengths, the base velocity uncertainties of spectra in this wavelength region are about 1/10 that of spectra in the NIR bands. This
means that even after accounting for the difference in received flux, M-dwarf spectra from 6000Å to 9000Å will give a lower velocity uncertainty than spectra in observed in the NIR at the same exposure time.

In a follow-on to this analysis, I plan to apply the formalism and the approximations I have derived to a notional multi-object RV survey. I will examine the expected yield of such a survey, and how best to optimize the observing parameters to maximize this yield.
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Note. — The complete table is available as online data.

Table 2.1. Chunk Velocity Uncertainties (km/s)
Fig. 2.1.— The exponent with which the velocity uncertainty $\sigma_{V}$ scales to the width $\Theta$ of an absorption line, i.e., $\sigma_{V} \propto \Theta^{n}$, as a function of the line depth relative to the continuum level.
The lines show how one would expect velocity uncertainty to vary with FWHM for absorption lines with a Gaussian, Lorentzian, or Voigt profile with $W = 0.2$ and $I_0 = 1$ based on Equations (2.13), (2.14), and (2.16). For the Voigt profile we set $\Theta_L/\Theta_G = 1$. The points are the directly calculated velocity uncertainties for these same absorption lines.
Fig. 2.3.— The different line broadening profiles we consider in Section 2.1, all drawn with the same FWHM ($\Theta = 1$) and equivalent width ($W = 0.2$).
Fig. 2.4.— The velocity uncertainty of 100Å chunks as a function of spectral resolution using Kurucz-based spectra. The top row shows the absolute uncertainty for the chunks in our wavelength regions, while the middle row shows the velocity uncertainty for each chunk normalized to $R=50,000$. The red line in the middle panel is the median of the relative velocity uncertainties. The bottom row shows the fractional difference between each individual chunk’s relative uncertainties and this median. The relative uniformity of the chunk behaviors is indicative of the external nature of spectral resolution, as an effect applied to all the chunks nearly equally.
Fig. 2.5.— The velocity uncertainty in a spectrum plotted as a function of spectral resolution. The black line shows how the uncertainty on an extremely finely spaced Kurucz-based spectra varies with resolution, and the green squares show our best fit. In addition, note the difference between the black Kurucz-based spectra and the BT-Settl-based spectra (blue line). The wavelength spacing of the BT-Settl spectra impose a resolution “floor” of R≈105,000. This is demonstrated by the orange line, which shows the behavior of the BT-Settl spectra if we assume that there is an R=105,000 base resolution. This transformed BT-Settl behaves similarly to the black Kurucz line. Alternatively, the blue line shows a Kurucz spectrum sampled with a wavelength spacing comparable to the BT-Settl spectra.
Fig. 2.6.— The median relative velocity uncertainty of a 5750K, log $g = 4.5$ star plotted against the width of the underlying Voigt profile, given by a combination of the average inherent line width in the spectrum and a changing spectral resolution, from R=$1 \times 10^6$ to R=10,000. While the calculated points deviate from $\sigma_V \propto \Theta^{3/2}$ on a small scale, the overall best fit to the median relative velocities is given by a $\Theta^{3/2}$ scaling.
Fig. 2.7.— The residuals to our fits to the median relative chunk uncertainty as a function of changing spectral resolution and effective temperature in the optical. The RMS across all resolutions and temperatures is 7%, and the peak value is about 20%.
Fig. 2.8.— The residuals to our fits to the median relative chunk uncertainty as a function of changing spectral resolution and effective temperature in the optical. The RMS across all resolutions and temperatures is 4%, and the peak value is about 10%.
Fig. 2.9.— The residuals to our fits to the median relative chunk uncertainty as a function of changing spectral resolution and effective temperature in the optical. The RMS across all resolutions and temperatures is 3.5%, and the peak value is about 10%.
Fig. 2.10.— The velocity uncertainty of 100Å chunks as a function of stellar rotation velocity from Kurucz-based spectra. The top row shows the absolute uncertainty for the chunks in our three wavelength regions, while the middle row shows the velocity uncertainty for each chunk normalized to $v_{\text{rot}}=5$ km s$^{-1}$. The red line in the middle panel is the median of the relative velocity uncertainties. The bottom row shows the fractional difference between each individual chunk’s relative uncertainties and this median.
Fig. 2.11.— How the measured FWHMs of specific isolated lines change as a function of effective temperature. We have normalized each to be unity at 5800K, so as to identify any broad trends. The lines are color-coded according to the atom responsible: red are iron lines at 5294.5Å, 5905.7Å, and 6078.5Å, blue is a silicon line at 6125.0Å, and green is a nickel line at 6482.7Å. While the FWHMs of all the lines generally trend smaller as temperature increases, one can see the large variation in how specific lines react.
Fig. 2.12.— The velocity uncertainty of 100Å chunks as a function of stellar effective temperature using BT-Settl-based spectra. The top row shows the absolute uncertainty for the chunks in our three wavelength regions, while the middle row shows the velocity uncertainty for each chunk normalized to 5800K. The red points in the middle panel is the median of the relative velocity uncertainties, while the orange overplotted line is our fit to the median. The bottom row shows the fractional difference between each individual chunk’s relative uncertainties and this median.
Fig. 2.13.— Velocity uncertainty as a function of effective temperature and surface gravity, relative to log\( (g) = 4.5 \) using BT-Settl spectra. The black lines are individual 100Å spectral chunks, and the red points are the medians of all the chunks at the effective temperatures of the model spectra. The overplotted orange lines are our fits to the chunk medians. Notice that at the extremes of the temperate range we consider, where we expect the stellar surface gravity to be most different from log\( (g) = 4.5 \), the effect of changing surface gravity is the least. Surface gravity therefore plays a minor (\( \sim 10\% \)) in setting velocity uncertainties.
Fig. 2.14.— The points show the measured median slopes used in Equation (2.34) to describe the change in velocity precision as a linear function of the change in surface gravity ($\log g$). The overplotted lines are the fits to these measurements, as per Equation (2.35).
Fig. 2.15.— The velocity uncertainty of 100Å chunks as a function of stellar metallicity using Kurucz-based spectra. These results are for a star with a fixed effective temperature of 5750K and a fixed log $g = 4.5$ in the optical (4000Å to 6500Å). The top panels shows the absolute uncertainty for the chunks, while the middle row shows the velocity uncertainty for each chunk normalized to [Fe/H]=0.0. The red line in the middle panel is the median of the relative velocity uncertainties. The bottom row shows the fractional difference between each individual chunk’s relative uncertainties and this median. As expected, a decrease in [Fe/H] reduces the number of lines in a spectrum, thus increasing the velocity uncertainty.
Fig. 2.16.— The radial velocity uncertainty of 100Å chunks in the optical, over the temperature range we consider. This assumes a continuum flux in each chunk equal to unity, or in terms of Equation (2.37), that $I_{0,i} = 1$. This is the same information as in Table 1.
Fig. 2.17.— The radial velocity uncertainty of 100Å chunks in the red, over the temperature range we consider. This assumes a continuum flux in each chunk equal to unity, or in terms of Equation (2.37), that $I_{0,i} = 1$. This is the same information as in Table 1.
Fig. 2.18.— The radial velocity uncertainty of 100Å chunks in the NIR, over the temperature range we consider. This assumes a continuum flux in each chunk equal to unity, or in terms of Equation (2.37), that $I_{0,i} = 1$. This is the same information as in Table 1. The sharp feature around 23000Å is the CO molecular bandhead.
Fig. 2.19.— Velocity uncertainty (red line) as a function of stellar mass in the optical, using a spectral resolution of 60,000 and a stellar age of 2.0 Gyr. We have decomposed the overall velocity uncertainty into its constituent parts (black lines), as described in the third paragraph of Section 5. The blue approximation line is from Equation (2.46), and has been normalized to match the calculated uncertainty at $0.5M_\odot$. In the optical, the approximated uncertainty is roughly proportional to the overall luminosity (L).
Fig. 2.20.— Velocity uncertainty (red line) as a function of stellar mass in the optical, using a spectral resolution of 60,000 and a stellar age of 2.0 Gyr. We have decomposed the overall velocity uncertainty into its constituent parts (black lines), as described in the third paragraph of Section 5. The blue approximation line is from Equation (2.46), and has been normalized to match the calculated uncertainty at 0.5$M_\odot$. In the red, the approximated uncertainty is roughly proportional to the overall luminosity (L) and temperature effects (T).
Fig. 2.21.— Velocity uncertainty (red line) as a function of stellar mass in the optical, using a spectral resolution of 60,000 and a stellar age of 2.0 Gyr. We have decomposed the overall velocity uncertainty into its constituent parts (black lines), as described in the third paragraph of Section 5. The blue approximation line is from Equation (2.46), and has been normalized to match the calculated uncertainty at 0.5$M_\odot$. In the NIR, the approximated uncertainty is roughly proportional to the overall luminosity (L), temperature effects (T), and the blackbody effect (BB).
Fig. 2.22.— The median base uncertainty values from Table 1 divided into M-stars (2600K to 4000K), K-stars (4000K to 5200K), G-stars (5200K to 6000K), and F-stars (6000K to 7600K). The two left panels show the medians for each 100Å wavelength chunk in Table 1, while the right panels show the median values after the chunks have been combined into 20 logarithmically-spaced bins. This is meant to replicate a spectrograph observing at fixed resolution. The top panels show the raw numbers from Table 1, while the bottom panels are normalized to according to the fraction of the overall bolometric luminosity that each chunk occupies.
Fig. 2.23.— The base uncertainty values from Table 1 for four different M-dwarf temperatures. As in Figure 2.22, the two left panels show the uncertainties for the 100Å wavelength chunks listed in Table 1, while the right panels show the uncertainties after the chunks have been combined into 20 logarithmically-spaced bins. This is meant to replicate a spectrograph observing at fixed resolution. The top panels show the raw numbers from Table 1, while the bottom panels are normalized to according to the fraction of the overall bolometric luminosity that each chunk occupies. The bottom right panel particularly illustrates that the most efficient wavelength range to observe M-dwarfs is generally shortwards of 10,000Å.
Chapter 3: The Discovery and Characterization of the Transiting Hot Jupiter KELT-2Ab

Individual giant planets transiting bright main sequence stars remain of prime scientific interest. While the multitude of hot Jupiters orbiting fainter (i.e., $V > 9$) stars provides an opportunity to learn about the statistical properties of giant planets, it is the hot Jupiters around the bright stars which provide us with specific information about planetary interiors and atmospheres (see, e.g., Winn 2010). Indeed, since their discovery, all of the transiting hot Jupiters orbiting bright ($V < 9$) stars have been observed repeatedly from space and the ground for precisely this reason (Seager 2010). Since there are currently only five transiting giant planets in this magnitude range, discovering even one more substantially increases the opportunities for these important, detailed, follow-up observations.

In this chapter I describe the discovery and characterization of a hot Jupiter transiting the bright primary component of the HD 42176 binary system, which I hereafter refer to as the KELT-2 system.

3.1. Discovery and Follow-up Observations

The KELT-North telescope consists of an Apogee AP16E ($4K \times 4K$ 9μm pixels) CCD camera attached to a Mamiya 645 camera lens with 42mm aperture and 80mm focal length (f/1.9). The resultant field of view is $26^\circ \times 26^\circ$ at roughly $23''$ per pixel. The telescope uses a Kodak Wratten #8 red-pass filter and the resultant bandpass
resembles a widened Johnson-Cousins R-band. The telescope is located at Winer Observatory in Sonoita, AZ. Pepper et al. (2007) give a more detailed description of the telescope and instrumentation.

KELT-2 is in KELT-North survey field 04, which we monitored from October 27, 2006 to March 31, 2011, collecting a total of 7,837 observations on 136,702 stars in the field. We reduced the raw survey data using a custom implementation of the ISIS image subtraction package (Alard & Lupton 1998; Alard 2000), combined with point-spread fitting photometry using DAOPHOT (Stetson 1987). Using the Tycho-2 proper motions and a reduced proper motion cut based on Collier Cameron et al. (2007), we identified 29,345 putative dwarf and sub-giant stars within field 04 for further post-processing and analysis. We applied the trend filtering algorithm (TFA, Kovács et al. 2005) to each dwarf star lightcurve to remove systematic noise, followed by a search for transit signals using the box-fitting least squares algorithm (BLS, Kovács et al. 2002). For both TFA and BLS we used the versions found in the VARTOOLS package (Hartman et al. 2008). A more detailed description of our data reduction, post-processing, and candidate selection can be found in Siverd et al. (2012).

KELT-2 passed our cuts based on the results of the BLS transit search, and was promoted to by-eye candidate vetting. The KELT-North lightcurve showed a strong transit-like signal from a relatively isolated star, with no significant power in either an Analysis of Variance (Schwarzenberg-Czerny 1989; Devor 2005) or a Lomb-Scargle periodogram (Lomb 1976; Scargle 1982; Press & Rybicki 1989; Press et al. 1992). To check for blending, we also examined if the center of light moved off the position of the star during transit, which it did not. At this point matching to the CCDM and WDS catalogs revealed that KELT-2 was a suspected close visual binary system. We considered KELT-2 a strong planet candidate, and passed it on
for follow-up observations. All of the KELT-2 data can be downloaded from the KELT-North website.¹

3.1.1. Follow-up Spectroscopy

Our follow-up spectroscopy was collected using the Tillinghast Reflector Echelle Spectrograph (TRES), on the 1.5m Tillinghast Reflector at the Fred L. Whipple Observatory (FLWO) at Mt. Hopkins, AZ. We specifically targeted KELT-2A for our follow-up; the typical seeing for TRES is 1.5″, which allowed us to exclude most of the light from KELT-2B 2.3″ away. The spectra had a resolving power of R=44,000, and were extracted following the procedures described by Buchhave et al. (2010).

Between UT 2012-02-01 and UT 2012-05-04 we obtained 18 spectra of KELT-2A. We have excluded one observation taken on the night of UT 2012-02-07 which suffered from higher than normal contamination from the Moon. The top panel of Figure 1 shows our radial velocity observations phased and overplotted to our final orbital fit. The residuals to this fit and the bisector spans are shown in the middle and bottom panels, respectively. Note that the bisectors do not show any phase structure, and the 24.42 m s⁻¹ RMS in the bisector values is 6.5 times smaller than the velocity amplitude of our final phased orbit: 161.1 m s⁻¹. This indicates that we are likely seeing true reflex Doppler motion in the spectra of the parent star, KELT-2A. The residuals to our orbital fit have a 22.74 m s⁻¹ RMS.

¹www.astronomy.ohio-state.edu/keltnorth/data/kelt2/
3.1.2. Follow-up Photometry

We obtained follow-up transit photometry of KELT-2A between late February and late March, 2012. Figure 2 shows the phased transit lightcurves. For all of the transit observations the observers deliberately defocused their telescopes to minimize the noise arising from interpixel variations coupled with pointing drift, and to prevent saturation. This meant that in none of our follow-up transit photometry were KELT-2A and -2B resolved. We find the same transit depths in our $g$, $i$ and $z$ follow-up lightcurves, which helps rule out stellar false positives\(^2\).

A complete transit in $g$ on UT 2012-02-20 and a partial transit with ingress in $z$ on UT 2012-03-29 were observed from Hereford Arizona Observatory (HAO), a private facility in southern Arizona. We observed with a 0.35m Meade Schimdt-Cassegrain equipped with a Santa Barbara Instrument Group (SBIG) ST-10XME camera using a Kodak KAF-3200E 2K×1.5K CCD. The field of view of the observations was 26.9′×18.1′ with a pixel scale of 0.74″ per pixel. We also used an SBIG AO-7 tip-tilt image stabilizer hold the target steady on the detector.

The Peter van de Kamp Observatory at Swarthmore College observed a partial transit with egress on UT 2012-03-21 in $i$. The Observatory uses a 0.6m RCOS telescope with an Apogee U16M 4K×4K CCD, giving a 26′×26′ field of view. Using 2×2 binning, this gives a pixel scale of 0.76″ per pixel. Fog arrived immediately after egress during these observations, so the out-of-transit baseline is shorter than usual.

We observed a partial transit with egress on UT 2012-03-25 with KeplerCam on the 1.2m telescope at FLWO. KeplerCam has a single 4K×4K Fairchild CCD with a pixel scale of 0.366″ per pixel, for a total field of view of 23.1′×23.1′. Observations\(^2\) After accounting for the small (~1%) but wavelength-dependent effect of KELT-2B’s added light on the transit depths.
were obtained in $z$. Thin clouds passed overhead for most of the egress; we have removed these observations from the lightcurve. The data were reduced using a light curve reduction pipeline outlined in Carter et al. (2011), which uses standard IDL routines.

In addition to transit lightcurves, Bruce Gary obtained absolute photometry of the KELT-2 system in $B$, $V$, $r$, $i$ and $z$ filters from HAO on UT 2012-04-16.

### 3.2. Planetary and Stellar System Fitting

KELT-2 is a binary system that is listed in several catalogs\(^3\) as a single star with a combined brightness of $V = 8.71$. As part of the fitting process described below we have determined that the KELT-2 system is composed of a $V = 8.77 \pm 0.01$, F7V primary in orbit with a $V = 11.9 \pm 0.2$, K2V secondary. The two stellar components are presently separated by 2.3″. While the two stars have been assumed to be associated based on their proximity (Couteau 1975), imaging gathered as part of our follow-up observations and discussed in Section 3.1 proves they are bound. Table 1 lists general system and catalog information for the KELT-2 system.

The presence of KELT-2B as an unresolved component in all of our follow-up photometry – both in the transit lightcurves and the broadband magnitudes – complicated our fitting. We wished to remove the light from B to deblend the transit lightcurves, and to measure the actual apparent $V$ magnitude of KELT-2A. The latter is important as it allowed us to make an independent estimate of KELT-2A’s radius in conjunction with the Hipparcos parallax.

The available magnitudes of the two stars (Couteau 1975), provided insufficient information on the properties of KELT-2B for us to accurately remove its light.

\(^3\)2MASS, NOMAD, UCAC3, GSC and ASCC
We therefore performed an iterative round of Markov Chain Monte Carlo (MCMC) fitting to the lightcurves and radial velocities (RVs) to determine the properties of KELT-2A, followed by fitting the spectral energy distribution (SED) of the binary to identify the contribution from KELT-2B. We conducted two rounds of these fits: a first MCMC fit to the lightcurves with no deblending and at the Hipparcos-derived distance to provide initial parameters, followed by an SED fit to determine the properties of KELT-2B. The SED fitting also provided us with an estimate of the reddening to KELT-2. We then used the results of this first SED fit to deblend the transit lightcurves and reran the MCMC fitting. At this point we also included the Hipparcos parallax as a constraint and added the system distance as an additional free parameter. The refined system parameters from the second MCMC fit then fed into a second SED fitting. This SED fit gave parameters for KELT-2B within 1% of the first SED fit, and so we judged the fitting process to have converged.

To perform the MCMC fits to the transit lightcurves and the radial velocity data we used the exofast package (Eastman et al. 2013). This is a suite of routines that performs MCMC fits to lightcurves and RV data simultaneously. exofast also fits the properties of the parent star using the relations from Torres et al. (2010). We modified exofast to include the distance prior from Hipparcos and account for KELT-2B’s flux. The inclusion of the Hipparcos-derived distance allowed us to also fit for the distance to KELT-2. For each transit, we fit a unique airmass detrending coefficient and baseline flux and subtracted the band-dependent companion flux. We assumed a constant period in our final fit.

As initial inputs for exofast we used values for the effective temperature (6146 ± 50 K), surface gravity (4.03 ± 0.1), projected rotational velocity (9.0 ± 2.0 km s$^{-1}$) and metallicity (0.06 ± 0.08) of KELT-2A derived from our follow-up spectroscopy. These were determined using the Spectral Parameter Classification
(SPC) procedure (Buchhave et al., in preparation). SPC cross-correlates synthetic spectra created from a grid of Kurucz model atmospheres against the observed TRES spectra to estimate the spectral parameters and their uncertainties.

For our SED fitting, we fit combined model spectra of KELT-2A and B to our measured $B$, $V$, $g$, $i$ and $z$ magnitudes for the KELT-2 system. We assigned KELT-2A the physical parameters from the exofast MCMC fit and considered several possible spectra for KELT-2B over a range of effective temperatures from 4200K to 5300K, all with the same metallicity as KELT-2A and a fixed log $g = 4.65$. We calculated a radius for each temperature and log $g$ using the Torres et al. (2010) relations. We also included reddening as a free parameter. We constructed our model spectra with the NextGen model atmospheres (Hauschildt et al. 1999).

Table 2 shows the stellar and planetary parameters derived from this iterative procedure. We include the results for a fit with eccentricity fixed to be zero, and another with eccentricity included as another free parameter. While formally the eccentricity for this second fit is non-zero ($e = 0.182^{+0.081}_{-0.084}$), note that the values for $e \cos \omega_*$ and $e \sin \omega_*$ are nearly exactly zero. We view this eccentricity as insignificant, considering the Lucy-Sweeney bias (Lucy & Sweeney 1971). We therefore have adopted the $e \equiv 0$ results as the true system parameters. Additionally, in both fits there is a small but non-zero slope in the RVs ($0.63 \pm 0.24$ m s$^{-1}$ day$^{-1}$). If real, this is too high to be accounted for by the binary orbit. An object in an edge-on circular orbit around KELT-2A should show a maximum RV slope of $0.63$ m s$^{-1}$ day$^{-1}$ ($m_p/2M_J$) ($a$/AU)$^{-2}$.

The planet KELT-2Ab has a mass of $M_P = 1.524 \pm 0.088M_J$ and radius of $R_P = 1.290^{+0.064}_{-0.050}R_J$. The radius is mildly inflated as compared to Baraffe et al. (2008)’s model for an similarly irradiated solar-composition giant planet (1 to 2 $M_J$).
at 3 to 4 Gyr (see §4), which would predict a radius of \( \sim 1.1 \) \( R_J \). Nevertheless, KELT-2Ab is in the middle of the observed radii for planets of this mass, and follows the trend of radius inflation versus planetary effective temperature described by Laughlin et al. (2011).

The star KELT-2A is a slightly evolved F7 dwarf. We find \( T_{\text{eff}} = 6148 \pm 48 \) K, \( \log g = 4.030^{+0.015}_{-0.026} \) and \([\text{Fe/H}] = 0.034 \pm 0.78\), with an inferred mass \( M_* = 1.314^{+0.063}_{-0.060} M_\odot \) and a relatively large radius \( R_* = 1.836^{+0.066}_{-0.046} R_\odot \). The distance we determine to the KELT-2 system, \( 128.6^{+5.2}_{-4.2} \) pc, is 1.2\( \sigma \) farther away than the \( 110 \pm 15 \) pc determined using the Hipparcos parallax alone.

For KELT-2B the results of our SED fits place the star as a K2V, with \( T_{\text{eff}} = 4850 \pm 150 \) K. Using the temperature-mass and temperature-radius relations from Torres et al. (2010) – and assuming \( \log g = 4.65 \) – we estimate KELT-2B has a mass of \( M_* \approx 0.78 M_\odot \) and radius of \( R_* \approx 0.70 R_\odot \).

### 3.2.1. The KELT-2 Binary System

As part of our follow-up observations we obtained five high resolution \( z \) images of the KELT-2 system from Spot Observatory, Nunnelly, TN on UT 2012-04-10 that resolved the two components. Spot Observatory is equipped with an RC Optics 0.6m telescope and an SBIG STL-6303E camera. For these observations, we used an SBIG AO-L transmissive adaptive optics system, which enabled a FWHM of \( \sim 1.35^" \).

In each of the five images we were able to measure the relative positions and fluxes of KELT-2A and -2B. We did so by fitting a two dimensional gaussian to KELT-2A, subtracting it, and then doing the same for KELT-2B. Figure 3 shows our measurements for the position of KELT-2B with respect to KELT-2A, along with those from Couteau (1975) and Argue et al. (1992). We also plot the
position KEL T-2B would have if it started at the 1973.96 average position and were unassociated with KEL T-2A. For this we assumed KEL T-2B had the average proper motion of K-giants between $11 < V < 13$ and within $1^\circ$ of the galactic coordinates of KEL T-2A. We determined this average proper motion using the Besançon galaxy model (Robin et al. 2003), which gave $\mu_\alpha = 0.04 \pm 0.27$ mas yr$^{-1}$ and $\mu_\delta = -0.26 \pm 0.39$ mas yr$^{-1}$. Note that under this assumption the effect of parallax on the relative position of the two stars at these epochs is ten times smaller than our uncertainties. We calculated the uncertainties on our average position and the 1973.96 average position by taking the standard deviation of the individual positions. From our observations, we exclude the possibility that these two stars are unassociated at the 8-$\sigma$ level.

Our measurement of the relative positions of KEL T-2A and B places KEL T-2B at $\Delta \alpha = 1.19'' \pm 0.03$ and $\Delta \delta = 1.95'' \pm 0.05$ relative to KEL T-2A for 2012.27. This corresponds to a position angle of 328.6$^\circ$ and a separation of 2.29''. From the distance we determine this is a projected separation of 295 $\pm$ 10 AU. At this semimajor axis, the binary orbital period would be $\sim$3,500 years.

3.3. Discussion

The final planetary parameters for KELT-2Ab place it in a region of mass-radius parameter space that is already well populated by other hot Jupiters. What is noteworthy about this system is the brightness of the primary star, the primary’s evolutionary state and the presence of a K2V common proper motion companion.

At an apparent magnitude of V=8.77, KELT-2A is the ninth brightest star with a known transiting planet, and the third brightest discovered by a ground-based
transit survey⁴. This makes KELT-2Ab an excellent candidate for both space- and ground-based follow-up work. In terms of the bright \((V < 9)\) transiting planets, KELT-2Ab, at 1.52 \(M_J\), allows access to a region of the mass-radius diagram otherwise unprobed by the known bright systems. HD 209458b, HD 149026b and HD 189733b are all less than 1.15 \(M_J\), while HD 17156b and HAT-P-2b are both over 3 \(M_J\). WASP-33b does not have a well-constrained mass, and 55 Cnc e and Kepler-21b are both super-Earths.

The star KELT-2A itself is in an interesting region of parameter space. Comparison to the Yonsei-Yale isochrones (Demarque et al. 2004) with our determined temperature, surface gravity, and metallicity suggests that KELT-2A has just left the main sequence and its convective core is in the process of halting hydrogen fusion and the star is transitioning to shell burning. This so-called ‘blue-hook’ transition (see, e.g., Exter et al. 2010), which occurs immediately prior to the star’s rapid transition across the Hertzsprung gap to the base of the red giant branch, only lasts a few tens of millions of years. Assuming the isochrones and our inferred stellar properties are correct in placing KELT-2A on the ‘blue-hook,’ we thus find a remarkably precise system age of 3.968 ± 0.010 Gyr.

The existence of the stellar companion KELT-2B raises the intriguing possibility that KELT-2Ab migrated inward to its present location through the eccentric Kozai mechanism (Lithwick & Naoz 2011). If this were true, then the orbit of the planet is likely misaligned with the spin axis of KELT-2A. Interestingly, the effective temperature of KELT-2A (6151K) places this system near the proposed dividing line between cool aligned and hot misaligned planetary systems noted by Winn et al. (2010). Future Rossiter-McLaughlin measurements of the system’s spin-orbit

⁴According to the Extrasolar Planets Encyclopedia at the date of writing.
alignment should provide insight into the efficiency of any mechanisms that might align planets around the cooler stars. We would expect, from equation (6) of Gaudi & Winn (2007), the amplitude of the Rossiter-McLaughlin anomaly to be $\sim 44 \text{ m s}^{-1}$.

Given the brightness and spectral type of KELT-2A, it is interesting to ask why this system was not observed by any of the RV surveys for exoplanets. In addition to KELT-2A being fainter than most of the targets for RV surveys, it may be that the RV surveys did not examine the KELT-2 system because it is listed as a binary in many of the available catalogs.
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Note. — 1=van Leeuwen (2007), 2=Cutri et al. (2003), 3=Zacharias et al. (2005)

Table 3.1. KELT-2 System Properties
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Planetary Parameters:

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<td>(K) RV semi-amp. (m s(^{-1}))</td>
<td>160.5 ± 8.7</td>
<td>161.1^{+7.6}_{-8.0}</td>
</tr>
<tr>
<td>(M_P\sin i) Minimum mass (M_J)</td>
<td>1.491 ± 0.096</td>
<td>1.523 ± 0.088</td>
</tr>
<tr>
<td>(\gamma_0) km s(^{-1})</td>
<td>47.5 ± 0.2</td>
<td>47.5 ± 0.2</td>
</tr>
<tr>
<td>(\gamma) RV slope (m s(^{-1}) day(^{-1}))</td>
<td>0.63 ± 0.26</td>
<td>0.63 ± 0.24</td>
</tr>
<tr>
<td>(e \cos(\omega_\star))</td>
<td>-0.04 ± 0.15</td>
<td>\equiv 0</td>
</tr>
<tr>
<td>(e \sin(\omega_\star))</td>
<td>0.00 ± 0.15</td>
<td>\equiv 0</td>
</tr>
</tbody>
</table>

Primary Transit Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value ((e \neq 0))</th>
<th>Value ((e \equiv 0), adopted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_P/R_\star) Radius ratio</td>
<td>0.0730 ± 0.0018</td>
<td>0.0723 ± 0.0018</td>
</tr>
<tr>
<td>(a/R_\star) Reduced semi-major axis</td>
<td>6.43^{+0.21}_{-0.26}</td>
<td>6.464^{+0.098}_{-0.21}</td>
</tr>
<tr>
<td>(b) Impact parameter</td>
<td>0.16^{+0.15}_{-0.11}</td>
<td>0.16^{+0.14}_{-0.11}</td>
</tr>
<tr>
<td>(\delta) Transit depth</td>
<td>0.00533 ± 0.00027</td>
<td>0.00522 ± 0.00026</td>
</tr>
<tr>
<td>(\tau) Ingress/egress dur. (days)</td>
<td>0.0151^{+0.0024}_{-0.0019}</td>
<td>0.01506^{+0.0012}_{-0.00062}</td>
</tr>
<tr>
<td>(T_{14}) Total dur. (days)</td>
<td>0.212^{+0.027}_{-0.022}</td>
<td>0.2156 ± 0.0022</td>
</tr>
</tbody>
</table>

Note. — The best parameters for the star KELT-2B are in the text at the end of §3.

Table 3.2. Median values and 68% confidence intervals for the Physical and Orbital Parameters of the KELT-2A System
Fig. 3.1.— TRES radial velocity measurements of KELT-2A. The top panel shows the observations phased to our best orbital model, shown in red. We have removed the system’s systemic velocity of 47.5 km s$^{-1}$ and the best fit slope. The middle panel shows the residuals of the radial velocity observations to our orbital fit, and the bottom panel shows the bisector span of each observations as a function of phase.
Fig. 3.2.— The KELT-North discovery photometry (top panel), and then the follow-up transit observations (bottom panel) of KELT-2. The red overplotted line in the bottom panel is the best fit transit model. Note that all the follow-up lightcurves have been deblended, to remove the light from KELT-2B.
Fig. 3.3.— The position of KELT-2B relative to -2A. The open blue points are observations by Couteau (1975) – the solid blue point with error bars is their average. The red point is from Argue et al. (1992). The open green points are the observations we made on UT 2012-04-10, and the solid green point with errors is their average. The black point shows the relative position of KELT-2B with respect to A if star B was an unassociated background star. The orange triangle shows where KELT-2B would be if it started at the 1973.96 position and were on a face-on circular orbit.
Chapter 4: Precision Secondary-Eclipse Follow-up Observations and Atmospheric Characterization of the Transiting Brown Dwarf KELT-1b

Among substellar objects, the relationship between giant planets and brown dwarfs is unclear. The generally acknowledged dividing line between these two classes of objects is based on mass. Specifically, objects above the minimum mass to burn deuterium are defined to be brown dwarfs, whereas objects less massive than this limit are defined to be planets. The deuterium burning limit is roughly $\sim 13 \, M_J$, although in detail this depends on one’s definition of “burning deuterium,” and on the detailed composition of the object (Spiegel et al. 2011).

On the one hand, distinguishing between objects below and above $13 \, M_J$ is clearly arbitrary, particularly since after roughly a billion years deuterium burning is over and any evidence of this initial internal energy source is largely gone, i.e., an old $\sim 50 \, M_J$ object that never fused deuterium would be difficult to distinguish from one that did (Spiegel et al. 2011; Bodenheimer et al. 2013; Mollière & Mordasini 2012). Therefore, giant planets and brown dwarfs can properly be thought of as a continuum of objects, with masses and surface gravities that vary accordingly. By studying how the observable properties of these objects vary as a function of mass and surface gravity for controlled samples with similar compositions and in similar environments, we can gain insight into the uncertain physics at work in these bodies. Such insights will in turn constrain the origin of these bodies. Particularly important in this regard are constraints on the atmospheric systems of brown dwarfs and giant
planets, as these systems present not only the most uncertain physics, but are also the most amenable to empirical constraints.

On the other hand, giant planets and massive brown dwarfs likely have distinct origins, at least for companions to sunlike stars. This is evidenced by the existence of the brown dwarf desert, the local minimum in the mass function of relatively close-in (≤10AU) companions to sunlike stars near ~30 to ~50 M_J (e.g., Marcy & Butler 2000; Grether & Lineweaver 2006; Sahlmann et al. 2011). Presumably, objects below the brown dwarf desert were formed in a circumstellar disk, whereas objects above were formed in a manner more analogous to stars. However, this hypothesis is relatively untested, and even within this interpretation many questions remain. For example: what is largest-mass object that can form in a circumstellar disk? What is the smallest mass companion that can form like a star? Do these masses overlap? Does the brown dwarf desert depend on the properties of the star, or the separation from the star? Why is there apparently no brown dwarf desert for isolated objects? By better understanding the physics of companions from the giant planet up through the brown dwarf regime, we may better understand their origins and thus provide answers to these questions.

Unfortunately, obtaining empirical constraints on giant planets and brown dwarfs in similar environments has proven difficult. The majority of our empirical constraints on brown dwarfs come from isolated brown dwarfs, brown dwarf binaries, or brown dwarfs as wide companions to stars (Luhman 2012). These systems are often amenable to detailed study of their atmospheres, including spectra and time series photometry. However, in the vast majority of cases, these objects do not have masses, radii, or age measurements. One exception is the eclipsing brown dwarf system 2M0535-05 (Stassun et al. 2006, 2007), which allows for a direct measurement of the masses and radii of the two brown dwarf components. Furthermore, analogous
isolated or wide-separation giant planets are considerably more difficult to study, due to their intrinsic faintness. It is only recently, and only for relatively massive and young planetary-mass objects, that the first such empirical constraints have been obtained (Marois et al. 2008; Lagrange et al. 2010; Liu et al. 2013, among others).

As expected, obtaining the kinds of measurements routinely acquired for more massive brown dwarfs for these first planetary-mass objects has already provided important insight into the physics at work in these bodies. For example, consider the strong J-band brightening that brown dwarfs undergo as they cross the L- to T-dwarf transition. This is widely interpreted as clouds clearing from the atmospheres of the L-dwarfs (Marley et al. 2010), though the precise mechanism for this process is not well understood (Burgasser 2013). Direct imaging observations of the HR 8799 planets, which are giant planets at or past the L/T transition, do not show a similar J-band brightening (Faherty et al. 2013). This indicates that the atmospheric dynamics responsible for the L/T transition is highly dependent on surface gravity (Bowler et al. 2010). If we are able to understand what the differences are between atmospheric flows and forcing mechanisms in brown dwarfs and exoplanets, we may be able to understand what is precisely occurring at the L/T transition.

In contrast to brown dwarfs, the majority of our empirical constraints on giant planets comes from transiting systems. These systems provide masses, radii, and crude ages for most systems, but because of selection biases, nearly all these planets are on short periods and so likely are tidally locked and subject to very strong stellar irradiation, dramatically altering and complicating their atmospheres and atmospheric dynamics. Therefore, the empirical constraints on these systems cannot be directly interpreted in the same context as isolated brown dwarfs, hampering the ability to define the relationship between these two types of objects. The existence
of the brown dwarf desert, and the resulting paucity of transiting brown dwarfs, has prevented any direct comparison of observations of brown dwarfs under similarly irradiated environments as close-in giant planets.

KELT-1b (Siverd et al. 2012) provides the best opportunity to directly compare a brown dwarf to giant exoplanets under the same environmental condition of strong external irradiation. The previously discovered transiting brown dwarf companions to stars, CoRoT-3b (Deleuil et al. 2008), CoRoT-15b (Bouchy et al. 2011A), Kepler-39b (Bouchy et al. 2011B), WASP-30b (Anderson et al. 2011), KOI-415b (Moutou et al. 2013), KOI-205b (Díaz et al. 2013), and LHS 6343C (Johnson et al. 2011), all orbit relatively faint stars (with the brightest being WASP-30 at $V = 11.9$ and the others being significantly fainter). On the other hand there are massive transiting giant planets like HAT-P-2b (Bakos et al. 2007), XO-3b (Johns-Krull et al. 2008), and WASP-18b (Hellier et al. 2009), all with masses around 10 $M_J$, that transit stars bright enough to allow for high quality follow-up observations, but their masses place them within the planetary regime.

KELT-1b is a $27 M_J$ object on a short 1.2 day orbit around a bright ($V = 10.8$) F5V star. The close orbit places KELT-1b in a highly irradiated environment, with an incident stellar flux of $7.8 \times 10^9$ erg s$^{-1}$ cm$^{-2}$, that places it forty times above the empirical threshold for inflated planets determined by Miller & Fortney (2011) and Demory & Seager (2011). In addition, based on the expected tidal synchronization timescale of $\sim 10$ Myr for KELT-1b (Guillot et al. 1996) we expect that KELT-1b is tidally locked to its orbital period. KELT-1b thus allows us to study a brown dwarf where we know the mass and radius, in an irradiation and tidal environment similar to hot Jupiters, and around a star bright enough to allow for precision follow-up observations. To take advantage of the opportunity offered by KELT-1b, I observed
several secondary eclipses of the KELT-1 system from the ground and from space during the fall of 2012.

4.1. Observations

4.1.1. Spitzer Observations

We observed secondary eclipses of KELT-1b in the 3.6 $\mu$m and 4.5 $\mu$m bands using the IRAC instrument on the Spitzer Space Telescope. We took the 3.6 $\mu$m observations over the course of UT 2012 September 10 and 11 and the 4.5 $\mu$m observations during UT 2012 September 11 and 12. Both sets of observations lasted for 6.57 hours and used the subarray mode with 2.0 second exposures. We additionally used the spacecraft’s peak-up mode, with KELT-1 as the peak-up target, to enhance the pointing stability. We executed two observing sequences in each band: the first was 0.5 hours long intended to allow for the detector ramp and for the spacecraft to settle its pointing. We discarded this first observing sequence, since it was dominated by these two effects. The second sequence in each band was the main science observing sequence. This lasted for 6.07 hours and provided us with 10,944 images in each band. We used the basic calibrated data (BCD) images from these two sequences for all our photometry.

We calculated the $\text{BJD}_{TDB}$ time of each image by using the FITS header entries as follows. The BCD images come in 64 image data cubes with header keywords that record the start time ($\text{MBJD}_{\text{OBS}}$) for each sequence containing 64 images, and the total sequence duration ($\text{AINTBEG}$ and $\text{ATIMEEND}$). We calculated the $\text{BJD}_{\text{UTC}}$ time at mid-exposure for each of the 64 images in each data cube by assuming the image sequences began at $\text{MBJD}_{\text{OBS}}$ and that the 64 images in each sequence were
evenly spaced between AINTBEG and ATIMEEND. To adhere to the timing system used in the KELT-1 discovery paper, we converted to BJD_{TDB} by adding 64.184 seconds to the derived BJD_{UTC} times (Eastman et al. 2010).

To calculate the background in each of the 32×32 pixel subarray images, we first excluded the light from the KELT-1 system by masking out a central circular area 12 pixels in radius. On each image, we then performed three rounds of 3σ clipping on the remaining non-masked area to remove outliers. We estimate the background flux for each image by fitting a Gaussian to a histogram of the values of the remaining pixels and used the fitted value for the mean as the background flux. We subtracted this fitted mean background value from each image. The mean background flux averaged over all the images used in the analysis was 0.05% of KELT-1’s flux at 3.6 μm and 0.02% at 4.5 μm.

We extracted lightcurves in each band using simple aperture photometry. To determine the position of the star in our images we fit a two-dimensional Gaussian to the stellar PSF using the entire image. We also tried finding the star’s position using flux-weighted centroiding following Knutson et al. (2008), but later found this provided inferior corrections for intrapixel sensitivity variations in our data. In doing the position determinations we used a modified set of our background-subtracted images: we replaced any hot pixels with median flux values for that pixel to prevent spurious centroid shifts. We identified pixels as “hot” in a particular image if their flux in the image was more than 3σ away from the median flux for that pixel over an entire 64 image data cube. We then replaced the hot pixel with the pixel’s 64 image median flux. For our aperture photometry we used the original background subtracted images without the hot pixels replaced, so as to remain as close to the raw data as possible. Instead, we used 5σ clipping on the aperture photometry results to remove images where a hot pixel occurred within the photometric aperture.
To extract photometry from the images we chose to use variable apertures scaled to the FWHM of the stellar PSF for both the 3.6\,\mu m and 4.5\,\mu m data. Other secondary eclipse measurements with Spitzer usually use a variable aperture for 3.6\,\mu m data and a fixed aperture for their 4.5\,\mu m observations (e.g., Knutson et al. 2012; Baskin et al. 2013), but we found that using a variable aperture for both bands gave the lowest RMS scatter in the residuals to the best fit lightcurves. We estimated the width of the stellar PSF in each image using the noise pixel parameter (following Knutson et al. 2012), which is defined in Section 2.2.2 of the IRAC instrument handbook as

$$\bar{\beta} = \frac{(\Sigma_i I_i)^2}{\Sigma_i I_i^2},$$

(4.1)

where $I_i$ is the intensity of the $i$th pixel. To calculate the noise pixel parameter in our data we summed over the pixels within a radius of three pixels of KELT-1’s position in the image, including fractional pixels. The FWHM of the stellar PSF is then $\sqrt{\bar{\beta}}$ (Mighell 2005). The third panels in Figures 1 and 2 show how the FWHM calculated from the noise pixel parameter (i.e. $\sqrt{\bar{\beta}}$) varied as a function of time. At 3.6\,\mu m the median FWHM over the 9,775 images we used was 2.15 pixels, with a standard deviation of 0.04 pixels. At 4.5\,\mu m the median over the 10,307 images we used was 2.03 pixels with a standard deviation of 0.02 pixels. At 3.6\,\mu m the noise pixel parameter was nearly perfectly ($r = 0.95$) correlated with the $y$-position of the stellar centroid.

We set our photometric aperture size to be $\sqrt{\bar{\beta}} + C$, which is the FWHM, $\sqrt{\bar{\beta}}$, in a particular image plus some constant, $C$. We chose the optimum aperture by extracting photometric timeseries for a range of values of $C$, fitting an eclipse model to each of these lightcurves, and then choosing the value of $C$ which resulted in the lowest RMS residuals with respect to the model. We tested values $C$ from $-0.5$
pixels to +2.0 pixels in steps of 0.05. This roughly corresponds to apertures with radii of 1.6 to 4.1 pixels (Figures 3 and 4), but we remind the reader that $\sqrt{\beta}$ varies with time. The approximate radii plotted in Figures 3 and 4 use the median values for $\sqrt{\beta}$ over all images. We fit the photometry using the first amoeba stage of the fitting procedure described in Section 4. Figures 3 and 4 show the fitted depth (black) and residual RMS (red) as a function of aperture size for the 3.6 $\mu$m and 4.5 $\mu$m data.

The lowest residual RMS occurred for an aperture size of $\sqrt{\beta} + 0.5$ pixels in the 3.6 $\mu$m data and $\sqrt{\beta} + 0.8$ pixels in the 4.5 $\mu$m data. We therefore utilized these aperture sizes to extract the photometry that we employ in our final analysis. The standard deviation on fitted depth as a function of aperture size in the 3.6 $\mu$m data was 0.003% over the entire trial range, which is below the 0.011% final uncertainty we find for the 3.6 $\mu$m eclipse. For the 4.5 $\mu$m data this was not the case, so we considered the behavior of this dataset in more detail.

At 4.5 $\mu$m we found the fitted eclipse depth varied by 0.06%, depending upon the aperture size used to extract the photometry (Figure 4), particularly for apertures smaller than $\sqrt{\beta} + 0.7$. This is significantly above the 0.012% final uncertainty we calculate for the 4.5 $\mu$m data. There is also a “bump” in the RMS and fitted depth around an aperture radius of $\sqrt{\beta} + 0.25$. For reference, we refer the reader to Figure 6 of Blecic et al. (2013) for an example of “well-behaved” 4.5 $\mu$m data. We were not able to satisfactorily determine the cause of the variation, or the reason for the bump. We first tried switching the lightcurve extraction to use a non-scaled aperture size, instead of an aperture scaled to the FWHM of the stellar PSF, but these lightcurves showed a similar variability in the eclipse depth and a “bump” at a radius of 2.3 pixels. Next, we tested to see if the bump was caused by a bad pixel by setting individual pixels in all the images to zero one-by-one. By setting
pixels (14,14), (14,15) and (14,16) to zero we were able to remove the “bump”, but this almost doubled the variability in the fitted eclipse depth for the photometry using both non-scaled and scaled aperture sizes. A visual inspection of these three pixels’ timeseries showed no obvious abnormalities. Zeroing out other pixels had no discernible effect.

We ultimately decided to use an aperture of \( \sqrt{\beta} + 0.8 \) pixels to extract the 4.5\,\mu m photometry from the unaltered BCD images. In Figure 4 one can see that for apertures larger than \( \sqrt{\beta} + 0.7 \) (\( \sim 2.7 \)) pixels the fitted eclipse depth is nearly constant, and the lowest residual RMS occurs at \( \sqrt{\beta} + 0.8 \). We therefore judged that whatever the cause of the systematic changes in the depth and RMS variation for smaller aperture size is mitigated for apertures larger than \( \sqrt{\beta} + 0.7 \), and thus, that the photometry at our chosen aperture of \( \sqrt{\beta} + 0.8 \) is representative of KELT-1b’s true eclipse depth at 4.5\,\mu m. If this is not the case, then the uncertainty on the 4.5\,\mu m depth we report is an underestimate.

We also trimmed out some of the initial images due to the remains of the initial photometric ramp. The first 1,000 images of the 3.6\,\mu m data and the first 600 4.5\,\mu m images displayed a clearly discernible ramp feature, so we excluded them from our analysis.

Finally, we removed points that were more than 5\,\sigma away from the median flux. The number of points that were clipped varied slightly depending on the exact aperture size (i.e. the value of \( C \)) used. In the apertures we chose to use for our final analysis, this clipping removed 169 (1.7\%) 3.6\,\mu m and 37 (0.4\%) 4.5\,\mu m images.

For our chosen photometric aperture for each data stream, our final lightcurve contained 9,775 images at 3.6\,\mu m covering -2.56 to +3.01 hours around the center of
secondary eclipse. The final 4.5μm lightcurve contained 10,307 images and spans from -2.98 to +2.82 hours around the center of secondary eclipse.

4.1.2. Ground-based Observations

Over the summer and fall of 2012 we observed seven secondary eclipses of KELT-1b in $z'$ at Moore Observatory, which is operated by the University of Louisville. We used the 0.6m RCOS telescope with an Apogee U16M 4K×4K CCD, giving a 26′×26′ field of view and a plate scale of 0”.39 pixel$^{-1}$. Since KELT-1 is separated from its nearest detectable neighbor in DSS2 imagery by ∼18", we were able to defocus the telescope to allow for longer exposures without the risk of blending from the neighbor star.

We used the same observing parameters for the ground-based observations across all nights. The exposure time was 240 seconds (plus a 20 second readout time), and we slightly defocused telescope to give a toroid shaped point spread function (PSF). The target and comparison stars were placed at the same detector locations, and the guiding maintained this placement within a few pixels across all nights. Our image calibration consisted of bias subtraction, dark subtraction, flat-field division, and detector non-linearity compensation. We extracted differential aperture photometry from the calibrated images using AstroImageJ (aij, Collins, et al., in preparation). The comparison stars were selected from sources on the detector which had $z'$ band brightness similar to KELT-1 and which produced relatively flat light curves (after airmass detrending) when compared to the other stars in the ensemble. The final comparison ensemble included four stars near KELT-1 (TYC 2785-2151-1, TYC 2781-2231-1, LTT 17089, and TYC 2785-1743-1). We chose to allow the photometric aperture radius to vary based on an aij estimate of the FWHM of the toroidal PSF. After testing values in the range of 1.0 to 1.4 times
the estimated FWHM, we found that a factor of 1.25 minimized the scatter in the light curves. This factor resulted in an aperture radius that varied between 20-30 pixels across the four nights. The sky background was estimated from an annulus with inner radius 40 pixels and outer radius 80 pixels. Iterative 2σ clipping was first performed to remove outliers and stars from the background annulus. The mean of the remaining pixels was adopted as the sky background value and subtracted from each pixel in the photometric aperture.

Three of the events, on UT 2012-09-07, 2012-10-05 and 2012-10-12, suffered from abnormally poor seeing or interruptions by clouds. We excluded these observations from consideration. The other four secondary eclipses, on UT 2012-07-30, 2012-08-16, 2012-11-18 and 2012-11-29, were high-quality, complete observations of the eclipses. The typical per point uncertainties on these nights were 0.10 to 0.13 percent. The top four panels in Figure 9 show the lightcurves from these four good nights plotted individually, after being detrended against airmass and time as described in Section 3.3.2.

4.2. Lightcurve Fitting and Results

4.2.1. Eclipse Model

We modeled the IR data as a combination of a Mandel & Agol (2002) eclipse lightcurve and a set of decorrelation parameters. To make the eclipse lightcurves we used the implementation of the Mandel & Agol (2002) lightcurves built into Exofast (Eastman et al. 2013). We modeled the eclipse by assuming KELT-1b was a uniformly bright disk, with no limb-darkening, being occulted by the much
larger KELT-1. Compared to a transit lightcurve, this has the immediate effect that $R_p/R_*$ and the eclipse depth are no longer directly related.

In both channels the data showed strong correlations between the flux and the $x$- and $y$-position of the star. These correlations persist regardless of the aperture size we used for the data reduction. These light curve systematics are a result of the well-known intrapixel sensitivity variation in the 3.6 $\mu$m and 4.5 $\mu$m detectors (e.g., Ballard et al. 2010). We fit for, and removed, these trends by including a decorrelation function that modifies the flux and contains three terms: a linear term each for the $x$- and $y$-pixel position of KELT-1, and a linear time term. We included this additional linear decorrelation against time as our initial fits using only the positional decorrelation showed a clear residual trend with time.

We considered using additional quadratic decorrelation terms in $x$- and $y$-position but found that the corresponding decorrelation coefficients varied substantially with choice of the aperture size used to extract the light curves (Figures 5 and 6). The linear terms, on the other hand, settled to specific values once the aperture size grew larger than approximately 2.6 pixels. We therefore considered the quadratic terms to be poorly unconstrained by the data. Furthermore, we found that the fitted eclipse depth was strongly correlated with the quadratic fit parameters, such that the eclipse depth was artificially suppressed when these terms were included. This is due to the relatively small amount of out-of-eclipse data, and the fact that the $x$- and $y$-position measurements are strongly correlated with time, which allowed the quadratic position terms to “fit out” the transit without significantly worsening the fit outside of eclipse.
The model we fit to the data was therefore

\[ F_m(t) = F_0 G(t, x, y; a_1, a_2, a_3) \times \]

\[ [1 - f(t; T_{C,\text{tran}}, P, \sqrt{e} \cos \omega, \sqrt{e} \sin \omega, \cos i, R_P/R_*, a/R_*, \delta)] \]

where

\[ G(x, y, t; a_1, a_2, a_3) = 1 + a_1 x + a_2 y + a_3 t \]

is the decorrelation function which describes the variation of the unocculted total flux due to systematic effects, and \( f(t) \) describes the fractional flux decrement during the eclipse as computed using the modified Mandel & Agol (2002) model.

In addition to a time dependence, the eclipse model’s exact form depends upon:

- the time of the previous transit (\( T_{C,\text{tran}} \)),
- the orbital period \( P \), \( \sqrt{e} \cos \omega \), \( \sqrt{e} \sin \omega \),
- the cosine of the orbital inclination (\( \cos i \)),
- the radius of the planet in stellar radii (\( R_P/R_* \)),
- the semi-major axis in units of the stellar radii (\( \log(a/R_*) \)),
- the baseline flux level (\( F_0 \)),
- and the eclipse depth (\( \delta \)).

Note that we do not explicitly fit for the time of secondary eclipse. Instead, we calculate the secondary eclipse time based on the time of the previous transit (\( T_{C,\text{tran}} \)), the orbital period \( P \), and \( \sqrt{e} \cos \omega \) and \( \sqrt{e} \sin \omega \). We begin by determining the eccentricity and orientation of the orbit via \( e = (\sqrt{e} \cos \omega)^2 + (\sqrt{e} \sin \omega)^2 \) and \( \omega = \tan^{-1}(\sqrt{e} \sin \omega/\sqrt{e} \cos \omega) \). This allows us to calculate the the mean anomaly of KELT-1b during transit (\( M_C \)) and eclipse (\( M_S \)). Then the eclipse time is

\[ T_S = T_{C,\text{tran}} + P(M_S - M_C), \]

We explain the motivation for using this parameterization below.
4.2.2. Fitting Parameters and Their Priors

We fit for a total of twelve parameters: the nine eclipse parameters and the three decorrelation parameters. For seven of these parameters ($T_{C,\text{tran}}$, $P$, $\sqrt{e}\cos\omega$, $\sqrt{e}\sin\omega$, $\cos i$, $R_P/R_*$, and $a/R_*$), we had a prior expectation for their values from the KELT-1b discovery paper. We did not have any prior expectations for the five remaining parameters ($F_0$, $\delta$, and the decorrelation terms). To incorporate our priors into the fitting process, we added a term for each parameter to the $\chi^2$ function of the form

$$
\Delta\chi^2_a = \left(\frac{a_i - a_0}{\sigma_a}\right)^2.
$$

(4.5)

Here $a_i$ is the trial value of an individual parameter $a$, $a_0$ is the prior value, and $\sigma_a$ is the 1$\sigma$ uncertainty in that prior value. Note that this does not consider any possible covariance between the parameters. We used central values and 1$\sigma$ uncertainties for $T_{C,\text{tran}}$, $P$, $\sqrt{e}\cos\omega$, $\sqrt{e}\sin\omega$, $\cos i$, $R_P/R_*$, and $a/R_*$ from the discovery paper fit that was based on a free eccentricity (see Tables 4 and 5 in Siverd et al. 2012), which we list for convenience in Table 1. To calculate $T_{C,\text{tran}}$, the time of the previous transit, we assumed no variation in the transit times, such that $T_{C,\text{tran}} = T_C + nP$ and $\sigma^2_{T_{C,\text{tran}}} = \sigma^2_T + n^2\sigma^2_P$. We also derive values and uncertainties for $\sqrt{e}\cos\omega$ and $\sqrt{e}\sin\omega$ by using the values and uncertainties for $e$, $e\cos\omega$, and $e\sin\omega$ in Tables 4 and 5 of Siverd et al. (2012) and dividing the two latter quantities by $\sqrt{e}$. We chose to use $\sqrt{e}\cos\omega$ and $\sqrt{e}\sin\omega$ as the prior parameters in our MCMC fits, rather than $e\cos\omega$ and $e\sin\omega$, as the former parameterization results in a uniform prior for $e$, while the latter leads to a prior that is proportional to $e$.

We chose to use $T_{C,\text{tran}}$, $P$, $\sqrt{e}\cos\omega$ and $\sqrt{e}\sin\omega$ to calculate the secondary eclipse time, instead of using the predicted time of secondary eclipse, $T_S$, from Table
5 of Siverd et al. (2012), so that we could properly allow for the possibility of a non-zero eccentricity and calculate appropriate uncertainties. If we were to only fit for the time of secondary eclipse \( T_S \), and calculate the orbital eccentricity based on the eclipse time and duration, then we would incorrectly be assuming a circular orbit in our modeling of the eclipse orbital geometry and lightcurve. This would mean that our priors on \( \cos i \) and \( a/R_\ast \), the two terms that would then completely set the eclipse duration, would potentially dominate our measurement of the eclipse duration and thence \( \sqrt{e} \sin \omega \). On the other hand, if we allowed for eccentric eclipse geometries in the lightcurve modeling by instead fitting for \( T_S, \sqrt{e} \cos \omega \) and \( \sqrt{e} \sin \omega \) we would be double-counting the uncertainties in \( \sqrt{e} \cos \omega \) and \( \sqrt{e} \sin \omega \); the uncertainty on \( T_S \) in Table 5 of Siverd et al. (2012) already includes the uncertainties in \( T_C, \sqrt{e} \cos \omega \) and \( \sqrt{e} \sin \omega \). Using \( T_{C,\text{tran}}, \sqrt{e} \cos \omega \) and \( \sqrt{e} \sin \omega \) therefore allows us to correctly model possible eclipse geometries and compute proper uncertainties. Note again, though, that this makes the assumption that KELT-1b has a fixed orbital period and does not display transit or eclipse timing variations. We do account for the 25 second Rømer delay the eclipse has relative to the transit ephemeris.

As we have no prior expectation for the values of \( F_0, \delta \), and the decorrelation terms \( (a_1, a_2 \text{ and } a_3) \), we do not include a \( \chi^2 \) penalty for these terms, and implicitly assume a uniform prior for all five parameters. Of the twelve fitting parameters, these five are therefore the only ones set entirely by our secondary eclipse observations.

Among the remaining seven terms, all of which had well defined priors, \( \sqrt{e} \cos \omega \) and \( \sqrt{e} \sin \omega \) are the only ones for which the data provide a tighter constraint than provided by their prior distributions. \( T_{C,\text{tran}} \) and \( P \) are completely determined by their priors, as the data from an individual eclipse provides no constraint on the nearest transit time or the orbital period. Similarly, our posterior constraints on \( \cos i, R_P/R_\ast \), and \( \log(a/R_\ast) \) are dominated by our priors. This is a result of the
fact that, for an eclipse lightcurve, the only constraint on $R_P/R_*$ comes from the ingress and egress durations, in contrast to a transit lightcurve, where $R_P/R_*$ is heavily determined by the depth of the transit. This means that the two major timing measurements an eclipse lightcurve provides (the FWHM and ingress/egress durations) now must be used to constrain three different parameters. As a consequence, unique values for $\cos i$, $R_P/R_*$, and $\log(a/R_*)$ are poorly constrained by the secondary eclipse data alone, and their values and uncertainties remain nearly identical to those from the discovery paper. In principle this degeneracy is resolved by the exact shape of the ingress and egress portions of the lightcurve, but this is below the precision of our data.

4.2.3. Fitting Process and Results

We fit our data using the AMOEBA and MCMC routines packaged with EXOFAST. We chose to fit the 3.6 $\mu$m and 4.5 $\mu$m data separately. Our fitting process began by using AMOEBA to find initial $\chi^2$ minima to use as estimates of the best fits. This is the same procedure we used previously, to fit the data for our determination of the optimum photometric aperture size, and allowed us to quickly find a likely minimum in the $\chi^2$ surface.

We then used MCMC to explore the parameter space around the $\chi^2$ minimum found by AMOEBA to determine uncertainties. EXOFAST uses the Differential Evolution Markov Chain (DEMC) implementation of the MCMC algorithm, which runs multiple, simultaneous, chains to determine the correct step size and direction. In addition to verifying that we had found the global $\chi^2$ minimum, the MCMC
analysis also provided us with appropriate uncertainties for all of the system and
eclipse parameters.

The final system parameters and errors determined by the MCMC analyses are
in Tables 2 and 3. Figure 7 shows our best fits to the detrended data.

We next conducted a prayer bead analysis on our data to assess the effects of
correlated noise in the data. The prayer bead analysis we conducted followed the
general description of Moutou et al. (2004) and Gillon et al. (2007). In each band
we took the residuals to our best fit model, shifted the residuals by one, added the
incremented residuals back onto the original best fit model and then refit using
amoeba. We shifted through the residuals to the entire lightcurve this way, such
that the \(i\)th model point had the \(i\)th+\(n\)th residual added to it, with \(n\) going from
one to the number of points in the lightcurve. The remainder at the temporal end of
the lightcurve was looped around and added to the beginning.

The goal of the prayer bead analysis is to appropriately account for the presence
of correlated noise in the data. This is done by examining the variation in the
fitted parameters as a function of the shifting residuals. For both the 3.6 \(\mu m\) and
4.5 \(\mu m\) observations the standard deviations in the lightcurve and decorrelation
parameters output by the prayer bead chains were within 10% of the 1\(\sigma\) error bars
from the MCMC analysis. We therefore chose to use the MCMC errors as our final
uncertainties for the Spitzer observations.

Another possible source of systematic uncertainty in our observations is the
possible stellar companion to KELT-1 discovered by Siverd et al. (2012). The
companion is located 558 mas to the southeast of KELT-1, and has \(\Delta H = 5.90 \pm 0.10\)
and \(\Delta K' = 5.59 \pm 0.12\). This separation places the companion 0.47 pixels away
from KELT-1 in our Spitzer observations and 1.4 pixels away in our ground-based observations. In both cases it is unresolved.

Assuming that the companion is associated with KELT-1, its luminosity difference and its $H - K'$ colors imply that it is a mid M-dwarf. If we extrapolate from $H$ and $K'$ to the Spitzer bandpasses using a Kurucz-based 6500K, $\log(g) = 4.25$, spectrum for KELT-1 and a 3200K blackbody spectrum for the companion, then we find that the companion contributes less than 1% of the total system light at 3.6 $\mu$m and 4.5 $\mu$m. In both bands this is substantially below the fractional uncertainty we calculate for the measured eclipse depths (about 10% in both cases). This is also faint enough that the companion does not affect our measurements of KELT-1’s pixel position in the images. We therefore chose to ignore its contribution to our observations.

**Ground-based**

We analyzed each of the four nights individually. We fit only for the depth of a possible eclipse using a trapezoidal eclipse model that had the eclipse time, total duration and ingress/egress duration fixed. We computed the expected eclipse times for each night by extrapolating from our measured 3.6 $\mu$m eclipse time and assuming a fixed period of 1.217514 days. The choice of the 3.6 $\mu$m eclipse time is arbitrary; we repeated our entire analysis of the $z'$ using the 4.5 $\mu$m eclipse time and found no difference in our results. The total duration and ingress/egress duration we set to the average of our 3.6 $\mu$m and 4.5 $\mu$m results. In addition to the eclipse model, we also included linear decorrelation terms for airmass and time. We scaled the errors on each night so that a baseline zero depth fit had a reduced $\chi^2$ of one. In all cases the scaling factor was within ten percent of unity. We used the baseline fit to calculate the $\Delta \chi^2$ values for the non-zero depth fits.
Combining the data from all four good nights, we find suggestive evidence for an eclipse depth of $0.049 \pm 0.023\%$ in $z'$. Figure 8 shows the $\Delta \chi^2$ as a function of eclipse depth for these four nights. The black line in Figure 8 is the $\Delta \chi^2$ of all the nights added together, and is valid under the assumption that our uncertainties are uncorrelated night to night. We have adopted this joint constraint as our final determination of the $z'$ eclipse depth of KELT-1b. Figure 9 shows the four complete $z'$ lightcurves individually, and combined, phased, and overplotted with our best fit eclipse model.

If the detection in $z'$ is real, then it is the result of thermal emission from KELT-1b, and not reflected light. In the case of an extreme Bond albedo of one, and assuming KELT-1b reflects as a Lambert sphere, then the eclipse depth due to reflected light alone would be $\sim 0.03\%$. A more realistic Bond albedo of 0.1 would reduce this depth by a factor of ten, and place it far below our precision in $z'$.

4.3. Results

We strongly detect the secondary eclipses of KELT-1b at both 3.6 $\mu$m and 4.5 $\mu$m, and weakly detect the eclipse in $z'$ (Figures 7, 9, and 10). We measure eclipse depths of $\delta_z = 0.049 \pm 0.023\%$ in $z'$, $\delta_{3.6} = 0.196 \pm 0.010\%$ at 3.6 $\mu$m and $\delta_{4.5} = 0.201 \pm 0.012\%$ at 4.5 $\mu$m. These depths correspond to brightness temperatures of 3300 K, 3150 K and 3000 K for the $z'$, 3.6 $\mu$m and 4.5 $\mu$m eclipses, respectively.

We derive the median and 68% confidence intervals for $e \cos \omega$, $e \sin \omega$, and $e$ from the final MCMC chain in both of the Spitzer bands. At both 3.6 $\mu$m and 4.5 $\mu$m we infer values of $e \cos \omega$ and $e \sin \omega$ that are consistent with zero, which signifies KELT-1b’s orbit is consistent with circular. While we formally calculate a value of $e$ that is greater than zero at $>1\sigma$ in both bands, this is a result of the well-known
Lucy-Sweeney bias (Lucy & Sweeney 1971). Our measurement of circular orbit lends credence to the strong circumstantial evidence that the orbit of KELT-1b has been tidally circularized and that the star KELT-1 has tidally synchronized to the orbital period (see Section 6.2 of the discovery paper).

4.4. Discussion

Overall, KELT-1b is a unique object: it is a relatively high mass and high surface gravity object that orbits only 3.6 stellar radii away from its host star. Among high mass sub-stellar objects, this places KELT-1b squarely in a radiation environment that until now has been populated solely by hot Jupiters. KELT-1b can therefore be interpreted in the context of a hot Jupiter with very high surface gravity, or in the context of a brown dwarf subject to strong external irradiation. We consider both perspectives.

4.4.1. From a Giant Planet Perspective

If considered as a planet, KELT-1b stands out due to its extremely high surface gravity ($\log(g) = 4.74$), which is thirty times higher than for a typical hot Jupiter. KELT-1b therefore allows us test theories of hot Jupiter atmospheres at a very high surface gravity. Of particular interest are the amount of heat redistribution and the presence of a stratospheric temperature inversion within the atmosphere of KELT-1b. Perez-Becker & Showman (2013) have noted that planets hotter than $\sim 2000$ K are observed to have extremely low amounts of heat redistribution from their day- to their nightsides, presumably because the shorter radiative timescales in hotter atmospheres cause these planets to reradiate the incident stellar flux, rather than advecting it through winds to the nightside. In KELT-1b, due to its
high surface gravity, the theoretical radiative timescale is relatively longer, and the theoretical advection timescale relatively shorter, than in most hot Jupiters.

Similarly, consider the presence of a stratospheric temperature inversion in the atmosphere of KELT-1b. Temperature inversions have been observed in several hot Jupiters, predominantly among those with equilibrium temperatures higher than 2000 K (e.g., Cowan & Agol 2011). Hubeny et al. (2003) and Fortney et al. (2008) have proposed that gas-phase TiO in an atmosphere causes temperature inversions, since it is a strong optical absorber and condenses between 1900 K to 2000 K, depending on the pressure. However, the ultimate cause of inversions, and their precise regulatory mechanisms, have not been definitively agreed upon.

Our observations show that KELT-1b does not have a high heat redistribution efficiency. The TiO, complete redistribution atmosphere model is strongly excluded by our observations, and the highest allowable redistribution efficiency, presuming the presence of TiO, would be for day-side redistribution. Following the notation of Seager (2010), it is probable that $f' > 1/2$, and possible that $f' \sim 2/3$ (instantaneous re-emission of the incident stellar radiation). The large difference between the equilibrium temperature of KELT-1b (2400 K) and the brightness temperatures we measure at all three wavelengths ($\sim 3100$ K) is also indicative of an extremely low amount of heat redistribution occurring in the atmosphere. This agrees with the trend noted by Perez-Becker & Showman (2013) that hotter planets have lower heat redistribution inefficiencies.

That being said, and as noted previously, the increased photospheric pressure in KELT-1b may complicate this interpretation. All other things being equal, the pressure level for the $\tau = 1$ surface in an atmosphere is proportional to surface gravity. Thus KELT-1b should have a photosphere at a pressure $\sim 30$ times deeper.
than on a typical hot Jupiter. Since radiative time constants tend to increase greatly with pressure (Iro et al. 2005; Showman et al. 2008), one would expect the radiative time constant at the photosphere would be larger for KELT-1b than for an otherwise identical low-mass hot Jupiter. Similarly, the advection timescale scales inversely with surface gravity and scale height (Perez-Becker & Showman 2013), which would make advection relatively quick in KELT-1b’s atmosphere. These changes in the radiative and advective timescales would allow more time for faster advection within KELT-1b’s atmosphere, thereby lessening the day-night temperature difference and causing a large hotspot offset from the substellar point, for a given set of irradiation conditions. For this reason it will be interesting to compare KELT-1b to lower-mass hot Jupiters that have similar irradiation levels. Specifically, orbital phase curve observations of KELT-1b would allow one to directly measure the day-night temperature difference and test the effect of KELT-1b’s greater photospheric pressure.

As a side note, our determination that $f' > 1/2$ would appear to be roughly inconsistent with the ground-based secondary eclipse observations undertaken for the KELT-1b discovery paper. Figure 14 of Siverd et al. (2012) implies that $f' > 1/2$ would have been detectable by at least 2σ in those data. However, when we examined this issue we discovered that the analysis in Siverd et al. (2012) incorrectly dealt with the observations taken using the Faulkes Telescope North (FTN). The FTN data were reported as differential magnitudes, but we treated them as differential fluxes, inverting the sense of the changes in intensity. If we recalculate Figure 14 of Siverd et al. (2012) correctly, then atmospheres with $f' > 1/2$ would at best be detectable at 0.8σ.

Our observations are not sufficient to conclusively determine whether a TiO inversion exists in the atmosphere of KELT-1b, but we can nonetheless provide
some useful constraints. Figure 10 shows our measured eclipse depths on top of atmosphere models from Fortney et al. (2008). The models without TiO (dashed lines) are for an atmosphere without an inversion, while the TiO models (solid lines) have an inversion. The best fit to the data is the no-TiO, strong hotspot model, with $\chi^2 = 2.23$ for three degrees of freedom. However, the no-TiO, mild hotspot model has a $\Delta \chi^2$ relative to the best model of only 1.44, while the TiO, day-side redistribution model has a $\Delta \chi^2 = 2.60$, and thus these models are also consistent with the data. On the other hand, the no-TiO, day-side redistribution model is marginally excluded with $\Delta \chi^2 = 11.31$, and while the TiO, complete redistribution model is strongly excluded with $\Delta \chi^2 = 60.07$. Since the $\sim 3100$K brightness temperature that we measure for the day side is much hotter than the 2000 K condensation temperature of TiO at KELT-1b’s photospheric pressure, the lack of a strong TiO signal raises the possibility that a day-night TiO cold trap exists in KELT-1b’s atmosphere.

A day-night cold trap occurs for TiO when a planetary day side is hot enough to allow for gaseous TiO, but the night side is below the condensation temperature. This allows for TiO to condense and settle out of the atmosphere on the planetary nightside, removing it from the upper atmosphere. This is distinct from the cold traps predicted by 1D atmosphere models, which exist between a hot upper atmosphere and a hot lower convective layer as a pressure band cold enough to allow for gaseous TiO to condense (Hubeny et al. 2003; Fortney et al. 2006). Day-night cold traps were suggested as an important mechanism in hot Jupiter atmospheres by Showman et al. (2009) and more thoroughly modeled by Parmentier et al. (2013), who specifically examined the role of a cold trap in HD 209458b. Parmentier et al. (2013) found that TiO could settle out of the nightside atmosphere rapidly enough to prevent an inversion if the condensate grain size was larger than a few microns, which they found unlikely due to the relative scarcity of TiO. Parmentier et al.
did allow that if the TiO combined and condensed with a more abundant gas, such as SiO, then sufficiently large grains were much easier to form. In the case of KELT-1b, its high surface gravity may aid the efficiency of a cold-trap by increasing the particle settling velocity, and hence the settling efficiency. The dynamics of a day-night TiO cold trap may be very different on KELT-1b than on a lower surface gravity hot Jupiter.

However, as an example of the complexity of the issue, consider the presence of temperature inversions in the two other planets with surface gravities higher than $\log(g) = 4.0$ that have been observed with Spitzer: WASP-18b and HAT-P-2b. WASP-18b does not show strong evidence for a temperature inversion (Nymeyer et al. 2011), while Lewis et al. (2013) find that HAT-P-2b does have a strong inversion. Both planets have large day to night temperature contrasts (Maxted et al. 2013; Lewis et al. 2013), and both have eclipse brightness temperatures at 3.6 $\mu$m and 4.5 $\mu$m above 2100K (Nymeyer et al. 2011; Lewis et al. 2013). This implies that both planets should have a day-night TiO cold trap that inhibits inversions, though the atmospheric dynamics of HAT-P-2b are complicated by its eccentric ($e=0.52$) orbit, which causes the stellar insolation to vary considerably.

Parmentier et al. (2013) makes the intriguing suggestion that one could test for the presence and efficiency of a day-night TiO cold trap by looking for a latitude dependence in the dayside temperature structure of a planet. Atmospheric gases at

\footnote{Nymeyer et al. (2011) concludes that WASP-18b probably does have an inversion, but the authors note that their model for an inverted atmosphere is only moderately better than their non-inverted model with regards to the data ($1\sigma$ versus $1.5\sigma$). Their conclusion is partly based on WASP-18b’s measured temperature of $\sim 3200$K, and the trend for planets hotter than 2000K to have a stratospheric temperature inversion.}
higher latitudes could have a shorter nightside crossing-time, lessening the amount of TiO depletion that occurs in the cold trap. If the high-latitudes retain TiO while the equator does not, this would create a latitude-dependent inversion on the dayside of the planet. A latitudinal variation in the dayside temperature could be directly observed by using the phase-mapping technique demonstrated by Majeau et al. (2012) and de Wit et al. (2012), though this requires previous knowledge of the the longitudinal temperature gradient of the planetary dayside. This argues for obtaining 3.6 μm and 4.5 μm orbital phase curve observations of KELT-1b using Spitzer, as those observations would be the only way to directly observe the longitudinal temperature gradient.

4.4.2. From a Brown Dwarf Perspective

As a brown dwarf, KELT-1b is unusual because of the strong stellar insolation it receives, which is far in excess of its own internal luminosity. If it were isolated, we estimate that KELT-1b’s surface flux from internal heat should be approximately $10^6$ to $10^7$ erg s$^{-1}$ cm$^{-2}$ (Burrows et al. 1997), assuming the discovery paper’s age measurement of $\approx 2$ Gyr. This heat flux is two to three orders of magnitude less than the incident stellar flux of $7.8 \times 10^9$ erg s$^{-1}$ cm$^{-2}$. The dayside energy budget of KELT-1b is therefore dominated by the incident stellar radiation. Indeed, our measured brightness temperature of about 3100 K corresponds to an mid M-dwarf, even though by its mass, age and surface gravity we would expect KELT-1b to be a mid T-dwarf if it were isolated, at about 700 K (Burrows et al. 2003).

The large amount of stellar insolation relative to the internal heat flux of KELT-1b probably means that the atmospheric circulation in KELT-1b is driven by thermal forcing, similar to hot Jupiters. This is in contrast to the atmospheres of cold brown dwarfs, whose circulation is expected to by primarily driven by the
breaking of upward-welling gravity waves generated at the radiative-convective boundary. Showman & Kaspi (2013) calculate that for internal energy fluxes of $\approx 10^8 \text{ erg s}^{-1} \text{ cm}^{-2}$, which is ten to one hundred times higher than what we expect for KELT-1b based on Burrows et al. (1997), these waves ought to generate atmospheric winds of tens to hundreds of meters per second. By comparison, the thermal forcing in a typical hot Jupiter atmosphere is expected, and observationally implied, to cause wind speeds of several thousand meters per second (Knutson et al. 2007; Snellen et al. 2010). Interestingly, since KELT-1b’s rotation is presumably tidally synchronized to its orbital period and thus relatively slow, the atmospheric Rossby number is $Ro = 0.25 \left( \frac{v_{\text{wind}}}{1 \text{ km s}^{-1}} \right) \frac{1 \text{ R}_J}{L}$, where $v_{\text{wind}}$ is the wind speed and $L$ is the characteristic length scale of atmospheric flows. This is very high compared to the expected Rossby numbers of cold brown dwarfs, which should range from $Ro \approx 0.0001$ to $Ro \approx 0.01$ (Showman & Kaspi 2013). We therefore expect the atmospheric dynamics of KELT-1b to be similar to hot Jupiters.

Despite the dominance of incident stellar fluxes over internal convective fluxes, the internal flux on KELT-1b is likely to exceed that of lower-mass ($\sim 1 \text{ M}_J$) hot Jupiters. This may lead to greater generation of small-scale gravity waves in the atmosphere of KELT-1b, as compared to a typical hot Jupiter, which may produce interesting dynamical effects. Thus, while the overall dynamical regime of KELT-1b is likely to resemble those of hot Jupiters, important differences may exist too. Future observations will be necessary to assess this possibility.

4.4.3. A Combined View?

Ultimately, we would like to join observations and theories of hot Jupiters and brown dwarfs, by using KELT-1b as one link in that chain. This sort of union is already occurring for brown dwarfs and directly imaged, internal energy dominated, giant
planets using the systems discovered around HR 8799 (Marois et al. 2008) and GJ 504 (Kuzuhara et al. 2013) as well as the isolated object PSO J318-22 (Liu et al. 2013). As of yet, however, there has been no chance to do the same with irradiated Jupiters and brown dwarfs.

A comparison of irradiated Jupiters and cold brown dwarfs already points to intriguing atmospheric differences between the two populations. Figure 11 shows the the equilibrium temperature and $[3.6]–[4.5]$ color of KELT-1b relative to other brown dwarfs and planets. We measure a $[3.6]–[4.5]$ color for KELT-1b of $0.07 \pm 0.11$. This is consistent with other brown dwarfs at a similar equilibrium temperature, which generally have $[3.6]–[4.5]$ colors near zero for $T_{\text{eq}} > 1400 \text{K}$.

The strongly irradiated Jupiters show no such clear behavior with temperature across any of the temperature range. This striking diversity for the planets could be due to a variety of factors, perhaps most strongly influenced by the presence or absence of dayside temperature inversions. Other factors such as non-standard abundance ratios and scatter in Bond albedos and dayside energy redistribution may be important as well. The clear trends in brown dwarf colors with $T_{\text{eff}}$ are now well understood in terms of atmospheric chemistry (Leggett et al. 2010). At high temperatures $\text{H}_2\text{O}$ and CO absorption bands dominate the infrared spectrum. When temperatures fall below $\sim 1400 \text{K}$ carbon transitions from CO to CH$_4$, which absorbs strongly in the Spitzer 3.6 $\mu$m band. This chemistry change combined with the ever redward shift of the Planck curve to longer wavelengths explains the trend to redder $[3.6]–[4.5]$ Spitzer colors in Figure 11.

The fact that KELT-1b so strikingly resembles other brown dwarfs, despite being more like a hot Jupiter in terms of external forcing of the atmosphere by stellar irradiation, suggests that surface gravity is a very important factor in governing
the ultimate atmospheric dynamics of these bodies. The further population of this
diagram across planet temperature and mass may help identify further differences or
similarities between irradiated planets and self-luminous brown dwarfs.

4.5. Summary and Conclusions

We have measured the secondary eclipse of the highly irradiated transiting brown
dwarf KELT-1b in three bands, and found that the object's high surface gravity,
and not the high stellar irradiation, dominates KELT-1b's atmosphere. This
makes KELT-1b's atmosphere appear more similar to field brown dwarfs at the
same effective temperature, rather than to strongly irradiated hot Jupiters. These
observations are the first constraints on the atmosphere of a highly irradiated
brown dwarf. Specifically, we measure secondary eclipse depths of $0.195 \pm 0.010\%$
at $3.6 \mu m$ and $0.200 \pm 0.012\%$ at $4.5 \mu m$. We also find tentative evidence for the
secondary eclipse in the $z'$ band with a depth of $0.049 \pm 0.023\%$. From these
measured eclipse depths, we conclude that KELT-1b does not have a high heat
redistribution efficiency, and does not show strong evidence for a stratospheric
temperature inversion. Importantly, our measurements reveal that KELT-1b has a
$[3.6] - [4.5]$ color of $0.07 \pm 0.11$, identical to that of isolated brown dwarfs of similarly
of $\sim 0.4$, with a very large range from $\sim 0$ to $\sim 1$.

KELT-1b gives us the chance to study a high surface gravity atmosphere using
all of the tools that have been developed to measure the dynamics in hot Jupiter
atmospheres (e.g, HD 189733b by Knutson et al. 2012). For the first time we will be
able to directly observe large scale atmospheric dynamics and flows in a high surface
gravity environment. Already, our secondary eclipse observations demonstrate that
there is almost no heat redistribution from the day to the night side of KELT-1b,
and suggest that there may be a global hotspot at the substellar point. This implies that the radiative timescale in the atmosphere is extremely short relative to the relevant dynamic timescale in the atmosphere.

By serving as a transitional object between cold brown dwarfs and hot giant planets, more detailed observations of KELT-1b will have a strong ability to illuminate the similarities and differences between these two populations. In the near-term, Spitzer observations of KELT-1b’s orbital phase curve would provide the best extension to our current understanding of the object’s atmosphere, by directly measuring the day-night temperature contrast and the presence of large scale flows in the atmosphere. Transmission spectroscopy of KELT-1b is currently impossible due to KELT-1b’s high surface gravity, though longer-term JWST may be able to conduct these observations. Finally, our tentative detection of the eclipse in $z'$ demonstrates the relatively unique opportunities for secondary eclipse observations from the ground that this system affords us – particularly with modest telescopes.
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### Planetary Parameters:

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<td>$e \sin \omega_*$</td>
<td></td>
<td>$0.0041^{+0.011}_{-0.0062}$</td>
</tr>
<tr>
<td>$\log g_P$</td>
<td>Surface gravity</td>
<td>$4.736^{+0.017}_{-0.025}$</td>
</tr>
<tr>
<td>$T_{\text{eq}}$</td>
<td>Equilibrium temperature (K)</td>
<td>$2432^{+34}_{-27}$</td>
</tr>
<tr>
<td>$\langle F \rangle$</td>
<td>Incident flux ($10^9$ erg s$^{-1}$ cm$^{-2}$)</td>
<td>$7.83^{+0.45}_{-0.34}$</td>
</tr>
</tbody>
</table>

### Primary Transit Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_P/R_*$</td>
<td>Radius of the planet in stellar radii</td>
<td>$0.07806^{+0.00061}_{-0.00058}$</td>
</tr>
<tr>
<td>$a/R_*$</td>
<td>Semi-major axis in stellar radii</td>
<td>$3.619^{+0.055}_{-0.087}$</td>
</tr>
<tr>
<td>$i$</td>
<td>Inclination (degrees)</td>
<td>$87.6^{+1.4}_{-1.9}$</td>
</tr>
<tr>
<td>$b$</td>
<td>Impact parameter</td>
<td>$0.150^{+0.11}_{-0.088}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Transit depth</td>
<td>$0.006094^{+0.000096}_{-0.000090}$</td>
</tr>
<tr>
<td>$T_{FWHM}$</td>
<td>FWHM duration (days)</td>
<td>$0.10645 \pm 0.00045$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Ingress/egress duration (days)</td>
<td>$0.00873^{+0.00049}_{-0.00020}$</td>
</tr>
<tr>
<td>$T_{14}$</td>
<td>Total duration (days)</td>
<td>$0.11526^{+0.00069}_{-0.00059}$</td>
</tr>
</tbody>
</table>

Table 4.1. Relevant Discovery Paper Values
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Measured Parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_0$</td>
<td>Baseline flux</td>
<td>$1.001042 \pm 0.000068$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>X-position linear coefficient</td>
<td>$0.0015 \pm 0.0037$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>Y-position linear coefficient</td>
<td>$-0.1629 \pm 0.0028$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>Time linear coefficient</td>
<td>$0.00623 \pm 0.00080$</td>
</tr>
<tr>
<td>$T_C$</td>
<td>Time of nearest transit (BJD$_{TDB}$)</td>
<td>$2456180.7969 \pm 0.0022$</td>
</tr>
<tr>
<td>$\log(P)$</td>
<td>Log orbital period (days)</td>
<td>$0.0854741 \pm 0.000053$</td>
</tr>
<tr>
<td>$\sqrt{e}\cos\omega$</td>
<td></td>
<td>$-0.025^{+0.032}_{-0.030}$</td>
</tr>
<tr>
<td>$\sqrt{e}\sin\omega$</td>
<td></td>
<td>$0.001^{+0.072}_{-0.085}$</td>
</tr>
<tr>
<td>$\cos i$</td>
<td>Cosine of inclination</td>
<td>$0.059^{+0.020}_{-0.023}$</td>
</tr>
<tr>
<td>$R_P/R_*$</td>
<td>Radius of planet in stellar radii</td>
<td>$0.07807 \pm 0.00058$</td>
</tr>
<tr>
<td>$\log(a/R_*)$</td>
<td>Log semi-major axis in stellar radii</td>
<td>$0.5671^{+0.0058}_{-0.0062}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Eclipse depth</td>
<td>$0.00195 \pm 0.00010$</td>
</tr>
<tr>
<td><strong>Derived Parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>Orbital period (days)</td>
<td>$1.217514 \pm 0.000015$</td>
</tr>
<tr>
<td>$T_S$</td>
<td>Time of eclipse (BJD$_{TDB}$)</td>
<td>$2456181.40403^{+0.00075}_{-0.00096}$</td>
</tr>
<tr>
<td>$a/R_*$</td>
<td>Semi-major axis in stellar radii</td>
<td>$3.691 \pm 0.051$</td>
</tr>
<tr>
<td>$i$</td>
<td>Inclination (degrees)</td>
<td>$86.6 \pm 1.2$</td>
</tr>
<tr>
<td>$b$</td>
<td>Impact Parameter</td>
<td>$0.220^{+0.070}_{-0.084}$</td>
</tr>
<tr>
<td>$T_{FWHM}$</td>
<td>FWHM duration (days)</td>
<td>$0.1037 \pm 0.0020$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Ingress/egress duration (days)</td>
<td>$0.00874^{+0.00029}_{-0.00024}$</td>
</tr>
<tr>
<td>$T_{14}$</td>
<td>Total duration (days)</td>
<td>$0.1125 \pm 0.0020$</td>
</tr>
<tr>
<td>$e\cos\omega$</td>
<td></td>
<td>$-0.0018^{+0.0020}_{-0.0031}$</td>
</tr>
<tr>
<td>$e\sin\omega$</td>
<td></td>
<td>$0.0000^{+0.0059}_{-0.0077}$</td>
</tr>
<tr>
<td>$e$</td>
<td>Orbital Eccentricity</td>
<td>$0.0050^{+0.0081}_{-0.0036}$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Argument of periastron (degrees)</td>
<td>$10^{+120}_{-140}$</td>
</tr>
</tbody>
</table>

Table 4.2. Median values and 68% confidence intervals for the 3.6 $\mu$m eclipse
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_0$</td>
<td>Baseline flux</td>
<td>$1.000885^{+0.000070}_{-0.000079}$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>X-position linear coefficient</td>
<td>$-0.0524 \pm 0.0044$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>Y-position linear coefficient</td>
<td>$0.0434 \pm 0.0039$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>Time linear coefficient</td>
<td>$0.0020 \pm 0.0013$</td>
</tr>
<tr>
<td>$T_C$</td>
<td>Time of nearest transit (BJD$_{TDB}$)</td>
<td>$2456182.0142 \pm 0.0023$</td>
</tr>
<tr>
<td>$\log(P)$</td>
<td>Log orbital period (days)</td>
<td>$0.0854739 \pm 0.0000053$</td>
</tr>
<tr>
<td>$\sqrt{e} \cos \omega$</td>
<td></td>
<td>$-0.038^{+0.034}_{-0.029}$</td>
</tr>
<tr>
<td>$\sqrt{e} \sin \omega$</td>
<td></td>
<td>$0.027^{+0.074}_{-0.082}$</td>
</tr>
<tr>
<td>$\cos i$</td>
<td>Cosine of inclination</td>
<td>$0.044 \pm 0.023$</td>
</tr>
<tr>
<td>$R_P/R_*$</td>
<td>Radius of planet in stellar radii</td>
<td>$0.07806^{+0.00058}_{-0.00060}$</td>
</tr>
<tr>
<td>$\log(a/R_*)$</td>
<td>Log semi-major axis in stellar radii</td>
<td>$0.5600 \pm 0.0065$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Eclipse depth</td>
<td>$0.00200 \pm 0.00012$</td>
</tr>
<tr>
<td>$P$</td>
<td>Orbital period (days)</td>
<td>$1.217514 \pm 0.000015$</td>
</tr>
<tr>
<td>$T_S$</td>
<td>Time of eclipse (BJD$_{TDB}$)</td>
<td>$2456182.62027^{+0.00099}_{-0.00115}$</td>
</tr>
<tr>
<td>$a/R_*$</td>
<td>Semi-major axis in stellar radii</td>
<td>$3.631 \pm 0.054$</td>
</tr>
<tr>
<td>$i$</td>
<td>Inclination (degrees)</td>
<td>$87.5 \pm 1.3$</td>
</tr>
<tr>
<td>$b$</td>
<td>Impact Parameter</td>
<td>$0.160^{+0.077}_{-0.082}$</td>
</tr>
<tr>
<td>$T_{FWHM}$</td>
<td>FWHM duration (days)</td>
<td>$0.1063 \pm 0.0022$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Ingress/egress duration (days)</td>
<td>$0.00877^{+0.00026}_{-0.00022}$</td>
</tr>
<tr>
<td>$T_{14}$</td>
<td>Total duration (days)</td>
<td>$0.1151 \pm 0.0023$</td>
</tr>
<tr>
<td>$e \cos \omega$</td>
<td></td>
<td>$-0.0032^{+0.0030}_{-0.0033}$</td>
</tr>
<tr>
<td>$e \sin \omega$</td>
<td></td>
<td>$0.0014^{+0.0096}_{-0.0055}$</td>
</tr>
<tr>
<td>$e$</td>
<td>Orbital Eccentricity</td>
<td>$0.0065^{+0.0081}_{-0.0045}$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Argument of periastron (degrees)</td>
<td>$97^{+43}_{-230}$</td>
</tr>
</tbody>
</table>

Table 4.3. Median values and 68% confidence intervals for the 4.5 μm eclipse
Fig. 4.1.— The X position, Y position, FWHM, and raw aperture photometry for the 3.6 μm observations. The photometry is for our chosen optimal aperture for the 3.6 μm data, as described in Section 2.1.
Fig. 4.2.— The X position, Y position, FWHM, and raw aperture photometry for the 4.5 μm observations. The photometry is for our chosen optimal aperture for the 4.5 μm data, as described in Section 2.1.
Fig. 4.3.— The variation of the measured eclipse depth (black, solid, and the left axis) and RMS of the residuals to the best fit (red, dashed, and the right axis) as a function of aperture size for the 3.6 $\mu$m data. The blue dotted lines show our final result for the eclipse depth at 3.6 $\mu$m and the 1$\sigma$ error bars. The vertical green dot-dashed line shows the median value of $\sqrt{\beta}$, the approximate stellar FWHM.
Fig. 4.4.— The variation of the measured eclipse depth (black, solid, and the left axis) and RMS of the residuals to the best fit (red, dashed, and the right axis) as a function of aperture size for the 4.5\(\mu\)m data. The blue dotted lines show our final result for the eclipse depth at 4.5\(\mu\)m and the 1\(\sigma\) error bars. We were not able to adequately determine the cause of the bump around 2.3 pixels (see the end of Section 2.1 in the text). The vertical green dot-dashed line shows the median value of \(\sqrt{\beta}\), the approximate stellar FWHM.
Fig. 4.5.— The best fit linear and quadratic decorrelation parameters for the 3.6 μm data as a function of aperture radius. The black line shows the variation in the fitted baseline flux (i.e., $F_0$ in Equation 4.2) and is measured on the right axis, while the dashed blue and red lines show the quadratic decorrelation parameters for x- and y-position, respectively, and are measured on the left axis. The variation in the quadratic parameters indicates that they are not well constrained by our data. We use a linear decorrelation for our fits.
Fig. 4.6.— The best fit linear and quadratic decorrelation parameters for the 4.5 $\mu$m data as a function of aperture radius. The black line shows the variation in the fitted baseline flux (i.e., $F_0$ in Equation 4.2) and is measured on the right axis, while the dashed blue and red lines show the quadratic decorrelation parameters for x- and y-position, respectively, and are measured on the left axis. The variation in the quadratic parameters indicates that they are not well constrained by our data. We use a linear decorrelation for our fits.
Fig. 4.7.— Our final, detrended, 3.6 μm and 4.5 μm lightcurves. The black overplotted points with error bars are the detrended data median binned into 50 points, while the red solid lines show our final best fit models to the eclipses.
Fig. 4.8.— Constraints on the eclipse depth in $z'$ from our ground-based observations. Though we observed seven eclipses, only four of the nights provided high-quality, complete observations. The black line shows the joint constraint on the eclipse depth from all four nights. This assumes that the observational errors are uncorrelated from night to night.
Fig. 4.9.— All four of the lightcurves used to calculate the constraint on the eclipse depth in $z'$. The bottom panel shows all four phased and overplotted. Each lightcurve has also been linearly detrended against airmass and time. The black points are binned versions of the individual and combined lightcurves, while the red line in the bottom panel is the best-fit eclipse model. We marginally detect the eclipse in $z'$ with a depth of $0.049 \pm 0.023\%$. 
The atmosphere models are based on Fortney et al. (2008), and are divided according to the presence of gaseous TiO and the amount of heat redistribution from the day to night side. The ‘TiO’ models have stratospheric temperature inversions, while the ‘no TiO’ models do not. The ‘hotspot’ models are scenarios wherein the heat from the stellar insolation is redistributed over only a portion of the planetary day side. In the ‘strong hotpot’ this redistribution area is smaller than in the ‘mild hotspot’ model. The colored triangles show the predicted flux ratios from each of the models in the three bandpasses.

Fig. 4.10.— Our measured planet-to-star flux ratios at 3.6 $\mu$m, 4.5 $\mu$m and in $z'$. The atmosphere models are based on Fortney et al. (2008), and are divided according to the presence of gaseous TiO and the amount of heat redistribution from the day to night side. The ‘TiO’ models have stratospheric temperature inversions, while the ‘no TiO’ models do not. The ‘hotspot’ models are scenarios wherein the heat from the stellar insolation is redistributed over only a portion of the planetary day side. In the ‘strong hotpot’ this redistribution area is smaller than in the ‘mild hotspot’ model. The colored triangles show the predicted flux ratios from each of the models in the three bandpasses.
Fig. 4.11.— Spitzer IRAC colors as a function of equilibrium temperature for planets and brown dwarfs. We calculated $T_{\text{eq}}$ for the brown dwarfs based on their spectral type and the empirical conversion from spectral type to temperature from Stephens et al. (2009). The brown dwarf colors and spectral types are from Patten et al. (2006), Leggett et al. (2010) and Kirkpatrick et al. (2011). The planetary colors are calculated using secondary eclipse depths listed on exoplanets.org and transforming those into fluxes using the $T_{\text{eff}}$ of the host star and assuming the host is a blackbody. The planetary $T_{\text{eq}}$ values are calculated assuming zero albedo and perfect heat redistribution. The downward arrow is the upper limit for GJ 436b, which has no detected 4.5 $\mu$m eclipse (Stevenson et al. 2010).
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