Systematic Optimization and Control Design for Downsized Boosted Engines with Advanced Turbochargers

THESIS

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Abstract

Downsized boosted engines are a promising solution to increase fuel economy and reduce CO$_2$ emissions. With the arrival of several advanced charging technologies, the complexity of matching an engine to charging systems to achieve high levels of boost pressure over a wide range of engine speed increases considerably. Besides, the air-path control of advanced boosting systems is more challenging due to the presence of multiple actuators. This research aims at developing a methodology for systematic optimization and control design for downsized boosted engines with advanced boosting systems.

The objective motivates the development of parametric models for families of compressors and turbines, considering the effects of geometric parameters on the flow and efficiency characteristics. The parametric modeling approach is used to replace an existing map-based model of the two-stage turbocharger of a Diesel engine simulator, to provide a validation against full engine test data.

Next, a study is conducted to assess the opportunity of controlling the air-path of Diesel engine with two-stage turbochargers to increase the exhaust gas energy upstream of the aftertreatment system. The problem is formulated as model predictive control tracking problem, and the controller and observer are designed on a piecewise affine system. The effects of different weights in cost function are compared, and a Pareto front is obtained. Finally, a heuristic control is developed based on MPC results.

As main contributions, the parametric model of turbocharger and model predictive control of air-path will serve as ground work for future research on systematic optimization and control design of engines with advanced boosting systems.
This document is dedicated to my parents.

I would not be who I am today without them.
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Chapter 1

Introduction

1.1 Background

The automotive industry is currently facing the challenge of meeting the conflicting goals of improving vehicle fuel economy, complying to stringent emission standards, satisfying the customer appeal for performance and keeping affordable costs. In the US, the Corporate Average Fuel Economy (CAFE) standards have been increased over time to 26.0 mpg by 1985, to 27.5 mpg by 1989, and to 37.8 mpg by 2016 in the passenger car fleet [1,2]. National Highway Traffic Safety Administration has recently developed new footprint-based CAFE standards for both passenger car and light-duty truck fleets [3]. These standards propose to double the required fleet fuel economy by 2025. As electric vehicles and fuel cell technologies present some deficiencies and are immature to be largely spread in mass production, internal combustion engines will remain the mainstay of growing mobility over the next 20 years. As a result, automotive companies will heavily rely on engine technology improvements to meet fuel economy improvement and emissions reduction goals.

These technology improvements have been realized in different engine fields. First, important progress has occurred in engine control. With the advance of electronic devices, the quantity and quality of sensors and actuators has progressively grown, and control strategies have become considerably sophisticated [4]. The injection and combustion processes have also been largely modified with the arrival of common rail systems [5]. These systems are able to realize multiple injections per engine stroke and present high flexibility to optimize the injection timings.
Aftertreatment systems have become necessary for Diesel engines [6]. Diesel oxidation catalyst (DOC) is implemented to reduce the emissions of CO, total hydrocarbon emissions, and volatile organic fractions [7]. Diesel particulate filter (DPF) is introduced to achieve extremely low particulate matters emissions [8]. Lean NO\textsubscript{x} traps (LNT) have proved to be a promising technique for reducing the NO\textsubscript{x} emissions from light-duty Diesel engines, while selective catalytic reduction (SCR) systems, especially urea-SCR systems, have become the leading devices to reduce tailpipe NO\textsubscript{x} emissions from medium- to heavy-duty Diesel engines [9]. Meanwhile, exhaust gas recirculation (EGR) has become common practice to meet the mandatory NO\textsubscript{x} emissions reductions [10]. EGR works by recirculating a portion of exhaust gas back to the intake manifold. Diluting the fresh air charge with exhaust inert gas lowers the peak combustion temperatures and the generation of NO\textsubscript{x}.

Air-path also has considerable improvements with the use of flexible valve actuation and turbocharging. Variable valve timing (VVT) is able to independently control the intake and exhaust valve opening to optimize the cylinder charge and residual gas fraction [11]. Similarly, variable valve actuation (VVA) provides more flexibility for engine performance optimization, due to the ability of controlling valve timing and lift [12]. Exhaust gas turbochargers increases the engine specific power by forcing even more air to enter into the cylinders. With higher air density in the combustion chamber, engine performance and specific fuel consumption have been improved. Variable geometry turbine (VGT) has gradually replaced fixed geometry turbine with a waste-gate. By varying the swallowing capacity of a turbine as the engine operating, VGT could increase useful engine operating speed range and reduce turbo-lag [13].

All these technological developments have been equipped in modern Diesel or SI engine to meet the strict emissions regulations requirements. However, their potential improvements to achieve the next standards (especially NO\textsubscript{x} and CO\textsubscript{2}) are insufficient and new technologies have to be developed.

Two technologies will be applied to reduce NO\textsubscript{x} emissions. First, more sophisticated
alternative combustion modes such as low temperature combustion (LTC), homogeneous charge compression ignition (HCCI), will be employed at low speed and low load regions [14, 15]. Second, low pressure (LP) EGR systems, extracting exhaust gases after the DPF to recirculate them at the compressor inlet, will arrive in complement or substitution of the conventional high pressure (HP) EGR systems [16].

To reduce CO₂ emissions, downsized boosted engines are the most promising solution currently followed by the major automakers. By reducing the engine displacement, lower fuel consumption is achieved while still making use of conventional aftertreatment systems. At the same time, engine peak performance and driveability are retained through the use of boosting solutions. Several studies have shown that 20% fuel economy improvement can be achieved with a 40% downsizing and turbocharging. The addition of advanced fuel management strategies, such as fuel cut off during idling and coasting, increases the fuel economy to about 30%, matching the performance of full hybrid vehicles without the added weight and cost penalties [17–19].

An important requirement for boosting system is to keep high pressure level over a wide range of engine speed. The high boost pressure required for the rated power can only be produced by a larger device, while the turbocharger design needs to be small to achieve a high boost pressure at low speeds. To solve this conflict, the state of art is the well-known method of designing the turbocharger with VGT. However, these systems are unable to provide the higher boost and wider operating range required by the downsized engine. So, advanced charging systems will be introduced for further engine development.

### 1.2 Motivations

In single-stage arrangements, turbochargers have been the dominant technology for engine charging system, taking almost 100% of the market share, because of its high efficiency and ability to recover waste energy. With the arrival of multistage systems, the chargers
can be arranged in series or parallel, working together or sequentially. Even though the main charger is generally a turbocharger, the second one can be turbocharged, mechanically assisted (supercharger) or electrically assisted. Variable geometry elements and an additional intercooler between stages can also be considered to enhance the performance. Depending on the control requirements, multiple actuators could be used to regulate the boost pressure. All these possibilities represent a large number of configurations and no standard solutions exist for all the different applications. In order to optimize the performance of the downsized engine, various boosting architectures should be analyzed and compared, including the component designs (sizing).

The system complexity and interactions between subsystems make the control of advanced boosting systems challenging. For instance, in a series/sequential arrangement, mode-switching control is required to improve the robustness and smoothness of response in transitions between one charger and the other. For the case of electrically assisted chargers, system-level energy optimization needs to be designed to optimize the energy utilization of the system. Therefore, there is a need to develop a systematic methodology for system optimization and control design for downsized engines with advanced boosting systems.

For typical automotive applications, the sizing and optimization of the engine system configuration and the control system optimization are usually performed in sequence. Although the approach of decoupling the engine hardware design from the control software design phase substantially simplifies the overall engineering process, a sequential procedure may not provide favorable performances, as the optimality for one single problem could be disrupted by another. Although more challenging, a combined optimization method, where the engine air-path system design and the control algorithm are simultaneously optimized, is more appealing. Such an approach would allow one to integrate individual problems (design and control) together, through multi-objective optimization.

To enable a coupled and systematic model-based optimization, parametric models for the boosting device components, such as compressor and turbine, are of critical importance.
These models will allow one to systematically evaluate the influence of design parameters on a set of performance objectives, particularly in cases where experimental data are not easily accessible. Moreover, considering the complexity of the optimization, the parametric models should be of low computational load, easy for the calibration and accurate enough at least within the range of interest for the geometric parameters.

1.3 Objectives

The content of this thesis is developed as part of a larger research project with General Motors Corporation oriented to explore model-based system optimization and control design for advanced engine charging systems, with a research approach that combines thermo-fluid modeling, engine/powertrain system simulation, and the use of estimation and control design methods for linear and nonlinear systems.

In this project, the model-based system-level optimization and control design methodology will be applied to a series/sequential two-stage turbocharger (2ST) system. The specific objectives which are pursued in the current period are:

a. Development of a parametric model that can represent the performance of a family of turbochargers based on key geometric parameters.

b. Control design of 2ST engine system for rapid warm-up of the aftertreatment system.

c. Co-optimization of system design and control for 2ST configurations to improve the system performance in steady-state and transient conditions.

d. Design of robust mode-switching control strategies between two stages to guarantee smooth boost response during transitions.

According to the progress of this ongoing project, this Master’s thesis will focus on the objectives a and b. The remaining tasks will be part of the PhD research program of the author.
1.4 Outline

This thesis is organized as follows. Chapter 2 contains an exhaustive literature review on several relative topics. First, the state of the art of advanced boosting systems is explored. Then, several analytical modeling approaches for turbochargers are evaluated. Next, various control approaches for engine air-path systems as well as integrated engine and aftertreatment systems are summarized. Finally, all existing works on combined design and control optimization are briefly introduced.

In Chapter 3, the development of parametric model for the compressor and turbine maps is described. First, a brief overview of basic turbomachinery theory is given, where the dimensionless parameters and relevant geometric parameters for the characterization of flow and efficiency maps are defined. Then, a formal derivation of the parametric maps for families of compressors and turbines is discussed together with the model results. The parametric modeling approach is applied to an engine model with 2ST, which is validated against steady-state and transient experimental data.

Chapter 4 presents the design and application of an air-path control strategy of the 2ST Diesel engine for rapid catalyst warm-up, based on model predictive control (MPC). First, thermal models for waste-gate and catalyst are introduced. Preliminary study on the FTP city driving cycle is conducted to analysis the influence of bypass opening on catalyst warm-up. Then, model order reduction and linearization are applied, resulting in a discrete-time piecewise affine (PWA) system. The rapid warm-up problem is formulated as MPC tracking control, and controller and state observer are designed on the PWA system. Finally, MPC and observer are implemented on the full order nonlinear engine model, and a heuristic control for bypass opening is extracted from the solution of MPC.

The thesis ends with conclusions and recommendations for future research.
Chapter 2

Literature Review

2.1 Advanced Boosting Systems

2.1.1 Single-Stage Turbocharging Systems

Turbocharger technology history can be presented through the evolution of compressor and turbine over the last decades. The exhaust-gas turbocharger with variable geometry turbine (VGT) has been widely adopted for Diesel engines. VGT is used to control a desired boost pressure over the engine operating range, which is achieved by guide vanes arranged upstream of the turbine rotor, as shown in Figure 2.1. Many research papers are available on the topic of the variable geometry turbines for Diesel engines [13,20–22]. Even though the spark ignition (SI) engine benefits from this technology, the high exhaust gas temperature have prevented the variable geometry turbine from being widely used [23].

![Figure 2.1: Variable Geometry Turbine [24]](image)

Figure 2.1: Variable Geometry Turbine [24]
Wide compressor operating ranges are a challenge for a single-stage compressor with high pressure ratio, since it is limited by surge at low flow rates and choke at high flow rates. More recently, variable geometry compressor (VGC) systems have been considered as a possibility to improve the compressor performance and stability. Centrifugal compressors can be equipped with either variable inlet guide vanes [25–29] or with a variable geometry diffuser [30,31]. Experimental and simulation results have shown improvements in efficiency and pressure ratio over fixed geometry compressor at low speed conditions, in conjunction with the opportunity to expand the operating range in terms of surge and choke limits.

2.1.2 Two-Stage Turbocharging Systems

Two-stage turbocharging is a generic term used to describe the installation of two turbines and compressors to boost a single engine installation. This technology has excellent transient response and it is capable of meeting the needs of current and future downsizing options. The variations of such installations are numerous and there is an issue with the nomenclature used by various OEMs which are explained below.

Parallel Boosting Systems

In parallel boosting systems, also called twin turbo, identically-sized turbochargers are used, each fed by a separate set of exhaust streams from the engine. The pressurized air from both compressors is added up in a common manifold and passed through an intercooler and then fed into intake manifold [32,33]. A typical layout of a twin turbo fitted on a V6 engine is shown in the left of Figure 2.2. The main advantage of twin turbochargers systems when compared with single turbocharger systems is the turbo-lag phenomenon diminishes considerably. But it does not improve the surge line and the torque at low speed operations.

One solution is to operate only with one turbo at low engine speed and operate both in parallel connection in high speed operation, which is called sequential parallel turbocharging as shown in Figure 2.2 on the right side. Since the sizes of two turbochargers are not
necessarily the same in parallel sequential case, this kind of system is able to improve Diesel engine transient response since low inertia turbocharger is used at low speed.

Galindo et al. [34–36] applied the parallel sequential boosting system on a 2.2L modern passenger-car Diesel engine to reach similar performance in terms of acceleration capability and low end torque as a 2.7L single-stage turbo equivalent engine. Turbocharger matching for the parallel sequential turbocharging system is discussed in [34]. A control system decides when to operate in each one of the modes, and when and how to perform the transition based on engine steady-state operation as well as transient conditions (acceleration, braking, gearbox shifts), is proposed in [35]. To smooth the torque oscillations during mode transition, fuel correction strategy and control valve pre-lift strategy are analyzed in [36].

Compared to serial arrangement, there is an advantage in carbon monoxide (CO) and hydrocarbon emissions because of a reduced gas contact surface and only one expansion stage. This allows lower heat losses in the turbochargers and a higher temperature at cata-
lyst entrance during the start up phase. A major disadvantage of the parallel arrangement is that, since the differently sized compressors share the same pressure ratio, the boost level is limited by the maximum pressure ratio of the smaller turbocharger \([37]\). Nevertheless, the parallel arrangement can cover a higher flow rate range. This system is suitable for SI engine which require wider flow range but lower boost pressure \([38]\).

**Series Boosting Systems**

Serial turbocharging systems were primary designed to increase the absolute boost pressure in power production and marine applications. For automotive applications, flexibility of operation rather than absolute gain in performance is generally preferred. In 1998, Pfliiger was the first to introduce a regulated 2-stage turbocharging system for commercial Diesel engines \([39]\). These objectives were not only to increase the engine rated power but also make available a very high maximum torque at very low engine speeds and over a wide speed range.

The basic set-up of series charging system, schematized in Figure 2.3, consists of two turbochargers connected in series with a bypass valve placed across the high pressure turbine. The larger low pressure (LP) turbine is matched to set the maximum power, while the
small high pressure (HP) turbine is matched to fulfill the low end torque requirements and to reduce the turbo-lag. At low engine speeds, the bypass is closed and the low exhaust gas energy is mainly used in the high pressure stage to compress fresh air. As the air flow rate increases, the capacity of the HP stage will be approached and the flow will consequently become choked. Therefore, in order to avoid excessive back pressures and overspeed of the turbocharger shaft, the HP stage is normally bypassed at high flow rates [39,41].

Variable geometry turbine can enhance even more the regulated 2-stage charging system performance. Lee et al. [41] replaced the HP fixed geometry turbine with its waste-gate by a VGT. They claims the system with VGT can improve both steady-state and transient performance. Canova et al. [42] compared two 2-stage turbocharger configurations, with different VGT locations. The results show that the proposed new configuration, which has a HP stage with bypass and LP stage with VGT, can achieve higher control flexibility.

For passenger car applications, Schmitt et al. [40,43,44] modified the regulated 2-stage system to improve the transient response. The system, schematized in the right of Figure 2.3, has been extended with a waste-gate across the LP turbocharger and an adequately large cross-section bypass valve across the HP compressor. The advantage of this arrangement is the flow can be fully diverted around the HP stage at high engine speeds, leaving the rotating parts to achieve just a basic speed level necessary for quick changes in load. Therefore these elements make possible the use of turbines with low absorption capacity and smaller compressors, allowing a high boost pressure to be achieved at a very early point and improving the transient response. More applications on passenger car can be seen in [45–48].

2.1.3 Electric Boosting Systems

Electric boosting systems have been used in two ways, either by having the electric motor attached directly on the turbocharger shaft, or by having an extra electrically driven compressor supporting the main turbocharger at low loads and during transients.
Electrically Assisted Turbochargers

The electrically assisted turbocharger, or EAT, is a turbocharger fitted with an electric motor/generator around the turbocharger shaft and between the compressor and turbine housings, as depicted in the left side of Figure 2.4. The impact of a EAT system has previously been shown to improve acceleration performance, reduced turbo-lag and therefore improve regulation of boost demand, reduce soot emissions, and improve fuel economy [49–53].

Using electrical energy, additional torque can be applied to the turbocharger shaft when there is not much exhaust gas energy available. This results in faster acceleration of the turbocharger during transient operations and increased boost pressure at low speed operation. When the available turbine power is higher than the power necessary to drive the compressor, the EAT system can be operated in generator mode using part of the turbine work to recover electric energy. This technique is called electric turbocompound.

Kolmanovsky et al. [49–51] applied optimal control techniques to a medium-size passenger car with a EAT system capable of bi-directional energy transfer between the turbocharger shaft and energy storage. The main objective is to optimize the transient response during full load accelerations. In [50], it is found that the system with 1.5kW EAT can achieve same performance and similar fuel consumption as with 10.3kW hybrid version. Similar conclusion is reported in [53].

Electric Boosters

The other way of using electric assistance is to have an extra centrifugal compressor driven by an electric motor in series with the main turbocharger. This electric supercharger, given in the left of Figure 2.4, is also called electric booster or eBooster. Since the eBooster is completely independent from the turbocharger, there is no way to utilize electric turbocompound. Depending on the amount of electrical energy available from the
Electric boosting systems have been used in two ways, either by having the electric motor attached directly on the turbocharger shaft or by having an extra electrically driven compressor installed in series with the turbocharger. Both arrangements are described in the following subsections.

### 2.4.1 Electrically Assisted Turbochargers

The electrically assisted turbocharger, generally called eu-ATL or EAT, consists of a standard wastegate or variable geometry turbine turbocharger with an additional high speed electric motor. Using electrical energy, additional torque can be applied to the turbocharger shaft when there is not much exhaust gas energy available. This results in either faster acceleration of the turbocharger during transient operations or increased boost pressure under low end operating conditions. Other functions such as maintaining turbocharger speed during gearshift to reduce emissions \[193\], or compressing air before cranking (intake air over 100 \(^\circ\)C) to improve cold starting can also be realized with the electrical assistance.

![Figure 2.32: The Turbostarter by Turbodyne System Inc. (Left) and Diagram of the Electrically Driven Turbocharger (Right)](image)

The first EAT system called Turbostarter was presented in 1995 by Turbodyne System Inc. \[253\]. The system was equipped with an external brushless d-c motor with forced oil cooling in the stator and forced air cooling in the rotor \[252\], see figure 2.35. Rated power was also slightly reduced at 2 kW. Results obtained on a 1.7l turbocharged engine showed engine torque could be increased by approximately 17% at 1000 and 1200 rpm with turbo spool-up time cut down by 33% \[164, 375\].

### 2.4.2 Electric Boosters

Another way of using electric assistance is to have an extra radial flow compressor driven by an electric motor in series with the turbocharger. This electric supercharger is called eBooster. It can be placed in the intake air path before or after the turbocharger, but placing it before provides more flexibility in terms of packaging and some minor advantages in terms of power. To keep the pressure losses as low as possible, a regulated bypass valve is added around the eBooster to completely divert the air mass flow when it is switched off. Valve actuation is supervised by appropriate control strategies to avoid flow recirculation during valve opening \[217\]. A diagram of the serial configuration with a view of an eBooster is given in figure 2.36.

![Figure 2.36: eBooster Charging System](image)

The eBooster is completely independent from the turbocharger and the thermal energy of the exhaust gases. Thermomechanical loads are relatively low and there is no way to utilize an energy recovery system due to the separation of the turbine and the electric motor. Depending on the amount of electrical power, the eBooster can be used at low engine speeds as a regulated two-stage charging system to increase the steady-state torque, or only in transient to reduce turbo-lag effects.

Lefebvre et al. \[55\] compared a 3-cylinder 1L turbocharged SI engine with eBooster to a baseline 1.8L naturally aspirated engine. With a 1.6kW eBooster, the transient response and low end torque are largely improved. Moreover, the eBooster can make the main compressor operate in areas of higher efficiency and higher surge margin.

Tavcar et al. \[56\] compared different electric boosting systems, including EAT, eBooster, and electrically split turbocharger. Results show that the engine utilizing an EAT can improve transient engine performance, and also possesses the potential to increase high-speed torque for a limited period of time. While eBooster system has highest low-speed torque and the fastest transient response during acceleration.

### 2.2 Analytical Model for Turbocharger

In control-oriented engine models, a turbocharger is usually modeled with a data-driven (black-box) approach. However, only a limited fraction of the compressor and turbine operating domain is typically available from a turbocharger supplier. Besides, standard table interpolation routines are not continuously differentiable, extrapolation is unreliable

![Figure 2.4: Electrically Assisted Turbocharger (Left) and eBooster Charging System (Right)](image)
and the table representation is not compact [57]. Several modeling approaches have been proposed, ranging from curve-fitting algorithms to more complex approaches in part based on physical considerations. Physics-based models have been also developed to predict the performance of turbocharger, but they are not well suited to control-oriented engine models.

### 2.2.1 Compressor Model

For compressor mass flow and efficiency models, dimensionless representation is often used for the performance characterization. Jensen-Kristensen method [57], which is widely adopted, used a hyperbolic function to fit the head parameter and flow parameter. This model provide high accuracy in interpolating characteristic maps, but the ability to extrapolate at surge and choke regions is limited. Eriksson et al. proposed an ellipsis model [58], which can be extended to describe reversed flow, surge, normal operation, choke and restriction [59,60]. In contrast, when the turbocharger is mainly operate at a particular range rather than the entire map, a simplified third-order polynomial model is developed in [61].

The compressor isentropic efficiency is typically fitted as a quadratic function of the dimensionless flow parameter. In [57], the coefficients of the quadratic function is fitted as hyperbolic function of blade Mach number, while a compact quadratic form is proposed in [58]. Similarly, a simplified linear relation of new defined efficiency and flow parameter is presented in [61]. These kind of models have limited extrapolation ability. A minimum constant efficiency is imposed to prevent negative value at low mass flow region. To address this issue, a new approach incorporating turbomachinery physics is developed in [62], where the extrapolation quality is ensured by physical laws.

### 2.2.2 Turbine Model

The mass flow rate through the turbine can be modeled as isentropic orifice flow, where the effective flow area is a function of the VGT position and pressure ratio [57,62,63]. In actual turbine, where the expansion is produced in two steps, critical flow conditions are
reached for an expansion rate of approximately 3, whereas a nozzle reaches choke conditions with an expansion rate of approximately 1.89. So the standard orifice flow model is not accurate in choked flow conditions. Although a model based on two nozzles in series [64, 65] can solve this issue, such wave action model is not suited to control-oriented engine models. Therefore, a modified orifice equation is proposed in [66], where a tunable polytropic coefficient is introduced to account for all the unmolded effects. Simplified turbine flow models can be found in [58,61,67].

The turbine efficiency characteristic map is the most difficult one to model, due to the dependence of the efficiency on pressure ratio, turbocharger speed, and VGT position, rendering the map four-dimensional [63]. Methods based on blade speed ratio are widely used, for instance see [57,58,68,69]. The dependence on speed prevents this type of model being used when the limited data is not sufficient for a reliable characterization. Therefore, power-based efficiency model is introduced in [24,70], which is independent on turbocharger speed. Other simplified approaches are found in literature, such as constant efficiency [67], efficiency independent on VGT opening [71].

2.3 Diesel Engine Air-Path Control

During the last decades, a considerable research effort has been dedicated to the control of modern Diesel engines, which are all equipped with an exhaust gas recirculation system and a variable geometry turbocharger. Key issue in Diesel engine air-path control is to coordinate the various actuators in meeting desired boost pressure and flow rates of air and residuals.

2.3.1 EGR-VGT Control of Diesel Air-Path System

The nonlinearity of the engine model and the coupling among actuators and controlled outputs make the Diesel engine air-path control challenging. Several methodologies have
been explored for the well-known EGR-VGT control problem. Coordinated decentralized and multivariable control strategies were first proposed to manipulate VGT and EGR openings for intake manifold pressure and air mass flow rate regulation [72–75]. Nonlinear control approaches were then developed, including constructive Lyapunov function approach [76], adaptive control [77], dynamic feedback linearization [76, 78, 79], and sliding mode control [79–81]. Furthermore, $H_\infty$ controller based on the linear parameter varying models is studied in [67, 82]. Other control designs, for instance, optimal control based on dynamic optimization and neural networks [69], motion-planning strategy [83], PID control for pumping work minimization [84], are also investigated in literature.

The choice of feedback variables defines the overall controller structure, and the most common choice in the literature are compressor air mass flow and intake manifold pressure [67, 72, 73, 79–82]. Other choices are exhaust manifold pressure and compressor air mass flow [76], intake manifold pressure and cylinder air mass-flow [77], or compressor air mass flow and EGR flow [83], oxygen/fuel ratio and EGR-fraction [84]. Even though oxygen/fuel ratio and EGR-fraction cannot be directly measured by production sensors, they are used for performance variables as they are directly related to the emissions.

2.3.2 Model Predictive Control for Diesel Air-Path System

More recently, the advance in model predictive control (MPC), in particular the development of explicit MPC schemes [85], and computational power, cause an growing interest for Diesel engine air-path control application. MPC is considered as a systematic approach to control development and calibration along with providing a multivariable control scheme that automatically accommodates actuator and state constraints [86].

In [87, 88], an explicit MPC is developed to regulate air mass flow and intake manifold pressure. In [89], an online MPC is applied to a real-world Diesel engine, by employing an extension of the online active set strategy. In [90, 91], nonlinear MPC approaches are proposed for the air-path control problem to solve the system nonlinearities and constraints.
The simulation results show improved performance, but the required computational power is far more than is available on production ECU. In [92], two MPC controllers using different outputs and a PID based controller are evaluated in simulation, showing that the proposed design has performance improvements. Furthermore, issues of practical importance were also discussed, including integral action of the EGR-fraction to handle model errors, prediction of engine load and speed, and prevention of input oscillations.

2.3.3 Integrated Control of Engine and Aftertreatment Systems

Integrated control of engine and aftertreatment systems is a novel concept, which exploits the synergy between engine and aftertreatment systems to optimize the overall system fuel economy and emissions. The previous research effort is mainly on two problems, namely active thermal management [93–95] and active NO\textsubscript{x} control [96–98].

In [93], a control-oriented temperature dynamic model for a modern Diesel engine equipped with a complete set of aftertreatment systems including Diesel oxidation catalyst (DOC), Diesel particulate filter (DPF), and selective catalytic reduction (SCR) is developed. The influences of in-cylinder post injection, both fuel injection rate and injection timing, on the temperature dynamics are investigated. In [94], different methods of temperature increase upstream catalyst are compared, including intake air throttling, retard of main injection, early and late post injection, and exhaust cam-phasing. In [95], 4 in-cylinder strategies and 1 in-pipe strategy were employed for the purpose of thermal management of a DPF. Experiments results show that at the low temperature of (150 °C), early injection strategies that create temperature rise from both combustion and light reductant exotherms are preferred.

In [96], a backstepping-based active NO\textsubscript{x} control method is proposed by treating the engine-out NO\textsubscript{x} concentration as a control input for the ammonia coverage ratio control of a two-cell SCR system. The engine brake specific fuel consumption (BSFC) can be reduced significantly by actively relaxing the constraint on the engine-out NO\textsubscript{x} concentrations. [97]
proposed an integrated emission management strategy based on Pontryagin’s minimum principle, which optimizes engine-out NOX emissions by online adjustment of EGR/VGT control settings based on the actual SCR NOX reduction capacity. In [98], a steady-state optimization and MPC is developed for a combined EGR/SCR control problem. The controller minimizes BSFC including urea cost by simultaneously optimizing engine out NOX and urea dosing, while maintaining emission levels.

2.4 Combined Design and Control Optimization

When optimizing a plant (artifact) and its controller, one could adopt a sequential strategy whereby the plant is optimized first, followed by the controller. However, the plant and controller optimization problems are coupled in the sense that their sequential solution is not guaranteed to be a combined optimum [99]. Design and control integration reported in literature are in diverse areas including flexible structures [100–102], vehicle suspensions [103, 104], electric motors [105, 106], robotics [107–109], and various types of mechanisms and machine tools [110–112].

Solving combined design and control optimization problems, or co-design problem, has received substantial attention from researchers. Fathy et al. [99] classified the various optimization strategies into sequential, iterative, bi-level (nested), and simultaneous strategies, and showed mathematically that system-level optimality is guaranteed with the nested and simultaneous strategies. Tanaka et al. [113] proposed a general formulation to the co-design problem and reduced the formulation to a bilinear matrix inequality (BMI) using a descriptor form. Shi et al. [114, 115] proposed an iterative redesign strategies, which have the advantage of guaranteeing a locally convergent solution to the optimization problem. Peters et al. [116] presented a modified sequential approach utilizing a control proxy function, which can provide near-optimal solutions to the co-design problem, while allowing the problem to be decomposed into an artifact design problem and a control design problem.
PARAMETRIC model for the compressor and turbine maps is developed in this chapter. First, a brief overview of basic turbomachinery theory is given, where the dimensionless parameters and relevant geometric parameters for the characterization of flow and efficiency maps are defined. Then, a formal derivation of the parametric maps for families of compressors and turbines is discussed together with the model results. The parametric modeling approach is applied to an engine model with two-stage turbocharger (2ST), which is validated against steady-state and transient experimental data. Preliminary analysis is conducted to show the influence of 2ST configurations on system performance.

3.1 Basic Turbomachinery Theory

3.1.1 Dimensionless Representation

For the compressor and turbine characteristic maps, the performance parameters are given in the form of pseudo-dimensionless variables, which is a standard representation method for turbomachinery. The variables defined in Table 3.1 are normalized by the inlet condition. Typically, turbocharger manufacturers provide data and performance maps in terms of corrected variables. For this reason, and the fact that the corrected variables are closer to the physical meaning of each parameter, the parametric turbocharger model will be developed using the corrected variables.

In addition to the above variables, the dimensionless parameters are often used for the performance characterization of both compressor and turbine, which are defined in Table 3.2.
Table 3.1: Pseudo-dimensionless Representation

<table>
<thead>
<tr>
<th></th>
<th>Corrected Variables</th>
<th>Reduced Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mass Flow</strong></td>
<td>$\dot{m}<em>{corr} = \dot{m} \sqrt{\frac{T</em>{in}}{T_{ref}}} \frac{p_{in}}{p_{ref}}$</td>
<td>$\dot{m}<em>{red} = \dot{m} \frac{\sqrt{T</em>{in}}}{p_{in}}$</td>
</tr>
<tr>
<td><strong>Pressure Ratio</strong></td>
<td>$\beta = \frac{p_{out}}{p_{in}}$ for compressor</td>
<td>$\epsilon = \frac{p_{in}}{p_{out}}$ for turbine</td>
</tr>
<tr>
<td><strong>Shaft Speed</strong></td>
<td>$N_{corr} = \frac{N}{\sqrt{\frac{T_{in}}{T_{ref}} \frac{T_{in}}{P}}}$</td>
<td>$N_{red} = \frac{N}{\sqrt{T_{in} \frac{P}{p_{in}}}}$</td>
</tr>
<tr>
<td><strong>Power</strong></td>
<td>$P_{corr} = \frac{P_{in}}{p_{in} \sqrt{T_{in} \frac{T_{in}}{P}}}$</td>
<td>$P_{red} = \frac{P}{p_{in} \sqrt{T_{in}}}$</td>
</tr>
</tbody>
</table>

Table 3.2: Dimensionless Representation

<table>
<thead>
<tr>
<th></th>
<th>Compressor</th>
<th>Turbine</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Flow Parameter</strong></td>
<td>$\Phi = \frac{\dot{m}<em>{corr}}{\rho</em>{in} \frac{\pi}{4} D_{c}^{2} U_{c}}$</td>
<td>$\Phi = \frac{\dot{m}<em>{corr}}{\rho</em>{in} \frac{\pi}{4} D_{t}^{2} U_{t}}$</td>
</tr>
<tr>
<td><strong>Head Parameter</strong></td>
<td>$\Psi = \frac{c_{p} T_{in} \left( \beta^{\frac{1}{\gamma}} - 1 \right)}{\frac{1}{2} U_{c}^{2}}$</td>
<td>$\Psi = \frac{c_{p} T_{in} \left( 1 - \epsilon^{\frac{1}{\gamma}} \right)}{\frac{1}{2} U_{t}^{2}}$</td>
</tr>
<tr>
<td><strong>Blade Tip Speed</strong></td>
<td>$U_{c} = \frac{\pi}{60} D_{c} N_{corr}$</td>
<td>$U_{t} = \frac{\pi}{60} D_{t} N_{corr}$</td>
</tr>
<tr>
<td><strong>Blade Mach Number</strong></td>
<td>$M = \frac{U_{c}}{\sqrt{\gamma R T_{in}}}$</td>
<td>$M = \frac{U_{t}}{\sqrt{\gamma R T_{in}}}$</td>
</tr>
<tr>
<td><strong>Blade Speed Ratio</strong></td>
<td>$BSR = \frac{U_{c}}{2 c_{p} T_{in} \left( \beta^{\frac{1}{\gamma}} - 1 \right)}$</td>
<td>$BSR = \frac{U_{t}}{2 c_{p} T_{in} \left( 1 - \epsilon^{\frac{1}{\gamma}} \right)}$</td>
</tr>
</tbody>
</table>

3.1.2 Similitude Theory

The theory of similitude (or similarity) allows one to compare the performance of different turbomachines that belong to the same family [117]. This theory, well-known in the general field of fluid mechanics, was originally used to predict performance of full-scale systems based on the analysis of scaled prototypes. The similarity theory recognizes three categories:
- Geometric similarity: it occurs when two turbomachines (model-m and prototype-p) differ only in scale, so corresponding dimensions have the same ratio.

\[
\left( \frac{L}{D} \right)_m = \left( \frac{L}{D} \right)_p \tag{3.1}
\]

- Kinematic similarity: it occurs when the streamline patterns in two machines are the same. This implies that the machines have the same velocity coefficients, hence similar velocity triangles.

\[
\Phi_m = \Phi_p \tag{3.2}
\]

- Dynamic similarity: it occurs if the ratios of force components at corresponding points in the flow through two machines are equal. This implies equality of the load coefficients.

\[
\Psi_m = \Psi_p \tag{3.3}
\]

This theory will be used here for interpolating turbocharger maps data and for the analysis of the relevant parameters.

### 3.1.3 Turbocharger Geometric Parameters

Traditionally, the performance of turbochargers is determined by three geometric parameters [118], namely diameter, Trim, and A/R (Area/Radius) as shown in Figure 3.1. The diameter is generally defined as the outer diameter of the wheel. In this sense, the compressor diameter refers to the exducer wheel diameter, while the turbine diameter refers to inducer wheel diameter. Since the flow capacity is approximately proportional to the size, the diameter is a major design parameter.

The Trim is defined as the ratio between the inducer and exducer areas of both turbine and compressor wheels:

\[
\text{Trim} = 100 \left( \frac{D_{in}}{D_{ex}} \right)^2 \tag{3.4}
\]
The Trim affects performance by shifting the airflow capacity. Under the same conditions, a higher Trim wheel will allow more flow than a smaller Trim wheel. However, it is very unlikely that a larger Trim can be obtained without changing any of the other geometric parameter and hence affecting the overall turbocharger performance. Hence, there is no guarantee that a larger Trim will allow more flow.

Another important parameter is the A/R, which is defined as the ratio between the inlet cross-sectional area and the distance between the axis of rotation and the centroid of that area. The A/R affects the swallowing capacity of a turbine. Larger A/R provides more flow through the turbine, hence improved boost at higher engine speed conditions by increasing the flow capacity and reducing the exhaust gas back pressure. Compared to the effects of A/R on the turbine performance, the sensitivity of the compressor performance to this parameter is rather limited.

### 3.2 Compressor Model

This section will describe the equations used to model a family of geometrically similar compressors relevant for the two-stage turbocharger system. In particular, the focus is on the scalability of the compressor mass flow rate and efficiency outputs, with respect to its design parameters, such as diameter, A/R and Trim.
Table 3.3: GT Compressors Specification

<table>
<thead>
<tr>
<th>No.</th>
<th>Model</th>
<th>Diameter [mm]</th>
<th>Trim [-]</th>
<th>A/R [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>GT2859R</td>
<td>59.4</td>
<td>56</td>
<td>0.42</td>
</tr>
<tr>
<td>2</td>
<td>GT2860R(1)</td>
<td>60</td>
<td>55</td>
<td>0.42</td>
</tr>
<tr>
<td>3</td>
<td>GT2871R(1)</td>
<td>71</td>
<td>52</td>
<td>0.60</td>
</tr>
<tr>
<td>4</td>
<td>GT2871R(4)</td>
<td>71</td>
<td>56</td>
<td>0.60</td>
</tr>
<tr>
<td>5</td>
<td>GT3071R(1)</td>
<td>71</td>
<td>56</td>
<td>0.50</td>
</tr>
<tr>
<td>6</td>
<td>GT3076R</td>
<td>76.2</td>
<td>56</td>
<td>0.60</td>
</tr>
<tr>
<td>7</td>
<td>GT3582R(2)</td>
<td>82</td>
<td>56</td>
<td>0.70</td>
</tr>
<tr>
<td>8</td>
<td>GT4088</td>
<td>88</td>
<td>54</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Based on the analysis of the most relevant publications in this field, several modeling approaches have been identified as potential candidates for characterizing and scaling the compressor maps. All these approaches are based on dimensionless representation of the flow, speed, pressure and efficiency variables. Among the various methods found in literature, it is necessary to select a model structure that maintains sufficient accuracy in representing a group of similar compressors when one of the geometric parameter is modified.

Based on the above considerations, eight sets of compressor flow and efficiency maps were selected from the Garrett GT Series, which is the same family as the two compressors of the two-stage turbocharger. Two approaches for the mass flow model and three approaches for the efficiency model are considered. A comparative study is proposed here to evaluate each modeling method.

3.2.1 Assumptions and Compressor Selection

To illustrate the advantages of adopting a dimensionless representation of the compressor performance maps, the corrected maps for the three sample compressors listed in Table 3.3 are shown in Figure 3.2, together with their dimensionless representation. Although the three compressor maps are quite different, their dimensionless representations condense to
the same region. Therefore, it is reasonable to assume that the kinematic similarity and dynamic similarity hold for a group of geometric similar compressors. Because the A/R has little effect on compressor performance, it is possible to reduce the number of design
parameter considered for the model development.

The specifications of the eight compressors in the Garrett GT family considered in this study are shown in Table 3.3, while their maps are shown in Figure 3.3 together with the maps of the original high and low pressure compressors of the two-stage turbocharger. The performance maps of the GT compressors are similar to the original two-stage compressors. Hence, they represent a relevant family for the application of the parametric modeling methodology.

3.2.2 Compressor Mass Flow

The first proposed compressor flow model is the Jensen-Kristensen (JK) model, based on the work presented in [57]. In this work, the head parameter is expressed as a hyperbolic function of the flow parameter. The function parameters are fitted as quadratic functions of the blade Mach number:

\[ \Psi = \frac{K_1 + K_2\Phi}{K_3 - \Phi}, \quad K_i = k_{i1}M^2 + k_{i2}M + k_{i3}, \quad i \in \{1, 2, 3\} \]

\[ P = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{21} & k_{22} & k_{23} & k_{31} & k_{32} & k_{33} \end{bmatrix} \]

The above modeling approach results into nine fitting parameters that are identified on the data extracted from the available compressor maps. The results of the curve fitting process using the JK model is shown in Figure 3.4 for the GT3071R(1) compressor.

An alternate approach that was used to fit the compressor flow maps is the Ellipse model, originally proposed in [61]. Here, a new flow parameter is defined to explicitly take into account the compressor speed. In this model, a third order polynomial is used for curve fitting. However, it has been shown that using this approach leads to numerical issues in the extrapolation. For this reason, an extension of the model was proposed in [58], where
the Ellipse model is used to correlate the flow parameter with the head parameter:

\[
\left( \frac{\Phi'}{k_1} \right)^2 + \left( \frac{\Psi}{k_2} \right)^2 = 1, \quad \Phi' = \frac{\Phi}{M^{\frac{\gamma - 1}{\gamma}}} \quad (3.6)
\]

\[
P = \begin{bmatrix} k_1 & k_2 \end{bmatrix}
\]

The above model results in only two regression parameters and the results of the curve fitting

Figure 3.4: Identification of the JK Flow Model

Figure 3.5: Identification of the SE Flow Model
process using the Shaver-Ellipse (SE) model is shown in Figure 3.5 for the GT3071R(1) compressor.

According to similarity theory, the dimensionless parameters for a turbomachine are expressed functionally as:

\[ f(\Phi, \Psi, M, \text{Re}, \gamma, \cdots) = 0 \]  

(3.7)

Each of the models presented has a unique set of parameters \( P \), which is identified through a nonlinear least square method. The parameter set is introduced to account for modeling error associated to neglecting the influence of non-dimension terms, including \( \text{Re} \), \( \gamma \), and other geometry parameters on the dimensionless characteristic curves. Therefore, above relationship can be reduced to:

\[ f(\Phi, \Psi, M) = 0 \]  

(3.8)

For a family of compressor listed in Table 3.3, a reasonable assumption is that the set of parameters \( P \) is the same for all compressors. To evaluate the above assumption, as well as the accuracy of the two modeling approaches, a comparative study is presented. First, the flow data from all compressors listed in Table 3.3 except for No. 6 are used to identify the parameter set for two models. The result of this procedure is one set of regression parameters \( P \) for each of the seven compressors. Finally, a single set of regression parameters representative of the compressor family is obtained by averaging among the seven sets.

The same procedure is applied to both the JK model and the SE model, and are then used to represent the flow characteristics of all compressors. Performance maps for all the eight compressors were generated, and results are shown in Figure 3.6 for two of the compressors in the calibration set (No. 2 and 4) and for the validation set (No. 6). The results show that the JK model is able to well predict the performance of the GT3076 compressor (No. 6), which is reserved for validation.

In general, there are two ways for implementing the compressor mass flow model in
an engine air-path system model, depending upon the causality relationships between the models of the system components. A first approach consists of modeling the mass flow rate through the compressor as function of the pressure ratio and turbo shaft speed:

$$\dot{m}_c = f_1(\beta, N_{tc})$$  \hspace{1cm} (3.9)

A different approach consists of considering the mass flow rate as the input to the compressor model, which is then used to compute the pressure ratio:

$$\beta = f_1(\dot{m}_c, N_{tc})$$  \hspace{1cm} (3.10)

When viewing compressor model in isolation, Type 2 model appears to be a better choice, because:

- In the region where the speed lines are almost flat, coinciding with a large part of
Table 3.4: Comparison of the Two Compressor Flow Models

<table>
<thead>
<tr>
<th></th>
<th>JK model</th>
<th>SE model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extrapolation</td>
<td>Low speed</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>Low flow</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>High flow</td>
<td>Low</td>
</tr>
<tr>
<td>Accuracy on data points</td>
<td>High</td>
<td>Low at high speed</td>
</tr>
<tr>
<td>Diameter sensitivity</td>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>

engine operating region, Type 2 model is insensitive to input errors. Conversely, the characteristic curves become very steep when pressure ratio is used as input.

- Type 2 model can be easily incorporated into a dynamic model that exhibits surge behavior [119].

When the compressor model is connected to the engine model, Type 2 model is equivalent to Type 1 model at steady-state, but more stable in transients, due to the fact that the pressure ratio changes much faster than the compressor flow rate.

The two modeling approaches are evaluated by analyzing the error on data points and extrapolation quality, particularly in the low speed region, low mass flow region (surge), and high mass flow region (choking). The results are summarized in Figure 3.7 to Figure 3.10, and in Table 3.4. It is evident that the JK model presents an overall higher accuracy of both interpolation and extrapolation and it appear to be less sensitive to the compressor diameter compared to the SE model. For this reason, the JK model is selected as the parametric compressor flow model. Moreover, Type 2 model is adopted due to its better prediction ability. The fully parameterized compressor flow model is validated on the eight compressor maps, as shown in Figure 3.11.

The implementation of this model in the engine air-path system is achieved through the introduction of a new state. Considering the inertia effects of the air mass $\dot{m}_c$ in the pipe connecting the compressor outlet and the volume after the compressor, as shown in
Figure 3.7: Prediction Error for JK Model (Type 1: Pressure Ratio as Input)

Figure 3.8: Prediction Error for SE Model (Type 1: Pressure Ratio as Input)
Figure 3.9: Prediction Error for JK Model (Type 2: Mass Flow as Input)

Figure 3.10: Prediction Error for SE Model (Type 2: Mass Flow as Input)

GT2860R(1)

GT2871R(4)

GT3076R

Mean = 6.7 [%]
Std = 7.73 [%]

Mean = −0.82 [%]
Std = 4.94 [%]

Mean = 4.21 [%]
Std = 9.45 [%]
Figure 3.11: Compressor Flow Parametric Model Validation (JK Model, Type 2)

![Graphs showing compressor flow parametric model validation](image)

Figure 3.12: Schematic Diagram for Momentum Equation

![Schematic diagram for momentum equation](image)

Figure 3.12, the momentum equation is given by:

\[ A(p_2 - p_c) = m\dot{v} = \rho l A\dot{V} = \frac{d\dot{m}_c}{dt} l \]  \hspace{1cm} (3.11)

hence

\[ \frac{d\dot{m}_c}{dt} = \frac{A}{l} (p_2 - p_c) \]  \hspace{1cm} (3.12)

where \( A \) and \( l \) are the cross sectional area and length of the connecting pipe.
3.2.3 Compressor Efficiency

Together with the modeling methodology for the compressor flow maps, [57] also introduces a modeling approach for the compressor efficiency maps. Application of this method, however, showed low accuracy and poor ability to extrapolate the efficiency maps to regions where experimental data are not available.

For this reason, an alternative approach is considered here, based on the study presented by Eriksson [71]. In this work, the efficiency is represented as a quadratic function of flow parameter and corrected speed:

\[
\eta_c = \eta_{max} - \chi^T Q \eta \chi = \eta_{max} - \left[ \phi - \phi_{max} \right]^T \begin{bmatrix} c_1 & c_2 \\ c_2 & c_3 \end{bmatrix} \begin{bmatrix} \phi - \phi_{max} \\ N - N_{max} \end{bmatrix}
\]

This modeling approach requires the determination of six parameters and sample results for one of the compressors considered in this study are shown in Figure 3.13.

Figure 3.13: Identification of the Eriksson Efficiency Model
A second approach is proposed by Shaver [61], where the new efficiency is defined as a linear function of the new flow parameter:

$$
\eta'_c = c_1 \Phi' + c_2, \quad \Phi' = \frac{\Phi}{M^{\gamma-1}}, \quad \eta'_c = \frac{\Psi}{\eta_c}
$$

$$
Q = \begin{bmatrix} c_1 & c_2 \end{bmatrix}
$$

Using this approach, the parameters to be identified are now reduced to two. The result of this fitting method for generating the efficiency maps is shown in Figure 3.14.

The last modeling approach considered in this study is based on a direct application of the conservation laws to the compressor, as proposed by Martin [62]. Here, the enthalpy rise is modeled as a linear function of the mass flow rate, and the parameters are assumed to
Table 3.5: Comparison of the Three Compressor Efficiency Models

<table>
<thead>
<tr>
<th>Extrapolation</th>
<th>Eriksson model</th>
<th>Shaver model</th>
<th>Martin model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low speed</td>
<td>High</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Low flow</td>
<td>Middle</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>High flow</td>
<td>High</td>
<td>Middle</td>
<td>Low</td>
</tr>
<tr>
<td>Accuracy on data points</td>
<td>Low at high speed</td>
<td>Insensitive to input</td>
<td>High</td>
</tr>
<tr>
<td>Diameter sensitivity</td>
<td>Low</td>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>

Figure 3.16: Extrapolation of 3 Efficiency Models

be a quadratic function of the corrected speed. The compressor efficiency is then calculated
Figure 3.17: Prediction Error for 3 Efficiency Models

as the ratio between the isentropic enthalpy rise and the actual enthalpy rise as follows:

$$\eta_c \triangleq \frac{\Delta h_{is}}{\Delta h}, \quad \Delta h_{is} = c_p T_{in} \left( \frac{\beta^{-1}}{\gamma} - 1 \right) = \frac{1}{2} \Psi U_c^2$$

$$\Delta h = a_1 \dot{m}_c + a_2, \quad a_i = c_{i1} N^2 + c_{i2} N + c_{i3}, \quad i \in \{1, 2, 3\}$$

$$Q = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{21} & c_{22} & c_{23} \end{bmatrix}$$

Since the shape of the efficiency curves is the same among different compressors (in light of the similitude theory), the set of parameters $Q$, determined through the fitting of the compressor efficiency maps, is assumed constant for all compressors listed in Table 3.3. To confirm this assumption and compare the accuracy of the three models, a comparative study similar to the one presented above, is conducted for the compressor efficiency. The
results are summarized in Figure 3.16, Figure 3.17, and Table 3.5.

Since the Eriksson model presents higher accuracy in both the interpolation and extrapolation of efficiency data with the least sensitivity to the compressor diameter, it will be selected as the parametric compressor efficiency model. The Eriksson model is validated on the eight compressor maps, as shown in Figure 3.18.

3.2.4 Surge and Choke Model

Surge is a dangerous instability and can occur during a gear shift under acceleration, when a throttle closing causes a fast reduction in mass flow. The reduction of mass flow produces an increase in pressure ratio over the compressor, which may lead to reversed flow. Such oscillating mass flow can cause undesired noise, and damage, or even destroy, the turbo.

Choke is a situation where the pressure ratio of a speed line drops rapidly (vertically) with little or no change in flow. In most cases the reason for choke is that close to Mach 1 velocities have been reached somewhere within the impeller and/or diffuser generating
a rapid increase in losses. Therefore, a maximum flow line is defined to avoid compressor
operate at low efficient choke region.

Since surge line and choke line represent the limit of compressor operation, their model
is important when investigating control design for turbocharged engines. In [59], a first
order polynomial is used to model maximum flow line:

\[
\dot{m}_{max} = k_{11}N + k_{12}
\]

where \(k_{12} > 0\) gives a positive flow also for \(N = 0\). Power functions are used to model surge
mass flow and pressure ratio lines:

\[
\dot{m}_{surge} = k_{21}N^{k_{22}} + 0 \\
\beta_{surge} = k_{31}N^{k_{32}} + 1
\]

where the 0 and 1 emphasize zero flow and unity pressure ratio for \(N = 0\).

To get a surge and choke model for a family of compressors, Mach number \(M\) is used
to replace corrected speed \(N\). The modified model becomes:

\[
\dot{m}_{surge} = (k_{11}D_c + k_{12})M^{k_{13}} \\
\beta_{choke} = k_{21}M^{k_{22}} + 1
\]

We only need to model surge mass flow or surge pressure ratio, the other variables is obtained
via compressor flow model. The reason of choosing mass flow rather than pressure ratio
for surge line model is that constant speed line is flat in surge region, so this model is less
sensitive to input or modeling errors. Similarly, pressure ratio is used to model choke line.
The results of the curve fitting process is shown in Figure 3.19. The integrated compressor
flow model and surge-choke model is validated on the eight GT Series compressor maps, as
shown in Figure 3.20.
Figure 3.19: Identification of the Surge and Choke Model

Figure 3.20: Integrated Compressor Flow Model and Surge-choke Model

3.3 Turbine Model

This section presents the approach and equations used to model a family of geometrically similar turbines, for the two-stage turbocharger system. The objective is to develop a
scalable flow model for fixed geometry turbine (FGT), which can predict the outputs based on a set of design parameters such as diameter, A/R and Trim. Methods to extend the model to variable geometry turbine (VGT) are also proposed.

In literature, the standard orifice equation \[120\] and its modified versions are widely used for turbine flow model, for instance see \[57\], \[71\], \[62\], \[67\], \[66\]. Among others, the modified orifice equation presented in \[66\] exhibits high accuracy in predicting the turbine mass flow rate in a wide range of operating conditions. This modeling methodology relies on the identification of two tuning parameters, which are representative of certain physical characteristics of the turbine flow. This approach is selected here to parameterize a family of turbine maps.

Similar to the previous section, a set of eight fixed geometry turbines was extracted from the Garrett GT Series, with performance map similar to the turbines of the original two-stage turbocharger. The parametric flow model for these FGTs is developed based on the assumption of geometric similarity. Then, three variable geometry turbines are analyzed with the objective of extracting a simplified model that represents the general relationship between the flow rate and the VGT opening position. The outcome of this analysis is a simple VGT scaling method that allows to estimate how the flow characteristic maps will change is a VGT actuation is considered.

For turbine efficiency, methods based on blade speed ratio are widely used, for instance see \[57\], \[71\], \[68\], \[69\]. Turbine efficiency is greatly affected by VGT opening, and there is limited data available, which make it challenging to get a fully parameterized turbine efficiency model. Some simplified approaches are seen in literature, such as constant efficiency \[67\], efficiency independent on VGT opening \[71\] or speed \[70\]. Since the manufacturer does not provide efficiency data for Garrett GT Series turbines, the original efficiency data is used, which include both FGT and VGT. Two modeling methods are developed and evaluated. The relation between efficiency and the VGT opening is obtained by interpolation.
3.3.1 Turbine Selection

Due to its critical effect on turbine performance, the A/R must be considered in the parametric model of the turbine. This introduces an additional degree of complexity to the problem, compared to the case of the compressor. A set of turbine data from the Garrett GT Series that match with the previous eight compressors are considered in this section, and their specifications are listed in Table 3.6.

Figure 3.21 shows that the performance maps of the family of GT turbines considered provides a good coverage of the operating range of the original two-stage turbine maps. Considering the typical range of operations of a turbine for engine charging applications, the set of turbines used for the parametric model development can be further reduced from eleven to eight (No. 2 to No. 9) without loss of accuracy.

<table>
<thead>
<tr>
<th>No.</th>
<th>Turbine model</th>
<th>Compressor model</th>
<th>Diameter [mm]</th>
<th>Trim [-]</th>
<th>A/R [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>GT28(1)</td>
<td>GT2859R</td>
<td>53.9</td>
<td>62</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GT2860R(1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>GT28(2)</td>
<td>GT2871R(1)</td>
<td>53.9</td>
<td>76</td>
<td>0.86</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>GT2871R(4)</td>
<td></td>
<td></td>
<td>0.64</td>
</tr>
<tr>
<td>4</td>
<td>GT30</td>
<td>GT3071R(1)</td>
<td>60</td>
<td>84</td>
<td>1.06</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>GT3076R</td>
<td></td>
<td></td>
<td>0.82</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.63</td>
</tr>
<tr>
<td>7</td>
<td>GT35</td>
<td>GT3582R(2)</td>
<td>68</td>
<td>84</td>
<td>1.06</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.82</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.63</td>
</tr>
<tr>
<td>10</td>
<td>GT40</td>
<td>GT4088</td>
<td>77.6</td>
<td>83</td>
<td>1.34</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.19</td>
</tr>
</tbody>
</table>
Figure 3.21: Turbine Maps of the Original Two-Stage Turbocharger and Similar Models of the GT Series

3.3.2 Turbine Mass Flow

Turbine Flow Model

The equation describing the flow of compressible fluids through a restriction (also known as the isentropic orifice equation) is defined as:

\[
\dot{m}_t = C_d A \frac{p_{in}}{\sqrt{RT_{in}}} \sqrt{f_1(\epsilon)}
\]

\[
f_1(\epsilon) = \begin{cases} 
\sqrt{\frac{2}{\gamma - 1} \left( \epsilon^{-\frac{2}{\gamma}} - \epsilon^{-\frac{\gamma+1}{\gamma}} \right)} & \text{if } \frac{1}{\epsilon} \geq \left( \frac{2}{\gamma + 1} \right)^{\gamma - 1} \\
\sqrt{\left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}}} & \text{if } \frac{1}{\epsilon} < \left( \frac{2}{\gamma + 1} \right)^{\gamma - 1}
\end{cases}
\]  

(3.19)

where \( \epsilon = \frac{p_{in}}{p_{out}} \) is the pressure ratio across the orifice.

In general, the flow through a turbine is considerably more complex than an orifice flow from a physical standpoint. The pressure ratio across the turbine at which chocked flow occurs is actually lower than the expected value of 1.8 (depending on \( \gamma \) only), because the turbine effectively behaves as a series of two nozzles (inlet nozzle vanes and rotor passages),
which each individually experience lower pressure ratios than the total across the turbine. Therefore, a modified orifice equation is considered, where a polytropic coefficient $m$ is introduced as a tunable parameter that accounts for all the unmolded effects. This leads to the following equation:

$$\dot{m}_t = C_d A \frac{p_{in}}{\sqrt{RT_{in}}} \sqrt{\gamma f_1(\epsilon)}$$

$$f_1(\epsilon) = \begin{cases} 
\sqrt{\frac{2}{\gamma - 1} \left( \epsilon - \frac{1}{m} - \epsilon^{\frac{m+1}{m}} \right)} & \text{if } \frac{1}{\epsilon} \geq \left( \frac{2}{m+1} \right)^{\frac{m}{m-1}} \\
\left( \frac{2}{m+1} \right)^{\frac{1}{m-1}} \sqrt{\frac{2}{\gamma - 1} \left( \epsilon - \frac{m-1}{m+1} \right)} & \text{if } \frac{1}{\epsilon} < \left( \frac{2}{m+1} \right)^{\frac{m}{m-1}}
\end{cases} \quad (3.20)$$

The critical pressure ratio, $\epsilon_{crit} = \left( \frac{2}{m+1} \right)^{-\frac{m}{m-1}}$, represents the limit conditions at which the flow through the turbine reaches sonic speed, therefore leading to the well-known choked behavior. Note that the critical pressure ratio can be approximated as a linear function of the parameter $m$ without loss of accuracy, as shown in Figure 3.22a. In this sense, the parameter $m$ can be interpreted as a scaling factor that linearly influences the choking limit of the turbine. This information can be useful to establish a set of constraints that allow one to simplify and automate the turbine flow model calibration process.

In (3.20), the product $C_d A$ is named the equivalent flow area, which generally relates to
the throat area of the turbine. As shown in Figure 3.22b, the throat area is defined as the cross-sectional area (normal to the direction of the flow) corresponding to the first point where the flow is in contact with the runner. Assuming that geometric similarity holds for the GT turbine family, the throat area results proportional to the turbine diameter and A/R parameters as follows:

\[
A = \left( \frac{A}{R} \right) R \propto \left( \frac{A}{R} \right) D \quad (3.21)
\]

The physical consistency of the two calibration parameters \(C_dA\) and \(m\) allows one to obtain a direct relation between the turbine geometric parameters and the model parameters, which facilitates the calibration and extrapolation of the results.

**Fixed Geometry Turbine Flow Parametric Model**

The procedure for fitting the turbine flow maps is similar to the one used for the compressor. As a first attempt, the eight GT turbines considered were fitted individually, without considering any constraint on the parameter \(m\). However, it was found that performing an unconstrained optimization led to values of \(m\) that ultimately made the critical pressure ratio unreasonably large. For this reason, the identification procedure was modified by considering the physical limits of the pressure ratio at which choking occurs. The results of
this regression procedure are shown in Figure 3.23. Once the parameters \( m \) and \( C_dA \) have been determined for each of the turbines considered, a linear fitting is used to schedule the two parameters as function of the turbine diameter and the A/R as follows:

\[
m = k_1 + k_2 \frac{A}{R} + k_3 D
\]

\[
C_dA = k_4 \frac{A}{R} D + k_5
\]  \hspace{1cm} (3.22)

Using the results of the fitting procedure, the turbine geometry parameters are used to generate the flow maps for the turbines listed in Table 3.6. The results for the eight GT fixed geometry turbines are obtained and shown in Figure 3.24.

The drawback of this modeling process is that the constraints of each turbine need to be tuned individually, which is time consuming. An alternative way proposed here is so-called global fitting, meaning that the data set \([\dot{m}_t \in D \ A/R]\) of 8 turbines can be fitted at one time, by combining (3.20) and (3.22). The extrapolation quality and error of two approaches are compared in Figure 3.25, noting that the coefficients of determination, denoted \( R^2 \), are almost the same.
Variable Geometry Turbine Flow Model

The parametric turbine flow model can be extended to the case of a turbine with variable geometry. The objective is to determine a parametric model that accounts for the influence of a VGT actuator on the output mass flow rate of a given fixed geometry turbine. As a starting point to develop the model, the data for the high pressure turbine (HPT) of the original two-stage turbocharger system were selected. The flow curve corresponding to each VGT position was interpolated using (3.20). This procedure leads to seven values for the parameters \( m \) and \( C_dA \) (one per VGT position), shown in the right plots of Figure 3.26. The two parameters were then obtained and fitted as quadratic functions of the VGT opening:

\[
m = k_{11}x_{vgt}^2 + k_{12}x_{vgt} + k_{13}
\]

\[
C_dA = k_{21}x_{vgt}^2 + k_{22}x_{vgt} + k_{23}
\]

As a verification step, the characteristic maps obtained through the model given by (3.20) and (3.23) are compared to the original data set, and the results are shown in the left
plot of Figure 3.26. It can be seen that the parametric VGT flow model preserves accuracy in interpolating the given data.

Figure 3.27 illustrates the results obtained when the same method is applied to other two variable geometry turbines, indicating that this approach can be in principle generalized to different VGT systems.
Scaling Method for Variable Geometry Turbine Flow Parametric Model

Since the availability of experimental data and geometric information for variable geometry turbines is rather limited, it is extremely challenging to generate a fully parametric VGT flow model that accounts for the real geometry of a VGT actuator (particularly, the relation between the throat area and VGT position). Instead, a different approach is proposed, namely consisting of finding a representation of the parameters $m$ and $C_dA$ as a function of the VGT position by averaging across several sets of data obtained from different turbines with VGT actuators. The ultimate goal of this modeling approach is to qualitatively predict how the flow map of a given fixed geometry turbine would be scaled if the turbine were equipped with a VGT actuator. This would enable one to conduct virtual design studies, evaluating and comparing performance of variable and fixed geometry turbines using simulation tools.

The proposed approach consists as follows. First, the parameters $m$ and $C_dA$ for the three VGTs considered above are summarized and shown in the left plots of Figure 3.28. The three curves are then ensemble-averaged to define a qualitative behavior representative

Figure 3.28: Turbine Flow Model Parameter as Function of VGT Position: 3 VGTs (left) and Normalization (Right)
of all the turbines considered. Defining the normalized parameter as:

\[
\overline{m} = \frac{m(x_{vgt})}{m(x_{vgt})|_{x_{vgt}=1}} \\
\overline{C_dA} = \frac{C_dA(x_{vgt})}{C_dA(x_{vgt})|_{x_{vgt}=1}}
\] (3.24)

the data from the three turbines will condensed to the same point at 100% VGT opening. A modified version of the quadratic function in (3.23) is then used to fit the normalized parameters:

\[
\overline{m} = k_{11}(x_{vgt} - 1)^2 + k_{12}(x_{vgt} - 1) + 1 \\
\overline{C_dA} = k_{21}(x_{vgt} - 1)^2 + k_{22}(x_{vgt} - 1) + 1
\] (3.25)

The complete parametric flow model for a family of variable geometry turbines is finally summarized as:

\[
m_{\text{scale}}(x_{vgt}, \frac{A}{R}, D) = \overline{m}(x_{vgt}) \cdot m_{\text{fix}}(\frac{A}{R}, D) \\
C_{dA_{\text{scale}}}(x_{vgt}, \frac{A}{R}, D) = \overline{C_dA}(x_{vgt}) \cdot C_{dA_{\text{fix}}}(\frac{A}{R}, D)
\] (3.26)

A verification of the parametric model was conducted by applying the model to two fixed geometry turbines taken from the GT family, and extrapolating the flow maps as if

![Figure 3.29: VGT Flow Parametric Model Validation (Extrapolated VGT Maps from Fixed Geometry Turbine Data)](image)

49
the turbines were equipped with VGT actuator. The results are illustrated in Figure 3.29, proving that the method leads to extrapolated maps with physically consistent behavior.

### 3.3.3 Turbine Efficiency

The first proposed turbine efficiency is based on blade speed ratio, which has the same form as Eriksson compressor efficiency model. The efficiency is represented as a quadratic function of blade speed ratio and Mach number:

\[
\eta_t = \eta_{max} - \chi^T Q \chi = \eta_{max} - \begin{bmatrix} BSR - BSR_{max} \\ M - M_{max} \end{bmatrix}^T \begin{bmatrix} c_1 & c_2 \\ c_2 & c_3 \end{bmatrix} \begin{bmatrix} BSR - BSR_{max} \\ M - M_{max} \end{bmatrix}
\]

\[
P = \begin{bmatrix} c_1 & c_2 & c_3 & \eta_{max} & BSR_{max} & M_{max} \end{bmatrix}
\] (3.27)

This modeling approach requires the determination of six parameters for one fixed geometry turbine. Figure 3.30 illustrates the result of the BSR-based turbine efficiency model. Since the data of original high pressure turbine (HPT) contains 7 VGT positions, the result of this procedure is one set of regression parameters \( P \) of each of the 7 VGT positions.

![Figure 3.30: BSR-based Turbine Efficiency Model Validation (HPT)](image)
Finally, efficiency at arbitrary VGT opening is obtained by interpolation among 7 efficiency submodels.

Although this model has high accuracy on turbine steady-state data points, it is not a good choice when connected to the engine model. Since pressure drop is faster than speed drop in simulation, and the speed line is very steep at low pressure region, in some cases efficiency will decrease to saturation points. Therefore, a robust turbine efficiency model is introduced, which is power-based and independent on speed. The corrected turbine power is defined as:

\[ P_{\text{corr}} = \eta_t \dot{m}_{\text{corr}} c_p T_{\text{in}} \left( 1 - \epsilon \frac{1 - \gamma}{\gamma} \right) \]  \hspace{1cm} (3.28)

which can be approximated as a power function of pressure ratio:

\[ P_{\text{corr}} = c_1 \epsilon^{c_2} + c_3 \]  \hspace{1cm} (3.29)

Then the efficiency is calculated as:

\[ \eta_t = \frac{c_1 \epsilon^{c_2} + c_3}{\dot{m}_{\text{corr}} c_p T_{\text{in}} \left( 1 - \epsilon \frac{1 - \gamma}{\gamma} \right)} \]  \hspace{1cm} (3.30)

The result of the power-based turbine efficiency model is shown in Figure 3.31.
Notice that in (3.30), $\dot{m}_{corr}$ is obtained from turbine flow model. Since the power-based efficiency model is coupled with flow model, it can not be used for scalable turbocharger model. In [121], a power function is used to model the turbine mass flow. Therefore, $\dot{m}_{corr}$ as well as constant term $c_p T_{in}$ can be removed from (3.30), which yields:

$$\eta_t = \frac{c_1 \epsilon^2 + c_3}{1 - \epsilon \gamma} \quad (3.31)$$

The result of the modified power-based turbine efficiency model, shown in Figure 3.32, is almost the same with that in Figure 3.31. The advantage is this efficiency model is decoupled with flow model, thus it can be used in the case the flow model is scaled with
geometry.

Similar as BSR-based model, one set of regression parameters \( P = [c_1 \ c_2 \ c_3] \) of each of the 7 VGT positions is obtained, and efficiency at arbitrary VGT opening is computed by interpolation. The efficiency map for variable geometry turbine is shown in Figure 3.33.

### 3.4 Mean-Value Model of Diesel Engine

In this section, the Mean-Value Model (MVM) of Diesel engine is introduced, focusing on the characterization of the engine air-path system, in order to study and control the two-stage turbocharger dynamics and its interaction with engine breathing dynamics. The engine model, based on the early work documented in [70], is updated with parametric model of turbocharger and thermal model of exhaust system.

The system considered is a GM Duramax medium-duty Diesel engine, equipped with a two-stage turbocharger, intercooler and high pressure cooled EGR. The air-path configuration is shown in Figure 3.34, while the engine characteristics are in Table 3.7.

The turbocharger system includes a smaller high pressure (HP) turbocharger arranged in series with a larger low pressure (LP) turbocharger. The HP stage includes a controlled bypass valve which allows for switching between single stage and two-stage operations. The fresh air is first compressed in the LP compressor and then fed to the HP compressor. In order to increase the air density, the air passes through an intercooler, and before entering the engine cylinders, it mixes with exhaust gases coming from the high pressure EGR system. The two turbines convert the exhaust gas energy into shaft work, through a common shaft drives the connected compressor. On the HP stage, the flow through the turbine is controlled through the VGT and the bypass actuator. The bypass, VGT, and EGR positions determine the operating points of the two-stage turbocharger and control the back pressure in the exhaust to drive the EGR flow.
Table 3.7: GM Duramax Engine Characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>6.599L</td>
</tr>
<tr>
<td>Valvetrain</td>
<td>OHV 4-V</td>
</tr>
<tr>
<td>Injection</td>
<td>Common-rail</td>
</tr>
<tr>
<td>Rated Torque</td>
<td>820 Nm @ 1800 rpm</td>
</tr>
<tr>
<td>Rated Power</td>
<td>231 kW @ 3200 rpm</td>
</tr>
</tbody>
</table>

The engine is divided into subsystems, as shown in Figure 3.35. The engine system model contains the intake and exhaust manifold dynamics coupled by EGR system, and model describing engine breathing, combustion, and torque production. The engine system model connected with the turbocharger model at the compressor side (mass flow rate and outlet temperature feeding the intake manifold model), and at the turbine side (the exhaust manifold states being the inputs of the high pressure turbine). The flow and energy outputs of the compressors and turbines are typically modeled as quasi-steady elements. In order to respect the cause and effect relationships, the two compressor stages must be separated by receivers in order to estimate the intermediate pressure (the same concept holds for the turbine stages).
3.4.1 Valves and Flow Restrictions

The flow restrictions are modeled as steady-state, adiabatic reversible flow through an orifice. The model can be applied to the EGR valve, HP bypass valve, and LP waste-gate.

\[
\dot{m} = C_d A \frac{P_{in}}{RT_{in}} \sqrt[\gamma]{f_1 \left( \frac{P_{out}}{P_{in}} \right)}
\]

\[
f_1 \left( \frac{P_{out}}{P_{in}} \right) = \begin{cases} 
\frac{2}{\gamma - 1} \left[ \left( \frac{P_{out}}{P_{in}} \right)^{\frac{2}{\gamma}} - \left( \frac{P_{out}}{P_{in}} \right)^{\frac{\gamma + 1}{\gamma}} \right] & \text{if } \frac{P_{out}}{P_{in}} \geq \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma + 1}} \\
\sqrt{\frac{2}{\gamma + 1}^{\frac{\gamma + 1}{\gamma}}} & \text{if } \frac{P_{out}}{P_{in}} < \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma + 1}}
\end{cases}
\]

(3.32)

where the effective area \(C_d A\) needs to be identified on experimental data for each component.

3.4.2 Receivers

The receivers are treated as mass and energy accumulators, and their dynamics is characterized by the conservation laws. In a mean-value engine model, the typical components
represented by receivers are intake and exhaust manifolds. Besides, intermediate volumes of the two-stages turbocharger should be modeled as receivers to maintain consistency with the dynamics of the engine system and the cause and effect relationships.

The isothermal approach is used to model the intermediate volumes of the two-stage turbocharger:

\[
\frac{d}{dt} p = \frac{RT}{V} (\dot{m}_{in} - \dot{m}_{out})
\]  

(3.33)

The adiabatic approach is applied to the intake manifold:

\[
\begin{align*}
\frac{d}{dt} p_{IM} &= \frac{\gamma R}{V_{IM}} \left[ \dot{m}_{HPc} T_{cc} + \dot{m}_{egr} T_{egr} - \dot{m}_{eng} T_{IM} \right] \\
\frac{d}{dt} T_{IM} &= \frac{\gamma R T_{IM}}{p_{IM} V_{IM}} \left[ \dot{m}_{HPc} T_{cc} + \dot{m}_{egr} T_{egr} - \dot{m}_{eng} T_{IM} - \frac{T_{IM}}{\gamma} (\dot{m}_{HPc} + \dot{m}_{egr} - \dot{m}_{eng}) \right]
\end{align*}
\]  

(3.34)

In exhaust manifold model, a heat loss term is introduced:

\[
\begin{align*}
\frac{d}{dt} p_{EM} &= \frac{\gamma R}{V_{EM}} \left[ (\dot{m}_{eng} + \dot{m}_{fuel}) T_{exh} - (\dot{m}_{egr} + \dot{m}_{HPt}) T_{EM} - \frac{hA(T_{EM} - T_{wall})}{c_p} \right] \\
\frac{d}{dt} T_{EM} &= \frac{\gamma R T_{EM}}{p_{EM} V_{EM}} \left[ (\dot{m}_{eng} + \dot{m}_{fuel}) T_{exh} - (\dot{m}_{egr} + \dot{m}_{HPt}) T_{EM} - \frac{T_{EM}}{\gamma} (\dot{m}_{eng} + \dot{m}_{fuel} - \dot{m}_{egr} - \dot{m}_{HPt}) - \frac{hA(T_{EM} - T_{wall})}{c_p} \right]
\end{align*}
\]  

(3.35)

3.4.3 Mass Flow through Engine

Speed-density equation is used to model the mass flow from the intake manifold into cylinders:

\[
\dot{m}_{eng} = \frac{\lambda_v p_{IM} V_d N_{eng}}{120RT_{IM}}
\]  

(3.36)

where \(V_d\) is the engine displacement, \(\lambda_v\) is the engine volumetric efficiency. The volumetric efficiency is modeled as:

\[
\lambda_v p_{IM} = k_1 p_{IM} + k_2
\]  

(3.37)
where $k_1$ and $k_2$ are constant parameters, identified on experimental data.

### 3.4.4 Engine-out Temperature and Torque Production

The application of MVM techniques to the engine combustion and torque production assumes that the processes occurring in the engine cylinders are cycle-averaged. Therefore, the in-cylinder processes is typically reduced to quasi-steady, algebraic relationships.

The exhaust gas temperature is estimated through the definition of a temperature increment $\Delta T$, which is a regression function of the charge composition:

$$T_{exh} = T_{IM} + \Delta T(AFR, EGR, T_{IM})$$  \hspace{1cm} (3.38)

The torque model is based on estimation of the indicated mean effective pressure (IMEP) from the experimental brake mean effective pressure (BMEP), and estimation of the friction mean effective pressure (FMEP) by the Willans line approach [122]. Then the indicated efficiency can be calculated as:

$$\eta_{ind} = \frac{IMEP \cdot V_d \cdot N_{eng}}{120 \cdot \dot{m}_{fuel} \cdot LHV_f}$$  \hspace{1cm} (3.39)

where $LHV_f$ is the fuel lower heating value. Based on the principles of Diesel combustion and on the analysis of engine data, a phenomenological model for engine indicated efficiency is proposed, which is a function of the engine speed, air fuel ratio, and the differential pressure:

$$\eta_{ind} = \eta_0(k_1 + k_2 N_{eng} + k_3 N_{eng}^2)(1 + k_4 AFR^{k_5})(1 + k_6(p_{EM} - p_{IM}))$$  \hspace{1cm} (3.40)

The FMEP is modeled as a quadratic function of engine speed:

$$FMEP = c_1 + k_c N_{eng} + c_3 N_{eng}^2$$  \hspace{1cm} (3.41)
Then the torque is calculated as:

\[
T_{\text{eng}} = \left[ \frac{120 \eta_{\text{ind}} \dot{m}_{\text{fuel}} LHV_f}{V_d N_{\text{eng}}} - (c_1 + k_c N_{\text{eng}} + c_3 N_{\text{eng}}^2) \right] \frac{V_d}{4\pi} \quad (3.42)
\]

3.4.5 Turbocharger Speed

The turbocharger speed is modeled using Newton’s second law for rotating systems:

\[
\frac{d}{dt} N_{tc} = \left( \frac{30}{\pi} \right) \frac{2}{J_{tc} N_{tc}} \left( P_t - P_c \right) \quad (3.43)
\]

where \( J_{tc} \) is the inertia, \( P_t \) is the power delivered by the turbine, and \( P_c \) is the power required to drive the compressor.

3.5 Integration and Validation of Two-Stage Turbocharged Engine Model

The parametric model of turbocharger is integrated with the Diesel engine air-path system model, and used to simulate the system behavior at steady-state and transient conditions and validate against a set of experimental data.

In order to identify the parameters of the engine system model, a sparse set of steady-state engine data is used, covering the engine operating range in terms of speed and torque, shown in the left plot of Figure 3.36. The validation of the model is done on the complete set of available data, considering the engine speed and torque as the inputs for the model. To this extent, the model was completed with the introduction of a simple control strategy, which operates the fuel, EGR, VGT and HP turbine bypass based on the desired engine speed and torque.

The structure of the control strategy is shown in Figure 3.37. The fuel mass flow rate is determined by a feedforward maps of desired torque and engine speed. The EGR, VGT
and bypass positions are controlled using a simple feedforward strategy, based on static maps depending on the desired engine speed and fuel mass flow rate. The positions of three actuators are shown in the right plots of Figure 3.36.

### 3.5.1 Steady-State Validation

The parametric model of turbocharger is used to replace the previously developed, look-up table based model of the two-stage turbocharger contained in the Diesel engine air-path
system. Note that, due to the modified causality of the compressor model (Type 2 model), the overall model results with a higher number of states.

Starting from the original maps, the parameters for the compressor and turbine models were identified by following the procedure highlighted in previous sections. The models were calibrated on the original two-stage turbocharger data, and then integrated with the air-path system model of the 6.6L Duramax Diesel engine.

Simulation results were conducted with respect to 115 steady-state points covering the entire engine operating range. The results obtained with the parametric turbocharger model are compared with the results from original look-up table based turbocharger model, as summarized in Figure 3.38 and Figure 3.39. From the analysis of the results it can be observed that the parametric model achieves the same accuracy as the original model. In particular, an analysis of the operating points on the compressors maps indicates that the two models are almost equivalent, hence showing the validity of the new parametric modeling approach in predicting the system behavior with full engine system simulation.

Figure 3.38: Steady-State Validation of the Engine Model
3.5.2 Simulation of FTP Cycle

After steady-state calibration and validation, the engine model is tested for transient simulations on a FTP cycle. The target profiles shown in Figure 3.40 for engine torque and speed are assigned to the model, while using the controller structure illustrated in Figure 3.37, regulate fuel, EGR, VGT and bypass commands. A feedback torque control is introduced, which adjust the fuel command to track the desired torque profile.

The left of Figure 3.41 shows the output of the torque controller, representing the feedforward command and the correction made by the feedback portion. The fuel flow to the engine is determined mostly by the open loop mapping, while the PI controller operates small changes around the set points to accommodate for plant errors. The right of Figure 3.41 shows the EGR, VGT and bypass actuators positions during the cycle. In particular, because of the low power demand during FTP operations, the HP turbine bypass valve is always closed.

Finally, Figure 3.42 shows the evolution of the intake and exhaust manifolds pressure, air mass flow, and EGR ratio during the FTP cycle. All the variables are in reasonable range, showing the validity of the integrated model in predicting the system behavior at transient operations.
Figure 3.40: Engine Speed and Torque Profile during FTP Cycle

Figure 3.41: Commands during FTP Cycle: Fueling Rate (Left) and Actuators Positions (Right)

Figure 3.42: Simulation Results of the Engine Model during FTP Cycle
Chapter 4

Air-Path Control of Diesel Engine for Rapid Warm-up

Legislated emission limits for medium and heavy duty trucks are constantly reduced while there is a significant drive for good fuel economy. In order to meet the stringent regulations, aftertreatment systems have become necessary for Diesel-powered vehicles. In specific, Diesel oxidation catalyst (DOC) is a key component for removing the engine-out CO, total hydrocarbon emissions, and volatile organic fraction. The chemical reactions inside DOC are suppressed under cold operating conditions until the light-off temperature (200 – 250°C) is reached. Therefore, active temperature control for rapid catalyst warm-up during cold start is necessary for emission reduction. Aftertreatment system thermal management based on fuel-path control can be found in [93–95]. However, the opportunity of controlling the air-path to achieve rapid catalyst warm-up has not been explored.

Therefore, an air-path control strategy, based on a two-stage turbocharged Diesel engine for rapid catalyst warm-up, is investigated in this chapter. As shown in Figure 4.1, the engine is equipped with a VGT and a high pressure EGR loop. A bypass valve is placed across the HP turbine, while a waste-gate is placed across the LP turbine, to regulate the available power to the corresponding compressor. The control actuators are the EGR valve, the VGT, the bypass, and the waste-gate. Typically, due to the relative low power demand during cold start, the bypass and waste-gate are fully closed. So all the exhaust gas energy is consumed by turbines for boosting. However, part of the exhaust energy could go through the waste-gate for catalyst warm-up, result in reduced boost performance. Meanwhile, the total amount of exhaust gas energy is affected by the boost performance, and the EGR
ratio should be kept at desired level. The nonlinearity of the engine system, the presence of multiple actuators, and the coupling effects, make the air-path control challenging. Since model predictive control (MPC) is considered as a systematic multivariable control scheme to accommodate actuators and state constraints, it is chosen for this Diesel engine air-path control study.

This chapter is organized as follows. First, thermal models for waste-gate and DOC are introduced. Preliminary study on the FTP city driving cycle is conducted to analysis the influence of bypass and waste-gate on catalyst warm-up. Then, model order reduction and linearization are applied, resulting in a discrete-time piecewise affine (PWA) system. The rapid warm-up problem is formulated as MPC tracking control, then the controller and state observer are designed on the PWA system. Finally, MPC and observer are implemented on the full order nonlinear engine model, and a heuristic control for bypass opening is extracted from the solution of MPC.
4.1 Thermal Model of Exhaust System

4.1.1 Bypass Valve Thermal Model

The bypass valve flow is modeled as steady-state, adiabatic reversible flow through an orifice. However, due to the sudden expansion after the throat, a large portion of kinetic energy is converted to internal energy, which makes the outlet temperature higher than the one determined from adiabatic approach. Therefore, a bypass valve thermal model is developed, which take temperature recovery and heat transfer into account.

As illustrated in the left side of Figure 4.2, bypass valve is divided into two parts: inlet to throat (1 to 2) is modeled as converging nozzle, where the flow is characterized by adiabatic orifice flow; throat to outlet (2 to 3) is modeled as diverging nozzle, where the kinetic energy is converted to internal energy. One widely adopted assumption is that the kinetic energy of 1 and 3 can be ignored ($c_1 = c_3 = 0$), since the cross sectional area of 1 and 3 are much larger than 2. To decouple the bypass valve thermal model and flow model, another assumption is made, such that the pressure in the diverging nozzle are roughly the same ($p_2 = p_3$).

Figure 4.2: Sketch of Bypass Valve Thermal Model: Converging-Diverging Nozzle (Left) and Turbine with Bypass (Right)

---

11, $p_{T_2}$, $p_{T_3}$, $p_{T_1}$

$T_{out}$, $T_{Turb.}$
The energy equation of converging nozzle 1 to 2 is:

\[ h_1 = h_2 + \frac{c_2^2}{2} \] (4.1)

where \( h \) stands for enthalpy and \( c \) stands for velocity. Since 1 to 2 is adiabatic process, the velocity of 2 is obtained as:

\[ c_2 = \sqrt{2c_pT_1 \left[ 1 - \left( \frac{p_3}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right]} \] (4.2)

The orifice equation is:

\[
\dot{m}_{wg} = C_d A \frac{p_1}{\sqrt{R T_1}} \sqrt{\gamma f_1 \left( \frac{p_3}{p_1} \right)}
\]

\[ f_1 \left( \frac{p_3}{p_1} \right) = \begin{cases} 
\sqrt{\frac{2}{\gamma - 1} \left[ \left( \frac{p_3}{p_1} \right)^{\frac{2}{\gamma}} - \left( \frac{p_3}{p_1} \right)^{\frac{\gamma+1}{\gamma}} \right]} & \text{if } \frac{p_3}{p_1} \geq \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma-1}} \\
\sqrt{\frac{2}{\gamma + 1}^{\frac{\gamma+1}{\gamma-1}}} & \text{if } \frac{p_3}{p_1} < \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma-1}}
\end{cases} \] (4.3)

where \( C_d A \) is a function of bypass valve opening. The energy equation of diverging nozzle 2 to 3 is:

\[ h_2 + \frac{c_2^2}{2} = h_3 + \frac{\dot{Q}_{23}}{\dot{m}_{wg}} \] (4.4)

where \( \dot{Q}_{23} \) is the heat transfer loss, \( \dot{m}_{wg} \) is the bypass valve mass flow determined above. The heat transfer term can be expressed as:

\[ \dot{Q}_{23} = hA(T_3 - T_w) \] (4.5)

where \( hA \) is the production of heat transfer coefficient and effective area, \( T_w \) is the wall temperature.
Assuming turbulent pipe flow, based on the Dittus-Boelter equation, the heat transfer coefficient can be expressed explicitly as:

\[
\frac{hD}{k} = 0.023\text{Re}^{0.8}\text{Pr}^{0.3}
\]  \(4.6\)

where \(D\) is the average inside diameter of the diverging nozzle, \(k\) is the thermal conductivity of exhaust gas, \(\text{Re}\) is the Reynolds number and \(\text{Pr}\) is the Prandtl number. The Reynolds number can be expressed as:

\[
\text{Re} = \frac{c_2D}{\nu}
\]  \(4.7\)

where \(\nu\) is the kinematic viscosity. Combining (4.6) and (4.7) yields:

\[
hA = Kc_2^{0.8}, \quad K = 0.023 \left(\frac{k\text{Pr}^{0.3}}{\nu^{0.8}}\right) \left(\frac{D^{0.8}}{D-\pi DL}\right)
\]  \(4.8\)

where \(K\) is a calibration parameter which depends on exhaust gas property and temperature (first brackets), and on bypass valve geometry (second brackets).

Finally, plugging (4.5) and (4.8) into (4.4), the outlet temperature is expressed as:

\[
T_3 = \frac{\dot{m}_{wg}c_p}{\dot{m}_{wg}c_p + Kc_2^{0.8}} T_1 + \frac{Kc_2^{0.8}}{\dot{m}_{wg}c_p + Kc_2^{0.8}} T_w
\]  \(4.9\)

It can be seen that bypass valve outlet temperature is a weighted average of inlet temperature and wall temperature.

Since the bypass valve and turbine are mounted in parallel, as shown in the right side of Figure 4.2, there is a mixing of the gases at the outlet. This mixing is modeled as an adiabatic process, which gives the following temperature for the mixture:

\[
T_{out} = \frac{\dot{m}_t T_t + \dot{m}_{wg} T_3}{\dot{m}_t + \dot{m}_{wg}}
\]  \(4.10\)
where $\dot{m}_t$ is the turbine mass flow, and $T_1$ is the turbine outlet temperature expressed as:

$$T_t = T_1 - \eta_t T_1 \left[ 1 - \left( \frac{p_3}{p_1} \right)^\frac{\gamma - 1}{\gamma} \right]$$

Both $\dot{m}_t$ and $\eta_t$ are obtained from turbine parametric model discussed in Section 3.3.

Given the inlet temperature $T_1$ and pressure $p_1$, the outlet temperatures are shown in Figure 4.3. It can be seen that the temperature $T_{is}$ obtained from adiabatic approach is lower than turbine outlet temperature $T_3$. In the proposed bypass thermal model, $T_3$ asymptotically decreases as pressure ratio approaches to 1, since trivial exhaust energy goes through bypass valve. At high pressure ratio region, due to the heat transfer, $T_3$ will slowly decrease. At a certain pressure ratio, the mass flow rates are the same but the velocity $c_2$ changes with bypass opening. Since $c_2$ affects heat transfer, larger bypass opening leads to higher $T_3$. 

---

*Figure 4.3: The Bypass Valve Outlet Temperature*
4.1.2 Catalyst Thermal Model

In literature, research conducted on control-oriented catalyst thermal model can be found in [122–124]. In the application of cold start warm-up, a reasonable assumption is that no chemical reaction, so a lumped catalyst thermal model is described by heat transfer model:

\[
(mc)_{cat} \frac{d}{dt} T_{cat} = \dot{m}_{ex} c_p (T_{ex} - T_{cat}) - hA(T_{cat} - T_{amb})
\]  

(4.12)

where \( \dot{m}_{ex} \) and \( T_{ex} \) stand for the mass flow and temperature of exhaust gas upstream of DOC, respectively. \((mc)_{cat}\) is the thermal inertia, and \(hA\) is the production of convective heat transfer coefficient and outer surface of a DOC, which can be identified experimentally.

4.2 Preliminary Analysis on FTP Cycle

4.2.1 Baseline Scenario

Since the study focuses on cold start warm-up, first 200 seconds of the FTP city driving cycle, during which the catalyst can reach the light-off temperature \((T_{thr} = 200^\circ C)\), is used to evaluate the system performance. Because of the low power demand during FTP cycle, the HP bypass valve and LP waste-gate valve are always closed. A preliminary analysis is conducted based on the original feedforward control strategy. The result of this baseline condition is shown in Figure 4.4. It can be seen that the catalyst is light off at 192s, and the peak boost pressure is about 2.5 bar. From the plots of turbocharger speed and pressure ratio, it can be concluded that the boost is mainly contributed by the HP turbocharger.

4.2.2 Influence of HP Bypass Valve

To analyze the effect of HP bypass valve on warm-up performance, a study is conducted considering several constant bypass openings, while keeping the original control strategy of EGR, VGT, and waste-gate. It can be seen in Figure 4.5, the boost pressure is very sensitive
to bypass opening, and the warm-up is improved by penalizing the boost pressure. To maintain certain level of boost, the maximum bypass valve opening should be constrained.

4.2.3 Influence of LP Waste-Gate Valve

Similarly, constant waste-gate openings are tested with original control strategy of other actuators, and the result is in Figure 4.6. Since LP turbocharger does not effectively
work in low power operations, waste-gate valve has trivial effect on warm-up temperature improvement. Therefore, the original waste-gate control strategy is reserved, which can reduce one input for control design.

4.3 Piecewise Affine System Development

The engine air-path model is a high order nonlinear system, which can be particularly troublesome for control design. Thus, approximations become necessary, very often in the form of local linear models. In this study, we propose a piecewise affine discrete-time approximation of the system dynamics, which is used for control and observer design. First, model order reduction techniques is applied to the high-order complex engine model. Then, the reduced order nonlinear model is linearized at 20 representative operating points. Finally, a discretized piecewise affine system with a polyhedral partition of the exogenous input space (engine speed and torque) is obtained.

4.3.1 Model Order Reduction

The engine air-path system is characterized by a 11-state dynamic model, where the states include catalyst temperature, intake/exhaust manifold pressures and temperatures, two-stage turbocharger shaft speeds, compressor mass flow rates, and intermediate volume
The compressor mass flow rates $\dot{m}_{HPc}$ and $\dot{m}_{LPc}$ can be reduced by using Type 1 model as discussed in previous report. The intake/exhaust manifold temperatures $T_{IM}$ and $T_{EM}$ are modeled as static correlations:

$$T_{IM} = k_1 + k_2p_{IM} + k_3z_{egr}$$
$$T_{EM} = k_4T_{exh} + k_5$$

Therefore, the original engine model can be approximated by a reduced order model with 7 states, which can simplify the control design:

$$x = \begin{bmatrix} T_{cat} & p_{EM} & T_{EM} & p_{IM} & T_{IM} & \dot{m}_{HPc} & \dot{N}_{HP} & p_{t,\text{int}} & \dot{m}_{LPc} & \dot{N}_{LP} & p_{c,\text{int}} \end{bmatrix}^T$$

The comparison of the reduced order model and the full order model are shown in

Figure 4.7: Validation of Reduced Order Model at Steady-State Conditions

pressures:

$$x = \begin{bmatrix} T_{cat} & p_{EM} & T_{EM} & p_{IM} & T_{IM} & \dot{m}_{HPc} & \dot{N}_{HP} & p_{t,\text{int}} & \dot{m}_{LPc} & \dot{N}_{LP} & p_{c,\text{int}} \end{bmatrix}^T$$

$$x = \begin{bmatrix} T_{cat} & p_{EM} & T_{EM} & p_{IM} & T_{IM} & \dot{m}_{HPc} & \dot{N}_{HP} & p_{t,\text{int}} & \dot{m}_{LPc} & \dot{N}_{LP} & p_{c,\text{int}} \end{bmatrix}^T$$
Figure 4.8: Validation of Reduced Order Model during FTP Cycle

Figure 4.7 and Figure 4.8. From the figures, it can be concluded that the reduced order model is capable to characterize the engine system behavior both in steady-state and in transient operations.

4.3.2 Linearization and Discretization

After model order reduction, the dynamics of the remaining 7 states are summarized as:

\[(mc)_{cat} \frac{dT_{cat}}{dt} = \dot{m}_{ex} c_p (T_{ex} - T_{cat}) - \dot{h}A (T_{cat} - T_{amb}) \tag{4.16a}\]

\[\frac{dp_{IM}}{dt} = \frac{\gamma R}{V_{IM}} (\dot{m}_{HPc} T_{cc} + \dot{m}_{egr} T_{egr} - \dot{m}_{eng} T_{1M}) \tag{4.16b}\]

\[\frac{dp_{EM}}{dt} = \frac{\gamma R}{V_{EM}} \left[ (\dot{m}_{eng} + \dot{m}_{fuel}) T_{exh} - (\dot{m}_{egr} + \dot{m}_{HPI}) T_{EM} - \frac{\dot{h}A (T_{EM} - T_{wall})}{c_p} \right] \tag{4.16c}\]

\[\frac{dp_{c,int}}{dt} = \frac{RT_{LPc}}{V_{c,int}} (\dot{m}_{LPc} - \dot{m}_{HPc}) \tag{4.16d}\]

\[\frac{dp_{t,int}}{dt} = \frac{RT_{HPI,mix}}{V_{t,int}} (\dot{m}_{HPI} + \dot{m}_{lp} - \dot{m}_{LPt} - \dot{m}_{wg}) \tag{4.16e}\]

\[\frac{dN_{HP}}{dt} = \left( \frac{30}{\pi} \right)^2 \frac{P_{HPl} - P_{HPC}}{J_{HP} N_{HP}} \tag{4.16f}\]

\[\frac{dN_{LP}}{dt} = \left( \frac{30}{\pi} \right)^2 \frac{P_{LPt} - P_{LPc}}{J_{LP} N_{LP}} \tag{4.16g}\]
The reduced order engine model is linearized at 20 representative operating points, resulting in a PWA system with a polyhedral partition of the exogenous input space, as illustrated in Figure 4.9. At each operating point, $\dot{m}_{\text{fuel}}$, $x_{\text{vgt}}$, $x_{\text{egr}}$ are determined by
Figure 4.11: Validation of Linearized Model at Region 6: Change of Operating Point

feedforward maps, and $\eta_{bp} = 3\%$. The continuous PWA system has form:

$$
\begin{aligned}
\dot{x} &= A_i(x - x_0) + B_i(u - u_0) + E_i(v - v_0) = A_i x + B_i u + E_i v + f_i \\
y &= C_i(x - x_0) + D_i(u - u_0) + y_0 = C_i x + D_i u + F_i v + g_i \\
\text{if } \begin{bmatrix} N_{\text{eng}} & T_{\text{eng}} \end{bmatrix}^T \in \mathcal{P}_i, \quad i = 1, \ldots, 20
\end{aligned}
$$

(4.17)

where states $x$, control inputs $u$, exogenous inputs $v$, outputs $y$ are defined as:

$$
\begin{aligned}
x &= \begin{bmatrix} T_{\text{cat}} & p_{\text{EM}} & p_{IM} & N_{\text{HP}} & N_{\text{LP}} & p_{c,\text{int}} \end{bmatrix}^T \\
u &= \begin{bmatrix} x_{\text{vgt}} & x_{\text{egr}} & x_{\text{bp}} \end{bmatrix}^T \\
y &= \begin{bmatrix} p_{IM} & \dot{m}_{\text{HP}c} & z_{\text{egr}} & T_{\text{cat}} \end{bmatrix}^T \\
v &= \begin{bmatrix} N_{\text{eng}} & \dot{m}_{\text{fuel}} \end{bmatrix}^T
\end{aligned}
$$

(4.18)

and $\mathcal{P}_i$ denotes the $i^{th}$ polytope on the engine map, which is given by:

$$
\mathcal{P}_i = \{x \in \mathbb{R}^2 : H x \leq k_i\}, \quad i = 1, \ldots, 20
$$

(4.19)

The quality of linearization is evaluated at each region, and the representative examples
at low load and high load regions are shown in Figure 4.10 to Figure 4.13. Overall, the linearized model captures the system dynamics well. However, in Figure 4.10, a dc-gain sign reversal from the VGT to the compressor mass flow channel around 5s is found in region 6. This sign reversal is an essential property of this nonlinear system, and is well discussed in [125].
The zero-order-hold method is used for discretization, and the sampling time is selected to be $T_s = 0.05$ s. Then the discrete-time PWA system is obtained, which has the form:

$$
\begin{align*}
    x(k+1) &= A_ix(k) + B_iu(k) + E_iv(k) + f_i \\
    y(k) &= C_ix(k) + D_iu(k) + F_iv(k) + g_i \\
    &\text{if } \left[ N_{eng}(k) \quad T_{eng}(k) \right]^T \in \mathcal{P}_i, \quad i = 1, \ldots, 20
\end{align*}
$$

(4.20)

### 4.3.3 Piecewise Affine System Validation

To verify the accuracy of the discrete PWA model, both full order nonlinear model and PWA model are tested on FTP cycle, where feedforward control strategy is used. The results in Figure 4.14 show that the PWA model is a reasonable approximation of the nonlinear plant model, thus will be used for control and observer design.

---

*Figure 4.14: Validation of Piecewise Affine System during FTP Cycle*
4.4 Model Predictive Controller Design

4.4.1 Introduction of Model Predictive Control

MPC is based on iterative, finite horizon optimization of a plant model. At time $t$, starting at the current state, an optimal control problem is solved over a finite horizon, as shown in the top of Figure 4.15. Only the first computed optimal control input is applied to the system during $[t, t+1]$. At time $t+1$, a new optimal control problem based on new measurements of the state is solved over a shifted horizon, as depicted in the bottom of Figure 4.15.

Consider the problem of regulating to the origin for the discrete time linear time invariant (LTI) system with polyhedral states and inputs constraints. The following N-horizon

![Figure 4.15: A Discrete MPC Scheme](image-url)
optimal control problem is solved at time $t$:

$$\min_{U_0} J(x(t), U_0) \triangleq J_f(x_N) + \sum_{k=0}^{N-1} l(x_k, u_k)$$

subj. to $x_{k+1} = Ax_k + Bu_k, \ k = 0, \ldots, N - 1$

$$A_x x_k \leq b_x, \quad A_u u_k \leq b_u, \quad k = 0, \ldots, N - 1$$

$$A_f x_N \leq b_f, \quad x_0 = x(t)$$

where $U_0 = \{u_0, \ldots, u_{N-1}\}$. Considering 2-norm cost function:

$$J(x(t), U_0) = x_N^T Q_f x_N + \sum_{k=0}^{N-1} (x_k^T Q x_k + u_k^T R u_k)$$

then the problem can be recast into the form of a quadratic programming (QP) problem:

$$\min_{U_0} J_N(x(t), U_0) = \begin{bmatrix} U_0 \\ x(t) \end{bmatrix}^T \begin{bmatrix} H & F^T \\ F & Y \end{bmatrix} \begin{bmatrix} U_0 \\ x(t) \end{bmatrix}$$

subject to $G_0 U_0 \leq w_0 + E_0 x(t)$

where $H, F, G_0, w_0, E_0$ can be obtained from the plant model and $Q, R$ (for details, see [85]). Since the optimization problem (4.23) depends on the current state $x(t)$, the implementation of MPC requires the online solution of a QP at each time step.

In [85], an approach to pre-compute the control law $U_0^*(z)$ for all possible state locations $x(t) = z$ is proposed. With polyhedral state and control constraints, explicit solution can be obtained through multiparametric quadratic programming (mp-QP), resulting in a
piecewise affine control law:

\[
U^*(x(t)) = \begin{cases} 
F_1 x(t) + f_1 & \text{if } H_1 x(t) \leq h_1 \\
\vdots & \vdots \\
F_M x(t) + f_M & \text{if } H_M x(t) \leq h_M 
\end{cases} \tag{4.24}
\]

As all \( F_i, f_i, H_i, h_i \) can be stored in look-up tables, the solution to the problem of \( (4.23) \) boils down to the search for the active partition and the use of corresponding control law given by \( (4.24) \). This makes the MPC usable for high-speed applications like engine control.

### 4.4.2 Model Predictive Control for the Engine Air-Path

The objective of air-path control for rapid catalyst warm-up is to improve warm-up performance with trade-off on boost pressure, while maintaining desired EGR ratio. This problem can be cast as MPC tracking problem, and the reference signals are defined as:

\[
r = \begin{bmatrix} T_{\text{cat},d} & p_{IM,d} & z_{\text{egr},d} \end{bmatrix}^T \tag{4.25}
\]

The proposed engine air-path control system is depicted in Figure 4.16. According to previous discussion, MPC is essentially state feedback control, while scheduling of linearized points according to operating region (see Figure 4.9) is essentially feedforward control. Since MPC needs the information of all 7 states in \( (4.15) \), observer is designed to estimate the unmeasured states.

For controller design, Matlab with the Multi-Parametric Toolbox (MPT) [126] is used. Since the current version of MPT does not support quadratic programming for PWA system,
the PWA system (4.20) is treated as 20 separated LTI systems:

\[
    x(k+1) = A_i x(k) + B_i u(k) + E_i v(k) \\
    y(k) = C_i x(k) + D_i u(k) + F_i v(k)
\]

if \[
    \begin{bmatrix} N_{eng}(k) & T_{eng}(k) \end{bmatrix}^T \in \mathcal{P}_i, \quad i = 1, \ldots, 20
\]

Since the partition of PWA system only depends on exogenous inputs, the switch between PWA regions can be easily converted to switch between LTI systems. A supervisory controller is added to select the active LTI system and corresponding controller.

It should be noted that (with a mild abuse of notation) all variables in LTI models represent deviations from the nominal operating condition. In this section, we will omit to distinguish between the variables representing deviations from steady-state and those representing actual values, as this will be clear from the context.

Because engine speed and EGR valve opening has direct effect on EGR ratio, there is
direct feedthrough from inputs to EGR ratio in (4.26). To address the feedthrough term, delta input ($\delta u$) formulation is introduced, the MPC scheme uses the following LTI model:

$$x(k+1) = A_i x(k) + B_i u(k) + E_i v(k)$$

$$u(k) = u(k-1) + \delta u(k)$$

$$y(k) = C_i x(k) + D_i u(k) + F_i v(k)$$

(4.27)

In order to use MPC, the engine air-path LTI model (4.27) has to be rewritten in a suitable form:

$$\begin{bmatrix}
x(k+1) \\
u(k) \\
v(k+1) \\
r(k+1)
\end{bmatrix} =
\begin{bmatrix}
A_i & B_i & E_i & 0 \\
0 & I & 0 & 0 \\
0 & 0 & I & 0 \\
0 & 0 & 0 & I
\end{bmatrix}
\begin{bmatrix}
x(k) \\
u(k-1) \\
v(k) \\
r(k)
\end{bmatrix} +
\begin{bmatrix}
B_i \\
I \\
0 \\
0
\end{bmatrix} \delta u(k)$$

(4.28)

$$y(k) =
\begin{bmatrix}
C_i & D_i & F_i & 0
\end{bmatrix}
\bar{x}(k), \text{ if } \begin{bmatrix} N_{eng}(k) \\
T_{eng}(k) \end{bmatrix}^T \in \mathcal{P}_1, \ i = 1, \ldots, 20$$

where $x, u, v$ are defined in (4.18), and $y$ is the EGR ratio $z_{egr}$.

Then the MPC tracking problem is formulated as:

$$\min_{\Delta U_0} \bar{x}_N^T Q_f \bar{x}_N + \sum_{k=0}^{N-1} (\bar{x}_k^T Q \bar{x}_k + \delta u_k^T R \delta u_k)$$

(4.29a)

subj. to

$$\bar{x}_{k+1} = \bar{A}_i \bar{x}_k + \bar{B}_i \delta u_k$$

(4.29b)

$$\text{if } \begin{bmatrix} N_{eng}(0) \\
T_{eng}(0) \end{bmatrix}^T \in \mathcal{P}_1, \ i = 1, \ldots, 20$$

(4.29c)

$$\bar{x}_{\min} \leq \bar{x}_k \leq \bar{x}_{\max}, \ k = 0, \ldots, N$$

(4.29d)
\[ \delta u_{\text{min}} \leq \delta u_k \leq \delta u_{\text{max}}, \quad k = 0, \ldots, N - 1 \quad (4.29e) \]

\[ u_{\text{min}} \leq u_{k-1} + \delta u_k \leq u_{\text{max}}, \quad k = 0 \quad (4.29f) \]

\[ \delta u_k = 0, \quad k = N_c, \ldots, N \quad (4.29g) \]

\[ \bar{x}_0 = \begin{bmatrix} x(t) & u(t-1) & v(t) & r(t) \end{bmatrix}^T \quad (4.29h) \]

where \( \Delta U_0 = \{ \delta u_0, \ldots, \delta u_{N_c-1} \} \) are the optimization variables, \( N \) and \( N_c \) denote the prediction horizon and the control horizon. The control input applied to the system is:

\[ u(t) = u(t - 1) + \delta u_0^* \quad (4.30) \]

The optimization (4.29) is repeated at time \( k + 1 \) based on the new state \( x(t + 1) \), exogenous inputs \( v(t + 1) \), reference signals \( r(t + 1) \) and current control inputs \( u(t) \), yielding a receding horizon control strategy.

In the MPC tracking problem (4.28)-(4.29), the following assumptions are used:

A1 The control signal is assumed constant for all \( N_c \leq k \leq N \) (4.29g). This reduces the computational complexity of the MPC.

A2 The exogenous inputs \( v \) are assumed constant over the horizon (4.28). If PWA prediction models for \( v(k) \) are available they could be included in the MPC formulation.

A3 Constant region \( i \) over the horizon (4.29c), which basically implies that over the prediction horizon only one linear MPC for one member of the set of LTI systems is implemented. Ideally, prediction of switches between affine dynamics over the horizon will improve the performance. But the problem becomes a mixed integer quadratic program, which is more complex than QP.
4.4.3 MPC Implementation Details

Reference Signals

The reference boost pressure and EGR ratio are maps of operating conditions based on steady-state simulation results, as shown in the left of Figure 4.17.

\[
p_{IM,d}(t) = f(N_{eng}(t), \dot{m}_{fuel}(t)), \quad z_{egr,d}(t) = f(N_{eng}(t), \dot{m}_{fuel}(t)) \quad (4.31)
\]

The map-based reference signals are compared with actual signals from full order engine model tested on FTP cycle. It can be seen that the reference one has a good match with baseline results, except EGR ratio transient during deceleration.

The reference temperature of catalyst not only relates to operating condition, but also relates to time, which makes it difficult to generate a map. Instead, the reference temperature will update at each sampling time, based on previous actual catalyst temperature plus an offset:

\[
T_{cat,d}(t) = T_{cat}(t - 1) + 5°C \quad (4.32)
\]

Figure 4.17: References Signal Maps (Left) and Comparison with FTP Baseline Results (Right)
Cost Function

Since a linear MPC is defined on a specific operating region shown in Figure 4.9, the cost function should be region dependent. Therefore, a normalized cost function, which normalize the weight on boost, is proposed as follows:

\[
\bar{x}^T Q \bar{x} = w_T (T_{cat} - T_{cat,d})^2 + \frac{w_p}{\bar{p}_{IM,i}} (p_{IM} - p_{IM,d})^2 + w_e (z_{egr} - z_{egr,d})^2
\]  

(4.33)

where \( \bar{p}_{IM,i} \) denotes the nominal boost pressure at \( i^{th} \) region. According to (4.28), \( Q \in \mathbb{R}^{15 \times 15} \) has the form:

\[
Q = \begin{bmatrix}
    w_T & 0 & 0 & \cdots & -w_T & 0 & 0 \\
    0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
    0 & 0 & w_p & \cdots & 0 & -\bar{w}_p & 0 \\
    & & & \ddots & & & \\
    -w_T & 0 & 0 & \cdots & w_T & 0 & 0 \\
    0 & 0 & -\bar{w}_p & \cdots & 0 & \bar{w}_p & 0 \\
    0 & 0 & 0 & \cdots & 0 & 0 & 0
\end{bmatrix} + w_e Q_{egr}, \quad \bar{x} = \begin{bmatrix}
    T_{cat} \\
    p_{EM} \\
    p_{IM} \\
    p_{IM,d} \\
    z_{egr,d}
\end{bmatrix}
\]

(4.34)

where \( \bar{w}_p \) is the normalized weight on boost, and \( Q_{egr} \in \mathbb{R}^{15 \times 15} \) is given by:

\[
Q_{egr} = \begin{bmatrix}
    \bar{C}^T_{1,i} \bar{C}_{1,i} & -\bar{C}^T_{1,i} \\
    -\bar{C}_{1,i} & 1
\end{bmatrix}
\]

(4.35)

where \( \bar{C}_{1,i} \in \mathbb{R}^{1 \times 14} \) is the first 14 columns of \( \bar{C}_i \in \mathbb{R}^{1 \times 15} \).
MPC Design Parameters

In the literature of Diesel engine air-path control application, the MPC parameters are typically in the following range:

\[
0.02 \leq T_s \leq 0.05, \quad 25 \leq N \leq 120
\]

\[
2 \leq N_c \leq 5 \text{ (online)}, \quad 1 \leq N_c \leq 2 \text{ (explicit)}
\]

(4.36)

Since the catalyst temperature dynamics is slow, a 0.05s sampling time is chosen. So far, \(v\) and \(r\) are assumed to be constant in prediction, then a larger prediction horizon does not necessarily improve the performance. The simulation results show that once \(N \geq 10\), prediction horizon has little effect on the performance, thus \(N = 10\) is used. Using constraints on the actuators and states as well as the control horizon 1 yields a polyhedral partition of the state space in about 300 regions for explicit MPC. A control horizon 2 would yield a partition into more than 2000 regions. Considering the memory occupancy, \(N_c = 1\) is retained.

Input Oscillation

In PWA system, there are discontinuities in the nominal control inputs (feedforward portion in Figure 4.16) when switching between operating regions. Although partition into smaller regions will decrease the magnitude of such discontinuities, it will result in more frequent switch and high memory occupancy. Slew rate constraints are not convenient to use for limiting these small oscillations since these input rate constraints have to be very tight, leading to a slow closed-loop system.

In [92], two different QP in different operating points are solved at each sampling instant \(k\). The resulting control inputs are a linear interpolation between the two solutions. In [127], similarly, a convex combination of two cost functions are proposed. Another way to prevent these oscillations is to use nonlinear MPC based on linear parameter varying models [128].
For the sake of simplicity, in this study, a moving average filter with 5th order is used to smooth the control inputs.

4.5 Observer Design

Since MPC is state feedback control which required either measurement or estimation of the 7 states defined in (4.15), observer is designed to estimate the unmeasured states. Starting from the discrete-time PWA system (4.20), check the observability when only MAF and MAP sensors are used. The system matrices $A_i \in \mathbb{R}^{7 \times 7}$ and $C_i \in \mathbb{R}^{2 \times 7}$ are given by:

$$A_i = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}, \quad A_{22} \in \mathbb{R}^{6 \times 6}$$

$$C_i = \begin{bmatrix} 0 \\ C_2 \end{bmatrix}, \quad C_{22} \in \mathbb{R}^{2 \times 6}$$

Then the observability matrix is expressed as:

$$\mathcal{O} = \begin{bmatrix} C_i \\ C_i A_i \\ \vdots \\ C_i A_i^6 \end{bmatrix} = \begin{bmatrix} 0 & C_2 \\ 0 & C_2 A_{22} \\ \vdots & \vdots \\ 0 & C_2 A_{22}^6 \end{bmatrix}$$

Since rank$(\mathcal{O}) = 6$, $(C_2, A_{22})$ is an observable pair, but $T_{cat}$ is unobservable. In fact, the catalyst temperature dynamics is connected to the rest of the system in a cascade structure, so it will not observable from the dynamics of the other states. Therefore, catalyst temperature sensor is needed, and observer is designed based on the reduced order PWA system:

$$x(k + 1) = A_i x(k) + B_i u(k) + E_i v(k) + f_i$$

$$y(k) = C_i x(k) + g_i$$

(4.39)
where \( x, u, v, \) and \( y \) are defined as:

\[
\begin{align*}
    x &= \begin{bmatrix} p_{EM} & p_{IM} & N_{HP} & p_{t,int} & N_{LP} & p_{c,int} \end{bmatrix}^T \\
    u &= \begin{bmatrix} x_{vgt} & x_{egr} & x_{bp} \end{bmatrix}^T \\
    v &= \begin{bmatrix} N_{eng} & \dot{m}_{fuel} \end{bmatrix}^T \\
    y &= \begin{bmatrix} p_{IM} & \dot{m}_{HPC} \end{bmatrix}^T
\end{align*}
\]

(4.40)

It should be noted that (with a mild abuse of notation) all matrices in (4.39) are submatrices of that in (4.20) with proper dimension. In this section, we will omit this distinction and this will be clear from the context.

### 4.5.1 Reduced Order Observer

The reduced order observer is designed based on the PWA system (4.39):

\[
\hat{x}(k+1) = A_i \hat{x}(k) + B_i u(k) + E_i v(k) + f_i + L_i (y(k) - C_i \hat{x}(k) - g_i)
\]

(4.41)

The error dynamics is:

\[
\begin{align*}
e(k+1) &= x(k+1) - \hat{x}(k+1) \\
&= A_i x(k) - A_i \hat{x}(k) - L_i (C_i x(k) + g_i - C_i \hat{x}(k) - g_i) \\
&= (A_i - L_i C_i) e(k)
\end{align*}
\]

(4.42)

Output injection matrix \( L_i \in \mathbb{R}^{6 \times 2} \) are designed such that the closed-loop system has faster convergence rate, and all the eigenvalues are in the unit circle:

\[
\text{eig}(A_i - L_i C_i) = 0.9 \cdot \text{eig}(A_i)
\]

(4.43)
The observer is first implemented on the PWA system and the states can be estimated accurately, as shown in Figure 4.18. Then the observer is applied to the nonlinear engine model. It can be seen in Figure 4.19 that estimation of $p_{c,int}$ has large error in high load region.
4.5.2 Observer Design Considering Model Uncertainties

It should be noted that the observer is designed based on PWA model (4.20). Due to model order reduction and linear approximation, the PWA model (4.20) has mismatch with the full order nonlinear engine model, even at linearized points. Consider the modeling error as additive disturbance, (4.39) should be rewritten as:

\[
x(k + 1) = A_i x(k) + B_i u(k) + E_i v(k) + f_i + \Delta x_i \\
y(k) = C_i x(k) + g_i + \Delta y_i
\]

(4.44)

Then the estimation error dynamics becomes:

\[
e(k + 1) = x(k + 1) - \hat{x}(k + 1) \\
= A_i e(k) + \Delta x_i - L_i (C_i x(k) + g_i + \Delta y_i - C_i \hat{x}(k) - g_i) \\
= (A_i - L_i C_i) e(k) + \Delta x_i - L_i \Delta y_i
\]

(4.45)

At steady-state, the estimation error is:

\[
e = (I - A_i + L_i C_i)^{-1} (\Delta x_i - L_i \Delta y_i)
\]

(4.46)

It can be seen that the output injection matrix \(L_i\) will affect steady-state estimation error due to the presence of \(\Delta y_i\). In the observer design, both the error convergence rate and the norm of \(L_i\) need to be considered.

<table>
<thead>
<tr>
<th>No.</th>
<th>(L_i)</th>
<th>(|L_i|)</th>
<th>(|I - (A_i - L_i C_i)|)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\text{eig}(A_i - L_i C_i) = 0.9 \cdot \text{eig}(A_i))</td>
<td>3.0006</td>
<td>2.9810</td>
</tr>
<tr>
<td>2</td>
<td>(\text{eig}(A_i - L_i C_i) = \text{eig}(A_i))</td>
<td>0.0582</td>
<td>1.1061</td>
</tr>
<tr>
<td>3</td>
<td>pole placement</td>
<td>0.3251</td>
<td>0.9157</td>
</tr>
</tbody>
</table>
Figure 4.20: Steady-State Estimation Error at Region 18: $\text{eig}(A_i - L_i C_i) = 0.9 \cdot \text{eig}(A_i)$

Figure 4.21: Steady-State Estimation Error at Region 18: $\text{eig}(A_i - L_i C_i) = \text{eig}(A_i)$
Three cases of $L_i$ are listed in Table 4.1 and their steady-state estimation errors are illustrated in Figure 4.20 to Figure 4.22. Comparing case 1 with case 2, the pole location is shrunk by the factor of 0.9, but $\|L_i\|$ increase more than 50 times, yielding larger steady-
state estimation error in Figure 4.20. By tuning the closed-loop pole location, smaller \( \| L_i \| \) is achieved with reasonably fast error convergence rate. The redesigned observer is implemented on the engine model and the FTP cycle results are shown in Figure 4.23. In this case, the estimation of \( p_{c,\text{int}} \) is much better than that in Figure 4.19.

4.6 Simulation Results and Discussion

4.6.1 Feedforward Control

In production engines, the desired actuator positions at different operating points are calibrated through experiments. The results are stored in lookup tables as feedforward control. For the engine considered, because of low power demand in FTP cycle, the bypass valve is always closed. However, opening bypass will increase the energy of exhaust gas which is transported to the catalyst. In a feedforward control strategy, VGT and EGR positions are obtained by lookup tables, while bypass opening is set to be constant regardless of operating condition. The results of feedforward control with different constant bypass openings are compared in Figure 4.24 and Table 4.2.

The catalyst warm-up performance index is defined as ratio between baseline final catalyst temperature and actual one

\[
I_T = \frac{T_{\text{cat, b}}(t = 200s)}{T_{\text{cat}}(t = 200s)}
\]  

(4.47)

<table>
<thead>
<tr>
<th>( x_{bp} ) [%]</th>
<th>( I_T )</th>
<th>( I_p )</th>
<th>( I_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.0258</td>
<td>0.0296</td>
</tr>
<tr>
<td>2</td>
<td>0.9817</td>
<td>0.0604</td>
<td>0.0375</td>
</tr>
<tr>
<td>4</td>
<td>0.9673</td>
<td>0.0993</td>
<td>0.0564</td>
</tr>
<tr>
<td>6</td>
<td>0.9577</td>
<td>0.1287</td>
<td>0.0753</td>
</tr>
</tbody>
</table>
The boost performance index is defined as the root mean square (RMS) error of desired boost pressure and actual one

\[ I_p = \sqrt{\frac{1}{n} \sum_{k=1}^{n} \left( 1 - \frac{p_{IM}(k)}{p_{IM,d}(k)} \right)^2} \]  

(4.48)

The EGR performance index is defined as the RMS error of desired EGR ratio and actual one

\[ I_e = \sqrt{\frac{1}{n} \sum_{k=1}^{n} \left( 1 - \frac{z_{egr}(k)}{z_{egr,d}(k)} \right)^2} \]  

(4.49)

It can be seen from the figure and table, in a small range, large bypass opening leads to higher catalyst temperature but less boost pressure and worse EGR ratio. Feedforward control is incapable to control the three actuators coordinately, which makes MPC an appealing solution.
4.6.2 MPC and Pareto Analysis

The controller developed in Section 4.4 and state observer developed in Section 4.5 are implemented on the engine model. First, set the weight on catalyst temperature \( w_T \) to be 0 to study the case when rapid catalyst warm-up is not considered, as depicted in Figure 4.25. It can be seen that boost pressure and EGR ratio are close to desired value, but no improvement on warm-up performance.

Then, the weights in the cost function (4.33) are tuned such that the EGR ratio is close to reference one, with a trade-off between warm-up and boost, as shown in Figure 4.26.

![Figure 4.25: MPC for Rapid Warm-up on FTP Cycle \((w_T = 0, w_p = 50, w_e = 50)\)](image1)

![Figure 4.26: MPC for Rapid Warm-up on FTP Cycle \((w_T = 2000, w_p = 10, w_e = 50)\)](image2)
Since the control inputs in MPC are defined as the deviations from the nominal operating condition, this controller consists of an integrated feedforward and feedback control. The scheduling of nominal control inputs according to operating region is the feedforward portion, and the deviation obtained from MPC is the feedback portion, which are shown in Figure 4.27. It is found that, in Figure 4.9, region 1, 2, 3 stands for deceleration, while region 4 represents idling. Since boost pressure is close to 1 bar and EGR ratio should be kept, the 3 actuators have very limited control authority in these regions. So MPC is not used in those regions, and the feedback portion is set to be 0.

The reduced order PWA observer is applied to estimate the unmeasured states, and
Table 4.3: Performance Indices of MPC

<table>
<thead>
<tr>
<th>$w_T$</th>
<th>$w_p$</th>
<th>$w_e$</th>
<th>$I_T$</th>
<th>$I_p$</th>
<th>$I_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>50</td>
<td>50</td>
<td>0.9932</td>
<td>0.0355</td>
<td>0.0288</td>
</tr>
<tr>
<td>2000</td>
<td>20</td>
<td>50</td>
<td>0.9824</td>
<td>0.0405</td>
<td>0.0310</td>
</tr>
<tr>
<td>2000</td>
<td>10</td>
<td>50</td>
<td>0.9696</td>
<td>0.0670</td>
<td>0.0344</td>
</tr>
<tr>
<td>2000</td>
<td>5</td>
<td>50</td>
<td>0.9568</td>
<td>0.1132</td>
<td>0.0399</td>
</tr>
</tbody>
</table>


![Figure 4.29: Comparison of MPC with Different Weights $w_T = 2000$, $w_e = 50$](image)

Figure 4.29: Comparison of MPC with Different Weights $w_T = 2000$, $w_e = 50$

The results are illustrated in Figure 4.28. It can be seen that the state estimator has high accuracy in the closed-loop system.

To analyze the trade-off between warm-up and boost, the MPC with different cost functions are compared in Figure 4.29 and Table 4.3. As the weight for boost $w_p$ decreases, catalyst warm-up is improved at the expense of boost pressure, while the EGR ratio, as a soft constraint, is roughly maintained at the desired level.

A comparison of feedforward control and MPC is depicted in Figure 4.30. Clearly,
MPC can achieve the same improvement of catalyst warm-up performance as feedforward control, with lower boost pressure and EGR ratio tracking errors. Therefore, a Pareto front is obtained.

4.6.3 Heuristic Control

Since this controller is dedicated for rapid catalyst warm-up during cold start and require large amount memory and computational effort, it is not cost-efficient to implement in production engines. In order to not violate the production engine control for the air-path system, a rule-based heuristic control is developed to only control the bypass opening, which is an “add-on” controller appended to the existing air-path control. From the Pareto front, the case with $w_T = 2000$, $w_p = 5$, $w_e = 50$ is chosen to be the candidate.

From the MPC results, it is found that the bypass opening can be approximated by a linear function of fuel flow rate in each region, as illustrated in Figure 4.31. In region 5, where desired boost pressure is low, the bypass are almost fully open, because the other two actuators can largely compensate the boost pressure. In other regions, bypass is partially open. When fuel injection increases, bypass opening should decrease to meet higher desired boost pressure.
The comparison of heuristic control and MPC is shown in Figure 4.32, Figure 4.33, and Table 4.4. Although it has some deviation from the MPC, the heuristic control can largely improve warm-up performance by penalizing boost, while keeping desired EGR ratio.
### Table 4.4: Comparison of Heuristic Control and MPC

<table>
<thead>
<tr>
<th></th>
<th>Warm-up Time [s]</th>
<th>Final Temp. [°C]</th>
<th>Boost RMS [%]</th>
<th>EGR RMS [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>192.2</td>
<td>212.0</td>
<td>2.6</td>
<td>3.0</td>
</tr>
<tr>
<td>MPC</td>
<td>169.5</td>
<td>233.9</td>
<td>11.3</td>
<td>4.0</td>
</tr>
<tr>
<td>Heuristic</td>
<td>170.6</td>
<td>229.2</td>
<td>9.6</td>
<td>3.4</td>
</tr>
</tbody>
</table>

**Figure 4.33: Comparison of Heuristic Control and MPC**
Chapter 5

Conclusions and Future Work

5.1 Conclusions

This thesis presents two main contributions to the research and development of systematic optimization and control design for downsized boosted engines with advanced charging systems. First, a parametric modeling approach for a family of geometrically similar turbochargers is proposed. Second, a model predictive control approach for a two-stage turbocharged Diesel engine air-path system is designed. The main conclusions of this study are summarized below.

5.1.1 Parametric Model of Turbocharger

Several modeling approaches has been evaluated to characterize the flow and efficiency of families of compressors and turbines in similar, with the objective of defining a fully scalable and parametric model that can represent the performance of a family of turbochargers based on key geometric parameters. From this study, the main conclusions are:

- Upon conducting a comparative analysis of several modeling approaches available in literature, the Jensen-Kristensen model for compressor flow and the Eriksson model for compressor efficiency are selected to develop a parametric compressor model. Specifically, the compressor performance maps (flow and efficiency) are shown to be scalable according to only one geometric parameter, namely the compressor diameter.
Implementation issues related to causality of the compressor performance map is also addressed. Since the compressor is ultimately modeled as a map, the causality can be invertible, assuming either the pressure ratio or the mass flow rate can be used as the input of the sub-model. Using the mass flow rate as the input to the compressor model (hence, calculating the pressure ratio as output) leads to a more stable model for transient simulation. However, coupling such model with the engine air-path system simulator requires the introduction of an additional state variable. Due to the better accuracy in predicting the compressor performance, this approach is ultimately selected.

Simplified surge and choke model is developed for a family of compressors, which represents the limit of compressor operation. The developed model will be used to integrate with compressor flow model to provide important information for system design and control optimization.

A fully parametric model for the turbine flow is also developed, starting from modified orifice flow equation. Based on the geometric similarity principle, a parametric flow model is obtained to characterize a family of fixed geometry turbines. Then, a scalable variable geometry turbine model is developed by representing the set of turbine flow parameters with respect to the VGT opening. Three different variable geometry turbines are considered, and their averaged behavior is assumed for defining a scalable model representing the effects of VGT actuation on turbine flow maps.

A modified power-based turbine efficiency model is obtained, which is decoupled with flow model and independent on speed, and thus can be used in the case the flow model is scaled with geometry. Since the set of turbine efficiency parameters is difficult to fit analytically as function of VGT opening, efficiency at arbitrary VGT opening is computed by interpolation.
To provide a preliminary verification, the parametric modeling approach is applied to a two-stage turbocharger and integrated with a Diesel engine air-path system simulation model, which is validated against steady-state and transient experimental data. The original two-stage turbocharger model, based on look-up tables, is used as benchmark for comparing the new parametric model. Good agreement on the performance of engine and compressors, and improved stability of the simulator demonstrate the advantages of adopting the new modeling approach.

5.1.2 Model Predictive Control of Diesel Engine Air-Path

Starting from the air-path model of a Diesel engine with two-stage turbocharger, an air-path control strategy based on model predictive control is designed, which can achieve rapid catalyst warm-up to improve cold start emission performance. From this study, the main conclusions that have been drawn are:

- Waste-gate thermal model is proposed, which take temperature recovery and heat transfer into account. The fluid temperature at waste-gate outlet is a weighted average of inlet temperature and wall temperature. Assuming no chemical reaction during cold start, a lumped catalyst thermal model is developed based on heat transfer model.

- According to the preliminary analysis on the FTP cycle, opening the HP bypass will improve the warm-up performance but deteriorate boost performance, while opening the LP waste-gate has trivial effect. To prevent large decrease of boost pressure, the maximum bypass opening should be constrained.

- Piecewise affine system is proposed, which is a reasonable approximation of nonlinear plant model and can be used for control and observer design. Starting from 11-state model for engine air-path system, the dynamics of compressor mass flow rates and manifold temperatures can be reduced. Then, the reduced 7th order model is
linearized at 20 operating points. Finally, a discrete-time piecewise affine system with a partition on the engine map is obtained.

- By properly defining the reference signals, the air-path control for rapid warm-up problem is formulated as MPC tracking control problem. Delta input formulation is introduced to address the feedthrough term from inputs to EGR ratio in the PWA system. The MPC design parameter is selected with the consideration of system performance and computational load. To address the discontinuities in control inputs, a moving average filter is proposed.

- The reduced order observer based on PWA system is designed to estimate the unmeasured states by production MAF and MAP sensors. Because catalyst is connected to the rest of the system in a cascade structure, catalyst temperature sensor is required. To minimize the estimation error caused by modeling error, both the error convergence rate and the norm of output injection matrix need to be considered in observer design.

- The developed model predictive controller and observer are implemented on the engine model. Compared to feedforward control strategy with constant bypass opening, MPC can achieve the same improvement of catalyst warm-up performance while keeping higher boost pressure and less deviation from desired EGR ratio. The effects of different weights in cost function are evaluated, and a Pareto front is obtained.

- After selecting the candidate MPC on the Pareto front, a rule-based heuristic control is developed. The bypass opening is controlled as a linear function of fuel rate in each region. The heuristic control is a good approximation of MPC and can be implemented as an “add-on” controller appended to the existing air-path control in production engines.
5.2 Future Work

Leveraging on the knowledge and results obtained in this thesis, the future work of system optimization and control design for Diesel engine with two-stage turbocharger (2ST) will focus on:

- Co-optimization of system design and control for 2ST configurations will be investigated to improve the system performance in steady-state and transient conditions. To enable a coupled and systematic model-based optimization, parametric turbocharger model will be used, which allows to systematically evaluate the influence of design parameters on system performance.

- Following the previous work on air-path control, a control strategy will be developed for Diesel engine with 2ST system with focus on improving robustness and smoothness of response during mode switching. This will be achieved by developing a reference governor to coordinate the control of the bypass valve and VGT to minimize the pressure excursion in the intake manifold.

In the next phase, the model-based synthesis work will be extended and generalized to investigate several advanced boosting technologies, such as electrically assisted turbochargers (EAT), electric superchargers (eBooster), or turbo/supercharging systems. The next phase of this research will focus on:

- Set up the engine in the test cell, and extend the engine air-path model in presence of EAT and eBooster systems.

- System optimization and control design approach will be extended to EAT and eBooster systems configurations, with the consideration of benefits, complexity vs. performance trade-off study.
- Energy-optimal control strategy will be developed for selected electric boosting system while minimizing turbo-lag, preventing compressor surge, and meeting emissions constraints.

This brings us to the conclusion of this thesis, it is hoped that this work will serve as the groundwork for future research in this fascinating area.
References


Appendix A

Nomenclature

A.1 Abbreviations

2ST  two-stage turbocharger
BMI  bilinear matrix inequality
BSFC brake specific fuel consumption
BSR  blade speed ratio
DOC  Diesel oxidation catalyst
DPF  Diesel particulate filter
EAT  electrically assisted turbocharger
EGR  exhaust gas recirculation
FGT  fixed geometry turbine
FTP  federal test procedure
HCCI homogeneous charge compression ignition
HP  high pressure
HPC  high pressure compressor
HPT  high pressure turbine
LNT  lean NO\textsubscript{x} traps
LP  low pressure
LPC  low pressure compressor
LPT  low pressure turbine
LTC  low temperature combustion
LTI  linear time invariant
MAF  mass air flow
MAP  manifold absolute pressure
MPC  model predictive control
MVM  mean-value model
PWA  piecewise affine
QP  quadratic programming
SCR  selective catalytic reduction
SI  spark ignition
VGC  variable geometry compressor
VGT  variable geometry turbine
VVA  variable valve actuation
VVT  variable valve timing

### A.2 Subscripts

- $amb$: ambient
- $b$: baseline
- $bp$: bypass
- $c$: compressor
- $cat$: catalyst
- $cc$: charge cooler
- $corr$: corrected
- $d$: displacement/desired
- $e$: EGR ratio
- $EM$: exhaust manifold
- $egr$: exhaust gas recirculation
- $eng$: engine
- $ex$: upstream of catalyst
- $exh$: upstream of exhaust manifold
- $HP$: high pressure
- $HT$: high pressure turbocharger
- $IM$: intake manifold
- $in$: inlet
- $int$: intermediate volume
- $is$: isentropic
- $LP$: low pressure
- $LT$: low pressure turbocharger
- $out$: outlet
- $p$: boost pressure
- $red$: reduced
- $ref$: reference
- $t$: turbine/throat
- $T$: catalyst temperature
- $tc$: turbocharger
- $thr$: threshold
- $vgt$: variable geometry turbine
- $w$: wall
- $wg$: waste-gate
## A.3 Variables and Constants

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>( A )</td>
<td>( m^2 )</td>
</tr>
<tr>
<td>Air fuel ratio</td>
<td>( AFR )</td>
<td>-</td>
</tr>
<tr>
<td>Blade speed ratio</td>
<td>( BSR )</td>
<td>-</td>
</tr>
<tr>
<td>Speed of gas</td>
<td>( c )</td>
<td>( m/s )</td>
</tr>
<tr>
<td>Specific heat at constant pressure</td>
<td>( c_p )</td>
<td>( J/kg/K )</td>
</tr>
<tr>
<td>Diameter</td>
<td>( D )</td>
<td>( m )</td>
</tr>
<tr>
<td>Specific enthalpy</td>
<td>( h )</td>
<td>( J/kg )</td>
</tr>
<tr>
<td>Heat transfer coefficient</td>
<td>( h )</td>
<td>( W/m^2/K )</td>
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<tr>
<td>Rotational inertia</td>
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<td>Thermal conductivity</td>
<td>( k )</td>
<td>( W/m/K )</td>
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<td>Length</td>
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<td>( m )</td>
</tr>
<tr>
<td>Mass flow rate</td>
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<td>( kg/s )</td>
</tr>
<tr>
<td>Blade Mach number</td>
<td>( M )</td>
<td>-</td>
</tr>
<tr>
<td>Speed</td>
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<td>( rpm )</td>
</tr>
<tr>
<td>Pressure</td>
<td>( p )</td>
<td>( Pa )</td>
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<tr>
<td>Power</td>
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<td>( W )</td>
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<tr>
<td>Prandtl number</td>
<td>( Pr )</td>
<td>-</td>
</tr>
<tr>
<td>Heat transfer rate</td>
<td>( \dot{Q} )</td>
<td>( W )</td>
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<td>Gas constant</td>
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<td>( J/kg/K )</td>
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<td>Reynolds number</td>
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<td>Time</td>
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<td>Volume</td>
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<td>( m^3 )</td>
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<td>Actuator opening</td>
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<td>EGR ratio</td>
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<td>Pressure ratio for compressor</td>
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<td>-</td>
</tr>
<tr>
<td>Pressure ratio for turbine</td>
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</tr>
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</tr>
<tr>
<td>Efficiency</td>
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<td>-</td>
</tr>
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<td>Density</td>
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<td>Volumetric efficiency</td>
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<td>-</td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>( \nu )</td>
<td>( m^2/s )</td>
</tr>
<tr>
<td>Flow parameter</td>
<td>( \Phi )</td>
<td>-</td>
</tr>
<tr>
<td>Head parameter</td>
<td>( \Psi )</td>
<td>-</td>
</tr>
</tbody>
</table>
Control

\begin{itemize}
  \item $e$ estimation error
  \item $N$ predictive horizon
  \item $N_c$ control horizon
  \item $\mathcal{P}$ polytope
  \item $r$ reference signals
  \item $T_s$ sampling time
  \item $u$ control inputs
  \item $v$ external inputs
  \item $x$ system states
  \item $y$ system outputs
\end{itemize}