Essays on the Aggregate and Heterogeneous Effects of Government Spending Shocks

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Rong Li

Graduate Program in Economics

The Ohio State University

2014

Dissertation Committee:

Dr. William Dupor, Advisor

Dr. Paul Evans, Advisor

Dr. Pok-Sang Lam

Dr. Byoung Hoon Seok
Abstract

This study focuses on the effects of unexpected changes in government spending. More specifically, I investigate the quantitative aggregate and heterogeneous effects of government spending shocks as well as the interaction between fiscal and monetary policies.

Existing studies demonstrate that the economy is responsive to unexpected changes in government spending, yet the majority emphasize the aggregate effects and treat government spending as consisting entirely of expenditure on goods. In Chapter 1, I show that distinguishing two components of government spending is important, because the shocks of these two components have opposing effects on the private sector. Quantitatively, a one dollar increase in government purchases on goods and services from the private sector leads to a $1.41 to $1.80 increase in the private sector output. However, a one dollar increase in government purchases on goods and services from the government sector results in a $0.45 to $0.47 loss of the private sector output. The policy implication is that in order to stimulate the private sector output, the government needs to spend money in the private sector instead of the government sector. Moreover, this paper can potentially reconcile the conflicts among different identification strategies regarding the response of private consumption. That is, the response of consumption varies with the structure of the identified government spending shocks.
Chapter 2 studies the heterogeneous effects of government spending shocks, and examines the welfare implications of postwar U.S. government spending shocks. Government spending shocks affect households by creating wealth effects, changing tax rates and changing factor prices. I distinguish between government goods purchases and government employee compensation because shocks to these two components have different impacts on the real interest rate and the real wage rate. Households responses and their welfare gain (or loss) depend on the type of government spending shocks and their wealth. Quantitative investigation is conducted using a calibrated heterogeneous-agents model with incomplete markets. My analysis reveals that the identified postwar U.S. government spending shock has reduced households welfare and amplified inequality.

Chapter 3 revisits the sizes of fiscal multipliers under a pegged nominal interest rate. The short-run effects of policy shocks also depend on expectations about future policy adjustment. I demonstrate how expected government spending reversals alter the impacts of fiscal policy under an interest rate peg. If the monetary-fiscal expansion is associated with an anticipated spending reversal, the output multiplier can be smaller than one and the inflation multiplier is in line with the empirical evidence in the literature.
Dedicated to my mother, Decun Li, and my wife, Xiaohui Tian
Acknowledgments

My deepest gratitude goes to my advisors, William Dupor and Paul Evans, for their valuable guidance, intellectual support, and constant encouragement during the five years. Dr. Dupor’s insights have inspired me throughout my research; Dr. Evans’ encyclopedic knowledge helps me greatly improved the quality of the essays. I would not have completed this dissertation without their supervision. Profound gratitude also goes to my committee members Pok-Sang Lam and Byoung Hoon Seok for their tremendous help during the years. I thank Dr. Lam not only for his invaluable comments and suggestions on my research, but also his warm support during stressful job market times. Dr. Seok’s sharp insights during discussions and seminars have always been great help improving my research and presentation skills.

I am also grateful for Hajime Miyazaki, who has offered me crucial advice during my graduate career. I also thank Jinhui Bai, Yili Chien, Paulina Restrepo-Echavarria, Zhigang Feng, David Wiczer, and Huanxing Yang for their stimulating discussions and valuable suggestions on my research. I would also like to thank seminar participants at the Ohio State University and a number of conference audiences for their many valuable comments. Great thanks also go to my friends at Ohio State, Carol Cui, Xiaoyi Han, Zheng Jiang, Huimin Shi, Chao Yang, Xin Yu, Fang Zhang, Lini Zhang, and many others that I cannot list all I should acknowledge.
I am deeply grateful to my mother, Decun Li, for her continuous love, support, and encouragement for every step in my life. Special thanks are given to my wife, Xiaohui Tian, for her unconditional love and support in countless aspects. She is the best blessing in my life.
Vita

April 18, 1986 .................................. Born – Shenyang, China

2009 .............................................. B.S. Mathematics, Zhejiang University

2010 .............................................. M.A. Economics, The Ohio State University

2010 to present ............................... Graduate Teaching Associate, Department of Economics, The Ohio State University

Fields of Study

Major Field: Economics
**Table of Contents**

Abstract ................................................................................................................................................. ii

Dedication .............................................................................................................................................. iv

Acknowledgments ................................................................................................................................... v

Vita ....................................................................................................................................................... vii

List of Tables ......................................................................................................................................... xi

List of Figures ......................................................................................................................................... xii

Chapter 1: Government Sector and Government Spending Multipliers .................. 1

1.1 Introduction ....................................................................................................................................... 1

1.2 Government Spending Multipliers in an Analytical Model ................................. 4

1.2.1 The two components in government spending ......................................................... 4

1.2.2 An analytical New Keynesian model ........................................................................... 6

1.2.3 Effects of government spending shocks ................................................................. 10

1.3 Identifying Government Spending Shocks ................................................................. 12

1.3.1 An extended version of the narrative approach .................................................... 13

1.4 Government Spending Multipliers in a Standard New Keynesian Model ....... 18
1.4.1 A standard New Keynesian model ........................................................... 18
1.4.2 Minimum distance estimation .............................................................. 25
1.4.3 Quantitative effects of different components in government spending........ 28
1.4.4 Total government spending multiplier .................................................. 30
1.5 Reconciling Conflicts .............................................................................. 32
1.6 Conclusion .............................................................................................. 35

Chapter 2: Government Employment and the Heterogeneous Effects of Government Spending ................................................................. 36

2.1 Introduction ............................................................................................. 36
2.2 Effects of Government Spending Shocks on Prices .................................... 39
2.3 A Simple Static Model ............................................................................ 42
  2.3.1 A shock to government goods purchases ............................................. 45
  2.3.2 A shock to government employment .................................................. 46
2.4 A Quantitative Model with Heterogeneous Agents .................................... 47
  2.4.1 The model economy ........................................................................... 48
  2.4.2 Calibration ......................................................................................... 54
2.5 The Heterogeneous Effects of Government Spending Shocks in the U.S. Economy ........................................................................................................ 57
  2.5.1 The government spending shocks ..................................................... 57
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The decomposition of additional government spending</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>Government Spending Multipliers</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>Estimation Results</td>
<td>28</td>
</tr>
<tr>
<td>4</td>
<td>Multipliers of different component in government spending</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>Total government spending multipliers on private sector</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>Identification and Multipliers</td>
<td>35</td>
</tr>
<tr>
<td>7</td>
<td>Identifying sign restrictions</td>
<td>41</td>
</tr>
<tr>
<td>8</td>
<td>Calibration</td>
<td>55</td>
</tr>
<tr>
<td>9</td>
<td>Transition matrix: working-age to working age (%)</td>
<td>55</td>
</tr>
<tr>
<td>10</td>
<td>labor productivity</td>
<td>56</td>
</tr>
<tr>
<td>11</td>
<td>Wealth distribution (%)</td>
<td>56</td>
</tr>
<tr>
<td>12</td>
<td>Consumption distribution (%)</td>
<td>56</td>
</tr>
<tr>
<td>13</td>
<td>Fiscal multipliers under a deterministic interest rate peg</td>
<td>75</td>
</tr>
<tr>
<td>14</td>
<td>Fiscal multipliers under a stochastic interest rate peg</td>
<td>81</td>
</tr>
<tr>
<td>15</td>
<td>Fiscal multipliers</td>
<td>83</td>
</tr>
</tbody>
</table>
# List of Figures

1. Distinction between government output and consumption .......................... 5
2. The responses to the defense news shock ................................................. 15
3. GDP and private output responses ......................................................... 17
4. Matching the IRFs ................................................................................. 27
5. Effects of different components in government spending .......................... 30
6. Total government spending multipliers on private sector ........................... 31
7. Comparison between Svar and the Narrative Approaches .......................... 34
8. Impulse responses to the government spending shocks .............................. 42
9. Fiscal variables ..................................................................................... 58
10. Impulse Responses .................................................................................. 61
11. Consumption equivalent variations ......................................................... 63
12. Impulse responses of interest rate, wage rate, private labor and capital .... 65
13. Comparison of consumption equivalent variations .................................. 66
14. Responses of inflation, private consumption and output ......................... 74
15. Sensitivity analysis ................................................................................ 76
16. Possible states ..................................................................................... 78
17. Responses of inflation, consumption and output in each possible state .... 79
18  Expected responses of inflation, consumption and output in the stochastic policy experiment ........................................ 80

19  Expected responses of inflation, consumption and output in the experiment with capital ........................................ 83
Chapter 1: Government Sector and the Government Spending Multipliers

1.1 Introduction

This paper studies the effects of shocks to different components in government spending. Many researchers assume that the goods and services purchased by the government are homogenous when estimating the effects of government spending shocks. For example, Blanchard and Perotti (2002), Mountford and Uhlig (2009) and Ramey (2011) identify government spending shocks using different approaches, but they all maintain the above assumption. However, there are two main components in government expenditure: the purchase of goods and services from the private sector, and the purchases of goods and services from the government sector. The latter one is the compensation to general government employees plus the general government consumption of fixed capital. In the National Income and Product Accounts (NIPA) table 1.3.5, this component is classified as the general government sector value added. Moreover, from 1947Q1 to 2012Q4, the average government spending share of purchases from the private sector is 0.40 and the average share of the other component is 0.60. So government expenditure in the government sector is a significant component in government spending. However households do not directly pay for the goods and services from the government sector, implying that the consumption bundles of the government and households are different.

Therefore, without distinguishing the two components of government spending, the
study of the effects of government spending shocks may lead to an inaccurate conclusion. Intuitively, an increase of spending in any component results in negative wealth effects due to the expansion of government’s usage of economy resources. This negative wealth effect will force the households to work more and consume less. However, an increase in purchases of goods from the private sector will expand the hours worked in the private sector while an increase in the demand for government sector goods leads to an increase in government sector hours. More importantly, the latter one will reallocate the labor from the private sector to the government sector due to the decrease in households consumption and hence the decrease in the demand for private sector output.  

To get a more precise analysis of the effects of government spending shocks, I distinguish these two components. I first construct an analytical two-sector New Keynesian model. The model illustrates that an increase in government purchases on private output has a positive impact on the private output, while an increase in government purchases of government output lowers private output.

I then study the quantitative effects of shocks to the two components. First, I identify the shocks to the two components of government spending using an extended version of the narrative approach and defense news series as in Ramey (2011). Second, I construct a standard New Keynesian model with habit formation, sticky prices, capital adjustment costs, and most importantly a distinction between government purchases of private and government output; and estimate the model’s parameters by matching the impulse responses of the model and the VAR. Third, I conduct counterfactual experiments by varying one type of government spending while keeping the other constant to illustrate how the isolated shocks affects the economy. I find that a one dollar increase in government purchases on goods and services from the private

---

1Note that the government demand of private output does not change.
sector leads to a $1.41 to $1.80 increase in the private sector output and a $1.45 to $1.60 increase in private consumption. However, a one dollar increase in government purchases on goods and services from the government sector results in a $0.45 to $0.47 loss of private sector output and lowers private consumption by about $0.02.

Based on the above findings, this paper can potentially reconcile the conflicts regarding the consumption response to a government spending shock. That is, the response of consumption varies with the structure of the identified government spending shock. If the identification method identifies a government spending shock with more money spent in the private sector, it tends to provide a positive impact on consumption, and vice versa. I find that the structural vector autoregression (SVAR) approach identifies a government spending shock with more money spent in the private sector, while the narrative approaches identify government spending shocks with more expenditure in the government sector. Hence the consumption response is positive with the SVAR approach; it is around zero with the Defense News approach; and it is negative with the Ramey and Shapiro (1998) War Dates approach.

The paper is organized as follows. Section 1.2 study the effects of different components in government spending analytically. Section 1.3 identifies the government spending shocks. Section 1.4 uses a New Keynesian model with the capability of matching the impulse responses from the data to study the quantitative effects. Section 1.5 provides empirical evidence on reconciling the conflicts regarding the consumption response to a government spending shocks. Section 1.6 concludes.
1.2 Government Spending Multipliers in an Analytical Model

In this section, I first discuss in detail how to distinguish between the two components of government spending. Then, I construct a simple analytical model to illustrate how different components of government spending affect private sector output.

1.2.1 The two components in government spending

Traditionally, researchers assume that the goods and services purchased by the government and households are homogenous. However, there are two components in government spending. According to the National Income and Product Accounts (NIPA), government purchases include both purchases of goods and services from the private sector and from the government sector. Since households do not directly pay for the goods and services provided by the government sector, the consumption bundles of government and households are different. Figure 1 illustrates the fact. The top panel divides GDP by producer. $Y_2$ represents the output by the government. Examples include defense services, public education services, and water and other sanitation services. In the NIPA 1.3.5, $Y_2$ is the general government sector value added which equals compensation to general government employees plus the general government consumption of fixed capital. $Y_1 = GDP - Y_2$ is the output of private sector. The middle panel divides GDP by purchaser, $G$ is the NIPA category of government purchases of goods and services; the rest of GDP is purchased by households, used as consumption, investments or net exports. The bottom panel combines these two categorizations. As it shows, government purchases $G$ include both the goods and services provided by private sector ($G_1$) and government sector ($G_2$), while house-
holds purchases do not include the goods and services from the government sector. Therefore, the goods and services purchased by the government and households are not homogenous.

Finn (1998) used a dynamic neoclassical model to show that increases in total government spending resulting from an increase in government employment and resulting from an increase in purchases of goods from the private sector have opposite effects on private sector output, employment, and investment. Cavallo (2005) showed that allowing for the distinction between the two main components of government consumption improves the quantitative performance of the neoclassical growth model.
Therefore, the analysis of government spending shocks without distinguishing the two sectors may lead to an inaccurate conclusion. Intuitively, an increase in $G_1$ or $G_2$ both have negative wealth effects because of the expansion of government’s usage of economy resources. This negative wealth effect will encourage the households to work more, and therefore total hours worked rise. However, an increase in $G_1$ will only expand the hours worked in the private sector while an increase in $G_2$ will generally lead to an increase in government sector hours. Moreover, more $G_2$ may also result in a sectoral employment reallocation which moves the labor time from the private sector to the government sector. Different from Finn (1998) and Cavallo (2005), I investigate the effects of shocks to different components in government spending in a New Keynesian economy and provide quantitative analysis.

### 1.2.2 An analytical New Keynesian model

There are two production sectors in the model: a private sector, denoted as Sector 1, and a government sector, denoted as Sector 2. Households consume goods from the private sector while the government purchases goods from both sectors. Moreover, Sector 1 is a monopolistic competitive sector with Calvo type of sticky prices. Sector 2 sells its output to the government at cost. The production functions are $Y_{it} = N_{it}$, for $i = 1, 2$, where $N_{it}$ is the labor in sector $i$. The total labor input is $N_t = N_{1t} + N_{2t}$, and the steady state ratio of $N_i/N$ is equal to $Y_i/Y$.

Households—The economy is populated by a representative household, whose lifetime utility, $U$, is given by

$$U = \sum_{t=0}^{\infty} \beta^t E_0 [C_t^{1-\sigma} - \Psi N_t^{1+\theta} N_t^{1+\theta}]$$

(1)

Here $E_0$ is the conditional expectation operator, and $C_t$ and $N_t$ denote consumption
and hours worked at time $t$, respectively. We assume $\sigma, \theta > 0$ and households do not value public consumption.

The household budget constraint is given by

$$P_tC_t + \frac{B_t}{R_t} = W_tN_t + B_{t-1} + D_t - T_t$$  \hspace{1cm} (2)

where $D_t$ denotes firms’ profits and $T_t$ denotes lump-sum taxes paid to the government. $B_t$ denotes the quantity of one-period bonds purchased by the household at time $t$. $P_t$ denotes the price level and $W_t$ denotes the nominal wage rate. Finally, the variable $R_t$ denotes the gross nominal interest rate in period $t$. The household’s problem is to maximize the lifetime utility given by (1) subject to the budget constraint (2) and the condition

$$E_0 \lim_{t \to \infty} B_t/[R_0R_1 \cdots R_t] \geq 0$$  \hspace{1cm} (3)

Firms.– The final good is produced by competitive firms using the technology

$$Y_{1t} = \left[ \int_0^1 y_{1t}(i) \epsilon^{-\frac{1}{\epsilon}} di \right]^\epsilon, \epsilon > 1$$  \hspace{1cm} (4)

where $y_{1t}(i), i \in [0, 1]$, denotes intermediate good $i$. $\epsilon > 1$ is the demand elasticity of differentiated goods.

Profit maximization implies the following demand function:

$$y_{1t}(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon}Y_{1t}$$  \hspace{1cm} (5)

where $P_t(i)$ denotes the price of intermediate good $i$ and $P_t$ is the price of the homogeneous final good.
The intermediate good \( y_{1t}(i) \), is produced by a monopolist in the private sector using the following production function:

\[
y_{1t}(i) = N_{1t}(i)
\]  

(6)

where \( N_{1t}(i) \) denotes the ith monopolist’s employment. There is no entry or exit into the production and the monopolist is subject to the Calvo-type price-setting. It can optimize its price, \( P_t(i) \), with probability \( 1 - \mu \), otherwise, the firm sets

\[
P_t(i) = P_{t-1}(i)
\]

The discounted profits of the ith intermediate production firm are given by

\[
E_t \sum_{j=0}^{\infty} \beta^{t+j} \lambda_{t+j}[P_{t+j}(i)y_{1t+j}(i) - (1 - v)W_{t+j}N_{1t+j}(i)]
\]  

(7)

as in Christiano et al. (2011), \( v = \frac{1}{\epsilon} \) denotes an employment subsidy that corrects the inefficiency created by the presence of monopoly power, in steady state. \( \lambda_{t+j} \) is the multiplier on the household budget constraint.

Firm \( i \) maximizes (7) subject to the Calvo-type price-setting friction, the demand function (5), and the production function (6).

Monetary policy.—The monetary authority uses the nominal interest rate, \( R_t \) as the instrument to conduct monetary policies. It determines \( R_t \) following a Taylor rule:

\[
R_t = R_{t-1}^{\rho_r} \pi^1_t (1 - \rho_r)^{\phi_1} (Y_{1t} + Y_{2t})^{\phi_2 (1 - \rho_r)}
\]  

(8)

where \( \rho_r \in (0, 1) \). The monetary authority sets the nominal interest rate based on the
last period’s rate, inflation $\pi_t$, output of private sector $Y_{1t}$, and output of government sector $Y_{2t}$.

Fiscal policy.–Government spending evolves according to:

$$G_{it+1} = G_{it}^\rho \exp(\varepsilon_{it+1})$$

for $i = 1, 2$.

$\varepsilon_{it}$ are i.i.d. shocks with zero mean. For simplicity, we assume that government spending and the employment compensation are financed with lump-sum taxes. Because Ricardian equivalence holds in this economy, the exact timing of these taxes is irrelevant.

The resource constraints are:

$$C_t + G_{1t} = Y_{1t}$$

and

$$G_{2t} = Y_{2t} = N_{2t}$$

The labor market clearing condition is

$$N_t = N_{1t} + N_{2t}$$

and $N_{1t} = \int_0^1 N_{1t}(i) \, di$.

Here $G_{it}$ is the level of government spending in sector $i$.

A "monetary equilibrium" is a collection of stochastic processes

$$\{C_t, N_t, N_{1t}, N_{2t}, W_t, P_t, Y_{1t}, Y_{2t}, P_t(i), y_{1t}(i), N_{1t}(i), B_t, \pi_t\}$$
such that for given \( \{G_{1t}, G_{2t} \} \) the household and firm problems are satisfied, the monetary and fiscal policy rules are satisfied, markets clear, and the resource constraints are satisfied.

The equilibrium can be described by the following log-linearized equations

\[
\frac{1}{1 - g_1} \hat{y}_{1t} - \frac{g_1}{1 - g_1} \hat{g}_{1t} = \frac{1}{1 - g_1} E_t \hat{y}_{1t+1} - \frac{g_1}{1 - g_1} E_t \hat{g}_{1t+1} - \frac{1}{\sigma} E_t [\beta (R_{t+1} - r) - \pi_{t+1}] \tag{9}
\]

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa [\sigma (\frac{1}{1 - g_1} \hat{y}_{1t} - \frac{g_1}{1 - g_1} \hat{g}_{1t}) + \theta (\frac{Y_1}{Y} \hat{y}_{1t} + \frac{Y_2}{Y} \hat{g}_{2t})] \tag{10}
\]

\[
R_{t+1} = r + \rho_r (R_t - r) + \frac{1 - \rho_r}{\beta} (\phi_1 \pi_t + \phi_2 (\frac{Y_1}{Y} \hat{y}_{1t} + \frac{Y_2}{Y} \hat{g}_{2t})) \tag{11}
\]

where \( g_{it} \) is the government purchases in sector \( i \) and \( g_i = \frac{G_i}{Y_i} \). \( \kappa \equiv \frac{(1 - \mu)(1 - \beta \mu)}{\mu} \). The first equation is the household’s intertemporal Euler equation. The second equation is the inflation Euler equation, and the third one is the monetary policy. A variable \( \hat{z}_t \) denotes the percentage deviation of \( Z_t \) from its nonstochastic steady-state value.

### 1.2.3 Effects of government spending shocks

I use the above model to study the effects of shocks in different components of government spending. Conjecturing a solution of the form:

\[
\pi_t = \theta_1 \hat{y}_{1t} + \theta_2 \hat{g}_{2t}
\]

\[
\hat{y}_{1t} = \alpha_1 \hat{g}_{1t} + \alpha_2 \hat{g}_{2t}
\]
There is a unique solution of this form, in which

$$\alpha_1 = \frac{(1 - \rho_1) + (\phi_1 - \rho_1) \frac{\kappa}{1 - \beta \rho_1}}{(1 - \rho_1) + \frac{1}{\sigma} \phi_2 \frac{Y_1}{Y} (1 - g_1) + (\phi_1 - \rho_1) \frac{\kappa}{1 - \beta \rho_1} (1 + \frac{1 - g_1}{\sigma} \theta \frac{Y_2}{Y})} \ g_1$$

$$\alpha_2 = (-1) \frac{\phi_2 + (\phi_1 - \rho_2) \frac{\kappa}{1 - \beta \rho_2} \theta}{(1 - \rho_2) + \frac{1}{\sigma} \phi_2 \frac{Y_1}{Y} (1 - g_1) + (\phi_1 - \rho_2) \frac{\kappa}{1 - \beta \rho_2} (1 + \frac{1 - g_1}{\sigma} \theta \frac{Y_2}{Y}) \sigma \ Y} (1 - g_1)$$

Under the calibration that ensures a stationary equilibrium\(^2\), we have $\alpha_1 > 0$ and $\alpha_2 < 0$. Therefore, shocks to $G_{1t}$ have a positive effect on private sector output while shocks to $G_{2t}$ have a negative effect on private sector output. Intuitively, suppose there is an increase in $G_{1t}$ and an unchanged level of $G_{2t}$. Then, there is a negative wealth effect because the government increases the usage of private resources. In response to this negative wealth effect, households increase $N_t$. Because $G_{2t}$ is fixed, $N_{2t}$ is constant. Therefore, the increase in $N_t$ occurs through the increase in $N_{1t}$. Consequently, $Y_{1t}$ increases. On the other hand, consider a positive increase in $G_{2t}$ and a constant level of $G_{1t}$. There is a negative wealth effect as usual. However, the increase in total labor supply $N_t$ occurs through the increase in $N_{2t}$ and the decrease in $N_{1t}$. The reason is that the negative wealth effect asks the households to increase labor supply $N_t$ but to reduce consumption $C_t$. And because $G_{1t}$ is fixed, the overall demand for private sector output $Y_{1t} = C_t + G_{1t}$ falls. Alternatively, from Eq. 10, an increase in $G_{2t}$ pushes up the real marginal cost and inflation. As a result, the nominal interest rate will be higher, then, from Eq. 9, households reduce consumption. Thus $Y_{1t}$ falls. Therefore, the effect of an increase in $G_{2t}$ is negative on private sector output.\(^3\)

\(^2\)A stationary equilibrium requires that $\rho_1 < 1$, $\rho_2 < 1$, $\phi_1 > 1$, and $\phi_2 > 0$.

\(^3\)In the appendix, I discuss the effects on wages and inflation.
I then study the quantitative effects of shocks to the two components on the private sector by the following way. First, I identify the shocks to the two components of government spending and get the impulse responses using an extended version of the narrative approach and defense news series in Ramey (2011). Second, I construct a standard New Keynesian model with a distinction between government purchases of private and government output; and estimate the model’s parameters by matching the impulse responses of the model and the VAR. Third, I conduct counter-factual experiments by varying one type of government spending while keeping the other constant to illustrate how the isolated shocks affect the economy. I also calculate the government spending multipliers on the private output and consumption for each component.

1.3 Identifying Government Spending Shocks

This section identifies the unexpected changes in government spending. I use the defense news series in Ramey (2011) to identify the government spending shocks. I find for each one dollar of total government spending increase, 44 cents are used as the compensation of general government employees or government consumption of fixed capital and 46 cents are spent in purchasing goods or services from the private sector.\footnote{10 cents are missing due to the use of average ratio of $G_i$ to $G$. During the period of 1947-2012, these ratios change appreciably.} In addition, I get the impulse responses of GDP, private output, investment, inflation, and the nominal interest rate which are used as the targets in the estimation.
1.3.1 An extended version of the narrative approach

In this subsection, I use the defense news series to identify government spending shocks and explore how private sector output responds to a defense news shock. Blanchard and Perotti (2002) identify government spending shocks using a Choleski decomposition in which the government spending is ordered first. Ramey (2011) argues that the standard SVAR misses the timing of the shocks, so she uses a narrative approach to identify the government spending shocks to avoid the potential anticipation effects. Following Ramey (2011), I use the defense news variables to identify government spending shocks in the VAR.\(^5\) This is an extended version of the narrative approach, because it measures the government spending changes due to exogenous foreign political events and uses more information. Another advantage of this approach is that it can provide the responses of each component in government spending to a single shock to the defense news, while other methods cannot. The defense news series was constructed by reading periodicals in order to measure the public’s expectations.\(^6\) In this approach, the defense news variable is ordered before other variables in the VAR.

The basic empirical specification is:

\[
A(L)Z_t = C + D_1 t + D_2 t^2 + U_t \tag{12}
\]

where \(Z_t\) is a \(6 \times 1\) vector of variables. The first 5 variables in \(Z_t\) are fixed including the defense news variable, the log of real per capita government spending, the log of real per capita GDP, the 3 month T-bill rate, and the Barro and Redlick (2011) average marginal income tax rate and the 6th in \(Z_t\) is a variable of interest. \(A(L) =\)

\(^5\)I use quarterly data from 1947 to 2008 for the defense news approach. All the data except for the government spending variables are taken from Ramey (2011).

\(^6\)See Ramey (2011) for details.
$A_0 + A_1 L + \ldots + A_4 L^4$, where $L$ is the lag operator. The 3 month T-bill rate and the average marginal income tax rate are used to control for monetary and tax policy.⁷

To the fixed five variables, I rotate a series of the sixth variables. This is a widely used strategy in the literature to avoid using too many variables in the VAR.⁸ The rotated variables are $\ln(G_1)$ government spending on private output, and $\ln(G_2)$ government spending on the government sector output. $G_2$ is taken from NIPA table 1.3.5 which is the gross value added by the general government including compensation of general government employees plus the general government consumption of fixed capital. And $G_1 = G - G_2$, where $G$ is the total government expenditure taken from NIPA table 1.1.5. That is, the government spending on private output is the difference between total government spending and its spending on the government sector. The average $G_1$ share of total government spending is 0.40 and the average $G_2$ share of total government spending is 0.60 in the period of 1947Q1-2012Q4.

Figure 2 shows the responses to a defense news shock. The responses are normalized such that the total government spending’s peak response is equal to one. After a defense news shock, total government spending and each of its components rise, peaking six quarters after the shock and returning to their normal level after 20 quarters. The peak response of $G_1$ is larger than the peak response of $G_2$ as well as $G$, meaning that the government spends more money in the private sector than it normally does. However, it can be seen from Table 1 that there is still a large amount of money that goes to the government sector. In Table 1, I use the cumulative share to explore the decomposition of the additional government spending in response to the shock. The cumulative share is calculated as $\frac{\sum_{j=0}^{20} \partial \ln G_{i,t+j}}{\sum_{j=0}^{20} \partial \ln G_{t+j}} G_t$, where $G_{i,t+j}$ is the government spending in sector $i$ at time $t + j$, $G_t$ is total government spending, $G$.

---

⁷Rossi and Zubairy (2011) emphasize analysis of fiscal policy should always control for monetary policy and vice versa.

⁸See Burnside et al. (2004).
and $\bar{G}_i$ are the average total government spending and its components over the entire time series. The result show, for each one dollar of additional government spending during the “stimulus” period, 44 cents are spent in the government sector and 46 cents are used to stimulate the private economy. That is roughly half of the extra spending goes to the private sector and half goes to the government sector. This finding implies that there is potentially a significant employment reallocation effect caused by the increase in $G_2$ that moves labor to the government sector from the private sector.

**Figure 2:** The responses to the defense news shock

The solid lines are the impulse response functions (IRFs) and the dashed lines are the 95% confidence interval. The peak response of $G_1$ is larger than the peak response of $G_2$ as well as $G$, meaning that government spends more money in the private sector than it normally does.
<table>
<thead>
<tr>
<th>Sector</th>
<th>Government</th>
<th>Private</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share</td>
<td>0.4368</td>
<td>0.4586</td>
</tr>
</tbody>
</table>

Table 1: The decomposition of additional government spending

For each one dollar of additional government spending during the “stimulus” period, 44 cents are spent in the government sector which is roughly the same amount of money used to stimulate the private economy. This finding implies that there is potentially a significant employment reallocation effect caused by the increase in $G_2$ that moves labor to the government sector from the private sector.

I then use the private sector output as the sixth variable in the VAR. Figure 3 shows the responses of GDP and private sector output. The responses are normalized such that the response of government spending is unity. This figure illustrates that the response of private sector output is less than GDP. The reason is that the overall output response includes the response of private sector output as well as the government sector output. The government sector output is compensation of general government employees plus general government consumption of fixed capital. It is actually a component of government spending. The changes of this component should be treated as shocks rather than responses. Therefore, to examine the effects of government spending shocks on output, we should focus on the response of private output.
I calculate two types of government spending multipliers: the peak multiplier and the cumulative multiplier. The peak multiplier is calculated as

\[ \left| \frac{\max_j \partial \ln Y_{i,t+j}}{\max_j \partial \ln G_{t+j}} \bar{Y}_i \right| \text{sign} \left( \frac{\partial \ln Y_{i,t+j}}{\partial \ln G_{t+j}} \right) \]

for \( i = p \) or \( t \), where \( Y_{i,t+j} \) is the output (value added) of private sector (denoted as \( p \)) or total output (denoted as \( t \)) at time \( t + j \), \( G_t \) is government spending, \( \bar{G} \) and \( \bar{Y}_t \) are the average government spending and sector output over the entire time series. The cumulative multiplier, following Drautzburg and Uhlig (2011), is calculated as

\[ \sum_{j=0}^{20} \frac{\partial \ln Y_{i,t+j}}{\partial \ln G_{t+j}} \frac{\bar{Y}_i}{\bar{G}} \].

Table 2 reports the government spending multipliers in private sector and total GDP. Although the total government spending multiplier is large, the multiplier of private sector output is much smaller. Therefore, the commonly used output multiplier overestimates the effects of government spending shocks on the private sector.
1.4 Government Spending Multipliers in A Standard New Keynesian Model

Since the empirical methods cannot isolate the responses of economic variables to a shock of a specific government spending component, I investigate the quantitative effects of each component of the government spending by the following method. I construct a standard New Keynesian model\(^9\) with habit formation, sticky prices, capital adjustment costs and a distinction between government purchases of private and government output. Then, I estimate the model’s parameters by matching the impulse responses of the model and the VAR. This step proves the capability of the model in matching the data. Finally, I conduct counter-factual experiments by varying one type of government spending each time to illustrate how the isolated shocks affect the economy. I also calculate the government spending multipliers for each component.

1.4.1 A standard New Keynesian model

There are two production sectors: a private sector, denoted as Sector 1, and a government sector, denoted as Sector 2. Households consumption comes only from the private sector, while government spends money on the goods and services from both

\(^9\)See Dupor et al. (2009).
sectors.

1.4.1.1 Households

A representative household maximizes its utility by choice of consumption $C_t$, nominal bonds $B_t$, and labor supply $N_t$. It maximizes:

$$\sum_{t=0}^{\infty} \beta^t E_0 \left[ \frac{(C_t - hX_{t-1})^{1-\sigma}}{1-\sigma} - \Psi_N N_{t}^{1+\theta} \right]$$  \hspace{1cm} (13)$$

subject to:

$$C_t + I_t + \frac{B_t}{P_t R_t} = \frac{W_t}{P_t} N_t + r^K_t K_t + \frac{B_{t-1}}{P_t} + \frac{D_t}{P_t} + \frac{T_t}{P_t}$$  \hspace{1cm} (14)$$

where, $P_t$, $W_t$, $D_t$, and $T_t$ are the nominal prices of private sector goods, the nominal wage level, nominal profits of private firms and a nominal lump sum tax/transfer from the government, respectively. $C_t$, $I_t$, and $K_t$ represent consumption, investment and the capital stock, respectively. $r^K_t$ is the real rental rate of capital. $h$ is the parameter of habit persistence; $\sigma$ is the inverse of intertemporal elasticity of substitution; $\theta$ is the inverse of the Frisch elasticity; and $\Psi_N$ denotes the disutility of labor.

The term $X_t$ is the level of habit accumulated by the household. Its law of motion is:

$$X_t = C_t + (1 - \delta)X_{t-1}$$  \hspace{1cm} (15)$$

where $\delta > 0$ is the depreciation rate of habit stock. In the above setting, let $\beta = 0.99$, $\sigma, \theta, \Psi_N > 0$, and $h, \delta \in (0,1)$. 

19
The capital accumulation equation is

\[ K_{t+1} = I_t + (1 - \delta_k)K_t - \frac{\sigma_t}{2} \left( \frac{I_t}{K_t} - \delta_k \right)^2 K_t \]  

(16)

The parameter \( \sigma_t \geq 0 \) governs the magnitude of capital adjustment costs and \( \delta_k \) is the depreciation rate of capital.

The household’s problem is to maximize lifetime expected utility (13), subject to (14), (15), (16) and (3).

### 1.4.1.2 Private Firms

In the private sector, there are final-good producers and a continuum of monopolistic competitive firms over \([0, 1]\). The problem of the final-good producers is the same as in Section 2. The discounted profits of the ith intermediate good producer are given by

\[
E_t \sum_{j=0}^{\infty} \beta^{t+j} \lambda_{t+j}[P_{t+j}(i)y_{1t+j}(i) - (1 - v)(W_{t+j}N_{1t+j}(i) + P_{t+j}r_{t+j}^kK_{t+j}(i))] 
\]  

(17)

The production function for firm i’s good is:

\[ y_{1t}(i) = [K_t(i)]^\alpha [N_{1t}(i)]^{(1-\alpha)} \]  

(18)

where \( N_{1t}(i) \) and \( K_t(i) \) denote the labor and capital input by the ith intermediate firm.

Each firm in the private sector optimally resets its price with probability \( 1 - \mu \) in each period. Let the optimally chosen price at \( t \) to be \( P_t^* \). If a firm optimized its price at
time $t$, then its time $t + j$ price is given by:

$$P_{t,j}^* = P_t^* \left( \frac{P_{t+j-1}}{P_{t-1}} \right)^\gamma$$

(19)

where, $\gamma \in [0, 1]$ is the degree of price indexation. If a firm has the opportunity to optimize its price at $t$, it maximizes the expected discounted value of real profits (17) subject to the production function (18), the demand curve (5), and the Calvo-type price-setting friction.

### 1.4.1.3 Government

The government in this model has two roles: a purchaser and a producer.

As a producer, it hires labor and produces output which is only purchased by the government. The production function of government sector is $Y_{2t} = N_{2t}$. Government purchases the output of this sector at cost, $W_t N_{2t}$.

As a purchaser, the government not only buys goods from the government sector, but also purchases goods and services from the private sector at the market price. All of the government spending is financed by the lump-sum taxes, $P_t G_{1t} + P_{2t} G_{2t} = T_t$.

Following Edelberg et al. (1999), I assume that the log-deviation of $G_{1t}$ and $G_{2t}$ have finite ordered $ARMA(p, q)$ representations, for $i = 1, 2$:

$$A_i(L) \hat{g}_{it} = B_i(L) \epsilon_{it}$$

(20)

where $A_i(L)$ and $B_i(L)$ are finite ordered polynomials in non-negative powers of the lag operator $L$. I assume $A_i(L)$ have only roots that lie outside the unit circle, and $\epsilon_{it}$ are iid shocks that are orthogonal to all model variables dated time $t - 1$ and earlier.

\[ ^{10} \text{Note } P_{2t} = W_t \]
1.4.1.4 Monetary Policy

The monetary policy is the same as in Section 2. As a reminder, the monetary authority uses the nominal interest rate, $R_t$ as the instrument to conduct monetary policies. It determines $R_t$ following a Taylor rule:

$$R_t = R_{t-1}^{\rho_r} \pi_t^{\phi_1(1-\rho_r)} (Y_{1t} + Y_{2t})^{\phi_2(1-\rho_r)}$$

where $\rho_r \in (0, 1)$. The monetary authority sets the nominal interest rate based on the last period’s rate, inflation $\pi_t$, output of private sector $Y_{1t}$, and output of government sector $Y_{2t}$.

1.4.1.5 Market Clearing

There are two goods market clearing conditions.

$$C_t + I_t + G_{1t} = Y_{1t}$$

and

$$G_{2t} = Y_{2t}$$

The labor market clearing conditions:

$$N_t = N_{1t} + N_{2t}$$

$$N_{1t} = \int_0^1 N_{1t}(i) di$$

and the bond market clears.
1.4.1.6 Equilibrium

A monetary equilibrium is a set of stochastic processes, \{C_t, I_t, N_t, W_t, P_t, Y_{1t}, Y_{2t}, N_{1t}, N_{2t}, K_t, r_t^k, R_t, B_t, T_t, \pi_t, P_t(i), y_{1t}(i), N_{1t}(i), K_t(i)\} such that for given \{G_{1t}, G_{2t}\} the households and firms problem are satisfied, the monetary and fiscal policy rules are satisfied, markets clear and the aggregate resource constraint holds.

1.4.1.7 Summary of Dynamic Equilibrium Model

The equilibrium of the model can be described by the following log-linearized equations.

\[
\frac{-\sigma}{\delta - h} (\delta \hat{c}_t - h \hat{x}_{t-1}) - (1 - \frac{\beta h}{1 - \beta(1 - \delta)}) \lambda_t - \frac{\beta h}{1 - \beta(1 - \delta)} \mu_t = 0
\]  
\tag{21}

\[
\frac{\sigma(1 - \beta(1 - \delta))}{\delta - h} (\delta \hat{c}_{t+1} - h \hat{x}_t) + \mu_t = \beta(1 - \delta) \mu_{t+1}
\]  
\tag{22}

\[
\hat{x}_t = (1 - \delta) \hat{x}_{t-1} + \delta \hat{c}_t
\]  
\tag{23}

\[
r_t - E_t \pi_{t+1} = \lambda_t - E_t \lambda_{t+1}
\]  
\tag{24}

\[
\lambda_t = \varphi_t - \sigma I_k (\hat{i}_t - \hat{k}_t)
\]  
\tag{25}

\[
\varphi_t = [1 - \beta(1 - \delta_k)](\lambda_{t+1} + r_{t+1}^k) + \beta(1 - \delta_k) \varphi_{t+1} + \beta \sigma I_k \delta_k^2 (\hat{i}_{t+1} - \hat{k}_{t+1})
\]  
\tag{26}
\[ \dot{k}_{t+1} = \delta_k \dot{k}_t + (1 - \delta_k) \dot{k}_t \tag{27} \]

\[ \dot{y}_{1t} = \alpha_k \dot{k}_t + (1 - \alpha) \dot{n}_{1t} \tag{28} \]

\[ \pi_t - \gamma \pi_{t-1} = \beta (E_t \pi_{t+1} - \gamma \pi_t) + \kappa [(1 - \alpha) (\theta \dot{n}_t - \lambda_t) + \alpha \dot{r}_t^k] \tag{29} \]

\[ \dot{r}_t^k = \theta \dot{n}_t - \lambda_t - \dot{k}_t + \dot{n}_{1t} \tag{30} \]

\[ r_t = \rho_r r_{t-1} + (1 - \rho_r)(\phi_1 \pi_t + \phi_2 (\frac{Y_1}{Y} \dot{y}_1 + \frac{Y_2}{Y} \dot{g}_{2t})) \tag{31} \]

\[ \dot{n}_{2t} = \dot{g}_{2t} \tag{32} \]

\[ \dot{n}_t = \frac{N_1}{N} \dot{n}_{1t} + \frac{N_2}{N} \dot{n}_{2t} \tag{33} \]

\[ \dot{y}_{1t} = \frac{C}{Y_1} \dot{c}_t + \frac{I}{Y_1} \dot{i}_t + \frac{G_1}{Y_1} \dot{g}_{1t} \tag{34} \]

where \( \lambda_t \) is log of the multiplier on household's budget constraint; \( \mu_t \) is log of the multiplier on habit equation; and \( \varphi_t \) is log of the multiplier on the capital formation equation. \( \kappa \equiv \frac{(1 - \nu)(1 - \beta \nu)}{\nu} \).

Eq. (21) is the first order condition with respect to current consumption. Eq. (22) is the first order condition with respect to \( x_t \). Eq. (23) and (27) are the accumulations of
habit stock and capital, respectively. Eq. (24) characterizes the optimal consumption-savings by households given the expectation of the real interest rate. Eq. (25) and (26) represent the first order condition with respect to \( i_t \) and \( k_{t+1} \), respectively. Eq. (28) is the production function of intermediate goods. Eq. (29) is the private firms’ optimal pricing behavior. Eq. (30) is the capital rental rate from intermediate good producer’s cost minimization problem. The monetary policy is given by Eq. (31). The government sector production is given by (32). Labor market and goods market clearing conditions are (33) and (34), respectively.

1.4.2 Minimum distance estimation

In this subsection, I discuss the methodology for estimating and evaluating my model. I partition the model parameters into two groups. The first group contains \( \beta, \alpha, \) and \( \delta_k \). \( \beta \) is calibrated as 0.99, which corresponds to a steady-state annualized real interest rate of 4 percent. I set \( \alpha = 0.36 \), which implies a steady-state share of capital income roughly equal to 36 percent. \( \delta_k \) is set to be 0.02, which implies an annual capital depreciation rate of 8 percent. The second group of parameters is \( \theta \equiv \{ \sigma, \theta, \kappa, h, \delta, \gamma, \sigma_I, \rho_r, \phi_1, \phi_2 \} \), I estimate these parameters\(^{11}\) by the minimum distance method. The minimum distance method is well known in the literature,\(^{12}\) which provides the estimates by minimizing a measure of distance between the impulse responses generated by the economic model and those estimated. The objective function is defined as:

\[
D = \min_{\theta} [\gamma(\theta) - \gamma^s] \Omega^{-1} [\gamma(\theta_0) - \gamma^s]
\]

\(^{11}\)I ignore the ZLB case in the estimation because it happens rarely in history.
\(^{12}\)See, for example, Dupor et al. (2009).
where $\theta = \{\ldots\}$ is the “deep” parameter vector. $\gamma^s$ is the impulse response function from the VAR model and $\gamma(\theta)$ is the impulse response function from the theoretical model. I use the impulse responses of private fixed investment, private sector outputs, inflation and the 3-month T-bill rates as my targets. The exogenous shocks used in the model are the responses of $G_{1t}$ and $G_{2t}$ taken from the VAR. $\Omega$ is the weighting matrix. Following Christiano et al. (2005), I set $\Omega$ to be a diagonal matrix with the sample variance of $\gamma^s$ on the main diagonal. The estimates, $\hat{\theta}$, is given by minimizing the objective function.

To simulate the model economy, I adopt Edelberg et al. (1999) parametrization of the government spending. That is: $A_i(L) = 1$ and $B_{ij}$ are the estimated response of real government purchases at $t + j$ to the defense news shocks at time $t$, for $i = 1, 2$ and $j = 1, 2, \ldots, 20$. The shocks are one time shocks to the two components of government spending at time $t = 1$ with a size of unity. The top two graphs of Figure 4 illustrate how government spending respond to these shocks, which are also depicted in Figure 3.

The solid lines in figure 4 reports the responses of investment, private sector output, inflation, and nominal interest rate to the defense news shock respectively, and the dotted lines are the 95% CI. The dash lines are the impulse responses generated by the two sector model given the exogenous government spending series. In general, the IRFs from the model lie in the 95% CI of the IRFs from the data.

Table 3 provides the point estimates and standard deviations.
Figure 4: Matching the IRFs

The solid lines report the responses of investment, private sector output, inflation, and nominal interest rate to the defense news shock respectively, and the dot lines are the 95% CI. The dash lines are the impulse responses generated by the two sector model given the exogenous government spending series. In general, the IRFs from the model lie in the 95% CI of the IRFs from the data.
### Parameter Meaning | DSGE estimation
--- | ---
$\hat{\sigma}$ | Inverse of the IES | 1.4464 (0.0061)
$\hat{\theta}$ | Inverse of the Frisch elasticity | 2.0583 (0.1740)
$\hat{\kappa}$ | Slope of NKPC | 0.1128 (0.0006)
$\hat{h}$ | Habit persistence | 0.7812 (0.0015)
$\hat{\delta}$ | Depreciation rate of habit stock | 0.5208 (0.0005)
$\hat{\gamma}$ | Degree of price indexation | 0.3836 (0.0053)
$\hat{\rho}_r$ | Smoothing coefficient in Taylor rule | 0.7496 (0.0020)
$\hat{\phi}_1$ | Response to inflation in monetary policy | 1.2538 (0.0062)
$\hat{\phi}_2$ | Response to output in monetary policy | 0.1544 (0.0013)
$\hat{\sigma}_I$ | Magnitude of adjustment costs | 10.1451 (0.1305)

Table 3: Estimation Results

**1.4.3 Quantitative effects of different components in government spending**

Given the point estimation of the model, I therefore conduct counter-factual experiments to explore the quantitative effects of different components in government spending.

First, I shut down the shocks to $G_{2t}$ and simulate the model using shocks to $G_{1t}$ only. I then shut down the shocks of $G_{1t}$ and simulate the model using shocks to $G_{2t}$ only. Figure 5 illustrates the simulations. The dot-dash line is the response with the shock in $G_{1t}$ only; the dash line is the response with the shock in $G_{2t}$ only; the solid line is the response with two shocks simultaneously. The upper panels illustrate that the $G_{1t}$ shock has positive effect on private sector output and hours worked, while the shock on $G_{2t}$ has negative impact on private sector output and hours worked. In response to the negative wealth effect caused by an increase in
either $G_{1t}$ or $G_{2t}$, households increase $N_t$ and decrease consumption. With shock in $G_{1t}$ only, $G_{2t}$ is fixed and $N_{2t}$ is constant. Therefore, the increase in $N_t$ occurs through the increase in $N_{1t}$. Consequently, $Y_{1t}$ increases. On the other hand, with shocks in $G_{2t}$ only, the increase in total labor supply $N_t$ occurs through the increase in $N_{2t}$ and the decrease in $N_{1t}$. The reason is that the negative wealth effect requires the households to increase labor supply $N_t$ but reduce consumption $C_t$. Because $G_{1t}$ is fixed, the overall demand of private sector output $Y_{1t} = C_t + G_{1t}$ falls. As a result, $N_{1t}$ decreases. Therefore, the shocks to $G_{2t}$ reallocates labor from the private sector to the government sector and lower the private sector output. The bottom panels show the responses of consumption and investment. An increase in $G_{1t}$ leads to an increase in private consumption, while a positive shock in $G_{2t}$ has no significant effect on private consumption. Both shocks reduce private investment.

I also calculate the implied multipliers which is reported in Table 4. The multipliers show that one dollar increase in the government purchases of goods and services from private sector leads to a $1.41$ to $1.80$ increase in private sector output and a $1.45$ to $1.60$ increase in consumption. However, one dollar increase in the government purchases of goods and services from government sector results in a $0.45$ to $0.47$ loss of private sector output and about $0.02$ loss of private consumption. Empirical evidence shows that a significant part of the unexpected changes in total government spending is used in the government sector output. Therefore, the overall multiplier of government spending shocks on private sector output can be small. The policy implication is that to stimulate private sector output, government needs to spend money in the private sector instead of the government sector.
Figure 5: Effects of different components in government spending

The dot-dash lines are the responses with the shock in $G_{1t}$ only; the dash lines are the responses with the shock in $G_{2t}$ only; the blue solid lines are the responses with two shocks simultaneously. This figure shows that the $G_{1t}$ shock has positive effect on private sector output, while the shock on $G_{2t}$ has negative effect on private sector output.

<table>
<thead>
<tr>
<th></th>
<th>Private Output Multipliers of:</th>
<th>Consumption Multipliers of:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_{1t}$</td>
<td>$G_{2t}$</td>
</tr>
<tr>
<td>Peak</td>
<td>1.4149</td>
<td>-0.4489</td>
</tr>
<tr>
<td>Cumulative</td>
<td>1.7990</td>
<td>-0.4736</td>
</tr>
</tbody>
</table>

Table 4: Multipliers of different component in government spending

1.4.4 Total government spending multiplier

I redistribute the series of $G_t$ in the VAR between the private and government sectors. That is, denote $\theta_g \in [0, 1]$ as the percentage of government spending shock spent
in the private sector. I then simulate the model economy using \( G'_{1t} = \theta_g G_t \) and \( G'_{2t} = (1 - \theta_g) G_t \) as the shocks.

Figure 6 and Table 5 show the total government spending multipliers as a function of the percentage of government spending shock spent in the private sector. Both the peak and cumulative multipliers increase as more unexpected change of total government spending are spent in the private sector. This confirms the finding that the government spending shocks that happen in the private sector have positive effects on the private output, while the effect of government spending shocks in the government sector is negative.

![Figure 6: Total government spending multipliers on private sector](image)

The total government spending multipliers as a function of the percentage of government spending shock spent in the private sector. The upper panel is the peak multiplier, and the bottom panel is the cumulative multiplier. The more spent in the private sector, the larger the total government spending multipliers on private sector.
<table>
<thead>
<tr>
<th>percentage spent in private sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>Peak</td>
</tr>
<tr>
<td>Cumulative</td>
</tr>
</tbody>
</table>

Table 5: Total government spending multipliers on private sector

### 1.5 Reconciling Conflicts

In the literature, there is no consensus regarding the effects of government spending shocks on private consumption. The SVAR approach\(^\text{13}\) developed by Blanchard and Perotti (2002) finds that a positive government spending shock raises consumption. In contrast, the narrative approaches developed in Ramey and Shapiro (1998) and Ramey (2011) typically find that government spending lowers consumption. Based on the above findings, this paper can potentially reconcile the conflicts regarding the response of private consumption among different identification strategies. That is, the response of consumption varies with the structure of the identified government spending shock. If a method identifies a government spending shock with more money spent in the private sector, it tends to provide a positive impact on consumption, and vice versa. In this section, I provide empirical evidence to support this finding.

I conduct three VAR exercises using different identification methods: the SVAR approach, the Ramey and Shapiro (1998) war dates approach, and the Ramey (2011) defense news approach. The fixed variables in the VAR are: Ramey (2011) defense news series or war dates\(^\text{14}\), real per capita government spending, real per capita GDP, total hours worked, real private fixed investment, the Barro and Redlick (2011) aver-

\(^{13}\)The SVAR approach is a widely used method in studying the government spending shocks. Blanchard and Perotti (2002) argue that government spending does not contemporaneously respond to total output. Thus, the identification of government spending shocks is identical to a Choleski decomposition where government spending is ordered first.

\(^{14}\)The SVAR approach does not use this type of variable.
age marginal income tax rate, and nominal compensation in private business divided by the deflator in private business. I rotate real non-durable and service consumption and total government hours worked\textsuperscript{15} as the last variable. All variables are in log term and the data is taken from Ramey (2011). These VAR exercises illustrate the response of private consumption as well as the structure of the identified government spending shocks. Figure 7 compares the impulse responses from the VARs. The responses are normalized such that the total government spending’s peak response is equal to one. As shown, in response to a defense related shock, the government increases government hours worked by more; while the SVAR approach identifies a much smaller increase in government hours. In other words, the SVAR approach identifies a government spending shock with more money spent in the private sector, while the narrative approaches identify government spending shocks with more expenditure in the government sector. This result is natural. The narrative approach shows how the government responds to a (or a potential) war, hence it is natural to see the government increase its hours worked, especially military hours. On the other hand, the consumption response is slightly positive or around zero with the SVAR approach; it is around zero or negative with the Defense News approach; and it is negative with the War Dates approach. This result is consistent with the findings in the above sections. That is, an increase in government purchases in the private sector can lead to an increase in private consumption, while an increase in government purchases in the government sector will lower private consumption. In sum, since the narrative approaches identify government spending shocks with more expenditure in the government sector, it typically finds a negative response of private consumption. Because the SVAR approach identifies a government spending shock with less money spent in the government sector, it finds an increase in private consumption.

\textsuperscript{15}The data of government hours worked including the armed force is the Francis-Ramey Updates taken from Ramey’s website.
Figure 7: Comparison between Svar and the Narrative Approaches

This figure compares the impulse responses identified by the SVAR approach and the Narrative approaches (with defense news series or war dates). The middle panel shows that the narrative identification approaches identify government spending shocks with larger increases in government hours. The bottom panel shows that the private consumption increases less with more increase in government hours.

Table 6 provides the peak and cumulative consumption multipliers of these three approaches. It confirms that the private consumption multipliers become smaller when the government hours worked variable increase more.
Table 6: Identification and Multipliers

<table>
<thead>
<tr>
<th></th>
<th>Gov hours elasticity</th>
<th>Private consumption multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Svar</td>
<td>Defense News</td>
</tr>
<tr>
<td>Peak</td>
<td>0.4317</td>
<td>0.7835</td>
</tr>
<tr>
<td>Cumulative</td>
<td>0.2924</td>
<td>0.7195</td>
</tr>
</tbody>
</table>

1.6 Conclusion

This paper studies the effects of shocks to different components in government spending. Shocks to the two components of government spending have opposite effects on the private sector. A one dollar increase in the government purchases of goods and services from the private sector leads to a $1.41 to $1.80 increase in the private sector output. However, a one dollar increase in the government purchases of goods and services from the government sector results in a $0.45 to $0.47 loss of the private sector output. Moreover, this paper can potentially reconcile the conflicts regarding the consumption responses among different identification strategies. That is, the response of consumption varies with the structure of the identified government spending shocks. The policy implication is that to stimulate the private sector output, the government needs to spend money in the private sector instead of the government sector.
Chapter 2: Government Employment and the Heterogeneous Effects of Government Spending

2.1 Introduction

Understanding how the economy responds to government spending shocks is important, since government spending is often used as a policy tool to smooth economic fluctuations. For example, the U.S. government announced the American Recovery and Reinvestment Act in February 2009 with the goal of combating the recent recession. However, there is a debate regarding the real effect of fiscal stimulus. Empirical studies such as Blanchard and Perotti (2002) and Nakamura and Steinsson (2011) argue that the government spending multiplier is greater than one. Eggertsson (2010), Christiano et al. (2011) and Woodford (2011) suggest that the government need to conduct counter-cyclical government spending policies to smooth the output fluctuations, especially when the nominal interest rate is constrained by the zero lower bound. In contrast, Edelberg et al. (1999), Barro and Redlick (2011), Ramey (2011), Drautzburg and Uhlig (2011) and Conley and Dupor (2013) show that government spending has negative or non-significant effects on private consumption or employment. Most of the existing studies have focused on the aggregate effects. On the other hand, empirical studies such as Anderson et al. (2012) and De Giorgi and Gambetti (2012) find that government spending shocks have substantially different effects on heterogeneous consumers. Moreover, Heathcote (2005) and Kaplan and Violante (2011) demonstrate that heterogeneity can be particularly relevant for fiscal policies.
Bachmann et al. (2013) show that the welfare gain of eliminating fiscal uncertainty is unevenly distributed among households.

This paper contributes to the literature on studying the heterogeneous effects of government spending shocks, taking into explicit account the role of government expenditure on employment. There are two channels that a government spending shock could affect households. An extended government expenditure requires a higher contemporaneous or future tax rate which creates negative wealth effects as well as substitution effects. In addition, the change in government spending would alter factor prices, i.e. the real interest rate and the real wage rate, which creates another channel to influence households. Moreover, this general equilibrium channel would have uneven impact on heterogeneous households. For example, an increase in government goods purchases pushes up the real interest rate while driving down the real wage rate. A lower wage may hurt households who rely on labor income, while a higher interest rate would benefit the households who have large capital incomes. In the empirical section, I examine the effects of government spending shocks on factor prices. The identification method applied is an agnostic approach of imposing sign restrictions on impulse response functions. In another aspect, Finn (1998), Cavallo (2005) and Li (2014) find that distinguishing between the wage and salary component versus expenditures on goods is important, because shocks to these two components can lead to very different aggregate effects. Hence, I distinguish the government employment compensation from its goods purchases in my empirical study. This analysis yields two important findings. First, shocks to government goods purchases and government employment have opposite effects on real wages. A positive government goods purchases shock would reduce real wages while a positive government employment shock tends to increase real wages on impact and in the medium term. Second, the government goods purchases shock has positive effects on the real interest rate in the
medium and long term while it has a negative effect initially. On the other hand, the government employment shock reduces the real interest rate.

The empirical findings encourage me to distinguish the two components in government spending in the theoretical models. In theoretical analysis, I also find that the changes in factor prices represent a channel through which a government spending shock can affect households unequally. Specifically, a shock to government goods purchases increases the demand of private sector output and therefore the labor demand in private sector. On the supply side, the negative wealth effect causes households to increase labor supply. As a result, private sector labor increases in equilibrium which leads to a reduction in wage rates and an increase in interest rates. The overall effect of this shock on households then depends on their wealth. Since high income households have relatively more capital incomes, the negative wealth effect is partially offset by the increase of the interest rate. On the other hand, the main source of income for lower income households is labor income, so the negative wealth effect is amplified by the decrease of the wage rate. On the contrary, a positive shock to government employment decreases the equilibrium labor in the private sector. In other words, a shock to government employment will reallocate labor from the private sector to the government sector. As a result, the equilibrium wage rate goes up while the interest rate goes down. The overall negative wealth effect is amplified on households whose main income source is capital income, while it is partially offset on households who rely on labor income. Accordingly, by distinguishing between the ways government spends, I am able to provide a more precise approach to evaluate government spending policies.

I quantitatively examine the heterogeneous effects and welfare implications of government spending shocks using a calibrated version of the stochastic growth model with heterogeneous agents and incomplete markets. In the model, there are a large number
of households who face idiosyncratic labor productivity shocks as well as aggregate productivity and government spending shocks. Following Castaneda et al. (2003), I calibrate the model such that its steady state values match the corresponding statistics in the data, including the wealth distribution. Then, I apply Ramey (2011)’s narrative approach to identify US government spending shocks and use the impulse responses of government goods purchases, government employment, and marginal income tax rate as the shocks of my model. I evaluate the aggregate and heterogeneous impacts of the identified government spending shocks using the general equilibrium model. In addition, the welfare implications to heterogeneous households are calculated. Finally, I illustrate the importance of distinguishing government employment from government goods purchases by comparing the welfare implications under different policy shocks.

The rest of this paper is organized as follows. Section 2.2 provides empirical evidence. Section 2.3 discusses the main mechanism of how government spending shocks affect different households with the help of a simple static heterogeneous agents model. Section 2.4 describes the full model and calibration. The main results and policy experiments appear in Section 2.5. Section 2.6 concludes.

### 2.2 Effects of Government Spending Shocks on Prices

The majority of the literature that studies the effects of government spending shocks overlook the employee compensation component, treating government spending as consisting entirely of goods purchases. However, in this section, I show that shocks to government goods purchases and employment have significantly different effects on interest rates and wage rates.

To identify government spending shocks, I use the method of imposing sign restric-
tions on impulse response functions, which was first developed in Uhlig (2005) and then extended by Mountford and Uhlig (2009). Specifically, I identify a government goods purchases shock as well as a government employment shock by imposing a positive reaction of impulse response of the appropriate government spending variable for quarters $k = 0, 1, 2, 3$ following the shock\(^{16}\) and imposing orthogonality to a business cycle shock as well as a monetary policy shock. The business cycle and monetary policy shocks are also identified with sign restrictions. The VAR has nine fixed variables including government revenue, real GDP, real wages, ex post real interest rate, adjusted reserves, the GDP deflator, the producer price index for crude materials, real private consumption and real private non-residential investment. The ex post real interest rate is constructed using the federal funds rate and CPI inflation. To the nine fixed variables, I rotate two fiscal variables: government goods purchases and government employment\(^{17}\). The VAR consists of these ten variables at a quarterly frequency from 1954Q3 to 2012Q4. It has six lags and uses the logarithm for all variables, except the real interest rate which enters in its level. The identifying sign restrictions on the impulse responses appear in Table 7. Instead of simultaneously identifying all three shocks subject to the orthogonality conditions, I identify the shocks sequentially. First, a business cycle shock is defined as a shock which jointly moves GDP, private consumption, private non-residential investment and government revenue in the same direction for four quarters following the shock. Second, a monetary policy shock moves ex post real interest rates up and reserves as well as prices down for four quarters after the shock. This is consistent with Mountford and Uhlig (2009), which imposes that the nominal interest rate rises and prices fall. The monetary policy shock is also required to be orthogonal to the business cycle shock.

\(^{16}\) The purpose of these tight restrictions is to rule out very transitory shocks to fiscal variables.

\(^{17}\) The data source of government employment is from Francis-Ramey updates in Ramey’s website. More about data is in Appendix B.3
The primary purpose of identifying the business cycle and monetary policy shocks is to filter out the effects of these shocks on the government spending variables. Finally, government spending shocks are identified only by restricting the impulse responses of the appropriate spending variable and the requirement that they are orthogonal to the business cycle and monetary policy shocks. I separately identify a government goods purchases shock and a government employment shock.

<table>
<thead>
<tr>
<th>fiscal variable</th>
<th>G. Revenue</th>
<th>GDP, cons, nonres.inv.</th>
<th>interest rate</th>
<th>Adj. res.</th>
<th>Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Busi. cycle</td>
<td>+</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monetary</td>
<td></td>
<td></td>
<td>+</td>
<td></td>
<td>−</td>
</tr>
<tr>
<td>G. goods pur.</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td>−</td>
</tr>
<tr>
<td>Gov. Emp.</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td>−</td>
</tr>
</tbody>
</table>

Table 7: Identifying sign restrictions

The sign restrictions on the impulse responses for each shock. A “+” means that the impulse response of the variable is restricted to be positive for four quarters. A “−” means a negative response for four quarters. Blank entries indicate no restrictions are imposed.

The impulse responses to a government goods purchases shock are shown in the left column of Figure 8 and the impulse responses to a government employment shock are displayed in the right column. These two shocks lead to substantially different consequences in factor prices. First, the real wage responds negatively to the goods purchases shock while it rises in response to a government employment shock. Second, the ex post real interest rate responds non-significantly in initial periods but negatively in the long run to the government goods purchases shock. It falls in early periods and rises in the long run in response to the employment shock. These two findings illustrate that different components in government spending have different general equilibrium effects. Starting from the next section, I study the distributional effects of shocks to the two components in government spending.
Figure 8: Impulse responses to the government spending shocks
The left column displays the IRFs to the government goods purchases shock and the right column displays the IRFs to the government employment shock. The medium, 32nd and 68th percentiles are plotted.

2.3 A Simple Static Model

In this section, I describe the main mechanism by which government spending shocks affect heterogeneous households with the help of a simple static model. Many researchers show that in a representative agent neoclassical growth model, private consumption drops and labor supply increases in response to an increase of government spending. I show that private consumption and labor supply responses to an unexpected change in government spending depend on the type of the shock (goods
purchases or employment) and households wealth.

The economy is populated by a continuum of households with potentially different capital stock endowments. A household values private consumption, \( c \), and leisure, \( 1 - l \). It is endowed with capital stock, \( k \), which has a proportional production cost of rate \( \delta \). Households supply labor to a labor intermediate which distributes the labor composite to both private sector and the government. The private production sector is competitive and uses a Cobb-Douglas constant-returns-to-scale technology. The government buys goods from the private sector and hires labor to work in the government to provide public services. The government offers the competitive wage rate to its employees, which is the same as the wage rate in the private sector. Government spending is financed by lump-sum taxes and the government budget is balanced.

A household with endowment \( k \) solves:

\[
\max_{c,l} u(c, l)
\]

s.t.

\[
c \leq w l + r k - T
\]

where \( w = (1 - \alpha) z K^{\alpha} L_p^{-\alpha} \), \( r = \alpha z K^{\alpha - 1} L_p^{1 - \alpha} \), and \( T = G + w L_g \). \( L_p \) is the total labor in the private sector, \( L_g \) is the government employment, and \( G \) is the government goods purchases. The government goods purchases and government employment are exogenous.

The first order condition is:

\[
u_l(c, l) + u_c(c, l) w = 0 \tag{35}
\]
The utility function satisfies: \( u_c > 0, \ u_{cc} < 0, \ u_l < 0, \ u_{ll} < 0, \) and \( u_{lc} = u_{cl} < 0 \) for non-separable utility or \( u_{lc} = u_{cl} = 0 \) for separable utility.

**Proposition I.** The aggregate private sector labor \( L_p(z, G, L_g) \) is strictly increasing in \( G \) and strictly decreasing in \( L_g \). One implication is that the wage rate (interest rate) falls (increases) when \( G \) increases and the wage rate (interest rate) increases (falls) when \( L_g \) increases.\(^{18}\)

The intuition is that when the government increases its goods purchases, it consumes more resource of the economy. Then, there is a negative wealth effect to households in the economy. The wealth effect leads households to reduce consumption while increasing the labor supply. On the other hand, the additional demand for goods by the government leads to an increase of labor demand in the private sector. As a result, the equilibrium labor increases in the private production sector. Consequently, from the constant return to scale production function, the wage rate goes down and the real interest rate goes up. On the contrary, an increase in government hours also creates negative wealth effects which lead to an increase in labor supply and a decrease in private consumption. However, private sector labor decreases in equilibrium on impact. That is because: i) private sector demand declines due to the decrease in private consumption; and ii) the shock directly creates additional labor demand in the government sector. As a result, some of the labor in the private sector has to be reallocated to the government sector. Consequently, because of the constant return to scale production technology, the wage rate increases and the interest rate falls.\(^{19}\)

This proposition shows that, besides the direct wealth effect, there is another channel through which a government spending shock can affect the economy. That is, it changes the prices. Next, I will show how the two government spending shocks affect

\(^{18}\)The proof is in Appendix B.  
\(^{19}\)This finding is consistent with the results in Finn (1998) and Cavallo (2005).
households with different levels of wealth.

2.3.1 A shock to government goods purchases

Household income is:

\[ i = wl + rk - G - wL_g \]

The response of income to a shock for government goods purchases is then:

\[ \frac{\partial i}{\partial G} = w \frac{\partial l}{\partial G} + \frac{\partial w}{\partial G} l + \frac{\partial r}{\partial G} k - 1 - \frac{\partial w}{\partial G} L_g \]

There are two components: the income effect, \( \frac{\partial w}{\partial G} l + \frac{\partial r}{\partial G} k - 1 - \frac{\partial w}{\partial G} L_g \), and the income change due to changes in labor supply, \( w \frac{\partial l}{\partial G} \).

There are three components in the overall income effect. \( \frac{\partial w}{\partial G} l < 0 \) is the change in labor income. \( \frac{\partial r}{\partial G} k > 0 \) is the change of capital income. \( -1 - \frac{\partial w}{\partial G} L_g \) is the change in taxes. The income effect is an increasing function in \( k \), i.e. \( \frac{\partial \text{income effect}}{\partial k} = \frac{\partial w}{\partial G} \frac{\partial l}{\partial k} + \frac{\partial r}{\partial G} > 0 \), because \( \frac{\partial r}{\partial G} > 0 \), \( \frac{\partial w}{\partial G} < 0 \), and \( \frac{\partial l}{\partial k} < 0 \). Since labor supply is a decreasing function of \( k \), i.e. \( \frac{\partial l}{\partial k} = -(u_{lc} + u_{cc}w)r[u_{lc}w + u_{cc}w^2 + u_{ll} + u_{cl}w]^{-1} < 0 \), we have \( l > L > L_g \) when \( k \) is low. Therefore, the income effect is negative when \( k \) is small and could be positive if \( k \) is large enough.

Intuitively, households have two sources of income: capital income and labor income. The increase of capital income, due to the increase in the interest rate, partially offsets the negative wealth effect. On the other hand, the decrease in the wage rate amplifies the negative wealth effect. The overall effect then depends on the wealth level of households. Since households have different endowments of capital, the overall wealth effects are different and depend on the capital endowment of households. If a household is poor, i.e. the capital stock endowment \( k \) is low, it has a large labor
income share but a small capital income share. In this case, the increase in capital income is not large enough to offset the decrease in labor income and the increase in taxes. The household reduces its consumption and increases its labor supply due to the negative wealth effect. As the capital stock increases, the capital income share is larger and the overall effect is smaller. The household responses of consumption and labor supply become smaller and could change signs if $k$ is large enough.

2.3.2 A shock to government employment

The response of income to a shock for government employment is then:

$$\frac{\partial i}{\partial L_g} = w \frac{\partial l}{\partial L_g} + \frac{\partial w}{\partial L_g} l + \frac{\partial r}{\partial L_g} k - \frac{\partial w}{\partial L_g} L_g - w$$

There are also two components: the overall income effect, $\frac{\partial w}{\partial L_g} l + \frac{\partial r}{\partial L_g} k - \frac{\partial w}{\partial L_g} L_g - w$, and the income change due to change in labor supply, $w \frac{\partial l}{\partial L_g}$.

Similarly, there are three components in the income effect. $\frac{\partial w}{\partial L_g} l > 0$ is the change of labor income. $\frac{\partial r}{\partial L_g} k < 0$ is the change of capital income. $-\frac{\partial w}{\partial L_g} L_g - w$ is the change in taxes. The income effect is negative when $k$ is large enough, and is a decreasing function of $k$, i.e. $\frac{\partial \text{income effect}}{\partial k} = \frac{\partial w}{\partial L_g} \frac{\partial l}{\partial k} + \frac{\partial r}{\partial L_g} < 0$, because $\frac{\partial r}{\partial L_g} < 0$, $\frac{\partial w}{\partial L_g} > 0$, and $\frac{\partial l}{\partial k} < 0$.

In this case, the changes in factor prices increase labor income while decreasing capital income. The absolute value of the negative income effect becomes larger as the capital stock becomes larger. That is because, with a higher level of capital stock, there is a larger income reduction due to the drop of the interest rate. The implication is that an increase in government employment tends to dampen the income and consumption inequalities; however, it achieves this by reducing the income and consumption of all
households.

From the analysis of the simple static model above, I have shown that there are heterogeneous effects of government spending shocks. The shocks affect households through two channels: the negative wealth effect and the changes of prices. The overall effects on different households depend on their wealth and the type of the shock.

2.4 A Quantitative Model with Heterogeneous Agents

In this section, I construct a heterogeneous-agent model and calibrate it such that it matches the U.S. wealth distribution as well as several other targets in the data. I then use this model to study the quantitative heterogeneous effects of shocks to the two components in government spending. The model is based on Castaneda et al. (2003). The key ingredients of the model economy are the following: (i) it has a unit mass of households with identical preferences; (ii) the households face an uninsurable idiosyncratic labor productivity shock; (iii) every household goes through the life cycle stages of working-age and retirement; (iv) there is a positive probability of dying for the retired households, and when the retired households die they are replaced by a working-age descendant; (v) the households care about their descendants’ utility as much as their own utility; (vi) there is a borrowing constraint; (vii) there are government goods purchases and government employment shocks.
2.4.1 The model economy

2.4.1.1 Labor productivity shocks

The model economy contains a unit mass of continuum of households. A household can either be of working-age or retired. A working-age household has an exogenous probability of retiring in the next period and faces an uninsured idiosyncratic stochastic process that determines their labor productivities. A retired household faces an exogenous probability of dying in the next period and has a labor productivity of zero. A retired household is replaced by a working-age descendant who inherits the household wealth when he dies. Following Castaneda et al. (2003), a one-dimensional shock, $s$, is used to denote the household’s random age and random labor productivity jointly. I assume that this is an independent and identically distributed process which follows a finite state Markov chain. The conditional transition probabilities are given by $\Gamma_{s's} = Pr\{s_{t+1} = s'|s_t = s\}$, where $s, s' \in S = \{\xi \cup R\}$. $\xi = \{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_h\}$ and $R = \{0, 0, 0, 0\}$ are two 4-dimensional sets containing the labor productivity of working-age households and retired people, respectively. I have four retirement states, because I use the last working-age labor productivity to keep track of the earnings ability of retired households in order to capture the inter-generational transmission of earning ability. The transition matrix can be partitioned into: 

$$
\Gamma_{s's} = \begin{bmatrix}
\Gamma_{\epsilon\epsilon} & \Gamma_{\epsilon R} \\
\Gamma_{R\epsilon} & \Gamma_{RR}
\end{bmatrix}
$$

where, $\Gamma_{\epsilon\epsilon}$ describes the changes in the labor productivity of working-age households that are still of working-age in the next period; $\Gamma_{\epsilon R}$ denotes the transition probabilities from the working-age states into the retirement states; $\Gamma_{R\epsilon}$ describes the transitions from the retirement states into the working-age states when a retired household dies.
and is replaced by its descendant; \( \Gamma_{RR} \) denotes the changes in the retirement states of retired households that are still retired in the next period. Next, I describe my assumptions with respect to these four submatrixes. 

\[ \Gamma_{\epsilon R} = p_r I, \]

where \( p_r \) is the probability of retiring, and \( I \) is the identity matrix. This means that every working-age household faces the same probability of retiring. I further assume that every retired household faces the same probability of dying, and \( \Gamma_{RR} = (1 - p_d) I \), where \( p_d \) is the probability of dying. Following Castaneda et al. (2003), \( \epsilon_l \) is normalized to be one. \( \Gamma_{\epsilon_3} \) and \( \epsilon_2, \epsilon_3, \epsilon_h \) are selected to match the wealth distribution in the data.

Moreover, following Castaneda et al. (2003), two parameters, \( \phi_1 \) and \( \phi_2 \), are used to pin down \( \Gamma_{R\epsilon} \) in order to determine the intergenerational persistence of earnings and measure the life cycle profile of earnings.\(^{20}\)

### 2.4.1.2 Preferences

Households value consumption and leisure, and they are altruistic towards their descendants. The households preferences can be described as:

\[
E \sum_{t=0}^{\infty} \beta^t \left[ c_t^{1-\sigma} \frac{1}{1-\sigma} + \chi \frac{(\bar{l} - l_t)^{1-\theta}}{1-\theta} \right]
\]

where, \( \beta \in (0, 1) \) is the discount factor; \( c_t \) is consumption; \( l_t \in [0, 1] \) is labor supply, \( \bar{l} \) is the endowment of hours in each period. \( \sigma \) and \( \theta \) are the curvatures of consumption and leisure, respectively. \( \chi \) is the relative share of consumption and leisure.

\(^{20}\)See the appendix in Castaneda et al. (2003) for details.
2.4.1.3 Households problem

Households can accumulate wealth $a_t$ in the form of real capital $k_t$ and real government debt $b_t$,\textsuperscript{21} to buffer their streams of consumption against the idiosyncratic labor productivity shocks as well as aggregate shocks. I further assume that the asset holdings belong to a compact set, and the lower bound of this set is a form of liquidity constraint.\textsuperscript{22} As shown in Huggett (1993), there exists an upper bound for the asset holdings as long as the after-tax rate of return to saving is smaller than the rate of time preference. The private production sector is assumed to be perfectly competitive, which implies that factor prices are given by their corresponding marginal productivities.

The individual states are, therefore, $(a, s)$. The aggregate states contain aggregate productivity, $z$, government goods purchases, $G$, government employment, $L_g$, and the distribution of households $\mu$. Households choose consumption $c$, labor supply $l$ and asset to maximize their utility in an infinite horizon.

The recursive formulation of a household’s problem is:

$$v(a, s; \mu, z, G, L_g) = \max_{\{c, k', l\}} \{u(c, l) + \beta E[v(a', s'; \mu', z', G', L'_{g}|z, s)]\}$$

s.t.

$$c + q = y - \tau(y) + a - T I_{s \in \xi}$$

$$y = r(K, L_p, \Omega)a + w(K, L_p, \Omega)l\epsilon + Tr I_{s \in R}$$

\textsuperscript{21}In equilibrium, both assets pay the same return rate.

\textsuperscript{22}I use zero as the lower bound.
\[ a' = \begin{cases} 
q - \tau_E(q) & \text{if } s \in \mathbb{R} \text{ and } s' \in \xi \\
q & \text{otherwise} 
\end{cases} \]

and

\[ \mu' = H(\mu, \Omega, \Omega') \]

\[ a' = f(a, s; \mu, z, G, L_g) \geq a \]

where \( \mu \) is the joint distribution on \((a, s)\), \( \Omega = \{z, G, L_g\} \); \( K \) is aggregate capital; \( L_p \) is the aggregate labor in the private sector; \( H \) is the law of motion of the households distribution; \( f \) is the decision rule for capital; \( y \) is household income including capital income, labor income that can be earned only by working-age households, and social security income, \( TrI_{s \in \mathbb{R}} \), that can be earned only by retired households; \( \tau(y) \) is the progressive income tax; \( TI_{s \in \xi} \) is the lump-sum tax that is only collected from working-age households; \( q \) is the capital choice; \( a' \) is the asset holding in next period; \( \tau_E \) is the estate tax, if possible; \( a \) is the lower bound of borrowing constraint; and

\[ r(K, L_p, \Omega) = \alpha z \left( \frac{K}{L_p} \right)^{\alpha - 1} - \delta, \quad w(K, L_p, \Omega) = (1 - \alpha) z \left( \frac{K}{L_p} \right)^{\alpha} \]

The index function \( I_{s \in \xi} \) and \( I_{s \in \mathbb{R}} \) mean that only the working-age households are paying lump-sum tax and retired households are getting transfer.

### 2.4.1.4 Taxes

This subsection describes the income and estate tax functions. The tax functions are taken from Castaneda et al. (2003):
\[ \tau(y) = a_0[y - (y^{-a_1} + a_2)^{-1/a_1}] + a_3y \]

and

\[ \tau_E(q) = \begin{cases} 0 & \text{for } q < q \frac{1}{2} \\ \tau_E(q - q) & \text{for } q > q \end{cases} \]

2.4.1.5 Government

The government in this model purchases private sector goods and hires workers to provide public services. It offers competitive wages which are the same as the wages in the private sector. The government consumption of goods and the public services do not yield utility to the households. The government also pays transfers to retired households and it collects income taxes, an estate tax, and a lump-sum tax to finance its spending. There also exists public debt that pays the same real interest rate as capital does.

The government budget constraint is

\[ G + w(\bar{k}, \bar{l}, \Omega)L_g + (1 + r)B + Tr = \int \tau(y)d\mu + \int \tau_E d\mu + T + B' \]

where, \( G \) is government goods purchases; \( L_g \) is government employment; \( Tr \) is the total amount of transfer; \( \tau(y) \) and \( \tau_E \) are income and estate taxes, respectively; \( T \) is lump-sum tax; \( Y \) is the total private output, \( B \) and \( B' \) are current and next period government debt, respectively.
2.4.1.6 Aggregation and markets clearing

The aggregate capital satisfies

\[ K = \int a d\mu - B \]

Households do not choose to work for the private sector or the government, instead they supply labor to a labor intermediate which then distributes total hours into the private sector and the government sector. Labor market clearing requires:

\[ L_g + L_p = \int l e d\mu \]

Private sector goods are used as households consumption, investment, and government goods consumption:

\[ Y = \int c d\mu + K' - (1 - \delta)K + G \]

2.4.1.7 Equilibrium

A recursive competitive equilibrium is then a law of motion \( H \), a pair of individual functions \( v \) and \( f \), pricing functions \( r \) and \( w \), the lump-sum tax \( T(z, G, L_g) \), and government debt \( B(z, G, L_g) \), such that (i) \((v, f)\) solves the household’s problem, (ii) \((r, w)\) are competitive, (iii) \( H \) is generated by \( f \), (iv) \( T(z, G, L_g) \) and \( B(z, G, L_g) \) solves the government budget constraint\(^{23}\), and (v) markets clear.

\(^{23}\)I assume, in steady state, the lump-sum tax is zero.
2.4.2 Calibration

I calibrate the economy such that its steady state statistics match the corresponding statistics in the data. The list of the parameters and their targets are summarized in Table 8. The values for $\theta, \alpha, \delta, p_r, p_d, \phi_1, \phi_2, a_0, a_1, a_2$ are the same as those used in Castaneda et al. (2003). $\sigma$ is set to be 2.0. $G_{ss}$ and $L_{ss}^g$ are selected such that the steady state total government expenditure is 20% of GDP, and the ratio between steady state government goods expenditure and employee compensation is 1. Steady state transfers are selected to match the target $Tr = 0.049Y$. $\tau_E$ and $z$ are selected to match $\tau_E = 0.002Y$. $a_3$ is selected to balance the steady state government budget constraint such that government debt and lump-sum tax are zero. $\beta, \chi$ and parameters in $\epsilon$ and $\Gamma$ are jointly endogenously determined by solving the steady state of the model and matching several targets including the steady state capital-output ratio, the normalized level of labor supply, and the wealth distribution in the data.

Table 9 displays the transition probabilities of the process on the labor productivity for working-age households that remain of working-age one period later, $\Gamma_{\epsilon\epsilon}$. All rows sum up to 97.78% because that a worker has a probability of 2.22% to be retired. This table illustrates that the labor productivity shocks are persistent. A household whose current productivity is $\epsilon_L$ is most likely to make a transition to $\epsilon_2$ than to any of the other levels. Households with productivity $\epsilon_2$ or $\epsilon_3$ are most likely to move to $\epsilon_L$. It is very hard for a household to move from any other state to $\epsilon_H$, and when a household draws a productivity of $\epsilon_H$, it is highly possible that it will draw back to $\epsilon_L$ in the near future.
Table 8: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Targets or Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>2.0</td>
<td>conventional</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1.016</td>
<td>Castaneda et al. (2003)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.376</td>
<td>Castaneda et al. (2003)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.059</td>
<td>Castaneda et al. (2003)</td>
</tr>
<tr>
<td>$p_r$</td>
<td>0.022</td>
<td>Castaneda et al. (2003)</td>
</tr>
<tr>
<td>$p_d$</td>
<td>0.055</td>
<td>Castaneda et al. (2003)</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.969</td>
<td>Castaneda et al. (2003)</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>0.525</td>
<td>Castaneda et al. (2003)</td>
</tr>
<tr>
<td>$G_{ss}$</td>
<td>0.5075</td>
<td>$\frac{G + wL^g_{ss}}{Y + wL^g_{ss}} = 0.2$</td>
</tr>
<tr>
<td>$L^g_{ss}$</td>
<td>0.3569</td>
<td>$\frac{G}{w^g_{ss}} = 0.5$</td>
</tr>
<tr>
<td>$T_r$</td>
<td>0.6473</td>
<td>$\frac{T_r}{Y} = 0.049$</td>
</tr>
<tr>
<td>$\tau_E$</td>
<td>0.0040</td>
<td>$\frac{\tau_E}{Y} = 0.002$</td>
</tr>
<tr>
<td>$\hat{z}$</td>
<td>32.002</td>
<td>$\frac{K}{Y} = 3.13$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.914</td>
<td>$L = 0.30$</td>
</tr>
<tr>
<td>$a_0$</td>
<td>0.258</td>
<td>Castaneda et al. (2003)</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.768</td>
<td>Castaneda et al. (2003)</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.491</td>
<td>Castaneda et al. (2003)</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0.1648</td>
<td>balance budget</td>
</tr>
</tbody>
</table>

Table 9: Transition matrix: working-age to working age (%)

<table>
<thead>
<tr>
<th>From $s$</th>
<th>$s' = \epsilon_L$</th>
<th>$s' = \epsilon_2$</th>
<th>$s' = \epsilon_3$</th>
<th>$s' = \epsilon_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s = \epsilon_L$</td>
<td>96.237</td>
<td>1.14</td>
<td>0.39</td>
<td>0.013</td>
</tr>
<tr>
<td>$s = \epsilon_2$</td>
<td>3.07</td>
<td>94.33</td>
<td>0.37</td>
<td>0.01</td>
</tr>
<tr>
<td>$s = \epsilon_3$</td>
<td>1.50</td>
<td>0.43</td>
<td>95.82</td>
<td>0.03</td>
</tr>
<tr>
<td>$s = \epsilon_H$</td>
<td>10.66</td>
<td>0.49</td>
<td>6.11</td>
<td>80.52</td>
</tr>
</tbody>
</table>

Table 10 displays the labor productivity set and the stationary distribution of working-age households. This table illustrates that more than half of the working-age households are of type $\epsilon_L$, followed by those with productivity $\epsilon_2$ and $\epsilon_3$. It also shows that households of type $\epsilon_H$ only counts 0.0773% of the working-age population.
Table 10: labor productivity

Table 11 displays the steady state wealth distribution of the model and data. The wealth distribution data is from Castaneda et al. (2003). The model matches the wealth distribution reasonably well, especially for the last two quintiles.

<table>
<thead>
<tr>
<th>Quintiles</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td>-0.39</td>
<td>1.74</td>
<td>5.72</td>
<td>13.43</td>
<td>79.49</td>
</tr>
<tr>
<td>model</td>
<td>0.22</td>
<td>2.04</td>
<td>2.57</td>
<td>14.58</td>
<td>80.59</td>
</tr>
</tbody>
</table>

Table 11: Wealth distribution (%)
Data source: Castaneda et al. (2003)

Table 12 displays the consumption distribution of in the steady state in the model and data. The first row shows the consumption distribution of non-durables in the U.S. data from Castaneda et al. (2003). The second row shows that the consumption distribution from the model using all households including working age and retired households. A comparison of the numbers reported in the table shows that those two rows are very similar. More importantly, since I have not used the consumption distribution as part of my calibration targets, any similarity between the model and data in this aspect should be considered a success in accounting for the U.S. wealth inequality.

<table>
<thead>
<tr>
<th>Quintiles</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td>6.87</td>
<td>12.27</td>
<td>17.27</td>
<td>23.33</td>
<td>40.27</td>
</tr>
<tr>
<td>model</td>
<td>6.34</td>
<td>13.46</td>
<td>14.02</td>
<td>19.69</td>
<td>46.50</td>
</tr>
</tbody>
</table>

Table 12: Consumption distribution (%)
Data source: Castaneda et al. (2003)
2.5 The Heterogeneous Effects of Government Spending Shocks in the U.S. Economy

This section provides the main results of this paper. In the first subsection, I identify government spending shocks using the narrative approach and defense news series in Ramey (2011). This method provides an opportunity to get the impulse responses (IRFs) of several fiscal variables to a single exogenous shock simultaneously. In contrast, other commonly used methods, such as the structural vector autoregression method and the sign restriction approach, can only identify one type of government spending shocks each time. I get the IRFs of government goods purchases, government employment, and marginal income tax rate from the VAR and use them as the impulses of my model. In the second subsection, I evaluate the aggregate and heterogeneous impacts of the identified government spending shocks using the model described in the last section. In addition, I calculate the welfare implications to heterogeneous households differing in their levels of wealth. Finally, in the last subsection, I evaluate the importance of distinguishing government employment from government goods purchases by comparing the welfare implications under different policy shocks.

2.5.1 The government spending shocks

Following Ramey (2011), I use the defense news series to identify government spending shocks. In this approach, the defense news variable is ordered before other variables in the VAR. The IRFs of other variables are the responses to the common shock in defense news. Hence, this method provides an opportunity to get the responses of all fiscal variables to an exogenous foreign political event simultaneously. Other
variables in the VAR are log of real per capita government goods purchases, log of per
capital government hours, log of real per capita GDP, the 3 month T-bill rate, the
Barro and Redlick (2011) average marginal income tax rate\(^{24}\), log of real per capita
consumption of non-durable goods and services, and a linear and quadratic trend. I
use US quarterly data from 1947 to 2008 and four lags in the VAR. The details of
data and empirical method can be found in the appendix.

The IRFs of government goods purchases, government employment, and marginal
income tax rate are provided in Figure 9.\(^{25}\) In response to a defense news shock,
both government goods purchases and government hours go up indicating that both
variables are used as fiscal tools. Moreover, the marginal income tax rate variable
increases correspondingly to finance the extended government expenditure. However,
the size of the tax rate response is not large enough to finance all of the extra spending
implying an increase in government debt.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fiscal_variables.png}
\caption{Fiscal variables}
\end{figure}

\(^{24}\)Following Ramey (2011), the annual tax rate series are converted to quarterly data by assuming
that the tax rate in each quarter of the year was equal to the annual rate for that year.

\(^{25}\)The IRFs are converted from quarterly responses to annual responses. Specifically, one year
response is the summation of four quarters responses.
2.5.2 The heterogeneous effects

2.5.2.1 Impulse responses

I use the IRFs of government goods purchases and government employment from last subsection as the impulses of the model economy and evaluate the aggregate and disaggregated impacts of fiscal shocks. The government funds the extended government expenditure by adjusting contemporaneous income tax rate, $a_3$, as well as government debt. The path of the marginal income tax rate is taken from the IRFs of the VAR\textsuperscript{26}. Any extra government deficit are funded by government debt. I assume that the government starts to reduce its debt in a rate of 2\% each year by adjusting the marginal tax rate when the values of the fiscal variables go back to steady state. The left column in Figure 10 displays the impulse responses of real interest rates, real wage rates and government debt. The real interest rate decreases initially. That is because the government increases its employment and does not change level of government goods purchases in time $t = 0$. This combination of fiscal changes reallocates labor hours from the private sector to the government sector in the initial period\textsuperscript{27}. With a lower level of labor, the marginal productivity of capital declines, which results in a lower real interest rate. On the other hand, a lower level of labor in the private sector increases the marginal productivity of labor and leads to an increase in the real wage rate. Starting from time $t = 2$, the real interest rate rises while the real wage rate declines. There are two driving forces of this result. First, the build up of government goods purchases requires more outputs and more labor hours from the private sector. Second, the increase in government debt and the decrease in private saving (as shown in the lower-right panel of Figure 10) result in a decline in capital stock. Hence, there are more labor hours in the private sector

\textsuperscript{26}See lower-left panel of Figure 9.
\textsuperscript{27}The IRF of private sector labor can be found in the lower-left panel of Figure 12.
and less capital, which results in a higher (lower) marginal productivity of capital (labor). The lower-left panel shows that government debt increases. That is because the increase in the contemporaneous income tax is not large enough to finance the entire extended government expenditure.

The right column of Figure 10 shows the impulse responses of consumption, labor supply and asset choice of the wealthiest quintile (the 5th quintile) and the least wealthy quintile of households. Consumption drops for both quintiles, and that is due to the negative wealth effect caused by the increase in government usage of private resources. Moreover, less wealthier households have to give up more consumption in percentage. That is due to the general equilibrium effects. Specifically, although there is an increase in the real wage rate and a decrease in the real interest rate in initial periods, there is a large and persistent decrease (increase) in the real wage (interest) rate in the long run. The decline in the real wage rate would amplify the negative wealth effect and hurt less wealthy households who rely on their labor income. On the other hand, the increase in the real interest rate would partially offset the negative wealth effect and benefit wealthier households who have a large amount of capital income.

Labor supply increases on impact which is also due to the negative wealth effect. But at time $t = 1$ and 2, the 5th quintile of households supply less labor and the 1st quintile only barely increase their labor supply. The reason is that, at time $t = 1$, the marginal income tax rate reaches its peak, which generates a large substitution effect. The substitution effect dominates the negative wealth effect at time $t = 1, 2$ and leads to a decline in labor supply. In addition, the labor supply of wealthier households has a wider fluctuation. That is due to the progressive taxation. Wealthier households have to pay more tax for their labor income, because their high capital income has

\[^{28}\text{Only working-age households' IRFs are plotted. That is because the retired households do not supply labor and receive transfers from the government.}\]
triggered a higher tax rate. As a result, richer households are more sensitive to tax changes.

In early periods, households have to choose a lower level of asset. That is due to the negative wealth effect and lower level of labor supply caused by the increase in the marginal income tax rate. In the medium and long run, private saving goes up in response to the increase in the real interest rate.

Figure 10: Impulse Responses

2.5.2.2 Welfare

Following Krusell and Smith Jr (1999), we measure the welfare change in terms of percentage change in life time consumption, i.e. the Consumption Equivalent Variation (denoted as $\lambda$). Given perfect foresight of the government spending shocks, we can calculate the consumption equivalent variation along the balanced growth path which makes households indifferent between the government spending shocks and the modified path. That is, we calculate $\lambda$, such that
$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(1 + \lambda) c_t}{1 - \sigma} + \chi \frac{(\bar{l} - l_t)}{1 - \theta} \right] = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\tilde{c}_{1-t}}{1 - \sigma} + \chi \frac{(\bar{l} - \tilde{l}_t)}{1 - \theta} \right]$$

where $c_t$ is consumption in the economy with government spending shocks, while $\tilde{c}_t$ is that in the economy without government spending shocks.

In order to see the importance of the general equilibrium effects, we also calculate the consumption equivalent variations in an economy with constant values of interest rates and wage rates. That is, in this partial equilibrium economy the fiscal variables have the same paths as in the general equilibrium model, but the factor prices are fixed at the steady state levels. Households are fully informed about the partial equilibrium feature of the economy. Figure 11 displays the consumption equivalent variations for each quintile of households in both general (benchmark) and partial equilibrium economies. The left (blue) bar is the $\lambda$ in the benchmark economy, while the right (red) bar is that with constant levels of factor prices.

Three findings need to be emphasized. First, consumption equivalent variations are positive for all quintiles implying no one benefits from the increase in government expenditure. Second, in the benchmark economy, the welfare loss is larger for less wealthier households, with a consumption equivalent variation of 0.12% for the 1st quintile compared to 0.04% for the 5th quintile. The size is comparable to the numbers from the literature regarding the welfare gain from the elimination of government spending cycles (see Bachmann et al. (2013)). Moreover, the calculated consumption equivalent variations reveal that government spending shocks would increase inequality in terms of welfare. Third, moving from the general equilibrium economy to the partial equilibrium one, the consumption equivalent variations vary significantly. The difference in the consumption equivalent variations between the general and partial equilibrium experiments represents the welfare change of households moving from the
general equilibrium economy to the other. This result indicates that the general equi-
librium effects on welfare are considerable. The welfare loss drops for all households
and lower quintiles enjoy larger decline in welfare loss. In the general equilibrium
economy, there is a persistent decrease (increase) in the real wage (interest) rate
which would amplify (partially offset) the negative wealth effect. If we fix the factor
prices to be constant, households could enjoy a welfare gain from the higher wage rate
and a welfare loss from the lower interest rate. The overall welfare change depends
on the levels of households’ wealth and labor supply. For the 1st quintile, since they
have the lowest level of asset and highest level of labor supply, their overall welfare
gain is the largest. For higher quintiles, as the households become more and more
wealthy and supply less and less labor, the overall welfare gain from eliminating the
general equilibrium effects is getting smaller.

Figure 11: Consumption equivalent variations

29Ferriere and Navarro (2014) argues that the general equilibrium effects are small. We believe that
result is due to that they use entirely contemporaneous taxes to financed the extended government
expenditure. However, without an increase in government debt (a decline in public saving), the
factor prices changes are only temporary. Moreover, since the general equilibrium effects do not
only work through current prices but also the expectation of future prices. Hence, whether or not
the households are fully informed about future price paths is crucial in calculating the consumption
equivalent variations.
2.5.3 Government employment, the general equilibrium effects and welfare

The importance of the general equilibrium effects and the empirical findings in this paper\textsuperscript{30} provide a rationale to distinguish the government employment compensation from government goods purchases. In this subsection, I further investigate the welfare implication of this differentiation. Two policy experiments are conducted. In the first experiment, all of the increase in government spending including the increase in government employment compensation is used to purchase more goods from the private sector. In the second one, the government uses all of the additional expenditure to hire more government labor hours. The paths of the marginal income tax rate are the same as in the benchmark economy in the above subsection. Therefore, these two experiments are comparable to the benchmark one.

The lower-left panel in Figure 12 shows the IRF of private sector labor. There are four driving forces that affect labor hours in the private sector. i) The substitution effect caused by the increase in the income tax rate discourages households labor supply. ii) The negative wealth effect caused by the increase in government usage of private resources encourages households to supply more labor. iii) The increase in government goods purchases requires more outputs and more labor hours from the private sector. iv) The increase in government labor hours reallocates labor hours from the private sector to the government sector. The response of private sector labor depends on the size of each driving force in each period. The substitution effect dominates in early periods leading to a decline in private labor hours, while in the long run the negative wealth effect dominates as the income tax rate returns to the steady state. The size of the private labor response varies in each experiment, which reflects the

\textsuperscript{30}As a reminder, the empirical part of this paper shows that shocks to different components in government spending lead to opposite effects on factor prices.
differences in the magnitude of each driving forces in each economy. In the first experiment, as the government spend all of the additional expenditure in purchasing goods from the private sector, it requires more labor hours in the private sector than the benchmark case. In contrast, if the government uses all the additional expenditure to employ more government labor, it drives down private sector labor hours. As a result, private sector labor declines by less (more) and reverses faster (slower) in the first (second) experiment. Moreover, the capital stock, as shown in the lower-right panel, decreases due to the negative wealth effect. Consequently, the real interest rate declines by less (more) and reverse faster (slower) in the first (second) experiment. Similarly, the real wage rate jumps by less (more) and reverses faster (slower) in the first (second) experiment. These two experiments illustrate that shocks to the two components in government spending have different effects on factor prices which is consistent with the empirical findings in this paper.

Figure 12: Impulse responses of interest rate, wage rate, private labor and capital

The differences in the IRFs of factor prices implying disparate general equilibrium effects. I calculate the consumption equivalent variations for each experiment and
compare them with the benchmark economy. Figure 13 displays the calculation results: the middle (green) bar is the benchmark result, the left (blue) bar is calculated from the first experiment, and the right (red) bar is from the second. In the first experiment, as the real interest rate decreases by less and reverses faster, it could benefit households through increasing the capital income. On the other hand, as the real wage rate increases by less and reverses faster, it would amplify the negative wealth effect. As a result, less wealthy households would suffer larger welfare loss in the first experiment while richer households would enjoy less welfare loss. In contrast, the second experiment results in larger (less) welfare loss for rich (poor) households through the general equilibrium channel.

To sum up, the experiments in this subsection illustrate the importance of distinguishing government employment from government goods purchases. As shown in the analysis, shocks to the two components in government spending lead to different consequences in aggregate factor prices as well as different welfare implications in the disaggregated level.

![Figure 13: Comparison of consumption equivalent variations](image-url)
2.6 Conclusion

This paper assesses the heterogeneous effects and welfare implications of government spending shocks. According to the empirical and quantitative studies, shocks to government goods purchases and government employment have dramatically different effects on prices and therefore on different households. Government spending shocks affect households by creating wealth effects, changing tax rates and changing factor prices. Households responses and their welfare gain (or loss) depend on the type of government spending shocks and their wealth. The quantitative analysis reveals that government spending shocks would reduce welfare and amplify inequality.

Taking into explicit account different components in government spending, rather than treating government spending as consisting entirely of goods purchases, provides a more precise approach to evaluate fiscal policy shocks. This approach extends the literature by bringing into attention the structure of government expenditure, which has been largely overlooked in existing studies and provides a new perspective to the debates over the effects of government spending. Given the better understanding on the impacts of different policy instruments, the immediate future research will be searching for the optimal policies to facilitate stable economic growth and reduce inequality.
Chapter 3: Spending Reversals and Fiscal Multipliers under an Interest Rate Peg

3.1 Introduction

There is a growing literature on the size of the government spending multiplier under a pegged nominal interest rate, especially pegged at the zero lower bound. Woodford (2011) and Christiano et al. (2011) argue that a large government spending multiplier is especially plausible when the zero lower bound on the nominal interest rate binds. Kim (2003) and Davig and Leeper (2011) show that, under passive monetary policy and active fiscal policy, government spending raises both output and consumption. On the other hand, Carlstrom et al. (2012) demonstrates that if the monetary-fiscal expansion is for a fixed duration instead of a stochastic duration, the output multiplier can be much smaller. Erceg and Lindé (2014) argues that although the multiplier is high for small increases in government spending in an endogenous liquidity trap, it may decrease substantially at higher spending levels.

These studies have posited that there is an expected inflation channel for government spending: a fiscal expansion drives up the current and expected future marginal cost, which leads to an increase in expected inflation and hence reduces the expected real interest rate. The reduction in the real rate leads households to shift consumption toward the present. This effect can be especially strong if the nominal interest rate is pegged at a constant level. According to this channel, a large output multiplier requires a large responses of expected inflation to a fiscal expansion. However, Dupor
and Li (2014) demonstrates that this large response is inconsistent with empirical evidence from the Federal Reserve's passive policy period as well as the 2009 Recovery Act period. Therefore, a theory that could generate a inflation response which is consistent with the empirical finding is needed. On the other hand, Corsetti et al. (2012) provides time series evidence for the US suggesting that an exogenous increase in government spending is followed by a reduction in spending below trend. It illustrates the importance of government spending reversals on fiscal multipliers but overlooks the possibility of interest rate peg.

The contribution of this paper is to demonstrate how anticipated government spending reversals alter the impacts of government spending shocks under an interest rate peg. It shows that an increase in government spending does not necessarily generate a large inflation response, nor a large output multiplier. Specifically, the private sector expects government spending reversals to reduce future inflation. This reduction in expected inflation, together with a pegged nominal interest rate, will lead to an increase in the real interest rate, discouraging private consumption. As a result, the output multiplier can be smaller than one even if the monetary policy is constrained at zero lower bound.

In addition, I investigate a policy experiment with a stochastic duration of interest rate peg. It is a realistic modification as the interest rate peg is usually a policy response to a stochastic exogenous shock. In this experiment, I find that a monetary-fiscal expansion with spending reversals generates a small inflation response which is consistent with the empirical finding in Dupor and Li (2014). Moreover, the output multipliers are below one.

The remainder of this paper is organized as follows. Section 3.2 presents the empirical evidence from the literature. Section 3.3 outlines the theoretical model and illustrates the mechanism through which spending reversals alter the short-run effects of fiscal
expansions. Section 3.4 demonstrates how the exit of the nominal interest rate peg can affect the impacts of a government spending shock. Section 3.5 shows this basic result extends to a generalized model in which capital accumulation is allowed. Section 3.6 concludes.

3.2 Empirical Findings from the Literature

3.2.1 The expected inflation channel

Dupor and Li (2014) show that a sticky price model calibrated to have a large output multiplier requires a large expected inflation response to an increase in government spending. Moreover, they show that this large response is inconsistent with the empirical evidence in the postwar US economy: i) there is no significant increase in inflation in the structural vector autoregression (VAR) evidence from the Federal Reserve’s passive policy period; ii) multiple expected inflation measures during the 2009 Recovery Act period exposed only insignificant responses.

Specifically, Dupor and Li (2014) follows Fisher and Peters (2010) and identifies government spending shocks using the total excess returns of a portfolio of corporations that are main defense contractors of the U.S.. They find that during the Federal Reserve’s passive policy period (1959-1979), the one-year inflation response is -0.08 to a government spending shock with a 90% confidence interval of (-1.39, 1.65).

Moreover, Dupor and Li (2014) uses several expected inflation measures including the inflation forecasts in the Survey of Professional Forecasters, a US-UK comparison of bond yields and fiscal policy news to examine the expected inflation responses to the 2009 American Recovery and Reinvestment Act. They find insignificant responses from these exercises.
3.2.2 The spending reversal

Corsetti et al. (2012) estimates a VAR model on U.S. data for the period 1983 to 2007 to study the evidence on the fiscal transmission mechanism. They apply two identification approaches: Blanchard and Perotti (2002) and Ramey (2011). One of their key results is that a positive government spending shock triggers a sizable buildup of public debt and is followed by a decline of government spending below trend. Corsetti et al. (2012) shows that anticipated spending reversals can alter the short-run impact of fiscal policy by affecting private agents’ expectations.

To sum up, empirical findings suggest that there is no large inflation response to a government spending shock under passive or constrained monetary policy; and a positive government spending is typically followed by a spending reversal. Therefore, a theory that could generate inflation responses consistent with the empirical finding is needed. Anticipated spending reversals that could alter private agents’ expectation is a potential candidate feature in such theory. Starting from the next section, I explore the role of expected spending reversals in a monetary-fiscal expansion where the nominal interest rate is pegged.

3.3 Spending Reversals with A Deterministic Interest Rate Peg

I consider the following log-linearized sticky price model\textsuperscript{31}:

\[ i_t - E_t \pi_{t+1} = \sigma (E_t c_{t+1} - c_t) \]  

\textsuperscript{31}The model set up and notation follow Carlstrom et al. (2012).
\[
\pi_t = \beta E_t \pi_{t+1} + \kappa (\sigma c_t + \nu y_t) \tag{37}
\]

\[
y_t = (1-s)c_t + sg_t \tag{38}
\]

where \(y_t, c_t, \pi_t, i_t\) and \(g_t\) are the log deviations of output, consumption, inflation, the nominal interest rate and government expenditure from steady-state values, respectively. The constant \(s = \frac{G_{ss}}{Y_{ss}}\) is the steady state government spending and output ratio. For simplicity, I assume the steady state net inflation is zero. I consider a policy experiment in which the central bank announces an interest peg \(i_t = 0\) for a deterministic duration \(T\). Government spending is set above steady state \(g_t = \bar{g} > 0\) for \(t = 1, ..., \frac{T}{2}\) and below steady state \(g_t = \frac{2-T}{T} \bar{g} < 0\) for \(t = \frac{T}{2} + 1, ..., T\).\(^{32}\) Hence, government expenditure increases by \(\bar{g}\) in total during these periods. After the interest rate peg period, the nominal interest rate is set according to a typical Taylor rule:

\[
i_t = \phi_\pi \pi_t + \phi_y y_t \tag{39}
\]

where \(\phi_\pi > 1\) and \(\phi_y \geq 0\). As there are no endogenous state variables nor exogenous shocks after the interest rate peg period, the unique equilibrium in the subsequent periods is given by \(\pi_t = y_t = c_t = 0\). Plugging \(i_t = 0\) and combining equations (36), (37), and (38), the equilibrium in the interest rate peg period can by solved by the following difference equation:

\(^{32}\)I use an even number for \(T\).
\[ \beta \sigma E_t \pi_{t+2} - \{ \kappa [\sigma + \nu (1 - s)] + \sigma (1 + \beta) \} E_t \pi_{t+1} + \sigma \pi_t + \sigma \kappa \nu s (E_t g_{t+1} - g_t) = 0 \]  

(40)

with two terminal conditions:

\[ \pi_T = \kappa \nu s g_T \]  

(41)

and

\[ \pi_{T-1} = \kappa \nu s g_{T-1} + \{ \kappa [1 + \nu (1 - s)] + \beta \} \pi_T \]  

(42)

### 3.3.1 Impulse responses and fiscal multipliers

The baseline parameter values are set as: \( \beta = 0.99, \kappa = 0.028, \sigma = 2, \nu = 0.5, s = 0.2 \) and \( T = 8 \). Figure 14 displays the path of government spending and, respectively, the responses of inflation, private consumption and output in the benchmark experiment. For comparison, I also plot the responses in an experiment without spending reversal. Current and future inflation drop in the experiment with a spending reversal while they increase in the comparison experiment. The two experiments also imply markedly different private consumption and output responses. The reversal case features a reduction in private consumption, while the comparison case has an increase in consumption. Without spending reversals, an increase in government spending drives up the current and future real marginal costs. If a firm may be unable to adjust its price for several periods, the increase in its expected real marginal costs leads the firm to increase its price today. This shift up will generate inflation which, together with a pegged nominal interest rate, reduces the expected real interest rate. The real rate reduction leads households to shift private consumption toward today, which implies
an increase in output larger than the size of government spending expansion. On the other hand, the spending reversal in our benchmark experiment drives down future real marginal costs and future inflation. There are two drivers in initial periods: i) the increase in government spending drives up the real marginal costs, which leads to upward pressure on current inflation; ii) the decrease in future inflation leads to downward pressure on current inflation. Therefore, the inflation responses in the initial periods depend on the sizes of these driving forces. If the downward pressure created by the reduction in future inflation dominates the other, the initial inflation would fall. As the nominal interest rate is pegged at zero the real interest rate would rise, which shifts consumption toward the future. As a result, the output increases by less than the size of government spending expansion.

Figure 14: Responses of inflation, private consumption and output

In this economy, the inflation and output responses during the interest rate peg are
time-varying. I define the impact and cumulative output multipliers as, respectively:

\[
\frac{\Delta Y_1}{\Delta G_1} \equiv \frac{1}{s} \frac{dy_1}{dg_1}
\]

\[
\frac{\Delta Y}{\Delta G} \equiv \frac{1}{s} \sum_{t=1}^{T} \frac{dy_t}{dg_t}
\]

Table 13 displays the fiscal multipliers. This result shows that the output multiplier is not necessarily larger than one even if the nominal interest rate is pegged at zero.

<table>
<thead>
<tr>
<th></th>
<th>Output Multipliers in:</th>
<th>reversal spending</th>
<th>non-reversal spending</th>
</tr>
</thead>
<tbody>
<tr>
<td>impact</td>
<td>0.9263</td>
<td>1.2246</td>
<td></td>
</tr>
<tr>
<td>cumulative</td>
<td>0.7154</td>
<td>1.2378</td>
<td></td>
</tr>
</tbody>
</table>

Table 13: Fiscal multipliers under a deterministic interest rate peg

### 3.3.2 Sensitivity

#### 3.3.2.1 Sensitivity to the size of the spending reversal

As the inflation responses in the initial periods depend on the relative importance between the increase in current marginal cost and the decrease in expected inflation, the size of the spending reversal may influence the inflation responses as well as the output multipliers. In this subsection, I document the impact of the size of the spending reversal on the fiscal multipliers. As shown in the upper panels of Figure 15, the impact and cumulative multipliers decrease as the size of the spending reversal falls. That is because as the size of the reversal becomes smaller the reduction in expected inflation declines. However, as long as the downward pressure on current inflation dominates, the output multipliers are smaller than one.
3.3.2.2 Sensitivity to price stickiness

Second, it is interesting to illustrate the effect of the price stickiness on the fiscal multipliers. In the model, $\kappa$ is the parameter that measures how sticky the price is. A smaller $\kappa$ represents a higher degree of price stickiness. The lower panels in Figure 15 display the output multipliers as a function of $\kappa$. The output multipliers decline when price become less sticky. This result is consistent with the literature such as Christiano et al. (2011), Drautzburg and Uhlig (2011) and Woodford (2011).

Figure 15: Sensitivity analysis
3.4 Spending Reversals with A Stochastic Interest Rate Peg

In this section I consider a modest change in the policy environment. The duration of nominal interest rate peg is no longer deterministic, but is instead stochastic. Each period there is probability $p$ that the central bank will continue pegging the interest rate. With probability $1 - p$ the interest rate peg ends and the monetary policy reverts to the Taylor rule. I assume that once the monetary policy returns to the Taylor rule, it does not go back. The rationale of this policy environment is that it is a simplified version of a realistic case. Specifically, the interest rate peg is not an intended policy by the central bank, but is a policy response to an exogenous shock. For example, Eggertsson (2010) uses a discount rate shock to drives down the shadow Taylor rule rate to be below zero. As the exogenous shock has the stochastic feature, it is more realistic to model a stochastic interest rate peg instead of a deterministic one. The fiscal policy expansion is still deterministic as in the last section. The deterministic fiscal expansion is more reasonable than a stochastic one, since a fiscal stimulus is often designed with certain schedule.

Figure 16 displays the possible monetary policy states in each period. At time $t = 1$, the central bank pegs the nominal interest rate at zero. Starting from $t = 2$, there are two possible states: pegged interest rate and non-pegged rate. In each period, if the interest rate remains pegged, there is probability $p$ such that the central bank keeps its pegged rate in the next period and with probability $1 - p$ the monetary policy follows the Taylor rule. If the monetary policy returns to the Taylor rule, it will follow the Taylor rule for sure in the next period. Figure 16 also displays the probability of each monetary policy state from the perspective of period $t = 1$. To calculate the

\[33\text{See Equation 39.}\]
expected paths of inflation, consumption and output, we need to calculate the values in each possible state from time $t = 1$ to $t = T$. I set $p = 1 - \frac{1}{T}$, $\phi_\pi = 1.5$ and $\phi_y = 0$.

![Figure 16: Possible states](image)

I first calculate the path of the “non-pegged” case which is deterministic. Denote $x_t^N$ as the variables in this case. Plugging Equation 39 into 36, and combining equations 36, 37 and 38, the “non-pegged” path is solved by the following difference equation:

$$
\beta \sigma \pi^N_{t+2} - [\sigma (\kappa + 1 + \beta) + (\kappa \nu + \beta \phi_y)(1-s)] \pi^N_{t+1} + [\sigma (\phi_\pi \kappa + 1) + (\phi_\pi \kappa \nu + \phi_y)(1-s)] \pi^N_t + \sigma \kappa s [\nu g_{t+1} + (\phi_y - \nu) g_t] = 0
$$

and two terminal conditions:

$$
\pi^N_T = \frac{[\phi_y(1-s) + \sigma] \kappa \nu s - \phi_y s}{\kappa [\sigma + \nu (1-s)] \phi_\pi + [\phi_y(1-s) + \sigma] g_T}
$$

and

$$
\pi^N_{T-1} = \frac{[\sigma (\kappa + 1 + \beta) + (\kappa \nu + \beta \phi_y)(1-s)] \pi^N_T + \sigma \kappa s [\nu g_T + (\phi_y - \nu) g_{T-1}]}{\kappa [\sigma + \nu (1-s)] \phi_\pi + [\phi_y(1-s) + \sigma]}
$$

As shown by the dash lines in Figure 17, this “non-pegged” case has the conventional
transmission dynamics in the literature. \(^\text{34}\) In this case, although the fiscal expansion drives up inflation, the nominal interest rate increases by more. Hence, the real interest rate goes up and private consumption falls in initial periods. When the spending reverses, it drives down inflation. Moreover, private consumption jumps due to the positive wealth effect as well as the reduction in the real rate.

![Graphs showing responses of inflation, consumption, and output](image)

**Figure 17:** Responses of inflation, consumption and output in each possible state

Next, I calculate the responses for each pegged interest rate state. If the economy is in a “pegged” state, it will have two possible states in the next period. So the inflation, consumption and output responses are affected by expected future values. Hence, we need to solve for the equilibrium backward using equation 36, 37 and 38. The expected inflation is \( E_t \pi_{t+1} = p \pi^0_{t+1} + (1-p) \pi^N_{t+1} \), and the expected consumption is \( E_t c_{t+1} = p c^0_{t+1} + (1-p) c^N_{t+1} \), where \( x^0_t \) denotes the value in the “pegged” state.

\(^{34}\)See, for example, Section 3.3 in Woodford (2011).
Hence, the responses in the “pegged” states are affected by the future “non-pegged” states. The solid lines in Figure 17 illustrate the dynamics. One remarkable result is that although the initial inflation responses are positive, private consumption still responds negatively. This is mainly due to the influence from the consumption fall in the “non-pegged” case.

The expected paths of inflation, consumption and output can be calculated as:

\[
E_t x = p_t^{t-1} x^0_t + (1 - p_t^{t-1}) x^N_t
\]

where \(E_t x\) denotes the expected value of a variable at time \(t\).

Figure 18: Expected responses of inflation, consumption and output in the stochastic policy experiment

Figure 18 displays the expected responses of inflation, consumption and output in
this stochastic policy experiment. Inflation increases in early periods, because the expected inflation increases in the “non-pegged” case. However, expected private consumption falls in the first four periods when there is a fiscal expansion. Expected output rises by less than the size of fiscal expansion.

Table 14 displays the impact multiplier and the expected cumulative multipliers

\[ E_1 \frac{\Delta Y}{\Delta G} \equiv \frac{1}{s} E_1 \frac{\sum_{t=1}^{T} \Delta y_t}{\sum_{t=1}^{T} \Delta g_t}. \]

I also calculate the impact and cumulative inflation elasticities:

\[ \frac{\Delta \pi_1}{\Delta G_1} \equiv \frac{d\pi_1}{dg_1} \quad \text{and} \quad E_1 \frac{\Delta \pi}{\Delta G} \equiv \frac{E_1 \sum_{t=1}^{T} d\pi_t}{\sum_{t=1}^{T} dg_t}. \]

The output multipliers are below one. Moreover, the inflation elasticity is small or even negative, which is consistent with the empirical finding in Dupor and Li (2014).

<table>
<thead>
<tr>
<th></th>
<th>Output Multiplier</th>
<th>Inflation Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact</td>
<td>0.9402</td>
<td>0.0034</td>
</tr>
<tr>
<td>1-year cumulative</td>
<td>0.9396</td>
<td>0.0008</td>
</tr>
<tr>
<td>2-years cumulative</td>
<td>0.8711</td>
<td>-0.0132</td>
</tr>
</tbody>
</table>

Table 14: Fiscal multipliers under a stochastic interest rate peg

3.5 A model with capital

In previous sections I use a simple model without capital to argue that the government spending multiplier is not necessarily larger than one even if the zero bound binds. Here I show that this basic result extends to a generalized model in which capital accumulation is allowed. As in Section 4, I focus on the case with a stochastic interest rate peg and a deterministic fiscal expansion.

The model is given by the following linearized equilibrium conditions:

\[ i_t - \pi_{t+1} = \sigma(c_{t+1} - c_t) \] (45)
\[ \pi_t = \beta \pi_{t+1} + \kappa [(1 - \alpha)(\nu n_t + \sigma c_t) + \alpha r_t^k] \] 

(46)

\[ r_t^k = \nu n_t + \sigma c_t - k_{t-1} + n_t \]

(47)

\[ -\sigma c_t + \sigma_I (k_t - k_{t-1}) = [1 - \beta (1 - \delta)](-\sigma c_{t+1} + r_{t+1}^k) + \beta (1 - \delta)[-\sigma c_{t+1} + \sigma_I (k_{t+1} - k_t)] \]

(48)

\[ \alpha k_{t-1} + (1 - \alpha)n_t = \frac{C}{Y} c_t + \frac{I}{Y \delta} (k_t - (1 - \delta)k_{t-1}) + \frac{G}{Y} y_t \]

(49)

Capital is denoted by \( k \) with depreciation rate \( \delta \). I use a capital adjustment cost specification used in Christiano et al. (2011), see their equation 40. \( \sigma_I \) and \( \alpha \) are calibrated as 17 and 0.3, respectively. As there exists an endogenous state variable we cannot obtain an analytical solution. Instead, I calculate the expected responses using a numerical method. Similar to the above section, I first calculate the paths of the “non-pegged” cases and then the “pegged” path. Due to the endogenous state variable, there are multiple “non-pegged” paths depending on the “exit” time.

Figure 19 displays the expected responses of inflation, consumption and output in this policy experiment. The responses are very similar to the ones in the model without capital. Inflation increases in the first two quarters and it falls below zero in the following periods. Expected private consumption falls in the first four quarters when there is a fiscal expansion and it rises when the government spending drops. Expected output rises by less than the size of fiscal expansion.

\footnote{The steady state ratios are set as \( \frac{G}{Y} = 0.2, \frac{I}{Y} = 0.29 \) and \( \frac{C}{Y} = 0.51 \). These values are the same as Christiano et al. (2011)’s calibration.}
Table 15 displays the impact multiplier and the expected cumulative output multipliers and inflation elasticities. The output multipliers are below one. The output multipliers are larger than the ones in the model without capital. That is because investment increases due to the increase in the real interest rate caused by the fall in the expected inflation. Again, the inflation elasticity is small or even negative, which is consistent with the empirical finding in Dupor and Li (2014).

<table>
<thead>
<tr>
<th></th>
<th>Output Multiplier</th>
<th>Inflation Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact</td>
<td>0.9794</td>
<td>0.0062</td>
</tr>
<tr>
<td>1-year cumulative</td>
<td>0.9845</td>
<td>-0.0019</td>
</tr>
<tr>
<td>2-years cumulative</td>
<td>0.9944</td>
<td>-0.0443</td>
</tr>
</tbody>
</table>

Table 15: Fiscal multipliers
3.6 Conclusion

This paper examines the sizes of fiscal multipliers under a pegged nominal interest rate. If the monetary-fiscal expansion is associated with an anticipated spending reversal, the output multiplier can be smaller than one. Moreover, if the duration of the interest rate peg is stochastic, the inflation multiplier is in line with the empirical evidence in the literature. The results extend to a generalized model in which capital accumulation is allowed.
References


Jing Han, and Yi-Chan Tsai, “What do technology shocks tell us about the New Keynesian paradigm?,” *Journal of Monetary Economics*, 2009, 56 (4), 560–569.


Appendix A: Technical Appendix for Chapter 1

In this appendix, I discuss what are the effects of these two shocks on marginal costs. In this model, marginal cost equals the wage. We can solve:

\[ \hat{mc}_t = \hat{w}_t = \left[ \sigma \left( \frac{\alpha - g_1}{1 - g_1} \right) + \theta \alpha_1 \frac{Y_1}{Y} \right] \hat{g}_1t + \left[ \frac{\sigma \alpha_2}{1 - g_1} + \theta \left( \frac{\alpha_2 Y_1}{Y} + \frac{Y_2}{Y} \right) \right] \hat{g}_2t \]

A shock to \( G_{1t} \) affects the marginal cost through two channels. First, it increases labor through the second term, \( \theta \alpha_1 \frac{Y_1}{Y} \hat{g}_1t \). The increase of labor demand puts upward pressure on the marginal cost. Second, it changes households consumption through \( \frac{\sigma (\alpha - g_1)}{1 - g_1} \hat{g}_1t \). This is actually the response of consumption to both the negative wealth effect and the New Keynesian mark-up effect. Overall, the negative wealth effect dominates, so consumption decreases and puts downward pressure on the marginal cost.

Similarly, a shock to \( G_{2t} \) affects the marginal cost through the labor and consumption channels as well. In this case, the negative wealth effect again dominates, so consumption drops. However, the sign of the response of equilibrium labor \( \theta (\alpha_2 \frac{Y_1}{Y} + \frac{Y_2}{Y}) \hat{g}_2t \) is ambiguous. The shocks to \( G_{2t} \) asks for more labor demand in the government sector, while it also reallocates some labor from private sector to government sector. The overall effects on total labor depends on the size of \( \alpha_2 \frac{Y_1}{Y} \) and \( \frac{Y_2}{Y} \).
Appendix B: Technical Appendix for Chapter 2

1. Proof of Proposition I

Use the labor-leisure condition (1), approximate $u_c(c, l)$ and $u_l(c, l)$ around $(C, L)$, the aggregate consumption and labor supply, using Taylor expansion. We have:

$$u_l(C, L) + u_{lc}(C, L)(c - C) + u_{ll}(C, L)(l - L) + w[u_c(C, L) + u_{cc}(C, L)(c - C) + u_{cl}(C, L)(l - L)] = 0$$

Integral over households. Since $\int_0^1 c_j dj = C$ and $\int_0^1 l_j dj = L = L_p + L_g$, we have:

$$u_l(C, L) + wu_c(C, L) = 0$$

Take derivative with respect to $G$, we have:

$$\frac{\partial L_p}{\partial G} = [u_{lc}w + u_l - \alpha \frac{w}{L_p} u_c + w(u_{cc}w + u_{cl})]^{-1}(u_{lc} + wu_{cc}) > 0$$

That is, the aggregate private sector labor $L_p(z, G, L_g)$ is strictly increasing in $G$.

Take derivative with respect to $L_g$, we have:

$$\frac{\partial L_p}{\partial L_g} = [u_{lc}w + u_l - \alpha \frac{w}{L_p} u_c + w(u_{cc}w + u_{cl})]^{-1}(-u_{ll} - wu_{cl}) < 0$$

That is, the aggregate private sector labor $L_p(z, G, L_g)$ is strictly decreasing in $L_g$. 

91
In addition, I show how the factor prices change with $G$ and $L_g$.

\[ \frac{\partial w}{\partial G} = -\alpha(1 - \alpha)K^\alpha L_p^{\alpha - 1} \frac{\partial L_p}{\partial G} < 0 \]

\[ \frac{\partial r}{\partial G} = \alpha(1 - \alpha)K^{\alpha - 1}L_p^{-\alpha} \frac{\partial L_p}{\partial G} > 0 \]

\[ \frac{\partial w}{\partial L_g} = -\alpha(1 - \alpha)K^\alpha L_p^{\alpha - 1} \frac{\partial L_p}{\partial L_g} > 0 \]

\[ \frac{\partial r}{\partial L_g} = \alpha(1 - \alpha)K^{\alpha - 1}L_p^{-\alpha} \frac{\partial L_p}{\partial L_g} < 0 \]

So, wage rate decreases (increases) in $G$ ($L_g$) while interest rate increases (decreases) in $G$ ($L_g$).

2. The narrative VAR approach

Following Ramey (2011), I use the defense news variables to identify government spending shocks in the VAR.\(^{36}\) This is an extended version of the narrative approach, because it measures the government spending changes due to exogenous foreign political events and uses more information. The defense news series was constructed by reading periodicals in order to measure the public’s expectations.\(^{37}\) In this approach, the defense news variable is ordered before other variables in the VAR.

The basic empirical specification is:

\[ A(L)Z_t = C + D_1t + D_2t^2 + U_t \] \hspace{1cm} (50)

\(^{36}\)I use quarterly data from 1947 to 2008 for the defense news approach.

\(^{37}\)See Ramey (2011) for details.
where $Z_t$ is a $7 \times 1$ vector of variables. $A(L) = A_0 + A_1 L + ... + A_4 L^4$, where $L$ is the lag operator. The 3 month T-bill rate and the average marginal income tax rate are used to control for monetary and tax policy.\textsuperscript{38}

3. Data

Following Mountford and Uhlig (2009), all the components of national income are in real per capita terms and are transformed from their nominal values by dividing them by the gdp deflator (NIPA table 7.1 Row 4) and the population measure (NIPA table 2.1 Row 35).

GDP: NIPA table 1.1 Row 1.

Private Consumption: NIPA table 1.1 Row 1.

Government Goods Purchases: This is government consumption expenditures taken from line 1 in NIPA table 3.10.5, minus line 4 in NIPA table 3.10.5 which is the compensation of general government employees.

Government Employment: This is from Francis-Ramey Updates on Ramey’s website.

Real Wages: This is 'Nonfarm Business Sector: Real Compensation Per Hour' Series COMPRNFB from the U.S. Department of Labor: Bureau of Labor Statistics.

Private Non-Residential Investment: This is 'Nominal Gross Private Domestic Investment', NIPA table 1.1 Row 6, minus private residential investment, NIPA table 1.1 Row 11.

Interest Rate: The ex post real interest rate is constructed using the federal funds rate and CPI inflation. Both of them are from the Federal Reserve Bank of St. Louis’ database.

\textsuperscript{38}Rossi and Zubairy (2011) emphasize analysis of fiscal policy should always control for monetary policy and vice versa.
Adjust Reserves: This series is taken from the Adjusted Monetary Base at the Federal Reserve Bank of St. Louis’ database.

PPIC: The Producer Price Index of Crude Materials is given by the ppicrm series at the Federal Reserve Bank of St. Louis’ database. PPIC and Adjust Reserves are arithmetically averaged of the monthly figures to get quarterly data.

GDP Deflator: NIPA table 7.1 Row 4.