Examining the Knowledge Domains Used in the Practice of Mathematics Teacher Educating

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Scott Alan Zollinger, B.S., M.A.

Graduate Program in Education: Teaching & Learning

The Ohio State University

2014

Dissertation Committee:

Professor Azita Manouchehri, Advisor

Professor Patti Brosnan

Professor Lin Ding

Professor Aina Appova
Abstract

Efforts to make mathematics accessible for all are driving mathematics education reform movements internationally. Successful educational reform necessitates quality instruction and thus teacher preparation programs and professional development opportunities play a significant role. Teacher educators and the practice of teacher educating lie at the heart of these learning opportunities. However, teacher educators in general, and mathematics teacher educators in particular, are an under-researched professional group that merit careful study.

My study attempted to address this gap conjecturing that with an absence of an understanding of the epistemic nature of mathematics teacher educators’ knowledge, efforts toward meeting reform goals might lead to limited success. I attempted to build a conceptual framework that characterizes mathematical knowledge for teacher educating by studying the work and thinking of three mathematics teacher educators. This study examined knowledge domains that three mathematics teacher educators used or expressed to be used when providing content specific pedagogical experiences for preservice and inservice teachers. Additionally, the study investigated how these knowledge domains influenced the activities of the mathematics teacher educators as they designed and implemented pedagogical experiences for their learners.
A qualitative, multiple case research design was used for this work. Data was collected using in-depth interviews, classroom and professional development session observations, and existing documents prepared and used by the participants when organizing activities around teacher educating. During observations, the intent was to document interactions of the educators and their respective students as they made in-the-moment decisions regarding how to facilitate learning among their teachers. Of particular interest was identifying the knowledge bases they referenced and used during these interactions. Brief pre- and post-observation interviews were conducted in order to elicit plans for the session and participants’ assessment of how their plans may have been enacted. Shulman’s (1986) categories of content knowledge for teaching, the practical knowledge of teachers as described by Elbaz (1983), and mathematical knowledge for teaching described by Ball, Thames, and Phelps (2008) provided the initial framework for coding of data.

Results indicated that pedagogical content knowledge was the primary source of knowledge of the three mathematics teacher educators while subject matter knowledge and curricular knowledge were secondary sources. Additionally, personal experiences as classroom teachers and teacher educators were prominent forces on their decision-making. Mathematics content was not the focus of their sessions but instead was used as a vehicle to address factors that influence the teaching and learning of mathematics. Some of the factors included instructional strategies, gaps in teacher knowledge, elements of sociocultural issues, and activity selection.
Acknowledgments

I would like to thank Dr. Azita Manouchehri and Dr. Patti Brosnan for believing in me from the start of this five-year journey. Without your knowledge, support, encouragement, and trust, none of this would have been possible. Only you truly know what a journey this has been and I am forever indebted to you for all you have done for me. I hope that someday, I also share with my students with the same concern and caring that you have shown me.

I want to thank my family, especially my mother, for all the love, encouragement, and support you provided during a very challenging time in my life. You were always available to help in any and every way possible and I am so thankful and blessed to have such a loving and caring family.

I wish to thank the three participants in my study, Alex, Tracy, and Luke. You gave freely of so much of your time to allow me the opportunity to conduct this research and I cannot thank you enough. I hope that, in some way, this experience was beneficial for you as well.

Finally, I want to thank all the MCP graduate students I have had the opportunity to work with during my time at The Ohio State University. You accepted me without hesitation even though I am considerably older than most of you and I feel truly fortunate to be able to call you colleagues and more importantly, friends.
Vita

1985.................................................................B.S. Petroleum Engineering, Marietta College

1991.................................................................M. A. Mathematics Education, The Ohio State University

1989 – 2008......................................................Mathematics Teacher, Lancaster High School, Lancaster, OH

2009 – 2013......................................................Graduate Research Associate, Mathematics Coaching Program, The Ohio State University

Summer 2102, 2013.................................Co-Instructor, Secondary Mathematics Methods for Preservice Teachers, The Ohio State University

Spring 2013.......................................................Instructor, Middle Childhood Mathematics Methods for Preservice Teachers, The Ohio State University

Fall 2013 – Spring 2104.................................College of Education and Human Ecology Graduate Dissertation Research Fellowship, The Ohio State University
Publications


Fields of Study

Major Field: Education: Teaching & Learning
Table of Contents

Abstract .............................................................................................................................. ii
Acknowledgments .......................................................................................................... iv
Vita ................................................................................................................................. v
List of Tables .................................................................................................................. xiv
List of Figures .................................................................................................................. xv
Chapter 1: Introduction ................................................................................................... 1
  Teacher Education and Teacher Educating: An Emerging Field of Inquiry .......... 3
    Teacher preparation programs ................................................................................... 8
    Professional development programs ......................................................................... 10
Purpose and Research Questions .................................................................................. 13
Framework of the Study ............................................................................................... 14
  Global framework ......................................................................................................... 14
  Initial analytical framework ........................................................................................ 15
Overview of the Methodology ..................................................................................... 17
Significance of the Study .............................................................................................. 18
Chapter 2: Literature Review and Theoretical Framework .......................................... 20
Research on Teacher Educating .................................................................................... 20
  Training and development of teacher educators ....................................................... 21
  Conceptualization of teacher educating ..................................................................... 24
  Teacher educators in practice ....................................................................................... 33
Teacher Educators and Teacher Educating ................................................................ 43
Theoretical grounding: Types of Teacher Knowledge .................................................. 45
  Shulman’s perspective ................................................................................................ 45
  Subject matter knowledge .......................................................................................... 49
  Pedagogical content knowledge ................................................................................ 53
  Mathematical knowledge for teaching ...................................................................... 58
Practical knowledge of self and surroundings ............................................ 254
Experiential and situational orientations .................................................. 255
Luke’s practical principle ........................................................................... 257
Summary ...................................................................................................... 258
Mathematics Methods Course Observations .............................................. 259
First Classroom Observation: Solving Algebraic Equations ...................... 260
  Code frequency data ................................................................................ 263
  Children’s mathematical thinking and teacher practices ....................... 263
  Pedagogical content knowledge and knowledge of instruction ............ 266
  Subject matter knowledge domains ....................................................... 269
  Knowledge of surroundings and a situational orientation .................... 271
  Knowledge of self and an experiential orientation ................................ 272
  Luke’s practical principle ........................................................................ 274
  Summary .................................................................................................. 275
Second Classroom Observation: Electronic Portfolio .............................. 276
  Code frequency data ................................................................................ 277
  Teacher practices and teacher learning ............................................... 279
  Pedagogical content knowledge and knowledge of instruction ............ 282
  Knowledge of surroundings and situational orientation ..................... 284
  Knowledge of self and experiential orientation ................................... 286
  Luke’s practical principle ........................................................................ 288
  Summary .................................................................................................. 289
Professional Development Observation: Three-Peg Puzzle ..................... 290
  Code frequency data ................................................................................ 292
  Teacher practices, teacher learning, and reflective practice ............... 295
  Pedagogical content knowledge and specialized content knowledge .... 297
  Knowledge of self and experiential orientation ................................... 300
  Luke’s practical principle ........................................................................ 301
  Summary .................................................................................................. 302
Knowledge Domain Frequency Totals ....................................................... 303
  Big idea themes ....................................................................................... 305
  Shulman’s categories of content knowledge ........................................ 306

xii
<table>
<thead>
<tr>
<th>Mathematical knowledge for teaching</th>
<th>309</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practical knowledge</td>
<td>310</td>
</tr>
<tr>
<td>Luke Case Profile Summary</td>
<td>314</td>
</tr>
<tr>
<td>Chapter 7: Knowledge Domains for Mathematics Teacher Educating</td>
<td>322</td>
</tr>
<tr>
<td>Cross-Case Analysis of Data of Three MTEs</td>
<td>323</td>
</tr>
<tr>
<td>Code frequency data</td>
<td>324</td>
</tr>
<tr>
<td>Big idea themes</td>
<td>326</td>
</tr>
<tr>
<td>Shulman's categories of content knowledge</td>
<td>327</td>
</tr>
<tr>
<td>Mathematical knowledge for teaching</td>
<td>329</td>
</tr>
<tr>
<td>Practical knowledge</td>
<td>331</td>
</tr>
<tr>
<td>Three MTEs knowledge domain summary</td>
<td>335</td>
</tr>
<tr>
<td>Unpacking the Knowledge Domains of Three MTEs</td>
<td>342</td>
</tr>
<tr>
<td>What is “content” for the MTEs?</td>
<td>343</td>
</tr>
<tr>
<td>Framework for discussion</td>
<td>344</td>
</tr>
<tr>
<td>Pedagogical content knowledge</td>
<td>345</td>
</tr>
<tr>
<td>Subject matter knowledge</td>
<td>355</td>
</tr>
<tr>
<td>Curricular knowledge</td>
<td>359</td>
</tr>
<tr>
<td>Practical knowledge</td>
<td>362</td>
</tr>
<tr>
<td>Summary</td>
<td>367</td>
</tr>
<tr>
<td>A Model to Study Mathematical Knowledge for Teacher Educating</td>
<td>368</td>
</tr>
<tr>
<td>The teacher educator knowledge tetrahedron</td>
<td>369</td>
</tr>
<tr>
<td>The teaching triad of mathematics teacher educators</td>
<td>371</td>
</tr>
<tr>
<td>The MTEs knowledge domains and the knowledge tetrahedron</td>
<td>374</td>
</tr>
<tr>
<td>The MTEs’ actions of teaching and the teaching triad</td>
<td>381</td>
</tr>
<tr>
<td>Chapter 8: Discussion and Implications</td>
<td>386</td>
</tr>
<tr>
<td>Knowledge Domains of MKTE</td>
<td>387</td>
</tr>
<tr>
<td>Learners and Content</td>
<td>389</td>
</tr>
<tr>
<td>Instructional Practices</td>
<td>390</td>
</tr>
<tr>
<td>Conceptualizing a Working Model for Teacher Educating Knowledge</td>
<td>391</td>
</tr>
<tr>
<td>Modeling Mathematics Teacher Educators’ Practice: A Disciplinary Framework</td>
<td>396</td>
</tr>
<tr>
<td>Levels of concern for MTEs</td>
<td>396</td>
</tr>
<tr>
<td>Levels of knowledge for MTEs</td>
<td>401</td>
</tr>
</tbody>
</table>
List of Tables

Table 1. Data Collection Summary – Alex ................................................................. 81
Table 2. Data Collection Summary – Tracy ............................................................... 82
Table 3. Data Collection Summary – Luke ............................................................... 83
Table 4. Codes and Indicators for Shulman’s Categories of Content Knowledge .......... 87
Table 5. Codes and Indicators for Categories of Practical Knowledge ....................... 88
Table 6. Codes and Indicators for Mathematical Knowledge for Teaching ................ 89
Table 7. Example of Code Frequency Totals............................................................ 92
Table 8. Data Collection Summary – Alex ............................................................... 101
Table 9. Initial Interview Code Frequency – Alex .................................................... 107
Table 10. First Classroom Observation Activity Sequence – Alex ............................ 116
Table 11. First Classroom Observation Code Frequency – Alex ............................... 118
Table 12. Second Classroom Observation Activity Sequence – Alex ....................... 128
Table 13. Second Classroom Observation Code Frequency – Alex ........................... 129
Table 14. Professional Development Observation Code Frequency – Alex ............... 142
Table 15. Data Collection Summary – Tracy ........................................................... 167
Table 16. Initial Interview Code Frequency – Tracy ................................................. 171
Table 17. First Classroom Observation Activity Sequence – Tracy ......................... 182
Table 18. First Classroom Observation Code Frequency – Tracy .............................. 185
Table 19. Second Classroom Observation Activity Sequence – Tracy ....................... 200
Table 20. Second Classroom Observation Code Frequency – Tracy .......................... 202
Table 21. Professional Development Observation Activity Sequence – Tracy .......... 213
Table 22. Professional Development Observation Code Frequency – Tracy .......... 215
Table 23. Data Collection Summary – Luke ............................................................. 243
Table 24. Initial Interview Code Frequency – Luke ................................................. 247
Table 25. First Classroom Observation Activity Sequence - Luke ............................ 262
Table 26. First Classroom Observation Code Frequency – Luke .............................. 264
Table 27. Second Classroom Observation Activity Sequence – Luke ....................... 278
Table 28. Second Classroom Observation Code Frequency – Luke .......................... 280
Table 29. Professional Development Observation Activity Sequence – Luke .......... 293
Table 30. Professional Development Observation Code Frequency – Luke ............ 294
List of Figures

Figure 1. The teaching triad. Jaworski (1992, p. 8) ................................................................. 25
Figure 2. Teaching triad of mathematics teacher educators. Zaslavsky and Leikin (2004, p. 8) ................................................................. 26
Figure 3. Teacher knowledge tetrahedron. Perks and Preststage (2008, p. 270) ........... 31
Figure 4. Teacher educator knowledge tetrahedron. Perks and Preststage (2008, p. 271) 32
Figure 5. Synthesis of models on teacher mathematical knowledge. Petrou & Goulding (2011, p. 21) .................................................................................................................. 47
Figure 6. Example of frequency totals for mathematical knowledge for teaching .... 93
Figure 7. Example of the frequency totals for the components of the analytical framework ................................................................. 94
Figure 8. Example of codes expressed as percentage of KDS .................. 95
Figure 9. Total code frequency based on the framework of analysis – Alex ........ 149
Figure 10. Big idea theme frequency – Alex ................................................................. 150
Figure 11. Big idea themes as percentage of KDS – Alex ........................................ 151
Figure 12. Shulman's categories of content knowledge frequency – Alex ........ 152
Figure 13. Shulman's categories of content knowledge as a percentage of KDS – Alex ................................................................. 153
Figure 14. Mathematical knowledge for teaching frequency – Alex ............. 154
Figure 15. Mathematical knowledge for teaching categories as a percentage of KDS – Alex .................................................................................................................. 155
Figure 16. Content of practical knowledge frequency – Alex .................. 156
Figure 17. Content of practical knowledge as percentage of KDS – Alex .... 157
Figure 18. Orientations of practical knowledge frequency – Alex .............. 158
Figure 19. Orientations of practical knowledge as percentage of KDS – Alex .... 159
Figure 20. Case profile summary of knowledge domains utilized by Alex .......... 160
Figure 21. Total code frequency based on the framework of analysis – Tracy ........ 224
Figure 22. Big idea theme frequency – Tracy ................................................................. 225
Figure 23. Big idea themes as percentage of KDS – Tracy ......................... 226
Figure 24. Shulman's categories of content knowledge frequency – Tracy .... 227
Figure 25. Shulman's categories of content knowledge as a percentage of KDS – Tracy ................................................................. 228
Figure 26. Mathematical knowledge for teaching frequency – Tracy ........ 229
Figure 27. Mathematical knowledge for teaching categories as a percentage of KDS – Tracy .................................................................................................................. 230
Figure 28. Content of practical knowledge frequency – Tracy .......... 232
Chapter 1: Introduction

Efforts to make mathematics accessible for all guide the mathematics education reform movements in many countries (Adler, Ball, Krainer, Lin, & Novotna, 2005). Increasingly, mathematics is seen as a necessity for critical citizenship and thus the need for quality teaching at all levels of schooling is evident. However, variables such as class sizes, material resource bases, range of learners, languages, and mathematical competencies make it difficult to define quality teaching in different contexts (Adler et al., 2005). Regardless of the variables involved, there is consensus that quality instruction depends on teachers’ knowledge and skills. Thus the preparation programs and professional development opportunities designed for them play a significant role in making the goals of mathematics education reform successful (Adler et al., 2005).

Hiebert, Morris and Glass (2003) asserted that if mathematics teaching and student learning showed signs of improvement, the concerns about the effectiveness of teacher preparation programs and professional development would be less urgent. However, they claim that the average classroom in the United States displays the same methods of teaching mathematics today as in the past and students continue to be deficient with respect to the competencies required to understand mathematics deeply and use it effectively (Hiebert, Morris, & Glass, 2003). Sowder (2007, p. 157) proposed “the key to increasing students' mathematical knowledge and closing the achievement gap is to put knowledgeable teachers in every classroom.” She further claims that awareness of the
need to change mathematics teaching and learning is international in scope but efforts to make this happen – especially with respect to professional development – often does not meet the teachers' needs.

For the goal of mathematical proficiency for all to be achieved, teachers need support which places significant, and often underappreciated, demands on teacher educators (Adler et al., 2005). Watson and Mason (2007, p. 208) suggest one key idea to keep in mind when considering teacher education; "… for teachers, learning and action are one and the same: their professional choices of actions are the manifestations of what they have learned or are learning." According to Adler et al. (2005), differences between teacher educators and both perspective and practicing teachers are increasing as these learners carry diverse mathematical histories. As a result, the focus of the field of mathematics education research has shifted from the curriculum to the learners to the teachers (Adler et al., 2005), and only recently, to teacher educators.

Research in teacher education is divided into two areas; preparation programs for prospective teachers and professional development programs for practicing teachers. Each area presents variables and challenges that may or may not be unique to that specific type of teacher education, but in most cases, preparation programs and professional development are addressed separately in mathematics education research. However, according to Watson and Mason (2007), teacher education programs and professional development projects throughout the world use similar methods of working with both prospective and practicing teachers. Most include prompting teachers to: engage in mathematical thinking, reflect on the experience of doing mathematics-related tasks oneself, or cooperatively; develop and try out frameworks for both pedagogy and
didactics; consider implications for teaching; observe and analyze teaching; experience opportunities to observe and listen to learners; challenge procedure-dominated approaches; challenge their memories of classroom experiences; support them in appreciating and making use of theoretical constructs (Watson & Mason, 2007, p. 207).

In addressing some of the challenges that teacher education programs face, Watson and Mason (2007) suggest that because teachers are actively learning beings, teacher educators need to design opportunities for them to "make sense actively of the tasks of teaching" (p. 210). They continue in claiming that unless teaching and learning can become real – referring to a person’s inner state and not necessarily to actions in the world – teachers have a difficult time making sense of them. Additionally, Watson and Mason, (2007) propose that often teacher education sessions do not occur within the relevant environment and the activities are not necessarily activities of teaching. How can teacher educators and the practice of teacher educating address these issues in an effort to move teacher education and thus mathematics education reform forward? Resolving this issue is not an easy task.

**Teacher Education and Teacher Educating: An Emerging Field of Inquiry**

Sztajin (2011), in her discussion of research reporting standards for mathematics professional development, asserts that within the last two decades, mathematics teacher education has become an emerging field of study. However, little is known about how teachers acquire, or fail to acquire, the mathematical knowledge necessary for teaching (Adler et al., 2005). Longitudinal studies are lacking and since teacher knowledge develops over time, understanding of how teachers learn and under what conditions is minimal. Additionally, there is an area absent from the literature pertaining to how
teachers learn from experiences and what type of support may facilitate their learning. It is also unclear how different approaches to helping teachers learn mathematics compare with each other (Adler et al., 2005). Based on these deficiencies, Adler et al. (2005) suggest directions for the field that include more action research where teachers investigate their practice and teacher educators investigate their support processes, development of better "local" – geographic, topic-specific, etc. – theories of teacher learning, and a more complete understanding of what it means to teach both mathematics and teaching simultaneously.

Sztajin (2011) reviewed seven articles from the *Journal of Research in Mathematics Education* with respect to professional development and found that all research reports contained information about mathematical concepts addressed, student thinking, or curriculum materials. Additionally, they included information about participants, statements about the goals of the mathematics professional development (MPD), and discussion of the need to connect professional development to teachers' practices. However, a clearly stated theory or model of adult learning was present in only one of the seven articles.

The lack of explicit attention to learning theories as they relate to teacher learning was surprising and deserves the attention of the field…This theoretical framework for the research refers to both teacher and student learning: the former shapes decisions made in developing a plan for the MPD, and the latter shapes decisions made in selecting the content of the MPD. Theories of teacher and student learning should be aligned and the latter should take into account issues that are particular to adult learners. Sztajin (2011, p. 228)
To further support this view, in a comparison of three lists of elements of successful professional development, each constructed from separate reviews of literature, only one made any mention of adult learning theories or models (Sowder, 2007, p. 170). Since all teacher education programs, whether designed for preservice or in-service teachers, focus on adult learners, consideration of adult learning theories or models seems necessary.

Speer and Star (n.d.) further argued that most studies of knowledge, beliefs, and practices have focused on elementary teachers to the extent that much less is known about the knowledge and beliefs of other populations of teachers. Moreover, relating teacher education to college teaching, Speer and Star argue that since TAs are responsible for a significant amount of undergraduate instruction and future faculty develop from this population of graduate students, this issue needs to be addressed with respect to teacher education. The research base to help TAs develop as effective teachers is very limited, and with respect to collegiate mathematics education, has not progressed in either quality or quantity (Speer, Smith, & Horvath, 2010; Speer & Star, n.d.). Additionally, the field of collegiate mathematics education research remains isolated from that at the K-12 level and thus has failed to benefit from their findings with regard to teaching and learning (Speer et al., 2010; Speer & Star, n.d.). Speer and Star (n.d., p. 2) have proposed research that examines the knowledge and beliefs of TAs and how their teaching practices are shaped accordingly. This same issue could be addressed with mathematics education faculty in regard to their work with prospective teachers and practicing teachers.

Other scholars support Speer and Star’s claim by asserting that formal programs that provide adequate training for mathematics teacher educators are few and research on becoming a mathematics teacher educator is very limited (Zaslavsky & Leikin, 2004).
Teacher educators lack a shared knowledge base from which to develop effective teacher preparation programs (Hiebert et al., 2003). Chauvet (2008, p. 83) posited: "There is limited research about the professional development of teacher educators and even less research specific to mathematics teacher educators.” Zaslavsky and Leikin (2004) conducted research relative to this issue by examining the process of becoming a mathematics teacher educator within a professional development program for secondary mathematics teachers. They analyzed the development of mathematics teacher educators through their practice, viewing the participants and educators as a community of practice. As part of their framework, they discussed the adult learning theory of reflective practice asserting that notions of reflection-on-action and reflection-in-action contribute to teacher knowledge development with regard to their practice. Goals of their study focused on what Zaslavsky and Leikin (2004) consider essential components for teacher education programs: 1) developing both mathematical and pedagogical teacher knowledge using methods that support a constructivist perspective of teaching; 2) providing opportunities for teachers to reflect on learning and teaching experiences; and 3) enhancing the socialization of teachers and teacher educators while developing a supportive professional community. One of the primary responsibilities of teacher educators is to provide opportunities for the participants to learn challenging mathematics similar to how they are expected to teach (Zaslavsky & Leikin, 2004).

Results of their study indicated that the process of designing these mathematical activities for teachers was a key for the professional growth of the mathematics teacher educators encouraging them to use appropriate mathematical knowledge, remain sensitive
to mathematics teachers as learners, and use innovative approaches for mathematics teacher knowledge development (Zaslavsky & Leikin, 2004).

In related research, Watson and Mason (2007) examined the use of mathematics-related tasks with teachers and the influence on their pedagogy together with how educators develop their own knowledge in the design of these tasks. The researchers described "activity" as actions and interactions of the learner with regard to other learners, other resources, and the teacher. They assert that the presentation, development, and the process of drawing to a conclusion are important in the development of the participants' beliefs about and attitudes towards mathematical learning and teaching (Watson & Mason, 2007) by encouraging reflection, discussion, and group work as methods to engage them in tasks. The authors warn however that teachers sometimes resist complexity and challenge and therefore tend to adapt the task to make it easier. Additionally, they offer that examples or example creation tasks that cause a disturbance in the teachers' current knowledge are more effective than those that do not. While engaging learners in an activity is necessary, learning from the activity requires transformation of their mathematical knowledge. For example, transformations may occur with respect to their ability to use mathematical procedures, representations, and formats or in their understanding of connections between mathematical topics (Watson & Mason, 2007). In doing so, Watson and Mason (2007) note the importance of situating teachers’ cognition in practice suggesting that teachers make sense of mathematics based on previous experience that is influenced by the institutional environment and culture of their school and classroom. Additionally, they stress the importance of listening to teachers as learners. In determining the focus, teacher educators must decide whether to
focus on key mathematical ideas, gaps in teachers' knowledge, or types of mathematical tasks that can be applied to a range of content (Watson & Mason, 2007).

**Teacher preparation programs.** Hiebert, Morris, and Glass (2003) argue that it is more realistic for teacher educators to help prospective teachers learn how to learn to teach mathematics effectively as opposed to focusing on the finished competencies of effective teaching. Based on this perspective they suggest two goals for teacher preparation programs. The first goal is to help teachers become mathematically proficient within five kinds of mathematical competencies: 1) conceptual understanding; 2) procedural fluency; 3) strategic competence – the ability to formulate, represent, and solve mathematical problems; 4) adaptive reasoning – capacity for logical thought, reflection, explanation, and justification; 5) productive disposition – seeing mathematics as sensible, useful, and worthwhile, together with the belief in one's own ability to be successful in mathematics (Hiebert et al., 2003, p. 20).

The second goal then becomes to prepare teachers to learn to teach for the same mathematical proficiency. Due to the relative brevity of teacher preparation programs and the complexity of teaching, prospective teachers are rarely, if ever, able to acquire the necessary knowledge to immediately become "highly qualified" for their first job. For this reason Hiebert et al., (2003) propose that the best possible environment for learning to teach is school classrooms and teacher preparation programs should help teachers to develop thinking skills and dispositions necessary to learn from their classroom experiences. Towards this end, they suggest course experiences may involve activities such as case studies and student interviews that present problematic teaching situations for which prospective teachers need to consider subject matter, student
thinking, and teacher-student interactions. "Preparing to learn to teach means knowing how to learn from classroom teaching experiences" (Hiebert et al., 2003, p. 206). Goals for teacher educators should be to learn to teach prospective teachers using methods that help them achieve their learning goals while at the same time generating a shared knowledge base for teacher education, (Hiebert et al., 2003).

The National Research Council (NRC) report, *Educating Teachers of Science, Mathematics, and Technology*, published in 2001 (as cited in Sowder, 2007, p. 200) discusses several recommendations for how mathematics teacher preparation programs might be improved. According to NRC, programs should be collaborative in nature, helping prospective teachers understand the fundamental content and concepts of mathematics while establishing connections among content courses, methods courses, and field experiences. Content should be taught using methods of inquiry and problem solving in ways to help develop an appreciation for the applications of mathematics. Prospective teachers should be provided opportunities to experience teaching in a variety of school contexts with diverse groups of children and be encouraged to use the process of reflective inquiry in their teaching (p. 200). Additionally, Sowder (2007) suggests prospective teachers need opportunities to reason in and about the practice of teaching as well as to observe and listen to students. She further asserts that the study of children's thinking is a necessary component of teacher preparation and notes that cognitively guided instruction offers a promising venue for raising teacher knowledge. Supporting Hiebert et al’s analysis, Sowder (2007) offers that methods courses should help prospective teachers develop "habits of mind" so that they can learn from their classroom experiences (p. 202).
Professional development programs. Previous discussion in this paper referred to studies that used different methods in professional development programs for practicing teachers. This discussion briefly examines types of knowledge addressed through professional development as well as different theories for how that knowledge is developed in teacher educating. Sowder (2007, p. 173) discusses three types of knowledge to be considered in professional development: 1) knowledge-for-practice; 2) knowledge-in-practice; and 3) knowledge-of-practice. Knowledge-for-practice is knowledge acquired by learning from formal professional development programs and university coursework. Knowledge-in-practice is knowledge acquired when teachers deliberate on their practice by consciously reflecting on events that occur in their classroom to the extent that transformation in perspectives may occur. Knowledge-of-practice is knowledge acquired when teachers use their own classrooms to investigate teaching and learning (Sowder, 2007).

With respect to teacher educating, Sowder (2007) discusses four approaches used in helping teachers acquire knowledge-for-practice. The first approach is to focus on student thinking such as in a program based on cognitively guided instruction (CGI). In a CGI program, teacher educators provide participants with a research-based analysis of a specific content domain, give teachers an opportunity to use this knowledge in their classrooms, and then provide them with a chance to reflect on what happens as a result of using the knowledge (Carpenter & Fennema, 1992). As teachers attempt to understand children's strategies, they are forced to examine their own understanding of these problems from a new perspective. This knowledge is used in the classroom to help teachers generate ideas about what their students know and provide direction for
instruction (Carpenter, Fennema, & Franke, 1996; Franke & Kazemi, 2001). The second approach is to focus on curriculum which can mean investigating student curriculum materials or possibly a set of materials designed to provide teachers with opportunities to explore new ways of learning (Sowder, 2007). Research suggests this approach can be beneficial for both preservice and in-service teachers as school curriculum can serve as a catalyst for thinking about mathematics content and students' mathematical thinking (Sowder, 2007).

The third approach focuses on classroom activities and artifacts whereby teachers may be asked to work collaboratively on a problem that could also be used in the classroom with their students (Sowder, 2007). As they work on the problem, they discuss and reflect on their own thinking and thinking of those with whom they work. The teachers think about and discuss how students may approach the same problem. For additional discussion purposes, they might also be asked to bring student work relative to the same or a similar problem to future professional development sessions. This approach could also include studying written cases of classroom events as well as analyzing video cases. Case studies allow teachers to talk about instruction without feeling threatened while preserving the complexity of teaching and providing opportunities for examination (Sowder, 2007). Discussion can focus on content and/or pedagogy with the lessons learned transferable to the classroom. The fourth approach utilizes formal coursework focused on the knowledge needed for effective teaching. Formal coursework can include Master’s degree programs, certificate programs, or courses focused on specific mathematical topics. Incentives for formal coursework
include university credit, continuing education credit, salary increase, and structured learning experiences (Sowder, 2007).

Sowder (2007) discusses six goals of professional development and thus goals of teacher educators and the practice of teacher educating: 1) a shared vision for mathematics teaching and learning; 2) a sound understanding of mathematics for the level taught; 3) an understanding of how students learn mathematics; 4) deep pedagogical content knowledge; 5) an understanding of the role of equity in school mathematics; and 6) a sense of self as a mathematics teacher (Sowder, 2007, p. 161). In summary, she claims professional development must provide opportunities for teacher growth as well as motivation to develop the knowledge and skills to effectively teach mathematics, altering teachers' understanding of mathematical knowledge so to effectively teach mathematics (Sowder, 2007).

According to Chauvot (2008), mathematics teacher educators represent a diverse group of professionals working in different but often overlapping fields as they perform multiple roles. They are university-based or school-based educators – sometimes both – who teach prospective teachers, conduct professional development for practicing teachers, contribute to research in education, and mentor future teacher educators and researchers (Chauvot, 2008). Adler et al. (2005) assert that they have the dual role of improving and investigating mathematics teacher education. Mathematics teacher educators are university faculty, adjunct instructors, and graduate students whose responsibilities include teaching courses for perspective and practicing teachers as well as providing supervision for student teaching experiences. Additionally, they are school leaders, department leaders, and mathematics curriculum supervisors who provide
support for the teachers in their school system (Chauvot, 2008). Results of the work of Chauvot (2008) suggest that collaboration and reflective analysis are key elements for the growth of mathematics teacher educators. Although the work of mathematics teacher educators is conceptually different from that of mathematics teachers the two professions can still inform one another. Chauvot (2008) asserts that as research about the professional development of mathematics teacher educators continues to progress, what is learned can be applied to develop a better understanding of the professional development of mathematics teachers. Likewise, what is known about the mathematical knowledge necessary for teaching mathematics at the K-12 level seems to have application for teacher educators and the practice of teacher educating.

Although there appears to be an agreement on the type of knowledge and dispositions teacher education programs must support, it is unclear what teacher educators may need to know to develop such learning opportunities for teachers. Moreover, studies that examine teacher educators practices and sources that contribute to the choices they make are absent from the literature.

**Purpose and Research Questions**

The purpose of this study was to contribute to research on mathematics teacher educators and the practice of teacher educating – areas of research currently lacking from the literature – by examining the knowledge domains from which teacher educators extract information when providing content specific activities for preservice and inservice teachers. In particular, this study examines ways these knowledge domains direct the activities of teacher educators as revealed in their interactions with learners. The following research questions guided this work.
1) What knowledge domains do mathematics teacher educators draw from and use when providing content specific pedagogical experiences for preservice and inservice teachers?

2) How do these knowledge domains influence the activities of mathematics teacher educators as they design and implement pedagogical experiences for preservice and inservice teachers?

3) How can these knowledge domains be incorporated into the developmental stages of a model to conceptualize mathematical knowledge for teacher educating?

**Framework of the Study**

The overarching framework for this study was one of situated learning as described by Brown, Collins, and Duguid (1989), Lave and Wenger (1991), and Mousley (2003). Within this umbrella, the analytical framework consisted of a combination of three additional components; Shulman’s (1986) categories of content knowledge, Elbaz’s (1983) practical knowledge of teachers, and mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008).

**Global framework.** Lave and Wenger (1991, p. 11) state "The situated nature of learning, remembering, and understanding is a central fact." They claim however, theories of knowledge representation and educational practice often do not address this issue. With respect to mathematics teacher educating, teacher development and use of mathematical concepts are mediated by social situations (Mousley, 2003). Teacher educators work with learners that bring a variety of knowledge and experiences to the table. Inservice teachers possess any number of classroom experiences from which to draw knowledge while preservice teachers have little or no classroom experience in their
background. Practicing teachers come to professional development programs and/or college-level courses from a variety of school settings with varying educational and professional backgrounds and experiences. Prospective mathematics teachers, especially those enrolled in Master’s programs, often arrive with different disciplinary foci in their undergraduate programs. Additionally, the teacher educators themselves rely on a wide variety of experiences that influence their perspectives on teacher education. Given that so many influential factors affect the practice of teacher educating, situated learning seems an appropriate overarching framework for this study.

**Initial analytical framework.** Within the global framework of situated learning, the initial analytical framework comprised Lee Shulman's (1986) categories of content knowledge, Freema Elbaz's (1983) categories of practical knowledge of teachers, and mathematical knowledge for teaching as described by Ball, Thames, & Phelps (2008). Shulman (1986) in his discussion of what he termed the “missing paradigm” (p. 7) or a blind spot with respect to content that was prevalent in most research on teaching as well as programs of teacher evaluation and certification, proposes a theoretical framework that consists of three categories of content knowledge: 1) subject matter content knowledge; 2) pedagogical content knowledge; and 3) curriculum knowledge. Shulman (1986) describes content knowledge as the amount of knowledge and how it is organized in the mind of the teacher. He defines pedagogical content knowledge as “subject matter knowledge for teaching…the ways of representing and formulating the subject that make it comprehensible to others” (Shulman, 1986, p. 9). Shulman (1986) identifies curriculum knowledge as knowledge of the programs designed for teaching subjects and topics, awareness of the instructional materials available to teach the
programs, and the ability to decide when and when not to use specific materials for instruction. Shulman suggests that teachers need these types of content knowledge to effectively teach in their subject areas and thus it seems reasonable to assume that teacher educators would draw from these types of knowledge in their practice of working with teachers. For this study, Shulman's categories of content knowledge provided one component of the analytical framework not specific to mathematics.

Elbaz (1983) refers to practical knowledge as all types of teacher knowledge "as integrated by the individual teacher in terms of personal values and beliefs and as oriented to her practical situation" (Elbaz, 1983, p. 5). Results from her research led to the categorization of practical knowledge into three major aspects: 1) content of practical knowledge; 2) orientations of practical knowledge; and 3) structure of practical knowledge. Within the aspect of content of practical knowledge, she discusses knowledge of subject matter, curriculum, instruction, self, and the environment of schooling. She also identifies five orientations of practical knowledge including situational, theoretical, personal, social, and experiential. Additionally Elbaz (1981) proposes three terms that she thought reflected “varying degrees of generality in teachers’ knowledge” (p. 49). The terms are 1) rule of practice; 2) practical principle; and 3) image. Each category is further explained in the theoretical framework section of the literature review. For the purpose of this study, Elbaz’s categories of practical knowledge of teachers served as the second component of the analytical framework not specific to mathematics.

Mathematical knowledge for teaching (Ball et al., 2008) formed the third component of the analytical framework and the one component that was specific to
Mathematics. Mathematical knowledge for teaching (MKT) is described as mathematical knowledge needed to effectively perform the job of teaching mathematics. Ball and her colleagues hypothesized that Shulman’s content knowledge could be subdivided into common content knowledge and specialized content knowledge and that his PCK could be divided into knowledge of content and students (KCS) and knowledge of content and teaching (KCT) (Ball et al., 2008). Their divisions of Shulman’s categories resulted in six domains of mathematical knowledge for teaching including 1) common content knowledge, 2) specialized content knowledge, 3) horizon knowledge, 4) knowledge of content and students, 5) knowledge of content and teaching, and 6) knowledge of content and curriculum (Ball et al., 2008). The first three domains are considered to be subject matter knowledge while the latter three domains are pedagogical content knowledge.

**Overview of the Methodology**

A qualitative, multiple case study was conducted to capture the knowledge domains used or expressed to be used by three mathematics teacher educators. Purposive and convenience sampling were used to select the participants. Data was collected using an in-depth initial interview, observations of classroom and professional development sessions, pre- and post-observation interviews, and artifacts of the teacher educators’ practices.

Data analysis consisted of three levels. The goal of the first level of analysis was to build a "big picture" view of the teacher educators’ practice based on themes obtained from a review of abstracts of research published from 2008 to 2012. The second level of analysis included the initial coding of data based on the components of the analytical framework with the intent of providing a more detailed analysis of the participants’
decision-making process and sources contributing to their institution. Once data was
coded, frequencies of codes were determined relative to the knowledge domains used or
expressed to be used by the participants. The frequency data was then used in the
development of individual case profiles that described the knowledge domain focus of
their practices and the factors influencing the decisions they made as they interacted with
their students. The final level of analysis consisted of a cross-case comparison of all
participants to generate a list of knowledge domains expressed to be used, or used in
practice by teacher educators. This research design was intended to identify knowledge
bases from which teacher educators drew, determine how these knowledge domains
influenced their interactions with learners, and provide information for the developmental
stages of a model for conceptualizing the practice of teacher educating.

**Significance of the Study**

Research relative to the practice of mathematics teacher educators has been
limited because most previous studies focused on the learning of teachers and not the
practice of mathematics teacher educators (Even, 2014). This study addressed that issue
as the focus was on the knowledge domains used or expressed to be used by the
mathematics teacher educators during their practice of teacher educating. Additionally,
empirical research relative to the practice of mathematics teacher educators mostly
consisted of self-reports of teacher educators on their own work (Even, 2014). This issue
was also addressed as I conducted my research as an outside observer in the natural
environments of the mathematics teacher educators’ practices. While it is presently
unclear what teacher educators may need to know to develop effective learning
opportunities for teachers, this research provided a global view of the knowledge
domains that were utilized, and how they were utilized, by mathematics teacher educators relative to their overall practice. This study also suggested a combination of teacher and teacher educator knowledge models that could be used in concert to further examine the practice of mathematics teacher educating.
Chapter 2: Literature Review and Theoretical Framework

The purpose of this research was to examine the sources of knowledge from which three mathematics teacher educators drew when organizing and implementing learning experiences for preservice teachers and inservice teachers. This chapter offers first an overview of perspectives pertaining to the nature, content and structure of knowledge of mathematics teacher educators as identified in the existing literature. Findings of the published studies on the practice of teacher educating and existing theoretical frameworks concerning mathematical knowledge for teaching are discussed. An explanation of the theoretical framework guiding the current study is then offered.

Research on Teacher Educating

For the purpose of reviewing the literature that is available with respect to teacher educators and the practice of teacher educating, an exhaustive review of published research, published conference proceedings, edited books, and dissertation abstracts led to identification of 14 reports. Collectively, these reports pertained to three major themes including: 1) the training and development of teacher educators; 2) the conceptualization of teacher educating; and 3) teacher educators in practice. These are reviewed in the following sections. The goal was to determine how teacher educators develop the knowledge and skills necessary to educate prospective and practicing teachers, how teacher educators view and/or conceptualize the practice of teacher educating, and how teacher educators approach the in-practice aspects of their profession.
Training and development of teacher educators. In an overview of the current difficulties of mathematics teacher education, Simon (2008) states the following: The challenge for mathematics teacher education is significant. The conceptual and cultural shift needed for traditionally educated teachers to participate effectively in envisioned reforms is profound and requires effective and sustained interventions… What do mathematics teacher educators need to know and who is prepared to conduct effective teacher education in line with reform goals? (p. 17)

Reform efforts have focused on changing instructional practices without making explicit the knowledge about teaching, learning, and mathematics. There is a lack of specificity of principles of teaching resulting in vague goals of mathematics teacher education (Simon, 2008). Additionally, literature describing important pedagogical concepts that might be promoted in teacher education is almost nonexistent. In directing his attention specifically to mathematics teacher educators, Simon (2008) asserts the following:

The inadequate knowledgebase is a serious impediment in the development of mathematics teacher educators. In the US, teacher educators are being prepared in doctoral programs without the conceptual frameworks that they require in order to work with perspective and practicing teachers. (p. 26)

Further, he generalizes that mathematics teacher education is more challenging than mathematics education because the former encompasses all of the latter and thus research in mathematics education is more challenging as well (Simon, 2008).

Even (2008) considered the education of mathematics educators in their work with practicing mathematics teachers suggesting it to be different from educating
mathematics educators to work with prospective teachers. Educating prospective teachers is usually part of a formal program that leads to teacher certification while educating practicing teachers is typically less formal and does not normally occur in a university setting (Even, 2008). She identified three gaps in the literature challenging mathematics educators in their work with practicing teachers: 1) almost no research exists regarding the education of mathematics teacher educators; 2) the field of educating practicing mathematics teachers is not very well defined; and 3) little information is available on the practice of mathematics teacher educators working with practicing teachers (Even, 2008, p. 58). Since the education of practicing teachers is expected to play a critical role in mathematics education reform, she argued for focused studies on understanding the educators of practicing teachers and their work.

Even (2008) suggests that mathematics teacher educators as a group are not well-defined, including both university faculty and school teachers. The difference lies in the fact that for some teacher educating is a full-time job while for others it is only a part-time activity. Additionally, some teacher educators work with both prospective teachers and practicing teachers while others only work with practicing teachers. She characterized professional development as random and fragmented and pointed that the terminology associated with educating practicing teachers is inconsistent (Even, 2008).

Little empirical research exists with regard to the actual practice of mathematics teacher educators who work with practicing teachers. A survey of research in mathematics teacher education conducted by Adler, Ball, Krainer, Lin, and Novotna (2005), as cited in Evan (2008), indicated that literature in this area tends to focus on the
learning of the teachers and not on the nature of the practice. Additionally, almost all the research consists of self-reports of teacher educators on their own work. The survey also suggested that almost all genre publications were from countries where English is the national language (Even, 2008, p. 60).

Consequently, Even (2008) developed and conducted the MANOR program in Israel for the preparation of educators of practicing secondary school mathematics teachers including resource materials for them to use in their practice. In developing the program, three areas were selected as professional knowledge base: 1) mathematics education; 2) mathematics; 3) teacher education (Even, 2008). Mathematics education included “current views of mathematics teaching, learning, and knowing” (p. 64) with the mathematics content being relative to secondary school mathematics and advanced mathematics. Teacher education refers to “current views of teaching teachers and of teacher learning” (p. 64). Practices of teacher education such as planning, conducting and assessing activities were also a part of the program. Files of resources were also developed as a part of the MANOR program with the major themes of the files being a historical view of the main topic of the file, selected mathematical aspects relevant to the topic, students' conceptions and ways of learning and thinking, and aspects of mathematics lessons and teaching (Even, 2008, p. 66). As she focused on the development of mathematics teacher educators, Even (2008) stated “Clearly we need to understand better what mathematics educators who work with practicing teachers do, in order to construct an informed understanding about the nature of adequate education for educators of practicing teachers” (p. 70).
Conceptualization of teacher educating. As the focus of the literature review of teacher educating moves from the training and development of teacher educators to the conceptualization of teacher educators and teacher educating, the works of Zaslavsky and Leikin (2004), Zaslavsky (2008), Tzur (2008), and Perks and Presstage (2008) stand out and are worthy of elaboration. Zaslavsky and Leikin (2004) conducted research relative to the professional development of teacher educators by examining the process of becoming a mathematics teacher educator within a professional development program for secondary mathematics teachers. In other words, they analyzed the development of mathematics teacher educators through their practice, viewing the participants and educators as a community of practice. As part of the development of a model of growth through practice for mathematics teacher educators, Zaslavsky and Leikin (2004) extended Jaworski’s (1992) teaching triad.

Jaworski’s (1992) teaching triad was developed to make sense of the practice of teaching mathematics. She felt that the essence of teaching mathematics fell within three domains; 1) the management of learning, 2) sensitivity to students, and 3) mathematical challenge. Management of learning referred to the creation of a learning environment through classroom organization, curriculum decisions, established ways of working, and classroom values and expectations (Jaworski, 1992). Sensitivity to students included developing knowledge of their individual characteristics and needs as well as an approach to accommodate those needs. Mathematical challenge manifested itself as stimulating mathematical curiosity and thinking which influenced activity selection and presentation. The challenging content for students is the mathematics. Each of the domains, though

24
distinct in theory, rarely emerged separately in practice but instead elements of each
domain were usually found in any teaching situation and most often they were in some
way related (Jaworski, 1992). Jaworski’s triad is represented in Figure 1 with double
arrows indicating a relationship between the domains.

![Figure 1. The teaching triad. Jaworski (1992, p. 8)](image)

Zaslavsky and Leikin (2004) extended the teaching triad to describe and analyze
the practice of mathematics teacher educators. The teaching triad of mathematics teacher
educators consists of challenging content for mathematics teachers, sensitivity to
mathematics teachers, and management of mathematics teachers’ learning. Zaslavsky
and Leikin (2004) used this model (See Figure 2) in their analysis of the practice of
mathematics teacher educators conducting professional development for inservice
teachers.

Just as mathematics is the challenging content for students, the teaching triad
represents challenging content for teachers. This model was used in combination with
Steinbring’s (1998) model (as cited in Zaslavsky and Leikin, 2004) of teaching and
learning to analyze the practice of mathematics teacher educators. Results of their study
indicated that a sorting activity challenged both the mathematics teachers and the mathematics teacher educators relative to the mathematics involved. The process of designing these mathematical activities was a key for the professional growth of the mathematics teacher educators (Zaslavsky & Leikin, 2004).

In later research, Zaslavsky (2008) suggested that teacher educators are self made in that they create their own transitions from experiences as mathematics teachers and/or researchers of mathematics learning and teaching. She states that "The demands on teacher educators, in terms of knowledge and qualities, are enormous and multifaceted. They need to constantly reflect on-action and in-action, in all phases of their work" (p. 94). Zaslavsky (2008) discusses the role and responsibility of teacher educators, the knowledge base they draw on, and the kind of practice and qualities they need to develop. She suggests examination of seven unifying themes that reflect goals for mathematics
teacher education. The themes include 1) developing adaptability, 2) fostering awareness to similarities and differences, 3) coping with conflicts, dilemmas and problem situations, 4) learning from the study of practice, 5) selecting and using appropriate tools and resources for teaching, 6) identifying and overcoming barriers to students' learning, and 7) sharing and revealing self, peer, and student dispositions (Zaslavsky, 2008, p. 95).

Zaslavsky (2008) explains how a teacher educator can use carefully designed tasks to accomplish these goals. In order to help teachers develop adaptability, the teacher educator must be adaptable and display this quality when working with teachers. To promote awareness among teachers, the teacher educator must involve them in activities that require noticing similarities and differences among mathematical concepts. As part of coping with conflicts, dilemmas and problem situations, the teacher educator needs a broad knowledge base relative to content and pedagogy and the confidence to engage teachers in open-ended problems which might lead to unanticipated questions (Zaslavsky, 2008). Zaslavsky (2008) proposes that learning from the study of practice requires that teacher educators be able to design and implement experiences such as videotaped study that demonstrate realistic classroom situations. Mathematics teacher educators must also be familiar with a wide range of available tools and how they can be used to accomplish mathematical learning. Additionally they must be aware of barriers to student learning, understanding the nature and causes of these barriers and possible interventions to overcome these barriers. Teacher educators must also be familiar with beliefs and dispositions and their various effects on learning and teaching mathematics in
order to help teachers become aware of their own beliefs as well as student dispositions (Zaslavsky, 2008).

Teacher educators have few resources to draw on directly. "The main sources that are specific for teacher educators (that is, in addition to the resources available for teachers) are professional journals and books, research papers and other accounts of practitioners, and personal experience, all related rather indirectly to tasks for teacher education” (Zaslavsky, 2008, p. 100). While the tasks teacher educators create might enhance teacher learning, their own learning is enhanced through design, implementation, and modification of these tasks.

Zaslavsky (2008) observed the work of both practicing and prospective teachers as they investigated three versions of what she termed "The Spiral Task" (p. 101). She began with an initial version of the task, and over time, modified the task so that it better accomplished her goals. Eventually, she realized that having the same group of teachers work through each version helped develop the teachers' adaptability and she further explained how various aspects of the task incorporated the seven themes for teacher education. Zaslavsky (2008) suggests that “the knowledge base of teacher educators consists of the knowledge for teaching mathematics (that is, mathematics teachers' knowledge base) as well as knowledge of how to enhance teacher learning” (p. 112). She asserts that knowledge of how to enhance teacher learning should include theoretical and practical knowledge of the seven themes discussed earlier.

In his work, Tzur (2008) introduced profound awareness of the learning paradox (PALP). He described the learning paradox as follows. "In order to assimilate and
complete tasks/activities in which a teacher engages her or his students to promote their learning of a new concept – the students need to have already established that concept prior to learning it” (p. 138). He suggested that in order to address the learning paradox, a shift in epistemological stances related to “teaching of mathematics, education of mathematics teachers, and mentoring of mathematics teacher educators” (p.138) might be necessary.

He characterized the shift through long-term collaborations with three classroom teachers indicating that this learning paradox was evident not only in teacher-student interactions, but also in teacher educator-teacher interactions (Tzur, 2008). Based on his personal experiences, Tzur (2008) focused on development of mathematics teacher educators’ awareness of how to move teachers towards teaching that is rooted in a profound awareness of the learning paradox (PALP) and an ability to address the learning paradox (LP). He claims that traditional and reform pedagogical approaches overlook the LP and that teaching rooted in PALP addresses it head-on. Additionally he claims that PALP is better able to move students forward in their learning as activities are based on what they know and what they are expected to learn. If the intended mathematics is not learned, PALP is better suited for figuring out why (Tzur, 2008).

In his development towards PALP, Tzur analyzed his progress by postulating four foci through which he claims an educator matures: mathematics learner, mathematics teacher, mathematics teacher educator, and mentor of mathematics teacher educators (Tzur, 2001 as cited in Tzur, 2008). His analysis is entirely based on personal experiences of learning mathematics through teaching it, learning pedagogy through
teaching teachers, learning teacher education through teaching perspective mathematics teacher educators, and learning to mentor mathematics teacher educators through scholarly collaboration in continual teaching (Tzur, 2008). He claims that mathematics teacher educators need to develop PALP in order to cultivate its development in teachers. Experiences that contributed to Tzur’s development of PALP focused on three types of reflection. The first consisted of a mathematics teacher educator’s reflection on his or her own activities when either doing or teaching mathematics. The second consisted of reflecting on teachers’ disbelief with respect to PALP-rooted lessons. And then the third type of reflection consisted of comparisons between the first and second types (Tzur, 2008, p. 152).

Perks and Prestage (2008) also describe how they themselves developed tools for their work with prospective teachers. The term tool in this instance refers to “the widest sense of Vygotsky’s use of the word” (p. 265). The authors consider tools as sources to generate learning about the teaching of mathematics as well as the tools they use to develop their own learning about being teacher educators. They suggest that three aspects of knowledge may need to combine in making decisions about classroom events and this contemplation transforms the teacher’s mathematical subject knowledge into knowledge of mathematics necessary for teaching the subject. They include: 1) practical wisdom – knowledge from being in classroom; 2) professional traditions – knowledge from existing school curriculum and practices and research; and 3) learner knowledge – the prospective teachers own knowledge (Perks & Prestage, 2008, p. 269). Together, these three aspects of knowledge form the teacher knowledge tetrahedron (See Figure 3).
Perks and Prestage (2008) described the same categories with regard to teacher educator knowledge that might be combined to transform their learning about teaching into learning about teacher education, including: 1) professional traditions – the existing teacher education course, ways of working, and research on mathematics teaching; 2) practical wisdom – the activities chose for sessions; and 3) learner knowledge – knowledge accrued from being teachers (p. 270). These three aspects of knowledge combined to form the teacher educator knowledge tetrahedron (See Figure 4).

The learner knowledge of mathematical subject knowledge for teacher educators is based on their own learner knowledge of mathematics, the practical wisdom of the experienced mathematics teacher and the professional traditions of curriculum, examination of syllabi and so forth. In other words, the teacher educator needs to begin with the teacher knowledge as previously described (Perks & Prestage, 2008). That knowledge together with prospective teacher education professional traditions and the
practical wisdom of teaching about teaching forms the basis for creating sessions for preservice teachers.

Therefore the teacher knowledge tetrahedron lies at the learner knowledge vertex of the teacher educator knowledge tetrahedron (Perks & Prestage, 2008, p. 271). Professional traditions arise from personal experiences, education and training, current government teaching training policies, national curricula for teacher education, and research. Practical wisdom results from considering what prospective teachers need to know and how sessions might be constructed to develop that knowledge (Perks & Prestage, 2008). Note that the experiential component of practical wisdom at the teacher knowledge level and the teacher educator knowledge level is similar to the experiential orientation of Elbaz’s (1983) practical knowledge. The theoretical component of professional traditions at both the teacher and teacher educator knowledge levels is likewise similar to the
theoretical orientation of practical knowledge described by Elbaz (1983). Sessions are formed based on their own teacher knowledge, literature and research, and writing about (publishing) and reflecting on sessions. Teacher knowledge acts as learner knowledge for teacher educators. In this sense, teacher knowledge can refer to how students react to various mathematical questions (Perks & Prestage, 2008).

**Teacher educators in practice.** With respect to teacher educators in practice, Bergsten and Grevholm (2008) discuss the knowledge of mathematics teacher educators and what is required of them as they work in teacher education programs trying to link theoretical coursework with teaching practice. The authors assert that mathematics teacher educators come from a background in mathematics, pedagogy, or as experienced schoolteachers. "Most mathematics teacher educators develop their own practices, either out of previous experiences as teachers in school or in university, or ‘anew’ based on research and experimentation is" (Bergsten & Grevholm, 2008, p. 233).

In their discussion of the learning and teaching activities of the primary participants involved in mathematics teacher education, Bergsten and Grevholm (2008) suggest that these activities take place in an "autonomous world of mathematics for learning and teaching, isolated from the world of applied mathematics and use of mathematical models and science and society in general" (p. 233). They call this world educational mathematics and that in order to deal with it, teachers and teacher educators develop educational knowledge of schools. Some of the practices of teacher educators include planning working sessions for students which requires knowledge of what is possible and necessary in terms of content, selecting the content, and teaching it. Teacher
educators need to be familiar with different forms of organizing their teaching such as student led lectures, lessons, seminars, group sessions, problem-based learning in small groups, school visits, lesson observations, interviews, videotaped recordings, and classroom students' work (Bergsten & Grevholm, 2008). Teacher educators also need to create working tasks for varying needs of student teachers. To do so knowledge about all kinds of tools and supporting material for mathematics teaching such as concrete material, graphics calculators, computers, video cameras, and multimedia presentations become necessary (Bergsten & Grevholm, 2008).

The authors define linking practices as "ways of working that aim at creating functional bridges between academic coursework in the home arena of the student teachers and teaching practice in the visiting arena, so that each of these may profit from experiences and reflections in the other" (Bergsten & Grevholm, 2008, p. 235). Examples of these practices include development of teachers’ professional language through learning opportunities in group work where they are given time to reflect on and discuss problem-solving situations and post-teaching observation meetings with student teachers. This also includes practices with an emphasis on writing such as using stories of practice, and viewing lessons as experiments. The linking practices described in their work focus on various aspects of bridging theory and practice (Bergsten & Grevholm, 2008).

Bergsten and Grevholm (2008) highlight the importance for student teachers to discover, experience and reflect on the role of language, specifically questioning and listening, so that they may teach according to their student's' needs. Through
documentation of stories of their own practice, they suggest that mathematics teacher educators can increase their own awareness of their students’ awareness and that these stories also provide rich sources of classroom events on which student teachers can reflect. Post teaching observation meetings allows the teacher educator to get involved in his or her own teaching, learning, and reflecting activities as well as those of the student teacher (Bergsten & Grevholm, 2008). Treating a lesson as an experiment is likely the most natural way to link theoretical coursework to practice that can be reflected on and discussed at the University.

The examples illustrate the need of mathematics teacher educators not only to have knowledge and experience in those competence areas involved in the work of mathematics teachers discussed above, but also to be knowledgeable both in the activities they design, such as the linking practices, and in such teaching and learning activities in which these practices are designed to make the student teachers knowledgeable (Bergsten & Grevholm, 2008, p. 242).

In a different study focusing on teacher educators in practice, McDuffie, Drake, and Herbel-Eisenmann (2008) worked with prospective elementary teachers in a “methods of teaching elementary mathematics” course. The three authors wrote and discussed their core beliefs in regard to courses they taught and came to the conclusion that developing understanding of children's mathematical thinking and learning was a central element in framing their courses for prospective teachers. They felt this conclusion was a result of their trust in cognitively guided instruction research. As a result, the unifying goal of the researchers was "to prepare prospective teachers to be
deliberate and reflective practitioners whose instructional decisions are grounded in children's mathematical thinking and learning” (McDuffie, Drake, & Herbel-Eisenmann, 2008, p. 249).

Many instructional decisions in each of their courses were similar, which they felt was a result of the belief among mathematics teacher educators that teachers need to develop skills for listening to children's ideas and thinking, for reflecting on their practice in light of student thinking, and for planning and adjusting instruction based on children's understanding and experiences. The mathematical content area most emphasized was number and operation and the mathematical process area was problem-solving and reasoning. McDuffie, Drake, Herbel-Eisenmann (2008) held the common belief that prospective teachers' learning should be situated in practice. As a result, they used video and written artifacts to illustrate children's mathematical thinking and learning. Prospective teachers solved problems and then later investigated, through video or written work of students, a similar problem and the method students used to solve the problem (McDuffie et al., 2008).

The researchers used the CGI framework to facilitate skills in planning, making instructional decisions, and reflecting on instruction. They presented research-based assignments such as an interview project and a reflective journal to develop prospective teachers understanding of the research (McDuffie et al., 2008). The interview project included the prospective teachers interviewing a student and then designing lessons based on what they learned from the interviews. Prior to the interviews teachers read research-based texts focusing on teaching and learning in a specific mathematics content area.
relative to their practice. Prospective teachers developed a series of interview questions which they administered to the students and through reflection and analysis then design lessons to meet the students needs (McDuffie et al., 2008). Additionally, prospective teachers read a CGI book and also took part in some professional development activities described therein and were required to develop a set of problem-solving tasks for the children and audio record sessions in which the children worked on the problems. They would then listen to the audio tape recording, write a reflective entry in their journal, and use what they learned to develop a new set of tasks. The reflective journal allowed them to record children's thinking and progress as well as their own thinking about the teacher-student interaction. The second part of the journal was for the teachers to collect data related to each child's content, processes, and attitudes. At the end of the course, the prospective teachers then wrote case studies of children with whom they worked. The third part of the journal was for personal reflection on their teaching experience (McDuffie et al., 2008).

Each of the instructors taught their first mathematics methods course while in graduate school and indicated that they were strongly influenced by their work with more experienced colleagues who also made children's ways of thinking about and doing mathematics central to the design of their courses (McDuffie et al., 2008). They were also influenced by the idea that CGI and other video materials helped teachers connect theory and research to classroom practice relative to students’ thinking and learning. "As we gained experience teaching the course, we continued to learn and develop our practices as our field developed" (p. 259). These instructors felt the need to inquire into
their own practices and find ways to promote students' learning and felt that inquiry practices were important in this regard. They utilized a number of published professional development materials for practicing teachers as resources from which to select videotape recordings and children's written work. These materials also provided ideas for leading discussions relative to the activities. Reading and conducting research in the field also influenced decisions in regard to assignments and class activities (McDuffie et al., 2008).

Similar to the methodology of McDuffie, Drake, Herbel-Eisenmann (2008), Sanchez and Garcia (2008) focused on their own learning that took place as they made decisions about what to teach and how to teach it in their mathematics teacher education program in Spain. These authors viewed the responsibilities of a mathematics teacher educator as "educating teachers to be self-sufficient professionals who are able to develop the tasks of their practice, be responsible for their actions, and reflect on these" (Sanchez & Garcia, 2008, p. 281). Sanchez and Garcia (2008) defined dilemma as "a situation in which a complicated choice has to be made between two different things you could do" (p. 282) and the conflict that arises with these dilemmas provides learning opportunities for the teacher educators. The only preparation these mathematics teacher educators had consisted of a degree in mathematics or other related scientific subject. They had received no education with regard to pedagogical or psychological issues or with respect to mathematics education (Sanchez & Garcia, 2008).

Initially, their decision of what to teach was based on curricular orientations of official documents and some textbooks. How to teach was based on personal experiences as a mathematics student and a mathematics teacher (Sanchez & Garcia, 2008). Based on
experiences in classrooms with student teachers, they felt the need to restructure what they were doing. As a result, Sanchez and Garcia (2008) began investigating other teacher education programs and developed the framework in which they could think about the content and organization of their own program. The framework consists of a theoretical background, content, and how student teachers could access the content. They then examined some of these programs and look for research that would help them reflect on the different options that might be used for their own program development. Through this process they began to categorize knowledge for teaching mathematics according to knowledge of and about mathematics, knowledge of the learners and learning processes, and knowledge of instructional processes (p. 284). Eventually they began to distinguish two different aspects of their program: mathematics as a science to be taught and learned, and a specific professional work involved in the teaching of mathematics (Sanchez & Garcia, 2008).

In organizing their ideas of what to teach in their mathematics methods course, Sanchez and Garcia (2008) focused on mathematical thinking and systems of mathematics teachers' activity. When developing the mathematical component, they considered activities of mathematical practices such as defining, justifying, and modeling and also considered mathematical content organized by subject areas. Research played a vital role in their determination of how to teach as they examined how knowledge could be constructed by student teachers. Eventually Sanchez and Garcia (2008) focused on the "constructed nature of knowledge and beliefs" as well as the "social and situated nature of cognition" (p. 286). Additionally, they included the ideas of communities of practice for
developing prospective teachers. "The different relationships between theory and practice take place as distinct dilemmas and correspond to the different teacher educators' learning moments" (Sanchez & Garcia, 2008, p. 292).

In a dissertation focused on the tasks of teaching mathematics to teachers, Zopf (2010) investigated the instructional practice of mathematics teacher educators to examine and hypothesize about their mathematical work and their necessary mathematical knowledge. As a precursor to her study, she suggested that mathematical knowledge for teaching and mathematical knowledge for teacher educators are likely different.

First, the mathematical content is different. Children are taught mathematics; teachers are taught mathematical knowledge for teaching. Second, the learners are different. Children come to school with some mathematical understandings. The goal of school mathematics is mathematical proficiency…Teachers possess mathematical knowledge learned during their school experiences. They know the multi-digit multiplication algorithm. However, it is the goal of teacher education to develop this mathematical knowledge into mathematical knowledge for teaching. (Zopf, 2010, p. 5-6)

Zopf (2010) conducted a pilot study in which she observed two teacher educators as they worked with prospective teachers in mathematics methods courses for elementary teachers. As a result of her pilot study, she identified 10 tasks which require the teacher educator’s use of mathematics; selecting the content, selecting tasks, modeling mathematics, selecting examples, managing discourse, responding to student teachers'
questions, selecting materials, creating assignments, analyzing and responding to student work, and designing and grading assessments (Zopf, 2010, p. 17).

Zopf’s (2010) multiple case studies included observing one teacher educator presenting two different lessons as part of professional development for practicing teachers and a different teacher educator presenting two separate lessons to prospective teachers in an undergraduate mathematics methods course. The four lessons dealt with different aspects of rational number arithmetic. Results indicated several types of mathematical knowledge used by the teacher educators: mathematical content knowledge, disciplinary knowledge of mathematics, mathematical knowledge for teaching, mathematical knowledge beyond mathematical knowledge for teaching that supports its teaching, knowledge of teachers as mathematics learners, and knowledge of pedagogical strategies for teachers (Zopf, 2010, p. 183).

Her findings suggest a domain of mathematical knowledge for teaching teachers that is "a more detailed and unpacked mathematical knowledge used to make visible the mathematical knowledge teachers use for teaching – unpacked, connected, language focused, and discipline oriented" (p. 185). Knowledge of interpretations and representations was used for the unpacking of mathematical knowledge for teaching. Mathematical knowledge within and across domains was evident in the teaching of connected mathematical knowledge. Knowledge of appropriate language that would maintain the integrity of the mathematics and be functional with the teachers was important in developing precise mathematical language (Zopf, 2010). A robust knowledge of mathematical structures, ways of reasoning, and methods of work also
appeared necessary for developing mathematical knowledge for teaching. Additionally, knowledge of teachers as learners was evident in this study. Summarizing her findings, Zopf (2010) states the following: "At the level of teaching mathematical knowledge for teaching, MKTT might be considered applied mathematical knowledge homed to address the special needs of teaching teachers" (p. 192)

Olanoff (2011) asserts that developing a knowledge base for teacher educators can provide a framework for guiding practice and may help to raise the level of perceived professionalism of teaching and teacher education. Through a qualitative study of teacher educators teaching mathematics content courses for preservice teachers, she attempted to characterize some elements of a framework for teacher educator knowledge specific to multiplication and division of fractions. Olanoff (2011) conducted a multiple case study of three teacher educators, one from a small, four-year private college, one from a four-year state university, and one from a two-year college. Because of the numerous tasks that teacher educators performed in their work with prospective teachers, she chose to focus on 1) introducing fraction multiplication; 2) helping students make sense of fraction division, and 3) assessing student understanding (Olanoff, 2011, p. 141).

With respect to fractions, Olanoff (2011) concluded that teacher educators must consider their students' preconceived understandings, building on them as well as clearing up any misconceptions. Additionally, they must be able to make connections between relationships with whole numbers and fractions. Teacher educators must also be able to determine which aspects of a topic need emphasis. As a result of her work, she
suggests four aspects that might be included in a framework for teacher educator knowledge relative to multiplication and division of fractions. Olanoff (2011) asserts that teacher educators must understand multiple ways of representing the concepts and how the representations are related to each other, to whole number concepts, and to corresponding algorithms. A second aspect of the framework includes making decisions about which aspects of the topic will help prospective teachers make the connections they need in order to teach the topic. She suggested that a third aspect is being able to set specific learning goals for the students. Finally, her fourth aspect of a framework for teacher educator knowledge was designing and using assessments for determining student understanding (Olanoff, 2011). She concluded that these four aspects do not provide a complete framework for the mathematical knowledge required by teacher educators to teach prospective teachers about multiplication and division of fractions though highlighting their importance.

Teacher Educators and Teacher Educating

Teacher educators and the practice of teacher educating are beginning to receive attention in mathematics education research (Adler et al., 2005; Chauvot, 2008; Hiebert et al., 2003; Sowder, 2007; Speer & Star, n.d.; Sztajn, 2011; Watson & Mason, 2007; Zaslavsky & Leikin, 2004). In order for the goal of mathematical proficiency for all to be achieved, teachers need support which places significant, and often underappreciated, demands on teacher educators and the practice of teacher educating (Adler et al., 2005). Jaworski (2008) expressed this sentiment clearly:
Mathematics teacher educators are professionals who work with practicing teachers and/or prospective teachers to develop and improve the teaching of mathematics. They are often based in university settings with academic responsibilities. The qualities required of teacher educators are in many respects the same as those required of mathematics teachers. They need to know mathematics, pedagogy related to mathematics, mathematics didactics in transforming mathematics into activity for learners and classrooms, elements of educational systems in which teachers work including curriculum and assessment, and social systems and cultural settings with respect to which education is located. In addition they need a knowledge of the professional and research literature relating to the learning and teaching of mathematics, knowledge of theories of learning and teaching, and knowledge of methodologies of research that inquires into learning and teaching in schools and educational systems. (p. 1)

She claims that knowledge and teacher education is categorized into three basic components: 1) educators knowledge of theory, research and systems; 2) knowledge shared by educators and teachers; 3) teachers' knowledge of students in schools. Jaworski (2008, p. 2) asserts that teacher educators draw on their knowledge from the first two components (1 & 2) to promote growth of knowledge in the last two components (2 & 3). However, very little research is available that reflects critically on the teacher education process, on the learning of teacher educators, or on programs designed to educate teacher educators (Jaworski, 2008).
The literature on teacher education and teacher educating highlights the reliance of studies on perspectives that account for teacher knowledge. This is sensible as teacher educators are also teachers. It is plausible that parallels might exist in their structure of thinking and work and those of the teachers. These connections informed the theoretical framework of this study.

**Theoretical grounding: Types of Teacher Knowledge**

Because research with respect to teacher educators and the practice of teacher educating, especially in the area of mathematics, is limited (Even, 2014), studies of mathematics teacher educator knowledge are often framed using research regarding teacher knowledge (Olanoff, 2011; Zaslavsky & Leikin, 2004; Zopf, 2010). Identifying the various knowledge domains that the mathematics teacher educators used when making decisions about mathematical concepts to pursue with teachers, instructional strategies they used, their knowledge of preservice teachers’ backgrounds and need, and knowledge of inservice teachers and their experiences thus necessitated grounding data analysis in perspectives pertaining to the structure of practitioner knowledge. The following sections describe each perspective, and the analytical structure they provided in this study.

**Shulman’s perspective.** Modern perspective on teacher knowledge began, seemingly, with Shulman (1986) and his identification of what he termed the “missing paradigm” (p. 7) or a blind spot with respect to content that was prevalent in most research on teaching as well as programs of teacher evaluation and certification. Shulman argued that too much emphasis had been placed on how teachers managed their
classroom, criticized their students, and planned lessons as opposed to a focus on content being taught, questions asked, and explanations offered (Shulman, 1986). As a result, he proposed a theoretical framework that consists of three categories of content knowledge: 1) subject matter content knowledge; 2) pedagogical content knowledge; and 3) curriculum knowledge. Shulman (1986) described content knowledge as the amount of knowledge and how it is organized in the mind of the teacher suggesting that to think correctly about content knowledge required understanding the structures of the subject matter including both substantive and syntactic structures (Shulman, 1986). Substantive structures include the organization of basic concepts, principles, and key facts while syntactic structures are the processes by which theories and models are established as valid (Petrou & Goulding, 2011; Shulman, 1986). In addition to providing the facts about a concept, Shulman (1986) argued that teachers must also be able to justify acceptance of the facts, explain their importance, and connect them to other concepts within the subject matter and outside the subject matter.

Shulman defined pedagogical content knowledge as “subject matter knowledge for teaching…the ways of representing and formulating the subject that make it comprehensible to others” (Shulman, 1986), p. 9). He emphasized that teachers must have an understanding of what makes a topic easy or difficult to learn as well as conceptions and/or misconceptions students may bring to the topic, along with a variety of strategies available to redirect student thinking when difficulties arise.

Shulman (1986) identified curriculum knowledge as knowledge of the programs designed for teaching subjects and topics, awareness of the instructional materials
available to teach the programs, and the ability to decide when and when not to use specific materials for instruction. Further, he distinguished between lateral curriculum knowledge – the teacher’s ability to relate a given concept to issues being discussed in other classes – and vertical curriculum knowledge – the teacher’s familiarity with concepts being taught within the same subject area in years prior and years following the current year.

Shulman’s work has been cited in almost every research report written in the area of teacher knowledge including that of Petrou and Goulding (2011) who attempted to synthesize the work of Shulman, Fennema and Franke, the Mathematics Teaching and Learning to Teach Project, and the Subject Knowledge in Mathematics Project in England and Whales. Their proposed synthesis of models on teacher mathematical knowledge distinguishes between subject matter knowledge and pedagogical content knowledge but also recognizes the interaction between the two (See Figure 5).

Figure 5. Synthesis of models on teacher mathematical knowledge. Petrou & Goulding (2011, p. 21)

Curriculum knowledge as defined by Shulman (1986) is central to understanding what teachers need to know to teach mathematics and Petrou’s and Goulding’s model indicates

47
that teachers’ subject matter knowledge and pedagogical content knowledge influences how teachers understand and use the curriculum and associated materials. The authors (2011) believe that “teacher knowledge can only be understood in the context in which they work” (p. 20) including the educational system, the aims of mathematics education, the curriculum and materials, and the assessment system. Additionally, they argued that understanding subject matter must include both an awareness of its facts as well as an understanding of its structure. They assert that teachers with limited experience of specific subject matter may need opportunities to work on it at their own level as learners of mathematics. For example, Petrou and Goulding (2011) suggest that generalizing patterns in number squares might be appropriate for primary preservice teachers while secondary geometry teachers might be exposed to why conditions for congruency of triangles hold and what can be deduced, once congruency is established.

Although the dividing lines are often blurred, Petrou and Goulding (2011) as well as Shulman (1986) indicate that distinguishing subject matter knowledge and pedagogical content knowledge is important for teacher development. Teachers need opportunities to work on mathematics relevant to the school curriculum, but at their own level and experiencing multiple methods of learning mathematics themselves could impact their view of how it can be learned (Petrou & Goulding, 2011). The researchers recommend switching focus from the individual teacher to the system, paying more attention to prior experiences of teachers and to resources available to them for their own use.

The curriculum and its associated materials can act as both a resource and a constraint on the teacher…We need to know more about how teachers use
curriculum materials to improve their teaching, and which curriculum materials are most effective in doing this (Petrou & Goulding, 2011, p. 23).

Shulman (1986), together with Petrou and Goulding (2011), offer bookend perspectives concerning 25 years of research in the area of mathematical teacher knowledge. The following segments of this literature review focus on the details of research associated with teacher knowledge for mathematics that occurred within the time frame of their work. The next section examines the work of Ball et al. (2008) and Hill et al. (2004 & 2008) relative to their expansion of Shulman’s generic subject matter content knowledge into knowledge specific to mathematics. Additionally, several studies regarding the development of teacher’s subject matter knowledge are reviewed.

Subject matter knowledge. In their work with the Mathematics Teaching and Learning to Teach Project, a group of researchers at the University of Michigan divided Shulman’s subject matter content knowledge into common content knowledge and specialized content knowledge (Ball et al., 2008; Hill, Ball, & Schilling, 2008; Hill, Schilling, & Ball, 2004). Initially, Hill, Schilling, and Ball (2004) were attempting to write, and pilot test, multiple-choice items that would represent mathematical knowledge for teaching elementary mathematics. They felt that one possible way of organizing the items necessitated a separation of content knowledge into common content knowledge – computing, solving problems, etc. and specialized content knowledge - for example, building or examining alternative representations or evaluating unconventional student methods. In later studies, the group continued with the division and further defined common content knowledge as mathematical knowledge and skill used in a variety of
settings that are not unique to teaching or in other words, in common with how it is used in other professions (Ball et al., 2008; Hill et al., 2008). They claim that this is probably what Shulman (1986) meant with his original version of subject matter knowledge. Their definition of specialized content knowledge included knowledge and skill that is unique to teaching such as how to represent mathematical concepts, explanations for rules and procedures, and evaluation of alternative solution methods (Ball et al., 2008; Hill et al., 2008). Their work is not in contrast to that of Shulman but instead supported and extended his original theories about teacher content knowledge.

Other scholars have relied on different descriptions and descriptors to identify what specialized mathematical knowledge might mean according to the concept under study. For instance, one case study of a fifth grade teacher’s instruction with regard to functions examined the relationship between knowledge of mathematics and instructional practices (Stein, Baxter, & Leinhardt, 1990). For this study, the authors conceptualized content knowledge as “knowledge of the content and organization of a specific mathematical topic” and teacher knowledge as “teacher beliefs and knowledge regarding why and how to teach the mathematical topic to elementary-school students” (p. 642). Their findings indicated that limited teacher content knowledge narrowed instruction in three ways by: 1) failing to lay the foundation for future learning in the area of functions; 2) overemphasizing a rule that was true only part of the time; and 3) ignoring opportunities to make connections between key concepts and representations. The teacher’s content knowledge lacked key ideas of function, most notably the idea of one-
to one correspondence. Because of this gap, the lessons focused on procedural rules instead of mathematical concepts (Stein et al., 1990).

A different case study of one student teacher explored a sixth grade lesson focused on division of fractions. This was part of the Learning to Teach Mathematics Study where researchers followed a group of eight novice teachers through their final year of teacher preparation and first year of teaching. The study investigated knowledge and beliefs related to all areas of mathematics instruction at the elementary and middle school levels (Borko et al., 1992). Researchers believed that the required mathematics methods course did not adequately prepare student teachers for the classroom, citing the instructor’s rapid coverage of material and student teachers’ limited command of verbal expressions as reasons for absence of conceptual understanding of the content. They also indicated that the mathematics courses the teachers had taken usually did not focus on meaningful learning of mathematics, emphasizing either rote learning or conceptual understanding at high levels of abstraction. Borko and her colleagues (1992) emphasize that prospective teachers must be given the opportunity to improve their content knowledge through conceptual development of important topics in elementary mathematics along with the language necessary to make connections between various representations through practice and reflection. They assert that courses that focus on the conceptual development of important topics in elementary mathematics and that challenge prospective teachers' knowledge of the topics are necessary. Additionally, Borko et al., (1992) claim that preservice teachers must be given the opportunity to talk
about and talk through their reasoning with others who are more proficient in mathematics education.

Two separate studies examined teacher content knowledge in relation to development of flexibility in teaching algebra (Yakes & Star, 2011) and proportional reasoning (Berk, Gorowara, Poetzl, & Taber, 2009). In this context, flexibility refers to the knowledge of multiple solution methods for a type of problem and the ability to choose the most efficient one. A flexible teacher has the ability to choose appropriate problems that provoke comparison conversations and recognize innovative approaches to problems during class that can be used for discussion. Less flexible teachers are likely to focus on one solution method and ignore students’ alternative solution strategies (Yakes & Star, 2011). Results from both studies indicated that limited content knowledge constrained teacher flexibility as well as their views of student flexibility.

Each study used comparison practices as the method for developing teacher flexibility by having teachers compare, either their own solution strategies, or those of the students’. Yakes and Star (2011) explored a one-day professional development program for middle and high school teachers across various algebra topics. First the teachers were introduced to three comparison strategies including side-by-side comparison of solution strategies, discussion of and comparison of multiple strategies, and creation of multiple solution methods to the same problem. Small groups of teachers were then given two similar mathematics problems along with two different strategies for solving the problems. The teachers were then asked to solve both problems using both strategies. Each group then presented their work to the remaining participants. Yakes and Star
(2011) found that using comparison in professional development provided teachers with a classroom instructional tool, a chance to explore their own flexibility in problem solving, and the opportunity to realize the importance of student flexibility. However, participants reported that, in the classroom, having comparison conversations with students was difficult due to the teachers’ tendency to lead the discussion as opposed to sharing it with students. The authors recommended that teachers should experience the opportunity to discuss their own ideas of what flexibility means for them and for their students. A significant amount of time should be devoted to questioning and discourse strategies in the classroom as well as opportunities for teachers to construct multiple solutions to a problem (Yakes & Star, 2011).

Berk and colleagues (2009) examined prospective elementary teachers’ in their understanding of proportional reasoning. Approximately 150 students enrolled in a teacher preparation program were divided into two groups and were presented with one of two types of comparison intervention. One group generated and compared their own solutions to contextual proportion problems while the other group compared worked solutions presented to them as middle school students’ solutions to the same problems. Prior to the study, the prospective teachers possessed little flexibility as they had limited knowledge of multiple solution methods to solve proportion problems (Berk et al., 2009). Results indicate that the participants became more flexible in the content domain of proportional reasoning after the intervention.

**Pedagogical content knowledge.** Pedagogical content knowledge (PCK) includes how to teach mathematics content combined with how to assess student thinking
while taking into account cultural backgrounds of students as well as teaching and learning style preferences or, in short, the knowledge of effective teaching (An, Kulm, & Wu, 2004). In a comparative study of PCK of mathematics teachers in China and the United States, An, Kulm, and Wu (2004) offer a somewhat broader view of PCK than that of Shulman in that they deconstruct PCK into three components. The first component is knowledge of content which they describe as broad mathematics knowledge and specific mathematics content knowledge at the grade level. Their second component, knowledge of curriculum, includes selecting and using appropriate curriculum materials while understanding the goals and ideas of the materials and curricula. This view certainly differs from that of Shulman (1986) in which curriculum knowledge is a distinct category separate from PCK. The third component includes knowledge of teaching which the authors describe as knowing students’ thinking, preparing instruction, and mastery of modes of delivering instruction. This component seems more in line with Shulman’s original designation of PCK, and An, Kulm, and Wu (2004) emphasize knowledge of teaching as the core component.

Silverman and Thompson (2008) suggest two models of PCK development in teachers, an integrative model and a transformative model. With the integrative model, the relative knowledge bases used in teaching are developed separately and the knowledge becomes integrated through the act of teaching. The transformative model of PCK provides teachers with integrated experiences that allow them to connect their mathematical and pedagogical understandings to create a “new” knowledge. Seemingly, the integrative model of development could lead to what An, Klum, and Wu (2004) call
divergent teaching or teaching based on content and curriculum knowledge but without a consideration of student thinking. Teachers tend to be satisfied with students remembering facts and skills but pay no attention to students’ thinking or misconceptions which leads to fragmented, disconnected knowledge. On the other hand, the transformative model seems more inclined to produce convergent teaching (An et al., 2004) that focuses on knowing student thinking and where understanding is internalized through connections to prior knowledge.

In their study comparing middle school teachers in the United States and China, An, Kulm, and Wu (2004) found that deep and broad PCK was necessary for effective teaching. Teachers need to connect prior knowledge and concrete models to new knowledge by focusing on conceptual understanding and procedural development. Additionally, they need to identify student misconceptions and correct them through probing questions and tasks. Findings also indicated that the Chinese teachers emphasized conceptual knowledge through traditional development of procedures while the U.S. stressed a variety of activities to promote creativity and inquiry but failed to make connections between manipulative and abstract thinking and between understanding and procedural development.

In their work, which focused on teacher knowledge, Biza, Nardi, and Zachariades (2007) engaged mathematics teachers in hypothetical classroom scenarios where the teachers had to reflect upon the learning objectives within a mathematical problem and solve it, examine a flawed student solution, and describe, in writing, feedback to the student. The mathematical content of the task for this report included solving equations
that involved absolute value. When designing these tasks, researchers emphasized the following principles: 1) the mathematical content of the task focuses on a topic known for its subtlety or for being difficult for students to learn; 2) the flawed student solution reflects the difficulty thus giving teachers an opportunity to think about and address the issue in terms of how they would help students overcome it; and 3) both the mathematical content in the flawed student response provide a context in which the teachers' mathematical, pedagogical, and didactical decisions emerged (Biza, Nardi, & Zachariades, 2007). They found that the use of this type of task revealed teacher tendencies and crucial aspects of teacher knowledge of specific concepts. Results also indicated that a diversity of tasks reveal different kinds of insight into teacher knowledge (Biza et al., 2007). The authors recommend the use of tasks such as classroom scenarios as tools for identification and exploration of mathematical issues regarding teacher knowledge, to encourage teacher reflection on these issues, and to help prepare preservice teachers for real classroom situations.

The tasks designed by Biza, Nardi, and Zachariades (2007) are similar to what Silver and his colleagues describe as professional learning tasks or “complex tasks that create opportunities for teachers to ponder pedagogical problems and their potential solutions through processes of reflection, knowledge sharing, and knowledge building” (Silver, Clark, Ghousseni, Charalambous, & Sealy, 2007, p. 262). In professional learning tasks, teachers participate in a sequence of four activities designed to help them learn mathematical content and pedagogy. The first activity consists of teachers solving a nontrivial mathematics problem both individually and then through discussion with other
participants. The authors suggest this activity gives teachers an opportunity to recognize the mathematical ideas within the problem without having to think about pedagogical issues (Silver et al., 2007). Next, teachers individually read and analyze a narrative case that describes student-teacher interaction with regard to a problem similar to the one they worked with in the first activity. Researchers provide participants with questions that help them focus on key aspects of the case. The teachers then move to the third activity in the sequence, whole-group discussion of the case, which provides teachers with the opportunity to clarify concepts and discuss issues related to mathematics content, pedagogy, and student learning. In the final stage of the activity, teachers engage with their colleagues in collaborative lesson planning and debriefing. Teachers select a topic, collaboratively plan a lesson around that topic, teach the lesson in their classrooms, reflect on their instructional practices in relation to evidence of student thinking and understanding, and then analyze their lessons with other participants in subsequent professional development sessions (Silver et al., 2007).

Overall results from the use of professional learning tasks indicate that teachers gained significant opportunities to learn mathematics. "We have seen that professional learning tasks... can provide opportunities for teachers to rethink and reorganize the mathematics that they encounter in their practice, thereby allowing them to render their mathematical knowledge more useful and usable” (Silver et al., 2007, p. 276). Teachers benefit from opportunities to make connections among related mathematical ideas, examine how students think about those ideas, and reflect on how instructional decisions influence student learning (Silver et al., 2007).
One final study in the area of PCK examined prospective secondary teachers in the final stage of their preparation, looking at the interrelations of their content knowledge with PCK in the context of teaching functions (Even, 1993). More specifically, the study focused on the author’s view of the essential features of the modern concept of function, arbitrariness and univalence. She describes arbitrariness as not having to exhibit some regularity and not having to be defined on any specific sets of objects. Univalence refers to the property of one-to-one correspondence between the domain and range. Her results indicated that most of the prospective secondary teachers had a limited view of the concept of function, one that was “similar to one from the 18th century” (Even, 1993). To develop better conceptual understanding in conjunction with “content-specific pedagogical preparation based on meaningful and comprehensive subject-matter knowledge” (p. 114), Even (1993) suggests that teachers need better content preparation through courses where mathematics is constructed in line with constructivist views on teaching and learning.

**Mathematical knowledge for teaching.** Mathematical knowledge for teaching (MKT) is described as mathematical knowledge needed to effectively teach mathematics. Some representations used by teachers help to students and others, although technically correct, do not effectively help students understand concepts (Ball et al., 2008). Researchers have been consistent in suggesting that, in the field of mathematics education, little progress has been made with respect to developing a framework or practice-based theory of mathematical knowledge for teaching (Ball et al., 2008; Silverman & Thompson, 2008).
…teaching for understanding is predicated on coherent and generative understandings of the big mathematical ideas that make up the curriculum…there is no commonly accepted theoretical framework for research in mathematics teacher education and, as such, a research base for prospective or practicing mathematics teacher development is emerging relatively slowly and is particularly sensitive to the influence of political agendas and current trends in related fields (Silverman & Thompson, 2008, p. 501).

Silverman and Thompson explain that developing MKT involves transforming key developmental understandings of a mathematical concept to an understanding of how they could benefit student learning of related ideas, the actions necessary to support student development in these areas, and why they might work. They propose a theoretical framework that extends a constructivist perspective to include the development of MKT. In their proposed framework, a teacher has the necessary knowledge to help students develop a conceptual understanding of a topic when he or she 1) has developed an understanding within which that topic exists; 2) has knowledge of the variety of ways students may understand the content; 3) understands how someone else might come to think of the mathematical idea in a similar way; 4) has an image of the kinds of activities and discussions about those activities that might support another person’s development of a similar understanding of the mathematical idea; and 5) has a vision of how students who have come to think about the mathematical idea in the specified way have the ability to learn other, related mathematical ideas (Silverman & Thompson, 2008).
Ball, Thames, and Phelps (2008) concur with Silverman and Thompson suggesting that the field has made little progress on developing a coherent theoretical framework for MKT claiming that key terms still lack definition, often blurring the boundaries between PCK and other forms of teacher knowledge. In addition to hypothesizing that Shulman’s content knowledge could be subdivided into common content knowledge and specialized content knowledge as previously discussed, they also offer that his PCK be divided into knowledge of content and students (KCS) and knowledge of content and teaching (KCT) (Ball et al., 2008).

Knowledge of content and students combines knowing about students and knowing about mathematics suggesting that teachers should be able to anticipate what students might think and what might confuse them. Knowledge of content and teaching combines knowing about teaching and knowing about mathematics. Examples of this type of knowledge include knowing the sequence of instruction for particular content or knowing which examples are appropriate and when they should be used (Ball et al., 2008). This division of PCK into KCS and KCT seems consistent with Silverman and Thompson’s (2008) framework as each of the five components of their model would fit into either KCS or KCT. Ball, Thames, and Phelps (2008) believe that their categories could be useful to help determine whether certain aspects of teachers’ content knowledge predicted student achievement. They also see a use in helping to determine whether different types of teacher development affect particular aspects of PCK thus potentially aiding in the design of teacher education programs and professional development. Possible limitations suggested by the authors include the effects of the variability of
teaching and learning on a practice-based theory, the difficulty in determining where one category divides from the next, and the extent to which their formulation of mathematical knowledge is dependent on teaching styles (Ball et al., 2008).

**Knowledge of content and students.** To provide a slightly more detailed picture of KCS, this section reports on three studies dealing with various aspects of what teachers should know about how students learn mathematics. The intent of the first study was to conceptualize, develop, and test measures of teachers’ KCS with the hope of encouraging large-scale studies of its potential contribution to students’ learning (Hill et al., 2008). For this research, Hill, Ball, and Schilling (2008) defined KCS as “content knowledge intertwined with knowledge of how students think about, know, or learn particular content” (p. 375). They further state that “a teacher might have a strong knowledge of the content itself but weak knowledge of how students learn the content or vice versa” (p. 378). As part of their study, they conducted an exploratory factor analysis on several items of teacher knowledge finding that KCS items were their own factor but that there seemed to be some overlap with content knowledge when items were allowed to load on either factor. They conclude that teachers’ familiarity with student thinking is an element of knowledge for teaching, however, measuring this knowledge is difficult as their scales are not reliable enough to be used in research. Findings indicate that what makes up knowledge of students’ thinking is multidimensional, not yet understood, and needs further development suggesting that future item development should progress to open-ended items as opposed to multiple choice items (Hill et al., 2008).
The second study dealing with aspects of KCS examined Turkish prospective teachers’ knowledge of common conceptions and misconceptions that sixth and seventh grade students have about multiplication of fractions (Isiksal & Cakiroglu, 2011). The authors hypothesize that teachers may have trouble understanding the conceptual structures underlying multiplication of fractions. Results from their study showed that preservice teachers’ knowledge on common misconceptions and their (sources) included algorithmically based mistakes (rote memorization), intuitively based mistakes (misinterpretation of division and multiplication), mistakes based on formal knowledge of fraction operations (inadequate knowledge on properties of operations), misunderstanding of the symbolism of a fraction (limited conceptions of notion of fraction), and misunderstanding of the problem (several reasons). Strategies they used to overcome the misconceptions are classified into three categories: 1) based on teaching – multiple representations, different methods of instruction, making students express their reasoning; 2) based on formal knowledge of fractions – focus on meaning of concept, focus on logical relationships; and 3) based on psychological constructs – developing positive attitudes toward mathematics (Isiksal & Cakiroglu, 2011). As a result of their findings Isiksal and Cakiroglu recommend that teacher education programs include discussions of various cognitive processes and how the processes may lead to various ways of thinking. Emphasis should be given to concepts and relationships among concepts and prospective teachers should be given opportunities to discuss these concepts and relationships as well as common conceptions and difficulties of students.
Additionally, preservice teachers should gain experience in working with cases where they can analyze student thinking.

The final study focused on preservice teachers and what they know in regard to students’ types of mathematical understanding together with instructional strategies that promote student construction of meaning of a particular topic (Graeber, 1999). According to Graeber (1999), it is important to understand children’s understanding in order to be able to help them reconstruct or expand their conceptions. “Wrong answers frequently have theoretical underpinnings that novice teachers do not anticipate” (p. 192). Teachers must be aware that knowledge gaps might exist at levels considered below that of the current concept being studied and instructional practices must involve continual assessment of student understanding. Through observation of tutoring sessions, the author found that preservice teachers frequently missed opportunities to address student misunderstanding thus failing to use the misunderstandings to guide their instruction.

Graeber (1999) asserts that teachers must recognize the difference between what students understand and what they can do and realize that because they have one form of knowledge does not necessarily mean they have other forms. Further, she emphasized that teachers should understand that student intuition is necessary for problem solving, but when based on misconceptions, often leads to incorrect solutions. The misconceptions must be addressed but treated in such a way as not to destroy the student’s confidence. Graeber (1999) suggested having preservice teachers read about and discuss common misconceptions as well as having them solve problems that often result in their own misconceptions leading to incorrect solutions. Asking preservice
teachers to explain their reasoning when solving a problem can help them to think about their own thinking. Preservice teachers should develop multiple approaches to teaching mathematical concepts, recognize and encourage multiple solution strategies, and be able to evaluate the validity of alternative solutions. Additionally, Graeber (1999) suggested that asking preservice teachers to interview students and report on the students' understanding can encourage teachers to listen to student ideas.

**Knowledge of content and teaching.** As previously discussed, knowledge of content and teaching (KCT) combines knowing about teaching and knowing about mathematics (Ball et al., 2008). For example, teachers must make choices about representations to use for a particular concept and they must sequence content to develop understanding. Hill, Schilling, and Ball (2004) aimed to measure growth in teachers’ content knowledge and learn how it contributes to student achievement. They also aimed to examine the ingredients of content knowledge necessary for teaching and learn about its organization and characteristics. Initially, they developed items that focused on two kinds of teacher knowledge: knowledge of content and KCS. Later, their research and that of their associates expanded mathematical knowledge for teaching to include KCT (Ball, Hill, & Bass, 2005; Ball et al., 2008). They believed KCT to consist of more than the knowledge of mathematics held by any educated adult and that teacher education programs and professional development should help teachers develop knowledge of mathematics that extends beyond that needed for everyday functioning. Teachers need to be able to evaluate student errors, explain, in understandable language, how and why an algorithm works, and use mathematical representations. “How well teachers know
mathematics is central to their capacity to use instructional materials wisely, to assess students’ progress, and to make sound judgments about presentation, emphasis, and sequencing” (Ball et al., 2005, p. 14). The authors claim that while many studies suggest teachers’ mathematical knowledge is related to student achievement, the type and extent of the knowledge remains elusive.

A study of first and third grade teachers and students indicated that their knowledge for teaching items significantly predict student gain scores. Their sample overrepresented high-poverty elementary schools in urban, urban fringe, and suburban areas and in controlling for socioeconomic status (SES), findings also indicated that improving this knowledge may provide a way to reduce the student achievement gap that seemed to widen as low-SES students progress through school. Additionally, results show that student minority status was negatively correlated with teacher knowledge indicating that higher-knowledge teachers usually teach non-minority students (Ball et al., 2005; Hill, Rowan, & Ball, 2005). Finally, teacher certification and number of content or methods courses did not necessarily indicate strong content knowledge for teaching mathematics. Hill, Rowan, and Ball (2005) suggest future research may consider 1) examining effects of mathematics instructional methods and curriculum materials on student achievement; 2) looking at the different distinctions in content knowledge, separately and in combination, in relation to student achievement; and 3) comparing instructional practices of mathematically knowledgeable and less knowledgeable teachers.
**Curriculum knowledge.** While content knowledge and PCK are concerned with more detailed aspects of teaching mathematics, Shulman (1986) seems to consider curriculum knowledge as more of the “big picture” of what mathematics teachers should know. Curriculum knowledge consists of knowledge of the programs designed for teaching subjects and topics, awareness of the instructional materials available to teach programs, and the ability to decide when and when not to use specific materials for instruction (Shulman, 1986). Further, curriculum knowledge is divided in to lateral curriculum knowledge and vertical curriculum knowledge. Lateral curriculum knowledge incorporates the teacher’s ability to relate a concept to issues being discussed in other subjects while vertical curriculum knowledge alludes to the teacher’s familiarity with concepts being taught within the same subject area in years prior and years following the current year (Shulman, 1986). In other words, teachers need to be able to connect the subject of mathematics and specific topics within the subject to other concepts within mathematics and in areas outside of mathematics. This is what Ball, Thames, and Phelps (2008) refer to as horizon knowledge. In Shulman’s original designation, curriculum knowledge is considered a category separate from content knowledge and PCK, however more recent research places curriculum knowledge within the umbrella of PCK (Ball et al., 2008; Hill et al., 2008).

Vale, McAndrew, and Krishnan (2011) conducted a study of grade 9-10 teachers in Australia where they analyzed teachers’ reflections on their own learning to explore their understanding of mathematical connections and their appreciation of mathematical structure. The researchers claim that awareness of mathematical structure, understanding
of mathematical connections, and knowledge of mathematics on the horizon should be the focus of professional development programs. They further state that “Making connections by observing and using similarities, differences, equivalences, relationships, and properties in various representations of mathematical objects is fundamental to structural or mathematical thinking” (p. 196). During professional development associated with this research, teachers worked on both mathematics and professional learning tasks. Participants worked collaboratively on closed tasks, investigative tasks designed to explore concepts, tasks focused on deriving or proving formulae or procedures, and problem-solving and modeling tasks (Vale, McAndrew, & Krishnan, 2011). Although the teachers in the study were from grades 9-10, the mathematical content used for development was from grades 11-12. By using the approach of developing teachers’ knowledge of mathematics needed for teaching at a higher grade level, their knowledge and understanding of mathematics at their own grade level was strengthened. Teachers also realized that emphasis on procedural knowledge and instructional thinking limited students’ understanding of mathematics. By giving them an opportunity to reflect on their own learning, teachers were able to make connections with more complex concepts and appreciated structure. As a result, they developed an understanding of student thinking and were able to design learning tasks to help students make relevant connections (Vale et al., 2011).

In contrast to results from Vale and colleagues’ study, Haimes (1996) found that one teacher’s pedagogical practices did not reflect the reform-based instructional practices suggested for the curriculum. In a qualitative case study, he compared a 9th
grade teacher’s actions with the type of instruction that had been expected of the teacher for the given curriculum. At the time of the study, Haimes (1996) believed reform ideas dictated that algebra instruction be moved from manipulative skills to development of conceptual understanding, with functions serving as a central theme. Student centered activities, student language, and student discovery were emphasized and the teacher was expected to be the facilitator of student learning as opposed to the transmitter of information. However, results indicated that the teacher implemented the curriculum as a set of individual content objectives making no connections to previous or future topics. The teacher emphasized the importance she placed on procedures instead of concepts and her pedagogical practices were teacher-focused. “Factors within the teacher’s environment along with her previous experiences, weighed more heavily on her actions than the intended curriculum” (Haimes, 1996), p. 600). Her experiences as both a teacher and a student of mathematics had reinforced the idea that methods and procedures were important in algebra. In this particular instance, horizon and/or curriculum knowledge either was not present or the teacher was not confident enough in her knowledge to make the necessary connections. The studies of Haimes (1996) and Vale, McAndrew and Krishnan (2011) suggested that teachers need to experience opportunities to make mathematical connections on their own before they can help students make the connections.

**Practical knowledge.** In 1976, Freema Elbaz began a case study of a Canadian, high school, English teacher with the purpose of investigating the role that practical knowledge played in the teacher’s practice (Elbaz, 1983). In her study, she viewed the
teacher as an agent whose role was shaped by her classroom experience and she attempted to illustrate and conceptualize this role emphasizing the knowledge that the teacher held and used in her work. She studied the teacher using a series of open-ended discussions and classroom observations.

Elbaz (1981, 1983) defined practical knowledge as all kinds of teacher knowledge "as integrated by the individual teacher in terms of personal values and beliefs and as oriented to her practical situation" (1983, p. 5). Included in her view of practical knowledge are firsthand experience of student learning styles, interests, needs, and supported with instructional strategies and classroom management skills. Additionally she included the teacher’s knowledge of the social structure of the school as well as the community to which the school belongs. She further asserted that the teacher’s experiential knowledge is informed by the teacher’s theoretical knowledge of subject matter, child development, learning and social theory (Elbaz, 1983).

One of Elbaz’s major concerns at the time of the study was the fact that classroom teachers were largely excluded from the work of curriculum development. She also claimed that research at the time viewed teachers through a negative lens where the teacher was an instrument that performed below expectations, a view commonly held by society as well (Elbaz, 1983). From her viewpoint, this negative perception of teachers was both incorrect and misleading. "The single factor which seems to have the greatest power to carry forward our understanding of the teacher’s role is the idea of teachers' knowledge" (Elbaz, 1983, p. 11) She further asserted that "the most basic assumption, of course, is simply that practical knowledge exists and the direct examination of the
thinking of teachers at work will make apparent to us the nature, defining characteristics and criteria of this knowledge" (Elbaz, 1983, p. 13). Through her work with teachers and her own teaching experiences, she developed the construct of practical knowledge and a number of assumptions regarding it. Her assumptions concerned the content of practical knowledge, the orientation of practical knowledge, the structure of practical knowledge, and its cognitive style (Elbaz, 1983, p. 14).

The content of practical knowledge is "the assumption that practical knowledge is knowledge of something, that it has content" (Elbaz, 1983, p. 14). Included within the domain of content practical knowledge is knowledge of self, of the milieu of teaching, subject matter, curriculum development, instruction. The orientations of practical knowledge refer to the way that practical knowledge is held and used including how it is oriented to situations, the personal character of teacher’s knowledge, the social dimension of teacher’s practical knowledge, the experiential base, and the theoretical orientation of teacher’s practical knowledge (Elbaz, 1983). The structure of practical knowledge refers to the internal structure of the teacher's knowledge. Cognitive style is associated with Elbaz’s belief that “teachers exhibit a particular style in the way they hold and use their practical knowledge" (Elbaz, 1983, p. 22). The focus of this work is on her first three assumptions of practical knowledge.

**The content of practical knowledge.** Elbaz (1983) posits five categories of the content of practical knowledge: 1) knowledge of self as a teacher; 2) knowledge of the surroundings in which the teacher works; 3) knowledge of the subject matter; 4) knowledge of instruction; and 5) knowledge of curriculum development (p. 45). Within
the content of practical knowledge, Elbaz (1983) investigated the origins of her participant's practical knowledge, the development and change in her practical knowledge, and the way that she maintained conflicting ideas simultaneously. With regard to knowledge of self, she studied the teacher's personal values and beliefs and how they informed her practical knowledge. She found that the teacher viewed herself as a resource and had knowledge of her skills and abilities with respect to the needs of the students in her classroom (Elbaz, 1983). The teacher also viewed herself in relation to others, as concerned for her colleagues and her students, and she displayed a strong sense of responsibility for the student as learner. Additionally, she viewed herself as a unique human being with various needs, talents, and limitations that influenced her work as a teacher although this was the least prominent of the three views (Elbaz, 1983).

With regard to knowledge of the surroundings, Elbaz (1983) offered evidence of her participant's recognition of the classroom setting, focusing primarily on teacher and students, relations with teachers and administrators, and the politics of being a teacher. She created social settings within the school that reflected her own values but at the same time allowed her to work within the primary structure and goals of the school. The subject had two views of English as a subject, a discipline and a medium for expression. For this teacher, English as an expressional medium took priority over English as a discipline (Elbaz, 1983). Part of the practical knowledge of content in the area of subject matter also included learning and study skills including teaching students to recognize thinking errors, organize information, develop memory, take notes, use the library, and communicate effectively in individual and group settings. In her view of reading and
writing as subject matter, she considered students who could learn through development of traditional skills as well as those who effectively attack problems in their own way (Elbaz, 1983).

The participant’s knowledge of curriculum was based on her experiences in program planning for her department, development of a learning course, and curriculum design for a reading center. In her curriculum development experiences, her strengths consisted of determining the needs of her students and predicting difficulties they might have (Elbaz, 1983). She considered subject matter and student to be equally important areas of concern when developing curriculum. Her view of learning did not seem to be based on theory but instead on practical knowledge about students and instruction. She felt the learning was an activity that needed to be relevant to life (Elbaz, 1983). Additionally, she felt the notion of reflexivity enhanced learning when the learner was "active, aware and involved" and in control of their own learning. Her practical knowledge of teaching centered around her beliefs about the act of teaching, organization of instruction, teacher-student interaction, and evaluation of the results of her instruction (Elbaz, 1983).

**Orientations of practical knowledge.** Elbaz, (1983) uses orientation "to indicate the way that practical knowledge is held in active relation to the world of practice" (p. 101). She examines five aspects of orientation, the first of which is situational orientation, or knowledge oriented to the practical situation the teacher encounters. The second is practical knowledge that is oriented to the owner of knowledge which is referred to as personal orientation. Third is social orientation or that which is shaped by
social constraints and used to “structure the social reality of the knower” (Elbaz, 1983, p. 101). Experiential orientation is the fourth aspect of the orientation of practical knowledge and it reflects and gives shape to the knower's experience. The fifth and final type of orientation is theoretical orientation or that which is oriented towards theory. In her view, theoretical orientation conditions all the others (Elbaz, 1983).

Elbaz’s teacher viewed theory as being situated above practice and difficult to use. Her use of theory was more geared toward practical application or theory of practice. Elbaz (1983) uses the term situation to refer to "that interaction of objective conditions with internal factors which constitutes experience" (p. 110). She situated her teacher in the classroom, in the school, and in the curriculum planning group. Knowledge of communication informed her participant’s practice with regard to the atmosphere she wanted to develop in her classroom – and atmosphere of open communication. Her knowledge of instruction also influenced the teaching situations that her participant encountered, or more clearly, the teaching situations helped determine the instructional knowledge from which the teacher drew (Elbaz, 1983).

With regard to personal orientation, the subject’s use of practical knowledge reflected her values and enabled her to accomplish goals that were in line with those values. She used her knowledge to make her teaching personally meaningful as she wanted no barriers between herself and her students and accepted responsibility for the welfare of her students (Elbaz, 1983).

*Structure of practical knowledge.* In examining the structure of practical knowledge, Elbaz (1983) characterized it according to relationship to practice, to the
teacher's experience, and to the personal dimension. She created three terms to represent these characterizations including the rule of practice, practical principle, and image (Elbaz, 1983, p. 132). The rule of practice is a statement of what to do and how to do it in a particular situation that frequently occurs in practice. These rules of practice may be very specific or applied to broader situations. The practical principle is "a more inclusive and less explicit formulation in which the teacher's purposes, implied in the statement of a rule, are more clearly evident" (Elbaz, 1983, p. 133). The practical principle is more representative of the personal dimension of practical knowledge. Practical principles may be a result of theoretical viewpoints, experience, or a combination of the two (Elbaz, 1983). Image refers to the combination of the teacher's feelings, values, needs and beliefs from which are formed the teacher's image of how teaching should be. A rule of practice is a guideline from which the teacher acts while an image is something to which the teacher responds (Elbaz, 1983). "The practical principle, however, may mediate between thought and action in both ways" (Elbaz, 1983, p. 134). The three levels of structure in practical knowledge are interrelated and serve one another. In the case of her subject, rules and principles usually expressed instructional knowledge while images ordered all aspects of practical knowledge. Practical knowledge was structured according to a few images which reflected the entire body of her knowledge while also holding together the principles and rules she used in bringing her knowledge to practice (Elbaz, 1983).

Summary

Based on the nature and diversity of the practice of teacher educating, specifically with regard to the vast array of experiences the teacher educators and learners bring to an
educational enterprise, the overarching framework for this study was one of situated learning as described by Brown, Collins, and Duguid (1989), Lave and Wenger (1991), and Mousley (2003). The observations were situated in the natural environment of the practice of the mathematics teacher educators to examine their cognition as they implemented lessons for their teachers. Within this broad framework, three analytical components, models of teacher knowledge, were selected to specifically examine the knowledge used or expressed to be used by the teacher educators. Shulman’s (1983) categories of content knowledge, mathematical knowledge for teaching as described by Ball, Thames, & Phelps (2008), and Elbaz’s categories of practical knowledge of teachers provided the initial theoretical framework for data analysis. Shulman's model identified the structure of teacher knowledge involved in the teaching of a nonspecific subject while mathematical knowledge for teaching identified the structure of mathematical knowledge involved in teaching mathematics. Both models of teacher knowledge are regularly referenced in the research community. Elbaz's practical knowledge grounded the sources that contribute to the structure of the professional practice of teaching and the structure of teacher thinking. This component was included in the framework to capture aspects of teacher educator knowledge that might not have been represented in Shulman’s (1986) generic model or in Ball et al.’s (Ball et al., 2008) framework. Note that teacher knowledge models for the analytical framework were used because of limited research with respect to the practice of teacher educating.
Chapter 3: Methods

According to Merriam (1998), the field of education offers ample opportunity for research regarding practice. She asserted that "…research focused on discovery, insight, and understanding from the perspectives of those being studied offers the greatest promise of making significant contributions to the knowledge base and practice of education" (p.1). Qualitative research design allows for the practice of educating to be examined through the perspective of the educators. To that end, I conducted a qualitative, multiple case study of three mathematics teacher educators and the practice of mathematics teacher educating. The following three research questions guided data collection and analysis.

1. What knowledge domains do mathematics teacher educators draw from and use when providing content specific pedagogical experiences for preservice and inservice teachers?

2. How do these knowledge domains influence the activities of mathematics teacher educators as they design and implement pedagogical experiences for preservice and inservice teachers?

3. How can these knowledge domains be incorporated into the developmental stages of a model to conceptualize mathematical knowledge for teacher educating?
Since research with respect to the practice of mathematics teacher educating is limited (Adler et al., 2005; Even, 2008, 2014), my research was exploratory in nature. Exploratory case studies are appropriate for "what" and "how" research questions when control of behavioral events is not necessary and when the focus is on contemporary events (Yin, 2009). The goal of this research was to develop hypotheses with respect to the knowledge domains mathematics teacher educators relied on so to develop a foundation for further inquiry into their practice. Specifically, the goal was for the results of this study to inform the design and implementation of teacher preparation programs and professional development for mathematics teachers as well as the preparation programs and professional development for mathematics teacher educators.

**Selection of Participants**

The participants for this research consisted of three experienced mathematics teacher educators – Alex, Tracy, and Luke – who taught methods courses at three different institutions and led PD sessions for the same research funded mathematics coaching professional development project. Alex had been university faculty for six years, Tracy for 18 years and Luke for 12 years. The criteria used to identify the participants included their involvement with both PTs and ITs, the level of PTs with which they worked, the types of institutions at which they were faculty members, and accessibility to the researcher.

To provide an overall view of the teacher educators’ practices, I wanted to observe them in University methods courses interacting with PTs and in a professional development setting working with ITs. Obviously the perspective of working with ITs
might be different from that of working with PTs due to the knowledge the ITs bring to the learning sessions from their classroom experiences. The PTs usually have minimal classroom experiences from which to draw to make sense of professional demands or instructional expectations. While the focus of this study was not to determine if the TEs interacted differently with the two groups of learners, the decision was made to include both groups so as not to bias the perspective toward one or the other.

In an effort to consider variability among the sample, participants were also selected based on the levels of PTs they taught in their University methods courses. The goal was to observe one MTE in an early childhood methods course, one in the middle childhood methods course, and one in secondary methods course. The participants chosen for this study satisfied the criteria but due to the timing of the data collection, I was unable to observe one MTE instruct a middle childhood methods course. Alex taught a middle childhood methods course the semester prior to data collection and would not teach it again until the following year. However, I was able to observe him in a methods course designed for special education and intervention specialists at the K-8 grade levels.

Since the participants were all involved with the same mathematics coaching professional development program, an effort was made to include MTEs from different types of institutions. Alex was a faculty member at a midsize Midwestern University located in Appalachia. Essentially, the University determined the population of the city in which it was located and the surrounding communities were rural. Tracy was a faculty member at a regional campus of a large research I, Midwestern University. While
the main campus of the University was located in a large Midwestern city, the regional campus was located in a community of less than 50,000 people. The third participant, Luke, was a faculty member at a midsize Midwestern University located in a city with a population of almost 300,000. All of the institutions offered master’s and doctoral programs.

The final criteria concerned accessibility to the MTEs. For convenience, I had to select participants whose universities were within an approximate 150 mile radius of my residence. Additionally, since I was a graduate research associate with the same mathematics coaching research project in which they participated, I was able to use the monthly PD sessions to conduct interviews and observations with the MTEs.

The Researcher’s Perspective

Prior to beginning my doctoral program, I taught secondary school mathematics at a public high school located in a Midwestern state for almost 20 years. I worked with students in grades 9-12 teaching subjects including Integrated Mathematics, Algebra I, Algebra II, Algebra III, and Geometry. I also provide individual instruction for students needing assistance with Precalculus and Calculus. I began my career using a teacher-centered approach to instruction and soon realized that this method was not effective for many of my students. As a result, I slowly began to incorporate problem based-activities to engage them in mathematical practices conducive to development of their mathematical thinking. By the time I left secondary school mathematics to pursue a doctoral degree, I had transitioned to an almost equal distribution of the two instructional methodologies.
As part of my doctoral program, I worked as a graduate research associate with a research funded mathematics coaching professional development project for four years. The program was designed to encourage coaches, and thus classroom teachers, to develop research-based and reform-based instructional strategies to ultimately help students better learn, understand and apply mathematics and improve student achievement. The structure of this program was such that during the academic year, coaches from across the state and a few from neighboring states participated in monthly, two-day professional development sessions at a central location within the state. At the time of this study, coaches also participated in two days of regional PD with a small segment of the coaches from schools relatively close geographically. In order to complete the entire program, coaches were expected to participate for three consecutive years resulting in PD sessions for first-year coaches, second-year coaches, and third-year coaches. My responsibilities included, but were not limited to, setting up technology for monthly PD sessions, leading or assisting with instruction, and collecting, documenting, and analyzing data.

Through this mathematics coaching professional development project and prior to the beginning of data collection for this study, I had many opportunities to observe and interact with each of the participants on both personal and professional levels. Alex and Tracy had participated in the coaching program since its inception with Tracy serving as co-director and facilitating PD sessions on issues of social justice in mathematics education. Alex led PD sessions and was a regional facilitator for a small segment of the coaches from schools near his University. Luke spent two years observing PD sessions and occasionally participated in the discussions. Although I had observed and/or assisted
each of them at PD sessions, my attention to detail was significantly greater during this study than had been the case with previous encounters.

**Data Collection**

Collection of data using multiple sources of evidence (Yin, 2009) began in March, 2012 and concluded in March, 2013. Methods of data collection included interviews, observations, and collection of documents pertaining to the methods courses. A total of 27 hours 48 minutes of audio-recorded interview and observational data were collected for this study. The data included 9 hours 15 minutes of interviews and 18 hours 33 minutes of observations. Total data collection for Alex consisted of 8 hours 21 minutes of interviews and observations. Table 1 offers a description of data collection for Alex.

Table 1. Data Collection Summary – Alex

<table>
<thead>
<tr>
<th>Data Collected</th>
<th>Date</th>
<th>Location</th>
<th>Approximate Length</th>
<th>Learners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Interview</td>
<td>12/7/12</td>
<td>Mathematics Coaching Program PD session</td>
<td>1 hr 15 min</td>
<td>NA</td>
</tr>
<tr>
<td>1st Classroom Observation</td>
<td>1/29/13</td>
<td>University Special Education &amp; Intervention Specialist Mathematics Methods Course</td>
<td>Pre-Int: 9 min Obs: 2 hr 14 min Post-Int: 23 min</td>
<td>5 graduate students (preservice teachers) seeking intervention specialist licensure</td>
</tr>
<tr>
<td>2nd Classroom Observation</td>
<td>2/5/13</td>
<td>University Special Education &amp; Intervention Specialist Mathematics Methods Course</td>
<td>Pre-Int: 7 min Obs: 2 hr 27 min Post-Int: 22 min</td>
<td>5 graduate students (preservice teachers) seeking intervention specialist licensure</td>
</tr>
<tr>
<td>Professional Development Observation</td>
<td>3/4/13</td>
<td>Mathematics Coaching Program PD session</td>
<td>Pre-Int: 8 min Obs: 1 hr 4 min Post-Int: 12 min</td>
<td>6 female mathematics coaches (inservice teachers) - 3rd year of participation in program</td>
</tr>
</tbody>
</table>
Total data collection for Tracy consisted of 10 hours 27 minutes of interviews and observations. Table 2 provides a detailed description of data collection for Tracy.

Table 2. Data Collection Summary – Tracy

<table>
<thead>
<tr>
<th>Data Collected</th>
<th>Date</th>
<th>Location</th>
<th>Approximate Length</th>
<th>Learners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Interview</td>
<td>3/6/12</td>
<td>University office at main campus</td>
<td>1 hr 22 min</td>
<td>NA</td>
</tr>
<tr>
<td>1st Classroom Observation</td>
<td>11/26/12</td>
<td>University Early Childhood Mathematics Methods Course (K-3)</td>
<td>Pre-Int: 5 min Obs: 3 hr 50 min Post-Int: 47 min</td>
<td>14 female students, 11 undergraduate and 3 graduate, 4 seniors &amp; 10 juniors, all preservice teachers</td>
</tr>
<tr>
<td>• Pre-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Obs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Post-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd Classroom Observation</td>
<td>12/3/12</td>
<td>University Early Childhood Mathematics Methods Course (K-3)</td>
<td>Pre-Int: 5 min Obs: 1 hr 37 min Post-Int: 21 min</td>
<td>14 female students, 11 undergraduate &amp; 3 graduate, 4 seniors &amp; 10 juniors, all preservice teachers</td>
</tr>
<tr>
<td>• Pre-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Obs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Post-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Professional Development</td>
<td>2/7/13</td>
<td>Mathematics Coaching Program PD session</td>
<td>Pre-Int: 3 min Obs: 2 hrs Post-Int: 17 min</td>
<td>20 mathematics coaches (inservice teachers) – 2nd year of participation in program</td>
</tr>
<tr>
<td>Observation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Pre-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Obs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Post-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total data collection for Luke consisted of 9 hours of interviews and observations. Table 3 provides a detailed description of data collection for Luke.

An initial semi-structured, in-depth interview was conducted with each participant. The interviews lasted between one and two hours and occurred prior to any classroom or PD session observations. Each interview was audio-recorded and then transcribed for analysis. The purpose of the initial interview was to acquire background information with regard to the MTEs’ educational and teaching experiences as well as to
gain insight into their thoughts and beliefs on their priorities when teacher educating. See Appendix A for the initial interview protocol. More specifically, I explored the MTEs' perspectives regarding issues such as content selection for methods courses and PD sessions and the knowledge they relied on as a school teacher in comparison to being a mathematics teacher educator. Additionally, I gathered information regarding their perspectives on the similarities and differences between teaching mathematics and teaching the methods of teaching mathematics as well as working with PTs as compared to ITs. Other topics were discussed as well.

Table 3. Data Collection Summary – Luke

<table>
<thead>
<tr>
<th>Data Collected</th>
<th>Date</th>
<th>Location</th>
<th>Approximate Length</th>
<th>Learners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Interview</td>
<td>3/17/12</td>
<td>Mathematics Coaching Program PD Session</td>
<td>1 hr 22 min</td>
<td>NA</td>
</tr>
<tr>
<td>Follow up to initial interview</td>
<td>5/16/12</td>
<td>Mathematics Coaching Program PD session</td>
<td>42 min</td>
<td>NA</td>
</tr>
<tr>
<td>1st Classroom Observation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Pre-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Obs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Post-Int</td>
<td>11/28/12</td>
<td>University Secondary Mathematics Methods Course</td>
<td>Pre-Int: 7 min Obs: 2 hr 8 min Post-Int: 36 min</td>
<td>2 male &amp; 2 female students, 3 undergraduates &amp; 1 graduate student, all preservice teachers</td>
</tr>
<tr>
<td>2nd Classroom Observation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Pre-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Obs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Post-Int</td>
<td>12/5/12</td>
<td>University Secondary Mathematics Methods Course</td>
<td>Pre-Int: 12 min Obs: 1 hr 40 min Post-Int: 16 min</td>
<td>2 male &amp; 2 female students, 3 undergraduates &amp; 1 graduate student, all preservice teachers</td>
</tr>
<tr>
<td>Professional Development</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Pre-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Obs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Post-Int</td>
<td>2/8/13</td>
<td>Mathematics Coaching Program PD session</td>
<td>Pre-Int: 6 min Obs: 1 hr 33 min Post-Int: 18 min</td>
<td>20 mathematics coaches &amp; 1 facilitator (inservice teachers) – coaches 2nd year of participation in program</td>
</tr>
</tbody>
</table>
As a means to examine pedagogical practices during teacher-learner interactions, participants were observed three times. Each was observed twice as they conducted lessons for a university methods course and once as they led a PD session for the mathematics coaching professional development project. I took field notes during the observations and each session was audio-recorded so to augment the field notes. All audio recordings were then transcribed for analysis. The intent was to document the interactions of the educators and their respective students and of particular interest was the knowledge domains from which they drew during these interactions.

Brief pre- and post-observation interviews were conducted immediately before and following each observation. These interviews were semi-structured and used to elicit plans for the session and participants’ assessment of how their plans may have been enacted. See Appendix B for the pre- and post-observation interview protocols. The pre-observation interview was used to determine the goals for each session, specific learning objectives of the MTEs, how they planned to assess their instruction, and problems or issues they anticipated. During the post-observation interview, participants were asked to assess the success of the session, reflect on any changes they would make, and discuss anything that surprised them. They were also asked to comment on specific events of the session which have been instrumental to determining the basis for their pedagogical decision making in action.

Relative to the methods course observations, documents were collected pertaining to both the course in general and specific sessions. Course syllabi were obtained from each MTE to extract information concerning course objectives and provide a “big
picture” view of the design of the course not available through just two observations. In addition to the course syllabi, any documents that were given to the students for a particular session or activity were also collected as data. The documents were used to help determine what the teacher educators believed to be essential components in the practice of teacher educating within a specific content domain. They served to support and supplement evidence from the interviews and observations (Yin, 2009).

**Data Analysis**

Interview and observational data were collected from each participant and transcribed verbatim. All transcriptions were then imported into the document section of MAXQDA qualitative data analysis software. The code system, described in the next section, was set up within MAXQDA to be used for data analysis. Each transcribed interview and observation was then coded using the software. Further details of the analytical procedure are provided in the next section. For this multiple case study, data was analyzed in two stages. Each case was first treated as a comprehensive case in and of itself in order to identify knowledge domains used by each MTE in their individual practices. After each individual case profile was constructed, data was then analyzed across cases to find commonalities among their practices (Merriam, 1998).

**Individual case analysis.** With respect to each individual case, data was analyzed at three levels. The first level of analysis was conducted with the intent to develop a "big picture" view of the teacher educators practice based on themes from the abstracts of research published from January of 2008 through December of 2012. Abstracts of three journals including 1) *Journal for Research in Mathematics Education*
(JRME), 2) *Journal of Mathematics Teacher Education* (JMTE), and 3) *Teaching and Teacher Education* (TTE) were reviewed with the prominent themes indicated in each abstract being recorded and subsequently tallied. These three journals were selected based on their differing perspectives with regard to mathematics teacher education. JRME offered a strong research base devoted to the interests of mathematics teachers and mathematics education while JMTE focused on all stages of professional development of mathematics teachers and mathematics teacher educators. JRME seemed more inclined to promote theory development whereas JMTE was concerned with the development of improved teaching methods. The third journal, TTE, was selected based on its strong international context together with a general focus (not content specific) on teaching and teacher education. Once the frequencies of the themes were determined, approximately 10 of the most prominent themes were identified as potential descriptors of each teacher educator’s overall practice. However, the frequencies of the two themes ranking 9th and 10th were significantly less than the theme that ranked eighth and the decision was made to reduce the list of potential descriptors from 10 to 8. Additionally, the most frequently occurring theme was teacher knowledge and since three models of teacher knowledge formed the theoretical framework and was the focus of this study, the decision was made to exclude it from the list of potential descriptors as well. Consequently, the resulting list of seven themes used to provide a broad view of the MTEs practices included; 1) reflective practice (RP), 2) teacher learning (TL), 3) teacher beliefs and attitudes (TBA), 4) technology (TECH), 5) teacher practices (TP), 6) social justice (SJ), and 7) children's
mathematical thinking (CMT). These themes offered a view of the major areas of concern in the teacher education process.

At the second level of analysis, the three components of the theoretical framework, Shulman’s (1986) categories of content knowledge, practical knowledge of teachers as described by Elbaz (1983), and mathematical knowledge for teaching as presented by Ball et al. (2008) were used to code the transcribed data from the interviews and observations. MAXQDA qualitative data analysis software was used to code the data. Initial coding of data according to the indicators of the theoretical framework provided a more detailed analysis of the focus and influences with regard to the knowledge base and decisions of participants. That is, participants’ comments were coded according to indicators of knowledge identified by each construct. Subject matter content knowledge, pedagogical content knowledge, and curricular knowledge as described by Shulman (1986) were viewed as the knowledge needed by teachers and thus potential knowledge bases from which teacher educators might draw. See Table 4 for the list of codes and potential indicators of Shulman’s categories of content knowledge.

Elbaz’s (1983) categories of practical knowledge of teachers – content of practical knowledge, orientations of practical knowledge, and structure of practical knowledge provided the lens from which to view the influences on teacher educators’ decision making. See Table 5 for the list of codes and potential indicators of practical knowledge.
Table 4. Codes and Indicators for Shulman’s Categories of Content Knowledge

<table>
<thead>
<tr>
<th>Knowledge Type</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject matter content knowledge (SMK)</td>
<td>Understanding structures - organization of basic concepts, principles, and key facts, understanding the processes by which theories and models are established as valid, ability to justify acceptance of the facts, explain their importance, and connect them to other concepts, understanding why a topic is central to discipline</td>
</tr>
<tr>
<td>Pedagogical content knowledge (PCK)</td>
<td>Most useful representations, most powerful analogies, illustrations, &amp; examples, making subject comprehensible, understanding what makes learning topic easy or difficult, knowledge of student conceptions and misconceptions, strategies to reorganize understanding of students</td>
</tr>
<tr>
<td>Curricular knowledge (CRK)</td>
<td>Knowledge of programs designed for teaching subjects and topics, awareness of instructional materials available to teach programs, ability to decide when and when not to use specific materials for instruction, lateral and vertical curriculum knowledge</td>
</tr>
</tbody>
</table>

Table 5. Codes and Indicators for Categories of Practical Knowledge

<table>
<thead>
<tr>
<th>Practical Knowledge</th>
<th>Indicators</th>
</tr>
</thead>
</table>
| Content of practical knowledge (CPK)            | Knowledge of:  
  Self as teacher (SLF) – personal values, beliefs, needs of students, others  
  Surroundings (SUR) – classroom, relation with others, goals of school  
  Subject matter (SUB) – views of content, teaching learning and study skills  
  Instruction (INS) – acts of teaching, organization, evaluation of results  
  Curriculum (CUR) – course and materials development, needs of students |
| Orientations of practical knowledge (OPK)       | Situational (SIT) – in classroom, in school, in relation to colleagues  
  Personal (PER) – reflects personal values, teaching personally meaningful  
  Social (SOC) – influenced by social conditions and constraints  
  Experiential (EXP) – how experiences develop teacher’s knowledge  
  Theoretical (THE) – theory as guide to practice, sets limits on work |
| Structure of practical knowledge (SPK)          | Rule of practice – what to do and how to do it in a particular situation, influences teacher’s method of instruction in situation  
  Practical principle – broader more inclusive statement than the rule, may be a result of theoretical viewpoint, experience, or both  
  Image – combinations of teacher’s feelings, values, needs, and beliefs which form image of how teaching should be |
Mathematical knowledge for teaching (Ball et al., 2008), like Shulman's categories of content knowledge, was viewed as the knowledge needed by teachers. However, unlike Shulman's categories of content knowledge, mathematical knowledge for teaching was specific to mathematics and also considered to be a knowledge domain from which MTEs might draw. See Table 6 for the list of codes and potential indicators of mathematical knowledge for teaching.

Table 6. Codes and Indicators for Mathematical Knowledge for Teaching

<table>
<thead>
<tr>
<th>Mathematical Knowledge for Teaching</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>Mathematical knowledge &amp; skills not unique to teaching</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>Mathematical knowledge &amp; skills unique to teaching, mathematical tasks of</td>
</tr>
<tr>
<td></td>
<td>teaching (presenting mathematical ideas, modifying tasks, evaluating student</td>
</tr>
<tr>
<td></td>
<td>claims, etc.), requires knowledge beyond that taught to students</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
<td>Awareness of how mathematical topics are related, vision useful in seeing</td>
</tr>
<tr>
<td></td>
<td>connections to later mathematical ideas</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
<td>Ability to anticipate what students might think, ability to predict what is</td>
</tr>
<tr>
<td></td>
<td>interesting and motivating to students, ability to hear and interpret students’</td>
</tr>
<tr>
<td></td>
<td>thinking, ability to anticipate what students will do with a task, knowledge of</td>
</tr>
<tr>
<td></td>
<td>students common conceptions and misconceptions</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
<td>Sequencing content for instruction, choosing examples and when to use them,</td>
</tr>
<tr>
<td></td>
<td>evaluate instructional advantages and disadvantages of representations,</td>
</tr>
<tr>
<td></td>
<td>understanding what different methods and procedures afford instructionally</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
<td>Full range of programs designed for teaching particular subjects and topics,</td>
</tr>
<tr>
<td></td>
<td>variety of instructional materials available, characteristics serving as both</td>
</tr>
<tr>
<td></td>
<td>indications and contradictions for use of particular curriculum or materials</td>
</tr>
</tbody>
</table>

The following is an example of how data were coded within the analytical framework. In Luke’s initial interview, he discussed a teaching experience in a work-based education program and how the background of the plant laborers influenced his...
instructional practices. In do so, his use of pedagogical content knowledge domains and practical *knowledge of instruction* was evident. Based on the location of the plant, most of the learners he worked with were hunters and were familiar with a parabolic path as it related to shooting a gun.

Luke: So for instance, when we talked about quadratic functions, I can remember a lot of guys, there was a group of guys that were taking this class and uh talking about the path of a quadratic if you, the trajectory. Right? And as soon as I used trajectory, it turned out everybody who was sitting there owned a gun and they were all hunters. And so then the conversation turned to ballistics and uh and different types of firearms and firearms that shot flat versus ones that had big parabolic paths and depending on how far away the deer was that you were aiming at, uh that kind of thing. (Initial interview, 3/17/12)

Luke’s use of bullet trajectories to relate parabolic paths to quadratic functions offered evidence of Shulman’s *pedagogical content knowledge* and representing mathematics in such a way to make it comprehensible to his learners (Shulman, 1986).

His willingness to allow the discussion to move toward bullet trajectories which was interesting and motivating to his learners (Ball et al., 2008) indicated Luke’s reliance on *knowledge of content and students* with respect to mathematical knowledge for teaching. Practical *knowledge of instruction* was also evident relative to teacher-student interaction and his knowledge of the importance of hunting to the learners (Elbaz, 1983). Note that the same data was coded within each of the components of the analytical framework.

Once the data was coded, I used the combination of the MAXQDA software and Microsoft Excel spreadsheet to examine the frequency of occurrence of knowledge domains referenced and/or enacted during each interview and observation (Jaworski & Potari, 2009). Specifically, I examined the frequency of occurrence of each individual code according to the indicators of the four components of the analytical framework; 1)
big idea themes, 2) Shulman's categories of content knowledge (Shulman, 1986), 3) mathematical knowledge for teaching (Ball et al., 2008), and 4), and practical knowledge for teachers (Elbaz, 1983). Table 7 is an example of frequency totals for an observation. Data from Alex’s first classroom observation and including the pre- and post-interviews were assigned 266 codes from within the analytical framework. The breakdown of frequencies for each individual code was also determined and included in the analysis. For instance, the big idea theme of teacher practices was identified and coded 16 times in data associated with Alex’s first classroom observation (See Table 7). Note that for this part of the analysis, practical knowledge was divided into content of practical knowledge and orientations of practical knowledge. Data was not coded relative to the components of the structure of practical knowledge. This category was instead used to describe the MTEs’ guiding beliefs with respect to the practice of mathematics teacher educating.

Each individual code frequency was determined for interviews, observations, and the combined total. Figure 6 represents an example the breakdown of code frequencies for mathematical knowledge for teaching. Using Tracy’s data as an example, within the component of mathematical knowledge for teaching, the code, knowledge of content and students (KCS) was assigned to data a total of 66 times with 27 occurring in interviews and 39 in observations.
## Table 7. Example of Code Frequency Totals

<table>
<thead>
<tr>
<th></th>
<th>Pre-Int</th>
<th>Obs</th>
<th>Post-Int</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observation Data Segmentation (ODS)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Knowledge Data Segments (KDS)</td>
<td></td>
<td></td>
<td></td>
<td>56</td>
</tr>
<tr>
<td><strong>Big Idea Themes (BIT)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflective Practice (RP)</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
<td>0</td>
<td>7</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Technology (TECH)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
<td>2</td>
<td>13</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Children's Mathematical Thinking (CMT)</td>
<td>3</td>
<td>7</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td><strong>Shulman Content Knowledge (SHU)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
<td>6</td>
<td>20</td>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
<td>2</td>
<td>21</td>
<td>0</td>
<td>23</td>
</tr>
<tr>
<td><strong>Mathematical Knowledge for Teaching (MKT)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
<td>2</td>
<td>14</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
<td>1</td>
<td>9</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td><strong>Content of Practical Knowledge (CPK)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self (SLF)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
<td>0</td>
<td>8</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Instruction (INS)</td>
<td>2</td>
<td>33</td>
<td>2</td>
<td>37</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
<td>2</td>
<td>19</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td><strong>Orientations of Practical Knowledge (OPK)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Situational (SIT)</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Personal (PER)</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Social (SOC)</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
<td>0</td>
<td>7</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>26</td>
<td>208</td>
<td>32</td>
<td>266</td>
</tr>
</tbody>
</table>
For each participant, total code frequencies were also determined for each of the components of the analytical framework; big idea themes (BIT), Shulman’s categories of content knowledge (SHU), mathematical knowledge for teaching (MKT), content of practical knowledge (CPK), and orientations of practical knowledge (OPK). For example, the total code frequency for mathematical knowledge for teaching was determined by combining the frequencies for each of its six components. Figure 7 is an example of the total code frequency for each of the components of the analytical framework and shows, for example, that domains within Shulman’s categories of content knowledge were coded a total of 142 times with 50 occurring in interviews and 92 in observations. Then total frequency of codes for each MTE was determined to be the sum of the total frequencies for the four components of the analytical framework. Using Tracy’s data in Figure 7, the total frequency of codes is 1176. Since most data segments
were coded within multiple categories of the analytical framework, the total code frequencies do not represent the number of data segments coded but instead represent the total frequency of knowledge domains within the analytical framework identified and coded.

Figure 7. Example of the frequency totals for the components of the analytical framework.

In addition to the frequency totals for the combined interviews and observations, I also examined frequencies with respect to observational data only. First I determined the total number of knowledge data segments (KDS) for each observation. Knowledge data segments were determined by divisions in the observation data based on the MTE’s interactions with learners that resulted in shifts in instruction during the sessions. For example, shifts occurred as a result of moving from one activity to another, transitioning within an activity, responding to learner questions, or even using questioning techniques
to raise the cognitive level of a discussion. Each time a shift occurred, the segment was identified as a knowledge data segments which could then be assigned one or more codes according to the analytical framework. For each MTE, the total number of knowledge data segments from the three observations was calculated and then the total frequency of each code was expressed as a percentage of the knowledge data segments. For example, a total of 215 knowledge data segments were identified in the data from Luke’s three observations. Among the 215 knowledge data segments, 64 (30%) were coded as representing a situational orientation to practical knowledge. Figure 8 is an example of the codes of orientations of practical knowledge expressed as a percentage of knowledge data segments. For the three MTEs combined, a total of 613 knowledge data segments were identified and coded.

![Figure 8](chart.png)

Figure 8. Example of codes expressed as percentage of KDS.
To conclude the second level of analysis, a knowledge domain profile was developed for each MTE. Within each of the three components of the theoretical framework, code frequencies were used to determine primary, secondary, and peripheral sources of knowledge for the MTEs in their practice of teacher educating.

For example, within Shulman's categories of content knowledge, curricular knowledge might be considered a primary, secondary, or peripheral source of knowledge. The same would apply for each of the other two categories. Primary sources of knowledge were those with the highest code frequencies. Secondary resources of knowledge had code frequencies substantially less than the primary sources but represented at least 10% of the total codes of the component of the theoretical framework to which it belonged. For instance, to be considered a secondary source, the code frequency of knowledge of content and students had to represent at least 10% of the total code frequency of mathematical knowledge for teaching. Peripheral sources of knowledge represented less than 10% of the total code frequency for the component to which it belonged.

**Cross-case analysis.** At the third level of analysis, a cross examination of all cases was conducted as a means to generate a comprehensive list of knowledge domains expressed to be used, or used in practice by the participants in order to isolate specifics according to the subject areas. Frequency totals were used in the cross-case analysis to develop a knowledge domain profile for the group of MTEs as a whole. These totals for the cross-case analysis were determined to be the sum of the frequencies of the three individual cases. The same criteria as in the individual cases were used to identify
primary, secondary, and peripheral sources of knowledge for the group. This cross analysis allowed for the isolation of unique aspects of knowledge for teacher educating.

**Trustworthiness**

To establish trustworthiness of the research included in this study, techniques suggested by Merriam (1998) and Yin (2009) were utilized. To enhance internal validity, two types of data triangulation were used. First, the three MTEs represented multiple data sources with varying levels of experience and backgrounds. The second technique involved triangulation of data by using multiple data collection methods such as interviews, observations and written documents (Merriam, 1998; Yin, 2009). Data collection methods for each MTE included an in-depth initial interview, three direct observations in the natural setting of their instruction, pre- and post-observation interviews, and collection of artifacts of their practice.

The interview protocol was reviewed by outside mathematics teacher educators and paralleled against instruments used in similar research conducted by Olanoff (2011) and Zopf (2010). As previously mentioned, the Olanoff (2011) study investigated the mathematical knowledge required by teachers of elementary mathematics content courses in the area of multiplication and division of fractions. Zopf (2010) investigated the mathematical knowledge needed for teaching both preservice and inservice teachers in the content areas of fraction multiplication, whole number division, and decimal multiplication. Initially, both used semi-structured interviews followed by data collection and analysis with the results of the analysis determining future interview questions. This study used a similar approach for data collection.
The technique of member checking was also used to enhance the trustworthiness of the research. Initial drafts of the individual case profiles were shared with each of the participants to ensure that my interpretation of the data accurately represented them as teacher educators and their practice of teacher educating (Merriam, 1998). A second round of member checking occurred following revisions to the individual case profiles. Frequency data used in the development of each case profile was included in each round of member checking. Additionally, peer review and debriefing (Glesne, 2010) was used repeatedly throughout the study to establish credibility.
Chapter 4: The Case Profile of Alex

The next three chapters develop the individual case profile for each of three mathematics teacher educators (MTE); Alex, Tracy, and Luke. These three participants were selected partially because of accessibility and more importantly due to the differing levels of preservice teachers (PTs) with which they work in their capacity as university faculty. Alex taught mathematics methods courses for early childhood and middle childhood PTs, Tracy worked primarily with early childhood PTs, and Luke’s area of focus was with secondary PTs.

As part of the data collection process, each MTE was interviewed and observed. Documents relative to their observed sessions were also collected. Data were then analyzed to address the following research questions:

1. What knowledge domains do mathematics teacher educators draw from and use when providing content specific pedagogical experiences for preservice and inservice teachers?

2. How do these knowledge domains influence the activities of mathematics teacher educators as they design and implement pedagogical experiences for preservice and inservice teachers?

3. How can these knowledge domains be incorporated into the developmental stages of a model to conceptualize mathematical knowledge for teacher educating?
The first section of the case profile provides a brief summary of the details of data collection for the MTE. Next offers an opportunity to learn about the MTE’s educational background and teaching experiences as well as their research interests. Data from the initial interview follows to provide the MTE’s own perspective with regard to mathematics teacher educating. Analyses of data from the two classroom observations and the professional development session are then presented. The last section summarizes the case profile and provides a visual model of the knowledge domains used or expressed to be important by the MTE relative to educating teachers. A cross case analysis of the three MTE's is provided in Chapter 7.

**Alex – Data Collection**

An initial semi-structured interview was conducted with Alex in early December of 2012. This interview provided information regarding his educational and teaching experiences as well as his research interests. It also allowed me to gain insight into his initial articulation of the knowledge domains from which he believed he drew in his practice of teacher educating. I then observed Alex once in January and once in February during his *Problems and Practices in School Mathematics* course for five graduate students seeking intervention specialist licensure. These two observations provided an inside look at his interactions with preservice teachers and content of these interactions. One final observation took place in March during a mathematics coaching program professional development session where Alex worked with a group of inservice teachers (IT) serving as third-year coaches for their school district. Pre-and post observation
interviews were also conducted for each observation as part of the data collection process. See Table 8 for further details of data collection.

Table 8. Data Collection Summary – Alex

<table>
<thead>
<tr>
<th>Data Collected</th>
<th>Date</th>
<th>Location</th>
<th>Approximate Length</th>
<th>Learners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Interview</td>
<td>12/7/12</td>
<td>Mathematics Coaching Program PD session</td>
<td>1 hr 15 min</td>
<td>NA</td>
</tr>
<tr>
<td>1st Classroom Observation</td>
<td>1/29/13</td>
<td>University Special Education &amp; Intervention Specialist Mathematics Methods Course</td>
<td>Pre-Int: 9 min Obs: 2 hr 14 min Post-Int: 23 min</td>
<td>5 graduate students (preservice teachers) seeking intervention specialist licensure</td>
</tr>
<tr>
<td>2nd Classroom Observation</td>
<td>2/5/13</td>
<td>University Special Education and Intervention Specialist Mathematics Methods Course</td>
<td>Pre-Int: 7 min Obs: 2 hr 27 min Post-Int: 22 min</td>
<td>5 graduate students (preservice teachers) seeking intervention specialist licensure</td>
</tr>
<tr>
<td>Professional Development</td>
<td>3/4/13</td>
<td>Mathematics Coaching Program PD session</td>
<td>Pre-Int: 8 min Obs: 1 hr 4 min Post-Int: 12 min</td>
<td>6 mathematics coaches (inservice teachers) - 3rd year of participation in program</td>
</tr>
</tbody>
</table>

Introducing Alex: Education, Experience, and Research

Alex is a Caucasian male and at the time of this study was a University faculty member at a midsized Midwestern institution in Appalachia. As part of his faculty duties he taught mathematics methods courses for early childhood, middle childhood, and special education and intervention specialist degrees and licensure. He also participated in a funded mathematics coaching professional development research project located in a
Midwestern state. His duties for the mathematics coaching program included leading professional development sessions for third-year coaches in the program as well as serving as a regional facilitator for a group of coaches from schools situated in or near Appalachia. Alex was licensed to teach mathematics in grades 5-12 and language arts in grades 5-8. He held a Master of Arts degree and PhD in mathematics education.

Alex had taught at a parochial school for six years where he instructed a variety of courses including seventh grade mathematics, seventh grade English language arts, eighth grade pre-algebra, geometry, algebra 2, and AP calculus all in one year. Although teaching such a variety of subjects in one day was taxing, Alex claimed during his initial interview that this was when he began to see the importance of helping students make connections as they progressed through various levels of mathematics.

Alex: As, as a teacher who was having an eight period school day with eight different preps, it was, it was a logistical nightmare. Just to kind of give you a sense, one, in one school year when we had that eight period day, I started off with algebra two honors with high school juniors then second period was pre-algebra with eighth-graders, third period was geometry with high school sophomores, fourth period was just a regular algebra two class with high school juniors, fifth period was AP calculus for seniors. Then after lunch, after lunch was another geometry class and then I finished the day that year with back-to-back seventh-grade classes. I had seventh graders for two periods back-to-back. Seventh period was seventh-grade English language arts and then I had that same group of seventh graders for mathematics at the very end of the day. [Z: Wow!] It was a nightmare. [Z: Oh man.] It was a planning and logistics nightmare but what grew out of that was hey if I do this geometry unit really, really well in pre-algebra, I've laid the foundation for when I have these kids as sophomores. If I do a really, really good job or if I get really, get students thinking and active in geometry in 10th grade, some of that will transfer into their trig and pre-Calc class. So that's when I started to think about the connections and it's almost like that Brunner spiral curriculum thing (Initial interview, 12/7/12).

Towards the end of his appointment at the parochial school, he began teaching remedial mathematics courses at night at a local community college which eventually led
to a full-time teaching position where he taught slightly higher level developmental mathematics courses. An unexpected occurrence where some of Alex’s former parochial school students were enrolled in his developmental courses alarmed him, provoking him to think more deeply about his instructional practices, specifically towards his understanding of the limitation of his teacher-centered instructional approach. “So that sent up a really big red flag for me that what I was, the way, the teacher centered way that I was presenting the algebra, the algebra two, the geometry wasn't getting through to all the kids” (Initial interview, 12/7/12). While teaching at the community college, Alex was also taking courses to complete his Master’s degree in Mathematics Education. However, dissatisfaction with the teacher educators who instructed courses he completed forced him again to contemplate his instructional practices and ultimately led him to pursue a PhD.

Alex: What really bothered me when I was teaching at the community college and taking my Masters classes was my education classes. And I, I would leave after my education classes, almost inevitably it was all teacher centered, it was three-hour lectures, it was painful for a lot of those courses. And I would come out of those classes thinking I could teach this class so much better. I could do so many different things with this particular education class. So that's what got the wheels spinning about looking for a doctoral program (Initial Interview, 12/7/12).

While pursuing his doctorate, Alex’s graduate assistantship included teaching developmental courses in the mathematics department as well as mathematics education methods courses in the college of education. Additionally, he served as a university supervisor for preservices teachers where he continually entertained thoughts about what he could do “to make these preservice teachers more problem-based, more student centered, more interactive, more, more of those process standards from our last round of
NCTM standards” (Initial interview, 12/7/12). It was during this time that Alex also witnessed a move in the mathematics department, an attempt to transform the “math for elementary teacher series” (Initial interview, 12/7/12) into a more student-centered problem-based approach. Coincidentally, he also began participating in the Mathematics Coaching Project at this time. The combination of experiences influenced the direction of Alex’s career steering him toward early and middle childhood mathematics education.

Alex: Once I got involved with that math for elementary teachers and the math coaching program, everything shifted towards early and middle and it just, the light bulb went on that there's so much work to be done in those early and middle grades and so few math educators wanting to have that focus (Initial interview, 12/7/12).

Following completion of his dissertation Alex began a University faculty position while pursuing research interests focused on nontraditional approaches to early childhood mathematics, the development of quantitative literacy in early and middle childhood mathematics, and meaningful professional development experiences for early and middle childhood mathematics teachers. Additionally Alex has had “several professional development programs funded by the (state) Board of Regents that are, that's targeted towards integrating mathematics and English language arts in grades K-3” (Initial interview, 12/7/12).

Alex’s Initial Interview.

Prior to observing Alex, an initial interview was conducted during which information about his guiding principles regarding teacher educating were elicited.

Code frequency data. Results from the analysis of Alex's initial interview showed that the total code frequency was 176. That is data segments from the interview
were coded within the categories of the theoretical framework of analysis a total of 176 times. This does not mean that there were 176 data segments as most were coded within multiple categories. \textit{Teacher learning} (18), \textit{teacher practices} (15), and \textit{children's mathematical thinking} (9) were the big idea themes coded most frequently. Shulman's categories of content knowledge were most frequently represented as \textit{pedagogical content knowledge} (22) while \textit{knowledge of content and teaching} (19) and the \textit{knowledge of content and students} (10) occurred most frequently for mathematical knowledge for teaching. Content of practical knowledge was most often represented as \textit{knowledge of instruction} (17) while orientations of practical knowledge were most often \textit{experiential} (20). See Table 9 for the initial interview coding frequency totals.

\textbf{Concern for teacher learning.} A recurring theme evident throughout Alex's initial interview was his concern for \textit{teacher learning}. He acknowledged his responsibility for helping teachers, whether preservice or inservice, to grow in their ability to help students learn mathematics. He felt that "...we as teacher educators, as classroom teachers have the personal responsibility to move them (classroom teachers) somewhere" (Initial interview, 12/7/12). In order to help the teachers move forward, Alex identified professional credibility with them as a key factor to influencing their thinking.

Alex: I think that's what has added to my credibility as a math educator that not only can I relate to students, I can also relate really well to any grade level teacher because I've, I've been there where they are and I, I, I think it gives me an inroad into working with in-service and preservice teachers. I can, especially in my methods classes, I can weave in the experiences that I've had in a classroom and it just makes it more relevant to the ones who are in a teacher preparation program to hear those stories, to hear what I struggled with, to hear the tough decisions that I had to make, and, and how I had to almost invent activities and structures to lead
kids to understanding rather than just memorize and here's the algorithm and go do it (Initial interview, 12/7/12).

Table 9. Initial Interview Code Frequency – Alex

<table>
<thead>
<tr>
<th>Alex - Initial Interview Code Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observation Data Segmentation (ODS)</strong></td>
</tr>
<tr>
<td>Knowledge Data Segments (KDS)</td>
</tr>
<tr>
<td><strong>Big Idea Themes (BIT)</strong></td>
</tr>
<tr>
<td>Reflective Practice (RP)</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
</tr>
<tr>
<td>Technology (TECH)</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
</tr>
<tr>
<td>Children's Mathematical Thinking (CMT)</td>
</tr>
<tr>
<td><strong>Shulman Content Knowledge (SHU)</strong></td>
</tr>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
</tr>
<tr>
<td><strong>Mathematical Knowledge for Teaching (MKT)</strong></td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
</tr>
<tr>
<td><strong>Content of Practical Knowledge (CPK)</strong></td>
</tr>
<tr>
<td>Self (SLF)</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
</tr>
<tr>
<td>Instruction (INS)</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
</tr>
<tr>
<td><strong>Orientations of Practical Knowledge (OPK)</strong></td>
</tr>
<tr>
<td>Situational (SIT)</td>
</tr>
<tr>
<td>Personal (PER)</td>
</tr>
<tr>
<td>Social (SOC)</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>
Alex believed that incorporating his prior classroom experiences with children into his instructional practices as a MTE allowed the teacher learners to relate to him.

The theme of his concern for teacher learning also appeared as he discussed what he believed to be the key components when planning for a PD session and the role that mathematics content should play in the design. Alex suggested that grade band specific PD works best because he thought it extremely difficult to determine appropriate mathematics content for a PD session with teachers from non-homogeneous backgrounds and teaching responsibilities.

Alex: And that's why I think grade band specific PD works best. For example, I, I just had to review a manuscript over the past couple days in which it was, it was a manuscript about a professional development program that had teachers from third grade to 12 in one program. The needs of that student population is so different that, that I don't see how you could do any mathematics justice by having such a wide grade band. So I, I think that grade band plays a key role in that and then once, for example, the grade band of K-3, what are the critical aspects, the critical elements at that grade level? (Initial interview, 12/7/12)

Consequently, Alex preferred a narrow grade band focus such as K-3, for example, that would then enable a more thorough examination of the critical elements for that level of mathematics content.

Regardless of the level of teacher or mathematics content, Alex pointed that teacher learning is best accomplished using problem-based, student-centered activities, and reflective practice. When asked to comment on how different levels of learners in PD and classroom sessions affected his work, he indicated that he used similar instructional practices regardless of whether he was working with a group of K-3 teachers or middle childhood teachers though his choice of problems for motivating discussions would be different.
Alex: To me, good teaching and learning is still good teaching and learning regardless of what grade level you're in. So from a teacher education standpoint, I don't think those knowledge bases are different. I'm still going, if I walk into a group of K-3 teachers for a PD or something like that, I'm still going to use the same approach that I would with a group of middle school teachers. Content is going to be different, the tasks are going to be different but it's still going to be that student centered, figure it out, talk about your thinking… (Initial interview, 12/7/12)

Alex’s stance of “student centered, figure it out, talk about your thinking” method of helping teachers learn is consistent with various aspects of the adult learning theory of reflective practice (a big idea theme) as described by Schon (1992). Schon suggests that in reflective teaching, the teacher and student engage in reflective conversation with a situation and they sometimes reach a conflict that makes them rethink their understanding of what's going on. Teachers reflect on their own and others ways of seeing things, and attempt to communicate about them (Schön, 1992). Even though this reference was not made within the context of adult learning theories, orientation towards nurturing reflective practice was certainly prevalent in his repertoire of instructional strategies as he expressed his views on teacher learning.

**Teacher practices and pedagogical content knowledge.** Another big idea theme evident in Alex’s initial interview was related to teacher practices. In his references to teacher practices, Shulman’s (1986) pedagogical content knowledge appeared frequently. Additionally, and not surprisingly, Elbaz’s content of practical knowledge was present as knowledge of instruction. In most cases, when the theme of teacher practices was evident among the data, knowledge domains related to pedagogical content knowledge and knowledge of instruction were also apparent. For example, drawing on his experience of teaching developmental math courses at a community
college, Alex explained how an encounter with students he had previously taught in high school alerted him to the fact that his instructional practices had not been effective for all of his students. He utilized *pedagogical content knowledge* and *knowledge of instruction* in describing and evaluating the teacher centered methodology upon which he had relied.

Alex: What really threw up a red flag for me was the beginning of one semester when I walked into my night class at (community college) and there were four of my high school students in that class... So that sent up a really big red flag for me that what I was, the way, the teacher centered way that I was presenting the algebra, the algebra two, the geometry wasn't getting through to all the kids... I was doing the mathematics in a, in a very hands-on, concrete way but I was still very teacher centered.

The big idea theme of *teacher practices* was represented in the above excerpt as Alex shared the teacher centered methods he used to teach high school mathematics at the parochial school. Shulman's *pedagogical content knowledge*, specifically knowledge of strategies needed to reorganize understanding of learners (Shulman, 1986), was also evident in the data. The same data segment also reflected his *knowledge of content and teaching* with respect to understanding the instructional benefits, or lack thereof, of different methods and procedures (Ball et al., 2008). Within the domain of content of practical knowledge, Alex’s ability to evaluate the results of his instruction demonstrated his *knowledge of instruction* (Elbaz, 1983). Finally, the orientation of his practical knowledge in this data segment was classified as *experiential* since his analysis rested upon the impact of his work on learners.

Alex’s exploratory and *experiential* approach to teaching was also manifested in how he elaborated on his decision to use graphing calculator laboratories in his teaching,
a practice with which Alex felt little comfort. He also discussed using discovery
activities to explore graphs of sign and cosine.

Alex: Then I was trying different things with the 100 and the 200 level
mathematics classes with problem-based learning with graphing calculator labs.
That was when I first started, I did some of these things at the high school but I
wasn't too comfortable with it. [Z: Yeah.] At the community college, I was a
little, I had a little bit more freedom in terms of what I could try and what I was
encouraged to do so I did a few more calculator labs, a few more discovery
activities with sine and cosine graphs and what do you, put this into your
calculator, what do you notice, those, those kinds of things. (Initial interview,
12/7/12)

The theme of teacher practices was evident in this data segment based on his discussion
of problem-based learning with graphing calculator labs. The problem-based learning
and discovery activity reference also indicated the presence of Shulman's pedagogical
content knowledge with respect to ways of formulating the subject to make it
understandable for students (Shulman, 1986). The same two references also indicated
knowledge of content and teaching relative to understanding instructional value of
different methods and procedures (Ball et al., 2008). With regards to content of practical
knowledge, knowledge of instruction was briefly indicated when he discussed asking
students to enter sine and cosine graphs into a graphing calculator and then make
observations regarding their behaviors. This portion of the data segment offered
evidence of organization and teacher-student interaction (Elbaz, 1983). Additionally,
Alex drew on his experiential orientation to practical knowledge as he referred to
instructional practices he used when teaching 100 and 200 level mathematics courses at a
community college.
For Alex, data segments referencing *teacher practices* also occurred with regard to practices that were not successful with students and how such knowledge could be used in the practice of mathematics teacher educating. Referring back to his discussion of the essential components necessary for design of a successful professional development, he articulated his knowledge of various methods of multiplication algorithms. Alex believed that by introducing teachers to the various non-standard algorithms for multiplication, he attempted to cause a "cognitive dissonance" (Initial interview, 12/7/12) among them (Shulman, 1986). He felt that these experiences were successful in shifting teachers’ perspective on teaching.

Alex: But it's not only the students in the classroom, it's also the teachers [Z: Yeah]. When, when I share with them like the lattice method, the partial products, the use of the distributive property, the use of counters and arrays and diagrams and we start talking about how there's over 98 different ways to multiply. I mean like the Russian peasant method and all these other different strategies but we funnel kids into this one algorithm, it has to be just like the teacher does and that's so detrimental. When we, when we create that cognitive dissonance for teachers, that's when I feel they start thinking differently about teaching mathematics. (Initial interview, 12/7/12)

With regard to mathematical knowledge for teaching, Alex drew on his *knowledge of content and students*, in-service teachers being the students in this case, suggesting that they have a misconception of one method for multiplication (Ball et al., 2008). Content of practical knowledge, specifically *knowledge of instruction*, was indicated when he asserted his assessment of teaching actions (i.e. guiding children toward one algorithm for multiplication is "so detrimental") (Initial interview, 12/7/12). Alex's concern for *teacher learning* also was evident again as he focused on finding ways to alter teachers’ thinking.
Practical principle and student mathematical thinking. Whether discussing effective mathematics instruction for school children or the preservice and inservice teachers in his classroom and professional development sessions, Alex's structure of practical knowledge, and his overall practice of teacher educating, seemed guided by what Elbaz (1983) refers to as a "practical principle." He demonstrated an “embodied purpose in a deliberate and reflective way” (Elbaz, 1981, p. 61) which may be a result of “theoretical viewpoints, experience, or a combination of the two.” He consistently referred to a practice that is learner centered, problem-based, and interactive. When he was asked to comment on how his work as a classroom teacher compared to what he did as a teacher educator, Alex highlighted the importance of giving children opportunities to think in mathematics classes and suggested he needed to offer teacher learners the same experiences in professional development.

Alex: The, the whole approach of letting kids think. Problem-based, student centered, how much more effective teaching and learning can be when you allow students to struggle within certain parameters, when you put students in the position of mathematician, when you put students, no matter what grade level, when we give them the time and the opportunity to think, they can blow us away [Z: Yeah]. So it was those elements that I started weaving into my methods class and in the, in the professional development program with the, with the mathematics and the literacy for early childhood, I was able to realize the importance of teachers actually trying this out for themselves and bringing in that action research project element into the professional development and making that a year-long expectation as part of that professional development. (Initial interview, 12/7/12)

This guiding principle was also consistent with the big idea theme of teacher learning. Alex’s initial thoughts demonstrated his belief that actively engaging preservice teachers and/or inservice teachers in problem solving activities and then
asking them to reflect on their own thinking was paramount to their knowledge development.

Alex: …Okay, key components of a meaningful professional development. Number one, number one it needs to be problem-based and interactive. What do I mean by that? I've learned over the years that if we want to impact classroom teaching than we have, we as teacher educators, have to put classroom teachers back into the role of learner…So what I, what I feel is critically important for professional development is to, they don't know what student centered teaching in mathematics looks like and sounds like. So the, the biggest key is to put the classroom teachers in the role of learner in those problem-based experiences. They have to solve actual problems and not exercises.

His purposeful and deliberate attempt (Elbaz, 1981) to put inservice teachers into the role of learners is consistent with his practical principle. Alex believed in order for in-service teachers to understand what student centered instruction is, they first had to experience it as a student. They needed to be placed in an environment that was different from their experiences as passive learners in elementary, middle, and high school.

**Summary.** Alex's initial interview provided insight into his past experiences as a classroom teacher and as a teacher educator and how those experiences have influenced his instructional practices as well as the knowledge domains he draws on as he interacts with preservice and in-service teachers. The primary big idea revealed through the initial interview data suggest a focus on teacher learning, teacher practices, and children’s mathematical thinking. The primary knowledge domains from which Alex drew include Shulman's pedagogical content knowledge, and knowledge of content and students and knowledge of content and teaching from the domain of mathematical knowledge for teaching. Alex's content of practical knowledge was focused on knowledge of instruction while the orientation of his practical knowledge was primarily experiential. His initial
interview also indicated a guiding practical principle that relies on problem-based, student centered, interactive instructional practices.

**Methods Course Observations**

Two classroom teaching sessions were observed in a course titled "Problems and Practices in School Mathematics." The course is a mathematics methods course for special education and intervention specialists with an overarching goal of helping learners “to acquire the pedagogical knowledge, skills, and disposition that value and support conceptual understanding of young learners in elementary mathematics environments through the explication of student thinking and interaction in the act of mathematics a rich, intellectually honest, problem solving learning experiences” (Course Syllabus, 2012, See Appendix C). The class consisted of five graduate candidates, two male and three female students, four of which were seeking intervention specialist degrees and licensure. One student was pursuing initial licensure in middle childhood math and science at the master's level. The students came to the class with a variety of bachelor's degrees. Observations took place during the third and fourth week of the course and each lasted approximately 2 hours and 20 min.

**First Classroom Observation: A Common Core Focus**

The first classroom observation took place during the third week of the course on January 29, 2013. The atmosphere was relaxed as Alex and the five graduate candidates sat around one table. In the two weeks prior to this period, they had compared and contrasted the traditional classroom with student-centered classrooms through a discussion of literature and their participation in student centered activities. Alex's
objectives for this class session were to familiarize his preservice teachers with the new Common Core Standards for Mathematics and to also help them understand how young children develop number sense.

Alex: But that’s goal number one is to get them comfortable with the language, the purpose, the scope, and the practices of the new Standards. And then my second goal for today is to get them into thinking about how students, young students, come to counting number, number sense, and place value. (Pre-observation interview, 1/29/13)

He mentioned that the second objective would flow into the next observation as well in anticipation of their first assignment of creating a children's counting book. Alex never made it to the second goal as all but about 10 minutes of class time was spent on a discussion of the Common Core Standards.

After a brief introduction where Alex stated his goals for the session, the students were asked to read and think about two scenarios related to the concept of one-to-one correspondence (See Appendix D). He then asked the students to hold their thoughts on the two scenarios and moved to a quick review of five shifts in mathematics education they had discussed the previous week. The remainder of the session was devoted to familiarizing students with the Common Core State Standards for Mathematics. Specifically, approximately one hour of the class was spent discussing the history, goals, language, and assessments of the Common Core and another hour was devoted to an activity to familiarize students with the content of the Standards for Mathematical Practice. See Table 10 for a more detailed description of the sequence of session activities.
Table 10. First Classroom Observation Activity Sequence – Alex

<table>
<thead>
<tr>
<th>Obs. Number</th>
<th>Course Week Number/ Class Date</th>
<th>Sequence of Topics (Time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Week 3 1/29/14</td>
<td>1) Introduction – Goals for class (2min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2) PTs read and thought about 2 scenarios; The Little Shepherd Boy &amp; Kids Fighting Over Legos (6 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. Each scenario deals with young children who cannot count to the level they need</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Little shepherd boy – needs a method to make sure the sheep he releases in the morning all return at night</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. Kid fighting over Legos – need a method to determine who has the most Legos</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3) Reviewed 5 shifts in mathematics education discussed in previous class (7 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4) Common Core Standards for Mathematics discussion (57 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. History of Common Core</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Goals of Common Core</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. Language of Common Core (domains, clusters, standards)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d. Overview of mathematics content for grades K-8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e. New assessments (PARCC)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5) Standards for Mathematical Practices activity (58 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. PTs (without using words) each represented one mathematical practice on chart paper. Each PT worked separately without knowing which mathematical practice others were doing</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. PTs worked as a group to match chart paper representations with name of mathematical practice</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. Alex led discussion about connecting mathematical practices to discussions from previous weeks</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6) Closing – Alex briefly discussed 2 upcoming assignments (creating a children’s counting book and watching an episode of Sesame Street (3 min)</td>
</tr>
</tbody>
</table>

During the first hour, Alex moved back and forth between presenting information using an LCD projector and sitting at the table with the students looking at and discussing the actual Standards document. The students were more actively engaged during the second hour as they first worked individually to visually represent one standard for mathematical practice on a piece of chart paper and then worked collectively to match the name of a mathematical practice with each of the visual representations. Alex was involved only to
answer clarifying questions and to help facilitate the matching process. The class session lasted approximately 2 hours and 20 minutes.

**Code frequency data.** Analysis of data from the first visit to Alex's classroom, including the pre-observation and post-observation interviews, resulted in a total frequency of 266 codes. That is, data segments from the observation were coded within the categories of the theoretical framework 266 times. Table 11 summarizes the coding frequency totals for the first classroom observation.

As shown in the table, *teacher practices* (16) and *children's mathematical thinking* (11) were the big idea themes that were coded most frequently. Shulman's categories of content knowledge were most frequently represented as *pedagogical content knowledge* (30) and *curricular knowledge* (23). Within the categories of mathematical knowledge for teaching, *knowledge of content and teaching* (18), *knowledge of content and students* (12), *knowledge of content and curriculum* (10), and *horizon knowledge* appeared most often. *Knowledge of instruction* (37), *knowledge of curriculum development* (21) and *knowledge of surroundings* (18) most frequently represented in Alex's content of practical knowledge.

The primary orientation of Alex’s practical knowledge was *experiential* (10). An obvious difference between data from the initial interview and the first classroom observation was the role that knowledge domains relative to curriculum played on the discussion of the Common Core Standards for Mathematics. Similar to the initial interview, *pedagogical content knowledge, knowledge of instruction, and experiential knowledge* were the domains Alex drew from in his interactions with students.
<table>
<thead>
<tr>
<th>Observation Data Segmentation (ODS)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge Data Segments (KDS)</td>
<td>56</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Big Idea Themes (BIT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflective Practice (RP)</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
</tr>
<tr>
<td>Technology (TECH)</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
</tr>
<tr>
<td>Children’s Mathematical Thinking (CMT)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shulman Content Knowledge (SHU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mathematical Knowledge for Teaching (MKT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Content Knowledge (CCK)</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Content of Practical Knowledge (CPK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self (SLF)</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
</tr>
<tr>
<td>Instruction (INS)</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Orientations of Practical Knowledge (OPK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Situational (SIT)</td>
</tr>
<tr>
<td>Personal (PER)</td>
</tr>
<tr>
<td>Social (SOC)</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Int</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>26</td>
</tr>
</tbody>
</table>

Table 11. First Classroom Observation Code Frequency – Alex
Teacher practices and pedagogical content knowledge. Similar to the initial interview, teacher practices and knowledge domains relative to pedagogical content knowledge as well as knowledge of instruction were evident in Alex's discussions and in his interactions with the preservice teachers. In the pre-observation interview, Alex was asked about problems or issues he anticipated in the upcoming session. He commented on difficulties his preservice teachers might experience in understanding the mathematical practices because their own school experiences were "antithetical" to the desired practices.

Alex: I anticipate more difficulty with the mathematical practices, what do these look like, sound like, because they are so antithetical to what these folks experienced in their prior schooling.

Z: What have they experienced, I mean, in terms of…

Alex: Very traditional and what I was able to discern about the conversations in our first week and in our second week was that most of them experienced a very teacher-centered, a sit down, shut up, and learn approach. I think we’re going to see some, not some high levels of mathematics anxiety but maybe some uncertainty with the mathematics that they’ll be doing. [Z: Yeah.] Being graduate students I don’t think they will have any mathematical difficulties with the activities but I’m thinking that this is probably their second full blown exposure to self-directed hands-on types of learning especially with numbers, something that we automatically take for granted in terms of kids going one, two, three, four, five and the multiple layers that lie underneath that. (Pre-observation interview, 1/29/13)

Alex's use of knowledge of instruction was manifested in his comments about the acts of teaching (Elbaz, 1983) and his preservice teachers’ prior experiences with mathematics instruction. His ability to recognize what would make learning and understanding the mathematical practices difficult, referencing the preservice teachers’ school experiences, evidenced Shulman's pedagogical content knowledge (Shulman,
1986) as well as knowledge of content and students (Ball et al., 2008). The big idea theme of teacher practices emerged in his references to “teacher-centered” instruction and “self-directed, hands-on types of learning.”

The same knowledge domains were evident at the beginning of the classroom observation as well. Alex utilized his pedagogical content knowledge and knowledge of instruction to provide the PTs with an activity they could use with children in order to develop the concept of one-to-one correspondence. He asked the teacher candidates to read and think about solutions to two scenarios exactly as he would with a class of school children. One of the scenarios, for example, depicted two children arguing over who had more Legos. The goal was for the PTs to offer a way to resolve the issue without actually counting the Legos. See Appendix D for the activity sheet.

Alex: Alright, if you look at our agenda, and like right behind our agenda there should be a sheet of paper with two different scenarios on it. The first one is about a little shepherd boy and the second one is about two kids fighting over Legos. So what I’m going to ask you to do is to read both of those paragraphs and then brainstorm or jot down some ideas about what you could do if you were in this situation to help the little shepherd boy and the kids fighting over Legos. So let’s take about three-four minutes to get that knocked out. (First classroom observation, 1/29/13)

Alex's selection of this activity, knowing what example to use and when to use it, provided evidence of his knowledge of content and teaching (Ball et al., 2008). By asking the PTs to brainstorm possible solutions, he evidenced Shulman's pedagogical content knowledge, a way to represent the topic of one-to-one correspondence that would make it understandable for learners (Shulman, 1986). Drawing from his knowledge of instruction relative to teacher-student interaction (Elbaz, 1983) and representing the big idea theme of teacher practices, Alex chose to ask the PTs to read the activity.
individually first and then to discuss their ideas with peers. He remained silent throughout these events.

**Experiential orientation of practical knowledge:** As the focus of the class session transitioned into a discussion of the Common Core Standards for Mathematics, Alex began to draw on his previous work with classroom teachers to help his PT's understand the difficulties teachers might encounter as they try to implement the Common Core Standards. The following dialogue was a result of a question about the critical areas of focus in second grade.

Alex: What else in grade 2?

PT: All figures with addition and subtraction is (inaudible) in the thousands, those go together.

Alex: So building fluency with addition and subtraction. Does it say that we teach them separately? No. This is another thing that’s kind of scaring early childhood teachers, is the idea of teaching these things together so they relate to each other so kids can make connections between addition and subtraction.

PT: That doesn’t seem that scary.

Alex: When I’ve taught addition and subtraction separately for 20 years, then this is a big change for some folks. (First classroom observation, 1/29/13)

Alex's response about teaching addition and subtraction together and how that approach might concern practicing teachers was likely drawn from conversations heard while supervising preservice teachers in the schools or working with in-service teachers at PD sessions. This data again suggested that the orientation of his practical knowledge was primarily **experiential**.

**Curricular knowledge domains.** Continuing his focus on the Common Core, Alex relied heavily on knowledge domains related to curriculum in an attempt to help
familiarize his learners with the curriculum they would likely be responsible for teaching as they begin their careers. In this data segment, he drew on current and future curricular knowledge to compare the current state academic content standards with the new mathematical standards to make the point that the new standards have fewer topics but the expected depth of understanding increases.

Alex: Just as a means of comparison, in what we currently have in (state), there are thirteen standards for number, number sense, and operation. There are seven standards for measurement. I’m sorry, these are indicators specifically. There are five indicators for geometry. There’s seven indicators for patterns, functions, and algebra. And there’s eight different indicators for data analysis and probability. Currently in the second grade, there’s eight, fifteen, twenty-two, thirty-five, thirty-five separate... That’s what we currently have. Now turn the page. So what the Common Core did, we said it was going to shrink the number of topics, so now in second grade, what we have is just right here. Limiting the number of standards but going deeper. In terms of what’s right here on this left-hand page, it’s like a two page spread. (First classroom observation, 1/29/13)

Shulman's curricular knowledge, specifically vertical curriculum knowledge (Shulman, 1986), and horizon knowledge (Ball et al., 2008) were evident in his knowledge of the number of second grade indicators and breadth and depth of mathematical understanding supported with them.

Later during the same discussion, Alex drew on the various domains of curricular knowledge in response to a PT's question. Relying on his middle childhood curriculum knowledge, Alex attempted to clarify the meaning of the term "number system" with respect to grades six, seven, and eight.

PT: What does it mean by the number system six, seven?

Alex: Where are you?

PT: In grades six, seven, and eight.
Alex: In grades six, seven, and eight, the number system extends beyond counting numbers and whole numbers. This gets, they’ve got fractions back in third, fourth, and fifth, so that brings in fractions, decimals. The number system talks about expanding to positives and negatives. It also talks about expanding into real numbers so expanding the number system beyond just zero and up and fractions. (First classroom observation, 1/29/13)

Alex’s knowledge of how the connections of various strands, specifically how the number system expands from fractions in the third, fourth, and fifth grade to including real numbers in grades six, seven, and eight evidenced Shulman's *vertical curriculum knowledge* (Shulman, 1986) and *horizon knowledge* of mathematical knowledge for teaching (Ball et al., 2008). Additionally, the content of practical knowledge was represented as *knowledge of curriculum development* in terms of the needs of students at various grade levels (Elbaz, 1983).

**Knowledge of surroundings.** The post observation interview provided evidence of how Alex, within the content of practical knowledge, utilized his *knowledge of surroundings*. In explaining why he chose to use giant playing cards to randomly assign a mathematical practice to each PT for a class activity, he concluded that the practice “also just exposes them to the manipulative wall. They have free reign if they want to borrow some of these to take out into their field placement classroom” (Post-observation interview, 1/29/13). His follow up comments further revealed how he relied on his knowledge of diversity among the PTs’ backgrounds.

Z: When do these guys get their field experience?

Alex: As they pick it up as they go along in their special ed courses.

Z: Ok, so how long do they spend in this master’s program then?

Alex: A year… Anywhere from a year and half to two years.
Z: Oh, okay. So they’ve got four years in terms of the undergraduate and then a year and a half to two years of the master’s.

Alex: But then again maybe their undergraduate degree is in something completely different. For example, Mary, down there on the end, her undergraduate was in communication disorders. [Z: Oh.] Smith’s undergraduate degree is from Hong Kong. I’m not sure what hers was in. Steve’s undergraduate was in social work. A wide range of academics, it’s just like a regular public school classroom. You never know who you’re going to get. And you can’t stack the deck in terms of I want these. You don’t get to pick and choose. (Post-observation interview, 1/29/13)

Alex drew on his content of practical knowledge, specifically his knowledge of surroundings, as he recognized that his classroom setting (Elbaz, 1983) in relation to the undergraduate degrees of his students affects how long it might take them to complete their masters program. The segment also offered evidence of a strong teacher-student relation (Elbaz, 1983) based on the fact that he had taken the time to learn about their educational background.

**Alex’s practical principle.** Alex's guiding practical principle was apparent in his initial interview and prevailed in this class session. The practical principle discussed in his initial interview of problem-based, student-centered, interactive instructional practices was again evident in how he engaged his PT's in exploring Standards for Mathematical Practice of the Common Core. The activity was not problem based in the sense that the PTs had to solve mathematical problems but they were asked to provide an imagery depicting one of the mathematical practices. The imagery was expected to offer, as a sole model, all relevant detail without a need for elaboration by the author. In the following data segment, Alex explains the activity to his PTs.
Alex: So what I’m going to ask you to do, we’ve got some markers over here. We’ve got some chart paper over here. We all have the number of a mathematical practice. What I’m going to ask you to do is take some chart paper and take some markers and without using words communicate your mathematical practice to everybody else in the room. Don’t label what number it is. Keep that to yourself for right now but we’ve got markers, we’ve got some chart paper. Let’s take a few minutes to read about your mathematical practice. I’ll let you find a spot anywhere in the room to create your visual, to create your chart paper. I’ll use this one since it ripped. And then once we get them all finished, we’ll put them all up here on the board and we’ll see if we can identify that mathematical practice. (First classroom observation, 1/29/13)

The problem the students had to solve was how to pictorially represent the mathematical practice they were randomly assigned. The activity was student-centered as each PT first had to read and internalize a mathematical practice. They had to consider what this mathematical practice would look and sound like in teaching and learning – without being too literal in their representation. The session then became interactive when the PTs collaborated to discuss and identify the mathematical practice that matched each representation. This matched his guiding practical principle.

**Summary.** Alex's first classroom observation confirmed his use of knowledge domains related to knowledge of instruction and pedagogical content knowledge. The fact that the Common Core was the primary focus for this class explains, in part, why Alex relied so heavily on the curriculum knowledge domains. The guiding principle of problem-based, student centered, interactive instructional activities was observed as well.

**Second Classroom Observation: Number and Number Sense**

The second classroom observation took place during the fourth week of the course on February 5, 2013. The focus of the second class was to help the PTs to understand how students make connections between number symbol, number word, and number
quantity. He also wanted them to think about how to help children move forward from counting to addition. Alex introduced his learners to SmartBoard technology during this classroom session as well.

The atmosphere was again relaxed but this week Alex began the class standing in front of a SmartBoard questioning the students about their familiarity with the technology. Following a brief discussion, he immediately used the SmartBoard to present what he called a "Number of the Day" activity. Alex suggested that this activity could be used to continually review number concepts. He asked his PTs to complete the activity just as he would with schoolchildren, starting with a two digit whole number and later considering a decimal number. Following completion of this activity, Alex returned to the one-to-one correspondence scenarios from the previous week. Both scenarios, The Little Shepherd Boy and Kids Fighting Over Legos (See Appendix D) presented a situation in which children could not count to the level they needed and it was the reader's responsibility to offer an alternative solution to this problem. Eventually, a discussion of rote counting versus meaningful counting ensued where Alex suggested that teachers often assume a child understands counting just because the child can say the numbers from one to ten. He posited that the teacher needs to determine if the child is simply repeating words or understanding the quantities represented. Following that discussion, Alex read the book Ten Black Dots by Donald Crews to his graduate students just as he would to a group of first graders. He wanted the PTs to see an example of a children's counting book because as their first assignment, they were going to be asked to create one. He also wanted them to notice how the author used number quantity, number
word, and number symbol throughout the book. Alex then moved to a discussion of, using his terminology, “bridges from counting to addition.” After the bridges segment, students then discussed an episode of Sesame Street they each had watched for their homework assignment explaining how number and counting had been presented. The final activity for this session revolved around the student "centers" Alex had created. The PTs moved around five different centers (tables) in the room and examined the materials related to number concepts at each center. Alex then closed the session by discussing expectations for their children's counting book assignment and then briefly discussed skip counting by three. Table 12 summarizes the sequence of activities during the second classroom observation.

Code frequency data. Analysis of data from Alex's second classroom observation resulted in a total code frequency of 447. Children’s mathematical thinking (22) and teacher practices (20) were the big idea themes coded most frequently. Within Shulman’s categories of content knowledge, pedagogical content knowledge (56) and subject matter knowledge (34) appeared most often. Knowledge of content and teaching (40), specialized content knowledge (39), knowledge of content and students (35) were most frequently coded within the domain of mathematical knowledge for teaching. Regarding content of practical knowledge, knowledge of instruction (60), knowledge of subject matter (29) and knowledge of surroundings (14) were represented most frequently.
Table 12. Second Classroom Observation Activity Sequence – Alex

<table>
<thead>
<tr>
<th>Obs. Number</th>
<th>Course Week Number/ Class Date</th>
<th>Sequence of Topics/Activities (Time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Week 4 2/5/14</td>
<td>1) Introduction – Discussed PTs familiarity with Smart Board (3 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2) Number of the Day activity (35 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. PTs worked through activity that reviewed number and place value presented on Smart Board</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Content – addition, divisibility by 2, 5, and 10, multiplication by 10, number sentences, expanded form, place value, prime and composite</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. Alex adjusted activity to decimal of the day – content included language, place value, division/multiplication by 1000, expanded form, model, number line location</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3) Little Shepherd Boy &amp; Lego Activity (15 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. Alex had PTs discuss possible solutions to the two scenarios they read in week 3.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Content – one-to-one correspondence</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4) Discussion of rote counting/meaningful counting (12 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. Number quantity, number word, and number symbol</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5) Ten Black Dots book by Donald Crews (7 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. Alex read book to PTs and discussed how author connected number quantity, number word, and number symbol</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6) Bridges from counting to addition (36 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. Number pattern recognition – Alex quickly showed PTs a paper plate with a group of colored dots on it. PTs had to determine quantity of dots and one more than quantity of dots and explain their reasoning</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Recognizing groups of 5 and 10 – Alex introduced manipulatives called 5 frames (5 connected red squares, each square may or may not have black dot in the center) &amp; 10 frames (two rows of connected 5 frames). He showed and discussed video clip of 2nd graders using 10 frames</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. Anchors to 5 and 10 – One PT held row of 5 snap cubes behind back, broke it into 2 groups, and showed other PT one of the groups. Other PT said how many left behind back</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d. Skip counting – Alex discussed skip counting by 5, 10, &amp; 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7) PTs discussed how number and counting were presented in an episode of Sesame Street watched as an assignment (7 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8) Activity centers – PTs examined and discussed materials/activities at 5 different “centers” in the room (22 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. Number boxes (boxes filled with homemade manipulatives), secret number puzzles, domino activity, hundreds chart activities, and children’s counting books</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9) Closing – Alex discussed expectations for children’s counting book assignment and skip counting by 3</td>
</tr>
</tbody>
</table>
The primary orientation of Alex’s practical knowledge was again *experiential* (28).

Table 13 presents the code frequency totals for the second classroom observation.

Table 13. Second Classroom Observation Code Frequency – Alex

<table>
<thead>
<tr>
<th>Alex - 2nd Classroom Observation Code Frequency</th>
<th>Pre-Int</th>
<th>Obs</th>
<th>Post-Int</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation Data Segmentation (ODS)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Knowledge Data Segments (KDS)</td>
<td></td>
<td></td>
<td></td>
<td>81</td>
</tr>
<tr>
<td>Big Idea Themes (BIT)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflective Practice (RP)</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Technology (TECH)</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
<td>0</td>
<td>18</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Children's Mathematical Thinking (CMT)</td>
<td>0</td>
<td>21</td>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>Shulman Content Knowledge (SHU)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
<td>1</td>
<td>30</td>
<td>3</td>
<td>34</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
<td>3</td>
<td>46</td>
<td>7</td>
<td>56</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
<td>1</td>
<td>6</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>Mathematical Knowledge for Teaching (MKT)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>2</td>
<td>34</td>
<td>3</td>
<td>39</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
<td>0</td>
<td>5</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
<td>1</td>
<td>28</td>
<td>6</td>
<td>35</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
<td>3</td>
<td>31</td>
<td>6</td>
<td>40</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Content of Practical Knowledge (CPK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self (SLF)</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
<td>0</td>
<td>6</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
<td>1</td>
<td>25</td>
<td>3</td>
<td>29</td>
</tr>
<tr>
<td>Instruction (INS)</td>
<td>2</td>
<td>52</td>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Orientations of Practical Knowledge (OPK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Situational (SIT)</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Personal (PER)</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Social (SOC)</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
<td>2</td>
<td>19</td>
<td>7</td>
<td>28</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>22</td>
<td>350</td>
<td>75</td>
<td>447</td>
</tr>
</tbody>
</table>
The big idea themes of *teacher practices* and *children’s mathematical thinking* were consistently represented in both classroom observations. Similar to the first observation and the initial interview, *pedagogical content knowledge*, *knowledge of instruction*, and *experiential knowledge* were knowledge domains from which Alex frequently drew in his interactions with students. Knowledge domains related to subject matter were coded considerably more frequently in this observation than in the initial interview or the first classroom observation. Given that a majority of class time was spent discussing and/or completing activities related to number and number sense, it is reasonable that Alex would draw from knowledge domains related to subject matter as he interacted with his preservice teachers.

**Teacher practices and children’s mathematical thinking.** The big idea themes of teacher practices and children’s mathematical thinking appeared frequently in the second classroom observation. Alex used a variety of *teacher practices* such as whole class discussion, analysis of a video clip, use of SmartBoard technology/manipulatives to enhance instruction, and centers to explore the concept of number sense. As he progressed through the activities with the PTs, Alex used questioning and teacher-student interaction to guide his preservice teachers towards focusing on children’s mathematical thinking.

Alex: Let’s come up here (T points to a representation of a number line without numbers). Why do you think I love this one?

PT: They have to put their own numbers in.

Alex: The end points on my number line aren’t labeled so the students would have to label them and think about the relative size of the decimal of the day. For example, if this were 0 and that were 100, where would my decimal of the day
go?

PT: Way down by the 0.

Alex: Way, way, way down here. Not on the 0 but...But if I got a wild hair maybe this is 0 and that’s 1, where does the decimal of the day go?

PT: A little over halfway towards the 1?

Alex: Right there?

PT: Yeah.

Alex: It gets kids to think about relative size. With this would be closer to this endpoint or would it be closer to this endpoint? (Second classroom observation, 2/5/13)

When Alex pointed to a blank number line and asked the PTs why they thought he loved that specific question on the activity, he wanted them to recognize that the number line had no numbers and thus the children would have to think mathematically in order to label it correctly. His final response was even more direct when he stated that “It gets kids to think about relative size.”

Later in the session, Alex had discussed pattern recognition when helping students understand groups of five and 10 through the use of five frames and ten frames (See Table 10, section 6b). As part of the discussion, he asked the PTs to view a video segment of a group of second graders using 10 frames and then began a discussion of the actions of both the students and the teacher.

Alex: I’ve got a little classroom video and these are second graders in a guided math lesson using a ten frame. Do I need to hit the lights or can you see it? Let me check my volume here, speaker on. [Class watches video.]

Alex: Okay let’s stop it right there (Alex stops video). So how did you see these kids making use of a ten frame or a double ten frame?
PT: They’re doing reflexive addition just by pattern recognition so they can immediately recognize that’s five. I will recognize that this plus…

Alex: Getting them to seek… Was he fixating on keeping all the numbers in columns? Was he fixating on the way the kids wrote the number sentence? (Second classroom observation, 2/5/13)

Obviously asking his PT's to watch and analyze video of a second grade classroom provided evidence of the big idea themes of teacher practices in this particular data segment. When Alex stopped the video he immediately directed his PT's to student mathematical thinking by asking if they noticed how the kids made use of the ten frame and double-team frame making it a point to note that the teacher was not concerned with the semantics of what the students were doing implying that he was focused on their thinking as well.

**Knowledge of instruction and pedagogical content knowledge.** As with previous data, evidence of Alex's use of the domains of knowledge of instruction and pedagogical content knowledge appeared throughout this observation period. Continuing with the "Number of the Day" lesson, he was able to anticipate difficulty that his PTs might experience as they worked through the activity and were asked to write the number of the day in expanded form. Realizing that the mathematical background of the PTs was not strong, Alex seized opportunity to reorganize their understanding.

Alex: This one is a little bit more open ended allowing kids that flexibility to think on their own. What in the world does this one mean?

PT1: I don’t know, I left that one blank.

PT2: Me too.

Alex: Great, great chance to talk about this one. When we’re talking about expanding a number, break it up into its place values. Ok? Each different math
curriculum series, each different math textbook series probably has a different way to do this or to write it in notation but when you see the words expand it, we’re talking about breaking it up into place values. How many thousands, how many tens, how many hundreds, how many ones? So in our number of the day, how many tens do we have? (Second classroom observation, 2/5/13)

When Alex asked "what in the world does this one mean,” he was anticipating that PTs would not remember what it meant to write a number in expanded form and in doing so, he drew from his knowledge of content and students, specifically his ability to anticipate what students might do with a task and whether they would find it easy or difficult (Ball et al., 2008). Similarly, he had a feel for the PTs preconceptions evidencing Shulman’s pedagogical content knowledge (Shulman, 1986). His knowledge of instruction was prevalent as he used teacher-student interaction to discuss expanded form (Elbaz, 1983).

Data suggesting Alex's use of the knowledge of instruction and pedagogical content knowledge appeared again in the form of an activity he used to help children to bridge the gap as they transition from counting to addition. He briefly showed the PTs a paper plate with a series of colored dots on it and asked them to show him the number of dots they saw. He also asked them to explain how they determined the total number of dots.

Alex: So our first bridge (Alex is writing on the board) is something called number pattern recognition. So what I’ve got, what I’ve got are just a bunch of paper plates with some dots on them. I’m going to mix these up a little bit and I’m going to show these to you and then I want you to show me on your fingers how many dots you see. Okay? We’ll start off with this one. Ready? Three, two, one, here it is, show me. (Alex showed a plate with five dots arranged as if on a die.)

PT: A die.

Alex: You saw what?
PT: Like a die.

Alex: Ok you automatically saw this “Oh that looks like five on a die” so it’s got be this many. Did anyone have the same idea or see it differently?

PT: Well there’s four on the outside and one in the middle.

Alex: Ok, so we haven’t talked about addition. We haven’t talked about number sentences but in getting kids to talk about how they see this, we’re planting seeds. Hey five is four and one. (Second classroom observation, 2/5/13)

Within the content practical knowledge, Alex utilized his knowledge of instruction, specifically teacher-student interaction (Elbaz, 1983), to get PTs to explain their thinking which led them to talk about combinations of groups of dots that formed the total. Alex drew on Shulman's pedagogical content knowledge by suggesting that getting kids to talk about what they see results in "planting seeds," understanding (Shulman, 1986) of children as they move from counting to addition. His understanding of what this method offers instructionally (Ball et al., 2008) provided evidence of his knowledge of content and teaching.

Subject matter knowledge domains. The nature of the activities in the second classroom observation resulted in frequent use of domains related to subject matter knowledge. Specifically, mathematics content played a far greater role in this class period than in the session devoted to the Common Core Standards. Although not as strongly represented in the data as pedagogical content knowledge, subject matter knowledge domains appeared far more frequently in this observational period than the previous one or in the interviews. Alex's reliance on knowledge domains relative to subject matter was evident as concept of divisibility was discussed during the "Number of the Day" activity. In this data segment, he used his knowledge of subject matter,
specifically divisibility, to help PTs clarify their understanding of divisibility by two and to use appropriate mathematical language.

Alex: I’m gonna talk about evenly divisible right here. There’s a reason why I selected 2, 5, and 10 that we’ll talk about in just a few moments.

Hi Steve (Steve just arrives in class). How are you doing?

PT: Good.

Alex: Good. We’re working with a number of the day activity up here on our Smart Board. Is it evenly divisible by 2? No. How do you know that?

PT: Because it’s an odd number.

Alex: It’s an odd number. What’s another way to say it? Or what’s another way that students could think about is it divisible by 2?

PT: If there’s one left over.

Alex: Ok, if there’s one left over. If a number is divisible by two, what does the ones digit have to be?

PT: Even.

Alex: Even, it’s got be a 2, 4, 6, 8, or a 0. (Second classroom observation, 2/5/13)

Alex drew on Shulman’s subject matter content knowledge to establish the validity (Shulman, 1986) of a number divisible by two being an even number. With respect to mathematical knowledge for teaching, he relied on specialized content knowledge to perform specific mathematical tasks of teaching including evaluating the PT's claim of "there's one left over" and critiquing language use (Ball et al., 2008). Alex's critique of the language that PTs used also offered evidence of his knowledge of subject matter within the content of practical knowledge domain as he encouraged more effective modes of mathematical communication (Elbaz, 1983).
Specialized content knowledge was also exhibited by Alex as he listened to, interpreted, and evaluated responses offered by one of his students. Additionally, he questioned the student in such a way as to move the group towards the use of more precise mathematical language. These actions focused on very specific mathematical tasks of teaching pertaining to subject matter knowledge. The following dialog resulted from an extension of the "Number of the Day" activity to "Decimal of the Day."

Alex: So we’ve got six-hundred twenty-five thousandths. Let’s start up here. And again this is for older grades once they’re more comfortable with multiplying, if I take our number of the day and multiply it by 10, what am I going to get?

PT: 6 point 25.

Alex: 6 point 25. Or more mathematically?

PT: Six and 25

Alex: Six and 25.

PT: Six and a quarter.

Alex: Six and a quarter because 25 hundredths is one fourth, another reference is six and a quarter, or six and 25 hundredths. If I take this and multiply it by 100.

PT: 62 and a half.

Alex: 62 and a half, 62.5, 62 and 5 tenths. And then if I multiply it by a thousand.

PT: 625.

His attempt to encourage the PTs to use more effective mathematical communication and build upon the mathematical practice for precision also suggested he relied on the practical knowledge of subject matter (Elbaz, 1983).
Alex's use of the subject matter knowledge domains became further evident in the post-observation interview as he responded to a question concerning the pattern of dots on the paper plates and how he decided what patterns to use or whether they were just random. He seemed to draw on all three domains of subject matter knowledge in his response.

Alex: Good question, good question. I usually try to mix up, again, for the different levels, I try to show them some easier ones, this is what it would look like with the early kids. But then I also want, as adults, I want them to see the more complex patterns and the different ways that it can be arranged. That’s why I kept going back to the eights and the nines. How many different ways can we arrange nine dots? And well, we’d kind of talked about, most of them saw “Well I counted by ones, I saw six and two, or I saw…” And none of them in the class that I had, did the plates with Monday, I showed them that exact same plate. Some of those undergrads saw, I just saw nine with one missing. They didn’t see it so that’s why I explicitly brought it and the addition and subtraction related to each other. (Post-observation interview, 2/5/13)

Shulman's subject matter content knowledge was evident in Alex's awareness of how addition and subtraction are related and the importance of bringing that connection to the PTs’ attention (Shulman, 1986). He again offered evidence of his use of specialized content knowledge relative to the mathematical tasks of teaching by suggesting a modification of the task (Ball et al., 2008) for the PTs. He showed them easier patterns he would use with early childhood students but kept going back to the eights and nines so the PT's would see more complex patterns. Alex’s view of the content (Elbaz, 1983), specifically the importance of the relationship between addition and subtraction demonstrated his use of practical subject matter knowledge as well. His reference to a group of undergraduates he had in class on the previous Monday and how they saw eight
dots as “nine with one missing” and then using that experience to guide his instruction with his group of PTs revealed his use of experiential knowledge.

**Alex’s Practical Principle.** In the second classroom observation, much like the initial interview and first classroom observation, Alex showed a preference for a disposition towards using learner centered, problem-based, and interactive instruction. This guiding practical principle was evident in his practice as Alex engaged his learners in activities that they might, in turn, use with classroom students so that they experienced what their future students might experience. This allowed the PTs to reflect on and discuss the problems from both a student and a teacher perspective. The following segment of dialogue refers to the Little Shepherd Boy scenario (See Appendix D) used to develop the concept of one-to-one correspondence. The PTs actually read and thought about possible solutions to the scenarios during the first classroom observation but were given the opportunity to discuss and reflect on their thinking during this class session.

After his initial instructions, Alex allowed for autonomous discussion by the PTs.

Alex: But the last time we were together, before we started talking about the mathematical practices, I had you consider two different situations. One was about the Little Shepherd Boy and the other one was about the kids fighting over the Legos. And I kind of left it out there hanging last week. We never took it anywhere so now I want to come back and take it somewhere. Let’s start with the Little Shepherd Boy. Talk amongst yourselves. What were some of the things that you suggested that we could do to help the Little Shepherd Boy?

PT1: I thought maybe he has names for all of his sheep so when he lets them out, that’s how he knows they all come back because he has a name for every one of them. So I thought we could replace the name with a number as like a nickname in case he’s real sensitive about the names.

PT2: That’s a good idea.

PT1: Yeah, so then he would know, maybe he has ten sheep, I don’t know I don’t
even remember what the thing was about. But if he had ten sheep then you know they all had Dasher, Dancer, Prancer, Vixen, Comet, Cupid, Donner, Blitzen, and another guy and another guy then maybe, I don’t know, then he would know they all came back and then he would understand numbers 1 through 10. (Second classroom observation, 2/5/13)

Alex: Ok, what are other ideas?

With little intervention by Alex, the PTs actively engaged in constructing their own solution and then explaining it to the class. This activity matched the theme of reflective practice, specifically reflective conversation (Schön, 1992) as an instructional method. In this particular situation, Alex put his PTs in the position of a school learner to model the activity as it could be completed with children.

**Summary.** Data from the second classroom observation again confirmed Alex’s reliance on knowledge of instruction and pedagogical content knowledge in his instructional practices. Domains related to subject matter knowledge were far more prevalent in this observation that in the first observation or initial interview. Given that the majority of the activities completed in this session centered around the development of specific mathematical content related to number and number sense, references to subject matter knowledge domains were not surprising. The guiding practical principle of problem-based, student centered, interactive and reflective instructional activities that appeared in previous observations and interviews was again evident in this observation.

**Professional Development Observation: Females in Mathematics**

Alex was observed a third time on March 4, 2013 during a mathematics coaching professional development session. This was the seventh of nine, two-day PD sessions that occurred monthly throughout the academic year. Alex was responsible for leading
PD sessions for mathematics coaches (all inservice teachers) in their third year of involvement in the program. One of his goals for these sessions was to keep the third-year coaches current on mathematics education research. Additionally, he also helped the third-year coaches design a PD session that they would conduct for the first and second-year coaches on the second day of the monthly PDs.

Due to the availability of both the researcher (I had responsibilities with first and second-year coaches) and Alex, the observed session revolved around Alex’s goal. Six female mathematics coaches from schools across a Midwestern state participated with Alex in a discussion of chapter 6, “Paying the Price for Sugar and Spice: How Girls and Women Are Kept Out of Math and Science” from Jo Boaler’s book *What's Math Got To Do With It? How Parents and Teachers Can Help Children Learn to Love Their Least Favorite Subject*. Although Alex had discussed this chapter with previous groups of third-year coaches, he had never before discussed it with only females so he was interested in learning how an-all-female audience might respond to this same article.

Alex: I’m kind of interested to see where this is going to go because I’ve never had a discussion about this particular chapter out of the Boaler book with only women. [Z: Yeah, that’s true.] And I am very open to let them, well, just over the past couple months, I’ve just tried to be open and let them take the conversation where they want to go with it. I’m interested to see because these are six pretty strong-willed women and they have achieved some level of success in mathematics. So I’m kind of interested to see and to hear some of their thinking about females in classrooms, about what they see from a coach perspective in terms of males and females being treated the same or differently, and how some of these things have played out in their own lives. (Pre-observation interview, 3/4/13)
When asked about specific learning objectives for the session, Alex felt that the main audience for learning was himself gaining insight into the perspectives of this particular group of mathematics coaches.

Alex: I think this just going to be more learning from my end because each time that we have one of these conversations about what we’re reading I get to know them a little bit better. I can kind of start piecing together the journey that they’ve taken and that’s led to them this point. (Pre-observation interview, 3/4/13)

Since the entire session was devoted to the book discussion and lasted only approximately an hour, a summary of the sequence of session activities for the lesson seemed unnecessary.

**Code frequency data.** A total code frequency of 113 resulted from analysis of data from Alex’s professional development session observation. Big idea themes most frequently observed were *reflective practice* (13) and *teacher learning* (11). Only *pedagogical content knowledge* (10) was coded within Shulman’s categories of content knowledge. No knowledge domains within mathematical knowledge for teaching were frequently observed. With respect to content of practical knowledge, *knowledge of instruction* (22) appeared most often while *knowledge of self* (7) and *knowledge of surroundings* (6) were evident less frequently. The primary orientation of Alex’s practical knowledge was again *experiential* (11) with an indication of a *social* (7) orientation as well. Table 14 offers the code frequency totals for the professional development session.
Table 14. Professional Development Observation Code Frequency – Alex

<table>
<thead>
<tr>
<th>Alex - Professional Development Observation Code Frequency</th>
<th>Pre-Int</th>
<th>Obs</th>
<th>Post-Int</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation Data Segmentation (ODS)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Knowledge Data Segments (KDS)</td>
<td></td>
<td></td>
<td></td>
<td>19</td>
</tr>
<tr>
<td>Big Idea Themes (BIT)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflective Practice (RP)</td>
<td>2</td>
<td>11</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
<td>4</td>
<td>0</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Technology (TECH)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Children's Mathematical Thinking (CMT)</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Shulman Content Knowledge (SHU)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
<td>4</td>
<td>0</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Mathematical Knowledge for Teaching (MKT)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Content of Practical Knowledge (CPK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self (SLF)</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Instruction (INS)</td>
<td>3</td>
<td>14</td>
<td>5</td>
<td>22</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Orientations of Practical Knowledge (OPK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Situational (SIT)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Personal (PER)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Social (SOC)</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>27</td>
<td>56</td>
<td>30</td>
<td>113</td>
</tr>
</tbody>
</table>
The nature of the professional development session was considerably different from the two PT classroom observations. Since the entire session was devoted to the discussion of a book chapter concerning females in mathematics, little to no mathematics content, per se, was evident. The discussion instead surrounded classroom environments as well as other environments in relation to children resulting in some code frequency differences. For the first time, the most frequent big idea theme was reflective practice. Since there was essentially no problem-solving activities or discussion of mathematics content, domains relative to mathematical knowledge for teaching were barely represented and even then only among the pre-and post-observation interview data. Knowledge of instruction and the experiential orientation of Alex's use of practical knowledge were again evident and consistent with previous interviews and observations.

**Reflective practice and teacher learning.** The big idea theme of reflective practice was evident in the observation while concern for teacher learning was more prevalent in the post observation interview. The entire book discussion pertained to reflective practice and reflective conversation (Schön, 1992). Alex continually asked he coaches to reflect on their professional and personal experiences in examining issues presented in the book. For example, Alex asked his learners to reflect back on classrooms in which they coached to consider the differential treatment that males and females tended to receive.

Alex: While Helen is digging up the back story of that, to what extent do you see this male-female thing… To what extent do you see it playing out in the classrooms that you’re in? Or do you even see it?

C1: Well yeah, I have a teacher where boys are smarter. She will recognize…
Alex: Even in the shadow of Title IX?

C1: You got to understand this woman. It’s more of a… not a blatant. She will you know…

C2: She’s a woman herself?

C1: Yeah she’s a woman herself. But there’s a definite bias, not a blatant that girls can’t do it, but a definite bias that the boys are going to excel better.

Alex: In just mathematics or across the board?

C1: Yeah, that’s all she teaches. (PD observation, 3/4/13)

Alex initially asked the teachers about the extent of the male-female bias they may have observed in their classrooms. This prompt motivated one coach to comment on a teacher who she believed treated boys as mathematically smarter than girls. The ensuing discussion uncovered Alex's social orientation to practical knowledge as he brought up the issue of Title IX. His acknowledgement of Title IX indicated his awareness of social conditions (Elbaz, 1983) under which classroom teachers and mathematics coaches must practice their craft.

Alex's concern for teacher learning, which was frequently evident in his initial interview, reappeared in the PD post-observation interview as did his use of Shulman's pedagogical content knowledge and knowledge of instruction. When asked if he would change anything about the session, Alex suggested that he could have improved the session through better questioning and alternative instructional practices. He suggested asking each coach to write a comment or question about the reading on an index card and using the cards to initiate discussion.

Alex: I think if I would go back and do that again, I think I would have some better questions in mind rather than just trying to direct it and make it up on the
fly. It’s still their book discussion but I think some of my questions in getting them to talk may have shut them off more than creating an opportunity to share their thinking and their ideas. So I think that’s one thing that I would have changed. The other thing is to maybe let them be a little bit more self-directive. I just had this idea while we were sitting here. I think it would have been interesting to have, give each of them an index card and have them write a response, or a question, or a comment based on what they had read and used those to get the ideas flowing. There were some sections of that discussion where I actually thought I was pulling teeth and it just kind of laid there. So I’m thinking maybe some more ownership on their part would have led to a more productive conversation. (Post-observation interview, 3/4/13)

The theme of teacher learning was evident when Alex suggested that he thought some of his questions may have hindered instead of igniting discussion. His use of Shulman's pedagogical content knowledge, specifically knowledge of strategies needed to reorganize understanding of learners (Shulman, 1986), was evident when he offered that it may have been a good idea to give the coaches an index card on which they could write a comment or question relative to what they had read and then using those cards to generate discussion. Alex's thoughts on starting the session with a better set of questions as well as the index card idea also suggested his use of the content of practical knowledge of instruction related to organization and teacher-student interaction (Elbaz, 1983).

Knowledge of Instruction and Experiential Knowledge. During the book discussion, Alex specifically focused the coaches’ attention to page 134.

Alex: Right after those bullet points on 133 and 134, something that kind of leapt out to me as we’re thinking about Common Core and we’re thinking about mathematical practices and connecting. That bottom paragraph on page 134. (Alex reading from the book) ”These and other striking findings led Brisandine to conclude that girl brain is a machine that is built for connection.” (PD observation, 3/4/13)
When asked in the post-observation interview why he decided to channel the discussion as he had, Alex’s response indicated his reliance on knowledge of instruction, suggesting his desire to meet his session objectives.

Alex: I was trying to, the conversation had drifted into other issues of race and socioeconomics and that was a blatant attempt to bring it back to gender in a math classroom and thinking about their own experiences with that. That was one of the lines that I had highlighted when I had read it. I just wanted to see an all-female participant response to that particular statement, that particular quote. So that’s why I brought it back. (Post-observation interview, 3/4/13)

Alex recognized that the discussion had drifted into issues of race and socioeconomics and used his knowledge of organization of instruction as well as his understanding of teacher-student interaction (Elbaz, 1983) to refocus the group on gender issues by focusing them on specific text from page 134.

As with Alex's consistent use of knowledge of instruction, his experiential orientation of practical knowledge appeared in the PD observation as well. During the discussion, Alex emphasized that most early childhood teachers are female. In doing so, he utilized his experience with teaching early childhood methods classes in stating that “I can count on two hands the number of males who have, that have been in my early childhood methods class in the last six years” (PD observation, 3/4/13). His orientation of practical knowledge as experiential (Elbaz, 1983) was evident throughout the interviews and observations.

**Alex’s practical principle.** Although the content of the PD observation was significantly different than either of the two classroom observations, Alex's guiding practical principle remained evident. The problem in this case, gender issues in mathematics education, required no special mathematical problem solving skills, but
instead focused the coaches on issues of classroom environment and culture in general, and to some extent, on student learning. The session was student-centered and interactive as the coaches led the discussion with Alex only offering occasional insights and sometimes suggestions to refocus the conversation. He even indicated that the coaches would lead the session when asked about specific learning objectives for the session and he replied “I don’t really have any specific goals as we have to talk about this, or we have to talk about that. I’m just interested to see where the conversation flows” (Pre-observation interview, 3/4/13). Additionally, reflective practice was the primary instructional methodology utilized throughout the PD session as the coaches referenced their experiences as students and teachers of mathematics. For example one coach suggested that she believed her mathematical background was not as strong as it should be due to her gender. Alex’s guiding practical principle remained intact even in a session where neither mathematics content nor curriculum was the focus.

**Summary.** Knowledge domains utilized by Alex in the PD session were somewhat different than those in previous interviews and observations primarily due to the topic of the session. There were no mathematical activities and no references to the Common Core Standards. Additionally, since mathematics content was absent from the session, domains pertaining to mathematical knowledge for teaching were infrequently observed as well. The big idea theme of teacher learning, Shulman’s pedagogical content knowledge, practical knowledge of instruction, and the experiential orientation of practical knowledge were evident in this PD observation as in previous interviews and observations. Alex’s practical principal of problem-based, student-centered, interactive,
and reflective instructional practices continued to guide him in his interactions with the coaches.

**Overall Knowledge Domain Data Analysis.**

For Alex, the total frequency of codes made according to the theoretical framework was 1002 with 39% coming from the interviews and 61% from the observations. It is important to remember that this does not mean that there were 1002 coded data segments because most data segments were coded within multiple categories. Instead, data segments from the interviews and observations were coded within the categories of the theoretical framework a total of 1002 times. It was expected that more codes would yield from the observation data than the interviews due to the amount of data corpus. To further break down the frequency, 193 codes represented big idea themes (BIT), 196 codes were considered within in Shulman's categories of content knowledge (SHU), 219 codes s fell into the categories of mathematical knowledge for teaching (MKT), 274 codes represented content of practical knowledge (CPK) and 120 codes indicated orientations of practical knowledge (OPK). In each of the five components of the analytical framework, the frequency of codes from the observations was larger than the frequency of codes from interviews. Figure 9 displays the total frequency of codes for each of the five components together with a breakdown of the frequency of codes that came from interviews versus observations.
Big idea themes. Analysis of the 193 big idea theme codes indicated that those most often represented in the Alex’s practice were teacher practices (TP), children’s mathematical thinking (CMT), teacher learning (TL), and reflective practice (RP). A total of 55 data segments were coded as teacher practices with 44% occurring among the interview data and 56% occurring in observations. With regard to children’s mathematical thinking, 44 segments were coded with 34% coming from interviews while 66% were coded in the observations. The researcher identified 40 teacher learning data segments almost exclusively from interviews (88%) while 29 segments were coded as reflective practice with 31% from the interviews and 69% from the observations. The remaining three categories, social justice, teacher beliefs and attitudes, and technology all had a frequency of coding of 10 or less. Figure 10 summarizes the frequency of codes for big idea themes.
In a closer look at the three observations only, the researcher identified a total of 156 knowledge data segments (KDS) from which big idea themes could possibly be drawn and analysis of these segments provided results consistent with previous data. Approximately 20% of the knowledge data segments were coded as teacher practices, 19% as children’s mathematical thinking and 13% as reflective practice. Only 3% of knowledge data segments were coded as teacher learning which is consistent with the fact that 88% of the codes in this category came from interviews. For each of the three remaining categories, the coded segments represented 5% or less of knowledge data segments (See Figure 11).
Shulman’s categories of content knowledge. Shulman's categories of content knowledge include subject matter content knowledge, pedagogical content knowledge, and curricular knowledge and all three appeared in Alex's practice with pedagogical content knowledge seemingly most significant. Of the 196 codes representing one of Shulman’s categories of content knowledge, 118 were coded as pedagogical content knowledge and were distributed almost evenly between interviews (44%) and observations (56%). Data segments were coded as subject matter content knowledge 41 times and heavily weighted toward observations (80%) while segments coded as curricular knowledge occurred 37 times and were also heavily weighted toward observations (73%). See Figure 12 below for a summary of the code frequency for Shulman's categories of content knowledge.
Results from examining only the observations indicated a similar emphasis on *pedagogical content knowledge*. Approximately 42% of the 156 knowledge data segments were coded as *pedagogical content knowledge* while 21% were coded as *subject matter content knowledge* and 17% as *curricular knowledge* (See Figure 13).

**Mathematical knowledge for teaching.** The six categories of mathematical knowledge for teaching are *common content knowledge* (CCK), *specialized content knowledge* (SCK), *horizon knowledge* (HK), *knowledge of content and students* (KCS), *knowledge of content and teaching* (KCT), and *knowledge of content and curriculum* (KCC). The data indicated that Alex relied most heavily on *knowledge of content and teaching*, *knowledge of content and students*, and *specialized content knowledge* as he prepared for and worked with PTs and mathematics coaches.
Figure 13. Shulman's categories of content knowledge as a percentage of KDS – Alex.

More than 40% of the 219 items pertaining to mathematical knowledge for teaching were coded as knowledge of content and teaching (79). Approximately 43% of these data segments occurred in the interviews and 57% in the observations. Sixty data segments were coded as knowledge of content and students with 45% coming from the interviews and 55% from the observations. Specialized content knowledge appeared in 45 data segments with the overwhelming majority (84%) resulting from the observations. The remaining categories of mathematical knowledge for teaching each had a frequency of 20 or less. Figure 14 displays the frequency of coding for mathematical knowledge for teaching.
A detailed look at the observation codes indicated that the same three categories were the most significant knowledge domains from which Alex drew. *Knowledge of content and teaching* represented the highest percentage of the 156 knowledge data segments at 29%. However, *specialized content knowledge* (24%) represented a slightly higher percentage of knowledge data segments than *knowledge of content and students* (21%) which was different than the results of the overall frequency when including interviews. Each of the three remaining categories represented less than 10% of the knowledge data segments. See Figure 15 for representation of the categories of mathematical knowledge for teaching as a percentage of knowledge data segments.
Practical knowledge. Data analysis for the practical knowledge of Alex was divided into three sections according to the primary classifications of content of practical knowledge, orientations of practical knowledge, and structure of practical knowledge. Code frequency data is presented for the content of practical knowledge and the orientations of practical knowledge while the structure of practical knowledge is developed based upon an overall view of Alex’s practice.

Content of practical knowledge. Knowledge of instruction was easily the most frequently coded type of knowledge utilized by Alex within the content of practical knowledge. Among the 274 segments within the domain of content of practical knowledge, 136 were coded as knowledge of instruction. As would be expected, a majority of the coded data segments were found throughout the observations (73%). The
frequencies of *knowledge of surroundings* (41), *knowledge of subject matter* (35), and *knowledge of curriculum development* (35) indicated that they were a secondary resource used by Alex. Data segments coded as *knowledge of surroundings* were more frequent in the interviews (59%) than in the observations (41%). Evidence of *knowledge of subject matter* was found mostly in the observations (80%) as were data segments for *knowledge of curriculum development* (69%). Figure 16 displays the coding frequency for content of practical knowledge.

![Alex - Content of Practical Knowledge Frequency](image)

Figure 16. Content of practical knowledge frequency – Alex.

Focusing on observations data only, similar results were found in that the greatest percentage of the 156 knowledge data segments were coded as *knowledge of instruction* (63%). Again *knowledge of surroundings* (11%), *knowledge of subject matter* (18%), and *knowledge of curriculum development* (15%) appeared to be secondary sources of
knowledge from which Alex drew. However, relative to the observations, *knowledge of surroundings* seemed to be the least significant of the three whereas the overall frequency totals, which included interviews, indicated that *knowledge of surroundings* was the most significant of the three. This seemed consistent with previous results in that categories related to *knowledge of subject matter* and *knowledge of curriculum development* were most often found in observations. Figure 17 summarizes the content of practical knowledge as a percentage of knowledge data segments.

![Content of Practical Knowledge](Image)

**Figure 17.** Content of practical knowledge as percentage of KDS – Alex.

**Orientations of practical knowledge.** *Experiential knowledge* was the primary knowledge base that influenced Alex's practice relative to the orientations of practical knowledge. *Theoretical knowledge, personal knowledge, and social knowledge* appeared to be far less influential and thus constituted secondary orientations of practical knowledge.
knowledge. Among the 120 codes as orientations of practical knowledge, 69 represented *experiential knowledge* and they were fairly evenly distributed between interviews (54%) and observations (46%). The frequency for *theoretical knowledge* was 18 and also was fairly evenly distributed with 44% from the interviews and 56% from the observations. The frequency of data segments coded as *personal knowledge* was 14 with the majority coming from interviews (79%). Data segments were coded as *social knowledge* 12 times with all of them occurring in the observations. *Situational knowledge* was evident in less than 10% of the coded data segments. Figure 18 is a graphical representation of the frequency of the orientations of practical knowledge.

![Figure 18. Orientations of practical knowledge frequency – Alex.](image)

When looking exclusively at the observations, *experiential knowledge* again was the primary source utilized by Alex. Approximately 21% of the 156 knowledge data
segments were coded as *experiential knowledge*. Each of the four remaining categories represented less than 10% of the knowledge data segments and thus appeared to be utilized peripherally in Alex’s interactions with PTs and mathematics coaches. See Figure 19 for a graphical representation of the orientations of practical knowledge as a percentage of knowledge data segments.

![Orientations of Practical Knowledge](image)

**Figure 19.** Orientations of practical knowledge as percentage of KDS – Alex.

**Alex case profile summary.**

At the time of data collection, Alex had been a mathematics teacher educator for approximately 8 years, had taught middle school and high school mathematics in most, if not all, subject areas and had taught developmental classes at a community college as well as at a large university. His additional university experiences included teaching mathematics methods courses for early childhood and middle childhood preservice
teachers in addition to special education and intervention specialists. He had also led professional development sessions for inservice teachers as part of a mathematics coaching program and in relation to his research interests in the area of connecting children's literature, English language arts, and early childhood mathematics.

Data suggested that the primary overarching themes of Alex’s practice were based on teacher practices, children’s mathematical thinking, and teacher learning while reflective practice was an instrumental secondary theme as well (See Figure 20).

![Diagram of ALEX - KNOWLEDGE DOMAIN CASE PROFILE]

Figure 20. Case profile summary of knowledge domains utilized by Alex.
These themes were consistent with the practical principle of problem-based, student centered learning activities that formed the foundation of his practical knowledge and seemed to guide his instructional practice. As a teacher educator, Alex was always concerned with two levels of learners; the preservice and in-service teachers in his university courses and professional development sessions as well as the school-age children teachers taught. In both cases, Alex believed that the learners needed to experience problem-based activities and have opportunities to reflect on solutions to the problems as well as their own thinking in regard to the problem. His preservice and inservice teachers (mathematics coaches) were always encouraged to think about these problems from the perspective of both the children with which they worked and the practice of teaching.

With respect to Shulman's categories of content knowledge, Alex relied heavily on pedagogical content knowledge as he interacted with his PTs and coaches. While his use of subject matter knowledge and curricular knowledge were certainly evident in the data, each appeared to be somewhat less prevalent than pedagogical content knowledge (See Figure 20). Alex relied extensively on curricular knowledge during discussion of the Common Core Standards for Mathematics but this knowledge domain was less evident during other classroom and professional development activities. Evidence of subject matter knowledge was apparent in his interactions with PTs as he worked through problem-based activities in his methods course. When they struggled with mathematical concepts during these activities, Alex always took the opportunity to use his subject matter knowledge in an effort to clarify their understandings. However, as a general rule,
subject matter was used more as a context to help his PTs develop ways of representing and formulating the subject to make it understandable for schoolchildren. Additionally, Alex used *subject matter knowledge* to help PTs anticipate conceptions and misconceptions with which schoolchildren might arrive (Shulman, 1986). These and other aspects of *pedagogical content knowledge* appeared to be the primary focus of these sessions.

Similarly, with regard to mathematical knowledge for teaching, data indicated that Alex again most frequently relied on aspects of pedagogical content knowledge, specifically *knowledge of content and teaching* and *knowledge of content and students*, while *specialized content knowledge* seemed more of a secondary source (See Figure 20). In interviews as well as observations, Alex provided evidence of his ability to predict what would motivate schoolchildren as well as his ability to anticipate what children might do with a task and how difficult that task might be for them (Ball et al., 2008). During his own sessions, he often drew on his ability to hear and interpret PTs’ and coaches’ thinking. His ability to sequence content for instruction was evident as was his understanding of the benefits and possibly drawbacks of certain methods and procedures (Ball et al., 2008). Alex's reliance on *specialized content knowledge* again occurred mostly during problem-based activities when he provided or evaluated mathematical explanations, discussed modification of tasks for different levels of schoolchildren, or selected representations for particular purposes (Ball et al., 2008).

*Knowledge of instruction* was easily the primary knowledge domain from which he drew in terms of content of practical knowledge. His use of the knowledge domains
with respect to curriculum development, subject matter, and surroundings, while clearly evident, were secondary in nature (See Figure 20). As would be expected, most of the data with regard to this particular category occurred in the classroom and professional development observations where his knowledge of organization of instruction as well as teacher-student interaction (Elbaz, 1983) was clearly evident in each activity. His practical knowledge of curriculum development appeared in discussions of the Common Core with regard to the needs of students as well as course development (Elbaz, 1983). Problem-based activities provided evidence of his practical knowledge of subject matter as he interacted with students during these sessions. Alex's use of practical knowledge of surroundings was evident in his ability to recognize the classroom and social setting of the poor, rural schools in Appalachia where his preservice teachers were placed for their field experiences. His knowledge of professional politics was also evident in his discussion of economic factors that influence resources in different school districts (Elbaz, 1983).

The orientations of Alex's practical knowledge were primarily experiential while theoretical and social orientations were secondary (See Figure 20). In both interviews and observations, he often referred to and discussed previous educational and or teaching experiences in reference to why specific activities were chosen or certain instructional practices were used. Often, he also seemed to rely on experiential knowledge to anticipate what school-age children might do with a specific task as well as the level of difficulty based on children’s age and maturation. His use of theoretical knowledge was less prevalent and appeared often to be based on theory in relation to his own research.
interests of connecting children's literature and English language arts to improve early childhood mathematics education. *Personal knowledge* seemed to influence Alex’s practice such that in order to make mathematics teacher education meaningful, he felt it necessary that the teachers he worked with experience problems as learners in an effort to improve not only their content knowledge but also improve their repertoire of strategies as well as their awareness of children’s mathematical thinking. While also a secondary aspect of Alex's orientations of practical knowledge, his use of *social knowledge* was evident in his ability to recognize the classroom and social setting of the poor, rural schools in Appalachia as well as in his discussion of economic factors that influence technology available in different school districts (Elbaz, 1983).

With respect to Shulman's categories of content knowledge and mathematical knowledge for teaching, the primary knowledge domain from which Alex drew was *pedagogical content knowledge* while *subject matter knowledge* and *curricular knowledge* appeared to be secondary sources. His use of practical knowledge was primarily situated in *knowledge of instruction* and *experiential knowledge* with several other categories providing secondary sources of knowledge. The guiding practical principle that formed the foundation of Alex’s instructional practice was his belief that all learners – preservice teachers, inservice teachers, and school children – benefited from problem-based, learner-centered, interactive activities with opportunities to discuss and reflect on their own thinking as well as the thinking of others. Specific to mathematics teacher educating, Alex felt it necessary that his preservice and inservice teachers experience problems first from the perspective of a child and then from the perspective of
a teacher. This dual focus was prominent throughout his practice. To Alex, mathematics teacher educating provided an avenue for guiding current and future teachers in such a way as to ultimately improve mathematics education of children.
Chapter 5: The Case Profile of Tracy

Tracy – Data Collection

Data collection began for Tracy in early March of 2012 with an initial semi-structured interview which provided information regarding her educational background, teaching experience, and her research interests. The interview also provided insight into Tracy's perspectives on teacher educating and the knowledge domains she utilized in her interactions when working with preservice teachers (PT) and in-service teachers (IT). She elaborated on what she assumed teaching and the practice of teacher educating should be. Two classroom observations were then conducted in late November and early December as she taught a course whose audience consisted of PTs including both undergraduate and graduate students. This course, Teaching and Learning of Mathematics in Grades PreK-3 was taught at a regional campus of a large Midwestern University. A third observation took place in early February during which Tracy lead a professional development session for mathematics coaches from across a Midwestern state. At the time of data collection, the coaches were in their second year of participation in the PD program. Data collection also included pre-and post-observation interviews immediately before and following each observation period. Table 15 offers data collection details.
Table 15. Data Collection Summary – Tracy

<table>
<thead>
<tr>
<th>Data Collected</th>
<th>Date</th>
<th>Location</th>
<th>Approximate Length</th>
<th>Learners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Interview</td>
<td>3/6/12</td>
<td>University office at main campus</td>
<td>1 hr 22 min</td>
<td>NA</td>
</tr>
<tr>
<td>1st Classroom Observation</td>
<td>11/26/12</td>
<td>University Regional Campus Early Childhood Mathematics Methods Course (K-3)</td>
<td>Pre-Int: 5 min Obs: 3 hr 50 min Post-Int: 47 min</td>
<td>14 female students, 11 undergraduate and 3 graduate, 4 seniors &amp; 10 juniors, all preservice teachers</td>
</tr>
<tr>
<td>• Pre-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Obs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Post-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd Classroom Observation</td>
<td>12/3/12</td>
<td>University Regional Campus Early Childhood Mathematics Methods Course (K-3)</td>
<td>Pre-Int: 5 min Obs: 1 hr 37 min Post-Int: 21 min</td>
<td>14 female students, 11 undergraduate &amp; 3 graduate, 4 seniors &amp; 10 juniors, all preservice teachers</td>
</tr>
<tr>
<td>• Pre-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Obs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Post-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Professional Development Observation</td>
<td>2/7/13</td>
<td>Mathematics Coaching Program PD session</td>
<td>Pre-Int: 3 min Obs: 2 hrs Post-Int: 17 min</td>
<td>20 mathematics coaches (inservice teachers) – 2nd year of participation in program</td>
</tr>
<tr>
<td>• Pre-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Obs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Post-Int</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Introducing Tracy: Education, Experience, and Research

Tracy is a Caucasian female and at the time of this study was a professor in mathematics education at a regional campus of a large Midwestern University. As part of her faculty duties, she taught a mathematics methods course for early childhood PTs as well as an action research course. Tracy was co-director of a funded mathematics coaching professional development research project. Her duties for the mathematics coaching program included leading professional development sessions related to social justice in mathematics for first- and second-year coaches as well as conducting informational sessions for regional facilitators and administrators. Tracy had a BS in
Applied Mathematics with a minor in Pure Mathematics from a midsize Midwestern university. After receiving her bachelor’s degree, she decided she wanted to teach “…because I didn’t like working with math by itself. I wanted to work with people…” (Initial interview, 3/6/12). She then earned a MA in Teaching (secondary education) and later secured a second MA and PhD in Mathematics Education.

Tracy began her teaching career as a long-term substitute teacher in a public school. Following this, she taught middle school mathematics (grades 7-8) for three years. Her teaching assignment included General Mathematics (grades 7-8), Pre-Algebra (grades 7-8), and Algebra 1 for eighth graders. During the final year of her middle school teaching experience, Tracy also began pursuing her PhD. She then worked for two years as a mathematics teacher leader in a K-5 public elementary school while continuing to pursue her doctoral degree. Tracy noted that this job as a teacher leader was “…where I started to get exposed to elementary mathematics” (Initial interview, 3/6/12). After her second year at the elementary school, Tracy devoted two years to full-time pursuit of her doctorate where she also had the opportunity to teach a secondary mathematics methods course. During her time as a doctoral student, she developed an interest in equity issues.

Upon completion of her PhD, she began an 18 year career as a faculty member. Tracy started as a full time lecturer in the elementary education program where she taught K-8 mathematics methods courses and supervised PT field experiences for one year. The following year, she served as a visiting assistant professor. During the following 16 years, Tracy progressed from tenure track assistant professor to a full professor rank in mathematics education while teaching PreK-3 mathematics methods courses,
mathematics education courses as professional development for area teachers, and an action research course. Her experiences in mathematics education provided Tracy with opportunities to work with K-8 school children, undergraduate and graduate teacher candidates, and K-12 teachers.

Initially, Tracy’s research interests related to gender in mathematics, specifically focusing on mathematical voice. This progressed to equity and social justice within the context of mathematics education. Her involvement with the mathematics coaching program originated as a result of her research interests in equity and social justice as well as her experience with professional development, particularly her mathematics teacher leader experience. In addition to the professional development, her methods course reflected this research interest as well.

**Tracy’s Initial Interview**

To collect information regarding Tracy's background as well as her initial thoughts concerning the practice of mathematics teacher educating, a semi structured interview (See Appendix A for interview protocol) was conducted in March of 2012. The interview took place in Tracy's project office with participants including this researcher, this researcher’s academic advisor, and Tracy. Since this interview was the first to be conducted by the researcher, his academic advisor participated to demonstrate effective interview techniques and offer guidance in conducting a semi-structured interview. As a result, this researcher was primarily an observer only occasionally directing a question to Tracy.
**Code frequency data.** Tracy's initial interview resulted in a total code frequency of 134 which means that data segments from the interview were coded within the categories of the theoretical framework a total of 134 times. Note that most data segments were coded within multiple categories. The big idea themes coded most frequently were teacher learning (14) and children’s mathematical thinking (11). Pedagogical content knowledge (9) was most frequently coded within Shulman's categories of content knowledge while knowledge of content and students (5) and knowledge of content and teaching (4) appeared most frequently with respect to mathematical knowledge for teaching. Categories of the content of practical knowledge most frequently coded were knowledge of self (14), knowledge of surroundings (13), and knowledge of instruction (8). Within the orientations of practical knowledge, experiential knowledge (16), theoretical knowledge (7), personal knowledge (7), and situational knowledge (6) appeared most often. See Table 16 for initial interview code frequency totals.

**Teacher learning and children’s mathematical thinking.** The big idea themes of teacher learning and children’s mathematical thinking were prevalent in Tracy's initial interview. The interview data provided evidence that children's mathematical thinking influenced her decisions regarding what type of learning she hoped to activate. Her experiences with early childhood teachers, whether PTs or ITs, had taught her that most teachers were not comfortable with the mathematics they needed to know to effectively teach children.
Table 16. Initial Interview Code Frequency – Tracy

<table>
<thead>
<tr>
<th>Observation Data Segmentation (ODS)</th>
<th>Knowledge Data Segments (KDS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big Idea Themes (BIT)</td>
<td></td>
</tr>
<tr>
<td>Reflective Practice (RP)</td>
<td>1</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
<td>14</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
<td>3</td>
</tr>
<tr>
<td>Technology (TECH)</td>
<td>1</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
<td>1</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
<td>3</td>
</tr>
<tr>
<td>Children's Mathematical Thinking (CMT)</td>
<td>11</td>
</tr>
<tr>
<td><strong>Shulman Content Knowledge (SHU)</strong></td>
<td></td>
</tr>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
<td>3</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
<td>9</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
<td>2</td>
</tr>
<tr>
<td><strong>Mathematical Knowledge for Teaching (MKT)</strong></td>
<td></td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>0</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>0</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
<td>2</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
<td>5</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
<td>4</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
<td>0</td>
</tr>
<tr>
<td><strong>Content of Practical Knowledge (CPK)</strong></td>
<td></td>
</tr>
<tr>
<td>Self (SLF)</td>
<td>14</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
<td>13</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
<td>1</td>
</tr>
<tr>
<td>Instruction (INS)</td>
<td>8</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
<td>2</td>
</tr>
<tr>
<td><strong>Orientations of Practical Knowledge (OPK)</strong></td>
<td></td>
</tr>
<tr>
<td>Situational (SIT)</td>
<td>6</td>
</tr>
<tr>
<td>Personal (PER)</td>
<td>7</td>
</tr>
<tr>
<td>Social (SOC)</td>
<td>1</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
<td>16</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
<td>7</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>134</td>
</tr>
</tbody>
</table>
As a result, Tracy felt it necessary to consider the teachers’ mathematical deficiencies as she was helping them to think about *children's mathematical thinking* and learning.

Tracy: When I design a course,…I’m trying to figure out how to get them started when I know they’re not likely to be comfortable with, I know there not comfortable with the mathematics and I know they don’t know it. And I’ve learned to never underestimate how little they know. So how do I get them to start thinking about what their students might be thinking if I also have to work them through their own math learning to some degree? (Initial interview, 3/6/12)

Tracy considered it a challenge to engage PTs to think about *children’s mathematical thinking* when she had to simultaneously help them through their own mathematical learning because she felt responsible for developing PT’s mathematical content knowledge.

Tracy: So I want them to find some way to be comfortable with the mathematics, admit that they don’t know it, be ok with learning that they can learn it...Then they have, since they’re not gonna know the math, so then we have to start dealing with math content in terms of what they know and in terms of what their students know or need to know. And they probably won’t know much more than their students, third grade level. It wouldn’t be unusual. (Initial interview, 3/6/12)

Her belief was that PTs probably would not “know much more than their students” and she had to consider that issue when making decisions about her instruction.

In setting up reasonable and realistic expectations for PTs, she relied on literature pertaining to children’s cognition.

Tracy: So if I know things in the literature about a certain way that students think about a certain math concept and I work with the teachers on that, we can look at student work, we can watch videos, we can do all kinds of things and they would learn it from talking about it. They would learn it from studying it. They would learn it from trying to analyze it somehow. (Initial interview, 3/6/12)
Tracy suggested that PTs could learn from analyzing and discussing student work, videos and other activities that address how children think about specific mathematical concepts. Her concern was again with helping PTs focus on *children’s mathematical thinking*.

Tracy used a baking analogy to describe her image of teaching. “I have a thing in my syllabus that I never give you fully baked cakes. I might give you some recipes to try, we’ll talk about a lot of ingredients, but it’s your job to make the cake” (Initial interview, 3/6/12). When asked how her conceptualization of baking should be different relative to the teacher’s knowledge and conceptualization of baking, she again referenced *teacher learning* in relation to *children’s mathematical thinking*.

Tracy: …So I don’t know the answer to this but maybe the teachers have a more complex recipe because theirs has to include some ingredients like how the students must be thinking about it not just how they’re [the teachers] thinking about it, not just what they need to know to make a cake but they have to incorporate things about the students. (Initial interview, 3/6/12)

Data indicated that *teacher learning* grounded in *children’s mathematical thinking* was a constant concern for Tracy and often guided her thoughts about her instructional practices.

**Pedagogical content knowledge.** Tracy often referred to past teaching experiences when offering thoughts on her use of mathematics as both a teacher and a teacher educator. Many of the examples she provided indicated a reliance on Shulman’s pedagogical content knowledge throughout her practice. Her references, categorically, pertained to a combination of knowledge of content and students, knowledge of content and teaching, and knowledge of instruction. When asked how her use of mathematics as a mathematics teacher was similar to or different from her use of mathematics as a MTE,
Tracy drew on her experience of teaching fractions to an 8th grade general math class. She suggested that her method of writing student responses on the board was organized in such a way as to help students expand conceptions of the mathematics.

Tracy: So, so like if I’m an a um, when I taught 8th grade general math, let’s say fractions on the table, my act of teaching that topic was about asking some general question like “What is a fraction?” and spending 40 minutes or so getting everything out of those students and up onto the chalkboard that I could, everything that they could say, get it up there. My job in terms of thinking about the mathematics was I didn’t just write it on the board. They didn’t know that. They thought I was just writing it on the board. But I would write it on the board in some structure that I knew eventually would make sense um in terms of the mathematics that I knew that they were bringing to that discussion. (Initial interview, 3/6/12)

Based on Shulman’s (1986) description of pedagogical content knowledge, Tracy structured the subject in such a way to help make it understandable for children. Writing student responses on the board in a “structure” (Initial interview, 3/6/12) served a purpose. Practical knowledge of instruction was also indicated by her organization (Elbaz, 1983) of the children’s responses.

In another reference to her experience with teaching 8th grade general math, Tracy’s use of pedagogical content knowledge and knowledge of instruction were evident as she completed an entire unit on ratio and proportion “in an application to the arts” (Initial interview, 3/6/12). She was asked how her teaching of teachers was similar to or different from her teaching of school children and Tracy suggested that current issues of accountability (testing) would force her teach differently than how she taught prior to the era of testing.

Tracy: Well can I just say that when I, the last year that I taught 8th grade General Math, I did a unit on ratio and proportion. I did the whole unit in an application to the arts and it was just a blast and they learned so much and they loved everything
they did because it was very relevant. It was, you know, perspective drawing and, and so I don’t know that I feel that I could do that with the accountability stuff that’s there now. (Initial interview, 3/6/12)

She utilized her *knowledge of content and students* by incorporating the arts to motivate the children and her understanding of what this method offered instructionally in terms of motivation. This indicated Tracy’s use of *knowledge of content and teaching* (Ball et al., 2008). Practical *knowledge of instruction* was suggested by the way she had to organize (Elbaz, 1983) lessons in such a way as to apply the arts to the entire unit. Knowledge domains with respect to pedagogical content knowledge and practical knowledge of instruction appeared frequently in Tracy’s initial interview.

**Practical knowledge of self and surroundings.** Tracy seemed comfortable making references to her own knowledge and abilities pertaining to mathematics teacher educating. She also made clear her displeasure with accountability issues. Evidence of practical *knowledge of self and surroundings* appeared often in the initial interview. For example, when asked to discuss similarities and differences between working with PTs and ITs, Tracy discussed how restricted she felt working with PTs because of university accreditation (NCATE) requirements and various required PT assessments (SPAs and TPAs).

Tracy:  Because with preservice teachers, um oh this is gonna really sound awful, I don’t, first of all I don’t feel like they know what to ask for. I mean they do but in my mind they don’t. But the other thing is I don’t, I just don’t, the last time I taught a methods course, I was really frustrated with knowing that there were things that I was responsible for as far as NCATE or assessments and things like that, that I just had to do. So I feel very restricted in the preservice setting in that way and I think that’s worse now than it was five years ago. (Initial interview, 3/6/12)

Her knowledge of NCATE and “assessments” (Initial interview, 3/6/12) required of PTs 175
indicated Tracy’s use of practical knowledge of surroundings, specifically related to the political context of teaching and the responsibility to adhere to national assessments. Her reference to knowledge of self was evident when she stated how “restricted” (Initial interview, 3/6/12) she felt in regard to the needs of her PTs (Elbaz, 1983).

Tracy relied on practical knowledge of self and surroundings when asked what mathematics her teachers need to know. Issues of accountability and the resulting pressure she felt were the focus of her response.

Tracy: So I could, there are different ways I could figure out what content. I’d like, if I didn’t have to worry about standards, I’d like to pick content that I think is rich and great but I feel the pressures of that. I’m not happy about that. I’m really not. I’m very disturbed by it. I was much gutsier when I was teaching in the middle school in terms of trusting the way I was teaching mathematics and what I know about mathematics to get these kids where they needed to be on the test that, at that time, was in 9th grade. (Initial interview, 3/6/12)

Tracy’s comment that she was much “gutsier” (Initial interview, 3/6/12) in the way she taught middle school mathematics spoke of her confidence in her abilities relative to meeting the learner needs (Elbaz, 1983). Tracy felt frustrated that assessments which measured the early childhood program did not include mathematics content.

Tracy: Well and here’s the big thing. They’re not even, I’d (inaudible) doing early childhood when we go to semesters next year. They don’t, they’re not even assessed on the math content in terms of pedagogy in mathematics. Everything is either literacy or child development. There’s no place in the assessments for content. (Initial interview, 3/6/12)

Her frustration with the state assessments provided evidence of her knowledge of surroundings, specifically the political context of teaching (Elbaz, 1983). The assessments did not meet her approval but she still had to use them.
**Experiential orientations of practical knowledge.** Tracy's initial interview revealed that her primary orientation to practical knowledge was *experiential*. Although *theoretical, personal, and situational* orientations were also evident, they were referenced less frequently. Throughout the interview, she continually referred to previous experiences when responding to a variety of questions. The interviewer brought up the idea of trimming, which she described as teaching in such a way as to “reduce the complexity of a concept without creating a scenario that would serve as a cognitive obstacle later” (Initial interview, 3/6/12). The example of trimming that Tracy provided drew on a combination of her *personal, situational, theoretical, and experiential* orientations as she discussed her work with PTs and how she does not expect them to have the knowledge of an experienced teacher.

Tracy:...if I think about how I would love to see a practiced, an experienced teacher teach. Think of...how you teach and, you know, you don’t have like your list of notes here, you know the students well enough and the mathematics well enough that you can go back and forth and you get them where they need to go. So in the practice of teaching about teaching, a novice in a preservice program, I can’t expect them to necessarily, to do that...So they have to start somewhere and I have to figure out how to take what the experienced good teacher does and how, how do I trim it down so that it’s, it doesn’t lose the essence of what is important but it’s something that they can actually digest and get...One of the things that I’ve found is one of the reasons I like early, working with early childhood is the, a lot of the videos and contexts and everything are actually less complex because the mathematics is less complex for what they’re trying to teach. So they can, that I can trim it down by showing a basic say CGI lesson, a classroom lesson and then we can talk about what’s this teacher doing and whatever. I always tell them it’s deceptively simple. What did she do? She posed a problem, she let them work, she asked them a couple questions. Yeah, it’s nowhere near that simple. (Initial interview, 3/6/12)

Tracy drew on her *personal* orientation of practical knowledge as she reflected on her personal values (Elbaz, 1983) in describing how she would want to see an experienced
teacher teach. She described an experienced teacher as someone who knows the students and mathematics well enough to comfortably negotiate between the two and utilized *situational* knowledge to recognize that she could not expect that of her PTs. She understood that communication and instruction would need to be adapted (trimmed) in that situation. Drawing on her experiences (Elbaz, 1983) with early childhood PTs and *theoretical* knowledge, she suggested that since the mathematics at that level is not overly complex, she could have PTs view and discuss a CGI (cognitively guided instruction) video focusing on the basic components of the teacher’s instructional methodology.

Further evidence of her orientations of practical knowledge appeared when she described the content of her methods courses. Tracy immediately offered evidence of how she made use of *theoretical knowledge* when she referenced what literature says about teaching and learning mathematics in designing content for the methods course (Elbaz, 1983).

Tracy: Um it’s based on what I know and continue to learn so like in math methods, it would be, you know, what the research literature is saying about teaching mathematics and learning mathematics and stuff. When I teach, I teach an action research course. I teach that entirely differently from the way I teach methods because I teach it as a project-based course. So the content in that course is based on their growing understanding of what research is and I’ve taught the course so many times now that I know, ok by that third class, these are the questions they’re gonna have and so I can now plan based on my anticipation of their growth trajectory in this. I’m not that good at that at the math methods and so in math methods, it’s more about, I actually make a grid for myself. When I said about the rows, I actually make a grid and then the columns are the days and think here are all the things that I know are important – equity and diversity, children’s thinking – you know, so those are all down this left hand column and then on any lesson as I’m going through mathematics content, “Ok now I did that content twice. I’m not doing that content again.” (Initial interview, 3/6/12)
She then drew on *experiential knowledge* to explain how she taught her action research class “entirely different” (Initial interview, 3/6/12) from her methods course and that she had taught the course so many times, she could anticipate PT’s questions and plan accordingly. However, reflecting on her *knowledge of self*, specifically her abilities with respect to the needs of her PTs (Elbaz, 1983), Tracy indicated that she knew exactly what content to include in her action research course but was not as comfortable with the content for the methods course. Because of this, she discussed how she used a grid to keep track of content that had been and needed to be addressed. Her personal values and beliefs, *personal knowledge* and *knowledge of self* (Elbaz, 1983), were reflected when she discussed the content listed for the columns of the grid and stated that “here are all the things that I know are important” (Initial interview, 3/6/12). Data indicated that Tracy’s orientations of practical knowledge were primarily experiential but theoretical, situational, and personal orientations were also evident.

**Summary.** Tracy’s initial interview provided insights into her beliefs about the knowledge domains she utilized in her practice and how they influenced her decisions. The most prevalent big idea themes included *teacher learning* and *children’s mathematical thinking*. Shulman’s *pedagogical content knowledge* together with *knowledge of content and students* and *knowledge of content and teaching* from mathematical knowledge for teaching were evident throughout the initial interview. Tracy’s content of practical knowledge appeared as *knowledge of self, knowledge of surroundings, and knowledge of instruction* and her orientations of practical knowledge
were primarily *experiential* but *personal, situational*, and *theoretical* orientations were present as well.

**Mathematics Methods Course Observations**

Two classroom sessions were observed in a course titled "Teaching & Learning of Mathematics in Grades PreK-3." The course was a mathematics methods course for undergraduate and graduate preservice teachers seeking an early childhood licensure. The course objectives were described according to Tracy’s “image” (Elbaz, 1983, p. 134) of teaching, the baking analogy she had described during the interview.

The focus of this course is **not** to give you fully baked cakes. Rather, the focus of this course is to teach you how to bake a cake. I will give you some ingredients, and direct you to more. I will give you the tools to help you put those ingredients together. I will expect you to design a few recipes, and to try a few cakes – some of which may flop. I will expect you to fix your recipes, making your cakes the most nutritious they can be, according to experts in the field of mathematics teaching and learning, include the NCTM, ODE, and researchers. I will also expect those cakes to be as tasteful as you can make them, both for you and your students. Toward that end, the objectives of this course are for each participant to develop, in the age 3 to grade 3 Early Childhood context, an informed, research-based understanding of:

1. Constructivist mathematics pedagogy;
2. Learner Responsive Pedagogy in mathematics;
3. Equity Pedagogy in mathematics; and

The class consisted of 14 female PTs, 10 juniors and four seniors, three of which were graduate students. Observations took place during the ninth and tenth weeks of the course which were the last two weeks prior to the final examination. The first observation lasted almost four hours and the second lasted approximately an hour and a half.

180
First Classroom Observation: Grandfather Tang and Henry “Box” Brown

The first classroom observation took place during the ninth session of the course on November 26, 2012. Fourteen female preservice teachers were present and forming groups of three or four sitting at tables and the atmosphere was relaxed. In the previous class session, Tracy had used children's literature to address number concepts and planned to focus specifically on two books to address geometry and measurement concepts. Additionally, she had a chart on the board from the previous class meeting that she planned to use to connect "different ways to look at equity with assessment, the challenges of it all, the purposes of it all and all of that in children’s literature" (Pre-observation interview, 11/26/12). Even though this was only the ninth session with her PTs, the session was the next to last meeting prior to the final examination and Tracy planned to start wrapping up the course as well.

The class began with a brief review of an assignment on assessment the PT's had completed prior to this session. See Table 17 for the sequence of activities conducted during the first classroom observation. The PTs watched a series of Annenberg videos and read an article from Teaching Children Mathematics and reflected on what they had seen and read relative to the topic of assessment. Following this discussion, Tracy asked the PTs to outline the defining characteristics, assessment connections, and challenges for equity pedagogy. She then began an activity using a book called Grandfather Tang’s Story. The story is about the legend of how a King’s servant dropped a square tile and it broke into seven pieces and it became a task of the Kings guests to put the pieces back together in the original square tile.
## Table 17. First Classroom Observation Activity Sequence – Tracy

<table>
<thead>
<tr>
<th>Obs. Number</th>
<th>Course Number/ Class Date</th>
<th>Sequence of Topics/Activities (Time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Week 9, 11/26/12</td>
<td></td>
</tr>
</tbody>
</table>

1) **Introduction – Discussion of assessment assignment (23 min)**
   a. Discussed flexible grouping & Annenberg videos

2) **Discussion of Equity, Diversity, Social Justice (10 min)**
   b. Completed first row of table for equity pedagogy

3) **Grandfather Tang’s Story – book activity (1 hr 50 min)**
   a. Tracy read book to class
   b. To complement story, each group of PTs given a different tangram assignment to work on “tangram centers.” Many table conversations with PTs at this time
   c. Once three groups finished their activities, they worked on the same activity as the fourth group (the most difficult activity)
   d. Tracy told PTs to get out CCSS and put on teacher hat
   e. PTs examined how each initial activity works at different grade levels using CCSS
   f. PTs wrote responses to activity on chart paper
   g. PTs discussed the activity – how they would adjust for different grade levels.
   h. Tracy led brief discussion of fractions and equivalence based on misconceptions she observed during activity
   i. Moved back to table on board to complete 2nd row (Cultural topics)

4) **Henry’s Freedom Box – book activity (1 hr 20 min)**
   a. Tracy handed out small toy characters for PTs to use with activity
   b. Tracy read true story of Henry “Box” Brown to class.
   c. PTs given construction paper to make a box for their character. Had to assume character would be in same position as Henry and replicated his box as closely as possible
   d. PTs constructed boxes while Tracy observed and offered suggestions about measurement – height, depth, width
   e. Tracy offered clarification on how to do box for school child and self, hinting at proportions
   f. Tracy walked around working with groups about mathematics
   g. Whole class discussion of activity
   h. Each group assigned one grade level (K-3) according to CCSS
   i. Group discussion about adaptation of activity for grade level assigned
   j. Whole group discussion of grade level adaptation.

5) **Questions & answer period about upcoming assignments (10 min)**
Illustrations in the book were made of the seven tangram pieces that formed the square. After reading the story to her PTs, Tracy assigned each group a different tangram activity to explore (See Appendix F for the activities). Mathematical concepts discussed as a part of these activities included area, perimeter, shapes, and fractions. Once the activities were completed, PTs used the Common Core Standards for Mathematics to discuss how each of the four different activities could be used at different grade levels. At the conclusion of this discussion, Tracy returned to the chart to address the purpose, defining characteristics, assessment connections, and challenges relative to cultural topics.

The class then moved to a second book, *Henry's Freedom Box*, to examine the concept of space and volume as well as race and sociocultural topics. Tracy read the story of Henry "Box" Brown, a slave in the 1800s who mailed himself to freedom in a wooden box. The PTs were then given a small toy figure and told to use materials provided to create a box for it. They were to assume that the toy figure would have to sit in the box exactly as Henry had to sit in his box. Once they determined the dimensions of the box for the toy, they then had to use mathematics to determine the dimensions of a box for a third-grader assumed to be 36 inches tall and for themselves. Discussion of the concept of ratio and proportion ensued as well as how the activity could be adapted for different grade levels.

**Code frequency data.** The length of the first classroom observation was significantly longer than the initial interview and either of the other two observations which resulted in code frequency totals much larger than those in the other data. Relative to the knowledge domains discussed in the theoretical framework of analysis, the total
frequency of codes noted in the first classroom observation was 572. The big idea themes most frequently coded were teacher practices (97), teacher learning (22), and children’s mathematical thinking (21). Within Shulman's categories of content knowledge, pedagogical content knowledge (36) and curricular knowledge (19) were observed most frequently. Knowledge of content and students (31), specialized content knowledge (30), and knowledge of content and teaching (24) were most frequently represented within the domain of mathematical knowledge for teaching. The categories of the content of practical knowledge coded most frequently were knowledge of instruction (122) and knowledge of curriculum development (19) while the orientations of practical knowledge most often observed were situational (20) and experiential (19). Continuous back and forth interaction between Tracy and the PTs during this observation likely resulted in the high frequency totals for teacher practices and knowledge of instruction. See Table 18 for the complete list of code frequencies.

Teacher practices, teacher learning, and student mathematical thinking.

Data indicated that the big idea themes that appeared most frequently in Tracy's first classroom observation were teacher practices, teacher learning, and children’s mathematical thinking. Most data with respect to teacher practices occurred when Tracy either modeled an instructional technique such as reading the children’s books out loud to the class or referred to a practice from her experiences. Teacher practices were evident in her organization of the activities to include group work as well as whole class discussion. Often when discussing specific experiences, she tied the chosen practices to children’s mathematical thinking.
Table 18. First Classroom Observation Code Frequency – Tracy

<table>
<thead>
<tr>
<th></th>
<th>Pre-Int</th>
<th>Obs</th>
<th>Post-Int</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observation Data Segmentation (ODS)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Knowledge Data Segments (KDS)</td>
<td></td>
<td>149</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Big Idea Themes (BiT)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflective Practice (RP)</td>
<td>0</td>
<td>11</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
<td>1</td>
<td>10</td>
<td>11</td>
<td>22</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Technology (TECH)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
<td>0</td>
<td>87</td>
<td>10</td>
<td>97</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
<td>1</td>
<td>10</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>Children’s Mathematical Thinking (CMT)</td>
<td>0</td>
<td>18</td>
<td>3</td>
<td>21</td>
</tr>
<tr>
<td><strong>Shulman Content Knowledge (SHU)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
<td>0</td>
<td>12</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
<td>1</td>
<td>28</td>
<td>7</td>
<td>36</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
<td>0</td>
<td>14</td>
<td>5</td>
<td>19</td>
</tr>
<tr>
<td><strong>Mathematical Knowledge for Teaching (MKT)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>0</td>
<td>27</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
<td>0</td>
<td>23</td>
<td>8</td>
<td>31</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
<td>3</td>
<td>16</td>
<td>5</td>
<td>24</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
<td>1</td>
<td>7</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td><strong>Content of Practical Knowledge (CPK)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self (SLF)</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
<td>0</td>
<td>8</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
<td>0</td>
<td>14</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>Instruction (INS)</td>
<td>3</td>
<td>110</td>
<td>9</td>
<td>122</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
<td>1</td>
<td>15</td>
<td>3</td>
<td>19</td>
</tr>
<tr>
<td><strong>Orientations of Practical Knowledge (OPK)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Situational (SIT)</td>
<td>0</td>
<td>19</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>Personal (PER)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Social (SOC)</td>
<td>0</td>
<td>11</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
<td>0</td>
<td>12</td>
<td>7</td>
<td>19</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>11</td>
<td>465</td>
<td>96</td>
<td>572</td>
</tr>
</tbody>
</table>
This was evident during the initial class discussion of assessment when one PT suggested that allowing children to work in groups and “self-assess” (First Classroom Observation, 11/26/12) each other was a good idea. Tracy drew on a previous middle school experience to explain her practice of teaching middle school children to work in groups and then her knowledge of their mathematical thinking to question whether they had the ability to assess each other. This implied the themes of teacher practices and children’s mathematical thinking.

Tracy: Being able to work in a group, I’ll tell you how important I think it is. When I taught middle school I spent the entire first grading period with middle schoolers doing nothing but teaching them how to work together. They would have preferred to fight…But I spent all of my time teaching them how to work together. So we didn’t make a lot of gains in mathematics but once they could work together then we were able to catch up very quickly because they could work together. But when you talk about self-assessing, can they self-assess their mathematics knowledge or, as a group, assess their mathematics understanding? (First Classroom Observation, 11/26/12)

The theme of children’s mathematical thinking appeared when she questioned whether school children had the ability to assess their own mathematical understanding.

As the class discussion of children’s ability to self assess continued, a PT suggested giving each child a paper with a list of questions they could use to assess each other. Each child would also answer the questions based on their own knowledge. Tracy and the PT eventually agreed that this paper would represent a type of post-lesson evaluation and the themes of teacher practices and children’s mathematical thinking again appeared in Tracy’s response to the PT’s suggestion of what information would be needed from the assessment.

PT: Yeah, it would be something along those lines because you want to see what they know but you don’t want necessarily what Ashley knows on my paper
necessarily as what I know…

Tracy: So if Cathy thinks she knows that mathematics, she can do that. If she’s wrong in what she thinks she’s knows, I can pick that up from what she’s written. Can Cathy also assess the mathematics understanding of her peers in this group? Can that be one of the questions in the whole group assessment? She can certainly tell me who’s being bossy in her group and she can tell me who’s not participating in the group. Can she tell me anything about what other people understand in the group? (First Classroom Observation, 11/26/12)

Tracy’s assertion that she could assess Cathy’s mathematical knowledge by reading what Cathy wrote on her paper evidenced of the theme of teacher practices and the use of formative assessment. Children’s mathematical thinking appeared as Tracy once again questioned Cathy’s ability to assess what others in the group understand.

An activity related to Grandfather Tang’s Story also revealed Tracy’s commitment to the themes of teacher practices, teacher learning, and reflective practice. After asking the PTs to discuss within their groups how the specific tangram activity they completed could differ across grade levels, Tracy brought them together for a whole class discussion of their ideas.

Tracy: There is so much overlap that happened in the doing of this that I think is fascinating. Everybody used all of the language of the geometric figures…So that sort of thing happened but then later I also heard after you did your original activities and then went to this chart, you took what you learned from your first activity and were able to apply it. All four of these were really kind of different though in different ways so I would like you each, I don’t care who wants to start, to tell us just briefly what your activity was and how that activity differs across the grade levels.

Evidence of knowledge of teacher practices occurred as Tracy transitioned the PTs from their group work with the four different activities to a whole class discussion of the mathematics of each activity and what she had observed about geometric language usage.

The theme of teacher learning appeared as Tracy described how the PTs used what they
learned from their group activity to complete a chart on which they were displaying arrangements of tangram pieces to form shapes. Reflective practice was indicated when she had the groups describe their activity and then share their thoughts on how the activity would vary at different grade levels. Although during this observation episode, reflective practice was not coded as frequently as teacher practices, teacher learning, and children’s mathematical thinking, Tracy motivated this practice in both book activities.

Pedagogical content knowledge and subject matter knowledge. As was experienced during Tracy's initial interview, pedagogical content knowledge domains often appeared in combination with other knowledge domains during the first classroom observation. This was not surprising since most mathematics teacher educators likely draw from multiple knowledge domains as they interact with PTs and ITs. During the sequence of activities associated with Grandfather Tang’s Story, Tracy observed a group that struggled to determine the fractional part of the total area that each tangram piece represented. As one PT was explaining how her group would use this activity with a specific grade level, she stated "...Then we had to use fractions to see what fractions of the tangrams we used. So to make a square we used two triangles and that would be 2/7” (First classroom observation, 11/26/12). Tracy relied on a combination of pedagogical content knowledge, subject matter knowledge, and knowledge of instruction in addressing what she had heard. By bringing this issue to the attention of the entire group as a way to share the relationship among fractions, area, and perimeter, she was addressing the misconception to reorganize the understanding of PTs, an indicator of Shulman's pedagogical content knowledge (Shulman, 1986). Additionally, Tracy utilized
her practical knowledge of instruction to organize (Elbaz, 1983), or in this case reorganize, her plan to address fraction misconceptions and even made it a point to make this apparent to the PTs when she suggested that children may even learn about content beyond the particular focus of a lesson "if you allow it to happen" (First classroom observation, 11/26/12). Knowledge of content and teaching was evident in the fact that Tracy recognized the instructional advantages of using tangrams to represent fractions, area, and perimeter (Ball et al., 2008).

Tracy:...One of the things I'd really like you recognize in here is how these mathematical concepts are interconnected across fractions and area and perimeter and the different shapes so even though your students, maybe your lesson is going to be about learning what these figures are, there are other things that they will be able to do that might not be the particular focus of your lesson but it’s still a part of it if you allow it to happen…What fractional part of the square is each of the triangles?  [PTs: Half.]  Ok so this is one half of the square and this is one half. Now that is if the square is your whole.  So these are two equal parts of the whole so the equal parts is what makes it fractional.  It has to be equal parts when you identify it as a fractional part.  So in the 2/7, can you tell me where you came from the 2/7.

PT:  We just used the 7 because of how many (inaudible).  Seven tangrams make up the whole.

Tracy: So 2 out of 7, those 7 pieces are not all the same amount of area, the same fractional part.  They are not equivalent.  So how do we figure out what the equivalent parts can be?

In the above exchange, Tracy’s subject matter knowledge was prevalent as she instigated how the group had arrived at 2/7. Shulman’s subject matter knowledge appeared in her ability to recognize the structure (Shulman, 1986) of fractional parts of area and a necessity for equivalent pieces. She relied on specialized content knowledge and knowledge of content and students in evaluating the claim (Ball et al., 2008) that 2/7 representing the fractional part of the total area of all seven pieces. Her views of the
content (Elbaz, 1983) and her desire for the students to recognize the interconnectedness of fractions, perimeter, and area suggested the use of practical knowledge of subject matter as well.

The simultaneous use of multiple knowledge domains became evident again as PTs completed activities associated with Henry’s Freedom Box. At a specific point in the sequence of activities, PTs were asked to utilize the dimensions of a box they had constructed for a toy figure and determine the dimensions of the box that would accommodate a third grader. In Henry's voyage, he had to sit in a position that was very compact and the PTs were to assume that the toy figure and the third grader had to sit in the same position. Tracy's plan was that they would use a proportion to determine the dimensions of the box for a third-grader but found that she had to address this issue when she observed a PT using measurement to determine the new dimensions. As a result, she drew on pedagogical content knowledge, subject matter knowledge, and knowledge of instruction to help the PT move forward.

Tracy: Henry did sit with his legs bent.

PT: That's what I was measuring.

Tracy: But you shouldn’t have to deal with that. That's not your issue right now. See one person here is listening to me. Everything now is done mathematically with the measurements. You have a proportion or a ratio between the action figure and your schoolchild. Those two boxes would be in the same proportion. You do the math. Then you do the same thing from the action child to you. (First classroom observation, 11/26/12)

Tracy’s suggestion for the use of proportions in place of measurement to solve the problem evidenced Shulman’s subject matter knowledge. Within the domain of mathematical knowledge for teaching, knowledge of content and students was indicated
by Tracy's ability to interpret the PT's incomplete thinking (Ball et al., 2008) and 

*knowledge of content and teaching* appeared in the way she sequenced the content (Ball et al., 2008) such that the PTs first had to use measurement with the toy and then could use proportions for the third-grader. Finally, practical *knowledge of instruction* was also evident in the organization of the activity as well as her interaction with the PT (Elbaz, 1983). Tracy again seemed to rely on multiple knowledge domains as she interacted with her PT's.

Tracy's use of pedagogical content knowledge domains and knowledge of instruction were also visible in the post observation interview. She made a reference to the term "centers" during the activities associated with Grandfather Tang’s Story when the interviewer asked her to explain what she meant by that term. Her explanation of centers indicated the use of *pedagogical content knowledge* and *knowledge of instruction*.

*Tracy:* …In primary grades they do a lot of centers where they teach kids how to work independently in a small area. So if this group were divided into centers groups, then I would have a center in every corner of the room and during centers time everybody goes to their center and then maybe the next day everybody goes to a different center. So for the whole week everybody gets through all the centers and a center would be like an activity they could work on generally independently although sometimes you may have an aid or teacher sit in on one of the centers. (Post-observation interview, 11/26/12)

Tracy's knowledge of the strategy of using centers so that students could experience multiple activities related to the same concept indicated Shulman's *pedagogical content knowledge* in the sense that it represented a strategy used to reorganize (Shulman, 1986) mathematical understanding of children. *Knowledge of content and teaching* was also indicated based on her understanding of what the method of centers afforded instructionally with respect to students experiencing each of the activities (Ball et al.,
Practical knowledge of instruction emerged as Tracy described how to organize and utilize centers for instruction (Elbaz, 1983). Both the classroom observation and the post-observation interview provided evidence of Tracy's use of multiple knowledge domains as she interacted with PTs in regard to mathematical content.

**Curricular knowledge domains.** Tracy’s curricular knowledge was noted based on her utilization of the Common Core Standards for Mathematics and her knowledge of the availability of a variety of materials available such as *Henry’s Freedom Box* and *Grandfather Tang’s Story* to enhance lessons focusing on particular content. Her use of curricular knowledge still seemed to play an important role in her instructional practice during the first classroom observation and was detected in pre-and post-observation interviews as well. Prior to the classroom observation, Tracy was asked about problems or issues she was anticipating with the session. She suggested that it was always difficult to get early childhood PTs to focus on the mathematics but that there use of a Common Core document seemed to help in that regard.

Tracy: I think the issue always with early childhood in particular is getting them to focus on the math and even to get any depth to it at all. Now this particular group has become very good with using the Common Core. I gave them a copy of the Common Core in hard copy because half of them can’t afford to print something so they have that and in almost every class and we’ll do that today with both texts that we’re using. They go back to the Common Core and they start to look at which objectives could be appropriately addressed and all that. (Pre-observation interview, 11/26/12)

Knowing which materials to use, when to use them and who to use them with is an indicator of both Shulman's *curricular knowledge* (Shulman, 1986) and *knowledge of content and curriculum* (Ball et al., 2008) and was reflected in her use of the Common Core document as a way to get her early childhood PTs to consider mathematics content.
Tracy's use of the document specifically to help PTs focus on the mathematics indicated her use of practical *knowledge of curriculum development* with respect to the needs of that group of students (Elbaz, 1983).

During the post-observation interview, Tracy was asked why, in both book activities, she asked the PTs to relate the activities to the Common Core and different grade levels. She emphatically stated “First of all, they are not getting out of my class without knowing the Common Core” (Post-observation interview, 11/26/12). Tracy believed it helped the PTs to recognize content they needed to teach as opposed to following a textbook. Additionally, she suggested review of standards across grade levels allowed for the development of their vertical curriculum knowledge.

Tracy: So I want them to focus on the Standards as opposed to what the textbook has. Like I said, they’re getting good at it and we do it every class for every content. And also, like today was the day for the first time that a same group had to look across multiple grade levels, the tangram one when they did that. That’s the first time I’ve done that in this class. Usually it’s a group gets a grade level [Z: Like you did the second… ] like I did, yea. That’s faster but I like the vertical articulation and I think they saw today even, this is the first time I think I ever worked with a bunch of teachers who had, or prospective teachers, who had content beyond what they were going to teach and they understood why. (Post-observation interview, 11/26/12)

Tracy's reference to "vertical articulation" and her suggestion that the PTs need to experience content “beyond what they were going to teach" indicated her use of both Shulman's vertical *curriculum knowledge* (Shulman, 1986) and *horizon knowledge* (Ball et al., 2008). Practical *knowledge of curriculum development* was again evident as Tracy focused on the needs (Elbaz, 1983) of this particular group of PTs.
Curricular knowledge relative to the availability of materials was indicated in the classroom observation when Tracy made it a point to mention to her PTs about using the internet to find activities.

Tracy: If you do a “search” [for Grandfather Tang’s Story], I have some materials for you that I printed off so you’ll have some stuff but if you do a search on the internet for Grandfather Tang’s Story or tangrams, they’ll be a million lessons, a million at least. (First classroom observation, 11/26/12)

Practical knowledge of curriculum development was indicated in relation to materials development (Elbaz, 1983) as well. Tracy's utilization of curricular knowledge domains was most often evident in her instructional practices as knowledge of the materials available for concept development as well as her reliance on the Common Core to help PT's focus on mathematical content.

**Situational, experiential, and social orientations.** Data from the classroom observation indicated that the orientations of Tracy's practical knowledge were most often situational, experiential, and social. Situational orientations were often a result of student questions related to particular end-of-course assignments while experiential and social orientations more often appeared within the equity pedagogy and social justice discussions. However, in the following discussion of the defining characteristics of equity pedagogy, data indicated that Tracy drew from practical knowledge that was socially oriented as she responded to a PT's discussion of a special needs student.

PT: If they had a special need, then it wouldn’t be equitable if say for instance, I have a kid who has an OT handicap where he has occupational therapy. If we have an assessment where he can’t really turn the chips over or, you know, if the chips were nice and flat and he can’t really handle them, then that’s not equitable for him because he’s going to have more difficulty.

Tracy: So you attend to these styles that it’s very specific to the needs of the
children not just the general needs of all children, very specific needs. (First classroom observation, 11/26/12)

A social orientation was indicated by her awareness of the social conditions and constraints (Elbaz, 1983) of working with special-needs children.

Towards the end of the observation period, situational and experiential orientations as well as knowledge of surroundings became evident in the following dialogue when Tracy responded to one PT's question about whether to include lessons for different grade levels on an upcoming assignment.

PT: I was just making sure (inaudible) because I know they had to fall into five different content areas but did you want them to be lessons for different grade levels?

Tracy: The choice is up to you if it’s in different grade levels but I will tell you it’s to your advantage to spread across grade levels just so you have the opportunity to think about it. This is pragmatic. You go for a job. Somebody’s going to say to you, what are you going to do in third grade and if all your lessons are written for kindergarten you’re going to struggle with that question. So have some experience of different grade levels. It’s not a requirement. (First classroom observation, 11/26/12)

Tracy's knowledge of surroundings was apparent in her recognition of the classroom setting (Elbaz, 1983) and that the PTs present would soon be looking for employment and offered advice in that regard. A situational orientation to her practical knowledge was indicated at the end of the dialogue when, knowing that incorporating multiple grade levels was not a requirement for the assignment, Tracy still used subtle communication influences (Elbaz, 1983) in suggesting that the PTs should indeed consider multiple grade levels. An experiential orientation was suggested when Tracy told the PT that she would struggle with an interview question about third-grade lessons if she had only prepared
lessons for kindergartners. Evidence indicated that she was drawing on interview experiences to guide her response to the PTs question (Elbaz, 1983).

The post-observation interview also provided evidence of Tracy's orientations of practical knowledge when she explained why she selected Grandfather Tangs Story for this particular session. This practical knowledge was both social and experiential.

Tracy: One is that cultural piece that they will do that stuff in their language arts and their social studies classes but never think about how to make a choice for a text that will take you to some mathematics. Like there’s a book Moja Means One I think is the title of it, it’s a Swahili counting book. So why use the M&M counting book when you can use that which also includes the picture of the Mancala game that the kids can learn how to play. I just think there are ways to recognize the contributions to mathematics across cultures and how people think about mathematics in different cultures based on a better choice of the literature that you use. (First classroom observation, 11/26/12)

The big idea theme of social justice was represented throughout Tracy's discourse as she discussed the selection of books that not only dealt with specific cultures, but also “recognize contributions to mathematics across cultures…” even suggesting a book that has a game called Mancala that children can play. Experiential orientation was indicated in her recognition of the Swahili counting book Moja Means One and knowing that it included a picture of the Mancala game and how she could use that to enhance her mathematical instruction. The social orientation of her practical knowledge was indicated when she discuss the social constraints (Elbaz, 1983) placed on children in their social studies and language arts classes by textbook choices that never lead to any mathematics.

Tracy’s image of teaching. Elbaz (1983), in her description of the three possible components of the structure of practical knowledge, describes “image” as a
teacher’s feelings, values, needs, and beliefs which form his or her view of how teaching should be. This image guides the teacher’s thinking and organization of knowledge relevant to the subject matter. As was discussed in Alex’s case profile, data were not coded to determine the structure of practical knowledge but instead was based on an overall view of each MTE’s instructional practice. Tracy's image of teaching in the first classroom observation appeared to be consistent with the “baking” analogy she had referenced in her initial interview as well is in her methods course objectives. Her goal was not to provide her PTs with fully baked cakes but instead to provide them with some of the ingredients and tools needed to bake a cake. The book activities offered evidence that this image of teaching guided Tracy's decisions but I will focus on Henry's Freedom Box for the purposes of this discussion. The following dialogue represents directions Tracy provided the PTs once she had finished reading the book to the class. She offered the tools (paper, scissors, tape, etc.) and little else other than, in building the box, the PTs were to replicate Henry’s experience.

Tracy: So you know the story of Henry and his travel to freedom. What I’d like you each to do is given some manila folder kind of paper and scissors and there should be tape for every table. I’d like you each to make a box for your little action figure. Some of your figures don’t move, don’t bend so you’ll have to measure and figure out. You want to replicate Henry’s experience in his box as best as you can. So you can’t just make a big old box so could he lay down in with straight legs. That’s not an option. So replicate the box for your figure. Any questions about that? (First classroom observation, 11/26/12)

True to her cake baking analogy, Tracy offers very few instructions related to the activity. She provides them tools (toy figure, paper, scissors, tape) and explains to them what they are supposed to make (box for toy figure). The only ingredient she provides is that they have to replicate Henry's experience and therefore cannot assume the toy figure
will be in the box with straight legs. The PT's were then left to come up with the rest of
the ingredients to make the cake. Tracy's image of what teaching should be was reflected
in how she directed this particular activity.

**Summary.** The big idea themes most often represented in Tracy's first classroom
observation were *teacher practices*, *teacher learning*, and *children's mathematical
thinking* backgrounded with *social justice*. Pedagogical content knowledge appeared to
be a primary source from which Tracy drew while subject matter knowledge domains and
curricular knowledge domains were observed less frequently and most often related to
discussions of mathematical content and the Common Core Standards for Mathematics,
respectively. With regard to content of practical knowledge, *knowledge of instruction*
was the primary knowledge domain she utilized while *knowledge of curriculum
development*, *knowledge of subject matter*, and *knowledge of surroundings* occurred far
less frequently. The orientations of Tracy's practical knowledge were divided between
*situational*, *experiential*, and *social*. Her image of teaching according to the cake baking
analogy was evident throughout this observation as well.

**Second Classroom Observation:**

The second classroom observation took place during the 10\textsuperscript{th} session of the course
on December 3, 2012. All fourteen female preservice teachers were present and again
sitting around tables in groups of three or four. Additionally, the chart remained on the
board from the previous class meeting which Tracy planned to use “…to start to kind of
finish bringing together the equity piece” and “do a little bit with social justice which
they’ve not really had…” (Pre-observation interview, 12/3/12). She also intended to
provide the PTs with two foundations for the final exam and allow them time “…to start
talking to each other across the course” (Pre-observation interview, 12/3/12). The last
half of the session was devoted to the PTs taking an assessment regarding mathematics
pedagogy. As a result, data were only collected during the first part of the session prior
to PTs taking the assessment.

The class began with a brief discussion of the agenda items followed by quick
review of cultural topics with respect to Grandfather Tang’s Story, augmented with
sociocultural topics in the context of Henry's Freedom Box. As part of the review, Tracy
showed the PTs pictures of a group of schoolchildren completing Henry's Freedom Box
activities as she described how the classroom teacher handled certain situations. They
then moved to a whole class discussion regarding the defining characteristics, assessment
issues, and challenges with respect to social justice pedagogy completing another row of
the chart as the discussion progressed. Tracy then gave PTs a one-page reading about
social justice and asked them to compare "just good teaching," equity pedagogy, and
social justice pedagogy. After the PTs discussed this in their groups, they had a whole
class discussion of the distinguishing characteristics of each. To finish the class prior to
the PTs taking the LAMP, Tracy provided a topical outline of the course and additional
information to help them prepare for the final examination. A complete sequence of
activities for the 2nd classroom observation is provided in Table 19.

**Code frequency data.** The total frequency of codes assigned in the second
classroom observation was 219. The big idea themes most frequently coded were social
justice (25) and teacher practices (16).
Table 19. Second Classroom Observation Activity Sequence – Tracy

<table>
<thead>
<tr>
<th>Obs. Number</th>
<th>Course Week Number/ Class Date</th>
<th>Sequence of Topics/Activities (Time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Week 10 12/3/12</td>
<td>1) Introduction – Agenda for the day (3 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. Discussion of equity, social justice with respect to Top 10 list of critical features of Learner Response Pedagogy (LRP) in mathematics education</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Topical outline in preparation for final exam</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. PTs take Learning About Mathematics Pedagogy instrument</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2) Reviewed table from week 9 – Grandfather Tang’s Story (cultural topics), Henry’s Freedom Box (race &amp; sociocultural topics) (13 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. Whole class discussion of classroom pictures of kids doing Henry’s Freedom Box problem from a mathematics coaching program coach</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Discussed how teacher did not let any words in discussion go unexplained, students had made a list of all the mathematics they thought Henry needed for his journey</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3) Discussion moved to categories of equity and social justice that were on the board from last session (9 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. Discussed defining characteristics of social justice pedagogy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Discussed issues with assessment in regard to social justice pedagogy.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. Referenced Freire in problem posing – noted that it’s about students posing the problems not teachers</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d. Moved to challenges with social justice pedagogy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4) Gave PTs one page reading about social justice and had them work in groups to compare “just good teaching”, equity pedagogy, social justice pedagogy (49 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. Whole class discussion comparison of “Just good teaching”, equity pedagogy, social justice pedagogy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Concluding statement about teaching progression with respect to social justice: Start with good teaching – become more aware – act, teacher then student as change agent</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5) Discussion of tying course together preparing for final exam (23 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. Gave PTs a topical outline of the course</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Discussed difference between summary &amp; synthesis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. PTs then given time in groups to discuss “new whole” for this class</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d. Whole class discussion of what groups talked about</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e. Tracy reminded PTs of her teaching motto – “I don’t give you fully baked cakes”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6) PTs took break and then returned to complete Learning About Mathematics Pedagogy instrument (No observation data)</td>
</tr>
</tbody>
</table>
Within Shulman's categories of content knowledge, only *pedagogical content knowledge* (20) appeared to occur with any frequency. With respect to the domain of mathematical knowledge for teaching, data indicated that only *knowledge of content and students* (12) was utilized with any frequency. The categories of the content of practical knowledge coded most frequently were *knowledge of instruction* (30) and *knowledge of surroundings* (12) while the orientations of practical knowledge most often coded were *experiential* (21), *social* (14), and *situational* (11). Since the data collection period for the first classroom observation was almost three times as long as that of the second classroom observation, the code frequencies of the second observation were significantly smaller. See Table 20 for the complete list of code frequencies.

**Social justice and teacher practices.** The entire portion of the second classroom observation that was used for data collection was devoted to equity pedagogy and social justice pedagogy resulting in evidence indicating the big idea theme of social justice. Just as in the first classroom observation, the theme of teacher practices was evident as well. Data from the pre-observation interview provided evidence of both themes when Tracy was asked about problems or issues she anticipated with the session. She projected that her PTs would expect definitive answers regarding equity pedagogy and social justice issues but indicated that definitive answers do not exist.

Tracy: I think one of the issues is both with the equity pedagogy and with the social justice. They do want some answers and I expect that because one of our tasks is how to distinguish and to unpack that little bit from last time and I think they’re going to want more definitive answers and there aren’t any and so the only way I can accommodate that is to help them understand that they’re really aren’t any. And to allow all their different perspectives to come out so that they understand that there are different perspectives and they can have different perspectives and its ok. (Pre-observation interview, 12/3/12)
Table 20. Second Classroom Observation Code Frequency – Tracy

<table>
<thead>
<tr>
<th>Tracing Code Frequency Table for Tracy’s 2nd Classroom Observation</th>
<th>Pre-Int</th>
<th>Obs</th>
<th>Post-Int</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observation Data Segmentation (ODS)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Knowledge Data Segments (KDS)</td>
<td></td>
<td>42</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Big Idea Themes (BIT)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflective Practice (RP)</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Technology (TECH)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
<td>0</td>
<td>12</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
<td>4</td>
<td>17</td>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>Children’s Mathematical Thinking (CMT)</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td><strong>Shulman Content Knowledge (SHU)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
<td>2</td>
<td>8</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td><strong>Mathematical Knowledge for Teaching (MKT)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
<td>2</td>
<td>2</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Content of Practical Knowledge (CPK)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self (SLF)</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Instruction (INS)</td>
<td>3</td>
<td>20</td>
<td>7</td>
<td>30</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td><strong>Orientations of Practical Knowledge (OPK)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Situational (SIT)</td>
<td>1</td>
<td>6</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>Personal (PER)</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Social (SOC)</td>
<td>0</td>
<td>14</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
<td>0</td>
<td>15</td>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>15</td>
<td>132</td>
<td>72</td>
<td>219</td>
</tr>
</tbody>
</table>
Evidence of the theme of *social justice* occurred when Tracy suggested that understanding equity pedagogy and social justice was about understanding the lack of definitive answers and that different perspectives exist and are acceptable. The theme of *teacher practices* was present when Tracy indicated that she would allow the PTs’ “different perspectives to come out” to enhance their understanding.

Themes of *social justice* and *teacher practices* also were indicated during the classroom observation when discussion of a context for mathematics and social justice led to the idea of children conducting a food drive for the local food bank at their school. In the discussion, a PT reflected on her knowledge of a program where children make sandwiches for the homeless. Tracy then used the PT’s suggestion and tied it back to a problem they had previously done in the course.

PT: A social injustice would be people who are hungry and don’t have food like you’re using the food bank example but trying to tie it back to math like I know there is a program where kids make sandwiches for homeless people. So like to tie it back to math, you can kind of say if we have 50 people how many pieces of bread do we need just so they’re like adding different things into it to tie it back to math.

Tracy: You know we did the ten black dots problem so you had to figure out how many dots you need for the whole class. What if that’s like the first lesson and the next lesson is connected to social studies when you’re talking about poverty and they have to similarly figure out how much stuff they need. They are taking the same processes and figuring out the problem but they have a purpose here. There would be mathematical stuff in terms of measuring and how old they are and how well they do that will depend on their development. (Second classroom observation, 12/3/12)

Tracy’s response provided evidence of the theme of *social justice* in mathematics when she talked about connecting the “ten black dots” problem to a social studies discussion of poverty and determining how much food the homeless would need. Her ability to make
the connection between the problem, the issue of poverty, and the context of food for the homeless was evidence of the theme of teacher practices in that by tying it all together, she felt the children would then have a “purpose” for the mathematics. Given that the focus of this entire session was equity and social justice, the most frequently occurring big idea themes of social justice and teacher practices were no surprise.

**Pedagogical content knowledge domains and instruction.** As with the first classroom observation, Shulman’s pedagogical content knowledge, knowledge of content and students from mathematical knowledge for teaching, and practical knowledge of instruction often appeared in an instructional combination. During the social justice discussion, a PT suggested that “…you could inspire change in students by making it personal. I guess authentic but more like they had to understand how Henry felt in that box in order to feel like something good happened from it” (Second classroom observation, 12/3/12). Tracy seemed to draw on her pedagogical content knowledge and knowledge of instruction to provide an example relative to kindergarteners.

Tracy: They are not going to put themselves in the box and ship themselves. But they can reenact that, so make it personal in terms on how their body fits in that space. For young children, sometimes that’ll mean you will have to come up with a context that really makes more sense to them. For instance, if you want to talk about fairness of food for instance, you might use graham crackers because you may not use some big study about how much food people eat, because a kindergartener might not grasp that. But they’ll really understand fairness in dividing up those. (Second classroom observation, 12/3/12)

When Tracy suggested using graham crackers for a kindergartner instead of a large study about food consumption, she drew on knowledge of a way to represent the mathematics that would make it understandable for young children (Shulman, 1986), an indicator of Shulman’s pedagogical content knowledge. Knowledge of content and students was also
evident in Tracy’s ability to predict that kindergartners would find the context of graham crackers more interesting and motivating (Ball et al., 2008) than a large scale study. Her practical knowledge of instruction, specifically her beliefs about the acts of teaching (Elbaz, 1983), in asserting that “sometimes that’ll mean you will have to come up with a context that really makes more sense to them.”

The post-observation interview offered insight into Tracy’s use of pedagogical content knowledge and knowledge of instruction. During the classroom session, she had made references to literary authors, specifically Paulo Freire and Gloria Ladson-Billings. Following up on those references in the post-observation interview, Tracy was asked about the role that literature and/or research played in how she designed her methods course. She explained that her work is grounded in her knowledge of the literature but that she had to utilize specific instructional tactics to get the PTs to read anything she assigned.

Tracy: Even though they have something to read from me, that is not the world according to Tracy and I really, I mean I like to ground my work in what I know about the literature. Now as far as what they’re going to get, they’re not going to read Freire because they don’t read anything and so I’ve had to give reading quizzes in here and that’s why we have part of the final is about the readings. They just don’t read. (Post-observation interview, 12/3/12)

Her knowledge of strategies such as reading quizzes and final exam questions to encourage the PTs to read and thus reorganize their understanding (Shulman, 1986) offered evidence of Shulman’s pedagogical content knowledge. The same strategies also indicated her ability to organize the readings and quizzes in such a way as to encourage the PTs to read (Elbaz, 1983). Tracy appeared to rely on pedagogical content knowledge
domains and practical *knowledge of instruction* to guide her instructional practices in the second classroom observation just as she had in the first.

**Practical knowledge of surroundings.** When considering issues of equity pedagogy and social justice, knowledge of the social climate in general and the background environment of learners are usually topics of concern. Both were evident in this observation and provided evidence of Tracy’s practical *knowledge of surroundings*. During her review of the previous class and the two book activities, the issue of women in mathematics became a point of discussion and Tracy reflected on her knowledge of how women who start in the field of mathematics eventually leave it for what she termed the “soft sciences.”

Tracy: They don’t stay in mathematics. They go into what we refer to as math using fields which might be technology, might be one of the sciences. Most of them are picking what are referred to as the soft sciences which are the ones for girls. So more are going into biology and fewer are going into physics. So it seems like every time we open that door a little bit then there’s another one that needs to be opened. (Second classroom observation, 12/3/12)

Her recognition of the current social setting (Elbaz, 1983) with respect to women in mathematics provided evidence of her use of the practical *knowledge of surroundings*. Her point was that women had made advancements in the field of mathematics but that more “doors” need to be opened.

*Knowledge of surroundings* with respect to school children and their background environment appeared during the whole class discussion following the PTs one-page reading on social justice. The subject of a food drive was discussed and one PT suggested that all children could be incorporated into a food drive but not necessarily in the same way. She asserted that “by knowing your students and their home environment
you can incorporate everyone the best way that they can” (Second classroom observation, 12/3/12). Tracy’s response confirmed the PT’s point and indicated her use of practical knowledge of surroundings with respect to children’s home environment.

Tracy: I think that’s a really critical point here because in the equity pedagogy, you could know the students home environment but when you accommodate that, is that a fair word, when you accommodate that and adjust your lessons or projects or goals accordingly, then you are allowing all the students to have that access.

She indicated the importance of gauging the classroom setting to the needs and backgrounds of children in accommodating them (Elbaz, 1983). Tracy provided evidence of her willingness to rely on her practical knowledge of surroundings to guide her instructional practices.

**Experiential, social, and situational orientations.** The most frequently noted orientations of practical knowledge utilized by Tracy during the second classroom observation were experiential, social, and situational with experiential seemingly most prominent. Situational and experiential orientations were indicated consistently throughout the interviews and observation while social and personal orientations seemed more prevalent in the observations during the equity and social justice activities. For example, in the discussion pertaining to the food drive that ensued following the reading on social justice, a PT reflected on an example of a project at her school where children contributed candy wrappers and how the teacher handled situations with children who did not bring any wrappers. Tracy’s response provided evidence of her use of personal and social knowledge as she referenced her granddaughter never having eaten candy and the social (economic) constraints placed on making the drive a competition. Tracy
immediately drew on a personal experience with her granddaughter who did not eat candy and used it to suggest that bringing in wrappers and bringing in cans of food were not the same (Elbaz, 1983). She then used the example to guide her conversation toward the idea of whether competition in this “collective effort towards a goal” is beneficial. The social orientation to her practical knowledge emerged in her questioning whether competition would benefit a food drive because of the social conditions and constraints (economic factors) (Elbaz, 1983) that may influence a child’s ability to contribute.

PT: …For example, we had a project at my school where kids were supposed to bring in candy wrappers. Some people brought in two cans, some people brought in, I mean like some people brought in five candy wrappers. Some kids didn’t bring any candy wrappers. So she was like “Oh we have extras. Who didn’t bring any candy wrapper? It wasn’t made like “Oh I don’t have any candy at my house,” like that...

Tracy: …First of all candy wrappers are very different. My granddaughter would never bring in a candy wrapper because she has never eaten candy. She’s five and there is no candy in her house…I think what she’s talking about here is something you have to think about in terms of if you’re going to have competition, does competition really contribute to a collective effort towards a goal? And if it’s going to be competitive, should it be competitive within the classroom or maybe your classroom with somebody else so you can have a community sense happening in the classroom and then there’s less pressure for every single child to bring something in? (Second classroom observation, 12/3/12)

Evidence of experiential and situational orientations to Tracy’s practical knowledge was indicated in the post-observation interview in a discussion of how she would like to use Paulo Freire’s book Pedagogy of the Oppressed in her methods course but is unable to find the time. Tracy indicated an experiential orientation to her practical knowledge as she reflected on her experiences teaching an equity and diversity course where she used the book and her methods course where she did not use the book and how that guides her decision to not use the book in her methods course now (Elbaz, 1983).
Tracy: Yes. So chapter 2 is the one that I would use if I wanted them to do that. When I taught the equity and diversity course and I taught the methods course, I could do that in the equity and diversity course. I can’t fit it in this course, but even if I could I don’t think they would read it. They don’t. This has been the biggest struggle for me this semester is getting them to read anything. I hate resorting to reading quizzes. I’ve had to do reading quizzes.

As she continued, Tracy also asserted that the PTs would not read the book even if she did include it providing evidence of a *situational* orientation to her practical knowledge. Time restrictions and student motivation influenced her instructional decisions (Elbaz, 1983). Data from the second classroom observation indicated that Tracy’s primary orientation of practical knowledge was *experiential* while *social* and *situational* orientations were evident as well.

**Tracy’s image of teaching.** Tracy’s view of teaching as providing some ingredients and tools but not fully baked cakes appeared again in the second classroom observation. Prior to the PTs’ group discussions about the distinguishing characteristics of good teaching, equity pedagogy, and social justice pedagogy, Tracy was transparent about the fact that the exercise would not provide all the answers.

Tracy: There’s an awful lot for you to do and think about. This will be one of those kinds of exercises. We don’t have a set definition for either one of these equity pedagogy or social justice pedagogy. So think about what it takes to be distinguished as equity pedagogy over just good teaching and then in your groups move to the social justice piece as best as you can. (Second classroom observation, 12/3/12)

She told the PTs up front that precise definitions for equity pedagogy and social justice pedagogy did not exist and that they would have to offer their own ideas as they discussed the characteristics in their groups. Tracy certainly did not try to provide them
with all the ingredients but instead allowed them to bring their own ingredients to the
table to begin to develop their own perspective of what the cake should be.

Towards the end of the class session, Tracy even made it a point to remind her
PTs of the cake baking analogy from the course syllabus. In preparation for the final
exam, she had asked the PTs to discuss in their groups how they would synthesize the
components of the course. As one PT was sharing what her group had discussed, Tracy
encouraged her to come up with a name for their method of synthesizing and made
reference to the cake analogy on her syllabus.

Tracy: So you could all do even the same sort of thing but what’s the name that
you do for it? Like you know on the syllabus, I had that thing about I’m not
giving you fully baked cakes. Do you remember that? That was a kind of
synthesis for me in terms of my teaching and my choices in this course too that
I’m not giving you fully baked cake as a teacher educator. (Second classroom
observation, 12/3/12)

Summary. Although the second classroom observation was significantly shorter
than the first, data were mostly consistent with regard to the themes and knowledge
domains indicated in Tracy’s interactions with the PTs. The big idea themes most often
represented were social justice and teacher practices while pedagogical content
knowledge domains were evident as well. With regard to content of practical knowledge,
knowledge of instruction and knowledge of surroundings were again prevalent. Tracy’s
orientations of practical knowledge were primarily experiential with social and
situational orientations also indicated. Given that this session included very little
mathematics content, the lack of data indicating domains of mathematical knowledge for
teaching as well as subject matter knowledge and curricular knowledge was to be
expected. Tracy’s guiding image of teaching as not providing fully baked cakes was
again evident as she specifically reminded her PTs of how she used it on the syllabus to describe course objectives.

**Professional Development Observation: Henry’s Freedom Box**

Tracy was observed for a third time on February 7, 2013 during a mathematics coaching professional development session. The observation occurred during the sixth of nine, two-day PD sessions that occurred monthly throughout the academic year. The observed session was selected for two reasons the first being the availability of both the researcher (I had responsibilities with first and second-year coaches) and Tracy. The second reason for selecting this particular session was to observe Tracy as she conducted the *Henry’s Freedom Box* book activity with a group of inservice teachers after having already observed her orchestrate this activity with preservice teachers. Twenty, second-year mathematics coaches from schools across a Midwestern state participated with Tracy in this session. She had done this activity with mathematics coaches in prior years but never using toy figures as the starting point for constructing the boxes and then scaffolding it to adults. Previously, coaches had only been asked to construct a box for themselves. Tracy felt that starting with the toy figures and progressing to adults would improve the session because “…part of the problem has always been, especially with the coaches, we never get them to move to a serious discussion about the mathematics or at least a more in depth discussion” (Pre-observation interview, 2/7/13). She believed that scaffolding the problem would bring out more mathematics than just measurement. Tracy also suggested this would provide the coaches an example of how to bring the mathematics and social justice out of a piece of children’s literature claiming that “…it’s
the issue of the field is that when people use a piece of children’s literature and integrate social justice, we lose the mathematics or we don’t have serious mathematics in the lesson…” (Pre-observation interview, 2/7/13).

The sequence of activities for this session was very similar to the session Tracy conducted with her PTs in the methods course. One noticeable difference was that after she read the book to the coaches, instead of immediately beginning to construct a box for the toy figure, they were asked to hold small group discussions and consider how they would use the activity in a classroom with children. The coaches were to focus on how they would use the book in general, through a social justice lens, and then with respect to mathematics. The remainder of the sequence of activities followed closely what Tracy had done with the PTs. Coaches constructed a box for the toy figure and were then asked to use mathematics to determine dimensions for a third grader and themselves. Additionally, they discussed how the activity could be adapted for multiple grade levels. For a complete list of the sequence of activities, see Table 21.

**Code frequency data.** A total code frequency of 251 resulted from analysis of data from Tracy’s professional development session. Big idea themes most frequently observed were *teacher practices* (29), *reflective practice* (18) and *social justice* (10). Only *pedagogical content knowledge* (28) was frequently coded within Shulman’s categories of content knowledge. Within the domains of mathematical knowledge for teaching, *specialized content knowledge* (21), *knowledge of content and students* (18), and *knowledge of content and teaching* (15) were frequently indicated.
Table 21. Professional Development Observation Activity Sequence – Tracy

<table>
<thead>
<tr>
<th>Obs. Number</th>
<th>Session Number/ Date</th>
<th>Sequence of Topics/Activities (Time)</th>
</tr>
</thead>
</table>
|             | 3  Session 6 2/7/13  | 1) Introduction – Tracy reminded Cs of previous discussions about their ideas of social justice & that no definitive answers were provided  
2) Henry’s Freedom Box – book activity (1 hr 20 min)  
a. Tracy read true story of Henry “Box” Brown to class  
b. Cs discussed in groups how they would use book in class at three levels  
   1) In general discussion of book  
   2) Through social justice lens  
   3) With focus on mathematics  
c. Whole class discussion of first two levels purposely avoiding mathematics  
d. Cs given a few more minutes to discuss book & mathematics  
e. Whole class discussion of book & mathematics  
f. PTs given toy character & materials to make a box for their character. Had to assume character would be in same position as Henry and replicated his box as closely as possible  
g. PTs constructed boxes while Tracy observed and offered suggestions about measurement  
h. Tracy asked Cs to use information from box they built to determine dimensions of a box for a 3rd grader 36 inches tall.  
i. Tracy walked around observing and listening to groups  
j. Whole class discussion of dimensions for 3rd grader  
k. Cs given 7 minutes to determine dimensions of box in which they could fit  
l. Whole group discussion of difference between task for 3rd grader and adult  
m. Tracy showed pictures of a previous coach using Henry’s Freedom Box with all 5th graders in her school  
n. Whole group discussion of grade level adaptation  
o. Tracy asked how mathematics could be as deeply addressed with this activity as with other activities Cs break for lunch |

Knowledge of instruction (45) was the only category frequently coded with respect to content of practical knowledge while evidence of an experiential (18) orientation to Tracy’s practical knowledge was observed as well. Given that this session involved the
same activity as one of the activities from the first classroom observation, a reasonable assumption would be that the PD data results would more closely resemble the first classroom observation than the second. See Table 22 for the professional development observation code frequency totals.

**Teacher practices, reflective practice, and social justice.** The entire PD session was devoted to activities revolving around the *Henry’s Freedom Box* book. The big idea themes most frequently indicated during Tracy’s session with the mathematics coaches were teacher practices, reflective practice, and not surprisingly, social justice. Each of these themes was evident in the instructions provided by Tracy after she finished reading the book to the coaches. Instead of having the coaches immediately begin constructing boxes for a toy figure she was prepared to give them, she allowed them an opportunity to first share thoughts on how they might use the book with children. Tracy modeled teacher practices by providing the coaches some direction in asking them to think about how they would use the book on different levels; focusing the discussion on the book itself, discussing it from the perspective of social justice, and with mathematics as the focus. She did not provide answers for the coaches but made suggestions for organizing their thoughts. Reflective practice, specifically reflective conversation (Schön, 1992), was evident as Tracy utilized small group discussion for coaches to either reflect on how they had already used the book or how they would use the book with children. The social justice theme was indicated by the choice of *Henry’s Freedom Box*, the story of a slave who mailed himself to freedom, for the session and her suggestion for
the PTs to think about how they would “bring a social justice lens to the discussion” (PD observation, 2/7/13).

Table 22. Professional Development Observation Code Frequency – Tracy

<table>
<thead>
<tr>
<th>Tracy - Professional Development Observation Code Frequency</th>
<th>Pre-Int</th>
<th>Obs</th>
<th>Post-Int</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation Data Segmentation (ODS)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Knowledge Data Segments (KDS)</td>
<td>51</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Big Idea Themes (BIT)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflective Practice (RP)</td>
<td>1</td>
<td>17</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Technology (TECH)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
<td>0</td>
<td>28</td>
<td>1</td>
<td>29</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
<td>2</td>
<td>8</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Children's Mathematical Thinking (CMT)</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Shulman Content Knowledge (SHU)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
<td>1</td>
<td>21</td>
<td>6</td>
<td>28</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Mathematical Knowledge for Teaching (MKT)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>0</td>
<td>20</td>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
<td>0</td>
<td>14</td>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
<td>2</td>
<td>9</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Content of Practical Knowledge (CPK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self (SLF)</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Instruction (INS)</td>
<td>1</td>
<td>39</td>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Orientations of Practical Knowledge (OPK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Situational (SIT)</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Personal (PER)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Social (SOC)</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
<td>4</td>
<td>12</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>16</td>
<td>202</td>
<td>33</td>
<td>251</td>
</tr>
</tbody>
</table>
Tracy further suggested to the coaches that they narrow their initial discussion to the first two levels of focus, the book itself through a social justice lens, and to temporarily ignore mathematics. Following a whole group discussion of ideas on the first two levels, coaches were then asked to focus on the mathematics.

Tracy: Anything else to add to this, because if not I’ll let you go off to think about the math part a little bit. Now you’re allowed to think about that, not that anybody listened to me because I heard the conversations, but if you can go ahead and focus a little on the math and then we’ll move on from that. (PD observation, 2/7/13)

As the session progressed into the box construction stage, Tracy expected that the coaches would use ratios and proportions to transition from the dimensions of the box for a toy figure to the dimensions of a box for a third grader. The themes of teacher practices and social justice appeared again as one coach suggested that “You can teach scale and proportion with a variety of texts” (PD observation, 2/7/13) and Tracy responded “Right, but my choice of this text was very deliberate because I want to approach this from the social justice perspective and have the discussion about slavery and about race and however else” (PD observation, 2/7/13). Tracy’s PD session continually highlighted the big idea themes of teacher practices, social justice, and reflective practice as the coaches learned all about the trials and tribulations of Henry “Box” Brown.

**Pedagogical content knowledge and specialized content knowledge.**

Tracy’s scaffolding of the box construction led to evidence of her reliance on both pedagogical content knowledge and specialized content knowledge. Her intent was to “force” (PD observation, 2/7/13) the coaches to use dimensions of the box they
constructed for the toy figure in combination with mathematics to determine the dimensions of a box that would accommodate a third grader and then an adult. As she provided instructions to direct the coaches, use of Shulman’s *pedagogical content knowledge, knowledge of content and teaching,* and *specialized content knowledge* were indicated.

Tracy: …So you have a 36 inch tall child. How would you go about, given what you know from your box, in determining what the dimensions should be for that box? You don’t have that kid right in front of you by the way. That kid’s not sitting there, you can’t measure this child.

C: That’s a problem.

Tracy: That’s a problem. It’s a problem. It’s a good problem if you ask me. So now you need to figure out what the dimensions of a box would be for a 36 inch tall kid…Figure that out and talk in your groups about how, the different ways in which you would figure that out and what some of the issues are as you are trying to transition from your figure’s box the 36 inch tall child. (PD observation, 2/7/13)

Tracy’s decision to ask the coaches to utilize the dimensions of the box they constructed to determine the dimensions of a box for a third grader demonstrated her knowledge of a strategy to reorganize the understanding (Shulman, 1986) transitioning coaches from using measurement to ratios and proportions, an indicator of Shulman’s *pedagogical content knowledge.* The same decision also suggested her reliance on *knowledge of content and teaching* as she sequenced the mathematics to progress from measurement to proportions (Ball et al., 2008). *Specialized content knowledge,* specifically mathematical tasks of teaching, was indicated by her selection of this activity to make the specific mathematical point of connecting measurement to proportions (Ball et al., 2008).
Further into the session, Tracy even made it transparent to her students as to why the activity had evolved into the lesson she presented that day. In particular, she alluded to the need to capture more substantive mathematics from the activity than had previously occurred when coaches were only directed to construct a box for themselves.

Tracy:…And the issue that I was always concerned with as we did that was that we could never get to any kind of substantive mathematics. People said oh you could use scale but they weren’t using scale. You could do something that was way different than actually using it. You applied proportional reasoning here in lots of different ways, very creatively even looking up things on the internet and what’s normal and what if the kid was proportioned like my poor little person with one seventh of his body being his legs. So you used it in a lot of different ways. One of the things that we did was to change to the figure and to try to scaffold this to kind of force people, my students refer to it as she gently manipulated, but to try to force the student into having to use this. (PD observation, 2/7/13)

Shulman’s *pedagogical content knowledge* was evident as Tracy used the scaffolding strategy to transition her coaches from measurement to proportions thus reorganizing their understanding of the necessary mathematics (Shulman, 1986). This strategy also indicated the use of *knowledge of content and teaching* based on her understanding that the different instructional strategy would lead to mathematical connections (Ball et al., 2008). *Knowledge of content and students* as well as specialized content knowledge was evident in Tracy’s ability to hear and interpret the coaches’ mathematical thinking and evaluate their claims as suggested by her recognition of the coaches’ different and creative uses of proportional reasoning (Ball et al., 2008). Additionally, data suggested Tracy utilized practical *knowledge of instruction* in her ability to evaluate (Elbaz, 1983) and determine that the previous instructional method used with this activity was not leading to “any kind of substantive mathematics” (PD observation, 2/7/13).
Evidence of *pedagogical content knowledge* and practical *knowledge of instruction* appeared again in the post-observation interview when Tracy was asked if she was surprised during the session. She immediately referred to her scaffolding strategy and that results were not necessarily what she expected.

Tracy: Some of, like I wasn’t, how I pictured the scaffolding really kind of surprised me in that I thought they’d make the box, I thought when they went from the little box for the figure to the imaginary three foot tall kid, that they would do more computation to get there but they didn’t, some of them didn’t. Although they were still using proportional reasoning, they weren’t doing it as abstractly as I expected…That surprises me, I don’t know if I’m necessarily distraught about or anything about it, but it does surprise me that going to the computation, symbol manipulation part wasn’t something that they really were that fond of… Instead of them each doing their own (toy figure) like they were supposed to, they didn’t pay attention to that rule and they chose the easy one, the one that was proportional or the one that wasn’t, you know, so they were able to avoid some of the contingencies that I wanted them to have to struggle with. (PD observation, 2/7/13)

Her discussion of the scaffolding strategy to reorganize the coaches’ mathematical understanding (Shulman, 1986) again indicated Tracy’s use of Shulman’s *pedagogical content knowledge* as well as knowledge of content and teaching. Evidence of her ability to hear and interpret the coaches’ mathematical thinking, *knowledge of content and students*, was indicated when she suggested that the coaches did use proportional reasoning but not in the abstract manner she had anticipated (Ball et al., 2008). Practical *knowledge of instruction* was again evident in terms of Tracy’s ability to evaluate the results of her instruction (Elbaz, 1983) when she suggested that the coaches picked easy toy figures to work with and avoided “some of the contingencies that I wanted them to have to struggle with” (PD observation, 2/7/13). As with previous activities in which mathematical content played a significant role, data indicated the Tracy simultaneously
utilized combinations of pedagogical content knowledge domains, specialized subject matter, and practical knowledge of instruction.

**Experiential orientation of practical knowledge.** Tracy’s reliance on previous experience to guide her instructional practice was present during this PD as in previous observations and interviews. Having taught the Henry’s Freedom Box activity twice before using the new scaffolding approach, she drew on knowledge gained from reflecting on those experiences to address a question in the pre-observation interview regarding problems or issues she anticipated in conducting the activity with mathematics coaches.

Tracy: As far as the actual teaching of it I think since I’ve taught it to pre-service teachers in this format and taught it at a conference workshop in this format, I’m hoping the bugs in the actual implementation are kind of worked out and I’ve tweaked the lesson to make it more specific in certain parts so I’m hoping that that fixes it. (Pre-observation interview, 2/7/13)

Tracy referred to having used this activity with PTs (first classroom observation) and with ITs (conference workshop) to suggest that she had “tweaked the lesson to make it more specific in certain parts” (Pre-observation interview, 2/7/13). Although not specific about the “tweaks,” this data indicated that she was using experiences to guide her knowledge (Elbaz, 1983) and thus her practice.

Her experiential orientation to practical knowledge was also evident during a segment of the session when Tracy was leading a whole class discussion about examining *Henry’s Freedom Box* from the perspective of social justice. When a coach stated that “We had some thoughts and I don’t know if I’m really following social justice as you had
intended here but…” (PD observation, 2/7/13), Tracy immediately drew on her experiences with the subject to comfort the coach.

Tracy: Well first thing you need to understand, there is no definition of social justice anywhere. You can ask Melissa (one of her graduate research associates), she’s doing her dissertation on it. It’s not out there. Carla (her colleague in social justice work) and I have written, it’s not out there. So part of what we’re all trying to do is figure out what this means, especially in mathematics education. So no apologies for not really knowing what social justice is because the field doesn’t know it. (PD observation, 2/7/13)

Tracy’s response indicated an experiential orientation to her practical knowledge as she relied on knowledge gained from dissertation work by one of her graduate research associates in the area of social justice as well as her own research (Elbaz, 1983). Evidence of Tracy’s experiential orientation of her practical knowledge from data relative to the PD observation was consistent with that found in previous observations and interviews.

**Tracy’s image of teaching.** As Tracy opened the PD session, she immediately offered insight into her beliefs about what teaching should be (Elbaz, 1983) and her image of teaching. She began by referring to what the group had done in previous sessions stating that “…we opened this year by talking about your ideas of what social justice was, we didn’t give you any answers if you remember correctly” (PD observation, 2/7/13). And although no visible reference was made to her cake baking analogy, she followed with “So if you’re thinking she didn’t give us any answers, you’re absolutely right” (PD observation, 2/7/13). This dialogue, as in previous observations and interviews, again verified Tracy’s analogy of baking. Even though part of the reason may
have been that not all the ingredients were available, Tracy certainly appeared comfortable with confirming that she had not provided the coaches answers.

The post-observation interview also offered evidence of her image in response to a question that referenced how one coach had used technology to solve a problem about ratios and proportions. Tracy suggested that allowing access to technology as a tool helped the coach and could help school children bake their cakes.

Z: Were you surprised at all, the one gentleman that did the, when they were talking about like the number of heads that make up a body, [Tracy: That was interesting.] the proportion, he had gotten on the computer and did the, talked about computer animation…

Tracy: And it was so interesting to me because if we have students, if we let students have access, free access like that, I can see that happening in a class and it was just so cool because I thought how different the presence of that technology, I mean look what it added right to that, in the moment…

Tracy responded in terms of what “free access” (PD observation, 2/7/13) and technology might mean for school children in the sense that, if accessible, they too could utilize technology (the tool) to approach problems in much the same way that the coach did. She was enthusiastic about providing children the freedom to attack problems in their own way again indicating her belief that teaching is not about providing all the answers.

**Summary.** Evidence of knowledge domains utilized by Tracy in the PD session was mostly consistent with those found in previous interviews and observations. Since the same activity was completed in both the first classroom observation and in the PD session, similar data, at least with respect to that activity, was to be expected. Additionally, mathematics content played a significant role in both activities of the first classroom observation as well as in the PD session. As a result, within the domains of
mathematical knowledge for teaching, *knowledge of content and teaching, knowledge of content and students,* and *specialized content knowledge,* were evident in Tracy’s practice. Additionally, Shulman’s *pedagogical content knowledge,* practical *knowledge of instruction,* and the *experiential* orientation of practical knowledge were evident in this PD observation as in previous interviews and observations. Curricular knowledge domains that were visible in the first classroom observation were not prevalent in the PD session mainly due to time constraints and Tracy’s instructional decision to eliminate the whole group discussion of grade level adaptations. The big idea themes of *teacher practices, reflective practice,* and *social justice* were reflective of the selection of Henry’s *Freedom Box* and Tracy’s image of allowing learners to discuss and develop their own thoughts regarding social justice in mathematics.

**Overall Knowledge Domain Data Analysis**

For Tracy, the overall frequency of codes was 1,176 with 32% of the codes coming from the interviews and 68% of the coding from the observations. Again this does not mean that there were 1,176 coded data segments because most data segments were coded within multiple categories. Instead, data segments from the interviews and observations were coded within the categories of the analytical framework a total of 1,176 times. More codes were expected to be from the observations than then interviews due to a significantly larger amount of data collected from the observations. To further break down the frequency, 340 codes represented big idea themes (BIT), 142 codes were considered within in Shulman's categories of content knowledge (SHU), 189 codes s fell into the categories of mathematical knowledge for teaching (MKT), 327 codes
represented content of practical knowledge (CPK) and 178 codes indicated orientations of practical knowledge (OPK). In each of the five components of the framework of analysis, the frequency of codes from the observations was greater than the frequency of codes from interviews. Figure 21 below displays the total frequency of codes for each of the five components of the framework of analysis together with a breakdown of the frequency of codes that came from interviews versus observations.

![Tracy - Total Code Frequency](image)

Figure 21. Total code frequency based on the framework of analysis – Tracy.

**Big idea themes.** Analysis of the 340 big idea theme codes indicated that those most often represented in the Tracy’s practice were teacher practices (TP), teacher learning (TL), social justice (SJ), children’s mathematical thinking (CMT), and reflective practice (RP). A total of 143 data segments were coded as teacher practices with 11% occurring in interviews and 89% occurring in observations. The researcher identified 52
teacher learning data segments with 73% and 27% from observations. The social justice code was assigned 51 times with 31% of the codes in interviews and 69% in observations. With regard to children’s mathematical thinking, 44 segments were coded with 39% coming from interviews while 61% were coded in the observations. Thirty-eight segments were coded as reflective practice and occurred almost exclusively in observations (87%). The remaining two categories, teacher beliefs and attitudes, and technology each represented less than 10% of the BIT codes. Figure 22 summarizes the frequency of codes for big idea themes.

Figure 22. Big idea theme frequency – Tracy.

Focusing on only the three observation sessions, the researcher identified a total of 242 knowledge data segments (KDS). Knowledge data segments represent divisions in the observation data based on Tracy’s interactions with learners that resulted in shifts
in instruction during the sessions. For example, shifts occurred as a result of moving from one activity to another, transitioning within an activity, responding to learner questions, or even using questioning techniques to raise the cognitive level of a discussion. Each time a shift occurred, the segment was identified as a knowledge data segments which could then be assigned one or more codes according to the framework of analysis.

Relative to the big idea themes, approximately 52% of the knowledge data segments were coded as teacher practices, 14 % as social justice, 14 % as reflective practice, and 11% as children’s mathematical thinking. Only 3% of knowledge data segments were coded as teacher learning which was consistent with the fact that 73% of the codes in this category came from interviews. For each of the remaining categories, the coded segments represented 1% or less of knowledge data segments (See Figure 23).

Figure 23. Big idea themes as percentage of KDS – Tracy.
Shulman’s categories of content knowledge. Shulman's categories of content knowledge include subject matter content knowledge, pedagogical content knowledge, and curricular knowledge (Shulman, 1986) and all three played a role in Tracy's practice but pedagogical content knowledge was indicated far more frequently than the other two. Of the 142 codes representing one of Shulman’s categories of content knowledge, 93 were coded as pedagogical content knowledge with interviews accounting for 39% of the codes and observations accounting for 61 % of the codes. Data segments were coded as curricular knowledge 26 times and with 35% from interviews and 65% from observations. Subject matter content knowledge was identified 23 times and was heavily weighted toward observations (78%). See Figure 24 for a summary of the code frequency for Shulman's categories of content knowledge.

![Tracy - Shulman Content Knowledge Frequency](image)

Figure 24. Shulman's categories of content knowledge frequency – Tracy.
Results from examining only the observations indicated a similar emphasis on pedagogical content knowledge. Approximately 24% of the 242 knowledge data segments were coded as pedagogical content knowledge while 7% were coded as subject matter content knowledge and 7% as curricular knowledge as well (See Figure 25).

![Shulman Content Knowledge](image)

Figure 25. Shulman's categories of content knowledge as a percentage of KDS – Tracy.

**Mathematical knowledge for teaching.** The six categories of mathematical knowledge for teaching are common content knowledge (CCK), specialized content knowledge (SCK), horizon knowledge (HK), knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). Data indicated that Tracy relied most heavily on knowledge of content and students, knowledge of content and teaching, and specialized content knowledge in her
instructional practices. More than one third of the 189 codes within mathematical knowledge for teaching were identified as *knowledge of content and students* (66). Approximately 41% of these codes occurred in the interviews and 59% in the observations. Forty-eight data segments were coded as *knowledge of content and teaching* with 46% coming from the interviews and 54% from the observations. *Specialized content knowledge* appeared in 52 data segments with the overwhelming majority (90%) occurring in observations. Each of the remaining categories of mathematical knowledge for teaching had a frequency representing less than 10% of the total codes. Figure 26 displays the frequency of coding for mathematical knowledge for teaching.

Figure 26. Mathematical knowledge for teaching frequency – Tracy.
A detailed look at the observational data also indicated that the same three categories were the most significant knowledge domains from which Tracy drew but the order was different. Considering observations only, *specialized content knowledge* represented the highest percentage of the 189 knowledge data segments at 19%.

*Knowledge of content and students* (16%) and *knowledge of content and teaching* (11%) ranked behind specialized content knowledge. Each of the three remaining categories represented less than 10% of the knowledge data segments. See Figure 27 for a graphical representation of the categories of mathematical knowledge for teaching as a percentage of knowledge data segments.

Figure 27. Mathematical knowledge for teaching categories as a percentage of KDS – Tracy.
Practical knowledge. Data analysis for the practical knowledge of Tracy, as with Alex, was divided into three sections according to the primary classifications of content of practical knowledge, orientations of practical knowledge, and structure of practical knowledge. Code frequency data is presented for the content of practical knowledge and the orientations of practical knowledge while the structure of practical knowledge is discussed in the summary based on an overall view of Tracy’s instructional practices.

Content of practical knowledge. Knowledge of instruction was the most frequently coded type of content of practical knowledge utilized by Tracy. Of 327 codes within the domain of content of practical knowledge, 205 were coded as knowledge of instruction. As would be expected, a majority of the coded data segments were identified within the observations (82%). The only additional code representing more than 10% of the total frequency for content of practical knowledge was knowledge of surroundings (43%). Data segments coded as knowledge of surroundings were almost evenly distributed between interviews (53%) and observations (47%). Knowledge of self, subject matter, and curriculum development each represented less than 10% of the total frequency for content of practical knowledge. Figure 28 displays the code frequency for content of practical knowledge.

Focusing on observations only, similar results were found in that the greatest percentage of the 242 knowledge data segments were coded as knowledge of instruction (70%). However, knowledge of subject matter (10%) was the only other category that
represented at least 10% of the knowledge data segments as *knowledge of surroundings* was identified in only approximately 8% of the knowledge data segments.

![Graph](image)

**Figure 28.** Content of practical knowledge frequency – Tracy.

Given that two of the observations involved significant mathematical content, practical *knowledge of subject matter* likely would play a more significant role in the observations. Figure 29 summarizes the content of practical knowledge as a percentage of knowledge data segments.

**Orientations of practical knowledge.** *Experiential* was the primary orientation of practical knowledge that informed Tracy's practice while *situational, social,* and *theoretical* orientations to practical knowledge seemed less influential and thus secondary in nature. Of the 178 codes of orientations of practical knowledge, 74 were coded as
experiential knowledge and they were fairly evenly distributed between interviews (47%) and observations (53%).

The frequency of data segments coded as situational knowledge was 40 with 38% occurring in interviews and 62% in observations. Social orientations to practical knowledge were coded 32 times and almost exclusively in observations (91%) while theoretical knowledge was coded 19 times and heavily weighted towards interviews (74%). Figure 30 is a graphical representation of the frequency of the orientations of practical knowledge.
When looking exclusively at the observations, *experiential knowledge* was the primary source utilized by Tracy. Approximately 16% of the 242 knowledge data segments were coded as *experiential knowledge* with *social knowledge* (12%) and *situational knowledge* (10%) also identified in the data. Each of the two remaining categories represented 2% or less of the knowledge data segments and thus appeared to be utilized peripherally, if at all, in Tracy’s interactions with PTs and mathematics coaches. See Figure 31 for a graphical representation of the orientations of practical knowledge as a percentage of knowledge data segments.
Figure 31. Orientations of practical knowledge as percentage of KDS – Tracy.

**Tracy Case Profile Summary**

At the time of this study, Tracy had been involved in the practice of mathematics teacher educating for approximately 18 years with experiences teaching elementary and secondary mathematics methods courses and an action research course as well as field experience supervision. She had taught elementary (K-5) and middle school (7-8) mathematics while serving as a teacher and math teacher leader in public schools. Additionally, she led professional development sessions for teachers as part of a mathematics coaching program and at professional conferences in relation to her research interests in the area of equity and social justice in mathematics education.

Data indicated that the primary big idea themes reflected in Tracy’s practice of mathematics teacher educating were *teacher practices, teacher learning, and social*
justice while children’s mathematical thinking and reflective practice were indicated as secondary themes (See Figure 32).

Figure 32. Case profile summary of knowledge domains utilized by Tracy.

Teacher practices were evident mostly as she modeled activities for her PTs and coaches in the same way she expected them to complete the activities with school children. Tracy made it transparent that she was concerned with the mathematical thinking and learning of both school children and the teachers with whom she worked. She often drew on past experiences to provide examples of children’s mathematical thinking with respect to the
content she discussed in session activities and also took advantage of available opportunities to address the PTs’ and coaches’ issues with mathematics content. The importance Tracy placed on children’s mathematical thinking was also evident on a topical outline she provided for the PTs in her methods course as the first objective of the second session stated “Begin work on thinking about student thinking” (Topical outline, second classroom observation, 12/3/12). Social justice played a significant role in all three observations as objectives of each session were based on equity and social justice pedagogy in the context of mathematics education. The timing of the classroom observations may have influenced the data in this regard as the topical outline suggested that equity and social justice pedagogy were a primary focus of the final three sessions of the course, and although evident, possibly not as specifically prominent in the first six sessions.

Reflective practice as a big idea theme was evident throughout the observations and corresponded with Tracy’s image of teaching as a process of baking a cake. The PTs and coaches were given numerous opportunities to engage in reflective conversation regarding children’s literature, problems posed, and equity and social justice pedagogy. Tracy provided them the necessary tools and limited direction to allow them the opportunity to develop their own knowledge with respect to mathematics content and other issues or topics associated with the activities. Her use of small group discussion followed by whole group reflection and sharing provided evidence of both reflective practice and Tracy relying on her image of teaching to guide her instructional practices.
Of Shulman’s categories of content knowledge, *pedagogical content knowledge* was indicated to be the primary knowledge domain from which Tracy drew while *subject matter content knowledge* and *curricular knowledge* were secondary (See Figure 32). Pedagogical content knowledge was evident in the methods she used to develop the mathematics and make it understandable as well as the strategies she implemented to reorganize the understanding of her learners (Shulman, 1986). Her use of scaffolding to encourage PTs and coaches to progress from measurement to proportions to determine box dimensions was an example of Shulman’s *pedagogical content knowledge*. *Subject matter content knowledge* was limited mostly to segments in activities when mathematics content was the focal point while *curricular knowledge* was limited to discussions of the Common Core Standards for Mathematics and availability of materials to use with specific activities.

With respect to mathematical knowledge for teaching, *knowledge of content and students*, *knowledge of content and teachers*, and *specialized content knowledge* all appeared to be primary knowledge domains from which Tracy drew as she interacted with PTs and coaches (See Figure 32). The two pedagogical content knowledge domains were evident in interviews as well as observations while *specialized content knowledge* was essentially limited to observations. *Knowledge of content and students* was visible in Tracy’s ability to predict what would motivate learners, whether children or adults, and her ability to listen to and interpret her PTs’ and coaches’ incomplete thinking and respond accordingly (Ball et al., 2008). Tracy’s understanding of affordances of different methods and her ability to efficiently sequence content for instruction provided evidence
of her use of *knowledge of content and teaching* (Ball et al., 2008). Specialized content knowledge was observed mostly during segments of activities where mathematics content was the primary focus and was recognized as a mathematical task of teaching. Tracy indicated use of this knowledge domain as she provided and evaluated mathematical explanations, asked productive mathematical questions, and connected mathematical topics (Ball et al., 2008). Since evidence was typically limited to mathematical tasks of teaching, *specialized content knowledge* was almost non-existent in the interviews.

*Knowledge of instruction* was the primary knowledge domain evident with respect to the content of practical knowledge. *Knowledge of surroundings* was secondary (See Figure 32). *Knowledge of instruction* often appeared as Tracy modeled instructional practices for the PTs and coaches by conducting the activities in the same manner as they would with children and as a result was evident mostly in observations. Her beliefs about the acts of teaching, ability to organize instruction, and willingness to evaluate results of her instruction indicated her use of the practical *knowledge of instruction* (Elbaz, 1983). Evidence of her *knowledge of surroundings* was more evenly distributed between interviews and observations and appeared primarily in Tracy’s recognition of the classroom setting and the social setting of the school, with respect to children and adults, and how they might influence learning (Elbaz, 1983). An example would be her awareness of the importance of teachers being familiar with children’s backgrounds and home environments in order to address specific needs.

Tracy’s orientations of practical knowledge were varied with the primary orientation indicated as *experiential* while *situational, social*, and *theoretical* orientations
appeared to be secondary (See Figure 32). In both interviews and observations, she frequently referred to previous experiences, especially with regard to children’s mathematical thinking, to provide examples for the PTs and coaches. Tracy drew from her experiences as a classroom teacher as well as her experiences as a mathematics teacher educator (Elbaz, 1983). A situational orientation to her practical knowledge was often reflected in her responses to PTs’ questions regarding specific assignments due at the end of the course or expectations for the final exam while the social orientation was usually indicated in the discussions about equity and social justice pedagogy. Although Tracy’s theoretical orientation of practical knowledge was not overwhelmingly evident in any of the observed sessions, initial interview and post-observation interview data verified its presence in her thinking. Additionally, the course syllabus and topical outline referred extensively to theoretical aspects of mathematics education instruction including Constructivist mathematics pedagogy, Learner Responsive Pedagogy in mathematics, Cognitively Guide Instruction, and Equity Pedagogy. Tracy also referenced her use of literature as a guiding force when she said “I like to ground my work in what I know about the literature” (Post-observation interview, second classroom observation, 12/3/12).

The research-based instructional practices that appeared in the objectives of the syllabus and the topical outline were consistent with Tracy’s view or image of how teaching should be. With respect to mathematics education, each is predicated on an instructional practice where learners construct their own knowledge through problem-based explorations, reflect on and discuss problems with other learners and the teacher, and teacher instructional decisions are guided by learner mathematical thinking. This
type of instructional practice was certainly modeled by Tracy and consistent with her image that she should not provide fully baked cakes but instead allow learners to use the tools provided and a few ingredients to create their own cake. Her concern for learning was ultimately focused on what was best for children and she believed that this instructional methodology allowed all children access to the mathematics.
Chapter 6: The Case Profile of Luke

Luke – Data Collection

Data collection for Luke began in March of 2012 with an initial semi structured interview conducted to gather information regarding his educational background, teaching experiences, and research interests. The interview also elicited Luke’s initial thoughts on teacher education and the knowledge domains that he thought he utilized in his interactions with PTs and inservice teachers (ITs). Following the initial interview, the researcher then conducted two classroom observations in late November and early December of 2012 at the main campus of a midsized Midwestern university. The observations occurred during the final two sessions of Luke’s mathematics methods course titled Teaching Methods for Secondary Mathematics and the class consisted of both undergraduate and graduate student preservice teachers. Luke was observed a third time in early February at a mathematics coaching program professional development session where he worked with mathematics coaches from across a Midwestern state that were in their second year of participation in the program. Data collection also included pre-and post-observation interviews corresponding to each observation. Table 23 summarizes the complete data collection process.
Table 23. Data Collection Summary – Luke

<table>
<thead>
<tr>
<th>Data Collected</th>
<th>Date</th>
<th>Location</th>
<th>Approximate Length</th>
<th>Learners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Interview</td>
<td>3/17/12</td>
<td>Mathematics Coaching Program PD Session</td>
<td>1 hr 22 min</td>
<td>NA</td>
</tr>
<tr>
<td>Follow up to initial interview</td>
<td>5/16/12</td>
<td>Mathematics Coaching Program PD session</td>
<td>42 min</td>
<td>NA</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; Classroom Observation</td>
<td>11/28/12</td>
<td>University Secondary Mathematics Methods Course</td>
<td>Pre-Int: 7 min Obs: 2 hr 8 min Post-Int: 36 min</td>
<td>2 male &amp; 2 female students, 3 undergraduates &amp; 1 graduate student, all preservice teachers</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; Classroom Observation</td>
<td>12/5/12</td>
<td>University Secondary Mathematics Methods Course</td>
<td>Pre-Int: 12 min Obs: 1 hr 40 min Post-Int: 16 min</td>
<td>2 male &amp; 2 female students, 3 undergraduates &amp; 1 graduate student, all preservice teachers</td>
</tr>
<tr>
<td>Professional Development Observation</td>
<td>2/8/13</td>
<td>Mathematics Coaching Program PD session</td>
<td>Pre-Int: 6 min Obs: 1 hr 33 min Post-Int: 18 min</td>
<td>20 mathematics coaches &amp; 1 facilitator (inservice teachers) – coaches 2&lt;sup&gt;nd&lt;/sup&gt; year of participation in program</td>
</tr>
</tbody>
</table>

Introducing Luke: Education, Experience, and Research

Luke is a Caucasian male and at the time of this study was an associate professor of mathematics education at a midsized Midwestern university. As part of his faculty duties, he taught content and methods courses for undergraduate and graduates students. Specifically, he was teaching a secondary mathematics methods course consisting of one graduate student and three undergraduate students. Luke also participated in a funded mathematics coaching professional development research project located in a Midwestern state. Although he had no formal responsibilities with the project, he often participated in
the PD sessions with the mathematics coaches. Luke’s educational background included a BS in Marketing and Economics, a MS in Mathematics and a PhD in Curriculum and Instruction with an emphasis in Mathematics Education. Additionally, he held a license to teach secondary mathematics.

Luke began his teaching career as a substitute teacher in a public school system and eventually progressed to full time teaching positions in public schools and on a Native American reservation. His school teaching experiences were primarily at the 9-12 grade levels. His teaching repertoire while on the reservation included instructing grades 7-12. Luke claimed that his experience teaching on the Native American reservation raised his awareness of the influence of cultural norms on decisions he made as an educator, acknowledging the need to adjust his lens when working with children there.

Luke: And then you get to the reservation and you realize well their world view is completely different. But if you don't have anybody there at the reservation who is going to share and say "Hey listen, you know you come from this Anglo world and this is how you do things but we don't do things around here like this and these are some of the cultural norms, social morays that you need to be aware of as an outside member." (Initial interview, 5/16/12)

Following his work on the reservation, Luke served as a graduate teaching assistant in a mathematics and statistics department and then taught mathematics at a series of community colleges. One of his community college teaching experiences was located at an industrial plant that manufactured dashboards and armrests for automobiles. The position was part of a work-based educational program for the plant employees and again encouraged Luke to consider the need to be responsive to backgrounds of learners when teaching mathematics.
Luke: So we spent a lot of time observing what was going on in the plant and trying to understand that and then try to incorporate that into the ideas of uh it was kind of a remedial program in mathematics, but not only ideas that were happening in the plant but also ideas that were related to the people that live there.

After the community college experiences, Luke transitioned to university settings where over an eight year period, he held an assistant professor position at three different institutions. During this time, he was positioned with the department of mathematics, the department of mathematics and statistics, and in a college of education. He secured tenure and promotion to the rank of associate professor in the department of mathematics and statistics while at the third university. At the time of this study, Luke had spent four years in his current position as an associate professor of mathematics education.

Luke’s research interest lies in the area of ethnomathematics and “specifically focused on improving mathematics education opportunities for Native Americans” (Initial interview, 3/17/12). He spent a considerable amount of time in Native American schools across the United States and published numerous articles with respect to both ethnomathematics and Native American mathematics education. His teaching experience on a Native American reservation likely ignited his research interests.

**Luke’s Initial Interview**

Data relative to Luke’s initial thoughts concerning knowledge domains utilized in the practice of mathematics teacher educating were collected in a two-part interview. The initial interview was conducted on March 17, 2012 and a follow-up interview took place on May 16, 2012. Both sessions occurred prior to any observations of Luke’s interactions with PTs or ITs in a classroom or professional development setting. The follow-up interview was necessary due to the researcher’s inexperience, time constraints,
and development of the interview protocol. The initial interview was the researcher’s first experience conducting interviews alone resulting in missed opportunities for appropriate and necessary questions. Additionally, the interview protocol was still being developed at the time of the initial interview therefore, the follow-up session allowed me to build a more complete picture of Luke’s views regarding his work as a teacher educator. For the purposes of reporting and discussion, the combination of the initial interview and the follow-up interview is simply referred to as the “initial interview.”

**Code frequency data.** The total code frequency for Luke’s initial interview was 257 which means that data segments from the interview were coded within the categories of the theoretical framework a total of 257 times. As with previous cases, most data segments were coded within multiple categories. The big idea themes coded most frequently were teacher learning (17) and children’s mathematical thinking (11). Pedagogical content knowledge (17) and subject matter content knowledge (10) were most frequently coded within Shulman's categories of content knowledge while knowledge of content and students (11) and specialized content knowledge (10) were indicated most frequently with respect to mathematical knowledge for teaching. Categories of the content of practical knowledge most often coded were knowledge of surroundings (28), knowledge of self (23), knowledge of instruction (16) and knowledge of subject matter (10). Within the orientations of practical knowledge, evidence of experiential knowledge (43) and situational knowledge (19) appeared most frequently. See Table 24 for initial interview code frequency totals.
Table 24. Initial Interview Code Frequency - Luke

<table>
<thead>
<tr>
<th>Code Area</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observation Data Segmentation (ODS)</strong></td>
<td></td>
</tr>
<tr>
<td>Knowledge Data Segments (KDS)</td>
<td></td>
</tr>
<tr>
<td><strong>Big Idea Themes (BIT)</strong></td>
<td></td>
</tr>
<tr>
<td>Reflective Practice (RP)</td>
<td>2</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
<td>17</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
<td>1</td>
</tr>
<tr>
<td>Technology (TECH)</td>
<td>1</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
<td>5</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
<td>2</td>
</tr>
<tr>
<td>Children's Mathematical Thinking (CMT)</td>
<td>11</td>
</tr>
<tr>
<td><strong>Shulman Content Knowledge (SHU)</strong></td>
<td></td>
</tr>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
<td>10</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
<td>17</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
<td>1</td>
</tr>
<tr>
<td><strong>Mathematical Knowledge for Teaching (MKT)</strong></td>
<td></td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>0</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>10</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
<td>1</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
<td>11</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
<td>7</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
<td>0</td>
</tr>
<tr>
<td><strong>Content of Practical Knowledge (CPK)</strong></td>
<td></td>
</tr>
<tr>
<td>Self (SLF)</td>
<td>23</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
<td>28</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
<td>10</td>
</tr>
<tr>
<td>Instruction (INS)</td>
<td>16</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
<td>1</td>
</tr>
<tr>
<td><strong>Orientations of Practical Knowledge (OPK)</strong></td>
<td></td>
</tr>
<tr>
<td>Situational (SIT)</td>
<td>19</td>
</tr>
<tr>
<td>Personal (PER)</td>
<td>9</td>
</tr>
<tr>
<td>Social (SOC)</td>
<td>4</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
<td>43</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
<td>8</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>257</td>
</tr>
</tbody>
</table>
**Teacher learning and children’s mathematical thinking.** In his discussion of the practice of mathematics teacher educating, Luke often related the ideas of *teacher learning* and *children’s mathematical thinking*. His beliefs about the teaching knowledge that his PTs and ITs needed were often tied to children’s conceptions and misconceptions about mathematical content. Specifically, Luke wanted to help his teacher learners understand, recognize, and anticipate difficulties children might encounter with certain mathematical subjects or topics. During the data collection process, a participant other than Luke had suggested that teaching methods of teaching was more difficult than teaching mathematics itself. Luke was asked whether or not he agreed with that belief which eventually led him to a discussion of *teacher learning* in relation to *children’s mathematical thinking*. He felt his lack of a deep understanding of high school mathematics content limited his ability to help his PTs recognize children’s (high school students) misconceptions.

Luke: And I don’t have a really rich background’ let’s just say in uh Algebra I, II, III, Analysis, Statistics, in the Calculus and Precalculus that happens in the high school. I don’t know those subject areas as well as I should to understand, for instance, students’ misconceptions and so to help my students, help my preservice people to become better at recognizing the misconceptions for what they are and not, and not recognizing the misconceptions of “Yeah when I did that I also had that same problem.” No, listen to this person, listen to this kid who’s talking to you about this and what is their misconception. Now it might, it might align with yours but I don’t have an anticipatory set for those because I just don’t have those experiences (Initial interview, 3/17/12).

Luke’s focus on *teacher learning* was evident when he specifically stated that he was not confident enough with his knowledge of the subject areas to “…help my preservice people to become better at recognizing the misconceptions for what they are…” His discussion of “students’ misconceptions” as well his reference to listening to children to
pick up on their misconceptions provided evidence that tending to children’s mathematical thinking was an important consideration in Luke’s practice as he repeatedly highlighted this point throughout the interview.

Luke was asked to compare the knowledge necessary to teach elementary teachers with the knowledge necessary to teach secondary teachers. He punctuated the difference in the mathematical content knowledge of elementary PTs compared to secondary PTs and offered how this knowledge influenced his task selection when working with each group. For example, he shared that he requires the PTs work in base five arithmetic so that they would experience some of the same difficulties and frustrations that children experience with base ten arithmetic. In the case of secondary teachers, Luke considered learning about content trajectory to be a priority for teachers. He believed building teachers’ capacity pertaining to this area was more difficult to achieve among elementary PTs due to their limited understanding of mathematics.

Luke:...And, on that, but on the elementary side, base 10 arithmetic, well what can I connect that to? Really nothing you know. I mean kids are counting and doing all kinds of activities away from school, ok well connect it to that but then see what they’re doing and see what kind of relationships we can build there. So I think that’s probably the biggest difference the, it’s a little more straight forward to put to secondary folks because of their mathematical sophistication and the lack of, I think as a society, the lack of, of a demand that we put on elementary folks for the amount of sophistication that they should have. (Initial interview, 3/17/12)

Consideration of children’s mathematical thinking was evident in Luke’s discussion of the difficulty in helping PTs find connections for base ten arithmetic to anything other than counting and activities children do away from school. His concern for teacher learning was evident with his suggestion that working with secondary teachers was easier because of their “mathematical sophistication.”
Pedagogical content knowledge and knowledge of instruction. One of Luke’s prior teaching experiences included a community college. There, he taught mathematics as part of a workplace education program in a plant that made dashboards and armrests for automobiles. Part of the incentive of the program was to allow plant employees the opportunity to earn a GED. As Luke discussed the background of the plant laborers and influences on his instructional practices, his use of pedagogical content knowledge domains and practical knowledge of instruction was evident. Based on the location of the plant, most of the learners he worked with were hunters and were familiar with a parabolic path as it related to shooting a gun.

Luke: So for instance, when we talked about quadratic functions, I can remember a lot of guys, there was a group of guys that were taking this class and uh talking about the path of a quadratic if you, the trajectory. Right? And as soon as I used trajectory, it turned out everybody who was sitting there owned a gun and they were all hunters. And so then the conversation turned to ballistics and uh and different types of firearms and firearms that shot flat versus ones that had big parabolic paths and depending on how far away the deer was that you were aiming at, uh that kind of thing. (Initial interview, 3/17/12)

Luke’s use of bullet trajectories to relate parabolic paths to quadratic functions offered evidence of Shulman’s pedagogical content knowledge and representing mathematics in such a way to make it comprehensible to his learners (Shulman, 1986). His willingness to allow the discussion to move toward a topic interesting and motivating to his learners (Ball et al., 2008) indicated Luke’s reliance on knowledge of content and students with respect to mathematical knowledge for teaching. Practical knowledge of instruction was also evident relative to teacher-student interaction and his knowledge of the importance of hunting to the learners (Elbaz, 1983). In most educational settings at the K-12 level,
discussion of guns would likely be discouraged but in this case, was certainly relevant and appropriate.

Pedagogical content knowledge and practical knowledge of instruction were also evident in his discussion of PTs completing mathematical tasks in base five arithmetic. Luke used this instructional strategy to deepen PTs’ understanding of base ten arithmetic while allowing them to experience the frustrations that some children endure when working in base ten arithmetic.

Luke: As soon as we start to do something in base five and I see this huge level of frustration with my students and, and some of them, are really just pissed off, “How long are we gonna do this?” “For about two weeks.” “Why? I will never do this in elementary school.” “I know.” “So why are we doing this?” I said “How good do you think your understanding is of base, base ten arithmetic?” “I think I’m expert at it.” I said “Well then show it to me using base five.” “I will never do this.” “Don’t really care.” I said “When you’re done with this experience, you’re gonna, you’re gonna run into some youngster in your classroom who has, who is struggling with base 10 and you’re gonna stand there and stare at him and say God I completely appreciate it. (Initial interview, 3/17/12)

Shulman’s pedagogical content knowledge was evident in the sense that Luke used base five arithmetic to reorganize his PTs’ understanding of base ten arithmetic (Shulman, 1986). With respect to mathematical knowledge for teaching, knowledge of content and students was indicated by his ability to anticipate that PTs would struggle with base five arithmetic finding it difficult and frustrating (Ball et al., 2008). Luke appeared to draw on practical knowledge of instruction relative to teacher-student interaction (Elbaz, 1983) in explaining to the PT that the exercise of completing tasks using base five arithmetic would help him or her understand the frustrations some of their future students might encounter with base ten arithmetic. Whether instructing laborers in factory or PTs in a
methods course, Luke’s examples provided evidence of his use of pedagogical content knowledge and practical knowledge of instruction.

**Subject matter knowledge domains.** Luke relied on stories of his practice to make visible his perspectives on teacher educating, drawing mostly on the type of mathematical activities he had used along with the goals he had intended to meet with them. Similar to his practice of having PTs complete tasks in base five arithmetic, Luke shared how he used a Cayley table with an undefined operator as a means for teachers to realize generalizability of the distributive and commutative properties for all operations. His references to Cayley tables, the commutative property, and the distributive law suggested he drew on subject matter knowledge when organizing these experiences.

Luke: Now every once in a while, I’ll reach into, you know, I’ll reach, I’ll grab my old Abstract Algebra book and I’ll put a Cayley table on the wall with some undefined operator and there’s the solutions and I’ll ask my students “Well if you really understand commutativity or distributive, or the distributive law, does it work?”… Maybe commutativity works and the, but then the distributive law doesn’t work… And I said “Ok so if you really understand this, now change the Cayley Table so that they both work.” I said “It’s the commutative law. It’s the distributive law.” I said “My child sees this in grades four and five. I’m just giving it to you in a different, through a different lens. (Initial interview, 3/17/12)

Luke indicated an understanding of the structure of mathematics relative to the commutative property and the distributive law. He recognized that Cayley tables could be used to determine the validity of the two properties for an undefined operation indicating his reliance on Shulman’s view of subject matter content knowledge (Shulman, 1986). Luke’s selection of an undefined operation instead of addition or subtraction to investigate the properties suggested the use of specialized content knowledge in regard to the selection of a representation for a specific purpose (Ball et al., 2008). This example
also indicated his use of practical *knowledge of subject matter* based on a unique rather than traditional approach (Elbaz, 1983) to exploring the properties.

Since Luke had been both a secondary mathematics teacher and a mathematics teacher educator, he was questioned about how the knowledge he used as a teacher educator compared to the knowledge he used as a secondary school teacher. He perceived his role as a teacher educator to help teachers develop their understanding of content trajectories and making connections between subject areas. He stated “I’m purposely grabbing this piece of geometry right here because I want to amplify what’s going on in algebra” (Initial interview, 3/17/12). Luke provided an example from his experiences as an MTE where issues with the Pythagorean Theorem led him to draw from subject matter knowledge domains to help teachers recognize connections between geometry and algebra.

Luke: The one thing that keeps rearing up in my classes, and this surprises me every time it happens, is \( a^2 + b^2 = c^2 \). And uh so I’ll ask people to draw, just give me a sketch of what that means and more times than not, they just sketch a right triangle. And I said “Ok so let’s go back. What does ‘a’ mean?” And they don’t think about ‘a’ being in that context about being the dimension of the side of the rectangle. And they don’t think about \( a^2 \) and moving into two dimensions or \( a^3 \) moving into three dimensions and drawing these kinds of pictures. Even my inservice teachers and that’s really troublesome because they’re teaching geometry. (Initial interview, 3/17/12)

*Specialized content knowledge* was evident as he linked the algebraic representation of the lengths of sides of a right triangle to the geometric representation of area, a mathematical task of teaching (Ball et al., 2008). Luke expressed his views of the content (Elbaz, 1983), an indicator of practical *knowledge of subject matter*, by emphasizing the importance of helping his learners make connections across subject areas.
Practical knowledge of self and surroundings. The initial interview provided insight into Luke’s knowledge of his strengths, weaknesses, and beliefs about his role as a mathematics teacher educator. His discussions offered examples of how he used his practical knowledge of self and practical knowledge of surroundings to formulate his instructional practices to utilize his strengths with respect to the needs of PTs, practicing teachers, workforce laborers, and even Native American children. As part of his response to the question concerning knowledge bases needed by teacher educators for the practice of teacher educating, Luke articulated the need for teacher educators to address gaps in teachers’ specialized mathematical knowledge for teaching. Luke asserted that some in the mathematics department were not concerned with students becoming teachers but more focused on preparing them to be mathematicians. This indicated his use of the practical knowledge of surroundings related to the social setting (Elbaz, 1983) of the campus with regard to department objectives. Further, he acknowledged awareness of his responsibility to offer the PTs ideas about teaching children that were not addressed in the mathematics department thus providing evidence of knowledge of self with respect to personal beliefs (Elbaz, 1983).

As was discussed earlier, Luke had spent time teaching children on a Native American reservation and his reference to the experience indicated how he recognized his weaknesses and adapted his practice based on practical knowledge of self and practical knowledge of surroundings. Luke claimed having been a “terrible teacher” until he familiarized himself with and adapted to their culture.

Luke: So it's safe to say, on the reservation initially, I was just complete disaster. I was, I was just terrible. But now all these years later, I've read a little bit about
the, a little of the history of how the US government has treated tribal people. I've talked to a lot of elders and I've talked to members of different tribes and, and I've learned that, and I've learned, I've actually learned from watching tribal members as classroom teachers. You know for instance the idea of singling out a child for their behavior is likely not important or, or not appropriate in a lot of reservations or singling out a child to get him or her to answer a question when you realize in a lot of these communities, the idea of one person rising about the group is inappropriate. (Initial interview, 5/16/12)

Luke specifically stated that “…on the reservation initially, I was just complete disaster.”

This acknowledgement indicated his use of practical knowledge of self in gauging his role and responsibilities as an educator (Elbaz, 1983). He claimed that through reading about the history of the Native American people and through observation of and discussion with tribal members, he familiarized himself with aspects of Native American culture in an attempt to better relate to the children in the classroom. Regardless of the level or culture of the learners with whom Luke worked, data indicated that he utilized knowledge of self and knowledge of surroundings to guide his practice.

**Experiential and situational orientations.** In continuing to examine Luke’s perspectives, the presence of experiential and situational orientations to his practical knowledge became more evident. Most of the data segments already presented were relived examples of past teaching experiences and often were specific to a given situation such as teaching on a reservation. Many of those data segments were coded as experiential and situational orientations of practical knowledge though for organizational purposes, not discussed as such at that time. Consequently, additional data segments are offered to present evidence of Luke’s use of practical knowledge that is experiential and situational. Continuing with his work on Native American reservations, Luke was asked how reservation teaching was similar to or different from his work as a teacher educator.
In his response, he referred to his ability to share experiences “...away from mainstream America” with teachers who wish to instruct in such communities.

Luke: You know when a, when a youngster tells me you know "I think I want to go teach in a community where I'm not a member." Let's, let's say it's a tribal community or something like that. The first thing I tell them, I say "Go look at the history and see how the government treated them." And you know "What's the situation for that tribe right now?" And you know "What's the environment look like? So you understand before you go in there. And then if there's any history written about the tribe, try to get an idea, you know was it a nomadic tribe? Like there's a few tribes that are in the United States, a couple that are still living on their homeland. (Initial interview, 5/16/12)

Luke appeared to draw on his past experiences to gauge his instruction when working with PTs encouraging them to first investigate the culture they wish to enter. Evidence of a situational orientation to his practical knowledge relative to communication and creating a functional classroom atmosphere (Elbaz, 1983) was suggested as Luke asserted that the PTs should examine both the history and the current situation of the particular community, gathering as much information as possible to understand the context.

In a sense, his experience teaching at a plant was not unlike teaching in a different culture. Luke acknowledged that he had much to learn when he began that teaching assignment and most of it he learned on his own. However, that experience helped him learn how to contextualize mathematics in such a way as to benefit his learners and offered evidence of his experiential orientation of practical knowledge.

Luke: Well yeah the knowledge, well the knowledge I needed for the first community college experience, I had to learn myself at the plant because I had nothing, I had no background or understanding of a manufacturing environment and what they did. Um so in that experience, you know we picked a couple of things that were happening in the plant and tried to understand that and tried to figure out ways of, to um thread that in to what would be, let’s say, a traditional algebra course um and to talk about that.

256
Additionally, evidence of his *situational* orientation of practical knowledge, specifically knowledge of instructional influences (Elbaz, 1983) was indicated as he suggested how he utilized things that were occurring in the plant and tried to figure out ways to incorporate them to contextualize an algebra course. Luke’s reliance on *experiential* and *situational* orientations of practical knowledge was evident throughout the examples he provided in his initial interview.

**Luke’s practical principle.** Throughout Luke’s initial interview, he continually referenced examples from his teaching experiences that provided insight into his guiding belief as a MTE. Using the terminology of Elbaz (1983), the “practical principle” or “embodied purpose in a deliberate and reflective way” of his instructional practice appeared to hinge on providing learners with opportunities to construct their own mathematical knowledge through investigation and discussion of mathematical tasks. He offered examples of his experiences with school children as well as the PTs in his methods courses to substantiate this belief. Luke illustrated an example of his practical principal as he applied it to school children through a math-science partnership between his university and local public schools. One of the conditions of the partnership placed him in school classrooms to observe and assist the teachers. After convincing a reluctant teacher to allow him to conduct an activity using Geoboards, Luke discovered that the children did not have a good understanding of the Pythagorean Theorem.

Luke: So I did the best I could, got them to do some things and the, and the, the, the lack of the grasp of the Pythagorean Theorem became evident when I said “Here’s a five by five Geoboard. Let’s find all the lengths that are available.” And so they were ok with the horizontal/vertical lengths but they weren’t ok with square root of two, square root of 13. (Initial interview, 3/17/12)
Luke allowed the children to investigate all the different lengths they could achieve on a five by five Geoboard and in doing so, he recognized they could not determine diagonal lengths. Allowing the children the opportunity to explore possible lengths not only provided them with an opportunity to construct their own knowledge but also made Luke aware of a conceptual gap he had not conceived before. These examples and others provided by Luke in the initial interview, indicated a belief that learners should be allowed to investigate and discuss mathematics, a practical principle that seemed to guide his instructional practice.

**Summary.** Throughout Luke’s initial interview, he illustrated numerous teaching experiences that offered insight into his thoughts regarding the knowledge domains he had relied on as both a classroom teacher and as a MTE. Additionally, he shared his beliefs regarding the knowledge domains he believed important for the practice of mathematics teacher educating. The big idea themes most frequently referenced were teacher learning and children’s mathematical thinking. Shulman’s pedagogical content knowledge and subject matter content knowledge were also evident throughout the interview. With respect to mathematical knowledge for teaching, knowledge of content and students and specialized content knowledge appeared most often. Content of practical knowledge was evident as knowledge of surroundings, knowledge of self, and knowledge of instruction while Luke’s orientations of practical knowledge emerged as experiential and situational. The initial interview also indicated that his instructional practice was based on the guiding belief that all levels of learners
should be provided opportunities to investigate mathematics in order to construct their own knowledge of the subject.

Mathematics Methods Course Observations

Luke was observed twice in a course titled “Teaching Methods for Secondary Mathematics.” The course was designed for graduate and undergraduate preservice teachers seeking licensure in secondary mathematics education. Overarching course objectives were to “explore what it means to think mathematically and to have mathematical power” (Course syllabus, 2012, See Appendix G) and then extend those ideas toward creating “a rich curriculum and environment that allows students to construct mathematical understanding for themselves” (Course syllabus, 2012). The course also placed a major emphasis on the Ohio Academic Content Standards in Mathematics, the NCTM Professional Teaching Standards, and the NCTM Principles and Standards (Course syllabus, 2012). In addition to course objectives, confirmation of the previously suggested practical principle that seemed to guide Luke’s instructional practices was evident on the first page of the course syllabus where he included the following paragraph and its reference.

It is assumed that learners have to construct their knowledge - individually and collectively. Each learner has a toolkit of conceptions and skills with which he or she must construct knowledge to solve problems presented by the environment. The role of the community - other learners and teacher - is to provide the setting, pose the challenges, and offer the support that will encourage mathematical construction. (Course syllabus, 2012)

Including these statements on the first page of the course syllabus offered strong evidence supporting the practical principle he indicated in the initial interview.

The class consisted of two male and two female PTs, one a graduate student and the remaining three all undergraduates. Observations took place during the 12th and 13th weeks of the semester which were the last two class sessions of the course. The first observation lasted slightly longer than two hours while the second observation lasted approximately one hour and forty minutes. The second classroom observation was atypical in that the entire session was devoted to helping PTs understand how to submit items for a university required teacher performance assessment electronic portfolio.

**First Classroom Observation: Solving Algebraic Equations**

The first classroom observation occurred during the 12th and next to the last session of the course on November 28, 2012. Two male and two female PTs were present and sitting in a group but in two consecutive rows in the center of a large room. The atmosphere was informal as Luke exchanged casual conversation with the PTs. The goal for the session was for PTs to consider the idea of assessment and think about “…how can that assessment inform their teaching, inform their practices after the assessment and to think about how the assessment might be aligned with objectives of their particular course” (Pre-observation interview, 11/28/12).

After a few minutes of casual conversation, Luke gave each PT an uncompleted copy of an algebra test consisting of 13 problems focused on solving first degree algebraic equations (See Appendix H for the assessment). Luke had been observing a secondary mathematics teacher and the handout was a test she had given to her Algebra 2
students. The PTs were first asked to solve all problems on the assessment and then encouraged to discuss their thoughts about what they believed the classroom teacher was trying to accomplish with the assessment. Following the discussion, Luke had the PTs work in pairs to create a rubric for grading the assessment giving them copies of actual student work to evaluate using the rubric. Once they finished scoring the student work, the PTs conducted an item analysis of the assessment. Luke then led a whole class discussion of the assessment, its blueprint, and the ways it could inform instruction. The segment of the class session devoted to the assessment activity was likely longer than necessary due to the PTs repeatedly asking questions concerning submission of items to an electronic portfolio required by the university. Luke willingly allowed and addressed their questions because the electronic portfolio requirement was new to the university and causing much confusion and stress among the PTs.

During the last half hour of the session, Luke and the PTs watched a video segment of a classroom session that occurred during the field experience of one of the PTs. Luke intermittently stopped the video to discuss the classroom interactions between the PT and the children often focusing on responses to children’s questions and the language children used. He also used the video to point out examples of items such as language artifacts that the PTs could use for their electronic portfolio. He then closed the session with a reminder of where the PTs could seek help with their electronic portfolios. See Table 25 for a list of the sequence of activities for the first classroom observation.
### Table 25. First Classroom Observation Activity Sequence - Luke

<table>
<thead>
<tr>
<th>Obs. Number</th>
<th>Course Week Number/ Class Date</th>
<th>Sequence of Topics/Activities (Time)</th>
</tr>
</thead>
</table>
| 1           | Week 12 11/28/12              | 1) Introduction – Casual discussion with PTs to start (3 min)  
  a. Discussion moved to how to convert from Pages to Microsoft Word  
  2) Luke gave PTs an Algebra 2 assessment that was given to high school students as part of a class & PTs completed assessment (9 min)  
  a. The assessment had to do with solving algebraic equations of degree 1 such as 7x-4=5x+18 and 5(x+2)-11= 2x+(9-7x)  
  b. Teacher who gave this assessment to high school students had 4-5 years experience  
  c. Luke began discussion of assessment by asking PTs “What was teacher trying to accomplish with this assessment?”  
  3) PTs created a rubric that would assess the worksheet (37 min)  
  a. PTs worked on rubric in pairs while Luke worked on computer to download a document discussed earlier  
  b. PT asked what to do about a problem that was written incorrectly on the worksheet and Luke told PT to make own decision about how to handle it  
  c. PTs scored actual student work on same assessment based on their rubric  
  4) PTs did item analysis on student work after they scored it (28 min)  
  a. Luke told PTs to distinguish between a problem that was wrong and one that was left blank  
  b. Luke moved to discussion of assessment and item analysis  
  c. Discussion of item analysis started with discussion of problems left blank  
  5) Luke then asked PTs “What would you do next?” (23 min)  
  a. Discussion moved to partial credit  
  b. Conceptual/procedural discussion  
  c. Luke then posed question “How might you engage students after these results on assessment?”  
  d. Discussion moved to politics of working within context of classroom teacher’s policies  
  e. Discussion of e-portfolio (teacher performance assessment) required of PTs intertwined throughout class  
  6) Class viewed a video clip of one of the PT’s field experience sessions with 9th grade Algebra 1 students regarding area of a triangle (27 min)  
  a. Luke stopped video several times to point out positive aspects of how PT handled student questions during episode  
  b. Luke emphasized language usage of students several times during clip  
  7) Closing – Luke reminded PTs about who to see if they had problems submitting item for e-portfolio (1 min) |
**Code frequency data.** With respect to the knowledge domains suggested on the analytical framework, the first classroom observation yielded a total code frequency of 367. Big idea themes most frequently coded were *children’s mathematical thinking* (19), *teacher practices* (15), *reflective practice* (13) and *teacher learning* (11). Within Shulman's categories of content knowledge, *pedagogical content knowledge* (27) and *subject matter content knowledge* (16) were observed most frequently. *Specialized content knowledge* (23), *knowledge of content and students* (14), and *knowledge of content and teaching* (14) were most frequently indicated within the domain of mathematical knowledge for teaching. The categories of the content of practical knowledge coded most frequently were *knowledge of instruction* (56), *knowledge of surroundings* (32), and *knowledge of self* (22) while the orientations of practical knowledge most often observed were *experiential* (46) and *situational* (24). See Table 26 for the complete list of code frequencies.

**Children’s mathematical thinking and teacher practices.** Data indicated that the big idea themes prevalent in Luke’s first classroom observation were varied with *children’s mathematical thinking and teacher practices* appearing most often. *Reflective practice* and *teacher learning* were also represented but observed less frequently. *Children’s mathematical thinking* and *teacher learning* were focal points in both the assessment activity and the viewing of the video clip while *teacher practices* and *reflective practice* were mostly observed in the way the Luke organized and conducted the activities. Even in the pre-observation interview, *teacher practices* and *children’s mathematical thinking* emerged as big idea themes.
Table 26. First Classroom Observation Code Frequency – Luke

<table>
<thead>
<tr>
<th>Observation Data Segmentation (ODS)</th>
<th>Pre-Int</th>
<th>Obs</th>
<th>Post-Int</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge Data Segments (KDS)</td>
<td></td>
<td></td>
<td></td>
<td>101</td>
</tr>
<tr>
<td>Big Idea Themes (BIT)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflective Practice (RP)</td>
<td>2</td>
<td>8</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Technology (TECH)</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
<td>2</td>
<td>11</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Children's Mathematical Thinking (CMT)</td>
<td>3</td>
<td>14</td>
<td>2</td>
<td>19</td>
</tr>
<tr>
<td>Shulman Content Knowledge (SHU)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
<td>0</td>
<td>13</td>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
<td>2</td>
<td>18</td>
<td>7</td>
<td>27</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Mathematical Knowledge for Teaching (MKT)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>0</td>
<td>19</td>
<td>4</td>
<td>23</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
<td>0</td>
<td>10</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
<td>2</td>
<td>6</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Content of Practical Knowledge (CPK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self (SLF)</td>
<td>0</td>
<td>21</td>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
<td>1</td>
<td>23</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
<td>0</td>
<td>5</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Instruction (INS)</td>
<td>4</td>
<td>49</td>
<td>5</td>
<td>58</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Orientations of Practical Knowledge (OPK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Situational (SIT)</td>
<td>0</td>
<td>23</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>Personal (PER)</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Social (SOC)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
<td>2</td>
<td>39</td>
<td>5</td>
<td>46</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>21</td>
<td>278</td>
<td>68</td>
<td>367</td>
</tr>
</tbody>
</table>
When Luke was asked about specific learning objectives for the session, he discussed the assessment activity and the questions he planned to pose to the PTs about how they would use results of the scored student work and item analysis to inform their instruction.

Luke: And I’ve said this to the students in the past, I want them to look at the assessment and I understand that they’re not always going to have time to do an item analysis but occasionally they should do an item analysis and understand what that information is that they get back from that, understand where they should go from that… What does it mean for your methods experience, what recommendations, if this was your assessment, would you make to yourself to address the students’ performance? Then we’ve got specific items from the rubric, anything from knowledge of students to inform your teaching and learning, after this assessment, how would you engage students in learning. (Pre-observation interview, 11/28/12)

The big idea theme of teacher practices was evident in Luke’s emphasis on how item analysis could guide instruction. Children’s mathematical thinking emerged as he discussed using the results of the assessment to make “recommendations” regarding how to “address the students’ performance.”

Teacher practices surfaced again, but this time in combination with reflective practice, during the portion of the classroom observation when the PTs and Luke viewed and discussed a video clip from one of the PT’s student teaching experience. As he transitioned from the assessment activity to the video clip, Luke provided evidence of both practices based on his decision to use a vignette from a real classroom to examine interactions.

Luke: So it’s a little bit after six but I really want to look at Utley’s video because I think there’s some really valuable stuff in here for us to think about. Hopefully it works. Sam you’re sure you don’t mind this?

PT: It's fine. (First classroom observation, 11/28/12)

Luke: I don’t know what your emotional state is but I want you to know up front I
thought this was really, really well done. Yeah I’m using it as an exemplar. (First classroom observation, 11/28/12)

Luke modeled the teacher practice of using classroom episodes as an instructional tool for PTs to observe and discuss teacher-child interactions. Additionally, he modeled the practice of acknowledging exemplary work turned in by one of his learners. The idea of reflective practice emerged when Luke asserted that “…there’s some really valuable stuff in here for us to think about” suggesting that he wanted the PTs to watch the video clip and then reflect on what they observed.

Data from the post-observation interview reinforced the big idea theme of children’s mathematical thinking and also offered evidence of teacher learning. Luke was asked about whether or not he felt the session was successful and he immediately focused on what he thought the PTs had learned with respect to assessing children’s thinking.

Luke: Yeah, I think this session went pretty well. I think they, well even Ned says “Geeze I’m learning something” because he’s thinking about, I think they all left here thinking about the assessment idea a little bit differently. They probably already analyzed the assessment they did in their unit and I’m hoping this might force them, if they already have, this might force them to stop and rethink that, understand that they can learn a tremendous amount from these assessments… Even the silly stuff, the girls were looking at and saying why would they even do this, where do they get those numbers? How did that suddenly become a decimal, those kinds of remarks. But they are going to see that at the high school level, they’re going to see kids do stuff and they’re like “What were they, how can we possibly understand what they were thinking?” (Post-observation interview, 11/28/12)

**Pedagogical content knowledge and knowledge of instruction.** Similar to the initial interview, pedagogical content knowledge domains often materialized in combination with practical knowledge of instruction. Specifically, Shulman’s
pedagogical content knowledge and two components of mathematical knowledge for teaching, knowledge of content and students and knowledge of content and teaching occurred in some combination with practical knowledge of instruction. The following two data segments, one from the observation and one from the post-observation interview, illustrate the presence of all four knowledge domains in Luke’s work. Toward the conclusion of the video analysis in his methods course, Luke utilized Shulman’s pedagogical content knowledge, knowledge of content and students, knowledge of content and teaching, and practical knowledge of instruction as he summarized a few of the PT-child interactions that he observed on the video. The video clip displayed a PT reviewing homework during his Algebra 1 class and Luke immediately relied on his practical knowledge of instruction with respect to organization of instruction as well as teacher-student interaction (Elbaz, 1983). He acknowledged that a student had asked for six problems to be addressed, but the PT made the instructional decision, likely for reasons of time, to review only two of them.

Luke: (Luke stops video) Good stuff and you’re just reviewing problems with kids. You’re just getting them started…She asked for problems ten through fifteen, you go through two. You get to a generalization about the area of a triangle. Was it David who asked you that question initially that motivated this problem or somebody did? So you get to a more convincing argument about the generalization for finding area of a triangle. You take this original triangle, this triangle right here, and you look at the decomposition of it into two right triangles so you give a different view of that. And if time would’ve allowed and you would have been able to circumscribe a rectangle about it, then you could have thought about it from that perspective and to look at the area relative to the rectangle and either extract those two right triangles from those imposed vertices, if you will. You got all kinds of artifacts going on in here, all kinds of engagement with the students. It’s a bit of problem solving but it’s a bit of procedural work. But your students are using the language correctly and you’re getting some insights into what they’re doing. (First classroom observation, 11/28/12)
Shulman’s *pedagogical content knowledge* emerged as knowledge of strategies needed to challenge the understanding of students (Shulman, 1986) when Luke recognized that the PT used a “more convincing argument” of dividing a triangle into two right triangles to help generalize finding the area of a triangle. His suggestion that the PT also could have circumscribed a rectangle about the triangle and approached area of the triangle from that perspective offered evidence of *knowledge of content and teaching* in terms of sequencing content for instruction choosing examples (Ball et al., 2008).

The same four knowledge domains again appeared during the post-observation interview. Luke had intended to pose two problems following the conclusion of the assessment activity but didn’t get to do so due to time constraints. His goal was to expose the PTs to alternatives to the traditional assessments.

Luke: And then the next problem I had was just starting with a 3-4-5 triangle and figuring out the largest square we could inscribe in the 3-4-5 triangle using the right angle as one angle of the square and then say what happens to the length of that square if I double the length of the Pythagorean triples. And then the classical question, although kinda hidden in there, was what happens to the area of that square when I double the lengths of the Pythagorean triple then look at the new square? What happens when I go from A 1 to A 2, what happens you know, what’s the multiplier of the area? They’d probably think “Well it sounds like four but I’m not absolutely sure. I know it’s not a two kind of thing.” The purpose of those problems was, okay here’s one assessment that you looked at with these thirteen items that’s just procedural, rote kind of garbage. But here’s another thing you can use for an assessment and there’s a tremendous amount of arithmetic in this base 9 (referring to the other problem) or there’s a tremendous amount of algebra and geometry in this little triangle 3-4-5. (Post-observation interview, 11/28/12)

Luke’s prediction of the PTs’ interpretation of the problem and how he planned to confront that interpretation indicated Shulman’s *pedagogical content knowledge* and *knowledge of content and students*. His ability to anticipate the PTs’ preconceptions
(Shulman, 1986), what they would likely think with respect to the problem (Ball et al., 2008) suggested his use of both domains. *Knowledge of content and teaching* appeared based on his evaluation of instructional advantages (Ball et al., 2008) of using the triangle problem as an assessment versus “thirteen items that’s just procedural, rote kind of garbage.” Practical *knowledge of instruction* was evident in terms of his intended organization and instruction (Elbaz, 1983) of having the PTs first examine the 13 problem, “solving algebraic equations” assessment which was a very procedural in nature and then providing them with two rich problem solving tasks for comparison. Even though time forced Luke to skip the problem solving activities, his practical *knowledge of instruction* was evident in his intent. The observation and post-observation interview both offered insight into how Luke draws from the pedagogical content knowledge domains in combination with practical *knowledge of instruction* to plan and conduct classroom activities.

**Subject matter knowledge domains.** In his interactions with the PTs, Luke often relied on Shulman’s *subject matter content knowledge* and the *specialized content knowledge* component of mathematical knowledge for teaching. He utilized both domains mostly in portions of the activities focusing on mathematical content. For example, during the assessment activity, Luke asked the PTs what mathematical ideas they thought the teacher was trying to assess. The PTs responded with mathematical ideas such as solving linear equations, isolating variables, and the distributive property. Luke’s confirmation of the PTs’ ideas in the following dialogue required knowledge of subject matter.

PT 1: You have one “check the solution.”

Luke: Yeah, ok, so one “check solution.”

PT 2: On one of, two of them use fractions or decimals. No three.


PT 3: They all ended up working out besides seven.

Luke: So pretty much all Algebra 1 terms. (First classroom observation, 11/28/12)

When Luke suggested that the ideas were “all Algebra 1 terms” he seemed to draw from Shulman’s *subject matter content knowledge* relative to the way in which he perceived Algebra 1 to be organized (Shulman, 1986). *Specialized content knowledge* emerged as a mathematical task of teaching, specifically evaluating the plausibility of the PT claims (Ball et al., 2008) about what mathematics the teacher was trying to assess, as Luke offered confirmation of their responses.

As the assessment activity continued, the PTs evaluated artifacts of children’s work. They recognized that many children had common mistakes. In reacting to these evaluations, Luke again drew on Shulman’s *subject matter content knowledge* and *specialized content knowledge* to address the type of feedback the PTs could provide.

Luke: How would you provide feedback to your students about this? Would you provide it to the individual on the assessment? Would you provide it collectively to the group because there’s these common errors that are being made? X plus four quantity squared isn’t x squared plus eight, or x squared plus sixteen, right. You know they’re missing that middle term. Would you talk to the individual about that? (First classroom observation, 11/28/12)

Luke’s acknowledgement that when multiplying quantities such as (x+4)^2, the product includes a “middle term” resembled an instance of Shulman’s *subject matter content*
knowledge related to the organization of basic concepts and principles (Shulman, 1986) such as the distributive property. Choosing that particular example to make a specific mathematical point (Ball et al., 2008) regarding the children’s work indicated he also drew from specialized content knowledge. The classroom observation provided ample data on the presence of Shulman’s subject matter content knowledge and the component of mathematical knowledge for teaching categorized as specialized content knowledge emerged in Luke’s work.

**Knowledge of surroundings and a situational orientation.** Throughout the discussion of the assessment activity, the PTs continually asked Luke questions about items they had to submit for the teacher performance assessment electronic portfolio. Since this was the first year for the university to require the end-of-course assessment, Luke was very receptive the PTs’ questions. He often drew on practical knowledge of surroundings and situational knowledge to respond. For example, one of the PTs had asked him about a long list of questions that appeared on the teacher performance assessment and whether or not he had to answer all of them. After addressing the initial question, Luke offered advice as to how the PTs could formulate their answers to the questions so to achieve the highest scores on the evaluation rubric.

Luke: The idea behind this again Ned is, I’m the only one who’s going to see this this semester but in the student teaching semester, I don’t know who is actually going to see it if anybody else will. You want the reader to attend to level four and level five in the rubric, I’m coaching him for this assessment. I thought I was being clever but I found out that everybody on the floor is coaching for this assessment. But you want them to see this language so then they are immediately drawn to okay this is a level four or level five work and by using this language right here, you are forcing them to look at that and you are just kind of stuffing what you did in between the words. (First classroom observation, 11/28/12)
Luke’s content of practical knowledge surfaced as *knowledge of surroundings*, specifically recognition of the social setting and politics of being a teacher (Elbaz, 1983) when he stated “I’m coaching for this assessment” and noted that other teacher educators were doing the same. As he guided the PTs on how they could secure high scores on items, Luke was drawing on *situational knowledge*.

Luke offered similar guidance when addressing how the PTs could negotiate expectations in their field placements.

Luke: Well you know maybe you pick one class in your student teaching experience that you do this with so she starts to see it. So this is what you could do. Not use her assessment, use your own and give them back to the students, right? And make sure she’s okay with that and make sure they don’t look anything at all like hers or have her make sure so you are kinda doing this blindly. Because we want your assessment to be aligned with your objectives for what you think you’re trying to accomplish in the classroom. (First classroom observation, 11/28/12)

*Knowledge of surroundings* emerged as recognition of the classroom setting (Elbaz, 1983) when Luke encouraged one of the PTs to create his own assessment and to make it considerably different from the cooperating teacher’s typical assessment so it could unfamiliar to school learners. The *situational* orientation to his practical knowledge manifested itself as knowledge of communication influences (Elbaz, 1983) as Luke suggested that the PT experiment with returning assessments in “one class” during his student teaching experience so that the cooperating teacher “starts to see it.” His suggestion implied that making wholesale changes might communicate a message to the cooperating teacher that would cause conflict between her and the PT.

**Knowledge of self and an experiential orientation.** The algebra assessment activity also provided insight into Luke’s use of practical *knowledge of self* and his
*experiential orientation* to practical knowledge. He readily offered information regarding how he chose to grade assessments and how he handled mistakes he made in creating assessments. In both situations, Luke suggested a method based on what was most fair and beneficial for the learners. When a PT asked him whether he graded an entire test at once or instead graded one question or one page at a time, Luke shared practical knowledge of self and experiential knowledge to explain why he graded one page at a time.

Luke: …I grade a page at a time because usually the front page has their name on it so I get that page out of the way and then I turn it over so the rest of the grading is anonymous. But I will admit, like you two are saying, how do they even come with this? How did they get…So if something ridiculous would happen on an assessment, I would stop and look at that name and “Why did he do this? What the hell was he thinking?” Or if they did something particularly clever that was arithmetically or mathematically correct, I would sneak a peek back and say “Oh wow they understand this at a much deeper level than I ever thought.” First classroom observation, 11/28/12)

Luke’s response indicated his personal values and beliefs in terms of respecting the anonymity of the person taking the assessment and thus provided evidence of his reliance on practical knowledge of self. Those same personal values and beliefs (Elbaz, 1983) also suggested a personal orientation to this practical knowledge. An experiential orientation to his practical emerged when he admitted that at times, he peeked to identify a student who provided an answer that was either “ridiculous” or “particularly clever.”

Luke again relied on practical knowledge of self and experiential knowledge as he offered the PTs advice about how to address having written a bad assessment or mistakes made while grading them. His personal values and beliefs (Elbaz, 1983) with respect to knowledge of self and a personal orientation to his practical knowledge emerged
immediately when Luke directed the PTs to not blame the students for the responses they provide on inadequate assessment items.

Luke: Now my advice to you is if you write a lousy assessment, don’t make it the blame of the students alright? Like on item number three. I think it tells you a little bit about the culture of the class that not one student brought it to the teacher’s attention and she evaluated the whole thing, put it all in the grade book, let me look at it, and she never caught the error herself. And then the fact that there was, yeah you caught it as soon as you looked at it right, and the idea, and we’re always going to make mistakes when we’re evaluating stuff. We are going to grade stuff incorrectly, Nina, I did the Jefferson high problem wrong the other day right? So we’re going to make those mistakes and that’s okay but don’t make it to the detriment of the student.

The experiential orientation to Luke’s practical knowledge appeared when he drew on his observation of the class in which the algebra assessment was administered. Item number three on the assessment had been written incorrectly by the teacher but she still graded despite her own mistake.

Luke’s practical principle. The organization and structure of the algebraic assessment activity, though not a problem solving activity per se, still reflected Luke’s belief that learners should investigate and discuss mathematical tasks to develop their knowledge base. The PTs were given the opportunity to take the test and then to discuss what they believed to be the purpose of the assessment. Working in pairs, they created their own rubric that could be used to evaluate the assessment and then used it to score actual student work. After completing an item analysis on the scored student work, the PTs were asked to comment on how the result could be used to inform instruction. Through each phase of the activity, they were provided an opportunity to develop and then share their own knowledge with respect to the assessment much like what Luke had suggested with the Pythagorean Theorem examples he provided in the initial interview.
Further evidence of his practical principle in relation to the assessment activity emerged during the post-observation interview where Luke discussed that he intended to pose two contrasting problems as alternative assessment ideas for the traditional (procedural) items included on the test. One was the 3-4-5 right triangle problem already discussed and the other was the traditional arithmetic problem of adding two WRONGs to make a RIGHT.

Luke: I wanted to purposefully pose a problem tonight that looked distinctly different than the assessment they did that was a legitimate problem. I had two problems in fact. The first problem was to use the word wrong, W, R, O, N, G and add it to itself so you get a right a right, R, I, G, H, T. (Post-observation interview, 11/28/12)

Luke had intended for the PTs to struggle with this problem just as school children would in an effort to provide them with an example of an alternative way of assessing mathematical knowledge. Although time constraints prevented him from posing these tasks, his intent to do so again indicated the practical principle guiding Luke’s practice. He wanted to provide the PTs with the opportunity to explore mathematics in a way that they could then use with their school children.

**Summary.** Four big idea themes materialized in Luke’s first classroom observation including *children's mathematical thinking, teacher practices, reflective practice,* and *teacher learning.* Pedagogical content knowledge domains including Shulman’s *pedagogical content knowledge, knowledge of content and students,* and *knowledge of content and teaching* were prevalent in Luke’s interactions with the PTs. Shulman’s *subject matter content knowledge* and the *specialized content knowledge* component of mathematical knowledge for teaching were also evident but most often
limited to discussions of specific mathematical content. Knowledge of instruction, knowledge of surroundings, and knowledge of self emerged relative to the content of practical knowledge while Luke’s orientations of practical knowledge were experiential and situational. His guiding belief of providing learners with opportunities to develop their own mathematical knowledge was reflected in his design of the algebraic assessment activity and the problems he intended to pose following the original assessment analysis.

Second Classroom Observation: Electronic Portfolio

The second classroom observation took place during the 13th and final session of the course on December 5, 2012 with all four preservice teachers present. Luke expressed two goals for the session. The first was to “Consider the human condition” (Pre-observation interview, 12/5/12). That morning, Luke had read an article in USA Today about a nearby city where one in six people lived in poverty and he wanted to share the article with his PTs in an effort to raise their awareness of social issues. “This semester, I failed in this course because we’ve been so busy with this (electronic portfolio) that we haven’t addressed these social issues” (Pre-observation interview, 12/5/12). The second goal was supporting the PTs’ portfolio development as Luke stated that “…students will be able to answer the rubric questions connecting their field experience of teaching a unit to student learning. The questions are all embedded in the electronic portfolio” (Pre-observation interview, 12/5/12). The electronic portfolio was part of a new university required teacher performance assessment and the system was unfamiliar to everyone including Luke. “This is a brand new experience for myself as
well. We are all trying to figure this out, and it is not easy” (Pre-observation interview, 12/5/12).

At the beginning of the session, the PTs read the USA Today article about poverty in a nearby city as they were eating pizza brought to class by one of the PTs. When they finished, Luke asked them what they thought it would be like to teach in that city. This question generated a brief discussion. Luke then shared a rubric for the teacher performance assessment that the PTs could use to guide the selection of materials they submitted to the electronic portfolio. The remainder of the class session was spent on examining the electronic portfolio site and trying to determine what needed to be submitted and how to complete the submissions. Luke projected his computer onto a screen, entered the site of one of the PTs who had already submitted some items, and used the PT’s site in combination with the rubric as an example for the class. At the end of the session, Luke and three of the PTs who would be student teaching the next semester had a brief discussion of when they would meet. A more detailed list of activities for the second classroom observation is provided in Table 27.

**Code frequency data.** Relative to the knowledge domains in the theoretical framework, the total frequency of codes assigned in the second classroom observation was 223. The big idea themes most frequently coded were *teacher practices* (19), *technology* (13), and *teacher learning* (11). Within Shulman's categories of content knowledge, only *pedagogical content knowledge* (14) appeared to occur with any frequency and no components of mathematical knowledge for teaching seemed prevalent.
Table 27. Second Classroom Observation Activity Sequence – Luke

<table>
<thead>
<tr>
<th>Obs. Number</th>
<th>Course Week Number/ Class Date</th>
<th>Sequence of Topics/Activities (Time)</th>
</tr>
</thead>
</table>
| 2           | Week 13 12/5/12                | 1) Introduction – Session began with PTs reading an article from USA Today about poverty in nearby city (15 min)  
  a. PTs read while eating pizza one student brought to class  
  b. PTs worked individually but stopped reading and asked questions about article every once in a while  
  c. Brief discussion of discipline problem one PT had with a student in his class during field experience  
  d. PTs finished reading and Luke asked “What do you think it would be like to be a teacher in Troy, OH?”  
  2) Luke moved to computer in front of room to play a video from Facebook page of a gay man (5 min)  
  a. Video started and then stopped playing  
  b. Luke and PTs tried to get video to play but were unable  
  3) Luke gave up on video and handed out rubric (TPAC document) PTs could use to guide the information they included on an e-portfolio required by the university (1st year university required this portfolio – this year like a pilot test) (1 hr 13 min)  
  a. Luke used one PT’s materials to use as an example to try to submit to test the site  
  b. PTs asked numerous questions about how to download materials  
  c. TE discussed assessment rubric for e-portfolio (TPAC document)  
  d. TE discussed his expectations of what would satisfy the rubric  
  e. Brief discussion of PTs including multiple forms of assessment  
  f. Discussed how e-portfolio would be graded and that PTs should strive for level 4 (highest level) on assessment rubric  
  g. Luke cautioned PTs about using same materials from this course for when they do student teaching  
  h. TE briefly inserted discussion of job opportunities that develop in the middle of the year  
  i. Moved back to discussion of other areas of input for portfolio and about uploading a movie clip of PTs’ teaching  
  j. PT brought up new “Blackboard” type web site called Edmodo?  
  4) Ended class with discussion of when to meet next semester for PTs that would be doing student teaching (3 min) |

The categories of the content of practical knowledge coded most frequently were

knowledge of instruction (29), knowledge of surroundings (25), and knowledge of self
The orientations of practical knowledge most often coded were *situational* (37) and *experiential* (15). Since three-fourths of the session was devoted to the submission of items for the electronic portfolio, evidence of the subject matter knowledge domains, the curricular knowledge domains as well as the pedagogical components of mathematical knowledge for teaching was almost nonexistent. See Table 28 for the complete list of code frequencies.

**Teacher practices and teacher learning.** Data indicated that the big idea themes most often appearing in Luke’s second classroom observation were *teacher practices*, *teacher learning*, and *technology*. The technology theme was prevalent due to the nature of the activities with almost the entire session devoted to the PTs’ electronic portfolios. In this case, technology had nothing to do with enhancing mathematical instruction but instead referred primarily to general knowledge of technology and the use of technology for exploration of the online teacher performance assessment site. As a result, the discussion of the big idea themes focuses only on *teacher practices* and *teacher learning*.

Even though the electronic portfolio site was going to be used as a teacher performance assessment with respect to mathematics education, the focus of this session was on the logistics of the site and how to submit items consistent with the rubric. Mathematics involved, in this case, was essentially limited to reading some of the critical performance requirements of the assessment and Luke offering general suggestions of what could be submitted to satisfy the requirement.
Table 28. Second Classroom Observation Code Frequency – Luke

<table>
<thead>
<tr>
<th>Observation Data Segmentation (ODS)</th>
<th>Pre-Int</th>
<th>Obs</th>
<th>Post-Int</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge Data Segments (KDS)</td>
<td>52</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Big Idea Themes (BIT)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflective Practice (RP)</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
<td>1</td>
<td>10</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Technology (TECH)</td>
<td>0</td>
<td>13</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
<td>2</td>
<td>16</td>
<td>1</td>
<td>19</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Children’s Mathematical Thinking (CMT)</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td><strong>Shulman Content Knowledge (SHU)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><strong>Mathematical Knowledge for Teaching (MKT)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
<td>4</td>
<td>0</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><strong>Content of Practical Knowledge (CPK)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self (SLF)</td>
<td>0</td>
<td>13</td>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
<td>5</td>
<td>18</td>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Instruction (INS)</td>
<td>2</td>
<td>24</td>
<td>3</td>
<td>29</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><strong>Orientations of Practical Knowledge (OPK)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Situational (SIT)</td>
<td>0</td>
<td>36</td>
<td>1</td>
<td>37</td>
</tr>
<tr>
<td>Personal (PER)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Social (SOC)</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
<td>0</td>
<td>12</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>32</td>
<td>161</td>
<td>30</td>
<td>223</td>
</tr>
</tbody>
</table>
However, most of the session was devoted to determining where and how to submit items. For example, evidence of the themes of *teacher practices* and *teacher learning* surfaced when a PT was unsure about whether once he submitted an item if he could go back and edit or change the submission. Luke responded that if the PT tried to go back on it and learned he could not, he would not be penalized. In fact, Luke added that “…we’ve all learned a valuable lesson.”

Luke: I think you probably can.

PT: Because I haven’t uploaded my video yet.

Luke: If you can’t go back on it, we’ve all learned a valuable lesson and you can’t be penalized for it. You’ve got three witnesses and it’s on tape. Where does it tell you that you can do it? Let’s submit this thing right now and see what the heck happens. (Second classroom observation, 12/5/12)

*Teacher practices* and *teacher learning* were evident as Luke asserted “Let’s submit this thing right now and see what the heck happens.” He wanted to show an example so that all PTs learned about the submission process.

Luke’s concern for *teacher learning* also materialized at the end of a brief discussion as the group was trying to determine where to download a video clip of the PTs’ classroom teaching and an appropriate length for the video.

Luke: Oh, here it is, video record your classroom teaching. Alright, so just stuff it in there under…

PT: (Inaudible) no more than 15 minutes in length but can I, well I guess it would go over 15 minutes if I just put what we watched last week?

Luke: Yeah, that would um… Well no, I think probably the reason they are saying that is the assessor away from here won’t watch more than 15 minutes, kinda like nine pages because they won’t read more than nine pages, so if you want to dump that in there, that would be alright. Okay. Do the four of you understand this any better? (Second classroom observation, 12/5/12)
At the end of the interaction, Luke asked the PTs if they had a better understanding of the process. He was concerned that they knew how, where, and what to submit for the electronic portfolio since it was required and an evaluation of their teaching performance.

**Pedagogical content knowledge and knowledge of instruction.** Given that most of the classroom observation focused on the electronic portfolio, data providing evidence of pedagogical content knowledge domains and practical knowledge of instruction with respect to mathematics education and mathematics teacher educating was more common in the pre- and post-observation interviews. Luke’s use of Shulman’s *pedagogical content knowledge, knowledge of content and teaching, and practical knowledge of instruction* emerged in the pre-observation interview when he was asked how he intended to assess the success of his instruction. He immediately referred to the required teacher performance assessment and stated “The success of this is when I read their electronic portfolios next week to see if they have actually done what I have suggested” (Pre-observation interview, 12/5/12). As he elaborated, Luke referenced a specific example from the video clip of a PT’s classroom teaching viewed collaboratively during the previous session. In particular, he addressed how the PT had sequenced his instruction to deepen the understanding of the students with respect to finding the area of a triangle.

Luke: The other thing, and this goes back to what Sam demonstrated last week, “Candidate uses strategically chosen representations in ways that deepen students’ understanding of mathematical concepts and procedures”. So he went from a right triangle to an acute triangle with an altitude. And then the way that particular class is broke up, and the way that the session went, we didn’t have time to further explore, but he would have, the next piece would have been to go to an obtuse triangle and see whether or not if the base-height over two would work. But he
was able to, based on the student’s question, he strategically chose that representation that deepened, I believe, deepened that student’s understanding of the concept of the area of a triangle. (Pre-observation interview, 12/5/12)

Luke relied on Shulman’s pedagogical content knowledge as well as knowledge of content and teaching in recognizing the strategy of transitioning among different classes of triangles would help students to generalize the formula for the area of a triangle (Ball et al., 2008). Shulman’s pedagogical content knowledge was evident as a way of representing and formulating the concept of area of a triangle to make it understandable for students (Shulman, 1986) and knowledge of content and students appeared relative to the way he sequenced content for instruction (Ball et al., 2008). Practical knowledge of instruction emerged relative to the sequence as well in terms of organization of instruction (Elbaz, 1983).

The post-observation interview also offered insight into Luke’s use of pedagogical content knowledge and practical knowledge of instruction. Our discussion had progressed to the teacher performance assessment rubric Luke had given the PTs and he was asked about what knowledge bases he expected to draw from when assessing the PTs. He initially expressed he would utilize content knowledge and his understanding of the language when observing their classroom performance. He then transitioned to pedagogical content knowledge stating that “I will draw on the pedagogy too because I’m interested in their discourse and how they are posing questions in the classroom” (Post-observation interview, 12/5/12). Luke followed that statement with a classroom example from one of his PTs concerned that his video segment did not show him standing in the front of the room lecturing. The PT had asserted that he had videotaped students working
on problems in small groups, assuming this to not be illustrative of his teaching. In
response to the PT, Luke relied on Shulman’s *pedagogical content knowledge, knowledge
of content and teaching*, and practical *knowledge of instruction* to suggest that the PT was
teaching.

Luke: I said, “They weren’t doing it haphazardly. It didn’t happen by accident.”
I said, “You set them up right?” He says, “Yeah.” I said, “Well what did you
do?” And he says, “Well we made sure that they were…” He was using matrices
leading up to an example where they had to do some coding and they had to use
matrices for decoding it. I said, “So your students were engaged with the
knowledge that you shared with them and the ideas that you shared with them and
now you’re listening to them negotiate and make sense out of that.” I said “I
don’t have any problem calling that teaching.” (Post observation interview,
12/5/12)

Shulman’s *pedagogical content knowledge* surfaced as ways of representing the subject
(Shulman, 1986) when Luke recognized that the PT had provided students with a
knowledge base about matrices and now they were working in groups on a decoding
exercise and had to “negotiate and make sense out of that.” His reference to that
instructional strategy also indicated *knowledge of content and teaching* with regard to
sequencing content for instruction (Ball et al., 2008) and practical *knowledge of
instruction* in terms of organization of instruction (Elbaz, 1983).

**Knowledge of surroundings and situational orientation.** Although Luke’s
knowledge of the teacher performance assessment requirements and the corresponding
rubric was not yet complete, the information he shared with the PTs indicated a
*situational* orientation to his practical knowledge. The assessment requirement was new
to the university and both faculty and students were unsure as to the precise working of it.
Luke was concerned that his PTs perform as well as possible on the assessment and
strongly emphasized the importance of submitting high quality work because it was not clear what would be considered as acceptable or the consequences for work deemed unacceptable.

Luke: You want to strive when you’re student teaching for your work and your responses to all fit nicely in the level four group because nobody in this building seems to have a clue what’s going to be acceptable and what’s not acceptable if it’s a two or a three. I think a two might get you, you might find yourself in some difficulty and depending on whoever’s supervising you doing your student teaching, if he or she doesn’t let you revisit it to get it to at least a three, you might get jammed up. Because there's something, and we don’t know when this is going to happen either but something is going to get sent to (external evaluator) and the idea is that (external evaluator) is going to evaluate it and then compare it to the evaluation at the (university) and if they’re not close and (external evaluator) grades you lower, then nobody seems to understand other than the fact that they might argue that, with the (university) suggestion that you be licensed to teach. (Second classroom observation, 12/5/12)

Luke’s situational orientation of practical knowledge emerged as knowledge of instructional influences (Elbaz, 1983) based on his familiarity with the scoring rubric as he emphasized the importance of PTs scoring in the “level four group” because evaluations of their materials that could affect licensure status.

Continuing with the assessment requirements, Luke encountered a statement concerning identification of students with special learning needs. He utilized his practical knowledge of surroundings with respect to the social setting of the school and confidentiality issues when he referenced IEP’s and FERPA to tell the PTs they were unlikely to have access to such information (Elbaz, 1983).

Luke: Yeah, so right here for instance, this sentence right here I think, you don’t know this. [S: I don’t have any of those.] And even if you did you wouldn’t know it unless somebody told you and they can’t tell you because you are not part of their IEP or you’re not part of their program. There’s a FERPA thing going on here, so for these morons to tell you that you got to have identified learning needs…
PT: I just asked my teacher and she told me.

Luke: Yeah, right, some teachers will tell you, right.

PT: My teacher told me that if they, the way that the school works that I’m at, they have a separate class.

Luke: Yeah, so you’re likely, [S: So I will not…] for the whole year that you’re there you are likely not to encounter in your class anyone with an identified learning need. [S: Yes] But yet they are telling you that you have to have somebody. That just isn’t going to happen. (Second classroom observation, 12/5/12)

The situational orientation to his practical knowledge again was evident as knowledge of instructional influences (Elbaz, 1983) when Luke told the PTs they probably would not have anyone in their class with an identified learning need and thus would not be able to address that particular assessment requirement. His knowledge of situations with special needs students guided his advice that the PTs to ignore that specific piece of the assessment. The emergence of evidence of Luke’s practical knowledge of surroundings and the situational orientation to his practical knowledge was likely enhanced during the second classroom observation due to the very specific nature of the teacher performance assessment requirements.

**Knowledge of self and experiential orientation.** As Luke and the PTs were in a discussion about uploading items for the teacher performance assessment, he utilized his practical knowledge of himself and the experiential orientation to his practical knowledge to warn his PTs about using the same items for both the current methods class and the following semester for student teaching. Luke drew on experiential knowledge to
Luke: I know students are uploading the material from the methods class during the student teaching experience and not dealing with all this stuff twice. Okay? And it will be interesting if whoever the faculty is that’s working with them is paying attention. Now, I will watch, I can promise you, next week I will watch your videos and I’ll go through this and I would recognize next semester when you go to through this exercise, wow that looks really familiar. (Second classroom observation, 12/5/12)

When Luke stated “I know students are uploading the material from the methods class during the student teaching experience and not dealing with all this stuff twice,” he was drawing on experiential knowledge to guide his instruction and warn his current group of PTs. His knowledge of self then manifested itself as knowledge of abilities with respect to students’ needs when he warned the PTs that he would recognize their work if they submitted the same or similar items the next semester (Elbaz, 1983).

Toward the end of the class session, one of the PTs asked Luke about the format for uploading the video and his response again offered evidence of practical knowledge of self and an experiential orientation to his practical knowledge. He drew on his experiences of working with iMovie and QuickTime to offer suggestions of how the PTs could get help and to indicate that he would be of little help.

Luke: I don’t know. Because everything I do in iMovie I got to upload, or export to QuickTime because of who’s using it after me and they don’t have iMovie and you might have to export it using QuickTime. It takes stupid long. So you might ask Josh that question Nina. Just go in and say “Hey my movie is MOD”, I think he’s got two or three students up there who are trained to know how to do this, so find out who’s trained in (electronic portfolio system) and say I’ve got an MOD I need an MOV and then I need advice on what I should be editing it with, unless you already have a tool in mind. And I’m here all next week, minus that, I will be no help, I might be helpful but on the movie side I won’t be any help.
Luke’s experiences with iMovie and QuickTime guided his knowledge (Elbaz, 1983) and influenced his instruction with respect to uploading videos for the assessment thus supporting an experiential orientation of practical knowledge. At the end of the monologue, Luke’s acknowledgement that he would not be of any help “on the movie side” indicated reliance on the practical knowledge of self manifested as skills and abilities with respect to student needs (Elbaz, 1983). Within the domain of content of practical knowledge, practical knowledge of self was evident throughout the second classroom observation. Additionally, data indicated the Luke relied on an experiential orientation to his practical knowledge as well.

**Luke’s practical principle.** Even though the teacher performance activity conducted during the second classroom observation did not lend itself to the type of instructional practices suggested by Luke’s practical principle, evidence of his guiding belief still emerged in the post-observation interview. During the class session, Luke had given the PTs a rubric to provide them direction relative to the items they needed to submit for the assessment. Following the session, he was asked to discuss the knowledge bases from which he expected to draw when assessing the PTs according to the rubric. As part of the discussion, Luke referenced a recent experience with one of the PTs who felt like a video clip he wanted to submit did not show enough of him actually teaching. Luke’s response was consistent with his practical principle providing learners opportunities to construct their mathematical knowledge.

Luke: So your students were engaged with the knowledge that you shared with them and the ideas that you shared with them and now you’re listening to them negotiate and make sense out of that.” I said “I don’t have any problem calling that teaching.” He says, “Yeah, but it’s not me standing at the board.” He says
He says “There’s only like 90 seconds of me up in the front.” I said, “I have to be perfectly honest. I’m not really interested in you. I’m interested in how you get these students engaged in doing something. So that 90 seconds up front is a reintroduction to the problem. Yeah, good, get out of the way and let’s see what the students do!” (Post-observation interview, 12/5/12)

Evidence emerged as Luke emphatically stated “Yeah, good, get out of the way and let’s see what the students do!” Although the class session did not offer support of Luke’s practical principle, his post-observation interview certainly did.

**Summary.** The difference in the foci of instruction between the first and the second classroom observation led to some variation in the knowledge domains revealed by Luke. The big idea themes of teacher practices and teacher learning were consistent in both observations but a technology theme that was not present in the first observation materialized in this observation. The nature of the teacher performance assessment created the disparity. With respect to Shulman’s categories of content knowledge, only pedagogical content knowledge was prevalent. Subject matter knowledge and curricular knowledge were absent due to the lack of mathematical content included in the session. No components of mathematical knowledge for teaching were consistently evident due to the lack of any mathematical content in the session. Knowledge of instruction, knowledge of self, and knowledge of surroundings from within the domain of content of practical knowledge appeared often just as in the first observation. Luke’s orientations of practical knowledge were situational and experiential also similar to the first observation. While the focus of the class session (on how and what to submit for the teacher performance assessment) limited Luke’s instructional options, the post-observation interview offered additional evidence supporting his practical principle.
Professional Development Observation: Three-Peg Puzzle

The third observation of Luke’s instruction took place on February 8, 2013 during a mathematics coaching professional development session. This was during the sixth of nine, two-day PD sessions that occurred monthly throughout the academic year. The observed session was scheduled based on the availability of both the researcher (I had responsibilities with first and second-year coaches) and Luke. Participants included twenty, second-year mathematics coaches and one regional facilitator from schools across a Midwestern state. Eighteen of the coaches worked exclusively with teachers in the K-8 grade levels, while two of the coaches spent some time with high school teachers. Luke’s goals for the session were to actively engage the coaches in solving what he called “The Three Peg Puzzle.” The Three Peg Puzzle consists of a game board with seven linear, adjacent, holes. The middle hole is empty and three game pieces (maybe golf tees) of one color fill in the three holes to one side of the empty space and game pieces of another color fill in the holes to the other side of the empty space. The objective of the puzzle is, in as few moves as possible, to switch both sets of game pieces from one side of the empty hole to the other. In doing so, only two types of moves are permitted; a slide to an adjacent, open hole or a jump over one peg to an empty hole.

For the activity, Luke used the coaches as the game pieces and asked them to physically solve the puzzle after which they investigated the mathematics of the puzzle. “I guess the overall objective is the students will be able to realize and determine an algebraic solution by looking at specific cases of a puzzle. Now within that algebraic solution for this audience, the objective was to use patterns to come up with a
generalization” (Pre-observation interview, 2/8/13). Based on a last minute decision, Luke had begun this activity with a subset of this group of coaches during the previous month’s PD session when coaches had time only to complete simulating the game by acting as game pieces, physically showing and documenting movements. Additionally, five coaches and one facilitator were not present at the previous session. Consequently, Luke decided to redo the entire activity using the six people not present at the previous session as the puzzle pieces. He instructed the other coaches that they were only allowed to provide small hints, if any at all, as the new group struggled with the puzzle. The algebraic and arithmetic solutions would be new to all participants.

The game board for the activity consisted of seven linear, adjacent, and connected square spaces taped on the floor, with each consecutive pair of squares sharing one side. The middle space was left open and three coaches filled in the three spaces on either side of the middle space. The objective of the puzzle was, in as few moves as possible, for the coaches on one side of the open space to exchange places with the coaches on the other side of the open space. Only two types of moves were permitted; a slide to an adjacent, open space or a jump over one person to an open space. The coaches in the audience (audience coaches) were responsible for counting the number of moves.

The activity began with the five coaches and one facilitator (the puzzle coaches) serving as game pieces trying to physically solve the puzzle. They discussed options and moved on the game board according to the rules until they felt they were stuck and needed to start over. The puzzle coaches started over numerous times while the audience coaches counted their moves. Eventually the puzzle coaches found a solution but not in
the fewest number of moves so they were encouraged to try solving the puzzle using less than six people. They then found solutions using only two people, four people, and finally six people. After physically determining the fewest number of moves for special cases, Luke asked them to generalize the result to \( p \) people. After allowing the coaches to work in small groups to determine a solution for \( p \) people, Luke led them through an algebraic solution using the combination of the number of people and the fewest number of moves followed by an arithmetic solution using numbers of jumps and slides. Mathematical concepts such as quadratic functions, finite differences, and figurate numbers were discussed and utilized to achieve mathematical solutions for the puzzle. See Table 29 for a detailed list of the sequence of activities for the PD session.

**Code frequency data.** Analysis of data from Luke’s professional development session resulted in a total code frequency of 316. Big idea themes most frequently observed were *teacher learning* (24), *teacher practices* (19), and *reflective practice* (16). Only *pedagogical content knowledge* (31) was repeatedly coded within Shulman’s categories of content knowledge. With respect to the domains of mathematical knowledge for teaching, *specialized content knowledge* (25), *knowledge of content and students* (21), and *knowledge of content and teaching* (16) were prevalent. *Knowledge of instruction* (52) and *knowledge of self* (30) emerged from within the content of practical knowledge while evidence of an *experiential* orientation (27) to Luke’s practical knowledge was also common. See Table 30 for the professional development observation code frequency totals.
Table 29. Professional Development Observation Activity Sequence – Luke

<table>
<thead>
<tr>
<th>Obs. Number</th>
<th>Session Number/ Date</th>
<th>Sequence of Topics/Activities (Time)</th>
</tr>
</thead>
</table>
| 3           | Session 6 2/8/13      | 1) Introduction – Session began with Luke reviewing the 3 peg puzzle problem partially completed during previous session (8 min)  
   a. Puzzle consists of 7 spaces with the middle space empty. The remaining 6 spaces each contain a person. The objective is for 3 people to the left of the empty to exchange places with the 3 people to the right of the empty space in the fewest number of moves. Only 2 types of moves are allowed, a slide to an adjacent empty space or a jump over 1 person to an empty space  
   b. Group of 5 coaches and 1 facilitator told Luke they were not present for previous session  
   c. Luke decided to recreate the puzzle and have 6 people not present for previous session act as puzzle pieces  
   d. Luke used masking tape to create game board on floor and reminded PTs of the rules  
   2) Coaches & facilitator acting as puzzle pieces began talking and moving to try to solve the puzzle (32 min)  
      a. Audience coaches were only allowed to offer small hints  
      b. Coaches got stuck and started over several times  
      c. Coaches solved puzzle but not in fewest number of moves so they tried again  
      d. Coaches eventually solved puzzle using only 2 people, then using only 4 people, then using 6 people  
   3) Luke asked coaches to determine a relationship for the number of moves necessary if p people are on the puzzle (23 min)  
      a. Coaches discussed possible solutions in small groups and displayed their thinking on chart paper  
      b. Luke told coaches to also think about and discuss the strategies they were using to solve the problem of p people  
   4) Whole class discussion of possible solutions & strategies (25 min)  
      a. Groups offered possible solutions and the term quadratic was discussed  
      b. Luke showed coaches a graphical representation of the data that he had made during the small group discussions  
      c. Luke lectured about the method of finite differences and using algebra to solve the puzzle  
      d. Luke suggested a method of counting the number of slides and jumps and using arithmetic to solve the puzzle  
   5) Closing – Luke summarized solving the puzzle using algebra with middle and high school students and using arithmetic with elementary students (4 min) |
Table 30. Professional Development Observation Code Frequency – Luke

<table>
<thead>
<tr>
<th>Observation Data Segmentation (ODS)</th>
<th>Pre-Int</th>
<th>Obs</th>
<th>Post-Int</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge Data Segments (KDS)</td>
<td>62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Big Idea Themes (BiT)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflective Practice (RP)</td>
<td>1</td>
<td>13</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>Teacher Learning (TL)</td>
<td>4</td>
<td>15</td>
<td>5</td>
<td>24</td>
</tr>
<tr>
<td>Teacher Beliefs &amp; Attitudes (TBA)</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Technology (TECH)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Teacher Practices (TP)</td>
<td>1</td>
<td>18</td>
<td>0</td>
<td>19</td>
</tr>
<tr>
<td>Social Justice (SJ)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Children's Mathematical Thinking (CMT)</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Shulman Content Knowledge (SHU)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject Matter Knowledge (SMK)</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
<td>2</td>
<td>22</td>
<td>7</td>
<td>31</td>
</tr>
<tr>
<td>Curricular Knowledge (CRK)</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Mathematical Knowledge for Teaching (MKT)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>5</td>
<td>15</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Horizon Knowledge (HK)</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Knowledge Content &amp; Students (KCS)</td>
<td>3</td>
<td>14</td>
<td>4</td>
<td>21</td>
</tr>
<tr>
<td>Knowledge Content &amp; Teaching (KCT)</td>
<td>2</td>
<td>11</td>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>Knowledge Content &amp; Curriculum (KCC)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Content of Practical Knowledge (CPK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self (SLF)</td>
<td>1</td>
<td>13</td>
<td>16</td>
<td>30</td>
</tr>
<tr>
<td>Surroundings (SUR)</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Subject Matter (SUB)</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Instruction (INS)</td>
<td>2</td>
<td>43</td>
<td>7</td>
<td>52</td>
</tr>
<tr>
<td>Curriculum Development (CUR)</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Orientations of Practical Knowledge (OPK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Situational (SIT)</td>
<td>0</td>
<td>5</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Personal (PER)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Social (SOC)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Experiential (EXP)</td>
<td>2</td>
<td>17</td>
<td>8</td>
<td>27</td>
</tr>
<tr>
<td>Theoretical (THE)</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>26</td>
<td>211</td>
<td>79</td>
<td>316</td>
</tr>
</tbody>
</table>
Teacher practices, teacher learning, and reflective practice. As Luke conducted the puzzle activity, his concern for teacher learning was evident in how he allowed the coaches to struggle with the problem and by the questions he asked to facilitate their progress. The manner in which Luke structured the activity allowed him to model a variety of teacher practices. The coaches were encouraged to discuss and reflect on their failed attempts to solve the puzzle in order to eventually arrive at a solution. In the following dialogue, big idea themes of teacher learning and reflective practice emerged as Luke suggested to a coach that he had advanced in his thinking relative to when he began trying to solve the puzzle.

C: I don't know I think we're gonna get stuck again.

Luke: So now you're in a different point because you've only moved three times and you immediately know that the next move is going to be incorrect.

C: All right so we did something wrong.

Luke: So your intuition is engaging and you're thinking about the problem differently than you were 15 min. ago. (PD observation, 2/8/13)

The theme of teacher learning was evident as Luke acknowledged that the coach was now able to recognize an incorrect move after making only three moves and that he was thinking about the problem differently than 15 minutes previous. Additionally, even though he was in dialogue with one coach, his comments were addressed to the entire group. Reflective practice was noted by the fact that the coaches, through discussion and reflection on previous attempts to solve the puzzle, were now able to recognize mistakes that they would not have recognized earlier.
As the puzzle activity progressed, Luke modeled teacher practices in conversations he maintained with the coaches. He often remained silent for long periods of time while the puzzle coaches discussed strategies. He chose to interject only when he heard a comment that provided him an opportunity to ask a question that would encourage the puzzle coaches to stop and think about what they had done.

Luke: Okay so, so Mark right now has said this is not right. How come, how come we think…

C1: I think I should have moved already.

C2: Yeah we haven’t moved.

C1: We can't, we always have to leave an open block somewhere like in the middle here so that we can be moving.

C2: Otherwise someone has to go backwards. (PD observation, 2/8/13)

Reflective conversation, a component of reflective practice (Schön, 1992), followed Luke’s question as the coaches recognized and discussed the fact that they were stuck and needed to leave an open block somewhere in the middle of the puzzle pieces so that no one had to move backward.

Toward the end of the activity when both physical and mathematical solutions to the puzzle had been discussed, one of the coaches commented that she did not have the mathematical background to produce an algebraic solution. Luke’s response provided evidence of the big idea themes of teacher practices and teacher learning as he explained why he chose not to lead the coaches in any particular direction.

Luke: You can do jumps and slides, right? So I was interested in you solving the problem. I didn't want to impose a new view on the puzzle. Okay? And so why didn't I do that?...I wanted you to explore the problem with the tools that you had
and then I just wanted to share some more thinking about this and I wanted to
share one of my student’s solutions to this problem. (PD observation, 2/8/13)

Luke’s method of allowing the coaches to develop their own mathematical solution to the
puzzle suggested the teacher practice of using problem-based, student-centered activities
to enhance mathematical understanding. His concern for teacher learning was also
evident in the instructional method he chose as well as his willingness to “share some
more thinking about this” referring to the method of finite differences. The puzzle
activity allowed Luke to model a variety of teacher practices, specifically engage the
coaches in reflective practice, and demonstrate his concern for teacher learning.

**Pedagogical content knowledge and specialized content knowledge.** At
any given time, Luke appeared to rely on a combination of Shulman’s pedagogical
content knowledge and components of mathematical knowledge for teaching such as
knowledge of content and students, knowledge of content and teaching, and specialized
content knowledge. Additionally, practical knowledge of instruction was often evident.

After Luke had instructed the coaches to work in small groups to develop a mathematical
solution for people on the puzzle, one coach arrived at a solution very quickly. Luke
subtly pulled her away from the groups so they would not see her solution and suspecting
that she was unsure of her method, quietly questioned her about how she had arrived at
the result. This interaction evidenced practical knowledge of instruction regarding
teacher-student interaction (Elbaz, 1983). During their conversation, Luke seemed to
draw on a combination of pedagogical content knowledge, specialized content
knowledge, and practical knowledge of instruction in asking her to explain her solution
and then interpreting and evaluating her explanation. Even though Luke’s initial
comment was partially inaudible, he wanted her to be able to explain her solution to the group. “It has to make sense to the rest of us.” His technique of wanting the coach to be able to explain her solution indicated use of a strategy that reorganizes the understanding of a learner (Shulman, 1986), an indicator of Shulman’s pedagogical content knowledge.

Luke: And now your test is to tell us how you (inaudible talking to coach who came up with solution). It has to make sense to the rest of us. Just because isn’t doesn’t support your theory.

C: Because I noticed it (inaudible). Because what I did was I just looked at the…

Luke: Because I noticed it. What does that mean?

C: I, I just noticed that, well let’s see. There were four people and it took, actually I couldn’t do it here, I went to four and I said well, half, two, actually I saw it there. I said six and three is nine plus six is 15. I tried to get (inaudible) somehow out of six.

Luke: Did you have any intuition that it was quadratic (inaudible)?

C: I had no idea it was a quadratic. I don’t know that much about higher level thinking. (PD observation, 2/8/13)

With respect to mathematical knowledge for teaching, Luke relied on knowledge of content and students and specialized content knowledge to hear, interpret, and evaluate the coach’s incomplete thinking (Ball et al., 2008) as she tried to explain what she meant by the statement “Because I noticed it.” His interpretation and evaluation of her explanation led him to believe that the coach was unaware of the quadratic relationship between the numbers which he then confirmed through questioning.

The session then progressed to a whole group discussion of the different problem solving strategies each group had used. Luke asked if anyone had graphed the data and one coach responded that she had tried but the data looked linear. Earlier, while the
coaches had been working in small groups, Luke sketched a graph of the data (number of people, number of moves) on chart paper for two, four, and six people. He then used the graph to help the coaches reorganize their understanding (Shulman, 1986) of the data transitioning from a linear relationship to a quadratic relationship, an indicator of Shulman’s *pedagogical content knowledge*.

Luke: All right. So that's another heuristic that we can think about. What does the data look like? All right. So if I've got two people, I've got three moves, four, I've got eight, six is 15, eight is, and it keeps going (Luke shows and already plotted set of data points on chart paper). And I'm not very good at drawing these, it's not to scale but it doesn't quite look like a straight line to me. It makes me question whether or not it's this y=mx+b stuff. So in my mind’s eye, I start to, I think well if I got nobody playing in the puzzle, all right (as Luke drills a smooth parabolic curve through the data). But I sort of kind of got something like that. Right? And that's related back to this idea, and somebody used this language and I'm just gonna abbreviate (as Luke moves back to a previous chart paper and begins to write). I'm gonna say the first difference, so $d_1$, difference one, and the second difference, the second difference was constant. All right? And that's really helpful if I, if I go out algebraically and I find where the, where the difference is constant, how many times did I have to go out, in this case two, and this is referred to as the method of finite differences. I go out twice so that tells me I, I bet I have something that looks like this; $ax^2+bx+c$. (PD observation, 2/8/13)

Luke’s progression from the graphical representation to the method of finite differences and then to the algebraic representation of $ax^2+bx+c$ suggested that he drew from *knowledge of content and teaching, specialized content knowledge*, and *knowledge of instruction*. His sequencing of the content (Ball et al., 2008) indicated *knowledge of content and teaching* while his decision to link the different representations (graphical, tabular, and algebraic) denoted a mathematical task of teaching (Ball et al., 2008) from the domain of *specialized content knowledge*. Luke’s ability to sequence his instruction to link the three representations offered evidence of practical *knowledge of instruction*.
with respect to organization of instruction (Elbaz, 1983).

Mathematics content became more of a focal point as this PD session progressed from the physical enactment of the game to formalizing mathematical solutions. As a result, evidence of the use of combinations of pedagogical content knowledge, both with respect to Shulman’s categories of content knowledge and mathematical knowledge for teaching was prevalent. Additionally, specialized content knowledge and practical knowledge of instruction appeared to be knowledge domains from which Luke drew.

**Knowledge of self and experiential orientation.** The post-observation interview offered insight into Luke’s use of his practical knowledge of self as well as an experiential orientation to his practical knowledge. When asked whether the events of the session surprised him, Luke immediately referenced the coach who had quickly developed a mathematical solution to the puzzle problem. His claim that she was the first person he could remember to have been able to achieve a solution without “some scaffolding” and that he was not prepared to handle the situation indirectly implied an experiential orientation to Luke’s practical knowledge.

Luke: She, she may very well have been the first person in the hundreds of people who have solved this puzzle that without some scaffolding was able to take the raw data in that form and do that. And so I haven't had that opportunity in the past and that's, that's why I was very surprised by that. And I didn't even have an anticipatory set because that hasn't happened before or at least if it's happened, it's been a very long time and so I didn't even anticipate somebody coming up with that and being able to have a set of questions to help lead them or maybe to give them a couple of, what do I want to say, a couple of cues so then they could, so I could start to understand better. I, I just wasn't prepared for it. (Post-observation interview, 2/8/13)

This example also suggests that Luke relied on experiences to decide how to organize instruction. Practical knowledge of self emerged as knowledge of abilities with respect to
student needs (Elbaz, 1983) when Luke admitted that he did not have an anticipatory set of questions he could use to help him better understand the coach’s thinking. Or simply put in Luke’s own words “I just wasn’t prepared for it.”

As the post-observation interview was concluding, the researcher asked Luke if he wished to make any additional comments on the session and/or the problem. His response highlighted again his reliance on experience when designing instruction. For instance, Luke quoted one coach who had offered her opinion of the algebraic solution by stating “This just makes me feel dumb. I haven’t done this in a long time” (Post-observation interview 2/8/13). He asserted that he did not intend to make people feel dumb but that he had done the activity numerous times and consequently had a comfort level in showing the algebraic solution first. He was relying on an experiential orientation to practical knowledge that guided (Elbaz, 1983) his instructional practices.

Luke: But, but, you know, through doing this problem so many times I, I've got a nice, a comfortable, I guess maybe a comfortable trajectory for me to explore the problem and show them okay this is a way that we can do this but let's use arithmetic to solve this. (Post-observation interview, 2/8/13)

His comfort with how he presented solutions to the “Three Peg Puzzle” indicated an awareness of his ability with respect to the needs of the learners (Elbaz, 1983), an indicator of knowledge of self. Luke’s discussion of the session and the results of the puzzle problem provided evidence of his use of knowledge of self and an experiential orientation to his practical knowledge.

Luke’s practical principle. The organization and structure of the entire PD session served to confirm previous indications of Luke’s guiding belief. The coaches were provided little more that the rules of the “Three Peg Puzzle” and then permitted to
investigate, attempt, and discuss possible physical solutions to the puzzle. Once physical solutions had been determined, coaches were then told to work in small groups and use the data to explore possible mathematical solutions. They were given little in the way of direction other than to move them forward when time limitation became a factor. In fact, one of the coaches commented that she might have solved the problem if Luke guided her toward the arithmetic solution, to which he responded “I wanted you to explore the problem with the tools that you had…” (PD observation, 2/8/13). He wanted the coaches to first investigate possible solutions using their own knowledge before he provided them with direction. In his discussion of the problem in the post-observation interview, Luke used an ice cream analogy to illustrate his guiding belief.

Luke: I like to think of the problem kind of like going to Baskin-Robbins. If you and I are going to Baskin-Robbins, I'm not gonna pick the flavor of ice cream you want. While there's all these different flavors for considering this problem, you pick the one you like, see what fits. (Post-observation interview, 2/8/13)

He did not want to point the coaches toward a particular solution but instead let them choose. The PD session offered a complete representation of Luke’s practical principle as he posed a problem and then allowed the PTs to discuss, reflect on, and develop possible solution strategies.

**Summary.** Data from the PD observation indicated that Luke utilized knowledge domains similar to those found in the first classroom observation. The puzzle problem from the PD observation and the algebraic assessment activity from the first classroom observation both led to fairly extensive discussions relative to mathematics content. As a result, within the domains of mathematical knowledge for teaching, specialized content knowledge, knowledge of content and students, and knowledge of content and teaching...
were evident in the PD session as they were in the first observation. Shulman’s *pedagogical content knowledge* was prevalent in the PD session as well but little evidence of curricular knowledge domains was observed. With respect to the domain of content of practical knowledge, *knowledge of instruction* and *knowledge of self* emerged as Luke interacted with the coaches. Additionally, consistent with all Luke’s interviews and observations, data indicated that his orientation of practical knowledge was primarily *experiential*. The problem-based, learner-centered nature of the puzzle activity allowed the big idea themes of *teacher learning*, *teacher practices*, and *reflective practice* to materialize as well. This PD session also provided confirming evidence of Luke’s practical principle by the way he conducted the activity so that the PTs constructed their own knowledge with respect to possible solutions to the puzzle.

**Knowledge Domain Frequency Totals**

Luke’s total code frequency including all interviews and observations was 1,163 with 44% of the codes coming from the interviews and 56% of the codes from the observations. Again this does not mean that there were 1,163 coded data segments because most data segments were coded within multiple categories. Instead, data segments from the interviews and observations were coded within the categories of the theoretical framework a total of 1,163 times. In other words, the total frequency of knowledge domains identified and coded within the framework was 1,163. The frequency of codes was expected be higher for the observations than the interviews due to a larger amount data collected from the observations. A closer examination of the code frequencies revealed that 237 codes represented big idea themes (BIT), 136 codes were
considered within Shulman's categories of content knowledge (SHU), 161 codes fell into the categories of mathematical knowledge for teaching (MKT), 366 codes represented content of practical knowledge (CPK) and 263 codes indicated orientations of practical knowledge (OPK). With the exception of mathematical knowledge for teaching, the frequency of codes from observations for each of the components of the framework of analysis was greater than the frequency of codes from interviews. The exception of mathematical knowledge for teaching was likely a result of limited mathematics content in the activities of the second classroom observation. Figure 33 displays the total frequency of codes for each of the five components of the analytical framework together with a breakdown of the frequency of codes that came from interviews versus observations.

Figure 33. Total code frequency based on framework of analysis – Luke.
**Big idea themes.** Analysis of the 340 big idea theme codes indicated that those most often represented in the Luke’s practice were *teacher learning* (TL), *teacher practices* (TP), *children’s mathematical thinking* (CMT), and *reflective practice* (RP). The researcher identified 63 *teacher learning* data segments with 54% from interviews and 46% from observations. A total of 58 data segments were coded as *teacher practices* with 78% occurring in observations. With regard to *children’s mathematical thinking*, 47 segments were coded with 55% coming from interviews while 45% were coded in the observations. Thirty-seven segments were coded as *reflective practice* with 41% from interviews and 59% from observations. The remaining three categories, *teacher beliefs and attitudes*, *technology*, and *social justice* each represented less than 10% of the BIT codes. Figure 34 summarizes the code frequency for big idea themes.

![Luke - Big Idea Theme Frequency](image)

Figure 34. Big idea theme frequency – Luke.
To further investigate the content of the three observation sessions, the researcher identified knowledge data segments (KDS) which represented divisions in the observation data based on Luke’s interactions with learners that resulted in shifts in instruction during the episodes. For example, shifts occurred as a result of moving from one activity to another, transitioning within an activity, responding to learner questions, or even using questioning techniques to raise the cognitive level of a discussion. Each time a shift occurred, the segment was identified as a knowledge data segments which could then be assigned one or more codes according to the framework of analysis.

For the three observation sessions, a total of 215 knowledge data segments were identified. Relative to big idea themes, approximately 21% of the knowledge data segments were coded as teacher practices and 13% as teacher learning. Reflective practice and children’s mathematical thinking each accounted for approximately 10% of the knowledge data segments while the three remaining categories, each accounted for 8% or less of the knowledge data segments (See Figure 35).

**Shulman’s categories of content knowledge.** Within the domain of Shulman’s categories of content knowledge, *pedagogical content knowledge* (PCK) and *subject matter content knowledge* (SMK) appeared to be prevalent in Luke’s practice while *curricular knowledge* (CRK) was less evident. Of the 136 assigned codes representing Shulman’s categories of content knowledge, 89 were coded as *pedagogical content knowledge* and evenly distributed between interviews (49%) and observations (51%). *Subject matter content knowledge* was identified 38 times with 42% from interviews and 58% from observations.
Figure 35. Big idea themes as percentage of KDS – Luke.

Data segments identified as evidence of *curricular knowledge* represented less than 10% of the assigned codes. Figure 36 represents a summary of the code frequency for Shulman’s categories of content knowledge.

Similar results were found when examining only the three observation sessions as *pedagogical content knowledge* emerged as the primary source of knowledge while *subject matter content knowledge* was secondary and *curricular knowledge* seemed to be a peripheral source. Approximately 21% of the 215 knowledge data segments were coded as *pedagogical content knowledge* while 10% were coded as *subject matter content knowledge*. Only about 2% of the knowledge data segments were coded as *curricular knowledge*. See Figure 37 for a graphical representation of that data.
Figure 36. Shulman’s categories of content knowledge frequency – Luke.

Figure 37. Shulman's categories of content knowledge as a percentage of KDS – Luke.
Mathematical knowledge for teaching. The six components of mathematical knowledge for teaching are common content knowledge (CCK), specialized content knowledge (SCK), horizon knowledge (HK), knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). Of these six components, data indicated that Luke relied primarily on specialized content knowledge, knowledge of content and students, and knowledge of content and teaching. More than one third of the 161 codes within mathematical knowledge for teaching were identified as specialized content knowledge (60). Approximately 43% of these codes occurred in the interview data and 57% in the observations. Forty-nine data segments were coded as knowledge of content and students with 51% coming from the interviews and 49% from the observations. Knowledge of content and teaching appeared in 44 data segments with 61% occurring in interviews and 39% in observations. Each of the remaining components of mathematical knowledge for teaching, common content knowledge, horizon knowledge, and knowledge of content and curriculum, had a frequency representing far less than 10% of the total codes. Figure 38 displays the frequency of codes for mathematical knowledge for teaching.

Careful analysis of the observational data revealed that the same three components were identified as primary knowledge domains from which Luke drew as he interacted with PTs and coaches. Considering observations only, specialized content knowledge represented the highest percentage of the 215 knowledge data segments at 16%. Knowledge of content and students (11%) and knowledge of content and teaching (8%) ranked behind specialized content knowledge.
Common content knowledge, horizon knowledge, and knowledge of content and curriculum each represented 1% or less of the knowledge data segments. See Figure 39 for a graphical representation of the categories of mathematical knowledge for teaching as a percentage of knowledge data segments.

**Practical knowledge.** Data analysis for the practical knowledge of Luke, as with Alex and Tracy, was divided into three sections according to the primary classifications of content of practical knowledge, orientations of practical knowledge, and structure of practical knowledge. Code frequency data is presented for the content of practical knowledge and the orientations of practical knowledge while the structure of practical knowledge is discussed in the summary based on an overall view of Luke’s instructional practices.
Content of practical knowledge. The categories of content of practical knowledge are knowledge of self (SLF), surroundings (SUR), subject matter (SUB), instruction (INS), and curriculum development (CUR). Knowledge of instruction represented the most frequently coded type of content of practical knowledge utilized by Luke. Within the domain of content of practical knowledge, 155 of the 366 assigned codes were identified as knowledge of instruction. As expected, a majority of the coded data segments were identified within the observations (75%). Other codes representing more than 10% of the total frequency for content of practical knowledge were knowledge of self (91) and knowledge of surroundings (91). Data segments coded as knowledge of self were almost evenly distributed between interviews (48%) and observations (52%). Similarly, codes for knowledge of surroundings were evenly distributed between interviews (52%) and observations (48%). Knowledge of subject matter and knowledge
of curriculum development each represented less than 10 of the total code frequency for content of practical knowledge. Figure 40 displays the code frequency for content of practical knowledge.

![Figure 40](image_url)

Figure 40. Content of practical knowledge frequency – Luke.

Similar results emerged when focusing on observations only as the greatest percentage of the 215 knowledge data segments was coded as knowledge of instruction (54%). Knowledge of self (22%) and knowledge of surroundings (20%) were also prevalent relative to the knowledge data segments. Knowledge of subject matter and knowledge of curriculum development each represented 4% or less of the knowledge data segments for content of practical knowledge. Figure 41 summarizes the content of practical knowledge as a percentage of knowledge data segments.
**Orientations of practical knowledge.** The components of orientations of practical knowledge are *situational* (SIT), *personal* (PER), *social* (SOC), *experiential* (EXP), and *theoretical* (THE). The primary orientation of practical knowledge that informed Luke’s practice was *experiential* while *situational* orientations to practical knowledge were also evident but less frequent. Almost half of the 263 codes of orientations of practical knowledge were identified as *experiential knowledge* and they were fairly evenly distributed between interviews (48%) and observations (52%). The frequency of data segments coded as *situational knowledge* was 88 with the majority emerging from observations (73%). Each of the three remaining components represented less than 10% of the codes determined to be orientations of practical knowledge. Figure 42 is a graphical representation of the frequency of the orientations of practical knowledge.
When looking exclusively at the observations, experiential knowledge again was the primary source utilized by Luke but situational knowledge was almost as prevalent. Approximately 32% of the 215 knowledge data segments were coded as experiential knowledge and 30% of the knowledge data segments as situational knowledge. The three remaining orientations of practical knowledge, personal, social, and theoretical, each represented 2% or less of the knowledge data segments. See Figure 43 for a graphical representation of the orientations of practical knowledge as a percentage of knowledge data segments.

Luke Case Profile Summary

At the time of this study, Luke had been involved in the practice of mathematics teacher educating for approximately 16 years, teaching undergraduate and graduate level content and methods courses for both elementary and secondary preservice teachers.
Prior to his work as a MTE, he had taught secondary level school mathematics at a combination of high schools and community colleges as well as on a Native American reservation. He had led numerous professional development sessions for inservice teachers and was involved with a mathematics coaching professional development program for coaches working with teachers in grades K-12. His primary research interests were in the area of ethnomathematics and focused on improving mathematics education opportunities for Native Americans.

The primary big idea themes that emerged from Luke’s practice of mathematics teacher educating were teacher learning and teacher practices while data indicated that children’s mathematical thinking and reflective practice were secondary themes (See Figure 44).
Figure 44. Case profile summary of knowledge domains utilized by Luke.

Luke’s concern for teacher learning was evident in both interviews and observations and usually linked to children’s mathematical thinking. He emphasized in the initial interview that his instructional practice often focused on helping PTs and practicing teachers recognize and interpret children’s conceptions and misconceptions with respect to specific mathematical content. The algebraic assessment activity completed during the first classroom observation also connected teacher learning with children’s mathematical thinking as the PTs scored actual student work and then used the results to suggest possible direction for future instruction. And finally, the puzzle activity
from the PD session was structured so that the coaches first had to determine physical and mathematical solutions to the puzzle as if they were school children so that when using the activity as a classroom teacher, they would have an anticipatory set of knowledge for what children might think and do. In other words, they approached the problem from the perspective of both children and teachers.

The big idea themes of teacher practices and reflective practice were mostly evident in the way Luke modeled his instructional practices. He utilized a variety of methods such as whole group, small group, and direct instruction to conduct activities during all three observational sessions. He specifically relied on reflective practice during the algebraic assessment activity and the puzzle problem activity. In both, Luke asked learners to complete specific tasks in pairs or small groups and then reflect on and discuss their thoughts with regard to the completed tasks. Additionally, he often reflected on his own actions and experiences.

With regard to Shulman’s categories of content knowledge, pedagogical content knowledge was the primary source of knowledge from which Luke grounded his interactions with learners while subject matter content knowledge materialized as a secondary source (See Figure 44). Evidence of pedagogical content knowledge was consistently apparent in both interviews and observations whereas subject matter content knowledge was slightly more favored in observations. Pedagogical content knowledge typically emerged as a method of representing and formulating the subject or a strategy used to reorganize the understanding of a learner (Shulman, 1986). While the nature of the activity had little effect on the frequency of pedagogical content knowledge, subject
matter content knowledge was most often identified relative to activities with a focus on mathematics content, specifically the organization of basic concepts and principles (Shulman, 1986). For example, subject matter content knowledge was almost nonexistent during the teacher performance assessment activity, but was prevalent in the activities associated with the algebraic assessment, the classroom video, and the puzzle problem. Concepts such as equivalence, area, and functions were nestled within those particular activities. Data also indicated that curricular knowledge was a peripheral knowledge source for Luke.

Specialized content knowledge, knowledge of content and students, and knowledge of content and teaching were the primary sources of knowledge identified from within the components of mathematical knowledge for teaching (See Figure 44). With the exception of the second methods course observation, all three knowledge domains appeared throughout the interviews and observations. The second methods course observation was an exception because of the focus of the session on learning both how and what to submit to the electronic portfolio system of the teacher performance assessment. As a result, the session was essentially void of any mathematics content. However, specialized content knowledge, knowledge of content and students, and knowledge of content and teaching were heavily represented in the activities with significant mathematics content.

Specialized content knowledge was most often evident as a mathematical task of teaching such as linking graphical, tabular, and algebraic representations in the puzzle problem or evaluating the plausibility of learner claims (Ball et al., 2008) in each activity
with a focus on mathematics content. *Knowledge of content and students* appeared as he anticipated what his learners were likely to think and what might be confusing to them. Luke’s ability to hear and interpret the learners’ incomplete mathematical thinking (Ball et al., 2008) was evident in the content focused activities and indicated *knowledge of content and students* as well. *Knowledge of content and teaching* often emerged from his practice in the way he chose to sequence content for instruction such as how organized the puzzle problem to have coaches transition from a physical solution to an algebraic solution to an arithmetic solution. His understanding of the advantages and disadvantages of different representations (Ball et al., 2008) with regard to the algebraic solution of the puzzle problem also suggested his use of *knowledge of content and teaching*. The remaining components of mathematical knowledge for teaching appeared to be peripheral sources of knowledge for Luke.

*Knowledge of instruction* was the primary domain evident with respect to the content of practical knowledge while *knowledge of self* and *knowledge of surroundings* were secondary (See Figure 44). *Knowledge of instruction* appeared for example, when Luke modeled the puzzle activity for coaches by having them investigate the problem in the same manner as they would have their school children investigate it. His organization of instruction as well as teacher-coach interaction indicated his reliance on practical *knowledge of instruction* (Elbaz, 1983). His *knowledge of self* typically materialized as knowledge of skills and abilities with respect to the needs of learners (Elbaz, 1983). For example, Luke often recognized and acknowledged his own weaknesses such as when he admitted to initially being a horrible teacher on the Native American reservation because
he was unfamiliar with the culture of the students. Evidence of his reliance on practical knowledge of surroundings usually appeared as recognition of the classroom and social setting of the school (Elbaz, 1983). Luke’s references to the influence of culture on his practice at the work-based education facility and on the Native American reservation suggested his use of knowledge of surroundings.

Data indicated that the orientations of practical knowledge for Luke were primarily experiential but situational orientations also played a part in his work (See Figure 44). Both interviews and observations indicated that past experiences strongly influenced his instructional practices as a MTE. In interviews for example, he repeatedly illustrated situations from past teaching experiences that influenced his knowledge and guided his instruction. His discussion of how the experience at the work-based education facility taught him to contextualize mathematics to make it relevant to the learner was an example of an experiential orientation to his practical knowledge. Although data indicated that situational knowledge was a secondary orientation of Luke’s practical knowledge, the second classroom observation likely skewed the results due to the nature of the session. Over 40% of the data coded as situational knowledge occurred during the second classroom observation where Luke continually relied on his knowledge specific only to the teacher performance assessment as he helped the PTs learn how and what to submit to the electronic portfolio. Situational knowledge emerged primarily as knowledge of instructional influences as he discussed what was or was not acceptable for the assessment (Elbaz, 1983). The session was almost entirely void of mathematics content thus limiting his use of subject matter knowledge domains, curricular knowledge
domains, and pedagogical content knowledge domains within mathematical knowledge for teaching.

The guiding belief in Luke’s instructional practices as a MTE were grounded in providing learners with opportunities to use the available tools to explore mathematics and construct their own knowledge relative to the subject matter. This practical principle as described by Elbaz (1983) formulates his structure of practical knowledge and influences his interactions with learners. The guiding belief was evident on the first page of the methods course syllabus as well as in how Luke orchestrated the algebraic assessment activity with PTs and especially the puzzle activity conducted with mathematics coaches. Additionally, his interviews offered references to numerous experiences, such as the Pythagorean Theorem examples, that reflected his practical principle as well. Evidence strongly supported Luke’s belief that one of his roles as a MTE was to provide all levels of learners with opportunities to construct their mathematical knowledge.
Chapter 7: Knowledge Domains for Mathematics Teacher Educating

As was discussed in Chapter 1, research with respect to the knowledge and practice of mathematics teacher educators is scarce (Even, 2014). Training for mathematics teacher educators as well as research devoted to becoming a mathematics teacher educator is also limited. Additionally, mathematics teacher educators lack a shared knowledge base at the instructional level from which to draw as they attempt to design of effective teacher preparation programs (Hiebert et al., 2003). There is also little consistency in preparation programs for the mathematics teacher educators themselves (Even, 2008). The purpose of this study was to contribute to the genre of research aimed at understanding mathematics teacher educators’ knowledge when engaged in the practice of mathematics teacher educating. To that end, I examined the practices of three MTEs as they provided content specific activities for preservice and inservice teachers. I investigated how MTEs knowledge domains directed their activities as revealed in their interactions with learners. The following research questions guided my inquiry:

1. What knowledge domains do mathematics teacher educators draw from and use when providing content specific pedagogical experiences for preservice and inservice teachers?

2. How do these knowledge domains influence the activities of mathematics teacher educators as they design and implement pedagogical experiences for preservice and inservice teachers?
3. How can these knowledge domains be incorporated into the developmental stages of a model to conceptualize mathematical knowledge for teacher educating?

Chapters 4, 5, and 6 provided individual case profiles for each of the three participants based on data acquired through interviews, observations, and artifacts of their practices. This chapter presents the cross-case analysis of the three subjects to answer the three research questions. Specifically, the first question will be addressed using the combined totals of the code frequency data relative to the theoretical framework. The second research question will address what each component of the theoretical framework looked like for mathematics teacher educators and how those components influenced their instructional practices with PTs and coaches. Information from responses to the first two questions will provide the basis for initial development of a model representing the knowledge of mathematics teacher educating.

**Cross-Case Analysis of Data of Three MTEs**

Data for each of the three MTEs was collected through an initial interview, three observations, pre-and post-observation interviews, and artifacts of their practices. During the data collection process, each MTE had one session that was distinctly different from the other two that were observed. For Luke and Tracy, the second classroom observation, which also happened to be the final class session of the semester, focused primarily on end-of-course activities with limited mathematics content included in the session. Alex's PD session consisted of the discussion of a book chapter regarding gender in mathematics and included limited mathematics content as well. In each case,
the session was likely atypical of the normal instructional practices of the MTEs but the data were analyzed and included in the cross-case analysis.

Code frequency data was used to address the first research question; *What knowledge domains do mathematics teacher educators draw from and use when providing content specific pedagogical experiences for preservice and inservice teachers?* The totals of the code frequency data for each of the MTEs were combined in the cross-case analysis. For each individual code, the frequency of occurrence was determined for interviews, for observations, and for the combined total. Total code frequencies were also determined for each of the four components of the analytical framework; 1) big idea themes, 2) Shulman's categories of content knowledge, 3) mathematical knowledge for teaching, and 4) practical knowledge. Practical knowledge, as with the individual case profiles, was divided into content of practical knowledge and orientations of practical knowledge. Additionally, the total number of knowledge data segments from observations was calculated and then each code was expressed as a percentage of that total. In the next section, numerical results of the cross-case analysis of the code frequency data are presented first, followed by a discussion of the results relative to the knowledge domains utilized by the MTEs.

**Code frequency data.** For the three MTEs combined, the total frequency of coded data according to the theoretical framework was 3,341 with 38% of the codes coming from the interviews and 62% from the observations. The frequency of assigned codes was expected to be higher for observations than for interviews due to a significantly larger amount data collected from the observations. To further break down
the frequency, 770 codes represented big idea themes (BIT), 474 codes were considered within in Shulman's categories of content knowledge (SHU), 569 codes fell into the categories of mathematical knowledge for teaching (MKT), 967 codes represented content of practical knowledge (CPK) and 561 codes indicated orientations of practical knowledge (OPK). In each of the five components of the theoretical framework, the frequency of codes from the observations was larger than the frequency of codes from interviews. Figure 45 displays the total frequency of codes for each of the five components of the framework together with a breakdown of the frequency of codes that came from interviews versus observations.

Figure 45. Total code frequency for 3 MTEs based on the theoretical framework.
Big idea themes. Analysis of the 770 big idea theme codes indicated that those most often represented in the trio’s practices were teacher practices (TP), teacher learning (TL), children’s mathematical thinking (CMT), and reflective practice (RP). A total of 256 data segments were coded as teacher practices with 79% coming from interviews and 21% occurring in observations. Among the 155 teacher learning data segments, with 69% came from interviews and 31% from observations. With regard to children’s mathematical thinking, 135 segments were coded with 43% coming from interviews while 57% were coded in the observations. Finally, 104 segments were coded as reflective practice with 28% from the interviews and 72% from the observations. The remaining three categories, social justice, teacher beliefs and attitudes, and technology each represented less than 10% of the big idea theme codes. Figure 46 summarizes the frequency of codes for big idea themes.

![3 MTEs - Big Idea Theme Frequency](image)

Figure 46. Big idea theme frequency for 3 MTEs.
A closer look at the three observations only, the researcher identified a total of 613 knowledge data segments (KDS) from which big idea themes could possibly be drawn. Approximately 33% of the knowledge data segments were coded as *teacher practices*, 13% as *children’s mathematical thinking* and 12% as *reflective practice*. Only 8% of knowledge data segments were coded as *teacher learning* which is consistent with the fact that 69% of the codes in this category came from interviews. For each of the three remaining categories, the coded segments represented 8% or less of knowledge data segments (See Figure 47).

**Figure 47.** Big idea themes as percentage of KDS for 3 MTEs.

**Shulman’s categories of content knowledge.** Of the 474 codes representing one of Shulman’s categories of content knowledge, 300 were coded as *pedagogical content knowledge* and were distributed almost evenly between interviews (44%) and
observations (56%). Data segments were coded as subject matter content knowledge 102 times with 28% occurring in interview data and 72% in observation data. Segments coded as curricular knowledge occurred 72 times with 33% resulting from interviews and 67% from observations. See Figure 48 for a summary of the code frequency for Shulman's categories of content knowledge.

![3 MTEs - Shulman Content Knowledge Frequency](image)

Figure 48. Shulman's categories of content knowledge frequency for 3 MTEs.

Results from examining only the observation data indicated a similar emphasis on pedagogical content knowledge. Approximately 27% of the 613 knowledge data segments were coded as pedagogical content knowledge while 12% were coded as subject matter content knowledge and 8% as curricular knowledge (See Figure 49).
Figure 49. Shulman's categories of content knowledge as percentage of knowledge data segments for 3 MTEs.

**Mathematical knowledge for teaching.** Within the domain of mathematical knowledge for teaching, data indicated that the three MTEs relied most heavily on *knowledge of content and students, knowledge of content and teaching, and specialized content knowledge* in their work with PTs and mathematics coaches. Within the 569 codes of mathematical knowledge for teaching, 175 were coded as *knowledge of content and students*. Approximately 45% of these data segments occurred in the interviews and 55% in the observations. One hundred and seventy-one data segments were coded as *knowledge of content and teaching* with and were evenly distributed between interviews (49%) and observations (51). *Specialized content knowledge* appeared in 157 data segments with the overwhelming majority (76%) resulting from the observations. The remaining categories of mathematical knowledge for teaching each represented less than
7% of the 569 codes. Figure 50 displays the frequency of codes for mathematical knowledge for teaching.

Figure 50. Mathematical knowledge for teaching frequency for 3 MTEs.

A detailed look at the observations indicated that the same three categories were the most significant knowledge domains from which the three MTEs drew but the ranking order was different. Specialized content knowledge represented the highest percentage of the 613 knowledge data segments at 19%. Knowledge of content and students (16%) represented a slightly higher percentage of knowledge data segments than knowledge of content and teaching (14%). Each of the three remaining categories represented 4% or less of the knowledge data segments. See Figure 51 for representation of the categories of mathematical knowledge for teaching as a percentage of knowledge data segments.
Practical knowledge. As with the individual cases, data analysis for the practical knowledge of the three MTEs combined was divided into three sections according to the primary classifications of content of practical knowledge, orientations of practical knowledge, and structure of practical knowledge. Code frequency data is presented only for the content of practical knowledge and the orientations of practical knowledge while the structure of practical knowledge is discussed based on an overall view of the practices of the MTEs.

Content of practical knowledge. Knowledge of instruction was easily the most frequently coded type of knowledge utilized by Alex within the content of practical knowledge. Of 967 codes within the domain of content of practical knowledge, 496 were coded as knowledge of instruction. As would be expected, a majority of the coded data
segments were found throughout the observations (77%). The frequencies of knowledge of surroundings (175) and knowledge of self (146) indicated that they were secondary resources used by the MTEs. Data segments coded as knowledge of surroundings were fairly evenly distributed between interviews (54%) and observations (46%) as were data coded as knowledge of self (53%) and (47%), respectively. Figure 52 displays the code frequency for content of practical knowledge.

Figure 52. Content of practical knowledge frequency for 3 MTEs.

Focusing on observations only, similar results were found in that the greatest percentage of the 613 knowledge data segments were coded as knowledge of instruction (63%). Again knowledge of surroundings (13%) and knowledge of self (11%) were prevalent with knowledge of subject matter (10%) also appearing in observations. Figure
53 summarizes the content of practical knowledge as a percentage of knowledge data segments.

![Content of Practical Knowledge](image)

Figure 53. Content of practical knowledge as percentage of KDS for 3 MTEs.

**Orientations of practical knowledge.** *Experiential knowledge* was the primary knowledge base that influenced the MTEs practices relative to the orientations of practical knowledge. *Situational knowledge* and especially *theoretical knowledge* appeared to be less influential and thus secondary orientations of practical knowledge. Of the 561 codes of orientations of practical knowledge, almost half were coded as *experiential knowledge* (274) and they were evenly distributed between interviews (49%) and observations (51%). The frequency for *situational knowledge* was 135 and mostly evident in the observations (70%). The frequency of data segments coded as *theoretical knowledge* was 57 with the majority coming from interviews (70%). *Social knowledge*
and *personal knowledge* were each evident in less than 10% of the coded data segments. Figure 54 is a graphical representation of the frequency of the orientations of practical knowledge for the MTEs.

![Figure 54 - Orientations of Practical Knowledge Frequency for 3 MTEs](image)

Figure 54. Orientations of practical knowledge frequency for 3 MTEs.

When looking exclusively at the observations, *experiential knowledge* again was the primary source utilized by the MTEs. Approximately 23% of the 613 knowledge data segments were coded as *experiential knowledge*. *Situational knowledge* accounted for 15% of the codes while each of the three remaining categories represented less than 8% or less of the knowledge data segments. See Figure 55 for a graphical representation of the orientations of practical knowledge as a percentage of knowledge data segments.
Figure 55. Orientations of practical knowledge as percentage of KDS for 3 MTEs.

Three MTEs knowledge domain summary. Alex, Tracy, and Luke had a wide variety of experiences as mathematics teachers and mathematics teacher educators. Each was chosen to participate in this study because of their accessibility through a mathematics coaching professional development program and the different levels of PTs with whom they could be observed in a classroom setting. Tracy was observed working with PTs seeking early childhood (PreK-3) licensure, Alex with PTs seeking special education and intervention specialist licensure for grades K-8, and Luke with PTs pursuing secondary mathematics education licensure. Individually, each brought unique qualities to their practice, but the big idea themes represented and the knowledge domains from which they drew as they interacted with PTs and coaches were similar.
The big idea themes most prevalent in the cross-case analysis were teacher practices, teacher learning, children's mathematical thinking, and reflective practice (See Figure 56).

Teacher practices were evident throughout the observations of MTEs primarily in the way that they modeled classroom instruction. They demonstrated a variety of instructional approaches, questioning techniques, and assessment strategies that their learners could use with school children. The MTEs conducted mathematical activities...
exactly as they would with school children. They analyzed the activities from the perspective of the children and teacher. The combination of the other three big idea themes, *teacher learning*, *children's mathematical thinking*, and *reflective practice* were evident in both interviews and observations and were consistent with Sowder’s (2007) discussion of effective approaches for helping teachers develop knowledge-for-practice. Sowder suggested that a focus on *children's mathematical thinking* often leads to opportunities for teachers to re-examine their own mathematical thinking. She further argued that allowing teachers to work collaboratively on mathematical activities that could also be used in the classroom grants teachers opportunities to discuss and reflect on their own mathematical thinking and the mathematical thinking of others as teachers.

Certainly, the MTEs in this study relied on both of these approaches with respect to their concern for *teacher learning*. *Reflective practice* was also evident with respect to the MTEs’ own practice as they often discussed how their previous experiences had led them to adapt an activity to better accomplish their learning objectives. For example, Tracy decided to scaffold the *Henry's Freedom Box* activity to encourage the use of ratios and proportions, mathematical content she had previously been unable to elicit.

With respect to Shulman's categories of content knowledge, *pedagogical content knowledge* was the primary domain from which the MTEs drew while *subject matter content knowledge* and *curricular knowledge* materialized as secondary resources (See Figure 56). *Pedagogical content knowledge* was evident in both interviews and observations while *subject matter content knowledge* and *curricular knowledge* were evident in both but weighted more heavily towards the observations. *Pedagogical*
content knowledge appeared relative to schoolchildren as learners as well as PTs and coaches as learners. Knowledge of strategies that can be used to reorganize the understanding of learners, recognition of the conceptions and preconceptions of learners, and ways of representing mathematics to make it understandable were the key indicators of pedagogical content knowledge (Shulman, 1986). Evidence of subject matter content knowledge was usually limited to activities conducted by the MTEs where the teachers were asked to complete a problem-solving task or an assessment task rich in mathematics content. Curricular knowledge usually surfaced as vertical curriculum knowledge (Shulman, 1986) in discussions that concerned the Common Core Standards for Mathematics or in terms of the knowledge of materials available to teach specific content.

Within the domain of mathematical knowledge for teaching, specialized content knowledge, knowledge of content and students, and knowledge of content and teaching were the primary components used by the MTEs (See Figure 56). Specialized content knowledge was revealed primarily in observation data while knowledge of content and students and knowledge of content and teaching were fairly evenly distributed between interviews and observations. Like the subject matter content knowledge component of Shulman's categories of content knowledge, specialized content knowledge was most often observed during activities where mathematics content was prevalent. Mathematical tasks of teaching such as modification of tasks, giving or evaluating mathematical explanations, asking productive mathematical questions, and selecting representations for specific purposes (Ball et al., 2008) were evident in the MTEs' practices and demonstrated their reliance on specialized content knowledge. Knowledge of content and
students emerged as the MTEs were able to predict what would motivate the PTs, coaches, and schoolchildren and anticipated what each learner group would think and what they would find confusing. Additionally, the MTEs were able to hear and interpret learners’ incomplete mathematical thinking and respond accordingly, while also recognizing common conceptions and misconceptions with regard to specific mathematical content (Ball et al., 2008). *Knowledge of content and teaching* was evident in the ways that the MTEs sequenced content for instruction, the examples they chose, their understanding of the advantages and disadvantages of various representations, and their knowledge of the instructional benefits of various methods and procedures (Ball et al., 2008). The three remaining components of mathematical knowledge for teaching, common content knowledge, horizon knowledge, and knowledge of content and curriculum, appeared infrequently and thus were considered as peripheral resources.

*Knowledge of instruction* was the primary knowledge within the content of practical knowledge used by the MTEs while *knowledge of self* and *knowledge of surroundings* appeared to be secondary resources (See Figure 56). Evidence of the *knowledge of instruction* was found primarily in observations while evidence of *knowledge of self* and *knowledge of surroundings* was fairly evenly distributed between interviews and observations. *Knowledge of instruction* often appeared in episodes where the MTEs modeled instructional practices for the PTs and coaches by conducting activities in a manner similar to what they would do with school children. These episodes demonstrated their beliefs about the acts of teaching, their knowledge of organization of instruction, their ability to interact with learners, and the desire and
knowledge to evaluate the results of their instruction (Elbaz, 1983). The MTEs knowledge of surroundings emerged with respect to their recognition of the classroom setting and/or social setting of the learning environment. They were aware of their learners’ backgrounds or current understandings, and in some instances, the environments in which the learners were expected to teach. Knowledge of surroundings also surfaced with regard to the political context of teaching as it related to local, state, and national assessment and curricular policies (Elbaz, 1983). Even though the frequency data suggested that knowledge of self was a secondary resource of the MTEs, this component of the content of practical knowledge was mostly prevalent in Luke's practice. For Tracy and Alex, knowledge of self was a peripheral resource. Within Luke's practice, knowledge of self emerged as he recognized and often discussed his skills and abilities with respect to the needs of his PTs and coaches (Elbaz, 1983).

The MTEs orientations of practical knowledge materialized as primarily experiential with situational knowledge and theoretical knowledge surfacing as secondary orientations (See Figure 56). Experiential knowledge was evident in both interviews and observations while situational knowledge was found primarily in observations. Theoretical knowledge was evident mostly in interviews. All three MTEs relied heavily on previous experiences to build and guide knowledge relative to their instructional practices (Elbaz, 1983). They referenced past experiences as both a mathematics teacher and a mathematics teacher educator during interviews. Experiential knowledge often appeared when MTEs were discussing children's mathematical thinking with respect to specific mathematics content. While situational knowledge was indicated
to be a secondary resource for the MTEs. Results may have been somewhat skewed due to the timing of the classroom observations of Tracy and Luke. Both were observed during the last two class sessions of the semester and each relied heavily on situational knowledge to answer PTs’ questions with respect to end-of-course assignments. For example, Luke's final class session was devoted entirely to helping the PTs learn how and what to submit electronically as a part of their teacher performance assessment. His knowledge of the assessment rubric and the electronic portfolio system was specific only to the teacher performance assessment. *Theoretical knowledge* was not overwhelmingly prevalent in any of the individual case profiles but appeared consistently enough, mostly in interviews, to barely emerge as a secondary orientation of practical knowledge. *Theoretical knowledge* typically occurred as a guide to the MTEs’ instructional practices relative to children's mathematical thinking and theory associated with, but not limited to, each of the MTE’s own research interests.

Although data were not coded according to the structure of practical knowledge, a big picture view of each MTE’s guiding belief was presented in the individual case profiles. For Alex and Luke, this guiding belief was presented as a practical principle and for Tracy, as an image (Elbaz, 1983). However the guiding practical principle that formed the foundation of all three MTEs’ instructional practices was their belief that all levels of learners – preservice teachers, inservice teachers, and school children – needed to experience problem-based, learner-centered, interactive activities with opportunities to discuss and reflect on their own thinking as well as the thinking of others. The first page of Luke's methods course syllabus suggested that learners should be provided with
opportunities to use the available tools to explore mathematics and construct their own knowledge relative to the subject matter. Tracy's course syllabus stated the objectives in terms of her image of not providing fully baked cakes but instead allowing learners to use tools provided and a few ingredients to create their own cake. In Alex's initial interview, he specifically discussed that all levels of learners should be provided opportunities to experience problem-based, learner centered, interactive activities. Each of the MTEs certainly modeled this type of instructional practice in the activities they conducted with the mathematics coaches and PTs.

With respect to Shulman's categories of content knowledge, the primary knowledge domain from which the MTEs drew was pedagogical content knowledge while subject matter knowledge and curricular knowledge appeared to be secondary sources. Within mathematical knowledge for teaching, specialized content knowledge, knowledge of content and teaching, and knowledge of content and students were primary sources of knowledge. Their use of practical knowledge was primarily situated in knowledge of instruction and experiential knowledge with several other categories providing secondary sources of knowledge. The next section in this chapter will examine what each of these knowledge domains looked like for MTEs, and how they influenced the MTEs instructional practices.

Unpacking the Knowledge Domains of Three MTEs

This section of the cross-case analysis closely examines the knowledge domains that data indicated were primary and secondary resources of knowledge for the three MTEs in this study and uses the findings to address the second research question; How do
these knowledge domains influence the activities of mathematics teacher educators as they design and implement pedagogical experiences for preservice and inservice teachers?

I attempted to unpack the knowledge domains either used or expressed to be used by the MTEs through examination of situations in which they became evident, how they were manifested, and the influence they had on the MTEs’ practices. For the purposes of this discussion, the big idea themes of teacher practices, teacher learning, children's mathematical thinking, and reflective practice will not be individually addressed but instead intertwined throughout the analysis.

What is “content” for the MTEs? Prior to the investigation of the knowledge domains, the issue of the “definition” of content for the MTEs must first be addressed. This study incorporated Mathematical Knowledge for Teaching suggested by Ball et al. (2008) as part of the theoretical framework with which to investigate mathematical knowledge for teacher educating. However, as suggested by Zopf (2010), the content for those two practices is different in that children are taught mathematics while teachers are taught mathematical knowledge for teaching. If teaching is considered to be a deliberate attempt to raise a learner’s awareness of a specific subject, then mathematics school teaching is a deliberate attempt to raise children's awareness of mathematics. The content is mathematics with respect to the curriculum, the children, instructional strategies, assessment, and all other dimensions of teaching. However, based on results of this study, the content for mathematics teacher educating was not mathematics itself but instead factors that influence mathematics teaching and learning.
For the study participants, teaching was a deliberate attempt to raise awareness of those factors. Mathematics itself was used as a vehicle to address the factors that influence mathematics teaching and learning. For example, Alex used mathematics as a means to highlight how gender differences influence mathematics teaching and learning while Tracy used it as a context for discussing social justice and equity pedagogy. Luke relied on mathematics as a way to examine content trajectories and raise awareness of connections between elementary and secondary mathematics. Hence, for the purposes of the following discussion, "content" for the practice of mathematics teacher educating is considered to be factors that influence mathematics teaching and learning.

**Framework for discussion.** The discussion of knowledge domains is organized into four broad categories derived from the theoretical framework: 1) *pedagogical content knowledge*, 2) *subject matter knowledge*, 3) *curricular knowledge*, and 4) *practical knowledge*. Since data segments were often assigned multiple codes from within these broad categories, each was selected to encompass multiple components of the theoretical framework so as not to have to address each individually. However within the broad categories, components that were found to be primary and secondary resources of knowledge for the MTEs are discussed.

Pedagogical content knowledge domains were evident throughout interviews and observations of all three MTEs and served as primary sources of knowledge in all three individual cases. Subject matter knowledge domains and curricular knowledge domains appeared less frequently, usually limited to specific situations or activities, and mostly were considered secondary sources of knowledge. With regard to practical knowledge,
specifically content of practical knowledge, knowledge of instruction was frequently identified in data from which components of pedagogical content knowledge also emerged. Therefore, knowledge of instruction is included in the discussion of pedagogical content knowledge. The remaining components of practical knowledge that were identified as primary or secondary sources are discussed within the domain of practical knowledge. Only data presented in the individual case profiles are referenced in this discussion and are identified according to whether they occurred in the initial interview, the first classroom observation, the second classroom observation, or the PD observation.

**Pedagogical content knowledge.** Components of the theoretical framework included in this domain are Shulman's *pedagogical content knowledge, knowledge of content and students, knowledge of content and teaching*, and practical *knowledge of instruction*. Relative to each MTE, the same data were usually coded with some combination of these four components and they were considered collectively to represent the domain of pedagogical content knowledge. Practical knowledge of instruction was included in this knowledge domain because the indicators most frequently evident in the data aligned with indicators of the other three components.

Two big idea themes that often emerged in the pedagogical decisions MTEs made were children's mathematical thinking and reflective practice. In first examining children’s mathematical thinking, data coded as one of the four components of pedagogical content knowledge were often also coded to represent children's mathematical thinking. Sensitivity to children's knowledge of mathematics was evident
throughout each MTE’s interviews and observations augmented with consideration of their teaching audiences. Even though they were only observed working with adults, the MTEs ultimately considered the learning of schoolchildren as well as the PTs and coaches in their work. They all identified children's mathematical thinking as a critical factor to influence teaching and learning of mathematics. This was frequently reflected in their pedagogical decisions.

Reflective practice, specifically reflective conversation (Schön, 1992), appeared as an instructional strategy for many of the activities the PTs and coaches were asked to complete. The MTEs modeled reflective practice and/or reflective conversation by having the PTs and coaches complete activities and mathematical problems and tasks from the perspective of a schoolchild. As the learners were engaged in a task, they were allowed opportunities to reflect on their own thinking as well as the thinking of others both from the perspective of a schoolchild and then from the perspective of a teacher. For instance, when Tracy was conducting the tangram activities associated with Grandfather Tang’s Story (First classroom observation, 11/26/12), one group of students expressed difficulty recognizing that the Tangram pieces were not of equal area and thus could not just be counted to determine the fractional part of the total area. Tracy overheard this difficulty as the PTs were engaged in reflective conversation and took the opportunity to address the issue with the entire class. Additionally, she emphasized that the same thing could happen in school classrooms with children "if you allow it to happen" (First classroom observation, 11/26/12). That is, if teachers provide
opportunities for children to discuss and explain their mathematical thinking, learning occurs for both the teacher and the children.

Data indicated that the pedagogical content knowledge directed the MTEs instructional practices relative to four factors that influence teaching and learning in mathematics; 1) instructional strategies used to address children's mathematical thinking and learning as well as teachers mathematical thinking and learning, 2) activity selection for developing specific mathematical topics, 3) gaps in teachers’ mathematical knowledge, and 4) sociocultural issues. Each of these factors is examined through data presented in the individual case profiles in Chapters 4, 5, and 6.

**Instructional strategies.** Pedagogical content knowledge was evident in the instructional strategies referenced or selected by the MTEs to address both children's, and teachers' mathematical thinking and learning. Focusing first on children's mathematical thinking and learning, instructional strategies were referenced or selected to promote or refute specific methodologies, provide context for instruction, or develop knowledge of a specific mathematical topic.

In promoting or refuting specific methodologies, the use of teaching "centers", reflective practice, and direct instruction were modeled or referenced. In his initial interview, Alex discussed teaching a developmental mathematics course at a community college. He learned that some of the students entering his class were students he had previously taught using direct instruction at a local parochial school. As a result, he began using problem-based, discovery activities in the developmental mathematics course allowing the learners to discuss and reflect on their own thinking and the thinking
of others. Pedagogical content knowledge appeared in this discussion based on his realization of the advantages and disadvantages of different instructional methods as well as his ability to evaluate the results of his instruction.

Tracy provided another example where a specific instructional methodology was promoted with respect to children's mathematical thinking and learning in her discussion of using teaching "centers" with elementary children (First classroom observation post-interview, 11/26/12). She suggested that centers allow children to experience multiple activities related to the same concept over a short period of time. She relied on pedagogical content knowledge and her understanding of what the method of centers afforded instructionally and how it could be used to reorganize the understanding of children as they investigate a concept from different perspectives (Ball et al., 2008; Shulman, 1986). Her knowledge of how to organize this type of instructional technique suggested the use of pedagogical content knowledge.

Tracy also relied on pedagogical content knowledge to suggest appropriate context that could be considered when developing activities for school children. Tracy and her PTs were in a discussion about examining mathematics through a social justice lens when one PT suggested making the activity personal for school children. Tracy's response indicated her knowledge of what would be interesting and motivating to school children as well as a way to represent the mathematics (Ball et al., 2008; Shulman, 1986) when she suggested that in a discussion about poverty and "fairness of food" (Second classroom observation, 12/3/12), using graham crackers would be more appropriate than a large study about the amount of food people eat.
In Luke's first classroom observation, he and the PTs reviewed a brief episode from a student teacher’s classroom. Luke’s reliance on pedagogical content knowledge was evident as he evaluated the instructional strategy (Elbaz, 1983) used by the student teacher and suggested a follow-up strategy that could be used to further develop the concept of area of a triangle. Luke's knowledge of strategies that reorganize the understanding of learners (Shulman, 1986) surfaced when he acknowledged that the student teacher had provided "a more convincing" (First classroom observation, 11/28/12) argument for dividing a triangle into two right triangles to help the children generalize finding the area of a triangle. His recommendation for follow-up instruction included examining the area of a triangle from the perspective of a rectangle circumscribed about the triangle. This comment suggested his pedagogical content knowledge with respect to sequencing content for instruction.

While the previous examples discussed pedagogical content knowledge in relation to instructional strategies for children, the discussion now focuses on pedagogical content knowledge in relation to instructional strategies for teacher learners. With respect to teacher learners, instructional strategies were referenced or selected to develop knowledge of a specific mathematical topic, raise awareness of connections between topics or subjects, illustrate and link mathematical representations, and enhance assessment strategies.

In Luke's initial interview, he provided an example from his teaching experiences where he asked PTs to complete mathematical tasks in base-five arithmetic. His use of pedagogical content knowledge was on display by selecting this instructional method to
reorganize and deepen the PTs’ understanding of base-10 arithmetic (Shulman, 1986). Additionally, he anticipated they would experience the same struggles with base-five arithmetic that school children experience with base-10 arithmetic thus raising their awareness of possible difficulties school children might have. Pedagogical content knowledge also materialized relative to teacher-student interaction (Elbaz, 1983) as he explained to one PT that the experience of working in base-five arithmetic would benefit him when he encountered children struggling with base-10 arithmetic.

Tracy used the instructional strategy of scaffolding to help mathematics coaches connect the concept of measurement to proportions (PD observation, 2/7/13). During the Henry's Freedom Box activity, she sequenced instruction such that the coaches would progress from using measurement to construct a box for a toy figure to using proportions to determine the dimensions of a box in which a third-grader could hide to eventually determining the dimensions for a box in which they themselves could fit. In previous sessions, Tracy had only asked learners to construct a box in which they themselves could hide and she was frustrated that the activity rarely led to substantive mathematics. As a result, she relied on pedagogical content knowledge to develop a strategy that would reorganize the understanding of the coaches (Shulman, 1986) so they would make the connection between measurement and proportions. Her knowledge of what the scaffolding method would afford instructionally as well as her ability to sequence the content for instruction was also evident (Ball et al., 2008).

The Three Peg Puzzle activity conducted by Luke demonstrated how he used pedagogical content knowledge to select instructional strategy that allowed him to
illustrate and link different mathematical representations (PD observation, 2/8/13). While the coaches were collaborating in small groups to discuss possible mathematical solutions to the puzzle, Luke had unknowingly sketched a graph of the data achieved through the physical solution to the puzzle. During a whole group discussion of possible solutions strategies, Luke questioned the coaches as to whether they had tried to graph the data. One coach responded that she had and that the data looked linear. Luke relied on pedagogical content knowledge to then present his sketch to reorganize the understanding of the coaches towards nonlinear data (Shulman, 1986). He then sequenced his instruction to transition from the graphical representation to a tabular form using the method of finite differences and finally an algebraic representation of a quadratic equation. This instructional strategy allowed him to both illustrate and connect the representations for the coaches.

The post-observation interview of Luke's first session provided insight into his intent to use an instructional strategy to develop PT’s knowledge of assessment. Although time constraints did not allow for this to occur, he had intended to have the PTs complete and then evaluate a traditional assessment for solving algebraic equations after which he would pose two problems for comparison as alternative assessments. The traditional assessment was actually used in a school classroom and had 13 typical "solve the algebraic equation" items. One of the problems he wanted to pose included determining the largest square that could be inscribed in a 3-4-5 triangle where the triangle and square shared an angle and then examining the area of the square if the lengths of the Pythagorean triple were doubled. Pedagogical content knowledge emerged
as an ability to anticipate the PTs preconceptions (Shulman, 1986) as he suggested that they might expect the area to be four times as large. He also recognized the instructional advantages of using the alternative assessment as a way to connect topics of algebra and geometry. Additionally, organization of instruction was evident in his intent to have the PTs first complete and discuss the original assessment and then examine the alternative problems (Elbaz, 1983).

**Activity selection.** Pedagogical content knowledge was evident in the activities the MTEs selected to address specific mathematical content. An example with respect activity selection was Alex's choice of the Lego scenario to develop the concept of one-to-one correspondence. The scenario was such that two children were unable to count to the level necessary to determine who had more Legos and the PTs were asked to provide alternative strategies to settle the argument. The PTs read the scenario, arrived at individual solutions strategies, and then shared strategies with the group. Alex's use of pedagogical content knowledge surfaced through his use of the Lego scenario to represent the concept of one-to-one correspondence in a way to make it understandable to the learner (could be the PTs or school children) assuming that one way to solve the problem was for the children to continue to match Legos until one ran out. This episode followed a scenario where one-to-one correspondence was used to determine if all the sheep a shepherd let out of a pen in the morning returned to the pen in the evening. Using the two different scenarios to demonstrate one-to-one correspondence offered evidence of Alex’s knowledge about which examples to choose (Ball et al., 2008), an indicator of pedagogical content knowledge.
**Gaps in teacher knowledge.** Opportunities for the MTEs to address gaps in teacher knowledge, some anticipated some unanticipated, often necessitated the use of pedagogical content knowledge. During one of the tangram activities associated with *Grandfather Tang’s Story* (First classroom observation, 11/26/12), a group of PTs struggled with understanding the concept of equivalence with respect to finding the fractional part of total area. One group was working with the seven traditional tangram pieces which were of different sizes and shapes and just counting pieces to determine the fractional part of the total area. They did not understand that they had to first determine equivalent unit pieces before determining the fractional parts of the area. As they worked, Tracy overheard their discussion and used the opportunity to address the issue with the entire class. Pedagogical content knowledge emerged when she used whole class discussion to reorganize the understanding (Shulman, 1986) of the PTs with respect to this concept. Additionally, her ability to use student-teacher interaction (Elbaz, 1983) to recognize the deficiency and then re-organize her planned instruction to address the deficiency also suggested pedagogical content knowledge.

Alex utilized pedagogical content knowledge to address anticipated gaps in PT knowledge. His *Number of the Day* activity provided him an opportunity to discuss the structure for writing a number in expanded form. The activity consisted of choosing a two digit whole number and then performing a series of mathematical tasks using that number. Rounding the number to the nearest 10 was an example of a task. When he instructed the PTs to write the number in expanded form, Alex anticipating they would struggle with that task, immediately initiated a whole class discussion by asking the PTs
what it might mean to write a number in expanded form. He also used teacher-student interaction to discuss and reorganize the understanding of the PTs (Ball et al., 2008; Elbaz, 1983; Shulman, 1986).

**Sociocultural issues.** The MTEs also relied on pedagogical content knowledge to address sociocultural factors influencing learning of mathematics. Alex's post-observation interview relative to his PD session provided evidence of his use of this knowledge domain with respect to raising awareness of gender issues in mathematics education. Prior to the session, the coaches had read the chapter “Paying the Price for Sugar and Spice: How Girls and Women Are Kept Out of Math and Science” from Jo Boaler’s book *What’s Math Got To Do With It? How Parents and Teachers Can Help Children Learn to Love Their Least Favorite Subject.* Alex's intent for the session was to allow the six female coaches to lead a discussion with respect to what they had read and what they had experienced in their classrooms. Following the session, Alex was asked if he would change anything about the session and he suggested that some of his questions to the coaches had limited instead of encouraging discussion with respect to the topic. He utilized his knowledge of sequencing content for instruction and his understanding of the advantages of different instructional strategies (Ball et al., 2008) to suggest that by first asking coaches to write a comment or question about what they had read on an index card, he would have encouraged more robust discussion.

Pedagogical content knowledge was a primary source of knowledge from which all three MTEs drew in their interactions with learners and in discussions of their instructional practices. Evidence suggested that they relied on the knowledge domain to
address four factors that influence teaching and learning in mathematics; 1) instructional strategies used to address children's mathematical thinking and learning as well as teachers mathematical thinking and learning, 2) activity selection for developing specific mathematical topics, 3) addressing gaps in teachers’ mathematical knowledge, and 4) raising awareness of sociocultural issues. Although likely not an exhaustive list of factors, these are important factors in mathematics teaching and learning.

**Subject matter knowledge.** Components of the theoretical framework included in this domain are Shulman's *subject matter content knowledge* and *specialized content knowledge* from within the domain of mathematical knowledge for teaching. As with pedagogical content knowledge data, the same data were often coded with both of these components and thus for this study, they were considered, collectively, to represent the domain of subject matter knowledge. Data indicated that the subject matter knowledge directed the MTEs instructional practices relative to four elements that influence teaching and learning in mathematics; 1) assessment of mathematical thinking, learning, and language usage, 2) activity selection for developing specific mathematical topics, 3) gaps in teachers’ mathematical knowledge, and 4) making connections between mathematical topics. An example from data presented in the individual case profiles in Chapters 4, 5, and 6 is presented for each of these factors.

**Assessment of mathematical thinking, learning, and language.** The MTEs used subject matter knowledge in their formative assessment of teacher mathematical thinking, learning, and language usage as well as in discussions relative to providing feedback for addressing children's mathematical thinking and learning. Referring again
to Alex's *Number of the Day* activity, he relied on subject matter knowledge to clarify understanding of divisibility by two. When one PT was asked whether the number 11 was divisible by two, he responded that it was not because it was an odd number. Alex then further inquired “What’s another way to say it? Or what’s another way that students could think about is it divisible by 2?” This time the PT responded that "there's one left over" (Second classroom observation, 2/5/13). Eventually the discussion progressed to divisibility by two necessitating the digit in the ones place being a 0, 2, 4, 6, or 8. Alex relied on subject matter knowledge, first as a mathematical task of teaching, to evaluate the PTs claims (Ball et al., 2008) that the number was an odd number and that dividing by two would result in a remainder of one. When Alex asked the PT for alternative language, he was critiquing the PTs response relative to what language might better help a schoolchild to understand divisibility by two. He also relied on subject matter knowledge to establish the validity of divisibility by two requiring a ones digit of 0, 2, 4, 6, or 8 (Shulman, 1986).

Subject matter knowledge emerged from Luke's practice during the algebraic assessment activity after the PTs had created their own rubric for the assessment and then used it to evaluate children's work. The PTs noticed common mistakes made by the schoolchildren. Luke used subject matter knowledge to provide an immediate example of a common algebraic mistake as he addressed how PTs might provide feedback on assessments. The mistake he referenced was neglecting the “middle term” when multiplying a quantity such as \((x+4)^2\). He chose that example to make a specific mathematical point regarding children's work and demonstrated his knowledge of basic
concepts and principles relative to the distributive property (Ball et al., 2008; Shulman, 1986), both indicators of subject matter knowledge.

**Activity selection.** Subject matter knowledge also played an important role in the selection of activities and tasks the MTEs chose to develop understanding of specific mathematical concepts. In Luke’s initial interview, he discussed previous experiences with PTs where he used a Cayley table with an undefined operator and the solutions and ask them to determine the validity of the commutative and distributive properties for that particular operator. In selecting this activity, he relied on his understanding of the structure of mathematics and how a Cayley table with an undefined operator could be used to investigate the two properties (Shulman, 1986). Additionally, with respect to the mathematical tasks of teaching, Luke chose a specific representation for particular purpose (Ball et al., 2008). Instead of using the traditional algebraic representation of the product of two quantities, he selected an unfamiliar representation to deepen the PTs understanding of the commutative and distributive properties.

**Gaps in teacher knowledge.** As with pedagogical content knowledge, the MTEs also relied on subject matter knowledge to address gaps in the teachers' mathematical knowledge. Referring again to the episode with tangram activities and the group of PTs who struggled with equivalence and fractional parts of area (First classroom observation, 11/26/12), Tracy’s use of subject matter knowledge was evident as she recognized and addressed the group's mathematical deficiencies. She first recognized their misconception by evaluating the claim (Ball et al., 2008) that two of the tangram pieces represented 2/7 of the total area of all seven tangram pieces even though the seven pieces
were of different sizes and shapes. She then relied on her knowledge of the structure of mathematics relative to fractional parts of area and the need for equivalence of unit pieces (Shulman, 1986). Tracy also expressed her views of the content as she expressed her desire for the PTs to recognize the interconnectedness of fractions, perimeter, and area. Subject matter knowledge was prevalent throughout this episode as she tried to close the gap in the PTs mathematical knowledge.

**Connecting mathematical topics.** Subject matter knowledge was also evident in the MTEs attempts to help their learners make connections among mathematical topics. Alex conducted an activity using paper plates with colored dots on them to demonstrate an instructional strategy that could be used with school children as they transitioned from counting to addition (Second classroom observation, 2/5/13). He briefly (2-3 seconds) showed the PTs a plate with a specific number of dots and they had to determine how many dots were on the plate and to explain their result. With respect to children, the intent of the activity was to begin to recognize combinations of groups of dots that determine a total. In the post-observation interview, Alex was asked how he decided which dot patterns to use. He drew from subject matter knowledge with respect to modification of a task (Ball et al., 2008) acknowledging that for the PTs, he primarily used plates with eight and nine dots so they would see more complex patterns than school children. Alex also referenced how eight dots could be seen as nine dots with one missing and stressed the importance of the relationship between addition and subtraction in bringing that connection to the PTs’ attention.
Data indicated that subject matter knowledge was used less frequently than pedagogical content knowledge for the MTEs. However, each relied on this domain in a variety of ways as they interacted with their learners. Subject matter knowledge assisted the MTEs with their assessment of mathematical thinking, learning, and language with respect to teacher learners and school children. Selection of activities to develop knowledge relative to specific mathematical concepts and making connections between those concepts was influenced by this knowledge resource as well. Additionally, subject matter knowledge was used to address gaps in teachers’ mathematical knowledge that became evident during instructional sessions.

**Curricular knowledge.** Components of the theoretical framework included in this domain are Shulman's *curricular knowledge* and *horizon knowledge* and *knowledge of content and curriculum* within the domain of mathematical knowledge for teaching. Since knowledge of content and curriculum is essentially defined to consist of components of Shulman's curricular knowledge, and horizon knowledge is very similar to Shulman's vertical curriculum knowledge, both components of mathematical knowledge for teaching are considered in this discussion. Similar to the two previous knowledge domains, the same data were usually coded with some combination of these three components and thus for this study, they were considered, collectively, to represent the domain of subject matter knowledge. Evidence suggested that curricular knowledge guided the MTEs instructional practices relative to sociocultural issues, connecting mathematical topics, and addressing teacher knowledge of the Common Core Standards for Mathematics.
Curricular knowledge with respect the availability of materials to address sociocultural issues in mathematics was evident in the practices of the MTEs. Tracy used two books, *Grandfather Tang’s Story* and *Henry’s Freedom Box* to address cultural and race issues while investigating mathematical topics including fractions, area, perimeter, measurement, and proportions. In fact, during the *Henry’s Freedom Box* activity, one of the mathematics coaches suggested that scale and proportion could be taught using a variety of texts and Tracy immediately responded "Right, but my choice of this text was very deliberate because I wanted to approach this from the social justice perspective and have the discussion about slavery and about race and however else" (PD observation, 2/7/13). Luke used an article from USA Today that addressed the issue of poverty in a city that was in close proximity to the University to lead a discussion with his PTs concerning what mathematics teaching might be like in that community. Alex used the book chapter, “Paying the Price for Sugar and Spice: How Girls and Women Are Kept Out of Math and Science” from Jo Boaler’s book *What's Math Got To Do With It? How Parents and Teachers Can Help Children Learn to Love Their Least Favorite Subject* to initiate a discussion with six female mathematics coaches with respect to gender issues in mathematics. Each of the MTEs relied on curricular knowledge specific to the availability of materials (Ball et al., 2008; Shulman, 1986) for addressing sociocultural issues in mathematics.

Curricular knowledge with respect to mathematics content was evident as the MTEs worked to develop the coaches’ and PTs’ knowledge of connections within and across mathematical topics. Luke used the *Three Peg Puzzle* to illustrate to the coaches
how the same activity could be used with elementary students to develop an arithmetic solution, and with secondary students to develop an algebraic solution (PD observation, 2/8/13). Alex relied on curricular knowledge to respond to a PT requesting clarification of the meaning of "number system" (First classroom observation, 1/29/13). In his reply, he offered that the number system expands from fractions in grades 3, 4, and 5 to include real numbers in grades 6, 7, and 8. Tracy's use of tangrams to connect fractions, perimeter, and area offered evidence of her use of curricular knowledge with respect to mathematics content (Ball et al., 2008; Shulman, 1986).

Familiarizing teachers with the new Common Core Standards for Mathematics also provided opportunities for the MTEs to use their curricular knowledge. Specifically, the MTEs used their knowledge of the proposed new curriculum to develop teachers' knowledge and understanding in this regard. Alex devoted almost an entire class session (First classroom observation, 1/29/13) presenting and discussing various aspects of the Common Core Standards for Mathematics. Curricular knowledge surfaced as he compared the existing curriculum with the new one. His familiarity with the language of the new curriculum as well as his knowledge of the standards for mathematical practices also provided evidence of his reliance on curricular knowledge. Tracy used the Common Core to force PTs to consider how the two book activities could be used across multiple grade levels and relied on her curricular knowledge to evaluate their suggestions. Her view of the importance of the Common Core curriculum was evident when she said "…I want them to focus on the standards as opposed to what the textbook has" (First classroom observation, 11/26/12).
Curricular knowledge, like subject matter knowledge, appeared to be a secondary knowledge resource for the MTEs. They used it in part to address sociocultural issues, help coaches make connections within and across mathematical topics, and familiarize their learners with the Common Core Standards for Mathematics. Curricular knowledge, unlike pedagogical content knowledge, was mostly evident only in instructional episodes that either emphasized the Common Core or focused on mathematics content. Sessions dominated by activities like Luke's teacher performance assessment activity and Alex's book discussion offered little evidence of the use of curricular knowledge.

**Practical knowledge.** Components of the theoretical framework included in this domain are categories of Elbaz’s (1983) practical knowledge that data indicated were primary or secondary resources of knowledge for the MTEs. This domain consists of knowledge of surroundings, knowledge of self, experiential knowledge, situational knowledge, and theoretical knowledge. Remember that practical knowledge of instruction was included within the pedagogical content knowledge domain. Evidence suggested that practical knowledge informed the MTEs practices relative to the following five factors that influence mathematics teaching and learning: 1) instructional strategies, 2) content selection and contextualization, 3) sociocultural issues, 4) assessment, and 5) materials selection.

**Instructional strategies.** Various aspects of practical knowledge informed the MTEs instructional strategies with respect to general method of delivery and within specific mathematical activities. With regard to general method of delivery for example, Alex's experience (Elbaz, 1983) with teaching a developmental mathematics course at a
community college led him to reconsider his teacher-centered method of delivery (Initial interview, 12/7/12). Students he had previously taught at a parochial school were taking his development math course because they were unprepared for college-level mathematics. Alex realized that his teacher-centered instructional practice was not effective for all students.

Tracy relied on previous experiences (Elbaz, 1983) teaching the *Henry's Freedom Box* activity to adapt the lesson for mathematics coaches to engage in more meaningful discussion of substantive mathematics (PD observation, 2/7/13). Specifically, she scaffolded the activity so that they had to transition from using measurement to construct a box for a toy figure to using proportions to determine the dimensions of the box for both a third-grade child and themselves. Previously she had only asked the coaches to construct a box for themselves and the mathematics discussion never progressed beyond the concept of measurement.

*Content selection and contextualization.* The MTEs considered many variables when selecting content for the methods courses and their PD sessions. Additionally, they struggled with how to contextualize the content to make it meaningful for their learners. Practical knowledge often informed their decisions regarding both of these issues. When asked questions concerning mathematics content she deemed important for her learners (Initial interview, 3/6/12), Tracy suggested her use of *theoretical knowledge* (Elbaz, 1983) asserting that research literature about teaching and learning mathematics informs her work. She also indicated her frustration that accountability issues often force content selection. Her practical knowledge of the
political context of teaching (Elbaz, 1983), specifically testing related to state and national standards, forced her to select content that she might not have otherwise. "I’d like, if I didn't have to worry about standards, I'd like to pick content that I think is rich in great but I feel the pressures of that."

Luke's experience with teaching in a work-based education program at a factory provided evidence of his use of practical knowledge to contextualize mathematics to make it relevant to learners. At the plant, he relied on his knowledge of instruction to make the classroom situation more manageable (Elbaz, 1983) by incorporating events in the plant into an algebra course. That experience and others similar to it ultimately lead Luke to consider the contextualization of mathematics for different groups of learners.

**Sociocultural issues.** With the exception of subject matter knowledge, each of the previously presented domains influenced the MTEs’ instructional practices with respect to sociocultural issues as did the practical knowledge domain. During Alex's book chapter discussion (PD observation, 3/4/13), the group was engaged in a conversation about the ratio of male to female in mathematics education. To make the point that very few male teachers work in the early childhood mathematics compared to the number that teach secondary mathematics, he relied on previous teaching experiences (Elbaz, 1983) to assert that in six years of teaching early childhood methods courses, he could count the number of males taking the course “on two hands” (II).

In his initial interview, Luke was asked how teaching on a Native American reservation was similar to or different from teacher educating. As part of his response, Luke referenced his ability to share experiences with PTs and ITs who wanted to teach in
those environments. He suggested that he relied on his experience teaching on the Native American reservation to advise PTs to first investigate the culture they were going to enter. He also indicated that he used his knowledge of instructional influences and making the situation more manageable (Elbaz, 1983) by telling PTs to gather as much information as possible about the history and current situation of the particular environment to understand their context. With regard to teaching in the Native American culture, Luke relied on his experiential and situational knowledge to advise PTs.

*Assessment.* Practical knowledge was used to inform the MTEs instructional practices with respect to assessment but in this case, not assessment of mathematical knowledge. For this discussion, assessment refers to course assessments required by the MTE and university required teacher performance assessments (TPA). When Tracy was asked by a PT about an upcoming assignment and whether she wanted lessons to be for different grade levels (First classroom observation, 11/26/12), she relied on knowledge of the classroom setting (Elbaz, 1983) in suggesting that the PTs would soon be looking for employment and should be knowledgeable of lessons for different grade levels. Continuing, and utilizing communication influences, Tracy subtly indicated that incorporating multiple grade levels was not a requirement but should be considered. Relying on past experiences she also suggested that the PTs would struggle with interview questions about third-grade lessons if they only prepared lessons for kindergartners.

Luke devoted an entire class session to university required teacher performance assessments and relied primarily on practical knowledge to address various aspects of
how and what to submit to the electronic portfolio. The teacher performance assessment was new to the University and Luke emphasized to the PTs the importance of submitting high quality work because no one knew exactly what would be considered as acceptable or the consequences for unacceptable work. He relied on his knowledge of instructional influences and making the classroom situation more manageable (Elbaz, 1983) by providing the PTs with the scoring rubric and specifically addressing the importance of securing scores of four. As the session continued, one PT asked a question about uploading a video and Luke relied on experiences with iMovie and QuickTime to provide as much instruction as he could in this regard. Additionally, he drew from his knowledge of students’ needs (Elbaz, 1983) in offering additional assistance as they uploaded videos.

Materials selection. Practical knowledge also influenced materials selection of the MTEs. For example, Tracy relied on experiential and situational practical knowledge to select materials for the social justice component of her methods course. In the post-observation interview (Second classroom observation, 12/3/12), Tracy discussed how she would like to use Paulo Freire’s book Pedagogy of the Oppressed but cannot due to time constraints. She drew on previous experiences teaching an equity and diversity course where she had the time to incorporate the book. As she continued, she used her knowledge of instructional influences and making the classroom situation more manageable (Elbaz, 1983) as she noted that the PTs would not read the book even if she did include it in the course. Her experiential and situational knowledge informed her decision to not use the book for this particular group.
Aspects of practical knowledge not included in the previous knowledge domains played an important role in MTEs’ instructional practices. *Experiential knowledge* strongly influenced their decisions while the other components acted as secondary sources of knowledge. The MTEs used practical knowledge to inform their practice with respect to instructional strategies, content selection and contextualization, sociocultural issues, assessment, and materials selection. Situational knowledge may have been overrepresented in this study relative to the MTEs’ typical practices due to the amount of time Luke devoted to addressing issues related to the university required teacher performance assessments.

**Summary.** This section of the chapter addressed the second research question; *How do these knowledge domains influence the activities of mathematics teacher educators as they design and implement pedagogical experiences for preservice and inservice teachers?* To address this question, the knowledge domains of the MTEs were categorized as pedagogical content knowledge, subject matter knowledge, curricular knowledge, and practical knowledge. Content for this discussion was not considered to be mathematics but instead factors that influence the teaching and learning of mathematics. Elbaz’s (1983) practical knowledge of instruction was included as part of the discussion with respect to pedagogical content knowledge. The remaining components from the Elbaz model were addressed as a group with respect to the domain of practical knowledge. Results from the data indicated that the four knowledge domains informed the MTEs instructional practices with respect to several factors that influence mathematics teaching and learning, including but not limited to, instructional strategies,
sociocultural issues, gaps in teacher knowledge, assessment, activity selection and materials selection, making mathematical connections, and content choices. The next section incorporates information from this section and the previous section to address the third and final research question.

A Model to Study Mathematical Knowledge for Teacher Educating

The final section of the cross-case analysis uses the results discussed in the first two sections to propose a possible model to be used in studying mathematical knowledge for teacher educating and thus address the third research question; How can these knowledge domains be incorporated into the developmental stages of a model to conceptualize mathematical knowledge for teacher educating? The proposed model is a combination of two autonomous models, the teacher educator knowledge tetrahedron proposed by Perks and Prestage (2008), and the teaching triad of mathematics teacher educators suggested by Zaslavsky and Leikin (2004). The work of Perks and Prestage (2008) relative to the teacher educator knowledge tetrahedron was briefly discussed in the literature review of this dissertation while Zaslavsky and Leikin’s (2004) use of the teaching triad of mathematics teacher educators was discussed in the statement of the problem in Chapter 1. To incorporate the findings from the first section of this chapter, the knowledge domains most frequently relied on by the mathematics teacher educators, the results were examined through the lens of the teacher educator knowledge tetrahedron (Zaslavsky & Leikin, 2004). The influences those knowledge domains had on the practice of the mathematics teacher educators were then analyzed and described using the teaching triad of mathematics teacher educators (Perks & Prestage, 2008). The intent
was that these two models, when used in combination, provided a tool to study mathematical knowledge for teacher educating.

**The teacher educator knowledge tetrahedron.** Perks and Prestage (2008) describe how they themselves develop tools for their work in prospective teacher education. The authors consider tools they use to develop other’s learning about the teaching of mathematics as well as the tools they use to develop their own learning about teacher educating. They suggest that three aspects of knowledge contribute to decisions about dealing with classroom events. They include: 1) practical wisdom – knowledge from being in classroom; 2) professional traditions – knowledge from existing school curriculum and practices and research; and 3) learner knowledge – the prospective teachers’ own knowledge (Perks & Prestage, 2008, p. 269). These aspects of knowledge transform the teacher’s mathematical subject knowledge into knowledge of mathematics necessary for teaching the subject. Together, they form the teacher knowledge tetrahedron (See Figure 57).

![Teacher knowledge tetrahedron](image)

*Figure 57. Teacher knowledge tetrahedron. (Perks & Prestage, 2008, p. 270)*
Perks and Prestage (2008) adapted the teacher knowledge tetrahedron to teacher educator knowledge that might combine to transform learning about teaching into learning about teacher education. The areas within teacher educator knowledge were considered to be; 1) professional traditions – existing teacher education courses, ways of working, and research on mathematics teaching; 2) practical wisdom – knowledge of teaching about teaching; and 3) learner knowledge – knowledge gained from being teachers (p. 270). These three aspects of knowledge combined to form the teacher educator knowledge tetrahedron (See Figure 58).

![Figure 58. Teacher educator knowledge tetrahedron. (Perks & Prestage, 2008, p. 271)](image)

Among other things, professional traditions emerge from personal experiences, education and training, teacher training policies and research. Practical wisdom develops as teacher educators consider what their learners need to know and how best to engage them in those ideas. The learner knowledge of mathematical subject knowledge for
teacher educators is based on their own learner knowledge of mathematics, the practical wisdom of the experienced mathematics teacher and the professional traditions of curriculum, practices, and research (Perks & Prestage, 2008). In other words, the teacher educator needs to begin with the teacher knowledge tetrahedron at the learner knowledge vertex of the teacher educator knowledge tetrahedron (Perks & Prestage, 2008, p. 271). Teacher knowledge acts as learner knowledge for teacher educators. That knowledge together with prospective teacher education professional traditions in the practical wisdom of teaching about teaching forms the basis for creating sessions for preservice teachers. Sessions are formed based on their own teacher knowledge, literature and research, and writing about (publishing) and reflecting on sessions. Though their use of the teacher educator knowledge tetrahedron was limited to work with preservice teachers, this study extended the use of the model to teacher educators who worked with both preservice and in-service teachers.

**The teaching triad of mathematics teacher educators.** Zaslavsky and Leikin (2004) conducted research relative to the professional development of teacher educators by examining the process of becoming a mathematics teacher educator within a professional development program for secondary mathematics teachers. They analyzed the development of mathematics teacher educators through their practice, viewing the participants and educators as a community of practice. To do so, they developed a three layer model of growth through practice for mathematics teacher educators. The model was a combination of two explanatory models of school mathematics teaching; Jaworski's (1992) teaching triad and Steinbring’s model of teaching and learning mathematics (as
cited in Zaslavsky & Leikin, 2004). For the purposes of this research, only the teaching triad is discussed.

In a study of the utility of investigative approaches to teaching mathematics, Jaworski (1992) developed a tool she referred to as the teaching triad to make sense of the practice of teaching mathematics. She felt that the essence of teaching mathematics fell within three domains; 1) the management of learning, 2) sensitivity to students, and 3) mathematical challenge. Management of learning referred to the creation of a learning environment through classroom organization, curriculum decisions, established ways of working, and classroom values and expectations (Jaworski, 1992). Sensitivity to students included developing knowledge of their individual characteristics and needs as well as an approach to accommodate those needs. Mathematical challenge manifested itself as stimulating mathematical curiosity and thinking which influenced activity selection and presentation. The challenging content for school children is the mathematics. Each of the domains, though distinct in theory, rarely emerged separately in practice but instead elements of each domain were usually found in any teaching situation and most often they were in some way related (Jaworski, 1992). The triad is represented and Figure 59 with a double arrows indicating a relationship between the domains.

Zaslavsky and Leikin (2004) extended the teaching triad to describe and analyze the practice of mathematics teacher educators, taking into account challenging content for mathematics teachers, sensitivity to mathematics teachers, and management of mathematics teachers’ learning.
For the teaching triad of mathematics teacher educators, the challenging content for mathematics teachers consisted of the teaching triad of mathematics teachers (See Figure 60). The authors used this model in their analysis of the practice of mathematics teacher educators when conducting professional development for inservice teachers.

Figure 59. The teaching triad. (Jaworski, 1992, p. 8)

Figure 60. Teaching triad of mathematics teacher educators. (Zaslavsky & Leikin, 2004, p. 8)
The MTEs knowledge domains and the knowledge tetrahedron. Perks and Prestage (2008) suggest that mathematics teacher educators draw from practical wisdom, professional traditions, and their own learner knowledge to design and implement mathematics education sessions. Their own learner knowledge essentially consists of knowledge of teachers and knowledge of teaching. Results from my study indicated the existence of primary and secondary resources of knowledge which three mathematics teacher educators used or expressed to be used in their instructional interactions with PTs and mathematics coaches. For the teacher educator tetrahedron to be useful in studying mathematical knowledge for teacher educating, the knowledge domains utilized by the MTEs in this study should be evident within the knowledge domains of the teacher educator tetrahedron. For example, pedagogical content knowledge should be evident within the practical wisdom, the professional traditions, or the learner knowledge of the mathematics teacher educator. For the purposes of this discussion, the four broad categories of pedagogical content knowledge, subject matter knowledge, curricular knowledge, and practical knowledge are addressed.

Pedagogical content knowledge was a primary resource for the three mathematics teacher educators and was determined to consist of Shulman’s pedagogical content knowledge, knowledge of content and students, knowledge of content and teaching, and practical knowledge of instruction. With respect to the teacher educator knowledge tetrahedron, pedagogical content knowledge appeared to exist within the teacher educators own learner knowledge of teaching (Perks & Prestage, 2008). Alex relied on knowledge of the PTs preconceptions when he anticipated that they would struggle with
writing a number in expanded form (Second classroom observation, 2/5/13). This knowledge was drawn from practical wisdom and learner knowledge at the teacher educator knowledge level. Tracy decided to scaffold the *Henry's Freedom Box* activity knowing that mathematics coaches would not progress to using proportions to determine the dimensions of a box for a third-grader (PD observation, 2/7/13). Pedagogical content knowledge was used to reorganize the understanding (Shulman, 1986) of the coaches, addressing the learner knowledge vertex of the teacher educator knowledge tetrahedron.

Luke asked his PTs to complete mathematical tasks using base five arithmetic to deepen their understanding of the base 10 number system. He allowed them to experience the same struggles schoolchildren might endure with base 10 arithmetic (Initial interview, 3/17/12). He anticipated that the PTs would experience difficulties using an alternative base and worked at the learner knowledge vertex of the teacher educator knowledge tetrahedron relying on practical wisdom, professional traditions and learner knowledge at the teacher knowledge level. In each case, the MTE relied on pedagogical content understanding and knowledge of teachers and teaching to create learning opportunities for PTs and ITs (Perks & Prestage, 2008).

The subject matter knowledge domain was considered to be a secondary source for the MTEs and consisted of Shulman's *subject matter content knowledge* and *specialized content knowledge*. As with pedagogical content knowledge, subject matter knowledge appeared to emanate from the learner knowledge vertex of the teacher educator knowledge tetrahedron. Perks and Prestage (2008) assert that one component of learner knowledge for mathematics teacher educators is their own learner knowledge of
mathematics (p. 271) combined with practical wisdom and professional traditions at the teacher knowledge level. Those components informed subject matter knowledge of the MTEs’ practices. For example, Luke used Cayley tables with an undefined operator to develop PTs' understanding of the commutative and distributive properties relying on a mathematical task of teaching where he selected a representation (undefined operator) for a specific mathematical purpose (Initial interview, 3/17/12). He relied on his own mathematical understanding and knowledge of teaching to address the commutative and distributive properties. Practical wisdom at the teacher educator knowledge level with respect to considering what the PTs needed to know and how to engage them in those ideas influenced his activity selection (Perks & Prestage, 2008). Tracy's use of tangrams to represent fractions, area, and perimeter suggested her use of specialized content knowledge with respect to mathematical tasks of teaching and connecting mathematical topics (First classroom observation, 11/26/12). Certainly the MTE’s knowledge of the teachers’ mathematics was evident in this interaction. Knowledge of the connections between fractions, area, and perimeter also suggested reliance on professional traditions at the teacher knowledge level with respect to the existing school curriculum (Perks & Prestage, 2008). Like pedagogical content knowledge, subject matter knowledge appeared to lie within the learner knowledge vertex of the teacher educator knowledge tetrahedron.

Curricular knowledge was also determined to be a secondary resource of knowledge for the MTEs and consisted of Shulman's curricular knowledge and knowledge of content and curriculum and horizon knowledge with respect mathematical
knowledge for teaching. Although curricular knowledge seemed to emerge from the learner knowledge vertex of teacher educators’ knowledge, professional traditions with respect to the existing school curriculum most closely reflected the curricular knowledge domain (Perks & Prestage, 2008). Tracy's selection of the *Henry's Freedom Box* book to address issues of race and diversity as well as mathematical topics such as measurement and proportions demonstrated her use of curricular knowledge with respect to materials available for instruction for the school curriculum at the teacher knowledge level (First classroom observation, 11/26/12). In a discussion of the Common Core Standards, Alex relied on Shulman's curricular knowledge and horizon knowledge to clarify the meaning of "number system," and discuss its expansion across 3-8 grade levels (First classroom observation, 1/29/13). This knowledge also appeared to be reflected in the professional traditions vertex regarding school curriculum at the teacher knowledge level. The curricular knowledge domain identified in this study materialized from the learner knowledge vertex at the teacher educator knowledge level aligning favorably with the professional traditions knowledge vertex at the teacher knowledge level (Perks & Prestage, 2008).

Components of practical knowledge found to be primary and secondary resources of knowledge for the MTEs included *knowledge of self, knowledge of surroundings, situational knowledge, experiential knowledge, and theoretical knowledge*. While pedagogical content knowledge, subject matter knowledge, and curricular knowledge appeared to lie mostly at the learner knowledge vertex of the teacher educator knowledge tetrahedron, the components of practical knowledge appeared to emanate more from
professional traditions and practical wisdom at both the teacher knowledge level and the teacher educator knowledge level. For example, *experiential knowledge* for the MTEs emerged at the teacher knowledge level from their own knowledge of being in the classroom. Additionally, *experiential knowledge* was manifested at the teacher educator knowledge level as they considered what their learners needed to know and the activities necessary to engage them in learning (Perks & Prestage, 2008). When Alex discussed that his students should experience problem-based, student-centered activities, he drew from practical wisdom at the teacher knowledge level to indicate that his teacher-centered approach at the parochial school was not effective for all learners (Initial interview, 12/7/12). However, when deciding which dot patterns to use with the paper plate activity (Second classroom observation post-interview, 2/5/13), Alex drew from practical wisdom at the teacher educator knowledge level to consider that the PTs would need to see more complex patterns than school children would need to see. He also highlighted additional concepts he wished for them to explore.

*Situational knowledge* appeared to emanate from practical wisdom at both the teacher and the teacher educator knowledge levels as well as from professional traditions at the teacher educator knowledge level. Contextualization of mathematics was an important consideration for Luke in his instructional practice. His *situational knowledge*, constructed from experience of working with Native American schoolchildren and plant workers guided his work (Initial interview, 3/17/12). With respect to professional traditions at the teacher educator knowledge level, specifically teacher training policies (Perks & Prestage, 2008), Luke relied on *situational knowledge* to address issues relative
to the University required teacher performance assessment (Second classroom observation, 12/5/12).

*Knowledge of self* and *knowledge of surroundings* both were reflected in the learner knowledge of the teacher educator in terms of practical wisdom at the teacher knowledge level and practical wisdom at the teacher educator knowledge level. Additionally, they were reflected in the professional traditions aspect of teacher educator knowledge. Tracy's *knowledge of surroundings* and *knowledge of self* was reflected at the professional traditions vertex of the teacher educator knowledge tetrahedron when she indicated her frustration of working with PTs because of University required assessments (Initial interview, 3/6/12). She felt restricted in her skills and abilities with respect to the needs of her PTs because she had to operate within the political context of teaching in addressing the University requirements. The University requirements were based on the national curricula (NCATE/NCTM) for teacher education (Perks & Prestage, 2008).

Luke's *knowledge of self* with respect to the needs of his teachers surfaced during a whole group discussion of the *Three Peg Puzzle* activity (PD observation post-interview, 2/8/13), where one coach announced finding an algebraic solution to the puzzle had made her feel dumb. The coach argued that Luke should have guided her towards the arithmetic solution. Luke responded that he was very comfortable with allowing the coaches to construct their own solutions and not guiding them toward what they would view as the easier solution. He felt it necessary for the coaches to rely on sophisticated tools when solving problems as a means to recognize connections among approaches. Luke drew on practical wisdom at the teacher educator level as he considered what the
coaches needed to know and how to engage them in constructing that knowledge (Perks & Prestage, 2008).

*Theoretical knowledge* for the MTEs, specifically knowledge of research with respect to teaching and learning of mathematics, was reflected in the professional traditions aspect of teacher educator knowledge (Perks & Prestage, 2008). In her initial interview, Tracy asserted that research literature about teaching and learning mathematics informed the content of her methods course. Although at a different point in the interview, she provided the example of PTs viewing and discussing a video about cognitively guided instruction to focus on instructional methodology.

**Summary.** The knowledge domains used by the study participants appeared to align favorably with the aspects of teacher educator knowledge suggested by Perks and Prestage (2008). Pedagogical content knowledge, subject matter knowledge, and curricular knowledge appeared to be primarily located at the learner knowledge vertex of the teacher educator knowledge tetrahedron. However, it is likely that knowledge domains are rarely distinctly individual as their dimensions often overlap. Practical wisdom and professional traditions also contributed to those three knowledge domains. The practical knowledge domains also appeared to emerge at the learner knowledge vertex of the teacher educator knowledge tetrahedron, specifically with respect to practical wisdom at the teacher knowledge level. However, the practical knowledge domains also appeared to align favorably with both practical wisdom and professional traditions at the teacher educator knowledge level. The teacher educator knowledge
tetrahedron suggested by Perks and Prestage (2008) offered confirmation of the knowledge domains determined to be used by the MTEs in this study.

**The MTEs’ actions of teaching and the teaching triad.** In addressing the second research question, "content” for mathematics teacher educators was considered understanding of factors that influence the teaching and learning of mathematics. The four knowledge domains that served as primary and secondary resources for the MTEs were unpacked to determine how they influenced their instructional practices relative to the content. Results suggested that four of the factors that influence teaching and learning of mathematics that were impacted by multiple knowledge domains were instructional strategies, activity or materials selection, addressing gaps in teacher knowledge, and attending to sociocultural issues. If we consider the teaching triad of mathematics teacher educators suggested by Zaslavsky and Leikin (2004) to represent the actions of teaching, the three components of the triad should be evident as the MTEs address their content. The three components of the triad include; 1) management of mathematics teachers' learning, 2) sensitivity to mathematics teachers, and 3) challenging content for mathematics teachers. Remember that challenging content for mathematics teachers consists of the teaching triad of mathematics teachers including mathematics content, management of student learning, and sensitivity to students (Zaslavsky & Leikin, 2004).

Data indicated that the MTEs use of pedagogical content knowledge and practical knowledge influenced the instructional strategies they employed. Tracy chose to scaffold the box construction for the *Henry's Freedom Box* activities for both the PTs and
mathematics coaches (First classroom observation, 10/26/12 and PD observation, 2/7/13) in order to help them progress from using measurement to relying on proportional reasoning to determine the dimensions of a box. She structured the problem such that they first had to use measurement to determine the dimensions of a box for a small toy figure and then using those dimensions, determine the dimensions of the box for a third-grader using proportions. Management of mathematics teachers’ learning was evident in the way she structured the problem to move them from measuring to proportional reasoning. Sensitivity to students emerged from her knowledge that the coaches, if not encouraged explicitly to do so, may not have progressed mathematically. Challenging content for mathematics teachers was reflected in the mathematics content necessary to construct the boxes (Zaslavsky & Leikin, 2004).

Activities/materials selection appeared to be influenced by a combination of pedagogical content knowledge, subject matter knowledge, and practical knowledge. Alex's choice of the Lego scenario (First & second classroom observations, 1/29/13/ & 2/5/13) to develop the concept of one-to-one correspondence incorporated all three elements of the teaching triad of mathematics teacher educators. Remember that the scenario involved two children who were not able to determine who had more Legos by counting and the PTs were asked to provide strategies to settle the argument. Once they had worked individually for a period of time, they then shared strategies with the group. Alex remained silent until the whole group discussion. The element of management of mathematics teachers learning was evident in how Alex structured the activity so that the PTs first worked individually and then participated in whole class discussion (Zaslavsky
Sensitivity to mathematics teachers appeared relative to Alex's decision to have the PTs develop solution strategies individually. There were only five students in this particular class and working in groups of two and three would limit the possible solutions strategies to be presented in the whole class discussion. Challenging content for mathematics teachers was reflected in how Alex influenced PTs to become engaged in mathematical thinking about one-to-one correspondence.

The MTEs relied on pedagogical content knowledge, subject matter knowledge, and curricular knowledge to address gaps in teacher knowledge. This was evident with the PTs and the mathematics coaches. In Luke's *Three Peg Puzzle* activity (PD observation, 2/8/13), one coach determined an algebraic solution to the puzzle very quickly and he anticipated that she was unsure of her method. To address what he believed to be a gap in her knowledge, he pulled her away from the group and quietly questioned her about her strategy. In fact, he initially asked her to explain her solution and when she failed to do so, he, in an attempt to help her, questioned her as to whether she was aware of the quadratic relationship of the data. This episode offered evidence of all three components of the teaching triad of mathematics teacher educators. He created an environment where all coaches could be comfortable discussing their knowledge (Zaslavsky & Leikin, 2004). The challenging content for mathematics teachers was the mathematics involved in determining a solution for the puzzle problem.

Data indicated that pedagogical content knowledge, curricular knowledge, and practical knowledge informed the MTEs practices with respect to elements of sociocultural issues they believed to be important. Prior to Alex’s PD session, the
coaches had read the chapter “Paying the Price for Sugar and Spice: How Girls and Women Are Kept Out of Math and Science” from Jo Boaler’s book *What's Math Got To Do With It? How Parents and Teachers Can Help Children Learn to Love Their Least Favorite Subject*. Alex’s intent for the session was to allow the six female coaches to lead a discussion about what they had read and what they had experienced in their classrooms. At one point in the discussion, Alex directed the PTs’ attention to a specific page to address a statement in the reading that suggested that the female brain is built for connection. When asked in the post-observation interview why he had made the decision to focus on that particular statement, Alex’s response offered insight into how the elements of the teaching triad of mathematics teacher educators are enacted in practice. He had noticed that the discussion had drifted into issues of race and socioeconomics and he wanted to refocus the discussion on gender. Management of mathematics teachers learning was evident as he redirected the discussion to gender in mathematics. Sensitivity to mathematics teachers emerged in the way that he suggested them to focus on a specific page to subtly redirect their discussion. The challenging content for mathematics teachers was gender in mathematics at the teacher level with respect to the management of student learning and sensitivity to students (Zaslavsky & Leikin, 2004).

In examining the knowledge domains and their influences on the practices of the MTEs, the teacher educator knowledge tetrahedron supports the knowledge domains found to be primary and secondary resources of knowledge for the MTEs. Components of pedagogical content knowledge, subject matter knowledge, curricular knowledge, and practical knowledge were reflected within the knowledge domains of knowledge domains
of practical wisdom, professional traditions, and learner knowledge at the vertices of the teacher educator knowledge tetrahedron. With respect to how those knowledge domains influenced the MTEs interactions with PTs and coaches, the teaching triad of mathematics teacher educators served as a tool to investigate the interactions. Results indicated that, in a sense, the teaching triad (the actions of teaching) was at the foundation (mathematics education sessions) of the teacher educator knowledge tetrahedron. The knowledge domains represented at each vertex of the tetrahedron, not unsurprisingly, directed the actions of teaching.
Chapter 8: Discussion and Implications

This study attempted to identify the knowledge domains used by MTEs in the practice of mathematics teacher educating and determine how those knowledge domains influenced the actions of MTEs as they designed and implemented pedagogical experiences for preservice teachers and practicing teachers. The following research questions guided data collection and analysis.

1) What knowledge domains do mathematics teacher educators draw from and use when providing content specific pedagogical experiences for preservice and inservice teachers?

2) How do these knowledge domains influence the activities of mathematics teacher educators as they design and implement pedagogical experiences for preservice and inservice teachers?

3) How can these knowledge domains be incorporated into the developmental stages of a model to conceptualize mathematical knowledge for teacher educating?

Due to a limited volume of research that unpacks the specialized knowledge needed or used in mathematics teacher educating, in building a theoretical frame for the study of the participants’ knowledge, Shulman’s categories of content knowledge (Shulman, 1986), mathematical knowledge for teaching (Ball et al., 2008), and the practical knowledge of teachers (Elbaz, 1983) were used. Based on and derived from this

386
framework, mathematical knowledge for teacher educating (MKTE) was categorized into four broad domains; 1) pedagogical content knowledge, 2) subject matter knowledge, 3) curricular knowledge, and 4) practical knowledge. Each of these broad categories consisted of components of the theoretical framework determined to be primary and secondary resources of knowledge for the MTEs.

Drawing from the cross analysis of three individual case studies, this chapter is organized to address key findings pertaining to: 1) knowledge domains of MKTE; 2) learners and content; 3) instructional practices; 4) and conceptualization of a working model for teacher educating knowledge. Additionally, I offer reflections on my research and discuss the limitations and implications of this study.

Knowledge Domains of MKTE

Components of the pedagogical content knowledge domain included Shulman's *pedagogical content knowledge, knowledge of content and students, knowledge of content and teaching, and practical knowledge of instruction*. For all three MTEs, pedagogical content knowledge was the primary knowledge domain used or referenced during interviews and observations. Pedagogical content knowledge was recognized as knowledge of strategies effective to reorganize the understanding of learners, recognition of learners’ conceptions and misconceptions, anticipating what learners would think and what they would find confusing, hearing and interpreting incomplete mathematical thinking, sequencing content for instruction, and choosing examples (Ball et al., 2008; Shulman, 1986).
Subject matter knowledge consisted of Shulman's *subject matter content knowledge* and *specialized content knowledge* from within mathematical knowledge for teaching. Subject matter knowledge was usually limited to activities conducted by the MTEs where the learners were asked to complete a problem-solving task or an assessment task rich in mathematical content. *Specialized content knowledge* was reflected in mathematical tasks of teaching such as modification of tasks, giving or evaluating mathematical explanations, asking productive mathematical questions, and selecting representations for specific purposes (Ball et al., 2008).

Curricular knowledge was determined to include Shulman's *curricular knowledge* and the combination of *horizon knowledge* and *knowledge of content and curriculum* with respect to mathematical knowledge for teaching. Curricular knowledge was a secondary resource of knowledge for the MTEs and usually only appeared in one of two ways. First, curricular knowledge emerged as vertical curriculum knowledge or horizon knowledge during discussions and activities relative to the Common Core Standards for mathematics. Second, curricular knowledge surfaced with respect to materials available to address specific mathematics content and/or factors that influence the teaching and learning of mathematics content (Ball et al., 2008; Shulman, 1986).

Components of the practical knowledge domain included *knowledge of self, knowledge of surroundings, experiential knowledge, situational knowledge*, and *theoretical knowledge*. Practical *knowledge of instruction* was a primary resource from within this knowledge domain but because of its similarity to many aspects of pedagogical content knowledge, was instead included within that category of MKTE.
Experiential knowledge was the primary source of practical knowledge relied on by the MTEs. They constantly drew on their previous K-16 teaching experiences as they designed and implemented activities for the PTs and coaches. Knowledge of self and situational knowledge surfaced as secondary resources of knowledge but the data was likely skewed due to classroom observations where significant time was devoted to end-of-course assignments. The MTEs knowledge of surroundings emerged with respect to their recognition of the classroom setting and/or social setting of the learning environment as well as with regard to the political context of teaching as it related to local, state, and national policy concerning assessments. Theoretical knowledge typically occurred as a guide during the MTEs’ instructional practices relative to children's mathematical thinking and theory associated with, but not limited to, the MTE’s own research interests.

Learners and Content

While MKTE’s domains of knowledge were somewhat similar to those characterized for teaching, two differences were revealed. First, the MTEs operated relying on knowledge of learners as it pertained to both school children and teachers with whom they worked. Children's mathematical thinking was a constant theme throughout the interviews and observations of the MTEs as it often informed their instructional strategies, activities used, and interactions during their sessions. They used children's mathematical thinking as a focal point for engaging PTs and coaches in learning about teaching and considered a particular mathematical concept to gauge teachers’ analysis and reflection. While the MTEs’ direct responsibility was to develop the PTs’ and
coaches’ knowledge about teaching mathematics, they believed their ultimate goal was to help improve mathematics learning of school children.

The second difference between MKTE and the MKT concerned both the treatment and use of content (mathematics) in instruction. For mathematics teachers, the content under scrutiny and study is mathematics as a discipline. For MTEs, the content was the study of factors that influence the teaching and learning of mathematics. Mathematics content was not the focus but was instead used as a vehicle to address factors such as pedagogy, sociocultural issues, and content trajectories.

**Instructional Practices**

The four knowledge domains that informed the MTEs instructional practices most noticeably influenced their work with respect to instructional strategies, elements of sociocultural issues, gaps in teacher knowledge, assessment, activity and materials selection, making mathematical connections, and content choices. The MTEs relied on MKTE to inform their instructional strategies relative to their overall practice and with respect to specific activities and content. Sociocultural issues such as equity, diversity, and social justice were addressed using MKTE. The MTEs relied on the four knowledge domains to anticipate, recognize, and address gaps in PTs’ and coaches’ mathematical content knowledge and deepen their understanding of mathematical concepts through mathematical connections and contextualization. Their instructional practices were deliberate and aimed at selecting appropriate materials to address specific mathematics content or pedagogical aspects of teaching it. Mathematical knowledge for teacher
educating, though different with respect to the learners and the content, in many ways, paralleled mathematical knowledge for teaching.

**Conceptualizing a Working Model for Teacher Educating Knowledge**

Research with respect to teacher educators and the practice of teacher educating, especially in the area of mathematics, has only recently become an area of focus (Even, 2014). As a result, studies of mathematics teacher educator knowledge are often framed using research regarding teacher knowledge (Olanoff, 2011; Zaslavsky & Leikin, 2004; Zopf, 2010). This study was similar in that two components of the analytical framework consisted of Shulman's categories of content knowledge (Shulman, 1986) and mathematical knowledge for teaching (Ball et al., 2008). Shulman's model identified the structure of teacher knowledge involved in the teaching of a nonspecific subject while mathematical knowledge for teaching identified the structure of mathematical knowledge involved in teaching mathematics. Both models of teacher knowledge are regularly referenced in the research community. This study was different in that the third component of the analytical framework consisted of Elbaz’s (1983) practical knowledge of teachers. Elbaz's practical knowledge grounded the sources that contribute to the structure of professional practice of guiding teacher thinking. This component was included to capture aspects of teacher educator knowledge that might not have been represented in Shulman’s (1986) generic model or in Ball et al’s (Ball et al., 2008) framework.

In analyzing data for this study, Shulman's categories of content knowledge resided at the intersection of mathematical knowledge for teaching and practical
knowledge (See Figure 61). In other words, data that was coded according to Shulman's categories was almost always coded with respect to some aspect of practical knowledge and/or mathematical knowledge for teaching. For example, Shulman's pedagogical content knowledge was often coded as practical *knowledge of instruction* (INS), *knowledge of content and students* (KCS), or *knowledge of content and teaching* (KCT). However, since mathematical knowledge for teaching was considered to be an expansion, and more detailed description of Shulman's knowledge domains (Ball et al., 2008), some data that were coded within the categories of mathematical knowledge for teaching, were not necessarily identifiable according to Shulman's categories.

Figure 61. Analytical relationship among the teacher knowledge models.

Aspects of *specialized content knowledge* (SCK), specifically mathematical tasks of teaching, were not readily identifiable using Shulman's model (See Figure 61). Examples included, but were not limited to, evaluating the plausibility of student claims, modifying
tasks, asking productive mathematical questions, and selecting representations for particular purposes (Ball et al., 2008).

One aspect of practical knowledge, *knowledge of instruction* (INS) appeared to exist at the intersection of the three models while other aspects were only identifiable as practical knowledge. For instance, the MTEs relied heavily on experiential knowledge (EXP) which could only be captured using the practical knowledge code. Additionally, practical knowledge components such as *knowledge of self* (SLF), *knowledge of surroundings* (SUR), *situational knowledge* (SIT), and *theoretical knowledge* (THE) were found to be secondary sources of knowledge and usually coded only as practical knowledge as well. Despite the overlap of the three teacher knowledge models of the initial analytical framework, in concert they allowed for the development of a more detailed description of the knowledge domains used by the MTEs in their practice.

Two additional models specific to the mathematics teacher educators, the teacher educator knowledge tetrahedron and the triad of mathematics teacher educators, were then incorporated to develop a coherent model that could be used to examine and study the practice of mathematics teacher educating (Perks & Prestage, 2008; Zaslavsky & Leikin, 2004). Aspects of these models also overlapped with the three teacher knowledge models but they distinguish components of teacher educator knowledge and the practice of teacher educating that were not visible using the initial analytical framework. Figure 62 represents the interactions of the four teacher/teacher educator knowledge models.
The teacher educator knowledge tetrahedron consists of three components; 1) mathematical learner knowledge of the teacher educator, 2) professional traditions of teacher education, and 3) practical wisdom of teaching about teaching. Note that the learner knowledge of the teacher educator consists of the entire teacher knowledge tetrahedron (Perks & Prestage, 2008). The components of the teacher knowledge tetrahedron include the learner knowledge of the teacher, practical wisdom derived from being in the classroom, and professional traditions with respect to school curriculum, practices, and research. Since the mathematical learner knowledge of the teacher educator included the teacher knowledge tetrahedron, most aspects of that knowledge exist at the intersections (A, B, and C) of the teacher educator models and the teacher educator knowledge tetrahedron.
models (See Figure 62). Knowledge of school curriculum, research with respect to mathematics teaching and learning, practical wisdom gained from experiences in the classroom, and mathematical knowledge were all identifiable using the teacher knowledge models as well. However, the professional traditions of teacher education and the practical wisdom about what teachers need to know was not identifiable using the teacher knowledge models. Knowledge of professional traditions emerged from personal experiences in teacher education, current government teacher training policies, education and training, and the national curricula for teacher education (Perks & Prestage, 2008). This type of knowledge was not always recognized by the teacher knowledge models but certainly impacted planning and instructional decisions of the MTEs.

The teaching triad of mathematics teacher educators was used to describe and analyze the practice of the MTEs and consisted of challenging content for mathematics teachers, sensitivity to mathematics teachers, and management of mathematics teachers' learning. The challenging content for mathematics teachers is included in the teaching triad of mathematics teachers including challenging mathematics content for students, sensitivity to students, and management of students' learning. Similar to learner knowledge for teacher educators, challenging content for mathematics teachers existed at the intersections (A, B, and C) of the teacher educator models and the teacher models. Mathematics content, sensitivity to schoolchildren, and management of school children's learning were all influenced and identifiable using the three teacher knowledge domains. Sensitivity to mathematics teachers and management of their learning were more likely
influenced and identifiable through the practical wisdom and professional traditions of teacher educator knowledge.

Each of the five models used in this study provided a unique lens to examine teacher educator knowledge and the practice of mathematics teacher educating. The teacher knowledge models of the initial analytical framework allowed for detailed identification of the knowledge domains used by the MTEs while the teacher educator models offered additional aspects of both knowledge and actions of teacher educators that were not necessarily identifiable using just the teacher knowledge models. This combination of theories provided greater analytical power when examining data and unpacking teacher educators’ work.

**Modeling Mathematics Teacher Educators’ Practice: A Disciplinary Framework**

**Levels of concern for MTEs.** In order to conceptualize a model of teacher educators’ practice, the concerns of the MTEs relative to their practice were examined first. While each had unique specific concerns relative to the situations in which they taught, three levels of concerns appeared to be prevalent across all three MTEs practices. Their practices revealed that two levels of the MTEs’ concerns formulated around children’s mathematical learning and teacher learning. The third level, a concern for professional competencies, emerged from the MTEs’ work with PTs during the methods course observations. See Figure 63 for a diagrammatic representation of the three levels of concerns.
Each MTE relied heavily on children’s mathematical learning as a focus to address their concerns for teacher learning. When discussions of children’s mathematical learning surfaced, the “content” of the episodes revolved around mathematics. For instance, when Alex conducted the activity using paper plates with colored dots, he was demonstrating a method that allowed children to recognize combinations of groups of dots that could be added or subtracted to determine a total thus helping children make the transition from counting to addition. In the video of the classroom episode that Luke showed to his PTs, he focused on the children’s learning with respect to area of triangles.
When the MTEs were at the level of concern for children’s mathematical learning, mathematics was more prevalent in their instruction.

When at the level of concern for teacher learning, the MTEs used mathematics as an instrument to raise awareness of the factors that influence the teaching and learning of mathematics. For example, Alex discussed his instructional strategy of placing teachers in the role of learner because he believed it helped them understand student-centered instruction. Specifically, he asked them to complete problem solving tasks exactly as he would expect school children to complete the tasks. Once the teachers had achieved solutions to the problems, they were then asked to discuss and reflect on the problem from the perspective of a school child and of a teacher. During her work with coaches, Tracy emphasized that her selection of *Henry's Freedom Box* was deliberate as she wanted to approach mathematics from a social justice perspective. In this instance, social justice was the focus of the session while mathematics only served as a context for addressing it. Alex and Tracy used mathematics as an instrument to raise awareness of instructional strategies, materials selection, and social justice, all factors that influence the teaching and learning of mathematics.

The third level, concern for professional competencies, emerged from the MTEs’ practices primarily in regard to teacher performance assessments. Tracy asserted that she felt both restricted and frustrated relative to instructional strategies and mathematics content selection for PTs because of university required teacher performance assessments and early childhood program assessments that did not include mathematics content. She felt that she had more freedom of choice in both regards when working with inservice
teachers. Luke devoted the entire final class session of the semester to helping his PTs learn how and what to submit to an electronic portfolio system used for teacher performance assessment. The system was new to the University and he chose to walk the students through the process of submitting items so that both he and they would be familiar with the system. While other professional competencies exist, the MTEs in this study were immediately concerned with teacher performance assessments.

Although each of the levels of concern has been discussed separately, they exist such that one level of concern can penetrate another and MTEs might begin an instructional session at any one of the levels. Luke, for instance, began the classroom video episode in his first classroom observation focused on concerns for teacher learning as he stopped the video and addressed how the classroom teacher (one of the PTs student teaching) decomposed the original triangle into two right triangles to provide a more convincing argument to generalize finding the area of a triangle. He then progressed into concerns for professional competencies suggesting that the PT had numerous artifacts from this video episode which he could download for his teacher performance assessment. Finally, Luke's concern for children's mathematical learning emerged when he suggested that the children were using correct mathematical language and that the PT was gaining insight into what they were thinking. Instructional sessions can begin at any one of the three levels of concern and transition into either or both of the others as the session progresses and thus the reason for dashed circles representing the levels and arrows suggesting movement from one level to another in the diagrammatic representation (See Figure 63).
In addressing the different levels of concern, pragmatic influences, theoretical influences, experiential influences, and personal influences guided MTEs instructional practices. These influences did not act independently as each could impact another and thus are connected with double arrows in a diagrammatic representation (See Figure 63). An example of a pragmatic influence occurred with respect to materials selection when Tracy chose not to include the Pedagogy of the Oppressed book in her methods course due to time constraints as well as her belief that the PTs would not read it. Alex's belief that problem-based, learner-centered instruction was appropriate for all levels of learners was a personal influence he used to address concerns for children's mathematical learning and teacher learning. Luke relied on past experiences teaching on a Native American reservation and at an industrial plant to stress the importance of contextualization of mathematics for school children. Tracy's practice was guided by theoretical influences such as her use of Cognitively Guided Instruction videos to address instructional methodologies with her PTs. These examples only provide a glimpse of the MTEs practices as they addressed the different levels of concerns.

Concerns for children's mathematical learning, teacher learning, and professional competencies surfaced in all three MTEs' practices. Their discussions and sessions often began at one level of concern and transitioned into one or both of the other levels. As these concerns were addressed, pragmatic, theoretical, experiential, and personal influences guided the MTEs practices with respect to materials selection, contextualization of mathematics, sociocultural considerations, and instructional strategies. Having identified the three primary levels of concern of the MTEs' practices,
the next section discusses the three levels of knowledge drawn from and used to address these concerns and the models appropriate for identifying each level of knowledge.

**Levels of knowledge for MTEs.** The MTEs relied on multiple aspects of knowledge to address the three levels of concern in their practices. As with the three levels of concern, the three levels of knowledge including 1) knowledge of school children, 2) knowledge of teachers, and 3) knowledge of the profession, acted in concert with one another as MTEs transitioned among the different levels of concern. Figure 64 indicates this interaction through the use of dashed lines showing that aspects of knowledge at each level can penetrate into another level. Knowledge of teachers and knowledge of the profession sometimes differed based on whether the MTE was working with PTs or ITs as indicated by the dashed arc through each of the levels.

Knowledge of school children consists of the knowledge necessary to effectively teach mathematics to school learners. Examples include knowledge of children's mathematical thinking, instructional strategies effective for advancing children’s acquisition of concepts, the school curriculum, theories regarding the teaching and learning of mathematics, sociocultural underpinnings, the nature of mathematics as a subject and a discipline, and assessment techniques. For example, all three MTEs believed that problem-based, student-centered, instructional activities provided school children with opportunities to discuss and reflect on their own mathematical thinking as well as that of others and thus construct mathematical knowledge that made sense to them.
The MTEs also relied on their knowledge of school curriculum; specifically the Common Core Standards for Mathematics, to address issues of content and help teachers connect mathematical topics across grade levels. Alex and Tracy drew from knowledge with respect to sociocultural considerations as they addressed issues of social justice and gender in mathematics. The MTEs used multiple aspects of knowledge with respect to school children to address their concerns for children's mathematical learning.
The four knowledge domains identified in this study as MKTE including 1) subject matter knowledge, 2) pedagogical content knowledge, 3) curricular knowledge, and 4) practical knowledge all were evident with respect to knowledge of children and their learning of mathematics. The three teacher knowledge models which comprised the initial analytical framework of this study all identified various aspects of this level of knowledge (See Figure 64). For instance, Shulman's (1986) subject matter knowledge and specialized content knowledge of mathematical knowledge for teaching identified Luke's knowledge of mathematics when he asserted that the product of \((x+4)^2\) has a middle term suggesting that school children often ignore it. Elbaz’s (1983) experiential orientation of practical knowledge identified Alex's knowledge of instructional strategies and his transition from teacher-centered instruction to student-centered instruction as he progressed from teaching at a parochial school to a community college. Additionally, the teacher educator learner knowledge aspect of the teacher educator knowledge tetrahedron could be used to identify multiple aspects within this level of knowledge as well. Knowledge of mathematics, knowledge of the school curriculum, and knowledge from being a classroom teacher are all evident at this level.

Knowledge of teachers consists of the knowledge necessary to teach PTs and ITs about how to effectively teach mathematics to school children. Examples of this type of knowledge include knowledge of teacher cognition, instructional strategies for teachers, university teacher education curriculum, theories of teacher learning, teacher knowledge gaps, and experience levels. Alex and Tracy, for example, used planned and unplanned activities to address gaps in their PTs’ mathematical knowledge. Alex specifically
included a question about writing a number in expanded form because he knew his PTs
would struggle with the concept. Tracy, on the other hand, overheard a discussion that
indicated her PTs had a gap in their knowledge with respect to fractional parts of area and
she chose to adapt her agenda to address the issue. All three MTEs again relied on
problem-based, learner-centered instructional strategies to develop their teachers’
mathematical content knowledge and also model the strategy for use with children.

Unlike the level of knowledge of school children, only aspects of practical
knowledge relative to MKTE were evident with respect to teacher knowledge. Luke
relied on practical knowledge of self to express his comfort level in regard to sequencing
instruction for addressing gaps in teachers’ knowledge with respect to determining an
algebraic solution for a puzzle activity. He felt secure in not guiding the coaches toward
an easier arithmetic solution but instead allowing them to use the tools available to
construct their own solution. Alex's experiential orientation to practical knowledge
identified his concern for teacher learning relative to experienced teachers when he
suggested that teaching addition and subtraction in a related fashion instead of as separate
concepts concerned practicing teachers. He asserted that teachers who had taught
addition and subtraction as separate entities for 20 years would struggle with this idea.

From the teacher educator knowledge tetrahedron, the teacher educator’s practical
wisdom and knowledge of professional traditions can also be used to identify knowledge
at this level. Practical wisdom is gained from determining what the teachers need to
know and how best to address those aspects of knowledge while the professional
traditions emerge from personal experiences, research on mathematics teaching and learning, and ways of working in the existing teacher education courses.

Knowledge of the profession refers to knowledge of the practice of mathematics teacher educating, the genre of research pertaining to this domain, and awareness of the professional competencies required of teachers. Professional competencies include but are not limited to teacher performance assessments, content and pedagogy assessments required for licensure, and national board certification requirements. In this study, knowledge of the profession was primarily evident relative to teacher performance assessments required of PTs. Luke demonstrated his knowledge at this level as he helped his group of PTs to determine how and what to submit to an electronic portfolio system that was being used as part of their teacher performance assessment. Additionally, he used the evaluation rubric for the teacher performance assessment to inform the PTs of how to achieve the highest possible scores with respect to the items they submitted. Tracy asserted that her knowledge of NCATE and assessments required of PTs restricted the creativity in her instructional strategies and in the activities she selected due to accountability issues. She claimed that she was much more confident and "gutsier" in how she used to teach middle school mathematics because she had more freedom of choice.

Like the teacher knowledge level, knowledge of the profession was identifiable using Elbaz’s (1983) practical knowledge model and the teacher knowledge tetrahedron model. Tracy’s practical knowledge of surroundings identified her knowledge of the existence of NCATE and the assessments required of PTs indicating her awareness of the
political context of teaching with respect to the national curriculum. Luke relied on his situational orientation to practical knowledge to express to his PTs the importance of submitting high quality work for their teacher performance assessment, because at the time, no one knew exactly what would be considered acceptable or the possible consequences for submitting unacceptable work. In addition to aspects of Elbaz’s (1983) practical knowledge, the professional traditions component of the teacher knowledge tetrahedron with respect to the national curricula for teacher education is appropriate for identifying knowledge at this level as well.

Concerns for children’s mathematical learning, teacher learning, and professional competencies formed the basis of the MTEs’ instructional practices. They typically began sessions focusing on one of the three levels and transitioned into one or both of the other levels. As the MTEs addressed different levels of concern, pragmatic, theoretical, experiential, and personal influences guided their decisions in areas such as instructional strategies, sociocultural considerations, activity and materials selection, and contextualization of mathematics. They relied on aspects of knowledge of children, knowledge of teachers, and knowledge of the profession of teacher educating to address the different levels of concern for both PTs and ITs. Shulman’s (1986) generic teacher knowledge model and the mathematical knowledge for teaching model of Ball et al. (2008), mostly identified aspects of the MTEs’ knowledge at the level of teaching school children mathematics. Elbaz’s (1983) practical knowledge for teachers model and the teacher educator knowledge tetrahedron identified aspects of knowledge with respect to teachers and the profession that were not visible using the lens of Shulman or
mathematical knowledge for teaching. The four models, when used in concert, allow for detailed analysis of MTEs’ practice.

**Reflections on My Research**

Research has suggested that teacher education programs need to help teachers develop a deep conceptual understanding of the mathematics they teach (Hiebert et al., 2003; National Council of Teachers of Mathematics, 2000; Petrou & Goulding, 2011; Zaslavsky & Leikin, 2004). The results of this study indicated that teaching mathematics content was not necessarily the main focus of what MTEs did, at least during the sessions they were observed, but instead their intent was to raise awareness of the factors that influence the teaching and learning of mathematics; relying on mathematical venues they perceived most useful in meeting this goal. Their sensitivity to these factors, and the need to assure knowledge of these factors was established, seemingly limited the amount of time available to instruct teachers on mathematical concept, but to raise their insights regarding mathematical concept development and in doing so, they taught them mathematics as well as tools for raising mathematical learning of children. That is not to say that when the opportunity presented itself, they failed to take advantage to focus on providing explicit instruction. Tracy noticed a gap in the PTs’ knowledge with respect to fractional parts of area and deviated from her intended agenda to address their misconceptions. Luke recognized that one of the coaches in his PD session was not aware of the quadratic relationship of data and he addressed the issue with her individually as well as with the entire group. Alex intentionally posed a question on his *Number of the Day* activity in anticipation of difficulties his PTs might have writing a
two digit whole number in expanded form. Each MTE recognized and took advantage of opportunities to address issues regarding mathematics content.

However, activities devoted specifically to discussions of mathematical content were limited and where content was prevalent, it had been chosen so to specifically address factors that influence the teaching and learning of the subject either horizontally or vertically. For example, Traci's selection of *Grandfather Tang’s Story* and *Henry's Freedom Box* was deliberate so that she could address cultural topics and issues of race. The *Three Peg Puzzle* used by Luke helped him to raise awareness of the connections between elementary mathematics and high school mathematics. Alex chose to devote almost an entire session to the Common Core Standards for Mathematics. This session relied heavily on mathematics content but the focus was on familiarizing PTs with the language and organization of the standards across grade levels as opposed to the content involved.

The range of decisions they made seem necessary for MTEs whose practice of teacher educating is influenced by their sensitivity to the needs of their teachers while forced to be responsive to time limitations. Mathematics teacher educators conceptualize their practice in light of multiple theories of teaching and learning and feel the responsibility to address issues pertaining to why certain topics are difficult or easy for children to learn, why common errors occur, and how children's mathematical thinking is enhanced or hindered by choice of instruction and curriculum. This knowledge base is fundamental to effective practice by teachers (Fosnot, 1993). The teaching triad of mathematics teacher educators (Zaslavsky & Leikin, 2004) suggested that sensitivity to
the mathematics teachers and management of their learning forms two thirds of actions they take while engaged in practice. Luke needed to devote his final classroom session of the semester to helping the PTs understand how and what to submit to the electronic portfolio system used for their teacher performance assessment. The system was new to the University and his sensitivity to the needs of the PTs necessitated that decision. Alex needed to devote time to language and structure of the Common Core Standards for Mathematics because that was the curriculum to be used in most schools. Tracy used a considerable amount of class time to discuss issues of equity pedagogy and social justice to raise awareness of how those issues can impact teaching and learning of mathematics.

Curricular matters, elements of sociocultural issues, assessment strategies, theories of teaching and learning, instructional strategies, course requirements, and teacher performance assessments all necessitated class time and were important to advancement of teachers. While mathematical content knowledge is obviously critical for effective teaching of school children, these MTEs were limited in the time they could devote to instructing teachers in mathematics, per se. Therefore, the structure of their own mathematical knowledge was less than obvious during the sessions they were observed. Further investigation is necessary to determine whether the results of this study with respect to mathematical content are indicative of most MTEs or whether this group was an exception. Might the results be different if the MTEs were observed teaching content courses designed specifically for teachers as opposed to the methods courses observed in this study?
Limitations of the Study

In this work, a qualitative multiple case research design was used to examine the domains the MTEs used or referenced while talking about and enacting their conceptualization of teacher educating. The goal was to provide a "rich and holistic" account of the practice of mathematics teacher educating in a real-life situation and advance the field’s knowledge base in this regard (Merriam, 1998, p.41). The qualitative design allowed for an in-depth investigation of the sources of knowledge relied on by MTEs in their practice and how those sources influenced their decisions as they interacted with teachers. As a result of using the qualitative design, there were limitations associated with data, the participants, and the timeframe in which some of the observations were conducted.

In order for this study to be feasible and for reasons of accessibility, the number of mathematics teacher educator participants was limited to three. Additionally, and for the same reasons, each participant was observed only three times and the observations occurred within a very short time span of approximately 4 months. While the interviews and observations provided a wealth of data, the limited number of observations provided only a snapshot of the MTEs instructional practices. Additionally, the participants represented a relatively small geographical area of the United States. Despite these, the findings could be used as a guide for development of a research based survey administered internationally as a means to determine commonalities among MTEs regarding their pedagogical decision making. For example, the survey could address the
role of mathematics content in the design and implementation of pedagogical experiences for teachers. This data can inform further clarification on the MKTE.

My familiarity with the participants raises issues relative to the trustworthiness of the results. Prior to their participation in my research, I had known each of them on a personal and a professional level. As a graduate research associate, I met each of them through a mathematics coaching professional development program. I assisted and observed both Alex and Tracy as they conducted professional development sessions and spent significant time in discussing professional and personal issues with Luke. So as not to avoid this issue, I addressed my familiarity with the participants as part of the methodology discussion in Chapter 3 and again in the introductory section of Chapter 4. Even so, it remains likely that my familiarity with the participants influenced questions that I asked in the interviews and my ability to objectively analyze observational data.

Another limitation of the study is the fact that all three MTEs were observed in the same PD environment as they conducted sessions for K-12 mathematics coaches. This choice was made for reasons of convenience and accessibility, for both the participants and the researcher. While the MTEs’ responsibilities were different in regard to the coaches with whom they worked and the “content” they were charged to teach, the research-based instructional practices promoted by the program were reflected in each of the MTE’s practices.

The timing of the classroom observations for Luke and Tracy were also an issue. Each was observed on consecutive weeks, specifically the last two weeks of the course prior to the final examination week. Consequently, segments of both classroom sessions...
for each MTE were devoted to end-of-course assignments. In Luke's case, the entire final classroom session dealt with logistical issues related to electronic submission of items for a university required teacher performance assessment. If instead, each been observed for two weeks in the middle of the semester, portions of the results may have been different. While the results of this study are not likely generalizable, the methodology used for data collection and analysis can be useful for further studies with respect to mathematical knowledge for teacher educating.

Implications for Practice and Research.

Implications for practice. Results from this study indicated that the knowledge domains used in the practice of mathematics teacher educating were somewhat similar to those used for teaching mathematics though they took on different form. Mathematics teacher educators relied on pedagogical content knowledge, subject matter knowledge, curricular knowledge, and practical knowledge comparable to classroom teachers. However, they also relied on the professional traditions and practical wisdom of teacher educating, knowledge domains not available to most classroom teachers. Typically, classroom teachers have little or no experience teaching about teaching mathematics and thus no familiarity with government teacher training policies or the national curriculum for teacher education. Additionally, their audience consisted of both teachers and school children. Mathematics content, instead of being the focus, provided a pathway or a context for the MTEs to address the factors that influence the teaching and learning of mathematics. Pedagogical content knowledge was determined to be a primary source of knowledge for the MTEs while subject matter knowledge was a secondary resource.
What role does the epistemology of mathematics play in how teacher educators organize their practice or define their roles and to what extent should mathematics content be an area of focus in development of programs for their preparation?

Hebert, Morris, and Glass (2003) contended that the average classroom in the United states displays the same methods of teaching as in the past and students continue to be deficient with respect to the competencies required to understand mathematics deeply and use effectively. Additionally, they suggested that one goal of teacher preparation programs should be to help teachers become proficient with five kinds of mathematical competencies (National Research Council, 2001) including; 1) conceptual understanding, 2) procedural fluency, 3) strategic competence – the ability to formulate, represent, and solve mathematical problems, 4) adaptive reasoning – capacity for logical thought, reflection, explanation, and justification, 5) productive disposition – seeing mathematics as sensible and worthwhile in the belief in one's own ability to be successful in mathematics. They indicated that mathematics content should play a significant role in the practice of mathematics teacher educating. Activities selected by the MTEs addressed mathematical competencies such as procedural fluency, strategic competence, and adaptive reasoning but these types of activities were limited. However, it is important to remember that the data collected for this research represented only a snapshot of the MTEs practices.

Even (2014) asserted that there is a need for empirical research on the practices of mathematics teacher educators to determine whether they "facilitate or impede the development of teachers and their own professional capacities" (p. 330). Part of this
empirical research of the practice of mathematics teacher educators might focus on the level of importance of mathematics content reflected in their practice. One of the results of the current study was that the MTEs relied on mathematical knowledge for teacher educating to address knowledge gaps relative to mathematics content for both PTs and ITs. Olanoff’s (2011) dissertation study revealed the mathematics teacher educators she observed had knowledge gaps relative to multiplication and division of fractions. Deeper understanding of mathematics content likely should be addressed both in the practice of mathematics teacher educating and in the development of mathematics teacher educators.

**Implications for research.** According to Even (2014), empirical research relative to the practice of mathematics teacher educators is limited for two reasons. The first problem is that the research focuses on the learning of teachers and not the practice of the mathematics teacher educators. This study addressed that issue as the focus was on the knowledge domains used or expressed to be used by the mathematics teacher educators as they designed and implemented pedagogical experiences for their teachers. While a theme for the practices of the mathematics teacher educators, teacher learning was not addressed in the data analysis. The second reason for limited empirical research relative to the practice of mathematics teacher educators is that almost all previous research conducted in this area consists of self-reports of teacher educators on their own work (Even, 2014). This issue was also addressed as I conducted my research as an outside observer in the natural environments of the mathematics teacher educators’ practices. They were only involved as participants with no input (other than member checks) regarding data collection and analysis.
This research provides a global view of the knowledge domains that were utilized, and how they were utilized, by mathematics teacher educators relative to their overall practice. While not generalizable, findings offer a foundation from which future research might develop. The combination of models used in this study appeared to capture the domains the MTEs relied on when engaged in practice. The characterization of these domains should be considered in future research when examining teacher educator knowledge.

Methodologically, the timing of methods course observations and the influence they might have on what educators choose to do must be carefully considered. Due to end-of-course issues, collecting observational data during the last two weeks of the semester can, in all likelihood, provide a snapshot that is atypical of the teacher educator’s practice as it happened to be the case in the present work. Additionally, participants from different professional development projects should be observed. The instructional practices of professional development providers reflect the goals of the program and thus observing multiple participants from the same professional development would likely result in similar instructional practices. To improve the trustworthiness of the study, characteristics of the participants should be as varied as possible (Merriam, 1998; Yin, 2009).

The three MTEs in this study possessed varied educational backgrounds and wide ranging teaching experiences. However, each brought a K-12 perspective to their practice. Mathematics teacher educators are university faculty, adjunct instructors, and graduate students whose responsibilities include teaching courses for perspective and
practicing teachers as well as providing supervision for student teaching experiences. Additionally, they are school leaders, department leaders, mathematics curriculum supervisors, and mathematics coaches who provide support for the teachers in their school system (Chauvot, 2008). Future research could examine how teacher educator backgrounds and their professional experiences and responsibilities may distinguish their specialized mathematical knowledge for teacher educating. To what extent might the model presented in this study for conceptualizing mathematics teacher educators’ practice vary according to the different populations involved in mathematics teacher educating?

Extensions of this research might also include subjects beyond mathematics to determine general and unique domains used by teacher educators. Specifically, research could examine how the model conceptualizing mathematics teacher educators’ practice might apply to other disciplines (science, engineering, technology, history, political science, etc.).

Conclusion

The practice of mathematics teacher educating is complex and a relatively young area of research. While the current study faces some limitations, the knowledge gained provides a basis from which future research could be developed. Education is the foundation of our society and those responsible for teaching our teachers assume a great responsibility. This study provided information that can guide the practice of mathematics teacher educators as they eagerly accept that responsibility. They can compare the knowledge domains found to be used in this study with the knowledge domains they use in their practice to raise awareness of possible areas of focus for their
preparation programs. These results also provide the impetus for mathematics teacher educators to examine their own treatment of mathematical content in their practice and determine whether they are meeting the needs of their teachers. Educational reform is multilayered with respect to the immediate participants as school children rely on teachers, teachers rely on teacher educators, and so ultimately school children rely on teacher educators. This work contributes to research on the practice of mathematics teacher educating and provides foundational information upon which MTEs can examine their own practice and reflect on their ability to help teachers, and ultimately children, learn.
References


421


Silver, E., Clark, L., Ghousesi, H., Charalambous, C., & Sealy, J. (2007). Where is the mathematics? Examining teachers’ mathematical learning opportunities in


Appendix A: Initial Interview Protocol

Questions for all participants.

1. Tell us a little about your background. What is your educational background? What have been your professional experience(s) including where you have worked and number of years at each position? What levels of students did you work with at each position?
2. What knowledge bases do you feel teacher educators need for the practice of teacher educating?
3. How is the knowledge base of a teacher educator similar to or different from the knowledge base of a K-12 teacher?
4. How is what you do as a teacher educator similar to or different from what is expected of a K-12 teacher? If you can, please provide an analogy that would help to illustrate your thoughts about this comparison.
5. What knowledge bases do you draw from when you make decisions about what to do in your interactions with students? Teachers?
6. Is there a difference between the types of knowledge you think are necessary when you work with elementary teachers versus secondary teachers?
7. How is your practice of teacher educating similar to or different from your work with school learners? Draw from whatever grade band of school learners you prefer.
8. When working with preservice and/or inservice teachers, do you draw from the knowledge base of adult learning theories?
9. Suppose you have to design a summer professional development program for inservice teachers. How does your knowledge base influence the plans? How is this similar to or different from the way you would design a university course for the same audience?
10. Assume you are observing your version of the ideal classroom. What is the teacher doing? What are the students doing? What curriculum is being taught? What knowledge do you need to effectively prepare teachers to teach in your ideal classroom?
11. Do you consider yourself an elementary teacher educator or a secondary teacher educator? Why?
12. Do you think your role or the function you play in interaction with teachers might be different according to grade bands? If so, how?
13. Describe some specific examples of when you feel you had success in the practice of teacher educating.
Questions for university faculty only.

14. How is teaching the methods of teaching a subject similar to or different from teaching the subject itself?
15. Suppose you have to design a methods course for your subject area. What do you feel are the essential components for the course?
16. How should teacher educators understand their subject matter to be able to effectively teach a methods course?
17. In your practice of teacher educating, how is working with preservice teachers similar to or different from working with inservice teachers?

Questions for mathematics coaches only.

18. How is the knowledge you need as a coach similar to or different from the knowledge you need as a classroom teacher?
Appendix B: Pre- and Post-Observation Interview Protocol

Pre-observation interview questions

1. What are your goals for this session?
2. What specific learning objectives do you have?
3. What are your plans for assessing the success of your instruction?
4. As this session progresses, what problems and/or issues do you anticipate?
5. How do you intend to address the problems and/or issues that you anticipate?

Post-observation interview questions

Specific questions based on the interactions during the session will be asked with the intent to understand the basis for decision making. Additionally, the following questions will be asked:

1. Do you feel the session was successful? Why or why not?
2. Would you change anything about the session and if so, what would you change?
3. Did anything occur during the session that surprised you? If so, why did you choose to handle it the way you did?
Appendix C: Course Syllabus Pages 1 & 2 – Alex

COURSE SYLLABUS

Teaching Mathematics in the Middle Grades

The Mission of the College of Education

We prepare leaders – talented, responsible, ethical educators – committed to diversity, technology proficiency, and the character of continuous assessment that brings learning to the center, for all.

- University Council for the Preparation of Education Professionals (U-PEP)

Course Description

Candidates engage with the principles and beliefs of reform-based efforts to increase the content-knowledge and mathematical thinking of students at the middle childhood level. University coursework will focus on the establishment of classroom practice for mathematics education that is student-centered and problem-based with purposeful and deliberate attention on the Common Core State Standards for Mathematics, and the NCTM Process Standards for School Mathematics. Candidates will create lesson plans and teaching units of mathematically-rich problem that encourage the development multiple solution paths, the use of manipulatives, the adaptation of instruction for multiple learning styles, the use of technology, the development of teaching skills and dispositions based upon student collaboration and interaction, and the explication and probing of students’ mathematical thinking through shared classroom discourse.

During an embedded field experience, preservice teacher candidates will work with groups of middle childhood students in the field-based teachers’ mathematics classrooms to implement the problems and lessons they design and implement. An opportunity for dialogue with inservice teachers will continue across the quarter as preservice teachers work towards gaining the knowledge, skills, and dispositions of mathematics teaching and learning for understanding.

Course Rationale

Teaching mathematics well is a complex endeavor, and there are no easy recipes or textbooks that can easily transform a preservice teacher candidate into an exemplary mathematics teacher in the course of ten weeks. However, current research and NCTM Standards suggest that when teachers allow students to investigate, to explore, to discuss, and to articulate and refine their mathematical thinking, mathematics becomes more meaningful and part of the students’ everyday world. Instead of learning mathematics as
rules and procedures, students learn to ‘mathematize’ their world through engaging and inquiry-based, problem-solving learning experiences. This course prepares preservice middle childhood mathematics teachers to become better students of mathematics and better problem-solvers. It also gives the candidates exposure and experience in creating unique, mathematically rich, learning experiences for their students.

**Overarching Goal**

To acquire the pedagogical knowledge, skills, and dispositions that value and support the conceptual understanding of young learners in middle childhood mathematics environments through the explication of student thinking and interaction in the act of mathematically-rich, intellectually-honest, problem-solving learning experiences.

**The Curriculum: Learning Outcomes**

As a result of this course, Candidates shall be able to:

1. Investigate and apply a wide-range of constructivist-based pedagogies applicable to the middle childhood mathematics classrooms and supported by NCTM, the (State) Department of Education, and the Common Core State Standards for Mathematics.

2. Actively engage within a mathematics community to create, solve, and refine mathematically-rich problems that integrate the concepts of number, number sense, operation, patterns, functions, algebraic thinking, measurement, geometry, data analysis, and probability.

3. Compare and contrast research-based teaching strategies that incorporate and extend student thinking.

4. View middle childhood mathematics as a discipline involving the processes of investigation, verification, exploration, explanation, discovery, conjecture testing, representation, reflection, and communication.

5. Design meaningful problem-based learning experiences and lesson plans that center on mathematical concepts that encourage the integration of technology and inquiry.

6. Clearly communicate their mathematical thinking to others in verbal and written forms, and to explicate these skills in their students.

7. Use cultural, historical, and scientific applications of mathematics to make a wide variety of connections within the field of mathematics and between mathematics and other aspects of the overall curriculum and with the everyday life of young children so that children learn to value mathematics.
Appendix D: One-to-One Correspondence Scenarios – Alex

The Little Shepherd Boy

Imagine a poor, rural shepherd boy. Due to the mountains and the landscape surrounding his family’s farm, he is very isolated from the rest of the world. The shepherd boy has never attended school, and has no mental construct of what it means to read, to write, or to count. Each day, his father entrusts him to take the sheep from their holding pen at the house to graze in the surrounding meadows. Since the family depends on the money they receive from the wool, the milk, and the meat that the sheep provide, keeping track of the herd is essential to the family’s way of life. How could our little shepherd boy develop a system to accurately know that the same number of sheep that he releases to the fields in the day is the same number of sheep that he brings back into the holding pen at night?

Kids Fighting Over Legos

One day, an argument arose between two four-year-olds as to who had the most Lego pieces. They each ‘counted’ the blocks that lay in front of them. One reported to the other, “I have a million and seven Legos,” and the other quickly responded that she had, “A thousandy hundred Legos.” Needless to say, a recount revealed that neither of them could reliably count high enough to count the Legos that each student had in his or her respective pile. How can they settle their argument without any adult intervention?
Appendix E: Course Syllabus Page 1 – Tracy

Teaching and Learning of Mathematics in Grades PreK-3

Autumn Semester, 2012

Course Expectations

I. Course Objectives: The focus of this course is not to give you fully baked cakes. Rather, the focus of this course is to teach you how to bake a cake. I will give you some ingredients, and direct you to more. I will give you the tools to help you put those ingredients together. I will expect you to design a few recipes, and to try a few cakes – some of which may flop. I will expect you to fix your recipes, making your cakes the most nutritious they can be, according to experts in the field of mathematics teaching and learning, include the NCTM, (State)DE, and researchers. I will also expect those cakes to be as tasteful as you can make them, both for you and your students. Toward that end, the objectives of this course are for each participant to develop, in the age 3 to grade 3 Early Childhood context, an informed, research-based understanding of:

1. Constructivist mathematics pedagogy;
2. Learner Responsive Pedagogy in mathematics;
3. Equity Pedagogy in mathematics; and

II. Attendance: (10%) This includes being mentally and physically present. Please let me know, in advance when possible, if you will not be in attendance on any particular day. Messages may be left 24 hours a day my office voicemail. Remember that if you are absent you miss what happens and the learning that goes with it. No amount of make-up will ever replace experience – and nothing you might get from your peers or from me will be the same as being present - mentally and physically.

III. Class participation: (10%) I expect active participation in all components of (course number) for all participants, student and instructor alike, including:

1) Being prepared for class,
2) Completing assignments on time,
3) Demonstrating engagement with course materials, activities, discussions, and readings,
4) Demonstrating thoughtfulness and reflective consideration of teaching mathematics in a diverse world,
5) Being knowledgeable about and an informed consumer of the Common Core State Standards (CCSS), their content and mathematical practices—go to: (State Department of Education website)
6) Being knowledgeable about and an informed consumer of the Ohio Pre-Kindergarten Content Standards – Mathematics (same site as #5)

Appendix F: Tangram Activities – Tracy

Center 1: Puzzle Page

One per student at the table. Distribute paper copy of numbered pieces, and chart of puzzles to complete. Group works together to complete the chart of tangram puzzles. Solutions should be drawn on individual charts, using numbered pieces. Then after a few minutes, hang the classroom chart of tangram puzzles for the group (and eventually other table groups) to document their solutions, again using numbered pieces.

Center 2: Tangram Puzzle Cards

- Work on these puzzles with a partner. Each puzzle consists of 4 cards; all four cards of one set are all one color. Select one set of 4 cards. The instructions on the cards are complete. They tell you whether to use one or two sets of tangrams. And they give you all of the necessary clues for the puzzles. Once you think you have completed a puzzle, check it by following the instructions one last time to see if all of the conditions are met with your solution. Once you have completed that puzzle, return it to the pile and take another set. Repeat the process again and again for solutions of the other puzzles.
- If completed before the rest of the class/groups, distribute paper copy of numbered pieces and work on the class chart puzzle draw them with as numbered pieces, on the classroom group chart.

Center 3: Tangram Fractions

- Using handout/worksheet:
  Explore with the tangrams to determine how pieces can be combined to make equivalent pieces. Document your explorations. AFTER documenting your discoveries, respond to the following questions:
  - How many small triangles equal one medium triangle?
  - How many medium triangles equal one large triangle?
  - How many small triangles equal one parallelogram?
  - How many small triangles equal one square?
  - If all of the pieces together equal 1, what fractional part of the whole tangram set is each of the 7 tangram pieces?
Center 4: Area, Perimeter, and Tangrams

- Have students create an animal (could be directly from the book) with their tangram set. As a table, predict the area and the perimeter of each of their table’s animals. Document on a chart – at least in terms of which is bigger (for example, the goldfish’s area is larger than the hawk, but the hawk’s perimeter is larger). Measure the area and perimeter of each and compare. Students will need to think and strategize how to measure their animal. (Note: areas may be measured using the small triangle).
- If completed before the rest of the class/groups, distribute paper copy of numbered pieces and work on the class chart puzzle draw them with as numbered pieces, on the classroom group chart.
Descriptor
Preparation for teaching in the secondary mathematics classroom, Techniques for motivating students, using questioning and critical thinking strategies and integrating technology are developed.

This course will focus on the following conceptual framework themes:
- **Content Knowledge**
- **Pedagogical Content Knowledge**
- **Student Learning**
- **Diversity**
- **Reflection & Self Assessment**
- **Technology**
- **Conceptual Learning**
- **Engagement w/Professional Practice**
- **Inquiry**

It is assumed that learners have to construct their knowledge - individually and collectively. Each learner has a toolkit of conceptions and skills with which he or she must construct knowledge to solve problems presented by the environment. The role of the community other learners and teacher - is to provide the setting, pose the challenges, and offer the support that will encourage mathematical construction.


The above statement is intended to give a feeling for the spirit of what mathematics and mathematics learning should encompass. In addition the NCTM outlines five basic goals: All students need to: Learn to value mathematics; become confident in their ability to do mathematics; become problem solvers; learn to communicate mathematically; learn to reason mathematically. (Curriculum and Evaluation Standards for School Mathematics, NCTM (1989), p. 5)
Course Objectives

In order to create an environment where these objectives are achievable we ourselves need to work toward understanding mathematics in this way. We will explore what it means to think mathematically and to have mathematical power. We will then extend these experiences toward your future classrooms by exploring how you might create a rich curriculum and environment that allows students to construct mathematical understanding for themselves. In this class we explore several aspects of mathematics teaching:

1. Knowledge of mathematical problem solving;
2. Knowledge of reasoning and proof;
3. Knowledge of mathematical communication;
4. Knowledge of mathematical representation;
5. Knowledge of technology;
6. Knowledge of mathematics pedagogy; and 7. Understanding your disposition to mathematics.

This course will address the following Professional/National Standards:

A major emphasis in this course will be on the (State) Academic Content Standards in Mathematics and the NCTM Professional Teaching Standards and the NCTM Principles and Standards. These standards outline what mathematics should be taught in grades 4-9 and also how mathematics should be taught.
Appendix H: Algebra Assessment Activity – Luke

Solve each equation

1. \[7x - 4 = 5x + 18\]

2. \[5(x + 2) - 11 = 2x + (9 - 7x)\]

3. \[4x - 17 = 3x - 11\]

4. \[3(5x + 2) = 36\]

5. \[7(x + 5) - 12 = 3(x + 2)\]

6. \[\frac{3}{4}x = 24\]

7. \[3x + 3.7 = 5x - 2.5\]

8. \[3x + 7 = 4\]

9. \[12x - 8 = 28\]

10. \[3\left(\frac{5x + \frac{1}{3}}{3}\right) - 4 = 7\]

11. \[4 - 3x = 10\]

12. \[2x + 3 = 3x - 5\]

13. Solve the equation and show how you would check to see if it is correct.

\[3x - 15 = 2x - 19\]