Assessment of Alternate Viscoelastic Contact Models for a Bearing Interface between an Axial Piston Pump Swash Plate and Housing

THESIS

Presented in Partial Fulfillment of the Requirements for the Degree Master of Science in the Graduate School of The Ohio State University

By

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2014

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ABSTRACT

Variable displacement axial piston pumps are commonly found on a wide variety of off-highway vehicles and industrial equipment. Typically, these pumps employ a swash plate supported by a hydrostatic bearing to control the pump's volumetric displacement. Previous vibro-acoustical studies have identified the bearing interface between the swash plate and pump housing as a potential source of error in predicting vibration and structure-borne noise. A better understanding of the physics of this interface is necessary to improve pump noise prediction. Therefore, several experiments are first designed to measure the interfacial properties under static (force vs. deflection), modal (motion transmissibility), and transient (step-like) conditions. The results of the three experiments are then used to develop tractable linear viscoelastic models, with minimal parameters. Parallel force transmission paths (structural and fluid) are required to adequately describe the observed transmitted forces in time domain. The interfacial stiffness and damping properties of the main models are quantified with varying mean loads in the presence or absence of oil. Of the models examined, a combination of Kelvin-Voigt and Maxwell formulations yields a better fit for vibration (modal) and transient (step-like) experiments, especially in the presence of oil within the interface. Nevertheless, the linear viscoelastic model parameters are effected by nonlinearities as their values depend on mean load, lubrication condition, and amplitude of excitation.
DEDICATION

I dedicate this thesis to my parents. Their never-ending love and support has allowed me to complete this work.
ACKNOWLEDGEMENTS

I owe my sincere gratitude to my advisor, Professor Rajendra Singh, for his expert knowledge and immeasurable guidance during my graduate research project. I thank Dr. Jason Dreyer for providing valuable time and knowledge to support my research. I would also like to thank The Smart Vehicles Concepts Center and Eaton for providing support for my graduate work, and the Ohio State University for providing me the opportunity to pursue my Master's degree. Finally, I would like to thank my fellow students in the Acoustics and Dynamics Laboratory for providing valuable feedback during the entire process.
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Miller, A. Investigation of Off-axis Excitation due to Component Asymmetry with a
         Non-Resonant Dynamic Stiffness Test Method. Undergraduate Honors Thesis, The Ohio
         State University, 2012.

FIELD OF STUDY

Major Field: Mechanical Engineering
Concentration: Automobile Noise, Vibration, and Harshness; System Dynamics
TABLE OF CONTENTS

ABSTRACT ...................................................................................................................... ii
DEDICATION ................................................................................................................. iii
ACKNOWLEDGEMENTS ................................................................................................. iv
VITA ................................................................................................................................. v
PUBLICATIONS ................................................................................................................ v
FIELD OF STUDY ............................................................................................................ v
TABLE OF CONTENTS ................................................................................................... vi
LIST OF FIGURES ...................................................................................................... viii
LIST OF TABLES .......................................................................................................... xi
LIST OF SYMBOLS ....................................................................................................... xii

1 INTRODUCTION ........................................................................................................... 1
  1.1 Background .............................................................................................................. 1
  1.2 Scope and Objectives ............................................................................................ 3

2 STATIC CHARACTERIZATION ................................................................................... 6
  2.1 Static Experiment Setup ......................................................................................... 6
  2.2 Static Experiment Results ....................................................................................... 7
  2.3 Analysis of Static Data .......................................................................................... 9

3 MODAL CHARACTERIZATION ............................................................................... 13
3.1 Modal Experiment Setup.................................................................13
3.2 Modal Test Results.........................................................................15
3.3 Alternative Contact Models ...........................................................21
4 TRANSIENT CHARACTERIZATION.........................................................30
  4.1 Transient Experiment Setup..........................................................30
  4.2 Transient Experiment Results.........................................................31
  4.3 Application of Alternate Contact Models to Transient Data.............33
5 CONCLUSION ...................................................................................41
  5.1 Summary......................................................................................41
  5.2 Contributions.................................................................................42
  5.3 Recommendations for Future Work...............................................42
REFERENCES .......................................................................................44
LIST OF FIGURES

Figure 1.1: Schematic of Swash Plate (Extracted from Manring (2000)) ......................3
Figure 1.2: Cross-Sectional Representation of Swash Plate Bearing (not to scale) ..........4
Figure 2.1: Fixturing of Pump in Elastomer Test Machine. ........................................7
Figure 2.2: Static Stiffness Curve of Bearing Interface without Oil with
Asymptotic Approximations .................................................................................................8
Figure 2.3: Static Stiffness Curve of Bearing Interface and Asymptotic
Approximations with Oil and without Oil. Key: - - - - , Without Oil; --- , With
Oil. ......................................................................................................................................9
Figure 2.4: Schematic of Elastic Foundation Concept for Illustrative Purpose ..........11
Figure 2.5: Effective Constant Loading for the Static Experiment. Key: ■ ,
Measured Loading; ■■ , Constant Loading Approximation .................................................12
Figure 3.1: Modal Experiment Setup with Accelerometers and Impulse Hammer .........14
Figure 3.2: Illustration of the Original (left) and the Modified (right) Bearings. ............15
Figure 3.3: Two Degree Of Freedom Representation of Modal Experiment .............16
Figure 3.4: Comparison of Measured Transmissibility for Low and High Preload
without Oil. Key: ——— , Low Preload; - - - , High Preload. .............................................17
Figure 3.5: Comparison Transmissibility for Low Preload with and without Oil.
Key: ——— , Without Oil; - - - , With Oil. ........................................................................18
Figure 3.6: Comparison of Transmissibility for Small and Full Sized Bearing with Oil. Key: ——, Full Sized Bearing; ······, Small Bearing.

Figure 3.7: Comparison of Transmissibility for Small and Full Sized Bearing without Oil. Key: , Full Sized Bearing; ······, Small Bearing.

Figure 3.8: Translational (left) and Translational plus Rotational (right) Motions of the Swash Plate due to Modal Excitation. Key: − − − , Initial Shape; , Displace Shape.

Figure 3.9: Two Degree of Freedom Model of Modal Experiment (Viscoelastic Model A).

Figure 3.10: Addition of Viscous Dampers (Representing Oil) to the Elastic Foundation Model of Figure 2.4.

Figure 3.11: Viscoelastic Model B With Structural and Fluid Paths.

Figure 4.1: Illustration of Transient Experiment in Terms of Measured Displacement (Input) and Transmitted Force (Output).

Figure 4.2: Comparison of Measured Transient Data with Static Curves without Oil (left) and with Oil (right). Key: ······, Static; ○ , Displacement Amplitude 1h; ○ , Displacement Amplitude 2h; ○ , Displacement Amplitude 4h.

Figure 4.3: Linear Viscoelastic Models Used to Describe Transient Cases. Model A is the Kelvin-Voigt Formulation; Model B, is a Combination of the Kelvin-
Voigt and Maxwell Formulations; Model C is the Standard Linear Solid

Figure 4.4: Comparison of Transient Model A with Measured Transmitted Force
Given Displacement Amplitude of about 0.02mm without Oil (left) and with Oil
(right). Key: ———, Simulated; ······, Measured.

Figure 4.5: Comparison of Transient Model B with Measured Transmitted Force
Given Displacement Amplitude of About 0.02mm (left) and 0.08mm (right) and
with Oil (right). Key: ———, Simulated; ······, Measured.

Figure 4.6: Comparison of Transient Model C with Measured Transmitted Force
Given Displacement Amplitude of About 0.02mm without Oil (left) and with Oil
(right). Key: ———, Simulated; ······, Measured.
LIST OF TABLES

Table 3.1: Interfacial Conditions for Modal Experiment..................................................14
Table 3.2: Simulation Parameters for Modal Models A and B........................................28
Table 4.1: Conditions of the Six Transient Test Cases......................................................30
Table 4.2: Measured Displacement and Force Values for Transient Experiments............32
Table 4.3: Expressions for Transmitted Force and Transfer Function for Linear Viscoelastic Models of Fig. 4.3 .................................................................35
Table 4.4: Best Fit Parameters for Transient Viscoelastic Models A, B, and C for Three Displacement Amplitudes (1h, 2h, and 4h), with and without Oil ..................39
# LIST OF SYMBOLS

## Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
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## Superscripts

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<td>~</td>
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1 INTRODUCTION

1.1 Background

Swash plate type variable displacement axial piston pumps are common components of off-highway vehicles, this is due in part to having a high power to size ratio and a continuously variable displacement (Eaton, 2013). The kinematics of axial piston pumps have been well studied (Manring, 1999). Figure 1.1 provides a simplified schematic to explain the basic operation of this type of pump. A finite number of pistons are contained by a cylinder block, the pistons have one degree of freedom in the axial x direction. The cylinder block is connected to a shaft and is free to rotate about the x axis. As this rotation occurs the pistons are moved into and out of the cylinder block as they follow along the surface of the swash plate. The angle of the swash plate controls the volumetric displacement of the pump. During operation significant dynamic forces are generated within the pump by the compression of the working fluid.

Simulations of axial piston pumps have been done to study the effect of various design parameters on the performance of the pump (Kim, 2012; Manring, 2013). The focus of these studies has been on the main contributors to structure borne and fluid borne noise. Kim (2012) notes that the structure borne noise is typically attributed to the oscillation of moments acting on the swash plate due to the discrete nature of the pump. The main source of fluid borne noise is the discharge flow ripple again caused by the finite number of pistons (Manring, 2000). A predictive model for the air borne noise
generated by a specific external spur gear motor was developed by Opperwall, and Vacca (2013). Their research concluded that the fluid borne noise was the main contributor to air borne noise, however this study did not consider the bearing performance in the acoustic model.

The development of vibro-acoustic models of hydrostatic transmission systems is a sparse area of research. A specific paper in this area focused on the noise prediction for a nine piston axial swash plate pump (Milind et al., 2010). In this study, the process involved a co-simulation between Adams multibody software (MSC, 2013) and AMESim code (SIEMENS, 2013) to determine the forces acting within the pump. The forces generated were used as input to a finite element model of the pump housing to predict displacements which were then used in a boundary element simulation to predict noise. The conclusion from this study was that there were a few components of the axial piston pump that warranted further study, namely the bearing interface between the swash plate and the pump housing. In the study this interface was treated as a revolute joint, no contact or lubrication models were considered.

Lim and Singh (1989) developed a six dimensional stiffness matrix to describe the vibration transmission through rolling element bearings. They assumed small deflections and no lubrication. The matrix consisted of stiffness elements that were linearized about an operating point and did not include damping. Lim and Singh (1989) used this formulation to calculate the stiffness values from the mean bearing displacements. In contrast, the bearing interface between the swash plate and housing is typically hydrostatic by design. This type of bearing requires a constant pressure source to
maintain the fluid film between the two surfaces (Rowe, 2012). In the event that the bearing is overloaded or sufficient pressure cannot be supplied; solid to solid contact, asperity contact or boundary layer lubrication conditions may exist.

Figure 1.1: Schematic of Swash Plate (Extracted from Manring (2000))

1.2 Scope and Objectives

The primary focus of this study is the bearing interface between the swash plate and the pump housing. This interface is depicted in Figure 1.2. The interface of the specific bearing being tested consists of a steel swash plate with a semi cylindrical cross-section resting on a composite material bearing insert that is positioned between the swash plate and the cylindrical groove of the pump housing. The interface will be characterized with and without lubricating fluid (Exxon Mobil DTE 24 Light Hydraulic
Oil) at ambient temperature. The material properties and geometry of the interface as well as the properties of the lubricating fluid are expected to contribute to the stiffness and damping of the interface, through elastic deformation and fluid flow.

During normal operation of the pump the swash plate bearing undergoes large oscillating loads in the downward radial direction. For this study it will be assumed that the displacement of the swash plate due to these loads does not affect the operation of the pump; thus the interface will be treated as a force transmission element. Only the properties of the bearing in the radial direction will be characterized. Only solid-to-solid contact and boundary layer lubrication will be considered (Bhushan, 2002). Additionally, only lower order viscoelastic models with linear time-invariant stiffness and viscous damping parameters will be considered.

![Cross-Sectional Representation of Swash Plate Bearing (not to scale)](image)

**Figure 1.2: Cross-Sectional Representation of Swash Plate Bearing (not to scale)**

The main goal of this study is to gain a better understanding of the contact mechanisms present in the bearing interface between the swash plate and pump housing of a swash plate type axial piston pump. Specific objectives include the following. 1.
Design several experiments to measure the interfacial properties under static (force vs. deflection), modal (motion transmissibility), and transient (step-like) conditions and characterize the interface under various loading and interfacial conditions to examine the effects of mean load, lubrication, and excitation amplitude. 2. Propose tractable viscoelastic contact models (a combination of Kelvin-Voigt and Maxwell formulations containing minimal linear time-invariant stiffness and damping elements) to explain the experimental results (Findley et al., 1989; Cyril, 1996). 3. Evaluate the viscoelastic contact models by comparison with the experimental results and suggest order of magnitude values of stiffness and damping elements with (or without) the oil within the interface under certain mean loads.
2 STATIC CHARACTERIZATION

2.1 Static Experiment Setup

An experiment is designed with the main goal of characterizing the properties of the swash plate bearing interface under quasi-static conditions. Due to the high stiffness of the interface as well as the large forces that would normally be acting on the swash plate the MTS 831.50 elastomer testing machine is selected for this experiment (MTS, 2013). The MTS 831.50 offers a high rate displacement transducer and load cell for data collection. The load cell in the machine is rated up to a maximum tensile and compressive force of 5000 N. The components of the pump are fixtured such that the swash plate is connected to the machine's displacement actuator and the pump housing is connected to the machine's load cell as shown by Figure 2.1.

The static experiment consists of a slow ramp up to 4000 N compressive force followed by a slow ramp down back to 0 N, the experiment is conducted once without oil and once with oil.
2.2 Static Experiment Results

The force-displacement curve of the bearing with no oil for the static test is shown in Figure 2.2. A nonlinear stiffening trend can be observed in both the loading and unloading direction. Under low load the asymptotic stiffness values in the loading and unloading directions are approximately 10 kN/mm. As load is increased the asymptotic stiffness increases to 50 and 100 kN/mm in the loading and unloading directions respectively.

Figure 2.3 shows the static force displacement curve for the interface with and without oil. The interface with oil still exhibits a stiffening nonlinearity and there is no discernible change to the area between the loading and unloading curves. It is also worth
noting that the low and high load asymptotic stiffness values agree well with the case without oil. However, there is an increase in the displacement of the swash plate under maximum load due to the oil.

Figure 2.2: Static Stiffness Curve of Bearing Interface without Oil with Asymptotic Approximations
2.3 Analysis of Static Data

The equation for Hertzian line contact is used to provide insight into the stiffness under lightly loaded conditions (Johnson, 1985). The contact stiffness \( k \) is a function of the effective elastic modulus \( E^* \) and the effective bearing width \( w \).

\[
k = \frac{\pi}{4} E^* w
\] (2.1)

The effective elastic modulus \( E^* \) is determined from the elastic modulus \( E \) and Poisson's ratio \( \nu \) of the two contacting surface materials steel (swash plate) and Teflon (bearing liner); \( E_{\text{steel}}, E_{\text{Teflon}}, \nu_{\text{steel}}, \nu_{\text{Teflon}} \) (Johnson, 1985).
The asymptotic stiffness at the beginning of the loading curve gives a stiffness near 10 kN/mm (Fig. 2.2). Assuming a bearing width \( w \) of 20-40 mm, elastic modulus for steel and Teflon \( (E_{steel} \text{ and } E_{Teflon}) \) of 207 GPa and 500 MPa respectively, and a Poisson's ratio for steel and Teflon \( (\nu_{steel} \text{ and } \nu_{Teflon}) \) of 0.3 and 0.45 respectively, Equations (2.1) and (2.2) predict a contact stiffness of 10-20 kN/mm for the lightly loaded case. It is possible to approximate the stiffness of the interface under light loading conditions using the linear Equation (2.1), however this equation does not explain the observed nonlinearity of the stiffness curve.

A hardening stiffness nonlinearity can often be attributed to asperity contact or non-conforming geometry (Johnson, 1985). It may be approximated using an elastic foundation model as shown in Figure 2.4. The elastic foundation model consists of a bed with a finite number of stiffness elements and a rigid indenter with a profile defined by the gap between the two contacting surfaces. As the rigid indenter moves into the foundation the number of springs in contact will increase, resulting in a higher stiffness. This is analogous to an increase in contact area due to non-conforming geometry, asperities, or a combination of both.
When comparing the stiffness curves in Figure 2.3, there is a noticeable difference between the displacements of the two curves under the maximum force. This difference of approximately 7.5 μm is explained conceptually by a special formulation of the Reynolds equation for squeeze film bearings (Bhushan, 2002) where $\Delta t$ is the change in time, $\mu$ is the dynamic viscosity, $l$ is the effective length of the bearing, $F$ is the mean load acting on the bearing, and $\delta_i$ and $\delta_f$ are the initial and final film thicknesses respectively.

$$
\Delta t = \frac{\mu \nu^2 l}{2F} \left[ \frac{1}{\delta^2_f} - \frac{1}{\delta^2_i} \right]
$$

Equation (2.3) describes the change in film thickness over a given time due to a constant loading, but the experiment involves a constantly changing force. To represent the time-dependent force ($F(t)$) used in the experiment, we consider an effective constant force with the same area under the force-time curve over the same time period, as shown in Figure 2.5. A change in time of 2.0 seconds, a constant force of 2000 N, an initial film thickness of 10 μm, a dynamic viscosity of 0.0197 Ns/m² (Exxon, 2013), an
effective bearing length of 124.5 mm, and an effective bearing width of 20.3 mm are utilized in Equation (2.3) to predict an initial film thickness of 8.4 μm.

![Graph showing F(t) vs. t][1]

**Figure 2.5: Effective Constant Loading for the Static Experiment.** Key: ■, Measured Loading; □, Constant Loading Approximation.

Equation (2.3) shows that the fluid film thickness in a bearing of this type will be reduced under a constant load over a finite amount of time, but it will never reach zero. This decrease in film thickness over time due to force would explain the displacement difference between the two measured curves seen in Figure 2.3.
3 MODAL CHARACTERIZATION

3.1 Modal Experiment Setup

A modal experiment was designed to understand the dynamic properties of the interface, a schematic is shown in Figure 3.1. The experiment uses six accelerometers (PCB356A15 with 100 mV/g sensitivity (PCB, 2013)) to record the accelerations at three points on the swash plate and three positions on the pump housing. An impulse hammer (PCB086C03 with 2.25 mV/N sensitivity (PCB, 2013)) is used to excite the system by striking the center of the swash plate in the positive x direction. Data is acquired with a multi-channel digital system (SCADAS III system and TestLab 13A (SIEMENS, 2013)).

The structure of the pump housing is modified to eliminate predicted elastic deformation (flexural) modes in the frequency range of interest. This allows the swash plate and pump housing to be treated as rigid bodies when developing lower order representative dynamic models. Because of the high stiffness of the interface, it is not possible to collect accurate modal data with the housing placed on a rigid base. Therefore, the modal experiment requires the housing to be placed on a compliant base.
The modal experiment is performed under 8 interfacial conditions. The preload, lubrication, and bearing size are varied as listed in Table 3.1.

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Figure 3.2 compares the original (full) bearing to the modified (small) bearing.

The arc length at the point of contact between the swash plate and bearing is
approximately 124 mm for the full bearing and 25 mm for the small. The width into the page is the same for both bearing sizes.

![Figure 3.2: Illustration of the Original (left) and the Modified (right) Bearings.](image)

3.2 Modal Test Results

Motion transmissibility will be used to compare the modal experiment cases. A two degree of freedom system used to represent the first two modes of experimental setup is shown in Figure 3.3. Where $\tilde{F}$ is the dynamic force applied by the impulse hammer, $m_s$ and $m_h$ are the masses of the swash plate and housing, $x_s$ and $x_h$ are the displacements of the swash plate and housing, $\tilde{k}_f$ is the unknown complex stiffness of the bearing interface (at a given frequency) that will be represented by viscoelastic models in future sections, and $\tilde{k}_r$ is the complex stiffness of the compliant base (at a given frequency).
The motions of the swash plate and housing are measured by the accelerometers and only the transmissibility magnitude will be considered. The complex-valued transmissibility ($T_{s,h}$) is defined as the amplitude of the swash plate motion ($\tilde{X}_s$) over the amplitude of the housing motion ($\tilde{X}_h$) at a given frequency.

$$T_{s,h}(\omega) = \frac{\tilde{X}_s}{\tilde{X}_h} \quad (3.1)$$

Figure 3.4 shows the measured motion transmissibility magnitude spectra between the swash plate and housing for modal cases 1 and 2. This comparison examines the effect of increasing preload on the interface. A preload of 26.4 N results in a peak frequency of 145 Hz, and a preload of 44.4 N results in a higher peak frequency of 213 Hz. These frequencies are verified to correspond to the motion of the swash plate and housing in the x-direction through observation of the recorded mode shapes. Since the
masses of the system remain unchanged, the increase in peak frequency suggests an increase in interfacial stiffness. An increase in stiffness with an increased load agrees with the results of static experiment.

Figure 3.4: Comparison of Measured Transmissibility for Low and High Preload without Oil. Key: , Low Preload; , High Preload.

Comparison of the low preload case with and without oil is shown in Figure 3.5. The introduction of a viscous fluid results a motion transmissibility much closer to 1.0 over the entire frequency range recorded. The oil is acting as an efficient transmitter of force over the range shown. The peak of the transmissibility occurs at approximately 145 Hz without oil and 600 Hz with oil. The oil appears to be effectively increasing the stiffness of the bearing under modal conditions.
Figure 3.5: Comparison Transmissibility for Low Preload with and without Oil.

Key: —, Without Oil; ——, With Oil.

Figure 3.6 shows the motion transmissibility between the swash plate and housing for both the small and full bearing sizes with oil. The peak of the motion transmissibility with the small bearing is more pronounced and occurs at a lower frequency (~400 Hz) than the full sized bearing (~600 Hz). Decreasing the size of the bearing controls the maximum contact area between the two surfaces and reduces the effective stiffness under modal conditions. The full bearing case with oil again shows values closer to 1.0 over the entire frequency range sampled than the small bearing case with oil, showing that with the reduction in bearing area there is also less damping in the interface.

The smaller bearing was also tested without oil, a comparison between the transmissibility of the small and full bearings without oil can be seen in Figure 3.7.
Unlike the tests with oil, there is no significant change in transmissibility peaks. There is however a noticeable difference in the amplitudes at which the peaks occur. This amplitude difference suggests that the outer portions of the bearing that have been removed have a damping effect in the interface.

Figure 3.6: Comparison of Transmissibility for Small and Full Sized Bearing with Oil. Key: , Full Sized Bearing; , Small Bearing.
Figure 3.7: Comparison of Transmissibility for Small and Full Sized Bearing without Oil. Key: —, Full Sized Bearing; —, Small Bearing.

In Figures 3.4 through 3.7 additional peaks are observed in the transmissibility magnitude spectra, typically in the 600 to 900 Hz range. Since multiple accelerometers are used on both the swash plate and housing it is possible to observe the mode shapes at each of the frequencies. The mode shapes corresponding to the second frequency peaks show coupled motion (in translational and rotational degrees of freedom), as conceptually displayed in Figure 3.8. The additional peaks corresponding to the coupled translational and rotational motions are not considered, because this study is only concerned with the translational motion in the x-direction of the swash plate and housing.
3.3 Alternative Contact Models

A two degree of freedom system shown in Figure 3.9 is used to simulate the modal experiment. The bearing interface is initially modeled with the Kelvin-Voigt formulation with stiffness \( k_{A1} \) and viscous damping \( c_{A1} \) connecting the swash plate and housing (designated Model A). The masses of the swash plate and housing are represented by \( m_s \) and \( m_h \). The motion of the swash plate and housing is given by \( x_s(t) \) and \( x_h(t) \). The housing rests on a compliant rubber base with stiffness \( k_r \) and structural damping coefficient \( \eta \). The input is a sinusoidal force of amplitude \( A \) at a circular frequency \( \omega \) [rad/s].
Figure 3.9: Two Degree of Freedom Model of Modal Experiment (Viscoelastic Model A)

The equations of motion for the swash plate and housing masses can be written in the time domain:

\[ m_s \ddot{x}_s = -k_{A11}(x_s - x_h) - c(A1_s - A1_h) + F \]  \hspace{1cm} (3.2)

\[ m_h \ddot{x}_h = k_{A11}(x_s - x_h) + c(A1_s - A1_h) - k_c x_h \]  \hspace{1cm} (3.3)

The compliant base is represented by a linear stiffness \( k_c \) in the time domain.

After taking the Laplace transform of equations (3.2) and (3.3) with zero initial conditions, the resulting equations (in Laplace domain) can be rearranged in matrix form.

\[
\begin{bmatrix}
    m_s s^2 + c_{A1} s + k_{A11} & c_{A1} s - k_{A11} \\
    -c_{A1} s - k_{A11} & m_h s^2 + k_c
\end{bmatrix}
\begin{bmatrix}
    X_s(s) \\
    X_h(s)
\end{bmatrix}
= \begin{bmatrix}
    F(s) \\
    0
\end{bmatrix}
\]  \hspace{1cm} (3.4)

Conversion to the frequency domain is accomplished by substituting \( j \omega \) for \( s \) in equation (3.4) where \( j \) is the imaginary number. Stiffness \( k_c \) is also replaced with the
complex-valued $k_r(1 + j \eta)$ term to allow for hysteretic damping of the compliant base material to be considered.

$$\begin{bmatrix}
-m_s \omega^2 + j \alpha \omega c_{11} + k_{11} & -j \alpha \omega c_{11} - k_{11} \\
-j \alpha \omega c_{11} - k_{11} & -m_h \omega^2 + k_r(1 + j \eta)
\end{bmatrix}
\begin{bmatrix}
X_s(j \omega) \\
X_h(j \omega)
\end{bmatrix}
= \begin{bmatrix}
F(j \omega) \\
0
\end{bmatrix}$$

(3.5)

For Model A, a heuristic method is used to determine the stiffness and damping properties of the interface ($k_{11}$ and $c_{11}$). The stiffness and damping parameters are varied and the motion transmissibility between the swash plate and housing ($T_{s,h}$) is calculated using equation (3.5) (Mathworks, 2013). The numerically simulated transmissibility can then be compared to the experimental results until a reasonable agreement is achieved. The stiffness and damping values ($k_{11}$ and $c_{11}$) that match the model prediction to the experimental measurements are listed in Table 3.2. Observe a good agreement between the estimated stiffness values without the oil (4.5 and 10 kN/mm) and the lower asymptote of the measured static stiffness curve (~10 kN/mm).

However, Model A does not provide an explanation for the observed increase in stiffness with oil. The elastic foundation model in Figure 2.4 will be considered to explain the effect oil has on the interface. The oil fills in the gaps where there was previously a separation between the two contacting surfaces. This is shown in Figure 3.10, where the viscous dampers are used to represent the oil.
The elastic foundation model shown in Figure 3.9 is used along with the Reynolds equation to determine what effect the oil may have on the interface. A single Maxwell element, consisting of a viscous damper ($c_e$) in series with a stiffness ($k_e$), is considered. The stiffness of the individual elements ($k_e$) will be determined by $k_e = \frac{k_{\text{max}}}{n}$, where $k_{\text{max}}$ is the maximum stiffness observed during static testing, 100 kN/mm, and $n$ is the total number of stiffness elements in the foundation.

The damping for each element ($c_e$) will be determined using the simplified Reynolds equation with the following assumptions: flow only occurs in the $z$ direction (Figure 3.10), the only motion between the surfaces is in the $x$ direction, density is time invariant, and density, dynamic viscosity, and film thickness are constant along the width of the bearing (Hamrock, 1991).

$$\frac{\partial^2 p(z)}{\partial z^2} = \frac{12\mu}{\delta(t)^3} \frac{\partial \delta(t)}{\partial t}$$  \hspace{1cm} (3.6)
Where $p$ is the fluid film pressure, $\mu$ is the dynamic viscosity of the lubricating film, and $\delta$ is the film thickness. To simplify the integration the right hand side of Equation (3.6) is replaced by $\dot{\lambda}(t)$.

$$\frac{12\mu}{\delta(t)^3} \frac{\partial \delta(t)}{\partial t} = \dot{\lambda}(t) \quad (3.7)$$

After substituting Equation (3.7) into Equation (3.6) the integral with respect to $z$ is taken, and appropriate boundary conditions are applied $p(z = 0) = 0$ and $p(z = w) = 0$ yielding:

$$p(z) = c_0 + c_1 z + \frac{\dot{\lambda}(t)}{2} z^2 = \frac{\dot{\lambda}(t)}{2} z^2 - \frac{\lambda(t)w}{2} z \quad (3.8)$$

The force ($F(t)$) due to $p(z)$ is found by integrating the pressure over the bearing width ($w$) and multiplying by the length of each foundation element ($l_c$). Substituting Equation (3.7) back into Equation (3.8) and letting $\partial \delta(t) / \partial t = \ddot{\delta}(t)$ gives:

$$F(t) = l_c \int_0^w p(z)dz = \frac{\dot{\lambda}(t)l_cw^3}{12} = \frac{\mu l_cw^3}{\delta(t)^3} \ddot{\delta}(t) \quad (3.9)$$

If the change in film thickness ($\dot{\delta}(t)$) is equivalent to the velocity of the swash plate ($\dot{x}(t)$), Equation (3.9) effectively provides a viscous damping term ($c_v$) that is heavily dependent on the film thickness ($\delta(t)$).

$$c_v = \frac{F(t)}{\delta(t)} = \frac{\mu l_cw^3}{\delta(t)^3} \quad (3.10)$$

For the modal test performed it is assumed that the film thickness has reached a nearly steady level and the excitation of the system results in small motions about this
mean film thickness level. As such, the time dependent film thickness \( (\delta(t)) \) is assumed to be a constant value given by the mean thickness \( (\bar{\delta}) \). The length of a single element \( (l_e) \) is found by \( l_e = l/n \).

The single element stiffness and damping terms \( (k_e \text{ and } c_e) \) can be used to predict the break-point frequency \( (\omega_{br}) \) of the Maxwell element.

\[
\omega_{br} = \frac{k_e}{c_e} = \frac{k_{\max} \bar{\delta}_m^3}{lw^3 \mu}
\]  

(3.11)

At frequencies greater than the break frequency, the damper of the Maxwell element will become nearly rigid. Using Equation (3.11) and assuming a maximum stiffness \( (k_{\max}) \) of 100 kN/mm, mean film thickness \( (\bar{\delta}_m) \) of 10 \( \mu \)m, effective bearing length \( (l) \) of 100 mm, effective bearing width \( (w) \) of 20 mm, and dynamic viscosity \( (\mu) \) of 0.0197 Ns/m\(^2\), a break-point frequency \( (\omega_{br}) \) of 6.35 rad/s is predicted. The low break-point frequency would explain the observed increase in stiffness as the oil is acting as an extremely efficient transmitter of force at any frequency above 1 Hz. The oil increases the effective contact area of the interface, which leads to a higher stiffness.

In the elastic foundation model with oil, two unique paths are expected. The first path is the structural contact path (with stiffness and damping \( (k_{B1} \text{ and } c_{B1}) \)) and the second is the fluid path consisting of viscous damper and stiffness elements in series ((\( k_{B2} \text{ and } c_{B2} \)) to account for the addition of oil. The simplified model is shown in
Figure 3.11, where a combined Maxwell and Kelvin-Voigt model is used to represent these paths. The compliant rubber base and masses are the same as in Model A.

Like Model A, the governing equations in frequency domain for Model B are represented in matrix form.

\[
\begin{bmatrix}
-m_a \omega^2 + j \alpha B_1 + k B_1 + k B_1 & -k_{B_21} & -j \alpha B_1 - k B_1 \\
-k_{B_21} & -m_a \omega^2 + j \alpha B_21 + k B_21 & -j \alpha B_21 \\
-j \alpha B_1 - k B_1 & -j \alpha B_21 & -m_h \omega^2 + k_h (1 + j \eta)
\end{bmatrix}
\begin{bmatrix}
X_s \\
Y_2 \\
X_h
\end{bmatrix} = \begin{bmatrix}
F_a \\
0 \\
0
\end{bmatrix} \quad (3.12)
\]

Equation (3.12) is solved similarly to Model A and the simulation parameters for Model B for the modal tests are found in Table 3.2.
Table 3.2: Simulation Parameters for Modal Models A and B

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Case Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model A: Kelvin-Voigt</td>
<td>$k_{A1}$ [kN/mm]</td>
<td>4.5</td>
<td>10</td>
<td>120</td>
<td>140</td>
<td>19</td>
<td>19</td>
<td>100</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>$c_{A1}$ [Ns/mm]</td>
<td>3.2</td>
<td>5.5</td>
<td>35</td>
<td>40</td>
<td>6</td>
<td>2.1</td>
<td>60</td>
<td>17</td>
</tr>
<tr>
<td>Model B: Structural and Fluid</td>
<td>$k_{B11}$ [kN/mm]</td>
<td>4.5</td>
<td>10</td>
<td>4.5</td>
<td>10</td>
<td>19</td>
<td>19</td>
<td>-</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>$c_{B11}$ [Ns/mm]</td>
<td>3.2</td>
<td>5.5</td>
<td>3.2</td>
<td>5.5</td>
<td>6</td>
<td>2.1</td>
<td>-</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>$k_{B21}$ [kN/mm]</td>
<td>-</td>
<td>-</td>
<td>110</td>
<td>120</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>$c_{B21}$ [Ns/mm]</td>
<td>-</td>
<td>-</td>
<td>32</td>
<td>32</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>20</td>
</tr>
</tbody>
</table>

A few conclusions are drawn from the simulation parameters of Table 3.2. For Model A, it is seen by comparing cases 1 and 2 that an increase in bearing mean load (from 26.4 to 44.4 N) results in an increase in interfacial stiffness ($k_{A1}$) from 4.5 to 10 kN/mm. This increase in stiffness with mean load is consistent with the stiffening nonlinearity in the quasi-static experiment (Fig. 2.2). For Model A, the addition of oil, as seen by comparing cases 1 and 3, results in a stiffness parameter ($k_{A1}$) significantly higher, 120 kN/mm, than the case without oil, 4.5 kN/mm. The effect of bearing area can be considered by comparing the parameters for cases 3 and 8, both cases are conducted with the smaller mean load (26.4 N) with oil. By decreasing the size of the bearing, Model A stiffness and damping parameters ($k_{A1}$ and $c_{A1}$) decrease from 120 kN/mm and 35 Ns/mm to 45 kN/mm and 17 Ns/mm. Also, the parameters of the Maxwell model in Model B ($k_{B11}$ and $c_{B11}$) decrease from 110 kN/mm and 32 Ns/mm to 35 kN/mm and
20 Ns/mm. The effect of oil on Model A or B is reduced when considering a bearing with lower (total) contact area.
4 TRANSIENT CHARACTERIZATION

4.1 Transient Experiment Setup

Transient experiments are conducted with the same setup (MTS 831.50) as the static experiment. The swash plate was attached to a displacement actuator, and the housing to a load cell.

In order to characterize the interface under a different type of loading condition, a step-like input is chosen. As opposed to the small motions about an operating point that the modal experiment considers, the transient experiment focuses on a rapidly changing mean load with much larger displacements. A total of six dynamic tests are run, one set of three tests (cases 1, 2, and 3) without any oil in the interface and another set of three tests (cases 4, 5, and 6) with oil in the interface. The lubricant and approximate displacements for each case are shown in Table 4.1. Only the data in the loading direction will be considered for this experiment.

<table>
<thead>
<tr>
<th>Transient Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lubricant</td>
<td>None</td>
<td>None</td>
<td>None</td>
<td>Oil</td>
<td>Oil</td>
<td>Oil</td>
</tr>
<tr>
<td>Approximate Displacement [mm]</td>
<td>0.02</td>
<td>0.04</td>
<td>0.08</td>
<td>0.02</td>
<td>0.04</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 4.1: Conditions of the Six Transient Test Cases.
4.2 Transient Experiment Results

Figure 4.1 shows a typical curve for the transient experiment. Each run consists of a displacement controlled step-like loading event followed by a displacement hold. The measured displacement ($x(t)$) is characterized by $x_m$ (mean displacement level), $x_a$ (displacement amplitude of the loading event), and $\Delta t$ (duration of loading). The measured force ($F(t)$) is characterized by $F_m$ (force corresponding to the mean displacement), $F_a$ (force corresponding to the change in displacement), and $F_{peak}$ (maximum force overshoot). Measured displacement and force values are summarized in Table 4.2 for all of the transient tests.

![Figure 4.1: Illustration of Transient Experiment in Terms of Measured Displacement (Input) and Transmitted Force (Output)]
Table 4.2: Measured Displacement and Force Values for Transient Experiments

<table>
<thead>
<tr>
<th>Case</th>
<th>$x_m$ [μm]</th>
<th>$x_a$ [μm]</th>
<th>$Δt$ [ms]</th>
<th>$F_m$ [N]</th>
<th>$F_a$ [N]</th>
<th>$F_{peak}$ [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-56</td>
<td>-23</td>
<td>62.5</td>
<td>-104</td>
<td>-171</td>
<td>-330</td>
</tr>
<tr>
<td>2</td>
<td>-57</td>
<td>-46</td>
<td>57.5</td>
<td>-103</td>
<td>-432</td>
<td>-592</td>
</tr>
<tr>
<td>3</td>
<td>-63</td>
<td>-80</td>
<td>61.3</td>
<td>-101</td>
<td>-1578</td>
<td>-1628</td>
</tr>
<tr>
<td>4</td>
<td>-77</td>
<td>-20</td>
<td>65.7</td>
<td>-149</td>
<td>-199</td>
<td>-462</td>
</tr>
<tr>
<td>5</td>
<td>-72</td>
<td>-43</td>
<td>64.8</td>
<td>-117</td>
<td>-488</td>
<td>-768</td>
</tr>
<tr>
<td>6</td>
<td>-54</td>
<td>-90</td>
<td>63.5</td>
<td>-93</td>
<td>-1266</td>
<td>-1409</td>
</tr>
</tbody>
</table>

Figure 4.2 compares mean and amplitude values from the six transient cases with the respective static force-displacement curves. The plots show a good correlation between the static and dynamic tests. The data points representing the dynamic cases follow the nonlinear static curves closely.

**Figure 4.2: Comparison of Measured Transient Data with Static Curves without Oil (left) and with Oil (right). Key: ------ , Static; ○ , Displacement Amplitude 1h; ○ , Displacement Amplitude 2h; ○ , Displacement Amplitude 4h.**
4.3 Application of Alternate Contact Models to Transient Data

The modal experiment provided justification for the use of alternate contact models to account for the effect of oil on the interface. Similar models, shown in Figure 4.3, will be used to evaluate the transient data. Transient Model A is based on the Kelvin-Voigt model, Model B is a combination of a Maxwell and Kelvin-Voigt model, and transient Model C is the standard linear solid model. To simulate the experimental test conditions the viscoelastic models are excited by a step-like input displacement \( x(t) \) like that shown in Figure 4.1 and the force transmitted into the base \( F(t) \) is calculated.

![Figure 4.3: Linear Viscoelastic Models Used to Describe Transient Cases. Model A is the Kelvin-Voigt Formulation; Model B, is a Combination of the Kelvin-Voigt and Maxwell Formulations; Model C is the Standard Linear Solid Formulation.](image)

Model A (Kelvin-Voigt) consists of a spring (with stiffness \( k_{A1} \)) in parallel with a viscous damper (with damping \( c_{A1} \)). The force transmitted to the base is:
\[ F(t) = k_{B1}x(t) + c_{B1}\dot{x}(t) \]  

(4.1)

Model B, from Figure 3.1, consists of a Maxwell model in parallel with a Kelvin-Voigt model. The first path consists of a stiffness parameter \( k_{B1} \) in parallel with a viscous damper \( c_{B1} \). The second path consists of a stiffness \( k_{B21} \) connected in series to a viscous damper with coefficient \( c_{B21} \) by the negligible mass \( m_{B2} \). The force transmitted to the base of Model B is:

\[ F(t) = k_{B1}x(t) + c_{B1}\dot{x}(t) + c_{B21}\dot{y}_2(t) \]  

(4.2)

Where \( \dot{y}_2(t) \) is the first derivative of displacement of the intermediate mass \( m_{B2} \).

The differential equation governing the motion of the intermediate masses of the second path \( m_{B2} \) is found to be:

\[ m_{B2}\ddot{y}_2(t) = k_{B21}(x(t) - y_2(t)) - c_{B21}\dot{y}_2(t) \]  

(4.3)

The Laplace transform with zero initial conditions is taken for Equations (4.2) and (4.3) and rearranged to yield:

\[ F(s) = X(s)\left[k_{B1} + c_{B1}s\right] + Y(s)c_{B21}s \]  

(4.4)

\[ Y(s)\left[m_{B2}s^2 + c_{B21}s + k_{B21}\right] = X(s)k_{B21} \]  

(4.5)

Equation (4.5) is used in Equation (4.4) to give the transfer function (in Laplace domain) between transmitted force and measured displacement for Model B.

\[ \frac{F(s)}{X(s)} = \frac{c_{B1}m_{B2}s^3 + [c_{B1}c_{B21} + k_{B1}m_{B2}]s^2 + [c_{B1}k_{B21} + k_{B1}c_{B21} + k_{B21}c_{B21}]s + k_{B1}k_{B21}}{m_{B2}s^2 + c_{B21}s + k_{B21}} \]  

(4.6)

Table 4.3 lists the transmitted force expression in the time domain and the Laplace domain transfer function \( F(s)/X(s) \). The transmitted force and transfer
function for Model C (of Figure 4.3) is found in a similar manner to Model B and is also shown in Table 4.3.

Table 4.3: Expressions for Transmitted Force and Transfer Function for Linear Viscoelastic Models of Fig. 4.3

<table>
<thead>
<tr>
<th>Model</th>
<th>Transmitted Force $F(t)$</th>
<th>Transfer Function $F(s)/X(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model A (Fig. 4.3)</td>
<td>$F(t) = k_Ax(t) + c_A\dot{x}(t)$</td>
<td>$\frac{F(s)}{X(s)} = k_{A11} + c_{A11}s$</td>
</tr>
<tr>
<td>Model B (Fig. 4.3)</td>
<td>$F(t) = k_{B1}x(t) + c_{B1}\dot{x}(t) + c_{B2}\ddot{x}(t)$</td>
<td>$\frac{F(s)}{X(s)} = \frac{c_{B1}m_{B2}s^2 + [c_{B1}c_{B2} + k_{B1}m_{B2}]s}{m_{B1}s^2 + c_{B2}s + k_{B21}}$</td>
</tr>
<tr>
<td>Model C (Fig. 4.3)</td>
<td>$F(t) = k_{C1}x(t) + c_{C2}\ddot{x}(t)$</td>
<td>$\frac{F(s)}{X(s)} = \frac{k_{C1}m_{C2}s^2 + [k_{C1}c_{C21} + k_{C21}c_{C21}]s}{m_{C2}s^2 + c_{C2}s + k_{C21}}$</td>
</tr>
</tbody>
</table>

To simulate Model A, the displacement vector measured during the experiment $x(t)$ and its derivative $\dot{x}(t)$, which is found numerically, are used to solve for the transmitted force using Equation (4.1). The force transmitted through Model B is solved as the sum of the transmitted force of the Maxwell and Kelvin-Voigt models. To calculate the force transmitted by the Maxwell model, Equation (4.3) is solved numerically with $x(t)$ known. The numerically determined velocity $\dot{x}(t)$, the measured displacement $x(t)$, and its derivative $\ddot{x}(t)$ are then used to solve for the force transmitted by Model B using the appropriate equation from Table 4.3.
A comparison between transient Model A and experiment is shown in Figure 4.4 given a displacement amplitude of approximately 1\( h \) (0.02mm) with and without oil. The parameters used for these simulations can be found in Table 4.4. The Stiffness \( k_{11} \) is chosen such that the value of the simulation matches the measured force after the loading event; the viscous damping term \( c_{11} \) is chosen so that the amplitude of the force peak during the loading event matches between simulated and measured data. The simulation of Model A offers a poor approximation of the measured force in the time domain. The force peak during the loading event has the correct magnitude value but occurs earlier in time in the simulation and is narrower than measured during the experiment. With Model A it is not possible to alter the time of occurrence, or width of the force peak. Additional elements are required to more accurately capture the viscoelastic behavior during the loading event.

![Figure 4.4: Comparison of Transient Model A with Measured Transmitted Force Given Displacement Amplitude of about 0.02mm without Oil (left) and with Oil (right). Key: \textcolor{red}{- - - - - - -}, Simulated; \textcolor{black}{- - - - - - -}, Measured.](image-url)
A comparison between transient Model B and experiment is shown in Figure 4.5 given a displacement amplitude of approximately $1h$ (0.02 mm) and $4h$ (0.08 mm) with oil. Simulation parameters for the cases with oil are in Table 4.4. Similar to modal Model B, the Kelvin-Voigt model of Model B is considered as the structural path. The stiffness ($k_{B1}$) is again chosen to match the force after the loading event, and the viscous damping term ($c_{B1}$) for the displacement amplitude $1h$, $2h$, and $4h$ cases with oil are equal to ($c_{A1}$) for the $1h$, $2h$, and $4h$ cases without oil respectively. The Maxwell model represents the oil path and its parameters ($k_{B2}^{1}$ and $c_{B2}^{1}$) are found heuristically to achieve a reasonable fit of the force peak. Model B provides an accurate representation of the viscoelastic behavior of the interface under transient loading for the lowest displacement case. However, under the largest transient displacement considered ($4h$), it is clear that the force after the loading event is not accurately modeled. This is due to the nonlinear stiffness of the interface that is observed during the static experiment (Fig. 2.2).
A comparison between the transient Model C and experiment is shown in Figure 4.6 given a displacement amplitude of approximately \( h \) (0.02 mm) with and without oil. Model C is considered to reduce the number of parameters required to fit the measured force data. Like Models A and B, stiffness \( k_{c1} \) is chosen to match the force value after the loading event, the stiffness and damping of the Maxwell model \( (k_{c21} \text{ and } c_{c21}) \) are then varied until the best fit of the peak force is obtained. From Figure 4.6, it is clear that this model provides a better fit of the measured force time history in the absence of oil. However, the fit in the presence of oil is not as good as predicted by Model B (Fig. 4.5). The simulated force shown for Model C with oil relaxes too quickly. This is better simulated by Model B (Fig. 4.5) because it has two time constants unlike Model C (Fig. 4.6) that has only has one time constant (Findley et al., 1989).
Figure 4.6: Comparison of Transient Model C with Measured Transmitted Force Given Displacement Amplitude of About 0.02mm without Oil (left) and with Oil (right). Key: —, Simulated; ……., Measured.

Table 4.4: Best Fit Parameters for Transient Viscoelastic Models A, B, and C for Three Displacement Amplitudes (1h, 2h, and 4h), with and without Oil.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Displacement Amplitude Without Oil</th>
<th>Displacement Amplitude With Oil</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1h  2h  4h</td>
<td>1h   2h  4h</td>
</tr>
<tr>
<td>A</td>
<td>$k_{A1}$</td>
<td>7360 9400 19000</td>
<td>9400 11000 13000</td>
</tr>
<tr>
<td></td>
<td>$c_{A1}$</td>
<td>203   180  180</td>
<td>360   330  300</td>
</tr>
<tr>
<td>B</td>
<td>$k_{B11}$</td>
<td>-     -   -</td>
<td>9400 11000 13000</td>
</tr>
<tr>
<td></td>
<td>$c_{B11}$</td>
<td>-     -   -</td>
<td>203   180  180</td>
</tr>
<tr>
<td></td>
<td>$k_{B21}$</td>
<td>-     -   -</td>
<td>6500  5000  3000</td>
</tr>
<tr>
<td></td>
<td>$c_{B21}$</td>
<td>-     -   -</td>
<td>535   300  180</td>
</tr>
<tr>
<td>C</td>
<td>$k_{C11}$</td>
<td>7360  9860 19400</td>
<td>9400  11330 13910</td>
</tr>
<tr>
<td></td>
<td>$k_{C21}$</td>
<td>7360  3974  956</td>
<td>13984 10740  4640</td>
</tr>
<tr>
<td></td>
<td>$c_{C21}$</td>
<td>187   99   99</td>
<td>426   310  150</td>
</tr>
</tbody>
</table>
Based on the best fit parameters for each case, the stiffness values of $k_{A1}$, $k_{B1}$, and $k_{C1}$ are very similar. As the transient excitation displacement amplitude ($h$) increases, so do these stiffness values. This increase is in agreement with the results from the static experiment and can be explained by an increase in contact area due to a higher load. The damping terms ($c_{A1}$ and $c_{B1}$) as well as the parameters of the Maxwell models in Models B and C ($k_{B21}, c_{B21}$, and $k_{C21}, c_{C21}$) all generally decrease with an increase in displacement amplitude ($h$). As seen in the modal experiment, this reduction in damping can be achieved by reducing the total bearing area. While the total geometric bearing area does not change during this experiment, the effective force area of the bearing that is contributing to the structural stiffness is increasing; this suggests a smaller area over which the additional contact mechanisms can act.

The differences between the stiffness parameters ($k_{A1}$) and ($k_{C1}$) with and without oil may simply be caused by slight differences between the initial (mean) loading on the bearing and the differences in actual displacement amplitudes. When combined with the nonlinear stiffness properties of the interface, these small differences could lead to a seemingly large effect. Nonetheless the increase in damping parameters ($c_{A1}$) and ($c_{C1}$) between cases with and without oil can only be attributed to the dissipative property of oil within the interface.
5 CONCLUSION

5.1 Summary

This thesis documents the development of three experiments to characterize the swash plate bearing interface of a variable displacement axial piston pump under various conditions. A quasi-static experiment is designed to capture the behavior of the interface under slowly increasing loads. The modal experiment is designed to characterize the bearing interface under low load, small displacement conditions. The transient experiment is designed to characterize the interface under higher loads and much larger displacements than the modal experiment. All experiments are considered with and without oil.

A viscoelastic model consisting of a Kelvin-Voigt and Maxwell model in parallel provides reasonable agreement for both the modal and transient experiments. The model parameters determined from the modal and transient experiments do not show much agreement. This is due to the nonlinearities of the interface that are highlighted by the significant differences in the conditions of the tests. The modal experiment consisted of very low displacements under low preload, while the transient tests considered much larger displacements under larger mean loads with a rapid change in mean due to the step-like input.
5.2 Contributions

The first contribution of this study is the design of three experiments to measure the interfacial properties between the swash plate and housing under various conditions. The quasi-static experiment is designed to examine the interface under a slowly changing mean load, the modal experiment focuses on very small oscillations about an operating point, and the transient (step-like) experiment examines the effect of a rapidly changing mean load with various displacement amplitudes. A second contribution is a better understanding of the physics of the interface accomplished through the development of tractable linear viscoelastic models. A combination of the Kelvin-Voigt and Maxwell formulations yields the best explanation for an increase in stiffness seen with the addition of oil for the modal cases as well as the best agreement with the measured transient force response. Through comparisons between measured and simulated results, suitable stiffness and damping parameters are identified for the models for several cases.

5.3 Recommendations for Future Work

The nonlinearities on the bearing interface (between the swash plate and pump housing) are evident from the quasi-static force-deflection curve, the differences between modal and transient best fit parameters, and amplitude-dependent parameters. During the course of this study, preliminary nonlinear models are indeed used to simulate the transient experiments, but these simulations are omitted from the work presented due to difficulties in properly quantifying their nonlinear parameters. Accurately incorporating such nonlinearities into the viscoelastic models is thus left for future work. The models in this work are insufficient in simultaneously capturing the transient (step-like) loading
and unloading behavior. This is thought to be due to additional mechanisms such as adhesion and dry friction elements. A more robust modeling of the bearing interface may require an accurate determination of the curvature of the contacting surfaces as this would allow for a more accurate calculation of the effect of contact area and fluid film using analytical and computational (finite element) methods. Finally, the hydrostatic bearing needs to be pressurized as this work is conducted under ambient pressure.
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