A TIME DOMAIN ANALYTICAL APPROACH TO PREDICT THE PRIMARY INTAKE RUNNER DYNAMICS OF A SINGLE CYLINDER ENGINE

A Thesis

Presented in Partial Fulfillment of the Requirements for

the Degree Master of Science in the

Graduate School of The Ohio State University

By

Shankar Kumar, B.E. (Hons.)

The Ohio State University
2006

Master's Examination Committee:
Dr. Ahmet Selamet, Adviser
Dr. Rajendra Singh
Keith D. Miazgowicz

Approved by
Adviser
Graduate Program in Mechanical Engineering
The wave dynamics in the intake system of internal combustion engines plays an important role in determining the engine performance characterized by the volumetric efficiency and torque. Numerous studies have been carried out to understand the nature of the wave action by investigating the pressure and velocity fluctuations in the intake system and their influence on engine performance. The complexities involved with the presence of multiple cylinders and plenums, have promoted the use of single cylinder (SC) engines as a simpler tool to measure and predict the behavior of acoustic waves in the intake system. Even with the SC engines, the acoustic phenomenon is complicated by factors such as the geometry of the intake port and runner, the intake valve lift profile, the pulsating fluid flow due to the piston motion, the backflow of fresh charge/exhaust gases into the intake duct, and the complex nature of the viscous and separation losses. Typically, the pressure fluctuations are studied computationally by solving the Navier-Stokes equations and validated experimentally. Pure analytical techniques to predict the dynamics of the intake system are rare, and can be grouped as time- and frequency-domain based methods. Both methods available in the literature rely heavily on the experimental data and are restricted to rather narrow engine geometry and operating conditions.
The present work is aimed at studying the acoustic characteristics of the induction system of a Ford SC research engine. A time-domain based analytical formulation has been developed to estimate the pressure field in the intake system. The objective is to capture the effect of time-varying piston and valve motion on the acoustics of the induction system. These results have been compared with the experimental data and the numerical predictions (performed using Ford Motor Company’s engine simulation code MANDY), to identify the applicability as well as the potential limitations of the analytical model. The analytical pressure predictions have subsequently been used to calculate the acoustic velocities in the intake duct, hence the volumetric efficiency at various engine speeds. The mathematical technique developed in the present work is expected to be useful because of its simplicity and the insights it could offer into the understanding of the physical processes.
Dedicated to my Family for their unflinching support, love and encouragement.
ACKNOWLEDGMENTS

I thank my adviser Prof. Ahmet Selamet for all his support, encouragement and guidance throughout the course of this research work. His knowledge and wisdom have contributed significantly not only to my technical learning, but also overall personality development during my association with him as an advisee. I am thankful to Prof. Rajendra Singh for his valuable time during the thesis defense. My thanks also to Ford Motor Company for their contribution to the single cylinder engine project: Keith Miazgowicz for kindly consenting to be a M.S. thesis committee member, Dr. Kevin Tallio for supporting the procurement of the single cylinder engine, Zafar Shaikh for his assistance as the supervisor of the Ford single-cylinder lab, Mike Magnan for designing and fabricating the intake configurations, Graham Hoare and Frank Fsadni for supporting the single-cylinder related Ford University Research Program (URP) proposal.

I acknowledge the contributions from the fellow members of my research team who have made my stay at Ohio State a profoundly enriching experience as well as a pleasant experience. I would like to specially thank Vincent Mariucci for his help with the MANDY simulation results and the experimental data, Dr. Iljae Lee for his guidance, and to my fellow graduate students Yuesheng He, Lauren Lecuru, and Asim Iqbal for their interest.
VITA

April 25th, 1981 .............................................. Born – India

June 2002 ......................................................... B.E.(Hons.) Mechanical Engineering,

Birla Institute of Technology and Science,

Rajasthan, India.

September 2004 - August 2005 ......................... University Fellow, The Graduate School

The Ohio State University,

Columbus, OH.

September 2005 – present ............................. Graduate Research Associate

The Ohio State University,

Columbus, OH.

FIELDS OF STUDY

Major Field: Mechanical Engineering
# TABLE OF CONTENTS

Abstract ........................................................................................................... ii
Dedication ........................................................................................................ iv
Acknowledgments ............................................................................................... v
Vita ..................................................................................................................... vi
List of Tables ..................................................................................................... ix
List of Figures ................................................................................................... x

Chapters:

1. Introduction .................................................................................................... 1
   1.1 Background and motivation ................................................................. 1
   1.2 Literature review .................................................................................. 3
      1.2.1 Numerical models ....................................................................... 3
      1.2.2 Experimental studies ................................................................. 4
      1.2.3 Analytical models ...................................................................... 6
   1.3 Scope and objectives ............................................................................ 10
   1.4 Outline ................................................................................................. 11

2. Experimental setup, theory and model requirements.................................... 13
   2.1 The experimental setup for the dynamometer tests ............................. 13
   2.2 Theory and complicating effects ......................................................... 18
   2.3 Model requirements .......................................................................... 24

3. Driels’ approach to predict the pressure in the intake pipe – Model 1 .......... 25


LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Specifications of the single cylinder engine</td>
<td>14</td>
</tr>
<tr>
<td>4.1</td>
<td>Listing of engine speeds corresponding to $q = 3 - 6$</td>
<td>75</td>
</tr>
<tr>
<td>4.2</td>
<td>Comparison of the experimental and theoretical dominant frequencies</td>
<td>79</td>
</tr>
<tr>
<td>4.3</td>
<td>Calculated vs. experimental tuning peaks</td>
<td>88</td>
</tr>
<tr>
<td>B.1</td>
<td>Friction factor $F$ at selected engine speeds</td>
<td>99</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Schematic of the single cylinder engine fitted with a straight intake (baseline) pipe (Mariucci, 2006)</td>
</tr>
<tr>
<td>2.2</td>
<td>Straight intake duct (baseline); (Mariucci, 2006)</td>
</tr>
<tr>
<td>2.3</td>
<td>Experimental acoustic pressure vs. CAD, along with valve events (location i2, 3500 rpm)</td>
</tr>
<tr>
<td>2.4</td>
<td>Experimental acoustic pressure vs. CAD at various engine speeds</td>
</tr>
<tr>
<td>2.5</td>
<td>Acoustic pressure vs. CAD (data measured at location i2 for 3500 rpm showing the first “valley” and the first “peak” during the IV open period)</td>
</tr>
<tr>
<td>2.6</td>
<td>Amplitude of the first “valley” and the first “peak” of the pressure (experimental) vs. N</td>
</tr>
<tr>
<td>2.7</td>
<td>Experimental $\eta$, vs. $N$</td>
</tr>
<tr>
<td>3.1</td>
<td>Simplified acoustical representation of the intake system of the SC engine</td>
</tr>
<tr>
<td>3.2</td>
<td>Acoustic pressure vs. CAD (location i2; 2500 rpm)</td>
</tr>
<tr>
<td>3.3</td>
<td>Acoustic pressure vs. CAD (location i2; 3500 rpm)</td>
</tr>
<tr>
<td>3.4</td>
<td>Acoustic pressure vs. CAD (location i2; 4500 rpm)</td>
</tr>
<tr>
<td>3.5</td>
<td>Acoustic pressure vs. CAD (location i2; 5500 rpm)</td>
</tr>
<tr>
<td>4.1</td>
<td>A simplified acoustical model of the SC engine with a straight intake pipe</td>
</tr>
</tbody>
</table>
4.2 An illustration of a simple Helmholtz resonator ........................................... 42
4.3 Force balance of the plug .................................................................................. 42
4.4 Valve lift profile of the single cylinder engine .................................................. 47
4.5 Frictional resistance $F$ vs. engine speed .......................................................... 49
4.6 The intake pipe treated as an open-open duct during the IV open period .......... 51
4.7 Acoustic pressure vs. frequency (baseline case, location i2, 2500 rpm) .......... 53
4.8 Acoustic pressure vs. frequency (baseline case, location i2, 3500 rpm) .......... 53
4.9 Acoustic pressure vs. frequency (baseline case, location i2, 4500 rpm) .......... 54
4.10 Acoustic pressure vs. frequency (baseline case, location i2, 5500 rpm) ......... 54
4.11 Acoustic pressure vs. CAD (location i2; 2500 rpm) ...................................... 58
4.12 Acoustic pressure vs. CAD (location i2; 2750 rpm) ...................................... 58
4.13 Acoustic pressure vs. CAD (location i2; 3000 rpm) ...................................... 59
4.14 Acoustic pressure vs. CAD (location i2; 3250 rpm) ...................................... 59
4.15 Acoustic pressure vs. CAD (location i2; 3500 rpm) ...................................... 60
4.16 Acoustic pressure vs. CAD (location i2; 3750 rpm) ...................................... 60
4.17 Acoustic pressure vs. CAD (location i2; 3850 rpm) ...................................... 61
4.18 Acoustic pressure vs. CAD (location i2; 4000 rpm) ...................................... 61
4.19 Acoustic pressure vs. CAD (location i2; 4250 rpm) ...................................... 62
4.20 Acoustic pressure vs. CAD (location i2; 4500 rpm) ...................................... 62
4.21 Acoustic pressure vs. CAD (location i2; 4750 rpm) ...................................... 63
4.22 Acoustic pressure vs. CAD (location i2; 5000 rpm) ...................................... 63
4.23 Acoustic pressure vs. CAD (location i2; 5250 rpm) ...................................... 64
4.24 Acoustic pressure vs. CAD (location i2; 5500 rpm) ...................................... 64
4.25 Acoustic pressure vs. frequency (location i2; 2500 rpm) ..............................................66
4.26 Acoustic pressure vs. frequency (location i2; 2750 rpm) ..............................................67
4.27 Acoustic pressure vs. frequency (location i2; 3000 rpm) ..............................................67
4.28 Acoustic pressure vs. frequency (location i2; 3250 rpm) ..............................................68
4.29 Acoustic pressure vs. frequency (location i2; 3500 rpm) ..............................................68
4.30 Acoustic pressure vs. frequency (location i2; 3750 rpm) ..............................................69
4.31 Acoustic pressure vs. frequency (location i2; 3850 rpm) ..............................................69
4.32 Acoustic pressure vs. frequency (location i2; 4000 rpm) ..............................................70
4.33 Acoustic pressure vs. frequency (location i2; 4250 rpm) ..............................................70
4.34 Acoustic pressure vs. frequency (location i2; 4500 rpm) ..............................................71
4.35 Acoustic pressure vs. frequency (location i2; 4750 rpm) ..............................................71
4.36 Acoustic pressure vs. frequency (location i2; 5000 rpm) ..............................................72
4.37 Acoustic pressure vs. frequency (location i2; 5250 rpm) ..............................................72
4.38 Acoustic pressure vs. frequency (location i2; 5500 rpm) ..............................................73
4.39 Variation of the function tanh(a+ib) for various values of ‘a’ ..............................................74
4.40 $S_n$ [as defined in Eq. Eq. (4.25)] w.r.t. $n$ for approximate integral values of $q$..............76
4.41 Volumetric efficiency vs. engine speed – analytical vs. experimental ..................................85
A.1 Schematic of the intake pipe – plug combination ..........................................................92
CHAPTER 1

INTRODUCTION

1.1 Background and motivation

Internal combustion (IC) engine performance has been an area of active investigation over the past hundred years. In contemporary times, efforts directed towards enhancing engine performance (without compromising fuel economy and emissions) are being made through several approaches. Evolving better engine designs through a more complete understanding of the underlying physics, and development of sophisticated control strategies, are two of them. The basic idea that the mass of air-fuel mixture in a Spark-Ignition (SI) engine (also referred to as ‘charge’) taken into the cylinder (during the induction stroke) dictates the work that can be extracted, has been traditionally the foundation of many design strategies aimed at optimizing engine performance. It is in this context that techniques to “force” extra charge into the cylinder have been developed. Additional charge can be forced into the cylinder by either exploiting the momentum of the charge as it enters the combustion chamber (ram effect) or through the use of intake tuning.
The motion of the piston and valves in an engine causes pressure pulsations in the intake system. The induction stroke of an engine cycle produces during the intake valve (IV) open period, a rarefaction wave that travels towards the ambient through the intake system. The wave gets partially reflected at the interface between the intake system and the ambient (or the intake plenum), and travels back towards the intake valve as a compression wave. If the compression wave is timed to arrive at the intake valve before the valve-closure, additional charge can be forced into the cylinder. This concept forms the basis of intake tuning. The “timing” of the pressure fluctuations is dependent on the dimensions of the intake system, the engine speed, and the valve timing – factors that basically decide the nature of the flow field in the air induction system. It is clear that the nature of pressure oscillations has a significant effect on engine performance characterized by volumetric efficiency:

$$\eta_v = \frac{\text{Actual volume of air inducted into the cylinder}}{\text{The volume of air, which would fill the displacement volume under ambient conditions}}$$

Engine performance is also characterized by indicated torque or indicated mean effective pressure, parameters with a direct relationship to the volumetric efficiency. Of relevance to an engine designer is thus the need to correlate engine performance, with the selection of design variables such as bore, stroke, compression ratio and intake system dimensions. This need has led to the development of techniques to predict acoustic pressure and volumetric efficiency. The present work focuses on developing an analytical technique to calculate oscillating pressures in the intake system of a single cylinder (SC) engine and extends the method to calculate volumetric efficiency.
1.2 Literature review

Numerous works have been published on the dynamics of IC engine intake systems. Many of these concentrate on predicting or measuring the pressure or velocity oscillations in the intake system and correlating them with the engine performance. Analytical, experimental, and computational investigations carried out in the past have all led to the now well-established conclusion, that depending on the geometry of the intake system the pressure waves can either improve or deteriorate the volumetric efficiency. Terms such as “non-mechanical supercharging”, “surge-phenomena”, and “resonant wave action”, have been used interchangeably in the literature with regard to the processes associated with intake tuning described briefly in Section 1.1.

1.2.1 Numerical models

Many studies in the past have used numerical approaches to solve the mass, momentum, and energy conservation equations by considering the breathing processes associated with the intake-exhaust systems, as transient gas-dynamic phenomena. Benson et al. (1964) proposed a time-domain approach to treat the unsteady conservation equations in the breathing system of engines using the method of characteristics. Chapman et al. (1982) used a one-dimensional (1-D) finite difference program to solve the continuity, Navier-Stokes (N-S), energy equations, and the equation of state to determine the total pressure (the sum of mean and acoustic pressures) field in the intake manifold of a four cylinder engine under wide-open throttle (WOT) conditions. Later, for example, Selamet et al. (2001a) used this program to study the effect of a silencing element (Helmholtz resonator) in the intake system by determining the insertion loss.
Their model predictions in time- and order-domains showed a reasonable comparison with the experimental data. Some have attempted multi-dimensional modeling of the engine intake system including Kim and Lee (2001), who solved the two-dimensional (2-D) N-S equations for unsteady compressible flow using finite volume discretization and a high accuracy “essentially non-oscillating” scheme. These references are merely few examples, and represent a small subset of the available literature.

1.2.2 Experimental studies

Due to the complex nature of physics associated with the induction process, experimental approaches have always been considered an attractive option. Boden and Schecter (1944) produced a systematic and extensive study to correlate the pressure oscillations in the intake system and indicated mean effective pressure using various combinations of engine speed, spark timing, and intake pipe length and diameter. They conducted two sets of experiments - on a four-stroke SC engine with a straight intake duct attached to the intake port, and also on a four-stroke six-cylinder engine with three of the cylinders attached to a common intake pipe (forming two sets of intake pipes). For tests performed on the SC engine, they concluded that the amplitude of the pressure pulses increases with a longer intake pipe for a given speed or with increasing engine speed at a fixed pipe length. Introducing a dimensionless “frequency factor”

\[ q = \frac{c_0 / 4l}{0.5(N/60)}, \]

(\(c_0\) [m/s] is the speed of sound, \(l\) [m] the intake pipe length, and \(N\) [RPM] the engine speed) defined as the ratio of the intake pipe fundamental quarter wave resonance
frequency to the first engine half order, they determined that when $q$ takes an integral value close to 3, 4 or 5, the volumetric efficiency (and hence, the indicated mean effective pressure) increases. It is clear that $q$ increases with both decreasing speed and shorter intake pipe lengths. Boden and Schecter, however, restricted their SC experiments to low engine speeds (less than 1600 rpm) and used pipe lengths ranging from 0.23-6.3 m. On experiments performed on the six-cylinder engine, they concluded that “increased volumetric efficiency at low speed was obtainable by using a longer pipe, but only with the sacrifice of volumetric efficiency at high speed”.

Mariucci (2006) in his Master’s thesis investigated the effect of intake pipe geometry on the performance of a SC engine. The intake system of the engine consisted a set of short pipes attached end-to-end serially to the intake port. Ducts having different geometries were then attached to the ambient end of the intake system. Several duct configurations such as a straight pipe, tapered-ducts with different half angles, ducts with bellmouth entries of various radii, and ducts with various bends, were studied through dynamometer experiments. Some of the parameters measured in the experiments were: instantaneous pressure at specific locations in the intake and exhaust pipes, in-cylinder pressure, volumetric efficiency, torque, power, mean effective pressure, and emissions. Numerical comparisons were performed using Ford Motor Company’s engine simulation code MANDY, which is essentially an improved version of the algorithm originally developed by Chapman et al. (1982).
1.2.3 Analytical models

Pure analytical treatments to understand the dynamics of the engine intake system are rare due to the complex character of the problem. Geometric similarities have tempted many investigators to use the well-studied acoustic element - Helmholtz resonator as an approximate acoustical representation of the engine cylinder-intake duct interaction. While all studies have treated the cylinder volume as the cavity of the Helmholtz resonator, the consideration of neck differs markedly. Some works (Morse et al., 1938) treat the intake valve as the neck of the Helmholtz resonator; others (Engelman, 1973 and Driels, 1975) treat the entire intake system as a single duct, which acts as the neck.

Boden (1936) in his PhD dissertation developed a mathematical expression to calculate the pressure and volumetric efficiency of a SC engine using linear acoustic theory. Ignoring the effect of cylinder volume and scavenging phenomena during the induction stroke, he derived expressions for instantaneous intake-port pressure and volumetric efficiency. His volumetric efficiency estimates however, involved the use of experimental data. The accuracy of his results, however, was poor and only predicted the general form of the pressure and volumetric efficiency measured by experiments. A significant work by Morse et al. (1938) investigated the effect of varying intake pipe length on pressure fluctuations at the intake port and volumetric efficiency of a SC engine. Under the consideration that air at the intake port acts as a lumped mass, and the varying volume as a spring of a simple spring-mass system, they derived an expression for the fluctuating pressure at a point just outside the intake valve of the engine. Additionally, they presented volumetric efficiency estimates for various pipe lengths at a fixed engine speed. Their pressure and volumetric efficiency results showed a reasonable
match with the experimental data. By changing the “frequency factor” $q$ [defined in Eq. (1.1)] using different intake pipe lengths and a fixed engine speed, Morse et al. determined that the volumetric efficiency shows peaks when $q$ approaches 3, 4, or 5, consistent with the conclusions reached later by Boden and Schecter (1944). However, the treatment of friction near the intake valve in their expression for pressure is unclear. Yet, their model presented an advancement in the analytical modeling of the intake system from an “acoustical” perspective.

Engelman (1973) suggested an analytical expression to estimate the engine speed at which the peak torque is obtained by simplifying the cylinder-intake system interaction as a constant volume Helmholtz resonator. The “Helmholtz resonator” comprised of the cylinder (at its mid-stroke) taken as the expansion volume, and the entire intake system as the neck. In SI units, the engine speed where peak torque is attained is given as

$$N \text{ [RPM]} = \frac{1}{K} \frac{60}{2\pi} C_0 \sqrt{\frac{A_p}{l V_m}},$$

(1.2)

where $A_p \text{ [m}^2\text{]}$ is the cross-sectional area of the intake pipe, $V_m = V_c + \frac{V_d}{2} \text{ [m}^3\text{]}$ the mean cylinder volume at mid-stroke, $V_c \text{ [m}^3\text{]}$ the clearance volume, $V_d \text{ [m}^3\text{]}$ the cylinder displacement volume, and $K$ is a constant equal to 2.1 for most conventional engines but can vary between 2.1-2.5 depending on the valve timing. His hypothesis states that whenever the natural frequency of the “Helmholtz resonator” equals approximately $K$ times the engine speed in rpm, the amplitude of the pressure fluctuations in the intake pipe and hence the volumetric efficiency at that speed increases. Engelman describes this condition as “resonance”. His method was thus intended to provide for an extremely
easy-to-use relation for the approximate location of the tuning peak. Unfortunately neither Engelman (1973), nor his coworkers – Eberhard (1971) and Thompson (1968) attempted to extend their Helmholtz resonator models to predict the pressure pulsations in the intake system.

Driels (1975) used a time-domain analytical model to capture the effect of varying cylinder volume on the pressure field in the intake pipe of a SC engine. One of his core assumptions was that the intake system could be construed as a Helmholtz resonator with the intake pipe treated as the neck combined with a varying volume as a result of the piston motion during the induction stroke. Assuming the intake valve to have a constant flow-loss coefficient and using simple duct acoustic expressions, he presented a model, the results of which replicated the overall behavior of the experimental pressure traces. But the accuracy of his predictions was rather poor – the amplitude, frequency and phasing of the predicted pressure showed large deviations from experimental data. Also, his treatment of the resonant frequency of the “Helmholtz resonator” and its connection with the frequency of the pressure pulses is unclear. Yet, his work remains useful since it is one of the original attempts to separate the behavior of pressure fluctuations during the IV open period from that of the closed period. Harrison and Stanov (2004a) presented a frequency-domain based technique to calculate the intake port pressure of a SC engine by modeling the intake valve as an acoustic source. Like many other earlier works described in this section, their success in terms of matching experimental results with model predictions was far from satisfactory despite the heavy use of experimental data.

The time-variant nature of the piston and the intake valve of an engine, make an analytical solution for pressure in the intake system a rather formidable challenge due
primarily to the inability to solve the relevant governing equations. Some approaches have attempted to ignore the effect of intake valve, and study the effect of piston motion alone, as a transient problem. Jang and Ih (2005) in their short communication used an electrical analogy to study the influence of a time-varying piston on the acoustical characteristics of a simple fluid machine. Considering a source-load system comprised of a single cylinder with a reciprocating piston and an exhaust duct, they present a mathematical model with the piston as the source, and the exhaust duct as the load. They also derived a second-order ordinary differential equation in time (referred to as Hill’s equation) containing time-varying coefficients to solve for the instantaneous mass present inside the cylinder. They admitted, however, that the equation couldn’t be solved for a closed form expression, unless further simplifying assumptions were made for the coefficients of the differential equation. While their paper does not directly relate to the intake systems of IC engines, its underlying approach offers a stimulating perspective.

From a theoretical standpoint, two simple approaches attempt to capture the nature of oscillations in the intake system. The first one suggests the oscillations as occurring due to “organ pipe” type behavior. The intake pipe behaves as an ‘open-open’ duct (open on both ends) during the IV open period, and a ‘closed-open’ one during the closed period. This approach is built on the basis of conclusions reached by Boden and Schecter (1944) as well as Morse et al. (1938). The second one views the gas pulsations as a result of the Helmholtz resonator-like behavior of the intake system (Engelman, 1973). However, as the volume of the intake system increases relative to that of the cylinder, the Helmholtz resonator consideration breaks down completely and the organ-pipe phenomena gains dominance (Thompson, 1968). Eberhard (1971) suggests that both
theories are relevant: the Helmholtz model predicts the speed corresponding to the peak volumetric efficiency well, whereas the organ pipe explains the secondary peaks of the volumetric efficiency, though this duality has not been clearly demonstrated in the literature due to the simple nature of two theories. The present work will use the ‘organ-pipe’ model of Morse et al. (1938) to calculate volumetric efficiency, and compare the location of tuning peaks to the ‘Helmholtz resonator’ approach of Engelman (1973), to understand the relevance of the two theories.

All analytical approaches described thus far are based on simple linear acoustic theory and have two major drawbacks: (1) their applicability is restricted to narrow engine operating conditions and geometry, and (2) they involve heavy use of experimental data. Driels’ approach works only at very low speeds (approx. 1500 rpm), whereas the model of Morse et al. considers exceedingly impractical lengths for the intake pipe (1-3 m), a low compression ratio of 5 and low speeds (approx. 1600 rpm). It is not yet clear if these models, would predict accurately the amplitude, frequency and the phasing of the pressure oscillations at significantly different operating conditions, such as higher engine speeds or shorter intake pipes.

1.3 Scope and objectives

The complexity of the breathing processes associated with multi-cylinder engines has promoted the use of single cylinder engines for the study of intake wave dynamics among other physics. SC engines are also easier to build relative to new multi-cylinder engines. The present work is aimed at studying the acoustic characteristics of the intake system of a Ford SC research engine. The limitations of analytical techniques available in
the literature as described in Section 1.2.3, motivate the development of an analytical model to analyze such intake systems, and explore its applicability across a wider range of operating conditions. It is yet to be seen if such perceived limitations are due to a lack of need to study higher speeds (possibly due to the timeframe in which most papers were written), or due to poor model performance at the high end. The objective of the present work therefore, is to develop a time-domain based analytical model to predict the pressure pulsations at a location in the intake pipe upstream to the intake valve (where experimental pressure recordings are available) and the volumetric efficiency at various engine speeds. The pressure and the volumetric efficiency will be calculated for a wide range of engine speeds from 2500-5500 rpm to assess the model performance. The experimental data for the current study is extracted from the work of Mariucci (2006). As discussed in Section 1.2.2, the experiments were conducted for various intake pipe geometries, and for each experiment, runs were carried out at speeds that include the range 2500-5500 rpm. Unlike some of the previous works (Morse et al., 1938), the intake pipes used here were relatively short (around 65 cm). The mathematical technique developed in the present work is expected to be useful because of its simplicity and the insights it could offer into the understanding of the physical processes.

1.4 Outline

Following the introduction in Chapter 1, Chapter 2 presents a brief description of the experimental setup used by Mariucci (2006), the theory behind the pressure fluctuations and the influence of various factors on the oscillating pressure in the intake system, and a summary of requirements for a robust model required to predict pressure.
Chapter 3 presents Model 1, which represents Driels’ method to predict the intake pressures in a SC engine. The solution technique along with its limitations will be discussed. Chapter 4 develops a much-improved Model 2 – a rigorous analytical approach to predict oscillating pressures. Chapter 4 also elaborates on a procedure to predict the volumetric efficiency of the SC engine, makes comparisons (in both time and frequency domains) between the analytical predictions, experimental data, and MANDY simulations, while also discussing the applicability and potential limitations of Model 2. Finally, the conclusions of the present study and recommendations for future work are provided in Chapter 5.
CHAPTER 2

EXPERIMENTAL SETUP, THEORY AND MODEL REQUIREMENTS

Before elaborating on the models, it is useful to describe the setup used for the SC engine experiments along with, the theory behind the pressure fluctuations and the complexities involved in the development of an analytical method. Since there are a number of approaches to model the dynamics of the intake system, the requirements of a robust technique aimed at predicting oscillating pressure from an analytical standpoint, is worthy of discussion. The present Chapter addresses these topics.

2.1 The experimental setup for the dynamometer tests

The engine used for the experiments is a 4-valve four-stroke spark ignition engine with specifications given in Table 2.1, essentially identical in design to one cylinder of Ford's Jaguar 3.0L V6 engine. The combustion chamber, bore and stroke, piston geometry, valve timing, and the intake and exhaust ports are the same as the Jaguar engine. These experiments were carried out under WOT conditions as part of another ongoing project (Mariucci, 2006) for Ford Motor Company. As indicated earlier in Section 1.3, the experiments studied the effect of eighteen different intake system
geometries (overall intake system shown in Fig. 2.1) on the intake pressure fluctuations, thereby their influence on engine performance. A particular intake system (baseline) is chosen here to provide experimental comparisons to the present analytical study.

<table>
<thead>
<tr>
<th>Geometric features</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bore Diameter</td>
<td>8.90 cm</td>
</tr>
<tr>
<td>Stroke Length</td>
<td>7.95 cm</td>
</tr>
<tr>
<td>Connecting Rod Length</td>
<td>13.81 cm</td>
</tr>
<tr>
<td>Maximum intake valve lift</td>
<td>0.914 cm</td>
</tr>
<tr>
<td>Maximum exhaust valve lift</td>
<td>0.937 cm</td>
</tr>
<tr>
<td>Valve timing (0 refers to the TDC of the combustion stroke)</td>
<td></td>
</tr>
<tr>
<td>Intake Valve opening</td>
<td>308 crank angle degree (CAD)</td>
</tr>
<tr>
<td>Intake Valve closing</td>
<td>594 CAD</td>
</tr>
<tr>
<td>Exhaust Valve opening</td>
<td>86.5 CAD</td>
</tr>
<tr>
<td>Exhaust valve closing</td>
<td>412.5 CAD</td>
</tr>
</tbody>
</table>

Table 2.1. Specifications of the single cylinder engine.
Figure 2.1 Schematic of the single cylinder engine fitted with a straight intake (baseline) pipe (Marucci, 2006).
A brief description of the experimental setup follows next. The intake system (Fig. 2.1) is divided into different sections – the split port, fuel rail block, adaptor, barrel throttle, and test piece. The split port is a short pipe, part of which is branched and connects the two intake ports with the fuel rail block. The fuel rail block, mounted on the cylinder head face, encloses the fuel injector. The adaptor contains two piezoresistive pressure transducers (referred to as i2 and i3), which are mutually perpendicular along their axes, and to the centerline of the intake pipe. The pressure transducers measure the absolute instantaneous air pressure at every CAD. A “barrel” straight duct is attached upstream of the adaptor to control the air flow when necessary during the induction stroke. A barrel is used instead of a conventional throttle plate to avoid any perturbation from the latter on the pressure measurements at transducers i2 and i3. Placed further upstream to the throttle is the test piece, which consists of a duct terminated by a flanged bellmouth facing the ambient end. The straight duct configuration of the test piece is called ‘baseline’. Figure 2.2 shows the geometry of the baseline. A third pressure transducer i1 is placed on the test piece near the opening of the intake pipe. The term “intake pipe” used in this thesis will refer to the entire intake system, starting from the intake port and terminating at the ambient.
A typical experiment for a selected intake pipe configuration varied from 1000-5500 rpm in increments of 250 (for a given experiment the measurements at different speeds are called ‘runs’, as mentioned in Section 1.2.2), while maintaining constant valve timing. Additional speeds were included near the peak torque. For example, in the case of baseline, while the peak torque occurs at 3750 rpm, runs were also performed at two other speeds – 3650 and 3850 rpm close to the peak torque. Every run involved measurement of a large number of engine parameters such as pressure-time histories at pre-selected locations in the intake and exhaust pipes, in-cylinder pressure, volumetric efficiency, torque, power, mean effective pressure, and emissions. For more information
on the experimental setup, procedure and the measurement systems used, the reader is referred to Mariucci’s thesis (2006).

2.2 Theory and complicating effects

The effectiveness of the pressure fluctuations in the engine intake system towards enhancing performance has been estimated by various studies, depending on, for example, the engine and intake system geometries, the valve and spark timings, and the operating conditions. Engelman (1973) indicates a 16% increase in volumetric efficiency with a proper selection of tuning length for a four-cylinder engine, whereas Boden (1936) claims a difference of 31% between the maximum and minimum power outputs for a SC engine fitted with intake pipes of different lengths. Given its impact, the need to develop a solid theoretical understanding of the induction process is clear.

The instantaneous pressure during the engine cycle can be separated into that of the IV open and closed periods (Driels, 1975). During the IV open period, Boden (1936) imagined the source of pressure fluctuations in the induction system as a “plug” of air oscillating at the intake valve. These fluctuations are sustained through the closed period due to repeated reflections at the intake valve and the open end. Boden adds that the fluctuations are periodic due to the regularity of the opening and closing events of the intake valve and that “standing waves” are set up in the intake pipe. Figure 2.3 shows a typical pressure-time history (and valve events) inferred from experiments at location t2 for a speed of 3500 rpm. Note that 0 CAD corresponds to TDC of the induction stroke. The figure shows the acoustic pressure component only, which fluctuates around 0 kPa. The pressure shows an initial drop after TDC, followed by a peak, which transitions into
periodic damped oscillations till the next intake valve opening (IVO) event. In view of the valve timings provided in Table 2.1, the intake valve closes at 594 – 360 = 234°, if 0° corresponds to CAD taken from TDC of breathing (see Fig. 2.3). After intake valve closing (IVC), the pressure takes the waveform of a damped circular function with an almost constant frequency. Since IVO occurs at 668 CAD, the duration from 234 – 668° corresponds to the IV closed period suggesting a behavior similar to that of a simple duct with "closed-open" ends.

![Graph showing acoustic pressure vs. CAD with valve events](image)

Figure 2.3 Experimental acoustic pressure vs. CAD, along with valve events (location i2; 3500 rpm).
Figure 2.4 shows the acoustic pressure measured at location i2 (of the intake pipe) at four different speeds of 2500, 3500, 4500, and 5500 rpm. At higher speeds, the first pressure peak after the induction stroke shifts to the right w.r.t. CAD. The figure also reveals that the amplitude of the pressure wave gradually increases with increasing engine speed. These traces show the same characteristics as those obtained by Boden and Schecter (1944) for their SC engine experiments.

![Figure 2.4 Experimental acoustic pressure vs. CAD at various engine speeds.](image)

Numerous factors affect the wave dynamics in the engine intake system, including the geometry of the ducts, and the valve lift profile and timing. Other factors are: (1) the
nature of the flow field and the viscous and separation losses in the intake system; (2) ‘backflow’ which refers to the flow of fresh charge and exhaust gas into the intake system; and (3) thermal effects which include the heat transfer between the charge and the walls of the intake pipe and cylinder. As a result, modeling intake phenomena using an analytical technique remains as a challenge.

For an illustration of the complexity, Fig. 2.5 shows the absolute amplitude of the first “valley” and the first “peak” of the pressure pulses (recorded experimentally) during the induction stroke for an engine speed of 3500 rpm. The complex variation of the “valleys” and “peaks” as measured over engine speeds of 2500-5500 rpm is shown in Fig. 2.6. Prior experience with the pressure measurements in the intake and exhaust systems of the SC engine points to some fluctuations at the very low end. Hence, the present work will use data from 2500-5500 rpm only.
Figure 2.5 Acoustic pressure vs. CAD (data measured at location i2 for 3500 rpm showing the first "valley" and the first "peak" during the IV open period).

Figure 2.6 Amplitude of the first "valley" and the first "peak" of the pressure (experimental) vs. $N$. 
An attempt will be made in this study to relate the acoustic pressures to volumetric efficiency $\eta_v$. Figure 2.7 shows the experimental results for $\eta_v$ as a function of engine speed $N$. The volumetric efficiency varies from around 80% at a speed of 5500 rpm to the highest value of 111% at 3750 rpm. There are distinct peaks at 3000, 3750, 4750 rpm, corresponding to speeds where turning effects dominate. Given the complex profile of $\eta_v$ in Fig. 2.7 and the limitations of the models described in the literature, extending the pressure predictions to volumetric efficiency also presents a challenge.

Figure 2.7 Experimental $\eta_v$ vs. $N$. 

23
2.3 Model requirements

As explained in Section 1.4, the primary objective of the present work is to develop a mathematical model to predict the instantaneous pressure at location i2 in the intake pipe, due to its proximity to the intake port hence its influence on the engine performance. It is now relevant to list some of the model requirements:

- The current work will concentrate on the acoustical characteristics of the baseline configuration. The proposed model must capture the effect of the time-varying piston motion and the intake valve on the acoustics of the induction system.

- The model must not use electro-mechanical and circuit-theory analogies, which have been already tried earlier (Eberhard, 1971 and Jang and Ih, 2005) with limited success.

- The predictive capability of the proposed model must extend across a wide range of engine speeds while capturing the trends properly, in order to remedy the perceived shortcomings of the available analytical models.

- The assumptions and approximations considered for the development and simplification of the model must be realistic. The dependence on experimental data must be kept to a minimum in order to justify a predictive tool. Also, in order to predict $\eta_c$ (or, at least the location of the peaks of $\eta_c$), the pressure calculations must capture the tuning effects.
CHAPTER 3

DRIELS’ APPROACH TO PREDICT THE PRESSURE IN THE INTAKE PIPE – MODEL 1

The present Chapter deals exclusively with the development of Driels’ analytical method to predict the acoustic pressure in the intake pipe of the SC engine. Driels (1975) separates the behavior of the pressure wave during the IV open period from that of the closed one. During the open period, he considers that the periodic pressure fluctuations in the intake system can be written as the sum of “steady oscillatory” and “transient” parts. The term “steady” is used here to remain consistent with Driels’ paper, and is not to be confused with the absence of time-dependency. The basis of splitting acoustic pressure into two components during the IV open period as done by Driels, will be clarified later in this chapter. For the IV closed period, he assumes the pressure to take the waveform of a simple damped harmonic function as discussed in Section 2.2. His simplified acoustical representation of the cylinder and intake duct combination for the SC engine is shown in Fig. 3.1. In the model, he assumes the intake valve opens and closes instantaneously and that the acoustic wave propagation in the intake pipe and cylinder is one-dimensional. He accounts for the effect of viscous damping of the pressure oscillations in the intake pipe,
while ignoring viscous effects in the cylinder. Heat transfer is ignored in the intake pipe and the cylinder. The derivation of the “steady oscillatory” and the “transient” pressures is described next following Driels (1975).

![Diagram of intake system](image)

Figure 3.1 Simplified acoustical representation of the intake system of the SC engine.

One-dimensional governing equations for mass and momentum conservation in the intake duct or the cylinder may be written as

\[
\frac{1}{\rho_0 c_0^2} \frac{\partial p}{\partial t} + \frac{\partial u}{\partial x} = 0, \tag{3.1}
\]

and

\[
\frac{\partial p}{\partial x} + \rho_0 \frac{\partial u}{\partial t} + R_0 u = 0, \tag{3.2}
\]

where \( p \) is the “steady state” pressure, \( u \) the velocity, \( \rho_0 \) the mean fluid density, \( c_0 \) the mean speed of sound, \( x \) the co-ordinate axis, \( t \) the time, and

\[
R_0 = \frac{32 \mu}{D^2} \tag{3.3}
\]
is the linearized viscosity term corresponding to frictional losses in the intake pipe (assuming laminar conditions), \( \mu \) is the dynamic viscosity of the fluid in the intake pipe and \( D \) the diameter. The solution of Eqs. (3.1) and (3.2) for pressure in the intake duct is

\[
p = e^{i\omega t} \left( A_1 e^{\alpha x} + A_2 e^{-\alpha x} \right),
\]

(3.4)

where \( x \) is the distance from the ambient end, \( A_1 \) and \( A_2 \) are constants dictated by the boundary conditions, and \( \alpha \) is a complex number. The \( \omega \) term in the oscillating component \( e^{i\omega t} \) represents the angular frequency corresponding to the engine speed:

\[
\omega = \frac{2\pi N}{60}.
\]

The velocity \( u \) in the duct can be determined by substituting Eq. (3.4) into Eq. (3.1) as

\[
u = -\frac{e^{i\omega t}}{Z_1} \left( A_1 e^{\alpha x} - A_2 e^{-\alpha x} \right),
\]

(3.5)

where \( Z_1 = \frac{\alpha c_0^2 \rho_0}{i\omega} \) is the characteristic impedance and

\[
\alpha = \frac{\sqrt{\omega \left( iR_0 \rho_0 - \omega \right)}}{c_0^2}.
\]

(3.6)

Equation (3.6) may alternatively be written as \( \alpha = \varepsilon + i\chi \), where

\[
\varepsilon = \frac{\omega}{\sqrt{2c_0}} \left( 1 + \frac{R_0^2}{\rho_0^2 \omega^2} \right)^{1/2},
\]

(3.7)

and

\[
\chi = \frac{\omega}{\sqrt{2c_0}} \left( 1 + \frac{R_0^2}{\rho_0^2 \omega^2} \right)^{1/2}.
\]

(3.8)
Assuming that the pressure at any point \( x_1 \) in the intake system at time \( t \) during the IV open period can be written as a sum of the “steady state” and “transient” parts, the acoustic pressure (during the IV open period) \( p_{\text{total}} \) is expressed as

\[
p_{\text{total}}(x_1, t) = p_{\text{steady state}} + p_{\text{transient}},
\]

where \( p_{\text{steady state}} \) and \( p_{\text{transient}} \) are components of the fluctuating pressure. The “steady state” pressure is a function of frequency \( \omega \) and follows from the assumption of Driels that “standing wave patterns” can be considered in the intake pipe and cylinder, provided the engine speed is low, so that “the piston appears almost stationary relative to the air column” in the cylinder. Therefore, the intake pipe and cylinder are treated as simple cylindrical ducts acoustically excited at one end. The derivation of the “steady state” pressure takes into account the viscosity of the fluid within the intake pipe as shown through the use of \( R_0 \) in Eqs. (3.3)-(3.8). The “transient” term accounts for the effect of piston motion on the wave dynamics of the intake pipe and its derivation is based on a more simplified model as developed by Driels.

The “steady state” pressure and velocity at any point \( x_1 \) in the intake system can be written from Eqs. (3.4) and (3.5) as

\[
P_{\text{a}} = P_A \cosh \alpha x_1 - Z_A u_A \sinh \alpha x_1,
\]

and

\[
u_A = [-P_A / Z_A \sinh \alpha x_1 + u_A \cosh \alpha x_1],
\]

where \( P_A \) and \( u_A \) are the pressure and velocity respectively at location A in Fig. 3.1.

The acoustic impedance at any section \( x_1 \) is
\[ Z_{s_i} = \frac{P_{s_i}}{u_{s_i}} = \frac{Z_A - Z_i \tanh \alpha x_i}{1 - (Z_A / Z_i) \tanh \alpha x_i}, \]  

(3.12)

where \( Z_A = \frac{P_A}{u_A} \) is the impedance at location A. Since \( P_A \) is zero due to the open duct boundary condition, \( Z_A = 0 \), reducing Eq. (3.12) to

\[ Z_{s_i} = -Z_i \tanh \alpha x_i. \]  

(3.13)

The "steady state" orifice equation can be used for the intake valve so that the pressure drop across the valve is written as

\[ P_B - P_C = \frac{1}{2} K_v \rho_v u_v^2 = \frac{1}{2} K'_v \rho_0 u_v, \]  

(3.14)

where \( P_B \) and \( P_C \) are the pressures at the locations B and C; and \( u_v, K_v \) are the velocity and flow-loss coefficient across the intake valve respectively. The numerical value of \( K_v \) is taken to be 0.8 (Taylor, 1960) and

\[ K'_v = K_v u_v. \]  

(3.15)

is a linearizing factor which can be assumed constant if a suitable approximation is chosen for \( u_v \). One good approximation for \( u_v \) which is a fluctuating quantity, would be its root mean square (RMS) value represented as \( u_{vRMS} \). Also, the value of \( u_v \) used in Eqs. (3.14) and (3.15), is approximated by Driels as an average

\[ u_v = \frac{1}{2} (u_C + u_B), \]  

(3.16)
where $u_c$ and $u_b$ are the velocities on either side of the intake valve. Equation (3.16) implies that $u_{\text{RMS}}$ can also be written as $u_{\text{RMS}} = \frac{1}{2} (u_{\text{CRMS}} + u_{\text{BRMS}})$. Hence, from Eqs. (3.15) and (3.14)

$$K_v = K_v u_v = K_v u_{\text{RMS}} = K_v \frac{1}{2} (u_{\text{CRMS}} + u_{\text{BRMS}})$$

(3.17)

and

$$P_B - P_C = \frac{1}{2} K_v \rho_0 u_v^2 = \frac{1}{2} K_v \rho_0 \frac{1}{2} (u_c + u_b).$$

(3.18)

Continuity across the intake valve gives

$$u_c S_C = u_b S_B,$$

(3.19)

or $u_b = u_c \frac{S_C}{S_B}$, where $S_B$ and $S_C$ are the cross-sectional areas at locations B and C respectively. Also, considering that the piston stroke is small, $u_c \approx u_d$ and $u_{\text{CRMS}} = u_{\text{DRMS}}$.

The value of $u_{\text{DRMS}}$ is not mentioned in Driels' paper, and may be approximated as

$$u_{\text{DRMS}} = \left( \frac{2SN}{60} \right),$$

(3.20)

where $S$ is the stroke length of the piston. Note that $\frac{2SN}{60}$ represents the average piston velocity. From Eqs. (3.17) and (3.19),

$$K_v = K_v u_{\text{CRMS}} \frac{1}{2} \left( \frac{u_{\text{BRMS}}}{u_{\text{CRMS}}} + 1 \right)$$

$$= K_v u_{\text{CRMS}} \frac{1}{2} \left( \frac{S_C}{S_B} + 1 \right) = K_v u_{\text{DRMS}} \frac{1}{2} \left( \frac{S_C}{S_B} + 1 \right).$$

(3.21)
And from Eq. (3.18),

\[ P_b - P_c = \frac{1}{4} K' \rho_0 u_c \left( \frac{1 + \frac{u_b}{u_c}}{1 + \frac{S_c}{S_b}} \right). \]  

(3.22)

Dividing Eq. (3.22) by \( u_c \) gives

\[ Z_c = \frac{S_c}{S_b} Z_b - \frac{1}{4} K' \rho_0 \left( \frac{1 + \frac{S_c}{S_b}}{1 + \frac{u_b}{u_c}} \right), \]  

(3.23)

where \( Z_c \) and \( Z_b \) are the impedances at the respective locations.

The acoustic pressure at any point \( x_i \) in the intake pipe can be found using the following procedure. The impedance at location B can be determined by setting \( x_i = l \) in Eq. (3.13) so that

\[ Z_b = -Z_i \tanh \alpha l, \]  

(3.24)

where Eqs. (3.7) and (3.8) can be substituted for \( \varepsilon \) and \( \chi \). From Eqs. (3.24) and (3.23), \( Z_c \) can then be readily evaluated. An impedance for the cylinder volume can be written similar to that developed for the intake pipe in Eq. (3.12) considering that the piston is a moving boundary \( [x_2 = x_2(t)]\):

\[ Z_D = \frac{P_D}{u_D} = \frac{Z_c - Z_2 \tanh \alpha x_2}{1 - (Z_c / Z_2) \tanh \alpha x_2}. \]  

(3.25)

Ignoring the effect of viscous losses in the cylinder eliminates the real parts of \( Z_2 \) and \( \alpha \), reducing \( Z_D \) to

\[ Z_D = \frac{P_D}{u_D} = \frac{Z_c - i \rho_0 c_0 \tan (\omega x_2 / c_0)}{1 - (i Z_c / \rho_0 c_0) \tan (\omega x_2 / c_0)}. \]  

(3.26)
Note
\[ x_2 = x_0 + \frac{1}{2} S (1 - \cos \theta) \]  \hspace{1cm} (3.27)

is a function of crank angle $\theta$, where $x_0$ is the clearance length at TDC. $Z_D$ can now be evaluated because $Z_c$ required for its calculation can be taken from Eq. (3.23). Differentiating Eq. (3.27) gives
\[ u_D = \frac{dx_2}{dt} = \frac{1}{2} S \omega \sin \omega t, \]  \hspace{1cm} (3.28)

where $\omega t = \theta$, and $t = 0$ corresponds to the TDC of the induction stroke. Since $u_D$ and $Z_D$ are known from Eqs. (3.26) and (3.28) respectively, $P_D$ can be calculated as
\[ P_D = Z_D u_D \]  \hspace{1cm} (3.29)

or, similar to Eq. (3.10),
\[ P_D = P_c \cosh \alpha x_2 - Z_c u_c \sinh \alpha x_2. \]  \hspace{1cm} (3.30)

Neglecting viscous effects in the cylinder reduces Eq. (3.30) to
\[ P_D = P_c \left[ \cos \frac{\alpha x_2}{c_0} \left( \frac{i \rho c_0}{Z_c} \right) \sin \frac{\alpha x_2}{c_0} \right]. \]  \hspace{1cm} (3.31)

or, solving for $P_c$,
\[ P_c = \frac{P_D}{\cos \frac{\alpha x_2}{c_0} \left( \frac{i \rho c_0}{Z_c} \right) \sin \frac{\alpha x_2}{c_0}}. \]  \hspace{1cm} (3.32)
Since $Z_C$ is known, $u_C$ can be determined from $u_C = \frac{P_c}{Z_C}$, then $u_B$ and $P_B$ can be calculated from $u_B = \frac{S_C}{S_B} u_C$ and $P_B = Z_B u_B$, respectively. Finally, using Eq. (3.10) and the value of $P_B$, the “steady state” pressure in the intake pipe becomes

$$P_i = P_B \frac{\sinh \alpha x_i}{\sinh \alpha l},$$

(3.33)

where $P_B$ is the pressure just outside the intake valve.

The total pressure during the IV open period is

$$P_{ni} = P_i + P_R,$$

(3.34)

where the “transient” pressure $P_R$, according to Driels, is of the form

$$P_R = e^{-\frac{1}{2 \rho_0 l}} (B_1 \cos \beta t + B_2 \sin \beta t),$$

(3.35)

with $B_1$ and $B_2$ being real constants. Driels also suggests that the value of $\beta$ in Eq. (3.35) can be approximated as the resonant frequency of the ‘Helmholtz resonator’ considered with the intake pipe as the neck and the varying cylinder volume (due to piston motion) as the cavity, leading to

$$\beta = c_0 \sqrt{\frac{S_B}{S_C x_l^2}}.$$

(3.36)

Combining Eqs. (3.33)- (3.35) gives

$$P_{ni}(t) = P_B \frac{\sinh \alpha x_i}{\sinh \alpha l} + e^{-\frac{1}{2 \rho_0 l}} (B_1 \cos \beta t + B_2 \sin \beta t).$$

(3.37)

Equation (3.37) must be evaluated for the entire duration of the IV open period. The crank angle at which the intake valve closes, is not specified in Driels’ model. If IVC is
assumed as the BDC of the induction stroke, Eq. (3.37) must be evaluated for all $t$ such that $\beta t \leq \pi$, where $t = 0$ corresponds to the TDC of the induction stroke. Using the simple initial conditions

$$P_{x_1}|_{t=0} = \frac{dP_{x_1}}{dt}|_{t=0} = 0,$$  

(3.38)

the constants $B_1$ and $B_2$ in Eq. (3.37) can be calculated as

$$B_1 = -P_{x_1}|_{t=0},$$  

(3.39)

and

$$B_2 = \frac{1}{\beta} \frac{dP_{x_1}}{dt}|_{t=0}.$$  

(3.40)

During the IV closed period, the pressure is assumed to behave as a damped circular function

$$P_L = e^{-\frac{1}{2} \rho_c t_i} \left( X_1 \cos \kappa t_i + X_2 \sin \kappa t_i \right),$$  

(3.41)

where $X_1$ and $X_2$ are real constants, $t_i = t - t_c$ is the time elapsed after the closing of the intake valve, with $t_c$ being the instant when the intake valve closes. Driels approximates $\kappa$ by the fundamental quarter wave resonance frequency of the intake duct given as

$$\kappa = 2\pi \frac{C_0}{4l},$$  

(3.42)

Constants $X_1$ and $X_2$ in Eq. (3.41) can be calculated by equating the instantaneous pressure and the slope of the pressure at IVC ($t_i = 0$), so that

$$X_1 = -P_{x_1}|_{t_i=0},$$  

(3.43)
\[ X_2 = -\frac{1}{\kappa} \left. \frac{dP}{dt} \right|_{t_\kappa=0} \]  \hspace{1cm} (3.44)

Having described Driels' technique, the pressure predicted by his model is compared next with the experimental data at location i2. Figures 3.2-3.5 show the variation of acoustic pressure vs. CAD for four different engine speeds of 2500, 3500, 4500, and 5500 rpm, respectively. Additionally, Fig. 3.2 shows the components of acoustic pressure – the "steady state" and the "transient" parts used in Driels' method.

![Graph of Acoustic Pressure vs. CAD](image)

Figure 3.2 Acoustic pressure vs. CAD (location i2; 2500 rpm).
Figure 3.3 Acoustic pressure vs. CAD (location i2; 3500 rpm).

Figure 3.4 Acoustic pressure vs. CAD (location i2; 4500 rpm).
From Figs. 3.2-3.5 substantial differences are observed in the amplitude, frequency and phasing of the pressure between the predictions and the experimental data, thereby reflecting upon the limitations of Driels' method. The deviations may partly be attributed to the assumptions used in the model. For example, the consideration represented by Eq. (3.38) is clearly invalid in light of the experimental data. Another shortcoming of Driels' model is that the variations in the amplitude of pressure pulsations are not accounted for across different engine speeds as illustrated in Fig. 2.7. Hence, there is a clear need to develop an improved technique to predict the pressures in the intake system. Such a technique is introduced and elaborated in the next chapter.
CHAPTER 4

AN IMPROVED METHOD TO PREDICT PRESSURE AND
VOLUMETRIC EFFICIENCY - MODEL 2

Following the limitations of Driels’ method, there is a need to develop a better analytical method capable of modeling the physics of the induction system. The baseline case of single cylinder experiments offers an opportunity for the development of such a model to predict acoustic pressures due to its relatively simple geometry, where the intake can be treated as a single long pipe attached to the port. Thus, the present chapter provides an improved method to calculate the oscillating pressure for the baseline case, and extends the technique to volumetric efficiency. As explained earlier in Section 1.2.3, the behavior of intake pressure during the IV open and closed periods is distinguished from each other, which has been utilized in the present chapter. The ‘organ-pipe’ approach of Morse et al. (1938) has been used to calculate the pressure during the IV open period, whereas Driels (1975) is followed for the IV closed period. The pressure predictions for these two periods are covered separately in Sections 4.1 and 4.2, followed by a discussion of results and the model in Sections 4.3 and 4.4, leading to the volumetric efficiency estimates presented in the subsequent Section 4.5. The tuning peak locations as
predicted by the present approach and the Engelman’s method (1973) will be compared with the experimental data in Section 4.6.

4.1 Calculating the instantaneous pressure during the IV open period

Figure 4.1 shows a simplified acoustical representation of the intake pipe, combustion chamber and piston of the SC engine connected to a straight intake duct (baseline case) of diameter \( d_p \) and cross-sectional area \( A_p \). Following Morse et al. (1938), the intake valve can be modeled as an oscillating “plug” of air (for simplicity the word “air” is used in this Chapter instead of “air-fuel mixture” or “charge”). This imaginary plug, which acts as the source of acoustic waves in the intake pipe has an effective cross-sectional area \( A_v \), and an effective volume \( V_v \). The cross-sectional area of the cylinder bore is \( A_c \) and the cylinder displacement volume is denoted by \( V_d \).

![Diagram of SC engine intake system](image)

Figure 4.1. A simplified acoustical model of the SC engine with a straight intake pipe.
The geometric length \( l \) of the intake pipe must be corrected by adding an end correction due to the effect of fluid inertia at the interface between the pipe and the ambient. Since the open end of the intake pipe terminates with a flanged bellmouth (see Fig. 4.1), an end correction corresponding to that case must be used. The value of such a correction depends on the Helmholtz number \( H = k_1 \left( \frac{d_p}{2} \right) \), where \( k_1 \) refers to the wave number of the acoustic waves in the intake pipe. For the case of small Helmholtz numbers in the range \( 0 - 0.1 \), the end correction can be assumed almost constant (Selamet et al., 2001b) and independent of the engine speed. If the highest possible values of \( k_1 \) can be shown to correspond to a Helmholtz number of 0.1, the end correction can be set to a constant irrespective of the engine speed. It will be mathematically shown later in this Chapter that the upper bound for the angular frequency (and hence wavelength) of the acoustic pulses at location i2 in the intake pipe (refer to Fig. 4.1) during the IV open period, will correspond to the dominant frequency of the pressure oscillations, which in turn, can be approximated by the fundamental quarter-wave resonance (angular) frequency \( \omega_i = 2\pi f_i \), where \( f_i \) is given as \( f_i = \frac{c_0}{4l} \).

Using the value of the dominant wavelength defined as \( k_1 = \frac{\omega_i}{c_0} = \frac{2\pi f_i}{c_0} \), and the intake pipe diameter \( d_p = 4.2 \text{ cm} \), \( H = k_1 \left( \frac{d_p}{2} \right) \) may readily be shown to be \( \approx 0.1 \). The end correction for this range \( (0 - 0.1) \) is nearly a constant \( 1.2 \frac{d_p}{2} \), Selamet et al., 2001b) and independent of frequency based on \( H \approx 0.1 \). This correction has to be added to the
geometric length of the pipe to account for the fact that the pressure oscillations extend
beyond \( l \). The value of \( l \) hereafter will refer to the corrected value of the pipe length.

The fact that the pressure pulses in the intake pipe have a predominantly planar
character can be demonstrated readily (Munjal, 1987) by

\[
f_c < \frac{1.84c_0}{\pi d_p},
\]

where \( f_c \) represents the cut-off frequency of the first diametral mode for wave
propagation in a circular duct of diameter \( d_p \). For an intake pipe of diameter \( d_p = 4.2 \text{ cm} \),
\( f_c \approx 4800 \text{ Hz} \), which is lot higher than the fundamental quarter wave resonance
frequency \( f_1 \approx 133 \text{ Hz} \). This shows that a one-dimensional model would be sufficient to
study the acoustic characteristics of the induction process for the geometry under
consideration.

Helmholtz resonators typically consist of a cavity volume connected to a neck.
Figure 4.2 shows a general representation of a simple Helmholtz resonator of cavity
volume \( V_m \) and neck cross-sectional area \( A_n \). Gas vibrations occur due to the inertia of the
air column at the neck and the compliance of the air volume \( V_m \). These vibrations in the
neck, can be thought as the reciprocatory motion of a plug of air. The cylinder-plug
interaction can be considered analogue to a Helmholtz resonator (compare Figs. 4.1 and
4.2): the cylinder (at mid-stroke) acts as the volume and the plug as the neck. The lumped
air-mass (or the plug) in the neck oscillates due to the piston and valve motion.
Figure 4.2. An illustration of a simple Helmholtz resonator.

Figure 4.3. Force balance of the plug.

Figure 4.3 indicates the forces acting on the plug imagined to oscillate at the intake valve. The pressure $p_p$ is due to the motion of the piston during the induction stroke, whereas $p_v^0$ is the pressure exerted by the gas column in the intake pipe (during the IV open period). $p_v^0$ also represents the pressure in the intake pipe just outside the plug. The development of analytical expressions for $p_p$ and $p_v^0$ follows next.

The mean density of air is $\rho_0$ and the effective mass of the air plug during the IV open period is given by $m_p = \rho_0 V_v$, where $V_v$ is the effective volume of air. Introduce the mean cylinder volume (at mid-stroke) as $V_m = V_c + \frac{V_d}{2} = \frac{V_d(r_c + 1)}{2(r_c - 1)}$, where $V_c$ is the clearance volume and
\[ r_c = \frac{V_c + V_d}{V_c} \]  

is the engine compression ratio. The mean stiffness constant for the plug due to the volume of air contained within the cylinder is \( K_p = \frac{\rho_o c_0^2 A_v^2}{V_m} \). Assuming \( \theta \) [CAD] is measured from the mid-stroke, that is \( \theta \) varies as \(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\) from TDC to BDC respectively, the instantaneous cylinder volume is given as

\[ V(\theta) = V_c + \frac{V_d}{2}(1 + \sin \theta) \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \quad (4.2) \]

\[ \Rightarrow \quad V(\theta) - V_c = \frac{V_d}{2}(1 + \sin \theta) = 2V_v \frac{r_c - 1}{2(r_c + 1)}(1 + \sin \theta) \]

\[ \Rightarrow \quad \frac{V(\theta) - V_c}{V_m} = \frac{r_c - 1}{r_c + 1}(1 + \sin \theta). \quad (4.3) \]

The left hand side of Eq. (4.3) represents the instantaneous strain of the cylinder volume due to the piston motion. Thus, the acoustic pressure associated with this volume strain, assuming an isentropic process for the wave motion in ideal gas may be written as

\[ p_p = \rho_o c_0^2 \left[ \frac{V(\theta) - V_c}{V_m} \right]. \quad (4.4) \]

From Eqs. (4.3) and (4.4), the fluctuating pressure exerted by the piston on the plug of air is given as

\[ p_p = \rho_o c_0^2 \frac{r_c - 1}{r_c + 1}(1 + \sin \theta) \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}. \quad (4.5) \]
Expressions for the acoustic velocity and the impedance will be developed next, so that
the fluctuating pressure just outside the plug may be determined.

Let $x$ represent the displacement of the air plug (see Figs. 4.1 and 4.3) and $R_p$
the mean resistance (the plug encounters resistance due to the frictional forces). The force
balance for the air plug (Morse et al., 1938) is then

$$m_p \ddot{x} + R_p \dot{x} + K_p x = A_v (p_p + p_v^0). \tag{4.6}$$

Substituting Eq. (4.5) into (4.6),

$$m_p \dddot{x} + R_p \dot{x} + K_p x = A_v \left[ \rho_0 c_0^2 \frac{(r_e - 1)}{(r_e + 1)} (1 + \sin \theta) + p_v^0 \right]. \tag{4.7}$$

For the angular frequency of the crankshaft $\omega = \frac{2\pi N}{60} = \frac{\theta \text{ rad}}{t \text{ s}},$ with $N \text{ [RPM]}$ being the
engine speed and $t = 0 \text{[s]}$ corresponding to mid-stroke piston position (instantaneous
volume of the cylinder during mid-stroke is $V_m$), Eq. (4.7) can be written as

$$m_p \dddot{x} + R_p \dot{x} + K_p x = A_v \left[ \rho_0 c_0^2 \frac{(r_e - 1)}{(r_e + 1)} (1 + \sin \omega t) + p_v^0 \right]. \tag{4.8}$$

Both $p_v^0$ and $x$ are function of time $t$. Morse et al. (1938) assumed that the mass and
resistance of the plug were negligible. Following the same assumption here, Eq. (4.8)
reduces to

$$K_p x = \frac{\rho_0 c_0^2 A_v^2}{V_m} x = A_v \left[ \rho_0 c_0^2 \frac{(r_e - 1)}{(r_e + 1)} (1 + \sin \omega t) + p_v^0 \right] \tag{4.9}$$

indicating an essentially “stiffness-controlled” plug motion. For further reading on the
relative importance of the individual terms in Eq. (4.8), the reader is referred to Morse
(1948). Even with the neglect of the mass and resistance terms, Eq. (4.9) contains two
unknowns $x$ and $p_v^0$. From the SC experimental data, $(p_v^0)_{\text{peak}} \sim 36$ kPa at an engine speed of 5500 rpm and $(p_v^0)_{\text{peak}} \sim 16$ kPa at 2500 rpm. By straightforward substitution,

$$\rho_0 c_0^2 \frac{(r_e - 1)}{(r_e + 1)} \approx 118 \text{ kPa. Though the second term on the right hand side of Eq. (4.9),}$$

$p_v^0(t)$, is not significantly smaller than $\rho_0 c_0^2 \frac{(r_e - 1)}{(r_e + 1)}$ at all engine speeds, it may still be ignored to simplify the calculation based on the work of Morse et al. (1938). Thus, $x$ is expressed as

$$x(t) = \frac{V_m (r_e - 1)}{A_v (r_e + 1)} (1 + \sin \omega t) \quad -\frac{\pi}{2} \leq \omega t \leq \frac{\pi}{2}. \quad (4.10)$$

Since $p_v^0$ is neglected in Eq. (4.10), this equation represents the motion of the air plug when the gas pressure in the intake pipe ($p_v^0$) is assumed not to influence the charge motion through the valve, that is, the plug is taken to oscillate purely due to the motion of the piston. Equation (4.10) is clearly an approximation used to simplify the problem and is not an exact solution of Eq. (4.9). Exact solution of Eq. (4.9) is complicated by the fact that $p_v^0$ and $x$ are both unknowns.

Assuming that the intake valve opens at TDC and closes at BDC of the induction stroke, Eq. (4.10) can be differentiated to yield the instantaneous velocity of the air plug. The validity of this assumption can be assessed by examining the intake and exhaust valve lift profiles. Figure 4.4 shows the valve profiles measured as a part of the experiments carried out on the SC engine (Mariucci, 2006). From his measurements, Mariucci determined that when the TDC of combustion is taken as $0^\circ$ CAD, the clocked
opening and closing of the intake valve occur at 308° and 594°, respectively. Note from
Fig. 4.4 that the maximum instantaneous valve flow area lies between the TDC and BDC
of the induction stroke (360° – 540°). While the valve area between 308° – 360° CAD in
Fig. 4.4 would not be completely negligible, such an approximation is necessary here for
the purpose of simplifying the pressure calculation, and the comparisons with the
experimental pressure (to follow later in Section 4.3) will show that this is a reasonable
approximation. The same argument may be applied to neglect the area between 540-594
CAD. However, the volumetric efficiency calculation will require a correction to account
for the valve open duration 308° – 360° CAD as will be described in Section 4.5. The
instantaneous plug velocity can be obtained by differentiating Eq. (4.10) for the IV open
period, and is set to zero for the closed period, so that

\[
x(t) = \frac{V_m (r_c - 1)}{A_c (r_c + 1)} \omega \cos \omega t \quad \frac{-\pi}{2} \leq \omega t \leq \frac{-\pi}{2}
\]

\[
= 0 \quad \frac{-\pi}{2} < \omega t < \frac{7\pi}{2}.
\]

Equation (4.11) represents the real part of acoustic velocity of the plug as a harmonic
function due to the presence of the circular function. In terms of the definition of \( \omega \),
rearranging the amplitude in Eq. (4.11) gives

\[
x(t) = \frac{V_m (r_c - 1) \pi N}{30 A_c (r_c + 1)} \cos \omega t \quad \frac{-\pi}{2} \leq \omega t \leq \frac{-\pi}{2}
\]

\[
= 0 \quad \frac{-\pi}{2} < \omega t < \frac{7\pi}{2}.
\]
\( x(t) \) in Eq. (4.12) can now be expanded to a Fourier series to determine the frequency components of the acoustic velocity:

\[
x(t) = \frac{V_0(r_e - 1)\pi N}{30 A_r(r_e + 1)} \sum_{n=0}^{\infty} B_n \cos \frac{n\omega t}{2},
\]

where \( B_n \) are the coefficients given by

\[
B_0 = 0.5 \quad B_2 = \frac{\pi}{4} \quad B_n = -\frac{4}{n^2 - 4}\cos\frac{n\pi}{4} \quad (0 \neq n \neq 2).
\]

Equation (4.13) illustrates an important aspect of the present approach that the imaginary plug of air oscillating at the intake valve acts as an acoustic driver with a fundamental (angular) frequency equal to \( \frac{\omega}{2} \), which is the same as the frequency with which the
intake valve actuates and equal to the first engine half-order $0.5(N/60)$. This feature can be clearly seen from the frequency domain data of the pressure pulses in the intake pipe, which will be presented later in this Chapter.

Acoustic impedance provides a link between acoustic velocity and pressure. Since acoustic impedance, in general, depends predominantly on known quantities such as geometry, frequency, density and speed of sound, it is easy to calculate the fluctuating pressure from the knowledge of acoustic impedance and velocity. The spectral acoustic impedance (the term ‘spectral’ refers to the fact that the acoustic impedance is a function of the frequency component $\frac{n\omega}{2}$) at the intake valve is given as (see Appendix A for the derivation)

$$Z_n = -\frac{A_v}{A_p} \rho_c c_o \tanh \left[ Fl + i \frac{n\omega}{2c_o} \right] = H_n e^{i\theta_n}, \quad (4.14)$$

where $F$ is the frictional resistance term due to viscous drag of the air in the plug and $Z_n$ is a complex number of magnitude $H_n$ and phase angle $\theta_n$. Considering that valve effects are complicated, the value of $F$ used for the current model can be calculated from the treatment of the shear term of the N-S equations used in MANDY, and this has been demonstrated in Appendix B. Setting the base value for $F$ as 0.165 at 2500 rpm, a linear interpolation has been performed to determine $F$ at various engine speeds, as derived in Appendix B and plotted in Fig. 4.5. Note that the wall friction factor (and hence the shear stress term) in MANDY applies to the entire intake system, whereas the friction factor $F$ refers specifically to the intake valve where the flow-field is quite complex. It is also relevant to point out that the $F$ used in the current model is much larger that that used by
Morse et al. (1938) who used 0.0484 [m⁻¹] for their model corresponding to an engine speed of 1600 rpm and pipe lengths varying from 1-3 m. The range used here $F_1 \leq F \leq F_2$, where $F_1=0.165$ and $F_2=0.4$ therefore, is based on experiments performed at much higher speeds and with a shorter intake pipe. A linear profile is regarded reasonable considering the complexity of the viscous and separation losses near the intake valve.

![Graph](image)

**Figure 4.5.** Frictional resistance $F$ vs. engine speed.

Using Eqs. (4.13)-(4.14) and the definition of impedance, the real part of acoustic pressure at the intake valve is expressed as

$$P_r^0\left(\frac{n\omega}{2},t\right) = \text{Re}\left[Z_n\left(\frac{n\omega}{2}\right)\tilde{u}_n\right],$$
where \( x(t) = \text{Re} \left[ \tilde{u} \right] \) with \( \tilde{u}_n \) being the \( n \)th frequency component of acoustic velocity (\( \tilde{u} \)).

Or, explicitly

\[
p_c^0(t) = \frac{-V_m(r_c - 1)N\rho_0c_0}{30(r_c + 1)A_p} \sum_{n=0}^{\infty} B_n H_n \cos(n\frac{\omega}{2}t + \theta_n).
\]

Equation (4.15) is an approximate expression for pressure just outside the plug when the intake valve is open (TDC to BDC). Pressure transducer i2 is located upstream of the plug and the pressure at that point can be calculated using duct acoustics relations.

Considering a case of an open-open duct (shown in Fig. 4.6) for the IV open period and ignoring the effect of spatial wave damping from the plug to location i2, the acoustic pressure in the intake pipe is written as

\[
p_c^0(\xi, t_2) = e^{i\Omega t_2} \left( C_1 e^{-\frac{i\Omega}{\rho_0} \xi} + C_2 e^{\frac{i\Omega}{\rho_0} \xi} \right),
\]

where \( \Omega \) is the angular frequency of the acoustic waves propagating in the intake pipe (\( \Omega \) is unknown at this point), \( \xi [m] \) the distance measured from the ambient end (see Fig. 4.6), and \( t_2 \) is the time [s]. A suitable approximation for \( \Omega \) is needed here, since the pressure in the intake pipe is comprised of different frequency components [Eq. (4.15)].
Figure 4.6 The intake pipe treated as an open-open duct during the IV open period.

The boundary conditions for the intake pipe are given as

\[ p_x^0(\xi = 0, t_2) = 0 \]
\[ \text{Re}[p_x^0(\xi = l, t_2)] = p_v^0(t) \]  \hfill (4.17)

where ‘Re’ refers to the real part of the acoustic pressure. The expression for pressure during the IV open period at any point \( \xi \) measured from the ambient end of the intake pipe, is then calculated as

\[ \text{Re}[p_x^0(\xi, t_2)] = p_v^0(t) \frac{\sin[(\Omega/c_0)\xi]}{\sin[(\Omega/c_0)l]} \]  \hfill (4.18)

The value of \( \xi \) in Eq. (4.18) corresponding to the location of the pressure transducer i2 can be determined from the geometry of the intake pipe and is taken to be 0.4013 m. The approximation for \( \Omega \), the frequency of wave-motion between the intake valve and the pressure transducer i2, is now discussed. Figures 4.7 – 4.10 show the frequency domain data of the pressure in the intake pipe at location i2 measured in the experiments (Mariucci, 2006), for four different speeds of 2500, 3500, 4500, and 5500 rpm. The
standard MATLAB function ‘FFT’ has been used to generate the data. The abscissa has been set in such a way that the dominant frequency components are clearly observable. Also, the first engine half-order is defined as \(0.5(N/60)\) and the various frequency components occur at integral multiples of this number \([0.5(N/60)]\).

The fundamental quarter-wave resonance frequency is \(f_i = \frac{c_0}{4l} = 133.14\) Hz. Figures 4.7 – 4.10 indicate that irrespective of the engine speed, the dominant frequency always lies close to \(f_i\) and that the magnitude of the higher frequency components of pressure is small. Thus, \(\Omega\) used in Eq. (4.18) is set to \(2\pi f_i\). This approximation essentially means that the pressure during the IV open period, comprised of different spectral components, is replaced by a single component. Further discussion on the frequency components of pressure will follow in Section 4.4. Thus, Eq. (4.18) reduces to

\[
\text{Re}\left[p_x^0(\xi, t_2)\right] = p_x^0(t) \frac{\sin[(\Omega/c_0)\xi]}{\sin[(\Omega/c_0)l]} \quad (4.19)
\]

and

\[
\text{Re}\left[p_x^0(\xi = \xi_t, t_2)\right] = p_x^0(t) \frac{\sin[(\Omega/c_0)\xi_t]}{\sin[(\Omega/c_0)l]} = p_x^0(t) \frac{\sin[(\Omega/c_0)\xi_t]}{1} = p_x^0(t), \quad (4.20)
\]

where \(\xi_t\) refers to the location of pressure transducer \(i2\) and \(p_i^0(t)\) the fluctuating pressure at that point.
Figure 4.7 Acoustic pressure vs. frequency (baseline case, location i2, 2500 rpm).

Figure 4.8 Acoustic pressure vs. frequency (baseline case, location i2, 3500 rpm).
Figure 4.9 Acoustic pressure vs. frequency (baseline case, location i2, 4500 rpm).

Figure 4.10 Acoustic pressure vs. frequency (baseline case, location i2, 5500 rpm).
4.2 Calculating the instantaneous pressure during the IV closed period

Following the work of Driels (1975), organ-pipe type damped oscillations of the form

\[ p_i^c(t_i) = e^{-\frac{1}{2} \beta t_i} (A \cos \omega t_i + B \sin \omega t_i) \] (4.21)

are considered during the IV closed period (at location i2) where \( A \) and \( B \) are real constants, \( \beta \) the damping factor, \( t_i \) the time, and \( \omega \) the frequency of the pressure pulses. Note that it is air motion near the intake valve that was modeled as the imaginary plug (refer to Section 4.1). The velocity of air molecules near the intake valve becomes zero after intake valve closing (IVC). However, pressure pulses persist during the IV closed period due to the fact that residual pressure from the open period set up standing waves in the intake pipe. Since the frequency of the pressure oscillations during IV open period can be approximated by the fundamental quarter-wave resonance (angular) frequency based on Section 4.1, \( \omega_i = 2\pi (c_0 / 4l) \). Time variable \( t_i = t - t_{ci} \) represents the time elapsed after IVC with \( t_{ci} \) being the instant when the intake valve closes. \( \beta \ [s^{-1}] \) indicates the wave damping. In view of the damping observed in the experimental pressure data and as established by Driels (1975), \( \beta \) can be set to a constant \( F_i c_0 \ [s^{-1}] \) for all engine speeds, where \( F_i = 0.165 \ [m^1] \) is the base value of friction at 2500 rpm (derived in Appendix B). Constants \( A \) and \( B \) in Eq. (4.21) can be calculated by equating the instantaneous pressures

\[ p_i^c(t_c) = \frac{-V_p (r_c - 1) N \rho_0 c_0}{30(r_c + 1)A_p} \sin[(\Omega / c_0)\xi_0] \sum_{n=0}^{\infty} B_n H_n \cos\left(\frac{\omega t_{ci}}{2} + \theta_n\right) \] (4.22)

\[ = p_i^c(t_i = 0), \]
and the slopes of the pressure at IVC

\[
\frac{dp^0}{dt}(t = t_c) = -\frac{V_m}{30(r_c + 1)} A_p \sin[(\Omega / c_0) \theta_i] \cdot \sin[(\Omega / c_0) \theta_i] \cdot \sum_{n=0}^{\infty} B_n H_n \cos\left(\frac{\text{not}}{2} + \theta_n\right) \bigg|_{t = t_c}
\]

\[
= \frac{dp^c}{dt_1}(t_1 = 0).
\]

(4.23)

Using Eqs. (4.22) and (4.23), the values of constants A and B in Eq. (4.21) can be easily calculated. This enables the complete description of the pressure at location i2 as

\[
p_i(t) = p_i^0(\omega t) \quad \frac{\pi}{2} \leq \omega t \leq \frac{\pi}{2}
\]

\[
= p_i^c(\omega t_1) \quad \frac{\pi}{2} < \omega t < \frac{7\pi}{2},
\]

(4.24)

for the IV open and closed periods respectively.

### 4.3 Results and discussion

Figures 4.11-4.24 show the acoustic pressure (at location i2) compared in time-domain with the experimental data, along with the predictions from the engine simulation code MANDY, for speeds ranging from 2500-5500 rpm. The experimental data and the simulation results are provided by Mariucci (2006). The IV open period has been shown first (in Figs. 4.11-4.24) followed by the closed one. Note that this period corresponds to the duration between TDC and BDC. The experimental data, as well as the MANDY predictions depict only the acoustic (oscillating) quantity, obtained by subtracting the total quantity from the mean value. The abscissa has been set such that 0° corresponds to the TDC of the induction stroke.

56
The overall behavior of the analytical estimates, at most speeds, matches reasonably well with the experimental data and the numerical predictions. The accuracy of the estimates, however, varies with engine speed. In general, the pressure calculated through the analytical model, shows a better match, in terms of amplitude and frequency with the experimental data for the IV closed period above 3000 rpm. The comparison between the analytical model and experiment is not satisfactory at 2500, 2750, and 3000 rpm. The phasing issues for speeds from 3250-5500 rpm may possibly be attributed to the neglect of areas under 308-360 CAD and 540-594 CAD in Fig. 4.4, and to the assumption of standing wave in the intake pipe through the use of Eq. (4.19). The reasonable comparison between the model results and the experimental data during the IV closed period at speeds higher than 3000 rpm, is primarily due to the fact that the frequency of the pressure pulses observed in the experimental data, is remarkably close to the fundamental quarter-wave resonance frequency used in the analytical predictions. The fundamental quarter-wave resonance frequency is approximately 133 Hz as mentioned earlier, and can be verified by observing the periodicity of the pressure wave in the experimental data. The pressure approximated by Eq. (4.21) is in essence a simple circular function of the form \( p(t) = e^{-\frac{1}{2} \rho_A} [C \sin(\omega t + \psi)] \), where \( C \) is the amplitude and \( \psi \) the phase. Therefore, this function provides a good fit to the waveform of the experimental data during the valve closed period.
Figure 4.11 Acoustic pressure vs. CAD (location i2; 2500 rpm).

Figure 4.12 Acoustic pressure vs. CAD (location i2; 2750 rpm).
Figure 4.13 Acoustic pressure vs. CAD (location i2, 3000 rpm).

Figure 4.14 Acoustic pressure vs. CAD (location i2, 3250 rpm).
Figure 4.15 Acoustic pressure vs. CAD (location i2; 3500 rpm).

Figure 4.16 Acoustic pressure vs. CAD (location i2; 3750 rpm).
Figure 4.17 Acoustic pressure vs. CAD (location i2; 3850 rpm).

Figure 4.18 Acoustic pressure vs. CAD (location i2; 4000 rpm).
Figure 4.19 Acoustic pressure vs. CAD (location i2; 4250 rpm).

Figure 4.20 Acoustic pressure vs. CAD (location i2; 4500 rpm).
Figure 4.21 Acoustic pressure vs. CAD (location i2; 4750 rpm).

Figure 4.22 Acoustic pressure vs. CAD (location i2; 5000 rpm).
Figure 4.23 Acoustic pressure vs. CAD (location i2; 5250 rpm).

Figure 4.24 Acoustic pressure vs. CAD (location i2; 5500 rpm).
The shortcomings of the present model are discussed next. The analytical results show deviations from the experimental data during the IV open period, partly due to complicated nature of the flow (and the acoustics) during induction. Since the pressure during the IV closed period depends on the open period, it is clear that the constants \( A \) and \( B \) used in Eq. (4.21) will have an impact on the pressure waveform of the closed period. The differences in the complete waveform (open and closed periods, combined) are more pronounced at the low end (2500-3000 rpm) and less visible at high speeds. The limitation of the model at low speeds is due in part, to its formulation and also to the failure to account for back-flow effects at the low end. This limitation will be explored further in Section 4.4. The deviations at the high end (5000-5500 rpm) may be attributed to nonlinearities and the simplifying assumptions such as zero mean flow and linear friction profile used in the current approach. Another drawback of the present model is the lack of continuity in the instantaneous pressure at \( \theta = 0^\circ \) and \( \theta = 719^\circ \), due to two different methods used to calculate the pressures during the IV open and closed periods. In light of the complex nature of the problem, the comparisons between theory and experiments for speeds ranging from 3250-5500 rpm is considered reasonable.

While time-domain information is important to observe the amplitude and phasing of the pressure waveform, frequency-domain data (generated using the ‘FFT’ command in MATLAB) is relevant to understand the magnitude of various components. Figures 4.25 - 4.38 show the comparisons between the analytical predictions for pressure and the experimental data, for engine speeds ranging from 2500-5500 rpm. The various frequency components occur at integral multiples of the first engine half-order as
mentioned in Section 4.2. The dominant frequency in the experimental data has been indicated in each figure.

Note from Figs. 4.25-4.38 that the magnitude of frequency components of approximately 200 Hz and higher (at all engine speeds) is small. Similar to the trends noticed in the time-domain results, the match between the experimental data and analytical predictions is poor at the low end (2500-3000 rpm) and improves with increasing speed. The magnitude of the dominant frequency component predicted by the model, closely matches the experimental data points at all speeds higher than 3000 rpm.

Figure 4.25 Acoustic pressure vs. frequency (location i2; 2500 rpm).
Figure 4.26 Acoustic pressure vs. frequency (location i2; 2750 rpm).

Figure 4.27 Acoustic pressure vs. frequency (location i2; 3000 rpm).
Figure 4.28 Acoustic pressure vs. frequency (location i2; 3250 rpm).

Figure 4.29 Acoustic pressure vs. frequency (location i2; 3500 rpm).
Figure 4.30 Acoustic pressure vs. frequency (location i2; 3750 rpm).

Engine Speed = 3750 rpm

125.0 Hz

Figure 4.31 Acoustic pressure vs. frequency (location i2; 3850 rpm).

Engine Speed = 3850 rpm

128.3 Hz
Figure 4.32 Acoustic pressure vs. frequency (location i2; 4000 rpm).

Figure 4.33 Acoustic pressure vs. frequency (location i2; 4250 rpm).
Figure 4.34 Acoustic pressure vs. frequency (location i2; 4500 rpm).

Figure 4.35 Acoustic pressure vs. frequency (location i2; 4750 rpm).
Figure 4.36 Acoustic pressure vs. frequency (location i2, 5000 rpm).

Figure 4.37 Acoustic pressure vs. frequency (location i2, 5250 rpm).
4.4 Mathematical basis of the current model

It would be helpful to study the interaction of variables in the current approach, in order assess the method's effectiveness, besides identifying the shortcomings. Hence, a discussion on the effect of friction factor $F$ and the frequency components of pressure follows. Also, the limitation of the present model at speeds of 2500-3000 rpm is worth investigating.

4.4.1 Effect of friction factor $F$

During the IV open period, the magnitude of pressure is decided by the coefficients of the summation term in Eq. (4.15). The coefficients $B_a$ are all constants and
independent of the engine geometry. Thus, the $H_n$ terms, as a function of $\tanh\left(FL + i\frac{no\ell}{2c_0}\right)$, have a significant impact on the nature of the instantaneous pressure.

It is important, therefore, to study the behavior of $\tanh\left(FL + i\frac{no\ell}{2c_0}\right)$ to compute the acoustic impedance at the intake valve as represented by Eq. (4.14). Figure 4.39 shows the variation of the magnitude of $\tanh(a + ib)$, plotted against $b$ for different values of $a$. As the parameter $a$ decreases, the magnitude of the function rises rapidly near $b = 1.5$. This condition can be compared to the use of the friction factor $F$ [m$^{-1}$] in $\tanh\left(FL + i\frac{no\ell}{2c_0}\right)$. As $F$ decreases, the magnitude of the function and hence the amplitude of pressure increases.

Figure 4.39 Variation of the function $\tanh(a + ib)$ for various values of ‘$a$’.
4.4.2 Frequency components of pressure and model limitations at low speeds

Having noted the influence of $F_1$ on the summation term
\[ \sum_{n=0}^{\infty} B_n H_n \cos(n \frac{\omega}{2} t + \theta_n) \] in Eq. (4.15), it is of interest to study the role of angular frequency $\omega$, and pipe length $l$ (imbedded within $H_n$), on pressure $p_0^0$. Here, it is of relevance to reintroduce the parameter $q = \frac{(c_0 / 4l)}{(N / 60)/2}$, originally defined in Chapter 1. A listing of engine speeds corresponding to the $q$ values close to 3-6 is given in Table 4.1, for a fixed length of intake pipe $l$. For instance, a speed of 3250 rpm results in $q \approx 4.91$, which is close to 5.

<table>
<thead>
<tr>
<th>S. No</th>
<th>Engine Speed N (RPM)</th>
<th>$q = \frac{(c_0 / 4l)}{0.5(N / 60)}$</th>
<th>Closest integer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2750</td>
<td>5.80</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>3250</td>
<td>4.91</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4000</td>
<td>3.99</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5250</td>
<td>3.04</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4.1 Listing of engine speeds corresponding to $q \approx 3 - 6$.

Boden (1936) in his PhD dissertation discusses the properties of the tanh function. From his work, it is inferred that the behavior of the tanh function used in Eq. (4.15) dictates that when $\frac{n}{q}$ is an odd integer, there is a sharp rise in the value of $H_n$. For example, with $q=1$, every odd value of $n$ satisfies this condition. Similarly for $q=3$ or 4, the $n=3$ and 4
terms in Eq. (4.15) take large values respectively. Writing Eq. (4.15) in a slightly different form:

\[
p_e^s(t) = \frac{-V_m (r_e - 1) N \rho c_0}{30 (r_e + 1) A_p} \sum_{n=0}^{\infty} B_n H_n \cos \left( \frac{\omega}{2} t + \theta_n \right)
\]

\[
= \frac{-V_m (r_e - 1) N \rho c_0}{30 (r_e + 1) A_p} \sum_{n=0}^{\infty} S_n,
\]

where \( S_n = B_n H_n \cos \left( \frac{\omega}{2} t + \theta_n \right) \) is the \( n \)th term of the series summation. Figure 4.40 shows the variation of \( S_n \) with respect to \( n \) for the four different engine speeds listed in Table 4.1, where the speeds correspond to the values of \( q \) close to 3, 4, 5, and 6.

Figure 4.40 \( S_n \) [as defined in Eq. (4.25)] w.r.t. \( n \) for approximate integral values of \( q \).
Figure 4.40 shows that any change in $q$ causes the $n^{th}$ ($n$ being the integer approximation of $q$) term of the summation [given by Eq. (4.25)] to peak, except when $q \approx 6$. As expected the $q \approx 3, 4,$ and $5$ conditions result in peaks at the corresponding $n$ values. However, for $q \approx 6$ this feature does not hold because $B_6=0$ in Eq. (4.13), and the $6^{th}$ term of the summation reduces to zero. This results in the corresponding frequency component of pressure during the IV open period, as calculated in the model to become very small. The result is a poor comparison between the analytical predictions and experimental data for pressure, at $q$ values of approximately 6 and higher. Fortunately this drawback is restricted to very few engine speeds of 2500-3000 rpm only, since the majority of the engine speeds that were tested lie in the range $q \approx 3-5$ where the model performs better.

From the behavior of the tanh function used in Eq. (4.15), as the index $n$ approaches the closest integral value of $q$, the corresponding frequency argument $n\frac{\omega}{2}$ in $\tanh\left(F \ell + i\frac{n\omega \ell}{2\ell_0}\right)$ causes a sharp rise in the value of $H_n$. Thus, the dominant frequency is $\frac{\text{int}(q)\omega}{4\pi}$, where $\text{int}(q)$ refers to the closest integer approximation of $q$. Hence $q$ would serve as a useful parameter to determine the dominant frequency of pressure. Table 4.2 shows the comparison between the dominant frequencies determined from the experimental pressure data (recall the frequency domain data plotted in Figs. 4.25 - 4.38) and those calculated from the integer approximations of $q$. The experimental observations for the dominant frequencies are listed in column A. The first engine half-order
0.5(N/60) is provided in column B. The subsequent column C shows the calculation for \( q \), and its integer approximation is indicated in column D. Column E shows the dominant frequency calculated using the integer approximation of \( q \). Columns A and E represent often identical values with the exception of 3500 rpm, where \( q \) is close to a half-integer. As a result, Table 4.2 illustrates the benefit of \( q \) in predicting the dominant frequencies in pressure pulses. The present model, therefore, shows satisfactory comparisons with the experimental results in frequency domain over a fairly large range of speeds, primarily because of its ability to predict the dominant frequency of pressure correctly, and the magnitude of the dominant frequency component with a reasonably good accuracy.
<table>
<thead>
<tr>
<th>S.No</th>
<th>Engine Speed (rpm)</th>
<th>Dominant Freq. [Hz] (from Expt. Data)</th>
<th>First half-order $0.5(N/60)$</th>
<th>$q = \frac{(c_o/4l)}{0.5(N/60)}$</th>
<th>$q$ rounded to the nearest integer [int(q)]</th>
<th>int(q)$\left(0.5\frac{N}{60}\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2500</td>
<td>125.0</td>
<td>20.83</td>
<td>6.38</td>
<td>6</td>
<td>125.0</td>
</tr>
<tr>
<td>2</td>
<td>2750</td>
<td>137.5</td>
<td>22.92</td>
<td>5.80</td>
<td>6</td>
<td>137.5</td>
</tr>
<tr>
<td>3</td>
<td>3000</td>
<td>125.0</td>
<td>25.00</td>
<td>5.32</td>
<td>5</td>
<td>125.0</td>
</tr>
<tr>
<td>4</td>
<td>3250</td>
<td>135.41</td>
<td>27.08</td>
<td>4.91</td>
<td>5</td>
<td>135.4</td>
</tr>
<tr>
<td>5</td>
<td>3500</td>
<td>116.7</td>
<td>29.17</td>
<td>4.56</td>
<td>5</td>
<td>145.9</td>
</tr>
<tr>
<td>6</td>
<td>3750</td>
<td>125.0</td>
<td>31.25</td>
<td>4.26</td>
<td>4</td>
<td>125.0</td>
</tr>
<tr>
<td>7</td>
<td>3850</td>
<td>128.3</td>
<td>32.08</td>
<td>4.15</td>
<td>4</td>
<td>128.3</td>
</tr>
<tr>
<td>8</td>
<td>4000</td>
<td>133.33</td>
<td>33.33</td>
<td>3.99</td>
<td>4</td>
<td>133.3</td>
</tr>
<tr>
<td>9</td>
<td>4250</td>
<td>141.7</td>
<td>35.42</td>
<td>3.76</td>
<td>4</td>
<td>141.7</td>
</tr>
<tr>
<td>10</td>
<td>4500</td>
<td>150.0</td>
<td>37.50</td>
<td>3.55</td>
<td>4</td>
<td>150.0</td>
</tr>
<tr>
<td>11</td>
<td>4750</td>
<td>118.8</td>
<td>39.58</td>
<td>3.36</td>
<td>3</td>
<td>118.7</td>
</tr>
<tr>
<td>12</td>
<td>5000</td>
<td>125.0</td>
<td>41.67</td>
<td>3.19</td>
<td>3</td>
<td>125.0</td>
</tr>
<tr>
<td>13</td>
<td>5250</td>
<td>131.3</td>
<td>43.75</td>
<td>3.04</td>
<td>3</td>
<td>131.3</td>
</tr>
<tr>
<td>14</td>
<td>5500</td>
<td>137.5</td>
<td>45.83</td>
<td>2.90</td>
<td>3</td>
<td>137.5</td>
</tr>
</tbody>
</table>

Table 4.2 Comparison of the experimental and theoretical dominant frequencies.
4.5 Volumetric efficiency calculation

The pressure estimates can be extended to calculate volumetric efficiency at various engine speeds. Equation (4.15) represents the pressure just outside the intake valve when the pressure exerted by the gas column \( p_v^0 \) in the intake pipe is neglected in comparison to that of the piston on the oscillating plug at the intake valve [recall the derivation of Eq. (4.10)]. Including \( p_v^0 \) in the calculations would lead to an expression for volumetric efficiency. This statement will become apparent when the deficiency of the pressure estimation technique described in Sections 4.1 and 4.2 is clarified.

Volumetric efficiency \( \eta_v \) is defined as the ratio between the actual volume of air drawn into the cylinder and the swept volume of the cylinder. Therefore, if \( x \) in Eq. (4.10) represents the displacement of air drawn into the cylinder and \( A_v \) the plug area, the instantaneous volume of air \( Q \) drawn into the cylinder during the IV open period will be

\[
Q = A_v x, \quad (4.26)
\]

where \( x \) is a function of \( \omega t \). At IVC, \( \omega t = \frac{\pi}{2} \), so the volumetric efficiency can be approximated as

\[
\eta_v \approx \left. \frac{Q}{V_d} \right|_{\omega t = \frac{\pi}{2}}, \quad (4.27)
\]

to represent the ratio between the volume of air trapped at IVC and the swept volume.

Since \( x = \frac{V_m(r_e - 1)}{A_v(r_e + 1)}(1 + \sin \omega t) \) is a simple sinusoidal function independent of \( q \), it is incapable of predicting the peaks of the volumetric efficiency. This problem can be
remedied if \( x \) were to include a \( q \) dependency - a consideration that forms the basis of the volumetric efficiency calculation described next.

As an approximation, and in order to incorporate the effect of \( q \), the pressure at the plug during the IV open period can be expressed as a simple circular function:

\[
p_v^0 \approx -P_k \cos(k \frac{\omega}{2} t - \varphi_k),
\]

(4.28)

where \( P_k \) is the amplitude of pressure, \( \varphi_k \) the phase, and \( k \) is an integer approximation of \( q = \frac{(c_o / A)}{0.5(N / 60)} \). Note that Eq. (4.28) is simply an approximation of Eq. (4.15) under the consideration that the pressure \( p_v^0 \) can be represented as a single frequency component of \( k \frac{\omega}{2} \). The importance of \( q \) and its integer approximation were described in Section 4.4. Rewriting Eq. (4.9),

\[
\frac{\rho_0 c_0^2 A_y^2}{V_m} x(t) = A_y \left[ \rho_0 c_0^2 \frac{(r_c - 1)}{(r_c + 1)} (1 + \sin \omega t) + p_v^0 \right] - \frac{\pi}{2} \leq \omega t \leq \frac{\pi}{2}
\]

\[
= 0
\]

\[
\frac{\pi}{2} < \omega t < \frac{7\pi}{2}
\]

(4.29)

and substituting Eq. (4.28) into Eq. (4.9) gives

\[
\frac{\rho_0 c_0^2 A_y^2}{V_m} x(t) = A_y \left[ \rho_0 c_0^2 \frac{(r_c - 1)}{(r_c + 1)} (1 + \sin \omega t) - P_k \cos(k \frac{\omega}{2} t - \varphi_k) \right] - \frac{\pi}{2} \leq \omega t \leq \frac{\pi}{2}
\]

\[
= 0
\]

\[
\frac{\pi}{2} < \omega t < \frac{7\pi}{2}.
\]

(4.30)
Defining the new variable \( \sigma = \frac{(r_e - 1)}{(r_e + 1)} \) and using Eq. (4.30),

\[
\frac{\rho_0 c_0^2 A_v}{V_m} x(t) = A_v \left[ \rho_0 c_0^2 \sigma (1 + \sin \omega t) - P_k \cos \left( k \frac{\omega}{2} t - \varphi_k \right) \right] \quad -\frac{\pi}{2} \leq \omega t \leq \frac{\pi}{2}
\]

\[
= 0 \quad \frac{\pi}{2} < \omega t < \frac{7\pi}{2}.
\]

(4.31)

Differentiating and rewriting Eq. (4.31) gives

\[
\dot{x}(t) = \frac{V_m \sigma N}{30 A_v} \left[ \pi \cos \omega t + \frac{k P_k}{2 \rho_0 \sigma c_0^2} \cos \left( k \frac{\omega}{2} t - \varphi_k \right) \right] \quad -\frac{\pi}{2} \leq \omega t \leq \frac{\pi}{2}
\]

\[
= 0 \quad \frac{\pi}{2} < \omega t < \frac{7\pi}{2}.
\]

(4.32)

Expanding the terms on the right hand side of Eq. (4.32) using the Fourier series and making the approximation that all terms except the \( k \)th one are negligible, yields

\[
\dot{x}(t) = \frac{V_m \sigma N}{30 A_v} \left[ B_k \cos \left( k \frac{\omega}{2} t \right) + \frac{P_k}{2 \rho_0 \sigma c_0^2} D_k \cos \left( k \frac{\omega}{2} t - \varphi_k \right) \right]
\]

(4.33)

where \( D_k = \frac{k \pi}{4} \). Using the relationship \( p_v^0 \left( \frac{n \omega}{2} , t \right) = \text{Re} \left[ Z_n \left( \frac{n \omega}{2} \right) \tilde{u}_n \right] \) (where \( x = \text{Re} \left[ \tilde{u} \right] \) with \( \tilde{u}_n \) being a component of acoustic velocity) between the impedance defined in Eq. (4.14) and the acoustic velocity in Eq. (4.33), gives an approximate relationship for the acoustic pressure at the plug during the IV open period

\[
p_v^0(t) = -\frac{\rho_0 c_0 V_m NH_k}{60 A_p} \left( \frac{2 \sigma B_k}{2 \rho_0 c_0^2} \cos \left( k \frac{\omega}{2} t + \theta_k \right) + \frac{P_k}{\rho_0 c_0^2} D_k \sin \left( k \frac{\omega}{2} t - \varphi_k + \theta_k \right) \right).
\]

(4.34)
Equating Eqs. (4.34) and (4.28) yields
\[
\cos\left(\frac{k \omega t}{2} - \varphi_k\right) = \left( C_k \cos\left[ \frac{k \omega t}{2} + \theta_k \right] + \delta_k \sin\left[ \frac{k \omega t}{2} - \varphi_k + \theta_k \right]\right),
\]  
(4.35)

where \( C_k = \frac{\rho c_w V_m N B_k H_k}{30 P_k A_p} \) and \( \delta_k = \frac{V_m N D_k H_k}{60 c_v A_p} \), are constants written in terms of engine parameters. Phase variable \( \varphi_k \) and amplitude factor \( P_k \) in Eq. (4.35) are unknowns, whereas \( \delta_k \) is a known quantity. The three circular functions in Eq. (4.35) can be expanded separately as follows:

\[
\cos\left(\frac{k \omega t}{2} - \varphi_k\right) = \cos\left(\frac{k \omega t}{2}\right) \cos \varphi_k + \sin\left(\frac{k \omega t}{2}\right) \sin \varphi_k,
\]  
(4.36)

\[
\cos\left(\frac{k \omega t}{2} + \theta_k\right) = \cos\left(\frac{k \omega t}{2}\right) \cos \theta_k - \sin\left(\frac{k \omega t}{2}\right) \sin \theta_k,
\]  
(4.37)

\[
\sin\left[ \frac{k \omega t}{2} + (\theta_k - \varphi_k) \right] = \sin\left(\frac{k \omega t}{2}\right) \cos(\theta_k - \varphi_k) + \cos\left(\frac{k \omega t}{2}\right) \sin(\theta_k - \varphi_k).
\]  
(4.38)

Using Eqs. (4.36)-(4.38) in Eq. (4.35) gives

\[
\cos\left(\frac{k \omega t}{2}\right)\left[\cos \varphi_k - C_k \cos \theta_k - \delta_k \sin(\theta_k - \varphi_k)\right] = 
\sin\left(\frac{k \omega t}{2}\right)\left[\delta_k \cos(\theta_k - \varphi_k) - C_k \sin \theta_k - \sin(\theta_k - \varphi_k)\right].
\]  
(4.39)

The two sides of Eq. (4.39) must be equal for any time instant \( t \). Hence the coefficients of the sine and the cosine terms on either sides of Eq. (4.39) containing the argument \( \left(\frac{k \omega t}{2}\right) \) must be zero, suggesting

\[
\cos \varphi_k - C_k \cos \theta_k - \delta_k \sin(\theta_k - \varphi_k) = 0,
\]  
(4.40)
and
\[
\delta_k \cos(\theta_k - \phi_k) - C_k \sin \theta_k - \sin(\theta_k - \phi_k) = 0. \tag{4.41}
\]
Simplifying and rearranging Eqs. (4.40) and (4.41) gives
\[
(\tan \phi_k) \delta_k \cos \theta_k - \left( \frac{C_k}{\cos \varphi_k} \right) \cos \theta_k = \delta_k \sin \theta_k - 1, \tag{4.42}
\]
and
\[
(\tan \phi_k) [1 - \delta_k \sin \theta_k] + \left( \frac{C_k}{\cos \varphi_k} \right) \sin \theta_k = \delta_k \cos \theta_k. \tag{4.43}
\]
Equations (4.42) and (4.43) can be written in a more compact form as
\[
\begin{bmatrix}
\delta_k \cos \theta_k & -\cos \theta_k \\
1 - \delta_k \sin \theta_k & \sin \theta_k
\end{bmatrix}
\begin{bmatrix}
\tan \phi_k \\
\frac{C_k}{\cos \varphi_k}
\end{bmatrix}
= \begin{bmatrix}
\delta_k \sin \theta_k - 1 \\
\delta_k \cos \theta_k
\end{bmatrix}. \tag{4.44}
\]
Thus, \( \tan \phi_k \) and \( \frac{C_k}{\cos \varphi_k} \), and hence \( \varphi_k \) and \( P_k \), can be easily calculated with a simple matrix inversion. Substituting \( x \) (required to determine \( \eta_v \)) from Eq. (4.31) into Eq. (4.26),
\[
Q = \sigma V_m \left[ (1 + \sin \omega t) - \frac{P_k}{\sigma \rho_o c_o^2} \cos \left( k \frac{\alpha}{2} t - \phi_k \right) \right]. \tag{4.45}
\]
Using \( V_d = 2\sigma V_m \) from Eq. (4.1), \( \eta_v \) becomes, in view of Eq. (4.27),
\[
\eta_v = \frac{\sigma V_m}{2\sigma V_m} \left[ (1 + \sin \frac{\pi}{2}) - \frac{P_k}{\sigma \rho_o c_o^2} \cos \left( k \frac{\pi}{4} - \phi_k \right) \right]
\]
\[
= \left[ 1 - \frac{P_k}{2\sigma \rho_o c_o^2} \cos \left( k \frac{\pi}{4} - \phi_k \right) \right]. \tag{4.46}
\]
Since volumetric efficiency depends on the time available for air to fill up the cylinder volume, it is heavily dependent on valve timing. In order to compare predictions using
Eq. (4.46) with the experimental results, the variable $\phi_k$ in Eq. (4.46) must be corrected to account for the actual valve timing used for the SC engine. Note that the current model assumes that the IV open period commences at TDC (or 360°) in Fig. 4.4, however in the SC experiments, the intake valve is set to open much before TDC (at 308°). The additional duration for which the intake valve stays open is given as $360° - 308° = 52°$. So $\phi_k$ must be replaced by $\phi_k + \zeta$ where $\zeta = \frac{52\pi}{180}$. Figure 4.41 shows the comparisons between volumetric efficiency calculated using Eq. (4.46) with the experimental measurements.

![Graph](image)

Figure 4.41 Volumetric efficiency vs. engine speed – analytical vs. experimental.
The magnitude of the calculated volumetric efficiency shows clearly noticeable deviations from the experimental data at certain engine speeds. The magnitudes are close near the speeds of 4500-5000 rpm, but show deviations near the peak volumetric efficiency at 3850 rpm, the low end (2500-2750 rpm), and the very high end (5250-5500 rpm). However, Eq. (4.46) predicts the overall form of the breathing curve, especially the location of the peak points. The deviation between the experiments and theory can be partly attributed to the approximations employed in the present model such as, the use of a single frequency component \( \frac{k \omega}{2} \) in Eq. (4.28) and one dominant term involving \( k \) in Eq. (4.34).

4.6 Location of the tuning peaks: Comparison with Engelman’s approach

The locations of the tuning peaks, as predicted by the present technique and Engelman’s method (1973) are compared here with the experimental data. Engelman (1973) suggested an expression [Eq. (1.2)] for calculating the speed where the highest volumetric efficiency is attained. Rewriting Eq. (1.2), the approximate speed for the highest tuning peak is

\[
N_E [\text{RPM}] = \frac{1}{K} \frac{60}{2\pi} c_0 \sqrt{\frac{A_p}{I V_m}}.
\]

(4.47)

Using \( K=2.1 \) as indicated by Engelman (1973) gives \( N_E = 4206.5 \) rpm. However, the experimental results show the location of the peak volumetric efficiency as 3750 rpm, which would correspond to
\[ K = \frac{1}{3750} \frac{60}{2\pi} c_0 \sqrt{\frac{A_p}{IV_m}} = 2.35. \]  

(4.48)

A value of \( K = 2.35 \) in Eq. (4.48) remains within the bounds of Engelman’s work, who suggested that \( K \) could vary between 2.1 and 2.5 depending on the valve timing. The exact dependency of \( K \) on the valve timing, however, is unclear.

Note that Eq. (4.47) is inherently confined to single peak. The experimental data for volumetric efficiency, as well as the results from the present model (refer to Fig. 4.41) indicate the location of the tuning peaks as 3000, 3750, and 4750 rpm. The values of \( q \), corresponding to these engine speeds can be extracted from Table 4.2 as 5.32, 4.26, and 3.36 respectively. These numbers are close but not equal to the integers 5, 4, and 3. Thus, the peaks of \( \eta_v \) are not exactly at integer \( q \) values. This observation is consistent with the conclusion reached by Morse et al. (1938) who state that the “condition of maximum supercharging is not necessarily exactly \( q = k \),” where \( k = 3, 4 \) or 5. His use of the term “maximum supercharging” refers to the tuning peaks. Using integer values of \( q \) (5, 4, and 3), provides a convenient method to calculate the location of the peaks of \( \eta_v \) through the use of

\[ N = \frac{120(c_0/4l)}{q}, \]

where \( c_0 = 345 \) [m/s] and \( l = 0.6478 \) [m]. A listing of engine speeds corresponding to integer values of \( q \) is compared in Table 4.3 with the experimental data. Differences are observed in the calculated and experimental values, with the largest deviation at \( q = 3 \).
<table>
<thead>
<tr>
<th>Integer values of $q$</th>
<th>Tuning peak (calculated) $N = \frac{q}{120(c_0/4l)}$ [RPM]</th>
<th>Tuning peak (experimental) [RPM]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3196</td>
<td>3000</td>
</tr>
<tr>
<td>4</td>
<td>3996</td>
<td>3750</td>
</tr>
<tr>
<td>3</td>
<td>5328</td>
<td>4750</td>
</tr>
</tbody>
</table>

Table 4.3 Calculated vs. experimental tuning peaks.
CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

Two time-domain based, one-dimensional analytical models to predict the intake dynamics of a SC engine have been investigated in the present study with the objective of understanding the wave action in the induction system. Both models (referred to as 1 and 2) ignore the influence of heat transfer, mean-flow and other complicating factors, and split an engine cycle into two periods -- the IV open and the closed. A mathematical treatment of the IV open period is more complicated than that of the closed one, due to the nature of flow and acoustics during induction. The methodologies used in both models are briefly described next.

Model 1, which represents Driels's approach, treats the pressure fluctuations during the IV open period as a sum of "steady state" and "transient" parts. During the IV closed period, the intake pipe is assumed to represent a 'closed-open' duct in which acoustic pulses propagate with the waveform of a damped circular harmonic function having a frequency close to that of the fundamental quarter-wave resonance. Model 1 has been shown in Chapter 3 to be unsatisfactory in its predictive accuracy for the acoustic pressures (at location i2) due to its simplistic nature and inherent
approximations. Model 2, on the other hand, is partly based on the ‘organ-pipe’ theory for the intake tuning originally developed by Morse et al. (1938), who proposed expressing the instantaneous pressure in the intake pipe as a Fourier series expansion of the frequency components of the engine half-order. However, they did not attempt to separate the behavior of the pressure during the IV open and closed periods. Model 2 further developed in the present study treats the IV open period as an ‘organ-pipe’, while relying on Driels’ approach of a ‘closed-open’ duct for the valve closed period. The results from this model for the acoustic pressure show a better agreement with the experimental data than Model 1. Model 2 also offers an opportunity to extend the pressure calculations to volumetric efficiency. Relative to the experiments, deviations are observed in the magnitude of predicted volumetric efficiency although the location of the tuning peaks is determined accurately. Following are the major conclusions and contributions of the present study:

1. Time-domain comparisons between the acoustic pressure predictions of Model 2 and the experiments reveal that the ‘organ-pipe’ model of the intake system during the IV open period, combined with the approach used by Driels (1975) for the closed period, provides a fair accuracy. The latter is guided by the observed correlation between the frequency components of the measured pressures with the fundamental quarter-wave resonances. Overall, Model 2 is an advancement over the technique employed by Driels because the results from the former provide for a reasonable correlation (with experiments) over a broad range of engine speeds (3250-5500 rpm), though the comparisons at the lower end (2500-3000 rpm) are poor.
2. Frequency-domain comparisons between Model 2 and experiments show a good match in the magnitudes of pressure at the dominant frequency. Also, a simple expression involving the use of frequency factor ‘\( q \)’ has been shown in Chapter 4 to be useful to predict the dominant frequency of intake pressures.

3. One of the concerns with the use of Morse’s ‘organ-pipe’ model was its applicability at higher speeds and shorter intake pipes. The approach outlined in Model 2 demonstrates that Morse’s concept could be extended for such operating conditions (similar to those of in the present study), provided the calibration variables (for example, friction factor ‘\( F \)’) are adjusted accordingly.

Model 2 offers a simple technique to study the air induction system of a SC engine from an acoustical perspective. Further research is needed into improving the results for volumetric efficiency because of its importance in engine design. The intake valve is a moving boundary that significantly influences the acoustics of the induction system. Hence, it is important to study the inertial effects near the valve. An experimental apparatus consisting of a cylinder and a stationary intake valve designed to represent the acoustic behavior of the intake system, is already available in the ‘Acoustic Research Laboratory’. Experiments to be conducted with this apparatus mounted on an impedance tube setup could help determine the end corrections near the valve. The same apparatus could also be used to measure the acoustic impedance across the valve. Extending this simple 1-D model for the SC engine to multi-cylinder engines would be another topic of interest for long-term research. Such a model should incorporate the effect of wave interactions in the intake plenum, the non-linearities, and the multi-dimensional effects.
Appendix A

ACOUSTIC IMPEDANCE AT THE INTAKE VALVE

Figure A.1 Schematic of the intake pipe – plug combination.

Figure A.1 represents the intake system of the single cylinder engine with the intake pipe modeled as a cylindrical duct (domain 1) connected to the intake valve represented as the plug (or neck) in Fig. 4.1. Introduce $A_v$ and $A_p$ as the cross-sectional areas of the plug and intake pipe respectively, and $l$ as the length of the intake pipe. The continuity and the Navier-Stokes (N-S) equation for a one-dimensional acoustic wave propagation within the intake pipe and the valve are written as

$$\frac{1}{\rho_0 c_0^2} \frac{\partial p}{\partial t} + \frac{\partial u}{\partial x} = 0,$$

(A.1)
\[ \frac{\partial p}{\partial x} + \rho_0 \frac{\partial u}{\partial t} + R_0 u = 0, \]  
\[ (A.2) \]

where \( p \) is the acoustic pressure, \( u \) the acoustic velocity, \( \rho_0 \) the mean fluid density, \( c_0 \) the mean speed of sound, \( R_0 \) the linearized viscosity term (Driels, 1975) of the N-S equation corresponding to the frictional losses at the plug (\( R_0 \) is unknown at this stage).

The solutions of Eqs. (A.1)-(A.2) for both domains 1 and 2 are of the form

\[ p_1 = e^{\frac{i\omega}{2}} \left( A_1 e^{\alpha_1 t} + A_2 e^{-\alpha_1 t} \right), \]  
\[ (A.3) \]

\[ p_2 = e^{\frac{i\omega}{2}} \left( B_1 e^{\alpha_2 t} + B_2 e^{-\alpha_2 t} \right), \]  
\[ (A.4) \]

where \( p_1 \) and \( p_2 \) are the acoustic pressures in the respective domains, \( t \) is the time, \( A_{1,2} \) and \( B_{1,2} \) are constants decided by the boundary conditions and \( \alpha \) a constant. \( \frac{\omega}{2} \) in the oscillating component \( e^{\frac{i\omega}{2}} \) represents the angular frequency corresponding the first engine half-order. This frequency corresponds to that of the acoustic driver considered at the intake valve. The acoustic velocities for the two domains can be determined by substituting Eqs. (A.3)-(A.4) into Eq. (A.1) as

\[ u_1 = e^{\frac{i\omega}{2}} \left( A_1 e^{\alpha_1 t} - A_2 e^{-\alpha_1 t} \right), \]  
\[ (A.5) \]

\[ u_2 = e^{\frac{i\omega}{2}} \left( B_1 e^{\alpha_2 t} - B_2 e^{-\alpha_2 t} \right), \]  
\[ (A.6) \]

where \( Z_i = \frac{2\rho_0 c_0^2}{i\omega} \) is the acoustic impedance and
\[ \alpha^2 = \frac{\omega}{2c_0^2} \left( \frac{iR_0}{\rho_0} - \frac{\omega}{2} \right). \]  
(A.7)

Equation (A.7) can be simplified using the binomial expansion:

\[ \alpha^2 = \frac{\omega}{2c_0^2} \left( \frac{iR_0}{\rho_0} - \frac{\omega}{2} \right) = -\frac{\omega^2}{4c_0^2} \left( -\frac{2iR_0}{\rho_0\omega} + 1 \right), \]  
(A.8)

\[ \alpha = \frac{i\omega \left( -\frac{2iR_0}{\rho_0\omega} + 1 \right)^{1/2}}{2c_0} = \frac{i\omega}{2c_0} \left( 1 - \frac{iR_0}{\rho_0\omega} \right) = \frac{i\omega}{2c_0} + \frac{R_0}{2c_0\rho_0} = \frac{R_0}{2c_0\rho_0} + \frac{i\omega}{2c_0}. \]  
(A.9)

\[ Z_1 = \frac{2\alpha c_0^2 \rho_0}{i\omega} \]  

\[ Z_1 = \frac{2c_0^2 \frac{\rho_0}{i\omega} \left( \frac{R_0}{2c_0\rho_0} + \frac{i\omega}{2c_0} \right)}{\omega} = \frac{-2c_0^2 \rho_0}{\omega} \left( \frac{R_0}{2c_0\rho_0} + \frac{i\omega}{2c_0} \right) \]  
\[ = -\rho_0 c_0 \left( \frac{2c_0 i\omega}{\rho_0} \left( \frac{R_0}{2c_0\rho_0} + \frac{i\omega}{2c_0} \right) = -\rho_0 c_0 \left( \frac{R_0}{\rho_0} \frac{i}{\omega} - 1 \right) \right). \]  
(A.10)

If \( \frac{R_0}{\rho_0} i \) \( \omega\) is ignored in comparison to unity (Boden, 1936), Eq. (A.10) reduces to

\[ Z_1 = \rho_0 c_0. \]  
(A.11)

In order to solve for the acoustic impedance of the intake valve, it is required to know the boundary conditions corresponding to the geometry shown in Fig. A.1. The relevant boundary conditions at \( x_1 = 0 \) and \( x_1 = l \) are given as:

\[ p_1(x_1 = 0) = 0 \]
\[ p_1(x_1 = l) = p_2(x_2 = 0) \]
\[ A_2 u_2(x_1 = l) = A_1 u_1(x_2 = 0). \]  
(A.12)

Using Eqs. (A.3)-(A.6) and (A.12),

\[ p_1 = e^{-\alpha x_1} \]  
(A.13)
\[ u_1 = -A_e \frac{i \omega}{2} \left( e^{\alpha_1 x_1} + e^{-\alpha_1 x_1} \right). \] (A.14)

Acoustic impedance at the intake valve

\[ Z = \frac{p_2}{u_2} \bigg|_{x=0} = \frac{p_1}{A_e u_1} \bigg|_{x=0} = \frac{A_e}{A_p} \frac{u_1}{u_1} \bigg|_{x=1}, \] (A.15)

where \( A_e \) and \( A_p \) are cross-sectional areas of the plug and the intake pipe respectively.

From Eqs. (A.9), (A.11) and Eqs. (A.13) - (A.15), the acoustic impedance becomes

\[ Z = -\frac{A_e}{A_p} \rho_0 c_0 \tanh(\alpha l). \] (A.16)

From Eq. (A.9), \( \frac{R_0}{2c_0 \rho_0} \) can be replaced with a single term \( F \) which represents the frictional losses and carries units of \( m^{-1} \). So the final expression for the acoustic impedance at the intake valve is given as:

\[ Z = -\frac{A_e}{A_p} \rho_0 c_0 \tanh(Fl + \frac{i \omega}{2c_0} l). \] (A.17)
THE CALCULATION OF FRICTION FACTOR \( F \)

The Navier-Stokes used to calculate the acoustic impedance at the intake valve (in Appendix A) is written as

\[
\frac{\partial p}{\partial x} + \rho_0 \frac{\partial u}{\partial t} + R_\mu \mu = 0.
\]  \( \text{(B.1)} \)

The momentum equation used in MANDY for a pipe of constant cross-sectional area is

\[
\frac{\partial}{\partial t} (\rho_T A U_T) + \frac{\partial}{\partial x} (\rho_T A U_T^2) + \frac{\partial (\rho_T A)}{\partial x} + \tau_w \phi = 0
\]

\[
\Rightarrow \quad \frac{\partial}{\partial t} (\rho_T U_T) + \frac{\partial}{\partial x} (\rho_T U_T^2) + \frac{\partial (\rho_T)}{\partial x} + \tau_w \phi = 0, \quad \text{(B.2)}
\]

where the subscript ‘T’ corresponds to the total quantities defined as

\[ U_T(x,t) = u_o(x) + u(x,t), \]
\[ p_T(x,t) = p_o(x) + p(x,t), \]
\[ \rho_T(x,t) = \rho_o(x) + \rho(x,t), \]

and A and \( \phi \) refer to the cross-sectional area and the perimeter of the pipe respectively. \( u_o(x), \ p_o(x), \) and \( \rho_o(x), \) are mean quantities representing the velocity, pressure, and the density of the charge in the intake pipe respectively. The shear stress is
\[ \tau_w = \frac{1}{2} f \rho_f U_f^2 \]
\[ \quad = \frac{1}{2} f (\rho_o + \rho)(u_o + u)^2 \]
\[ \quad = \frac{1}{2} f (\rho_o + \rho)(u_o^2 + 2u_o u + u^2) \]
\[ \quad = \frac{WF}{\text{Re}_D^{0.25}} (\rho_o + \rho)(u_o^2 + 2u_o u + u^2), \]

where \( f \) is a dimensionless constant representing turbulent frictional losses, \( WF \) the wall friction factor and \( \text{Re}_D \) the Reynolds number. Equating the shear terms in Eqs. (B.1) and (B.2) and using the definition \( R_0 = 2F \rho_o c_o \),

\[ \frac{WF}{\text{Re}_D^{0.25}} (\rho_o + \rho)(u_o^2 + 2u_o u + u^2) \frac{\varphi}{A} = R_0 u \]

\[ \Rightarrow \quad F = \frac{1}{2c_o} \frac{WF}{\text{Re}_D^{0.25}} \left( 1 + \frac{\varphi}{\rho_o u^2} \right) \frac{u_o^2}{u} \frac{2u_o u + u}{A}. \quad (B.3) \]

The perimeter to area ratio in Eq. (B.3) is simply

\[ \frac{\varphi}{A} = \frac{\pi D}{(\pi D^2 / 4)} = \frac{4}{D}, \]

where \( D \) is the diameter of the intake pipe. Mean flow velocity \( u_o \) in Eq.(B.3) near the intake valve is approximated as

\[ u_o \sim 2L \frac{N}{60}, \quad (B.4) \]

where \( L \) is the piston stroke length, with a corresponding Reynolds number of

\[ \text{Re}_D = \frac{\rho_o u_o D}{\mu}, \]
where \( \mu = 2 \times 10^{-5} \text{ kgm}^{-1}\text{s}^{-1} \) is the dynamic viscosity of the fluid near the intake valve, \( \rho_0 = 1.2 \text{ kgm}^{-3} \), and \( D = 4.2 \text{ cm} \). The acoustic velocity \( u(x,t) \) of the plug [used in Eq.(B.3)] corresponds to the instantaneous plug velocity \( \dot{x}(t) \) previously determined to be

\[
\dot{x}(t) = \frac{V_m (r_e - 1)}{A_v (r_e + 1)} \omega \cos \omega t \quad -\frac{\pi}{2} \leq \omega t \leq \frac{\pi}{2} \\
= 0 \quad \frac{\pi}{2} < \omega t < \frac{7\pi}{2},
\]

(B.5)

where \( A_v \) is the maximum valve flow area calculated as

\[ A_v = 2\pi r_v h_v, \]

where \( r_v \) is the radius of the valve curtain and \( h_v \) the maximum valve lift. For the SC engine, these dimensions are 3.5 cm and 0.95 cm respectively. Acoustic velocity \( u \) used in Eq. (B.3) can be substituted by the amplitude value \( |u| \) taken from Eq. (B.5) so that

\[ |u| = \frac{V_m (r_e - 1)}{A_v (r_e + 1)} \omega = \frac{V_m (r_e - 1)\pi N}{30 A_v (r_e + 1)}. \]

(B.6)

Since the terms on the right hand side of Eq. (B.3) (except \( \text{Re}_{D_e} \)) scale linearly with \( N \) as seen from Eqs. (B.4) and Eq. (B.6), \( F \) can be made to vary linearly from 2500-5500 rpm. Using \( c_0 = 345 \text{ m/s} \) and \( \text{WF} = 0.3 \), the values of \( F \) at various engine speeds can be easily calculated with the corresponding \( F \) at the lowest and highest engine speeds listed in Table B.1.

98
| Engine Speed | $u_*$ (m/s) | $Re_d$ | $|u|$ (m/s) | $F \approx$ |
|--------------|-------------|--------|-------------|-----------|
| N (rpm)      |             |        |             |           |
| 2500         | 6.58        | 16581.6| 30.81       | 0.165     |
| 5500         | 14.48       | 36489.6| 67.79       | 0.3       |

Table B.1. Friction factor $F$ at selected engine speeds.

The experimental data shows a good match with the analytical estimates, with the use of $0.165 \leq F \leq 0.4$ as the friction profile for speeds ranging from 2500-5500 rpm. This range $0.165 \leq F \leq 0.4$ is very close to the one in Table B.1.
Appendix C

NOMENCLATURE

\[ A \]
\text{cross-sectional area of the duct}

\[ a, b \]
\text{constants}

\[ A_{1,2}, B_{1,2} \]
\text{constant coefficients}

\[ A_c \]
\text{cross-sectional area of the piston}

\[ A_p \]
\text{cross-sectional area of the intake pipe}

\[ A_v \]
\text{effective flow area of the intake valve}

\[ B_n \]
\text{coefficients of the Fourier Series expansion}

\[ C \]
\text{amplitude of the wave}

\[ c_0 \]
\text{mean speed of sound}

\[ C_k \]
\text{parameter as defined in Eq. (4.35)}

\[ D \]
\text{diameter of the duct}

\[ D_k \]
\text{parameter as defined in Eq. (4.33)}

\[ d_p \]
\text{diameter of the intake pipe}

\[ F \]
\text{frictional resistance term}

\[ f \]
\text{dimensionless constant representing turbulent frictional losses}

\[ f_l \]
\text{fundamental quarter-wave resonance frequency of the intake pipe}

\[ F_{1,2} \]
\text{lower and upper limits of frictional resistance}

\[ f_c \]
\text{cutoff frequency for the first diametral wave propagation}

\[ H \]
\text{Helmholtz number}

\[ H_n \]
\text{absolute value of the spectral acoustic impedance}
$h_v$ maximum valve lift

$i$ imaginary unit ($=\sqrt{-1}$)

$K$ constant as defined in Eq. (1.2)

$k$ integer

$k_l$ wave number corresponding to the fundamental quarter-wave resonance frequency of the intake pipe

$K_p$ stiffness constant of the air plug considered at the intake valve

$K_v$ flow loss coefficient

$K_v'$ linearized flow loss coefficient

$l$ length of the intake pipe

$m_p$ mass of the air plug considered at the intake valve

$N$ engine speed [rpm]

$n$ index (0,1,2,3…)

$N_E$ location of the highest tuning peak as defined in Eq. (4.47)

$p$ acoustic pressure

$p_0$ mean pressure

$P_l$ acoustic pressure in domain 1

$P_2$ acoustic pressure in domain 2

$P_{A-D}$ pressures at locations A-D in Fig. 3.1

$p_{i2}$ complete description of acoustic pressure at location i2 (IVO+IVC)

$p_{i2}^0$ instantaneous pressure at location i2 during the IV open period

$p_{i2}^C$ instantaneous pressure at location i2 during the IV closed period

$P_k$ amplitude of the $k^{th}$ frequency component of pressure

$P_L$ acoustic pressure during the IV closed period as defined in Eq. (3.41)

$p_p$ pressure exerted by the piston

$P_R$ "transient" pressure

$P_{x_i}$ total pressure at location $x_i$ in the intake pipe

$p_v^0$ instantaneous pressure just outside the intake valve during the IV open period

$p_{\xi}^0$ instantaneous pressure at location $\xi$ during the IV open period
$P_x$  “steady state” pressure  
$q$  frequency factor  
$Q$  volume of air drawn into the cylinder during the induction stroke  
$R_0$  linearized viscosity  
$r_c$  compression ratio of the engine  
$Re_D$  Reynolds number  
$R_p$  mean frictional air resistance at the plug  
$r_v$  radius of the valve curtain  
$S$  piston stroke  
$S_B$  cross-sectional area at location B in Fig. 3.1  
$S_C$  cross-sectional area at location C in Fig. 3.1  
$S_n$  n$^{th}$ term of the Fourier Series expansion  
$t, t_2$  time  
$t_1$  = (t-$t_c$), time elapsed after intake valve closure  
$t_C$  intake valve closing time  
$u$  acoustic velocity  
$u_0$  mean velocity  
$U_1$  acoustic velocity in domain 1  
$U_2$  acoustic velocity in domain 2  
$u_{A-D}$  velocities at locations A-D in Fig. 3.1  
$U_T$  total velocity  
$u_v$  velocity across the valve  
$u_{vRMS}$  root mean square velocity across the valve  
$u_{n_i}$  component of acoustic velocity  
$V$  instantaneous volume of the cylinder  
$V_c$  clearance volume of the engine  
$V_d$  displacement volume of the cylinder  
$V_m$  mean swept volume of the cylinder  
$V_v$  effective volume of the air at the plug
\[ WF \quad \text{wall friction factor in MANDY} \]
\[ x \quad \text{instantaneous displacement of air plug} \]
\[ \tilde{u} \quad \text{acoustic velocity near the air plug} \]
\[ x_0 \quad \text{clearance length at TDC} \]
\[ X_{1,2} \quad \text{constants} \]
\[ x_{1,2} \quad \text{coordinate axes} \]
\[ y \quad \text{coordinate axis} \]
\[ y_i \quad \text{location of the pressure transducer i2} \]
\[ Z \quad \text{acoustic impedance at the intake valve as defined in Eq. (A.16)} \]
\[ Z_{1,2} \quad \text{acoustic impedance at the respective locations} \]
\[ Z_{A-D} \quad \text{acoustic impedance at locations A-D in Fig. 3.1} \]
\[ Z_n \quad \text{acoustic impedance of the n\textsuperscript{th} harmonic component} \]
\[ Z_{\eta_i} \quad \text{impedance} \]

**Greek Symbols**

\[ \alpha \quad \text{constant} \]
\[ \beta \quad \text{damping factor during the IV closed period} \]
\[ \delta_k \quad \text{constant defined in terms of engine variables in Eq. (4.35)} \]
\[ \varepsilon \quad \text{real part of } \alpha \]
\[ \eta_v \quad \text{volumetric efficiency} \]
\[ \theta \quad \text{CAD} \]
\[ \theta_n \quad \text{phase angle of the spectral acoustic impedance} \]
\[ \kappa \quad \text{angular frequency} \]
\[ \mu \quad \text{dynamic viscosity of the fluid} \]
\[ \varphi \quad \text{perimeter of the duct} \]
\[ \zeta \quad \text{a constant } = (52\pi/180) \]
\[ \rho_0 \quad \text{mean density of fluid} \]
\[ \rho_T \quad \text{total density} \]
\[ \sigma \quad \text{constant defined as } (r_c-1)/(r_c+1) \]
\( \tau_w \) shear stress
\( \varphi_k \) phase angle of the \( k^{th} \) frequency component of pressure
\( \chi \) imaginary part of \( \alpha \)
\( \psi \) phase angle
\( \omega \) angular frequency of the crankshaft (\( = \frac{2\pi N}{60} \))
\( \Omega \) angular frequency of the acoustic pulses in the intake pipe

**Acronyms**

IV intake valve
1-D one-dimensional
2-D two-dimensional
BDC bottom dead center
CAD crank angle degree
IC internal combustion
IVC intake valve closing
IVO intake valve opening
N-S Navier-Stokes
RMS root mean square
SC single cylinder
SI spark ignition
TDC top dead center
WOT wide open throttle
REFERENCES


