OPTIMIZATION OF PREHEATING SCHEDULES FOR NICKEL BASE SUPERALLOY INGOTS USING FINITE ELEMENT ANALYSIS

A Thesis

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by

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Dedicated to
Appa and Amma
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# TABLE OF CONTENTS

ACNOWLEDGEMENTS ................................................................. iii

VITA ......................................................................................... iv

LIST OF FIGURES ................................................................. ix

LIST OF TABLES ................................................................. xii

## CHAPTER PAGE

I. INTRODUCTION ............................................................... 1

II. LITERATURE REVIEW ....................................................... 6

2.0 Introduction ........................................................................ 6
2.1 Convective heat transfer in cylinders .................................. 6
2.2 Thermal processes in furnaces .......................................... 10
2.3 Heating of ingots in furnaces ........................................... 20
2.4 Quench treatment of ingots .............................................. 32
2.5 Variation of material properties ........................................ 34

III. A REVIEW OF SUPERALLOYS ........................................ 37

3.0 Introduction ........................................................................ 37
3.1 Introduction to Superalloys .............................................. 37
3.2 Classification of Superalloys ............................................. 41
3.3 Physical metallurgy of superalloys .................................... 43
   3.3.1 Ni- base superalloys ............................................... 47
   3.3.2 Fe-Ni- base superalloys ......................................... 47
   3.3.3 Co- base superalloys ............................................. 48
3.4 Microstructure of superalloys .......................................... 48
   3.4.1 Effect of precipitate volume fraction and morphology .... 48
   3.4.2 Effect of grain size ................................................ 50
   3.4.3 Role of carbides .................................................. 50
   3.4.4 Effect of c" precipitation ....................................... 51
3.5 Nickel base superalloys .................................................. 52
   3.5.1 Composition ........................................................ 53
   3.5.2 Phases .............................................................. 53
3.6 Heat Treatment of Super alloys ....................................... 56
   3.6.1 Heat treatment for stress relief ............................... 56
   3.6.2 Heat treatment for high strength ............................. 59
   3.6.3 Precautions during heat treatment ........................... 62
   3.6.4 Heat Treatment of Incoloy 901 and Inconel 718 .......... 63
IV. PRIMARY AND SECONDARY MELT PROCESSING AND DEFORMATION MECHANISMS IN SUPERALLOYS................................. 65

4.0 Introduction............................................................................. 65
4.1 Primary and secondary melt processing of superalloys ........... 66
  4.1.1 Vacuum Induction Melting ( VIM ) ................................. 67
  4.1.2 Vacuum Arc Remelting ( VAR ) ..................................... 67
  4.1.3 Electro Slag Refining ( ESR ) ....................................... 75
  4.1.4 Electron Beam Cold Hearth Refining ( EBCHR ) ............. 75
  4.1.5 Plasma Cold Hearth Refining ( PCHR ) .......................... 75
  4.1.6 Comparison of VAR and ESR ..................................... 76
  4.1.7 Summary of melt processing techniques ....................... 76
4.2 Defects .................................................................................. 80
  4.2.1 Microsegregation .......................................................... 80
  4.2.2 Macrosegregation ......................................................... 82
  4.2.3 Sonic Defects ................................................................ 85
4.3 Prevention of defects ............................................................ 91
  4.3.1 Prevention of freckles .................................................... 91
  4.3.2 Prevention of white spots ............................................. 91
  4.3.3 Surface Quality Considerations ...................................... 92
4.4 Effect of ingot homogenization ............................................ 92
4.5 Deformation behavior ........................................................... 95
4.6 Fracture ................................................................................. 97
  4.6.1 Types of fracture .......................................................... 97
  4.6.2 Metallographic aspects of fracture ................................. 98
4.7 Deformation mechanisms in superalloys ................................ 99
  4.7.1 Hot deformation behavior of as-cast superalloy ingots ........ 99
  4.7.2 Mechanisms of deformation in wrought components .......... 101
  4.7.3 Summary of deformation mechanisms in superalloys ......... 105

V. FINITE ELEMENT MODELING OF INGOT HEATING IN FURNACES.... 107

5.0 Background............................................................................. 107
5.1 ANSYS .................................................................................. 111
  5.1.1 Finite Element Analysis using ANSYS ......................... 113
5.2 Thermal Analysis ( KAN = -1 ) ........................................... 113
  5.2.2 STIF 75 ........................................................................ 116
5.3 Elements used in Structural Analysis ................................... 119
  5.3.1 STIF 42 ........................................................................ 119
5.4 Pre- and post processing ....................................................... 121
  5.4.1 PREP7 .......................................................................... 121
  5.4.2 POST 1 ......................................................................... 121
  5.4.3 POST 26 ....................................................................... 121
VI. APPLICATION OF FINITE ELEMENT ANALYSIS TO THE FURNACE HEATING OF AN OCTAGONAL STEEL INGOT .................................. 137

6.0 Introduction ........................................... 137
6.1 Experiment ............................................ 138
  6.1.1 Description of furnace ......................... 138
  6.1.2 Measurements on the furnace ............... 140
  6.1.3 Temperature measurements in the ingot ... 141
  6.1.4 Description of the Heating cycle .......... 142
6.2 Finite Element Modeling of Experiment 1 .... 142
  6.2.1 Material properties .......................... 145
  6.2.2 Boundary conditions ......................... 145
  6.2.3 Modeling the heating schedule .......... 145
  6.2.4 Postprocessing ................................ 149
  6.2.5 Case 1 .................................... 149
    6.2.5.1 Approach ................................ 149
    6.2.5.2 Mesh generation ........................ 150
  6.2.6 Case 2 .................................... 150
    6.2.6.1 Approach ................................ 150
    6.2.6.2 Mesh generation ........................ 150
6.3 Results and Discussion ............................. 151
  6.3.1 Case 1 .................................... 151
  6.3.2 Case 2 .................................... 154
  6.3.3 Comparison of results for the two cases .. 157
6.4 Scope of the Analysis ............................... 158

VII. OPTIMIZATION OF HEATING SCHEDULES FOR NICKEL BASE SUPERALLOY INGOTS ............................................. 161

7.0 Background ......................................... 161
7.1 Furnace heating ................................... 163
  7.1.1 Experiments .................................. 163
7.2 Finite Element Modeling of furnace heating . 166
  7.2.1 Approach .................................... 166
  7.2.2 Preprocessing ................................ 168
  7.2.3 Material properties ........................ 168
  7.2.4 Boundary conditions ......................... 168
  7.2.4 Solution and post processing ............. 171
7.3 Optimization of heating schedules .................................................. 172
  7.3.1 Formulation of the problem .................................................... 173
  7.3.3 Safety criterion ....................................................................... 175
  7.3.4 Optimization procedure .......................................................... 176
7.4 Results and Discussion ................................................................. 177
  7.4.1 Thermal Analysis ..................................................................... 177
  7.4.2 Optimization of heating schedules .......................................... 181
    7.4.2.1 Incoloy 901 ................................................................. 183
    7.4.2.2 Inconel 718 .................................................................. 191
    7.4.2.3 Discussion of the optimization procedure ......................... 198
  7.4.3 Comparison of material-on-heating behavior ............................ 198

VIII. CONCLUSIONS AND FUTURE WORK ............................................. 203

LIST OF REFERENCES ............................................................................ 205

APPENDIX A ......................................................................................... 210
  ANSYS INPUT FILE FOR THERMAL STRESS ANALYSIS ................. 210

APPENDIX B ......................................................................................... 214
  THERMOCOUPLE READINGS ON THE ALLOY 901 INGOT ................. 214
  [SOURCE: TELEDYNE ALLVAC, MONROE, NORTH CAROLINA]
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Fig. No.</th>
<th>Title</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Sketch of a batch annealing furnace for steel sheets</td>
<td>12</td>
</tr>
<tr>
<td>2.2</td>
<td>Radial temperature distribution in the coil at three time intervals</td>
<td>14</td>
</tr>
<tr>
<td>2.3</td>
<td>Model of a batch type combustion furnace</td>
<td>16</td>
</tr>
<tr>
<td>2.4</td>
<td>Computational grids for two types of flame shapes</td>
<td>18</td>
</tr>
<tr>
<td>2.5</td>
<td>Calculated flow patterns and temperature distributions for the two burners</td>
<td>19</td>
</tr>
<tr>
<td>2.6</td>
<td>Vertical section of a cylindrical ingot</td>
<td>24</td>
</tr>
<tr>
<td>2.7</td>
<td>The time-temperature plot of the ingot for a heating rate of 51.5 K per hour</td>
<td>26</td>
</tr>
<tr>
<td>2.8</td>
<td>The time-temperature plot of the ingot for a heating rate of 128.76 K per hour</td>
<td>27</td>
</tr>
<tr>
<td>2.9</td>
<td>The time-temperature plot of the ingot for a heating rate of 257.5 K per hour</td>
<td>28</td>
</tr>
<tr>
<td>2.10</td>
<td>The time-temperature plot of the ingot for a constant furnace temperature of 810 K</td>
<td>29</td>
</tr>
<tr>
<td>3.1</td>
<td>Primary hot working routes for converting ingots into wrought products</td>
<td>41</td>
</tr>
<tr>
<td>3.2</td>
<td>Typical mechanical properties of standard 901 38 - 816 C</td>
<td>50</td>
</tr>
<tr>
<td>4.1</td>
<td>Schematic of VIM processing of superalloys</td>
<td>68</td>
</tr>
<tr>
<td>4.2</td>
<td>Comparison of heat flux profiles in VAR and ESR processing</td>
<td>69</td>
</tr>
<tr>
<td>4.3</td>
<td>Comparison of hardware used for VAR and ESR processing</td>
<td>70</td>
</tr>
<tr>
<td>4.4</td>
<td>Corresponding IN-100 macrostructures produced by VAR and VADER methods</td>
<td>72</td>
</tr>
<tr>
<td>4.5</td>
<td>Microstructural evidence of nitrides/oxides that have been segregated to the edge of a VAR ingot</td>
<td>73</td>
</tr>
<tr>
<td>4.6</td>
<td>Examples of solute lean shelf area in an alloy 718 VAR product</td>
<td>74</td>
</tr>
<tr>
<td>4.7</td>
<td>Comparison of longitudinal microstructures produced in a) VAR and b) ESR</td>
<td>77</td>
</tr>
<tr>
<td>4.8</td>
<td>Processing parameters widely used to produce superalloy components.</td>
<td>78</td>
</tr>
<tr>
<td>4.9</td>
<td>Processing parameters typically used for conventional cast/wrought superalloy components</td>
<td>79</td>
</tr>
<tr>
<td>4.10</td>
<td>Cleanliness of Rene 95 as a result of various processing methods.</td>
<td>81</td>
</tr>
<tr>
<td>4.11</td>
<td>Examples of freckles in alloy 718</td>
<td>83</td>
</tr>
<tr>
<td>4.12</td>
<td>Example of a white spot in 6 inch diameter alloy 718 billet</td>
<td>86</td>
</tr>
<tr>
<td>4.13</td>
<td>Large oxide cluster found in a superalloy billet</td>
<td>89</td>
</tr>
<tr>
<td>4.14</td>
<td>Carbide/nitride cluster found in alloy 718</td>
<td>90</td>
</tr>
<tr>
<td>4.15</td>
<td>Defect size comparison with other metallurgical parameters</td>
<td>93</td>
</tr>
</tbody>
</table>
5.1 Operations in each phase of finite element analysis

5.2 Main routines and analysis types associated with each phase of ANSYS FEA

5.3 Thermal analysis flow solution chart

5.4 STIF 75 element

5.5 STIF 42 element

5.6 Basic PREP7 data flow diagram

5.7 2-D representation of a cylindrical ingot

5.8 General form of a heating schedule

5.9 Typical heating schedule employed in industry

6.1 Bogie hearth furnace

6.2 Experimental heating schedule

6.3 Cross-section of ingot

(a) circular cross section (case 1)

(b) octagonal cross section (case 2)

6.4 Variation of specific heat capacity (1022 steel) with temperature

6.5 Variation of thermal conductivity (1022 steel) with temperature

6.6 Variation of surface heat transfer coefficient with ingot skin temperature

6.7 Variation of ingot skin temperature with time for cylindrical cross section steel ingot

6.8 Variation of ingot center temperature with time for cylindrical cross section, steel ingot

6.9 Variation of ingot skin temperature for octagonal cross section, steel ingot

6.9 Variation of ingot center temperature for octagonal cross section, steel ingot

6.10 Comparison of FEM results for circular and octagonal cross sections for ingot skin temperature values

6.11 Comparison of results for circular and octagonal cross sections for ingot center temperature values

7.3 Ingot geometry and location of thermocouples

7.3 Location of burners in furnace

7.3 Experimental heating schedule (Teledyne Allvac)

7.4 Comparison of top and bottom surface temperatures measured on the Alloy 901 ingot during the furnace heating operation

7.5 Variation of surface heat transfer coefficient with ingot skin temperature for Alloy 901 ingot

7.6 Variation of ingot skin temperature with time for Alloy 901 ingot

7.7 Variation of ingot center temperature with time for Alloy 901 ingot

7.8 Predicted (FEM) variation of ingot skin and center temperatures for Alloy 718 ingot
7.9 Effect of initial furnace temperature on the maximum temperature difference between center and skin for Alloy 901 ingot.................. 184
7.10 Effect of t2 on the total heating time for Alloy 901 ingot................. 186
7.11 Effect of t2 on the maximum temperature difference between center and skin for Alloy 901 ingot........................................... 187
7.12 Optimum heating schedule for 20” diameter Alloy 901 ingot........... 188
7.13 Predicted variation of ingot skin and center temperatures for a 20” Alloy 901 ingot subject to the optimum heating schedule............. 189
7.14 Predicted variation of the maximum thermal stress within the ingot during the heating period for Alloy 901 ingot.......................... 190
7.15 Effect of initial furnace temperature on the maximum thermal stress for Alloy 718 ingot............................................................. 192
7.16 Effect of t2 on the maximum temperature difference between the ingot center and the skin for Alloy 718 ingot.............................. 193
7.17 Effect of t2 on the total heating time for Alloy 718 ingot.................. 194
7.18 Optimum heating schedule for alloy 718 ingot............................... 195
7.19 Predicted (FEM) variation of ingot skin and center temperatures in the optimum heating schedule for Alloy 718 ingot...................... 196
7.20 Predicted variation of the maximum thermal stress within the ingot during the heating period for Alloy 718 ingot.............................. 197
7.21 Comparison of ingot skin temperatures between Alloy 901 and Alloy 718 ingots subject to the same heating schedule..................... 199
7.22 Comparison of ingot center temperatures between Alloy 901 and Alloy 718 ingots subject to the same heating schedule..................... 200
7.23 Comparison of room temperature and elevated temperature yield strengths between Alloy 901 and Alloy 718................................. 202
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table No.</th>
<th>Title</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Typical mechanical properties of superalloys</td>
<td>40</td>
</tr>
<tr>
<td>3.2</td>
<td>Forgeability ratings of selected superalloys</td>
<td>39</td>
</tr>
<tr>
<td>3.3</td>
<td>Nominal compositions of selected superalloys</td>
<td>42</td>
</tr>
<tr>
<td>3.4</td>
<td>List of alloying additions and their effects on superalloy properties</td>
<td>44</td>
</tr>
<tr>
<td>3.5</td>
<td>Typical stress relieving and annealing cycles for wrought superalloys</td>
<td>55</td>
</tr>
<tr>
<td>3.6</td>
<td>Typical solution treating and aging cycles for wrought superalloys</td>
<td>58</td>
</tr>
<tr>
<td>3.7</td>
<td>Effect of intermediate aging on typical properties of alloy 901</td>
<td>63</td>
</tr>
<tr>
<td>3.8</td>
<td>Causes, effects, prevention and correction of contamination during heat treatment</td>
<td>64</td>
</tr>
<tr>
<td>4.1</td>
<td>Positive segregation in alloy 718 (wt%)</td>
<td>84</td>
</tr>
<tr>
<td>4.2</td>
<td>White spot analysis in alloy 718 (wt%)</td>
<td>87</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

A survey of the energy requirements in the iron and steel industry reveals that the reheating of ingots in batch type furnaces is one of the most thermally inefficient processes in the industry. The energy used in forging represents a vitally important factor in the economics of many firms devoted solely to forging, because the cost of heating represents a considerable proportion of the cost of the forging process.

The furnaces are among the important facilities in the forging operation. The ingots are heated in the furnaces to the uniform temperature profiles required for upsetting. As is obvious, a certain amount of energy is consumed in the reheating of ingots to these desired temperature profiles. It is apparent that the thermal energy consumed in raising these ingots to the final temperatures is heavily dependent upon the thermal state of the ingot at the time of charging, the combustion control and thermal efficiency of the furnaces, the nature of the ingot material itself and a number of other factors. The heating pattern of the ingot in the furnace, i.e. the trajectory of the furnace temperature from the time of charging the ingot to the time when the ingot is taken out, controls the fuel consumption rate of the furnace. In general, the heating schedules or patterns are determined by experience and/or the consideration of the metallurgical reasons, especially in the case of superalloys, in the industry today.
Since the energy crisis exploded worldwide, the attention of researchers has been focussed on the development of optimal ingot heating strategies for reducing fuel consumption. The inverted L-type heating pattern proposed by Yooichi et al [1979] and the modified L-type heating pattern developed by Lu et al [1983] are regarded as typical research results which demonstrate the relationships between the heating strategies and the fuel consumption in the soaking pits. The development of these methods, however, was based on the conservation of energy and lacking strict proofs for their optimality in the *optimal control theory*.

Because of these problems, the optimization of furnace heating schedules is still largely dependent upon the visual inspection of the problem and based on a common-sense approach. The advent of the digital computers made it possible to simulate a large number of the industrial processes to an acceptable degree of accuracy. This coupled with a common-sense based approach, can lead to tremendous improvements in industrial processes, enhanced performance and efficiency of the process, which are reflected in better economics and savings in terms of lesser energy consumption.

The performance of a forging furnace is dependent upon the ease with which heat is transferred from the furnace to the skin and from the ingot skin to its center. The heating and soaking times necessary for ingots of different size can only be assessed from a knowledge of the temperature distribution within the ingot during the heating cycle. It is impractical in most cases to drill the ingot with holes, in order to insert thermocouples to record the internal temperature distribution so that the solution of heating rates and soaking times lie in theoretical calculations. The calculation of the internal temperature distribution within an ingot during heating is a problem in unsteady state conduction. Numerous
mathematical texts are available so that the programming of heating and cooling schedules has passed from an empirical to a calculable procedure.

During the reheating stage, large stresses can develop and may lead to "cracking" in the ingot. The cracking that occurs has been observed to be catastrophic in the form of large bursts through the entire ingot rather than small surface cracks [Sun, 1971]. It is believed that this cracking is due to the large thermal stresses that build up during the reheating stage.

As a result of this, the reheating is normally carried out very slowly as not to cause large thermal stress build-up in the ingots. This leads to lower utilization of furnaces and as a result, higher operating costs. In this context, it is necessary to optimize the efficiency of the furnaces and to evaluate the thermal stresses which develop during the heating of the ingots. The optimization of the heating schedules leads to savings in energy costs as a result of lesser heating times, lesser queueing and as a consequence, higher utilization of the furnaces in reheating. The optimization of the heating schedules entails a thorough investigation of the heat transfer processes in furnaces, heat transfer in cylindrical ingots and the application of a pre-defined methodology for the optimization of the heating schedules.

There are commercially available software, which are applicable in the thermal stress determination in the heating of ingots for forging. A prominent example is ANSYS, a commercial Finite Element Analysis software, marketed by Swanson Analysis Inc., which is extensively used in structural analysis and heat transfer applications.
The complexity of the calculations of thermal stresses has ensured the pre-eminence of the finite element method in the most recent developments in the subject. The extension of the finite element method to problems of progressively increasing complexity has accompanied the increase in the capacity and speed of computers. Even so, the calculation of thermal stress in heat treated components is amongst the most complicated yet undertaken by the method. Indeed, even now, the geometry of the shapes considered is relatively simple and other factors of possible importance such as viscous flow etc. are usually ignored. A limiting factor has been the time required and the hence the cost of three dimensional calculations.

It is the aim of the present work to develop improved pre-heating schedules for Incoloy 901 and Inconel 718 ingot pre-heating. Chapter two outlines the past research carried out in the area of laminar convective heat transfer in furnaces and flow across cylindrical bodies. Also included in this chapter are descriptions of previous approaches to the problem of studying heat transfer in cylindrical bodies, including the application of various numerical methods. Chapter three is included to provide an understanding of the materials under study, which is essential for successful solution of the problem of optimizing pre-heating operations. Chapter four provides a background to the applications of the finite element method in the analysis of heat transfer to ingots in furnaces. Also included is a description of ANSYS, a popular commercial software, used in the present study. A methodology for the improvement of the pre-heating schedules is presented in chapter five. Chapter six describes experimental studies and simulation of furnace heating of steel ingots. ANSYS is verified as an accurate and reliable tool in the present study. The optimization methodology is applied to the furnace heating operations of Teledyne Allvac, North Carolina and the
optimized heating schemes for Inconel 718 and Incoloy 901 ingots is presented in chapter seven. The results are discussed and conclusions are presented in chapter eight. Suggestions and recommendations for future work in this area are presented in the final chapter.
CHAPTER II

LITERATURE REVIEW

2.0 Introduction

This chapter contains a literature review of the past research in the areas relevant to the present work like convective heat transfer across cylinders, radiant heat transfer in furnaces, thermal analysis of ingot heating in furnaces and so on.

Also presented here is a background and a review of some of the past work done in the application of numerical as well as experimental techniques in the analysis of convective and radiative mechanisms of heat transfer and in the thermal analysis of forge furnaces to study the effect of the furnace parameters in the pre-heating for forging.

2.1 Convective heat transfer in cylinders

Presented here is a brief description of the past research on the role of convection in heat transfer to a body from the environment and the determination of surface heat transfer coefficient for heating conditions in a furnace. The discussion is confined to bodies of cylindrical geometry subjected to laminar convection.

Numerous researchers have studied the convective heat transfer processes to cylinders covering the entire range from pure forced to pure natural convection. With the exception of
papers by Lee et al [1986, '87] and by Lin and Chen [1988], the previous studies considered pure free convection, pure forced convection, or mixed convection for a limited range of Richardson numbers. Lee et al [1987] analyzed mixed convection along slender cylinders with transformation parameters covering the entire range of thermal convection. But their analysis was limited to (Prandtl number) Pr < 100. Model accuracy checks were few and fluid mechanics properties were not discussed. Wang et al [1988] in their study on laminar mixed convection on slender cylinders have numerically analyzed two cases for the constant wall-heat flux and the isothermal wall using a set of axisymmetric boundary layer equations. The effects of the mixed convection, cylinder heating mode, the transverse curvature parameter and the Prandtl number on the temperature distributions and the heat transfer coefficient have been studied. Their results revealed that the magnitude and direction of the buoyancy force in natural convections can have a significant effect on the thermal flow around the cylinders. Strong variations of the skin coefficient are produced with an increase in the buoyancy force becomes stronger in aiding the flow. The skin friction coefficient increases with increasing transverse curvature and Prandtl numbers. Recent papers in which mixed thermal convection on cylinders is treated include studies by Bui and Cebecci [1985] for Newtonian fluids and Wang and Kleinstreuer [1988] for power-law fluids.

Measurements and predictions of average mixed convection Nusselt numbers for horizontal cylinders in cross-air flow ( Pr = 0.7 ) have been reported by Oosthuizen and Madan [1970, '71], Badr [1983, '84], Hatton et al [1970] and Nakai and Okazaki [1975] among others for assisting flow ( in which the forced flow is along the same direction as the buoyancy force), opposing flow ( in which the forced flow is exactly in the opposite
direction to the buoyancy force) and cross flow, covering a wide range of Reynolds and Grashof numbers.

Paolina et al [1985] have studied the unsteady flow and heat transfer to a vertical cylinder in cross flow. The calculations start with an impulsive motion of the free stream and a step change of the cylinder temperature. The solution was advanced in time until steady state conditions were achieved. Experimental studies in obtaining heat transfer coefficient over the surface of the cylinder were first reported by Schmidt and Wenner [1943] for air flowing over a cylinder with constant wall temperature. Giedt [1949, '51] later reported reported results for a vertical cylinder in air for an extended range of Reynolds numbers. A variation in the behavior of the local Nusselt number was found at the backside of the cylinder, where separated flow is encountered. This was attributed to the presence of free stream turbulence and the failure of the stream to behave as though it were infinite in extent. Seban [1960] later confirmed the dependence of the local heat transfer coefficient on the free stream turbulence.

All the above studies indicate that the local heat transfer coefficients are sensitive to the physical nature of the viscous flow field. Jain and Goel [1976] and Chang [1983] have used various techniques including experimental and numerical techniques to obtain the heat transfer coefficients for different ranges of Reynolds numbers with air as the surrounding fluid medium.

Although experimental procedures are constantly improving, it is difficult for measurements to reveal the finer details of flow behavior in the region where the flow field separates from the body, such as on the back side of a cylinder. Experimental results are
sensitive to factors such as free stream turbulence, wind-tunnel blockage and end effects. Furthermore, fluid flow in enclosures such as gas fired furnaces is influenced by the geometry and volume of enclosure as well as the nature of the fluid medium which is employed for heating purposes. Large amount of data and numerical models are available for air (Pr = 0.7) as a fluid medium, but application of these models in the study of other fluid mediums and in the study of convective mechanisms in such cases is limited by the non availability of data. Also, the heating employed varies from situation to situation in the industry and the heat transfer coefficients have to be experimentally or numerically obtained for each case under study. It is often difficult to estimate the complex dependence of the local heat transfer coefficients on the transverse curvature of a cylindrical body and for all practical purposes, it may be pragmatic to ignore this effect. It is therefore reasonable to expect that the experimental and numerical results agree qualitatively.

The study of the literature revealed little or almost no information on the specific convective transport mechanisms that are present in gas fired furnaces. However, considerable amount of literature is available on the analysis of general situations where forced convection is present. A detailed study of the research in the convective flow over cylinders provided an insight into the convective heat transfer mechanisms at work, but did not provide quantitative information with respect to the variation of the local surface heat transfer coefficients with curvature or with respect to the dependence of the convection on fluid medium.

The literature and tests on radiant heat transfer contain a large amount of data and models on convective and radiant heat transfer, but upon close examination, little of this information is useful for furnace analysis. In the case of radiant heat transfer, the
emissivities should be obtained for surfaces which have been exposed to different oxidation conditions and over different temperature conditions.

2.2 Thermal processes in furnaces

Research has been carried out in the study of the thermal processes in furnaces. The work presented by Jaluria [1984] and Nakamura et al [1987] are noteworthy and are dealt with detail as the results presented in these papers provide insight into the heat transfer processes that occur in a forge furnace.

Jaluria [1984] presented a numerical study of the thermal and fluid flow processes in a batch annealing furnace for steel sheets and the consequent numerical simulation of this system. His approach was to develop a mathematical model of the physical processes undergone by the components of a system, solve the governing equations using finite difference techniques and obtain a numerical representation of the system, focussing on the temperature decay of the gases as they flow from the inlet to the outlet and the resulting transient increase in the temperature level in the furnace.

The industrial system considered here is a batch annealing furnace for steel sheets. Annealing is done to relieve residual stresses which are due to the rolling operations. Figure 2-1 shows a sketch of the furnace. Flat steel sheets are rolled in the form of annular cylindrical coils and stacked vertically separated by convector plates. A stainless steel cover encloses the coils. The burners are located circumferentially and the the flow enters tangentially, causing swirl in the flow.
The steel coils are heated to 723 °C, which is the annealing temperature and then slow cooled and finally rapid cooled. The heating, slow cooling and fast cooling stages are studied to determine the temperatures at various locations in the system as functions of time. The system consists of the coils, the convector plates, the cover, the walls and the inert and flue gases.

Each coil was assumed to be a cylindrical annulus with inner and outer diameters and height. The material properties were taken as functions of temperature. The initial boundary conditions included both convective and radiative components. The heat conduction in the wall is a two dimensional transient problem with the boundary condition at the inner surface due to convection and radiation, including gas radiation. The gas temperature is taken as a function of time. The present problem has several unknown parameters such as the surface emissivities, conductivity ratio \( k_r/k_z \) (\( k_r \) and \( k_z \) are radial and longitudinal conductivities) and the flow distribution between the convector plates. The physical inputs are the gas flow rates, initial temperature, inflow temperature of the gas and energy input at the burners. The adiabatic flame temperature at the burners may be determined based on the gas composition and employed as thermal input.

The governing equations, with the corresponding boundary conditions, were solved numerically using finite difference methods. The grid size, time step and convergence criterion were varied to remove the dependency of results on the values chosen. For the numerical simulation of the furnace, the actual values as obtained from experiments were input. The results were compared with experimental results.
Figure 2-1
Sketch of a batch annealing furnace for steel sheets
[Jaluria, 1984]
The first case was the numerical analysis of the complex, coupled fluid and thermal processes involved in the furnace, and concerned the transient response of the individual components of the system. These were studied initially, with suitable approximations for boundary conditions, as uncoupled problems. Then the coupled problem was considered and the time dependent thermal field was obtained. The coil had, as is to be expected, a much slower response than the gases or the walls. It was observed that a large central region in the core of the coil had essentially uniform temperature. Figure 2-2 shows the radial temperature distribution in the top coil at its mid height and at the three time intervals. The temperature level is lower at a lower value of $k_r/k_z$, indicative of lower inflow of thermal energy into the coils. Two values of $k_r/k_z$ were employed. The temperature in the cover, wall and the gases were found uniform near the burners and this was attributed to the large heat transfer coefficient resulting from the mixing and the turbulence in this region.

The various results were compared with the experimental results. The differences in the results is attributed to lack of adequate information on $k_r/k_z$, inert gas flow between plates, which were adjusted according to the experimental results. The numerical simulation also incorporated the control thermocouple by simulating an on-off mechanism for gas flow. After soaking process, the gas flow is turned off and the cooling is done by means of an overhead fan under actual conditions.

Nakamura et al [1987] presented another approach to the analysis of the thermal processes in a forge furnace. Of the several methods to analyze heat transfer in combustion furnaces, the zone method and the monte carlo method are well known. The zone method is applicable only to furnaces of simple geometry because of the complexity in procedure
Figure 2-2
Radial temperature distribution in the coil at three time intervals
[Jaluria, 1984]
when extended to complicated geometries. On the other hand, the monte carlo method is not restricted and is more flexible to three dimensional complex geometry, especially the absorption coefficient. In this method, radiant energy is represented in the form of discrete packets and evaluated by statistical procedures. Large amounts of such 'bundles' are required and an iterative procedure is adopted to converge the radiant energy terms. Since this results in large computational time, this problem was by modified applying the monte carlo method to determine the total exchange areas and then apply the zone method to analyze the heat transfer. They adopted a similar approach in the study of the flow pattern, temperature distribution and heat fluxes into the heating materials in three dimensional complex geometry. The flow pattern was determined as as isothermal flow by the k-e turbulence model. The field test results are also described.

The model of the batch type combustion furnace studied is shown in figure 2-3. Steel billets are charged through the front door and heated to 1200 C (2192 F ). Some of the important assumptions made in this analysis are:

- the flow pattern is considered incompressible, isothermal and steady
- gas absorption coefficients and the surface emissivities are assumed constant.
- thermal conduction between the gas zones is neglected.
- steel billets are heated from their upper surfaces.
- initial billet temperature is 20 degrees C.

The energy equations for the walls, gas zone and the billet surface were obtained on the basis of these assumptions.
Figure 2-3
Model of a batch type combustion furnace
[Nakamura et al, 1987]
The total exchange areas were determined by the monte carlo method. The radiant bundles emitted from each gas zone and surface zone have their own emitting point, direction and path length. The emitting points are distributed uniformly throughout the gas zone. The direction was described in terms of cumulative distribution functions which were in turn determined by random numbers ranging from 0 to 1. The path length \( L \) was also calculated in a similar fashion using a random number generation and the absorption coefficient. The radiant bundle is absorbed in the gas zone when \( L \leq 1 \) where \( L \) is the distance from the emitting point to interface of the next zone. The radiant bundle is absorbed or reflected if it reaches the next zone. The remaining path length is calculated based on the absorption coefficient of the next zone. The total exchange areas between two zones were then calculated based on number of radiant bundles emitted from one zone to the other, gas absorption coefficient, surface emissivity and areas of these zones.

The computational grids for two types of flame shapes are show in figure 2-4. They are cylindrical and fish-tail type flames. Some of the zones were blocked off to deal with the roundness of the furnace ceiling. The radiant bundles are not absorbed in the blocked off zones. The distribution of heat generation was assumed as follows: 20% is generated inside the burner outside the furnace, 50% in the zones just after burner outlet and 30% in other flame zones.

The calculated flow pattern and temperature distributions for the two burners are shown in figure 2-5. While the combustion gas flow from the burner 2 spreads widely in the furnace, the gas flow from burner 1 hits the opposite wall like a jet and spreads and returns to the burner side. About 30% of the heat input is transferred to the billets and 90% of the
Figure 2-4
Computational grids for two types of flame shapes
[Nakamura et al, 1987]
Figure 2-5
Calculated flow patterns and temperature distributions for the two burners
[Nakamura et al, 1987]
effective energy is transferred by radiative heat transfer. A new type of flat flame was developed from these results and tested.

A numerical method of analyzing radiative heat transfer was developed, which can be applied to three dimensional complex geometry. This was used to compare the performances of two types of burners. A new type of burner was developed and tested in a practical forge furnace.

Both the papers described above have employed mathematical models in the analysis and evaluation of the heat transfer to the body from the furnace. The effect of the flame type, orientation of the body with respect to the heat source have been dealt with. It is constructive to note that though, these analyses have been restricted in their utility by lack of data, they represent an increasing interest in the analysis of heating furnaces for improvement of their efficiency.

Bevans [1961] analyzed the radiant heat transfer in a combustion furnace used in the petroleum industry. The physical system with interchange of radiant flux was modeled using the network analogy, with less complex procedures. A mathematical model has been presented, but has not been mathematically validated by the demonstration of an example. This work is noteworthy for the in-depth analysis of the radiative heat transfer that is dominant in a combustion furnace.

2.3 Heating of ingots in furnaces

Presented here is a review of the past research in the determination of transient temperature fields and thermal stresses in the heating of cylindrical ingots for forging. The work done
by Reddy et al [1988] and Sun [1971] have been dealt with in some detail due to the close resemblance of their research to the present work.

In order to determine the thermal stress levels in a body where the temperature is changing, it is necessary to know the temperature distribution within the body and the effect of time on the temperature distribution. In the steady state heat transfer analysis however, there is no effect of time. If the temperature distribution within the body is affected by time, the body is said to be in a transient state. It is the transient state of heat transfer which is of interest in the calculation of thermal stress during heat treatment.

The temperature distribution within the body may be experimentally measured by the use of thermocouples or other suitable measuring devices, at appropriate positions in the body and this data may be used in thermal stress calculations by means of a polynomial approximation. Alternately, the temperature distribution may be obtained as a solution of the governing differential equation for transient heat heat transfer, by applying appropriate boundary conditions. Classical calculus methods may be used to solve the differential equations where the boundary conditions are few and simple. However, in practical situations, the heat transfer is complex and requires the use of powerful and efficient numerical methods. The labor involved in the application of these numerical methods restricted their use to limited situations until the advent of high speed digital computers has served to remove this constraint. The classical methods of calculus, however, are still restricted to relatively simple heat transfer analyses. In addition, these methods also require that the initial temperature be constant and the boundary conditions be either constant or a simple function of time.
The various drawbacks to the available methods of calculation meant that much of the earlier work depended on the actual, experimental measurements of temperatures in the bodies for the calculation of thermal stresses. Since the 1960s, the use of numerical methods has been a practical proposition. The influence of temperature on the physical properties and the surface heat transfer coefficient has placed further severe restrictions of the efficacy of classical calculus techniques in providing accurate solutions and today the use of the finite difference and the finite element methods is almost universal.

In addition to the thermal strains that are generated during the heat treatment, the transformations themselves produce a change in volume that is seen as a change in the length of dilatometer specimens as they undergo phase transformations. This change in dimensions is dependent upon the type of transformation and the temperature at which these transformations occur.

The work presented by Reddy et al [1988] basically deals with the study of the heat transfer within a forging ingot considering it as an axisymmetric field problem. The boundary elements exchange heat with the surroundings by a combination of convection and radiation processes. The heat transfer was modeled to take place from the curved surfaces as well as the top and bottom flat surfaces, which is in contrast to the earlier work in this area. The material properties were assumed to have a non linear dependence on temperature.

The differential equation for heat transfer within the ingot is given by

\[ k_r \frac{\partial^2 T}{\partial r^2} + \frac{k_r}{r} \frac{\partial T}{\partial r} + k_z \frac{\partial^2 T}{\partial z^2} = \rho c \frac{\partial T}{\partial t} \]
subject to the boundary conditions

\[ k_r \frac{\partial T}{\partial r} l_r + k_z \frac{\partial T}{\partial z} l_z + h(T - T_F) + \sigma \epsilon (T^4 - T_F^4) = 0 \]

where 'T_F' represents the furnace temperature, 'k_r' and 'k_z' represent the thermal conductivities in the 'r' and 'z' directions, 'p' and 'c' represent the density and specific heat capacity of the ingot material, 'h' represents the surface heat transfer coefficient, e the emissivity and 'l_r' and 'l_z' represent the conducting lengths in the r and z directions respectively.

The equations for the transient temperature distribution within the ingot due to radiative and convective heat flux from the furnace were obtained from a combination of the finite element and finite difference methods. The nodal temperatures were obtained employing the Gauss-Seidel iteration technique.

Figure 2-6 shows the cross section of the ingot. The shaded region represents the portion modeled. The material of the ingot was assumed to be 1% chromium steel. Linear triangular elements were used and the mesh was refined towards the edges where the gradient is high. A program was written to calculate the temperatures based on the mathematical formulation described earlier.

The ingot was subjected to heating linearly from 295 K to 810 K adopting different heating rates. The temperature distributions were obtained for the various cases and are shown in figures 2-7, 2-8, 2-9 and 2-10.
Figure 2-6
Vertical section of a cylindrical ingot
The point A in figure 2-6 shows the slowest response while the fastest temperature rise is shown at point C. The temperature difference per unit length is maximum between A and B. This temperature difference (between A and B) is also plotted as a function of time. This quantity controls the maximum thermal stresses in the ingot and is controlled not to exceed a certain value to prevent cracking. The value of \( \Delta T_{\text{max}} \) shifts to the left as the heating rates are increased. The magnitude of \( \Delta T_{\text{max}} \) also increases with increased heating rates.

The work of Richard Sun [1971] at the Cabot Corp. presents an approach to the problem of determining thermal stresses in ingots subjected to pre-heating for forging / heat treatment in the optimization of the heating schedule for the ingot. There were two types of ingots forged at the Cabot Corp, Stellite division: 1) air-temed 2) consumable-electrode melted. The consumable electrode melted ingots are cooled to room temperature before they are reheated for forging. During the reheating stage, large stresses can develop and may lead to "cracking". The cracking that occurs has been observed to be catastrophic in the form of large bursts through the entire ingot rather than small surface cracks. It is believed that this cracking is due to the large thermal stresses that build up during the reheating stage. As a result of this, the reheating is normally carried out very slowly as not to cause large thermal stress build-up in the ingots. This leads to lower utilization of furnaces and as a result, higher operating costs. This paper deals with the modeling of the ingot heating process.

The analysis was performed for the heating of 40 inch diameter ingots of Hastellox alloy X. The heat transfer calculations were performed using the explicit finite difference method (Dusinberre method). For the given case, the authors expressed the heat transfer coefficient as
Figure 2-7
The time-temperature plot of the ingot for a heating rate of 51.5 K per hour
[Reddy et al, 1988]
Figure 2-8
The time-temperature plot of the ingot for a heating rate of 128.76 K per hour
[Reddy et al, 1988]
Figure 2-9
The time-temperature plot of the ingot for a heating rate of 257.5 K per hour
[Reddy et al, 1988]
Figure 2-10
The time-temperature plot of the ingot for a constant furnace temperature of 810 K
[Reddy et al, 1988]
\[ h = \xi \mathrm{A} f F \left( \frac{T_f^4 - T_s^4}{T_f - T_s} \right) + 0.27 \left( \frac{T_f - T_s}{D} \right) \]

where

\[ F = \frac{1}{F_{BR}} + \left( \frac{1}{\xi f} - 1 \right) + \frac{A_f}{A_c} \left( \frac{1}{\xi c} - 1 \right) \]

The thermal stresses due to the non-uniform temperature distribution inside the ingot when the Youngs modulus is taken as a function of temperature and the ingot is allowed to expand along the Z axis while heating, in the cylindrical coordinate system is given by:

\[ \sigma_r = \frac{\alpha}{1-\nu} \left( \frac{1}{b^2} \int_0^b E T_r \, dr - \frac{1}{r^2} \int_0^r E T_r \, dr - T \right) \]

\[ \sigma_\theta = \frac{\alpha}{1-\nu} \left( \frac{1}{b^2} \int_0^b E T_r \, dr - \frac{1}{r^2} \int_0^r E T_r \, dr - T \right) \]

\[ \sigma_z = \frac{\alpha}{1-\nu} \left( \frac{1}{b^2} \int_0^b E T_r \, dr - \frac{1}{r^2} \int_0^r E T_r \, dr - T \right) \]

The stresses in these equations are the principal stresses in the r, \( \theta \) and z directions.

The mathematical model was applied to a 27 inch diameter ingot and the calculated results were compared to experimental results. The numerical results were in reasonable agreement with the experimental results. The heat transfer coefficient of the suggested heating practice for the 40 inch diameter ingot was also calculated. It had a value of 5 to 75 Btu per sq. ft. hr F.
The thermal stress along the \( r \) direction at the surface region was zero. This is to be expected since the surface is free to expand in the \( r \) direction. Also, the thermal stress was tensile at the center due to the center being heated slower than the surface. The stresses in the \( \theta \) and \( z \) directions at the surface were compressive. The thermal stress at the center continues to increase until the center temperature starts to rise and the temperature gradient between the center and the skin decreases. In the given heating schedule, the maximum stress occurred after approximately 4.2 hours after heating.

The yield criteria for ductile metals was calculated at the point in time when maximum thermal stresses were generated. The maximum values of the shear stresses \( r_{rz} \) and \( r_{r\theta} \) were developed at the surface. The average stress was tensile at the center and compressive at the surface with zero stresses occurring at about 5 in. below the surface. The maximum shear stress in the 40 in. diameter ingot was found to have the same value from the center to about 16 from center and increased to high point at the surface. The maximum distortion energy was found at the surface and the minimum was found at about 8 in. away from the center.

It was assumed that fracture occurs when the maximum stress exceeds 90 percent of the 0.2 percent yield stress. The stress along the \( z \) direction at the center and the average stress were considered key parameters in determining the fracture criterion. Since tensile and average stress are directly proportional to each other, only the tensile stress along the \( z \) direction at the center was taken.

Using this approach, for the case of a 40 inch diameter ingot cold charged in to the furnace at 2240 F, a maximum tensile stress based on elastic behavior, of 200,000 psi along the \( z \)
direction at the ingot center was found. This was found to be in excess of 90 percent of the 0.2% yield stress. This meant that the ingot cannot be heated up using the given schedule without cracking.

A new heating practice was recommended by which the maximum thermal stresses was kept at 44,000 psi. This is about 90 percent of the yield stress of the material. Using these procedures, the ingots can be heated up safely, without the risk of internal bursting or cracking.

2.4 Quench treatment of ingots

The problem of calculating thermal stresses in quenching treatment of steels and other materials is another field of interest and considerable work has been carried out in this area. The manner by which stress is generated during quench is complex and the factors that dominate in this situation are different from those in the heating situation. The rates of heat transfer during a quench operation are greater than those during heating in orders of magnitude. Moreover, the rates of stress and strain generation are tremendously high that much more stringent control is required over the heat transfer that is taking place. When a cylinder is quenched, the shrinkage of the surface relative to the center causes the generation of a tensile axial stress in the surface region and a compressive stress at the center. As the cooling proceeds, stress reversal takes place that, if residual stresses are present at the end of the quenching, the residual stress is compressive at the surface and tensile at the center.

Since it is the purpose of the quenching operation to induce certain phase transformations, the volume change associated with this phase change has to be taken into account. This
transformation induced stress depends upon the type of phase transformation that is occurring and the temperature of transformation. The effect of the transformation stress is dependent on the size and shape of the body being subjected to quenching. Although a number of authors have detailed quantitative descriptions of the stress levels generated during the quenching of cylinders, it is not possible to evaluate the results due to the non-availability of experimental data on the same. A discussion of the experimental techniques, available to measure thermal stresses in bodies, is presented elsewhere. However, the calculated and measured values of residual stresses seem to agree fairly well in some of the models presented that there may be reason to extend these models to stress generation during the quench treatments.

Rammersdorf et al [1981] calculated the thermal stresses during the quenching of a 50 mm diameter steel cylinder that was completely transformed to martensite during the quench. Plastic flow was produced in the first second of the quench due to the phase transformation involved. The transformation stress produced a substantial compressive stress at the surface. A second stress reversal occurred when the center reached the $M_s$ temperature and a compressive stress was now formed at the center. Thus, both the center and the surface possessed residual compressive stresses at the end of the quench. The model employed to calculate the thermal stresses included transformation plasticity and kinematic work hardening.

Sjosdor [1982] obtained data for the relationship between the stress generated and the temperature at the surface during quenching. The main difference in these two approaches was that Sjosdor's model included isotropic hardening while Rammersdorf employed kinematic hardening. This led to the prediction of a smaller residual stress at the center by
the Sjosdorff model. There is a considerable amount of past research by researchers such as Inoe and Wang [1985], Yu et al [1978, '79, '80] on the transformation plasticity induced by various phase transformations in steels. As mentioned earlier, the information that is available cannot be evaluated because of the lack of experimental data.

The effect of internal stress on the kinetics of these transformations has also been investigated. Denis et al [1982] have studied the relation between the stress and strain during quenching and their relation to the transformation kinetics and found that there was very insignificant effect on the transformation kinetics.

The various models of generation of stress during the quench treatments have produced results for residual stresses which agree fairly well with the experimentally measured values. The concepts of transformation induced plasticity and kinematic hardening have been introduced very recently. There is no universal agreement about the relative effect of these phenomena and until more data is available, this question will remain unanswered.

2.5 Variation of material properties

The mechanical and physical property data are of the greatest importance in any calculations involving the determination of thermal stress and strain generated during heat treatment. Obviously, the results obtained can be no better than the accuracy that the property data allow. Where metastable structural changes are involved and metastable phases are present for a substantial part of the treatment, it may be difficult to obtain the necessary data. Of the data, the most difficult problem relates to determination of the surface heat transfer coefficients as a function of temperature.
The surface heat transfer coefficient and the related surface heat flux rate are the most difficult to obtain of the physical properties used in the calculation of the temperature gradients in heat treated material. These problems arise from the marked variation of the surface heat transfer coefficient during the heat treatment procedures and its sensitivity to small variations in the heating conditions and the state of specimen surface. Specimen shape and possibly size too, affect this property. This problem has forced many of those involved in the calculation of thermal stress to rely upon temperature measurements at the surface and center of the component, which have been used as boundary conditions in the calculation of the temperature gradient.

Of all the properties that affect the generation of thermal stress, it is the surface heat transfer coefficient that can be readily changed to control the generation of stress and distortion.

The Young's modulus \( (E) \) presents less problems than those encountered in the case of other mechanical properties. The specific heat capacity \( (C) \) varies with both the structure and composition. It tends to show anomalously high values near phase transformation temperatures. The density does not vary significantly with temperature and for all practical purposes is assumed to be a constant. The thermal conductivity is a function of the structure of the material, so considerable changes may occur in this property. The level of change in this property much smaller than that in the surface heat transfer coefficient and therefore Davies [1972] has suggested that the variations in this property produced by temperature may be ignored. But this simplification has not been made by those involved in the calculation of thermal stress during heat treatment. Determination of the linear coefficient of
expansion requires a knowledge of the coefficients of the various phases that participate in transformations during heat treatment.

Apart from the above properties, there exist other properties which are of theoretical interest in heat treatment of metallic materials and which have been referred to occasionally. These include the latent heat of transformation, viscous effects and the memory of previous deformation. Very little information is available on these properties and they have not been discussed here.

However, much as the finite element and finite difference methods are theoretically capable of providing very accurate results, the quality of the output from these methods is largely dependent on the accuracy and completeness of the physical data available. There is still a considerable uncertainty over the data available, especially so in the case of heat transfer coefficients and emissivities for furnace heating conditions.
CHAPTER III

A REVIEW OF SUPERALLOYS

3.0 Introduction

This chapter provides a background of superalloys and a description of the classification, physical metallurgy and the special features of the superalloys. Emphasis is placed on the nickel base superalloys, which have been discussed separately in 3.6. Also included is a brief description of the heat treatment of superalloys again with special attention given to Incoloy 901 and Inconel 718.

3.1 Introduction to Superalloys

'Superalloy' is a generic term that can be applied for a wide range of materials. To narrow the focus of this discussion, the superalloys can be defined as a group of Fe-Ni-base, Ni-base or Co-base alloys which provide useful strength capabilities at high temperatures exceeding 538 C (1000 F). Superalloys possess high strength, excellent corrosion resistance and good creep and fatigue resistance, which make them suited for elevated temperature applications. The primary application of the super alloys has been in air breathing jet engines for critical components such as turbine discs, blades, vanes and burner cans. Other elevated temperature applications of superalloys are found in the nuclear industry, in steam power generation, in coal gasification, in pollution control and in
hydrogen production equipment. For all these applications, elevated temperature properties are critical and prime importance is given to them.

Most of the superalloys used in these applications are Ni-base superalloys, but Fe-Ni-base and Co-base superalloys are also employed in severe elevated temperature service. Recent studies indicate that the Ni-base superalloys have high strength and toughness at cryogenic temperatures, thus making them suitable for applications in design of superconducting machinery.

Within the broad field of technology, the need for high temperature, high strength materials in aircraft engines has been the driving force behind the development and better understanding of the superalloy material systems. For the use of the high temperature steels and superalloys in electrical power plants, in gas turbines, in chemical plants, in high temperature nuclear reactors or in aircraft turbine engines, the required properties are as follows:

(i) high creep resistance at application temperatures
(ii) high resistance to thermal and mechanical fatigue
(iii) high structural stability
(iv) high resistance to corrosion in the working environment

Typically, the superalloys have moduli of elasticity of $30 \times 10^3$ ksi ($207$ GPa) although the moduli may vary from $25$ to $35 \times 10^3$ ksi ($172$ to $241$ GPa) at room temperature depending upon the alloy system. The physical properties, electrical conductivity, thermal conductivity and thermal expansion tend to be low (relative to other metal systems). These properties are influenced by the nature of base metals (transition elements) and the presence of
refractory metal additions. Table 3-1 provides the typical mechanical properties of selected superalloys.

The superalloys are relatively ductile, although the ductility of the Co-base system is less than that of the Ni-, Fe-Ni-base systems. Fe-Ni- and Ni-superalloys are available in extruded, forged or rolled form. Hot deformation is the preferred process, cold forming is usually restricted to thin sections (sheets) [Donachie, 1984]. Cold rolling may be used to increase short time strength properties for applications at temperatures below the lower temperature level of 1000°F. Table 3-2 below provides the forgeability ratings for selected superalloys.

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Forging temp.</th>
<th>Forgeability</th>
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</thead>
<tbody>
<tr>
<td>A-286</td>
<td>1950 °F</td>
<td>1339 °C Excellent</td>
</tr>
<tr>
<td>Inconel 901</td>
<td>2000 °F</td>
<td>1366 °C Good to excellent</td>
</tr>
<tr>
<td>Hastelloy X</td>
<td>2000 °F</td>
<td>1366 °C Excellent</td>
</tr>
<tr>
<td>Waspaloy</td>
<td>1975 °F</td>
<td>1353 °C Good</td>
</tr>
<tr>
<td>Inconel 718</td>
<td>1950 °F</td>
<td>1339 °C Excellent</td>
</tr>
<tr>
<td>Astroloy</td>
<td>2000 °F</td>
<td>1366 °C Fair to good</td>
</tr>
</tbody>
</table>

In an aircraft engine, there are components, such as turbine blades, which need to be extremely creep resistant, and components such as sheet materials for combustion chambers which must be extremely corrosion resistant. The optimum combination of properties cannot be achieved in a single alloy and must be achieved by alloy
# Table 3-1

Typical mechanical properties of superalloys

*The Source Book of Superalloys, 1972*

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>°F</th>
<th>Tensile strength MPa</th>
<th>ksi</th>
<th>Yield strength MPa</th>
<th>ksi</th>
<th>Elongation %</th>
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<td>20</td>
<td>140</td>
<td>20</td>
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</tr>
<tr>
<td><strong>Inconel 625, bar</strong></td>
<td></td>
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</tr>
<tr>
<td>21</td>
<td>70</td>
<td>855</td>
<td>124</td>
<td>490</td>
<td>71</td>
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</tr>
<tr>
<td>540</td>
<td>1000</td>
<td>745</td>
<td>108</td>
<td>405</td>
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<td>50</td>
</tr>
<tr>
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<td>1200</td>
<td>710</td>
<td>103</td>
<td>420</td>
<td>61</td>
<td>35</td>
</tr>
<tr>
<td>760</td>
<td>1400</td>
<td>505</td>
<td>73</td>
<td>420</td>
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<tr>
<td>870</td>
<td>1600</td>
<td>285</td>
<td>41</td>
<td>475</td>
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<td>125</td>
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<td><strong>Inconel 706, bar</strong></td>
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<td>21</td>
<td>70</td>
<td>1300</td>
<td>188</td>
<td>980</td>
<td>142</td>
<td>19</td>
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<td>1120</td>
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<td>895</td>
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<td>650</td>
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<td>1010</td>
<td>147</td>
<td>825</td>
<td>120</td>
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</tr>
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<td>760</td>
<td>1400</td>
<td>690</td>
<td>109</td>
<td>675</td>
<td>98</td>
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</tr>
<tr>
<td><strong>Inconel 718, bar</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>70</td>
<td>1430</td>
<td>208</td>
<td>1190</td>
<td>172</td>
<td>21</td>
</tr>
<tr>
<td>540</td>
<td>1000</td>
<td>1280</td>
<td>185</td>
<td>1060</td>
<td>154</td>
<td>18</td>
</tr>
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<td>650</td>
<td>1200</td>
<td>1235</td>
<td>178</td>
<td>1020</td>
<td>148</td>
<td>19</td>
</tr>
<tr>
<td>760</td>
<td>1400</td>
<td>950</td>
<td>138</td>
<td>740</td>
<td>107</td>
<td>25</td>
</tr>
<tr>
<td>870</td>
<td>1600</td>
<td>340</td>
<td>49</td>
<td>330</td>
<td>48</td>
<td>88</td>
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<td><strong>Inconel 718, sheet</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>70</td>
<td>1280</td>
<td>185</td>
<td>1050</td>
<td>153</td>
<td>22</td>
</tr>
<tr>
<td>540</td>
<td>1000</td>
<td>1140</td>
<td>166</td>
<td>945</td>
<td>137</td>
<td>26</td>
</tr>
<tr>
<td>650</td>
<td>1200</td>
<td>1030</td>
<td>150</td>
<td>870</td>
<td>126</td>
<td>15</td>
</tr>
<tr>
<td>760</td>
<td>1400</td>
<td>675</td>
<td>98</td>
<td>625</td>
<td>91</td>
<td>8</td>
</tr>
<tr>
<td><strong>Inconel X 750, bar</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
composition and thermo-mechanical heat treatment. The influence of heat treatment on properties is closely connected to the microstructural features of the alloy. The primary hot working routes for converting ingots into wrought products are shown below in figure 3-1.

![Diagram of forging, extruding, and rolling processes](image)

**Figure 3-1**  
Primary hot working routes for converting ingots into wrought products  
[L.A. Jackman, Source Book of Superalloys, 1972]

### 3.2 Classification of Superalloys

The superalloys are divided into three categories:

1) Nickel base superalloys  
2) Iron-nickel base superalloys  
3) Cobalt base superalloys

Table 3-3 provides the nominal composition of selected superalloys that are in commercial use today. The Fe-Ni base superalloys are extension of the stainless steel technology and generally are wrought, while the Ni- and Co-base superalloys may be
Table 3-3
Nominal compositions of selected commercial superalloys
[The Source Book of Superalloys, 1972]

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Cr</th>
<th>Ni</th>
<th>Co</th>
<th>Mo</th>
<th>W</th>
<th>Nb</th>
<th>Al</th>
<th>Fe</th>
<th>C</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe-Ni-base</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In 701DL</td>
<td>19.0</td>
<td>1.0</td>
<td>—</td>
<td>—</td>
<td>1.25</td>
<td>1.25</td>
<td>0.4</td>
<td>0.3</td>
<td>—</td>
<td>66.8</td>
</tr>
<tr>
<td>Inconel 700</td>
<td>21.0</td>
<td>22.5</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.38</td>
<td>0.34</td>
<td>45.7</td>
<td>0.05</td>
</tr>
<tr>
<td>Inconel 626</td>
<td>15.8</td>
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<td>—</td>
<td>1.25</td>
<td>—</td>
<td>2.0</td>
<td>0.2</td>
<td>55.6</td>
<td>0.04</td>
<td>0.005 B; 0.3 V</td>
</tr>
<tr>
<td>V-57</td>
<td>14.8</td>
<td>27.0</td>
<td>—</td>
<td>1.25</td>
<td>—</td>
<td>—</td>
<td>3.0</td>
<td>0.25</td>
<td>18.6</td>
<td>0.08 max</td>
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<tr>
<td>Inconel 901</td>
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<td>—</td>
<td>6.0</td>
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<td>—</td>
<td>2.7</td>
<td>32.1</td>
<td>0.10 max</td>
</tr>
<tr>
<td>Inconel 718</td>
<td>19.0</td>
<td>52.5</td>
<td>—</td>
<td>3.0</td>
<td>5.1</td>
<td>0.9</td>
<td>0.5</td>
<td>18.5</td>
<td>0.08 max</td>
<td>0.15 max Cu</td>
</tr>
<tr>
<td>Hastelloy X</td>
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<td>49.0</td>
<td>1.15 max</td>
<td>9.0</td>
<td>0.6</td>
<td>—</td>
<td>—</td>
<td>2.0</td>
<td>15.8</td>
<td>0.15</td>
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<tr>
<td>Ni-base</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Waspaloy</td>
<td>19.5</td>
<td>57.0</td>
<td>13.5</td>
<td>4.3</td>
<td>—</td>
<td>—</td>
<td>3.0</td>
<td>1.4</td>
<td>20 max</td>
<td>0.07</td>
</tr>
<tr>
<td>M265</td>
<td>19.0</td>
<td>59.9</td>
<td>10.0</td>
<td>10.0</td>
<td>—</td>
<td>—</td>
<td>2.6</td>
<td>1.0</td>
<td>&lt;0.75</td>
<td>0.15</td>
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<tr>
<td>Udiment 500</td>
<td>19.0</td>
<td>58.0</td>
<td>19.0</td>
<td>4.0</td>
<td>—</td>
<td>—</td>
<td>3.0</td>
<td>3.0</td>
<td>4.0 max</td>
<td>0.08</td>
</tr>
<tr>
<td>Udiment 700</td>
<td>15.5</td>
<td>53.0</td>
<td>19.5</td>
<td>5.0</td>
<td>—</td>
<td>—</td>
<td>3.4</td>
<td>3.4</td>
<td>&lt;1.0</td>
<td>0.07</td>
</tr>
<tr>
<td>Astroloy</td>
<td>15.0</td>
<td>56.5</td>
<td>15.0</td>
<td>5.25</td>
<td>—</td>
<td>—</td>
<td>3.6</td>
<td>4.5</td>
<td>&lt;0.3</td>
<td>0.06</td>
</tr>
<tr>
<td>Rene 60</td>
<td>15.0</td>
<td>60.0</td>
<td>15.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>3.0</td>
<td>2.0</td>
<td>—</td>
<td>0.17</td>
</tr>
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<td>IN-100</td>
<td>10.0</td>
<td>60.0</td>
<td>15.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>3.0</td>
<td>2.0</td>
<td>—</td>
<td>0.17</td>
</tr>
<tr>
<td>Rene 95</td>
<td>14.0</td>
<td>62.0</td>
<td>8.0</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
<td>&lt;0.3</td>
<td>0.16</td>
<td>0.015 B; 0.05 Zr</td>
</tr>
<tr>
<td>Mar-M247</td>
<td>8.25</td>
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<td>10.0</td>
<td>0.7</td>
<td>1.0</td>
<td>0.7</td>
<td>1.0</td>
<td>0.5</td>
<td>&lt;0.5</td>
<td>0.15</td>
</tr>
<tr>
<td>IN MA-774</td>
<td>20.0</td>
<td>78.5</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.5</td>
<td>0.3</td>
<td>—</td>
<td>0.05</td>
</tr>
<tr>
<td>IN MA-600E</td>
<td>15.0</td>
<td>68.5</td>
<td>—</td>
<td>2.0</td>
<td>4.0</td>
<td>2.5</td>
<td>4.5</td>
<td>—</td>
<td>—</td>
<td>0.05</td>
</tr>
<tr>
<td>Co-base</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Haynes 22 (L-605)</td>
<td>20.0</td>
<td>10.0</td>
<td>50.0</td>
<td>—</td>
<td>15.0</td>
<td>—</td>
<td>—</td>
<td>3.0</td>
<td>0.10</td>
<td>1.5 Mn</td>
</tr>
<tr>
<td>Haynes 188</td>
<td>22.0</td>
<td>22.0</td>
<td>37.0</td>
<td>—</td>
<td>14.5</td>
<td>—</td>
<td>14.5</td>
<td>0.10</td>
<td>0.90 La</td>
<td></td>
</tr>
<tr>
<td>S-416</td>
<td>20.0</td>
<td>20.0</td>
<td>42.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>—</td>
<td>4.0</td>
<td>0.38</td>
<td>—</td>
</tr>
<tr>
<td>N-40</td>
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<td>20.0</td>
<td>42.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>—</td>
<td>4.0</td>
<td>0.38</td>
<td>—</td>
</tr>
<tr>
<td>W-52</td>
<td>21.0</td>
<td>10.0</td>
<td>57.5</td>
<td>—</td>
<td>1.5</td>
<td>—</td>
<td>1.5</td>
<td>0.5</td>
<td>50</td>
<td>0.05</td>
</tr>
<tr>
<td>Mar-M302</td>
<td>21.5</td>
<td>58.0</td>
<td>—</td>
<td>10.0</td>
<td>—</td>
<td>—</td>
<td>0.3</td>
<td>0.85</td>
<td>9.0 Ta; 0.005 B; 0.2 Zr</td>
<td></td>
</tr>
<tr>
<td>Mar-M509</td>
<td>23.5</td>
<td>10.0</td>
<td>54.5</td>
<td>—</td>
<td>7.0</td>
<td>—</td>
<td>0.2</td>
<td>0.6</td>
<td>0.5 Zr; 3.5 Ta</td>
<td></td>
</tr>
<tr>
<td>J-1570</td>
<td>20.0</td>
<td>10.0</td>
<td>46.0</td>
<td>—</td>
<td>—</td>
<td>4.0</td>
<td>2.0</td>
<td>0.2</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
wrought or cast depending upon the application/composition involved. The more highly alloyed compositions are normally processed as castings. Appropriate compositions may be rolled, forged or otherwise formed into a variety of shapes. Properties can be controlled by chemistry and processing (including heat treatment) and excellent elevated temperature strengths are available in finished products.

3.3 Physical metallurgy of superalloys

Superalloys consist of the austenitic FCC matrix phase gamma (γ') plus a variety of secondary phases. The principal secondary phases are the carbides MC, $M_{23}C_6$, $M_6C$ and $M_7C_3$ in all superalloy types and gamma prime (γ') FCC ordered Ni$_3$(Al,Ti) intermetallic compound in Ni- and Fe-Ni-base superalloys. The superalloys derive their strength from the solid solution hardeners and precipitating phases. Carbides may provide limited strengthening directly through dispersion hardening or indirectly through by stabilizing grain boundaries against excessive shear.

In addition to elements that provide solid solution hardening promote carbide formation and gamma prime formation, other elements such as B, Zr, Hf, Ce are added to enhance mechanical properties. The table 3-4 give a list of alloying additions and their effect on superalloy properties.

In some systems where Cb or (Cb + Ta) is present, gamma double prime phase (γ''), body centered tetragonal (BCT) is the principal strengthenner. In many systems, undesirable phases such as delta (δ) orthorhombic Ni$_3$Cb, sigma (σ'), Laves and eta (η) HCP
Table 3-4
List of alloying additions and their effects on superalloy properties
[The Source Book of Superalloys, 1972]

Effects of Several Elements in Ni-Base Superalloys

<table>
<thead>
<tr>
<th>Element</th>
<th>Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chromium</td>
<td>Oxidation and hot corrosion resistance; solid solution strengthening</td>
</tr>
<tr>
<td>Molybdenum: tungsten</td>
<td>Solid solution strengthening; form ( \text{M}_2\text{C} ) carbides</td>
</tr>
<tr>
<td>Aluminum: titanium</td>
<td>Form ( \gamma' ), ( \text{Ni}_3(\text{Al}, \text{Ti}) ), hardening precipitate; ( \text{Ti} ) forms ( \text{MC} ) carbides as well; ( \text{Al} ) enhances oxidation resistance</td>
</tr>
<tr>
<td>Cobalt</td>
<td>Raises ( \gamma ) solvus temperature</td>
</tr>
<tr>
<td>Boron: zirconium: hafnium</td>
<td>Improve rupture life through increases in ductility; ( B ) also forms borides; ( \text{Hf} ) forms ( \text{MC} ) carbides and also promotes eutectic ( \gamma - \gamma' ) formation in cast alloys</td>
</tr>
<tr>
<td>Carbon</td>
<td>Forms ( \text{MC}, \text{M}_2\text{C}, \text{M}_2\text{C}_3 ) and ( \text{M}_4\text{C} ) carbides</td>
</tr>
<tr>
<td>Columbium</td>
<td>Forms ( \gamma' ), ( \text{Ni}_4\text{Cb} ), hardening precipitate; forms ( \delta ) orthorhombic ( \text{Ni}_4\text{Cb} )</td>
</tr>
<tr>
<td>Tantalum</td>
<td>Solid solution strengthening; forms ( \text{MC} ) carbides, enhances oxidation resistance</td>
</tr>
</tbody>
</table>

(a) Not all these effects necessarily occur in a given alloy.
Ni$_3$Ti may be present due to processing or exposure effects.

3.3.1 Ni-base superalloys

The most important class of Ni-base superalloys is that strengthened by intermetallic compound precipitation in a FCC matrix. The precipitate is the gamma prime (\(\gamma'\)) phase and this class of alloys is typified by Waspaloy or Udimet 700. Another class of Ni-base superalloys, represented by Hastelloy X, is essentially solid solution strengthened and also derives strengthening from carbide precipitation. A third class includes oxide dispersion strengthened (ODS) alloys such as IN MA - 754 which are strengthened by presence of inert particles such as yittria coupled in some cases with gamma prime precipitation.

Ni-base superalloys are utilized in both cast and wrought forms. Special processing such as powder metallurgy/isothermal forging is used to produce wrought forms of alloys such as Astroloy, IN -100 in some cases.

3.3.2 Fe-Ni-base superalloys

The most important class of Fe-Ni-base superalloys contains those alloys which are strengthened by intermetallic compound precipitation in the FCC matrix. The most common precipitate is gamma prime (\(\gamma'\)) typified by A-286, Incoloy 901. Some alloys precipitate gamma double prime (\(\gamma''\)) typified by Inconel 718. Another class of Fe-Ni-base superalloys is typified by the CRM series, which is hardened by carbides, nitrides and carbonitrides. Other Fe-Ni-base superalloys consist of modified stainless steels primarily strengthened by solution hardening. The Fe-Ni-base superalloys are used in the wrought condition while the CRM series was primarily developed for casting applications.
3.3.3 **Co-base superalloys**

The Co-base superalloys are strengthened by a combination of carbides and solid solution hardeners. Cast alloys are typified by X-40 and wrought forms are typified by Haynes 25. No intermetallic compound precipitation such as gamma prime in Ni- and Fe-Ni-base systems, has been reported in Co-base systems.

3.4 **Microstructure of superalloys**

The principal microstructural variables of superalloys are

a) the precipitate amount and morphology  
b) grain size and shape  
c) carbide distribution

3.4.1 **Effect of precipitate volume fraction and morphology**

Fe-Ni- and Ni-base systems are controlled by all three variables, while the first variable is essentially absent in Co-base systems. Structure control is achieved by composition selection, modification and by processing. For a given nominal composition, there are both advantages and disadvantages for the structure produced by processing or by casting. Cast superalloys have coarser grain sizes, increases alloy segregation and improved creep and rupture characteristics. Wrought alloys have more homogeneous and finer grain sizes and improved tensile and fatigue properties.

For the Ni- and Fe-Ni-base systems, the strength is a function of the volume fraction ($V_p$) of the gamma prime phase present in the gamma matrix. The lowest volume fractions are
present in the Fe-Ni- and first generation Ni- base systems, where the $V_f$ is less than 0.25 (25%). The gamma prime is spheroidal in lower $V_f$ phases and cuboidal in the higher $V_f$ phases. Satisfactory properties are achieved by optimizing the $V_f$ and morphology and obtaining a dispersion of discrete globular carbides along the grain boundaries. Discontinuous carbide at the grain boundaries increases surface area and drastically reduces the rupture strength.

Strength increases as $V_f$ Y' increases, while the ductility is reduced. Normally, the Y' size is optimized in the 0.2-0.3 micron range, although the strength is increased with decrease in the Y' size. Creep rupture strength is related to the $V_f$ of fine Y'. In cast alloys, the coarse Y' is replaced with fine Y' with special heat treatments. For lower temperature applications, where the tensile or ultimate strengths are critical, fine Y' is produced. Duplex Y'( uniform coarse and fine) are preferred as they disperse slip and reduce notch sensitivity.

Complex heat treatment procedures referred to as yo-yo heat treatments have been developed to produce appropriate dispersions of Y' along with a suitable carbide dispersion in wrought alloys. Although standard heat treatments generally consist of successive steps at decreasing temperatures, some heat treatments include one or more pairs of aging temperatures where the lower aging temperature precedes the higher aging temperature. The allowable $V_f$ of Y' is set by the composition, but the actual distribution of Y' is influenced by the number, level and sequence of the pre-service heat treatments. The size of Y' is also influenced by the cooling rates employed in the heat treatments as well as the time at the aging temperatures [Muzyka, 1979].
3.4.2 Effect of grain size

Grain size also affects the alloy strength. A uniform grain size is preferred, but is difficult to achieve in castings or conventional forging operations. Grain sizes obtained through isothermal forgings are found to be most uniform [Muzyka, 1979]. The achievements of fine grain sizes in wrought alloys have made yield strengths of 150 to 160 ksi (1030 to 1100 MPa) possible at room temperatures. The grain size has a pronounced effect on the fatigue strength. The type of application influences the improvement of fatigue properties in a particular direction. Generally, for a given composition, high cycle fatigue (HCF) capabilities are improved with finer grain size [Antolovich & Campbell, 1982]. Optimum fracture toughness, as evidenced by \( \frac{da}{dN} \) is achieved by balancing the grain size in wrought superalloys as it tends to become poorer at higher temperatures for finer grain size.

3.4.3 Role of carbides

The role of carbides in the strengthening of superalloys is not as well defined for Fe-Ni-base superalloys as it is for Ni-base alloys. Carbides in Co-base superalloys may act in a manner as to inhibit sliding and migration of grain boundaries. In the highest C content Co-base superalloys, the carbides may support the load much as strengthening is achieved in a composite material. Generally, the carbides exert a profound influence on the properties by precipitating in a discrete globular form at the grain boundaries. In most superalloys, \( M_{23}C_6 \) forms at the grain boundaries after a post casting or a post solution treatment thermal cycle such as aging. In contrast, if the carbides precipitate as a continuous film, the properties are severely degraded. As another extreme, if no grain boundary carbide
precipitates are present, grain boundary movement is essentially unrestricted and leads to cracking at grain boundary triple points.

Another effect produced by the grain boundary $M_{23}C_6$ carbide precipitation is the occasional formation on either side of the grain boundary of a zone depleted in $\gamma'$ precipitate. If this zone should become wider or weaker than the matrix, deformation concentrates here leading to failure at lower loads.

Carbides may also provide strengthening by precipitation in the matrix. These matrix carbides in Ni- and Fe-Ni- base superalloys may also be solutioned. The distribution of the carbides in the matrix can be modified by heat treatment. The matrix carbides contribute a very small increment of strengthening to the Ni- base and the Fe-Ni- base systems.

A negative role of the matrix carbides (also shared by the grain boundary carbides) is their participation in the fatigue cracking process by prematurely cracking or by oxidizing at the surface of uncoated alloys to cause a notch effect. Oxidized carbides or pre-cracked carbides resulting from machining or thermal stresses may initiate fatigue cracks. Carbide size is important, and reduced volumes and sizes lead to reduction in pre-cracked carbides.

3.4.4 Effect of $\gamma''$ precipitation

The practical use of $\gamma''$ precipitation is restricted to Fe-Ni- base superalloys with Cb additions. An excellent example of this class of alloys is the Inconel 718. The $\gamma''$ phase is disk shaped and the $V_f$ of $\gamma''$ is in excess of the $V_f$ of $\gamma'$ in Inconel 718. The most significant feature of the $\gamma''$ phase is the ease with it forms at relatively moderate
temperatures. Alloys strengthened by the $\gamma''$ phase possess high tensile strengths and excellent creep rupture properties at lower temperatures.

3.5. Nickel base superalloys

The Ni-base superalloys are compositionally complex, but microstructurally simple compared to steels or titanium alloys. The microstructure consists of an austenitic matrix which is solid solution strengthened, precipitates that are coherent with the matrix and various types of carbides and other phases which are distributed throughout the matrix and along the grain boundaries. These alloys may be produced in the cast, wrought and powder forms, with each process either improving key properties or resulting in advantages for specific applications. Figure 3-2 provides the typical tensile properties of standard Incoloy 901 in the range 38 to 816 C.

![Graph showing mechanical properties of Incoloy 901](image)

**Figure 3-2**
Typical mechanical properties of standard 901: 38 - 816 C
[Wilkinson, 1977]
3.5.1 Composition

The compositions of $\gamma'$ strengthened Ni-base superalloys are extremely complex, as they may contain up to 15 elements. Typical compositions of nickel base superalloys are shown in Table 3-3. Each element included in the superalloy has one or more functions, which can be categorized as follows:

a) Solid solution strengtheners: e.g. V, Cr, Mo, W, Fe and Co.
b) Gamma prime formers: Al (Ti, Nb and Ta can be substituted for Al)
c) Carbide formers: V, Ti, Mo, Cr, Nb and Ta in decreasing order of effectiveness.
d) Oxide formers: Al and Cr
e) Grain boundary modifiers: Mg, B, C, Zr and Hf

3.5.2 Phases

Austenite (FCC) matrix

The matrix phase is FCC and contains solid solution strengtheners, whose effects are proportional to the difference in atom size between the matrix and solute atom.

Gamma prime precipitates

The $\gamma'$ prime phase is based on the ordered $Ni_3Al$ structure with nickel atoms at face centers and aluminium atoms at the cube corners. The strengthening is achieved in two ways. First, the coherency strains make it difficult for the dislocations to penetrate the precipitates and second, when dislocations do penetrate the $\gamma'$, antiphase boundary (APB) energy
must be created because of the ordered structure. Besides, the strength of the alloys is believed to obey an equation of the following form [Paris & Erdogan, 1963]:

\[ \tau_c = \frac{\gamma_0}{2b} - \frac{T}{br_0} + \frac{1}{2} \left( \tau_0 + \tau_p \right) \]

where
\[ \tau_c \] = critical resolved shear stress
\[ \gamma_0 \] = APB energy
\[ \tau_0 \] = precipitate radius
\[ b \] = Burgers vector
\[ T \] = dislocation line tension
\[ \tau_p \] = lattice friction stress
\[ \tau_p \] = lattice friction stress of the particle

The \( \gamma' \) phase is believed to be quite stable with respect to temperature. The shape and structural stability of \( \gamma' \) depend on the misfit parameter \( \delta' \) [Decker & Mihalisin, 1969], which is given by

\[ \delta = \frac{a_p - a_m}{\bar{a}} = 2 \frac{a_p - a_m}{a_p + a_m} \]

where
\[ a_p \] = lattice parameter of precipitate
\[ a_m \] = lattice parameter of the matrix
\[ \bar{a} \] = average lattice parameter.
The morphological stability of the \( \gamma' \) phase depends upon the sense of applied stress and the misfit parameter.

**Carbides**

Most Ni-base superalloys contain carbides, both at the grain boundaries and in the matrix. The most frequently observed types are MC, \( M_{23}C_6 \) and \( M_6C \).

The MC carbides have a blocky morphology and an FCC structure and are believed to form below the freezing temperature. The prototypes that form at freezing temperatures and decompose at higher temperatures are TaC, NbC, TiC and VC.

The \( M_{23}C_6 \) carbides have a complex cubic structure, tend to form along grain boundaries and are relatively abundant in alloys with high chromium content. They tend to improve creep properties by inhibiting grain sliding. They may be sites for temperature creep cracking.

\( M_6C \) carbides too have a complex cubic structure and generally occur at the grain boundaries. The high temperature stability of these carbides is used to control grain size during high temperature heat treatments. They may react with other carbides, the reactions occurring gradually over long periods of time. Some of the reactions that are important are [Decker & Sims, 1972]:

\[
\begin{align*}
\text{MC} + \text{Y} & \rightarrow \text{M}_{23}\text{C}_6 + \text{Y}' \\
\text{MC} + \text{Y} & \rightarrow \text{M}_6\text{C} + \text{Y}' \\
\text{M}_6\text{C} + \text{Y} & \rightarrow \text{M}_{23}\text{C}_6 + \text{M}''
\end{align*}
\]
Other phases

In addition to the phases mentioned earlier, there may be other phases such as eta, sigma, mu or Laves phases which are generally deleterious. Boron leads to the formation of borides which are of the composition $M_3B_2$. The borides are hard, refractory particles that delay the onset of grain boundary tearing during creep.

3.6 Heat Treatment of Super alloys

The first part of this discussion is confined to the heat treatments that are applied to the superalloys and the later deals with specific heat treatment procedures for alloys of interest.

3.6.1 Heat treatment for stress relief

Stress relieving of heat resistant materials often entails a compromise; the desirability of maximum relief of residual stresses is weighed against possible effects deleterious to high temperature properties and corrosion resistance. True stress relieving is usually confined to alloys that are not age hardenable. Thus the time and temperature cycles may vary considerably, depending on the metallurgical characteristics of the alloy and on the type and magnitude of the residual stresses developed in the component by previous treatments or fabricating processes [ASM Committee on Heat-resisting Alloys, 1981].

Stress relieving temperatures are below the annealing or recrystallization temperatures. Typical stress relieving and annealing cycles for wrought alloys are shown in the Table 3-5. Some heat resistant alloys are placed in service in the as-cast condition. However, castings are relieved because
### Table 3-5
Typical stress relieving and annealing cycles for wrought superalloys

[The Source Book of Superalloys, 1972]

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Stress relieving</th>
<th>Annealing</th>
<th>Holding time per inch of section, h</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Temperature</td>
<td>Temperature</td>
<td></td>
</tr>
<tr>
<td></td>
<td>°C</td>
<td>°F</td>
<td>°C</td>
</tr>
<tr>
<td>Iron-base and iron-nickel-chromium alloys</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RA-330</td>
<td>980</td>
<td>1828</td>
<td>1100</td>
</tr>
<tr>
<td>9-9 DL</td>
<td>975</td>
<td>1775</td>
<td>1050</td>
</tr>
<tr>
<td>A-286</td>
<td>980</td>
<td>1800</td>
<td>1000</td>
</tr>
<tr>
<td>Dincool</td>
<td>1035</td>
<td>1914</td>
<td>1000</td>
</tr>
<tr>
<td>Nickel-base alloys</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Astaloy</td>
<td>1175</td>
<td>2155</td>
<td></td>
</tr>
<tr>
<td>Hastaloy B</td>
<td>1175</td>
<td>2155</td>
<td></td>
</tr>
<tr>
<td>Hastaloy C</td>
<td>1235</td>
<td>2275</td>
<td></td>
</tr>
<tr>
<td>Hastaloy W</td>
<td>1175</td>
<td>2155</td>
<td></td>
</tr>
<tr>
<td>Hastaloy X</td>
<td>1175</td>
<td>2155</td>
<td></td>
</tr>
<tr>
<td>Incoloy 600</td>
<td>1175</td>
<td>2155</td>
<td></td>
</tr>
<tr>
<td>Inconel 601</td>
<td>980</td>
<td>1798</td>
<td></td>
</tr>
<tr>
<td>Inconel 625</td>
<td>980</td>
<td>1798</td>
<td></td>
</tr>
<tr>
<td>Inconel 701</td>
<td>900</td>
<td>1650</td>
<td></td>
</tr>
<tr>
<td>Inconel 718</td>
<td>955</td>
<td>1753</td>
<td></td>
</tr>
<tr>
<td>Nibral 80A</td>
<td>1175</td>
<td>2275</td>
<td></td>
</tr>
<tr>
<td>Nibral 90</td>
<td>1175</td>
<td>2275</td>
<td></td>
</tr>
<tr>
<td>Milor 41</td>
<td>1175</td>
<td>2275</td>
<td></td>
</tr>
<tr>
<td>Colbalt-chromium-nickel-base alloys</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L-605 HS-25</td>
<td>1230</td>
<td>2250</td>
<td></td>
</tr>
<tr>
<td>N-155 HS-35</td>
<td>1175</td>
<td>2155</td>
<td></td>
</tr>
<tr>
<td>S-316</td>
<td>1205</td>
<td>2196</td>
<td></td>
</tr>
<tr>
<td>Refractory metal(s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ta-10W</td>
<td>1205/ksi</td>
<td>2200/ksi</td>
<td>1</td>
</tr>
<tr>
<td>FB-10</td>
<td>1056/ksi</td>
<td>1897/ksi</td>
<td>1</td>
</tr>
<tr>
<td>FS-62</td>
<td>1056/ksi</td>
<td>1897/ksi</td>
<td>1</td>
</tr>
<tr>
<td>M-15Ti</td>
<td>1205/ksi</td>
<td>2196/ksi</td>
<td>1</td>
</tr>
<tr>
<td>TLM</td>
<td>1205/ksi</td>
<td>2196/ksi</td>
<td>1</td>
</tr>
</tbody>
</table>

**Notes:**
- Minimum hardness is achieved by quenching rapidly from the annealing temperature, to prevent precipitation of hardening phases. Water quenching is preferred, and is usually necessary for heavy sections. Air cooling is preferred for heavy sections at 1550°F. Inconel 601 and Inconel 718 require air quenching causes cracking. However, for complex parts subject to severe distortion, an air quench is safer and more practical. Maximum cooling velocity is adequate for parts formed in hot stamp or weld. Rapid cooling from the annealing or solution treating temperature does not improve the Aging resistance of some alloys such as Astaloy. These alloys require quenching to avoid cracking.
- Normalizing temperature, 1025 to 1150°F, is commonly used. An extended time is required for prevention of grain coarsening.
- Nominal temperature, 1200°F, is 1200°F; 1300°F is 2300°F. Full annealing is recommended because intermediate temperatures cause aging to occur. A delay for stress relieving after quenching and before finishing, to avoid aging.
- The aging temperatures are given as those most frequently used for cold worked alloy. All aging must be done at temperatures above 1200°F; 1300°F is more usually done for 1300°F to 1350°F. When quenched, a second aging may be desirable to complete the aging.
- The cooling rates of quenching and annealing temperatures are commonly used for a given application:
  - 100°F per minute to 200°F per minute
  - 200°F per minute to 300°F per minute
  - 300°F per minute to 400°F per minute
  - 400°F per minute to 500°F per minute

For vacuum-arc-materials with a minimum of 30% cold work.
a) the castings, if they are of complex geometric shape, may crack during the initial heating up period in service

b) the dimensional tolerances are stringent

c) a post-weld treatment is necessary, if welding has been done.

Stress relief cycles may be developed by empirical studies of stress decay with time, as determined by non-destructive techniques such as x-ray diffraction for many alloys. However, this is not an effective technique for superalloys, where extensive material testing of critical properties and subsequent data analysis has to be done to determine the efficacy of the cycle.

*Annealing*

When applied to the superalloys, annealing implies full annealing, i.e. complete recrystallization and the attainment of maximum softness. The practice is normally applied to wrought alloys of the non-hardening type. Annealing is mainly done to improve ductility (and reduce hardness) and to facilitate forming or machining, prepare for welding, relieve stresses after welding, produce specific microstructures or soften age-hardened structures by re-solution of second phases. Solution treating is done to dissolve second phases to produce maximum corrosion resistance and to prepare for aging. Also it is done to homogenize microstructure before aging treatments.

Annealing practices vary considerably in the industry. Experience with specific parts for known requirements often indicates advantageous modifications of temperature, time or cooling method. Annealing of weldments should immediately follow welding, where
restrained joints are involved. If the configuration of the joint does not permit annealing, aging may be done to relieve the stresses.

Reheating for hot working is an annealing practice whose aim is to promote adequate formability of the alloy being deformed. Temperatures vary widely depending upon the alloy and working practice. Control of temperature may be critical to the resultant properties, as varying degrees of recrystallization may be desired. In most standard operations, reheating for hot working is a full annealing step, with recrystallization and dissolution of all or most secondary phases. Occasionally, reheating is done to limit grain growth and restricted to temperatures that do not dissolve all secondary phases.

3.6.2 Heat treatment for high strength

Solution treating

The solution treating temperatures depend on the properties desired. For optimum creep and creep-rupture properties, high solution treating temperatures are required; a lower temperature for optimum tensile strengths at elevated temperatures. The higher temperature solution treating results in greater grain growth and and more extensive solution of carbides in wrought alloys. The principal objective is to put gamma prime phase into solution and dissolve some carbides. Typical solution treating and aging cycles for wrought heat-resistant alloys are shown in Table 3-6.
### Table 3-6
Typical solution treating and aging cycles for wrought superalloys

*The Source Book of Superalloys, 1972*

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Temperature C</th>
<th>Solution treating time h</th>
<th>Cooling procedure</th>
<th>Temperature C</th>
<th>Aging time h</th>
<th>Cooling procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Iron-base alloys</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IN-718</td>
<td>1240</td>
<td>1</td>
<td>Oil quench</td>
<td>1200</td>
<td>125</td>
<td>Air cool</td>
</tr>
<tr>
<td>Discialloy</td>
<td>1010</td>
<td>1</td>
<td>Oil quench</td>
<td>1000</td>
<td>125</td>
<td>Air cool</td>
</tr>
<tr>
<td>N-155</td>
<td>1175</td>
<td>1</td>
<td>Water quench</td>
<td>1150</td>
<td>125</td>
<td>Air cool</td>
</tr>
<tr>
<td><strong>Nickel-base alloys</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hastelloy B</td>
<td>1175</td>
<td></td>
<td></td>
<td>1150</td>
<td>125</td>
<td>Air cool</td>
</tr>
<tr>
<td>Hastelloy B-2</td>
<td>1175</td>
<td>1</td>
<td>Rapid quench</td>
<td>1150</td>
<td>125</td>
<td>Air cool</td>
</tr>
<tr>
<td>Hastelloy C-276</td>
<td>1175</td>
<td>1</td>
<td></td>
<td>Rapid quench</td>
<td>1150</td>
<td>125</td>
</tr>
<tr>
<td>Hastelloy V</td>
<td>1175</td>
<td>1</td>
<td>Rapid quench</td>
<td>1150</td>
<td>125</td>
<td>Air cool</td>
</tr>
<tr>
<td>Hastelloy X</td>
<td>1175</td>
<td>1</td>
<td></td>
<td>Rapid quench</td>
<td>1150</td>
<td>125</td>
</tr>
<tr>
<td>Inconel 701</td>
<td>1095</td>
<td>2</td>
<td></td>
<td>Water quench</td>
<td>1000</td>
<td>125</td>
</tr>
<tr>
<td><strong>Inconel 600</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inconel 601</td>
<td>1120</td>
<td>2</td>
<td>Air cool</td>
<td>1100</td>
<td>125</td>
<td>Air cool</td>
</tr>
<tr>
<td><strong>Inconel 617</strong></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inconel 625</td>
<td>1150</td>
<td>2</td>
<td>Air cool</td>
<td>1100</td>
<td>125</td>
<td>Air cool</td>
</tr>
<tr>
<td><strong>Inconel 706</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inconel 708</td>
<td>925-1010</td>
<td>1700-1850</td>
<td>Air cool</td>
<td>925-1010</td>
<td>1700-1850</td>
<td>Air cool</td>
</tr>
<tr>
<td><strong>Inconel 718</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inconel X-750</td>
<td>925-1010</td>
<td>1700-1850</td>
<td>Air cool</td>
<td>925-1010</td>
<td>1700-1850</td>
<td>Air cool</td>
</tr>
<tr>
<td><strong>Monel 400</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monel 400</td>
<td>1100</td>
<td>2</td>
<td>Air cool</td>
<td>1095</td>
<td>125</td>
<td>Air cool</td>
</tr>
<tr>
<td><strong>Waspaloy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Waspaloy</td>
<td>1100</td>
<td>2</td>
<td>Air cool</td>
<td>1095</td>
<td>125</td>
<td>Air cool</td>
</tr>
</tbody>
</table>

**Note:** Alternate treatments may be used to improve specific properties. 1) To provide an adequate quench after solution treating, it is necessary to cool below about 650°C (1200°F) fast enough to prevent precipitation in the intermediate temperature range. For some metal parts in most alloys, rapid air cooling will suffice. Oil or water quenching is frequently required for heavier sections that are not subject to cycling. 2) Aging occurs in service at elevated temperatures.
The lower solution treating temperature dissolves the principal aging phases, that are present after aging, without grain growth or significant carbide dissolution.

Quenching

The purpose of quenching is to maintain, at room temperature, the supersaturated solid solution obtained during solution treating. Quenching permits a finer gamma prime particle size to be achieved on aging.

Aging

These treatments strengthen alloys by causing precipitation of one or more phases from the supersaturated solid solution that is developed during the solution treating treatment and retained by rapid cooling from the solution treating temperature.

Factors that influence the selection of the aging temperature and the number of aging steps are

(a) type and number of precipitating phases available
(b) anticipated service temperature
(c) precipitate size
(d) combination of strength and ductility desired.

Principal aging phases that are available are gamma prime (Ni$_3$Al) or Ni$_3$ (Al,Ti), eta (Ni$_3$Ti), gamma double prime (Ni$_3$Nb) and so on. These phases occur mainly in the Ni-base superalloys. The selection of a single aging temperature may result in obtaining optimum amounts of multiple precipitating phases. Alternately, a double aging treatment
producing different sizes and types of precipitates may also be employed. The aging temperature determines the size and the type of precipitates that are obtained.

Exposure to temperatures above the aging temperature may result in decrease in strength, re-solution and coarser gamma prime precipitates. Lower final aging temperatures are employed to obtain creep-rupture strengths desired. Care must be taken to ensure correct carbide distributions in the matrix and to control grain boundary carbide morphology.

3.6.3 Precautions during heat treatment

Surface Contamination

To prevent oxidation during the heat treatments, some alloys may require surface coatings, particularly blades for turbojet engines and compressors, though most superalloys possess good oxidation resistance at high temperatures. However the alloys may be susceptible to inter granular oxidation, which adversely affects the thermal fatigue resistance. The oxidation resistance may be enhanced by additions of chromium and aluminium and other elements.

Carbon pickup may occur in a carburizing atmosphere. The carbon picked up may form TiC, thus removing Ti from the solid solution and preventing normal precipitation hardening in the surface layers. TiN may also form in a similar manner. As a result of these occurrences, a protective atmosphere is maintained in the furnace to prevent the alloy from external contaminants. Vacuum atmosphere ( below 2x10-3 torr ) is usually employed for heat treating superalloys above 815 C. Air is the most commonly used atmosphere for
aging. The use of gases containing hydrogen and carbon monoxide is extremely dangerous for aging cycles below 716 C because of the explosion hazards involved.

3.6.4 Heat Treatment of Incoloy 901 and Inconel 718

Stress relieving and Annealing

The superalloy systems which rely upon gamma prime phase for strengthening, like Incoloy 901, cannot be stress relieved because the intermediate temperatures of stress relief result in aging. Consequently, the restoration of ductility and reduction of stresses in formed parts and weldments is achieved by rapidly heating to the annealing temperature. In the forging of this alloy, the finishing temperature is above 925 C (1700 F) and stress relieving is not required for as-forged parts. These alloys are solution treated and aged after forging.

The age-hardenable Inconel 718 is more crack sensitive than Incoloy 901 and must be annealed during fabrication to relieve forming and welding stresses. Multiple shaping and forming operations may require numerous in-process anneals.

Casting

The castings are normally put in service in the as-cast condition. If stress relieving is required, the castings are slowly heated 980 to 1040 C (1800 to 1900 F) and then furnace cooled. The castings are not normally annealed and sometimes stress relief operations are performed to relieve residual stresses.
The Inconel 718 castings too are seldom stress relieved or annealed in practice before they are put in service. Such parts are used in the as-cast, solution treated and aged condition.

**Solution treating and Aging**

Incoloy 901 shows relative insensitivity to the rate of cooling from the solution treating temperature. The gamma prime transforms to eta phase if the aluminum to titanium ratio is too low. Cold working after aging affects the properties. The higher aging temperatures also improve the structural stability of the part in service. Table 3-7 shows the effect of intermediate aging on typical properties of Incoloy 901.

Solution treating temperature of both the alloys are determined based on the properties desired after aging. Protective atmosphere or vacuum is necessary. Table 3-8 shows the causes, effects, prevention and correction of contamination during heat treatment. Tight control of furnace temperatures is critical to obtaining the desired properties.
Table 3-7
Effect of intermediate aging on typical properties of Incoloy 901
[The Source Book of Superalloys, 1972]

<table>
<thead>
<tr>
<th></th>
<th>Ultimate tensile strength MPa</th>
<th>Yield strength(MPa) ksi</th>
<th>Elongation in 50 mm (2 in.), %</th>
<th>Reduction in area, %</th>
<th>Creep-rupture life, h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tested at 20 °C (70 °F)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No intermediate aging(b):</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat A</td>
<td>1050</td>
<td>152</td>
<td>790</td>
<td>115</td>
<td>12</td>
</tr>
<tr>
<td>Heat B</td>
<td>1080</td>
<td>157</td>
<td>790</td>
<td>114</td>
<td>17</td>
</tr>
<tr>
<td>With intermediate aging(c):</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Heat A</td>
<td>1040</td>
<td>151</td>
<td>730</td>
<td>106</td>
<td>12</td>
</tr>
<tr>
<td>Heat B</td>
<td>1040</td>
<td>151</td>
<td>710</td>
<td>103</td>
<td>12</td>
</tr>
<tr>
<td>Tested at 650 °C (1200 °F)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>No intermediate aging(b):</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat A</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>1.0</td>
<td>...</td>
</tr>
<tr>
<td>Heat B</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>1.5</td>
<td>...</td>
</tr>
<tr>
<td>With intermediate aging(c):</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Heat A</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>11</td>
<td>...</td>
</tr>
<tr>
<td>Heat B</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>7</td>
<td>...</td>
</tr>
</tbody>
</table>

(a) At 0.2% offset. (b) Heat treatment: 1120 °C (2050 °F) for 2 h, water quench; 745 °C (1375 °F) for 24 h, air cool. (c) Heat treatment: 1120 °C (2050 °F) for 24 h, water quench; 815 °C (1500 °F), 4 h; air cool; 745 °C (1375 °F), 24 h; air cool.
### Table 3-8
Causes, effects, prevention and correction of contamination during heat treatment
[The Source Book of Superalloys, 1972]

<table>
<thead>
<tr>
<th>Type and quantity</th>
<th>Contaminants</th>
<th>General effect</th>
<th>Preventive action</th>
<th>Corrective action</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unalloyed molybdenum or tungsten: TZM alloy</td>
<td>Oxygen and nitrogen in all quantities, including minute concentrations</td>
<td>Formation of surface oxides and nitrides. Diffusion into surface causes embrittlement.</td>
<td>Use of hydrogen atmosphere, inert gas, or vacuum</td>
<td>Pickle in molten caustic or caustic solution to remove surface contamination. Acid pickle to remove subsurface contamination zone.</td>
<td>Dry hydrogen is virtually always the atmosphere selected for heat treating molybdenum and tungsten.</td>
</tr>
<tr>
<td>FS-80; FS-82; Ta-10 W: unalloyed tantalum</td>
<td>Oxygen, nitrogen and hydrogen in all quantities, including minute concentrations in inert gases or moderate vacuum</td>
<td>Formation of surface and subsurface oxides, nitrides and hydrides. Rapid diffusion at elevated temperature causes embrittlement.</td>
<td>Use of high vacuum (10-mm Hg, mini or high-purity inert gas preferably argon)</td>
<td>Remove hydrogen contamination by high-vacuum thermal treatment. Acid pickle to remove surface layers contaminated by oxygen and nitrogen. No corrective action is possible when excessive diffusion of oxygen or nitrogen occurs.</td>
<td>Diffusion of oxygen, nitrogen and hydrogen is rapid. Observable surface effects are negligible, but extreme loss of ductility occurs when impure inert atmosphere or poor vacuum is used.</td>
</tr>
</tbody>
</table>
CHAPTER IV

PRIMARY AND SECONDARY MELT PROCESSING AND
DEFORMATION MECHANISMS IN SUPERALLOYS

4.0 Introduction

Superalloys possess high strength, excellent corrosion resistance and good creep and fatigue resistance, which make them suited for elevated temperature applications. The primary application of the super alloys has been in air breathing jet engines for critical components such as turbine discs, blades, vanes and burner cans. Most of the superalloys used in these applications are Ni-base superalloys, but Fe-base and Co-base superalloys are also employed in severe elevated temperature service. However the fracture mechanics data relating to elevated temperature service are generally limited to fatigue crack propagation rates and sustained load crack growth rates in Ni-base superalloys.

In order to improve the performance of the superalloy component, there is general consensus that one must follow one or more of the following options:

1) Decrease the size of the initial defects that are present.
2) Develop alloy microstructures that have greater crack growth resistance.
3) Develop alloy chemistries that have greater crack growth resistance.
4) Develop NDT techniques to guarantee smaller defect levels.

Superalloy processing is important to all these options. Processing superalloy melts with fewer and smaller non-metallic inclusions is the first priority. The next priority is casting the cleaner melt into a uniform macro/micro structure in order to develop optimum crack
resistance in standard alloys or new alloys with chemistries specifically designed for improved crack resistance.

This chapter contains a brief description of the primary and secondary melt processing techniques employed in the development of superalloys. Also contained is a discussion of the defects produced in these techniques, prevention of these defects and the hot deformation behavior of the superalloys, with special emphasis on alloy 718.

4.1 Primary and secondary melt processing of superalloys

The superalloy melt processing techniques are aimed at obtaining better metal purity and greater structural uniformity. Earlier, development of new alloy compositions was successful only if melt processing techniques were advanced enough to produce commercial size quantities. Melt processes were developed which reduced ingot segregation and allowed sufficient ingot sizes such that subsequent thermal and mechanical processing were adequate to homogenize the micro and macro structures. The powder metallurgy superalloys arose as a result of conventional melt processing methods proving inadequate for the more highly alloyed compositions. Ingots cast from these advanced compositions would either self destruct from thermal stress or have severe dendritic segregations, which was impractical to homogenize [Maurer.G.E., 1989].

The goal in superalloy melt processing is to consolidate cost effective raw materials into a product which meets all chemistry specifications, requirements of mechanical properties, microstructure standards and ultrasonic inspection requirements. Very high strength superalloys that cannot be physically melted using conventional processes due to segregation and ingot cracking are processed by powder metallurgy. The following is a
brief discussion of the various techniques employed in melt processing of superalloys in relation to the defects produced in these processes.

4.1.1 Vacuum Induction Melting (VIM)

This process converts raw materials and scrap alloy into homogeneous alloy chemistries. The main justification for this process is to prevent alloying elements such as Al, Ti, Hf and Zr from oxidizing. Gaseous elements such as hydrogen can also be readily removed while refining of nitrogen is limited. Figure 4-1 shows the schematic of vacuum induction melting (VIM) processing of superalloys.

The VIM process is a key process since the oxide/nitride content of the melt is influenced by the charge materials and the reactions that occur in the crucible. Melt additions can also be affected by melt additions made to remove or decrease amounts of elements such as sulfur. Some of the approaches to cleaner VIM material are cleaner raw materials, melt stirring, argon purging and improved poring and casting techniques.

4.1.2 Vacuum Arc Remelting (VAR)

This process converts VIM process electrodes into ingots with greater chemical and physical homogeneity. The VAR process maintains a liquid pool that extends down to the mushy region (Figure 4-2), which is the transition zone to the fully solidified ingot. The size and geometry of this pool influence the micro and macro structure and are responsible for the formation of many harmful defects. As illustrated in the figure 4-2, the VAR process maintains a liquid pool that extends down to a mushy region, which is the transition zone to the fully solidified ingot. Figure 4-3 illustrates a comparison of the
Figure 4-1
Schematic of VIM processing of superalloys
Source: Maurer, G. E., Superalloys, Supercomposites and Superceramics, 1989
Figure 4-2
Comparison of heat flux profiles in VAR and ESR processing.
Source: Maurer G.E., Superalloys, Supercomposites and Superceramics, 1989
Figure 4-3
Comparison of hardware used for VAR and ESR processing
Source: Maurer, G.E., Superalloys, Supercomposites and Superceramics, 1989
hardware used for VAR and ESR processing. Cooling of the pool is accomplished by the water-cooled copper skin that forms the crucible. As the ingot cools, normal thermal contraction causes a gap to be created between the solidified ingot and the crucible. Cooling across this gap is very inefficient when a partial vacuum exists. Many producers replace this gap with a positive pressure of helium that significantly improves thermal conductivity. Figure 4-4 shows corresponding IN-100 macrostructures produced by VAR and VADER methods.

Greater pool depths result in increased ingot segregation and coarseness may occur to the point that homogenization may not be possible by further thermal mechanical processing. This can also lead to the formation of freckles in the ingot. Figure 4-5 shows microstructural evidence of nitrides/oxydes that have been segregated to edge of a VAR ingot [Maurer.G.E., 1989]. Figure 4-6 illustrates examples of solute lean shelf area in an alloy 718 VAR product [Maurer.G.E., 1989].

If during the melt sequence, any condition causes the shelf material or the crown to become detached from the crucible wall, it can lead to the shelf material collapsing and falling into the molten pool. This may lead to conditions which leave an oxide/nitride cluster in the mushy zone and the resulting ingot, and may leave a localized area that is mechanically inferior to the rest of the ingot [Maurer.G.E., 1989]. Forging can crack open these areas and subsequent ultrasonic inspection can reveal these areas down to the limits of NDT. The situation leaves a solute-lean area, which is a mechanically inferior white spot, in the ingot. Apart from the shelf falling mechanism, electrode fall-in and electrode torus fall-in may also cause these consequences.
Figure 4-4
Corresponding IN-100 macrostructures produced by VAR and VADER methods
Source: Maurer, G.E., Superalloys, Supercomposites and Superceramics, 1989
Figure 4-5
Microstructural evidence of nitrides/oxides that have been segregated to edge of a VAR ingot. Source: Maurer G.E., Superalloys, Supercomposites and Superceramics, 1989
Figure 4-6
Examples of solute lean shelf area in an alloy 718 VAR product
Source: Maurer G.E., Superalloys, Supercomposites and Superceramics, 1989
4.1.3 Electro Slag Refining (ESR)

The ESR process is similar to the VAR process in its geometry (figure 4-2) but is different in many ways. This process operated in air under a molten slag instead of under vacuum. The molten pool is less turbulent and is in contact with the molten slag and as a result, heat losses from the surface and sides are prevented.

Slag entrapment may occur and new defects may form. Efforts to reduce defects in ESR process ingots include improving electrode quality, reduction of ingot diameters and improving ingot cooling by introducing helium between the ingot and the crucible.

4.1.4 Electron Beam Cold Hearth Refining (EBCHR)

This process offers the most dramatic improvements seen in the oxide refining of superalloys. This process requires a relatively higher investment per unit and the production rate for a given furnace is less than a VAR or ESR unit. Some of the other shortcomings are the difficulty in controlling the melt rate, disruption occurring by the electrode falling into the hearth and variations in chemistry of the ingot. The formation of freckles in the ingot is possible, but the process has many opportunities to control the critical parameters.

4.1.5 Plasma Cold Hearth Refining (PCHR)

This process is very similar to the EBCHR process, but this process allows better chemistry control than EBCHR. The chamber and the associated hardware are much less expensive. The ability of PCHR to produce ultra-clean metal has not been documented extensively yet.
4.1.6 Comparison of VAR and ESR

VAR and ESR are the only two established remelting methods. ESR has been the dominant process in Europe whereas VAR has been the dominant process in Europe. Accordingly, process control technology for ESR is probably most advanced in Europe. Evidence of this can be seen at Inco Alloys Ltd. where the first and still the only fully computer controlled ESR furnace for the production of superalloys in the world operates [Siddall.R.J., 1989].

Results of the comparison of the two processes by [Siddall.R.J., 1989] reveal the following: ESR produces material of the highest integrity which is demonstrably free of inclusions that are otherwise inherent in VAR. But ESR is not as effective as VAR in controlling segregation. Also, control of magnesium and sulfur in ESR ensures that higher ductility material is always produced. This degree of control is not possible in VAR. Figure 4-7 shows a comparison of longitudinal microstructures produced in a) VAR and b) ESR.

4.1.7 Summary of melt processing techniques

There are many alternatives in choosing the melt processing for a given high-strength superalloy. Figure 4-8 shows the processing parameters widely used to produce superalloy components. Process parameters typically used for conventional cast/wrought superalloy products are shown in figure 4-9. The final choice is driven by the cost effectiveness. All the processes described earlier can be used in various combinations. Standard practice in North America for wrought alloy 718 components is (VIM + VAR). Results of investigations indicate the efficiency of a VIM, ESR and VAR triple melt process for selected high pressure turbine disks. The intermediate ESR step not only provides a
Figure 4-7
Comparison of longitudinal microstructures produced in a) VAR and b) ESR.
Source: Maurer, G.E., Superalloys, Supercomposites and Superceramics, 1989
Figure 4-8
Processing parameters widely used to produce superalloy components
Source: Maurer G.E., Superalloys, Supercomposites and Superceramics, 1989
Figure 4-9
Processing parameters typically used for conventional cast/wrought superalloy components
Source: Maurer. G.E., Superalloys, Supercomposites and Superceramics, 1989
product with fewer and smaller inclusions, but also provides sound consistent electrodes for better controlled VAR melting and solidification. Figure 4-10 illustrates the cleanliness of Rene 95 as a result of various processing methods.

4.2 Defects

Defects in superalloys include micro/macro structural occurrences in a superalloy product that can lead to a reduction in performance. Some of the defects are:

4.2.1 Microsegregation

This is the most common indigenous defect and is in fact, natural to the dendritic solidification mode. This is marked by larger primary dendrite spines and secondary dendrite arms and increased interdendritic spacing. Microsegregation is caused by increased local solidification time. Increased interdendritic spacing contains the lower melting precipitate-a primary phase eutectic-a material enriched in niobium.[Yu.K.O and Domingue.J.A., 1989]. More explicit names for microsegregation are "remnant dendritic pattern" and "residual dendritism". It is not feasible to inspect routinely for segregation in wrought products at cast ingot stage. Therefore, no statistical capability analysis is known to have been done and no standards are in effect for as-cast dendrite arm spacing. At billet level, no attempt is made to quantify the extent of remnant pattern [Yu.K.O and Domingue.J.A., 1989]. The initial distribution of niobium in alloy 718 has been shown [R.P.Singh et al, 1989] to have direct effects on the microstructure of the converted billet and it must be controlled more carefully to optimize alloy 718 properties.
Figure 4-10
Cleanliness of Rene 95 as a result of various processing methods.
4.2.2 Macrosegregation

The primary purpose of VAR and ESR processing is to improve ingot compositional and structural homogeneity as well as forgeability by adjusting ingot structure. Three types of macrosegregation have been reported in billets that converted from VAR and ESR processed ingots. A brief outline of these defects has been outlined below:

*Tree rings*

These are concentric rings in the transverse macrostructure associating from a minor gradient in chemical composition or fluctuation in dendritic orientation. No observed detrimental effects on the material mechanical properties due to the presence of tree rings has been reported.

*Freckles*

These are the most common type of macrosegregation to which VAR and ESR are prone. These appear as isolated areas of rich chemical composition and occur during solidification. The defect has a high aspect ratio and is aligned parallel to the liquid pool profile and if near center of ingot, parallel to the growth direction. [Maurer, 1979]. Figure 4-11 illustrates examples of freckles in alloy 718. Corresponding chemical analyses (Table 4-1) illustrate enriched areas of Cb and Ti. It has been shown [Fleming et al, 1967,1977,1978] that freckles result from flow of solute-rich interdendritic liquid in the mushy zone during solidification. For alloy 718, [Domingue et al,1984,1985] have shown that freckles are dark etching spots, rich in niobium, that appear in the center to mid-radius of forged billets. The size of the freckles are much larger than the dendrite arms of as-cast ingots. The size factor coupled the negligible slid state diffusion rate of niobium, makes freckles virtually unmovable by any amounts of thermo-mechanical processing. The effect of freckles on
Figure 4-11
Examples of freckles in alloy 718.
Source: Maurer, G.E., Superalloys, Supercomposites and Superceramics, 1989
Table 4-1
Positive segregation in alloy 718 (wt %)
Source: Maurer G.E., Superalloys, Supercomposites and Superceramics, 1989

<table>
<thead>
<tr>
<th>Element</th>
<th>Inside Freckle</th>
<th>Outside Freckle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>0.43</td>
<td>0.67</td>
</tr>
<tr>
<td>Si</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td>Ti</td>
<td>1.33</td>
<td>0.97</td>
</tr>
<tr>
<td>Cr</td>
<td>17.36</td>
<td>18.58</td>
</tr>
<tr>
<td>Fe</td>
<td>15.23</td>
<td>17.62</td>
</tr>
<tr>
<td>Ni</td>
<td>52.55</td>
<td>53.19</td>
</tr>
<tr>
<td>Cb</td>
<td>9.43</td>
<td>5.46</td>
</tr>
<tr>
<td>Mo</td>
<td>3.51</td>
<td>3.38</td>
</tr>
</tbody>
</table>
properties is to reduce ductility and yield strength [Eiselstein. H. L., 1965]. Ingots processed at higher melting rate and ingots of larger diameters are much more often affected by freckle formation.

White Spots

Macrostructural defects that are lean in the hardening elements of the particular alloy are referred to as white spots. Shown in figure 4-12 is an example of a white spot in a 6 inch diameter alloy 718 billet. Corresponding (Table 4-2) illustrates areas that is lean in Cb and Ti. For alloy 718, white spots have been shown to be randomly distributed, discrete non-etching shiny areas, which are lean of niobium [Yu et al, 1985]. White spots are referred to as "dirty" white spots if clusters of oxides or slag are associated with them, while those with clean interfaces with the matrix are referred to as "clean" white spots. Surface white spots have adverse effect on component mechanical properties and dirty white spots at any location reduce the low cycle fatigue life. Ingots processed at a relatively low melting rate are more often affected and in general, the problem is localized in the portion of the ingot influenced by hot topping [Yu, K. O and Domingue, J. A., 1989].

4.2.3 Sonic Defects

During VAR and ESR, non-metallic particles can be trapped by the rising solidification front. In VAR, inclusion removal is by transport and adhesion of non-metallics to the ingot-mold wall interfacial region. Ultrasonic indications in wrought alloy 718 are found to be minute cracks associated with magnesium and aluminum oxides and titanium nitrides [Yu, K. O and Domingue, J. A., 1989]. Aerospace ultrasonic rejection standards range
Figure 4-12
Example of a white spot in 6 inch diameter alloy 718 billet.
Source: Maurer, G.E., Superalloys, Supercomposites and Superceramics, 1989
Table 4-2
White Spot Analysis in alloy 718 (wt %)
Source: Maurer G.E., Superalloys, Supercomposites and Superceramics, 1989

<table>
<thead>
<tr>
<th>Element</th>
<th>Alloy</th>
<th>White Spot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>0.49</td>
<td>0.41</td>
</tr>
<tr>
<td>Si</td>
<td>0.20</td>
<td>0.19</td>
</tr>
<tr>
<td>Ti</td>
<td>0.81</td>
<td>0.62</td>
</tr>
<tr>
<td>Cr</td>
<td>17.50</td>
<td>17.71</td>
</tr>
<tr>
<td>Fe</td>
<td>17.75</td>
<td>19.21</td>
</tr>
<tr>
<td>Ni</td>
<td>54.97</td>
<td>55.70</td>
</tr>
<tr>
<td>Cb</td>
<td>5.10</td>
<td>2.96</td>
</tr>
<tr>
<td>Mo</td>
<td>3.18</td>
<td>3.20</td>
</tr>
</tbody>
</table>

$T_{\text{Solidus}}$ 2269 °F  2389 °F
1242 °C  1309 °C

$T_{\text{Liquidus}}$ 2459 °F  2503 °F
1348 °C  1373 °C
between 2 to 0.8 mm (5/64 in. to 2/64 in.) cavity, as the stress applied to the component increases.

Another mechanism for sonic defect formation is provided by white spots. In fact, non metallic inclusions causing ultrasonic indications in VAR billets are typically associated with "dirty" white spots.[Yu.K.O and Domingue.J.A., 1989].

**Oxides**
This class includes oxides such as Al\textsubscript{2}O\textsubscript{3}, MgO, SiO\textsubscript{2}, HfO, TiO and Y\textsubscript{2}O\textsubscript{3}. The sources of these non-metallic inclusions are raw materials and melt environments. While individual oxide particles may be harmless, a cluster of oxides can create macrodefects which, if undetected, may lead to unpredicted failure of the component. Figure 4-13 shows a large oxide cluster found in a superalloy billet. Aluminum-rich area corresponds to Al\textsubscript{2}O\textsubscript{3} particles. Titanium-rich area corresponds to TiN particles.

**Carbides/Nitrides/Borides**
During final solidification process, coarse precipitates such as TiC, TiN, Ti(C,N) can readily form. While homogenization of some carbides/borides/nitrides is possible, certain stable carbides of columbium and tantalum are difficult to dissolve in the solid state during homogenization and will remain in their as solidified state through out processing. Figure 4-14 shows a carbide/nitride cluster in alloy 718.

**Grain size non-uniformity**
Due to non-uniform dendritic segregation occurring because of cooling conditions or solidification interruption, chemical non-uniformity in an ingot can result in grain size non-
Figure 4-13
Large oxide cluster found in a superalloy billet.
Source: Maurer, G.E., Superalloys, Supercomposites and Superceramics, 1989
Figure 4-14
Carbide/nitride cluster found in alloy 718.
Source: Maurer, G. E., Superalloys, Supercomposites and Superceramics, 1989
uniformity. Variations in grain size can lead to areas with inferior mechanical properties and in a sense, can be considered defects.

4.3 Prevention of defects

4.3.1 Prevention of freckles

[Yu et al, 1986] indicated that, although a high melting rate results in a high cooling rate and short local solidification time, the associated long mushy zone and strong interdendritic fluid flow often cause the formation of freckles. On the other hand, low melting rates lead to coarse dendritic structure (exaggerated microsegregation, which cannot be broken under open-die forging) and poor ingot surface. A better technique suggested by [Yu.K.O and Domingue.J.A., 1989] is to improve the heat extraction rate. This results in ingots possessing a shorter mushy zone, less intensive interdendritic fluid flow and lower freckle formation tendency.

4.3.2 Prevention of white spots

The formation of white spots can be minimized by reducing the frequency at which exogenous material falls into the pool and by increasing the probability of remelting before entering the mushy zone [Yu et al, 1986]. This can be achieved by reducing the extent of the primary shrinkage cavity (pipe) in the electrode. Relative high power at the main burn off region to facilitate remelting of exogenous materials accompanied by an improved heat extraction rate to avoid freckle formation, is also recommended [Yu.K.O and Domingue.J.A., 1989].
4.3.3 Surface Quality Considerations

One of the characteristics of remelted ingots is that they have a surface structure which is controlled by the form of the initial contact of the liquid metal with the chill of the water-cooled crucible. For processes where there is relative movement between the mold and the ingot, the surface layers are complex and involve some degree of "leakage" of interdendritic liquid during the micro-tearing which accompanies the movement, giving a surface in which we find high local concentrations of eutectic. This effect causes embrittlement of the surface layer and can give rise to cracking in the breakdown steps [Mitchell A., 1989].

Shown in figure 4-15 is defect size comparison with other metallurgical parameters. The goal for maximum defect size in superalloy billet and the ultrasonic inspection limit for fine-grain material are illustrated for comparison.

4.4 Effect of ingot homogenization

Current applications for nickel base superalloys, especially alloy 718, are placing increasing emphasis on improvement of alloys' properties and quality. In the case of alloy 718, processing conditions are controlled with the objective to control the precipitation of the delta phase (Ni₃Nb) which has significant impact on the tensile properties, stress rupture properties, structure of the alloy, etc. This is achieved by thermo-mechanical processing (TMP). During solidification, as-cast alloy 718 suffers extreme segregation of niobium and is not amenable to TMP. This problem is corrected by employing an ingot homogenization practice with the objective to reduce the niobium gradient. Studies were
<table>
<thead>
<tr>
<th>Powder mesh size</th>
<th>100</th>
<th>90</th>
<th>80</th>
<th>70</th>
<th>60</th>
<th>50</th>
<th>40</th>
<th>30</th>
<th>20</th>
<th>ASTM grain size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Powder mesh size used for critical components</td>
<td>10</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>µm (Diameter)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Goal for superalloy billet</td>
<td>10</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>µm² (Area)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ultrasonic inspection limit for fine grain material</td>
<td>10</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>µm² (Area)</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Figure 4-15
Defect size comparison with other metallurgical parameters.
Source: Maurer G.E., Superalloys, Supercomposites and Superceramics, 1989
conducted by [Poole et al, 1989] on 15kg cast alloy 718 lab ingots. A sample was cut for analysis and ingots were homogenized. After homogenization, another sample was cut for evaluation and all the ingots were heated to 1121 C/2hr and forged in an identical fashion. Room temperature tensile testing, combination smooth notch bar stress tensile testing and metallographic analysis were performed on the wrought bars in the direct aged and annealed plus aged temperature. Differential thermal analysis was performed on annealed material. It was observed that the macrostructure of the as-cast lab ingots showed a typical structure with equiaxed and columnar regions. Macrostructures did not appreciably change after homogenization in the cases of the extreme high and low initial homogenization temperature. Intermediate starting homogenization temperatures (below 1177 C) produced grain growth. Room temperature tensile tests showed little or no effect of homogenization practice. Ingots with higher homogenization temperatures tended to have better stress rupture life and anneal plus aged rupture ductility. For the cast and homogenized material, the homogenization practice has a profound impact on the microstructure. As the homogenization temperature is increased from 1150 C to 1191 C, the dendritic appearance fades, the degree of porosity increases and the carbide structure is altered. Ingots homogenized by higher initial homogenization temperatures (1218 C) produced poor results like low melting point, niobium rich grain boundary phase and could not be hot worked. This study also indicated that the best practice seemed to be a two step homogenization procedure (1177 C/2hr + 1204 C/6hr) which produced the most uniform niobium distribution, most easily amenable for TMP. Also, it was observed that the homogenization practice influenced the primary carbide size and morphology. The carbide morphology also changes from clustered to a more uniform dispersion. Studies by

4.5 Deformation behavior

The general behavior of materials under load can be classified as ductile or brittle depending upon whether or not the material exhibits the ability to undergo plastic deformation. A completely brittle material would fracture almost at the elastic limit while a brittle material, like white cast iron, would show some measure of plasticity before fracture. When localized stresses at notches and other accidental stress concentrations do not have to be considered, it is possible to design for static situations on the basis of average stresses. However, with brittle materials, localized stresses build up to the point when there is no local yielding. Finally, a crack forms at one or more points of stress concentrations, and it spreads rapidly over the section. Even if no stress concentrations are present in a brittle material, fracture will occur suddenly because the yield stress and tensile strength are practically identical. Also, it is important to remember that a ductile material at room temperature may become brittle at elevated temperatures.

Structural components may fail in three general ways:

1) Excessive elastic deformation
2) Yielding or excessive plastic deformation
3) Fracture

For different types of failure, different significant parameters will be important. Excessive elastic deformation may occur by excessive deflection under stable equilibrium or by sudden buckling under unstable equilibrium. Yielding or excessive plastic deformation
occurs when the elastic limit of the material has been exceeded. This produces permanent change of shape and may prevent further proper functioning. In a ductile material at room temperature, yielding rarely results in fracture, because the material strain hardens as it deforms, and an increased stress is required to produce further deformation. For more complex loading conditions, the yield strength is still the significant parameter, but must be used with a suitable failure criterion. At temperatures significantly greater than room temperature, metals no longer exhibit strain hardening.

The formation of a crack which can result in complete disruption of continuity of a member constitutes fracture. Fracture can occur in three ways: (1) sudden brittle fracture (2) fatigue or progressive fracture (3) delayed fracture. A change from ductile to brittle type of fracture is promoted by a decrease in temperature, an increase in the rate of loading and the presence of complex state of stress due to a notch. The general method of analyzing brittle fracture problems is the technique called fracture mechanics. Fatigue failures occur in parts subject to alternating stresses and occurs at nominal or average stresses well below the tensile strength of the material. A type of delayed fracture is the stress-rupture failure, which occurs when the material has been statically loaded at elevated temperature for a long period of time.

Most engineering materials show a certain variability in mechanical properties, which can be influenced by changes in heat treatment or fabrication. Also, uncertainties may exist in magnitude of applied loads and approximations are required in calculation of allowable stresses. Allowance must be made for the possibility of accidental loads of high magnitude. Thus, it is necessary that the allowable stresses be of magnitude smaller than the stresses
that can cause failure. This factor of safety is dependent upon whether the loading is static or dynamic and upon the ductile-brittle nature of the material.

4.6 Fracture

Fracture is the separation or fragmentation of a solid body into two or more parts under the action of stress. The process of fracture can be divided into (1) crack initiation and (2) crack propagation. Fractures can be classified into (1) ductile fractures and (2) brittle fractures. A ductile fracture is characterized by significant plastic deformation prior to and during crack propagation. Brittle fractures are characterized by a rapid rate of crack propagation, with no gross deformation and very little microdeformation. It is to be avoided at all costs, because it occurs without warning and causes catastrophic failure.

4.6.1 Types of fracture

Fracture occurs in characteristic ways, depending upon the state of stress, the rate of application of stress and the temperature. The two broad categories of ductile and brittle fractures have already been considered. A brittle fracture is characterized by separation normal to the tensile stress. Outwardly, there is no evidence of deformation, although with x-ray analysis, it is possible to detect deformation at the fracture surface. Brittle fractures have been observed in bcc and hcp metals, but not in fcc metals, unless there are factors contributing to grain boundary embrittlement.

Ductile fractures can take several forms: Single crystals of hcp metals may slip on successive basal planes until separation occurs by shear. Polycrystalline specimens of very ductile metals may be drawn to a very fine point before they rupture. In the tensile fracture
of moderately ductile metals, the plastic deformation eventually produces a necked region. This results in the "cup-and-cone" fracture.

Fractures are classified with respect to several characteristics, such as strain to fracture, crystallographic mode of fracture and the appearance of the fracture. In the crystallographic mode of fracture, the terms used to describe fracture are shear and cleavage. Fibrous and granular are used to describe the appearance of fractures, while ductile and brittle are terms employed when the strain to fracture behavior is considered.

A shear fracture occurs as a result of extensive slip on the active slip plane. This type of fracture is promoted by shear stresses. The cleavage mode of fracture is controlled by tensile stresses acting normal to a crystallographic cleavage plane. A fracture surface caused shear appears at low magnification to be gray and fibrous, while a cleavage fracture appears bright and granular. Fracture surfaces frequently consist of a mixture of fibrous and granular fracture and it is customary to report the percentage of surface area represented by one of the categories. Based on metallographic examination, fractures in polycrystalline samples are classified as either transgranular, where the crack propagates through the grains, or as intergranular, where the crack propagates along the grain boundaries. A ductile fracture is one which exhibits a considerable degree of deformation. The boundary between ductile and brittle fracture is arbitrary and depends on the situation being considered.

4.6.2 Metallographic aspects of fracture

Detailed experiments demonstrate that the cracks responsible for brittle-cleavage type fracture are not initially present in the material but are produced by the deformation process
[Hahn et al, 1959]. The process of cleavage fracture consists of plastic deformation to produce dislocation pile-ups, crack initiation and crack propagation. The initiation of microcracks can be influenced by the presence and nature of second-phase particles [Decker, R.F., 1973]. It is common for the particle to crack during deformation. If the particle is well bonded to the matrix, there is greater resistance to cracking. If the dispersion of second phase particles is readily cut by the dislocations, then there will be no planar slip and relatively large dislocation pile-ups will occur. This will lead to high stresses, easy initiation of microcracks, and brittle behavior. If the dispersion is fine and impenetrable, the slip distance is reduced and therefore the number of dislocations sustained in a pileup is also reduced. Thus fine dispersions of particles can lead to increased toughness under the proper circumstances. The ductile phase must be thick enough to yield before large dislocation pile-ups are created against it.

Most brittle fractures occur in a transgranular manner. However, if the grain boundaries contain a film of brittle constituent, the fracture will occur in an intergranular manner. The character of the slip band can also affect the fracture behavior.

4.7 Deformation mechanisms in superalloys

4.7.1 Hot deformation behavior of as-cast superalloy ingots

The necessity for uniform microstructures and properties in industrial components requires that importance be given to the choice of the mechanical hot deformation techniques which are utilized at all stages of processing. During initial ingot breakdown by rolling or forging, the primary processor must use reduction schedules based on considerations such as economical viability and efficiency in transforming the inhomogeneous cast structure into a
uniform structure. Despite the general consensus on the important role of the processing procedures from the cast ingot to the final part in influencing final material structure and properties, most of the research has been focused on the final processing stage and limited literature is available on the development of structure and properties from the initial cast ingot.

In general, due to the size of material being formed, deformation requirements during primary breakdown are significantly different from those for the final component fabrication. Typically, average strains imposed in each cycle during breakdown of large ingots are typically low and non-uniform and result in non-uniform recrystallized microstructure. During working, as the size of the worked product decreases, the potential for imposing higher and more uniform strain fields, which correspondingly produce more uniform recrystallized microstructures, increases. Investigations performed by [Weis et al, 1989] on the hot deformation behavior of as-cast alloy 718 provides information on this aspect of processing and is described briefly here. Longitudinal and transverse sections of a 406 mm diameter as-cast alloy 718 vacuum induction melted ingot were examined. While the transverse sections showed relatively uniform structure, the longitudinal section showed that grain structure varied with radial portion and is not symmetric. Hot compression tests at constant true strain rates were performed in air on cylindrical samples. Microstructural development, during deformation and as a result of hold time after deformation, was evaluated with light and transmission electron microscopy.

It was seen that the hot compression stress-strain behavior of as-cast alloy 718 varied systematically with strain rate and temperature. Extensive strain softening was observed at 950 C at higher strain rates. Strain softening decreased with decrease in strain rate and an
increase in temperature. The mechanisms responsible for softening were primarily dynamic recovery, adiabatic heating and shear band formation. Based on this study, it is anticipated that hot working in the temperature range of 1050 C to 1150 C (1922 F to 2102 F). Working in this range was shown to provide microstructural refinement as a result of this static and dynamic recrystallization.

4.7.2 Mechanisms of deformation in wrought components

The total fatigue life of a specimen consists of a crack initiation phase and a crack propagation phase. There has been a substantial amount of observations on how cracks initiate in superalloys. A type of cracking observed is along slip bands and is termed stage I cracking. Since this requires the presence of slip bands, it would seem that shear stresses are important to move dislocations within the bands and produce cracking along the such planes. Normal and shear stresses appear to be important in the forming of the stage I crack. The basic idea is that shear stresses drive the dislocations, which in turn reduce the integrity of the slip plane. Normal stresses are required for the complete separation of the damaged slip plane and to overcome the effect of surface roughness which can 'lock' the cracks. Once stage I cracks form, they can link up with one another, propagating from grain to grain to form a macrocrack. This process has been documented for Waspaloy [Lerch.B.A., 1982].

As the temperature is increased, damage becomes more complex since creep and environmental effects become more pronounced. Since the creep resistance of superalloys is due in part to the carbides along the grain boundaries, the most important damage mechanism seems to be environmental attack. When environmental attack is important in
the cracking process, cracks usually initiate at the surface connected boundaries as seen in Udimet [Wells.C.H. and Sullivan.C.P., 1968]. Associated with these cracks were 'ridges' which were shown to be oxide intrusions that had penetrated the grain boundaries. The matrix on either side can be plastically deformed through the stresses generated by the change in volume of the oxide. The stresses at the intersection of these ridges with the grain boundaries are high enough to cause local fracture in the boundary.

It has been shown in recent work [McMahon.C.J., 1973] that the dominant failure mechanism in nickel base superalloys tested in creep or low cycle fatigue in air in the range of 815 C to 927 C is stress assisted oxidation of grain boundaries and subsequent grain boundary cracking. This was also observed in turbine blades fabricated from wrought Udiment 700. The cracks are quite branched, which is indicative of some kind of environmental influence. In all of these the cracking occurred by the sequence: stress assisted penetration of grain boundaries by oxygen, selective oxidation of certain grain boundary phases, formation of massive oxygen spikes or edges and cracking of the oxide. Failure also results from low cycle fatigue imposed by the differential thermal expansion which occurs during the start up and speed changes. Here the leading edges of the turbine blades are heated faster than the thicker parts. It has been shown McMahon.C.J., 1973] that failure mechanism is essential to stress corrosion cracking and the selective oxidation along grain boundaries is sensitive to stress level.

The low cycle fatigue behavior of wrought Waspaloy, alloy 718 and alloy 901 has been examined at room and elevated temperatures (1000 F) [Merrick.H.,F., 1973]. Optical micrographs revealed that deformation of all three alloys occurred by planar slip under low cycle fatigue. The fact that these slip lines could be revealed on an etched, polished surface
indicates that they are associated with some permanent structural change. The slip lines seem to occur gradually during the softening stage.

Surface replica electron micrographs[ Merrick.H.,F., 1973] revealed that the slip lines were associated with the extensive shear of the fcc \( \gamma' \) precipitate in alloy 901 and Waspaloy and bct \( \gamma'' \) in alloy 718. All three alloys displayed similar fatigue fracture characteristics and little difference was apparent between fracture at room temperature and 1000 F. Crack propagation occurred by a mixture of transgranular and intergranular modes. These cracks originated at the surface and are associated with persistent slip lines. The crack path deviates along grain boundaries leading to mixed mode crack propagation.

Studies [Antolovich et al, 1973] in correlating microstructure with FCP rates in Waspaloy, Astroloy and Rene 95 indicate that above 0.2 microns, the \( \gamma' \) particle size has little or no effect on the residual life, whereas below this size residual life increases rapidly especially for a coarse grained structure. The materials having fine c' and coarse grains have vastly superior FCP resistance. Consequently, the differences in FCP behavior is likely to be slip-mode related rather than a strength effect.

Another mechanism for enhancement of FCP properties in planar glide materials has been proposed in terms of the surface roughness. The underlying theory is that, for a planar glide material the fracture surface will show crystalline facets. While the individual facets are smooth, the facets that make up the surface will contribute to an over-all jagged appearance. This has been shown for Waspaloy. In computing the experimental FCP rate, the distance between the crack tip and a point ahead of it is divided by the number of cycles required for the crack to propagate this distance. If the surface is jagged, there could be
considerable difference between the actual distance and the straight line distance between the two points.

The size of the precipitate as well as other microstructural features influence the initiation of such cracks. Crack initiation in a wavy slip material is more difficult than in a material that deforms by planar slip [Hornbogen et al, 1976]. In alloy 738, initiation was associated with micropores.[Jianting.G and Ranucci.D., 1983]. The observations suggest that defects such as carbides or micropores which are either intentionally added to the alloy or a consequence of processing techniques, are very influential in the cracking process and as such reduce the useful life of components. As service temperatures approach the aging temperature, phase instabilities may occur. This may be beneficial or deleterious to the alloy depending upon their morphology and location within the grain.

Coarsening may have a dual role in affecting the material properties. If the precipitates are originally fine (i.e. shearable), and after coarsening they are still sheared, this may lead to strengthening of the alloy. However if the mechanism changes from weak to strong dislocation coupling, then the material will become weaker. The coarsened structure is associated with a stress decrease and enhanced ductility. This leads to increased fatigue life. In alloy 718, studies have shown that coarser grains tend to have better FCP resistance. The fracture mechanisms were found to depend on the testing temperature, K and to some extent, on heat treating. No detailed information is provided on the nature of plastic deformation.
The variations in FCP resistance in superalloys cannot be attributed to a single microstructural effect. However, metallurgical control through processing procedures, melting practices and/or heat treatment can be used in manipulating the FCP resistance.

4.7.3 Summary of deformation mechanisms in superalloys

It can be stated in summary that cracks in superalloys initiate on slip bands, carbides and micropores at room temperature. Alloys containing small precipitates show intense planar slip and cracks initiate in these alloys sooner than in those containing large precipitates at equivalent strains. At elevated temperatures, instabilities such as γ' coarsening and precipitation of carbides on slip bands and grain boundaries occur. Cracks initiate at these carbides, micropores and grain boundaries. Crack formation and initial crack growth occur as a result of complex interactions between beneficial (coarsening) and harmful (carbide precipitation) structural changes and the environment.

Similar to deformation mechanism maps, fracture mechanism maps use axes of normalized stress and temperature to show the fields of dominance of various fracture mechanisms such as cleavage, ductile fracture, transgranular and intergranular creep fracture, rupture with dynamic recrystallization and so on. These maps are constructed by tabulating and plotting observation of each mechanism. Deformation maps can be used to help identify the mechanisms of deformation and fracture that are likely to occur in material in a given engineering application or in deformation processing.

However, for a number of reasons, the deformation, fracture and processing types of map mentioned have found little application in the selection of deformation processing of
engineering alloys. The evaluated expressions for strain rate are conditions of steady state and are most valid for pure polycrystalline materials and simple alloys. The deformation behavior of an alloy under a given set of conditions will depend upon its current microstructure and its prior thermomechanical history. Therefore, the locations of boundaries on the maps may vary. A number of material constants must be determined. In sum, the response of precipitation-hardened engineering alloys in processing is complex and not easily described by simple mechanistic models.

It is also to be noted that it is difficult to quantify the precise point at which a heavily deformed slip band becomes a crack at low temperatures. The kinetics of coarsening, carbide precipitation and oxidation of carbides are essentially unknown. This information is required for accurate life prediction schemes for superalloy components.
CHAPTER V

FINITE ELEMENT MODELING OF INGOT HEATING IN FURNACES
AND OPTIMIZATION OF FURNACE HEATING SCHEDULES

5.0 Background

The performance of a forging furnace is dependent upon the ease with which heat is transferred from the furnace to the skin and from the ingot skin to its center. The heating and soaking times necessary for ingots of different size can only be assessed from a knowledge of the temperature distribution within the ingot during the heating cycle. It is impractical in most cases to drill the ingot with holes, in order to insert thermocouples to record the internal temperature distribution so that the solution of heating rates and soaking times lie in theoretical calculations. The calculation of the internal temperature distribution within an ingot during heating is a problem in unsteady state conduction. Numerous mathematical texts are available so that the programming of heating and cooling schedules has passed from an empirical to a calculable procedure.

During the reheating stage, large stresses can develop and may lead to "cracking" in the ingot. The cracking that occurs has been observed to be catastrophic in the form of large bursts through the entire ingot rather than small surface cracks [Sun, 1971]. It is believed that this cracking is due to the large thermal stresses that build up during the reheating stage.

107
As a result of this, the reheating is normally carried out very slowly as not to cause large thermal stress build-up in the ingots. This leads to lower utilization of furnaces and as a result, higher operating costs. In this context, it is necessary to optimize the efficiency of the furnaces and to evaluate the thermal stresses which develop during the heating of the ingots.

There are commercially available software, which are applicable in the thermal stress determination in the heating of ingots for forging. A prominent example is ANSYS, a commercial Finite Element Analysis software, marketed by Swanson Analysis Inc., which is extensively used in structural analysis and heat transfer applications.

*Heat Transfer in cylindrical ingots*

The heat transfer in a cylindrical ingot can be represented in the cylindrical coordinate system as:

\[
k_r \frac{\partial^2 T}{\partial r^2} + \frac{k_r}{r} \frac{\partial T}{\partial r} + k_z \frac{\partial^2 T}{\partial z^2} = \rho c \frac{\partial T}{\partial r}
\]

subject to the boundary conditions

\[
k_r \frac{\partial T}{\partial r} l_r + k_z \frac{\partial T}{\partial z} l_z + h(T - T_F) + \sigma \varepsilon (T^4 - T_F^4) = 0
\]

where 'T_F' represents the furnace temperature, 'k_r' and 'k_z' represent the thermal conductivities in the 'r' and 'z' directions, '\rho' and 'c' represent the density and specific heat capacity of the ingot material, 'h' represents the surface heat transfer coefficient, e the
emissivity and \( l_r \) and \( l_z \) represent the conducting lengths in the \( r \) and \( z \) directions respectively.

**Elastic relationships between the stress and strain**

The elastic relationships between stress and strain are the generalized form of Hooke's Law modified to take into account the dimensional changes associated with a change in temperature.

\[
\begin{align*}
e_{xx} &= \frac{1}{E} \left[ (1 + \nu)\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz}) \right] + \alpha \Delta T \\
e_{yy} &= \frac{1}{E} \left[ (1 + \nu)\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx}) \right] + \alpha \Delta T \\
e_{zz} &= \frac{1}{E} \left[ (1 + \nu)\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy}) \right] + \alpha \Delta T
\end{align*}
\]

Where \( e_{xx}, e_{yy} \) and \( e_{zz} \) are the thermal strains in the \( X \), \( Y \) and \( Z \) directions in the Cartesian coordinate system.

Here \( \Delta T \) is the value above the level of reference temperature at which thermal strain is assumed zero. If the material is assumed isotropic, the thermal strain is \( \alpha \Delta T \) in all directions. When the temperature is raised by \( \Delta T \). During the heat treatment operations, the material properties may undergo changes. In particular, changes may occur in the coefficient of expansion (\( \alpha \)), Poisson's ratio (\( \nu \)) and the Young's modulus (\( E \)). The classical techniques of calculation of elastic thermal stress generated required these properties to be constant [Carslaw & Jaeger, Conduction of heat in solids, 1959]. However, with the advent of numerical methods, there is provision by which the physical
and thermal properties can be taken as functions of temperature, which is essential in the calculations of thermal stresses.

*The Finite Difference Method*

The classical analytical solutions are unsatisfactory when the physical properties vary with temperature, since these methods cannot deal with irregular variations in the boundary conditions [Carslaw & Jaeger, *Conduction of heat in solids*, 1959]. The finite difference method overcomes this problem and has become extremely popular in solving problems in transient heat transfer to obtain temperature distributions. This is due to the fact that finite difference techniques can handle the very complex variations in physical properties and boundary conditions that occur during many heat treatment operations.

*The Finite Element Method*

Although, this method is best known as a stress analysis technique, it is equally well used to tackle problems in fluid mechanics and heat transfer, such as transient heat transfer in heating or quenching. The advantages of the finite element method are its

1) suitability to complex geometry.

2) suitability for coupled problems involving heat transfer and subsequent thermal stress calculations.

3) suitability to model complex boundary conditions.

Though this method is more prevalent in analysis of structural problems, this method is less popular than the finite difference method where heat treatment is concerned. In those cases, where it is used, it is usually used in conjunction with a subsequent thermal stress
calculation. The whole operation is carried out by means of a commercial package. In future, as more complex components are considered, the finite element method will be extensively used in determining transient temperature in heat treatment.

An essential feature of the finite element method is the division of the material into small elements with appropriate nodes. A continuous function, in this case of temperature, is replaced by a set of functions each of which is continuous in each specific element. These functions, which are represented by polynomials, are written in terms of their values at the element nodes and shape functions. The shape functions have the property of being unity at the relevant node and zero elsewhere (Lagrangian elements).

The complexity of the calculations of thermal stresses has ensured the pre-eminence of the finite element method in the most recent developments in the subject. The extension of the finite element method to problems of progressively increasing complexity has accompanied the increase in the capacity and speed of computers. Even so, the calculation of thermal stress in heat treated components is amongst the most complicated yet undertaken by the method. Indeed, even now, the geometry of the shapes considered is relatively simple and other factors of possible importance such as viscous flow etc. are usually ignored. A limiting factor has been the time required and the hence the cost of three dimensional calculations.

5.1 ANSYS

The ANSYS program is a self-contained general purpose finite element program developed and maintained by Swanson Analysis Systems, Inc. The program contains many routines,
all inter-related, and all for the main purpose of achieving a solution to an engineering
problem by the finite element method.

Since 1970, the Swanson Analysis Systems, Inc. [ SASI ] has developed, maintained and
supported the ANSYS FEA program. The ANSYS-PC products were introduced in 1985
with the advent of the PC technology. The choice of ANSYS for the current study was
prompted by the advantages of this program over the other commercially available FEA
programs like the versatility of the program, its ability to be implemented on any computer
system matching any requirements or budget and the comprehensive nature of the program,
which is a requirement for the industry environment. Access to 'big' machines is no longer
required and engineers can now perform finite element analysis right on desktop PCs. To
provide additional flexibility, the ANSYS program interfaces with major CAD packages
and several FEA modeling codes. Also the powerful pre- and post- processing facilities of
ANSYS were contributing factors in the selection of this computer program.

The ANSYS program is available on most commercial computers used for engineering
analysis ranging from personal computers (PCs) through large mainframe computers.
Special ANSYS PC versions ( with preprocessing, solution and postprocessing capability )
are available for PC products. On minicomputers and mainframes, the full program may be
installed or in some cases, reduced versions may be installed. Preprocessing output files
are coded and may be transmitted directly from machine to machine. Solution output files
are machine dependent binary and must be coded before being transmitted.
5.1.1 Finite Element Analysis using ANSYS

An engineering problem may be solved in three phases: 1) pre processing 2) solution phase and 3) post processing. Figure 5-1 shows some of the operations in each phase.

The main routines and analysis types associated with these phases are shown in the figure 5-2. Standard analysis types include heat transfer, statics, dynamics, magnetics and piezo electrics. Fluid flow, acoustic, and composite capabilities are also provided.

Once the preprocessing phase has been completed, the user may progress through various analyses (on the same model) in the solution phase. For example, a thermal analysis may be done to determine temperature distributions for a succeeding static or dynamic stress analysis. Once the solution phase is completed, the user may progress through various post processing operations on the same solution output.

The ANSYS program is designed to run fully interactive, fully batch or a combination of both. It is recommended that the preprocessing be done interactively, the solution phase be submitted as a batch job, and then the postprocessing be done interactively.

5.2 Thermal Analysis (KAN = -1)

The thermal analysis is mainly used to solve for the steady-state or transient temperature distribution in a body. Conduction, convection, radiation and internal heat generation may be included. Material properties may be orthotropic and temperature dependent. Output temperature may be stored and used for structural analyses. A time step optimization procedure is available for transient solutions.
Figure 5-1
Operations in each phase of finite element analysis
Source: ANSYS User's Manual
Figure 5-2
Main routines and analysis types associated with each phase of ANSYS FEA
[ANSYS Users Manual]
The basic equation for the thermal analysis is Poisson's equation with temperature as the primary unknown. The equation may be written as

\[
[ C ] \{ T \} + [ k ] \{ T \} = \{ Q \}
\]

where \([C]\) = specific heat matrix

\([k]\) = thermal conductivity matrix

\([T]\) = nodal temperature vector

\([Q]\) = heat flow rate vector (including applied convection, heat flow, internal heat generation)

The basic equation is solved by an implicit integration scheme based on a modified Houbolt method. Figure 5-3 depicts the thermal analysis solution flow chart.

5.2.2 STIF 75

This element is used as an axisymmetric ring element with a three dimensional thermal conduction capability. The element has four nodal points with a single degree of freedom, temperature at each node (figure 5-4). The element is a generalization of the axisymmetric version of STIF 55 in that it allows axisymmetric loading.

The element is applicable to a two dimensional axisymmetric, steady state or transient thermal analysis. If the model containing the element is also to be analyzed structurally, the element should be replaced by equivalent structural element (STIF 25). MODE and ISYM are used to describe the type of temperature loading present. The data input is essentially the same as for STIF 55. The face area and the heat flow rate are on a per radian basis.
Figure 5-3
Thermal analysis solution flowchart
[ANSYS Users Manual]
Figure 5-4
STIF 75 element
[ANSYS Users Manual]
The temperature distribution for the element is obtained from the numerical solution of the following form

\[ \rho C_p \left( \frac{\partial T}{\partial t} \right) = k_{xx} \left( \frac{\partial^2 T}{\partial x^2} + \frac{1}{x} \frac{\partial T}{\partial x} \right) + k_{yy} \frac{\partial^2 T}{\partial y^2} + k_{zz} \frac{\partial^2 T}{\partial z^2} + \bar{q} \]

The assumptions and restrictions for this element are similar to that of STIF 55. Material properties dependent on temperature are considered axisymmetric even if the temperature varies harmonically. Thermal transients require a fine integration time step and and a severe thermal gradient at the surface will require a fine mesh at the surface.

5.3 Elements used in Structural Analysis

5.3.1 STIF 42

The thermal stress (structural) analysis was performed by replacing the thermal solid element with an equivalent element (STIF 42). This element is used for 2-D modeling of solid structures. The element can be used either as a biaxial plane element or an axisymmetric element. The element is defined by four nodes with two degrees of freedom at each node, translations in the X and Y directions (figure 5-5).

The material properties required to be input are the Youngs modulus, coefficient of linear expansion, density and the Poisson's ratio. The element loading may be input as any combination of nodal temperatures, nodal fluences and face pressures.
Figure 5-5
STIF 42 element
[ANSYS Users Manual]
5.4 Pre- and post processing

5.4.1 PREP7

PREP 7 is used to prepare the data for an ANSYS analysis. It prepares all data necessary for the solution. In addition to the extensive mesh generation capability, the material properties, nodal constraints, loading conditions, etc. can also be prepared. All PREP 7 generation is done in memory with results stored on various files.

5.4.2 POST 1

This routine is a database post processor for sorting, printing and displaying selected results from any ANSYS analysis. Geometry displays corresponding to selected results may be made. Results may be scaled, added, multiplied etc. and safety factors may be calculated.

5.4.3 POST 26

The POST 26 routine allows the cross displaying or listing of any variable for an element or nodal point against any other variable for an element or nodal point. This post processor is used in displaying the time history results of an analysis. Operations include the sum, product, quotient, absolute value, square root, time derivative etc.

5.5 Thermal Stress Procedure

The structural solution (thermal stress) is obtained from two ANSYS analyses in series. The first analysis solves for the temperature distribution within the model from the given
Figure 5-6
Basic PREP 7 data flow diagram
thermal boundary conditions. Temperature data sets are written consecutively on a separate file for each iteration of the thermal analysis.

This file is input to the ANSYS structural analysis with the elements converted to structural element types. Interchangeable element types have the same geometry and element orientation may be important in some cases. The model developed for the thermal problem should be defined to satisfy the structural requirements also.

The input data required for an analysis of this type are the following:

i) thermal conductivity ($k_{xx}$, $k_{yy}$)
ii) specific heat capacity ($C_p$)
iii) density ($\rho$)
iv) Young's modulus ($E$)
v) Coefficient of linear expansion ($\alpha$)
vi) Poisson's ratio ($\nu$)

and the heat transfer coefficients for the given ingot geometry and material as a function of the ingot skin temperature.

The furnace heating schedule is simulated with the boundary conditions described above and the temperature distributions are obtained. The values obtained are compared with experimental observations. The thermal stress analysis is also performed for the given heating schedule towards optimizing the heating schedule. The application of the finite element modeling procedure is described in chapters VI and VII for the heating of steel and nickel base superalloy ingots respectively.
5.6 Transient Optimization Procedures

The transient thermal time step optimization procedure was employed in all the analyses. This minimizes the number of iterations required within a load step to reach the final load step time and yet adequately follow the thermal response of the structure. The optimization procedure is activated within a load step of the Thermal Analysis with a negative value of NITTER on the ITER command and a non-zero value of TIME. Boundary conditions may be ramped or stepped within the load step.

In the transient thermal analysis, the integration time step size is related to the "conducting length" of the element. The larger the thermal gradient to be resolved, the smaller both the integration time step (ITS) and the element length should be. The following guideline [ANSYS Users Reference] was used to select the ITS for a given model:

\[(\text{ITS})_j = \frac{\delta^2}{4\alpha}\]

where (ITS)\(_j\) is the initial integration time step size, \(\alpha\) is the material thermal diffusivity and \(\delta\) is the conducting length of the element in the region over which the largest gradient acts. Smaller ITS sizes may cause temperature oscillations in the large temperature gradient region. The presence of heat transfer mechanisms other than conduction may require additional guidelines for selecting the ITS.

5.7 Optimization Methodology

A survey of the energy requirements in the iron and steel industry reveals that the reheating of ingots in batch type furnaces is one of the most thermally inefficient processes in the
industry. The energy used in forging represents a vitally important factor in the economics of many firms devoted solely to forging, because the cost of heating represents a considerable proportion of the cost of the forging process.

The furnaces are the among the important facilities in the forging operation. The ingots are heated in the furnaces to the uniform temperature profiles required for upsetting. As is obvious, a certain amount of energy is consumed in the reheating of ingots to these desired temperature profiles. It is apparent that the thermal energy consumed in raising these ingots to the final temperatures is heavily dependent upon the thermal state of the ingot at the time of charging, the combustion control and thermal efficiency of the furnaces, the nature of the ingot material itself and a number of other factors. The heating pattern of the ingot in the furnace, i.e. the trajectory of the furnace temperature from the time of charging the ingot to the time when the ingot is taken out, controls the fuel consumption rate of the furnace. In general, the heating schedules or patterns are determined by experience and/or the consideration of the metallurgical reasons, especially in the case of superalloys, in the industry today.

Since the energy crisis exploded worldwide, the attention of researchers has been focussed on the development of optimal ingot heating strategies for reducing fuel consumption. The inverted L-type heating pattern proposed by Yooichi et al [1979] and the modified L-type heating pattern developed by Lu et al [1983] are regarded as typical research results which demonstrate the relationships between the heating strategies and the fuel consumption in the soaking pits. The development of these methods, however, was based on the conservation of energy and lacking strict proofs for their optimality in the optimal control theory.
Although, the minimum fuel control systems, based on the modern control theory, have been successfully implemented in the aero-space industry, only few reports involving the applications of optimal control strategies for complicated industrial processes were presented during the past years, the major reason being that most of the industrial processes involve the high-dimensional, time-variant, non-linear models with very complex constraints. Besides, the absence of the effective optimal control algorithms for solving such kinds of problems is another significant factor.

Because of these problems, the optimization of furnace heating schedules is still largely dependent upon the visual inspection of the problem and based on a common-sense approach. The advent of the digital computers made it possible to simulate a large number of the industrial processes to an acceptable degree of accuracy. This coupled with a common sense based approach, can lead to tremendous improvements in industrial processes, enhanced performance and efficiency of the process, which are reflected in better economics and savings in terms of lesser energy consumption.

Presented here is an optimization strategy for the industrial reheating operations in forging, where the ingots are heated to a final temperature profile and then subject to upsetting. As mentioned earlier, the determination of heating schedules are largely based upon previous experience and not upon any methodology. The objective of the optimization procedure is to minimize the costs of heating, which is solely dependent upon the total heating time. The problem is essentially one of minimizing the total heating time required in the reheating operations. The minimization of the heating time leads to savings in energy and higher furnace capacity utilization rates.
Because of the dangers involved in the cracking of the ingots, the ingots have to be heated according to strictly controlled heating pattern. If the ingots are heated too fast, the large temperature gradients which are built up lead to internal stresses and cracking or ‘clinking’ may occur. Therefore the optimization procedure is subject to the constraint of keeping the thermal stress levels in the ingot below a safe level to prevent the cracking the cracking of the ingot during the heating operation.

To summarize, the optimized heating schedule for preheating of ingots for forging achieves the following objectives:

i) Minimizes the total heating time for the ingot.

ii) Maximizes the thermal stress build up within the ingot and maintains these stress levels in the tolerable range.

The optimization of the heating schedules leads to savings in energy costs as a result of lesser heating times, lesser queueing and as a consequence, higher utilization of the furnaces in reheating.

Simply stated, the objective of the optimization procedure is to minimize the total time required to raise the ingot to the upsetting temperature without cracking it. This is subject to the constraint of maximizing the thermal stresses which build up in the ingots as a result of heating and maintaining these stresses below permissible levels. The thermal stress build up in the ingots is maximum along the radial direction at the center of the ingot. This is directly proportional to the temperature gradient between the center and the surface at this position. This corresponds to the direction along A-B shown in the figure 5-7. Hence the
problem is one of maximizing the temperature difference between A and B (ΔT) by adopting appropriately fast heating rates.

5.7.1 General formulation of the problem

The problem can be expressed for a heating schedule of a general form shown in the figure 5-9, by using the optimal theory as

\[ \text{Min } Z = t_1 + t_2 + t_3 + t_n \]

Subject to

\[ (\Delta T_{\text{max}})_{t_1} \leq (\Delta T_{\text{max}})_{\text{allowed}} \]
\[ (\Delta T_{\text{max}})_{t_2} \leq (\Delta T_{\text{max}})_{\text{allowed}} \]
\[ (\Delta T_{\text{max}})_{t_3} \leq (\Delta T_{\text{max}})_{\text{allowed}} \]

......

......

\[ (\Delta T_{\text{max}})_{t_n} \leq (\Delta T_{\text{max}})_{\text{allowed}} \]

where \( t_1, t_2, t_3 \ldots t_n \geq 0 \)

Here \((\Delta T_{\text{max}})_{t_n}\) refers to the maximum value of the temperature difference between points A and B (figure 5-7) that occurs during the time period \(t_n\) in the heating schedule shown in figure 5-8.
Figure 5-7
2-D representation of a cylindrical ingot
Figure 5-8
General form of a heating schedule
And, the parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4 ... \alpha_n$ are given by

$$\tan \alpha_i = \left( \frac{dT}{dt_i} \right)$$

$$\tan \alpha_2 = \left( \frac{dT}{dt_2} \right)$$

.....

$$\tan \alpha_n = \left( \frac{dT}{dt_n} \right)$$

where $T$ refers to the furnace temperature at any given time. Therefore, $\tan \alpha_n$ represents the heating rate (degree F/hour) during the heating period $t_n$.

Before proceeding further, it would be worthwhile to note the following points based on observations and principles of heat transfer in cylindrical shapes:

- The maximum value of thermal stress in the entire heating cycle occurs in the initial stage of heating during $t_1$. This is because the temperature difference between the ingot surface and the ambience is maximum during this stage, the ingot being charged cold into the furnace. Also, the maximum permissible stress decreases with increasing temperature (i.e. with time) thus prohibiting any values greater than the value in the first stage of heating.

  i.e. $$(\Delta T_{\text{max}})_{t_1} > (\Delta T_{\text{max}})_{t_2} > \ldots \ldots \ldots > (\Delta T_{\text{max}})_{t_n}$$

- The value of $(\Delta T_{\text{max}})_{t_1}$ is independent of $t_1$. The position in time at which $(\Delta T_{\text{max}})_{t_1}$ occurs is shifted inwards (in decreasing X direction) as $t_1$ is decreased.
- The value of $\left( \Delta T_{\text{max}} \right)_n$ is a function of $t_1$ and $\alpha_1$
- The value of $\left( \Delta T_{\text{max}} \right)_2$ is a function of $t_2$ and $\alpha_2$
- The value of $\left( \Delta T_{\text{max}} \right)_n$ is a function of $t_{n-1}$ and $\alpha_n$

and so on.

Since adopting a safety criterion based on a stress value rather than a temperature difference value is easier to implement, the $\left( \Delta T_{\text{max}} \right)$ terms in the constraints can now be replaced by stress terms.

The problem now bears the form:

$$\text{Min } Z = t_1 + t_2 + t_3 + \ldots + t_n$$

Subject to

$$\begin{bmatrix} \frac{(\sigma_{\text{max}})}{(\sigma_{\text{allowable}})}_{t_1} \\ \frac{(\sigma_{\text{max}})}{(\sigma_{\text{allowable}})}_{t_2} \\ \ldots \\ \frac{(\sigma_{\text{max}})}{(\sigma_{\text{allowable}})}_{t_n} \end{bmatrix} \leq 1$$

where $t_1, t_2, t_3, \ldots, t_n > 0$
Here \((\sigma_{\text{max}})_n\) or \((\sigma_{\text{max}})_n\) refers to the maximum thermal stress build up that occurs during the heating period \(t_n\) and \((\sigma_{\text{allowable}})_n\) refers to the maximum permissible stress in the heating period. \((\sigma_{\text{allowable}})_n\) is dependent upon the temperature and hence varies between the heating periods.

### 5.8 Solution of the optimization problem

An approach to solving this problem is to break it up into individual cases and obtain the solutions for these individual cases. The individual solutions can be combined, in the final step, to obtain the final solution. The problem can be broken up thus:

For the heating period \(t_1\):

\[
\text{Max } Z_1 = (\sigma_{\text{max}})_1
\]

where
\[
(\sigma_{\text{max}})_1 = f(T_{\text{init}})
\]

where \(T_{\text{init}}\) is the furnace temperature when the ingot is charged into the furnace.

Subject to:

\[
\left[\begin{array}{c}
(\sigma_{\text{max}}) \\
(\sigma_{\text{allowed}}) \\
\end{array}\right]_1 \leq 1
\]

where
\[
(\sigma_{\text{max}})_1 \geq 0
\]

For the heating period \(t_2\):

\[
\text{Max } Z_2 = (\sigma_{\text{max}})_2
\]

where
\[
(\sigma_{\text{max}})_2 = f(t_1, \alpha_2)
\]
Subject to

\[ \left[ \frac{(\sigma_{\text{max}})}{\sigma_{\text{allowed}}} \right]_2 \leq 1 \]

where \( (\sigma_{\text{max}})_2 \geq 0 \)

Similarly, for the heating period \( t_n \):

\[ \text{Max } Z_n = (\sigma_{\text{max}})_n \]

where \( (\sigma_{\text{max}})_n = f(t_{n-1}, \alpha_n) \)

Subject to

\[ \left[ \frac{(\sigma_{\text{max}})}{\sigma_{\text{allowed}}} \right]_n \leq 1 \]

where \( (\sigma_{\text{max}})_n \geq 0 \)

5.9 Procedure for optimization of heating schedule

The optimization of the heating schedules is done by studying the effect of the various parameters involved in the heating operation such as \( T_{\text{init}}, t_1, t_2 \) and \( \ldots \), \( t_n \) on the total heating time and the thermal stresses within the ingot. The heating schedule for reheating of ingots for forging, generally adopted in industry today is of the form shown in figure 5-9. This inverted L-type heating schedule contains three heating steps. The heating schedule is varied for different values of \( T_{\text{init}}, t_1, t_2 \) and \( t_3 \). The knowledge of the effect of these parameters on the total heating times and stresses enables the construction of the optimized heating schedules.
It is not feasible in practice to study the effects of these parameters because of the time and costs involved in heating operations and unavailability of furnaces for experimental studies. Computer simulation of the heating schedules provides an answer to this problem. The heating schedules for various values of $T_{init}$, $t_1$, $t_2$ and $t_3$ can be simulated using the finite element method and the temperatures within the ingot can be determined during the various stages of heating. The knowledge of the temperature distribution enables calculation of thermal stresses in the ingot which is essential in the optimization procedure. The finite element simulation of heating schedules involves a sound understanding of the various mechanisms of heat transfer involved in the gas fired heating of ingots in furnaces. It also involves an accurate knowledge of the material thermal and mechanical properties at various temperatures that exist during operation.

The heating schedule is simulated using ANSYS, a finite element analysis software. The thermal stresses obtained from the simulation are compared with allowable stress values and the procedure is repeated until total heating time is minimized subject to the constraints.

Described in chapter VII is the application of the optimization methodology to the heating schedules adopted in the reheating of nickel base superalloy ingots. Optimum heating schedules have been developed and presented for Incoloy 901 and Inconel 718.
Figure 5-9
Typical heating schedule employed in industry
CHAPTER VI

APPLICATION OF FINITE ELEMENT ANALYSIS TO THE FURNACE HEATING OF AN OCTAGONAL STEEL INGOT

6.0 Introduction

The objective of this analysis was to verify ANSYS as a reliable and efficient Finite Element Analysis tool to predict temperature distributions and stresses which arise in an ingot as a result of heating. The experimental studies, described in [Finlayson & Schofield, 1959], on a batch type forge furnace were simulated and used to verify the results of the Finite Element Analysis using ANSYS.

The results of a preliminary study of a batch type forge furnace of modern design have been presented. The shape, size and material of construction of the furnace influences the efficiency of the furnace. The way the furnace is operated, application of pressure and temperature control are other factors that affect the efficiency of the furnace. The furnace should be loaded to its designed capacity to allow maximum permissible heating rates and minimum soaking times.

Measurements taken of the temperatures within the ingot during the heating cycles was used to verify the temperature distributions obtained through the use of the commercial Finite Element Analysis software ANSYS.
6.1 Experiment

6.1.1 Description of furnace

The heating was carried out on a bogie hearth furnace at the Walter Sommers works. The furnace is oil fired and has the capacity to heat ingots up to 65 tons in weight. Some of the features of the furnace are the use of high temperature refractories, water cooled doors and jambs. The furnace can be seen in figure 6-1. The hearth dimensions are 31 ft. 6 in. x 7 ft. 6 in. The height to the bottom of the furnace arch is 9 ft 0 in. The heat capacity of the furnace is reduced to minimum by the use of hot-face insulating bricks in the construction. The side and back walls consist of 9 in. of Mg high-temperature insulating bricks, 4.5 in of P₃ insulating firebricks, 3/8 in. of asbestos boarding, and 1/4 in. of welded steel casing. The furnace arch is constructed of 9 in. thickness of Mg high-temperature insulating brick. The door and jambs are water cooled.

The furnace brickwork does not extend below the bogie level, a free space being left for ventilation. It can be seen from the figure 6-1 that the bogie is driven by two electric motors mounted directly below it.

The furnace is fired by 28 Schioldrop oil burners recessed in side walls. On each side of the furnace, there are 8 burners burning above the furnace and 6 below the ingot. The burners are grouped in four independently controlled zones. The temperature in each zone is measured by a thermocouple located in the furnace walls. The oil is preheated to a temperature of 210 F and circulated in a ring main surrounding the furnace at a pressure of 15 lb/in². The Schioldrop burners are each provided with a valve to control the air/oil
Figure 6-1
Bogie hearth forge furnace
[Finlayson, P. C. & Schofield, J. S., 1959]
mixing ratio. The combustion air is supplied by four fans. Individual fans supply air to each zone thereby reducing interaction between the zones.

The end zones are smaller and temperature fluctuations occur in these end zones. There are four waste gas offtakes, two on each side of the furnace. There is no waste heat recovery. The furnace pressure is automatically controlled using two water cooled dampers located in the underground flues. The instrument panel contains pneumatically controlled temperature indicating controllers, for each zone, pneumatically operated pressure controller, and an eight point recorder giving an indication of the temperature conditions in the roof and walls of the furnace.

6.1.2 Measurements on the furnace

The following is a description of the various measurements taken during the heating of a 52.5 ton steel (1022 steel) to the forging temperature. The measurements made were

i) fuel input rate

ii) oil and combustion air temperatures

iii) waste gas temperatures

iv) carbon dioxide and oxygen content in waste gas

v) inside and outside temperatures of the furnace structure

vi) internal and skin temperatures of the ingot.

The fuel input was measured using an integrating flowmeter, situated at the inlet to the oil heater. The readings were taken at half hourly intervals. The oil was preheated to a temperature of 210 °F to ensure satisfactory atomization. The heavy fuel oil has a net
calorific value of 17400 Btu/lb. (Gross C.V. 18600 Btu/lb) and a specific gravity of 0.95 at 60 F. The temperatures of the waste gases were measured by 5%Rh-Pt /20%Rh-Pt thermocouples situated in the waste gas offtakes. Four Pt/Rh thermocouples sheathed in Inconel tubes were placed permanently in the side walls of the furnace. These were used for automatic control of the furnace temperatures in the four zones. The outside temperature of the furnace was measured at regular intervals using 'Tempilstik' crayons.

6.1.3 Temperature measurements in the ingot

Thermocouples were inserted at various depths in the ingot to determine the temperature distributions at these points in the ingot during the heating cycles. For the test on the 52.5 ton ingot (58 in. octagon), 5%Rh-Pt/20%Rh-Pt thermocouple wire was used. The wire was protected by twin bore insulators and was enclosed in Inconel tubing. Specially designed mild-steel thermocouple plugs were used so that the thermocouples could be firmly located at various depths in each of the holes drilled in the ingot.

Four holes were drilled at various depths, perpendicular to one face of the ingot. One thermocouple was inserted in each of these holes providing temperature measurements at radial distances from the center of 7.2, 15.5, 22.9 and 27.9 in. The Inconel tubing was inserted into the holes and the ends of the couples were held firmly in place by means of tightly fitting plugs. The temperatures were continuously recorded during the heating cycle. The skin temperature of the ingot was measured at lower temperatures using a chromel/alumel thermocouple welded to the surface of the ingot. The couple was protected from the flame by an Inconel sheath. At temperatures above 950 C, the temperatures at the skin were measured using an optical pyrometer, sighted on the shoulder of the ingot.
6.1.4 **Description of the Heating cycle**

The heating schedule is seen in figure 6-2. The furnace temperature is taken up rapidly at the rate of about 110 °C/hour to 650 °C and held at 650 °C for about 6 hours. The fuel input rate is then increased and the furnace temperature raised to 1275 °C. The temperature is maintained at 1275 for 14 hours. The total heating time of the cycle is 32 hours.

Only 10 burners were used in the first 12 hours of heating. After this stage, further burners were lit and the furnace temperature was allowed to rise to 1275 and held at this temperature until the end of the soaking stage by the use of temperature controllers.

It can be seen that the heat input rate to the furnace is constant in the first 12 h of heating at 61 therms/h (36 gal/h). The heat input rate is then raised to a level of about 151 therm/h (89 gal/h) for the remainder of the heating period.

6.2 **Finite Element Modeling of Experiment 1**

The calculation of the temperature distribution within the ingot is a case of unsteady state heat conduction. It is possible to deduce the temperatures within the ingot from a knowledge of furnace temperature and heat transfer coefficients at the ingot surface.

The analysis was based on two kinds of approaches. The initial approach was to model the ingot as cylinder with a circular cross section (figure 6-3 (a)). The second approach was closer to the actual situation, in which the ingot was modeled as a cylinder of octagonal cross-section with a 58 in. diagonal (figure 6-3 (b)). Some of the information which was common to the two approaches is described below.
Figure 6-2
Experimental heating schedule (variation of furnace temperature with time)
[Finlayson P.C. & Schofield J.S., 1959]
Figure 6-3
(a) circular cross section (case 1)
(b) octagonal cross section (case 2)
6.2.1 Material properties

The material (1022 steel) properties used in this analysis are shown in figure 6-4 and figure 6-5. The thermal conductivity and specific heat capacity of steel exhibit strong variation with temperature, while the density can be assumed practically constant over the range of temperatures in consideration. The appendix contains a comprehensive list of the data used in the analysis.

6.2.2 Boundary conditions

In the early stages of heating, the ingot is receiving heat from the convection of hot gases and flame alone and the heat transfer coefficients are low. Towards the end of heating, the radiation from the furnace walls contributes to the heating of the ingot. The heat transfer coefficients were obtained from [Finlayson & Schofield, 1959]. The variation of the heat transfer coefficient with the ingot skin temperature is shown in figure 6-6. A rapid increase in the surface heat transfer coefficient is seen as the ingot skin temperature rises. The design of the furnace should provide for rapid circulation of hot gases around the ingot and ensure uniform heating of the ingot. This will obviate the necessity to turn the ingot while heating.

6.2.3 Modeling the heating schedule

The heating schedule was broken into several smaller linear steps. There were seven load steps in the analysis.
Figure 6-4
Variation of specific heat capacity with temperature
(1022 steel)
Figure 6-5
Variation of thermal conductivity with temperature
(1022 Steel)
Fig. 6-6
Variation of surface heat transfer coefficient with ingot skin temperature
(1022 Steel ingot)
[Finlayson P.C. & Schofield J.S., 1959]
6.2.4 Postprocessing

The results obtained at the end of the solution procedure were analyzed using the POST 1 and POST 26 post processors of ANSYS. The POST 26 post processor which is a time-history processor was used to obtain the variation of the ingot skin and center temperatures with time.

6.2.5 Case 1

6.2.5.1 Approach

As an initial approach, the ingot was assumed to be a cylinder of infinite length. The heat input to the ends of the ingot was ignored in the calculations. From the knowledge of the weight, radius and density of the ingot, the height of the ingot was calculated. The height of the ingot was calculated to be 11.773 ft.

The problem was formulated as an axisymmetric, two dimensional transient thermal analysis. Convective boundary conditions were employed at the ingot surface. A number of assumptions, based on the study of the furnace heating processes, are involved in the analysis, some of which are:

i) the whole of the ingot surface is uniformly at a temperature \( T_0 \) F.

ii) the absorption of heat is axisymmetric

iii) the primary mode of heat transfer to the ingot is convection

Since the heat transfer was assumed to be axisymmetric, the heating was modeled using two dimension elements. The top right quadrant was modeled since symmetry is present.
6.2.5.2 **Mesh generation**

The PREP7 preprocessor of ANSYS was used to create the finite element mesh. The 2-D thermal solid element STIF 55 with the axisymmetric option was employed in the analysis. A coarse mesh was initially employed and then refined in the later stages to obtain better accuracy and convergence. The mesh was also refined towards the ingot surface as the temperature gradients are largest in this region. The mesh density employed was of the order of 160 nodes and 135 elements.

6.2.6 **Case 2**

6.2.6.1 **Approach**

In this approach, the ingot was modeled as a cylindrical ingot of octagonal cross section as shown in figure 6-3 (b). This was done to obtain improvement in the accuracy of the predicted values because the initial case was an approximation of the actual situation. The analysis could no longer be treated as axisymmetric heat transfer because of the geometry of the ingot involved. Also, the heat transfer was modeled in 3-D due to the non-axisymmetry present.

6.2.6.2 **Mesh generation**

The mesh was generated using 3-D isoparametric thermal solid elements (STIF 77). The mesh was refined towards the edges, where the temperature gradients are the largest. The mesh density was of the order of 600 nodes and 400 elements. The mesh was, again initially coarse to start with, and was refined for better accuracy in later runs.
6.3 Results and Discussion

The heating of the steel ingot was simulated and the temperatures in the ingot were determined using ANSYS. The variation of the ingot skin and center temperatures were determined as a function of the heating time. Simulations were performed for the two cases described earlier, namely for circular as well as octagonal cross-sections.

6.3.1 Case 1

Figure 6-7 shows a comparison of experimental and simulation results for the variation of the ingot skin temperature with heating time for the circular cross section ingot (case 1). Figure 6-8 shows a similar comparison for the variation of the temperature at the center of the ingot with heating time. The initial mode of heat transfer is primarily convection to the ingot from the ambient hot gases and flame. The surface of the ingot gets heated rapidly due to the large initial temperature difference between the ingot skin and the furnace atmosphere ($T_F - T_S$). The heat absorbed on the surface by convection is transferred to the interior towards the center by conduction. As the results show, the ingot skin temperature rises faster than the center temperature in the initial stages of heating. Indeed, the largest temperature gradients during the entire heating operation are observed in the first few hours of heating.

As the skin temperature ($T_S$) approaches the furnace temperature ($T_F$), which is held constant during this period, the value of ($T_F - T_S$) decreases and the center temperature
Figure 6-7
Variation of ingot skin temperature with time
(cylindrical cross section, steel ingot)
Experimental Results Source: [Finlayson P.C. & Schofield J.S., 1959]
Figure 6-8
Variation of ingot center temperature with time
(cylindrical cross section, steel ingot)
Experimental Results Source: [Finlayson P.C. & Schofield J.S., 1959]
starts approaching the skin temperature. As a result, a decrease in the temperature difference between the ingot skin and the center \((T_s - T_c)\) is observed during the later half of the first stage of the heating.

At this point, the furnace temperature is steadily ramped from its initial value of 1500 F to 2200 F at the rate of 200 F/hr. This results again, in a faster increase of the ingot skin temperature. As the rate of heating is not very high, we do not see any spectacular increase in the values of \((T_s - T_c)\).

The furnace temperature is now held constant at 2200 F for a period of 16 hours. This holding period allows the ingot center to attain the outside temperature. The ingot attains a uniform temperature of 2200 F at the end of the heating period.

As figures 6-7 and 6-8 show, there is a reasonably good agreement between the experimentally measured values and the values of ingot temperatures predicted by ANSYS simulations for the case of the circular cross section.

6.3.2 Case 2

Figures 6-9 and 6-10 show a comparison of the experimental and predicted values of the ingot skin and center temperatures respectively for the case of the octagonal cross section. These results reveal a similar behavior of the ingot temperatures during the various stages.
Figure 6-9
Variation of ingot skin temperature
(Octagonal cross section, steel ingot)
Experimental Results Source: [Finlayson, P.C. & Schofield, J.S., 1959]
Figure 6-10
Variation of ingot center temperature
(Octagonal cross section, steel ingot)
Experimental Results Source: [Finlayson, P. C. & Schofield, J. S., 1959]
of heating. Also, it can be seen that there is closer agreement between the experimental and simulation values for the case 2. This is due to the more accurate representation of the ingot geometry in this case. The ingot cross section was modeled as a 58 in. octagon which was the case in the experimental studies.

It was seen that the temperature variation from the center to the surface of the ingot is steepest during the first few hours of heating and gradually decreases to a small value towards the final stages of the heating. These predicted results are in line with the theoretical principles of heat transfer and are seen to be in excellent agreement with the experimental results. There was a noticeable difference between the experimental and simulation predicted values at $t = 18 \text{ h}$. This is attributed to the lack of availability of accurate data on the surface heat transfer coefficient above 2000 F. The surface heat transfer coefficient values were extrapolated in the region of temperatures between 2000 F and 2200 F, and this might have contributed to the discrepancy in results in this temperature range. As the ingot skin approaches the furnace temperature, the effects of convection are diminished.

6.3.3 **Comparison of results for the two cases**

As seen in figure 6-11 and 6-12, there is closer agreement between the experimental values and simulation results for case 2 (octagonal cross section) than for case 1 (circular cross section). This is due to the more accurate representation of the ingot geometry.

It is pointed out that, since there is no spectacular improvement in the simulation results in case 2, it would be justifiable to approximate the ingot as a circular cross sectional ingot in
heat transfer analyses. The modeling of the octagon was performed in 3-D and the model contained twice as many elements as in case 1 for similar levels of accuracy. The computing time and hence, costs are increased in case 2 without any corresponding increase in the accuracy of the temperatures predicted by this model.

Caution should however be exercised in modeling of the ingot geometry, if the thermal analysis is to be followed by a thermal stress calculation procedure. The thermal stresses that are built up in the ingot during the heating are sensitive to the ingot geometry and configuration and approximations in the ingot geometry may lead to erroneous results.

6.4 Scope of the Analysis

It has been demonstrated that the analysis outlined is justified, in spite of the assumptions made its derivation. It was also instrumental in verifying ANSYS as an accurate and reliable finite element analysis tool in measuring temperature distributions within an ingot during furnace heating operations. This is borne out by the excellent agreement of the ANSYS simulation results with the experimentally measured values of ingot temperatures. It has also been demonstrated that where it is not convenient to measure the ingot skin temperature during heating, the temperature distribution within the ingot can still be calculated from a knowledge of the variation of furnace temperature with time and the values of surface heat transfer coefficients.
Figure 6-11
Comparison of results for circular and octagonal cross sections
(skin temperatures)
Figure 6-12
Comparison of results for circular and octagonal cross sections (center temperatures)
Experimental Results Source: [Finlayson, P.C. & Schofield, J.S., 1959]
CHAPTER VII

OPTIMIZATION OF HEATING SCHEDULES FOR NICKEL BASE
SUPERALLOY INGOTS

7.0 Background

Basic equipment considerations for furnace heating of nickel base superalloy ingots almost
differ from those for heat treating steel ingots. In general, the temperatures are
controlled at 14 C (25 F) with a maximum temperature range up to about 1290 C (2350
F). Belt conveyor furnaces, although widely used for production annealing, are less gas-
tight than the roller hearth furnaces resulting in higher atmosphere costs for a roller hearth
furnace of the same volume. Batch heating for annealing or solution treating is done in box
furnaces. These may have provisions for purging, preheating and quenching, if the high
temperature compartment is supplemented by other chambers.

Aging of heat resisting alloys, commonly in the range of 650 to 900 C (1200 to 1650 F)
is usually done in box furnaces, with or without protective atmospheres. The usual
operating-temperature tolerance is 14 C (25 F) for wrought alloys and 8 C (15 F)
for casting alloys. Continuous furnaces are seldom used because of the long aging cycles.
Vacuum furnaces are used for heat treatment of niobium and tantalum and other heat
resisting alloys. Heating may be accomplished by resistance elements or by induction.
Furnace design dictates a batch operation. Cooling can be done in a vacuum retort pressurized with inert gas that provides conductive cooling after heating is discontinued.

Fixtures for holding finished parts of assemblies during heat treatment may be of support type or the restraint type. Minimum fixturing is used for alloys that must be cooled rapidly from the solution treating temperature during the solution treating or quenching operations. Dimensional control is maintained by the use of restraining fixtures during the aging treatment.

Support fixtures are used when the part itself provides self-restraint. Parts that have a flat surface can be placed on a furnace tray or plate. Examples of such parts are cylinders, rings and disks. For parts of non-symmetrical shape, special supports are built up from the flat tray. These supports are welded and stress relieved before use.

Restraint fixtures are generally more complicated than the support fixtures and may require machined grooves, lugs or clamps to hold parts to a given shape. Usually the coefficient of expansion of both the part and the fixture should be nearly the same. However, in some cases, the fixture is deliberately chosen to be made from a material with different characteristics to apply pressure to the part when the temperature increases.

The optimization of the heating schedules entails a thorough investigation of the heat transfer processes in furnaces, heat transfer in cylindrical ingots and the application of a pre-defined methodology for the optimization of the heating schedules.
7.1 Furnace heating

Experiments were conducted at Teledyne Alivac on an Incoloy 901 ingot. At first simulations using ANSYS are performed for the Teledyne Alivac case. The simulation results are compared with experimental observations for model verification. Then optimization procedures are applied to the Incoloy 901 case. In addition, optimization procedure is applied to the case of Inconel 718 ingot heating. Though experimental results are not available for Inconel 718, the modeling was carried out because IN 718 is most commonly used nickel base superalloy.

7.1.1 Experiments

The furnace heating of a 20" diameter, 98" long Incoloy 901 ingot (figure 7-1) was carried out at the Teledyne Alivac facilities at Monroe, North Carolina. The furnace was gas fired and contained 6 burners. The burners were located three on each side. Figure 7-2 shows the relative location of the burners in the furnace. The furnace dimensions are 67" high, 216" wide and 18" deep. The thermocouples in the furnace were located on the left front wall (24" from front and 30" from floor) and back center wall (24" from floor).

The temperatures were measured at various locations in the ingot using thermocouples. Holes were drilled into the ingot at the various locations to the center and mid radius to insert thermocouples. 0.5" deep holes were drilled on the surface to measure the ingot skin temperatures. The locations of the thermocouples on the ingot are shown in figure 7-1. Locations A, B and C correspond to the ingot center, skin and mid-radius locations respectively. The holes for the thermocouples were drilled at various points along the ingot length and diameter. These locations were labelled 101, 102, etc. and temperatures were
Figure 7.2
Location of the burners in the furnace

TOP

FRONT

2''

#3

7''

4.5''

#2

7''

#1

BACK

2.5''

216''

BOTTOM
Figure 7.2
Location of the burners in the furnace
recorded every 0.5 hour for all the locations. The location 101 in figure 7-1, for example corresponds to the mid-radius location on the face of the ingot. Location 106 corresponds to the center of the ingot at the mid-height and so on.

The ingot is cold-to-touch when it is charged into a furnace maintained at an initial temperature of 1500 F. The ingot is kept in the furnace at 1500 F for 10 hours after which the furnace temperature is raised at a rate of 200 F every 2 hours to 2175 F. The furnace is then maintained at 2175 F for the next 10 hours to allow the ingot center to reach the outside furnace temperature. The total heating time is 26 hours. The experimental heating schedule, give by the variation of the furnace temperature with time, is shown in figure 7-3.

7.2 Finite Element Modeling of furnace heating

The furnace heating schedule employed in the reheating of an Incoloy 901 ingot was modeled to obtain the temperature distributions and subsequently the thermal stresses in the ingot.

7.2.1 Approach

The problem was formulated as a two dimensional transient thermal analysis problem with non axisymmetric loading. Convective boundary conditions were employed at the ingot surface. The assumptions in the analysis were:

i) the entire ingot surface is at a uniform temperature $T_s$.

ii) the primary mode of heat transfer to the ingot is modeled as convection (radiation is modeled as a convective boundary condition)
Figure 7-3
Experimental heating schedule
Source: Teledyne Allvac.
The heating was modeled using two dimensional axisymmetric ring elements with non-axisymmetric boundary conditions.

7.2.2 Preprocessing

The mesh was generated using 2-D thermal solid STIF 75 elements with the capability to accept non-axisymmetric boundary conditions. For thermal stress analysis, an equivalent structural element (STIF 25) was used. The mesh refinement was performed in line with earlier defined objectives of better accuracy and shorter simulation time. The transient time-step optimization procedure was employed.

7.2.3 Material properties

The ingot materials under study are Incoloy 901 and Inconel 718 which are classified under nickel base superalloys. The material properties were input as functions of temperature. The heating schedule was incorporated in the form of five load steps.

7.2.4 Boundary conditions

The measurements of the ingot skin temperatures at the bottom and the top of the ingot reveal little or negligible difference in their values. This is evident from figure 7-4. This can be attributed to the uniform heat transfer occurring over the curvature of the cylindrical surface as a result of the location of burners on either side of the ingot. The ingot skin temperature and therefore, the surface heat transfer coefficient can be assumed to be constant over the curvature and uniform boundary conditions can be employed over the
Figure 7-4
Comparison of top and bottom surface temperatures measured on the Incoloy 901 ingot during the furnace heating operation. "Top" and "bottom" refer to locations 107 and 109 in figure 7-1. Results Source: Teledyne Allvac, NC.
curved surface area. The ends of the ingots are however special cases, which are modeled as vertical planes in cross flow, as suggested by [Wang et al., 1988] in their study on convective heat transfer in cylinders. The heat transfer coefficients are suitably modified to include the end effects.

Thermal loading was incorporated in the form of nodal convections on the surface. The heating of the ingot is primarily due to convection from hot gases in the initial and middle stages of heating. Towards the end of the heating, when the ingot temperature approaches the final temperature, the furnace is turned off and the radiation from the walls is the primary source of heating. The surface heat transfer coefficient is obtained empirically from the equation suggested by [Sun, 1971] for the heating of cylindrical ingots of heat resisting alloys:

$$h = \xi A_f F \left( \frac{T_f^4 - T_s^4}{T_f - T_s} \right) + 0.27 \left( \frac{T_f - T_s}{D} \right)$$

where

$$F = \frac{1}{F_{BR}} + \frac{1}{\xi_f - 1} + \frac{A_f}{A_c} \left( \frac{1}{\xi_c - 1} \right)$$

where $T_f$ and $T_s$ are the furnace and ingot skin temperatures respectively.

The two modes of heat transfer, namely convection and radiation, are combined in an empirical relationship and have been used in the thermal stress analysis of cylindrical Hastelloy alloy X ingots [Sun, 1971]. Since this expression yielded satisfactory results, the expression can be extended in its application to a similar class of materials and geometry.
The variation of the heat transfer coefficient with the ingot skin temperature is shown in the figure 7-5. This equation was implemented in the form of nodal convections on the ingot surface. Both stepped and ramped boundary conditions were employed appropriately, depending on the heating stage. For the thermal stress analysis, symmetry boundary conditions were enforced along the lines of axes. The thermal stress procedure was performed as described earlier in section V. The assumptions made in the thermal stress analysis were:

1) No residual stresses in the ingot prior to reheating.
2) Phase transformation induced changes in dimensions are ignored.

7.2.4 Solution and post processing

The input data was prepared according to the conditions described above and the simulations were performed using ANSYS. All simulations were run on the VAX 8550/VMS operating system at the Robinson Computer Graphics Laboratory, the Ohio State University. Post processing was done using the POST 1 and the POST 26 processors of ANSYS.

The output from a typical analysis included the temperature distributions within the ingot at various stages of the heating and the thermal stresses (principal stresses) at the corresponding time periods. The results of the thermal and thermal stress analyses have been presented and discussed in section 7.4. The results of the finite element analysis procedure were coupled with the optimization procedure to obtain the optimized heating schedules for the two ingot materials under study, namely Incoloy 901 and Inconel 718.
7.3 Optimization of heating schedules

The figure 7-3 shows the form of the heating schedule generally adopted in the industry today. This is the form of the heating schedule adopted at Teledyne Allvac, North Carolina in their furnace re-heating of as-cast ingots for upsetting. The ingots are initially cold-to-
touch when they are loaded into the furnace operating at 1500 F \( (T_{\text{init}}) \). The furnace temperature is raised to 2175 F (which is the final temperature desired) and held until the temperature is uniform throughout the ingot.

7.3.1 *Formulation of the problem*

Applying the optimization criteria presented in chapter 5, this problem can be formulated as

\[
\text{Min } Z = t_1 + t_2 + t_3
\]

Subject to

\[
\left[ \frac{(\sigma_{\text{max}})}{(\sigma_{\text{allowable}})} \right]_{t_1, t_2, t_3} \leq 1
\]

where \( t_1, t_2, t_3 > 0 \)

It can be seen that

\[
\alpha_1 = 0
\]

\[
\alpha_3 = 0
\]

\[
\tan \alpha_2 = \frac{(2175 - T_{\text{init}})}{t_2} \text{ F/hour}
\]

The problem is solved by dividing it into two parts. In the first part, the value of \( T_{\text{init}} \) is varied so as to obtain the maximum value of thermal stress allowed for a given value of initial furnace temperature. This problem can be formulated as

\[
\text{Max } Z_1 = (\sigma_{\text{max}})
\]

Subject to the constraint
\[
\left[ \frac{\sigma_{\text{max}}}{\sigma_{\text{yield}}} \right]_{t_1} \leq 1
\]

where \( t_1 > 0 \) and \( \sigma_{\text{max}} = f(T_{\text{init}}) \)

On completion of this stage, the value of \((T_{\text{init}})^{\text{opt}}\) is obtained. Now the value of \( \alpha_2 \) and \( t_1 \) are varied to obtain the maximum stress value in the second stage of heating. This can be done by assuming a maximum value of \( \tan \alpha_2 = 150 \, \text{F/hr} \) and varying the value of \( t_1 \) to obtain the maximum stresses. The value of \( \tan \alpha_2 \) is based on the furnace heating operations at Teledyne Allvac, North Carolina.

This problem is represented as

\[
\text{Max } Z_2 = \sigma_{\text{max}}
\]

Subject to the constraints

\[
\left[ \frac{\sigma_{\text{max}}}{\sigma_{\text{yield}}} \right]_{t_2} \leq 1
\]

where \( \sigma_{\text{max}} \geq 0 \) and \( \sigma_{\text{max}} = f(t_1, \alpha_2) \)

Intuitively, it can be seen that it is more advantageous to decrease \( t_1 \) and increase \( \alpha_2 \). This is because the ingot is heated at a higher temperature for a greater percentage of time, which leads reduces the total heating time. However, this may not be possible in actual situations because a value of 90 for \( \alpha_2 \) corresponds to a value of zero for \( t_2 \). It is not practically possible to raise the temperature of the furnace by a large value instantaneously. Therefore it can be seen that the value of \( \alpha_2 \) has to be determined based on practical considerations.
7.3.3 Safety criterion

The safety criterion adopted in this problem is based on the criticality of the application, the material being heated and the availability of data to implement the criterion. It was decided to use the 0.2% yield stress as the maximum permissible stress value ($\sigma_{\text{allowable}}$) for the thermal stress in the ingot. The ingot is assumed to fracture at stress levels beyond this value. A similar approach towards applying a safety criterion has been described in [Sun, 1971] in the heating of Hastelloy alloy X ingots. In that case however, a value of 90 percent of the yield stress was used as the 'safe' value for thermal stresses within the ingot. The yield stress safety criterion is a conservative approach to implementing safe heating rates for ingots.

The maximum principal stress within the ingot was calculated for the three steps of heating and controlled so as not to exceed the yield strength values at the corresponding temperatures. The stress analysis was based on an elastic model and the safety criterion implemented was in line with this approach.

It is pointed out that the elastic-plastic models for stress evaluation have been adopted in the thermal stress analysis of quenching treatments of steels [Inoue & Tanaka, 1975, Sjostrom, 1982, etc.], where large temperature gradients are generated in extremely small periods of time. The stresses generated in quenching treatments typically exceed the yield strength and plastic deformation has been observed to occur.

In cases of heating in furnaces, where relatively modest temperature gradients are generated, the elastic model of stress evaluation and the yield strength safety criterion suit
the purposes admirably, taking into account the ease and speed with which the analyses can be performed.

7.3.4 Optimization procedure

The ingot materials under study are Incoloy 901 and Inconel 718, which are nickel base superalloys used primarily in aerospace applications. A background of the materials under study can be found in chapter 2. The heating schedules for these two materials, adopted in industry, is identically the same. The present optimization methodology has been used to refine heating practices so that alloys such as 718 and 901 have different heating times. A comparison of the materials' thermal behavior was done and has been discussed in [7.4.4] under results and discussion. Since these two materials are of similar chemical composition, a similar optimization procedure was adopted for both.

The maximum principal stress for the various values of the starting furnace temperature are calculated by simulation and compared with the value of the yield stress at that temperature. The values of these maximum stresses are checked to see if they exceeded the yield stress values. This is the basis upon which the initial furnace temperature \( T_{\text{init}}^{\text{opt}} \) is chosen. The value of the starting furnace temperature at which the thermal stresses are just below the yield stress values is determined by simulating these cases. A value of \( T_{\text{init}}^{\text{opt}} \) is found satisfactory for the furnace starting temperature.

With the value of the starting furnace temperature \( T_{\text{init}}^{\text{opt}} \) thus established, the value of \( t_1 \) is varied next. The objective of the second stage of simulations is to maximize the rate of heating from the starting furnace temperature to the final temperature desired. The maximum possible heating rate, theoretically, is achieved at the value of \( \alpha_2 = 90 \), i.e. the
furnace temperature is stepped up directly from the value of $T_{\text{init}}$ to the final temperature. But under most practical conditions, stepping up the furnace temperature directly is not possible. As a result, there exists a constraint upon the maximum value of the rate of heating in the second stage. A practical value of 150 F/hr can be assumed for the heating rate ($\tan \alpha_2$). Based on the above considerations, a number of cases at $(T_{\text{init}})_{\text{opt}}$ F and $\tan \alpha_2 = 150$ F/hr were simulated.

Again, the maximum values of principal stresses during the second stage of heating are determined and compared to the yield strengths at the appropriate temperatures. The value of $t_1$ is increased or decreased such that the thermal stresses are maintained below the tolerable values. An acceptable value for $t_1$ is thus determined.

Given the values of $t_1$ hours at $(T_{\text{init}})_{\text{opt}}$ F and $\tan \alpha_2$ F/hr, the value of $t_3$ is determined by simulating the entire heating operation until the center and skin temperatures of the ingot are nearly the same. The total heating time is thus found for the entire operation.

This procedure was repeated for both Incoloy 901 and Inconel 718 to determine the optimum heating schedules for the materials. The results of the optimization procedure are presented and discussed in the following section.

### 7.4 Results and Discussion

The results of the thermal stress analyses and the optimization procedures for the two materials are presented in this section. Also presented are improved heating schedules for the materials and a comparison of the materials' thermal behavior.

#### 7.4.1 Thermal Analysis
The furnace heating schedule was simulated for the case of the Incoloy 901 ingot. Figure 7-6 shows a comparison of the experimental values and simulation results for the variation of the ingot skin with heating time. Ingot center and skin locations correspond to points A and B on the ingot shown in figure 7-1. Figure 7-7 shows a similar comparison for the variation of the ingot center temperature. As can be seen, the skin temperature rises rapidly at first causing large temperature gradients. The center shows the slowest response and the center temperature gradually rises towards the final stages of the heating to the outside furnace temperature. At this point in the heating, the temperature gradients are completely absent. Initially, due to the different heating rates at the ingot skin and center, temperature gradients are built up within the ingot. The temperature gradient reaches its peak value at about 2.5 hours after the heating is commenced. The temperature difference decreases from the peak value and then we see another increase in the value of this temperature difference towards the initial stages of the ramped heating.
Figure 7-6
Variation of ingot skin temperature with time
(Incoloy 901 ingot) Experimental results source: Teledyne Alvac
Figure 7-7
Variation of ingot center temperature with time
(Incoloy 901 ingot) Experimental results source: Teledyne Allvac
It can be seen from these figures that there is agreement between the experimental and simulation results to an acceptable level of accuracy. This agreement between the predicted and experimental values justifies the assumptions made in the analysis and provides basis for further studies in this direction.

It was decided to simulate the same heating schedule for a different material, namely Inconel 718, in an effort to understand the heating behavior of this nickel base superalloy. Figure 7-8 shows the predicted variation of the ingot skin and center temperatures for an Inconel 718 ingot subject to the heating schedule described in figure 7-3. The variations in the ingot temperatures for Inconel 718 are found to be largely similar to the those of Incoloy 901 as expected. The results of the thermal analyses were used to determine the thermal stresses within the ingot and in the optimization procedure for the two materials.

7.4.2 Optimization of heating schedules

The various cases based on the considerations described in [7.3.4] were simulated for both Incoloy 901 and Inconel 718 ingot heating operations. Though experimental results were available for 20" diameter, 98" long ingot, the optimization procedure was employed for 20" diameter x 60" long ingots because the latter are typical dimensions of as-cast ingots being reheated for upsetting in industrial practice.
Figure 7-8
Predicted (FEM) variation of ingot skin and center temperatures
(Inconel 718 ingot)
7.4.2.1 **Incoloy 901**

*Effect of $T_{\text{init}}$*

Figure 7-9 shows the effect of the initial furnace temperature ($T_{\text{init}}$) on the largest temperature difference ($\Delta T_{\text{max}}$) between the center and the surface. The temperature difference, which is directly related to the thermal stress, rises rapidly as the furnace initial temperature is increased. It was observed that the time of holding at the initial furnace temperature (t₁) did not affect the temperature difference value. As t₁ is decreased, the time at which ($\Delta T_{\text{max}}$) occurs is seen to move inwards, i.e. it occurs earlier in time. These phenomena have also been reported by [Reddy et al, 1984] and [Sun, 1971]. On comparison of the maximum principal stress values with the yield stress, it was determined that 1700 F was the highest acceptable value for $T_{\text{init}}$. Any value of furnace starting temperature above 1700 F would lead to an undesirable build up of thermal stresses beyond the yield stress level, even if the ingot were subject to heating for a very short period of time.

*Determining the value for t₁*

With the value of ($T_{\text{init}}$)$^{\text{opt}}$ established, the next step was determine the time of holding at this starting furnace temperature and the subsequent rate of heating to the final temperature. While the value of t₁ does not influence the thermal stresses in the initial stage, it does, along with tan $\alpha_2$ influence the build up of thermal stresses during the next (ramped) stage of heating. It should be noted that it is advantageous to raise the furnace temperature as fast as possible to the final temperature in order to achieve low heating times. The maximum heating rate of 150 F/hr ($\tan \alpha_2$) leads to a value of about 3 hours for t₂. With
Figure 7-9
Effect of initial furnace temperature on the maximum temperature difference between center and skin (Incoloy 901 ingot)
the value of $t_2$ thus established, simulations were performed for various values of $t_1$ to
determine its minimum value.

Effect of $t_2$

The effect of $t_2$ on the total heating time and $\Delta T_{\text{max}}$ is shown in figures 7-10 and 7-11. It
can be seen that as the rate of heating increases (as $t_2$ decreases), the total heating times
are rapidly decreased and the thermal stresses are increased. The values of the furnace
heating rates in the ramped stage of heating are typically predetermined taking into account
the fastest heating rates achievable that could be withstood by the ingot. It is the aim of the
optimization procedure to obtain the fastest heating time without exceeding tolerable stress
levels.

Based on the approach described earlier and the results of the simulations, the following
heating schedule was found to be the optimal heating schedule:

1) Charge ingot cold into furnace at 1700 F.
2) Hold at 1700 F for 3 hours.
3) Raise furnace temperature to 2175 F in a ramped manner in the next 3 hours.
4) Hold at 2175 F for 8 hours.

The total heating time is 14 hours. The revised heating schedule is shown in figure
7-12. The predicted variations of the ingot center and skin temperatures with heating time
for the new schedule are shown in figure 7-13. Figure 7-14 shows the variation of the
maximum principal stress in the ingot as heating progresses along with the yield stress of
the material. The maximum principal stress is maintained very close to the value of the yield
Figure 7-10
Effect of $t_2$ on the total heating time
(Incoloy 901 ingot)
Figure 7-11

Effect of $t_2$ on the maximum temperature difference between center and skin (Incoloy 901 ingot)
Figure 7-12
Optimum heating schedule for 20" diameter Incoloy 901 ingot
Figure 7-13
Predicted variation of ingot skin and center temperatures for an Incoloy 901 ingot subject to the optimum heating schedule
Figure 7-14
Predicted variation of the maximum thermal stress within the ingot during the heating period (Incoloy 901 ingot)
stress taking care at the same time not to exceed it. The thermal stresses in the ingot are observed to be maximum at the center of the ingot acting in the radial 'r' direction of the cylindrical coordinate system. The stress is observed to rapidly increase to the peak value, which occurs at about 1.2 hours after the heating is started. The stresses then decrease within the ingot as a result of the center temperature rising at a faster rate. The stress once again shows an increase at the beginning of the second stage of heating and then decreases to a very low value towards the end of the heating operation. This procedure provides for the maximum usage of tolerable stress levels in the ingot, which directly leads to a minimization of heating time.

7.4.2.2 **Inconel 718**

Figure 7-15 shows the effect of $T_{\text{init}}$ on the maximum principal stress within the ingot. Figures 7-16 and 7-17 show the effect of $t_2$ on the maximum thermal stress and the total heating times, for Inconel 718, respectively. A similar optimization procedure was adopted for Inconel 718 and the following optimized heating schedule was obtained:

1) Charge ingot cold into furnace at 1900 F.
2) Hold at 1900 F for 3 hours.
3) Raise furnace temperature to 2175 F in a ramped manner in the next 2 hours.
4) Hold at 2175 F for 8 hours.

The total heating time for the ingot to reach a uniform temperature of 2175 F is 13 hours. Figure 7-18 shows the optimum heating schedule and 7-19 shows the predicted variations of the ingot temperatures with time for the new schedule. Figure 7-20 shows the variation of the maximum principal stresses in the ingot during the period of heating.
Figure 7-15
Effect of initial furnace temperature on the maximum thermal stress (Inconel 718 ingot)
Figure 7-16
Effect of $t_2$ on the maximum temperature difference between the ingot center and the skin (Inconel 718 ingot)
Figure 7-17
Effect of $t_2$ on the total heating time
(Inconel 718 ingot)
Figure 7-18
Optimum heating schedule for alloy 718 ingot
Figure 7-19
Variation of ingot skin and center temperatures in the optimum heating schedule
(Inconel 718 ingot)
Figure 7-20
Predicted variation of the maximum thermal stress within the ingot during the heating period
(Incconel 718 ingot)
7.4.2.3 Discussion of the optimization procedure

The heating schedules obtained through this procedure lack strict proof of optimality, the major reason being that the heating operations involve time-variant, non-linear models with complex constraints. Besides, the absence of the effective optimal control algorithms for solving such kinds of problems is another significant factor.

The optimization procedure is based on a 'common sense' approach coupled with finite element simulations and a knowledge of the material systems. It has yielded heating schedules which are a major improvement over the current heating schedules being employed in the industry. The total heating time in the case of Incoloy 901 is 14 hours and in the case of Inconel 718 is 13 hours, which reflect 45 to 50 percent savings in time over the current heating times.

This methodology of optimization can be extended universally to any class of materials of any geometric shape. The results from the optimization procedure will be recommended to the industry for implementation and further improvements can be based on the feedback received.

7.4.3 Comparison of material-on-heating behavior

Figures 7-21 and 7-22 show a comparison of the ingot center and skin temperatures for Incoloy 901 and Inconel 718. These results were obtained for the same heating schedule (figure 7-3), ingot geometry and dimensions. The Incoloy 901 ingot showed faster response compared to the Inconel 718 ingot due to its higher thermal diffusivity. However, being similar materials, there is not a major, though noticeable, difference in their heating
Figure 7-21
Comparison of ingot center temperatures between Incoloy 901 and Inconel 718 ingots subject to the same heating schedule.
Figure 7-22
Comparison of ingot center temperatures between Incoloy 901 and Inconel 718 ingots subject to the same heating schedule.
behaviors. This, coupled with difference in yield strengths at higher temperatures, has contributed to the difference in the optimized heating schedules. As a matter of fact, it is to expected that the total heating time would be lesser for Incoloy 901 because of its higher thermal properties. But Inconel 718 exhibits yield higher strengths (shown in figure 7-23), and as a result higher thermal stress tolerance levels, at elevated temperatures thereby facilitating the heating of the ingot at higher starting temperatures and faster heating rates. This explains its shorter heating time in the optimum heating schedules.
Figure 7-23
Comparison of room temperature and elevated temperature yield strengths between Incoloy 901 and Inconel 718.
CHAPTER VIII

CONCLUSIONS AND FUTURE WORK

The present work has outlined an optimization methodology for pre-heating operations in furnaces. The heating schedules obtained through this procedure lack strict proof of optimality, the major reason being that the heating operations involve time-variant, non-linear models with complex constraints. Besides, the absence of the effective optimal control algorithms for solving such kinds of problems is another significant factor.

The optimization procedure is based on a 'common sense' approach coupled with finite element simulations and a knowledge of the material systems. It has yielded heating schedules which are a major improvement over the current heating schedules being employed in the industry. The total heating time in the case of Incoloy 901 is 14 hours and in the case of Inconel 718 is 13 hours, which reflect 45 to 50 percent savings in time over the current heating times. This methodology of optimization can be extended universally to any class of materials of any geometric shape. The results from the optimization procedure will be recommended to the industry for implementation and further improvements can be based on the feedback received.

Although, the minimum fuel control systems, based on the modern control theory, have been successfully implemented in the aero-space industry, only few reports involving the applications of optimal control strategies for complicated industrial processes were presented during the past years, the major reason being that most of the industrial processes involve the high-dimensional, time-variant, non-linear models with very complex
constraints. Besides, the absence of the effective optimal control algorithms for solving such kinds of problems is another significant factor.

Because of these problems, the optimization of furnace heating schedules is still largely dependent upon the visual inspection of the problem and based on a common-sense approach. The advent of the digital computers made it possible to simulate a large number of the industrial processes to an acceptable degree of accuracy. This coupled with a common sense based approach, can lead to tremendous improvements in industrial processes, enhanced performance and efficiency of the process, which are reflected in better economics and savings in terms of lesser energy consumption.

In considering the application of optimization methods in design and operations, it is to be kept in mind that optimization is but one step in the overall process of arriving at an optimal design or an efficient operation. Generally, the overall process will consist of an iterative cycle, involving synthesis or definition of the structure of the system, model formulation, model parameter optimization and analysis of the resulting solution. The final optimal design will be obtained only after solving a series of optimization problems, the solutions of which will serve to generate new ideas for further system structures.

Future work in this area is recommended in evolving an algorithm for the solution of the optimization problem, applying optimal control theory. Algorithms may be developed which can be readily applied to batch processes, governed by linear or non-linear, high dimensional, time varying, state space models. This will lead to more efficient solution of such types of problems and also contain strict and more reliable proof of optimality.
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209
APPENDIX A

SAMPLE INPUT DATA FILE FOR THERMAL STRESS ANALYSIS USING ANSYS

/prep7
/title, heating of ALLOY 901 billet
kan,-1
et,1.55,,1
tunif,106

C*** ---- Thermal Conductivity data --------------------------

mpdata,1,30,1000,1100,1200,1300,1400
mpdata,kxx,1,1,5.63325,8.99856,9.30096,9.6685,10.008,10.26864
mpdata,7,1500,1600,1700,1800,1900,2000
mpdata,kxx,1,7,10.26864,10.8,11.15,11.5992,11.84976,12.19968
mpdata,13,2100,2165
mpdata,kxx,1,13,12.49776,12.71808

C*** ---- Specific. Heat Data -------------------------------

mpdata,1,30,1000,1100,1200,1300,1400
mpdata,c,1,1,,10684,1078,,114,,115,,116,,12
mpdata,7,1500,1600,1700,1800,2000,2100,2165
mpdata,c,1,1,128,,138,,16,,236,,208,,164

C*** ---- Heat Transfer Coefficient Data -----------------

mpdata,1,0,212,392,572,752,1112
mpdata,hf,1,,94,1,176,1.882,2.588,3.764,6.352
mpdata,7,1292,1472,1652,1832,2012,2192
mpdata,hf,1,,8,0,10.352,13.176,16.706,21.412,31.76

C***-------- Density Data ---------------------------------

mp,deas,1,1513.216

C*** --- Defining Ingot Geometry ---------------------------

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ngen,21,10,1,10,1,,2057
e,11,1,2,12

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egen, 9, 1, 1
egen, 20, 10, 1, 9

C*** ---- Load Step 1 ----

  cv, 201, 202, -1, 70, 210
  cv, 10, 20, -1, 70, 210, 10
  kbc, 0
  iter, -10, 0, 1
  time, 0.002
  lwrite

C***--- Load Step 2 ------

  cv, 201, 202, -1, 1500, 210
  cv, 10, 20, -1, 1500, 210, 10
  kbc, 1
  iter, -20, 0, 1
  time, 9.998
  lwrite

C***--- Load Step 3 -------

  cv, 201, 202, -1, 1700, 210
  cv, 10, 20, -1, 1700, 210, 10
  kbc, 0
  iter, -10, 0, 1
  time, 12.000
  lwrite

C*** --- Load Step 4 -----

  cv, 201, 202, -1, 1900, 210
  cv, 10, 20, -1, 1900, 210, 10
  kbc, 0
  iter, -10, 0, 1
  time, 14.000
  lwrite

C*** --- Load Step 5 --------

  cv, 201, 202, -1, 2175, 210
  cv, 10, 20, -1, 2180, 210, 10
  kbc, 0
  iter, -10, 0, 1
  time, 16.000
  lwrite
### Load Step 6

iter., 20, 0, 1  
time, 26.0  
lwrite

### Load Step 7

iter., -5, 0, 1  
time, 26.5  
lwrite

### Solution Procedure

afwrite  
finish  
/input.27  
finish

### Thermal Stress Procedure

/prep7  
resume  
kan, 0  
et, 1, 42, .., 1  
tref, 68  
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mp, nuxy, 1, 0.30  
mp, dens, 1, 513.216  
mp, temp, 1, 0, 200, 400, 600, 800, 1000  
mpdata, alp, x, 1, .., 6.5e-6, 7.1e-6, 7.8e-6, 8.0e-6, 8.1e-6, 8.3e-6  
mp, temp, 7, 1200, 1400, 1600, 1800, 2000, 2200  
mpdata, alp, x, 1, .., 8.6e-6, 9.2e-6, 9.5e-6, 9.8e-6, 10e-6, 10.2e-6  
cpsize, 300  
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nall  
nsel, x, 0.0  
d, all, ux, 0  
nall  
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cp, ngen, 8, ux, 12, 202, 10  
cpsgen, 20, 10, 7  
cpsgen, 9, 1, 8  
ktemp, .., 10
C***----------Solution Procedure -------------
afwrite
finish
/input,27
finish

C***-------- End of File ------------------------
### APPENDIX B

**THERMOCOUPLE READINGS ON THE ALLOY 901 INGOT**

[SOURCE: TELEDYNE ALLVAC, MONROE, NORTH CAROLINA]

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