The Differential Effects of Three Variations of Cover-Copy-Compare on Fluency, Generalization, and Maintenance of Basic Division.

Dissertation

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By

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Abstract

This study examined the differential effects of three variations of the Cover-Copy-Compare intervention method on elementary students’ fluency, maintenance, and generalization of basic division skills. The participants consisted of seven fourth-grade students attending a small urban district who had difficulty with basic division facts as measured by division fact probes and a standardized measure of academic achievement. An intervention was employed which utilized three variations of the Cover-Copy-Compare intervention method to improve participants’ basic division fluency and generalization. In the first condition, participants completed a typical Cover-Copy-Compare method for basic division facts. In the second condition, participants completed a Cover-Copy-Compare method but were required to complete an extra repetition of each division fact. In other words, participants completed each division fact twice, even if their initial response was correct. In the third condition, participants again completed a typical Cover-Copy-Compare method; however, presentation of the division facts was varied across four formats (i.e., obelus, fraction, division bracket, written words). An alternating treatment design was used to measure the effects of the intervention conditions. In general, participants improved basic division fluency and were able to generalize division skills to story problems at levels greater than baseline across all intervention conditions.
For most participants, there was considerable overlap in their performance across the three intervention conditions. Maintenance data obtained two weeks following the conclusion of the intervention indicated a slight drop in performance across participants and intervention conditions, but overall, rates remained higher than baseline levels. Implications for practitioners and future directions are discussed.
Dedication

This work is dedicated to elementary students and staff, in hopes that you will always know how valuable you are.
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Chapter 1: Introduction

Academic intervention strategies that promote generalization of skills need to be identified. This study examines the effectiveness of three variations of an intervention designed to increase fourth-grade students’ mathematical fluency, maintenance, and generalization, specifically with basic division. Math, in one way or another, is an integral part of societal life for the majority of adults. In fact, some level of math proficiency is required in most jobs, and the ability to apply mathematical skills is essential for successful independent living both at home and in the greater community (Codding et al., 2007). Mathematical foundations can be applied to everyday situations such as using vending machines and public transportation, and managing money and time, as well as more complex tasks such as abstract thinking and problem solving (Codding et al., 2007; Hodge, Riccomini, Buford, & Herbst, 2006).

Recent legislation, such as No Child Left Behind (Public Law, 2002), has emphasized the need for accountability within school systems and for increases in positive student outcomes across academic areas, including math. Yet, a gap exists between legislation, research and practice, and thus, student performance in mathematics continues to be poor and progress, slow. The National Center for Education Statistics indicated that in 2011, only 33% of 4th grade students and 27% of 8th grade students performed at the Proficient level on National Assessment of Educational Progress.
(NAEP) mathematics assessment. These percentages were not significantly different from those attained in 2009 (National Center for Education Statistics, 2012). Thus, it is essential that schools incorporate evidence-based interventions into the curriculum and prepare all students to use mathematical skills outside of the classroom.

Mathematical difficulties can manifest themselves at almost any point in a child’s academic career, and the seriousness of the difficulties can vary from temporary difficulties in one specific domain, to severe learning disabilities affecting multiple domains (Kroesbergen & Van Luit, 2003). Failure in fundamental mathematical skills places a student at-risk in higher order mathematical material, as well as for overall school failure given the number of other curricular areas that rely on arithmetic skills (Codding, Eckert, Fanning, Shiyko, & Solomon, 2006; Hodge, Riccomini, Buford, & Herbst, 2006).

Significant implications exist for those who are not able to master basic computational skills. Research has shown that individuals with poor mathematical skills are more likely to disengage from society, experience financial difficulties, hold prejudicial views of other ethnic and social groups, feel less secure in their employment, and be at higher risk for health problems (Methe et al., 2011). Often an inability to master mathematical concepts and procedures is present early on.

Being successful in math requires a variety of skills. Not only must a student possess the appropriate prerequisite skills and content knowledge, but also the appropriate procedural and strategic knowledge in order to compute or solve a mathematical problem (Montague, 1997). However, mathematical skills continue to build
upon each other as a student progresses through their academic career; therefore, a student who has difficulty calculating and retrieving basic math facts is at a great disadvantage when it comes to long-term mathematical competency.

Education today emphasizes problem solving and meaningful instruction; therefore, it is critical that students appropriately organize, calculate, and evaluate their work (Meltzer, 2007), especially in mathematics. In other words, students need instruction not only in math, but also in self-management techniques.

Although many evidence-based interventions exist, such as response repetition, error correction, peer tutoring, response cards, and traditional flashcard drill and practice, which educators can use to remediate students’ math skills, few incorporate a self-management technique to assist students in gaining knowledge of basic math facts. Even fewer can be applied to other academic skills and areas as well. One such intervention is cover-copy-compare.

Cover-copy-compare (CCC) is a relatively simple intervention, aimed at helping students self-manage their academic skills. CCC has an extensive literature base and has been shown to be an effective technique to help students improve their spelling (e.g., Hansen, 1978), reading (e.g., Conley, Derby, Roberts-Gwinn, Weber, & McLaughlin, 2004), geography (e.g., Skinner, Belfiore, & Pierce, 1992), science (e.g., Smith, Dittmer, & Skinner, 2002), and math fact acquisition and fluency (e.g., Bolich, Kavon, McLaughlin, Williams, & Urlacher, 1995). Additionally, CCC has been implemented in a variety of settings, such as in the home (e.g., Stading, Williams, & McLaughlin, 1996), general education classrooms (e.g., Schermerhorn & McLaughlin, 1997), and in special
As a math intervention, CCC is designed to advance both accurate and fluent responses across a range of math calculation skills (Skinner, McLaughlin, & Logan, 1997). CCC provides students with a series of learning trials within a short amount of time. Students self-manage the intervention by checking the accuracy of their work immediately upon completion of the task (Mong & Mong, 2010; Skinner, McLaughlin, & Logan, 1997). The CCC procedure includes several components of effective instruction, including modeling, practice opportunities, and feedback (Skinner, Bamberg, Smith, & Powell, 1993), through a sequence of specific steps. Typically, CCC involves five steps: (a) looking at the model problem; (b) covering the problem; (c) emitting a response, such as writing a math fact; (d) uncovering the model; and (e) evaluating the emitted response (Skinner, Turco, Beatty, & Rasavage, 1989; Joseph, Konrad, Cates, Vajcner, Eveleigh, & Fishley, 2012). If the student’s answer does not match the original problem, the student should repeat the steps. Prior to implementation, CCC requires a thorough explanation of the intervention process and creation of necessary materials, most likely a paper or skill worksheet (Mong & Mong, 2010). Students are able to implement the intervention independently once the intervention procedures have been learned, a clear advantage of this procedure. Additionally, both students in the acquisition phase, as well as students in the fluency phase of math skills can utilize CCC as it can be modified to meet different needs.
Among the first to use CCC with math facts were Skinner, Turco, Beatty, and Rasavage (1989). Four students with behavior disorders were taught to use the CCC procedure with multiplication facts. Specifically, students would look at a multiplication fact (problem and answer) on the left side of a piece of paper, cover the fact, write the entire fact on the right side of the paper, and then uncover and check their response. If correct, students proceeded to the next fact. If incorrect, students rewrote their response so that their final response was a correct one. The researchers were able to demonstrate that CCC was an effective and efficient procedure for increasing students’ multiplication fact fluency.

Since this study, CCC has been frequently utilized as an intervention to help students learn basic math facts. For example, Codding et al. (2007) examined the effects of CCC on digits correct per minute for basic math facts with second and third grade students who were either at an instructional or frustration level prior to the intervention. Although CCC was most effective for students who began the intervention at a frustration level for solving basic math facts, students who began at an instructional level also showed improvements. In 2010, Mong and Mong used an alternating treatments design to compare the effects of CCC and another intervention, Math to Mastery, on basic addition and subtraction accuracy and fluency. Participants were three second-grade students in a suburban school general education classroom. Again, CCC was shown to be an effective procedure for increasing students’ digits correct per minute and results were maintained 6 and 18 days following intervention termination. Finally, in a study conducted by Poncy, Skinner, and McCallum (2012) students were taught to use the CCC procedure with math
fact families. In other words, students would use a math fact family triangle, cover it, and then write one of the possible facts, its reciprocal, and then uncover and evaluate their answer. The third-grade participants in the CCC group were found to have outperformed their control group counterparts based upon digits correct per minute.

The CCC procedure can not only be used with mixed math facts, but also can be used to target one specific skill. For instance, Poncy, Skinner, and Jaspers (2007) evaluated the CCC procedure for basic addition problems with a student diagnosed with a cognitive disability. Results indicated that CCC was successful in helping the student increase basic addition fluency, which the student maintained two weeks after the intervention ended. In another study, Grafman and Cates (2010) targeted subtraction fluency with 47 students in second-grade classrooms. They compared CCC with a modified version of the procedure called Copy, Cover, Compare using a within-subjects design. In the modified version, students looked at the subtraction problem, copied it, covered the problem, wrote the problem from memory, and then uncovered and evaluated. In both conditions, if the student was incorrect, they were asked to copy the problem and answer once more. The researchers found that students had significantly more digits correct per minute under the CCC procedure than the modified procedure. Further, students reported that they preferred the CCC procedure to the modified one.

Similarly, Becker and colleagues (2009) used a simple ABC design to compare the effects of CCC with CCC and an additional error correction procedure on a fourth-grade special education student’s multiplication fluency. The additional error correction procedure consisted of an extra column on the CCC worksheet in which the student
would re-write any incorrect multiplication facts three times. Results indicated that the student’s digits correct per minute increased significantly and digits incorrect per minute decrease significantly under the CCC procedure. When the additional error correction procedure was implemented, digits correct again improved and digits incorrect dropped even farther. In other words, CCC combined with additional repetition may produce gains that are even more significant.

One mathematical calculation skill that has received less attention than others with regards to CCC, is division. Lee and Tingstrom (1994) used CCC with a small group of five middle school students to increase digits correct per minute across sets of division facts. Even though subjects began the intervention at varying levels of accuracy, results showed an increase in rate and accuracy of responding for all participants. This study also demonstrated that CCC can be an effective intervention for a small group of students struggling with division fluency. More recently, Cieslar, McLaughlin and Derby (2008) used CCC in a case study of a high school student with a behavioral disorder. Using a multiple baseline across sets of division problems design, the researchers aimed to evaluate the effectiveness of the CCC procedure on the student’s division and fraction calculation accuracy. Using typical CCC procedures, the student was asked to complete a sheet of six problems each day. The student’s performance across all sets of division problems was significantly improved compared to baseline, and high rates of responding were maintained as new sets were introduced.

Despite the fact that each of the studies previously mentioned provides evidence for the effectiveness of CCC as a tool for increasing math fact calculation accuracy and
fluency, generalization data are absent from virtually all of these studies. Although many studies assessed the maintenance of skills gained through CCC (e.g., Mong & Mong, 2010; Cieslar, McLaughlin, & Derby, 2008), none of the above-mentioned actually programmed for generalization.

Programming for generalization is an instructional strategy beneficial for both educators and students. Today’s teachers must utilize their instruction time effectively and efficiently. Programming for generalization allows teachers to instruct students how to apply their knowledge and skills in other situations. It also saves teachers from having to re-teach forgotten skills or how to apply skills in new circumstances (Konrad, Helf, & Joseph, 2011).

According to Cooper, Heron, and Heward (2007), three types of generalization exist: maintenance-- exhibiting skills even after instruction has stopped; setting/situation generalization-- using skills in a setting or situation different from the instructional setting; and stimulus generalization-- exhibiting skills that were not explicitly taught but that accomplish the same goal as those for which the student received instruction. Students who are taught skill generalization better understand how and in what situations to use their skills and knowledge. In other words, they are better prepared to apply this information in real-world situations (Bottge, 2001). In mathematics, this is critical because students will encounter math in the real world nearly every day. It is not uncommon for empirical studies to examine an intervention’s effects on maintenance of a particular skill. Less common is programming for and examining an intervention’s effects on generalization of a particular skill.
Purpose of the Study

The purpose of this study was to examine the effects of three variations of the Cover-Copy-Compare intervention on fourth-grade students’ basic division fluency, maintenance, and generalization. Although CCC had been researched frequently with mathematics, there was limited research on its applicability to division facts in isolation. Programming for generalization using CCC had not been examined in studies. Moreover, a comparison of CCC, CCC+ repetition, and CCC+ programming for generalization had not been conducted with regards to their differential effects on student performance math fluency probes.

Significance of the Study

This study sought to extend the current literature on CCC. It makes an important contribution by evaluating the effectiveness of three variations of the CCC intervention on basic division fact fluency, maintenance, and generalization in fourth-grade students. Limited research on the CCC intervention in isolation with basic division facts exists. No previous study utilized repetition and programming for generalization as variations of the CCC intervention. The variations of CCC used in this study could easily be adapted for a variety of circumstances as they could be used one-on-one, in a small group, in a large group, and across academic subjects and participant demographics.

Research Questions

This study sought to answer the following questions:
Research question 1: Are there differential effects of three variations of CCC conditions (CCC, CCC+ repetition, CCC+ generalization) on division fact calculation fluency?

Research question 2: Are there differential effects of three CCC conditions (CCC, CCC+ repetition, CCC+ generalization) on maintenance?

Research question 3: Are there differential effects of three CCC conditions (CCC, CCC+ repetition, CCC+ generalization) on generalization?

Research question 4: Do participants express a preference for one intervention treatment condition?

Research question 5: Do participants’ teachers find one intervention treatment condition to be more feasible to implement than the other treatment conditions?

Definition of Terms

Fluency. According to the National Council of Teachers of Mathematics (2000), computational fluency is possessing and using efficient and accurate methods for computing. In other words, a student is fluent when s/he has knowledge of the appropriate mathematical procedures, a knowledge and flexibility of when and how to use them, and is skilled in calculating accurately and quickly (National Research Council, 2001).

Maintenance. Maintenance is the extent to which the student continues to perform a learned skill after the intervention has been completed (Cooper, Heron, & Heward, 2007).
**Generalization.** For the purposes of this study, generalization is the extent to which the student is able to solve untrained division problem formats (story problems) that are functionally equal to the division problem for which the student received intervention (basic division facts in isolation) (Cooper, Heron, & Heward, 2007).
Chapter 2: Literature Review

This chapter presents a review of literature that is most closely related to the current study. It begins with a discussion of the acquisition of fundamental math skills, particularly for elementary-aged students. Next, a discussion of effective instructional practices, such as programming for generalization, is presented. Following, mathematical interventions targeting basic skill fluency are discussed. This is followed by a description of the Cover-Copy-Compare intervention method and a discussion of the research on the effectiveness of this method.

Fundamental Math Skills

Unlike the clear prerequisite and precursive skills of reading’s “big ideas” (e.g., phonological awareness, alphabetic knowledge), much less is known about the specific building blocks and skills required for numeric literacy, early numeracy or number sense (Missall, Mercer, Martinez, & Casebeer, 2012; Methe & Riley-Tillman, 2008). In math, several domains interweave and evolve as information, skills and operations become more complex. This poses a challenge to researchers attempting to define mathematical competence. It also has resulted in a lack of consensus regarding the specific mathematical domains necessary for mathematical competence, the types of tasks included within each domain, and at which age particular skills should be developed (Methe et al., 2011). For example, some researchers have theorized math domains to
include operations, word problems, fractions, algebra and general math proficiency (Gersten et al., 2009). Others have posited counting, comparing and ordering, equal partitioning, composing and decomposing, grouping and ordering, and adding to/taking away as the critical math domains (Clements et al., 2004, as cited in Methe, 2009).

Most experts do agree, however, that both conceptual and procedural knowledge are critical to mathematical learning (Hiebert, 1986). Conceptual knowledge is relationship-based: individual facts and pieces of information are linked together into a larger “network” of information. To develop conceptual knowledge, previously acquired information can be linked together, and new information can be linked to previously learned information. Procedural knowledge, on the other hand, is composed of the formal language, or system of mathematics, as well as the rules, or process, for completing mathematical tasks (Hiebert, 1986). To illustrate the difference between conceptual and procedural knowledge, consider the following example adapted from Hiebert (1986): If asked, most adults would understand the concept of “square root,” but would lack the procedural knowledge to calculate the square root of 622,521 with only a paper and pencil.

The foundation for fundamental conceptual and procedural math skills necessary for competence throughout life must be developed early on. Frequently used, the term, early numeracy, thus refers to the scope and sequence of mathematical learning that can aid further learning of exactly what numerals represent, as well as the rules specifying how numerals can be combined and applied (Methe et al., 2011).
Incorporating conceptual and procedural knowledge, as well as early numeracy logic and research, the National Council of Teachers of Mathematics (NCTM, 2000) developed *Standards for School Mathematics* in order to present a common mathematical foundation for all students. Frequently referenced by researchers (Codding et al., 2007; Methe & Riley-Tillman, 2008; Methe, Hintze, & Floyd, 2008; Methe, 2009; Methe et al., 2011), this document suggests five content and five process standards students must acquire for mathematical competency. These ten standards are relevant across a student’s academic career from kindergarten through grade twelve; however, each standard is also divided into four grade-level sections progressing in difficulty (prekindergarten though grade 2, grades 3-5, grades 6-8, and grades 9-12) in order to provide more specific expectations for student performance (NCTM, 2000).

According to the NCTM (2000), the five content standards clearly illustrate the five strands of critical math content that students should acquire. As previously stated, the standards are divided into four grade-level bands, allowing for more thorough indication, discussion, and examples of strand expectations. Although applicable across all grade-levels, the relative emphasis placed on particular strands will vary across the grade-level bands (NCTM, 2000). These five standards, or math strands, represent a basic set of foundational concepts that support later mathematical learning (Methe, 2009). One such content standard is *Number and Operations*.

The *Number and Operations* standard lays the foundation for the other four Content Standards as well as all five Process Standards. Essential to this standard is the development of number sense, which is most predictive of later math success (Missall,
Mercer, Martinez, & Casebeer, 2012). Students who exhibit number sense are able to understand numbers and number systems, ways of representing numbers, and relationships among numbers. Students with number sense also understand the meanings and relationships among operations. Computational fluency should develop concurrently with number sense. Examples of mathematical tasks included in this standard are decomposing numbers (e.g., 35 is composed of 3 tens and 5 ones), mental calculations and estimations (NCTM, 2000). In other words, students experiencing difficulty acquiring basic math skills, or completing them fluently, may need instruction and/or intervention related to the Number and Operations content standard, allowing them to acquire the ability to perform basic calculations mentally and fluently.

**Instructional Practices**

A direct approach has typically been used in mathematics instruction. In this approach to math instruction, highly organized tasks are presented in a clear, accurate, unambiguous manner in order to promote student learning (Hodge, Riccomini, Buford, & Herbst, 2006; Przychodzin, Marchand-Martella, Martella, & Azim, 2004). The direct instruction approach provides comprehensive instruction that follows the previously mentioned instructional hierarchy so that students not only acquire new information, but retain, generalize and adapt information as well (Przychodzin, Marchand-Martella, Martella, & Azim, 2004). In addition, research has consistently demonstrated direct instruction to be effective for students exhibiting mathematical difficulties (Kroesbergen & Van Luit, 2003).
Traditionally, math curriculum content has been presented in a differentiated—different areas, or domains, are separately identified and taught—manner (Choat, 1978), but math instruction is slowly becoming more integrated. Mathematical domains are being connected not only to other mathematical domains, but also to a variety of other contexts. Yet, improvement is still needed. Many instructional programs are inadequate, not allowing students to progress through an instructional hierarchy of skill acquisition, fluency, generalization, and adaptation (Methe & Riley-Tillman, 2008), and many teachers are not fully prepared to meet the diverse needs of today’s student population (Stein, Kinder, Silbert, & Carnine, 2006). Consequently, math instruction and intervention that is both direct and allows for integration of other skills may be of most benefit to students in the acquisition and fluency stages of mathematical learning.

Programming for generalization during an intervention would meet both of these criteria.

**Programming for Generalization.** Mathematical competency does not always transfer to situations and settings other than those in which instruction was received. In other words, many students experience difficulty generalizing math skills. As previously stated, according to Cooper, Heron, and Heward (2007), three types of generalization exist: maintenance—exhibiting skills even after instruction has stopped; setting/situation generalization—using skills in a setting or situation different from the instructional setting; and stimulus generalization—exhibiting skills that were not explicitly taught but that accomplish the same goal as those for which the student received instruction. However, for many students, generalization of math skills does not spontaneously occur
and educators must program instruction to promote generalization (Duhon, House, & Stinnett, 2010).

Instructors who assist students with skill generalization help them to understand how and in what situations to use their skills and knowledge and thus, to be better prepared to apply this information in real-world situations (Bottge, 2001). Promoting generalization of math skills is beneficial for both educators and students. Teachers are often pressed for time and must find the most efficient and effective instructional strategies possible. Programming for generalization not only allows educators to teach students how to apply skills and content to other situations, but saves educators the time of reteaching forgotten skills, or how to apply the skills in new situations (Konrad, Helf, & Joseph, 2011). Programming for skill generalization is not necessarily something that educators can do spontaneously, but rather, successful generalization instruction requires forethought and planning.

Educators should never assume that students will be able to appropriately and independently make generalizations. In order to ensure that this is indeed the case, Cooper, Heron, and Heward (2007) recommend systematic planning for generalization instruction. First, they recommend that educators choose a target behavior that will elicit reinforcement naturally instead of only being reinforced in contrived situations. They caution that the ultimate goal of a behavior is functionality, that is, the behavior should produce reinforcement for the student. Second, they recommend that educators determine the acceptable variations of the target skill as well as the situations in which it should (or should not) occur. In other words, which variations of the mathematical skill are desirable
and which are not. With this information, instructors will be best prepared to determine which strategies and tactics to implement to ensure successful and appropriate generalization of the targeted math skill (Cooper, Heron, & Heward, 2007).

Numerous methods for promoting generalization exist and these methods are often transferable across academic subjects. For example, Alber-Morgan, Hessler, and Konrad (2007) suggested strategies to promote generalization when teaching written expression. However, many of these strategies can also be used to promote generalization in mathematics. For example, teacher modeling of how to solve a variety of problems, mnemonic strategies, intermittent reinforcement, computer-assisted instruction, peer-assisted instruction, using common stimuli, self-management strategies, and making instruction personally relevant to students are all educators can use to promote skill generalization.

These tactics fall under one of five categories according to Cooper, Heron, and Heward (2007): teaching the full range of relevant conditions and response requirements; making the instructional setting similar to the generalization setting; maximizing contact with reinforcement in the generalization setting; mediating generalization; or training to generalize. When considering mathematical interventions for students experiencing difficulty acquiring and becoming fluent with basic math facts, teaching the full range of relevant conditions and response requirements may assist students in not only acquiring, but also generalizing math facts.

In order for a given mathematical skill to be especially useful, it must be utilized in a variety of ways and under a wide range of conditions. However, it would be
impossible for an educator to teach every possible way and scenario under which to use a specific mathematical skill. Therefore, Cooper, Heron, and Heward (2007) suggest a procedure educators can use to promote math generalization within the time constraints of the educational system: teach sufficient stimulus examples.

**Teach sufficient examples.** It would be incredibly difficult for a student to not only master a mathematical concept, but also understand when and how to apply that concept having been given only one example. This generalization tactic reminds educators that students need varying examples in order to understand the full scope of how to use a particular math skill. Educators must provide students with a sufficient number and range of examples to assist the student in successful application of the skill when an untrained situation is encountered. More specifically, teachers can vary mathematical examples across four dimensions: the specific item, stimulus context, setting, or person (Cooper, Heron, & Heward, 2007). When varying the specific item for instance, an educator might present specific math facts such as 4+3, or 7x9. Providing various stimulus contexts could mean that an educator instructs students on how to calculate problems presented in a vertical format, or in a horizontal format (Cooper et al., 2007). This tactic may be especially beneficial when combined with a direct intervention targeting math fact acquisition and fluency.

**Mathematical Interventions**

An intervention typically involves a specific instruction to teach a particular skill for a certain amount of time (Kroesbergen & Van Luit, 2003; Hodge, Riccomini, Buford, & Herbst, 2006). The goal is for the student(s) to acquire the knowledge and skills and be
able to apply the information independently. Most children with math difficulties who require intervention are unable to master one or all of the four basic operations (addition, subtraction, multiplication, division) (Kroesbergen & Van Luit, 2003). However, understanding what skill(s) a student is having difficulty mastering is only part of the equation. An educator must also determine which stage of mastery the student is currently in: acquisition, fluency, generalization, or adaptation (Codding et al., 2007). This can have significant implications on the effectiveness of an intervention as some may be more suited to particular skills and stages than others. Educators must make informed choices regarding which intervention may best suit a particular student in order for the chosen intervention to be as effective as possible.

**Self-management.** Self-management is personally applying a behavior change strategy in order to achieve some desired change of behavior (Cooper, Heron, & Heward, 2007). Self-management skills are critical to academic success, yet many students do not possess the ability to apply these skills to their learning (Joseph & Konrad, 2009). These skills allow students more flexibility and the opportunity to take more responsibility for their learning. All students can benefit from gaining the ability to self-manage their academic work, as these skills can promote study habits, improve task quantity and quality, and allow students to evaluate their outcomes and modify their strategies based upon whether their goals were met (Joseph & Konrad, 2009).

Self-management, or student-administered, interventions may be more attractive to teachers and educators. These interventions may require less planning and administration time, requiring less of the teacher’s already limited instructional time.
Moreover, self-managed academic interventions are typically student-paced, meaning heterogeneous groups of students can learn at their individual rates across different academic subjects (McLaughlin & Skinner, 1996).

Students may also prefer student-administered interventions. These types of interventions allow students to work at their own pace, giving them more independence and control over their learning. They may serve to boost students’ confidence as well. With self-management interventions, students’ initial evaluations of their responses are not usually made public. Students may feel as though they are spared any negative evaluations of their responses from others (McLaughlin & Skinner, 1996). This may come as a particular relief to students who have previously experienced academic failure and those with learning difficulties. Those students who may need additional extrinsic reinforcement still have access to teacher reinforcement for appropriately completing the intervention strategy. Finally, students may find that they are learning skills which can easily be transferred to other situations and academic subjects (McLaughlin & Skinner, 1996).

Interventions that allow students to self-manage their learning may be particularly effective in mathematics. Self-management interventions assist students in acquiring skills, and regulate and generalize their learning across tasks, settings, and time (Smith, Dittmer, & Skinner, 2002). They can be used to support, not replace, more explicit rule-based instruction (Moser, Fishley, Konrad, & Hessler, 2012), allowing teachers to provide initial instruction and use self-management interventions as follow-ups to instruction. Additionally, self-management interventions often utilize clear instructional
cues, immediate error correction, and encourage high rates of responding, making these interventions well suited to mathematical tasks.

**Cover-Copy-Compare.** One of the most versatile interventions is Cover-Copy-Compare (CCC). CCC incorporates a self-management technique to assist students in gaining targeted knowledge and can be applied to a variety academic skills and areas. CCC is designed to advance both accurate and fluent responses across a range of skills (Skinner, McLaughlin, & Logan, 1997). Typically, the CCC process includes five steps: (a) looking at the model problem; (b) covering the problem; (c) emitting a response, such as writing a math fact; (d) uncovering the model; and (e) evaluating the emitted response (Joseph, Konrad, Cates, Vajcner, Eveleigh, & Fishley, 2012; Skinner, Turco, Beatty, & Rasavage, 1989). If the student’s answer does not match the original problem, the student should repeat the steps. Once the intervention procedures have been learned, students are able to implement the intervention independently, making this intervention an attractive choice for educators.

Research has shown CCC to be effective in a variety of settings, such as in the home (e.g., Stading, Williams, & McLaughlin, 1996), general education classrooms (e.g., Schermerhorn & McLaughlin, 1997), and in special education classrooms (e.g., Conley et al., 2004). It has also been an effective intervention for individuals or groups, as well as students with and without learning problems (Cieslar, McLaughlin, & Derby, 2008). CCC has been used in a variety of academic subjects including, spelling (e.g., Hansen, 1978), reading (e.g., Conley, Derby, Roberts-Gwinn, Weber, & McLaughlin, 2004),
geography (e.g., Skinner, Belfiore, & Pierce, 1992), and science (e.g., Smith, Dittmer, & Skinner, 2002).

Skinner, Turco, Beatty, and Rasavage (1989) were among the first researchers to use CCC as an intervention targeting mathematics. Subjects included one female fourth grader, and two male tenth graders for a total of three participants. Each participant was considered to be behaviorally disordered. The CCC intervention was chosen due frequent opportunities to respond and immediate corrective feedback. The intervention targeted multiplication problem fluency, specifically single-digit by single-digit facts which, when calculated, produced a two-digit product. All multiplication facts were divided into three mutually exclusive sets (i.e., Set A, Set B and Set C). In order to allow for repetitions of any functional relationship between the independent variable and the dependent variable, Skinner and colleagues chose to implement a within subjects multiple baseline design.

CCC worksheets were generated containing ten multiplication problems from one specific set, on the left side of the paper, with room for students to complete the CCC procedures, as well as the experimenter to list the subject, date, number of seconds it took to finish the worksheet, correct number of digits, and percentage of items correct.

Skinner and colleagues (1989) taught students to complete the five-step CCC process initially, and students were expected to complete this process independently for the remainder of the study. For each session, students completed a CCC worksheet and then a CCC assessment sheet which was identical to the CCC worksheet but without answers provided, at the beginning of the next session (at least 18 hours later). A set was considered to be mastered when a student reached 40 digits correct per minute and 90%
or better accuracy. Multiplication facts from the next set would then be introduced and the previous set would be assessed daily as a follow-up. This continued for the duration of the study until mastery was achieved for all three sets, at which point weekly maintenance assessments were conducted.

All three subjects showed improvement in multiplication fluency as a result of the CCC intervention. High rates of responding were maintained across time for two subjects, although the third subject showed some regression at the conclusion of the intervention. The percentage of problems correct for each subject was as follows: Subject 1 baseline = 94%, Intervention = 94%, Maintenance = 94%; Subject 2 baseline = 90%, Intervention = 91%, Maintenance = 94%; Subject 3 baseline = 96%, Intervention = 98%, Maintenance = 97%. Skinner and colleagues found the CCC procedure to be effective for all three subjects, although they noted increasing trends in baseline and follow-up data. They contributed this to reactivity in the assessment condition. In other words, the repeated practice during the assessment condition may have lead to increases in untaught items. Skinner and colleagues (1989) recommended additional research on CCC in other content areas and across a variety of learners.

More recently, Codding, Shiyou, Russo, Birch, Fanning, and Jaspen (2007) compared the effects of CCC and explicit timing (ET) on the subtraction fluency of 98 second and third grade students. According to Codding and colleagues (2007), ET is a procedure requiring students to mark their progress during one-minute timings when completing an academic task. In this fashion, students and educators are able to receive visual feedback regarding the total completed problems within each interval.
The goal of comparing ET with CCC was determining which intervention would produce the greatest amount of change in the shortest amount of classroom time for students who began the interventions in either the frustration or instructional level of fluency (Codding et al., 2007). Participants were recruited from 16 general education classrooms in one elementary school.

A curriculum-based assessment consisting of single-digit addition, single-digit subtraction, and single-digit multiplication was administered to participants in order to determine which participants were beginning the interventions at a frustration level and which were at an instructional level. Digits correct per minute (DCPM) was calculated. Students were considered to be mastery level if their DCPM was equal to or better than 20, instructional level DCPM was between 10 and 19, and frustration level DCPM was 9 or below. Equal numbers of students were then assigned to a control, ET, or CCC group which did not vary significantly. Intervention sessions took place twice a week for six weeks. Worksheets consisting of single-digit subtraction problems were generated for both the ET and CCC conditions.

Worksheets in the CCC condition were arranged where the problem and answer was on the left-side of the page, and the same problem was on the right-side of the page without an answer. Students were provided instruction in completing the five CCC steps during the first three sessions. Instructions were then omitted from all subsequent sessions. During each CCC session, students were given a packet of CCC worksheets and asked to continually solve problems in the packet for a total of five minutes.
Worksheets in the ET condition consisted of single-digit subtraction problems. Students in the ET condition were given worksheets and asked to complete subtraction facts for five minutes. The experimenter notified students, using a stopwatch, at the end of each one minute interval, and students were instructed to circle the subtraction problem they were completing at that moment. No feedback about their performance was given.

Results indicated that participants’ initial level of fluency significantly impacted their overall performance. Overall, students in both the ET and CCC conditions improved their subtraction fluency. However, students initially in the frustration level of computation fluency showed greater improvement in the CCC condition than the ET condition. Codding and colleagues (2007) attributed this to the benefits of more opportunities to practice subtraction skills, as opposed to a focus on timing.

Also in 2007, Poncy, Skinner, and Jaspers compared the effects of Cover-Copy-Compare and taped problems (TP) on an elementary student with a cognitive disability’s math fact accuracy and fluency. The student was a ten-year old female student attending a rural public school in the Midwest. The student was reported to have a Full Scale IQ of 44, consistent with a cognitive disability, and received the majority of her services in a special education classroom. Prior to the intervention, the experimenters divided basic addition facts into three different, functionally equivalent, sets containing four problems each. For each set, the researchers constructed six different assessment probes, consisting of 24 problems. Each set was then randomly assigned to either the CCC intervention condition, the TP intervention condition, or a no-treatment condition. Packets were then
created for each intervention condition containing an intervention sheet, a sprint sheet, and an assessment probe.

In the CCC condition, the student completed a CCC worksheet with four basic addition problems following the general five-step procedure, and then completed a sprint sheet, followed by an assessment probe. In the TP condition, the student completed a TP worksheet containing four basic addition problems by listening to a tape recording of the problem being stated, then a four second pause, then listening to the answer. In the four second pause, the student was expected to write her answer; when listening to the answer, the student was expected to compare her answer with the verbalized answer. As with the CCC condition, upon completion of the intervention worksheet, the student then completed a sprint sheet, followed by an assessment probe.

Using an adapted alternating treatments design and a multiple probe design simultaneously, Poncy and colleagues (2007) measured the student’s percentage of digits correct (DC) and the number of DC completed per minute on the assessment probes. Immediately upon receiving intervention, the student’s accuracy on basic addition problems increased to 100% in the TP condition, and 90% in the CCC condition. These levels were maintained for the duration of the intervention, and follow-up sessions. The student’s accuracy in the control condition remained within a range of 27-44%. In analyzing the time spent on each intervention, the researchers noted that the student spent an average of 6 minutes, 36 seconds daily on the CCC intervention, as opposed to 4 minutes, 41 seconds daily on the TP intervention (Poncy, Skinner, & Jaspers, 2007). Consequently, the researchers concluded that the TP intervention was superior based
upon the fact that it took less time to complete. However, these results are not surprising given that the student was essentially guided through the TP condition by listening to the taped recordings, as opposed to completing the CCC intervention sheets at a completely independent pace. Nevertheless, results from this study demonstrated that CCC can not only be used to increase accuracy and fluency in basic addition problems, but can be used with special education students, even those with IQs in the cognitive disability range.

In a similar study, Poncy, Skinner, and McCallum (2012) used an adapted alternating treatments design to again compare Cover-Copy-Compare (CCC) with Taped Problems (TP). Unlike the previous study, participants in this study were 20 general education students from an intact third-grade classroom. The study took place in North-Central Iowa and the class consisted of 9 boys and 11 girls ranging in age from 8 to 10 years. The goal of the study was to extend the previous line of research through a comparison of class-wide CCC and class-wide TP on participants’ subtraction fact fluency.

Poncy, Skinner, and McCallum (2012) generated three sets of subtraction problems, with each set containing 21 or 22 one-digit minus one-digit and two-digit minus one-digit problems. For each set, the researchers created six different assessment probes, with each assessment containing 48 problems. Each set was then randomly assigned to either a control condition, CCC condition, or TP condition.

Assessment probes in the CCC condition contained a grid of 42 boxes (6 X 7) for the student to practice the 24 target problems in the CCC condition. Fact family triangles for the problems were placed in columns 1 and 4 and were used as the CCC stimuli
Probes in the TP condition were constructed under the same guidelines but contained subtraction problems assigned to the TP condition.

In contrast to the traditional CCC procedures, Poncy and colleagues (2012) gave students in the CCC condition 6 minutes to complete as many fact family triangles as possible. In other words, if given the numbers 7, 12, and 5, students could write 12-7=5 and the corresponding reciprocal fact prior to moving on to the next fact family. In the TP condition, students were given two probe sheets, instructed to listen to an audio recording of the subtraction fact being read aloud, and attempt to write the answer to the problem during a two-second pause, prior to hearing the answer on the tape. As with the CCC intervention, the duration of the TP intervention was 6 minutes.

Results revealed little initial change from baseline in the CCC condition and the control condition, but an increasing trend in digits correct per minute (DCPM) in the TP condition. However, over as the study continued, participants showed an increasing DCPM in the CCC and control conditions, whereas the DCPM in the TP condition appeared to level off (Poncy, Skinner, & McCallum, 2012). Overall, TP yielded a 13.5 DCPM increase, CCC yielded a 6.6 DCPM increase, and the control condition yielded a 5.3 DCPM increase for the group. Individual student results varied. The researchers concluded that, overall, TP was superior to CCC because it was more efficient in a specified amount of time. Yet, they noted the fact that students completed significantly more writing in the CCC condition, and each individual student’s writing capability, attention, and prior math-fact knowledge would influence performance in the CCC condition more so than in the TP condition. This study illustrates how critical it is for
educators to choose appropriate interventions for the participants. As both interventions were implemented class-wide, TP may have been more efficient due only to the fact that it led students at a constant pace, whereas CCC is expected to be completed independently.

The effects of Cover-Copy-Compare on math fact fluency and accuracy have also been compared with a modified version of Cover-Copy-Compare with an additional error correction drill (Becker, McLaughlin, Weber, & Gower, 2009). The participant was a fourth-grade, female elementary student with a learning disability who was exhibiting difficulty in math. The goal of this study was to determine if an additional repetition error correction procedure would produce higher gains in basic multiplication fact accuracy and fluency than the traditional CCC procedures.

Becker and colleagues (2007) used an ABC design to compare the CCC condition with the CCC plus error drill. In the CCC condition, the student completed a traditional CCC worksheet containing ten multiplication facts. Immediately following, the student completed a one minute timing worksheet containing 90 problems. The experimenter recorded the number of correct and incorrects on the timing, and then reviewed any errors with the student. This condition was implemented for a total of seven school days.

In the additional error correction condition, the same procedures as in the CCC condition were followed; however, any errors made on the timing sheet were verbally modeled by the experimenter. The student was then asked to repeat the entire fact aloud and then write the entire fact on a separate sheet of paper. After all incorrect problems were reviewed in this manner, the student was then asked to repeat aloud all problems
written on the separate sheet of paper (those that were initially incorrect). This condition was implemented for a total of ten school days.

Results yielded a significant increase in correct responses and a gradual decrease in errors under both conditions when compared to baseline performance. In baseline, the student averaged 34 digits correct per minute (DCPM) and 56 errors per minute. In the CCC condition, the participant averaged 54.5 DCPM and 35 errors. In the CCC plus error correction condition, the student averaged 83 DCPM and 6.6 errors. The researchers concluded that this data not only supports CCC as an effective intervention for basic multiplication accuracy and fluency, but supports error correction repetition in areas such as reading. However, the researchers noted that the error correction procedure utilized multiple modes of learning (i.e., visual, verbal) as opposed to the CCC condition. Additionally, Becker and colleagues (2007) noted that the treatments were not counterbalanced and the length of treatment was equal in each condition. They recommended additional research comparing CCC with CCC and an additional error correction procedure.

In another study, an alternating treatments design was used to compare two interventions designed to increase basic math fluency in three elementary students (Mong & Mong, 2010). The Cover-Copy-Compare (CCC) intervention was compared with the Math to Mastery (MTM) curriculum-based intervention. According to Mong and Mong (2010), MTM is a structured package including previewing mathematical problems, recurring practice, instant corrective feedback, summative and formative feedback, as well as progress self-monitoring. Both CCC and MTM include modeling, practice,
immediate feedback, and reinforcement aspects; yet, CCC requires additional repetition of a problem only if an initial response is incorrect, whereas MTM requires additional repetition for incorrect and slow responses (Mong & Mong, 2010).

Participants included three second-grade students from a suburban school in the Southeast: a 7-year-old Caucasian male, an 8-year-old Latin-American female, and an 8-year-old African-American male. All students were general education students who, per teacher report and district-level benchmarking, were performing in the lowest quartile in their respective classrooms in math.

The researchers used a computer-based program to generate worksheets with randomized math problems (e.g., addition and subtraction of one- and two-digit numbers with and without regrouping) based upon the grade level. MTM worksheets consisted of 24 problems of a single targeted skill. Procedures during MTM intervention sessions included: (a) the experimenter modeled each problem, (b) the student practiced each problem in a series of one-minute trials until 32 digits correct per minute (DCPM) was reached (mastery), (c) during the one-minute timing the experimenter tallied incorrect answers and provided immediate corrective feedback, (d) following each one-minute trial, the experimenter notified the student of their overall performance and provided praise, (e) the student completed a self-monitoring chart of his/her performance.

CCC worksheets consisted of four problems with correct answers on the left side of the paper, and the same four problems without the answers on the right side of the paper. In a session, six CCC worksheets (i.e., 24 total problems) were completed by the participant following the traditional five step procedure. One intervention was
implemented per day, and the order of intervention presentation was counterbalanced. Both the CCC and MTM conditions were implemented an equal number of sessions with the intervention phase lasting 28 days after baseline.

Results indicated that all students improved their DCPM on assessment worksheets under both conditions. For one student, CCC and MTM were equally effective (CCC Percent Non-overlapping Data [PND] = 100%, MTM PND = 100%) , and for the other two students, MTM was slightly more effective; however, divergence in data between the CCC and MTM conditions was very minimal as the two students performed marginally better in the MTM condition (CCC PND = 79%, MTM PND = 86%) (CCC PND = 67%, MTM PND = 100%). Mong and Mong (2010) attributed these results to the repeated practice present in the MTM condition that was absent from the CCC condition. They also noted that the CCC condition was more efficient, as it required less time to implement, produced significant results, and was independently implemented by the students.

Grafman and Cates (2010) provided additional support for the Cover-Copy-Compare method in the area of mathematics. They compared the math fluency and error rates of second-grade students under a Cover-Copy-Compare (CCC) condition and under a modified CCC condition (MCCC). The goal of the study was to investigate if one strategy was more effective for basic subtraction problems than the other, as well as to determine which procedure students and their teachers preferred.

A total of 47 second-grade students from two general education classrooms participated in the study. The study was conducted at two middle class suburban schools
in the Midwest. Students ranged in age from 7 to 8-years-old with 60% of students identifying themselves as Caucasian.

A within-subjects design was used to evaluate the effects of the CCC and MCCC self-directed procedures. Each student was initially administered a pretest, a CCC worksheet, an MCCC worksheet, and a posttest worksheet. Worksheets were generated for each respective condition. Pretest and Posttest worksheets consisted of 40 two-digit by one-digit subtraction problems presented in 10 rows of four problems. CCC and MCCC worksheets were similar in that they contained 25 subtraction problems similar to those generated for pretests and posttests. Each CCC and MCCC worksheet was labeled clearly in order to direct student was to which procedures to follow.

On CCC worksheets, students were asked to follow a traditional CCC procedure: (a) look at problem and solution, (b) cover problem and answer, (c) write the problem and answer from memory, (d) uncover the problem and answer, (e) compare their response to the original. On MCCC worksheets, students followed a slightly different procedure: (a) look at the problem and answer, (b) copy the problem and answer, (c) cover the problem, (d) write the problem and answer on the right side of the paper, (e) uncover the problem and answer, (f) compare their response to the model. Both conditions followed a standard error correction procedure of repeating the process an additional time; however, the experimenters hypothesized that the extra repetition of the math fact under the MCCC procedure would be more beneficial to participants (Grafman & Cates, 2010).
Digits correct per minute (DCPM) was scored on each worksheet for all participants. At the conclusion of the intervention, participants showed a significant increase in the number of DCPM from pretest to posttest, and maintained accuracy despite an increase in their speed after exposure to both conditions. Results further indicated a significantly higher number of DCPM on the CCC worksheets as compared to the MCCC worksheets. An effect size of 1.30 was calculated using a dependent t test and the researchers determined this to be a large effect size. Interestingly, teachers indicated that they would prefer to use the MCCC procedure to teach students basic math facts, but students preferred the CCC procedure. Consequently, the experimenters concluded that teachers, not unlike their initial hypothesis, felt that extra repetition of the problem would be of more beneficial to a student attempting to improve subtraction accuracy and fluency than a single trial (Grafman & Cates, 2010). Yet, the results of this study suggest that the extra copying may not be necessary to improve student subtraction fluency and accuracy. Thus, the CCC procedure, without any modifications, is a successful and efficient approach to increase accurate rates of responding on subtraction problems in little time.

Noticeably, there has been a lack of research examining CCC targeting basic division facts solely. One such study was conducted by Lee and Tingstrom (1994). The CCC intervention was modified and implemented in a small, middle school math class. Participants included 5 fifth-grade students (3 girls and 2 boys) in a middle school in the Southeastern United States. Students ranged in age from 10 to 12 years, and all identified as African-American.
Lee and Tingstrom (1994) generated three sets of division facts, each consisting of 10 division problems, and labeled them as Sets A, B, and C. All division facts were a two-digit dividend and a one-digit divisor, resulting in a one-digit quotient. Training worksheets and assessment worksheets were generated for each set of problems. These worksheets were identical with the exception of answers to the problems being provided on the training worksheets but not the assessment worksheets. Problems (and answers on the training worksheets) were listed on the left side of the paper, with space on the right side for the student to write the problem and answer.

Using a multiple baseline design across the three sets of division facts, the researchers (Lee & Tingstrom, 1994) examined the number of digits correct per minute (DCPM) for each participant across the sets. Procedures for CCC were traditional including: (a) silently reading the problem and answer, (b) covering the problem and answer with an index card, (c) writing the problem and answer from memory on the right side of the paper, (d) uncovering the problem and answer, and (e) repeating the process for all remaining division problems. During all sessions, students would complete an assessment sheet prior to a training sheet. All students began with Set A until they reached mastery (40 DCPM and 90% accuracy on three consecutive assessment sheets). When mastery for a set was achieved, the student would be introduced to the following set.

Results of this study (Lee & Tingstrom, 1994) indicated significant improvement from baseline with the application of the intervention across all participants; the extent to which subjects improved varied. Additionally, subjects showed continued increases in
DCPM on follow-up data following the withdrawal of the intervention. More specifically, the percentage of items correct per subject across all sets were as follows: Subject 1 (Baseline-89%, Intervention-98%, Follow-up-99%), Subject 2 (Baseline-91%, Intervention-96%, Follow-up-97%), Subject 3 (Baseline-97%, Intervention-98%, Follow-up-99%), Subject 4 (Baseline-71%, Intervention-90%, Follow-up-98%), Subject 5 (Baseline-97%, Intervention-99%, Follow-up-100%) (Lee & Tingstrom, 1994). This study provides clear support for CCC as an effective strategy to increase students’ speed and accuracy of completing division facts.

More recently, Cieslar, McLaughlin, and Derby (2008) examined the effects of Cover-Copy-Compare (CCC) for a 16-year-old high school, special education student. Although they utilized CCC for spelling as well, they also targeted mathematics, specifically division facts. Cieslar, McLaughlin, and Derby (2008) chose to implement CCC as it allows students more independence in the intervention process (e.g., they can self-correct their work and engage in an error correction procedure if necessary). As this student was performing far below academic grade level, the goal of the study was to use one intervention (CCC) across two subject-matter areas (spelling and math) to increase skills in both areas.

The study took place in a large suburban area in the Pacific Northwest. A 16-year-old male high school student was the sole participant in the study. He had a diagnosis of behavioral disorder, and received special education services in the areas of reading, math, and writing language (Cieslar, McLaughlin, & Derby, 2008). Prior to the intervention, the student was administered the Wechsler Individual Achievement Test- Second Edition
(WIAT-II; Wechsler, 2005) and his overall math performance was found to be at a third grade equivalency. For this particular student, single- and double-digit division and fractions were the areas of most difficulty.

Based upon these results, the student was administered a pretest consisting of 55 division/fraction problems. Errors that were made on the pretest were equally divided into three separate sets (i.e., Set A, B, and C). Each set had a total of six problems; 18 total errors on the pretest. These sets were then used to generate the CCC and daily assessment sheets (Cieslar, McLaughlin, & Derby, 2008).

Cieslar and colleagues (2008) used a multiple-baseline, single-case design across the three different division sets in order to evaluate the effectiveness of the CCC procedure. CCC intervention sessions were conducted three days a week and the dependent variable for math was the number of target problems answered correctly (Cieslar, McLaughlin, & Derby, 2008). For each session, the student completed a CCC worksheet with the six division/fraction problems on the left and a space to write the problem and answer on the right.

Basic CCC procedures were taught to the student in order for him to complete the CCC sheets independently, with the exception for an additional repetition of each problem. In other words, the student (a) read the problem and answer, (b) copied it in the next column, (c) covered the model and copied problem, (d) wrote the problem again from memory, and (e) uncovered the original and copy to compare. If the response was incorrect, the process was repeated prior to moving on to the next item. After completing the CCC sheet, the participant was then administered the daily assessment worksheet,
containing the same six problems in a test format. The participant was required to answer the problems correctly on the daily assessment for two consecutive sessions in order for an item to be considered mastered. Mastery had to be achieved prior to the next set of problems being introduced (Cieslar, McLaughlin, & Derby, 2008). A posttest of the 18 target problems was administered at the conclusion of the study.

Results indicated significant improvement in the number of correct answers on division/fraction problems as compared to the student’s baseline. For Set A, the average number correct during baseline was 2; during intervention the participant averaged 6. For Set B, the average number correct during baseline was 1; during intervention the participant reached mastery after only one session and averaged 6 correct problems. For Set C, the average number correct during baseline was 2; during intervention the average number was 6 and the participant again reached mastery after only one session. All improvements were maintained throughout the duration of the study. In addition, the student’s pretest score of 0% increased to a posttest score of 89% (Cieslar, McLaughlin, & Derby, 2008).

Cieslar and colleagues (2008) were able to demonstrate a clear functional relationship between the CCC intervention and the increase in division calculation. They noted that CCC was efficient, practical and cost-effective. Additionally, CCC was implemented with a high school student, whereas prior to the study, most CCC interventions were implemented with elementary students. Also, despite the student’s behavior challenges, he was able to stay on-task and complete all CCC interventions and
sessions (Cieslar, McLaughlin, & Derby, 2008). This may be due to the fact that CCC allows students independence and responsibility for their learning and success.

**Current Study**

The current study intends to contribute to this literature by examining the differential effects of three variations of CCC on students’ basic division fact fluency, generalization, and maintenance. Successful outcomes will further expand the range of potential use of the CCC intervention, modifications to the CCC intervention, and strategies to incorporate generalization into the CCC intervention method. Teachers and other practitioners need to be made aware of evidence-based interventions for increasing students’ math fluency and maintenance, specifically in the area of division, and learn of strategies to program for generalization of math skills.
Chapter 3: Method

This chapter describes the methods and procedures employed during the execution of this study. Description of the process of obtaining research approval, the research design, as well as detailed information regarding the participants, setting, experimenter, dependent variables, and independent variables are provided.

Institutional Review Board

Due to the nature of this study, approval from the Institutional Review Board (IRB) for Research with Human Subjects at The Ohio State University was required prior to the start of data collection. Following submission of necessary information, IRB approval was received for the procedures and forms described below.

Participants

The participants of this study consisted of seven, fourth grade students who attended a small urban school district in a large metropolitan city in the Midwest. Based upon prior test scores on state and district-wide assessments, participants were all considered to be at-risk for mathematical difficulties and were already identified and selected to receive Tier 2 intervention services for mathematics by the school district. Furthermore, all participants were performing below their same-aged peers on curriculum-based measurements of basic division, and when administered a standardized
assessment of mathematical applications and fluency, all but one participant had at least one standard score below 90 (below average). All participants spoke English as their only language. Participant demographic information including, age, gender, ethnicity, and pre-intervention scores are presented in Table 1.

Table 1. Participant Demographic Information with Woodcock-Johnson (3rd Edition) (Woodcock, McGrew, & Mather, 2001) mathematical subtest and pre-intervention basic division fact probe scores.

<table>
<thead>
<tr>
<th>Name</th>
<th>Age</th>
<th>Gender</th>
<th>Ethnicity</th>
<th>WJ-III Applied Problems</th>
<th>WJ-III Math Fluency</th>
<th>Basic Division Fact Probe</th>
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<tbody>
<tr>
<td>Adrianna</td>
<td>10 years, 6 months</td>
<td>Female</td>
<td>African-American</td>
<td>88</td>
<td>73</td>
<td>7</td>
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<tr>
<td>Bailey</td>
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<td>Female</td>
<td>Multiracial</td>
<td>93</td>
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<td>Jordan</td>
<td>9 years, 5 months</td>
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<td>99</td>
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</tbody>
</table>

Note: The basic division fact probe score was out of a possible 50 points.

**Sampling procedure.** Student assent to participate was requested of all potential participants selected by the district to receive Tier 2 math interventions (see Appendix G for student assent form). Parental permission was also requested for all potential students via a letter sent home by the researcher (see Appendix H for parent consent form). Once permission was received, each student was considered for participation in the study based
on prerequisite criteria: (a) currently enrolled in fourth grade, (b) current performance on curriculum-based measurements of basic division below that of same-aged peers per teacher report, (c) below average (standard score below 90) on a standardized assessment of mathematical fluency and/or applied math problems, (d) less than 75% correct on pre-intervention basic division fact probe, and (e) teacher and principal recommendation of those students who possessed enough fundamental mathematical knowledge (i.e., conceptual and procedural knowledge of addition, subtraction, multiplication, and division) to be successful with an intervention targeting basic division skills, and those students that were likely to cooperate with the investigator, and who had strong attendance.

Seven participants were recruited for this study and all seven participants assented to participate. Parent permission was also granted for all seven students allowing for a desirable number of participants. Once assent and consent was obtained for each participant, the participants’ general education teachers were also asked to consent to complete a questionnaire upon completion of the study. Three general education teachers were recruited. All three teachers consented to participate.

**Setting**

Assessment, intervention, and data collection took place in a computer lab, separate from the classroom and mostly free from noise and other distractions, at the elementary school. The room was separated into computers on one side of the room, and a white board and work desk on the opposite side of the room. Although other students would complete work at the computers at the same time as intervention, because the
students wore headphones and there was little speaking, the presence of other students at the computers was unobtrusive. The researcher and participants sat at a table on the opposite side of the room during each session. Data was collected over the course of three months from September 25 until December 13, 2013.

**Experimenter**

The experimenter was a fifth-year doctoral candidate enrolled in the School Psychology program at The Ohio State University. She held a Master’s Degree in Special Education/Applied Behavior Analysis and was a Board Certified Behavior Analyst. She had previously served as a graduate teaching associate at The Ohio State University, an Intervention Specialist at a charter school, and an independent provider for children with autism in the Columbus, Ohio vicinity. The researcher was the only interventionist and completed this study as partial fulfillment of the requirements for the Doctor of Philosophy in School Psychology degree.

**Pre-Assessments**

Prior to intervention implementation, participants were administered math fluency and applied problems portions of the standardized academic achievement assessment, the Woodcock-Johnson Tests of Achievement (3rd ed.) (WJ-III ACH) (Woodcock, McGrew, & Mather, 2001). The WJ-III ACH is considered a valid and reliable assessment of academic achievement. The assessment was normed with individuals between the ages of 2 and over 90 years old. According to the technical manual, scores produced by the WJ-III ACH are valid and the test correlates highly with other tests measuring the same abilities. For instance, it is reported to have a correlation of .65 with the Wechsler
Individual Achievement Test and .79 with the Kaufman Test of Education Achievement. Additionally, the median reliability coefficients on the standard battery range from .81 to .94 (Mather & Woodcock, 2001).

The math fluency portion of the WJ-III ACH required students to compute a variety of basic math problems in isolation. Problems included addition, subtraction, multiplication and division, use single or double digits, and became progressively more difficult. Students were asked to complete as many problems as they could within a three minute time limit. In the applied problems subtest of the WJ-III ACH, students were asked to solve mathematical story problems. As with the previous subtest, problems began at a very basic level and increased in difficulty. Problems may or may not have included pictures, and consisted of simple one-step problems, or more complex, multi-step problems. In this fashion, the experimenter obtained a measure of each participant’s computational skill and knowledge of mathematical concepts and vocabulary prior to beginning any intervention.

Additionally, a basic division fact probe was administered to all participants prior to intervention in order to determine participants’ levels of instruction in division skills prior to baseline. This probe contained 50 problems and consisted exclusively of basic division facts (0 through 12) presented across all four division formats.

**Materials**

Materials for this study consisted of sets of generated CCC intervention sheets, fluency probe sheets containing randomized division fact problems of equal difficulty,
word problem generalization probe sheets containing previously taught facts but in word story format, as well as pencils, index cards and a stopwatch.

**Intervention Script.** A script was created to ensure standardized administration of the CCC intervention, including all conditions, probes and data collection (See Appendices A, B, C).

**Random Assignment of Division Problems to Sets.** After facts were randomly assigned to the sets, the sets were randomly assigned to one of the three experimental conditions (CCC, CCC+R, or CCC+G). In this fashion, each intervention condition had a separate set of 48 randomized division facts, which were of equivalent difficulty.

**Study Variables**

**Dependent Variables.** Dependent variables assessed included division fact calculation fluency, generalization and maintenance, as well as social validity.

**Fluency.** Division fact fluency was assessed through generated CCC probe sheets and expressed in terms of facts correct per minute (FCPM). Three sets of division facts were created, one for each intervention condition. Each set contained an equal number of problems and equivalent in range of difficulty. Fluency probes contained 40 randomized problems covering previously taught items and were administered following each intervention session (See Appendix I for a sample probe).

**Generalization.** Generalization was assessed through the administration of story problems. Story problems were administered on a weekly basis following each intervention condition, and contained versions of the division facts that participants had been taught through the intervention that week, using a variety of phrases signaling
participants to divide. Participants earned points for successfully solving the division word problem as follows: 0 points for no written problem and no answer, 1 point for a correctly written division problem or only the correct answer, 2 points for a correctly written problem and answer (See Appendix J for a sample probe).

**Maintenance.** Maintenance measures mirrored the fluency probes, contained randomized division facts according to each set, and were taken two weeks after the intervention concluded (See Appendix K for a sample probe).

**Social Validity.** A social validity measure was also included. This evaluated the usefulness and preferences of participants for the CCC intervention conditions, (See Appendix E), as well as participants’ teachers’ judgments as to the usefulness and feasibility of the CCC intervention conditions (See Appendix F). Items included on these scales asked participants and teachers which condition they preferred, if they felt the intervention helped participants, and if they would likely use this intervention in the future.

**Independent Variable.** The independent variable in this study is the Cover-Copy-Compare (CCC) intervention, which was used to teach basic division skills, and ultimately, improve division calculation fluency and generalization. This intervention included the CCC worksheets, fluency probes, and generalization probes across all three intervention conditions.

**Experimental Conditions**

**Baseline Condition.** Baseline data consisted of the administration of division fact fluency probes and word story generalization probes. These probes were of the same
skills used throughout the intervention and corresponded to the three previously mentioned division fact sets generated for the intervention. Division fact problems were chosen at random from Set A, Set B, and Set C in order to create separate baseline fluency and generalization probes for each set. The fluency probes were timed (one minute), and contained 40 basic division fact problems presented in horizontal format for Sets A and B (corresponding to the CCC and CCC+R conditions), and in both horizontal and vertical formats (i.e., a division bracket, obelus, a fraction, and the words, “divided by”) in Set C (corresponding to the CCC+G condition). Word stories using a variety of phrases signaling the students to divide, were generated for each set of division facts in order to determine students’ pre-intervention story-solving capabilities. No intervention was implemented during baseline sessions. When administering each baseline probe to students, the experimenter explained to students that the probe would assist the experimenter in better understanding what they already knew or did not know. It was stated that some items might be easy and other items might be difficult. Students were encouraged to attempt as many problems as possible, try their best and make a guess if they did not know an answer. As with intervention probes, students were given exactly one minute to complete baseline fluency probes, and generalization word story probes were read aloud to all students.

**Intervention Conditions.** The intervention conditions include CCC, CCC + additional repetition (CCC+R), and finally, CCC + generalization (CCC+G) strategy in which division facts are presented in an alternate but equivalent format.
Cover-Copy-Compare (CCC). In this condition, students practiced solving division facts from Set A. In this condition, CCC intervention sheets were created with division facts on the left side and space for students to write the division facts on the right side of the paper. CCC consisted of five steps: (a) students look at the division fact with the answer, (b) students cover the problem with hand or index card, (c) students write the fact with answer, (d) students uncover the problem and answer, (e) students compare original problem to written problem. If the student was correct, s/he moved on to the next problem. If the student was incorrect, s/he repeated the steps prior to moving to the next problem. Students received intervention on eight division facts for each CCC intervention session, and immediately following the intervention session, students were asked to complete a timed (one minute) probe sheet and then a word story generalization probe of the target division facts (See Appendix A).

CCC + Repetition (CCC+R). The procedures described above for CCC were administered under this condition as well; however, students solved division facts from Set B twice even if the first response was correct. In other words, students would follow the five step procedure for the CCC condition for the given problem, repeat the five step procedure for the same problem, regardless of whether they were initially correct, and then move on to the next problem. As with the CCC only condition, upon completion of eight new division facts in each CCC+R session, students were asked to complete a timed probe sheet and then a word story generalization probe of target division facts (See Appendix B for more details of the implementation procedures for this condition).
CCC + Generalization (CCC+G). Again, students followed the procedures described for CCC but solved division problems from Set C; however, under this condition, the formats of the facts were different. For example, facts in this condition were presented in four different formats: division bracket, obelus, fraction, and words. As with both previous conditions, students again completed a timed probe sheet and word story generalization probe, immediately following each intervention session with eight new division facts from Set C (See Appendix C for more details on the implementation procedures).

Experimental Design and Procedures.

Following baseline, an alternating treatments design was used. This design was chosen in order to compare the effects of all three treatment conditions in a single subject, however, in order to control for potential practice effects, the treatments were presented in a counterbalanced order (See Appendix L for counterbalanced intervention schedule). In other words, each participant experienced all three treatment conditions, but in a different order each week. The order of treatments was randomly assigned to each week. Only one intervention was implemented per day, and each intervention was implemented an equal number of times.

The experimental conditions were implemented by the author. Sessions occurred three times a week for six weeks, for a total of eighteen intervention sessions. Within the six week timeframe, participants received intervention on eight division facts per each intervention condition (n = 3 instructional conditions) per week. Thus, the total number
of division facts presented across all three conditions combined was 24 (i.e., 8 per instructional condition). Sessions lasted approximately 20-30 minutes.

Prior to the intervention, all 144 division facts (0-12) were grouped according to divisor (e.g., facts divided by 1, facts divided by 2, facts divided by 3 and so on) and randomly assigned to one of three sets (i.e., Set A, Set B, and Set C.). In this fashion, Set A, Set B and Set C each contained the same number of problems according to divisor, and the same number of facts overall. Therefore, each set of division facts contained and equal number of problems equivalent in difficulty.

After facts were randomly assigned to the sets, the sets were randomly assigned to one of the three experimental conditions (CCC, CCC+R, or CCC+G). In this fashion, each intervention condition had a separate set of 48 randomized division facts, which were of equivalent difficulty. Additionally, participants did not experience overlap in division facts from condition to condition. Within each fact set, the problems were further be broken down into weeks. Since the expected duration of the implementation of each condition was six weeks, eight facts were taught per condition, per week. In other words, each set of 48 facts (Set A, Set, B and Set C) that will be randomly assigned to the intervention conditions (CCC, CCC+R, or CCC+G) were segmented into six groups of eight facts, one for week 1, one for week 2, one for week 3 and so on up through week 6. More specifically, facts divided by 1’s and 2’s were taught during week 1, facts divided by 3’s and 4’s were taught during week 2, facts divided by 5’s and 6’s were taught during week 3 and so on through facts divided by 12’s in week 6 (See Appendix M for schedule according to week).
Also prior to implementing the intervention, the weekly sets of 8 facts per condition were used to generate the fluency and generalization probes. Fluency probes were administered immediately following each session, or after a set of 8 facts had been taught in an intervention condition. For example, after a participant received the CCC+R in week 2, s/he was immediately administered a fluency probe containing the 8 facts just learned and randomized previously taught division facts (those from week 1) in that particular condition. The same procedure was followed for this participant and for all the participants for each of the eight facts taught in each of the other two conditions on week 1 and subsequent weeks. Fluency probes were cumulative but focused on the eight most recently taught facts. Each of the most recently taught eight facts were included three times on that session’s fluency probe for a total of 24 repetitions of that session’s facts. The remaining facts consisted of previously taught facts randomized for the remaining items on the probe to equal a total of 40 problems. This method continued for the duration of the study. Therefore, fluency probes were administered immediately following each intervention session for each condition. In other words, three fluency probes were given per week, one per intervention condition.

As with fluency probes, generalization probes were administered following each intervention condition. Generalization probes consisted of the eight facts for which the participant has just received instruction. As previously stated, generalization was assessed through the administration of story problems. Story problems were administered immediately following fluency probes, contained versions of the division facts that participants had been taught in that intervention condition that day, and used a variety of
phrases signaling participants to divide. As with fluency probes, three generalization probes were given per week, one per intervention condition. Generalization probes were not timed.

**Maintenance.** Two weeks following the conclusion of the intervention, students were administered three follow-up timed fluency probe sheets only, in order to determine if any gains from the intervention had been maintained. Each fluency probe sheet consisted entirely of division facts from either Set A (CCC condition), Set B (CCC+R condition), or Set C (CCC+G). In other words, one fluency probe consisted of problems from Set A which were problems taught in the CCC intervention condition. A second fluency probe consisted of problems from Set B which were problems taught in the CCC+R intervention condition. A third fluency probe consisted of problems from Set C which were problems taught in the CCC+G intervention condition. As with fluency probes during intervention, maintenance fluency probes consisted of 40 previously learned, randomized basic division facts. Division fact on the maintenance probes from the CCC and CCC+R fact sets were presented in horizontal format only. Division facts on the maintenance probe from the CCC+G fact set were presented across the four different previously mentioned formats. Maintenance probes were timed (one minute).

**Procedural Integrity.** Procedural integrity was assessed by an independent observer through the use of a checklist. The checklist contained the intervention steps, the probe steps, and materials required. A graduate student acted as an independent observer and recorded a checkmark for the presence of any required materials, as well as the
correct implementation of the intervention and probe steps by the experimenter (See Appendix D).

**Interobserver Agreement.** For fluency data, Interobserver agreement was assessed for facts correct per minute (FCPM) by having the independent observer graduate student independently score division probe sheets. Comparisons were made between the experimenter’s calculated FCPM and the independent observer’s calculated FCPM. Agreement was calculated by dividing the number of agreements per fact by the number of agreements per fact, plus disagreements per fact and multiplying by 100 \[
\left(\frac{\text{Agreements}}{\text{Agreements} + \text{Disagreements}}\right) \times 100.
\] Interobserver agreement was also calculated for generalization data by having an independent observer graduate student independently score generalization probe sheets. Comparisons were made between the experimenter’s calculated total generalization score and the independent observer’s calculated generalization score. Agreement was calculated according to the equation mentioned above.

**Social Validity.** Social validity of the CCC intervention was assessed via a questionnaire generated by the experimenter. Upon the conclusion of the study, student participants were asked to rate and compare the usefulness of the intervention and their preferences of the different intervention conditions within the intervention, using a social validity rating scale (See Appendix E). Participants’ teachers were also given a questionnaire and asked to rate their preferences, the usefulness, and the feasibility of implementing the intervention and the different conditions within the intervention (See Appendix F).
Threats to Validity

With an alternating treatment design in which each participant served as his or her own control, threats to internal validity such as confounding variables, history, and maturation are not a concern. Testing or practice effects have been controlled for with the use of a counterbalanced design in which division facts were assigned to only one condition, not used across multiple conditions.

The design’s small sample size is a threat to external validity. Findings are representative of the seven participants and may not be generalized to other populations, or children of other backgrounds, ages, or abilities.
Chapter 4: Results

This chapter presents results of the study, which examined the differential effects of three variations of Cover-Copy-Compare (CCC) on the fluency, maintenance, and generalization of basic division facts of seven elementary school students. First, inter-observer agreement and procedural integrity data are reported. Next, results illustrating the effects of the three CCC conditions on participants’ fluency are reported, including change from baseline, and cumulative performance totals. Following, results of the three CCC conditions on each participant’s basic division skill generalization are presented. In addition, each participant’s basic division fact maintenance data under each CCC condition are reported. Finally, the social validity of the study from participant and teacher perspectives is described.

Procedural Integrity

Procedural integrity data were collected to ensure that the intervention was implemented with fidelity. These data were collected for 28% of intervention session and data collection assessments.

Intervention procedural integrity. During intervention, a graduate student trained in intervention and data collection assessed the fidelity with which the intervention was implemented. Given the intervention script, the graduate student
indicated a checkmark for each line of the intervention script correctly implemented. Mean adherence to the script was 99% (range: 95-100%).

**Inter-observer Agreement**

Inter-observer agreement (IOA) was calculated for the basic division facts and generalization probes during data collection. A graduate student trained in intervention and data collection scored 28% of the basic division fluency facts, and 28% of the generalization story problem probes.

**Data collection.** Items were analyzed for IOA using the following point-by-point formula: number of agreements divided by the sum of agreements plus disagreements, multiplied by 100. The mean IOA for the fluency probes was 97% (range: 86-100%).

**Generalization.** Items were analyzed for IOA using the following point-by-point formula: number of agreements divided by the sum of agreements plus disagreements, multiplied by 100. The mean IOA for generalization story problems was 98% (range: 86-100%).

**Participants**

The participants of this study consisted of seven, fourth grade students. Participant ages, at the start of the study, ranged from nine years and zero months to ten years and six months. As evidenced by pre-intervention data, all participants were performing below their same-aged peers on curriculum-based measurements of basic division, and when administered a standardized assessment of mathematical applications and fluency, all but one participant had at least one standard score below 90 (below average).
Summary of Baseline Performance for all Students. Baseline fluency data collected prior to intervention were similar to pre-intervention curriculum-based data (less than 75% correct on pre-intervention basic division fact probe), and were similar across participants. Adrianna averaged 5.8 (range 4 to 9) correct division facts per minute across all sets of problems in baseline, Bailey averaged 9.2 (range 6 to 15) correct division facts per minute, Jordan averaged 2 (range 1 to 3) correct division facts per minute, Terrell averaged 5.5 (range 1 to 9) correct division facts per minute, Keith averaged 9.3 (range 7 to 13) correct division facts per minute, Jason averaged 1.8 (range 0 to 8) correct division facts per minute, and Yvette averaged 5.7 (range 1 to 7) correct division facts per minute.

Baseline generalization data collected prior to intervention were similar to pre-intervention probes, fluency data, and were similar across most participants. Adrianna obtained a mean of 3.5 (range 1 to 5) correct responses on math story problems. Bailey obtained a mean of 13 (range 8 to 15) correct responses on story problems, Jordan obtained a mean of 2.7 (range 0 to 5) correct responses on story problems, Terrell obtained a mean of 7 (range 4 to 11) correct responses on story problems, and Keith obtained a mean of 3.7 (range 3 to 4) correct responses on story problems. Jason obtained a mean of 7.2 (range 5 to 11) correct responses on story problems, and Yvette obtained a mean of 9.8 (range 6 to 14) correct responses on math story problems.

Intervention Effectiveness

The effectiveness of the three conditions of the Cover-Copy-Compare (CCC) intervention was evaluated using fluency probes and generalization story problems
administered to each participant at the end of each intervention session. Fluency probes consisted of targeted and previously targeted basic division facts practiced during respective CCC intervention conditions. Generalization story problem probes consisted of eight story problems utilizing basic division facts targeted in the respective CCC intervention conditions. A total of 40 correct facts could be earned on fluency probes, and a total of 16 points for correct responses on written story problem equations could be earned on the generalization probes.

**Percentage of Non-Overlapping Data.** Percentage of non-overlapping data (PND) was calculated between the baseline phase and each CCC intervention condition for each participant. The PND reveals the magnitude of the intervention and is calculated by identifying the highest data point during baseline, counting all intervention data points higher than that highest baseline point, and calculating the proportion of non-overlapping data to total number of intervention points. As a general rule, an intervention demonstrates a strong effect if PND is greater than 70%. PND is included in the raw data below for each participant.

**Summary of All Students’ Performance Under Type of CCC Intervention Condition.**

**Visual analysis.** Graphs were created for visual analysis of reported raw data. The number of sessions each participant engaged in each condition and the mean performance for fluency and generalization within each condition are reported in Tables 2 and 3 respectively. Cumulative fluency and generalization totals across all conditions for each participant are presented in Table 4. It should be noted that cumulative fluency and
generalization graphs represent performance totals for participants as opposed to cumulative skill acquisition.

Table 2: Baseline Mean, Number of Sessions, and Mean Fluency Performance Across Participants Within Each Condition

<table>
<thead>
<tr>
<th>Participant</th>
<th>CCC Baseline</th>
<th>CCC Sessions</th>
<th>CCC Mean</th>
<th>CCC+R Baseline</th>
<th>CCC+R Sessions</th>
<th>CCC+R Mean</th>
<th>CCC+G Baseline</th>
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Table 3: Baseline Mean, Number of Sessions, and Mean Generalization Across Participants Within Each Condition

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<th>CCC Sessions</th>
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Table 4: Cumulative Fluency and Generalization Totals Across Participants Within Each Condition

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<td>Jason</td>
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<td>51</td>
</tr>
<tr>
<td>Yvette</td>
<td>73</td>
<td>93</td>
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</table>

When compared to baseline, all participants improved on correct division facts per minute across all three CCC conditions. Average fluency performance was greatest for Adrianna (tied with CCC+G), Bailey, Jordan, and Jason in the CCC condition. Terrell’s average fluency was highest in the CCC+R condition, while Keith and Yvette averaged the highest number of correct division facts per minute in the CCC+G condition. Said another way, four participants (57%) had the highest mean fluency performance in the CCC condition, one participant (14%) had the highest mean fluency performance in the CCC+R condition, and two participants (29%) had the highest mean fluency performance in the CCC+G intervention condition.

When compared to baseline, all participants improved on the number of correct story problem across all three CCC conditions, with the exception of Bailey, whose average performance on story problems was lower in the CCC condition than baseline. No participants averaged the highest number correct on story problems under the CCC condition. Adrianna’s average correct on story problems was highest in the CCC+R
condition. Bailey, Jordan, Keith, Jason, and Yvette averaged the highest number correct on story problems in the CCC+G condition. Terrell’s average number of correct responses on story problems was equivalent across all conditions. In other words, one participant (14%) averaged the highest number correct on story problems in the CCC+R intervention condition and five participants (71%) averaged the highest number correct on story problems in the CCC+G condition. One participant (14%) averaged equivalent scores across all three intervention conditions.

**Individual Student Performance on All Outcome Variables Across Intervention Conditions.**

*Adrianna.* Adrianna exhibited immediate improvement in the number of correct division facts per minute immediately upon introduction of the intervention across all three conditions. Baseline scores ranged from four to nine. Mean baseline was 5 correct division facts per minute in the CCC condition, 9 correct facts per minute in the CCC+R condition, and 4.5 correct division facts per minute in the CCC+G condition. Adrianna averaged 12.4 correct division facts per minute in the CCC+R condition and 13.8 correct division facts per minute in the CCC and CCC+G conditions. Percent Non-Overlapping Data (PND) was 100% (highly effective) in the CCC and CCC+G conditions, and 80% (moderately effective) in the CCC+R condition. Figure 1 displays her performance on math fluency across baseline and intervention conditions. Generally, there appears to be variability and overlap in performance within and across intervention conditions.

Cumulative fluency totals of all sessions across all conditions can be seen in Figure 2 below. Initially, Adrianna exhibited overlap in cumulative performance across
intervention conditions. Adrianna’s cumulative math fluency performance on the final intervention sessions was highest in the CCC condition.

Adrianna averaged 3.5 correct responses on story problems in baseline (range one to five). As with her fluency, she exhibited immediate gains upon introduction of the intervention. She averaged 13.7 correct responses in the CCC condition, 15 correct responses in the CCC+R condition, and 14.4 correct responses in the CCC+G condition. PND was 100% (highly effective) across all conditions. Figure 3 displays performance on generalization measures across baseline and intervention conditions. Generally, there was overlap in her performance across intervention conditions, especially between performance on CCC+R and CCC+G. There was less overlap between CCC and the other two intervention conditions (i.e., CCC+R and CCC+G).

Cumulative generalization totals of all sessions across all conditions can be seen in Figure 4. Consistent with her fluency, Adrianna exhibited continued cumulative improvement across all conditions, yet, cumulative generalization effects were again highest in the CCC condition. It should be noted that Adrianna’s performance across sessions was generally higher in the CCC+R condition. However, because she completed six sessions within the CCC intervention condition, as compared to only five sessions in the CCC+R intervention condition, her final cumulative total was highest in the CCC condition. This may not have been the case had Adrianna’s attendance allowed her to complete all sessions.
Figure 1: Number of Facts Correct Per Minute Across Conditions for Adrianna

Figure 2: Number of Correct Division Facts Per Minute Cumulative Totals Across Conditions for Adrianna
Figure 3: Correct Generalization Responses Across Conditions for Adrianna

Figure 4: Cumulative Correct Generalization Responses Across Conditions for Adrianna
Bailey. Bailey began the intervention with a higher baseline than all other participants with the exception of Keith. Upon introduction of the intervention, Bailey immediately showed improvement in her fluency, however, as intervention continued, her performance leveled off. Bailey’s baseline fluency scores ranged from six to fifteen correct division facts per minute. Her mean baseline performance in the CCC condition was 9.5 correct division facts per minute, 10.5 correct division facts per minute in the CCC+R condition, and 7.5 correct division facts per minute in the CCC+G intervention condition. During intervention, Bailey averaged 12.2 correct division facts per minute in the CCC+R condition, 15 correct division facts per minute in the CCC+G condition, and 18 correct division facts per minute in the CCC condition. Percent Non-Overlapping Data (PND) was 83% (moderately effective) in the CCC, 17% (ineffective) in the CCC+R condition, and 100% (highly effective) in the CCC+G condition. Figure 5 displays Bailey’s performance on division fact fluency across baseline and intervention conditions. Initially, there was overlap in performance across all three conditions. As the intervention continued, Bailey exhibited more differentiation between the intervention conditions with higher performance in the CCC condition, followed by the CCC+G and CCC+R conditions.

Cumulative fluency totals of all sessions across all conditions can be seen in Figure 6 below. Although Bailey exhibited continued cumulative improvement across all conditions, cumulative fluency effects were highest in the CCC condition. Cumulative fluency totals initially showed overlap, yet as the intervention continued, there was less overlap between the CCC condition and the other two conditions (CCC+R and CCC+G).
Bailey averaged 13 correct generalization story problem responses in baseline (range eight to fifteen). Due to her high baseline average and an obtainable ceiling of 16 generalization points, Bailey did not experience significant gains immediately upon introduction of the intervention. Instead, Bailey showed slight improvement in the CCC+R and CCC+G conditions and maintained consistent performance. However, Bailey was the only participant to have a lower generalization correct response average than her baseline comparison. She averaged 11 correct responses in the CCC condition, 14.4 correct responses in the CCC+R condition, and 14.8 correct responses in the CCC+G condition. PND was 0% (ineffective) in the CCC condition, and 33% (ineffective) in the CCC+R and CCC+G conditions. Figure 7 displays Bailey’s performance on generalization measures across baseline and intervention conditions. Initially, Bailey’s performance on generalization measures was highest in the CCC+G condition with lower performance on the CCC+R and CCC intervention conditions. As the intervention continued, there was overlap and consistency across all three conditions.

Cumulative generalization totals of all sessions across all conditions can be seen in Figure 8 which reveals overlap across all three conditions. As with her fluency, Bailey exhibited continued cumulative improvement across all conditions, however, unlike her fluency, Bailey’s cumulative generalization effects were highest in the CCC+G condition.
Figure 5: Number of Facts Correct Per Minute Across Conditions for Bailey

Cumulative Fluency-Bailey

Figure 6: Number of Correct Division Facts Per Minute Cumulative Totals Across Conditions for Bailey
Figure 7: Correct Generalization Responses Across Conditions for Bailey

Figure 8: Cumulative Correct Generalization Responses Across Conditions for Bailey
Jordan. Jordan began the intervention with a baseline fluency mean of 1.5 correct division facts per minute in the CCC condition, 2 correct division facts per minute in the CCC+R condition, and 2.5 correct facts per minute in the CCC+G intervention condition. Jordan’s baseline fluency scores ranged from one to three correct division facts per minute. Upon introduction of the intervention, Jordan showed immediate improvement in fluency. As intervention continued, his performance lowered slightly but was still above baseline levels. Towards the end of the intervention, Jordan’s performance increased again. In the CCC+R intervention condition, Jordan averaged 7.3 correct division facts per minute. In the CCC+G and CCC intervention conditions, Jordan averaged 8.2 and 9.4 correct division facts per minute respectively. Percent Non-Overlapping Data (PND) was 83% (moderately effective) in the CCC+R intervention condition, and 100% (highly effective) in the CCC and CCC+G conditions. Figure 9 shows Jordan’s basic division fluency across baseline and intervention conditions. With the introduction of the intervention, Jordan’s fluency was highest in the CCC+R condition, but as the intervention continued and his performance decreased, there was overlap and convergence across all three intervention conditions. Then, as the intervention progressed there was less overlap among the three conditions with CCC being the highest.

Cumulative fluency totals of all sessions across all conditions can be seen in Figure 10 below. Throughout the intervention, Jordan’s cumulative fluency scores were highest in the CCC intervention condition. When analyzing the cumulative effects of the
intervention conditions on Jordan’s division fluency, there was less overlap between the CCC condition and the other two conditions. Cumulative fluency totals were consistently highest in the CCC intervention condition.

Jordan’s mean correct in generalization baseline was the lowest of all participants (range zero to five). He averaged 4.5, 2, and 1.5 correct generalization responses in baseline across the CCC, CCC+R and CCC+G conditions respectively. Immediately upon introduction of intervention, Jordan’s ability to generalize and solve generalized story problems significantly improved. Jordan’s performance in the CCC and CCC+R conditions continued to improve throughout the intervention; however, Jordan’s performance in the CCC+G condition was highest at the start of the intervention but gradually decreased as the intervention continued. He averaged 11 correct responses in the CCC+R condition, 11.4 correct responses in the CCC condition, and 13 correct responses in the CCC+G condition. PND was 100% (highly effective) across all conditions. Jordan’s correct generalization responses can be seen in Figure 11 below. Overall, there was some overlap among the three conditions with CCC+G responses generally being the highest. However, by the end of the intervention, Jordan’s correct generalization responses were highest in the CCC condition.

Cumulative generalization totals of all sessions across all conditions can be seen in Figure 12. Unlike Jordan’s cumulative fluency totals, Jordan’s cumulative generalization correct responses overlapped throughout the intervention. At the culmination of the intervention, Jordan’s highest cumulative total was in the CCC+R intervention condition.
Figure 9: Number of Facts Correct Per Minute Across Conditions for Jordan

Figure 10: Number of Correct Division Facts Per Minute Cumulative Totals Across Conditions for Jordan
**Generalization-Jordan**

![Generalization-Jordan](image1)

*Figure 11: Correct Generalization Responses Across Conditions for Jordan*

**Cumulative Generalization-Jordan**

![Cumulative Generalization-Jordan](image2)

*Figure 12: Cumulative Correct Generalization Responses Across Conditions for Jordan*
**Terrell.** Terrell began the intervention with an average fluency baseline of 5.5 correct division facts per minute (range 1 to 9). More specifically, his average correct facts per minute in baseline was 5 in the CCC condition, 6 in the CCC+R condition, and 5.5 in the CCC+G condition. Upon introduction of the intervention, Terrell showed immediate improvement in fluency, but only in the CCC+R condition. Terrell’s initial fluency in the CCC and CCC+G intervention conditions decreased from baseline upon introduction of the intervention. Figure 13 below shows little overlap initially across the three conditions with CCC+R having the highest response rates. By the third session for each condition, his performance increased and continued to increase across all conditions for the duration of the intervention. Here, Figure 13 indicates significant overlap across the three conditions as Terrell’s fluency was consistent across the conditions. Overall, in the CCC+G intervention condition, Terrell averaged 5.2 correct division facts per minute. When compared to his average fluency baseline of 5.5 correct division facts per minute, Terrell was the only participant to have a lower average fluency performance in an intervention condition. In the CCC and CCC+R intervention conditions, Terrell averaged 10 and 10.7 correct division facts per minute respectively. Thus, by the end of the intervention Figure 13 shows less overlap between the conditions with the highest rate of correct division facts per minute in the CCC+R condition. Percent Non-Overlapping Data (PND) was 16% (ineffective) in the CCC+G intervention condition, 33% (ineffective) in the CCC intervention condition, and 50% (minimally effective) in the CCC+R intervention condition.
Cumulative fluency totals of all sessions across all conditions can be seen in Figure 14 below. Throughout the intervention, Terrell’s cumulative fluency scores were lowest in the CCC+G intervention condition. There was little overlap between this condition and the CCC and CCC+R conditions. More overlap can be seen between the CCC and CCC+R conditions. However, by the end of the intervention, his cumulative fluency scores were highest in the CCC intervention condition.

Terrell’s mean generalization correct responses in baseline was 8.5 in the CCC condition, 4.5 in the CCC+R condition, and 8 in the CCC+G conditions, with scores ranging from four to eleven. Immediately upon introduction of intervention, Terrell’s correct responses increased, and continued to do so for approximately three sessions in each condition. Terrell then experienced a downward trend in his generalization performance prior to improvement again towards the end of the intervention in the CCC and CCC+R conditions. Overall, Figure 15 shows Terrell’s correct generalization responses across baseline and all intervention conditions. For the duration of the intervention, there was overlap and some variability in Terrell’s performance across all three conditions. Terrell was the only participant to average the same number of correct generalization responses across all three conditions. Terrell averaged 13 correct responses during intervention across all three conditions, and PND was 80% in CCC, 83% in CCC+G, and 100% in the CCC+R condition. Said another way, the CCC and CCC+G intervention conditions were moderately effective and the CCC+R intervention condition was highly effective.
Cumulative generalization totals of all sessions across all conditions can be seen in Figure 16. Unlike Terrell’s cumulative fluency totals in which his overall performance was highest in the CCC condition, Terrell’s cumulative generalization correct responses were highest in the CCC+R and CCC+G conditions. At the culmination of the intervention, Terrell’s cumulative correct responses were equivalent in these two conditions. Visual analysis of cumulative generalization totals in Figure 16 shows more overlap between the CCC+R and CCC+G condition, and little overlap with the remaining condition, CCC.

**Fluency-Terrell**

![Fluency-Terrell Graph](image)

*Figure 13: Number of Facts Correct Per Minute Across Conditions for Terrell*
Figure 14: Number of Correct Division Facts Per Minute Cumulative Totals Across Conditions for Terrell

Figure 15: Correct Generalization Responses Across Conditions for Terrell
Figure 16: Cumulative Correct Generalization Responses Across Conditions for Terrell

**Keith.** Keith began the intervention with the highest average correct division facts per minute when compared to all other participants. Keith averaged fluency 9.3 correct division facts per minute (range 7 to 13) in baseline. He averaged 10 correct facts in baseline in the CCC condition, 8 correct facts in the CCC+R condition, and 10 correct division facts in the CCC+G condition. As in Terrell’s case, Keith showed immediate improvement upon introduction of the intervention, but experienced a decrease in performance across all intervention conditions in the middle of the intervention (session 10). However, as the intervention continued, Keith’s fluency improved across all intervention conditions. By the end of the intervention, Keith’s average fluency was highest in the CCC+G condition. Figure 17 displays his performance on math fluency across baseline and intervention conditions. There appears to be some variability across conditions but little overlap. In general, Keith performance was highest in the CCC condition, followed by the CCC+G condition, and finally the CCC+R condition. He
averaged 19.5 correct basic division facts per minute in the CCC+G condition. Keith averaged 18.8 correct facts per minute in the CCC condition, and 13.5 in the CCC+R condition. Percent Non-Overlapping Data (PND) was 100% in the CCC+G condition indicating a highly effective intervention, and 80% in the CCC and CCC+R intervention conditions indicating moderately effective interventions.

Cumulative fluency totals of all sessions across all conditions can be seen in Figure 18 below. Throughout the intervention, Keith’s cumulative fluency totals were highest in the CCC+G condition and lowest in the CCC+R condition with little variability or overlap within or across conditions.

Keith’s baseline correct response average on generalization measures was 3.7, with scores ranging from three to four. Keith’s baseline performances were stable across all three conditions. Interestingly, upon introduction of intervention, Keith’s generalization scores increased and stabilized for the duration of the intervention across all conditions. As can be seen in Figure 19, there was little variability or overlap in Keith’s correct responses within or across conditions. Overall, Keith averaged 15.2 correct responses in the CCC+G condition, and 12.8 correct responses in the CCC and CCC+R conditions. Each intervention condition was highly effective as PND was 100% across all intervention conditions.

Cumulative generalization totals of all sessions across all conditions can be seen in Figure 20. Nearly identical to his cumulative fluency totals, Keith’s cumulative generalization totals were highest in the CCC+G condition and lowest in the CCC+R condition for most of the intervention. However, by the end of the intervention, his
cumulative generalization point totals were equivalent in the CCC and CCC+R conditions, and remained highest in the CCC+G condition. Similar to his cumulative fluency totals, there was little variability or overlap within or across conditions. Keith’s performance was consistently highest in the CCC+G intervention condition.

**Figure 17: Number of Facts Correct Per Minute Across Conditions for Keith**
Figure 18: Number of Correct Division Facts Per Minute Cumulative Totals Across Conditions for Keith

Figure 19: Correct Generalization Responses Across Conditions for Keith
Jason. Jason had the lowest average correct division facts per minute in baseline when compared to all other participants. Jason averaged 1.8 correct division facts per minute (range 0 to 8) in baseline. His average baseline in the CCC intervention condition was 4 correct facts per minute. In the CCC+R condition he averaged .5 correct facts per minute, and in the CCC+G condition his baseline average was 1 correct fact per minute. As with other participants, Jason showed immediate improvement in basic division fluency with the introduction of the intervention. Although Jason’s data shows increasing trends across all intervention conditions, differentiation among the three conditions is lacking. Jason’s division fluency across baseline and intervention conditions can be seen below in Figure 21. Jason’s performance overlapped across all three intervention conditions with variability within and between intervention conditions. By the end of the intervention, Jason’s average fluency was highest in the CCC condition. It should be noted though, that Jason was only able to complete four intervention sessions in the

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Figure 20: Cumulative Correct Generalization Responses Across Conditions for Keith
CCC+G condition due to poor school attendance. Overall, Jason averaged 12 correct basic division facts per minute in the CCC condition. In the CCC+R and CCC+G intervention conditions, Jason averaged 8.5 and 8.3 correct basic division facts per minute respectively. The Percent Non-Overlapping Data (PND) was 100% in the CCC+R and CCC+G conditions, indicating highly effective intervention, and 83% in the CCC condition, indicating moderately effective intervention.

Cumulative fluency totals of all sessions across all intervention conditions can be seen in Figure 22 below. Initially there appeared to be overlap in cumulative fluency totals across intervention conditions. Yet, a clear differentiation between cumulative fluency in the CCC condition and cumulative fluency in the CCC+R condition existed as soon as Jason was unable to attend any further CCC+G intervention sessions. Overall, Jason’s cumulative fluency totals were highest in the CCC condition and lowest in the CCC+R condition; however, since Jason was only able to complete four CCC+G conditions because of attendance issues, it is possible that his cumulative fluency totals would have been different had he completed all sessions in this condition.

Jason averaged 7.2 correct responses to baseline generalization story problems, with scores ranging from five to eleven. More specifically, Jason averaged 9.5 correct responses in the CCC baseline condition, and 6 correct responses each in the CCC+R and CCC+G baseline conditions. As with his fluency data, Jason’s data shows slight increasing trends across all intervention conditions, but differentiation among the three conditions is lacking as can be seen in Figure 23 below. Again, since Jason was only able to complete four sessions within the CCC+G condition, a comparison between all three
intervention conditions is difficult. Generally, there appears to be variability and overlap within and across intervention conditions. Overall, Jason averaged 13.3 correct generalization responses in the CCC+G condition, and 11.7 correct generalization responses in the CCC and CCC+R conditions. PND was indicative of a highly effective intervention (100%) in the CCC+R and CCC+G conditions while PND was indicative of a moderately effective intervention (83%) in the CCC intervention condition.

Cumulative generalization totals of all sessions across all conditions can be seen in Figure 24. In general, Jason’s cumulative generalization totals indicated consistent improvement with overlap across intervention conditions. Jason’s cumulative generalization totals were equivalent in the CCC and CCC+R conditions; however, because Jason was only able to complete four CCC+G conditions, it is difficult to determine if his cumulative generalization totals would have been different if he had completed all sessions in this condition.

**Figure 21: Number of Facts Correct Per Minute Across Conditions for Jason**
Figure 22: Number of Correct Division Facts Per Minute Cumulative Totals Across Conditions for Jason

Cumulative Fluency-Jason

Figure 23: Correct Generalization Responses Across Conditions for Jason

Generalization-Jason
Yvette. Prior to intervention, Yvette had an average fluency of 5.7 correct division facts per minute (range 1 to 7) in baseline. In the CCC condition she averaged 6 correct facts per minute in baseline, in the CCC+R condition she averaged 7 correct facts, and in the CCC+G intervention condition she averaged 4 correct division facts per minute. Even though Yvette’s overall baseline average was higher than many participants, she showed immediate and significant improvement in basic division fluency with the introduction of the intervention. Her performance decreased in the second and third implementations of each intervention condition, but immediately thereafter, her fluency data showed an increasing trend throughout the remainder of the intervention across all intervention conditions. These results can be seen in Figure 25 below. Generally speaking, Yvette’s performance showed variability and overlap within and across all intervention conditions.
By the end of the intervention, Yvette’s average performance was highest in the CCC+G condition as she averaged 17.8 correct basic division facts per minute in this condition. In the CCC+R and CCC intervention conditions, Yvette averaged 15.5 and 14.6 correct basic division facts per minute respectively. Yvette’s Percent Non-Overlapping Data (PND) was 83% in the CCC+G condition, and 100% in the CCC and CCC+R conditions. Moreover, the CCC and CCC+R intervention conditions were highly effective for Yvette while the CCC+G condition was moderately effective with regards to basic division fluency.

Cumulative fluency totals of all sessions across all conditions can be seen in Figure 26 below. Throughout the intervention, Yvette’s cumulative fluency totals were lowest in the CCC condition. Conversely, her cumulative fluency totals were highest in the CCC+R condition initially. However, by the end of the intervention, Yvette’s cumulative fluency totals were highest in the CCC+G condition, followed by the CCC+R condition. Generally, there was very little overlap or variability within or across the intervention conditions.

Yvette averaged 9.8 correct responses to baseline generalization story problems, with scores ranging from six to fourteen. Yvette demonstrated immediate improvement from baseline with the introduction of the intervention. Her correct responses achieved the ceiling, or near ceiling level of 16, across all intervention conditions. She maintained this performance throughout the duration of the intervention as can be seen in Figure 27, with scores not significantly deviating from ceiling levels. Due to Yvette’s performance reaching ceiling levels, there was considerable overlap across intervention conditions.
with little variability. Overall, Yvette averaged 15.3 story problem points in the CCC+G condition, 14.5 points in the CCC+R condition, and 14 points in the CCC condition. PND was 40% in the CCC indicating an ineffective intervention. By way of comparison, Yvette’s PND was 100% in the CCC+R and CCC+G conditions, indicative of highly effective interventions.

Cumulative generalization totals of all sessions across all conditions can be seen in Figure 28. Similar to her cumulative fluency totals, Yvette’s cumulative generalization correct responses were lowest in the CCC condition throughout the intervention. Consistent with her cumulative fluency, by the end of the intervention, Yvette’s cumulative generalization totals were highest in the CCC+G intervention condition. Generally, there was consistent improvement and overlap across CCC+R and CCC+G intervention conditions. There was no overlap between these conditions and the remaining condition, CCC.

**Fluency-Yvette**

![Fluency-Yvette graph](image)

*Figure 25: Number of Facts Correct Per Minute Across Conditions for Yvette*
Figure 26: Number of Correct Division Facts Per Minute Cumulative Totals Across Conditions for Yvette

Figure 27: Correct Generalization Responses Across Conditions for Yvette
Figure 28: Cumulative Correct Generalization Responses Across Conditions for Yvette

**Maintenance.** In order to obtain maintenance data, maintenance measures were taken two weeks after the conclusion of the intervention. Maintenance measures mirrored the fluency probes. In other words, they contained randomized division facts according to each set across all previously targeted basic division facts. One maintenance probe was taken per intervention condition. Maintenance results can be seen below in Table 5.
As can be seen in the table above, two participants (Adrianna and Jason) maintained the highest fluency in the CCC condition, two participants (Bailey and Terrell) maintained the highest fluency in the CCC+R condition, and two participants (Keith and Yvette) maintained the highest fluency in the CCC+G condition. Jordan’s highest maintenance score was obtained on math problems taught in both CCC and CCC+R conditions.

**Social Validity.** Social validity questionnaires evaluated the usefulness and preferences of participants for the CCC intervention conditions (See Appendix E), as well as participants’ teachers’ judgments as to the usefulness and feasibility of the CCC intervention conditions (See Appendix F). Items included on these scales asked participants and teachers which condition they preferred, if they felt the intervention helped participants, and if they would likely use this intervention in the future.

**Participants.** When completing the questionnaire, all participants (86%) except for one (14%), indicated that division was difficult prior to beginning the intervention.

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<th>CCC</th>
<th>CCC+R</th>
<th>CCC+G</th>
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More specifically, two participants (29%) somewhat agreed and four participants (57%) completely agreed that division was difficult. Following the intervention, 100% of participants indicated that division was easier when compared to the level of difficulty prior to the intervention. Three participants (43%) found division somewhat easier, and four participants (57%) found division completely easier.

Six out of seven participants (86%) of participants indicated that they completely enjoyed the CCC intervention condition. The final participant indicated that s/he somewhat enjoyed the CCC condition. When asked about the CCC+R condition, three participants (43%) indicated that they enjoyed this condition (2 participants somewhat enjoyed; 1 participant completely enjoyed). The remaining four participants (57%) disagreed. One participant completely disagreed and three participants somewhat disagreed. All participants (100%) indicated that they enjoyed the CCC+G intervention condition. Two participants (29%) indicated that they somewhat enjoyed the CCC+G condition, while five participants (71%) indicated that they completely enjoyed the CCC+G intervention condition.

When participants were asked to their opinions on the usefulness of each intervention condition, one participant (14%) somewhat disagreed that the CCC condition was helpful for learning basic division facts. The remaining six participants (86%) indicated that they completely agreed the CCC condition was helpful. Similarly, one participant (14%) somewhat disagreed that the CCC+R condition was helpful for learning basic division facts. The remaining six participants (86%) indicated that the CCC+R intervention condition was helpful for learning basic division facts. More specifically,
one participant (14%) felt the CCC+R condition was somewhat helpful and five participants (71%) felt it was completely helpful. All participants (100%) indicated that the CCC+G intervention condition was helpful when learning basic division facts. Three participants (43%) somewhat agreed that the CCC+G intervention condition was helpful. Four participants (57%) completely agreed that CCC+G condition was helpful for learning basic division facts.

When asked if they would utilize a Cover-Copy-Compare method for other types of math facts, not simply division, all participants (100%) responded affirmatively. One participant (14%) somewhat agreed and six participants (86%) completely agreed. When asked if they would utilize a Cover-Copy-Compare method for other academic subjects in addition to math, again, all participants (100%) responded affirmatively. Three participants (43%) somewhat agreed and four participants (57%) completely agreed.

Finally, participants were asked which intervention condition they preferred. Two participants (29%) most preferred the CCC condition. The remaining five participants (71%) most preferred the CCC+G intervention condition.

**Teachers.** All three participant’s homeroom teachers completed the social validity questionnaire for teachers. However, because each homeroom teacher specialized in a particular academic area (e.g. reading, math, science/social studies), at times, a particular teacher was not able to answer specific questions due to a lack of knowledge.

When asked if their students struggled with division prior to the intervention, all three teachers agreed completely (100%). When asked if division was easier for their students following the intervention, one teacher somewhat disagreed (33%), one teacher
somewhat agreed (33%), and one teacher did not feel adequately knowledgeable to answer.

Teachers were asked if they would enjoy teaching each individual intervention condition. All three teachers (100%) responded neutrally regarding the CCC intervention condition. Two teachers (66%) responded neutrally regarding the CCC+R condition, while one teacher (33%) somewhat agreed that she would enjoy teaching the CCC+R intervention. The teachers responded in the same fashion to the CCC+G intervention condition.

Next, teachers were asked how beneficial they felt each intervention condition to be. All three teachers (100%) somewhat agreed that the CCC intervention condition was a helpful strategy for their students to learn basic division facts. These results were mirrored exactly for the CCC+R and the CCC+G intervention conditions.

All three teachers (100%) somewhat agreed that they would utilize a Cover-Copy-Compare intervention method to teach other types of math facts, in addition to basic division facts. All three teachers (100%) also somewhat agreed that they would utilize a Cover-Copy-Compare intervention strategy to teach other academic subjects in addition to mathematics.

When asked to provide their opinions regarding the feasibility of implementing each intervention condition respectively, one teacher (33%) indicated that the CCC+R condition would be the most feasible to implement, one teacher (33%) indicated that either the CCC or CCC+R intervention conditions would be the most feasible to implement, and one teacher (33%) was unsure and felt she was unable to respond to the
question. Finally, the participants’ teachers were asked how likely they were to implement the intervention condition in the aforementioned question. Two teachers (66%) responded that they were somewhat likely, and the third teacher (33%) indicated that she was not likely to implement the intervention, as she did not teach math.
Chapter 5: Discussion

This chapter presents a discussion of the results of the study. First, results are discussed according to each of the following research questions: (1) *Are there differential effects of three variations of CCC conditions (CCC, CCC+ repetition, CCC+ generalization) on division fact calculation fluency?*, (2) *Are there differential effects of three CCC conditions (CCC, CCC+ repetition, CCC+ generalization) on maintenance?*, (3) *Are there differential effects of three CCC conditions (CCC, CCC+ repetition, CCC+ generalization) on generalization?*, (4) *Do participants express a preference for one intervention treatment condition?*, and (5) *Do participants’ teachers find one intervention treatment condition to be more feasible to implement than the other treatment conditions?*. Obstacles and limitations, implications for practitioners, and suggestions for future research are also described.

**Dependent Variables**

**Research question 1.** *Are there differential effects of three variations of CCC conditions (CCC, CCC+ repetition, CCC+ generalization) on division fact calculation fluency?* This study was structured such that improvement would be observed if a participant’s performance on timed division fact assessment probes was higher in any intervention condition after the introduction of the intervention than it was during
baseline. It was predicted that following the implementation of the intervention, a participant’s fluency on timed basic division fact calculation probes would improve.

Results of this study indicated that all three conditions improved students’ basic division fluency. This supports previous research regarding the effectiveness of the Cover-Copy-Compare intervention method (e.g., Skinner, Belfiore, & Pierce, 1992; Cieslar, McLaughlin, & Derby, 2008; Bolich, Kavon, McLaughlin, Williams, & Urlacher, 1995). All participants’ PND exceeded 70% in at least two intervention conditions, with the exception of Bailey’s fluency in the CCC+R condition (PND = 17%) and Terrell’s fluency across all three conditions (PND = 33%, 50%, 16%). Generally, PND suggests moderate to strong effects across Cover-Copy-Compare intervention conditions and participants’ basic division fluency. Five out of seven students exhibited the highest PND (100%) in the CCC+G intervention condition, suggesting the largest effects on fluency were within the CCC+G condition. More specifically, the CCC intervention was equally effective as the CCC+G condition for Adrianna (PND=100%) and Jordan (PND=100%), and was equally effective as the CCC+R condition for Yvette (PND=100%). The CCC+R intervention condition produced the greatest effect on Terrell’s division fluency (PND=50%) compared to the other intervention conditions, although this is still considered to be an ineffective intervention. Finally, PND was highest in the CCC+G condition for fluency for Bailey (100%) and Keith (100%), and was equivalent to the CCC+R condition (100%) for Jason.

With regard to Terrell’s lower-than-expected PND, one possible explanation is unstable baselines. In each baseline condition, Terrell performed higher on one baseline
point than the other. Fluency data across all three intervention conditions indicates a slow upward trend; however, one high score in baseline may yield lower PND scores despite continued growth. This is a limitation when calculating PND. Additionally, Terrell’s lowest fluency scores across all three intervention conditions were during the week when facts were divided by 3’s and 4’s. It is quite possible that these division facts are an area of weakness for Terrell, he had more difficulty with these problems, and may have benefitted from more exposure, more direct instruction, and more opportunities to solve these problems than was allowed for within this intervention.

Similarly, Bailey obtained a PND less than 70%. Whereas Terrell’s PND was lower than 70% across all three intervention conditions, Bailey’s PND was only lower than 70% in the CCC+R condition. As with Terrell, Bailey also had one baseline data point in this condition significantly higher than the other. Again, this lowered her calculated PND. Unlike Terrell, Bailey’s performance in the CCC+R condition indicated a downward trend. There are numerous possibilities for the difference in her fluency performance in the CCC+R condition as compared to the CCC and CCC+G condition. First, it is possible that the randomized facts in the CCC+R condition were genuinely more difficult for Bailey. Second, outside ecological factors could have affected her performance in this condition; however, as her performance in this condition continued to decrease throughout the intervention, where her performance in the other two conditions gradually increased, it is unlikely that this is the case. Finally, the extra repetition required in the CCC+R condition may simply have been perceived as aversive to Bailey resulting in decreased performance through the study.
In contrast, only two students displayed the highest cumulative fluency totals in the CCC+G intervention condition: Keith and Yvette. Interestingly, both of these students had higher baseline fluency scores than other participants. The remaining five students had the highest cumulative fluency total score in the CCC intervention condition. Differences between PND and cumulative scores (i.e., higher PND in CCC+G but higher cumulative scores in CCC) may be due to the fact that some participants had one or two data points lower than baseline in a given intervention condition (CCC or CCC+R). This lowered the PND for that respective intervention condition resulting in higher PND in the CCC+G condition. However, growth from intervention session to intervention session (represented as cumulative totals) compounded to yield highest cumulative scores, or performance totals, in a separate condition (CCC).

Another possibility is that participants may have had lower baseline scores in the CCC+G condition due to initial difficulty when calculating basic division facts in different formats. Unfamiliarity with the formats may have required more cognitive exertion, and thus, more time, resulting in lower baseline scores. As a result, participants may have had fewer intervention points below that of their initial baseline leading to higher PNDs in the CCC+G condition when compared to the other conditions. Indeed there is some support for this as Codding, Shiyko, Russo, Birch, Fanning, and Jaspen (2007), noted that participants performances on two different interventions (CCC and Explicit Timing) was, in part, a result of their initial level of subtraction fluency.

**Research question 2. Are there differential effects of three CCC conditions (CCC, CCC+ repetition, CCC+ generalization) on maintenance?** As can be seen
previously in Table 4, two participants (Adrianna and Jason) maintained the highest fluency in the CCC condition, two participants (Bailey and Terrell) maintained the highest fluency in the CCC+R condition, and two participants (Keith and Yvette) maintained the highest fluency in the CCC+G condition. One student, Jordan, tied his highest maintenance score in the CCC and CCC+R conditions.

Across participants, maintenance data revealed a decrease in performance two weeks following the conclusion of the intervention. Nevertheless, maintenance data for all participants was still above baseline levels and tended to be commensurate with the last intervention data point in each respective intervention condition. These results were commensurate with previous research indicating that the Cover-Copy-Compare intervention method produced lasting results (e.g., Skinner, Turco, Beatty, & Rasavage, 1989; Mong & Mong, 2010; Poncy, Skinner, & Jaspers, 2007; Cieslar, McLaughlin, & Derby, 2008). For example, Yvette’s final fluency score in the CCC condition was 15 and her maintenance follow-up score was 11. In the CCC+R condition, Yvette correctly solved 13 basic division facts in her final assessment, and maintenance data yielded 12 correct facts. In the CCC+G intervention condition, Yvette’s final intervention assessment score was 24, and her maintenance score for that condition was 18 correct facts.

Jason was the only participant to actually improve his correct division facts per minute as assessed on maintenance measures when compared to his fluency during the intervention. During intervention, Jason’s final facts correct per minute score in the CCC condition was 15. During maintenance he was able to calculate 17 facts correctly per
minute on facts from this condition. Jason’s final intervention score in the CCC+R condition was 8 correct facts, but on a maintenance measure he correctly calculated 9 facts per minute. Despite low attendance and missing data from the CCC+G intervention condition, Jason was able to maintain his rate of 13 correct facts per minute from intervention to maintenance.

When maintenance scores are compared across all participants, differential effects seemed to be dependent, to an extent, on the individual participant’s performance during the intervention. In other words, if the participant had the highest ending fluency score in the CCC condition, maintenance scores tended to be highest in this condition as well. If the participant had the highest ending fluency score in the CCC+G condition, maintenance scores tended to be highest in that condition.

**Research question 3. Are there differential effects of three CCC conditions (CCC, CCC+ repetition, CCC+ generalization) on generalization?** As with participants’ basic division fluency, this study was structured such that improvement would be observed if a participant’s performance on story problem generalization probes was higher in any intervention condition after the introduction of the intervention than it was during baseline.. It was predicted that following the implementation of the intervention, a participant’s generalization of basic division facts to story problems would improve.

Results of this study indicated that all three conditions improved students’ basic division generalization. All participants’ PND exceeded 70% across all three intervention conditions, with the exception of Bailey’s generalization across all three conditions (PND = 0%, 33%, 33%) and Yvette’s generalization in the CCC condition (PND = 40%).
Overall, PND suggests strong effects between all three variations of the Cover-Copy-Compare intervention method and participants’ generalization of basic division facts. Surprisingly, four out of seven students exhibited a PND equal to 100% across all three intervention conditions. One student, Terrell, had a PND equal to 100% only in the CCC+R condition.

As previously stated, PND scores tend to be lowered when an individual has a high baseline point. This was the case for Bailey across all three intervention conditions, and for Yvette in the CCC condition. Both girls had high baseline points in these conditions (near ceiling levels), making it difficult to show significant improvement from baseline as these students maintained their high level of performance for the duration of the study. Further, these scores seem to indicate that these two participants were already capable of generalizing basic division facts to story problems at the onset of the intervention.

For all remaining participants, high PND scores indicate an effective intervention leading students to generalize basic division facts to story problems. That being said, PND was significant for most participants across all three intervention conditions, making it difficult to determine if one intervention condition was more effective at teaching students to generalize than the other intervention conditions. In addition, when considering participants’ highest cumulative generalization totals, much variation existed. One student (Adriana) had the highest cumulative generalization score in the CCC condition, three students (Bailey, Keith, Yvette) had the highest cumulative generalization score in the CCC+G condition, and one student (Jordan) had the highest
cumulative generalization score in the CCC+R condition. Furthermore, Terrell’s highest cumulative generalization score was tied between the CCC+R and CCC+G conditions, and Jason had the same cumulative generalization score between the CCC and CCC+R conditions.

Clearly, no single intervention condition demonstrated the most effective generalization results. This lack of differentiated among intervention conditions, as it relates to generalization, may be due to spill-over effects. In other words, once a participant learned how to generalize basic division facts in any one intervention condition, s/he was also able to then transfer those skills to the other intervention conditions, resulting in improve generalization performance across all three conditions. Despite a lack of prior research evaluating the effects of programming for generalization in a Cover-Copy-Compare method, the improvement in generalization and transfer of skills is consistent with the benefits of programming for generalization during instruction according to Cooper, Heron, and Heward (2007).

**Research question 4.** *Do participants express a preference for one intervention treatment condition?* In order to answer this question, social validity questionnaires were administered to participants at the conclusion of the intervention. These questionnaires evaluated the usefulness and preferences of participants regarding various CCC intervention conditions. It was hypothesized that students would prefer the intervention condition in which their performance improved the most.

Following the intervention, 100% of participants indicated that division was easier when compared to the level of difficulty prior to the intervention. This is consistent
with participants’ PND across fluency and generalization scores and supports the effects of the intervention.

When asked about their level of enjoyment, six out of seven participants (86%) of participants indicated that they completely enjoyed the CCC intervention condition. The final participant indicated that s/he somewhat enjoyed the CCC condition. When participants were asked to their opinions on the usefulness the CCC condition, one participant (14%) somewhat disagreed that the CCC condition was helpful for learning basic division facts. The remaining six participants (86%) indicated that they completed agreed the CCC condition was helpful. As five out of seven participants had the highest cumulative fluency, or performance totals in the CCC condition, this response is not surprising. As participants made fluency gains within this condition, division facts may have become easier and it seems reasonable then that participants would enjoy this condition more and find it helpful.

As opposed to the CCC condition, only three participants (43%) indicated that they enjoyed the CCC+R condition (2 participants somewhat enjoyed; 1 participant completely enjoyed). The remaining four participants (57%) disagreed. One participant completely disagreed and three participants somewhat disagreed. As previously mentioned, these results were consistent with those obtained by Grafman and Cates (2010) in which participants reported disliking additional repetition of facts. Therefore, it is possible that students did not like having to repeat each division fact regardless of whether or not their initial response was correct. Participants may have found this repetition requirement aversive, leading to a dislike for this intervention condition, and
consequently, lower performance within this condition. Despite the general dislike for the CCC+R condition, only one participant (14%) somewhat disagreed that the CCC+R condition was helpful for learning basic division facts. The remaining six participants (86%) indicated that the CCC+R intervention condition was helpful for learning basic division facts. In other words, even though the participants may not have enjoyed the CCC+R condition, it is possible they recognized the value of the repetition.

All participants (100%) indicated that they enjoyed the CCC+G intervention condition. This supports the hypothesis of participant preference based upon results. As nearly all participants’ PND was above 70% for fluency and generalization within this condition, it is logical that participants would enjoy this condition given that they made significant gains. Likewise, all participants (100%) indicated that the CCC+G intervention condition was helpful when learning basic division facts. In other words, participants not only enjoyed the varied division fact formats, but found it aided their learning as well.

When asked if they would utilize a Cover-Copy-Compare method for other types of math facts, not simply division, all participants (100%) responded affirmatively. In fact, numerous participants reported using a Cover-Copy-Compare method in mathematics classes previously in their academic career. When asked if they would utilize a Cover-Copy-Compare method for other academic subjects in addition to math, once more, all participants (100%) responded affirmatively. Again, a few participants reported having previously used a Cover-Copy-Compare method for spelling.
Finally, participants were asked which intervention condition they preferred. Two participants (29%) most preferred the CCC condition. The remaining five participants (71%) most preferred the CCC+G intervention condition. This is important for educators to note, as not only is the Cover-Copy-Compare intervention method evidence-based, but in isolation or in different formats, students enjoy using this method.

**Research question 5. Do participants’ teachers find one intervention treatment condition to be more feasible to implement than the other treatment conditions?** It was predicted that participants’ teachers would find the CCC in isolation condition the most feasible to implement simply due to time constraints.

Commensurate with participant responses, all three teachers (100%) indicated that their students struggled with division prior to the intervention. Unlike participant responses, when asked if division was easier for their students following the intervention, one teacher somewhat disagreed (33%), one teacher somewhat agreed (33%), and one teacher did not feel adequately knowledgeable to answer. As responses were anonymous, it is impossible to know whether the teacher that agreed was the math teacher or not, making it difficult to interpret these results. Still, it is reasonable to assume that there was at least some transfer and generalization of basic division skills to other settings as at least one teacher saw improvement in the general education classroom setting.

Teachers were asked if they would enjoy teaching each individual intervention condition. All three teachers (100%) responded neutrally regarding the CCC intervention condition. Two teachers (66%) responded neutrally regarding the CCC+R condition, while one teacher (33%) somewhat agreed that she would enjoy teaching the CCC+R
intervention. The teachers responded in the same fashion to the CCC+G intervention condition. These results lend support to the CCC+G intervention condition, as both participants and at least one teacher stated that they would enjoy the CCC+G intervention condition.

When asked how beneficial they felt each intervention condition to be. All three teachers (100%) somewhat agreed that all three intervention conditions were helpful strategies for their students to learn basic division facts. Again, these responses were similar to the participants themselves lending support to the social validity of all three intervention conditions.

When asked to provide their opinions regarding the feasibility of implementing each intervention condition respectively, one teacher (33%) indicated that the CCC+R condition would be the most feasible to implement, one teacher (33%) indicated that either the CCC or CCC+R intervention conditions would be the most feasible to implement, and one teacher (33%) was unsure and felt she was unable to respond to the question. These responses partially supported the initial hypothesis as one teacher indicated that the CCC intervention condition may be the most feasible to implement. However, two teachers indicated that the CCC+R intervention condition may be the most feasible to implement. Although this was in contrast to the original hypothesis, it is possible that the teachers felt the extra repetition would be the best intervention condition for their students. Results are similar to those obtained by Grafman and Cates (2010) in which teachers, in contrast to participants, indicated that they would prefer using a
modified method with extra repetition of facts as opposed to a traditional Cover-Copy-Compare method.

**Obstacles and Limitations**

Threats to internal validity were minimal as results demonstrated effective intervention across students and across intervention conditions. Though results cannot be generalized to the larger population, they offer considerable support for the value of these intervention conditions. Results also provide direction for future research to implement these intervention conditions with larger samples, across settings and populations in order to provide additional support for these intervention conditions and established them as evidence-based practices.

A second limitation of this study is the potential ceiling effect in the generalization probes. The number of story problems generated on each generalization probe equaled the number of basic division facts targeted per intervention session. Consequently, the number of possible points earned on story problem generalization probes was capped at 16. Therefore, some students displayed low PND as their generalization baseline scores were high initially and they could not score more than 16 correct points on the generalization probes. In other cases, participants displayed PND signifying large effects; however, their progress reached a plateau when they reached the ceiling of 16 correct points. This made it difficult to determine any significant differential effects between the three intervention conditions as they related to generalization of basic division facts. Therefore, when designing materials, potential ceiling effects should be taken into consideration.
Another possible obstacle for this study was the CCC+R intervention condition. It was hypothesized that an extra repetition of each basic division fact would be the most beneficial for students as it was for students in a study conducted by Becker, McLaughlin, Weber, and Gower (2009). In actuality, students strongly disliked the additional repetition and this, in turn, may have affected participants’ performances within this intervention condition, as they generally performed better in the other two intervention conditions. In retrospect, it may not have been necessary for students to complete the additional repetition after an initially correct response. Interestingly, these results are similar to those obtained by Grafman and Cates (2010) in which students performed better on a traditional Cover-Copy-Compare intervention than on a modified condition in which students were required to copy each problem twice. Noticeably, Grafman and Cates (2010) also reported that students indicated they preferred the traditional method to the modified method. Again, this is consistent with results obtained in this study.

Scheduling and duration of the intervention was another limitation in this study. As the intervention was implemented within a public school setting, implementation of the intervention was dependent upon an allotted amount of time during the school day, week, and year provided by the school. For example, the school’s daily schedule only allowed for the intervention to be implemented for approximately 30 minutes in total, three times a week. Also, as the participants’ schedules were set to change upon returning from winter break, it was essential that the entire intervention be completed prior to winter break. Although scheduling conflicts are sometimes unavoidable when
implementing interventions within the school setting, this impacted the number of possible sessions, number of weeks in which to implement the intervention, and the number of maintenance probes that could be administered following the conclusion of the intervention.

Poor attendance was a significant obstacle to the intervention implementation. This was compounded by the limited amount of time available to implement the intervention. Consequently, when a student missed a baseline or intervention session, as many did, there was no time available to make up any missed sessions. This was the case for Adrianna, who missed sessions during baseline and Keith who missed an intervention session. However, attendance was a significant issue for Jason. Unfortunately, Jason missed the final two sessions within the CCC+G intervention condition. This made it nearly impossible to compare any effects across intervention conditions. It is difficult to determine if PND, cumulative performance data, and general effects would have been different had Jason completed the final two intervention sessions in the CCC+G condition. In addition, it is possible that Jason’s maintenance data may have been different had he completed the final two CCC+G intervention sessions.

**Future Research**

This study used three variations of the Cover-Copy-Compare method for teaching basic division facts. No other mathematical processes were incorporated, nor were more complex division facts. Future research may consider incorporating mixed methods, such as multiplication in order for students to more fully comprehend the necessary number
families and relationships among numbers when dividing. Moreover, incorporating more complex division problems may be appropriate for some participant demographics.

When considering the design of the present study, time and scheduling factors were a limitation. Although there was little differentiation in results across intervention conditions for participants, other researchers have successfully utilized an alternating treatments design to compare interventions to the Cover-Copy-Compare method (Mong & Mong, 2010). Future researchers may want to implement variations of the Cover-Copy-Compare method across a longer period of time. For example, instead of targeting division facts by 1’s and 2’s within the first week, future researchers may consider targeting division facts by one number per week. In other words, target division facts by 1’s the first week, division facts by 2’s the second week, division facts by 3’s the following week, and so on. It may also be possible to schedule targeted facts according to perceived level of difficulty. For example, division facts by 1’s may be the easiest, followed by division facts by 2’s, but then some students may find division facts by 5’s or 10’s easier than division facts by 3’s or 4’s for example. Future research may want to examine if the order in which division facts are targeted influences students fluency when learning basic division facts. Targeting only one type of division fact per week would effectively lengthen the duration of the intervention from six weeks as in the present study, to potentially 12 weeks. This extended time may be of benefit to some participants and allow researchers to address any attendance issues. Consequently, it is conceivable that researchers may find more differentiation between variations of the Cover-Copy-Compare method given longer study duration.
With regard to experimental design, future researchers may want to consider utilizing designs other than alternating treatments in order to further examine the effects of these intervention conditions. Typically, intervention conditions implemented in an alternating treatments design are substantially different from each other. This was not the case in the current study as all three intervention conditions were very similar to each other. This may have impacted results and future researchers should consider alternate experimental designs.

As previously discussed, one limitation of this study was the potential ceiling effects during the generalization story problem probes. Future researchers interested in programming for generalization may want to consider increasing the number of story problems available to students as they show growth and progress. This would avoid any potential plateau in generalization scores. Further, as this study only targeted basic division facts, story problems were also based around basic division facts. In the future, studies should be designed with more complicated story problems and facts in order to determine skill generalization to other types of problems.

When further examining generalization, it may also be helpful to examine the third type of generalization. This study was designed to measure two types of generalization: maintenance and stimulus generalization. Researchers may want to program for generalization within their intervention and examine whether students can achieve generalization in other settings outside of the intervention setting (Cooper, Heron, & Heward, 2007). Given time and scheduling limitations, this was not possible within the scope of this study. There is some likelihood that this occurred given the
teacher responses on the social validity questionnaire, but no data were collected in order to support teacher responses.

Maintenance measures in this study were only able to be collected once per intervention condition, two weeks following the conclusion of the intervention. It would be interesting to schedule additional follow-up maintenance probes in order to determine the length of time in which students maintain results from a Cover-Copy-Compare intervention. Researchers may gain beneficial information from conducting maintenance measures at two weeks, one month, or even six months following the conclusion of the intervention. Such follow-ups may show more differentiation between Cover-Copy-Compare intervention conditions.

Additionally, future research may want to further consider participants’ initial levels of fluency. Studies examining the effects of the various Cover-Copy-Compare intervention conditions based upon students’ initial math calculation fluency levels may be beneficial. It may be possible that students with higher initial math calculation fluency levels may benefit more from one intervention condition than others. For example, students with higher initial fluency baselines may benefit more from a CCC intervention condition or a CCC+G intervention condition as they already have the necessary prerequisite skills to complete these tasks more fluently than other students. Further, do students with lower initial math calculation scores make greater gains in a CCC+R intervention condition as they need more opportunities to practice? Is it possible that students with lower initial math calculation fluency scores make the least amount of gains in a CCC+G intervention condition as it requires more mental concentration on the
formats of the problems, thus impeding their fluency? As baseline measures within this study were limited, and some participants were absent, it is difficult to analyze this information in the current study. However, these are questions that future researchers should consider.

Finally, replication of this study, with more participants and diverse populations may be advantageous. This may be based upon the mathematical facts targeted and the introduction of said facts within the curriculum. For this specific study within this specific school, the target grade for introduction of basic division facts was fourth grade. This may vary across schools and parts of the country. Replicating this study with students of higher or lower age, grade and ability level will provide valuable information. Participants in this study were primarily of Caucasian or African-American descent. Recruiting participants of other, more diverse, background may yield different results. Moreover, participants in this study were general education students who were exhibiting documented math difficulties, specifically in division. Future studies may want to recruit participants with a variety of educational needs, diagnoses, and/or educational placements.

**Implications for Practice**

The results of this study provide many implications for practitioners in work with elementary students, specifically with regard to math. First, it supports previous research which concluded that the Cover-Copy-Compare intervention could be used to improve students’ math skills. This study demonstrates that elementary students can experience significant gains in basic division fluency in a relatively short amount of time when a
self-management intervention such as Cover-Copy-Compare is implemented. Basic division fluency is critical to the developmental of fundamental math skills required for higher order mathematical skills such as fractions and algebra (NCTM, 2000). Therefore, targeting this skill as early as possible when students show signs of difficulty, is vital.

More specifically, this study added to the literature base for Cover-Copy-Compare by showing that variations of this method are also effective in improve students’ basic division fact fluency. Variations with a required repetition component and varying formats of division facts were also shown to have a significant impact on student’s division fact fluency. Although additional research is needed in this area, practitioners can modify the Cover-Copy-Compare method as appropriate for individual students and still feel confident that students may make fluency gains.

Perhaps most significant are results from this study indicating that practitioners can program for generalization within a Cover-Copy-Compare intervention and participants can successfully generalize their skills to other response types, and maintain their high levels of response after the intervention is withdrawn. This is advantageous for practitioners as it creates efficient and long-lasting effects with minimal time and preparation required on the part of the practitioner.

Simple alterations could be made to this intervention that would make it suitable for use with larger groups, a whole class, individuals, and individuals with varied needs. After teaching students how to implement the Cover-Copy-Compare method, along with any variations to this method, students can effectively self-manage the intervention. Instructors would need to modify materials to target skills per individual or group of
students. Within a large group or whole class setting, it may be more difficult for an instructor to monitor students as they complete the necessary steps.

The inclusion of an additional reinforcement or motivational component could easily be utilized by an instructor. For example, instructors could provide students with individual graphs to chart their improvement, or a class-wide chart could be created for students who enjoy a more competitive intervention. The competition with oneself or others in itself may be motivating for some students. Other students may require additional motivation for “beating” their own scores or having the highest scores within the group.

This intervention is appropriate for classroom implementation as it is inexpensive, requires few materials, can be performed quickly, allows for simple progress monitoring, and allows students independence and responsibility within their own learning. Additionally, since students self-manage their own intervention, this allows practitioners to focus their planning time on instruction and other interventions. Because of the self-management, ease of implementation, immediate feedback, and potential variations within the intervention, this intervention can be enjoyable and engaging for students. It should be noted though, that despite demonstration of its efficacy at significantly improving students’ fluency, maintenance, and generalization of basic division facts, this intervention alone is not adequate for creating mathematical proficiency within students. Effective math instruction will incorporate many evidence-based practices.

The implementation of the Cover-Copy-Compare intervention method, or any variations of this method, in conjunction with other similar evidence-based interventions
with elementary students, is necessary for identifying and assisting students with math difficulties. Students who do not exhibit expected gains after the implementation of intervention may require more intensive intervention. It is critical to identify these students early on in order to provide extra support and address any potential deficits. If more intensive, individualized interventions also do not yield positive results, a team should consider whether or not the individual should be evaluated for a math disability such that intensive, specially designed instruction can be incorporated.

Conclusions

Despite the critical importance of mathematics skills across academic subjects and in daily life, there is a lack of research examining fluency, maintenance, and generalization of basic division skills. Coupled with the fact that there is a lack of consensus regarding math’s “big ideas”, and the need for effective, evidence-based interventions for elementary students struggling with division is great. This study was designed to evaluate the effectiveness of three variations of the Cover-Copy-Compare intervention on elementary students’ fluency, maintenance, and generalization of basic division facts. Elementary students’ responsiveness to all three intervention conditions not only demonstrates their ability to rapidly improve, maintain, and generalize basic division facts when provided intervention, but also reveals our responsibility to implement such interventions when students’ division skills are weak. Effective, self-management interventions in mathematical skills are particularly important for elementary students who are at-risk for mathematical difficulties.
References


Appendix A: Cover-Copy-Compare (CCC) Condition Intervention Script

Directions:
1. Participants should be seated in appropriate location(s).
2. Pass out all necessary materials (e.g., intervention sheets, pencils).
3. Introduce intervention to participants: “Today we are going to use a strategy to help you practice your division skills. I am going to show you a way to help you practice division that you could also use for other math problems or other subjects, such as spelling. It is called Cover-Copy-Compare. Cover-Copy-Compare has five steps. First, I will show you the five steps, then you will use the Cover-Copy-Compare sheet I have given you to do it on your own.” (This may be modified upon subsequent trials.)
4. Narrate the five steps [(a) look at the division fact with the answer, (b) cover the problem with hand or index card, (c) write the fact with answer, (d) uncover the problem and answer, (e) compare original problem to written problem] while demonstrating.
5. Explain to participants steps for correct (move to next problem) and incorrect (repeat the CCC steps) responses.
6. For initial introduction to intervention, guide participants through completion of first problem. (This step may be removed during subsequent trials.)
7. Instruct participants to complete CCC sheet independently.
8. Monitor participants and provide assistance when needed.
9. Collect CCC sheets upon completion.
10. Pass out division fact probe sheets and instruct participants not to begin until told to do so.
11. Tell participants they will have one minute to complete as many problems as possible, when the timer goes off they must put their pencils down and stop working, and that they must wait until told to start.
12. Set timer for one minute and instruct participants to, "Begin."
13. Continue timing for one minute.
14. When timer goes off, instruct participants to, “Stop. Put pencils down.”
15. Collect fluency probe worksheets.
17. Read story problems aloud to all students.
18. Monitor participants and provided assistance when needed.
19. Upon completion of the story problems, gather all participants’ worksheets, thank them for participation, and dismiss them.
Appendix B: Cover-Copy-Compare + Repetition (CCC+R) Condition Intervention Script

Directions:
1. Participants should be seated in appropriate location(s).
2. Pass out all necessary materials (e.g., intervention sheets, pencils).
3. Introduce intervention to participants: “Today we are going to again use the Cover-Copy-Compare strategy to help with your division facts. Remember, Cover-Copy-Compare has five steps.” (This may be modified upon subsequent trials.)
4. Narrate the five steps (a) look at the division fact with the answer, (b) cover the problem with hand or index card, (c) write the fact with answer, (d) uncover the problem and answer, (e) compare original problem to written problem.
5. Explain to students that today they will complete each fact twice for extra practice. In other words, students follow the five CCC steps twice for each individual problem prior to moving on to the next problem.
6. Instruct participants to complete CCC sheet independently.
7. Monitor participants and provide assistance when needed.
8. Collect CCC sheets upon completion.
9. Pass out division fact probe sheets and instruct participants not to begin until told to do so.
10. Tell participants they will have one minute to complete as many problems as possible, when the timer goes off they must put their pencils down and stop working, and that they must wait until told to start.
11. Set timer for one minute and instruct participants to, "Begin."
12. Continue timing for one minute.
13. When timer goes off, instruct participants to, “Stop. Put pencils down.”
15. Pass out CCC+R story problem generalization sheets to students.
16. Read story problems aloud to all students.
17. Monitor participants and provided assistance when needed.
18. Upon completion of the story problems, gather all participants’ worksheets, thank them for participation, and dismiss them.
Appendix C: Cover-Copy-Compare + Generalization (CCC+G) Condition Intervention Script

Directions:
1. Participants should be seated in appropriate location(s).
2. Pass out all necessary materials (e.g., intervention sheets, pencils).
3. Introduce intervention to participants: “Today we are going to again use the Cover-Copy-Compare strategy to help with your division facts. Remember, Cover-Copy-Compare has five steps.” (This may be modified upon subsequent trials.)
4. Narrate the five steps (a) look at the division fact with the answer, (b) cover the problem with hand or index card, (c) write the fact with answer, (d) uncover the problem and answer, (e) compare original problem to written problem.
5. Explain to participants steps for correct (move to next problem) and incorrect (repeat the CCC steps) responses.
6. Explain to students that today the division facts will look different. Instruct students to write the division facts the same way they are written on the sheet and follow the same CCC steps as usual.
7. Instruct participants to complete CCC sheet independently.
8. Monitor participants and provide assistance when needed.
9. Collect CCC sheets upon completion.
10. Pass out division fact probe sheets and instruct participants not to begin until told to do so.
11. Tell participants they will have one minute to complete as many problems as possible, when the timer goes off they must put their pencils down and stop working, and that they must wait until told to start.
12. Set timer for one minute and instruct participants to, "Begin."
13. Continue timing for one minute.
14. When timer goes off, instruct participants to, “Stop. Put pencils down.”
15. Collect fluency probe worksheets.
16. Pass out CCC+G story problem generalization sheets to students.
17. Read story problems aloud to all students.
18. Monitor participants and provided assistance when needed.
19. Upon completion of the story problems, gather all participants’ worksheets, thank them for participation, and dismiss them.
Appendix D: Procedural Integrity Checklist

Verify:
- Participants seated in appropriate location(s).
- Experimenter passed out all necessary materials (e.g., intervention sheets, pencils).
- Experimenter introduced intervention to participants (e.g., “Today we are going to again use the Cover-Copy-Compare strategy to help with your division facts. Remember, Cover-Copy-Compare has five steps.”)
- Experimenter narrated the five steps (a) look at the division fact with the answer, (b) cover the problem with hand or index card, (c) write the fact with answer, (d) uncover the problem and answer, (e) compare original problem to written problem.
- Experimenter explained to participants steps for correct (move to next problem) and incorrect (repeat the CCC steps) responses.
- Experimenter explained to students any differences in the current intervention conditions from previous intervention conditions (e.g., that today the division facts will be repeated, or will look different) and instructed students to write the division facts the same way they are written on the sheet and follow the same CCC steps as usual.
- Experimenter instructed participants to complete CCC sheet independently.
- Experimenter monitored participants and provided assistance when needed.
- Experimenter collected CCC sheets upon completion.
- Experimenter passed out division fact probe sheets and instructed participants not to begin until told to do so.
- Experimenter told participants they will have one minute to complete as many problems as possible, when the timer goes off they must put their pencils down and stop working, and that they must wait until told to start.
- Experimenter set timer for one minute and instructed participants to, "Begin.”
- Experimenter continued timing for one minute.
- When time expired, experimenter instructed participants to, “Stop. Put pencils down.”
- Experimenter collected one minute timed probe sheets.
- Experimenter passed out story problem generalization sheets to students.
- Experimenter read story problems aloud to all students.
- Experimenter monitored participants and provided assistance when needed.
- Upon completion of the story problems, experimenter gathered all participants’ worksheets, thanked them for participation, and dismissed them.
Appendix E: Social Validity Questionnaire for Students

Questionnaire for participants in the Cover-Copy-Compare and Division study.

Directions:
Please circle the answer the best represents your feelings about each question.

1. Disagree completely
2. Somewhat disagree
3. Neutral
4. Somewhat agree
5. Agree completely

1. Division was hard for me before starting the Cover-Copy-Compare strategy.
   1  2  3  4  5

2. Division is easier after using the Cover-Copy-Compare strategy.
   1  2  3  4  5

3. I enjoyed using the Cover-Copy-Compare strategy.
   1  2  3  4  5

4. I enjoyed the repetition part of the Cover-Copy-Compare strategy.
   1  2  3  4  5

5. I enjoyed having the division facts in different forms in the Cover-Copy-Compare strategy.
   1  2  3  4  5

6. The Cover-Copy-Compare strategy is helpful for learning division facts.
   1  2  3  4  5

7. The repetition of the Cover-Copy-Compare strategy is helpful for learning division facts.
   1  2  3  4  5

8. Having the division facts in different forms in the Cover-Copy-Compare strategy is helpful for learning division facts.
   1  2  3  4  5
9. I will use the Cover-Copy-Compare strategy for other math facts, not just division.

10. I will use the Cover-Copy-Compare strategy for other subjects, not just math.

11. The strategy I liked the best was:
   
a. Cover-Copy-Compare by itself
b. Cover-Copy-Compare and repeating problems
c. Cover-Copy-Compare with division written in four ways
Appendix F: Social Validity Questionnaire for Teachers

Questionnaire for teachers in the Cover-Copy-Compare and Division study.

Directions:
Please circle the answer the best represents your feelings about each question.

1. Disagree completely
2. Somewhat disagree
3. Neutral
4. Somewhat agree
5. Agree completely

1. My students struggled with division before starting the Cover-Copy-Compare strategy.
   1  2  3  4  5

2. Division is easier for my students after using the Cover-Copy-Compare strategy.
   1  2  3  4  5

3. I would enjoy teaching the Cover-Copy-Compare strategy.
   1  2  3  4  5

4. I would enjoy teaching the repetition part of the Cover-Copy-Compare strategy.
   1  2  3  4  5

5. I would enjoy teaching the division facts in different forms in the Cover-Copy-Compare strategy.
   1  2  3  4  5

6. The Cover-Copy-Compare strategy is beneficial for students learning division facts.
   1  2  3  4  5

7. The repetition of the Cover-Copy-Compare strategy is beneficial for students learning division facts.
   1  2  3  4  5

8. Having the division facts in different forms in the Cover-Copy-Compare strategy is beneficial for students learning division facts.
   1  2  3  4  5
9. I would teach the Cover-Copy-Compare strategy for other math facts, not just division.
   1  2  3  4  5

10. I would teach the Cover-Copy-Compare strategy for other subjects, not just math.
    1  2  3  4  5

11. Which strategy (CCC, CCC+ repetition, CCC+ generalization) do you think is the most feasible for teachers to implement?

12. How likely are you to implement the strategy you chose in Question 11 in the future?

   1. Not likely at all
   2. Somewhat likely
   3. Definitely likely
Appendix G: Verbal Assent Script for Children to Participate in Research

Study Title: The Differential Effects of Three Variations of Cover-Copy-Compare on Fluency, Generalization, and Maintenance of Basic Division

Researchers: Laurice M. Joseph, Ph.D. & Rachel L. Lee, M.A.

Researcher: “You are being asked to participate in a research study. First, I’m going to tell you more information about the study to help you decide whether or not you want to participate. You can ask any questions you have before making up your mind. You can also think about it, and discuss it with your family or friends before you decide. It’s completely okay to say ‘No’ if you don’t want to be in the study. If you say, ‘Yes’, you can change your mind and quit being in the study at any time without getting into trouble. If you decide that you want to be in the study, an adult (your parent or guardian) will also have to give their permission for you to be in the study. All of your information will be kept confidential and your name will not be used on any documents associated with this study.

This study wants to see the best way to help students learn to calculate division problems quickly and learn how to solve division words problems. If you are in the study, the first thing that will happen is I’ll give you a math assessment to see how much math you already know. This will not be graded and will not be calculated or figured into your math classroom grade. Then you will learn how to look at division facts, cover them, write them, and compare your answers. You will also learn how to write your answers correctly and quickly, as well as practice word problems with division. You will be in the study for about six weeks and we will work together three times a week for no more than 20 minutes. About two weeks after we finish the study, I will follow-up with you. This means that you’ll complete some more random division facts that you’ve already learned. It will take about 5 minutes, and you’ll only complete this follow-up once. You can stop being in the study at any time. No bad things will happen to you if you are in the study. The study will not be much different from what you experience in the classroom. I will work with you so that you hopefully will not be bored or frustrated.
Good things that might happen to you if you are in the study are, learning division facts, being able to do them quickly, understanding word problems, and hopefully making math easier for you. However, you will not be given anything for being in the study.

If you ever have any questions about the study, you can contact me, Rachel Lee. You can call me at 614-288-1144 and your parents or guardians will be given my contact information as well.”
Appendix H: Parental Permission

The Ohio State University Parental Permission
For Child’s Participation in Research

The Differential Effects of Three Variations of Cover-Copy-
Study Title: Compare on Fluency, Generalization, and Maintenance of Basic
Division

Researcher: Laurice M. Joseph, PhD & Rachel L. Lee, MA

Sponsor: n/a

This is a parental permission form for research participation. It contains important
information about this study and what to expect if you permit your child to participate.

Your child’s participation is voluntary.
Please consider the information carefully. Feel free to discuss the study with your friends
and family and to ask questions before making your decision whether or not to permit
your child to participate. If you permit your child to participate, you will be asked to sign
this form and will receive a copy of the form.

Purpose:
The purpose of this study is to compare the effects of three different types of an
intervention on students’ division fact fluency: Cover-Copy-Compare, Cover-Copy-Compare
with repetition, and Cover-Copy-Compare with generalization. The study will help educators
determine the best way to help students learn to calculate division facts accurately and fluently, as well as to generalize their skills to multiple formats.
Procedures/Tasks:
Prior to any intervention, children will be administered math portions of a standardized academic achievement assessment in order to determine eligibility and pre-intervention math knowledge. This assessment will not be graded and will not factor into your child’s math classroom grade in any way. A graduate student from The Ohio State University will then work with each eligible child for a total of 18 sessions. Children will be presented with 8 division facts each session and will be asked to look at the problem and answer, cover it, write it from memory, uncover the original and compare their problem to the original problem. In the first condition, children will follow this procedure for each problem. If correct, they will move to the next problem. If incorrect, they will repeat the problem and procedures before moving to the next problem. In the second condition, the process will be the same except that children will repeat each problem regardless of whether they were correct or incorrect on the first attempt. The third condition will also be identical to the first condition, but in this condition the format of the problems will vary (e.g., division box, fraction, division symbol (obelus), or words.) In all conditions, students will then be asked to complete a word story using the division facts, and every other week will be asked to complete a one-minute time fluency worksheet. Upon completion of the study, your child will also be asked to participate in a short follow-up session approximately two weeks after the study. During this session, your child will be asked to complete random division facts which were previously learned in the study.

Duration:
Each child will participate in intervention sessions lasting no more than 20 minutes, three times a week over approximately 6 weeks, with a total time commitment for participants of 360 minutes. A one-time follow-up session will also be completed approximately two weeks after completion of the study. This session should take no more than 5 minutes.

Your child may leave the study at any time. If you or your child decides to stop participation in the study, there will be no penalty and neither you nor your child will lose any benefits to which you are otherwise entitled. Your decision will not affect your future relationship with The Ohio State University.

Risks and Benefits:
There are no risks associated with this study. Any challenges presented to the children will be typical of their experience in the classroom. Possible benefits include enhanced ability to fluently calculate division facts and apply them to multiple formats at sufficient levels, thus increasing their chance to be successful in future academic endeavors.

Confidentiality:
Efforts will be made to keep your child’s study-related information confidential. However, there may be circumstances where this information must be released. For example, personal information regarding your child’s participation in this study may be disclosed if required by state law. Also, your child’s records may be reviewed by the following groups (as applicable to the research):

- Office for Human Research Protections or other federal, state, or international regulatory agencies;
- The Ohio State University Institutional Review Board or Office of Responsible Research Practices;
- The sponsor, if any, or agency (including the Food and Drug Administration for FDA-regulated research) supporting the study.

**Incentives:**

There are no incentives for participating in this study.

**Participant Rights:**

You or your child may refuse to participate in this study without penalty or loss of benefits to which you are otherwise entitled. If you or your child is a student or employee at Ohio State, your decision will not affect your grades or employment status.

If you and your child choose to participate in the study, you may discontinue participation at any time without penalty or loss of benefits. By signing this form, you do not give up any personal legal rights your child may have as a participant in this study.

An Institutional Review Board responsible for human subjects research at The Ohio State University reviewed this research project and found it to be acceptable, according to applicable state and federal regulations and University policies designed to protect the rights and welfare of participants in research.

**Contacts and Questions:**

For questions, concerns, or complaints about the study you may contact Dr. Laurice Joseph at 614-688-4992.

For questions about your child’s rights as a participant in this study or to discuss other study-related concerns or complaints with someone who is not part of the research team, you may contact Ms. Sandra Meadows in the Office of Responsible Research Practices at 1-800-678-6251.
If your child is injured as a result of participating in this study or for questions about a study-related injury, you may contact Dr. Laurice Joseph at 614-688-4992.

**Signing the parental permission form**

I have read (or someone has read to me) this form and I am aware that I am being asked to provide permission for my child to participate in a research study. I have had the opportunity to ask questions and have had them answered to my satisfaction. I voluntarily agree to permit my child to participate in this study.

I am not giving up any legal rights by signing this form. I will be given a copy of this form.

---

Printed name of subject

Printed name of person authorized to provide permission for subject

Signature of person authorized to provide permission for subject

AM/PM

Relationship to the subject

Date and time
**Investigator/Research Staff**

I have explained the research to the participant or his/her representative before requesting the signature(s) above. There are no blanks in this document. A copy of this form has been given to the participant or his/her representative.

________________________________________  ______________________________________
Printed name of person obtaining consent  Signature of person obtaining consent

AM/PM

________________________________________
Date and time
Appendix I: Sample Fluency Probe

CCC

Week 2
Date:

<p>| | | | | |</p>
<table>
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<th></th>
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</thead>
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<td>24</td>
<td>4</td>
<td>32</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>3</td>
<td>21</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>4</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td>2</td>
<td>22</td>
<td>4</td>
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<td>4</td>
<td>40</td>
<td>3</td>
<td>30</td>
<td>4</td>
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<tr>
<td>1</td>
<td>6</td>
<td>3</td>
<td>24</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>4</td>
<td>32</td>
<td>3</td>
</tr>
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</table>
## Appendix J: Sample Generalization Probe

**CCC+Generalization**  
**Generalization Probe**  
**Week 5**  
**Date:**

<table>
<thead>
<tr>
<th>READ</th>
<th>WRITE FACT</th>
<th>POINTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dontay had ten different paint colors. He divided the paint colors by ten different art canvases. How many colors did each canvas get?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ciara spent 45 minutes writing nine pages for an essay. If she spent the same amount of time on each page, how many minutes did she spend writing each page?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Devonte saved $20. He wanted to buy a shirt that cost $10. How many shirts could Devonte buy?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kelly split 18 pieces of pizza evenly among nine girls. How many pieces of pizza did each girl get?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elijah wanted to save up $80 to buy video games. He earns a weekly allowance of $10. If he saves all of his allowance each week, how many weeks will it take him to reach $80?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eric counted a total of 70 toothpicks. If the toothpicks were in bundles of</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
How many bundles were there?

Adrianna divided 36 cards into nine groups. How many cards were in each group?

Camille made 81 muffins. She sold the same number of muffins to nine different people. How many muffins did each person buy?
Appendix K: Sample Maintenance Probe

**CCC+G**

**Maintenance**

**Date:**

<table>
<thead>
<tr>
<th>11 ÷ 1 =</th>
<th>4 ÷ 4 =</th>
<th>42 ÷ 6 =</th>
<th>60/5 =</th>
<th>12 divided by 6=</th>
</tr>
</thead>
<tbody>
<tr>
<td>63 divided by 7 =</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>45/9 =</td>
<td></td>
<td></td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>96 ÷ 8 =</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 ÷ 10 =</td>
<td>132/12 =</td>
<td>33 divided by 3 =</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18/9 =</td>
<td>8 divided by 8 =</td>
<td>48 divided by 4 =</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>10/10 =</td>
<td>20 divided by 4 =</td>
<td>30/6 =</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21 divided by 7 =</td>
<td>88 ÷ 11 =</td>
<td>36 ÷ 9 =</td>
<td>18/2 =</td>
<td>48 divided by 8=</td>
</tr>
<tr>
<td>80 ÷ 10 =</td>
<td>108/12 =</td>
<td>84 ÷ 12 =</td>
<td>12 ÷ 1 =</td>
<td>33 divided by 11=</td>
</tr>
<tr>
<td>20/2 =</td>
<td>15 divided by 3 =</td>
<td>44 divided by 4 =</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3/3 =</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix L: Counterbalanced Intervention Schedule

A = CCC
B = CCC+R
C = CCC+G

Schedule:

Week 1: B, A, C
Week 2: A, B, C
Week 3: C, B, A
Week 4: A, C, B
Week 5: B, C, A
Week 6: C, A, B
Appendix M: Weekly Division Fact Schedule

Weekly Targeted Division Facts

Week 1: Division facts 1-12 divided by 1’s and 2’s.
Week 2: Division facts 1-12 divided by 3’s and 4’s.
Week 3: Division facts 1-12 divided by 5’s and 6’s.
Week 4: Division facts 1-12 divided by 7’s and 8’s.
Week 5: Division facts 1-12 divided by 9’s and 10’s.
Week 6: Division facts 1-12 divided by 11’s and 12’s.
### Set A-CCC

**Week 1**

**Date:**

<table>
<thead>
<tr>
<th>COVER</th>
<th>COPY</th>
<th>COMPARE</th>
<th>REPEAT?</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1)10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1)1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1)6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1)8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2)22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
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<td>2)14</td>
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<td></td>
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<td>3</td>
<td></td>
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<td>2)5</td>
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<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2)2</td>
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Set B-CCC+Repetition
Week 5
Date:

<table>
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<tr>
<td>10</td>
<td>9</td>
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<td>9</td>
<td>108</td>
<td>□</td>
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<tr>
<td>3</td>
<td>9</td>
<td>27</td>
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</tr>
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<td>11</td>
<td>9</td>
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<td>□</td>
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<td>720</td>
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<td>10</td>
<td>30</td>
<td>□</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>710</td>
<td>□</td>
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</table>
Appendix P: Sample CCC+G Worksheet

Set C-CCC+Generalization
Week 1
Date:

<table>
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<tr>
<th>COVER</th>
<th>COPY</th>
<th>COMPARE</th>
<th>REPEAT?</th>
</tr>
</thead>
<tbody>
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<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>1)11</td>
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<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 = 3</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 ÷ 1 = 7</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>12 divided by 1 =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2)16</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>2 = 9</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 ÷ 2 = 6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 divided by 2 =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
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