A STUDY OF QUASI-STATIC AND DYNAMIC BEHAVIOR OF DOUBLE-HELICAL GEARS

DISSERTATION

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By

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ABSTRACT

The quasi-static and dynamic behaviors of a double-helical gear pair are investigated both experimentally and theoretically. The impact of the key design and manufacturing parameters associated with double-helical gears, including nominal right-to-left stagger angle, the stagger angle deviation (error) from the nominal stagger angle, and axial gear supporting conditions, is the main focus of this study.

On the experimental side, a state-of-the-art experimental double-helical test set-up consisting of a test machine, test specimens, and various measurement systems is developed for operating a double-helical gear pair under realistic torques and speed ranges. A test gear pair formed by novel three-piece double-helical gears is fabricated to allow adjustable (i) right-to-left stagger angles, (ii) intentional stagger errors, and (iii) axial support conditions. Separate measurement systems are developed and implemented simultaneously to measure three-dimensional vibratory motions and dynamic transmission error of the gears under high-speed conditions, the static transmission error and axial motions of the gears under low-speed conditions, and gear root strains to determine right-to-left load sharing and dynamic stress factors. Test matrices that included a number of tests with various combinations of key system parameters (various stagger angles, intentional stagger errors, and axial support conditions) under realistic torque values within a wide speeds range are implemented to establish an extensive database.

On the modeling side, first the measured quasi-static behavior of double-helical gear pairs is simulated by using an existing quasi-static double-helical load distribution model. Direct
comparison of the measurements and predictions of loaded static transmission error, axial play, root stresses and right-to-left load sharing factors are used to validate the quasi-static model. For the simulation of the dynamics experiments a new linear time-invariant dynamic model of a double-helical system is developed. This three-dimensional model includes gear mesh compliances as well as flexibilities of the shafts and their bearing, and allows any stagger angle between the right and left sides of a double-helical gear pair through proper definition of the phasing between the two gear mesh excitations. The predictions of the model to the measurements indicate that the model is accurate in predicting three-dimensional gear motions and dynamic transmission error.

Main contribution of this study on the experimental front is the proposed methodology to investigate the double-helical gear pair behavior. This experimental study sets itself apart from earlier gear experiments in terms of its abilities in introducing all key gear parameters and errors in a user-defined manner and measuring all relevant performance parameters simultaneously under realistic operating conditions. The database that resulted from this study is the first extensive of its kind in terms of simultaneous measurements of quasi-static and dynamic motions and stresses. The qualitative matching of the sensitivities of the static and dynamic transmission errors to various design parameters and gear errors as well as the overall shapes of the measured forced response curves establish that dynamic behavior of a double-helical gear set can be characterized as a linear, time-invariant one. Other main contribution on the modeling side is the first validated three-dimensional dynamic model of a double-helical gear pair as established through direct comparisons to experiments.
DEDICATION

This dissertation is dedicated to my dear family.
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TABLE OF CONTENTS

ABSTRACT .................................................................................................................. ii

DEDICATION .............................................................................................................. iv

ACKNOWLEDGMENTS ............................................................................................. v

VITA .......................................................................................................................... vi

PUBLICATION .......................................................................................................... vi

FIELDS OF STUDY .................................................................................................. vi

TABLE OF CONTENTS ............................................................................................... vii

LIST OF TABLES ....................................................................................................... xi

LIST OF FIGURES .................................................................................................... xii

NOMENCLATURE .................................................................................................... xxi

CHAPTER 1: INTRODUCTION .................................................................................. 1

1.1 Background and Motivation .................................................................................. 1

1.2 Literature Search ................................................................................................ 4

1.2.1 Double Helical Gears .................................................................................... 4

1.2.2 Measurement of Gear Motions ..................................................................... 7

1.3 Dissertation Objectives ....................................................................................... 8
1.4 Dissertation Outline ........................................................................................................... 10

References for Chapter 1 ........................................................................................................ 12

CHAPTER 2: EXPERIMENTAL METHODOLOGY ................................................................. 18

2.1 Introduction ...................................................................................................................... 18

2.2 Test Machine .................................................................................................................. 19

2.3 Test Gears and Test Shafts ............................................................................................ 27

2.3.1 Method of Setting up a Desired Stagger Angle ........................................................... 27

2.3.2 Application of Various Axial Support Conditions ....................................................... 32

2.3.3 Application of Intentional Stagger Errors ................................................................... 37

2.4 Measurement Systems, Instrumentation, and Data Acquisition ...................................... 38

2.4.1 Static Transmission Error Measurement System ....................................................... 38

2.4.2 Axial Motion Measurement System ........................................................................... 45

2.4.3 Dynamic Transmission Error and 3D Gear Motion Measurement System ............... 48

2.4.4 Root Stress Measurement System ............................................................................. 56

References for Chapter 2 ........................................................................................................ 62

CHAPTER 3: QUASI-STATIC BEHAVIOR OF A DOUBLE HELICAL GEAR PAIR ............... 64

3.1 Introduction ...................................................................................................................... 64

3.2 Quasi-static DH Gear Pair Measurements ..................................................................... 65

3.2.1 Static Transmission Error Measurement ................................................................. 65
3.2.2 Axial Motion Measurement ......................................................... 74

3.2.3 Root Stress Measurements and Right-to-Left Load Sharing............... 78

3.3 Simulation of Quasi-static Double-Helical Gear Pair Experiments and Model Validation
.................................................................................................................. 90

3.3.1 Simulation of the Static Transmission Error Measurements ................. 93

3.3.2 Simulation of the Axial Motion Measurements ........................................ 97

3.3.3 Simulation of the Root Stress and Load Sharing Measurements ............ 106

3.4 Summary ............................................................................................. 119

References for Chapter 3 ........................................................................ 119

CHAPTER 4: DYNAMIC BEHAVIOR OF A DOUBLE-HELICAL GEAR PAIR ........ 121

4.1 Introduction .......................................................................................... 121

4.2 Experimental Results ........................................................................... 121

4.2.1 Measured Dynamic TE and 3D Gear Vibration Results ......................... 122

4.2.2 Measured Dynamic Stress Factors ....................................................... 139

4.3 A Dynamic Model of a Double-Helical Gear-Shaft-Bearing System ......... 144

4.3.1 Model Assumptions ........................................................................... 144

4.3.2 Single Gear Pair Model ..................................................................... 149

4.3.3 DH Gear Pair Sub-System Model ......................................................... 154

4.3.4 Formulation of the Shafts and Bearings Supports ............................... 157

4.3.5 Overall System Equations .................................................................. 160
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 2.1</td>
<td>Basic design parameters of the DH gear pair used in this study</td>
<td>29</td>
</tr>
<tr>
<td>Table 4.1</td>
<td>Harmonic amplitudes and phase angles of the transmission error excitation of the tested gear pair of Table 2.1</td>
<td>166</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1.1</td>
<td>(a) An example double helical gear pair [1.21] and (b) a double-helical planetary gear set from a jet engine turbofan application [1.22].</td>
</tr>
<tr>
<td>Figure 1.2</td>
<td>Schematics of (a) in-phase and (b) out-of-phase DH stagger configurations.</td>
</tr>
<tr>
<td>Figure 2.1</td>
<td>The gear dynamics test machine used in Refs. [2.1-2.7].</td>
</tr>
<tr>
<td>Figure 2.2</td>
<td>Double-helical gear test machine used in this study.</td>
</tr>
<tr>
<td>Figure 2.3</td>
<td>Top view solid model layout of the DH test machine.</td>
</tr>
<tr>
<td>Figure 2.4</td>
<td>Views of the DH test machine (a) with and (b) without the safety guards.</td>
</tr>
<tr>
<td>Figure 2.5</td>
<td>Solid models depicting (a) a DH test gear pair and (b) a test shaft.</td>
</tr>
<tr>
<td>Figure 2.6</td>
<td>An image of the DH gear pair-shaft assembly.</td>
</tr>
<tr>
<td>Figure 2.7</td>
<td>Cross-sectional view of the test gear-shaft assembly.</td>
</tr>
<tr>
<td>Figure 2.8</td>
<td>Illustration of the right-to-left stagger conditions in a double helical gear pair; (a) $\gamma_s = 0$, (b) $\gamma_s = \pi/4$, (c) $\gamma_s = \pi/2$, (d) $\gamma_s = 3\pi/4$, and (e) $\gamma_s = \pi$.</td>
</tr>
<tr>
<td>Figure 2.9</td>
<td>Oblique views of the gear pair at five different stagger angles.</td>
</tr>
<tr>
<td>Figure 2.10</td>
<td>(a) A schematic showing a set-up with aligned 34T and 31T gears with a zero axial shift, and (b) depiction of the resultant zero stagger error condition.</td>
</tr>
<tr>
<td>Figure 2.11</td>
<td>(a) A schematic showing a set-up with aligned 34T and 31T gears with a positive axial shift, and (b) depiction of the resultant positive stagger error condition.</td>
</tr>
<tr>
<td>Figure 2.12</td>
<td>(a) A schematic showing a set-up with aligned 34T and 31T gears with a negative axial shift, and (b) depiction of the resultant negative stagger error condition.</td>
</tr>
</tbody>
</table>
Figure 2.13  A schematic of a gear pair showing the base and pitch circles and the line of action.................................................................................................................................................. 43

Figure 2.14  An image of the static transmission error measurement system using optical encoders .................................................................................................................................................................. 43

Figure 2.15  Flowchart of the static transmission error measurement system instrumentation and data acquisition .................................................................................................................................................................. 44

Figure 2.16  (a,b) Solid model image showing the proximity probe placement for axial motion study and (c) an actual image of the axial motion measurement system .............. 46

Figure 2.17  Flowchart of the axial motion measurement system instrumentation and data acquisition .................................................................................................................................................................. 47

Figure 2.18  (a) A view of Solid model depicting the tri-axial accelerometers and their adapters for the instrumentation, and (b) the actual arrangement ......................... 49

Figure 2.19  A schematic showing the coordinates of the gears, and the locations and orientations of the accelerometers [2.11] .................................................................................................................................................................. 51

Figure 2.20  Flowchart of the 3D motion and DTE measurement system instrumentation and data acquisition .................................................................................................................................................................. 55

Figure 2.21  A schematic showing locations of the strain gauges on a tooth..................... 57

Figure 2.22  (a) An illustration depicting the array of strain gauges and their labeling scheme and (b) the installed strain gauges mounted on the roots of the actual right gear mesh of the 34-tooth gear .................................................................................................................................................................................................. 58

Figure 2.23  (a) An illustration depicting the array of strain gauges and their labeling scheme and (b) the installed strain gauges mounted on the roots of the actual left gear mesh of the 34-tooth gear .................................................................................................................................................................................................. 59

Figure 2.24  Flowchart of the root stress measurement system instrumentation and data acquisition .................................................................................................................................................................................................. 61
Figure 3.1  (a) An example of a measured time history of the transmission error and (b) the corresponding Fourier spectrum. $\Omega_p = 100$ rpm, $T_p = 400$ Nm and $\gamma_s = 0$. ..... 66

Figure 3.2  (a) High-pass filtered measured transmission error and (b) the corresponding Fourier spectrum. $\Omega_p = 100$ rpm, $T_p = 400$ Nm and $\gamma_s = 0$, and filter cut-off frequency of 10 Hz ................................................................. 68

Figure 3.3  High-pass filtered TE time histories for stagger angles of (a) $\gamma_s = 0$, (b) $\gamma_s = \pi/4$, (c) $\gamma_s = \pi/2$, (d) $\gamma_s = 3\pi/4$, and (e) $\gamma_s = \pi$. $\Omega_p = 100$ rpm, and $T_p = 400$ Nm ................................................................. 69

Figure 3.4  Variation of (a) the fundamental (first) and (b) second harmonic amplitudes of measured transmission error at different torque levels. $\Omega_p = 100$ rpm .......... 72

Figure 3.5  Three-dimensional representations of the data of Figure 3.4 showing combined influence of torque and stagger angle on (a) fundamental and (b) second harmonic amplitudes of the measured transmission error. $\Omega_p = 100$ rpm ...... 73

Figure 3.6  A measured tooth surface topographies of (a) the left-hand and (b) right-hand flanks of the 34-tooth test gear ................................................................. 75

Figure 3.7  A measured tooth surface topographies of (a) the right-hand and (b) left-hand flanks of the 31-tooth test gear ................................................................. 76

Figure 3.8  Measured axial motion time histories of the 34-tooth gear for (a) $\gamma_s = 0$, (b) $\gamma_s = \pi/4$, (c) $\gamma_s = \pi/2$, (d) $\gamma_s = 3\pi/4$ and (e) $\gamma_s = \pi$. $T_p = 100$ Nm, and $\Omega_p = 100$ rpm ................................................................. 77

Figure 3.9  Measured axial motion time histories of the 34-tooth gear for $\gamma_s = \pi$ at (a) $T_p = 100$ Nm, (b) 200 Nm, (c) 300 Nm, and (d) 400 Nm ................................................................. 79
Figure 3.10  Measured root time histories for the gear pair having $\gamma_s = 0$ and axially floating gear at $T_p = 400$ Nm.  (a) Left deck gauges L7, L8 and L9, and (b) right deck gauges R7, R8 and R9.......................... 81

Figure 3.11  Measured root stress root time histories for the gear pair having $\gamma_s = \pi/2$ and axially floating gear at $T_p = 400$ Nm (a) Left deck gauges L7, L8 and L9, and (b) right deck gauges R7, R8 and R9 ........................................ 82

Figure 3.12  Measured root stress time histories for the gear pair having $\gamma_s = \pi$ and axially floating gear at $T_p = 400$ Nm.  (a) Left deck gauges L7, L8 and L9, and (b) right deck gauges R7, R8 and R9.......................... 83

Figure 3.13  Comparison of measured root stresses from gauges R8 and L8 at $T_p = 400$ Nm for $\epsilon_s = 0$.  (a) $\gamma_s = 0$ and (b) $\gamma_s = \pi$ ................................................................. 85

Figure 3.14  Comparison of measured root stresses from gauges R8 and L8 at $T_p = 400$ Nm for $\gamma_s = \pi$.  (a) $\epsilon_s = -72$ $\mu$m and (b) $\epsilon_s = 72$ $\mu$m........................................ 86

Figure 3.15  Comparison of maximum root stress values from the respective right and left deck gauges for $T_p = 400$ Nm and $\gamma_s = \pi$ under the axially fixed conditions.  (a) Gauges R7 and L7, (b) gauges R8 and L8, and (c) gauges R9 and L9........... 88

Figure 3.16  Comparison of maximum root stress values from the respective right and left deck gauges for $T_p = 200$ Nm and $\gamma_s = \pi$ under the axially fixed conditions.  (a) Gauges R7 and L7, (b) gauges R8 and L8, and (c) gauges R9 and L9........... 91

Figure 3.17  Comparisons of the measured and predicted fundamental harmonic amplitudes of TE presented as a function of the stagger angle $\gamma_s$ at (a) $T_p = 100$ Nm, (b) 200 Nm, (c) 300 Nm, (d) 400 Nm, and (e) 500 Nm .................................................. 94
Figure 3.18 Comparisons of the measured and predicted second harmonic amplitudes of TE presented as a function of the stagger angle $\gamma_s$ at (a) $T_p = 100$ Nm, (b) $200$ Nm, (c) $300$ Nm, (d) $400$ Nm, and (e) $500$ Nm ................................................................. 98

Figure 3.19 Three-dimensional representations of the predictions of Figures 3.17 and 3.18 showing combined influence of torque and stagger angle on (a) fundamental and (b) second harmonic amplitudes of the measured transmission error.............. 101

Figure 3.20 The difference between measured and predicted (a) fundamental harmonic amplitudes and (b) second harmonic amplitudes of TE as a function of $T_p$ and $\gamma_s$ ................................................................. 102

Figure 3.21 Comparisons of the measured and predicted axial motions for (a) $\gamma_s = 0$, (b) $\pi / 4$, (c) $\pi / 2$, (d) $3\pi / 4$, and (e) $\pi$. $T_p = 100$ Nm................................. 103

Figure 3.22 Predicted root stress distributions of left deck of the 34-tooth axially floating gear $\gamma_s = \pi$ and $T_p = 400$ Nm. (a) The 3D plot, and (b) the contour plot with nominal gage locations marked by squares ...................................................... 107

Figure 3.23 Predicted root stress distributions of right deck of the 34-tooth axially floating gear $\gamma_s = \pi$ and $T_p = 400$ Nm. (a) The 3D plot, and (b) the contour plot with nominal gage locations marked by squares ...................................................... 108

Figure 3.24 Measured versus predicted root stress time histories at $T_p = 400$ Nm for the axially floating gear pair having $\gamma_s = \pi$. (a) Gauge L7, (b) gauge L8, and (c) gauge L9 ........................................................................................................ 110

Figure 3.25 Measured versus predicted root stress time histories at $T_p = 400$ Nm for the axially floating gear pair having $\gamma_s = \pi$. (a) Gauge R7, (b) gauge R8, and (c) gauge R9 ........................................................................................................ 112
Figure 3.26 Measured versus predicted root stress time histories at $T_p = 400$ Nm for the axially fixed gear pair having $\gamma_s = \pi$ and $\varepsilon_s = 0$ $\mu$m. (a) Gauge L7, (b) gauge L8, and (c) gauge L9 ................................................................. 114

Figure 3.27 Measured versus predicted root stress time histories at $T_p = 400$ Nm for the axially fixed gear pair having $\gamma_s = \pi$ and $\varepsilon_s = 0$ $\mu$m. (a) Gauge R7, (b) gauge R8, and (c) gauge R9 ................................................................. 116

Figure 3.28 Comparison of measured and predicted maximum root stresses measured by gauges L8 and R8 as a function of stagger error. $T_p = 400$ Nm and an axially fixed gear pair having $\gamma_s = \pi$ ................................................................. 118

Figure 4.1 Example frequency spectra of $\delta_d$ measured at
(a) $f_m = 567$ Hz, (b) 1133 Hz, (c) 1700 Hz, (d) 2267 Hz, (e) 2833 Hz, and (f) 3400 Hz. $\gamma_s = 0$ and $T_p = 400$ Nm .................................................................................. 124

Figure 4.2 Measured root-mean-square values of the dynamic transmission error $\hat{\delta}_d$ as a function of the gear mesh frequency $f_m$ at various stagger angle $\gamma_s$ values. $T_p = 400$ Nm .................................................................................. 125

Figure 4.3 Example frequency spectra of $z_p$ measured at
(a) $f_m = 567$ Hz, (b) 1133 Hz, (c) 1700 Hz, (d) 2267 Hz, (e) 2833 Hz, and (f) 3400 Hz. $\gamma_s = \pi$ and $T_p = 400$ Nm .................................................................................. 128

Figure 4.4 Measured root-mean-square values of axial vibration of (a) the pinion and (b) the gear as a function of the gear mesh frequency $f_m$ at various stagger angle $\gamma_s$ values. $T_p = 400$ Nm .................................................................................. 129
Figure 4.5  Measured root-mean-square values of rocking vibration of (a) the pinion and (b) the gear as a function of the gear mesh frequency $f_m$ at various stagger angle $\gamma_s$ values. $T_p = 400$ Nm.................................

Figure 4.6  Measured root-mean-square values of line of action translational vibration of (a) the pinion and (b) the gear as a function of the gear mesh frequency $f_m$ at various stagger angle $\gamma_s$ values. $T_p = 400$ Nm.................................

Figure 4.7  Measured root-mean-square values of off line of action translational vibration of (a) the pinion and (b) the gear as a function of the gear mesh frequency $f_m$ at various stagger angle $\gamma_s$ values. $T_p = 400$ Nm.................................

Figure 4.8  Comparison of measured (a) $\dot{\delta}_d$ , (b) $\dot{z}_p$ , (c) $r_p \ddot{y}_p$ , (d) $\dot{q}_{LAp}$ , and (e) $\dot{q}_{OLAp}$ at $T_p = 200$, 400, and 600 Nm. $\gamma_s = \pi$ .................................................................

Figure 4.9  Dynamic factors of the right and left decks of the gear pair measured by using gauges R8 and L8 at (a) $T_p = 200$ Nm and (b) 400 Nm for $\gamma_s = 0$ ..................

Figure 4.10 Measured root stresses of gauge L8 at $T_p = 400$ Nm and (a) $f_m = 567$ Hz, (b) 1133 Hz, (c) 1700 Hz, and (d) 2267 Hz for $\gamma_s = 0$. Here, quasi-static stress curves at $f_m = 567$ Hz represent the quasi-static values.................................

Figure 4.11 Measured root stresses of gauge R8 at $T_p = 400$ Nm and (a) $f_m = 567$ Hz, (b) 1133 Hz, (c) 1700 Hz, and (d) 2267 Hz for $\gamma_s = 0$. Here, quasi-static stress curves at $f_m = 567$ Hz represent the quasi-static values.................................

Figure 4.12 Dynamic factors of the right and left decks of the gear pair measured by using gauges R8 and L8 at (a) $T_p = 200$ Nm and (b) 400 Nm for $\gamma_s = \pi$ ..................
Figure 4.13  Measured root stresses of gauge L8 at $T_p = 400$ Nm and (a) $f_m = 567$ Hz, (b) 1133 Hz, (c) 1700 Hz, and (d) 2267 Hz for $\gamma_s = \pi$. Here, quasi-static stress curves at $f_m = 567$ Hz represent the quasi-static values........................................ 146

Figure 4.14  Measured root stresses of gauge R8 at $T_p = 400$ Nm and (a) $f_m = 567$ Hz, (b) 1133 Hz, (c) 1700 Hz, and (d) 2267 Hz for $\gamma_s = \pi$. Here, quasi-static stress curves at $f_m = 567$ Hz represent the quasi-static values........................................ 147

Figure 4.15  A three-dimensional dynamic model of a double-helical gear pair ............... 150

Figure 4.16  A three-dimensional dynamic model of the left side helical gear pair .......... 151

Figure 4.17  (a) Geometry of a double-helical gear, and (b) three-piece (left and right side gears, and two connecting beam elements) model of the double-helical gear... 156

Figure 4.18  A view of the overall system with the shaft beam elements and bearing supports........................................................................................................... 158

Figure 4.19  Illustration of the coupling of the DH gear $j$ and its shaft......................... 161

Figure 4.20  Comparison of predicted and measured rms dynamic transmission error amplitudes $\hat{\delta}_d$ for a gear pair having $\gamma_s = 0$ and the axially floating conditions at $T_p = 400$ Nm............................................................................................................ 168

Figure 4.21  The shape of the normal mode A at $f_A = 2153$ Hz. (a) A side view and (b) a top view of the model of the double-helical gear pair. Dotted lines represent the equilibrium (zero displacement) position ................................................................. 169

Figure 4.22  Comparison of predicted and measured (a) $\hat{y_p}$, (b) $\hat{x_p}$, (c) $\hat{y_g}$, and (d) $\hat{x_g}$ for a gear pair having $\gamma_s = 0$ and the axially floating conditions at $T_p = 400$ Nm ........................................................................................................ 170
Figure 4.23  Comparison of predicted and measured (a) $\hat{z}_p$ and (b) $\hat{z}_g$ for a gear pair having $\gamma_s = 0$ and the axially floating conditions at $T_p = 400 \text{ Nm}$............................... 173

Figure 4.24  The shape of the normal mode B at $f_B = 510 \text{ Hz}$. (a) a side view and (b) a top view of the model of the double-helical gear pair. Dotted lines represent the equilibrium (zero displacement) position .................................................................................. 174

Figure 4.25  Comparison of (a) predicted and (b) measured $\hat{\delta}_d$ forced response curves of gear pairs having $\gamma_s = 0, \pi/2$ and $\pi$ at $T_p = 400 \text{ Nm}$................................. 175

Figure 4.26  Comparison of (a) predicted and (b) measured $\hat{\gamma}_g$ forced response curves of gear pairs having $\gamma_s = 0, \pi/2$, and $\pi$ at $T_p = 400 \text{ Nm}$................................................................. 176
**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{iT}^i$</td>
<td>Tangential acceleration of $i$-th ($i \in [1,3]$) accelerometer of gear $j$ ($j = p, g$)</td>
</tr>
<tr>
<td>$a_{LA}^i$</td>
<td>Axial acceleration of $i$-th accelerometer of gear $j$</td>
</tr>
<tr>
<td>$a_{IR}^i$</td>
<td>Radial acceleration of $i$-th accelerometer of gear $j$</td>
</tr>
<tr>
<td>$f$</td>
<td>Forcing vector</td>
</tr>
<tr>
<td>$f_m$</td>
<td>Gear mesh frequency</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Cut-off frequency</td>
</tr>
<tr>
<td>$\hat{g}$</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>$H_i$</td>
<td>$i$-th harmonic amplitude of transmission error</td>
</tr>
<tr>
<td>$\dot{H}$</td>
<td>Root-mean-square amplitude of $H$</td>
</tr>
<tr>
<td>$I$</td>
<td>Diametral mass moment of inertia</td>
</tr>
<tr>
<td>$J$</td>
<td>Polar mass moment of inertia</td>
</tr>
<tr>
<td>$k$</td>
<td>An average gear mesh stiffness value</td>
</tr>
<tr>
<td>$K$</td>
<td>Stiffness matrix</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
</tr>
</tbody>
</table>
\( \mathbf{M} \)  
Mass matrix

\( n_{sj} \)  
Number of shaft element of gear \( j \)

\( N_j \)  
The number of teeth of gear \( j \)

\( \mathbf{q} \)  
Displacement vector

\( \mathbf{Q} \)  
Mode shape

\( q_{LA} \)  
Transverse motion along the line-of-action direction

\( \dot{q}_{LA} \)  
Root-mean-square amplitude of the transverse motion along the line-of-action direction

\( q_{OLA} \)  
Transverse motion along the off line-of-action direction

\( \dot{q}_{OLA} \)  
Root-mean-square amplitude of the transverse motion along the off line-of-action direction

\( r_{cc} \)  
Outside radius of a beam element of a connecting structure

\( r_i \)  
Inner radius of a beam element of a connecting structure

\( r_j \)  
Base radii of gear \( j \)

\( T \)  
Transmitted torque

\( x \)  
Motion in the \( x \) direction

\( \dot{x} \)  
Root-mean-square amplitude of the motion in the \( x \) direction

\( y \)  
Motion in the \( y \) direction

\( \dot{y} \)  
Root-mean-square amplitude of the motion in the \( y \) direction

\( z \)  
Axial motion
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{z}$</td>
<td>Root-mean-square amplitude of the axial motion</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Intentional axial shift</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Position angle between pinion and gear</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Helix angle</td>
</tr>
<tr>
<td>$\delta_d$</td>
<td>Dynamic transmission error</td>
</tr>
<tr>
<td>$\hat{\delta_d}$</td>
<td>Root-mean-square amplitude of dynamic transmission error</td>
</tr>
<tr>
<td>$\delta_s$</td>
<td>Static transmission error</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>Intentional stagger error</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Transverse pressure angle</td>
</tr>
<tr>
<td>$\varphi_i$</td>
<td>Phase angle of $i$-th harmonics of transmission error</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>Stagger angle</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Dynamic compliance</td>
</tr>
<tr>
<td>$\theta_j$</td>
<td>Angular displacement of gear $j$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Radius between the centers of the sensor and the shaft</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Root stress</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>Dynamic root stress</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>Quasi-static root stress</td>
</tr>
<tr>
<td>$\tau_m$</td>
<td>Gear mesh period</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Rocking motion</td>
</tr>
</tbody>
</table>
\( \hat{\psi} \) Root-mean-square amplitude of the rocking motion

\( \bar{\psi} \) Angle made by plane of action with vertical \( y \) axis

\( \Omega \) Rotational speed

\( \zeta \) Modal damping ratio

Subscript

\( b \) Bearing

\( c \) Connecting structure of right and left side gear

\( e \) Beam element

\( g \) Gear

\( L \) Left side of a double-helical gear

\( m \) Gear mesh

\( p \) Pinion

\( R \) Right side of a double-helical gear

\( x \) \( x \) direction

\( y \) \( y \) direction

\( z \) \( z \) direction

\( \theta_x \) \( \theta_x \) direction

\( \theta_y \) \( \theta_y \) direction
\( \theta_z \) \hspace{1cm} \theta_z \text{ direction}

Superscript

\( L \) \hspace{1cm} \text{Left side}

\( R \) \hspace{1cm} \text{Right side}
CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

Helical (or single-helical) gears are preferred to spur gears in most of the automotive, aerospace, industrial, and wind turbine gearboxes primarily due to their noise and vibration performance. Spur gears often exhibit large vibration amplitudes with strongly nonlinear behavior [1.1-1.6]. Meanwhile helical gear pairs have much lower amplitudes of gear mesh excitations in the form of loaded motion transmission error and mesh stiffness fluctuations, typically resulting in up to an order of magnitude lower dynamic response amplitudes compared to their spur counterparts [1.7-1.13]. However, helical gear pairs require bearing supports with a capability to react to the axial thrust created at the gear mesh due to the helix angle. In addition, gear bodies and support shafts must be sufficiently stiff to counter the tilting moments caused by the axial component of the gear mesh force. With these, helical gear motions, while smaller in amplitude, were observed to be three-dimensional [1.7-1.12, 1.14] while spur gear pairs often have purely torsional or two-dimensional transverse-torsional motions [1.14-1.20].

One hybrid helical gear concept, called double-helical (DH), as illustrated in Figure 1.1(a), addresses many of these issues associated with helical gears. Assembling two nominally
Figure 1.1. (a) An example double helical gear pair [1.21] and (b) a double-helical planetary gear set from a jet engine turbofan application [1.22].
identical helical gears of opposite hand into one gear, the axial thrust created by one helical gear mesh is balanced (neutralized) by the axial thrust of the other. In addition to cancellation of axial forces, positions of gear teeth of right and left sides can be staggered in an attempt to further minimize the gear mesh dynamic excitations. This arrangement also allows use of journal bearings in one of the gears, which makes DH gears very desirable for high-speed aerospace applications such as jet engine turbofan gearboxes (Figure 1.1(b)) and rotorcraft drive trains as well as high-torque marine and gas turbine applications.

In this research, experimental and theoretical studies will be performed to investigate the quasi-static and dynamic behavior of DH gear pairs focusing on a number of issues specific to DH gear pairs that are yet to be fully understood. Three primary issues of DH gears are described below:

(i) Right-to-left Load Sharing. The apparent advantage of DH gear pairs is based on the assumption that the right and left gear meshes of DH pair carry equal amounts (one-half) of the total load. However, there is an array of manufacturing errors and tolerances that might prevent such ideal “equal” load sharing. For instance, the tooth profile and lead modifications of right and left side teeth might be different causing different load distributions. There might be a mismatch between the stagger angles (angle between the teeth right and left sides) of the driving and driven gear. The axial support conditions impact such right-to-left load sharing of the DH gear pair as well. While such load sharing problems of DH systems are apparent, very little is known about the quality and tolerance requirements needed to avoid them, or to account for such errors in the design calculations. One motivation of this study is to shed light into load sharing behavior of DH gear pairs as a function of right-to-left position error ($\varepsilon_\alpha$) and axial boundary conditions.
(ii) Right-to-left Stagger Angle. As stated above, right and left side gear teeth can be positioned in relation to each other at any arbitrary (designer defined) angle. Figure 1.2 illustrates solid models of a DH gear pair having in-phase (0% stagger $\gamma_s = 0$, i.e. right and left side teeth are mirror images) and $180^\circ$ out-of-phase (50% stagger $\gamma_s = \pi$, i.e. tip of a tooth on the right hand side aligned with a root center on the left hand side) stagger conditions. Influences of the stagger angle $\gamma_s$ on the right-to-left load sharing, gear motions and the dynamic response are not known. This study aims at describing the impact of the stagger angle on these.

(iii) Dynamic Behavior of DH Gear Pairs. As it will be described in the literature survey below, there are only a small number of published dynamic models of DH gear systems, with no validation due to lack of good experimental data. This study aims at producing such data and developing a validated DH gear pair dynamics model.

In view of above motivation, a family of experimental and theoretical studies on a DH gear pair will be performed in this work. These studies will include both quasi-static (low speed with little or no dynamic effects) and dynamic (high-speed) conditions focusing on investigating the DH specific issues outlined above.

1.2 Literature Search

1.2.1 Double Helical Gears

To date, very little research has been performed on double helical gears. Most of the published work on DH gears was on the theoretical side. Among them, Thomas and Houser [1.23] developed an analytical model for predicting the load distribution and the motion transmission error (TE) in DH gears under quasi-static conditions and at various stagger conditions, concluding that introduction of a stagger between the two halves can reduce TE.
Figure 1.2. Schematics of (a) in-phase and (b) out-of-phase DH stagger configurations.
Later, Clapper and Houser [1.24] utilized this earlier model in order to investigate the root stresses of DH gears along the face width. While comparisons to limited number of experiments [1.24] were done, this static analysis model was not fully validated with experiments. Moreover, these quasi-static modeling efforts were not concerned with other gear motions such as axial play of the gears.

There is also very little published work on the dynamic behavior of DH gears. Zhang et al [1.25] performed a noise optimization of double helical parallel shaft gearbox by developing a three-dimensional FE model. This study was focused not on DH gear itself but the external factors such as the different thickness of internal bearing supporting panels and walls of gear box to reduce noise level. Wang et al [1.26] presented a study about tooth modification of a DH gear pair to reduce TE. Experiments were very limited to gearbox acceleration measurements. Jauregui and Gonzalez [1.27] developed a single degree-of-freedom model to study only axial vibrations of a double helical gear pair due to manufacturing errors. Some partial comparisons were made to experiments. Quasi-static and dynamic analyses of a DH gear pair was carried out by Ajmi and Velex [1.28]. A 12-DOF model of the left side of a gear pair was combined with another 12-DOF model of the companion right side pair using Euler beam elements. The effect of a floating pinion and staggering of teeth on the quasi-static and dynamic behavior of a DH gear pair was investigated without experimental validations. Anderson et al [1.29] conducted experiments on DH planetary gear set to measure efficiency, vibration amplitudes and stress level. They measured the vibration amplitude indirectly by mounting accelerometers on the gear box. Recently, Sondkar and Kahraman [1.30, 1.31] developed a three-dimensional dynamic model of a DH planetary gear set including all gear mesh, bearing, and support structure compliances. They presented the theoretical demonstration of the influences of planet phasing, stagger configurations, and gear support conditions on the forced responses. Most of these published dynamic studies on double helical gear have been theoretical. Some of them were indirectly correlated to noise and
gearbox vibration measurements instead of direct comparisons to gear vibration measurements. Also, there are no published works of experimental investigation of root stresses under dynamic conditions. Due to this apparent lack of experimental validation of dynamic models, 3D vibrations and load sharing issues of double helical gear are not understood well.

1.2.2 Measurement of Gear Motions

The gear literature is rather rich in terms of measurement of gear motion transmission error (TE) under low-speed conditions. One early method proposed by Munro [1.32] and Gregory et al [1.33] used optical techniques to measure the TE of unity-ratio gear pairs. A more general method was developed by a number of researchers [1.34-1.36]. In this method, the two optical encoders are attached to the pinion and gear shafts to measure their rotational positions. Then, these two signals are compared to each other to determine the deviation of the rotation of one gear relative to the other. Such encoder based TE measurement systems were employed successfully by Boguski et al [1.37] for planetary gear sets and by Makam [1.38] for hypoid gear sets. The same type of a TE measurement system will be employed here.

Measurement of TE under dynamic conditions cannot be realized using optical encoders. Tordion et al [1.39] and Houser et al [1.40] employed an accelerometer-based measurement method to measure dynamic motion transmission error at higher speed conditions. The same accelerometer based measurement technique was used in a group of studies [1.2-1.4], which investigated the nonlinear dynamic behavior of a geared system in the form of dynamic transmission error. The same investigators relied on the same method to investigate the influence of involute contact ratio [1.5] and tip relief [1.6] on the torsional vibrations for a spur gear pair. In these studies, two pairs of diametrically-opposed linear accelerometers that are tangentially mounted at certain radius next to the gears were employed. The signals were processed to obtain
the rotational (torsional) acceleration of the gears, from which the dynamic TE was calculated.

This method was further enhanced by Kang and Kahraman [1.14] to replace the uni-axial accelerometers with tri-axial ones to measure not only the dynamic TE, but also the other gear motions as well. For the measurement of dynamic TE and other 3D vibratory motions of DH gear, the tri-axial accelerometer based measurement method of Ref. [1.14] will be adapted here.

Measurement of root stresses of gear pair under static or dynamic conditions has been another significant task considered by several investigators. Early experiments on the dynamic factors for both spur and helical gears were carried out by Houser and Seireg [1.40, 1.41]. A group of recent studies [1.42-1.46] used similar strain gauge instrumentation to measure root stresses of spur, helical and hypoid gears under both quasi-static and dynamic conditions. This research will apply the same methodology to DH gears.

1.3 Dissertation Objectives

The review of the current literature on DH gear pairs in relation to the motivation of this study reveals major gaps in understanding of DH gear behavior. While spur and helical gear literature contains detailed experimental databases to guide the modeling efforts, lack of such database for DH gear pairs appears to be the major gap, hampering modeling efforts, and preventing development of physics-based design and analysis tools for DH systems. Accordingly the first group of technical objectives of this study is towards filling this gap. Specific experimental objectives of the proposed study are as follows:

(i) Design, fabricate and run-off a DH test set-up for operating a DH gear pair under realistic torque and speed conditions.
(ii) Design and develop DH test articles, which allow adjustable (1) right-to-left stagger angles, (2) intentional manufacturing and alignment errors, and (3) axial support conditions.

(iii) Develop and incorporate measurement and data acquisition systems to quantify root stresses, 3D gear motions and motion transmission error.

(iv) Define and execute test matrices to investigate DH specific issues under quasi-static and dynamic conditions.

With the experimental database for DH gear pairs established, second group of objectives focus on development and validation of mathematical models to predict quasi-static and dynamic behavior of DH gear systems. Specifically:

(i) Simulate the quasi-static experiments using an existing DH load distribution model [1.23] and compare its predictions to measurements of loaded static transmission error, axial play, root stresses and right-to-left load sharing under different stagger, manufacturing error and axial support conditions. Access the accuracy of the DH load distribution model [1.23].

(ii) Develop a linear, time-invariant dynamic model of a DH gear-shaft-bearing system based on the formulations of Sondkar and Kahraman [1.30]. Use the model to simulate the high-speed DH gear pair experiments under different stagger and axial support conditions. Through comparisons to the experimental data, arrive at a validated DH gear pair dynamic model.
1.4 Dissertation Outline

In Chapter 2, the experimental set-up of a double helical gear pair will be presented. A test machine that is capable of operating a DH gear pair under high-torque and high-speed conditions will be described in detail. In addition, a new concept of a DH gear pair will be devised to allow adjustable stagger angles, right-to-left stagger error, and axial support conditions. Four separate measurement systems developed for this study will be described in detail. They are (i) an accelerometer-based system to measure 3D vibratory motions of gears and dynamic TE under high-speed conditions, (ii) an angular encoder-based system to measure static TE of the DH gear pair under low-speed condition, (iii) a non-contact probe system to measure axial motions of the gears under low-speed conditions, and (iv) a root strain measurement system to measure right-to-left load sharing and dynamic stress factors of the DH gear pair. Furthermore, the data acquisition and processing systems will be presented.

Chapter 3 presents the quasi-static behavior of a DH gear pair. A test matrix that includes a number of loaded DH experiments with different right-to-left stagger angles, tightly controlled stagger errors, and different support conditions (axially floating or fixed) will be presented and executed to collect sets of static transmission error, axial gear motions, and root strain data to study the DH issues of right-to-left stagger angle, stagger errors and axial support conditions. An existing DH load distribution model [1.23] will be used to simulate the same tests to explain experimental results and validate the model predictions.

Chapter 4 proposes another test matrix to quantify the impact of the different right-to-left stagger angles, stagger error, and different support condition (axially floating or fixed) on the DH gear motions under wide ranges of speed and torque. This test matrix will be executed and the measured dynamic TE, 3D vibratory gear motions (transverse, axial, and rocking motions) as well as dynamic stress factors will be presented. A dynamic model of a DH gear pair will be proposed.
next. This discrete dynamic model of a DH gear pair will include shafts and bearing compliance, user-defined stagger angles, stagger errors and axial support conditions. The dynamics experiments will be simulated using this model. Predicted and measured values of dynamic transmission error and 3D gear motions will be compared directly to validate the DH dynamic model formulation.

Chapter 5 provides a summary of this study. It lists the major conclusion in terms of the suitability of the test set-up and experimental technique as well as accuracy of the predictions. A list of recommendations for future work is also included in this chapter in an attempt to guide future work on this topic.
References for Chapter 1


CHAPTER 2

EXPERIMENTAL METHODOLOGY

2.1 Introduction

A new experimental methodology is developed in this study to investigate issues or parameters that are unique to DH gear systems including the influences of the nominal right-to-left stagger angle ($\gamma_s$), deviations from the nominal $\gamma_s$ ($\epsilon_s$) and axial supporting conditions on both quasi-static and dynamic behavior. The experimental set-up designed and developed for this purpose consists of a test machine, test specimens, and numerous measurement systems. An existing test machine used for various spur and helical gear dynamics investigations [2.1-2.7] was modified heavily (i) to accommodate a DH gear pair, (ii) to increase the maximum operating speed and torque ranges, (iii) to improve the lubrication system capabilities, and (iv) to incorporate various measurement systems in a simultaneous manner.

In practice, the right and left sides of a DH gear are connected the each other through a rigid flange; i.e. they are machined from a single blank. If such one-piece DH gears were to be used for investigating the issues specified above, a large number of DH gears having different $\gamma_s$ and $\epsilon_s$ would need to be designed and procured. This is not only very expensive, but also is a very challenging task in terms of manufacturing of DH gear sets. This appears to be main reason
for the lack of experimental data on DH gearing as well. In order to overcome this challenge, a novel three-piece DH gear assembly is developed in this study to allow adjustability in terms of both $\gamma_s$ and $\varepsilon_s$. Through special design of supporting shafts, different axial supporting conditions are achieved. Instrumentation and data analysis systems to measure static TE, axial motions, three-dimensional gear motions, and root stress under quasi-static and dynamic conditions are also developed and incorporated with the test machine.

In this chapter, details about the DH gear test machine with its main component, as well as its capabilities are described first. The novel three-piece adjustable DH gear concept to have a desired stagger angle value is introduced next. The methods for applying various axial support conditions and intentional stagger errors to the test gear pairs are described. Finally, four separate measurement systems used in this study are introduced. They are (i) an optical encoder based system to measure loaded static TE, (ii) a non-contact probe system to measure axial motions of the gears under low-speed and axially floating conditions, (iii) a tri-axial accelerometer-based system to measure 3D vibratory motions of gears and dynamic TE under high-speed conditions, and (iv) a root strain measurement system to measure right-to-left load sharing and dynamic stress factors of the DH gear pair.

2.2 Test Machine

An existing test machine from a family of spur and helical gear studies [2.1-2.7] was heavily modified to develop a test machine suitable for this study. Figure 2.1 shows this test machine in the form used by those previous studies. This machine used a “four-square” or “back-to-back” arrangement where two gearboxes, test gearbox and the reaction gearbox, that have the same center distance and same (1:1) gear ratio were connect to each other by long, compliant
Figure 2.1. The gear dynamics test machine used in Refs. [2.1-2.7].
shafts and elastomer couplings. A large flywheel was placed on each shaft axis to further isolate the test gearbox (together with the compliant shafts and elastomer couplings) from the vibrations that might originate from the reaction gearbox. In this form, this machine was capable of achieving maximum speed of 4,000 rpm at a maximum torque of 400 Nm (corresponding to a maximum transmitted power of 167.5 kW). As the required maximum torque and speed values for this DH gear pair study were 600 Nm and 7,000 rpm, respectively, amounting to a maximum transmitted power of 440 kW (590 HP), various structural changes had to be made to the rotating components of the test machine shown in Figure 2.1. These included the following:

- The existing elastomer couplings were not rated for the required speed and torque levels. As such, they were replaced by a high-speed and higher torque capacity elastomer coupling design.

- The massive flywheels mounted on flexible shafts posed dynamics problems at desired higher speed ranges. They were removed from the shafts and the added torsional compliances of the new elastomer couplings were used to compensate for them.

- All of the bearings of the test and reaction gearboxes were replaced by bearings having higher dynamic ratings to achieve desired high speeds at higher load conditions.

- A center support bearing box with high-speed support bearings were placed at the mid-span of the connecting shafts to ensure high-speed operation with these long shafts.

- Lubrication of gears and bearings of the reaction gearbox was originally done though dip-lubrication method. This was modified with the forced (jet) lubrication method to be able to remove heat at high power test conditions effectively. This required incorporating a high-capacity lubricant system.
• The drive unit of the original set-up was a 10 HP DC motor. This was found to be insufficient to meet the torque and speed requirements of this study. A new 30 HP AC drive motor with a new computerized speed controller was implemented in the new machine.

Figure 2.2 shows the test machine in its modified form. In Figure 2.3, a top-view solid model of the machine is displayed to identify its main components and highlight the differences from the older machine shown in Figure 2.1. The application of the torque transmitted was done through the split coupling placed on one of the connecting shafts. With the bolts of the coupling loosened, one flange of the coupling was held fixed and other torque applied to the other flange through a torque arm and calibrated weights. In this loaded condition, bolts of the coupling were tightened, torque arm was removed and the fixed coupling was released, in the process trapping a constant magnitude of torque within the closed back-to-back loop to be transmitted by the gears of both gearboxes. As such, the drive motor had only the task of overcoming the power losses of the drivetrain to operate it at a desired speed level.

The speed controller of the new machine was designed to operate the gears at any programmable speed schedules including step-wise incremental speed changes to collect data at steady-state conditions or speed ramp-up and ramp-down conditions to collect waterfall data.

The test machine, in its new form, used a test gear box that houses a DH gear pair having a 31:34 gear ratio. To go with this, a wide face width single helical gear pair of the same ratio and the same center distance (150 mm) was designed and procured for the reaction gear box.

In order to comply with safety regulations, proper safety guards were fabricated to cover all rotating components of the test machine. Figure 2.4(a) shows the test machine with all of its safety guards, painted in yellow.
Figure 2.2. Double-helical gear test machine used in this study.
Figure 2.3. Top view solid model layout of the DH test machine.
Figure 2.4. Views of the DH test machine (a) with and (b) without the safety guards.
While this back-to-back arrangement is the most common in gear durability test machines, use of the same concept for dynamic measurements successfully requires careful design of the components connecting the two gearboxes. Here, the one of the primary goals was to measure 3D gear motion and dynamic TE of the DH gear pair under high-speed conditions. However, any vibrations resulted from the reaction gear pair mesh could affect the dynamic behavior of the test gear pair if test gear and reaction gearboxes are rigidly connected. For this reason, a reasonably good dynamic isolation between test and reaction gear pairs had to be employed. As shown in Figure 2.3 and 2.4(b), there main measures were taken to isolate test and reaction gear pairs:

(i) The sufficiently longer shafts connecting the test and reaction gearboxes had torsional stiffnesses that are much softer than gear mesh stiffnesses. This is critical in providing sufficient compliance between the two gearboxes.

(ii) Elastomer couplings connecting the test gear shafts to the connecting shafts increased compliance between the test and reaction gear boxes further, in the process, eliminating any adverse effects of the misalignments of the connecting shafts on the test gearbox.

(iii) The new reaction gear pair had a very high total contact ratio with optimized tooth profile and lead modifications to minimize its vibration excitations within the operating range of torque.

The test gears and reaction gears were both jet lubricated using a stand-alone lubrication system for safe operation of gears as well as bearings under relatively high-torque and high-speed conditions. A set of thermocouples on the bearings of the center support module and the reaction gear box was in place to monitor temperatures at these critical locations.
2.3 Test Gears and Test Shafts

A DH test gear pair-shaft assembly was specifically designed for the experiments of this study. The test gears were capable of having adjustable stagger angles while the test shafts were suitable for applying different axial support conditions as well as intentional profile position errors to the test gears for the proposed purposes. Solid models of the test gears and test shafts are shown in Figure 2.5. The design parameters of the test DH gear pair are listed in Table 2.1. This gear pair mimics a jet engine sun-planet gear pair in terms of radial sizes, center distance and gear ratio while other parameter such as face width, module pressure and helix angles as well as tooth modifications were altered significantly based on the limitations of the test machine. The gear pair was designed to operate at a fixed center distance of 150.0 mm. The total theoretical contact ratio of each side of this gear pair was 2.65 (an involute contact ratio of 1.63 and a face contact ratio of 1.02). A picture of the DH gear pair assembled on the test shafts is shown in Figure 2.6.

2.3.1 Method of Setting up a Desired Stagger Angle

As mentioned in Chapter 1, the stagger angle $\gamma_s$ is defined as the angular position difference between the left and right gears of a DH gear pair due to intentional nominal stagger of the teeth. In order to implement any user-defined stagger angle, the right and left sides of the DH gears were designed as individual separate gears to be piloted and mounted on a center flange that itself was a separate part from the test shafts. The cross-sectional view of the gear-shaft assembly shown in Figure 2.7 illustrates this three-piece arrangement. Right and left side helical gears of the DH gear were connected to each other through this center flange that had details (dowel pin holes) to align the two gears in certain stagger angle positions. This novel concept of adjustable DH gear specimens allowed the same gear pair to be operated at different nominal
Figure 2.5. Solid models depicting (a) a DH test gear pair and (b) a test shaft.
Table 2.1. Basic design parameters of the DH gear pair used in this study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pinion</th>
<th>Gear</th>
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<tr>
<td>Number of teeth</td>
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<td>31</td>
</tr>
<tr>
<td>Normal module (mm)</td>
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<td></td>
</tr>
<tr>
<td>Normal pressure angle (deg.)</td>
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<td></td>
</tr>
<tr>
<td>Center distance (mm)</td>
<td>150.0</td>
<td></td>
</tr>
<tr>
<td>Helix angle (deg.)</td>
<td>$\pm 35.0$</td>
<td>$\mp 35.0$</td>
</tr>
<tr>
<td>Base diameter (mm)</td>
<td>145.03</td>
<td>132.23</td>
</tr>
<tr>
<td>Major diameter (mm)</td>
<td>166.52</td>
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<tr>
<td>Circular tooth thickness (mm)</td>
<td>4.68</td>
<td>4.81</td>
</tr>
</tbody>
</table>
Figure 2.6. An image of the DH gear pair-shaft assembly.
Figure 2.7. Cross-sectional view of the test gear-shaft assembly.
stagger conditions shown in Figure 2.8(a-c). For instance, Figure 2.8 (a) shows the DH gear pair in its in-phase condition ($\gamma_s = 0$ or 0% staggars, i.e. right and left side teeth are mirror images of each other). In order to assemble right and left gear with the center flange at this stagger position, a pin was placed in the first dowel pin hole as shown in Figure 2.8(a) and multiple bolts were used to clamp the right and left side gears to each other through the center flange. Using the other four dowel pin holes placed next to the $\gamma_s = 0$ hole circumferentially, four other stagger conditions were also achieved: $\gamma_s = \frac{\pi}{4}$ (12.5% stagger) in Figure 2.8(b), $\gamma_s = \frac{\pi}{2}$ (25% stagger) in Figure 2.8(c), $\gamma_s = \frac{3\pi}{4}$ (37.5% stagger) in Figure 2.8(d), and $\gamma_s = \pi$ (50% stagger) in Figure 2.8(e). Figure 2.9 shows oblique views of the gear pair in these five stagger angle conditions.

2.3.2 Application of Various Axial Support Conditions

The center flange that held a pair of right and left gears according to a certain stagger angle was slide-fit on a test shaft. While a pair of rectangular keys prevented any rotation of the gear-flange sub-assembly about its shaft, it allowed it to slide freely in the axial direction unless restricted. The upper DH gear in Figure 2.7 shows the condition where the gear can move freely on its shaft, which will be called here the axially floating support condition. Meanwhile, the lower DH gear in the same figure is clamped axially against a shaft flange through a pair of spacers and a lock nut. This represents the case of an axially fixed gear. It is noted in the same figure that two different types of rolling element bearings support each shaft. The bearing to the left was cylindrical roller type with no axial thrust capability while the double-row angular contact ball bearing on the right hand side was designed to provide axial reaction to the gear-shaft assembly. As such, the axially fixed condition referred to above is indeed a flexibly supported condition defined by the axial stiffness of the double-row ball bearing.
Figure 2.8. Illustration of the right-to-left stagger conditions in a double helical gear pair; (a) $\gamma_s = 0$, (b) $\gamma_s = \pi/4$, (c) $\gamma_s = \pi/2$, (d) $\gamma_s = 3\pi/4$, and (e) $\gamma_s = \pi$. Continued
Figure 2.8. Continued.

(left) $l = 0.5l$

(right) $l = 1.5l$

$\gamma_s = \pi / 2$

(d) $\gamma_s = 3\pi / 4$

(e) $\gamma_s = \pi$

(left gear teeth) (right gear teeth)
Figure 2.9. Oblique views of the gear pair at five different stagger angles.
\[ \gamma_s = \frac{3\pi}{4} \]

\[ \gamma_s = \pi \]
In a DH gear pair, only one of the gears can be axially floated as the gear contacts limit the rigid body motions of this floating gear. If both gears were to be floated axially, they would travel along the shafts to hinder the operation of the gear pair. On the other hand, if both gears were to be axially fixed, then their relative axial position as well as the errors associated with the stagger angle would become critical parameters defining right-to-left load sharing of the gear pair.

Additionally, the space allocated to the spacers in Figure 2.7 can be utilized to mount leaf spring washers to achieve any other spring supported axial conditions desired. This type of support condition was not included in this the test matrix.

2.3.3 Application of Intentional Stagger Errors

As mentioned before, the manufacturing errors might prevent equal load sharing between the right and left sides of the DH gear pair, in the process eroding the main advantage of a DH gears significantly. In this study, any mismatch between the nominal stagger angles of the driving and driven gear is simulated by regulating the relative axial position of the gears.

The positions of the shafts in relation to each others are established accurately by the bearing supports while the positions of the gear-flange sub-assemblies on the shaft are defined by the thickness of the spacer placed between the shaft flange and the gear sub-assembly. With one gear in its fixed position establishing the axial reference position, the other gear can be axially floated and gears can be loaded to establish the “no error” position where right and left sides should carry equal amount of load. In this position, the gap between the floating gear and the shaft flange can be measured and a spacer of the same thickness can be placed to have a fixed-fixed gear pair with no stagger error (ε = 0) and zero axial shift, i.e. \( \overline{z} = 0 \). This condition is
shown in Figure 2.10 where both gears are shown to align perfectly in axial direction such that $\varepsilon_s = 0$.

Any stagger error between right and left gear pairs can be created by manipulating axial position of one of the gears in this fixed-fixed configuration with respect to the other gear. When a left-side spacer that is thicker than that of in Figure 2.10 by an amount of $\bar{z}$ is used, the position of lower gear is shifted by the same amount to the right as shown in Figure 2.11(a). Likewise, in Figure 2.12, the lower gear is positioned to the left of the other gear via a spacer that is thinner than the nominal spacer, resulting in a negative $\bar{z}$. The resultant stagger error of $\varepsilon_s$ is positive in Figure 2.11 for $\bar{z} > 0$ (i.e. there is a gap between the teeth of the right side when the left side is in contact) and $\varepsilon_s$ is negative in Figure 2.12 for $\bar{z} < 0$ (i.e. there is a gap between the teeth of the left side when the right side is in contact). A mathematical relationship between $\varepsilon_s$ and $\bar{z}$ is found from basic gear geometry as

$$\varepsilon_s = 2\bar{z}\tan|\beta|$$

(2.1)

where $\beta$ is a helix angle of a DH gear. Sets of spacers at different thicknesses were designed according to Eq. (2.1) to achieve different $\varepsilon_s$ values.

2.4 Measurement Systems, Instrumentation, and Data Acquisition

2.4.1 Static Transmission Error Measurement System

The static TE has been recognized as one of the main excitations leading to gear vibrations and gear noise. In general, TE is defined as the “the difference between the actual position of the output gear and the position it would occupy if the gears were perfectly conjugate”
Figure 2.10. (a) A schematic showing a set-up with aligned 34T and 31T gears with a zero axial shift, and (b) depiction of the resultant zero stagger error condition.
Figure 2.11. (a) A schematic showing a set-up with aligned 34T and 31T gears with a positive axial shift, and (b) depiction of the resultant positive stagger error condition.
Figure 2.12. (a) A schematic showing a set-up with aligned 34T and 31T gears with a negative axial shift, and (b) depiction of the resultant negative stagger error condition.
[2.8]. Referring to Figure 2.13, static TE of a gear pair operating at very low (quasi-static) speeds under loaded conditions along its line of action can be defined as

$$\delta_s(t) = r_p \theta_p(t) + r_g \theta_g(t)$$

(2.2)

where \( r_p \) and \( r_g \) are the base circle radii and \( \theta_p \) and \( \theta_g \) are angular displacements of the pinion (\( p \)) and gear (\( g \)) forming the pair (both defined positive in the same rotational direction). TE can be classified as a displacement excitation applied at the gear mesh between the contacting gear teeth. It is typically a periodic function with a fundamental frequency equal to the gear mesh (tooth passing) frequency. The TE excitation is amplified under dynamic conditions, causing a typically larger motion transmission error, called dynamic transmission error (\( \delta_d \)). It is noted here that any \( \delta_s \) measured from a DH gear pair is a composite of two TE values associated with the right and left-side gear pairs.

A pair of precision optical encoders were employed here to measure \( \theta_p(t) \) and \( \theta_g(t) \). As shown in Figure 2.14, an encoder (18,000 lines per revolution with an angular resolution of 1.7 micro-radian) was placed at the end of each shaft. Since no tangible torsional wind-up should be expected between the open end of the shaft where the encoder was placed and the location where the gear mounted, the angular position measured by the encoder was safely assumed to represent the angular position of the gear as well. In Figure 2.15, the instrumentation and data acquisition system set-up to process the encoder signals to compute \( \delta_s \) is shown.

The measured encoder signals were first conditioned by the encoder conditioners (Heidenhain, IBV600). The conditioned encoder signals were fed into a commercial transmission error analyzer (Transmission Error Measurement System, TEMS). The analyzed output data
Figure 2.13. A schematic of a gear pair showing the base and pitch circles and the line of action.

Figure 2.14. An image of the static transmission error measurement system using optical encoders.
Figure 2.15. Flowchart of the static transmission error measurement system instrumentation and data acquisition system.
from TEMS included the measured raw time-domain $\delta_s(t)$ signal as well as the high-pass and low-pass filtered time-domain $\delta_s(t)$ (short-term and long-term) signals. These filtered time domain signals were processed in the frequency domain to determine the harmonic content of static TE.

### 2.4.2 Axial Motion Measurement System

A pair of non-contact proximity probes (Bently Nevada XL 3300 8mm series) was employed to measure the axial motions of the DH gears. Each proximity probe was mounted on a bracket that was secured on the test bed, positioned at a certain gap on one side of a gear, as shown in Figure 2.16(a). The brackets allowed the probes to be positioned at a desired height and gap with respect to a gear as shown in Figure 2.16(b). The probes utilized the eddy current principle to deliver voltage signals proportional to the measured open air gap within a resolution of 0.5 μm. The linear range of these probes was 2 mm, and the sensitivity was 7.87 V/mm. The initial gap between the end of the probe and the side surface of the gear shown was set to 1.27 mm (9V DC), as recommended by the manufacturer, using bolts shown in Figure 2.16(c).

As shown in Figure 2.17, the probes were connected with shielded extension cable to two Bently Nevada proximitor sensors. The outputs of the proximitor sensors were then connected via BNC cables to an NI PXI-4472 input module housed in an NI PXI-1042 data acquisition chassis. The chassis was connected to the data acquisition computer through an NI 6052 data acquisition PCI card. LabView was implemented to process the raw data from the probes to obtain the axial displacement. In a virtual interface (VI) of Labview, the sensitivity of the proximitor sensor, the sampling rate, the test duration, and the cut-off frequency were defined for this application. The
Figure 2.16. (a,b) Solid model image showing the proximity probe placement for axial motion study and (c) an actual image of the axial motion measurement system.
Figure 2.17. Flowchart of the axial motion measurement system instrumentation and data acquisition system.
data from VI was recorded in a text file. In this study, the sampling frequency of 2.5 kHz and a cut-off frequency of 1 kHz were used.

### 2.4.3 Dynamic Transmission Error and 3D Gear Motion Measurement System

An accelerometer-based measurement system was devised for measuring gear vibrations under high-speed conditions over the past decades. This is not a new concept as tangentially mounted uni-axial accelerometers were used commonly to measure torsional vibrations of each gear and dynamic TE [2.2-2.5, 2.9-2.10]. For the measurement of components of 3D gear vibrations, a metrology that was developed for single helical gears by Kang and Kahraman [2.11] and Kang [2.12] will be adapted here to the DH gear pair in hand. Only the basic principles of this method will be provided here for completeness purposes as details of it can be found in References [2.11, 2.12].

This tri-axial accelerometer based method allows computation of motions of various directions including

1. **dynamic TE through measurement of the motions in the torsional directions,**
2. **transverse motions in the transverse plane of the gears along the line-of-action (LA) and off-line-of-action (OLA) directions,** as defined in Fig. 2.13,
3. **axial motions in the direction of gear rotation axes,** and
4. **rocking motions that are in the directions perpendicular to the transverse plane of gears.**

This method used sets of tri-axial accelerometer mounted on an adapter that was attached to the gears at a certain radius as shown in Figure 2.18(a). The adapters of the shafts of the pinion and
Figure 2.18. (a) A view of Solid model depicting the tri-axial accelerometers and their adapters for the instrumentation, and (b) the actual arrangement.
gear were machined to mount three tri-axial accelerometers (PCB Piezotronics, Model: 354C10, sensitivity: nearly 10 mV/g, frequency range: 0 to 8 kHz) of each shaft tangentially at 0°, 90° and 180° angles as shown in Figure 2.18(b).

Schematic of Figure 2.19 defines the coordinates of the gears, and the locations of the accelerometers. Here, three tri-axial accelerometers are mounted tangentially at angles of 0°, 90° and 180° on the adapter next to the pinion at a radius ρ_p. In this arrangement, each sensor i (i ∈ [1,3]) measures tangential a^{(p)}_{iT}(t), radial a^{(p)}_{iR}(t) and axial a^{(p)}_{iA}(t) acceleration components at that position while rotating with the pinion. Likewise, another nine channels of accelerations a^{(g)}_{iT}(t), a^{(g)}_{iR}(t) and a^{(g)}_{iA}(t) associated with the motions of the gear can be measured at a radius ρ_g.

In order to measure dynamic TE, two tangential acceleration signals having gravitational and rotational terms were considered on the gear j (j = p,g where p and g denote pinion and gear, respectively)

\[ a^{(j)}_{iT}(t) = \rho_j \ddot{j}(t) + \hat{g}\sin(\omega_jt), \quad (2.3a) \]

\[ a^{(j)}_{3T}(t) = \rho_j \ddot{j}(t) - \hat{g}\sin(\omega_jt) \]

where \( \hat{g} \) is the gravitational acceleration and \( \omega_j \) is the angle from the horizontal reference position. By simply adding \( a^{(j)}_{iT}(t) \) and \( a^{(j)}_{3T}(t) \), the angular acceleration of gear j can be found as

\[ \ddot{j}(t) = \frac{1}{2\rho_j} \left[ a^{(j)}_{iT}(t) + a^{(j)}_{3T}(t) \right], \quad j = p,g. \quad (2.4) \]
Figure 2.19. A schematic showing the coordinates of the gears, and the locations and orientations of the accelerometers [2.11].
The processed signals \( \ddot{\theta}_p(t) \) and \( \ddot{\theta}_g(t) \) from Eq. (2.4) were added to each other and integrated twice with respect to time to obtain the dynamic TE as

\[
\delta_d(t) = \int \left[ r_p \ddot{\theta}_p(t) + r_g \ddot{\theta}_g(t) \right] dt^2
\]

(2.5)

where \( r_p \) and \( r_g \) are the base circle radii of the pinion and the gear.

For obtaining the transverse vibrations of gear \( j \) in both the line-of-action (LA) and off-line-of-action (OLA) directions of the gear meshes, two radial acceleration signals from two locations that are 90° apart were required. Considering \( a_{2R}^{(j)}(t) \) and \( a_{3R}^{(j)}(t) \) for this purpose, these accelerations are written in terms of their components as

\[
a_{2R}^{(j)}(t) = -\ddot{x}_j(t) \sin \omega_j t - \dot{y}_j(t) \cos \omega_j t + \ddot{g} \cos \omega_j t,
\]

(2.6a)

\[
a_{3R}^{(j)}(t) = -\ddot{x}_j(t) \cos \omega_j t + \dot{y}_j(t) \sin \omega_j t - \ddot{g} \sin \omega_j t
\]

(2.6b)

where \( \ddot{x}(t) \) and \( \ddot{y}(t) \) are the horizontal and vertical accelerations of gear \( j \). By solving Eq. (2.6a) and Eq. (2.6b) for \( \ddot{x}(t) \) and \( \ddot{y}(t) \) yields

\[
\ddot{x}_j(t) = -a_{2R}^{(j)}(t) \sin(\omega_j t) - a_{3R}^{(j)}(t) \cos(\omega_j t),
\]

(2.7a)

\[
\ddot{y}_j(t) = \ddot{g} - a_{2R}^{(j)}(t) \cos(\omega_j t) + a_{3R}^{(j)}(t) \sin(\omega_j t).
\]

(2.7b)

Equations. (2.7a) and (2.7b) require that the instantaneous angular position \( \omega t \) of the sensors be determined. This was done by generating a once-per-revolution tachometer signal using an optical pick-up. A reflective tape was attached one of the shafts to produce an impulse at every instant when the first sensor on the pinion is at its horizontal position. With this tachometer
signal, the $\sin(\omega t)$ and $\cos(\omega t)$ terms were defined. Further details about generating these angular positions are described in Kang [2.12].

Since the axes of both gears were contained by a common horizontal plane, Eq. (2.7a) and Eq. (2.7b) were used to obtain the vibration of the gear $j$ along the LA and OLA directions, respectively, as

$$2(\int \ddot{x}_j(t)\sin\phi + \ddot{y}_j(t)\cos\phi)dt^2,$$  \hspace{1cm} (2.8)

$$2(\int \ddot{x}_j(t)\cos\phi - \ddot{y}_j(t)\sin\phi)dt^2$$  \hspace{1cm} (2.9)

where $\phi$ is the transverse pressure angle of the gears.

The axial accelerations from the first and third tri-axial accelerometers of gear $j$ was used to determine the rocking (rotational) motions of the gear about the $x$ and $y$ axes. The rotational acceleration of gear $j$ about the rotating diametral axis connecting the points at which accelerometers 1 and 3 are located is given as

$$\ddot{\psi}_j(t) = \frac{1}{2\rho_j} \left[ a_{1A}^{(j)}(t) - a_{3A}^{(j)}(t) \right].$$  \hspace{1cm} (2.10)

Double integration of this signal results in the rocking vibration amplitude of

$$\psi_j(t) = \int \ddot{\psi}_j(t)dt^2.$$  \hspace{1cm} (2.11)
This rocking motion can be decomposed further into two angular acceleration components
\[ \ddot{\psi}_x(t) = \dot{\psi}_x(t) \cos(\alpha_j t) \quad \text{and} \quad \ddot{\psi}_y(t) = \dot{\psi}_y(t) \sin(\alpha_j t) \]
about the x and y axes, respectively, that are integrated twice to obtain rocking vibration amplitudes about these axes.

While the rocking motions result in the acceleration components that are opposite of each other, the average of \( a^{(j)}_{1,tt} \) and \( a^{(j)}_{3,tt} \) simply represents the axial acceleration of gear \( j \) as

\[ \ddot{z}_j(t) = \frac{1}{2} \left[ a^{(j)}_{1,tt}(t) + a^{(j)}_{3,tt}(t) \right]. \quad (2.12) \]

Double integration of Eq. (2.12) with respect to time gives the axial displacement of gear \( j \)

\[ z_j(t) = \int \int \ddot{z}_j(t) dt^2. \quad (2.13) \]

Figure 2.18(b) shows this accelerometer-based system implemented on the DH test gears. As shown schematically in Figure 2.20, the data from the rotating accelerometers were transmitted to the fixed frame through the end-of-shaft type slip rings (Michigan Scientific-SR10M). Signals from the output of the slip rings were fed into a multi-channel signal conditioner (PCB Piezotronics ICP 483M92) to condition and amplify the acceleration data. Then, the signals were transmitted to an A/D converter (NI PXI-4472) that digitized the analog signals at a user-defined sampling rate. The digitized data was fed into a general-purpose chassis for PXI (NI PXI-1042) and then sent to the PC for the analysis and processing of the data. Here, a LabView program was used primarily for the data acquisition purposes while the bulk of the data analysis was done using MATLAB.

The steady-state response of the test gear pair was the main focus of this study. As such, constant speed data were collected within a gear rotational speed range of 500 to 6,000 rpm with
Figure 2.20. Flowchart of the 3D motion and DTE measurement system instrumentation and data acquisition system.
an increment of 100 rpm. Finally, all the data was written in a very large file including tachometer signals for the pinion and gear as well as all acceleration data. In the MATLAB program, this large file was separated in individual steady-state segment by using the tachometer signal since it gives speed information of the test gears. After this, each segment was processed to compute $\delta_d(t)$, $q_{LA}(t)$, $q_{OLA}(t)$, $\psi(t)$ and $z(t)$ according to the formulations given above.

Fast Fourier Transforms (FFT) was applied at the end to analyze the data in the frequency domain. This provided the amplitudes of the harmonics of each vibratory motion as well as the root-mean-square value that was defined by using the first three gear mesh harmonics.

2.4.4 Root Stress Measurement System

For quantifying right-to-left load sharing and dynamic stress factor of the DH gear pair, one of the test gears was instrumented using two sets of strain gauges. Figure 2.21 specifies the intended location of the three gauges at a given root fillet. The three strain gauges were installed evenly on the root along the face width at $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ of the active face width. All gauges were placed at the same radius, $R_{gauges}$, corresponding to point at 11.39 mm distance from the tip as shown in Figure 2.21. The radial positions of gages were such that the location was close to where the maximum tensile stress is expected. Figure 2.22(a) is a schematic showing the array of 9 strain gauges and their labels on the right side of the 34-tooth gear. Three consecutive teeth from each side were strain-gauged at three root locations to quantify tooth-to-tooth variations as well as the variations along the face width of the gear. Figure 2.22(b) shows an image of the installed strain gauges mounted on the roots of the actual right gear of the 34-tooth gear. Likewise, Figure 2.23 shows the left side gauges mounted at the same locations as the right side gauges.
Figure 2.21. A schematic showing locations of the strain gauges on a tooth.
Figure 2.22. (a) An illustration depicting the array of strain gauges and their labeling scheme and (b) the installed strain gauges mounted on the roots of the actual right gear mesh of the 34-tooth gear.
Figure 2.23. (a) An illustration depicting the array of strain gauges and their labeling scheme and (b) the installed strain gauges mounted on the roots of the actual left gear mesh of the 34-tooth gear.
With such instrumentation, a total of 18 gauge signals were captured and compared to determine right-to-left load sharing factors and dynamic stress factors. All 18 gauges were normally identical (Vishay Micro-Measurements, Model: CEA-XX-015UW-120) with a gauge factor of 2.05 and a resistance of 120 ohms.

A 36 channel slip ring (Michigan Scientific, model SR36M) as shown in Figure 2.24 was installed at the end of shaft of 34-tooth gear. The outputs of the strain gauges were fed into a terminal block unit (NI SCXI-1314) where the gauges were configured as a quarter Wheatstone bridge with 120 ohm dummy resistor manually. The terminal block was connected to the universal strain gauge conditioning module (NI SCXI-1520) where the signal was amplified and filtered. The conditioning module was capable of collecting the data in the multiplexing mode with a maximum rate of 333,000 [samples/sec] to capture the 18 strain signals simultaneously. The data was then sent to the multifunction DAQ board (NI PCI-6052E) mounted in a PC and recorded by a LabView program.

The LabView program allowed configuring the quarter Wheatstone bridge with a gage factor, a gain factor, a dummy resistor and lead resistance. It also provided control of sampling frequency and a cut-off frequency. In this study, the sampling frequency is 2.5 kHz and the cut-off frequency is 1 kHz was used to measure right-to-left load sharing under quasi-static condition. Null compensation was applied the strain signals when the gauges has off-set from the unloaded strain signal of the gear mesh. This helped greatly in eliminating any residual strain signal in the measurement system.
Figure 2.24. Flowchart of the root stress measurement system instrumentation and data acquisition system.
References for Chapter 2


3.1 Introduction

This chapter presents the results of various measurements performed on the double-helical (DH) gear pair shown in Figure 2.4 under loaded, quasi-static conditions. As specified in Table 2.1 and shown in Figure 2.6, the DH gear was formed by a 34-tooth gear (called the pinion here and denoted by subscript $p$) and a 31-tooth gear (called the gear and denoted by subscript $g$).

Such quasi-static conditions were obtained by operating the gear set at an extremely low pinion speed of $\Omega_p = 100$ rpm corresponding to a gear mesh (tooth passing) frequency of $f_m = \frac{34}{60} \Omega_p = 56.67$ Hz. As it will be demonstrated later in Chapters 4 and 5, this speed value is too low for any dynamic behavior to take effect.

The quasi-static measurements consisted of

(i) the static TE ($\delta_s$) of the gear pair under various gear torque ($T_p$) and nominal stagger angle ($\gamma_s$) values,

(ii) axial motions of the gears at various $T_p$ and $\gamma_s$ values, and

(iii) tooth root stresses at various $T_p$ and stagger error ($\varepsilon_s$) values with different axial (fixed or floating) gear support conditions.
These experimental results presented in Section 3.2 will be simulated in Section 3.3 by using the double-helical load distribution model of Thomas [3.1] to describe the measured behavior better while, at the same time, assessing the accuracy of the model.

3.2 Quasi-static DH Gear Pair Measurements

3.2.1 Static Transmission Error Measurements

Static TE (\(\delta_s\)) measurements were made by using the optical encoder-based measurement system introduced in Section 2.4.1 and illustrated in Figures 2.15. The measured transmission error signal represents the deviation of the transmitted motion from its kinematic nominal position as defined by Eq. (2.2). Here, all types of gear errors, tooth profile and lead modifications as well as the deflections of the gear teeth contribute to the measured \(\delta_s(t)\) signal. Nominal tooth corrections (profile and lead modifications imposed by the designer on the teeth as a deviation from the perfect involute shape) and tooth deflections result in TE fluctuations at the gear mesh frequency \(f_m\), i.e. at the gear mesh period \(\tau_m = 1/f_m\). These are often referred to as “short-term” or high-frequency components of TE. Other errors such as tooth indexing or spacing errors, pitch line run-out errors, gear eccentricities and helix angle variations (lead wobble) typically repeat themselves with each rotation of the corresponding gear, causing “long-term” fluctuations to TE at shaft rotational frequencies of \(f_m/N_p\) and \(f_m/N_g\) where \(N_p = 34\) and \(N_g = 31\) are the numbers of teeth.

Figure 3.1(a) shows an example measured TE time history covering one complete rotation of the 34-tooth pinion that exhibits both long-term and short-term components. 34
Figure 3.1. (a) An example of a measured time history of the transmission error and (b) the corresponding Fourier spectrum. $\Omega_p = 100$ rpm, $T_p = 400$ Nm and $\gamma_s = 0$. 

66
periods of low-amplitude fluctuations (the gear mesh component) are superimposed on a shaft frequency variation that moves δs signal within ±15 μm. In Figure 3.1(b), the Fourier spectrum of same data is presented. The horizontal (x axis) frequency axis is normalized by \( f_m \) in this figure such \( \bar{f} = f / f_m = 1, 2, 3, \ldots \) represent the fundamental (first) harmonic and the higher harmonics of δs. In this figure, the tangible TE harmonic amplitudes at \( \bar{f} = 1 \) and 2 are the prominent gear mesh orders. Other low-frequency components are also noted at \( \bar{f} = \frac{1}{31}, \frac{2}{31}, \cdots, \frac{1}{34}, \frac{2}{34}, \cdots \).

As they do not impact to noise performance of the gear pair significantly, it is customary to filter out the long-term components by applying a high-pass filter to \( \delta_s(t) \) of Figure 3.1(a). Figure 3.2(a) shows the high-pass filtered version of \( \delta_s \) of Figure 3.1(a). Here a filter cut-off frequency of \( f_c = 10 \) Hz is used (\( f_c \approx \frac{1}{2} f_m \)) such that the mesh-frequency components are preserved in the process. It is evident from Figure 3.2(a) that mesh harmonic amplitude of \( \delta_s \) is within ±2.5 μm, repeating itself at the gear mesh period of \( \tau_m = 0.018 \) sec. The corresponding Fourier spectrum show a fundamental harmonic amplitude of \( H_1 = 2.1 \) μm and the second harmonic amplitude of \( H_2 = 0.6 \) μm while other higher harmonics are insignificant and the shaft harmonics are completely filtered out. This filtered form of \( \delta_s \) will be considered throughout the rest of this section focusing primarily on the variation of \( H_1 \) and \( H_2 \) with the torque \( T \) and stagger angle (\( \gamma_s \)) values.

Figure 3.3 shows the high-pass filtered \( \delta_s(t) \) measured at \( \Omega = 100 \) rpm, \( T_p = 400 \) Nm using the gear pairs having stagger angle values of \( \gamma_s = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4} \) and \( \pi \). The pinion was allowed to float axially about its shaft (i.e. not supported axially) while the gear was fixed on its
Figure 3.2. (a) High-pass filtered measured transmission error and (b) the corresponding Fourier spectrum. $\Omega_p = 100$ rpm, $T_p = 400$ Nm and $\gamma_s = 0$, and filter cut-off frequency of 10 Hz.
Figure 3.3. High-pass filtered TE time histories for stagger angles of (a) $\gamma_s = 0$, (b) $\gamma_s = \pi/4$, (c) $\gamma_s = \pi/2$, (d) $\gamma_s = 3\pi/4$, and (e) $\gamma_s = \pi$. $\Omega_p = 100$ rpm, and $T_p = 400$ Nm.
shaft. Here $\gamma_s = 0$ represents the in-phase condition as shown in Figure 2.8(a) where right and left decks of the DH gear set are mirror images of each other. The case of $\gamma_s = \pi$ is when the teeth of one deck are half a base pitch off the teeth of the other deck (Figure 2.8(e)) while the other three $\gamma_s$ values are in between these limiting values. It is clear from Figure 3.3 that the amplitude of $\delta_s$ is impacted by $\gamma_s$ significantly. The largest peak-to-peak (p-p) amplitude is observed in Figure 3.3(a) for $\gamma_s = 0$ while an increase in $\gamma_s$ results in sizable reductions in p-p $\delta_s$ values. In Figure 3.3(e), the lowest p-p $\delta_s$ amplitude is obtained when $\gamma_s = \pi$. This can be explained heuristically at this point as a cancellation or neutralization of the transmission error of right and left decks. Assume that the left-deck $\delta_s$ is a periodic function

$$\delta_L(t) = \sum_{i=1}^{N} H_i \cos(i\omega_m t + \varphi_i). \quad (3.1a)$$

where $\omega_m = 2\pi f_m$. With the right deck tooth geometry being identical to that of the left deck, transmission error of the right deck is obtained by shifting $\delta_L(t)$ by angle $\gamma_s$ as

$$\delta_R(t) = \sum_{i=1}^{N} H_i \cos(i\omega_m t + \varphi_i + i\gamma_s). \quad (3.1b)$$

For the gear pair having $\gamma_s = 0$, $\delta_R(t) = \delta_L(t)$. As such, the overall TE of the DH gear pair is composed by the both components acting in unison. For the case of $\gamma_s = \pi$, the phase angle term is $i\gamma_s = i\pi$ such that the fundamental terms ($i = 1$) in Eq. (3.1a,b) have a phase difference of $\pi$, potentially cancelling out each other while the second harmonic terms ($i = 2$) with $2\gamma_s = 2\pi = 0$ will still add up. Using the same argument, one can expect that the second harmonic terms to cancel out for $\gamma_s = \frac{\pi}{2}$ where $2\gamma_s = \pi$. 

70
The measured TE harmonic amplitudes $H_1$ and $H_2$ are plotted in Figures 3.4(a) and 3.4(b), respectively, as a function of $\gamma_s$ for five different $T_p$ values of $T_p = 100, 200, 300, 400$ and 500 Nm. In Figure 3.4(a), measured $H_1$ values reduce significantly with increased $\gamma_s$ regardless of torque. For instance, at $T_p = 100$ Nm, $H_1 = 3.7 \mu m$ for $\gamma_s = 0$ while it is only $0.5 \mu m$ for $\gamma_s = \pi$, representing a significant reduction. Meanwhile, the maximum $H_2$ value of about $0.6 \mu m$ is obtained at $\gamma_s = 0$ and $\pi$ for the same torque value while its minimum value is at $\gamma_s = \frac{\pi}{2}$. All these quantitative observations are in support of the earlier heuristic argument regarding the influence of the stagger angle on TE.

On the other hand, it is observed in Figure 3.4 that an increase in $T_p$ causes the TE amplitudes (especially the $H_1$ component) to be reduced. This is more obvious in Figure 3.5 presents the same data in 3D format to illustrate the combined influence of $\gamma_s$ and $T_p$. For instance, in Figure 3.5(a), the gear pair with $\gamma_s = 0$ has $H_1 = 3.7 \mu m$ at $T_p = 100$ Nm while $H_1 = 1.7 \mu m$ at $T_p = 500$ Nm. This reduction is stemmed from the fact that the teeth of the test gears used in this study were “corrected,” i.e. their profiles were intentionally modified to reduce TE [3.2- 3.4] as well as minimizing any adverse effects of gear alignment errors [3.4]. Per Refs. [3.2] for spur gears and [3.3, 3.4] for helical gears, a gear pair having purely involute profiles would have zero TE amplitudes at $T_p = 0$ with TE amplitudes increasing with increasing torque due to the increases in tooth deflections. Meanwhile, if the gear tooth profiles are modified by removing small amounts of material from the involute profiles, the TE value at $T_p = 0$ represents the kinematic error caused by the profile modifications. It reduces with increasing torque reaching its minimum at a certain “design torque” value when the tooth deflections are cancelled out by the profile modifications. This happens to be the case here as well. As shown in the
Figure 3.4  Variation of (a) the fundamental (first) and (b) second harmonic amplitudes of measured transmission error at different torque levels. $\Omega_p = 100$ rpm.
Figure 3.5. Three-dimensional representations of the data of Figure 3.4 showing combined influence of torque and stagger angle on (a) fundamental and (b) second harmonic amplitudes of the measured transmission error. $\Omega_p = 100$ rpm.
measurements of the teeth in Figures 3.6 and 3.7, both test gears (34-tooth and 31-tooth) were modified heavily using a bias modification scheme [3.3]. With $z=0$ plane representing purely involute surface, all tooth surfaces are seen to deviate from the involute profile, at some corner regions as much as 50 $\mu$m. As a result, the design load for this gear pair can be expected to have a value much larger than the maximum torque value afforded by the test set-up.

3.2.2 Axial Motion Measurements

As stated in Chapter 2, the gear pair assembly was devised with a capability to allow one of the gears to float axially (i.e. not restricted) on its shaft. This represented some of the DH applications in various aerospace systems. The axial play of this axially free gear was measured next as a function of the stagger angle and the torque transmitted. For these measurements, the non-contact probe system shown in Figure 2.17 and described in Section 2.4.2 was used.

Figure 3.8 shows the measured axial motions of the 34-tooth gear of the test pair under all five stagger angle values of $\gamma_s = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ and $\pi$ at $T_p = 100$ Nm, and $\Omega_p = 100$ rpm. These time histories cover five mesh cycles corresponding to $\frac{5}{34}$ rotation of the gear. It is seen that the lowest value of axial play takes place when $\gamma_s = 0$. Peak-to-peak axial motion amplitudes in Figure 3.8(a) for $\gamma_s = 0$ are very small, about 4 $\mu$m, that is in the order of roughness amplitudes of the gear surface where these measurements taken from. Axial play amplitudes are seen to increase with increased $\gamma_s$ with a maximum p-p axial motion amplitude of 12 $\mu$m for $\gamma_s = \pi$. Also evident here is that the axial motions measured are periodic at the gear mesh frequency, indicating that they are caused by right-to-left mesh variations. The relationship between the axial motions with $\gamma_s$ exhibited here is the opposite of those presented in Figure 3.3 for $\delta_s$ amplitudes. A DH gear pair with 180 degrees out-of-phase meshes causes
Figure 3.6. A measured tooth surface topographies of (a) the left-hand and (b) right-hand flanks of the 34-tooth test gear.
Figure 3.7. A measured tooth surface topographies of (a) the right-hand and (b) left-hand flanks of the 31-tooth test gear.
Figure 3.8. Measured axial motion time histories of the 34-tooth gear for (a) $\gamma_s = 0$, (b) $\gamma_s = \frac{\pi}{4}$, (c) $\gamma_s = \frac{\pi}{2}$, (d) $\gamma_s = \frac{3\pi}{4}$ and (e) $\gamma_s = \pi$. $T_p = 100$ Nm, and $\Omega_p = 100$ rpm.
the lowest TE values (relative torsional motions of the gears) accompanied by the highest axial motion amplitudes. Conversely, a DH gear pair with in-phase meshes results in maximum TE values with little or no axial motions.

Another set of axial motion measurements are presented in Figure 3.9 to illustrate the influence of the torque transmitted. Four time histories of the gear pair having $\gamma_s = \pi$ are presented at $T_p = 100, 200, 300$ and $400$ Nm. As in the TE measurements, the axial play amplitudes are reduced here with increased $T_p$. The p-p axial motion amplitude at $T_p = 400$ Nm is about three times lower than those at $T_p = 100$ Nm. This is partly because there is less TE at higher $T_p$ that must be compensated by the axial motions of the gears. The increased axial friction forces at the shaft-gear interface are also likely to contribute to this trend.

### 3.2.3 Root Stress Measurements and Right-to-Left Load Sharing

As described in Section 2.4.4 and illustrated in Figure 2.22 to 2.23, three consecutive teeth on both right and left decks of the 34-tooth test gear were instrumented using root gauges. Gauge groups R1-R2-R3, R4-R5-R6 and R7-R8-R9 were placed on the right deck (three gages per tooth) while the corresponding gauge groups L1-L2-L3, L4-L5-L6 and L7-L8-L9 were placed on the left deck in identical gauge locations. With three gauges at each respective location on three consecutive teeth (e.g. gages L1, L4 and L7 or R2, R5 an R8) were intended to be redundant in case certain gauges do not function well. Measurements from these respective redundant gauges were observed to be consistently close. As such, measurements from only one set of gauge groups L7-L8-L9 and R7-R8-R9 will be presented here.

The test matrix for root stress measurements included tests within a range of torque ($T_p = 100 – 500$ Nm) and at a single speed value ($\Omega_p = 100$ rpm). Measurements were first
Figure 3.9. Measured axial motion time histories of the 34-tooth gear for $\gamma_s = \pi$ at (a) $T_p = 100$ Nm, (b) 200 Nm, (c) 300 Nm, and (d) 400 Nm.
performed for the condition when there is no stagger error $\varepsilon_s = 0$ considering three different stagger angles of $\gamma_s = 0, \frac{\pi}{2},$ and $\pi$ with one of the gears is allowed to float axially. Additional tests on the influence of the right-to-left stagger error $\varepsilon_s \in [-108, 108]$ $\mu$m on the right to left load sharing were performed at $\gamma_s = 0$ and $\pi$ with both gears fixed axially on their respective shafts.

Figures 3.10, 3.11 and 3.12 show the measured root stresses from gauge groups L7-L8-L9 and R7-R8-R9 for $\gamma_s = 0, \frac{\pi}{2}$ and $\pi$, respectively, at $T_p = 400$ Nm and $\Omega_p = 100$ rpm. All these measurements were performed with the 34-tooth gear floating axially. In these figures, the measured strain values are converted to stresses assuming a uni-axial stress state and using Hooke’s law with a modulus of elasticity value of $E = 207$ GPa for a carbon steel. Null compensation was applied to the strain signals when the gauges were far from the mesh and the gear was unloaded.

Several general observations can be made from the measurements presented in Figures 3.10 to 3.12:

- All strain gauges exhibit tensile stresses rather than compressive stresses, indicating that efforts to move the gauges to a location as close to start of active profile as possible was successful.

- All gauges are observed to be loaded for nearly 1.5 mesh cycles representative of the actual contact ratio of the gear pair.

- The measurements from the gauges mounted on the same tooth were seen to be different in both shape and maximum value. This is expected since the gears are
Figure 3.10. Measured root stress time histories for the gear pair having $\gamma_s = 0$ and axially floating pinion at $T_p = 400$ Nm. (a) Left deck gauges L7, L8 and L9, and (b) right deck gauges R7, R8 and R9.
Figure 3.11. Measured root stress time histories for the gear pair having $\gamma_s = \pi/2$ and axially floating pinion at $T_p = 400$ Nm. (a) Left deck gauges L7, L8 and L9, and (b) right deck gauges R7, R8 and R9.
Figure 3.12. Measured root stress time histories for the gear pair having $\gamma_s = \pi$ and axially floating pinion at $T_p = 400$ Nm. (a) Left deck gauges L7, L8 and L9, and (b) right deck gauges R7, R8 and R9.
helical type [3.5] and teeth are heavily modified as shown in Figures 3.6 and 3.7. Also in agreement with single-helical gear experiments of Ref. [3.5] is the fact that gauges located in the middle (R8 and L8) measure the highest levels of stresses.

- There is very little difference amongst the measurements at different stagger angles under these axially floating conditions.

The measured stresses from the respective gauges on the right and left decks are compared next to investigate the impact of the stagger error $\epsilon_s$ on right-to-left load sharing under the condition when both gears are axially fixed. In Figure 3.13, measured stresses from gauges R8 and L8 are compared when there is no stagger error (i.e. $\epsilon_s = 0$) for $\gamma_s = 0$ and $\pi$. Signals from gauges R8 and L8 are rather close each other in terms of both amplitudes and overall shapes. In Figure 3.13(b) for $\gamma_s = \pi$, the signal from gage L8 is shifted by an half base pitch ($\frac{1}{2}$ mesh cycles) so that both signals can be overlaid for a better quantitative comparison. These measurement indicate that there is a very good 50-50 load sharing between the left and right decks regardless $\gamma_s$ in the absence of any stagger error $\epsilon_s$.

In Figure 3.14(a), a similar comparison is made between the measurements from gauges L8 and R8 of a gear pair having $\gamma_s = 0$ under a stagger error of $\epsilon_s = -72$ $\mu$m (i.e. the right deck tooth surfaces are lagging their left deck counterparts by 72 $\mu$m). In this case, the left side gauge L8 reads a maximum root stress of $\sigma_{left} = 145$ MPa, while the maximum stress registered by gage R8 is $\sigma_{right} = 95$ MPa, in comparison to the nominal maximum stress value of 120 MPa in Figure 3.13(b) for both gages. This indicates that the left deck carries nearly 21% more load while the right deck carries 21% less than it should. In other words, there is a 60-40 split between the right and left decks. In Figure 3.14(b), $\epsilon_s = 72$ $\mu$m in dicating that right deck tooth
Figure 3.13 Comparison of measured root stresses from gauges R8 and L8 at $T_p = 400$ Nm for $\varepsilon_\alpha = 0$. (a) $\gamma_s = 0$ and (b) $\gamma_s = \pi$. 

85
Figure 3.14. Comparison of measured root stresses from gauges R8 and L8 at $T_p = 400$ Nm for $\gamma_s = \pi$. (a) $\varepsilon_s = -72$ $\mu$m and (b) $\varepsilon_s = 72$ $\mu$m.
surfaces lead their left deck counterparts by that amount, the roles are switched completely to a 40-60 split between the ring and left decks.

Figure 3.15 compares the maximum $\sigma_{\text{right}}$ and $\sigma_{\text{left}}$ values measured by the respective gages for the axially fixed gear pair having $\gamma_s = \pi$ for a torque value of $T_p = 400$ Nm. Here Figures 3.15(a), 3.15(b) and 3.15(c) represent the comparisons of pairs R7-L7, R8-L8 and R9-L9, respectively, all as a function of the stagger error $\varepsilon_s$ that was varied between -108 and 108 $\mu$m. The $\sigma_{\text{right}}$ and $\sigma_{\text{left}}$ values are expected to be equal here for $\varepsilon_s = 0$. This is true for gauges R8 and L8 in Figure 3.15(b) where $\sigma_{\text{right}} = \sigma_{\text{left}} = 120$ MPa at $\varepsilon_s = 0$ while they are not equal for gauge pairs R7-L7 in Figure 3.15(a) and to a certain extend for R9-L9 in Figure 3.15(c). In Figure 3.15(a), gauge R7 reads 118 MPa while gage L7 reads only 91 MPa. The likely primary reason for these differences is the unavoidable mounting errors in gauges along the desired root locations. It was reported in earlier work on spur [3.6], helical [3.5] and hypoid [3.7] gears that the stress gradient is very steep in the root region of a gear such that even very small deviations from the intended mounting location can result in sizable differences in measured strains. The following main observations are made from Figure 3.15.

- A positive stagger error $\varepsilon_s$, meaning the profiles of the right deck teeth lead the profiles of the left deck teeth by this error amount, the right deck carries more load. Likewise, a negative $\varepsilon_s$ results in more loads on the left deck.

- The maximum stresses carried by the left and right deck teeth vary linearly with $\varepsilon_s$, at least nominally, crossing each other at $\varepsilon_s = 0$.

- From the slope of the curves in Figure 3.15, one can determine the amount of stress that must be accounted for as a function of $\varepsilon_s$. At the center of the teeth, for instance as
Figure 3.15. Comparison of maximum root stress values from the respective right and left deck gauges for $T_p = 400$ Nm and $\gamma_s = \pi$ under the axially fixed conditions. (a) Gauges R7 and L7, (b) gauges R8 and L8, and (c) gauges R9 and L9.
Figure 3.15 Continued.

![Graph showing the relationship between stagger error and maximum stress](image)

- **σ_{max} [MPa]**
- **Gauge R9**
- **Gauge L9**

stagger error, \( \varepsilon_s [\mu m] \)
depicted in Figure 3.15(b), the slope for the maximum stress of the right flank versus error curve is 0.23 error. With this, a 100 \( \mu \text{m} \) of stagger error resulting in 23 MPa additional stress. Likewise, the slope for the maximum stress versus \( \varepsilon_s \) for the left deck is the negative of the other curve, i.e. \(-0.23 \text{ MPa/\( \mu \text{m} \)}\).

Another set of measured error sensitivity curves are shown in Figure 3.16 for the same gear set except at \( T_p = 200 \text{ Nm} \). All of the above observations apply to this set of data as well including the same slope values. The only difference between these curves and those in Figure 3.15 is that they are shifted downward such that the nominal stresses at \( \varepsilon_s = 0 \) are lower. For instance, the nominal stress value along the middle gages is 60 MPa for \( \varepsilon_s = 0 \). This indicates that a stagger error sensitivity for a given DH gear pair can be established to be valid for all load values. It is also noted that here for this torque level that with the established slopes, one would need \( 60 / 0.23 = 260 \) \( \mu \text{m} \) of stagger error to prevent one deck completely from carrying any load while the other deck carries twice the load.

### 3.3 Simulation of Quasi-static Double-Helical Gear Pair Experiments and Model Validation

In this section, some of the experiments presented in the previous section will be simulated by using a double helical gear load distribution model [3.1]. This model was developed as an extension of a single mesh helical gear load distribution model [3.8] by performing simultaneous load distribution analysis of both right and left side gear meshes. This model included only axial and torsional degrees of freedom of the DH gear pair. A semi-analytical compliance formulation that included tooth bending, Hertzian contact, shear deformations and base rotation effects are combined with a modified simplex algorithm to predict
Figure 3.16. Comparison of maximum root stress values from the respective right and left deck gages for $T_p = 200$ Nm and $\gamma_s = \pi$ under the axially fixed conditions. (a) Gauges R7 and L7, (b) gauges R8 and L8, and (c) gauges R9 and L9.
Figure 3.16 Continued.

![Graph showing the relationship between stagger error and maximum stress.](image)

- The graph shows the maximum stress ($\sigma_{max}$) in MPa plotted against stagger error ($\varepsilon_s$) in micrometers.
- Two lines are shown: one for Gauge R9 (blue line) and one for Gauge L9 (red line).
- The maximum stress decreases as the stagger error increases for Gauge R9, while it increases for Gauge L9.
contact load distributions. With this information in hand, the static TE, contact stresses and root stresses as well as axial motions of gears are calculated.

The measured tooth profile shapes shown in Figure 3.6 and 3.7 are used in the model for simulation of TE, axial play and root stresses under the same torque conditions. As the model is a quasi-static one (i.e. no dynamic effects are included), the low-speed experiments of the previous section are suitable here.

3.3.1 Simulation of the Static Transmission Error Measurements

In this section, some of the TE measurements presented in Section 3.2.1 are simulated. In Figure 3.17, the predicted and measured fundamental harmonic amplitudes ($H_1$) are compared as a function of the stagger angle value are shown under five different torque values of $T_p = 100, 200, 300, 400$ and $500$ Nm. It is clear from these figures that the influence of $\gamma_s$ is predicted accurately by the model with the minimum $H_1$ value predicted to occur for the gear pair having $\gamma_s = \pi$ while $\gamma_s = 0$ results in the highest $H_1$ value. With the exception of the values at $\gamma_s = \pi$, the model over-predicts the $H_1$ values. This is expected since the semi-analytical model does not include some of compliances associated with gear blanks and support structures, i.e. the model is more rigid that the actual system. Similar differences were reported by Tamminana and Kahraman [3.9] who compared the single mesh version of this model [3.8] to a finite element based-deformable body contact model. Also noted in these figures that the influence of $T_p$ on $H_1$ is captured by the model.
Figure 3.17. Comparisons of the measured and predicted fundamental harmonic amplitudes of TE presented as a function of the stagger angle $\gamma_s$ at (a) $T_p = 100$ Nm, (b) 200 Nm, (c) 300 Nm, (d) 400 Nm, and (e) 500 Nm.
Figure 3.17 Continued.

(c) $H_1$ [μm]

(d) $H_1$ [μm]

stagger angle, $\gamma_s$ [rad]
Figure 3.17 Continued.

\[ H_1 \] measured
\[ H_1 \] model [3.1]

stagger angle, \( \gamma_s \) [rad]
The corresponding figures for the second harmonic amplitude $H_2$ of TE are presented in Figure 3.18. The predicted $H_2$ values are minimum when $\gamma_s = \frac{\pi}{2}$, which is in agreement with the measurements.

Figure 3.19 is presented next using the model to compare directly to the measurements of Figure 3.5 in the same format. This further confirms the ability of the model in capturing the effects of $T_p$ and $\gamma_s$ on the $H_1$ and $H_2$ amplitudes. The differences $|H_{1,\text{predicted}} - H_{1,\text{measured}}|$ and $|H_{2,\text{predicted}} - H_{2,\text{measured}}|$ between Figures 3.5 and 3.19 are presented in Figure 3.20 as a function of $T_p$ and $\gamma_s$. Differences between the measured and predicted values of $H_1$ and $H_2$ are less than 1 $\mu$m and 0.2 $\mu$m, respectively.

Overall, the comparisons presented in Figures 3.17 to 3.20 indicate that the DH load distribution model of Thomas [3.1] is sufficiently accurate for engineering purposes in predicting the static transmission amplitudes with torque and stagger error effects captured accurately.

### 3.3.2 Simulation of the Axial Motion Measurements

The same model is used here to simulate some of the experiments from Section 3.2.2 in order to predict the axial motion of the 34-tooth gear that was allowed to slide on its shaft freely. Here, the measurements from five DH gear pairs each representing a different stagger angle value (as presented in Figure 3.8 earlier) are simulated. Figures 3.21(a-e) compares axial motion predictions to measurements at $T_p = 100$ Nm. It is evident in these figures that the model predictions are rather accurate. The peak-to-peak amplitudes and the overall ‘triangular’ shapes of the time histories are predicted accurately. The experimentally observed decay in axial play with a reduction in $\gamma_s$ is also predicted accurately. Only obvious difference in these comparisons
Figure 3.18. Comparisons of the measured and predicted second harmonic amplitudes of TE presented as a function of the stagger angle $\gamma_s$ at (a) $T_p = 100$ Nm, (b) 200 Nm, (c) 300 Nm, (d) 400 Nm, and (e) 500 Nm.
Figure 3.18 Continued.

H_2 [μm] vs. stagger angle, \(\gamma_s\) [rad].

- (c) Measured data and model [3.1].
- (d) Measured data and model [3.1].
Figure 3.18 Continued.

\[ H_2 [\mu m] \]

\( \gamma_s \) [rad]

- Blue square: measured
- Red square: model [3.1]
Figure 3.19. Three-dimensional representations of the predictions of Figures 3.17 and 3.18 showing combined influence of torque and stagger angle on (a) fundamental and (b) second harmonic amplitudes of the measured transmission error.
Figure 3.20. The difference between measured and predicted (a) fundamental harmonic amplitudes and (b) second harmonic amplitudes of TE as a function of $T_p$ and $\gamma_s$. 
Figure 3.21. Comparisons of the measured and predicted axial motions for (a) $\gamma_s = 0$, (b) $\pi/4$, (c) $\pi/2$, (d) $3\pi/4$, and (e) $\pi$. $T_p = 100$ Nm.
Figure 3.21 Continued.

(c) measured model [3.1]

(d) measured model [3.1]
Figure 3.21 Continued.

Axial Motion [μm] vs. mesh cycles
is in Figure 3.21(a) for \( \gamma_s = 0 \) where amplitudes of measured axial play are within \( \pm 2 \mu m \) while predicted amplitudes are nearly half of it. While the predictions exhibit periodicity at the gear mesh period, measurements reveal higher frequency fluctuations that are attributable to the micron level surface undulations and machining marks. These effects could be filtered out from the experimental data to improve on the match between the predictions and measurements further.

### 3.3.3 Simulation of the Root Stress and Load Sharing Measurements

Predicted maximum root stress distributions of the left and right deck teeth of the 34-tooth gear of the DH gear pair are shown in Figures 3.22 and 3.23, respectively, at a torque value of 400 Nm with the gears staggered at \( \gamma_s = \pi \). Here, the root zone ranging from the start of the active profile (SAP) to the root center (RC) are covered within the radius range of 74.0 mm (SAP) and 71.7 mm (RC) along the y axis while the x axis range covers the face width of the gear. Only the stresses of the 34-tooth gear are provided since this was the gauged gear in the experiments. While quite similar (and should be nominally identical), there are minor differences between the stress distributions of the right and left sides. This is primarily due to the tooth surface deviations (modifications) present on these respective tooth surfaces as shown in Figures 3.6 and 3.7. The lower contour plots in these figures mark the designed (nominal) locations of the three gauges along the root. Looking at the locations of these gauges, it is obvious that the middle gauge should register higher stress values. It is also evident that the stress gradient along the radial direction (and to a certain extent, along the face width) is very steep. As such, minor mounting errors for these gauges would result in sizable differences in measured stresses. For instance, the maximum root stress of the center gauge in Figure 3.22 at its nominal position (at radius of 73.1 mm) is 120 MPa while an upward shift of the gauge by 0.1 mm (to radius of 73.2 mm) would
Figure 3.22. Predicted root stress distributions of left deck of the 34-tooth axially floating gear \( \gamma_s = \pi \) and \( T_p = 400 \) Nm. (a) The 3D plot, and (b) the contour plot with nominal gage locations marked by squares.
Figure 3.23. Predicted root stress distributions of right deck of the 34-tooth axially floating gear $\gamma_s = \pi$ and $T_p = 400$ Nm. (a) The 3D plot, and (b) the contour plot with nominal gage locations marked by squares.
correspond to 106 MPa. This indicates that the root stress measurements must be viewed with such sensitivity in mind.

Figure 3.24 compares the measured and predicted root stress time histories at left deck locations of gauges L7, L8 and L9 for the axially floating pinion having $\gamma_s = \pi$ at $T_p = 400$ Nm. The same is presented in Figure 3.25 for the right deck locations of gages R7, R8 and R9. It is evident from these comparisons that the model [3.1] is in good agreement with the measurements. The exception to this is the root stress time history of gauge R7 (Figure 3.25(a)), which shows about 20% difference between the measured and predicted maximum stress values, most likely due to inherent mounting error associated with this gage. Overall shapes of the signals as well as the duration within which the gauge is loaded are all predicted reasonably well.

Similar comparisons are presented in Figure 3.26 and 3.27 for the left and right deck locations of gauges for the axially fixed gear pair having $\gamma_s = \pi$ at $T_p = 400$ Nm. This gear pair has zero stagger error. Again with the exception of gage R7 (Figure 3.27(a)), all predicted root stress time histories are in good agreement with the model [3.1].

As a final set of simulations, the experiments of Figure 3.15(b) are simulated to assess the ability of the DH load distribution model [3.1] in predicting right-to-left load sharing. In Figure 3.28, measurements from gauges R8 and L8 are compared to predictions within a range of $\varepsilon_s \in [-108, 108] \mu$m the axially fixed gear pair operating at $T_p = 400$ Nm. While some deviations are present, the model is seen to capture the influence of $\varepsilon_s$ on the stresses experienced by the respective right and left deck gauges rather well, suggesting that the load sharing predictions of this model can also be trusted.
Figure 3.24. Measured versus predicted root stress time histories at $T_p = 400$ Nm for the axially floating pinion having $\gamma_s = \pi$. (a) Gauge L7, (b) gauge L8, and (c) gauge L9.
Figure 3.24  Continued.

![Graph showing stress vs. mesh cycles](image)

- **σ [MPa]**
  - Measured
  - Model [3.1]

**mesh cycles**
Figure 3.25. Measured versus predicted root stress time histories at $T_p = 400$ Nm for the axially floating pinion having $\gamma_s = \pi$. (a) Gauge R7, (b) gauge R8, and (c) gauge R9.
Figure 3.25  Continued.

![Graph showing stress (σ) vs. mesh cycles. The graph compares measured data with the model [3.1].]
Figure 3.26. Measured versus predicted root stress time histories at $T_p = 400$ Nm for the axially fixed gear pair having $\gamma_s = \pi$ and $\varepsilon_s = 0$ μm. (a) Gauge L7, (b) gauge L8, and (c) gauge L9.
Figure 3.26 Continued.

![Graph showing stress (σ) vs. mesh cycles](image)

- **σ [MPa]**
- mesh cycles

- Blue line: measured
- Red line: model [3.1]
Figure 3.27. Measured versus predicted root stress time histories at $T_p = 400$ Nm for the axially fixed gear pair having $\gamma_s = \pi$ and $\varepsilon_s = 0$ μm. (a) Gauge R7, (b) gauge R8, and (c) gauge R9.
Figure 3.27 Continued.

![Graph showing stress (σ) vs. mesh cycles. The graph compares measured and model [3.1] data.]
Figure 3.28. Comparison of measured and predicted maximum root stresses measured by gauges L8 and R8 as a function of stagger error. $T_p = 400$ Nm and an axially fixed gear pair having $\gamma_s = \pi$. 
3.4 Summary

In this chapter, sets of measurements from DH gear pairs operated under loaded, low speed (quasi-static) conditions were presented. They included the static transmission error, axial motions of gears, root stresses and stress-based right-to-left load sharing. Torque transmitted, stagger angle, stagger error as well as the axial gear boundary conditions were varied in these tests to quantify their impact on the measured performance parameters. Some of these experiments were simulated by using the model of Thomas [3.1] to better describe the measured trends as well as assessing the accuracy of the model. The comparisons presented in the previous section indicate that the model is indeed accurate in predicting all of these measured parameters.

References for Chapter 3


[3.8] LDP Gear Load Distribution Program, 2011, Gear and Power Transmission Research Laboratory, The Ohio State University, USA.

CHAPTER 4

DYNAMIC BEHAVIOR OF A DOUBLE-HELICAL GEAR PAIR

4.1 Introduction

This chapter investigates the dynamic behavior of double-helical (DH) gear pairs focusing on the influences of the stagger angle and the axial support conditions on the dynamic response. In the next section, results of sets of dynamics experiments performed using the methodologies and instrumentation described in Chapter 2 are presented. A dynamic model is proposed in Section 4.3, followed by Section 4.4 that compares measured and predicted response parameters.

4.2 Experimental Results

The results of the experimental study on the dynamic behavior of DH gear pairs are presented in this section. They include dynamic transmission error $\delta_d$ (defined by Eq. (2.5)), translations of each gear $j$ along the line-of-action ($q_{LAj}$ defined by Eq. (2.8)) and off-line-of-action ($q_{OLAj}$ defined by Eq. (2.9)) directions based on the data collected and analyzed through the accelerometer-based measurement system described in Section 2.4.3 and shown in Figure 2.18(b). Additional gear vibration amplitudes in the axial ($z_j$ defined by Eq. (2.13)) and the
rocking (\(\psi_j\) as defined by Eq. (2.11)) directions are also studied. Also included are dynamic factors based on root stresses measured through the strain gauge measurement system shown in Figure 2.22 and 2.23.

The test matrix considered in the dynamics experiments was designed to quantify the impact of the right-to-left stagger angle \(\gamma\) and the applied torque \(T_p\) within a range of the pinion speed (\(\Omega_p = 500\) to 6000 rpm). This speed range was deemed to be sufficiently wide to capture the main resonance activity exhibited by the gear pair.

### 4.2.1 Measured Dynamic TE and 3D Gear Vibration Results

No vibration data was presented at speeds below 500 rpm since the computation the vibration amplitudes is based on double integration of the measured acceleration signals. Each vibration amplitude at any given steady-state speed increment was processed in the frequency domain to compute the root-mean-square (rms) amplitude. For this, instead of using the true rms of a given vibration signal \(H\), a gear-mesh-harmonics-based rms amplitude was adapted. By considering the first three mesh harmonic amplitudes \(H_1\), \(H_2\) and \(H_3\), the rms amplitude of \(H\) is defined here as

\[
\hat{H} = \sqrt[3]{\sum_{i=1}^{3} H_i^2}
\]

(4.1)

where an over-cap indicates the rms amplitude.
Dynamic Transmission Error: Figure 4.1 shows an example set of measured $\delta_d$ for a DH gear pair having $\gamma_s = 0$ at $T_p = 400$ Nm and various speed increments within $\Omega_p \in [500, 6000]$ rpm. This corresponds to a gear mesh frequency range of $f_m \in [283, 3400]$ Hz. It is seen that the measured $\delta_d$ signals are defined predominantly by the gear mesh fundamental harmonic (marked as $f_m$) and its higher harmonics ($2f_m$, $3f_m$, …). With that, the root-mean-square value $\hat{\delta}_d$ calculated according to Eq. (4.1) can be used to quantify the dynamic TE amplitude at a given speed.

Within the speed (mesh frequency) range of $\Omega_p \in [500, 6000]$ rpm ($f_m \in [285, 3400]$ Hz) with an increment of 100 rpm (56.7 Hz), the measured $\hat{\delta}_d$ values of the gear sets having $\gamma_s = 0$ (in-phase), $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$ and $\pi$ (180° out of phase) and axially floating conditions are compared in Figure 4.2. The steady-state forced response of the DH gear pair was the main focus here. As such, $\hat{\delta}_d$ values presented in Figure 4.2 were measured under the steady-state (constant speed and torque values) condition. It is customary in spur gear experiments of this kind (e.g. [4.1]) to perform tests with a negative speed increment to capture the regimes of double steady-state motions due to softening nonlinearity caused by tooth separations. This was not needed here since the DH gear pairs tested did not exhibit such nonlinearities, an observation that is in line with other single-helical gear pair experiments (e.g. [4.2]).

Several key observations can be made from Figure 4.2. First of all, a prominent resonance peak at 2150 Hz (about 3800 rpm) is evident regardless of the value of the stagger angle $\gamma_s$. As it will be confirmed later, this is due to a natural mode (called mode A here whose mode shape of it will be presented in Section 4.4) at frequency $f_A$ excited by the fundamental mesh harmonic of the gear mesh excitations, leading to the resonance at $f_m \approx f_A$. It is clear
Figure 4.1. Example frequency spectra of $\delta_d$ measured at (a) $f_m = 567$ Hz, (b) $f_m = 1133$ Hz, (c) $f_m = 1700$ Hz, (d) $f_m = 2267$ Hz, (e) $f_m = 2833$ Hz, and (f) $f_m = 3400$ Hz. $\gamma_s = 0$ and $T_p = 400$ Nm.
Figure 4.2. Measured root-mean-square values of the dynamic transmission error $\delta_d$ as a function of the gear mesh frequency $f_m$ at various stagger angle $\gamma_s$ values. $T_p = 400$ Nm.
from the data that the resonance peak is the largest for the gear pair having $\gamma_s = 0$ (in-phase) and the lowest for the gear pair having $\gamma_s = \pi$ (180° out of phase) with the $\delta_d$ amplitudes for other three gear pairs having $\gamma_s = \frac{\pi}{4}, \frac{\pi}{2}, \text{and} \frac{3\pi}{4}$ lining up in between in this order. This is in agreement with Figure 3.4(a) that showed measured relation between the fundamental harmonic amplitude ($H_1$) of the static transmission error $\delta_s$ and the stagger angle $\gamma_s$. In Figure 3.4(a), $H_1 = 2.1, 1.8, 1.3, 0.7$ and $0.4$ μm for $\gamma_s = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ and $\pi$, respectively. In Figure 4.2, meanwhile, the amplitudes of the dynamic transmission error $\delta_d$ at $\gamma_s = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ and $\pi$ are $7.2, 7.0, 5.0, 2.4, 1.8$ μm, respectively, indicating a near proportionality between $\delta_d$ and $\delta_s$. This is a significant observation as it indicates that the system behaves linearly and $\delta_s$ can be considered as the excitation for $\delta_d$. The same level of sensitivity of $\delta_s$ and $\delta_d$ to $\gamma_s$ is also important from the design point of view.

The second, somewhat smaller, resonance peak observed at 1075 Hz (1900 rpm) is associated with the second harmonic of gear mesh excitations and the same mode A. This resonance condition is $2f_m \approx f_A$ (or $f_m \approx \frac{1}{2} f_A$). The amplitudes of $\delta_d$ at this peak are influenced by $\gamma_s$ the same way the second harmonic amplitude ($H_2$) of $\delta_s$ were shown to be influenced by $\gamma_s$ in Figure 3.4(b). Both $H_2$ and amplitudes of the $2f_m \approx f_A$ resonance peak are the highest when $\gamma_s = 0$ and $\pi$. Likewise, they are both minimum when $\gamma_s = \frac{\pi}{2}$. In fact, there is no sign of a resonance at this speed for $\gamma_s = \frac{\pi}{2}$ in Figure 4.2. As such, the conclusions drawn in Chapter 3 between the harmonic amplitudes of $\delta_s$ and $\gamma_s$ can be extended to $\delta_d$ as well.
Some other resonance activity is also evident in Figure 4.2 below a mesh frequency of 700 Hz (1200 rpm). These resonances will be investigated further in Section 4.4 with the help of the model predictions.

**Axial Vibrations**: The same methodology used to obtain the frequency response curves for $\hat{\delta}_d$ is applied next to the axial vibration signals. Figure 4.3 shows example frequency spectra of the axial vibrations of the pinion $z_p(t)$ at pinion speed values of 1000 to 6000 rpm with an increment of 1000 rpm (567 to 3400 Hz mesh frequency with an increment of 567 Hz). While not as clean as the $\delta_d$ spectra in Figure 4.1, these spectra can also be characterized as line spectra dominated by the gear mesh harmonic amplitudes, primarily the fundamental order amplitude $H_1$ at frequency $f_m$. Variation of $H_1$ with speed from spectrum to spectrum indicates that these axial motions are impacted by the dynamics of the gear pair. Figure 4.4 shown the measured steady-state rms $z_p$ and $z_g$ amplitudes ($\hat{z}_p$ and $\hat{z}_g$) as a function of mesh frequency for the same five stagger angles of $\gamma_s = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \text{ and } \pi$. In Figure 4.4(a), $\hat{z}_p$ is observed to be consistently the lowest when $\gamma_s = 0$ and the largest when $\gamma_s = \pi$ with $\hat{z}_p$ for other three stagger angles falling in between the envelope of these two curves. This again is in total agreement with the quasi-static axial motion measurements presented in Figure 3.8 and 3.9 (and predicted by the quasi-static DH load sharing model in Figure 3.21). In those figures, the quasi-static $\hat{z}_p$ values were shown to be the largest at $\gamma_s = \pi$ and the lowest for $\gamma_s = 0$. This is again very useful
Figure 4.3. Example frequency spectra of \( z_p \) measured at (a) \( f_m = 567 \) Hz, (b) \( f_m = 1133 \) Hz, (c) \( f_m = 1700 \) Hz, (d) \( f_m = 2267 \) Hz, (e) \( f_m = 2833 \) Hz, and (f) \( f_m = 3400 \) Hz. \( \gamma_s = \pi \) and \( T_p = 400 \) Nm.
Figure 4.4. Measured root-mean-square values of axial vibration of (a) the pinion and (b) the gear as a function of the gear mesh frequency $f_m$ at various stagger angle $\gamma_s$ values. $T_p = 400$ Nm.
information in regards to employment of static transmission error as the primary excitation for the
dynamic model of a DH gear pair.

The largest resonance peak in Figures 4.4 occurs at a frequency of \( f_m = 510 \) Hz (a pinion
speed of 900 rpm). The model proposed in the next section will be used to describe what kind of
a mode (called mode B at \( f_m = f_B \)) is excited to form this resonance.

**Rocking Vibrations:** Figure 4.5 shows the measured frequency response in terms the rms
amplitudes of the rocking motions of the pinion and gear (\( \hat{\Psi}_p \) and \( \hat{\Psi}_g \)). As these motions are
rotational with units in radians, it is not easy to compare them in magnitude to other motion
amplitudes (\( \hat{\delta}_d, \hat{z}_j, \hat{q}_{LAj} \) and \( \hat{q}_{OLAj} \)) that are translations. In order to circumvent this, these
amplitudes were multiplied by the respective base circle radii to convert them to equivalent
translations at the gear mesh normal to the transverse plane of the gears. Both \( r_p \hat{\Psi}_p \)
and \( r_g \hat{\Psi}_g \) in Figures 4.5(a) and (b) exhibit magnitudes that are less than 1.5 μm regardless of \( f_m \) and \( \gamma_s \),
suggesting that support conditions where the test gears were prohibitive of such motions.
However it is still clear that the \( r_p \hat{\Psi}_p \) and \( r_g \hat{\Psi}_g \) amplitudes are typically the largest for \( \gamma_s = \pi \)
and the lowest for \( \gamma_s = 0 \). This confirms further that choosing an out-of-phase gear set to
minimize transmission error comes with the side effects of maximized axial and rocking motions.

**Line-of-Action and Off-Line-of-Action Vibrations:** Measured rms line-of-action (\( \hat{q}_{LAj} \)) and off-
line-of-action (\( \hat{q}_{OLAj} \)) translational vibration amplitudes of each gear \( j (j = p, g) \) are presented
next in Figures 4.6 and 4.7 for the same conditions and frequency range as Figures 4.2, 4.4 and
4.5. In Figure 4.6, \( \hat{q}_{LAj} \) are plotted against \( f_m \) for all five \( \gamma_s \) values. The
qualitative shapes of these frequency response curves resemble those for \( \hat{\delta}_d \) in Figure 4.2.
Figure 4.5. Measured root-mean-square values of rocking vibration of (a) the pinion and (b) the gear as a function of the gear mesh frequency $f_m$ at various stagger angle $\gamma_s$ values. $T_p = 400$ Nm.
Figure 4.6. Measured root-mean-square values of line of action translational vibration of (a) the pinion and (b) the gear as a function of the gear mesh frequency $f_m$ at various stagger angles $\gamma_s$ values. $T_p = 400$ Nm.
Figure 4.7. Measured root-mean-square values of off line of action translational vibration of (a) the pinion and (b) the gear as a function of the gear mesh frequency $f_m$ at various stagger angle $\gamma_s$ values. $T_p = 400$ Nm.
Resonance peaks occur at the same frequencies as the same modes are excited in these curves as well. Specifically, mode A is excited by the fundamental and second harmonic amplitudes of the gear mesh excitation at \( f_m = f_A = 2150 \text{ Hz} \) and \( 2f_m = f_A = 1075 \text{ Hz} \). Furthermore, the same resonance frequencies below 700 Hz are also evident. This indicates that the modes excited to form the resonance peaks in the frequency responses of \( \hat{\delta}_d \) and \( \hat{q}_{LAj} \) are primarily transverse-torsional. Mode A that is more prominent in Figure 4.2 is likely to be dominated by torsional motions to yield large peaks in \( \hat{\delta}_d \) response and the modes below 700 Hz are likely to be more translational such that \( \hat{q}_{LAj} \) resonances associated with them are more significant.

The sensitivity of \( \hat{q}_{LAj} \) in Figure 4.6 to \( \gamma_s \) is essentially the same as the sensitivity of \( \hat{\delta}_d \) to \( \gamma_s \). Gear having \( \gamma_s = 0 \) appears to cause the largest \( \hat{q}_{LAj} \) while an increase in \( \gamma_s \) seemingly reducing the \( \hat{q}_{LAj} \) amplitudes. Yet, the overall line-of-action translations are rather small, less than 2 \( \mu \text{m} \) for \( \hat{q}_{LAp} \) and less than 3 \( \mu \text{m} \) for \( \hat{q}_{LAG} \). This indicates that the support structure of the test gears, namely the shafts and the bearings are rigid enough to limit bouncing motions of the gears to a few microns.

A similar behavior is observed for the off-line-of-action vibration amplitudes represented by \( \hat{q}_{OLAj} \) as shown in Figure 4.7. This is true both in terms of overall vibration amplitudes as well as the sensitivity to \( \gamma_s \).

**Influences of Torque Transmitted:** Measured frequency response curves presented up to this point were taken at a pinion torque value of \( T_p = 400 \text{ Nm} \). In this section, these frequency
response curves will be compared to those at $T_p = 200$ and 600 Nm to quantify the impact of torque on the forced response amplitudes.

In general, there are two ways the torque transmitted can impact the dynamic response of a gear pair. One has to do with its impact on the excitation, namely the static transmission error $\delta_s$. This was demonstrated in Figure 3.5 experimentally and then in Figure 3.19 theoretically. If $\delta_s$ is the excitation of the dynamics of the gear set, then the influence of the torque transmitted on $\delta_s$ will be reflected on its influence on $\delta_d$. With the tooth modifications shown in Figures 3.6 and 3.7, a design torque (the torque level at which $\delta_s$ is minimum) for the gear pair tested is at a torque value beyond 500 Nm. As such, increase in $T_p$ up to 500 Nm is seen to reduce the fundamental harmonic amplitude $H_1$ of $\delta_s$. The forced response curves shown in Figure 4.8 reflect this effect accurately. The response amplitudes $\hat{\delta}_d$ in Figure 4.8(a), $\hat{\delta}_p$ in Figure 4.8(b), $r_p \hat{\psi}_p$ in Figure 4.8(c), $\hat{q}_{LAp}$ in Figure 4.8(d) and $\hat{q}_{OLAp}$ in Figure 4.8(e) all have the largest amplitudes at $T_p = 200$ Nm while the curves for $T_p = 600$ Nm represent the lowest values.

The second way the torque transmitted impacts the dynamic response is through its influence on the average gear mesh stiffness. Increase in torque brings more areas along the gear mesh in, to contact, in the process increasing the effective contact ratio (average number of tooth pairs in contact) and hence the average stiffness of the gear mesh. With this, most of the natural frequencies can be expected to increase with an increase in torque transmitted. This is quite obvious in Figure 4.8. For instance, in Figure 4.8(a), the resonance peak at $f_m = 2070$ Hz for $T_p = 400$ Nm is shifted to $f_m = 1925$ Hz for $T_p = 200$ Nm (a 7% reduction in resonance
Figure 4.8. Comparison of measured (a) $\hat{\delta}_d$, (b) $\hat{z}_p$, (c) $r_p\hat{\Psi}_p$, (d) $\hat{q}_{LAp}$, and (e) $\hat{q}_{OLAp}$ at $T_p = 200, 400, \text{ and } 600 \text{ Nm}$. $\gamma_s = \pi$. Continued
Figure 4.8. Continued.
Figure 4.8. Continued.

(e)
frequency) and to $f_m = 2240$ Hz (an 8% increase in resonance frequency) for $T_p = 600$ Nm. The same behavior can be seen in other response curves and other resonance peaks as well.

Overall conclusion from Figure 4.8 is that, even with a linear system, the influence of the torque transmitted cannot be neglected. The values of the gear mesh stiffness and the static transmission error excitation must be evaluated at each torque value individually.

### 4.2.2 Measured Dynamic Stress Factors

Measurements from the strain gauge system made under dynamic conditions are presented in this section. Here a quasi-static ($\Omega_p = 100$ rpm) root stress signal ($\sigma_s$) from any particular gauge at a given torque $T_p$ is compared to the corresponding dynamic stress ($\sigma_d$) at a higher speed to calculate a stress-based dynamic factor ($DF$) for the right and left decks of the DH gear pair. $DF$ is defined here as the ratio of the maximum values of $\sigma_d$ and $\sigma_s$ such that

$$DF = \frac{(\sigma_d)_{\text{max}}}{(\sigma_s)_{\text{max}}}.$$  

Here, signals from two root center gauges L8 and R8, one from the left side and other from the right side of the DH gear pair, were considered. A low-pass filter of 25 kHz was applied in order to remove any high frequency noise. The gear mesh frequency $f_m$ was kept below 2830 Hz ($\Omega_p = 5000$ rpm) throughout these measurements. A sampling frequency of $f_s = 75$ kHz was applied to provide sufficient resolution regardless of the pinion speed, allowing at least 25 data points per mesh cycles.

Figure 4.9(a) shows the measured steady state $DF$ data for $\gamma_s = 0$ and $T_p = 200$ Nm under axially floating pinion condition. The $DF$ response curves of left and right deck exhibit
Figure 4.9. Dynamic factors of the right and left decks of the gear pair measured by using gauges R8 and L8 at (a) $T_p = 200$ Nm and (b) 400 Nm for $\gamma_s = 0$. 
resonance peaks at $f_m = 1925$ Hz due to the same resonance condition exhibited in Figure 4.8(a) at $T_p = 200$ Nm. Here the maximum $DF$ value reaches 2.1. Considering $(\sigma_s)_{\text{max}} = 74.5$ MPa at $T_p = 200$ Nm, this maximum $DF$ value corresponds to $(\sigma_d)_{\text{max}} = 153.4$ MPa at $f_m = 1925$ Hz. Similar levels of $DF$ are measured at the right and left decks of the gear pair with the minor differences originating mostly from the differences of their tooth modifications shown in Figures 3.6 and 3.7.

In Figure 4.9(b), $DF$ values at $T_p = 400$ Nm exhibit a resonance peak at $f_m = 2070$ Hz (3700 rpm) in line with Figures 4.2 and 4.8(a). The maximum value of $DF$ of gauge L8 reaches 1.5. Given $(\sigma_s)_{\text{max}} = 117.5$ MPa at $T_p = 400$ Nm, this maximum $DF$ value corresponds to $(\sigma_d)_{\text{max}} = 168.9$ MPa at $f_m = 2267$ Hz. As such, while $DF$ values are lower for the higher torque case, absolute values of $(\sigma_d)_{\text{max}}$ are significantly larger, indicating that use of $DF$ as a durability measure might be misleading.

A representative number root stress time histories of the left deck at various pinion speeds are shown in Figure 4.10 for $T_p = 400$ Nm. In Figure 4.10(a), the dynamic root stress at $f_m = 567$ Hz ($\Omega_p = 1000$ rpm) exhibit small oscillations about the static root stress. These oscillation (two per mesh cycle) increase slightly in amplitude where the pinion speed reach at $f_m = 1133$ Hz ($\Omega_p = 2000$ rpm). This indicates that the second harmonic of the gear mesh excitation causes these oscillations. In Figure 4.10(d) for $f_m = 2267$ Hz ($\Omega_p = 4000$ rpm), one period of such oscillation is evident per mesh cycle since the pinion speed is close to the first harmonic of the gear mesh excitation. The same phenomenon can also be found at the right deck, as shown in Figure 4.11.
Figure 4.10. Measured root stresses of gauge L8 at $T_p = 400$ Nm and (a) $f_m = 567$ Hz, (b) 1133 Hz, (c) 1700 Hz, and (d) 2267 Hz for $\gamma_s = 0$. Here, quasi-static stress curves at $f_m = 57$ Hz represent the quasi-static values.
Figure 4.11. Measured root stresses of gauge R8 at $T_p = 400$ Nm and (a) $f_m = 567$ Hz, (b) $1133$ Hz, (c) $1700$ Hz, and (d) $2267$ Hz for $\gamma_s = 0$. Here, quasi-static stress curves at $f_m = 57$ Hz represent the quasi-static values.
Figure 4.12 to 4.14 present the same type of steady state $DF$ data for the case of $\gamma_s = \pi$. In Figure 4.12(a) and 4.12(b), the $DF$ curves of the DH gear pair at $\gamma_s = \pi$ shows the similar behavior in terms of resonances frequencies. The main difference here is that $DF$ values are significantly lower for $\gamma_s = \pi$ in comparison to the gear pair having $\gamma_s = 0$. In Figures 4.12(a) and (b), $DF < 1.5$ for $T_p = 200$ Nm and $DF < 1.15$ for $T_p = 400$ Nm, respectively. This indicates that $\delta_d$ and $DF$ correlate each other in DH gear pairs. Similar correlations were reported earlier for spur gears [4.3, 4.4] gears. In addition, the conclusion reached earlier regarding the influence of $\gamma_s$ as the $\delta_d$ is valid for $DF$ as well.

Selected dynamic root stress time histories at $\gamma_s = \pi$ are compared to their quasi-static counterpart in Figures 4.13 and 4.14. Figures 4.13(d) and 4.14(d) at for $f_m = 2267$ Hz ($\Omega_p = 4000$ rpm) both exhibit one period of oscillation due to the excitation of the first harmonic of $\delta_s$ while their peak values of the oscillations are significantly reduced.

4.3 A Dynamic Model of a Double-Helical Gear-Shaft-Bearing System

4.3.1 Model Assumptions

A three-dimensional dynamic model of a DH gear pair is developed here to be correlated to the measurements presented in the previous section. This discrete (lumped parameter) model employs a number of modeling assumptions. The same assumptions were used by Sondkar and Kahraman [4.5, 4.6] to study the dynamics of a planetary gear set formed by double-helical gears. They include the following:
Figure 4.12. Dynamic factors of the right and left decks of the gear pair measured by using gauges R8 and L8 at (a) $T_p = 200$ Nm and (b) 400 Nm for $\gamma_s = \pi$. 
Figure 4.13. Measured root stresses of gauge L8 at $T_p = 400$ Nm and (a) $f_m = 567$ Hz, (b) 1133 Hz, (c) 1700 Hz, and (d) 2267 Hz for $\gamma_s = \pi$. Here, quasi-static stress curves at $f_m = 57$ Hz represent the quasi-static values.
Figure 4.14. Measured root stresses of gauge R8 at $T_p = 400$ Nm and (a) $f_m = 567$ Hz, (b) 1133 Hz, (c) 1700 Hz, and (d) 2267 Hz for $\gamma_s = \pi$. Here, quasi-static stress curves at $f_m = 577$ Hz represent the quasi-static values.
(i) The blanks (bodies) of the pinion and the gear are assumed to be rigid with the gear mesh compliance being represented by a linear spring acting on the plane of action in a direction normal to gear tooth surfaces (i.e. inclined by helix angle $\beta$).

(ii) Time-varying component of mesh stiffness due to fluctuation of number of tooth pairs in contact is neglected as this was shown to be a good assumption for helical gear pairs [4.2]. Likewise, any nonlinearities associated with backlash (tooth separations) are also neglected as helical gears fail to exhibit such phenomena [4.2] in contrast to spur gears that are strongly nonlinear [4.3, 4.4, 4.7, 4.8]. The same is evident from the data presented in the previous section as well. With these two assumptions, the proposed model can be characterized as a linear, time-invariant one dynamically.

(iv) Frictional forces due along the contact interfaces are neglected since to focus is the prediction of torsional, axial and the line-of-action motions of the gear pair. As shown in a study by Kahraman et al [4.9], the impact of tooth fiction on these types of motions is negligible. On the other hand, Li and Kahraman [4.10] showed using a tribo-dynamic model of a spur gear pair that the off-line-of-action motions are dictated solely by tooth traction, requiring a transient elastohydrodynamics lubrication model for its prediction. As such behavior is beyond the scope of this work, sliding friction taking place at the gear mesh contact zones will be neglected.

(v) Left and right gear pairs forming of double-helical gears are considered to have the same nominal geometry while the hands of the teeth are opposite and the teeth are staggered by a certain angle with respect to each other. Meanwhile, the differences in profile modifications (or manufacturing errors) of the right and left side gear teeth are captured through their impact on their static transmission error excitations.
(vi) A constant modal damping ratio of $\zeta$ is employed for the entire simulation work.

4.3.2 Single Gear Pair Model

The discrete dynamic model of the double-helical gear pair is shown in Figure 4.15. Here two separate helical gear pairs (one right-side gear pair and one left side gear pair) are connected to each other through a rigid flange. Focusing on the one of these pairs (say the left side) and assuming the gear is positioned at an angle $\alpha$ (measured positive in the counterclockwise direction from the $x$ axis), a 12-degree-of-freedom subsystem model can be developed for this pair as shown in Figure 4.16. Similar three-dimensional helical gear pair models were proposed earlier by Kahraman [4.11, 4.12] and Kubur et al [4.2] and adapted to planetary gear arrangements by Kahraman [4.13]. Both gears are assumed to have rigid blanks that are connected to each other by a linear gear mesh spring $k$ on the plane of the action in the normal direction determined by the helix angle $\beta$. Also, a displacement excitation in the form motion transmission error $\delta(t)$ is applied in the same direction of the linear spring $k$ as shown in Figure 4.16. Here, the plane of action makes an angle $\psi$ with positive vertical $y$-axis. As the plane of action changes direction depending on the direction of the applied external torque acting on the pinion, $\bar{\psi}$ is defined as

$$
\bar{\psi} = \begin{cases} 
\phi - \alpha, & T_p \text{ : counterclockwise} \\
-\phi - \alpha, & T_p \text{ : clockwise}
\end{cases}
$$

(4.3)

where $\phi$ is the transverse pressure angle of the gear pair and $T_p$ is the external torque exerted on the pinion. The sign of the helix angle $\beta$ is dependent on the hand of the pinion. i.e. $\beta$ is positive if the pinion has left hand teeth and negative if the pinion has right hand teeth.
Figure 4.15. A three-dimensional dynamic model of a double-helical gear pair.
Figure 4.16. A three-dimensional dynamic model of the left side helical gear pair.
Both gears are allowed to translate in $x$ and $y$ direction (on the transverse plane) and in the axial $z$ direction. In addition, they are allowed rotate about these three axes by $\theta_x$, $\theta_y$, and $\theta_z$, respectively. With this, the gear pair has a total of 12 degrees of freedom to define the coupling between the two shafts holding the gears. The equations of motion of the pinion (subscript $p$) of helical gear pair of Figure 4.16 are given as

$$m_p \ddot{y}_p(t) + k \beta \cos \psi_p(t) = 0, \quad (4.4a)$$

$$m_p \ddot{x}_p(t) + k \beta \sin \psi_p(t) = 0, \quad (4.4b)$$

$$m_p \ddot{z}_p(t) - k \sin \beta p(t) = 0, \quad (4.4c)$$

$$I_p \ddot{\theta}_{yp}(t) + kr_p \sin \beta \cos \psi_p(t) = 0, \quad (4.4d)$$

$$I_p \ddot{\theta}_{xp}(t) + kr_p \sin \beta \sin \psi_p(t) = 0, \quad (4.4e)$$

$$J_p \ddot{\theta}_{zp}(t) + kr_p \cos \beta p(t) = T_p \quad (4.4f)$$

where a double overdot indicates double derivative with respect to time (e.g. $\ddot{x} = \frac{d^2 x}{dt^2}$). Since damping will be introduced in modal form, these equations do not have damping terms.

Equations of motion of the gear (subscript $g$) of the same system are defined as

$$m_g \ddot{y}_g(t) - k \beta \cos \psi_p(t) = 0, \quad (4.5a)$$

$$m_g \ddot{x}_g(t) - k \beta \sin \psi_p(t) = 0, \quad (4.5b)$$

$$m_g \ddot{z}_g(t) + k \sin \beta p(t) = 0, \quad (4.5c)$$

$$I_g \ddot{\theta}_{yg}(t) + kr_g \sin \beta \cos \psi_p(t) = 0, \quad (4.5d)$$

$$I_g \ddot{\theta}_{xg}(t) + kr_g \sin \beta \sin \psi_p(t) = 0, \quad (4.5e)$$

$$J_g \ddot{\theta}_{zg}(t) + kr_g \cos \beta p(t) = 0 \quad (4.5f)$$
In these equations, \( m_j \), \( I_j \), and \( J_j \) are the mass, the diametral mass moment of inertia and the polar mass moment of inertia of one side of gear \( j \) (\( j = p, g \)), and \( r_j \) is base radius of gear \( j \). \( k \) is the average value of the gear mesh stiffness predicted by a gear load distribution model [4.14]. In these equations, \( p(t) \) represents the relative displacement at the gear mesh in the direction normal to contact surfaces. It has the form

\[
p(t) = [(y_p - y_g) \cos \Psi + (x_p - x_g) \sin \Psi + r_p \theta_{zp} + r_g \theta_{zg}] \cos \beta
\]

\[
+ [(r_p \theta_{zp} + r_g \theta_{zg}) \cos \Psi + (r_p \theta_{zg} + r_g \theta_{xzg}) \sin \Psi - z_p + z_g] \sin \beta - \delta(t).
\] (4.6)

It is noted that the static transmission error of the gear pair \( \delta(t) \) is included in \( p(t) \) as a displacement excitation. Equations (4.4)–(4.6) are written in matrix form as

\[
\begin{bmatrix}
M_p & 0 \\
0 & M_g
\end{bmatrix}
\begin{bmatrix}
\dot{q}_p(t) \\
\dot{q}_g(t)
\end{bmatrix} + k
\begin{bmatrix}
K^{11} & K^{12} \\
K^{12} & K^{22}
\end{bmatrix}
\begin{bmatrix}
q_p(t) \\
q_g(t)
\end{bmatrix} = \begin{bmatrix}
f_p(t) \\
f_g(t)
\end{bmatrix}
\] (4.7a)

where

\[
q_j(t) = \begin{bmatrix}
y_j(t) \\
x_j(t) \\
z_j(t) \\
\theta_{yj}(t) \\
\theta_{xj}(t) \\
\theta_{zj}(t)
\end{bmatrix}, \quad M_j = \text{Diag}[m_j \ m_j \ m_j \ I_j \ I_j \ J_j], \quad j = p, g \] (4.7b,c)

\[
K^{11} =
\begin{bmatrix}
c^2 \Psi \beta \ c \bar{\Psi} \bar{\beta} & -c \bar{\Psi} \bar{\beta} & r_p c^2 \Psi \beta \bar{\beta} & r_p c \bar{\Psi} \bar{\beta} & r_p c \Psi \beta \bar{\beta} & r_p c \bar{\Psi} \beta \bar{\beta} \\
-s^2 \Psi \beta \ -s \bar{\Psi} \bar{\beta} & r_p s^2 \Psi \bar{\beta} & r_p s \bar{\Psi} \beta \bar{\beta} & r_p s \Psi \beta \bar{\beta} & r_p s \bar{\Psi} \beta \bar{\beta} \\
-s^2 \bar{\Psi} \beta \ -s \Psi \bar{\beta} & r_p s^2 \bar{\Psi} \beta & r_p s \Psi \beta \bar{\beta} & r_p s \bar{\Psi} \beta \bar{\beta} & r_p s \Psi \bar{\beta} \bar{\beta} \\
-s^2 \beta & -r_p \Psi \beta \bar{\beta} & r_p \bar{\Psi} \beta \bar{\beta} & -r_p \Psi \bar{\beta} \bar{\beta} & r_p \bar{\Psi} \beta \bar{\beta} \\
r_p^2 c^2 \Psi \beta \bar{\beta} & r_p^2 \bar{\Psi} \bar{\beta} & r_p^2 s^2 \Psi \beta \bar{\beta} & r_p^2 s \bar{\Psi} \beta \bar{\beta} & r_p^2 s \Psi \beta \bar{\beta} \\
r_p^2 s^2 \bar{\Psi} \bar{\beta} & r_p^2 \bar{\Psi} \bar{\beta} & r_p^2 s \Psi \beta \bar{\beta} & r_p^2 s \bar{\Psi} \beta \bar{\beta} & r_p^2 s \Psi \beta \bar{\beta}
\end{bmatrix}
\]

\[\text{Sym.}\] (4.7d)
\[
\mathbf{K}^{12} = \begin{bmatrix}
-c^2 \psi \psi^2 \beta & -c \psi s \psi \psi^2 \beta & c \psi c \beta \beta & r_g c^2 \psi \psi \psi^2 \beta & r_g c \psi s \psi c \beta \beta & r_g c \psi \psi^2 \beta \\
-c \psi s \psi \psi^2 \beta & -s^2 \psi \psi^2 \beta & s \psi c \beta \beta & r_g c \psi s \psi \psi^2 \beta & r_g s^2 \psi \psi^2 \beta & r_g s \psi \psi^2 \beta \\
c \psi c \beta \beta & s \psi c \beta \beta & -s^2 \beta & -r_g c \psi s^2 \beta & -r_g s \psi^2 \beta & -r_g c \beta \beta \\
-r_p c^2 \psi \psi c \beta \beta & -r_p \psi s \psi \psi c \beta \beta & r_p c \psi s^2 \beta & r_p r_g c \psi s \psi \psi^2 \beta & r_p r_g s \psi c \beta \beta & r_p r_g \psi \psi \psi^2 \beta \\
r_p c s \psi c \beta \beta & -r_p s^2 \psi c \beta \beta & r_p \psi c \psi \psi^2 \beta & r_p r_g \psi \psi \psi^2 \beta & r_p r_g s \psi c \beta \beta & r_p r_g \psi \psi \psi^2 \beta \\
r_p c \beta \beta & -r_p s \psi c \beta \beta & r_p c s \beta \beta & r_p r_g c \psi s \psi \psi^2 \beta & r_p r_g s \psi c \beta \beta & r_p r_g \psi \psi \psi^2 \beta \\

\end{bmatrix}, \quad (4.7e)
\]

\[
\mathbf{K}^{22} = \begin{bmatrix}
c^2 \psi \psi^2 \beta & c \psi s \psi \psi^2 \beta & -c \psi c \beta \beta & -r_g c^2 \psi \psi \psi^2 \beta & -r_g c \psi s \psi c \beta \beta & -r_g c \psi \psi^2 \beta \\
-s^2 \psi \psi^2 \beta & s \psi c \beta \beta & -s^2 \beta & r_g c \psi s^2 \beta & -r_g s \psi^2 \beta & -r_g c \beta \beta \\
& & & -r_g c \psi s^2 \beta & -r_g s \psi^2 \beta & -r_g c \beta \beta \\
& & & r_g c \psi c \beta \beta & r_g \psi c \beta \beta & r_g c \psi \psi^2 \beta \\
& & & & & \text{Sym.} \\
& & & & & \\
\end{bmatrix}, \quad (4.7f)
\]

\[
f_p(t) = k \delta(t), \quad f_g(t) = k \delta(t).
\]

In Eq. (4.7d-h), \(c\psi \equiv \cos \psi\), \(s\psi \equiv \sin \psi\), \(c\beta \equiv \cos \beta\), and \(s\beta \equiv \sin \beta\).

### 4.3.3 DH Gear Pair Sub-System Model

The experimental setup shown in Figures 2.4(b), (2.6), and (2.7) consists of a DH gear pair (with right and left side gear pair connected to each other rigidly), two flexible shafts and two pairs of bearings which support the gear pair. As such, two additional formulations are required, one to connect the right and left gear pairs to each other to form a DH gear pair and one to account of the shaft and bearing flexibilities.
An actual double helical gear is usually created with one-piece of left and right sides of gears. In this dynamic model, each double helical gear is modeled as a three-piece assembly to utilize the single helical gear pair model described in the previous section. As done earlier by Ajmi and Velex [4.15] and Sondkar and Kahraman [4.5, 4.6], left and right sides of a gear are connected by using Euler beam elements. Figure 4.17 show that a DH gear consists of three pieces: the left side gear, right side gear, and a connecting structure consisting of beam elements. In Figure 4.17(b), the connecting structure between left side and right side gears span between gear face width mid-points. Here, partitioning is done such that the total mass and inertia of the one-piece DH gear equals the sum of individual masses and inertias of the three pieces forming it.

Representing the connecting structure by two beam elements, each of length \( L_{ec} \) and outside and bore diameters of \( r_{ec} \) and \( r_i \), the stiffness and mass matrices of each element \( e_i \) are given as

\[
\begin{bmatrix}
K_{ii}^{11} & K_{ij}^{12}
\end{bmatrix}_{\text{sym.}}
\]

\[
\begin{bmatrix}
M_{ii}^{11} & M_{ij}^{12}
\end{bmatrix}_{\text{sym.}}
\]

(4.8)

Components of these 12x12 Euler beam element matrices are defined in Appendix A. With this, the structure connecting right and left sides of gear \( j (j = p, g) \) can be given in matrix form as

\[
K_{ej} = \begin{bmatrix}
K_{ej}^{11} & K_{ej}^{12} & 0 \\
K_{ej}^{11} & K_{ej}^{12} + K_{ej}^{21} & K_{ej}^{22} \\
K_{ej}^{11} & K_{ej}^{12} & K_{ej}^{22}
\end{bmatrix}_{\text{sym.}}
\]

\[
M_{ej} = \begin{bmatrix}
M_{ej}^{11} & M_{ej}^{12}
M_{ej}^{21} & M_{ej}^{22} + M_{ej}^{11} & M_{ej}^{12}
M_{ej}^{22} & M_{ej}^{22}
\end{bmatrix}_{\text{sym.}}
\]

(4.9a,b)

where subscript \( eij \) \( (i = 1, 2, j = p, g) \) denotes beam element \( i \) of the connection of gear \( j \).

Writing right and left side versions of Eq. (4.7a) and combining with Eq. (4.9), the stiffness and mass matrices of the DH gear pair shown in Figure 4.15 are given as
Figure 4.17. (a) Geometry of a double-helical gear, and (b) three-piece (left and right side gears, and two connecting beam elements) model of the double-helical gear.
These 36x36 matrices of Eq. (4.10) represent stiffness and mass matrices of the DH gear pair shown in Figure 4.15.

4.3.4 Formulation of the Shafts and Bearing Supports

Next, the shaft and bearing matrices of the overall system shown in Figure 4.18 must be defined. The shaft of the pinion (shaft $sp$) is discretized into $n_{sp}$ number of elements with nodes created at the exact positions of the bearings and the mid node of the connecting structure of each gear. Likewise, a $n_{sg}$ number of beam elements are used to define the shaft of gear (shaft $sg$). Using the element Euler beam element matrices (Eq. (4.8)), individual shaft stiffness and mass matrices of each shaft $sj$ ( $j = p, g$) are given as
Figure 4.18. A view of the overall system with the shaft beam elements and bearing supports.
These symmetric matrices have the dimension of $6(n_{sj} + 1)$. For instance, if 10 finite beam elements are used to discretize each shaft, then the number of degrees of freedom required to represent the shafts through Eq. (4.12) is equal to 132.

Each of the rolling elements bearings shown in Figure 4.18 to support the shafts is represented by a $6\times6$ diagonal stiffness matrix as

$$K_{bi} = \text{Diag} \begin{bmatrix} k_{yi} & k_{xi} & k_{zi} & k_{\theta yi} & k_{\theta xi} & 0 \end{bmatrix}.$$  

(4.13)
These bearing stiffness matrices are added to \( \mathbf{K}_s \) in Eq. (4.11a) in respective nodal coordinates to account for bearings as well.

4.3.5 Overall System Equations

It is noted from the above formulations that separate sets of beam element nodes were defined for gears in Eq. (4.9) and shaft in Eq. (4.12). The reason for this is to allow the simulation of the case when the shaft-gear interface is other than a rigid connection. Specifically, when any of the gears is allowed to float axially about its shaft, as it was the case in many of the experiments, two sets of nodes are required to define such condition. As shown in Figure 4.19, the node \( r \) of the gear \( j \) and node \( s \) of the shaft can be coupled by defining a stiffness matrix

\[
\mathbf{K}_{aj} = \begin{bmatrix}
  k_y & 0 & 0 & 0 & 0 & -k_y & 0 & 0 & 0 & 0 \\
  k_x & 0 & 0 & 0 & 0 & -k_x & 0 & 0 & 0 & 0 \\
  k_z & 0 & 0 & 0 & 0 & -k_z & 0 & 0 & 0 & 0 \\
  k_{\theta_y} & 0 & 0 & 0 & 0 & -k_{\theta_y} & 0 & 0 & 0 & 0 \\
  k_{\theta_x} & 0 & 0 & 0 & 0 & -k_{\theta_x} & 0 & 0 & 0 & 0 \\
  k_{\theta_z} & 0 & 0 & 0 & 0 & -k_{\theta_z} & 0 & 0 & 0 & 0 \\
  k_y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  k_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  k_z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  k_{\theta_y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  k_{\theta_x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  k_{\theta_z} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Here when the numerical values of all elements are defined at least two order of magnitude higher than the gear mesh stiffness \( k \), a rigid connection between the gear \( j \) and its shaft is achieved. Meanwhile, an axially floating condition is obtained by setting \( k_z = 0 \).

The shaft matrices \( \mathbf{K}_s \) and \( \mathbf{M}_s \) are expanded to \( \mathbf{K}_s \) and \( \mathbf{M}_s \) by adding 36 zero rows and column to accommodate the gear matrices. Likewise the gear matrices \( \mathbf{K}_m \) and \( \mathbf{M}_m \) are
Figure 4.19. Illustration of the coupling of the DH gear $j$ and its shaft.
expanded to $\mathbf{K}_m$ and $\mathbf{M}_m$ by adding $6(n_{sp} + n_{sg} + 2)$ zero rows and column to accommodate the shaft matrices. These expanded matrices are added and combined with the gear-shaft coupling matrices $\mathbf{K}_{ap}$ and $\mathbf{K}_{ag}$ to obtain the overall system stiffness and mass matrices, $\mathbf{K}$ and $\mathbf{M}$. With this, the equations of motion of the overall system are given in matrix form as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{F}(t), \quad (4.15)$$

Here the displacement vector $\mathbf{q}(t)$ contains all $6(n_{sp} + n_{sg} + 8)$ degrees of freedom for the gears and shafts. The forcing vector $\mathbf{F}(t)$ contains four non-zero gear mesh force vectors, $\mathbf{f}_p^R(t)$, $\mathbf{f}_p^L(t)$, $\mathbf{f}_g^R(t)$ and $\mathbf{f}_g^L(t)$, as defined by Eq. (4.7g,h). These gear mesh force vectors are given as a function of the static transmission excitations of the left and right side gear meshes, $\delta_L(t)$ and $\delta_R(t)$, respectively. Choosing the left side gear mesh as the reference mesh, $\delta_L(t)$ is written in Fourier series form as

$$\delta_L(t) = \sum_{i=1}^{N} H_i^L \cos(i\omega_m t + \phi_i^L), \quad (4.16a)$$

where $H_i^L$ and $\phi_i^L$ are the amplitude and phase angle of $i$-th harmonic term of $\delta_L(t)$. With a given right to left stagger angle $\gamma_s$, $\delta_R(t)$ is written as

$$\delta_R(t) = \sum_{i=1}^{N} H_i^R \cos(i\omega_m t + \phi_i^R + i\gamma_s). \quad (4.16b)$$
4.3.6 Solution Methodology

The natural modes of the linear time-invariant system defined by Eq. (4.15) are found by performing free vibration analysis by setting $F(t) = 0$ and assuming an undamped condition such that Eq. (4.15) reduces to

$$M\ddot{q}(t) + Kq(t) = 0.$$  

(4.17)

The corresponding Eigen value problem is defined as

$$KQ = \omega^2 MQ,$$  

(4.18)

which is solved to find the undamped natural frequencies $\omega_\lambda$ and the corresponding mode shapes $Q_\lambda$ ($\lambda \in [1, N_{dof}]$ where $N_{dof}$ is the total number of degrees employed by the model).

The excitations defined by Eq. (4.16a) and (4.16b) at the left and right side meshes are in general out-of-phase of each other excitations. In addition, each individual harmonic term $i$ of each excitation has a different phase angle. First, the response of the system to each harmonic term $i$ ($i \in [1, N]$) of each gear mesh transmission error excitation is determined. Since the system is linear, the individual responses to each harmonic force term are then combined using the Modal Summation Technique to obtain the total steady state response. With the expansion theorem and the superposition principle being its foundation, the Modal Summation Technique determines the response from $\omega_\lambda$ and mass-matrix-normalized $Q_\lambda$. The forcing vector in Eq. (4.15) is written as the sum of two vectors, each representing the excitation at one gear mesh as

$$F(t) = F_L(t) + F_R(t).$$  

(4.19)
Response to each individual forcing vectors $F_L(t)$ and $F_R(t)$ are obtained, respectively, by modal summation as

$$q_L(t) = \hat{F}_L k \sum_{i=1}^{N_{dof}} \sum_{\lambda=1}^{N_{dof}} \Theta_{\lambda i}(j\omega_m) H_i^L \cos(i\omega_m t + \varphi_i^L),$$

(4.20a)

$$q_R(t) = \hat{F}_R k \sum_{i=1}^{N_{dof}} \sum_{\lambda=1}^{N_{dof}} \Theta_{\lambda i}(j\omega_m) H_i^R \cos(i\omega_m t + \varphi_i^R + i\gamma),$$

(4.20b)

Here, $j = \sqrt{-1}$, $\hat{F}_L$ and $\hat{F}_R$ are the amplitudes of $F_L(t)$ and $F_R(t)$, and $\Theta_{\lambda i}(j\omega_m)$ is dynamic compliance matrix given by

$$\Theta_{\lambda i}(j\omega_m) = \frac{Q_\lambda Q_\lambda^T}{(\omega^2 - i^2\omega_m^2) + j(2i\zeta \omega_m \omega_m)}$$

(4.21)

where $Q_\lambda$ is the $\lambda$-th normalized mode shape. Finally, $\zeta$ is the modal damping ratio applied uniformly to each mode $\lambda$. With this, the total response of the system is obtained by the linear superposition as

$$q(t) = q_L(t) + q_R(t).$$

(4.22)

With the steady state displacement amplitudes given above, composite quantities such as dynamic transmission error, and line-of-action and off-line-of-action motions can be computed by using the formulations defined in Section 2.4.3. Furthermore, the root-mean-square amplitudes can also be obtained according to Eq. (4.1).
4.4 Comparison of the Model to Experiments

In this section, some of the experiments presented in Section 4.2.1 are simulated by using the double-helical gear dynamic model proposed in Section 4.3 to (i) assess the accuracy of the dynamic model and (ii) bring a better understanding to the experimental data. Direct comparisons between the model predictions and measurements will be provided wherever possible including natural frequencies and the force response amplitudes. In addition, the sensitivity of the model to key parameters such as the stagger angle will be verified through comparisons to the measured sensitivities.

Diagonal bearing stiffness values of the matrix $K_{hi}$ in Eq. (4.13) were taken to be $k_{xi} = k_{yi} = 6.0(10)^7$ N/m, $k_{zi} = 2.5(10)^7$ N/m, and $k_{\theta_i} = k_{\theta_{zi}} = 6.0(10)^5$ Nm/rad in these simulation. The average gear mesh stiffness value $\bar{k}$ in Eq. (4.4-5) for each side of the test double-helical gear pair at $T_p = 400$ Nm was predicted by LDP [4.14] as $2.2(10)^8$ N/m. A constant modal damping ratio of $\zeta = 0.07$ (7%) was used. The harmonic components of the transmission error excitation (amplitudes $H_i^L$ and $H_i^R$, phase angles $\phi_i^L$ and $\phi_i^R$ in Eq. (4.16) are listed in Table 4.1. The measure tooth surface variations shown in Figures 3.6 and 3.7 were used for prediction of TE values for the right and left side gear pairs. As such, the harmonic amplitudes $H_i^L$ and $H_i^R$ differ slightly. The first two harmonics of the TE excitation (i.e. $i \in [1, 2]$) were included in the simulations as the amplitudes of other higher harmonics were predicted to be negligibly small.

The shafts of the test gear pair used in the experiments (Figure 2.5 (b)) spanned 247 mm between the two bearings and had a diameter of 41 mm. Furthermore, each shaft was extended by 135 mm at one side to connect to its flexible coupling, as depicted by Figure 4.18. A total of
Table 4.1. Harmonic amplitudes and phase angles of the transmission error excitation of the tested gear pair of Table 2.1

<table>
<thead>
<tr>
<th>Harmonic index, $i$</th>
<th>$H_i^L$ (μm)</th>
<th>$\phi_i^L$ (deg)</th>
<th>$H_i^R$ (μm)</th>
<th>$\phi_i^R$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.46</td>
<td>-45.77</td>
<td>2.48</td>
<td>-58.58</td>
</tr>
<tr>
<td>2</td>
<td>0.44</td>
<td>82.70</td>
<td>0.38</td>
<td>64.15</td>
</tr>
</tbody>
</table>
14 finite beam elements were used to model each shaft in these simulations, i.e. \( n_{sp} = n_{sg} = 14 \) in Figure 4.18.

Figure 4.20 shows the rms dynamic transmission error amplitudes \( \hat{\delta}_d \) of the test gear pair having \( \gamma_s = 0 \) at \( T_p = 400 \) Nm. They are shown to compare rather as both the overall shape of the response curve as well as the resonance frequencies are captured by the model well. In Section 4.2, the resonance peaks at about 2150 Hz and 1075 Hz were hypothesized to be due to a mode (called Mode A) being excited by the first two harmonics of the static TE, i.e. \( f_m = f_A = 2150 \) Hz and \( 2f_m = f_A = 1075 \) Hz. The model predicts such a mode at \( f_A = 2150 \) Hz. The predicted shape of this mode is shown in Figure 4.21 through side and top views of the gear pair. It is seen from this figure that, while this mode shape is three-dimensional exhibiting torsional, translations, rocking and axial motions, the torsional and translations (on the transverse plane \( x-y \)) dominate this mode. Gears translate in opposite directions in Figure 4.21(a) combined with torsional motions that are opposite as well. As such, a large relative gear mesh displacement is evident in this mode, such that it is excited by the transmission error excitation rather efficiently. This type of transverse-torsional mode was called in many earlier gear-shaft dynamics models as the primary gear mesh mode. Here, since \( \gamma_s = 0 \), both first and second harmonics of the excitation excite this mode A, leading to predicted resonance amplitudes of \( \hat{\delta}_d = 8.4 \) and \( 2.8 \) \( \mu \)m at \( f_m = 2153 \) and \( 1076.5 \) Hz, in comparison to the measured peak amplitudes of 7.2 and 2.2 \( \mu \)m, respectively.

Figures 4.22(a-d) compare measured and predicted values of the transverse vibration amplitudes of the pinion and gear along the \( x \) (horizontal) and \( y \) (vertical) directions. In the experimental setup shown in Figure 2.19, both shaft axes were on the same horizontal plane such that the \( x_p \) and \( x_g \) axes coincide since the shaft angle \( \alpha = 0 \) in Figure 4.16. With the exception
Figure 4.20. Comparison of predicted and measured rms dynamic transmission error amplitudes $\hat{\delta}_d$ for a gear pair having $\gamma_s = 0$ and the axially floating conditions at $T_p = 400$ Nm.
Figure 4.21. The shape of the normal mode A at $f_A = 2153$ Hz. (a) A side view and (b) a top view of the model of the double-helical gear pair. Dotted lines represent the equilibrium (zero displacement) position.
Figure 4.22. Comparison of predicted and measured (a) $\hat{y}_p$, (b) $\hat{x}_p$, (c) $\hat{y}_g$, and (d) $\hat{x}_g$ for a gear pair having $\gamma_s = 0$ and the axially floating conditions at $T_p = 400$ Nm.
Figure 4.22. Continued.
of $\hat{y}_p$ in Figure 4.22(a) that shows more than 1 μm difference at the resonance peak amplitude, measured and predicted $\hat{x}_p$, $\hat{y}_g$ and $\hat{x}_g$ values are seen to agree well. This is true not only in terms of amplitudes but also in terms of the resonance frequencies.

In Figure 4.23, predicted and measured $\hat{z}$ amplitudes of the double-helical gear pair at $\gamma_s = 0$ and $T_p = 400$ Nm are compared. In contrast to the forced responses of $\hat{\delta}_d$, $\hat{x}_j$, and $\hat{y}_j$, the axial motions $\hat{z}_j$ ($j = p,g$), as shown Figure 4.22 (a, b), have limited resonance activities associated with the mode at 2153 Hz, while having their largest resonance peak at a frequency of $f_B = 510$ Hz mentioned in Section 4.2. The shape of this natural mode B at 510 Hz is shown in Figure 4.24. The torsional and translational ($x$ and $y$) motions in Figure 4.24(a) are insignificant in this mode while axial and rocking motions dominate.

With this reasonable agreement between the model predictions and measurements exhibited in Figures 4.20, 4.22 and 4.23 for the gear pair having $\gamma_s = 0$, the ability of the model in capturing the influence of the stagger angle $\gamma_s$ is checked next. Root-mean-square dynamic transmission error amplitudes predicted by the model for of $\gamma_s = 0$, $\frac{\pi}{2}$, and $\pi$ are compared in Figure 4.25(a). Meanwhile, Figure 4.25(b) shows the corresponding measured values, reproduced from Figure 4.2. These two sets of curves are rather similar characteristically and quantitatively. They show that the model is accurate in capturing of the effect of $\gamma_s$ on the dynamic response. As such, it predicts the lowest $\hat{\delta}_d$ amplitudes at resonances caused by the first harmonic of the excitation when $\gamma_s = \pi$ while the resonances caused by the second harmonic of the excitation are absent in predictions when $\gamma_s = \frac{\pi}{2}$. Similar level of agreement was observed in terms of other response parameters as well. For instance, in Figure 4.26 predicted and
Figure 4.23. Comparison of predicted and measured (a) $\hat{z}_p$ and (b) $\hat{z}_g$ for a gear pair having $\gamma_s = 0$ and the axially floating conditions at $T_p = 400$ Nm.
Figure 4.24. The shape of the normal mode B at $f_B = 510$ Hz. (a) a side view and (b) a top view of the model of the double-helical gear pair. Dotted lines represent the equilibrium (zero displacement) position.
Figure 4.25. Comparison of (a) predicted and (b) measured $\hat{\delta}_d$ forced response curves of gear pairs having $\gamma_s = 0$, $\pi/2$ and $\pi$ at $T_p = 400$ Nm.
Figure 4.26. Comparison of (a) predicted and (b) measured \( \hat{y}_g \) forced response curves of gear pairs having \( \gamma_s = 0, \pi/2, \) and \( \pi \) at \( T_p = 400 \) Nm.
measured $\hat{y}_g$ values of the gear pairs having $\gamma_s = 0, \frac{\pi}{2},$ and $\pi$ are displayed to show such agreement.

4.5 Summary

In this chapter, the dynamic behavior of a double-helical gear pair is investigated in detail using experimental and theoretical means. These investigations encompassed wide ranges of operating speed and torque as well as key system parameters such as the stagger angle. All possible means to measure all key output parameters were explored in this effort. They included all components of three dimensional gear motions as well as the dynamic transmission error; a quantity that is defined by all of the motions of the gears forming the pair relative to each other and that is proportional to dynamic gear mesh forces. They also included the dynamic stress measurements that resulted in root-stress based dynamic factors that are done in direct relation to the gear motion and dynamic TE measurements.

The measurements presented in this chapter indicate clearly that a double-helical gear pair behaves linearly. Furthermore, vibration amplitudes that it exhibits follow the same dependency to various parameters such as torque as the static transmission error, indicating that the static TE acts as the excitation for the dynamic system. As such, the proposed linear time-invariant model with static TE as the excitation was shown to represent the dynamic response of the system accurately in terms of resonance frequencies, response amplitudes as well as the impact of key design parameters such as the stagger angle.

The model predictions and the experiments collectively point to a response that is three-dimensional with natural modes defined by torsional, translational and rotational motions. This points out the necessity to use three-dimensional models which capture the shaft and bearing
conditions accurately. Overall, the proposed double-helical gear pair dynamic model can be deemed validated such that it can be used with confidence to predict the dynamic response of any such system.

References for Chapter 4


CHAPTER 5

CONCLUSIONS

5.1 Summary

In this study, the quasi-static and dynamic behavior of a double-helical gear pair was investigated both experimentally and theoretically, focusing specifically on the impact of the three key design and manufacturing parameters, namely nominal right-to-left stagger angle, the stagger angle deviation (error) from the nominal stagger angle, and axial gear supporting conditions.

On the experimental side, a new experimental double-helical test set-up consisting of a test machine, test specimens, and various measurement systems was designed and fabricated for operating a double-helical gear pair under realistic torques and speed ranges. A test gear pair formed by three-piece double-helical gears was designed and developed, allowing adjustable (i) right-to-left stagger angles, (ii) intentional stagger error, and (iii) axial support conditions. Four separate measurement systems were developed and implemented. They consisted of an accelerometer-based system to measure three-dimensional vibratory motions of gears and dynamic TE under high-speed conditions, an angular encoder-based system to measure static TE of the DH gear pair under low-speed condition, a non-contact probe system to measure axial motions of the gears under low-speed conditions, and a root strain measurement system to
measure right-to-left load sharing and dynamic stress factors of the double-helical gear pairs. A multi-channel, digital signal acquisition and analysis systems were devised for collection and analyzing signals of four different measurements. Test matrices that included a number of tests with various combinations of key system parameters (various stagger angles, intentional stagger errors, and axial support conditions) under realistic torque values within a wide speeds range were implemented to establish an extensive database.

With the experimental database for the DH gear pairs established, development and validation of mathematical models to predict quasi-static and dynamic behavior of double-helical gear systems were performed. The quasi-static experiments were simulated using an existing double-helical load distribution model. Then, measurements of loaded static transmission error, axial play, root stresses and right-to-left load sharing were compared to its predictions for validating the quasi-static model. Through the comparisons, the model was also shown to accurately capture the impact of the key system parameters on the quasi-static behavior of a double-helical gear pair.

For the simulation of the dynamic experiments performed, a linear time-invariant dynamic model of a double-helical system was developed. This three-dimensional model included gear mesh compliances as well as flexibilities of the shafts and their bearing. The model allowed any stagger angle between the right and left sides of a double-helical gear pair through proper definition of the phasing between the two gear mesh excitations. In addition, different axial supporting conditions were also accounted for in the model. With the natural mode shapes calculated from the corresponding Eigen value solution for the system, the modal summation technique was used to calculate the forced response of the double-helical gear pair. Using this model, the dynamic experiments of the gear pair under different stagger angles were simulated to
validate the dynamic model, showing that the predictions match well with the experiment in terms of resonances frequencies and their amplitudes.

5.2 Conclusions and Contributions

This study resulted in extensive experimental and theoretical results to enhance the understanding of DH gear behavior. Some of the main conclusions and contributions of this study can be listed as follows:

- The experimental set-up proposed to apply key system parameters and errors such as stagger angle, intentional stagger error, and axial support conditions was shown to be able to capture the effects of these parameters on the quasi-static and dynamic behavior of the double-helical gear pair effectively. The test set-up is novel and can be considered as a significant contribution to the experimental gear literature.

- An experimental database established in this study is very extensive and comprehensive, even when it is compared to the benchmark date available in the literature for spur, single-helical or hypoid gear types. This database not only covers both quasi-static and dynamic conditions but also includes both vibration and stress-based parameters while other databases focused only on one sub-set, say vibratory motions or quasi-static stresses. In this sense, this data base represents the most complete experimental data on gears.

- It was shown both experimentally and theoretically that the right-to-left stagger angle impacts the quasi-static and dynamic response of a double-helical gear pair significantly. An out-of-phase stagger angle that minimizes the static transmission error amplitude does the same to the dynamic response parameters such as dynamic transmission error and
line-of-action translations while maximizing the motions in the axial direction. Meanwhile, a zero (in-phase) stagger angle corresponds to minimized axial vibrations at the expense of dynamic transmission error and line-of-action vibrations.

- The stagger angle error was observed to be detrimental to the right-to-left load sharing of a double-helical gear pair unless one of the gears forming the pair can be floated (not supported) axially.

- A linear, time-invariant dynamic model which does not allow tooth separation induced nonlinearities and assumes constant gear mesh stiffness was shown to correlate to the experiments well, forming the baseline for future dynamic modeling of gearboxes containing this type of gears. It also has shown clearly that the dynamic behavior of double-helical gear pairs are truly three-dimensional that cannot be captured by a reduced-order (torsional or torsional-translational) model.

5.3 Recommendations for Future Work

The following are listed as potential future studies to enhance and complement the work presented in this dissertation:

- The test set-up can be modified to have bearing support conditions that represent fixed axial boundary conditions better. Likewise, axial springs can be implemented to study the effect of axial support conditions in a more detailed manner.

- The dynamic model presented did not predict tooth root stresses for computation of dynamic factors. Deformable-body formulations can be devised to predict stress-based dynamic factors of double-helical gear pairs.
The friction forces and shuttling moments that are present in the gear mesh interface can be included in the dynamic model formulations for more accurate prediction of rocking and off-line-of-action motions of double-helical gear pairs. This would also provide a between modeling of the gear mesh damping beyond simple constant modal damping approach used in this study.


LDP Gear Load Distribution Program, *Gear and Power Transmission Research Laboratory*, The Ohio State University, Columbus, OH, 2011.


APPENDIX A

A Finite Beam Element

The formulation of the finite beam element employed to form a pinion and a gear shafts as well as to connect left and right side gears are defined here. Considering a finite element $n$ on one of the shafts defined by two nodes $n$ and $n+1$ as shown in Figure A.1, the stiffness of the this $n$-th shaft (rotor) element are defined as a summation of bending, torsional, and axial components as

$$
K_{ei} = (K_{ei})_{bending} + (K_{ei})_{torsional} + (K_{ei})_{axial} \quad (A1)
$$

where

$$
(K_{ei})_{bending} = \frac{EI_{ge}}{\ell^3} \begin{bmatrix}
12 & 0 & 12 \\
0 & 0 & 0 \\
0 & 6\ell & 0 & 4\ell^2 \\
-6\ell & 0 & 0 & 4\ell^2 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-12 & 0 & 0 & 6\ell & 0 & 12 \\
0 & -12 & 0 & -6\ell & 0 & 0 & 0 & 12 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 6\ell & 0 & 2\ell^2 & 0 & 0 & 0 & -6\ell & 0 & 4\ell^2 \\
-6\ell & 0 & 0 & 2\ell^2 & 0 & 6\ell & 0 & 0 & 0 & 4\ell^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4\ell^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

Sym.
Figure A.1. A finite shaft element.
Here, the subscript \(a\) represents the property related to area. \(\ell\) and \(A\) are the length and the cross-sectional area, \(I_{ae}\) and \(J_{ae}\) are diametral and polar area moments of inertia of the beam element. \(E\) and \(G\) are the Young’s modulus and the shear modulus of elasticity, respectively.

The mass/inertia matrices of the this \(n\)-th shaft (rotor) element are similarly calculated as a summation of bending, axial, torsional, and rotational components as

\[
M_{ei} = (M_{ei})_{bending} + (M_{ei})_{axial} + (M_{ei})_{torsional} + (M_{ei})_{rotational}
\]  

(A5)
where

\[
\begin{bmatrix}
156 & & & & & & \\
0 & 156 & & & & & \\
0 & 0 & 140 & & & & \\
0 & 22\ell & 0 & 4\ell^2 & & & \\
-22\ell & 0 & 0 & 0 & 4\ell^2 & & \\
0 & 0 & 0 & 0 & 0 & 0 & \\
54 & 0 & 0 & -13\ell & 0 & 156 & \\
0 & 54 & 0 & 13\ell & 0 & 0 & 0 & 156 & \\
0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 & 140 & \\
0 & -13\ell & 0 & -3\ell^2 & 0 & 0 & 0 & -22\ell & 0 & 4\ell^2 & \\
13\ell & 0 & 0 & 0 & -3\ell^2 & 0 & 22\ell & 0 & 0 & 0 & 4\ell^2 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
\end{bmatrix}
\]

\( (M_{el})_{bending+axial} = \frac{m\ell}{420} \), \hspace{1cm} (A6)

\[
\begin{bmatrix}
36 & & & & & & \\
0 & 36 & & & & & \\
0 & 0 & 0 & & & & \\
0 & 3\ell & 0 & 4\ell^2 & & & \\
-3\ell & 0 & 0 & 0 & 4\ell^2 & & \\
0 & 0 & 0 & 0 & 0 & 0 & \\
-36 & 0 & 0 & 0 & 3\ell & 0 & 36 & \\
0 & -36 & 0 & -3\ell & 0 & 0 & 0 & 36 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 3\ell & 0 & -\ell^2 & 0 & 0 & 0 & -3\ell & 0 & 4\ell^2 & \\
-3\ell & 0 & 0 & 0 & -\ell^2 & 0 & 3\ell & 0 & 0 & 0 & 4\ell^2 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
\end{bmatrix}
\]

\( (M_{el})_{rotational} = \frac{J_{me}}{30\ell} \), \hspace{1cm} (A7)

\[
\begin{bmatrix}
0 & & & & & & \\
0 & 0 & & & & & \\
0 & 0 & 0 & & & & \\
0 & 0 & 0 & 0 & & & \\
0 & 0 & 0 & 0 & 0 & 1 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
\end{bmatrix}
\]

\( (M_{el})_{torsional} = \frac{J_{me}\ell}{3} \), \hspace{1cm} (A8)

194
Here, the subscript $m$ represents the property related to mass. $m$ is the mass of the beam element per unit length, $I_{me}$ and $J_{me}$ are the diametral and polar mass moments of inertia of the element per unit length.