A MATHEMATICAL MODEL OF A POWER STEERING SYSTEM
FOR IMPLEMENTATION IN A DRIVING SIMULATOR

A Thesis

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by

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To My Family
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CHAPTER I

INTRODUCTION

1.1 General

The automobile steering system permits a driver to control the direction of the vehicle and aim it in the desired path. The steering system also affects the directional stability and control of the vehicle. Among the driver's primary inputs; steering, braking and throttle, the steering input provides a vital link for the driver of an automobile to supply information cues to control the vehicle.

A driving simulator steering system should provide correct information back to the driver based on his input decisions. To achieve a high level of realism in the
steering model, torque feedback is essential for an advanced driving simulator.

Since the torque feedback to the driver provides vital information to aid in controlling an automobile, the actual steering system dynamics need to be determined. Steering “feel” is defined as the torque reaction of the handwheel during vehicle maneuvering. Therefore, the “feel” of the steering system is very important in determining the correlation between the driver characteristics and the vehicle.

In previous studies, when investigating the vehicle handling behavior, the steering system dynamics have not been considered in many simulations and vehicle models. Instead, the front wheel displacement input, which can be determined via the kinematic gear ratio, has been considered as the steering wheel input in many automotive simulations and models. A power steering system model in this paper assumes the steering system to be connected with the rest of the vehicle dynamics such as yaw and roll of the body and the lateral forces on the tire.

Ever since Segel [1] described the lateral responses combining the mathematical steering system model with the vehicle equations of motion [2,3], his work has been used in other studies for the past three decades. Segel found that
when the steering system model is incorporated into the vehicle equations of motion in handling studies, a more precise theoretical computation of the front wheel angular displacement is obtained. He also demonstrated how an unrestrained steering system and its dynamic behavior affects the static directional stability of the vehicle.

Recently, Mabrouka [4] incorporated a mathematical model of the steering mechanism into the vehicle’s equations of motion to obtain a more suitable study of the vehicle directional control and stability and to investigate the influence of the steering system characteristics on the handling degrees of freedom transient responses. She showed steering system vibration combining a discrete steering system model with a modified laterally flexible tire model. Rupp [5] represented a full model of an automotive steering system with both manual and power assisted configurations to better understand the interactions of the steering system dynamics with the vehicle dynamics. The steering dynamics model was implemented into a Vehicle Dynamics Analysis - Non-Linear(VDANL) simulation.

Since most driving is done in the on-center region, this becomes an important area where a simulator steering system must “feel” like an actual vehicle. These characteristics can change significantly for different driving conditions and steering
configurations. A mathematical model of the power-assisted rack-and-pinion steering system will be developed as part of this work. A “fixed” control scheme is used by inputting an angular displacement into the steering wheel and the appropriate torque back to the driver is calculated. The correct feel the driver will receive will be attained by using a dynamic approach of a total steering system to calculate the torque back to the handwheel.

The focus of this research is to set up and describe the mathematical derivation of an automobile power-assisted rack and pinion steering system dynamics. The developed mathematical model will try to provide enhanced driver realism to a Systems Technology, Inc. driving SIMulator (STISIM) [6] which will be used at the Vehicle Research and Test Center (VRTC) of the National Highway Traffic Safety Administration (NHTSA) so that the effects of the man-machine link can be studied by connecting the steering system with the rest of the vehicle dynamics. STISIM is designed as a fixed-base, IBM PC-based system which is used in human factors studies and driver training. The device can be expanded into a simulator much like one used by the Federal Highway Administration, with the fixed base consisting of an actual vehicle cab facing a large projection screen.
A driving simulator provides a projected, computer-generated color image of the road scene, which dynamically responds to driver control inputs as a function of programmable vehicle model parameters. The simulator also provides researchers or test drivers with high precision. One very significant advantage of the use of simulators when studying the correlation of vehicle/driver performance is that no matter how seriously degraded the driver is (e.g., by drugs, alcohol, age, fatigue, etc.), there is no danger to the driver or to other road users. In addition, an important advantage of simulation with a Man-In-The-Loop (MITL) [7] is that it allows engineers to determine vehicle response and to study the man-machine interface before the start of a vehicle test program. The MITL system provides many advantages in that it can maintain a constant steering feel over a wide variety of operating conditions.

It has been apparent that present-day simulation technology has matured to a high level of sophistication and it has significant advantages for automotive research and training drivers. Computer simulations allow studies of vehicle/driver characteristics that are difficult or impossible to measure for an actual vehicle. Simulations also permit driver participation in judging vehicle handling characteristics and in evaluating the ‘feel’ of the steering, suspension, braking, and power drive train, including the driver’s environment, beginning with the initial vehicle design stages. In addition, using simulations can lead to a reduced number
of actual tests. An important aspect of computer simulation is the validation of the model. Because of the advantages of simulations, significant effort has been put into the development of both simple and complex computer simulations. Computer simulation has become a standard tool for analyzing mechanical systems. When it comes to vehicle dynamics, such investigations might consist of the vehicle itself and its handling or only a specific part like the steering system or drive line. For the engineer, the problem is to set up a mathematical model which must be sufficiently detailed to describe the dynamics involved.

One of the most critical issues for vehicle modeling is the model parameters describing the vehicle characteristics. Models should be flexible enough to accommodate different vehicles through simple parameter changes. A technique to measure some parameters values relating to steering system and vehicle response was described by Normaa [8] and Tandy et.al. [9]. This research relies on both measured and estimated steering parameter values. The mathematical model of the power steering system dynamics with a 5 degree-of-freedom linear vehicle model will be used in the implementation of the computer simulation. Eventually, to evaluate the steering system model the computer simulation results will be compared with experimental results from full-scale vehicle tests.
1.2 Objectives

The major objectives of this research are to:

1. Develop a full model of an automotive power-assisted rack-and-pinion steering system
2. Derive the mathematical equations of the power steering system dynamics.
3. Implement the power steering system model with a comprehensive vehicle model to better understand the characteristics of the steering system dynamics as distinct from the rest of the vehicle dynamics.
4. Implement this steering system model into a computer simulation for model validation.
5. For use with the VRTC Driving Simulator, evaluate this developed steering system model comparing with the experimental results.

1.3 Thesis Overview

This thesis is divided into six chapters. The first chapter describes the major research problem of this thesis. The second chapter introduces the driving simulator
by introducing the VRTC driving simulator. Chapter II also deals with the motivation of the power steering system model.

The third chapter shows the mathematical derivation of the power steering system model and a mathematical model of the inclusion of 'free play' is obtained. This chapter also describes a 5 degree-of-freedom linear vehicle model that includes lateral tire dynamics. The fourth chapter deals with the parameter measurement and estimation which are used in the computer simulation.

The fifth chapter demonstrates the evaluation of the mathematical model by comparing the computer simulation results with the experimental results. The sixth and final chapter, discusses the evaluation of the steering system model and implications of future research areas.
References


CHAPTER II

BACKGROUND INFORMATION

AND LITERATURE REVIEW

2.1 Introduction

Though most models and vehicle simulations have not accounted for the steering system dynamic behavior, one of the most subjective evaluations of an automobile’s handling performance has been derived in the steering system. Recently, improvement of "steering feel" - a driver perception of steering characteristics in normal driving - has become an important issue. In addition, power steering design logic has a significant effect on overall vehicle ‘stiffness’ and ‘feel’ characteristics.
This chapter deals with the background information of the steering system and overview of driving simulators. Section 2.2 introduces the general concept of driving simulators and the VRTC driving simulator. Section 2.3 describes the vehicle dynamics simulation by showing the steering system mechanical model and tire model which is very important to the directional control and stability of a vehicle. This section also presents how the driver perceives the actual dynamics in high fidelity real-time dynamics and the relationship between the driver and the vehicle response. Section 2.4 accounts for the two main issues: 'fidelity and realism' for the simulator's design. An important cue to the driver, handwheel torque feedback is discussed in Section 2.5.

2.2 Introduction to Driving Simulators

Advances in aircraft-pilot interactive simulation have led ground vehicle manufacturers and researchers to consider the potential of ground vehicle driving simulators for evaluations of alternatives in vehicle design and safe use. These advances have also created great interest in development and use of driving simulators for the purpose of training drivers of ground vehicles, in both normal and hazardous environments. Thus, the interest and investment in vehicle simulations have been increasing over the past three decades and there has been considerable
effort to improve the technology of vehicular simulators. In addition, the role of
driving simulators for research and training is steadily increasing. The driving
simulator is a computer-controlled tool to study the interface between driver and
vehicle under simulated traffic conditions. It also provides researchers or test drivers
high precision and reliability. At present, advanced driving simulators including
motion bases [1,2] are very useful and future designs of driving simulations have also
been proposed [3].

This next section describes the function and applications of simulators, the
development transition in chronological order or group research and the conception of
the simulator design. The last section will introduce a Systems Technology, Inc.
SIMulator (STISIM), and thus the developed mathematical model of the power
steering system will be used for the STISIM at VRTC.

2.2.1 Applications of Simulators

Simulations allow studies of vehicle/driver characteristics that are difficult or
impossible to measure for an actual vehicle. Simulations also permit driver
participation in judging vehicle handling characteristics, and in evaluating the ‘feel’
of the steering, suspension, braking, and drive train, including the driver’s
environment, beginning with the initial vehicle design stages. In addition, using simulations can lead to a reduced number of actual tests.

As previously mentioned, the applications of simulators have been in use for many years for training the operating personnel of complex systems such as power stations, aircraft, ships, tanks and spacecraft. While the training simulator places the emphasis on operator-training on a simulated system, the purpose of the development simulator is to analyze and develop functions of the system itself by a team of experienced personnel working with the simulator. Human factors researchers in the Office of Safety and Traffic Operations R&D of the Federal Highway Administration use the fully interactive Highway Driving Simulator (HYSIM) [4] for a variety of critical research efforts to help improve highway safety and efficiency.

In some applications, such as the space shuttle simulator, there has never been a good alternative to simulator training. In other areas, however, such as vehicle engineering, the cost of MITL simulation testing has always been an important consideration, and as a result, has only recently been low enough to justify simulation on a large scale.
2.2.2 Historical Perspective

Many vehicle manufacturers and research centers have worked on the development of the simulators due to the usefulness of the simulations. In 1962, MIT first used the CRTs in driving simulation [5] supported in part by the US Air Force and in 1966 a refined implementation was developed by W. W. Wierwille, G. A. Gagne, and J. Knight at Cornell Aeronautical Laboratory [6]. In the early 1970’s, both Volkswagen and the Swedish Road and Traffic Institute(VTI) [7,8,9] built driving simulators with visual and motion subsystems. Systems Technology, Inc. also had experiments on a fixed base operation using the UCLA Driving Simulator [10,11]. The driver was seated in a 1965 Chevrolet sedan which was mounted on a chassis dynamometer. HYSIM [4] became a fully interactive human factors research simulator at the Federal Highway Administration in 1983, capable of investigating the driver's performance capabilities and limitations in a wide field of view.

General Motors, Corp. [12,13] has developed three driving simulators, as well as sponsored several university facilities to explore driver-vehicle performance since the early 1960’s. To study the benefits of utilizing driving simulation in the vehicle design process [13] a fourth in-house driving simulator development project was built by GM in the 1980’s. In March of 1985, over 20 years after the MIT project, Daimler-Benz introduced its state-of-art driving simulator in Berlin [2,14,15].
Various series of tests have been conducted at the Daimler-Benz driving simulator shown in Figure 2.1, which has been fully operational since March, 1985 [15,16]. This system was the first to use a 180-degree-wide image on a dome in a driving simulator shown in Figure 2.2. Daimler-Benz driving simulator that created a full six degree of freedom motion base is the best driving simulator in existence to date and is an effective tool for many human factors and design studies.

Figure 2.1 Daimler-Benz Driving Simulator [Ref. 14]
1. Projection Dome
2. Six color video projectors
3. Passenger car changeover cabin
4. Truck changeover cabin
5. Six degree-of-freedom motion system
6. Extended transverse motion system
7. Retractable entrance
8. Hydraulic station
9. Computer center
10. Data station
11. Electronic lab

**Figure 2.2 Daimler-Benz In-house Driving Simulator Overview** [Ref. 16,17,18]

The Daimler-Benz driving simulator is a development simulator for vehicle dynamic characteristics and driver behavior research. Its digital sound system was also designed to generate the realistic noise for an engine [17]. The kernel for this system is a digital signal processor. It has also been used for road design studies concerning traffic regulations and safety[6] and for the assessment of active closed-loop driving with the stochastic lane change maneuver [19].
In late 1988, Greenberg and Park [20] introduced the design and development of the Ford Driving Simulator (FDS). FDS is a laboratory instrument that integrates the three most important elements of an automotive system: the environment, the car and the driver.

Figure 2.3 Schematic of the Ford Driving Simulator [Simplified from Ref. 20]

Figure 2.3 shows a simple representation of the FDS. Although a road vehicle consists of both a vehicle and its human driver, the laboratories operated by vehicle manufacturers have traditionally been concerned almost exclusively with the testing and measurement of electro-mechanical components.
The Iowa Driving Simulator, a new, state-of-the-art simulator at the University of Iowa, incorporates complex vehicle dynamics developed by Tsai and Haug [21,22]. The proposed National Advanced Driving Simulator (NADS) [23,24] combines the technologies of all types of simulators, enabling many aspects of automotive research to be pursued. The driving simulator consists of a vehicle cab, display dome and motion system housed in a two-story simulation bay.

2.2.3 Simulator Design

The simulator is a type of device which provides real-time, interactive feedback to the driver through a combination of visual, auditory and tactile cues. Most of the time, the simulator has been used in studies where the driver plays the central part. Several of these studies have shown the effects of alcohol and drugs on driver performance.

Two main issues, "fidelity and realism," should be considered for the simulator's design, the relationship between which is key to effective simulator design. Realism is defined to mean the human perception that real world experience is replicated in the simulation. Fidelity refers to the simulator's quantitative performance correlation with the real world.
Simulator design can be categorized into two groups. One is Moving-base like NADS. The other is Fixed-base, i.e., there is no motion system to simulate the inertial cues that a driver would receive in an actual vehicle. The moving base full-scale and fixed-base simulator differences are primarily the motion cues present on the road course rather than to any visual field differences. Existing simulators have up to six degrees of freedom. The VTI system [9], for example, has three main degrees of freedom (i.e., lateral, roll and pitch) and can simulate acceleration up to 0.4g. The cabin itself is mounted on three actuators producing vibrations, roll and pitch. In a fixed-base simulator the sensory environment is somewhat limited, (i.e., no motion), so that elaborate vehicle dynamics models are not justified. In a fixed-base simulator, the operator also experiences the lack of motion cueing. The visual cueing delay ideally should be small in comparison with vehicle response lags. Significant additional delays can influence control performance and also induce simulator sickness symptoms [25].

Modern vehicle simulators [26] are controlled by digital computer systems which are needed to perform massive numbers of calculations. Some part of these calculations are used to mimic the control activities and instructor or experimenter inputs, provide feedback data to an instructor-operator station, and provide computing-image generation (CIG) for a visual simulation of the external-vehicle environment.
Simulators require cost considerations. Low-to moderate-cost driving simulators [27,28] have been used in the past and have proven successful. Low-cost simulators use microcomputer technology found in desktop personal computers (PCs). The general approach for simulation development on PC-computers has been described by Allen et. al. [28]. Microcomputer technology can meet all of the driving simulation functional requirements for visual and auditory feedback, as discussed by Allen et al. Driver control inputs (i.e., steering, throttle and brake) are processed by a vehicle dynamics model, which computes vehicle angular and transitional motions. Thus, a lower-fidelity, fixed-base simulator may have some additional virtues in addition to low cost, particularly in applications requiring extended exposure.

2.2.4 Introduction to STISIM

Stimulus response dynamics involve reactions of the feedback cueing to driver control inputs including steering, throttle and brakes. Most simulators can be categorized by the driver feedback systems which are employed. For example, the Ford Driving Simulator [20] is a fixed-base design since it includes all but the body motion cueing systems. The simulators which do not include all of the main cueing systems are called “part-task simulators.”
The cost of driving simulators has been an important consideration. Over the years, low cost simulation technology has been employed by STI to present part task driving simulator for automobile and truck research. The current simulator, STISIM, includes vehicle dynamics, visual and auditory displays and a performance measurement system. This simulator [28] provides a reasonably low cost and portable driving simulator and it has been implemented using low cost personal computer hardware, and is intended to be a general purpose device for the study of the full range of driver behavior, including psychomotor, divided attention and cognitive capabilities. A unique Scenario Definition Language (SDL) allows driving scenarios to be easily programmed with roadway, traffic and other task features, and for the convenient specification of performance measurement.

At the Vehicle Research and Test Center (VRTC) of the National Highway Traffic Safety Administration (NHTSA), the STI SIMulator (STISIM) is used to gain knowledge in the use of simulators prior to testing at the University of Iowa and NADS. Figure 2.4 represents a simple schematics of the VRTC driving simulator system. STISIM is designed as a fixed-base, PC-based system for use in human factors studies and driver training. The device can be expanded into a simulator much like the one used by the Federal Highway Administration, with the fixed base consisting of an actual vehicle cab facing a large projection screen.
A single program written in BASIC controls the vehicle dynamics calculations and driving scene perspective. Control input and proprioceptive feedback are linked to the main program through an analog-to-digital converter. The base code is rather simplified for minimizing calculation time, but may be modified to include a more complex vehicle dynamics model.

The STI software is installed in a desktop computer using the 90 MHz Intel Pentium Processor. Although the basic program can run on a slower speed processor, future additions to the software computational workload may necessitate the use of the more powerful machine. The video projection system consists of a high speed and resolution video board driving a 3-beam projector. The roadway scene is projected from behind an 8 ft by 6 ft screen, allowing a 60 degree horizontal field of view. Proprioceptive cueing is limited to a steering torque feedback system connected to the upper steering shaft of the test cab. Driver control devices consist of steering, brake, and throttle inputs sensed through controls installed in the test vehicle cab.
Figure 2.4 - VRTC Driving Simulator System Feedback and Control [29]
2.3 Vehicle Dynamics Simulation

2.3.1 Steering Compliances

A complete description of the dynamics of the vehicle can be explained in the sense that the motion of the vehicle is determined if the initial conditions and certain control variables are specified. These variables (i.e., Steering wheel angle, Brake, Throttle, Gear, Clutch) are controlled by the driver.

Jacksh [30] of Volvo proposed that steering torque gradient as well as steering angle gradient (amount of steering input per g of lateral acceleration) determines drivers’ perception of steering sensitivity. He termed the inverse of the product of these gradients “steering sensitivity.” He also found that the total compliance at the front axle consists of the following contributions:

1. The tire : 64 %
2. Steering compliance : 31.7%
3. Roll-steer and camber : 4.3 %
4. Suspension compliance : 0 % (negligible)
Figure 2.5 - A Schematic of Steering System Compliances Model

Figure 2.6 shows a reasonably complete steering system mechanical model. The steering system, including power boost, can have significant delay between steering wheel deflection and front wheel deflection. This delay is a function of the dynamics of the boost system and the basic mechanical response of the overall steering system. Steering compliance is also an important factor in vehicle understeer.
2.3.2 Tire Modeling

The vehicle dynamics include tire and wheel spin models, drive train, braking and steering systems, inertial vehicle dynamics and kinematics which create visual feedback for the human operator. The tire model is critical to the overall vehicle dynamics since the primary physical forces acting on the vehicle emanate from the tires. This is important for steering and braking inputs and is also critical in a MITL vehicle dynamics sense, because vehicle motions induce tire forces, which in turn further influence the vehicle motion. In this sense, tire forces provide effective compliance and damping to vehicle motion, and thus become an integral part of the vehicle dynamics.

![Diagram of tire dynamics in MITL operation](image)

**Figure 2.6 Tire Dynamics in MITL Operation [31]**
The tire dynamics are very important to the directional control and stability of a vehicle. The objective of tire/roadway interaction model [32] is to provide a useful force-producing element for the simulator simulation dynamics. The forces are ultimately caused by the contact between the tire and roadway, the interaction between the tire and roadway surface must also be taken into account. Therefore, it is important to properly model maneuvering forces including the interaction of longitudinal and lateral forces generated at the tires. The effects of the first-order tire dynamics for the lagged lateral forces with a linear three degree-of-freedom vehicle model are studied in this paper and all the equations will be discussed in detail in Section 3.6.

2.3.3 Real Time Simulation

Recent technological advances for the computer simulation of vehicle dynamics have brought high fidelity vehicle dynamics simulation into real time. Real time simulation enables automotive engineers to place human operators in a loop concerning various investigations. Because of the advantages of simulations, significant effort has been put into the development of both simple and complex computer simulations. Computer simulation has become a standard tool for analyzing mechanical systems. An important aspect of computer simulation is the validation of
the model. Allen, et. al. [33] described the validation work of VDANL. General vehicle testing requirements and validation procedures have been defined for vehicle dynamics computer simulations [17,18]. These approaches are relevant to the simulation of specific real world vehicles, and require not only that the vehicle model be formulated correctly, but also that the model parameters be correctly specified for a given vehicle. When the computer simulation is used in the vehicle dynamics, such investigations might consist of the vehicle itself and its handling or only a specific part like the steering system or drive line. For the engineer, the problem is to set up a mathematical model which must be sufficiently detailed to describe the problems involved.

A solution to real-time engineering simulation requires both cost-effective computational capacity and the adaptability of general purpose computers and software. Two significant approaches to real-time vehicle dynamics modeling, albeit at very different levels, are those implemented by the Swedish Road and Traffic Research Institute (VTI) [1,7] and Daimler-Benz [2,15]. Nordmark, et. al. [1] described a new approach to real-time vehicle dynamics, exploiting natural parallelism in the vehicle system graph, how graphics and dynamics must work together in driving simulation to provide realistic operator environmental cueing.
The extremely dynamic environment and kinematic complexities of a ground vehicle, however, create much greater computational challenges than those encountered in aircraft flight simulators. Relatively high frequency physical motion feedback to the driver of a ground vehicle is essential to create the required perception of realism. This means that the physical motion of the vehicle must be imparted to the cab mock-up in which the driver sits, creating the need for sophisticated computer simulation and motion platform dynamics that create a realistic Man-In-The-Loop (MITL) dynamic simulation.

Figure 2.7 Driving Simulator MITL Operation [Ref. 33,34]
Within the last 15 years, MITL simulation using computer-generated real-time graphics has played an important role not only in pilot training, but also in engineering applications for land, sea, air and space vehicles. MITL simulation shown in Figure 2.7 uses a MITL of a simulation to provide feedback to the system operator. In a real time simulator, the dynamics model is continually evaluated to give feedback to the operator. On the basis of this data, the simulator provides realistic environmental cues for the operator to allow him to control the simulated vehicle in the same way that he would in the real world.

Computer image generation technology has brought significant advances in vehicle dynamic simulation. Advanced computer graphics systems for the speed and processing power have been supported by defense, aerospace, and commercial aircraft applications of flight simulation. A major development in the field of computer image generation since 1986 has been the emergence of texture graphics. Increasing levels of texturing detail in individual polygons of graphics display devices permit the display of three orders of magnitude more imaged detail than possible in the mid 1980's. With high resolution textured graphics, extraordinarily realistic rendering scenes and roadways are now possible. Most importantly, such graphics create realistic visual displays, thus providing realistic ground vehicle driving simulation for the ground vehicle driver.
The first serious attempt at a high fidelity ground vehicle driving simulator was initiated in the early 1980's by Daimler-Benz in Germany. The principle reason for interest in a very high-fidelity (and high-cost) driving simulator is that it incorporates accurate vehicle dynamics into the real-time simulation. Driving safety research using driving simulators has often been criticized because the laboratory simulations have been too different from actual driving of cars for the results of studies to apply or generalize to that situation.

Until recently, many simulation devices have employed gross simplifications of vehicle dynamics. Although these simulators give an operator some impressions of actual driving, the results obtained have not always held up when attempts were made to apply them to driving safety. This is not surprising, since no "real-time" simulator has been based upon accurate models of vehicle dynamics of the kind that are used as engineering tools for the development of equipment. This is even true of the Daimler-Benz device, currently the world's most realistic driving simulator, and certainly true of less expensive, lower-fidelity machines. The recent computing breakthroughs [35] have now made it possible to perform computations that promise to provide for more accurate vehicle modeling in real-time. The use of these new techniques in the NADS should greatly increase the realism of the driving experience and the applicability of results to real world situations.
2.4 Human Factors

The technology applied to driving simulation must account for two different types of issues that may affect the simulator's design: the measure of performance fidelity that is attained in replicating natural cues for each cueing subsystem and the measure of realism that is generated by the combined subsystems in application scenarios. ‘Fidelity and realism’ are two separate issues, the relationship between which is key to effective simulator design. **Realism** is defined to mean the human perception that real world experience is replicated in the simulation. **Fidelity** refers to the simulator’s quantitative performance correlation with the real world.

The term fidelity refers to the level of correspondence of a simulated system to the real world system that it is attempting to simulate. Perfect fidelity in a simulator would mean that an observer would be unable to distinguish the simulator from reality. **Fidelity** comes in several forms (physical system, environmental and situational) and has many different levels. **Fidelity in a simulator** is an important issue because in training, assessment, or research the intent is to improve the performance of the subject in the real world. Fidelity can be divided in four areas as shown in Table 2.1.
Table 2.1 Type of Fidelity [Ref. 36]

<table>
<thead>
<tr>
<th>TYPE</th>
<th>Fidelity Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical Fidelity</td>
<td>The physical resemblance of the simulator to the real object.</td>
</tr>
<tr>
<td>System Fidelity</td>
<td>The level of detail in the models used by the simulator to represent the real system.</td>
</tr>
<tr>
<td>Environmental Fidelity</td>
<td>The amount of correspondence to the outside world as portrayed by the simulator.</td>
</tr>
<tr>
<td>Situational Fidelity</td>
<td>The fidelity of the situation being simulated.</td>
</tr>
</tbody>
</table>

The amount of fidelity necessary for simulator depends on three factors. These are (1) the role determined for the simulator, (2) the current state-of-the-art of the available components and (3) the money available (cost) to build the simulator.

![Diagram](image)

**Figure 2.8 The Process of Human Perception**
The process of human perception as shown in Figure 2.8 is the key that links the fidelity and realism together. Human perception connects the quantitative measures of performance with qualitative effects. Each human's perception processes one's sense and measures one's fidelity in emulating real-world conditions and interprets the results to be a measure of realism of the simulation.

These issues are extremely important in that increasing realism is the principal means of improving the effectiveness of a simulator. Increased realism reduces the need for adaptation in the simulation. Inadequate realism increases driver adaptation, which decrease accuracy (and therefore validity) of driver responses. Experiments in existing simulations have shown that adaptation also leads to unwanted and unnatural side-effects, such as simulation sickness [37,38]. Conversely, little or no adaptation leads to natural behavior and natural driver vehicle interaction.

Vehicle design issues associated with human factors and the driver-vehicle interface include: understanding how design integration tradeoff decisions affect driver comfort and usability, flagging problematic driver-system interactions with and between other interface systems in a newly proposed design, or understanding how increased system complexity affects usability under high driving demand.
2.5 Torque Feedback

It is a well known fact that the driver receives much information concerning the road and the maneuver ("road feeling") from the reaction torque at the steering wheel. This feedback mechanism is thus important to model rather carefully. Steering system torque feedback to the driver in vehicles comes from the wheel aligning torque arising from lateral tire forces and power steering assist force. To generate the proper steering wheel forces, a torque motor is connected in-line with the steering wheel. The torque is controlled by the steering feel subsystem and reflects the tire forces calculated in the vehicle dynamics equations. A servo disk motor connected to the steering wheel shaft provides tactile feedback. The tachometers are also connected on each wheel to sense rotational rate to provide information to the driver. Steering angle is sensed with a potentiometer. The steer angle is transmitted from the steering wheel to the front wheel via the steering system kinematic gear ratio. The calculation of the feedback torque and power assist force will be detailed in Chapter III.
2.6 Motivation of the Power Steering System Model

"Steering feel" was defined as the derivative relating steering input torque and steering input angle by Norman [39]. Norman also describes a technique to measure some of the parameters relating to ‘steering feel’ and vehicle response. Deppermann [40] addressed the measurement of some of the parameters describing handwheel activity and proposed a method for quantifying ‘steering feel.’ A large number of evaluative expressions that have been applied to driver perception in driving were analyzed and categorized into ten groups by Koide and Kawakami [41].

Among the primary driver control inputs as shown in Figure 2.9: steering, brake and throttle, this research focuses on improving the modeling of the power assisted steering system. The steering system of a vehicle obviously has a major impact on its handling characteristics. As previously mentioned, some vehicle dynamics models and simulations have considered front roadwheel steer angle as the input control variable, thus completely eliminating the steering system from the analysis.
<table>
<thead>
<tr>
<th>Expression</th>
<th>Typical Language used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steering effort</td>
<td>Heavy, light, high, low, shoulders become stiff, arms grow tired, right-left difference, high power assistance, heavy at low speed but light at higher speed</td>
</tr>
<tr>
<td>Steering holding effort</td>
<td>Heavy, light, high, low, responsive, not very responsive</td>
</tr>
<tr>
<td>Sense of solidity</td>
<td>Feels solid, lacks solidity, feels rigid, lacks rigidity, tight, flabby, secure, uncertain</td>
</tr>
<tr>
<td>Smoothness</td>
<td>Smooth, not smooth, linear, lacks linearity, stiff, natural, smooth linkage, linkage not smooth</td>
</tr>
<tr>
<td>Steering effort (in the steering wheel neutral area)</td>
<td>Force required (not required) in the steering wheel neutral area; steering wheel unsteady at start of turn; vague neutral area</td>
</tr>
<tr>
<td>Response of vehicle (in the steering wheel neutral area)</td>
<td>Sharp movement, brisk movement, feeling of backlash, no backlash, sluggish</td>
</tr>
<tr>
<td>Reluctant feel</td>
<td>Reluctant feeling of friction, lack of friction, well-defined</td>
</tr>
<tr>
<td>Sticky feel</td>
<td>Feels sticky, viscosity, lack of viscosity, rubbery feeling</td>
</tr>
<tr>
<td>Centering</td>
<td>Straight-ahead position clear (not clear), stable, lack of stability, strong (weak) centering, stable straight-ahead position</td>
</tr>
<tr>
<td>Steering wheel returnability</td>
<td>Steering wheel returns (does not return) returns fast(slowly, strongly, weakly)</td>
</tr>
</tbody>
</table>
Figure 2.9 A Block Diagram of Driver Control Inputs in Driving Simulator [32,33]

Many studies of the power steering system have been introduced recently. Nishikawa, et.al. [42] described the variable assistance power steering system which was used in the 1978 Honda Accord LX. Adams [43] discussed the hydraulic power steering characteristics. Baxter [44] modeled a power-assisted rack-and-pinion steering gear mathematically. Miyamamoto, et.al. [45] reported the driver’s steering
control characteristics, describing a transfer function to various kinds of vehicles used in a driving simulator.

The advantage of the power steering system has become a positive necessity on many large modern cars. The power steering system allows the car to be controlled in unexpected situations such as when a front wheel strikes an obstruction or if the tire bursts when the vehicle is at high speed. Good power steering reduces driver fatigue and contributes to safety. Power steering system provides enough assistance to make parking maneuvers easy and good "road feel" at high speeds. Therefore, this research examines the steering system characteristics and sets up a mathematical model for the steering system dynamics which will be used in the implementation of the computer simulation.

References


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CHAPTER III

POWER STEERING SYSTEM MODEL

3.1 Introduction

The steering system allows the driver to steer the front wheels of an automobile to control and aim it in the desired path. Although the steering system plays an important role in the directional stability and control of the vehicle, only the front wheel angle by the kinematic steering gear ratio and steering compliance is often regarded as the steering angle when studying vehicle handling behavior. This chapter models the physical behavior of each component of the typical power steering system configuration. The dynamic behavior of the power steering system is discussed by expressing the mathematical relationships useful in analytical analysis and digital
simulation studies. In addition, the steering system dynamics is studied using a 5
degree-of-freedom automobile handling model.

3.2 Steering Assembly

A steering system is a device to control the direction of an automobile. The
steering system for a gear box and pitman design consists of the steering wheel, the
steering-column (shaft), the steering gear, pitman arm, tie-rod, drag link, knuckle-arm,
and front wheels. Rack and pinion steering system contains the rack and pinion
instead of the steering gear box.

The steering wheel composed of hub, spoke and rim as shown in Fig. 3.1 is
connected to an upper steering column shaft. The column is connected to a universal

Figure 3.1 Steering Wheel
joint for attachment to the lower steering column shafts and rubber vibration isolator connections.

Figure 3.2 Gear box and pitman arm steering configuration [1]

The steering column shaft is connected to a gearbox in order to convert the angular input motions into transitional motions along a rack or relay linkage. In the automotive industry, the steering gear configurations are various, for example, worm sector type, worm-sector roller type, recirculating ball type, variable ratio type, and rack-and-pinion type, but all are functionally the same. The two main types within the automotive industry are the rack-and-pinion and the gearbox and pitman arm configuration. The gearbox and pitman arm configuration, as shown in Fig. 3.2, is
more commonly found in older passenger cars and light trucks. This steering system type consists of a steering gearbox which rotates a lever arm known as the pitman to control the motion of the relay linkages and tie rods to the wheels.

Figure 3.3 Conventional Rack-and-pinion type steering configuration [2]
The steering linkage consists of pitman arm, drag link knuckle arm and tie-rod. The linkage connected to the shaft and knuckle arm transmits the motion of the gear. The conventional rack-and-pinion steering system, as shown in Fig. 3.3, is becoming more common. It has many advantages over the gearbox and pitman arm configuration in that it has a much simpler design and higher accuracy placement and is more compact and integrated as well as smaller and lighter. This paper models the rack-and-pinion steering system. The pump provides the driver with power assist through the steering valve. Figure 3.4 shows a conventional steering valve.

The ‘Input Shaft’ connects to the steering wheel, the ‘sleeve’ connects to the pinion gear, and are connected by the ‘Torsion Bar.’ Twisting of the torsion bar causes the steering valve to open, and thus steering wheel torque causes rotary valve displacement which produces steering pressure. A constant fluid flow $Q$ from the flow control valve in the pump enters the steering gear valve through the inlet. A transverse cross section through the conventional steering valve at the inlet is shown in Figure 3.4. Four identical steering valve sections are equally spaced around the O.D. of the shaft and I.D. of the sleeve, in parallel in the hydraulic circuit.
Longitudinal Cross-Section of Valve

Transverse Cross-Section of Valve (Right Turn Start)

Figure 3.4 - Conventional Steering Valve Motion [2]
When the valve is on-center (straight ahead driving) as shown in Figure 3.4, torsion bar deflection and steering torque are zero, all chamfer areas are equal each other. Thus, $A_c$ is maximum and $P$ is minimum. During a turn, the input shaft turns a small amount $\theta$ relative to the sleeve. This relative valve motion distributes fluid to the “right turn” side of the piston and vents the left turn side. The steering valve “chamfer areas,” $A_c$, gradually close as valve travel ($\theta$) from center increases, throttling the constant flow. As $A_c$ decreases, $P$ increases according to the “Orifice Equation”:

$$\text{Steering Pressure } P = \frac{6(Q)^2}{(100 \times A_c)^2}$$

where

$P = \text{Steering Pressure (psi)}$

$Q = \text{Flow from pump to Gear (gpm)}, \text{(constant)}$

$A = \text{Combined effective area of Chamfers = Throttling Area, in}^2$
3.3 Nomenclature

Table 3.1 shows the vehicle parameters description used for the 5 degree-of-freedom vehicle handling model. Dynamic variables are shown in Table 3.2. The measured steering system parameters are listed in Table 3.3. The symbols are the same as those used in the implementation of the SIMULINK of MATLAB [3] shown in the Appendix. The parameters and all the variables are used for the derivation of the steering model in Section 3.5 and Section 3.6.

Table 3.1 Measured Vehicle System Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Vehicle Parameter Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vx</td>
<td>Vehicle Forward Speed</td>
<td>m/s</td>
</tr>
<tr>
<td>M</td>
<td>Total Vehicle Mass</td>
<td>kg</td>
</tr>
<tr>
<td>Ms</td>
<td>Vehicle Sprung Mass</td>
<td>kg</td>
</tr>
<tr>
<td>a</td>
<td>Longitudinal distance from front axle to vehicle C.G.</td>
<td>m</td>
</tr>
<tr>
<td>b</td>
<td>Longitudinal distance from rear axle to vehicle C.G.</td>
<td>m</td>
</tr>
<tr>
<td>Ixxs</td>
<td>Sprung mass roll inertia about vehicle roll axis</td>
<td>kg-m²</td>
</tr>
<tr>
<td>Izz</td>
<td>Total vehicle yaw mass moment of inertia</td>
<td>kg-m²</td>
</tr>
<tr>
<td>Ksw</td>
<td>Steering ratio</td>
<td>rad/rad</td>
</tr>
<tr>
<td>e</td>
<td>Distance from roll axis to sprung mass C.G.</td>
<td>m</td>
</tr>
<tr>
<td>Calphaf</td>
<td>Front axle tire cornering stiffness</td>
<td>N/rad</td>
</tr>
<tr>
<td>Calphar</td>
<td>Rear axle tire cornering stiffness</td>
<td>N/(rad-sec)</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration due to gravity</td>
<td>m²/s²</td>
</tr>
<tr>
<td>dLdPhi</td>
<td>Total vehicle roll stiffness</td>
<td>N-m/rad</td>
</tr>
<tr>
<td>dLdPhiD</td>
<td>Total vehicle roll damping</td>
<td>N-m/(rad-sec)</td>
</tr>
<tr>
<td>ef</td>
<td>Front axle roll steer</td>
<td>rad/rad</td>
</tr>
<tr>
<td>er</td>
<td>Rear axle roll steer</td>
<td>rad/rad</td>
</tr>
<tr>
<td>RL</td>
<td>Relaxation Length</td>
<td>m</td>
</tr>
<tr>
<td>X_T</td>
<td>Overall trail length</td>
<td>m</td>
</tr>
<tr>
<td>tau</td>
<td>Time constant(RL/Vx)</td>
<td>sec</td>
</tr>
</tbody>
</table>
Table 3.2 Dynamic Simulation Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Dynamic Parameter Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{sw}$</td>
<td>Torque at hand wheel</td>
<td>N-m</td>
</tr>
<tr>
<td>$\delta_{SWDD}$</td>
<td>Angular Acceleration at hand wheel</td>
<td>rad/s$^2$</td>
</tr>
<tr>
<td>$\delta_{SWD}$</td>
<td>Angular Velocity at hand wheel</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\delta_{SW}$</td>
<td>Angular displacement of hand wheel</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_{SC}$</td>
<td>Angular displacement of steering column</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_{T}$</td>
<td>Angular displacement of torsion bar</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_{p}$</td>
<td>Angular displacement of pinion</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_{\text{VALVE}}$</td>
<td>Rotational displacement of input shaft to power steering unit</td>
<td>rad</td>
</tr>
<tr>
<td>$Y_{RDD}$</td>
<td>Transitional Acceleration on the steering rack</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td>$Y_{RD}$</td>
<td>Transitional Velocity on the steering rack</td>
<td>m/s</td>
</tr>
<tr>
<td>$Y_{R}$</td>
<td>Transitional displacement on the steering rack</td>
<td>m</td>
</tr>
<tr>
<td>$\delta_{SLU1}$</td>
<td>Rotational displacement of upper steering linkage on the passenger side kingpin</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_{SLU2}$</td>
<td>Rotational displacement of upper steering linkage on the driver side kingpin</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_{SLL1}$</td>
<td>Rotational displacement of lower steering linkage on the passenger side kingpin</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_{SLL2}$</td>
<td>Rotational displacement of lower steering linkage on the driver side kingpin</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_{FW1DD}$</td>
<td>Angular Acceleration of the passenger side road wheel</td>
<td>rad/s$^2$</td>
</tr>
<tr>
<td>$\delta_{FW1D}$</td>
<td>Angular Velocity of the passenger side wheel</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\delta_{FW1}$</td>
<td>Angular displacement of the passenger side road wheel</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_{FW2DD}$</td>
<td>Angular Acceleration of the driver side road wheel</td>
<td>rad/s$^2$</td>
</tr>
<tr>
<td>$\delta_{FW2D}$</td>
<td>Angular Velocity of the driver side road wheel</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\delta_{FW2}$</td>
<td>Angular displacement of the driver side road wheel</td>
<td>rad</td>
</tr>
<tr>
<td>$rD$</td>
<td>Yaw Acceleration</td>
<td>rad/s$^2$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Roll angle of vehicle</td>
<td>rad</td>
</tr>
<tr>
<td>$T_{K1}$</td>
<td>Torque at passenger side kingpin</td>
<td>N-m</td>
</tr>
<tr>
<td>$T_{K2}$</td>
<td>Torque at driver side kingpin</td>
<td>N-m</td>
</tr>
<tr>
<td>$AT_1$</td>
<td>Aligning torque of the passenger side road wheel</td>
<td>N-m</td>
</tr>
<tr>
<td>$AT_2$</td>
<td>Aligning torque of the driver side road wheel</td>
<td>N-m</td>
</tr>
<tr>
<td>$F_{PS}$</td>
<td>Power steering assist force on the rack</td>
<td>N</td>
</tr>
</tbody>
</table>
### Table 3.3 Measured Steering System Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Measured Parameter Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{SW}$</td>
<td>Upper Steering Column viscous damping coefficient</td>
<td>(N-m)/(rad/s)</td>
</tr>
<tr>
<td>$I_{SW}$</td>
<td>Moment of Inertia of handwheel</td>
<td>(N-m)/(rad/sec²)</td>
</tr>
<tr>
<td>$K_{SC}$</td>
<td>Steering column rotational stiffness</td>
<td>N-m/rad</td>
</tr>
<tr>
<td>$K_T$</td>
<td>Rotational stiffness of power steering torsion spring on variable flow valve</td>
<td>N-m/rad</td>
</tr>
<tr>
<td>$N_G$</td>
<td>Steering gear ratio of rotation of upper column to pinion shaft (unity in the rack and pinion system)</td>
<td>rad/rad</td>
</tr>
<tr>
<td>$R_P$</td>
<td>Radius of pinion</td>
<td>m</td>
</tr>
<tr>
<td>$M_R$</td>
<td>Steering rack mass</td>
<td>kg</td>
</tr>
<tr>
<td>$N_L$</td>
<td>Front steering linkage rate on driver side from displacement to rack road wheel angle</td>
<td>m/rad</td>
</tr>
<tr>
<td>$K_{SL1}$</td>
<td>Front passenger side steering rotational stiffness due to linkage and bushing</td>
<td>(N-m)/rad</td>
</tr>
<tr>
<td>$K_{SL2}$</td>
<td>Front driver side steering rotational stiffness due to linkage and bushing</td>
<td>(N-m)/rad</td>
</tr>
<tr>
<td>$I_{FW1}$</td>
<td>Inertia of right wheel and rotation mass about steering displacement</td>
<td>(N-m)/(rad/sec²)</td>
</tr>
<tr>
<td>$I_{FW2}$</td>
<td>Inertia of left wheel and rotation mass about steering displacement</td>
<td>(N-m)/(rad/sec²)</td>
</tr>
<tr>
<td>$B_{SL1,2}$</td>
<td>Viscous damping at each steering linkage bushing</td>
<td>(N-m)/(rad/s)</td>
</tr>
<tr>
<td>$B_R$</td>
<td>Viscous damping of steering rack linear motion</td>
<td>(N-m)/(rad/s)</td>
</tr>
<tr>
<td>$C_{FR}$</td>
<td>Coulomb friction breakout force on steering rack</td>
<td>N</td>
</tr>
<tr>
<td>$C_{FW}$</td>
<td>Coulomb friction breakout force on roadwheel</td>
<td>N</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Roll steer coefficient</td>
<td>rad/rad</td>
</tr>
<tr>
<td>$\eta_F$</td>
<td>Gear ratio efficiency of forward torque transmission</td>
<td>%</td>
</tr>
<tr>
<td>$\eta_B$</td>
<td>Gear ratio efficiency of backward torque transmission</td>
<td>%</td>
</tr>
<tr>
<td>$\eta_{PS}$</td>
<td>Efficiency of the power steering system due to hydraulic losses</td>
<td>%</td>
</tr>
<tr>
<td>$A_{PIST}$</td>
<td>Power steering pump effective piston area</td>
<td>m²</td>
</tr>
</tbody>
</table>
3.4 Power Steering System Model Overview

This model is derived for fixed control input using an angular displacement input at the handwheel. By inputting an angular displacement at the handwheel, the feedback torque to the driver is obtained. The roadwheel angular input created by the gear ratios are transmitted backward to the handwheel through a tire model. The tire forces function will be discussed in Section 3.6.

The standard SAE tire coordinate convention as shown in Figure 3.5 is used for the model development. Both the clockwise directional input at the handwheel and the torques and angles in the model are assumed to be positive. Conversely, the counter-clockwise directional torques are assumed to be negative. The aligning torques from the tire dynamics play an important role in the driver’s “feel.”

Figure 3.6 shows the schematic of the power steering system dynamic model. The full dynamic mathematical model of this power steering system configuration consists of a lumped mechanical system with four degrees of freedom; the rotational displacement of the handwheel, transitional displacement of the steering rack, and the rotational displacement of both roadwheels about their each kingpin axis. The differential equations of motion are derived from the input at the handwheel to the roadwheel in Section 3.5.
Figure 3.5 SAE Standard Conventional Tire Axis [4]
Figure 3.6 Schematic of the Dynamic Power Steering System Model
The equations of motion that will be developed for the steering system use the basic concept of compliance elements to transmit torque information to each successive element in the steering model. This model is composed of four lumped masses and inertia elements. These lumped terms are made up of the inertia at the steering wheel, inertia at the right and left wheel assemblies about their respective kingpin axis, and the effective mass of the steering rack having transitional motion. A combined total of nine rotational and one translational dynamic displacements are used in the developed steering model. These are made up of the angular displacements of the steering wheel, steering column shaft, torsion bar, upper steering linkage on the left and right side, lower steering linkages on the left and right sides, and left and right wheel assembly, and the transitional displacement of the steering rack. The rotational motion of the torsion bar is converted into transitional motion with information about the pinion radius.

The torsion bar is put in line with the upper steering column spring to be representative of modern rack and pinion systems. The steering rack provides for an easy means of implementing a Coulomb friction element into the model, which is necessary in all steering systems to maintain stability as discussed by Segel in his early work the directional response. A viscous damping element is also found on the rack to represent the mounting of a typical steering assembly to reduce shock transmission and wheel shimmy inputs.
The viscous damping term, $B_{sw}$, is representative of the rubber bushings and drag characteristics found in the upper steering column. A viscous damper is also used on the rack along with the Coulomb friction element. The viscous damping terms, $B_{SL1}$ and $B_{SL2}$ are representative of the rotation damping of the wheel assembly about its kingpin axis rotation.

The torque transmitted back to the steering wheel in a power steering equipped vehicle results from the summation of the compliance force from torsional bending of the steering system elements, pump assist force, and the aligning torques at the wheels. Because of the added assist force on the rack, the power steering equipped vehicle has an artificial "feel" to the road. The force applied on the rack is calculated from the measured piston area times the corresponding differential pressure applied to the piston. The differential pressure is defined as the difference between inlet and outlet pressure on either side of the piston.

The aligning torques are very important in the overall stability of the steering system because they tend to pull the wheel back to its straight line position due to the caster effects of the front steering geometry. The geometry of the upper steering linkages will not be discussed in this research. The three dimensional nature of this geometry creates forces and moments that are applied to the steering system that can become quite complex and are therefore not discussed in this research.
3.5 Mathematical Derivation of the Power Steering System Model

To obtain the differential equation of motion of the steering system model shown in Figure 3.6 we apply Newton’s second law to each of the inertias.

\[ \sum \text{(External Torques)} = (\text{Inertia}) \times (\text{Angular Acceleration}) \]  \hspace{1cm} (3.1)

Summation of the dynamic torques at the steering wheel yields

\[ I_{sw} \ddot{\delta}_{sw} = T_{sw} - K_{sc} (\dot{\delta}_{sw} - \dot{\delta}_{sc}) - B_{sw} (\dot{\delta}_{sw} - \dot{\delta}_{sc}) \]  \hspace{1cm} (3.2)

Summing the dynamic torques on each front wheel

\[ I_{FW1} \ddot{\delta}_{FW1} = K_{SL1} (\delta_{SLU1} - \delta_{SLL1}) - AT_1 - B_{SL1} \dot{\delta}_{FW1} - CF_{FW} \times \text{sgn}(\dot{\delta}_{FW1}) \]  \hspace{1cm} (3.3)

\[ I_{FW2} \ddot{\delta}_{FW2} = K_{SL2} (\delta_{SLU2} - \delta_{SLL2}) - AT_2 - B_{SL2} \dot{\delta}_{FW2} - CF_{FW} \times \text{sgn}(\dot{\delta}_{FW2}) \]  \hspace{1cm} (3.4)

where the \( \text{sgn}(\dot{\delta}_{FW}) \) terms in Equations 3.3 and 3.4 accounts for the sign change of the front wheel angular velocity \( \dot{\delta}_{FW} \); and \( AT_1 \) and \( AT_2 \) stand for the tire aligning torque
produced by tire forces which are generated at the tire-road interface during steering and rolling action.

Figure 3.7 Theoretical Tire Model [5]

As shown on Figure 3.7, the overall aligning torque AT generated at the ground tire interface of interest here is defined as the resultant lateral force factored by the sum of the pneumatic trail and the mechanical trail distance. Hence, AT can be expressed as
\[ AT = (X_p + X_c)y = X_T y = X_T F_y \]  \hspace{1cm} (3.5)

where \( X_p \), \( X_c \), and \( X_T \) denote the pneumatic trail length, the mechanical trail length and the overall trail length, respectively. The lateral force \( Y(F_y) \) is related to the slip angle and cornering stiffness. Thus,

\[ Y(F_y) = C \times \alpha \]  \hspace{1cm} (3.6)

Substituting Equation 3.6 into 3.5

\[ AT = X_T \times C \times \alpha \]  \hspace{1cm} (3.7)

Each of the summed torque equations in the rack motions can be represented by applying Newton's second law and dividing by the linkage rate, \( N_L \), or pinion radius values. The front steering linkage rate on the driver side is the same as the one on the passenger side, since the steering linkage is assumed to be symmetric in the model.

The summation of all the forces is the following
\[ M_R \dot{Y}_R = \eta_f \frac{T_p}{R_p} - \eta_b \frac{T_{k1}}{N_L} - \eta_b \frac{T_{k2}}{N_L} - B_R \dot{Y}_R - \eta_{ps} F_{ps} - CF_R \times \text{sgn}(\dot{Y}_R) \]  

(3.8)

where \( \eta_f \) represents the torque-transmission efficiency of the gearbox which can be set equal to unity in the rack-and-pinion steering gear, \( \eta_b \) the backward torque-transmission efficiency and \( \eta_{ps} \) the efficiency of the power steering system due to hydraulic losses. \( F_{ps} \) accounts for a power steering assist force on rack, and \( \dot{Y}_R \) the transitional acceleration of the rack mass. Equations 3.2, 3.3, 3.4, and 3.8 represent the four degrees of freedom for the steering system model. Equating the torques between the steering column and the torsion bar assuming unity gear ratio for a rack-and-pinion system, we obtain the following,

\[ K_{sc}(\delta_{sw} - \delta_{sc}) = K_T(\delta_{sc} - \delta_T) \]  

(3.9)

The relationship between the torsion bar angular displacement and the pinion angular displacement is expressed by

\[ \delta_T = \delta_p \]  

(3.10)
The torque across the torsion bar is

\[ T_r = K_T (\delta_{sc} - \delta_r) \]  

(3.11)

The rotational motion of the pinion is converted into translational motion with the pinion radius

\[ \delta_p = \frac{Y_R}{R_p} \]  

(3.12)

The transitional motion of the rack mass is related to the upper steering linkage angular displacement, \( \delta_{SLU1} \), which is related to the lower steering linkage rate by

\[ Y_R = N_L \delta_{SLU1} \]  

(3.13)

\[ Y_K = N_L \delta_{SLU2} \]  

(3.14)

Considering the relationship between the angular displacement of torsion bar and transitional motion of steering rack mass, all the torques \( T_{k1}, T_{k2} \) and \( T_p \) need to be converted into force terms. The torques in both steering linkage
\[ T_{K_1} = K_{SL1} (\delta_{SLU1} - \delta_{SLL1}) \]  \hspace{1cm} (3.15)

\[ T_{K_2} = K_{SL2} (\delta_{SLU2} - \delta_{SLL2}) \]  \hspace{1cm} (3.16)

The front wheel angular displacements, \( \delta_{FW1}, \delta_{FW2} \), are related to the lower steering-linkage angular displacement by

\[ \delta_{FW1} = \delta_{SLL1} + \epsilon \phi \]  \hspace{1cm} (3.17)

\[ \delta_{FW2} = \delta_{SLL2} + \epsilon \phi \]  \hspace{1cm} (3.18)

where \( \epsilon \phi \) is an additional front wheel steer angle caused by the vehicle body roll.

Equations 3.2 through 3.18 represent the complete set of equations describing the steering system model shown in Figure 3.6. Upon eliminating the intermediate displacement variables \( \delta_{SC}, \delta_P, Y_R, \delta_{SLU1}, \delta_{SLU2}, \delta_{SLL1} \) and \( \delta_{SLL2} \), some relationships among the steering wheel angular displacement, the transitional motion of the rack mass and each front wheel angular displacement are obtained. These four variables are the main degrees of freedom of the steering system.
A steady state analysis useful for the computer simulation is discussed below. At steady state, all time rates of change are equal to zero. Therefore, Equations 3.2, 3.3, 3.4 and 3.8 become as follows, respectively.

\[ T_{sw} = K_{sc} (\delta_{sw} - \delta_{sc}) \quad (3.19) \]

\[ AT_1 = K_{SL1} (\delta_{SLU1} - \delta_{SLL1}) \quad (3.20) \]

\[ AT_2 = K_{SL2} (\delta_{SLU2} - \delta_{SLL2}) \quad (3.21) \]

\[ \frac{K_T (\delta_{sc} - \delta_T)}{R_p} = \frac{\eta_B K_{SL1} (\delta_{SLU1} - \delta_{SLL1})}{N_L} + \frac{\eta_B K_{SL2} (\delta_{SLU2} - \delta_{SLL2})}{N_L} + \eta_{ps} F_{ps} \quad (3.22) \]

Equation 3.6 can be rewritten as

\[ T_{sw} = K_{sc} (\delta_{sw} - \delta_{sc}) = K_T (\delta_{sc} - \delta_T) \]

\[ = \frac{\eta_B K_{SL1} (\delta_{SLU1} - \delta_{SLL1})}{N_L} + \frac{\eta_B K_{SL2} (\delta_{SLU2} - \delta_{SLL2})}{N_L} + \eta_{ps} F_{ps} \quad (3.23) \]
From Equation 3.20 and 3.21, substituting for $K_{SL1}(\delta_{SLU1} - \delta_{SLL1})$, and $K_{SL2}(\delta_{SLU2} - \delta_{SLL2})$ respectively, we obtain the steady-state steering torque applied by the driver

$$T_{SWSS} = -\frac{\eta_B R_p}{N_L} AT_1 - \frac{\eta_B R_p}{N_L} AT_2 + \eta_{PS} R_p F_{PS}$$ (3.24)

The steering wheel angular displacement $\delta_{sw}$ can be derived from Equation 3.23

$$\delta_{sw} = \frac{\eta_B R_p K_{SL1}(\delta_{SLL1} - \delta_{SLU1})}{N_L K_{SC}} + \frac{\eta_B R_p K_{SL2}(\delta_{SLU2} - \delta_{SLL2})}{N_L K_{SC}} + \frac{\eta_{PS} R_p F_{PS}}{K_{SC}} + \delta_{sc}$$ (3.25)

Upon substituting for $\delta_{SLU1}$, $\delta_{SLL1}$, $\delta_{SLU2}$ and $\delta_{SLL2}$ from Equations 3.13, 3.14, 3.17, and 3.18 respectively, substituting for $Y_r$ from Equations 3.10 and 3.12, and $\delta_T = \delta_{sc} + \frac{K_{SC}}{K_T} \delta_{sc} - \frac{K_{SC}}{K_T} \delta_{sw}$ from Equation 3.9, Equation 3.25 can be rearranged as
\[
\left[ \frac{\eta_B R_p^2 K_{SL1} K_{SL2}}{N_L^2 K_{SC}} \left( \frac{1}{K_{SL1}} + \frac{1}{K_{SL2}} + \frac{K_{SC}}{K_T K_{SL1}} + \frac{K_{SC}}{K_T K_{SL2}} \right) + 1 \right] \delta_{SC}
\]

= \left[ \frac{\eta_B R_p^2}{N_L^2 K_T} (K_{SL1} + K_{SL2}) + 1 \right] \delta_{SW}

+ \frac{\eta_B R_p}{N_L K_{SC}} (K_{SL1} \delta_{FW1} + K_{SL2} \delta_{FW2}) - \frac{\eta_B R_p e_0}{N_T K_{SC}} (K_{SL1} + K_{SL2}) - \frac{\eta_p R_p}{K_{SC}} F_{PS}

(3.26)

Grouping the terms of \( \delta_{SC} \) on the right hand side, Equation 3.26 yields,

\[
\delta_{SC} = \left[ \frac{\eta_B R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right] \delta_{SW}
\]

\[
+ \left[ \frac{\eta_B R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_B R_p^2 (K_{SL1} + K_{SL2})}{N_T^2 K_T} + 1 \right] \delta_{FW1}
\]

\[
+ \left[ \frac{\eta_B R_p K_{SL1}}{N_L K_{SC}} \right] \delta_{FW2}
\]

\[
+ \left[ \frac{\eta_B R_p K_{SL2}}{N_L K_{SC}} \right] \delta_{FW3}
\]

\[
+ \left[ \frac{\eta_B R_p e_0 (K_{SL1} + K_{SL2})}{N_T K_{SC}} \right] \delta_{FW4}
\]

\[
+ \left[ \frac{\eta_p R_p}{K_{SC}} F_{PS} \right] \delta_{FW5}
\]
\[
\begin{align*}
    \delta_{FW2} &= \left( \frac{\eta_p R_p K_{SL2}}{N_L K_{SC}} \right) + \frac{\eta_b R_F^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_b R_F^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \\
    \delta_{FS} &= \left( \frac{\eta_p R_p}{K_{SC}} \right) - \frac{\eta_b R_F^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_b R_F^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \\
\end{align*}
\]

(3.27)

Next, we substitute Equation 3.27 into Equation 3.2, 3.3, 3.4 and 3.8 to eliminate \( \delta_{SC} \) in favor of \( \delta_{SW} \), \( \delta_{FW1} \) and \( \delta_{FW2} \). Equation 3.2 becomes
\[ I_{sw} \delta_{sw} = T_{sw} - B_{sw} (\delta_{sw} - \delta_{sc}) \]

\[ + \begin{bmatrix} \frac{\eta_B R_P K_{sl,1}}{N_L} \\ \frac{\eta_B R_P^2 (K_{sl,1} + K_{sl,2})}{N_L^2 K_{sc}} + 1 \end{bmatrix} \delta_{fw1} \]

\[ + \begin{bmatrix} \frac{\eta_B R_P K_{sl,2}}{N_L} \\ \frac{\eta_B R_P^2 (K_{sl,1} + K_{sl,2})}{N_L^2 K_{sc}} + 1 \end{bmatrix} \delta_{fw2} \]

\[ + \begin{bmatrix} K_{sc} \left[ \frac{\eta_B R_P^2 (K_{sl,1} + K_{sl,2})}{N_L^2 K_T} + 1 \right] \\ \frac{\eta_B R_P^2 (K_{sl,1} + K_{sl,2})}{N_L^2 K_{sc}} + 1 \end{bmatrix} - K_{sc} \delta_{sw} \]

\[ - \begin{bmatrix} \frac{\eta_B R_P (K_{sl,1} + K_{sl,2})}{N_L} \\ \frac{\eta_B R_P^2 (K_{sl,1} + K_{sl,2})}{N_L^2 K_{sc}} + 1 \end{bmatrix} \varepsilon \phi \]
\[
\begin{align*}
\left( \frac{\eta_p R_p}{\left[ \frac{\eta_B R_p^2 (K_{SL,1} + K_{SL,2})}{N_L^2 K_{SC}} + \frac{\eta_B R_p^2 (K_{SL,1} + K_{SL,2})}{N_L^2 K_T} + 1 \right] } \right) I_{PS} &= \frac{\eta_p R_p}{\left[ \frac{\eta_B R_p^2 (K_{SL,1} + K_{SL,2})}{N_L^2 K_{SC}} + \frac{\eta_B R_p^2 (K_{SL,1} + K_{SL,2})}{N_L^2 K_T} + 1 \right] } F_{PS} \\
&= \left( \frac{R_p K_{SL,1}}{N_L} \left(1 + \frac{K_{SC}}{K_T} \right) \times \left( \frac{\eta_B R_p K_{SL,1}}{N_L^2 K_{SC}} \right) \right) - K_{SL,1} \delta_{FW1} \\
&+ \left( \frac{R_p K_{SL,2}}{N_L} \left(1 + \frac{K_{SC}}{K_T} \right) \times \left( \frac{\eta_B R_p K_{SL,2}}{N_L^2 K_{SC}} \right) \right) - K_{SL,2} \delta_{FW2} \\
&+ \left[ \frac{\eta_B R_p (K_{SL,1} + K_{SL,2})}{N_L^2 K_{SC}} + \frac{\eta_B R_p (K_{SL,1} + K_{SL,2})}{N_L^2 K_T} + 1 \right] \\
\end{align*}
\]

Upon eliminating \( \delta_{SL,1} \) and \( \delta_{SL,1} \) from Equation 3.3, and substituting for Equation 3.27, Equation 3.3 becomes

\[
I_{FW1} \delta_{FW1} = -AT_1 - B_{SL,1} \delta_{FW1} - CF_{FW} \times \text{sgn}(\delta_{FW1})
\]
\[
\begin{align*}
\frac{R_P K_{SL1}}{N_L} (1 + \frac{K_{SC}}{K_T}) \times \left[ -\frac{\eta_B R_P^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right] \\
\left[ \frac{\eta_B R_P^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_B R_P^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right] \\
\\frac{R_P K_{SC} K_{SL1}}{N_L K_T} \delta_{SW} \\
\frac{R_P K_{SL1}}{N_L} (1 + \frac{K_{SC}}{K_T}) \times \left[ \frac{\eta_B R_P (K_{SL1} + K_{SL2})}{N_L K_{SC}} \right] \\
\left[ \frac{\eta_B R_P^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_B R_P^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right] \\
\\frac{K_{SL1}}{N_L} \varepsilon \phi \\
\frac{R_P K_{SL1}}{N_L} (1 + \frac{K_{SC}}{K_T}) \times \left[ \frac{\eta_{PS} R_P}{K_T K_{SC}} \right] \\
\left[ \frac{\eta_B R_P^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_B R_P^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right] \\
\\frac{F_{PS}}{N_L} \end{align*}
\]

(3.29)

Using the same method as shown above, Equation 3.4 becomes

\[
I_{FW2} \ddot{\delta}_{FW2} = -AT_2 - B_{SL2} \dot{\delta}_{FW2} - CF_{FW} \times \text{sgn}(\dot{\delta}_{FW2})
\]
\[
\begin{align*}
&\left( \frac{R_p K_{SL2}}{N_L} (1 + \frac{K_{SC}}{K_T}) \times \left( \frac{\eta_B R_p K_{SL1}}{N_L K_{SC}} \right) \right) \delta_{FW1} \\
&+ \left\{ \frac{\eta_B R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_B R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right\} \\
&\left( \frac{R_p K_{SL2}}{N_L} (1 + \frac{K_{SC}}{K_T}) \times \left( \frac{\eta_B R_p K_{SL2}}{N_L K_{SC}} \right) \right) - K_{SL2} \delta_{FW2} \\
&+ \left\{ \frac{\eta_B R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_B R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right\} \\
&\left( \frac{R_p K_{SL2}}{N_L} (1 + \frac{K_{SC}}{K_T}) \times \left( \frac{\eta_B R_p K_{SL1} + K_{SL2}}{N_L K_{SC}} \right) \right) \delta_{SW} \\
&+ \left\{ \frac{\eta_B R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_B R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right\} \\
&\left( \frac{R_p K_{SL2}}{N_L} (1 + \frac{K_{SC}}{K_T}) \times \left( \frac{-\eta_B R_p (K_{SL1} + K_{SL2})}{N_L K_{SC}} \right) \right) + K_{SL2} \phi \\
&+ \left\{ \frac{\eta_B R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_B R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right\}
\end{align*}
\]
Representing Equation 3.8 in favor of $\delta_{SW}, \delta_{FW1}$, and $\delta_{FW2}$, it becomes

$$M_R \ddot{Y}_R = -B \dot{Y}_R - CF_r \times \text{sgn}(\dot{Y}_R)$$

$$\left\{ \frac{\eta_p R_p (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_p R_p K_{SC} (K_{SL1} + K_{SL2})}{N_L^2 K_T} + \frac{K_{SC}}{R_p} \right\}$$

$$\times \left[ \frac{\eta_p R_p K_{SL1}}{N_L K_{SC}} \right] \delta_{FW1}$$

$$\left\{ \frac{\eta_p R_p (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_p R_p (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right\} \frac{\eta_p K_{SL1}}{N_L}$$
\[-\left\{ \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2} + \frac{\eta_B R_P K_{SC} (K_{S11} + K_{S12})}{N_L^2 K_T} + \frac{K_{SC}}{R_P} \right\} \]

\[\times \left\{ \frac{\left( \frac{\eta_B R_P K_{S12}}{K_{SC}} \right)}{\frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_{SC}}} + \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_T} + 1 \right\} \delta_{FW2} \]

\[-\left\{ \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2} + \frac{\eta_B R_P K_{SC} (K_{S11} + K_{S12})}{N_L^2 K_T} + \frac{K_{SC}}{R_P} \right\} \]

\[\times \left\{ \frac{\left( \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_{SC}} + 1 \right)}{\frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_T} + \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_T} + 1} \right\} \delta_{SW} \]

\[\frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2} + \frac{\eta_B R_P K_{SC} (K_{S11} + K_{S12})}{N_L^2 K_T} + \frac{K_{SC}}{R_P} \]

\[\times \left\{ \frac{\left( \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_{SC}} \right)}{\frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_T}} + \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_T} + 1 \right\} \delta_{SW} \]

\[\frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2} + \frac{\eta_B R_P K_{SC} (K_{S11} + K_{S12})}{N_L^2 K_T} + \frac{K_{SC}}{R_P} \]

\[\times \left\{ \frac{\left( \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_{SC}} \right)}{\frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_T}} + \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_T} + 1 \right\} \delta_{SW} \]

\[\frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2} + \frac{\eta_B R_P K_{SC} (K_{S11} + K_{S12})}{N_L^2 K_T} + \frac{K_{SC}}{R_P} \]

\[\times \left\{ \frac{\left( \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_{SC}} \right)}{\frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_T}} + \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_T} + 1 \right\} \delta_{SW} \]

\[\frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2} + \frac{\eta_B R_P K_{SC} (K_{S11} + K_{S12})}{N_L^2 K_T} + \frac{K_{SC}}{R_P} \]

\[\times \left\{ \frac{\left( \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_{SC}} \right)}{\frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_T}} + \frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2 K_T} + 1 \right\} \delta_{SW} \]

\[\frac{\eta_B R_P (K_{S11} + K_{S12})}{N_L^2} + \frac{\eta_B R_P K_{SC} (K_{S11} + K_{S12})}{N_L^2 K_T} + \frac{K_{SC}}{R_P} \]
\[
\left[ \frac{\eta_B R_p (K_{SL1} + K_{SL2})}{N_L^2} + \frac{\eta_B R_p K_{SC} (K_{SL1} + K_{SL2})}{N_L^2 K_T} + \frac{K_{SC}}{R_p} \right] \\
\times \left[ \frac{\eta_{PS} R_p}{K_{SC}} - \frac{\eta_{PS} R_p (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + 1 \right] - \eta_{FS} F_{FS} \]

(3.31)

We now turn our attention to Equations 3.28, 3.29, 3.30 and 3.31. These four equations represent the four degrees of freedom of the steering system, where $\ddot{Y}_R$ can be expressed as a function of the steering wheel angular displacement $\delta_{sw}$, and each road wheel displacement, $\delta_{fw1}$ and $\delta_{fw2}$. The above four equations are very useful for handling studies where the steering system dynamics are connected to the vehicle body dynamics.
3.6 Vehicle Handling Model

Up to now, the mathematical model for power steering system dynamics has been considered. This section will include a three degree-of-freedom vehicle handling model: lateral perturbation velocity, $v$, yaw rate, $\dot{r}$ and sprung mass roll angle, $\phi_s$. Segel [6] derived a 3 D.O.F. automotive model for lateral dynamic properties of the vehicle to steering control. He also showed that the model given by three differential Equations was substantiated by experimental testing of a 1953 Buick Sedan. The work for handling lateral dynamics responses to steering input has been reviewed by many researchers and represented in the literature [7,8,9].

The three equations describing the directional response of the vehicle are given below. These equations, which assume small angles, and are the same as those presented by Allen, et. al. [10], except that here, all products of inertia are assumed to be zero. No pitch or bounce degrees-of-freedom are modeled and the vehicle speed, $U$, is modeled as constant. The coordinate system used for the vehicle model is shown on Figure 3.8. The equations of motions for the vehicle body are written for an orthogonal coordinate system centered at the vehicle sprung mass center of gravity. Figure 3.9 shows the tire side forces and the tire angle relationships.
Figure 3.8 - Vehicle Rear View Coordinate System [7]
Figure 3.9 - Vehicle Top View Schematic Showing Tire Side Forces and Tire Angle Relationship [7]
These models were developed to study the effects of tire dynamics and not to necessarily provide an accurate representation of vehicle response. With this in mind, several simplifying assumptions were made in the development of the equations for the external loads acting on the vehicle, $\Sigma Y$, $\Sigma N$, and $\Sigma L_s$. Some of the expressions used in these equations are different from those presented elsewhere because of these simplifying assumptions and sign convention. The notation used in the following equations is defined in Table 3.4.

**Side Force Equations**

$$\sum Y = m(\dot{v} + V_x r) + m_x e \dot{p}$$  \hspace{1cm} (3.32)

**Yaw Moment Equation**

$$\sum N = I_{zz} \dot{i}$$  \hspace{1cm} (3.33)

**Sprung Mass Roll Moment**

$$\sum L_s = I_{xx} \dot{p} + m_x e (\dot{v} + V_x r)$$  \hspace{1cm} (3.34)

where

$$\sum Y = -2(F_{yR} + F_{yF})$$  \hspace{1cm} (3.35)

$$\sum N = -2(aF_{yF} - bF_{yR})$$  \hspace{1cm} (3.36)

$$\sum L_s = (m_x e - \frac{\partial L}{\partial \phi_s}) \phi_s - \frac{\partial L}{\partial p} \dot{p}$$  \hspace{1cm} (3.37)
Table 3.4 Nomenclature and Parameters Used In Linear Model

<table>
<thead>
<tr>
<th>Nomenclature for Linear Model:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Denotes differential operator, d/dt</td>
</tr>
<tr>
<td>Fy</td>
<td>Tire lateral force (N)</td>
</tr>
<tr>
<td>Ls</td>
<td>Roll moment applied to the sprung mass (N-m)</td>
</tr>
<tr>
<td>N</td>
<td>Yawing moment (N-m)</td>
</tr>
<tr>
<td>p</td>
<td>Roll rate (rad/sec) ( p = \dot{\phi}_s )</td>
</tr>
<tr>
<td>r</td>
<td>Yaw rate (rad/sec)</td>
</tr>
<tr>
<td>U</td>
<td>Vehicle longitudinal steady-state velocity (m/sec), ((U=V_x))</td>
</tr>
<tr>
<td>Y</td>
<td>Lateral force (N) ((Y=F_y))</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Tire slip angle (rad)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Vehicle slip angle (rad)</td>
</tr>
<tr>
<td>(\delta_{sw})</td>
<td>Steering wheel angle (rad)</td>
</tr>
<tr>
<td>(\delta_{fw})</td>
<td>Roadwheel angle (rad)</td>
</tr>
<tr>
<td>(\phi_s)</td>
<td>Roll angle (rad)</td>
</tr>
</tbody>
</table>

Parameters for the 1995 Ford Taurus GL:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>1542</td>
<td>Total vehicle mass</td>
</tr>
<tr>
<td>(m_s)</td>
<td>1356</td>
<td>Vehicle sprung mass</td>
</tr>
<tr>
<td>a</td>
<td>0.92</td>
<td>Longitudinal distance from front axle to C.G.</td>
</tr>
<tr>
<td>b</td>
<td>1.77</td>
<td>Longitudinal distance from rear axle to C.G.</td>
</tr>
<tr>
<td>e</td>
<td>0.454</td>
<td>Distance from roll axis to sprung mass C.G.</td>
</tr>
<tr>
<td>g</td>
<td>9.8</td>
<td>((m/s^2)) Acceleration due to gravity</td>
</tr>
<tr>
<td>(I_{zz})</td>
<td>2786</td>
<td>Total vehicle yaw mass moment of inertia</td>
</tr>
<tr>
<td>(I_{xxs})</td>
<td>670</td>
<td>Sprung mass roll inertia about vehicle roll axis</td>
</tr>
<tr>
<td>(K_{ew})</td>
<td>16.0</td>
<td>Steering ratio</td>
</tr>
<tr>
<td>ef</td>
<td>0.01</td>
<td>((rad/rad)) Front axle roll steer</td>
</tr>
<tr>
<td>er</td>
<td>0.01</td>
<td>((rad/rad)) Rear axle roll steer</td>
</tr>
<tr>
<td>(\partial L / \partial p)</td>
<td>3757</td>
<td>((N-m/sec/rad)) Total vehicle roll damping</td>
</tr>
<tr>
<td>(\partial L / \partial \phi_s)</td>
<td>147588</td>
<td>((N-m/rad)) Total vehicle roll stiffness</td>
</tr>
<tr>
<td>(\partial F / \partial \alpha)</td>
<td>(N/rad)</td>
<td>Tire cornering stiffness</td>
</tr>
<tr>
<td>(Calpha)</td>
<td>70000</td>
<td>((N/rad)) Front axle tire cornering stiffness</td>
</tr>
<tr>
<td>(Calphar)</td>
<td>51000</td>
<td>((N/rad)) Rear axle tire cornering stiffness</td>
</tr>
<tr>
<td>RL</td>
<td>1</td>
<td>((m)) Relaxation length</td>
</tr>
</tbody>
</table>
As previously mentioned, to better understand the actual dynamics characteristics, we need to consider the tire model. Tire dynamics has been an important contributor to the vehicle body dynamics. Guenther et. al. [11] and Loeb. J.S. [12] developed a simple first order model for the dynamic response of tire side force to variations in slip angle. Loeb showed the good agreement for the tire amplitude ratio \( F_{yl}/F_y \) by testing eight tires.

Heydinger [8] included the first-order tire dynamics which improved the predictions of the VDANL simulation. The first-order lag on the tire side force, based on tire relaxation length, is described by the following equation.

\[
\frac{F_{yl\text{(D)}}}{F_y} = \frac{1.0}{\tau_{TL}D + 1.0} \quad (3.38)
\]

\[
\frac{F_{yr\text{(D)}}}{F_y} = \frac{1.0}{\tau_{TL}D + 1.0} \quad (3.39)
\]

Where \( F_{yl\text{(D)}} \) is the lagged front (rear) side force, \( F_{yr\text{(D)}} \) is the side front (rear) force from the quasi-static tire model, \( \tau_{TL} \) is the tire lag time constant and \( D \) is the differential operator \((d/dt)\). The time constant is expressed by the following
\[ \tau_{TL} = \frac{C}{KU} \quad (\text{sec}) \]  

Where \( C \) is cornering stiffness of the tire, \( K \) lateral stiffness of the tire and \( U \) the vehicle speed.

As shown in Figure 3.9, the slip angles, \( \alpha \)'s, can be expressed as

\[ \alpha_F = \left[ \frac{v + ar}{U} - \delta_{FW} \right] \]  

\[ \alpha_R = \left[ \frac{v + ar}{U} - \delta_{RW} \right] \]  

The wheel steer equations given below include roll steer but no bounce steer effects. For the front wheels, the steer angle, \( \delta_{FW} \), is a function of the input steering angle, \( \delta_{SW} \), roll steer and the compliance steer. For the rear wheels, the steer angle, \( \delta_{RW} \), is caused by roll steer and compliance. For this model, both front wheel were assumed to steer the same amount. The same is true for rear wheels. This model is a so called bicycle model, or two wheel model, since no lateral weight transfer was modeled into the equations (that is, no \( F_z \) effects on the lateral tire forces were modeled).
\[
\delta_{FW} = \frac{\delta_{SW}}{K_{SW}} + \epsilon_F \times \phi + K_{SCF} \times F_{yr}
\]

(3.43)

\[
\delta_{RV} = K_{SCR} \times F_{yr} + \epsilon_R \times \phi
\]

(3.44)

Substituting Equations 3.43 and 3.44 into Equations 3.41 and 3.42 gives the following expressions for the slip angles.

\[
\alpha_F = \left[ \frac{v + ar}{U} - \left( \frac{\delta_{SW}}{K_{SW}} + \epsilon \times \phi + K_{SCF} \times F_{yr} \right) \right]
\]

(3.45)

\[
\alpha_R = \left[ \frac{v + ar}{U} - \left( K_{SCR} \times F_{yr} + \epsilon \times \phi \right) \right]
\]

(3.46)

where \( \epsilon \) equals to the front and rear roll steer.

3.7 Numerical Solution for the Combination of Two Models

From the previous two sections, we have obtained the differential equations for the power steering system dynamics and a linear vehicle handling model with a first order lagged lateral force on the tire. Figure 3.10 shows the block diagram to
approach to the solutions to the linear vehicle model's equations. The slip angles from the Equation 3.45 and 3.46 can be calculated from the given steering wheel angle input and vehicle responses (yaw rate, lateral velocity, lateral force by steering compliance). Multiplying the tire cornering stiffness with the slip angles produce the lateral forces which are the inputs to the vehicle model.

Figure 3.10 A Representation of the Blockdiagram for the Linear Vehicle Model
In the linear vehicle model, the roadwheel angle was considered with the steering compliance and the steering wheel angle was divided by the steering gear ratio. However, in the steering system model, it is not necessary to consider the steering compliance and the steering gear ratio to calculate the slip angles. It is because the steering system has already included the steering gear ratio and the compliances in the model. Thus, Figure 3.11 provides the different methodology to solve the slip angles. Figure 3.11 shows the combined blockdiagram of the power steering system model and the 5 D.O.F. linear vehicle model.

Figure 3.11 A Representation of the Combined Blockdiagram for the Power Steering System Model and the 5 D.O.F. Linear Vehicle Handling Model
References


CHAPTER IV

PARAMETER MEASUREMENT

4.1 Introduction

This chapter introduces the measurement of the parameters used in the vehicle model. The methodology to measure the compliance’s will follow the similar step which was used by Rupp [1], Tardy [2] and Heydinger [3]. In addition, the information about the power steering pump boost curves will be described in detail.

The developed model uses an extensive number of parameters to describe the steering system. Though the parameters used in the computer simulation are representative of physically measurable quantities of a typical power steering system,
the accuracy of the simulation model is largely dependent upon the vehicle parameter measurements that are obtained. The ability to easily measure these parameters was a top priority to simplify the implementation into the computer simulation. Thus, this chapter discusses the acquisition and estimation of the parameters which are used into the computer simulation model.

4.2 Compliances Measurement

Figure 4.1 shows a schematic of the steering system compliance measurement procedure which shows an effective steering column torsional spring, \( K_{SC} \); torsion bar stiffness, \( K_T \); two effective steering linkage springs, \( K_{SL1} \) and \( K_{SL2} \); and the tire aligning moments, \( M_{Z1} \) and \( M_{Z2} \). As shown, a moment, \( M_{Z1} \), is applied about wheel 1 using a lever arm apparatus attached to wheel 1. A load cell at the end of the moment arm is used to measure the force applied to the moment arm, from which the moment, \( M_{Z1} \), can be determined. During this test, the steering wheel is held fixed, and the rotations of both wheel 1, \( \delta_{FW1} \), and wheel 2, \( \delta_{FW2} \), are recorded. Both front wheels are on air bearings which allow them to rotate freely about their steer axes. Moments are applied to cause wheel 1 to rotate between about ±3 degrees. The vehicle engine is running during the tests, and the power steering system will be active. This will
keep the deflections in the torsion bar very small, so the torsion bar stiffness can be neglected in this measurement procedure.

Figure 4.1 - A Schematic of Compliances Measurement [Adapted from 1, 3]
Since wheel 2 is allowed to rotate freely, there is no deflection in $K_{SL2}$, and all rotation of wheel 2 is due to the compliance of $K_{SC}$. Using the procedure the effective steering column stiffness, $K_{SC}$, is defined by the relationship between $M_{Z1}$ and $\delta_{FW2}$.

$$K_{SC} = \frac{\Delta M_{Z1}}{\Delta \delta_{FW2}}$$  \hspace{1cm} (4.1)

The relationship between $M_{Z1}$ and $\delta_{FW1}$ defines the stiffness of $K_{SC}$ and $K_{SL1}$ added together in series, $K_{TOTAL}$.

$$K_{TOTAL} = \frac{K_{SC} \cdot K_{SL1}}{K_{SC} + K_{SL1}} = \frac{\Delta M_{Z1}}{\Delta \delta_{FW1}}$$  \hspace{1cm} (4.2)

With $K_{SC}$ and $K_{T}$ known, $K_{SL1}$ can be computed, from the equation above, as

$$K_{SL1} = \frac{K_{TOTAL} \cdot K_{SC}}{K_{SC} - K_{TOTAL}}$$  \hspace{1cm} (4.3)

For this research $K_{SL1}$ and $K_{SL2}$ were assumed to be equal, thus requiring measurement to be taken on only one side of vehicle. For a vehicle with a power assisted steering system, the vehicle's engine must be running. Typical hydraulic
power steering systems contain a rotary servo valve which is supplied with fluid supply pressure, from the power steering pump, and is typically not engine speed dependent. Table 4.1 shows the values for the effective steering system stiffness for the Ford Taurus, GL.

<table>
<thead>
<tr>
<th>Steering Stiffness</th>
<th>Effective Steering Stiffness Value (N·m/rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ksc</td>
<td>42079</td>
</tr>
<tr>
<td>Kt</td>
<td>83</td>
</tr>
<tr>
<td>KS1</td>
<td>14878</td>
</tr>
</tbody>
</table>

The compliance of the torsion bar was measured using a specially designed test rig [4]. The torsion bar needs to be removed from the vehicle and mounted in the test rig, so the compliance can be measured.

4.3 Power Assist Force

The information of the piston area and differential pressure developed as the valve opens determines the effective power assist force that the power steering system applies to the rack. A constant speed vehicle test with a slowly increasing steering
wheel input causes the deflection of the upper steering column compliances elements including the torsion bar. The deflection of the torsional bar is the difference in the steering column angular displacement and the pinion gear angular displacement. That is, the equation for the valve angular displacement is the following:

\[
\delta_{\text{VALVE}} = \delta_{SC} - \delta_T
\]  

(4.4)

The power steering boost curve was determined by measuring the torque at the torsion bar and tierod forces on both linkages as the torsional bar valve is twisted during a vehicle test. By slowly increasing the steering wheel angle, while recording the steering wheel torque and tierod forces, the pressure boost curve can be obtained from the following relationship:

\[
\frac{T_p}{R_p} = \text{Tierod Forces} + F_{PS} \text{ (Power Assist Force)}
\]  

(4.5)

where \( R_p \) is the pinion gear pitch radius, and \( T_p \) is the measured torque in the steering column.

Differential Pressure = \( F_{PS} / \text{Piston Area} \)  

(4.6)
Figure 4.2 shows the pressure boost curve as the steering wheel is turned against the rack. The displacement of the torsional bar angle needs to be zeroed from the initial point of contact against the rack. The valve angle is defined as shown in Equation 4.1.

![Pressure Boost Curve at 22 & 32 m/sec](image)

Legend: Experiments (Dashed) Curve Fit (Solid)

**Figure 4.2 - Power Pressure Boost Curve**

Using the mathematical relationship from Section 3.5, the valve angle at steady state can be expressed by:
\[ \delta_{VALVESS} = \]
\[ + \left( \frac{\eta_b R_p K_{SL1}}{N_L K_{SC}} \right) \delta_{FW1} \]
\[ + \left( \frac{\eta_b R_p K_{SL2}}{N_L K_{SC}} \right) \delta_{FW2} \]
\[ + \left( \frac{\eta_b R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_b R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right) \delta_{SW} \]
\[ - \left( \frac{\eta_b R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_b R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right) \phi \]
\[ - \left( \frac{\eta_b R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_b R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right) \frac{\eta_{PS} R_p}{K_{SC}} \]
\[ - \left( \frac{\eta_b R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_{SC}} + \frac{\eta_b R_p^2 (K_{SL1} + K_{SL2})}{N_L^2 K_T} + 1 \right) F_{PS} \]

\[ (4.7) \]
Equation 4.7 is an important equation to calculate the torque by multiplying the torsion bar stiffness. The scale of pressure should be zeroed from its initial contact pressure at the rack stop. With this information, the curve can be replotted with the zeroed pressure versus the zeroed valve displacement. A eighth order polynomial curve fit was used to provide sufficient match of the experimental data. Figure 4.2 shows a curvefit of the experimental data to form the pressure boost curve for torsion bar valve displacement. The implementation of the power assist force, \( F_{PS} \), on the rack is shown in the next equation:

\[
F_{PS} = (C_1 \delta_v^8 + C_2 \delta_v^7 + C_3 \delta_v^6 + C_4 \delta_v^5 + C_5 \delta_v^4 + C_6 \delta_v^3 + C_7 \delta_v^2 + C_8 \delta_v + C_9) \cdot A_{\text{piston area}}
\]

(4.8)

where

\( C_1 = 4.7097 \times 10^11 \)
\( C_2 = 5.6380 \times 10^{10} \)
\( C_3 = -1.1864 \times 10^{10} \)
\( C_4 = -1.1362 \times 10^{-5} \)
\( C_5 = 8.0934 \times 10^7 \)
\( C_6 = 6.8698 \times 10^{-8} \)
\( C_7 = 3.4791 \times 10^5 \)
\( C_8 = -9.8992 \times 10^{-11} \)
\( C_9 = -115.9779 \)
4.4 Coulomb Friction Force Measurement

The coulomb friction or 'dry friction' is important to the overall steering model for its contribution to the overall stability of the automobile. This section will show the development of the Coulomb friction model and its implementation into the simulation algorithm.

The classic Coulomb friction model uses a constant friction force that opposes the motion when the velocity does not equal zero. This means that friction is independent of velocity. A simple mass-spring system can be used to represent this in the following figure.

![Diagram of a mass-spring system with friction](image)

**Figure 4.3 - A Simple Representation of Coulomb Friction of Mass Spring System**
For zero velocity, the 'stiction' will oppose the motion as long as the forces applied are smaller in magnitude than the 'stiction' force. When the applied force is greater than the 'stiction' force or breakout force, the linear friction force will be applied to the dynamic motion. The computer simulation will implement the following logic for Coulomb friction:

\[ F = CF \times \text{Sign(Velocity)} \]  \quad (4.9)

Figure 4.4 - Experimental Hysteresis plot of the Steering Wheel Torque and Steering Wheel Displacement (22 m/sec On-Center Weave Test - 1994 Ford Taurus, GL)
The Coulomb friction model in the steering system simulation is lumped with the linear motion of the steering rack for easy modeling. Figure 4.4 shows the overall steering dynamics with a plot of the applied steering wheel torque versus the steering wheel displacement to reveal the Coulomb friction characteristics (half the width of the hysteresis in N-m). This paper uses two Coulomb friction forces, 261 N at the vehicle speed 22 m/sec, and 169 N at 33 m/sec which were provided by the Ford Motor Company.

4.5 Steering System Damping

The values for damping used in the simulation were mainly taken from published literature because no direct measures were performed. The viscous damping proved to be the most difficult of the parameters to obtain. The viscous damping values were based on reasonable estimates of the percentage of critical damping from the time domain field test data. Each damping coefficient was then calculated in relation to the overall steering kinematics and its location in the steering model. A proportionally higher viscous damping term is used on the steering rack to prevent the nonlinear coulomb element from becoming unstable. The power steering
equipped automobile is assumed to have a higher degree of viscous damping because of the hydraulic assistance element. The power steering rack is filled with hydraulic fluid, which acts as viscous damping and helps to reduce road shock inputs from being transmitted to the steering wheel. This also reduces the road feel back to the driver as compared to a manual steering vehicle.

After the values of viscous damping were proportioned throughout the steering model, numerical sensitivity on the individual viscous elements was performed to see how each affected the overall model.

4.5 Measured and Estimated Parameter Values

This section shows the parameter values which are used in the computer simulation. Table 4.2 shows the parameter values used in the vehicle model. Table 4.3 shows the measured and estimated parameter values to run the power steering system simulation. Table 4.3 shows the test maneuver codes used in VRTC and also provides each vehicle speed, steering wheel angle, lateral acceleration and coulomb friction forces provided by the Ford Motor Company. To avoid the numerical instability from the Coulomb friction, the measured deadband, 6.1°, was used in the computer simulation.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Vehicle Parameter Description</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vx</td>
<td>Vehicle Forward Speed</td>
<td>10,22,33</td>
<td>m/s</td>
</tr>
<tr>
<td>m</td>
<td>Total Vehicle Mass</td>
<td>1542</td>
<td>kg</td>
</tr>
<tr>
<td>m_s</td>
<td>Vehicle Sprung Mass</td>
<td>1356</td>
<td>kg</td>
</tr>
<tr>
<td>a</td>
<td>Longitudinal distance from front axle to vehicle C.G.</td>
<td>0.92</td>
<td>m</td>
</tr>
<tr>
<td>b</td>
<td>Longitudinal distance from rear axle to vehicle C.G.</td>
<td>1.77</td>
<td>m</td>
</tr>
<tr>
<td>I_xxs</td>
<td>Sprung mass roll inertia about vehicle roll axis</td>
<td>670</td>
<td>kg-m²</td>
</tr>
<tr>
<td>I_zz</td>
<td>Total vehicle yaw mass moment of inertia</td>
<td>2786</td>
<td>kg-m²</td>
</tr>
<tr>
<td>K_sw</td>
<td>Steering ratio</td>
<td>16</td>
<td>rad/rad</td>
</tr>
<tr>
<td>e</td>
<td>Distance from roll axis to sprung mass C.G.</td>
<td>0.454</td>
<td>m</td>
</tr>
<tr>
<td>Calphaf</td>
<td>Front axle tire cornering stiffness</td>
<td>70000</td>
<td>N/rad</td>
</tr>
<tr>
<td>Calphar</td>
<td>Rear axle tire cornering stiffness</td>
<td>51000</td>
<td>N/(rad)</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration due to gravity</td>
<td>9.8</td>
<td>m/s²</td>
</tr>
<tr>
<td>dL_dPhi</td>
<td>Total vehicle roll stiffness</td>
<td>147588</td>
<td>N-m/rad</td>
</tr>
<tr>
<td>dL_dPhi</td>
<td>Total vehicle roll damping</td>
<td>3757</td>
<td>N-m/(rad-sec)</td>
</tr>
<tr>
<td>ef</td>
<td>Front axle roll steer</td>
<td>0.01</td>
<td>rad/rad</td>
</tr>
<tr>
<td>er</td>
<td>Rear axle roll steer</td>
<td>0.01</td>
<td>rad/rad</td>
</tr>
<tr>
<td>RL</td>
<td>Relaxation Length</td>
<td>1</td>
<td>m</td>
</tr>
<tr>
<td>X_T</td>
<td>Overall trail length</td>
<td>-0.0305</td>
<td>m</td>
</tr>
<tr>
<td>tau</td>
<td>Time constant(RL/Vx)</td>
<td>Vx/RL</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Measured Parameter Description</td>
<td>Values</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------------------------------------------------------------------------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>$B_{SW}$</td>
<td>Upper Steering Column viscous damping coefficient</td>
<td>0.36042</td>
<td>(N-m)/(rad/s)</td>
</tr>
<tr>
<td>$I_{SW}$</td>
<td>Moment of Inertia of handwheel</td>
<td>0.0344</td>
<td>(N-m)/(rad/sec$^2$)</td>
</tr>
<tr>
<td>$K_{SC}$</td>
<td>Steering column rotational stiffness</td>
<td>42079</td>
<td>N-m/rad</td>
</tr>
<tr>
<td>$K_T$</td>
<td>Rotational stiffness of power steering torsion spring on variable flow valve</td>
<td>83</td>
<td>N-m/rad</td>
</tr>
<tr>
<td>$R_P$</td>
<td>Radius of pinion</td>
<td>0.007367</td>
<td>m</td>
</tr>
<tr>
<td>$M_R$</td>
<td>Steering rack mass</td>
<td>2</td>
<td>kg</td>
</tr>
<tr>
<td>$N_L$</td>
<td>Front steering linkage rate on driver side from displacement to rack road wheel angle</td>
<td>0.11816</td>
<td>m</td>
</tr>
<tr>
<td>$K_{SL1}$</td>
<td>Front passenger side steering rotational stiffness due to linkage and bushing</td>
<td>14878</td>
<td>(N-m)/rad</td>
</tr>
<tr>
<td>$K_{SL2}$</td>
<td>Front driver side steering rotational stiffness due to linkage and bushing</td>
<td>14878</td>
<td>(N-m)/rad</td>
</tr>
<tr>
<td>$I_{FW1}$</td>
<td>Inertia of right wheel and rotation mass about steering displacement</td>
<td>0.61463</td>
<td>(N-m)/(rad/sec$^2$)</td>
</tr>
<tr>
<td>$I_{FW2}$</td>
<td>Inertia of left wheel and rotation mass about steering displacement</td>
<td>0.61463</td>
<td>(N-m)/(rad/sec$^2$)</td>
</tr>
<tr>
<td>$B_R$</td>
<td>Viscous damping of steering rack linear motion</td>
<td>88.128</td>
<td>(N-m)/(rad/s)</td>
</tr>
<tr>
<td>$B_{SL1,2}$</td>
<td>Viscous damping at each steering linkage bushing</td>
<td>25-65</td>
<td>(N-m)/(rad/s)</td>
</tr>
<tr>
<td>$C_{FR}$</td>
<td>Coulomb friction breakout force on steering rack</td>
<td>169; 261</td>
<td>N</td>
</tr>
<tr>
<td>$C_{FW}$</td>
<td>Coulomb friction breakout force on roadwheel</td>
<td>169; 261</td>
<td>N</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Roll steer coefficient</td>
<td>0.01</td>
<td>rad/rad</td>
</tr>
<tr>
<td>$\eta_F$</td>
<td>Gear ratio efficiency of forward torque transmission</td>
<td>0.985</td>
<td>%</td>
</tr>
<tr>
<td>$\eta_B$</td>
<td>Gear ratio efficiency of backward torque transmission</td>
<td>0.985</td>
<td>%</td>
</tr>
<tr>
<td>$\eta_{PS}$</td>
<td>Efficiency of the power steering system due to hydraulic losses</td>
<td>0.95</td>
<td>%</td>
</tr>
<tr>
<td>$A_{PIST}$</td>
<td>Power steering pump effective piston area</td>
<td>0.0010645</td>
<td>m$^2$</td>
</tr>
</tbody>
</table>
Table 4.4 Test Codes and Parameters Inputs Used in the Computer Simulation

<table>
<thead>
<tr>
<th>Maneuver Code</th>
<th>Maneuver Description</th>
<th>$V_x$ (m/sec)</th>
<th>$C_{F_R,CF_{PW}}$ (N)</th>
<th>Steering Input (rad)</th>
<th>$A_y$ (m/sec²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>Constant Speed I-Turn</td>
<td>11.2</td>
<td>261</td>
<td>0.75</td>
<td>2.0</td>
</tr>
<tr>
<td>CN</td>
<td></td>
<td>22.7</td>
<td>261</td>
<td>-0.297</td>
<td>-2.0</td>
</tr>
<tr>
<td>CP</td>
<td></td>
<td>22.3</td>
<td>261</td>
<td>0.58</td>
<td>4.0</td>
</tr>
<tr>
<td>CV</td>
<td></td>
<td>33</td>
<td>169</td>
<td>0.27</td>
<td>2.0</td>
</tr>
<tr>
<td>CW</td>
<td></td>
<td>33.5</td>
<td>169</td>
<td>-0.42</td>
<td>-4.0</td>
</tr>
<tr>
<td>XC00</td>
<td>Slowly Increasing Steer (negative)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>XC01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>XC02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>XC03</td>
<td>Slowly Increasing Steer (Positive)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>XC04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>XC05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>XI</td>
<td>On-Center : Transition</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>XH</td>
<td>On-Center : Weave</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PA</td>
<td>Pulse Steer</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PE</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

References


CHAPTER V

RESULTS AND DISCUSSIONS

5.1 Introduction

This chapter focuses on the validation of the steering system modeling using the program SIMULINK which is part of MATLAB[1]. One of the most important aspects of computer simulation is the validation of the model. The derived mathematical model representation of the power steering system must be compared to the field test results to see in what circumstances the results can be relied upon. Therefore, several effective field test maneuvers must be arranged to show the model performance. This chapter is divided into six main sections; steady state time domain results, time domain responses to slowly increasing steer input, time domain
responses to on-center transition maneuvers, time domain responses to on-center weave tests, time domain responses to pulse input, and frequency domain (frequency response) results.

All of the simulation results presented in this chapter are from simulation runs using the "as measured" vehicle parameters. That is, no parameter adjustments were made to improve the simulation's predictions. Many vehicle responses to steering inputs can be measured and simulated. However, for the purposes of evaluating the validity of a vehicle handling simulation, only a few responses, which define the major motions of the vehicle, need be studied. These include; roadwheel angular displacement, lateral acceleration and yaw rate. To evaluate the dynamic characteristics from the power steering system, the responses for handwheel torsion bar deflection, power assist force and handwheel torque need be obtained.

5.2 Time Domain Comparison to Step Steer Inputs

The results for the steady-state time domain comparison are generated using J-turn maneuvers. The simulated results were obtained using the same angular displacement step input as was experimentally measured during the field test to generate the desired lateral acceleration level. All the results are appended in the end
of this section. All of the roadwheel angular displacements and steering handwheel inputs are shown in the first figure of all figures. The second figure in each figure set shows the comparison of lateral acceleration responses and the third figure in each figure set shows the comparison of yaw rate. The torsion bar deflections of each maneuver are also shown on the fourth figure in all figure sets. The simulation predictions of the torsion bar deflection represent an important characteristic for the handwheel torque because handwheel torque can be calculated from the multiplication between torsion bar stiffness and torsion bar deflection. The fifth figure in each figure set provides the power assist force in the computer simulation. The final and sixth figure in all figure set compares the handwheel torque predictions with experimental results.

The results for the lateral acceleration deviate for all cases for the 1994 Ford Taurus GL. At low speed, the lateral acceleration became close to the experimental result and this also happens in the responses to slowly increasing steer inputs. At higher speeds, the lateral acceleration values as shown in Figures 5.1, 5.2 and 5.3 are a little higher than the experimental values. Notice that the lateral acceleration is a function of yaw rate and forward speed. Thus, we are going to look at the yaw rate responses in detail.
**Yaw Rate Response Comparison**

The yaw rate prediction of the vehicle is an important characteristic for validation of the developed steering model, because it is largely dependent on the developed steering model. Simulation yaw rate response to a steering input can only accurately predict actual vehicle behavior if many vehicle attributes are correctly modeled. The field tests provide a good match with the yaw rate predictions from the simulation runs for several cases.

As part of the thorough evaluation of a simulation, it is necessary to examine simulation’s predictions at various levels of maneuver severity. For instance, a simulation which accurately predicts yaw rate gains at lateral acceleration levels of 0.2 g’s may not do a good job of prediction yaw rate at 0.6 g’s lateral acceleration. Thus, Figure 5.1 through Figure 5.5 represent a good agreement between the simulation predictions and experimental results.

Table 5.1 is provided to show the variability of the simulations’ steady state yaw rate predictions over the range of lateral accelerations tested as shown in Figures 5.1 through Figure 5.5. This table shows the relative difference (RD) between the simulated predictions of steady state yaw rate and the experimental values of yaw rate. The computed relative differences are the difference between the simulation
Table 5.1 Yaw Rate Response, Peak Response Time, and Percent Overshoot -
(1994 Ford Taurus GL)

<table>
<thead>
<tr>
<th>Lateral Accel.</th>
<th>Handwheel Angle (rad)</th>
<th>Vehicle Speed (m/s)</th>
<th>EXPER. Value</th>
<th>Simulation Model (RD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.4 g</td>
<td>-0.42</td>
<td>33.5</td>
<td>-0.1065</td>
<td>-0.1104 (-4)</td>
</tr>
<tr>
<td>-0.2 g</td>
<td>-0.297</td>
<td>22.7</td>
<td>-0.089</td>
<td>-0.0731 (18)</td>
</tr>
<tr>
<td>+0.2 g</td>
<td>+0.75</td>
<td>11.2</td>
<td>0.1402</td>
<td>0.1358 (-0.3)</td>
</tr>
<tr>
<td>+0.2 g</td>
<td>+0.27</td>
<td>33</td>
<td>0.0477</td>
<td>0.0677 (42)</td>
</tr>
<tr>
<td>+0.4 g</td>
<td>+0.58</td>
<td>22.3</td>
<td>0.1539</td>
<td>0.1521 (-0.01)</td>
</tr>
</tbody>
</table>

Peak Response Time (sec)

<table>
<thead>
<tr>
<th>Lateral Accel.</th>
<th>Handwheel Angle (rad)</th>
<th>Vehicle Speed (m/s)</th>
<th>EXPER. Value</th>
<th>Simulation Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.4 g</td>
<td>-0.42</td>
<td>33.5</td>
<td>0.450</td>
<td>0.402</td>
</tr>
<tr>
<td>-0.2 g</td>
<td>-0.297</td>
<td>22.7</td>
<td>0.380</td>
<td>0.342</td>
</tr>
<tr>
<td>+0.2 g</td>
<td>+0.75</td>
<td>11.2</td>
<td>0.245</td>
<td>0.282</td>
</tr>
<tr>
<td>+0.2 g</td>
<td>+0.27</td>
<td>33</td>
<td>0.360</td>
<td>0.371</td>
</tr>
<tr>
<td>+0.4 g</td>
<td>+0.58</td>
<td>22.3</td>
<td>0.390</td>
<td>0.326</td>
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</tbody>
</table>

Percent Overshoot (%)

<table>
<thead>
<tr>
<th>Lateral Accel.</th>
<th>Handwheel Angle (rad)</th>
<th>Vehicle Speed (m/s)</th>
<th>EXPER. Value</th>
<th>w/ Steering Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.4 g</td>
<td>-0.42</td>
<td>33.5</td>
<td>35.8</td>
<td>21</td>
</tr>
<tr>
<td>-0.2 g</td>
<td>-0.297</td>
<td>22.7</td>
<td>10.2</td>
<td>8.4</td>
</tr>
<tr>
<td>+0.2 g</td>
<td>+0.75</td>
<td>11.2</td>
<td>0.35</td>
<td>14.1</td>
</tr>
<tr>
<td>+0.2 g</td>
<td>+0.27</td>
<td>33</td>
<td>30</td>
<td>21.6</td>
</tr>
<tr>
<td>+0.4 g</td>
<td>+0.58</td>
<td>22.3</td>
<td>8.32</td>
<td>9.2</td>
</tr>
</tbody>
</table>
steady state values and experimental steady state mean values divided by the experimental steady state mean values. The positive RD value represents experimental quantities whose absolute values are less than the corresponding simulated values. Conversely, negative RD differences represent experimental quantities whose absolute values are greater than the simulated values.

**Handwheel Torque Comparison**

In general, for the handwheel torque predictions it is found that the transient responses are a little dependent on the damping coefficient value of the steering linkage bushing and the damping on the rack and also these values can affect the magnitude of the steady state handwheel torque response. This is because the damping coefficient value should not be a constant. When simulating the actual damping coefficient, its value should be considered or estimated based on the vehicle speed and handwheel input magnitude. Figure 5.1 through 5.5 show the good transient response for the handwheel torque response. For the simulation of these maneuvers, the damping values for the steering linkage bushing, from 25 (N-m)/(rad/sec) to 65 (N-m)/(rad/sec) were used. The steady state handwheel torque predictions showed a small difference about 0.2 (N-m). When a big damping coefficient for the steering linkage bushing, 125 (N-m)/(rad/sec) or 225
(N-m)/(rad/sec) was used in the computer simulation, overshoot value became much bigger shown in Figure 5.3 though this change does not affect the lateral acceleration and yaw rate.

One of the factors which can affect the handwheel torque are the steering system stiffness values. Though the stiffness values were obtained from experiments, 'engine off test' and 'engine on test', the stiffness values can be softer in the actual world and the point is that the stiffness value can be smaller in the field test. The model did not include the amount of free play. This may be another factor to produce the bigger handwheel torque predictions.

The most important factor for the prediction of handwheel torque is the aligning torque prediction. The aligning torque feedback has directly had effect the handwheel torque predictions. With the constant data of tire cornering stiffness, the pneumatic trail length ($X_T$) which was obtained from the experiment in a test is assumed to be changed in the real world. The value of the pneumatic trail length in the experiment was -3.05 cm. Although the value was a little changed from -3.05 to 3 cm, due to the big value of tire cornering stiffness ($\alpha$), 70,000 N/rad, the aligning torque feedback can be changed a lot. Thus, this kind of change can produce the different predictions. The inspection of the handwheel torque will be looked at from the other manuevers in detail.
Table 5.2 shows the simulation’s handwheel torque predictions over the range of lateral accelerations tested as shown in Figure 5.1 through Figure 5.5. This table shows the RD between the simulated values of steady state handwheel torque and the experimental values of handwheel torque. Experimental values show that the handwheel torque ranges between ±4.4 N-m and the simulation prediction changes within ± 4.5 N-m.

Generally, it is found that the valve angle deflects within ± 0.07 rad. The power assist force to keep a certain handwheel torque provides about 1400 N. As shown in Figure 5.3 and Figure 5.4, these figures imply that the actual power assist force are in the range of 800 N.

<table>
<thead>
<tr>
<th>Lateral Acceleration (g)</th>
<th>Handwheel Angle (rad)</th>
<th>Vehicle Speed (m/s)</th>
<th>EXPER. Value</th>
<th>Relativity Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.4 g</td>
<td>-0.42</td>
<td>33.5</td>
<td>-3.3851</td>
<td>-4.0042 (18.3)</td>
</tr>
<tr>
<td>-0.2 g</td>
<td>-0.297</td>
<td>22.7</td>
<td>-3.0824</td>
<td>-3.6785 (9.5)</td>
</tr>
<tr>
<td>+0.2 g</td>
<td>+0.75</td>
<td>11.2</td>
<td>3.062</td>
<td>4.990 (63)</td>
</tr>
<tr>
<td>+0.2 g</td>
<td>+0.27</td>
<td>33</td>
<td>3.325</td>
<td>4.412 (32.7)</td>
</tr>
<tr>
<td>+0.4 g</td>
<td>+0.58</td>
<td>22.3</td>
<td>4.371</td>
<td>5.047 (15.5)</td>
</tr>
</tbody>
</table>
Roadwheel Angular Displacement (Dashed) and Steer Input (Line)

Lateral Acceleration (m/sec²)

Yaw Rate (rad/sec)

Legend: Experimental Mean Value(Line) Simulation(Dashed)

Figure 5.1 - Vehicle Dynamics Responses from +0.4 g J-Turn Maneuver at 22.3 m/sec (1994 Ford Taurus, GL)
(Figure 5.1 Continued)

Torsion Bar Deflection (rad)

Power Assist Force (N)

Handwheel Torque (N-m)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
Roadwheel Angular Displacement (Dashed) and Steer Input (Line)

Lateral Acceleration (m/sec^2)

Yaw Rate (rad/sec)

Legend: Experimental Mean Value (Line)   Simulation (Dashed)

Figure 5.2 - Vehicle Dynamics Responses from +0.2 g J-Turn Maneuver at 33.5 m/sec (1994 Ford Taurus, GL)
(Figure 5.2 Continued)

Torsion Bar Deflection (rad)

Power Assist Force (N)

Handwheel Torque (N-m)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
Figure 5.3 - Vehicle Dynamics Responses from +0.2 g J-Turn Maneuver at 11.2 m/sec (1994 Ford Taurus, GL)
(Figure 5.3 Continued)

Torsion Bar Deflection (rad)

Power Assist Force (N)

Handwheel Torque (N-m)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
Roadwheel Angular Displacement (Dashed) and Steer Input (Line)

Lateral Acceleration (m/sec$^2$)

Yaw Rate (rad/sec)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)

Figure 5.4 - Vehicle Dynamics Responses from -0.2 g J-Turn Maneuver at 22.7 m/sec (1994 Ford Taurus, GL)
(Figure 5.4 Continued)

Torsion Bar Deflection (rad)

Power Assist Force (N)

Handwheel Torque (N-m)

Legend: Experimental Mean Value (Line) Simulation (Dashed)
Roadwheel Angular Displacement (Dashed) and Steer Input (Line)

Lateral Acceleration (m/sec²)

Yaw Rate (rad/sec)

Legend: Experimental Mean Value (Line)   Simulation (Dashed)

Figure 5.5 - Vehicle Dynamics Responses from -0.4 g J-Turn Maneuver at 33.5 m/sec (1994 Ford Taurus, GL)
(Figure 5.5 Continued)

Torsion Bar Deflection (rad)

Power Assist Force (N)

Handwheel Torque (N-m)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
5.3 Time Domain Responses to Maneuver Severity

In this section, the time domain predictions for several maneuvers will be discussed. All the figures are appended on Section 5.4 to Section 5.7. The time domain responses to slowly increasing negative steer input are shown in Figure 5.6 through Figure 5.7 in Section 5.4. Conversely, Figure 5.8 to Figure 5.11 show the responses to slowly increasing positive steer input.

The yaw rate predictions to the lateral acceleration from 0 g to 0.4 g in Figure 5.6 and Figure 5.11 demonstrate that the simulation runs of the power steering system combined with a vehicle handling model shows a good match with the experiment. In addition, the improved predictions of yaw rate represent that the developed steering system model is reliable.

Considering that the differential pressure boost curves were obtained from the vehicle speed of 32 m/sec, the simulation predictions for handwheel torque could be close to the experimental data at higher speed. Thus, the handwheel torque responses provide a good prediction with the Figures 5.8, 5.10 and 5.11. At low vehicle speed, 11.2 m/sec, the handwheel torque prediction shows a bad prediction. This is due to the differential pressure boost curve from the 32 m/sec tests being used (The differential pressure boost curve was not computed for the 11 m/sec runs). It is because the differential pressure would be much bigger at the same torsion bar
deflection, as mentioned in Chapter IV. Conversely, at higher vehicle speeds the handwheel torque predictions provide a good match with the experimental values. The results shown on Figures 5.6 through 5.8 when changing the vehicle speeds from 11.2 m/sec to 32.3 m/sec, and Figures 5.7 through 5.9 from 11.2 m/sec to 32.9 m/sec are the good example to show the good handwheel torque predictions at high vehicle speed.

Section 5.5 shows the comparison between the simulation predictions and experimental data for the on-center transition test. Two input cases were generated from negative increasing steer input to a sudden large positive input and vice versa. The experimental handwheel torque values in Figures 5.13 and 5.14 show a better agreement of prediction than Figure 5.12. In addition, the predictions for lateral acceleration and yaw rate provide good agreement in Figures 5.12 and 5.14. This on-center transition test and on-center weave test (Section 5.6) can explain the characteristics of the torsion bar twist and power assist force. As shown in Figure 5.12 through Figure 5.16, the torsion bar deflection follows the shape of the steering input. The power assist force shows that the steering system provides the positive force in all cases since when the valve angle opens being affected by the torsion bar deflection. The power assist force provide 0 N when there is a switch between the torsion bar deflections as shown in Figure 5.14 and Figure 5.15.
Next, the on-center weave test results are shown from Figure 5.15 through Figure 5.16. In general, the simulation predictions show a good match with the experimental data like the on-center transition test in Section 5.5. Figures 5.16 and 5.17 show how the power assist force works in each maneuver test. Due to the constant term, $C_9$, from the power pressure boost curve in Chapter IV, in the case of the torsion bar deflection, is smaller than the vail opening angle, the power assist force became negative value. However, it does not affect the vehicle dynamic responses. The negative power assist force are just to be equal to zero.

Section 5.7 presents the responses for a simple steering pulse input to see what predictions can be obtained. This section also shows a good prediction of the responses. The responses of lateral acceleration, yaw rate and handwheel torque are very close to the experimental results. The torsion bar deflections in each figure are shown well and they are bigger than those resulted from the other maneuvers. This is because the deflection of torsion bar is affected by the handwheel input and the differential pressure does not provide enough power assist force to control the torsion bar deflection. Thus, the torsion bar deflection ranges between $\pm 0.1$ rad.
5.4 Time Domain Results to Slowly Increasing Steer Input

Legend: Experimental Mean Value (Line)  Simulation (Dashed)

Figure 5.6 - Vehicle Dynamic Responses from 0 g to 0.4 g Slowly Increasing Steer Maneuver at 11.2 m/sec (1994 Ford Taurus, GL)
(Figure 5.6 Continued)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
Legend: Experimental Mean Value (Line) (Dashed) Simulation

Figure 5.7 - Vehicle Dynamic Responses from 0 g to 0.4 g Slowly increasing Steer Maneuver at 21.8 m/sec (1994 Ford Taurus, GL)
(Figure 5.7 Continued)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
Legend: Experimental Mean Value (Line) (Dashed)  Simulation

Figure 5.8 - - Vehicle Dynamic Responses from 0 g to 0.4g Slowly Increasing Steer Maneuver at 32.3 m/sec (1994 Ford Taurus, GL)
(Figure 5.8 Continued)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
Figure 5.9 - Vehicle Dynamic Responses from 0 g to 0.4g Slowly Increasing Steer Maneuver at 11.2 m/sec (1994 Ford Taurus, GL)
(Figure 5.9 Continued)

Legend: Experimental Mean Value (Line) Simulation (Dashed)
Legend: Experimental Mean Value (Line)  Simulation (Dashed)

Figure 5.10 - Vehicle Dynamic Responses from 0 g to 0.4g Slowly Increasing Steer Maneuver at 22.9 m/sec (1994 Ford Taurus, GL)
(Figure 5.10 Continued)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
Legend: Experimental Mean Value (Line)  
Simulation (Dashed)

Figure 5.11 - Vehicle Dynamic Responses from 0 g to 0.4g Slowly Increasing  
Steer Maneuver at 32.9 m/sec (1994 Ford Taurus, GL)
(Figure 5.11 Continued)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
5.5 On-Center Transition Test Results

Roadwheel Angular Displacement (Dashed) and Steer Input (Line)

Lateral Acceleration (m/sec^2)

Yaw Rate (rad/sec)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)

Figure 5.12 - Vehicle Dynamic Responses from On-Center Transition Test at 22 m/sec (1994 Ford Taurus GL)
(Figure 5.12 Continued)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
Figure 5.13 - Vehicle Dynamic Responses from On-Center Transition Test at 22 m/sec (1994 Ford Taurus GL)
(Figure 5.13 Continued)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
Figure 5.14 - Vehicle Dynamic Responses from On-Center Transition Test at 22 m/sec (1994 Ford Taurus GL)
(Figure 5.14 Continued)

Legend: Experimental Mean Value (Line)   Simulation (Dashed)
5.6 On-Center Weave Test Results

Roadwheel Angular Displacement (Dashed) and Steer Input (Line)

Lateral Acceleration (m/sec^2)

Yaw Rate (rad/sec)

Legend: Experimental Mean Value (Line) Simulation (Dashed)

Figure 5.15 - Vehicle Dynamic Responses to On-Center Weave Test at 22 m/sec (1994 Ford Taurus, GL)
(Figure 5.15 Continued)

Legend: Experimental Mean Value (Line) Simulation (Dashed)
Roadwheel Angular Displacement (Dashed) and Steer Input (Line)

Lateral Acceleration (m/sec^2)

Yaw Rate (rad/sec)

Legend: Experimental Mean Value (Line) Simulation (Dashed)

Figure 5.16 - Vehicle Dynamic Responses to On-Center Weave Test at 22 m/sec (1994 Ford Taurus, GL)
(Figure 5.16 Continued)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
5.7 Time Domain Responses to Pulse Input

Roadwheel Angular Displacement (Dashed) and Steer Input (Line)

Lateral Acceleration (m/sec^2)

Yaw Rate (rad/sec)

Legend: Experimental Mean Value (Line) Simulation (Dashed)

Figure 5.17 - Vehicle Dynamic Responses to Pulse Steering Input at 33.7 m/sec (1994 Ford Taurus GL)
Figure 5.17 Continued

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
Roadwheel Angular Displacement (Dashed) and Steer Input (Line)

Lateral Acceleration (m/sec^2)

Yaw Rate (rad/sec)

Legend : Experimental Mean Value (Line) Simulation (Dashed)

Figure 5. 18 - Vehicle Dynamic Responses to Pulse Steering Input at 22.8 m/sec (1994 Ford Taurus GL)
(Figure 5.18 Continued)

Legend: Experimental Mean Value (Line)  Simulation (Dashed)
Figure 5.19 - Vehicle Dynamic Responses to Pulse Steering Input at 22.7 m/sec
(1994 Ford Taurus GL)
(Figure 5.19 Continued)

Legend: Experimental Mean Value (Line) Simulation (Dashed)
5.8 Frequency Domain Results

For this research, all the frequency response functions were computed from pulse steering maneuvers. The frequency response functions were computed using the Fast Fourier-Transform (FFT) techniques. To reduce the transient end effects in the experimental and simulated data, a hamming window was applied to the input and output channels of interest from beginning to endpoint of interest. The channels of interest were then FFT to obtain their spectral components.

The frequency domain comparison provides a means to measure the model’s performance across the frequency spectrum. In general, good dynamic performance in the frequency domain will yield correct time domain results for maneuvers in the vehicle’s linear handling range. The model will only be validated for steering wheel input frequencies up to 20 (rad/sec). The Bode plots for the frequency response functions are presented at the end of this section.

The responses for the torque to the driver across the frequency domain at two different vehicle speed are represented in Figure 5.20 and 5.21. The results show a phase discrepancy above approximately 8 rad/sec. The gain across the frequency spectrum in Figure 5.20 shows the simulation results are similar to the experimental but, the gain in Figure 5.21 is slightly greater than the experimental.
The lateral acceleration frequency response as shown in Figure 5.22 and 5.23 have a similar magnitude ratio up to 8 (rad/sec). Generally, the phase angles show good agreement up to 10 (rad/sec). This means that the model of the power steering system would be reliable. The results show good yaw rate predictions to steering wheel angle for the two test conditions for the phase component of the response functions. The phase angle of the yaw rate is very good for both higher speeds.
Figure 5.20 - Handwheel Torque Frequency Response from Pulse Steer Test at 33.5 m/sec (1994 Ford Taurus. GL)

Figure 5.21 - Handwheel Torque Frequency Response from Pulse Steer Test at 22.7 m/sec (1994 Ford Taurus. GL)
Figure 5.22 - Lateral Acceleration Frequency Response from Pulse Steer Test at 33.5 m/sec (1994 Ford Taurus. GL)

Figure 5.23 - Lateral Acceleration Frequency Response from Pulse Steer Test at 22.7 m/sec (1994 Ford Taurus. GL)
Figure 5.24 - Yaw Rate Frequency Response from Pulse Steer Test at 33.5 m/sec (1994 Ford Taurus. GL)

Figure 5.25 - Yaw Rate Frequency Response from Pulse Steer Test at 22.7 m/sec (1994 Ford Taurus. GL)
CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The project for the mathematical model of the power steering system dynamics has been completed. The derived mathematical model which describes major components of the power steering systems will be implemented into an advanced driving simulator. The derived model was implemented using the computer simulation program SIMULINK which resulted in very good predictions. Since the model was completely quantitative in its approach, this model can be used for different vehicles based on vehicle system parameters.
The developed mathematical model for the steering system dynamics has shown good simulation predictions and advanced the state-of-the-art of vehicle dynamics simulation. This research has provided for a better understanding of vehicle handling dynamics and the mathematical equations considering the steering system nonlinearities.

A methodology developed by Heydinger [1] to evaluate the simulations was used in the computer simulation evaluation by using the repeated experimental runs at each test condition to improve the confidence of the experimental data. Ensembling averaging, computing mean values, the confidence limits on the mean values, for experimental data in both the time and frequency domains are much reliable. For all simulation cases, the experimentally measured handwheel angle and averaged vehicle speed were used as inputs to this simulation. This simulation with the experimental measured inputs provides the best validation of the model during the full scale vehicle testing.

A power steering system model developed was used for the fixed control simulation of vehicle handling dynamics. The effects of power assisted steering on steering system behavior were studied. For the fixed control steering model developed, power steering effects are included into the model by measuring the
steering stiffnesses with the vehicle engine running. Including power steering effects improved the steady state and dynamic yaw rate predictions of the SIMULINK simulation.

The prediction of driver feedback cues, which will be required for the National Advanced Driving Simulator (NADS) and STI Simulator in VRTC, were found to be very sensitive to the differential pressure boost curve. When a curve fit for a third order polynomial pressure boost curve from experimental data was used, the handwheel torque has been increased up to 18 N-m. A curve fit for a eighth order polynomial pressure boost curve which is very close to the experimental data made the handwheel torque reduced shown in Chapter V. Thus, the experimental torque ranges within about ± 4.4 (N-m), and the simulated one ranges within ± 5.0 (N-m) to step steering handwheel input with the same condition.

For steady state conditions, the simulations provide similar levels of agreement with the experimental results. The simulation does a good job of predicting vehicle dynamic responses to any steering handwheel input. The simulation runs for slowly increasing steer input conditions showed much improved predictions for yaw rate and handwheel torque as the vehicle speed increased. This is because the differential pressure boost curve for the power assist force was obtained
from the high vehicle speed. For on-center transition test, the predictions of yaw rate (and lateral acceleration) provided a good agreement. In case of handwheel torque, it shows a good shape for a certain range of handwheel input. Generally, the predictions to the on-center weave test provide a good level of agreement for handwheel torque and yaw rate (and lateral acceleration).

Time domain responses to pulse steer inputs also provided good predictions of vehicle responses. Unlike the handwheel torque to step steer input, the amplitudes of both handwheel torque prediction and experimental results range between ± 8 N-m.

6.2 Recommendations

Generally, it was found that the handwheel torque prediction is very sensitive to the power assist force. However, other simulated vehicle responses are not as sensitive to power assist force; lateral acceleration, yaw rate, roll angle, lateral forces on the tire. For the simulation runs, the pressure boost curve was obtained from the experimental data which was calculated from the slowly increasing steer test. Thus, this test does not consider the coulomb friction forces which will be difficult to model, in connection with the calculation of power assist force. It is needed to
consider the coulomb friction force at each different vehicle speed to get each differential pressure boost curve. Since three types of tests at three different vehicle speeds have been done, three different pressure boost curve need to be obtained.

As a five degree-of-freedom linear vehicle handling model was used, the simulation predictions have a certain range of limitation within the lateral acceleration $\pm 0.5 \, g$. When attempting to simulate maneuvers with a greater than $+0.6 \, g$ the simulation becomes numerically unstable. The simulation has a linear relationship between the lateral force and slip angle, though with actual tires there is a nonlinear relationship. Thus, the simulation needs a simple nonlinear tire model to predict higher vehicle lateral accelerations.

In the simulation implementation of the developed power steering system model, only the steering system dynamics alone could be observed without the consideration of a Man-In-The-Loop (MITL). Therefore, this research does not account for human factors. The implementation of the developed model with a MITL driving simulator is needed to refine the model and to test the fidelity of the model. Thus, we will study which components and parameters in the model are most important from a human factors (driver feel) point of view.
Until now, we have only focused on the developed steering model without disturbances; road disturbance and aerodynamics as well as the other driver control inputs; throttle and braking. The developed model was driven with the handwheel input, vehicle speed and some limited experimental data; differential pressure, coulomb friction force, etc. The implementation of the model has provided a similar agreement on flat road. Thus, the combination of the brake model or ABS will be a good example to refine the model and to test the fidelity of the model when we have cornering or straight line braking or miscellaneous tests.

References

LIST OF REFERENCES CITED


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APPENDIX

MATLAB (SIMULINK) Programs
Figure A.1 - SIMULINK Program for Vehicle Handling Model
Figure A.2 - SIMULINK Program for Power Steering System and Vehicle Handling Model
clear, clc

load v0dxc02.mat

% The unit vectors in rack local coordinates
% They relate the directions of the tie rods
% with respect to the tie rod
% The forces in that direction are F = |F|\hat{U}

Ei = 0.985;
Er = 0.985;

% The 'Efficiency' the y-component of the unit vector varies from 0.981-7
% even when extreme positions are considered

pr = 0.007366; % pinion radius (m)
Ar = 0.0010645; % piston area (m^2)
Kt = 83; % tortional bar stiffness, N-m/rad

% Force applied from the handwheel
% Adjust the offset of the torque

torque(1:200) = zeros(1:200);
torque(200:length(V_MZ_HW_AVG)) = V_MZ_HW_AVG(200:length(V_MZ_HW_AVG)) - mean(V_MZ_HW_AVG(150:180));

% Torque = V_MZ_HW_AVG -
% sign(mean(V_MZ_HW_AVG(1:100)))*abs(mean(V_MZ_HW_AVG(1:100)))*force_t = torque/pr;
% for the right side

dum = abs(V_FRC_TIEROD_1_F*Er)+abs(V_FRC_TIEROD_2_F*El)+261;
    % along the rack y-direction + Coulomb Friction Force 261 N;

force_rack = dum - mean(dum(1:200));

% The boost pressure
pressure_b = (force_rack - force_t')/Ar/1000;  % pressure in KP

% The valve angle
% compute the deflection (valve angle) of the steering column in RADIANS

% for the left side
defl = (torque)/Kt;  % rad

% set the polynomial coefficient
a = 287;
b = 379;
%
deflection = zeros(size(1:500));
pressure = zeros(500,1);
%
deflection(1,1:93) = defl(1:1:a);
pressure(1:93,1) = pressure_b(1:1:a);
%
deflection(1,94) = -0.0319;
deflection(1,95) = -0.0315;
deflection(1,96) = -0.0311;
deflection(1,97) = -0.0307;
deflection(1,98) = -0.0303;
deflection(1,99) = -0.0301;
deflection(1,100:1:250) = -0.030:0.03/150:0.0;
%
pressure(94,1) = 31.84;
pressure(95,1) = 26.84;
pressure(96,1) = 21.84;
pressure(97,1) = 16.84;
pressure(98,1) = 11.84;
pressure(99,1) = 6.84;
pressure(100:1:250,1) = zeros(151,1);
%
figure(1)
cig,
plot(deflection', pressure,'g-')

% coefficient calculations
coef(1) = -6.4020e+04;
coef(2) = -2.0596e+03;

xl= -0.1119: 0.0001:deflection(1);
yl = coef(1)*xl+coef(2);
hhold on
plot(xl,yl,'y')

% zero initialization
deflection = zeros(size(1:1600));
pressure = zeros(1600,1);

deflection(1,1:550) = xl(1:1:550);
pressure(550,1) = yl(1:1:550);%
deflection(1,551:643) = def(1:-1:a);
deflection(551:643,1) = pressure_b(1:-1:a);
deflection(1,644) = -0.0319;
deflection(1,645) = -0.0315;
deflection(1,646) = -0.0311;
deflection(1,647) = -0.0307;
deflection(1,648) = -0.0303;
deflection(1,649) = -0.0301;
deflection(1,650:1:800) = -0.030:0.03/150:0.0;

% pressure values
pressure(644,1) = 31.84;
pressure(645,1) = 26.84;
pressure(646,1) = 21.84;
pressure(647,1) = 16.84;
pressure(648,1) = 11.84;
pressure(649,1) = 6.84;
preser(650:1:800,1) = zeros(151,1);
hold on
plot(deflection(1,1:800),pressure(1:800,1),'c')

% deflection(1,801:1:951) = 0.0:0.03/150:0.030;
deflection(1,952) = 0.0301;
deflection(1,953) = 0.0303;
deflection(1,954) = 0.0307;
deflection(1,955) = 0.0311;
deflection(1,956) = 0.0315;
deflection(1,957) = 0.0319;

% pressure(801:1:951,1) = zeros(151,1);
pressure(952,1) = 6.84;
presure(953,1) = 11.84;
presure(954,1) = 16.84;
presure(955,1) = 21.84;
presure(956,1) = 26.84;
presure(957,1) = 31.84;

clg
plot(deflection(1,1:957)', pressure(1:957,1),'y')

% the valve angle
% compute the deflection (valve angle) of the steering column in RADIANS
% for the right side

defl = abs(torque)/Kt;       % rad

% deflection(1,958:1050) = defl(a:1:b);
pressure(958:1050,1) = pressure_b(a:1:b);

clg
plot(deflection(1,1:1050)', pressure(1:1050,1),'g')

coef(1) = 6.4020e+04;
coef(2) = -2.0596e+03;

x_r = -deflection(550); 0.0001 : 0.1119;
y_r = coef(1)*x_r+coef(2);
hold on
plot(xr, yr, 'c')

% deflection(1,1051:1600) = xr(1,1:550);
% pressure(1051:1600,1) = yr(1:550)
%
figure(2)
clg
plot(deflection(1,1:1600)', pressure(1:1600,1)', 'g')
%

n = 8;

% get the coefficients

if n == 2
    coef = polyfit(deflection', pressure, n);
elseif n == 3
    coef = polyfit(deflection', pressure, n);
elseif n == 4
    coef = polyfit(deflection', pressure, n);
elseif n == 5
    coef = polyfit(deflection', pressure, n);
elseif n == 6
    coef = polyfit(deflection', pressure, n);
elseif n == 7
    coef = polyfit(deflection', pressure, n);
elseif n == 8
    coef = polyfit(deflection', pressure, n);
end

x = deflection(1): deflection(1600)/1000: deflection(1600);

if n == 2
    y = coef(1)*x.^2+coef(2)*x + coef(3);
elseif n == 3
    y = coef(1)*x.^3+coef(2)*x.^2+coef(3)*x + coef(4);
elseif n == 4
    y = coef(1)*x.^4+coef(2)*x.^3+coef(3)*x.^2+coef(4)*x + coef(5);
elseif n == 5
    y=coef(1)*x.^5+coef(2)*x.^4+coef(3)*x.^3+coef(4)*x.^2+coef(5)*x+coef(6);
elseif n == 6
    y=coef(1)*x.^6+coef(2)*x.^5+coef(3)*x.^4+coef(4)*x.^3+coef(5)*x.^2
    +coef(6)*x+coef(7);
elseif n == 7
    y=coef(1)*x.^7+coef(2)*x.^6+coef(3)*x.^5+coef(4)*x.^4+coef(5)*x.^3
    +coef(6)*x.^2+coef(7)*x.+1+coef(8);
elseif n == 8
    y=coef(1)*x.^8+coef(2)*x.^7+coef(3)*x.^6+coef(4)*x.^5+coef(5)*x.^4
    +coef(6)*x.^3+coef(7)*x.^2+coef(8)*x+coef(9);
end

% %-----------------------------------------------------------------------
hold on
plot(x',y',r'-')
title('Pressure Boost Curve at 32 m/sec')
xlabel('T-bar deflection (rad)')
ylabel('Pressure Differential (Kpa)')
% %-----------------------------------------------------------------------
%
% %-----------------------------------------------------------------------
% the fourth polynomial coefficients are
% coef=[1.4517e+08 7.0470e-10 7.9611e+04 -4.4971e-12 -42.1240]
% roots([coef]) ans = 0.01805 rad point where the curve touches zero
% %-----------------------------------------------------------------------
%
% the eighth polynomial coefficients are
%
c1 = 4.7097e+11;
c2 = 5.6380e-04;
c3 = -1.1864e+10;
c4 = -1.1361e-05;
c5 = 8.0934e+07;
c6 = 6.8698e-08;
c7 = 3.4791e+05;
c8 = -9.8992e-11;
c9 = -115.9779;
% data file for simulation runs with a SIMULINK file
% ------------------------------------------------------
% written by Jeff Christos
% matlab file v0d3l.m in VRTC
% May 20, 1995.
% ------------------------------------------------------

% This file contains the data for the parameter values of a vehicle,
% 1994 Ford Taurus, GL
% Define all vehicle parameters as global
%
global m ms a b lx xs i zz Ksw e Calpha f Calpha r g ef er lx Kr Cr RL
%
% ------------------------------------------------------

% Vehicle System Parameter Definitions:
% ------------------------------------------------------
%
% M = 1542; % Total Vehicle Mass, kg
Ms = 1356; % Vehicle Sprung Mass, kg
Calpha f = 70000.0; % Front axle tire cornering stiffness, N/rad
Calpha r = 51000.0; % Rear axle tire cornering stiffness, N/rad
g = 9.8; % Acceleration due to gravity, m/sec^2
dLdPh i = 147588; % Total vehicle roll stiffness, N-m/rad
dLdPh i D = (2694+1063); % Total vehicle roll damping, (N-m-sec)/rad
ef = 0.01; % Front axle roll steer, (rad/rad)
er = 0.01; % Rear axle roll steer, (rad/rad)
a = 0.92; % Longitudinal distance from front axle vehicle C.G., m
b = 1.77; % Longitudinal distance from rear axle vehicle C.G., m
lx xs = 670; % Sprung mass roll inertia about vehicle roll axis, kg-m^2
lzz = 2786; % Total vehicle yaw mass moment of inertia (kg-m^2)
Ksw = 16.0; % Steering ratio (rad/rad)
e = 0.454; % Distance from roll axis to sprung mass C.G., m
RL = 1; % Relaxation Length, m
tau = RL/Vx;
%
% Pre-Computed constants
%
lx = (Ms*e)^2 - M*lx xs;
Kr = Ms*g*e - dLdPh i;
Cr = dLdPh i D;
%
% For the state-space matrix of 5.D.O.F. car handling model
%
% psv = [M 0 0 Ms*e 0 0; 0 lzz 0 0 0 0; 0 0 1 0 0 0; ...  
% Ms*e 0 0 lxxs 0 0; 0 0 0 0 1 0; 0 0 0 0 0 1];
%
% PSA = [0 -M*Vx 0 0 -2 -2; 0 0 0 0 -2*a 2*b; 0 0 0 1 0 0; ...  
% 0 -Ms*e*Vx Kr -Cr 0 0; 0 0 0 0 -1/tau 0; 0 0 0 0 0 1/tau];
%
% PSB = [0 0; 0 0; 0 0; 1/tau 0; 0 1/tau];
%
% VPA = inv(psv)*PSA;
%
% VPB = inv(psv)*PSB;
%
% VPC = eye(6);
%
% VPD = [0 0 ;0 0; 0 0; 0 0; 0 0];
%
% %

% Vehicle: 1994 Ford Taurus GL
% Parameter definition file and Vehicle Handling Model: car_data
% Matlab data file: data.m in VRTC
% written by Bong-Choon Jang

% Parameter definitions for Steering System Dynamics
%
% Isw = 0.03444;    % kg-m^2,  
%      % Steering wheel moment of inertia, 0.30454 in-lb-s^2

Bsw = 0.36042;    % N-m/(rad/s),  
%      % Steering Wheel viscous damping, 3.19 (in-lb)/(rad/sec)
%
% Ng = 1.0;        % rad/rad
A = 0.0010645;    % m^2, %piston area, 1.65 in^2
Ksw = 16;         % rad/rad
GR = Ksw;         % m, %Gear Ratio, 46.4 mm/rev = 0.024228 ft/rad
FPsc = 0.001517;  % Free Play of Steering column, rad
FPsl = 0.001517;  % Free Play of steering linkage, rad
%
lw = 0.61463;  % kg-m^2, %Front wheel moment of inertia...
% about their steering axes, 0.45333333 ft-lbf-s^2
Hr = 88.128;  % N-m/(rad/s), % 65 (ft-lb)/(rad/sec)
Br = Hr;  % N-m/(rad/s), % (ft-lb)/(rad/sec)
Ef = 0.985;  % Efficiency, %
Eb = 0.985;  % Efficiency, %
Eps = 0.95;  % Efficiency, %
%
Xt = -0.1*0.3048;  % Overall trail distance, m
C = Calphaf;  % N/rad, %Front axle Conering Stiffness, kgf/rad
%
NI = 0.11816;  % m, NI = 0.3876528 ft
Rp = 0.007367;  % m, Pinion Radius, Rp = 0.024228 ft
Mr = 2;  % kg, %0.25 lbf-s^2/ft
%
Kt = 83;  % N-m/rad, Torsion-bar stiffness,% 1146 (in-lb)/rad
Ksc = 42057;
Ksl = 14878;  % N-m/rad,= Ksl(left), % Ksl(right) = 15140 (N-m/rad)
%Ksl = 42079;
%Ksc = 75830;
%
% Parameter Variables:

TA = Eb*Rp^2*Ksl/(NI^2*Ksc);
TB = Eb*Rp^2*Ksl/(NI^2*Kt);
TC = Eb*Rp^2*Ksl/(NI^2*Ksc);
TD = Eb*Rp^2*Ksl/(NI^2*Kt);

TL = Eb*Rp*Ksl/(NI*Ksc);
TJ = Eb*Rp*Ksl/(NI*Ksc);
TK = -(TL+TJ)*ef;

T_DEN = TA+TB+TC+TD+1;

VA = (TB+TD+1)/T_DEN;
VB = TL/T_DEN;
VC = TJ/T_DEN;
VD = TK/T_DEN;
VH = Rp*Ksl*(1+Ksc/Kt)/Nl;
VI = Rp*Ksc*Ksl/(Kt*Nl);

% PVA = VH*VB - Ksl;
PVB = VH*VC;
PVC = VH*VA - VI;
PVD = VH*VD + Ksl*ef;

% VJ = Rp*Ksl*(1+Ksc/Kt)/Nl;
VK = Rp*Ksc*Ksl/(Kt*Nl);

% PVE = VJ*VB;
PVF = VJ*VC - Ksl;
PVG = VJ*VA - VK;
PWH = VJ*VD + Ksl*ef;

% VL = Eb*Rp*(Ksl+Ksl)/Nl^2;
VM = Eb*Rp*Ksc*(Ksl+Ksl)/(Nl^2*Kt);
VN = (Ksc/Rp) + VM;
VP = Eb*ef*(Ksl+Ksl)/Nl;
VQ = -(VL + VM + Ksc/Rp);

% PVI = VQ*VB + Eb*Ksl/Nl;
PVI = VQ*VC + Eb*Ksl/Nl;
PVK = VQ*VA + VN;
PVL = VQ*VD - VP;

% PAF = -(Eps*Rp/Ksc)/T_DEN;
FPS1 = VH*PAF;
FPS2 = VJ*PAF;
FPS3 = (VQ*PAF - Eps);

% % State Space Matrix for the power steering system dynamics
% %------------------------------------------------------------------------
% %
tempa = [1 0 0 0 0 0;...
         0 1 0 0 0 0;...
         0 0 1 0 0 0;...
         0 0 0 1 0 0;...
         0 0 0 0 1 0;...
         0 0 0 0 0 Mr];
\[ TPSSA = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ PVA & PVB & 0 & -Bfw & 0 \\ PVE & PVF & 0 & 0 & -Bfw \\ PVI & PVJ & 0 & 0 & 0 \\ \end{bmatrix}; \]

\[ TPSSB = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ PVC & PVD & -1 & 0 & FPS1 & -CF \\ PVG & PVH & 0 & -1 & FPS2 & 0 \\ PVK & PVL & 0 & 0 & FPS3 & 0 \\ \end{bmatrix}; \]

\[ PSSA = \text{inv(tempa)} \ast TPSSA; \]

\[ PSSB = \text{inv(tempa)} \ast TPSSB; \]

\[ PSSC = \text{eye(6)}; \]

\[ PSSD = \text{zeros(6,8)}; \]
clear all,clc

% load v0dpgt.mat
% This file shows the experimental results in the time domain.
% Steering input : PULSE INPUT (Amp.: -0.38 rad)
% Vehicle Speed : Vx: 33.5 m/sec

% The results of the simulation of an Automotive Power Steering System
% Model can be obtained from simulation run, a_new.m SIMULINK file.
% %
% % To generate a pulse input from Experiment

load v0dpgt.mat

Vx = 33.5; % Vehicle forward speed, m/sec

% CF = 169; % N, Coulomb Friction breakout force on steering rack
% CF = 261; % N, when Vx = 22 m/sec
%
% To insert the experimental steering input

h = 0.01;
S_time = min(V_TIME);
E_time = max(V_TIME);
tt = S_time:h:E_time;

n = abs(S_time)*100 + abs(E_time)*100 + 1;
st = abs(S_time)*100 + 1;
st2 = abs(S_time)*100 + 11;
nn = length(THETA_HW_m);

U = zeros(1,(E_time-S_time)/h+1);
U(1:n) = THETA_HW_m(1:n);
U(1:st) = THETA_HW_m(1:st)- mean(THETA_HW_m(1:st));
U(1:st) = U(1:st) - mean(U(1:st));
U(1:st) = U(1:st) - mean(U(1:st));
U(st2:nn) = U(st2:nn) - mean(U(st2:nn));
U(st2:nn) = U(st2:nn) - mean(U(st2:nn));
U(st2:nn) = U(st2:nn) - mean(U(st2:nn));

% THETA_HW_m(1:st) = THETA_HW_m(1:st) - mean(THETA_HW_m(1:st));
THETA_HW_m(1:st) = THETA_HW_m(1:st) - mean(THETA_HW_m(1:st));
THETA_HW_m(1:st) = THETA_HW_m(1:st) - mean(THETA_HW_m(1:st));

% THETA_HW_m(st2:nn) = THETA_HW_m(st2:nn) -
mean(THETA_HW_m(st2:nn));
THETA_HW_m(st2:nn) = THETA_HW_m(st2:nn) -
mean(THETA_HW_m(st2:nn));
THETA_HW_m(st2:nn) = THETA_HW_m(st2:nn) -
mean(THETA_HW_m(st2:nn));

% To check out the generated pulse input, U
%
pload('V_TIME, THETA_HW_m', 'Y', 'U', 'c--',
   title('Handwheel Steering Input & Radial Wheel Output'))
   xlabel('Time (sec)'), ylabel('Handwheel Input (rad)'))
%
% Parameter Definitions: data.m % data file
% This file contains the data for the parameter values of Ford Taurus GL,
% 1994.
% Steering System Model Parameter Definitions
%
% Steering model variables for steering system dynamics
%
% The pressure boost polynomial coefficients
%
% c1 = 4.7097e+11;
c2 = 5.6380e-04;
c3 = -1.1864e+10;
c4 = -1.1361e-05;
c5 = 8.0934e+07;
c6 = 6.8698e-08;
c7 = 3.4791e+05;
c8 = -9.8992e-11;
c9 = -115.9779;
Bfw = 65; % N-m/(rad/s),
% Front wheel viscous damping, 150 (ft-lb)/(rad/sec)

% data 
% data file

% XT = -0.0177; % m, ranges from -0.0305 to -0.0177
%
% %-----------------------------------------------------------------------------------
% % Vehicle Responses Comparison with the Experimental Results
% %-----------------------------------------------------------------------------------

% Handwheel Steer Input
%
figure(1)
clear,
plot( S_t,Dsw_s,'g-', S_t, Dfwl_s, 'g-',' V_TIME, THETA_HW_m,'r')
xlabel('Time (sec)'), ylabel(' Amplitude (rad)')
title(' Handwheel Steer Input & Road wheel Output')
legend(' Simulation Input (Dsw) ',' Simulation Output (Dfw) ', ...
' Experimental ')
%
% Lateral Acceleration,
%
AY_CHAS_CG_m = AY_CHAS_CG_m-mean(AY_CHAS_CG_m(1:st));
AY_CHAS_CG_m = AY_CHAS_CG_m-mean(AY_CHAS_CG_m(1:st));
AY_CHAS_CG_m = AY_CHAS_CG_m-mean(AY_CHAS_CG_m(1:st));
%
AY_CHAS_CG_m(1:st) = AY_CHAS_CG_m(1:st)-
mean(AY_CHAS_CG_m(1:st));
AY_CHAS_CG_m(1:sti) = AY_CHAS_CG_m(1:sti)-
mean(AY_CHAS_CG_m(1:sti));
AY_CHAS_CG_m(1:sti) = AY_CHAS_CG_m(1:sti)-
mean(AY_CHAS_CG_m(1:sti));
%
AY_CHAS_CG_m(st+1:st2-1) = AY_CHAS_CG_m(st+1:st2-1)-
mean(AY_CHAS_CG_m(1:st));
AY_CHAS(CG)_m(st2:nn) = AY_CHAS(CG)_m(st2:nn) -
    mean(AY_CHAS(CG)_m(st2:nn));
AY_CHAS(CG)_m(st2:nn) = AY_CHAS(CG)_m(st2:nn) -
    mean(AY_CHAS(CG)_m(st2:nn));
AY_CHAS(CG)_m(st2:nn) = AY_CHAS(CG)_m(st2:nn) -
    mean(AY_CHAS(CG)_m(st2:nn));

figure(1)
clf
plot( S_t, Ay_s,'g-','V_TIME, AY_CHAS(CG)_m','r')
xlabel("Time (sec)")
ylabel('Mag. - ( m/sec^2 )')
title(' Lateral Acceleration Responses at 33.5 m/sec ')
legend(' Car Model ',' w/ Steering Model ',' Experimental ')

% % Yaw Rate
%
figure(2)
clf,
plot( S_t,r_s,'g-',' V_TIME, RVZ_CHAS_m','r')
xlabel("Time (sec)")
ylabel('Yaw Rate - (rad/sec)')
title(' Yaw Rate Responses at 33.5 m/sec ')'
legend(' w/ Steering Model ',' Experimental ')

% % Handwheel Torque
%
MZ_HW_m = MZ_HW_m - mean(MZ_HW_m(1:st));
MZ_HW_m = MZ_HW_m - mean(MZ_HW_m(1:st));
MZ_HW_m = MZ_HW_m - mean(MZ_HW_m(1:st));

figure(3)
clf,
subplot(311)
plot( S_t, T_HW_s,'g-',' V_TIME, MZ_HW_m','r')
xlabel("Time (sec)", ylabel(' Handwheel Torque (N-m) ')
title(' Handwheel Torque ')
legend(' Steering Model ',' Experimental ')

%
subplot(312)
plot(S_t, Dv_s,'g--')
ylabel('Torsion Bar Twist - (rad)')
xlabel('Time (sec)')
title('Torsion Deflection Response ')
%
subplot(313)
plot(S_t, F_s*1000,'g--')
xlabel('Time (sec)'),ylabel('Power Assist - (N)')
title('Power Assist Force ')
%

% -----------------------------------------------------------------------
% Frequency Responses at Vx = 33.5 m/sec
%
% Oct. 10, 1995
% Oct. 26, 1995 % Correction inp_vector (rad)
%
% % Originally modified from Jeff's program.
% Bong-Choon Jang
%
% -----------------------------------------------------------------------
	%TREXP Computes and plots transfer function of output to input(Dsw)
% by dividing the cross spectral estimation by the
% power spectral estimation of the input
%
% Dsw_inp input vector
% Dfw_out output vector
% DT time spacing of data (seconds)
% wf maximum frequency to plot on x-axis (rad/sec)
% mag transfer function magnitude
% ph transfer function phase angle (degrees)
% coherence
%
% -----------------------------------------------------------------------
% compute csd and psd
% From Simulation of the a_final.m
% -----------------------------------------------------------------------
%
inp = Dsw_s;
DT = 0.001;
wf = 1000;
%
 n = length(inp);
%
PSD = psd(inp, n, 1/DT);
%
out_Dfw_s = Dfwl_s;           % Roadwheel Ang. Disp., rad
out_Ay_s = Ay_s;              % Lateral Accel. output m/sec^2
out_r_s = r_s;                % Yaw Rate output, rad/sec
out_T_s = T_HW_s;             % Handwheel Torque, N-m/rad
%
CSD_Dfw_s = csd(inp, out_Dfw_s, n, 1/DT);
CSD_Ay_s = csd(inp, out_Ay_s, n, 1/DT);
CSD_r_s = csd(inp, out_r_s, n, 1/DT);
CSD_T_s = csd(inp, out_T_s, n, 1/DT);
%
%---------------------------------------------------------------------
% compute transfer function magnitude and phase angle
%---------------------------------------------------------------------
%
%mag_Dfw_s = abs(CSD_Dfw_s) ./abs(PSD);
mag_Ay_s = abs(CSD_Ay_s) ./abs(PSD);
mag_r_s = abs(CSD_r_s) ./abs(PSD);
mag_T_s = abs(CSD_T_s) ./abs(PSD);
%
%ph_Dfw_s = (angle(CSD_Dfw_s) - angle(PSD)) *180/pi;
ph_Ay_s = (angle(CSD_Ay_s) - angle(PSD)) *180/pi;
ph_r_s = (angle(CSD_r_s) - angle(PSD)) *180/pi;
ph_T_s = (angle(CSD_T_s) - angle(PSD)) *180/pi;

% Generate frequency vector
%
w = [1/(DT*n)*(0:n/2-1)*2*pi];  %rad/sec

% load v0fpff.mat (data file)
% This file shows the experimental results in the frequency domain.
% Steering input : PULSE INPUT ( Amp.: 0.35 rad )
% Vehicle Speed  : Vx: 33.74 m/sec
% compute csd and psd
% From Experimental data

% inpe = THETA_HW_m ;  % Steering input
DTc = 0.00001;
wf = 10000;
%
ne = length(inpe/.01);
%
out_TH_e = MZ_HW_m;
% Exp. Handwheel Torque, N-m/rad
%
CSD_TH_e = csd(inpe, out_TH_e, ne,1/DTc);
%
PSDe = psd(inpe, ne,1/DTc);
%
% compute transfer function magnitude and phase angle
% %
% ph_TH_e = (angle(CSD_TH_e) - angle(PSDe)) *180/pi;
%
% Generate frequency vector
%
we = [1/(DTc*ne)*(0:ne/2-1)*2*pi]/1000; %rad/sec
%
load v0fpgf.mat

% % Handwheel Torque Frequency Responses at Vx = 33.74 m/sec
% %
figure(1)
cig,
subplot(211)
semilogx(w, mag_T_s(1:length(w)),'g-';we,
(mag_TH_e(1:length(we))),'r-';we, (mag_TH_e(1:length(we))),'co')
grid on
ylabel('Mag. - (N-m/rad)'),xlabel('Frequency (rad/sec)')
axis([1 20 0 30])
subplot(212)
    semilogx(w, ph_T_s(1:length(w)),'g-', we, ph_TH_e(1:length(we)),'r',...
    we, ph_TH_e(1:length(we)),'co')
    grid on
    ylabel('Phase (deg.)'), xlabel('Frequency (rad/sec)')
    axis([1 20 -100 100])
    title('Handwheel Torque Frequency Responses (Vx: 33.5 m/s) ')
    legend( ' w/ Steering Model', ' Experimental')

% --------------------------------------------------------------------------------------------------------
% Lateral Acceleration Frequency Responses at Vx = 33.74 m/sec
% --------------------------------------------------------------------------------------------------------
%
figure(2)
clo
subplot(211)
    semilogx(w, (mag_Ay_s(1:length(w))),'g-', AY_CHAS_CG_f,
    (AY_CHAS_CG_mag_m), 'r',...
    AY_CHAS_CG_f, (AY_CHAS_CG_mag_m), 'ro')
    grid on
    ylabel('Mag. - (m/sec^2/rad) '), xlabel('Frequency (rad/sec)')
    axis([1 20 0 20])

subplot(212)
    semilogx(w, ph_Ay_s(1:length(w)),'g-',
    AY_CHAS_CG_f, AY_CHAS_CG_ph_m,'r',...
    AY_CHAS_CG_f, AY_CHAS_CG_ph_m,'ro')
    grid
    ylabel('Phase (deg.)'), xlabel('Frequency (rad/sec)')
    axis([1 20 -200 100])
    title('Lateral Acceleration Frequency Responses (-0.38 rad)')
    legend(' w/ Steering', ' Experimental')

%--------------------------------------------------------------------------------------------------------
% Yaw Rate Frequency Responses at Vx = 33.74 m/sec
%--------------------------------------------------------------------------------------------------------
figure(3)
clo
subplot(211)
    semilogx(w, (mag_r_s(1:length(w))),'g--', ...
    RVX_CHAS_f, (RVZ_CHAS_mag_m), 'r',...
    RVX_CHAS_f, (RVZ_CHAS_mag_m), 'ro')
grid
ylabel('Mag. (rad/sec/rad)'), xlabel('Frequency (rad/sec)')
axis([1 20 0 1])

RVX_CHAS_ph_m=RVX_CHAS_ph_m+126;

subplot(212)
   semilogx(w, ph_r_s(1:length(w)),'g--',...
           RVX_CHAS_f, RVX_CHAS_ph_m,'r',...
           RVX_CHAS_f, RVX_CHAS_ph_m,'ro')
grid on
ylabel('Phase (deg.)'), xlabel('Frequency (rad/sec)')
axis([1 20 -300 100])
title('Yaw Rate Frequency Responses (-0.38 rad)')
legend('w/ Steering    ', ' Experimental    ')