SPATIAL VISUALIZATION, FIELD DEPENDENCE/INDEPENDENCE,
VAN HIELE LEVEL, AND ACHIEVEMENT IN GEOMETRY: THE
INFLUENCE OF SELECTED ACTIVITIES FOR
MIDDLE SCHOOL STUDENTS

DISSERTATION

Presented in Partial Fulfillment of the Requirement
for the Degree of Doctor of Philosophy in the
Graduate School of The Ohio State University

By

Noraini Idris, B.Sc.Ed(Hons),M.Ed.,M.A.

* * * * *
The Ohio State University
1998

Dissertation Committee

Prof. Sigrid Wagner, Adviser
Prof. Janet L. Henderson
Prof. Joseph Ferrar

Approved By

Sigrid Wagner
Adviser
College of Education
© Copyright by
Noraini Idris
1998
ABSTRACT

Studies have shown that cognitive variables of spatial visualization, field dependence/independence, van Hiele levels of geometric thought seem important in learning mathematics (Ben-Chaim, Lappan, & Houng, 1988; Carment, 1989; Lourdusamy, 1982; Satterly, 1976; Senk, 1989). Thus a knowledge of the cognitive abilities of students would definitely help teachers in understanding the cognitive developmental level of their students. The purpose of this study was to investigate the relative importance of the cognitive variables of spatial visualization, field dependence/independence, and van Hiele level of geometric thought in predicting achievement in geometry of middle school students and to determine the effect of selected instructional activities on improving these cognitive variables and achievement in geometry.

In this study the researcher did not assign subjects randomly to the treatment. Instead, a total sample of subjects in six intact classes in a public middle school was used, that is, one experimental class
and one control class from each of grades 6, 7, and 8. Four instruments were used for data collection: The Middle Grades Mathematics Project Spatial Visualization Test, the Group Embedded Figures Test, the Van Hiele Geometry Test, and the Geometry Test developed for this study. This study was a quasi-experimental, nonequivalent control group design (Tuckman, 1978). The four instruments were administered before and after the treatment. For the experimental group, all students received the instructional activities. The researcher developed the instructional activities in such a way that students were given the opportunity to visualize geometric constructions, relate properties, and disembed simple geometric figures from complex designs in order to be successful in geometry. The treatment lasted about three weeks. Regular teachers taught and used the instructional activities in the experimental classrooms.

Most of the middle school students who took the Van Hiele Geometry test in fall 1997 were at level 1. Two 8th grade students were at level 4. Spatial visualization had a correlation of .245 with field dependence/independence (p < .01). Van Hiele level had a Spearman rank correlation with spatial visualization of r = .313. Van Hiele level had a Spearman rank
correlation with field dependence/independence of
r = .281 (p < .01). Geometry achievement had a
correlation coefficient of .566 with field
dependence/independence, .351 with spatial
visualization, and .366 with van Hiele levels of
geometric reasoning (p < .01). Multiple regression
analysis showed that field dependence/independence is
the best single predictor of geometry achievement. The
findings of this study suggest that among middle school
students grades 6 through 8 using selected
instructional activities, there is a statistically
significant increase in each of the cognitive variables
tested and in geometry achievement.

One significance of this study was that the
instructional activities can facilitate teachers in
designing, selecting, and preparing instructional
activities appropriate to the cognitive levels of the
students. The study also helped in determining whether
the selected instructional activities improve students' ability to communicate and represent geometry
information in order to optimize student achievement in
geometry.
Dedicated to my husband, Alias and our children, Nor Iskandar and Nurul Nadzirah with love
ACKNOWLEDGMENTS

I wish to express my sincere appreciation and gratitude to many people who made this study possible. My very special gratitude is expressed to Prof. Sigrid Wagner, my adviser, for her continuous encouragement, support, and advice throughout my study. Her constant concern, encouragement, time, and effort she spent on guiding me in my research will always be remembered.

My special sense of gratitude is also extended to Prof. Joseph Ferrar for so carefully and patiently advising me on the geometry instructional activities of my project, and Prof. Janet L. Henderson for her valuable suggestions and advice.

I would also like to extend a special thanks to Prof. Glenda Lappan, Michigan State University and Prof. Zalman Usiskin, University of Chicago, for their generous permission to use their instruments. And also my deep thanks to Prof. David Fuys, National Council of Teachers of Mathematics, and others to give me permission to use their activities in my project.
In addition, I would like to thank the principal and other school authorities who participated in the study for allowing me to administer the tests and carried out the instructional activities in their schools. Without their generous assistance, this study could not have been conducted.

Finally, deep thanks go to my wonderful husband, Alias, for his support, understanding, encouragement, and sacrifices during our study together in the United States. And also thanks to my two children, Nor Iskandar and Nurul Nadzirah for their sacrifices in many ways, patience, love, and support that have given me the peace of mind to complete this project. Deep thanks also go to my father, mother (who passed away before this project finish), and all relatives who helped me during my study. Their prayers have sustained me throughout the years. May Allah bless you all.
VITA

1982 .........................B.Sc.Ed (Hon), University of Malaya, Malaysia


1983............................Teacher of Mathematics & Physics, Ulu Klang Secondary School, Malaysia

1986............................Lecturer, Mathematics Education, Teacher Training College, Malaysia

1988............................Study Leave, University of Malaya

1990............................M.Ed., Mathematics Education, University Malaya. Assistant Director, Textbook Division, Ministry of Education, Malaysia


1993............................Certificate of Writing, Institute of Education, University of London
1993............................................Lecturer, Department of Mathematics & Science Education, Faculty of Education, University of Malaya.

1997.............................................M.A., Mathematics Education, The Ohio State of University, Columbus, Ohio.

PUBLICATIONS


FIELDS OF STUDY

MAJOR FIELD: EDUCATION

Studies in:

Mathematics Education with Dr. Sigrid Wagner, Dr. Douglas T. Owens, and Dr. Patricia Brosnan.

Instructional Design and Technology with Dr. Suzanne Damarin, Dr. Allen Shaffer, and Dr. Jeffery Smith.

Mathematics with Dr. Joseph Ferrar, Dr. Joseph R. Fiedler, Dr. J. Philip Huneke, Dr. Mike Snell, and Dr. Bostwick Wyman.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter/Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>DEDICATION</td>
<td>v</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>vi</td>
</tr>
<tr>
<td>VITA</td>
<td>viii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xiv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xvi</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>1 STATEMENT OF THE PROBLEM</td>
<td>1</td>
</tr>
<tr>
<td>Background</td>
<td>1</td>
</tr>
<tr>
<td>Spatial Visualization</td>
<td>5</td>
</tr>
<tr>
<td>Field Dependence/Independence</td>
<td>7</td>
</tr>
<tr>
<td>Van Hiele Levels</td>
<td>9</td>
</tr>
<tr>
<td>Effect of Instruction Instructional Activities</td>
<td>13</td>
</tr>
<tr>
<td>Statement of the Problem</td>
<td>14</td>
</tr>
<tr>
<td>Definitions</td>
<td>15</td>
</tr>
<tr>
<td>Conceptual Framework</td>
<td>17</td>
</tr>
<tr>
<td>Conceptual Model</td>
<td>21</td>
</tr>
<tr>
<td>Limitations of the Study</td>
<td>23</td>
</tr>
<tr>
<td>Significance of the Study</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>
2 REVIEW OF THE LITERATURE ...................... 27

Spatial Visualization and Achievement.................. 27

Studies Showing a Positive Relationship.................. 28

Studies Showing a Negative Relationship.................. 33

Field Dependence/Independence and Achievement............ 35

Studies Showing a Positive Relationship.................. 36

Is Field Dependence/Independence a style or an ability?......... 40

Spatial Visualization, Field Dependence/Independence, and Achievement....... 43

Van Hiele Levels of Geometric Thinking.................. 45

Procedures to Measure the van Hiele levels.................. 46

Van Hiele Levels and Achievement/Performance.............. 50

The Effect of Instruction.......................... 52

Spatial Visualization.......................... 52

Field Dependence/Independence................. 59

Van Hiele Level.......................... 60

Significance of the Study.......................... 65

3 METHODS AND PROCEDURES.......................... 68

Site.......................... 69

Sample .......................... 70

Instrumentation.......................... 72

Spatial Visualization Test ........ 72
Purpose of the Study........................142
Research Design..........................143
Findings....................................144
Conclusions and Discussion...............149
Implications and Recommendations.......154
   Implications for Teaching and Learning..........................154
   Recommendations for Future Research.161
APPENDICES
   A  Spatial Visualization Test...........164
   B  Van Hiele Geometry Test.............180
   C  Geometry Test.......................192
   D  Instructional Activities.............205

BIBLIOGRAPHY..............................................260
LIST OF TABLES

Table                                      Page

1.1 Characterization of Van Hiele Level........ 11

3.1 Description of All Students in the School by Grade, Sex, and Ethnicity........ 69

3.2 Number and Percentage of Participants in the Study................................. 71

3.3 Categorization Scheme of GEFT Score........ 74

3.4 Content Classification and Number of Items for Geometry Test.................... 77

3.5 Reliability Analysis-Scale(Alpha)........... 80

3.6 Description of Instructional Activities.... 84

4.1 Numbers and Percentages of Van Hiele Levels Among Middle School Students on Pretest of Van Hiele Geometry Test.................................102

4.2 Means, Standard Deviations, Minima, and Maxima of SVT and GEFT Among Middle School students..................................................105

4.3 Correlation Matrix: Spatial Visualization, Field Dependence/Independence, and Van Hiele Levels.............................................106

4.4 Means, Standard Deviations, Minima, and Maxima of Pretest Achievement in Geometry....109

4.5 Correlation Matrix: Spatial Visualization, Field Dependence/Independence, Van Hiele Levels of Geometry Reasoning with Geometry Achievement..........................110
4.6 One-way Analysis of Variance for Geometry Achievement by Field Dependent/Independent Groups........................................113

4.7 One-way Analysis of Variance for Geometry Achievement by Van Hiele Levels..............115

4.8 Multiple Regression Summary Table for Prediction of Geometry Achievement of Middle School Students.................................118

4.9 Means and Standard Deviation of Spatial Visualization Pre- and Posttest..................121

4.10 Effects of Instructional Activities on Spatial Visualization.................................123

4.11 Means and Standard Deviation for Experimental and Control Groups on Pre- and Posttest Spatial Visualization.................................124

4.12 Means and Standard Deviations for GEFT..............126

4.13 Effects of Instructional Activities on GEFT..................................................128

4.14 Means and Standard Deviation for Experimental and Control Groups on Pre- and Posttest GEFT..................................................129

4.15 Numbers and Percentages of Van Hiele Levels Among Middle School Students on Posttest of VHGT..................................................131

4.16 Comparison of Students' Pre- and Posttest Van Hiele Levels (Experimental, Control).....132

4.17 Change in Van Hiele Levels by Groups..............133

4.18 Rank Change in Van Hiele Levels by Group..............135

4.19 Means and Standard Deviations for Achievement in Geometry........................................136

4.20 Effects of Instructional Activities on Geometry Achievement........................................138

4.21 Means and Standard Deviation for Experimental and Control Groups on Pre- and Posttest on Geometry........................................139

xv
LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Triangle EFG</td>
<td>18</td>
</tr>
<tr>
<td>1.2 Illustration to Identify Triangle</td>
<td>19</td>
</tr>
<tr>
<td>1.3 Model of Variables of their Relationship</td>
<td>22</td>
</tr>
<tr>
<td>3.1 Finding Cross Section</td>
<td>81</td>
</tr>
<tr>
<td>3.2 Illustration for the Pythagorean Theorem</td>
<td>88</td>
</tr>
<tr>
<td>4.1 Correlation Among Cognitive Variables</td>
<td>107</td>
</tr>
<tr>
<td>4.2 Profiles for Pretest-Posttest on Spatial Visualization Test Scores</td>
<td>122</td>
</tr>
<tr>
<td>4.3 Profiles for Pretest-Posttest on GEFT Scores</td>
<td>127</td>
</tr>
<tr>
<td>4.4 Profiles for Pretest-Posttest on Geometry Test Scores</td>
<td>137</td>
</tr>
<tr>
<td>5.1 Relationship Between Cognitive Variables and Geometry Achievement</td>
<td>147</td>
</tr>
<tr>
<td>5.2 The Influence of Instructional Activities</td>
<td>153</td>
</tr>
<tr>
<td>5.3 Illustration for Spatial Visualization Activity</td>
<td>157</td>
</tr>
<tr>
<td>5.4 Illustration for Field Dependence/Independence Activity</td>
<td>158</td>
</tr>
<tr>
<td>5.5 Illustration for van Hiele Level of Geometric Reasoning</td>
<td>159</td>
</tr>
</tbody>
</table>
CHAPTER 1

STATEMENT OF THE PROBLEM

This chapter presents an overview of the study. First, background supporting the study is presented followed by identification of the problem for the study. This is followed by operational definitions for terms of the study. Then, the conceptual framework is discussed, conceptual model is formed, and several hypotheses are put forward. This is followed with a presentation of the limitations of the study. The chapter closes with the significance of the study.

Background

The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and other important literature in the area of reform in mathematics education (Mathematical Sciences Education Board, 1990; National Research Council, 1989) call for change in emphasis and content in geometry at all levels. In Everybody Counts: A Report to the Nation on the Future of Mathematics Education (1989), a plan of action is
outlined, that is, students learn mathematics well only if they are actively involved in doing mathematics at every grade level. The nature of what geometry content should be learned and how it should be learned were embedded in these recommendations.

Twenty years ago, The National Council of Supervisors of Mathematics (1978) established a task force to produce a list of ten basic skill areas in mathematics, and one of them was geometry.

Students should learn the geometric concepts they will need to function effectively in the three-dimensional world. They should have a knowledge of concepts such as point, line, plane, parallel, and perpendicular. They should know basic properties of simple geometric figures, particularly those properties that relate to measurement and problem-solving skills. They also must be able to recognize similarities and differences among objects. (p. 149)

*Reshaping School Mathematics: A Philosophy and Framework For Curriculum* (1990) provides a rationale for the inclusion of geometry in the mathematics curriculum. Geometry is a unifying theme to the entire mathematics curriculum and as such is a rich source of visualization for arithmetical, algebraic, and statistical concepts. For example, geometric regions and shapes are useful for developmental work with the meaning of fractional numbers, equivalent fractions,
ordering of fractions, and computing with fractions (Sherard, 1981, p. 20).

In spite of its importance in the curriculum, geometry is an area in which students experience little success. For example, there were markedly low achievement patterns in geometry for eighth grade students on both national and international assessments over the past two decades (McKnight, Travers, Crosswhite, & Swafford, 1985; Mullis, Dossey, Owen, & Phillips, 1993; Mullis, Martin, Kelly, and Smith, 1996). Little improvement has been noted for eighth grade students in geometry for the National Assessments of Educational Progress in mathematics (Mullis et al., 1996).

Third International Mathematics and Science Study (TIMSS) data reveal that the performance of U.S. students in the areas of geometry and measurement was disappointing; the international rank of U.S. students' average percent score was below the international average. TIMSS conducted a video study of eighth-grade mathematics lessons in Germany, Japan, and the United States and the expert analysis of the content indicated that U.S. classrooms in the study did not focus on high-quality mathematical thinking and reasoning. All TIMSS countries focus on computation in the early elementary grades, and at later grades most shift to
other topics like algebra and geometry except U.S. For too many middle school students, algebra, geometry, and statistics never become part of their curriculum. Yet geometry is an integral part of the mathematics curriculum in the middle grades (MSEB, 1991; NCTM, 1989). Achievement of students in geometry has often been a concern for both mathematics educators and the public.

Geometric concepts are often neglected in elementary and middle schools in favor of teaching computational skills (Cox, 1985). Various reasons related to the mathematical system itself, curricular materials, instructional practices, and cognitive development have been proposed to explain students' difficulties with geometry. Cognitive factors as variables influencing achievement in geometry have been a major concern of mathematics educators. Such factors include spatial visualization, field dependence/independence, and van Hiele level of thought. This study suggested that the use of instructional activities be emphasized in the classroom to help facilitate the improvement in the cognitive factors and then geometry achievement at the middle school geometry.

Middle school students were chosen in this study because many mathematics educators perceive that the
weakest part of the precollege mathematics curriculum is at the middle school level (Posamentier, 1989). If more effort were put into overcoming the obstacles to learning about geometry in middle school, then many of the problems which children have with geometry itself could be resolved (Hoffer, 1981). Then, the children will be much more ready for formal geometry in high school.

Research shows that only those students who enter high school geometry at van Hiele Level 2 or higher have a good chance of becoming competent with proof - Level 4 (Senk, 1989). Thus, the middle school is where the critical visualization and analysis work must be done to move students within reach of more advanced learning later.

**Spatial Visualization**

Spatial ability is one of the three primary factors of mathematics aptitude which Guilford (1967) factored into two independent components: spatial visualization and spatial orientation. In this present study, the researcher will focus on spatial visualization. McGee (1979) describes spatial visualization, a particular subset of spatial skills, as "the ability to mentally manipulate, rotate, twist, or invert a pictorially presented stimulus object" (p. 5).
Bertoline et al. (1995) describe visualization as "the mental understanding of visual information" (p. 196).

Because of geometry's visual nature, spatial visualization has been linked with geometry achievement. For example, Rolen (1985) found a statistically significant positive correlation between spatial skills, measured at the end of the year, and final grade in geometry. Spatial visualization is related to the content of mathematics, such as geometry. Every geometry course taught calls on logical reasoning and spatial ability (Mitchelmore, 1974).

The lack of basic visualization skills sometimes results in insecurity which causes many students not to do well in geometry (Hoffer, 1983). Thus, investigating spatial visualization is important because of correlational and logical-intuitive support for its relationship to geometry.

Among the curricular recommendations in the NCTM's Agenda for Action (1980) are the following:

- Mathematics programs of the 1980s must be designed to equip students with the mathematical methods that support the full range of problem solving, including ...
the use of imagery, visualization, and spatial concepts (p. 3)

- There should be increased emphasis on such activities as ... using concrete representations and puzzles that aid in improving the perception of spatial relationships (p. 7)

Mathematics achievement generally correlates with spatial visualization in the range of .30 to .60 (Ben-Chaim, Lappan, & Houang, 1988; Fennema & Tartre, 1985; Harris, 1981; Johnson & Meade, 1987). Instruction in spatial visualization tasks significantly affects the students' spatial visualization performance in the middle school grades (Ben-Chaim, Lappan, & Houang, 1988; Brinkman, 1966; Wolfe, 1970). Piaget and Inhelder (1967) stated that visualization is a learned trait. Thus, this study also attempted to evaluate the effect of the instructional activities in the area of spatial visualization skills in order to improve geometry achievement.

Field Dependence/Independence

Mathematics educators acknowledge the influence of cognitive styles in the learning of mathematics. Field dependence/independence as a dimension of cognitive
style has been studied by many researchers (Abraham, 1994; Berenson, 1986; Berry, 1994; Lourdusamy, 1982, Mullally, 1993). Field dependence/independence is a cognitive style defined as a measure of one's ability to disembed relative information from an irrelevant background and to analyze and cognitively restructure information (Witkin & Goodenough, 1981).

Field independent learners have more highly developed cognitive restructuring ability, exhibit better personal autonomy, are more skilled at extracting the salient features of a problem, are more analytical, and are more successful in mathematics (Davis & Frank, 1979; Moore & Moore, 1984). Field independent persons are able to remember spatial information and accurately place map features that they recalled during the reconstruction task (Shaha, 1982).

On the other hand, field dependent learners are more global in their perceptions, are more influenced by the complex design; that is, the situation is perceived as presented (Witkin, Moore, Goodenough, & Cox, 1977).

Investigations by Witkin et al. (1977) have indicated that field dependence/independence, the ability to disembed common geometric shapes embedded within a larger geometric design, is a significant
contributor to achievement in science, mathematics, engineering and architecture. Field dependence/independence can be considered a possible factor affecting the ability to do well in geometry, for example in a formal geometry course where the ability to find hidden figures is needed. The ability to disembend figures has been shown to vary from person to person (Witkin, 1962). This raises the question of how well the ability to disembend figures influences geometry achievement and whether the instructional tasks used in this study would improve students' ability to disembend figures.

**Van Hiele Levels**

Pierre Marie van Hiele and his wife Dina van Hiele (1957/1984) developed a model of geometric thought. The van Hieles were greatly concerned about difficulties their students encountered with school geometry. The van Hiele model has three main components: insight, phases of learning, and thought levels (Hoffer, 1983). "Insight exists when a person acts in a new situation adequately and with intention. The Gestalt psychologists and I say the same thing with different words" (p.24). To gain insight into a geometry problem, a student must first perceive a structure.
The second component of the van Hiele model, the phases of learning, describes the stages through which students progress in order to attain the next higher level of thinking. Basically these stages constitute an outline for organizing instruction.

The third component of the van Hiele model grew out of the concern the van Hieles felt when their geometry students repeatedly encountered difficulties with parts of the subject matter even after being given various explanations. Their joint interest in wanting to improve teaching outcomes led to the development of a theoretical model involving five levels of geometric thinking.

The van Hieles were first to characterize five levels of geometric thinking (1957/1984). The levels are recognition, analysis, ordering, deduction, and rigor. The van Hieles characterized their model in terms of five levels of thought as in Table 1.1.
Table 1.1
Characterization of Van Hiele Level

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
</table>
| Level 1       | **Recognition**  
Student can recognize geometric figures by appearance alone, but properties are not distinguished. Student merely identifies the figure as a rectangle because it looks like other objects called rectangles. |
| Level 2       | **Analysis**  
Student analyzes figures by their geometric properties with activities such as observation, measuring, cutting, and folding. |
| Level 3       | **Ordering**  
Student can logically order figures and understands relationships among properties of figures. The student is able to see that a square is a rectangle; but a rectangle may not be a square. |
| Level 4       | **Deduction**  
Student can understand the roles of postulates and theorems. |
| Level 5       | **Rigor**  
Student can understand the need for axioms, definitions, theorems, and proof. |

The five discrete levels have a hierarchical arrangement through which students seem to move sequentially (Denis, 1987; Senk, 1989), but not all at the same rate. The van Hiele model was formulated to
describe the geometric thinking of elementary and secondary students (Burger & Shaughnessy, 1986; Fuys, Geddes & Tischler, 1985; Usiskin, 1982). Instruction rather than maturation appears to be the most significant factor contributing to progression through the levels (van Hiele, 1986).

The transition from one level to the next is a learning "process that has to be done by the pupils themselves" (p. 62). Teachers can give guidance to the students during this complicated exercise. "Transition from one level to the following is not a natural process; it takes place under the influence of a teaching-learning program" (p. 50). The teachers' choice of lessons and activities is critical in the transition from one level to the next. In this manner, teachers help students find ways to ascend to the next higher level.

Thus, this present study investigated the van Hiele levels for middle school students, to examine whether van Hiele levels of geometric thought relate to student's performance in geometry, and to evaluate the instructional activities developed for improving van Hiele levels of geometric reasoning.
**Effect of Instructional Activities**

The essence of this study was to evaluate the effectiveness of selected instructional activities on improving spatial visualization ability, field dependence/independence, and van Hiele level of geometric thinking and thus on geometry achievement. Experience with instructional activities that emphasize on spatial visualization skills, disembedding figures, and van Hiele levels of geometric reasoning, regardless of the developmental level of the student aids in providing a strong foundation for conceptual understanding (Ben-Chaim, 1988; Fuys et al., 1988; Hoffer, 1983).

Dina van Hiele (1957) developed a series of 70 lessons which successfully moved secondary students from the first to the third van Hiele levels. Since then instruction has been developed and tested in the classroom (Wirszup, 1976) and in a laboratory setting (Fuys, Geddes & Tischler, 1985) which raised the van Hiele levels of thinking for elementary and secondary students.

Thus, the effects of using instructional activities on spatial visualization, van Hiele level, field dependence-independence, and achievement, and the
interrelationships between these factors should help to provide meaningful data supporting an alternative to traditionally taught geometry classes which better addresses the cognitive needs of the students.

**Statement of the Problem**

The purpose of this study is to investigate the relative importance of the cognitive variables of spatial visualization ability, field dependence/independence, and van Hiele level of thought in predicting achievement in geometry of middle school students and to determine the effect of selected instructional activities on improving the cognitive variables and achievement in geometry.

Specifically this study will seek to answer the following questions:

1. What van Hiele levels of reasoning do middle school students exhibit;
2. What are the relationships among spatial visualization, field dependence/independence, and van Hiele levels of thought?
3. What relationships exist between spatial visualization ability, field dependence/independence, van Hiele levels, and achievement in geometry among middle school
students?

4. Do students who receive selected instructional activities show significantly greater improvement on the Spatial Visualization Test, Group Embedded Figures Test, Van Hiele Geometry Test, and Geometry Test than students in a control group?

Definitions

The operational definitions of important terms used in this study are as follows:

**Spatial Visualization Ability (SVA)**

The ability of a student to rearrange a visual representation. In this study, SVA will be measured by the total score earned on the Middle Grades Mathematics Project (MGMP) Spatial Visualization Test.

**Field Dependence/Independence**

Refers to the cognitive ability of an individual as measured by the Group Embedded Figures Test (GEFT) developed by Witkin et al. (1977). A score of 0 to 9 on the GEFT designates the person as field dependent whereas a score of 14 to 18 indicates field independence. People who are heavily influenced by the surrounding field are called field dependent (FD).
Those who are relatively uninfluenced by the surrounding field are referred to as field independent (FI).

**Van Hiele Level**

An indicator of a subject's geometric maturity; one of the four levels forming a hierarchy of geometric sophistication; assessed by a *Van Hiele Geometry Test* developed at the University of Chicago during 1979-1980. Van Hiele level also refers to different types of thinking that students pass through as they move from a global perception of geometry figures to an understanding of postulates and theorems.

**Geometry Achievement**

Refers to a quantitative measurement indicating the ability of an individual to determine the outcome of a problem or a set of problems. The geometry achievement score in this study will be based on the number of correct responses to the paper-and-pencil *Geometry Test* developed by the researcher. The items test students' understanding and application of mathematics concepts in solving geometry problems.
Conceptual Framework

For the last two decades, it has become increasingly apparent that many of the problems students have in trying to learn geometrical ideas are rooted in their conceptualizations of the spatial world, (Battista, Wheatley, & Talsma, 1982; Hoffer, 1981). This means that instead of learning the definitions of a circle, a square, and a rectangle, the child needs to develop ideas about "circularity" - e.g., it allows objects to roll; about "squareness" - a four-sided symmetrical feeling; and about "rectangularity" - the "box" shape, as Freudenthal calls it, with a symmetry of sorts, but basically unbalanced (Bishop, 1986, p. 146).

Spatial Visualization

Spatial visualization appears to account for some of the variance in ability to solve mathematical problems (Battista, Wheatley, & Talsma, 1982; Fry, 1987; Moses, 1980; Schoenberger, 1976, 1981). Begle (1979) noted that "there is a small but significant correlation between measures of spatial visualization and mathematics achievement" (p. 92). To successfully learn geometry, students need to develop spatial visualization skills. Horton (1982) indicated that
schools emphasize the verbal approach to learning even though we live in a visually oriented world. Students are given little or no instructional activities in spatial visualization.

Field Dependence/Independence

According to Guay and McDaniel (1977), the Embedded Figures Test is used to test "low-level" spatial abilities, which need "the visualization of two-dimensional configurations, but no mental transformations of these visual images" (p. 211). Hoz (1981) describes several aspects of geometrical rigidity, caused, in many cases, by the students being unable to "see" a diagram in different way. For example, the student unable to use the line EH (Figure 1.1) as a "side", because it is seen only as the "height" of triangle EFG.

![Diagram of triangle EFG]

Figure 1.1: Triangle EFG
This rigidity is also tested in the test of "embedded figures", where a simple geometric figure is embedded in a complex design. According to Hoffer (1986), "Complex geometric figures which uses circles, chords, angles, etc., can present severe obstacles to students if they are not able mentally to disentangle and recompose the various geometric components in order to demonstrate or uncover the relationship" (p. 152).

Geometry deals with the properties and relationships of points, lines, angles, surfaces, and solids. There are many concepts in geometry that cannot be recognized or understood, unless the students can disembend figures from complex design. For example, if students are given an illustration like Figure 1.2 and asked to identify as many triangles as they can,

![Figure 1.2: Illustration to Identify Triangle](image)

19
they must scan the design and disregard irrelevant shapes while looking for the triangle in the midst of a large area with similar shapes.

**Van Hiele Levels**

The van Hiele research suggests that students should be given more experiences with physical representations of shapes so that they explore the properties of shapes before going on to proofs and formulas (Fuys, Geddes, and Tischler, 1988). Thus, this research not only focused on van Hiele levels of geometric reasoning exhibit by middle school students but also on helping students make the transition from one level to the next and thus improving geometry achievement.

Most of the studies pertaining to prediction of achievement have generally been concerned with using one or two predictors. While a range of variables needs to be taken into account by researchers, the relative importance of variables such as spatial visualization and field dependence/independence as predictors of learning has been questioned (Battista, 1989; Blurton, 1985; Herron, 1978; Lawson, 1979).

Fuys, Geddes, and Tischler (1988) found that geometry was often taught by rote memorization or in a manner which required very little student explanation.
This approach prevents students from involving themselves in appropriate thinking about geometry. They tend to forget or confuse what they have memorized. Despite this finding, many teachers still use rote memorization along with heavy reliance on the textbook when teaching geometry (Driscoll, 1988). Thus, this present study looked at all variables together and evaluated the effect of instruction with special tasks.

**Conceptual Model**

Figure 1.3 is a conceptual model of the significant variables in this study and how they may relate to each other. An arrow going from one variable to another indicates that the variable on the receiving end is affected in some way by the variable from where the arrow originates. For example, there is literature supporting the fact that achievement in geometry varies according to spatial skills, so an arrow goes from spatial skills to achievement in geometry.

Double arrows indicate the influence goes both ways. This model is not illustrating all of the variables at work in a mathematics classroom, only the important variables being considered in this study.
Figure 1.3 Model of Variables and Their Relationship
Limitations of the Study

The following limitations restrict the generalization of findings of the study to different classes, situations, and settings:

1. The study sample was not randomly selected. Instead, a total sample of subjects in six intact classes was used in the study, that is, one experimental class and one control class from each of grades 6, 7, and 8.

2. The sample in this study were from predominantly black, middle-class families, students in an inner city public middle school. Thus, the generalization of the findings of the study is limited to similar schools with similar students.

3. The instruments used in this study were all paper-and-pencil tests. More in-depth personal interviews with subjects attempting the problems could have enhanced the research by triangulating the modes of data collection.
Significance of the Study

The Curriculum and Evaluation Standards for School Mathematics published by the National Council of Teachers of Mathematics (NCTM) in 1989 state "equally important is the continued development of students' skills in visualization, pictorial representation, and the application of geometric ideas to describe and answer questions about natural, physical, and social phenomena" (p. 160).

Geometry is a branch of mathematics concerned with the study of spatial properties of various figures abstracted from the concrete world of physical objects. The geometric content consists of many visual components. Experiences with visualizing and making pictorial representations at middle school level form the foundation for higher study of Euclidean and non-Euclidean geometries and calculus. Gaulin (1985) stresses the need of re-establishing the development of spatial intuition as one major goal for teaching geometry.

Studies (eg. Speer, 1979) have shown that field independent students may have an advantage over field dependent students when using mathematics textbooks, especially geometry textbook, because of the heavy
reliance on pictures or diagrams. Also, geometry appears to require the ability to be analytical and be able to extract relevant information in interpreting figures and shapes, it will therefore be important to find out whether field dependence/independence does influence achievement in geometry.

The van Hieles were concerned about the difficulties their students encountered with geometry. Several research projects have investigated and verified aspects of the van Hiele model in relation to the middle grades (Burger & Shaughnessy, 1986; Fuys, Geddes, & Tischler, 1988). But the question now is, do van Hiele levels of reasoning play an important role in achievement of geometry in middle school? It will therefore be beneficial to find out whether the van Hiele levels of reasoning is a significant predictor of achievement in geometry.

The major significance of the study resides in determining whether selected instructional activities are optimally suited to help in improving students' ability to communicate and represent geometry information in order to optimize student achievement in geometry. Specifically, the result of this study can have a bearing on how instructional designers or instructors optimize learning achievement in geometry.
by using activities designed to enhance students' spatial visualization, field dependence/independence, and van Hiele levels of geometric thought.
CHAPTER 2

REVIEW OF THE LITERATURE

In the previous chapter, the purpose of this research was described, and its background briefly discussed. This chapter will look at what the literature provides for influencing and improving mathematics/geometry learning. Specifically, what role do spatial visualization ability and field dependence/independence play, and how do the van Hiele levels of geometric thought relate to what is learned. In the last section of this chapter, studies that examined instructional intervention will be reviewed.

**Spatial Visualization and Achievement**

Spatial visualization enables the student to manipulate mentally, rotate, twist, or invert stimuli that are presented pictorially (Fennema & Behr, 1982; McGee, 1982) and is related to geometry (Bishop, 1986, Hoffer, 1981)). According to Hoffer (1981), learning
geometry involves five types of skills, that is, visual, verbal, drawing, logical, and applied.

Spatial visualization ability has often been found to be significantly related to mathematics achievement. However, results have been quite inconclusive. Some researchers have found a positive relationship between spatial visualization and achievement, while others have reported little or no correlation between the two variables.

**Studies showing a Positive Relationship**

Moses (1978) in an initial study involving 145 fifth grade students, administered a battery of six tests, five of which were *Spatial Tests* (Punched Holes, Card Rotations, Form Board, Figure Rotations, Cubes Comparisons). The dependent variable was a problem-solving inventory. She found that spatial ability was a good predictor of mathematics problem-solving performance and that although individuals with high spatial ability usually did well on pencil-and-paper problem-solving exercises, their written solutions did not give a proper indication of the extent visual solution processes had been used.

Sherman (1980) found that spatial ability was a good predictor of success for high school students in
geometry. In subsequent studies, spatial visualization ability was found to be not only related to mathematics performance, but also to achievement in science, engineering, and chemistry. Spatial visualization ability has also been shown to differentiate between girls who take more mathematics courses in high school and girls who do not (Sherman, 1980).

Middaugh (1980) investigated the nature of spatial ability and its relationship to the mathematical performance of 357 junior high school students in a midwestern United States industrial city. Results of the findings revealed significant and positive relationships between spatial ability and mathematical performance in five areas: mathematical computations, mathematical concepts, mathematical applications, graphical skills, and mathematics grades.

Fennema (1983) used a longitudinal design to investigate how girls and boys who were discrepant in their spatial and verbal performance used spatial visualization skills in solving word problems and fraction problems. The subjects, 36 girls and 33 boys, were interviewed annually in grades six, seven, and eight. Each student was asked to read a problem, draw a picture, solve the problem, and then explain how the picture was used in the solution. Students who differed
in spatial visualization skill did not differ in their ability to find correct problem solutions, but students with a higher level of spatial visualization skill tended to use spatial skills in problem solving more often students with a lower level of skill. Girls tended to use pictures more during problem solving than boys did, but this did not enable them to get as many correct solutions. Low spatial visualization skill may be more debilitating to girls' mathematical problem solving than to boys'.

A study done by Landau (1984) investigated how spatial visualization, problem presentation format, and their interaction influence middle school students' performance on mathematical problems which vary in difficulty and in the extent to which a diagrammatic representation is likely to facilitate solution. Sex-related differences in performance were also examined. Subjects were 384 middle school students representing a wide range of ability, SES, racial, and ethnic groups. The instruments used were the Monash Space Visualization Test (MSVT) and an investigator-made parallel 17-item problem-solving test (Pretest and Posttest). Subjects were blocked using MSVT scores and randomly assigned to four treatment groups.
Analyses of covariance were conducted for the posttest and four subtests on two spatial ability levels, the four treatment groups and sex, with the pretest, grade, school, and enrollment in algebra as covariates. The findings showed there was a strong correlation between spatial ability and problem-solving performance.

Battista, Wheatley, and Talsma (1989) investigated the relationship between the strategies used by preservice elementary teachers in geometric problem solving and the two primary mental abilities — spatial visualization and formal reasoning. This study provided insight about strategies used to solve geometry problems and spatial visualization. The subjects were students enrolled in five sections of a geometry course for preservice elementary teachers. In order to investigate the strategies utilized by the students as they solved problems of a geometric nature, eight problems were constructed. Each of these problems could be solved using some type of "visual strategy — a drawing could be made or a mental picture formed. Seven of the problems dealt with obvious geometric content. These researchers found that spatial visualization, formal reasoning, and problem-solving performance were significantly related to geometry course grade, and that spatial visualization and formal
reasoning were significantly related to problem-solving performance.

Tartre (1990) examined the role of spatial skill in mathematical problem solving. The Gestalt Completion Test and a tenth-grade mathematics achievement test were administered to 97 tenth-grade students with discrepant spatial skills. Students who scored in the top and bottom third were interviewed to elicit more information about their thinking processes. Results indicated that students who possessed higher spatial skills could estimate more accurately, were more able to analyze problems, organize their thinking, and relate new problems with previous knowledge.

A study done by Gum (1997) determined whether spatial intelligence contributes to a student's success in a computer science program. The subjects consisted of 15 computer science majors, enrolled in a computer science class, and 15 non-computer science majors, enrolled in a statistics class. The instruments used were — Card Rotations Test to determine spatial orientation ability; the Maze Tracing Speed Test to determine spatial scanning ability; and the Surface Development Test to determine visualization ability. Results indicated that experience in computer
programming increased visualization ability and spatial scanning ability, while decreasing spatial orientation ability.

**Studies showing Negative Relationship**

Battista (1990) investigated the role that spatial thinking plays in learning, problem solving, and gender differences in high school geometry. Spatial thought was examined along with its counterpart, verbal-logical thought. Data were collected on 145 high school geometry students from a middle-class, midwestern community. Analyses were performed only on data from students who completed all the tests: 75 males and 53 females. Paper-and-pencil tests were administered in four areas: spatial visualization, logical reasoning, knowledge of geometry, and geometric problem-solving strategies. A modified version of the Purdue Spatial Visualization Test: Rotations was used to measure students' ability to mentally visualize rotations of objects in space. An experimenter-constructed test of logical reasoning was used to assess students' ability to draw conclusions in logical syllogisms. The results suggested that, whereas males and females differed in spatial visualization and in their performance in high school geometry, they did not
differ in logical reasoning ability or in their use of geometric problem-solving strategies.

A study done by Pandisco (1994) investigated a theoretical relationship between mental rotation of visually presented objects and proficiency in certain geometric tasks. The subjects in this study were high school students. A standardized test of mental rotations and a standardized geometry achievement test were administered to all participants. The results did not support that spatial ability is a strong predictor of achievement in geometry. This result and information are very helpful to my study, that is, to examine whether spatial ability is a strong predictor of achievement in geometry for middle school students.

Gould (1996) examined the performance of adolescents, aged 15–19, on a series of tasks with a strong spatial component. Seven female-female, seven male-male, and seven male-female pairs were video-taped to show how each pair approached the problems, the time spent on each problem, and their drawings as they worked on eight tasks incorporating spatial visualization. For each problem, scoring rubrics were used to evaluate each pair's performance. Gould found female-female pairs spent more time on fundamental
definitions, details, and concepts for the initial exercises and experienced little success with the remaining exercises.

The studies cited seem to indicate the importance of spatial visualization skills to achievement in mathematics/geometry (Middaught, 1980; Robinson, 1994; Sherman, 1980; Tartre, 1990). However, the relevant literature reported little or no positive correlation between geometric ability and spatial visualization (Bishop, 1980; Gould, 1996; Lean & Clements, 1981; Pandisco, 1994). In conclusion, the results of these studies are inconclusive and the field is still open for further research. Most of the studies used high school or college students for their sample (Middaught, 1980; Pandisco, 1994; Robinson, 1994) and very few studies used middle school (Ben-Chaim, 1988). Thus, there is a special need to investigate further with middle school students especially using instructional activities to improve their geometry achievement, and possibly, their spatial visualization.

Field Dependence/Independence and Achievement

Field dependence/independence involves the extent of the influence of the complex design or the field on the individual's performance on perceptual,
intellectual, social, and interpersonal tasks. Field independent people have the ability to distinguish and coordinate items extracted from a complex stimulus context that may be confusing to others, while field dependent people tend to preserve the holistic nature of the stimulus and conform to the prevailing field (Roberge & Flexer, 1983).

**Studies Showing a Positive Relationships**

A number of studies have shown that in general field independent students are better problem solvers (Ballinger et al., 1984; Squire, 1977). Studies by Satterly (1976), Bien (1974), Buriel (1978), Vaidya and Chansky (1980) reveal that field independence is significantly related to achievement in mathematics.

Roberge and Flexner (1983) examined the relationship between FDI and mathematical ability in a sample of 450 sixth, seventh, and eighth grades students with above-average IQ scores. They found that on tests of mathematical problem solving, the FI students did significantly better than FD students. They believe that "further attention should be given to examining variables that might influence the mathematics achievement of field dependent students
(e.g., problem structure, explicitness of instructions, task-related experience)" (p. 350).

In a sample of one hundred fourth-graders, Bien (1974) found that FI students outperformed FD students on both word and numerical problems. She also found that providing cognitive structuring techniques for the FD children resulted in an increase in problem-solving success.

Threadgill-Sowder and Sowder (1982) investigated the effect on performance of two problem formats — the usual words-and-numerals-only (verbal) format versus a line-drawing-with-numerals-and-minimal-verbiage (drawn) format — and explored how field dependence/independence, spatial visualization, and general reasoning ability related to performance. The problems used were called "routine" story problems which do not require extensive analysis. The sample included 262 fifth-grade students. They used the Hidden Figures Test to measure field dependence/independence, Punched Holes Test to measure spatial visualization, and the Arithmetic Reasoning Test to measure general reasoning ability. The sets of problems in verbal format were given to the students in the verbal treatment group, while the problems in drawn format were given to
drawing treatment group for about 5 - 6 weeks for them to practice. Pre- and posttests were given to the students. Informal interviews with both groups showed that students preferred the drawn format. Students who practiced on drawn format problems received higher posttest scores then students who practiced on verbal format problems. Result indicated that only the Hidden Figures Test score appeared to interact with treatment, that is, only field independence is related to performance.

Hitchens (1992) investigated middle-school-aged students' preference toward field dependence or field independence and toward learning in a cooperative or a traditional classroom. Two 2-week poetry units were presented to four sections of seventh grade students in a predominately white, middle class setting. Each section was taught in both a cooperative learning setting and in a traditional classroom setting. After 2-week instruction and testing, students were given a questionnaire to ask for their preferences and the Group Embedded Figures Test. Results indicated that there was a correlation between students' levels of preference toward learning in a cooperative classroom and the students' tendency toward field independence. Though this study was not conducted in geometry, it
does gives the researcher an idea about 7th grade students' preference toward way of learning in the classroom. The researcher shared this idea with the teachers that used the instructional activities. In a sample of 537 college students enrolled in an introductory education course Wieseman (1992) found that students who were more field independent performed better academically in six courses required for education majors than students who were field dependent. Of the 537 students, 448 were female and 89 were male. No statistically significant differences were found between male and female GEPT scores or course grade scores.

Field independence has been found to relate to problem-solving ability, and good problem solvers tend to be field independent (Maher, 1982). Reiff (1992) indicated that field independent students have been found to be more flexible in their problem-solving approaches and were more task oriented and able to focus on the relevant aspects of a task. On the other hand, Fritz (1992) found that field dependent students are also able to solve problems, but have been found to use different processes to reason about events.

Waktins and Astilla (1981) found that field independence was associated with higher first year
grades in engineering. In a multiple regression equation, scores on the *Hidden Figures Test* added significantly to score on the college entrance exam scores in predicting first-year grades of male students. On the other hand, Thomas (1983) found that field independence was not uniformly related to performance in different courses within mechanical engineering.

Several researchers have hypothesized that mathematical behaviors exhibited by individuals as they solve problems or learn mathematics may depend on more fundamental or "primary" mental abilities. Kulm and Bussman (1980) argue that students may fail to use a certain problem-solving process at a particular stage of problem solving because they lack a specific primary mental ability that is necessary for learning to apply that process.

**Is Field Dependence/Independence a Style or an Ability?**

There has also been discussion of whether field dependence/independence is a style or an ability. Cronbach (1970, 1977), interpreting the data of his experiments, suggests that field dependence is not a distinctive style but a deficiency in ability. Ability is usually measured by performance and quantity.
Canelos and Taylor (1981) conducted an experiment in which a networking learning strategy was given to field dependent students. It was found that field dependent students' achievement was no higher than the achievement of field independent students' achievement even though no strategy was given to the field independent students.

Ausburn et al. (1978) suggest a concept of supplantation with relation to differences in cognitive styles. Supplantation is defined by them as "the explicit and overt alteration or performance of a task requirement which a learner would otherwise have to perform covertly for himself." They further suggest two kinds of supplantation to fill a "gap" between a learner and a learning task. Supplantation, for example, could be giving different colors to particular parts on a complex map or diagram for clarification to help field dependent persons who are not good at disembedding simple figures from a complex design. Another type of supplantation is conciliatory supplantation, in which instructional modes of presentation are varied, as in providing oral or written instructions.

It has long been recognized that field dependence/independence is likely to have important
educational implications (Ausburn, 1978; Threadgill-Sowder, Sowder, 1985). Tinajero and Páramo (1997) investigated the relationship between academic achievement and field dependence/independence for a sample of 408 students aged between 13 and 16 years. Field dependence/independence was evaluated using the Embedded Figures Test, the Group Embedded Figure Test (Witkin et al., 1971), and the Portable Rod and Frame Test (Oltman, 1968). Results indicate that field dependence/independence is positively related to achievement.

If field dependence/independence is not a style and field dependence is a deficiency in ability as Cronbach suggests, there should be some help to be gained in providing field dependent persons with tasks that allow them to practice disembedding necessary information from the complex background. Thus, field dependence/independence can be considered a possible factor affecting the ability to do well in geometry. There are many examples in geometry where the ability to disembed is needed. This raises the question of how well the ability to disembed figures from complex design predicts geometry achievement.
**Spatial Visualization, Field Dependence/Independence, and Achievement**

Spatial visualization ability and field dependence/independence are widely thought of as distinct psychological dimensions. There is, however, evidence that the two constructs appear to be linked (McLeod, Jackson, & Palmer, 1986). Spatial visualization ability is measured by skill in manipulating two-dimensional or three-dimensional, or by skill in performing mental manipulations. Field dependence/independence is indexed by success in isolating a particular geometric figure/shape inside a complex design.

Hozaki (1987) investigated the interaction between cognitive styles (field dependence/independence) and visualization on a paper-folding task on a group of 109 field dependent and independent male and female college students of Ohio State University. Hozaki found that field independence seems to play an important role in perceptual disembedding skill and visualization skills in physical performance of a moderately complex paper-folding task. Thus, from the Ausburns and Hozaki's point of view, providing different modes of presentation in instructional tasks as in this study,
could be one effective way for field dependent persons to help process visual information, van Hiele geometric thought and improve in geometry achievement among middle school students.

Carment (1989) examined the effects of field dependence/independence, spatial visualization, and cerebral dominance on mathematics achievement. The sample (N=240) was chosen from tenth grade students at Muskogee High School during the school year 1986-87. Four measuring instruments — Group Embedded Figures Test, Card Rotations Test, Cube Comparision Test and Your Styles Of Learning questionnaire — were administered to the sample. The dependent variable, mathematics achievement, was operationally defined as the mathematics subtest raw score of the Metropolitan Achievement Test which was made available from the school records. The reliable predictors of mathematical achievement were three dimensional spatial visualization and field dependence-independence. Cerebral dominance and two-dimensional spatial visualization were found not to be reliable predictors of mathematics achievement scores.

The strength of the relationships between field dependence/independence, spatial visualization, and problem solving of 100 adolescents in grade eight was
determined by Arrington (1987) using the Group Embedded Figures Test and Purdue Perception Screening Test. Results indicated that cognitive style and spatial visualization were positively related to problem solving. Field independent subjects with high visualization ability scored higher than field dependent, low spatial visualization ability adolescents.

A review of the literature shows that there are not many studies that have looked at the relationship between spatial visualization and field dependence/independence and geometry achievement. This suggested that it was necessary to study the importance of spatial visualization and field dependence/independence in influencing geometry achievement for middle school students.

**Van Hiele Levels of Geometric Thinking**

The theory developed by Marie and Pierre van Hiele in their research described five levels of thought. At Level 1, students recognize figure by looking at shape alone. At Level 2, students discovered basic properties of figures. At Level 3, students logically interrelate properties/rules and are able to follow simple informal argument. At Level 4, students prove
theorems and establish interrelationships among theorems. When students reach Level 5, they establish theorems in different postulate systems and analyze/compare these systems (Fuys, Geddes, & Tischler, 1988).

**Procedures to Measure the van Hiele Levels**

Collis (1975) suggests that an individual's way of learning mathematics is related to their cognitive level of functioning. Students can be assigned to a particular van Hiele level or learning period by analyzing their responses to specific geometric tasks.

To measure the van Hiele levels of preservice elementary teachers, Mayberry (1981) developed an interview script with appropriate written problems. Topics chosen were squares, isosceles and right triangles, circles, parallel lines, similar and congruent figures. At each van Hiele level for each topic Mayberry developed questions to evaluate the student's understanding of geometric concepts. To record the answer, a check sheet was used. The hypothesis was that "for each geometric concept, a student at level N would answer all the questions at a level below N to criterion but would not meet the criterion on questions above level N" (p. 58). The
sample consisted of 19 preservice teachers. Each was interviewed for two hours during which 128 questions were posed. A Guttman scalogram analysis was used to test the validity of the hypothesis. The hypothesis that the van Hiele levels form a hierarchy was not rejected. Mayberry, however, cited several reasons for limits placed on the generalizability of the study. These included (i) the choice of representative questions for each level of thought, (ii) the homogeneous nature of the sample, (iii) the subjective nature of the success criterion, and (iv) the sample size of 19 subjects.

The study conducted by Fuys and Geddes (1984) investigated geometric thinking of sixth and ninth grades students in inner city schools. The conceptual framework of their study was built on van Hiele levels of thought development in geometry. Their findings showed that there was a wide range in levels of thinking among the subjects, that is, from some who were consistent by Level 1 thinkers to some who were able to give informal deductive explanations. The entry level of the sixth graders was mainly Level 1 or at times Level 2 and for ninth graders, it was mainly Level 1-2 or Level 2. This study was helpful in planning for the instructional activities in the
present study because I could anticipate the entry levels of the middle school students.

During 1979 - 1982, the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project at the University of Chicago developed a paper-and-pencil Van Hiele Geometry Test for identification of van Hiele levels. This van Hiele test was structured as a 25 multiple-choice question, with five questions at each level. Usiskin (1982) later modified the test to assess van Hiele level. The modified test contains 20 items, also with five questions at each level. For subjects to be classified at van Hiele level N, they must succeed at all levels below N and at level N but not above level N.

Burger and Shaughnessy (1986) conducted a study in Oregon. Their approach was to use a structured interview format as opposed to the multiple choice paper-and-pencil test used by Usiskin. The subjects were students from grades K — 12 plus a university mathematics major. The tasks included drawing shapes, identifying and defining shapes, sorting shapes, determining a mystery shape, establishing properties of parallelograms, and comparing components of a mathematical system. Each task performed by the students was assigned a van Hiele level using a
protocol analysis form based on descriptions of the level. They found that students could be assigned to levels, but they did experience difficulty in assigning levels to some students whom they believed to be in transition between levels.

Fuys, Geddes, and Tischler (1988) designed interview scripts involving activities, games, and manipulatives to evaluate the level of thought of individual students. For example, a student could be asked to explain why a given figure is a rectangle. If she/he said that it looked like a rectangle, the student was at the Level 1. If she/he was able to recognize the rectangle had four right angles and four sides, she/he was at Level 2. If she/he said that figure was a parallelogram or that the figure was a quadrilateral, she/he was at Level 3 or higher. Their results supported the hierarchical nature of the van Hiele level.

Investigations by Burger and Shaughnessy, Fuys et al. and Mayberry have utilized formal student interviewing to identify van Hiele levels. Such limited approaches, however, do not extend easily to a natural classroom setting. Thus, the most comprehensive formal instrument useful for whole classes is Van Hiele
Geometry Test developed at University of Chicago, and this was the instrument used in the present study.

**Van Hiele Levels and Achievement/Performance**

Many researchers have attempted to relate students' academic performance to levels of cognitive development. Research supports the accuracy of the van Hiele model for assessing student understandings in geometry (Burger, 1985; Burger & Shaughnessy, 1986; Fuys, Geddes, & Tischler, 1988; Hoffer, 1981). Wirszup (1976) reported on the success of the Soviets in implementing a curriculum based on the van Hiele levels of learning.

Usiskin (1982) conducted a study which involved 2,699 students over a one-year period in geometry classes. The students came from 13 high schools that met certain socio-economic criteria and that offered a high probability of success in obtaining reliable data consistent with the testing requirements. Usiskin found it easy to classify individual students into a van Hiele level but the reliability was low for students who were in transition from one level to another. Correlations between the van Hiele levels and concurrent geometry achievement were .61. Both Usiskin
(1982) and Mayberry (1983) have concluded that many students never attain the level of formal deduction.

Senk (1989) investigated relations between Van Hiele levels, achievement in writing geometry proofs, and achievement in standard geometry content. This study used a pre- and posttest design. The Van Hiele Geometry Test and the Entering Geometry Student Test were administered to all students in the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) sample. Proof-writing achievement also varied significantly with van Hiele level when either entering knowledge of geometry or geometry achievement in the spring was used as a covariate. The predictive validity of the van Hiele test written at the University of Chicago was supported. Senk concluded that students who enter a formal geometry course at Level 1 have a probability less than .35 of succeeding in doing proofs in geometry, those who enter at Level 2 have a chance ranging from .38 to .6, and those who enter at Level 3 have a probability greater than .75 of succeeding to construct proofs.

In an analysis of van Hiele levels of thinking of geometry material in three major U.S. textbook series for grades K – 8, Fuys et al. (1988) found most geometry material at Level 1. The small amount at
Level 2 had in most cases been reduced to Level 1 by the use of rote exercises. These texts provide very little opportunity for students to progress to a higher level even though some sixth grade students are capable of Level 2 and Level 3 thinking (Fuys, 1985). A unified seventh grade mathematics program, using a text series adapting exercises to the van Hiele phases for promoting growth through the levels, promoted thinking leading to Level 2 (Joyce, 1984).

The Effect of Instruction

Can spatial visualization, field dependence/independence, van Hiele level, and ultimately, achievement in geometry be improved?

Spatial Visualization

Can spatial visualization skill be taught/trained? Krutetskii (1976) considers spatial visualization as a mathematical ability. According to him, spatial visualization abilities are not inborn but are constructed as a result of development. "They are formed and developed during activity, instruction, and training" (p.60). Bertoline (1988) suggests that human beings are not born with spatial visualization
abilities and these abilities are developed through experiences.

Several studies of training programs to improve spatial visualization are reported in the literature. Among them, inconsistent results are found. Bishop (1980) theorized that spatial training could probably help students organize their mental pictures to solve mathematical problems, and thus recommended the inclusion of spatial training in mathematics curricula. He encouraged more studies be carried out before any conclusive relationship could be drawn between spatial visualization and mathematics performance.

Ferrini-Mundy (1987) investigated the effects of spatial training upon calculus achievement, spatial visualization ability, and the use of visualization in solving problems on solids of revolution. The treatment groups of the study participated in a spatial-training program that included work on (a) two-dimensional spatial tasks, (b) three-dimensional spatial tasks, (c) rotations from two- into three-dimensional space, (d) area estimation, and (e) the development of three-dimensional images from two-dimensional representations. Spatial ability was assessed using the Space Relations Subtest, Form T of the Differential Aptitude Test as both the pretest and
posttest. The findings indicated the spatial training program was not statistically significant in improving spatial ability scores, though the scores on the posttest did favor the treatment groups. However, the spatial training program was effective in improving the scores on the solid-of-revolution problems.

Tillotson (1985) investigated how instruction in spatial visualization affected a student's level of spatial visualization and examined its correlation to problem-solving performance. Three aptitude tests — Card Rotation, Cube Comparison, and Punched Holes — were selected to measure spatial visualization. A problem-solving inventory was used in addition to the spatial tests to determine the relationship between problem solving and spatial visualization. The problems were equally divided among three categories: spatial problems, analytical problems, and problems equally spatial and analytical.

The ten-week instructional program was administered to 102 sixth grade students. The first and the last weeks of the quarter were reserved for testing students on the spatial battery and the problem-solving inventory. Students manipulated three dimensional models, imagined the movement of those models, practiced transformations with two dimensional
drawings, and participated in some problem-solving activities. The results of the study suggested three major conclusions. First, spatial visualization is composed of at least two component parts. The Punched Holes Test was not significantly correlated to the other two spatial tests and appeared to be measuring a different skill. Second, spatial visualization is a good predictor of general problem-solving performance. Third, spatial visualization is a trainable attribute. Students in the experimental classes made significant gains in spatial scores.

Ben-Chaim (1982) investigated the effects of instruction on the spatial visualization skills and the performance of a sample of sixth, seventh, and eighth grade students. The instruments used included a Spatial Visualization Test developed by the Middle Grades Mathematics Project staff, in the Mathematics Department of Michigan State University, and semantic differential scales to measure attitudes toward mathematics. The spatial visualization instruction material included ten sequenced activities which required two to three weeks of instructional time. After the instruction, sixth, seventh, and eighth grade
boys and girls performed significantly higher on the *Spatial Visualization Test*.

A study done by Robinson (1994) examined the relationships among the students' spatial visualization ability, mathematical ability, and problem-solving strategies with and without the availability of The Geometer's Sketchpad. Participants for the study consisted of 158 geometry students from seven classes at one high school. Three measures of spatial visualization were used — *Card Rotations*, *Cube Comparisons*, and *Paper Folding Test*. Two groups were formed — one had the computer software, while the other group did not. The availability of the computer was not a significant factor in performance. Results suggest that strategies learned with technology are transferable to paper and pencil situations, and that active participation in instructional activities is important to successful performance. During the interview Robinson also found that language and communication were critical to expanding the student's zone of proximal development.

With a sample of thirty-six students in grades 6 — 8 in a summer school mathematics project, Sunderg (1994) investigated the comparative effects of spatial training and geometry instruction on spatial
performance and mathematics achievement. Students were randomly placed in one of four groups. Spatial Groups 1 and 2 were given concrete materials to develop spatial visualization ability while Geometry Groups 1 and 2 received traditional instruction from an eighth grade mathematics book. All students received twenty-five hours of instruction in geometric concepts. The 3-R's Test on Mathematics Achievement and Quantitative Abilities (1982), and the Middle Grades Mathematics Project Spatial Visualization Test (1981) were given to all students before and after the instruction. It was found that spatial ability increased more for the Spatial Groups than for the Geometry Groups. However, none of the groups increased their score on the test of mathematics achievement.

The objective of the study by Gillespie (1995) was to examine the effect that solid modelling tutorials have on the development of the three-dimensional spatial visualization skills of engineering graphics students. The study was classified as quasi-experimental by design, since the research was conducted within existing classes at the University of Idaho. The data used in this study included those 41 students who successfully completed all measuring instruments in the experimental and control groups.
The treatment, which incorporated the solid-modeling tutorials, took place over about 10 weeks. Three instruments were used for evaluation of the subjects in this study, that is the Mental Rotation Test, the Paper-Folding Test, and the Rotated Blocks Test. The study suggested that a significant improvement of visualization skills occurred in the experimental group as compared to the control groups.

Pillay (1997) investigated the effect of four different instructional formats — physical model, isometric drawing, isometric drawing plus a model, an orthographic drawing. The sample consist of 40 secondary school students with mean age of 14.2 years. The instructional materials were developed using cognitive load and spatial information processing theories. Results indicated that model format produced less cognitive load compared to the isometric and the orthographic formats. The researcher found that there was no significant difference between the model and the isometric-plus-model formats on all measures.

In conclusion, the mixed results of studies on training in spatial visualization and individual differences in this area leave the field open for further research.
Field Dependence/Independence

Several studies have shown that field dependence/independence had a significant effect on the mathematics achievement (Burriel, 1978; Roberge & Flexer, 1983; Vaidya & Chansky, 1980; Wieseman, 1992). These significant influence of field dependence/independence on students' achievement suggests the need for investigations that examine the feasibility of using instructional activities that are improving individual field dependence/independence.

According to Cronbach (1977), field dependence/independence is not a distinctive style but a deficiency in ability, that is, there should be some help to improve field dependence/independence persons in the teaching and learning situation. Providing a specific instructional activity could be one of the ways to help the learning of field dependence/independence persons to be enhanced because they have some difficulty in disembedding simple geometric figure from complex design. Thus, it is important to try to improve field dependence/independence using selected instructional activities for geometry students because disembedding figures is so necessary in geometry.
Van Hiele level

The van Hiele model of geometric thought proposes a means for identifying a students' level of intellectual maturity in geometry and suggests ways to help the student progress from one level to the next. Progress from one level to the next, asserts van Hiele (1959/1984), is more dependent upon instruction than on age or biological maturation, and types of instructional experiences can affect progress.

In a sample of 190 high school students, Corley (1990) investigated students' geometric thinking as related to geometric achievement, and determined whether certain instructional activities were related to students' levels of geometric thinking. The first semester traditional geometry instruction was conducted largely as determined by the Addison-Wesley textbook. During the second semester the treatment used instructional activities specially designed by the investigator to increase the student's van Hiele levels. The Van Hiele Geometry Test was administered three times during the school year. Students' final semester grades given by the geometry course instructors were the data used as a measure of student achievement in geometry. The results of the study

60
showed that "traditional" instruction used in the study did raise students' level of geometric thinking. The specially designed activities used in the second semester did not affect the van Hiele levels of the students who participated in the study.

Yusuf (1990) in his study investigated whether middle school students taught geometry concepts with Logo Based Instruction (LBI) would score significantly higher than students taught the same concepts by the teacher through lecture and paper-and-pencil activities. The sample consisted of 67 seventh and eighth graders, and they were divided into experimental and control groups. Students in the experimental group were taught the basic turtle commands of Apple II Logo. They were taught the concepts of points, rays, lines, and line segments through LBI, a set of tutorial modules written by the researcher. Achievement tests, Likert and Semantic Differential Scales, and interview sheets were developed and used to determine students' achievements, attitudes, and conceptualization of geometry concepts respectively. The results indicated that students in the experimental group had a better conceptualization of the geometry concepts after the LBI and also had significant increases in their van Hiele levels compared to the control group.
Fitzsimmons (1995) investigated the relationship between cooperative student pairs' van Hiele levels and success in solving specific three-dimensional geometric problems in the calculus course. The training laboratories consisted of nine hour-long sessions, constructed to bring the students through the five phases of learning. The control group studied the same topics in the traditional manner. The subjects were 76 students enrolled in second semester calculus at a private suburban college in New York State. Pretreatment data were gathered on the Scholastic Aptitude Test, high school grade point average, and a mathematics placement examination. Subjects were pretested on van Hiele levels and spatial visualization, and posttreatment scores were gathered on spatial visualization, area, and volume problems. The findings showed that there were significant differences in achievement favoring the experimental groups over the control group in the scores on a volume of a solid revolution problem on the final examination (cf. Ferrini-Mundy, 1987).

The purpose of a McClendon (1990) study was to evaluate a sixty-hour summer institute designed to teach geometry concepts to selected elementary in-service teachers. Activities were sequenced according
to the van Hiele model. A van Hiele level test was used to evaluate the level of understanding geometry, and an attitude survey was used to evaluate the attitude toward teaching geometry. The five phases of the van Hieles' attainment model were used to develop the activities to teach content topics. A comparison group was tested prior to and eight weeks following the treatment. A significant difference was found for participants in the Summer Institute as to improvement in the level of understanding in geometry and attitude toward the teaching of geometry.

Frerking (1994) examined the relationships among van Hiele levels, proof-writing achievement, and conjecturing by secondary geometry students. Fifty-eight students were taught using the inductive conjecturing method. Students were given the opportunities to conjecture about properties of geometric figures using Geometer's Sketchpad. The control class of 27 students were taught using traditional deductive approach. Van Hiele Geometry Test and the Entering Geometry Test were administered for pre- and posttest. Achievement in geometry was significant by related to abilities to write conjectures and to posttest van Hiele levels. Scores
on conjecturing and justification activities were affected by the design of the activity worksheets.

Swafford et al. (1997) investigated the effects on instruction of an intervention program designed to enhance teachers' knowledge of geometry and their knowledge of research on student cognition in geometry. The sample consisted of 49 middle-grade (4 - 8) teachers. The teachers participated in a 4-week program including a content course in geometry and a research seminar on van Hiele Theory. The pretest and posttest results showed significant increases in content knowledge and in van Hiele level.

Geddes and Fortunato (1993) suggest that "teachers need to have students spend more time on developing geometric concepts concretely and using multiple embodiments and principles in varied settings and not merely focus on practice involving algorithmic procedures" (p.217). The NCTM Standards also state that a student should be actively involved both mentally and physically in constructing his or her own mathematical knowledge (NCTM, 1989, p.17). Thus, the present study examined the use of special instructional activities to improve spatial visualization, field dependence/independence, van Hiele level, and achievement in geometry among middle school students.
Significance of the Study

From the review of the literature, it appears that no study has been conducted on the relative importance among spatial visualization, field dependence/independence, and van Hiele level and achievement in geometry among middle school students.

Spatial visualization abilities have a complex role in understanding geometry. Students are given little or no instruction in spatial visualization skill. In order to be successful at deductively proving what seems to be pictorially obvious, students must become aware and visually literate. Engaging students in experiencing and visualizing geometric constructs are important before moving on to prove geometric theorems.

Similarly, here in the U.S., the research and recommendations for reviving the geometry curriculum are following much the same path. The van Hiele model of geometric thought has had the potential to address some of the problems that beset the teaching and learning of geometry. Research has validated most of the principles behind the van Hiele theory, but there
have been repeated calls for further refinement of the characterization of the van Hiele levels.

Previous studies on field dependence/independence and visual learning were reviewed to look for linkages. Field dependence/independence could be a very important factor in learning from visuals as more and more visual materials for instruction are used in the teaching and learning situation, especially geometry. How field dependence/independence might relate to spatial visualization and van Hiele levels of reasoning and improve geometry achievement have not been intensively studied yet.

Thus, this study used selected instructional activities that provide strong visual elements creating learning situations with the potential to enhance visual skills, facilitate learning specific geometric concepts, and develop thinking process among middle school students.

The present research provides information about the relationship between spatial visualization ability, field dependence/independence, van Hiele levels, and geometry achievement among middle schools students. Also, this study investigated whether specially designed activities related to geometry can improve students' spatial visualization ability, field
dependence/independence, van Hiele levels, and geometry achievement.
CHAPTER  3

METHODS AND PROCEDURES

The primary objective of this study was to investigate the relative importance of the cognitive variables of spatial visualization ability, field dependence/independence, and van Hiele level of thought in predicting achievement in geometry of middle school students and to determine the effect of special instructional activities on improving these cognitive variables and achievement in geometry.

This chapter presents a comprehensive description of the methodology, including the design, procedures, data analysis, and the materials used in this study. The first section describes the sample for this study, and the second section describes the instruments. The third section explains the instructional activities, while the fourth section describes the design, procedures and the data analysis.
The study took place in one of the public middle schools in Franklin County. This school has 437 students, and is located in Columbus, Ohio. This school has held an important place in United States history. This school was founded in 1909 and it was the first junior high school in the United States of America.

**Table 3.1**
Description of All Students in the School by Grade, Sex, and Ethnicity

<table>
<thead>
<tr>
<th>Grade</th>
<th>Female</th>
<th>Male</th>
<th>Black</th>
<th>Asian</th>
<th>White</th>
<th>Hispanic</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Six</td>
<td>59</td>
<td>70</td>
<td>83</td>
<td>1</td>
<td>44</td>
<td>1</td>
<td>129</td>
</tr>
<tr>
<td>Seven</td>
<td>60</td>
<td>61</td>
<td>84</td>
<td>9</td>
<td>22</td>
<td>6</td>
<td>121</td>
</tr>
<tr>
<td>Eight</td>
<td>63</td>
<td>53</td>
<td>73</td>
<td>10</td>
<td>30</td>
<td>3</td>
<td>116</td>
</tr>
<tr>
<td>Special</td>
<td>51</td>
<td>20</td>
<td>50</td>
<td>-</td>
<td>20</td>
<td>1</td>
<td>71</td>
</tr>
<tr>
<td>Total</td>
<td>233</td>
<td>204</td>
<td>290</td>
<td>20</td>
<td>116</td>
<td>11</td>
<td>437</td>
</tr>
</tbody>
</table>

In Table 3.1, the "Special" grade means students from grade sixth, seventh, or eighth who were not promoted to the next grade because they did not achieve
a certain percentage in the school assessment. This group represents 16.2% of the students in the school. Of the 437 in the total number of students in the school, 290 (66.36%) were Black, 116 (26.54%) were White, and 31 (7.09%) were others.

This school serves a diverse academic, social, economic, and cultural population. The school's major goal is to enable all of its students to be successful in life by preparing them for high school.

**Sample**

The participants in this study were students (male and female) in the above middle school. The students in this school were from the middle social-economic-status. The average ages of the students were between 12 to 14 years. There were five classes in each grade. According to the Principal each class was assigned with mixed ability — high, average, and low. After discussion with the Principal and teachers, two intact classes were identified from each of 6th, 7th, and 8th grades for this study. In each grade, one class was assigned to be the experimental group and the other class was assigned to be the control group. Both groups had comparable socio-economic and ethnic
backgrounds as well as comparable mathematics grades, according to the teachers' grade book.

The sample studied consists of 137 students. Of the 137 in the total sample, 71 (51.8%) were male and 66 (48.2%) were female. The eleven students not participating were either absent on the days of the testing or transferred to other school. Table 3.2 illustrates a breakdown of the participants in the study.

Table 3.2
Number and Percentage of Participants in the Study

<table>
<thead>
<tr>
<th>Grade</th>
<th>Experimental N (%)</th>
<th>Control N (%)</th>
<th>Total N (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>21 (15.3)</td>
<td>22 (16.1)</td>
<td>43 (31.4)</td>
</tr>
<tr>
<td>7</td>
<td>27 (19.7)</td>
<td>25 (18.2)</td>
<td>52 (37.9)</td>
</tr>
<tr>
<td>8</td>
<td>20 (14.6)</td>
<td>22 (16.1)</td>
<td>42 (30.7)</td>
</tr>
<tr>
<td>Total</td>
<td>68 (49.6)</td>
<td>69 (50.4)</td>
<td>137 (100.0)</td>
</tr>
</tbody>
</table>
**Instrumentation**

Four instruments were used for data collection: the Middle Grades Mathematics Project (MGMP) *Spatial Visualization Test* (SVT), the *Group Embedded Figures Test* (GEFT), the *Van Hiele Geometry Test*, and a *Geometry Test* constructed by the researcher as a criterion measure of geometry achievement of the students.

**Spatial Visualization Test (SVT)**

The SVT consists of 32 multiple-choice items, each with five options. Each item consists of one correct answer and four distractors. The test is an untimed power test which in practice generally takes 20-30 minutes (Ben-Chaim, Lappan, & Houang, 1988). The types of representation used are two-dimensional flat views, three-dimensional corner views, and "mat plan" which includes the description of the base of a building by squares and numbers in each square to tell how many cubes are to be placed on that square, (e.g., see item 3 in Appendix A). Each type of representation is illustrated in sample items on the introductory page of the test.

The SVT includes items that deal with finding either flat or corner views of "buildings" given the
other view, adding and removing cubes, combining two solids together, and applying the ideas of a "mat plan" (refer to No. 2, 3, 8 and 29 from the test in Appendix A). The Cronbach's Alpha reliability coefficients for the MGMP Spatial Visualization Test from various groups of middle school students ranged from .72 to .86 (Ben-Chaim, Lappan, & Houng, 1988). The MGMP SVT test-retest effect was examined by Ben-Chaim et al. (1988), based on a sample of 73 middle school students, one class from each of grades 6, 7, and 8. The test was given twice within three weeks without any intervention. The test-retest reliability was .79 for each grade level.

**Group Embedded Figures Test (GEFT)**

The GEFT is a paper-and-pencil test developed by Oltman, Raskin, and Witkin (1977). This test was used to measure the degree of field dependence/independence of the subjects. This test consists of seven practice items and 18 test items that require the subject to locate and trace the outline of a simple geometric shapes in a more complex figure in which the simple shape has been obscured by extraneous lines and shading. The subjects' scores are calculated by summing the number of correct answers. Scores can range from 0 (extreme field dependence) to 18 (extreme
field independence). Scores are distributed into the
categories shown in Table 3.3.

Table 3.3
Categorization Scheme of GEFT Scores

<table>
<thead>
<tr>
<th>Cognitive Style</th>
<th>GEFT Score (# correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field-dependent</td>
<td>0 - 9</td>
</tr>
<tr>
<td>Intermediate</td>
<td>10 - 13</td>
</tr>
<tr>
<td>Field-independent</td>
<td>14 - 18</td>
</tr>
</tbody>
</table>

This test is a valid and reliable alternative to
individually administered measures of field dependence/
independence (Witkin et al., 1977). The Split-half
reliability coefficient on this test for a sample of
college students is reported as .82 in the test manual.
Although Witkin et al. presented no validity and
reliability data for younger than college-aged samples,
Flexer and Roberge (1983) reported a test-retest
reliability coefficient for a one-year interval of .78
and .79 for sixth and seventh graders.
**Van Hiele Geometry Test**

The test modified by Usiskin (1982) to assess van Hiele level based on the van Hieles' descriptions of the five levels of geometric thought was adapted for this study. The test contains 20 items (see Appendix B) which students can complete in about 25 minutes. The first five questions are designed to measure student achievement in level 1, recognition of geometric figures. Items six through ten measure knowledge of geometric properties, which reflect level 2. The next five items are designed to test level 3 by measuring students' understanding of relations between classes of figures. The next five items measure students' attainment in level 4. No questions to measure level 5 were included in this study because students in middle school have most likely not attained this level. Senk (1983) also indicated that the van Hieles' writings have not described level 5 behaviors clearly.

Usiskin (1982) reported that reliabilities of each of those five subsets was .56 for the first five levels using Kuder-Richardson Formula. Bobango (1987) also checked the test-retest reliability in a school different from the one in which the study took place. The resulting coefficient of stability was .60.
The test was graded according to the following rule: For subjects to be classified at a particular van Hiele level, they had to meet the "3 out of 5" criterion; that is, they have to complete three of the five problems at that and previous levels successfully and fail to complete three of the five problems at succeeding levels.

**Geometry Test**

The geometry test was constructed by the researcher and contained 25 multiple-choice items covering geometry facts and concepts, such as point, line, segment, diameter, angle, parallel, circle, similar, area, perimeter, volume, triangle, congruence, and symmetry — that are presented in most middle schools texts and curricula. The content classification and number of items in the geometry test were as shown in Table 3.4.

The student's score on this test was be the number of items answered correctly during the allocated 25-30 minutes time period for taking the test.
<table>
<thead>
<tr>
<th>Classification of Content</th>
<th>Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points and segments</td>
<td>2</td>
</tr>
<tr>
<td>Properties of triangles</td>
<td>2</td>
</tr>
<tr>
<td>Properties of quadrilaterals</td>
<td>2</td>
</tr>
<tr>
<td>Areas</td>
<td>3</td>
</tr>
<tr>
<td>Perimeter</td>
<td>2</td>
</tr>
<tr>
<td>Circles</td>
<td>2</td>
</tr>
<tr>
<td>Angle relations and measures</td>
<td>2</td>
</tr>
<tr>
<td>Congruent</td>
<td>2</td>
</tr>
<tr>
<td>Similarity</td>
<td>3</td>
</tr>
<tr>
<td>Circumference and area of circles</td>
<td>2</td>
</tr>
<tr>
<td>Volume</td>
<td>3</td>
</tr>
</tbody>
</table>

Consultations and discussions between the researcher and the classroom teachers were conducted concerning format, the wording, and the content validity of each item on the test. The method used to test the internal reliability of the instrument was to correlate scores of each item against the total test.
scores. For testing the reliability of the instrument, Cronbach's Alpha method was used.

Pilot Study

A pilot study was conducted to test the internal reliability of the geometry achievement test written by the researcher. The participants in the pilot study were sixth, seventh, and eighth grade middle school students at a public middle school in Columbus, Ohio different from the sample school. Three classes — one 6th grade, one 7th grade, and one 8th grade for a total of forty five students — were chosen as the participants in the pilot study. A stratified random sampling was used to select five above-average students, five average students, and five below-average students in each grade level. Stratified random sampling increases the likelihood of representativeness and ensures that any key characteristics of individuals in the population are included in the same proportions in the sample.

Validity. Two colleagues from mathematics education, one instructor from the Mathematics Department of The Ohio State University, and two instructors from Mathematics, Science, and Technology Education at the
Ohio State University who were considered to be knowledgeable about the materials were selected. Consultations and discussions between the researcher and these colleagues were conducted concerning the format, the wording, and the content validity of each item. This panel of experts agreed that the content of the instrument was valid for measuring geometry achievement with suggested revisions of several of the items.

Reliability. The next step of the pilot study was to test the internal reliability of the geometry test. The revised instrument was administered to the pilot participants. For testing the reliability of the instrument, Cronbach's Alpha method was used. The result of the reliability test for the overall instrument (see Table 3.5) was .77. This result indicated that the instrument was sufficiently reliable for measuring student geometry achievement.
Table 3.5
Reliability Analysis—Scale (Alpha)

<table>
<thead>
<tr>
<th></th>
<th>Scale Mean if Item Deleted</th>
<th>Scale Variance if Item Deleted</th>
<th>Corrected Item-Total Correlation</th>
<th>Alpha if Item Deleted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>18.7674</td>
<td>14.7065</td>
<td>.2338</td>
<td>.7638</td>
</tr>
<tr>
<td>Q2</td>
<td>18.7907</td>
<td>14.1694</td>
<td>.4281</td>
<td>.7545</td>
</tr>
<tr>
<td>Q3</td>
<td>18.7442</td>
<td>15.0997</td>
<td>.0768</td>
<td>.7697</td>
</tr>
<tr>
<td>Q4</td>
<td>18.7907</td>
<td>14.2647</td>
<td>.3878</td>
<td>.7565</td>
</tr>
<tr>
<td>Q5</td>
<td>18.9767</td>
<td>12.8328</td>
<td>.6822</td>
<td>.7334</td>
</tr>
<tr>
<td>Q6</td>
<td>18.9535</td>
<td>13.8549</td>
<td>.3727</td>
<td>.7556</td>
</tr>
<tr>
<td>Q7</td>
<td>18.8605</td>
<td>14.7420</td>
<td>.1399</td>
<td>.7693</td>
</tr>
<tr>
<td>Q8</td>
<td>18.7674</td>
<td>15.1827</td>
<td>.0222</td>
<td>.7725</td>
</tr>
<tr>
<td>Q9</td>
<td>18.7209</td>
<td>14.6822</td>
<td>.3628</td>
<td>.7604</td>
</tr>
<tr>
<td>Q10</td>
<td>18.8372</td>
<td>13.4729</td>
<td>.6227</td>
<td>.7423</td>
</tr>
<tr>
<td>Q11</td>
<td>19.0233</td>
<td>14.1185</td>
<td>.2674</td>
<td>.7629</td>
</tr>
<tr>
<td>Q12</td>
<td>18.9070</td>
<td>13.0864</td>
<td>.6632</td>
<td>.7368</td>
</tr>
<tr>
<td>Q13</td>
<td>18.9302</td>
<td>13.4950</td>
<td>.5026</td>
<td>.7470</td>
</tr>
<tr>
<td>Q14</td>
<td>18.8837</td>
<td>13.7719</td>
<td>.4513</td>
<td>.7511</td>
</tr>
<tr>
<td>Q15</td>
<td>18.8605</td>
<td>14.4086</td>
<td>.2530</td>
<td>.7629</td>
</tr>
<tr>
<td>Q16</td>
<td>18.8605</td>
<td>13.7896</td>
<td>.4703</td>
<td>.7503</td>
</tr>
<tr>
<td>Q17</td>
<td>18.9302</td>
<td>13.5426</td>
<td>.4870</td>
<td>.7481</td>
</tr>
<tr>
<td>Q18</td>
<td>18.8837</td>
<td>14.0100</td>
<td>.3702</td>
<td>.7560</td>
</tr>
<tr>
<td>Q19</td>
<td>19.0465</td>
<td>14.3111</td>
<td>.2025</td>
<td>.7676</td>
</tr>
<tr>
<td>Q20</td>
<td>19.0000</td>
<td>14.7143</td>
<td>.1047</td>
<td>.7737</td>
</tr>
<tr>
<td>Q21</td>
<td>18.8372</td>
<td>15.1395</td>
<td>.0141</td>
<td>.7754</td>
</tr>
<tr>
<td>Q22</td>
<td>19.0233</td>
<td>14.4042</td>
<td>.1867</td>
<td>.7685</td>
</tr>
<tr>
<td>Q23</td>
<td>19.0698</td>
<td>14.6379</td>
<td>.1156</td>
<td>.7737</td>
</tr>
<tr>
<td>Q24</td>
<td>18.9797</td>
<td>15.0233</td>
<td>.0224</td>
<td>.7786</td>
</tr>
<tr>
<td>Q25</td>
<td>18.7442</td>
<td>14.9092</td>
<td>.1730</td>
<td>.7662</td>
</tr>
</tbody>
</table>

Reliability Coefficients
N of items = 25

Alpha = .7677
**Instructional Activities**

The goal of the instructional activities in this study was to improve spatial visualization ability, field dependence/independence, van Hiele level, and achievement in geometry.

Bishop (1986) emphasizes that geometrical learning should be full of activities with containers of different shapes, tiling patterns, tessellations, paper folding, model building, sketching, and mapping. And these activities should be structured in ways that emphasize geometrical features, properties, and phenomena (p. 151). For example, ask students to look for cross sections of a solid, such as a rectangular solid (Fig. 1). Can they find a cross section that has the shape of a triangle?

![Fig. 3.1: Finding Cross Sections](image)

A problem like this encourages students to review the word triangle, to think about properties of triangle, and to see how triangles relate to other figures. The van Hieles formulated a structure of
thought levels and principles designed to help students gain insight into geometry (see Chapter 1). The use of instructional activities is strongly justified at the lower levels of thought to assist the student in progressing to higher levels of geometric understanding.

It is important for students to be able to visualize geometric constructions and to disembed simple geometric figures from complex designs in order to be successful with deductive reasoning in Euclidean geometry. Geometry is the branch of mathematics that connects mathematics with the real, physical world (Usiskin, 1997). Thus, there is a need to train and expose students to these activities in middle school so that they can succeed in the geometry course in high school.

The researcher selected and/or designed the instructional activities for the teachers to use during the 3-week treatment following the suggestions and examples from published research and published articles. The activities involve investigating geometric objects and properties to deepen students' geometric concepts. The activities offer students the opportunity to speculate, explore, criticize, and justify. The student also constructs various figures
using geoboard, models, cardboard strips, cubes or other materials.

The instructional activities were divided into six units as summarized in Table 3.6. Activity I consisted of three lessons, Activities II and IV consisted of four lessons each, Activities V and VI consisted of two lessons each, and Activity III consisted of one lesson.

Brief summaries of lessons (see Appendix D for complete descriptions) are as follow:

**Lesson 1** was designed as introductory activities to engage students in the idea of visualization, disembedding simple geometric figures from complex designs, and addressing concepts at the first van Hiele level. In this lesson both examples and nonexamples of geometric concepts are shown. The task was to recognize the basic geometric shapes.
Table 3.6
Description of Instructional Activities

<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson</th>
<th>Topics</th>
<th>SVA</th>
<th>FDI</th>
<th>VHL</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>• Basic geometric concepts</td>
<td>/</td>
<td>/</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Shape observation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>• Properties of triangles, Quadrilaterals</td>
<td>/</td>
<td>-</td>
<td>1,2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>• Spatial visualization</td>
<td>/</td>
<td>/</td>
<td>1,2,3</td>
</tr>
<tr>
<td>II</td>
<td>4</td>
<td>• Angle measurement/construction</td>
<td>/</td>
<td>/</td>
<td>1,2,3</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>• Alternate angles and corresponding angles</td>
<td>/</td>
<td>/</td>
<td>1,2,3,4</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>• Triangle angle sum</td>
<td>/</td>
<td>/</td>
<td>1,2,3</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>• Pythagorean Theorem</td>
<td>/</td>
<td>-</td>
<td>1,2,3</td>
</tr>
<tr>
<td>III</td>
<td>8</td>
<td>• Perimeter</td>
<td>/</td>
<td>/</td>
<td>1,2</td>
</tr>
<tr>
<td>IV</td>
<td>9</td>
<td>• Area of rectangles</td>
<td>/</td>
<td>/</td>
<td>1,2,3</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>• Area of triangles</td>
<td>/</td>
<td>/</td>
<td>1,2,3,4</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>• Area of parallelogram</td>
<td>/</td>
<td>/</td>
<td>1,2,3,4</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>• Area of circle</td>
<td>/</td>
<td>/</td>
<td>1,2,3,4</td>
</tr>
<tr>
<td>V</td>
<td>13</td>
<td>• Volume of rectangular box</td>
<td>/</td>
<td>/</td>
<td>1,2,3</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>• Volume of other shapes</td>
<td>/</td>
<td>/</td>
<td>1,2,3</td>
</tr>
<tr>
<td>VI</td>
<td>15</td>
<td>• Congruence</td>
<td>/</td>
<td>/</td>
<td>1,2,3,4</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>• Similarity</td>
<td>/</td>
<td>/</td>
<td>1,2,3,4</td>
</tr>
</tbody>
</table>
Lesson 2 was adapted from *Geometry in the Middle Grades*, NCTM Addenda Series, part of Activity 13B & 13C (1992, pp. 56–57). These activities enable students to think about shapes in terms of properties rather than by appearance only. The tasks were to identify figures, to characterize shapes in terms of properties, and to explore subclass relationships among quadrilaterals. Students should be able to visualize, as well as recognize and distinguish similarities and differences between objects/figures.

Lesson 3 was based on a unit entitled *Spatial Visualization*, developed as part of the Middle Grades Mathematics Project, Michigan State University. Students need to visualize by building, drawing, and evaluating three dimensional figures. These three kinds of tasks are used in various combinations throughout the worksheets. Lesson 3 also deals with the properties and relationships of points, lines, angles, surfaces, and solids. There are many concepts in geometry that cannot be recognized or understood, unless the students can visually perceive examples and identify figures and/or properties. Students need to delete mentally the extraneous things surrounding the object/figure and not be distracted by irrelevant visual stimuli. Making 3-D models from the 2-D
representations tests the student's ability to perceive spatial relationships and construct in three dimensions that which is seen in two dimensions.

Lesson 4 was adapted from Geometry in the Middle Grades, NCTM Addenda Series, part of Activity 3A & 3B (1992, pp. 28-29). This activity enables students to develop the concept of angle measurement and to develop an intuitive concept of angle by having students construct, rotate, and use the angle models in a variety of settings. This task requires students to identify the various angles formed from the intersection lines. Fine visual discrimination is needed as students learn to recognize different kinds of angles. Otherwise, students will not be able to pick out details from drawings or illustrations.

Lesson 5 (adapted from Geometry in the Middle Grades, NCTM Addenda Series, Activity 8A, 1992, p.49) requires students to identify certain angles (Level 1), see connections (Level 2), and deduce conclusions (Level 3). One task involves recognizing and applying the principle: If lines are parallel, alternate interior/corresponding angles are congruent (NCTM, p.43). The other task requires a two-step argument involving the transitive principle, that is, angle 1 ≅ angle 2, angle 2 ≅ angle 3, hence angle 1 ≅ angle 3.
This task also provides students with practice in visualization and recognizing spatial patterns. Students must scan the design and disregard irrelevant shapes and sizes while looking for certain angle. **Lesson 6** (adapted from *Geometry in the Middle Grades, NCTM Addenda Series*, Activity 8C, 1992, p. 51) encourages students to "read the grid" — to make observations and formulate conjectures that they can verify by examples from the grid. The tasks are to discover that (a) the sum of the angle measures of a triangle is equal to the measure of a straight angle and (b) the measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles. **Lesson 7** was adapted from *Geometry in the Middle Grades, NCTM Addenda Series*, part of Activity 5E (1992, pp. 38-39). The activity gives students experience in the use of inductive reasoning by having them gather data in a variety of geometric settings, find patterns in the data, and make generalizations. From the visual representations of the triangles, students should conjecture that $a^2 + b^2 = c^2$ (see Figure 3.2).
Students were also given real-world applications of the Pythagorean Theorem. This task required students to visualize and to pick out details from an illustration in order to understand ideas, concepts, and relationships.

Lesson 8 (adapted from Geometry and Visualization, NCTM, 1984) was designed to develop the concept of perimeter of geometric shapes which are drawn on grid paper and then to transfer to situations where they need to know "the distance around." Learning the concept of perimeter in this activity comes through visual experiences and the ability to recognize and discriminate among distracting irrelevant stimuli.
Lesson 9 was adapted from the Brooklyn College Project. This activity begins with Level 1 experiences on areas, namely, counting how many square inches cover a rectangle. Students need to visualize from the activity and be able to develop the concept of "area" as space inside, "measure of area" as how many units cover a figure, and a procedure for finding the area of rectangles — length times width.

Lesson 10 was adapted from the Brooklyn College Project. This activity leads students to discover a procedure for finding the area of a right triangle in terms of the area of a related rectangle, that is, "area = base x height/2." The question then is, "What can we do with other triangles?" Students need to decide which side of a triangle is to be the base, and then there are two possible situations that can occur — either the line perpendicular to the base and passing through the opposite vertex hits the base of the triangle (5a), or it lies outside the triangle and hits the line which includes the base, but not the base (segment) itself (5b). Students need to understand that either situation leads to the same formula for area.

Lesson 11 was adapted from the Brooklyn College Project. The task of the student is to discover
procedures for finding the area of a parallelogram by (a) using a grid and counting squares and (b) cutting off a right triangle and moving it to form a rectangle with the same base and height as the parallelogram. Students are guided to discover the second method, to explain it, and then to relate the parallelogram area rule to those for rectangle and right triangle.

Lesson 12 (adapted from Manual Math 105/106, Math Department, The Ohio State University) asks students to visualize an important relation between the area of a circle and its perimeter (circumference). To estimate the area of a circle students sequentially draw diameters through the circle in such a way as to make each new angle formed congruent to its predecessor. Students need to focus their attention on a figure and be able to disregard the extraneous things surrounding it.

Lesson 11 (adapted from Geometry and Visualization, NCTM, 1984) was designed for students to model a picture with cubes so that it could help students interpret the volume of objects correctly. Students can count the cubes in their models and then check the picture again. Sometimes students need to examine the cross section of an object. The task is to look at drawings of three-dimensional objects and find the
volume. The volume of a geometric figure refers to the amount of space the figure occupies or encloses. Lesson 14 has students look at solid geometric figures (cylinder & triangular prism), and visualize the shapes of the top and bottom of the figures. The task is to discover that the volume of the cylinder and triangular prism is found by simply multiplying the area of the base of the cylinder or prism by the height. Students need to experience both the concrete and abstract representations of volume.

Lesson 15 (adapted from Manual 105/106, Math Department, The Ohio State University) encourages students to use triangles in a tessellation and be able to observe congruent angles, congruent segments, and similar triangles. What are the various angles/shapes formed from the intersection of transversals with parallel lines? Students need to visualize, disembed the hidden figure, and to relate different kinds of angles/figures.

Lesson 16 (adapted from Geometry in the Middle Grades, NCTM Addenda Series, Activity 8B, 1992, pp.50) has students discover as many geometric ideas as possible in the grids by visualizing, disembedding hidden figures, and using van Hiele levels of geometric reasoning. The task is to read from the grid and
discover ideas such as alternate interior angles, corresponding angles, congruent angles, parallelograms, congruent triangles, trapezoids, hexagons, similar triangles, etc. This activity summarizes those concepts that have been introduced to students.

The instructional activities were designed to require approximately three weeks due to the fact that most middle schools in Columbus teach geometry for about three to four weeks.

The content validity of the instructional activities was judged by a group of experts which consisted of colleagues who are mathematics educators, or mathematicians in the Department of Mathematics at The Ohio State University. The instructional activities were also pilot-tested with 4 students. Following pilot testing, the instructional activities were revised and improved.

The activities range from those requiring little experience to those that maximize open-ended discovery. Besides enriching the student's thinking at his/her level, these activities should be moving him/her toward the next higher level of thinking. For example, for students at level 0, the aim of this instructional activities is to give them experiences oriented toward level 1 and higher. The activities also encourage
students to discover patterns, make conjectures, investigate and use models, materials, and manipulatives. Also, included are tasks designed to help students explore a topic further, independently or in groups.

Each lesson consists of a teachers' guide that includes introduction about the activity, objective(s) of the lesson, the materials needed, teachers' notes, and student activity sheets. Teachers' notes outline instructions for activities, mathematical concepts covered, and ways to involve students.

**Research Design**

This study was a quasi-experimental Nonequivalent Control Group Design (Tuckman, 1978). In this study the researcher did not assign subjects randomly to treatments. Instead, a total sample of six intact classes was used in this study.

Pre- and post-measures were administered to determine: (1) changes in spatial visualization ability scores; (2) changes in field dependence/independence scores; (3) changes in van Hiele Geometric scores; and (4) changes in geometry achievement.
**Procedures.** In November/December 1997 all students in the experimental and the control groups were given four pretests: *Spatial Visualization Test, Group Embedded Figures Test, Van Hiele Geometry Test,* and *Geometry Test.* All four tests were administered by classroom teachers and monitored by the investigator. The scores of these tests were used in analyzing relationships among spatial visualization, field dependence/independence, van Hiele levels, and achievement in geometry.

For the experimental group, all students received the instructional activities. The instructional activities were carried out by the three classroom teachers of the experimental groups. The treatment took approximately three weeks in the month of January 1998. The classroom teachers were provided with all the instructional materials. Prior to instruction, the researcher trained the teachers in a group and also met with the teachers individually to discuss the instructional activities, recommend instructional strategies, and present a more in-depth view of the geometrical concepts involved. The teachers agreed to follow strictly what had been written in the geometry lesson plans.
With my active involvement as a Chairman in the Community Advisor Board, Parent-Teacher Association, and other school activities with the teachers and students, I had good cooperation from the teachers, students, and the administrator in collecting my data. Teachers and students in the experimental group showed an encouraging attitude toward using the instructional activities.

Since three different teachers were involved in teaching experimental groups, the researcher was present in the classroom and observed the teaching during the instructional activities to make sure that all teachers carried out the instructional activities as the researcher wanted them to do, so that there would be consistency across all the three teachers.

Students in the experimental classes completed activities and exercises designed by the researcher following the suggestions and examples from published research and published articles addressing spatial visualization, field dependence/ independence, and van Hiele levels. Students worked either individually or in groups of two, three, or four, depending on the types of activities. The teacher played a major role in giving students directions for the day's tasks and in guiding discussions. Materials required students to
use an inductive approach throughout the treatment period, forming and testing conjectures regarding the properties of triangles and quadrilaterals and their relationships.

To encourage students to reflect upon their activities the teacher instructed students to keep a record in their notebooks of what they did each day. After completing the activities in class, the students constructed a general statement that describes the result of what they learned on a particular task to reflect on the work they had done. In the set of exercises, the teacher allocated time for the students to do as many problems as they could. Throughout the treatment period, the teacher and the researcher continued to discuss the instructional activities.

For the control groups no treatment was given. Students used their own textbook for geometry and no material was provided by the researcher. The main teaching tool was a textbook which presents three-dimensional concepts through two-dimensional pictures. The course of study was largely determined by the Macmillan/McGraw-Hill text entitled Mathematics Applications and Connections, Course 1-3.

When the instructional activities were completed, all students in both experimental and control groups
took four posttests in February 1998: retest of Spatial Visualization Test; retest of Group Embedded Figures Test; retest of Van Hiele Geometric Test; and retest of the Geometry Test.

**Data Analysis**

To analyze the data collected during the study, analyses of means, standard deviations, Pearson product-moment correlations, Spearman correlations, analyses of variance, ANCOVA, Multiple Regression, Tukey-b test for pairwise comparison procedure were conducted. A detailed description of the hypotheses tested and the statistical procedures used for each research question of the study will be given in the next chapter, followed by a description of the findings.

The analysis of covariance (ANCOVA) with a single covariate was used to analyze the effects of instructional intervention on spatial visualization, field dependence/independence, and geometry achievement. The pretest scores were used as a covariate to provide an adjustment for initial differences between the groups. This adjustment was made to increase the precision of results (Kerlinger, 1973).
The Kruskal-Wallis test of significance was used to compare the experimental group and the control group using the van Hiele level pre- and posttest data. All the analyses were carried out using Statistical Package for the Social Science (SPSS) programs.

Significance

The study is designed to analyze relationships between the cognitive factors — spatial visualization, field dependence/independence, and van Hiele level of geometric thought — and achievement in geometry and to determine the influence of instructional activities on improving the cognitive variables and achievement in geometry.

According to Mitchelmore (1975), appropriate instructional activities are the greatest need for development of practical geometric and spatial teaching programs. Sherman (1979) argues that "methods for improving spatial skill need to be devised, and their feasibility and advisability evaluated" (p. 26-27). Bishop (1983) also emphasized in his review of research on space and geometry that more training programs are needed to improve spatial visualization, field
dependence/independence, and van Hiele level of geometric thought.

There is a growing body of data from teachers that indicates that pupils can perform geometric tasks very well when the tasks are related to spatial abilities (Del Grande, 1990; Swafford et al., 1997). However, activities that involved disembedding simple geometric figures from complex designs in order to be successful with geometry do not seem to be available in the literature. Several researchers also found that instructional activities can be developed and helped to raise the van Hiele levels of thinking (Fitzsimmons, 1995; Fuys, Geddes, & Tischler, 1985; McBride, 1996; Usiskin, 1982). Thus, this study was a comprehensive effort to examine the effects of selected instructional activities on spatial visualization, field dependence/independence, van Hiele levels, and ultimately geometry achievement.
CHAPTER 4

DATA ANALYSIS AND RESULTS

This chapter presents the findings that correspond to the four research questions of this study. The first section discusses the data analysis and results which answer the questions regarding the van Hiele levels of geometric thought exhibited by middle school students. The second section addresses the relationships between spatial visualization, field dependence/independence, and van Hiele levels of thought. And the third presents the results relationships between spatial visualization, field dependence/independence, van Hiele levels, and achievement in geometry. The fourth section discusses the effect that selected instructional activities had on students' scores on the Spatial Visualization Test, Group Embedded Figures Test, Van Hiele Geometry Test, and Geometry Test.
Question 1: What van Hiele levels of geometric thought do middle school students exhibit?

In this study, students were assigned to van Hiele level if they meet the "3 out of 5" criterion; they have to complete three of the five problems at that and previous levels successfully and fail to complete three of the five problems at succeeding levels.

Table 4.1 shows the distribution into van Hiele levels for all students in grade sixth, seventh, and eighth who took the Van Hiele Geometry Test (VHGT) in November 1997. There were 5 of the 137 students who could not be classified according to the 3 out of 5 criterion. One satisfied VHGT at levels 1 and 3 but not at level 2. This student obtained a score of "1 out of 5" at level 2 and was assigned to level 2. Another three students were satisfied VHGT at levels 1, 2, and 4 but not at level 3. These students obtained a score of "1 out of 5" at level 3 and were assigned to level 3. One student was assigned to level 4 because this student was satisfied VHGT at levels 1, 3, and 4 but not at level 2 and obtained a score of "2 out of 5" at level 2.
Table 4.1

Numbers and Percentages of van Hiele Levels Among Middle School Students on Pretest of Van Hiele Geometry Test.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Level</th>
<th>0 (%)</th>
<th>1 (%)</th>
<th>2 (%)</th>
<th>3 (%)</th>
<th>4 (%)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
<td>15(11.0)</td>
<td>20(14.6)</td>
<td>8(5.8)</td>
<td>0(0.0)</td>
<td>0(0.0)</td>
<td>43(31.4)</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>11(8.0)</td>
<td>34(24.8)</td>
<td>5(3.7)</td>
<td>2(1.5)</td>
<td>0(0.0)</td>
<td>52(38.0)</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>6(4.4)</td>
<td>20(14.6)</td>
<td>10(7.3)</td>
<td>4(2.9)</td>
<td>2(1.4)</td>
<td>42(30.6)</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>32(23.4)</td>
<td>74(54.0)</td>
<td>23(16.8)</td>
<td>6(4.4)</td>
<td>2(1.4)</td>
<td>137(100.0)</td>
</tr>
</tbody>
</table>
Most of the middle school students who took the VHGT in fall 1997 were at Level 1. There were only two students at level 4 and those were from Grade 8. Table 4.1 shows that more than half of the middle school students in this study were at level 1 — able to identified geometric figures on the basis of their visual appearance as a whole — prior to using selected instructional activities. Students at this level can recognize a rectangle but do not readily recognize that the figure has right angles or parallel sides. On the other hand, 23.4% of the students were unable to recognize geometric figures by appearance alone.

**Spatial Visualization, Field Dependence/Independence & van Hiele Level**

**Question 2:** What is the relationship among spatial visualization, field dependence/independence, and van Hiele levels?

The null hypotheses associated with this question are:

**Ho 2.1** The correlation between spatial visualization and field dependence/independence is equal to zero.

**Ho 2.2** The correlation between spatial visualization and van Hiele levels is equal to zero.
Ho 2.3 The correlation between field dependence/independence and van Hiele levels is equal to zero.

To answer this question, the investigator used the scores from the pretest. The researcher will present the descriptive statistics for the independent variables and then analyze correlation coefficients relating the cognitive variables. The correlation coefficient is a measure of the strength of association between two variables. It reflects how closely scores on two variables go together. The scores of the subjects of the study for the spatial visualization (SVT) and field dependence/independence (GEFT) scores were examined in terms of means, standard deviations, minima and maxima as shown in Table 4.2.


Table 4.2
Means, Standard Deviations, Minima and Maxima of Spatial Visualization Test and Group Embedded Figures Test Among Middle School Students

<table>
<thead>
<tr>
<th>Variable</th>
<th>Grade</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minima</th>
<th>Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial Visualization Test (SVT)</td>
<td>6</td>
<td>4.65</td>
<td>1.77</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>5.33</td>
<td>2.97</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>7.84</td>
<td>4.08</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>All</td>
<td>5.94</td>
<td>3.37</td>
<td></td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>Group Embedded Figures Test (GEFT)</td>
<td>6</td>
<td>5.07</td>
<td>4.21</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>5.41</td>
<td>4.15</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>7.40</td>
<td>4.93</td>
<td>1</td>
<td>17</td>
</tr>
<tr>
<td>All</td>
<td>5.96</td>
<td>4.52</td>
<td></td>
<td>0</td>
<td>17</td>
</tr>
</tbody>
</table>

Relationships between spatial visualization, field dependence/independence, and van Hiele levels among middle school students were then examined. A Pearson product-moment correlation between spatial visualization and field dependence/independence was computed to see the strength of the correlation. Results of the inter-correlation analyses, shown in Table 4.3, indicate the correlation between
Table 4.3

Correlation Matrix: Spatial Visualization, Field Dependence/Independence, and Van Hiele Levels

<table>
<thead>
<tr>
<th>Variable</th>
<th>SVA</th>
<th>FDI</th>
<th>VHL</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVA</td>
<td>1.000</td>
<td>.245**</td>
<td>.313**</td>
</tr>
<tr>
<td>FDI</td>
<td></td>
<td>1.000</td>
<td>.281**</td>
</tr>
<tr>
<td>VHL</td>
<td></td>
<td></td>
<td>1.000</td>
</tr>
</tbody>
</table>

**. Correlation is significant at the 0.01 level (2-tailed).

spatial visualization ability (SVA) and field dependence/independence (FDI) is statistically significant. Thus, the null hypothesis, Ho 2.1 was rejected. Spatial visualization has a correlation coefficient of .245 with field dependence/independence (ρ < 0.01).

Spearman rank correlations were calculated to determine if there were significant correlations between van Hiele Level (VHL) and spatial visualization ability (SVA) and between VHL and FDI because of the
ordinal nature of the van Hiele levels. In the first pair (VHL and SVA), the Spearman rank correlation, $r = .313$, indicates a moderate relationship in the sample between the subjects' van Hiele levels and spatial visualization scores and it is statistically significant at the .01 level. The second pair (VHL and FDI) is statistically significant at the .01 level with the Spearman rank correlation, $r = .281$. Thus, both of the null hypotheses, Ho 2.2 and 2.3, were rejected. These results seem to indicate that there is a significant link between SVA and FDI, VHL and SVA, and between VHL and FDI as indicated by this part of the model from Chapter 1.

![Figure 4.1: Correlation Among Cognitive Variables](image-url)
Cognitive Variables and Achievement in Geometry

**Question 3:** What relationships exist among spatial visualization, field dependence/independence, van Hiele levels, and achievement in geometry?

The null hypotheses associated with this question are:

Ho 3.1 The correlation between spatial visualization and achievement in geometry is equal to zero.

Ho 3.2 The correlation between field dependence/independence and achievement in geometry is equal to zero.

Ho 3.3 The correlation between van Hiele level and achievement in geometry is equal to zero.

The relationships between the cognitive factors and achievement in geometry were examined in two ways. First, the relationships between the subject variables spatial visualization ability (SVA), field dependence/independence (FDI), van Hiele levels (VHL), and achievement in geometry were analyzed in a correlational study. The means and standards deviations for pretest achievement in geometry used in these correlations are listed in Table 4.4.
Table 4.4
Means, Standard Deviations, Minima and Maxima of Pretest Achievement in Geometry

<table>
<thead>
<tr>
<th>Variable</th>
<th>Grade</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minima</th>
<th>Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry Achievement</td>
<td>6</td>
<td>9.95</td>
<td>3.37</td>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>11.35</td>
<td>3.61</td>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>13.18</td>
<td>4.09</td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>All</td>
<td>11.51</td>
<td>3.90</td>
<td>3</td>
<td>22</td>
</tr>
</tbody>
</table>

Secondly, to determine the relative importance of the variables as predictors of geometry achievement, a multiple regression analysis was carried out.

Correlation Analyses of Cognitive Variables and Geometry Achievement

Results of the inter-correlation analyses are shown in Table 4.5. Since scores on the Spatial Visualization Test (SVT), Group Embedded Figures Test (GEFT), and Geometry Test (GT) are interval data, a Pearson product-moment correlation coefficient was computed to determine their relationship. In the first pair, a Pearson $r$ of .351 between SVT and GT shows that
spatial visualization ability and achievement in geometry are significant by correlated ($p < .01$). In the second pair, GEFT and GT, the Pearson $r = .566$ also shows that field dependence/independence and achievement in geometry is significant by correlated ($p < .01$).

Table 4.5
Correlation Matrix: Spatial Visualization, Field Dependence/Independence, Van Hiele Level of Geometry Reasoning with Geometry Achievement

<table>
<thead>
<tr>
<th>Variable</th>
<th>Independent Variable</th>
<th>Dependent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SVT</td>
<td>GEFT</td>
</tr>
<tr>
<td>SVT</td>
<td>1.000</td>
<td>.245**</td>
</tr>
<tr>
<td>GEFT</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>VHGT</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>GEOMETRY ACHIEVEMENT</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**. Correlation is significant at the 0.01 level (2-tailed)
For the relationship between van Hiele levels and achievement in geometry, there is a moderate association in the sample between van Hiele levels and geometry achievement with a Spearman rank correlation, \( r = .336 \) which is statistically significant at the .01 level.

Thus, geometry achievement has correlation coefficient of .351 with spatial visualization, .566 with field dependence/independence, and .336 with van Hiele levels of geometric reasoning, all statistically significance (\( p < .01 \)). A positive linear correlation between each of the cognitive variables and achievement in geometry indicated that students with high cognitive ability (SV, FDI, VHL) would also have higher achievement scores in geometry and those with lower cognitive ability would have lower scores in geometry.

In particular, the results of the correlation analyses in Table 4.5 indicate that the better achievers on the geometry test are likely to be those students who scored higher on GEFT. As mentioned in Chapter 3, the subjects' field dependence/independence scores are distributed according to scoring norms into three groups, that is, field dependent (FD), Intermediate (Int), and field independent (FI). Additional evidence of the correlation between field
dependence/independence and achievement in geometry is given by the results of the analysis shown in Table 4.6.

When the geometry achievement is compared across different field dependent/independent groups, the F-ratio of 13.342 is found to be significant at \( p < 0.05 \) (see Table 4.6). The results of the Tukey-b test show that the mean geometry achievement of the field independent group is significantly higher than that of the field dependent students. However, there is no significant difference in geometry achievement between the intermediates and the field independent group. These results demonstrate that generally the field independent students perform better on the geometry test than their field dependent peers.
### Table 4.6

One-Way Analysis of Variance for Geometry Achievement by Field Dependent/Independent Groups

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>343.797</td>
<td>2</td>
<td>171.898</td>
<td>13.342</td>
<td>.000</td>
</tr>
<tr>
<td>Within Groups</td>
<td>1726.437</td>
<td>134</td>
<td>12.884</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2070.234</td>
<td>136</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Multiple Range Test
Tukey-b Procedure

(* Denotes pairs of groups significantly different at the .05 level)

<table>
<thead>
<tr>
<th>Mean</th>
<th>FDI Group</th>
<th>FD</th>
<th>Int</th>
<th>FI</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.75</td>
<td>FD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.25</td>
<td>Int</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.00</td>
<td>FI</td>
<td></td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>
From Table 4.1, there were five van Hiele levels of geometric thought for all subjects who took the van Hiele geometry test. Level 0 is the lowest and level 4 is the highest following the modified van Hiele levels as defined by Usiskin, that is without level 5. The correlation analyses as shown in Table 4.6 also suggest that the more advanced van Hiele levels tend to have higher achievement in the geometry test. Further evidence of this finding is furnished by the results of analysis shown in Table 4.7.

As shown in Table 4.7, the one-way analysis of variance, using geometry achievement as the dependent variable and the van Hiele levels of reasoning as the independent variable, gives an F-ratio of 5.529 which is statistically significant at p < 0.05 level. The results of the Tukey-b test show that the mean geometry achievement of the higher van Hiele levels is significantly higher than those of the lower van Hiele levels. These results show that students with higher van Hiele level perform better in the geometry test. Table 4.6 and 4.7 reaffirm Usiskin's finding that there is a significant relationship between student van Hiele level as established by the van Hiele test and student achievement in geometry.
Table 4.7

One-way Analysis of Variance for Geometry Achievement by Van Hiele Levels

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>297.103</td>
<td>4</td>
<td>74.276</td>
<td>5.529</td>
<td>.000</td>
</tr>
<tr>
<td>Within Groups</td>
<td>1773.130</td>
<td>132</td>
<td>13.433</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2070.234</td>
<td>136</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Multiple Range Test
Tukey-b procedure

(*) Denotes pairs of groups significantly different at the .05 level

<table>
<thead>
<tr>
<th>Mean</th>
<th>VHL Group</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.27</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.88</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.31</td>
<td></td>
<td>2</td>
<td>*</td>
<td></td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>15.67</td>
<td></td>
<td>3</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.68</td>
<td></td>
<td>4</td>
<td>*</td>
<td></td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>
Multiple Regression Analyses of Achievement in Geometry

Of the three independent variables considered in this study, field dependence/independence was more strongly correlated with geometry achievement than either spatial visualization or van Hiele level of geometric reasoning. The strength of the field dependence/independence correlation might be attributed to the fact that the Geometry Test items involved considerable disembedding of simple geometric figures from complex designs, as do many problems in geometry. Students who are highly field independent will probably be better at disembedding figures and thus more successful in geometry.

To determine the amount of variance accounted for by the best linear prediction equation of the three predictor variables as well as to determine the amount of geometry achievement score variance accounted for by each predictor variable, multiple regression analyses was performed, using the geometry achievement score as dependent variable and spatial visualization, field dependence/independence, and van Hiele levels as independent variables.
The following is a model equation which relates a dependent variable $Y$ to three independent predictor variables $X_1$, $X_2$, and $X_3$:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3,$$

where, $Y =$ geometry achievement

$X_1 =$ spatial visualization

$X_2 =$ field dependence/independence

$X_3 =$ van Hiele Level

$\beta_0$ is the intercept (or constant), and $\beta_1$, $\beta_2$, and $\beta_3$ are slopes. To test whether or not the multiple regression model is valid, the researcher first stated the null hypothesis and the alternate hypothesis.

The null hypothesis $H_0$ is:

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$

The alternate hypothesis is:

$$H_1: \text{Not all the } \beta's \text{ are 0.}$$

If the null hypothesis $H_0$ is true, it implies that the regression coefficients are all zero and logically of no use in predicting the dependent variable. To test the null hypothesis that the multiple regression coefficients are all zero, the researcher applied the $F$ test, analysis of variance, and using the .05 level of significance (see Table 4.8).
Table 4.8 presents the results which have been found to be significant in the regression analysis.

**Table 4.8**

Multiple Regression Summary Table for Prediction of Geometry Achievement of Middle School Students

<table>
<thead>
<tr>
<th>Dependent Variable: Geometry Achievement (GT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables Entered: 1. Spatial Visualization (SVT)</td>
</tr>
<tr>
<td>2. Field Dependence/Independence (GEFT)</td>
</tr>
<tr>
<td>3. Van Hiele Level (VHGT)</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple R</td>
<td>.646</td>
</tr>
<tr>
<td>R Square</td>
<td>.417</td>
</tr>
<tr>
<td>Adjusted R Square</td>
<td>.404</td>
</tr>
<tr>
<td>Standard Error</td>
<td>3.0113</td>
</tr>
</tbody>
</table>

**Analysis of Variance**

<table>
<thead>
<tr>
<th></th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>3</td>
<td>864.182</td>
<td>288.061</td>
</tr>
<tr>
<td>Residual</td>
<td>133</td>
<td>1206.052</td>
<td>9.068</td>
</tr>
<tr>
<td>Total</td>
<td>136</td>
<td>2070.234</td>
<td></td>
</tr>
</tbody>
</table>

F = 31.767  
Significance F = .000

------------------------Variables in the Equation------------------------

<table>
<thead>
<tr>
<th>Variable</th>
<th>B</th>
<th>SE</th>
<th>Beta</th>
<th>t</th>
<th>Sig t</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Constant)</td>
<td>6.752</td>
<td>.606</td>
<td>11.144</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>GEFT</td>
<td>.433</td>
<td>.060</td>
<td>.502</td>
<td>7.272</td>
<td>.000</td>
</tr>
<tr>
<td>SVT</td>
<td>.215</td>
<td>.081</td>
<td>.181</td>
<td>2.593</td>
<td>.011</td>
</tr>
<tr>
<td>VHGT</td>
<td>.816</td>
<td>.309</td>
<td>.182</td>
<td>2.637</td>
<td>.010</td>
</tr>
</tbody>
</table>
The F statistic is statistically significant and lies in the rejection region. The null hypothesis that all the multiple regression coefficients are zero is therefore rejected. The alternate hypothesis H1 is accepted, meaning that not all the regression coefficients are zero.

As a guide regarding useful predictors, the researcher looked for t values well below −2 or above +2. In this study, the t's are 7.272, 2.637, and 2.593, so all independent variables meet the guideline. Table 4.8 shows that, field dependence/independence (GEFT) is the best single predictor of geometry achievement with a for t value equal to 7.272. The second best predictor is van Hiele level of geometric reasoning (VHGT) with a t value equal to 2.637. Spatial visualization (SVT) is the third best predictor of geometry achievement. Both VHGT and SVT are more marginal than GEFT.
**Effect of Selected Instructional Activities**

**Question 4:** Do students who receive selected instructional activities show significantly greater improvement on the Spatial Visualization Test, GEFT, Van Hiele Geometry Test, and Geometry Test than students in a control group?

The null hypotheses associated with this question are:

Ho 4.1 There is no significant difference between the two groups (Experiment and Control) in improvement on Spatial Visualization Test scores.

Ho 4.2 There is no significant difference between the two groups (Experiment and Control) in improvement on GEFT scores.

Ho 4.3 There is no significant difference between the two groups (Experiment and Control) in improvement on Van Hiele Geometry Test scores.

Ho 4.4 There is no significant difference between the two groups (Experiment and Control) in the improvement on Geometry Test scores.
A nonequivalent control group design was used to collect data. To answer question 4, an analysis of covariance (ANCOVA) was used.

**Treatment Effects on Spatial Visualization**

Table 4.9 provides pre- and posttest means and standard deviations of the MGMP *Spatial Visualization Test* (SVT) scores by grade and by experimental and control groups.

**Table 4.9**

Means and Standard Deviation of Spatial Visualization Pre- and Posttest

<table>
<thead>
<tr>
<th>Grade</th>
<th>Pretest</th>
<th></th>
<th>Posttest</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard Deviation</td>
<td>Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Experimental</td>
<td>4.23</td>
<td>1.69</td>
<td>11.18</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>4.05</td>
<td>1.68</td>
<td>5.19</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Experimental</td>
<td>4.75</td>
<td>1.86</td>
<td>9.13</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>3.96</td>
<td>1.88</td>
<td>5.20</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Experimental</td>
<td>7.00</td>
<td>4.20</td>
<td>8.17</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>7.48</td>
<td>4.40</td>
<td>8.62</td>
</tr>
<tr>
<td>All</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Experimental</td>
<td>5.76</td>
<td>3.28</td>
<td>10.07</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>5.09</td>
<td>3.26</td>
<td>6.26</td>
</tr>
</tbody>
</table>
The MGMP SVT means from Table 4.9 are used in Figure 4.2 to show profiles for pretest-posttest by experimental and control groups.

![Graph showing pretest-posttest profiles](image)

Figure 4.2: Profiles for Pretest-Posttest on Spatial Visualization Test scores

To answer the question whether students in the experimental group using selected instructional activities achieve significantly greater improvement on spatial visualization scores compared to students in the control group who do not use the instructional activities, an analysis of covariance was used. The independent variable was use or no-use of the instructional activities, and the dependent variable
was the spatial posttest, with the spatial pretest as the covariate.

Table 4.10 presents the result for an analysis of covariance. The ANCOVA shows a main effect for instructional activities, $F(1,134)=63.88$, $p < .05$, indicating significant difference on the improvement scores in spatial visualization between the experimental and the control groups.

Table 4.10

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate</td>
<td>667.555</td>
<td>1</td>
<td>667.555</td>
<td>186.261</td>
<td>.000</td>
</tr>
<tr>
<td>Main Effects</td>
<td>228.951</td>
<td>1</td>
<td>228.951</td>
<td>63.882</td>
<td>.000</td>
</tr>
<tr>
<td>Model</td>
<td>1162.609</td>
<td>2</td>
<td>581.305</td>
<td>162.196</td>
<td>.000</td>
</tr>
<tr>
<td>Residual</td>
<td>480.252</td>
<td>134</td>
<td>3.584</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1642.861</strong></td>
<td><strong>136</strong></td>
<td><strong>12.080</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To further examine the data for differences between the two groups, the adjusted mean scores of the achievement posttest of the two groups were determined.
Table 4.11 provides a summary of the adjusted means of the experimental and control groups of subjects.

Table 4.11

Means and Standard Deviations for Experimental and Control Groups on Pre- and Posttest Spatial Visualization

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Covariate (Pretest)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>70</td>
<td>67</td>
</tr>
<tr>
<td>Mean</td>
<td>5.7571</td>
<td>5.0896</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>3.2812</td>
<td>3.2647</td>
</tr>
<tr>
<td>Dependent Variable (Posttest)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>70</td>
<td>67</td>
</tr>
<tr>
<td>Mean</td>
<td>10.0714</td>
<td>6.2687</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2.9553</td>
<td>2.8740</td>
</tr>
<tr>
<td>Adjusted Means</td>
<td>9.8054</td>
<td>6.4621</td>
</tr>
</tbody>
</table>

There were spatial visualization scores for 70 subjects in the experimental group and 67 in the control group. The mean for the experimental group was 5.7571 (SD=3.2812) compared to the control group mean of
5.0896 (SD=3.2647). The posttest means for both groups increased from the pretest, with the experimental group showing the greater increase. On the posttest, the mean of the experimental group was 10.0714 (SD= 2.9553), and 6.2687 (SD= 2.8740) for the control group. Table 4.11 shows that the adjusted mean of the experimental group was significantly higher than the adjusted mean of the control group.

Based on the results in Table 4.10 and Table 4.11, the hypothesis that there was no significant difference, at the p < .05 level, between the experimental and control groups was rejected. In other words, the results of the analysis of the effects of the instructional activities indicated that students in experimental group showed significantly higher on the Spatial Visualization Test than those in the control group.
Treatment Effects on Field Dependence/Independence

Table 4.12 provides pre- and posttest means and standard deviations of the GEFT scores by grade and by experimental and control groups.

Table 4.12
Means and Standard Deviations for GEFT

<table>
<thead>
<tr>
<th>Grade</th>
<th>Pretest</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Experimental 4.50</td>
<td>4.01</td>
</tr>
<tr>
<td></td>
<td>Control      5.66</td>
<td>4.43</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Experimental 5.63</td>
<td>3.95</td>
</tr>
<tr>
<td></td>
<td>Control      5.00</td>
<td>4.37</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Experimental 7.21</td>
<td>5.08</td>
</tr>
<tr>
<td></td>
<td>Control      7.62</td>
<td>4.86</td>
</tr>
<tr>
<td>All</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Experimental 5.89</td>
<td>4.46</td>
</tr>
<tr>
<td></td>
<td>Control      6.03</td>
<td>4.61</td>
</tr>
</tbody>
</table>
GEFT means from Table 4.12 are used in Figure 4.3 to show profiles for pretest-posttest by experimental and control groups.

![Graph showing pretest-posttest means for experimental and control groups.](image)

Figure 4.3: Profiles for Pretest-Posttest on GEFT scores

To answer the question whether students in the experimental group using instructional activities achieve significantly greater change in GEFT scores compared to students in the control group who do not use the instructional activities, an analysis of covariance was used. The independent variable was use or non-use of the instructional activities, and the dependent variable was the GEFT posttest, with the GEFT pretest as the covariate.

127
The results of the ANCOVA is presented in Table 4.13.

Table 4.13

Effects of Instructional Activities on GEFT

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate</td>
<td>1729.866</td>
<td>1</td>
<td>1729.866</td>
<td>638.555</td>
<td>.000</td>
</tr>
<tr>
<td>Main Effects</td>
<td>155.818</td>
<td>1</td>
<td>155.818</td>
<td>57.518</td>
<td>.000</td>
</tr>
<tr>
<td>Model</td>
<td>1869.599</td>
<td>2</td>
<td>934.779</td>
<td>345.060</td>
<td>.000</td>
</tr>
<tr>
<td>Residual</td>
<td>363.010</td>
<td>134</td>
<td>2.709</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2232.569</td>
<td>136</td>
<td>16.416</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The ANCOVA table (Table 4.13) shows a main effect for instructional activities, $F(1,134) = 57.518$, $p < .05$, indicating significant differences in the improvement on field dependence/independence between the experimental and control groups.

To further examine the data for differences between the two groups, the adjusted mean scores on the posttest of the two groups were determined.
Table 4.14 provides a summary of the adjusted means for the two groups of subjects.

Table 4.14
Means and Standard Deviations for Experimental and Control Groups on Pre- and Posttest GEFT

<table>
<thead>
<tr>
<th>Test</th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (Pretest)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>70</td>
<td>67</td>
</tr>
<tr>
<td>Mean</td>
<td>5.8857</td>
<td>6.0299</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.4642</td>
<td>4.1080</td>
</tr>
<tr>
<td>Dependent Variable (Posttest)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>70</td>
<td>67</td>
</tr>
<tr>
<td>Mean</td>
<td>8.8857</td>
<td>6.8657</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>3.7669</td>
<td>4.1080</td>
</tr>
<tr>
<td>Adjusted Means</td>
<td>8.8657</td>
<td>6.7865</td>
</tr>
</tbody>
</table>

Table 4.14 shows that the mean for the treatment group was 5.8857 (SD= 4.4642) compared to the control group mean of 6.0299 (SD=4.1080). The posttest means for both groups increased from the pretest, with the experimental group showing the greater increase. Table 4.14 shows that the adjusted mean of the experimental group was significantly higher than the adjusted mean of the control group.
Based on the results in Table 4.13 and Table 4.14, the hypothesis that there was no significant difference, at the p < .05 level, in improvement on field dependence/independence between the experimental and control groups was rejected. In other words, the selected instructional activities had a significant effect on field dependence/independence for the students in this study.

**Treatment Effect on van Hiele Level of Geometric Reasoning**

Table 4.15 shows the number and percent of students at level 0 through level 4 for all students in grade sixth, seventh, and eighth who took the Van Hiele Geometry Test (VHGT) in February 1998. These data illustrate there were gains in van Hiele levels of geometric thought. The greatest change from pre-test (see Table 4.1) to posttest (see Table 4.15) was in the decrease in number of students at level 1 and the increase in number of students at level 2.
Table 4.15
Numbers and Percentages of van Hiele Levels Among Middle School Students on Posttest of Van Hiele Geometry Test

<table>
<thead>
<tr>
<th>Grade</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N (%)</td>
<td>N (%)</td>
<td>N (%)</td>
<td>N (%)</td>
<td>N (%)</td>
<td>N (%)</td>
</tr>
<tr>
<td>6</td>
<td>13(9.5)</td>
<td>19(13.9)</td>
<td>11(8.0)</td>
<td>0(0.0)</td>
<td>0(0.0)</td>
<td>43(31.4)</td>
</tr>
<tr>
<td>7</td>
<td>8(5.8)</td>
<td>26(19.0)</td>
<td>15(11.0)</td>
<td>3(2.2)</td>
<td>0(0.0)</td>
<td>52(38.0)</td>
</tr>
<tr>
<td>8</td>
<td>4(2.9)</td>
<td>17(12.5)</td>
<td>13(9.4)</td>
<td>6(4.4)</td>
<td>2(1.4)</td>
<td>42(30.6)</td>
</tr>
<tr>
<td>Total</td>
<td>25(18.2)</td>
<td>62(45.4)</td>
<td>39(28.4)</td>
<td>9(6.6)</td>
<td>2(1.4)</td>
<td>137(100)</td>
</tr>
</tbody>
</table>

To answer the question whether subjects in the experimental group using instructional activities achieved significantly greater change in van Hiele levels compared to subjects in the control group who
did not use instructional activities, a Kruskal-Wallis Test was performed on the data. The subjects were sorted by group and by change in van Hiele level from pre- to posttest. After posttesting, all subjects either remained at the same level or were assigned to a level one or two above their pretest level. Table 4.1 and Table 4.15 did not indicate individual students' change in van Hiele levels. Thus, the researcher constructed Table 4.16 and Table 4.17 to show the students' changes in level from pretest to posttest for experimental and control groups.

Table 4.16

Comparison of Students' Pre- and Post—Van Hiele Levels (Experimental, Control)

<table>
<thead>
<tr>
<th>Pre</th>
<th>Pre</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>(5,20)</td>
<td>(20,9)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>(8,25)</td>
<td>(17,1)</td>
<td>(1,0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>(11,10)</td>
<td>(2,0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>(4,2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(2,0)</td>
</tr>
</tbody>
</table>
As shown in Table 4.16, the ordered pair (5,20) under the level 0 column indicates that there were five students in the experimental group and twenty students in the control group who remained the same from pretest to posttest. The ordered pair (17,1) under level 2 indicates that seventeen students in the experimental group and one in the control group moved from level 1 to level 2. These data show that all the students either showed gains or remained the same from pretest to posttest.

Table 4.17 shows the number of subjects remaining at the same level or gaining one or two levels by the end of the treatment period, with the corresponding percentages.

<table>
<thead>
<tr>
<th>van Hiele Level</th>
<th>Group</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Control</td>
<td></td>
</tr>
<tr>
<td>No Gain</td>
<td>30 (42.86%)</td>
<td>57 (85.07%)</td>
<td></td>
</tr>
<tr>
<td>Gained 1 Level</td>
<td>39 (55.71%)</td>
<td>10 (14.93%)</td>
<td></td>
</tr>
<tr>
<td>Gained 2 Levels</td>
<td>1 (1.43%)</td>
<td>0 (0.0%)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>70 (100.0%)</td>
<td>67 (100.0%)</td>
<td></td>
</tr>
</tbody>
</table>
The analysis of change in van Hiele levels between pretest and posttest for the experimental group revealed that more than half of the subjects (55.71%) gained a level compared to 42.86% showing no gain, while the opposite situation occurred in the control group. Only 10 of 67, or 14.93%, gained one level in the control group, while 85.07% showed no gain. Only one experimental subject gained two levels, beginning at Level 1 and achieving Level 3 on the posttest.

To test the hypothesis Ho 4.3, a Kruskal-Wallis Test statistic was calculated on the data. The procedure requires converting all changes in van Hiele levels to ranks. Table 4.18 shows the sum of the ranks for the treatment and control groups, the expected sums and standard deviations if there were no difference between the two groups, and the mean scores of the groups.
Table 4.18

Rank Change in van Hiele Levels by Group

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Sum of Scores</th>
<th>Expected under Ho</th>
<th>SD under Ho</th>
<th>Mean Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>70</td>
<td>377.97</td>
<td>413.97</td>
<td>45.71</td>
<td>5.40</td>
</tr>
<tr>
<td>Control</td>
<td>67</td>
<td>309.97</td>
<td>345.97</td>
<td>45.71</td>
<td>4.63</td>
</tr>
</tbody>
</table>

χ² = 18.637

Table 4.16 provides the result of the Kruskal-Wallis Test with χ² = 18.637, df = 1, indicates a significant difference (p < .05) between treatment and control groups on subjects' change in rank on van Hiele levels from pretest to posttest.

**Treatment Effect on Geometry Achievement**

Table 4.19 provides pre- and posttest means and standard deviations on the *Geometry Test* (GT) scores by grade and by experimental and control groups.
Table 4.19
Means and Standard Deviations for Achievement in Geometry

<table>
<thead>
<tr>
<th>Grade</th>
<th>Pretest</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td>9.91</td>
<td>3.51</td>
</tr>
<tr>
<td>Control</td>
<td>9.90</td>
<td>3.69</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td>9.95</td>
<td>3.51</td>
</tr>
<tr>
<td>Control</td>
<td>9.80</td>
<td>3.37</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td>12.58</td>
<td>4.72</td>
</tr>
<tr>
<td>Control</td>
<td>13.85</td>
<td>3.21</td>
</tr>
<tr>
<td>All</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td>11.90</td>
<td>3.94</td>
</tr>
<tr>
<td>Control</td>
<td>11.10</td>
<td>3.83</td>
</tr>
</tbody>
</table>

The Geometry Test means from Table 4.19 are used in Figure 4.4 to show profiles for pretest-posttest improvement by the experimental and control groups.
Figure 4.4: Profiles for Pretest-Posttest on Geometry Test scores

To answer the question whether students in the experimental groups using instructional activities achieved significantly greater improvement on Geometry Test scores compared to students in the control group, an analysis of covariance was used. The independent variable was use or non-use of the instructional activities, and the dependent variable was the geometry posttest, with the geometry pretest as the covariate.
Table 4.20 provides the summary for the analysis (ANCOVA).

Table 4.20

Effects of Instructional Activities on Geometry Achievement

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate</td>
<td>949.062</td>
<td>1</td>
<td>949.062</td>
<td>226.670</td>
<td>.000</td>
</tr>
<tr>
<td>Main Effects</td>
<td>213.222</td>
<td>1</td>
<td>213.222</td>
<td>50.925</td>
<td>.000</td>
</tr>
<tr>
<td>Model</td>
<td>1267.586</td>
<td>2</td>
<td>633.793</td>
<td>151.372</td>
<td>.000</td>
</tr>
<tr>
<td>Residual</td>
<td>561.056</td>
<td>134</td>
<td>4.187</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1828.642</td>
<td>136</td>
<td>13.446</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The ANCOVA table (see Table 4.20) shows a main effect for instructional activities, $F(1,134) = 50.925$. The null hypothesis was tested using an analysis of covariance ($p < .05$). The results indicate significant differences in the improvement scores on geometry between the experimental and control groups.

To further examine the data for differences between the two groups, the adjusted mean scores on the achievement posttest of the two groups were determined.
Table 4.21 provides a summary of the adjusted means for the two groups of subjects.

Table 4.21

Means and Standard Deviations for Experimental and Control Groups on Pre- and Posttest Geometry

<table>
<thead>
<tr>
<th>Test</th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Covariate (Pretest)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>70</td>
<td>67</td>
</tr>
<tr>
<td>Mean</td>
<td>11.9000</td>
<td>11.1045</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>3.9495</td>
<td>3.8381</td>
</tr>
<tr>
<td><strong>Dependent Variable (Posttest)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>70</td>
<td>67</td>
</tr>
<tr>
<td>Mean</td>
<td>15.5429</td>
<td>12.4925</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>3.3780</td>
<td>3.3092</td>
</tr>
<tr>
<td>Adjusted Means</td>
<td>15.5029</td>
<td>12.4525</td>
</tr>
</tbody>
</table>

The pretest mean for the experimental group was 11.9000 (SD= 3.9495) compared to the control group mean of 11.1045 (SD=3.8381). The posttest means for both groups increased from the pretest, with experimental group showing the greater increase. Table 4.21 shows that the adjusted mean of the experimental group was
significantly higher than the adjusted mean of the control group.

Based on the results in Table 4.20 and Table 4.21, the hypothesis that there was no significant difference, at the $p < .05$ level, between the two groups was rejected. In other words, the results of the analysis of the effects of the instructional activities indicated that students in the experimental group showed significantly greater improvement on geometry achievement than students in the control group.

**Summary of Data Analysis**

The results of the study seem to confirm that the entry level of middle school students were at level 1 (Fuys et al., 1988). There was significant correlation between spatial visualization and field dependence/independence, between spatial visualization and van Hiele levels, and between field dependence/independence and van Hiele level ($p < .01$). The results also shows that there was a significant correlation between cognitive variables and geometry achievement. The findings of this study indicate that among middle school students grade 6 through 8 using the selected instructional activities, there is an increase in
achievement of cognitive variables and geometry. Thus, it is possible to improve students' spatial visualization ability, field dependence/independence, and van Hiele level of geometric thinking, and ultimately increased geometry achievement in a natural classroom setting by providing selected instructional activities.
CHAPTER 5
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This chapter attempts to synthesize the results of the study which were reported in the previous chapters in an effort to interpret more deeply the present study. In the first section of this chapter, a brief summary of the research is presented which explains about this study and its findings. The second section discusses major conclusions. The third section discusses the implications for educational issues and the fourth section discusses the recommendations for further research.

Summary

Purpose of the Study

This study had four related purposes: (1) to determine the van Hiele levels of reasoning exhibited by middle school students, (2) to study the relationships among spatial visualization, field dependence/independence, and van Hiele levels of
thought, (3) to examine the relationships between spatial visualization ability, field dependence/independence, van Hiele levels, and achievement in geometry among middle school students, and (4) to analyze the effect of special instructional activities involving spatial visualization tasks, disembedding simple geometric figures from complex design tasks, and van Hiele levels of geometric thought on students' cognitive abilities and achievement in geometry.

**Research Design**

This study took place during the school year at a public middle school. It took place in a natural school setting. The student population in this middle school was approximately 437 students. In this study one experimental class and one control class from each of grades 6, 7, and 8 were selected. Four instruments were used for data collection: The Middle Grades Mathematics Project *Spatial Visualization Test*, *Group Embedded Figures Test*, *Van Hiele Geometry Test*, and the *Geometry Test* developed for this study.

Pretests were given in November/December 1997 and the posttests were completed in February 1998. During January 1998, the classroom teachers used the selected instructional activities (please refer to Chapter 3 for full description) for the experimental groups.
The van Hiele levels of the middle school students were determined using scores from the pretest of the *Van Hiele Geometry Test*. Correlations among spatial visualization, field dependence/independence, van Hiele levels, and geometry achievement were calculated on the pretest data. Analyzing posttest data, with the pretest as covariate, the effect of using instructional activities on SVA, FDI, VHL, and achievement were determined (please refer to Chapter 4).

**Findings**

The questions dealing with the research themes guiding this investigation were addressed as follows:

1. **What van Hiele levels of reasoning do middle school students exhibit?**

To answer this question, the researcher used students' van Hiele levels before treatment. This research question was answered in Table 4.1 in Chapter 4. Most of the middle school students in this study were at Level 1 (Recognition). They were able to identify geometric figures on the basis of their visual appearance as a whole, but properties were not distinguished. About 23.4% of the students were at Level 0 (Prerecognition) and were still unable to recognize geometric shapes based on visual appearance.
There were only two students at Level 4 and those were from grade 8.

2. What are the relationships among spatial visualization, field dependence/independence, and van Hiele levels of thought?

This research question centered on the correlation among the cognitive variables. A significant positive correlation existed between spatial visualization (SVA) and field dependence/independence (FDI) for the total sample of 137 students (see Table 4.3). Rank correlations between spatial visualization and van Hiele level (VHL) were also positive and significant for the total sample (see Table 4.3).

A significant positive correlation also existed between field dependence/independence and van Hiele level (see Table 4.3). Correlation between spatial visualization and van Hiele level was higher than either the correlation between FDI and SVA, or between FDI and VHL. This seems to indicate that spatial visualization is more closely related to van Hiele levels prior to using selected instructional activities.
3. What relationships exist between spatial visualization, field dependence/independence, van Hiele levels, and achievement in geometry among middle school students?

Results of the inter-correlation analyses are shown in Table 4.5. A significant positive correlation was found for field dependence/independence and achievement in geometry and between spatial visualization and achievement in geometry for the total sample. There was a moderate association in the sample between van Hiele levels and geometry achievement. For the students in this study field dependence/independence was more closely related with geometry achievement than either spatial visualization or van Hiele level.

Figure 5.1 is a model of the significant variables in this study and how they relate to each other as discussed in Chapter 1.
Figure 5.1 Relationship Between Cognitive Variables and Geometry Achievement
4. Do students who receive selected instructional activities show significantly greater improvement on the Spatial Visualization Test, Group Embedded Figures Test, Van Hiele Geometry Test, and a Geometry Test than students in a control group?

The following findings are related to the result of the effects of the instructional activities involving spatial visualization, disembedding simple geometric figures from complex designs, and van Hiele levels of geometric reasoning. The results of the analysis of the effects of the instructional activities indicated that students in the experimental group showed significantly higher improvement scores on the spatial visualization test than in the control group (please see Table 4.10). There was also a significant difference in students' improvement scores in GEFT between the experimental and control groups.

The experimental subjects' change in rank on van Hiele levels from pretest to posttest was significantly different from the control subjects' change (see Table 4.16). There was also a significant difference in students' improvement in geometry achievement between the experimental and control groups.
Conclusions and Discussion

Why is it so difficult for students to learn and understand geometry concepts? This is a question with which many geometry teachers have grappled. Hoffer (1981, p. 11) wrote, "Each year we ask many of our first year students at the University of Oregon to list the mathematics subjects or topics that they liked best and topics they liked least in their precollege classes. Although several subjects were 'favorites,' the subject that was almost universally disliked was geometry in high school." Making geometry enjoyable for students seems to be a universal dilemma.

In this research, the analysis of the data gathered and presented in the preceding chapter suggest a number of conclusions. The findings of this study prior to the instructional intervention suggest that among middle school students there is a trend toward an increase in van Hiele levels (as measured by CDASSG VHGT) with an increase in grade level. This observation might be expected because of the increased maturity and background of the subjects. This result is consistent with other findings of significant grade effects for this cognitive variable (Fuys et al., 1988; van Hiele, 1957; Usiskin, 1982).
For both the *Spatial Visualization Test* and the *Group Embedded Figures Test*, the mean scores increased by grade level. This might imply that students' ability increases with time in both these skill areas. In this study field dependence/independence was more strongly correlated with geometry achievement than was either spatial visualization or van Hiele levels of geometry reasoning. The regression analysis confirmed that field dependence/independence was the best single predictor of achievement in geometry. No previous research was found that investigated the relationship between field dependence/independence and achievement in geometry for middle school students. However, these results are in accordance with previous studies of achievement in other school subjects (Moore & Dwyer, 1991; Roach, 1985, Vaidya & Chansky, 1980).

The current study produced evidence to support the position that selected instructional activities improved subjects' spatial visualization, field dependence/independence, van Hiele levels, and ultimately achievement in geometry. The experimental group after using the selected instructional activities scored significantly better than the control group on all three cognitive variables. Based on these results, it seems possible to provide instructional activities
that benefit students more than the traditional
textbook experiences.

This study appears to support Ben-Chaim's (1982)
claim that sixth, seventh, and eighth grade students
profited from instructional activities involving
spatial visualization tasks. The effect of selected
instructional activities to increase spatial
visualization skills provides evidence to support the
notion that these skills are teachable and can be
learned (Ben-Chaim et al., 1988, p.66).

The results of this study also support Cronbach
and Snow's (1977) claim that field dependence/
independence is an ability, that is, there was a
significant difference in students' improvement scores
in GEPT between the experimental and control groups.

It is also interesting to note for the van Hiele
level of geometric reasoning there was an increasing
from level to level within each of the three grades in
this study after the treatment. This result is in
accordance with previous studies of van Hiele levels.
For example, the van Hieles designed approximately 70
lessons to move students from one level to another. It
also supports the van Hieles' claim that the levels are
not developmental and are influenced by instruction.
The majority of the subjects in this study began at
Level 0 and 1, thus the selected instructional
activities help students achieve higher levels by focussing on those level. Based on these results, it seems possible to provide the selected instructional activities in a natural classroom setting so as to raise students' levels of reasoning.

Geometry is identified as area in which eighth graders in the United States do less well than their counterparts in other countries. Collier (1998, p.387) stated, "We are described as teaching 'how to do' rather than 'how to understand'." In this study, selected instructional activities — involving hands-on experiences, such as model building, visualization, disembedding, and developing levels of thinking — have an impact on students' learning and enhance achievement in geometry for middle school students. These activities also help students with specific skills, such as integrating several concepts and finding hidden figures in a pattern.

To conclude, the selected instructional activities in this study did improve the cognitive variables — spatial visualization, field dependence/independence, and van Hiele level — and ultimately improved the geometry achievement, as illustrated in Figure 5.2.
Figure 5.2 The Influence of Instructional Activities
Implications and Recommendations

The following implications and recommendations are based on the investigator's interpretation of the analysis of data collected in this study, on the conclusions reached, and on the review of the related literature. General and specific implications are discussed in terms of their educational impact and action needed. Recommendations will be made regarding possible extension of this study and future research in the area of spatial visualization, field dependence/independence, and van Hiele levels of geometric thought.

Implications for Teaching and Learning

The common belief is that the more a teacher knows about a subject's cognitive abilities (spatial visualization, field dependence/independence, van Hiele Level in this study) and the way students learn, the more effective that individual will be in nurturing mathematical understanding.

The results of the present study demonstrate that using certain instructional activities can have significant positive effects upon cognitive behaviors of students in learning geometry. It has been recommended in the Standards (NCTM, 1989) that
instructional activities in geometry classrooms should provide perspectives from which students can analyze and solve problems and interpret an abstract representation easily.

The review of the related literature in Chapter 2 indicated the existence of evidence to support the notion that spatial visualization ability, field dependence/independence, van Hiele levels of geometric reasoning are an important consideration in most engineering, technical-scientific fields, and especially in the study of science and mathematics. It is also obvious that geometry is needed by anyone who works with physical objects, such as carpenters, plumbers, sculptors, and designers (Usiskin, 1997). Thus, the results of this study suggest that teachers need to be aware of these cognitive factors while planning lessons and remediation.

The additional finding is that middle school students can benefit significantly from the instructional activities. Mathematics educators need to structure the activities to include spatial visualization, according to the developmental level of the child, and to emphasize disembedding simple figures from complex designs. Improving geometry achievement requires mathematics educators' best efforts to upgrade classroom instruction and the use of a variety of
teaching materials. The rapid pace of technological progress necessitates a revised set of priorities for geometry instruction. To improve their understanding of geometry and their ability to solve problems, students need the benefit of instruction that emphasizes practical experiences in solving geometry problems.

The following examples are offered to illustrate the kinds of features that should be included in instructional activities:

**Spatial Visualization**

Spatial visualization plays a vital role in the understanding of geometric shapes and concepts. If the students are having trouble with visualization, they will experience frustration and failure when trying to recognize figures and patterns in geometry. For example, in this study one of the activities to improve spatial visualization asked students to first build a solid from cubes, copy the drawing, count the number of cubes used in the drawing, and then check the count from the solid, as shown in Figure 5.3. We can also ask students to add/take away cube(s) and then draw the new solid. This activity helps the student learn to interpret the 2-D representations of 3-D objects — an
important skill in visualization. The activity suggests that having good visual spatial visualization abilities would be a definite advantage in the study of geometry to lead students through investigations into concepts such as volume and area.

Figure 5.3: Illustration for Spatial Visualization Activity

Field Dependence/Independence

The analytic ability that is required on geometry achievement tests involves disembedding and developing strategies that depend on reorganizing and
restructuring information. For example, an illustration like Figure 5.4 asks students to identify geometric shapes and ideas in the grid. The student who is looking for a parallelogram must scan the design and disregard irrelevant shapes while looking for the parallelogram in the midst of a complex design. This activity helps students differentiate an object, shape, or form from its background, a skill that is needed to improve geometry achievement as shown in the research.

![Diagram of a grid with points labeled A, B, C, and T.]

Figure 5.4: Illustration for Field Dependence/Independence Activity

Van Hiele Level of Geometric Reasoning

The van Hiele model of thinking, designed to help students gain insight into geometry, uses five levels
to describe student behaviors. The van Hiele approach to learning and teaching geometry focuses on developing student's insight and higher levels of thinking. For example, the illustration in Figure 5.5 engages students in identifying certain angles (Level 1), seeing connections (Level 2), and deducing conclusion (Level 3).

Figure 5.5: Illustration for Van Hiele Level of Geometric Reasoning
In this study, it was found that middle school mathematics teachers successfully taught sixteen lessons on instructional activities to engage students in the idea of visualization, disembedding simple geometric figures from complex designs, and development through the van Hiele levels. These activities also help students learn to recognize a diagram that may contain the information needed for the solution of a problem.

Another implication is the timing of the instruction (Ben-Chaim, 1982). The Second International Mathematics Study data indicated that U.S. students in the eighth and twelfth grade scored at the 25th percentile or below in geometry (McKnight, Travers, & Dossey, 1985). The performance of U.S. students in the area of geometry was below the international average (TIMSS Report, 1996).

In the present study, all three grade levels — sixth, seventh, and eighth — gained significantly from the instructional activities. This suggest that middle school is an optimal time for teaching spatial visualization, disembedding figures from complex designs, and van Hiele levels of geometric reasoning tasks.
It is also important that teacher education programs reflect the significance of spatial visualization, field dependence/independence, and van Hiele levels of geometry reasoning and their relationship to achievement in geometry. Preservice teachers of mathematics need to be aware of the important cognitive variables at work in the classroom. Teacher education programs should promote the use of activities to enhance spatial visualization, field dependence/independence, and van Hiele levels especially for middle school students.

**Recommendations For Future Research**

Recommendations regarding the extension of this study and future research in the area of spatial visualization, field dependence/independence, van Hiele levels of geometric thought are as follows:

1. In this study, the presence of the researcher in the experimental classroom and the active involvement of the researcher in the school might increase the *Hawthorne effect*. Since the implications of the results obtained in this study are important, thus it is recommended that the study should be replicated with other samples of middle school students.
students to verify the generalizability of the results.

2. Students can actively participate in geometry learning by working together to review concepts and work in small groups to test out conjectures. Although many of the activities in this study involve cooperative learning, this study did not look at how cooperative learning influenced in geometry achievement. Thus, it is recommended that future studies examine the effects of cooperative learning activities on students' geometry achievement.

3. This study only used the *Van Hiele Geometry Test* (VHGT) to determine the van Heile level of students. It is recommended that students be interviewed to identify their van Hiele level and also to identify particular points that students do not understand and then test whether there is a significant correlation between students' van Heile levels of geometric thought as determined by the VHGT and the levels determined by interview evaluators.

4. It is recommended that a comparison be made between schools within a county or district
with different demographics in order to discover other possible sources (e.g. learning geometry in a language which is different from their home language) and determinants of differences in cognitive achievement.

5. This study focused on the van Hiele level of middle school geometry students. However, further research might examine/determine the van Hiele level of the middle school geometry teacher.

6. Further research is required to elucidate the nature and characteristics of those underachieving students in geometry with poor attitudes.

7. Other than SV, FDI, and VHL, several types of individual difference variables were possible factors related to achievement in geometry. It is recommended that other variables such as quantitative aptitude, reading comprehension, and previous knowledge of geometry be studied to examine their role in geometry achievement.
APPENDIX A

SPATIAL VISUALIZATION TEST
SPATIAL VISUALIZATION TEST

Do not write on this test booklet.
Read questions carefully.
Select the answer to the question.
Mark your answer on the answer sheet.

Example  A  B  C  D  E

Be sure to fill the circle completely.
Erase completely when necessary.
Mark only in the response circles provided.
Make no stray marks on the answer sheet.
Stop: Wait for instructions.

Copyright 1981 MSU Mathematics Department
Do these sample items and then wait for further instructions.

This is an example of the mat plan of a building. The number in each square tells how many cubes are to be placed on that square.

Use the information in the mat plan to answer the two sample items.

**Sample Item 1**
This is a corner view of the building above. Which corner was it drawn from?

A B C D
FRONT-RIGHT BACK-RIGHT BACK-LEFT FRONT-LEFT

Sample 1. A B C D

Sample 2
These are the views of the same building, when seen straight on from the sides. Which is the FRONT VIEW?

A B C D

Sample 2. A B C D

STOP: Wait until you are told to begin.
1. You are given a picture of a building drawn from the FRONT-RIGHT corner. Find the RIGHT VIEW.

![Diagram of a building with the FRONT-RIGHT corner labeled]

A B C D E

2. You are given a picture of a building drawn from the FRONT-RIGHT corner. Find the BACK VIEW.

![Diagram of a building with the FRONT-RIGHT corner labeled]

A B C D E

3. You are given the mat plan of a building. Find the RIGHT VIEW.

![DIagram of a building with the FRONT-RIGHT corner labeled]

A B C D E
4. You are given a picture of a building drawn from the FRONT-RIGHT corner. Find the LEFT VIEW.

5. You are given the RIGHT VIEW of a building. Find the LEFT VIEW.

6. You are given the mat plan of a building. Find the BACK VIEW.
7. You are given the **front** plan of a building. Find the **front view**.

```
    3
   1 1
  2 1
FRONT

A  B  C  D  E
```

8. You are given the **back** view of a building. Find the **front view**.

```
FRONT VIEW

A  B  C  D  E
```

9. You are given the **front** view of a building. Find the **back view**.

```
FRONT VIEW

A  B  C  D  E
```
10. How many cubes are needed to build this rectangular solid?

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>18</td>
<td>24</td>
<td>26</td>
<td>36</td>
<td>52</td>
</tr>
</tbody>
</table>

11. You are given the BASE, FRONT VIEW, and RIGHT VIEW of a building. Find the mat plan that can be completed to fit the building.

BASE

FRONT VIEW

RIGHT VIEW

A

B

C

D

E

12. How many cubes are needed to build this rectangular solid?

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>36</td>
<td>47</td>
<td>60</td>
<td>72</td>
<td>94</td>
</tr>
</tbody>
</table>
13. You are given the BASE, FRONT VIEW, and RIGHT VIEW of a building. Find the mat plan that can be completed to fit the building.

BASE  FRONT VIEW  RIGHT VIEW

14. How many cubes touch the indicated cube face to face?
15. You are given the BASE, FRONT VIEW, and RIGHT VIEW of a building. Find the mat plan for the building that uses the greatest number of cubes and also fits the given base and views.

![Base, Front View, Right View Diagrams]

A

\[
\begin{array}{c}
2 \\
3 \\
1 \\
\end{array}
\]

B

\[
\begin{array}{c}
1 \\
2 \\
1 \\
\end{array}
\]

C

\[
\begin{array}{c}
2 \\
3 \\
1 \\
\end{array}
\]

D

\[
\begin{array}{c}
1 \\
3 \\
1 \\
\end{array}
\]

E

\[
\begin{array}{c}
2 \\
1 \\
3 \\
\end{array}
\]

16. You are given the BASE, FRONT VIEW, and RIGHT VIEW of a building. Find the mat plan for the building that uses the least number of cubes and also fits the given base and views.

![Base, Front View, Right View Diagrams]

A

\[
\begin{array}{c}
3 \\
2 \\
1 \\
\end{array}
\]

B

\[
\begin{array}{c}
3 \\
2 \\
1 \\
\end{array}
\]

C

\[
\begin{array}{c}
1 \\
2 \\
3 \\
\end{array}
\]

D

\[
\begin{array}{c}
1 \\
1 \\
3 \\
\end{array}
\]

E

\[
\begin{array}{c}
1 \\
1 \\
3 \\
\end{array}
\]
17. How many cubes touch the indicated cube face to face?

```
     A B C D E
    1 2 3 4 5
```

18. If a cube were added to the shaded face of the given building, what would the new building look like?

```
A
B
C
D
E
```

19. If the shaded cubes were removed from the given building, what would the new building look like?

```
A
B
C
D
E
```
20. Find the view from the **FRONT-RIGHT** corner.

21. Find another view of the first building.

22. Which of these buildings can be made from the two pieces given?
23. If the shaded cubes were removed from the given building, what would the new building look like?

24. Find another view of the first building.
25. Find the view from the **BACK-RIGHT** corner.

![Diagram of building views](image)

26. If a cube were added to each shaded face of the given building, what would the new building look like?

![Diagram of building views](image)
27. Find the view from the FRONT-LEFT corner.

28. Which of these buildings can be made from the two pieces given?

29. Find another view of the first building.
30. Find another view of the first building.

31. Find another view of the first building.

32. Find another view of the first building.
September 11, 1997

Noraini Idris  
1893-B Fiesta Court  
Columbus, OH 43229

Dear Noraini:

I am very happy to give you permission to use the Spatial Visualization Test. You certainly may copy it for your research use. In addition, you may use the parts of the Middle Grades Mathematics Project unit on Spatial Visualization that are useful to you in your research.

I wish you well in your work and look forward to seeing what you find in your research.

Yours truly,

Glenda Lappan  
Professor

\[ \phi \mu ^2 \]
APPENDIX B

VAN HIELE GEOMETRY TEST
VAN HIELE GEOMETRY TEST

Directions

Do not open this test booklet until you are told to do so.

This test contain 20 questions. It is not expected that you know everything on this test.

When you are told to begin:

1. Read each question carefully.

2. Decide upon the answer you think is correct. There is only one correct answer to each question. Mark your answer on the answer sheet.

3. Do not mark anything on this test booklet.

4. If you want to change an answer, completely erase the first answer.

5. If you need another pencil, raise your hand.

Wait until your teacher says that you may begin.

Copyright © 1980 by The University of Chicago. This test may not be reproduced without the permission of the CDASSG project at the University of Chicago, Zalman Usiskin, Director.
1. Which of these are squares?

(A) K only.
(B) L only.
(C) M only.
(D) L and M only.
(E) All are squares.

2. Which of these are triangles?

(A) None of these are triangles.
(B) V only.
(C) W only.
(D) W and X only.
(E) V and W only.

3. Which of these are rectangles?

(A) S only.
(B) T only.
(C) S and T only.
(D) S and U only.
(E) All are rectangles.
4. Which of these are squares?

(F) [Rectangle]
(G) [Square]
(H) [Parallelogram]
(I) [Rhombus]

(A) None of these are squares.
(B) G only
(C) F and G only
(D) G and I only
(E) All are squares.

5. Which of these are parallelograms?

(J) [Parallelogram]
(M) [Rectangle]
(L) [Rhombus]

(A) J only
(B) L only
(C) J and M only
(D) None of these are parallelograms.
(E) All are parallelograms.

6. PQRS is a square.

Which relationship is true in all squares?

(A) PR and RS have the same length.
(B) QS and FR are perpendicular.
(C) PS and QR are perpendicular.
(D) PS and QS have the same length.
(E) Angle Q is larger than angle R.
7. In a rectangle GHJK, GJ and HK are the diagonals.

Which of (A) – (D) is not true in every rectangle?

(A) There are four right angles.
(B) There are four sides.
(C) The diagonals have the same length.
(D) The opposite sides have the same length.
(E) All of (A) – (D) are true in every rectangle.

8. A rhombus is a 4-sided figure with all sides of the same length.

Here are three examples.

Which of (A) – (D) is not true in every rhombus?

(A) The two diagonals have the same length.
(B) Each diagonal bisects two angles of the rhombus.
(C) The two diagonals are perpendicular.
(D) The opposite angles have the same measure.
(E) All of (A) – (D) are true in every rhombus.
9. An isosceles triangle is a triangle with two sides of equal length. Here are three examples.

Which of (A) – (D) is true in every isosceles triangle?

(A) The three sides must have the same length.
(B) One side must have twice the length of another side.
(C) There must be at least two angles with the same measure.
(D) The three angles must have the same measure.
(E) None of (A)-(D) is true in every isosceles triangle.

10. Two circles with centers P and Q intersect at R and S to form 4-sided figure PRQS. Here are two examples.

Which of (A)-(D) is not always true?
(A) PRQS will have two pairs of sides of equal length.
(B) PQRS will have at least two angles of equal measure.
(C) The lines PQ and RS will be perpendicular.
(D) Angles P and Q will have the same measure.
(E) All of (A)-(D) are true.
11. Here are two statements.

Statement 1: Figure F is a rectangle.

Statement 2: Figure F is a triangle.

Which is correct?

(A) If 1 is true, then 2 is true.
(B) If 1 is false, then 2 is true.
(C) 1 and 2 cannot both be true.
(D) 1 and 2 cannot both be false.
(E) None of (A)-(D) is correct.

12. Here are two statements.

Statement S: \( \triangle ABC \) has three sides of the same length.

Statement T: In \( \triangle ABC \), \( \angle B \) and \( \angle C \) have the same measure.

Which is correct?

(A) Statements S and T cannot both be true.
(B) If S is true, then T is true.
(C) If T is true, then S is true.
(D) If S is false, then T is false.
(E) None of (A)-(D) is correct.
13. Which of these can be called rectangles?

(A) All can.
(B) Q only
(C) R only
(D) P and Q only
(E) Q and R only

14. Which is true?

(A) All properties of rectangles are properties of all squares.
(B) All properties of squares are properties of all rectangles.
(C) All properties of rectangles are properties of all parallelograms.
(D) All properties of squares are properties of all parallelograms.
(E) None of (A)-(D) is true.

15. What do all rectangles have that some parallelograms do not have?
(A) opposite sides equal
(B) diagonals equal
(C) opposite sides parallel
(D) opposite angles equal
(E) none of (A)-(D)
16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.

![Diagram of a right triangle with additional constructions]

From this information, one can prove that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common. What would this proof tell you?

(A) Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.

(B) In some but not all right triangles, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.

(C) In any right triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.

(D) In any triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.

(E) In any equilateral triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.

Property S: It is a square.

Property R: It is a rectangle.

Which is true?

(A) D implies S which implies R.

(B) D implies R which implies S.

(C) S implies R which implies D.

(D) R implies D which implies S.

(E) R implies S which implies D.
18. Here are two statements.

I. If a figure is a rectangle, then its diagonals bisect each other.

II. If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

(A) To prove I is true, it is enough to prove that it is true.
(B) To prove II is true, it is enough to prove that I is true.
(C) To prove II is true, it is enough to find one rectangle whose diagonals bisect each other.
(D) To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.
(E) None of (A)-(D) is correct.

19. In geometry:

(A) Every term can be defined and every true statement can be proved true.
(B) Every term can be defined but it is necessary to assume that certain statements are true.
(C) Some terms must be left undefined but every true statement can be proved true.
(D) Some terms must be left undefined and it is necessary to have some statements which are assumed true.
(E) None of (A)-(D) is correct.
20. Examine these three sentences.

(1) Two lines perpendicular to the same line are parallel.
(2) A line that is perpendicular to one of two parallel lines is perpendicular to the other.
(3) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?

(A) (1) only
(B) (2) only
(C) (3) only
(D) Either (1) or (2)
(E) Either (2) or (3)
Ms. Noraini Idris  
333 Arps Hall  
The Ohio State University  
1945 North High Street  
Columbus, OH 43210

Dear Ms. Idris:

In response to your e-mails of August 11th and September 23rd, I am happy to give permission for you to copy and use the van Hiele test developed by the CDASSG project in your dissertation.

Best wishes for success.

Sincerely,

Zalman Usiskin  
Professor of Education
APPENDIX C

GEOMETRY TEST
GEOMETRY TEST

DIRECTION:

DO NOT WRITE ON THIS TEST BOOKLET.
READ QUESTIONS CAREFULLY.
SELECT THE ANSWER TO THE QUESTION.
MARK YOUR ANSWER ON THE ANSWER SHEET.

Example:
How many faces do a triangle has?
A. 6           C. 9
B. 3           D. 12
1. How many faces does this figure have?

A. 4  
B. 6  
C. 8  
D. 12

2. Which pair of figures is most likely congruent?

A. 1 and 3  
B. 1 and 4  
C. 2 and 3  
D. 2 and 4
3. What is the area of the following figure?

A. 29 ft$^2$
B. 81 ft$^2$
C. 90 ft$^2$
D. 45 ft$^2$

4. Which line segment names a diameter of circle?

A. CE
B. AB
C. DE
D. FE
5. Which angle in this figure is acute?

A. $\angle EDC$
B. $\angle AED$
C. $\angle DCB$
D. $\angle CBA$

6. What is the area of the shaded region?

A. $56 \text{ ft}^2$
B. $44 \text{ ft}^2$
C. $76 \text{ ft}^2$
D. $54 \text{ ft}^2$
7. Which of these is an isosceles triangle?

A.  
\[ \begin{array}{c}
5 \text{ in} \\
3 \text{ in} \\
\end{array} \]

B.  
\[ \begin{array}{c}
5 \text{ in} \\
5 \text{ in} \\
7 \text{ in} \\
\end{array} \]

C.  
\[ \begin{array}{c}
2 \text{ in} \\
2 \text{ in} \\
2 \text{ in} \\
\end{array} \]

D.  
\[ \begin{array}{c}
3 \text{ in} \\
5 \text{ in} \\
7 \text{ in} \\
\end{array} \]

8. What is the volume of water that will fill a rectangular trough that is 7 feet long by 5 feet wide by 3 feet deep?

A. 105 ft$^3$

B. 135 ft$^3$

C. 100 ft$^3$

D. 120 ft$^3$
9. Smith wants to paint the four walls and ceiling of his room. How many square feet will he cover with paint if the room measures: 12 feet long by 8 feet wide by 10 feet high?

A. 592 square ft.
B. 960 square ft.
C. 472 square ft.
D. 120 square ft.

10. Which sides of this figure are parallel?

A. QP and PO
B. RQ and QP
C. OM and MR
D. OM and RQ
11. Triangles ACB and QPR are congruent. Which statement is true about these triangles?

A. CB = PR  
B. AC = PR  
C. BA = QP  
D. AC = RQ

12. What is the perimeter of this shape?

A. 47.5 mm  
B. 58.5 mm  
C. 144.0 mm  
D. 180.0 mm
13. ABCD is a square and PQRS is a rectangle. If \( AB = 8 \text{ cm} \), what is the area of the shaded region?

\[ \text{Area} = 10 \text{ cm}^2 \]

A. \( 10 \text{ cm}^2 \)  
B. \( 28 \text{ cm}^2 \)  
C. \( 36 \text{ cm}^2 \)  
D. \( 64 \text{ cm}^2 \)

14. The volume of the cylinder is \( \_ \_ \_ \_ \).

\[ \text{Volume} = 168\pi \]

A. \( 168\pi \)  
B. \( 504\pi \)  
C. \( 126\pi \)  
D. \( 184\pi \)
15. Which two points should be connected in order to form an obtuse angle with UW?

A. U and Z  
B. U and X  
C. U and Y  
D. U and W

16. If \( \angle PQS \) measures \( 22^\circ \) and \( \angle TQR \) measures \( 35^\circ \), find the measure of \( \angle SQT \).

A. \( 57^\circ \)  
B. \( 123^\circ \)  
C. \( 112^\circ \)  
D. \( 68^\circ \)
17. Triangles ABC and XYZ are similar. What is the length of YZ?

A. 3 ft
B. 5 ft
C. 7 ft
D. 10 ft

18. Triangle PQR is a right triangle. What is the length of PQ?

A. 4 in.
B. 8 in.
C. 16 in.
D. 64 in.

19. What is the volume of a cylinder that has a base area of 78 square centimeters and a height of 12 centimeters?

A. 468 cm³
B. 312 cm³
C. 936 cm³
D. 808 cm³
20. What is the base of a parallelogram that has a height of 15 inches and an area of 360 square inches?
   A. 12 inches
   B. 24 inches
   C. 54 inches
   D. 121 inches

21. Which of these is a measure of an obtuse angle?
   A. 27°
   B. 57°
   C. 87°
   D. 97°

22. A quadrilateral MUST be a rectangle if it has
   A. two pair of opposite sides congruent
   B. one pair of parallel sides
   C. no pair of opposite sides congruent
   D. no pair of parallel sides

23. What is the area of triangle ABC, if AD = 4 inches, AC = 8 inches, BD = 3 inches, and AB = 5 inches?
   A. 6 square inches
   B. 10 square inches
   C. 12 square inches
   D. 20 square inches
24. Find the circumference of the circle shown. Round your answer to the nearest whole number.

A. 226 m  
B. 254 m  
C. 28 m  
D. 46 m

![Diagram of a circle with a radius of 9 m]

25. Mr. Michael has a square garden. He used 250 feet of fencing around the outside of the garden. What are the dimensions of his garden?

A. 60.25 ft.  
B. 500 ft.  
C. 1000 ft.  
D. 62.50 ft.
LESSON 1 - INTRODUCTION

Introduction:

This activity assesses what students already know about some basic geometric shapes such as rectangle, square, triangle, parallelogram, trapezoid, and circle.

Objective:

(1) To discover what aspects of their past experiences they will use to complete these activities.
(2) To recall the names, shapes, and components of rectangle, square, triangle, parallelogram, and circle.

Materials:

Student notebook, copies of the worksheets for each student.

Notes:

Before getting started with Activity 1, have student identify shapes in the classroom - i.e., the clock is a circle, the chalkboard is a rectangle, etc. Ask the student to list these objects and then their shapes. Then, tell the students the objectives for the day.
Activity 1

Look at the pictures, identify geometric shapes in them, and name the shapes.
Activity 2

Give the name of each of these figures (for example rectangle, square, quadrilateral, triangle, circle, etc.)

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12. 
13. 
14. 
15. 
16. 
17. 
18. 

Name ____________________
LESSON 2 - PROPERTIES OF TRIANGLES AND QUADRILATERALS

Introduction:

This activity is designed to assess a student's ability to think about shapes in terms of properties rather than by appearance only.

Objectives:

(1) Student will be able to characterize shapes in terms of properties.

(2) Student can identify, explain, and write subclass relations — for example, all square are rectangles, all rectangle are parallelograms.

Materials:

Student notebook, one set of worksheets for each student.

Notes for teachers:

In Activity 1, investigation could be done with movable models or with cut-out models that can be folded and measured. Students need to construct and examine properties of various types of triangles and quadrilaterals. Working in cooperative groups, questions should be raised, alternatives strategies might be posed for consideration, and explanations and justification of students, conjectures should be sought.

Activity 2 will help student to identify properties of shapes and explain subclass relation — for example, all rectangles are parallelograms.
Activity 1

To begin this activity, let's construct some triangles.

1. Make two triangles and compare them with your neighbor's. How are the triangles alike? Write a description in your notebook. How are they different? Write a description in your notebook.

2. a. Make a triangle that has three congruent sides.
   b. Make a triangle that has exactly two congruent sides.
   c. Make a triangle that has no congruent sides.

3. Select the figures that satisfy each of the sets of properties below and state their names. (from NCTM Addenda Series/Grades 5-8).

<table>
<thead>
<tr>
<th>Has 4 sides.</th>
<th>Has 4 sides.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opposite sides are congruent.</td>
<td>Only two sides are parallel.</td>
</tr>
<tr>
<td>Opposite sides are parallel.</td>
<td>Name:</td>
</tr>
<tr>
<td>Opposite angles are congruent.</td>
<td>Name:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Has 4 sides.</th>
<th>Has 4 sides.</th>
</tr>
</thead>
<tbody>
<tr>
<td>All sides are congruent.</td>
<td>Has two pairs of congruent adjacent sides.</td>
</tr>
<tr>
<td>Opposite sides are parallel.</td>
<td>Name:</td>
</tr>
<tr>
<td>All angles are right angles.</td>
<td>Name:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Has 4 sides.</th>
<th>Has 4 sides.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opposite sides are congruent.</td>
<td>All sides are congruent.</td>
</tr>
<tr>
<td>Opposite sides are parallel.</td>
<td>Opposite sides are parallel.</td>
</tr>
<tr>
<td>All angles are right angles.</td>
<td>Opposite angles are congruent.</td>
</tr>
<tr>
<td>Name:</td>
<td>Figures:</td>
</tr>
<tr>
<td>Name:</td>
<td>Figures:</td>
</tr>
</tbody>
</table>
Activity 2

Select names from the following list to fill in the blanks that complete the family tree correctly:
- rhombus
- square
- kite
- rectangle
- trapezoid

Write in your notebook why you arranged them in such a way.
LESSON 3 - SPATIAL VISUALIZATION

Introduction:

In this activity, students will improve their ability to visualize by building, drawing, and evaluating three-dimensional figures. These three tasks are used in various combinations throughout the worksheets.

Objectives:

(1) Student will see how a solid and a drawing of it are related to each other.
(2) Student will be able to build a solid, and draw a two-dimensional representation of it.

Materials:

10 to 12 small cubes for each student or small group of students, tape, one set of worksheets for each student, and a set of transparencies for discussion of solutions. Answers can be found on the next two pages.

Notes for teachers:

• Distribute the activity worksheets one at a time to each student.
• Discuss solutions for each sheet before going on to the next one.

Activity 1 is designed to acquaint students with representations made on isometric dot paper. The students build simple solids from cubes, copy isometric drawings of the solids, and answer questions about the numbers of cubes needed to build the solids.

Activity 2, students look at a drawing, build solid from the drawing, add or remove cubes, then draw the modified solid.

Activity 3, students look at a solid from a corner to see the solid as it can be drawn on isometric dot paper. This matching requires a great deal of eye movement back and forth from solids to drawings.

Activity 4, students put together two simple solids to match a drawing of a solid. This open-ended problem allows the student to create, represent, and then evaluate the picture against the solid object.
Activity 1

For each solid shown, do the following:
- Build the solid from cubes.
- Copy the drawing.
- Count the number of cubes used in the drawing.
- Check your count from the solid.

1. Number of cubes _____

2. Number of cubes _____

3. Number of cubes _____

4. Number of cubes _____

5. Number of cubes _____
Activity 2

For each of the four solids 1–4, do the following:
- Build the solid.
- Take away the shaded cube or cubes and then draw the remaining solid.

For each of the solids 5–8, do these steps:
- Build the solid.
- Add a cube to each shaded face and then draw the new solid.
Activity 3

Build a solid on a piece of paper using the following plan:
- Label the corners of the paper A, B, C, and D.
- Position the paper as shown, with corners A and B at the bottom.
- Build the solid using cubes. The numbers tell you how high each stack of cubes should be.

The following drawings represent the four corner views of the solid you built. Turn the paper on which your solid is built and look at the solid from each corner. Match the letter of each corner to the appropriate drawing.

Corner ______  Corner ______  Corner ______  Corner ______

Build the solid shown below and draw views of it from two different corners. For each drawing, indicate the letter of the corner that it represents.
Activity 4

For parts A and B, construct the puzzle pieces indicated from cubes attached with tape.
- Use the two puzzle pieces to build each solid.
- Show how you built them by shading one puzzle piece in each drawing.

Find a different way to make a solid from the two puzzle pieces. Draw your solid here. Ask a friend to solve your puzzle. Build and draw a different solid.
Lesson 4 - Angle Measurement/Construction

Introduction:

These activities are designed to develop the concept of angle measurement and to develop an intuitive concept of angle by having students construct, rotate, and use the angle models in a variety of settings.

Objectives:

1. To be able to measure angles and recognize how angle measures of adjacent angles can be added.

2. To be able to construct angles in a variety of settings.

Materials:

Tracing paper, copies of activity sheets.

Notes for teachers:

- **Activity 1** is designed to develop the concept of angle measurement. As in all initial measurement lessons, students should begin with nonstandard units to estimate and measure and then discover the need for a standard units.

- **Activity 2** is designed to develop an intuitive concept of angle by having students construct angle models in a variety of settings. Descriptions of the "clock" angle and strips should lead to developing the meanings of right, acute, obtuse, and straight angles.
Activity 1

(From NCTM Addenda Series)
Here are three circles, each divided into a different number of equal wedges.

Circle 1: 12 A wedges  Circle 2: 24 B wedges  Circle 3: 36 C wedges

For each angle below, estimate how many wedges of each kind (A, B, or C) make up each angle. Check your estimate by tracing each angle, using tracing paper on each circle and count the number of wedges that fit in the angle.

Use or make a model of an equilateral triangle and measure its angles in terms of A, B, and C wedges. What is the sum of the wedges? What have you found? Will this always be true?

Draw a rectangle. Repeat the same measurements and questions as above.
Activity 2

Form the following angles (from a clock face) and draw a sketch of each angle:

3:00 angle  
2:00 angle  
2:15 angle  
6:00 angle  
8:00 angle  
10:00 angle

Next to each angle, write a word used to describe its size.

Are any of your angles congruent? Describe them.

2. Which angle(s) are acute, obtuse, right angle, straight angle, adjacent angle.
3) Find and list pairs of acute, obtuse, right, straight, and adjacent angles in the figure below:

4) If you were to describe acute, obtuse, right, straight, and adjacent angles to a friend over the telephone, what would you say?
LESSON 5 - ALTERNATE INTERIOR ANGLES & CORRESPONDING ANGLES

Introduction:

The purpose of this activity is to have students identify angles, see connections among types of angles, and deduce conclusions about these angles.

Objective:

Student will be able to identify alternate interior angle and corresponding angles, and to realize that certain facts about angles and lines can be established by deduction.

Materials:

Copy of Activity Sheets 8A from NCTM Addenda Series — Geometry in the Middle Grades, colored pencils or crayons.

Notes for teacher:

Before doing this activity, students need to be familiar with and have had some experience in identifying alternate interior angles and corresponding angles. If lines are parallel, alternate interior/corresponding angles are congruent. Examples of alternate interior angles and corresponding angles are given in the activity.
Activity 1

PARALLELOGRAM GRID AND MINDEDUCTIONS

This grid is formed by sets of parallel lines.

What shapes do you see in the grid?

Using one color, color in one set of corresponding angles in the grid. Are the angles congruent? Why or why not?

Using a different color, color in a different set of corresponding angles. Are the angles congruent? Why or why not?

In the grid, now use a third color to color in a set of alternate interior angles. Are the angles congruent? Why or why not?

Using a fourth color, color in another set of alternate interior angles. Are the angles congruent? Why or why not?

The following diagrams have been taken out of parallelogram grids. Examine each diagram and explain how to obtain the conclusion from the given information.

(a) Show angle 1 = angle 2; angle 3 = angle 4
(b) Show angle a = angle c; angle r = angle b
(c) Show angle r = angle s
LESSON 6 - TRIANGLE ANGLE SUM

Introduction:

The purpose of this lesson is to have students identify certain angles (Level 1), see connections among types of angles (Level 2), deduce certain facts about angles and lines (Level 3), and establish interrelationships (Level 4). The cluster of activities focuses on student investigation and development of the relationships of properties of shapes and angle sums.

Objective:

Students should be able to discover the following:
- The sum of the angles measures of a triangle is equal to the measure of a straight angle.
- The measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles.

Materials:

Student notebook, colored pencils or crayons, copy of Activity Sheet 8C adapted from NCTM Addenda Series.

Notes for Teachers:

"Reading from the grid" permits the students to discover and investigate all the relationships as mentioned in the objective above. This activity establishes the angle sum for a triangle. **Straight angle** and **exterior angle** of a triangle are shown below:
Activity

This grid is formed by three sets of parallel lines.

1) Outline triangle CAT on this grid.

2) Choose three different color crayons.
   (i) With the first color, color in angle C and all
       the angles near points C, A, and T that are
       congruent to angle C.
   (ii) With the second color, color in angle A and
        all the angles near points C, A, and T that
        are congruent to angle A.
   (iii) With the third color, color in angle T and all
        the angles near C, A, and T that are congruent
        to angle T.

3) What do you observe about the angles around C?
   Around A? Around T?

4) Find a straight angle in the colored diagram. What
   colors are the angles that make up the straight
   angle?

5) What conjecture can you make about the angle sum of
   triangle CAT?

Do you think this conjecture is true for all triangles? Why?
6) In the diagram, $\angle$CTS is called an exterior angle of triangle CAT. Is $\angle$ACE an exterior angle of triangle CAT? Why? Name another exterior angle.

7) Examine the size of an exterior angle in your colored diagram above in relation to interior angles of the triangle. Do this for several exterior angles. Do you notice any pattern? What conjecture might you make?

Justify your conclusion.
LESSON 7 - PYTHAGOREAN THEOREM

Introduction:

In Activity 5E (from NCTM Addenda Series/Grades 5-8), students can go on to discover some properties of even and odd numbers under addition. Students can discover properties by experimentation and by informally "proving" rules by using geometric representations for odd and even numbers.

Objective:

Student should be able to use inductive reasoning by gathering data in a variety of geometric settings, finding patterns in the data, and making generalizations:
• \( a^2 + b^2 = c^2 \)

Materials:

Student notebook, rulers, sheets of grid paper, strips of the same grid graph paper, geoboards, and rubber bands.

Notes for teacher:

In Activity 5E (from NCTM Addenda Series/ Grades 5-8), from the visual representations of triangles, students should conjecture that \( a^2 + b^2 \) is equal to \( c^2 \) for a triangle. This activity also gives real-world applications of the Pythagorean theorem in two and three dimensions.
**Activity**

**PYTHAGOREAN THEOREM**

1. On graph paper, create five right triangles with legs of the following lengths:
   
   a. 3 and 4  
   b. 5 and 12  
   c. 7 and 24  
   d. 6 and 8  
   e. 8 and 15

   Find the length of the hypotenuses of each of these triangles (use a strip of graph paper) and record the data in the first three columns of the chart below:

<table>
<thead>
<tr>
<th>leg 1</th>
<th>leg 2</th>
<th>hyp</th>
<th>(leg 1)^2</th>
<th>(leg 2)^2</th>
<th>(hyp)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>5</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f.</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Now complete the last three columns of the chart.

   Do you see any patterns? Describe.

   What conjecture would you make concerning the lengths of the three sides of a right triangle?

2. a. In the middle of a geoboard, create a small right isosceles triangle and construct a square on each side of the triangle. Record the result on dot paper. Is there a relationship among the areas of the three squares? Describe.

   b. Repeat the activity above on a geoboard, using a right triangle with legs of 1 and 2 units.

   c. From a set of tangram pieces, select the middle-sized right triangle. Now build squares on each side of the triangle using the following tangram pieces: two large triangles, two middle-sized triangles, and four small triangles. Is there a relationship among the areas of the three squares? Describe it.
3. Real-world applications:

a. If two joggers want to go from A to B in a square-shaped open field, what possible paths could they take (without retracing any direction)? What is the length of the shortest path? What is the longest path? Explain.

b. Can a circular table top with diameter 2.7 meters long fit through a doorway 2.5 meters high and 1 meter wide? Why or why not?

c. How far up on a wall of a building will a 10-meter ladder reach if the foot of the ladder is 6 meters from the wall? Explain.

d. What is the length of the longest pole you could put in a rectangular storage room 12 units long, 9 units wide, and 8 units high? Explain.
LESSON 8 – PERIMETER

Introduction:

This activity assesses what students already know about the word "perimeter" and solve problems which involve the concept of perimeter.

Objective:

Student should be able to find the perimeter of geometric shapes which are drawn on paper and then to transfer to situations where they need to know "the distance around."

Materials:

Student notebook, copies of the worksheets for each student.

Notes for teachers:

When the word "perimeter" appears in a problem, students might not remember its meaning. We can set up a series of activity to measure an actual objects. Organize the students into small groups. As the activities are being worked try to talk with groups to see that they understand the concepts involved.
Activity 1

Find the perimeter of each figure below.
Activity 2

I Count to find the perimeter of each polygon.

Perimeter: _______ Perimeter: _______ Perimeter: _______

II Find and write the length of the unmarked sides next to each segment. Find the perimeter of each polygon.

a) 7 1 7-2-
   3 2 3-1-

Perimeter: _______

b) 6 7 12 9
   10 6 20

Perimeter: _______ Perimeter: _______

d) 7 9 6 4 10
   2 7 25

Perimeter: _______

III Find each perimeter.

a) 2 3 3 3 3 13 2
   4 2

Perimeter: _______

b) 9 7 8 11 24
   30

Perimeter: _______

c) 9 3 4 4 3

Perimeter: _______

d) 10 8 4 3 2
   13 10 18

Perimeter: _______
LESSON 9 – AREA OF RECTANGLE

Introduction:

This activity will assesses the student's understanding of "area" (as "space inside"), measure of area (as "how many units cover a figure"), and procedures for finding the area of rectangles.

Objectives:

1. Student will be able to understand the meaning of area and measure the area.
2. Student will be able to discover a procedure for finding the area of the rectangle – multiplying the number of square inches in a row by numbers of rows in a rectangle.
3. Student will be able to summarized the area rule "length x width".

Materials:

student notebook, cardboards with length 3" x 5" and 4" x 6", square inch papers, and a ruler.

Notes to Teacher:

Activity 1 is begin by first asking students to cover cardboard by strips of square inches paper and then count how many squares inches cover a rectangle. Then it leads student to discover a procedure for finding the area of a rectangle – length x width.

Activity 2 need student to find the area of each figure given.
Activity 1

1) Cover cardboard 3" x 5" with strips of square inches paper as follow:

2) Count how many square inches cover the rectangle?

3) How many squares in a row?

4) How many rows?

5) What do you get when you multiply your answers in (3) and (4)?

6) Measure length and width of rectangle?

7) What do you get when you multiply length x width?

8) Compare your answer in (2), (5), and (7). What can you say?

9) Repeat (1) to (8) for cardboard 4" x 6".

10. What conjecture can you make about the area of a rectangle?

Justify your conclusion.
Activity 2

Find the area of each figure below.

1) \[ \text{Area} = \text{length} \times \text{width} = 12 \times 8 = 96 \]

2) \[ \text{Area} = \text{length} \times \text{width} = 5 \times 3 = 15 \]

3) \[ \text{Area} = \text{length} \times \text{width} = 12 \times 5 = 60 \]

4) \[ \text{Area} = \text{length} \times \text{width} = 9 \times 3 = 27 \]

5) \[ \text{Area} = \text{length} \times \text{width} = 13 \times 6 = 78 \]

6) \[ \text{Area} = \text{length} \times \text{width} = 21 \times 13 = 273 \]

7) \[ \text{Area} = \text{length} \times \text{width} = 6 \times 6 = 36 \]

8) \[ \text{Area} = \text{length} \times \text{width} = 6 \times 3 = 18 \]
LESSON 10 - AREA OF TRIANGLES

Introduction:

This activity will assess the student's understanding of area and procedure for finding the area of triangles.

Objectives:

1. Student will be able to discover a procedure for finding the area of a right triangle.
2. Student will be able to discover ways to find the area of any triangle.

Materials:

Student notebook, square inch papers, ruler.

Notes for Teachers:

Students are led to discover a procedure for finding the area of a right triangle in terms of the area of a related rectangle. Then, they relate the area rules for rectangle and right triangle.
Activity

1) Given the triangles on a grid marked off in square centimeters. Cut-out the triangles and then match them to form a rectangle.

2) What is the sides of triangle in cm?

3) What is the sides of rectangle in cm?

4) What is the area of the rectangle? Compute the area of the triangle? How do you get the area?

5) Given triangles below and find the area of each triangles.

(a)

(i) Can you break into right triangle? What's the area of each right triangle? What can you say about the whole triangle?

(ii) Compare your answer in (i) to the area of the right triangle?

(iii) What can you say about area of triangle in 5(a).
(b) Refer to the figure given and answer the questions.

(i) What is the area of triangle ABC?
(ii) What is the area of triangle ADC?
(iii) What can you say about the area of the triangle ABD?
(iv) What is the base and height of the triangle?

(c) From 5(a) and (b), compute the area of the triangle below.
Lesson 11 - Area of Parallelograms

Introduction:

In this activity students are asked to discover procedures for finding the areas of a parallelogram. The activity opens with a discussion of the base and height of cut-out parallelograms.

Objectives:

Student will be able to find three ways to find area of a parallelogram:
(a) using a grid and counting squares,
(b) cutting off a right triangle and moving it to form a rectangle with the same base and height as the parallelogram.

Materials:

Student notebook, copy of worksheet, grid paper

Notes for Teachers:

Students will be guided to discover the second method, to explain it, and then to relate the parallelogram area rule to those for rectangle and right triangle.
Activity

Name___________

1. Count the squares to cover the figure of a parallelogram.

2. Cut off a right triangle and move it to form a rectangle (refer diagram) with the same base and height as the parallelogram.

   What is the base?
   What is the height?

   What do you get when you multiply base and height?
   Compare your answer in (1) and (2).

   What do you think about the formula for the area of the parallelogram? Why?
LESSON 12 - AREA OF A CIRCLE

Introduction:

This activity will assess the student's understanding of area of a circle and procedures for finding the area of circle.

Objective:

Student will be able to find the area of a circle.

Materials:

Student notebook, copy of the worksheet.

Notes for teacher:

- Providing concrete experiences with circumference might give students a better understanding of area of a circle. For example, have students find the circumference of cylinder tin and others.

- To estimate the area of a circle we sequentially draw diameters through the circle in such a way as to make each new angle formed congruent to its predecessor.

- Area of a circle = base x height (see diagram)
  = 1/2 x circumference x radius
  = 1/2 x C x r
  = 1/2 x 2 x \pi x r x r
  = \pi x r x r
  = \pi x r^2

- For the irrational number \pi use in the area of the circle, teacher could mention and explain to the student.
Activity

1) Cut-out the figure on the next page and fold in such a way as to make its two halves fit perfectly on top of each other as shown below:

2) Continue draw diameters through the circle in (1) as shown below:

3) Now, cut the circular region along diameter to make two semicircular regions.

Then, cuts along the line as shown in the picture.
Reassembling the pieces in the form of pictured below:

5) What conjecture can you make about the area of both figures below.

What can you say about the height and the base of the figure?
What do you think about the diameter of the circle?
Circumference of the circle?

What can you say about the area of the circle by filling the space below:

Area of a circle = ___ ___
= ___ ___ ___
= ___ ___ ___ ___ ___
= ___ ___
LESSON 13 - VOLUME OF RECTANGULAR BOX

Introduction:

This activity is designed to assess a students' ability to interpret drawings of three-dimensional objects, or when to apply formulas.

Objective:

Student should be able to find the volume of a geometric figure by referring to the amount of space the figure occupies or encloses.

Materials:

Student notebook, copies of the worksheets, set of 100 cubes

Notes for teacher:

Providing concrete experiences with volume (before abstracting to paper and pencil activities or formulas) might give students a better understanding of volume. For example, have students find the volume of open boxes by filling them with unit cubes.
Activity 1

1) Build each of these models. Record the number of cubes needed for each model. Record the length, width and height below each model.

a)  

b)  

c)  

d)  

2) The dimensions of four models are given below. Build each model and record the number of cubes needed.

<table>
<thead>
<tr>
<th>LENGTH</th>
<th>WIDTH</th>
<th>HEIGHT</th>
<th>NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>4</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>b</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>c</td>
<td>5</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>d</td>
<td>8</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

3) The number of cubes needed to build a box model are given below. Record possible dimensions for each model, then build the model.

<table>
<thead>
<tr>
<th>CUBES</th>
<th>LENGTH</th>
<th>WIDTH</th>
<th>HEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Compare your models with a friend's. Are they the same or different?

4) The number of cubes needed to build a box model are given below. Use the clues to help you find the dimensions. No models can have a dimension of 1 unit. Build each model.

a) 64 cubes (all dimensions the same)

b) 64 cubes (all dimensions different)

c) 36 cubes (all dimensions different)

d) 48 cubes (two dimensions the same)

Length  Width  Height

Name__________________
Activity 2

A) How many unit cubes would be needed to fill this box?

To find out:

1) How many unit cubes would be needed to cover the base? _____

2) How many layers would it take to fill the figure? _____

3) The figure has a volume of _____ cubic units.

B) 1) How many unit cubes would cover the base? _____

2) How many layers to fill the figure? _____

3) The volume is __________

C) 1) Base ______

2) Layers ______

3) Volume ______

D) 1) Base ______

2) Layers ______

3) Volume ______

E) 1) Base ______

2) Layers ______

3) Volume ______

Did you discover a rule for finding the volume? ______
LESSON 14 - VOLUME OF OTHER SHAPES

Introduction:
This activity is design for students to look at solid geometry, the study of figures with three dimensions, length, width, and height. It will assess student ability to find volume of cylinder and triangular prism.

Objective:
Student will be able to discover the formula for the volume of cylinder and triangular prism.

Materials:
Student notebook, model of cylinder, copies of worksheets.

Notes for teachers:
Students already know how to find area of squares, triangles, rectangles, and circles. To find the volume, student simply multiply the area of the bottom of a figure by the height of the figure. Revisit the idea to find the volume of rectangular box – i.e.,

Area of rectangle(base)

\[ \text{Volume} = \text{(Area of rectangle})(\text{height}) \]
Activity 1

1. What are the shapes of the top and bottom of a cylinder in the picture?

2. What is the area needed to fill the base of the cylinder?

3. How many layers would it take to fill the figure?

4. The figure has a volume of ______ cubic units.

<table>
<thead>
<tr>
<th>1. a) Area of the base ______</th>
<th>2. a) Area of the base ______</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) Layers ______</td>
<td>b) Layers ______</td>
</tr>
<tr>
<td>c) Volume ______</td>
<td>c) Volume ______</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. a) Area of the base ______</th>
<th>4. a) Area of the base ______</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) Layers ______</td>
<td>b) Layers ______</td>
</tr>
<tr>
<td>c) Volume ______</td>
<td>c) Volume ______</td>
</tr>
</tbody>
</table>

Did you discover a rule for finding the volume? ________________________________
Activity 2

1. What are the shapes of the top and bottom of the figure?

2. What is the area needed to fill the base of the figure?

3. How many layers would it take to fill the figure?

4. The figure has a volume of ____ cubic units.

<table>
<thead>
<tr>
<th>1. a) Area of the base ____</th>
<th>2. a) Area of the base ____</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) Layers ____</td>
<td>b) Layers ____</td>
</tr>
<tr>
<td>c) Volume ____</td>
<td>c) Volume ____</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. a) Area of the base ____</th>
<th>4. a) Area of the base ____</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) Layers ____</td>
<td>b) Layers ____</td>
</tr>
<tr>
<td>c) Volume ____</td>
<td>c) Volume ____</td>
</tr>
</tbody>
</table>

Did you discover a rule for finding the volume?

_____________________________________________________

249
Lesson 15 - Congruence & Similarity

Introduction:

This activity is design for students to use triangle in the tessellation and be able to observe congruent angles, congruent segments, and similar triangles.

Objectives:

Student will be able to observe and discover the following:
- Pairs of congruent angles
- Pairs of congruent segments
- Similar triangles

Materials:

Student notebook, copy of worksheet, colored pencils or crayon.

Notes for teachers:

Before getting started with activity, review and discuss with students the meaning of congruent and similar.
1) State whether each pair of figures below are or not congruent. Explain your answers.

2) (From Manual Math 105/106, Math Department, Ohio State University)

Look at the angles \( \angle ABC \) and \( \angle CDA \) in the shaded rectangle \( ABCD \). Each angle is made up of a pair of smaller angles. Identify them.

\( \angle ABC \) is made up of _______ and _______

\( \angle CDA \) is made up of _______ and _______
Each angle of one pair is congruent to one in the second pair:
\[ \angle \_ \_ \_ \_ \_ \_ \_ \_ \_ = \angle \_ \_ \_ \_ \_ \_ \_ \_ \_ , \text{ and } \angle \_ \_ \_ \_ \_ \_ \_ \_ \_ = \angle \_ \_ \_ \_ \_ \_ \_ \_ \_
\]
What does this say about \( \angle ABC \) and \( \angle CDA \)?

What does this activity suggest about the opposite angles of any parallelogram?

2)

In this triangular tessellation, two large triangles are outlined in boldface, \( \triangle XYZ \) and one other. Label the vertices of the other triangle \( X', Y', \) and \( Z' \)
so that \( \angle X \equiv \angle X' \), etc.

Call \( XY \) and \( X'Y' \) corresponding sides of the triangles. What are the other pairs of corresponding sides?

Does it appear that \( XY \) and \( X'Y' \) are congruent?

What does appear to be the relationship between the lengths of these two sides?

Similarly, compare the other two pairs of corresponding sides. What do you observe?
Pairs of triangles related as these are called similar triangles. Find other pairs of similar triangles in the tessellation. Do you note the same relationship between lengths of corresponding sides?
LESSON 16 - SIMILARITY & CONGRUENCE

Introduction:

This activity is design for students to discover as many geometry ideas as possible in the grids especially about similarity and congruent.

Objective:

Student will be able to discover:
- Similar triangles, trapezoids, hexagon
- Congruent triangles

Materials:

Student notebook, copy of worksheet 8B, p. 50 from NCTM Addenda Series/Grades 5-8.

Notes for teachers:

Many geometry ideas can be discover in the grids as in Figure 18 - NCTM Addenda Series/Grades 5 - 8, p. 43.
This grid is formed by sets of parallel lines.

**Activity**

1. What geometry shapes and ideas do you see in the grid? List them.

2. Did you find any trapezoids? Outline one. Any hexagons? Outline one. Any different-shaped parallelograms? Outline a pair. Do they have equal areas? Why or why not?

3. Did you find any similar triangles? Outline a pair. What is the ratio of their sides? What is the ratio of their areas? Find another pair of similar triangles whose sides are in the ratio of 1:3. Outline them. What is the ratio of their areas? If the ratio of the sides of two similar triangles is 1:4, what would you predict for the ratio of their areas? What conjecture might you make concerning the ratio of the areas of similar triangles?
May 12, 1998

Ms. Idris Noraini  
Mathematics Education 
The Ohio State University 
333 Arps Hall 
1945 North High Street 
Columbus, OH 43210

Dear Ms. Noraini:

This message is meant to confirm that you are permitted to reproduce short passages from NCTM copyrighted publications in your dissertation without express written permission from NCTM. Please make sure to identify the original source, page number, publisher, and copyright date for each passage you use.

Thank you for contacting us.

Sincerely,

Denise Baxter 
Publications Staff Assistant 
DBaxter@nctm.org
Date: Thu, 18 Sep 1997 11:59:44 -0500
From: Permissions <Permissions@nctm.org>
To: idris.3@postbox.acs.ohio-state.edu
Subject: Permission letter to use some part of nctm publish activity
         -Reply
Content-Disposition: inline

Dear Ms. Idris:

This message is meant to confirm that you are permitted to reproduce short passages from NCTM copyrighted publications in your dissertation without express written permission from NCTM. Please make sure to identify the original source, page number, publisher, and copyright date for each passage you use.

Thank you for contacting us.

Jacqui Olkin
Assistant Permissions Editor
permissions@nctm.org
To: Noraini Idris
From: Joe Ferrar, Coordinator of Math 105/106
Date: May 2, 1998

Re: Permission to use excerpts from Laboratory Manual

You have my permission as course coordinator and editor of the OSU Math 105/6 Supplements and Labs, to excerpt any materials from the Lab Manual for use in your educational research.
September 25, 1997

Ms. Noraini Idris  
333 Arps Hall  
Ohio State University  
1945 North High Street  
Columbus Ohio

Dear Ms. Idris:

As requested in your recent email, I am giving you permission to use our materials on the van Hiele levels of thinking in your research project. Please let us know about the kinds of activities or questions that you are using from our materials. If possible, send us a report about your research when you complete it.

Sincerely,

[Signature]

Professor David J. Fuys  
Mathematics Education
BIBLIOGRAPHY


Battista, M.T., Wheatley, G.W., & Talsma, G. (1982). The importance spatial visualization and formal reasoning for geometry learning in pre-service


261
Montreal, Canada: Universite' de Montreal.


Denis, L.P. 91987). Relationships between stage of cognitive development and van Hiele level of geometric thought among Puerto Rican adolescents. *Dissertation Abstracts International*, 48, 859A.


264


Computers in Mathematics and Science Teaching, 10(1), 85-90.


Marriot, P. (1978). Fractions, now you see them, now you don't. In D. Williams (Ed.), Learning and applying mathematics. Melbourne Australia: Australia
Association of Mathematics Teachers.


Wirszup, I. (1976). Breakthroughs in the psychology of learning and teaching geometry. In J.L. Martin & D.A. Bradbard (Eds.), *Space and geometry: Papers from a research workshop* (pp. 75-97). Columbus, OH: ERIC Center for Science, Mathematics and Environmental Education.


