THE MUSICA ENCHIRIADIS AND SCHOLIA ENCHIRIADIS:

A TRANSLATION AND COMMENTARY

DISSETERATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

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* * * * *

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This dissertation is gratefully dedicated to Dr. Norman Phelps of the Ohio State University School of Music, a person whose love of knowledge, inquisitive spirit, and deep concern for his students has greatly influenced all of those fortunate to know him, and whose generous assistance, advice, and unfailing support made this dissertation possible.
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The Sources and Contents of the Musica enchiriadis and the Scholia enchiriadis. BIBLIOGRAPHY
INTRODUCTION

Few medieval theoretical tracts on music have occupied the minds and scholarly output of so many people over the centuries as have the *Musica enchiriadis* and the *Scholia enchiriadis*. Some forty manuscripts dating from as early as the end of the ninth century have thus far been identified as containing them. If nothing more, these numbers attest to the great literary importance of these works in the field of music.\(^1\) Based upon the manuscript evidence, this importance was firmly established during the eleventh century. About fifty-five percent of all existing copies were completed by that time, with the eleventh century alone producing about thirty percent of the entire number of manuscripts preserved today.

In the course of a discussion of any historical document, one must distinguish carefully between the words *importance* and *influence*. The literary importance of these two works was in no doubt due in part to the doctrine of *auctoritas*. There was simply no other earlier work which dealt with polyphony in such a detailed manner (at least none

\(^1\) Only two other medieval music treatises have left more manuscript copies: the *Micrologus* of Guido of Arezzo, and the *Dialogus* which has been attributed to Odo. See *Johannis Affligemensis De musica cum tonario*, ed. J.Smits van Waesberge, *Corpus Scriptorum de Musica* I (Rome: American Institute of Musicology, 1950), 3.
have survived), and which included musical examples and incorporated the relevant and essential mathematical explanations of Boethius. The more astute medieval reader might have also recognized the opening paraphrase in the *Musica enchiriadis* taken from Chalcidius' commentary on Plato's *Timaeus*, which linked these works to one of the most important scientific tracts in the Middle Ages. This helped to give the *Musica enchiriadis* literary influence, establishing it in the mainstream of literary thought, and endowing it with what might be rationalized as a kind of dual function, that of a detailed explanation and adjunct to part of Chalcidius' text, and perhaps more realistically, that of its commonly accepted role as a separate treatise, with its separate identity, perfectly able to stand on its own.

The practical influence and importance of the *Musica enchiriadis* and *Scholia enchiriadis* is more difficult to ascertain. Although its literary popularity remained, its practical use was short-lived. The tremendous innovations which were described in the tenth and eleventh centuries by such theorists as Odo, Guido of Arezzo, and Hermannus Contractus culminated in the establishment of modern letter notation, a solimization and hexachordal system, as well as a definite consciousness and techniques for the formation and identification of the modes.

See Commentary, 212-220.
All of these contributed to the desuetude of the *Musica enchiriadis* as a practical text. One important medieval theorist, Hermannus Contractus, scathingly attacked it. His criticism centered around its underdeveloped system for the recognition of the modes, and its unique system of Dasian notation.

Dasian notation seems to have generated much confusion from the very start, or at least by the time Hermannus got around to it. Some of his remarks show a basic misunderstanding of it (perhaps contrived to aid his cause, especially in light of his keen perception regarding other aspects of it). One of the major problems with the system is the insistence at the very beginning of the *Musica enchiriadis* on the rigid tetrachord structure of tone, semitone, tone. This rule led to certain inconsistencies when one tried to form a perfect octave or fourth above the third note of each tetrachord, or below the second note of each tetrachord. In order to decipher the modern alphabetic interpretation of the basic system, certain statements in the text have to be carefully considered. The first such modern interpretation was published by Phillip Spitta in 1889, and is now considered definitive by most scholars. His interpretation is as follows:

\[
\begin{align*}
\text{C} & \quad \text{D} & \quad \text{E} & \quad \text{F} & \quad \text{G} \\
\text{A} & \quad \text{B} & \quad \text{c} & \quad \text{d} & \quad \text{e}
\end{align*}
\]

When one tries to form a perfect octave between $l (F)$ and $e (f#)$ for example, one of the Dasian symbols must be altered. In so doing, the basic tetrachordal structure of tone, semitone, tone would also be altered, creating a major contradiction of the rule regarding the system's basic intervallic organization. For example, if one altered $e (f#)$, and sang an $f#$ instead for the Dasian symbol $e$, the intervallic structure of that tetrachord would be changed to semitone, tone, tone. Ambiguity is also created from the fact that there is no change in the Dasian symbol. Although both treatises give the rule that octaves, fourths, and fifths must be perfect, and they can be altered to make them so, or that one should better yet avoid the third note of each tetrachord altogether, these rules tended to make the system very cumbersome.

Although many copyists also tried to include letters next to the Dasian notes in their manuscripts, none were correct according to all of the statements in the text. Obviously some of these are later attempts, but it is still convincing evidence that the system generally was not understood.

Hermannus Contractus gives us further evidence. His interpretation of the basic system is as follows:

---

It was little wonder that Dasian notation soon fell victim to the more sensible letter notation, without its cumbersome and contradictory rules and exceptions.

Of all of the internal complexities, none have created more difficulties than the effort of musicologists to establish the identity of the author, as well as the place and date of these works. Martin Gerbert, who published his Latin edition in 1784, attributed them, along with several other treatises, to Hucbald. This view was generally held until 1884, when Hans Müller established that of all of the works which Gerbert attributed to Hucbald, only the *De institutione harmonica* could be definitely ascribed to him. In his research, Müller carefully studied all of the manuscripts which he found containing the *Musica enchiriadis*, particularly those giving the name of an alleged author. After attempting to match names given in the manuscripts with actual historical figures, Müller concluded:


"Until further notice, we have no new name to introduce into music history, and, as a result, we must mark the musica enchiriadis as the work of an Anonymous." 8

Seven years after Müller's book, G. Morin published an article based upon his research to establish the identity of the author. Once again the manuscripts were consulted and compared. Morin's hypothesis centered around two names which he found more than any other in the sources. The first was Otger (or its related forms Notger and Hoger), and the second was Odo (or its related forms Obdonis, Oddonis, Ottonis, Odon, and Otho). In a few manuscripts, this Odo was listed as Odo of Cluny, but none of these were copied earlier than the fifteenth century. 10 Two theorists, Guido of Arezzo, and Wilhelm of Hirsaug (died 1091), also cite an "Enchiridion [or Enchiriadis] Oddonis." 11 All of this information led Morin to search

8 "Wir haben also bis auf weiteres keinen neuen Namen in die Musikgeschichte einführen und müssen die musica enchiriadis für die Folge als das Werk eines Anonymus bezeichnen...", Müller, op.cit., 98.


10 Morin, op.cit., 348-349.

11 Guido in his Epistola de ignoto cantu (GS II, 43-50), and Wilhelm in his Musica (Willhelmi Hirsaugensis Musica, ed. D. Harbinson, as Vol. 4 of Corpus Scriptorum de Musica, 45).
for a historical figure, who was an abbot (since nearly all of the inscriptions indicated he was), and who lived after Hucbald but before Guido. He soon came across the name of one Otgarius, the first abbot of the monastery Saint Pons de Tomieres, and who died in 940. The fact that Otgarius' name is found in two different ways, both as Otgarius (or Otgerius) and also as Odo, provided Morin with what he felt to be strong evidence to support his theory, and evidence which could help explain the use of both forms of the name in various manuscripts which contained the Musica enchiriadis.

Morin's research also led him to an earlier conclusion that Odo of Cluny was not the author of the Dialogus de Musica, which was also known as the Enchiridion musices. Based upon the literary style of the work and the "authority of the Manuscripts," both he and T. Nisard came to the similar conclusion that Guido of Arezzo was the true author.12 This also supported Morin's view that Guido and Wilhelm were not referring to Odo of Cluny as the author of one Enchiridion, but were in fact referring to Otgarius.13

The appearance of two library manuscript catalogues resulted in Morin suddenly changing his position in regards to the authorship of


13 Morin, op.cit., 350-351.
the *Musica enchiriadis* in an article which was published in 1895. One catalogue, that for the Municipal Library at Valenciennes, gave the date for the Manuscript 337 as the ninth century. Morin had previously believed that it was written in the tenth century. The other catalogue, that for Chartes, listed another copy of the *Musica enchiriadis*, which was written between the ninth and tenth centuries. Further research now led Morin to believe, based upon these two early manuscripts, that one Abbot Hoger of Werden (who died in 902) was the author.

Today, Morin's work seems to have done more for the argument that the *Musica enchiriadis* and *Scholia enchiriadis* would best be left anonymous, than to provide us with a definite, undisputed author. And in these regards, Jacques Handschin has also further complicated the question.

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15 *Catalogue général des manuscrits des bibl. publ. de France. Départements*, Tome XXV (Poitiers-Valenciennes), 1894. This date is now felt to be incorrect, so that the tenth century would be more correct.

16 Morin, "L'Auteur de la...", 352, referred to as ms. 325 by Morin, and 359 by Müller, it is now catalogued as 337.

17 Chartes Bibliothèque Municipale 130, which was destroyed in 1944. *Catalogue gén. des man. des bibl. publ. de France. Dépts.*, XI (1889).

18 Morin, "Un essai...", 394.
Based upon passages which are similar in Erigena's *De divisione natura*, Handschin feels that Erigena must have known the *Musica enchiriadis* directly. This would mean that it must have been written before 867 (or as early as 847 if he had known the work while he was in Ireland). Handschin believes that it was written in Ireland, or at least somewhere in the British Isles.\(^{19}\)

Even more recent has been the contribution of J. Smits van Waesberghe. His research concludes that Otger of Laon, Abbot of Saint Amand (920, or 924 until around 952) was the author of these works.\(^{20}\)

Despite all of these attempts, no one has yet been able to conclusively show who wrote the *Musica enchiriadis* and *Scholia enchiriadis*, or when they were written. Many names have been proposed, some more plausible than others, yet none without some doubt. The same is true regarding a possible date. Although many feel that a safe assumption would be to place them around the year 900, evidence has been presented that could place them as early as 847. In any event,

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\(^{19}\) Jacques Handschin, "Die Musikanschauung des Johannes Scotus (Erigena)," *Deutsche Vierteljahrsschrift für Literaturwissenschaft und Geistesgeschichte*, V (1927), 316-341.

all of these names and dates remain as mere suggestions—all of which must be assessed carefully as to their validity. As time passes, however, the prophetic words of Hans Müller come closer to becoming a reality:

"The most probable contention is that from the beginning the Musica enchiriadis has appeared and remains anonymous, and that later on, one will strive in vain to discover its author." 21

21 "Die meiste Wahrscheinlichkeit hat sohin die Behauptung, dass die Musica enchiriadis von vornherein anonym erschienen ist und bleib, und dass man sich späterhin vergeblich bemühte, ihren Verfasser zu entdecken." Müller, op.cit., 98.
PART I

INVENTORY OF MANUSCRIPTS
INTRODUCTORY NOTES

The following inventory and description of manuscripts was compiled from various sources for the purpose of providing information regarding the context within which the Musica enchiridis and Scholia enchiridis are found, as well as showing the wide dissemination of these texts throughout Europe and Great Britain. Most of them have been described in their individual library catalogues, in various critical Latin editions in the Corpus Scriptorum de Musica series,\(^1\) or in the two-volume RISM, The Theory of Music manuscript catalogues.\(^2\) The entries here are greatly simplified, and all of the information presented represents the opinions and scholarship of the various editors and compilers consulted. No attempt was made to correct or supplement these descriptions for two reasons: all of the manuscripts consulted were in microfilm form which greatly limited accurate and detailed assessments of them;

\(^1\) Published by the American Institute of Musicology.

such a project would have required a major effort more appropriate to
a critical Latin edition, rather than to a translation. Since a crit-
ical edition is near publication, a similar project at this time would
result in unnecessary duplication with little added benefit to the
translation at hand.

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3

By Dr. Hans Schmid in Emmering, West Germany.
BAMBERG, Bayerische Staatsbibliothek ms. class. 9 (HJ-IV-19)

10th c.; cod. membr.; 170 folios.

fol. 1r-15v  [Musica enchiriadis].
fol. 15v-48v  [Scholia enchiriadis].
fol. 49r-123r [Boethius, De institutione musica].

BAMBERG, Bayerische Staatsbibliothek Var l (HJ-IV-20)

11th c.; cod. membr.; 66 folios.

fol. 2r-38v  [Scholia enchiriadis].
fol. 40v-41v  "quod in aliquibus rationis huius profundus..."
"...sequens opusquam aliquod continebit excerptum. Huiska orationcula ponamus hic finem." [Musica enchiriadis, Ch.XIX. GS I, 172-173.].
fol. 41v-47v  [Commemoratio brevis].
fol. 47v-58r  "Quid sit inter phthongos et sonos..." [Musica enchiriadis, Ch.X-XXIII, GS I, 159-171a].
fol. 58v  [Bamberg Dialogue De organo].
BERLIN, Staatsbibliothek Ms. lat. oct. 265

12th c.; cod. membr.; 95x140 mm; 24 folios; also known as the Codex Rosenthal.

Contains fragments from the *Musica enchiridias* (GS I. 159, 160), on fol. 1r.

For a complete description of this manuscript see J. Smits van Waesberghe, *Guidonis Aretini Micrologus* (CSM, 4), 1955, 4-6.

BOLOGNA, Civico Museo Bibliografico Musicale, Cod. 33

Dates from the Renaissance. Not consulted.

Contains two manuscripts, both of which include the *Musica enchiridias*.

BRUGGE, Stadtbibliothek 531

11th c.; cod. membr.; 294x223 mm; 61 (68) folios; from the Abbey of Ter Duyn.

Descriptions:


RISM, The Theory of Music I, p.52 (see below).

R. Bragard, Hommage à Charles van den Borren, Antwerpen, 1945, pp.103ff, 120-123.

fol. 1r  "Liber Abbatiae Dunensis ordinis Cisterciensis iuxta Neoportum Flanàriae."


fol. 59r-61v  "De Organo. Dictis autem prout potuimus... in signo sunt "Æel." [GS II, pp.74-78; cf. Paris B.N. lat. 7202].
BRUXELLES, Bibliothèque Royale 10078/95

11th-12 c.; cod. membr.; 206x280 mm, 116 folios.

Descriptions:


fol. 1-42v Non agunt de musica.
fol. 43r-44v "Epistola Hieronymi ad Dardanum..."
fol. 46r-56r [Musica enchiriadis] Inc. "De musicis notis et consonantiarum modis Otgeri abbatis liber incipit. Sicut vocis articulatae..."
fol. 56r-74v [Scholia enchiriadis].
fol. 84v-92r [De harmonica institutione].
fol. 93r-96v [Aureliani Reomensis Musica Disciplina].
BRUXELLES, Bibliothèque Royale 10114/6

11th c.; cod. membr.; 155x200 mm; 110 folios.

Descriptions:


RISM, The Theory of Music I, pp. 57f (see below).

fol. 1r Vacat.
fol. 1v Figura.
fol. 2r [The melody of the Hymn "Ut queant laxis...Sancte Johannes" with neumes. Cf. GS I, 45].
fol. 75v Vacat.
fol. 108r Figura.
fol. 108v Vacat.
fol. 109r "Carolus Longius commodarat Andraeae Papio. eidem postea moriens legavit."
fol. 109v Vacat.
CAMBRIDGE, Trinity College 944 (R. 15. 22)

12th c.; cod. membr.; 140 folios. From Christ Church, Canterbury.

Descriptions:


fol. 1-101v [Boethius, *De institutione musica*].
fol. 103-140r [Guido, *Micrologus*].
fol. 129r-131r [Fragments from the Scholia enchiriadis].
CESENA, Biblioteca Malatestiana S.XXVI.1

15th c.; cod. membr.; c.340 x c.240 mm; 1+250 folios; written for Malatesta Novello Malvesti (1418-1465), Signore di Cesena, founder of the Library.

Descriptions:

RISM, The Theory of Music II (BIII2), pp.21f. (see below).

Gius. Maria Muccioli, Catalogus codicum manuscriptorum...Malaste-
testianae Caesenatis Bibliothecae, I. II, Caesena, 1784, pp.167-
172.

Raimondo Zuzzeri, Sui codici e libri a stampa della Biblioteca Malatestiana di Cesena, Cesena, 1887, pp.465-470.

Hans Müller, op. cit., pp.30-31, 51, 56, 58, 63.

Corpus Scriptorum de Musica, Vol. 21. Aureliani Recensens
Musica Disciplina edidit Lawrence Gushee, American Institute of Musicology, Rome, 1975, pp.43f.

fol. 1r-6lr Non agunt de musica.
fol. 61v-132v [Boethius, De institutione musica libri v].
fol. 194r-195v [Excerpta ex Aureliano]. Inc. "Argumentatio cuius-
dam de musica." Expl. "...quia nec usus in sese sancte retinet Ecclesie."

fol. 196v  "De cimbalorum ponderibus." [GS I, p.149].

fol. 196v-197r  "De modis. Autentus auctoralis...Virgines Domini." [GS I, pp.149-150].

fol. 197rv  "De quinque symphoniis, tribus simplicibus, et tribus compositis, Diapason...Dupla." [GS I, pp.149-150].

fol. 197v-198r  "De consonantibus tribus." [GS I, p.150].

fol. 198v  "Si eiusdem...dividere in XVicim partes." [GS I, pp.150-151].


fol. 200-250  [Boethius, Geometria; De ratione abaci].
CHARTES, Bibliothèque Municipale 130 (390)

9th and 10th (fol.50, 11th)c.; cod.membr.; c.250x205 mm; 50 folios; destroyed in 1944; copy of fol. 1-29 at Abbey of Solesmes; microfilm of entire manuscript at the Instituut voor Middeleeuwse Muziekwetenschap, Amsterdam.

Descriptions:

*RISM, The Theory of Music I, pp.84-85. (see below).*


Hans Müller, *op.cit.*, p.25.


fol. 28rv [Berno, fragments from De musica].

fol. 29v-31v [Cassiodorus]. "Veniamus ergo ad Musicam."

fol. 31v-49v Non agunt de musica.

fol. 49v [fragments]. "Quattuor principales qui autentidicuntur."

fol. 50rv Prosae cum musica.
DÜSSELDORF, Heinrich-Heine-Institut, Hs. H3 (olim Stadtbibliothek Ms. H3)

9th c.; cod. membr.; 4 folios. From the Abbey of St. Liudger in Werden.

Descriptions:


fol. 1-4 [Fragments from the Scholia enchiriadis, Part I].
EINSIEDELN Klosterbibliothek 79

10th c.; cod. memb.; 190x248 mm; 110 pag. bound in the following order: 1-45, 66-109, 46-65.

Descriptions:

*RISM, The Theory of Music I*, pp.74-75. (see below).


p. 2-27  [Musica enchiriadis].
p. 27-102  [Scholia enchiriadis]. Sequitur cum aliquibus notis dasianis: "Noanoeane, Noeagis..."
p. 103-109  Tonarium [sine tit.]. Inc. "Igitur primi toni neuma regularis haec est Noanoeane" [cum notis dasianis]. Expl."...Iudicare...et pauperes."
FIRENZE, Biblioteca Medicea Laurenziana, Ashburnham 1051

12th-13th c.; cod. membr.; 265x190 mm; 171 folios.

Descriptions:


fol. 2r-41v [St. Augustine, *De musica*].
fol. 50r-57v [Guido, *Micrologus*].
fol. 57v-59v [Guido, *Regulae rhythmicae*].
fol. 60v-73v [Musicae artis disciplina].
fol. 74r-75r [Guido, *Epistola ad Michaelem*].
fol. 75v-88v [Johannis Afflighemensis musica].
fol. 96r-135v [Boethius, *De institutione musica*].
fol. 147r-150v [Isidore, *De musica*].
fol. 151r-156v [Musica enchiriadis].
fol. 156v-168v [Scholia enchiriadis].
FIRENZA, Biblioteca Nazionale II.I.406 (olim Magliab. XIX.19)

15th c.; paper; 328x225 mm; I+43 folios; first and last parts of ms. lost.

Descriptions:

RISM, The Theory of Music, II (BIII/2), 33-34.

Hans Müller, op.cit., 4-5.

fol. [I] Annotations by Andrien de la Fage regarding the defects of this codex.
fol. 4r-5r [Tonale dictum Sancti Bernardi].
fol. 6r-39v [Boethius, De institurione musica].
KÖLN, Historisches Archiv Hs W 331

10th c.; cod.membr.; 258 folios.

Descriptions:


fol. 2v-189v [Boethius, De institutione musica].
fol. 190r-216v [Musica enchiriadis].
fol. 217r-258r [De organo. See "Ein Gladbuche Orgeltraktatus aus dem Jahre 1037," in Kumb, XL (1937), 16-22].
LONDON, British Museum, add. 17808

11th c.; cod. membr.; 55 folios.

Descriptions:


fol.1v-11v [Guido, *Micrologus*].
fol.11v-16v [Guido, *Regulae rhythmicae*].
fol.17r-18v [Prefatio Guidonis in Antiphonarium].
fol.18v-22v [Epistola Guidonis ad Michaeli].
fol.23r-49v [Musica enchiriadis].
fol.50r-53v [Two anonymous treatises on music].
fol.56r-57r [De abaco].
LONDON, British Museum, Arundel 77

11th c.; cod.membr.; 320x230 mm; 98 folios.

Descriptions:


fol. 1-3 [Fragments from Aurilianus, Musica disciplina].
fol. 6v-62 [Boethius, De institutione musica].
fol. 62v-87v [Scholia enchiriadis].
fol. 87v-98 [Berno, Prologus in Tonarium and Tonarius].
LONDON, British Museum, Harl. 3199

13th c.; cod. membr.; 94 folios.

Descriptions:


fol. 1 Vacat.
fol. 2r-55v Non agunt de musica.
fol. 56v-58v [Prefatio Guidonis in Antiphonarium].
fol. 58v-65r [Guido, Epistola ad Michaeli].
fol. 65r-69v [Fragments from Musica enchiriadis].
fol. 79r-88v [Guido, Micrologus].
MADRID, Biblioteca Nacional, Mss. 9088 (Aa. 53)

12th c.; cod. membr.; 333x242 mm; 150 folios.

Descriptions:


fol. 1v-40v [Boethius, De arithmetica].
fol. 41v-94 [Boethius, De institutione musica].
fol. 94v-103v [Musica enchiriadis, altered text with additions, etiam in Paris, BN lat. 7202 fol. 50r-56v; Brugge 531, fol. 52r-61v; Firenza, BN II.I.406, fol.39v-43v].
fol. 104-110v [Musica enchiriadis].
fol. 110v-123 [Scholia enchiriadis].
fol. 130-149r "Quaedam Geometrie excerptiones..."
MILANO, Biblioteca Ambrosiana D.5.inf.

14th-15th c.; cod. membr.; 255x193 mm; f.1-121 folios.

Descriptions:

Gaetano Cesari, Biblioteca Ambrosiana, in Bolletino dell' Associazione dei Musicologi Italiani, 1910-1911, III. Seria, La Punta, 5, 12-16.

J. Smits van Waesberghe, Guidonis Aretini Micrologus ..., 42-43.


Franconis de Colonia Ars cantus mensurabilis, ed. G. Reaney and A. Gilles (Corpus Scriptorum de Musica, 18), 1974, 12-14.

Corpus Scriptorum de Musica, 6, pp.9-10.

fol. 1r-11v [Guido, Micrologus].
fol. 12v-13v [Guido, Prologus in Antiphonarium].
fol. 13v-17v [Guido, Epistola ad Michaelum].
fol. 18v-21r [Guido, Regulae rhythmicae].
fol. 21v-30v [Musica enchiriadis].
fol. 30v-49r [Scholia enchiriadis].
fol. 52r-77v [Marchettus of Padua, Lucidarium].
fol. 78r-110r [Marchettus of Padua, Pomerium].
fol.110v-118v [Franco, Ars cantus mensurabilis].
MONTE CASSINO, Biblioteca Abbaziale 318

11th c.; cod. membr.; 263x165 mm; 300 pages; from S. Maria de Albaneto.

Descriptions:

Adrien de la Fage, *Essais de diptherographie musicale*, 392-408.


*RISM, The Theory of Music*, II (B III/2), 64-69.

A compendium of music theory consisting of two parts. An incomplete copy of the *Musica enchiriadis* can be found in the first part, Chapters LI-LXXIX (pp.70-90). Inc. "Sicut vocis articulatae..." Expl. "...in his designichil (??) decoris commisit (?) [illegible] (GS I, 152-172).

For a detailed description of the contents of this manuscript, see *RISM*, loc.cit.
MÜNCHEN, Bayerische Staatsbibliothek, cod. lat. 6409

10th-11 c.; cod. membr.; 30 folios. (Olim Fris. 209)

Descriptions:


fol.1-10 [Musica enchiriadis].
fol.10-30 [Scholia enchiriadis].
MÜNCHEN, Bayerische Staatsbibliothek, cod. lat. 14272

10th-11th c.; cod. membr.; 192 folios. From St. Emmeram (Em. C.91).

Descriptions:


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MÜNCHEN, Bayerische Staatsbibliothek. cod. lat. 14372

11th c.; cod. memb.; 193 folios. From St. Emmeran's Abbey, Regensburg (Em. D 97).

Descriptions:

C. Halm and G. Laubmann, comp., op.cit., V (Tom. IV, Pars II), 163.

fol. 1-8          [Musica enchiriadis].
fol. 8-25         [Scholia enchiriadis].
fol. 26-193       Non agunt de musica.
MÜNCHEN, Bayerische Staatsbibliothek, cod. lat. 14649

13th c.; cod. membr.; 67 folios; from St. Emmeran's Abbey, Regensburg (Em. G 33).

Descriptions:


fol. 1-32     [Musica enchiriadis].
fol. 32-33    [Commemoratio brevis].
fol. 33-34    "Super unum concavum lignum in una linea..."
fol. 37-61    [Scholia enchiriadis].
fol. 61-67    Non agunt de musica.
MÜNCHEN, Bayerische Staatsbibliothek, cod. lat. 18914

12th c.; cod. membr.; 42 folios; from the Abbey of Tegernsee (Teg. 914).

Descriptions:

C. Halm and G. Laubmann, comp., op.cit., VI (Tomi IV, Pars III), 219.


fol. 1-29v  [Musica Enchiriadis cum scholiis].
fol. 31-32   [Epistola Adelboldi ad Silvestrum Papam].
fol. 38v-39  [Eberhardi Frisingensis Tractatus de mensura fistularum (GS II, 181)].
fol. 40-41   [Versus ad mensuram monochordi secundum Guidonem (GS II, 181f)].
NÜRNBERG, Erlangen Universitätsbibliothek, lat. cod. 66

13th–14th c.; cod. membr.; 305x225 mm; 119 folios.

Descriptions:

*Katalog der Handschriften der Universitätsbibliothek Erlangen*,

fol. 1-34  Non agunt de musica.
fol. 35-85  [Boethius, De institutione musica].
fol. 85-95  [Musica enchiriadis].
fol. 95-100  [Scholia enchiriadis].
fol. 102-109  [Tractatus complures de Musica].
fol. 110-119  [Musica enchiriadis].
OXFORD, Bodleian Library Ms. Canonici Misc. 212

c.a. 1400; cod. membr.; 300x210 mm; I+55 folios.

Descriptions:


Aureliani Reomensis Musica disciplina, ed. Lawrence Gushee (Corpus Scriptorum de Musica, 21), 1975, 45-46.

fol. 1-2 Vacat.
fol. 3-11v [Musica enchiriadis].
fol. 11v-30 [Scholia enchiriadis].
fol. 30-31 Inc. "Super unum concavum lignum..."
Expl. "...auricularis gravi tetrardo notabilis."
fol. 31v-39v [Hucbald, De harmonica institutione]
fol. 39v-40 [De modis (GS I, 149)].
fol. 40-52 [Tractatus similis textui Aureliani].
fol. 53-55 Vacat.
PARIS, Bibliothèque Nationale lat. 7202

11th c.; cod. membr.; 338xc.210 mm; 56 folios.

Descriptions:


Catalogus codicum manusciptorum bibliothecae regiae, Paris, 1744, Pars III, Tomus IV, 326.

Hans Müller, op. cit., 3-4.

fol. 1-50r [Boethius, De institutione musica].
fol. 50r-56v [Musica enchiriadis. Variant text and additions, etiam in Brugge 531, fol. 52r-59r; Madrid, BN 9088, fol. 94v-103v; Firenza, BN II.I.406, fol. 39v-43v. See H. Sowa, Textvariationen zur Musica Enchiriadis in ZfMw, XVII (1935), 194-207.]

Persequitur tractando de organo "Dictis autem prout potuimus..." Expl. "...He littere in signo sunt R A E I."
PARIS, Bibliothèque Nationale lat. 7210


Descriptions:

Catalogus codicum manuscriptorum bibliothecae regiae, Paris, 1744, Pars III, Tomus IV, 327.


pag. 71 and 1–21 [Musica enchiriadis].
pag. 21–65 [Scholia enchiriadis].
pag. 65–67 "Super unum concavum lignum..."
PARIS, Bibliothèque Nationale 7211

13th c. (fol. 1-72v); early 12th c. (fol. 73-151v); cod. membr.; 280x180 mm; 151 folios.

Descriptions:

Andrien de la Fage, op.cit., 180ff.

Hans Müller, op.cit., 18-19.


fol. 1r-12r [Musica enchiriadis].
fol. 12r-16v [Scholia enchiriadis].
fol. 49r-51r [Commemoratio brevis].
fol. 54r-71r [Alia musica].
fol. 73r-89r [Guido, Micrologus].
fol. 89r-95r [Regulae rhythmicae Guidonis].
fol. 95r-97v [Prologus Guidonis in Antiphonarium].
fol. 97v-105r [Epistola Guidonis ad Michaelem].
fol. 105v-115r [Dialogus Oddoni adscriptus].
fol. 115v-125v [Liber Argumentorum].
fol. 134v-144v [Aureliani Reomensis De musica].

For a more detailed description of the contents see RISM, loc.cit.
PARIS, Bibliothèque Nationale lat. 7212

Late 11th or 12th c.; cod. membr.; 335x230 mm; 52 folios.

Descriptions:


Catalogus...bibliotheca regiae, Paris, 1774, Pars I, Tomus IV, 327.


fol. 1r               Non agunt de musica.
fol. 1v-12v           [Musica enchiriadis].
fol. 12v-36r          [Scholia enchiriadis].
fol. 36r-37v          [Commemoratio brevis (incomplete)].
fol. 38rv             Mensura monochordi. Inc. "Super unum concavum
                      lignum..."
fol. 39r-50v          [Hucbald, De musica (fragments)].
fol. 51r-52v          Vacant.
PARIS, Bibliothèque Nationale lat. 13955

9th, 10th (fol. 3v-4r), and 11th (?) c.; cod. membr.; c.190x216 mm; 169 folios.

Descriptions:


fol. 1-3r Non agunt de musica.
fol. 3v-4r [Musica enchiriadis (fragments, Chs. I-IV].
fol. 4v Vacat.
fol. 5-59 Non augunt de musica.
fol. 60r-105v [Boethius, De institutione musica].
fol. 106r-158r Non augunt de musica.
fol. 158v-165v [St. Augustine, Compeneium musicae].
fol. 166-169 Non augunt de musica.
ROMA, Biblioteca Apostolica Vaticana Pal. lat. 1342

11th c.; cod. membr.; 158 folios.

fol. 1r-104r [Boethius, De institutione musica].
fol. 105r-121v [Musica enchiriadis].
fol. 121v-158v [Scholia enchiriadis].
ROMA, Biblioteca Apostolica Vaticana Reg.lat. 1315

14th c.; cod. membr.; c.375x c.264 mm; 110 folios.

Descriptions:

RISM, The Theory of Music, II (BIII/2), 116-117.

fol. 1r-66r [Boethius, De institutione musica]
fol. 66r-71v [Divisio monochordi et diagrammata tonorum inter-vallorum, eorumque proportionum].
fol. 72r-78r [Macrobius, Fragmentum de musica].
fol. 78v-80v [Fulgentii sententia de musica].
fol. 81r-88r [Isidori sententiae de musica].
fol. 97v-110v [Scholia enchiriadis].

This entire manuscript corresponds to Firenza, Biblioteca Medicea Laurenziana, Ashburnham 1051, fol. 96-169.
VALENCIENNES, Bibliothèque Municipale 337 (olim 359, 325)

10th c. in.; cod. membr.; 252x151 mm; 79 folios written in various hands; from the Abbey of St. Amand.

Descriptions:


fol. 1-41 Non agunt de musica.


Vacant.

[Scholia enchiriadis]. Titulus saeculo duodecimo adiectus est. "Incipit scola Enchiriadis de musica" loco alii tituli erasi. Tractatus in fine inventur mutilus. Inc. "Musica quid est?..."
Expl. "...positionem non invent aut alieno."
(GS I, 173-212).
WIEN, Nationalbibliothek Cpv 55

10th c.; cod.membr.; 326x255 mm; 208 folios (cf. Tabulae codicum manu scriptorum...in Bibl. Palatina Vindobonensi asservatorum T.I, Wien, 1868, p.8).

Descriptions:

RISM, The Theory of Music I, pp.36-37 (see below).


fol. 1r-92v [Boethius, De institutione geometrica; De institutione arithmetica].
fol. 93r-167v [Boethius, De institutione musica].
fol. 167r [Alia manu]. "Quinque sunt consonantiae musicae..." [cf., GS I, 338-339].
fol. 167v Figura [manu poster.] in qua litterae et nomina tonorum, etc.
fol. 168r-182r [Musica enchiriadis]. Inc. "Incipit Musica. Siccut vocis articulatae..."
fol. 182r-208v [Scholia enchiriadis].
fol. 208v "In primo diapason mensura..." (GS I, 121-122). In calce folii: "Liber fratum praedicatorum de buda."
WIEN, Nationalbibliothek Cpv 2231

10th c.; cod. membr. in 4°; 116 folios; to the inside of the flyleaves two folios have been glued; these folios, taken from a 12th century manuscript, are numbered afterwards fol. lrv and 117rv; the first folio of the manuscript itself is numbered 2.

Descriptions:

RISM, The Theory of Music I, p.40 (see below).

[Musica enchiriadis, fragments (GS I, 181b-2)]. Folium antepositum lrv et folium 117rv unum faciunt, permutato vero ordine.

fol. 117rv
"[p]ositio vergat, et particula a quo sono...
decem et octo námque sonis ponimus id est"
(fol.117v, cuius regula prima: "tetrachordis...
quidem" abscissa est) "quod est humilium grave
tetrachordum nuncupantes...sine adiectione
alterius soni ad." (GS I, 181b-182a).

fol. lrv
(Regula prima: "quem naturali...ad aliquid"
abscissa est). "per se non intelligitur, ita
cum aliquot soni...sed quod haec spaci ei sui sit
po" (fol.1v, cuius prima regula "-testas. Iam
...systemate trans-" abscissa est.) "ierit,
maiori tono deputari solet...Ac inprimum videndum
ut numerose quodlibet." (GS I, 182).
WOLFPENHÜTTEL, Cod.-Guelf. 72 Gud. Lat.

10th c., fol. 50r-51v 11th c.; cod. membr.; 290x260 mm; 87 folios.

Descriptions:

Hans Müller, op.cit., 29.


fol. 1-50 [Boethius, De institutione musica].
fol. 50 Carmen, 16 verse. "Qui cupias priscum modulandi noscere nisum."
fol. 50v [Notkeri Labeonis De mensura fistularum organi-
carum institutio].
fol. 51-51v [Divisio monochordi. "Super unum concavum lignum."].
fol. 51v Decem nomina dei.
fol. 52-61v [Musica enchiriadis].
fol. 62-82 [Scholia enchiriadis].
fol. 82v-87 [Commemoratio brevis].
fol. 87v Literae Noeane cum musicis.
PART II

TRANSLATION
MUSICA ENCHIRIADIS

Chapter I

Just as letters are the primary and indivisible elements of articulated speech, consisting of syllables from which arise verbs and nouns ultimately forming the text of perfect discourse, so too are pitches [phthongi] which are called soni in Latin, fundamental elements, and the entire essence of music ultimately can be reduced to them. From the combination of sounds [soni] arise intervals [diastemata], and from the combination of intervals are formed systems [systemata]. Sounds are, indeed, the primary elements of song. However, not all pitches [phthongi] are called sounds [soni]. Only those which are arranged in a lawful succession are suitable for a melody. The arrangement of these sounds continues naturally upwards and downwards so that four sounds and four groups of the same always follow one another. But these four, by means of their own individuality, are all dissimilar. They do not differ merely in height or depth, yet even in height or depth their nature has a particular quality which the lawful distance upwards or downwards determines for each separate one. For example, these notes in order: $\text{F} F / \text{F}$. The first and lowest is called Protus or Archoc by the Greeks. The second is called Deuterus and is a tone
above Protus. The third is called Tritus and is a semitone above Deuterus. The fourth is called Tetrardus and is a tone above Tritus. An endless succession of these sounds is created by their continuous accretion, and they follow one another in the same arrangement of four groups of four until they run out by ascent or descent, in this manner:

\[
\text{FFIF/FFFF/FFFF/FFFF/FFFF/FFFF}
\]

As this short representation shows, whether ascending or descending, these sounds follow in order such that the succession of tetrachords does not stop until the voice fails. The perfect quality of these four sounds creates the quality of the eight modes, as will be demonstrated later. The entire harmony is united by the interrelated [socialis] diversity of these sounds.

Yet, as was said, because their accretion leads to infinity, knowledge has selected eighteen sounds from this confusing multitude, in which are contained the four groups of four sounds along with the two remaining sounds. Among these is the first and lowest tetrachord Graves. Next is the tetrachord Finales. After this follows the tetrachord Superiores. And next, the tetrachord Excellentes. At the end two sounds remain. All of these are shown in this representation:
Chapter II

The symbols for the pitches and why there are eighteen.

Because, as was said, the four sounds and four tetrachords of the same arrangement are established by nature, so also are the symbols within each tetrachord nearly the same. The differences between the various tetrachords are indicated only by the way the characters are turned. The first Finales or Terminales sound is indicated by the Dasian ḳ with an S leaning back on top of it, in this manner: ḳ. The second Finales is indicated with a backwards C on top, in this manner: ḳ. The third Finales is indicated with a simple and inclined I, in this manner: I. The fourth Finales is indicated with a half-C on top, in this manner: ḳ. The Graves sounds are indicated by the characters of the Finales reversed, in this manner: ḳ. The Superiores are like those of the Finales, but with their heads hanging down, in this manner: ḳ. The Excellentes are like the Graves with
hanging heads, in this manner: ☪ ☪ ☪. The exception is the third sound, which in Graves has an inclined N as its mark: ☪. In Superior it has a backwards N: Ʌ. In Excellent it has an X with tips. The remaining two sounds have the signs: ☪ ☪, Protus and Deuterus lying down. All together there are eighteen which individually reach their utmost symphonia, that is a sound [sonus] of the fifteenth,¹ which will be discussed later.² There are many other signs invented by the ancients for the many sounds which have been avoided here with care, in order to introduce these easier ones.

Chapter III

Where the tetrachord Finales gets its name.

These sounds are called Terminales or Finales because every melody must end on one of these four. A melody of the first tone [tonus] and its lower [subiugalis] tone is governed by and ends with

¹ [Actually in this eighteen tone system, only the first four notes of the Graves tetrachord can form a fifteenth without exceeding the notes of the system, and the interval would not be a perfect one if the strict rules given in Ch. I were followed. It is possible that this passage refers to the fact that any of these sounds can form a fifteenth, the utmost (i.e., greatest) of the symphonias described in these two treatises, even though they would exceed the upper limits of given notes, a practice which is described in Chs. XI, XIV, and XV. See Commentary, pp. 227, 229.]

² [Musica enchir. Ch. XL.]
the first sound Archoos †. A melody of the second tone and its lower tone is governed by and ends with the sound † Deuterus. A melody of the third tone and its lower one is governed by and ends with the sound / Tritus. A melody of the fourth tone and its lower one is governed by and ends with the sound ‡ Tetrardus. In any event, any greater [maior] tone is called authentic, and any lesser [minor] tone is called plagal [plagis seu plagius].

Chapter IV

Why only a single tetrachord stands

below the Finales tetrachord

yet two stand above it.

The tetrachord Finales or Terminales has one tetrachord below it, which is called Graves. However, above it are two, that is, Superiores and Excellentes, with two pitches remaining. The simple and lawful cantus does not descend lower than to the fifth pitch below its final. In the first and second modes below the final † Archoos or Protus it descends only as far as to the same Graves †. In the third and fourth modes below the final † Deuterus, it descends as far as to Graves Deuterus ‡. In the fifth and sixth modes below the final / Tritus it descends as far as to Graves Tritus ‡. In the seventh and
eighth modes from the final Tetrardus \textsuperscript{1}, it descends as far as to Graves Tetrardus \textsuperscript{2}. However, one can ascend from whatever its final, all the way up to the third sound of the same name, that is, all the way up to the Excellentes tetrachord. \textsuperscript{3}

Chapter V

How to distinguish between the authentic and the lesser tones.

Because each authentic tone, and those which are below them, are governed by and end on the same sound, and are therefore held by a single tone [tonus], they still differ, for with the lesser tones there are fewer ascending intervals, and each lower [inferior] tone ascends only up to the same sound a fifth from its final. But in fact, this seldom occurs.

\textsuperscript{3} [This, of course, results in a total range of a ninth, which fits Dasian notation perfectly (since the same form of the symbols recur at the fifth and the ninth), but is not compatible with the later letter notation. Hermannus Contractus bitterly attacks this aspect of Dasian notation, under the incorrect assumption that somehow there is an octave within the nine Dasian symbols. Compare the similar statements in the Scholia, Pt. I, 133, with Musica Hermanni Contracti, trans. and ed. Leonard Ellinwood. Rochester: Eastman School of Music, University of Rochester, 1936, 23-24: "...on whatever tone you begin...you will find its duplicate not in the octave, as it should be, but rather in the Ninth." See also the Commentary, 228-229.]
Chapter VI

The peculiarity of the pitches and how many steps separate pitches of the same quality.

He who finds pleasure in this study takes pains to discern the peculiarity of each of the sounds effectively, then to grasp quickly among the mingled sounds just what each pitch is, whether low or high, so that it can be judged clearly from its effect and its notation how far one sound is from another. Every musical sound has in both directions a sound of the same quality at the fifth position, and has on either side, the same sound at the third position. On the side which has a second, the opposite side will be a fourth. Something also must be ascribed to those which are few in these exercises up to now, by which one learns to distinguish the proper nature of the sounds according to the notes of the melodies, or to investigate an unknown melody by means of the notes, through the quality and order of their signs. Truly, one does not easily progress towards this investigation

4 [That is, every fifth position has the same name (Protus, Deuterus, etc.).]

5 [E.g., both have the same name (Tritus).]

6 [E.g., both are Tetrardus.]
unless one sings by means of single neighboring pitches in order with their Greek names, in this manner:

\[ \text{Archoos Deuterus Tritus Tetrardus} \]

In the following example, the written text is the lowest, over which the written musical notes can be sung. Above the notes are written-out the names of the notes, in this manner:

Chapter VII

A short description of the properties of the sounds through an exercise.
Te humiles fasmuli modulis venerrando pilis
Se iubebas flagiant variis liberare malis

Rex sueili Domini maria unico

Ti-terius ni-ti-ili sque-li-di-quique ali

Te humiles Samueli modulis venerrando pilis
Se iubebas flagiant variis liberare malis
Chapter VII

How all six tones are produced from the quality of the four sounds.

It will now be shown how this force of the four pitches \( phthongi \) is controlled, and how such an orderly arrangement is made by the modes, which we improperly call the tones. Strewn out in order are certain strings, as it were, with the single notes of the sounds \( soni \) placed in order at the opposite side. Thus, the strings are like the sounds \( vocum \) which these notes signify. A neume is then placed anywhere one wishes between these strings, as in this manner, for example:

\[
\begin{align*}
\text{T} & : Alx
\text{T} & : le
\text{T} & : y
\text{T} & : lu
\text{s} & : i
\end{align*}
\]

Thus, in order that what has been said is acknowledged both by hearing and seeing, a short example, using the same neumes, is given again in another way. One draws the lines in a similar manner from one side to the other, with the strings written in a succession of four rows together, so that each series is marked by its own conspicuous character. The first series of neumes begins from the sound \( \text{\textcircled{-}} \) and ends with the sound \( \text{\textcircled{\text{\textcircled{-}}}} \). The second starts with the sound \( \text{\textcircled{-}} \) and ends with
the sound $\mathcal{F}$. The third begins with the sound $\mathfrak{S}$ and ends with the sound $\mathcal{F}$. The fourth is begun with the sound $\mathfrak{S}$ and ends with the sound $\mathcal{F}$, in this manner:
These four short examples, while separated alternately only by a semitone or a tone, that is by a harmonic interval, are transferred by this sound [sonus] from one genus to another. If you sing the first group, you can recognize that the quality [vis] of the first sound produces the quality of the first tone [tonus] which is called protus autentus. If you sing the second, you perceive the second tone, governed by the second sound. If you take the third, you see similarly the quality of the third tone to exist in the third sound. If you sing the fourth, you will comprehend the genus of the fourth tone to arise from the fourth sound. Thus, any melodies of the first tone and its lower one [subiugalis] can be fit to the first song, similarly to the second song, similarly to the third song, and similarly to the fourth song, according to each example, which I endeavored for better understanding to write in two ways—both by means of liaes used as strings, and as individual notes situated according to the syllables.

A melody according to the first principal and secondary mode:

Al le lu / / ia / /. Lau da te / Do mi / num de / coe lis. 

---

7 [The terms "the lower one," and "secondary" both refer to the plagal form of the mode.]
Coe li f coe flo rum f lauf te f.

All lu -- ia  Landa te Dominum de. Coelis; Coel i Coelegum Lau da te Deum.

A melody according to the second principal and secondary mode:

Al le lu f fia f. Con fi s te bor f Dom in no f ni s mis i

in fo f i re f me f o f.

Lauf da f lo De lu m me f um f in vi ta f me f a f.
A melody according to the third principal and secondary mode:

Alleluia. In te lingua clausula remedium.

Do mi ne.

Intellige clamorem meum. Domine, sancte, rex, Deus.

Mi se fons me Deus.

A melody according to the fourth principal and secondary mode:

Alleluia. Sit nomen Domini benedicat.

dicens in secula seculorum.

8 [This melody is also used by Guido in Ch.XVIII of the Micrologus (Waesberghe ed., 198-199).]
In 7 aeternum et 7 in seculum seculum 7 seculum li 7.

In this manner, one makes use of the ordinary small spaces [modulus] for the purpose of investigating the quality of any tone, with the same orderly reasoning which, in the principal modes begins from the Superior(es) and ends on the Finales sound, while in the minor modes both commences and ends on the Finales and does not reach to the place of the Superior(es), such as Noannoeane and Noeaglis, etc., which we judge as not indicating words, but rather as being syllables alloted to the singing.
Chapter IX

What the difference is between Phthongus
and Sonus, and between Tonus and
Epogdoos. What Tones and Modes
or Tropi, and Particulae are.
What Diastema and Systema are.

After the previous discussion, and the minor elements have been
learned as a kind of preliminary exercise, we shall proceed through
fairly easy steps to the harmonic rules.

Harmony is the appropriate uniting of diverse sounds [voces] into
one, of whose sounds which we frequently hear the terms sonus, phthongus,
tonus, and epogdoos used, the quality of these having never been made
known. Sonus is the general name of each and every sound [vox], but we
call the sounds [soni] of the sounds [voces] of song phthongus. Tonus
is the lawful size of an interval from one sound [sonus] to another, and
this musical sound, which is in the sesquioctave proportion, is called
Epogdoos in Greek, whereas a proportion in which the larger number
exceeds the smaller by half of the latter is called a sescupla, or
sesquialter, or hemiola proportion.

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9 [9:8].
10 [3:2].
Sesquitertius is where the larger part surpasses the smaller part by a third; \(^{11}\) sesquiartus, where the larger portion exceeds the smaller by a fourth; \(^{12}\) sesquiquintus, where one part is a fifth larger than the other; \(^{13}\) sesquisextus, where it is larger by a sixth; \(^{14}\) sesquiseptimus, where it is larger by a seventh. \(^{15}\) Thus, the sesquioctavus is where a sound is in the ration to another where the larger part is an eighth greater than the smaller. The semitone is that non-perfect interval of a tone, which sometimes is called limma, or disson. Modes, or tropes are species of song which are mentioned above—-as the first authentic or plagal, or the second authentic or plagal, or the modes Dorius, Phrygius, Lydus, etc., which have obtained their names from mighty people.

Phrases [particulae] are the semicolons [cola], or commas [commata] of song, which divide the song within their boundaries. The semicolons originate appropriately from two or more commas, although there are instances where it is uncertain if one should say semicolon or comma. Yet the comma originates through arsis and thesis, that is, the raising and lowering of the voice. At times, the voice raises and lowers in a

\[
\begin{align*}
11 & \quad 14 \\
[4:3] & \quad [7:5] \\
12 & \quad 15 \\
13 & \quad [6:5].
\end{align*}
\]
simple arsis and thesis only once, at other times, more often.

The distance between the highest and lowest voices of a comma is called an interval [diastema]. The intervals are sometimes smaller, as that which we call a tone [tonus], sometimes larger, that is to say, an interval which comprises two, three, or several tones. In addition, as the semicolon consists of commas, we can also call the distances between the commas intervals [diastemata]: those of the colon or the entire melody we call systems [systemata].

Chapter X
The Symphonias

Not all written sounds [praemissae voces] combine in an equally pleasant way, yet, in every kind of union they produce a harmonious effect in song. Just as if you were to join letters at random with each other, quite often neither syllables nor words would be produced in these unions. Thus, in music there are certain intervals, symphonias, which are appropriate to produce. A symphonia is, therefore, a pleasant harmony [concentus] of the mutual joining of different sounds [voces].

There are three simple and principal symphonias from which the remainder are constructed. The first is called the diatessaron, the
second is called the diapente, and the third is called the diapason. Diatessaron is translated "through four," because it either resounds mutually with the fourth sound, or because it is a successive arrangement of four sounds, as you observe in the following diagram if you descend to the fourth tone, or if you count four in order, in this manner:

Description of the diatessaron symphonia:

In this way they proceed with variety, four in a group in either direction, and they return back in a different direction.
Diapente is translated "through five," because it either consists of five sounds (soni) joined together, or because the sounds (voces) answer harmoniously with the interval of the fifth, according to the following description:

Description of the diapente symphonia:

In regards to this description, the diapente symphonia can be sung from each of the four sounds to the fifth, which has the same name, through arsis and thesis, or you can lead each in order to either side. Furthermore, if you sing one of the groups of four in the following short representation, the same will answer harmoniously with the fifth line, which yields the characteristic diapente:
Another description of the same diapente:

So it is with the diapason, which is translated "through eight," from the eighth to the eighth is a consonance, containing two greater ones (that is, the diatessaron and the diapente) in its system. This symphonia is therefore called "through all," because the ancients used no more than eight strings. In this way, then, they cannot be called harmonious sounds, but rather, equal-sounding, and in the diapason the sound is regenerated. Now this can be seen on musical instruments, but if they are not available, hold another voice on any sound you please, and take another up and down in order, four by four, and hear it replace the perfect consonance from the
first tone, while it makes the newest sound, that is eighth to eighth. Therefore, as these equal-sounding tones are sung, they are brought together in this way:

16

[The use of a letter notation to illustrate the diapason (and later the double diapason) is necessary here because there is no way of notating the chromatic alteration of some of the dasian notes which would be necessary to make them perfect octaves (i.e., diapasons). The letters here do not indicate exact pitch (they would be backwards as a comparison of this example with the previous example illustrating the diapente will show), but are used here only in a geometrical sense to indicate a diapason (compare this with that given in Ch.XI). The example in modern notation was derived by starting each series on each note of the Finales tetrachord. All chromatic alterations are noted with an asterisk. These examples, containing seven letters, the eighth of which is the same as the first, could well have been the impetus which led to modern letter notation.]
Chapter XI

How the other symphonias are composed
from the simple symphonias.

From these simple symphonias the other symphonias are com-
posed, such as the diapason with diatessaron [eleventh], and
the diapason with diapente [twelfth]. The disdiapason [fif-
teenth], which is also called the disdiaplasion, is formed
if a third equal-sounding boy's voice is joined with two male
voices singing together at the diapason. In this way, the
highest and the lowest voices mutually render the disdia-
pason. The voices make this symphonia harmonious at the
fifteenth interval. The middle voice, however, answers at
the diapason on either side, as this diagram shows:
Consonance does not really arise from any one particular sound at the fourth, fifth, or eighth position, but rather, as I may say, individual sounds both low and high respond with the diatessaron at the fourth sound from themselves, and individual sounds respond with
the diapente at the fifth sound from themselves, and any sound which resounds on either side of itself with the eighth sound is a diapason. Each, in turn are at the eighth position on either side, as there is a diapason at the middle. Thus, as has been said, individual sounds which resound mutually at the fifteenth interval from themselves comprise a disdiapason. As always, the diapason is made complete by the diatessaron and the diapente. In fact, there are two middle sounds in this space where the diatessaron and the diapente resound on either side, and it so happens that if one of them on one side is a diapente, on the other side it will be a diatessaron, and the side which is a diatessaron, the other side will be a diapente. In this way, the greatest symphonia is made complete by two lesser symphonias. Moreover, the diapason is called the greatest symphonia because within its exteriors a consonance is made perfect. Whether you begin from a higher or lower one, the sound which you would change at the eighth space [ordine] into either the higher or the lower would be held as the same sound [unisonus] by the first voice singing in this manner:
Thus, indeed, the natural sequence progresses into an infinity of sounds when the sound is regenerated at the eighth step, so that in this manner a new order emerges with the voice, and according to the custom of the day, the eighth is the first and the first is the eighth. And so in Vergil, Orpheus in Elysium sings to seven distinct sounds,
because, of course, the order of sounds is contained in seven distinct sounds, but at the eighth it is changed into a new one. ["""] Even if something is added to the number ten, that is, placed within it, it is preserved whole and inviolate. Since it happens this way in the least other things, so it happens this way in musical harmony, for if you add two to three, you always get five, and the species of the number is not changed. If you add the same number to ten, you get twelve, and the quality of the two is preserved even though it has been joined to ten. The same is true for three, and so on, in the same manner. In this way, the diapason symphonia preserves the other consonances which it receives: it neither changes it nor gives back dissonance for consonance. ["""] 17 Hence it is with the fifteenth, because there are two eighths combined consonantly with themselves. In this same way, each pitch resounds through the diapason to the twelfth, just as through the diapente at the fifth; and in the same manner with the fourth, so that at the eleventh you get the diatessaron. A marvelous bit of reasoning must be mentioned, that is, although both in singing absolutely [absolute canendo], or in examining the sounds in their order, in either case, the same ninths are found at the ninth places, but the same eighths are not found at the eighth places. 18

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17 [A direct quote from Boethius, De institutione musica, Bk. V, Ch. X (Friedlein ed., 360). Compare with GS I, 163-164.]

18 [i.e., not all of the octaves in the system are perfect.]
symphonia, however, not only the diapason, which occurs at the eighth place, but also the double diapason, the eighth [sounds] and the eighth [places] by a miraculous change, are the same.

Chapter XII
Also regarding these
same symphonias.

While the four little spaces in which the first symphonia is found, which is called the diatessaron, each differ with their own concordant property as was already stated, their harmonic quality is woven by the elementary tetrachord, and just as for the fourths and the fourths continually following each other in the natural sequence, it is necessary to respond the symphonia of this arrangement or quality to themselves, which is called the diapente. Although something has already been said regarding the diapente as it has been regarding other things, in order that those things which must be said in succession can be explained more clearly, a diagram of lines has again been appended and an explanation of this diagram is given for consideration. Just as before, there is a progression of lines with the symbols of the sounds put in place, and the diapente symphonia is arranged between the lines as follows:
When, according to the reasoning of this diagram, one after another is sung at the fifth interval, you can hear that it harmonizes with itself at the diapente symphonia with a symbol of the same quality. In truth, the natural division into four parts distinguishes by its own nature the positions of tones and semitones, and it is not able to bring about that which is in these four sounds, and those single sounds which are united. And so no melody is able to remain in this very arrangement of tones and semitones if it is transposed from one sound to another. We have touched upon this and other small matters in the above, and now, by repeated examples, it must be undertaken

19 [i.e., two adjacent symbols with the same name always form a diapente with themselves.]
again more fully. Let there be put forth as before the series of tetra-
chords, and let the same melody depicted in the diapente symphonia be
represented as in the fourth or fifth description, and you will see
that the same form of the melody is not able to remain in its own
transposition, but through the distance of a tone or semitone it is
transformed into another mode, in this manner:
If you make the first representation, which begins and ends on the sound $\mathcal{F}$ and is considered the first mode, higher by an entire space so that it designates a little space thrown in between the line, it is quickly changed into the second mode which begins with the sound $\mathcal{G}$ and ends with the same. Make it higher by two, which begins at the sound $\mathcal{I}$, and it will be the third mode. If you make it higher by three, the fourth mode arises. If it is carried one space higher from here it will be at the fifth place and once again in the first mode. And so, in other respects, it reverts back from the fifth to the first series.

Chapter XIII

The particular nature

of the symphonias.

It has already been demonstrated that any one sound in the tetra-chord is different because of its own position from another sound, and that different modes or tropes may come forth because of the concordant diversity. And it has been stated also in which places united voices, that are equally distant, reproduce particular symphonias. Now, let us pursue how symphonias are properly defined and named, that is, when
these voices position themselves by singing together. This is what we call diaphonic song, or customarily, organum. It is called diaphonia because it does not remain in a uniform melody, but is in an agreeable concordant difference. Whatever is common to all symphonias in the diatessaron and diapente has this name. First, an example of an organum melody is placed at the diatessaron, as in the appended diagram, where two sounds have been interposed; each voice, by singing as one at the fourth interval, corresponds to the other, in this way:

\[
\begin{align*}
	\text{F} & \quad \text{es} \\
	\text{G} & \quad \text{pa} \\
	\text{E} & \quad \text{fi} \\
	\text{D} & \quad \text{es li} \\
\text{Tu} & \quad \text{tris sempiternus us.} \\
	\text{pa} & \quad \text{fi} \\
	\text{li} & \quad \text{us.} \\
\end{align*}
\]

Thus, indeed, by combining two or more voices as one, in so far as it is done with moderate and concordant capriciousness, because that is the nature of this melody, you will see a delightful harmony arise from
this mixture of sounds.

The simple voice, not only in this collection, but also in simple organum, if it is added to another voice responds to the duplum, or the duplum responds to the simple voice, or else, if you double both at the diapason, you will hear that the voices in these proportions sound smoothly together, as is shown in the following diagram:

Chapter XIV

The higher diaphony at the diatessaron, and its description. 20

```
     es
t

     tris sempiternus/ \ fi /
     pa/  

     Tu  es  us.

     tris sempiternus/ \ fi /
     pa/  

     Tu  es  li

     tris sempiternus/ \ us.

     pa/  

     Tu  es  li

     tris sempiternus/ \ us.

     pa/  

     Tu  es  us.

     tris sempiternus/ \ fi /

     pa/  

     Tu  us.
```

20

[It seems more sensible that the heading for this chapter should go right before the preceding paragraph.]
Explanation of this diagram:

Whether you add a duplex organum to a simple song as is shown in the first and third lines, which take the position of the organum, or whether a simple organum is added to a duplex song which the second and fourth lines show, containing an organum in the middle, or if you double both the organum and the song, or even if you triple both of them, all harmonize mutually according to the rationale which has been described. In both human voices and in some instruments, not only two and two, but also three and three at a time can be mixed together in this arrangement, while as if by one impulse the three voices are either engaged as one, or else all of the voices together respond to the organum. Notice that whenever that happens, the middle
voice between the two others may not be an equal distance from either of them, since when at the eighth there can be no middle of the unity. If it is answered at the lower side by the song at the diatessaron, it will be answered at the higher side by the diapente. And as this is clearly inserted for the uninitiated, without fastidious knowledge, if it is organized with a man's voice and a boy's voice together, these two voices are consonant with each other at the diapason. To the voice which they hold between them in the middle, to which of course either organum responds, the higher one, which is the boy's, projects higher, from the fifth; that of the man's projects lower, from the fourth. Therefore the symphonias tie each other with a similar mutual bondage, so that whenever a voice is at the fourth with another, another part at the diapason from the first voice will be at the fifth from the part which is at the fourth from the first voice.

Chapter XV

A description of higher diaphony

by means of the diapente.

When at the diapente, diaphony is sung by three or four voices, and it is necessary that the voice which is at the fifth below answer the other voice which is at the diapason above it with a fourth from the middle voice. This is accomplished in the following manner:
Explanation of this diagram:

The four-part series of this diagram is divided with the same arrangement, in which the superior part in the first and the third lines, responds to the organum in the middle, in the second and fourth lines. 21 The third line, which is in the middle, mixes with the simple organum at the consonant diapason, and thus, indicates twin organum with a

21 [The organal voice must be altered at Deuterus ∨ to avoid the tritone with Tritus / in the principal part.]
double cantus. You also observe that the third line from the lower part
is combined with the second line at the fifth interval, and that on the
other side, the third line is combined at the fourth interval, and that
the first line is consonant with the second line at the fourth. Follow-
ing these explanations, these two symphonias mix varied and sweet songs.

Chapter XVI

What Boethius relates regarding Ptolemy's

thoughts on these matters: consonance,

particularly at the eleventh.

At this point, we are able to observe the truer assertion of Ptolemy
brought forth, an assertion in which, as Boethius recalls, Ptolemy
disagreed with the Pythagoreans who said that the eleventh is not con-
sonant. Certainly whenever you sing the description of these with both
double cantus and organum, thus a double voice is joined with a double
voice, just as if a simple one were added to a simple voice. Likewise,
if singing as one we join the second and third lines of the previous
example at the fourth interval below, with a fifth line, in the usual
way the diatessaron will harmonize simply with the diapason. Let
us sing the fourth and second lines of the previous description, while
joining them with the first, and in a similar manner the same consonance is harmonious with the diapason. The same thing follows from the opinion of Ptolemy. The magnificent teacher [i.e., Boethius] foretold it in the fifth volume of his musical rules by placing the higher sounds at the bottom and the lower sounds at the top, in the following manner:

22 [That is, the diatessaron harmonizes with the diapason forming an eleventh.]

23 [De institutione musica, Book V, Chapter IX. There is no chart like the one given here, in Chapter IX of the Boethius text. Compare Friedlein ed., 358-360, with GS I, 167-169. There is a similar chart, however, in Boethius, Book IV, Chapter XIV, Friedlein ed., 341.]
He said that the diapason consonance makes such a total joining of sound that it appears to be one and the same string. However, that is what the Pythagoreans believe as well. Wherefore, if any consonance
be added to it, it would remain whole and inviolate. In this way the
diatessaron is added to the diapason consonance as if it were added
to one string. Let the diapason consonance be that which is confined
between hypate meson and nete diezeugmenon. Each of these thus harm-
onizes with itself, and so is joined in sound as if it were one voice
like one string, not like a mixture of two, and thus it plays what is
heard. Therefore, whenever we join a consonance to this diapason
consonance, the whole is preserved because it is joined in such a way
as to one soft voice and to one string. If, therefore, the hypate
meson and the nete diezeugmenon were joined together at the high dia-
tessaron, then the nete diezeugmenon would be joined to nete hyper-
bolaeon. The hypate meson, however, is that which is in the middle,
and each will harmonize with each, that is, both the mese with nete
diezeugmenon, and the same mese with hypate meson. Likewise the nete
hyperbolaeon will harmonize with nete diezeugmenon and with hypate meson.
Likewise if at the lower consonant diatessaron, the diatessaron conson-
ance will be retained between hypate meson and hypate hypaton. With
nette diezeugmenon it harmonizes with paramese, and both hypate hypaton
and hypate meson will harmonize with nete diezeugmenon. The diatessaron
retains a consonance at paramese with nete diezeugmenon and hypate
meson in the following way: since it is lower, it retains the consonance which is closer to itself. At the farther side is the eleventh, since hypate hypaton is a diatessaron with hypate meson, and an eleventh with neta diezeugmenon. The same with neta hyperbolaeon, which is the higher, the neta diezeugmenon closest to itself is a diatessaron consonance. With hypate meson it is an eleventh, et cetera. So spoke Boethius.24

Chapter XVII

The order of the consonances;

consonance and inconsonance.

There is no need for further explanation regarding the diapason and the double diapason because they occur so naturally in the singing of every era, and since they do not lack the means of being handed down. It suffices to say in regards to the double diapason symphonia that the middle is conferred upon each at the eighth interval, and that these extremes answer at the fifteenth. It must be said, however, what the proper standard of measure is for the individual symphonias. The diatessaron symphonia encompasses two tones and a semitone, [the diapente symphonia encompasses three tones and a semitone,] and the diapason symphonia is made from these two symphonias. Therefore, most emphatically, in the diapason symphonia beyond all others, the different

24 [This entire chapter is taken from Book V, Chapter IX of the De institutione musica.]
voices mutually resound with a greater perfection.

Next is the diapente symphonia. But, on the other hand, in the diatessaron, because the pitches are not smoothly concordant with themselves in the entire series of sounds at each fourth place, as they are in the other symphonic songs,\textsuperscript{25} therefore, in this kind of song, the voices are divinely adapted to the other voices by an intrinsic law. Throughout the entire series of sounds, only Tritus at the lower fourth is not consonant with Deuterus above it. It is made inconsonant because the diatessaron exceeds the proper measure of a diatessaron symphonia, and is stretched out into three whole tones where it is a fourth below Deuterus. Therefore, the organal voice usually accompanies the other voice, which is called the principal, in such a way that in any tetrachord, or in any phrase [particula], it neither descends below Tetrardus, nor raises up at the beginning from below it, since the inconsonant sound Tritus is in the way. In order that these things may become clearer, an example is set up in a diagram for your inspection:

\[\text{[Diagram]}\]

\textsuperscript{25} [That is, in parallel organum at the diapente and diapason.]
It is easily understood from singing this example in the diagram, how, in a two-part example, the organal voice is not able to begin the response below the sound \( \mathfrak{X} \) Tetrardus, and it is likewise not able to move up from below the same sound. Because of this, the final position is occupied by the principal voice, so that both come together as one, because the organum seeks the higher position sometimes, and sometimes it seeks the lowest.

Chapter XVIII

How the organum seeks the higher position,

and how it seeks the lower position.

Therefore, because \( \mathfrak{X} \) of the organal voice of the previous example is confined by the short opposite interval, and the vocal range is only four spaces, it changes the position and the places of the phrase, following the position of the moving phrase. The song proceeds in the higher part and the phrase first takes its position around the Superiores sounds, next around the Finales sounds, and then
around the Graves sounds, for the organal voice, by its own law conforms to the final position, so that the Tetradus sound below, in which either end of the phrase arrives, or which is below the final which is closest to itself, can neither begin by rising from below the final, nor end below the final. For example:

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In the same way as in the previous two part Rex coeli Domine maris undisoni, and Tytanis nitidi, the first three syllables, which sound Tetradus †, Archoos †, Deuterus †, do not have an organal response below Tetradus. This clearly is because the Deuterus sounds have inconsonance with the sound Tritus, which must be followed by Tetradus. The same is true for the examples here, Te humiles famuli, and Se iūbeas flagitant, which are conspicuous in their high range in Excellentes, and also in Superiores, where the organum is also cut short by this rule. Similarly at the three principal sounds Tetradus †, Archoos †, and Deuterus †, the organal voice, according to the rule, cannot descend below Tetradus, and pauses at Tetradus, and although it closely follows it, it does not reach the established response.

And as it is correctly distinguished in a clear voice, as much as the inconsonance of the aforementioned two sounds in the diatessaron symphonia is diminished, let us establish a song with four transpositions, so that this song may demonstrate how one voice follows another in different transpositions—indeed, in a different manner, but not with different rules applying to each. Thus is given the first diagram, the top part of which is arranged so that the sound begins and ends on †, while overall, the organal voice is patterned exactly after the principal voice:

26

[Chapter XVII.]
You can understand this by singing, since the dissonance [absonia] of the Deuterus $\ell$ and Tritus $\ell$ sounds was not interfering. The lowering of the organal voice at the fourth below was imposed consonantly, but at the beginning, where the same dissonance was impeding the Tetrardus sound, the position of the organal voice did not go to the fourth below.

Let us undertake a second transposition which begins and ends on the sound $\ell$:
You can understand by singing, just how the organal response is lacking in the lowering of it and at the position at the Deuterus $f$ sound. Thus, it does not descend below Tetrardus $g$.  

Let us undertake a third transposition which begins and ends on the sound Tritus:

This transposition, since it advances principally to the sound Deuterus $f$, appropriately does not have an organal response.

Let us undertake a fourth transposition which begins and ends on the sound Tetrardus $f$:

27

[There is no mention of the tritone which occurs on "es," which has been altered in the organal voice to make it a perfect fourth.]
Thus you can understand by singing just how the organal voice begins at Tetrardus $\frac{4}{5}$, and ends there also. Nor is it able to progress beneath Tetrardus—naturally not, with the sound Tritus $\frac{4}{5}$ in the way, which, as I have often said, exhibits inconsonance with Deuterus.

A certain superficiality of ars musica is somehow shown in these things regarding the ornamentation of ecclesiastical songs. Certainly, this does not exhibit less reverence and contemplation of the more profound. For why some things agree with so sweet a mutual mixing, and other sounds being hesitant to mix with one another are inharmonious, is a matter of profound divine thought, and in some ways is among the most secret of hidden nature. Many things remain beyond this written tract of the ancients, into the labors of which, and in this part, God allows us to enter. For it is constructed by the best arguments, because this
thinking, which governs the harmony of voices and regulates the nature of mortals, follows the same rule, and because this thinking uses the same parts of numbers, thus, unequal sounds harmonize after they have been gathered together, and souls harmonize with bodies, and all the world joins in an eternal harmony.

Chapter XIX

The fact that in some of this reasoning the profundity may be less penetrable.

There is a fictitious story from the ancients regarding Aristeus, who fell in love with Eurydice, the young wife of Orpheus. While she was fleeing from the advances of Aristeus, she was bitten by a snake. We understand that the name Orpheus comes from Oreophone, that is, the best voice, such as a skilled singer of sweet sounding songs. Eurydice represents divine judgement, and Aristeus represents a good man who pursues her because of love. She is taken away by the serpent, as if by divine wisdom, lest she be possessed [by Aristeus]. But while she is recalled from her own solitude in the underworld by Orpheus, that is, by the best sound of a song, she is led forth as a shade into the world of the living, and although she appears to be seen, actually she is absent. And because, among other things which we discern here in part
and enigmatically, this discipline hardly has a comprehensible rationale in this life. If, indeed, we are able to understand, indeed, if there are these explanations of song, and if we are able to comprehend the qualities of the sounds and modes, and the other things of this art, likewise, we are able to deduce the spaces of musical sounds or the symphonias of voices by the reasoning of numbers, and so, we are able to produce certain rules of consonance and inconsonance. In truth, in this way, if we know what alteration and association music may have, once we have agreed on a certain similarity, we are not able to speak about it in a clear voice; not only are we able to discern melodies according to every nature of the sounds, but also according to the nature of other things. It is fitting for the disposition of things which are sung that the disposition of song be imitated, as in tranquil things, the neumes will be tranquil, they will be happy in happy things, sad in sad things. Those things which are spoken or done harshly should be portrayed with harsh neumes, with sudden, noisy, excited, and according to the other qualities of the events. Likewise, the phrases of the neumes and of words should end together.

Although our good judgement prevails in such things, many other things, however, lie hidden; things which are due to causes hidden from us. While there are some things which are received harmoniously by both this tone [tonus] and that tone as well, so that they are able to be sung, there are other things also, which attach to this or that tone.
with ill effect, so that if they are transposed, either they do not preserve the original sweetness, or else they are harsh to the senses. Beasts and birds are said to be pleased by certain modes more than other modes. But how or why these others exist is not easily investigated. Therefore, what we know in this art, by the gift of God, we use in the praise of God, and those things which have reached us because of the laborious investigation of the ancients, let us receive in rejoicing, celebrating, and singing, which in previous generations were not known to the sons of men, but now have been revealed by the holy ones. The most eminent author Boethius revealed many miracles of the art of music, clearly proving all things by the use of numbers, an excerpt of which, God willing, the following work will contain. With this we close this short treatise.
Part I

Student: What is music?

Master: The skill of good singing.

Student: What is good singing?

Master: Controlling a melody by a pleasant sound. But this is to a great extent according to art. I consider one not well sung if the pleasantness of art is abused in a false way. Art, used where it is proper is not new, however, any pious heart sings sweetly before God.

Student: I believe that is so.

Master: You believe correctly. Only through good practice do songs become sweet. On the other hand, it is not good if sacred songs are revealed unpleasantly without discipline. For this reason, these ecclesiastic songs require strict discipline. So they are not spoiled by negligence or ignorance, we look to the means of good singing.
Student: I see that there is much which a singer must watch out for, which, if he does not alter he will not be skillful. But it is for you to explain these things precisely.

Master: There are some things which the quality of sounds requires for itself; others, which reason demands of rhythms; others, with which the discipline of singing closely shapes itself to occurrences on the outside.

Student: What are these sounds?

Master: We call these sounds pitches \([phthongi]\), that is, soft notes in concordant harmony. For just as speech has letters, so too does harmony exist in pitches.

Student: What are the things which the quality of the sounds require?

Master: That anything not become a discordant error in them.

Student: How does this discord occur in pitches?

Master: It occurs if they are either lazily or harshly sung. This error in the human voice first spoils not only the quality of the pitch, but also the whole song. This happens when through laziness, the pitch falls or is not properly raised. This error cannot occur with a musical instrument because
in the disposition of intervals each note has its own pitch. Another error occurs when the interval from one note to another is measured falsely, that is, one note instead of another. The third error arises when a sound does not respond to another sound by the proper interval. These last two errors arise from the same source, but they differ in that they are in the same neume, but vary in presentation and response.

Student: Explain how these last two errors arise.

Master: Four diverse sounds coincide to form harmony, which in their proper order retain a certain quality, and they are not moved from their fixed places. I believe that you know this note series.

Student: I am committed completely to your instruction. I would like to hear you sing these sounds.

Master: Here, I shall sing them:

\[ \text{\image{diagram.png}} \]
We have received the names of them from the ancients. The first, that is, the lowest, is Protus or Archoos. The second is Deuterus, which differs from Protus by a tone. The third is Tritus, which is a semitone distant from Deuterus. The fourth is Tetrardus, which is distinguished from Tritus by a tone. The signs are notated in this way: the first is the dasian note \( \text{\textbackslash,} \), written with an inclined \( \text{s} \) at the top, in this manner: \( \text{\textbackslash,} \). The second has an inverted \( \text{c} \) at the top, in this manner: \( \text{\textbackslash,} \). The third is a simple and inclined \( \text{i} \), written in this manner: \( \text{\textbackslash,} \). The fourth has a supine \( \text{c} \) at the top, in this manner: \( \text{\textbackslash,} \).

Student: How is harmony composed from these four sounds, and not instead composed from many sounds?

Master: It is clear that the sounds of songs are innumerable, but from the raising and lowering of these four and four of the same quality in the natural order of sounds, a multitude increases. I will sing them properly:

\[
\text{\textbackslash,} \text{\textbackslash,} \text{\textbackslash,} \text{\textbackslash,} \text{\textbackslash,} \text{\textbackslash,} \text{\textbackslash,} \text{\textbackslash,} \text{\textbackslash,} \text{\textbackslash,} \text{\textbackslash,} \text{\textbackslash,} \]
Therefore, in this way, in all concordant sounds, a song advances in ascent and descent through these tetrachords, and the four interrelated diverse sounds are related to the different sounds by combining in a continuous sequence until either they run out by ascending too high, or fall silent by descending too low. Furthermore, single sounds, from the previously mentioned four, command the tetrachords. Likewise, single sounds command the pentachords. Tetrachords, of course, are individual sounds placed in order by fours, in this way:
Pentachords, on the other hand, have one note added to the higher part, the same note becoming the first and the last note, in this way:
In this way, we call that which begins and ends with the sound *Archoos* the first pentachord, and it consists of two tones, a semitone, and a tone. That which begins and ends with the sound *Deuterus* is the second pentachord and consists of three tones and a semitone. That which begins and ends with the sound *Tritus* is the third tetrachord and consists of a semitone and three tones. That which begins and ends on the sound *Tetrardus* is the fourth pentachord and consists of a tone, a semitone, and two tones.

**Student:** What is a semitone?

**Master:** We call the semitone, or *limmata*, that which is not a full interval of sound. However, it is joined at the appropriate place and is assigned properties of sound, and it insures the sweetness of harmony in sounds. If it is not joined at the proper place it causes the melody to be different. One must know, then, in a series of sounds, the usual natural qualities, while they measure mutually the natural interval. When the distance from one sound to another is not measured correctly, it later changes it into another quality and changes the order.
Student: Will you give me clear examples of how they possess such discords [absconia] with themselves?

Master: I will attempt it as well as I can. Pay close attention. The sound Deuterus $\mathcal{F}$ is always connected to the sound Tritus $\mathcal{J}$ by the interval of a semitone. Tritus is connected on its higher side by Tetrardus $\mathcal{F}$, and Deuterus $\mathcal{F}$ has Protus on its lower side in this manner:

$\mathcal{F}$ [G]
$\mathcal{J}$ [F]
$\mathcal{F}$ [E]
$\mathcal{F}$ [D]

Note that the two pitches $\mathcal{F}$ Tetrardus and $\mathcal{F}$ Protus can be in any tetrachord you chose. If in ascending, the next sound after $\mathcal{F}$ Protus is measured as Tritus $\mathcal{J}$, as after $\mathcal{F}$ Deuterus,\(^1\) it will be discordant. The same is true if descending in order, if the nearest sound after $\mathcal{F}$ Tetrardus is measured $\mathcal{F}$ Deuterus, as after $\mathcal{J}$ Tritus, this will also be inharmonious.\(^2\)

---

\(^1\) [I.e., if Deuterus (E) is read as a half step from Protus (D), making Deuterus sung as $E^\flat$.]

\(^2\) [If it is sung as a half step instead of a step: $F\to F\#$.]
Student: How?

Master: Recite the pentachord by counting upwards from $\mathfrak{f}$ Tetradus, and descending the same by steps.

Student: I will recite it:

Master: I will also recite the same, and then change the given order, namely at the third place, which will be measured Tritus instead of Deuterus, in this way:

Now, do you perceive this pair of pentachords as being mutually the same?
Student: I hear and perceive clearly that the second pentachord does not return the same order of sounds with which it had begun.

Master: It is thus: beginning from *Tetrardus* 7 and ending with 8 *Archoos*, those on the first side do not reach to the measure of *Deuterus* 6, but the sound 7 *Tritus* is measured with a short interval at the position of *Deuterus*, because the line does not designate an interjected *paginula line*. Recite also the tetrachord from 8 *Protus*.

Student: I have recited it.

Master: I will also recite the same; next, I will deviate a little from this order so that on the second side the sound 6 *Deuterus* follows *Tetrardus* 7 like *Tritus* 8, in this way:
Do you perceive that this pentachord deviates from the original order and does not return or agree with the original order?

Student: I vaguely perceive it.

Master: Do you see that it begins with the sound $\text{Protus}$, and ends with the sound $\text{Tetrardus}$?

Student: I certainly do see that.

Master: Do you also notice how, if the short interval is used on both sides, it returns to the sound from which it began? This is true in the pentachord of Tetrardus:
Is this clear to you?

Student: It is certainly clearly revealed visually and aurally that in neither side does the order remain unchanged.

Master: The limmata here is usually called "not a full interval," and through it, the mode is occasionally changed or restored. This may be observed fully in singing, for example.

Student: Can we actually consider these things as defects?

Master: Certainly they are defects. However, just as many mixed figures are measured carelessly and incorrectly, the
liammatata occasionally results from diligent singing.

In addition, we will also see another. The third dissonance, for instance, is the opposite of the first. That is, if the interval from Deuterus $\sharp$ is measured incorrectly on the first side and properly on the other side, it returns to the Tetrarius pentachord creating this species of error:

[Diagram of musical intervals]

Student: I understand this dissonant one also.

Master: Consider also the fourth form of dissonance in this manner, which is contrary to the third. That is, if, on the second side another Tritus $\flat$ is measured from Tritus $\natural$, as if it were from Tetrarius $\natural$. This is the pentachord from Protus:
Student: I recognize it because in place of Deuterus ⊃, the sound / Tritus is measured—a false Tritus from / Tritus, although the space was measured longer from / Tetrardus than is correct.

Master: If, for instance, we connect such pentachords, because these errors break each side in both directions, it becomes dissonant. It may be the Tetrardus pentachord:
Student: It is certainly a dissonance and not a pleasant thing you resound.

Master: Consequently, by perceiving this, as a sound is measured from a false sound, we see how it happens, so that if it is correct that sounds do not respond to sounds from these places, concordant melodies do not resonate.

Student: But I request it at any rate.

Master: Come see and also hear how, in tetrachords or pentachords the order of the four differences is distributed, so that you will clearly observe that any sound from another sound correspond. Just as in colors, if there are four and four and they are arranged from the order and arranged in order, such as red, green, yellow, and black. It is necessary to determine how three others inserted over them produces a fifth place. Also, in regards to sounds, it occurs that something new always follows repetition, which corresponds to the fifth place where it is joined.
Student: How do you compare these that are divided high or low?

Master: Indeed, the high and low are different, but, however, they are certainly related concords. For example, each sound, perhaps the second on one side, may be the fourth on the other side. The third on one side may be the third on the other side. But this indicates which of the four varieties is arranged in the continuation of the tetra-chord.

Student: I would also like to get to know the particular qualities of each in order to distinguish which sound is which.

Master: You are right to ask. When the particular quality of the first sound is clear, one can err only slightly. This knowledge is obtained from an easy exercise. Select a song which ends on the sound Archoos, for example. Next, another is joined which begins either with the same tone or with its relative either higher or lower—either it begins from the sound Tetrardus, or from the sound Deuterus. Therefore, unless it is united and begins with the sound Archoos, or you set down the equal to the final sound of the preceding melody to the same place, strictly speaking to Archoos, or to the fifth place
higher, or to the fifth place lower. It, however, which begins from the sound \( \Upsilon \) *Tetrardus* begins from the second sound below, or from the fourth above. And that which begins from the sound \( \Upsilon \) *Tritus*, begins from the third place below, or the third place above. On the other hand, it, which begins from the sound \( \Upsilon \) *Deuterus*, begins either at the second place higher, or the fourth place lower, as is shown in the following example of sounds.

\[
\begin{array}{cccccccc}
\Upsilon & \Upsilon & \Upsilon & \Upsilon & \Upsilon & \Upsilon & \Upsilon & \Upsilon \\
\end{array}
\]

There is very little which follows which can be concordant with that which precedes it. And thus it happens that in all sounds, as namely in one agreeable body, that which precedes and that which follows, if not able to be joined, the first above or below is able to be joined mutually because the end of one and the beginning of the other is properly measured. Therefore, when it is required that this joining occur, one must ascertain the arrangement so that the beginning sounds mutually follow the closing sounds in order naturally. When this is neglected, or the work is not observed, this requirement will not be met. Certainly songs are able to proceed harmoniously within themselves, but they will not unite with each other to

\[3\]

[Read quartio for Gerbert's tercio.]
form a harmonious whole. Therefore, it is the duty of experienced singers to know where joined harmonies come together and where it is not necessary. Enough has been said about this discrepancy.

One should know that the first concord is that which makes the melody from the above method. Another concord, a little less used, where one wishes to avoid the difficulty of high or low, is to transpose it either a fifth higher or lower. The third concord is that at the eighth region of sound, that is, when a young or high voice transforms the melody. And with some combinations of songs in which the unanimity can be observed, and others where it cannot be observed, it is firmly changed by transposing the entire melody into the other mode. When one transposes a melody in the same series of sounds one, two, or three intervals higher or lower, at the same time, the mode moves to another species.

Student: Give me an example of this moving.

Master: I will sing the first five tetrachords or pentachords in the following manner:
In this manner, by singing the pentachord of the first mode one space higher, it is transposed into the pentachord of the second. The second is shifted to the third, the third into the fourth, and the fourth returns again to the first. Thus, the further it is transposed entirely by one, two, or three tones higher or lower, the same it is changed into another mode. For example, we accept this as the usual note pattern for the first mode:

\[
\text{No an no } \varepsilon \alpha \text{ ne}
\]
When the entire order of this pattern of neumes is moved totally one space higher or lower, the tone [mode] Protus becomes Deuterus. Similarly transposed from Deuterus, it becomes Tritus. If it is raised from the tone Tritus by one space, Tetrardus follows. When it is taken one space higher, Protus is produced again. When you are shown a short example, will this become clearer?

Student: Yes, it certainly will.

Master: Well then, we will arrange short examples of these in a line so that they are connected step by step in order to clearly show the transpositions of the tones, in this manner:
The first example is the mode Protus, the second Deuterus, the third is Tritus, and the fourth Tetrardus. The fifth is the same again as the first. So, in this manner, as was said, whenever you transpose a melody, it is transformed into another species of mode. If the mutation of the trope is major, the trope will become major also; into whatever one it was changed. The tropes or scales are improperly called tones. I believe that you have already separated the differences of them individually and together.

Student: Nevertheless, separate them.

Master: Indicate songs of the first mode, the second, and those remaining.

Student: Here is one in the first mode, here is one in the second. This is in the third, and this is in the fourth.

Master: Now tell how they are judged by the ears with a certain diversity to be mutually different and what the cause is of the diversities.
Student: One may indeed perceive the mutual differences up to a point by the properties of each species. One must admire the cause of these differences.

Master: So that this can begin to be clear, sing the tetrachord, or better, the first pentachord.

Student: Here, I shall sing it:

\[
\begin{align*}
\text{f} & \quad [A] \\
\text{g} & \quad [G] \\
\text{f} & \quad [F] \\
\text{e} & \quad [E] \\
\text{f} & \quad [D]
\end{align*}
\]

Master: Apply the mode Protus to this which you have sung, and you will see all which are sung to the mode Protus end with the sound Protus.

Student: Yes, I clearly see this.

Master: Now sing another pentachord which is set one step higher, beginning with the sound \text{f} Deuterus, and proceeding in the same manner.

Student: This is sung as follows:
Master: Again sing another melody in the mode Deuterus, and compare it with the sound $f^\uparrow$ Deuterus, which it perhaps ends with.

Student: It does indeed.

Master: Now sing the third pentachord, which is two steps higher and, of course, begins on the sound $f^\uparrow$ Tritus and ends in the same manner.

Student: This is the manner: $f^\uparrow [C]$

Master: Apply the mode Tritus to this and you will discover that it ends with the sound $f^\uparrow$ Tritus.

Student: It certainly does.
Master: Now sing three steps higher, that is, the fourth pentachord from the sound $\overrightarrow{\text{Tetrardus}}$.

Student: It is sung in this manner:

$\overrightarrow{\text{D}}$
$\overrightarrow{\text{C}}$
$\overrightarrow{\text{B}}$
$\overrightarrow{\text{A}}$
$\overrightarrow{\text{G}}$

Master: Now look at the mode $\text{Tetrardus}$. Do you perceive these to consist in the sound $\overrightarrow{\text{Tetrardus}}$?

Student: I perceive this most clearly.

Master: Now sing the fifth pentachord and the new pentachord reverts back to the same order of sounds and of mode.

Student: This is certainly amazing:

$\overrightarrow{\text{E}}$
$\overrightarrow{\text{D}}$
$\overrightarrow{\text{C}}$
$\overrightarrow{\text{B}}$
$\overrightarrow{\text{A}}$

Master: As I was saying, the individuality of the sounds modify all melodies. Observe the admirably variable effect of
these four pitches and the special manner whereby each differs from the others and forms the differences of the modes.

Student: I don't understand, however, why from these four sounds only four modes result, and why from these four we usually count eight.

Master: We usually count eight modes, so that two modes, the major and the minor are singularly guided by a sound of the tetrachord. Therefore, ♮ Archos or Protus guide the modes Protus authentic and plagal. ♭ Deuterus guides the modes Deuterus authentic or plagal. ♯ Tritus guides the modes Tritus authentic and plagal. ♮ Tetrardus guides the modes Tetrardus authentic and plagal. As you know, we call the higher one authentic, and the lower one plagal. Therefore, the characteristics of each belong to its individual tetrachord or pentachord, and, as was already stated, they distinguish the tones or modes.

Student: Is it therefore solely the individuality of the final sound which makes a trope or mode to be one or another because in it the melody terminates with the final?

Master: Certainly the individuality of each trope appears in the final sound because with them the trope ends. Yet this
must be added, that the same final and its related sounds are found frequently at the end of the commata or *cola*. Its related sound occurs not only at the fifth, but also at the fourth. It also seeks other relatives: this is the position of the third symphonia. You will find in fragments or in parts of melodies that *cola* and *commata* attain arsis and thesis in an ascending or descending relationship to the other sounds. Melodies offer enough examples according to the rules. In the following, containing two *commata*, consider the neume at the end:

```
\begin{array}{c}
\text{\textit{N}} \text{\textit{a}} \text{\textit{n}} \text{\textit{e}} \\
\text{\textit{a}} \text{\textit{n}} \text{\textit{e}} \text{\textit{a}} \text{\textit{n}}
\end{array}
```

Notice how both *commata* turn to the fourth sound at the end and the passage closes with the same sound with which it began. We call the larger interval which contains two or three *commata*, *cola*. Furthermore, *commata* connected together in ascent or descent create a *cola*. However, it sometimes happens that one may use the terms *cola* or *commata* without distinction.

**Student:** How are the major and minor tones different when they terminate with the same sound and are regulated by the same sound?
Master: We shall easily deal with what you have asked if we describe the specific notes of sound. We have set down notes for eighteen sounds, that is, four and one-half tetrachords. The first tetrachord, the lowest, is called Graves. The second is called Finales. The third is called Superiores. The fourth is called Excellentes. Naturally, these soft tones which we call the Finales we designate with the notes which have been described above, in this manner: $\text{♭♭} / \text{♭♭}$. Graves uses the same figures reversed, as: $\text{♭♭} \text{♭♭}$. The Superiores has the figures upside down and backwards, as: $\text{♭♭} \text{♭♭}$, and Excellentes has them upside down. An exception is the sound Tritus, which, in Graves has an $\text{♭♭}$ inclined; in Superiores it is reversed and inclined $\text{♭♭}$, and in Excellentes it is a transfixed $\text{♭♭}$. The remaining two sounds are expressed with reclining symbols: $\text{♭♭}$, all of which are placed here in order, in this manner:

$$\text{♭♭} \text{♭♭} \text{♭♭} \text{♭♭} \text{♭♭} \text{♭♭} \text{♭♭} \text{♭♭} \text{♭♭} \text{♭♭} \text{♭♭} \text{♭♭}$$

Student: How may I distinguish which are Finales, which are Superiores, or what order this or that one may be?
Master: It is well established that all music exists with respect to something, for a musical sound cannot exist without the combination of the sound to another to which it resounds music with the natural interval. Although the something may not be understood by itself; yet with some the sounds are counted perfectly, neither can they be judged Superiores, nor Finales, or the others of any order. But it is made necessary, to whatever extent it is justly sung, that one end higher and lower on one of the same four; on whatever of them it has ended, it keeps its name with the other sounds of the Finales tetra-chord. And from this one, all of the tetrachords are ordered. From this sound is also interpreted by measure whether the tone is major or minor. For since we are not able to arrange the higher tone at the fifth place lower, as has been demonstrated above, because, in fact, it is not termed in relation to higher or lower because the higher or lower is sung one with the other; but, in such relationships, the tones on both sides are held by the final sound; but they, on the other hand are held with this measure.

The minor tone is equally strong above and below the Finales sound, that is, it extends all the way to the fifth sound on both sides. This does not always happen,
but it is able to control these spaces. If it is changed to the fifth higher sound of the system, it is usually reckoned as the major tone. The major, therefore, has two spaces in the higher tones, that is, all the way to the ninth sound of the System.

Student: What is a System?

Master: With *cola* and *commata* we relate *diastemata*, and System in relation to a complete phrase [*particula*] or to an entire period. For a *diastemata* is any interval of sounds which comprise a phrase, that is, which include a high and a low sound. The System is the entire quantity of the melody. The species of tetrachords, pentachords, and octochords are the same Systems which its own species give the single modes.

Student: Why does one have a single tetrachord below *Finales* and two tetrachords above *Finales*?

Master: This is because in any simple and lawful melody, whether it is sung high or low, the sound may only descend to the fifth sound below its own final, or ascend only to the ninth above.

At this point we conclude what has been said above about avoiding the various kinds of discrepancies. Now we
will next proceed, with God's help, to speak about the embellishment of melodies. In particular, it will first be demonstrated how any melody can be brought to light rhythmically.

Student: What is measured singing [numerose canere]?

Master: Observe where the lengthening and the shortening of the note [morulis] is to be used. Observe also to what extent which syllables are short and which are long. In this way somehow sounds are bound to the long ones and somehow sounds are bound to the short ones, so that those that are long unite lawfully to those that are short, and the song is measured with the beat of the foot. We will sing the exercise, I will tap my foot the first time, and you will imitate me afterward.

\[Ego \ sum \ vi-a \ veritas \ et \ vi-ta \ al-le \ lu \ ia \ alleluia\]

Only the last note in each of the three sections is long, the others are short. This, therefore, is called measured singing—measuring the long and short sounds by proportioned
notes, and not to drag them out or shorten them more than is proper, but to keep them together below the ascending lawful cantus, so that the melody ends with the value with which it began. You will here and there alter the value, that is, near the beginning or the end employ a slower or faster speed. You will make it double, that is, alter it so that the lengthened time is twice the short time, or the shortened time is twice as short as the long.

**Student:** I believe that it is advantageous to test these things, and to alter them in practice.

**Master:** You believe correctly. Now we will take the above melody and sing it first shortened, then lengthened, so that in this way, the values are lengthened by the shortening of them, and are likewise shortened in value, which shall have lengthened them. We sing in this manner: first by shortening the values, followed by the lengthening, thereupon shortening them again:

```
Ego sum vi-a veritas et vi-ta al-le-lu-ia alleluia
Ego sum vi-a veritas et vi-ta al-le-lu-ia alleluia
Ego sum vi-a veritas et vi-ta al-le-lu-ia alleluia
```
This rhythmic ratio is always proper with skilled singing, as it adorns them with the greatest dignity, whether sung slowly or quickly, whether sung by one or by many. Also, while singing rhythmically, one does not lengthen or shorten one more or less than the others do, so that the voices of the crowd are heard as if from one mouth. Likewise, in alternate responsorial singing in the same rhythmic manner, the union [concordia] conforms no less to the value than to the sounds.

Student: In what way is it proper so that songs agree in value?

Master: It happens with the harmonious coupling of the song through the proper foundation of the sounds, which has already been discussed above. The agreement of values arises when it, which is joined, responds with an equal
value, or through a coming together with a value twice as long or twice as short.

Student: It depends on singers to observe all that is necessary to sing \textit{modulation} well.

Master: Observe also what I say about the consideration of the differences, that is, as you know, what one brings together to combine, and what one separates. One must also consider what value coincides with this or that melody. It is proper, on the other hand, that this melody be sung quickly, and that the pronunciation is made sweet slowly. One can easily recognize this from the formation of the melody whether the composition of the neume is ascending or descending. Therefore, you render any suitable value you please, of which and to which the melody agrees, and also render the height to the consistency of the values with clear and pleasant neumes. And in this manner, with caution, you will control respectable and well-mannered music. Moreover, the agreeable joining of the symphonias adds the greatest pleasantness to the songs.

Here ends Part I.
Part II

Symphonias

Student: What is a Symphonia?

Master: A sweet mixing of certain sounds [voces] of which three are simple—the diapason, diapente, and the diatessaron. Three are composite—the double diapason, diapason with diapente, and the diapason with diatessaron.

Student: What is the diapason symphonia?

Master: That which is sung at the eighth place with six sounds interposed.

Student: What is a diapente, and what is a diatessaron?

Master: A diapente occurs at the fifth place, a diatessaron occurs at the fourth place; just as in pentachords and tetrachords the extremes combine with themselves.

Student: Why is it called the diapason?

Master: The Greek word diapason is translated in Latin as ex omnibus [through all], because the ancient cithera contained only eight strings.
Student: Why is one named diapente, and why is one named diatessaron?

Master: The diapente is translated "through five," because it contains five sounds. The diatessaron is translated "through four," because it includes four sounds.

Student: How is the diapason sung?

Master: Whenever a sound is changed in ascent or descent by another, so that the higher and the lower are not so much consonant as equal-sounding, the concordant ones harmonize the diapason with this agreement. Just as if it is sung lower from H to A, or if it is raised from H to P, according to the representation below:

```
A B C D E F G H I K L M N O P
```

relaxed diapason tensed diapason

For whether it is taken one after another at the eighth place, or it is sung as one with two equal voices, with this rationale the singing of the simple diapason arises. But let us sing three-in-one with the three described sounds, and the singing of the two-fold diapason arises in this manner. And if it is sung at the fifteenth place, with the middle sound removed, nevertheless, it will still be the double diapason. Let us sing them in the manner written above:
Therefore, this symphonia, because it is easier and more evident, is named the greatest and first.

The Diapente

The diapente symphonia follows. This, however, is where it is taken at the fifth line, or one after another, or where both are
led as one, in the manner written below:

Therefore, it is simple to compose the diapente in this way. The first orderly arrangement of the diapente exists if the voice of the organum is both doubled at the diapason, and the principal voice is in the middle, as the fifth between the first and the eighth. I call the perfect singing the principal voice; the organal voice is that which is added to it in the manner of a symphonia. Let us sing in the manner written below:
The second arrangement of it exists if the voice, which we call the principal, is both doubled at the diapason, and the organal voice is in the middle between the fifth and the twelfth.$^1$ Let us sing this in the manner written below:

1 [Read "...inter quintam ac duodecimam." for Gerbert's (and many mss.) "...inter quartam ac undecimam." GS I, 186.]
The third arrangement of the diapente exists if the organal voice is doubled at the diapason below, so that the principal voice is the highest, as the twelfth opposite the eighth and the first. This consonance is concordant at the twelfth place, and the middle voice is removed.

The fourth arrangement of the diapente exists if the principal voice is doubled at the diapason higher, and the organal voice is at the lowest place, as at the first place below the fifth and the twelfth. This is similarly concordant and the middle voice is removed.

The fifth arrangement of the diapente exists if it is sung with four diverse voices with two pairs of voices doubled at the diapason, as namely, when the first and eighth places respond as organum with the fifth and the twelfth places. All are sung in the manner written below:
The sixth arrangement of the diapente exists if the organal voice is the highest, as the fifteenth with the twelfth and the fifth, according to the manner written below:
Many species of this same symphonia can be made different with the voices tripled at the double diapason at either one or both places.

The Diatessaron Symphonia

The symphonia of the diatessaron follows. This exists where it is fixed as one voice at the fourth place. But it should be known that it is not as simple as in
the others, which are larger; but it is derived accordingly by another natural law, which is discussed later. Nevertheless, if it has been produced with a restrained fastidiousness, which is peculiar to it, and administered with careful concords, there will be a most virtuous pleasantness of the song.

The diatessaron symphonia is arranged in the same manner as with those of the diapente. The first arrangement of it exists if the organal voice is both doubled at the diapason, and the principal one is in the middle, as the fourth between the first and the eighth, in the manner written below:
The second arrangement of it exists if, on the other hand, the principal voice is both doubled at the diapason, and the organal voice is in the middle, as the eighth between the fourth and the eleventh.
The third arrangement of the diatessaron exists if the organal voice is doubled at the diapason below, so that the principal one is the highest, as the eleventh above the eighth and the first.

Likewise, the fourth arrangement of the diatessaron exists if the principal voice is doubled at the diapason higher, so that the organal voice is the lowest, as the first below the fourth and eleventh.

Likewise, the fifth arrangement of the diatessaron exists if doubled voices, namely the principal as well as the organal, are both doubled at the diapason, so that the first and the eighth places respond as organum with the fourth and eleventh places.
The sixth arrangement of the diatessaron exists if the organal voice is highest, as the fifteenth to the eleventh as well as the fourth. This is sung in the manner below:
It should be noted that whether the principal or organál or both voices are doubled at the diapason, the voice of a youngster can always fill the position of the highest voice.

Student: What, I ask is the difference between the first arrangement of the diapente and the second one of the diatessaron, when the extreme voices are divided here and there at the middle
with the same interval? Similarly, what is the difference between the second arrangement of the diapente and the first arrangement of the diatessaron?

Master: If you investigate why in the first arrangement of the diapente the middle voice is the principal rather than the organal voice; in the second arrangement of the diatessaron, however, the middle voice is the organal rather than the principal, while the middle voice has the same intervals with itself here and there at the extremes; and, on the contrary, in the second arrangement of the diapente, why the middle voice is called the organal one, but in the first arrangement of the diatessaron the middle voice is called the principal: realize that this indeed is the rule, that in the diatessaron symphonia the organal voice does not under these circumstances follow the principal voice simply and absolutely as in the diapente symphonia, but, in fact, with its own certain natural law it stops at specific places and is not able to proceed further consonantly, as it was revealed with the description and statements above how it does not descend below the sound

2 [I.e., in both the middle voice is a diapason above the lowest voice, and a diatessaron below the highest voice.]

3 [Where in both the middle voice is a diatessaron above the lowest voice, and a diapente below the highest voice.]
To trandus. Yet it should be known that in the earlier mentioned arrangements the diatessaron as well as the diapente differ with another certain quality. Since there is always a recurrence of the tropes and tones while at the fifth and the eighth place it is necessary that the lower voice respond to the higher voice with the same trope in the manner of the diapente; it responds back on both sides to the voice at the eighth place with its own same trope. It happens that, when the organal voice is doubled at the diapason and the organal voice is divided, being at the fifth place below and the fourth above the middle voice, with the principal voice at the middle, this is the first arrangement of the diapente. However, with the principal voice doubled at the diapason, and the organal voice in the middle, the principal voice is separated at the fourth place below the middle voice, this is the second arrangement of the diapente. On the other hand, while in the diatessaron arrangement, the lower voice does not respond with the higher voice with the same trope at the fourth place, so it is necessary for it to respond to the principal and the organal voices not with the same trope, but with its own individual trope at its own eighth place. It happens that when the organal voice is doubled
at the diapason and the principal voice is in the middle, the organal voice is at the fourth place below and the fifth place above the middle, this is the first arrangement of the diatessaron. The principal voice, however, doubled at the diapason with the organal voice in the middle, with the principal voice separated from the middle at the fifth place below and the fourth place above: this is the second arrangement of the diatessaron, of which all the above descriptions point out. But what more do you ask?

Student: Because you divulge that the diapente symphonia indeed harmonizes with the same tropes, and the diatessaron indeed does not, therefore, in the previously mentioned arrangements of the symphonias, the middle voice is separated with the same interval, while neither here nor there is it the same trope; here in fact it is the principal, there it is the organal. I ask what is the difference between the principal and the organal voice at the diapente symphonia where the tropes are not dissimilar?

Master: Consider now everything that has been discussed, for when the organal voice has been doubled in the diapente, if the middle voice, namely the principal, was the same interval with the extremes, nothing would appear to be different between the principal and the organal voice. Now, however,
since the organal voice extends to the principal voice at the fifth place below, but through the diapason at the fourth place above;\(^4\) just as in the diatessaron arrangement the organal voice extends to the principal voice at the fourth place below, but through the diapason, to the fifth place above;\(^5\) similarly, if the principal voice has been doubled at the diapason, since the middle voices are not at the same interval with the extreme voices; it can clearly be perceived as the strength of the diapason symphonia, that it multiplies the voices in both directions and also arranges which voices are principal and which are organal, through certain intervals.

Student: Why is the organal voice not able to unite as absolutely with the principal voice in the diatessaron symphonia as in the other symphonias?

Master: Because, as has been said, the same tropes cannot be obtained at the fourth region, and the modes of the different tropes are not able to proceed through their entirety or to proceed at the same time. Therefore, in the diatessaron symphonia, the principal voice and the organal voice

\(^4\) [I.e., in the first arrangement of the diapente.]

\(^5\) [I.e., in the first arrangement of the diatessaron.]
do not agree totally at the fourth region.

Student: I wish to distinguish how the genus at the fourth region is dissimilar.

Master: That is easy for you to perceive, for whether it is transposed one tone higher, or four places lower, the mode of the different tropes may be discerned with a clear sense of hearing. This is sung in the manner written below:

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Student: I clearly distinguish with this transposition that the authentic tone Protus was transformed into the authentic tone Deuterus. But, however, will you now give the reason why at some places the sounds are consonant, while at others, however, either there are discrepancies, or they do not exactly agree.
Master: Truly it is proper to consider what rules God has granted, with which we somewhat sweetly discover the reasons for the agreement and the reasons for the discrepancies of the sounds, and the nature of the diverse tropes, and why in transposing they change into the other species or return once more to themselves. For just as in counting absolutely, the series of numbering is simple and easy, and is accessible even to boys, such as 1, 2, 3, 4, and so on. However, one having been contrasted unequally with another, falls into various species of inequality. Thus, pitches in music, whose mother is Arithmetic, that is the knowledge [scientia] of number, are counted with an easy order. They give to the others revealed not only the species of pleasant agreement, but also the most pleasant rules of the same agreement.

Student: How is harmony begotten from mother Arithmetic, or what is both harmony and music?

Master: Harmony is considered the mixture of concordant sounds [voces]. Music is the method of the concord itself. As this is joined to the rule of numbers through everything, together with the other disciplines of mathematics, it is thus necessary that it be known.

Student: What are the disciplines of mathematics?
Master: Arithmetic, Geometry, Music, and Astronomy.

Student: What is Mathematics?

Master: The knowledge of learning [doctrinalis scientia].

Student: Why is it the knowledge of learning?

Master: Because it examines abstract quantities.

Student: What are abstract quantities?

Master: Those without material, that is, corporeal admixture, handled by the intellect alone. In quantities, and even multitudes, magnitudes, paucity, smallness, forms, equalities, appearances, et cetera, which, to speak in the words of Boethius, are by their own nature incorporeal, and of immutable substance flourishing by reason, completely changed by the participation of the corporeal, and with a variable touch of matter are transformed into changeability and inconsistency.⁶ These quantities, furthermore, are considered differently in Arithmetic, Music, Geometry, and Astronomy. For these four disciplines are not the skills [artes] of human invention, but the considerable investigations of divine works, and they direct, with

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⁶ [De institutione arithmetica, Bk.I, Ch.1 (Friedlein ed., 8).]
ingenious minds, by the most miraculous of reasons, understanding in the creatures of the world. Thus, they, who through these things learn to know God and his everlasting divinity are inexcusable if they do not glorify God and give thanks.

Student: What is Arithmetic?

Master: The discipline of numerical quantities according to themselves.

Student: What is Music?

Master: The rational discipline of the agreement as well as the discrepancies of sounds according to numbers, which to some things are to those which are discovered in sounds.

Student: What is Geometry?

Master: The discipline of immobile magnitudes, and of forms.

Student: What is Astronomy?

Master: The discipline of mobile magnitudes, which contemplates the course of heavenly constellations and all figures, and the appearances of the stars around themselves and around the earth, with an investigating rationale.
Student: How do the three others correspond through numerical knowledge?

Master: Because all things which are comprehended through these disciplines are formed with the rationale of numbers, for they cannot be either understood or related without numbers. For how can it be made known what a triangle or a quadrangle is, et cetera, which are from Geometry, unless it is first known what 3 or 4 are?

Student: In no way.

Master: What value is to be understood in the rationale of Astronomy without number? Whence do we learn the risings and the settings, the slowness and the speed of the wandering constellations? Whence do we recognize the phases of the moon and the multiple variations of it, or what part of the Zodiac the sun or moon hold, or any other planets you wish? Is it not that everything is led with certain laws of number, so that they are unknown without number?

Student: Yes, indeed.

Master: What is the reason in Music that at the eighth place the sounds are equal-sounding, and consonant at the fifth and fourth? And, likewise, why do they respond as equal-sounding at the fifteenth region, and as consonant at the twelfth
and eleventh?

Student: Certainly it is amazing that there are commensurabilities of the sounds in this manner with which the symphonias also agree so pleasantly in sound with themselves, and the sounds remaining are coupled in order so suitably with themselves. But it is for you to explain these things you have proposed.

Master: I say, because it is the same at the eighth place, that is, it is equal-sounding as the diapason, because these two sounds are intertwined with this relationship, as 6 to 12, or 12 to 24. Similarly, it is equal-sounding at the fifteenth place, which is called the double diapason, because it corresponds to the quadruple proportion, as 6 to 24. Therefore, they respond consonantly with themselves at the fifth region, which is the diapente, because they are in the sesquialter relation. The sesquialter is, therefore, where the lesser number contains two parts, and the greater contains three, such as 6 to 9, or 8 to 12. Sounds at the fourth are consonant because it is the diatessaron, in which they are at the epitritus relation. The epitritus, or sesquiterius, is where the smaller number has three parts, and the larger one has four, such
as 6 to 8, or 9 to 12. Therefore, in bringing them together at the twelfth they are concordant because the diapente responds with the diapason, that is, the sesquialter with the duple, such as 18 to 12, or 8 to 12 and 24, or, because at the twelfth place they are made triple, as 18 to 6, or 24 to 8. Also, they are consonant at the eleventh region because the diatessaron resounds with the diapason, that is, the epitritus with the duple, such as 6 to 12 and 16, or 9 to 12 and 24.

The same sounds, which make the preceding symphonias resound, as the diatessaron relates to four, and the diapente to five, are concordant by this quality, because the diatessaron and diapente are connected mutually, namely to the epogdoos, that is, the sesquioctavus: for the difference between the sesquialter and the sesquitertius is always the epogdoos; which are, in fact, at the proportion of 8 to 9, making 16 to 18, making 32 to 36, and so on, into infinity.

A description of these things which were discussed:
Moreover, the symphonias in the diapason and double diapason are more perfect than those in the diatessaron and diapente, because the former are a multiplex inequality, and the latter are superparticular. The multiplex inequality is more perfect than the superparticular inequality. The sounds of these previously mentioned proportions, that is, the duple, triple, quadruple, sesquialter, sesquitertius, and sesquioctavus, stand together for that reason, as consonant, or equal-sounding, because, of course, these relative numbers alone are commensurate and connumerate in all disciplines, and, because of this, they are reckoned with the symphonias and with the rest of musical sounds, and in fact, melodious sounds are produced by means of these numbers. Do you therefore realize that Music cannot be explained, except by arithmetic relations?

**Student:** I clearly realize that for Music to be understood, Arithmetic is necessary.

**Master:** By all means necessary, since Music is completely formed in the likeness of numbers. For if you make a string or a pipe twice as long as a string or pipe of equal thickness, such as 12 to 6, or 24 to 12, they mutually resound the diapason. If you make a string or a pipe hollowed
equally, longer by a third part of the smaller one, such
as 8 to 6, or 16 to 12, or make one smaller by a fourth
part of the larger one, such as 9 to 12, or 18 to 24, it
will, in fact, be the diatessaron consonance 8 to 6, and
16 to 12. Similarly, 9 to 12, or 18 to 24. Moreover,
9 to 6, and 8 to 12, and similarly 18 to 12, and 16 to
24 will be the diapente. And, in this manner, just as
the sesquialter and the sesquitertius are contained in the
duple, as between 6 and 12 is contained 16 to 18, namely,
in an alternate way, so that it is a sesquialter to the
smaller number, as 9 to 6, or 18 to 12, the subsesqui-
tertius is made to the larger, as 9 to 12, or 18 to 24.

So, between two sounds you resound the diapason
mutually, the symphonias of the fourth and fifth place
are always arranged naturally, so that which is a dia-
tessaron at the fourth place on this side, is the diapente
at the fifth place on the other side, and that which re-
sounds the diatessaron at the fourth place, resounds the
diapente.

Furthermore, just as in the sesquioctavus proportion
9 exceeds 8 as 18 exceeds 16, and 36 exceeds 32, so, if the
larger pipe or string exceeds the smaller by an eighth
part, they will mutually resound the tone. Therefore, whatever is pleasant in singing [modulatione] works with number through the fixed dimensions of sound. Whatever exhibits delightful rhythms, or in singing, or in whatever rhythmic motion you wish, are caused totally by numbers. And sounds, indeed, pass away quickly, but numbers, which are discolored by the corporeal material of the voice and movements, remain. Wherefore, as St. Augustine relates:

"Thus reason has perceived that numbers govern and make all perfect and all that is in rhythms (called "numbers" in Latin) and in song itself, has examined them diligently, and has found them to be eternal and divine. Next surveying heaven and earth, reason has also perceived that in them only beauty pleases it, in beauty only figures, in figures dimensions, and in dimensions numbers. These things, separated and ordered, reason has brought together in a discipline which it calls Geometry. Profoundly impressed by the movements of the heavens, reason has been further prompted to inquire diligently into this. Through the endless succession of the seasons, through the harmonious and regular courses of the stars, through the orderly extent of the intervening distances, reason has perceived that, here too, only dimension and numbers hold sway. Similarly putting these things in order by defining and dividing, reason brought forth Astronomy. In this way, then, all things present themselves in the mathematical disciplines as harmonious, as having to do with the immortal
numbers which are apprehended by reflection
and study, those which are perceived by the
senses being mere shadows and images."7

Who, therefore, would say that the changeable reason-
ing of numbers, or of any art could be in existence with-
out it?

Student: It is now sufficently understood that not only Music, but
also all three other disciplines are in existence through
the authority of numbers. But, I ask, begin to discuss:
the nature of numbers more fully, and repeat the single
points which have gone before, so that I can arrive at
the innermost contemplation of the musical reasoning of
numbers through your leadership.

Here ends Part II.

7 De ordine, II, xiv-xv; see also his De libero arbitrio, II, xvi,
as cited by Oliver Strunk, Source Readings in Music History, Vol.I,
(Antiquity and the Middle Ages) (New York: Norton, 1965),137. English
translation by Richard McKeon in Selections from Medieval Philosophers,
I (New York, 1929), 58-61, as cited by Strunk, loc.cit.]
Part III

Student: I ask you, proclaim this first: what is quantity?

Master: We speak of quantity both in numbers and in movements. The quantity of numbers is properly called multitude, that which is gathered together from units. The special quantity which is in movement is called magnitude, which is divided into units. Multitude increases with an infinite increase, magnitude diminishes with an infinite division. That is, multitude, stretching to one from several, is increased in space; magnitude, stretching from one into several, is decreased in space. In as much as a stone, a tree, the less it is divided in number, the larger the spaces of divisions become. The more, however, things are separated into several, the more the spaces themselves diminish. For example, 24 divided into two parts is 12, into three parts is 8, into four is 6; 12 divided into three is 4, and into four is 3. Therefore, with the similitude of either kind of quality, arithmetic yields music, and arranges contrary melodies and voices in sweet concordance. Indeed, when a voice arises
upon another voice, either doubly or triply or quad-
ruply, it gives the diapason, the diapason with diapente,
and the double diapason, and it preserves the essence of
the numerical quantity. Or indeed, whether 1/2 of the
larger voice remains, or 1/3, or 1/4, or 1/8, that is, the
diapente, the diatessaron, and the tone, it holds the
likeness of the unbroken quantity. Since, therefore,
varieties of voices in quantity occur in this manner,
it follows that the double contrary natures of quantity
agree in a mutual sweet mixture.

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If these things are now clear, let us proceed to what
remains.

Student: It is clear indeed. What about this kind of numerical
quantity, in which the increases themselves grow in the
order of the numbers? For that reason, this combines
in a continuous quantity, which equally diminishes itself
in the order of the numbers in diminution. For if it
increases by 2, it gets cut in half; if it gets bigger by
3 or 4, it gets smaller by a third or a fourth. It remains for you to describe in detail how and in what order consonance occurs throughout these double-natured qualities of sound.

Master: In order that these things may be explained by a giving God, let us put forth a few things regarding the nature of the numbers, as we proposed. We have said that there are two species of quantity, multitude and magnitude. Again, of either multitude, or magnitude, some things are inherent, such as the populace, a chorus, a mountain, 1,000, 2,000, 3,000. Others do not themselves remain unchanged inherently, but are repeated. For example, double, triple, quadruple. But Geometry handles the immobile; Astronomy indeed, treats the mobile quantity. And so, Arithmetic treats that which is inherent. We contemplate that which exists in relation to something, because through it sweet harmony is divinely controlled. Every number, and whatever is said to be in compliance with number, is called either that number inherent, or in some other relationship. A number inherent is one which is without a relation, such as 1, or 2, or 3, or 4, etc. Therefore, whatever is said to be in relation to something else is either equal or unequal. Equal is that which when compared to another is
is neither less on either side, or greater, for example, 1/10 to 1/10, or 1/4 to 1/4, or a cubit to a cubit, or a foot to a foot, and so on. It is agreed that this part which is by relation to something of quantity, that is, equality, is indefinable. No one is able to define it, because such is the nature of equality; truly it is this way.

Student: That is for sure. What species, then is able to be another, when they are supposed to be different?

Master: Consider what the divisions of an unequal relation are. First of all, it is divided into a greater and a smaller, and there are five species of the greater. The first is called multiplex, the second is called super-particular, the third is called superpartiens, the fourth is called multiplex superparticular, and the fifth is called multiplex superpartiens. For these five parts of the greater inequality there are five of the lesser inequality, and as they are named with the same names, they are distinguished only by the prefix "sub," that is, submultiplex, subsuperparticular, subsuperpartiens, multiplex subsuperparticular, multiplex subsuperpartiens. These are the species of either inequality, which we will examine one by one.
The Multiplex Inequality

The multiplex is an inequality when a larger number has a smaller number within itself two or three, or four times, or more. For example, when 2 is compared to 1, it is duple. 3 to 1 is triple, 4 to 1 is quadruple, and so on.

Next is the submultiplex, which is just less than the multiplex, when a number is contained in it two, three, four, or more times. For example, 1 is contained twice in 2, and is called subduple. 1 is contained three times in 3 and is called subtriple, four times in 4, and is called subquadruple, and so on. To go further would be a little too involved for us.

Student: I understand that this inequality is of that kind of quantity whose greater part is able to grow into infinity, as was stated above.

The Superparticular

Master: Pay attention to another inequality of this kind of quantity, the greater part of which, on the contrary, decreases into infinity. That is, the superparticular, in which a
number, when compared to another number holds a whole smaller number within it, and another part of it. If it holds half of the smaller number it is called sesquialter. If it holds a third part it is called sesquitertius. If it holds a fourth it is called sesquiquartus. If it holds a fifth it is called sesquiquintus. And so, as the forms of the superparticular progress into infinity, so do their names. The lesser numbers which follow and are held both in their entirety as well as in some part are called subsesquialter, subsesquitertius, subsesquiquartus, subsesquiquintus, and so on. And now, you can distinguish this genus enough to differentiate it from the one above.

Student: I can distinguish between them already, because in the first kind the lesser was not lumped with the whole quantity of greater numbers. In any one part, whether divided in half, for example, 2, or divided into thirds, such as 6:9, 12:18, or divided into fourths, such as 4:5, 8:10, 12:15, the greater, in relation to the lesser is the middle part to the third, the third to the fourth, the fourth to the fifth, and so on: the part decreases from the greater with the same denominator into infinity.
The Superpartiens

Master: Pay attention also to the third kind of inequality, which is called the superpartiens, which occurs when one number is compared to a smaller number and the first contains in itself the entire smaller number. It also contains other parts, two, or three, or four, or five, or any others. For example, three is contained in five with two left over and is called superbipartiens. Five is contained in nine with four parts of its own and is called superquadripartiens, and so on. The lesser number, which is hidden by the greater number is called the subsuperpartiens. Enough has been said regarding this kind of inequality.

Student: I think it is enough.

The Multiplex Superparticular

Master: After these three simple manifestations, two follow, which are comprised of the previous ones, of which the first is called the multiplex superparticular, which is comprised of both of the others, as 2 is to 5, 3 to 6, 4 to 8, and 5 to 11. Indeed, because that which holds a number which was
compared more than once is multiplex, and that which
transcends the lesser number in the part which is held
is a superparticular. A number which contains some other
number twice and a third as well is a duple sesquitetius;
two times and a quarter is a duple sesquiquartus, and so
on. If one number contains some other number three times
and one half of it, or three times and one third of it, or
three times and one fourth of it, it is called a triple
sesquialter, a triple sesquitertius, a triple sesquiquartus, and so on. That is enough regarding this kind of
inequality.

Student: Yes, it is enough.

The Multiplex Superpartiens

Master: The fifth inequality is called multiplex superpartiens.
This occurs whenever a number, which has been compared to
another number, has in itself the entire other number more
than once, and two or three or however many are left over,
according to the figure of the superpartiens number, and
these are named according to the number of parts they con-
tain. For example, 8 to 3 is a duple superbipartiens,
since it contains the threes two times, with two left over.
Likewise, 16 to 7. In the same way there are duple supertripartiens, triple supertripartiens, and triple superquadpartiens.

It is agreed that the multiplex inequality, as was stated, pertains to numerical quantity, and there is quantity in the other four categories. Likewise, musical voices, that is, those sweetly harmonizing, occur according to the multiplex superparticular kind: unharmonizing voices, however imitate the remaining inequalities. While there may be an innumerable and infinite blending of the unequals, an infinity of disharmonies follows also the infinitely blended inequality. Two also, I say, which we discussed among the first pertain to music, the three remaining are set aside.

Student: How are these two pertinent to music?

Master: Because every pitch is formed by its coming together according to these numbers, which are either multiplex or superparticular. And unharmonizing voices arise according to the remaining inequalities.

Student: By what rule do either consonant or inconsonant voices imitate these or those rules?

Master: Because by this rule diverse sounds agree with one another,
and differ according to these inequalities.

Student: In what lies this difference?

Master: In the highness and lowness; in the raising and lowering.

Student: How do they differ among themselves, according to the aforementioned numbers, and in differing, how do they agree?

Master: All pitches, that is all sounds [vocabulae] harmonious with themselves, are distant from one another either doubly, triply, or quadruply, which are species of the multiplex inequality, or through the sesquialter, sesquitertius, or sesquioctavus, which are species of the superparticular inequality. The interval between pitches, indeed, is not one of silence, but of space, in which one goes beyond another.

Student: Which pitches are distant by the duple?

Master: Always those which are at the eighth interval, that is, the diapason.

Student: Which ones are distant by the triple?

Master: Always those which are at the twelfth interval, which is the diapason with the diapente.
Student: Which ones are distant by the quadruple?

Master: Always those which are at the fifteenth interval, which is called the double diapason.

Student: Which sounds are distant by the sescuple?

Master: Always those which are at the fifth interval, which is called the diapente.

Student: What are the sesquiterti?

Master: Those which are separated by the fourth interval, which is called the diatessaron.

Student: Which pitches hold the sesquioctavus interval?

Master: Any one pitch next to another, such as a string to a string, which is called a tone. And to these is added a semitone, which is an interval which is not a full tone. Divinity governs the harmonious diversity of sounds with these moderations of distances, and with these, a pleasant harmony is regulated. Whatever sounds, high or low, which the standards of measure for these proportions demand, fall into whatever categories there are of the remaining inequalities, and dissonance occurs, which is unsuitable for the song.
Student: How is it known whether the musical pitches hold the
aforementioned standards of measure, when these pitches
cannot be seen or touched?

Master: First of all, it is recognized from this, that only
these appearances, that is, multiplexes and superparticulars, are reckoned with one another and are commensurate,
and somehow cognate by their own commensuration. Nor are
any other things in the world either actual or potential,
which are joined harmoniously with each other, unless the
appearance or proportions connects these related things.
These also are the only proportions through which, accord-
ing to the different feet, the verses run harmoniously.
In as much as the joinable nature of these remains, so the
opposing characteristics of the four elements are joined
in a meeting of these proportions, just as it is revealed
in the Timaeus, and with other philosophers

"you tie the elements with numbers,
so that cold things are suitable for
flames, and dry things are suitable
for liquids,"

because of which musical sounds do not combine into one
mode so sweetly unless commensurability associates them
according to the congruent and consonant intervals of
proportions, which were mentioned before.
Student: What is this commensurate or connumerate quality?

Master: When there is a certain common measure of larger and lesser, for example, 2 to 4, 2 to 6, 2 to 8. Does not the binary measure the quaternary two times, or measure the senary three times, and the octonary four times?

Student: Indeed.

Master: In this manner, then, as many times as you bring a simple number into a multiplex, so many times that same number yields a larger sum. So that 3 doubled gives 6, tripled 9, quadrupled 12, etc. What about in the superparticulars, as 4 to 6? Does not a binary measure the lesser number twice, and the greater number three times, which is a sesquiusplus? Likewise, in 6 to 8, 2 measures the lesser number three times, and the greater number four times. Likewise in 8 to 10, 2 measures the lesser number four times and the larger one five times, which is the sesquiquartus. And so into infinity, by whatever part the larger number is larger, by such a part both are measures to each other. For example, if an 8 is succeeded by a 9, that is, by one more, that same 1, by which they differ, is the distance between them both. And now you understand, that these inequalities grow by the same increase as they lessen by the same decrease.
Student: I understand, indeed, what these commensurates or con-
numerates are. But tell me how the same connummeration
or commensuration is missing from other inequalities.

Master: Let us contemplate this, since you inquire about it.
Take, for example, 3 to 5, which is a superbipartiens
comparison, and see how much the 5 surpasses the 3.

Student: I can't find, however I should state it, whether the 5
surpasses the 3 by a whole or a part of the whole.

Master: Certainly not, for the greater part is not measured in
this kind of case, nor is it measured by the quality of
the lesser, as it is in the multiplexes. Nor is it
measured by whatever part is the difference between them,
as in the superparticulars. When, in the sesquialter
proportion, as 4 to 6, 2, which is the difference between
them, measures both of them. In a sesquitertius pro-
portion, such as 3 to 5, 2, which is the difference,
measures neither. It is also likewise in multiplex super-
particulars, as 2 to 5, which is a duple sesquialter, or,
for example, 2 to 7, which is a triple sesquialter, or,
3 to 10, which is a triple sesquitertius. And likewise, in
multiplex superpartiens, 3 to 8, which is a duple super-
bipartiens, 3 to 11, which is a triple superbipartiens, or
4 to 11, which is a duple supertripartiens. You therefore see, that proportions of this kind are measured neither by a simple quantity, nor by their own differences; nor by those parts by which they differ are they increased or decreased. And so, therefore, they are rightly called incommensurates and incommunerable. Furthermore, if we discern with our eyes both crooked and straight, and we discern whatever things lie under our vision, thus, since our nature remains intact in the aforementioned parts, voices coming together as commensurate intervals please the listener, others are dissonant. But do you want to know more about this commensurability?

Student: It's up to you.

Master: We have said how like quantity measures multiplexes, unlike quantity superparticulars, now let us examine the multiplexes with the superparticulars, which they hold cognate. Posis the agreed order of a multiplex, and at the same time the order of a superparticular is woven—whether they are principals, between which a unity is the difference, which are the bases of the multiplex and superparticular numbers, or whether they are greater, which are derived from this compilation. The principals are
1, 2, 3, 4. The fourth number, indeed, finishes all of
the symphonias perfectly. 2 to 1 is the duple, which is
a diapason. 3 to 1 is triple, which is a twelfth, 4 to 1
is quadruple, which is a double diapason. 3 to 2 is a
sesquialter, which is a diapente. 4 to 3 is a sesqui-
tertius, which is a diatessaron. Derivations are 6, 12,
18, 24, etc., for 12 to 6 is duple, 18 to 6 is triple, 24
to 6 quadruple. A triple to a duple, however, is a sesqui-
alter. A quadruple to a triple is a sesquitertius. Like-
wise, a quintuple to a quadruple is a sesquiquartus. A
sextuple to a quintuple is a sesquiquintus. A septuple
to a sextuple is a sesquisextus, and so on into infinity.
Superparticulars accompany multiplexes, so that music is
said to be created, not undeservedly, from both at the
same time.

Student: Both connumeration and cognition seem, without a doubt
to be of the same kind of number.

Master: Because, therefore, it is clear, as multiplexes are
arranged in order to one another they become superpartic-
ular. So, indeed, it is clear to you how duple or
quadruple multiplexes are composed of superparticular
sesquialteri and sesquiterti. So, let us begin from a
6. The sesquialter is added to 6 and becomes 9. The
sesquitertius is added to the 9 and becomes 12. The 12 thus is double 6, and is filled out by the sesquialter and the sesquiteri. Likewise, the sesquialter is added to the 12 and becomes 18. The sesquiteri is added to the 18 and becomes 24. But 24 is 2 times 12, and is filled out by the sesquialter and sesquiteri. And, alternately, the sesquiteri is added to the 6, which is 8. The sesquialter is added to the sesquiteri, which is 12. 12, therefore, is 6 doubled, and is filled out by the sesquiteri and the sesquialter. Likewise, the sesquiteri is added to the 12, which is 18. The sesquialter is added to the sesquiteri, which is 24. And so, 24 is double 12, and is filled out by the sesquiteri and the sesquialter. And so, finally, one duple is always made whole by a sesquialter and a sesquiteri increment.

Therefore, the sesquialter and sesquiteri are then mutually connected by an epogdoos¹ measure, so that, if you take the epogdoos away from the sesquialter it becomes sesquiteri, and again, having been added to the sesquiteri, it reconstitutes the sesculus.² It is agreed,

¹ [Epogdoos=9:8 proportion, or a tone.]
² [Sesculus=the sesquialter.]
therefore, that sounds coming together with themselves from
numbers that come together with themselves create harmony.

Student: This is made clear from your reasoning.

Master: Pay attention now, because harmonizing voices are not ones
that come together in any old way, but, for example, it is
agreed that 24 is double 12, triple 8, quadruple 6; it is
the sesquialter at 16, the epitritus\textsuperscript{3} at 18, the subduplus
at 48, the subepitritus at 32, the subsesculus at 36.
Thus, any pitch sounds the diapason at a certain other
place; at yet another, the diapason with diapente; at
another, the double diapason; at another, the diapente; at
another, the diatessaron; at another, the subdiapason; at
another, the subdiatessaron; at another, the subdiapente.
This would not be, unless according to the rule of the
appearance of the multiplex and superparticular they would
be cognate in commensurability, whence it is known according
to which proportion any symphonia is formed.

Student: Since this has been demonstrated sufficiently, because the
role of commensurability binds music and voices to one another,
whence is it possible to be known to what proportion a

\textsuperscript{3} [Epitritus=sesquitertius.]
certain symphonia is thought to belong? How, indeed, is it known when the diapason is thought to be of a double appearance, the diapente of a sesquiplus, the diatessaron of an epitritus, the diapente with diapason of a triple, and the double diapason of a quadruple?

Master: It is the first proof of this matter, because one duple is always filled out by a sesquialter and a sesquitertius, as was said before, thus, two smaller symphonias, that is a diapente and a diatessaron, complete the diapason, which contains eight sounds. Indeed, whether you measure the fourth interval from a lower part, or a higher part, or you measure the fifth from the fourth, or the fifth is marked with the fourth, the one is always filled out by two symphonias, because it could not occur that either there are others which fill it out, or others which are filled out.

[Note: The diagram shows the relationship between the sesquialter and sesquitertius, and the diapason.]

4 [I.e., the diapason.]
Concerning these things, Boethius also, among all, said that those consonances which we have related must be judged both by listening and by reasoning as to which is the better. For the ear is affected in this way by the sounds, either by observation, which action is the judgement of numbers, or by a continuous quantity. With the number of lines having been proposed, nothing is easier than to see its duple either by the eye or the mind, for example, 12 as opposed to 6. Likewise, after comprehension of the duple follows comprehension of the half, after the half, of the third, and after the third, of the quarter. And so, because the description of the duple is easier, the duple is imputed correctly to that consonance which is easier, and the one which the sense comprehends more sharply. This, therefore, is the first and sweet consonance which occurs in the eighth interval and which is called the diapason. After the duple, which is prominent, is the sesquialterus, because it prevails in the middle of the subduple. For example, 9, as opposed to 6; and that, which exceeds two times in quantity, that is, three times, such as 18 as opposed to 6. Therefore, when these two adjacent proportions follow after the duple, it
is permitted with an opposing division, and symphonias are rightly ascribed to these, symphonias which we sense as following the diapason, that is the diapente, and likewise, the diapason with diapente.

While surely there are consonant sounds because they hold the diatessaron space, we give this elegance with good reason to this proportion, when the larger number transcends in the third part of the lesser, as in 8 as opposed to 6. Because, in truth, the sounds which are at the elegant double diapason are the most greatly distant, we rightly assign them to this proportion which breaks off at the quadruple interval, for example, 24 as opposed to 6.

And finally, the mode of symmetries stands, which is neither able to be extended beyond the quadruple, nor to be shortened less than the third part. And thus, consonances take up this order, whose increases yield the multiplex numbers and whose subtractions are in the superparticular region. And also, an argument of equal worth is given, the value of which is attested not only through hearing, but also through seeing. Strings or pipes, if they are equal, and if the larger is twice as long as the smaller, sound consonantly with each other at the diapason, as was said before. If the larger is three times the smaller, they
harmonize at the double diapason with diapente. If the larger is four times the smaller, they harmonize at the double diapason. If the larger is one and a half times the smaller, a consonанance occurs at the diapente. If the larger is one and a third times the smaller, consonance at the diatessaron occurs. If the larger is one and one-eighth times the smaller, the tone occurs.

Student: How no more than three species of the multiplex and no more than three species of the superparticular pertain to music has now been proven by this fruitful endeavor: not only does music arise from connumeration or commensuration, but also from those connumerate numbers through which symphonias are contiguous. It is marvelous, indeed, but what is the reason that no more than three forms of multiplexes and no more than three forms of superparticulars are admitted to music: only the duple, triple, and quadruple forms, and only the sesquialter, and sesqui-tertius with the sesquioctavus produce consonances?

Master: It may not be extended beyond the double diapason, i.e., beyond the quadruple because the nature of sound assigns this rule. If, for example, with the individual distinctions of the seven sounds in the diapason, individual consonances are assigned at the double diapason, then double symphonias are unable to progress further. Therefore, indeed, they
are not less than the third part, that is, less than the epitritus interval, because then the third part would be less than the second, the fourth less than the third, the fifth less than the fourth, the sixth less than the fifth, and so on. Then the sesquialter, which is the interval in the middle, would take three tones with a semitone; an epitritus, which is the interval of the third part, would take two tones with a semitone; it would happen that the smaller intervals would not have room for two tones and a semitone, nor would they be commensurate with two tones, or one and a half tones. Only the sesqui-octavus interval, because it measures a tone, is allowed.

Student: Why does the whole measure fall into only this proportion?

Master: Because this proportion is created only by a comparison of the sesquialter and the sesquitertius.

Student: How does it arise from comparisons of sesquialters and sesquiterti?

Master: Is it not easily understood that the number 6, for example,

\[5\]

[I.e., the diatessaron.]
grows by half of itself into its own sesquialter, that is, 9? That it grows by one third of itself into its own epitritus, that is 8? But if 9 is the sesquialter to 6, then it is the sesquioctavus to the epitritus. Likewise, 12 yields, by traversing one half of the distance of itself, its own sesquialter, which is 18, its epitritus by one third of itself, which is 16. But 18, which is the sesquialter for 12, thus becomes the sesquioctavus for the epitritus.

Student: I understand.

Master: Then understand at the same time that the interval of the whole tone between one sound and another sound can be no more and no less than what at the same time is the difference between the sesquialter and the sesquitertius. Therefore, there is commensurability in only the sesquioctavus intervals with the sesquialters and sesquitertii: the intervals of which, as was previously stated, are equally filled out by a tone with a semitone. Again, only the sesquialters and the sesquitertii intervals are commensurate with the duples and quadruples, with which their intervals are filled out. There is no commensuration
for the sesquiquarti, or for the sesquiquinti, or the other
decreasing intervals with either the duple, triple, quad-
ruple, or sextuple, and therefore, they are excluded from
music. But even if it is possible to prove cognate the
commensurability of strings and pipes by the dimensions of
the aforementioned proportions which are admitted to music,
you would make string to string and pipe to pipe longer by
the sescuplus, placing the epitritus with the sescuplus, and
the duple in the same way. For that which is the epitritus
to the second, extends longer at the duple than to the
first. Again, conversely, if you measure the epitritus
string to string and pipe to pipe, you would place the
sesquialter at the epitritus, and it would be made duple.
Place these in order, as 6, 8, 9, and 12, and you will
properly find the epogdoos between the two middles, that
is, the greater string or pipe is greater by one eighth of
the smaller. Likewise, in another way: measure a pipe
longer than another pipe by the duple. At the shorter pipe
you will measure the epitritus, at the longer pipe you will
measure the subepitritus, and will put in order the sub-
duple epitritus and the duple epitritus.
And thus the epogdoos naturally falls between two middles. The sescuplus will be on the farther side, and the epitritus on the closer side. And furthermore, if you double the epitritus and the sescuplus, at the same time you will make another diapason in the same order as before.

**Student:** In what way do the other superparticular forms not have commensurability with the duple and the quadruple?  

**Master:** So that this may become clear to you, I shall start from 12, which is the smallest number which can be used as an example. This number, measured with its own quantity, becomes 24. When it is measured with a quantity 1/2 of itself, it becomes 18; with a number 1/3 of itself, 16; with a number 1/4 of itself, 15. But 18, which is a sesquialter to 12, occurs at 24 through the epitritus rule of commensurability. Therefore, the number 16, which is the epitritus at 12, occurs at 24 by the sescuplus rule of commensuration. The number 15, which is the sesqui-quartus at 12, is not commensurate with 24, but rather, is an inconsonant supertripartiens figure.
Likewise, the number 24 becomes 48 when measured with its own quantity, 36 when measured with 1/2 of itself; when with 1/3 of itself it becomes 32; when with 1/4 of itself, becomes 30. But the number 36, which is the sescuplus of 24, becomes 48 through the sescuplus rule of commensuration. 30, which is the sesquiquartus for 24, is not able to be commensurate with 48. You see, therefore, how as soon as they begin to go between the intervals of the sesquiterti, the superparticulars do not have commensuration with the duples, and, therefore, do not have a consonant mixture.

Student: By what rule does it happen that in a series of sounds, at the fourth place, for example, the sounds are epitriti, at the fifth place are sesquialters, at the eighth duples, and are certain others in other positions?

Master: It is agreed that the duple interval is filled out by either the epitritus or the sesquialter, or by two epitriti and in the middle an epogdoos, and the intervals of the epitritus are themselves filled out by the epogdoi. If indeed there are two smaller intervals in the middle of the diapason, i.e., the fourth and the fifth, while individual sounds respond on one side at the diatessaron, on the other side the diapente, and both are distant from the epitritus
on their own sides. But again, the epogdoi accompany either of the epitriti by the same reasoning, so that three intervals are cut into four little sounds, such that two of the individual intervals can precisely take individual epogdoi, i.e., tones, and the third interval can be filled out exactly by a limmata, which is a semitone. Thus it happens that the sesquiterius sounds are found in the fourth positions, the sesquialter sounds are found in the fifth positions, the duple occur in the eighth positions, the bisduple [quadruple] sounds occur in the fifteenth, and the triple in the twelfth.

Student: I ask by what reason or rule in the series of sounds are tones mixed in with semitones?

Master: You will be able to understand more clearly if I first show you what the arithmetic mean is, what the geometric mean is, and what the harmonic mean is.

Student: Please do.

Master: After proportions, proportionalities are considered, for a proportion is a form with two limits, a proportionality is a form with no less than three. Thus, a proportionality is joined with either an arithmetic, geometric, or harmonic
mean.

We speak of a mean when the two extremities of two items are bound in concordance by the bond of a lesser limit. Limits, however, we call the highest of numbers. Therefore, the arithmetic mean is when there are equal differences between the limits, but these themselves are not of the same proportion, as for example, 1.2.3.

Student: I see, 1 is a duple for 2, 2 is a sesquialter for 3.

Master: Likewise, the geometric mean is one in which not the equal differences, but the equal proportions are considered, as you see in 1.2.4.

Student: I see. As 2 is double 1, so 4 is double 2.

Master: But the harmonic mean is one in which neither the same proportions, nor the same differences are sought, but as the extreme limits alternate, so do the differences of those limits, as for example 3.4.6.

Student: I hardly understand: both the extreme limits and their differences are distant by a duple.\(^6\)

Master: Thus it is necessary that these things be put forth for

\[^6\] 3:4:6, 3:6 is duple; the differences of the extremes are \(-\frac{3}{1}\) and \(-\frac{4}{2}\), thus 1:2 is also duple.
you to see, according to a mean which the measures of tones hold for themselves, although individual measures, indeed, have individual means, nevertheless, the music seems created from all together. The geometric mean ties all limits of the duples to each other, for example if 12 is to 6, then 24 is to 12. And again, if 24 is to 12, then 48 is to 24. These intervals of duples then are completed by two limits, one by an arithmetic mean, the other by a harmonic mean to link the limits to each other. Indeed, by the same number that 9 is greater than 6, so 12 is greater than 9. And by the same number that 18 is greater than 12, so 24 is greater than 18: strictly speaking, by an arithmetic mean. Again, by as much as 8 is greater than 6, by the same part of 12, 8 is lesser than 12. You see then, that by 1/3 of 6, 3 is larger than 6, and by 1/3 of 12, 4 is larger than 3, and likewise, 12 to 16, and 16 to 24. This is all according to the harmonic mean.

Student: Indeed, I understand, because, although by the part of 6 by which 8 is greater, by the same part of 12, 12 is greater than 8, and it happens that there are differences in this proportion by which occur the extreme limits themselves. As 6 is to 12, so 2 is to 4. Between 6 and 8 the
difference is binary, and between 8 and 12 it is quaternary.

Master: You understand well. Now listen to how these tones themselves complete the intervals of the previously mentioned limits; not by an arithmetic mean, as days to days diminish and grow with equal parts for each other; not at the harmonic mean of dissimilar proportions; but at a geometric mean as they complete the epitritus and sesquialter intervals. Just as the second sesquioctavus is to the first, so the third is to the second. Just as at 64 the sesquioctavus is 72, so at 72 the sesquioctavus is 81.

Student: I understand how by means of these numbers, tones are measured from tones. But tell, why do you choose these numbers for examples?

Master: Because it cannot be shown by a smaller number than an 8 times an 8. This, therefore, is 64. This is because the second is an octuple producing two sesquioctaves from itself. For 1/8 of the number 64 added to itself is 72. One eighth of 72 added to itself is 81. Thus, there are two sesquioctaves, 64:72:81. Indeed, because it was proposed to explain further how in the epitritus interval there are two tones and a semitone, and in the entire series of sounds semitones are placed with tones, the fourth limit ought to be set, which would be
the epitritus to the first, it being placed between the first and the third limit of sounds. But while 64 cannot be divided into three equal parts, the epitritus is not attached to it. When it is multiplied by three then, the epitritus can be attached. And what is 3 times 64?

Student: 192.

Master: Yes, and we shall begin from this number to explain other things. Double it.

Student: 2 times 192 equals 384.

Master: Yes, and so you have these two numbers in the duple interval, 192 and 384. It is fitting now that we give the two epitriti for this duple interval. Therefore, give the epitritus for the number 192.

Student: 1/3 of 192, that is 64, added to it gives 256. When this number becomes the epitritus at the subduple limit, the sesquisalter appears at the duple.

Master: Give the sescplus of 192.

Student: 1/2 of 192, that is, 96, added to itself is 288. When this number becomes the sescuple at the subduple limit, the epitritus appears at the duple.
Yes, that is perfect. These four limits are now placed in order: 192.256.288.384. Therefore, the epitritus is between the first and second limit, likewise, between the third and fourth limit. Now let us fill these intervals of the epitritus with tones and semitones. I take 1/8 of 192, which is 24, and it becomes the number 216 by the addition of a tone, the second tone. I take 1/8 of 216, which is 27, and it becomes the number 243, the third tone, by the addition of this tone [27]. I am not able to take 1/8 of 243, because between this and the fourth limit, which is 256, is the epitritus of the first, and is found in 13 units, which distance is according to the rule of semitones. And the result is the diatessaron mode, computed with two tones and a semitone. Again, because the sescuplus is greater according to the epogdoos rule, I shall take up the fourth sound, that is, I shall take 1/8 of 256, which is 32, and it becomes 288, by the addition of this [32] which [288] is the fifth sound. I take 1/8 of the fifth sound, which is 36, and it becomes, by addition, the sixth sound 324. I take 1/8 of the sixth sound, which is 40, and it becomes, by addition, the seventh sound [364]. From this seventh sound comes 384, which is the eighth sound, and there remains, by the rule of

7 [Actually, it is 40.5.]
semitones, 20 units, and the result is the diapente, computed as three tones and a semitone.

**A Description of these things:**

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But if you desire proof of the quantities, make a string greater than another, and a pipe greater than another at the epitritus, which must be placed at the fourth place. And make it longer at the sesquialter, which is located at the fifth place. Make it longer by the duple, which you put in the eighth place. Likewise, taking the eighth part of the first string or pipe, you designate the second. Taking 1/8 of the second, you designate the third. You are not able to take 1/8 of the third to make the fourth, which now takes its measure from the first, against which the epitritus is hardly made. Likewise, it
is not possible to measure the fifth by repetition at the fourth because while it becomes the sescuplus at the first, the sesquioctavus is made at the epitritus. Again, taking 1/8 of the fifth, you measure the sixth; taking 1/8 of the sixth, you measure the seventh. You cannot measure the eighth by the seventh, because while the eighth, in relation to the fifth, becomes the epitritus, the distance of the seventh to the eighth will be a semitone.

And so, in the manner of intervals, different sounds agree in sweet joining.

Student: Marvelous, certainly, that voices are not able to harmonize otherwise, except sometimes between epitriti. Sometimes between the sesquialter the spaces of the semitones are diminished, so that thus they do not come together at the middle of the preceding tone.

Master: Truly this happens by a miraculous and divine rationale. But since you discern all things in a clear voice, let us try to take up the division of the monochord by these certain rules. Therefore, I recognize this first: the longer strings or pipes produce lower sounds, and the shorter they are the higher the sounds, so that, on the other hand, the middle gives the duple, the fourth part gives the quadruple. Therefore, take, for example, a high
string, as I take from a. to z., and in this space take 1/2, as from h. to z., and the middle sounding at the diapason answers to the whole. I take the middle of the half, as from h. to p., and it resounds the double diapason. Likewise, the lesser symphonias are arranged on either side of the diapason, as from a.z.to a.h., and again from h.z. to h.p. I take 1/4 of the a.z. space, and it answers d.z. at the diatessaron. I take 1/3 of the a.z. and e.z. responds at the diapente. Likewise, in the h.p. space, when 1/4 is taken away, it sounds at the diatessaron; when 1/3 is taken away, it will be at the diapente.

Now we arrange the diatessaron and diapente spaces with the epogdoi:. Indeed, with 1/9 taken from the a.z. space, b.z. will be the tone. When 1/9 is taken from the b.z. space, c.z. will be the tone, and d.m. the semitone. When the difference of the diatessaron and the diapente is a tone, there will be also d.z. at the tone e.z.: 1/9 taken away from the e.z. space and f.z. will be a tone; if 1/9 is taken away from the f.z. space, g.z. will be a tone and h.z. a semitone, and the final pitch [h.z] is a diapason. In this manner, as in other things, you
will modulate the diapason space, that is, from $h.z.$ to $h.p.$ at the diatessaron and diapente intervals. You will arrange tones with semitones according to the diatonic rule.

**Student:** When there are four species of tetrachords following each other, how are individual sounds able to agree with certain tones, as well as with the position of the semitones?

**Master:** They are not able to. For when there is one tetrachord which exists through two tones and a semitone, and the nearest is higher by one tone, then it happens that it is composed of a tone, a semitone, and a tone. That which follows in the third place is higher by a semitone, and consists of the necessary semitone and two tones. Then, that which follows in the fourth place, and is one tone away from the previous, happens to be of three tones. Again, that which is in the fifth place will be made up by returning to the first order of two tones and a semitone.\(^8\)

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\(^8\) [This description is clarified by the example on p.204.]
Student: Are these individual positions appropriate to the individual species of tetrachords?

Master: They certainly are. Thus, at the fifth place, the same quality of the song is given, because the order of this position is turned around.

Student: By what rule does the semitone fall between certain sounds?

Master: Give the form of Tritus to a higher one, and the form of Deuterus to the lower one, and you have between them a tone which is distant by a semitone. For one tone from that sound [i.e., Tritus] would be above, and two [Protus] and Deuterus] would be below, including the semitone, and this would be the series which we call Archoops or Protus. That which at the higher part would have two tones, and at the lower part would have one tone, is called Tetrardus. And so now you can easily distinguish between tetrachords, because when it is placed ascending, with two tones and a semitone, it gives this order of sounds: Tetrardus, Protus, Deuterus, Tritus. That which is arranged with a tone, a semitone, and a tone occurs in the sounds Protus, Deuterus, Tritus, Tetrardus. That which is arranged with a semitone and two tones occurs in the sounds Deuterus, Tritus, Tetrardus, Protus. That which is arranged with three tones
occurs in the order * Tritus, Tetrardus, Protus, and Deuterus.* This becomes more clear in the following description of the individual sounds:

In this description of the four tetrachords with spaces interjected, the distances of the sounds are designated, the lines interplaced between the semitone sound show how they differ, and the properties of the individual sounds are made clear.

**Student:** That is clear, but how can there be one tetrachord with three tones?

**Master:** That is understood also from things stated before. When two tetrachords complete one diapason, each tetrachord contains two tones and a semitone, and a median whole tone
divides the tetrachords into two, so that there is
the appearance of four tetrachords, and when it exceeds
the diatessaron, that is, two tones and a semitone, it
is made complete by three whole tones. This is also
apparent by the diagram.

Student: Now I understand both the description and the explanation
as well, and, as I see it, this transposition of semitones
becomes a change of tropes.

Master: That is hardly doubtful, for here the discernible quality
for each pitch and for the tetrachords and the pentachords
and, as you said, the forms of all the tropes and the semi-
tone itself, lie in their own species, and in this way,
the heart and spirit of the melody is created.

The strength of the semitone survives to deform the
appearance of the pitches, so that if you put a different
order of semitones beneath two different pitches, they
change form at the same time. For example, make one string
longer than another at the epitritus, so that they can
scarcely respond to each other at the diatessaron. This
consonance, when it contains two other tones within it,
is taken up by the lower part, that is, by the longer
string or pipe, and a tone is placed between the first
and the second, and likewise, between the second and third.
Let us separate the third from the fourth with a semitone, and there will be a Tetradus arranged with these sounds: Tetradus, Protus, Deuterus, Tritus. Again, if it is begun from the higher part, a tone divides the first and the second, and likewise the second and the third. Between the third and the fourth is a semitone, consequently it will be another species of the tetrachord, as with the position of the tones, thus, likewise, with the nature of the sounds, and it becomes the first, Protus, which is higher, then Tetradus, then Tritus, and Deuterus at the fourth place, as this description shows:

Moreover, it must be investigated why the sounds Tritus and Deuterus have the discernible quality of their own nature between them, by which things the difference is clearly only that of a semitone, and the remaining sounds are separated by a whole tone, not to the point that the quality per se would be divided, but rather, it is joined by the order; it is easily understood from circumstance
which it would be. Thus, by the introduction of this art, we modulate what the Greeks call *phthongus* to the point of raising or lowering the third pitches, by which the quality of the sounds is distinguished, in this way:

![Diagram of musical notation]

Student: Then everything is explained, that is, either what causes, or in what way the positions of the tetrachords are varied, or how, through individual variations of the species of modes or tropes are generated. It is now time for investigation. You said that in the fifth places a certain kind of trope returns, because it is reverted to the order of the same position.

Master: I said, clearly, that both a symphonía harmonizes in the fifth place because of the sesquialter form, and that a trope of this kind, because of this order, reverts to the position. And so, as that which is taken up at another
place, or else, retaining the quality of the trope
decreases from that which went before, so it does not reach
the position of its own order, or else yielding to another
order of trope, it changes its quality by transposition.

But state that which you ask about.

Student: Since the mode of a certain trope does not seem to be
another, although there is the same order in a certain
melody of tones with semitones, and since in certain
orders the position does not return at the eighth place,
I ask, how do sounds at the eighth place harmonize in the
same trope?

Master: It is true that that which is in the fifth place thus
occurs in the ninth, rather than in the eighth place, and
in the same sequence of sounds returns to the same position:
so that if the first and the second are distinguished by a
tone, then the tone occurs between the ninth and the
tenth sound: if a tone occurs between the second and third
sounds, then there is a tone between the tenth and eleventh;
if there is a semitone between the third and fourth, then
it too divides the eleventh and twelfth. But look, the
sound in this greatest symphonia, which agrees with the
higher and lower sounds in the eighth places, does not
follow the order of its own place, but of that to which it consonantly responds. Nor indeed, is a consonance preserved if either a tone clashes with a semitone, or a semitone with a tone in that place.

Student: How do tones clash with semitones?

Master: If a pitch in the eighth place distinguishes distances that are sometimes semitones and sometimes tones. For example, if there is a tone between the first and second, as opposed to the eighth and ninth sound, there is a tone between the second and third as opposed to the ninth and tenth, then there will be semitones between the third and fourth sounds, and a tone, on the contrary, between the tenth and the eleventh; a tone, likewise, between the fourth and the fifth, and a semitone, on the contrary, between the eleventh and twelfth sounds. Thus, actually, with semitones conflicting against tones, it is possible neither among the fourth and eleventh sound, nor among the fifth and the twelfth sound, either for the condition of duples, or the consonance of the same trope, to exist. Now when there is a tone between the eighth and the ninth sounds as opposed to the first and the second, a tone between the ninth and tenth as opposed to the second and third, there is a semitone between the tenth and the eleventh
as between the third and the fourth. In this arrangement of tones and semitones, the duple proportion preserves the harmony and the symphonia, and retains the mode of the trope.

Here ends the *Scholia enchiriadis*.
PART III

COMMENTARY
The presentation of the practical aspects of music supported by a pervasive philosophical and speculative framework is one of the most evident and important characteristics of the *Musica enchiridias* and *Scholia enchiridias*. Two closely related works, the first is a concise handbook presenting the practical and devoid of the speculative aspects of music, and the second volume is cast in the manner of a Platonic dialogue, giving a scholastic, speculative justification and amplification of the contents of the first volume. It is most important to first establish what appear to be the basic influences upon the adoption of this dual literary structure, and to establish the philosophical framework of these two works before proceeding to a discussion of the subject matter of the texts.

The analogy between language and music at the beginning of the *Musica enchiridias* has been identified by William Waite as a paraphrase of a passage from the Commentary on Plato's *Timaeus* by Chalcidius.⁴ The *Timaeus* was an enormously influential work during the Middle Ages, and it was carefully studied and quoted. It existed in two forms, that transmitted by Chalcidius, along with his commentary, which was the

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¹ *Journal of the American Musicological Society* IX/2, (Summer, 1956), pp.147-148. A similar analogy transmitted through the *Musica enchiridias* is also found in *De Musica* of Odo (GS I, 275ff.), and in Guido's *Micrologus*.  

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most influential, and a fragment of a commentary by Cicero. Most medieval libraries contained at least the Chalcidius version.2

The dialogue stresses, among other things, the importance of mathematics to explain the universe, coupled with belief in a divine creator who fashioned the soul of man and the universe from a mathematically divided whole. There are many other aspects of this fascinating work which were important to the study of science and theology during the Middle Ages, but this section on the fashioning of the soul and the universe is particularly vital to the *Musica enchiriadis*. After dividing this whole substance into intervals, the creator cut it into two equal strips, formed it into an X and joined the ends into two circles. Adding motion to these circles and further dividing them into seven and making them revolve in different directions and speeds, he thus formed the universe so that these elements were related proportionately. Going on, Plato states:

"And when the whole structure of the soul had been finished to the liking of its framer, he proceeded to fashion the whole corporeal world within it, fitting the two together centre to centre: and the soul was woven right through from the centre to the outermost heaven, which it enveloped from the outside and, revolving on itself, provided a divine source of unending and rational life for all time. The body of the heaven is visible, but

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the soul invisible and endowed with reason and harmony, being the best creation of the best of intelligible and eternal things. And because it is compounded of Same, Different, and Existence as constituent parts, and divided up and bound together in proportion and is revolving upon itself, whenever the soul comes into contact with anything whose being is either dispersed or indivisible, it is moved throughout and calculates similarity and difference, that is, in exactly what respects and in what way and how and when it happens that a particular thing is or is qualified by these terms in respect of anything either in the realm of change or in relation to things eternally the same. And whenever reasoning that is true, whether about the different or about the same, takes place, being carried on without speech or sound in the self-moved, if it concerns the sensible world, and the circle of Different, running straight, reports it to the whole soul, then there arise opinions and beliefs that are sure and true: but if it concerns the world apprehended by reason, and the circle of Same, running smoothly declares it, then the result must be apprehension and knowledge. And if anyone calls that in which this pair take place anything but soul he is speaking anything but truth."

The link between the Timaeus and the Musica enchiriadis occurs in the section which precedes the above, where Plato describes the division

3 The three constituent parts of the whole substance which was previously divided into intervals.

of the whole by the creator in detail:

"He began the division as follows. He first marked off a section of the whole, and then another twice the size of the first, next a third, half as much again as the second and three times the first, a fourth twice the size of the second, a fifth three times the third, a sixth eight times the first, a seventh twenty-seven times the first. Next he filled in the double and treble intervals by cutting off further sections and inserting them in the gaps, so that there were two mean terms in each interval, one exceeding one extreme and being exceeded by the same fraction of the extremes, the other exceeding and being exceeded by the same numerical amount. These links produced intervals of 3/4 and 4/3 and 9/8 within the previous intervals, and he went on to fill all intervals of 4/3 with 9/8; this left, as a remainder in each, an interval whose terms bore the numerical ratio of 256 to 243. And at that stage the mixture from which these sections were being cut was all used up."\(^5\)

Although Chalcidius compares other sections of the *Timaeus* with music, it is his discussion of this section which the author of the *Musica enchiriadis* paraphrases to begin his treatise. Both of the Latin texts are presented here for comparison:

<table>
<thead>
<tr>
<th>Chalcidius</th>
<th><em>Musica enchiriadis</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Quid est quod vult intelligi?</td>
<td>&quot;Sicut vocis articulatae</td>
</tr>
<tr>
<td>Dicam sed quae ante dicenda sunt</td>
<td></td>
</tr>
<tr>
<td>prius explicebo. Etenim quem ad</td>
<td></td>
</tr>
</tbody>
</table>

modum articulatae vocis principales sunt et maximae partes
nomina et verba, horum autem syllabae, sylabarum litterae, quae sunt
primae voces individuae atque elementariae, --ex his enim totius constituitur continentia et
ad postremas eadem litteras dissolutio pervenit orationis--: ita
etiam canorae vocis, quae a
Graecis emmeles dicitur et est
modis numerisque composita, principales quidem sunt hae, quae a
musicis adpellantur systemata.
Haec autem ipsa constant ex certo
tractu pronuntiationis, quae dicuntur diastemata. Diastemata
porro ipsorum partes sunt phthongi, qui a nobis vocantur soni. Hae
autem soni prima sunt fundamenta cantus. Est autem in sonis differen
tia iuxta chordarum intentionem: siquidem acuti soni
vehementius et citius percusso
adre excitantur, graviores autem,
quotiens leniores et tardiores
pulsus erunt. Et accentus quidem
existunt ex nimo inciatoque
pulsu, succentus vero leni et
tardiore. Ex accentibus porro et
succentibus variata ratione musicae
cantilenae symphonia dicitur.
Prima igitur symphonia, quae in
quattuor primis modulis inventitur,
diessarova dicitur. Secunda vero,
quae ex quinque primis modulis
constat, diapente cognomita est."

6
GS I, 152a. Translation, p. 54.

7
Platonis Timaeus interprete Chaldicio cum eiusdem commentario, ed.
Johannes Wrobel (Leipzig: B.C. Teubneri, 1876), 110-111.
From this passage, the *Musica enchiriadis* goes on to describe the Dasian scale which forms the basis of its musical system. Although Plato's scale is easily reduced to the numerical series 1, 2, 3, 4, 9, 8, 27 from which can be deduced the intervals of the octave (2:1), the twelfth (3:1), the fifth (3:2), the fourth (4:3), the whole tone (9:8), and other intervals, it remains ambiguous in other important details, and has led to a number of different interpretations. One such interpretation, worked out by Jacques Handschin, renders a scale much like the Dasian scale of the *Musica enchiriadis*, with the division of the scale by fourths, each tetrachord having the identical intervallic structure tone, semitone, tone. The successive tetrachords are each separated by a tone, and these four disjunct tetrachords produce the same pitches that are in the Dasian scale.  

Although Handschin discussed the contributions of many of the famous commentators of the *Timaeus*, he did not mention Chalcidius, which could have further substantiated the direct influence of the *Timaeus* on the *Musica enchiriadis*.  

Two other items in the *Musica enchiriadis* also illustrate the influence of the *Timaeus*. The first is found in three of four closely related manuscripts which are unique in their depiction of the so-called  

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9 Also pointed out by Waite, *op.cit.*, 148.
"Dasiapropeller." This illustration occurs right after the *Timaeus* paraphrase:

![Diagram](image)

This is an obvious reference to the passage in the *Timaeus* which describes the $X$ formed by the creator from the two strips of the proportionately divided whole substance. This figure, however, is not present in the main body of preserved manuscripts. The three which do include it are characterized by other textual variations of the better known form of the *Musica enchiriadis*.

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10 As coined by Heinrich Sowa. The four manuscripts are Paris, B.N lat. 7202, Madrid, B.N. Mss. 9088, Firenze, B.N.C., II.1.406, and Brugge, Stadsbibliotheek, 531. All but Brugge contain the figure, and Brugge does not contain any musical examples. All four differ from the text of the *Musica enchiriadis* which is common in the rest of the manuscript tradition. Three of them are copies of the Paris manuscript, and this manuscript, along with the variant text is printed in Latin with German translation by Heinrich Sowa in "Textvariationen zur Musica Enchiriadis," Zeitschrift für Musikwissenschaft, XVII/5 (May, 1936), 194-207. The interrelation of Madrid with the Paris manuscript has not been previously pointed out. This manuscript contains both versions of the text, side by side, and dates from the 12th century, making it the oldest copy of Paris which includes musical examples (Brugge and Paris are both from the 11th century, and Firenze is from the 15th century). The copy of the *Musica enchiriadis* in the Madrid manuscript is an almost exact duplication of that in the Paris manuscript, down to the placement of the musical examples.
The second item which shows the direct influence of the *Timaeus* is characteristic of almost all of the preserved manuscripts. This is the statement, which is found at the end of the next to the last chapter of the *Musica enchiridias* (Ch. XVIII, Translation, 102). This passage seems somewhat out of context until it is rightfully placed within the framework of the *Timaeus*:

"...the thinking which governs the harmony of voices and regulates the nature of mortals, follows the same rule, and because this thinking uses the same parts of numbers, thus, unequal sounds harmonize after they have been gathered together, and souls harmonize with bodies, and all the world joins in an eternal harmony."

All of the direct and indirect references to the *Timaeus* are uncited in the *Musica enchiridias*, which is not an unusual practice in medieval writing. In this case, it might have been unnecessary since the *Timaeus* was well known at the time. The Scholia enchiridias has a direct reference in Part III (p. 177), which reads: "...as it is revealed in the *Timaeus*, and with other philosophers." Here, the student has just asked how it is possible that musical pitches can be measured by the various arithmetic proportions, when they can not be seen or touched. The master answers that only multiplex and superparticular ratios are suitable for joining together, and that nothing in the world
can be joined without the appearance of proportions. Going on, he states:

"These are also the only proportions through which, according, to the different feet, the verses run harmoniously. In as much as the joinable nature of these remains, so the opposing abilities of the four elements are joined in a meeting of these proportions, just as it is revealed in the Timaeus, and with other philosophers: 'you tie the elements with numbers, so that cold things are suitable for flames, and dry things are suitable for liquids.' Because of which musical sounds do not combine into one mode so sweetly unless commensurality associates them according to the congruent and consonant intervals of proportions, which were mentioned before."

There remains one further circumstance that possibly influenced the author of the Scholia, which, although difficult to substantiate, is, nevertheless, important to point out. It seems quite possible that the choice of the dialogue form in the Scholia, and the use of the Scholia as a commentary on the Musica enchiriadis was directly influenced by the dialogue form of the Timaeus and the resultant commentary of Chalcidius. Thus, the Musica enchiriadis is tied closely to Chalcidius' commentary, itself amplifying certain parts of it, and the Scholia in effect becomes a commentary on this commentary, with the literary forms of the models, that of dialogue and prose, become reversed. The
word *scholium* (σχολία) is actually first found in Cicero (*Epistulae ad Atticum*, 16.7.3) and is defined as consisting of an individual *lemma*, that is, a word or phrase repeated directly from a text, along with an interpretation of that word or text. Its plural form, *scholia*, has come to mean an entire body of marginal notes, or glosses of a text. It is often difficult to rigidly distinguish between *scholia* and commentaries, although a commentary is usually a continuous exposition and is generally transmitted as a separate work.\(^1\) Thus, the *Scholia enchiriadis* is more a commentary, in the classical sense, than a *scholia*, particularly in view of the dialogue form, the absence of a continuous *lemma*, and the fact that it is a separate work. Although it is seldom found without the *Musica enchiriadis*, it could be used as a completely self-contained and separate treatise.

Perhaps the most important aspect of the *Timaeus* influence is that there seems to have been a conscious, yet inconspicuous attempt to integrate the subject matter of the *Musica* and *Scholia enchiriadis* into its Platonic framework: to make polyphonic practice a part of the medieval *scientia* by means of a most influential and well known work. By establishing this framework, even though done in a most subtle way

without citing his source, the author was, in effect, adding a prestigious element to his subject matter. This integration could have been one of the reasons for its preservation in numerous manuscript copies, even though the obscure and often misunderstood Dasian notation had become obsolete soon after its appearance.

Another reason for the lasting influence, and perhaps the most important reason, is its close theoretical association with the De institutione musica, and parts of the De institutione arithmetica of Boethius. It is Boethius, above all others, who is quoted and paraphrased, and whose mathematics provide the speculative support for the polyphonic practice presented in these works. This influence is most evident in Part III of the Scholia, which incorporates most of the mathematical explanations, and is drawn almost entirely from Book V of the De institutione musica. However, even in the Musica enchiriadis, which includes only a minimum of mathematics, Boethius is drawn upon, but primarily as a citing of an "authority" to give credence to various aspects of subject matter. In Chapter XI, an analogy between the number ten and the diapason is taken verbatim and uncited from Book V, Chapter X of the De institutione musica, to help explain the regeneration of the sound in the octave. Our author then goes on to paraphrase, from the same chapter, Boethius' statements regarding the consonances

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Although in several manuscripts the copyist, or a later person has provided Boethius' name in the margin.
of the twelfth and the eleventh.\textsuperscript{13} Chapter XVI consists entirely of a cited paraphrase from Chapter IX of Book V,\textsuperscript{14} which discussed Ptolemy's view that the diatessaron with diapente (the eleventh) was consonant, including an illustration not found in Boethius to help clarify the relationships between the Greek names of the notes spoken of in the text.\textsuperscript{15} And finally, at the close of the \textit{Musica enchiriadis}, we find this statement:

"The most eminent author Boethius revealed many miracles of this art of music, clearly proving all things by the use of numbers, an excerpt of which, God willing, the following work will contain." \textsuperscript{16}

Although direct references to Boethius are not found in Parts I and II of the \textit{Scholia}, these parts still present indirectly, the already assimilated theory of consonance as transmitted in the \textit{De institutione musica}.\textsuperscript{17} Almost all of Part III of the \textit{Scholia}, which deals primarily


\textsuperscript{14} Compare Translation, p. 91-4, with Bower, \textit{op.cit.}, 307-309.

\textsuperscript{15} Translation, p.92. \textsuperscript{16} \textit{Ibid.}, p. 104.

\textsuperscript{17} Boethius role as translator, rather than theorist, and all of his Greek sources which together make up the theory presented in \textit{De institutione musica}, are thoroughly discussed in Bower, \textit{op.cit.}, 333-369.
with proportions and the placing of the consonant intervals within a mathematical context, as previously mentioned, comes from Boethius directly.

It seems important to realize that even though the influence of Boethius may seem extensive, he is still drawn upon only to provide the mathematical support needed for the practical theory which is presented in these treatises. By the tenth century, most of the *De institutione musica* was held as highly impractical, and it seems quite obvious that the borrowed data from Boethius in both the *Musica enchiriadis* and *Scholia enchiriadis* were carefully selected to provide some measure of practicability. In Book III of the *Scholia*, most of the theoretical concepts are first presented in arithmetic terms, and then followed with a further explanation using the practical application of pipes or strings to aid the reader in putting the abstract into musical reality. Part III of the *Scholia* fulfills the hopes expressed at the end of the *Musica enchiriadis*, for, despite its repetitive explanations, it is truly an excerpt of useful mathematics, distilled from a mass of complex and largely philosophical speculation.

Although the theoretical support given to the practical aspects of these works through Boethius may still seem quite impractical to the modern mind, they were necessary to establish these works within the scholastic framework, as exemplified through the *Timaeus* and its commentary by Chalcidius, and its basic tenet that all things can be
explained by means of numbers. But even more importantly, it helped to place polyphonic practice within the established Boethian framework, using it to justify this practice, and in so doing, establishing itself as a logical extension of a basically impractical, yet highly esteemed treatise.

The corpus of extant manuscripts gives us some indication of the establishment of this extension, for in many of them the Musica enchiriadis and Scholia enchiriadis come directly after the De institutione musica. One copyist added the subtitle at the beginning of the Musica enchiriadis: "Commentum musicæ artis ex opusculis Boetii exerptum et a venerabili abbate Hogero elaboratum." 18

One of the most problematic aspects of the Musica enchiriadis and the Scholia enchiriadis regards the deciphering of the Dasian notation into modern pitch notation, particularly important in that all of the musical examples are presented in this system. As there is no specific reference to alphabetic equivalents in the treatises, we can only glean these through a careful examination of the text. 19 By doing just that,

18 Valenciennes, Bibliothèque Municipale 337 (olim 359, 325), fol. 42r.

19 Letters found next to the symbols in some manuscripts never completely conform to textual statements and are more confusing than helpful.
Phillip Spitta made the first successful clarification of the system in 1889. This study is now considered fundamental to any understanding of the system.

Although the alphabetic system established through the works of Odo, Guido and Hermannus Contractus was much easier to understand, being less complicated, Dasian notation still fulfilled a need, even if it was short-lived. The biggest problem with the system was that it often contradicted itself, whereas the alphabetic system was much more precise, more versatile, and did not require the contradictory rules inherent in the Dasian system.

Immediately following the Timaeus analogy at the beginning of the Musica enchiriadis, the system is first described by giving the forms of the symbols, their names, and the names and intervallic structure of the tetrachords which combine to make up the entire system. The four basic symbols which form the system are:

\[ F, \text{ named Protus, or Archoos (i.e., the first)} \]
\[ \underbrace{F}_{\text{named Deuterus (the second)}} \]
\[ \underbrace{F}_{\text{named Tritus (the third)}} \]
\[ \underbrace{F}_{\text{named Tetrardus (the fourth)}} \]

These four sounds are arranged in order with a tone separating Deuterus from Protus, a semitone separating Tritus from Deuterus, and a tone separating Tetrardus from Tritus, in the following manner: \[ \frac{\text{F }}{\text{F}} \frac{\text{F}}{\text{F}}. \]

20 "Die Musica Enchiriadis und ihr Zeitalter," Vierteljahrsschrift für Musikwissenschaft, V (1889), 443-482.
The example in Chapter I illustrates how these four pitches are formed into four groups of four notes each, with two notes left over at the top. It is somewhat confusing in that the author uses only the symbols for the basic tetrachord, having not introduced the others, but probably also to help get the point across that each tetrachord has the same basic intervallic structure of tone, semitone, tone: ²¹

\[ FF/F \cdot FF/F \cdot FF/F \cdot FF/F \]

Next, the names of the four tetrachords are given, along with an example of the other symbols for the entire system:

\[ \begin{array}{cccc}
\text{Graves} & \text{Finales} & \text{Superiores} & \text{Excellentes} \\
\end{array} \]

Chapter II describes these altered forms and states that there are eighteen sounds which individually reach the symphonia of a fifteenth ²².

Chapter III gives the first clue as to the specific pitches that should be assigned to the Dasian symbols: "The Finales tetrachord is so named because every note is governed by and ends on one of these sounds." ²³

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²¹ Translation, 55.

²² Ibid., 57. That is, a fifteenth form each of the notes of the Graves tetrachord only, which accounts for the two "extra" notes at the top of the system.

²³ Translation, 57.
Thus, the symbols for the finales tetrachord and their corresponding letter names would be interpreted as follows: \( \text{\textit{f} f^\# \textit{f} g} \)

Chapter IV gives a few vague rules regarding the modes, although one does not know which specific mode is meant, or whether the rule can apply to any melody. Starting from its final, it is said that a cantus can descend only to the note which is a fifth below its final.\(^{24}\) This accounts for the Graves tetrachord, and provides us with the letter names of these symbols as well. Thus,

\[
\begin{array}{cccccc}
C & A & B & C & D & E \\
(\text{Graves}) & (\text{Finales}) \\
\end{array}
\]

The upper range for each is limited "up to the third sound of the same name, that is, up to the tetrachord Excellentes." The "third sound of the same name" refers to the third occurrence of the symbol name, that is, Protus, Deuterus, Tritus, or Tetrardus. Thus, if the cantus started on Protus, its upper range would be limited to the third occurrence of Protus, that is, Protus excellentes. A similar statement in the Scholia states that the sound "may only descend to the fifth sound below its own final, or ascend only to the ninth above."\(^{25}\) Thus, the range for each

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\(^{24}\) In later modal theory the plagal form only descends to the fourth below the final. Whether the author is consciously changing practice to conform with the fact that the symbols duplicate themselves at the fifth position, or whether the author is adding a subfinal to fill out the system, or whether he is just in error can not be ascertained.

\(^{25}\) Scholia, Part I, 133.
mode is a ninth, instead of an octave. Since the Dasian symbols repeat themselves at the fifth and the ninth, these rules for the modal range fit the system perfectly. These statements point out another problem, as can be seen from the example below. If the strict tone, semitone, tone arrangement of the tetrachords is maintained, as is ordered in Chapter I, any octave above any Tritus, or below any Deuterus will not be perfect (note that the same is true of fourths above Tritus, or below Deuterus):

![Diagram](image)

The first clarification of this dilemma is not offered until Chapter X, which deals with the symphonias. Here it is stated that through the octave the original sound is regenerated, and thus, they are called "equal-sounding." Chapter XI sheds further light on the matter with the statement that the same ninths are found at the ninth positions, but the same eighths are not found at all eighth positions, but in the symphonia, "the eighth sounds and the eighth positions by a miraculous change, are the same." In other words, the octaves are made perfect

26 Translation, 54-56.

27 P. 74

28 80-81.
by simply changing them where necessary.

Chapter VI contains several observations meant to help one keep the relationships between the individual symbols clear. He states that from any sound, the following relationships exist:

1) There is a sound (or rather, a Dasian symbol) of the same quality at the fifth position. Thus, a fifth from Protus Finales yields another Protus, either Protus Superiores at the fifth above, or Protus Graves at the fifth below. The same is true of Deuterus, Tritus, or Tetrardus:

\[
\begin{align*}
\text{Prot.} & \quad \text{Deut.} & \quad \text{Trit.} & \quad \text{Tetr.} \\
\begin{array}{c}
\text{F.} \\
5 \\
4 \\
3 \\
2 \\
1 \\
\end{array} & \quad & \begin{array}{c}
\text{F.} \\
5 \\
4 \\
3 \\
2 \\
1 \\
\end{array} & \quad & \begin{array}{c}
\text{F.} \\
5 \\
4 \\
3 \\
2 \\
1 \\
\end{array} \\
\text{Finales} & \quad & \text{Finales} & \quad & \text{Finales} \\
\end{align*}
\]

2) The same sound will occur at the third position above and below any sound:

\[
\begin{align*}
\text{Tritus} & \quad \text{Tritus} \\
\begin{array}{c}
\text{F.} \\
5 \\
4 \\
3 \\
2 \\
1 \\
\end{array} & \quad & \begin{array}{c}
\text{F.} \\
5 \\
4 \\
3 \\
2 \\
1 \\
\end{array} & \quad & \begin{array}{c}
\text{F.} \\
5 \\
4 \\
3 \\
2 \\
1 \\
\end{array} \\
\text{Tritus} & \quad & \text{Tritus} & \quad & \text{Tritus} \\
\end{align*}
\]

3) If a sound is at the second position above another, that same sound will also occur at the fourth below, and, on the contrary, if it is at the second below, the same sound will occur at the fourth above: 29

\[
\begin{align*}
\text{Deut.} & \quad \text{Deut.} \\
\begin{array}{c}
\text{F.} \\
5 \\
4 \\
3 \\
2 \\
1 \\
\end{array} & \quad & \begin{array}{c}
\text{F.} \\
5 \\
4 \\
3 \\
2 \\
1 \\
\end{array} & \quad & \begin{array}{c}
\text{F.} \\
5 \\
4 \\
3 \\
2 \\
1 \\
\end{array} \\
\text{Deut.} & \quad & \text{Deut.} & \quad & \text{Deut.} \\
\end{align*}
\]

In Chapter VIII, musical examples that help to explain the modes are presented. 30 Each example consists of three parts: an Alleluia,
to establish the authentic form of the mode, consisting of a perfect fifth in range; a phrase in the authentic form of the mode; a phrase in the plagal form of the mode. In the example for Mode I, the authentic phrase has a range of a perfect fifth from its final. Its plagal form extends a third above the final, with one note below the final in the middle of the phrase. Mode II has a total compass of a minor sixth in the authentic example, with a subfinal near the cadence. The plagal phrase has a total range of a fifth, from a fourth above the final to a single note below the final in the middle of the phrase. Mode III extends only to a fifth above the final, and the plagal example begins and ends on the final, extending only a major third above it. Mode IV extends a major sixth above the final, and its plagal form extends only a perfect fourth above the final with a subfinal at the cadence. The rules at the end of the chapter seem to have been violated in the examples themselves, at least in the plagal ones. He says that the principal modes begin in the Superiores and end on their final, which they all do, and that the plagal should start and end on their finals and should not reach into the Superiores tetra-chord. Only the example for Mode III both starts and ends on its final, and, if one were to take the Superiores place to include any pitch above a, then only the example for Mode I conforms to this rule. If however one were to stretch the definition of Superiores in this context so that it would mean a fifth above the final, then all of the examples conform,
and the statement itself conforms to earlier ones made in Chapter V. 31

It should be pointed out that in all of the examples and explanations regarding the modes, the octave species are only vaguely alluded to, and most of the examples do not extend more than a fifth above the final and a third below. Half of the examples contain a tone below the final at the cadence, but this is never discussed in the text. Also, all of the modes seem to be reckoned from the highest note (a fifth above the final) downwards, instead of the usual low to high arrangement we find later.

Chapter IX of the Musica enchiriadis includes several definitions of terms, some of which were also mentioned in the Timaeus analogy in Chapter I. Of particular importance is the fine distinction between the words sonus and phthongus. Here, sonus is the general name given to any sound, whereas phthongus is reserved for the sounds of song. 32 The same definition is given in Part I of the Scholia. In Chapter I of the Musica enchiriadis, however, the definitions are reversed. Here it is stated that:

"Not all phthongi are called soni, only those arranged in a lawful succession are suitable for a melody." 33

A tone is defined in Chapter IX as a musical interval which is in the sesquioctavus (9:8) proportion. 34 In previous chapters, "tone" was

31 P. 59  32 Ch. IX, 69.  33 Ch. I, 54.  34 P. 69.
also used to mean "mode," and even though it is stated in Chapter VIII that the modes are improperly called the tones, at the end of the same chapter bad habits return, and once more the modes are referred to, quite improperly, as tones. 35

After the naming of several of the different proportions, the modes are referred to as tropes, and only three of their Greek names are given: "Dorius Phrygius Lydius etc.,..." 36 In all other places in these works they are referred to as Protus, Deuterus, Tritus, and Tetrardus, named, of course, after their finals.

Distinctions are also made between the divisions of melodies, adopting terms used to describe textual divisions: phrases (particulae); semicolons (cola); commas (commata). Commas function in music much like their textual counterparts, and originate through the raising (arsis) and lowering (thesis) of the voice. A semicolon consists of two or more of these commas. Intervals (diastemata) are defined as the distance between the highest and lowest sounds within a comma, and those of an entire melody are called systems (systema). 37

Essentially, the first nine chapters of the Musica enchiriadis establish the basic principles of Dasian notation, the eighteen pitch scale, the modes, and the introduction of a few musical definitions.

35 Cf. 63 and 68.
36 Ch. IX, 70.
37 Pp. 70-71.
Part I of the *Scholia enchiriadis* corresponds to the first nine chapters of the *Musica enchiriadis*, but in the *Scholia*, exercises are provided that assist the singer to both read the notation as well as to hear correctly. After explaining that good singing requires discipline and practice, the major part of this section is then devoted to various errors (called *absonia*) which can occur in singing. Some errors occur through laziness, or when the intervals are measured incorrectly, or when an interval does not respond to another by the proper interval. To explain the difference between these last two, the entire system of Dasián notation along with the tetrachords and their intervallic structure is again presented. 38 Pentachords are introduced and then are used in place of tetrachords almost entirely throughout the *Scholia*. When any one of the intervals, which should follow in the "natural order," is not measured correctly, it changes the quality of the sound, and the first dissonance of the last two errors is produced. This concept is reinforced through many exercises in Dasián notation, which switch or omit various pitches from the pentachord thereby altering the whole and half step order of the "natural order," i.e., of the original tetrachords (tone, semitone, tone). This section points out the relative pitch aspect of Dasián notation, where the symbols stand as representing intervallic relationships rather than simply exact pitches, and misreading one symbol, will change the others. Due to the fact that the author continually refers to these as errors, and as ruining the song, it seems

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quite obvious that he is not establishing a kind of medieval transposition system, used to introduce chromatic tones not a part of the regular diatonic system. A transposition system is described later in this section, using several examples of pentachords. This transposition system is a device for changing mode, and at the fifth, the original order of the first mode is once again resumed. Further definitions of the modes are also given here, all of which closely parallel those given in the Musica enchiriadis. At the end of the first part of the Scholia is a section dealing with the measured singing which is not discussed in the Musica enchiriadis. The basic tenet of this discussion consists of two rhythmic values, a long and a short. These are not notated, and follow very simple rules. A leader guides the rhythm by tapping his foot, and all of the values of a melody are short, with the exception of the last note of each section which is twice as long as the others. Also, an entire tempo can be made twice as fast or twice as slow by altering all of the values of the melody. It appears from the comments at the end of this section that this method of rhythmic singing was meant for the purpose of synchronizing the singers of a group, or a chorus, rather

39 Ibid., 122-128

40 Compare the Scholia, 122-128 with the Mus. enchir., 63-68.

41 Scholia, 134-137.
than as a system of varied durational values.

The rest of the *Musica enchiriadis*, from Chapter X to the end, and Part II of the *Scholia enchiriadis*, deal with the symphonias and their various combinations and arrangements which are termed Diaphony, or Organum. This material presents the earliest surviving detailed account of polyphony.

A symphonia is described as a "pleasant harmony of the mutual joining of several sounds," and three of them, the diatessaron, the diapente, and the diapason, are classified as "simple and principle." These three are illustrated as both harmonic and melodic intervals, although it is their harmonic forms which are given prominence.

Other symphonias can be formed by combining the simple ones with the diapason. The diatessaron with the diapason forms an eleventh, the diapente with the diapason forms a twelfth, and two diapasons form the double diapason, or the fifteenth. Thus, the diapason holds special significance, for it has the power of maintaining consonance when another symphonia is added to it. It also has the power, or rather,

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44 Chapter XI, 76.

45 Chapter XI, 77-81. The section in question is included in a paraphrase from Boethius' *De institutione musica* Bk.V, Ch.X (Bower, *op.cit.*, 309) which forms the greatest part of this chapter and is uncited. Boethius is discussing Ptolemy's theories.
the particular quality of regenerating a sound at its diapason. In this respect, it is more correct to use the term "equal-sounding," rather than "harmonious," which is used to describe the other symphonias. The diapason is also called the "greatest and first symphonia," because the other simple symphonias can be found within it. This concept, that the diapason is made up of a diapente and a diatessaron, is stated many times in these final chapters and also in the Scholia.

A most important point in the discussion of the diapason is the statement that it makes a consonance perfect by changing the note at the diapason to correspond with the first sound. This solves a major problem of Dorian notation: the augmented octaves which occur above any Tritus, or below any Deuterus:

\[ \text{G A B}^\flat \text{ C D E F G A B C D E F}^\# \text{ G A B C}^\flat \]

47 Musica enchiriadis, Ch.X, 74, again stemming from Boethius, Bk.V, Ch.XI (Bower, op.cit., 310-311), and also originating from Ptolemy.

48 Musica enchiriadis, Ch.XI, 78.

49 Ch.XI, 76; Ch.XIV, 87-8; Ch.XVII, 91. The Scholia also shows through numbers that the duple proportion (the diapason) consists of a sesquiquarteta proportion (a diatessaron), and a sesquiquarter proportion (a diapente). See the graph in Part II, 161, and the text, 162-163.

50 Ch.X, 74-75 alludes to this, but it gets fuller treatment in Ch. XI, 77-81.
This same rule can also apply to the double diapason. The musical examples provided in the *Musica enchiriadis* to illustrate the quality of the diapason and double diapason employ a letter notation rather than Dasion notation. This notation consists of seven different letters which duplicate themselves at the eighth. Thus, they provide an excellent indication of the equal-sounding and regenerative qualities of these two symphonias in a manner similar to the way they are described in the text. Obviously not intended to indicate specific pitches as later letter notation, they are arranged the same way that the Greek system is indicated, that is, with the lowest pitch at the top, and the highest at the bottom.

When comparing the melodic contour of the example of the Alleluia at the diapason (Ch.X, 75) with the earlier example at the diapente (p.74) as well as the example *Tu patris sempiternus* at the diapason and double diapason (Ch.XI, 79) with the same melody at the diapente (Ch.XII, 82), it can be seen clearly that these letters are only a device for showing that the symphonia at the diapason or double diapason duplicated itself.

When comparing this section with corresponding sections in the *Scholia*, we find an incorporation of both systems of notation in an

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51 This was also described briefly above.

52 *Musica enchiriadis*, Ch.X, 73; Ch.XI, 77 and 79.
ingenious fashion. There is first laid out the alphabetic series A–P, with A–H indicating the lower diapason, H–P the higher diapason, and A–P indicating the double diapason:

A B C D E F G H I K L M N O P
relaxed diapason

tensed diapason

After this is described, the Scholia includes an example of three-part organum at the diapason and double diapason. The music here is different than in the Musica enchiriadis, and all three parts are written using the same Dorian symbols (only those of the Finales tetra-chord) stacked on top of each other. The octave separation of each part is indicated with the letters A, H, and P at the beginning of each appropriate part. The letter A does correspond here to the Dorian but the letters are used here only to indicate the octave transposition of the various parts, and not any specific pitches. The fact that there

53 No doubt brought about by the inherent limitations of augmented octaves discussed above.

54 Also found in Boethius, Bk.IV, Ch.XVII (Bower, 289), whose letters stem from Boethius’ own interpretation of Ptolemy (Harmonics, ii.9.63) regarding the hypermixolydian mode. See Bower, footnote 56, p.289.

55 Scholia, Part II, 139-140.

56 Chapter XI, 79.
are no Dasian symbols to indicate the very high range of the upper voice at the fifteenth, being almost an octave above the system, also contributes to the necessity of the employment of this kind of letter notation, both in the Scholia and in the *Musica enchiriadis*. 57

The diapente is illustrated next in the *Musica enchiriadis*. This illustration uses the same melody (*Tu patris sempiternus*) with the added organal part at the fifth below. 58 It is then shown that a melody can be changed by transposing it by steps, but at the fourth transposition by step (at the interval of the fifth above), the original order of the melody is restored. Only after this discussion is the concept of diaphony, or organum introduced, beginning with organum at the diatessaron below, and using the same music as was used both for the diapente as well as for the diapason and double diapason. 59 Although there seems to be a conscious attempt to play down organum at the diapente by not specifically including it in this chapter, there is the statement that "whatever is common to all symphonias in the diatessaron and diapente has this name" (i.e., either diaphony or organum). 60

From the simple organum at the diatessaron, a four-part organum is illustrated—formed by doubling the principal voice an octave lower, and doubling the organal voice an octave higher. 61 There are also other

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57 Ch.XI, 79. 58 Ch.XII, 82. 59 P.82 and 79.
60 Ch.XIII, 85. 61 Ch.XIV, 86-87.
arrangements of voices given, but not illustrated. It is also stated that instruments might be used, with boy's voices employed for the highest parts. As with other examples of diaphony, all of the voices must harmonize with concordant symphonias again implying that the natural order determined by the recurring interval series tone, semitone, tone, within each tetrachord would have to be altered in order to change the dissonant augmented octaves and tritones to perfect octaves and fourths.

In Chapter XV another example of four-part diaphony is included, based upon the diapente—in this case with an organal voice doubled an octave above, and the principal voice doubled at an octave below. Once again there follows the admonition regarding concordant symphonias. The question as to which voice should be altered is never stated in either treatise, probably because the writer believed it to be self-evident that the organal voice, the one being added to an existing principal voice, would be the one made to conform.

Chapter XVI presents Ptolemy's description of the consonance of the eleventh, as related by Boethius. After presenting a previous musical example to point out the elevenths, a graph of the Greek names of the notes is given with the elevenths clearly marked in each instance. The rest of the chapter is a verbatim rendering of the

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62 Ibid., 87-88. 63 88-90. 64 89-90. 65 90-94.

chapter in the *De institutione musica*, in which this discussion occurs. 67

Chapters XVII and XVIII of the *Musica enchiriadis* present an entirely new species of organum quite different from the rigid parallel organum previously described. After reestablishing the "greater perfection" of the diapason symphonia, since it contains both the diatessaron and the diapente, it goes on to say that with the diatessaron, since the fourths are not always concordant (i.e., because of the inherent tritone), these voices are "divinely adapted to the other voices by an intrinsic law." 68 It is then pointed out that between Deuterus and the Tritus a fourth below (*Tritus subquartus*), the diatessaron exceeds the proper measure of the symphonia, being three whole tones, rather than the two whole tones and a semitone as was explained at the beginning of the chapter. Thus, a rule is established that one must not allow the organal voice to descend below the Tetradus of any tetrachord in any phrase, nor ascend from below Tetradus at the beginning, since the cause of the inconstance, Tritus, stands just before Tetradus. 70

To illustrate this rule, the musical example *Rex coeli* is presented in two parts (the organal voice being the lower one). It begins

67 P. 92. 68 Ch.XVII, 94-95. 69 P. 95.

70 P. 95.
on *Tetrardus Graves*, and the organal voice remains on *Tetrardus* (without dropping below it in conformity with this rule) as the principal voice ascends. When a diatessaron is reached between the voices, the organal voice then is free to move in parallel motion, but as the principal voice descends, the organal voice, once again, can never drop below *Tetrardus*. Ironically, the limitations of the rule results in what has been termed as "free" organum, starting on a unison, then proceeding in oblique motion, continuing in parallel motion when the fourth is reached and until *Tetrardus* approaches, at which time the organal voice returns in oblique motion and closes on a unison with the principal voice. The next section of the *Rex coeli* sequence is given in Chapter XVIII, and occurs in a higher range. The last part of this chapter gives the correct way to harmonize the *Tu patris sempiternus* stanza when transposed by steps through the four modes, using the rule for avoiding the tritone. In one transposition, that starting on *Tritus*/(F), diaphony at the diatessaron is not possible because the melody proceeds to *Deuterus* (b°F), which would require the organal part to be on *Tritus*/(F) thus forming a tritone.

These two chapters contradict the earlier examples of strict organum at the diatessaron, particularly in Chapter XIII, where the same *Tu partrs sempiternus* melody is harmonized throughout at the
diatessaron in strict parallel motion, completely disregarding the later rule of the organal voice not being able to descend below Tetrardus. Compare this example with the later example in Chapter XVIII:

Chapter XIII:

\[
\begin{align*}
& \text{es} \\
& \text{pa/} \\
& \text{fi\}/} \\
& \text{es \ li\}/} \\
& \text{Tu \ tris sempiternus/ \ us.} \\
& \text{pa/} \\
& \text{fi\}/} \\
& \text{Tu \ us.} \\
& \text{Tu \ pa-tais sempiteanus es fi-li. us}
\end{align*}
\]

Chapter XVIII: \(^{76}\)

\[
\begin{align*}
& \text{es} \\
& \text{pa/} \\
& \text{fi\}/} \\
& \text{es \ li\}/} \\
& \text{Tu \ tris sempiternus/ \ us.} \\
& \text{pa/} \\
& \text{fi-li/} \\
& \text{Tu \ pa-tais sempiteanus es fi-li-us}
\end{align*}
\]

\(^{76}\) Pp. 98-99.
In the first example, from Chapter XIII, the tritone between Tritus Graves $N(B^b)$ and Deuterus Finales $P(E)$ at 1i of filius, according to the rule that all symphonias must be concordant, the organal voice (the lower one) would require a chromatic alteration of Tritus Graves $N(B^b)$ up a half step to $B^b$. In the example from Chapter XVIII, according to the Tetrardus rule, one must start from a unison, since a diatessaron below Protus $A(D)$ would extend below Tetrardus Graves $A(C)$ down to Deuterus Graves $A(A)$. Even though no tritone would be involved here, as in the first example, the rule still forbids starting the organal voice below Tetrardus. At 1i of filius, the organal voice again cannot continue in parallel fourths, for in so doing, it would descend below Tetrardus $A(C)$.

In light of these inconsistencies and contradictions in the presentation of these two distinct kinds of organum, it seems most possible that the use of free, oblique organum starting from a unison is here being justified on the basis of the avoidance of the tritone. In other words, it must have already been in practice and in need of some kind of theoretical justification. As Gustav Reese points out, since the Micrologus also employs a lower limit for the organal voice (the third sound of the tetrachord, rather than the fourth as advocated in the Musica enchiriadis), and the avoidance of the tritone is no longer alleged to be the reason for the limit, particularly since Guido's gamut, unlike that of the Musica enchiriadis, did not contain the
inherent tritones,

"The continued appearance of a limit in spite of this is rather striking and subjects the explanations of the Musica enchiriadis to suspicion." 77

Thus, the Musica enchiriadis presents two kinds of organum with differing rules for each species. That the strict, parallel kind was in use up to the time of Guido is witnessed in the Micrologus, although he favors the free, oblique style. 78 In order to present both species, both of which were being practiced, the author of the Musica enchiriadis establishes an inconsistent and contradictory theoretical system, yet one which still was able to give theoretical justification to the singing in strict parallel organum, and the singing in free, oblique organum.

The Scholia enchiriadis is more comprehensive in its examples of organum. Using another musical example, NOS qui vivimus, as the basis for all of the examples, organum is first presented at the diapente below the principal voice, and then given six different arrangements using the diapente with three or four voices. 79 All of the examples for organum at the diapente are in the strict, parallel style, and none of the arrangements correspond exactly to that given in the Musica enchiriadis. 80

77 Gustav Reese, Music in the Middle Ages (New York: W.W. Norton, 1940), 259.

78 79 80 
Ibid. Scholia, Part II, 140-45. Ch. XV, p. 89.
The examples of organum at the diatessaron, unlike those of the diapente, employ the rule of avoiding the tritone by not descending below Tetrardus, and thus, the strict, parallel style is impossible in some instances. All of the examples begin at the diatessaron, rather than starting at the unison, although several of them end on a unison. The proper interpretation of these examples of organum at the diatessaron in the Scholia enchiriadis presents several difficulties, mostly because of scribal errors in the manuscripts (in several manuscripts the musical examples are missing entirely), but also from the nature of Nos qui vivimus itself. This melody is drawn from the last verse of the 113th Psalm (the 27th verse, right before the Gloria Patri), "In exitu Israel de Aegypto" ("When Israel came forth out of Egypt"). This last verse reads: "Sed nos qui vivimus, benedicimus Domino, ex hoc nunc et usque in saeculum" ("But we will bless the Lord from this day forth and forever"). The music exists today as the psalmody of the Tonus Peregrinus for Psalm 113 (Liber Usualis, 160-161):

\[ \text{\begin{center} \includegraphics[scale=0.5]{image.png} \end{center} } \]

\text{Sed nos qui vivimus benedicimus Domino Ex hoc nunc et usque in saeculum} 

\[ \text{\footnote{Scholia, 145-150. They are arranged the same as those of the diapente.}} \]
The *Tonus Peregrinus* is an additional tone to the regular eight which includes two tenors (one on A, another on G, in the modern form), a characteristic $A-B^b-A$ opening, and a close on D. It was probably derived to harmonize with several Antiphons, all of which belonged to a similar group which defied modal organization. 82 These troubled Aurelianus, and he classified them under the eighth mode, as a variant, although the eighth tone did not fit them. He termed the tone *neophytus* (new-fangled). 83 The *Commemoratio brevis* also lists it under the name *Tonus novissimus*. 84 These names would seem to indicate that it was new (or perhaps rediscovered) in the ninth century, as opposed to the belief held by some scholars that it was derived from Hebraic or Byzantine chant, and was one of the oldest surviving forms of chant linking Western and Eastern practice. 85

One complication which arises from the earlier notated examples of the *Tonus Peregrinus* or *Novissimus*, is the opening formula $A-B^b-A$, the common characteristic of the tone as well as the antiphons to which


83 GS I, 52a, as cited by Apel, *loc.cit.*

84 GS I, 218.

85 Apel, *op.cit.*, 213
they correspond as they are used today. In the Dasian notation of the Scholia enchiriadis and the Commemoratio brevis, it is impossible to tell if $b$ was intended, and the notation implies a $b^\sharp$. Compare the differences of the tone as given in the Scholia with that of the Commemoratio brevis and the modern version:

Scholia Enchiriadis

Nós qui vivimus benedíctimus Domínium Ex hoc nunc et usque in sæculum.

Commemoratio Brevis:

Affecte Domino filii Dei Affecte Domino filios antequam

The fact that the tenors of these examples differ in the second phrase has led Apel to believe that the distinctive trait of the tone was not the two tenors, but rather the special intonation and termination formulae which was made to conform to the melodic outline of the
Antiphons.

The question of $B^\flat$ or $B^\natural$ remains an open one. The Antiphons cited in the *Commemoratio brevis* also do not indicate a $B^\flat$, and there are no indications in the texts of the *Musica enchiriadis* or *Scholia enchiriadis* that the note in question ($J$) should be altered.

Whether a $B^\flat$, or a $B^\natural$ was sung would also determine the organal voice at the fourth below in the examples of diaphony at the diatessaron in the *Scholia*, particularly at the second note, at which a tritone would normally occur between Tritus $I$ ($F$) of the organal voice, and Deuterus $J$ ($B^\natural$) of the principal voice. These examples are interesting because they incorporate both styles of organum—the strict, parallel style for the majority of the examples, with the oblique, free style occurring at the end of each example, following the rule that the organal voice may not descend below Tetradus. In any event, in actual practice, one of the voices must be altered to correct the tritone at the beginning. It seems odd that this tritone was left, while at the end the rule of Tetradus was used to avoid it. Yet it is correctable, and lacking any comment about it in the texts it seems that the organal voice would be chromatically altered to $F^\#$ to correct it.

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86. Ibid.

87. Part II, 145-150.

88. The manuscript tradition is varied in this regards. Many show the examples as being parallel throughout, others mix the styles, and others have some examples in free and others in strict style.
In the last section of Part II of the *Scholia*, the subject shifts from musical practice to the realm of philosophy and the speculative aspects of music, which are later presented in detail in Part III of the *Scholia*. This section of Part II serves as a transition from the practical to the mathematical by first presenting the concept of the quadrivium, explaining music's proper place within the structure of philosophy, and emphasizing the absolutely essential role of arithmetic in any understanding of music. The major source for this section is Book I, Chapter I or Boethius' *De institutione arithmetica*, with many passages used verbatim, only slightly altered to fit the form of the dialogue.

Part III of the *Scholia enchiriadis* delves deeply into musical mathematics. Compiled almost entirely from various chapters of Boethius' *De institutione musica*, again many times verbatim, the discussion begins with a detailed explanation of the various kinds of inequality and how they are expressed in arithmetic proportion. The proportions most relevant to music are next discussed in detail. The ordering and justification of consonance is presented in great detail, and are most important to the chapters which dealt with polyphony in the *Musica enchiriadis* and Part II of the *Scholia*.

It will, of course, be noted that from the mass of data found in the *De institutione musica*, the author has distilled only those pertinent facts needed to establish the philosophical support and justification

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89 Pp. 156-165.  
90 Friedlein, *op.cit.*, 7-12.
for the musical practice described—support and justification which was particularly necessary to sanction, at least theoretically, the already accepted practice of polyphony.

After a careful examination of the contents of the Musica enchiriadis and Scholia enchiriadis, it is most evident that their importance is not limited to the fact that they are the earliest remaining detailed account of polyphony. They represent a description of an established style of music—one which was practiced but not yet codified. With few resources at his disposal, our author devised (or at the very least adapted) a system of notation and molded a theoretical explanation, an explanation which, even with all of its inconsistencies could best be described as ingenious. Obviously a learned man, he incorporated the knowledge and descriptions of others to help clarify his system, as well as to give it needed support, all the while striving to bridge what had become a chasm between the philosophical and the practical aspects of music. Without a doubt they remain an important link between philosophical speculation and practical description—innovative and essential, and deserving of our most careful reflection and scrutiny.
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