ULTRASONIC CHARACTERIZATION OF MULTILAYERED COMPOSITES

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By

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ABSTRACT

An efficient algorithm has been proposed for simulation of ultrasonic waves with multilayered composites. The method is based on combining of global matrix method with accounting for the system periodicity. The results show this method is stable and efficient especially for cross-ply composites. Using this algorithm, the behavior of wave transmission through a laminate immersed in water have been discussed and explained by Floquet wave pass and stop bands. Based on Floquet wave representation, a homogenization procedure has been proposed to model the multilayered composite as an anisotropic homogeneous medium. To describe experimental conditions for an impulse regime interrogation of composites a beam model has been developed. Also the time-domain response in an acoustic microscopy has been investigated and a procedure to measure the elastic constant using the acoustic microscopy signature has been discussed.
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CHAPTER 1

INTRODUCTION

With the increasing use of advanced composites in a variety of modern applications, it has become necessary to employ reliable and effective nondestructive evaluation (NDE) methods for characterization and interpretation of their properties. The most promising is the ultrasonic method. Over the past 15 years, extensive investigations have been made in solving wave propagation and scattering in composites [1-3]. In these studies, the fibrous-enforced composites are considered as anisotropic media. The solutions are obtained by extending the results for isotropic solids to anisotropic media.

Research on wave propagation in an isotropic medium has a long history. In 1885 Lord Rayleigh [4] studied surface wave propagation along the free surface of a semi-infinite elastic solid. For a finite-thickness plate, Lamb [5] derived the characteristic equation for symmetric and anti-symmetric modes known as Lamb waves in 1917. The interface wave traveling along the interface between two semi-infinite solids was investigated by Stoneley in 1924 [6]. For multilayered structures, Thomson [7] first introduced the well-known transfer matrix method in 1950. Later, Haskell [8] and Gilbert [9] improved computational aspects of this method. Using this method surface and Lamb waves and acoustic beam propagation in a multilayered structure have been investigated [10-13]. But this solution is limited to low frequency and small thickness due to numerical instability. Dunkin [14] proposed a delta operator technique to improve the
stability of the method. In this technique, the subdeterminants in the Thomson-Haskell transfer matrix form elements in a new transfer matrix which does not suffer from loss of precision. But it loses the simplicity of the Thomson-Haskell formulation. Another approach which does not have numerical instability is the global matrix method proposed by Knopoff [15]. In this method, the displacement and stress continuity conditions on all the interfaces are assembled in one matrix. Although this method is robust and can be implemented simply, the dimension of the global matrix become very large when applied to laminates with many laminas and solving the corresponding linear equation system needs extensive computing resources [16].

The cross-ply composites which are manufactured by layup of orthotropic laminas with different orientations are the most difficult for ultrasonic study. This is because the media should be considered as multilayered anisotropic media, and one needs consider ultrasonic wave propagation in a wide frequency range and wide range of incident angles. The techniques developed for analysis of multilayered isotropic solids are also applicable to multilayered anisotropic media [17-19]. To remove the so-called "precision problem" in the transfer matrix method, Hosten and Castaings [17] have generalized the delta matrix method. However, for anisotropic solids, the dimensions of the delta matrix are increased to 20 and the formulation for each term in the delta matrix obtained by symbolic computer software is complicated. This method becomes not only complex but also computationally inefficient. Mal [20] has applied the global matrix method to obtain the solution for a layered anisotropic composite.

The multiply composites usually are made by repetition of selected layups of laminas with different fiber orientations, for example \([0/45/-45/90]_n\), which is repeated \(n\) times.
Such composites may be considered as repetitions of a unit cell and described as anisotropic periodical media. The free wave propagating in a periodical medium is the Floquet wave. Floquet wave propagation in a periodic media have been intensively investigated by combining transfer matrix or global matrix techniques with Floquet’s theory. Rytov, Sve and Delph [21-23] utilized the global matrix method to derive the characteristic equations of a periodic laminate. The Floquet wave characteristic equation also can be derived by the transfer matrix method [3, 25-28]. In this method, the Floquet wave numbers correspond to the eigenvalues of the global transfer matrix. When the z direction is the symmetry direction, Braga and Herrmann [3] showed that six real or complex Floquet wave numbers appear in pairs of opposite sign. The stop and pass bands in frequency or wave number domains have been investigated. At low frequency, periodic solids can be considered as effectively homogenous. Postma [30] and White and Angoma [31] proposed the effective modulus theory in which the composites are modeled as a homogeneous anisotropic medium. The elastic constants of this medium are a geometrically weighted average of the properties of the constituents. In this theory, the wave velocities of the homogeneous media are independent of frequency. Achenbach, Sun and Herrmann [33] developed effective stiffness theories for laminated media.

To study wave propagation in composites, we have to know the properties of these materials. Rokhlin and Wang [35-36] used the phase velocity and critical angle to determine the elastic constant of a composite. Acoustic microscopy has been used to measure material properties since the 1980s [37-39]. By measuring the leaky surface wave velocity from the \( V(z) \) curve, material properties (elastic constants and mass density) can be determined. Time-resolved acoustic microscopy [40-42] has also been used to
determine material properties by measuring the phase velocity from the time delay. For graphite epoxy composite materials, due to their low density and significant fluid loading, the acoustic microscopy response is significantly different from that for higher density materials.

In this work, a new efficient algorithm exploring the repetition and symmetry properties in a typical composite and combining the stiffness matrix and recursive methods has been proposed. Using this method, the Floquet wave propagation in a periodic anisotropic medium will be discussed. We will also propose a new homogenization method based on Floquet waves in a periodic medium. The application of these theories in composite bond characterization and interpretation of acoustic microscopy signatures for composites will be discussed.

The thesis is organized as follows: in chapter 3, the transfer matrix and global matrix methods are briefly reviewed and the new algorithm is formulated. The comparison of the efficiency of these solutions is provided. In chapter 4, the stop and pass bands of Floquet waves in an anisotropic medium in frequency, rotation angle and incident angle domains are discussed. A new homogenization method has been proposed using the Floquet wave slowness curve. In chapter 5, a beam model has been used to interpret the experimental results for a bonded composite sample. In chapter 6, the application of acoustic microscopy in composite characterization has been discussed.
CHAPTER 2

OBJECTIVES

Usually for high-resolution ultrasonic measurements one utilizes a short pulse with wide frequency bandwidth. Application of this method to the cross-ply composite which is made from a significant number of laminas resulted in large value of parameter \( fh \) (\( f \) is frequency and \( h \) is composite thickness). Thus due to computational difficulty, both the transfer matrix and global matrix are not suitable for modeling and interpretation of the ultrasonic measurements for composites.

The objective of this work is three folds, first is to develop an efficient algorithm applicable to composites, demonstrated its applicability and several applications. The algorithm with improved efficiency and stability is based on global matrix method incorporating a recursive matrix algorithm.

Second, since the behavior of wave transmission through a laminate immersed in water is very complicated and not well understood, we will use the new algorithm to obtain physical insights of the mechanisms of wave transmission through a composite. Third a beam model will also be developed to interpret the reflections from a bonded composite sample and will be used as a quantitative tool for composite bond characterization in the future. As illustration, a new property measurement method based on acoustic microscopy will be developed for lamina property determination.
CHAPTER 3

ANALYSIS OF ULTRASONIC WAVE PROPAGATION IN MULTILAYERED COMPOSITES

3.1 Background

There are two methods for modeling wave propagation in multilayered anisotropic medium. One is the transfer matrix method. Though this method is conceptually simple and the dimension of the total transfer matrix is low, it becomes unstable as frequency and thickness increase. To remove this so-called “precision problem”, the delta matrix method has been introduced. However, for anisotropic solids, the equations for the delta matrix elements become very complicated and dimensions of the matrix significantly increased. Another method is the global matrix method. The disadvantage in this method is that the dimensions of the global matrix for a laminate with a large number of laminas become very large and solution requires large computer memory and long computational time. Since most fiber-enforced composites include several repetitions of an identical unit element and are symmetrical about the centerline of the laminate, we resolve the problem by utilizing this material structure, and put forward a new efficient algorithm by combing the global stiffness matrix and recursive methods.
3.2 Transfer matrix method

Let us consider a composite plate, consisting of $N$ arbitrarily oriented orthotropic layers stacked normally to the $z$-axis as illustrated in Fig.3.1. In anisotropic media, the stress $\sigma_{ij}$ and displacement $u_i$ satisfy the equation

$$\sigma_{ij} - \rho \ddot{u}_i = 0,$$

(3.1)

Figure 3.1 A multilayered anisotropic solid and the coordinate system.
For fiber-reinforced composites, each lamina may have orthotropic symmetry. For the orthotropic lamina arbitrarily oriented with respect to the plane of incidence with axis of symmetry "3" coinciding with the plate normal direction and, the constitutive relation may be represented as

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{12}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\
C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\
C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\
0 & 0 & 0 & C_{44} & C_{45} & 0 \\
0 & 0 & 0 & C_{45} & C_{55} & 0
\end{bmatrix}
\begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
\epsilon_{33} \\
\gamma_{23} \\
\gamma_{12}
\end{bmatrix},
\]

(3.2)

where \( C_{ij} \) represent the elastic constants. We used the contracted subscript notations \( 1 \rightarrow 11, 2 \rightarrow 22, 3 \rightarrow 33, 4 \rightarrow 23, 5 \rightarrow 13, 6 \rightarrow 12 \) to relate \( C_{mn} \) to \( C_{ijkl} \). In order to consider absorption, we will use complex elastic constants, which are obtained from the real elastic constants using the method in [43]. The relation between real and complex elastic constants may be written as:

\[
C'_{11} = C_{11} / (1 + i\alpha \sqrt{C_{55} / C_{11}}), \quad C'_{22} = C_{22} / (1 + i\alpha \sqrt{C_{55} / C_{22}}),
\]

\[
C'_{44} = C_{44} / (1 + i\alpha \sqrt{C_{55} / C_{44}}), \quad C'_{55} = C_{55} / (1 + i\alpha),
\]

\[
C'_{12} + C'_{55} = (C_{12} + C_{55}) / (1 + i\alpha \sqrt{C_{55} / (C_{12} + C_{55})}).
\]

\( \alpha \) is the attenuation factor which may be frequency-dependent.

The displacement and phase velocity of the plane waves can be determined from the Christoffel equation

\[
B(k_z)U = 0,
\]

(3.3)

where the matrix elements of \( B \) can be written as
\[ b_{11} = C_{11}k_z^2 - \rho \omega^2 + C_{55}k_z^2, \]
\[ b_{12} = C_{16}k_z^2 + C_{45}k_z^2, \]
\[ b_{13} = (C_{13} + C_{55})k_zk_z, \]
\[ b_{22} = C_{66}k_z^2 - \rho \omega^2 + C_{44}k_z^2, \]
\[ b_{23} = (C_{16} + C_{45})k_zk_z, \]
\[ b_{33} = C_{55}k_z^2 - \rho \omega^2 + C_{33}k_z^2. \] (3.4)

There are three independent plane waves (a quasi-longitudinal wave and two quasi-transverse waves) which are determined by Eq. (3.3) in an anisotropic infinite medium.

We express the displacements of the plane waves in the form
\[ \mathbf{U}^\pm = \begin{bmatrix} \pm u_n \\ \pm v_n \\ w_n \end{bmatrix} \exp(i(k_z x + \omega t)), \] (3.5)

where
\[ u_n = b_{12}b_{23} - b_{13}b_{22}, \]
\[ v_n = b_{12}b_{13} - b_{11}b_{23}, \]
\[ w_n = b_{11}b_{22} - b_{12}^2. \] (3.6)

where \( k_z = \omega \sin \theta / c_f, \) \( c_f \) is the acoustic velocity in the fluid and \( \theta \) is the incident angle. \( k_z^n \) is determined from the characteristic equation as a condition for vanishing of the determinant of Eq.(3.3). The sign + is selected for \( +k_z^n \) and for \( -k_z^n \). The stress in the 1-2 plane can be obtained from the constitutive relation
\[ \mathbf{T}^\pm = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{bmatrix} = \begin{bmatrix} \pm T_{33} \\ T_{12} \\ T_{21} \end{bmatrix} = \begin{bmatrix} C_{55}(u_n k_z^n + w_n k_z) + C_{45}v_n k_z^n \\ C_{45}(u_n k_z^n + w_n k_z) + C_{44}v_n k_z^n \\ C_{13}u_n k_z + C_{36}v_n k_z^n + C_{36}v_n k_z \end{bmatrix} \exp(i(k_z x + \omega t)), \] (3.7)

In multilayered media, the displacement in the \( m \)th layer can be written as
\[ \mathbf{U}_m = \sum_{n=1}^{1} (A_n^+ \mathbf{U}^+ e^{i(k_z x + \omega t)}) + A_n^- \mathbf{U}^- e^{-i(k_z x + \omega t)}), \] (3.8)
$A_n$ represents the magnitude of the corresponding wave. The displacements and stresses on the upper surface of the $m$th layer, i.e. $z=z_{m-1}$, can be obtained from Eqs. (3.5-3.8) in the matrix form

$$
P_n^+ = \begin{bmatrix}
  u_1 & -u_1 g_1 & u_2 & -u_2 g_2 & u_3 & -u_3 g_3 \\
  u_2 & v_1 & v_2 & -v_2 g_2 & v_3 & -v_3 g_3 \\
  u_3 & w_1 & w_2 & -w_2 g_2 & w_3 & -w_3 g_3 \\
  \sigma_{13} & T_{11} & T_{12} & -T_{12} g_2 & T_{13} & -T_{13} g_3 \\
  \sigma_{23} & T_{21} & T_{22} & -T_{22} g_2 & T_{23} & -T_{23} g_3 \\
  \sigma_{33} & T_{31} & T_{32} & T_{32} g_2 & T_{33} & T_{33} g_3 
\end{bmatrix} = X_n^+ A_m, \quad (3.9)
$$

The displacements and stresses on the layer bottom surface, i.e. $z=z_m$, can be written as

$$
P_n^- = \begin{bmatrix}
  u_1 & -u_1 g_1 & u_2 & -u_2 g_2 & u_3 & -u_3 g_3 \\
  v_1 & v_1 g_1 & v_2 & v_2 g_2 & v_3 & v_3 g_3 \\
  w_1 & w_1 g_1 & w_2 & -w_2 g_2 & w_3 & -w_3 g_3 \\
  \sigma_{13} & T_{11} g_1 & T_{12} & -T_{12} g_2 & T_{13} & -T_{13} g_3 \\
  \sigma_{23} & T_{21} g_1 & T_{22} & T_{22} g_2 & T_{23} & T_{23} g_3 \\
  \sigma_{33} & T_{31} g_1 & T_{32} & T_{32} g_2 & T_{33} & T_{33} g_3 
\end{bmatrix} = X_n^- A_m, \quad (3.10)
$$

where $g_m = e^{ikz_m} = e^{-ikz_m}$, $h_m$ is the thickness of the $m$th layer. In order to make the algorithm stable for large frequency and layer thickness, we can select the origin at the layer top ($z=z_{m-1}$) for a wave propagating along the -$z$ direction and at layer bottom ($z=z_m$) for a wave propagating along the $+z$ direction. Therefore $g_m$ is always an exponentially decreasing function of frequency and thickness. From Eq. (3.9) and (3.10), the transfer matrix $M_m$ which relates $P_n^+$ to $P_n^-$ can be obtained from

$$
M_m = X_n^+(X_n^-)^{-1}. \quad (3.11)
$$
Using the displacement and stress continuity at all interfaces, the global transfer matrix for the laminate with $N$ laminas is obtained from multiplication of all the transfer matrices for each lamina

$$\mathbf{M} = \mathbf{M}_N \mathbf{M}_{N-1} \cdots \mathbf{M}_2 \mathbf{M}_1. \quad (3.12)$$

For a composite defined as $[\theta_1 / \theta_2 / \cdots / \theta_{n-1} / \theta_n]_m$, the laminate is made of $n$ repetitions of cell $[\theta_1 / \theta_2 / \cdots / \theta_{n-1} / \theta_n]$ and symmetrical about the centerline of the laminate. In this case, the global matrix $\mathbf{M}$ becomes

$$\mathbf{M} = (\mathbf{M}_n \mathbf{M}_{n-1} \cdots \mathbf{M}_2 \mathbf{M}_1)^* (\mathbf{M}_1 \mathbf{M}_2 \cdots \mathbf{M}_{n-1} \mathbf{M}_n)^* \quad (3.13)$$

If the laminate is immersed in fluid, using the boundary conditions of equal normal displacement in the fluid and solid, pressure in the fluid equal to the stress $\sigma_{33}$ in the solid and zero tangential stress $\sigma_{13}$ and $\sigma_{23}$ at the top and bottom surfaces of the laminate, the reflection $R$ and transmission coefficients $T$ can be obtained from

$$R = \frac{i k_x \rho_f \omega^2 (M_{22} M_{42} - M_{23} M_{41} - M_{31} M_{42} + M_{32} M_{41}) + k_x^2 (M_{32} M_{42} - M_{31} M_{42} - M_{23} M_{41} + M_{21} M_{43})}{i k_x \rho_f \omega^2 (M_{21} M_{42} - M_{22} M_{41} + M_{31} M_{42} - M_{32} M_{41}) + \rho_f^2 \omega^4 (M_{32} M_{41} - M_{31} M_{42} + M_{23} M_{41} - M_{21} M_{43})} \quad (3.14)$$

$$T = \frac{i 2 k_x \rho_f \omega^2 (M_{22} M_{42} - M_{23} M_{41} + M_{31} M_{42} - M_{32} M_{41}) M_{33} + (M_{32} M_{41} - M_{31} M_{42}) M_{23} + (M_{32} M_{41} - M_{31} M_{42}) M_{33}}{i k_x \rho_f \omega^2 (M_{21} M_{42} - M_{22} M_{41} + M_{31} M_{42} - M_{32} M_{41}) + \rho_f^2 \omega^4 (M_{32} M_{41} - M_{31} M_{42} + M_{23} M_{41} - M_{21} M_{43})} \quad (3.15)$$

The transfer matrix method is numerically unstable. For large frequency and thickness, the reflection and transmission amplitudes calculated by Eqs. (3.14) and (3.15) are incorrect due to loss of precision. Similar instability arises for layered isotropic media. Thus this instability is inherent to the transfer matrix method. In order to improve stability for the isotropic case, Dunkin introduced a delta operator technique. In Eq.
(3.14) and (3.15), the $R$ and $T$ are functions of $3 \times 3$ or $2 \times 2$ subdeterminants of the $M$ matrix. An important property of the delta operator is the distributivity for the matrix product; i.e., if $C = AB$, one has $C^\Delta = A^\Delta B^\Delta$, where $C$, $A$ and $B$ are $N \times N$-order square matrices and $C^\Delta$, $A^\Delta$ and $B^\Delta$ are square matrices made up of all $n \times n$ subdeterminants of $C$, $A$ and $B$ respectively. The dimension of $C^\Delta$, $A^\Delta$ and $B^\Delta$ is $C_N^n$. Using this property, one can obtain the delta matrix $M^\Delta$ for the global transfer matrix $M$ from the delta matrices $M_i^\Delta$ for each lamina

$$M^\Delta = (M_m^\Delta M_{m-1}^\Delta \cdots M_1^\Delta M_0^\Delta)^n (M_{m-1}^\Delta M_{m-2}^\Delta \cdots M_0^\Delta M_0^\Delta)^n.$$

(3.16)

The dimension of these matrices is $C_0^n = 20$. For stable numerical computation using the delta matrix, the analytical expression for each term in $M_i^\Delta$ has to be derived. The transfer matrix is given by Eq. (3.11). Expansion of Eq. (3.11) leads to very complicated equations. Software which can manipulate the mathematical symbolic computation such as Maple and Mathcad have to be used to obtain the explicit equation for each term in $M_i^\Delta$.

Summary: in the ordinary transfer matrix method the global transfer matrix can easily be obtained from the product of the transfer matrices for each lamina (Eq. 3.13) and this leads to a simple formulation for the reflection and transmission coefficients (Eqs. 3.14 and 3.15). It is very useful at low frequency and thickness. As the thickness and frequency increase, this algorithm becomes unstable and the delta matrix method may eliminate this numerical problem. However, in the delta matrix method, the dimension of the matrix is increased to 20 and the global delta matrix involves the product of all delta matrices for each lamina; in addition, the equations for each term in

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the delta matrix $M^\Delta_k$ are very complicated. Therefore, the delta matrix method becomes not only complex but also computationally inefficient for anisotropic multilayered solids.

3.3 Global matrix method

Another solution for multilayered solids is the global matrix method. Consider the same laminate illustrated in Fig.1. From Eqs.(3.9) and (3.10), the displacements in the upper surface ($z=z_m$) $U_m^+$ and lower surface ($z=z_m$) $U_m^-$ are given by

$$
\begin{bmatrix}
U_m^+ \\
U_m^-
\end{bmatrix} =
\begin{bmatrix}
W^+ & W^- \\
W^+ & W^- H
\end{bmatrix}
\begin{bmatrix}
A_m^+ \\
A_m^-
\end{bmatrix} = E_m^u A_m,
$$

(3.17)

and the stresses in the $m$th lamina top surface ($z=z_m$) $T_m^+$ and bottom surface ($z=z_m$) $T_m^-$ are given by

$$
\begin{bmatrix}
T_m^+ \\
T_m^-
\end{bmatrix} =
\begin{bmatrix}
D^+ & D^- \\
D^+ & D^- H
\end{bmatrix}
\begin{bmatrix}
A_m^+ \\
A_m^-
\end{bmatrix} = E_m^\sigma A_m
$$

(3.18)

where $D^\pm = [T_1^\pm, T_2^\pm, T_3^\pm]$, $W^\pm = [U_1^\pm, U_2^\pm, U_3^\pm]$ and $H = \text{Diag}[g_1, g_2, g_3]$. From Eqs. (3.17) and (3.18), the relation between the stresses and displacements of the upper and lower surfaces of the $m$th lamina can be obtained

$$
\begin{bmatrix}
T_m^+ \\
T_m^-
\end{bmatrix} = E_m^\sigma E_m^{-1} \begin{bmatrix}
U_m^+ \\
U_m^-
\end{bmatrix} = K_m \begin{bmatrix}
U_m^+ \\
U_m^-
\end{bmatrix} = \begin{bmatrix}
K_m^{11} & K_m^{12} \\
K_m^{21} & K_m^{22}
\end{bmatrix} \begin{bmatrix}
U_m^+ \\
U_m^-
\end{bmatrix},
$$

(3.19)

$K_m$ is the stiffness matrix for the $m$th lamina. The interface between the $(m-1)$th and $m$th lamina is assumed to be perfect; i.e., the displacements and stresses are continuous across this interface. Using this boundary condition, we may rewrite Eq. (3.19) as

$$
X_m^{p_{m-1}} + X_m^{p_m} = 0,
$$

(3.20)
where \( X_m^+ = \begin{bmatrix} K_{m1} & 0 \\ K_{m2} & I \end{bmatrix}, \quad X_m^- = \begin{bmatrix} K_{m1} & 0 \\ K_{m2} & I \end{bmatrix}, \quad P_{m-1} = \begin{bmatrix} U_{m-1}^+ \\ T_{m-1}^+ \end{bmatrix} \) and \( P_m = \begin{bmatrix} U_m^+ \\ T_m^+ \end{bmatrix} \), \( I \) is the identity matrix.

As shown in Fig. 3.1, the anisotropic solid is immersed in fluid. The stresses and displacements in the top and bottom surfaces generated by the reflected (\( R \)) and transmitted (\( T \)) waves may be written as

\[
\begin{bmatrix} U \end{bmatrix}_{\text{top}} = \begin{bmatrix} -\cos \theta \\ 0 \\ 0 \\ \rho_f c_f^2 \end{bmatrix}, \quad R = X_m R, \quad \begin{bmatrix} U \end{bmatrix}_{\text{boucem}} = \begin{bmatrix} \cos \theta \\ 0 \\ 0 \\ \rho_f c_f^2 \end{bmatrix}, \quad T = X_m T.
\]

(3.21)

where \( \rho_f \) is the fluid density, and \( c_f \) is the acoustic velocity. \( \theta \) is the incident angle. \( R \) and \( T \) represent the reflection and transmission amplitude respectively. The requirement of displacement and stress continuity at all interfaces leads to the global stiffness matrix

\[
\begin{bmatrix} X_m & I \\ X_m^+ & X_m^- \\ X_2^+ & X_2^- \\ \vdots & \vdots \\ X_N^+ & X_N^- \\ I & X_d \end{bmatrix} \begin{bmatrix} R \\ P_0 \\ P_1 \\ \vdots \\ P_{N-1} \\ P_N \end{bmatrix} = \begin{bmatrix} A_{in} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},
\]

(3.22)

\( A_{in} = [\cos(\theta), 0, 0, \rho_f c_f^2]^T \) represents the displacements and stresses generated by the incident wave. Solving this linear equation using Gaussian elimination we can obtain the reflection and transmission coefficients. For laminates made from large number of
laminas, the global stiffness matrix is very big and solution becomes computationally inefficient.

3.4 A special algorithm for composites

For a periodic composite defined as \([\theta_1/\theta_2/\cdots/\theta_{n-1}/\theta_n]\)\_t, an efficient global matrix method can be formulated. The cell includes \(n\) laminas and the laminate has \(n\) number of repetition of the cell and is symmetrical about the center plane. The stress and displacement at the top and bottom surfaces of the cell are denoted as \(T^+, T^-, U^+, U^-\). The requirements of displacement and stress continuity at all interfaces inside the cell lead to the global stiffness matrix for the cell. Using Eq. (3.22), we obtain

\[
\begin{bmatrix}
X_{12}^- & X_1^- & 0 & 0 \\
X_2^- & X_2^+ & 0 & P_1 \\
\vdots & \vdots & \ddots & \vdots \\
0 & X_{n-1}^- & X_{n-1}^+ & P_n \\
0 & 0 & X_n^- & X_n^+ \\
\end{bmatrix}
\begin{bmatrix}
T^+ \\
P_1 \\
\vdots \\
P_n \\
T^- \\
\end{bmatrix}
= 
\begin{bmatrix}
X_{11}^+ U^+ \\
0 \\
\vdots \\
0 \\
X_{n1}^+ U^- \\
\end{bmatrix},
\]

(3.23)

where \(X_{i1} = \begin{bmatrix} K_{i1}^{11} \\ K_{i1}^{21} \end{bmatrix}\), \(X_{i2} = \begin{bmatrix} 0 \\ I \end{bmatrix}\) and \(X_{n1} = \begin{bmatrix} K_{n1}^{11} \\ K_{n1}^{21} \end{bmatrix}\), \(X_{n2} = \begin{bmatrix} 0 \\ I \end{bmatrix}\). The matrix \(K_c\) relating \(T^+, T^-\) to \(U^+, U^-\) is defined as the cell stiffness matrix.

\[
\begin{bmatrix}
T^+ \\
T^- \\
\end{bmatrix}
= 
\begin{bmatrix}
K_c^{11} & K_c^{12} \\
K_c^{21} & K_c^{22} \\
\end{bmatrix}
\begin{bmatrix}
U^+ \\
U^- \\
\end{bmatrix},
\]

(3.24)
Its elements $\mathbf{K}^\theta_c$ are obtained by solving Eq. (3.23).

For two cells bonded together, the cell stiffness matrix for each cell is calculated by Eq. (3.23). We assume that the first cell stiffness matrix is given by

\[
\begin{bmatrix}
T_1 \\
T_2
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{21} \\
A_{12} & A_{22}
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2
\end{bmatrix},
\]  

(3.25)

and the second cell stiffness matrix is given by

\[
\begin{bmatrix}
T_2 \\
T_3
\end{bmatrix} =
\begin{bmatrix}
B_{11} & B_{21} \\
B_{12} & B_{22}
\end{bmatrix}
\begin{bmatrix}
U_2 \\
U_3
\end{bmatrix},
\]  

(3.26)

Then the stiffness matrix for these two cells (i.e., the relation between $T_1$, $T_3$ and $U_1$, $U_3$) may be calculated as

\[
\begin{bmatrix}
T_1 \\
T_3
\end{bmatrix} =
\begin{bmatrix}
A_{11} + A_{12} (B_{11} - A_{22})^{-1} A_{21} & -A_{12} (B_{11} - A_{22})^{-1} B_{12} \\
B_{21} (B_{11} - A_{22})^{-1} A_{21} & B_{22} - B_{21} (B_{11} - A_{22})^{-1} B_{12}
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_3
\end{bmatrix}.
\]  

(3.27)

In the case of periodicity, the two cells are identical, therefore the stiffness matrices for the two cell are also identical ($A_{ij} = B_{ij}$), so Eq. (3.27) may be rewritten as

\[
\begin{bmatrix}
T_1 \\
T_3
\end{bmatrix} =
\begin{bmatrix}
A_{11} + A_{12} (A_{11} - A_{22})^{-1} A_{21} & -A_{12} (A_{11} - A_{22})^{-1} A_{12} \\
A_{21} (A_{11} - A_{22})^{-1} A_{21} & A_{22} - A_{21} (A_{11} - A_{22})^{-1} A_{12}
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_3
\end{bmatrix},
\]  

(3.28)

If the two cells are symmetrical about their bond line, then the two cell stiffness matrices have the relations $A_{12} = -I_2 B_{21} I_2$ and $A_{11} = -I_2 B_{21} I_2$, where $I_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

Therefore in the case of symmetry, Eq. (3.27) may be rewritten as
\[
\begin{bmatrix} T_1 \\ T_3 \end{bmatrix} =
\begin{bmatrix} A_{11} - A_{12} (I_2 A_{22} I_2 + A_{22})^{-1} A_{21} \\ I_2 A_{12} I_2 (I_2 A_{22} I_2 + A_{22})^{-1} A_{21} \end{bmatrix}
\begin{bmatrix} A_{12} (I_2 A_{22} I_2 + A_{22})^{-1} I_2 A_{21} I_2 \\ I_2 (A_{11} + A_{12} (I_2 A_{22} I_2 - A_{22})^{-1} A_{21}) I_2 \end{bmatrix}
\begin{bmatrix} U_1 \\ U_3 \end{bmatrix}
\]

(3.29)

Using the global matrix method to calculate the cell stiffness matrix for the cells (Eqs. 3.23) and the recursive algorithm to calculate the effects of repetition and symmetry (Eqs. 3.28 and 3.29), one can obtain the stiffness matrix for the entire laminate. The procedure for this algorithm is shown in Fig. 3.2. We denote the stiffness matrix for the whole laminate as \( \mathbf{K} \left( \begin{bmatrix} K^{11} & K^{12} \\ K^{21} & K^{22} \end{bmatrix} \right) \) and its inverse \( \mathbf{S} \left( = K^{-1} = \begin{bmatrix} S^{11} & S^{12} \\ S^{21} & S^{22} \end{bmatrix} \right) \), we define it as the global

---

**Figure 3.2 Procedure for recursive global matrix method**

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compliance matrix. Combining this cell compliance matrix with the reflected and transmitted waves in the fluid, one may write the transmission coefficient for laminates immersed in fluid as

\begin{equation}
T = 2\Lambda S_{33}^{21} / \left[ \left( S_{33}^{11} + \Lambda \right) S_{33}^{22} - \Lambda \right] - S_{33}^{21} S_{33}^{12},
\end{equation}

(3.30)

and the reflection coefficient as

\begin{equation}
R = \left[ \left( S_{33}^{11} - \Lambda \right) S_{33}^{22} - \Lambda \right] - S_{33}^{21} S_{33}^{12} / \left[ \left( S_{33}^{11} + \Lambda \right) S_{33}^{22} - \Lambda \right] - S_{33}^{21} S_{33}^{12},
\end{equation}

(3.31)

where \( S_{33}^{ij} \) are the elements in the global compliance matrix \( S^{ij} \) and \( \Lambda = \cos \theta / \rho_f c_f^2 \),

\( \rho_f \) is the fluid density, and \( c_f \) is the acoustic velocity in the fluid. \( \theta \) is the incident angle.

3.5 Numerical and experimental results

We will compare the four algorithms described above, using the wave propagation in a cross-ply and quasi-isotropic composites as examples. Table 3.1 shows the properties for one lamina. The lamina is assumed to be transverse isotropic. Figure 3.3 shows the reflection coefficient for a \([0/45/90/-45]_{2s}\) composite calculated by the transfer matrix method (dashed line) and the global matrix method without attenuation in the lamina at incident angle 52°. As can be seen, the transfer matrix method becomes unstable when the frequency is above 3.8MHz, while the global matrix method is always robust.
Elastic constants of one lamina (GPa)

\begin{align*}
C_{11} & \quad 143.2 \\
C_{22} & \quad 15.8 \\
C_{12} & \quad 7.5 \\
C_{23} & \quad 8.2 \\
C_{55} & \quad 7.0 \\
\text{Density (g/cm}^3\text{)} & \quad 1.6 \\
\text{Thickness (mm)} & \quad 0.18 \\
\text{Attenuation} & \quad 0.03
\end{align*}

| Table 3.1 Properties of one lamina |

Figure 3.3 Reflection coefficient calculated by different methods for composite $[0/45/90/-45]_2$. The dashed line is calculated by transfer matrix method and the solid line calculated by global matrix method. Transfer matrix method becomes unstable when frequency reaches 3.8MHz.

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Figure 3.4 and 3.5 presents the efficiency of different algorithms. The time in second used to calculate 10000 reflection points on a computer with Pentium II CPU for composites \([0/45/90/-45]_n\) is shown in Fig. 3.4. As the repetition number \(n\) increases, the required CPU time increases only slightly for transfer matrix, delta matrix and recursive global matrix methods. In these methods, the repetition operation is simple matrix multiplication as shown in Eqs. (3.13), (3.16) and (3.28). The transfer matrix method seems to be the most efficient due to its simplicity as shown in Eq. (3.13). The repetition operation in the delta matrix method is also very simple as shown in Eq. (16). However, each element in the delta matrix which has to be obtained using symbolic software (Mathcad or Maple) becomes complicated, the time required for one cell becoming 20 times larger than that required for the transfer matrix method. The dimension of the global matrix given in Eq. (3.22) is \(6*4*N+2\) for composite \([0/45/90/-45]\) and \(6*2*N+2\) for composite \([0/90]\). The CPU time required to solve Eq. (3.22) increases exponentially as repetition number increases. The recursive global matrix method has almost the same efficiency as the transfer matrix method. But the transfer matrix method has numerical problems in some conditions and can only be applied in the low frequency range. The recursive global matrix method seems to be the best solution for anisotropic composites.
Figure 3.4 Efficiency of different methods for a quasi-isotropic composite \([0/45/90-45]_{ns}\),
(1) transfer matrix method, (2) recursive global matrix method, (3) delta matrix method
(4) global matrix method

Figure 3.5 Efficiency of different methods for cross-ply composite \([0/90]_{ns}\), (1) transfer
matrix method, (2) recursive global matrix method, (3) delta matrix method (4) global
matrix method
Figure (3.6) shows the experimental transmission coefficient versus incidental angle at frequency 2.5MHz. The layout of the composite is [0/45/90/-45]_2s. The orientation angle is 25°. Figure 3.7 shows the corresponding theoretical result. As shown in the two figures, large transmission amplitude appears at both small (0°-10°) and large incident angles (40°-60°). For medium incident angles (20°-40°), the transmission amplitude is almost zero. Figures (3.8 and 3.9) show the transmission amplitude at all rotation angles. The gray level represents the transmission amplitude; higher brightness corresponds to larger transmission amplitude. The laminate used in Fig. 3.8 has four cell repetitions [0/45/90/-45]_4 and the symmetrical laminate used in Fig.3.9 has two repetitions. The transmission amplitude is almost independent of rotation angle for the laminate with four repetitions at large incident angle. For incident angles from 20°-40°, the transmission amplitude is close to zero at all rotation angle. At small incident angles, the transmission amplitude forms a bright circle surrounded by an ellipse. For the symmetrical composite, the rotation angle has larger affects on transmission amplitude. At large incident angles, transmission amplitude is maximum around 70° and minimum around 110°. The transmission amplitude at small incident angles still forms a bright circle but the ellipse surrounding the bright circle in Fig. 3.9 becomes a hyperbola. The characteristics of the transmission amplitude distribution will be explained in the next chapters.
Figure 3.6 Experimental transmission coefficient at rotation angle 25° for a quasi-isotropic composite [0/45/90/-45]_{2s} (from X. Qiang).

Figure 3.7 Theoretical transmission coefficient at rotation angle 25° for a quasi-isotropic composite [0/45/90/-45]_{2s}
Figure 3.8  Transmission rotation angle at all rotation angle for the composite [0/45/90/-45]_4.

Figure 3.9  Transmission rotation angle at all rotation angle for the composite [0/45/90/-45]_2π.
CHAPTER 4

FLOQUET WAVE AND HOMOGENIZATION

4.1 Background

Composite laminates are constructed by stacking a number of laminas in the thickness direction. Most of the laminates, such as cross-ply and quasi-isotropic composites, include a number of repetitions of an identical unit. These materials can be considered as anisotropic periodic media. Recently, much research has been done to investigate wave propagation in anisotropic periodic solids[25-28]. At low frequency, periodic solids can be considered as effectively homogenous materials. At high frequency, the behavior of wave propagation in a periodic medium is significantly different from that in a homogeneous medium. The free wave propagating in a periodic medium is called a Floquet wave. Two principle approaches are applied to obtain the characteristic equation for Floquet waves. One is the global matrix method, in this method; all the interface continuity conditions and the requirements of the Floquet theory are solved simultaneously, therefore it leads to a characteristic matrix whose dimension depends on the number of lamina in the cell. Another method is the transfer matrix approach. The Floquet wave characteristic equation is governed by the global transfer matrix whose dimension is always six. The Floquet wave numbers corresponds to the eigenvalues of the global transfer matrix. When the $z$ direction is a symmetry direction, Braga and
Herrmann showed that the six real or complex Floquet wave numbers appear in pairs of opposite signs. From the Floquet wave number, the stop and pass bands in the frequency or wave number domains have been investigated. In this chapter, we will derive the Floquet wave characteristic equation using the cell stiffness matrix obtained in chapter 3. Based on the similarity between Floquet wave and partial wave for a homogeneous medium, a new homogenization procedure will be proposed.

4.2 Floquet wave

Consider an infinite solid stacked from unit cells that repeats periodically. The stiffness matrix for this unit cell can be obtained by Eq. (3.23). We write this cell stiffness matrix as

\[
\begin{bmatrix}
T^+ \\
T^-
\end{bmatrix} = \begin{bmatrix}
K^{11}_s & K^{12}_s \\
K^{21}_s & K^{22}_s
\end{bmatrix} \begin{bmatrix}
U^+ \\
U^-
\end{bmatrix},
\] (4.1)

The generalized Floquet theory requires

\[
\begin{bmatrix}
T^+ \\
T^-
\end{bmatrix} = e^{i\zeta h} \begin{bmatrix}
U^-

\end{bmatrix},
\] (4.2)

where \(\zeta\) represents the Floquet wave number and \(h\) is the thickness of the unit cell.

Combining Eqs. (4.1 and 4.2) gives

\[
(e^{i\zeta h}K^{21}_s - e^{-i\zeta h}K^{12}_s + K^{22}_s - K^{11}_s)U^- = 0.
\] (4.3)

The determinant of the system of Eq. (4.3) gives the characteristic equation for the Floquet wave

\[
A_3 \cos(3\zeta h) + A_2 \cos(2\zeta h) + A_1 \cos(\zeta h) + A_0 = 0.
\] (4.4)
where $A_3 = |K_{s}^{21}|$, $A_2 = (|E + K_{s}^{21}| + |E - K_{s}^{21}|)/2 - |E|$, 

$$A_1 = (|K_{s}^{12} + K_{s}^{21}| + |K_{s}^{12} - K_{s}^{21}| - |K_{s}^{12} + E| - |K_{s}^{12} - E|)/2 + |K_{s}^{12}| - |K_{s}^{21}|,$$

$$A_0 = 2|E| + (|E + K_{s}^{12} - K_{s}^{21}| + |E + K_{s}^{12} - K_{s}^{21}| - |K_{s}^{12} + E| - |K_{s}^{12} - E| + |K_{s}^{21} + E| - |K_{s}^{21} - E|)/2$$

$E = K_{s}^{22} - K_{s}^{11}$, $|E|$ represents the determinant of matrix $E$.

Eq. (4.3) has six solutions for $\zeta$. We will assume the $z$ axis coincides with a direction of elastic symmetry of the anisotropic laminas in the unit cell; then these solutions satisfy $\zeta_1 = -\zeta_2, \zeta_3 = -\zeta_4, \zeta_5 = -\zeta_6$. For each Floquet wave number, one can obtain a displacement vector by solving the linear equation (4.3). We will use the symbol $W_{j}^{f\pm}$ ($j=1,2,3$) to denote their displacement vectors. Then the displacement at the $m$th cell bottom surface may be represented by

$$U^- = \sum_{j=1}^{3} (A_j^{f+} W_j^{f+} e^{im\zeta h} + A_j^{f-} W_j^{f-} e^{-im\zeta h}) e^{i(k_x x + \omega t)}, \quad (4.5)$$

where $A_j^{f\pm}$ represents the corresponding Floquet wave amplitude. From Eq. (4.1 and 4.5), one can obtain the associated stress

$$T^- = \sum_{j=1}^{3} (A_j^{f+} D_j^{f+} e^{im\zeta h} + A_j^{f-} D_j^{f-} e^{-im\zeta h}) e^{i(k_x x + \omega t)}, \quad (4.6)$$

where $W_j^{f\pm}$ are related to $D_j^{f\pm}$ by $D_j^{f\pm} = (K_{j}^{11} + e^{\pm i\zeta h} K_{j}^{12}) W_j^{f\pm}$. Comparing Eq. (4.5 and 4.6) with Eq. (3.4 and 3.7), one can see that the stress and displacement fields have the same form as for homogenous solids. The polarization vector and wave number in a periodic medium are determined by Eq. (4.3) while they are determined by Eq. (3.3) in a homogenous medium.
Figures 4.1, 4.2 and 4.3 show the Floquet wave slowness curve in the x-z plane at different frequencies. The wave number of the Floquet wave is calculated by finding the roots of Eq. (4.3). At low frequency, the behavior of the slowness of the Floquet wave is similar to that of the partial wave in a homogeneous anisotropic medium. As frequency increases, distortion begins in directions close to the z axis. Figure 4.4 shows the Floquet wave slowness curve in the x-y plane at 2MHz.

![Image: Slowness curve for Floquet wave for composite [0/90] in x-z plane at 2MHz]

Figure 4.1 Slowness curve for Floquet wave for composite [0/90] in x-z plane at 2MHz
Figure 4.2 Slowness curve for Floquet wave for composite [0/90] in x-z plane at 2.5MHz.

Figure 4.3 Slowness curve for Floquet wave for composite [0/90] in x-z plane at 3MHz
Figure 4.4 Slowness curve for Floquet wave for composite [0/90] in x-y plane at 2MHz.

Figures (4.5, 4.6 and 4.7) shows the Floquet wave slowness curve in the x-z plane for a quasi-isotropic composite [0/45/90/-45] at different frequencies. As for the [0/90] composite, at low frequency, the slowness of the Floquet wave is similar to that of the partial wave in a homogeneous anisotropic medium. When the frequency is above 1.3MHz, distortion begins in directions close to the z axis. Figure 4.8 shows the Floquet
wave slowness curve in the x-y plane at 1MHz. Because it is a quasi-isotropic composite, the slowness is independent of the rotation angle at this frequency.

Eq. (4.3) has six solutions for $\zeta$ which can be real or complex. Real wave numbers correspond to propagating Floquet waves, while complex wave numbers are associated with waves whose amplitude decays exponentially along either the positive or negative direction of the $z$ axis. The Floquet wave numbers are related to $\omega$, $k_x$, rotation angle ($\phi$) and stacking sequence. The values ($\omega$, $k_x$, $\phi$) for which $\zeta$ is real form the pass bands and regions in which $\zeta$ is complex are called stop bands. Figure 4.9 shows the stop and pass bands of the three Floquet waves in the $k_x$ and $\omega$ domains for composite [0/90] at 0° rotation angle. The gray level represents the number of Floquet waves in the pass band. Therefore white indicates that all three Floquet waves have real wave number and black corresponds to the nonpropagating zone for all three Floquet waves. Colors between white and black represent that one or two of the Floquet waves have real wave numbers. At low frequency, the behavior of the laminate is similar to a homogeneous solid. The three Floquet waves can be considered as the three partial waves in a homogenous anisotropic medium. As the incident angle increases, there is a critical angle corresponding to each Floquet wave. The largest critical angle decreases as the frequency increases. When the frequency increases, the behavior of the three Floquet waves becomes distinct from that of the partial waves in a homogenous medium. Total stop bands in which no Floquet wave can propagate appear at intermediate incident angles. Figure 4.10 shows the stop and pass bands of the three Floquet waves in the $k_x$ and rotation angle domains for composite [0/90] at 2MHz. The total nonpropagation zone
is independent of the rotation angle. One of the Floquet waves depends significantly on
the rotation angle, which forms the cross shape. Figure 4.11 shows the stop and pass
bands of the three Floquet waves in $k_x$ and frequency for composite [0/45/90/-45] at $0^\circ$
rotation angle. As can be seen, the first and second critical angles increase as frequency
increases while the third critical angle decreases as frequency increases. At low
frequency the three Floquet waves are similar to the three partial waves in a homogenous
medium. A zone of total nonpropagation only appears when incident angle is larger than
the third critical angle. As frequency increases, the stop band distribution becomes
complicated. Figure 4.12 show the stop and pass bands of the three Floquet waves in $k_x$
and rotation angle domain for composite [0/45/90/-45]. For this quasi-isotropic composite
the three frequency waves are insensitive at low frequency to rotation in the composite $x$-
$y$ plane.
Figure 4.5 Floquet wave slowness curves for composite [0/45/90/-45] in x-z plane at 1MHz.

Figure 4.6 Floquet wave slowness curves for composite [0/45/90/-45] in x-z plane at 1.5MHz.
Figure 4.7 Floquet wave slowness curves for composite [0/45/90/-45] in x-z plane at 2MHz.
Figure 4.8 Floquet wave slowness curve for composite [0/45/90/-45] in x-y plane at 1MHz.
Figure 4.9 Pass and stop bands of the three Floquet waves in $k_x$ and $\omega$ domains for composite [0/90] at 0° rotation angle. The gray levels show the number of Floquet waves whose wave number is real. White indicates the three Floquet waves are all within the pass band and black indicates the three Floquet waves are all within the stop band.

Figure 4.10 Pass and stop bands of the three Floquet waves in $k_x$ and rotation angle domains for composite [0/90].
Figure 4.11 Pass and stop bands of the three Floquet waves in $k_x$ and $\omega$ domains for composite [0/45/90/-45].

Figure 4.12 Pass and stop bands of the three Floquet waves in $k_x$ and rotation angle domains for composite [0/45/90/-45].
4.3 Homogenization

As discussed above, the three Floquet waves are similar to the three partial waves in a homogeneous anisotropic medium at low frequencies. In this case, the periodic medium may be

\[ \text{Floquet wave characteristic equation} \]

\[ \det(e^{i\omega h} K^{21} - e^{-i\omega h} K^{12} + K^{22} - K^{11}) = 0. \]

\[ \text{Relation between Floquet wave velocity and propagation direction} \]

\[ V_F(\theta, \omega, cell) \]

\[ \text{Relation between partial wave velocity and propagation direction} \]

\[ V_A(\theta, C_{ijkl}(\omega)) \]

\[ \text{Christoffel equation} \]

\[ \det(C_{ijkl} k_j k_l - \rho \omega^2 \delta_{ik}) = 0 \]

Figure 4.13 Homogenization procedure.

considered as a homogeneous anisotropic medium with effective elastic constants. Figure 4.13 shows a procedure to obtain the effective elastic constants of a homogenized periodic medium from the Floquet wave slowness surface. The Floquet wave is governed

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by the Floquet wave characteristic equation; from this equation, the Floquet wave velocity in all directions can be determined. The partial wave velocity in a homogeneous medium is determined by the Christoffel equation. If a periodic medium is replaced by a homogeneous medium, then the Floquet wave is equivalent to the partial wave. In order to obtain the effective elastic constants, one can first generate the slowness curve from the Floquet wave characteristic equation, then fit this slowness curve using the Christoffel equation to obtain the effective elastic constants. For a cross-ply composite [0/90], the effective homogeneous medium should satisfy $2\pi/4$ symmetry and be symmetrical about the three planes perpendicular to 1, 2 and 3 axial. Therefore this medium has tetragonal symmetry and can be described by six elastic constants. Comparison between the slowness curve of the Floquet wave and the partial wave in the corresponding medium is shown in Figure 4.14 and 4.15. As can be seen, at low frequency the cross-ply composite can be well described by a tetragonal homogeneous medium. The equivalent elastic constants are given in Figure 4.16 and 4.17. $C_{11}$ decreases as frequency increases, while $C_{12}$ and $C_{13}$ increase as frequency increases. Other elastic constants are almost independent of frequency.

At low frequency, a quasi-isotropic composite can be considered as a solid with hexagonal symmetry. Therefore it is a transversely isotropic medium which can be described by five independent elastic constants. Figures 4.18 and 4.19 give the comparison of the slowness in the $x$-$y$ and $x$-$z$ plane. The equivalent elastic constants are given in Fig. 4.20. $C_{11}$ also decreases as frequency increases and other parameters depend slightly on frequency. Tables 4.1 and 4.2 show the effects of noise in the determination of the elastic constants.
Figure 4.14 Slowness surfaces for partial waves in a homogeneous medium (solid lines) and Floquet waves (open circle) in a periodic medium [0/90] in the x-z plane.

Figure 4.15 Slowness surfaces for partial waves in a homogeneous medium (solid lines) and Floquet waves (open circle) in a periodic medium [0/90] in the x-y plane.
Figure 4.16 Effective elastic constant $C_{11}$ for [0/90].

Figure 4.17 Effective elastic constant $C_{ij}$ for [0/90]
Figure 4.18 Slowness surfaces for partial waves in a homogeneous medium (solid lines) and Floquet waves (open circle) in a periodic medium [0/45/90/-45] in the x-z plane.

Figure 4.19 Slowness surfaces for partial waves in a homogeneous medium (solid lines) and Floquet waves (open circle) in a periodic medium [0/45/90/-45] in the x-y plane.
Fig. 4.20 Effective elastic constant $C_{11}$ for $[0/45/90/-45]$

Fig. 4.21 Effective elastic constant $C_{ij}$ for $[0/45/90/-45]$
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<th>C23</th>
<th>C55</th>
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<tr>
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<tr>
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<td>0%</td>
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<td>0%</td>
<td>0%</td>
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<tr>
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Table 4.1 Convergence and noise effect [0/90]

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<th>C12</th>
<th>C23</th>
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<td>15.8</td>
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<tr>
<td>Initial guess</td>
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<td>5.6</td>
</tr>
<tr>
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<td>2.55%</td>
<td>1.56%</td>
<td>2.69%</td>
</tr>
</tbody>
</table>

Table 4.2 Convergence and noise effect [0/45/90/-45]
CHAPTER 5

ANGLE BEAM CHARACTERIZATION OF COMPOSITES

5.1 Beam model

For ultrasonic testing of composites, one usually applies finite aperture ultrasonic transducers in impulse regime. As an example, Figure 5.1 shows two ultrasonic in a pitch catch mode. Figure 5.2 also shows a coordinate system. The output voltage at varying $z_0$ (defocus) can be written as [50]

$$ V_{\text{out}}(z, t) = \int_{-\infty}^{\infty} f_T(\omega) f_R(\omega) e^{i\omega t} d\omega \int_{-\infty}^{\infty} P_T(\theta) P_R(\theta) R(\theta, \omega) e^{-\frac{z_0}{c}} e^{-\frac{z}{c}} \sin x_0 \sin \theta \, d\theta , 
$$

(5.1)

where $f_T(\omega)$ and $f_R(\omega)$ are the frequency responses of the transmitter and receiver respectively. $P_T(\theta)$ and $P_R(\theta)$ represent the angular responses of the transmitter and receiver respectively. $z_0$ is the vertical position of the transducer and $x_0$ is the horizontal position of the transducers. $R(\theta, \omega)$ is the reflection coefficient.

5.2 Numerical and experimental examples

In our experiments, the distance between the two oblique incident transducers is 76mm. The transducer has center frequency 5MHz and incident angle 52°. The composite sample has [0/45/90/-45]_2s lay-up. The thickness of the laminate is 3.075 mm. The reflections from the sample bottom are shown in Figs. 5.3-5.7. Figure 5.3 shows the
normal incident reflection. In the z-direction, all laminas have identical properties and the reflection is the same as that of a homogeneous medium. Three multiple reflections within the laminate can be observed. An example of reflection at rotation angle 60° is given in Figure 5.6. As can be seen, two separate reflections can be observed at this frequency. Figures 5.4 show the reflection signal at all rotation angles at defocus $z_d=20\text{mm}$ and figure 5.5 shows the computational results. The reflection depends significantly on rotation angle. The time delay of the first reflection forms an eclipse. Amplitude and time delay reach maximum value at 75° and become minimum at 25°. Figure 5.7 shows the reflection at larger defocus. As defocus increases, the first reflection disappears while other reflections from the bottom surface appear. Due to the filtering effect of the multilayered structure, only lower frequencies can reach the laminate bottom surface and be reflected back.

As one can see, the model compared well with the experimental results. Further development of the model for application to adhesive bonds of composite will be addressed in future.
Figure 5.1 Transducer configuration

Figure 5.2 Coordinate system
Figure 5.3 Normal incident reflection for composite $[0/45/90/-45]_{2s}$. 
Figure 5.4 Experimental oblique reflection signals at different rotation angles for composite [0/45/90/-45]_2s.
Figure 5.5 Theoretical oblique reflection signals at different rotation angles for composite [0/45/90/-45]_2s.
Figure 5.6 An example of the oblique reflection at rotation angle 60°.
Figure 5.7 An oblique reflection at large defocus at rotation angle 60° for composite.
CHAPTER 6

ACOUSTIC MICROSCOPY OF MULTILAYERED COMPOSITES

6.1 Background

Acoustic microscopy has been used to measure material properties since the 1980s [37-42]. The velocity of the leaky surface wave can be accurately determined from the $V(z)$ curve which is formed by interference between the leaky surface wave and the specular reflection. By fitting the leaky wave velocity or the $V(z)$ curve itself, Kim et al. [39] reconstructed the material properties (elastic constants and mass density). Another approach is time-resolved acoustic microscopy [41-42]. In this method, the leaky surface wave and the specular reflection are separated in the time domain and the velocity is determined from the time of flight. For a graphite/epoxy composite, due to the complexity of the reflected signal and the absence of Rayleigh wave excitation, it is impractical to determine material properties from the $V(z)$ curve. In time-resolved acoustic microscopy, the different reflection signals are separated in the time domain and the velocity measurement is simplified. For graphite epoxy composite materials, due to their low density and significant fluid loading, the acoustic microscopy response is significantly different from that for higher density materials.
6.2 Acoustic microscopy model

Figure 6.1 Configuration of a line focus transducer and a composite plate.

Figure 6.1 shows the configuration of a line focus transducer and an anisotropic multilayered composite. The acoustic probe consists of a transducer and a cylindrical lens. The coupling fluid between the lens and specimen is usually water. The output voltage at different $z_0$ (defocus) has been analyzed by ray and wave methods. A widely used equation can be written as [44]

$$V_{out}(z,t) = \int_{-\infty}^{\infty} f(\omega)e^{i\omega t} d\omega \int_0^{\theta_s} P(\theta)R(\theta,\omega)e^{-2iz_0/c}e^{i\omega z_0 \cos \theta} d\theta,$$  \hspace{1cm} (6.1)

where $f(\omega)$ and $P(\theta)$ are the frequency and angular responses of the transducer respectively. They are characteristic functions of the acoustic lens. $R(\theta,\omega)$ is the reflection coefficient which is calculated by the solution described in chapter 3.
6.3 Numerical and experimental results

Two kinds of graphite/epoxy composites are used: one is a unidirectional and another is a multilayered composite which has stack sequence [0/-45/90/45]_2s. In our experiment, the specimen is put on a rotation table which is controlled by a computer. The reflection signal at different rotation angles and defocus z₀ will be recorded by an oscilloscope. The line focus transducer has been developed at NIST and provided to us for the experiments. It has center frequency 6MHz, focus length 24.5mm and half aperture angle 32°.

Figure 6.2. Reflection signals at different rotation angles for the unidirectional composite plate with thickness h=2.4mm. The cylindrical transducer with PVDF film has center frequency 6MHz and focus length 24.5mm.
Figure 6.3 Experimental reflection signals for all orientations. The signal amplitude is represented by the gray level in the polar coordinate system. The radius direction is time of flight. The sample and experiment parameters are the same as in Fig. 6.2.

Figure 6.4 Theoretical reflection signals for all orientations.
Figure 6.2 presents the time domain signals at several rotation angles. By converting the amplitude to a gray level, we plot the time domain signal in Fig. 6.3 for all rotation angles. In general, these signals can be included in three categories: 1. specular reflections which correspond to normal incident rays ($L_1$ and $L_2$); 2. lateral waves which correspond to rays incident at critical angles ($L_a$); 3. bulk wave reflections which correspond to rays focusing on the bottom surface ($LS_2,S_1S_1,S_1S_2$). The time delay of the specular reflection is determined by $C_{22}$. Therefore these signals ($L_1,L_2$) are almost independent of the rotation angle. Because the half aperture angle of our transducer is $32^\circ$, only the longitudinal lateral wave ($L_a$) has been observed and there are no surface waves. The time delay and amplitude of this lateral wave are significantly dependent on
the rotation angle. As the rotation angle decreases from 90° to 0°, the time delay decreases due to the longitudinal wave velocity along the surface decreasing. The appearance of the bulk wave reflected from the bottom surface is dependent on: 1. the incident angle at which the ray reflected from the plate bottom returns to the transducer; 2. the wave conversion coefficient between the incident and reflection rays being large. Therefore these signals are dependent on the rotation angle, defocus $z_0$ and specimen thickness. $S_1S_1$ always appears at small rotation angles; $LS_2$ appears at large rotation angles; $S_1S_2$ appears at medium rotation angles.

Figure 6.6 shows the time domain signals for different orientations of the specimen with sixteen different oriented layers with stack sequence $[0^-45/90/45]_{2s}$. This material is quasi-isotropic in the plate plane at low frequency. As can be seen, the composite does not behave as an isotropic material for waves propagating obliquely to the plate normal in the plate cross section. There are multiple reflections at the interface between each layer and the reflection signal becomes much more complicated than those for the unidirectional composite. The results show the model describes experimental observations well.

6.4 Elastic constant determination

6.4.1 Wave decomposition

As shown in Fig. 6.2, for unidirectional composites the most significant signals are the reflection from the top surface and the first reflection from the bottom surface; other multi-reflections in the plate are almost unobservable. Therefore it will be convenient and efficient to use geometrical acoustics and to express the reflection coefficient as the
The summation of the corresponding signals in the time domain. This formulation can easily be obtained by the corresponding signals in the time domain. This formulation can easily be obtained by multiplying the reflection and transmission coefficients and the phase delay along the corresponding rays [45]. For example, the reflection coefficient for signal LS2 is $T_{nl} R_{ls_i} T_{sl} e^{i(k_l^1 + k_l^2) \lambda}$. $T_{nl}$ is the transmission coefficient from the incident wave in the fluid to the longitudinal wave in the plate at the fluid-solid interface. $R_{ls_i}^2$ is the reflection coefficient from the longitudinal wave to the slow shear wave at the solid-fluid interface. $T_{sl}^1$ is the transmission coefficient from the slow shear wave to the wave in the fluid at the solid-fluid interface. $e^{i(k_l^1 + k_l^2) \lambda}$ is the phase delay.
Figure 6.7 Comparison between the reflection signals calculated by Eqs. (6.23) and (6.2).

Therefore, the reflection coefficients corresponding to these important signals in Fig. 6.2 (L₁, Lₐ, LS₂, S₁S₂, S₁S₁) can be written as

\[
R = R_{ff} + T_{f1} R_{ls}^2 T_{ls}^3 e^{i(k_{1}^l + k_{1}^s)l} T_{ls} R_{ls}^3 T_{ls}^1 e^{i(k_{1}^l + k_{1}^s)l} + T_{ls}^1 R_{ls}^2 T_{ls}^3 e^{i(k_{1}^l + k_{1}^s)l} + \ldots.
\]

(6.2)

\(R_{ff}\) is the reflection coefficient at the fluid-solid interface, representing signals L₁ and Lₐ. Other terms represent L₂, LS₂, S₁S₂, S₁S₁ respectively.

Figure 6.7 shows the comparison between the reflection signals calculated by Eq. (3.22) and the leading terms in Eq. (6.2). As expected, Eq. (6.2) gives the same results within the first reflection. By using Eq. (6.2), we can clearly identify the character of the

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reflection signals and it is also more convenient and efficient to reconstruct the elastic constants using Eq. (6.2). The same procedure can also be applied to multilayered composites to identify the different reflection components in the complex reflection signal.

6.4.2 Surface wave

The leaky surface wave plays an important role in formulation of the contrast in acoustic microscopy. However, there is no surface wave pattern in Fig. 6.2. To observe it one should use a transducer with appropriate aperture angle for the surface wave excitation. The small half aperture angle of our transducer is 32° and this is not sufficient. For graphite epoxy composites, the surface wave velocity is always about 2mm/μs and the corresponding excitation angle will be 50°. Figure 6.8 shows the calculated reflection signal using a cylindrical transducer with aperture angle 80°. The sample is assumed to be a semi-space, therefore the multi-reflections do not exist. The three signals shown in this image correspond to longitudinal lateral (Lₐ), fast shear (Lₛₗ) and slow shear (Lₛₚ) lateral waves respectively. There is no surface wave observed for water loaded composites in this case also. This phenomenon also has been observed experimentally by Wolf and Vines [46,47] and theoretically by Every and Briggs [48]. They emphasize this as an unresolved issue. The disappearance of the surface wave signal in composite materials is due to their low density and as a result the fluid loading significantly influences the reflection signature and surface wave propagation. Figure 6.9 shows the surface wave velocity versus fluid density. Compared to aluminum, the surface wave velocity for the composite is much more strongly dependent on the fluid loading. As the
fluid density increases, the surface velocity increases. At a certain density value, the surface wave velocity will become larger than the velocity of the slow shear wave in the composite. Therefore, as the density continues to increase, the surface wave becomes a nonpropagating mode. Figure 6.10 shows the calculated reflection signatures at different densities. The surface wave signal (S) can be clearly identified. As the density increases, the surface wave signal disappears, the amplitude of the specular reflection decreases and the amplitude of lateral waves (L_a, L_fs and L_ss) increases. At large fluid density, the lateral wave becomes more significant as shown in Fig. 6.2.

Figure 6.8 Simulated reflection signal for a transducer with half aperture angle 80°. The specimen is a unidirectional half space composite with the property shown in Table 6.1.
Figure 6.9. The dependence of surface wave velocity on fluid density. The specimen has the same properties as in Figure 6.8.

Figure 6.10. The reflection signal at different fluid densities. The specimen has the same properties as in Figure 6.8.
6.4.3 General strategies

As shown above, the time delay and the appearance of the signal depend on the experimental setup and the material properties. When the characteristic functions $f(\omega)$ and $P(\theta)$ of the transducer have been determined, the reflection signal will depend only on the material property $R(\theta, \omega)$ and the defocus $z_0$. The defocus $z_0$ can easily be determined by controlling the $z$ motor movement. The general procedure in elastic constant determination is illustrated in Fig. 6.11. The unknown material properties can be found by minimizing the sum of the squares of the deviations between the experimental and calculated time delays

$$\min_{C_2} \sum_{i=1}^{N} (t_{\theta_i}^T - t_{\theta_i}^{M})^2,$$

(6.3)

where $t_{\theta_i}^M$ is the measured time delay of the microscope pattern at rotation angle $\theta_i$ with respect to the first surface reflection; $t_{\theta_i}^T$ is the same predicted by the model.

$C_{22}$ can be directly determined from the normal specular reflection $C_{22} = (2d / \Delta \tau)^2 \rho$, where $\Delta \tau$ is the time difference between signal $L_1$ and $L_2$ as shown in Fig.6.2 and $\rho$ is the specimen density.
6.4.4 Elastic constant measurement from lateral waves

As discussed above, the time delay of the lateral wave depends on the rotation angle, i.e., the anisotropic property of the composite sample. Because the half aperture angle of our transducer is $32^\circ$, only the longitudinal lateral wave was observed. Because the longitudinal wave velocity is independent of $C_{23}$, only three elastic constants can be measured from the lateral wave time delay. Figure 6.12 shows the experimental and theoretical time delay; the points are experimental time delays and the solid curves are time delays fitted by the model with corresponding reconstructed elastic constants shown in the first line in Table 6.1.
If one increases the half aperture angle, the shear lateral waves can be excited. Therefore, it is possible to measure all elastic properties using a large aperture angle transducer.

![Graph showing time delay vs. angle]

Fig. 6.12. Fitting the time delay of the lateral wave. The time delay is measured from Fig. 6.3.
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<td>7.5</td>
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</tbody>
</table>

Table 6.1. Reconstructed elastic constants by different methods.

6.4.5 Elastic constant determination from bulk wave reflection

As shown in Fig. 6.2, the appearance and time delay of the bulk wave reflected from the plate bottom depend on the rotation angle. The elastic constant can be determined from the time delay for these bulk waves. In this reconstruction, we have to select a set of signals which can be described by Eq. (6.2) and rotation angles at which the amplitude of the signal will be large enough to be experimentally measured. The elastic constants measured from S1S2 and LS2 are shown in Table 6.1. The data shown in the third row in Table 6.1 are measured by the double through transmission method [49] on the same sample. The results measured by the three methods have good consistency except for C11. Because only the rotation angle from 60° to 90° has been used in the lateral wave method, C11, which is the property along the fiber direction, may not be exactly reconstructed by this method.
CHAPTER 7

CONCLUSION

An efficient algorithm has been proposed for simulation of ultrasonic waves with multilayered composites. The method is based on combining of global matrix method with accounting for the system periodicity. The results show this method is stable and efficient especially for cross-ply composites. Using this algorithm, the behavior of wave transmission through a laminate immersed in water have been discussed and explained by Floquet wave pass and stop bands. Based on Floquet wave representation, a homogenization procedure has been proposed to model the multilayered composite as an anisotropic homogeneous medium. To describe experimental conditions for an impulse regime interrogation of composites a beam model has been developed. Also the time-domain response in an acoustic microscopy has been investigated and a procedure to measure the elastic constant using the acoustic microscopy signature has been discussed.
REFERENCE


