MODEL OF HEAT GENERATION EFFECTS DURING UNIAXIAL TENSILE TEST

A Thesis

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By

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To My Parents,
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LIST OF SYMBOLS

A = cross section area; constant in constitutive equation

A₀ = original cross section area

A₁ = surface area of an element

B = constant in constitutive equation

c = specific heat

C, D, E, d = constants in function ε = f(x,t)

F = 2h t / cw, constant

H = heat source

H = dH/dt, heat source rate

h = heat convection coefficient

h₀ = 2h/ρcw, effective convection coefficient

i = 0, 1, ...., n

j = 0, 1, ...., m

K = strength coefficient in constitutive equation

k = thermal conductivity

L₀ = original normal deformation length

L = deformation length; Laplace transformation operator

m = strain rate sensitivity exponent; number of divisions of time axis

n = strain hardening exponent; number of divisions of x axis

P = load

x
p = parameter in Laplace transformation
pt = path parameter
Q = amount of heat
q = heat flux; parameter in Laplace transformation
R = \( \alpha \frac{\Delta t}{\Delta x^2} \), Fourier number
r = plastic anisotropic parameter
s = general structure parameter
t = time
T = temperature difference between current and initial temperature, \( \theta - \theta_0 \)
T = temperature difference after Laplace transformation

\( V_x \) = velocity

\( V_0 \) = crosshead speed

V = volume of an element

w = thickness of the specimen

x = coordinate axis along the specimen

\( \alpha \) = \( k/\rho c \), thermal diffusivity

\( \sigma \) = true stress

\( \bar{\sigma} \) = effective stress

\( \sigma_{ij} \) = flow stress tensor

\( \sigma_0 \) = strength coefficient in constitutive equation

\( \epsilon \) = true strain

\( \bar{\epsilon} \) = effective strain

\( \epsilon_{ij} \) = plastic strain tensor

\( \epsilon_u \) = uniform strain

\( \dot{\epsilon} \) = strain rate

\( \xi \)
\( \dot{\varepsilon}_{ij} \) = strain rate tensor
\( \eta \) = fraction of the plastic work converted to heat
\( \rho \) = density of material
\( \beta_0, \beta \) = temperature coefficient
\( \theta \) = deformation temperature
\( \theta_0 \) = initial temperature
I. SUMMARY

The temperature rise in a sheet tensile specimen during constant extension rate testing has been investigated using several techniques: analytic solutions of an approximate heat flow equation, finite difference calculations both before and after necking, and by limited experimentation to assess the validity of the models.

Analytical solutions to the second-order partial differential equation require knowledge of the form of the heat generation function: \( H(x,t) \). Forms of this function were found by fitting of curves to experimental data describing the strain distribution at each time in the tensile test, and by assuming that all plastic work (or a fixed fraction thereof) is converted into heat. Two forms for \( H(x,t) \) were found appropriate: \( H_1(x,t) \) was accurate but not amenable to closed-form solution, and \( H_2(x,t) \) was less accurate but allowed closed-form solution using LaPlace transforms. The purpose of the approximate, analytical solutions is to gain physical insight into the relative importance of conduction, heat generation, and convection at a variety of strain rates and heat transfer conditions. The analytical solution treats the finite tensile bar as an infinite specimen.

Finite difference methods were used to solve the heat flow equation with boundary conditions more closely
reflecting the physical situation. The large metal grips act as heat sinks during deformation of the specimen. This boundary condition is treated as a constant temperature at a fixed distance from the center of specimen. Prior to necking, a uniform heat generation function \( H(t) \) is used while the post-uniform region must still rely on an empirical heat generation function for modeling.

The uniform strain (i.e. strain at maximum load point) has been calculated by taking into account strain hardening, strain-rate hardening, and deformation-induced heating. Simple predictions are possible for the extreme heat transfer cases: isothermal testing and adiabatic testing. Intermediate heat transfer conditions require the numerical modeling described above in order to evaluate the temperature rise term.

The results of all of these calculations show that significant deformation heating can occur during tensile testing of a plain-carbon steel in air at "normal" strain rates near \( 10^{-2}/s \). The critical strain-rate range falls between approximately \( 5 \times 10^{-4}/s \) and \( 10^{-2}/s \), the lower rate nearly approximating isothermal conditions while the higher rate approaches adiabaticity. Immersion in a stirred water bath provides much higher convective losses during deformation and the critical strain-rate range is increased to greater than \( 10^{-1}/s \). The experiments confirm the magnitude of the temperature rise in air for several strain rates.
Adiabatic temperature rises in the plane-carbon steel can exceed $50^\circ C$ at the center of the specimen. This magnitude of temperature can be shown to influence the uniform strain significantly and will have a pronounced effect on total elongation. Therefore, formability under non-isothermal conditions can be expected to differ from isothermal conditions. Decreased forming speeds or forced cooling during forming should therefore promote formability and improve the quality of the operation.
II. INTRODUCTION

The most widely used theoretical criterion for plastic instability is due to Considère [1] and shows the clear relationship between the uniform strain in a uniaxial tensile test of a rate-insensitive material and strain-hardening exponent, $n$. When the materials obey the constitutive equation

$$\sigma = k\varepsilon^n$$  \hspace{1cm} (1)

an instability will occur if the load reaches its maximum value at

$$\varepsilon_u = n$$  \hspace{1cm} (2)

where $\sigma$ is the true stress and $\varepsilon$ is true strain. Numerous experimental results show that this criterion is closely obeyed for strain-rate insensitive materials.

In 1967, Hart [2] presented a theory which showed that the strain-rate sensitivity of a material might delay the commencement of instability beyond maximum load. Hart proposed a criterion that the behavior of a cross-sectional inhomogeneity $\Delta A$ be monitored. Following Hart's treatment on the tensile test, several other authors [3-6] chose to consider the behavior of a relative strain inhomogeneity $\Delta \varepsilon$. Jonas et al [7] distinguished two types of defects on a tensile specimen. They called the defects either "mechanical defect" for $(d\varepsilon_0/dx) \neq 0$ and "geometric defect"
for $({\text{dln}A_0}/dx) \neq 0$. Here, $\varepsilon_0$ and $A_0$ are the initial true tensile strain and initial cross-section area. As mentioned by Kocks, Jonas and Mecking [8], these defects have been considered separately; the deformation defect being treated by Hart [1] and Duncombe [4] while the geometric defects have been treated by Campbell [3] and Ghosh [9]. Lin, Hirth, and Hart [10] showed that Hart's criterion [2] for the onset of plastic instability as expressed in terms of a cross-sectional inhomogeneity $\Delta A$ may apply for either geometrical or deformation defects.

According to Hart, for example, instability starts when the cross-section at a pre-existing imperfect site begins to decrease at a faster rate than the rest of the specimen (homogeneous section). On this basis, he derived a stability criterion for rate-sensitive materials

$$\varepsilon_u = n/(1-m) \quad (\Delta A = 0) \quad (3)$$

Ghosh [11] re-examined Hart's criterion and concluded that Hart's definition of stability was wrong. Ghosh argued that tensile deformation is stable as long as it is accompanied by a load rise, even though an imperfection may be growing during this stage. Thus, the onset of instability is at the maximum load which, for a perfect specimen of strain-rate insensitive materials constant strain-rate, occurs when the well-known Considere condition [1] is satisfied. For tests at constant crosshead speed, Ghosh derived the instability strain to be

$$\varepsilon_u = n/(1 + m) \quad (\Delta P = 0) \quad (4)$$
Recent analyses of the onset instability have mainly been concerned with the effect of strain rate sensitivity. The effect of heat generation has been discussed by a few authors [12-17]. Korhonen and Kleemola [12] applied Ghosh's criterion [11] and considered the thermal effect on the instability of the material. They derived a general criterion for flow instability and found that premature instability of the material occurs by the thermal effect. Generally, the plastic strain rate at a certain applied stress is larger at higher temperatures and therefore it becomes larger in the central portion. The heat generation, at the same time, increases in the central portion because the generation rate is proportional to the strain rate and applied stress. Ferron [13] showed this effect by comparing the localizations of failure in the cases of presence of heat sinks and absence of heat sinks.

Ferron [13] also derived new criteria for flow instability of the material from Hart's criterion and Ghosh's criterion by considering two cases: (1) presence of heat sinks at the ends of the specimen and (2) absence of heat sinks at the ends of the specimen. He found that for the first condition, temperature gradients resulting from heat flow toward the ends of the specimen play a prominent part in the development of geometric defects in different positions along the tensile specimen. Therefore, the necking and failure will occur at the center of the specimen because of both geometric defect and higher thermal effect at the center of the specimen. For the second condition,
however, since heat flow towards the ends of the specimen can be reduced drastically, the necking and failure will occur in the most defective region caused by both geometric defect and thermal effect. This position could be random along the tensile specimen, depending on the initial geometric defect.

Ferron also stated that ductility will decrease with an increase in strain rate for the non-isothermal condition. The effect of strain rate is to induce adiabaticity of plastic flow: a lack of time for heat transfer allows nonuniform heating and generation is highest in the regions of high strain rate. As the result, the effect of thermal softening, which competes with strain hardening and strain rate hardening for the onset of necking, are stronger in the hot, high strain rate regions, and they enhance the plastic flow in these regions.

When a metal is deformed, most of the strain energy absorbed is converted to heat. Farren and Taylor [18] measured the heat evolved as percentage of the work expended in the tensile deformation of various metals. They found that for steel, 86.5% of the work appeared in the form of heat energy, 90.5 - 92% for copper, 92 - 93% for polycrystalline aluminium and 95 - 95.5% for aluminium single crystals. During laboratory tensile testing, temperature increases are rarely noticed because the strain rates are low and sufficient time is available for heat exchange between the specimen and surroundings, including the heat conduction and heat convection. At higher strain rate,
however, the effect of this temperature gradient can no longer be ignored because less time is available for heat exchange. Zener and Hollomon [19] first recognized the effect of thermal softening in reducing the slope of the stress-strain curve in nearly adiabatic deformation. The effect of deformation heating has been analyzed in tensile testing by several authors [12][20–23], and in compression testing by Lahoti and Altan [24]. The simultaneous effect of strain rate and deformation heat on the flow stress has been experimentally investigated by Kleemola and Ranta-Eskola [25] for A.K. steel and by Lin and Wagoner [26] for I.F. steel and stainless steel 310. They developed a constitutive equation for flow stress

$$\sigma = k \varepsilon^p \dot{\varepsilon}^m (1 - \beta T)$$

(5)

The distribution of temperature rise has been studied numerically by Wada and Nakamura [14], Korhonen and Kleemola [12], and Ayres [22][27]. Wada and Nakamura assumed a relatively simple relation for the strain rate given by

$$\dot{\gamma} = \dot{\gamma}_0 (\tau/\tau_0)^m \exp(-E/RT)$$

(6)

where $\dot{\gamma}_0$, $\tau_0$, and $m$ are constants, $E$ is the activation energy and $R$ is the gas constant. In their calculation, Wada and Nakamura used a constant activation energy which means that the strain rate is controlled by self-diffusion at high temperatures, and that deformation is controlled by a thermally activated dislocation motion at lower temperatures. The calculated results showed that at high temperature, the temperature rises varied with strain rate and initial and
boundary temperature. When strain rate was fixed, the temperature rise would increase with a decrease of the initial temperature. The strain distribution was very inhomogeneous at low boundary temperatures, and was nearly uniform at high temperatures.

Korhonen and Kleemola [12] applied the constitutive equation from Kleemola and Ranta-Eskola [25] for flow stress in the analysis of temperature increase. They also considered two kinds of heat losses: heat conduction to the specimen grips and free convection into air. Bishop's method [28] was modified to predict temperature distribution before necking. According to the calculation, they concluded that strain rate and deformation heating influenced the flow stress and the uniform strain even at moderate forming speeds. So in order to get more accurate predictions of formability, the thermal effect should be always considered. The temperature rise in the calculation was greater than that measured. Nearly the same results were obtained by Ayres et al. [22] Ayres et al analyzed the temperature rise by considering high strain rate, and showed that for moderate high strain rate (6x10^-2 s^-1), the simulation results were over-estimated compared with measurements but for high strain rate (1.5x10^-1 s^-1), the simulation results were under-estimated when deformation approached the failure.

In the present work, a criterion of instability is derived by considering the thermal effect. One-dimensional heat flow is also considered. An analytical solution is
present for prediction of the temperature distributions during non-uniform deformation. A more accurate prediction of temperature is calculated during the whole uniaxial tensile test by a numerical method, considering the effects of heat conduction and heat convection. Both ends of the specimen are kept at a constant initial temperature of the specimen. Several materials are used in the calculation for comparison.
III. THEORY

In this chapter, a general constitutive equation will be discussed and a criterion for instability will be derived by considering thermal effects. It is shown that the temperature rise caused by deformation heating plays an important part in determining instability. During deformation, the temperature increase varies with heat transfer conditions, especially for high strain-rate deformation. The heat transfer equation is derived by considering a small element in deformed specimen. The Finite Difference Method is used to obtain a numerical formula for solving the heat transfer equation.

A. Constitutive Equations

For a general material, the flow stress during deformation is known to depend at least on the strain, strain rate, temperature, anisotropy, structure, and strain or stress path. The flow stress in terms of true stress can be written generally as

\[ \sigma_{kl} = \sigma_{kl}(\varepsilon_{ij}, \dot{\varepsilon}_{ij}, T, s, r, pt) \]  \hspace{1cm} (7)

where

- \( \varepsilon_{ij} \) = plastic strain tensor
- \( \dot{\varepsilon}_{ij} \) = true strain rate tensor
- \( T \) = temperature difference between current and room, \( \theta - \theta_0 \)
- \( s \) = general structure parameter
\[ r = \text{plastic anisotropic parameter} \]

\[ pt = \text{path parameter} \]

Eq. (7) does not represent a state function since the flow stress at a point is not a state function and depends on the loading history. Rewriting Eq. (7) in differential form, we may have, for a given material state,

\[
\begin{align*}
\frac{\partial \sigma_{kl}}{\partial \varepsilon_{ij}} d\varepsilon_{ij} + \frac{\partial \sigma_{kl}}{\partial \dot{\varepsilon}_{ij}} d\dot{\varepsilon}_{ij} + \frac{\partial \sigma_{kl}}{\partial T} dT + \frac{\partial \sigma_{kl}}{\partial s} ds \\
+ \frac{\partial \sigma_{kl}}{\partial r} dr + \frac{\partial \sigma_{kl}}{\partial p} dp \\
\end{align*}
\]

\[ \ldots \ldots \quad (8) \]

This equation does not represent an exact differential because the stress is not a state function of these parameters, but should be valid for two successive points sufficiently close to each other in the multidimensional space represented by this equation.

In general, the structure and nature of anisotropy may change during the deformation. For simplicity, however, it will be assumed that the structure is unchanged, \( r \)-value is constant, and a proportional path is selected during the deformation. In the present work, the material is assumed to be isotropic, i.e. \( r = 1 \), and the material hardens isotropically so that strain, strain rate, and stress can be reduced to an effective or scalar form. [29-30] Equations (7) and (8) can then be written as

\[
\sigma = \bar{\sigma} (\bar{\varepsilon}, \dot{\varepsilon}, T) \quad (9)
\]

and

\[
\frac{d\sigma}{d\varepsilon} + \frac{d\bar{\sigma}}{d\dot{\varepsilon}} + \frac{d\bar{\sigma}}{dT} \quad (10)
\]
where $\bar{\sigma}$ is effective stress, $\bar{\varepsilon}$ effective strain and $\dot{\bar{\varepsilon}}$ effective strain rate. Effective properties relate a complex stress and strain state to an equivalent tensile state. For Von Mises' theory, the effective stress in 2-dimensions, is represented as follows

$$\bar{\sigma} = \left[ \sigma_1^2 + \sigma_2^2 + \sigma_1 \sigma_2 \right]^{1/2}$$  \hspace{1cm} (11)

where $\sigma_1$ and $\sigma_2$ are principal stresses. A similar relation can be derived from the principle of equivalent plastic work defining incremental effective strain:

$$d\bar{\varepsilon} = 2/\sqrt{3} \left[ d\varepsilon_1^2 + d\varepsilon_2^2 + d\varepsilon_1 d\varepsilon_2 \right]^{1/2}$$  \hspace{1cm} (12)

where $d\varepsilon_1$ and $d\varepsilon_2$ are principle strain increments. For proportional loading, the path dependence disappears and the effective strain becomes

$$\bar{\varepsilon} = 2/\sqrt{3} [ \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_1 \varepsilon_2 ]^{1/2}$$  \hspace{1cm} (13)

Under one dimensional loading (uniaxial tension) Eq.(11) reduces to

$$\bar{\sigma} = \sigma_1 = \sigma$$  \hspace{1cm} (14-A)

and effective strain is

$$\bar{\varepsilon} = \varepsilon_1 = \varepsilon$$  \hspace{1cm} (14-B)

Although Eq.(9) is not a complete representation of material behavior, it may provide useful results in a certain temperature, strain, and strain-rate range, particularly under nearly proportional stress and strain paths. For different materials, the constitutive equation for stress may be different in form.
Kleemola and Ranta - Eskola [25] found that for a ferritic A.K. steel uniaxial tensile test, Eq.(3) may be written as

$$\sigma = k \varepsilon^n \dot{\varepsilon}^m (1 - \beta T) \quad (15)$$

where

$$T = \theta - \theta_0,$$

the temperature difference between deformation temperature, $\theta$, and the room temperature, $\theta_0$.

$n = \text{the strain hardening exponent}$

$m = \text{the strain rate sensitivity coefficient}$

$k, \beta = \text{constants}$

Lin and Wagoner [26] also found that Eq.(15) could be employed for I.F. steel (interstitial free steel), and they found that for stainless steel 310, Eq.(9) can be written in the form

$$\sigma = \sigma_0 (1 - Ae^{-B\dot{\varepsilon}}) \dot{\varepsilon}^m (\theta/\theta_0)^{-\beta_0} \quad (16)$$

Where

$\theta$ ---- the absolute temperatures during deformation

$\theta_0$ ---- the absolute temperature at initial state

$m$ ---- the strain rate sensitivity coefficient

$A, B, \beta_0$ and $\sigma_0$ ---- constants. These constitutive equations show the effect of strain rate and temperature. The stress will increase with increasing of strain rate, while it will decrease with increasing temperature. Therefore, this equation can be used to analyze the thermal effect on stress and uniform strain.
B. Criteria For Uniform Tensile Strain

For a specified constitutive behavior, the maximum uniform strain, $\varepsilon_u$, (the true strain at the maximum load) in a tensile test can be found by a differential of total load under external constraints such as constant elongation rate and isothermality. If non-isothermality develops, such as at high strain rate in the air, the maximum uniform strain will depend on a host of conditions. For materials with low thermal conductivity ($k$), such as stainless steels, the effect is enhanced. Ferron [13] made several uniaxial tension tests in air for stainless steel at different strain rates. He found that the uniform elongation was much smaller at a higher strain rate than that at a lower strain rates. This is presumably caused by the temperature gradient as well as a uniform heating effect.

1. General Construction—Isothermal case — In this section, it is assumed that 1) the strain throughout the entire specimen length is initially uniform and remains uniform until $d$(load) = 0, 2) the temperature is uniform throughout the specimen during uniform deformation, and 3) the volume of the specimen is constant during deformation.

The well-known Considère criterion may be written as follows:

$$\frac{d\sigma}{d\varepsilon} = \sigma \quad \text{at} \quad \varepsilon = \varepsilon_u$$  \hspace{1cm} (17)

This equation denotes that the tangent of the true stress–true strain curve at the maximum strain is equal to the true stress at that point. So the maximum uniform strain could
also be determined by this equation if the stress is known. Combining the equations (10), (15), and (17) for the nonisothermal tensile test, the maximum uniform strain (maximum load point) is obtained (see Appendix A) for I.F. steel

\[ \varepsilon_u = n/[1 + m + (\beta/1 - \beta_0)(\partial T/\partial \varepsilon)] \]  \hspace{1cm} (18)

For the isothermal condition, Eq.(18) reduces to

\[ \varepsilon_u = n / (1 + m) \]  \hspace{1cm} (19)

Eq.(18) shows the effect of temperature rise on the maximum uniform strain. When temperature rise increases, the maximum uniform strain will drop. If the material is strain-rate insensitive, Eq.(19) reduces to

\[ \varepsilon_u = n \]  \hspace{1cm} (20)

where \( n \) is strain hardening rate coefficient. This is a well-known Considère criterion [1] for necking under the isothermal constant strain rate conditions.

For stainless steel, the flow stress can be described by Eq.(16). For small value of \( \beta_0 \), analysis (Appendix A) shows that the maximum uniform strain for stainless steel may be written

\[ \varepsilon_u = (1/B)\ln A \left\{ 1 + \frac{B}{1 + m + (\beta/1 - \beta_0)(\partial T/\partial \varepsilon)} \right\} \]  \hspace{1cm} (21)

Under the isothermal testing, Eq.(14) reduces to

\[ \varepsilon_u = (1/B)\ln A \left\{ 1 + B/(1+m) \right\} \]  \hspace{1cm} (22)
Eqs.(18) and (21) represent the uniform strain condition in terms of given constitutive equations (for I.F. steel and 310 stainless steel, respectively) and in terms of an unknown temperature vs. strain function. For isothermality, this term disappears and Eqs.(19) and (22) result. For non-isothermality, additional information is needed to solve Eqs.(18) and (21).

2. **Adiabatic Case** - For very high strain rates, heat transfer conditions approach adiabatic, i.e. no heat transfers out of the system during the deformations, so \( \text{d}Q = 0 \). According to the first law of the thermodynamics, one may write conservation of energy as

\[
\rho c \text{d}T = \eta \sigma \text{d}\varepsilon
\]

(23)

Where

\( \eta \) ----- fraction of the plastic work converting to heat.

\( \rho \) ----- density of the material

\( c \) ----- specific heat

According to the measurements of Farren and Taylor [18], depends on the material during the uniaxial tension test. For steel, is about 0.86.[18]

Rewriting Eq.(23), we obtain

\[
\frac{\text{d}T}{\text{d}\varepsilon} = \frac{\eta \sigma}{\rho c}
\]

(24)

where \( T \) is temperature increase. By substituting Eq.(24) into Eq.(18), the uniform strain for I.F. steel becomes

\[
\varepsilon_u = \frac{n}{1+m+\left(\frac{\psi}{1-\beta}T\right)\left(\frac{\eta \sigma_u}{\rho c}\right)}
\]

(25)
where \( \sigma_u \) is true stress at strain \( \varepsilon_u \). Introducing Eq. (15) for stress, we obtain uniform strain

\[
\varepsilon_u = n/[1 + m + (\beta n/\rho c)(km^n)(\varepsilon_u^n \varepsilon^m)]
\]  
(26)

This is a transcendental for uniform strain \( \varepsilon_u \). In order to get the solution for uniform strain, a trial-and-error method must be used in the calculation. For approximate estimation of the uniform strain, one may assume that on the right side of Eq. (26)

\[
\varepsilon_u \approx n
\]

\[
\dot{\varepsilon} = (V_0/L_0)\exp(-\varepsilon_u) \approx (V_0/L_0)\exp(-n)
\]

Thus Eq. (26) reduces to

\[
\varepsilon_u = n/[1 + m + (\beta n/\rho c)(kn^n)(V_0/L_0)^m\exp(-nm)]
\]  
(27)

This expression is valid (with the approximations listed above) for materials like I.F. steel which obey the constitutive equation shown in Eq. (15). Similarly for stainless steel 310, the constitutive equation of Eq. (16) and the stability equation of Eq. (21) can be combined to obtain the uniform, adiabatic strain:

\[
\varepsilon_u = (1/B)\ln A \left\{ 1 + \frac{B}{1 + m + (\beta /1 - \beta T)(\eta_0 \varepsilon_u /\rho c)} \right\}
\]  
(28)

Introducing constitutive equation for \( \sigma_u \) [Eq. (A-17)], we obtain the uniform strain

\[
\varepsilon_u = (1/B)\ln A \left\{ 1 + \frac{B}{1 + m + (\beta n_0 /\rho c)(1 - A\exp(-B\varepsilon_u)\varepsilon^m)} \right\}
\]

\[
................
\]  
(29)
This is also an implied equation for uniform strain, so the solution for uniform strain could be obtained numerically. An approximate solution is not as straightforward as for the I.F. steel case.

3. Intermediate Heat Transfer Condition - In general, significant heat transfer can occur during deformation when strain rate is not too high. If we consider an infinitely long specimen subject to uniform deformation, heat conduction can be ignored since no gradients can develop. The heat transfer equation for a one-dimensional specimen depends solely on convection to the surrounding medium (ignoring radiation) and may be expressed as follows:

$$\rho c_d \frac{dT}{dt} = \eta \sigma \frac{d\varepsilon}{dt} - 2hT \frac{dt}{w}$$  \hspace{1cm} (30)

where $T$ is temperature increase (relative to the surrounding medium), $h$ is the heat convection coefficient and $w$ the thickness of the specimen. Rewriting Eq.(30), we can obtain the required derivative of $T$ with respect to strain:

$$\frac{dT}{d\varepsilon} = \frac{\eta \sigma}{\rho c} - \frac{(2hT/\rho c w)(dt/d\varepsilon)}{dt/d\varepsilon}$$  \hspace{1cm} (31)

where

$$dt/d\varepsilon = (L_0/V_0) \exp(\varepsilon)$$

from Eq.(A-6). Substituting Eq.(31) into Eq.(18) and Eq.(21), respectively, the uniform strain becomes

$$\varepsilon_u = \frac{n}{\left\{1 + m + \frac{\beta}{1 - \beta T} \left( \frac{\eta \sigma u}{\rho c} - \frac{2hT L_0}{\rho c w V_0} \exp(\varepsilon_u) \right) \right\}}$$  \hspace{1cm} (32)

for I.F. steel and
\[ \varepsilon_u = (1/B) \ln A \left\{ \frac{1+\beta}{1+\beta T} \left( \frac{\eta \sigma u}{\rho c} - \frac{2hTL_0}{\rho c w V_0} \exp(\varepsilon_u) \right) \right\} \]

for stainless steel 310.

These two equations cannot be solved because of the unknown, but coupled, temperature rise term. So we have to solve simultaneously equations (30) and (32) for uniform strain of I.F. steel, and equations (30) and (33) for that of stainless steel.

C. Heat Transfer During Nonuniform Straining

During nonuniform straining, temperature gradients develop because of the different heat generation rates at different points in the material. Even an adiabatic experiment will involve heat transfer by conduction. Because temperatures are not very high during the tensile test, the heat losses by radiation can be safely ignored. According to the concept of energy conservation, when we consider a small volume element in the deformation region, we can derive the heat transfer equation for one - dimension as (see Appendix B)

\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} - V_x \frac{\partial T}{\partial x} - \frac{2hT}{\omega \rho c} + \frac{\dot{H}}{\rho c} \]

(34)

Where

- \( T \) = temperature rise (ºC)
- \( \dot{H} = \eta \sigma \dot{\varepsilon} \), heat source (cal/sec cm³)
- \( \alpha = \) thermal diffusivity (k/\( \rho c \)) (cm²/sec)
- \( \omega = \) thickness of the sample (cm)
\[ h = \text{heat convection coefficient } (\text{cal/sec} \ 0^\circ C \ cm^2) \]
\[ \rho = \text{density } (\text{gm/cm}^3) \]
\[ c = \text{specific heat } (\text{cal/gm} \ 0^\circ C) \]
\[ V_x = \text{velocity } (\text{cm/sec}) \]

This equation is not solvable analytically, in general, because of the heat generation term. A numerical method is therefore used to obtain a solution.

D. Analytic Solutions

Heat transfer phenomena during deformation can be expressed mathematically by a partial differential equation. In order to understand directly the heat transfer phenomena, one may want to solve the heat transfer equation analytically rather than numerically. In general, the partial differential equation of heat transfer is difficult to solve if a heat source is considered. In most cases, it is impossible to obtain a closed solution. In the present work, a simple formulae for heat source derived from Eq.(39) is used to solve heat transfer equation.

For non-static state problems, the heat transfer equation is usually transformed to static state equation which is easier to solve analytically. For this purpose, Laplace transformation method is used because time space is finite. The final results can be obtained by inverting Laplace transformation of the solutions for static state equation. The whole procedure for solving heat transfer equation is shown in Appendix C.
E. Numerical Modeling

In order to obtain more accurate results, a numerical method is used. In the present work a total Lagrangian frame of reference is used for the heat transfer, neglecting the movement and deformation of the element during the deformation. Eq.(34) therefore reduces to (with \( V_x = 0 \))

\[
\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} - \frac{2hT}{\rho_0 c} + \frac{\dot{H}}{\rho c}
\]  

(35)

The average convection coefficient, \( h \), is assumed to remain unchanged although in fact the effective \( h \) will change based on the change of surface area and the temperature of the specimen. Because the effective heat convection coefficient, \( h \), is a weak function, for small deformations and temperature rise, of the length and the temperature, the error from this assumption is negligible. The thickness is also assumed to remain unchanged. This assumption leads to an error of only a few percent for the deformations studied.

The initial and boundary conditions for Eq.(35) are

\[
T(x,0) = 0 \quad \text{for all } x
\]  

(36-A)

and

\[
T(x,t) = 0 \quad \text{at } x = \pm L/2, \text{ for all } t > 0
\]  

(36-B)

\[
\frac{\partial T}{\partial x} = 0 \quad \text{at } x = 0, \text{ for all } t > 0
\]  

(36-C)

respectively. \( L \) is the length of specimen.

Eq.(35) with the boundary conditions of Eq.(36) can be solved by a computer using the finite difference method. [31]

A two-dimensional mesh is made in the present work by dividing the \( x \) axis into \( n \) elements with an interval length
Fig. 1 - a) The mesh used in calculation, and b) an example showing how temperature at a point can be calculated by three temperatures of the preceding row.
of $\Delta x$ and the time axis into $m$ steps with a time interval of $\Delta t$, shown in Fig.(l). There are a total of $n \times m$ nodes which represent different positions and times. For example, the temperature rise $T_{i,j}$ at the node $(i,j)$ represents the temperature rise at the position $x = i \times \Delta x$ from the center and the time $t = j \times \Delta t$. Therefore, the numerical solution is found by solving the temperature rise at each of the nodes. By using the C-N second level method [32] we obtain the finite difference equation from Eq.(35) (see Appendix D)

$$-(R/2)T_{i-1,j+1} + (1+R)T_{i,j+1} - (R/2)T_{i+1,j+1} = \frac{(R/2)T_{i-1,j} + (1-R-F)T_{i,j} + (R/2)T_{i+1,j} + Q_{i,j}}{.................} \quad (37)$$

Where

$$R = \frac{\alpha \Delta t}{(\Delta x)^2}, \text{ the Fourier number}$$

$$F = 2h \Delta t / (\rho c w)$$

$$Q_{i,j} = \frac{n}{\rho c} \sigma_i, j (\varepsilon_i, j+1 - \varepsilon_i, j)$$

In order to guarantee convergence, $R$ must be less than unity, $R < 1$. [33-34] Introducing the boundary conditions of Eq.(36-C), Eq.(37) can be rewritten in matrix form (see Eq.(D-6)). It can be seen that if $T$ is known at three points of the $j$th row, $T$ at one point in the $(j+1)$th row can be determined. From the initial and boundary conditions, i.e., equations (36-A) and (36-B) we know the values of $T$ on the base line and the right side of the mesh and thus the temperature rise at each node of the mesh can be computed.
Using the proper equation of stress for the different materials, and considering the effects of strain rate and deformation heating on the strain hardening, our problem can be solved.
IV. EXPERIMENTAL PROCEDURE

A. Materials

Commercial quality interstitial-free steel (I.F. steel) [35] sheet and stainless steel 310 sheet with a thickness of 0.9 mm were used in the experiments. The compositions of I.F. steel and stainless steel 310 are given in the Table I. Tensile samples were made from the sheets with the tensile axis parallel to the rolling direction. The tensile samples were made according to ASTM - 8 with 12.7 mm width x 50.8 mm gage length. In order to ensure repeatability of failure at the center of the samples, each sample was given a 1% taper at the center over a 50 mm length. The tensile tests were performed in an Instron Universal servo-hydraulic testing machine at constant cross-head velocity. The elongation-load curve for each test was recorded by x-y plotter. Standard mechanical properties of I.F. steel sheets and stainless steel 310 sheets are presented in the Table II.

B. Testing

Two groups of I.F. steel and S.S. 310 tensile samples were strained to failure at crosshead speeds of 0.48 mm/s to 480 mm/s, corresponding to initial strain rates of $1 \times 10^{-4}$ /s to $1.0 \times 10^{-1}$/s at initially room temperature. Both isothermal (i.e. in stirring water) and non-isothermal (in air) tension tests were performed in the present work. The maximum
TABLE I - Chemical Composition Of I. F. Steel And S. S. 310 (w.t.%)

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Si</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. F. Steel</td>
<td>0.008</td>
<td>0.29</td>
<td>0.008</td>
<td>0.012</td>
<td>0.011</td>
<td>0.049</td>
</tr>
<tr>
<td>N</td>
<td>0.0066</td>
<td>Nb</td>
<td>Ti</td>
<td>Fe</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.055</td>
<td>0.13</td>
<td></td>
<td>Balance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS 310 [36]</td>
<td>0.25</td>
<td>0.2</td>
<td>1.5</td>
<td>25</td>
<td>22</td>
<td>Balance</td>
</tr>
</tbody>
</table>
### TABLE II - Standard Mechanical Properties Of Selected Alloys

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile Strength (MPa)</td>
<td>326</td>
<td>660</td>
<td>340</td>
<td>331</td>
<td></td>
</tr>
<tr>
<td>Strain Hardening Exponent ( n )</td>
<td>0.22</td>
<td>0.4</td>
<td>0.2</td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td>Strain Rate Coeff. ( m )</td>
<td>0.018</td>
<td>0.0098</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strength Coeff. (MPa)</td>
<td>566.3</td>
<td>969.9</td>
<td>556</td>
<td>755.2</td>
<td></td>
</tr>
<tr>
<td>Constants ( A )</td>
<td></td>
<td>0.727</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B )</td>
<td></td>
<td>3.766</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.0015</td>
<td>0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
uniform strains (the strains at the maximum load) were obtained from the load - elongation curves. For the isothermal tests, a stirred water bath was used to maintain isothermality.

1. **Photogridding** - In order to follow the progress of flow localization, the tensile specimens were photogridded using Kodak [39] photofabrication techniques. The procedure of photogridding can be divided into several steps. The specimen was first immersed into photo-resist solution in a Kepro Bench-Top Coater machine [39] to apply a thin, uniform film. The thickness of coat was controlled by the drawing speed, a higher speed resulting in a thinner the film. After air drying, the specimen was covered with a photographic mask of circles of 1 mm diameter and exposed in a Kepro Ultra-Violet Exposure Frame. The exposed specimen was then immersed into the Kodak Photo Resist Developer for one minute in which the non-exposed area of coated film was dissolved, leaving invisible circles on the specimen corresponding to the mask pattern. The Kodak Photo Resist Dye was used to make the remaining grids visible. Finally, the specimen was cleaned with methanol to remove extra dye and to increase the contrast. This procedure allows very good contrast, high spatial resolution, and good film adherence.

2. **Photographic Recording** - The contrast between the circles and the background of surface allows measurement of surface strains over small regions, thus allowing measurement of
localization of deformation by taking photographs during tensile tests. This technique was used for the tensile tests at the strain rates $1.0 \times 10^{-3}/s$, $1.0 \times 10^{-2}/s$, and $1.0 \times 10^{-1}/s$. High resolution photography requires rapid shutter speeds, particularly at the faster strain rates. These high speeds in turn require intense illuminations. In order to protect the sample from being heated by the light, two plastic plates were put in the front of the light. There was virtually no heat absorbed by the sample during tensile test from the light. A Nikon FD-3 high speed camera was used to take photographs because when strain rate was very high, only very short time was available after necking. The speed of 3 photographs per second was used for a strain rate of $10^{-2}/s$, and that of 6 photographs per second for a strain rate of $10^{-1}/s$. The circle diameters on the negatives of the photographs were directly measured by means of a travelling microscope, which was connected to IBM-PC by an IEEE interface. All the data were stored in IBM-PC for further analysis. Because only one dimension is considered in the present work, only the diameters along the specimen axis were measured. The true strain at each circle was calculated from initial and current diameters. The accuracy of measurement of the diameter of the circles is estimated to be about ±0.002 mm. This resulted in an accuracy in the measurement of strain of ±0.005 (0.5%).
V. RESULTS

A. Uniform Deformation

The maximum uniform strains as shown in Fig.(2-a) for I.F. steel and in Fig.(2-b) for S.S. 310, were calculated from Eq.(18) and (21) respectively. In Fig.(2-a), the isothermal line is from Eq.(19), considering constant crosshead velocity, and the adiabatic line is from Eq.(27). The uniform strain under adiabatic conditions varies only slightly with crosshead speed. For intermediate heat transfer cases, the uniform strains were calculated by solving the partial differential Eq.(35) and Eq.(18) simultaneously, considering different heat convection conditions and including one-dimensional conduction (i.e. by assuming that the temperature at the two grips is kept constant). The thermal constants in calculation are given in Table III. The heat convection coefficients were estimated from Table IV [40] for the conditions of the natural convection in the air, forced convection in the air, natural convection in water, and forced convection in water. A program was developed for solving the partial differential equations (35), as shown in Appendix E. The curves in Fig.(2-b) give the uniform strains for stainless steel under the same conditions as that for I.F. steel. Fig.(2-a) shows that the uniform strain for the forced convection case in
Fig. 2 - Uniform strain as a function of the strain rate at different heat transfer conditions
a) for I. P. steel, b) for stainless steel 310.
<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g/cm³)</th>
<th>Specific Heat (cal/g °C)</th>
<th>Thermal Diffusivity (cm²/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.F. Steel</td>
<td>7.8</td>
<td>0.111</td>
<td>0.147</td>
</tr>
<tr>
<td>S.S Steel</td>
<td>7.8</td>
<td>0.11</td>
<td>0.04</td>
</tr>
<tr>
<td>2036-T4 Al alloy</td>
<td>2.7</td>
<td>0.211</td>
<td>0.666</td>
</tr>
<tr>
<td>Brass (70/30)</td>
<td>8.52</td>
<td>0.092</td>
<td>0.34</td>
</tr>
<tr>
<td>mode</td>
<td>heat trans. coeff. ( h ) (cal/cm(^2)°C sec) ( \times 10^{-4} )</td>
<td>( h ) used in paper</td>
<td></td>
</tr>
<tr>
<td>--------------</td>
<td>-------------------------------------------------</td>
<td>---------------------</td>
<td></td>
</tr>
<tr>
<td>free air</td>
<td>( 1 - 6 )</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>forced air</td>
<td>( 2 - 100 )</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>water</td>
<td>( 25 - 3500 )</td>
<td>167</td>
<td></td>
</tr>
<tr>
<td>boiling water</td>
<td>( 600 - 6000 )</td>
<td>1300</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 3 - Comparison of the calculated uniform strain with experimental data. a) for I. F. steel, b) for stainless steel 310.
1. Isothermal
2. In air

Lines are predicted values
Points are experimental data
Fig. 4 - The calculated temperature distributions for I. F. steel at engineering strain 24% a) initial strain rate: $10^{-3}/s$, b) initial strain rate: $10^{-2}/s$
STRAIN RATE : \(10^{-2}/s\)

- END EFFECT
- IN AIR
- IN FORCED AIR
- IN WATER
- IN FORCED WATER

ENG STRAIN : 24% ADIABATIC

(b)
Fig. 5 - The calculated temperature distributions for stainless steel 310 at engineering strain 40%  
(a) initial strain rate: $10^{-3}/s$  
(b) initial strain rate: $10^{-2}/s$
STRAIN RATE: $10^{-2}/s$

- END EFFECT
- IN AIR
- IN FORCED AIR
- IN WATER
- IN FORCED WATER

ENG STRAIN: 40%  ADIABATIC
water is very close to the value in isothermal case except at very high strain rates, while the uniform strain for natural convection in the air at the strain rate $1.0 \times 10^{-2}$ or higher approaches the value in adiabatic case. Between these two cases, the uniform strains will decrease with an increase of the initial strain rate.

Maximum uniform strains in Fig.(3) were measured from the load-elongation curves for various initial strain rates. Mid-points on the plateau at the maximum load were chosen for uniform strain. The dashed lines in Fig.(3) were calculated from equations (18) and (21) respectively. The heat convection coefficient for our tests is chosen as $8 \times 10^{-4}$ (cal/cm$^2$ sec °C). The curves in Fig.(3) show that the calculated values agree with the results corresponding to the test in the air. Under isothermal conditions the strain rate does not affect the uniform strain. [41] Deformation heating decreases the uniform strain of all materials compared with that corresponding to the isothermal testing. This is particularly true for S.S. 310 because of its lower thermal conductivity.

The temperature distributions calculated by the numerical solution of Eq.(35) are given in Fig.(4) for I.F. steel and in Fig.(5) for stainless steel 310. In these calculations, we consider the temperature effect on the stress, that is, we use equations (15) and (16) to calculate the stress. The parameters in equations (15) and (16) are given in the Table II. Fig.(4-a) shows that for I.F. steel,
at the lower speed, the maximum temperature rise in the blowing air is only about 1°C, so that this deformation could be considered the isothermal deformation. At higher speeds, the maximum temperature rise given by Fig. (4.b) is high, so this deformation is clearly non-isothermal. However, if the tensile test is conducted in the stirring water, the deformation may still be called isothermal deformation although the speed is very high.

Stainless steel 310 has a lower thermal conductivity and higher strength than I.F.steel, so the end effect (heat conduction at two grips) is less important, even at an initial strain rate of 1.0x10⁻³/s. Fig.5 shows the temperature distribution for stainless steel 310 at different strain rates and at different convection conditions. The tensile test in the stirred water is still nearly isothermal.

For other materials such as brass and 2036-T4 Al alloy, we can predict the temperature distributions under various conditions as shown in Figures (6) and (7). Because the data for the thermal effect on the stress are unknown for these materials, the predictions are made by neglecting the thermal softening on the stress. The flow stresses are described by the Hollomon equation [42], and the parameters are also given in Table II.

The maximum temperature rise depends mainly on the heat convection and the initial strain rate if the initial strain rate is higher than 1.0x10⁻² s⁻¹ because thermal conduction becomes less important at high strain rates. The results are
Fig. 6 - The calculated temperature distributions for brass at engineering strain 40%
Fig. 7 - The calculated temperature distributions for 2036-T4 Al alloy at engineering strain 22%
Fig. 8 - Temperature rise as a function of ratio of effective convection coefficient and strain rate at engineering strain 20%
Fig. 9 - Relative temperature decrease due to heat conduction vs. thermal diffusivity
shown in the Fig.(8). It can be seen that when the ratio of the effective heat convection coefficient (h/cpw) and the initial strain rate is larger than 100, the conditions approximate isothermal deformation, at least for the initial strain rate range of $1.0 \times 10^{-4}/s - 1.0 \times 10^{-1}/s$ used to generate Fig.(8).

If we ignore the effect of heat convection, the temperature rise at the center of the specimen will depend on the thermal diffusivity, and the initial strain rates. The relation between the thermal diffusivity and temperature decrease from the adiabatic case by end effect is plotted in Fig.(9). It is interesting to note that the temperature drop due to conduction is nearly linear with $z$ value and that for I.F. steel and S.S. 310, very little conduction effect is apparent except for rates slower than $10^{-3}$.

B. Non-uniform Deformation

As described in section IV.E, flow localization in the I.F. steel specimen was recorded by means of photographs taken during testing. The diameters of circles on the negative films were measured. In Fig.(10.a-e), photographs of the specimen at different stages of flow localization are shown. It is evident that the deformation is nearly uniform prior to the maximum load point although the specimen was slightly reduced in width by about 1% at the center of initial specimen. The relations between strain at the specimen center and time at different strain rates are shown in Fig.(11), compared with analytical solutions, Eq.(A-5).
Fig. 10 - Evolution of flow localization

(a) initial

(b) uniform strain
(c) near necking  (d) post uniform strain

(e) fracture
Fig. 11 - Uniform true strain vs time for I. F. steel
Fig. 12 - Strain distributions as a function of x and t for I.F. steel
STRAIN RATE: $10^{-2}/s$

ENG STRAIN:
- 36 %
- 33 %
- 27 %
- 22 %
- 16 %

TRUE STRAIN

X (MM)
STRAIN RATE: $10^{-3}/s$

ENG STRAIN:
- 41 %
- 39 %
- 36 %
- 33 %
- 29 %
- 24 %
- 18 %
STRAIN RATE: $10^{-2}$/s

ENG. STRAIN:
- 38 %
- 33 %
- 27 %
- 22 %
- 16 %

TRUE STRAIN

X (MM)
Fig. 13 - Strain distributions as a function of x and t for stainless steel 310 at a) strain rate: $10^{-1}/s$, b) strain rate: $10^{-2}/s$, c) strain rate: $10^{-3}/s$
STRAIN RATE: $10^{-2}/s$

ENG. STRAIN:

\begin{itemize}
  \item [0] 39.2\%
  \item [*] 34.3\%
  \item [+] 30.2\%
  \item [-] 26.5\%
\end{itemize}

(b)
STRAIN RATE: $10^{-3}/s$

ENG. STRAIN:

- $43.8\%$
- $42.3\%$
- $40.7\%$
- $37.5\%$
- $33.1\%$
Fig. 14 - Analytical solutions of temperature distribution in air for a) strain rate: $10^{-3}$/s, b) strain rate: $10^{-2}$/s
STRAIN RATE: $10^{-2}/s$
IN AIR: $h = 1.9 \times 10^{-4}$ (cal/cm$^2$°c sec)
ENG. STRAIN:

(b)
Fig. 15 - Numerically calculated temperature distributions in air by using empirical equation (39) at a) strain rate: $10^{-3}/s$, b) strain rate: $10^{-2}/s$
STRAIN RATE: $10^{-2}$/s
IN AIR: $h = 1.9E-4$ (cal/cm$^2$°C sec)
ENG STRAIN:

![Graph showing temperature change with strain rate and temperature difference $T - T_0$ over $X$ (MM)].

(b)
Fig. 16 - Numerically calculated temperature distributions in air by using more accurate equation (38) at a) strain rate: \(10^{-3}/s\) b) strain rate: \(10^{-2}/s\)
STRAIN RATE: \(10^{-2}/s\)
IN AIR: \(h = 1.6 \times 10^{-4} \text{cal/cm}^2\text{C sec}\)
ENG STRAIN: 

\(T - T_0 \text{ (C)}\)

\(X \text{ (MM)}\)

38%
33%
27%
22%
16%
10%

(b)
In Fig.(10), it can be seen that the neck developed in a small region, and that after necking, the position away from the necking region is nearly static. The true strain along the specimen length was calculated from the local diameters after successive time increments. Strain distributions along one half of the specimen length are shown in Fig.(12) for I.F. steel and in Fig.(13) for S.S. steel. The simplex method [43] was employed to fit two kinds of equations to these data for I.F. steel. One equation is given in the form

$$\varepsilon = C \exp[Dt/(E+x^d)]$$  \hspace{1cm} (38)

Where C, D, E, and d are constants, t is time, and x is the position from the failure. This equation has very small deviation and fit the data very well as shown in Fig.(12-a) and (12-b). Standard error of the curve fitting is $6 \times 10^{-3}$ for strain after necking. Another equation is given by

$$\varepsilon = C \exp(-Dx + Et)$$  \hspace{1cm} (39)

Where C, D, and E are constants. Standard error is about $1 \times 10^{-2}$ for strain after necking. Although this equation does not fit the data as well as the first one near failure, as shown in Fig.(12-c) and (12-d), the equation gives a particularly simple form for solution of the heat transfer equation (35) for non-uniform deformation. A complete analytic solution for the temperature distribution is given in equation (C-24) in which we neglect the end effect (i.e. infinite specimen). The temperature distribution along the
whole deformation zone as derived in Appendix D is shown in Fig.(14-a) and (14-b). It is seen that the maximum temperature rise for an initial strain rate of $1.0 \times 10^{-3}$/s is about $40 \, ^{\circ}C$ and for the strain rate $1.0 \times 10^{-2}$/s is about $55 \, ^{\circ}C$. If we consider the end effect, we can solve equation (35) by the finite difference method. The calculated temperature distribution solutions are given in Fig.(15) by using Eq.(39) and Fig.(16) by using Eq.(38) which is more accurate than Eq.(39). It is evident that there is not much difference between the curves in the specimen center in the Figures (14), (15) and (16). The temperature calculated by the numerical method is compared with experimental data in Fig.(17). In this calculation, we choose the heat convection coefficient to be $8.0 \times 10^{-4}$ (cal/cm$^2$ sec $^{\circ}C$). It can be seen that the predicted temperature is very close to the measured data in this experiment. [44]

From the photographs taken at known time intervals, the true strain at each circle each time can be calculated. When circles corresponding to an initial length of 50 mm along the center line were measured in each photograph, the engineering strain for each time could be obtained. The average true strain rate ($\dot{\varepsilon} / \Delta t$) at the specimen center in air was considered as a function of engineering strain, as shown in Fig.(18-a) for initial strain rate $10^{-3}$ and in Fig.(18-b) for initial strain rate $10^{-2}$. The data points are shown at three initial positions: the center, 5 mm from the center and 20 mm from the center. Strain rates for each
Fig. 17 - Comparison of calculated temperature rise with experimental data from ref. (44)
Fig. 18 - Strain rate vs. engineering strain for I. F. steel at three particular points for a) initial strain rate: $10^{-3}/s$, b) initial strain rate: $10^{-2}/s$
STRAIN RATE: $10^{-2}$/s
AT THE CENTER
5 MM FROM CENTER
20 MM FROM CENTER
position were found initially to be nearly constant until the maximum uniform strain was reached, that is until necking occurred. After that, strain rate at the position far away from the necking was dropped gradually to zero with further strain. The strain rates near the neck (5 mm) increase at first and then decrease. The strain rate at the neck increased continuously and the slope of the curve increased more quickly, especially near the fracture strain. This property is nearly the same for different initial strain rates.
VI. DISCUSSION

Uniform strain is an important material property in stretch forming and in deep drawing of complex parts. Uniform strains of I.F. steel and S.S. 310 remain constant in the isothermal tests, independent of the strain rate. In non-isothermal tests, the effect of strain rate and deformation heating on the uniform strain is important, as shown in Fig.(3). The results agree very well with Kumpulainen's results [38] for the materials of AISI 304, AISI 316, Al and A.K. steel and Korhonen's results [12] for the A.K. steel. Because the stress is dependent on the testing temperature, the onset of diffuse necking occurs earlier. The uniform strains are therefore reduced. From Eqs.(18) and (21), the uniform strains are determined by both the temperature increase rate and the temperature rise. The higher the temperature increase rate and temperature rise are, the smaller the uniform strains become. Since the thermal conductivity of S.S. 310 is smaller than that of I.F. steel, the heat loss to the grips is lower than that for I.F. steel. The uniform deformation of S.S. 310 is much larger than that of I.F. steel. Therefore, compared with an I.F. steel specimen, the temperature increase rate and temperature rise are higher. The reduction of maximum uniform strain is therefore greater for S.S. 310 under similar heat transfer conditions.
The amount of heat transferred depends on the available time and surroundings. When the strain rate is high, the available time for heat transfer is short. Thus the deformation at high strain in air (h value is lower in air) will approach the adiabatic deformation. On the other hand, although the strain rate is very high such as $1 \times 10^{-4}/s$, the deformation in the stirring water could still be considered as isothermal deformation because the stirring water has very strong heat transfer ability, as shown in Fig.(2).

The maximum temperature rise during uniform deformation depends mainly on the heat conduction and heat convection. For the standard specimen, the end effect (or thermal conduction at the grips) at the lower crosshead speed is significant, compared with the effect of heat convection in the air. At high speeds the end effect can be neglected. In order to show the thermal conduction effect and the heat convection effect on the temperature increase, it is better to consider these two factors separately. The effect of heat convection for different materials is shown in the Fig.(8). It can be seen that when the ratio of the effective heat convection coefficient ($h/\text{cpw}$) and the initial strain rate is larger than 100, the tensile test could be characterized as isothermal, at least for the initial strain-rate range of $10^{-4}/s - 10^{-1}/s$. For strain rate sensitive materials, the temperature rise at the same (eff $h$/strain rate) ratio may vary with the strain rate (strain rate range $10^{-4}/s - 10^{-}$)
Fig. 19 - Effect of specimen length on the temperature distribution
1/s), as shown by the error bars in Fig.(8). In Fig.(8),
temperature increase is highest for 2036-T4 Aluminum alloy
because more plastic work has been done at engineering strain
20%.

In general, the predicted temperature rise is greater than
that measured. [12][23][22] In the present problem, if we
choose the proper heat convection coefficient, the calculated
value is very close to the measured value shown in Fig.(17).
This indicates that at high strain rate deformation, the
proper h value could be used to predict correctly the
temperature distribution during deformation.

The end effect is also complicated because the end effect
depends not only on the property of material, but also on the
dimension of the specimen. For the standard specimen, the
temperature increase varies with thermal diffusivity. For
various strain rates, the percentage of temperature drop due
to the end effect from adiabatic case is shown in Fig.(9).
Because the thermal conductivity is very high for 2036-T4 Al
alloy about 95% of heat is lost at the grips at low strain
rate. If the strain rate is increased, the importance of end
effect diminishes very rapidly. The dimension of specimen is
also an important factor to estimate the end effect.
Fig.(19) shows the temperature distribution for smaller
specimens. The temperature gradient diffuses to the center
of the specimen. While in Fig.(4) and (5) for a standard
specimen, the temperature gradient penetrates to nearly the
same distance, x = (k/cp)t , but far from the center. This
result agrees with Wada's result [14]. This result may explain why small specimens for some material has lower uniform strain than larger specimens.

As shown in Fig.(10), after necking, deformation localizes in a small region and the gradient of strain is very steep. More heat is generated in this zone and the corresponding temperature rise will be much higher than that outside of the zone. Furthermore, because of the high temperature in this area, the thermal softening causes the flow to be more localized in the zone, or neck. Consequently, the gradient in temperature becomes sharper. In this sense, the temperature gradient may make the deformation unstable. Ferron (13) performed several tension tests, using different kinds of grips. He found that 1) the failure localization with metallic grips was near the center of the specimens, and 2) the failure localization with Araldite grips in air (or with metallic grips in water) is random. This indicates that in the case one, there is a temperature gradient along the specimen because of the thermal conduction. At the center of the specimen, the temperature is higher than other positions. So thermal softening is larger in the center. When the thermal softening suppresses strain hardening, deformation of specimen becomes unstable at the center. Therefore, the failure localization is near the center. On the other hand, in case 2, there is no thermal gradient. That is, thermal softening is the same along the specimen. If there is no
geometrical defect in the center of the specimen, the failure will occur at some material defect area which is random. This effect will occur even in an ideally uniform tensile specimen, solely on the basis of heat conduction from the ends.

The effect of thermal conductivity and heat convection on the temperature increase after necking is a complicated problem. The approximate analytical solution Eq.(C-24) can serve to estimate roughly the thermal and convection effects. The increase in thermal diffusivity and $h$ values will decrease the temperature rise.

Empirical equation (38) can be employed to estimate the temperature rise. The predicted results and experimental data are shown in Fig.(17). Because the strain rate is high, the heat convection term is more important than the thermal conductivity term. In our case, the $h = 8.0 \times 10^{-4}$ (cal/cm$^2$ sec $^\circ$C) is chosen to calculate the temperature rise. It is clear that the predicted value is a little higher than the experimental data. The error may come from both experiment and empirical equation. It is important to note that although the empirical equation has no physical meaning, it is a useful form to show the local flow of strain and to predict the temperature increase during deformation.
VII. CONCLUSIONS

The effect of strain rate and deformation heating on uniform strain has been calculated numerically. The predicted results agree well with experimental data. When material is deformed at high strain rate, causing a temperature increase, formability of material will be reduced.

Deformation in air approaches adiabatic condition at strain rates of $10^{-2}$/s or higher. For forced water cooling, deformation will be isothermal even at strain rates as high as $10^{-1}$/s. Formability of material will be improved by cooling.

When I.F. steel is deformed at strain rate $10^{-2}$/s, temperature increases about 18 °C at uniform strain, and about 55 °C at necking region near fracture. The temperature rise for lower strain rate will be lower because of heat transfer.

The analytical solutions for temperature distribution is the same as numerical results for the simple formula of post-uniform strain except for region near two ends. The analytical solutions is also close to the numerical results obtained by using more accurate formulae of strain. The difference between these two results in necking region is only a few degrees. For the position away from necking
region, analytical results are higher than numerical results.

Strain rate will concentrate at necking region, and increase very rapidly. The strain rate at 5 mm away from the necking point keeps nearly constant and then diminishes when deformation approaches fraction.
VIII. REFERENCE


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IX. APPENDICES

A. Uniform Strain For Uniaxial Tension

Assume that 1) the strain throughout the entire specimen length is initially uniform and remains uniform until \(d(\text{load}) = 0\), 2) the temperature is uniform throughout the specimen until \(d(\text{load}) = 0\), 3) the volume of the specimen is constant during deformation, and 4) the flow stress in terms of true stress can be written as

\[
\sigma = \sigma(\varepsilon, \dot{\varepsilon}, T) \quad (A-1)
\]

where

\[
\begin{align*}
\varepsilon &= \text{true strain} \\
\dot{\varepsilon} &= \text{true strain rate} \\
T &= \text{temperature rise}
\end{align*}
\]

The differential form of Eq.(A-1) is for a given material state

\[
\frac{d\sigma}{d\varepsilon} = \frac{\partial \sigma}{\partial \varepsilon} \frac{d\varepsilon}{d\varepsilon} + \frac{\partial \sigma}{\partial \dot{\varepsilon}} \frac{d\dot{\varepsilon}}{d\varepsilon} + \frac{\partial \sigma}{\partial T} \frac{dT}{dT}
\]

or

\[
\frac{d\sigma}{d\varepsilon} = \left( \frac{\partial \sigma}{\partial \varepsilon} \right)_{T} + \left( \frac{\partial \sigma}{\partial \dot{\varepsilon}} \right)_{T} \frac{d\dot{\varepsilon}}{d\varepsilon} + \left( \frac{\partial \sigma}{\partial T} \right)_{\varepsilon} \frac{dT}{d\varepsilon} \quad (A-2)
\]

For uniform strain, \((d/d\varepsilon)\) term has an analytical form.

By the definition of true strain,

\[
d\varepsilon = dL/L \quad \text{and} \quad L = \nu_0 t + L_0 \quad (A-3)
\]

Where \(L\) is the current specimen length and \(L_0\) initial length.
length. Using assumption 1, we know

\[ \dot{\varepsilon} = \frac{V_0 \, dt}{V_0 \, t + L_0} \quad (A-4) \]

where \( V_0 \) is crosshead speed. Integrating Eq.(A-4), the true strain is obtained as

\[ \varepsilon = \ln(1 + V_0 \, t / L_0) \quad (A-5) \]

By differentiating Eq.(A-5) with respect to the time, we may get the strain rate

\[ \dot{\varepsilon} = \frac{V_0}{L_0} \exp(-1) \quad (A-6) \]

and we also may get the derivative

\[ \frac{d \dot{\varepsilon}}{d \varepsilon} = -\left(\frac{V_0}{L_0}\right) \exp(-\varepsilon) = -\dot{\varepsilon} \quad (A-7) \]

For different materials, Eq.(A-1) has different forms. Two kinds of particular constitutive equations of flow stress may be written as follows

\[ \sigma = k \varepsilon^n \dot{\varepsilon}^m (1 - \beta T) \quad (A-8) \]

for I. F. steel and

\[ \sigma = \sigma_0 (1 - A \exp(-B \varepsilon)) \dot{\varepsilon}^m (\theta / \theta_0)^{-\beta_0} \quad (A-9) \]

for S. S. 310

where

- \( \sigma \) = flow stress
- \( \varepsilon \) = true strain
- \( \dot{\varepsilon} \) = true strain rate
- \( n \) = strain hardening exponent
- \( m \) = strain rate sensitivity coefficient
- \( K, \beta, A, B, \) and \( \sigma_0 \) = constants
- \( T \) = temperature difference between current absolute
temperature $\Theta$ and the initial absolute temperature $T_0$

For I. F. steel, we obtain from Eq.(A-8)

$$\left( \frac{\partial \sigma}{\partial \varepsilon} \right) = n \sigma / \varepsilon$$ \hspace{1cm} (A-10)

$$\left( \frac{\partial \sigma}{\partial \dot{\varepsilon}} \right) = m \sigma / \dot{\varepsilon}$$ \hspace{1cm} (A-11)

$$\left( \frac{\partial \sigma}{\partial T} \right) = -\beta \sigma / (1 - \beta T)$$ \hspace{1cm} (A-12)

Substituting equations (A-10), (A-11), (A-12) and (A-7) into Eq.(A-2), one obtains

$$\frac{d\sigma}{d\varepsilon} = \frac{n \sigma}{\varepsilon} + \frac{m \sigma}{\dot{\varepsilon}} (-\dot{\varepsilon}) - \frac{\beta \sigma}{1 - \beta T} \frac{\varepsilon}{\partial T / \partial \varepsilon}$$ \hspace{1cm} (A-13)

The well-known Considere criterion may be written as

$$\left( \frac{d\sigma}{d\varepsilon} \right) = \sigma \text{ at } \varepsilon = \varepsilon_u$$ \hspace{1cm} (A-14)

By solving Eqs.(A-13) and (A-14) for uniform strain, we obtain a criterion of instability, considering thermal effect:

$$\varepsilon_u = n / \left\{ l + m + \frac{\beta}{1 - \beta T} \varepsilon / \partial T / \partial \varepsilon \right\}$$ \hspace{1cm} (A-15)

For SS 310, because the $\delta_0$ in Eq.(A-9) is small and temperature increase is not high, the term $(\Theta / \Theta_0)$ can be reduced by Taylor's theorem to

$$\left( \frac{\Theta}{\Theta_0} \right)^{-\beta_0} = 1 - \beta T$$ \hspace{1cm} (A-16)

Where $\beta = \delta_0 / \Theta_0$ and $T = \Theta - \Theta_0$. Therefore Eq.(A-9) may be rewritten as

$$\sigma = \sigma_0 (1 - A \exp(-B \varepsilon)) \dot{\varepsilon}^m (1 - \beta T)$$ \hspace{1cm} (A-17)

Thus

$$\left( \frac{\partial \sigma}{\partial \dot{\varepsilon}} \right) = \sigma A \beta \exp(-B \varepsilon) / (1 - A \exp(-B \varepsilon))$$ \hspace{1cm} (A-18)
\[
\left( \frac{\partial \sigma}{\partial \varepsilon} \right) = m\varepsilon \quad \text{(A-19)}
\]

\[
\left( \frac{\partial \sigma}{\partial T} \right) = -\beta \sigma / (1 - \beta T) \quad \text{(A-20)}
\]

Substituting the equations (A-18), (A-19), (A-20) and (A-7) into Eq. (A-2), we obtain

\[
\sigma \left\{ \frac{A\exp(-B\varepsilon)}{1-A\exp(-B\varepsilon)} \right\} = \frac{m - \beta}{1-\beta T} \left( \frac{\partial T}{\partial \varepsilon} \right) = \frac{d\sigma}{d\varepsilon} \quad \text{(A-21)}
\]

Substituting into Considere's criterion, Eq. (A-14), the following equation may be obtained

\[
\frac{A\exp(-B\varepsilon)}{1-A\exp(-B\varepsilon)} - m - \frac{\beta}{1-\beta T} \left( \frac{\partial T}{\partial \varepsilon} \right) = 1 \quad \text{(A-22)}
\]

Solving this equation for uniform strain, the maximum uniform strain is

\[
\varepsilon_u = (1/B) \ln A \left\{ 1 + \frac{B}{1+m+(\beta/1-\beta T)(\partial T/\partial \varepsilon)} \right\} \quad \text{(A-23)}
\]

The thermal effect terms of Eq. (A-23) are discussed in Section III.B, where various heat transfer conditions are considered.
B. Heat Transfer Equation

The heat transfer equation expresses the energy balance by considering heat generation, heat conduction and heat convection in a spatial element. The strain rate and stress in the element are assumed to remain uniform. Also the temperature, $\theta$, will only depend on the position $x$ and time $t$, and any geometric variation in the element is neglected.

The heat generation due to plastic work is

$$\Delta Q_1 = \eta \sigma \, d\varepsilon \, dV \quad \text{(B-1)}$$

Where $\eta$, $dV$ denote the fraction of the plastic work converted into heat and the volume of element, respectively and $d\varepsilon$ is the increment of plastic strain.

The heat change in the element due to the conduction can be described as

$$\Delta Q_2 = \left\{(q(x) - q(x+dx))\right\} A \, dt \quad \text{(B-2)}$$

where $q(x)$ is conductive flux at point $x$, and $A$ is the cross-sectional area of the element. Taylor's expansion for $q(x+dx)$ is

$$q(x+dx) = q(x) + (3 q / 3 x) dx + \text{higher orders} \quad \text{(B-3)}$$

Neglecting the higher orders, Eq.(B-3) reduces to

$$q(x+dx) = q(x) + \left( \frac{3 q}{3 x} \right) dx \quad \text{(B-4)}$$
Thus

\[ \Delta Q_2 = -\left( \frac{\partial q}{\partial x} \right) dx A \, dt \quad (B-5) \]

Fourier's law [45] states that the heat flux is proportional to \(- k(\partial \Theta / \partial x)\), where \(k\) is the thermal conductivity (assumed to be constant here). That is

\[ q = - k(\partial \Theta / \partial x) \quad (B-6) \]

Therefore the heat change due to the conduction is

\[ \Delta Q_2 = k(\partial^2 \Theta / \partial x^2) dv dt \quad (B-7) \]

Another contribution to the heat change in the element interested is that due to heat convection, which means that the heat flux \(q\) at the interface is proportional to the difference at the temperature of the surface and the temperature of the surroundings, \(\Theta_0\). Thus the heat change due the convection is equal to

\[ \Delta Q_3 = -2h(\Theta - \Theta_0)A_1 dt \quad (B-8) \]

Where \(A_1\) and \(h\) are the surface area of the element exposed to the surrounding medium, and heat convection coefficient, respectively.

The final term is heat storage, which causes the temperature to change during a time interval \(dt\),

\[ \Delta Q_4 = c \rho \partial \Theta / \partial t + V_x (\partial q / \partial x) dt \, dv \quad (B-9) \]

Where \(\rho\) and \(c\) are the density and specific heat of the material, respectively.
Because of the energy balance in the element, the heat transfer equation is obtained by summing all the energy terms except the energy lose due to radiation

$$\Delta Q_4 = \Delta Q_2 + \Delta Q_3 + \Delta Q_1 \quad (B-10)$$

That is

$$c \rho \frac{\partial \theta}{\partial t} dV dt = k \left( \frac{\partial^2 \theta}{\partial x^2} \right) dV dt - c \rho V \frac{\partial \theta}{\partial x} dV dt - 2h(\theta - \theta_0) A_1 dt + \eta \sigma \varepsilon dV \quad (B-11)$$

Dividing both sides by $c \, dV dt$, the equation becomes

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2} - V_x \frac{\partial \theta}{\partial x} - \frac{h}{cw \rho} (\theta - \theta_0) + \frac{\eta \sigma \varepsilon}{\rho c} \quad (B-12)$$

Where $\alpha = k/\rho c$, is the thermal diffusivity of the material, and $w$ is thickness of the specimen.

Initial and boundary conditions are required for solving any kind of the differential equation. In our problem, the initial condition could be considered as the uniform temperature ($\theta_0$: room temperature) along the specimen at time $t = 0$, and the boundary conditions are assumed that the temperature at the grips remains constant ($\theta_0$) and the temperature gradient is zero at the center. That is

Initial condition:

$$\theta = \theta_0 \quad \text{at } t = 0, \text{ for all } x \quad (B-13)$$

Boundary condition:

$$\theta = \theta_0 \quad \text{at } x = \pm L/2, \text{ for all time} \quad (B-15)$$

$$\frac{\partial \theta}{\partial x} = 0 \quad \text{at } x = 0, \text{ for all time} \quad (B-15)$$
C. Analytical Solution of Temperature Distribution

Heat transfer phenomena can be expressed by a partial differential equation mathematically. In order to understand the heat transfer phenomena directly during deformation, it is better to obtain an analytical solution instead of a numerical result. In general, the partial differential equation of heat transfer is difficult to solve if heat source is considered. In present work, a simple form of heat source from Eq.(39) is used.

The heat transfer equation (35), and initial and boundary conditions (36.a–c) can be written as

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} - \varepsilon_0 T + \frac{\dot{H}}{\rho c} \quad \text{(C-1)}$$

- $T(0,t) = 0$ for all $x$
- $T(x,t) = 0$ for all time, at $x = \pm L/2$
- $\frac{\partial T}{\partial x} = 0$ for all time, at $x = 0$ \quad \text{(C-2)}$

Where $T$ is temperature increase.

If the strain is known as a function of $x,t$ (as from experimental determination), empirical equation (39) for true strain can be written as

$$\varepsilon = C \exp(-Dx + Et) \quad \text{(C-3)}$$

where $C$, $D$, and $E$ are constants. The equation has a simple form which shows the relationship between strain and spatial
position, and time. Hollomon equation is used for flow
stress, as shown

\[ \sigma = k \varepsilon^n \]  \hspace{1cm} (C-4)

Where \( k \) and \( n \) are constants. Heat source comes from plastic
work, and heat source rate can be written as

\[ \dot{H} = \eta \sigma \varepsilon \]  \hspace{1cm} (C-5)

Thus by substituting Eqs. (C-3) and (C-4) into (C-5), the heat
generation term on the right side of Eq. (C-1) becomes

\[ \left( H/c \rho \right) = C_0 \exp(-D_0 x + E_0 t) \]  \hspace{1cm} (C-6)

Where

\[ C_0 = KC(n + 1)E/\rho c \]

\[ D_0 = (n + 1)D \quad \text{and} \]

\[ E_0 = (n + 1)E \]

where \( C_0, D_0 \) and \( E_0 \) are constants. So the Eq. (C-1) may be
rewritten as

\[ (\partial T/\partial t) = \alpha(\partial^2 T/\partial x^2) - h_0 T + C_0 \exp(-D_0 x + E_0 t) \]  \hspace{1cm} (C-7)

where \( h_0 = 2h/\rho c w \). The heat convection coefficient is
assumed a constant in the present work.

Introducing Laplace transformation, we have the following
relations:

1). \( L\{\partial T/\partial t\} = \int_0^\infty e^{-pt}(\partial T/\partial t)dt \]
\[ = -\frac{T_0}{\bar{T}} + \frac{p}{\bar{T}} \]  \hspace{1cm} (C-8)

where

\[ \bar{T} = \int_0^\infty e^{-pt} T dt \]

Referring to the boundary condition, Eq. (C-8) reduces to
\[ L\{\partial T/\partial t\} = p\bar{T} \]  
(C-9)

2). \[ L\{ \alpha \partial^2 T \over \partial x^2 \} = \int_0^\infty e^{-pt} \left( \alpha \partial^2 T \over \partial x^2 \right) dt \]
\[ = (\alpha \partial^2 T \over \partial x^2 ) \int_0^\infty e^{-pt} T \, dt \]
\[ = (\alpha \partial^2 T \over \partial x^2) \]  
(C-10)

3). \[ L\{ h_0 T \} = \int_0^\infty h_0 e^{-pt} T \, dt = h_0 \bar{T} \]  
(C-11)

4). \[ L\{ C_0 \exp(-D_0 x + E_0 t) \} = \int_0^\infty C_0 \exp(-D_0 x + E_0 t) e^{-pt} dt \]
\[ = C_0 \exp(-D_0 x) \int_0^\infty e^{-(p - E_0)t} dt \]
\[ = C_0 \exp(-D_0 x)/(p - E_0) \]  
(C-12)

Summing equations (C-9), (C-10), (C-11) and (C-12), the equation (C-7) and the conditions of Eq.(C-2) reduce to

\[ \alpha \left( \partial^2 \bar{T} / \partial x^2 \right) - (p+h_0)\bar{T} + C_0 \exp(-D_0 x)/(p - E_0) = 0 \]  
(C-13)

and

\[ T = 0 \quad x = \infty, \text{for} \; t > 0 \]  
(C-14)
\[ d\bar{T}/dx = 0 \quad x = 0, \; \text{for} \; t > 0 \]

The solution for Eq.(C-13), considering the conditions of Eq.(C-14), are

\[ \bar{T} = 2q Me^{-D_0 X}/(q^2 - D_0^2) - 2MD_0 e^{-qX}/(q^2 - D_0^2) \]  
(C-15)

Where

\[ M = C_0/(2 \alpha q(p - E_0)) \]
and
\[ q = ((h_0 + p)/\omega)^{1/2} \]

We refer the first term on the right side of Eq. (C-15) as
\[ \bar{T}_1 = C_0 \exp(-D_0 x)/[(p - E_0)(p + h_0 - \alpha D_0^2)] \quad (C-16) \]

and the second as
\[ \bar{T}_2 = C_0 D_0 e^{-qx}/(\alpha q(p - E_0)(q^2 - \alpha D_0^2)) \quad (C-17) \]

It is noticed that if \( T_1 \) tends to zero as \( p \) tends to infinit, the inverse Laplace transformation of \( \bar{T}_1 \) may be obtained by the method of residuals. [46] Fortunately, \( \bar{T}_1 \) satisfies this condition, and there are two one-order poles:

\[ p_1 = E_0 \quad \text{and} \quad p_2 = \alpha D_0^2 - h_0 \]

Thus the inverse Laplace transformation of \( T_1 \) can be calculated by the residual method as shown
\[
L^{-1}(\bar{T}_1) = \text{Res}\{C_0 e^{-D_0 x}/(p - E_0)(p + h_0 - \alpha D_0^2), E_0\} \\
+ \text{Res}\{C_0 e^{-D_0 x}/(p - E_0)(p + h_0 - \alpha D_0^2), \alpha D_0^2 - h_0\} \\
= C_0 \exp(-D_0 x + E_0 t)/(E_0 + h_0 - \alpha D_0^2) + \\
- C_0 \exp(-D_0 x + (\alpha D_0^2 - h_0) t)/(E_0 + h_0 - \alpha D_0^2) \\
= \frac{C_0}{E_0 + h_0 - \alpha D_0^2} \{\exp(E_0 t) - \exp[(\alpha D_0^2 - h_0) t]\} \exp(-D_0 x) \\
\text{...............} \quad (C-18) \\
\]

The inverse Laplace transformation of \( \bar{T}_2 \) is very complicated. For simplicity, we use the standard transform
tables [45] to calculate $L^{-1}(\bar{T}_2)$. $\bar{T}_2$ may be divided into two parts:

$$\bar{T}_2 = \bar{T}_{21} - \bar{T}_{22}$$  \hspace{1cm} (C-19)

where

$$\bar{T}_{21} = \frac{C_0 D_0}{E_0 + h_0 - \alpha D_0^2} \frac{\exp(-q x)}{q(p - E_0)}$$  \hspace{1cm} (C-20)

$$\bar{T}_{22} = \frac{C_0 D_0}{E_0 + h_0 - \alpha D_0^2} \frac{\exp(-q x)}{q(p + h_0 - \alpha D_0^2)}$$  \hspace{1cm} (C-21)

From the tables, [45] we find

if $f(p) = e^{-q x/[q(p - a)]}$, then

$$F(t) = (1/2)e^{at} \left( \frac{q}{\alpha} \right)^{1/2} \exp[-x(\alpha/\alpha)^{1/2}]\text{erfc}[x/2(\alpha t)^{1/2} - (\alpha t)^{1/2}] - \exp[x(\alpha/\alpha)^{1/2}]\text{erfc}[x/2(\alpha t)^{1/2} + (\alpha t)^{1/2}]$$

where $\alpha$ is thermal diffusivity, and a is a constant. Thus

$$\bar{T}_{21} = \frac{1}{2} N(\alpha/E_0)^{1/2}\exp(E_0 t)\left\{\exp[-x(E_0/\alpha)^{1/2}]\text{erfc}(S_1) - \exp[x(E_0/\alpha)^{1/2}]\text{erfc}(S_2)\right\}$$ \hspace{1cm} (C-22)

$$\bar{T}_{22} = \frac{1}{2} N \left( \frac{\alpha}{\alpha D_0^2 - h_0} \right)^{1/2} \exp[(\alpha D_0^2 - h_0)t] \left\{ \exp[-x(D_0^2 - h_0/\alpha)^{1/2}] \text{erfc}(S_3) - \exp[x(D_0^2 - h_0/\alpha)^{1/2}] \text{erfc}(S_4) \right\}$$ \hspace{1cm} (C-23)

where

$$N = \frac{C_0 D_0}{(E_0 - h_0 - \alpha D_0^2)}$$

$$S_1 = x/2(\alpha t)^{1/2} - (E_0 t)^{1/2}$$

$$S_2 = x/2(\alpha t)^{1/2} + (E_0 t)^{1/2}$$

$$S_3 = x/2(\alpha t)^{1/2} - [(\alpha D_0^2 - h_0)t]^{1/2}$$

$$S_4 = x/2(\alpha t)^{1/2} + [(\alpha D_0^2 - h_0)t]^{1/2}$$
Therefore, a complete solution of Eq.(C-1) is

$$T = T_1 - (T_{21} - T_{22})$$  \hspace{1cm} \text{(C-24)}

The solution has closed form although it is very complicated. This solution is valid for any material and for any heat convection condition if strain distribution equation (39) is satisfied. Thus the solution can be used to predict the temperature distribution for other materials when we know the constants in Eq.(39).
D. Finite Difference Formulation For Heat Transfer Equation

Using the conventional method of finite differences, the derivatives in Eq.(35) can be written by the finite differences

\[ \frac{\partial T_{i,j}}{\partial t} = \frac{T_{i,j+1} - T_{i,j}}{\Delta t} \quad (D-1) \]

and

\[ \frac{\partial^2 T_{i,j}}{\partial x^2} = \frac{1}{2} \left[ \frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{\Delta x^2} \right. \]

\[ \left. + \frac{T_{i-1,j+1} - 2T_{i,j+1} + T_{i+1,j+1}}{\Delta x^2} \right] \quad (D-2) \]

Where \( T_{i,j} \) is the temperature rise at the node \((i,j)\), shown in Fig.(1-a), in which \( i \) represents the position interval number and \( j \) represents the time interval number. By replacing the derivatives in Eq.(35) by Eqs.(D-1) and (D-2), we get

\[ -RT_{i-1,j+1/2} + (1+R)T_{i,j+1} - RT_{i+1,j+1} = RT_{i-1,j} \]

\[ + (1-R-F)T_{i,j} + RT_{i+1,j} + H_{i,j} \quad (D-3) \]

where

\[ R = \alpha \frac{\Delta t}{\Delta x^2} \]

\[ F = (2h/\rho c)\Delta t \]

\[ H_{i,j} = (\eta/\rho c)\sigma_{i,j}(\epsilon_{i,j+1} - \epsilon_{i,j}) \]

The initial and boundary conditions are replaced by
\[ T_{i,0} = 0 \quad \text{(D-4-A)} \]
\[ T_{n,j+1} = 0 \quad \text{(D-4-B)} \]
\[ \frac{T_{1,j+1} - T_{-1,j+1}}{2^\Delta x} = 0 \quad \text{(D-4-C)} \]

Substituting Eq. (D-4-C) into Eq. (D-3), we have
\[ (1+R)T_{0,j+1} - RT_{1,j+1} = (1-R-F)T_{0,j} + RT_{1,j} + H_{0,j} \quad \text{............... (D-5)} \]

Therefore, it is possible to solve Eq. (D-3) by considering the initial and boundary conditions numerically. Rewriting Eq. (D-3) by considering B.C. in matrix form, we have

\[
\begin{pmatrix}
(1+R) & -R & 0 & \ldots & 0 & 0 & 0 \\
-R/2 & (1+R) & -R/2 & 0 & \ldots & 0 & 0 \\
0 & -R/2 & (1+R) & -R/2 & 0 & \ldots & 0 \\
0 & \ldots & 0 & -R/2 & (1+R) & -R/2 \\
0 & \ldots & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
T_{0,j+1} \\
T_{1,j+1} \\
T_{2,j+1} \\
\vdots \\
T_{n-1,j+1} \\
T_{n,j+1}
\end{pmatrix}
= 
\begin{pmatrix}
(1-R-F) & R & 0 & \ldots & 0 & 0 \\
R/2 & (1-R-F) & R/2 & 0 & \ldots & 0 \\
0 & \ldots & 0 & R/2 & (1-R-F) & R/2 \\
0 & \ldots & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
T_{0,j} \\
T_{1,j} \\
\vdots \\
T_{n-1,j} \\
T_{n,j}
\end{pmatrix}
= 
\begin{pmatrix}
H_{0,j} \\
H_{1,j} \\
\vdots \\
H_{n-1,j} \\
H_{n,j}
\end{pmatrix}
\]
E. A Program

C NUMERICAL METHOD TO CALCULATE TEMP. DISTRIBUTION
DIMENSION X(70), T(200), A(90,90), B(90,90), Q(90), D(90), TT(150,20),
* A2(90), S(100), SS(100)

C
C*******************************************************************************
C
C * I. F. STEEL IS TAKEN AS AN EXAMPLE IN THE PROGRAM *
C *
C*******************************************************************************
C
C******************************************************************************* LIST OF SOME SYMBOLS ******************************************************************************
C
C H IS CONVECTION COEFF., B0 STRAIN RATE, DT TIME STEP, C1 HEAT
C SPECIFIC, P DENSITY, E STRAIN HARDENING EXPONENT, AL THERMAL
C DIFFUSIVITY, K TRENCH STRENGTH COEFF., W WIDTH, FR FRACTION OF WORK
C CONVERT TO HEAT, A1, B1, C1, D1 CONSTANTS IN STRAIN EQUATION,
C BATA TEMPERATURE COEFF.

DO KK=1,4
READ(5,*)H
B0=1.E-2
DT=.2
C1=.111
P=7.8
E=.22
AL=.147
K=566
T0=0
W=.08
FR=.86
A1=.226
B1=3.43
C2=20.98
D1=1.816
BATA=0.0011
AA=K*FR*(B0/.002)**.01B/(P*C1*4.1B)
AC=AA*A1**(1+E)
BC=B1*E
HH=DT*H/(W*C1*F)

C
C POSITION STEP
DX2=1.
R2=AL*Dt/(DX2*2)

97
SET THE VALUE OF X COORDINATE
DO I=1,46
X(I)=I-1.
END DO

INITIAL TEMPERATURE
DO I=1,46
T(I)=T0
END DO

SET THE VALUES FOR A AND B MATRIX
A(I,1)=1+R2/2.
A2(I)=A(I,1)
A(I,2)=-R2/2.

B(I,1)=1,-R2/2,-HH
B(I,2)=R2/2.
DO I=2,45
A(I,I-1)=-R2/2
A(I,I)=1+R2
A2(I)=A(I,I)
A(I,I+1)=-R2/2
B(I,1-I)=R2/2
B(I,I)=1-R2-HH
B(I,I+1)=R2/2
END DO
A(46,46)=1
A2(46)=1
B(46,46)=1

DO I=2,46
A(I,I)=A(I,1)-A(I-1,I)*A(I,I-1)/A(I-1,I-1)
END DO

CALCULATE TEMP.

L=0
DO I=1,46
Q(I)=0
END DO
DO J=1,140
K=10*(L+1)
DO I=1,46
B(I)=0.
END DO

CALCULATE RIGHT HAND SIDE OF EQUATIONS (D=-6)
110 TIME STEPS BELOW SEPARATES UNIFORM DEFORMATION AND
POST-UNIFORM DEFORMATION

IF (J.GT.110) THEN

UNIFORM DEFORMATION

DO I=1,46
  F=AC*(EXP(BC*(J-111)*DT/(C2+X(I)**D1)))*E*(1-BATA*T(I))
  G=EXP(B1*(J-110)*DT/(C2+X(I)**D1))
  GG=EXP(B1*(J-111)*DT/(C2+X(I)**D1))
  Q(I)=F*(G-GG)
  DO N=1,46
    D(I)=D(I)+B(I,N)*T(N)
  END DO
  D(I)=D(I)+Q(I)
  END DO
  GOTO 200
ELSE

POST-UNIFORM DEFORMATION

DO I=1,46
  F=AA*(LOG(1+BO*(J-1)*DT))**E*(1-BATA*T(I))
  FF=LOG(1+BO*J*DT)-LOG(1+BO*(J-1)*DT))
  Q(I)=F**FF
  DO N=1,46
    D(I)=D(I)+B(I,N)*T(N)
  END DO
  D(I)=D(I)+Q(I)
  END DO
ENDIF

USE GASSIAN ELIMINATION TO SOLVE THE TRIDIAGONAL MATRIX EQUATION

200 DO I=2,46
  D(I)=D(I)-A(I,I-1)*D(I-1)/A2(I-1)
END DO

CALCULATE T(J+1)
T(46)=0.0
DO I=2,46
  N=47-I
  T(N)=D(N)/A(N,N)-A(N,N+1)*T(N+1)/A(N,N)
END DO
IF (J.NE.K) GOTO 10
L=L+1
T1=J*DT
PRINT OUT THE RESULTS

WRITE(4,20)TI
FORMAT(1X,'TIME = ',5X,F10.5)
WRITE(4,30)
FORMAT(3X,'POSITION',10X,'TEMPERATURE')
TT(46,L)=T(I)
S(46)=0.
DO LK=1,45
S(46-LK)=-LK
S(46+LK)=LK
TT(LK+46,L)=T(LK+1)
TT(46-LK,L)=T(LK+1)
WRITE(4,**X(LK),T(LK)
END DO
END DO

-----------------------------

PLOT TEMPERATURE VS POSITION USING QWIKPLT

CALL QP_START
CALL QP_POS(-0.75,-0.6,1.5,1.2)
CALL QP_AX(-45.,45.,9,0.,1,0,5,1)
CALL QP_TTL('TEMP VS POSITION','X (MM)','TEMP')
DO I=1,4
DO L=1,91
SS(L)=TT(L,I)
END DO
CALL QP_LSTYL(I)
CALL QP_(5,SS,91,2,0)
END DO
CALL QP_LG(0.,15.,0.,'STRAIN RATE: 2.E-3')
CALL QP_LG(0.,11.5.,0.,'ENG STRAIN 24%')
CALL QP_LG(0.,3.,0.,'END EFFECT')
CALL QP_LG(0.,2.,0.,'IN AIR')
CALL QP_LG(0.,1.,0.,'IN FORCED AIR')
CALL QP_LG(0.,0.,0.,'IN WATER')
CALL QP_LG(0.,-.1.,0.,'IN FORCED WATER')
CALL QP_END
END