Control of Arc Weld Thermal Cycles

A Dissertation
Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of the Ohio State University

by

Dave F. Farson

* * * * *

The Ohio State University
1987

Dissertation Committee:
Prof. R.J. Mayhan (Advisor)
Prof. K.A.K. Ossman
Prof R.W. Richardson
Prof. S. Yurkovich

Approved by:

[Signature]
Adviser
Department of Electrical Engineering
Acknowledgements

First and foremost, I would like to thank Professor Richard Richardson for his years of advice, guidance and support. Likewise, Professor Robert Mayhan has been very helpful in guiding my Ph.D. research with insightful criticism and suggestions. Others on the reading committee, Professors Steve Yurkovich and Kathy Ossman patiently and skillfully read and amended the manuscript and provided enlightening comments during the latter phases of my research. Thanks are also due to Choong Yoo for his thorough reading of this manuscript, to Phil Nease for his expert laboratory assistance and to other Welding Process Control Lab colleagues for help in many ways. Finally, I would like to thank Mary Kay for her forebearance and support during the years spent in my graduate study.
Vita

September 10, 1958 .................. Born — Marietta, Ohio

1980 .................................. B.S., Department of Welding Engineering, The Ohio State University, Columbus, Ohio

1980 – 1982 ............................ Graduate Research Associate, Graduate Teaching Associate, Graduate Research Fellow, Department of Welding Engineering, The Ohio State University.

1982 .................................. M.S., Department of Welding Engineering, The Ohio State University, Columbus, Ohio

1982 – 1987 ............................ Graduate Research Associate, Doctoral Candidate, Department of Electrical Engineering, The Ohio State University

Publications and Presentations


iii


Fields of Study

Major Field: Control Systems
Minor Fields: Digital Systems
Mathematics
Welding Engineering
# TABLE OF CONTENTS

Acknowledgements iii

Vita iv

LIST OF FIGURES ix

LIST OF TABLES xiv

I. Introduction 1

II. Background and Past Work 6
   2.1 Arc Welding Processes .......................... 6
   2.2 Welding Thermal Cycles .......................... 11
   2.3 Analysis of Welding Heat Flow .................. 15
   2.4 Extensions of The Classical Solutions ........... 24
   2.5 Numerical Simulation Techniques ................ 26
   2.6 The Effect of Thermal Cycles Upon Weld Properties .. 30
   2.7 Control of the Welding Thermal Cycle ............ 32
      2.7.1 Dynamic Response of Temperature Distributions and Weld
            Pool Dimensions .................................. 33
      2.7.2 Welding Temperature Distribution Controls .......... 37
   2.8 Distributed Parameter State Estimation ............ 43
   2.9 Interpretation of Background Information .......... 48
## III. Approaches To The Control of Thermal Cycles

### 3.1 Selection of Thermal Cycle and Weld Pool Dimension Measures

### 3.2 Approaches to the control of cooling rate

## IV. Formulation of Dynamic Welding Heat Flow Observers

### 4.1 Formulation of a Conventional Welding Heat Flow Observer

### 4.2 Formulation of an Adaptive Observer

### 4.3 Formulation of a Welding Heat Flow Simulation

### 4.4 Numerical Solution of the Observer Equations

## V. Experimental Testing of the Heat Flow Simulation

### 5.1 Experimental procedure

### 5.2 Experimental Results

### 5.3 Comparison With Simulation Results — Quasi-Steady State

### 5.4 Experimental Results and Comparisons — Transient

### 5.5 Summary

## VI. Formulation of Static Observers

### 6.1 An example from Ideal Heat Flow Relationships

### 6.2 Empirical Weld Pool Dimension/Cooling Rate Relationships

### 6.3 Discussion of The Empirical Relations

### 6.4 Summary

## VII. Control Formulation and Simulation

### 7.1 Control Formulation and Testing — Dynamic Observers
7.1.1 System Dynamic Response Characteristics and Proposed Control Structure .................. 133
7.1.2 System Modeling and Compensator Design .................. 145
7.1.3 Determination of Observer Feedback Gains and Control Response Simulation Testing .................. 151
7.1.4 Dynamic Observer Robustness .................. 159
7.1.5 Summary — Dynamic Observer Control Design and Testing 167
7.2 Control Design and Testing — Static Observers .................. 168
7.2.1 Dynamic Response Characteristics and Proposed Control Structure .................. 170
7.2.2 System Modeling and Compensator Design .................. 175
7.2.3 Simulation Testing of Control Algorithms .................. 181
7.2.4 Summary — Static Observer Control Design and Testing 191
7.3 Multivariable Control of Cooling Rate and Pool Width ........ 192

VIII. Summary, Conclusions and Topics For Further Investigation 200
8.1 Summary .............. 200
8.2 Conclusions .............. 203
8.3 Topics for Further Investigation .............. 205

A. Derivation of Green’s Function with Gaussian Heat Input Distribution 209

B. Heat Simulation and Observer Programs 212

C. ARMA Model-fitting Program 257

References 268
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Arc Welding Fundamentals</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>GTAW Process Fundamentals</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>Weld Metallurgical Structure</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>Welding Thermal Cycle at a Point</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>Welding Thermal Cycle Measures</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>Moving and Fixed Coordinate Systems</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>Grid Superimposed on the (w,y) Domain</td>
<td>28</td>
</tr>
<tr>
<td>9</td>
<td>Sensitivity Index Distribution [1]</td>
<td>34</td>
</tr>
<tr>
<td>11</td>
<td>Time Constant vs. Final Travel Speed [2]</td>
<td>37</td>
</tr>
<tr>
<td>12</td>
<td>Time Constant vs. Initial Travel Speed [2]</td>
<td>38</td>
</tr>
<tr>
<td>13</td>
<td>Predicted and Actual Temperatures for ((n,m) = (2,2)) [3]</td>
<td>40</td>
</tr>
<tr>
<td>14</td>
<td>Proposed Adaptive Control Scheme [3]</td>
<td>41</td>
</tr>
<tr>
<td>15</td>
<td>Adaptive Observer Block Diagram</td>
<td>48</td>
</tr>
<tr>
<td>16</td>
<td>Feedback Cooling Rate Control Approach</td>
<td>55</td>
</tr>
<tr>
<td>17</td>
<td>Welding Heat Flow Observer</td>
<td>56</td>
</tr>
<tr>
<td>18</td>
<td>Location of the array of points used to simulate weld pool width</td>
<td>79</td>
</tr>
<tr>
<td>19</td>
<td>Welding Fixture</td>
<td>86</td>
</tr>
<tr>
<td>Page</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>20</td>
<td>Thermocouple–Chart Recorder Instrumentation Schematic</td>
<td>88</td>
</tr>
<tr>
<td>21</td>
<td>Typical Thermal Cycle</td>
<td>89</td>
</tr>
<tr>
<td>22</td>
<td>Microcomputer thermal-cycle data-logging system</td>
<td>90</td>
</tr>
<tr>
<td>23</td>
<td>Weld Bead Step Responses</td>
<td>96</td>
</tr>
<tr>
<td>24</td>
<td>Comparison of Experimental and Simulation Results</td>
<td>103</td>
</tr>
<tr>
<td>25</td>
<td>Variation of $h_s$ with Arc Current</td>
<td>109</td>
</tr>
<tr>
<td>26</td>
<td>Variation of $h_s$ with Travel Speed</td>
<td>110</td>
</tr>
<tr>
<td>27</td>
<td>Comparison of Experimental and Simulation Results</td>
<td>112</td>
</tr>
<tr>
<td>28</td>
<td>Comparison of Experimental and Simulated Pool Width Step Responses</td>
<td>115</td>
</tr>
<tr>
<td>29</td>
<td>Comparison of Empirical Relationship to Data</td>
<td>122</td>
</tr>
<tr>
<td>30</td>
<td>Cooling Rate vs. Arc Current, Travel Speed and Preheat Temperature</td>
<td>124</td>
</tr>
<tr>
<td>31</td>
<td>Comparison of Equation (6.15) to Experimental Data</td>
<td>125</td>
</tr>
<tr>
<td>32</td>
<td>Comparison of Equation (6.16) to Experimental Data</td>
<td>126</td>
</tr>
<tr>
<td>33</td>
<td>Comparison of Equation (6.17) to Experimental Data</td>
<td>126</td>
</tr>
<tr>
<td>34</td>
<td>Cooling Rate Control Structure</td>
<td>130</td>
</tr>
<tr>
<td>35</td>
<td>Cooling Rate Response to Travel Speed Perturbations</td>
<td>134</td>
</tr>
<tr>
<td>36</td>
<td>Comparison of ARMA Model Response with Actual Response: $V_0 = 4$ in./min.</td>
<td>137</td>
</tr>
<tr>
<td>37</td>
<td>Comparison of ARMA Model Response with Actual Data, $V_0 = 5$ in./min.</td>
<td>138</td>
</tr>
<tr>
<td>38</td>
<td>Rudimentary Feedback Cooling Rate Control</td>
<td>139</td>
</tr>
<tr>
<td>39</td>
<td>Root Locus of The Rudimentary Control System</td>
<td>140</td>
</tr>
<tr>
<td>40</td>
<td>Comparison of a Small Signal Model Output to Data: $V_0 = 5$in./min.</td>
<td>142</td>
</tr>
<tr>
<td>41</td>
<td>Cooling Rate Control With Feedforward Term</td>
<td>144</td>
</tr>
<tr>
<td>Page</td>
<td>Section Title</td>
<td>Page</td>
</tr>
<tr>
<td>------</td>
<td>-------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>42</td>
<td>Comparison of System and Model Responses for Operating Point 5</td>
<td>148</td>
</tr>
<tr>
<td>43</td>
<td>Rudimentary Control System Root Locus For Operating Point 5</td>
<td>149</td>
</tr>
<tr>
<td>44</td>
<td>Sample Root Locus of Compensated System</td>
<td>150</td>
</tr>
<tr>
<td>45</td>
<td>Conventional Observer Error Convergence, $V = 4$ in./min.</td>
<td>152</td>
</tr>
<tr>
<td>46</td>
<td>Conventional Observer Error Convergence, $V = 5$ in./min.</td>
<td>153</td>
</tr>
<tr>
<td>47</td>
<td>Adaptive Observer Error Convergence, $V = 3$ in./min.</td>
<td>154</td>
</tr>
<tr>
<td>48</td>
<td>Adaptive Observer Error Convergence, $V = 4$ in./min.</td>
<td>155</td>
</tr>
<tr>
<td>50</td>
<td>Closed-Loop System Response, Op. Pt. 2</td>
<td>157</td>
</tr>
<tr>
<td>51</td>
<td>Closed-Loop System Response, Op. Pt. 3</td>
<td>158</td>
</tr>
<tr>
<td>52</td>
<td>Closed-Loop System Response, Op. Pt. 4</td>
<td>158</td>
</tr>
<tr>
<td>53</td>
<td>Diffusivity Robustness</td>
<td>161</td>
</tr>
<tr>
<td>54</td>
<td>Thickness Robustness</td>
<td>162</td>
</tr>
<tr>
<td>55</td>
<td>Efficiency Robustness</td>
<td>162</td>
</tr>
<tr>
<td>56</td>
<td>Surface Heat Transfer Robustness</td>
<td>163</td>
</tr>
<tr>
<td>57</td>
<td>Preheat Robustness</td>
<td>163</td>
</tr>
<tr>
<td>58</td>
<td>“Open-Loop” Observer Robustness Test</td>
<td>165</td>
</tr>
<tr>
<td>59</td>
<td>Control Simulation Used To Evaluate Effect of Measurement Error</td>
<td>166</td>
</tr>
<tr>
<td>60</td>
<td>Adaptive Observer-Based System Step Responses</td>
<td>171</td>
</tr>
<tr>
<td>61</td>
<td>Modified Adaptive Observer-Based System Step Responses</td>
<td>171</td>
</tr>
<tr>
<td>62</td>
<td>Comparison: Adaptive Observer-Based System Model and Data</td>
<td>173</td>
</tr>
<tr>
<td>63</td>
<td>Comparison: Modified Adaptive Observer-Based System Model and Data</td>
<td>174</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>64</td>
<td>Comparison: Modified Adaptive Observer-Based System Model and Data</td>
<td>175</td>
</tr>
<tr>
<td>65</td>
<td>Proposed Structure for Static Observer-Based Control</td>
<td>176</td>
</tr>
<tr>
<td>Page</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>66</td>
<td>Adaptive Observer-Based Control Root Locus</td>
<td>179</td>
</tr>
<tr>
<td>67</td>
<td>Modified Adaptive Observer-Based Control Root Locus</td>
<td>180</td>
</tr>
<tr>
<td>68</td>
<td>Adaptive Observer-Based System Response</td>
<td>182</td>
</tr>
<tr>
<td>69</td>
<td>Adaptive Observer-Based System Response</td>
<td>182</td>
</tr>
<tr>
<td>70</td>
<td>Adaptive Observer-Based System Response</td>
<td>183</td>
</tr>
<tr>
<td>71</td>
<td>Adaptive Observer-Based System Response</td>
<td>183</td>
</tr>
<tr>
<td>72</td>
<td>Modified Adaptive Observer-Based System Response</td>
<td>185</td>
</tr>
<tr>
<td>73</td>
<td>Modified Adaptive Observer-Based System Response</td>
<td>185</td>
</tr>
<tr>
<td>74</td>
<td>Modified Adaptive Observer-Based System Response</td>
<td>186</td>
</tr>
<tr>
<td>75</td>
<td>Modified Adaptive Observer-Based System Response</td>
<td>186</td>
</tr>
<tr>
<td>76</td>
<td>Diffusivity Robustness</td>
<td>187</td>
</tr>
<tr>
<td>77</td>
<td>Thickness Robustness</td>
<td>188</td>
</tr>
<tr>
<td>78</td>
<td>Arc Efficiency Robustness</td>
<td>188</td>
</tr>
<tr>
<td>79</td>
<td>Surface Heat Trans. Robustness</td>
<td>189</td>
</tr>
<tr>
<td>80</td>
<td>Preheat Temperature Robustness</td>
<td>189</td>
</tr>
<tr>
<td>81</td>
<td>Multivariable Control Block Diagram</td>
<td>193</td>
</tr>
<tr>
<td>82</td>
<td>Multivariable Dynamic Adaptive Observer-Based Control Cooling Rate Responses</td>
<td>195</td>
</tr>
<tr>
<td>83</td>
<td>Multivariable Dynamic Adaptive Observer-Based Control Pool Width Responses</td>
<td>195</td>
</tr>
<tr>
<td>84</td>
<td>Multivariable Static Adaptive Observer-Based Control Cooling Rate Response</td>
<td>196</td>
</tr>
<tr>
<td>85</td>
<td>Multivariable Static Adaptive Observer-Based Control Pool Width Response</td>
<td>197</td>
</tr>
<tr>
<td>86</td>
<td>Multivariable Static Modified Adaptive Observer-Based Control Cooling Rate Response</td>
<td>198</td>
</tr>
</tbody>
</table>
LIST OF TABLES

1  Welding Parameters for 4130 Steel .............................................. 85
2  Welding Parameters for 2219 Aluminum Welds ............................... 86
3  Welding Conditions for Step Response Tests ................................ 92
4  AISI 4130 Steel Quasi-steady State Test Results .......................... 94
4  AISI 4130 Steel Quasi-steady State Test Results—cont. ................. 95
5  2219 Aluminum Quasi-Steady State Test Results .......................... 97
6  AISI 4130 Steel HAZ Hardness Results ......................................... 98
7  Simulation Parameter Values for AISI 4130 Steel ......................... 100
8  Simulation Parameter Values for 2219 Aluminum .......................... 100
9  Comparison: Experimental vs. Simulated Outputs for Steel .......... 101
10 Comparison: Experimental vs. Simulated Outputs for Aluminum ...... 102
11 Comparison of Experimental and Simulated Cooling Rates With ...
    Surface Heat Transfer Preheat Decay Correction ..................... 105
12 Comparison of Pool Width Step Responses ................................. 114
13 ARMA Parameters, \( V_0 = 4 \text{ in./min} \) ..................................... 136
14 ARMA Parameters, \( V_0 = 5 \text{ in./min} \) ..................................... 137
15 Nominal Operating Points .................................................. 146
16 Inequalities for Sorting of Inputs ........................................... 146
17 ARMA Parameters at Nominal Operating Points ......................... 147
18 Compensator Parameters .................................................. 151
<table>
<thead>
<tr>
<th></th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>Operating Points for Control Tests</td>
<td>156</td>
</tr>
<tr>
<td>20</td>
<td>Observer and Simulation Parameter Variations</td>
<td>161</td>
</tr>
<tr>
<td>21</td>
<td>Closed-Loop Control Measurement Error Robustness</td>
<td>167</td>
</tr>
<tr>
<td>22</td>
<td>ARMA Parameters, $V_0 = 4$ in./min.</td>
<td>173</td>
</tr>
<tr>
<td>23</td>
<td>ARMA Parameters: Adaptive Observer-Based System</td>
<td>177</td>
</tr>
<tr>
<td>24</td>
<td>ARMA Parameters: Modified Adaptive Observer-Based System</td>
<td>178</td>
</tr>
<tr>
<td>25</td>
<td>Static Adaptive Observer-Based Control-Test Operating Points</td>
<td>181</td>
</tr>
<tr>
<td>26</td>
<td>Static Modified Adaptive Observer-Based Control Test Operating Points</td>
<td>184</td>
</tr>
<tr>
<td>27</td>
<td>Measurement Error Robustness</td>
<td>190</td>
</tr>
</tbody>
</table>
CHAPTER I

Introduction

Arc welding, traditionally a manufacturing process which is manual-labor intensive, is being automated at an increasing pace. One factor which is promoting this trend is the availability of affordable, efficient industrial robots. Modern computer controlled robots possess sufficient flexibility of motion and ease of programming to perform arc welding on relatively complex parts that formerly could only be welded manually. Use of such robots in arc welding applications promises to increase both productivity and quality. However, the current generation of welding robots still possess limitations which make their application difficult in some situations. The arc welding process is relatively complex, having a number of variables which are subject to change, adversely affecting the properties of the finished weld. Unfortunately, most modern welding robots are incapable of detecting the effects of such variables upon the process. Furthermore, programming of welding robots is often a highly time-consuming task requiring skilled personnel and iterative trial-and-error techniques.

It has been generally recognized that equipping welding robots with the ability to monitor the welding process in “real time” (i.e. as welding progresses) and to perform feedback control of process inputs would increase weld uniformity. For example, one feedback control function which has been widely developed is seam tracking, wherein the position of the weld seam is measured by a sensing system

1
and the robot path is modified so that it follows the seam. This control technique is useful in situations where a number of nearly identical parts are to be welded, and minor adjustments to the nominal robot path are needed to make it conform to the inconsistencies of each individual part. There are other arc welding variables besides robot path which one might wish to control. For example, the size of the weld is apt to vary due to local variations in heat sinking caused by weldment or fixturing geometry. In situations where such heat sinking variations exist, a feedback control over weld pool or weld bead size would be helpful in maintaining weld uniformity.

Another impetus for the application of feedback control to the welding process will come from the drive to implement industrial CAD/CAM systems. In such systems, the arc welding robot program is generated by the CAD system at the same time that a part is designed. This robot program includes both the path program and the values of the arc welding process inputs (e.g. arc current and travel speed) that are to be used to weld the part. Programs generated in this manner will likely be somewhat nominal, requiring adjustment for the production of proper welds. For example, the CAD-generated welding program may contain parameters which produce a weld bead which is too large. Feedback control of the arc welding process promises to provide the welding robot with the ability to make adjustments so as to produce a more optimal weld.

There has been relatively little practical application of welding feedback controls which perform functions other than seam tracking. There are several reasons for this. One major obstacle has been the difficulty in developing practical techniques for measuring arc welding process outputs. The environment near the welding arc is quite harsh; energetic infrared, visible and ultraviolet radiation, molten metal spatter and fumes all limit the lifetime of delicate electronic sensors.
Constraints upon the physical size and construction of sensors are also restrictive. Sensors located on the robot end effector must be relatively compact and rugged so as not to restrict access to complex part geometries and to withstand accidental collisions with the workpiece.

An arc welding sensing method developed at The Ohio State University, known as “coaxial viewing” promises to overcome many of the difficulties outlined above. A special welding torch permits acquisition of an image of the arc welding process from a convenient, sheltered vantage point located inside the welding torch. When coupled with a machine vision system, this sensing method allows measurement of weld pool dimensions and the position of the weld pool with respect to the weld seam. Coaxial viewing has been used in seam tracking applications and to a limited extent in feedback control of weld pool width. However, the practical advantages of the sensing method have led to interest in determining how the sensor can be used in the implementation of controls which provide further regulation of the welding process.

One type of welding feedback control which is of practical importance and has been the subject of past investigation is thermal cycle control. As will be discussed in more detail in the next chapter, the thermal cycles (or thermal histories) induced in the part during welding often play an important role in determining the mechanical properties (e.g. strength and toughness) of the finished weld. Hence, a feedback control which regulates some measure of the thermal cycle would tend to regulate weld mechanical properties. Such a capability would be desirable in applications where weld mechanical properties are critical.

The objective of this dissertation is to investigate via simulation how to implement feedback control over weld thermal cycles and in particular, one measure of the thermal cycle known as centerline cooling rate. Measurement and control
of thermal cycles using weld pool geometry/size information (obtainable from a coaxial sensor) will be of primary interest.

In Chapter II, a general discussion of arc welding processes and heat flow, the influence of arc welding thermal cycles upon base metal mechanical properties and a summary of past investigations of thermal cycle control will be presented.

In Chapter III, two methods of estimating centerline cooling rate from weld pool dimensional measurements are presented. One method is based upon feedback observers of the welding heat flow process while the other uses empirical relationships between weld dimensions and cooling rate. The first method is capable of providing estimates of cooling rates during transient periods such as at the beginning of welds, and is referred to as “dynamic observation”. Dynamic observers are found to be relatively computation-intensive. The other method, which uses empirically derived relationships, is shown to be computationally simple, but capable of providing accurate estimates only of steady state cooling rates. Because two of the empirical relationships correspond to steady state versions of the dynamic observers, the empirical relationships are referred to collectively as “static observers”.

Chapter IV describes the formulation and implementation of two types of dynamic heat flow observers. These observers are used in subsequent cooling rate control tests, and as the basis for the construction of a simulation of welding heat flow. The ability of the heat flow simulation to accurately predict weld cooling rates and pool dimensions is tested experimentally. The experimental procedures and measurements are described in Chapter V, along with comparisons of the experimental and simulation results. Some subsequent modifications to the observers are also discussed in this chapter.

In Chapter VI, three empirical static observer relationships between weld pool
dimensions and cooling rate are formulated. These static observer expressions are derived using multilinear regression techniques.

Cooling rate control investigations performed via numerical simulation are described in Chapter VII. In the first section of this chapter, the convergence of cooling rate estimates of the two dynamic heat flow observers is studied. The results indicate that the observation error decreases rapidly enough that closed-loop control performance would not be adversely affected by the use of observers to estimate cooling rate. Next, the dynamic response of the travel-speed/cooling-rate system is approximated using linear, low-order models. These models are used in the design of compensators which improve the closed-loop response of dynamic observer-based controls. The robustness of the dynamic observer-based controls with respect to observer parameter errors is studied. One of the dynamic observers is found to have advantages over the other in this regard.

In the second section of Chapter VII, closed loop cooling rate controls utilizing two of the static observer relations of Chapter VI are designed and simulated. The procedures used were very similar to those used in the previous section. Again, system responses are approximated with linear models and compensators are designed. The modeling-error robustness of the two static observer-based controls is found to be comparable.

In the third section, simultaneous feedback control of both cooling rate and weld pool width is studied via simulation. This multivariable control approach is shown to be feasible, but requires further investigation to correct stability deficiencies.

Finally, a summary of the research and recommendations for future research are given in Chapter VIII.
CHAPTER II

Background and Past Work

The objective of this chapter is to provide an introductory description of arc welding processes, a short review of past work concerning the nature of thermal cycles in the base metal during arc welding and their effect upon weldment properties, and a somewhat more thorough examination of past literature dealing with various aspects of the control of welding thermal cycles. The first two topics will be dealt with in somewhat summary fashion because they underlie, but are not central to, the topic of this dissertation. The last topic, thermal cycle control, is the main concern of this dissertation and the literature regarding this subject will be reviewed and discussed in more detail.

2.1 Arc Welding Processes

Arc welding processes are characterized by the use of an electrical arc to obtain melting of abutting metals (called the base metals or sometimes base plates) so that they coalesce. The fundamental aspects of arc welding processes are illustrated in Figure 1. The arc is maintained between a welding electrode, which may or may not melt, and the base metal using electrical current derived from a welding power supply. Base metal melting is obtained only in the immediate vicinity of the welding arc, making it necessary to traverse the welding electrode along an extended interface which is to be welded. If the base metal to be joined is relatively
Figure 1: Arc Welding Fundamentals

thick, the abutting edges may be beveled or similarly prepared and several passes of the welding arc along the joint combined with the addition of a filler metal are usually required to complete the weld.

Arc welding processes are classified as consumable electrode processes if the welding electrode is melted by the arc heat and becomes part of the molten weld pool, or as a nonconsumable electrode process if the electrode does not melt. In the case of consumable electrode processes, the electrode takes the form of a wire which is continuously fed into the welding arc, either mechanically or manually, and acts to fill in the joint preparation gap between the pieces being welding. In non-consumable electrode processes, the additional metal needed to fill the joint preparation gap between the base metals is added to the molten pool via a separate wire feed mechanism.

Several elements present in the atmosphere adversely effect the arc welding
process by destabilizing the welding arc and reacting with the molten weld pool in undesirable ways. For this reason, it is necessary to provide for some means of shielding the welding arc and molten weld metal from the surrounding air. This is commonly accomplished by introducing a shielding gas (often an inert gas such as argon) or solid flux into the vicinity of the welding arc to displace the atmosphere. Solid welding flux is commonly a low melting point material which, when melted and partially vaporized by arc heat, produces a gas which shields the welding arc from the atmosphere. The molten flux also interacts chemically with the molten weld metal to mitigate the deleterious effects of atmospheric contaminants upon weld metal properties.

Some commonly used consumable electrode, flux-shielded welding processes are the submerged arc welding (SAW) process, the shielded metal arc welding (SMAW) process, and the flux cored arc welding (FCAW) process. A commonly used consumable electrode, gas-shielded arc welding process is the gas metal arc welding (GMAW) process. The interested reader is referred to [4] for further information concerning these and other arc welding processes.

The welding process which is of interest in this dissertation is the gas tungsten arc welding (GTAW) process — a non-consumable electrode, gas-shielded process. It is by nature a relatively controllable, stable and clean welding process, making it a desirable choice for laboratory work. The GTAW process, shown schematically in Figure 2, utilizes a non-consumable tungsten rod as one electrode of the welding arc, the other electrode being the base metal. Shielding gas (usually argon or helium) protects the arc and molten weld pool from the atmosphere, and is introduced into the arc/weld pool area from a nozzle which surrounds the electrode. Filler metal is added to the weld pool using a separate wire feed mechanism. In a robotic arc welding process, the GTA welding torch is traversed along the weld.
Figure 2: GTAW Process Fundamentals
joint at a programmed speed by a robotic manipulator. The distance from the tungsten tip to the workpiece, which is approximately equal to the welding arc length, is determined by the location of the programmed robot path with respect to the location of the workpiece. The electrical current which is needed to maintain the welding arc is obtained from a constant current power supply, the output current of which is adjustable.

The electric arc functions as a compact, intense and adjustable source of heat with which to melt the base and filler metal in the GTAW process. The magnitude of the energy generated in the GTAW arc is usually adjusted by variation of the arc current although the rate at which energy is produced, \( P_a \), is actually a function of both arc current and voltage:

\[
P_a = EI
\]  

(2.1)

where \( P_a \) = arc power,

\[ E \] = arc voltage,

and \( I \) = arc current.

The amount of power which is transferred from the arc to the base metal is less than the total amount generated in the arc. To account for this fact, the arc power transferred to the base metal, \( \dot{q} \), is often expressed as a fraction of the total amount generated using an efficiency factor which depends upon arc voltage and other welding parameters:

\[
\dot{q} = \eta EI
\]  

(2.2)

where \( \dot{q} \) = arc power transferred to base metal
\[ \eta = \text{efficiency factor, } 0 \leq n \leq 1 \]

The efficiency has been shown by Niles and Jackson [5] to depend heavily upon the GTAW welding parameters. They found that efficiency tended to increase slightly as travel speed was increased, but decreased as arc voltage and current were increased.

The voltage, \( E \), of the typical welding arc at fixed current is an affine function of arc length and may be expressed as

\[ E = E_0 + \frac{dE}{dl} \cdot l \quad (2.3) \]

where \( E_0 = \text{arc voltage at } l \approx 0^+ \),
\[ \frac{dE}{dl} = \text{arc voltage/length gradient}, \]
and \( l = \text{arc length} \)

The arc voltage/length gradient and \( E_0 \) are dependent upon shielding gas composition, base and filler metal composition and other factors. However, arc voltage is relatively invariant with respect to arc current in the range of currents usually employed for welding. Hence, arc voltage and arc length are directly related, and arc voltage adjustments may be effected through corresponding adjustments in arc length. Mechanisms known as automatic voltage controls are commonly employed in automatic welding equipment to adjust arc length so as to maintain a constant arc voltage, thus maintaining constant heat input to the base metal.

2.2 Welding Thermal Cycles

The primary goal of arc welding is the joining of separate base metals, this being accomplished by melting and subsequent coalescence. However, the heat
input from the arc has significant effects upon the metallurgical structure of the materials being joined. The metallurgical structure of a typical weld bead is shown in cross-section in Figure 3. In broad terms, three zones can be identified: 1) a weld metal zone (WMZ), 2) a surrounding heat affected zone (HAZ), and 3) a base metal zone (BMZ). The weld metal zone contains a mixture of base metal and filler metal which has been melted by arc heat and subsequently solidified. The heat affected zone consists of base metal which, while not melted by the heat of the welding arc, has been heated to a sufficient temperature that the metallurgical properties of the material are altered. The base metal zone is practically unaffected by the arc heat. The relative sizes of the weld metal and heat affected zone are shown roughly to scale in this illustration.

The properties of material in the heat affected zone and weld metal zone, while dependent upon the alloy composition and past processing history of the base and
filler materials, are also a function of the time-temperature history experienced by these regions during welding. If one examines the temperature variations which occur at a selected point in the weld zone or heat affected zone near the midpoint of an extended weld, one finds that the temperature of the point generally increases as the welding arc approaches then decreases as the arc passes by (Fig. 4). This time-temperature history is known as the welding thermal cycle at the selected point.

Various measures, depicted in Figure 5, have been used to describe the welding thermal cycle. The cooling rate at temperature $T'$, denoted by $\frac{\delta T}{\delta t} |_{T'}$, is the slope of the cooling portion of the temperature cycle curve measured when temperature equals $T'$. The time to cool from $T'$ to $T''$ (often simply referred to as “the time to cool” when $T'$ and $T''$ have been previously specified), denoted by $\Delta t |_{T'-T''}$,
Figure 5: Welding Thermal Cycle Measures
is the time required for the temperature at the selected point to decrease from \( T' \) to \( T'' \). The time above temperature \( T' \), denoted \( \Delta t \rvert_{T'} \), is the total time during the thermal cycle for which the temperature of the selected point is above \( T' \). The peak temperature, \( T_p \), is the maximum temperature attained at the selected point during the thermal cycle.

2.3 Analysis of Welding Heat Flow

Past investigators have found that certain WMZ and HAZ material properties of interest (e.g. tensile strength and toughness) may be related to selected measures of the welding thermal cycle (e.g. peak temperature and time above a critical temperature). Hence, quantitative knowledge of the thermal cycles experienced by the weld zone and heat affected zone are of great practical interest. The mathematical modeling and prediction of the thermal cycles which occur in the base metal as a result of the arc welding processes have been studied for many years, the first major work on the topic being published by Rosenthal[6]. This seminal treatment sparked interest in a research area which continues to be important to this day. Generally speaking, most of the work carried out during the 1940’s and 1950’s was analytical in nature and was concerned only with the state distribution of heat in the weldment. This work has come to be known as the classical analysis of welding heat flow. From the early 1960’s onward, there continued to be important analytical work on welding thermal cycles, but the numerical approach to the problem was also vigorously pursued. The analytical work through the late 1960’s has been presented in a summary paper by Myers et al.[7]. To the knowledge of this author, there is no summary paper which treats numerical solutions of the welding heat flow problem. In this section, the analytical approach to the welding heat flow problem will be briefly described, after which numerical techniques will
be discussed.

Heat flow in the weld pool and surrounding base metal occurs as a result of conduction, convection and radiation. Of these, conduction in the base plate is usually considered to be the dominant mode of heat transfer in most welding situations. However, Heiple and Roper [8] have shown that liquid convection in the molten weld pool is important in some cases. Additionally, in the welding of thin material where the cross-sectional area for base metal thermal conduction is relatively small, air convection and radiation can cause significant heat losses from the surface of the material [9]. Nevertheless, the classical treatments of welding heat flow are concerned primarily with conduction in the base metal. In the following discussion of the welding heat flow problem, a somewhat general approach will be taken at the outset but the problem will be simplified by a number of restrictions, leading to the classical formulation and solutions.

A necessary preliminary to the discussion of welding heat flow is the discussion of the conventions used in assigning coordinate systems to the base metal. It is usual practice in welding heat flow analysis to express relationships in terms of a moving coordinate system \((w,y,z)\), centered on the welding heat source as is illustrated in Figure 6. An \((x,y,z)\) coordinate frame (which is referred to as the fixed frame) is customarily centered at the point where welding is considered to have begun; its position does not vary with time. The heat source is assumed to move in the positive \(x\) direction with a constant velocity, \(V\), so given an \(x\) and \(V\), \(w\) can be written as

\[
w = x - V \cdot t
\]

(2.4)

where \(V\) = welding-arc travel speed.
Figure 6: Moving and Fixed Coordinate Systems

For purposes of describing welding heat flow, the relationships are usually formulated in either two or three dimensions, the first corresponding to thin base material and the second corresponding to thick material. In general, base metal is considered to be “thin” if the welding temperature distribution is approximately the same when measured from either side of the material. The base metal is considered “thick” if the temperature of the back side (opposite to the welding arc) is not significantly affected by the arc heat input. Heat flow in thin material is referred to as two-dimensional heat flow while the thick-plate case is called three-dimensional heat flow. In this section, the welding heat flow problem will be formulated in three dimensions. The two-dimensional formulation is easily obtained by eliminating the z dimension.

The conduction of heat in solids is described by the so-called heat equation:

$$\rho(T)C(T) \frac{\partial T(x,y,z,t)}{\partial t} = \nabla \cdot (K(T)\nabla T(x,y,z,t))$$  \hspace{1cm} (2.5)
where $\rho(T) =$ base metal density

$C(T) =$ base metal mass specific heat

$K(T) =$ base metal thermal conductivity

$\nabla =$ gradient operator

Since arc welding is characterized by the presence of a molten weld pool, there are two phases present in the base metal: solid and liquid. The fact that the molten phase (the pool) moves as the arc traverses along the weld joint is also important in the analysis of welding heat flow. A condition for movement of a phase boundary is given by Stephan's equation:

$$ L \frac{\partial \hat{n}}{\partial t} = K_s(T_m^-) \frac{\partial T_m}{\partial \hat{n}} |_{T_m^-} - K_l(T_m^+) \frac{\partial T_m}{\partial \hat{n}} |_{T_m^+} $$

(2.6)

where $L =$ base metal latent heat of fusion

$\hat{n} =$ normal vector to phase boundary

$K_s =$ solid base metal conductivity at $T = T_m^-$

$K_l =$ liquid base metal conductivity at $T = T_m^+$

$T_m =$ base metal melting temperature

$T_m^+ = T_m + \Delta T, \Delta T \ll 1$

$T_m^- = T_m - \Delta T$

Boundary conditions are needed to make the welding heat flow problem well posed. The first condition, representing heat loss from the surface of the material is written as

$$ K(T(x, y, 0, t)) \frac{\partial T(x, y, z, t)}{\partial z} |_{z=0}= q_s(x, y) $$

(2.7)
where \( q_s(x, y) \) = energy flux from the base metal surface.

A second condition represents the thermal state of the edges of the base metal, and is written as

\[
\lim_{R \to \infty} T(x, y, z, t) = 0
\]  

(2.8)

where \( R = \sqrt{x^2 + y^2 + z^2} \).

Note that in this condition, the assumption has been made that the base metal is practically infinite in the \( \pm x, \pm y \) and \( +z \) dimensions.

Applying the transformation \( w = x - Vt \) yields the transformed heat equation

\[
\rho(T)C(T) \left( \frac{\partial T}{\partial t} - V \frac{\partial T}{\partial w} \right) = \nabla \cdot \left( K(T) \nabla T \right)
\]  

(2.9)

where \( T = T(w, y, z, t) \).

The boundary conditions for the moving coordinate formulation are the same as Equations (2.7) and (2.8) with \( w \) substituted for \( x \) and are written as

\[
\lim_{R \to \infty} T(w, y, z, t) = 0.
\]  

(2.10)

where \( R = \sqrt{w^2 + y^2 + z^2} \) and

\[
K(T(w, y, 0, t)) \frac{\partial T(w, y, z, t)}{\partial z} \bigg|_{z=0^+} = q_s(w, y)
\]  

(2.11)

A number of assumptions — listed below — are usually made in order to obtain closed form solutions to Equations (2.9), (2.10) and (2.11). Solutions to the welding heat flow problem which are based upon these assumptions are often referred to as classical welding heat flow solutions.

1. The thermal properties of the base material are approximately constant over the range of temperatures experienced during welding.
2. The latent heat of fusion of the material is insignificant. This assumption allows Stefan's equation to be ignored.

3. The welding arc may be modeled as an infinitesimally small source of heat flux (i.e. a point source of heat on the plate surface in three dimensions or a line source of heat through the plate thickness in two dimensions).

4. Heat transfer in the base metal is by conduction only. Losses of heat from the base metal surface via convection and radiation are negligible. Furthermore, heat transfer by convection in the liquid pool is neglected.

5. Only the quasi-steady state solution to the equations is desired. A quasi-steady state temperature distribution is defined as one which is non-time varying with respect to the moving \((w, y, z)\) frame.

6. Resistance heating of base metal due to the flow of welding arc current is negligible.

Under these assumptions, the welding heat flow equation (2.9) may be written as

\[
\rho C V \frac{\partial T(w, y, z)}{\partial w} + KV^2 T(w, y, z) = 0 \tag{2.12}
\]

Another boundary condition is added to account for the point heat source. In three dimensions, this source boundary condition is written as

\[
\lim_{R \to 0} 2\pi R^2 K \frac{\partial T}{\partial r} = \dot{q} \tag{2.13}
\]

The surface heat transfer condition becomes

\[
\frac{\partial T(w, y, z)}{\partial z} \bigg|_{z=0+} = 0 \tag{2.14}
\]

The three-dimensional solution from Equations (2.10), (2.12), (2.13) and (2.14) is written as [6]:

20
\[ T(w, y, z) - T_0 = \frac{\dot{q}}{w\pi K} \exp \left( -\frac{uw}{2\alpha} \right) \frac{\exp \left( \frac{VR}{2\alpha} \right)}{R} \] (2.15)

A plot of Equation (2.15) for typical welding conditions is shown in Figure 7. Note that the temperature distribution is asymmetric: w-axis gradients are steeper in front of the welding arc and shallower behind. This shape is a result of the motion of the arc heat source. If a temperature profile is taken along a line parallel to the w axis (y = constant), a curve similar to the thermal cycle shown in Figure 4 is obtained. This result is not a coincidence: it arises from the relation \( w = x - Vt \).

The temperature history at a point \((x', y')\) may be obtained by scaling the distance vs. temperature profile along \( y = y' \). However, note that the temperature cycles at points located on the arc path (\( y = 0 \)) are anomalous due to the fact that the temperature at the origin of the moving coordinate system (corresponding to the location of the point heat source) is infinite. This obviously inaccurate result arises from the point heat source assumption. Due to this inaccuracy, the classical solution can only be used to predict temperatures at points sufficiently removed from the welding arc.

The above formulation may be modified slightly to represent heat flow in a relatively thin base metal (i.e. two-dimensional heat flow). The heat conduction equation and associated boundary conditions appear as:

\[ \rho CV \frac{\partial T(w, y)}{\partial w} + K \nabla^2 T(w, y) = 0 \] (2.16)

\[ \lim_{r \to \infty} T(x, y, t) = 0 \] (2.17)

where \( r = \sqrt{x^2 + y^2} \)

and

\[ \lim_{r \to 0} 2\pi rhK \frac{\partial T}{\partial r} = \dot{q} \] (2.18)
Figure 7: Dimensionless Welding Temperature Distribution [11]

(a) Vertical Sections parallel to the $\lambda$ axis
(b) Vertical sections parallel to the $\psi$ axis
(c) Isotherms in the $\lambda - \psi$ plane at $\zeta = 0$
(d) Isotherms in the $\psi - \zeta$ plane at $\lambda = 0$

Note: $\lambda = \frac{V_w}{2\alpha}, \psi = \frac{V_y}{2\alpha}, \zeta = \frac{V_z}{2\alpha}, \rho = \frac{V_R}{2\alpha}, \eta = \frac{\delta V}{4\pi\alpha^2 C_7(T_c - T_0)}, \theta = \frac{T - T_0}{T_c - T_0}, T_c = \text{a temperature of interest}$
where $h =$base metal thickness.

The solution of the two-dimensional temperature distribution as determined by Equations (2.16),(2.17) and (2.18) may be written as [6]:

$$T(w, y) - T_0 = \frac{\dot{q}}{2\pi K h} \exp\left(\frac{-vw}{2\alpha}\right) K_0\left(\frac{wr}{2\alpha}\right)$$  \hspace{1cm} (2.19)

where $T_0 =$ initial base metal temperature

and $K_0 =$ Bessel function of the second kind of $0^{th}$ order.

These solutions, when plotted, are very similar in shape to those depicted in Figure 7 with the exception that the temperature is constant through the thickness of the base metal.

It has been found convenient to manipulate the classical welding heat flow solutions to obtain relationships involving weld pool dimensions, centerline cooling rate and other useful parameters. Wells [10] approximated the maximum weld pool width in the two-dimensional heat flow case as

$$d = \frac{4\alpha}{V} \left(\frac{\dot{q}}{8K(T_m - T_0)h} - \frac{1}{5}\right)$$  \hspace{1cm} (2.20)

where $d =$ maximum weld pool width.

Christenson et al. [11] found a number of similar relationships for the three-dimensional heat flow case, including expressions for maximum weld pool width and length. However, many relationships (e.g. for maximum weld pool width) could not be expressed in closed form, so numerical techniques were used to obtain solutions for particular values of welding input parameters.

Adams [12] calculated the centerline cooling rate behind the line source for two-dimensional heat flow to be

$$\left.\frac{\partial T}{\partial t}\right|_{T=T'} = 2\pi K \rho C h^2 \left(\frac{V}{\dot{q}}\right)^2 (T' - T_0)^3$$  \hspace{1cm} (2.21)
where \( T' \) = temperature at which cooling rate is calculated.

The corresponding relationship for 3-dimensional heat flow was given as

\[
\frac{\partial T}{\partial t} \bigg|_{T=T'} = 2\pi K \left( \frac{V}{\dot{q}} \right) (T' - T_0)^3
\]  

(2.22)

The ratio \( \dot{q}/V \) which appears in both relationships represents the effect which the welding inputs (arc current, arc voltage and travel speed) have upon centerline cooling rate. \( \dot{q}/V \) is often referred to as the "welding heat input" in the literature.

The accuracy of the classical solutions to the welding heat flow problem have been the subject of much study. Christensen et al [11] gave a thorough comparison of analytical and experimental results for the three-dimensional heat flow case. Graville [13] examined the accuracy of both two-dimensional and three-dimensional results. In general, one finds that the classical theory of welding heat flow produces results which agree with experimental data to well within an order of magnitude or better over a wide range of welding conditions.

2.4 Extensions of The Classical Solutions

Various attempts have been made to modify the classical formulation of the welding heat flow problem to provide better correlation between experiment and theory. In general, the thrust of such work has been to obtain solutions in the absence of one or more of the six assumptions listed in the formulation of the classical problem. In this section, some of the more important results which have been obtained will be described briefly.

Convective losses from the plate surface may be easily added. It has been found that such a modification brings the predictions of the two-dimensional classical theory closer to experimental results [13]. When surface convection is added, the solution to the two-dimensional heat flow problem is
\[ T(w, y) - T_0 = \frac{q}{2\pi K h} \exp \left( \frac{-Vw}{2\alpha} \right) K_0 \left( r \left( KV^2 + \frac{2H}{K_h} \right)^{\frac{1}{2}} \right) \] (2.23)

where \( H = \) surface heat transfer coefficient.

Another way in which the classical formulation may be made more realistic is by modeling the arc as a spatially distributed rather than point heat flux source. Tsai [14] obtained a three-dimensional conduction solution for an arc heat flux distribution having a two-dimensional Gaussian shape which was skewed, or elongated, in the \( w \) direction. This alteration eliminates the infinite temperature at the heat source center which resulted from the point heat source assumption, thus increasing the accuracy of the solution at points near to the heat source.

Some progress has been made in obtaining solutions to the welding heat flow problem in which the thermal properties are not assumed to be constant with temperature. In particular, Lee [15] has obtained closed form solutions for the case where the thermal properties of the base material vary with temperature in a specified way.
2.5 Numerical Simulation Techniques

More realistic solutions of the welding heat flow equations may be obtained by solving the conduction equations numerically. In general, the numerical techniques which have been applied in the simulation of welding heat flow fall into one of three categories: numerical integration of an ideal instantaneous source solution, finite difference approximation of the heat conduction equations, and finite element approximation of the heat conduction equations. In this section the ideal instantaneous source solution and the finite difference approximation simulation techniques will be discussed. The former was used as the basis for a simulation described later in this dissertation and the latter provides background for a discussion of welding temperature distribution dynamic response.

As the name suggests, the first method cited above depends upon an solution given by Carslaw and Jaeger [16] for the rise in temperature at a point in thin plate due to an instantaneous source of heat located at another point and operating at a previous time. An instantaneous source of heat corresponds to the release of a finite quantity of energy over a vanishingly small time interval at a specified point (a "delta function"). The solution in two dimensions is

\[ T(x, x', y, y', t, t') - T_0 = \frac{\dot{q}}{4\pi K h(t - t')} \exp \left( -\frac{(x - x')^2 + (y - y')^2}{4\alpha(t - t')} \right) \]  \hspace{1cm} (2.24)

where

\((x,y,t) = \text{spatial and time coordinates of the point where temperature rise is calculated}\)

\((x',y',t') = \text{spatial and time coordinates of the instantaneous heat source}\)
If the point source travels along the y axis with constant speed \( V \), the change of coordinates

\[
(x - x') = w + V(t - t')
\]

may be made to transform the relationship into the moving coordinate, and the expression may be integrated over time to yield the following solution for a moving, continuous source of heat input:

\[
T(w, y, t, t') - T_0 = \frac{q}{4\pi K h(t - t')} \int_0^t \frac{\exp \left( -\frac{(w+V(t-t'))^2 + y^2}{4\alpha(t-t')} \right)}{t - t'} \, dt'
\]  

(2.25)

The integral may be approximated numerically (e.g. using gaussian quadratures) to obtain a solution of the two-dimensional welding heat flow problem. Note that transient as well as quasi-steady state results may be obtained in this way: the latter is found by integrating Equation (2.25) over a relatively long time period with constant inputs. Eager and Tsai [17], using a similar relationship for the three-dimensional case, incorporated a gaussian heat input distribution into a simulation of this type. Predictions of weld width and depth produced by this method have been found by Eager and Tsai [17] and Bates and Hardt [18] to compare favorably with experimental results. Hou and Tsai [19] describe a heuristic method of accommodating temperature varying thermal properties in a similar simulation.

Another simulation technique, the finite difference method, utilizes an approximation of the welding heat conduction equation which, repeated in a slightly different form is

\[
\frac{\partial T(w, y)}{\partial t} = \alpha \nabla^2 T(w, y) + V \frac{\partial T(w, y)}{\partial w} + S(w, y)q
\]  

(2.26)
The term $S(w, y)q$ is added to account for heat input from a distributed source, and will be discussed below. For simplicity of exposition, only the two-dimensional case will be treated.

Consider the temperatures at a number of points forming a regularly spaced array about the welding heat source center. It is customary to represent the locations of these points as the intersection points of a grid such as the one shown in Figure 8. Note that since a finite grid must be used, it is no longer possible to use infinite boundary conditions as was done to obtain the classical solutions. In the boundary conditions usually used with the finite grid, it is commonly assumed that all edges of the grid are insulated (adiabatic condition), all edges are held at fixed temperature, or some of the edges are held at fixed temperature temperature while the remainder are insulated. The adiabatic conditions are written as
\[
\frac{\partial T}{\partial w} \bigg|_{z=z_{\text{max}}} = 0 \tag{2.27a}
\]

and
\[
\frac{\partial T}{\partial w} \bigg|_{z=z_{\text{min}}} = 0 \tag{2.27b}
\]

The fixed-temperature boundary conditions are written as
\[
T \bigg|_{w=w_{\text{max}}} = 0 \tag{2.28a}
\]

and
\[
T \bigg|_{w=w_{\text{max}}} = 0 \tag{2.28b}
\]

The spatial derivatives found in Eq. 2.21 may be approximated at each grid point in terms of the temperature at the grid point of interest and its nearest neighbors. In particular,
\[
\frac{\partial T(w,y)}{\partial w} \approx \frac{T(w + \Delta w,y) - T(w - \Delta w,y)}{2\Delta w} \tag{2.29}
\]

and
\[
\frac{\partial^2 T}{\partial^2 w} \approx \frac{T(w + \Delta w) + T(w - \Delta w) + T(y + \Delta y) + T(y - \Delta y) - 2T}{\Delta w^2} \tag{2.30}
\]

If the temperatures at the grid intersections are arranged into a vector \( Z \), then Equation (2.26) may be represented by the following vector-matrix relationship:
\[
\frac{dZ}{dt} = AZ + VB_1Z + qB_2 + C \tag{2.31}
\]

where \( Z = (T(x_1,y_1), T(x_1,y_2), \ldots, T(x_1,y_m), T(x_2,y_1), \ldots, T(x_n,y_m)) \cdot \)

\( A = \text{matrix of coefficients of the discrete Laplacian operator} \)

\( B_1 = \text{matrix of coefficients of the discrete first difference} \)

\( B_2 = \text{matrix of coefficients of discrete gaussian arc heat source} \)
C = matrix of coefficients from discrete boundary conditions

The thermal flux from the arc may be modeled as a gaussian distribution or any other general distribution using the finite difference numerical technique. For example, if the heat source is assumed to be gaussian, then $S(w, y)$ (in Eq. (2.26)) is written as

$$S(w, y) = \frac{1}{2\pi \sigma^2}e^{\frac{-w^2 + y^2}{\sigma^2}}$$  \hspace{1cm} (2.32)

The thermal flux at a node point $(w', y')$ may be approximated to the first order by evaluating the gaussian flux distribution $S(w, y)$ at the point $(w', y')$ and integrating over the grid element area using Euler's method. The result is entered into the corresponding position in the vector $B_2$ (see Eq. (2.31)).

Equation (2.31) is an ordinary differential equation in the time variable, which may be solved numerically by several standard methods. For welding heat flow simulation, the Crank-Nicholson technique is very commonly used, although Euler integration will also produce satisfactory results if sufficiently small time steps are taken [20].

2.6 The Effect of Thermal Cycles Upon Weld Properties

One driving force for the study of welding thermal cycles has been the fact that, for many materials of practical interest, weld properties are dependent upon thermal cycles. In this section, a short review of past experimental studies of this dependency is given. The principle goal is to set forth relationships which have been found to exist between measures of welding thermal cycles and material properties. These relationships will be used later when selecting a particular thermal cycle measure to be feedback regulated.
Many of the studies of the relationship between welding thermal cycles and material properties are concerned with medium alloy steels. In one such steel (ASTM A515 Gr. 70), Graville [13] found that both heat affected zone and weld metal zone maximum hardnesses could be related to centerline cooling rate at a selected temperature. The temperature at which the cooling rate was measured was found to affect the correlation: better correlation could be obtained for relatively low cooling rate conditions by selecting a higher temperature for cooling rate measurement \((T' \approx 425^\circ C)\). For higher cooling rate conditions, the best temperature of measurement was lower \((T' \approx 300^\circ C)\).

Connor et al. [21] found that the centerline cooling rate measured at 538 C (1000 F) was correlated to the tensile strength of HY-130(T) weld metal and was largely uncorrelated with weld metal toughness. However, Signes and Baker [22] produced data from welds on carbon-manganese base metal which tend to indicate that HAZ impact toughness is influenced by the time for a centerline point to cool from 800 C to 500 C.

Jackson [23] studied the time-temperature effects of arc welding upon 2219 aluminum alloy. Partial penetration welds were made in samples in which thermocouples were imbedded to measure the thermal cycles experienced at the centerline of the weld near the bottom surface of the plate. It was found that the maximum temperature obtained at the point of measurement, and the length of time for which the point was above a critical temperature \((T' \approx 232^\circ C)\) in this case) had influence upon the yield strength, tensile strength and toughness (as measured by elongation of tensile specimens).

Glover et al. [24] studied the effects of welding thermal cycle upon base metal and weld metal microstructures in Grade 516 and 515 pressure vessel steel and X70 HSLA plate. Two experimental procedures were used: in the first, bead on
plate welds were made in the 516 and 515 steels using the SAW process and a variety of heat inputs. Thermal cycles were measured using thermocouples. In the second procedure, two pass-welds were made in the HSLA steel, and samples of the resulting weld metal were removed and subjected to controlled thermal cycles using a weld thermal cycle simulation device known as a Gleeble. The specimens from both procedures were examined metallographically, and the microstructures which were found were related to measures of the thermal cycle which the specimen had experienced. The measure of the thermal cycle which was utilized was the time to cool from 800 to 500 C. A thorough discussion of the results is beyond the scope of this paper, but in short, it was found that HAZ and WMZ microstructures and mechanical properties depended heavily upon the time to cool from 800 to 500 C.

2.7 Control of the Welding Thermal Cycle

The control of welding thermal cycles has been the subject of a number of past investigations. The welding thermal cycle control literature may be conveniently separated into two broad divisions: 1) investigations of the dynamic response of the base metal temperature distribution (including weld pool dimensions) and 2) investigations of the control of the temperature distribution and weld pool dimensions. A majority of the investigations which have been described in the welding literature are primarily concerned with the sensing rather than the control aspects of the problem. This chapter emphasizes results dealing with the control aspects of the thermal cycle control problem and contains two sections: one dealing with the dynamic response of the welding temperature distribution and the other with its control.
2.7.1 Dynamic Response of Temperature Distributions and Weld Pool Dimensions

Kisilevskii and Butakov [1] investigated the dynamic response of the temperature fields in the vicinity of the welding arc using a numerical simulation of welding heat flow. The simulation utilized the finite difference technique to approximate the three-dimensional heat conduction equations and included the non-linear effects of temperature-varying thermal properties, phase change in the base metal and radiative and convective surface heat losses. The arc heat input was taken to be a two-dimensional Gaussian distribution. The boundary conditions in the \((w, y)\) plane were taken to be adiabatic.

Step changes in welding arc current and travel speed were made about nominal operating points, and the speed with which temperatures at points in the base metal surrounding the welding arc responded to this step input were evaluated using a sensitivity factor. The sensitivity factor at a selected point was defined as the magnitude of the slope of the temperature-time response curve at that point shortly after the time of the step perturbation in input. That is,

\[
P_l(w, y) = \frac{\partial T(w, y)}{\partial t} \bigg|_{t=t_1^+} \tag{2.33}
\]

where \(P_l(w, y)\) = sensitivity factor at \((w, y)\) for a step perturbation

\(l =\) input \((V\) or \(q)\)

\(t_1 =\) time of perturbation

If the response is modeled as a simple exponential, the sensitivity corresponds to the time constant of the response. The goal of the work was said to be the identification of those locations having fastest dynamic response so thermal sensors could be located at these points for subsequent control implementation.
Figure 9: Sensitivity Index Distribution [1]

The main result of the work, shown in Figure 9, was a plot depicting the locus in the plane \((w, y, 0)\) of constant sensitivity factors. The \((w, y, z)\) coordinates had their origin at the heat source center and the positive \(w\) direction corresponded to the direction of travel of the welding arc; \(z = 0\) corresponded to the top surface of the base metal. The solid lines in Figure 9 correspond to sensitivity factors for step changes in arc current, while the dashed lines represent sensitivities to changes in travel speed.

The sensitivity index distribution for arc current perturbations indicated that points further away from the arc had slower response characteristics (i.e. smaller sensitivity factors). The distribution for travel speed perturbations was more complicated but points behind the arc generally responded faster than points in front. Also, points closer to the origin had faster response times than those further away. However, it is interesting to note that there is a locus of points (curve #9) whose
temperature did not respond to the travel speed perturbation. This fact would be of significance if one were designing a control to regulate the temperature distribution in the vicinity of the weld pool using the travel speed as an input. The locus of points whose temperature does not vary with travel speed would not be controllable under such a control scheme.

Ilenko and Baumann [25] studied the dynamic response of weld pool maximum width and depth to changes in arc current in the GTAW process. Experiments were carried out in very thin (0.2 mm to 1.0 mm thick) steel sheet. The necessary measurements were taken after welding was completed; the plates were cross-sectioned along the weld centerline for the depth measurements. It was found that the response of weld width and depth to arc current perturbations were approximately exponential and had time constants of about .12 second for the 0.2 mm. sheet and 0.2 sec for the 1 mm. sheet.

Hardt, Garlow and Weinert [2] developed a simplified lumped model of the pool width dynamics in the 2-dimensional heat flow case. The weld pool was assumed to have a circular shape, and an energy balance between energy input from the welding arc and energy expended through melting of base metal and conduction to the base metal from the pool periphery was formulated as is indicated in Figure 10. The resulting relationship was

\[ \rho L 2\pi hr \frac{dr}{dt} + 2\pi K hr \frac{dT}{dr} = q \]  \hspace{1cm} (2.34)

where \( r \) = weld pool radius.

The temperature gradient surrounding the weld pool was assumed to be radially symmetric and of magnitude directly proportional to the travel speed, and
Figure 10: Simplified Welding Energy Balance [2]

(2.34) was modified accordingly, yielding

$$\frac{r}{q - (2\pi \rho L h r) \dot{r}} = \frac{1}{V} \cdot \frac{1}{2\pi h K k_s}$$

(2.35)

where $\dot{r} = \frac{dr}{dt}$. This equation for the pool radius dynamics is highly non-linear. No experimental or simulation verifications of its accuracy were given.

Experimental results presented later in the paper indicate that the response of weld pool width to step changes in arc current and travel speed could be modeled as exponential. The steady state input-output gains between arc current/weld pool width and travel speed/weld pool width were found to be heavily dependent upon the nominal operating points of current and travel speed and upon base metal thickness and initial temperature. From analysis of travel speed step perturbation test data, the time constant of the weld pool width response was found to depend upon the final value of the travel speed (after the step perturbation) and base
metal initial temperature and be relatively independent of the initial (pre-step perturbation) travel speed (Figs. 11 and 12).

2.7.2 Welding Temperature Distribution Controls

In this section, a review will be given of the literature dealing with the problem of feedback control of the welding temperature distribution. Most of the control methods which appear under this broad subject heading are actually concerned with the somewhat more restricted problem of weld pool dimension control. In general, the focus of these papers is upon the sensing methods rather than control. Feedback controls are typically implemented with digital proportional-integral (P-I) compensation which modify arc current or travel speed to regulate the measured pool dimension to a desired value. Sensors for weld pool dimension measurement
Figure 12: Time Constant vs. Initial Travel Speed [2]
are almost exclusively optical devices sensitive in the visible or infra-red spectrum. Examples of such papers are thoses by Vroman and Brandt [26] and Richardson et al. [27].

A few papers which focus upon various aspects of the control of the welding temperature distribution have appeared. Several of these are discussed below.

Dornfeld et al. [3] considered the feedback control of the temperature of the underside of thin base metal at a point 0.5 in. behind the center of the welding arc. The temperature at this point was measured using an infra-red sensitive photodetector. The dynamic response of the measured temperature with respect to travel speed variations was identified empirically, and an adaptive scheme for controlling temperature was proposed.

An autoregressive moving average (ARMA) relationship (2.30) was used to relate backside temperature to welding travel speed. The ARMA model had the form

\[ T(k) = \sum_{i=1}^{n} a_i T(k - i) + \sum_{i=0}^{m} b_i V(k - i) \]  

(2.36)

where \( T(k) \) = measured temperature at time \( k \)

\( V(k) = \) travel speed at time \( k \)

The parameters of the ARMA model were identified directly from a process input/output record obtained by perturbing the welding travel speed about a nominal operating point with a pseudo-random binary sequence. A typical input/output record is shown in Figure 13. The algorithm used for identification was a recursive least squares method. The order of the model \((n, m)\) was increased from \( (2,2) \) to \( (2,9) \), and the output predicted by the ARMA model was compared to the actual output to determine the accuracy of the model. A model of order
Figure 13: Predicted and Actual Temperatures for \((n, m) = (2, 2)\) [3]

\(2, 2\) was found to yield satisfactory performance, as is shown in Figure 13. Some non-linearity in the process behavior was noted; the change in temperature due to negative travel speed perturbations was found to be greater than the temperature change observed for positive speed perturbations of the same magnitude.

Dornfeld et al. proposed a model reference adaptive control of the backside temperature. A block diagram of the proposed control structure is shown in Figure 14. The control design algorithm and controller structure were not described in detail. The control variable in this proposed scheme was the wire feed rate, which is directly related to the welding arc current in the welding process under consideration (GMAW). Note that the model identification results described above were for the travel speed/temperature system, rather than for the wire speed/temperature system. The proposed control scheme was not implemented, hence test results were not available.
Kissilevskii et al. [28] have described a feedback algorithm for controlling the temperatures at a number of selected points in the vicinity of the welding arc. At each iteration of the discrete-time control method, a numerical search was performed to determine input (i.e., arc current and travel speed) values which minimized predicted temperature deviations from desired values at the next iteration. A simulation was used to study the effectiveness of the control.

The key feature of this algorithm was a simplified version of the weld heat conduction equation which, given the temperatures of selected points at some time, allowed the temperatures at a future time to be predicted with a minimum of computation. This was important because the numerical searching nature of the control algorithm necessitates many such predictions at each control iteration.

The algorithm is best described by tracing the steps which are performed for each control step. Assume that the temperatures at selected points in the vicinity
of the welding arc are given. The next step is the prediction of temperatures of
the points at the next control time using the simplified heat conduction equation
along with default values of welding inputs. The predicted temperatures are then
entered into a quadratic cost function which provides a positive measure of the error
between the predicted and desired temperature values. The welding inputs are then
varied in such a way as to decrease the cost (using a gradient searching technique
which was not explained) and new next-time-step temperatures are predicted. The
last step is repeated until the cost has been minimized. The resulting inputs are
then used in the simulation to generate temperatures at the selected points at the
next time step and the process is repeated.

The cost function, $J$, which the control minimized at each step was a quadratic
function of the temperature error, written as

$$ J^2(\xi(t), t) = \sum_{i=1}^{n} (T^*(i, t) - T(i, t))^2 $$  \hspace{1cm} (2.37)

where $T^*(i, t)$ = desired temperature distribution

$T(i, t)$ = measured temperature distribution

$\xi(t)$ = control input vector (e.g. $\xi(t) = (V(t)q(t))$)

$n$ = number of points at which temperature is controlled.

The simplified heat flow equation allowing the prediction of $T_{k+1}$ given $T_k$ was
given as

$$ T_{k+1} = T_k + a T_k + \Delta V_k \Delta t \frac{\partial T_k}{\partial x} $$  \hspace{1cm} (2.38)

where $a = q_{k+1}/q_k$,

$\Delta V_k = V_{k+1} - V_k$,

$V_k = $ travel speed at time $k$. 

42
\[ q_k = \text{arc power at time } k, \]

and \( \Delta t = \text{control iteration interval}. \)

Results from simulations of this control method are sketchy. However, the statement is made that "the developed algorithm ... reduces [by three times] the value of integral temperature deviation from the set value in the workpiece /ldots [with] gap width variations from 0 to 2 mm in butt joint welding." The authors also noted that, in spite of the simplified heat conduction equation used in conjunction with the numerical search technique, the control execution time was still unacceptably slow for real-time control.

2.8 Distributed Parameter State Estimation

The cooling rate control algorithms to be proposed in this dissertation all involve the use of some form of observer of welding thermal distributions. The transfer of heat in an extended solid is in general classified as a distributed parameter system because the state of such a system (i.e. the temperature distribution) depends upon both space and time. In this section, a very brief introduction to the underlying structure of observers of distributed parameter systems is presented. The idea of adaptive observation of dynamic systems is also introduced.

Recall that welding heat conduction in a fixed coordinate system may be modeled by the equation

\[
\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \dot{q} G(x - x_0(V, t), y)
\]

(2.39)

where \( G(\cdot, \cdot) \) is a normalized two-dimensional Gaussian distribution.

The boundary and initial conditions are

\[
\lim_{x \to \pm \infty} T(x, y, t) = T_0
\]

(2.40)

43
\[
\lim_{y \to \pm \infty} T(x, y, t) = T_0
\]  
\[
\lim_{x \to \pm \infty} \frac{\partial T}{\partial x} = 0
\]  
\[
\lim_{y \to \pm \infty} \frac{\partial T}{\partial y} = 0
\]  
\[
T(x, y, 0) = T_0
\]  

Again, the fact that the state \( T(x, y, t) \) is a function of both time and space distinguishes the system as a distributed parameter one. Also note that the travel speed input enters into the equations in a nonlinear fashion by appearing in the Gaussian distribution. However, if travel speed is regarded as a constant and \( \dot{q} \) as the sole system input, the system is linear with a time-varying input operator \( G(\cdot, \cdot) \). In either case the application of standard linear distributed parameter observation techniques is possible because the homogenous system (i.e. the system with both inputs set to zero) is linear and time-invariant.

The topics of state estimation and, to a lesser extent, observation of distributed parameter systems have been a research topic for a number of years. In this dissertation, state estimation (also known as filtering) will be used to refer to methods of inferring the value of the state of a system from system outputs using techniques which take into account noise in the measurements of system outputs. State observation will be used to refer to techniques which infer the value of the state from output measurements while not explicitly considering measurement noise. The topic of observability of distributed parameter systems was studied in an early paper by Wang and Tung [29] and further developments were made by Goodsen [30], Sakawa [31], Liu and Lapidus [32] and others. Two survey papers describing research and applications of distributed parameter state estimation/observation have been published by Ray [33,34].

A major difference between the welding heat flow problem formulation as
stated above and the formulation of the problems treated in the vast majority of
the distributed parameter observer/estimator literature lies in the definition of the
domain upon which the system acts. In the welding case, the use of a domain of
infinite extent in the x and y coordinate directions is common; in most control-
oriented literature, domains of finite extent are specified. This difference does
not impact upon the basic observer structure; the observer has the same form
for both problems. However, the assumption of infinite boundary conditions does
have significant impact upon the methods which may be applied to the analysis
of convergence of observation error and related questions. Most such analysis
depends upon the existence of a countable set of orthogonalizable basis functions
for the domain of interest. Functions defined upon the domain may be expanded
in terms of such a basis. For the infinite domain, no countable basis exists. Some
basic results concerning observability of distributed parameter systems defined on
infinite domains are found in Wang [29] and Goodsen [30].

At this point, it would seem wise to question the necessity and usefulness of
formulation of the welding heat flow problem upon an infinite domain. There are
some points which can be made in defense of this approach. Perhaps the most
important argument is that the infinite domain assumption yields results which
are more generally applicable than those based upon finite domain assumptions.
Results based upon an assumption of infinite domain may be applied to any weld-
ment geometry which is of large enough extent that the edges do not significantly
influence the temperature distribution. This encompasses a large number of weld-
ing applications. Another advantage of the infinite domain assumption is that the
boundary conditions of the heat flow equations expressed in a moving coordinate
system are much simpler than is the case for the finite domain. The derivation
of the classical point-source welding heat flow relationships depends upon this
fact. Although the latter of these two arguments is not particularly relevant to the goals of this dissertation, the former is very relevant. For this reason, the infinite-domain version of the welding heat flow problem will be utilized exclusively in this dissertation.

To present the basic structure of the distributed parameter observer, a formulation given by Sakawa [31] will be sketched below. This formulation is concerned with systems defined on finite domains, so the presentation below will emphasize the basic structure of the distributed parameter observer rather than associated conditions for observation error convergence and system observability. We begin by considering the homogeneous linear distributed parameter system

$$\frac{\partial u(x,t)}{\partial t} = \nabla^2 u(x,t) - q(x)u(x,t)$$  \hspace{1cm} (2.45)

Here, $u(x,t)$ is the state of the system and $q(x)$ satisfies certain mild continuity conditions which will not be stated. We assume that $x \in \Omega$, a bounded set in Euclidian N-space having piecewise $C^3$ boundary denoted by $\partial \Omega$. The system has initial condition $u(x,0) = u_0(x)$ and homogeneous boundary conditions. Assume that $m$ independent measurements of the system state $u(x,t)$ are available and may be written as

$$z_k(t) = \langle H_k(x), u(x,t) \rangle \quad k = 1 \text{ to } m \hspace{1cm} (2.46)$$

where $\langle \cdot , \cdot \rangle$ denotes an inner product on $\Omega$. Here, $H_k(x)$ are known output operators satisfying certain boundedness conditions which ensure that the inner product given above exists.

An observer of this distributed parameter system is defined by

$$\frac{\partial \hat{u}(x,t)}{\partial t} = \nabla^2 \hat{u}(x,t) - q(x)\hat{u}(x,t) + \sum_{k=1}^{m} g_k(x)(z_k(t) - \hat{z}_k(t))$$  \hspace{1cm} (2.47)
where \( \hat{u}(x,t) \) = estimated system state

\[ g_k = \text{feedback gains (to be determined)} \]

\[ \hat{\sigma}(t) = \langle H_k(x), \hat{u}(x,t) \rangle \]

The boundary conditions are assumed to be the same as for (2.45) and the initial condition is \( \hat{u}(x,0) = \hat{u}_0(x) \), not necessarily equal to \( u_0(x) \). This observer structure is analogous to that of the well-known finite-dimensional observer structure.

The error system for the given observer, \( \hat{u}(x,t) \) is defined as

\[ \hat{u}(x,t) = u(x,t) - \hat{u}(x,t) \] (2.48)

Subtracting (2.47) from (2.45) yields

\[ \frac{\partial \hat{u}(x,t)}{\partial t} = \nabla^2 \hat{u}(x,t) - q(x)\hat{u}(x,t) - \sum_{k=1}^{k=m} g_k(x) \langle H_k(x), \hat{u}(x,t) \rangle \] (2.49)

The boundary conditions are the same as in (2.45) and the initial condition is \( \hat{u}(x,0) = \hat{u}_0 \). If the observer feedback gains \( g_k(x) \) are such that Equation (2.49) is an asymptotically stable system, then (2.47) is said to be an asymptotic observer.

The above discussion describes the development of what might be termed a traditional observer. Recently, there has been some interest in adaptive observation algorithms for lumped systems. In simple terms, these observers differ from the traditional ones in that the model parameters of the observer are adjusted based upon the measured input/output behavior of the process. A block diagram of such an observer is shown in Figure 15. Adaptive observers for linear lumped systems have been studied in [35], [36], [37], [38] and elsewhere.

As a final remark, we note that an alternate formulation of the welding heat flow equations results in a bilinear distributed parameter system. Recall that
formulation of the welding heat flow equations in a moving coordinate system $(w, y)$ results in the equation

\[
\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{\partial T}{\partial w} \cdot V(t) + \dot{\theta} G(w, y)
\]  

(2.50)

This system is classified as a bilinear distributed parameter system due to the fact that the travel speed input multiplies a linear function of the system state. Application of observers to the control of such systems have been described in [39] and [40].

2.9 Interpretation of Background Information

This chapter has provided a broad discussion of topics related to arc welding processes and thermal cycles in arc welding and a more detailed discussion of past work which has been published dealing specifically with control of arc welding thermal cycles. In this section, some interpretation of these past thermal cycle
control investigations is provided, the intention being to indicate how some of the results relate to thermal cycle control via weld pool dimensional measurements.

The discussion of the studies which have attempted to determine the correlation of various thermal cycle measures to weldment properties showed that a number of different thermal cycle measures and materials have been considered with mixed results. It is likely that attempts to determine thermal–cycle/mechanical–property relationships have been hindered by the fact that thermal cycles are not the sole factor determining weld mechanical properties. For example, weld metal properties are known to be influenced by processes occurring during solidification of the weld metal. The conclusion which can be drawn from these considerations is that, while thermal cycle control will provide control over some mechanical properties in many welding applications, it will not result in control of all mechanical properties in all welding situations.

The most fundamental conclusion which can be drawn from the temperature-distribution dynamic response studies is that the temperature fields and weld pool dimensions in arc welding respond in an over-damped, approximately exponential way to step perturbations in arc current and travel speed. All of the pertinent references discussed provide either simulation or experimental results which support this conclusion. This conclusion does not relate directly to the response of the weld centerline cooling rate or other thermal cycle measures, but it does suggest that the step response of these parameters might also be of a similar nature. Also of particular interest is the result in [1] which indicates that the response of points removed from the center of the welding arc in general have slower response speeds than those which are near to the arc. This tends to indicate that attempts to control thermal cycle measures which are derived from temperatures at points far from the arc will result in a system having slow response.
An equally important fact drawn from past investigations is that the dynamic response of many important measures of welding temperature fields (e.g. weld pool dimensions) are inherently non-linear, this conclusion being based upon the findings in [28,3,2]. In particular, in [2] it was found that the steady-state input/output gain and dynamic response of the travel-speed/bottom-bead-temperature system was dependent upon the nominal value of travel speed and other welding conditions. This is not surprising, since it has been pointed out in the previous section that the welding temperature field represents a bilinear system with respect to the travel speed input. This result suggests that centerline cooling rate or other thermal cycle measures might also be expected to have similar response characteristics.

In the area of temperature distribution control, the paper which has the most direct relationship to the topic of interest is [28]. A few observations on the general approach taken in this paper are in order. The temperature distribution control was formulated so as to incorporate measurements of the process, meaning that the control is feedback rather than open loop. This is desirable given the difficulties encountered in attempting to formulate accurate models of the welding process. However, the approach may be criticized on several points. First, the nature of the control algorithm was such that there were more outputs than inputs. Hence, in the steady state, there existed no set of inputs which would produce exact regulation of the outputs. Minimization of a cost function of the error at all points was the best that could be done. Secondly, the control utilized the travel speed as an input, but no corresponding adjustments in the desired temperature field were made. This is undesirable because, as has been pointed out in a previous section, cooling rates are often quite influential in determining final weld properties. When travel speed is varied, but the desired temperature distribution is not, cooling rates vary and hence material properties are not maintained constant. Finally, it is not
clear that the control could be implemented in real time due to the numerical searching which needed to be performed at each control iteration.

2.10 Summary

Background material drawn from a broad range of research areas has been presented in this chapter. Basic descriptions of arc welding processes were intended to introduce the topic to the reader not already familiar with the subject. This was also the goal of the discussions dealing with the classical welding heat flow relationships. The literature describing weld mechanical–property/thermal–cycle relationships was reviewed to determine which thermal cycle measures, if any, had been found to correlate to weld mechanical properties. Results showed that centerline cooling rate, time above a selected temperature and time to cool from one selected temperature to another could all be correlated to mechanical properties under different welding conditions. Several papers describing investigations of the dynamic response of the base metal temperature distribution to variations in arc welding inputs were reviewed. Response characteristics of the temperature at selected points to process input perturbations were found to vary depending upon the location of the selected point with respect to the welding arc and upon the nominal values of process input. In general, all responses could be characterized as over-damped. Review of the literature dealing with the control of base metal temperature distribution during arc welding revealed relatively few investigations upon the topic. Two papers were discussed: the first proposed an adaptive control approach to the problem while the second provided simulation results from a model–based numerical searching technique. The second paper, which is more relevant to this dissertation is described in more detail and some disadvantages of the control approach proposed in this paper are pointed out. Finally, to provide
an introduction to ideas which will be utilized later in the dissertation, the basic structure of the distributed parameter observer was presented.
CHAPTER III

Approaches To The Control of Thermal Cycles

In this Chapter, approaches to the implementation of thermal cycle controls using weld pool dimensional measurements are explored. First, the scope of the discussion is narrowed by the selection of one particular measure of the welding thermal cycle to be regulated. Particular weld pool dimensions which are candidates for use in implementation of such controls are also selected. Next a control approach based upon techniques of estimating cooling rate from weld dimension measurements are proposed and discussed.

3.1 Selection of Thermal Cycle and Weld Pool Dimension Measures

The final goal of a thermal cycle control is the production of a weld having specified mechanical and/or physical properties. Clearly, in formulating a thermal cycle control method, one should attempt to regulate a measure of the thermal cycle which has been found to correlate well with the mechanical or physical properties of interest. As was shown in Chapter II, such correlation has been sought for a number of different thermal cycle measures with varying success.

It is important to note that all measures of the welding cycle which have been considered are interrelated to some order of approximation. For example, a first order approximation of time-to-cool from 800 C to 500 C may be expressed in terms of the cooling rate measured at 500 C by the equation
\[ \Delta t |_{800 \rightarrow 500} = \frac{300}{\frac{dT}{dt} |_{T=500}} \] (3.1)

A similar expression approximating the time above a selected temperature in terms of the centerline cooling rate may also be written. These relationships suggest that the choice of exactly which thermal cycle measure is to be controlled may not be critical from a materials properties standpoint.

Primarily because it is the thermal cycle measure most commonly used in the literature, weld–centerline cooling rate at a specified temperature will be used as the measure of the thermal cycle in the remainder of this dissertation.

The next issue to be addressed is the selection of a weld pool dimension or dimensions for use in the control of cooling rate. This question can be decided largely by practical considerations. Although it is feasible to identify the entire weld pool periphery in certain cases, the only weld pool dimension which has been proven to be reliably measureable in real time is maximum weld pool width. For this reason alone, weld pool width will be the measured welding process output in most subsequent discussions. However, since the potential for the extraction of other pool dimensions exists, some results which assume the measurement of other dimensions will be given. For example, weld pool length is another dimension which will be of particular interest because knowledge of it and weld pool width allow the approximation of a third dimensional measure, weld pool area.

3.2 Approaches to the control of cooling rate

We now turn to a discussion of proposed methods for control of centerline cooling rate. The proposed methods will be of basically two types, both utilizing the control philosophy illustrated in Figure 16. The principle feature of this control structure is that, in accordance with the considerations discussed in Chapter I, the
centerline cooling rate of the welding process is not directly measured. Instead, a selected weld pool dimension is measured by a sensor connected to the process, and is input to a welding process model. This model, using the weld pool dimension measurement and values of welding process inputs, generates a cooling rate estimate which is subsequently used in a feedback control of the process.

The key concept embodied in this control approach is the use of a model of the welding process to generate cooling rate estimates. The difference between the two types of controls to be proposed lies in the nature of this model. In the first type, cooling rates are estimated by dynamic observers of the welding heat flow process similar to the types discussed in Chapter II. In the second type, cooling rates are estimated using static relationships which provide correct approximations only when the welding process is operating in the quasi–steady state.

A block diagram of a dynamic observer of the welding heat flow process is
shown in Figure 17. The main component is a dynamic model of the welding process which is capable of generating predictions of temperatures at selected points in the weldment given values of welding process inputs. From these temperature predictions are generated two outputs: a selected weld dimension and centerline cooling rate at a predetermined temperature. The pool dimension output of the model is compared to the value of the corresponding dimension measured from the welding process and the error is then used to correct the process model in some manner. The objective of this correction is to make the weld dimension estimate and measurement correspond more closely and thereby cause the process model to correspond more closely to the actual process. If the output error corrections do indeed improve the accuracy of the process model, the result will be that the cooling rate output of the model will correspond more closely to the actual pro-
cess cooling rate. The development of two observers which operate on this general principle will be described in Chapter IV.

The observer-based approach to cooling rate control described above is certain to be quite computationally intensive regardless of the manner in which it is implemented. Most of the observer computation burden is associated with the welding heat flow model needed to build the observer. As was pointed out in Chapter II, the numerical solution of the welding heat flow equations typically requires a large amount of computation, and may be difficult to implement in a real time control.

Motivated by this consideration, a less complicated observation approach, based upon the formulation of an observer which provides only quasi-steady state estimates of the cooling rate is also proposed. The advantage of this approach would be that the model of the process, being a static relationship involving process inputs and weld pool dimensions, would be much less computationally burdensome and therefore easier to implement as part of a real-time feedback control algorithm. Such an estimator would provide incorrect cooling rate estimates at times when the thermal state contains transient components (e.g. at start up and after process input changes) but would converge to a correct estimate in the quasi-steady state. This approach might be considered adequate for relatively long welds having few changes in process inputs or time-varying disturbances.

In essence, to construct models of the welding process which produce quasi-steady state cooling rate estimates based upon welding process and weld dimension measurements, quasi-steady state versions of the observers described above are needed. Such relationships could be derived from analysis of heat flow relationships. However, estimators derived from empirical measurements of the process would undoubtedly be a more accurate. The development of several static cooling rate estimation relationships by both methods is described in Chapter VI.
Before closing this chapter, it is well to note some of the limitations of the proposed control approach. Of particular import is the fact that both of the proposed approaches make use of welding process models to generate approximations of weld centerline cooling rate. Even though the model is adjusted during operation so as to produce weld pool dimension outputs which are close to those measured from the process, the accuracy of the control is very much dependent upon the accuracy of the process model. If the model does not correspond well to the process, it will generate cooling rate estimates which do not correspond to those experienced by the process, and control accuracy will suffer. Essentially, centerline cooling rate is "outside of the control loop"; its value is not directly measured. This fact is emphasized in the block diagram of the control structure (Fig. 16). The conclusion to be drawn from these facts is that for the proposed control approach to be effective, an accurate model of the welding process must be used.
CHAPTER IV

Formulation of Dynamic Welding Heat Flow Observers

In this chapter, dynamic observers of the welding heat flow process are formulated and the resulting equations are expressed into a form convenient for numerical solution. Basically, two types of dynamic observers will be considered: the first is based upon a formulation very similar to that presented in Chapter II while the formulation of the second falls under the heading of an adaptive observer. To distinguish it from the adaptive observer, the first of the two will be referred to in the following as a conventional observer. To begin with, the formulation and some convergence characteristics of the conventional observer will be presented. Next, the motivation for considering an adaptive observation approach will be discussed, followed by a formulation and discussion of some characteristics of one particular type of adaptive observer. Finally, the conventional and adaptive observer formulations are solved using the method of Green's functions, and the solutions are expressed in a form suitable for numerical calculation.

4.1 Formulation of a Conventional Welding Heat Flow Observer

We begin by considering a model of welding heat flow, written as

\[
\frac{\partial T}{\partial t} = \alpha \nabla^2 T - \frac{2h_s}{\rho C h_0} (T - T_0) + \frac{\dot{q}}{\rho C h} G(x - x_0(V, t),y)
\]

(4.1)

where \( T = T(x,y,t) \)
\[ T(x, y, 0) = T_0 \]

\[ G(\cdot, \cdot) = \text{two-dimensional normalized Gaussian distribution} \]

\[ h_s = \text{surface heat conduction coefficient} \]

The boundary and initial conditions are defined to be

\[
\lim_{x \to \pm \infty} T(x, y, t) = T_0 \quad (4.2a)
\]

\[
\lim_{y \to \pm \infty} T(x, y, t) = T_0 \quad (4.2b)
\]

\[
\lim_{x \to \pm \infty} \frac{\partial T(x, y, t)}{\partial t} = 0 \quad (4.2c)
\]

\[
\lim_{y \to \pm \infty} \frac{\partial T(x, y, t)}{\partial t} = 0 \quad (4.2d)
\]

\[
T(x, y, 0) = T_0 \quad (4.2e)
\]

These equations represent the conduction of heat in a two-dimensional infinite solid from a mobile gaussian-distributed energy source and include convective surface heat transfer. The latter effect will be shown by experimental data (discussed in Chapter V) to be necessary to bring the model into correspondence with real welding processes, so it is included in this development.

Initially, it will be assumed that the temperature of a point fixed in the moving coordinate (i.e. fixed with respect to the moving heat source) may be measured. This assumption will be modified to accommodate measurement of weld pool width later in the development. This system output is expressed by the relation

\[ z(x(t), t) = T(w_1, y_1, t)) \quad (4.3) \]

where \( z(x(t), t) = \text{system output} \)

\( (w_1, y_1) = \text{moving-coordinate point} \)
We emphasize the fact that the point at which temperature is measured, \((w_1, y_1)\) is not stationary in the \((x, y)\) coordinate frame. For convenience of notation, the notation \(z(x(t), t)\) will be abbreviated to \(z(t)\) or just \(z\) in the remainder of this chapter.

Following the discussion of Chapter II, a feedback observer for this system may be represented by the equations

\[
\frac{\partial \hat{T}}{\partial t} = \alpha \nabla^2 \hat{T} - \frac{2h_s}{\rho C_h} (\hat{T} - \hat{T}_0) + \frac{\dot{q}}{\rho C_h} G(z - x_0(V, t), y) + k_o(z - \hat{z})
\]  

(4.4)

where \(k_o\) = output error feedback gain

As in the original system, the observer output is defined to be

\[
\hat{z}(t) = \hat{T}(w_1, y_1, t)
\]  

(4.5)

The initial temperature and boundary conditions for the observer are defined by

\[
\lim_{x \to \pm \infty} \hat{T}(x, y, t) = \hat{T}_0
\]  

(4.6a)

\[
\lim_{y \to \pm \infty} \hat{T}(x, y, t) = \hat{T}_0
\]  

(4.6b)

\[
\lim_{x \to \pm \infty} \frac{\partial \hat{T}(x, y, t)}{\partial t} = 0
\]  

(4.6c)

\[
\lim_{y \to \pm \infty} \frac{\partial \hat{T}(x, y, t)}{\partial t} = 0
\]  

(4.6d)

\[
\hat{T}(x, y, 0) = \hat{T}_0
\]  

(4.6e)

where \(\hat{T}_0\) is not in general equal to \(T_0\).

Note that in both the process and the observer equations, the initial condition is constant over the entire spatial domain. This (not unrealistic) assumption is nearly always employed in the analysis of welding heat flow and will play a very important role in establishing convergence properties of the observer.

We now wish to examine the convergence properties of the observer formulation given above. This will be done by studying the manner in which the state (or
temperature distribution) predicted by the observer converges to the state of the process. Recall that in the control approach outlined in Chapter III, the function of the observer is to produce estimates of the cooling rate of the process. Hence, one is really interested in determining properties of convergence between the estimated and actual cooling rate. However, since the cooling rate is a nonlinear function of the state, this difficult to do analytically. For this reason, we study convergence of the states under the assumption that if the state error is made small enough, the cooling rate error will be likewise reduced.

The error between the process and observer states is defined by $\tilde{T} = T - \hat{T}$ and is given by the differential equation

$$\frac{\partial \tilde{T}}{\partial t} = \alpha \nabla^2 \tilde{T} - \frac{2h_s}{\rho C_h} (\bar{T} - \bar{T}_0) - k_o \cdot \tilde{z}(t)$$  \hspace{1cm} (4.7)

where $\bar{T} = \bar{T}(x, y, t)$

$\bar{T}_0 = T_0 - \hat{T}_0$

$\tilde{z}(t) = z(t) - \hat{z}(t)$

The initial and boundary conditions for the error system are

$$\lim_{x \to \pm \infty} \tilde{T}(x, y, t) = \tilde{T}_0 \hspace{1cm} (4.8a)$$

$$\lim_{y \to \pm \infty} \tilde{T}(x, y, t) = \tilde{T}_0 \hspace{1cm} (4.8b)$$

$$\lim_{x \to \pm \infty} \frac{\partial \tilde{T}(x, y, t)}{\partial t} = 0 \hspace{1cm} (4.8c)$$

$$\lim_{y \to \pm \infty} \frac{\partial \tilde{T}(x, y, t)}{\partial t} = 0 \hspace{1cm} (4.8d)$$

$$\tilde{T}(x, y, 0) = \tilde{T}_0$$  \hspace{1cm} (4.8e)
It will now be shown that the norm of the state of the error system given in Equation (4.7) may be decreased to an arbitrarily small value at an arbitrarily fast rate by adjustment of the feedback gain, $k_o$. It is important to note at this point that the initial condition of the error system (Eq. (4.7)) is not spatially varying. Furthermore, there are no spatially-varying forcing terms in the equation. Therefore, the error system equation reduces to a simple ordinary differential equation and the value of the output error $\bar{z}$ is not dependent upon the point $(w_1, y_1)$ at which it is measured. The error system equation may be written as

$$\frac{d\bar{T}(t)}{dt} + (h'_s + k)\bar{T}(t) = h'_s\bar{T}_0$$  \hspace{1cm} (4.9)

where $h'_s = \frac{2h'}{\rho C h}$

Note that the Laplacian operator which was present in Equation (4.7) does not appear in Equation (4.9); the fact that the observation–error temperature distribution is constant with respect to spatial variables causes it to vanish. The solution of Equation (4.9) may be written as

$$\bar{T}(t) = e^{-(h'_s + k_o)t} \left( 1 - \frac{h'_s}{h'_s + k_o} \right) \bar{T}_0 + \frac{h'_s}{h'_s + k_o} \bar{T}_0$$ \hspace{1cm} (4.10)

The rate of convergence of the state error, determined by the exponent $(h'_s + k_o)$ in the first term, may be adjusted to be arbitrarily fast by increasing the observer feedback gain, $k_o$. This also has the effect of decreasing the steady state value of the error, which is expressed by the latter term in Equation (4.10).

In practice, it is usually desired that the error of the state estimates produced by an observer approach zero over time. The observer of Equation (4.4) may be modified to exhibit this property by feedback of the integrated output error. Such an observer structure is represented by the equations

$$\frac{\partial \bar{T}}{\partial t} = \alpha \nabla^2 \bar{T} - h'_s(\bar{T} - \bar{T}_0) + \frac{\bar{q}}{\rho C h} G(x - x_0(V, t), y) + k_o \cdot \xi(t)$$ \hspace{1cm} (4.11)
where $\zeta(t)$ is the integrated output error, given as

$$\dot{\zeta}(t) = z(t) - \tilde{z}(t)$$

(4.12)

The boundary and initial conditions of Equation (4.11) are identical to those in Equation (4.6a). The initial condition for Equation (4.12) is arbitrarily chosen to be

$$\zeta(0) = 0$$

(4.13)

The error system generated by this observer is described by the differential equations

$$\frac{d\tilde{T}(t)}{dt} + h'_s \tilde{T} = h'_s \tilde{T}_0 - k_0 \zeta(t)$$

(4.14)

and

$$\dot{\xi}(t) = \tilde{T}(t)$$

(4.15)

Again, the assumption of spatially invariant initial temperature has allowed the error–system equations to be reduced to ordinary differential equations. The error system may be rearranged to show that

$$\ddot{\zeta}(t) + h'_s \dot{\zeta}(t) + k_0 \zeta(t) = h'_s T_0$$

(4.16)

This system is guaranteed to be stable if the homogeneous system has no right half–plane poles, which is equivalent to the requirement that

$$Re \left( -h'_s \pm \frac{h'_s^2 - 4k_0}{2} \right) < 0$$

(4.17)

which is true for $k_0 > 0$. If this is the case, the steady state solution may be written as
\[ \lim_{t \to \infty} \xi(t) = \frac{h'_s}{k_o} \tilde{T}_0 \]  \hfill (4.18)

Since \( \tilde{T}_0 \) is a constant, this implies (by virtue of Eq. (4.15)) that

\[ \lim_{t \to \infty} \tilde{T}(x, y, t) = 0 \]  \hfill (4.19)

Note that the dynamics of the error system are more complicated in comparison to the previous case, being represented by a second-order, rather than first-order, ordinary differential equation. However, the dynamics of this system may be still be adjusted by variation of the observer feedback gain.

The observer formulation presented above uses the temperature of a point fixed in the moving coordinate as the measured system output. In keeping with the goals of this dissertation, it is desirable that this output be replaced with a weld pool dimension measurement. A proposed formulation for such an observer is written as

\[ \frac{\partial \hat{T}}{\partial t} = \alpha \nabla^2 \hat{T} - h'_s (\hat{T} - \tilde{T}_0) + \frac{\dot{q}}{\rho C} G(x - x_0(V, t), y) + k(\xi(t)) \]  \hfill (4.20a)

\[ \dot{\xi}(t) = w(t) - \dot{w}(t) \]  \hfill (4.20b)

where \( \hat{T} = \hat{T}(x, y, t) \)

\( w(t) \) = weld pool width

\( \dot{w}(t) \) = observer weld pool width estimate

The next question to be addressed is whether or not the convergence properties of the observer of Equation (4.20a) are similar to those of the observer which was analyzed earlier. Although simulation results which will be presented in Chapter VII will show that the observation error for Equation (4.20a) is convergent to zero,
the convergence analysis given above may also be applied. To do this assume that for small preheat error, weld width error may be approximated as

$$\tilde{w}(t) = k_w \tilde{T}(t) + O(\tilde{T}^2) \quad \tilde{T}(t)\text{small}$$

(4.21)

where $k_w$ is a positive constant.

In words, we assume that for small observation errors, the difference between observer and process pool widths is approximately proportional to the difference in observation state error.

The above relationship allows the same demonstration of convergence which was presented above to be applied to the observer of Equation (4.20a) with obvious minor modifications. The demonstration is valid only for small $\tilde{T}(t)$. As was stated once before, simulation results will be relied upon for final demonstration of observation error convergence.

4.2 Formulation of an Adaptive Observer

In this section, an adaptive observer of the welding heat transfer process is formulated and discussed. Before setting out on this task, however, the motivation for investigating adaptive observers is outlined.

The basic assumptions which are made when (non-adaptive) observers are applied in control problems are that an accurate model of the process is available, but that the initial condition of the process is poorly known. Essentially, the underlying principle upon which observers operate is that by correction of the initial condition of an accurate process model, the state of the process model is made to converge to the state of the process. Although it may not be obvious from the formulation given in the preceding section, the observer which was formulated essentially does what was just stated; corrects the initial temperature (or preheat
temperature) of a model of the welding heat flow process so as to cause output error between observer and process to decrease.

Whether or not adjustment of the preheat of a welding heat flow model will cause the model to generate good estimates of weld centerline cooling rate (which is, after all, the goal to be achieved) depends upon several considerations. Most importantly, it depends upon the accuracy with which the welding process is modeled. The heat transfer processes which occur during welding are notoriously difficult to model, and it is virtually guaranteed that any model which might be used to construct an observer will not accurately represent some aspect of the process. If it happens that the initial temperature of the weldment is known well (which is often the case), but that errors between process and model weld pool widths occur because of other modeling errors (such as inaccurate knowledge of thermal properties, etc.), adjusting the initial temperature of the process model may actually produce worse estimates of cooling rate than would result if no correction at all were performed. Thus, the question which is raised is one of robustness of the observer cooling rate estimates to errors in process modeling.

In fact, there is some evidence to indicate that the observer formulation which was presented in the previous section may result in cooling rate estimates which are very sensitive to modelling errors. To illustrate this, we make use of the ideal heat flow relationships representing weld pool width and centerline cooling rate in terms of welding process inputs, thermal properties and initial temperature. These relationships were discussed in Chapter II. Although it is known that these relationships do not accurately model the welding process, it has been found that they correctly predict general trends and the qualitative effects of process variables. Thus, they may be used to discuss in general terms, the operation of the observer which was formulated in the previous chapter.
Of particular interest are Equation (2.20), which is an expression for weld pool width and Equation (2.21) which expresses weld cooling rate. For convenience, these two equations are repeated below

\[
d = \frac{4\alpha}{V} \left( \frac{\dot{q}}{8K(T_m - T_0)h} - \frac{1}{5} \right) \tag{4.22}
\]

\[
\frac{\partial T}{\partial t} \bigg|_{T=T'} = 2\pi K \rho C h^2 \left( \frac{V}{\dot{q}} \right)^2 (T' - T_0)^3 \tag{4.23}
\]

Note that weld width and cooling rate depend upon, for example, thermal conductivity \( K \) in approximately an inversely proportional and proportional fashion, respectively. Assume that in the implementation of the observer, a value of thermal conductivity \( K \) which does not exactly match that of the process is used. By virtue of this fact, the pool width and cooling rate predictions of the observer will not match those of the process. To emulate the operation of the closed-loop observer, we then adjust the preheat temperature of the observer to produce a weld pool width estimate matching that of the process. The effect of this preheat temperature adjustment upon the accuracy of the preheat prediction of the observer can then be assessed using the second equation. Note that this expression indicates that cooling rate depends upon initial temperature to the third power. Thus, cooling rate is much more sensitive to preheat temperature variation than is pool width. Hence, adjusting the preheat temperature to make estimated and measured pool widths equal would cause what was initially a small cooling rate estimation error to drastically increase.

The above considerations suggest that construction of an observer algorithm which adjusts some model parameter other than preheat may produce an observer which is more robust with respect to modelling errors. This would result in (as it is known in the literature) an adaptive observer.
Again using the above ideal quasi-steady state heat flow relationships, the argument can be made that adjustment of arc efficiency represents a plausible alternative to preheat adjustment. Note that in Equation (4.22), \( \dot{q} \), which represents the product of arc efficiency, current and voltage, appears to the first power. In Equation (4.23), it appears raised to the negative-second power. Hence, arc efficiency correction would represent a compromise between adjustment of thermal properties and initial temperatures. If preheat errors did in fact exist between model and process, adjustment of arc efficiency would have more of an effect than would variation of a thermal parameter such as conductivity. On the other hand, if errors in thermal characteristics or other parameters existed, correction of arc efficiency would not cause as severe sensitivity problems as initial temperature adjustment.

Aside from the above arguments, it is also well to note that arc efficiency is known to vary with welding process inputs such as travel speed, arc current and electrode geometry. That this is the case was briefly discussed in Chapter II. Thus, an adaptive welding heat flow observer which adjusts arc efficiency based upon output error measurements would likely be applicable over a wider range of process input values than would be the case otherwise.

For the reasons discussed above, an adaptive observer algorithm in which arc efficiency is adjusted in response to weld dimensional output errors is proposed. In particular, the observer which is proposed is the following:

\[
\frac{\partial \hat{T}}{\partial t} = \alpha \nabla^2 \hat{T} - h_s'(\hat{T} - \hat{T}_0) + (\hat{n} + k_0 \xi(t)) \frac{\dot{q}_t}{\rho C h} G(x - x_0(V, t), y) \tag{4.24a}
\]

\[
\dot{\xi}(t) = w(t) - \hat{w}(t) \tag{4.24b}
\]

where \( \hat{T} = \hat{T}(x, y, t) \)
\[
\dot{q}_t = EI
\]
\[ \hat{\eta} = \text{observer arc efficiency} \]

Here, \( \hat{\eta} \), represents the total rate of energy expenditure in the welding arc. Recall that in Chapter II, the effective arc energy \( \hat{\eta} \) was defined as \( \hat{\eta} = \eta EI = \eta \hat{q}_T \). The initial conditions for the observer are defined as

\[
\lim_{x \to \pm \infty} \tilde{T}(x, y, t) = T_0 \quad (4.25a)
\]

\[
\lim_{y \to \pm \infty} \tilde{T}(x, y, t) = T_0 \quad (4.25b)
\]

\[
\lim_{x \to \pm \infty} \frac{\partial \tilde{T}(x, y, t)}{\partial t} = 0 \quad (4.25c)
\]

\[
\lim_{y \to \pm \infty} \frac{\partial \tilde{T}(x, y, t)}{\partial t} = 0 \quad (4.25d)
\]

\[
\tilde{T}(x, y, 0) = T_0 \quad (4.25e)
\]

This observer differs from the one considered previously in several respects. Note that the observer arc efficiency is not assumed to be equal to process arc efficiency, and that a correction term proportional to the integrated observer output error is added to the observer arc efficiency. Also, the initial conditions of the observer are taken to be equal to those of the process.

The next issue to be addressed is whether or not the same convergence properties which were demonstrated for the conventional observer hold for this adaptive observer. An analytical demonstration of such properties could not be found, but the convergence will be studied by simulation as described in Chapter VII. However, a few points concerning this question are discussed in the following.

As before, the observation error can be defined as \( \tilde{T}(x, y, t) = T(x, y, t) - \hat{T}(x, y, t) \). Using Equations (4.1) and (4.24a), we obtain the error system
\[ \frac{\partial \tilde{T}}{\partial t} = \alpha \nabla^2 \tilde{T} - k_s'(\tilde{T} - \tilde{T}_0) + \frac{\dot{q}_t}{\rho C_h} (\eta - \dot{\eta} - k_o \xi(t)) G(x - x_0(V(t), t), y_0) \quad (4.26) \]

It is difficult to establish that the state of this error system converges to zero in norm as time grows large. Note that this system may not be simplified to an ordinary differential equation as was the case previously since this error system contains a spatially-varying forcing term. However, if it is conjectured that the system does have a steady state, then it follows that

\[ \lim_{t \to \infty} \frac{\partial \tilde{T}}{\partial t} = 0 \quad (4.27) \]

It is reasonable to conjecture that since the system forcing term has time-varying components which affect the system in fundamentally different ways, and since the travel speed is not generally equal to zero, that if a steady state does exist then the following facts are true:

1. \[ \lim_{t \to \infty} \xi(t) = \text{constant} \]

2. \[ \lim_{t \to \infty} (\eta - \dot{\eta} - k_o \xi(t)) = 0 \]

The first limit indicates that the pool width error approaches zero so that its integral approaches a constant, and the second requires that the effective efficiency of the observer (\(\dot{\eta} + \text{a correction term}\)) equal the efficiency of the process. If these two expressions converge relatively quickly, the forcing term is removed from the error system which then decays at the rate naturally determined by the conduction of heat by the plate and convective heat loss to the atmosphere. Thus, it is reasonable to suppose that if a value of feedback gain can be found for which the effective observer efficiency quickly converges to a constant, then the observation error of the adaptive observer will decay to zero, but at a rate determined by the physical nature of the system rather than one determined by the feedback gain.
4.3 Formulation of a Welding Heat Flow Simulation

Before embarking upon a description of numerical solution of the dynamic observer formulations, we note that a principle component of the distributed parameter observer is a model of the process. A feedback correction term involving the error between the outputs of the model and process is added to form an observer. Hence, a simulation of the welding process may be obtained from either of the implementations of observers presented in this chapter by removal of the feedback correction term (or by setting the observation–error feedback gain $k_o$ to zero, which has the same effect). The simulation thus obtained will be used in the studies described in Chapter VII where the design and performance of feedback cooling rate controls are investigated.

4.4 Numerical Solution of the Observer Equations

Having established observer formulations, the next issue to be addressed is the numerical implementation of these systems.

At least three methods have been applied to the solution of the welding heat flow equations: finite element approximation, finite difference and numerical integration of the Green’s function. Each of these techniques has unique advantages and disadvantages. In this dissertation, a solution is required to fulfill both the roles of heat flow observer and simulation for control testing. Therefore, the criteria which are of importance in the selection of a solution technique are:

1. reasonably accurate simulation of the full time-varying welding temperature distribution response

2. ability to accommodate time varying (more precisely, piece-wise constant) welding inputs
3. ability to provide weld pool dimensions and centerline cooling rate as outputs

4. reasonable execution time and computer memory demands

After consideration of the literature and advice from an expert in the field [41], a simulation based upon integration of Green's functions was chosen. As was noted in Chapter II, this simulation is not capable of accounting for non-linearities as easily as the finite element and finite difference types. However, it does have the important advantage of allowing faster execution times. The excellent correlation with experimental data that Eager and Tsai [17] and Bates and Hardt [18] were able to obtain using the ideal solution also argued for the selection of this simulation method.

In the remainder of this section, the solution and implementation of the observer and simulation equations is presented. The solution to the conventional observer equations represented by Equation (4.20a) will be obtained in two steps. First, a solution which takes into account the arc heat input term but does not include the output error feedback term present in the conventional observer formulation will be obtained. Next, a solution which includes the error feedback term of the conventional observer but omits the arc heat input term will be determined. The two solutions are then added to to yield the complete solution to the conventional observer equations. Coincidentally, the first part of the conventional observer solution represents a simulation of welding heat flow and will be used as such in Chapter VII. The first part of the conventional observer solution can also be modified to account for time-variation of arc energy and hence may also be used to provide a solution of the adaptive observer equations. A solution of the adaptive observer equations obtained in this way will be presented.
The basis of the first portion of the solution of the conventional observer equations is the Green's function for the flow of heat from a point source in a two-dimension solid with surface heat loss. This is given by Carslaw and Jaeger [16] as

$$T(x, x', y, y', t, t') - T_0 =$$

$$\frac{q}{4\pi hK(t - t')} \exp \left( -h_s'(t - t') - \frac{(x - x')^2 + (y - y')^2}{4\alpha(t - t')} \right)$$  \hspace{1cm} (4.28)

where \((x, y, t) = \text{space/time coordinates of the point of interest}\)

\((x', y', t') = \text{space/time coordinates of the point of heat release}\)

The first step in the formulation is the inclusion of a gaussian arc heat distribution. Such a distribution may be represented as

$$G(x' - x_0, y' - y_0) = \frac{1}{2\pi \sigma^2} \exp \left( -\frac{(x' - x_0)^2 + (y' - y_0)^2}{2\sigma^2} \right)$$  \hspace{1cm} (4.29)

where \((x_0, y_0) = \text{center of the gaussian distribution}\)

\(\sigma^2 = \text{the variance of the distribution}\)

An equation analogous to (4.28), but for a Gaussian heat source rather than a point heat source, may be obtained by substituting (4.29) for \(q\) in (4.28) and integrating over the \((x', y')\) coordinates. The resulting expression is

$$T(x, y, t, t') - T_0 = \int_\infty^\infty \int_\infty^\infty \frac{q \exp \left( -\frac{(x' - x_0)^2 + (y' - y_0)^2}{2\sigma^2} \right)}{8\pi^2 hK\sigma^2(t - t')} \exp \left( -h'_s(t - t') - \frac{(x - x')^2 + (y - y')^2}{4\alpha(t - t')} \right) dx'dy'$$  \hspace{1cm} (4.30)
Performing the integration (shown in Appendix A) results in the solution:

\[ T(x, y, t') - T_0 = \]
\[ \frac{\dot{q}}{\pi \rho C (2\sigma^2 + 4\alpha(t - t'))} \exp \left( -h^'s(t - t') - \frac{(x - x_0)^2 + (y - y_0)^2}{2\sigma^2 + 4\alpha(t - t')} \right) \]

(4.31)

This solution may be modified to account for the motion of the Gaussian heat source and integrated with respect to time to obtain the time–varying temperature distribution. If the travel speed is constant, the motion of the heat source is accounted for by setting \((x_0, y_0) = (Vt', 0)\). However, in order to preserve the option of using travel speed as a control input to the process, the observer solution will be obtained for the case where travel speed is assumed to be piecewise constant over regular intervals, \(\Delta t\). This is the type of travel speed input which is obtained when travel speed is computed via a digital feedback algorithm and input to the welding process through an analog to digital converter with a zero-order hold. Under this assumption, an expression for \(x_0\) at time \(t'\) is

\[ x_0(t') = \sum_{k=0}^{N-2} V_k \Delta t' + \int_{(N-1)\Delta t'}^{t'} V_{N-1} \, dr \]

or

\[ x_0(t') = \sum_{k=0}^{N-2} V_k \Delta t' + V_{N-1}(t' - (N - 1)\Delta t) \]

where \(t' \in [(N - 1)\Delta t', N\Delta t')\)

and \(V_k = V(t)\) for \(t \in [(k)\Delta t', (k + 1)\Delta t']\)

Finally, we insert the above expression into Equation (4.31) and integrate with respect to time, obtaining the following expression for the time–varying temperature distribution at a point:
\[ T(x, y, t) - T_0 = \frac{q}{\pi \rho C \Delta t} \sum_{j=0}^{N-1} \int_{j\Delta t}^{(j+1)\Delta t} \exp \left( -h'_e(t - t') - \frac{(x - \sum_{i=0}^{j-1} V_i \Delta t' - V_i(t' - j\Delta t))^2 + (y - y_0)^2}{2\sigma^2 + 4\alpha(t - t')} \right) dt' \]

where \( t \in [(N - 1)\Delta t, N\Delta t) \).

As it is written, Equation (4.32) expresses the temperature of a point in the fixed coordinate system at a given time \( t \) after the start of welding. For calculation of weld pool dimensions, the temperatures of points fixed with respect to the heat source (i.e. fixed in the moving coordinate frame) are needed. The temperatures at points \((w, y)\) in the moving frame were simulated by calculating their corresponding fixed frame location \((x_k, y_k)\) at each simulation iteration, \( k \). This was accomplished using the relation \((x_k, y_k) = (w + \sum_{j=0}^{k-1} V_i \Delta t, y)\).

where \((x_k, y_k) = \text{fixed frame location corresponding} \)

\[ \text{to the point } (w, y) \text{ at simulation iteration } k. \]

This completes the solution of the first portion of Equation (4.20a).

To obtain the second portion of the solution of Equation (4.20a), we need to solve

\[ \frac{\partial \hat{T}}{\partial t} = \alpha \nabla^2 \hat{T} - h'_e(\hat{T} - \hat{T}_0) + k_c \xi(t) \]

\[ \xi(t) = w(t) - \dot{w}(t) \]

under the boundary and initial conditions

\[ \lim_{x \to \pm \infty} \hat{T}(x, y, t) = 0 \]

\[ \lim_{y \to \pm \infty} \hat{T}(x, y, t) = 0 \]
\[ \lim_{x \to \pm \infty} \frac{\partial \hat{T}(x, y, t)}{\partial t} = 0 \]  
(4.34c)

\[ \lim_{y \to \pm \infty} \frac{\partial \hat{T}(x, y, t)}{\partial t} = 0 \]  
(4.34d)

\[ \hat{T}(x, y, 0) = T_0 \]  
(4.34e)

\[ \xi(0) = 0 \]  
(4.34f)

Due to the lack of spatial variation in the initial conditions and boundary conditions of Equation (4.33a), the equations may be simplified to

\[ \frac{d\hat{T}}{dt} + h_\lambda(T - T_0) = k_0 \cdot \xi(t) \]  
(4.35a)

\[ \dot{\xi}(t) = w(t) - \dot{\omega}(t) \]  
(4.35b)

The solution to these is written as

\[ T(x, y, t) = k \int_0^t e^{-h_\lambda(t-\tau)} \xi(\tau) d\tau \]  
(4.36a)

\[ \xi(t) = \int_0^t w(\tau) - \dot{\omega}(\tau) d\tau \]  
(4.36b)

This completes the solution of the second portion of Equation (4.20a).

As was noted earlier, the solution of the conventional observer equations is obtained by simply adding Equations (4.36a) and (4.32).

To obtain the adaptive observer solution from the above solution, one first must make provision for the time-variation of the arc energy. Assuming that arc energy is to vary in a piece-wise constant fashion, this is accomplished very simply by moving total arc energy term to the inside of the integral sign and multiplying it by the corrected observer arc efficiency. This results in the following system of equations

\[ T(x, y, t) - T_0 = \frac{1}{\pi h_\rho c} \sum_{j=0}^{N-1} \int_{j\Delta t}^{(j+1)\Delta t} (\hat{\eta} + k_0 \xi_k) q_i \]
\[
\exp \left( -h_s'(t - t') \cdot \frac{(x - \sum_{k=0}^{j-1} V_k \Delta t' - V_j(t' - j \Delta t))^2 + (y - \delta_0)^2}{2\sigma^2 + 4\alpha(t - t')} \right) dt' \tag{4.37a}
\]
\[
\dot{x}(t) = w(t) - \dot{w}(t) \tag{4.37b}
\]

The solution to Equation (4.37b) is the same as expressed in Equation (4.36b).

In order to implement the solutions represented by the above equations, the time integrals contained in them were approximated numerically. There are a large number of methods for numerically approximating integrals. In this case, the integrals contained in Equation (4.32) and (4.37a) were approximated using a three-point Gaussian quadrature algorithm [42] because of its computational efficiency and accuracy. The integrals of Equation (4.36a) and (4.37b) were approximated using Euler integration.

In the Gaussian quadratures integration method, the integral of a function \( f(x) \) is approximated by the formula
\[
\int_{-1}^{1} f(x) dx = .555 \cdot f(-.77459667) + .888 \cdot f(0) + .555 \cdot f(.77459667) \tag{4.38}
\]
When the integration is between arbitrary real numbers \( x = a \) and \( x = b \), the change of variables
\[
x = \frac{(b - a)z + b + a}{2} \tag{4.39}
\]
is applied. Note that \( x \in (a, b) \Rightarrow z \in (-1, 1) \). The Gaussian quadrature method was used to integrate Equation (4.32) by approximating each of the integrals under the sum.

The above solutions allow calculation of the temperatures at selected points. However, to be useful as observers, the final implementations of the solutions must be capable of calculating outputs of interest to this dissertation, namely weld pool...
Figure 18: Location of the array of points used to simulate weld pool width dimensions and centerline cooling rate. We now discuss how these outputs can be calculated from the temperatures at selected points in the heat flow domain.

An estimate of the maximum weld pool width was derived from the temperatures of a rectangular array of points, fixed in the moving frame and situated to one side of and behind the heat source center. The location of this array of points is shown in Figure 18. The pool width was calculated from the temperatures of points in the array using the following method. At every time iteration of the simulation, each row of the array was searched for the y-coordinate location of the point having temperature equal to the melting temperature for the material under consideration. The y-coordinate location was found using linear interpolation when it did not correspond exactly to one of the simulated points. After this process was repeated for every row in the array, the y-coordinate locations for each row were compared, and the largest one was found and doubled, the result being
the weld pool width estimate.

The weld pool length was calculated in a manner similar to that used to calculate the weld pool width. The temperatures of two columns of points lying on the moving frame y-axis ahead of and behind the heat source center were simulated. The locations of these points, which were fixed in the moving frame, are shown in Figure 18. At each simulation iteration, the columns of points ahead of and behind the heat source center were searched for the y-coordinate location of the points having temperature equal to the melting temperature of the material under consideration. This was done using linear interpolation when this point did not correspond to one of the simulated points. When the location of the melting temperature point was found for both columns, the two values were added together to yield the weld pool length.

The centerline cooling rate at a specified temperature was calculated from the temperatures of a column of points lying behind the heat source center on the moving frame y-axis. The location of this column of points is shown in Figure 18. The cooling at a temperature $T'$ was calculated from a finite difference approximation of the derivative

$$\frac{\partial T(x, y)}{\partial t} \bigg|_{T'}$$

(4.40)

At iteration $k$, the column of points was searched for the y-coordinate, $y'_k$ of the location having temperature $T'$. The point $y'_k$ was found using linear interpolation when it fell between the location of simulated points as shown in Fig. 18. This location was saved for use in the subsequent iteration. Next, the temperature of the point having y-coordinate location $y'_{k-1}$ was found. The time derivative in Equation (4.40) was then approximated by a backwards difference of the present- and previous-iteration temperature at the point $(w, y) = (0, y'_{k-1})$.  

80
\[ \frac{\partial T(w, y)}{\partial t} \bigg|_{T'} \approx \frac{T_k(w, y'_{k-1}) - T'}{\Delta t} \]  

(4.41)

The location of the column of points used to calculate the cooling rate was not fixed with respect to the moving frame: its position was adjusted automatically by the simulation software if conditions warranted. If, during the search for \( y' \), (the point having temperature equal to \( T' \), it was found to lie outside of the bounds of the simulated column of points, then the position of the column was shifted in the appropriate direction by the value \( \Delta y \). This method reduced the number of points which were required in the column of points used for the cooling rate calculation.

The welding heat flow simulation and observer formulated above were implemented on a VAX 11/785 digital computer in the Fortran programming language (listings of this program are shown in Appendix B). The simulation time increment, \( \Delta t \) was taken to be two seconds. The steps performed at each simulation iteration \( k, k = 1,..N \) are summarized as follows:

1. Calculate the \( k^{th} \)-iteration fixed-frame locations of points which are specified in the moving frame using \( (x, y)_k = (w + \sum_{i=0}^{k-1} V_i \Delta t, y) \).

2. Calculate the \( k^{th} \)-iteration temperatures at all points using Gaussian quadratures integration.

3. Calculate the outputs of interest, such as weld pool width and length and centerline cooling rate.

4. Obtain the travel speed and welding arc current for the \((k + 1)^{th}\) iteration, for example from a control algorithm or an open loop law.

5. Increment a time counter by \( \Delta t \) seconds and repeat the above steps
The time required to compute the above algorithm is not constant, but increases linearly with the number of iterations. This must be regarded as a disadvantage for an algorithm proposed for real-time execution. However this problem may be circumvented by "forgetting" old input values. It is clear that after a sufficient amount of time has passed, there is no need to continue to include the first values of the welding inputs in the summation. From simulation results, it was found that forgetting of inputs more than 250 seconds old had no influence upon the weld pool dimensional or cooling rate outputs, so the simulation and observer were programmed to forget any input older than this.

Under this modification, the simulation/observer was found to execute at the rate of approximately 2 seconds per iteration. Since each simulation iteration also corresponds to the passage of 2 seconds of process time (recall that $\Delta t$ was chosen equal to 2 seconds), the simulation was capable of running in "real time". This was accomplished while calculating both the weld pool width and length dimensions and cooling rate; reductions in computing time of approximately 30 percent would be achieved by eliminating any one of these calculations.

In most cases, it would be unreasonable to require that the computing power of a VAX 11/785 be installed in a process controller (although due to advances in microprocessor technology, this is perhaps not as extreme as it sounds). However, it is important to note that the observation algorithm as implemented above is highly parallel. The temperature of each point can be computed completely independently of the other. Moreover, each time iteration step of the calculation of the temperature at each point can be performed independently of the others. This fact suggests that the observer could be implemented to run in a sufficiently small amount of time using relatively inexpensive microcomputers configured in a parallel processing arrangement.
CHAPTER V

Experimental Testing of the Heat Flow Simulation

As has been pointed out previously, a simulation of welding heat flow was obtained as part of the solution of the conventional observer equations presented in the previous chapter. This simulation represents the "heart" of the observers, and their accuracy is in large part determined by the accuracy of the simulation. For this reason, the simulated values of certain welding process outputs were compared to experimentally measured values to help determine the degree to which the simulation represents actual welding processes. In this section, the procedures by which the experimental measurements were obtained are described.

The general thrust of the comparisons were to verify that the simulation provided reasonable approximations of the outputs of interest in this dissertation (i.e. weld pool dimensions and centerline cooling rates). The following are the welding process outputs which were measured experimentally and compared to simulation outputs:

1. quasi-steady state weld pool width
2. quasi-steady state weld pool length
3. quasi-steady state centerline cooling rate
4. dynamic (step input) weld pool width response
The choice of the outputs which were experimentally measured was somewhat restricted by experimental practicalities. For example, it would have been desirable to compare predicted and actual time-varying cooling rate responses. However, this was not done due to the experimental difficulties encountered in the measurement of time-varying cooling rates. Time varying weld pool length responses were not compared for the same reason. Nevertheless, it was felt that comparison of the listed outputs provided a sufficient test of the accuracy of the simulation.

5.1 Experimental procedure

Experimental welds were performed on a medium alloy steel (AISI 4130) and in lesser numbers on a precipitation hardening aluminum alloy (2219). The steel was obtained in the form of 0.090 inch-thick strip in a subcritical annealed condition. In this state, the steel is relatively soft but possesses maximum hardenability. The aluminum was supplied in a precipitation hardened state in the form of 0.125 inch-thick plates. In the hardened state, the aluminum is at or near its maximum attainable hardness.

Since steel and aluminum have drastically different thermal properties, performing test welds on both of these materials allowed the simulation to be tested over a wider range of conditions than would have been possible if only one material had been used. AISI 4130 steel was chosen because its hardness is quite sensitive to cooling rate. It was hoped that this sensitivity would allow cooling rate to be measured indirectly through WMZ or HAZ hardness. The particular alloy of aluminum used (2219) was chosen because it was readily available.

All welds performed for these experiments were made using the GTAW process, and welding parameters were selected to ensure that all welds were full penetration, thus maintaining two-dimensional heat flow conditions. The welds were
all autogeneous (i.e. no filler wire was used). The AISI 4130 steel welds were made using the parameters listed in Table 1. The parameters used for the aluminum welds are shown in Table 2.

The fixture which was used to hold the specimens during welding is shown in Figure 19. This fixture was constructed of aluminum to minimize magnetic welding arc deflection. The specimens were lightly clamped between 0.25 inch-thick sheets of Ceramfab™ insulating material to minimize surface heat loss. Due to their narrowness, two of the steel specimens were butted tightly along their length in the fixture and the weld was made along this seam. The aluminum specimens, being wider, were welded singly with a bead down the center. The base metal electrical connection necessary for the flow of welding current was made through a copper clamp located at the stop end of the piece (the end opposite from where welding was started). The area of contact of the ground clamp with the welding
Table 2: Welding Parameters for 2219 Aluminum Welds

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>arc current</td>
<td>40 amperes</td>
</tr>
<tr>
<td>arc voltage</td>
<td>18.5 volts</td>
</tr>
<tr>
<td>travel speed</td>
<td>4 - 5 in./min</td>
</tr>
<tr>
<td>shielding gas</td>
<td>He @ 30 cu. ft./min.</td>
</tr>
<tr>
<td>back shield gas</td>
<td>Ar @ 30 cu. ft./min.</td>
</tr>
<tr>
<td>electrode</td>
<td>W / 2% Th., 0.125 in. diameter</td>
</tr>
<tr>
<td>wire feed</td>
<td>60 degrees included tip angle</td>
</tr>
<tr>
<td>nominal specimen size</td>
<td>6.0 in. x 12.0 in. x 0.125 inch-thick</td>
</tr>
</tbody>
</table>

Figure 19: Welding Fixture
plate was minimized so as to prevent excessive heat loss.

For some of the trials, the base metal was preheated prior to welding. This was accomplished using two 500-watt photographic flood lamps trained on the base metal from a distance of approximately 6 inches. The surface temperature of the plates was monitored using a digital thermometer and welding was commenced when all points on the plate were greater than or equal to the desired temperature. By adjusting the position of the lamps and monitoring the temperature distribution in the plate, the preheat was kept as uniform as possible. Welding was begun as soon as possible after the removal of the preheat source, typically within 30 seconds.

Weld bead width and weld pool length were measured for each of the specimens. Using a 6X optical comparator, the bead width was measured at five points along the length of each bead with an estimated accuracy of ± 0.0025 in. These measurements were taken at points sufficiently removed from the start end of the plates so that quasi-steady state dimensions were obtained. The five width measurements were averaged to obtain the recorded value of weld bead width for each bead. The pool length was measured from the pool outline visible in the solidified crater at the end of the weld using the 6X comparator. Since the weld crater was only visible at the "stopping" end of the weld, it was not possible to obtain multiple measurements of weld pool length from each weld.

The thermal cycles which occurred on the centerline of each weld at a point four inches from the starting end were obtained by plunging a Tungsten+5% Rhenium/Tungsten+26% Rhenium thermocouple into the weld pool from the backside. The thermocouple was inserted into a hole in the bottom of the welding fixture (shown in Fig. 19) prior to welding, and when the welding arc and molten pool passed over the thermocouple location, the junction was manually inserted approximately 1 mm. into the pool from the bottom side. The weld metal subsequently
solidified around the thermocouple junction, ensuring an intimate, low thermal-
resistance connection. Two different methods of recording the thermocouple out-
put voltage were used. In the first, the thermocouple junction was connected via
extension wires to a ice point reference junction (constructed in accordance with
ASTM specifications [43]) and then to the differential inputs of a Western Digital
X-Y recorder. The details of this circuit are shown in Figure 20. The X-Y recorder
X axis was connected to a time base (generated within the recorder) and the Y-axis
to the thermocouple output.

A sketch of a thermal cycle measured using the setup described above is
shown in Figure 21. This sketch shows thermocouple output millivoltage (denoted
by $E_{TC}$ versus time. For all traces, the time-scale factor was 0.5 cm./sec. and
the voltage-scale factor was either 1.0 mv./cm. or 0.5 mv./cm. The method used
to calculate cooling rate from the thermal cycle plot is illustrated in the figure.
Figure 21: Typical Thermal Cycle

Using a thermocouple millivolt to temperature conversion table, the thermocouple output voltage corresponding to the temperature at which the cooling rate was to be measured was found. The point corresponding to this thermocouple voltage was found on the thermal cycle and the time derivative at this point was approximated using the difference equation:

\[ \frac{dT}{dt}|_{T'} \approx \frac{E_{TC}(t' + \Delta t) - (E_{TC}(t' - \Delta t))}{2 \cdot \Delta t} \cdot K_1 \]  \hspace{1cm} (5.1)

where \( \Delta t \) = a selected time interval

\( t' \) = time corresponding to \( T = T' \)

\( E_{TC} \) = thermocouple EMF

\[ K_1 = \frac{dT}{dE_{TC}}|_{T'} \]

\[ \frac{dT}{dE_{TC}}|_{T'} \) = slope of the thermocouple EMF vs. temperature characteristic at \( T' \)
Figure 22: Microcomputer thermal-cycle data-logging system

In this calculation, $\Delta t$ was taken to be 2.5 seconds, and the critical temperature $T'$ was set to 500 C for the steel tests and 300 C for the aluminum tests. The error in calculating cooling rate in this manner was estimated to be $\approx 0.5$ C/sec., solely due to measurement errors in reading thermocouple voltages from the X-Y recordings for the slope calculation in Equation (5.1). Measurement errors due to thermocouple nonlinearities, inaccuracies in instrument calibration and other factors are not included in this estimate.

The second method used to record weld thermal cycles via thermocouple output voltage measurements was based upon a microcomputer data-logging system. This method of measurement was instituted to eliminate the inconvenience of maintaining an ice point reference, to expedite cooling rate calculation and to reduce errors associated with the slope calculation referred to above. A schematic of this system is shown in Figure 22. The thermocouple junction was connected to an
electronic ice-point reference junction compensating device, the output of which was fed through a precision isolation amplifier and then to the single-ended inputs of a 12-bit analog to digital (A/D) converter system. The A/D converter resided in an Apple IIe microcomputer.

Microcomputer software was written to automatically sample and store the thermal cycle at 0.1 second intervals, to calculate cooling rate and to store the thermal cycle information on floppy disk for future reference. The listings of this software are shown in Appendix C. The method of cooling rate calculation used in the programs was essentially the same as the manual method used to calculate cooling rate from X-Y recordings. The point in the sampled thermal cycle corresponding to the critical temperature \( T' \) was identified, and samples one second on either side of this were converted to temperatures using the known gain of the A/D converter isolation amplifier system along with a thermocouple conversion table stored in computer memory. The cooling rate was then calculated using the central difference approximation of Equation (5.1).

The HAZ hardnesses of the specimens were tested using a Wilson tester with a Braille indenter. For the steel specimens, the Rockwell "C" scale was employed; the Rockwell "A" scale was used to measure aluminum HAZ hardnesses. The hardness was taken as the average of three readings made at a point approximately 4 inches from the start end of the bead and within 1 mm. of the fusion line.

The dynamic response of the weld pool width was studied by introducing step perturbations in arc current at several different travel speeds. The corresponding changes in top-side weld width were measured using the optical comparator at 0.125 in. intervals along the weld length. These tests were performed only on AISI 4130 steel. The nominal values of travel speed and arc current as well as the step perturbation magnitudes are summarized in Table 3.
Table 3: Welding Conditions for Step Response Tests

<table>
<thead>
<tr>
<th>travel speed (in./min.)</th>
<th>current perturbation (amperes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>95 → 110</td>
</tr>
<tr>
<td>3</td>
<td>110 → 95</td>
</tr>
<tr>
<td>4</td>
<td>95 → 105</td>
</tr>
<tr>
<td>4</td>
<td>105 → 95</td>
</tr>
<tr>
<td>5</td>
<td>95 → 105</td>
</tr>
<tr>
<td>5</td>
<td>105 → 95</td>
</tr>
</tbody>
</table>
5.2 Experimental Results

In this section, the results from the experiments described in the previous section are summarized.

Tables 4 and 5 list the results from the quasi-steady state welding tests. Included are the welding conditions, weld pool width, weld pool length and cooling rate for each weld. The entry N/A indicates that the corresponding output could not be measured accurately due to an obvious experimental error. An example of such an experimental error in the case of cooling rate measurement is failure to insert the thermocouple fully into the molten weld pool. In the case of pool length measurements, N/A indicates that the outline of the rear edge of the weld pool in the solidified "crater" at the end of the weld was not sufficiently well-defined to allow accurate measurement.

The results of the weld bead width step response tests are summarized in Figure 23. There, the weld bead front and back width responses to current step perturbations at three different travel speeds are plotted. The widths were measured at 0.125 in. intervals; to facilitate comparison, the 0.125 in. intervals have been converted to equivalent time intervals for plotting. For example, for a travel speed of 4 in./min., the time equivalent of 0.125 in. is the time required for the arc to traverse this distance, given as

\[
\Delta t = \frac{(0.125 \text{ in.})}{(4 \text{ in./min.}) \cdot (60 \text{ sec./min.})} = 1.875 \text{ sec.} \tag{5.2}
\]

The HAZ hardnesses of the quasi-steady state test specimens were measured and are summarized in Table 6.

In general, the AISI 4130 steel quasi-steady state weld pool width test results appeared to be relatively repeatable and consistent. In the first 7 tests listed in
Table 4: AISI 4130 Steel Quasi-steady State Test Results

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Arc Current</th>
<th>Travel Speed</th>
<th>Arc Voltage</th>
<th>Pre-Heat</th>
<th>Pool Width</th>
<th>Pool Length</th>
<th>Cooling Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-18-1</td>
<td>100</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.292</td>
<td>.50</td>
<td>6.5</td>
</tr>
<tr>
<td>-2</td>
<td>100</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.283</td>
<td>N/A</td>
<td>4.8</td>
</tr>
<tr>
<td>-3</td>
<td>100</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.28</td>
<td>.5</td>
<td>5.5</td>
</tr>
<tr>
<td>-4</td>
<td>100</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.277</td>
<td>.58</td>
<td>5.9</td>
</tr>
<tr>
<td>-5</td>
<td>100</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.288</td>
<td>.53</td>
<td>5.5</td>
</tr>
<tr>
<td>6-23-1</td>
<td>100</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.305</td>
<td>.51</td>
<td>N/A</td>
</tr>
<tr>
<td>-2</td>
<td>100</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.30</td>
<td>.54</td>
<td>5.1</td>
</tr>
<tr>
<td>-3</td>
<td>100</td>
<td>5.0</td>
<td>11.0</td>
<td>25</td>
<td>.257</td>
<td>.39</td>
<td>7.8</td>
</tr>
<tr>
<td>-4</td>
<td>100</td>
<td>4.5</td>
<td>11.0</td>
<td>25</td>
<td>.287</td>
<td>.48</td>
<td>6.7</td>
</tr>
<tr>
<td>-5</td>
<td>100</td>
<td>3.5</td>
<td>11.0</td>
<td>25</td>
<td>.327</td>
<td>.52</td>
<td>5.2</td>
</tr>
<tr>
<td>-6</td>
<td>100</td>
<td>3.0</td>
<td>11.0</td>
<td>25</td>
<td>.357</td>
<td>.69</td>
<td>N/A</td>
</tr>
<tr>
<td>-7</td>
<td>100</td>
<td>3.0</td>
<td>11.0</td>
<td>25</td>
<td>.355</td>
<td>.64</td>
<td>4.8</td>
</tr>
<tr>
<td>-8</td>
<td>100</td>
<td>4.0</td>
<td>11.0</td>
<td>100</td>
<td>.297</td>
<td>.5</td>
<td>5.7</td>
</tr>
<tr>
<td>-9</td>
<td>100</td>
<td>4.0</td>
<td>11.0</td>
<td>150</td>
<td>.295</td>
<td>.49</td>
<td>4.3</td>
</tr>
<tr>
<td>Test No.</td>
<td>Arc Current</td>
<td>Travel Speed</td>
<td>Arc Voltage</td>
<td>Pre-Heat</td>
<td>Pool Width</td>
<td>Pool Length</td>
<td>Cooling Rate</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
<td>--------------</td>
<td>-------------</td>
<td>----------</td>
<td>------------</td>
<td>-------------</td>
<td>--------------</td>
</tr>
<tr>
<td>6-24-1</td>
<td>100</td>
<td>4.0</td>
<td>11.0</td>
<td>200</td>
<td>.317</td>
<td>.51</td>
<td>4.5</td>
</tr>
<tr>
<td>-2</td>
<td>87</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.242</td>
<td>.39</td>
<td>6.9</td>
</tr>
<tr>
<td>-3</td>
<td>108</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.342</td>
<td>.7</td>
<td>5.5</td>
</tr>
<tr>
<td>-4</td>
<td>93</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.268</td>
<td>.5</td>
<td>6.2</td>
</tr>
<tr>
<td>-5</td>
<td>110</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.347</td>
<td>.57</td>
<td>6.4</td>
</tr>
<tr>
<td>-6</td>
<td>110</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.322</td>
<td>.75</td>
<td>5.9</td>
</tr>
<tr>
<td>6-25-1</td>
<td>100</td>
<td>4.0</td>
<td>11.0</td>
<td>100</td>
<td>.315</td>
<td>.58</td>
<td>3.5</td>
</tr>
<tr>
<td>-2</td>
<td>100</td>
<td>4.0</td>
<td>11.0</td>
<td>100</td>
<td>.300</td>
<td>.54</td>
<td>4.6</td>
</tr>
<tr>
<td>-3</td>
<td>90</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.25</td>
<td>.42</td>
<td>5.6</td>
</tr>
<tr>
<td>-4</td>
<td>90</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.262</td>
<td>.41</td>
<td>5.6</td>
</tr>
<tr>
<td>-5</td>
<td>110</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>.318</td>
<td>.69</td>
<td>5.9</td>
</tr>
<tr>
<td>7-18-1</td>
<td>110</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>N/A</td>
<td>N/A</td>
<td>5.8</td>
</tr>
<tr>
<td>-2</td>
<td>110</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>N/A</td>
<td>N/A</td>
<td>6.1</td>
</tr>
<tr>
<td>-3</td>
<td>110</td>
<td>4.0</td>
<td>11.0</td>
<td>25</td>
<td>N/A</td>
<td>N/A</td>
<td>5.9</td>
</tr>
</tbody>
</table>
Figure 23: Weld Bead Step Responses
Table 5: 2219 Aluminum Quasi-Steady State Test Results

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Arc Current</th>
<th>Travel Speed</th>
<th>Arc Voltage</th>
<th>Pool Width</th>
<th>Pool Length</th>
<th>Cooling Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>8-14-1</td>
<td>40</td>
<td>4.0</td>
<td>18.5</td>
<td>.255</td>
<td>.31</td>
<td>3.15</td>
</tr>
<tr>
<td>-2</td>
<td>40</td>
<td>3.0</td>
<td>18.5</td>
<td>.310</td>
<td>.38</td>
<td>1.53</td>
</tr>
<tr>
<td>-3</td>
<td>40</td>
<td>5.0</td>
<td>18.5</td>
<td>.210</td>
<td>.24</td>
<td>6.94</td>
</tr>
<tr>
<td>-4</td>
<td>40</td>
<td>4.5</td>
<td>18.5</td>
<td>.225</td>
<td>.26</td>
<td>4.87</td>
</tr>
</tbody>
</table>

Table 4, all welded using the same conditions, the pool width measurements varied from 0.305 in. to 0.277 in., an uncertainty of 0.028 in. or about 10 percent. The steel weld pool length measurements were not as repeatable, varying from 0.58 to 0.50 in. for the same tests, an uncertainty of approximately 15 percent. The steel weld centerline cooling rates were also relatively variable, ranging from 4.8 to 6.5 C/sec for the first 7 tests. The resulting uncertainty of 1.7 C/sec. is about 30 percent of the average cooling rate value for these trials. The HAZ hardmesses obtained from the same tests ranged from 30.7 R_c to 39.3 R_c. This was as large as the variation for all of the trials, indicating that the hardness measurements were not repeatable enough to use as an indirect indicator of cooling rate.

The repeatability of the quasi-steady state aluminum tests was not investigated due to a shortage of material; the results appeared to vary consistently with the changes in welding process inputs. For example, increases in travel speed caused the pool width and length to decrease and the cooling rate to increase in each case.
Table 6: AISI 4130 Steel HAZ Hardness Results

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Hardness $R_c$</th>
<th>Test No.</th>
<th>Hardness $R_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-18-1</td>
<td>31.3</td>
<td>6-24-1</td>
<td>30.7</td>
</tr>
<tr>
<td>-2</td>
<td>39.3</td>
<td>-2</td>
<td>37.3</td>
</tr>
<tr>
<td>-3</td>
<td>36</td>
<td>-3</td>
<td>33</td>
</tr>
<tr>
<td>-4</td>
<td>37.7</td>
<td>-4</td>
<td>36</td>
</tr>
<tr>
<td>-5</td>
<td>35.3</td>
<td>-5</td>
<td>35</td>
</tr>
<tr>
<td>6-23-2</td>
<td>37.7</td>
<td>-6</td>
<td>33</td>
</tr>
<tr>
<td>-3</td>
<td>37</td>
<td>6-25-1</td>
<td>31.3</td>
</tr>
<tr>
<td>-4</td>
<td>35.7</td>
<td>-2</td>
<td>33</td>
</tr>
<tr>
<td>-5</td>
<td>36</td>
<td>-3</td>
<td>34.7</td>
</tr>
<tr>
<td>-6</td>
<td>33</td>
<td>-4</td>
<td>36</td>
</tr>
<tr>
<td>-7</td>
<td>31.7</td>
<td>-5</td>
<td>33.3</td>
</tr>
<tr>
<td>-8</td>
<td>31.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-9</td>
<td>31.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.3 Comparison With Simulation Results — Quasi-Steady State

In this section, the experimental results presented in the previous section are compared to results obtained from the welding heat flow simulation. Heat flow simulation modifications leading to better correlation between experimental and simulation values are also described.

Before running the welding heat flow simulation, the values of various material thermal constants and other process parameters had to be determined. To the extent possible, these parameters were assigned values which were found to yield satisfactory results in past simulations as described in the literature. Based upon trial and error experimentation, some of the parameter values were subsequently modified to bring simulation results into closer agreement with experimental data. Base-metal thermal properties (diffusivity and conductivity) were taken from [44] and were not subsequently adjusted. As in [9], conductivity and diffusivity were taken equal to the values at the melting point of the material in question. The value of the surface heat transfer coefficient was taken equal to that listed in [13], although this value was subsequently altered to obtain better agreement with experimental cooling rates. Values of arc efficiency and arc heat concentration factor were adjusted by trial and error so as to minimize the weld pool width and centerline cooling rate errors. The values of the physical properties and welding process constants were used for the first round of simulation tests are listed below in Tables 7 (for 4130 steel) and Table 8 (for 2219 aluminum).

Simulation outputs are compared to experimental results in Table 9 (for 4130 steel) and Table 10 (for 2219 aluminum). Where more than one experimental trial was run using the same inputs, the result shown in these tables is for the average of the results.

99
Table 7: Simulation Parameter Values for AISI 4130 Steel

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency</td>
<td>0.56</td>
</tr>
<tr>
<td>Heat Concentration Factor</td>
<td>0.02 cm²</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>0.312 W/cm²°C</td>
</tr>
<tr>
<td>Thermal Diffusivity</td>
<td>0.063 cm²/sec</td>
</tr>
<tr>
<td>Surface Heat Transfer Coeff.</td>
<td>0.02 W/cm²°C</td>
</tr>
</tbody>
</table>

Table 8: Simulation Parameter Values for 2219 Aluminum

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency</td>
<td>0.95</td>
</tr>
<tr>
<td>Heat Concentration Factor</td>
<td>0.02 cm²</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>1.3 W/cm K</td>
</tr>
<tr>
<td>Thermal Diffusivity</td>
<td>0.5 cm cm/sec</td>
</tr>
</tbody>
</table>
Table 9: Comparison: Experimental vs. Simulated Outputs for Steel

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>amps</td>
<td>in./min.</td>
<td>C</td>
<td>cm.</td>
<td>cm.</td>
<td>cm.</td>
<td>cm.</td>
<td>C/sec.</td>
<td>C/sec.</td>
</tr>
<tr>
<td>100</td>
<td>3.0</td>
<td>25</td>
<td>0.90</td>
<td>0.94</td>
<td>1.69</td>
<td>1.29</td>
<td>4.8</td>
<td>3.4</td>
</tr>
<tr>
<td>100</td>
<td>3.5</td>
<td>25</td>
<td>0.83</td>
<td>0.80</td>
<td>1.32</td>
<td>1.11</td>
<td>5.2</td>
<td>4.3</td>
</tr>
<tr>
<td>100</td>
<td>4.0</td>
<td>25</td>
<td>0.73</td>
<td>0.71</td>
<td>1.34</td>
<td>0.96</td>
<td>5.6</td>
<td>5.3</td>
</tr>
<tr>
<td>100</td>
<td>4.5</td>
<td>25</td>
<td>0.73</td>
<td>0.64</td>
<td>1.22</td>
<td>0.83</td>
<td>6.7</td>
<td>6.4</td>
</tr>
<tr>
<td>100</td>
<td>5.0</td>
<td>25</td>
<td>0.65</td>
<td>0.57</td>
<td>0.99</td>
<td>0.73</td>
<td>7.8</td>
<td>7.6</td>
</tr>
<tr>
<td>87</td>
<td>4.0</td>
<td>25</td>
<td>0.61</td>
<td>0.57</td>
<td>0.99</td>
<td>0.62</td>
<td>6.9</td>
<td>6.6</td>
</tr>
<tr>
<td>90</td>
<td>4.0</td>
<td>25</td>
<td>0.65</td>
<td>0.60</td>
<td>1.05</td>
<td>0.74</td>
<td>5.6</td>
<td>6.2</td>
</tr>
<tr>
<td>93</td>
<td>4.0</td>
<td>25</td>
<td>0.68</td>
<td>0.64</td>
<td>1.27</td>
<td>0.81</td>
<td>6.2</td>
<td>5.9</td>
</tr>
<tr>
<td>110</td>
<td>4.0</td>
<td>25</td>
<td>0.84</td>
<td>0.81</td>
<td>1.70</td>
<td>1.18</td>
<td>6.0</td>
<td>4.6</td>
</tr>
<tr>
<td>100</td>
<td>4.0</td>
<td>100</td>
<td>0.76</td>
<td>0.76</td>
<td>1.37</td>
<td>1.07</td>
<td>4.6</td>
<td>3.4</td>
</tr>
<tr>
<td>100</td>
<td>4.0</td>
<td>150</td>
<td>0.75</td>
<td>0.80</td>
<td>1.24</td>
<td>1.15</td>
<td>4.3</td>
<td>2.5</td>
</tr>
<tr>
<td>100</td>
<td>4.0</td>
<td>200</td>
<td>0.81</td>
<td>0.85</td>
<td>1.30</td>
<td>1.25</td>
<td>4.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

101
Table 10: Comparison: Experimental vs. Simulated Outputs for Aluminum

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>amps</td>
<td>in./min.</td>
<td>C</td>
<td>cm.</td>
<td>cm.</td>
<td>cm.</td>
<td>cm.</td>
<td>C/sec.</td>
<td>C/sec.</td>
</tr>
<tr>
<td>40</td>
<td>4.0</td>
<td>25</td>
<td>0.65</td>
<td>0.80</td>
<td>0.79</td>
<td>0.81</td>
<td>3.1</td>
<td>4.0</td>
</tr>
<tr>
<td>40</td>
<td>3.0</td>
<td>25</td>
<td>0.79</td>
<td>1.08</td>
<td>0.97</td>
<td>1.10</td>
<td>1.5</td>
<td>2.7</td>
</tr>
<tr>
<td>40</td>
<td>5.0</td>
<td>25</td>
<td>0.53</td>
<td>0.50</td>
<td>0.61</td>
<td>0.53</td>
<td>6.9</td>
<td>6.0</td>
</tr>
<tr>
<td>40</td>
<td>4.5</td>
<td>25</td>
<td>0.57</td>
<td>0.65</td>
<td>0.66</td>
<td>0.56</td>
<td>4.9</td>
<td>4.9</td>
</tr>
</tbody>
</table>

A graphical presentation of the data given in the above tables is shown in Figure 24. In these plots, the experimental values are sorted into increasing order and plotted with their corresponding simulated value. The correlation between experimental and simulated pool widths appears to be acceptable. Correlation between the pool length and cooling rate results is not satisfactory.

Regarding lack of agreement between cooling rates, closer examination revealed that the experimental trials using base-metal preheat were most at variance with the simulation results. Also relatively poorly matched were those trials where relatively slow travel speeds or high arc currents were used. All of these tests had simulated cooling rates consistently lower than the experimental results.

The errors in the tests involving preheat will be discussed first. The errors in agreement for these tests were thought to be due to the fact that the experimental cooling rates were increased by the decay of the preheat of the entire plate. Recall that for the experimental trials which involved preheat, the preheating lamps were removed before welding was begun. Thus, even if no welding were performed at
Figure 24: Comparison of Experimental and Simulation Results
all, a thermocouple in contact with the plate would measure a cooling rate due to
the decay in temperature of the plate as a whole.

The following describes the method was used to include the effects of base-
metal preheat decay in the simulation. Assuming that the entire plate is at a
preheat temperature of $T_0$ and that heat is lost through convective action to an
environment with temperature $T_a$ with a Newtonian (surface heat transfer) coef-
cient of $H$, a simple energy balance for the plate yields the following differential
equation:

$$\rho C_l w h \frac{dT}{dt} = H l w 2 (T - T_a)$$  \hspace{1cm} (5.3)

or

$$T - \frac{\rho Ch}{H^2} \cdot \frac{dT}{dt} = T_a$$  \hspace{1cm} (5.4)

The solution of this differential relationship gives an equation for the decay of the
preheat of the plate after the preheat source is removed. The exponential solution
is:

$$T = (T_0 - T_a) \exp \left( -\frac{2H}{\rho Ch} t \right) + T_a$$  \hspace{1cm} (5.5)

Under the assumption of linear heat flow, this preheat temperature decay may
be added to the temperature variations caused by welding. The simulation was
modified accordingly; a value of 0.06 W/cm\(^2\)C (arrived at by trial and error) was
used for the surface heat transfer coefficient.

Results from the modified simulation showed that the temperature decay increased the simulated cooling rates, bringing the simulation and experimental results into better agreement as is shown in Table 11. In this table, only the results from the runs with preheat were compared; the results from the
other runs were not altered by the modification.
Table 11: Comparison of Experimental and Simulated Cooling Rates With Surface Heat Transfer Preheat Decay Correction

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>amps</td>
<td>in./min.</td>
<td>C</td>
<td>cm.</td>
<td>cm.</td>
<td>cm.</td>
<td>cm.</td>
<td>C/sec.</td>
<td>C/sec.</td>
</tr>
<tr>
<td>100</td>
<td>4.0</td>
<td>100</td>
<td>0.76</td>
<td>0.74</td>
<td>1.37</td>
<td>1.02</td>
<td>4.6</td>
<td>4.9</td>
</tr>
<tr>
<td>100</td>
<td>4.0</td>
<td>150</td>
<td>0.75</td>
<td>0.76</td>
<td>1.24</td>
<td>1.07</td>
<td>4.3</td>
<td>4.5</td>
</tr>
<tr>
<td>100</td>
<td>4.0</td>
<td>200</td>
<td>0.81</td>
<td>0.78</td>
<td>1.30</td>
<td>1.12</td>
<td>4.5</td>
<td>4.1</td>
</tr>
</tbody>
</table>

The remaining discrepancy between the experimental and simulated cooling rate values appear to be primarily due to those runs made at currents higher than 100 amperes and at travel speeds less than 4 in./min. Regarding the former, the experimental data indicates that an increase in welding current from 100 amperes to 110 amperes caused an increase in cooling rate from 5.6 to 6.0 C/sec. This trend is opposite to that predicted by the simulation: it indicates that the cooling rate value should decrease with increasing arc current. For travel speeds lower than 4 in./min, the experimental cooling rate decreased somewhat, to a value of 4.8 C/sec. at 3.0 in./min. This is not nearly as large a decrease as predicted by the simulation which output a value of 3.4 C/sec. at this travel speed.

The values of cooling rate quoted above for tests at 100 amps and 110 amperes are actually averages of results from a number of trials at each current level. Inspection of the first 7 entries in Table 4 reveals that, for identical welding conditions with arc current equal to 100 amps, the measured centerline cooling rate varied from 4.8 to 6.5 C/sec., a total of 1.7 C/sec. Since the total variation of cooling rate over all of the experiments was only from 3.8 to 7.5 C/sec, a total
change of 3.7 C/sec., it is evident that the possibility for experimental error in
the results is large. This raises the question as to whether the anomalous rise
in cooling rate with arc currents above 100 amperes could arise solely from the
effects of experimental measurement error. Some investigations of this question
are presented below.

The experimental data relevant to this question, presented in Table 4, are
runs 6-18-1 to 6-18-5, 6-23-1, 6-23-2, 6-24-5, 6-24-6, 6-25-1 and 7-18-1 to 7-18-
3. The first 7 of these trials correspond to input conditions of \( I = 100 \) amps,
\( V = 4.0\text{in./min} \) and \( T_0 = 25\text{C} \), and will be referred to below as the "medium
current data set". The input conditions for the remaining 6 trials were the same
as for the former with the exception of arc current which was equal to 110 amperes.
This set of data will be referred to as the "high-current data set".

We now wish to perform an analysis to determine whether or not it is rea-
sonable to suppose that, given the amount of measurement error inherent in the
test, the mean cooling rate of the high current data set is actually less than or
equal to that of the medium current data set. Put another way, assuming that the
experimental cooling rate value of 5.5 C/sec. at 100 amperes is "correct" (since it
is fairly closely matched by the simulation result), we wish to assess the probability
that the reason for obtaining a cooling rate value higher than 5.5 C/sec. for tests
at 110 amperes is measurement error alone. It is expected that the chance of this
will be fairly low since none of the 6 tests at 110 amperes resulted in cooling rates
less than 5.5 C/sec.

Although the number of samples in the data sets are not large enough to allow
accurate statistical analysis of the data, we can obtain a rough idea of whether
or not measurement errors can account for the anomaly pointed out above. If it
is assumed that, for a fixed set of welding inputs, the measured cooling rate data
has a normal (i.e. Gaussian) distribution, then we may approximate the mean and variance of this distribution as [45]

\[
\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} CR_i \\
\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (CR_i - \hat{\mu})^2
\]

(5.6) (5.7)

where \( \hat{\mu} \) = estimate of the mean

\( n \) = number of measurements

\( CR_i \) = individual cooling rate measurement

\( \hat{\sigma}^2 \) = estimate of the variance

From the medium current data set, we obtain \( \hat{\mu}_{mc} = 5.55C/\text{sec.} \) and \( \hat{\sigma}^2_{mc} = 0.30 \).

From the high current data set, we calculate \( \hat{\mu}_{hc} = 6.00C/\text{sec.} \) and \( \hat{\sigma}^2_{hc} = 0.04 \). The subscripts “mc” and “hc” refer to medium current and high current, respectively.

Accordingly, we wish to test the following hypotheses:

\[ H_0: \mu_{hc} \leq 5.55 \]

\[ H_1: \mu_{hc} > 5.55 \]

If \( H_0 \) is found to be false, then it is unlikely that random measurement error accounts for the anomalous cooling rate behavior. A uniformly most powerful test (UMP) of this hypothesis exists; the exact details of the test procedures may be found in [45, pg. 399]. Basically, the output of the UMP test is a critical value \( c \) to which the high-current data-set mean-estimate \( \hat{\mu}_{hc} \) is compared. If the test if performed for a selected level of significance (5 % is common), then we reject the hypothesis \( H_0 \) if \( \hat{\mu}_{hc} \geq c \). Performance of the UMP test for a 5 % significance level
with a conservative variance estimate of 0.30 (the larger of \( \hat{\sigma}_{mc}^2 \) and \( \hat{\sigma}_{hc}^2 \)) results in a critical value of \( c = 5.9 \). Since \( \hat{\mu}_{hc} \) is greater than this critical value, we conclude that there is at least a 95 percent probability that \( H_0 \) is false. The implication of this test result is that random measurement error is probably not sufficient to account for the anomalous cooling rate behavior.

The above analysis suggests that either the cooling rate did in fact increase as current increased or that there was systematic instrumentation error which caused cooling rate measurements to increase as current increased. An example of the latter might be induction of voltages in the thermocouple circuitry by welding arc current electro-magnetic fields. Experiments designed to detect effects of this kind failed to confirm any electro-magnetic interference. As a result, we must conclude that the anomalous results probably do, to some extent, reflect the actual behavior of the system.

A physical mechanism which would cause the cooling rate to increase with increased arc current or to not decrease enough with decreasing travel speed is not immediately obvious. However, the fact that both high current and low travel speed are relatively high "heat input" conditions, coupled with the fact that simulated weld pool widths agreed relatively well with experiment under all conditions provides a clue as to how the simulation might be modified to reflect the behavior. It was noted during calibration of the simulation that adjustment of the surface heat transfer coefficient \( h_s \) caused significant variation in the simulated cooling rate while hardly altering the pool dimension values. This fact suggests that simulation cooling rates might be made to better match the experimental results if the value of \( h_s \) used in the simulation is made a function of travel speed and arc current. Such variations in \( h_s \) would not significantly affect the pool width or pool length correlations.
Figure 25: Variation of $h_s$ with Arc Current

No specific physical mechanism causing variation of surface heat transfer in the manner proposed above will be sought in this dissertation. However, it is reasonable to speculate that, under conditions of high heat input, the high temperatures present over a large surface area of the plate might cause increases in convection currents rising from the plate surface. The effect of increased convective air flow would be increased surface heat transfer.

Further simulations confirmed that variation of the surface heat transfer coefficient as a function of arc current and travel speed produced simulated cooling rates which agreed more closely with experimental cooling rates without significantly disturbing the weld pool width correlation. The surface heat transfer variation shown in Figure 25 was found to produce adequate correlation with the cooling rate data from those experimental trials where arc current was varied but all other inputs remained fixed. The surface heat transfer variation shown in Figure 26 was found
Figure 26: Variation of \( h_s \) with Travel Speed

to produce adequate correlation with those experimental trials where travel speed was varied, but all other inputs remained fixed. The effect of all of these variations upon the simulated weld pool width were minimal, being less than 0.01 cm. in all cases.

Questions as to how to implement the above variations of surface heat transfer coefficient \( h_s \) in the simulation had to be resolved. Two issues were considered. The first was how the alterations in \( h_s \) were to be implemented as travel speed varied with time (recall that travel speed was assumed to be a piece-wise constant time-varying input in the formulation of the simulation). This was resolved by using the relationship shown in Figure 26 to calculate a value of \( h_s \) appropriate for the travel speed value in effect at each time-iteration of the simulation. More precisely, at each iteration \( k \) of Equation (4.32), the value of \( h_s \) used in the simulation was calculated using the relationship of Figure 26 with travel speed \( V_k \).
The second issue considered was how the surface heat transfer coefficient was to be adjusted when travel speed was less than 4 in./min. and arc current was greater than 100 amperes at the same time. None of the experimental trials corresponded to such a case. In the implementation decided upon, values of \( h_s \) are independently calculated from the relationships shown in Figures 25 and 26, and then the larger of these two values is used in the simulation. To what extent results obtained with this procedure match experiment will have to be determined in work subsequent to this dissertation.

The modifications described above brought the cooling rate into closer agreement with the experimental values, but the pool length predictions remained relatively poor. The average error was 0.31 cm., indicating underestimation of pool length by the simulation. Communications with other researchers [46] who have had experience with the simulation technique used in this dissertation revealed that this problem had been previously encountered. It has been conjectured to be due to pool convection effects or effects due to the latent heat of fusion of the material. Both of these effects are difficult to add to the simulation method, so the decision was made to compensate for them heuristically, by adding the average value of the prediction error to the estimate. When this was done, the resulting estimates had an average error of 0.02 cm. Note that this correction was only to the simulation of steel weld pool lengths; the aluminum welds did not need this correction. This could be due to the fact that the latent heat of fusion of aluminum is less than that of steel. Thus, neglecting latent heat effects may cause less modeling error for aluminum than for steel.

A final graphical comparison of the simulated and experimental results are shown in Figure 27. In each of these plots, the experimental data was sorted in increasing order and plotted with the corresponding simulation result.
Figure 27: Comparison of Experimental and Simulation Results
Before closing this section, a few comments upon the generality of the method of simulation calibration used above are in order. Recall that the simulation surface heat transfer coefficient was made a function of current and travel speed. In principle, by allowing $h_s$ to vary with these two inputs, any set of experimental cooling rate data obtained by variation of current and travel speed can be fit as closely as desired. Whether or not this is desirable depends partly upon the reliability of the experimental data. If the data is badly contaminated with measurement noise, causing the simulation fit it closely would not necessarily produce desirable results.

It is also of particular interest to note that the method of simulation calibration above suggests a more general method by which the simulation can be made to exactly match any two welding process outputs of interest. For example, in this dissertation, in addition to centerline cooling rate, weld pool width is also an important output. The simulation could be made to exactly match any set of cooling rate data obtained by variation of arc current and travel speed by making not only $h_s$, but also $\eta$ (arc efficiency) functions of current and travel speed. A viable calibration procedure would be to vary $\eta$ so that, at each current/travel speed input combination, the experimental and simulation pool widths match. Then, for each input combination, $h_s$ would be varied to produce a match between cooling rates. The latter adjustment would not significantly alter the pool width correlation. The result of this procedure would be values of $h_s$ and $\eta$ needed to provide correlation of weld width and cooling rate at each combination of inputs. Determination of $h_s$ and $\eta$ values for input combinations not used in the calibration could be found by using, for example, linear interpolation and/or extrapolation.

The above calibration procedure was not employed in this chapter for two reasons. First, for the range of current and travel speed which were employed, the
Table 12: Comparison of Pool Width Step Responses

<table>
<thead>
<tr>
<th>Weld No.</th>
<th>Arc Current Change</th>
<th>Travel Speed</th>
<th>Rise Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>amperes</td>
<td>in./min.</td>
<td>Exp.</td>
</tr>
<tr>
<td>7-10-1</td>
<td>110 → 95</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>95 → 110</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>7-10-3</td>
<td>105 → 95</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>95 → 105</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>7-10-6</td>
<td>110 → 95</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>95 → 110</td>
<td>3</td>
<td>14</td>
</tr>
</tbody>
</table>

simulation provided acceptable correlation with experimental weld widths with a constant value of arc efficiency. Secondly, the cooling rate data was thought to be sufficiently uncertain that fitting each point exactly was unwise. However, the method could prove useful in subsequent work involving a wider range of inputs.

5.4 Experimental Results and Comparisons — Transient

After the simulation quasi-steady state tests and modifications described above were completed, the dynamic performance of the simulation was tested. The step response tests described previously were simulated and the weld pool width responses were compared to the experimental results. This comparison is presented in Table 12 where the rise times of the simulated and experimental results are tabulated. In each case, the rise time was taken to be the time required for the response to change from the initial value ±0.01 in. to the final value ±0.01 in. For example, if the step response was a pool width increase from 0.30 in. to 0.40 in.,
the rise time was the time required for the pool width to increase from 0.31 in. to 0.39 in. Similarly, for a pool width decrease from 0.40 in. to 0.30 in., the rise time was the time required for the pool width to change from 0.39 in. to 0.31 in. The number tabulated is the average of the rise times calculated from the front width and for the back width of the weld bead.

The correlation between simulated and experimental rise times appears to be acceptable, the maximum error being approximately one second. However, this good result is subject to some uncertainty due to noise present in some of the experimental pool width step response measurements. The magnitude of this noise is clearly evident in the plots of Figure 23.

A comparison between a typical experimental step response and a simulation step response is shown in Figure 28. The correspondence between the two appears to be acceptable.
5.5 Summary

In summary, the procedures by which experimental results have been obtained and analyzed were presented in this chapter. The data was summarized and compared to simulation results. In general, the pool length and cooling rate data were found to contain more measurement uncertainty than the pool width data. It was found that the simulation predictions agreed quite well with experiments in the case of weld pool width, but not as well for centerline cooling rate and weld pool length. The correlation with cooling rate was improved by the addition to the simulation of plate preheat decay due to convective heat loss effects. The cooling rate correlation at currents above 100 amperes and travel speeds less than 4 in./min. was improved by varying the surface heat transfer coefficient $h_s$ as a function of travel speed and arc current. Communication with other researchers indicated that the under-prediction of weld pool length was inherent in the type of simulation which was used. Weld pool length was brought into closer correspondence with experiment by the addition of a constant to the predicted values. The dynamic tests results indicated that the step response of the weld pool widths obtained from simulation matched the experimentally determined responses to an acceptable degree of accuracy.
CHAPTER VI

Formulation of Static Observers

In this chapter, relationships which express quasi-steady state centerline cooling rate in terms of weld pool dimensions are studied. As was mentioned in Chapter III, the relationships derived in this chapter may be thought of as corresponding to steady state versions of the dynamic observers which were discussed previously. They will be referred to hereafter as static observers. In the first section of this chapter, examples of static observer relationships are derived using the ideal welding heat flow results of Chapter II. In the following two sections, empirical relationships of a similar nature are derived from the experimental and simulation data presented in Section 5.2 and discussed. The relationships derived from simulation data will be utilized in Chapter VII when cooling rate control design is studied. The relationships obtained from the actual experimental data will be of use in implementations of actual cooling rate control, performed subsequent to the work of this dissertation.

6.1 An example from Ideal Heat Flow Relationships

In this section, relationships which express centerline cooling rate in terms of weld pool width and welding process inputs are formulated from the ideal welding heat flow results discussed in Chapter II. This concept was first proposed by Stewart [47] who is responsible for the first example shown below.
We begin with Equations (6.1) and (6.2), which express weld width and centerline cooling rate in terms of welding inputs and base metal thermal properties.

\[ W = \frac{4\alpha}{V} \left( \frac{\dot{q}}{8K(T_m - T_0)h} - \frac{1}{5} \right) \quad (6.1) \]

\[ \frac{\partial T}{\partial t} \bigg|_{T=T'} = 2\pi K\rho Ch^2 \left( \frac{V}{\dot{q}} \right)^2 \left( T' - T_0 \right)^3 \quad (6.2) \]

An equation which expresses centerline cooling rate as a function of weld pool width and welding inputs may be obtained by combining (6.1) and (6.2). There are many ways to combine the two equations so as to arrive at the desired result. In general, one may solve Equation (6.1) for some selected parameter common to both equations and substitute the resulting expression into Equation (6.2). Stewart [47] solved Equation (6.1) for the arc power input, \( \dot{q} \) and substituted the result into (6.2), yielding

\[ \frac{\partial T}{\partial t} \bigg|_{T'} = \frac{\pi V^2(T' - T_0)^3}{32\alpha \left( \frac{Yw}{4\alpha} + \frac{1}{5} \right)^2 (T_m - T_0)^2} \quad (6.3) \]

This equation expresses the quasi-steady state centerline cooling rate in terms of weld pool width, welding travel speed, base metal preheat temperature and thermal diffusivity. Note that no knowledge of the welding arc power input or material thickness is needed to calculate cooling rate; these values have been replaced by the weld pool width measurement.

As another example, consider the case where equation (6.1) is solved for preheat temperature, \( T_0 \), the result being substituted into (6.2) yielding

\[ \frac{\partial T}{\partial t} \bigg|_{T=T'} = 2\pi K\rho Ch^2 \left( \frac{V}{\dot{q}} \right)^2 \left( T' - T_m + \frac{\dot{q}}{8K \frac{Vw}{4\alpha} + \frac{1}{5}} \right)^3 \quad (6.4) \]

This represents an equally valid approach, but results in a cooling rate relationship having different properties than the first one.
In the first example, heat input, $\dot{q}$, is replaced by weld width while in the second, the preheat temperature is replaced. In this respect, the latter example is similar in philosophy to the conventional dynamic observer formulated in Chapter IV. In fact, Equation (6.4) approximates the quasi-steady state response of the conventional dynamic observer. Although why this is so is perhaps not immediately obvious, the procedure of solving the weld pool width equation for the preheat temperature and substituting it into the cooling rate relationship emulates the process of adjusting preheat temperature so that a desired pool width output is obtained. A discussion involving two "ideal" welding processes, identical but for differing preheats clarifies the procedure. Since each process is described by Equations (6.1) and (6.2), and since pool width is a one-to-one function of preheat temperature in Equation (6.1), a measurement of weld width from one of the "ideal" welding processes allows calculation of the preheat temperature of that process. Altering the preheat temperature of the second process to equal this value then causes the second process to output the same cooling rate and weld pool width as the first.

In a similar way, the first relationship derived above (Eq. (6.3)) approximates the steady state response of the dynamic adaptive observer. Again, the procedure of solving for arc power and substituting into the cooling rate equation emulates the process of adjusting arc efficiency until observer and process weld pool width are equal.

It is important to note that the above relationships, being derived from ideal heat flow results, do not exactly represent the steady state behavior of the dynamic observers. The dynamic observers are based upon assumptions of gaussian arc power distribution and non-constant, non-zero surface heat transfer. The ideal relationships are based upon assumptions of a delta function arc power distribution
and negligible surface heat transfer. It follows that the static observers derived above would not accurately represent the welding process. Derivation of quasi-steady state weld pool width and centerline cooling rate relationships for any of the assumptions used in the formulation of the dynamic observers is difficult. For this reason, an empirical approach will be taken to the formulation of more accurate static observer relationships. These empirical relationships are presented below.

6.2 Empirical Weld Pool Dimension/Cooling Rate Relationships

To begin the process of derivation of empirical static observers, the general form of the empirical relationships to be used must be determined. In many past works concerned with the development of empirical relationships describing various aspects of the arc welding process, power-law relationships have been successfully applied. In the experiments described in Chapter V, three process inputs were varied, these being arc current, travel speed and base metal preheat temperature. A power-law relationship which expresses cooling rate in terms of these three welding inputs has the form

\[ CR = kI^xV^yT_0^z \] (6.5)

where \( k, x, y, \) and \( z \) are constants which must be determined so that the relationship fits the data in some sense. Such relationships may be reasonably applied in cases where the dependency of the output and the first derivative of the output with respect to the inputs is continuous and monotonic.

A curve-fitting procedure known as multilinear regression is commonly used to fit power expressions such as the above to experimental data. This technique fits linear equations of the form:

\[ f(x_1, x_2, ..., x_n) = K_0 + \sum_{i=1}^{n} K_i \cdot x_i \] (6.6)
to a given set of data by finding $K_i$, $i = 0, 1, ..., n$ so as to minimize a quadratic measure of the error between $f(x_1, x_2, ..., x_n)$ and the data points. To apply multilinear regression to an equation such as Equation (6.5), one first takes the log of both sides, yielding

$$\log(f(I, V, T_0)) = \log K + x \log(I) + y \log(V) + z \log T_0$$  \hspace{1cm} (6.7)

Application of the linear regression curve fitting technique is then straightforward. A parameter known as the correlation coefficient is commonly used to describe the "goodness of fit" of the fitted curve to the data. This coefficient is a number which varies between 0 and 1, representing respectively a very poor fit and a perfect fit.

To investigate the ability of power relationships to fit the simulation data presented in Section 5.2, the linear regression technique was used on this data to find an expression for cooling rate in terms of $I, V$ and $T_0$. This yielded the relationship:

$$CR = 2.7245 \cdot I^{-0.6841} V^{0.9074} (1530 - T_0)^{0.9013}$$  \hspace{1cm} (6.8)

Preheat temperature appears in this equation in the term $(T_m - T_0)$ because it appears in the weld pool width analytical relationships in this form and because slightly better correlation coefficients were obtained in this way. The correlation coefficient for this relationship was 0.964 indicating a good fit. The curve is compared to the data graphically in Figure 29. In this plot, the cooling rates calculated from Equation (6.8) are labelled RES1 while the simulated cooling rate data are labeled CR. This figure shows that the largest error between the curve fit and the data was an error of about 0.4 C/sec, occurring at point number 6 (the sixth point from the top of the plot).
Figure 29: Comparison of Empirical Relationship to Data

The accuracy of the empirical relationship was slightly improved by "lagging" the terms used in the curve fit. A power-law relationship with lagged variables has the form

\[ CR = k \cdot (I - I_l)^a (v - v_l)^b (T_m - T_0)^c \]  

(6.9)

where \( I_l \) = current lag factor
\( v_l \) = travel speed lag factor

Note that the preheat was essentially already lagged in the original expression (Eq. (6.8)). Some statistics packages have provisions for determining optimal lag factors using numerical search techniques, but the program being used in this work (SPEAKEZ) did not provide such facilities. Consequently, lag factors were deter-
mined by repetitively fitting relationships while varying lag values until satisfactory values were arrived at. The resulting relationship was

$$CR = 1.2683 \times 10^{-8} (I - 65)^{-2.512} (v + 0.4)^{3.3357} (1530 - T_0)^{3.1071} \quad (6.10)$$

This relationship had a correlation coefficient of 0.978, slightly better than the 0.964 of the previous relationship. The maximum error between the relationship and the data was about 0.3 C/sec., also slightly improved.

It is important to note that the power-type of curve cannot be expected to fit the given cooling rate data precisely. For a perfect fit, the relationship between the cooling rate and each input (considered individually) would have to be monotonic, and also be monotonic in all derivatives with respect the input. This is obviously not true in the case of cooling rate as a function of arc current. This relationship is decreasing up until arc current equals approximately 100 amps, after which it is an increasing relationship. Considering these facts, it is fortunate that the power-type relationship fits as well as it does.

The same lagged power-law relationship described above was next fit to the experimental data. The resulting expression was

$$CR = 2.8653 \times 10^{-8} \cdot (I - 65)^{-0.1341} (v + 0.4)^{3.2716} (1530 - T_0)^{2.9339} \quad (6.11)$$

The correlation coefficient for the relationship was 0.87, indicating fair correlation. A plot comparing actual cooling rate to that predicted by Equation (6.11) is shown in Figure 30.

To generate empirical versions of the static observers, the general principle illustrated in the preceding section of eliminating one variable from the cooling rate relationship with pool width was employed. Each of the three inputs (current, travel speed and preheat) was in turn replaced by pool width and a power
Figure 30: Cooling Rate vs. Arc Current, Travel Speed and Preheat Temperature relationship was determined using multilinear regression. The result of this procedure was a total of 6 cooling rate relationships, 3 from the experimental data and 3 from the simulation results. In the following, these relationships are presented and the relative merits of each are discussed.

Regression fitting of relationships to simulation data yields

\[ CR = 1.9284 \times 10^{-8} \cdot (W - 0.4)^{-0.2263} (V + 0.4)^{1.749} (1530 - T_0)^{2.7542} \quad (6.12) \]

\[ CR = 3.4511 \times 10^{-8} \cdot (I - 65)^{0.2955} (W - 0.4)^{-0.4789} (1530 - T_0)^{2.3736} \quad (6.13) \]

\[ CR = 2.003 \times 10^{-6} \cdot (I - 65)^{1.9516} (V + 0.4)^{-10.229} (W - 0.4)^{-1.9349} \quad (6.14) \]

Correlation coefficients for the above 3 relationships were respectively 0.982, 0.985 and 0.977.

The first of the above relationships corresponds to a static version of the adaptive observer and the last to a static conventional observer. The second rela-
Figure 31: Comparison of Equation (6.15) to Experimental Data

tionship corresponds to an observer formulation wherein travel speed is adjusted to eliminate pool width error.

Regression fitting of expressions involving pool width to the experimental data yielded

\[ CR = 1.8864 \times 10^{-8} \cdot (W - 0.4)^{-0.1002} (V + 0.4)^{2.8278} (1530 - T_0)^{2.8777} \quad (6.15) \]

\[ CR = 1.0374 \times 10^{-8} \cdot (I - 65)^{0.5304} (W - .4)^{-0.6411} (1530 - T_0)^{2.4061} \quad (6.16) \]

\[ CR = 9.5972 \times 10^{-2} \cdot (I - 65)^{0.7336} (V + 0.4)^{-0.8679} (W - 0.4)^{-0.9264} \quad (6.17) \]

The correlation coefficients for these relationships are respectively 0.864, 0.805 and 0.534. In Figures 31, 32 and 33, the fit of the above equations to the data is displayed graphically. In each plot, the result of the appropriate regression expression, labelled RES1, is plotted against the experimental data, labeled CR for each input point.
Figure 32: Comparison of Equation (6.16) to Experimental Data

<table>
<thead>
<tr>
<th>RES1</th>
<th>Cr</th>
<th>3.9741</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.80679</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>5.99409</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>6.14293</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td>6.71144</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td>5.18976</td>
<td>5.2</td>
<td></td>
</tr>
<tr>
<td>6.14659</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>6.14996</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>5.84504</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td>6.25778</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td>6.14996</td>
<td>6.7</td>
<td></td>
</tr>
<tr>
<td>6.4229</td>
<td>6.9</td>
<td></td>
</tr>
<tr>
<td>7.34758</td>
<td>7.8</td>
<td></td>
</tr>
</tbody>
</table>

---

Figure 33: Comparison of Equation (6.17) to Experimental Data

<table>
<thead>
<tr>
<th>RES1</th>
<th>Cr</th>
<th>4.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.61711</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>6.85129</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>5.47242</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td>6.71658</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td>4.79704</td>
<td>5.2</td>
<td></td>
</tr>
<tr>
<td>5.99357</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>5.47318</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>6.46401</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td>5.84403</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>5.74682</td>
<td>6.7</td>
<td></td>
</tr>
<tr>
<td>6.04264</td>
<td>6.9</td>
<td></td>
</tr>
<tr>
<td>7.20855</td>
<td>7.8</td>
<td></td>
</tr>
</tbody>
</table>

126
6.3 Discussion of The Empirical Relations

In general, the fit of curves to the experimental data was fair (with one notable exception) while the fit to simulation data was good for all three observers. The maximum error for the first experimental relationship (shown graphically in Figure 31 was about 0.65 C/sec., while that of the second relationship was 0.5 C/sec. Both represent errors of about 10 percent of the corresponding cooling rate. Very poor fits to experimental data were obtained with the static version of the conventional observer, Equation (6.17). This fact was likely due to the combined effects of experimental measurement error and the small percentage of runs made at preheats other than room temperature. Also contributing to the problem were some poor robustness properties of this type of observer. This effect will not be elaborated upon further until the following chapter, where extensive robustness testing is done. It is not likely the static version of the conventional observer represents the welding process well enough to be applied in practical cooling rate controls.

The decision as to which of the remaining two types static observers are best applied in practice depends mainly upon the prevailing welding conditions and the results of the control robustness tests which will be described in the next chapter. The conditions under which the adaptive observer might be considered for application have been outlined in Chapter IV. Basically, the ability of this observer to automatically correct for arc efficiency variations would appear to make it an attractive choice in many situations. It is not clear under what conditions it would be appropriate to apply the other static observer. In this formulation, the travel speed term is replaced by pool width. Since it is hard to imagine an automatic welding situation in which travel speed would not be known, the formulation would not appear to be of much use. However, as will be discussed in the next chapter...
on control design, this formulation does have the redeeming quality of greatly simplifying the implementation of cooling rate control, and so would appear to be of some utility.

6.4 Summary

In this section, multilinear regression was used to determine empirical relationships between weld pool width and weld centerline cooling rate. Power-type relations were found for both experimental data and simulation results. The relationships which were developed expressed quasi-steady state cooling rate in terms of weld pool width, arc current and preheat temperature. It was found that power relations of the form

\[ CR = k \cdot (I - 65)^x (V + 0.4)^y (W - 0.4)^z \] (6.18)

were not sufficiently reliable to be used in the implementation of cooling rate controls, while those of the form

\[ CR = k \cdot (W - 0.4)^x (V + 0.4)^y (1530 - T_0)^z \] (6.19)

\[ CR = k \cdot (I - 65)^x (W + 0.4)^y (1530 - T_0)^z \] (6.20)

provided an adequate fit to the data.
CHAPTER VII

Control Formulation and Simulation

In this chapter, cooling rate control algorithms which utilize the observers discussed in Chapters IV and VI are formulated and tested by simulation. The goals of this chapter are two: to design feedback cooling rate controls having desired dynamic and steady-state responses and to study the robustness of those feedback controls.

The structure of the feedback cooling rate control proposed in Chapter III is illustrated in Figure 16. This block diagram is repeated here for convenience as Figure 34. In the proposed control scheme, the error between desired cooling rate and estimated cooling rate is operated on by a compensator, the result being a new travel speed command for the process. One of the goals of this chapter is to design compensators which yield satisfactory dynamic and steady-state closed loop performance. The design process described below involves approximation of the cooling-rate system by linear reduced-order models and design of compensators using root locus techniques.

The second goal of this chapter is to characterize the robustness of the closed-loop cooling rate controls implemented using the various observers. One facet of control robustness which is discussed is modeling-error robustness: the effect upon output regulation accuracy of mismatch between the welding process and the observer process model. Another aspect of control robustness which is investigated
is measurement error robustness: the effect of weld pool width measurement errors upon cooling rate regulation accuracy.

All control design and testing and robustness testing described in this chapter were performed using a numerical simulation. The entire control system depicted in Figure 34 was simulated on a digital computer, the block labeled “welding process” being replaced by the heat flow simulation described in Chapter IV. The pool dimension measurement block was eliminated in this simulation: the pool dimensions produced by the heat flow simulation were input directly to the observer algorithm. The fact that there are two cooling rate outputs of interest in the diagram creates the possibility of confusion. In the following discussion, the cooling rate output of the observer will be referred to as the “estimated cooling rate” or the “observer cooling rate”. The cooling rate output of the welding process simulation
will be referred to as the "simulated cooling rate" or the "process cooling rate". The latter should be thought of as the cooling rate which might be measured experimentally from an actual welding process using a thermocouple.

The principle reason for choosing a simulation-based approach was the increased ease with which control performance could be measured. As was pointed out earlier, measurement of time-varying cooling rates in actual welding processes is difficult and would hinder the control design process. The dynamic response of pool dimensions produced by the simulation were found in Chapter V to match that of the welding process reasonably well. This provides basis for the belief that the simulated control performances resemble those of the actual process. However, the final evaluation of the performance of the controls discussed in this chapter will need to be performed experimentally in work subsequent to this dissertation.

We also remark that the control algorithms described in this chapter are designed for the welding conditions described in Chapter V for tests run on AISI 4130 steel. Although the values of compensator parameters arrived at in this chapter apply specifically to this particular set of conditions, it is hoped that the approaches and algorithms which will be presented are general enough to be applied to a broad range of welding conditions.

This chapter consists of three sections, one treating compensator design and robustness of feedback control algorithms using dynamic cooling rate observers, one discussing compensator design and robustness of feedback controls using the quasi-steady state (or static) observers and a third discussing multivariable control of static and dynamic observer-based systems. The motivation and goals of the first two sections have been discussed. The motivation for the last section will come mostly as a result of the investigations described in the first two. Basically, the multivariable controls approach allows simultaneous control of both weld size
and cooling rate.

7.1 Control Formulation and Testing — Dynamic Observers

In this section, the design of centerline cooling rate controls utilizing the conventional and adaptive dynamic observers is discussed. In outline, the steps which were taken in this design process are:

1. Study of the welding process step response from travel speed (input) to centerline cooling rate (output) at several nominal operating points.

2. Approximation of the step response by reduced-order models valid in the neighborhood of the nominal operating points.

3. Design of compensators yielding closed-loop pole locations conducive to desirable dynamic and static control responses.

4. Determination of observer feedback gains providing adequate rate of decay of observation error.

5. Simulation of the closed-loop dynamic response of the simulation/observer combination.

6. Simulation testing of the robustness of the cooling rate control algorithms with respect to modeling and pool dimension measurement errors.

Note that the response of the travel-speed/cooling-rate system is approximated at various operating points by linear models. Analysis of the system dynamic response characteristics (item 1) demonstrates that the system is nonlinear and has a variable input delay. As a consequence, linear models valid in the neighborhood of a number of nominal operating points (as defined by the input values) are
formulated and used in the design of compensators. The effectiveness of this control design approach is then confirmed by simulation testing.

7.1.1 System Dynamic Response Characteristics and Proposed Control Structure

We now proceed with the study of the dynamic characteristics of the travel-speed/cooling-rate system. An initial investigation was performed by examining step travel speed perturbations of +0.5 in./min at nominal travel speed levels of 3, 4 and 5 in./min. For example, the step perturbation at a nominal speed of 3 in./min was performed by commencing simulation with a travel speed of 2.75 in./min., allowing the cooling rate output to reach steady state, then introducing a step perturbation of 0.5 in./min., resulting in an increase of the travel speed to 3.25 in./min. The resulting responses for all three tests, sampled at 2 second intervals, are shown in Figure 35. The time of application of the step perturbation is indicated by the vertical dashed line. The arc current was 100 amps for all of these tests and the preheat temperature was 25 C.

The step responses appear to be over-damped and approximately exponential, leading to the conjecture that they are dominated by a small number of real poles. Note that the system is nonlinear: it responds more slowly and has lower input/output gain at lower nominal travel speeds. In general, all of the responses are relatively slow, having rise times ranging from approximately 20 to 60 seconds. All of the responses also exhibit a significant amount of pure delay from the time of input perturbation (indicated by the vertical dashed line) to the time the output reflects this input change. This delay time also appears to vary with the nominal travel speed, being about 10 seconds at a nominal speed of 5 in./min. and about 30 seconds at 3 in./min.
Figure 35: Cooling Rate Response to Travel Speed Perturbations
The small oscillations in cooling rate which are evident in some of the step responses were found to be due to errors induced by the use of linear interpolation in the simulation cooling rate calculation. These errors, being of relatively small magnitude, did not interfere with system modeling and control.

To provide a basis for a preliminary investigation of some aspects of the control of the system, the dynamic response of the system was approximated by reduced-order linear models. One model was fit to the step response data generated with a nominal travel speed input of 4 in./min. The values of arc current and travel speed selected for this operating point are near the middle of the range of feasible values for the welding conditions under consideration.

The least squares technique was used to fit a discrete-time autoregressive moving average (ARMA) model of the form

$$y(k) + a_1 \cdot y(k - 1) = b_0 \cdot u(k - n_d) \quad (7.1)$$

to the selected data. This technique requires that Equation (7.1) be expressed in vector-matrix notation, yielding

$$Y = \Phi \Theta \quad (7.2)$$

The terms in Equation (7.2) are defined as

$$Y = (y(n_d + 1), y(n_d + 2), ..., y(n_d + r + 1))^T \quad (7.3)$$

$$\Phi = \begin{pmatrix}
    y(n_d) & u(1) \\
    y(n_d + 1) & u(2) \\
    ... & ... \\
    y(n_d + r) & u(r + 1)
\end{pmatrix} \quad (7.4)$$

and
Table 13: ARMA Parameters, $V_0 = 4$ in./min

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>-0.9439</td>
</tr>
<tr>
<td>$b_0$</td>
<td>1.9126</td>
</tr>
<tr>
<td>$n_d$</td>
<td>15</td>
</tr>
</tbody>
</table>

\[
\Theta = (a_1, b_0)^T
\]  
(7.5)

The variable $r$ which appears in the above expressions is defined as the number of data points minus one.

The least squares fit of the parameter vector $\Theta$ to the data is determined by the relationship

\[
\Theta_{ls} = (\Phi^T\Phi)^{-1}\Phi^TY
\]  
(7.6)

Equation (7.6) was implemented in a Fortran program, a listing of which is shown in Appendix C.

The least-squares-fit ARMA parameters for the selected step response data are shown in Table 13. The ARMA model may be re-arranged into transfer function form, yielding

\[
\frac{y(k)}{u(k)} = \frac{1.9126}{z^{15} - 0.9439z^{14}} = \frac{1.9126}{z^{14}(z - 0.9439)}
\]  
(7.7)

Note that the input delay of 15 time steps generates 14 system poles at the point $(0,0)$.

A plot comparing the step response predicted by the above ARMA model with that of the original data for the same step input sequence is shown in Figure 36. The ARMA model output appears to match the original data satisfactorily.
Figure 36: Comparison of ARMA Model Response with Actual Response: \( V_0 = 4 \) in./min.

Increasing the order of the ARMA model slightly improved agreement between the model output and the data.

An ARMA model was also fit to the step response data for the nominal travel speed of 5 in./min. The resulting ARMA parameters are shown in Table 14 and the ARMA model output is compared to the original data in Figure 37.

The ARMA model output does not match the original data and the poor fit was not significantly improved by increasing the model order. From Tables 13

| Table 14: ARMA Parameters, \( V_0 = 5 \) in./min |
|---|---|
| \( a_1 \) | -0.9305 |
| \( b_0 \) | 2.6147 |
| \( n_d \) | 12 |
Figure 37: Comparison of ARMA Model Response with Actual Data, $V_0 = 5\text{ in./min}$.

and 14, it is evident that the gain of the system changes with travel speed. This leads to the conclusion that a linear model is not capable of providing a completely adequate description of the system behavior at this operating point.

The ARMA model defined by the parameters in Table 13 represents the system well enough that it is used as a basis for the following — somewhat qualitative — discussion of the control characteristics of the system.

The relatively large delay associated with the system response and the slow system time constant make "tight" feedback regulation of the system difficult. This fact may be illustrated by study of the rudimentary control system depicted in Figure 38. Note that this control system employs unity feedback of a cooling rate measurement from the welding process to control the travel speed input. The compensator is simply an adjustable gain element. Assuming that the system is operating in the vicinity of $V = 4\text{ in./min}$, the ARMA model of Table 13 may
be used to describe the welding process. A root locus of this system is shown in Figure 39. There are 14 locus branches — generated by input delay — emanating from the point (0,0). Note that the positions of the “delay poles” move towards the unit circle as gain is increased, signifying more oscillatory closed-loop response. However, the maximum closed-loop gain for system stability is determined by the two branches near the point (1,0). These branches extend outside of the unit circle (indicating system instability) at a gain of approximately 0.08.

The above results suggest, even without simulation, that the response of a simple unity feedback control with adjustable compensator gain would be unsatisfactory. The position of the closed loop poles forces the use of relatively small feedback gains, which result in unacceptably large steady state errors. The closed-loop steady state error may be calculated from the closed loop transfer function by setting $K_c = 0.06$, approximately the largest value of gain for which the system
Figure 39: Root Locus of The Rudimentary Control System
is stable. The closed loop transfer function is given as

\[ \frac{y(k)}{u(k)} = \frac{1.9126K_c}{z^{14} - 0.94391z^{13} + 1.9126K_c} \]  
(7.8)

The closed-loop steady-state gain, calculated by putting \( z = 1 \), yields \( y_{ss}/u_{ss} = 0.64 \) (the subscript "ss" refers to steady state values). This value of steady state gain would generate large (approximately 35%) steady state errors.

The steady state error problem demonstrated above may be alleviated by a slightly more complex compensator design. By introducing an integral term (or terms) into the compensator transfer function, steady state error may be significantly reduced. Although integral compensation has the disadvantage of destabilizing the closed-loop performance, thereby reducing maximum allowable closed-loop gains, the reductions in steady state error are judged to be worth the cost in this case.

Before compensator design can proceed further, the system response must be modeled over the entire range of feasible operating conditions. However, the nonlinear system behavior exhibited in the above step responses makes modeling complicated. The fact that the dynamic characteristics and the input delay varies with the inputs implies that no single linear model can describe the system over a range of operating conditions. Furthermore, it was demonstrated above that a linear model could not describe the system response even in the vicinity of some operating points. However, it is important to note that the linear model which failed was a large-signal model. The step response data used in the least-squares fit represents a perturbation about a nominal operating point. By subtracting the nominal travel speed from the input data and the nominal cooling rate from the output data, a small signal response of the system may be obtained. As is shown in Figure 40, a linear model is capable of fitting the small signal response.
Figure 40: Comparison of a Small Signal Model Output to Data: $V_0 = 5\text{ in./min.}$ much more accurately than was the case with the large signal response. This plot compares the output of a small-signal ARMA model to the small-signal response for a nominal travel speed of 5 in./min.

The above result suggests a method of modeling and controlling the system response over a wide range of input values. A number of operating points which span the range of expected inputs can be selected, and small signal models can then be used to describe the system response in the neighborhood of these operating points. Compensators which provide desirable system response at each of the operating points can then be designed, and a control scheme which selects appropriate compensator parameters depending upon the regime of system operation can be implemented.

The use of small signal models for compensator design makes it necessary that the cooling rate control algorithm provide some means of generating nominal
process inputs (i.e. nominal travel speeds). In other words, by using small signal models, one is designing a feedback control which is intended to provide "small" corrections to a nominal travel speed value based upon "small" errors from a nominal cooling rate. This presents a practical problem since, given a nominal cooling-rate set-point, one does not generally know the corresponding nominal travel speed input (i.e. the travel speed which will cause the welding process to output the nominal cooling rate). However, this information can be obtained from an inverse model of the travel-speed/cooling-rate system. The inverted model may be either static (i.e. steady state) or dynamic. Recall that a steady state model of the welding heat flow system was formulated in Chapter VI. This relationship is repeated here for convenience as

\[ CR = 1.2683 \times 10^{-8} (I - 65)^{-2512} (v + 0.4)^{3.3357} (1530 - T_0)^{3.1071} \]  

(7.9)

To use this relationship as a basis for a control implementation, it may be rearranged to express travel speed in terms of cooling rate set point, current and preheat, i.e.

\[ v = \frac{CR(I - 65)^{-2512}}{1.2683 \times 10^{-8} (1530 - T_0)^{3.1071}} - 0.4 \]  

(7.10)

Based upon these considerations, the control system structure shown in Figure 41 is proposed. Given a desired centerline cooling rate, a steady state inverse model of the system is used to obtain an estimate of the steady-state travel speed which will produce the set point cooling rate. This portion of the control is commonly referred to as feedforward control. The feedback control algorithm makes adjustments about the nominal (or feedforward) travel speed. The value of the feedforward travel speed, along with the nominal arc current and preheat temperature are used to select appropriate compensator parameters at the beginning of
control operation. Also note that a saturation element has been added to limit the travel speed command to the welding process within certain reasonable limits. This is necessary to ensure that weld pools of unreasonably large or small sizes are not generated by the control algorithm as it manipulates travel speed to regulate cooling rate.

Before leaving this section, we note that the findings of Hardt et. al. [2] provide justification for implementing compensator adaptation by using a static inverse-model. They found that the speed of response of weld pool dimensions to step travel speed perturbations was a function of the post step-perturbation value of travel speed. Based upon these results, it is reasonable to propose that the compensator parameters for any given control “run” be chosen based upon the final value of system inputs (particularly travel speed) which will prevail when steady state conditions are attained. Of course, if the behavior of the process being
controlled is not accurately represented by the inverse model, then the travel speed estimate obtained in this way will be in error. However, it is expected that the value will be close enough that an appropriate control response can be selected.

7.1.2 System Modeling and Compensator Design

We now proceed with the task of modeling the system response in the vicinity of a number of selected operating points, and designing compensators which provide acceptable dynamic response in the vicinity of these operating points. The design approach employed is based upon a separation principle. Compensators are designed using input/output models which do not include the observer response. The observer gains are then adjusted so that the observation error decays significantly faster than the closed-loop control error. When the final observer-based control is then implemented, the effect of the observer response upon the control closed-loop response is then negligible.

For modeling purposes, the operating range of the system was divided into six different regions, based upon nominal values of travel speed, arc current and preheat temperature (Table 15). These operating points were selected to span the range of acceptable inputs for the welding conditions under consideration.

A given set of welding inputs must be associated with one of the operating points listed in Table 15 so that an appropriate compensator can be selected. The inequalities shown in Table 16 were used for this purpose.

To obtain system models corresponding to the nominal operating points listed in Table 15, travel speed step perturbation response data was collected at each of the operating points. Each input/output data set was then modified by subtraction of the nominal travel speed and cooling rate, and the small signal data was fit with an ARMA model using the least squares technique. The ARMA parameters which
Table 15: Nominal Operating Points

<table>
<thead>
<tr>
<th>Pt. No.</th>
<th>current</th>
<th>travel speed</th>
<th>preheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>3</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>5</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>110</td>
<td>5</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 16: Inequalities for Sorting of Inputs

<table>
<thead>
<tr>
<th>Pt. No.</th>
<th>current</th>
<th>travel speed</th>
<th>preheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$I &lt; 100$</td>
<td>$V &lt; 4$</td>
<td>$T_0 &lt; 100$</td>
</tr>
<tr>
<td>2</td>
<td>$I &lt; 100$</td>
<td>$V &lt; 4$</td>
<td>$T_0 \geq 100$</td>
</tr>
<tr>
<td>3</td>
<td>$I &lt; 100$</td>
<td>$V \geq 4$</td>
<td>$T_0 \geq 100$</td>
</tr>
<tr>
<td>4</td>
<td>$I \geq 100$</td>
<td>$v &lt; 4$</td>
<td>$T_0 &lt; 100$</td>
</tr>
<tr>
<td>5</td>
<td>$I \geq 100$</td>
<td>$v \geq 4$</td>
<td>$T_0 &lt; 100$</td>
</tr>
<tr>
<td>6</td>
<td>$I \geq 100$</td>
<td>$V \geq 4$</td>
<td>$T_0 \geq 100$</td>
</tr>
</tbody>
</table>
Table 17: ARMA Parameters at Nominal Operating Points

<table>
<thead>
<tr>
<th>Pt. No.</th>
<th>$a_1$</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>zero</th>
<th>$n_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.9159</td>
<td>1.0427</td>
<td>0.5239</td>
<td>-0.5111</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>-0.9204</td>
<td>0.2239</td>
<td>0.0249</td>
<td>-0.1112</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>-0.9300</td>
<td>1.2170</td>
<td>1.0493</td>
<td>-0.8622</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>-0.9426</td>
<td>0.6143</td>
<td>0.3133</td>
<td>-0.5100</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>-0.9132</td>
<td>2.6520</td>
<td>1.7857</td>
<td>-0.6333</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>-0.9334</td>
<td>0.8622</td>
<td>0.5796</td>
<td>-0.6772</td>
<td>21</td>
</tr>
</tbody>
</table>

resulted from this procedure are listed in Table 17.

The equivalent transfer function of each ARMA model in Table 17 has the form

$$ CR(z) = \frac{b_0 z + b_1}{V(z) \ z^{n_d} (z + a_1)} $$

(7.11)

Note that the negative of the ARMA parameter "$a_1$" corresponds to one open loop pole of the system transfer function. There are also $n_d$ poles at $(0,0)$ generated by the input delay of each model and one zero at $-\frac{b_1}{b_0}$. The latter is tabulated in Table 17.

A sample comparison of the system and model step perturbation responses for a nominal operating point of $v = 3$ in./min., $I = 100$ amperes and $T_0 = 25$ C is shown in Figure 42. This model corresponds to operating point 5. As can be seen, the small-signal response of the system is well represented by a linear model. The standard deviation of the error between the model outputs and the data was less than 0.05 C/sec.
Figure 42: Comparison of System and Model Responses for Operating Point 5

Having generated models of the dynamic response of the system, compensators were designed to improve closed loop system response at each operating point. The design process was similar for compensators for each of the nominal operating points. As an example, the design process for operating point 5 of Table 15 is described below.

We begin by considering a rudimentary feedback control system similar to the one depicted in Figure 38. In this system, the compensator consists of an adjustable gain element. The root locus for such a system, operating in the vicinity of point 5 of Table 17 is shown in Figure 43. To decrease steady state error, a compensator having integral action is desired. This corresponds to the addition of a pole at (1,0) to the open loop transfer function. From Figure 43, it is obvious that the addition of this pole will have adverse effects upon the system dynamic response. With only a small amount of compensator gain, branches generated by the poles at
Figure 43: Rudimentary Control System Root Locus For Operating Point 5

(1,0) and (0.9132,0) would migrate outside of the unit circle, causing instability. To prevent this, a compensator zero was added on the real axis at (0.9132,0). This zero effectively cancels the open loop pole at this location. Hence the compensator transfer function for operating point number 5 is

\[
\frac{V_k}{e_k} = \frac{K_c(z - 0.9132)}{(z - 1)}
\]

(7.12)

The root locus of the closed loop system with the compensator described above is shown in Figure 44. Using the root locus as a guide, a compensator gain of 0.002 was chosen as a value which would yield stable and reasonably damped closed loop system response. This gain corresponds to the gain at which the dominant pole near (0.9,0) breaks away from the real axis. Simulations (discussed below) confirmed that use of this somewhat heuristic design rule produced acceptably damped closed-loop system responses over the range of operating conditions mapped into operating point 5.
A disadvantage of the compensation scheme outlined above is that it guarantees a slow, real closed loop pole somewhere in the vicinity of $(0.90, 0)$. Unfortunately, the reason for this undesirable circumstance is the placement of a compensator pole at $(1,0)$. Since this pole is needed for the integral action it provides, the undesirable outcome is not easily avoided.

At this point, further improvement of the system response by compensator pole placement would require attempts to diminish the effects of the poles generated by the system delay. Although this could done, the compensator would be increased in complexity, and the increased performance would be marginal. The main reason for the marginal increase in performance lies in the saturation element depicted in the control block diagram of Figure 41. This element performs a crucial function, limiting the travel speed to values which ensure reasonable weld pool sizes. However, it has a retarding effect upon system dynamic performance.
Table 18: Compensator Parameters

<table>
<thead>
<tr>
<th>Pt. No.</th>
<th>zero</th>
<th>pole</th>
<th>gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.9159</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>-0.9204</td>
<td>1</td>
<td>0.08</td>
</tr>
<tr>
<td>3</td>
<td>-0.9300</td>
<td>1</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>-0.9426</td>
<td>1</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>-0.9132</td>
<td>1</td>
<td>0.002</td>
</tr>
<tr>
<td>6</td>
<td>-0.9334</td>
<td>1</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Compensator designs which attempt to achieve control performance significantly faster than that produced by the above design tend to generate travel speeds which are limited by the saturating element, and thus result in no real advantages.

The design procedure which is described above was followed to determine compensator pole placements and gains for the remainder of the operating points. The compensator poles and gain for each operating point are summarized in Table 18 below.

7.1.3 Determination of Observer Feedback Gains and Control Response Simulation Testing

After compensator poles and gains were selected for all of the nominal operating points, the next step in the control design process was the determination of observer feedback gains providing rates of convergence of observation error significantly faster than the control response designed above. This was done by trial-and-error adjustment based upon simulation tests. It was found that the cooling rate observation error rates which were obtained by adjustment of the observer
feedback gain, $k_o$, were fast enough so as not to impact significantly upon the compensator designs presented above.

We begin with the conventional observer. A pool width feedback error gain of $k_o = 600$ was found to produce satisfactorily fast cooling rate observation error convergence in the conventional observer. Simulation results showing convergence of observer and simulation cooling rate outputs are displayed in Figure 45. The plot compares cooling rate outputs of the simulation and observer for two sets of operating conditions. In both tests, the initial temperature of the observer was taken to be 25 C, that of the simulation was 100 C. For the first plot, travel speed was 4 in./min. while for the second, travel speed was 5 in./min. Arc current was 100 amperes for both tests. The initial few iterations for which the cooling rate outputs are both zero is an artifact of the numerical technique used to calculate cooling rate. It represents the time taken by the "base metal" to warm up to a
temperature sufficient that the centerline cooling rate could be calculated. As is evident in the plots, the difference in initial temperature between the observer and plant is practically eliminated in approximately 6 iterations (12 seconds); after this amount of time, the cooling rate error was less than 5 percent of its initial value. If one makes the assumption that the observation error system can be approximated by a first order model, the convergence rate quoted above is represented by a real pole at 0.61. Since this is significantly faster than the dominant system pole near (0.90,0), it is reasonable to suppose that the observer pole will not significantly affect system closed-loop response.

Next, the observer feedback gain of the adaptive observer was set, again by trial and error experimentation. It was determined that a gain setting of $k_o = 18$ provided suitably fast convergence of the weld width observation error. Plots illustrating the convergence between pool width and cooling rate observation error
for two operating points are shown in Figures 47 and 48. Here, the arc current was 100 amperes, the efficiency of the process model was $\eta = 0.45$ and that of the observer was $\hat{\eta} = 0.56$. The first plot corresponds to travel speed of 3 in./min. while the second corresponds a travel speed of 4 in./min. The cooling rate observation error converges to less than 5 percent of its initial value in approximately 8 iterations (16 seconds) in both instances. Again, assuming that the observation error system is first order, this response is characterized by a real pole at 0.69. This pole is significantly faster than the dominant closed loop pole at 0.90.

We now present results from simulation tests of closed loop system control performance. For all of the trials shown below, the response is of the closed loop system implemented using the adaptive observer. The adaptive observer was chosen for these tests because it was found in the tests described above to have a slightly slower rate of observation error convergence than the conventional
Figure 48: Adaptive Observer Error Convergence, $V = 4$ in./min.
Table 19: Operating Points for Control Tests

<table>
<thead>
<tr>
<th>Test No.</th>
<th>current</th>
<th>preheat</th>
<th>Cooling Rate Set Point</th>
<th>Figure No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>25</td>
<td>4.5</td>
<td>Fig. 49</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>25</td>
<td>7.5</td>
<td>Fig. 50</td>
</tr>
<tr>
<td>3</td>
<td>110</td>
<td>25</td>
<td>5.5</td>
<td>Fig. 51</td>
</tr>
<tr>
<td>4</td>
<td>110</td>
<td>25</td>
<td>7.5</td>
<td>Fig. 52</td>
</tr>
</tbody>
</table>

observer. Hence, use of the adaptive observer constitutes a "worst-case test" as far as the effect of the observer upon the closed-loop performance.

As was evident in the observation-error convergence responses shown above, the start-up transient of the system represents a large perturbation. The closed-loop response characteristics of the control system will be judged by its response to this perturbation. The response of the closed-loop system (including adaptive observer) to the initial start-up transient was simulated for a total of 4 different operating points, shown in Table 19. The operating points for current and preheat were taken from Table 15. In all of the tests, the observer was given an initial arc efficiency estimate of $\hat{\eta} = 0.45$ while the efficiency of the plant was set at the usual $\eta = 0.56$.

The results of the closed-loop control simulations are shown in Figures 49, 50, 51 and 52. The operating point to which each of these responses correspond is indicated in Table 19.

The response of the system was clearly slower for lower values of cooling rate set point and higher values of nominal arc current. This phenomenon is attributable to the fact that the system has larger input delay and slower poles under these conditions. The responses exhibit a moderate amount of overshoot.
Figure 49: Closed-Loop System Response, Op. Pt. 1

Figure 50: Closed-Loop System Response, Op. Pt. 2
Figure 51: Closed-Loop System Response, Op. Pt. 3

Figure 52: Closed-Loop System Response, Op. Pt. 4
and relatively long settling times. For the selected values of compensator poles and zeros, adjustment of the compensator gain allows variation of these two dynamic response characteristics. Increasing gain produces faster response but more oscillation, whereas decreasing the gain has the opposite effect. The simulation results shown above verify that the gains which were arrived at from the root locus analysis described earlier provided adequate response.

7.1.4 Dynamic Observer Robustness

We now turn to a discussion of the robustness of observer-based cooling rate control with respect to modeling errors and measurement errors. In Chapter IV, convergence of cooling rate observation error was demonstrated under the assumption that an exact system model was known. In actual welding situations, this is not likely to be the case. One of the more troubling aspects of the welding process is the fact that it is difficult to model accurately. Hence, the question of the robustness of cooling rate controls which rely upon the use of a welding heat flow model is an important one.

A related question which will also be discussed here is the effect of pool width measurement errors upon control accuracy. The resolution of optical devices used to measure weld pool dimensions is necessarily limited, but is to some extent adjustable by variations in optical magnification. Using a reasonable estimate of measurement resolution, simulation results will be used to estimate cooling rate control accuracies which might be achieved by controls implemented using the two different observers.

We first consider the robustness of the observer-based controls with respect to modeling errors. Robustness investigations were carried out using a simulation of the cooling rate control system depicted in Figure 41. As in the above control tests,
the welding process block in this diagram was replaced by a simulation of welding heat flow for the robustness tests. In a series of trials, selected physical parameters of the welding process heat flow simulation were varied individually while the corresponding parameters in the observer were left unchanged. Parameters which were varied were base metal thickness, thermal diffusivity, preheat temperature, surface heat transfer coefficient and arc efficiency. For each parameter alteration, the time response of feedback control systems utilizing the conventional and adaptive observers were simulated until steady state conditions were attained. The steady state control error, defined as the difference between simulation (i.e. plant) cooling rate and setpoint cooling rate, was used as a measure of the robustness of the control.

The inputs to the simulation for the robustness tests were constant: cooling rate setpoint was 6.0 C/sec. and arc current was 90 amperes. The parameter variations which were used are summarized in Table 20. The magnitude of the parameter variations used for the robustness testing were arbitrarily taken to be ±10% of the nominal value for all of the parameters except preheat temperature. In the absence of hard data, it is assumed that variations of ±10% approximate the uncertainty of the respective parameters under welding conditions. The preheat temperature variations were taken to be larger than 10 % because it is felt that much larger variations of this parameter are commonly experienced in multipass welding using customary procedures.

Results of the robustness tests are displayed graphically in Figures 53 through 57. These figures show the steady state value of simulation cooling rate for the various conditions of parameter mismatch.

For all but two of the parameter variations considered, the conventional observer was much less robust than the adaptive observer. For efficiency, thickness
Table 20: Observer and Simulation Parameter Variations

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Parameter</th>
<th>Observer Value</th>
<th>Simulation Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Diffusivity</td>
<td>0.063</td>
<td>0.057</td>
</tr>
<tr>
<td>2</td>
<td>Diffusivity</td>
<td>0.063</td>
<td>0.069</td>
</tr>
<tr>
<td>3</td>
<td>Thickness</td>
<td>0.2286</td>
<td>0.2086</td>
</tr>
<tr>
<td>4</td>
<td>Thickness</td>
<td>0.2286</td>
<td>0.2486</td>
</tr>
<tr>
<td>5</td>
<td>Arc Efficiency</td>
<td>0.56</td>
<td>0.50</td>
</tr>
<tr>
<td>6</td>
<td>Arc Efficiency</td>
<td>0.56</td>
<td>0.60</td>
</tr>
<tr>
<td>7</td>
<td>Sur. Heat Trans.</td>
<td>0.001</td>
<td>0.0009</td>
</tr>
<tr>
<td>8</td>
<td>sur. Heat Trans.</td>
<td>0.001</td>
<td>0.0011</td>
</tr>
<tr>
<td>9</td>
<td>Preheat Temp.</td>
<td>25</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>Preheat Temp.</td>
<td>25</td>
<td>150</td>
</tr>
</tbody>
</table>

Figure 53: Diffusivity Robustness
Figure 54: Thickness Robustness

Figure 55: Efficiency Robustness
Figure 56: Surface Heat Transfer Robustness

Figure 57: Preheat Robustness
and diffusivity variations, the total control error of conventional observer-based controls was more than 5 C/sec. for each case while that of the adaptive observer-based controls was less than 0.7 C/sec. One exception was preheat temperature variation: the conventional observer, being designed for precisely this condition, performed perfectly. The corresponding maximum adaptive observer-based control error was 3.2 C/sec. The other exception was surface heat transfer coefficient variation, where the two observers were nearly perfectly robust. Modeling errors of only 10% in arc efficiency and base metal thickness caused the conventional observer to produce such inaccurate cooling rate estimates that the travel speed variation was limited by the saturation element in the control loop. This occurred in spite of the fact that the set point cooling rate of 6.0 C/sec should have required a travel speed near the middle of the allowable range.

The poor performance of the conventional observer algorithm leads one to speculate that this algorithm might perform better if no observer feedback were used. In this case \( k_0 = 0 \), the observer is essentially nothing more than a welding heat flow simulation, identical in form to the one used to represent the welding process. Rerunning the "diffusivity variation" robustness test with \( k_0 = 0 \) produced the result shown in Figure 58. Comparison of this result and the one shown in Figure 53 reveals that the conventional observer-based control was substantially less robust than a control based upon a model of the process with no "observer correction". On the other hand, the corresponding adaptive observer result shows that the adaptive observation algorithm was comparably robust.

The above results indicate that, while the conventional observer is probably not robust enough to be applied in practice, the adaptive observer appears to be much more desirable from this standpoint. The adaptive observer performance does, however, have one significant disadvantage: for cases where the preheat tem-
Figure 58: “Open-Loop” Observer Robustness Test

...temperature of the observer model does not match that of the process, the control error was only slightly improved by the adaptive observer algorithm. This implies the preheat temperature of the base metal must be fairly well known for the adaptive observation technique to be successfully applied.

We now discuss investigations of the effects of pool width measurement error upon the accuracy of cooling rate controls implemented using the conventional and adaptive observers. Investigations were performed using a simulation of the cooling rate control system depicted in Figure 59. The pool width measurement was modified by the addition of a small constant perturbation, $\Delta w$, before being input to the observer algorithm. This perturbation represents the maximum pool width measurement error which would be produced by a device having a resolution equal to $\Delta w$. For the tests described below, a resolution of 0.5 mm is chosen as a value being readily attainable with current technology [48].
A constant cooling rate setpoint of 5.3 C/sec. was input to the simulation, along with a constant arc current of 100 amperes and preheat of 25 C. The time response of the system was simulated until steady-state conditions were attained. The control error was evaluated as the difference between the steady-state simulation cooling rate output and the setpoint cooling rate (5.3 C/sec.). This procedure was repeated with both the conventional and adaptive observers.

The results of the measurement-error robustness tests are displayed in Table 21. Again, it is obvious that the adaptive observer is much more robust than the conventional observer. For the same pool width measurement error (0.5 mm.), cooling rate control implemented using the conventional observer produced a cooling rate error of about 2 C/sec. while the adaptive observer-based control yielded an error of only 0.4 C/sec.

166
Table 21: Closed-Loop Control Measurement Error Robustness

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Observer</th>
<th>Set Point Cooling Rate</th>
<th>Simulation Cooling Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Conventional</td>
<td>6.5 C/sec.</td>
<td>8.5 C/sec.</td>
</tr>
<tr>
<td>2</td>
<td>Adaptive</td>
<td>6.5 C/sec.</td>
<td>6.9 C/sec.</td>
</tr>
</tbody>
</table>

7.1.5 Summary — Dynamic Observer Control Design and Testing

The design of controls utilizing conventional and adaptive dynamic observers was studied via numerical simulation. From investigations of the dynamic behavior of the travel-speed/cooling-rate system, it was determined that the system had nonlinear dynamics and a variable input delay. A compensator design approach based upon linearization of the system in the vicinity of selected operating points, and formulation of compensators yielding adequate steady-state and dynamic response in the vicinity of these operating points was followed. Simulation results showing system response over a variety of operating conditions were presented. The robustness of the cooling rate control using the two types of observers was also investigated. The conventional observer was found to lack robustness to the extent that application of the algorithm in practical situations would appear to be impossible. In comparison, the adaptive observation algorithm exhibited superior robustness characteristics: for all types of parameter variations except cooling rate variations, adaptive observer-based controls maintained cooling rate to within 0.7 C/sec. of the desired value. However, in the case of preheat temperature variation, maximum adaptive observer-based control error was 3.2 C/sec., leading to the conclusion that application of the adaptive observer to situations where preheat is not well known would be unsuitable.
7.2 Control Design and Testing — Static Observers

In this section, cooling rate controls which utilize the static observation algorithms described in Chapter VI are discussed. As in the previous section, compensators which provide desirable dynamic and steady-state closed loop performance are designed, and the robustness of the controls is studied.

The observation algorithms of interest in this section are those expressed by Equations (6.12), (6.15), (6.13) and (6.16). For convenience, these relationships are repeated here as Equations (7.13) through (7.16).

\[
CR = 1.9284 	imes 10^{-8} \cdot (W - 0.4)^{-0.2263} (V + 0.4)^{1.749} (1530 - T_0)^{2.7542} \tag{7.13}
\]

\[
CR = 1.8864 \times 10^{-8} \cdot (W - 0.4)^{-0.1002} (V + 0.4)^{2.8278} (1530 - T_0)^{2.8777} \tag{7.14}
\]

\[
CR = 3.4511 \times 10^{-8} \cdot (I - 65)^{0.2955} (W - 0.4)^{-0.4789} (1530 - T_0)^{2.3736} \tag{7.15}
\]

\[
CR = 1.0374 \times 10^{-8} \cdot (I - 65)^{0.5304} (W - 0.4)^{-0.6411} (1530 - T_0)^{2.4061} \tag{7.16}
\]

Equations (7.13) and (7.14) represent steady state versions of the dynamic adaptive observer discussed in Chapter IV. The first is derived from experimental data while the second is derived from simulation data. Both will be referred to as adaptive observers in this chapter.

Equations (7.16) and (7.15) represent the steady state of an observer formulation wherein travel speed is modified so as to eliminate pool width observation error. Again, the first is derived from experimental data while the second is derived from simulation data. These will be referred to in this chapter as (for want of a better name) "modified adaptive observers".

It is important to stress that all of the control design and testing results presented in this chapter are obtained from simulations. The control block diagram,
Figure 34, was simulated on a digital computer. The block labeled "welding process" was replaced by the welding heat flow simulation of Chapter IV. The block labeled "weld pool dimension measurement" was eliminated: the weld pool width output of the heat flow simulation was input directly to the static observation algorithms.

Before proceeding with control design and testing, some points concerning the philosophy of the control approach to be taken with the static observers should be noted. There is a fundamental difference in the dynamic behavior of the cooling rate estimates produced by the static and dynamic observers. The dynamic response of the cooling rate estimate of either of the dynamic observers is nominally the same as that of the cooling rate output of the process. Hence, even during periods when the temperature distribution is undergoing transients, the estimated cooling rate at least approximates the process cooling rate. In the case of the static observers, the dynamic response of the cooling rate estimate is determined by the dynamic response of the pool width term in the observer formula. Since, in general, the dynamic response of the weld width is much different from that of the centerline cooling rate, the dynamic responses of the static cooling rate estimates are different from that of the process cooling rate. As a result, when the welding temperature distribution is undergoing transients, the static cooling rate estimate does not, in general, equal the process cooling rate.

The above considerations raise the question of how to best design feedback cooling rate controls using the static observation algorithms. In this dissertation, the following approach is taken. The feedback control response is designed to provide rapid convergence of the cooling rate estimate to the desired cooling rate. Then, within the limits of the accuracy of the observation algorithms, the process cooling rate will converge to the set point cooling rate as steady state conditions
are achieved. In this approach, the convergence rate of the process cooling rate error is limited by the physics of the process. For example, even if a deadbeat control were used to cause convergence of the estimated cooling rate to the desired value in one control iteration, the process cooling rate would converge at a rate determined by the dynamics of the heat flow system.

A second difference between the static and dynamic observer-based control designs arises from the fact that static observers were derived both from experimental and from empirical data. In the case of the dynamic observers, the same observation algorithm that would be used in implementation of cooling rate control of an actual welding process was also used in the simulation design and testing. Although both “actual-data” and “simulation-data” static observers share the same form, there are differences in the values of the coefficients in the relationships. To achieve the best control results with an actual welding process, the “actual-data” observers would be employed. However, since all testing described in this section was done using a simulation of welding heat flow, the “simulation-data” observers were used. Hence, the parameters of the control algorithms derived in this chapter may need to be somewhat altered to be applied in an actual welding process.

7.2.1 Dynamic Response Characteristics and Proposed Control Structure

We now proceed with a study of the dynamic characteristics of the cooling rate estimates produced by the static-adaptive and modified-adaptive observers. We begin by considering the step perturbation responses of cooling rate estimates of the two observers.

The plots in Figures 60 and 61 show the response of the respective cooling rate estimates to step perturbations of +0.5 in./min in travel speed. Three nominal travel speeds were used: 3, 4 and 5 in./min. Figure 60 displays the adaptive
Figure 60: Adaptive Observer-Based System Step Responses

Figure 61: Modified Adaptive Observer-Based System Step Responses
observer responses while Figure 61 shows the modified adaptive observer response. Note that the responses of the two observers are quite similar. They contain essentially no input delay and are approximately exponential, leading to the conclusion that they may be approximated by a relatively low-order model. However, the responses do exhibit the expected nonlinearity: the gain and the "response speed" appear to increase with increasing travel speed.

Next, the ability of linear models to fit the responses was studied. A nominal travel speed of 4 in./min. was perturbed by the addition of a pseudo-random binary sequence uniformly distributed on the interval [-0.5,0.5] in./min.¹ This travel speed sequence was input to the system and the resulting estimated cooling rate (from both observers) was recorded. Using least squares, ARMA models were fit to this I/O data. Models of the form

\[ y(k) + a_1 y(k-1) = b_0 u(k-1) + b_1 u(k-2) \]  

(7.17)

were found to fit the data adequately. The ARMA parameters for each I/O model are shown in Table 22 In Figures 62 and 63, the fit of the models are compared graphically to the data.

The data is relatively well-fit by the ARMA models. However, since the responses are not linear, the same model may not be used to represent the system over a wide range of travel speeds. As an example, a pseudo-randomly perturbed nominal travel speed of 5 in./min was input to the modified adaptive observer ARMA model of Table 22 and to the simulation. The output of the model is

¹In the previous section, system modeling was performed using step perturbation responses. This was particularly convenient because it allowed easy measurement of the large, unknown input delays inherent in the systems being considered. In this section, the systems being modeled have minimal delay (1 iteration), so the more persistently exciting pseudo-random perturbation is used.
Table 22: ARMA Parameters, $V_0 = 4$ in./min.

<table>
<thead>
<tr>
<th>Param</th>
<th>Adaptive</th>
<th>Mod. Adaptive</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>-0.7487</td>
<td>-0.7278</td>
</tr>
<tr>
<td>$b_0$</td>
<td>20.9549</td>
<td>7.9711</td>
</tr>
<tr>
<td>$b_1$</td>
<td>-12.1066</td>
<td>1.5723</td>
</tr>
<tr>
<td>$n_d$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 62: Comparison: Adaptive Observer-Based System Model and Data
Figure 63: Comparison: Modified Adaptive Observer-Based System Model and Data

compared to the corresponding simulation cooling rate estimate output in Figure 64. The simulation data taken at a nominal speed of 5 in./min. is relatively poorly fit by the ARMA model for $V_0 = 4$ in./min.

Based upon the above findings, a control approach similar to the that employed in the previous section for the dynamic observer-based control is proposed. A block diagram of the proposed control structure is shown in Figure 65. As in the dynamic observer case, the compensator parameters are selected based upon the values of welding process inputs (i.e. arc current, travel speed and preheat temperature). An estimate of the travel speed for use in this calculation is obtained from the set point cooling rate and a static inverse-model of the heat flow process. Slight simplification over the dynamic observer-based case comes from the fact that the cooling rate estimate data appears to be fit reasonably well by a large-signal model. Recall that in the previous case, a small signal modeling ap-
Figure 64: Comparison: Modified Adaptive Observer-Based System Model and Data

An approach was required. The impact of this is that, for static observer-based control, no feed forward control structure is needed.

7.2.2 System Modeling and Compensator Design

In this section, we form ARMA models of the travel-speed/cooling-rate-estimate system for each of the two static observation algorithms under consideration. These models are fit to data taken from selected operating points. Next, using root locus techniques, compensators which yield adequate response in the vicinity of each operating point are designed.

The operating points for which ARMA models were obtained are listed in Table 15. These are the same as those used for dynamic observer-based systems. At each nominal operating point, the travel speed input was perturbed with a pseudo-random binary sequence having magnitude in the range -0.5 to 0.5 in./min.
Figure 65: Proposed Structure for Static Observer-Based Control
Table 23: ARMA Parameters: Adaptive Observer-Based System

<table>
<thead>
<tr>
<th>Pt. No.</th>
<th>$a_1$</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$n_d$</th>
<th>zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.8743</td>
<td>18.8724</td>
<td>-13.6703</td>
<td>1</td>
<td>0.7243</td>
</tr>
<tr>
<td>2</td>
<td>-0.8942</td>
<td>16.9987</td>
<td>-12.8506</td>
<td>1</td>
<td>0.7560</td>
</tr>
<tr>
<td>3</td>
<td>-0.4722</td>
<td>29.0526</td>
<td>-8.9895</td>
<td>1</td>
<td>0.3094</td>
</tr>
<tr>
<td>4</td>
<td>-0.9022</td>
<td>15.9650</td>
<td>-12.3359</td>
<td>1</td>
<td>0.7727</td>
</tr>
<tr>
<td>5</td>
<td>-0.5370</td>
<td>24.0476</td>
<td>-8.8422</td>
<td>1</td>
<td>0.3673</td>
</tr>
<tr>
<td>6</td>
<td>-0.6182</td>
<td>22.2935</td>
<td>-9.8504</td>
<td>1</td>
<td>0.4419</td>
</tr>
</tbody>
</table>

The resulting cooling rate estimate was recorded, and the input/output data was fit with an ARMA model.

ARMA model parameters for the adaptive observer-based system are shown in Table 23. ARMA models of the modified adaptive observer-based system responses are shown in Table 24. The above ARMA models have the equivalent transfer function

$$
\frac{CR(z)}{V(z)} = \frac{b_0 z + b_1}{z(z + a_1)}
$$

(7.18)

This transfer function has 1 open loop pole at the point (0, 0) and another at the point $(-a_1, 0)$. There is one open loop zero at the point $(-\frac{b_1}{b_0}, 0)$. The value of $-\frac{b_1}{b_0}$ for each operating point is listed in Table 24.

Having obtained models of the system responses over the operating range, the design of compensators which provide desirable closed loop dynamic and static response for adaptive observer- and modified adaptive observer-based systems is begun. We start with the adaptive observer-based system and, as an example, describe the design of a compensator for the system when operating in the vicinity
Table 24: ARMA Parameters: Modified Adaptive Observer-Based System

<table>
<thead>
<tr>
<th>Pt. No.</th>
<th>( a_1 )</th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>( n_d )</th>
<th>zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.8385</td>
<td>5.1450</td>
<td>1.3532</td>
<td>1</td>
<td>-0.2630</td>
</tr>
<tr>
<td>2</td>
<td>-0.8634</td>
<td>3.0075</td>
<td>1.8943</td>
<td>1</td>
<td>-0.6299</td>
</tr>
<tr>
<td>3</td>
<td>-0.4737</td>
<td>15.8049</td>
<td>2.8665</td>
<td>1</td>
<td>-0.1814</td>
</tr>
<tr>
<td>4</td>
<td>-0.8755</td>
<td>2.8890</td>
<td>1.7240</td>
<td>1</td>
<td>-0.5967</td>
</tr>
<tr>
<td>5</td>
<td>-0.5781</td>
<td>11.8192</td>
<td>2.0793</td>
<td>1</td>
<td>-0.1759</td>
</tr>
<tr>
<td>6</td>
<td>-0.6538</td>
<td>7.7711</td>
<td>2.8040</td>
<td>1</td>
<td>-0.3608</td>
</tr>
</tbody>
</table>

of operating point 1. The corresponding ARMA model has an open-loop pole at 0.8743 and a zero at 0.72. Placing a compensator pole at (1,0) provides an integral term, ensuring small steady state error for constant set points. Placement of a compensator zero at (0.87,0) and a pole at (0.72,0) provides approximate cancellation of the system pole and zero at these locations. The transfer function of the compensator may then be written as

\[
\frac{V_c}{E_{cr}} = \frac{k_c(z - 0.87)}{(z - 1.0)(z - 0.72)}
\]  

A root locus of the resulting closed-loop system is shown in Figure 66. From this root locus, a system gain of 0.02 is chosen as a value compatible with fast system response with minimal overshoot.

The design of compensators for the remaining operating points proceeded along similar lines.

We next consider the modified adaptive observer-based system and as an example, describe the design of a compensator for the system when operating in the vicinity of operating point 1. This system has an open-loop pole at 0.8385 and
Figure 66: Adaptive Observer-Based Control Root Locus
a zero at -0.263. Again, placing a compensator pole at (1,0) provides an integral term, ensuring small steady state error for constant set points. A compensator zero placed over the system pole at 0.8385, provides approximate cancellation of the corresponding system pole. A root locus of the resulting closed-loop system is shown in Figure 67.

From this root locus, a system gain of 0.06 is chosen as a value compatible with fast system response with minimal overshoot. At this gain, the locus branches
near (0.4,0) are just breaking from the real axis.

As before, the design of compensators for the remaining operating points proceeded along similar lines.

7.2.3 Simulation Testing of Control Algorithms

In this section, the control algorithms designed in the previous section are tested via simulation to determine dynamic response and robustness characteristics. As in the previous section dealing with dynamic observers, the transient response of the system to the start-up perturbation is used as a measure of the dynamic response of the system. Robustness with respect to modeling errors and measurement errors will also be studied.

The adaptive observer-based control systems are considered first. Responses were simulated at a variety of operating conditions; the operating points and the corresponding figure in which the response appears are summarized in Table 25.

The responses show that estimated cooling rate converged to within 5 percent of the desired value in approximately 20 seconds or less in all cases. The 5 percent settling time of process cooling rates was longer, requiring from 60 to 120 seconds.
Figure 68: Adaptive Observer-Based System Response

Figure 69: Adaptive Observer-Based System Response
Figure 70: Adaptive Observer-Based System Response

Figure 71: Adaptive Observer-Based System Response
Table 26: Static Modified Adaptive Observer-Based Control Test Operating Points

<table>
<thead>
<tr>
<th>Figure number</th>
<th>Arc Current</th>
<th>Set Point Cooling Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>90</td>
<td>4.5</td>
</tr>
<tr>
<td>73</td>
<td>90</td>
<td>7.5</td>
</tr>
<tr>
<td>74</td>
<td>110</td>
<td>5.5</td>
</tr>
<tr>
<td>75</td>
<td>110</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Due to the fact that the adaptive static observer relationship did not match the simulation response exactly, small steady state errors were evident in some of the control runs.

The modified adaptive observer-based control responses were simulated next. The simulation operating points and the corresponding Figure in which the response appears are summarized in Table 26.

The response show that estimated cooling rate convergence to within 5 percent of the desired value occurred in approximately 20 seconds or less in all cases. The 5 percent settling time of process cooling rates ranged from 60 to 120 seconds. As in previous tests, small steady state errors due to mismatch between the process simulation and observer relationship were evident in some of the control runs.

The model-error robustness of the adaptive observer- and modified adaptive observer-based controls were tested in a manner similar to that employed in the previous section for dynamic observer-based controls. The values of selected physical parameters of the process simulation were varied from their nominal values by 10 percent and the cooling rate control was then run until steady state conditions were reached. The value of the process cooling rate was then compared to the
Figure 72: Modified Adaptive Observer-Based System Response

Figure 73: Modified Adaptive Observer-Based System Response
Figure 74: Modified Adaptive Observer-Based System Response

Figure 75: Modified Adaptive Observer-Based System Response
Figure 76: Diffusivity Robustness

set point value to evaluate the robustness of the control to the selected parameter variation.

The model-error robustness results for the adaptive and modified-adaptive observer-based control are shown in Figures 76 through 80. The adaptive control was relatively robust with respect to thickness, diffusivity and surface heat transfer errors: the control error over the range of parameter variation was less than or equal to 0.5 C/sec in all of these cases. The adaptive control was much less robust with respect to preheat temperature errors. The control also exhibited some lack of robustness with respect to efficiency errors, contrary to expectations. Recall that the dynamic adaptive observer was able to correct for efficiency modeling errors exactly. Since the static adaptive observer approximates the steady state response of the dynamic adaptive observer, we would expect if to compensate perfectly for efficiency modeling errors also. However, the static observer relationships only
Figure 77: Thickness Robustness

Figure 78: Arc Efficiency Robustness
Figure 79: Surface Heat Trans. Robustness

Figure 80: Preheat Temperature Robustness
Table 27: Measurement Error Robustness

<table>
<thead>
<tr>
<th>Observer type</th>
<th>Set Point Cooling Rate</th>
<th>Simulation Cooling Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adaptive</td>
<td>6.0</td>
<td>6.1</td>
</tr>
<tr>
<td>Modified Adaptive</td>
<td>6.0</td>
<td>6.5</td>
</tr>
</tbody>
</table>

approximate the simulation behavior. Particularly at higher heat input conditions, there is some mismatch between the observer and simulation cooling rates. The less-than-perfect robustness with respect to efficiency errors is attributable to this error.

The results show that the modified-adaptive observer-based control was slightly less robust with respect to thickness and diffusivity errors than the adaptive observer-based control, but was more robust with respect to efficiency errors. The modified adaptive observer-based control was about equivalent to the adaptive observer-based control with respect to preheat errors and surface heat transfer errors.

The robustness of the adaptive and modified adaptive observer-based controls with respect to weld pool width measurement errors was also tested using the simulation. The pool width measurement produced by the heat flow simulation was perturbed by addition of a constant value of 0.05 cm. before being input to the static observer relationships. The cooling rate control was then operated until steady state conditions were obtained, and the robustness was evaluated by comparison of the resulting simulation cooling rate with the set point cooling rate. The results are shown in Table 27. The modified adaptive observer-based control was not as robust as the adaptive observer-based control with respect to weld pool width measurement errors.
7.2.4 Summary — Static Observer Control Design and Testing

In the foregoing section, design and testing of static observer-based cooling rate controls were discussed. Both the adaptive and modified adaptive observers of Chapter VI were considered. A study of the dynamic characteristics of the travel-speed/cooling-rate-estimate step responses revealed that systems implemented with either observer exhibited approximately exponential but nonlinear step responses. Based upon this finding a control structure similar to that employed in the case of dynamic observer-based controls was proposed. The main feature of the control is a compensator whose parameters are adjusted depending upon the operating point of the system. The dynamic responses of the adaptive observer and modified adaptive observer-based systems were linearized about selected input operating points and appropriate compensators designed for each of these responses. The dynamic responses of cooling rate controls were tested via simulation. The modeling-error robustness of the two types of controls was also tested using the simulation. Both the adaptive and modified adaptive observer-based controls were found to be relatively robust with respect to efficiency, diffusivity, surface heat transfer and base metal thickness parameter variations, having maximum controls error of less than or equal to 0.6 C/sec. for these conditions. The controls were less robust with respect to preheat errors, having maximum control errors of more than 2.6 C/sec. The adaptive observer-based control was more robust with respect to weld pool width measurement errors. For a measurement error of 0.05 cm., it yielded a control error of 0.1 C/sec. On the other hand, the modified adaptive observer-based control exhibited a control error of 0.5 C/sec for the same measurement error.
7.3 Multivariable Control of Cooling Rate and Pool Width

In this section, some simulation results from multivariable controls of weld centerline cooling rate and weld pool width are presented. The objectives are to illustrate the desirability of such controls, to demonstrate that they are possible and to point out some aspects of this control problem which warrant further investigation.

The motivation for considering multivariable control of the welding process comes from the desire to maintain weld pool size constant while performing cooling rate control. Recall that in the control schemes which have been studied in the previous two sections, the travel speed is varied to regulate the value of estimated cooling rate without regard for the resulting weld pool size variations. The only mechanism built into the control algorithms to safeguard against excessively large or small weld pool size is the travel speed saturation element, depicted in the block diagram of Figure 41.

In this section, an alternate control approach wherein both cooling rate and weld pool size are regulated is proposed. The basic premise underlying the proposed control method is that by varying both arc current and travel speed, cooling rate and weld pool width may be independently controlled to desired values. A block diagram of the proposed control structure is shown in Figure 81. Both weld width and estimated cooling rate are compared to set points and the errors are input to a compensator. The compensator generates command values of arc current and travel speed. These are sent to the welding robot which adjusts its outputs accordingly. With this compensator structure, the cooling rate and weld width feedback loops basically operate independently of one another. Weld pool width errors are corrected for by adjustments in arc current and cooling rate errors are
corrected by adjustments in travel speed.

To demonstrate the feasibility of a multivariable control, the control system shown in Figure 81 was simulated. The results demonstrate simultaneous regulation of weld pool width and estimated cooling rate for dynamic adaptive observer-based, static adaptive observer-based and static modified adaptive observer-based controls. For each the three types of observers, the compensator used to implement the cooling rate control loop of the algorithms was the same as that discussed in the previous sections of this chapter. The implementation of the weld pool width control loop was identical in all of the simulations. A compensator having the following transfer function was found to yield satisfactory weld pool width response:

$$\frac{I_c(z)}{E_w(z)} = \frac{350(z - 0.43)}{z - 1.0}$$  \hspace{1cm} (7.20)

where $E_w$ = pool width control error. Preliminary simulations in which only weld pool width was regulated demonstrated that this compensator provided adequate
response. The compensator pole at $z = 1.0$ was chosen to add integral action to the compensator. As will be demonstrated in the subsequent simulations, the fact that both the weld pool width and cooling rate control loop compensators had integral terms allowed regulation of both of these outputs to desired set point values with essentially no steady state error.

Simultaneous regulation of cooling rate and pool width was demonstrated using simulation results presented in previous sections as a starting point. For each observer type, one set of cooling rate control simulation conditions was chosen from those used in the previous sections. The steady state weld pool width corresponding to the selected simulation conditions (referred to hereafter as the "nominal pool width") was then determined by re-running the simulation (with cooling rate control only). The multivariable control simulation was then performed with the same value of cooling rate setpoint and two pool width set points: one greater than the nominal pool width and one less than the nominal width.

We begin with the dynamic adaptive observer. Operating point 2 in Table 19 was selected. This corresponds to conditions of $I = 90$ amperes, $T_0 = 25$ C and $CR_{sp} = 7.5$. Re-running the simulation revealed that a steady state weld pool width of 0.57 cm. was attained for the given conditions. Running the multivariable simulation with a cooling rate set point of $CR_{sp} = 7.5 \text{C/sec.}$ and pool width set points of $W_{sp} = 0.52 \text{ cm.}$ and $W_{sp} = 0.65 \text{cm.}$ yielded the pool width and cooling rate responses shown in Figures 82 and 83. The horizontal dashed lines in these figures show the magnitude of the set point for the various runs.

Note that steady state values of pool width and estimated cooling rate were regulated to their respective set points with no steady state error. However, comparison with Figure 50, for the case where only cooling rate control was used, reveals that the 5 percent settling time of the multivariable cooling rate control
Figure 82: Multivariable Dynamic Adaptive Observer-Based Control Cooling Rate Responses

Figure 83: Multivariable Dynamic Adaptive Observer-Based Control Pool Width Responses
Figure 84: Multivariable Static Adaptive Observer–Based Control Cooling Rate Response

loop was about 50 seconds slower in the case where $W_{sp} = 0.65$ cm.

Multivariable control using the static adaptive observer was investigated next. Operating point 2 from Table 25 was selected for the test. This operating point corresponds to input values of $I = 90$ amperes, $T_0 = 25$C and $CR_{sp} = 7.5$C/sec. Rerunning the simulation resulted in a steady state weld pool width of 0.57 cm. The multivariable control was run with a cooling rate set point of 7.5 C/sec., pool width set points of 0.52 and 0.65 cm. The resulting cooling rate and pool width responses are shown in Figures 84 and 85. The simulation run with $W_{sp} = 0.65$cm. had cooling rate settling times comparable to the cooling-rate-control-only result (shown in Figure 69). The run having a pool width set point of 0.52 cm. showed that the system behaved in an unstable manner. Both the pool width and cooling rate values oscillated with increasing amplitude until the growth was checked by the saturation of several simulation parameters.
Figure 85: Multivariable Static Adaptive Observer-Based Control Pool Width Response

Finally, multivariable control using the static modified adaptive observer was investigated. Operating point 2 from Table 26 was selected for the test. This operating point corresponds to input values of $I = 90$ amperes, $T_0 = 25C$ and $CR_{sp} = 7.5C/sec$. Rerunning the simulation revealed that a steady state weld pool width of 0.57 cm. was obtained with these inputs. The multivariable control was then run with a cooling rate set point of 7.5 C/sec. and pool width set points of 0.52 and 0.65 cm. The resulting responses are shown in Figures 86 and 87.

The responses are similar to those obtained for the static adaptive observer-based control. The cooling rate settling times are comparable to the previous cooling-rate-control-only simulations. However, the responses for the run having the lower pool width set point exhibited some oscillation which were not present in the previous runs.

The above results demonstrate that multivariable control of weld centerline
Figure 86: Multivariable Static Modified Adaptive Observer–Based Control Cooling Rate Response

Figure 87: Multivariable Static Modified Adaptive Observer–Based Control Pool Width Response
cooling rate and weld pool width is possible. They also illustrate some of the
problems associated with this type of control. Most prominent is the unstable
behavior of the static adaptive observer-based control. More work is needed to
determine the cause of this instability and what steps are needed to prevent it.
CHAPTER VIII

Summary, Conclusions and Topics For Further Investigation

8.1 Summary

In this dissertation, control of weld cooling rates using measurements of weld pool width has been studied. An observer-based approach was taken to the implementation of such controls. In this approach, weld pool width measurements from a welding process are compared to weld width predictions from a welding heat flow model and any error between the two is used to "adjust" the heat flow model. The heat flow model also produces estimates of weld centerline cooling rate which is then used in a feedback control implementation. The underlying assumption upon which this approach is based is that, by adjusting the heat flow model so as to minimize error between predicted and measured weld pool width, the model will produce centerline cooling rates which correspond more closely with those which are actually experienced by the welding process.

Several types of observer-based cooling rate controls were formulated and simulation-tested in this dissertation. Two classes of observers were studied: dynamic and static. The dynamic observers were based upon a dynamic model of the welding heat flow process while the static observers used a steady state welding heat flow model. The former are capable of producing cooling rate estimates while the welding process is undergoing transients; the latter provide accurate estimates only in the steady state. Two types of dynamic observers were formulated, one
termed a conventional observer and the other an adaptive observer. Three static observers were formulated. Two, termed conventional and adaptive observers, are steady state versions of the dynamic observers bearing the same name. The third one was referred to as a modified adaptive observer.

The dynamic observers were formulated using a mathematical model of the welding flow process. The static observers were empirically derived from experimental and simulation data using power-law relationships. The dynamic welding heat flow model upon which the dynamic observers are based was tested by comparing various model outputs (i.e. centerline cooling rate, weld pool width and length) with measured steady-state values of these outputs from welding tests. These tests were performed using as wide a range of variation in arc current, travel speed and preheat temperature as was possible while maintaining two-dimensional heat flow conditions. The experimental values of centerline cooling rate were found to be subject to a large amount of uncertainty relative to the total range of cooling rates which could be experimentally produced. This measurement uncertainty was to some extent compensated for by making multiple measurements at many of the input conditions used in the tests. By modifying the welding heat flow model so as to make surface heat transfer from the weldment to the atmosphere dependent upon travel speed and arc current, predicted values of steady-state weld width and cooling rate which closely matched the experimental values were produced. The tests also determined that the predicted response of the weld pool width to step perturbations in arc current agreed well with those measured experimentally.

Empirical derivation of static observer relationships from the experimental data revealed that reasonably accurate adaptive and modified adaptive observers could be formulated. However, no acceptably accurate conventional static observer could be derived from the data. This finding is thought to be due to the fact
(noted below) that the dynamic conventional observer displayed poor robustness properties.

Observer-based feedback cooling rate controls were designed with the aid of a welding heat flow simulation. In the feedback control approach which was used, cooling rate estimates from the observer are compared to a cooling rate setpoint (assumed to be constant) and the error between the two is input to a compensator which produces a command value of travel speed for the welding process. The compensator design approaches for both dynamic observer-based controls were identical. The dynamic and steady-state input/output characteristics of the travel-speed/cooling-rate system was studied with the aid of the welding heat flow simulation. The system was found to have a significant amount of pure input delay and had nonlinear gain and dynamic response characteristics. Because of these properties, the system dynamic response was modeled in piece-wise linear fashion with linear ARMA models, and compensators providing desireable dynamic responses at selected operating points were designed using root locus techniques. A method of selecting appropriate compensator parameters based upon cooling rate set point, arc current and preheat temperature was developed.

The response and robustness of dynamic observer-based controls was tested at a variety of operating points. Of particular interest were the robustness tests: simulation experiments designed to test the accuracy of the feedback control when various physical parameters of the observer welding process model did not match those of the welding process being controlled and when weld pool width measurement errors existed. These tests showed that the dynamic adaptive observer-based control produced relatively robust control under most conditions whereas the dynamic conventional observer-based control failed under most conditions.

The approach taken to the design of static observer-based controls was similar
to that described above. However, one difference in philosophy was that the control dynamic responses were designed using models of the welding heat flow system from travel speed input to cooling rate estimate output. Again, the responses were found to be nonlinear. As a consequence, a piecewise linear control approach similar that described above was employed. The system response was modeled in the vicinity of a number of operating points, and compensators appropriate for each operating point were designed. A method of choosing compensator parameters based upon cooling rate setpoint, arc current and preheat temperature was then determined.

The responses of the static observer-based cooling rate controls were simulated at a variety of operating points, and the robustness properties of these controls was tested. The robustness tests showed that both the adaptive and modified adaptive observer-based controls produced acceptably accurate cooling rate regulation in the presence of process physical parameter variation and weld width measurement error.

Finally, some results which demonstrated multivariable control of the welding process were presented. In these tests, weld pool width and centerline cooling rate were feedback-controlled simultaneously by varying arc current and travel speed. The tests showed such control to be feasible using both static and dynamic heat flow observers in the cooling rate control loop, but also revealed absolutely unstable system behavior under one set of operating conditions. The settling time of the cooling rate output was also increased over that of the cooling-rate-only control in some cases.

8.2 Conclusions

In this section, some conclusions are drawn regarding the significance of the previously-summarized results. In particular, the bearing of the results upon the
question “Will observer-based cooling rate control work?” is discussed.

The answer to the question “Will observer-based cooling rate control work?” is a qualified “Yes”. A better understanding of this answer requires a review of the basic goals and objectives of this research. The impetus for investigating cooling rate controls was the desire to provide real-time control over the mechanical properties of arc welds. In this dissertation, HAZ hardness was selected as the mechanical property of interest. Unfortunately, the results which were obtained for the particular welding conditions used in this dissertation do not allow one to conclude that such control would be feasible. The principle reason for this is that the magnitude of variability inherent in the HAZ hardness measurements for one fixed set of welding process inputs was as large as the total variation in hardness which could be attained with all feasible inputs. The range of feasible inputs (i.e. current, travel speed and preheat temperature) is restricted by the need to maintain full weld pool penetration of the base metal — two-dimensional heat flow conditions — without producing such a large pool that gravity causes the molten metal to drop through the plate. These results suggest that either a base material which provides more consistent hardenability or a different welding conditions (e.g. base metal thickness) allowing a wider range of feasible inputs would be needed in order to experimentally demonstrate control of weld mechanical properties. The latter topic is discussed further in the following section.

In spite of difficulties encountered in demonstrating control of base metal thermal properties, it appears that experimental demonstration of cooling rate control is possible. One may estimate the accuracy of controls based upon the adaptive and modified-adaptive observer algorithms from the experimental and simulation results presented earlier. The simulation results indicate that, given reasonable values of pool width measurement resolution, cooling rate control errors on the
order of 0.5 C/sec. could be expected. If one assumes that base metal preheat temperature is reasonably well known, the robustness results indicate that observer modeling errors, taken singly, could cause additional control inaccuracy of the order of 0.5 C/sec. This yields a total possible control error of approximately 1 C/sec. However, it is possible that if several modeling errors were present simultaneously, the control error might be somewhat larger.

Control errors due to modeling and measurement errors would appear to be slightly less significant than the experimental errors encountered in measuring cooling rates. Recall from Section 5.2 that experimental cooling rate measurements varied by 1.7 C/sec with no change in welding inputs. This effect, whether due to measurement techniques or to variations in uncontrolled process inputs, would probably be the largest source of error encountered in attempting to experimentally demonstrate cooling rate control. Some further study as to the sources of this variability might be warranted.

Finally, it should be pointed out that the dynamic observation algorithms which were developed as part of this work can be readily adapted for use in other welding thermal cycle control approaches. For example, consider a system wherein sensors measure the temperature at one or more points which are fixed with respect to the welding torch. Observers which would allow estimation of the temperature at other points (and hence, the approximation of thermal outputs such as cooling rate) could be formulated using methods similar to those employed in this dissertation.

8.3 Topics for Further Investigation

The most logical extension of the work which has been discussed in this dissertation is the implementation and testing of cooling rate controls on actual welding processes. As was emphasized throughout Chapter VII, all control design and
testing in this dissertation was performed using simulation (although the heat flow model upon which the simulation and dynamic observers are based was tested experimentally). The static observer-based control algorithms are less demanding in terms of computing power requirements than the dynamic observer-based controls, and so should probably be implemented first. Furthermore, to minimize additional control design, the controls should be implemented using the same experimental conditions as were used to collect the data presented in this dissertation.

The nature of the controls tests which should be performed is a somewhat open-ended question. Logically, it seems that the ability of the controls to regulate cooling rate to desired values in the absence of disturbing inputs should be verified first. Either prior to or in conjunction with these tests, investigations into the sources of variability in experimental cooling rate measurements might also be carried out.

The cooling rate regulation tests might be followed by experiments designed to determine the accuracy of the control in the presence of disturbances such as variations in thickness or clamping heat-sinks.

Experimental implementation and testing of the dynamic observer-based controls would likely depend upon the availability of the sufficiently powerful computer hardware, but would otherwise probably proceed along lines similar to those described above.

The multivariable control approach which was proposed would greatly enhance the utility of the cooling rate control, and hence represents an attractive topic for further investigation. More simulation testing of this control technique is needed to determine the cause of the instability which was present in some of the preliminary simulation tests and to determine solutions to the problem. It seems likely that a scheme of choosing compensator parameters based upon both
set point cooling rate and pool width might produce a stable control. Following this work, the multivariable controls could then be tested experimentally. Again, the static observer-based controls would be simplest to implement experimentally.

Another attractive extension of this dissertation work would the extension of the observer formulations to three-dimensional and finite-thickness heat flow conditions. The latter term is meant to refer to conditions where the base metal is too thick to produce two-dimensional heat flow, but too thin for three-dimensional relationships to apply. Maintenance of two-dimensional heat flow conditions requires that arc current, travel speed and other parameters be limited to relatively narrow ranges. This in turn limits total cooling rate variation to a small range. In fact, in this work, the allowable range of inputs was not sufficient to allow cooling rate to be varied enough to cause significant material properties (i.e. hardness) variations. Experiments involving larger base metal thicknesses would allow investigation of the effect of cooling rate control upon weld mechanical properties.

Extension of the static observer relationships to either condition presents no fundamental problem: more extensive experimentation is all that is needed. The simplest approach would likely be derivation of a relationship of the form:

\[ CR = f(I, V, T_0, t) \]  \hspace{1cm} (8.1)

where \( t \) = base metal thickness.

This relationship could then be employed for a variety of finite thicknesses.

Extension of the dynamic observer relationships to the three dimensional case would also appear to be relatively simple. A three-dimensional counterpart of the two-dimensional Green's function used in this dissertation is available. Observer formulation with this relationship would proceed along the same lines as in Chapter IV and Section V. A finite-dimensional Green's function is also available, but it
is not as computationally attractive as the two- and three- dimensional formulas; it is basically an infinite sum of modified three dimensional Green's functions. However, it may be that a sufficiently accurate solution could be obtained using only a few terms of the summation, resulting in an observer implementation which could be practically implemented.

Finally, we note that, with some modification, the welding heat flow observers which have been formulated in this dissertation are generically applicable to other welding temperature measurement and control problems. One example might be an application in which it is desirable to estimate temperature at some point in the base metal given a temperature measurement at another point. In such a setting, the conversion of the observer algorithms to filtering or estimation algorithms (where temperature measurement "noise" is included in the formulation) is possible.
APPENDIX A

Derivation of Green's Function with Gaussian Heat Input Distribution

In this appendix, the derivation of Equation (4.31) from Equation (4.30) is given. Repeated here for convenience, Equation (4.30) is:

\[
T(x, y, t) - T_0 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{q} \exp \left( -\frac{(x' - x_0)^2 + (y' - y_0)^2}{2\sigma^2} \right) \frac{1}{8\pi^2 h K(t - t') \alpha^2} \exp \left( -\frac{(x - x')^2 + (y - y')^2}{4\alpha(t - t')} \right) dx' \, dy' \tag{A.1}
\]

The following changes of variables are made to simplify the expression:

\[
\hat{x} = x' - x_0 \\
\hat{y} = y' - y_0 \\
\hat{x} = x_0 - x \\
\hat{y} = y_0 - y
\]

Equation (A.1) then becomes:

\[
T(x, y, t) - T_0 = \left[ \frac{\hat{q} \, dt'}{8\pi^2 h K(t - t') \alpha^2} \right] \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left( -\frac{(\hat{x} - \hat{x})^2 + (\hat{y} - \hat{y})^2}{4\alpha(t - t')} \right) \exp \left( -\frac{\hat{x}^2 + \hat{y}^2}{2\sigma^2} \right) d\hat{x} \, d\hat{y} \tag{A.2}
\]
To begin simplifying this equation, the quadratic exponential terms are expanded, yielding the following:

$$T(x, y, t) - T_0 = \left[\frac{\dot{q}dt'}{8\pi^2 hK(t - t')\alpha^2}\right] \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(-\frac{\hat{x}^2 - 2\hat{x}\hat{x} + \hat{y}^2 - 2\hat{y}\hat{y} + \hat{y}^2}{4\alpha(t - t')}\right) \exp\left(-\frac{\hat{x}^2 + \hat{y}^2}{2\sigma^2}\right) dx'dy'$$

Using the change of variables

$$a = 4\alpha(t - t')$$

and

$$b = 2\sigma^2$$

and moving the terms not involving the variable of integration outside of the integral yields:

$$T(x, y, t) - T_0 = \left[\frac{\dot{q}dt'}{8\pi^2 hK(t - t')\alpha^2}\right] \exp\left(-\frac{\hat{x}^2 + \hat{y}^2}{a}\right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(-\left[\frac{a + b}{ab}\right]\left[\hat{x}^2 - \hat{x}\hat{x}\frac{ab}{a(a + b)} + \hat{y}^2 - \hat{y}\hat{y}\frac{ab}{a(a + b)}\right]\right) d\hat{x}d\hat{y}$$

The square is then completed on the two terms inside of the integral, which results in the expression:

$$T(x, y, t) - T_0 = \left[\frac{\dot{q}dt'}{8\pi^2 hK(t - t')\alpha^2}\right] \exp\left(-\frac{\hat{x}^2 + \hat{y}^2}{a}\right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(-\left[\frac{a + b}{ab}\right]\left[\left(\hat{x} - \frac{b}{a + b}\right)^2 - \left(\frac{b\hat{x}}{a + b}\right)^2 + \left(\hat{y} - \frac{b}{a + b}\hat{y}\right)^2 - \left(\frac{b\hat{y}}{a + b}\right)^2\right]\right) d\hat{x}d\hat{y}$$

Again removing terms not involving the variable of integration to the outside of the integral and simplifying.
\[ T(x, y, t) - T_0 = \left[ \frac{\dot{q} dt'}{8 \pi^2 hK(t - t') \alpha^2} \right] \exp \left( -\frac{\hat{x}^2 + \hat{y}^2}{a} \right) \]
\[
\cdot \exp \left( \left[ \frac{a + b}{ab} \right] \left[ \left( \frac{b}{a + b} \right)^2 \left( \frac{b}{a + b} \hat{x} \right)^2 + \left( \frac{b}{a + b} \hat{y} \right)^2 \right] \right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left( -\left[ \frac{a + b}{ab} \right] \left[ \left( \frac{b}{a + b} \hat{x} \right)^2 + \left( \frac{b}{a + b} \hat{y} \right)^2 \right] \right) d\hat{x} d\hat{y}
\]

The identity
\[
\int_{-\infty}^{\infty} \exp \left( a(x - b)^2 \right) = \sqrt{\frac{\pi}{a}}
\]

may now be used to reduce the integral, the result being:

\[ T(x, y, t) - T_0 = \left[ \frac{\dot{q} dt'}{8 \pi^2 hK(t - t') \alpha^2} \right] \exp \left( -\frac{\hat{x}^2 + \hat{y}^2}{a} \right) \]
\[
\cdot \exp \left( \left[ \frac{a + b}{ab} \right] \left[ \left( \frac{b}{a + b} \hat{x} \right)^2 + \left( \frac{b}{a + b} \hat{y} \right)^2 \right] \right) \left( \frac{\pi ab}{a + b} \right)
\]

After some algebraic manipulation, and substituting back for \( a, b \) and \( \hat{x} \), the final expression results:

\[ T(x, y, t) - T_0 = \frac{\dot{q} dt'}{\pi h \rho C(2\sigma^2 + 4\alpha(t - t'))} \exp \left( -\frac{(x - x_0)^2 + (y - y_0)^2}{2\sigma^2 + 4\alpha(t - t')} \right) \]
APPENDIX B

Heat Simulation and Observer Programs

This appendix contains the listings of the Fortran programs which implement the dynamic heat flow simulation and observers. The simulation program, called GAUSHTSM is shown first. The next program, called SMHTSM, implements the dynamic conventional and adaptive observers.
2-D HEAT SIMULATION PROGRAM
INTEGRATES THE ANALYTICAL SOLUTION USING 3 PT GAUSSIAN QUADRATURE

MAIN ROUTINE

THIS SECTION OF CODE INITIALIZES CODE AND WILL EVENTUALLY BE REPLACED BY A ROUTINE WHICH READS STUFF OUT OF A DISK FILE OR FROM THE USER INPUT

DIMENSION V(251), AMPS(251), CAPPA(50), COND(50), THET(50)
DIMENSION X(50), Y(50), TEM(50), CA(50), CO(50), PWDEL(15)
DIMENSION CRHIS(10), ARM(10), TAM(10)
INTEGER*2 PWI, MOVFL
INTEGER*2 GQI, GQP, GQD, PWREL, PWSRL, PWI, PWPBRT
INTEGER*4 ISEED

SET UP TIME COUNTERS

ISTTM = 0
IFINTM = 1

GET THE DELTIME

WRITE(6,*) 'INPUT THE DELTIME'
READ(5,*) DELTM

IENDTM = 250

STUFF FOR RANDOM INPUT

ISEED = 62233+3
IRANIN = 0

ASSIGN STARTING LOCATIONS TO POINTS
ALL POINTS EXCEPT THE LAST MOVE WITH THE SOURCE CENTER

NUMPTS = 50

POOL WIDTH LOCATOR POINTS

DO 464 I=1,4
DO 464 J=1,4
X(I*(I-1)+J) = .1*(1-I)
Y(I*(I-1)+J) = .1*J+.1

464

POOL BACK EDGE LOCATOR POINTS

AI = -.4
DO 470 I=17,26
X(I) = AI
Y(I) = 0.
AI = AI - .1

POOL FRONT EDGE LOCATOR POINTS

AI = 0.
DO 474 I = 27, 32
X(I) = AI
Y(I) = 0.
AI = AI + .1

COOLING RATE CALCULATOR POINTS

XIT = .25
AI = 8.
DO 465 I = 33, 46
X(I) = AI
Y(I) = 0.0
AI = AI - XIT

THREE POINTS CLOSER TO POOL FOR COOLING RATE APPROXIMATION

X(47) = -2.0
Y(47) = 0.
X(48) = -2.25
Y(48) = 0.
X(49) = -2.5
Y(49) = 0.

FIXED POINT. THIS POINT DOES NOT MOVE WITH SOURCE

X(50) = 15.
Y(50) = 0.0

SEE IF RANDOM INPUT IS TO BE USED

WRITE(6, *) 'USE RANDOM INPUT? , 0=NO, 1= YES'
READ(5, *) IRANIN

SEE IF OUTPUT TO DISK (AUTOMATIC IF IRANIN = 1)

WRITE(6, *) 'OUTPUT TO DISK? , 0=NO, 1= YES'
READ(5, *) IDISKO

SEE IF PID OR ONE-STEP CONTROL IS TO BE USED

WRITE(6, *) 'PID(0), ONE-STEP CONTROL(1), STEP INPUT(2)'
WRITE(6, *) 'COMPENSATOR(3), gain switch(4), none(5)'
READ(5, *) ICON

GET THE INIT TEMP

WRITE(6, *) 'INPUT THE PREHEAT TEMP'
READ(5, *) PHTEM

GET THE MATERIAL TYPE

WRITE(6, *) 'STEEL (1) OR ALUMINUM (2)?'
READ(5, *) IMAT

GET THE CONTROL GAINS, FIRST CURRENT, THEN TRAV. SPEED

WRITE(6, *) 'INPUT GQP, GQI, GQD, PWD (6 1 5 .6)'
READ(5, *) GQP, GQI, GQD, PWD
WRITE(6, *) 'INPUT GVP, GVI, GVD, CRD (.004 .001 .001 32)'

214
READ(5,*) GVP,GVI,GVD,CRD

INIT A BUNCH OF CONTROL PARAMETERS

PWERSM = 0
CRERSM = 0.
OLTHFT = 0.
OLTHSC = 0.
OLTHMC = 0.
CREROL = 0.
PWEROL = 0
ICOCO = 0

INIT ONE-STEP CONTROL PARAMETERS

ARM(1) = -.35094
ARM(2) = -.38740
MAM(1) = -40.4940
MAM(2) = 27.65411
MAM(3) = 21.41389
ALAM = 80

PI = 4. * ATAN(1.)

GET THE WELDING PARAMETERS

WRITE(6,*) 'AMP,VOLT,TRAV,EFF,HCFS,SHTC(100,11,4,.56,.02,.001)'
READ(5,*) CUR, VOLTS, TRAV, EFF, HCFS, SHTC

GET SURFACE HEAT TRANS COEFF. FOR PRIHEAT DECAY RATE

WRITE(6,*) 'INPUT PLATE SHT (.006)'
READ(5,*) SHTP

write(6,*) 'input thickness,diffusivity (.2286 .063)'
read(5,*) hu,difffu

write(6,*) 'input coolrate crfta5(5),crfta6(6),crfta7(7)'
read(5,*) icr

SET UP THE THERMAL PROPS FOR EACH POINT

DO 10 I=1,NUMPTS
IF (IMAT .EQ. 1) THEN
   CAPPA(I) = difffu
   COND(I) = .312
   THET(I) = PTHET
ENDIF
IF (IMAT .EQ. 2) THEN
   CAPPA(I) = 0.5
   COND(I) = 1.3
ENDIF
10 CONTINUE

THET(I) = PTHET

MAKE THINGS METRIC,MISC CALCS.

TRAV = TRAV*2.54/60.
V(1) = TRAV
AMPS(1) = CUR

ND CAN BE USED TO DELAY POOL WIDTH MEASUREMENT IF DESIRED
ND = 0
\[ ND = (0.5 \times 2.54) / \text{TRAV} \]

DEFINE THERMAL AND OTHER PROPS FOR STEEL

\[ \text{IF (IMAT .EQ. 1) THEN} \]

\[ \text{PLATE THICKNESS IN CM.} \]
\[ H = \text{hu} \]

\[ \text{ITM} = 6 \]

\[ \text{MELTING TEMP} \]
\[ \text{TMEL} = 1530. \]

\[ \text{TEMP TO CALC COOLING RATE AT} \]
\[ \text{TCRIT} = 500. \]

\[ \text{TEMP VARYING THERMAL PROPS} \]

\[ \text{TEM(1)} = 200. \]
\[ \text{CO(1)} = 0.42 \]
\[ \text{CA(1)} = 1 \]
\[ \text{TEM(2)} = 704 \]
\[ \text{CO(2)} = 0.29 \]
\[ \text{CA(2)} = 0.046 \]
\[ \text{TEM(3)} = 1429. \]
\[ \text{CO(3)} = 0.312 \]
\[ \text{CA(3)} = 0.063 \]
\[ \text{TEM(4)} = 1431. \]
\[ \text{CO(4)} = 0.312 \]
\[ \text{CA(4)} = 0.044 \]
\[ \text{TEM(5)} = 1529. \]
\[ \text{CO(5)} = 0.312 \]
\[ \text{CA(5)} = 0.044 \]
\[ \text{TEM(6)} = 1531. \]
\[ \text{CO(6)} = 0.9312 \]
\[ \text{CA(6)} = 0.0463 \]

\[ \text{ENDIF} \]

\[ \text{IF (IMAT .EQ. 2) THEN} \]

\[ \text{CONSTANT THERMAL PROPS FOR AL.} \]

\[ \text{CO(1)} = 1.3 \]

\[ \text{CA(1)} = 0.5 \]

\[ \text{MELTING TEMP FOR AL.} \]
\[ \text{TMEL} = 660. \]

\[ \text{PLATE THICKNESS IN CM. FOR AL.} \]
\[ H = 0.3175 \]

\[ \text{TEMP TO CALC COOLING RATE AT} \]
\[ \text{TCRIT} = 300. \]

\[ \text{ENDIF} \]

CALC. SURFACE HEAT TRANS. FROM COEFF. WHEN USED. CURRENTLY
SURFACE HEAT TRANS. IS INPUT DIRECTLY

\[ \text{SHT} = 2 \times \text{SHTC/COND(1)/H} \]
\[ \text{SHT} = \text{SHTC} \]

CALCULATE THE HEAT INPUT USING VARIABLE EFFICIENCY FACTOR
QDOT = ETA(AMPS(1),V(1),VOLTS,EFF,PHTEM,THEL)*
6 AMPS(1)*VOLTS*EFF

DISK OUTPUT IS DEVICE #1. FILENAME DEFAULTS TO FOR001.DAT
OPEN(1)

END OF INITIALIZATION SECTION

******************************************************************************
*                                                                      *
*                   START OF TEMP CALC SECTION                           *
*                                                                      *
******************************************************************************

DO 5 WHILE (IFINTM .LE. IENDTM)

LOOP 5 IS REPEATED AT EACH TIME STEP FOR IENDTM TIMES
THE HEAT EQUATION IS INTEGRATED 'STARTING AT ZERO' AND
GOING UP TO IFINTM EACH TIME STEP. THIS PROCESS TAKES
LONGER TO ACCOMPLISH EACH TIME AS THE NUMBER OF TIME STEPS
INCREASES. THE SOURCE CENTER LOCATION IS SET BACK TO ZERO
EACH TIME AND THE POINTS WHICH TRAVEL ALONG WITH IT ARE
MOVED AT EACH TIME STEP BY A C DISTANCE DETERMINED BY
THE TRAVEL SPEED.

THIS LOOP UPDATES THE LOCATION OF EACH POINT FOR
THE NEXT TIME STEP

DO 20 NPTCNT = 1,NUMPTS-1
   X(NPTCNT) = X(NPTCNT) + V(IFINTM)*DELTIM
   CONTINUE

   THIS CALL RESETS THERMAL PROPERTIES
   CALL RETHPR(NUMPTS,CAPPA,COND,THET,PHTER,IMAT,difu)

DO 30 NPTCNT = 1,NUMPTS

SET THE SOURCE CENTER BACK TO ZERO FOR NEXT PASS THRU

   XPRIM = 0
   YPRIM = 0

   THIS LOOP INTEGRATES HEAT EQU UP TO CURRENT
   TIME

DO 40 ITCNT = 1,IFINTM
   T0 = (ITCNT-1) * DELTIM
   T1 = ITCNT * DELTIM
   T = IFINTM * DELTIM

   THIS SUBROUTINE CALL DOES THE 3-PT GAUSSIAN
   QUADRATURE INTEGRATION
   QDOT = ETA(AMPS(ITCNT),V(ITCNT),VOLTS,EFF,PHTEM,THEL)*
   AMPS(ITCNT)*VOLTS*EFF

   call fnshlt(amps(ITCNT),v(ITCNT),shf,cond(1),h,shtc,
   eff,volts)
   WRITE(6,*),'shtc',shtc
   CALL GAQUIN(PI,T0,T1,T,AMPS(ITCNT),V(ITCNT),

217
CALL CAPP(A(NPTCNT),COND(NPTCNT),H,XPRIM,X(NPTCNT),
Y(NPTCNT),YPRIM,SUM,HCSF,HCSFT,SDOT)

ADD THE NEW PART TO THE SUM
THET(NPTCNT)=THET(NPTCNT)+SUM

MOVE SOURCE CENTER
XPRIM = XPRIM + V(ITCNT) * DELTIM

FIND TEMP VARYING THERMAL PROPS. NOT USED NOW
CALL FNDTHM(THET(NPTCNT),CA,CO,TEM,CAPP(A(NPTCNT),
COND(NPTCNT),ITM,NPTCNT)

CONTINUE

INCLUDE CONVECTION COOLING OF PLATE AS A WHOLE
ASSUME AMBIENT TEMP IS 25 C

WRITE(6,*) (PHTEM-25)*(1-EXP(-SHTP*IFINTM*DELTIM))
THET(NPTCNT)=THET(NPTCNT)-(PHTEM-25)*(1-EXP(-SHTP*
IFINTM*DELTIM))

CONTINUE

THIS SUBROUTINE CALL CALCULATES THE VALUES OF THE OUTPUTS
SUCH AS POOL DIMENSIONS AND COOLING RATES

CALL CAOUPU(PW,IFINTM,THET,X,Y,V,OLTHFT,OLTHSC,OLTHMC,
DELTIM,CRFT,CRSC,CRMC,TCRIT,OLICFT,TIM800,OLWTC,CRFTAP,PL,
CAPP,COND,PI,CRFTA2,AMPS,H,PWDEL,ND,PW1,TMEL,MOVEFL,
PL1,QDOT,CRFTA3,CHRIS,CRFTA4,CRFTA5,crfta6,crfta7)

THIS SUBROUTINE CALL DOES THE CONTROL CALCULATIONS,
ADJUSTING THE VALUES OF AMPS AND V

CALL CONCAL(IFINTM,GQI,GQP,GQD,GVI,GVP,FWDES,CRD,THET,CRFT,
CRSC,CRMC,V,AMPS,FWERSM,CRERSM,PW,CREROL,GVD,DELTIM,
OLWTC,CRFTAP,PL,CRFTA2,VLTS,FFP,PWEROL,PW1,ISEED,
IRANIN,QDOT,PHTEM,TMEL,CRFTA3,CHRIS,ARM,ANM,ALAM,ICON,
CRFTA4,CRFTA5,crfta6,COOLD,CREROL1,CREROL0,ICOCO,CROL,crfta7
,cur,icr)

THIS SUBROUTINE CALL DISPLAYS THE OUTPUT ON THE CRT, OR
TO DISK. IF RANDOM OUTPUT IS SPECIFIED, OUTPUT IS TO DISK,
OTHERWISE TO CRT

CALL OUTDIS(V,AMPS,NMPTS,IFINTM,DELTIM,THET,CRFT,CRSC,
CRMC,PW,CRFTAP,PL,CRFTA2,PW1,IRANIN,X,PL1,QDOT,CRFTA3,
CHRIS,IDISO,CRFTA4,CRFTA5,crfta6,crfta7,icr)

THIS CALL MOVES THE POINTS USED TO CALCULATE THE COOLING
RATE SO THAT TCRT REMAINS WITHIN THEM

CALL MVCCOO(MOVEFL,X,XTI,OLICFT)

UPDATE TIME COUNTERS
ISTTM = ISTTM + 1
IFINTM = IFINTM + 1

218
CONTINUE
CLOSE(1)
END

******************************************************************************
*  SUBROUTINE FNDTHM
*  THIS SUBROUTINE FINDS THE APPROPRIATE THERMAL
*  PROPS FOR 4130 STEEL GIVEN THE TEMP AT SOME POINT
* ******************************************************************************

SUBROUTINE FNDTHM(THET, CA, CO, TEM, CAPPA, COND, ITM, NPTCNT)
DIMENSION CA(50), CO(50), TEM(50)

CAPP = CA(1)
COND = CO(1)
RETURN
IF (THET .LT. TEM(1)) THEN
  CAPP = CA(1)
  COND = CO(1)
ENDIF

IF (THET .GE. TEM(ITM)) THEN
  CAPP = CA(ITM)
  COND = CO(ITM)
ENDIF

DO 50 K = 2, ITM
  L = K - 1
  IF ((THET .GE. TEM(L)) .AND. (THET .LT. TEM(K))) THEN
    CAPP = CA(L) + (CA(K) - CA(L)) / (TEM(K) - TEM(L)) * (THET - TEM(L))
    COND = CO(L) + (CO(K) - CO(L)) / (TEM(K) - TEM(L)) * (THET - TEM(L))
  ENDIF
  CONTINUE
  50
WRITE(6, *) NPTCNT, THET, CAPP, COND
RETURN
END

******************************************************************************
*  SUBROUTINE RETHPR
*  THIS SUBROUTINE RESETS THE THERMAL PROPERTIES AT
*  A POINT TO THE INITIAL ONES
* ******************************************************************************

SUBROUTINE RETHPR(NUMPTS, CAPP, COND, THET, PHTEM, IMAT, diffu)
DIMENSION CAPP(50), COND(50), THET(50)

DO 60 I = 1, NUMPTS
  IF (IMAT .EQ. 1) THEN
    CAPP(I) = diffu
  ENDIF
  60
COND(I) = .312
THET(I) = PHTEM
ENDIF
IF (IMAT .EQ. 2) THEN
CAPPA(I) = 0.5
COND(I) = 1.67
THET(I) = PHTEM
ENDIF

C 60 CONTINUE
C RETURN
END

********************************************************************************
)*
SUBROUTINE GAQUIN
(*
THIS SUBROUTINE PERFORMS 3 PT GAUSSIAN QUAD INTEGRATION OF THE INTEGRAND IN SUB. EVALIN
********************************************************************************
)*
SUBROUTINE GAQUIN(P1,T0,TL,T,AMPS,V,CAPPA,COND,H,XPRIM,
4 X,Y,YPRIM,SUM,HCFS,SHT,QDOT)
C C
CONST = QDOT/( PI * H * cond/cappa)
SUM = 0.
C DO 70 I = 1,3
  IF (I .EQ. 1) THEN
    A = -.77455667
    B = .555555555
  ENDIF
C
  IF (I .EQ. 2) THEN
    A = 0.
    B = .888888888888888
  ENDIF
C
  IF (I .EQ. 3) THEN
    A = .77455667
    B = .555555555
  ENDIF
C
TP = ((T1 - T0) * A + T1 + T0)/2.
C CALL EVALINT TO EVALUATE THE INTEGRAND
C CALL EVALIN(T0,XPRIM,YPRIM,X,Y,CAPPA,T,TP,V,CONST,
6 VALU,HCFS,SHT)
C SUM = SUM + VALU * B * ((T1 - T0)/2.)
C 70 CONTINUE
C RETURN
END
C
C234567890123456789012345678901234567890123456789012345678901234567
C
220
SUBROUTINE EVALINT

THIS SUBROUTINE EVALUATES THE INTEGRAND
AT THE POINT TP PROVIDED BY THE CALLING ROUTINE

SUBROUTINE EVALIN(T0,XPRIM,YPRIM,X,Y,CAPPA,T,TP,V,CONST
1 ,VALU,HCFS,SHT)

AA = (X - XPRIM - (V * (TP-T0))) ** 2.
BB = (Y - YPRIM) ** 2.
DD = CAPPA * SHT * (T-TP)
CC = EXP(-DD +((-AA-BB)/((4.*CAPPA * (T - TP))+ 2.* hcfS))
VALU = CC / (4. * cappa * (T - TP) + 2.* hcfS) * CONST

RETURN
END

SUBROUTINE CAOUPU

THIS SUBROUTINE CALCULATES THE COOLING
RATE OUTPUT AND POOL DIMENSIONS NEEDED TO
do the control

SUBROUTINE CAOUPU(PW,IFINTM,THET,X,Y,V,OLTHFT,OLTHSC,
6 OLTHMC,DELTIM,CRFT,CRSC,CMHC,TCRIT,OLICTF,TIM800,OLWTC,
6 CRFTAP,PL,CAPPA,COND,PI,CRFTA2,AMPS,H,PWDEL,ND,PWI,TWEL
6 ,MOVEF,PL,TDOT,CRFTA3,CHIS,CRFTA4,CRFTA5,CRFTA6,CRFTA7)

DIMENSION THET(50),X(50),Y(50),V(251),PW(4),CAPPA(50)
6 ,COND(50),AMPS(251),PWDEL(15),CHIS(10)
INTEGER*2 PWI

MOVEF = 0.

IF (THET(43) .GT. TCRIT) THEN
ICT = 46
CRFT = 0.
MOVEF = -1
ELSE
IF (TCRIT .GT. THET(34)) THEN
ICT=33
CRFT = 0.
MOVEF = 1
OLICTF = 33
ELSE

THE FOLLOWING Computes COOLING RATE AT A FIXED TEMP
TCRIT, BY SEARCHING FOR TCRIT AND THEN GETTING
SLOPE THERE, ETC Pts. 27-46

DO 777 ICT=33,43
IF ((THET(ICT).GE.TCRIT).AND.(THET(ICT+1).LT.TCRIT)) THEN
ICT=ICT
XCR=X(ICT)-(((THET(ICT)-TCRIT)/(THET(ICT)-
6 THET(ICT+1))))*(X(ICT)-X(ICT+1))
ENDIF

CONTINUE

6

TNS1 = THET(ICT+1)-(THET(ICT+1)-THET(ICT+2))*
       (X(ICT)-XTCR)/(X(ICT)-X(ICT+1))

6

SLOPE = (TCRIT - TNS1)/(X(ICT)-X(ICT+1))

find the temp at the place having temp. equal to tcrit
last time so d(theta)/dt can be found

6

OLTHFT = OLTHFT + (V(IFINTM) * DELTIM)

757

do 757 icnt = 33,43
   if ((x(icnt) .ge. olthft) .and. (x(icnt+1) .lt. olthft)) then
      icnt = icnt_endif
   continue

6

TLTIT = THET(OLICFT) -
       (THET(OLICFT) - THET(OLICFT+1)) * (X(OLICFT) - OLTHFT)

6

/(X(OLICFT) - X(OLICFT+1))

6

TLTIT = THET(ict1) -
       (THET(ict1) - THET(ict1+1)) * (X(ict1) - OLTHFT)

6

/(X(ict1) - X(ict1+1))

CRFT = SLOPE * (V(IFINTM) + (XTCR - OLTHFT))

6

+(TCRIT-TLTIT)/DELTIM

write(6,*) (xcr-olthft)

6

adjust tholft back to the previous iteration value and
use it to find another approximation of cooling rate
which will be delayed by one iteration, but otherwise
an accurate backward difference calculation of the
cooling rate at TCRIT at a fixed point.

6

OLTHFT = OLTHFT - (v(ifintm)*deltim)

758

do 758 icnt = 33,43
   if ((x(icnt) .ge. olthft) .and. (x(icnt+1) .lt. olthft)) then
      icnt = icnt_endif
   continue

6

TLTIT = THET(ict1) -
       (THET(ict1) - THET(ict1+1)) * (X(ict1) - OLTHFT)

6

/(X(ict1) - X(ict1+1))

CRFT6 = (TCRIT - TLTIT)/DELTIM

WRITE(6,*) 'ICT1',ICT1

set up variables for the next time thru, move the array
of points used to calculate cooling rate if necessary

6

OLTHFT = XTCR

OLICFT = ICT

IF (ICT . LT. 36) THEN
**THE FOLLOWING APPROXIMATES THE COOLING RATE
* AT A FIXED TEMPERATURE, TCRIT, USING MEASUREMENTS
* AT FIXED POINTS IN THE MOVING COORD. 47-49

**

\[ SL1 = (\text{THT}(47) - \text{THT}(49))/((X(47) - X(49))) \]
\[ SL2 = (\text{THT}(47) - 2 \times \text{THT}(49)) / ((X(47) - X(48))) \]
\[ TWT = X(48) + (1/(SL1 + SL2)) \times (THT - \text{THT}(48)) \]
\[ DTDWTC = SL1 + SL2 \times (TWT - X(48)) \]

**

\[ \text{THOWTC} = \text{THT}(48) + SL1 \times ((\text{OLWT} + V(\text{FINX}) \times \text{DELTM}) - X(48)) \]
\[ \text{DTDITC} = (\text{THOWTC} - \text{TCRIT}) / \text{DELTM} \]

**

\[ \text{CRIPTA} = \text{DTDITC} \times (V(\text{FINX}) - \text{DTDITC}) \]
\[ \text{OWTC} = \text{WT} \]

**

\[ \text{CALC COOL RATE AT PT. FIXED IN MOVING COORD PT. 47} \]
**

\[ \text{CRTC} = (\text{THT}(48) - \text{THT}(49)) / ((X(48) - X(49))) \times (1 + V(\text{FINX}) \times \text{DELTM}) \]
\[ \text{OLTHMC} = \text{THT}(48) \]

**

* CALCULATE TIME TO COOL 800 C TO 500 C AT PT 50
**

IF (((\text{THT}(50) \leq 800 \text{.}) \text{.AND.} (\text{OLTHSC} \geq 800 \text{.})) \text{.THEN}}
\text{TIM800 = IFINX}
\text{ENDIF}

IF (((\text{THT}(50) \leq 500 \text{.}) \text{.AND.} (\text{OLTHSC} \geq 500 \text{.})) \text{.THEN}}
\text{CRTCSC = IFINX - TIM800}
\text{ENDIF}

\[ \text{OLTHSC} = \text{THT}(50) \]

**

**

\[ \text{CALCULATE THE POOL WIDTH ON EACH OF 4 SCAN LINES} \]
\[ \text{POOL WIDTH IS THE MAX OF THESE FOUR VALUES PTS. 1-16} \]
**

\[ \text{DO 11 J=1,4} \]
\[ \text{DO 11 I=1,4} \]

IF ((\text{THT}(I-1) \times 4 + 1) < \text{TMEL}) \text{.THEN}
\text{PWA(J) = .2}
\text{ENDIF}

IF ((\text{THT}(J) \times 4) \geq \text{TMEL}) \text{.THEN}
\text{PWA(J) = .5}
\text{ENDIF}

\[ \text{K} = (J-1) \times 4 + 1 \]

IF (((\text{THT}(K) \geq \text{TMEL}) \text{.AND.} (\text{THT}(K+1) \leq \text{TMEL}) \text{.THEN}}
\text{PWA(J) = (Y(K+1) - (\text{TMEL} - \text{THT}(K+1)) \div (\text{THT}(K) - \text{THT}(K+1)))}

223
*$(Y(K+1)-Y(K))$

**ENDIF**

**CONTINUE**

**FIND THE MAX OF THE FOUR VALUES BY BUBBLE SORT**

**PW=0.**

**DO 12 I=1,4**

**IF (PWA(I) .GT. PW) THEN**

**PW=PWA(I)**

**ENDIF**

**CONTINUE**

**PW = 2.*PW**

**DELAY THE POOL WIDTH BY # OF DELAY CYCLES (ND)**

**DO 13 I=15,2,1**

**IF (PWA(I) .GT. PW) THEN**

**PW=PWA(I)**

**ENDIF**

**CONTINUE**

**PW = PWDEL(ND+1)**

**PRI = INT(PW*132)**

**DO 13**

**CONTINUE**

**= CALCULATE THE POOL LENGTH IN A MANNER SIMILAR**

**TO POOL WIDTH**

**FIND THE DISTANCE FROM THE SOURCE CENTER TO THE BACK**

**OF THE POOL**

**IF (THET(17) .LT. TMEL) THEN**

**PL = X(17)**

**ELSE**

**IF (THET(26) .GE. TMEL) THEN**

**PL = X(26)**

**ELSE**

**DO 19 J=17,25**

**IF ((THET(J) .GE. TMEL) .AND. (THET(J+1) .LT. TMEL)) THEN**

**PL = X(J) - ((THET(J)-TMEL)/(THET(J+1)-THET(J))**

***(X(J)-X(J+1))**

**ENDIF**

**CONTINUE**

**ENDIF**

**NOW FIND THE DISTANCE FROM THE SOURCE CENTER TO THE FRONT**

**OF THE POOL**

**IF (THET(27) .LT. TMEL) THEN**

**PL1 = X(27)**

**ELSE**

**IF (THET(32) .GE. TMEL) THEN**

**PL1 = X(32)**

**ELSE**

**DO 219 J=27,31**

**IF ((THET(J) .GE. TMEL) .AND. (THET(J+1) .LT. TMEL)) THEN**

**PL1 = X(J) + ((THET(J)-TMEL)/(THET(J+1)-THET(J))**

***(X(J+1)-X(J))**

**ENDIF**

**CONTINUE**

**ENDIF**

**ENDIF**

**ENDIF**
ADD THE TWO DISTANCES TO GET THE TOTAL POOL LENGTH
ADD .31 FOR A KLUDGE SO THAT RESULTS MATCH WITH EXPERIMENTS

\[ PL = PL_1 - PL + .31 \]

*******************************************************************************
* CALCULATE THE FIXED TEMPERATURE COOLING * RATE USING THE POOL WIDTH (ANALYTICAL RELATIONSHIP
* ASSUME: CONSTANT THERMAL PROPS * 2-D INFINITE DIM. HEAT FLOW
* POINT HEAT SOURCE
*******************************************************************************/

\[ TEM_1 = \frac{1}{(V(\text{IFINTM}) \cdot PW/(4. \cdot \text{CAPPA}(1))) + .2)} \]

*******************************************************************************
* ADDING 50 IS A KLUDGE * T01 = 40 + TREL - QDOT/(H*\text{CAPPA}(1)*\text{COND}(1)) * TEM1
* WRITE(6,*) T01, \text{CAPPA}(1), QDOT
C23456789012345678901234567890123456789012345678901234567890123456789012
C6 QDOT**2/((TCRIT - T01)**3)
*******************************************************************************/

*******************************************************************************
* APPROXIMATE COOLING RATE USING EMPIRICAL * FORMULA FROM MULT. LIN. REG.
* CR = CR(1,V,PW)
*******************************************************************************

WRITE(6,*) PW,V(\text{IFINTM})
CRFT3 = EXP(-7.284)*\text{AMPS(IFINTM)}**3.2822/V(\text{IFINTM})**1.541/
6 PW**3.2069

*******************************************************************************
* APPROXIMATE COOLING RATE USING EMPIRICAL * FORMULA FROM MULT. LIN. REG.
* CR = CR(1,PL,T0)
*******************************************************************************

CRFT4 = (EXP(-7.664))*\text{AMPS(IFINTM)}**1.818/(PL**1.761)*
6 ((500 - PHTEM)**0.222)

*******************************************************************************
* APPROXIMATE COOLING RATE USING EMPIRICAL * FORMULA FROM MULT. LIN. REG.
* CR = CR(1,PW,T0)
*******************************************************************************

CRFT5 = exp(-20.24)*\text{AMPS(IFINTM)}**.9353/(PW-.3995)**.48081*
6 (1530 - PHTEM)**2.3459
CRFT5 = EXP(-5.295)*\text{AMPS(IFINTM)}**.862/PW**1.593*
6 (500 - PHTEM)**0.383

*******************************************************************************
* APPROXIMATE COOLING RATE USING EMPIRICAL * FORMULA FROM MULT. LIN. REG.
* CR = CR(PW,1,T0)
*******************************************************************************

CRFT7 = EXP(-17.645)/ (PW-.39995)**.2263 * (V(\text{IFINTM})**.4)**1.749
6 *(1530 - phtem)**2.7542
CRFTA7 = 0.443 / PW**1.014 * v(IFINTM)**.556 *
6 *(500 - 25)**.508

SHIFT ARRAY OF COOLING RATES USED FOR CONTROL BACKWARDS
TO MAKE ROOM FOR NEW ONE
DO 473 I=1,9
473 CRHIS(10-I+1) = CRHIS(10-I)
CRHIS(1) = CRFTA3
RETURN
END

**********************************************************************
* SUBROUTINE CONCAL
* THIS SUBROUTINE IS USED TO CALCULATE THE
* ADJUSTMENTS TO CURRENT AND VOLTAGE SO
* TO REGULATE THET(2) AND CR TO DESIRED VALUES
* **********************************************************************
C234567890123456789012345678901234567890123456789012345678901234567890
SUBROUTINE CONCAL(IFINTM,GQI,GQP,GQD,GVI,GVP,FWDES,
1 CRD,THET,CRFF,CRSC,CRM,C,VRMS,FWERSM,CRESM,PR,CREROL,GVD,
2 DELTM,OLTDC,CRFTAP,PL,CRFTA2,OLTVS,EFF,PWERI,PWI
3 ,ISEED,IRANIN,QQOT,PDTHM,TEL,CRFTA3,CRHIS,ARM,MAM,ALAM,ICON,
4 CRFTA4,CRFTA5,crf6,COOLD,CREROL1,CREROL0,ICOCO,CRSOL,crf7,
cur,icr)
C
DIMENSION THET(20),V(251),AMPS(251),CRHIS(10),ARM(10),MAM(10)
C INTEGER=2 PWERS,FWDES,FWERSM,FWDER,PWERI,GQI,GQP,GQD,DI,1
INTEGER=4 ISEED
C
IF(IFINTM=DELTIM .LE. 4) THEN
if((icone .eq. 0).or.(icone .eq. 4)).and.(icr.eq.6)) then
off = (((7.0893)/cur)**1.5584*(500-phtem)**.72507
5 /crd)**(1./.5246)
off = (crd/1.2683E-8*(cur-65)**.2512)/((1530-phtem)**3.1071)
6 **(1./.3357)-.4
off = l/off
if (icr.eq.6) then
v(IFINTM+1) = 1./off
else
v(IFINTM+1) = v(1)
off = 1/off
endif
else
V(IFINTM + 1) = V(IFINTM)
endif
AMPS(IFINTM + 1) = AMPS(IFINTM)
C
SET UP SOME STUFF FOR COMPENSATOR
CREROL0 = 0
CREROL1 = 0
COOLD = COOLD
CROL-CRFTA4
ENDIF
C
DO CONTROL IF TIME IS MORE THAN 50

226
IF (IFINTM*DELTIM .GE. 4) THEN

C IF (IRANIN .EQ. 0) THEN
IF (ICON .EQ. 0) THEN
PWERR = PWDES - PW
CRERR = CRD - CRFTA5
PWERSM = PWERSM + PWERR
CRERSM = CRERSM + CRERR
PWDER = -(PWERR - PWEROL)/DELTIM
CRDER = -(CRERR - CREROL)/DELTIM
DI = GGI*PWERSM/2 + GGP*PWERR/2 + GGD*PWDER/2
DV = GVI*CRERSM + GVP*CRERR + GVD*CRDER
AMPS(IFINTM+1) = AMPS(1) + DI
QDOT = ETA(AMPS(IFINTM+1),V(IFINTM+1),VOLTS,EFF,PHTEM,TMEL)*
6 AMPS(IFINTM+1)*VOLTS*EFF
off = ((exp(7.0853)/cur**1.5584*(500-phtem)**.72507)
6 /crd)**(1.1.5246)
off = (crd/1.2603e-8*(cur-65)**.2512/(1530-phtem)**.71071
6)**(1/3.3357)-.4
off = 1./off
write(6,*,'off',off)
V(IFINTM+1) = off + DV
IF (OFF + DV).GE. 0.25 THEN
V(IFINTM+1) = .25
CRERSM = CRERSM - CRERR
ELSEIF ((OFF+DV) .LT. .1) THEN
V(IFINTM+1) = .1
CRERSM = CRERSM - CRERR
ENDIF
PWEROL = PWERR
CPREROL = CRERR

C ELSEIF(ICON .EQ. 1) THEN

C DO WEIGHTED ONE-STEP CONTROL AS IN GOODWIN AND SIN
"ADAPTIVE FILTERING, PREDICTION AND CONTROL", PAGE 122
C
6 V(IFINTM+1) = gvp*(1. / (7.01 + .816*crfta5))*(CRD-.638*CRFTA5)
+ v(ifintm)
AMPS(IFINTM+1)=AMPS(1)
I = AMPS(IFINTM+1)
QDOT = I*VOLTS*EFF*
6 ETA(AMPS(IFINTM+1),V(IFINTM+1),VOLTS,EFF,PHTEM,TMEL)
CROL=CRFTA4
C
C DO STEP INPUT IF ICON .EQ.2
C
ELSEIF (ICON .EQ. 2) THEN

C IF (IFINTM*DELTIM .LT. 220.) THEN
AMPS(IFINTM+1) = AMPS(1)
V(IFINTM+1)=V(1)
ELSEIF ((IFINTM*DELTIM .GE. 220).AND.(IFINTM*DELTIM .LT. 400)) THEN
6 AMPS(IFINTM+1) = AMPS(1)
V(IFINTM+1)=V(1)+(.5*2.54/60)
elseIF (IFINTM*DELTIM .GE. 400) THEN
AMPS(IFINTM+1) = AMPS(1)
V(IFINTM+1) = V(1)
ENDIF
I = AMPS(IFINTM+1)
QDOT = I*VOLTS*EFF*
6 ETA(AMPS(IFINTM+1),V(IFINTM+1),VOLTS,EFF,PHTEM,TMEL)
6
C
C IF ICON = 3 THEN DO A COMPENSATOR CONTROLLER FOR TRAVEL SPEED

227
ELSEIF (ICON .EQ. 3) THEN
ICOCO = ICOCO+1
IF (ICOCO .EQ. 1) THEN
ICOCO = 0
CRERR = CRD - CRFTA4
WRITE(6,'(A)') CRERR,C
ENDIF
CO = (-GVP)*(CRERR-2.0*CREROL1+.9975*CREROLOL)+COOLD
V(IFINTM+1)=V(1)+CO
AMPS(IFINTM+1) = AMPS(1)
I = AMPS(IFINTM+1)
QDTE = ETA*(AMPS(IFINTM+1),V(IFINTM+1),VOLTS,EFF,PTEM,TREL)*
6 AMPS(IFINTM+1)*VOLTS*EFF
COOL1 = COOLD
COOLD = CO
CREROLOL = CREROL1
CREROL1 = CRERR
ELSEIF (ICON .EQ. 4) THEN
IF icon is 4, do gain switched control
IF ((CRFTA6 .LT. 1) .OR. (CRFTA6 .GT. 20)) THEN
off = (Crd/1.2683e-8*(cur-65)**2.512/(1530-phtem)**1.1071)
6 **(1/3.3357)-.4
off = 1./off
if (icr .eq. 6) then
v(ifintm+1) = 1./off
else
v(ifintm+1) = v(1)
off = 1/off
ENDIF
AMPS(IFINTM + 1) = AMPS(IFINTM)
ELSE
off = ((exp(7.0893)/CUR**1.5584*(500-phtem)**.72507)
6 /crd)**(1/1.5246)
6 off = (Crd/1.2683e-8*(cur-65)**2.512/(1530-phtem)**3.1071)
6 **(1/3.3357)-.4
ENDIF
WRITE(6,*)) 'off',off
if (off .lt. 4.*.254/60.) then
if (amps(1) .le. 100) then
if (phtem .lt. 100) then
z1 = .97
z2 = .68
z1 = .9159
g = .01
za7 = .874
g = .01
pa7 = .724
za5 = .839
g = .06
else
z1 = .98
z2 = .65
z1 = .9204
g = .08
za7 = .894
g = .01
pa7 = .756
za5 = .863
g = .08
END IF
endif
else
  z1=.98
  z2=.65
  zl = .9426
  g=.02
  za7 = .902
  pa7 = .773
  ga7 = .01
  za5 = .876
  ga5 = .09
endif
else
  if (amps(1) .le. 100) then
    z1=.96
    z2=.89
    zl = .93
    g=.02
    za7 = .472
    pa7 = .308
    ga7 = .008
    za5 = .474
    ga5 = .03
  else
    if (phtem .le. 100) then
      z1=.91
      z2=.78
      zl = .9132
      g=.006
      za7 = .537
      pa7 = .367
      ga7 = .01
      za5 = .578
      ga5 = .04
    else
      z1=.96
      z2=.89
      zl = .9334
      g=.02
      za7 = .618
      pa7 = .442
      ga7 = .01
      za5 = .654
      ga5 = .05
    endif
  endif
endif
endif
if (icr .eq. 6) then
  gvd=g*zl+z2
  gvp=g*(zl-z2-2.*z1*z2)
  gvi=g-gvp-gvd
  gvd=gvd+deltim
  gvp = zl*g
  gvi = g - gvp
  gvd = 0.
elseif (icr .eq. 5) then
  gvp = za5*ga5
  gvi = ga5 - gvp
  gvd = 0.
else
  gvp = za7*ga7
  gvi = ga7 - gvp
  gvd = 0.
endif
write(6,*) gvp,gvi,gvd
PKERR = FWDES - FW
if (icr .eq. 6) then
  CRERR = CRD - CRFTA6
elseif (icr .eq. 5) then
  crr = crd - crfta5
else
  crr = crd - crfta7
endif

******************************************************************************

C do a compensator for crfta7 because it is not realizable with pid algorithm
C
C IF (ICr .EQ. 7) THEN
C
CO = coold*(pa7+1) - coool*pa7
+ (ga7)*(crercl1 - creniol*za7)

V(IFINTM+1)=V(1)+CO

coolol = coold
COOLD = CO
CREROLOL = CREROLL
CREROLL = CRERR
endif

******************************************************************************

PWSRM = PWSRM + PWERR
PWDER = -(PWERR - PWEROL)/DELTIM
DI = GQI*PWSRM/2 + GQP*PWERR/2 + GQD*PWDER/2
AMPS(IFINTM+1) = AMPS(1) + DI
QDOT = ETA(AMPS(IFINTM+1),V(IFINTM+1),VOLS,EFF,PHTEM,TMEF)*
AMPS(IFINTM+1)*VOLTS*EFF

if (icr .ne. 7) then
  CRERSM = CRERSM + CRERR
  CRDER = -(CRERR - CREROL)/DELTIM
  DV = GV1*CRERSM + GVP*CRERR + GVD*CRDER
off = ((exp(7.0893)/amps(ifintm)**1.5584*(500-phtem)**.72507)/crd)**(1./.5246)
off = 1./off
write(6,*) off
if (icr .eq. 6) then
  V(IFINTM + 1) = off + DV
else
  v(ifintm + 1) = 0.16933 + dv
endif
if (v(ifintm+1) .ge. 0.25) THEN
  V(IFINTM+1) = .25
CRERSM = CRERSM - CRERR
ELSEIF (v(ifintm+1) .lt. .1) THEN
  V(IFINTM+1) = .1
if (icr .eq. 6) then
  CRERSM = CRERSM - CRERR
endif
ENDDIF
PWEROL = PWERR
CREROL = CRERR
ENDDIF

no control

elseif (icon .eq. 5) then
  V(IFINTM+1)=V(1)
AMPS(IFINTM+1) = AMPS(1)
I = AMPS(IFINTM+1)
QDOT = ETA(AMPS(IFINTM+1),V(IFINTM+1),VOLS,EFF,PHTEM,TMEF)*
AMPS(IFINTM+1)*VOLTS*EFF

230
ENDIF
ENDIF

DO RANDOM INPUT IF IT IS DESIRED

IF (IRANIN .NE. 0) THEN
DV = (RAN(IIMAX)-.5)*2.54/60
I = AMPS(1)
amps(ifintm+1) = amps(1)
QDOT = I*VOLTS*EFF*
ETA(AMPS(1),V(IFINTM+1),VOLTS,EFF,PHTEM,TMEL)
V(IFINTM+1) = V(1)+DV
ENDIF
ENDIF

RETURN
END

******************************************************************************

*    SUBROUTINE OUDIS
*
*    THIS SUBROUTINE OUTPUTS THE TEMPERATURE
*    THE(1) AND THE COOLING RATE AS WELL AS
*    TIME TO THE CRT
*
******************************************************************************

SUBROUTINE OUDIS(V,AMPS,NUMPTS,IFINTM,DELTIM,THET,CRFT,
CRSC,CRMC,FW,CRFTAP,PL,CRFTA2,PWI,IRANIN,X,PL1,QDOT,
CRFTA3,CRHIS,IDISKO,CRFTA4,CRFTA5,crfta6,crfta7,icr)

DIMENSION THET(50),V(251),AMPS(251),X(50),CRHIS(10)
INTEGER*2 PWI

WRITE TO CRT AND TO DISK if requested

if (icr .eq. 5) then
WRITE(6,*)(IFINTM = DELTIM,amps(ifintm),v(ifintm),FW,crfta5
,crfta7,crfta6
eelseif (icr .eq. 6) then
WRITE(6,*)(IFINTM = DELTIM,amps(ifintm),v(ifintm),FW,pl,crfta6
else
WRITE(6,*)(IFINTM = DELTIM,amps(ifintm),v(ifintm),FW,crfta7,
crfta6
endif

IF (IDISKO .EQ. 1) THEN
if (icr .eq. 5) then
WRITE(1,*)(IFINTM = DELTIM,v(ifintm),FW,crfta6,CRFTA5
,crfta7
eelseif (icr .eq. 6) then
WRITE(1,*)(IFINTM = DELTIM,v(ifintm),FW,crfta6
else
WRITE(1,*)(IFINTM = DELTIM,v(ifintm),FW,CRFTA6,crfta7
endif
ENDIF

WRITE(6,*)(I
I-I+1

231
C IF ((IRANIN .NE. 0).AND.(FINITM .GE. 50)) THEN
    WRITE(1,*) V(FINITM), Fw, CRFT95, CRFT87
ENDIF
C
300  RETURN
END
C
************************************************************************************
*  *  SUBROUTINE MOVCOO
*  *  DAVE FARSON 6-86
*  *  THIS SUBROUTINE MOVES THE POINTS USED TO CALCULATE THE FIXED TEMP COOLING RATES SO THAT THE TC CRIT REMAINS WITHIN THEM.  IT ALSO ADJUSTS THE VALUE OF OLICFT SO THAT IT IS CORRECT
*  ************************************************************************************

SUBROUTINE MOVCOO(MOVEFL, X, XIT, OLICFT)
C
DIMENSION X(50)
INTEGER*2 MOVEFL
C
IF (MOVEFL .EQ. 0) THEN
    RETURN
ENDIF
C
IF (MOVEFL .EQ. -1) THEN
    DO 400 I=13,46
        X(I) = X(I) - 4*XIT
    CONTINUE
400    OLICFT = OLICFT - 4
    ENDIF
C
IF (MOVEFL .EQ. 1) THEN
    DO 405 I=31,46
        X(I) = X(I) + 4*XIT
    CONTINUE
405    OLICFT = OLICFT + 4
    ENDIF
C
RETURN
END
C
************************************************************************************
*  FUNCTION ETA  
*  *) TO CALCULATE THE EFFICIENCY OBTAINED BY COMPARING SIMULATIONS TO EXPERIMENT
*  ************************************************************************************

FUNCTION ETA(AMPS, V, VOLTS, EFF, PHEM, TMEL)
C
WRITE(6, *) 'INP EXPS V, TO, I (.29418, .28517, .2101)'
READ(5, *) VPWR, TOPWR, CPWR
ETA = (EXP(-3.7553)/EXP(-3.2058))*V**.294
6 (TMEL-PHEM)**.285/AMPS**.21
C
************** DISABLE VARIABLE EFFICIENCY **************

ETA = 1.
C
WRITE(6, *) ETA, AMPS, V, PHEM, TMEL
RETURN
END

232
subroutine fnsh.ft(amps, v, sht, cond, h, shtc, eff, volts)

gdot = eff*amps*volts

if (gdot .gt. 616) then
  if (v .lt. 0.1693333) then
    teml = shtc + (0.1693333 - v)*(.0016-shtc)/0.423333
    tem2 = shtc + (gdot - 616.)*(0.0016-shtc)/61.6
    if (tem1 .ge. tem2) then
      shtctem = tem1
    else
      shtctem = tem2
    endif
  else
    shtctem = shtc + (gdot - 616.)*(0.0016-shtc)/61.6
  endif
else
  if (v .lt. 0.1693333) then
    shtctem = shtc + (0.1693333 - v)*(.0016-shtc)/0.423333
  else
    shtctem = shtc
  endif
endif

SHR = 2.*SHTCTEM/COND/H

return

de
**2-D HEAT SIMULATION PROGRAM**

**INTEGRATES THE ANALYTICAL SOLUTION**

*using 3 pt GAUSSIAN QUADRATURE*

*copied from gaushtsm.for on 11-7-86*

*and reduced in complexity for use as*

*an observer*

*also added simultaneous observer/simulation functions at this time*

******************************************************************************

* MAIN ROUTINE *

******************************************************************************

THIS SECTION OF CODE INITIALIZES variables, etc

non-global dimensions

dimension AMPS(251), AMPSp(251)

generic variable dimensions

DIMENSION V(251)
DIMENSION X(50), Y(50), THET(50)
DIMENSION XP(50), YP(50), THETP(50)

control variable dimensions and defs

DIMENSION ARM(10), MAM(10), pws(4)
DIMENSION CHRH(10), PWSDEL(15)
DIMENSION CHRHP(10), PWSDELP(15)
INTEGER=4 ISEED

common /generic/ v, imat,
6 volts, tmei, ifintm, pi, deltim, itmcnt
6
common /tempcal/ t0, t1, t, tp, xprim, yprim, nptcnt, sum, shpt,
6 itcnt, costnt
6
common /contrl/ olthsc, olthmc, crsc, drcsc, tcr, tcr2, nd, pil,
6 gqi, gqp, gqg, gvi, gvp, gvd,
6 pwdesi, crd, pweroi, iseed, iranin, arm, mamp, alam, icon, coold,
6 ceroil, ceroi1, cico, co1, idisko, xit, pwa

SET UP TIME COUNTERS

ISTTM = 0
IFINTM = 1
deltim = 2
IENETM = 150
itmcnt = 1

STUFF FOR RANDOM INPUT

ISEED = 62233+3
IRANIN = 0

ASSIGN STARTING LOCATIONS TO POINTS
ALL POINTS EXCEPT THE LAST MOVE WITH THE SOURCE CENTER

NUMPTS = 46
NUMTPP = 46

POOL WIDTH LOCATOR POINTS

DO 464 I=1,4
DO 464 J=1,4
X(4*(I-1)+J)=.1*(1.-I)
XP(4*(I-1)+J)=.1*(1.-I)
YP(4*(I-1)+J)=.1*J+.1
Y(4*(I-1)+J)=.1*J+.1

POOL BACK EDGE LOCATOR POINTS

A1 = -.4
DO 470 I=17,26
X(I) = AI
XP(I) = AI
YP(I) = 0.
Y(I) = 0.
A1 = A1 - .1

POOL FRONT EDGE LOCATOR POINTS

A1 = 0.
DO 474 I=27,32
X(I) = AI
XP(I) = AI
YP(I) = 0.
Y(I) = 0.
A1 = A1 + .1

COOLING RATE CALCULATOR POINTS

XIT = .25
A1=4.
DO 465 I=33,46
X(I)=AI
XP(I) = AI
YP(I) = 0.
Y(I)=0.0
A1=A1-XIT

THREE POINTS CLOSER TO POOL FOR COOLING RATE APPROXIMATION

X(47)=2.0
Y(47)=0.
X(48)=2.25
Y(48)=0.
X(49)=2.5
Y(49)=0.

XP(47)=2.0
YP(47)=0.
XP(48)=2.25
YP(48)=0.
XP(49)=2.5
YP(49)=0.

FIXED POINT. THIS POINT DOES NOT MOVE WITH SOURCE

X(50)=15.

235
Y(50) = 0.0
XP(50) = 15.
YP(50) = 0.

**init various parameters**

iranim = 0
****** no random trav. speed prf.
iddisco = 1
****** do disk output
icon = 0
****** no feedback control
phtem = 25.
PHTEMP = 25.
phtennom = 25.
****** preheat temps.
imat = 1
****** material is steel

**INIT A BUNCH OF CONTROL PARAMETERS**

PWERSM = 0.
CREERM = 0.
OLTHFT = 0.
OLTHFP = 0.
OLTHSC = 0.
OLTHMC = 0.
CREROL = 0.
PWEROL = 0.
ICOCO = 0.
GQP = 0.
GQI = 0.
GQQ = 0.
PWDES = 0.
GVP = 0.
GVI = 0.
GVD = 0.
GDP = 0.
Gsum = 0.
sum = 0.
pwact = .6
XPRIMO = 0.
YPRIMO = 0.

**INIT ONE-STEP CONTROL PARAMETERS**

ARM(1) = -.35094
ARM(2) = -.38740
MAM(1) = -.40.4940
MAM(2) = 27.65421
MAM(3) = 21.41389
ALAM = 80

PI = 4. * ATAN(1.)

**init a bunch of welding parm's.**
cur = 100.
volts = 11.
trav = 4.
eff = .56
eftp = .56
hcfs = 0.02
hcftsp = .02
shtc = .001
shtcp = .001
shtp = .006
cappapu = .063

236
cappap = .063
condpu = .312
condp = .312
************ observer feedback gain
og = 500.
************ base met. thick.
hp = .2286
hpu = .2286
************ use adapt. obs. algo.
isc = 1
************ pool width meas. error
delw = 0.

ND CAN BE USED TO DELAY POOL WIDTH MEASUREMENT IF DESIRED

ND = 0
ND = (.5*2.54)/TRAV

ISC APPLIES TO DYN. OBS. ONLY
write(6,*) 'servo preheat(0) or current (1)',isc
read(5,*) isc

if (isc .eq. 1) then
  og = 15
endif

the following section has a bunch of user input queries

WRITE(6,*) 'CHANGE PARAMETERS 0=NO 1=YES'
READ(5,*) IJUNK

IF (IJUNK .EQ. 0) GOTO 932

GET THE DELTIME

WRITE(6,*) 'INPUT THE DELTIME'
READ(5,*) DELTIM

SEE IF RANDOM INPUT IS TO BE USED

WRITE(6,*) 'USE RANDOM INPUT?, 0=NO,1=YES'
READ(5,*) IRANIN

IF (IRANIN .EQ. 1) GOTO 934

SEE IF OUTPUT TO DISK (AUTOMATIC IF IRANIN = 1)

WRITE(6,*) 'OUTPUT TO DISK?, 0=NO,1=YES'
READ(5,*) IDISKO

SEE IF PID OR ONE-STEP CONTROL IS TO BE USED

WRITE(6,*) 'PID(0),ONE-STEP CONTROL(1), STEP INPUT(2)'
write(6,*) 'COMPENSATOR(3), gain switch(4) or none(5)''
READ(5,*) ICON

GET THE INIT TEMP

WRITE(6,*) 'INPUT THE PREHEAT TEMP'
READ(5,*) PHTEMP
phtemnom = phtemp

GET THE MATERIAL TYPE

WRITE(6,*) 'STEEL (1) OR ALUMINUM (2)''
READ(5,*) IMAT
GET THE CONTROL GAINS, FIRST CURRENT, THEN TRAV. SPEED

WRITE(6,*),'INPUT GQP,GQI,GOQ,FWD (200,150,0,.72)'
READ(5,*),GQP,GQI,GOQ,FWDES
WRITE(6,*),'INPUT GVQ,GVQI,GVP,FWD (.004 .001 .001 32)'
READ(5,*),GVQ,GVQI,GVP,FWDES

GET THE WELDING PARAMETERS

WRITE(6,*),'AMP,VOLT,TRAV,EFF,HCFS,SHTC(100,11,4,0.56,0.02,0.001)'
READ(5,*),CUR,VOLTS,TRAV,EFFP,HCFS,HPC

GET SURFACE HEAT TRANS COEFF. FOR PREHEAT DECAY RATE

WRITE(6,*),'INPUT PLATE SHT (.006)'
READ(5,*),SHTP

WRITE(6,*),'input cappa and cond (.063 .312)'
READ(5,*),cappapu,condpu

OG = 500 FOR CONV. DYN. OBS, 18 FOR ADAP. DYN. OBS
WRITE(6,*),'input observer gain (500), thickness (.2826)'
READ(5,*),og,hpu

WRITE(6,*),'input delw (0)'
READ(5,*),delw

********** which cooling rate to use cfrta5,6,7
5 = static mod. adap. obs.
6 = dynamic conv. obs.
7 = static adap. obs.
WRITE(6,*),'input icr (5),(6),(7)'
READ(5,*),icr

CONTINUE

set up thermal props and init temps

IF (imat.eq.1) THEN
CAPPA = .063
COND = .312
cappap = cappapu
condp = condpu
ELSE IF (imat.eq.2) THEN
CAPPA = .5
cappap = .5
COND = 1.3
condp = 1.3
END IF

DO 10 I=1,NUMPTS
IF (IMAT.EQ.1) THEN
THET(I) = PHTEM
ELSE IF (IMAT.EQ.2) THEN
THET(I) = PHTEM
END IF
CONTINUE

DO 192 I=1,NUMTP
IF (IMAT.EQ.1) THEN
THETP(I) = PHTEMP
ELSE IF (IMAT.EQ.2) THEN
THETP(I) = PHTEMP
END IF
CONTINUE

238
DEFINE THERMAL AND OTHER PROPS FOR STEEL

IF (IMAT .EQ. 1) THEN
  PLATE THICKNESS IN CM.
  H = .2286
  hp = hpu
  MELTING TEMP
  TMEL = 1530.
  TEMP TO CALC COOLING RATE AT
  TCRT = 500.

else IF (IMAT .EQ. 2) THEN
  MELTING TEMP FOR AL.
  TMEL = 660.
  PLATE THICKNESS IN CM. FOR AL.
  H = .3175
  hp = .3175
  TEMP TO CALC COOLING RATE AT
  TCRT = 300.

END IF

make things metric, etc

TRAV = TRAV*2.54/60.
V(1)=TRAV
AMPS(1)=CUR
AMPSp(1)=cur

CALC. SURFACE HEAT TRANS. FROM COEFF. WHEN USED. CURRENTLY
SURFACE HEAT TRANS. IS INPUT DIRECTLY

SHT = 2.*SHTC/COND/H
SHTP = 2.*SHTCP/CONDp/hp

CALCULATE THE HEAT INPUT USING VARIABLE EFFICIENCY FACTOR

QDOT = ETA(THTR,amps)*AMPS(1)*VOLTS*EFF
QDOTP = ETA(THTR,ampsp)*AMPSp(1)*VOLTS*EFFp

DISK OUTPUT IS DEVICE #1,FILENAME DEFAULTS TO FOR001.DAT
OPEN(unit = 1, status = 'old')

END OF INITIALIZATION SECTION

******************************************************************************
*
*               START OF TEMP CALC SECTION
*
******************************************************************************
do WHILE (IFINTM .LE. IENDTM)

THIS LOOP IS REPEATED AT EACH TIME STEP FOR IENDTM TIMES
THE HEAT EQUATION IS INTEGRATED 'STARTING AT ZERO' AND
GOING UP TO IFINTM EACH TIME STEP. THIS PROCESS TAKES
LONGER TO ACCOMPLISH EACH TIME AS THE NUMBER OF TIME STEPS
INCREASES. THE SOURCE CENTER starting point IS SET BACK
EACH TIME STEP BY A DISTANCE DETERMINED BY THE TRAVEL SPEED.

THIS UPDATES THE LOCATION OF THE HEAT SOURCE STARTING
POINT FOR THE NEXT TIME STEP. Don't do this if time is getting
large, just start forgetting points.
if (ifintm*deltim .le. 400) then
  XPRIMO = XPRIMO - V(IFINTM)*DELTIM
else
  xprimo = xprimo - V(ifintm)*deltim
xprimo = xprimo + V(i)*deltim
  call shift(v,ifintm)
call shift(amps,ifintm)
call shift(amps,ifintm)
  ifintm=ifintm-1
endif

servo preheat to eliminate difference in pool width
  from observer to plant
if (isc .eq. 0) then
  ousum = ousum + (pwP - pw) + delw
  suml = ousum * exp(sht*(itmcnt-1)*deltim)
  offset = og * exp(-sht*(itmcnt-1)*deltim)*suml
  phtem = 25 + offset
else
  servo current to eliminate difference in pool width
  from observer to plant
  ousum = ousum + (pwP - pw) + delw
  suml = ousum * exp(sht*(itmcnt-1)*deltim)
  offset = og * exp(-sht*(itmcnt-1)*deltim)*suml
  amps(ifintm) = amps(l)+offset
  WRITE(6,*) 'AMPS',AMPS(IFINTM)
endif

******* DO TEMP/OUTPUT CALCS. FOR PLANT ********

THIS CALL RESETS THERMAL PROPERTIES
CALL RETHPR(THEP,XP,YP,NUMTP,PTEMP)

THIS SUBROUTINE CALL INTEGRATES THE H.E. UP TO IFINTM
FOR EACH OF THE POINTS (X,Y)
CALL CALTEM(THEP,XP,YP,NUMTP,PTEMP,XPRIMO,YPRIMO,QDOTP,
  cappap,condp,effp,hp,shtpp,hcfsp,amps,phtemp)

INCORPORATE PREHEAT DECAY FOR PLANT
DO 723 J=1,NUMTP
  THEP(J)=THEP(J) - (PTEMP-25.)*(1-EXP(-SHT*ITMCNT*DELTIM))
723

THIS SUBROUTINE CALL CALCULATES THE VALUES OF THE OUTPUTS
SUCH AS POOL DIMENSIONS AND COOLING RATES
CALL CAOUFU(THEP,XP,YP,NUMTP,PTEMP,QDOTP
  ,pwP,polthfp,crtfp,olicfp,olwtcP,pLp,pwelf)
  ,crtfp,critsP,crtfp4P,crits5P,crft6P,
  cappap,condp,effp,hp,shtpp,hcfsp,lweflp,amps,crft6p,
  phTEMn)
  WRITE(6,*') 'lweflp',lweflp

******* DO TEMP/OUTPUT CALCS FOR THE OBSERVER *********

THIS CALL RESETS THERMAL PROPERTIES
CALL RETHPR(THEP,XY,NUMPTS,PTEMP)

THIS SUBROUTINE CALL INTEGRATES THE H.E. UP TO IFINTM
FOR EACH OF THE POINTS (X,Y)
CALL CALTEM(THEP,XY,NUMPTS,PTEMP,XPRIMO,YPRIMO,QDOTP,
  cappp,condp,effp,h,sht,hcfs,amps,shtc)

THIS SUBROUTINE CALL CALCULATES THE VALUES OF THE OUTPUTS
SUCH AS POOL DIMENSIONS AND COOLING RATES

240
CALL CACUPU(THET,X,Y,NUMPTS,PHTEM,QDOT
6,pw,olthf,crft,olicft,olvtc,pl,pwdel
6,crfta3,crhia,crfta4,crfta5,crfta6,
cappa,cond,eff,h,sht,hsfs,imvefl,amps,crfta7,
6 phtem)
write(6,*),'imvefl',imvefl

*********** END OF TEMP/OUTPUT CALCULATIONS ***********

THIS SUBROUTINE CALL MODES THE CONTROL CALCULATIONS,
ADJUSTING THE VALUES OF AMPS AND V
CALL CONCAL(THET,X,Y,NUMPTS,PHTEM,QDOT
6,pw,olthf,crft,olicft,olvtc,pl,pwdel
6,crfta3,crhia,crfta4,crfta5,crfta6,
cappa,cond,eff,h,amps,phtemnom,cur,crfta7,icr)

this code gives the simulation the value of the
observer current. if servoeing current to make observer
converge to simulation, this offset is added up above
and should not affect the control value

ampsp(ifintm+1)=amps(ifintm+1)

*********** DISPLAY AND MOVE COORDS FOR PLANT ***********

THIS SUBROUTINE CALL DISPLAYS THE OUTPUT ON THE CRT AND DISK
CALL OUTDIS(THETP,XP,YP,NUMTP,PHTEMP,QDOTP
6,pwp,olthfP,crftP,olicfP,olvtp,plP,pwelP
6,crft3P,crhiaP,crft4P,crft5P,crft6P
6,cappa,condp,effp,hp,amsp,crft7p)

THIS CALL MOVES THE POINTS USED TO CALCULATE THE COOLING
RATE SO THAT TCRT REMAINS WITHIN THEM
CALL MOVCOO(THETP,XP,YP,NUMTP,PHTEMP
6,pwp,olthfP,crftP,olicfP,olvtp,plP,pwelP

*********** DISPLAY AND MOVE COORDS FOR OBSERVER ***********

THIS SUBROUTINE CALL DISPLAYS THE OUTPUT ON THE CRT AND DISK
CALL OUTDIS(THET,X,Y,NUMPTS,PHTEM,QDOT
6,pw,olthf,crft,olicft,olvtc,pl,pwdel
6,crfta3,crhia,crfta4,crfta5,crfta6,
cappa,cond,eff,h,amps,crfta7)

THIS CALL MOVES THE POINTS USED TO CALCULATE THE COOLING
RATE SO THAT TCRT REMAINS WITHIN THEM
CALL MOVCOO(THET,X,Y,NUMPTS,PHTEM
6,pw,olthf,crft,olicft,olvtc,pl,pwdel
6,crfta3,crhia,crfta4,crfta5,crfta6,1,imvefl)

*********** END OF DISPLAY AND MOVE COORDS SECTION ***********

UPDATE TIME COUNTERS

ISTTM = ISTTM + 1
IFINMT = IFINMT + 1
itmcnt = itmcnt + 1

end do

CLOSE(1)

END
SUBROUTINE RETHPR

* THIS SUBROUTINE RESETS THE THERMAL PROPERTIES AT
* A POINT TO THE INITIAL ONES
* now just resets temperatures

SUBROUTINE RETHPR(THET,X,Y,NUMPTS,PTEM)

generic variable dimensions

DIMENSION V(251)
DIMENSION X(50), Y(50), THET(50)

common /generic/ v,imat,
6 volts,tmel,ifihtm,pi,deltim,itmctnt

DO 60 I = 1,NUMPTS

IF (IMAT.EQ.1) THEN
THET(I) = PTEM
else IF (IMAT.EQ.2) THEN
THET(I) = PTEM
END IF

60 CONTINUE

RETURN
END

SUBROUTINE CALTEM()

* THIS SUBROUTINE INTEGRATES THE HEAT EQ. SOLUTION
* UP TO THE CURRENT TIME FOR EACH POINT
* OF INTEREST

SUBROUTINE CALTEM(THET,X,Y,NUMPTS,PTEM,XPRIMO,YPRIMO,QDOT,
cappa,cond,eff,h,sht,hcfs,amps,shtc)

6 dimension AMPS(251)

generic variable dimensions

DIMENSION V(251)
DIMENSION X(50), Y(50), THET(50)

common /generic/ v,imat,
6 volts,tmel,ifihtm,pi,deltim,itmctnt

common /temct/ t0,t1,t,tp,xprim,yprim,nptcnt,sum,shtp,
6 itmctnt,coastnt

DO 30 NPTCNT = 1,NUMPTS

SET THE SOURCE CENTER BACK TO ZERO FOR NEXT PASS THRU

XPRIM = XPRIMO
YPRIM = YPRIMO

242
THIS LOOP INTEGRATES HEAT EQU UP TO CURRENT TIME

DO 40 ITCNT = 1, IFINTM

T0 = (ITCNT - 1) * DELTIM
T1 = ITCNT * DELTIM
T = IFINTM * DELTIM

THIS SUBROUTINE CALL DOES THE 3-PT GAUSSIAN QUADRATURE INTEGRATION

QDOT = ETA(PHTEM,amps)*AMPS(ITCNT)*VOLTS*EFF

CALL FNNH(SHT(amps(ittcnt),v(ittcnt),sht,cond,h,shtc, eff,volts)

CALL GAQUIN(THET,X,Y,NUMPTS,PHTEM,QDOT, cappa,cond,eff,h,sht,hcis)

ADD THE NEW PART TO THE SUM

THET(NPTCNT) = THET(NPTCNT) + SUM

MOVE SOURCE CENTER

XPRIM = XPRIM + V(ITCNT) * DELTIM

CONTINUE

RETURN

END

*************************************************************************

*************************************************************************

SUBROUTINE GAQUIN

THIS SUBROUTINE PERFORMS 3 PT GAUSSIAN QUAD INTEGRATION OF THE INTEGRAND IN SUB. EVALIN

*************************************************************************

SUBROUTINE GAQUIN(THET,X,Y,NUMPTS,PHTEM,QDOT, cappa,cond,eff,h,sht,hcis)

generic variable dimensions

DIMENSION V(251), X(50), Y(50), THET(50)

common /generic/ v,imat,

volts,tmel,ifintm,pi,delim,itmctn

common /temcal/ t0,t1,t,tp,xprim,yprim,nptcnt,sum,shtp,

40 itcnt,cosnt

COSTnt = QDOT/( PI * H * cond/cappa)

SUM = 0.

243
DO 70 I = 1,3
   IF (I .EQ. 1) THEN
      A = -.77459667
      B = .5555555555
   ELSEIF (I .EQ. 2) THEN
      A = 0.
      B = .88088888888888
   ELSEIF (I .EQ. 3) THEN
      A = .77459667
      B = .5555555555
   ENDIF
   TP = ((T1 - T0) * A + T1 + T0)/2.
   CALL EVALINT TO EVALUATE THE INTEGRAND
   CALL EVALINT(VALU,X,Y,cappa,cond,eff,h,sht,hcfs)
   SUM = SUM + VALU * B * ((T1 - T0)/2.)
  70 CONTINUE
RETURN
END

******************************************************************************
SUBROUTINE EVALIN

THIS SUBROUTINE EVALUATES THE INTEGRAND
AT THE POINT TP PROVIDED BY THE CALLING ROUTINE
******************************************************************************

SUBROUTINE EVALIN(VALU,X,Y,cappa,cond,eff,h,sht,hcfs)

generic variable dimensions

DIMENSION V(251),
DIMENSION X(50),Y(50)
common /generic/ v,imat,
6 volts,tmel,fintrm,deltrm,itcnt
common /temcal/ t0,tl,t,tp,xprim,yprim,nptcnt,sum,shtp,
6 itcnt,costnt
   AA = (X(nptcnt) - XPRIM - (V(itcnt) * (TP-T0))) ** 2.
   BB = (Y(nptcnt) - XPRIM) ** 2.
   DD = CAPPA * SHT * (T-TP)
   CC = EXP(-DD +((-AA-BB)/((4.*CAPPA * (T - TP)) + 2.*hcfs))
   VALU = CC / (4. * cappa * (T - TP) + 2.*hcfs) * COSTnt
RETURN
END

******************************************************************************

SUBROUTINE CAGUPU

******************************************************************************

244
* THIS SUBROUTINE CALCULATES THE COOLING
* RATE OUTPUT AND POOL DIMENSIONS NEEDED TO
* DO THE CONTROL
*
***********************************************************************

SUBROUTINE CAOUUPU(THET,X,Y,NUMPTS,PHTEM,QDOT
6,pw,olthft,crft,olcft,olwtec,pl,pwdel
6,crfta3,crhis,crfta4,crfta5,crfta6,
6,capp,cond,eff,h,sht,hcfs,imvefl,amps,crfta7
6,phtemmom)

dimension AMPS(251)

generic variable dimensions

DIMENSION V(251)
DIMENSION X(50),Y(50),THET(50)

control variable dimensions and def s

DIMENSION ARM(10),MAM(10),pwa(4)
DIMENSION CHIS(10),PWDEL(15)
INTEGER*4 ISEED

common /generic/ v,imat,
6 volts,tmel,ifintpi,pi,deltim,itmctn

cm m /temcal/ t0,t1,t,tp,xprim,yprim,nptcnt,sum,shtp,
6 itcnt,cosntn

cm m /contr1/ olthsc,olthmc,crsc,drmc,tcrit,
6 tim800,crftap,crfta2,nd,pl1,
6 gqi,gqp,gq3,gv1,gvp,gvd,
6 pwdesi,crd,pwerol,iseed,iranin,arm,mam,alam,icon,coold,
6 crrol,crrolol,icoco,crRol,disko,xit,pwa

IMVEFL = 0

IF (THET(43) .GT. TCRIT) THEN
ICT = 46
CRFT = 0.
IMVEFL = -1
ELSE
IF (TCRIT .GT. THET(34)) THEN
ICT=33
CRFT = 0.
IMVEFL = 1
OLICFT = 33
ELSE

* THE FOLLOWING COMPUTES COOLING RATE AT A FIXED TE P
* TCRIT, BY SEARCHING FOR TCRIT AND THEN GETTING
* SLOPE THERE, ETC PTS. 27-46
* use crfta6 instead of this one, it better

DO 777 ICTN=33,43
   IF (((THET(ICTN).GE.TCRIT).AND.(THET(ICTN+1).LT.TCRIT)))THEN
      ICT=ICTN
      XCTR=X(ICTN)-((THET(ICTN)-TCRIT)/(THET(ICTN)-
6 THET(ICTN-1)))*(X(ICTN)-X(ICTN+1))
   ENDIF
777 CONTINUE

TNSIT = THET(ICT+1)-(THET(ICT+1)-THET(ICT+2))*
(X(ICT)-XCTR)/(X(ICT)-X(ICT+1))

SLOPE = (TCRIT - TNSIT)/(X(ICT)-X(ICT+1))

find the temp at the place having temp. equal to tcrit
last time so d(theta)/dw can be found

OLTHFT=OLTHFT+(V(IFINTM)*DELTIM)

do 757 icnt = 33,43
   if ((x(icnt).ge.OLTHFT).and.(x(icnt+1).lt.OLTHFT)) then
      ictl = icnt
      endif
      continue

TLTIT = THET(OLICFT) -
       (THET(OLICFT)-THET(OLICFT+1))*(X(OLICFT)-OLTHFT)

TLTIT = THET(ictl) -
       (THET(ictl)-THET(ictl+1))*(X(ictl)-OLTHFT)

CRFT = SLOPE*V(IFINTM)+(TCRIT-TLTIT)/DELTIM

adjust tholhtf back to the previous iteration value and
use it to find another approximation of cooling rate
which will be delayed by one iteration, but otherwise
an accurate backward difference calculation of the
cooling rate at TCRIT at a fixed point = CRFTA6

OLTHFT = OLTHFT - (V(IFINTM)*DELTIM)

do 758 icnt = 33,43
   if ((x(icnt).ge.OLTHFT).and.(x(icnt+1).lt.OLTHFT)) then
      ictl = icnt
      endif
      continue

TLTIT = THET(ictl) -
       (THET(ictl)-THET(ictl+1))*(X(ictl)-OLTHFT)

CRFTA6 = (TCRIT - TLTIT)/DELTIM

set up variables for the next time thru, move the array
of points used to calculate cooling rate if necessary

OLTHFT = XCTR
OLICFT = ICT
IF (ICT .LT. 36) THEN
  IMVEPL = +1
ENDIF
IF (ICT .GT. 39) THEN
  IMVEPL = -1
ENDIF
ENDIF

ENDF

**************************************************************************

CALCULATE THE POOL WIDTH ON EACH OF 4 SCAN LINES
POOL WIDTH IS THE MAX OF THESE FOUR VALUES PTS. 1-16
**************************************************************************

DO 11 J=1,4

DO 11 I=1,4

246
IF (THET((J-1)*4+1) .LT. TMEL) THEN
    PWA(J) = .2
ENDIF
IF (THET(J*4) .GT. TMEL) THEN
    PWA(J) = .5
ENDIF

K=(J-1)*4+1

IF ((THET(K) .GT. TMEL) .AND. (THET(K+1) .LE. TMEL)) THEN
    PWA(J) = (Y(K+1)-(TMEL-THET(K+1))/((THET(K)-THET(K+1)))
        *(Y(K+1)-Y(K)))
ENDIF
CONTINUE

FIND THE MAX OF THE FOUR VALUES BY BUBBLE SORT

PW=0.
DO 12 I=1,4
    IF (PWA(I) .GT. PW) THEN
        PW=PWA(I)
    ENDIF
12 CONTINUE
PW = 2.*PW

DELAY THE POOL WIDTH BY # OF DELAY CYCLES (ND)

DO 13 I=15,2,-1
    PWDEL(I)=PWDEL(I-1)
    CONTINUE
13 PWDEL(1)=PW
PW = PWDEL(ND+1)

*****************************************************************************
*                      CALCULATE THE POOL LENGTH IN A MANNER SIMILAR          *
*                      TO POOL WIDTH                                      *
*****************************************************************************

FIRST, FIND THE DISTANCE FROM THE SOURCE CENTER TO THE BACK
OF THE POOL

IF (THET(17) .LT. TMEL) THEN
    PL = X(17)
ELSE
    IF (THET(26) .GE. TMEL) THEN
        PL = X(26)
    ELSE
        DO 19 J=1,25
            IF (((THET(J) .GE. TMEL) .AND. (THET(J+1) .LT. TMEL)) THEN
                PL = X(J)-((THET(J)-TMEL)/((THET(J+1)-THET(J)))
            ENDIF
        CONTINUE
    ENDIF
ELSE
    IF (THET(32) .GE. TMEL) THEN
        PL = X(32)
    ELSE
        DO 19 J=1,25
            IF (((THET(J) .GE. TMEL) .AND. (THET(J+1) .LT. TMEL)) THEN
                PL = X(J)-((THET(J)-TMEL)/((THET(J+1)-THET(J)))
            ENDIF
        CONTINUE
    ENDIF
ENDIF

NOW FIND THE DISTANCE FROM THE SOURCE CENTER TO THE FRONT
OF THE POOL

IF (THET(27) .LT. TMEL) THEN
    PL1 = X(27)
ELSE
    IF (THET(32) .GE. TMEL) THEN
        PL1 = X(32)
    ELSE
        DO 19 J=1,25
            IF (((THET(J) .GE. TMEL) .AND. (THET(J+1) .LT. TMEL)) THEN
                PL1 = X(J)-((THET(J)-TMEL)/((THET(J+1)-THET(J)))
            ENDIF
        CONTINUE
    ENDIF
END IF
ELSE
DO 219 J=27,31
IF ((THT(J) .GE. TMEL).AND.(THT(J+1) .LT. TMEL)) THEN
PL1 = X(J) + ((THT(J) - TMEL)/(THT(J) - THT(J+1))
1 * (X(J+1) - X(J))
ENDIF
CONTINUE
ENDIF
ENDIF

ADD THE TWO DISTANCES TO GET THE TOTAL POOL LENGTH
ADD .31 FOR A FLUDGE SO THAT RESULTS MATCH WITH EXP
ERIMENTS

PL = PL1 - PL + .31

******************************************************************************
**
** APPROXIMATE COOLING RATE USING EMPERICAL
** FORMULA FROM MULT. LIN. REG.
** CR = CR(I,V,PW)
******************************************************************************

CRFTA3 = EXP(-17.284)*AMPS(IFINTM)**3.2822/\V(IFINTM)**1.541/PW**3.2069

******************************************************************************
**
** APPROXIMATE COOLING RATE USING EMPERICAL
** FORMULA FROM MULT. LIN. REG.
** CR = CR(I,PL,T0)
******************************************************************************

CRFTA4 = (EXP(-7.66))*(AMPS(IFINTM)**1.818)/(PL**1.761)*
((500 - PHTEM)**0.222)

******************************************************************************
**
** APPROXIMATE COOLING RATE USING EMPERICAL
** FORMULA FROM MULT. LIN. REG.
** CR = CR(I,PW,T0)
******************************************************************************

CRFTA5 = EXP(-5.295)*AMPS(IFINTM)**.882/PW**1.593*
(500 - 25)**0.383

******************************************************************************
**
** APPROXIMATE COOLING RATE USING EMPERICAL
** FORMULA FROM MULT. LIN. REG.
** CR = CR(PW,I,T0)
******************************************************************************

CRFTA7 = 0.443 / PW**1.014 * V(IFINTM)**.556 *
(500 - phtenom)**0.508

SHIFT ARRAY OF COOLING RATES USED FOR CONTROL BACKWARDS
TO MAKE ROOM FOR NEW ONE
DO 473 I=1,9
CRHIS(10-I+1) = CRHIS(10-I)
CRHIS(1) = CRFTA3
RETURN
END

248
SUBROUTINE CONCAL

THIS SUBROUTINE IS USED TO CALCULATE THE
ADJUSTMENTS TO CURRENT AND VOLTAGE SO
TO REGULATE THE(2) AND CR TO DESIRED VALUES

C23456789012345678901234567890123456789012345678901234567890
SUBROUTINE CONCAL(THET,X,Y,NUMPTS,PH TEM,QDOT
6,pw,olthf,crft,olificr,olwcc,p1,pwde1
6,crfta3,crfns,crfta4,crfta5,crfta6,
6 cappw,cond,eff,h,amps,phtemnom,cur,crfta7.ict)

C dimension AMPS(251)

generic variable dimensions

DIMENSION V(251)
DIMENSION X(50),Y(50),THET(50)

control variable dimensions and defs

DIMENSION ARM(10),MAM(10),pwa(4)
DIMENSION CRHIS(10),PWDEL(15)
INTEGER*4 ISEED

C common /genetic/ v,imat,
6 volts,teml,iifinm,p1,deltim,itmcnt

common /control/ olthsc,olthmc,crsc,drmc,tcrit,
6 tim800,crttap,crfta2,nd,p11,
6 ggi,qQP,gqq,gvi,gvp,gvd,
6 pwdesi,crd,perol,iased,iramin,erm,mam,aiam,icon,coold,
6 creroll,crerolol,icoco,crol,idiiko,xit,pwa,

IF(IIFINM*DELTIM .LT. 8) THEN
  IF (.NOT. ICON .EQ. 0 .OR. (ICON .EQ. 4)) THEN
    off = ((exp(7.0897)/cur**1.5584*(500-ph temnom)**.72507)
6 /crd)**(1/1.5246)
    V(IIFINM+1) = 1./off
  ELSE
    V(IIFINM+1) = V(I)
ENDIF
AMPS(IIFINM+1) = AMPS(I)

SET UP SOME STUFF FOR COMPENSATOR
CREROLOL = 0
CREROL1 = 0
COOLD = 0.
CROL = CRFTA6
ENDIF

DO CONTROL IF TIME IS MORE THAN 50
C

IF (IIFINM*DELTIM .GE. 8) THEN
  IF (IRAMIN .EQ. 0) THEN
    IF(ICON .EQ. 0) THEN
      *************************KLUDGE***************
pweit = pwdez - pw
CRERR = CRD - CRFTA6
PWERSH = PWERSH + PWERR

249
CRESM = CRERSM + CRERR
PDWR = - (PWERR - PWEROL)/DELTIM
CADER = -(CRERR - CREROL)/DELTIM
DI = GQ1*PWERSM/2 + GQF*PWERR/2 + GQD*PDWDER/2
DV = GVI*CRESM + GVF*CRERR + GVD*CADER
AMPS(IFINTM+1) = AMPS(1) + DI
QDOT = ETA(PHTEM,amps)*AMPS(IFINTM+1)*VOLTS*EFF
V(IFINTM+1) = V(1) + DV
PWEROL = PWERR
CREROL = CRERR

ELSEIF(ICON .EQ. 1) THEN

DO WEIGHTED ONE-STEP CONTROL AS IN GOODMAN AND SIN
"ADAPTIVE FILTERING, PREDICTION AND CONTROL", PAGE 122

V(IFINTM+1)=(-1/./.785)*(CRD-1.5529+CRTFTA4+.5253*CROL)
AMPS(IFINTM+1)=AMPS(1)
I = AMPS(IFINTM+1)
QDOT = I*VOLTS*EFF*ETA(PHTEM,amps)
CROL=CRFTA4

DO STEP INPUT IF ICON .EQ. 2.

ELSEIF (ICON .EQ. 2) THEN

IF (IFINTM .LT. 30.) THEN
AMPS(IFINTM+1) = AMPS(1)
V(IFINTM+1)=V(1)
ELSEIF ((IFINTM .GE. 30.) .AND. (IFINTM .LT. 70)) THEN
AMPS(IFINTM+1) = AMPS(1)
V(IFINTM+1)=V(1)+2.54/60.
ENDIF

IF (IFINTM .GE. 70.) THEN
AMPS(IFINTM+1) = AMPS(1)
V(IFINTM+1) = V(1)
ENDIF
I = AMPS(IFINTM+1)
QDOT = I*VOLTS*EFF*ETA(PHTEM,amps)

ELSEIF (ICON .EQ. 3) THEN

IF (ICOCO .EQ. 1) THEN
ICOCO = 0
CRERR = CRD - CRFTA4
C
CO = (-GVP)*(CRERR-2.0*CREROL1+.9975*CREROLOL)+COOLD
V(IFINTM+1)=V(1)+CO
AMPS(IFINTM+1) = AMPS(1)
I = AMPS(IFINTM+1)
QDOT = ETA(PHTEM,amps)*AMPS(IFINTM+1)*VOLTS*EFF
COOLD = CO
CREROLOL = CREROL1
CREROL1 = CRERR
ENDIF

ELSEIF (ICON .EQ. 4) THEN

if icon is 4, do gain switched control

IF ((CRFTA6 .LT. 1) .OR. (CRFTA6 .GT. 20)) THEN
-offset = ((exp(7.0893)/cur**.5584*(500-phtem)**.2507)
/cur)**(1/.5246)

250
off = (crd/1.2683e-8*(cur-65)**.2512/(1530-phtem)**3.1071
)**(1/3.3357)-.4
off = 1./off
if (icr .eq. 6) then
  v(ifintm+1) = 1./off
else
  v(ifintm+1) = v(1)
  off = 1./off
endif
AMPS(IFINTM + 1) = AMPS(IFINTM)
ELSE
  off = ((exp(7.0893)/CUR**1.5584*(500-phtem)**.72507)
  /crd)**(1./1.5246)
  off = (crd/1.2683e-8*(cur-65)**.2512/(1530-phtem)**3.1071
)**(1/3.3357)-.4
ENDIF

write(6,*)'off',off
if (off .lt. 4.*2.54/60.) then
  if (amps(1) .le. 100) then
    z1=.97
    z2=.68
    zl = .9159
    g=.01
    za7 = .874
    ga7 = .04
    za5 = .839
    ga5 = .06
    else
    z1=.98
    z2=.65
    zl = .9204
    g=.08
    za7 = .894
    ga7 = .04
    za5 = .863
    ga5 = .08
    endif
else
  z1=.98
  z2=.65
  zl = .9426
  g=.02
  za7 = .902
  ga7 = .05
  za5 = .876
  ga5 = .09
endif
else
  if (amps(1) .le. 100) then
    z1=.96
    z2=.89
    x1 = .93
    g=.01
    za7 = .472
    ga7 = .03
    za5 = .474
    ga5 = .03
  else
    if (phtem .le. 100) then
      z1=.91
      z2=.78
      zl = .9132
      g=.002
      za7 = .537
    else
      ...
ENDIF

no control

elseif (icon .eq. 5) then

V(IFINTM+1)=V(1)
AMPS(IFINTM+1) = AMPS(1)
I = AMPS(IFINTM+1)
QDOT = ETA(AMPS(IFINTM+1),V(IFINTM+1),VOLTS,EFF,PHTEM,TMEL)*
6
AMPS(IFINTM+1)*VOLTS*EFF
ENDIF
ENDIF

DO RANDOM INPUT IF IT IS DESIRED

IF (IRANIN .NE. 0) THEN

DV = (RAN(ISEED)-.5)*2.*1.*2.54/60
I = AMPS(I)
QDOT = I*VOLTS*EFF*ETA(PHTEM,amps)
V(IFINTM+1) = V(1)+DV
ENDIF
ENDIF

RETURN
END

**************************************************************************************************************************

SUBROUTINE OUTDIS

THIS SUBROUTINE OUTPUTS THE TEMPERATURE
THET(1) AND THE COOLING RATE AS WELL AS
TIME TO THE CRT

**************************************************************************************************************************

SUBROUTINE OUTDIS(THET,X,Y,NUMPTS,PHTEM,QDOT)
6
,pw,olthsc,olthmc,crsc,crmc,pl,pwdel
6
crfa3,cchis,crfa4,crfa5,crfa6,
6
cappa,cond,eff,h,amps,crfa7)

dimension AMPS(251)

generic variable dimensions

DIMENSION V(251)
DIMENSION X(50),Y(50),THET(50)

ccontrol variable dimensions and def

DIMENSION ARM(10),MAN(10),pwa(4)
DIMENSION CRFIS(10),PWDEL(15)
INTEGER*4 ISEED

c
common /generic/ v,imat,
6
volts,tmel,ifinmtm,pl,deltim,itmcent

common /control/ olthsc,olthmc,crsc,crmc,tcrit,
6
tim800,crfata2,nd,p11,
6
sqi,qpp,qpd,qvi,gvp,vd,
6
pwe,cri,cmd,percol,iseed,iranin,arm,mam,alam,icon,cool,
6
creroll,crnroll,icoro,corol,disko,xit,pwa
WRITE TO CRT AND TO DISK IF RANDOM INPUT

WRITE(6,*) itmcnt, * DELTIM, v(ifintm), pw, crfta6, crfta7

IF ((IDISKO.EQ.1).AND.(IRANIN.EQ.0)) THEN
  WRITE(1,*) itmcnt, * DELTIM, v(ifintm), pw, crfta6
ENDIF

IF ((IRANIN.NE.0).AND.(IFINTM.GE.50)) THEN
  WRITE(1,*) V(IFINTM), PL, CRFTA4, CRFT
ENDIF

RETURN
END

******************************************************************************

*  
* SUBROUTINE MOVCOO
*  
* DAVE FARSON 6-86
*  
* THIS SUBROUTINE MOVES THE POINTS USED TO CALCULATE THE FIXED TEMP COOLING RATE SO THAT THE TCRIT REMAINS WITHIN THEM. IT ALSO ADJUSTS THE VALUE OLFCT SO THAT IT IS CORRECT, BUT ONLY IF IOBFL IS SET TO 1, INDICATING THE OBSERVER COORDINATE OF MOVE.

******************************************************************************

SUBROUTINE MOVCOO(THET, X, Y, NUMPTS, PHTEN
  pw, olthft, crft, olicft, olwtc, pl, pwdel
  crfta3, crhisi, crfta4, crfta5, crfta6, iobfl, imvefl)

  generic variable dimensions

  DIMENSION V(251)
  DIMENSION X(50), Y(50), THET(50)

  control variable dimensions and defs

  DIMENSION ARM(10), MAM(10), pwa(4)
  DIMENSION CRHIS(10), PWDEL(15)
  INTEGER*4 ISEED

  common /generic/ v, imat,
  volts, tmel, ifintm, pi, deltim, itmcnt

  common /ctrl/ olthsc, olthnc, crsc, drmc, tcrit,
  tim800, crftap, crfta2, nd, pli,
  gqi, gqp, gqd, gvi, gvp, gvd,
  pwde5, crd, pwerol, iseed, iranin, arm, mam, alam, icon, coold,
  crerol, crerolol, icoco, crol, idisko, xit, pwa

IF (IMVEFL.EQ.0) THEN
  RETURN
ENDIF

IF (IMVEFL.EQ.-1) THEN
  DO 400 I=33, 46
    X(I) = X(I)-4*XIT
  CONTINUE
  IF (IOWFL.EQ.1) THEN
    OLICFT = OLICFT - 4
  ENDIF
400  CONTINUE
ENDIF
IF (IIMVEFL .EQ. 1) THEN
  DO 405 I=33,46
    X(I) = X(I) + 4*XIT
    CONTINUE
  if (iofbfl .eq. 1) then
    OLICFT = OLICFT + 4
  endif
ENDIF
RETURN
END

**********************************************************************
* FUNCTION ETA
* TO CALCULATE THE EFFICEINCY OBTAINED BY COMPARING
* SIMULATIONS TO EXPERIMENT
* this variable efficiency function is disabled.
* it now just returns a value of 1.0

FUNCTION ETA(PHTEM,amps)
  dimension AMPS(251)
  generic variable dimensions
  DIMENSION V(251)
  DIMENSION X(50),Y(50),THET(50)
  common /generic/ v,imat,
  6 volts,tme1,ifintm,pi,deltm,itmct
  ETA = (EXP(-3.7553)/EXP(-3.2058))*V(ifintm)**.294*
  6 (TMTL-PHTEM)**.285/AMPS(ifintm)**.21

************ DISABLE VARIABLE EFFICIENCY ************
ETA = 1.
RETURN
END

**********************************************************************
* subroutine shift(vec,i)
* this subroutine will shift all elements down
* one slot. vec(1) is lost

subroutine shift(vec,i)
  dimension vec(251)
  do 10 j = 1,i
    vec(j) = vec(j+1)
  continue
return
end

**********************************************************************
* subroutine findsh
* this subroutine finds the value of sht appropriate for
* the values of amps and travel speed input to it.
* ***************************************************

subroutine ndsht(amps,v,sht,cond,h,shtc,eff,volts)

gdot = eff*amps*volts

if (gdot .gt. 616) then
   if (v .lt. 0.1693333) then
      tem1 = shtc + (0.1693333 - v)*(0.0016-shtc)/0.0423333
      tem2 = shtc + (gdot - 616.)*(0.0016-shtc)/61.6
      if (tem1 .ge. tem2) then
         shtc.tem = tem1
      else
         shtc.tem = tem2
      endif
   else
      shtc.tem = shtc + (gdot - 616.)*(0.0016-shtc)/61.6
   endif
else
   if (v .lt. 0.1693333) then
      shtc.tem = shtc + (0.1693333 - v)*(0.0016-shtc)/0.0423333
   else
      shtc.tem = shtc
   endif
endif
SHT = 2.*(SHTC.tem)/COND/H

return
end
APPENDIX C

ARMA Model-fitting Program

This appendix contains Fortran programs which fit an ARMA model of variable size to a given set of input/output data and then generate ARMA model outputs which may compared to the original output data. The model-fitting function is performed by the program DARMFIT. The program which uses the ARMA model along with the input data to generate output data is called DARSIM. Also shown is MATLAB, a module which contains a number of useful vector/matrix algebra routines.
**-----------------------------**
** DARMFIT.FOR **
** EARLY MAY, 1986 D.F.F. **
** THIS PROGRAM IS INTENDED TO FIT A DARMA **
** MODEL TO A SET OF I/O DATA READ FROM DISK **
** FILE FOR001.DAT. THE DATA IS ASSUMED TO BE **
** IN THE FORM: INPUT(K) JUNK OUTPUT(K+1) JUNK **
** THE MODEL IS FIT USING PSEUDO INVERSE ASSUMING **
** RELATION Y1 = P*TH6 **
** Y1(K) = -A1*Y(K-1)-A2*Y(K-2)...-AN*Y(K-N)+Q**(-IN6)**
** (B0*U(K)+B1*U(K-1)+...-BN1*U(K-N)) **
**-----------------------------**

******** DEFINE ARRAYS, VARIABLES ********

REAL*8 U(200), Y(200), YL(200), P(200, 200), TH6(200)
REAL*8 A(200, 200), FT(200, 200), YERR(200), N(200, 200)
REAL*8 Z(200)

******** INPUT NEEDED PARAMETERS FROM USER ********

WRITE(6,*) 'INPUT NUMBER OF ROWS IN PHI MATRIX (78)', K
READ(5,*) K

WRITE(6,*) 'INPUT DELAY FROM INPUT TO OUTPUT (1)', ND
READ(5,*) ND

WRITE(6,*) 'INPUT NUMBER OF POLES-1 (2-10)', N
READ(5,*) N

WRITE(6,*) 'INPUT NUMBER OF ZEROS+1 (2-10)', N1
READ(5,*) N1

****** GET READY TO BUILD THE PHI MATRIX ******

IF ((N+ND) .GT. N+1) THEN
IT = N+ND+1
ELSE
IT = N+2
ENDIF

IT = IIMAX(N+ND, N+1)+1

****** OPEN FOR001.DAT, FOR002.DAT ************

OPEN(1)
OPEN(2)
OPEN(8)

****** BUILD THE PHI MATRIX ********

DO 11 I=1,100
  READ(1,*) JUNK, JUNK1, JUNK2, JUNK3

DO 10 I=1, K+IT+1

****** GET I/O DATA FROM FOR001.DAT ********

  READ(1,*) JUNK, U(I), JUNK, Y(I)
  WRITE(2,*) U(I), Y(I)

  ASSUMED 2 JUNK VALUES

10 CONTINUE

258
average data?
write(6,*) 'average data? (0=no,1=yes-avev,2=yes-endpts'
read(5,*) iadat

if (iadat .eq. 1) then
  AVERAGE U AND Y, PUT BACK INTO U AND Y
  CALL AVEV(U,Y,AVU,K+it+1)
  CALL AVEV(Y,Y,AVY,K+it+1)
  write(6,*) 'AVEU,AVEY',AVU,AVY
elseif (iadat .eq. 2) then
  tem = (y(i)+y(it+k+1))/2.
  teml = (u(i)+u(it+k+1))/2.
  write(6,*) tem,teml
  do 39 i=1,k+it+1
       u(i) = u(i) - teml
       y(i) = y(i) - tem
     continue
39 endif

do 20 i=1,k
  do 30 j=1,n
    P(i,j)=y(it-j)
  do 40 j=1,n1
    P(i,n+j)=u(it-nd-j+2)
    y1(i)=y(it)
    it=it+1
20 continue

********* DO THE PSUEDO INVERSE TO FIND PARAMETER MATRIX *****

CALL TRPM(P,PT,K,N+N1)
CALL MULNM(PT,P,A,N+N1,K,N+N1)
CALL MINV(A,N+N1)
CALL MULNM(A,PT,B,N+N1,N+N1,K)
CALL MULNV(B,Y1,TH0,N+N1,K)

write(2,*) n,n1,nd,(th0(i),i=1,n+n1)
write(6,*) n,n1,nd,(th0(i),i=1,n+n1)

********* CALL SUBROUTINE TO SIMULATE SYSTEM,COMPARE OUTPUT TO KNOWN OUTPUT FROM PHI MATRIX ***********

ifin = 100
it1 = 60

see if want to input a new th0
write(6,*) 'input new th0 matrix? 1=yes, 0=no'
read(5,*) ijunk
if (ijunk .eq. 0) goto 333
write(6,*) 'n=',n,'n1=',n1,'nd=',nd
write(6,*) 'input a(1),...,a(n),b(0),...,b(n1-1)'
  do 334 i=1,n+n1
    334 read(5,*) ajunk
    th0(i)=ajunk

333 call darsim (k,n,n1,nd,p,th0,y1,yerr,var)
DO 300 I=1,100+1

**********CLOSE FOR0D1.DAT **********

CLOSE(1)
CLOSE(2)
CLOSE(8)

STOP
END
**SUBROUTINE DARSIM.FOR**

**THIS PROGRAM IS DESIGNED TO SIMULATE**

**A DARMA MODEL WHICH IS PASSED TO THE**

**THE ROUTINE IN THE TH0 MATRIX**

**THE INITIAL CONDITIONS FOR THE SIMULATION**

**ARE PULLED FROM THE PHI (OR P) MATRIX**

**WRITTEN early may, 1986 D.F.P.**

**P** : PHI MATRIX, PASSED

**K** : NUM OF ROWS IN P,Y

**TH0** : PARAMETER MATRIX, PASSED

**N** : NUMBER OF POLES IN TH0, PASSED

**N1** : NUMBER OF ZEROS IN TH0, PASSED

**Y1** : OUTPUT MATRIX, PASSED

**YERR** : DIFFERENCE BETWEEN Y AND SIMULATED

**OUTPUT, RETURNED**

**SUBROUTINE DARSIM(K,N,N1,ND,P,TH0,Y1,YERR,VAR)**

REAL*8 P(200,200),TH0(200),Y1(200),YERR(200)

REAL*8 A(200),PHI(200),B(200)

OPEN(4)
OPEN(9)

WRITE(6,*) 'REPLY OLD DATA? (0=NO, 1=YES)'
READ(5,*) L

IF THE USER WANTS TO REPLAY THE OLD DATA, THEN THE OUTPUTS
AND INPUTS CONTAINED IN THE P MATRIX, PASSED FROM THE CALL-
NING ROUTINE ARE USED WITH THE TH0 MATRIX TO CALCULATE THE OUTPUT
IF THE OLD DATA IS NOT TO BE REPLAYED, THEN THE VALUES
IN THE PHI MATRIX ARE USED ONLY TO GET THE SIMULATION
STARTED (AS I.C.'S) AND THE OUTPUTS CALCULATED IN PAST ITERATIONS
ARE USED AS SIMULATION PROGRESSES. THE LATTER METHOD
IS A MORE STRINGENT TEST OF THE CURVE FIT VALIDITY.

IF (L.EQ.0) THEN

HERE, USE THE VALUES OF PAST ITERATIONS TO DETERMINE NEXT OUTPUT

N2 = N+N1

GET IC'S FROM P MATRIX, PASSED FROM CALLER

DO 10 I=1,N2
   PHI(I) = P(I,1)
10

DO 20 I=1,K

CALCULATE NEXT OUTPUT

CALL MULV(PHI,TH0,B(I),N+N1)

INCLUDE NEW OUTPUT IN PHI FOR NEXT ITERATION, GET NEXT INPUT
FROM P MATRIX

DO 30 J=N+1,N+N1
   PHI(J) = P(I+1,J)
30

DO 40 J=N,2,-1
   PHI(J) = PHI(J-1)
40

PHI(1) = - B(1)

261
CALCULATE ERROR AT THIS ITERATION

YERR(I) = Y1(I) - B(I)
CONTINUE

ELSE

HERE. SIMPLY REPLAY OLD OUTPUTS AND INPUTS FROM THE
P MATRIX PASSED BY THE CALLER

CALL MULMV(P,TH0,A,E,N+N1)
CALL MULVS(A,-1.,B,K)
CALL ADDV(Y1,E,YERR,K)
CALL MULVS(B,-1.,B,K)

ENDIF

WRITE 'TRUE' AND SIMULATED OUTPUTS TO FILE FOR PLOTTING

DO 50 I=1,K
WRITE(4,*) I,Y1(I),B(I)
WRITE(4,*) N,N1,ND,(TH0(I),I=1,N+N1)
WRITE(4,*) N,N1,ND,(TH0(I),I=1,N+N1)

GET THE STATISTICS OF THE ERROR

MAKE A VECTOR OF 1'S FOR DOING ERROR AVERAGE

DO 55 I=1,K
B(I)=1.

GET SUM OF ERRORS, DIVIDE BY K TO GET AVERAGE

CALL MULMV(YERR,B,AV,F)
AV = AV/K

GET SUM OF ERRORS SQUARED (SES), DIVIDE BY K ADD TO AV**2 TO GET VARIANCE

CALL MULMV(YERR,YERR,SES,K)
VAR = SES/K - AV**2
STDEV = SQRT(VAR)

WRITE(6,*) 'ERROR AVERAGE IS',AV
WRITE(6,*) 'SUM OF SQUARED ERRORS',SES
WRITE(6,*) 'ERROR VARIANCE IS',VAR
WRITE(6,*) 'ERROR STANDARD DEVIATION IS ',STDEV

WRITE(9,*) (YERR(I),I=1,K)

CLOSE(4)
CLOSE(9)

RETURN
END
**CALCULATE A inverse**

**THIS SUBROUTINE WILL DESTROY THE CONTENTS OF THE A MATRIX, SINCE THE INVERSE IS RETURNED THERE. HENCE, IT SHOULD BE CALLED WITH A COPY OF THE MATRIX TO BE INVERTED IN THE COMMON VARIABLE A.**

**THIS VARIABLE WILL THEN BECOME THE INVERSE IN THE CALLING ROUTINE AND HENCE SHOULD BE CALLED SOMETHING LIKE A1 IN THE COMMON STATEMENT IN THE CALLING MODULE**

"BORROWED" FROM JAKE GLOWER DEC. 1985

**SUBROUTINE minv(A,N)**

**REAL*8 A(200,200),B(200,200)**

**do 1 i=1,N**
**do 1 j=1,N**
**b(i,j)=0**
**do 2 i=1,N**
**b(1,1)=1**
**do 100 i=1,N**
**WRITE(6,*)(i)**
**if(a(i,1),ne.0) goto 20**
**k=0**
**do 10 j=n,i,-1**
**if(a(j,i),eq.0) goto 10**
**k=j**
**continue**

**if(k,ne.0) goto 12**
**write(6,*),********** FAIL IS SOLVE **********
**if(k.eq.0) goto 100**
**do 15 j=1,n**
**b(i,j)=b(i,j)+b(k,j)**
**a(i,j)=a(i,j)+a(k,j)**
**continue**

**x1=a(i,1)**
**do 30 j=1,n**
**b(i,j)=b(i,j)/x1**
**a(i,j)=a(i,j)/x1**
**do 100 j=i+1,n**
**x1=a(j,1)**
**do 90 k=1,n**
**b(j,k)=b(j,k)-x1*b(i,k)**
**a(j,k)=a(j,k)-x1*a(i,k)**
**continue**

**do 200 i=n,2,-1**
**x1=a(i,1)**
**if(x1.eq.0) goto 200**
**do 110 j=1,n**
**b(i,j)=b(i,j)/x1**
**a(i,j)=a(i,j)/x1**
**do 150 j=i-1,-1**
**x1=a(j,1)**
**do 140 k=1,n**
**b(j,k)=b(j,k)-x1*b(i,k)**

263
A(J,K) = A(J,K) - X1*A(J,K)

CONTINUE

WRITE(6,*), I

CONTINUE

DO 201 I=1,N
DO 201 J=1,N

A(I,J) = B(I,J)

RETURN

END

********************************************************************************

*                         ADD MATRIX SUBROUTINE ADDM(A,B,C,M,N)
*                         A,B : INPUT MATRICES
*                         C : OUTPUT MATRIX
*                         M,N : ROW,COL SIZE OF A,B

********************************************************************************

SUBROUTINE ADDM(A,B,C,M,N)

REAL*8 A(200,200),B(200,200),C(200,200)

DO 10 I=1,M
  DO 10 J=1,N
    C(I,J) = A(I,J) + B(I,J)

RETURN

END

********************************************************************************

*                         MUL MATRIX BY MATRIX
*                         SUBROUTINE MULMM(A,B,C,M1,N1,N2)
*                         A : INPUT MATRIX SIZE OF M1,N1 ROW X COL
*                         B : INPUT MATRIX SIZE OF N1,N2 ROW X COL
*                         C : OUTPUT MATRIX SIZE OF M1 X N2

********************************************************************************

SUBROUTINE MULMM(A,B,C,M1,N1,N2)

REAL*8 A(200,200),B(200,200),C(200,200)

DO 10 I=1,M1
  DO 10 J=1,N2
    C(I,J) = 0.
  DO 10 K=1,N1
    C(I,J) = A(I,K) * B(K,J) + C(I,J)

RETURN

END

********************************************************************************

*                         MULTPLY MATRIX BY SCALAR
*                         SUBROUTINE MULMS(A,B,C,M,N)
*                         A : INPUT MATRIX SIZE IS M X N ROW BY COL
*                         B : SCALAR REAL
*                         C : OUTPUT MATRIX; MAY BE A

********************************************************************************

SUBROUTINE MULMS(A,B,C,M,N)

REAL*8 A(200,200),C(200,200)

DO 10 I=1,M
DO 10 J=1,N
  C(I,J)=A(I,J)*B
RETURN
END

***********************************************
* TRANSFER MATRIX SUBROUTINE
* SUBROUTINE TRAM(A,B,N,M)
* A : INPUT MATRIX
* B : OUTPUT MATRIX
***********************************************
SUBROUTINE TRAM(A,B,M,N)
REAL*8 A(200,200),B(200,200)
DO 10 I=1,M
  DO 10 J=1,N
    B(I,J)=A(I,J)
10 RETURN
END

***********************************************
* ADD VECTOR SUBROUTINE ADDV(A,B,C,M)
* A,B : INPUT MATRICES
* C : OUTPUT MATRIX
* M,N : ROW OF A,B,C
***********************************************
SUBROUTINE ADDV(A,B,C,M)
REAL*8 A(200),B(200),C(200)
DO 10 I=1,M
  C(I)=A(I)+B(I)
10 RETURN
END

***********************************************
* MUL MATRIX BY VECTOR
* SUBROUTINE MULMV(A,B,C,M1,N1)
* A : INPUT MATRIX SIZE OF M1,M1 ROW X COL
* B : INPUT VECTOR SIZE OF N1,1 ROW X COL
* C : OUTPUT VECTOR SIZE OF M1 X 1
***********************************************
SUBROUTINE MULMV(A,B,C,M1,N1)
REAL*8 A(200,200),B(200),C(200)
DO 10 I=1,M1
  C(I)=0.
  DO 10 K=1,N1
    C(I)=A(I,K)*B(K)+C(I)
10 RETURN
END
**MULTIPLY VECTOR BY SCALAR**

SUBROUTINE MULVS(A,B,C,M)
* A : INPUT VECTOR SIZE IS M X 1 ROW BY COL
* B : SCALAR, REAL
* C : OUTPUT MATRIX; MAY BE A
**********************************************************

SUBROUTINE MULVS(A,B,C,M)
C
REAL*8 A(200),C(200)
C
DO 10 J=1,M
  C(I)=A(I)*B
10 RETURN
END

**TRANSFER VECTOR SUBROUTINE**

SUBROUTINE TRAV(A,B,M)
* A : INPUT VECTOR
* B : OUTPUT VECTOR
**********************************************************

SUBROUTINE TRAV(A,B,M)
C
REAL*8 A(200),B(200)
C
DO 10 I=1,M
  B(I)=A(I)
10 RETURN
END

**TRANSPOSE MATRIX SUBROUTINE**

SUBROUTINE TRPM(A,B,M,N)
* A:INPUT MATRIX, SIZE M BY N
* B:OUTPUT MATRIX, SIZE N BY M
**********************************************************

SUBROUTINE TRPM(A,B,M,N)
REAL*8 A(200,200),B(200,200)
C
DO 10 I=1,M
  DO 10 J=1,N
    B(J,I)=A(I,J)
10 RETURN
END

**MULTIPLY VECTOR,VECTOR SUBROUTINE**

SUBROUTINE MULV(A,B,C,N)
* A : INPUT VECTOR
* B : INPUT VECTOR
* C : INNER PRODUCT OF A,B
* N : SIZE OF A,B
**********************************************************

SUBROUTINE MULV(A,B,C,N)
C
REAL*8 A(200),B(200),C(200)
C
DO 10 I=1,N
  C(I)=A(I)*B(I)
10 RETURN
END
SUBROUTINE MULV(A,B,C,N)
REAL*8 A(200),B(200),C
C
C = 0.
DO 10 I = 1,N
   C = C + A(I) * B(I)
10 RETURN
END

*******************************************************************************

* AVEV(A,B,C,N)
* A: INPUT VECTOR(N)
* B: OUTPUT VECTOR(N) = A - [AVE(A)]
* C: OUTPUT SCALAR = SUM(AI)/N
* N: INPUT SIZE OF VECTORS
*******************************************************************************

SUBROUTINE AVEV(A,B,C,N)
REAL*8 A(200),B(200),D(200)
C
DO 10 I = 1,N
   D(I) = 1.
10 CALL MULV(A,D,C,N)
C
C = C/N
C
DO 20 I = 1,N
   D(I) = -C
20 CALL ADDV(A,D,B,N)
C
RETURN
END
References


269


270