PERMANENT MAGNET SYNCHRONOUS MACHINE BASED TRACTION DRIVE DESIGN FOR HYBRID SCOOTER CONSIDERING CONTROL NONLINEARITIES AND COMPENSTIONS

DISSERTATION

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ABSTRACT

For the past several decades, the development of modern power electronics devices and motor control technologies makes Variable Speed Drive (VSD) popular in various application areas, such as on-road vehicles, home appliances as well as renewable energy field. Permanent Magnet Synchronous Machines (PMSM), one of the most promising motor technologies for its high power density, wide speed operation range and fast torque-speed response, has attracted great interest among various VSD applications. In this work, a PMSM based Integrated Start Generator (ISG) system is developed for a 50cc gasoline scooter to achieve higher efficiency and better road performance while still reasonably priced.

The classical control theory based on root locus analysis and frequency response analysis is used to study a linear Single Input and Single Output (SISO) system. However, the PM machine as well as its drive inverter is a highly nonlinear, tightly coupled, multi-variable dependent system, which the classical control theory cannot be applied to directly. This work focuses on the analysis and compensation of these nonideal terms with an emphasis mainly on two perspectives: 1) Current control loop modeling with transfer function and the effect of various coupling terms and their compensations; 2) Inverter dead time investigation, its effect on the voltage output and new dead-time compensation method.
The application of PM machine on an ISG scooter has a number of requirements and specifications of both the PM machine design and control algorithm implementation. One is the contradiction between mounting space limitation and high starting torque requirement. Another is the control challenge of speeding the motor to more than 10K rpm with only low resolution hall position sensors.

A cost effective 1.5 KW Interior Permanent Magnet (IPM) machine and its drive inverter are designed based on these requirements. This IPM machine, its matching inverter and a 2 Ah 48V lithium-ion battery are well packaged on a prototype 50cc four stroke ISG scooter. Experiments are carried out to test multi-modes of this ISG system including starting, acceleration, and regeneration.
DEDICATION

This document is dedicated to my parents and my wife.
VITA

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FIELDS OF STUDY

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CHAPTER 1

INTRODUCTION

1.1. Background Introduction

The electric motor began its history in the 19th century, accompanied by the discovery and wide usage of electricity. For centuries, the AC synchronous machine was designated as an electricity generator; the AC induction machine was for large power scale, fix speed drive application; and the DC machine was for VSD application. Though, the DC motor was born to be fit for VSD for its proportional relation between the speed and terminal voltage, the mechanical commutator is its innate defect, which cannot meet the safety requirement in a harsh environment. Other than the DC machine, the other two AC machines are plugged into the utility line without any control. This situation has maintained for hundreds of years without too much advancement. Until late 1950s, the breakthrough in the semiconductor technology triggered a new global technology revolution. The wide usage of high power electronics devices in motor drive has substantially enforced the development of modern motor control algorithm. The VSD is no longer exclusive for DC machine, but applicable to all AC machines with an even higher torque density and almost zero maintenance effort than DC machine. The Permanent Magnetic (PM) machine, with an unbeatable high efficiency, high power
density and high control precision, receives tremendous attention for various applications [5,6,7].

1.1.1. Industrial Application of PM Machines

Owing to its incredible high power density and high efficiency, PM machines became famous for its application on Hybrid Electric Vehicle (HEV) and Pure Electric Vehicle (PEV). Toyota first released its strategic hybrid car “Prius” in 1997, which kicked off the car revolution and soon became the icon and the synonym of a green car with its startling mileage per gallon index. This figure is far beyond the scope of any car on the market and can even compete with that of a motorcycle. Its two Interior Permanent Magnet (IPM) synchronous machines, one mainly for traction and one mainly for regeneration, collaborate with each other to either drive the vehicle or to absorb the brake energy or to balance the peak and niche of gasoline engine output. Though induction machine and switched-reluctance machine can do the same job, permanent magnet brushless machines offer a much higher efficiency and torque density [6].

Home appliance is another area that needs a large number of different types of motors. The efficiency and price are two of the major concerns for this application. The motors can account for up to 43 percent of the total electricity consumption in an average American home [7]. Figure 1.1 cited from [7] illustrates all the locations of a home appliance that requires motors as its power source. In order to maximize energy efficiency, the US appliance industry has increasingly adopted VSD technology. PM machine is one of the best candidates to fulfill this task. One successful case is the usage of PM machine in the direct driven washing machine, which couples with the washing
chamber directly without belt or chain connection. This saves up to 30 percent of the total energy consumption as compared to the traditional AC induction machine. Moreover, it reduces the noise, minimizes the mechanical friction and extends its life span.

Figure 1.1. Various motors in an average American home.

The above examples of the application of PM machines are mainly of commercial products. While, for industrial products, the control accuracy, ripple torque component as well as reciprocating movement bandwidth are the major concerns. Servo drive is the general nomenclature for this type of application. PM machine, control based on accurate rotor positioning, can meet this most critical dynamic performance expectation.

In all, PM motor drive is becoming the state-of-the-art technology and shows its flexibility in all kinds of applications. It has established its predominance over other types
of machine drives. In recent years, the PM drive is still expanding its application area toward both high power areas such as heavy duty truck, steel rolling mill; and low power areas such as electric bicycle and various actuators.

The IPM machine, one category of PM machines based on geometric classification, is superior to other types of PM machines because it combines both the characteristics of the Surface-mount PM Machine (SPM) and the Synchronous Reluctance Machine (SynRM). Its performance is enhanced by both the PM torque at low speed and reluctance torque at high speed flux-weakening region [8]. The work in this thesis concentrates on the development based on this machine structure.

1.1.2. PM Machine Development

A. Permanent Magnet Materials

The permanent magnet motor develops with the discovery of new permanent magnet materials. One of the most important specifications to measure the quality of PM materials is its maximum achievable energy product. The larger this parameter is, the less PM material needed for the same magnet force.

Alnico magnets, which are aluminum, nickel and iron composite, were the first largely used PM material in history. It has high residual flux density, excellent temperature stability and a strong corrosion resistance level. However, it can easily be demagnetized and its maximum energy product is not very high.

Ferrite magnets are another significant PM material discovery. Since it doesn’t contain these noble metal elements, it is one of the cheapest PM materials ever found. It has relatively high intrinsic coercive force, which shows higher resistant demagnetization.
capability. Its lower mass density reduces the weight of its product. Its main drawback is the limited residual flux density and thus its lower maximum energy density.

Rear earth magnets are one of the most significant findings in last century, which finally lead to the wide application of PM motor. It has high coercive force as well as large residual flux density. There are two types of rear earth magnets: one uses high cost Samarium and Cobalt (SmCo) material. Another is NdFeB, though with lower limited working temperature, but with even higher residual flux density and energy density. Table 1.1 summarizes all the specifications for different PM materials addressed above.

Table 1.1. Typical permanent magnet material properties.

<table>
<thead>
<tr>
<th>Materials</th>
<th>Br [T]</th>
<th>Hc [kA/m]</th>
<th>BHmax [kJ/m³]</th>
<th>Tc [°C]</th>
<th>Tw-max [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alnico</td>
<td>1.2</td>
<td>10</td>
<td>6</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Ferrite</td>
<td>0.43</td>
<td>10</td>
<td>5</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>SmCo</td>
<td>Up to 1.1</td>
<td>Up to 820</td>
<td>Up to 240</td>
<td>Up to 820</td>
<td>Up to 350</td>
</tr>
<tr>
<td>NdFeB</td>
<td>Up to 1.5</td>
<td>Up to 1033</td>
<td>Up to 422</td>
<td>Up to 380</td>
<td>Up to 200</td>
</tr>
</tbody>
</table>

B. The PM Machine Geometries

There are various types of PM motors in terms of different application requirements. They can be grouped into different categories based on different criteria. For example, slot stator vs. slotless stator, distributed winding vs. concentrated winding, radial flux vs. axial flux. In general, the PM motor can be classified into surface mounted PM motor, buried PM motor and spoke-type PM motor in terms of PM location on the rotor. Figure 1.2 shows a collection of these three types of structures.
The permanent magnet of surface mounted PM motor is either glued or bonded on the rotor surface. It has the largest flux density and, accordingly, the largest torque output. However, since the permeability of PM material is close to that of air, the inductance is relatively low and the $d$- and $q$- axis inductance is nearly the same. This means the armature reaction is small and the torque output at high speed is very limited. Since the PM is facing the stator tooth directly, high Total Harmonic Distortion (THD) is expected in the stator winding and also it is easy to be demagnetized.

As for the buried PM motor in Figure 1.2(b), its PMs are protected by an iron sleeve, thus it presents better mechanical strength and less demagnetization risk. The spoke-type PM motor in Figure 1.2(c) has a relatively large rotor because of the circumferential PMs orientation. It takes the advantages of larger air gap flux density or the so-called flux-concentration capability and low-cost fabricated design [4].

If the PM motor geometry is classified based on its saliency ratio, its sorting in Figure 1.3 is obtained.
C. Motor Design Development

After the selection of the PM material and rotor geometry, optimizing the rough
design could be most cumbersome, since every nuance could have a different result. Is
there any optimization guideline to direct the design of PM motor in terms of its design
specifications? How to select the parameters, such as inductance, flux density and salient
ratio, such that the motor delivers its best performance? The advancement of motor
design can be shown in terms of enhanced flux weakening capability [17,18,19],
minimized cogging torque ripple [11,12], reduced eddy current losses and harmonic
losses [13,14]. A lot of details and research topics are presented in this area, which is out
of the scope of my work.

1.1.3. Drive Development

The drive technologies are evolving gradually in the 20th century. The control
methodology development and the switching device upgrade are stepping forward
together. The DC VSD has dominated the market for the first half century before the
introduction of the gate-controlled power switch device. Eventually, these power
electronics devices greatly improve the performance and reduce the cost of an AC drive. The modern power electronics device switching time and switching loss has been eliminated considerably, which increases the system’s overall efficiency, reduces system package size, and improves system reliability.

For the inverter stage, modern IGBT and MOSFET power devices can switch as fast as tens of kilohertz without any performance compromise. The traditional snubber circuit, suppressing the voltage stress but also causing complex current circulation, is no longer necessary. The DC capacitors, either electrolytic or film type, become smaller, enhance ripple current absorption capability, and have a longer life span. The digital real time controller becomes more and more powerful with enhanced computational capability and integrated peripheral functions for drive implementation [15].

All the advancements above drive the technology toward the way of higher fundamental frequency, larger volume, and smarter self-diagnostic ability, which makes the following dissertation work possible.

1.2. Importance of the Work

The electrification of transportation vehicles brings the merits of less fuel consumption and consequently less noxious emission. However, the extra embedded electrical system makes the system sophisticated and more expensive than a traditional gasoline engine. Basically, the output power ratio of electric motor to gasoline engine determines the balance point between the cost and the energy saving performance. For the full-scale hybrid technology such as the Toyota Prius, it can reduce the gasoline
consumption to half of that of the traditional car for the same traveling mileage. Of course, people have to pay more for these electrical parts, which has the same output power rating as the gasoline engine. On the other hand, the mild hybrid is less electrified and offers a cheaper option which could be a good candidate right now.

Integrated Starter Generator (ISG) is a mild hybrid technology with the smallest change to the original gasoline system. If we compare the gain and cost of the electrification of a car to that of a motorcycle, a motorcycle has much less energy to be recycled during regeneration because of its lighter frame. Moreover, the motorcycle has very limited space to fit a full-scale electric system. And its operating condition is harsher than that of a car because the power train of a motorcycle is exposed to the ambient environment. Therefore, ISG system is an ideal technical advancement for a motorcycle. The original DC starter motor and AC brushless magnetic generator can be omitted and their functions can be embedded in this new hybrid drive train and, at the same time, power the motorcycle when accelerating. Honda reports on its website that its ISG motorcycle can improve the fuel economy and reduce emission by certain percentages, listed in Figure 1.4.

Figure 1.4 Percentage of fuel economy boost and exhaust emission reduction.
(Honda Hybrid Model: PCX150)
The wide RPM range of the gasoline engine in a motorcycle requires the co-axial coupled ISG motor to be able to run over the same speed range, which pushes the fundamental drive frequency to be relatively high. The high ratio between fundamental frequency to the switching frequency requires careful tuning of the current loop including the current regulator parameters as well as the compensations for nonlinearities. The nonlinearity inside the current control loop doesn’t affect the system performance at low frequency but could certainly mess things up at high running frequency. All these delays and couplings inside the loop should be taken care of to make the system robust and responsive. The bandwidth of the controller should be able to sustain the periodic disturbance coming from the engine side.

The nonlinear characteristic of the inverter could cause problems. The primary part is the dead time, which slices off part of the output voltage and makes the system sluggish. The modeling of dead time is always not an easy job, since it is dependent on multiple working parameters. In this work, space vector notation will be used to associate the dead time with voltage vectors. Other than the mostly used simple current polarity determined dead time voltage modeling, the device paralleled snubber and parasitic capacitor is also considered in the modeling. Based on this modeling, a novel dead-time compensation method is investigated [66].
1.3. Literature Reviews

1.3.1. Flux-Weakening Operation of PM Machines

The deep flux-weakening capability of an IPM motor is its greatest advantage over other machine types. This capability should be explored and examined at both the machine design stage and machine control stage. In literature, the wide speed operation of an IPM machine has been investigated extensively [20,21,22,23,24,25,26].

From control aspects, Jahns and Sneyers first demonstrated the flux-weakening capability of an IPM machine in constant power region back in the 1980s [21,22]. S. Morimoto examined the theoretical control limit of an IPM machine. The optimal current trajectory on $d$- and $q$- axis planes is plotted to maximize the torque output capability both below and above the base speed [23,24,25]. He also made a comparison of the IPM’s performance with regard to different sets of motor parameters. Given these optimal working points on the current plane, Seung-Ki Sul proposed a new self-regulating flux-weakening algorithm to automatically track those points in the flux weakening region [26]. An extra PI regulator is added to adjust the flux-weakening current based on the feedback of the used voltage.

As for the IPM machine itself, the set of parameters should be designed properly so that both the low speed torque and high speed power meet the design criterion. In order to generalize the design criterion to different power-scale machines, a per-unit scaled model is performed. The nominal flux and salient ratio are selected to be the indices to characterize different PM machines [9,10]. R. Schiferl and T. A. Lipo first analytically associated the torque and power capability of a motor with a key set of motor parameters.
as $d$ axis inductance $L_d$, $q$ axis inductance $L_q$ and back-emf $E_0$ [18]. Later on, W. L. Soong developed this topic by combining the SPM machine, the SynRM machine, and the IPM motor into the same theory frame [17]. By doing this, the optimal design region on the IPM parameter plane (flux linkage vs. salient ratio plane) can be found with maximized constant power value over wide speed range. All the above papers try to get the best understanding of the IPM machine from its known parameters. N. Bianchi and S. Bolognani explain how to derive those optimal parameters from the known speed-torque requirement, which follows the machine design procedure [19].

### 1.3.2. Current Loop Control

The space vector field orientation control requires the precise control of its phase current. Its bandwidth determines how fast it is going to react to external disturbance. Its root locus decides how large it’s fundamental frequency could be before it crashes. A lot of research has been carried out to really understand how to design the current controller so that its performance is neither too aggressive, nor too sluggish.

Early literature shows current control is implemented in the stationary reference frame with sine-triangle comparison CRPWM inverter. However, the base operation frequency changes with the changing of rotating speed, which requires a much higher bandwidth controller. In the 1980s, Schauder first demonstrated a more excellent control by implementing the controller in a synchronous rotating reference frame rather than the stationary reference controller [32]. T. R. Rowan compared the transient performances of both of the controllers in the frequency domain comprehensively [33]. The synchronous
regulator is independent of the load and operating point. Phase compensation or adjustable gain settings are no longer viable as the regulator in the stationary reference.

The conventional classical control theory is only applied to scalar system with single input and single output (SISO). However, the vector controlled three phase motor is a multi-input multi-output system, which means the traditional method is not applicable. The matrix based MIMO analysis is useful but not always easy to be combined with frequency analysis. The complex vector notation, a powerful tool to translate the MIMO system into SISO system, exhibits a formal correspondence to the description using matrices [1,27,28,39]. It uses single complex eigenvalues, different from conventional complex conjugate eigenvalues, to distinguish between system engenfrequencies and the angular velocity of a reference frame.

Current sensing is the first and foremost part of current control loop. Different sampling moment and sampling rate will exhibit different results. The ideal case is the sampling method can make the sampled signal tracking its fundamental frequency component with minimum sampling delay. V. Blasko in his paper [29] compares different sampling techniques and PWM modulation strategies to epitomize the one with maximum current control bandwidth. S. H. Song focuses his analysis on hardware delay caused by current sensing and conditioning circuits [30]. The current sampling moment is usually aligned to the midpoint of zero vectors to get the fundamental component of the current. However, the delay associated with filter circuit before A/D sampling breaks this alignment. The incorrect sampling points result in a difference of measured current from its fundamental value. This error could be minimized by delaying the sampling moment the same length as the time of the delay caused by filter circuit.
The design of PI controller parameters based on its frequency response is summarized in [29] with the notation of “technical optimum” and “symmetrical optimum.” A simple RL load is used as an illustration. This method is effective in dealing with the system with simple first order or second order transfer function. However, a real PM motor is a strongly coupled system and the method used to deal with one-to-one mapping is not applicable. The issue of cross-coupling brought by synchronous speed rotation is addressed in Paper [39]. For the same PI controller parameters, the performance would be degraded as the synchronous speed increases.

Delays and cross-coupling terms seem to be the chief culprit that deteriorates the overall system reliability at high synchronous frequency. They are originated from the digital implementation of a motion control system such as digital sampling, arithmetical calculation and PWM modulation. B. H. Bae and S. K. Sul treated the PWM delay to be the rotation of the synchronous reference at the length of the delay time [31]. This delay is designated as one and a half PWM cycle and the compensation improves its stability based on the root locus analysis.

Paper [41] evaluates the performances of different discretization methods of the current regulator operating at high fundamental-to-sampling frequency ratio. It compares the different transfer functions and stability issues of those different discretization methods in Z-domain. The direct design complex vector regulator was evaluated as the best design choice.

All the above investigations use the same techniques, transforming the PM machine in stationary three phase system to the synchronous two phase coordinates, such that all the tricks to analyze a DC machine are also applicable to an AC machine. Instead of
treating everything in synchronous $d,q$ frame, Paper [34] does the reverse job by
designing a controller in synchronous $a,b,c$ frame with exact the same performance as its
counterpart in $d,q$ synchronous frame.

1.3.3. Dead Time and Compensation

In a power inverter, the dead time is necessary to prevent the shoot through of power
devices. But it also leads to the distorted output voltage and current. The nonlinearity of
this output voltage is very detrimental to the performance of the inverter and ultimately
leads to large output current harmonics as well as torque ripple.

Remarkable efforts have been made to investigate the dead-time problem by both the
industry and academia [44,45,46,49,60,64]. The output voltage distortion caused by dead
time has all been examined with respect to different load power factor angle, modulation
index and carrier frequency [46,60]. The nonlinearities of inverter caused by the nonideal
characteristic of the power device as well as the parasitic elements have also been studied
in the literature [51,59].

To mitigate the negative effect of dead time, various compensation strategies have been
proposed [41-59]. Two major types of compensation methods are summarized in terms of
compensation objects. For the first type, the dead-time is modeled as an average voltage
loss where the voltage-second is averaged over a PWM cycle. The lost voltage-second
due to dead time is then added directly to the reference voltage [41,42,43,44,45]. The
second type models the dead-time effects as a pulse shift error and compensated in each
PWM period [46,47]. Several on-line compensation methods are also proposed to
accommodate the modeling errors [48,49,50,51]. In [52,53,54], single switch
commutation of each inverter leg within PWM period is proposed with additional hardware modification. Paper [58] adapts the similar method as in [59] for hardware implementation.

However, most previous work assumes that the turn-on and turn-off of power switching devices are infinitely fast and few literatures examine the effects of parallel capacitance to dead-time compensation. The nonlinearity between phase current and dead-time voltage error is tested and compensated based on offline measurement [59]. In [60], the parasitic capacitor is treated as a linear fictitious resistor only when the phase current is below certain value. In [61], the finite response of the power switch device during turn-on and turn-off intervals is studied and its effect on flux estimation is investigated. In [62], numerous parasitic effects on voltage source inverter are identified and qualified with limited compensation discussion. Paper [63] delivers the similar statement of the effect of parasitic capacitance on dead time. However, the inaccuracy of compensation is not well summarized and compared thoroughly. Moreover, most of the compensations are gauged based on offline measurement without detailed modeling.

1.4. Organization

The whole dissertation is organized as follows

Chapter 2 reviews the mathematic model of permanent magnet motor and the general control method over its entire speed range.
Chapter 3 presents the general specifications of an ISG hybrid motor and drive system. A prototype IPM motor is designed based on these preliminary requirements. Various tests are conducted to validate the prototype machine with the simulation.

Chapter 4 reviews the control of this IPM motor based on the requirement. The main focus is to tune a current regulator based on its transfer function and frequency domain analysis. Cross-coupling and its compensation is analyzed both in time domain and with frequency response analysis. Finally, the engine start control and flux weakening control are implemented to fulfill its function.

Chapter 5 analyzes the dead time of a conventional three phase inverter based on space vector model. A modified compensation strategy is proposed to take the snubber capacitor into account.

Chapter 6 concludes the whole thesis with the suggestions of future works.
CHAPTER 2

MODELING AND CONTROL OF A PERMANENT MAGNET MACHINE

This chapter is going to review the basics of permanent magnet motor. The mathematical model of a permanent magnet motor is presented with its reference frame first oriented in the stator $a,b,c$ stationary frame then in stator $a,\beta$ stationary frame and finally in rotor $d,q$ synchronous frame. Basic operation principle of PM machine is also discussed.

2.1. Reference Frame Theory

The intuitive way of expressing the mathematical model of a three phase PM machine is in the stationary reference frame. However, the self and mutual inductances in stationary reference system have position dependent term, which is a function of the time and speed. Then, all the mathematical equations would be time-variant, which cannot be analyzed using the Linear Time Invariant (LTI) method. In order to make these linear system analysis tools still be applicable to an AC motor, coordination transformation is applied to get rid of these time dependent terms.
Generally, all the variables in three phase stationary coordinates can be transferred to any two phase coordinates with arbitrary angular speed. Any real transformation is obtained by assigning the speed of the rotation of reference frame \[ 2 \]. In Figure 2.1, the \( \alpha,\beta \) axis and \( d,q \) axis are two particular cases of the arbitrary reference frame with the rotation speed of the coordinate equal to zero and synchronous speed respectively.

![PM motor sketch picture.](image)

In Figure 2.1, the vector \( f \), a general notation for either voltage, current, or magnetic flux, in \( a,b,c \) coordinate can be denoted as \( f_{abc} \) or \([f_a, f_b, f_c]\). This synthesized vector can be expressed in equation (2.1) with axis \( a \) as the real axis and its perpendicular axis as imaginary axis.
\[ f_{abc} = f_a + e^{\frac{2\pi}{3}} f_b + e^{\frac{4\pi}{3}} f_c \quad (2.1) \]

If the arbitrary angular speed is selected to be zero with its real axis aligned to phase axis in three phase system, this forms the stationary \(\alpha, \beta, 0\) axis. The above vector can be transferred to stationary \(\alpha, \beta, 0\) axis with its notation as \(f_{\alpha\beta0}\) or \([f_\alpha, f_\beta, f_0]\). The 0 axis represents the common mode component, which is zero for a balanced three phase system. Thus,

\[ f_{\alpha\beta} = f_\alpha + jf_\beta \quad (2.2) \]

The transformation from \(a,b,c\) coordinate to \(\alpha, \beta, 0\) coordinate is,

\[ f_{\alpha\beta} = C_{3s/2s}f_{abc} \quad (2.3) \]

Where,

\[
C_{3s/2s} = \frac{2}{3} \begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]

\[ (2.4) \]

\(C_{3s/2s}\) means conversion from three phase stationary frame to two phase stationary frame. The same naming rule can be used for various transformation matrix bellows.

The inverse process transfers vector from \(\alpha, \beta, 0\) coordinate back to \(a,b,c\) coordinate, which can be expressed as,

\[ f_{abc} = C_{2s/3s}f_{\alpha\beta} \quad (2.5) \]

Where,
Similarly, if the synchronous speed is assigned to be the reference frame, this comes with the \(d,q,0\) coordinate. The same vector above can be expressed in \(d,q,0\) coordinate as \(f_{dq0}\) or \([f_{dq}, f_q, f_0]\). If we ignore the zero sequence component, the transformation from \(\alpha, \beta, 0\) coordinate to \(d,q,0\) coordinate can be written as,

\[
f_{dq} = C_{2s/2r} f_{\alpha\beta}
\]  

(2.7)

Where,

\[
C_{2s/2r} = \begin{bmatrix} \cos \theta_r & \sin \theta_r \\ -\sin \theta_r & \cos \theta_r \end{bmatrix} 
\]  

(2.8)

\(\theta_r\) represents the angle of synchronous rotating coordinate frame from stationary \(\alpha, \beta\) coordinate frame. Usually, the \(d\) axis of the synchronous coordinate is aligned to the center line of permanent magnetic flux, which eliminates any extra angle-dependent term in the equations.

The inverse transformation from \(d,q\) synchronous reference frame to \(\alpha, \beta\) stationary reference frame is expressed as,

\[
f_{\alpha\beta} = C_{2r/2s} f_{dq}
\]  

(2.9)

Where,

\[
C_{2r/2s} = \begin{bmatrix} \cos \theta_r & -\sin \theta_r \\ \sin \theta_r & \cos \theta_r \end{bmatrix} 
\]  

(2.10)
The scaling of these transformation matrixes is not unique but flexible depending on
the user’s preference. But the same scaling should be used for all the transformations in
the system. The listed transformation matrix above adopts the widely applied magnitude
equality based transformation, which is also designated as $\frac{2}{3}$ transformation. Of course,
the calculated power would be different using this scaling factor because the power under
$a,b,c$ coordinate has one more axis component than that in $a,\beta$ coordinate. The power
equality or flux equality based transformation is also widely used which is named as $\sqrt{\frac{2}{3}}$
transformation. As known by its name, the calculated power in $a,b,c$ coordinate is exactly
the same as that in $a,\beta$ coordinate with this scaling coefficient. For the same vector using
this transformation, the amplitude of $a,\beta$ coordinate components should be larger than
that in $a,b,c$ coordinate, which the increased amplitude is to make up for the less
coordinate numbers in $a,\beta$ frame.

2.2. Electrical Modeling of a Permanent Magnet Machine

After having all the transformation matrixes ready, the equations of a PM motor are
summarized below. The summary includes all the equations under a different reference
frame, which follows the same sequence of the description of above transformation
matrix.

2.2.1. Modeling in Stationary $a,b,c$ Reference Frame

The three-phase voltage equation of a PM motor can be expressed as,
Where $v_a, v_b, v_c$ are three-phase stator voltage; $i_a, i_b, i_c$ are three-phase stator current; $\lambda_a, \lambda_b, \lambda_c$ are three-phase stator flux; $p$ is the differential operator; $R_s$ is the stator winding resistance matrix. It can be expanded as,

$$ R_s = \text{diag} [r_r, r_s, r_t] $$

(2.12)

The stator flux is a combination of stator armature reaction flux and rotor permanent magnet flux and is expressed as,

$$ \begin{bmatrix} \lambda_{sa} \\ \lambda_{sb} \\ \lambda_{sc} \end{bmatrix} = \begin{bmatrix} L_{abc} & \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} \end{bmatrix} + \begin{bmatrix} \lambda_{ma} \\ \lambda_{mb} \\ \lambda_{mc} \end{bmatrix} $$

(2.13)

Where $\lambda_{ma}, \lambda_{mb}, \lambda_{mc}$ are three-phase rotor permanent magnet flux. $L_{abc}$ is stator inductance matrix and can be written in matrix format as,

$$ L_{abc} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} $$

(2.14)

Where $L_{aa}, L_{bb},$ and $L_{cc}$ are stator-self inductances and $L_{ab}, L_{ba}, L_{ac}, L_{ca}, L_{bc},$ and $L_{cb}$ are mutual inductances. For each of these inductance elements, it can be further expanded as the addition of DC constant with even harmonic components,

$$ L = L_c + L_{h2} \cos(2\theta) + L_{h4} \cos(4\theta) + L_{h6} \cos(6\theta) + L_{h8} \cos(8\theta) $$

(2.15)

Since this work is not going to cover harmonics analysis, only the DC component and rotor position dependent second order harmonic component in above inductance
expression are studied. Therefore, for a general salient pole PM machine, its self-inductance in matrix (2.14) can be written as,

\[
L_{oa} = L_{sr} + L_{sl} - L_{g2} \cos 2\theta_r
\]

\[
L_{bh} = L_{sr} + L_{sl} - L_{g2} \cos 2(\theta_r - \frac{2}{3}\pi)
\]

\[
L_{cc} = L_{sr} + L_{sl} - L_{g2} \cos 2(\theta_r + \frac{2}{3}\pi)
\]

(2.16)

And the mutual-inductance can be expressed as,

\[
L_{cb} = L_{ba} = -\frac{1}{2} L_{sr} - L_{g2} \cos 2(\theta_r - \frac{\pi}{3})
\]

\[
L_{sc} = L_{cb} = -\frac{1}{2} L_{sr} - L_{g2} \cos 2\theta_r
\]

\[
L_{ac} = L_{ca} = -\frac{1}{2} L_{sr} - L_{g2} \cos 2(\theta_r + \frac{\pi}{3})
\]

(2.17)

Where, \(L_{sr}\) is space fundamental self-inductance; \(L_{sl}\) is armature leakage inductance; \(L_{g2}\) is rotor position dependent inductance; \(\theta_r\) is rotor electric angle.

The rotor permanent magnet flux in equation (2.13) can also be decomposed into the combination of first order fundamental component and odd harmonic components, which is,

\[
\lambda_m = \lambda_{h1} \cos(\theta) + \lambda_{h3} \cdot 3 \cos(3\theta) + \lambda_{h5} \cdot 5 \cos(5\theta) + \lambda_{h7} \cdot 7 \cos(7\theta) + \lambda_{h9} \cdot 9 \cos(9\theta) + \lambda_{h11} \cdot 11 \cos(11\theta)
\]

(2.18)

Only the fundamental component is studied here and the rotor permanent magnet induced stator flux can be expressed as,

\[
\begin{bmatrix}
\lambda_{ma} \\
\lambda_{mb} \\
\lambda_{mc}
\end{bmatrix} = \begin{bmatrix}
\cos\theta_r \\
\cos(\theta_r - \frac{2}{3}\pi) \\
\cos(\theta_r + \frac{2}{3}\pi)
\end{bmatrix} \lambda_m
\]

(2.19)

Substituting equation (2.16), (2.17) and (2.19) into stator flux equation of (2.13), we can get,
\[
\begin{bmatrix}
\lambda_{as} \\
\lambda_{bs} \\
\lambda_{cs}
\end{bmatrix} = \left(L_{al} + \frac{3}{2}L_{ar}\right)\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} - L_{g2} \begin{bmatrix}
\cos 2\theta_r & \cos 2(\theta_r - \frac{\pi}{3}) & \cos 2(\theta_r + \frac{\pi}{3}) \\
\cos 2(\theta_r - \frac{\pi}{3}) & \cos 2(\theta_r + \frac{\pi}{3}) & \cos 2\theta_r \\
\cos 2(\theta_r + \frac{\pi}{3}) & \cos 2\theta_r & \cos 2(\theta_r - \frac{\pi}{3})
\end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix}
\cos \theta_r \\
\cos(\theta_r - \frac{2}{3}\pi) \\
\cos(\theta_r + \frac{2}{3}\pi)
\end{bmatrix} \lambda_m
\]

(2.20)

For a neutral point floated three-phase machine, the summation of all three phase currents would be equal to zero,

\[
i_{as} + i_{bs} + i_{cs} = 0
\]

(2.21)

Equation (2.20) can be further simplified as,

\[
\begin{bmatrix}
\lambda_{as} \\
\lambda_{bs} \\
\lambda_{cs}
\end{bmatrix} = \left(L_{al} + \frac{3}{2}L_{ar}\right)\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} - \frac{3}{2}L_{g2} \begin{bmatrix}
\sqrt{3}/3 \sin 2\theta & -\sqrt{3}/3 \sin 2\theta & \sqrt{3}/3 \sin 2\theta \\
-\sqrt{3}/3 \sin 2\theta & \sqrt{3}/3 \sin 2\theta & -\sqrt{3}/3 \sin 2\theta \\
-\sqrt{3}/3 \sin 2\theta & \sqrt{3}/3 \sin 2\theta & \sqrt{3}/3 \sin 2\theta
\end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix}
\cos \theta_r \\
\cos(\theta_r - \frac{2}{3}\pi) \\
\cos(\theta_r + \frac{2}{3}\pi)
\end{bmatrix} \lambda_m
\]

(2.22)

At this point, the stator voltage equation (2.11) can be formalized by substituting its stator flux vector with equation (2.22). Now, the voltage equation comes to the simplest format with only machine parameters and phase currents.
2.2.2. Modeling in Stationary $\alpha, \beta$ Reference Frame

The vector in three-phase $a,b,c$, coordinate can be decomposed into two orthogonal $\alpha, \beta$ coordinate using the transformation matrix in equation (2.3). Apply this transfer matrix to the voltage equation in equation (2.11) results,

$$
C_{3s/2s} \begin{bmatrix} v_{s\alpha} \\ v_{s\beta} \\ v_{s0} \end{bmatrix} = R_s C_{3s/2s} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ i_{s0} \end{bmatrix} + C_{3s/2s} P \begin{bmatrix} \lambda_{s\alpha} \\ \lambda_{s\beta} \\ \lambda_{s0} \end{bmatrix}
$$

(2.23)

Since the transformation matrix $C_{3s/2s}$ is a constant matrix and can be put inside the differential operator. Equation (2.23) can be converted to,

$$
\begin{bmatrix} v_{\alpha s} \\ v_{\beta s} \\ v_{0s} \end{bmatrix} = R_s \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{0s} \end{bmatrix} + P \begin{bmatrix} \lambda_{\alpha s} \\ \lambda_{\beta s} \\ \lambda_{0s} \end{bmatrix}
$$

(2.24)

Where the flux linkage in $\alpha, \beta, 0$ can be expressed as,

$$
\begin{bmatrix} \lambda_{\alpha s} \\ \lambda_{\beta s} \\ \lambda_{0s} \end{bmatrix} = C_{3s/2s} \begin{bmatrix} \lambda_{s\alpha} \\ \lambda_{s\beta} \\ \lambda_{s0} \end{bmatrix}
$$

$$
= C_{3s/2s} \left( L_{abc} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{0s} \end{bmatrix} + \begin{bmatrix} \lambda_{ma} \\ \lambda_{mb} \\ \lambda_{mc} \end{bmatrix} \right)
$$

$$
= C_{3s/2s} L_{abc} C_{3s/2s}^{-1} \cdot C_{3s/2s} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{0s} \end{bmatrix} + C_{3s/2s} P \begin{bmatrix} \lambda_{ma} \\ \lambda_{mb} \\ \lambda_{mc} \end{bmatrix}
$$

(2.25)

$$
= L_{\alpha\beta0} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{0s} \end{bmatrix} + \begin{bmatrix} \lambda_{ma} \\ \lambda_{mb} \\ \lambda_{mc} \end{bmatrix}
$$

Where, $L_{\alpha\beta0}$ is stator inductance matrix in $\alpha, \beta$ coordinate and can be defined as,
\[ L_{\alpha\beta0} = C_{3s/2r} L_{abc} C_{3s/2s}^{-1} \]
\[ = \begin{bmatrix} L_{sl} + \frac{3}{2} (L_{sr} - L_{sg} \cos 2\theta_r) & \frac{3}{2} L_{sg} \sin 2\theta_r & 0
                      \frac{3}{2} L_{sg} \sin 2\theta_r & L_{sl} + \frac{3}{2} (L_{sr} + L_{sg} \cos 2\theta_r) & 0
                      0 & 0 & L_{sl} \end{bmatrix} \]

(2.26)

For a balanced three-phase system, the zero-sequence component does not exist. The stator flux linkage and stator voltage in \( \alpha, \beta \) axis can be simplified as,

\[
\begin{bmatrix} \lambda_{\alpha s} \\ \lambda_{\beta s} \end{bmatrix} = \begin{bmatrix} L_{\alpha\alpha} & L_{\alpha\beta} \\ L_{\alpha\beta} & L_{\beta\beta} \end{bmatrix} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix} + \begin{bmatrix} \cos \theta_r \\ \sin \theta_r \end{bmatrix} \lambda_m \quad (2.27)
\]

\[
\begin{bmatrix} v_{\alpha s} \\ v_{\beta s} \end{bmatrix} = \begin{bmatrix} r_s + pL_{\alpha\alpha} & pL_{\alpha\beta} \\ pL_{\alpha\beta} & r_s + pL_{\beta\beta} \end{bmatrix} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix} + \begin{bmatrix} -\sin \theta_{re} \\ \cos \theta_{re} \end{bmatrix} \lambda_m \quad (2.28)
\]

### 2.2.3. Modeling in Synchronous \( d,q \) Reference Frame

The equations in \( \alpha,\beta \) stationery reference frame can be further transferred into \( d,q \) synchronous reference frame for easy modeling and decoupled control of PM machine based on equation (2.7) and transformation matrix (2.8).

The \( d,q \) axis voltage equation is derived by applying the transformation matrix (2.8) to the \( \alpha,\beta \) axis voltage equation (2.29), which is,

\[ C_{2s/2r} \begin{bmatrix} v_{\alpha s} \\ v_{\beta s} \end{bmatrix} = R_s C_{3s/2s} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix} + C_{3s/2r} P \begin{bmatrix} \lambda_{\alpha s} \\ \lambda_{\beta s} \end{bmatrix} \quad (2.30) \]

Since \( C_{2s/2r} \) contains time dependent rotor angle information, it cannot be included inside the derivative operator straightforward. So, the above equation turns out to be,

\[ \begin{bmatrix} v_{ds} \\ v_{qs} \end{bmatrix} = R_s \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + C_{2s/2r} \frac{d}{dt} \begin{bmatrix} \lambda_{\alpha s} \\ \lambda_{\beta s} \end{bmatrix} \quad (2.31) \]
We can use the rule of derivative to further simplify the flux term in equation (2.31) as,

\[
C_{2s/2r}\frac{d}{dt} \lambda_{s}\nabla = \left[ \begin{array}{c}
\lambda_{ds} \\
\lambda_{qs}
\end{array} \right] = \frac{d}{dt} \left[ \begin{array}{c}
\lambda_{ss} \\
\lambda_{ss}
\end{array} \right] - \frac{d}{dt} \left[ \begin{array}{c}
\lambda_{js} \\
\lambda_{js}
\end{array} \right]
\]

= \frac{d}{dt} \left[ \begin{array}{c}
\lambda_{ds} \\
\lambda_{qs}
\end{array} \right] - \frac{d}{dt} \left( C_{2s/2r}^{-1} \right) C_{2s/2r} \left[ \begin{array}{c}
\lambda_{ss} \\
\lambda_{ss}
\end{array} \right]
\]

= \frac{d}{dt} \lambda_{ds} - \omega \frac{d}{d\theta} \left( C_{2s/2r}^{-1} \right) C_{2s/2r} \left[ \begin{array}{c}
\lambda_{ss} \\
\lambda_{ss}
\end{array} \right]
\]

= \frac{d}{dt} \lambda_{ds} - \omega \left[ \begin{array}{c}
0 \\
-1
\end{array} \right] \left[ \begin{array}{c}
\lambda_{ds} \\
\lambda_{qs}
\end{array} \right]
\]

\tag{2.32}
\]

By substituting the flux term in (2.31) with expression (2.32), the voltage equation in \(d,q\) axis synchronous rotating frame can be written as,

\[
\begin{bmatrix}
v_{ds} \\
v_{qs}
\end{bmatrix} = R_s \begin{bmatrix}
i_{ds} \\
i_{qs}
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix}
\lambda_{ds} \\
\lambda_{qs}
\end{bmatrix} - \omega \begin{bmatrix}
0 \\
-1
\end{bmatrix} \begin{bmatrix}
\lambda_{ds} \\
\lambda_{qs}
\end{bmatrix}
\]

\tag{2.33}
\]

Where the \(d,q\) axis flux linkage is obtained by applying the transformation matrix to equation (2.27), which gives,

\[
\begin{bmatrix}
\lambda_{ds} \\
\lambda_{qs}
\end{bmatrix} = C_{2s/2r} \begin{bmatrix}
\lambda_{ss} \\
\lambda_{ss}
\end{bmatrix}
\]

= \frac{d}{dt} \lambda_{ds} - \omega \frac{d}{d\theta} \left( C_{2s/2r}^{-1} \right) C_{2s/2r} \left[ \begin{array}{c}
\lambda_{ss} \\
\lambda_{ss}
\end{array} \right]
\]

= \frac{d}{dt} \lambda_{ds} - \omega \left[ \begin{array}{c}
0 \\
-1
\end{array} \right] \left[ \begin{array}{c}
\lambda_{ds} \\
\lambda_{qs}
\end{array} \right]
\]

\tag{2.34}
\]

\[
L_{dq} \begin{bmatrix}
i_{ds} \\
i_{qs}
\end{bmatrix} + \begin{bmatrix}
1 \\
0
\end{bmatrix} \lambda_m
\]

Where \(L_{dq}\) is stator inductance matrix in \(d,q\) reference frame and has the format of,

\[
L_{dq} = C_{2s/2r} L_{\alpha\beta} C_{2s/2r}^{-1}
\]

= \begin{bmatrix}
\frac{3}{2} (L_{sr} - L_{g2}) + L_{sl} & 0 \\
0 & \frac{3}{2} (L_{sr} + L_{g2}) + L_{sl}
\end{bmatrix}
\]

\tag{2.35}
\]

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The \(d\) and \(q\) axis inductances are the two most important parameters to bridge the gap between electric circuit and magnetic circuit. And it is strongly correlated to the mechanical geometry of the motor. It is given as,

\[
L_q = \frac{3}{2}(L_{sr} + L_{sg2}) + L_{sl} \\
L_d = \frac{3}{2}(L_{sr} - L_{sg2}) + L_{sl}
\]  

(2.36)

With stator current as its independent arguments, the \(d,q\) axis voltage equations can be summarized as follows,

\[
\begin{bmatrix}
v_{ds} \\
v_{qs}
\end{bmatrix} = \begin{bmatrix}
r_s + L_d p & 0 \\
0 & r_s + L_q p
\end{bmatrix} \begin{bmatrix}
i_{ds} \\
i_{qs}
\end{bmatrix} + \omega_r \begin{bmatrix}
-L_q i_{qs} \\
L_d i_{ds} + \lambda_m
\end{bmatrix}
\]

(2.37)

Since \(L_d, L_q\) have very clear mechanical definition and are able to be measured directly. Both the flux equations in \(a,b,c\) reference frame and \(\alpha,\beta\) reference frames are modified with \(L_d, L_q\) as the arguments, which gives,

\[
\begin{bmatrix}
\lambda_{\alpha s} \\
\lambda_{\beta s}
\end{bmatrix} = \frac{L_d + L_q}{2} \begin{bmatrix}
i_{\alpha s} \\
i_{\beta s}
\end{bmatrix} - \frac{L_q - L_d}{2} \begin{bmatrix}
\sqrt{3}/3 \sin 2\theta & \sqrt{3}/3 \sin 2\theta & -\sqrt{3}/3 \sin 2\theta \\
-\sqrt{3}/3 \sin 2\theta & \cos 2\theta & \sqrt{3}/3 \sin 2\theta
\end{bmatrix} \begin{bmatrix}
i_{\alpha s} \\
i_{\beta s}
\end{bmatrix} + \begin{bmatrix}
\cos \theta_r \\
\cos(\theta_r - \frac{2\pi}{3})
\end{bmatrix} \lambda_m
\]

(2.38)

\[
\begin{bmatrix}
\lambda_{\alpha 3s} \\
\lambda_{\beta 3s}
\end{bmatrix} = \begin{bmatrix}
\frac{L_d + L_q}{2} + \frac{L_d - L_q}{2} \cos 2\theta \\
\frac{L_d - L_q}{2} \sin 2\theta
\end{bmatrix} \begin{bmatrix}
i_{\alpha 3s} \\
i_{\beta 3s}
\end{bmatrix} + \begin{bmatrix}
\frac{L_d - L_q}{2} \sin 2\theta \\
\frac{L_d + L_q}{2} - \frac{L_d - L_q}{2} \cos 2\theta
\end{bmatrix} \begin{bmatrix}
i_{\alpha 3s} \\
i_{\beta 3s}
\end{bmatrix} + \begin{bmatrix}
-\sin \theta_r \\
\cos \theta_r
\end{bmatrix} \lambda_m
\]

(2.39)
2.3. Mechanical Modeling of PM Machines

The mechanical modeling of a PM motor is mainly about the definition of the torque equation. The essential definition of torque is originated from the energy conversion principle. A conventional PM machine is used to convert the energy from either electrical port to mechanical port (which is regarded as motoring mode) or from mechanical port to electrical port (which is recognized as generating mode). The magnetic field inside the machine serves as an energy reservoir, offering continuous active power flow from these two ports back and forth.

![Energy flow inside a machine.](image)

Figure 2.2 shows the energy flow inside a machine from electric port to mechanic port (red curve) and vice versa (green curve). Losses always exist on both the stator and the rotor of a machine. It is not a significant part compared to the total flowing energy and is omitted for easy analysis. Then, the energy variations of electric port, mechanic port and inside the magnetic field are always in a balanced state, which can be expressed as,

30
\[ dW_{\text{fld}} = dW_{\text{elec}} - dW_{\text{mech}} \]
\[ = i d\lambda - T d\theta_m \] (2.40)

The torque is obtained by applying the law of conservation of energy [3]

\[ T_e = i \frac{d\lambda}{d\theta_m} - \frac{dW_{\text{fld}}}{d\theta_m} \] (2.41)

Since \( \lambda \) has nothing to do with \( \theta \), the torque now can be expressed as,

\[ T_e = -\frac{dW_{\text{fld}}}{d\theta_m} \] (2.42)

As for the magnetic field, its total energy can be expressed with respect to the stator current and flux linkage as,

\[ W_{\text{fld}} = \int_0^\lambda i(\lambda') \cdot d\lambda' \] (2.43)

The magnetic co-energy is defined as the integral of magnetic flux with respect to current,

\[ W'_{\text{fld}} = \int_0^{i'} \lambda'(i') \cdot di' \] (2.44)

By adding equation (2.43) and (2.44) together, we can get,

\[ W'_{\text{fld}} + W_{\text{fld}} = \lambda \cdot i \] (2.45)

If we further differentiate the above equation, we can get,

\[ dW'_{\text{fld}} - \lambda \cdot di = i \cdot d\lambda - dW_{\text{fld}} \] (2.46)

By substituting equation (2.46) into equation(2.41), the torque equation based on co-energy can be expressed as,

\[ dW_{\text{fld}}' = T_e \cdot d\theta_m + \lambda \cdot di \] (2.47)
The torque on the shaft is the partial derivative of electromagnetic co-energy with respect to the rotational angle, which is,

\[ T_e = \frac{\partial W_{\text{fld}}}{\partial \theta_m} \]  
(2.48)

As for a PM machine, its magnetic co-energy can be expressed in a three-phase coordinate as,

\[ W_{\text{fld}} = \frac{1}{2} i_{\text{abc}}^T L\text{abc} \theta_i_{\text{abc}} + \frac{1}{2} i_{\text{abc}}^T \lambda_{\text{abc}} \]  
(2.49)

If equation (2.49) is plugged into equation (2.48), we can have the magnetic torque as,

\[ T_e = \frac{\partial W_{\text{fld}}'}{\partial \theta_m} = n_p \frac{\partial W_{\text{fld}}'}{\partial \theta_r} 
= \frac{1}{2} n_p (i_{\text{abc}}^T \frac{dL_{\text{abc}}(\theta_r)}{d \theta_r} i_{\text{abc}} + i_{\text{abc}}^T \frac{d\lambda_{\text{abc}}}{d \theta_r}) \]  
(2.50)

In the above equation, the torque is expressed in three-phase stationary coordinates. We would like to simplify it by transforming it to \(d,q\) synchronous frame. By inserting the unity matrix into equation(2.50), we can get,

\[ T_e = n_p \left( \frac{1}{2} i_{\text{abc}}^T C_{3s/2r}^{-1} C_{3s/2r} C_{3s/2r}^{-1} \frac{dL_{\text{abc}}(\theta_r)}{d \theta_r} C_{3s/2r}^{-1} i_{\text{abc}} + \frac{1}{2} i_{\text{abc}}^T C_{3s/2r}^{-1} \frac{d\lambda_{\text{abc}}}{d \theta_r} \right) \]  
(2.51)

Equation (2.51) contains the inductance derivative term, which can be simplified as,
\[ C_{3/2r} \frac{dL_{abc}(\theta_r)}{d\theta_r} C_{3/2r}^{-1} = C_{3/2s} C_{3/2s} \frac{dL_{abc}(\theta_r)}{d\theta_r} C_{3/2s}^{-1} C_{3/2s}^{-1} \]
\[ = C_{3/2r} \frac{dL_{a\beta}(\theta_r)}{d\theta_r} C_{3/2r}^{-1} \]
\[ = C_{3/2r} \cdot 3L_{g2}^2 \begin{bmatrix} \sin 2\theta_r & \cos 2\theta_r \\ \cos 2\theta_r & -\sin 2\theta_r \end{bmatrix} C_{3/2r}^{-1} \]
\[ = \frac{3L_{g2}^2}{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{2.52} \]

The flux derivative term can also be simplified as,
\[ C_{3/2r} \frac{d\lambda_{abc}}{d\theta_r} = C_{3/2r} \frac{d\lambda_{a\beta}}{d\theta_r} \]
\[ = C_{2/2r} \frac{d\lambda_{a\beta\ell m}}{d\theta_r} \]
\[ = C_{2/2r} \begin{bmatrix} -\sin \theta_r \\ \cos \theta_r \end{bmatrix} \lambda_m \]
\[ = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \lambda_m \tag{2.53} \]

Finally, by substituting equation (2.52) and (2.53) into equation(2.51), the magnetic torque of the PM machine turns out to be,
\[ T_e = \frac{1}{2} n_p \left( -\frac{3L_{g2}^2}{2} (2i_d i_q) + i_q \lambda_m \right) \tag{2.54} \]

\( L_{g2} \) is the inductance due to rotor position dependent flux. It is directly related to the \( d,q \) axis inductances based on equation (2.36), which gives,
\[ L_q - L_d = 3L_{g2} \tag{2.55} \]

Finally, the magnetic torque \( T_e \) is obtained based on pure \( d,q \) axis variables,
\[ T_e = \frac{1}{2} n_p (i_q \lambda_m - (L_q - L_d) i_d i_q) \tag{2.56} \]
There are two separated terms in equation (2.56); the first term is generated between the stator current and rotor permanent flux, which is called electromagnetic torque. The second term is the reluctance torque, which is the mutual effect of stator current and rotor saliency. The surface PM machine has same $d, q$ axis inductances due to the uniform air gap structure. Thus, there is no reluctance torque.

2.4. Basic Operation Principle of PM Machines

The particular structure of the PM machine makes the control different from other types of AC machine. This section will go over the basic operation principles of a PM machine.

An induction machine can operate in four quadrants but the operation zone in all the four quadrants is not symmetrical. Its torque cannot reverse its direction instantaneously without the change of speed. Let’s take a look at the torque-speed curve in Figure 2.3(a). Suppose a machine is operating at point $A$ in first quadrant as a motor, there is no mirror point $A'$ with opposite torque output in the fourth quadrant. Instead, the machine has to accelerate to point $B$ in order to get the same reserve torque.
As for a PM machine, all the torque speed-curves in the four quadrants are symmetrical. The four points $A, A', B, B'$ are with the same absolute speed and torque value but different signs. Consequently, the inverter can push the motor to output the same torque and speed based on control purposes. The utmost profile on the torque-speed curve determined the achievable region of the motor.
2.4.1. Constraint of Voltage and Current

The PM motor output capability is restricted by its maximum allowable electric stress and magnetic stress as well as the designated DC bus voltage as shown in Figure 2.4. Theoretically, the output torque can go to infinite at zero speed, if the stator winding is superconductive and the lamination has pure linear B-H curve without saturation and losses. Similarly, if the DC bus voltage is considered to be infinite without isolation problems, the running speed with zero torque can be infinitive also. These two hypotheses above are certainly not practical.

![Operation boundary of a PM machine.](image)

Let’s first consider the voltage constrain effect of the inverter to the PM machine.

The stator voltage equation of a PM machine is summarized in equation (2.37) in the synchronous reference frame, which rewrites here as,
For a three phase DC/AC inverter with SVPWM modulation method, the maximum achievable phase voltage is \( \frac{V_d}{\sqrt{3}} \). Therefore, the inverter output voltage in \( d,q \) axis can be written as,

\[
v_{ds}^2 + v_{qs}^2 \leq \left( \frac{V_d}{\sqrt{3}} \right)^2 \tag{2.58}
\]

By plugging equation (2.57) into equation (2.58), the voltage constrain of the inverter is converted to that on the current plane by omitting the voltage drop on stator resistor,

\[
(\omega_r L_q i_q)^2 + (\omega_r L_d i_{ds} + \omega_r \lambda_m)^2 \leq \left( \frac{V_d}{\sqrt{3}} \right)^2 \tag{2.59}
\]

In Figure 2.5, the voltage constrain equation (2.59) can be drawn on the \( d,q \) axis current plane as a series of voltage limit ellipses.
The current constrain of a inverter to the PM machine is simply expressed by limiting the current magnitude inside the maximum achievable current, which is,

\[ i_{ds}^2 + i_{qs}^2 \leq i_{\text{max}}^2 \]  

(2.60)

This is drawn in Figure 2.5 as the current limit circle.

After reviewing all the operation constrains, the different operation region of the control trajectory on the current plane can be investigated.
2.4.2. Constant Torque Region (MTPA)

The best advantage of field oriented control of a PM motor is its decoupled control between torque and flux. When the speed is below the base speed \( n_{\text{base}} \), the used voltage is below the DC bus constrain which is named constant torque region in Figure 2.6. On either \( T_a \) or \( T_b \) constant torque curve in Figure 2.5, there are infinite current pairs \((i_{ds},i_{qs})\) achieving the same output torque. Among all these current pairs, the one with the smallest magnitude is preferred because of its minimum loss. This current vector can be calculated by performing derivative calculation over the toque equation (2.56) in reference to either the stator \( d \)-axis current \( i_{ds} \) or \( q \)-axis current \( i_{qs} \). Since there are two unknown variables in equation (2.56), the stator current magnitude equation \( i_{ds}^2 + i_{qs}^2 = i_s^2 \) is used to make the torque equation only dependent on one variable.

\[
T_v = \frac{3}{2} \frac{P}{2} \left( \lambda_m i_{qs} + (L_{ds} - L_{qs}) i_{qs} \sqrt{i_s^2 - i_{qs}^2} \right)
\]  
(2.61)
By taking derivative of the torque equation, the maximum torque per phase current Ampere can be found as,

\[
\begin{align*}
    i_{ds} &= \frac{\lambda_m}{4(L_q - L_d)} - \sqrt{\frac{\lambda_m^2}{16(L_q - L_d)^2} + \frac{i_s^2}{2}} \\
    i_{qs} &= \sqrt{i_s^2 - i_{ds}^2}
\end{align*}
\]

(2.62)

By changing the current magnitude \(i_s\) from zero the its maximum limit \(i_{max}\) and connecting all the vectors together, the maximum torque per ampere (MTPA) curve is obtained as in Figure 2.5. The efficiency of the motor would be maximized if running the motor along this curve bellow base speed.

2.4.3. Constant Power Region

Constant power region is a commonly used term to describe the region, where the motor is forced to leave the MTPA curve because of the voltage limitation. Equivalently, it means extra flux weakening current is necessary to further push the motor to deliver torque in higher speed range. There is a common mistake that the motor has to deliver constant power at the “constant power region.” The “constant power region” means the maximum achievable power in Figure 2.6 has almost constant power profile. The maximum power profile is decided by the machine parameters, e.g. saliency ratio, per unit flux linkage, and is not an absolute constant in most cases. A machine is still able to deliver constant torque, e.g. \(T_a\) and \(T_b\) in Figure 2.5, in the constant power region using proper flux weakening control. Figure 2.6 shows that the maximum achievable speed for different torque value is different. We still designate the maximum achievable speed with maximum torque output as the base speed \(n_{base}\).
When the machine accelerates with constant torque output, the bus voltage will be fully utilized eventually. The machine then has to leave the $MTPA$ curve and enter into flux weakening region. Let’s take a look at point $A$ and point $B$ in both Figure 2.5 and Figure 2.6 for example. After the speed ramps up to $\omega_a$ at point $A$ in both the figures, the DC bus runs out. More flux weakening current $i_d$ is injected into the inverter to ensure the current regulator stays actively tuning the $d, q$ axis current. Of course, the $i_q$ current must decrease as the speed goes higher. In order to keep the motor current vector moving along the $AB$ curve as the speed increases, it is necessary to set the proper command current vector. The current vector point should be at the cross point between constant torque curve and voltage limit ellipse. These two equations can be put together as,

$$
\begin{align*}
T_e &= \frac{3}{2} \frac{P}{2} (L_{m} i_{qs} + (L_{d} - L_{q}) i_{d} i_{qs}) \\
\left(\omega_{e} L_{q} i_{qs}\right)^2 + \left(\omega_{e} L_{d} i_{ds} + \omega_{e} \lambda_{m}\right)^2 &= \left(\frac{V_{dc}}{\sqrt{3}}\right)^2 \\
\end{align*}
(2.63)
$$

By solving equation (2.63) with respect to $i_{ds}$ and $i_{qs}$, the current vector is obtained.

The above equation represents a fourth order system and its explicit expression is pretty intricate and will not be shown over here.

After the current vector reaches point $B$, the current constraint will be effective. The current vector cannot continue moving along the constant torque $T_a$ curve any more. It diverts to track the maximum current circle from point $B$ over point $C$ to point $D$. The current vector point now is at the crossing point between current constrain circle and voltage constrain ellipse. By putting current constrain equation and voltage constrain equation together, we will have,
\[
\begin{align*}
\left(\omega_L i_{qs}^2\right)^2 + \left(\omega_L i_{ds} + \omega_L \lambda_m \right)^2 &= \left(\frac{V_{dc}}{\sqrt{3}}\right)^2 \\
\therefore \quad i_{ds}^2 + i_{qs}^2 &= i_{max,s}^2
\end{align*}
\]  

(2.64)

This gives the current reference point as,

\[
\begin{align*}
\begin{cases}
i_{ds} = -\frac{\lambda_m}{L_d} + \frac{1}{L_d} \sqrt{\frac{V_{dc}^2}{3\omega_r^2} - (L_q i_{qs})^2} \\
i_{qs} = \sqrt{i_{max,s}^2 - i_{ds}^2}
\end{cases}
\end{align*}
\]  

(2.65)

After reaching point \(D\), there are two cases that need to be considered separately depending on the motor specifications. If the maximum speed point \(E\) \(\left(\frac{\lambda_m}{L_d}, 0\right)\) locates outside of the maximum current circle, this indicates the motor cannot reach the infinite speed. Then the current vector shall keep moving along the current limit circle until to its maximum achievable speed at the cross point of current circle and negative \(d\)-axis. While, if point \(E\) is inside the current limit circle, the control switches to the next region.

### 2.4.4. Maximum Power per Voltage (MPPV) Region

As mentioned above, for different machine specifications, there could be different control strategies. The theoretical infinite speed point on the current plane is \(\left(\frac{\lambda_m}{L_d}, 0\right)\) without considering the current limitation. Nonetheless, this point is either achievable, \(\frac{\lambda_m}{L_d}\) is smaller than the maximum current limit, or is beyond the operational scope, which the \(\frac{\lambda_m}{L_d}\) is larger than the maximum current limit, depending on the design.
specifications. Suppose this point is inside the current circle, then the maximum power points at different speeds are no longer lying on the crossing point between the current limit circle and voltage limit ellipse, but lying on the tangential point between voltage ellipse and constant torque curve. When the motor speed goes beyond $\omega_3$ in Figure 2.5, the current vector corresponds to the maximum output power can be obtained by taking derivative of the output power over the d-axis current to be zero, which is

$$\frac{dP}{di_d} = 0 \quad (2.66)$$

The output power of the machine can be obtained based on the torque and speed of the motor, which is,

$$P_e = \omega_3 \frac{3}{2} \frac{P}{2} (\lambda_m i_{qs} + (L_{ds} - L_{qs}) i_{ds} i_{qs}) \quad (2.67)$$

Since the output power is dependent on both the d, q axis current, the q axis current is substituted by using the voltage constrain in equation(2.59), which gives the next equation set,

$$\begin{align*}
P_e &= \omega_3 \frac{3}{2} \frac{P}{2} (\lambda_m i_{qs} + (L_{ds} - L_{qs}) i_{ds} i_{qs}) \\
\left(\omega_L L_{qs} i_{qs}\right)^2 + \left(\omega_L L_{ds} i_{ds} + \omega_L \lambda_m\right)^2 &= \left(\frac{V_d}{\sqrt{3}}\right)^2
\end{align*} \quad (2.68)$$

By substituting equation(2.68) into equation(2.66) and ignoring the stator resistance, the stator current vector at maximum output power point will be,
\[
\begin{aligned}
\begin{cases}
    i_{ds} = -\frac{\lambda_m - \Delta i_{ds}}{L_{ds}} \\
    i_{qs} = \sqrt{\frac{(V_{dc} / (\sqrt{3}\omega_r))^2 - (L_{ds}\Delta i_{ds})^2}{L_{qs}}}
\end{cases}
\end{aligned}
\] (2.69)

Where,
\[
\Delta i_{ds} = \frac{L_{qs}\lambda_m - \sqrt{(L_{qs}\lambda_m)^2 + 8L_{qs}L_{ds}(V_{dc} / (\sqrt{3}\omega_r))^2}}{4(L_{qs} - L_{ds})L_{ds}}
\] (2.70)

This curve segment is shown in Figure 2.5 from point \(D\) to point \(E\).

### 2.4.5. Optimal Control of Stator Current

Optimal current control is to extract the maximum power from the PM machine at both electrical and mechanical transients within the limitation of the inverter. Suppose we already have a well-tuned current regulator (the actual current tracks command current tightly) with flux weakening capability. We hope to get the fastest response from one steady state to another during speed acceleration or deceleration. Equivalent speaking, it is to tune the speed controller to have the highest bandwidth.

There are multiple-paths to achieve the same steady state point on the current plane. Figure 2.7 shows three of them: 1, 2, and 3 respectively. Path 1 represents the mildest acceleration path with constant torque output, which is almost equal to the load. Path 2 shows a slightly aggressive one, but with constant terminal voltage vector. Path 3 illustrates the stiffest one with the full inverter capability being utilized.

For control with the best inverter utilization, both the voltage and current would hit the limit of the inverter, and the whole system would offer the highest control bandwidth. We will investigate this in more details in the later chapter.
2.5. Summary

In this chapter, the modeling of a standard PM machine is reviewed based on the theory of reference frame transformation. Both the electrical and mechanical equations of the machine are illustrated. Based on these modeling equations, the operating constraints associated with the drive inverter are inspected. The maximum power operation of the motor at different regions on the \( i_d-i_q \) plane is explained.

Figure 2.7. Multi-current path to achieve the same steady state point.
CHAPTER 3

PM MACHINE DESIGN AND CHARACTERIZATION

The electric machine for traction application has its own unique requirements than these standard industrial drives. It cares more about efficiency and it prefers high torque and power density to cut the weight and save the space. Also, its cost is another big concern on the premise of 100 percent safety assurance. Fault tolerant design offers the safety solution for both the machine and inverter. It is pretty challenging to fulfill all these requirements in a single drive, since they are sometimes contradicted with each other. That’s why the best design could never happen but a better design could. Among all types of PM machines, the IPM motor, with high energy density, high efficiency and wide speed operation range, offers the best solution for the application.

3.1. Application Requirement

Before we start to plan the blueprint of the motor, the characteristic of its coupling counterpart, the 50cc gasoline engine, should be first learned. The torque and power output of a conventional 50cc engine is show in Figure 3.1.
The engine and the wheel are detached with each other bellow 3500rpm by Continuous Variable Transmission (CVT) clutch. So, the curve in Figure 3.1 doesn’t stretch bellow speed 3500rpm. Based on the engine curve, the machine is expected to output a slightly small profile as that of the engine. The prototype design is to output 1.5KW power at the constant power region, with maximum torque around 3Nm. Since the machine torque will decrease above the base speed, a minimum of 1Nm torque is required at 7000rpm. The engine no load rotational speed can surpass 10000 rpm and the maximum loaded speed is around 9600 rpm. So the designed maximum speed for the machine is 12000rpm. Another important parameter of the designed machine is its start torque, which should be at least as larger as the maximum resistive torque of the engine at compression stroke. The compression stroke resistive torque can vary considerably, ranging from 5Nm to 10Nm depending on ambient temperature. The higher the
temperature is, the smaller the viscosity of the engine oil, then the smaller the resistive torque. However, even the smallest resistive torque is out of the range of the rated output torque. A special control strategy is implemented to solve this problem. The detailed description is addressed in the following chapter.

The mechanical scale of the mounting rack is the major factor to size the machine. The machine together with the engine power train is shown in Figure B.2 in Appendix B. The diameter of the mounting chamber is around 100mm. The depth is around 60mm. Based on these two figures, the rough torque density and power density for the total volume can be obtained as,

\[
T_{e,\text{den.}} = \frac{T_{e,\text{rated}}}{\pi D^2 L} = 0.006366 (N \cdot m / cm^3)
\]

(3.1)

\[
P_{e,\text{den.}} = \frac{P_{e,\text{rated}}}{\pi D^2 L} = 3.18 (W / cm^3)
\]

(3.2)

3.2. Design Specifications

After having all the constraints ready, the general specifications of the machine are presented in Table 3.1.
Table 3.1. Specifications of the machine.

<table>
<thead>
<tr>
<th>Items</th>
<th>Abbreviation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>$P_n$</td>
<td>1.5kW</td>
</tr>
<tr>
<td>Base Speed</td>
<td>$n_{base}$</td>
<td>4800rpm</td>
</tr>
<tr>
<td>Max Speed</td>
<td>$n_{max}$</td>
<td>12000rpm</td>
</tr>
<tr>
<td>Rated DC Bus Voltage</td>
<td>$V_{dc}$</td>
<td>48V</td>
</tr>
<tr>
<td>Rated Current</td>
<td>$I_n$</td>
<td>30A</td>
</tr>
<tr>
<td>Pole of Pairs</td>
<td>$n_P$</td>
<td>4</td>
</tr>
</tbody>
</table>

Next, the materials and dimensions of various parts are selected based on the general size and FEA simulation adjustment.

A. *Permanent Magnet.*

Since the machine needs to deliver large torque at low speed, strong rear-earth magnetic material is preferred for its high flux density. After evaluating both the price and performance, the Nd-Fe-B material N35UH is selected. Figure 3.2 presents the B-H curve of N35UH material under different operation temperatures. The characteristic of rear-earth material is susceptible to temperature variation. So, the maximum demagnetizing current should be calibrated in terms of the rotor temperature to make sure the PM material will not be permanently demagnetized.
Figure 3.2. N35UH permanent magnet B-H curve.

Figure 3.3. Size of N35UH permanent magnet material
B. Lamination.

In this design, the stator has 12 slots and the rotor has 8 poles. The SPP value is 0.5, which is a popular number for PM machine. Double-layer fractional slot concentrated winding (FSCW) structure is applied to the stator. The 0.35mm BaoSteel B35A270 is used both for the stator and rotor laminations. Its B-H curve is shown in Figure 3.4,

![B-H curve of steel B35A270.](image)

The dimensions of the stator as well as the rotor are listed in Table 3.2. And its lamination is shown in Figure 3.5.
Table 3.2. Dimensions of stator and rotor lamination.

<table>
<thead>
<tr>
<th>Items</th>
<th>Abbreviation</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>Stator Core Outer Diameter</td>
<td>$D_{so}$</td>
<td>100mm</td>
</tr>
<tr>
<td>Stator Core Inner Diameter</td>
<td>$D_{si}$</td>
<td>70mm</td>
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<tr>
<td>Stator Core Length</td>
<td>$L_{e}$</td>
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</tr>
<tr>
<td>Air Gap Length</td>
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<td>Rotor Core Outer Diameter</td>
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<td>68.8mm</td>
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</tr>
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<tr>
<td>Core Lamination Thickness</td>
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</tr>
<tr>
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</tr>
<tr>
<td>Slot Opening</td>
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</tbody>
</table>

Figure 3.5. Stator and rotor lamination.
One important parameter for a PM machine is its winding slot ratio. The larger this parameter is, the higher the electric load density would be, then the larger its inductance would be. The slot area is either calculated analytically or measured using the measuring tool in Ansoft. The total cross area of the copper winding is calculated by doing a summation of all the wires.

Stator Slot Area: 70mm$^2$

Copper Wire Area:

$$A_{cu} = 16 \times 10 \times \pi \times \left(\frac{0.62}{2}\right)^2 = 48.305mm^2$$  \hspace{1cm} (3.3)

Then, the slot ratio would be around 69%.

3.3. Finite Element Analysis

The Ansoft Maxwell Finite Element Analysis tool is used to simulate the prototype machine based on the above modeling dimensions. The main objective during design iteration was to achieve a reasonable flux and current density distribution at various load points. In addition, the design principle for a PM machine is to aim for the highest torque and torque density by properly sizing the PM material as well as d-axis to q-axis saliency ratio, so that a reasonable solution is obtained by balancing the low speed and high speed torque requirements. Meanwhile, the flux weakening current constraint to prevent the permanent demagnetization of PM material and saturation point of the iron lamination should also be carefully validated.

Figure 3.6 shows the flux density distribution and rotor airgap flux density along one pole pair. Three cases are simulated: (a) no load test, (b) 30A q-axis torque current, (c)
30A $d$-axis flux weakening current. The stator and rotor lamination is beyond the saturation point of the iron material at the rated stator current.

Figure 3.6. Flux density magnitude and airgap flux density. 
FEA provides a convenient design verification tool before final prototype fabrication. However, the design of a machine still needs accurate modeling by using analytical equations. Rich experience and solid background knowledge are essential. Since this is not the main part of this work and will not be analyzed in-detail.

3.4. Prototype Machine and Parameters Verification

3.4.1. PM Machine Prototype

The PM machine above is fabricated and its prototype is shown in Figure 3.7.

(a) (b)

Figure 3.7. Prototype IPM machine. (a) Stator assembly. (b) Rotor assembly.

The 3-D exploded view of the parts assembly in a motor housing are shown in Figure 3.8,
After having both the FEA model and realistic prototype, the design parameters are studied comparatively.

### 3.4.2. Stator Resistance

Stator resistance can be calculated based on the definition of resistance. Since the length of the coil, the diameter of the wire and its material are all known, the resistance is calculated as,

\[
r_s = \frac{4\rho Q n_s l_h}{3\pi n_p d^2}
\]  
(3.4)

Where, \(\rho\) is the resistance coefficient, \(Q\) is total number of stator slots, \(n_s\) is number of wires per slot, \(l_h\) is the half of the length of a coil, \(n_p\) is the number of parallel wires per strand, \(d\) is the diameter of the wire,
If we plugging all the parameters into equation (3.4), the stator resistance turns out to be,

$$r_s = \frac{4 \times 1.85 \times 10^{-5} \times 12 \times 16 \times 0.059}{3 \pi \times 10 \times (0.62 \times 10^{-3})^2} = 23.13 \text{m}\Omega$$

(3.5)

The stator resistance is also measured by injecting line-to-line DC current as,

Table 3.3. Measured winding DC resistance.

<table>
<thead>
<tr>
<th>Line to Line</th>
<th>Value (m\Omega)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>50.3</td>
</tr>
<tr>
<td>BC</td>
<td>53</td>
</tr>
<tr>
<td>CA</td>
<td>52.4</td>
</tr>
</tbody>
</table>

The actual tested phase resistance is slightly higher than the calculated value. This is mainly because the length of the winding end across the stator tooth is not included in above calculation.

### 3.4.3. Back-EMF

The Back-EMF waveforms through testing as well as FEA simulation are presented in Figure 3.9. Figure 3.10 compares the magnitude of different harmonic orders. The tested Back-EMF waveform has almost the same fundamental frequency magnitude as the simulated one. But its harmonics for different orders are much larger than the simulated model. This is most likely because the Ansoft 2-D modeling cannot take the armature end connections into account. However the large ratio between the size of the
stator radius to the stack length of this design makes the terminal winding accounting for large effect for the overall performance.

![Tested and Simulated Back-EMF Voltage](image1)

**Figure 3.9.** Simulated and tested back-EMF waveform.

![Back-EMF Harmonics Comparison](image2)

**Figure 3.10.** Harmonics of simulated and tested back-EMF.
3.4.4. Inductance Verification

Inductance is a critical parameter for motor design, because it affects the control performance in large. Therefore, precise measuring of inductance is necessary for a better control.

A. Inductance Calculated by Ansoft Simulation

The inductance is a parameter to measure the magnetic circuit reluctance. This reluctance is inverse proportional to the magnetic permeability of the material. While, the permeability of any material is constant at low flux density range and begins to decrease when the material is saturated. So, the inductance of a magnetic circuit will change with respect to injected current level.

Figure 3.11 and Figure 3.12 show \( d \)-axis and \( q \)-axis inductance variation curves with respect to the injected current on that axis brought by Ansoft simulation. The \( d \)-axis inductance increases when a flux-weakening current (negative) is injected into the stator winding; it decreases when a flux-strengthening current (positive) is injected. Similarly, the \( q \)-axis inductance decreases when the injected current along \( q \)-axis increases.
B. Inductance Measured through Test

There are multiple ways to test inductance of a machine, either with inverter or without inverter. Basically, the measurement of machine inductance is still based on its
terminal voltage equation. When using the inverter to calibrate the machine inductance, the nonideal of the inverter, including its voltage drop and digital time delay, may cause the actual machine terminal voltage deviate from its command voltage [66]. Large analog amplifier with high current rating is a feasible solution since it won’t cause any phase shift. Since we don’t have that in the lab, we have to come up with another method. A simple stand-alone DC power supply is able to do the job by injecting current into the winding and measuring the phase voltage on its terminal. This job includes several steps:

1. Inject current to align the rotor to a certain direction. Tune the output voltage back and forth so that the current is around its rated value or at least at the same quantity level. Write down the steady state voltage value.

2. Tune the DC power supply voltage to the same voltage value in step (1) without connecting this power supply to the winding.

3. Hook up the machine winding to the power supply instantly and record both the machine terminal voltage and current waveforms and wave data points.

4. Calculate the current waveform based on the above voltage waveform and FEA simulated inductance. If this final current overlaps with the current waveform obtained in step (3), the FEA result is verified by the experiment.

First, the d-axis inductance is tested based on the above method. The d-axis or center line of the N pole of the rotor is aligned to Phase A axis of the stator winding by injecting Phase A current vector. This is realized by connecting motor Phase A terminal to the positive terminal of the DC power source and Phase B, Phase C together to the negative supply of the DC source. Repeat these steps (1)-(4) above and we can get the voltage and current waveforms in Figure 3.13.
Figure 3.13. Current and voltage response for testing d-axis inductance.
(Phase A connected with positive knob, Phase B and C connected with negative knob)
Trace 1: DC power supply voltage. Trace 4: DC power supply output current.

The instantaneous voltage equation of the above experiment is expressed as,

\[ u^* = \frac{3}{2} r_{sa} i_a + L_d \frac{di_a}{dt} \]  \hspace{1cm} (3.6)

So, the Phase A current is calculated and given by,

\[ i_a = \frac{u^*}{1.5 \times r_s} \left( 1 - e^{-\frac{r_s}{L_d} \frac{1}{u^*}} \right) \]  \hspace{1cm} (3.7)

Equation (3.7) is used to predict the current \( i_a \) with the real voltage waveform \( u^* \) in Figure 3.13 and the FEA simulated inductance \( L_d \). According to step (4), both the currents from the test in Figure 3.13 as well as the prediction using equation (3.7) are plotted together in Figure 3.14. These two curves are overlapping with each other pretty well, which verifies that the simulation result is consistent with the test result.
Similar to the characterization to the d-axis inductance $L_d$, the same job can be carried out to calibrate the q axis $L_q$. The only difference is that the injected d-axis current is going to align the rotor to its stable equilibrium point from mechanical aspect. However, the q-axis current will lead to an unstable equilibrium point and the rotor tends to move away from this point soon after applying the q-axis current. We will come up with a method to figure this out later on. The same four test steps are implemented. Step (1) is carried out the same as that for d-axis inductance testing, the rotor is aligned to Phase A axis. Then, a large clamp is used to fix the rotor after the above alignment. Then, in step (3), the motor terminals will be hooked up to the power supply. However, in this case, Phase B terminal is connected to positive supply and Phase C is connected to negative supply, Phase A is left open. By doing so, the injected current vector would stay along the q-axis of the rotor.
Figure 3.15 records the instantaneous voltage and current waveforms. Small oscillation in the current waveform is observed on its rising slope. This is mainly because the clamp cannot reject the increasing torque and keep the rotor motionless.

![Figure 3.15 - Current and voltage response for testing q-axis inductance.](image)

(Phase B is connected with positive nob, Phase C is connected with negative nob, Phase A is left open).

Trace 1: DC power supply voltage. Trace 4: DC power supply output current.

The voltage equation for this case is different from equation (3.6) for their different connections, which gives,

\[ u^* = 2r_i b_{\phi} + \frac{3}{2} L_q \frac{di_{b_{\phi}}}{dt} \]  \hspace{1cm} (3.8)

Therefore, the current, conducting through Phase B and Phase C, is obtained by solving the above differential equation, which gives,
When comparing the current obtained through equation (3.9) using the FEA predicted q-axis inductance with the tested result in Figure 3.15, Figure 3.16 shows a perfect match.

![q-axis Theoretical and Real Current](image)

Figure 3.16. q-axis current response to step voltage.

### 3.4.5. Efficiency Test

Efficiency is one of the most important criterions to evaluate the designed motor for traction application. However, its measurement is also the most difficult one and needs precision testing equipment support and lots of repeated work. The measuring accuracy depends both on the testing equipment and testing method. In this experiment, the YOKOGAWA newly released WT1800 power analyzer is used to draw the efficiency map. Almost all the current points on the second quadrature of $i_d-i_q$ plane with fix
intervals are swept. The measuring of electric power is acquired by using three voltage and current measuring terminal elements. The mechanic power is measured by torque and speed measuring element.

Two identical PM machines mounted on the dyno bench are coupled together by their shafts. Two inverters--one to drive the machine in motoring mode and another to drive the machine in generator mode--are used with their DC bus tied together. The test setup is shown in Appendix B.4.

Finally, the efficiency map of the prototype machine is drawn in Figure 3.17. If the frequency above 94.5 percent is regarded as high efficiency area, it is located at the speed from 4800rpm to 7000rpm and the torque from 0.2Nm to 1.5Nm of this particular motor.

![Figure 3.17. Efficiency map of the prototype machine.](image-url)
3.4.6. Output Torque and MTPA Curve

The machine is supposed to run on the MTPA curve below the base speed. Its output torque to absolute current curve and current vector angle for each point on MTPA curve are studied comparatively.

Figure 3.18 shows the torque-current curves of both the simulated result and tested result. They are matching with each other very well. The error is around 2.8 percent if the tested result is used as its base.

![Torque Output Comparison](image)

Figure 3.18. Torque to current curves.

The MTPA curve is shown in Figure 3.19. The simulated and tested results are not matching with each other pretty well. In the actual test, it is not easy to identify the angle of the current vector with the maximum torque output, because the torque doesn’t change too much when changing the current angle at a certain area. The accuracy of the torque
transducer, the mechanical alignment, and the friction and coupling inside can affect the testing results.

![MTPA curve](image)

Figure 3.19. MTPA curves of both the simulation result and test result.

### 3.5. Summary

In this chapter, the design and testing of a prototype PM machine is presented. Based on the application requirements, the rough dimension of the PM machine is obtained. Ansoft simulation model is build and its detailed scaling is simulated by using the trial and error method. A prototype machine is fabricated using Nd-Fe-B permanent magnetic material. This PM motor is calibrated in terms of its various parameters and output characteristics.

Other than the work addressed above, a lot of detailed work is expected to optimize this motor. For example, the optimized design of machine parameters based on its per
unit flux and saliency ratio plane [17]. Efficiency targeted motor structural optimization is expected to minimize motor harmonics, reduce cogging torque, and relocate its highest efficiency area to the frequent operation region during city driving. PM machine design requires multi-discipline background knowledge for its highest design flexibility. Each of these optimization points can be one specific research topic and is out of the focus of this work.
CHAPTER 4
PM MACHINE CONTROL CONSIDERING COUPLINGS

The frequency response based classical control principle serves as the design criterion for the dynamic system. However, it only applies to linear systems with single input single output (SISO). A PM machine is a multi-input multi-output (MIMO) system with strong nonlinearity. Therefore, the classical control theory is not applicable. The matrix based nonlinear MIMO control theory is complex and seems not intuitive. Literature shows that the frequency response analysis can still be applied if the system is properly decoupled and linearized. The control of a PM machine using classical frequency response analysis is presented in this chapter.

4.1. Space Vector Field Orientation Control

Space-vector field-oriented control has become the most popular machine control method for induction machines and PM machines with three-phase voltage-source inverters (VSI). The decoupled control of flux and torque in a synchronous reference frame makes the control of a three phase machine as simple as a DC motor.
The position signal is the main reference for the vector control of a PM machine. However, it is also the most vulnerable part in the drive system. It needs multiple components including the sensor, the mechanical housing, signal transmission and shielding cable, and signal conditioning circuit, which eventually degrade system reliability. EMI noise is easily coupling into the system because of the long cable. Maintenance work is indispensable to inspect or replace the sensor because of its high damage risk due to its harsh running environment. A lot of research works are accomplished, trying to extract position signal from these sensed voltage and current variables with acceptable accuracy and dynamic response. However, due to the higher torque demand of the traction drive at standstill, position sensor is still vital for the control.

4.1.1. Position Acquisition

There are various types of principles that may be used to detect the position such as the most popular hall principle, the photoelectric principle or the electromagnetic principle. The most popular sensors are resolver, encoder, and hall sensors. Here, three discrete hall sensors are used to extract position signal. The hall sensors may use the main magnetic field as its detection source or use an extra mounted co-axil magnet plate. The former one calls less but the armature field will affect the detection angle, while the latter one is vice versa. The latter method is adopted in our system for the high rotation speed requirement.
4.1.2. Position Acquisition Circuit

TI 28035 DSP chip only accepts one capture signal input. However, there are three hall signals coming out of the motor. We only care about the switching edges of each channel; a digital circuit is implemented to combine all three hall sensor outputs into one channel.

![Diagram of hall sensor feedback signals](image)

(a) 60 degree hall sensor. (b) 120 degree hall sensor.

Figure 4.1. Different hall sensor feedback signal.

For three digital signals, there are eight states combinations total. There are generally two types of configuration, the 60 degree type in Figure 4.1(a) and 120 degree type in Figure 4.1(b).

So, we have to design a logic circuit to have each edge captured in the combined output channel. Table 4.1 shows the truth table of the input and output digital signal and it can be translated into Karnaugh map in Table 4.2.
Table 4.1. Truth table of Hall signal conversion.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.2. Karnaugh map.

<table>
<thead>
<tr>
<th>A</th>
<th>BC</th>
<th>B̅C</th>
<th>BC̅</th>
<th>B̅C̅</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>×(1)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>×(0)</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

By filling the null state in Table 4.2 with the same digit around it, the Karnaugh map can be further simplified using analytical expression of equation (4.1) and equation (4.2).

\[
\overline{A}B + AC + \overline{B}C \quad (4.1)
\]

\[
\overline{\overline{A} + B + \overline{A} + C + B + \overline{C}} \quad (4.2)
\]

Based on equation (4.2), the logic circuit is obtained as in Figure 4.2,
After having all the circuit logic, we made position sensor mounting PCB and rack. The mechanical installation angles of hall chips for the 60° and 120° electrical angles are,

\[
\frac{k \times 60}{n_p} \quad k = 1, 2, 4, 5, \ldots \quad (4.3)
\]

\[
\frac{k \times 120}{n_p} \quad k = 1, 2, 4, 5, \ldots \quad (4.4)
\]

Where, \( k \) is the coefficient of hall sensor mounting angle and can be any number except 3 and integral times of 3. \( k \) can be as small as possible, only limited by the chip package and conditioning circuit.
4.1.3. Position Signal Calibration with Back-EMF

For each output edge of the hall sensor, there will be an absolute electrical position value. Before we can extract the position information from these hall signals, the calibration of these hall edges are necessary. Theoretically, we only need to know the edge of one hall sensor, if all these hall sensors are positioned exactly 60° apart from each other. However, these are certainly mechanical assembly error and cannot be very accurate at prototype development. It may not make any difference for a low speed drive but will result in an inaccurate field orientation for a motor running at super high speed. So, all the six edges are precisely calibrated.
4.1.4. Position Acquisition Algorithm

A. Position Acquisition at Motor Start-up

The motor has to start from zero speed with tough loading. The position closed loop field oriented control is engaged from the beginning. However, the position sensor can only tell which section the rotor is positioning without any speed information at start-up. If a fix angle is used as the decoupling angle, the output torque could be either positive or negative depending on the rotor stop position. A start-up method is used here to sweep the angle in the first section repetitively until the controller captures two consecutive hall edges. In Figure 4.5, E1 and E2 are two consecutive edges, and the field orientation angle and actual rotor position angle may have different changing rate initially. After the rotor passing over E2, the field angle can be calculated based on the average speed from E1 to E2 and time passed by after E2. Now, the fired angle would gradually be close to the actual angle and they would finally overlap with each other after several cycles.
**B. Position Acquisition at Motor Running**

After the motor is started, its speed can easily be extracted using consecutive edges passing by dividing the known distance over the time period.

### 4.2. Control Parameters Tuning for a Simple RL Load

For the hybrid traction drive, its torque needs to be actively controlled. Usually, this is done by active-controlled current feedback loop. Figure 4.6 illustrates the general current loop with different blocks shown different parts of the system. The main challenge is to design a proper current controller (usually a PI controller) in reference to the control plant, so that its steady-state and dynamic performance (e. g. steady state error, rise time, settling time and overshoot height) are all qualified.
4.2.1. Modeling of Current Control Loop

Generally, the modeling of an electromechanical control plant $G_p(s)$ can be divided into electrical model $G_{pe}(s)$ and mechanical model $G_{pm}(s)$. Since in our analysis below, we only focus on current control loop. $G_p(s)$ is used to designate the electrical model of the plant. A three phase PM machine is a typical example. However, the modeling of a PM motor is intricate because of its nonlinearity and various coupling inside it. So initially, a simple $RL$ load is used to generalize the controller design.

For a typical $RL$ plant, it can be treated as a first order inertial system. Digital implementation delays, mainly including sample and hold delay, digital output delay, are all unified as another first order inertial system. A typical plant considering delays can be summarized with a second order transfer function,

$$G_p = \frac{K_p}{(1+sT_1)(1+sT_2)} \quad (4.5)$$

Where, $T_1$ is the main constant of the plant; $T_2$ is the cascade connection of multiple delay elements. They can be expressed as

$$T_1 = T_{rl} = \frac{L_s}{R_s} \quad (4.6)$$


\[ T_2 = T_{d\text{PWM}} + T_{fi} \]  \hspace{1cm} (4.7)

Where, \( L_s \) is motor stator inductance; \( R_s \) is motor stator resistance; \( T_{d\text{PWM}} \) is the delay associated by PWM inverter, and \( T_{fi} \) is the feedback current filter delay.

If we plug in all the known transfer function into Figure 4.6, the system block diagram is shown as,

![Figure 4.7. Current loop block diagram of a RL load.](image)

The open loop transfer function is given by,

\[ G_c G_p = \frac{K_r K_p (1 + s T_r)}{s T_r (1 + s T_1)(1 + s T_2)} \]  \hspace{1cm} (4.8)

Our goal is to choose the proper PI parameters, \( K_r \) and \( T_r \), such that the dynamic response of the system can meet the requirements. Generally, there are two standard methods used to determine the parameters: the technical optimum (TO) and the symmetric optimum (SO) [29].

### 4.2.2. Technical Optimum
A standard procedure to simplify the open loop transfer function (or design the controller parameters) is to set the time constant of PI controller to be equal to the dominant time constant of the plant. So, we can get the integral parameter as,

\[ T_r = T_{rd} = T_1 \]  \hspace{1cm} (4.9)

As a result, equation (4.8) can be reduced to,

\[ G_{open} = G_c G_p = \frac{K_r K_p}{sT_1(1 + sT_2)} \]  \hspace{1cm} (4.10)

The closed loop transfer function of the current feedback loop is,

\[ G_{close} = \frac{G_c G_p}{1 + G_c G_p} = \frac{K_r K_p}{s^2 + \frac{2}{T_2} s + \frac{K_r K_p}{T_1 T_2}} \]  \hspace{1cm} (4.11)

Equation (4.11) is a second order low pass filter with per unity DC gain. By making an analogy to the equation of a standard second order system definition, these parameters can be associated with the basic coefficients of a second order system as,

\[ \omega_n^2 = \frac{K_r K_p}{T_1 T_2} \]  \hspace{1cm} (4.12)

\[ 2\xi \omega_n = \frac{1}{T_2} \]  \hspace{1cm} (4.13)

Thus, the eigenvalues of the closed-loop system can be located on the s plane as in Figure 4.8.
If a damping factor $\xi = \sqrt{2}$ is preferred, the gain of the PI regulator would be

$$K_p = \frac{1}{4\xi^2} = \frac{1}{K_p \frac{T_1}{2T_2}}$$

By substituting equations (4.6), (4.7) and (4.14) into equation (4.8), the following closed loop transfer function is obtained,

$$G_{\text{close}} = \frac{T_{rl}}{T_{d\text{pwm}} + T_{fi}} \frac{1}{4\xi^2} \frac{T_1}{T_{d\text{pwm}} + T_{fi}} + T_{rl}^2 + T_{rl} + \frac{T_{rl}^2}{4\xi^2}$$

The cut-off frequency of the system is calculated based on the

$$\omega_c = \frac{1}{T_2} \sqrt{\frac{1 + \frac{1}{4\xi^2} - 1}{2}} = \frac{1}{T_2} \sqrt{\frac{\sqrt{2} - 1}{2}} \approx 0.455 \frac{T_2}{T_2}$$

Figure 4.8. Eigenvalues of the closed-loop system.
By adopting the classical second order system design methodology, the current loop can be tuned to have the ideal characteristic of a second order system.

### 4.2.3. Symmetrical Optimum

The technical optimum follows exactly the theoretical way of designing a controller based on its frequency response. But sometimes, more emphasis lies on some specific target over other parameters. Then a common procedure for the determination of regulator parameters is symmetrical optimum (SO). The gain of regulator is selected as

$$K_r = \frac{1}{K_p} \frac{T_1}{a T_2}$$  \hspace{1cm} (4.17)

The integral time constant is

$$T_r = a^2 T_2$$ \hspace{1cm} with \hspace{0.2cm} $$a = \frac{1 + \cos \psi}{\sin \psi}$$  \hspace{1cm} (4.18)

Where, \( \psi \) is the phase margin, 45 degrees is recognized as a moderate value for \( \psi \) for its dynamic performance. Thus, \( a \) equals 2.4 based on this criterion.

The frequency responses of both TO and SO methods are illustrated in Figure 4.9. The gap between the plant time constant \( T_1 \) and delay time constant \( T_2 \) determines the difference of frequency response of TO and SO method. The smaller the gap is, the closer these two methods become, which is shown in Figure 4.9(a). If \( T_2 \) is much smaller than \( T_1 \), then the frequency response would be different as in Figure 4.9(b). Generally, the SO method tends to increase the gain of the system at low frequency at the expense of degraded phase response.
4.3. Cross Coupling and Compensation

The real plant, a three phase PM motor, is a nonlinear system with the rotor spinning at the synchronous speed. In order to simplify the system such that the classical frequency response methodology can be applied to it, the $d,q$ decoupling transformation is employed to make the flux and torque control separate. However, any system internal elements that have phase shift effect can cause residual coupling. The most prominent case is those delays associated with digital implementation. Next, the effects of cross coupling will be analyzed and its compensation will be implemented all based on its frequency response.

Figure 4.9. Frequency response of the TO and SO methods.
4.3.1. Why Cross Coupling Analysis?

Cross coupling is not prominent at low rotational speed and doesn’t affect the control performance too much. However, as the speed increases, the fundamental frequency also increases, which the cross coupling effect would cause system stability issue.

Figure 4.10 shows the system reaction as its speed ramps up to around 9000rpm. The current begins to oscillate at certain point and becomes out of control when the current regulator is saturated. Finally, the current touches its protection limit and trips the shutdown of the system.

Figure 4.10. System oscillation and trip protection at 9000rpm.
Trace 1: DC bus voltage; Trace 2: Phase current; Trace 3: Output voltage command magnitude; Trace 4: q-axis current reference.
4.3.2. Complex Vector Modeling

Complex vector [1, 27, 28], an effective way to simplify a multiple-input/multiple-output system to an equivalent single-input/single-output system, is used to deduce the root locus and frequency-response function of the system with strong couplings. The primary goal of this chapter is to use the complex root locus and complex frequency response to analyze a Permanent Magnet machine in such a way that the same controller design methodology for a simple RL load can be applied.

Usually, the complex vector can be represented using two forms, the Cartesian coordinate form of equation (4.19) or the Polar coordinate form of equation (4.20).

\[
f_{dq} = f_d + j f_q \tag{4.19}
\]

\[
f_{dq} = F e^{j \theta} \tag{4.20}
\]
4.3.3. Modeling of Cross Coupling

The biggest coupling term is not delays but the mechanical rotation of the machine, which the PM rotational back-emf is presented on the $q$ axis, if the field is accurately oriented toward the rotor flux. Usually, the mechanical time constant is considerably larger than electromagnetic time constant. If we take a look at the pole-zero map of the transfer function, the pole brought by this mechanical rotation on the left $s$ plane is far away from the image axis, which can be easily ignored. The only exception is the servo drive, which requires faster dynamic performance. Usually, the diameter of the servo motor rotor is pretty small and the same as its rotor inertia. This can lead to strong coupling of its mechanical constant to the current control loop and is not considered here.

Figure 4.13 shows a real motor drive system block diagram with combined stationary and synchronous system reference frame. Since there are two sets of reference frame in Figure 4.13, a common approach is to unify the whole system into synchronous reference
frame with complex notation. Therefore, those blocks in stationary reference frame needs to be transferred into synchronous reference frame.

\[
G_c(s) = \frac{K^*}{sT_r + 1}
\]

\[
G_p(s) = \frac{K^*}{1 + sT_{dPWM}}
\]

\[
G_f(s) = \frac{1}{1 + sT_{fi}}
\]

\[
G_{PWM}(s) = \frac{K_{PWM}}{1 + sT_{dPWM}}
\]

\[
G_{RL}(s) = \frac{K_{RL}}{1 + sT_{RL}}
\]

Figure 4.13. Original reference system with hybrid stationary and synchronous reference system.

The delays associated with PWM inverter and current feedback filter can be approximated by a first-order delay function in stationary reference axis, which is shown as \(G_{PWM}\) and \(G_f\) in Figure 4.13. They can be converted into differential equations in the time domain as,

\[
T_{dPWM} \frac{dv_{dq}^s}{dt} + v_{dq}^s = v_{dq}^{s*}
\]  \(4.21\)

\[
T_{fi} \frac{di_{dq}^s}{dt} + i_{dq}^s = i_{dq}^{s*}
\]  \(4.22\)

The electrical modeling of a plant \(G_p\) can be depicted as,

\[
T_{RL} \frac{di_{dq}^s}{dt} + i_{dq}^s = K_{RL} v_{dq}^r
\]  \(4.23\)
Equations (4.21), (4.22), and (4.23) are all multiplied by the stationary to synchronous vector rotator $e^{-j\omega t}$. Let’s first focus on PWM inverter modeling equation which can be transferred as,

$$e^{-j\omega t} (T_{dPWM} \frac{dv_{dq}^e}{dt} + v_{dq}^e) = e^{-j\omega t} v_{dq}^e$$ (4.24)

If we move the vector rotator inside and combine with these voltage vectors in the stationary reference frame, it becomes,

$$T_{dPWM} \frac{dv_{dq}^e}{dt} + (1 + j\omega T_{dPWM}) v_{dq}^e = v_{dq}^e$$ (4.25)

And, the transfer function in synchronous reference frame becomes,

$$G_{PWM}(s) = \frac{v_{dq}^e(s)}{v_{dq}^e(s)} = \frac{K_{PWM}}{T_{dPWM}s + (1 + j\omega T_{dPWM})}$$ (4.26)

Similarly, the plant and current filter delay can be written as,

$$G_{p}(s) = \frac{i_{dq}^e(s)}{v_{dq}^e(s)} = \frac{K_{RL}}{T_{RL}s + (1 + j\omega T_{RL})}$$ (4.27)

$$G_{f}(s) = \frac{i_{dq}^e(s)}{i_{dq}^e(s)} = \frac{1}{T_{f}s + (1 + j\omega T_{f})}$$ (4.28)

Being different from the common definition of transfer function with all real coefficients and either real poles or a pair of conjugate poles, these transfer functions under synchronous reference frame have only one single complex pole on the s plane.

Accordingly, the closed loop signal flow diagram can be depicted in Figure 4.14.
The open loop transfer function is obtained by multiplying all the four blocks together, which comes to,

\[
G_0 = \frac{K_r(1+sT_r)}{sT_r} \frac{1}{T_f s + (1+j\omega T_f)} \frac{e^{j\omega T_{PWM}}}{T_{dPWM} s + 1} \frac{K_{PWM}}{T_{RL} s + (1+j\omega T_{RL})} \tag{4.29}
\]

If we change Figure 4.14 into \(d\)-, \(q\)- axis scalar form, all these transfer function with complex poles evolve into the coupling terms between \(d\)- and \(q\)- axis, which is shown in Figure 4.15.
4.3.4. Cross Coupling Compensation

In general, two main cross-coupling items need to be compensated; one is the cross-coupling inside the plant, the other is the cross-coupling the results from delays. In the next part, the frequency response and the root locus curve are employed to compare the system response before and after compensation. Our final target is to convert the system in Figure 4.15 to that in Figure 4.16, so that all the techniques used to design the controller for a RL load can be implemented to the design of the motor controller.
Figure 4.16. Signal block diagram of decoupled controlled system in scalar format.

If all the cross-couplings are compensated, the open loop transfer function becomes,

\[
G_0(s) = \frac{G_c(s)}{1+G_p(s)} \frac{1}{1+G_{PWM}(s)} \frac{1}{K_{PWM} \cdot \frac{1}{1+sT_{PWM}}} 
\]

\[
= \frac{K_c (1+sT_r)}{sT_r} \frac{K_{RL}}{1+T_{RL}s+(1+j\omega_c T_{RL})} \frac{1}{1+K_{PWM} \cdot \frac{1}{1+sT_{PWM}}} 
\]

(4.30)

A. Plant Feed Forward Compensation

The coupling terms inside a plant can be compensated by subtracting the same coupling terms in the control loop based on the sensed current. Then the signal diagram of the compensated system turns out to be,
Figure 4.17. Signal diagram after plant feed forward compensation

The open loop and close loop transfer function can be written as,

\[ G_{0\_comp} = \frac{G_c G_{PWM} G_p G_{fi}}{1 - G_{PWM} G_p G_{fi} j\omega L_s} \]  

\[ G_{c\_comp} = \frac{G_c G_{PWM} G_p}{1 - G_{PWM} G_p G_{fi} j\omega L_s + G_c G_{PWM} G_p G_{fi}} \]  

**B. Delay Caused Coupling Compensation**

Theoretically, delays cannot be compensated because people cannot predict the status of the system for the next step. The one need to be compensated is the coupling terms brought by delays. Figure 4.18 shows the system diagram after delay compensation. It is pretty much the same as that in Figure 4.17 with only the elimination of these coupling terms in \( G_{PWM} \) and \( G_{fi} \).

Figure 4.18. Signal diagram after both feed forward and delay coupling compensation.
The coupling terms of the system in Figure 4.18 is still not fully compensated. If the full decoupled system is expected, we may have to move the plant decoupling term $j\omega_L s$ to the plant side rather than keep it in the controller side. Figure 4.19 shows the completely decoupled system with its open loop transfer function to be,

$$G_0 = \frac{K_r (1 + s T_r)}{s T_r} \frac{K_{PWM}}{T_{dPWM}s + 1} \frac{K_{RL}}{T_{RL}s + 1} \frac{1}{T_p s + 1}$$

(4.33)

However, this is not achievable if the time delay constant in block $G_{fi}(s)$ is large enough.

**Figure 4.19. Fully decoupled system.**

### 4.3.5. Time Domain Compensation of Cross Coupling

After reviewing all the coupling terms and its compensation methods inside the control loop, the next part is to evaluate the compensation in both time domain and frequency domain.

In time domain, the exact elements representing these time delay constants are examined. The effect of these delays on both the $d$- and $q$- axis current vector plane and
the PWM carrier signal timing are shown. The decoupling compensation is performed in the real experiment.

A. Time delay caused by current sampling

Current sampling time delay can be separated into two parts: the current sensor inherent delay and the delay associated with current signal conditioning circuit. The system is operating with the current closed-loop controlled in $d,q$ synchronous reference frame. Figure 4.20 shows the reference current vector $I_{s,\text{ref}}$ and the actual output current vector $I_{s,\text{act}}$. The current PI controller will always keep the feedback current vector the same as its reference vector. However, the feedback current vector is always lagging the actual machine terminal current vector because of the sampling delay. If the speed of the machine is almost constant, then the delay angle $\theta_1$ can be estimated by multiplying the speed with that time delay. If replacing the $a,b,c$ to $d,q$ transformation angle $\theta$ with $\theta-\theta_1$ on purpose, the coupling caused by current sampling delay can be compensated. Instead of having the output current $I_{s,\text{act}}$ as in Figure 4.20, which deviates from the reference current $I_{s,\text{ref}}$, we are able to have the output current $I_{s,\text{act\_comp}}$ after this compensation, which tends to overlap with the reference current $I_{s,\text{ref}}$.

Other than analysis with current vector, the current sampling delay can also be analyzed in the time domain, which is shown in Figure 4.21. The triangle waveform is PWM carrier waveform. Usually, the microprocessor will sample three phase current at point A. However, this current after the current sampling and conditioning circuit is not the instantaneous current at the inverter side because of the delay. It is the inverter current before the time delay at point B. When doing transformation of the current from a,b,c
coordinate to $d,q$ coordinate, the position angle at point B as ($\theta - \theta_1$) is used instead of using the position angle at point A as ($\theta$).

![Figure 4.20. Current vectors and its compensation in orthogonal coordinates.](image)

Figure 4.20. Current vectors and its compensation in orthogonal coordinates.

![Figure 4.21. Current sampling time delay in one PWM period.](image)

Figure 4.21. Current sampling time delay in one PWM period.

Figure 4.22 shows the real experiment before and after this compensation. This test is done on two identical mechanically coupled PM motors with a torque sensor mounted between them. Suppose Motor-B is the prime mover and is set to running at 1200rpm.
Motor-A is the drive system being tested. Look at the first curve in Figure 4.22(a), a negative d-axis current is injected into Motor-A. Ideally, the torque on the shaft shouldn’t change if only d-axis current is loaded. However, in the same figure, both the output from the torque sensor (the second curve) and the Motor-B current (the fourth curve) change, which indicates setting d-axis current of Motor-A will bring torque output. This is mainly because of the delay brought by current sampling. If we bring the angle back as that in Figure 4.20, the reference and actual current of Motor-A would be perfectly matched and no torque would be generated as that in Figure 4.22(a).

(a). Without current sampling compensation.

Figure 4.22. d-axis current step response(20A) at constant speed(1200rpm). continued
B. Time delay caused by PWM voltage update

Different from analog controller, all the digital microprocessors have a fixed time interval for signal sampling and algorithm calculation. The sampled signal will be used to generate the control signal for the next step. This is the same story in the motor drive controller. This time interval can be either short or long depending on the setting. It is common to have one and a half the PWM period as the time delay for the motor controller.

The PWM delay is essential for digital controlled power electronics systems. If calculating the output voltage based on the last sampled current and position, error is expected because the real position changes at the voltage updating moment. So instead of using last sampled position, we have to predict the position at the voltage updating moment on the average speed perspective. Figure 4.23 shows this delay in the time
domain in reference to the carrier signal. The sampling moment is at point A, the voltage output register update is at point B, the real voltage output in average aspect is at point C. This is the reason for one and a half PWM period delay.

If we look at the voltage vector in Figure 4.24, the synchronous reference frame is $d,q$ axis at point A in Figure 4.23. The $d,q$ axis moves to $d',q'$ axis after the PWM output Delay $\theta_2$. The controller will output voltage vector $v_q$ at point C, since it is calculated based on the angle at point A. So, the required voltage vector at point C is not the output voltage from the inverter, then unexpected current is generated for the voltage difference. Of course, the current regulator will function at this scenario to output correct voltage $v_q'$, but this voltage vector will have both $d,q$ axis component, which is supposed to only have $q$ axis component. This actually shows the coupling.

The similar verification experiment as in Figure 4.22 will be implemented in Figure 4.26. If we set the reference current of Motor-A to be zero and ramp up the speed of Motor-B, the gradually increasing back-emf voltage inside Motor-A will always reside on a $q$ axis base and no voltage component is on $d$-axis. However, the $d$-axis voltage in Figure 4.25(a) is decreasing toward the negative side as the speed goes up, which matches that in Figure 4.24. After applying the compensation, the $d$-axis voltage in Figure 4.25(b) maintains zero as the speed goes up.
Figure 4.24. PWM output delays in the time domain.

Figure 4.25. PWM output delay and its compensation in d-q coordinates.
Figure 4.26. d-axis command voltage variation with speed acceleration from 600rpm to 3600rpm.

(a) Without PWM update compensation.

(b) With PWM update compensation.
4.3.6. Frequency Analysis of Cross Coupling

With the help of complex vector notation, the PM motor can be analyzed with conventional root locus method and frequency response function method. Thus, all the frequency domain criterions can be used to gauge the system performance.

A. Uncompensated System Response

If we leave both the machine coupling term and delay coupling term uncompensated, the open loop transfer function is shown in equation (4.29). If the synchronous frequency is treated as the varying parameter of the root locus curve, its close-loop root locus is shown in Figure 4.26. Since the fourth pole of the system is far away from image axis in the left plane, only three poles are shown in the figure. When the frequency increases from 0Hz to 1kHz, one of the poles moves across the image axis to the right half plane.

![Figure 4.27. Complex root locus of the current regulator without decoupling compensation.](image)
This dominant pole closest to the imaginary axis is canceled out by the pole of the controller at 0Hz. However, as the frequency increases, the pole of the system moves to the right half plane of the s domain. The transition happens to be around 500Hz. If we take a look at its open loop frequency response in Figure 4.27, both the gain and phase response deteriorate as the frequency increases.

![Bode Diagram](image)

Figure 4.28. Bode plot with no compensation (0Hz, 50Hz, 200Hz, 500Hz, 1000Hz).

### B. Frequency Response with Plant Feed Forward Compensation

The coupling between $d$-, $q$- axis of the plant can be partially compensated by doing feed forward compensation based on the sampled current. One of the poles far away from image axis in Figure 4.26 is pulled back to be another dominant pole. Two of its poles still move into unstable region when the frequency going as high as 450Hz to 500Hz.
basically, the plant decoupling doesn’t improve the system stability. If we take a look at its frequency response in Figure 4.27, it does improve the phase response curve at around 200Hz, but it gets a worse phase response at higher synchronous frequency.

![Pole-Zero Map](image)

Figure 4.29. Complex root locus of the current regulator after plant coupling compensated.
C. Frequency Response after Delay Coupling Compensation

The last step is trying to remove all the couplings including the plant coupling and the delay couplings. If we take a look at the root locus in Figure 4.30, the dominant pole is compensated by the zero of the controller, and all the poles are at the left hand-side of the s plane, which indicates the system is a stable system all up to 1KHz. Figure 4.31 shows its frequency response. The phase response no longer drops below -180 degree at low synchronous frequency. The gain response doesn’t change a lot when the synchronous frequency goes higher because the synchronous frequency dependent terms in equation (4.29) are compensated.
Figure 4.31. Complex root locus of the current regulator after all the coupling compensated.

Figure 4.32. Bode plot with all the coupling compensated (0Hz, 50Hz, 200Hz, 500Hz, 1000Hz).
4.4. Simulation

All the analysis above is based on system average mathematical model. The SimPowerSystem library in Matlab supports the modeling of the real inverter with PWM switching, which is pretty close to the actual system. This power model is used to examine the accuracy of the modeling based on average signal perspective.

4.4.1. Step Response

Step response is used to calibrate the controller response in the time domain. Figure 4.32 shows the $d$, $q$-axis current step response of both the average model and power switching model. For exactly the same PI parameters, the current waveforms of power model and average model overlap with each other, which demonstrated the correctness of the average model. Thus, the average model based frequency analysis in section 4.3 is reasonable.
4.4.2. Stability Comparison

The frequency response in Figure 4.31 after coupling compensated shows a better stability margin than that in Figure 4.30. This is further demonstrated by simulation results. Figure 4.33 and Figure 4.34 show the simulation without and with cross-coupling compensation. The sub-figures listed from top to bottom are motor speed, $d$-axis PI output voltage, $q$-axis PI output voltage, $d$-axis reference and feedback current, and $q$-axis reference and feedback current. In Figure 4.33, when the speed ramps up to around 7700rpm, the system becomes unstable. However, the system response in Figure 4.34 is stable for the same test with cross-coupling compensation. When examining the coupling by stepping up and down the $q$-axis current, the $d$-axis current in Figure 4.33 has large oscillation at the stepping edge while the $d$-axis current in Figure 4.34 isn’t affected.
Figure 4.34. Simulation results without coupling compensation.

Figure 4.35. Simulation results with coupling compensation.
4.4.3. Cross Coupling Elimination

The main focus of this chapter is to investigate the cross coupling inside a PM motor. Time domain results provide a good method to show the effect of compensation intuitively. Figure 4.35 shows how the current control can react on a commanded change without any compensation. The reference current vector moves from (0,0) to (-20,0) and then to (-20,20). The actual current trajectory shows $q$-axis component when only $d$-axis current command changes and shows $d$-axis component when only $q$-axis current command changes.

![Current Trajectory](image)

Figure 4.36. Without Coupling Compensation at 50Hz fundamental frequency.
If all the decoupling methods above are implemented, Figure 4.36 shows the current reaction. The current still shows coupling but is considerably smaller than that in Figure 4.35. This can be explained by section 4.3.4.

![Current Trajectory](image)

Figure 4.37. After Coupling Compensation 50Hz fundamental.

### 4.5. Integrated Control on a Motorcycle

After the motor is mounted on a motorcycle, the control of the motor has to comply with the application requirement. Some operational strategies are implemented to start the engine and to boost the power when accelerating.
4.5.1. Swing Back Control to Start Engine

The starter of a traditional scooter uses a permanent magnet brush DC motor. More than 100 amperes of DC current is injected to crank the engine. It makes a harsh noise, causes EMI issues and downgrades the system reliability. The aim of this design is to improve its startability by embedding smart technology.

The “running start” method proposed in [67] is adopted here to start the motor with minimum power. The low resolution position sensor makes it challenging to detect the rotation direction and velocity at pretty low speed. Therefore, a successful start requires properly tackling the problem of rotation direction detection and command torque selection.

As explained early in section 4.1, the electric angle will sweep in one section continuously to get the motor started. This can be demonstrated by the initial current saw tooth waveform in Figure 4.37. After catching two consecutive switching edges, the average speed is obtained based on the known distance and the elapsed time. Based on this speed, the motor angle in the next section is obtained by linear prediction.

Figure 4.37 shows the phase current, motor angle and rotation direction for the start process. The swing-back control is implemented to start the engine. Initially, the motor outputs backward rotating torque by setting the command current negative value and decreasing the angle from $2\pi$ to 0. The initial absolute command current is set high enough to overcome the starting friction and decreases gradually to prevent reverse rotation over the reverse compression stoke continuously. When the crankshaft runs backward toward reverse compression top dead center, it is either bounced back by the resistive torque or stuck somewhere beneath the highest resistive torque point. The
program detects the above action at moment $A$ in Figure 4.37 and switches to forward rotation with constant torque output. Now, the engine has enough acceleration distance to run over the compression stroke and is accelerated to the speed for normal ignition. After touching the idling speed, the ISG motor finishes its start process.

![Figure 4.38. Phase current and motor angle at “running start.”](image)

**4.5.2. Flux-Weakening Operation at High Speed**

The operation of an ISG motor over a wide speed range with full torque output should first check all the operation points on the speed-torque plane. For every achievable speed and torque combination, an optimal point on the $d$-, $q$- axis plane exists with the highest overall system efficiency [23,24,25]. Usually, this optimal current can be reached either by an extra regulator [26] or by an experimental test. In this research work, each of the operation points is examined by testing to make sure that it is reachable and there is
no mechanical problem or heat problem. Below the base speed, the maximum torque points for each current magnitude was obtained by sweeping the current vector linearly in the second quadrant. Then the MTPA curve is obtained by connecting all the current points with maximum torque value for each current magnitude. For the speed above base speed, the DC bus voltage is used out. By injecting flux weakening current, the torque range is expanded considerably. Similarly, for each torque and speed combination, an $i_d$, $i_q$ current pair is derived. Then, the chart in Figure 4.38 is obtained by plotting all the data points on the $i_d$-$i_q$ plane.

![Figure 4.38. MTPA and flux weakening curves in the second quadrant of id-iq plane.](image)

There are two main problems I want to look into. One is how to translate the command torque and sensed speed into $d$-, $q$- current command. The other is to find what steps should be taken if the DC bus voltage fluctuates.
Flux weakening control is all about how to set the proper $d$, $q$-axis current command after the current loop is well-tuned. Remarkable research work has been done to achieve this purpose. Look-up table is widely used in industrial application when the nonlinearity cannot be simplified or the system output is bounded. Basically, Look-up table is an open-loop control method using preliminary test data. The tricky part is determining how to make this table and how to implement the look-up method.

In section 2.4, the control of a PM machine has been reviewed. For the same speed and torque, we always can find the optimal current vector with the minimum magnitude but deliver the highest efficiency. The first step is to sweep all the achievable points on the second quadrature and get the whole mapping between current pair $(i_d, i_q)$ and torque speed command. This is the preliminary test data to make the table.
Figure 4.40. Two dimension table indexing example.

An example is illustrated in Figure 4.39 to show how the current vector is indexed based on the data in the table and the reference torque and feedback speed. For a designated torque and speed point $E$, its position in the table can be pinpointed between two rows and two columns. The cross point of these two rows and two columns are denoted as points $A, B, C, D$ in Figure 4.39. The current vector at position $A, B, C, D$ are all known from the test. The unknown current pair $(i_{dij}, i_{qij})$ can be deduced based on these known data point using two dimension linear interpolation method. The following equation shows the calculation $d$-axis current by using this method. The same equation can be applied to get the $q$-axis current $i_{qij}$. 
When making this two dimension table, the torque on the sample speed curve needs to be unified based on the maximum torque at that speed, because an IPM motor has different maximum torque value at different speeds in flux weakening region. If the absolute torque and speed are used to index the table, the corner above base speed with high torque index is not achievable.

The next problem we are going to investigate is how to adapt this method to DC bus fluctuation. The preliminary testing data is obtained under fixed DC bus voltage. However, the actual DC bus may not be the one for testing or it may decrease slowly due to the discharging of a battery. So it is necessary to take this into account when indexing the table. The voltage equation (4.35) is used to explain the detail method.

\[
\omega^2 \left[ (L_q i_q)^2 + (L_d i_d + \psi_f)^2 \right] = \left( \frac{V_{dc}}{\sqrt{3}} \right)^2 \quad (4.35)
\]

If DC bus voltage \( V_{dc} \) sags in equation (4.35), one solution is to decrease speed \( \omega \) accordingly, so the current as well as the torque is kept constant. However, this is not practical because the time constant of mechanical speed is much larger than electrical time constant of DC bus. No matter how sophisticated the tracking algorithm is, the narrow bandwidth of the mechanical system cannot match the time constant of the electrical system. Since, for the same torque and speed output, there are theoretically infinite d-,q- axis current combinations. We can still find current vector with the highest efficiency after the change of DC bus voltage. Instead of changing the speed, the
reference current vector $i_{dq}$ is adjusted to be adaptive to the DC bus voltage. We can rewrite equation (4.35) into equation (4.36)

$$\left[(L_q i_q)^2 + (L_d i_d + \psi_f)^2\right] = \left(\frac{V_{dc}}{\sqrt{3} \omega}\right)^2$$ \hspace{1cm} (4.36)

The above equation would be the same if we consider the drop of DC bus equivalent to the increase of speed. As for the real implementation, a fictitious speed $\omega_{new}$ is used to index the table, which can be expressed as,

$$\omega_{new} = \frac{V_{DC}}{V_{DC_{new}}} \omega$$ \hspace{1cm} (4.37)

With all data points programmed into DSP, the continuous operation of IPM motor over wide speed range is realized. Figure 4.40 shows the acceleration and deceleration of an IPM motor over wide speed range at the same time changing the load torque back and forth.

Figure 4.41. Dual-motor power test at different speed and torque commands.
4.5.3. Scooter Driving Test

Usually, a typical driving cycle is used to calibrate the performance of the vehicle. After the ignition of the engine, the ISG motor can be commanded to assist the running of the scooter or to generate electricity back to the battery. Figure 4.41 demonstrates a simple drive cycle with all the running modes included,

![Figure 4.41. Simple drive cycle with all running modes included.](image)

The upper level control design is strategic to achieve better performance and higher efficiency. The switch over different running modes is dependent on various operational conditions and is beyond the scope of this work.

![Figure 4.42. Different operation modes of ISG motor.](image)
4.6. **Summary**

A well-tuned current control loop with fast response will always be the first priority of designing the controller for a PM motor. This chapter deals with the tuning of the current control loop. It contains the selection of these control parameters based on frequency response, the investigation of the compensation of these coupling factors in both the time domain and frequency domain. The complex vector modeling of a PM machine makes this classical control design theory applicable to a PM motor with large cross-coupling terms.

After the controller is well-tuned, the machine is mounted on the motorcycle and integrated control is carried out. The swing back control strategy enables the machine to run over the highest resistive torque without increasing the size of the machine. Finally the motor was running over its full speed range with flux weakening control and a simple driving cycle is tested.
CHAPTER 5

DEAD TIME AND DEAD-TIME COMPENSATION

Dead time is an effective method to prevent conflict when toggling between two events, not only for power electronics, but also for daily life. A common example of the usage of dead time in daily life is the traffic light, which are programmed with an interval that stops traffic in all directions. Dead time in power electronics is used to prevent the conduction overlap at the instant of switchover between upper and lower switches across the DC link.

Though dead time is a small period of time compared to the normal conduction period, it makes the output voltage distorted and introduces nonlinearity into the system. This chapter continues to investigate the detail modeling of dead time to the three phase inverter. Then a modified dead time compensation method is proposed considering device parallel snubber capacitance.

5.1. Space Vector Modeling of Dead Time

5.1.1. Dead-Time Effect and Space Vector PWM

The effect of dead time on the performance of the inverter can mainly be grouped into two categories—the voltage error and the zero current clamping effects. Dead time
must exist for all the voltage source inverter with switches connected across DC bus in series. This paper focuses on a conventional three phase inverter connected to a PM motor as an example.

Figure 5.1. Schematic of the drive system.

Figure 5.1 shows the schematic of the entire drive system. Figure 5.2(a) shows the space vector representation of voltage vector $\overline{V}_s$. The SVPWM modulation strategy is used here to develop the PWM switch pattern from voltage command. Any voltage vector on this plane can be realized by designating a combination of six active vectors and two zero vectors. Figure 5.2(b) shows the switch pattern for the voltage and current vector in Figure 5.2(a). $T_a$, $T_b$, $T_c$ represents the symmetrical command pulses. The solid lines indicate the switching signals for upper switches. The dashed lines show the actual phase voltage pulse with dead time considered for the specific current vector in Figure 5.2(a). The solid blocks of phase A and phase B are the voltage gain due to dead time.
(a). Space vector diagram of a voltage vector.

(b). SVPWM switch pattern.

Figure 5.2. Switch vector and PWM pattern for a general six switch inverter.

This work mainly focuses on the nonlinear effect cause by the dead time. Other inherent nonlinear characteristics of the switching device such as voltage drop, and turn on/off delay time are not considered here.

Several remarks are expected from Figure 5.2

1. The ideal dead time voltage loss is related to the current polarity (phase angle of current vector).

2. The effect of dead time on the system is proportional to the ratio of dead time length to modulation index.

3. The PWM output voltage is no longer symmetrical and extra harmonics are expected from the output voltage.
5.1.2. Space Vector Analysis of Dead Time

Space vector analysis is always performed for a three phase system to evaluate its fundamental components and variation trajectory as well. In this paper, the dead time voltage is also modeled using vectors. By doing so, the detailed nonlinear model of the dead-time voltage is predictable.

A. Space Vector Analysis Considering Dead Time

For the system with ideal dead time, Figure 5.3 illustrates the vector correlation for two different scenarios. The developed voltage vectors are given in equation (5.3) and equation (5.6).

\[
\vec{V}_s = \frac{T_1}{T_{pum}} \vec{V}_1 + \frac{T_5}{T_{pum}} \vec{V}_5
\]  

(5.1)
\[
\begin{align*}
T'_1 &= T_1 - 2T_{dt} \\
T'_5 &= T_5
\end{align*}
\] (5.2)

\[
\vec{V}'_s = \frac{T'_1}{T_{pwm}} \vec{V}_1 + \frac{T'_5}{T_{pwm}} \vec{V}_5 = \vec{V}'_s - \frac{2T_{dt}}{T_{pwm}} \vec{V}_1
\] (5.3)

\[
\vec{V}'_s = \frac{T_5}{T_{pwm}} \vec{V}_5 + \frac{T_4}{T_{pwm}} \vec{V}_4
\] (5.4)

\[
\begin{align*}
T'_4 &= T_4 \\
T'_5 &= T_5 - 2T_{dt}
\end{align*}
\] (5.5)

\[
\vec{V}'_s = \frac{T'_4}{T_{pwm}} \vec{V}_4 + \frac{T'_5}{T_{pwm}} \vec{V}_5 = \vec{V}'_s - \frac{2T_{dt}}{T_{pwm}} \vec{V}_5
\] (5.6)

In Figure 5.3, the dead-time voltage vector \( \vec{V}_{dt} \) is determined by the location of current vector. Figure 5.4 slices the entire vector plane into six sections considering different current polarities.

Figure 5.4. Different sections considering current polarities.
B. Space Vector Analysis Considering Limited Voltage Variation Slope

For the real implementation of a PWM inverter, the phase voltage cannot ramp up or ramp down instantaneously due to the parasitic and snubber capacitor in parallel to the switching device [65]. Apart from the ideal deal-time voltage vector noted above, the limited variation slope of the phase voltage should also be taken into account. Figure 5.5(b) shows the phase voltage waveforms in one PWM period with different slopes for different phases.
Figure 5.5. Space vector representation of dead time with limited voltage variation slope.

Since the voltage slope in Figure 5.5(b) cannot be depicted in space vector form, the timing of phase voltage edge is modified based on average voltage-second criteria. The enlarged figure at the right side of Figure 5.5(b) shows the specific case here. By equating the two section block, the average equivalent time varies due to limited voltage variation slope is $T_{al}$, $T_{bl}$ and $T_{cl}$ for all three phases respectively. And their space vector
representations are $V_{al}$, $V_{bl}$ and $V_{cl}$ shown in Figure 5.5(a). Next, for this specific case, $V_{al}$, $V_{bl}$ and $V_{cl}$ are calculated as follows,

1). Three phase instantaneous current $I_a$, $I_b$, $I_c$ calculation based on current vector $I_s$.

2). Equivalent time calculation due to limited voltage variation slope,

$$T_i = \begin{cases} \frac{V_{dc} C}{i} & \text{if } |i| > i_{thres} \\ \frac{1}{2} T_{dt} & \text{if } |i| = i_{thres} \\ T_{dt} = \frac{T_{dt}^2 i}{4CV_{dc}} & \text{if } |i| < i_{thres} \end{cases}$$ \hspace{1cm} (5.7)

$$T_{al} = (T_{dt} - \frac{T_{dt}^2 I_a}{4CV_{dc}})$$ \hspace{1cm} (5.8)

$$T_{bl} = \frac{V_{dc} C}{|I_b|}$$ \hspace{1cm} (5.9)

$$T_{cl} = \frac{V_{dc} C}{|I_c|}$$ \hspace{1cm} (5.10)

3). Equivalent space vector representation due to limited voltage variation slope,

$$\vec{V}_{al} = \frac{T_{al}}{T_{pwm}} \vec{V}_1 - \frac{T_{al}}{T_{pwm}} \vec{V}_5 = \text{sign}(I_a) \frac{T_{al}}{T_{pwm}} \vec{V}_4 = \text{sign}(I_a) \frac{T_{dt}}{T_{pwm}} \vec{V}_4 = \frac{T_{dt}^2 I_a}{4CV_{dc} T_{pwm}} \vec{V}_4$$ \hspace{1cm} (5.11)

$$\vec{V}_{bl} = \text{sign}(I_b) \frac{T_{bl}}{T_{pwm}} \vec{V}_2 = \frac{V_{dc} C}{T_{pwm} I_b} \vec{V}_2$$ \hspace{1cm} (5.12)

$$\vec{V}_{cl} = \text{sign}(I_c) \frac{T_{cl}}{T_{pwm}} \vec{V}_1 = \frac{V_{dc} C}{T_{pwm} I_c} \vec{V}_1$$ \hspace{1cm} (5.13)

$$\vec{V}_{dt, \text{actual}} = \vec{V}_{dt, \text{ideal}} + \vec{V}_{al} + \vec{V}_{bl} + \vec{V}_{cl}$$ \hspace{1cm} (5.14)

C. Dead Time Associated Parameters
Apart from dead-time length, the output voltage deviation caused by dead time is associated with several parameters. Each of those parameters exerts different effects toward the final output voltage vector. Besides the load power factor, modulation index and carrier to fundamental frequency ratio addressed in early publications, the snubber capacitance, dead-time length and load type (inductor, PM motor, induction motor) can be investigated and synthesized together to formalize the final picture of dead time associated parameters. Due to limited time, this section hasn’t been studied in-depth and will be published in the future paper.

5.1.3. Simulation and Experimental Verification of the Model

The system model is implemented in Matlab/Simulink using SimPowerSyste’s toolbox to verify the analytical vector variation waveforms. Finally, with the same inverter and machine model, the analytical model, the simulation and the experimental results are verified to match each other.

A. Phase Voltage and Current Waveform

The simulation and experiment phase current and the vector trajectories of phase current and voltage are shown in Figure 5.6.
Figure 5.6. Simulated and experimental results.

B. Phase Voltage Waveform Comparison

The above dead time model is programmed in Matlab to estimate the real motor terminal voltage from command phase voltage and sampled phase current. The system block diagram is shown in Figure 5.7. Three voltages from the command, the dead-time model and the sampled motor terminal are compared in Figure 5.8. The actual phase
(solid green line) voltage deviates from the command voltage due to the dead-time voltage loss. The calculated phase voltage based on the dead-time model tracks the actual sampled phase voltage from the motor terminal tightly, which verified the proposed model.

Figure 5.7. System block diagram for the simulation.
5.2. Dead-Time Compensation

Various dead time compensation methods have already been reviewed in the literature reviews in section 1.3. The Pulse Based Dead-Time Compensator (PBDTC) method proposed in Paper [49] is effective to compensate the dead-time effect without considering the parallel capacitance. This section will investigate the same method but with this parallel capacitance being considered.

5.2.1. Dead Time Considering Snubber and Parasitic Capacitance

As denoted in the above modeling section, the real voltages in each PWM cycle change with slopes, especially when an additional snubber capacitor is added. In Figure 5.8, the Alpha axis voltage waveforms from simulation are shown.

Figure 5.8. Alpha axis voltage waveforms from simulation.
5.9, the current commutation process in one of the phase legs is examined with snubber capacitor added.

Figure 5.9. One phase leg of an inverter with snubber and parasitic capacitance.

A. Current Switchover Considering Snubber and Parasitic Capacitance

Considering the different phase current signs and switching edges, four possible switching cases are separately described below. Figure 5.10 shows the switching signals and phase voltage waveforms.

Case I: \(i_u\) is positive at the instant when \(T_{lo}\) turns OFF and \(T_{up}\) turns ON. \(v_u\) stays low till the end of dead time. At this moment, \(C_{up}\) holds the DC bus voltage and \(C_{lo}\) is at zero. After \(T_{up}\) transits from OFF to ON, \(i_u\) switches from \(D_{lo}\) to \(T_{up}\) and now \(C_{lo}\) hooks up to the DC bus through \(T_{up}\). It pumps up the voltage on \(C_{lo}\) due to the large charging current. The upper capacitor \(C_{up}\) discharges itself through \(T_{up}\). In this case, the phase voltage edge is very sharp as is shown in Figure 5.10.
Case II: $i_u$ is positive at the instant when $T_{up}$ turns OFF and $T_{lo}$ ON. After $T_{up}$ turns OFF, $i_u$ begins to charge the upper capacitor $C_{up}$, which decreases the phase voltage $v_u$. The voltage falling slope of $v_u$ is directly proportional to the value of phase current. There can be two different phase voltage profiles in terms of different voltage falling slopes. In Figure 5.11(a), the phase voltage is higher than zero at the end of dead time. Then, the voltage is forced to pull down to zero when $T_{lo}$ begins to conduct. In Figure 5.11(b), the phase voltage reaches zero before the end of dead time. In this scenario, lower diode $D_{lo}$ will conduct the phase current till the end of dead time.

Case III: $i_u$ is negative at the instant when $T_{lo}$ turns OFF and $T_{up}$ turns ON. This switching process is the reverse process of that in case II. There are also two possible voltage profiles depending on $i_u$.

Case IV: $i_u$ is negative at the instant when $T_{up}$ turns OFF and $T_{lo}$ turns ON. Opposite to that in case I, the phase voltage jumps down instantaneously with a sharp edge.

In all, the snubber and parasitic capacitance slows down the voltage falling edge when $i_u$ is positive and the voltage rising edge when $i_u$ is negative. This is actually what the snubber capacitor is supposed to do, offering another current path to limit the di/dt voltage when the main switch is turning off. However, the altered voltage waveform during dead time as well as its asymmetry to the rising and falling voltage edges makes the modification of the existing dead-time compensation method necessary.
Figure 5.10. Gating signals and voltage variation of one phase leg in one PWM cycle with parasitic and snubber capacitance considered.

Figure 5.11. Different voltage profiles for positive current at voltage falling edge. (a). Phase voltage is higher than zero at the end of dead time. (b). Phase voltage reaches zero before the end of dead time.

B. Problems Associated with PBDTC Method

When including the parallel capacitance in the circuit, the PBDTC method no longer behaves effectively. Figure 5.12 shows the phase current testing waveforms with and without the PBDTC method under exactly the same modulation index and reference frequency. It is evident that when the PBDTC compensation method is applied, the
current peak-to-peak magnitude is increased. However, the current waveform is serious distorted.

(a). Without PBDTC method. (b). With PBDTC method.

Figure 5.12. Inverter phase current output with large snubber capacitor.

$V_{dc} = 48V; \text{Modulation Index} = 0.04; \text{Frequency} = 120Hz$

Theoretically, for any current distortion, there must be an abnormal voltage component associated with that. The irregular voltage ramps during dead time mentioned in Section A are most likely responsible. Therefore, the dead-time voltage alone in each PWM cycle is analyzed. Figure 5.12(a) shows the phase voltage waveform along with the dead-time voltage in one PWM cycle without any compensation. Figure 5.12(b) shows the one with compensation. In each figure, the topmost curve is the reference voltage $v_u^*$ with the real voltage curves plotted below for both the negative and positive current directions. The right half of the figure shows the dead-time voltage. For the positive current in Figure 5.13(a), the average dead-time voltage is negative. While, for the negative current, it
shows positive. Therefore, the dead-time voltage always holds the opposite sign as that of the phase current and has the tendency to push the current toward zero. This tendency is aggravated when the magnitude of the current becomes larger, which can be designated as negative feedback of the dead-time voltage on the phase current. After the dead-time voltage has been compensated by PBDTC method in Figure 5.13(b), the dead-time voltage now has the same sign as that of the phase current, which reverses the feedback feature of the signal from a negative to a positive relationship. The system runs into an unhealthy condition, which is demonstrated by the abnormal waveforms in Figure 5.12(b).

Figure 5.13. Phase voltage and its difference between the ideal voltage and the actual voltage.
(a) Without compensation. (b) With PBDTC compensation.
5.2.2. Proposed Dead-Time Compensation Method

A. The Compensation Principle of the Proposed Method

The analysis above clearly points out the way to the correct solution: Trim the excessive compensation time introduced by PBDTC method so that the final system is neither over-compensated nor under-compensated. The fully compensated voltage in Figure 5.13(b) is now partially filled by dead-time compensation voltage $v_{dt,c}$ in Figure 5.14. Finally, the dead-time voltages of both the rising and falling voltage edges in one PWM cycle cancel out each other.

![Figure 5.14. Phase voltage and its dead-time voltage with proposed dead-time compensation method.](image)

B. Derivation of Compensation Time

Now, the compensation time of the proposed method can be derived based on the above principle. First, the voltage-current relation on a general capacitor is examined here. The voltage variation on a capacitor with constant current is shown from equation(5.15) to equation(5.17), where, $v_1$ is the initial voltage at $t_1$ and $v_2$ is the final voltage at $t_2$. Therefore, for the predetermined dead-time interval, the phase voltage
changing speed is proportional to phase current level and is inversely proportional to parallel capacitance.

\[
\frac{dv}{dt} = \frac{i}{C} \quad (5.15)
\]

\[
v = \int_{t_1}^{t_2} \frac{i}{C} \, dt \quad (5.16)
\]

\[
v_2 - v_1 = \frac{i}{C} (t_2 - t_1) \quad (5.17)
\]

Two cases are considered separately for different phase current levels. If the phase current absolute value \(|i|\) is less than \(V_{dc} C / T_{dt}\) (\(V_{dc}\) --- DC bus voltage; \(C\) --- parallel capacitance; \(T_{dt}\) --- dead time), the dead time elapses before the phase voltage reaches \(V_{dc}\). Otherwise, the phase voltage hits \(V_{dc}\) before the end of dead time. \(V_{dc} C / T_{dt}\) is recognized as the threshold current \(i_{thres}\). Figure 5.15 shows the dead-time voltage in one PWM period for different current magnitude after compensation. Finally, the area \(ABCD\) is made to be equal to the area \(A'B'C'D'\) in Figure 5.15. Table 5.1 summarizes all the equations based on voltage-second balance principle for the negative current.

![Diagram showing phase voltage difference in one PWM period for different phase current.](image)

Figure 5.15. Phase voltage difference in one PWM period for different phase current.
Table 5.1. Dead-time compensation for calculation for different cases

<table>
<thead>
<tr>
<th>Condition</th>
<th>Voltage-Second Balance Principle</th>
<th>Compensated Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-i_{\text{thres}} &lt; i &lt; 0$</td>
<td>$\frac{1}{2}(-V_{dc} + v_i) \times T_{di} = -(V_{dc} \times (T_{di} - t_{di,c}))$</td>
<td>$-\frac{i}{2C V_{dc}} T_{di}^2$</td>
</tr>
<tr>
<td>$i = -i_{\text{thres}}$</td>
<td>$\frac{1}{2}(-V_{dc}) T_{di} = (-V_{dc})(T_{di} - t_{di,c})$</td>
<td>$\frac{1}{2} T_{di}$</td>
</tr>
<tr>
<td>$i &lt; -i_{\text{thres}}$</td>
<td>$\frac{1}{2}(-V_{dc}) \times (t_1 - t_i) = (-V_{dc})(T_{di} - t_{di,c})$</td>
<td>$T_{di} + \frac{C}{2i} V_{dc}$</td>
</tr>
</tbody>
</table>

5.2.3. Experiment Investigation and Results

For the verification of the proposed method, an inverter with a controller based on TMS320LF28035 is used to drive an 8 kW induction machine. Fig. 12 shows the control block diagram of the drive system. The compensation algorithm is embedded in the PWM calculator to modify the duty cycles of all the three phases. The specifications of the inverter, the motor and the operation conditions are listed in Table 2. The dead time is set to 6μs in order to clearly show the changes after applying the proposed dead-time compensation method.
Figure 5.16. Experiment control block diagram.

To highlight the effect of the phase current on the dead-time voltage, the phase voltage and current are tested and shown in Fig. 13. The switching signals of both the upper and lower switches are shown in channel 1 and channel 2. The third channel is the phase voltage and the fourth channel is the phase current. The phase current stays at zero initially and the phase voltage maintains its value during the dead time and flips over right after the dead time. This is mainly because none of the parallel diodes are conducting during dead time when the current is near zero. When the phase current becomes negative slowly, the rising edge of the phase voltage changes its rising slope and falling edge still keeps straight, which agrees with the analysis in Fig. 3 exactly.
Figure 5.17. Gating signals and phase voltage and current waveforms. 
\(S_{up}\): Upper gating signal. \(S_{lo}\): Lower gating signal. \(v_u\): Phase voltage waveform. \(i_u\): Phase current waveform.

(a) Gating signals. (b) Phase voltage for different current level. (c) Phase Current. 
Figure 5.18 Rising edge of output voltage for different current level with 100nF snubber capacitor at dead time. 
Figure 5.19 Falling edge of output voltage for different current level with 100nF snubber capacitor at dead time.
The problem is further investigated by comparing the voltage waveforms at different current magnitude of both the rising and falling edges shown in Fig. 14 and Fig. 15. In Fig. 14, the rising edges of the output phase voltage are illustrated. For V₁ in Fig. 14(b), its current I₁ is around -14 ampere in Fig. 14(c). After the turning off of the lower switch, the large phase current, charging the lower switch parallel capacitance, makes the phase voltage rise really fast. As for V₂ and V₃ in Fig. 14(b), the corresponding current magnitude of I₂ and I₃ in Fig. 14(c) are relatively small, the ramp up speed of the phase voltage becomes slow. The I₄ and I₅ currents deviate further toward the positive direction, the voltage changing behavior above stressed. Finally, the voltage stays low until the turning on of the upper switch. For the falling edge of output voltage, the similar results can be obtained in Fig. 15. Therefore, the effect of current magnitude to the voltage waveform has also been approved.

In the real experiment, the machine is operated with constant current reference. The output current and command voltage waveforms are shown from Fig. 16 to Fig. 19 for both the cases with and without proposed dead-time compensation. The improvements can be demonstrated in three aspects:

A. Phase current distortion improved

Fig. 16 and Fig. 17 show the phase current and its spectrum without and with proposed dead-time compensation method. The reference current is set to 30 Ampere. In Fig. 16, the current is distorted and zero current clamping effect is obvious. In Fig. 17, the current maintains sinusoidal and the x-y plot exhibits round shape. The frequency domain analysis shows the same story. The 5ᵗʰ, 7ᵗʰ, 11ᵗʰ current harmonics in Fig. 16(c) are larger than that in Fig. 17(c).
B. Command voltage distortion improved

The command voltage is obtained from the output of the current controller. Fig. 18 and Fig. 19 show this command voltage without and with the proposed dead-time compensation method. In Fig. 18, the voltage is distorted and its $x$-$y$ plot shows regular hexagon shape. The voltage $x$-$y$ plot shrinks considerably after compensation in Fig. 19.

C. Command voltage amplitude reduced

With this proposed compensation method, the amplitude of command phase voltage is greatly reduced for the same reference current. This can be demonstrated by comparing the instantaneous voltage amplitude waveform in Fig. 18(c) before compensation and Fig. 19(c) after compensation.

![Figure 5.20. Phase current without compensation. (a) $x$-$y$ plot of current. (b) Current waveforms in stationary frame. (c) Frequency spectrum of the phase current.](image-url)
Figure 5.21. Phase current with compensation.
(a) x-y plot of current. (b) Current waveforms in stationary frame. (c) Frequency spectrum of the phase current.

Figure 5.22. Phase voltage without compensation.
(a) x-y plot of voltage. (b) Voltage waveforms in stationary frame. (c) Inverter output voltage magnitude.
Figure 5.23. Phase voltage with compensation. 
(a) x-y plot of voltage. (b) Voltage waveforms in stationary frame. (c) Inverter output voltage magnitude.

Figure 5.24 shows the experimental result using the same setup as before. This figure emphasizes the transition process before and after the proposed control. In region (A), the compensation is disabled and the system is operating with distorted phase current and phase command voltage. The required phase command voltage amplitude is comparatively high. In region (B), a weighting function is increasing from zero to unity in linear mode. The performance of the system became better with this proposed method gradually applied. In region (C), the proposed control is fully activated and the system is operating with minimum voltage requirement.
5.3. Summary

In this chapter, the dead time and its effect to the inverter are studied, which includes the modeling of the dead time in space vector notation and the modified dead-time compensation method.

Dead time is an intuitive time domain notation and it causes inverter output voltage distortion. However, space vector is used to analyze a three-phase machine. This chapter applied space vector analysis to inverter dead time so that dead time voltage can be incorporated into the machine model. By doing so, various nonlinear aspects affecting the dead-time voltage can be incorporated based on average voltage perspective. These help to make dead-time compensation more accurate.
In the second part, a modified dead-time compensation method based on PBDTC method is investigated considering inverter snubber and parallel capacitance. The experimental results verify its effectiveness in reducing the harmonics current as well as minimizing the command voltage.
CHAPTER 6

CONCLUSIONS AND FUTURE WORK

6.1. Conclusions

The major content of this dissertation is inspired by the issues encountered when developing an IPM machine based ISG system. The inconsistency between the mathematical model and real experiment result stimulate me to uncover the mystery beneath it. This works goes over the IPM machine and its drive from the mathematical model and general operation principle to the general description of the designed IPM motor. It focuses on the investigation of these small nonlinear elements inside the current loop, which could result in the failure of a high speed drive.

- A prototype IPM motor is designed and fabricated. The parameters of this IPM motor are calibrated on the dyno. The testing result is compared to the FEA result in terms of Back-EMF waveform, stator resistance and inductance. These parameters match tightly between FEA result and the prototype. Finally, efficiency test was carried out to exhibit the region with the highest efficiency on the speed-torque map.

- Field orientation vector control of the IPM machine requires the perfect tuning of the current control loop. However, the internal nonlinearity and cross-coupling inside a machine makes the frequency response analysis inapplicable. Complex

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vector notation is used to simplify the analysis of the PM machine. All the cross-coupling terms are analyzed and compensated in both the time domain and frequency domain. The simulation result shows the correctness of the modeling and the effectiveness of the coupling compensation.

- Other than the coupling terms, the nonlinearity brought by dead time is another detrimental factor to the current loop. Conventional dead-time modeling idealizes the switching process without considering these parasitic elements in a power converter. So, the detailed modeling of dead time is pursued in space vector notation by considering all these nonlinear factors. A modified dead-time compensation method is implemented to take these nonlinear effects into account.

### 6.2. Future Work

Due to the limited time and facilities, future research has to first implement some of this modeling and simulations in real experiments. Then research would be interested in the following:

- The optimization of IPM motor design.

The preliminary IPM prototype machine only fulfills the basic function of the ISG system. Tremendous optimization works are needed to further optimize the design. These works include the parameters optimization in reference to the optimized design curve of IPM machine on flux-saliency plane, the consideration of cogging torque minimization, the proper location of the motor’s highest efficiency area to match the hybrid working zone, etc.
- Investigate the speed loop design based on well-tuned current loop

After getting the perfect current control loop, the speed loop design is also substantially important for some of the applications. The speed loop design is different from the current loop design because the mechanical modeling of the plant is involved. The frequency response analysis is still applicable to tune the speed controller to make the system either rigid or elastic.

- Current ripple analysis associated with dead time

The dead time voltage can result ripple in current output at low modulation index. The ripple current can cause torque ripple and harmonic loss. These ripples won’t affect large inertial systems but could certainly degrade the control performance in a position controlled servo area. Research on this part is currently ongoing and results are expected to be released in future publications.
APPENDIX. A

SIMULATION MODEL

Figure A.1. PM machine simulation model.
APPENDIX. B

HYBRID POWER TRAIN AND TEST SETUP

Figure B.1 shows the main parts of the hybrid power train. It includes:

Mechanical Parts: tyre, gear box, continuous variable transmission, transmission belt gasoline engine, crankshaft.

Electrical Parts: ISG PM motor, Li-ion battery, inverter, vehicle controller.

Figure B.1. ISG scooter configuration diagrams and real scooter on a chassis dyno.
Figure B.2 and Figure B.3 show a close-up view of the whole power train and inverter. This ISG power train in Figure B.2 is pretty much the same as the pure gasoline power train. The differences are the removal of DC start motor and the change of outer rotor generator to inner rotor ISG motor.

Figure B.2. ISG hybrid drive power train.
Figure B.3. Inverter prototype.
Figure B.4 shows the test dynamometer for the PM machine.

In order to test the acceleration performance of this ISG scooter, three scooters with same displacement volume but different powertrain structures were used to compare the acceleration performance. Figure B.5 shows the acceleration time to distance curves. Apparently, the 4-stroke scooter shows better performance than the 2-stroke scooter initially but soon is surpass by 4-stroke scooter. While, after adding the ISG system on the 4-stroke scooter, its acceleration performance is enhanced remarkably. It beats the 2-
stroke scooter quite easily. Since the ISG system only boosts the power during acceleration, the 2-stroke scooter catches up with the ISG power train eventually.

Figure B.5. Acceleration performance comparison between different systems.


