Improving accuracy in gravitational weak lensing measurements of clusters

Dissertation

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Abstract

Measuring the distribution of galaxy clusters provides a powerful constraint on cosmological parameters. Currently, the largest challenge for using the observed abundance of clusters to constrain cosmology is to measure their mass accurately. The best tool to measure the mass of clusters is weak gravitational lensing, which measures the baryonic and dark matter present in galaxy clusters by observing the distortion of the shape of sources behind the gravitational lens called shear (γ). Weak lensing measurements are technically challenging to measure due to the distortion in the shape of sources from the atmosphere and telescope optics called the Point Spread Function or PSF.

To measure shear, images are processed by various software programs called lensing pipelines which correct for the distortion due to the PSF. Using image simulations with sources of known characteristics and known shear the systematic error of different lensing pipelines can be compared. In this dissertation the results of the Cluster Shear TEsting Program (CSTEP), a test of lensing pipelines on simulated images, is presented. CSTEP was developed to accurately measure the systematic bias on weak lensing measurements of clusters expected by the Dark Energy Survey (DES). The systematic error from lensing pipelines is then used to predict the error on mass measurements of galaxy clusters observed by DES.
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Chapter 1

INTRODUCTION

Cosmology within the last twenty years has been undergoing a dramatic transition. In 1993 large cosmological surveys such as the CfA2 redshift survey and the Southern Sky Redshift Survey (SSRS2) had sample sizes of around 12,000 galaxies while today the Baryon Oscillation Spectroscopic Survey (BOSS) has a sample size of 1.5 million galaxies (Marzke et al. (1995), Dawson et al. (2013)) and the upcoming Dark Energy Survey (DES) will have a sample size of $3 \times 10^8$ galaxies. With the increase in the amount of cosmological data the statistical errors of measurements has decreased. As statistical errors shrink, systematic sources of errors become increasingly important. As in all experiments, systematic uncertainties are typically harder to predict and control.

Interest in large cosmological surveys has grown tremendously with the discovery of the acceleration in the expansion of the universe (Riess et al., 1998; Perlmutter et al., 1999). This acceleration implies that either the universe is filled with some form of “dark energy” that supplies a negative pressure or that general relativity breaks down on large scales. A number of cosmological surveys including the Kilo Degree Survey (KiDS) (de Jong et al., 2013), the Dark Energy Survey (DES) (Annis et al., 2005), BigBOSS (Schlegel et al., 2011), and the Large Synoptic Survey Telescope (LSST) (LSST Dark Energy Science Collaboration, 2012) have been designed to perform the precise measurements required to study the expansion history of the universe. In order to place the strongest constraints on cosmology these surveys will use information from multiple cosmological probes. The four most promising methods are Type Ia Supernova, baryon acoustic oscillations (BAO), weak gravitational
lensing (cosmic shear) and the abundance of galaxy clusters. Each probe is best suited to measure slightly different and complimentary cosmological information. Since multiple probes are used it is also possible to cross-check the results, confirming that sources of systematic error are correctly assigned.

This dissertation focuses on systematic error in the mass measurement of clusters which has a large impact on cluster cosmology. Cosmology from cluster abundances is a powerful probe to measure the expansion history of the universe. Matter overdensities in the early universe develop into clusters of galaxies. The number and distribution of these structures is predicted by theoretical models of cosmology. By comparing predictions of the expected number of clusters above a mass threshold to those observed, it is possible to constrain the expansion of the universe and test General Relativity on large scales.

Accurate mass measurements are the greatest challenge of precision cosmology with clusters. There are a number of ways to measure the mass of clusters including X-ray temperature (Mantz et al., 2010), Sunyaev-Zel’dovich (SZ) integrated flux (Vanderlinde et al., 2010), and weak gravitational lensing. Since both X-ray temperature and SZ flux are sensitive only to the temperature, and thus the inferred gas mass (\( M_{\text{gas}} \)) present in the center of galaxy clusters, it requires simulations to relate the measured \( M_{\text{gas}} \) to the total mass of the clusters. In contrast, gravitational weak lensing directly measures all of the mass present in a galaxy cluster but is very technically challenging to measure.

According to General Relativity mass concentrations induce a curvature in space-time which deflects photons emitted from background sources. This effect, known as gravitational lensing, causes the background sources to appear distorted. Several shape measurement algorithms, or weak lensing pipelines, are designed to measure the slight distortion in shape caused by weak lensing after removing the distortion on the observed images from the atmosphere and telescopic optics. Lensing pipelines must be tested on simulated images to determine that they are capable of measuring shear with the required degree of accuracy (Kitching et al., 2012). The dominant source of error in the mass measurements of clusters observed by DES is expected to be bias in the measured shear from lensing pipelines (Weinberg et al., 2012).
This dissertation presents the results of the Cluster Shear TEsting Program (CSTEP) which tested lensing pipelines on simulated images that model the distribution of galaxy properties that will be present for sources observed by DES. CSTEP was developed to determine the accuracy of lensing pipelines for DES. Using the shape measurement bias from lensing pipelines measured by CSTEP, the systematic error on the cluster mass measurement is compared to the expected statistical error.

The outline of this dissertation is as follows: Chapter 2 discusses the cosmology relevant to gravitational weak lensing, Chapter 3 discusses gravitational weak lensing as applicable to clusters, Chapter 4 describes the Cluster Shear TEsting Program and the main results, Chapter 5 is the interpretation of the CSTEP results and a calibration of one of the contributing lensing pipelines, Chapter 6 discusses the weak lensing analysis of the Abell 611 galaxy cluster and Chapter 7 contains the conclusions and a discussion of future work.
Chapter 2

Cosmology and Cosmological Probes

This chapter provides background information on the cosmological model and structure formation that is relevant to gravitational weak lensing and cluster abundance cosmology. The outline of this chapter is as follows, first there is a brief description of the history of the universe as currently understood, then a section on the standard model of the universe (which describes how the universe evolves over time), followed by a brief discussion of cosmological probes and a more detailed description of cluster cosmology.

The density of matter and radiation in the early universe influenced the expansion of the universe and the distribution of matter at late times. An accurate model of the early conditions of the universe is needed to describe the universe at late times, when large structures such as galaxy clusters have formed. The expansion of the universe is described by the standard cosmological model. The standard cosmological model, commonly called ΛCDM, describes a universe that contains “dark energy” (Λ), radiation, baryonic matter and cold dark matter. Dark energy is the component of the universe which drives the acceleration of the expansion of the universe, while dark matter only interacts gravitationally. The standard cosmological model predicts relationships between the density of the components of the universe and its expansion.
2.1 History of the Universe

The accepted model of cosmology begins with a hot dense universe that expands and cools. The first observational evidence for the expansion of the universe was that observers (including Edwin Hubble) saw that distant sources have velocities directed away from the earth. The farther away a source, for instance a galaxy, is the faster it seems to be moving away from us, as evidenced by a shift in its observed wavelength. This shift in the observed wavelength is caused by the expansion of the universe, as the space expands the wavelength of light increases. The farther away a source is from earth, the earlier the light observed was emitted, and the more its wavelength has expanded. This change in wavelength is known as an object’s redshift.

Another piece of observational evidence that supports an expanding universe is the Cosmic Microwave Background. The Cosmic Microwave Background (CMB) consists of photons observed isotropically in all directions that are at a current temperature of around 2.725 K (Dodelson, 2003). At the earliest of times the Universe was very hot and dense and the distance particles, including photons, were likely to travel before interacting with other particles was quite small. As the universe expands and cools the radiation which has previously been in thermal equilibrium with matter decouples, or stops interacting and begins to expand freely. When the baryons are no longer coupled to photons, the baryons collapse due to their gravitational attraction and that of dark matter. Dark matter is matter that neither emits nor absorbs light, and only interacts gravitationally.

The universe we observe today is filled with regions of overdensity, such as our galaxy, and regions of underdensity called voids. It is only on large scales (roughly 200 Mpc or $6 \times 10^{24}$ m) that the universe appears homogenous and isotropic. These areas observed with different densities today evolve from initial fluctuations in density of dark matter in the universe. The amplitude of these initial fluctuations are predicted to be Gaussian in distribution. Since the size of the fluctuations compared to the average density is small, the evolution of these can be modeled using linear perturbation theory. When the amplitude of fluctuations is small and they can be described with linear theory, the linear growth function
$G(z)$ or $\delta(z)$ describes the growth of density perturbations. The evolution of fluctuations is roughly as follows, small matter overdensities attract more matter which increases the size of the overdensity. As the region becomes more dense, the increasing pressure causes the area to lose matter. As the region then becomes less dense, the attractive force of gravity is again stronger than the repulsive force of the pressure. This leads to oscillations in the density field still observable today referred to as Baryon Acoustic Oscillations.

At late times and small scales the growth of the fluctuations due to gravity can no longer be treated with linear theory, however there are several tools that model the non-linear growth of structure. These tools predict the frequency for regions of different size to collapse and form bound structures (such as galaxy clusters). For an object to collapse the region must be overdense enough that the local forces of gravity are strong enough that its mass no longer expands with the Hubble flow, or the motion of galaxies due only to the expansion of the universe. From these predictions, the number of collapsed objects in a given mass bin as a function of redshift can be derived. This prediction of the number of expected galaxy clusters as a function of mass and redshift is called the halo mass function.
2.2 Standard cosmological model

The standard model of the universe, commonly called ΛCDM, describes a universe that contains “dark energy” (Λ), radiation, baryonic matter and cold dark matter. Current observations support that dark energy is around 70 percent, dark matter is around 27 percent and baryonic matter is around 3 percent of the mass-energy density of the universe at present (Planck Collaboration et al., 2013). This model provides the simplest explanation for a wide range of observational evidence but creates a puzzle for current physics. There is a wide range of evidence supporting the presence of both dark matter and dark energy, but no currently accepted theory that predicts why they occur. Both dark matter and dark energy have only become commonly accepted within the last 15 years, as stronger observational evidence of their presence has been shown.

General relativity predicts that a universe that contains matter and radiation may be expanding, but that the expansion should be slowing due to the attractive force of gravity. Contrary to expectations, in 1998 two independent surveys of supernovae found that the expansion of the universe is accelerating (Riess et al., 1998; Perlmutter et al., 1999). This acceleration may be due to a component of the universe which creates a repulsive force or due to general relativity breaking down on large scales. It has yet to be determined what the precise magnitude of this acceleration is, and if its strength has changed over time.

The cosmological constant (Λ) was first introduced by Einstein as a repulsive force to create a static universe that was neither collapsing or expanding. This Λ is an energy density that did not change over time and is evenly distributed in space. After the discovery of the expansion of the universe, this concept was rejected but now is considered a possible explanation of dark energy. Another possible explanation for dark energy is that it is a form of repulsive fluid. It is possible that the equation of state of this fluid has changed over time and thus the repulsive pressure may have varied over time. It is also possible that general relativity may not be valid on large scales.

To measure dark energy, the density of dark matter must also be understood since it influences the expansion of the universe. Dark matter does not emit or absorb light and is
thought to only interact gravitationally. The first evidence of the presence of dark matter was found in the rotation curves of galaxies. When observers studied galaxies they noticed that the rotational speed of stars present in the galaxy did not decrease as a function of distance to the galaxy center as predicted by theory. This indicated that there must be mass extending out to larger radius in a dark matter halo or that the laws of newtonian gravity should be modified.

Recent observations of the bullet cluster provide further evidence for the existence of dark matter. As described in [Clowe et al., 2006] the bullet cluster consists of two galaxy clusters which have collided and passed through each other. When the two clusters collided, the galaxies present in the clusters continued moving along the direction that they were initial traveling, and passed through each other. In current observations the galaxies of the two clusters are located in two regions separated by around 2.0 Mpc. X-ray observations show that the gas of the both clusters interacted, slowing down its velocity relative to that of the galaxies, so that the gas is now located between the two separated regions. Weak lensing measurements are strongest around these two separated regions, demonstrating that the mass of the clusters is mostly present near the galaxies, which indicates that the majority of the mass of galaxy clusters is dark matter.

2.2.1 Robertson-Walker metric

Modern cosmology is based on the cosmological principle which states that the universe is homogeneous and isotropic on large scales. In other words that the earth is not in a special location with respect to the rest of the universe. This assumption of isotropy on scales larger than 200 Mpc is consistent with current observations. Assuming the cosmological principle holds true, it is possible to define both the Robertson-Walker metric and the Friedmann equations. The Robertson-Walker metric defines separation for spacetime under the assumption the universe is homogenous and isotropic and distances can change as a function of time:

\[ ds^2 = c^2dt^2 - a(t)^2(dr^2 + f_K^2(r)(d\theta^2 + \sin^2 \theta d\phi^2)) \]  

(2.1)
where $t$ is time, $r$ is the radial coordinate, $\theta$ and $\phi$ are the angular coordinates, $a(t)$ is the scale factor and $f_K(r)$ is the angular diameter distance defined by

$$f_K(r) = |K|^{-1/2} \sinh(|K|^{1/2}r) \quad (2.2)$$

where $K$ is the curvature parameter. The scale factor is introduced so that the physical distance between objects can change while they maintain the same comoving distance. The comoving distance can be thought of as the distance between objects on an expanding grid. Even though the grid itself is expanding, the comoving distance between two objects fixed to the grid remains constant. The standard convention is that $a(t_0) = 1.0$ where $t_0$ is the current time.

**2.2.2 Redshift**

Observational evidence shows that the scale factor $a(t)$ is increasing meaning that the universe is expanding. Spectral lines from distant sources are observed to have a shift from the values measured on earth. We measure an increase in the wavelength in the light of distant sources and thus an increase in the scale factor. The redshift $z$ is defined

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{a(t_0)}{a(t_{\text{emitted}})} = \frac{1}{a} \quad (2.3)$$

Sources at large redshift emitted light earlier and are farther away than sources at small redshift. The rate of change of the scale factor at current time is

$$H_0 \equiv \frac{\dot{a}(t_0)}{a(t_0)} \quad (2.4)$$

or the Hubble constant which describes the speed at which the universe is expanding at the current time. The Hubble parameter $H(z)$ describes the expansion rate at a redshift $z$ and is defined as

$$H(z) \equiv \frac{\dot{a}}{a} \quad (2.5)$$

The most recent measurement of the Hubble constant is $H_0 = 67.3 \pm 1.2$ km s$^{-1}$ Mpc$^{-1}$ from the Planck Collaboration (Planck Collaboration et al. (2013)).
2.2.3 Einstein equations

The expansion of the universe is determined by the Einstein field equations which relate the geometry of the universe to the stress-energy tensor. The Einstein tensor $G_{\mu \nu}$ is related to the stress-energy tensor $T_{\mu \nu}$ by

$$G_{\mu \nu} \equiv \frac{8\pi G}{c^2} T_{\mu \nu} + \Lambda g_{\mu \nu} \quad (2.6)$$

where $G$ is Newton’s constant and $g_{\mu \nu}$ is the metric tensor and $\Lambda$ is the cosmological constant. With $\rho(t)$ as the energy density and $p(t)$ as the pressure, the two Einstein equations are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho}{3} - \frac{Kc^2}{a^2} + \frac{\Lambda}{3} \quad (2.7)$$

and

$$\frac{\ddot{a}}{a} = -\frac{4}{3} \pi G (\rho + \frac{3p}{c^2}) + \frac{\Lambda}{3} \quad (2.8)$$

As mentioned previously the scale factor $a(t)$ evolves over time. Here we use $a = 1$ at the present epoch $t_0$ and $K$ is the curvature. The first equation is called *Friedmann’s equation*. This equation can also be expressed as

$$\frac{H^2(z)}{H_0^2} = \Omega_m (1 + z)^3 + \Omega_r (1 + z)^4 + \Omega_k (1 + z)^2 + \Omega_\Lambda (1 + z)^3(1+w) \quad (2.9)$$

$\Omega_m, \Omega_r, \Omega_k$ and $\Omega_\Lambda$ are the present day energy densities of matter, radiation, curvature and dark energy and $w$ defines the equation of state of dark energy. A standard parameterization defines $w$ as

$$w(a) = w_0 + w_a (1 - a) \quad (2.10)$$

If dark energy was a cosmological constant, then $w_0 = -1$ and $w_a = 0$. Observational evidence shows that the curvature of the universe is likely to be close to zero, or $\Omega_k \approx 0$. The density parameters can be expressed as

$$\Omega_m \equiv \frac{8\pi G \rho_0}{3H_0^2} \quad (2.11)$$
<table>
<thead>
<tr>
<th>Constraints from Planck</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_\Lambda$</td>
<td>0.686 ± 0.02</td>
</tr>
<tr>
<td>$\Omega_m$</td>
<td>0.314 ± 0.02</td>
</tr>
<tr>
<td>$H_0$</td>
<td>67.4 ± 1.4</td>
</tr>
</tbody>
</table>

| Constraints from CFHTLenS + WMAP7               |          |
| ACDM                                            |          |
| $\Omega_m$                                     | 0.274 ± 0.013 |
| $w_0$                                          | -1       |

| wCDM                                            |          |
| $\Omega_m$                                     | 0.325 ± 0.08 |
| $w_0$                                          | -0.86 ± 0.32 |

Table 2.1: Recent measurements of cosmological parameters from Planck, CFHTLenS and WMAP7 at 68 percent confidence intervals (Planck Collaboration et al. (2013), Kilbinger et al. (2013), Komatsu et al. (2011))

and

$$\Omega_k \equiv 1 - \Omega_m - \Omega_r - \Omega_\Lambda$$

(2.12)

(where $\Omega_k = 0$ for a spatially flat universe as is supported by current observations). The matter energy density is made up of cold dark matter, baryons and non-relativistic neutrinos. The current measurements of cosmological parameters from two recent surveys CFHTLenS and Planck are shown in Table 2.1. The CFHTLenS results include information from WMAP7 which measured anisotropies in the CMB.

### 2.2.4 Distance equations

The comoving distance between two objects remains constant if the two objects are moving with the Hubble flow, or moving only due to the expansion of the universe. The line of sight comoving distance from an observer to a distant object is

$$D_C(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{H(z')}$$

(2.13)

The transverse comoving distance between two events at the same redshift but at an angle $\delta \theta$ from each other on the sky for $\Omega_k = 0$ is

$$D_M = D_C$$

(2.14)
The angular diameter distance $D_A$ is the ratio of an object's comoving size $l$ to its angular size in radians $\theta$ ($D_A = \frac{l}{\theta}$). Angular diameter distance is related to comoving distance by

$$D_A = \frac{D_M}{1 + z}. \quad (2.15)$$

For gravitational lensing it is often useful to find the angular diameter distance $D_{A12}$ between two objects at redshift $z_1$ and $z_2$. The formula for a universe with $\Omega_k \geq 0$ is

$$D_{A12} = \frac{1}{1 + z_2} \left[ D_{M2} \sqrt{1 + \Omega_k \frac{D_{M1}^2}{D_H^2}} - D_{M1} \sqrt{1 + \Omega_k \frac{D_{M2}^2}{D_H^2}} \right] \quad (2.16)$$

where $D_H \equiv \frac{c}{H_0}$ is the Hubble distance. The luminosity distance $D_L$ is the relationship between the bolometric flux $S$ and the bolometric luminosity $L$ is

$$D_L = \sqrt{\frac{L}{4\pi S}} \quad (2.17)$$

this is related to the comoving distance by

$$D_L = (1 + z)D_M. \quad (2.18)$$

The comoving volume $V_c$ describes the volume in which the number of objects in the Hubble flow are constant.

$$dV_c = D_H \frac{(1 + z)^2D_A^2H_0}{H(z)}d\Omega dz \quad (2.19)$$

for $\Omega_k = 0$ to redshift $z$ is

$$V_c = \frac{4\pi D_M^3}{3}. \quad (2.20)$$
2.3 Cosmological probes

To measure the strongest constraints on dark energy, multiple cosmological probes are needed. Cosmological probes are sensitive to the energy density of the universe at different ages and on different scales. Even probes which do not provide the strongest constraints can be useful as a cross-check since they have different potential systematic problems. The observables that cosmological surveys try to measure are often $D(z)$ the proper distance as a function of redshift and $G(z)$ the growth function. These are then used to constrain other parameters such as $w_0$. 

Figure 2.1: This figure shows the expected 68 percent forecasts on constraints from the DES survey and Planck (DES (2006)). The red center shows the forecast on constraints from combing Baryon Acoustic Oscillations, cluster abundance, weak lensing (cosmic shear) and Type Ia Supernova.
An example of how surveys can use information from multiple probes is the Dark Energy Survey. As shown in Figure 2.1, the best constraints on the evolution of dark energy from the Dark Energy Survey requires information from the CMB, BAO, Type Ia supernova, weak lensing (cosmic shear) and cluster abundance. Since the Dark Energy Survey (DES) is a large optical survey, it will place strong constraints on cosmological parameters measured from weak lensing and cluster abundance.

2.3.1 Cosmic Microwave Background (CMB)

The Cosmic Microwave Background as discussed previously is the radiation (photons) observed today isotropically in all directions roughly at a temperature of 3 K. By studying the fluctuations in the temperature or anisotropies of the CMB it is possible to constrain multiple cosmological parameters. The statistical properties of the anisotropies in the CMB are described by the correlation function

\[ C(\theta) = \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l + 1)C_l P_l(\cos(\theta)) \]  

(2.21)

where \( P_l \) are the Legendre polynomials. The anisotropy of the CMB temperature can thus be represented as a series of peaks in a power spectrum, for example the Planck survey power spectrum is defined as

\[ D_l \equiv l(l+1)C_l/2\pi \]  

(2.22)

where \( l \) is the multipole moment (Planck Collaboration et al. (2013)). The fluctuations in the CMB observed today arise from:

- Intrinsic temperature fluctuations in the cosmic plasma
- Velocity fluctuations at the time of last scattering
- Fluctuations in gravitational potential at the time of last scattering. If photons are in an area of overdensity at the time of last scattering they are hotter on average and if they are in an area of underdensity they are cooler than average. This is known as the Sachs Wolfe effect.
• Time-dependent fluctuations in gravitational potential after last scattering also can affect the observed temperature of CMB photons. If a photon passes through a region of overdensity which decays (becomes less dense) before the photon leaves, the photon will be slightly hotter than previously. This is known as the integrated Sachs Wolfe effect.

The highest peak in the temperature power spectrum of the CMB shows that the universe is mostly flat. In addition the CMB places constraints on $\Omega_m$, $\Omega_\Lambda$ and $\Omega_b$ by the shape and height of the peaks in the power spectrum. For more details of how the power spectrum depends on degenerate parameters, the Planck cosmology paper is a good resource (Planck Collaboration et al. (2013)).

2.3.2 Type Ia supernova

One of the primary ways to measure distance (with the goal of measuring the expansion of the universe as function of distance) is to take objects of known absolute luminosity and compare that to the measured luminosity as seen on earth. Type Ia supernova have a predicted intrinsic luminosity and are thought to occur when a white dwarf star acquires mass from a binary partner and goes over the Chandrasekhar limit. This limit is set by the maximum amount of gravitational pressure from the stars own mass that can be supported by the pressure of electron degeneracy. The extra mass added pushes the star to become unstable and sets off a thermonuclear explosion. Although there is some variation in the absolute luminosity of type Ia supernova, the shape of the light curve is related to its peak luminosity. Supernovae that peak and decline rapidly are less luminous, than supernovae that rise more slowly. By finding a large number of supernova and calibrating them it is possible to place strong constraints on $D_L(z)$ (the luminosity distance) (Riess, 1996). It was using type Ia supernova that the first compelling proof of Dark Energy was obtained (Riess et al., 1998; Perlmutter et al., 1999).

There are a number of potential problems in using Type Ia supernova as a cosmological probe. Supernovae at a known distance ($z \leq 0.1$) are used to calibrate more distant
supernova requiring the assumption that the luminosity of supernovae does not change as a function of redshift. Another problem with using supernovae as a cosmological probe is that dust clouds between the source and observer change the observed luminosity, which creates a systematic source error that is difficult to correct for.

2.3.3 Baryon Acoustic Oscillations (BAO)

In the early universe there was a fluid composed of photons, electrons, and protons. This fluid had regions both of overdensity and underdensity. The cold dark matter was attracted into regions of overdensity, while the photon-baryon fluid experiences pressure and the fluid is pushed apart. Once the fluid is more diffuse gravitational attraction pulls the fluid back into a clump. This process of expanding, collapsing, expanding and so forth is referred to as the acoustic oscillations, which creates a spherical shell that is overdense compared to the overall density of matter approximately 150 Mpc comoving. Since the size of this overdensity is known, it is possible to measure the angular diameter distance as a function of redshift \( D_A(z) \) and the Hubble parameter \( H(z) \) using BAO. One way BAO is commonly measured is by the two-point autocorrelation function \( \xi(r) \), which quantifies the probability, of finding a pair of galaxies, separated by a distance \( r \), compared to a random distribution. To compute \( \xi(r) \) requires large spectroscopic surveys that can accurately measure the redshift of millions of galaxies. The major difficulty involved in using BAO is that the overdensity is a weak signal that requires a large survey to measure. It is likely that measurements of BAO will be most strongly constrained by statistical rather than systematic effects (Weinberg et al. (2012)) however there are systematic effects that will need to be considered. For example the nonlinear growth of structure complicates the shape of the BAO peak.

2.3.4 Gravitational weak lensing (cosmic shear)

The path of light from distant sources is bent when it travels past matter between the source and the observer. The original sources have their observed size, shape, position and magnitude distorted by the foreground structure. If there is a massive structure such as a cluster then strong lensing can occur. Strong lensing can lead to multiple images of the
same source or an Einstein ring. Strong lensing can also lead to arcs as seen in Figure 2.2.

Weak lensing, specifically cosmic shear refers to the more common distortion of images on the order of 1 percent. Since the distortion is small and the original shape and position of sources is unknown this effect can only be measured statistically. In addition, observed images are distorted by the atmosphere, and the telescope optics, which complicates the measurement. By averaging over large numbers of galaxies it is possible to measure the matter power spectrum. By measuring the matter power spectrum at different redshifts one can extract a measurement of $G(z)$, the growth function. It is also possible to use weak lensing for measurements of $D(z)$.

The major difficulty in weak lensing as a cosmological probe is limiting the possible sources of error. The main sources of systematic error for cosmic shear are shape measurement error, and photometric redshift errors. Shape measurement error occurs when the shape distortion of sources due weak lensing is incorrectly measured. Often shape measurement error occurs when the distortion of the observed images from the atmosphere and telescopic optics is not corrected for properly.
The redshift and solid angle depend on the angular diameter distance $dA(z)$ and the Hubble parameter $H(z)$ at the redshift. The abundance is an integral over the mass function of clusters ($dn/dM$: number density per unit mass) times an electron function $f(M,z)$, which describes the sensitivity of the survey to clusters of mass $M$ at redshift $z$.

$\frac{d^2N}{dz d\Omega} = c H(z) d^2A(z)(1+z)^2 \int_0^\infty dM \frac{dn}{dM}(M,z) f(M,z)(1)$

The evolution of the halo mass function $dn/dM$ can be understood within the context of the linear growth of density perturbations (i.e. Press & Schechter 1974; Sheth & Tormen 1999). This evolution is now well studied using N-body simulations, and further improvements are expected (e.g. Jenkins et al. 2001; Hu & Kravtsov 2003).

Figure 2.3: The expected redshift distribution (top) and quantified differences (bottom) for models where only the dark energy equation of state parameter $w$ is varied. These models are all normalized to produce the same local abundance of galaxy clusters. Note that for models with higher $w$ there is less volume, reducing the number of clusters at $z \sim 0.5$, and structures grow less rapidly, increasing the number of clusters at higher redshift.

The sensitivity of the cluster redshift distribution to $w$ is illustrated in Figure 1. Three redshift distributions for a large Sunyaev-Zel’dovich Effect (SZE) cluster survey are shown; each corresponds to a different value of $w$. The statistical significance of the differences among the models appears in the lower panel. The volume sensitivity dominates at intermediate redshifts $z \sim 0.5$, and the growth rate sensitivity dominates at high redshift.

For large solid angle cluster surveys there are additional observables, including the clustering power spectrum for the galaxy clusters (Majumdar & Mohr 2004; Lima & Hu 2004). Galaxy clusters, like other cosmic objects, act as tracers of the underlying dark matter. In the case of galaxy clusters, the most massive collapsed objects in the universe, the exact relationship between the cluster power spectrum and the dark matter power spectrum is well understood theoretically (i.e. Mo & White 1996), and this relationship or biasing is

**Figure 2.3:** The predicted number of clusters as a function of redshift for several different cosmological models for a survey that detects clusters in a Sunyaev-Zeldovich survey (Mohr (2005)).

It is prohibitively expensive to measure spectroscopic redshifts, which very accurately determines the redshift of sources, for all the galaxies present in a cosmic shear measurement. For large optical surveys the redshift of a galaxy sources is determined through a comparison of the photometry, or brightness of the source, in several filter bands. If the method that relates the comparison of the photometry of sources to their redshift is not accurate, a portion of the lensing catalog will be assigned incorrect redshifts. When sources are assigned incorrect redshifts, this leads to systematic error in the cosmic shear measurement.
2.3.5 Cluster cosmology

Clusters can be used to constrain cosmology through the halo mass function. The abundance of clusters as a function of mass and redshift is predicted by cosmological models. By comparing the predicted and measured mass function it is possible to place constraints on cosmological parameters. Using clusters to constrain cosmology therefore requires, an accurate prediction of the halo mass function and a well understood relationship between the observed and true halo mass function. The observed mass function depends on the true halo mass function, the selection function of the clusters detected and an accurate mass measurement of the clusters detected.

Predicting the halo mass function as a function of cosmological parameters can be done in several different ways. As shown in Figure 2.3, the number of clusters as a function of redshift depends on the expansion rate of the universe and thus dark energy. A recent study of numerical simulations to predict the halo mass function, $dn/dM$, shows discrepancies between methods of less than 5 percent (Weinberg et al., 2012). Current research indicates the greatest difficulty for predicting the halo mass function will be in understanding the evolution of the baryonic mass in halos.

Relating the true and observed halo mass function by understanding the relationship between existing and detected clusters, or the selection function, is a challenge for cluster abundance cosmology. Different methods chosen to detect clusters, have different selection functions, for example optical surveys are more likely to detect clusters that are aligned along the line of sight. The first large cluster catalog was detected by George Abell who examined photographic plates visually (Abell et al. 1989). The criteria for objects to be classified as a cluster was to have a certain number of galaxies in a given radius in a magnitude range. Today clusters are identified by looking for observables that are related to cluster mass in optical, X-ray or CMB surveys. In principle clusters could be found by looking for peaks in the weak lensing signal, which is an ongoing area of research.

To detect clusters in optical surveys software programs analyze data catalogs that contain the location and observed properties of galaxies, and look for dense clumps of galaxies.
of similar luminosity and color. The larger the cluster is, the more galaxies are expected to exist within its dark matter halo. The number of galaxies that exist in a halo brighter than the faintest defined cluster member is referred to as its optical richness. One example of an optical cluster finder that will be used for the DES survey is redMaPPer as described in (Rykoff et al. 2013). The redMaPPer algorithm starts with a training set of red galaxies that have a spectroscopic redshifts to identify the red sequence. The red sequence refers to the fact that the galaxies in rich clusters tend to be early-type (elliptical) and exhibit a strong relationship between color and magnitude (Gladders & Yee 2000). Next redMaPPer seeks overdensities of galaxies that match the colors training set. These new cluster galaxies are used to improve the training set, then the algorithm searches for overdensities again. The strength of using optical searches to locate clusters is that it is possible to detect clusters of low mass. The weakness of using an optical search to locate clusters is that multiple weak halos aligned along the line of sight can be mistaken for one large halo. Another possible selection effect is that since most modern cluster finders look for red-sequence galaxies, any cluster that does not have these objects will not be detected.

Methods that search for clusters using X-ray observations are only able to find large galaxy clusters but they do not have false detections. Clusters give off X-rays from the baryonic gas heated at the center of clusters. The larger a cluster is, the brighter an X-ray source is expected because the gas is the center of a deeper potential well and thus hotter. Current studies show a tighter correlation between the mass of a cluster measured by weak lensing and its optical richness, then X-ray luminosity of a cluster and its mass measured by weak lensing. As discussed in (Hoekstra et al. 2011) it is possible that both the lensing signal and the optical richness are boosted if there are multiple structures along the line of sight, which would account in part for the tighter correlation. To use X-ray luminosity to determine mass, a cluster must be in hydrostatic equilibrium. For systems that consist of multiple interacting or merging clusters, (such as the bullet cluster), the assumption of hydrostatic equilibrium is incorrect.

Another way to detect clusters is to look for distortions in the CMB. When a photon passes through the center of a cluster, it can scatter off an electron and gain energy. This
is known as the Sunyaev-Zeldovich or SZ effect. One strength of looking for clusters using the SZ effect is that it is independent of redshift. SZ surveys are best able to detect massive clusters but similar to X-ray surveys the total number of clusters found will be much smaller than those detected optically. Another difficulty is that a sensitive SZ survey will have significant numbers of clusters whose signal overlaps and complicates the mass measurement of individual halos (Voit (2005)).
Chapter 3

Gravitational weak lensing

When light rays travel past gravitational fields their path is distorted. How the light ray is deflected is independent of the type of matter present which makes lensing a very powerful tool to measure the total mass of structures which contain both the dark and baryonic matter. There are four main applications that gravitational lensing is useful for:

- **Measuring mass distributions.** Lensing is able to measure both the mass and the mass distribution in clusters. With deep observational data, lensing also can measure substructures such as the two mass peaks present in the bullet cluster (Clowe et al. (2006)).

- **Providing estimates of cosmological parameters.** Lensing has been used to measure $H_0$, the magnitude of density fluctuations in the early universe, and the bias parameter or the relationship between the distribution of dark matter and galaxies. When lensing information is combined with information from the CMB it places constraints on the dark energy equation of state (Oguri et al. (2012)).

- **Magnifying distant background sources.** Since gravitational lenses can magnify background sources they have been used to study high redshift galaxies. Lensing can turn a galaxy into an extended arc making it is possible to study its features in detail, or make galaxies at a high redshift $z > 7$ detectable (Wong et al. (2013)).

- **Detecting planets.** Microlensing refers to small objects, the size of planets or stars, causing a background source to increase in brightness. Microlensing is one of the best
ways to find planets. If a star with an orbiting planet passes in front of a source, the light curve it produces will be different then if there was no orbiting planet.

This dissertation will discuss the first two applications as they are relevant to cluster abundance cosmology.

Often when weak lensing is discussed, what is actually being discussed is *cosmic shear*. Cosmic shear refers to lensing by large-scale structure. Unlike cluster lensing the lensing is not due to a single lens but caused by a 3-D matter distribution. Cosmic shear creates distortion in the shape of intrinsic sources an order of magnitude smaller than the distortion caused by large galaxy clusters, and thus to measure this effect shear pipelines often focus on controlling different systematics.

Gravitational lensing from massive clusters can cause both strong and weak lensing. Strong lensing refers to strong distortions in the shape of background sources such as arcs and multiple images. Arcs and multiple images can provide information about the distribution of mass in a galaxy cluster but they are not present for all clusters. Weak lensing from clusters causes a distortion in the shape of background sources which causes them to appear tangentially aligned. Background sources are galaxies which are at a higher redshift than the lens (galaxy cluster) and have their apparent size and shape changed by the mass of the lens. An idealized depiction of lensing from a cluster is shown in Figure 3.1 with circular background sources. In reality background sources have a wide range of intrinsic shapes, which means to measure the shear signal one must measure an average shape from a large number of galaxies. Shape noise refers to the scatter in the measured shear, due to the distribution of intrinsic ellipticity present in sources which are averaged. A depiction of lensing from a cluster on sources with a distribution of intrinsic shape is shown in Figure 3.2. The error on the measured shear can be decreased by increasing N (averaging over more galaxies).
Figure 3.1: This figure depicts an ideal case of lensing from a cluster on circular background sources.
Figure 3.2: This figure depicts a case of lensing from a cluster on sources with shape noise.
3.1 Deflection of light

In General Relativity light propagates or travels along the null geodesics of the metric. A light ray that travels past a sphere of mass M where the impact parameter $\xi$ (the perpendicular distance from the light ray to the center of the mass if the light ray were not deflected) is much larger than the Schwarzschild radius $\xi >> R_s$ has a deflection

$$\alpha = \frac{4GM}{c^2\xi}$$

(3.1)

where $G$ is the gravitational constant. The Schwarzschild radius defines the radius for a sphere that contains a mass $M$ at which the escape speed from the surface equals the velocity of light.

$$R_s = \frac{2GM}{c^2}$$

(3.2)

Objects that have a radius smaller than their Schwarzschild radius are black holes. For most astrophysical situations, like that of a galaxy cluster deflecting light from a background source, the angle of deflection is small $\alpha << 1$ and can be predicted from equation [3.1]. If the gravitational field that the light ray passes through is relatively weak, then the deflection angle from a group of points is the sum of their individual deflections. For a galaxy cluster with a volume density described by $\rho(r)$ it can be divided into regions of a given size $dV$ and mass $dm$. If a light ray passes this system but has a small angle of deflection, the mass causing the deflection is a thin lens. Figure 3.3 shows the path of light from a background source as changed by a gravitational lens.

3.2 The Single Isothermal Sphere

A lens model which is relevant for galaxy clusters is that of a single isothermal sphere (SIS). The density distribution for a SIS lens is

$$\rho(r) = \frac{\sigma_v^2}{2\pi Gr^2}$$

(3.3)
Gravitational weak lensing

When photons travel past gravitational fields their path is distorted. This leads to a distortion in the size, position and shape of the observed source.

Figure 3.3: This figure depicts a gravitational lens. Here $D_l$ is the distance to the lens, $D_{ls}$ is the distance between the source and the lens and $D_s$ is the distance from the source to the observer.
where $\sigma_v$ is the velocity dispersion. This model of the density of galaxy clusters is correct to first order. Although more accurate models of the density profile of clusters exist, this model is still used to measure the mass of galaxy clusters experimentally [Radovich et al. (2008)]. This model does have an infinite mass so a cut off radius must be chosen. The mass of a SIS lens within a radius $r$ is

$$M(r) = \frac{2\sigma_v^2 r}{G} \quad (3.4)$$

where $\sigma_v$ is the velocity dispersion [Wright & Brainerd 2000]. The surface mass density for a SIS is

$$\Sigma(\xi) = \frac{\sigma_v^2}{2G\xi} \quad (3.5)$$

(Schneider et al. 2006). Moving to angular coordinates $\xi = D_l\theta$, where $D_l$ is the distance from the lens to the observer, the deflection angle is

$$\alpha(\theta) = \frac{1}{\pi} \int_{R^2} d^2\theta' \kappa(\theta') \frac{\theta - \theta'}{|\theta - \theta'|^2} \quad (3.6)$$

For clusters, strong lensing occurs for background galaxies close to the cluster center, while weak lensing occurs for background galaxies at a larger radius. Strong lensing occurs for convergence $\kappa(\theta) \geq 1$ where

$$\kappa(\theta) \equiv \frac{\Sigma(D_l\theta)}{\Sigma_{cr}} \quad (3.7)$$

and the critical surface mass density is

$$\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{D_s}{D_l D_{ls}} \quad (3.8)$$

where $D_s$ is the distance from the observer to the source, $D_l$ is the distance from the observer to the lens, and $D_{ls}$ is the distance from the lens to the source. For some source positions lensing masses with $\kappa(\theta) \geq 1$ can produce multiple images of the same source as shown in Figure 2.2. For a SIS lens the convergence is

$$\kappa(\theta) = \frac{\theta_E}{2|\theta|} \quad (3.9)$$
Figure 3.4: This figure shows the profile of four clusters stacked and fit with a NFW profile. Figure taken from [Silva et al. (2013)].

where the Einstein radius $\theta_E$ is

$$\theta_E = 4\pi(\frac{\sigma_v}{c})^2 D_{ls} D_s$$

(3.10)

The shear ($\gamma$) for a SIS as a function of $\theta$ is

$$\gamma(\theta) = \frac{\theta_E}{2|\theta|}$$

(3.11)

### 3.3 Navarro Frank White (NFW) profile

It is an ongoing area of research to measure the density profile of galaxy clusters and develop an analytic model. One model that fits both N-body simulations and observations well is called the Navarro Frank White (NFW) profile ([Navarro et al. (1996)]). Recent observations have shown that this profile fits the stacked shear signal as measured from four large clusters with $M \geq 10^{15}M_\odot$ ( $M_\odot$ is the mass of the sun or $\approx 2 \times 10^{30}$ kg ) at $z \approx 0.3$ ([Silva et al. (2013)]). The stacked lensing profile is shown along with an NFW fit in Figure 3.4. The
NFW density profile is given by

\[ \rho(r) = \frac{\delta_c \rho_c}{(r/r_s)(1 + r/r_s)^2}, \]

where \( \rho_c = \frac{3H^2(z)}{8\pi G} \) is the critical density for redshift \( z \), \( H(z) \) is Hubble’s parameter, \( G \) is Newton’s constant, \( r_s = r_{200}/c \), \( c \) is the concentration and

\[ \delta_c = \frac{200}{3} \frac{c^3}{\ln(1 + c) - c/(1 + c)}. \]

from [Wright & Brainerd (2000)]. A relevant variable for a NFW profile is the virial radius, \( r_{200} \). This is defined as the radius inside which the mass density of the cluster is equal to \( 200\rho_c \). Defining \( x = R/r_s \), the shear profile \( \gamma(x) \) from a NFW halo is

\[ \gamma(x) = \frac{\Sigma(x) - \Sigma(x)}{\Sigma_c} = \frac{\Delta \Sigma(r)}{\Sigma_c} \]

where

\[ \Sigma_c = \frac{c^2 D_s}{4\pi G D_l D_{ls}} \]

\( c \) in the above equation is the speed of light, \( D_s \) is the distance to the source, \( D_l \) is the distance to the lens and \( D_{ls} \) is the distance from the source to the lens. The \( \Sigma(x) \) is the surface density for a NFW lens.

For \( x < 1 \) \( \Sigma(x) \) is defined as

\[ \Sigma(x) = \frac{2}{(x^2 - 1)} r_s \delta_c \rho_c [1 - \frac{2}{\sqrt{1-x^2}} \text{arctanh} \left( \frac{\sqrt{1-x}}{1+x} \right) ] \] (3.13)

for \( x = 1 \) \( \Sigma(x) \) is defined as

\[ \Sigma(x) = \frac{2r_s \delta_c \rho_c}{3} \] (3.14)

and for \( x > 1 \) \( \Sigma(x) \) is defined as

\[ \Sigma(x) = \frac{2}{(x^2 - 1)} r_s \delta_c \rho_c [1 - \frac{2}{\sqrt{x^2 - 1}} \text{arctanh} \left( \frac{x-1}{1+x} \right) ] \] (3.15)

as derived in [Wright & Brainerd (2000)]. The \( \overline{\Sigma(x)} \) is the mean surface density that is interior to the radius \( x \).
For \( x < 1 \) \( \Sigma(x) \) is defined as
\[
\Sigma(x) = \frac{4}{x^2} r_s \delta_c \rho_c \left[ \frac{2}{\sqrt{1 - x^2}} \arctanh \left( \sqrt{\frac{1 - x}{1 + x}} \right) + \ln \left( \frac{x}{2} \right) \right]
\] (3.16)

For \( x = 1 \) \( \Sigma(x) \) is defined as
\[
\Sigma(x) = 4 r_s \delta_c \rho_c \left[ 1 + \ln \left( \frac{1}{2} \right) \right]
\] (3.17)

and for \( x > 1 \) \( \Sigma(x) \) is defined as
\[
\Sigma(x) = \frac{4}{x^2} r_s \delta_c \rho_c \left[ \frac{2}{\sqrt{x^2 - 1}} \arctanh \left( \sqrt{\frac{x - 1}{x + 1}} \right) + \ln \left( \frac{x}{2} \right) \right]
\] (3.18)

The reduced shear from a NFW halo is
\[
g = \frac{\gamma}{1 - \kappa} = \frac{\Delta \Sigma/\Sigma_c}{1 - \Sigma/\Sigma_c}
\] (3.23)

### 3.4 Distortion of background sources

The shape of a galaxy can be defined in terms of its second order brightness moments:
\[
Q_{ij} = \frac{\int d^2 \theta_i \theta_j I(\theta)}{\int d^2 I(\theta)}, \quad i, j \in 1, 2,
\] (3.19)

with the surface brightness distribution \( I(\theta) \) centered at \( \theta \). The two commonly used complex interrelated ellipticity definitions are
\[
\chi = \chi_1 + i \chi_2 = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22}}
\] (3.20)

\[
\epsilon = \epsilon_1 + i \epsilon_2 = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2\sqrt{Q_{11}Q_{22} - Q_{12}^2}}
\] (3.21)

with
\[
\chi = \frac{2\epsilon}{1 + |\epsilon|^2}, \quad \epsilon = \frac{\chi}{1 + \sqrt{1 - |\chi|^2}}
\] (3.22)

Defining reduced shear as,
\[
g = \frac{\gamma}{1 - \kappa}
\] (3.23)
with $\kappa$ the dimensionless convergence and $\gamma$ the shear the observed ellipticity $\epsilon^{(o)}$ is related to the intrinsic ellipticity $\epsilon^{(i)}$ by

$$
\epsilon^{(i)} = \frac{\epsilon^{(o)} - g}{1 - g^* \epsilon^{(o)}}
$$

(3.24)

where $|g| \leq 1$ (Schneider et al., 2006). In the weak lensing regime, where ($\kappa \ll 1, |\gamma| \ll 1$), the expectation value of $\epsilon^{(o)}$ is given by:

$$
< \epsilon^{(o)} > \approx \frac{1}{2} < \chi > \approx g \approx \gamma
$$

(3.25)
Cluster abundance cosmology measures the number of galaxy clusters observed in a given volume as a function of mass and redshift. When the observed distribution of massive clusters, or the halo mass function, is compared to the distribution of clusters predicted by simulations this places constraints on cosmological parameters. The strongest constraints from cluster cosmology are on $\sigma_8$ which describes the amplitude of fluctuations in the early universe and $\Omega_M$ the energy density of matter in universe.

There are three types of observations used to measure the mass of clusters, X-ray, CMB and optical. X-ray and CMB surveys are not able to measure the mass of clusters directly, but measure observables which correlate with mass. X-ray cluster surveys are able to measure the X-ray luminosity, the temperature and the inferred gas mass ($M_{gas}$) of galaxy clusters. CMB surveys measure the change in temperature of CMB photons caused by galaxy clusters through the Sunyaev-Zel’dovich effect. This effect occurs when CMB photons scatter off hot electrons present in gas at the center of galaxy clusters and gain energy. Since X-ray and CMB surveys are sensitive to the temperature of intra-cluster gas, they measure mass in inner regions of galaxy clusters. These observations require assumptions about how the mass distribution at the center of the cluster is related to the mass distribution at its outer regions to measure the total mass. The relationship between the temperature of the gas present in the cluster center, and the total mass of the cluster is only well predicted for clusters in hydrostatic equilibrium. Clusters which are merging, such as
the bullet cluster, are not in hydrostatic equilibrium and create a systematic source of error in CMB and X-ray cluster surveys.

Unlike both CMB and X-ray observations, which require assumptions on how the temperature of the gas present in the cluster center is related to the total cluster mass, optical observations are capable of measuring the entire cluster mass using weak lensing directly. Since weak lensing measures the gravitational deflection of light on background sources, it is sensitive to the distributions of both the dark and luminous matter present in the cluster. Weak lensing from massive clusters is described in greater detail in Chapter 3 of this dissertation.

The most challenging technically problem in weak lensing is to measure shear in an unbiased way. The shape distortion of sources due to the telescope optics and atmosphere called the Point Spread Function (PSF), makes the distortion of sources due to weak lensing very difficult to measure. The distortion from the PSF is typically the same size as the distortion due to gravitational lensing that surveys are trying to measure. To measure the lensing signal, the distortion from the PSF must be measured from stars and then removed from galaxies.

A lensing pipeline refers to a specific set of software programs and data processing methods that takes input data in the form of images and object catalogs, creates a PSF model, tries to remove the effect of the PSF on the galaxy objects, and produces a lensing measurement. Calibrating lensing pipelines so that they produce unbiased measurements is very difficult. Even lensing pipelines which have the same basic algorithm can produce different results on the same data set, depending on details of implementation. One example of an implementation choice that can produce a large difference in the final shear measurement, even when the same lensing algorithm is used, is how objects are rejected as too poorly measured to include. If pipelines are not carefully tested they introduce systematic bias in the weak lensing measurement or shape measurement bias.

Applegate et al. (2012), hereafter A12, performed the largest comparison of weak lensing masses measured from clusters by different groups and demonstrated the importance of testing for shape measurement bias. A12 compared the weak lensing mass measured for
several clusters to measurements on the same clusters by four other groups. A troubling conclusion was that A12 showed different systematic offsets between their results and the mass measured by the four other studies. A plot that compares the mass measurements between studies is shown in Figure 4.1.

![Figure 4.1: A comparison of the mass measured by several different groups on the same cluster sample. The x axis shows the mass of the clusters as measured by A12, while the y axis shows the mass of the clusters as measured by, in the top left corner Okabe et al. 2010, in the top right corner Mahdavi et al. 2008, in the bottom left corner Bardeau et al. 2007 and in the bottom right corner Pendersen and Dahle 2007. This plot is taken from (Applegate et al. (2012)) .](image-url)
After a detailed examination of the methods used to measure the cluster mass by previous groups, A12 concluded that shape measurement bias is the largest source of the discrepancy for the masses measured. Since the accuracy of the lensing pipelines of the various groups has never been directly compared, it is currently unclear which group has the most accurate lensing pipeline, and thus the most accurate mass measurements.

Testing multiple lensing pipelines on simulated images with a known input shear is the best way to distinguish which pipelines are the most accurate. There have been several lensing challenges which created images simulations with several values of known shear and a range of PSF models. The groups which created these lensing challenges made the images publicly available to other groups interested in testing their shear pipelines, but did not release the input shear values until after the lensing pipelines submitted their analysis. The shape measurement bias present in different pipelines was next evaluated and made public so that the accuracy of different lensing pipelines could be directly compared. There have been four publicly available image simulation challenges: The Shear TEsting Programme (Heymans et al. (2006)), The Shear TEsting Programme 2 (Massey et al. (2007)), The GRavitational lEnsing Accuracy Testing 2008 challenge (Bridle et al. (2010)), and The GRavitational lEnsing Accuracy Testing 2010 challenge (Kitching et al. (2012)). These previous lensing challenges were designed to test different aspects of how accurately lensing pipelines were able to measure cosmic shear $|\gamma| < 0.06$. Although no lensing pipeline has been able to meet the systematic requirements for large cosmic shear surveys, each challenge showed significant improvement in accuracy by the best performing lensing pipelines.

Shape measurement bias is typically modeled as

$$\gamma_m = \gamma_t M + C$$

where $\gamma_m$ is the measured shear and $\gamma_t$ is the true shear. In this model the multiplicative shear calibration bias is $M$, and the residual shear offset is $C$.

The challenge of accurately measuring the weak lensing signal from clusters $|\gamma| < 0.15$ has not been tested in previous lensing challenges. For the larger shears present around
galaxy clusters, the model

$$\gamma_m = (\gamma_t)^2 Q + \gamma_t M + C$$

(4.2)

may be more accurate as a nonlinear response to shear \( Q \) may be present. This nonlinear response to shear is expected at large shears since lensing pipelines experience increasing difficulty in measuring highly elliptical objects \cite{Bernstein & Jarvis 2002}

The Cluster Shear TEsting Program (CSTEP) is the first lensing challenge to test the shape measurement bias present in measurements of weak lensing from clusters. The first goal of CSTEP was to validate implementations of lensing pipelines for the Dark Energy Survey by measuring their systematic error. The shape measurement bias as measured in CSTEP is an important benchmark and provides a good estimation of the systematic error due to lensing pipelines that can be expected for clusters from the observed data of the Dark Energy Survey. CSTEP tested the shape measurement bias from pipelines on images that contain constant shear values ranging from 0.0 to 0.15 and 6 different PSF types. The CSTEP image simulations, unlike previous image simulations, are created with distributions of galaxy properties similar to those expected to be observed by DES. Since the amount of shape measurement bias depends on the properties of the source population, using simulated images with realistic distributions of galaxy properties better tests the accuracy of lensing pipelines for a specific survey. In addition to average performance, lensing pipelines are evaluated for how they perform in areas relevant to their suitability for DES. These areas are the stability of the shape measurement bias for different PSF conditions, the shape measurement bias as a function of redshift, the shape measurement bias as a function of noise, the overall pipeline efficiency and the contributions of selection effects to the shape measurement bias.

Pipelines which are suitable for the DES survey must be able to measure shear accurately for a wide range of PSF types. The cosmological constraints from weak lensing by DES depend on shear being accurately measured for a range of atmospheric conditions, so PSF dependent bias is important to control.

The shape measurement bias as a function of redshift is also important for the accuracy
of cosmological constraints from weak lensing by DES. To create the CSTEP simulated images, empirical data from two surveys the Sloan Digital Sky Survey (Abazajian et al. (2009)), and HST/GEMS (Caldwell et al. (2008)) were combined with a mock catalog from ADDGALS, adding density-determined galaxies to lightcone simulations (R. H. Wechsler et al. 2013 in prep) to populate sources on the simulated images. Since the source populations are designed to be the same, the level of shape measurement bias measured as a function of redshift should be similar in CSTEP and DES.

The shape measurement bias as a function of noise is an additional aspect of pipeline behavior that is studied in CSTEP. Pipelines which are relatively accurate for sources with significant noise are more valuable for some lensing applications with clusters. There are individual clusters of interest such as the bullet cluster (Clowe et al. (2006)) which require a high density of sources to study. If a pipeline is able to measure shear on objects with a high level of noise relative to the strength of the signal, more background sources can be included in the lensing measurement. For studies on individual clusters it can be more important to use a lensing pipeline able to run on sources with a high level of noise, than a pipeline which has greater accuracy for objects with the typical levels of noise present in the lensing catalog.

It is also important to characterize the pipeline efficiency, or the number of objects on which a lensing pipeline is able to measure shear, compared to the total number of objects it tries to analyse. Most lensing pipelines fail to return a shape measurement for some fraction of the source population; in some cases this fraction can be quite large, damaging the statistical power of the weak lensing measurement. Rejecting sources can introduce systematic errors, as well. Any dependence of the efficiency on the ellipticity of background objects will introduce a selection bias, as the mean intrinsic ellipticity of the source population no longer vanishes. This effect is important to study since it separates the bias due to incorrectly measuring the shear of objects from the bias due to rejecting objects which is important for future lensing pipeline development.

The second goal of CSTEP was to compare the systematic errors on the stacked weak lensing mass measurement due to lensing pipeline bias to the statistical errors expected
for DES. A major science result of the Dark Energy Survey is the stacked weak lensing analysis of galaxy clusters, which will be used to measure the halo mass function, and contribute to the cosmological constraints from DES. Stacked weak lensing of clusters combines the lensing measurement of clusters with similar properties or observables, such as X-ray temperature or number of cluster members. Since the background galaxies behind a large number of clusters are included, the statistical error on the shear profile measured is smaller. This improvement in statistical errors allows the mass to be measured on average for small clusters which do not have enough background galaxies to be measured individually. Cluster stacking also improves the level of systematic error on the cluster mass measurement since it is unbiased by the presence of an incorrect elliptical PSF correction. Since the observations of all the clusters are taken at different times and the PSF direction is randomly orientated, the average residual PSF bias should be zero. Cluster stacking can introduce systematic error if clusters of different mass are included in the same bin. Since the average mass of clusters will be binned by a number of observables (such as total cluster luminosity and number of galaxies present in the cluster) the systematic error due to improper binning should be small.

The statistical error and systematic error for cluster stacking due to shape measurement bias are compared by choosing a representative redshift, \( z = 0.5 \), and comparing the two types of errors for four different cluster mass bins. The statistical error expected for each bin depends on number of expected background sources, which is calculated based on the number of clusters of that given mass and redshift predicted by N-body simulations. The systematic error calculation begins by determining the shear as a function of distance from the cluster center based on the mass present in each bin. The error due to shape measurement bias is then modeled by transforming the predicted shear with the measured shape measurement bias. This biased shear profile is fit to determine the mass that would be measured and compared to the original mass. The amount that the shape measurement biased mass changes compared to original mass determines the systematic error.

The third goal of the CSTEP project is to develop a noise dependent calibration factor for a lensing pipeline and test its robustness. If pipelines are able to calibrate for the effect
of noise they can use more sources and decrease the statistical errors on measurements. A high level of noise present in the observed images creates a bias in the measured lensing, by causing elliptical objects to be measured as slightly more round. This effect is present for all lensing pipelines, but the amount of the effect varies [Melchior & Viola 2012]. Using simulated images with known input shear it is possible to measure shape measurement bias as a function of noise. A calibration factor that boosts the measured shear signal as a function of noise can then be derived. If a robust noise dependent calibration factor can be derived, it will significantly improve the cluster lensing results.

4.1 Cluster STEP Simulations

Figure 4.2: A 1 arcmin$^2$ portion of a simulated DES image from PSF1. The color scale is logarithmic.
The Dark Energy Survey (DES) is a project to image 5000 square degrees of the southern hemisphere using the Dark Energy Camera (DECam) (Honscheid et al. (2008)) at the Blanco telescope. DES will measure 300 million galaxies up to a redshift of 1.4 in five filters and begin taking data August 31st, 2013. The DES image focal plane has 62 charge-coupled device (CCD) chips each of which is 2000 by 4000 pixels with a 0.27 arcsecond per pixel scale. Before the survey began taking data the DES collaboration created a suite of software to simulate images that model the expected real data as observed by DECam as described in (Lin et al. (2010)). A section from an image is included in Figure 4.2 The CSTEP uses a modified version of this image simulation software.

The DES simulation software was modified for CSTEP in several ways. CSTEP images are designed to represent the image quality that is expected for the DES survey after the individual images taken by the camera are combined to form a coadded image. A coadded image has a higher level of signal to noise than the individual images and removes some image problems like dead pixels present in the camera and cosmic rays. The DES simulation software was also modified to remove the edge dependent effects that modeled the behavior of the DECam CCD chips, so that the CSTEP data would be similar in quality to the typical processed images that lensing pipelines run on.

For the CSTEP simulated images mock galaxy catalogs were created with the ADDGALS method that reproduced observed correlations for galaxies among clustering, luminosities, and colors. The parent N-body simulations used by ADDGALS are the “Carmen” Las Damas N-body simulation box (1 Gpc/h) with flat $\Lambda CDM, \Omega_M = 0.25, \Omega_\Lambda = 0.75, \sigma_8 = 0.8, z < 1.35$ (McBride et al. (2009)). In each image 140,000 galaxy objects are present. Star and galaxy objects are rendered using a software package created by Huan Lin, N. Kuropatkine and C. Stoughton, where the intensity of each pixel is calculated by drawing random samples of photons from the theoretical light profile of the source modified by the point spread function (PSF).

Stars are generated for all PSFs by throwing photons with coordinates given by

$$x = \text{starGauss}.x + \text{psfFx}.x + x_o$$  \hspace{1cm} (4.3)
\[ y = \text{starGauss}.y + \text{psfF}.y + y_o \]  

(4.4)

Here \textit{starGauss} is a Gaussian random generator with the standard deviation defined by the diffusion scale. The \textit{psfF} is a random generator for the corresponding PSF profile.

The galaxy objects are created with Sersic profiles defined by:

\[ F(R) = \frac{P^{-1}(0.5, p)^{2p}}{2\pi r_e^2 p \gamma(2p)} e^{-k(R/r_e)^{\frac{1}{p}}} \]  

(4.5)

where \( P^{-1} \) is an inverse gamma function; \( r_e \) is the effective radius; \( p \) is the power factor.

The Sersic profile reproduces a DeVaucouleurs profile for \( p = 4 \), an exponential profile with \( p = 1 \) and a Gaussian profile for \( p = 0.5 \). Similarly to the stars the galaxies are simulated by photon shooting with

\[ x = \text{diffusionGauss}.x + \text{Profile}.x + \text{psfF}.x + x_o \]  

(4.6)

\[ y = \text{diffusionGauss}.y + \text{Profile}.y + \text{psfF}.y + y_o \]  

(4.7)

Here \textit{diffusionGauss} is a Gaussian random generator with the standard deviation defined by the diffusion scale; \textit{psfF} is a random generator for the shape of the PSF and \textit{Profile} are random deviates calculated for the corresponding profile.

An example of PSF variation observed across a DES image is shown in Figure 4.3. The CSTEP project uses simulated images which contain a constant PSF which separates the technical challenge of how to best model a varying PSF from that of determining the best lensing pipeline.
Figure 4.3: An example of variation in the PSF ellipticity as present in a coadded image taken by the Dark Energy camera. This coadded image was created by taking individual images and combining them to create a deeper and larger image. The x axis shows the x position of the stars in pixels, the y axis shows the y position of the stars in pixels. The orientation of the ellipticity of each star is shown as a line or whisker.

Once a method to interpolate a PSF is chosen, the PSF can be evaluated at the location of all the galaxy objects across the focal plane. PSF interpolation is lensing pipeline independent and the total contribution to the systematic errors is not studied by the CSTEP challenge.

For CSTEP there are two branches in the image simulation sets, those that include a circular Gaussian PSF and those that include an elliptical Gaussian PSF. The elliptical Gaussian PSF has \( \epsilon = 0.03 \). For both the Gaussian and elliptical Gaussian PSF the simulated images contain a convolution kernel that mimics the range of isotropic distortion due to the atmosphere that DES will likely observe. This isotropic distortion simulates seeing conditions, or atmospheric blurring, of 0.7 – 0.9 arcsec. A summary of the PSF
<table>
<thead>
<tr>
<th>PSF Number</th>
<th>Type</th>
<th>Seeing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>circular Gaussian</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>circular Gaussian</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>circular Gaussian</td>
<td>0.9</td>
</tr>
<tr>
<td>4</td>
<td>elliptical Gaussian</td>
<td>0.75</td>
</tr>
<tr>
<td>5</td>
<td>elliptical Gaussian</td>
<td>0.8</td>
</tr>
<tr>
<td>6</td>
<td>elliptical Gaussian</td>
<td>0.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shear Number</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.03</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>0.06</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>0.09</td>
</tr>
<tr>
<td>4</td>
<td>0.0</td>
<td>0.15</td>
</tr>
<tr>
<td>5</td>
<td>0.03</td>
<td>0.0</td>
</tr>
<tr>
<td>6</td>
<td>0.06</td>
<td>0.0</td>
</tr>
<tr>
<td>7</td>
<td>0.09</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>0.15</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4.1: A summary of the PSF and shear sets for the CSTEP simulated images.

The simulated galaxy images are designed to have similar properties to the galaxies that the DES survey will observe by combining a mock catalog with empirical data from the HST/GEMS catalog for galaxies with magnitude $r > 23$, and from the SDSS stripe 82 coadded image for galaxies with magnitude $r < 23$. The distributions of various galaxy properties in the simulation are shown in Figure 4.4. All simulated galaxies are Sersic profiles with an index that ranges from 0.5 to 5.
Figure 4.4: This figure shows distributions of various galaxy properties in the CSTEP simulation. The top left corner shows the distribution in redshift, the top right corner shows the distribution in size (flux radius in pixels), the bottom left corner shows the distribution in magnitude, and the bottom right corner shows the distribution in intrinsic ellipticity.

The CSTEP simulations contain sources with a variety of signal to noise ratio. The
signal to noise ratio (SNR) of a source galaxy is defined as

\[
SNR = \frac{\text{Flux}}{\text{Flux Error}} \tag{4.8}
\]

The flux error is defined as

\[
\text{Flux Error} = \sqrt{(\text{source area} \ast (\text{background rms})^2) + \text{Flux}/\text{gain}} \tag{4.9}
\]

It is difficult to measure the shape accurately of objects with a low SNR, which increases systematic errors. Some examples of objects at various SNR are shown in Figure 4.5.

<table>
<thead>
<tr>
<th>SNR</th>
<th>SNR</th>
<th>SNR</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>35</td>
<td>50</td>
</tr>
</tbody>
</table>

Figure 4.5: This figure shows examples of objects with various levels of SNR in the CSTEP simulation.
4.2 Shear Measurement Pipelines

The CSTEP challenge includes a broad range of pipelines. The eight submitted lensing pipelines are DEIMOS, ksbm, IMCAT, im3shape, Gaussian Mixtures, PKSB, PFDNT and MJ. DES members who worked on developing these lensing pipelines include Peter Melchior, Mike Jarvis, Erin Sheldon, Barnaby Rowe, Kenneth Patton, and Daniel Gruen. The shear measurement pipelines can roughly be divided into groups based on the algorithms used to measure the shape of galaxies and remove the effects of the PSF. A more detailed explanation of pipelines as submitted by the relevant contributor is included in Appendix A.

4.2.1 Moment Based

The moment based methods are three KSB+ (PKSB, ksbm, IMCAT) implementations and DEIMOS. KSB+ parameterises galaxies according to their weighted quadropole moments

\[ Q_{ij} = \frac{\int I(x, y) x_i x_j W(x, y) dx dy}{\int I(x, y) W(x, y) dx dy} \]  

where \( W \) is a Gaussian weight function of scale length \( r_g \) where \( r_g \) is the half-light radius, \( (\text{Kaiser et al., 1995}) \). The quadropole moments are then used to create an ellipticity measurement for both stars and galaxies. The first widely adopted lensing pipeline was KSB+ and it is still often used to measure the mass of clusters \( (\text{Applegate et al., 2012}) \). KSB+ assumes that the PSF can be described as an ellipticity distortion convolved with a large circular symmetric function. KSB+ pipelines have been some of the best performing lensing pipelines in a number of lensing challenges, but implementation details can cause the results to vary widely. The results of CSTEP show PKSB and ksbm are two of the best performing lensing pipelines.

4.2.2 Model Based

The model based methods are im3shape and Gaussian Mixtures. Model based methods fit expected galaxy profiles such as exponential or de Vaucouleurs profiles to each galaxy in the image and generate an ellipticity estimate. These methods required a probability
distribution of the intrinsic ellipticity as input. These pipelines assume an ideal galaxy model, so it is important to test that they do not have systematic bias on galaxies not described well by their model type.

### 4.2.3 Shapelet Based

These pipelines describe each galaxy through an orthonormal set of basis functions called shapelets. The shapelet based pipeline present for CSTEP is the MJ pipeline. A weighted sum of these basis functions is created for each galaxy. One difficulty with this technique is that for reasons of processing speed it is necessary to truncate the number of basis functions allowed which introduces error. The MJ implementation attempts to form a shapelet model that is circular. This model is then sheared and convolved until it matches the observed galaxy. Like the model based approach, this method also requires a probability distribution of galaxy ellipticity.

### 4.2.4 PFDNT

FDNT is a promising new lensing pipeline as proposed by Gary Bernstein which works in the Fourier domain unlike all other lensing pipelines. PFDNT is based on FDNT but uses an additional KSB+ pipeline to provide an initial guess of the intrinsic galaxy ellipticity. Working in the Fourier domain is advantageous since both the effect of the PSF and the shear are easier to describe.

### 4.3 Shear bias results

For gravitational weak lensing the error on the measured shear from lensing pipelines can be roughly divided into two categories: multiplicative bias combined with an additive bias (Heymans et al. (2006)). An illustration of a multiplicative and additive bias is shown in Figure 4.6. Some lensing pipelines also exhibit a quadratic shear bias as well (Kitching et al. (2012)).
The additive bias that is sometimes observed from lensing pipelines is generally due to a failure in the PSF deconvolution. If an elliptical PSF is present in the observation, errors can be introduced if the lensing pipeline either over or under corrects for the PSF on source galaxies.

The presence of additive lensing bias can be tested by correlating the shape of stars with the shape of galaxies after the lensing pipeline has deconvolved the PSF. The ellipticity distortion from the PSF for most observations should vary gradually across the field, thus correlation in shape of stars and galaxies should initially be larger for objects that are located closer together on the image. The stars prior to PSF correction and the galaxies after PSF correction should not be more correlated for close objects if the lensing pipeline has worked correctly. For weak lensing with galaxy clusters the shear is significantly larger.
than the additive shape measurement bias, which means it is not a large source of systematic error, but CSTEP reports the additive bias since this is of interest for cosmic shear.

Multiplicative errors introduced by the lensing pipeline cause the shear to be incorrectly measured as a function of the strength of the shear. For lensing pipelines with multiplicative errors, in general the $|\gamma|$ measured will be less than the true $|\gamma|$. Two sources of multiplicative error in lensing pipelines are selection effects and noise bias. As has been demonstrated in Melchior & Viola (2012) all lensing pipelines will be biased for objects with a low SNR. Selection bias can also cause multiplicative error. More elliptical objects are often difficult for pipelines to measure and can be preferentially rejected, which leads to a biased shear measurement result.

In order to evaluate the performance of each pipeline we use the following metric

$$\gamma_{\text{measured}} = \gamma_{\text{truth}}^2 Q + \gamma_{\text{truth}} M + C$$

$Q$ is the quadratic shear bias, $M$ is the multiplicative bias and $C$ is the shear offset. A plot showing a fit to some of the CSTEP data for the DEIMOS lensing pipeline using this metric is shown in Figure 4.7. To be consistent with previous lensing challenges

$$\gamma_{\text{measured}} = \gamma_{\text{truth}} M + C$$

is reported as well. An ideal lensing pipeline would have $Q = 0$, $M = 1$ and $C = 0$. 

50
Figure 4.7: This figure shows a polynomial fit to the data in order to model the error in each shear measurement pipeline.

### 4.4 Average Shear Measurement Bias

The lensing catalog of DES is expected to include sources with SNR $> 20$. The results for the Q,M and C fit is included in Table 4.4 and displayed in Figure 4.8.
Table 4.2: The Q, M, C results for objects SNR > 20.

<table>
<thead>
<tr>
<th>Pipeline</th>
<th>Q</th>
<th>M</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEIMOS</td>
<td>-0.094 ± 0.009</td>
<td>0.924 ± 0.001</td>
<td>-0.0004 ± 0.0001</td>
</tr>
<tr>
<td>PFDNT</td>
<td>-0.058 ± 0.038</td>
<td>0.963 ± 0.005</td>
<td>-0.0008 ± 0.0003</td>
</tr>
<tr>
<td>GaussianMix</td>
<td>0.153 ± 0.034</td>
<td>0.991 ± 0.004</td>
<td>-0.0001 ± 0.0003</td>
</tr>
<tr>
<td>MJ</td>
<td>0.106 ± 0.043</td>
<td>0.867 ± 0.006</td>
<td>0.0016 ± 0.0004</td>
</tr>
<tr>
<td>PKSB</td>
<td>-0.022 ± 0.016</td>
<td>0.945 ± 0.002</td>
<td>0.0003 ± 0.0001</td>
</tr>
<tr>
<td>im3shape</td>
<td>-0.051 ± 0.017</td>
<td>0.982 ± 0.002</td>
<td>0.0001 ± 0.0002</td>
</tr>
<tr>
<td>IMCAT</td>
<td>-0.035 ± 0.066</td>
<td>1.098 ± 0.009</td>
<td>-0.0080 ± 0.0006</td>
</tr>
<tr>
<td>ksbm</td>
<td>-0.040 ± 0.010</td>
<td>0.973 ± 0.001</td>
<td>0.0004 ± 0.0001</td>
</tr>
</tbody>
</table>

Table 4.3: The Q, M, C results for objects SNR > 20.

<table>
<thead>
<tr>
<th>Pipeline</th>
<th>M</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEIMOS</td>
<td>0.911 ± 0.001</td>
<td>-0.0002 ± 0.0001</td>
</tr>
<tr>
<td>PFDNT</td>
<td>0.956 ± 0.005</td>
<td>-0.0007 ± 0.0003</td>
</tr>
<tr>
<td>GaussianMix</td>
<td>1.012 ± 0.004</td>
<td>-0.0003 ± 0.0003</td>
</tr>
<tr>
<td>MJ</td>
<td>0.881 ± 0.006</td>
<td>0.0015 ± 0.0004</td>
</tr>
<tr>
<td>PKSB</td>
<td>0.942 ± 0.002</td>
<td>0.0004 ± 0.0001</td>
</tr>
<tr>
<td>im3shape</td>
<td>0.975 ± 0.002</td>
<td>0.0002 ± 0.0002</td>
</tr>
<tr>
<td>IMCAT</td>
<td>1.093 ± 0.008</td>
<td>-0.0079 ± 0.0006</td>
</tr>
<tr>
<td>ksbm</td>
<td>0.967 ± 0.001</td>
<td>0.0005 ± 0.0001</td>
</tr>
</tbody>
</table>
Figure 4.8: This figure shows the average Q and M results for all pipelines for objects SNR > 20 in the top panel and the average M and C results for all pipelines for objects SNR > 20 in the bottom panel.

The best performing pipelines for the CSTEP challenge were im3shape, ksbm, PFDNT and PKSB. These pipelines are able to measure shear with a shape measurement error that is within the statistical limits of the stacked weak lensing analysis for DES with \(|M| < 0.05\).
and \(|Q| < 0.2\). The allowed statistical limits on \(M\) and \(Q\) will be discussed in chapter 6 of this dissertation.

The CSTEP results show that a quadratic shape measurement bias or \(Q\) is not significant for the four best performing pipelines. The only three pipelines to show a significant \(Q\) dependance are IMCAT, MJ and Gaussian Mixtures. Both IMCAT and MJ are not within the statistical limits allowed and are not suitable for DES if sources with \(\text{SNR} > 20\) are used. Gaussian Mixtures shows a strong \(Q\) dependance, although it performs well for \(M\) and \(C\), and it is not suitable for a cluster lensing analysis.

Three of the lensing pipelines that ran on the CSTEP data, im3shape, ksbm and DEIMOS also competed in the previous lensing challenge GREAT10 \citep{Kitching2012}. A plot that compares the measured \(Q, M\) and \(C\) for CSTEP and GREAT10 is shown in Figure 4.9. CSTEP is able to measure the quadratic bias \(Q\) of the three lensing pipelines with significantly more accuracy than GREAT10 since it used images with larger shear. CSTEP shows that the quadratic bias can be neglected for im3shape, ksbm and DEIMOS since it is within the limits of statistical precision.

The bias measured in the two simulations CSTEP and GREAT10 is different because GREAT10 ran on a significantly different galaxy population. GREAT10 did not try to create a realistic distribution of galaxy types and instead focused on testing how pipelines performed when galaxies included both a galaxy bulge and a galaxy disk. Two of the lensing pipelines, ksbm and im3shape perform with \(|M| \leq 0.05\), the allowed systematic limit, in both the CSTEP challenge and GREAT10. These two pipelines show a similar level of multiplicative bias and have been shown to be robust overall. The CSTEP results indicate the both ksbm and im3shape perform well for a wide range of source types and that are able to produce a lensing catalog with small systematic errors for DES.
Figure 4.9: This figure shows the average Q and M results for three pipelines for objects SNR > 20 comparing the GREAT10 and CSTEP simulations in the top panel. The bottom panel shows the average M and C results for three pipelines for objects SNR > 20 comparing the GREAT10 and CSTEP simulations.
A large source of error for some lensing pipelines is an inability to remove an elliptical PSF. In Figure 4.10 the circles show the distribution of $Q$ and $M$ for each pipeline measured on each PSF separately. The area of each circle shown includes roughly 95 percent of the measured points. The pipelines that are the least effected by the different types of PSFs are im3shape and ksbm. These two pipelines are able to deconvolve the PSF for a variety of atmospheric seeing conditions and for PSFs which have an elliptical component. Three of the lensing pipelines that are most unstable are Gaussian Mixtures, IMCAT and MJ. These pipelines are not able to perform a lensing deconvolution without shape measurement bias for a PSF with a elliptical component.
Figure 4.10: This figure shows the Q and M results for all pipelines for objects SNR > 20 in the top panel and the M and C results for all pipelines for objects SNR > 20 in the bottom panel. The area of the circle includes the Q,M and C bias measured on each PSF and γ type individually.
4.5 Shear bias as a function of redshift

A important criteria of a lensing pipeline to be used for a cluster abundance study is that it does not have a shear bias that varies strongly as a function of redshift. A redshift dependent lensing bias would impact the observed cluster halo mass function and distort the cosmological results. A redshift dependent lensing bias would also effect a number of other lensing applications such as the shear ratio test, which measures the effect of dark energy by using the lensing strength behind large structures as an independent distance measurement. The CSTEP simulations have a distribution of SNR, galaxy morphology, intrinsic ellipticity and size that are modeled for galaxies as a function of redshift. By testing the bias as function of simulated redshift, CSTEP provides a test of this possible source of systematic error.

The multiplicative shear as a function of redshift is shown in Figure 4.11. The most accurate lensing pipelines do not show a multiplicative bias that varies as a function of redshift in the CSTEP simulations. The pipelines that show the same level of multiplicative bias for all redshifts are ksbm, im3shape, PKSB, and DEIMOS. The fact that the lensing bias is constant as a function of redshift is an important result of CSTEP, since this makes these lensing pipelines suitable to produce the lensing catalog for DES.
4.6 Shear bias as a function of SNR

The signal to noise ratio of sources effects the ability of pipelines to accurately measure shear. With noise present, pipelines preferentially measure elliptical objects as round and the lensing results display a multiplicative bias. The SNR ratio at which pipelines are able to measure shear within the systematic requirements is different for each lensing pipeline. The lensing bias is displayed as a function of SNR in Figure 4.12 and Figure 4.13.

The SNR at which each pipeline is within the systematic requirements will impact the final statistical error on the measurement. The higher the SNR cut, the less sources in each image may be used in the lensing analysis. The density of sources used in the lensing measurement effects the shape noise on the lensing measurement and thus the statistical
It may be possible to characterize the lensing bias as a function of SNR and provide a calibration factor. For the DES survey there are several ongoing areas of research in alternative methods to correct for noise bias. One method being developed and studied is that of direct calibration. This method works by applying a simulated shear to observed data. By studying the lensing bias with a known shear on the observed data it may be possible to correct for noise bias. Another ongoing area of research is the possibility of using multiple shear pipelines to develop a calibration factor. The difference in the measured shear results of several shear pipelines may be able to quantify the noise bias expected.

Figure 4.12: This figure shows the M results for all pipelines as a function of SNR.
Figure 4.13: This figure shows the C results for all pipelines as a function of SNR.

4.7 Lensing pipeline efficiency and selection effects

For lensing with clusters, only sources that are located behind a cluster can be used in the lensing analysis. Therefore clusters that are located at a higher redshift will have a smaller number of surviving background galaxies. A figure that shows the original number of galaxies in a square arc minute, and the number that return a shear measurement for two pipelines PFDNT and DEIMOS for three different SNR cuts is shown in Figure 4.14.
Figure 4.14: This figure shows the pipeline efficiency for DEIMOS and PFDNT.

A table of the surviving number of galaxies for each pipeline is included below for sources with SNR > 5 in Table 4.7, for sources SNR > 20 in Table 4.7 and for sources with SNR > 50 in Table 4.7.

The pipelines that return the highest efficiency of successful measurements are PKSB, DEIMOS and ksbm. Gaussian mixtures uses a weighting system therefore the total number of sources that return a shear measurement is not comparable since many of these sources are downweighted. The results seen in CSTEP are similar to those seen in previous lensing challenges in regards to which class of pipeline type is the most robust. The moment based pipelines DEIMOS, PKSB, ksbm are more robust. With the current quality cuts in place DEIMOS has a higher survival rate, but is less accurate than im3shape.

For lensing applications where the number of background sources are small the two best pipelines are ksbm and PKSB. Both of these pipelines are fairly accurate and return a reasonable number of measured objects.
### Table 4.4: The Ngals for each pipeline for objects SNR > 5.

<table>
<thead>
<tr>
<th>Redshift</th>
<th>Total</th>
<th>DEIMOS</th>
<th>PFDNT</th>
<th>GaussMix</th>
<th>MJ</th>
<th>PKSB</th>
<th>im3shape</th>
<th>IMCAT</th>
<th>ksbm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>14.50</td>
<td>13.26</td>
<td>11.49</td>
<td>13.30</td>
<td>10.66</td>
<td>12.36</td>
<td>10.64</td>
<td>6.18</td>
<td>13.00</td>
</tr>
<tr>
<td>0.30</td>
<td>13.74</td>
<td>12.61</td>
<td>10.95</td>
<td>12.64</td>
<td>10.11</td>
<td>11.73</td>
<td>10.08</td>
<td>5.82</td>
<td>12.36</td>
</tr>
<tr>
<td>0.40</td>
<td>12.63</td>
<td>11.63</td>
<td>10.14</td>
<td>11.65</td>
<td>9.29</td>
<td>10.79</td>
<td>9.27</td>
<td>5.34</td>
<td>11.39</td>
</tr>
<tr>
<td>0.50</td>
<td>11.23</td>
<td>10.37</td>
<td>9.04</td>
<td>10.38</td>
<td>8.27</td>
<td>9.58</td>
<td>8.23</td>
<td>4.77</td>
<td>10.15</td>
</tr>
<tr>
<td>0.60</td>
<td>9.44</td>
<td>8.76</td>
<td>7.60</td>
<td>8.74</td>
<td>6.98</td>
<td>8.02</td>
<td>6.92</td>
<td>4.06</td>
<td>8.55</td>
</tr>
<tr>
<td>0.70</td>
<td>7.33</td>
<td>6.82</td>
<td>5.85</td>
<td>6.78</td>
<td>5.44</td>
<td>6.19</td>
<td>5.35</td>
<td>3.20</td>
<td>6.65</td>
</tr>
<tr>
<td>0.80</td>
<td>5.59</td>
<td>5.20</td>
<td>4.40</td>
<td>5.16</td>
<td>4.15</td>
<td>4.68</td>
<td>4.06</td>
<td>2.48</td>
<td>5.07</td>
</tr>
<tr>
<td>0.90</td>
<td>4.23</td>
<td>3.94</td>
<td>3.30</td>
<td>3.90</td>
<td>3.14</td>
<td>3.51</td>
<td>3.05</td>
<td>1.89</td>
<td>3.83</td>
</tr>
</tbody>
</table>

### Table 4.5: The Ngals for each pipeline for objects SNR > 20.

<table>
<thead>
<tr>
<th>Redshift</th>
<th>Total</th>
<th>DEIMOS</th>
<th>PFDNT</th>
<th>GaussMix</th>
<th>MJ</th>
<th>PKSB</th>
<th>im3shape</th>
<th>IMCAT</th>
<th>ksbm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>8.43</td>
<td>7.47</td>
<td>6.73</td>
<td>7.57</td>
<td>6.56</td>
<td>7.21</td>
<td>6.33</td>
<td>4.44</td>
<td>7.43</td>
</tr>
<tr>
<td>0.20</td>
<td>8.06</td>
<td>7.17</td>
<td>6.49</td>
<td>7.26</td>
<td>6.28</td>
<td>6.91</td>
<td>6.07</td>
<td>4.23</td>
<td>7.13</td>
</tr>
<tr>
<td>0.30</td>
<td>7.41</td>
<td>6.62</td>
<td>6.05</td>
<td>6.71</td>
<td>5.79</td>
<td>6.38</td>
<td>5.60</td>
<td>3.87</td>
<td>6.59</td>
</tr>
<tr>
<td>0.40</td>
<td>6.53</td>
<td>5.86</td>
<td>5.40</td>
<td>5.94</td>
<td>5.10</td>
<td>5.63</td>
<td>4.94</td>
<td>3.40</td>
<td>5.83</td>
</tr>
<tr>
<td>0.50</td>
<td>5.47</td>
<td>4.93</td>
<td>4.56</td>
<td>5.00</td>
<td>4.28</td>
<td>4.72</td>
<td>4.15</td>
<td>2.86</td>
<td>4.89</td>
</tr>
<tr>
<td>0.60</td>
<td>4.18</td>
<td>3.79</td>
<td>3.51</td>
<td>3.84</td>
<td>3.29</td>
<td>3.61</td>
<td>3.19</td>
<td>2.23</td>
<td>3.76</td>
</tr>
<tr>
<td>0.70</td>
<td>2.82</td>
<td>2.56</td>
<td>2.37</td>
<td>2.59</td>
<td>2.23</td>
<td>2.43</td>
<td>2.16</td>
<td>1.54</td>
<td>2.54</td>
</tr>
<tr>
<td>0.80</td>
<td>1.87</td>
<td>1.70</td>
<td>1.56</td>
<td>1.71</td>
<td>1.48</td>
<td>1.60</td>
<td>1.43</td>
<td>1.04</td>
<td>1.68</td>
</tr>
<tr>
<td>0.90</td>
<td>1.26</td>
<td>1.14</td>
<td>1.04</td>
<td>1.15</td>
<td>1.00</td>
<td>1.07</td>
<td>0.96</td>
<td>0.71</td>
<td>1.13</td>
</tr>
</tbody>
</table>

63
Table 4.6: The N_gals for each pipeline for objects SNR > 50.

<table>
<thead>
<tr>
<th>Redshift</th>
<th>Total</th>
<th>DEIMOS</th>
<th>PFDNT</th>
<th>GaussMix</th>
<th>MJ</th>
<th>PKSB</th>
<th>im3shape</th>
<th>IMCAT</th>
<th>ksbm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>3.15</td>
<td>2.68</td>
<td>2.28</td>
<td>2.70</td>
<td>2.40</td>
<td>2.63</td>
<td>2.27</td>
<td>1.54</td>
<td>2.68</td>
</tr>
<tr>
<td>0.20</td>
<td>2.89</td>
<td>2.48</td>
<td>2.14</td>
<td>2.49</td>
<td>2.21</td>
<td>2.43</td>
<td>2.09</td>
<td>1.40</td>
<td>2.48</td>
</tr>
<tr>
<td>0.30</td>
<td>2.46</td>
<td>2.14</td>
<td>1.88</td>
<td>2.15</td>
<td>1.89</td>
<td>2.09</td>
<td>1.79</td>
<td>1.17</td>
<td>2.13</td>
</tr>
<tr>
<td>0.40</td>
<td>1.92</td>
<td>1.68</td>
<td>1.52</td>
<td>1.69</td>
<td>1.48</td>
<td>1.64</td>
<td>1.40</td>
<td>0.91</td>
<td>1.68</td>
</tr>
<tr>
<td>0.50</td>
<td>1.35</td>
<td>1.19</td>
<td>1.11</td>
<td>1.20</td>
<td>1.04</td>
<td>1.16</td>
<td>0.99</td>
<td>0.65</td>
<td>1.19</td>
</tr>
<tr>
<td>0.60</td>
<td>0.75</td>
<td>0.66</td>
<td>0.63</td>
<td>0.66</td>
<td>0.58</td>
<td>0.64</td>
<td>0.55</td>
<td>0.38</td>
<td>0.66</td>
</tr>
<tr>
<td>0.70</td>
<td>0.31</td>
<td>0.27</td>
<td>0.26</td>
<td>0.27</td>
<td>0.24</td>
<td>0.26</td>
<td>0.23</td>
<td>0.16</td>
<td>0.27</td>
</tr>
<tr>
<td>0.80</td>
<td>0.12</td>
<td>0.10</td>
<td>0.10</td>
<td>0.11</td>
<td>0.09</td>
<td>0.10</td>
<td>0.09</td>
<td>0.06</td>
<td>0.10</td>
</tr>
<tr>
<td>0.90</td>
<td>0.06</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Pipelines may reject objects in an ellipticity dependent way. To test for this selection effect bias, the $\gamma$ measured is corrected for the intrinsic ellipticity of the sources as measured by each lensing pipeline. After correcting for selection bias some pipelines, im3shape and DEIMOS have the same level of shape measurement bias. The pipeline that is most affected by selection bias is PFDNT. Q, M and C as measured for each lensing pipeline after correcting for selection effects is included in Table 4.7 and Table 4.7. The average Q,M and C after correcting for selection effects is shown in Figure 4.15. The distribution of Q,M and C after correcting for selection effects, when measured from each PSF individually is shown in Figure 4.16.
Table 4.7: The Q, M, C results for objects SNR > 20 after correcting for selection effects.

<table>
<thead>
<tr>
<th>Pipeline</th>
<th>Q</th>
<th>M</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEIMOS</td>
<td>-0.088 ± 0.017</td>
<td>0.928 ± 0.002</td>
<td>-0.0014 ± 0.0002</td>
</tr>
<tr>
<td>PFDNT</td>
<td>-0.033 ± 0.040</td>
<td>0.988 ± 0.005</td>
<td>-0.0004 ± 0.0003</td>
</tr>
<tr>
<td>GaussianMix</td>
<td>0.236 ± 0.032</td>
<td>0.944 ± 0.004</td>
<td>0.0006 ± 0.0003</td>
</tr>
<tr>
<td>MJ</td>
<td>0.083 ± 0.035</td>
<td>0.901 ± 0.004</td>
<td>0.0015 ± 0.0003</td>
</tr>
<tr>
<td>PKSB</td>
<td>-0.029 ± 0.016</td>
<td>0.946 ± 0.002</td>
<td>0.0006 ± 0.0001</td>
</tr>
<tr>
<td>im3shape</td>
<td>-0.042 ± 0.016</td>
<td>0.983 ± 0.002</td>
<td>0.0000 ± 0.0001</td>
</tr>
<tr>
<td>IMCAT</td>
<td>-0.020 ± 0.058</td>
<td>1.275 ± 0.008</td>
<td>-0.0066 ± 0.0005</td>
</tr>
<tr>
<td>ksbm</td>
<td>-0.050 ± 0.010</td>
<td>0.981 ± 0.001</td>
<td>0.0007 ± 0.0001</td>
</tr>
</tbody>
</table>

Table 4.8: The M, C results for objects SNR > 20 after correcting for selection effects.

<table>
<thead>
<tr>
<th>Pipeline</th>
<th>M</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEIMOS</td>
<td>0.915 ± 0.002</td>
<td>-0.0013 ± 0.0002</td>
</tr>
<tr>
<td>PFDNT</td>
<td>0.983 ± 0.005</td>
<td>-0.0004 ± 0.0003</td>
</tr>
<tr>
<td>GaussianMix</td>
<td>0.976 ± 0.004</td>
<td>0.0003 ± 0.0003</td>
</tr>
<tr>
<td>MJ</td>
<td>0.913 ± 0.004</td>
<td>0.0014 ± 0.0003</td>
</tr>
<tr>
<td>PKSB</td>
<td>0.942 ± 0.002</td>
<td>0.0007 ± 0.0001</td>
</tr>
<tr>
<td>im3shape</td>
<td>0.977 ± 0.002</td>
<td>0.0001 ± 0.0001</td>
</tr>
<tr>
<td>IMCAT</td>
<td>1.272 ± 0.007</td>
<td>-0.0066 ± 0.0005</td>
</tr>
<tr>
<td>ksbm</td>
<td>0.975 ± 0.001</td>
<td>0.0008 ± 0.0001</td>
</tr>
</tbody>
</table>
Figure 4.15: This figure shows the average Q and M results for all pipelines for objects SNR > 20 after correcting for selection effects in the top panel. This figure shows the average M and C results for all pipelines for objects SNR > 20 after correcting for selection effects in the bottom panel.
Figure 4.16: This figure shows the Q and M results for all pipelines when measured on each PSF individually for objects SNR > 20 after correcting for selection effects in the top panel. This figure shows the M and C results for all pipelines when measured on each PSF individually for objects SNR > 20 after correcting for selection effects in the bottom panel.
4.8 Conclusions

The CSTEP results show that quadratic bias lensing is not present in the best performing pipelines. The quadratic bias is more precisely measured in CSTEP than in previous lensing challenges and shown to not be a large source of systematic error. CSTEP shows that the systematic bias as a function of redshift is stable for most of the lensing pipelines. CSTEP also shows that a number of lensing pipelines are able to perform robustly, and measure shear accurately even at low SNR.
Chapter 5

Cluster Shear Testing Program II:

In this chapter of the dissertation we compare the systematic errors due to shape measurement error to the statistical errors from shape noise. A major goal for the weak lensing group of DES is to have the systematic errors from shape measurement error, smaller than the statistical errors from shape noise.

The second half of this chapter contains the results of deriving a calibration factor for ksbm. The lensing pipeline ksbm performs very well in the CSTEP challenge overall and shows a negligible multiplicative and additive bias for all sources. The robust performance of ksbm makes it an ideal pipeline to test a SNR dependent $M$ calibration.

5.1 Effects of shear measurement bias on stacked lensing measurements

For optical surveys like DES, clusters are detected using various algorithms and grouped by redshift and observables like richness and luminosity. The clusters are then stacked by combining the tangential weak lensing signal from the cluster center of all the clusters in each bin. By combining many clusters a high $S/N$ shear profile is used to measure cluster weak lensing. There are a number of sources of systematic error, for example orientation biases, but the error from shape measurement is expected to be dominant (Weinberg et al., 2012).

Here we compare for the DES data sample the statistical error expected, to a systematic
error introduced by bias in the measurement of $\gamma$ from weak lensing pipelines. The statistical error we model combines the scatter due to shape noise in background galaxies, and the scatter due to cluster halos not being spherical mass distributions as described in (Weinberg et al., 2012; Becker & Kravtsov, 2011). In this dissertation clusters at $0.45 > z > 0.55$ in four bins of mass $[1.0 \times 10^{14} M_{\odot}, 2.0 \times 10^{14} M_{\odot}, 4.0 \times 10^{14} M_{\odot}, 8.0 \times 10^{14} M_{\odot}]$.

To estimate the number of background galaxies per arcmin$^2$ ($\bar{ngal}$) and their mean redshift for a survey like DES we use a redshift distribution of galaxies

$$f(z) = z^m \exp(-(z/z_*)^\beta)$$

where $m = 2.0$, $z_* = 0.5$ and $\beta = 2.0$ as given in (Weinberg et al., 2012). To determine the number of background sources at a given redshift the background fraction of galaxies is

$$F_{bg} = \frac{\int_{z}^{\infty} dz' f(z')} {\int_{0}^{\infty} dz' f(z')}$$

which is then used to determine

$$\bar{ngal} = 10 F_{bg}(z)$$

for the DES survey (Rozo et al., 2011). Using this equation for clusters at $z = 0.5$, the background sources have $\bar{ngal} = 5.72$ and mean redshift of the background is $z = 0.725$. In the CSTEP images selecting for galaxies at $z > 0.5$ with a $S/N > 20$ we have $\bar{ngal} = 5.8567$ and a mean redshift of sources $z = 0.766$. These are similar numbers and we pick $\bar{ngal} = 5.72$ and mean redshift of the background as $z = 0.725$ for our further calculations.

The statistical error or mass uncertainty $\Delta lnM$ as described in Weinberg et al. (2012) for stacked weak lensing is:

$$\Delta lnM = \sqrt{(\Delta lnM_{shape})^2 + (\sigma_{wl})^2}$$

which combines the error due to the shape noise and the error due to scatter based on clusters not being spherical. From (Becker & Kravtsov, 2011) we take

$$\sigma_{wl} = \frac{0.3}{\sqrt{N}}$$
where $N$ is the number of clusters in a given bin. To calculate the shape noise we use the equation

$$
\Delta \ln M_{\text{shape}} \approx 6.0 \times 10^3 \left( \frac{N}{4000} \right)^{1/2} \left( \frac{\sigma_e}{0.3} \right)^2 \left( \frac{2 \times 10^{14} M_\odot}{M} \right)^{-2/3} \left( \frac{\overline{n_{\text{gal}}}}{2 \times 10^{14} \text{arcmin}^{-2}} \right)^{-1/2} \left( \frac{D_{ls}/D_s}{0.5} \right)^{-1},
$$

from (Weinberg et al., 2012) with $\sigma_e = 0.4$, $\overline{n_{\text{gal}}} = 5.72$ and $D_{ls}/D_s = 0.27$ which yields the statistical limits shown in Table 5.1. To model the concentration we expect for clusters at this redshift we assign an initial concentration $c$

$$
c = A \times \left( \frac{m_{200}}{(2.0 \times 1.12)} \right)^B (1 + z_{\text{cluster}})^C
$$

Where $A = 7.85$, $B = -0.081$, $C = -0.71$ and $m_{200}$ is the mass within $r_{200}$ from (Oguri & Takada, 2011).

### Table 5.1: $\Delta \ln M$

<table>
<thead>
<tr>
<th>Mass Bin</th>
<th>$1.0 \times 10^{14} M_\odot$</th>
<th>$2.0 \times 10^{14} M_\odot$</th>
<th>$4.0 \times 10^{14} M_\odot$</th>
<th>$8.0 \times 10^{14} M_\odot$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N clusters</td>
<td>10000.0</td>
<td>3000.0</td>
<td>500.0</td>
<td>50.0</td>
</tr>
<tr>
<td>$\Delta \ln M$</td>
<td>0.034</td>
<td>0.040</td>
<td>0.062</td>
<td>0.128</td>
</tr>
<tr>
<td>$c$</td>
<td>4.29</td>
<td>4.06</td>
<td>3.83</td>
<td>3.62</td>
</tr>
</tbody>
</table>

### 5.1.1 Systematic bias

To measure the effect of shape measurement bias, a Navarro, Frenk and White (NFW) density profile is created for each mass bin. This density profile is used to calculate the reduced shear $g$ from the theoretical predication as described in Wright & Brainerd (2000).
The NFW density profile is given by

$$\rho(r) = \frac{\delta_c \rho_c}{(r/r_s)(1 + r/r_s)^2}$$  \hspace{1cm} (5.8)$$

where $\rho_c = \frac{3H^2(z)}{8\pi G}$ is the critical density, $H(z)$ is Hubble’s parameter, $G$ is Newton’s constant, $r_s = r_{200}/c$, $c$ is the concentration and

$$\delta_c = \frac{200}{3} \frac{c^3}{ln(1+c) - c/(1+c)}$$  \hspace{1cm} (5.9)$$

from (Wright & Brainerd, 2000). The reduced shear from a NFW halo is

$$g = \frac{\gamma}{1 - \kappa} = \frac{\Delta \Sigma/\Sigma_c}{1 - \Sigma/\Sigma_c}$$  \hspace{1cm} (5.10)$$

details of which are described in chapter 3 of this dissertation. The reduced shear as measured by the current measurement pipelines is modeled as

$$g' = g^2 * Q + g * M$$  \hspace{1cm} (5.11)$$

We then fit this $g'$ distribution to get a $M_{200}$ and $c$ value.

In addition to a NFW profile, the mass of clusters is often measured using a SIS profile. The SIS profile does not require assumptions about the concentration of clusters as a function of redshifts. The shear from a SIS profile is related to the cluster mass by

$$M_{200} = \sqrt{\frac{48}{200\pi \rho_c}} \left(\frac{\sigma_v^2}{2G}\right)^{3/2}$$  \hspace{1cm} (5.12)$$

(Abate et al., 2009) and

$$\gamma(\theta) = \frac{2\pi \sigma_v^2 D_{ls}}{\theta \ c^2 \ D_s}$$  \hspace{1cm} (5.13)$$

where $\sigma_v$ is the velocity dispersion, $D_{ls}$ is the distance between the lens and the sources and $D_s$ is the distance between the observer and the source. The SIS profile is described in detail in chapter 3 of this dissertation. For a SIS profile $\gamma = \kappa$ and the reduced shear becomes

$$g = \frac{\gamma}{1 - \gamma}$$  \hspace{1cm} (5.14)$$
we again model the shear errors as

\[ g' = g^2 \ast Q + g \ast M \]  \hspace{1cm} (5.15)

and fit this modified profile to model the expected error in mass due to shear measurement error.

5.1.2 Bias in M and Q

In stacked cluster weak lensing the strength of the lensing signal is such that the shear offset \( C \) contributes negligible error compared to a multiplicative shear error \( M \). Under the assumption that a survey does not have a preferred orientation for the elliptical portion of the PSF, a shear bias \( C \) will add scatter to the mass measurement but it will not bias the stacked results. In contrast a shear bias in \( Q \) and \( M \) will effect all the cluster mass measurements. Since the lensing profile depends on the mass and redshift of the average cluster in each bin, a shape measurement bias in \( Q \) and \( M \) will bias the mass for each bin of mass and redshift differently.

A bias in \( M \) is of the most concern for stacked cluster weak lensing. A \( Q \) bias does influence the concentration of NFW clusters, but it is unclear how important this is for future cosmological constraints. The comparison of the statistical limits to a systematic bias in \( M \) is shown in Figure 5.1 and in Figure 5.2. The comparison of the statistical limits to a systematic bias in \( Q \) is shown in Figure 5.3 and in Figure 5.4.
Figure 5.1: This figure compares the bias in mass and concentration due to a M bias for the $1.0 \times 10^{14} \, M_\odot$ and $2.0 \times 10^{14} \, M_\odot$ mass bins with the expected statistical errors. The shaded band in the above plots shows the expected statistical errors on the mass.
Figure 5.2: This figure compares the bias in mass and concentration due to a M bias for the $4.0 \times 10^{14} \, M_\odot$ and $8.0 \times 10^{14} \, M_\odot$ mass bins with the expected statistical errors. The shaded band in the above plots shows the expected statistical errors on the mass.
Figure 5.3: This figure compares the bias in mass and concentration due to a Q bias for the $1.0 \times 10^{14} \, M_\odot$ and $2.0 \times 10^{14} \, M_\odot$ mass bins with the expected statistical errors. The shaded band in the above plots shows the expected statistical errors on the mass.
Figure 5.4: This figure compares the bias in mass and concentration due to a $Q$ bias for the $4.0 \times 10^{14} \, M_{\odot}$ and $8.0 \times 10^{14} \, M_{\odot}$ mass bins with the expected statistical errors. The shaded band in the above plots shows the expected statistical errors on the mass.
The results show that to be within the statistical limits a lensing pipeline must have a multiplicative bias $|M| < 0.95$ and a quadratic bias $|Q| < 1.2$.

5.1.3 Shape measurement bias from lensing pipelines

It is clear that for several pipelines including MJ and IMCAT the expected errors due to shear measurement are greater than the expected statistical limits. The robust lensing pipelines which are closest to the allowed statistical limits are ksbm and im3shape. The plots for the NFW mass and concentration for the first mass bin are shown in Figure 5.5 and Figure 5.6. They show that the strongest statistical constraints are on the low mass clusters for DES. This is due to the much larger number of small mass clusters observed. The plots for the NFW mass and concentration for remaining mass bins are shown in Figure 5.7, Figure 5.8, Figure 5.9, Figure 5.10, Figure 5.11 and Figure 5.12. All of the mass bins show that ksbm and im3shape are the two best performing pipelines.
Figure 5.5: This figure compares the bias in mass and concentration due to a shape measurement for each pipeline for the first mass bin with the expected statistical errors. The top plot shows the average performance and the bottom plot shows the behavior for each PSF, where the area of the ellipse includes the Q and M bias measured on each PSF, $\gamma_1$ and $\gamma_2$ separately.
Figure 5.6: This figure compares the bias in mass for both a SIS and NFW profile of the first mass bin with the expected statistical errors.
Figure 5.7: This figure compares the bias in mass and concentration due to a shape measurement for each pipeline for the second mass bin with the expected statistical errors. The top plot shows the average performance and the bottom plot shows the behavior for each PSF, where the area of the ellipse includes the Q and M bias measured on each PSF, \( \gamma_1 \) and \( \gamma_2 \) separately.
Figure 5.8: This figure compares the bias in mass for both a SIS and NFW profile of the second mass bin with the expected statistical errors.
Figure 5.9: This figure compares the bias in mass and concentration due to a shape measurement for each pipeline for the third mass bin with the expected statistical errors. The top plot shows the average performance and the bottom plot shows the behavior for each PSF, where the area of the ellipse includes the Q and M bias measured on each PSF, $\gamma_1$ and $\gamma_2$ separately.
Figure 5.10: This figure compares the bias in mass for both a SIS and NFW profile of the third mass bin with the expected statistical errors.
Figure 5.11: This figure compares the bias in mass and concentration due to a shape measurement for each pipeline for the fourth mass bin with the expected statistical errors. The top plot shows the average performance and the bottom plot shows the behavior for each PSF, where the area of the ellipse includes the Q and M bias measured on each PSF, $\gamma_1$ and $\gamma_2$ separately.
5.2 Calibrated ksbm

The goal of a SNR dependent calibration term is to increase the number of sources in a given area that ksbm is able to accurately measure and therefore decrease the statistical error without increasing the systematic error. The improvement of statistical error allowed for a stacked weak lensing measurement for the calibrated ksbm pipeline at a representative redshift is shown in table 5.2. The ksbm pipeline was shown to perform as one of the top pipelines and to be a robust overall. This ksbm pipeline has a number of advantages that make it well suited for a SNR calibration, it has a high lensing pipeline efficiency and it is very stable even under varying PSF conditions. Describing the shape measurement bias for ksbm as a multiplicative bias $M$, accounts for almost all of the error which makes it well suited for a $M$ calibration term.
Table 5.2: This table includes the improved statistical errors on the stacked weak lensing results after a SNR dependent M calibration is applied. These results are for a representative redshift $z = 0.5$ and for the number of returned objects for ksbm. For the uncalibrated results an assumed SNR cut at 20 results in 4.72 galaxies per $\text{arcmin}^2$. For the calibrated results an assumed SNR cut at 5 results in 10.15 galaxies per $\text{arcmin}^2$.

<table>
<thead>
<tr>
<th>Mass Bin</th>
<th>$\Delta \ln M$ uncalibrated</th>
<th>$\Delta \ln M$ calibrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.0 \times 10^{14} M_\odot$</td>
<td>0.0370</td>
<td>0.0257</td>
</tr>
<tr>
<td>$2.0 \times 10^{14} M_\odot$</td>
<td>0.0427</td>
<td>0.0299</td>
</tr>
<tr>
<td>$4.0 \times 10^{14} M_\odot$</td>
<td>0.0667</td>
<td>0.0473</td>
</tr>
<tr>
<td>$8.0 \times 10^{14} M_\odot$</td>
<td>0.1370</td>
<td>0.0999</td>
</tr>
</tbody>
</table>

To derive a calibration factor the multiplicative bias $M$ for the ksbm pipeline is measured for all PSFs and all shears for the first 31 chips in the DES simulated focal planes. A plot that shows the fit to the $M$ as measured on 31 chips is shown in Figure 5.13. The fit to the average multiplicative error for ksbm results is

$$M(SNR) = 0.013 \times SNR + 0.6811 \tag{5.16}$$

where

$$\gamma_{\text{calibrated}} = \frac{\gamma_{\text{measured}}}{M(SNR)} \tag{5.17}$$

and $SNR$ is measured from each source.
Figure 5.13: This figure shows the measured M bias from ksbm fit as a function of SNR.

This M calibration is applied as a function of SNR for all sources SNR < 25 to two areas, area one which consists of 4 chips (chips 32-36) and area two which consists of 4 chips (chips 58-62). These two regions are chosen as a random representative area and the behavior across the rest of the CSTEP simulated focal plane and in DES data should be similar. As shown in Figure 5.14 both sample areas show better ksbm performance when it is calibrated as a function of SNR. These results demonstrate that it is possible to decrease the statistical error of the ksbm pipeline, while keeping the systematic error at a similar level with a M calibration.
Figure 5.14: This figure shows the measured M bias from ksbm for both calibrated and uncalibrated $\gamma$ as a function of SNR. The top panel shows the results of the calibrated ksbm pipeline. The bottom panel shows the results of the uncalibrated ksbm pipeline.

To determine the M calibration has not introduce additional sources of systematic error the behavior of the calibrated and uncalibrated ksbm are compared as a function of redshift. The behavior of both samples is shown in Figure 5.15. The results show that both samples,
both calibrated and uncalibrated are stable as a function of redshift and thus suitable to be used in the DES survey. Since the calibrated sample returns many more objects it is better for high redshift clusters and should improve the statistical errors on the measurement.

Figure 5.15: This figure shows the measured M bias from ksbm for both calibrated and uncalibrated $\gamma$ as a function of redshift.
Chapter 6

Weak lensing on LBT and DES data

6.0.1 Observations and Data Reduction

The images of Abell 611 was observed in March 2007, during the Science Demonstration
Time (SDT) for the Large Binocular Camera (LBC) built for the prime foci of the LBT.
The observations consist of several sets of exposures of 5 minutes in the Uspec, SDSS g and
r band filters. A color image 2.82’ across and centered around the brightest cluster galaxy is
shown in Figure 6.1. The images were de-biased and flat fielded using iraf. The astrometric
solution and co-addition was done using SCAMP and SWarp.

The LBC focal plane is four CCDs each of which are 2048 x 4608 pixels with 0.2”/pixel.
The top chip is rotated 90 degrees with respect to the other chips, and is positioned across
the top of the other three chips. The chip area is equivalent to a field of view of 23’ x
23’. The chip structure creates a complicated PSF pattern, so the PSF is interpolated
across each chip, rather than across the full focal plane. The observations were taken with
seeing conditions that created a FWHM of 0.6 arc second seeing, or atmospheric smearing,
which is suitable for a weak lensing analysis. The photometric calibration is performed by
matching objects observed in these images to stars observed in the same field by the Sloan
Digital Sky Survey as shown in Figure 6.2.

These images taken at the LBT were used in the previous analysis LBCA611 (Romano
et al., 2010). The previous analysis used both a KSB and a Shapelets lensing pipeline and
compared the two lensing catalogs results (Romano et al., 2010). In this study of the data
Figure 6.1: A three color image of A611 LBC data, 2.82' across and centered around the brightest cluster galaxy at $\alpha = 8:00:56:760$, $\delta = 36:03:22:86$.

Table 6.1: Filter limiting magnitudes

<table>
<thead>
<tr>
<th>Filter</th>
<th>Exptime (s)</th>
<th>Magnitude $5\sigma$ detection</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDSS g</td>
<td>3600</td>
<td>28.3</td>
</tr>
<tr>
<td>SDSS r</td>
<td>900</td>
<td>26.9</td>
</tr>
<tr>
<td>Uspec</td>
<td>1200</td>
<td>27.2</td>
</tr>
</tbody>
</table>
both the r-band and g-band images are used in the shape measurement analysis, while in the previous analysis only the g-band data was used. This data is now analysed using a SNR dependent calibration of PKSB based on the results of the CSTEP challenge. LBCA611 used a different astrometric solution, SWarp and the previously published zero points from (Giallongo et al., 2008). Their photometric solution gave them a 5σ limiting magnitude in g of 28.0, in r of 26.0, and in Uspec of 27.2. In contrast we find a 5σ limiting magnitude in g of 28.3, in r of 26.9, and in Uspec of 27.2 as described in Table 6.1.

6.0.2 Star and galaxy selection

For a star to be selected to model the PSF, it had to be within the following criteria:

1. It was detected in all three bands, g, r and Uspec,

2. It had $1.6 < \text{FLUX\_RADIUS (pixels)} < 1.9$ in g
3. It had $20.5 < \text{MAG\_AUTO} < 23.0$ in $g$

This initial star selection is shown in Figure 6.3. This star selection is also shown as plotted as whiskers across the focal plane in Figure 6.4. The star selection was then run through the PSFex pipeline to create a polynomial function across each chip. The order of the polynomial function was determined by using the two-point autocorrelations of ellipticity residuals (D1) and cross-correlation of residuals and measured ellipticity (D2) as described in [Rowe, 2010]. These statistics are used to prevent either over or underfitting of the polynomial used to model the PSF across each of the chips. Our D1, and D2 statistics show a successful model of the PSF. To perform a more direct comparison to previously measured masses we take the background selection as described in LBCA611. This background selection consists of galaxies with $g > 23.0$ based on redshift distributions from the Canada–France–Hawaii Telescope Legacy Survey (CFHTLS). It was determined that this gives a contamination of likely foreground and cluster galaxies of around 10 percent. We then used all surviving objects on which a shape measurement was possible and calibrated as a function of Signal to Noise Ratio. For this shape measurement analysis both the r-band and g-band data was
Figure 6.4: Ellipticity plot of stars across focal plane. The stars from the top chip are shown in green. The galaxies which we select for our shear analysis are shown in yellow.
used. If an object had a SNR > 10 in the relevant band the shape measurement had an multiplicative bias M correction factor derived from the PKSB results on the CSTEP data applied.

### 6.0.3 Mass Measurements

We measure a $r_{200}$ value of $1504 \pm 120 / -140$ kpc and a c value of $4.1 \pm 1.8 / -1.5$ from PKSB. In contrast LBCA611 measures a $r_{200}$ value of $1545 \pm 345 / -306$ kpc and a c value of $3.9 \pm 5.6 / -2.1$ from their KSB+ pipeline and a $r_{200}$ value of $1570 \pm 177 / -170$ kpc and a c value of $3.7 \pm 2.2 / -1.3$ from their Shapelet pipeline.
Figure 6.6: Likelihood contours of the NFW fit of A611.
The results of the CSTEP study and the A611 project show that systematic errors in shear bias for lensing are of concern for large upcoming surveys. Although the level of systematic error due to shape measurement bias is within the limits required for the DES survey, it is not within the error required for the next generation of large surveys such as LSST. The overall results of CSTEP are promising but illustrate the challenges in developing a lensing pipeline that performs with the required systematic error for cluster weak lensing.

7.1 Conclusions and Summary of Results

A comprehensive test suite CSTEP was developed to test eight lensing pipelines, some of which had not competed in previous lensing challenges, on realistic distributions of galaxy properties as expected in DES data in the cluster shear regime. At least two of the pipelines ksbm and im3shape performed robustly, were able to perform within the statistical requirements imposed by DES, exhibited good lensing pipeline efficiency and were shown to have the same level of systematic error for sources independent of redshifts. These qualities make ksbm and im3shape suitable for a DES weak lensing analysis of clusters.

The CSTEP challenge also demonstrated the two pipelines, MJ and IMCAT were not suitable for a weak lensing catalog with a typical signal to noise ratio for DES, which exclude objects with a signal to noise ratio less than 20. The performance of IMCAT was shown to be poor overall and it is not suitable even for objects of high SNR. A surprising result of the CSTEP challenge was that DEIMOS performed less competitively than in had
in previous lensing challenges (Kitching et al., 2012).

The performance of two new pipelines, PFDNT and Gaussian Mixtures, was another positive result. Both PFDNT and Gaussian Mixtures performed competitively, and with further development may improve even more. The amount of shape measurement error from PFDNT will substantial improve if the bias due to selectively rejecting objects can be eliminated. The calibration of ksbm as a function of SNR developed using the CSTEP simulated images also shows considerable promise. When sources with a signal to noise ratio, $5 < \text{SNR} < 20$, are included, rather than limiting a lensing analysis to sources with a signal to noise ratio, $\text{SNR} > 20$, the statistical errors on the cluster mass measurement significantly improve. It was validated that the calibrated ksbm pipeline with more sources also does not increase the systematic bias, compared to original ksbm results only using sources $\text{SNR} > 20$.

The CSTEP challenge was able to measure the quadratic lensing bias, with greater precision than previous lensing challenges (Kitching et al., 2012). CSTEP determined that for a representative redshift, $z = 0.5$, all of the lensing pipelines tested do not have a quadratic bias of concern. In general CSTEP determined that a quadratic lensing bias most effects the concentration of the measured NFW profile.

CSTEP also determines that the multiplicative shape measurement bias $M$ allowed for lensing pipelines to remain within the statistical limits of 0.95 for DES. This level of precision can be achieved for most pipelines if sources with SNR > 50 are chosen, but only im3shape, ksbm, and Gaussian Mixtures achieve this level of precision for SNR > 20. Using sources of SNR > 50 rejects too many objects for the main lensing analysis of DES, but may be suited for other applications.

### 7.2 Future projects

There are a number of future projects that can build on the results of CSTEP. In CSTEP the $M$ calibration as a function of SNR significantly improved the statistical error for ksbm. The lensing pipeline ksbm was well suited for a $M$ calibration, but there are other developing
methods which may improve the statistical error for other pipelines. Direct calibration applies a small shear to the galaxies present in an image, and measures the shear response to determine a correction factor. The advantage of this method is that it acts on the data directly, so the source population is identical. In certain areas of the sky, such as around filaments and multiple clusters, the source population has different characteristics than in more isolated regions. Direct calibration will be more accurate in this instance since it does not rely on the distribution of the average properties of sources, but rather on the properties of the sources present in each image.

Combining the information from multiple shear measurement pipelines is another promising area of research. In CSTEP, when the shear was measured well, multiple lensing pipelines returned the same result. The sources that were measured badly, were measured badly for multiple pipelines, and the lensing pipelines returned different results. Studying the amount that multiple lensing pipelines disagree, places constraints on the accuracy of the lensing measurement. It requires more study to determine if the results of lensing pipelines diverge in a predictable way that can be used to determine the true shear, thus improving the systematic error. To improve the statistical error rather than comparing the lensing catalogs of multiple pipelines, the results are combined. All lensing pipelines reject some objects thus by combining multiple lensing catalogs, increases the total number of sources with successful measurements.

The CSTEP results also provides indications that for stacked cluster weak lensing, a statistic other than the mean may be more accurate. The presence of noise on images means that \( <\epsilon> = \gamma \) is only approximately true. It requires further examination to determine if alternative statistics, such as the median, are more accurate for some lensing pipelines.
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Appendix A

PIPELINE DESCRIPTION

A.0.1 im3shape

im3shape is a modular shape measurement code that performs a maximum likelihood (ML) fit of a de Vaucouleurs bulge plus Exponential disc galaxy model to noisy images, incorporating an applied PSF (see Zuntz et al 2013/in prep.). The software is written primarily in C with supporting Python infrastructure, and is publicly available. For the ClusterSTEP data the PSF is modelled as an elliptical Moffat profile (1969), assumed to be constant across each chip. Moffat profiles were fit (using the Levenberg-Marquardt algorithm) to stellar images in the ClusterSTEP data, and the chipwise model estimated from these individual fits using an inverse variance weighted average of the resulting best-fitting parameters. For the subset of fields in which the PSF was known to be Gaussian, a $\beta$ slope parameter of 1000 was fixed in the profile fitting process (the Moffat approaches the Gaussian profile for large $\beta$).

For galaxy shape measurement using parametric profiles, ML estimators are known to be biased due to the presence of noise (e.g., Refregier et al 2012; Kacprzak et al 2012). For the tests in this paper a suite of noise bias-calibrating simulations were not conducted, due to resource constraints and the challenge of producing a representative calibration suite for data with realistic distributions of size and signal-to-noise such as ClusterSTEP: the noise calibration schemes presented by Kacprzak et al (2012) and Zuntz et al (2013) were for far simpler distributions of galaxy properties. Understanding how to build such suites is an active field of research, and of great relevance to the many methods with known noise bias
issues (see also Bernstein and Jarvis 2002; Hirata et al 2004; Melchior and Viola 2012).

The performance of im3shape shear estimates is therefore expected to degrade somewhat as signal-to-noise decreases. Objects with an im3shape-determined signal-to-noise (in total flux) of lower than 10 were assigned a weight of zero in the final catalogue, along with catastrophic outliers in the value of the best likelihood, and objects for which any pixel value in a model-minus-data residual image was found to be greater than the peak pixel flux in the data.

A.0.2 KSBP

The KSB+ pipeline is based on the method as initially described in (Kaiser et al. (1995)). For KSB+ stars and galaxies the first and second moments are measured using a Gaussian weight function \( w(|\theta|) \),

\[
Q_{ij} = \int d^2\theta I(\theta)w(|\theta|)\theta_i\theta_j
\]

, where \( I(\theta) \) is the surface brightness distribution of the galaxy. For any KSB+ pipeline an assumption is made that the PSF can be described with a isotropic component (seeing) and an anisotropic part. KSB+ implementations been shown to perform competitively against other pipelines in large image simulation challenges such as (Kitching et al. (2012)).

A.0.3 FDNTP

For FDNTP we prepare the image postage stamp and PSF model in the same way as KSBP. Shear is estimated by applying roundness tests on the anti-sheared, deconvolved Fourier transform of the image. To this end, we use a weight function limited to frequencies where the Fourier transform of the PSF is above zero according to (Bernstein (2010)). We use the ellipticity estimate from the PKSB pipeline as a starting point and sample ellipticities on a hexagonal grid to find the shear estimate as the probability-weighted integral over ellipticity space.