A UNIFIED APPROACH TO OPTIMAL ENERGY MANAGEMENT
IN HYBRID VEHICLES

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ABSTRACT

In recent years, the need for alternative transportation has grown stronger due to concerns about environmental pollution and the realization that fuel reserves are not endless. Hybrid-electric vehicles are fast emerging as a viable form of alternative transportation. Hybrid-electric vehicles offer the prospect of better fuel economy and reduced emissions without compromising the performance of contemporary gasoline or diesel-powered vehicles. Hybrid vehicles differ from conventional vehicles in two important respects: the greater number of design degrees of freedom presents a more complex design optimization problem and a greater complexity in the powertrain imposes a greater challenge in control design.

Efficient control and coordination of powertrain components is critical to the success of hybrid technology. The primary function of a supervisory control strategy is energy management in the hybrid powertrain. The scheduling of the various power providers and consumers in the powertrain is an energy management function.

In the present work, a methodology is proposed for the optimal solution of the energy management problem in a unified context. The energy management strategy is optimized with respect to energy consumption. An abstracted model of hybrid powertrains is used to
cast the energy management problem in a power-efficiency based format. Dynamic programming is then used to solve the problem. In the operation of a hybrid vehicle, decisions at different time instants are not independent of each other due to the presence of integral constraints such as charge sustenance. Moreover, the quantity to be optimized is an integral quantity. Therefore, the nature of the problem makes the use of such a method necessary. The abstract model highlights some essential similarities between different hybrid powertrain systems. It is expected that the proposed unified methodology will allow the solution of the optimal energy management problem for different classes and types of hybrid powertrains using the same general problem structure. The methodology is also suitable for the two different operational contexts: on-line and off-line. Online optimization is important for economic operation. Offline optimization is important from the point of comparing different designs during the design process, which is essential for design optimization.
Dedicated to my parents and to Lord Shiva and Ma Durga
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NOMENCLATURE

$a_j$ - $j^{th}$ arc cost

$A_F$ - frontal area ($\text{m}^2$)

ADVISOR - Advanced Vehicle Simulator

BSFC - brake-specific fuel consumption

$C_D$ - coefficient of drag (unitless)

$C_R$ - coefficient of rolling resistance (unitless)

CARB - California Air Resources Board

CVT - continuously variable transmission

$E$ - primary or secondary energy consumed in a state transition

EM - electric motor

$F_A$ - aerodynamic resistance (N)

$F_G$ - grade resistance (N)

$F_R$ - rolling resistance (N)

$F_T$ - tractive force (N)

FUDS - Federal Urban Driving Schedule

HEV - hybrid-electric vehicle

ICE - internal combustion engine
LEV - low emission vehicle
N - normal force (N)
P\text{input} - input pressure (Pa)
P\text{output} - output pressure (Pa)
P_{\text{output, o}} - output pressure when input pressure is zero (Pa)
q - power flow appearing at the end of a power path
QSS-TB - QSS-Toolbox
S - optimal value function
SOC - state of charge, often used in an absolute sense as the secondary energy level
SUV - sport utility vehicle
T - torque (N-m)
TLEV - transitional low emission vehicle
ULEV - ultra-low emission vehicle
US - United States
VCU - vehicle control unit
V-ELPH - Versatile Electrically Peaking Hybrid
VPSim - Vehicle Performance Simulator
z - power load on powertrain
ZEV - zero emission vehicle
e - slope of Willans curve approximation
kW - kilowatts
m - mass (kg)
$m_f$ - mass-flow rate (kg/s)

mpg - miles per gallon

mph - miles per hour

s - second

$\alpha$ - weight for secondary cost term in arc cost

$\beta$ - time based penalty on state of charge variation

$\gamma$ - state of charge based penalty on state of charge variation

$\rho$ - air density (kg/m$^3$)

$\theta$ - grade angle (rad)

$\omega$ - rotational speed (rad/s)

$\eta$ - path efficiency
Chapter 1

INTRODUCTION AND MOTIVATION

1.1 Introduction

1.1.1 The motivation for hybrid vehicles

The gasoline motorcar has come to stay. In a hundred years since its first mass manufacture, the car has gone from being a rich man's gadget to an object of utility, status and entertainment, all at the same time. In the United States, it is a defining aspect of human existence. In other parts of the world, it is the commonest object of aspiration and one of the most prized possessions a person has. Modern day lives are built around cars: the desire to own one, the need to use one, the pleasure of flaunting one. It is therefore quite natural to expect that any form of alternative transportation in the immediate future would have to have a similar form to be successful. The question to be asked is: with everything going so well, where is the need to even consider alternative transportation?

A 1996 survey in The Economist [1] suggests four factors that may contribute to this need. Named in the survey, these are: oil depletion, global warming, urban pollution and urban congestion. The latter three factors have already started prompting legislation and government regulations for their control. The most notable among these are the California Air Resources Board's regulations. CARB has introduced four categories of vehicle based on environmental cleanliness: transitional low emission vehicles (TLEV),
low emission vehicles (LEV), ultra low emission vehicles (ULEV) and zero emission vehicles (ZEV). It has begun enforcing quotas for each type to be manufactured by the industry. Other initiatives are the US Tier 2 proposal and the European D3 norms.

Even though the prospect of an oil-depleted world seems to be well beyond our lifetimes, it is a reality that is inexorable. Moreover, the possibility of rising oil prices is definitely realistic to expect. It behooves us as an intelligent species to consider solutions before the problem arrives.

1.1.2 Where do hybrid vehicles stand?

In the arena of alternative propulsion, it is instructive to investigate where hybrid vehicles stand. The survey cited earlier seems to take the view that hybrid vehicles will be the long-range cousins of short range electric cars. This is also a view that is prevalent in parts of the hybrid vehicle community; hence the ‘range extender’ concept.

The other point of view is that hybridization is a means to alleviate the ill effects of the tight powerplant-load coupling in conventional vehicles. This school of thought seems to believe that hybridization allows the opportunity to operate engines in zones favourable to fuel economy and that the elimination of transients is a means to achieving lower emissions.
1.2 Hybrid Vehicles

1.2.1 Definitions

In a recent conference on Hybrid Vehicle technology [26] that the author was fortunate to attend, a very workable definition of the term hybrid vehicle was given by a DaimlerChrysler executive. This is adopted as the standard definition for this thesis. A hybrid vehicle is defined as an automobile that has two sources of energy, one consumable and one rechargeable.

The term component sizing implies the selection of the power capacities for the corresponding component. It does not imply anything about the actual physical size.

The term architecture denotes the physical configuration of the powertrain. It suggests the topological connections between the components.

The terms control policy or operational policy denote the overall operational sequencing of the power producing components in a hybrid powertrain.

1.2.2 Categorization

Hybrid vehicles are categorized on the basis of components, architecture and control policies. Depending on the nature of the energy storage mechanism, they can be called mechanical or electric hybrids. Mechanical hybrids are of different varieties such as flywheel, hydraulic or pneumatic. Electric hybrids or hybrid-electric vehicles are probably the most popular subject of current research, and most likely to be successful. Hybrid vehicles have a petroleum-based source as the consumable fuel and an electrochemical mechanism as the storage source.
Classification on the basis of control policy is mainly of two types: charge sustaining and charge depleting. The former have enough onboard electricity generating capacity so as to preclude charging from external sources such as the grid. Of course, these classes apply only to hybrid-electric vehicles.

There are two main classes of architectures that can be found in the hybrid vehicle domain. These are the series and parallel hybrids. Grossly speaking, in a series hybrid, only one type of power plant is directly coupled to the load. In a parallel hybrid there may be multiple couplings. However, systems like the Toyota Prius can have multiple power paths and do not fall strictly into any one type. In fact, as will be later found in this thesis, there can be a lot of similarities between series and parallel operation. A parallel hybrid can be operated as a series and vice versa.

Figure 1.1 Toyota Prius power flows [26]
At this point it is relevant to mention a recent approach that has been published on hybrid-electric vehicles classification [2]. This paper is a description of an approach to the classification of hybrid vehicle taken by Peugeot-Citroen. It defines a system to be used for hybrid vehicle classification and is motivated by the emergence of complex hybrids. The basis of the system is the definition of three key elements: the drivetrain, the drive system and the linking element. Hybridization is defined to be at the levels of the energy source, the drive system and the linking element. A quantification of hybridization is also given. Quantification of the level of hybridization can also be found in [3].

1.3 Motivation

A generic hybrid powertrain supervisory control unit can be thought of as being composed of two fundamentally distinct parts: the controller and the control strategy. Figure 1.2 represents this notion graphically by demonstrating the hierarchical control structure of a typical hybrid powertrain.

![Diagram of hybrid vehicle hierarchical supervisory control structure](image)

**Figure 1.2** A generic hybrid vehicle hierarchical supervisory control structure
The controller is the “brawn” of the VCU. Its function is to make the individual powertrain components act in accordance with the operational or control strategy. Controller architectures can vary widely depending on suitability to the system. Current popular thinking is that event based architectures such as PLC or Fuzzy Logic would prove to be most suitable to HEV applications. To repeat, the main considerations for the choice of a particular controller architecture would be suitability and ease of implementation rather than higher level issues such as fuel economy and emissions.

The operational or control strategy, on the other hand, is the “brain” of the VCU. It consists of the laws that the controller refers to, to issue the appropriate control commands. This is also where the operational philosophy of the hybrid powertrain is embedded. A typical example of the meaning of the term “operational philosophy” is the question, “given a certain engine speed, what torque should the engine develop such that fuel consumption (instantaneous or global) is minimized while satisfying overall load requirements”. Of course, one immediately realizes that this question is a direct consequence of asking “given a certain power demand on the hybrid powertrain, what should be the split between the different power sources such that some specific objectives are achieved”.

An example of the above distinctions can be had from a Fuzzy logic based controller architecture. The inference mechanism, fuzzifier and defuzzifier are parts that fall into the controller block. The fuzzy rule base is a part of the control strategy.

Specific system level objectives that are desired to be achieved dictate the choice of control strategy. Examples of system level objectives are: satisfactory fulfillment of load requirements (driveability), minimization of fuel consumption and minimization of tailpipe emissions. A control strategy may or may not be designed to meet every conceivable objective. However, control strategies designed to meet different objectives may differ significantly.
At this point, the impact of hybridization on control strategies needs to be mentioned. Hybridization allows us the opportunity to have more system objectives than what a conventional powertrain can possibly handle. This is made possible in hybrid powertrains by virtue of the decoupling of the load from the power sources.

Along with the opportunity comes a challenge. The challenge is to understand what can or cannot be achieved by hybridization and to design control strategies that can meet those goals (optimal control strategies). This is where the motivation for this work lies.

Next, particular aspects of the control policy element of a hybrid powertrain will be discussed. These aspects are desirable from the point of view of design optimization and others. These discussions will serve to put the goals of this work into a proper perspective and will point the way towards the formulation of the problem that is tackled in this thesis.

1.3.1 Design Optimization of Hybrid Powertrains, the big picture

Design of an engineering system entails the computation of numerical values for entities that characterize the system. These entities are called design variables. For example, if we wanted to design a box, we would have to choose the exterior and interior dimensions and the material. It follows that the selection of design variables is affected by the intended objective of the design. In the context of the box, if the objective were only to save space, we need only be concerned about the box dimensions. If the objective included saving weight, we would have to take the box material as a design variable. If aesthetics were to be considered, then we would have to include additional variables like shape and colour – variables that may not necessarily be characterized by numbers.

Therefore, the starting point of design optimization is the clarification of design objective(s) and the identification of the design variables that affect them. The design
objective(s) is/are quantified by the cost function(s). The design variables become the parameters of the cost function(s).

Having said this, let us now consider the hybrid powertrain design space. Compared to the conventional design space, it represents a quantum leap in variety and complexity [x]. A closer look reveals four primary aspects on which hybrid powertrains differ: component types, powertrain architecture, component sizing and control strategy.

The meaning of the term control policy was discussed at the beginning of this section. The term component sizing refers to the power ratings of the powertrain components and not their physical size. Physical size or weight is a consequence of the power rating and component type. Architecture refers to the topological configuration of the powertrain such as series or parallel. Component type dictates operational efficiencies of the hybrid powertrain.

Being the key aspects of a hybrid powertrain design, these can be assumed to be the four basic design variables in the hybrid powertrain design space. Design optimization of hybrid powertrains would then imply selecting these design variables to optimize one or more design objectives.

The following table shows a list of possible system level objectives and the design variables that affect them:

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<th>Design Variables</th>
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</tr>
<tr>
<td></td>
<td>• Component sizing</td>
</tr>
<tr>
<td></td>
<td>• Operational policy (control policy)</td>
</tr>
<tr>
<td></td>
<td>• Architecture</td>
</tr>
</tbody>
</table>
| Performance (Acceleration and Gradeability) | • Component sizing  
• Shape and Interaction Parameters  
• Control Policy  
• Architecture |
| Handling and Dynamics | • Architecture  
• Shape and interaction parameters  
• Internal physical location of components |
| Environmental Cleanliness (Emissions) | • Component type selection  
• Control policy |
| Cost | • All of the above |

**Table 1.1** Design objectives and design variables

Why is the design optimization problem for the hybrid design space so difficult? Design optimization of HEV powertrains is a complex issue due to factors such as: interdependencies amongst design objectives, interrelationships amongst design variables and the difficulty in quantifying the functional dependence of objectives on the design variables. Interrelationships amongst hybrid design variables are difficult because they are reversible, like the chicken and egg syndrome. There is a causal loop among the design variables that must be resolved by the designer. For example, selection of control policy depends on the powertrain architecture, but so does architecture on control policy. So where does a designer start? There is another kind of dependency. To compare two different architectures, a designer must implicitly assume some control policies. This is because the parameters on which two architectures will be commonly compared involve the operation of the system.
It must be clear now that the problem of control policy optimization is only a part of the overall design optimization problem. Moreover, this part is not separate from the optimization of component sizing, powertrain architecture and component type selection. Design optimization would be greatly benefited if either of the following two things were to happen:

1. A control policy was found to exist that was globally optimal, i.e., its optimality was not sensitive to variations in the other design parameters, or,
2. A method was known by which the best control policy for a given architecture and set of components could be determined. There would then be no problems in comparing designs, because the best possible performance of each design would be guaranteed.

The second method appears to be a more feasible path to take and is also suggested in [55].

1.3.2 The need for a parametrization framework

Due to the complexity and unique nature of the hybrid design space, traditional optimization techniques are probably not a very effective way of solving the design optimization problem. A mixture of heuristic reasoning and formal mathematical techniques must be employed to make the optimization process practicable.

One of the techniques proposed is the Large Design Space Search by Chandrasekharan and Josephson [x]. This technique utilizes the advancements in computing power of modern day computers and couples it with traditional Pareto optimality paradigms.

This method requires scalable and composable models of hybrid powertrains for automatic data generation. Work in parametrizing component sizing and architecture is
currently in progress at OSU. One of the biggest hurdles still facing the effective application of this method to the hybrid design space is the lack of a formal parametrization framework for control policies. Therefore, it would also be useful to have a method to parametrize control policies.

1.4 Aims and objectives

It was mentioned earlier that the primary motivation for this work lies in the design of control policies that can meet certain system level goals. These control policies can be called optimal with respect to the goals. It is now time to go back to the discussion about what factors guide the choice of a control policy for a hybrid powertrain. In other words, what are those goals that we want our control policies to meet?

The highest level control problem existing in the hybrid domain is the resource allocation problem. Resource allocation or energy management in hybrid vehicles refers to the definition of the relative usage pattern of the various energy sources within the system. The simplest hybrid powertrains typically have two energy types in their systems: one rechargeable and one consumable. An important issue for the control policy is its ability to apportion this energy efficiently, so that overall energy consumption is minimized. Among other things, a control policy must be able to split the power from various sources available in an efficient way.

Another issue for a control policy is its ability to operate the powertrain with minimum emissions. A third issue is to enable the powertrain to produce good acceleration and performance when demanded. These three together make up the bulk of goals that we would want our control policy to optimize. The first step towards the solution of this problem is to optimize the design with respect to one goal at a time. Fuel efficiency seems the obvious first choice.
The primary aim of this work is to develop a unified methodology to derive hybrid powertrain control policies that optimize fuel economy. This will be done by formulating and solving the resource allocation problem in a unified framework. The effort will be to formulate the resource allocation problem in such a manner so that optimal solutions for different hybrid powertrain designs can be derived from the same general problem structure or by using the same methodology. A solution of the resource allocation problem for a particular combination of architecture, component sizes and types will then be the optimal control strategy for that particular hybrid powertrain design.

It is hoped that the unified approach to the development of a framework to derive optimal control policies will reveal the essential similarity among different control policies for different designs. Conversely speaking, the essential similarities will be exploited in the development of this framework.

In the previous section, concerns were raised as to the sensitivity of control policies to variations in other design variables (especially architecture) and about the inability to parametrize policies. The unified approach adopted here is an effort to address these concerns.
CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter is a review of current literature in the field of hybrid vehicles, with a special focus on literature related to control and energy management of hybrid powertrains. It begins with a brief overview of literature in modeling, simulation and computer aided engineering tools for hybrid powertrains. It continues onto a review of work done in the field of hybrid powertrain design and design optimization. It concludes with a review of published literature related to control and energy management issues in hybrid powertrains.

2.2 Modeling, Simulation and Computer aided engineering tools

Simulators

The need for computer aided engineering tools such as simulators or simulation programs cannot be overstated in the field of hybrid powertrain development. Design and prototyping are expensive propositions for the automotive industry, and hence there is a industry wide demand for reliable tools that can accurately mimic the performance of real systems. The hybrid powertrain community has seen the development of a number of
such simulators. Miller [4] in his thesis describes the development of VPSim, a brand new simulation tool based on the novel concept of scalable and composable models for powertrain components. In this work is also available a review of the development of other simulators. Notable among these is the development of OSU HEVsSim [5], [6]. The review in [4] mentions the development of QSS Toolbox, which many consider to be the first truly conceptual level simulator. It was developed at ETH Zurich, and is a backward facing simulator incorporating scalable models for powertrain components. In the context of simulators, the ADVISOR program from DOE’s NREL is worthwhile of a special mention [8]. It has seen a rapid development in the last two years from a rather barebones simulator to one with a significant level of sophistication. It is a backwards-forwards simulator with a quasistatic approach. The current version of the simulator has thermal models and emissions data. Combined with its ease of availability, its easy to use GUI makes it a powerful starting point in the analysis and design of hybrid powertrains.

Another simulator available in the market is V-Elph [9]. It is a forward simulator developed by Texas A&M to simulate electric and hybrid electric powertrains. Little is known about PNGVSat, a simulator developed by SWRI for USCAR companies [10]. It is considered to be very sophisticated with very detailed models of powertrain components.

Apart from these established simulators, one often comes across various smaller programs written for a specific purpose. An example of this is Marr and Walsh [11]. Their work describes a computer program written for establishing optimal battery requirements for hybrid electric powertrains. This program uses optimality criteria and analyses to identify the optimum battery sizing and battery/engine combinations for different vehicle types and vehicle missions. This program has its focus on the battery system, and as such there is a comprehensive battery system model in the program.

Models

Linked closely with simulation programs is the development of mathematical models of hybrid powertrains. A review of published literature reveals that a significant amount of
work has been done in the development of reliable models of for hybrid powertrains. The purpose of model development is not unitary. Sometimes, models have been developed primarily for performance evaluation of hybrids, and at other for control development. The flavour of the models have depended on the intended purpose. Thus, one finds models whose primary purpose is energy performance evaluation to be mainly quasistatic, whilst those intended for control development to be dynamic. However, this distinction is rather synthetic and one often finds cross utilization of the developed models.

A first example of the latter type of model is found in the work of Hubbard and Youcef-Toumi [12]. These authors describe a Bond Graph model of a hybrid electric powertrain. Of necessity, the model is dynamic and has a system level approach. The paper presents dynamic models for various powertrain components and simulation results for the models.

A logical progression of dynamic models for various hybrid powertrains is found in the works of Powell et al. [13]–[17]. In [13], the authors describe a dynamic model for a range extender series hybrid electric powertrain. In [15] is described the development of a dynamic model for a parallel hybrid powertrain. More recently, [16] presents a comprehensive development and validation of dynamic models for a conventional powertrain. It also presents dynamic models and control law developments for series and parallel hybrid powertrains. In all of these works, the emphasis has been on highlighting issues related to the development of dynamic control laws for hybrid powertrains.

A description of models for hybrid powertrains would be half complete without the mention of the approach taken by Rizzoni et al. at OSU [18]. Rizzoni, in collaboration with Guzzella at ETH Zurich has hosen a unique road to modeling and simulation of hybrid powertrains. Their philosophy is one of scalability and composability. Scalability implies the ability of components to be characterized completely by a few chosen parameters, such that describing different instances of the same component type amounts
only to a simple variation in these parameters. This approach is extremely valuable in reducing computation time while performing exhaustive searches of the hybrid vehicle design space in order to discover optimal designs. They have currently been able to develop scalable models for IC Engines and Electric Machines (some types). Validation of these models and development of similar models for other components such as fuel cells and transmissions is underway. As mentioned before, the offshoots of this approach are the QSS Toolbox, and most recently VPSim.

2.3 Development, System Design, and Design Optimization

*System Development*

A wealth of information on actual hybrid powertrain engineering and system development can be found in the post competition reports released by the FutureCar competition authority [19]. Among these reports can be found the description of the development of the 1999 OSU FutureCar [20], along with the design reports of twelve other participating schools.

Among other literature published related to system design and engineering, [21]–[25] are worthy of mention. In [21], the parametric design of an electrically peaking hybrid powertrain is described. This is a hybrid philosophy followed by Texas A&M, and has seen some success. Davis [22] describes the engineering of an automated manual transmission for the Lawrence Tech FutureCar. The paper includes a brief description of the overall vehicle design and a detailed discussion of the justifications behind the design of the automated manual transmission.

The Uni1 concept car developed jointly by four German companies is described in [24]. The paper includes a detailed description of the design process, starting with the choice of a powertrain architecture. The paper describes the trade offs involved in the choice of
an architecture and the method used to resolve the trade offs. Design considerations for each individual powertrain component are described.

The development of the Toyota Hybrid System is described in [25]. The paper focuses mainly on the 1.5 L inline gasoline engine, reporting the design considerations and justifications behind the choice of the engine. It has a brief overview of the overall Prius configuration. More importantly, it has the Toyota philosophy towards energy optimal engine operation strategies.

The proceedings of the Hybrid Vehicles Toptec [26] contain development reports for quite a few hybrid initiatives, including American, European and Japanese. These include the Fiat system, the Peugeot-Citroen System, the Ford system, the Toyota system and the Nissan system. Each has its own unique points and is worth going through as an educational exercise.

*System level studies on hybrid technology*

Literature related to system level studies on the viability of hybrid vehicle technology can be found as far back as 1968 [27]. In this paper, the authors attempt to examine electric and a series hybrid powertrain from a systems standpoint. The operational problems and demands of the powertrains are evaluated. Road loads are investigated in detail in order to draw conclusions regarding component sizing and ratings in the powertrain. However, the fact that the control policy is an important design variable in hybrid powertrain design is not emphasized strongly. A simulation is conducted and some general conclusions are drawn about road loads and battery sizing, and the general viability of hybrid technology.

Liddle [28], in his 1973 paper analyzes the impact of hybridization on vehicle emissions. It is an interesting study in which series and parallel hybrid configurations were simulated over an FTP cycle. The effect of transients was not considered in the computation of the emissions. Conclusions were drawn as to the emissions reduction
potential of hybrid vehicles, which were found to even have worse emissions in some cases.

More recently, Cuddy and Wipke [29], have analyzed the fuel economy benefit of drivetrain hybridization. The study was conducted using the ADVISOR package and showed promising results for hybrid technology, especially for parallel hybrid technology.

System Design Issues and Design Optimization

Schwarz [30], deals with the design considerations for a hybrid vehicle in one of the early attempts at an organized approach to the solution of the hybrid vehicle design problem. The paper describes a hybrid vehicle design program undertaken by the author's organization and the approaches adopted therein. In the paper, we can see the emergence of design concepts such as a design goal, system constraints and the process of selection of design variables and system parameters. The awareness that the three main design variables in the hybrid design problem are powertrain architecture, component rating or sizing and control policy, can be felt to exist. The justifications and the selection processes of some of the design variables (architecture and control policy) are rather heuristic. There is an absence of a description of an overall design procedure, and the procedure itself is a little simplistic. Despite these issues, the paper represents a very good overall attempt at design optimization, and is a necessary first read for anyone about to be involved in design and design optimization.

Moore [31], in his widely referenced work takes another approach towards hybrid powertrain design optimization. The author maintains that the fuel savings potential of hybrid technology is highly dependent upon component optimization within the drive system. To this end, a series hybrid architecture is assumed after some qualitative arguments and analyses conducted towards what constitutes component and drivesystem optimization. Considerations for the selection and sizing of powertrain components are
presented. A design philosophy and specification methodology for individual components is suggested. The recommendation of appropriate control strategies is also qualitatively given. Here again, there is an absence of a definite procedure to determine optimal designs for different vehicle mission requirements and usage patterns.

A design methodology for electric and hybrid electric systems is presented in [32]. The method is based on designing the vehicles based on certain velocity parameters dictated by vehicle operation regimes such as acceleration and cruising. These regimes impose certain constraints on the system variables, which are then calculated to fulfill the design constraints, which are primarily the ICE and the electric motor sizes for the hybrid configuration. Even though it appears that the component sizing problem is decoupled from the control policy, a closer look reveals that a control policy has implicitly been assumed, which involves pure ICE operation at rated cruise speed and a pure electric launch, with the electric motor operating in the constant power region of its map. Weinstock et al. [33] present a design process for the optimal sizing and selection of hybrid vehicle components. In their paper they describe the design of a series hybrid powertrain using their design philosophy.

A decomposition based approach to the optimal design problem for hybrid powertrains is presented in [34]. A parallel type powertrain architecture is assumed which includes only a battery and a flywheel as power sources. An operational policy is also assumed and the problem is then solved for remaining design variables. In a similar study, Fellini et. al. [35] tackle the design optimization problem. This work describes both a software environment for design optimization and an application to a specific optimal design problem. The design problem tackled is an optimal component sizing problem given an architecture, a drive cycle and a control strategy. Various mathematical optimization algorithms are used for numerical solution of the problem.
The OSU approach to design optimization

Researchers at OSU recognize the complexity of the design optimization problem for the hybrid vehicle design space. The enormity of the hybrid powertrain design space prevents solution through brute force. The complexity of the problem stems not only from this enormity, but also from the interactive nature of the design variables, and a lack of understanding of the causality among these interactions. More importantly, the complexity of the problem is compounded when real time optimization is considered. How does one optimize for situations that are not known? Currently, efforts are being made to tackle the problem in all its complexity. Due to the nascency of this effort, tangible results are not yet available. Till now, however the OSU approach has been of a large design space search [36], [37] using artificial intelligence techniques. In this approach, the Pareto surface of the design space is mapped out, and optimal design chosen from this set. This approach is more general than others in the sense that it does not limit itself to a single architecture or drive cycle.

2.4 Control Issues

Controller architectures and control paradigms

A finite state description of the hybrid vehicle is found in [38]. The paper gives a state language from which microcomputer software can be produced using a translator. This state language is then used in a hybrid powertrain sequencing application. In the process, the hybrid powertrain is described in terms of a finite number of states, meta states conditionals and actions. Another state based approach to hybrid driveline management is presented in [39]. Here too, a finite state description of the hybrid drive mode is mentioned. A power-speed based operating strategy is assumed for the operation of the electric motor and IC Engine. A process oriented representation of the control strategy for the ika hybrid is described. The paper also includes a detailed look at the hardware and real time software characteristics.
Hierarchical control structures for series and parallel hybrid powertrains are presented in [40]. Powertrains are represented through a particular system description language. Specific operational strategies of the IC Engine are assumed. Powertrains are simulated for short term and long term behaviour and the strategies are optimized at different levels of hierarchy, and for different timespans. Details of controller structure for another hybrid vehicle are presented in [41]. This paper also describes a process based specification procedure for the formal definition of the control system for the powertrain. In this method, environment objects are again represented as finite state machines. Once again, operating strategies and drive modes are assumed.

Dynamic control laws

Control laws are presented for a dynamic powertrain model in [42]. An hierarchical control structure is proposed which comprises of a supervisory controller and component level controllers. Control laws are synthesized for individual components. The supervisory controller uses an adaptive identifier to generate commands for the low level controllers. Dynamic control laws for hybrid powertrains are synthesized in [13]-[17].

Mayer [43]-[45], presents the derivation of dynamic control laws for a special CVT based drivetrain. The control laws are based on an assumed engine operating strategy, which will be discussed later. The paper also contains a discussion of the various operational modes of the drivetrain, a derivation of its dynamic model, and a proposed hierarchical controller structure. The design of a robust drivetrain controller for this particular drivetrain is discussed in [45]. The controller uses sliding mode control as a means for tracking the commanded output. A slight modification is made in the algorithm to yield a continuous control law instead of a discontinuous one at the expense of a very small tracking error. The controller forces the engine to follow the assumed operating strategy. Simulation results for the developed control algorithm are presented and discussed.
Fuzzy logic/Rule based energy management

Baumann [3], presents in detail the design and development of an intelligent controller for a parallel hybrid powertrain. The controller uses neural networks to estimate road loads, battery state of charge and optimal engine operating points based on vehicle speed. Fuzzy logic is used to ensure that the drivetrain follows the commanded outputs. An operating strategy for the IC Engine is also presented in this thesis. The basis of the operating strategy is the operation of the engine on or near its minimum BSFC point.

A fuzzy logic based torque control strategy for a parallel hybrid powertrain is discussed by Lee et. al. [46]. The fuzzy controller produces a torque command for the electric machine in response to the accelerator pedal stroke and vehicle speed. The details of the fuzzy controller including inputs, output and rulebase are given in detail. Experimental results indicate a reduction in NOx and good charge balance through the use of this strategy.

Jalil et. al.[47] discuss a rule based control and energy management strategy for a series hybrid powertrain. This work is unique in the fact that an attempt is made to apply a power split strategy to a series powertrain. Traditionally, one finds the power split paradigm to be more popular for the parallel configuration, and the thermostat for the series case. The power split is determined through the use of a rule base. Improvements over the thermostat strategy are reported.

Another work that is found worthy of mention is the paper by Cerruto et. al. [48]. This paper also describes a fuzzy logic based energy management system for a series hybrid vehicle. The paper gives a concise description of the available power paths in a series powertrain. A detailed description of the fuzzy inputs, output and rulebase is given. The unique feature of this controller is that it performs energy management in the system by forcing the batteries to track a reference state of charge profile.
Operational strategies

Even though they are closely linked, operational strategies deserve a discussion separate from a discussion on controllers. It will have caught the reader's notice that almost each paper discussed till now has had an assumption of an engine or electric machine control strategy. A review of operating strategies found during the literature search is now presented.

As mentioned before, an engine operating strategy was assumed for the development of a dynamic control in [45]. This operating strategy involves the control of engine operation around an operating line. The line of power specific best efficiencies on the engine line was taken to be the desired operating trajectory of the engine. The same strategy was assumed by Schlueter [40]. Engine operating lines are defined differently but in a similar spirit in Wallentowitz [39]. Baumann [3] mentions an engine operating strategy for a parallel hybrid where the engine is always operated on or near its point of best efficiency on the map. One can find this idea to be prevalent in a few other places as well, such as Moore [31].

Optimal energy management, optimal control strategies, control optimization

Research on optimal operating strategies for hybrid vehicles appears to be by far the most attractive and intriguing aspect of hybrid vehicle research. The field is flooded with opinions. In what follows, published literature related to operational strategies that are optimal or claim to be so is presented. It will be found that approaches differ widely, from qualitative reasoning to mathematical rigour.

A work that needs to be mentioned in this context is the literature review conducted by Sweet [49] on control systems for automotive fuel economy. Though most of the paper deals with optimal control of conventional IC Engine vehicles, there is a section on
hybrid vehicles. Research related to the energy optimal operation of flywheel energy storage hybrid vehicles is reported in this section. Here it is mentioned that work has been done on the derivation of an optimal controller that minimizes quadratic energy losses in a particular flywheel hybrid drivetrain.

In another early work, Wouk [50] reports an operating strategy for a battery dominant parallel hybrid. The operating system is designed with the intention of minimizing fuel consumption. The operating strategy precludes charging of the batteries from the engine. The engine operation is on its maximum torque curve for most of its operation. The transition from electric to hybrid is dependent on a critical pedal angle, which is set heuristically. In the results, a better specific fuel consumption than conventional vehicles is reported as a consequence of this control policy.

A switching logic operating strategy for a parallel hybrid drivetrain is reported in [51]. It is claimed that the goal of the operating strategy is to optimize some system performance measure. In the current work, the state of charge is the measure chosen to be optimized. The procedure through which the control variables are chosen is based more on qualitative argument than on a definite mathematical procedure. Vehicle operation is broken into regimes characterized by the commanded torque level. A separate control logic is used for each of these regimes. The overall operation is such that the battery state of charge is maximized.

The fuel optimal operation of a specific parallel hybrid drivetrain is discussed in Guzzella et. al. [52]. The authors use a quasi static simulator to pose and solve several optimization problems. Optimal recharge conditions, optimal emission conditions and the effect of the value of a transition threshold on energy consumption are investigated. An operational strategy that optimizes fuel economy for CVT based parallel hybrid drivetrains is proposed in Kim et. al. [53]. A new metric of energy consumption called effective specific fuel consumption is defined which takes into account the battery power. Based on the metric, an operating line is defined on the engine map which is essentially the trajectory of minimum esfc. Maps of the control variables are defined as functions of
vehicle power demand and battery state of charge, and run time control variables are chosen from these maps.

Paganelli et. al. [54] present a mathematically rigorous approach to the derivation of an energy optimal operational strategy for a parallel hybrid drivetrain. The control input profiles are calculated for a given run profile. The global minimization problem is treated as a local problem, and the control variables are optimized for each instant. The charge sustaining constraint is implemented by transforming it into an equivalent fuel flow map, the derivation of which is not explained in detail but which appears to be conceptually similar to the approach in [53].

Having read the review up to now, it will come as a surprise to learn that scientific approaches to optimal energy management were published as far back as 1979, 1980 and 1988. In their 1979 paper, Willis et. al. [55] describe a simulation process developed by the Ford Motor Co.. It appears that at that time, the types of hybrids under investigation were primarily mechanical hybrids, i.e., hybrids with a flywheel, hydraulic, pneumatic or spring storage as the secondary energy source. Simulation of a hydraulic hybrid is reported in the paper. The significance of the paper lies in the fact that it gives a scientific method for optimizing the usage of the secondary source. For a given drive cycle, the optimal usage pattern is derived using sequential decision theory, which bears some similarities to the current work. The simulation also involves optimization of the engine calibration for the generated speed-load map. Optimization of the engine calibration was done using dynamic programming, and the process is reported in detail in [56].

The earliest published application of dynamic programming to hybrid vehicle control was found to be by Mosbech [57] in 1980 (if the previous work is not considered). In this work, a ‘power-split’ hydraulic hybrid is modeled, and an optimal control strategy derived for it for a drive cycle. The objective of the control policy was to minimize fuel consumption. The hybrid architecture considered in this paper is a combination of series and parallel architectures. The recursive algorithm uses a future fuel consumption concept, the use of which is not clearly explained or justified. The control variables are
the transmission mode, the combustion engine torque, the combustion engine speed and the hydraulic pump displacement. It appears that the minimum arc cost is obtained by using an iterative method through a computer routine.

Another use of dynamic programming to hybrid vehicle control was found in the work of Oprean et. al. [58] in 1988. Here again, the optimal secondary usage pattern is derived for a hydraulic hybrid for a given drive cycle. In this case, engine always operates at the point of best efficiency on its entire operating map. Using this knowledge, arc costs are calculated for each transition of the secondary source. Dynamic programming is then utilized to derive the best secondary energy trajectory from the possible trajectories. The definition of arc costs is not very clearly given and the paper is a little hard to follow.

Dynamic programming is also the method of choice for the current thesis, and will be used here within a more general framework.

The final paper that will be reviewed in this chapter is a very recently published work by Zoelch and Schroeder (1998) [59]. This is considered by the author to be currently the most advanced approach to the solution of the optimal energy management problem for hybrid vehicles. In this work, the authors derive the optimal engine operating trajectories for a dynamic model of a special parallel drivetrain with a CVT. This drivetrain has been mentioned before in [43] and [45]. The control inputs for the system are the engine and electric machine torques, and the CVT ratio. The optimal control inputs are derived dynamically for a given drive cycle, while ensuring a charge sustaining operation of the powertrain. A dynamic optimal control problem is formulated and solved numerically using an available SQP based algorithm mentioned in the paper. After the optimal control has been derived, they are used as control inputs to a simulation model of the driveline to verify their accuracy. The optimal engine operating trajectory is very close to the line of best power specific efficiencies, which would seem to justify the assumptions in [40] and [45]. In course of the current work, a similar justification will also be found.
2.5 Conclusions

Form the above survey, the following conclusions may be drawn regarding the published research in various aspects of hybrid vehicle technology:

1. A good amount of progress has been made in the development of both dynamic and quasi-static models for hybrid powertrains. A number of simulators are now available that can emulate vehicle operation to varying levels of accuracy and for various purposes. However, none of the simulators known to the author incorporate dynamic models and emissions calculations at the same time. This observation does not include PNGV Sat, which the author is not familiar with.

2. There is a lack of established or proven procedures related to either hybrid powertrain design or design optimization. This aspect seems to be still in a nascent state. A holistic approach attacking the hybrid powertrain design optimization problem in all its complexity is yet to be established or standardized. Approaches differ widely in the detailed specification of design variables for the hybrid powertrain design problem. The author also did not come across an eloquent study of the tradeoffs involved in hybrid powertrain optimization.

3. A process or state machine based approach seems to be popular for the real time supervisory level controller architectures. Rule based controller architectures are also becoming popular because of the complexity of hybrid powertrains.

4. Till now, with the exception of only a few instances, approaches to optimal energy management in hybrid vehicles have tended to be heuristic. The optimal energy management problem has begun to see mathematically rigorous treatment only in the very recent past. Surprisingly, systematic mathematical treatment of the problem was emerging around twenty years ago [55]–[58], but its progress has not been as consistent in the last decade. These approaches were linked to specific
systems, but could have easily evolved into generic approaches. It is as if they were lost only to be rediscovered now. There was also difficulty in finding literature in which generalized principles of optimal energy management in hybrid vehicles have been formally stated and justified. The author hopes that this thesis will fill these gaps.
CHAPTER 3

PROBLEM FORMULATION
PART 1

3.1 Philosophy

It is our belief that at a certain level, all systems share a certain degree of similarity. An example of this is the Bond Graph modeling paradigm, which treats all dynamic systems from a power flow viewpoint. If a suitable language can be found, apparently dissimilar systems can be described in the same unified manner. Finding a unifying description is contingent upon abstracting the problem to the proper level. An abstraction that is made for this problem is described next.

In chapter 1 it was mentioned that a generic VCU could be decomposed into a controller part and a control strategy part. The abstraction proposes the logical partitioning the control strategy block itself. It is imagined to be composed of two logical layers: the basis and the implementation. The basis is the kernel of the control strategy. It is the fundamental purpose(s) that the control commands try to fulfil. The implementation layer is the interface between the basis and the system. It is responsible for translating system objectives into control commands that the system can understand. The controller interfaces with the implementation and forces the system to behave in a certain way. Figure 3.1 shows the abstraction discussed above.
Figure 3.1 The logical partitioning of the control strategy into a basis and an implementation.

Going back to the fuzzy system example, the fuzzy rule base represents the implementation layer of the control strategy. It contains rules for control inputs specific to a given system such as throttle angle, gear ratio, motor torque etc. *However, the purposes for which the rules were derived need not be system specific.* For example, let us assume that the rules try to minimize emissions. Then, the phrase “minimum emissions” represents the fundamental purpose for which the rules were derived. In the framework that is being proposed here, the *basis* would be a set of physical and operational principles that *all* systems would have to follow to achieve low emissions. The basis would apply to all systems. Of course, a suitable language would have to be found first using which the basis would be written. Implementations for specific systems would then be derived from the basis, case by case.
The implementation layer of a control policy is specific to a given system. It depends upon the nature of control hardware and the control inputs available. The same control policy has to be implemented differently in a PLC system than in a Microcontroller based architecture. Differences in powertrain components cause an implementation to vary from powertrain to powertrain. For example, in a diesel engine with a common rail injection system, injection timing can be controlled whereas in a conventional diesel, it cannot. The gear shift sequence in a manual transmission can be infinitely varied, whereas in an automatic transmission it cannot. Differences in hybrid powertrain architecture are also a significant cause for the variability of control policies.

It is futile to try to find a unified description of control policies at the implementation level because of their specificity to individual systems. Till now, very little has been published on treating the supervisory control problem from a systems perspective. The review in the previous chapter gives us a feeling that the problem must be tackled at a higher level. The philosophy here is to seek a unified description of the basis, instead of the implementation. This is done by casting the supervisory control problem in as abstract a way as possible, and then obtaining specific solutions for specific cases.

3.1.1 Nature of the model

A modeling paradigm where power flow is the primary variable of interest is adopted here. This seems to be a logical choice in light of the philosophy stated above. The emphasis is on capturing the power loss characteristics of components rather than their dynamic behaviour, as in Bond Graphs. The power loss characteristics of components are obtained from their steady state efficiency data. Powertrains are modeled as power flow networks where each component is a node in the network. Different components fall into different generic types. Each type of component has different power loss characteristics.

Even though treating the problem as a quasistatic one can seem to be a bit oversimplified, it can actually generate a depth of information as to whether a deeper look into the
problem is warranted or not. A number of simulators including NREL's ADVISOR have used this quasi-static approach with success. The utility for the quasi static approach is also emphasized in [52].

3.1.2 Nature of the solutions

Solutions will be given in terms of power flow values to and from the components as functions of time. As stated before, the primary control and controlled variables will be power flows. Specific control input values such as throttle angle, gear ratio, motor torque can then be derived individually for specific systems. In other words, the solutions would be the basis for energy optimal operation of hybrid vehicles.

3.2 The abstract model

3.2.1 Qualitative development

The abstract model and the abstraction procedure used for the solution of the generalized supervisory control problem is now described.

A powertrain is a system of machines that converts energy stored in some form into useful mechanical work, which is then used to propel a vehicle. In conventional powertrains, energy is stored in the form of fuel as chemical energy. A heat engine converts this chemical energy into mechanical power. The rest of the components in the powertrain are concerned with regulating or distributing the power flow. Regulation can be of two types: modulation of the magnitude of the power flow or adjustment of the relative magnitudes of the flow and effort components of the power flow while maintaining the magnitude constant. Rizzoni, Guzzella and Baumann [18] have
developed a generic classification system for powertrain components on the basis of their functionality. They classify powertrain components as:

1. Energy Storage devices
2. Power converters
3. Power transformers

Examples of powertrain components that fall into each generic class are given in the table below [18].

<table>
<thead>
<tr>
<th>From To</th>
<th>Mechanical</th>
<th>Chemical</th>
<th>Electrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical</td>
<td>Gearbox, CVT</td>
<td>IC Engines</td>
<td>Electric motors</td>
</tr>
<tr>
<td></td>
<td>Flywheels</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemical</td>
<td></td>
<td>Reformers</td>
<td>Electrolysis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Flow Amplifier</td>
<td>Batteries</td>
</tr>
<tr>
<td>Electrical</td>
<td>Electric Generators</td>
<td>Fuel Cells</td>
<td>Power Amplifier</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Batteries</td>
<td>Super Caps</td>
</tr>
</tbody>
</table>

Table 3.1 Generic classification scheme of Rizzoni et. al.

A powertrain may then be abstractly visualized as being a network of these components. A description of a conventional powertrain using this language is shown in Figure 3.2.
Figure 3.2 Conventional powertrain description using the generic elements

Power flow has a definite directionality in this network. For example, in the conventional powertrain, it flows from the source to the sink. The source is obviously the fuel in the fuel tank. But what is the sink? Is there only one or are there more?

The answer to these questions lies in our viewpoint. Obviously, each component of the powertrain is a sink because each consumes some power during operation, which is reflected in its mechanical or thermal efficiency. However, here we define the sink to be the vehicle. To be more exact, the primary sink in the powertrain is the sum of the following:

1. The kinetic energy of the vehicle (translational and rotational),
2. The potential energy of the vehicle and
3. The work done to overcome the nonconservative forces that act on the vehicle.

This is why the vehicle is modeled as an energy storage device in Figure 3.2. This is a slightly inexact representation because it has included in it the losses due to the dissipative forces of friction and drag.

Why are we discussing all this? The point that this discussion is trying to make is that at a sufficiently abstract level, all conventional powertrains are chains that convert chemical energy to mechanical work, irrespective of the individual details of each
powertrain. Moreover, from an energy or power flow standpoint, all components could be replaced by their net effect on the system, which is to cause losses. Power losses in each component are represented by an associated component efficiency. The overall efficiency of the chain is the product of the efficiencies of the components appearing in the chain. Therefore, all conventional powertrains could be represented by a power flow from a chemical source to a mechanical sink. Their differences would be reflected in the power and efficiency characteristics of the path. This higher abstraction is shown in figure 3.3.

![Diagram](Image)

**Figure 3.3** A generic representation of all conventional powertrains.

The terms used in the model will now be mathematically defined. There are three kinds of quantities involved in the model. They are:

1. The quantity of energy stored at a storage node. These are the states of our system.
2. The instantaneous power flow appearing at the destination of a power flow path, and
3. The overall instantaneous path efficiency which characterizes the losses in the path.

Let us call the chemical storage Node A and the vehicle Node B in Figure 3.3. Let \( e_1(\tau) \), \( e_2(\tau) \), represent the quantity of energy contained in nodes A,B at time \( \tau \). At time \( t \), let a
power flow \( q \) appear at node B from node A. If the path AB has efficiency \( \eta \), then the following relations hold:

\[
\dot{e}_1 = -(q/\eta) \\
\dot{e}_2 = q
\]  

(3.1)

How are the effects of power transformers such as gearboxes reflected in this framework? They can be thought of as controlling path efficiency. Similarly, power regulators such as fuel injectors can be thought of as controlling the power flow magnitude. These two observations are the basis of this formulation.

This discussion can now be extended to cover hybrid powertrains. Figures 3.4 and 3.5 show the two levels of abstraction for a generic hybrid powertrain.

![Diagram of a parallel hybrid powertrain](image)

**Figure 3.4** Representation of a parallel hybrid powertrain using the generic elements
Figure 3.4 is the representation of a parallel hybrid powertrain in the language of Rizzoni et. al.. This model is abstracted further in Figure 3.5 using the logic presented before.

Figure 3.5 Parallel hybrid model abstracted further

As before, A, B and C are the node names and $e_1$, $e_2$ and $e_3$ are the quantities of energy stored in each node.

Figure 3.5 above deserves a more detailed explanation. The circles represent the two energy stores and the energy sink. The arrows represent the power flow paths from the storage elements to the sink. $q_1$ represents the net (instantaneous) power flow in the mechanical path. $q_2$ represents the net power flow in the electrical path. $q_3$ represents the net power supplied to the vehicle from the powertrain. The power flow path from the chemical storage is unidirectional, because it represents the irreversible heat engine. The power flow path from the electrical storage is bidirectional, because power can flow
either from or to it. However, the efficiencies in both directions need not be the same. Power in branch 3 is the net power supplied to the vehicle from the powertrain. It is bidirectional to allow for power recovery during regenerative braking.

The double-headed arrows in the bidirectional power paths signify that it is possible for power to flow in both directions in the path. This does not mean that power is flowing in both directions at the same time. An appropriate sign convention can denote the actual sense of flow. Henceforth, the following convention is adopted: a positive flow always signifies power flow towards the sink or away from the storage element. For example, \( q_2 \) represents the net power flow in the electrical path. Its sign determines the direction of flow. If it is positive, it means that power is flowing out of the electrical storage and into the summing junction.

The small circle in the middle is a lossless summation element or power splitter. It is a conceptual representation of the power summation that eventually must occur in any parallel drivetrain. The summation can physically occur at different places. For a parallel hybrid with an integrated starter/alternator, it occurs at the engine propshaft. In a parallel hybrid where the electric drive and IC Engine are on different axles, the summation occurs at the tire/road interface. In a system like the Toyota Prius where there may simultaneously be two power paths to and from the electrical storage, the path between the electrical storage and the summer represents the net of the two power flows. With a little reflection, it can be seen that Figure 3.5 is an abstract representation of all possible parallel hybrid powertrains with two energy stores, one of which is consumable and the other rechargeable.

At any instant, it must have at least one port as the output port. The flow in the output port is the algebraic sum of the flows in the other two ports. The port connected to the chemical storage is always an input port. The summer implies that the following relationship holds at all times between the power flows:

\[
q_3(t) = q_1(t) + q_2(t) , \quad \forall t
\]  

(3.2)
Series architecture

Figure 3.6 shows the composition of a generic series powertrain.

Figure 3.6 Generic representation of a series powertrain

To justify the statement that our model in Figure 3.5 also represents a series powertrain, let us investigate the allowable power flow paths in the above topology. The allowable paths are:

1. To and from the batteries to the vehicle through the power bus. SOC ±
2. From the APU/Generator set to the vehicle through the power bus. SOC =
3. From the batteries and the APU/Generator set to the vehicle at the same time. SOC-
4. From the APU/Generator set to the batteries and the vehicle at the same time. SOC+
If we now consider the summing junction in our simplified model to take thye place of the power bus, it is immediately obvious that all of the above paths are represented faithfully in the model. $q_1$ would then represent the power flow in the APU/Generator branch. $q_2$ would represent the power flow in the battery branch and $q_3$ the power flow in the motor branch. The efficiency of the motor branch can be lumped together with the efficiencies of the APU and the battery branches. The behaviour of the battery state of charge is mentioned in each case. A + indicates increase and a – indicates decrease. = indicates no change.

Here again, the summer implies the following relationship among the variables:

$$q_3(t) = q_1(t) + q_2(t) , \quad \forall t$$  \hspace{1cm} (3.3)

The similarities between the power flow paths in the series and parallel architectures are surprising. The difference lies in the ratings. In a series powertrain, the motor path must be rated to handle the peak power load as well, because it is the sole connection of the system to the environment.

### 3.2.2 Mathematical definition of the supervisory control problem

With the model and mathematical language defined, we are now ready to formulate the supervisory control problem. For this purpose, the model is shown again in Figure 3.7.
In the present framework, the supervisory control problem involves the determination of the time profiles of the power flows $q_1$ and $q_2$. We can also call this to be the power split between the conventional and electric sides. The control problem also involves the determination of the path efficiencies $\eta_1$ and $\eta_2$ as a function of time. The determination of the power split is subject to constraints, which are as follows:

1. The operation of the vehicle represents some net work that must be done to overcome the inertia of the vehicle and the resistive forces of friction and drag. This work manifests itself as a power demand on the system over time. For proper operation, the power supplied to the vehicle must be at least equal to the power demand. If we represent the power demand on the system by a function $z(t)$, then this constraint can be mathematically stated as
\[ q_3(t) = z(t) \] for all times of operation \( (3.4) \)

We follow the same sign convention for \( z(t) \) as for others: Positive power demand represents power flow away from the vehicle. Negative \( z(t) \) represents power that can be recuperated back into the vehicle. Therefore, negative power profiles represent braking regimes. From (1), the above equation becomes

\[ q_1(t) + q_2(t) = z(t) \] \( (3.5) \)

In this work, the power demands will be derived from existing driving cycle data for vehicles with typical characteristics for their class. Power demands are determined on a second by second basis using a quasi static approach that is standard for these applications. It is hoped that in the future, appropriate prediction techniques will be available for real time estimation of power demands. The methods in this work can then be seamlessly applied for optimization.

2. For our control policy to be optimal, a suitable index of optimality must be defined. This becomes the second condition that the control policy must satisfy. In this work, optimization of the control policy will be done only with respect to fuel economy. In order to maximize fuel economy, we must minimize fuel consumption. Fuel consumption can be quantified by the total flow out of the consumable (chemical) energy source during a trip. The criterion to be minimized is

\[ J = \int_{t_0}^{t_f} \left( \frac{q_1}{\eta_1} \right) dt \] \( (3.6) \)

Note that the criterion is an integral quantity and not a function. Currently, most methods approach the problem from an instantaneous optimization standpoint, with the assumption that the solution to the local problem will also be the solution to the global one. This may not necessarily be true. Secondly, these methods may not be
mathematically appropriate, as the optimality criterion in question is a functional and not a function.

3. A third constraint may be imposed on the system that is purely an operational characteristic of hybrid powertrains. This pertains to the quantity of energy in the rechargeable source, before and after a trip. In hybrid electric terms, this refers to charge sustaining or charge depleting. Even though “charge sustaining” is a concept that is intuitively easy to understand, it is extremely difficult to have an exact mathematical definition of the term. This is because the term refers to two different aspects at the same time: it refers to both the general design philosophy and the exact control strategy for a hybrid powertrain.

As an indicator of design philosophy, the term charge sustaining refers to a hybrid vehicle which is completely self sufficient in terms of rechargeable energy. The powertrain has the capacity for onboard replenishment and does not need an extra input port. In other words, the only thing that you would put into a charge sustaining gas-electric hybrid would be gasoline. The Toyota Prius is an example of that. Compare this to a charge depleting gas electric hybrid, where the batteries need to be charged from the wall at the end of the day. The UC Davis FutureCar is a prototype example.

Making the distinction between sustaining and depleting is not only important from the marketing viewpoint, it is also important from the point of view of comparability to conventional vehicles. These are the easiest of the hybrids that can be compared to conventional vehicles, because they act like one. An additional energy accounting does not have to be done for charge sustaining vehicles and their energy performance can be directly compared to that of conventional vehicles. It is easy to be misled by energy efficiency data for charge depleting vehicles because a different framework must be adopted to describe the energy efficiency of charge depleting hybrids. Charge sustaining hybrids, on the other hand, can fool no one. It would not be an exaggeration to say that the true test of hybrid technology or of a hybrid control
strategy lies in the performance of the charge sustaining species. That is why they are of so much interest to us.

Coming up with a definition of “charge sustaining” as a control strategy that is practical and mathematically exact at the same time is a very difficult process. SAE has its own definition of charge sustaining, where it defines it as the ability of a hybrid vehicle to maintain the state of charge of the batteries at the start and end of a trip. The term trip is also formally defined. However, the opinion here is that these definitions are artificial and break down in real operational circumstances. For the average motorist, the term trip could mean a million different things, depending on time of day and month of year. Secondly, it does not mean much to say that the batteries should be at the same exact state of charge? This is of course absurd. The tolerance range will have as many definitions as there are regulators.

Not only is it difficult to define in this way, it is also difficult to implement. This definition implies the application of an integral constraint on the control inputs. The integral constraint may be formulated as follows:

\[
C = \int_{t_0}^{t_f} \left( q_4 + \frac{q_5}{n_2} \right) dt = 0, \quad q_4 = q_2, \quad q_2 \leq 0,
\]

\[
q_5 = q_2, \quad q_2 > 0
\]

(3.7)

The existence of an integral constraint poses very difficult problems in the formulation and computation of solutions. All these considerations lead us to think about an alternative definition of the term.

In the Toyota Prius, the state of charge of its battery system is monitored continuously and is controlled within a band of values. This, to the author’s mind, is the best working definition of charge sustaining that one can find, and will be used in this work.
The first constraint is a universal constraint that all control policies should satisfy to have meaning. The third one is unique to the hybrid domain. The second criterion is the one that makes the investigation of hybrid vehicles relevant in the context of control policies. Truly speaking, the term “optimal control policy” only has relevance in the hybrid domain, because it is only here that we have the freedom to optimize. In a conventional powertrain, we have to do what the load demands. In a hybrid powertrain, we have the ability to modulate the load on the primary power source using a control input. This control input is the power available from the secondary source. Therefore, the formulation of an optimal control policy for hybrids is basically an investigation of how this modulation can be made useful, if at all.

In summary,

1. The control inputs are: $q_1, \eta_1, q_2, \eta_2$

2. The state equations are:

   \[
   \dot{e}_1 = -(q_1/\eta_1) \tag{3.8}
   \]

   \[
   \dot{e}_2 = -(q_2/\eta_2) \tag{3.9}
   \]

   \[
   \dot{e}_3 = q_3 \tag{3.10}
   \]

3. The constraints are:

   \[
   q_3(t) = q_1(t) + q_2(t) \text{, } \forall t \tag{3.11}
   \]

   \[
   q_3(t) = z(t) \text{, } \forall t \tag{3.12}
   \]
\[ \bar{q}_1 \leq q_1 \leq \underline{q}_1 \]  
\[ \bar{q}_2 \leq q_2 \leq \underline{q}_2 \]  
\[ \bar{\eta}_1 \leq \eta_1 \leq \underline{\eta}_1 \]  
\[ \bar{\eta}_2 \leq \eta_2 \leq \underline{\eta}_2 \]  
\[ \bar{C} \leq C = \int_{t_0}^{t_f} \left( q_4 + \frac{q_5}{\eta_2} \right) dt \leq C, \quad q_4 = q_2, \quad q_2 \leq 0, \quad q_5 = q_2, \quad q_2 > 0 \]  
\[ t_0 \leq t \leq t_f \]  

4. The system objective to be minimized is:

\[ J = \int_{t_0}^{t_f} \left( \frac{q_1}{\eta_1} \right) dt \]  

\[ (3.13) \]  

\[ (3.14) \]  

\[ (3.15) \]  

\[ (3.16) \]  

\[ (3.17) \]  

\[ (3.18) \]  

Series architecture

The structure of the optimal control problem essentially remains the same for the series case. The differences are as follows:

1. The power ratings of the power paths would have to be different in the series case. Thus, the upper and lower bounds on the power flow constraints (12), (13) would change.
2. The efficiency bounds are different.

### 3.3 Impact of the model on objectives

It is now time to take a first look at how the model just developed relates to our stated objectives. It is also an appropriate time to note what new insights into the problem of hybrid powertrain control are provided by this methodology.

1. This model reflects the fundamental structure of all hybrid powertrains. It tells us that at the most bare-bones level, all of them are simply chains of energy conversion from at least two energy sources to an energy sink.

2. This model allows us to present the supervisory control problem in a unified manner. The problem is converted to the determination of the power flows and path efficiencies for the primary and secondary power paths in a hybrid powertrain. Implementation level control policies can be obtained for different powertrains using the same basic solution. The solutions can also serve another important purpose. They can serve as scientific guidelines in the development of heuristic control approaches, such as fuzzy control.

3. This method presents a unified way to represent control strategies. Let a function called Power Split be defined as follows:

\[
\begin{align*}
PS(t) &= \frac{q_2(t)}{q_i(t)}, \quad q_3(t) \neq 0 \\
&= \frac{q_2(t)}{q_i(t)}, \quad q_3(t) = 0
\end{align*}
\]

(3.19)

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This function can be said to parametrize the overall control policy. Any control policy can be represented in terms of this function.

4. This methodology allows us the possibility of online real time optimization. The solutions are a function of the power demand. In this work, power demand is assumed known \textit{a priori} on the basis of a driving cycle. However, this is not necessary for the method to be successful. Work in online estimation and/or prediction of power demand for vehicle systems will enable these methods to be applied in real time with greater accuracy.

The uncluttered view offered by the model has also presented us with some insights into the business of hybrid powertrain supervisory control.

1. It is realized that hybridization adds a degree of freedom to the system by introducing an additional control variable, namely, the power available from the secondary power source. This causes the load to be decoupled from the primary source. The additional power available serves as an instrument that allows us to modulate the load on the primary (consumable) source. Formulation of an optimal control policy is directly correlated to the investigation of the manner in which load modulation can be useful.

2. Load modulation, and therefore the optimality of the control strategy, is a direct function of the state of charge trajectory of the secondary energy storage. This then defines a new hybrid powertrain control paradigm. This paradigm can be called “state of charge based control”. It means that state of charge is treated as a control variable instead of as a controlled variable in the framework of the supervisory control policy.

Next, a method of solution will be presented for the optimal control problem posed in this chapter.
CHAPTER 4

PROBLEM FORMULATION
PART 2

4.1 Choice of solution method

In the last chapter, the resource allocation problem for a hybrid powertrain with two energy stores was formulated as an optimal control problem. In this chapter, a solution method will be identified and the problem will be cast in a form compatible to that method.

We will recall from the last chapter that the quantity that we are trying to minimize with the available control inputs is fuel consumption (in terms of energy). Fuel consumption is an integral quantity. This means that we are trying to optimize a trajectory, and not an instantaneous quantity. Secondly, the operation of a vehicle is a staged process, and the decisions made at each stage affect the decisions made at a latter stage. Bellman’s Dynamic Programming method seems to be a logical choice of solution method in this case. Dynamic Programming is a well established optimization procedure which is very well suited for application to problems requiring a sequence of interrelated decisions. It can be easily recognized that the current problem fits this description very well.

The problem can also be viewed as the optimization of the state of charge trajectory of the electrical storage with respect to energy consumption. Dynamic Programming is a
method that is directly suited to the optimization of trajectories. In this work, the discrete version of the method is used. The quasi-static approach to modeling the physical system makes the discrete version well suited to the problem. It is also convenient from the point of view of computation. A detailed explanation of the method is available in the [61]. Previous instances of application of this method to hybrid vehicle control can be found in [57] and [58].

In the next section, the problem will be set up in terms of standard Dynamic Programming terminology and problem structure. The terminology used here is taken mainly from [61], but it is general enough for someone with knowledge of the method to understand.

4.2 Dynamic Programming Formulation

4.2.1 Stage Variable

In the current formulation, time is taken to be the stage variable. This may not be the only one that is appropriate. Powertrain simulations often use crank angle based approaches. Since the focus here is on the determination of the power split, it was felt that a time based approach would be sufficient.

Time is discretized into one second intervals. The solutions consist of second by second profiles of the power flows and path efficiencies. This approach is similar to the approach used in current quasistatic simulators such as ADVISOR. It is hoped that the approach used here will enable the results to be incorporated into the simulator control strategies with ease.
4.2.2 State Variables

The control inputs \( q_1, \eta_1, q_2, \eta_2 \) are our state variables. This leads to a four dimensional dynamic programming formulation. However, application of certain considerations reduce the problem to just one dimension. These are described next.

In the early stages of solution of this problem, dynamic programming schemes using path efficiencies as control inputs were implemented. However, it was found that the solutions consisted of only the best efficiency points in the data. For example if the efficiency of the electrical path was allowed to vary between .6 and .8, it was found that the minimum cost would always involve an efficiency of .8, irrespective of the power levels or power split involved. This observation can be intuitively justified as follows: for every power and efficiency combination that yields a certain cost, if a higher efficiency exists, a lower cost is obtained. From the point of optimality, therefore, only the highest efficiencies matter.

Physically, this means that the engine or the electric machine must be operated on its line of best power specific efficiencies to optimize energy consumption. For a CVT based system, the optimal operating line (OOL) of the engine is the line of best power specific efficiencies as shown in Figure 4.1. It may be recalled that this result is similar to the ones assumed or obtained in [45] and [59], respectively. For a system with fixed gear ratios, optimal operation implies the selection of the highest gear that is possible.
Figure 4.1 The line of best power specific efficiencies for a specific IC Engine [60]

Therefore, the quantities of importance to the formulation are only the best efficiencies that the power paths are capable of. These are functions of the power levels in the corresponding paths. For example, for a given powertrain and for say a power level of 30kW, the mechanical path can at best achieve an efficiency of .28. The best efficiencies for each power level must be supplied as data to the algorithm.

Due to these considerations, we can eliminate efficiencies as independent variables. Only the best efficiencies are used. This reduces the dimension of the problem apparently to two independent variables, $q_1$ and $q_2$. However, these are coupled through a constraint given by equation 3.4 repeated here:
\[ q_1(t) + q_2(t) = z(t) \]  \hspace{1cm} (4.1)

Assuming a value for one variable automatically fixes the value for the other. Therefore we are left with one independent variable that becomes the state variable for this formulation.

The net power flow in the electrical path, \( q_2 \), is chosen to be the single independent state variable in this formulation. The direct result of this algorithm is the power flow profile in the electrical path. The power flow profile of the mechanical path is the difference of the power demand profile and the electrical power flow profile. Dynamic Programming is used to optimize this power profile with respect to energy consumption.

The dependent variable is the net power flow in the mechanical path, \( q_1(t) \). Given \( q_2(t) \) and \( z(t) \), it is calculated according to

\[
q_1(t) = z(t) - q_2(t), \quad \begin{array}{l}
q_1(t) \geq 0 \\
= 0, \quad q_1(t) < 0
\end{array}
\]  \hspace{1cm} (4.2)

This definition is necessary to account for the fact that the mechanical path cannot absorb any power. It must be noted that 1 and 2 follow a strict order of causality. The electrical power flow cannot be derived from 2.

An electrical power flow profile is directly correlated to a corresponding state of charge profile of the rechargeable energy storage. The problem may also be formulated in terms of the state of charge as the independent state variable. In other words, instead of the electrical power flow profile, the state of charge trajectory is optimized. This should yield identical results.

The state variable is discretized into a number of discrete states. Jumps are allowed from one allowable power level (or SOC) to another. It is obvious that the finer our discretization is, the more accurate our solutions will be. However a trade off must be
made in terms of computation time while increasing the number of discrete levels the state variable can take.

This leads us to the formation of a typical DP path problem. The problem is to choose the appropriate path in the network given the initial final and boundary conditions. A network representation of the path problem is shown in Figure 4.2 with electrical power flow as the state variable.

![Network representation of supervisory control problem](image)

**Figure 4.2** Network representation of the supervisory control problem

The horizontal axis represents time, the stage variable. The vertical axis represents allowable electrical power flow, the state variable. It can be seen here that the state variable has been discretized into five allowable power levels. A node in the network represents a particular combination of time and power level. The arcs (arrows) joining the nodes represent jumps from one state to the other. There is a definite cost associated with
each arc (called arc cost), that must be defined on the basis of the overall cost function. The initial and final conditions can be represented as a starting node and an ending node or stage. A power flow trajectory is the path defined by a sequence of nodes from the start to the finish. The cost of the path is the sum of the individual arc costs. The problem is to find the path from the initial state to the final state(s) that minimizes our cost function. This would be the optimal time profile for the electrical power flow. In the figure above, the light arrows represent allowable paths. The heavy arrow sequence represents an EM power profile that could be a solution to some problem.

4.2.3 The optimal policy function

The optimal policy function tells us the optimal decision for each node. In the present context, the optimal decision represents a target power level in the next time step, or the power level in the previous stage in the case of a forward formulation. In program written to implement the algorithm, the optimal policy function is a matrix, whose every element is mapped to a node in the network. The number in each element points to (or from) the node that represents the optimal decision.

4.2.4 The optimal value function and recurrence relations

The optimal value function is the function keeps track of the best possible solution to the problem at any stage and state. Its arguments are usually the stage and state variable. The optimal value function for a particular stage state combination is the best cost that can be achieved at that point. The definition of the optimum value function decides the appropriate recursive relation to use.
In a backward dynamic programming formulation, we define the optimal value function $S$ as

$$S(t, q_2) = \text{the least energy consumption from the current node to the end of the process}$$

The appropriate recursive relation is then

$$S(t,q_2) = \min_{j} \left[ a_j(t,q_2) + S(t+1,q_2) \right] \quad (4.3)$$

$a_j(t,q_2)$ represents the arc cost associated with the $j^{th}$ decision at node $(t, q_2)$. $S(t+1,q_2)$ is the optimal value function at node $(t+1, q_2)$, which is the consequence of the $j^{th}$ decision at node $(t, q_2)$.

In a forward formulation, the optimal value function would be defined as

$$S(t, q_2) = \text{the least energy consumption to the current node $(t, q_2)$ from the beginning of the process}$$

The appropriate recursive relation is then

$$S(t,q_2) = \min_{j} \left[ a_j(t,q_2) + S(t-1,q_2) \right] \quad (4.4)$$

$a_j(t,q_2)$ represents the arc cost associated with the $j^{th}$ decision at node $(t-1, q_2)$. $S(t-1,q_2)$ is the optimal value function at node $(t-1, q_2)$, from which we arrive at node $(t, q_2)$ as a consequence of the $j^{th}$ decision.
4.2.5 Definition of arc costs

Until now, the development has been a generic one applicable to all dynamic programming formulations. However, the definition of arc costs is specific to the physics of each individual problem and depends on the objective function. The objective function for the current problem is the overall energy consumption. Therefore, the arc costs should represent the energy consumed by the system in making a state transition. First, methods to calculate energy consumption for the IC engine and the electric machine must be defined. For the conventional path, the energy consumed at each state transition is defined as

$$E_{ic}(t) = \frac{1}{2} \left[ \frac{q_1(t)}{\eta_1(t)} + \frac{q_1(t \pm 1)}{\eta_1(t \pm 1)} \right] \Delta t$$ \hspace{1cm} (4.5)

The energy consumed by the electric path is calculated as

$$E_{em}(t) = \frac{1}{2} \left[ \frac{q_2(t)}{\eta_2(t)} + \frac{q_2(t \pm 1)}{\eta_2(t \pm 1)} \right] \Delta t$$ \hspace{1cm} (4.6)

It may be recalled that $q_1$ and $q_2$ are related by equation (4.2), given a certain power demand. Therefore, energy consumption in one path determines energy consumption in the other.

The arc cost must represent the energy consumed by the system in going from one timestep to another. What we define as energy consumption is totally up to us, however. For a charge depleting strategy, accounting for the chemical energy consumed alone is not enough. Account must be made for the net electrical energy consumed also. This leads us to the first definition of an arc cost, as used in this work:

$$a_j(t, q_2) = E_{ic_j}(t, q_2) + \alpha E_{em_j}(t, q_2)$$ \hspace{1cm} (4.7)
\( \alpha \) is an empirical weighing parameter that is used to adjust the relative value of the electrical energy consumed. The subscript \( j \) denotes the \( j^{th} \) arc cost. The \( j^{th} \) arc represents the trajectory of the system as a result of the \( j^{th} \) decision at the node \((t, q_2)\).

For a charge sustaining strategy, it is enough to account for only the consumable energy. This is because charge sustaining implies that net electrical energy usage for is zero (except for the losses that occur in the electrical path). This leads to a definition of arc costs for charge sustaining strategies:

\[
a_j(t, q_2) = E_{ic,j}(t, q_2)
\]  

(4.8)

### 4.2.6 Implementation of a charge sustaining strategy

The usage of a weighing parameter as in 7 to account for the electrical energy consumed should not be confused with the implementation of a charge sustaining strategy. Even though the term charge sustaining implies accountability of the electrical energy used by the system, the manner in which it is done is intrinsically different. In a charge sustaining strategy, the accounting is done implicitly by forcing the net electrical energy usage to be zero over a cycle. Making a system to be charge sustaining is an operational issue separate from energy auditing issues. This distinction must be kept in mind because the method of implementation of a charge sustaining strategy in the algorithm used here looks very similar to the use of a weighing parameter as before.

Ideally speaking, a charge sustaining process could be implemented by the algorithm by looking only at charge sustaining profiles for electrical power, and choosing the one that gives the least cost. All profiles will not be charge sustaining. The ones that are not are automatically eliminated from consideration by the algorithm. Even though this is simple as a concept, it is difficult to program. Other methods must be used for this purpose.
There are two methods by which a charge sustaining condition is implemented here. These are described next.

1. The first is to penalize the optimal policy function on the basis of the difference of the current SOC and the original SOC. This is done as follows:

   \[
   S = \min_j [a_j + S_j + \beta(t) \Delta SOC]
   \]

   \[
   \Delta SOC = SOC_0 - SOC_t
   \]  \hspace{1cm} (4.9)

   Note that it is not a true integral constraint, because it also acts during the progress of the process, not just at the end. That is why the weighing parameter \( \beta \) is made a function of the stage variable. It has a small value at the beginning of the process to simulate a relaxed constraint. Towards the end, it is made larger to enforce the penalty more strongly.

2. The second method utilizes the concept discussed in Chapter 3 about the charge sustaining process used in the Toyota Prius. This is implemented here as follows:

   \[
   S = \min_j [a_j + S_j + \gamma \Delta SOC]
   \]

   \[
   \Delta SOC = SOC_0 - SOC_t
   \]

   \[
   \gamma = f(\mid \Delta SOC \mid)
   \]  \hspace{1cm} (4.10)

   \( \gamma \) is either a linear or nonlinear function of \( \mid \Delta SOC \mid \). The algorithm can be made to simulate an on off thermostat charge sustaining strategy by 'switching' \( \gamma \) on and off depending on the magnitude of \( \mid \Delta SOC \mid \) and the tolerance allowed.
4.2.7 Data and inputs

The final items left to describe are the data and inputs to the algorithm. The first kind of data that the algorithm needs are the power versus efficiency data for the power paths in the hybrid powertrain. The best efficiencies for different power levels have to be specified for the conventional and the hybrid paths in the powertrain. Therefore, the actual powertrain must be cast in the form of the model in Chapter 3, and power versus efficiency data derived from published efficiency data for the powertrain components.

The primary input to the algorithm is the system load or power demand profile, as denoted by \( z(t) \) previously. The current work derives this from established drive cycles, such as FUDS and the federal highway cycles. For real time optimization, the load profile has to be estimated. This requires some amount of look ahead on the part of the estimator. Prediction of load profiles is a separate subject in itself, and is beyond the scope of the present work. However, once available, this algorithm could be used seamlessly to give much improved results.

4.3 Alternative formulations

The problem could also be formulated in terms of the state of charge or secondary energy level as the state variable. Conceptually it is similar to the previous formulation, but the problem with this formulation is computational. The variation in the secondary storage energy level over a whole drive cycle is an order of magnitude greater than the its maximum variation over a time step. Over a whole drive cycle, the variation is in mega Joules, whereas a 30kW electric motor can at most cause a variation in the order of kilojoules over a one second time step. To accommodate this, the number of discrete levels of the state variable must be very large, and this imposes a direct computational overhead.
The advantage of this formulation is that the charge sustaining constraint can be imposed almost exactly, even if it would not make much sense terms of real situations. This alternative formulation is now described in some detail. Even though it is not used here, it is given so that other researchers can use it for reference.

### 4.3.1 Stage and state

The stage variable is time, discretized into 1-second intervals. The stage variable is the secondary energy level, $e_2$.

### 4.3.2 Optimal policy function

In this case, the optimal decision represents a target secondary energy level or state of charge in the next time step, or the secondary energy level or state of charge in the previous stage in the case of a forward formulation. In program written to implement the algorithm, the optimal policy function could be represented as a matrix, whose every element is mapped to a node in the network. The number in each element points to (or from) the node that represents the optimal decision.

### 4.3.3 Optimal value function and recurrence relations

In a backward dynamic programming formulation, the optimal value function $S$ is defined as before:

$$S(t, e_2) = \text{the least energy consumption from the current node to the end of the process}$$

The appropriate recursive relation is then
\[ S(t,e_2) = \min_j [a_j(t,e_2) + S(t+1,e_{2,j})] \]  \hspace{1cm} (4.11)

\( a_j(t,e_2) \) represents the arc cost associated with the \( j^{th} \) decision at node \((t, e_2)\). \( S(t+1,e_{2,j}) \) is the optimal value function at node \((t+1, e_{2,j})\), which is the consequence of the \( j^{th} \) decision at node \((t, e_2)\).

In a forward formulation, the optimal value function would be defined as

\[ S(t, e_2) = \text{the least energy consumption to the current node } (t, e_2) \text{ from the beginning of the process} \]

The appropriate recursive relation is then

\[ S(t,e_2) = \min_j [a_j(t,e_2) + S(t-1,e_{2,j})] \]  \hspace{1cm} (4.12)

\( a_j(t,e_2) \) represents the arc cost associated with the \( j^{th} \) decision at node \((t-1, e_{2,j})\). \( S(t-1,e_{2,j}) \) is the optimal value function at node \((t-1, e_{2,j})\), from which we arrive at node \((t, e_{2})\) as a consequence of the \( j^{th} \) decision.

### 4.3.4 Arc cost calculation

As before, the objective functional for the current problem is the overall energy consumption. Therefore, the arc costs should represent the energy consumed by the system in making a state transition. The only difference between the previous and current formulations is the method by which the energy consumed for each state transition is calculated. For the conventional path, the energy consumed at each state transition is defined as
\[ E_{ic}(t) = \left[ \begin{array}{c} q_1(t) \\ \eta_1(t) \end{array} \right] \Delta t \]  \hspace{1cm} (4.13)

The energy consumed by the electric path is calculated as

\[ E_{em}(t) = \left[ \begin{array}{c} q_2(t) \\ \eta_2(t) \end{array} \right] \Delta t \]  \hspace{1cm} (4.14)

The power flows \( q_1 \) and \( q_2 \) are calculated from the following relationships:

\[ q_2(t) = \frac{e_2(t) + e_2(t+1)}{\Delta t} \]  \hspace{1cm} (4.15)

\[ q_1(t) = \begin{cases} z(t) - q_2(t), & q_1(t) \geq 0 \\ 0, & q_1(t) < 0 \end{cases} \]  \hspace{1cm} (4.16)

There does not have to be any change in the arc cost formulation, because the definition of energy consumption does not depend on the formulation. As before, the arc cost must represent the energy consumed by the system from one timestep to another. This leads us again to the definition of an arc cost:

\[ a_j(t, q_2) = E_{icj}(t, q_2) + \alpha E_{emj}(t, q_2) \]  \hspace{1cm} (4.17)

\( \alpha \) is an empirical weighing parameter that is used to adjust the relative value of the electrical energy consumed. The subscript \( j \) denotes the \( j^{th} \) arc cost. The \( j^{th} \) arc represents the trajectory of the system as a result of the \( j^{th} \) decision at the node \((t, q_2)\).

For a charge sustaining strategy, it is enough to account for only the consumable energy. This is because charge sustaining implies that net electrical energy usage for is zero.
(except for the losses that occur in the electrical path). This leads to a definition of arc costs for charge sustaining strategies:

\[ a_j(t,q_2) = E_{ic_j}(t,q_2) \]  \hspace{1cm} (4.18)

4.4 Summary

In this chapter, an algorithm was formulated to solve the problem posed in the last chapter. This algorithm outputs the energy optimal power profiles for a hybrid powertrain, given certain data and inputs. The data are in the form of power versus efficiency characteristics of the powertrain power paths. The input is the load profile on the powertrain. In the next chapter, this algorithm is executed for typical cases of powertrains and load profiles.
CHAPTER 5

RESULTS AND ANALYSES

5.1 Physical Arguments

5.1.1 Introduction

In the previous chapters, the problem was cast as one of finding an electrical power profile (or a state of charge trajectory) that optimizes the energy consumed by the system. In this section, we try to find the optimal solution to our problem using simple physical energy and power related arguments. This is done by analyzing the effect of Load Modulation or Load Leveling on the energy consumption of a charge sustaining parallel hybrid electric vehicle without regenerative braking. Any nontrivial electrical power profile amounts to a form of load modulation on the primary power source. This is because the two are coupled together through the power demand. Therefore, this analysis should yield the nature of electrical power profiles that can optimize energy consumption.

5.1.2 Definitions

Load Modulation or Load Leveling

A particular drive cycle represents a certain time profile of the power required at the wheels for an automobile. This directly translates to a load profile for the primary power
source (the engine) for a conventional vehicle. In a hybrid vehicle, we have the ability to artificially modulate this load profile by, say, charging or discharging the batteries. This is what we call load leveling. Therefore,

**Definition:** Load Leveling is the artificial modulation of the required load profile by the use of some mechanism. Please note that the modulation may be both positive and negative, i.e., we may have a higher load at some instants and lower loads at others.

**Regenerative Braking or Regeneration**

It is particularly important to define this term as used here. In a drive cycle, there are regions where the acceleration of the vehicle is negative, i.e., the velocity of the vehicle must decrease. It is obvious that it is the job of the braking system and not the power producing side to enforce the vehicle to follow the velocity profile in these regimes. These regimes represent energy that can be brought back into the system through some mechanism, e.g., regenerative braking. Therefore,

**Definition:** Regeneration implies the recovery of part or whole of the energy available to the system during regimes of negative acceleration. These regimes are usually represented as a 'negative power' load on the system.

**Charge Sustaining Parallel Hybrid**

**Parallel hybrid:** A hybrid in which there is direct coupling of the load from each power source.

**Charge Sustaining:** A hybrid in which the average usage of the secondary source is zero over the period of one cycle time. For example, for an HEV it means that the battery's state of charge is the same at the end of a cycle as it was at the beginning.
Cycle time: It is up to us to define this. Here, we take it to be the time to complete one standard drive cycle like FUDS or NEDC.

5.1.3 Analysis

The system that is analyzed is a charge sustaining parallel hybrid electric vehicle without regenerative braking. We go about the exercise by proving two independent statements, and then combining them to get our final conclusion.

Statement 1

A cycle that is load leveled requires a higher energy to complete than the original one. This in turn results in a higher average power load during the cycle.

Justification

Let $E_R$ be the total energy required over a period of one cycle time to complete a given drive cycle. This is the integral of the power demand over the cycle time period. Let a policy of load modulation be implemented so that the load is higher during some of the cycle time and lower during the remaining time. Let this result in some energy $E_H$ being supplied to the modulating mechanism and some energy $E_L$ being taken from it. Then, the total energy required for the system to complete the load-leveled cycle is given as

\[ E_{Load\ Leveling} = E_R + E_H - E_L \]

Now consider the difference $E_H - E_L$. If there are no losses in the modulation process, $E_L$ is equal to $E_H$. If there are losses, as is always the case, $E_L$ is always less than $E_H$. Therefore, for all practical purposes, we can state that

\[ \Delta = E_H - E_L > 0 \quad always, \quad which \ implies \]

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\[ E_{Load\ Leveling} > E_R \]

Proved

A higher energy requirement during the same cycle time also means a higher average power demand

Proved

Graphical Explanation

In order to make the above conclusions geometrically explicit, let us analyze a drive cycle which has a constant power demand. Figure 5.1 shows such a cycle. The area under the curve represents the total energy demand \( E_R \).

![Graphical representation of energy demand](image)

**Figure 5.1.** The energy required during the cycle is the area under the curve \( P \).
Figure 5.2 shows a load leveling policy in which the batteries are charged for half the time and discharged in the other half. This results in a higher engine load $P_1$ during the first half and a lower load $P_2$ during the second half.

Figure 5.2 The area of the leftmost graph is the algebraic sum of the areas of the other three.

The engine load profile is shown in Figure 5.3. Since there are losses in the electric drivetrain area $B$ is always less than area $A$. In an ideal case, it can at most be equal to $A$. Therefore, the height $X$ is greater than $Y$, which means that the average power load is higher after implementation of the load leveling policy.
Figure 5.3 X is always greater than Y, which means $P_{LL\text{avg}}$ is always greater than $P$.

Since the proof was given without assuming any particular shape of the power demand profile, there is no loss of generality in assuming a constant power demand.

Statement 2

For typical engines, an increase in load represents an increase in chemical energy spent, even if the resulting efficiency is higher.

Justification

Let $P_1$ be a load which the engine can supply with a maximum efficiency of $\eta_1$. Let $P_2$ be another load greater than $P_1$ which the engine can supply with a maximum efficiency of $\eta_2$, which is greater than $\eta_1$. The cost or chemical energy spent for the two processes are then respectively, $P_1/\eta_1$ and $P_2/\eta_2$. For

$$\frac{P_2}{\eta_2} < \frac{P_1}{\eta_1}$$

we must have

$$\frac{\eta_2}{\eta_1} > \frac{P_2}{P_1}.$$  However, for typical engines the best efficiency ratio $\frac{\eta_2}{\eta_1}$ over the engine operating range is much less than the corresponding power ratio.
\( \frac{P_2}{P_1} \). Therefore, the previous condition can never hold over the range of operation of a typical, present day engine. Therefore, the statement that an increase in load implies an increase in cost or energy spent is justified.

The above statement can also be verified by plotting the ratio \( \frac{P}{\eta} \) along any direction (of increasing power) in the map of a typical engine. It always slopes upwards.

**Statement 3**

*For a charge sustaining parallel hybrid electric vehicle without regenerative braking, any load leveling strategy that has a region in which the engine load is more than what is required is not viable.*

**Justification**

This follows from combining statements 1 and 2.

### 5.1.4 Implications

The foregoing analyses yield the following implications for load leveling:

1. The load modulation process is not free—it requires energy to execute. If we supply this energy from the same power source that supplies our motive need, the net energy requirement would be higher.

---

\(^1\) Lines of constant \( P/\eta \) are quite similar to fuel consumption (g/s) lines on an engine map.
2. To be effective a load modulation process should *reduce* the average load over a cycle. This implies that the *only* part of a load leveling strategy that is actually *beneficial* to fuel economy is the part where load is reduced. The energy needed to execute the load leveling process should not come from the engine itself but from an external source such as regeneration.

3. The above analyses were done assuming quasi static conditions. They would not hold if the load leveling process actually reduces the energy requirement (by eliminating some dynamic transients, for example). For a load leveling strategy to be effective in a system without regeneration, the reduction in overall energy requirement should be greater than the energy consumed by the load leveling process itself.

4. Finally, it is now obvious that regenerative braking is absolutely essential to the success of hybrid technology. Through regenerative braking, we can recuperate the energy that would otherwise have been lost, and use it to reduce the total energy demand on the vehicle. The net energy demand on the primary power source is reduced via load leveling. The question that remains to be answered is at what times do we use the recuperated energy for maximum benefit? This question is answered through the numerical solution of the problem posed in the previous chapters.
5.2 Numerical Results

5.2.1 Drive cycles

Two common drive cycles were chosen for the purpose of deriving numerical results. These were the FUDS and the FTP Highway cycles. The speed vs time profiles of these cycles are shown next.

![FUDS Cycle](image)

![Federal Highway Cycle](image)

**Figure 5.4** The Federal Urban Driving Schedule (top) and Federal Highway cycles
5.2.2 Road Load profiles

For the purpose of deriving road load profiles for the drive cycles, the following vehicle parameters were chosen:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>1900 kg</td>
</tr>
<tr>
<td>Drag Coefficient $C_d$</td>
<td>.45</td>
</tr>
<tr>
<td>Rolling resistance coefficient $C_r$</td>
<td>.013</td>
</tr>
<tr>
<td>Wheel Radius</td>
<td>14”</td>
</tr>
<tr>
<td>Frontal Area $A_f$</td>
<td>2.58m²</td>
</tr>
</tbody>
</table>

|Table 5.1 Vehicle parameters for the calculation of road loads |

The road load (in terms of power) at any instant of time is given by equation 1.

$$P_{load} = m\frac{dv}{dt} + \frac{1}{2}\rho C_d A_f v^3 + vC_r mg \cos \theta + mgv \sin \theta$$  \hspace{1cm} (5.1)

The road load profiles calculated according to 5.1 for the FUDS and Federal Highway cycles are shown next.
Figure 5.5 Road load profile for the FUDS cycle

Figure 5.6 Road load profile for the Federal Highway cycle
5.2.3 Powertrain Characteristics

As mentioned in Chapter 4, data in the form of power vs best efficiency are needed for the powertrain. For this exercise, the following values were assumed:

<table>
<thead>
<tr>
<th>Mechanical Power Level, $q_1$ (kW)</th>
<th>Best Efficiency</th>
<th>Electrical Power Level, $q_2$ (kW)</th>
<th>Best Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>.25</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>.29</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>.36</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>.42</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>.42</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>.4</td>
<td>30</td>
<td>.8</td>
</tr>
<tr>
<td>20</td>
<td>.35</td>
<td>20</td>
<td>.77</td>
</tr>
<tr>
<td>10</td>
<td>.2</td>
<td>10</td>
<td>.75</td>
</tr>
<tr>
<td>0</td>
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<td>-</td>
</tr>
<tr>
<td>-10</td>
<td>0</td>
<td>-10</td>
<td>1.25</td>
</tr>
<tr>
<td>-20</td>
<td>0</td>
<td>-20</td>
<td>1.3</td>
</tr>
<tr>
<td>-30</td>
<td>0</td>
<td>-30</td>
<td>1.33</td>
</tr>
</tbody>
</table>

Table 5.2 Powertrain specifications

Using these values, the cost for any power level can be calculated by the expression

$$C_{\text{mechanical/ electrical}} = \frac{P}{\eta} \tag{5.2}$$
Note that the efficiencies for negative power levels for the electric path are greater than one because the power flow direction is reversed. This means that the cost for a negative power flow is less than the power flow, which reflects the losses during the charging cycle.

The arc costs were calculated using 5.2 and the above efficiency data and power demand profiles. A dynamic programming algorithm was then used to determine the optimal power split between the mechanical and electrical paths. The optimal power split and State of Charge profiles for different cases are shown and discussed next.

5.2.4 Numerical Output

Demonstration of the algorithm

In order to demonstrate the algorithm and the effects of the formulation of the arc cost function, results are shown for a hypothetical power transient. The power transient is symmetrical and rises to 30kW in the first half and falls to −30kW in the second half. The whole process is divided into seven stages. The hypothetical transient is shown in Figure 5.7.
Figure 5.7 Hypothetical load for demonstration of the effect of cost function formulation

For the charge depleting case, the appropriate arc cost is be taken to be a sum of the mechanical energy consumed and the electrical energy consumed/recuperated over the transition.

\[ a_j(t,q_2) = E_{ic_j}(t,q_2) + \alpha E_{em_j}(t,q_2) \]

A weight \( \alpha \) is applied to the electrical energy term. Different runs were made using different values of \( \alpha \) to demonstrate its effect on the results. Figure 5.8 shows the optimal power split profiles for three different values of \( \alpha \): 0, 2 and 3.
Figure 5.8 Effect of the parameter $\alpha$ on the optimal power split. CW from top left, $\alpha=0$, $\alpha=2$, $\alpha=3$

For $\alpha=0$, the algorithm favours a full electric use which is natural to expect. As the weight on the electric energy term increases, the electric usage during the positive load phase decreases and in the extreme even becomes negative. In all three cases, the algorithm favours regeneration.
Another question that might be raised about the algorithm is the effect of the fineness of the discretization of the states. In the results above, the electric machine state was discretized into 11 levels from −25 to 25 kW. Another run was made for α=0 using 51 discretized levels (1 kW increments) from −25 to 25 kW, and the results compared. The results are the same. This goes on to show that it is not necessary to have a very fine discretization. The discretization must be fine enough just to capture the optimal profile. Of course there is a trial and error element inherent in this process. Figure 5.9 shows the result of using the 1 kW discretization.

![Optimal Power Split Profile](image)

**Figure 5.9** Effect of the discretization on the optimal power split. This is the result for a 1 kW discretization.
City cycle, charge depleting

For the charge depleting case, the appropriate arc cost was be taken to be a sum of the mechanical energy consumed and the electrical energy consumed/recuperated over the transition.

\[ a_j(t,q_2) = E_{ic_j}(t,q_2) + \alpha E_{em_j}(t,q_2) \]

A weight \( \alpha \) was applied to the electrical energy term. Different runs were made using different values of \( \alpha \) to see its effect on the results. It was found that the algorithm favours a purely mechanical power supply when the weight is above a critical value. Below that, a mix of mechanical and electrical power is favoured. Figure 5.10 shows the optimal State of Charge profiles for three different values of \( \alpha \).

![Optimal State of Charge Profiles](image)

**Figure 5.10** Variation of optimal SOC profiles with \( \alpha \)
The optimal power split between the mechanical and electrical side also varies with $\alpha$. Figure 5.11 shows snapshots of the power split for varying values of $\alpha$.

**Figure 5.11** Optimal Power Split profiles for $\alpha$ equal to 0 and $\alpha$ equal to 2.4.
City cycle, charge sustaining

As mentioned before, charge sustaining is implied to mean a continuous modulation of the state of charge within a tolerance band. Figure 5.12 shows optimal state of charge profiles for varying widths of the band.

Figure 5.12 Variation of optimal SOC profiles with tolerance bandwidth $\gamma$. 
Figure 5.13 An optimal power split profile for $\gamma$ equal to .2.

Figure 5.14 Closer view of the power split between $t=700$ and $t=1000s$. 

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From Figure 5.14 it can be seen that the overall cycle can be divided into two distinct phases: electric dominant and mechanical dominant. In both these phases, the electric side is regenerating to the maximum extent. Figure 5.14 is a close up view of the transition from the first phase to the second.

Finally, Figure 5.15 shows an optimal SOC profile for the case where the SOC is not allowed to fall below its initial value. This constraint ensures that the electric side uses only the regenerated energy. The peaks in the figure represent instances when energy is regenerated and used.

![Optimal SOC Profile](image)

**Figure 5.15** An optimal SOC profile for the purely regenerative case

An optimal power split profile for the purely regenerative case is shown in Figure 5.16.
Figure 5.16 A snapshot of an optimal power split profile.

The total chemical energy spent in running the system in this mode is about 80% of the energy spent in running it conventionally. This implies that there is an approximate saving of 20% of the energy through regeneration.

The exercise is now repeated for the highway cycle.
Highway cycle, charge depleting

As before, for the charge depleting case, the arc cost was be taken to be a sum of the mechanical energy consumed and the electrical energy consumed/recuperated over the transition.

\[ a_j(t,q_2) = E_{ic,j}(t,q_2) + aE_{em,j}(t,q_2) \]

Different runs were made using different values of \( \alpha \) to see its effect on the results. It was found that the algorithm favours a purely mechanical power supply when the weight is above a critical value. Below that, a mix of mechanical and electrical power is favoured. Figure 5.17 shows the optimal State of Charge profiles for three different values of \( \alpha \).

![Variation of Optimal SOC Profiles with alpha](image)

**Figure 5.17** Variation of optimal SOC profiles with \( \alpha \)
The optimal power split between the mechanical and electrical side also varies with $\alpha$. Figure 5.18 shows snapshots of the power split for varying values of $\alpha$.

**Figure 5.18** Optimal Power Split profiles for $\alpha$ equal to 0 and $\alpha$ equal to 2.4.
Highway cycle, charge sustaining

As before, charge sustaining is implied to mean a continuous modulation of the state of charge within a tolerance band. Figure 5.19 shows optimal state of charge profiles for varying widths of the band.

Figure 5.19 Variation of optimal SOC profiles with tolerance bandwidth $\gamma$. 
Figure 5.20 An optimal power split profile for $\gamma$ equal to 0.2.

Figure 5.21 Closer view of the power split between $t=150$ and $t=550s$. 
From Figure 5.20 it can be seen that the overall cycle can again be divided into two distinct phases: electric dominant and mechanical dominant. In both these phases, the electric side is regenerating to the maximum extent. Figure 5.21 is a close up view of the transition from the first phase to the second.

Finally, Figure 5.22 shows an optimal SOC profile for the case where the SOC is not allowed to fall below its initial value. This constraint ensures that the electric side uses only the regenerated energy. The peaks in the figure represent instances when energy is regenerated and used.

**Figure 5.22** An optimal SOC profile for the purely regenerative case

An optimal power split profile for the purely regenerative case is shown in Figure 5.23.
Figure 5.23 A snapshot of an optimal power split profile for the purely regenerative case.

The total chemical energy spent in running the system in this mode is about 90% of the energy spent in running it conventionally. This implies that there is an approximate saving of 10% of the energy through regeneration.
5.3 An Application

Dynamic Programming was applied to devise an optimal controller for the simulator VPSim, developed at OSU. VPSim is a simulator that incorporates scalable and composable models of powertrain components. The motivation behind its development is the need for a scalable and composable tool to aid design optimization of hybrid powertrains. A detailed description of VPSim is available in [4]. Figure 5.24 shows the main interface of VPSim [4].

![Diagram](image)

**Figure 5.24** VPSim main interface

The simulator consists of models of individual components from which different powertrain architectures can be configured according to a plug and play philosophy. Figure 5.25 shows a parallel hybrid architecture using a hard coupling between the electric motor and the engine propshaft. This typically reflects a rear wheel drive vehicle.
**Figure 5.25** Parallel hybrid powertrain model top level view in VPSim

### 5.3.1 VPSim Controls

VPSim is unique in that it has static models for the powertrain components but a dynamic model for the vehicle. The static powertrain models are based on the Willans line concept, which is an innovative use of a concept that was originally meant to be applied only to IC engines. According to this method, the components are classified on the basis of an input output efficiency relationship. Moreover, the component parameters are nondimensionalized appropriately so that the models are scalable.

The vehicle model in VPSim includes a driver module that simulates the human element in a vehicle. The driver module is modeled as a PID controller that forces the dynamic vehicle module to track a prescribed velocity profile in time. The outputs of the driver module are the accelerator and brake commands. These are the inputs to the vehicle control unit (VCU) in the powertrain module.

The function of the VCU is to send command signals to the IC Engine, the electric machine, the brake and the gear box, depending on the accelerator and brake commands,
the SOC of the battery and the current vehicle speed. The VCU interprets the accelerator/brake command as a torque request and proportions the requested torque among the engine, electric machine and brake. The gearshift is independent of the torque request and is based solely upon vehicle speed. It simulates a fixed ration automatic transmission.

5.3.2 Intended Objective

As mentioned above, the VCU interprets the accelerator/brake command as a positive/negative torque request and apportions the torque between the torque providers. Currently, the apportionment of the torque request is heuristic and does not have any particular basis. The aim was to derive an apportionment based on optimal energy management in the system.

The dynamic programming method was applied to derive the engine/electric machine/brake torque split for a given drive cycle and gearshift profile. Some specific issues related to the application such as control variables and calculation of arc costs will be discussed next. The results will be presented after that and finally conclusions will be drawn.

5.3.3 Stage and State variables

As before, time was taken as the stage variable. The engine, electric machine and brake torques were taken to be the state variables. The state variables are not independent, but are linked together through the torque demand constraint. The constraints can be expressed as follows:

\[ T_{ic} + \frac{1}{\lambda_c} T_{en} = T_{prop.shap}, \quad T_{load} \leq 0 \]  \hspace{1cm} (5.3)

and

95
\[
\frac{1}{\lambda_g} T_{\text{brake}} + \frac{1}{\lambda_c} T_{\text{em}} = T_{\text{propshaft}}, \quad T_{\text{load}} \leq 0
\]  

(5.4)

where,

\[
T_{\text{propshaft}} = \frac{1}{\lambda_g} T_{\text{load}}
\]  

(5.5)

\(\lambda_c\) = Speed ratio of hard coupling between IC Engine and Electric Machine \((\omega_{em}/\omega_{ic})\)

\(\lambda_g\) = Overall gear ratio between propshaft and axle

Due to the existence of these constraints, the number of independent state variables is reduced to one, and that state variable was chosen to be the electric machine torque, \(T_{\text{em}}\). The results were obtained in the form of an optimal electric machine torque profile. The controller was programmed to command this torque profile from the electric machine. The remaining torques were derived using the above relations.

### 5.3.4 Calculation of arc costs

The arc costs were taken to be sums of the chemical energy spent and electrical energy spent/recuperated in a state transition, as before. For the charge sustaining case, the arc costs were taken to be only the chemical energy consumed by the system. Given an IC Engine torque level, the chemical energy consumed was calculated according to the following relation:

\[
E_{\text{chemical}} = P_{\text{input}} \left( \frac{V_d}{k\pi} \right) \omega,
\]  

(5.6)

where,

\[
P_{\text{input}} = \frac{P_{\text{output}} - P_{\text{output,0}}}{e},
\]  

(5.7)
\[ P_{\text{output}} = T \left( \frac{k \pi}{V_d} \right) \]  \hspace{1cm} (5.8)

Further,

\[ e = -2.75 \times 10^{-6} \omega^2 + 0.0015 \omega + 0.2585, \text{ and,} \]  \hspace{1cm} (5.9)

\[ P_{\text{output},e} = -200 \omega - 122230 \]  \hspace{1cm} (5.10)

Similarly, for the electric machine, the electric energy was calculated from

\[ E_{\text{em}} = P_{\text{input}} V_b \omega, \text{ where} \]  \hspace{1cm} (5.11)

\[ P_{\text{input}} = \frac{P_{\text{output}}}{e}, \text{ and} \]  \hspace{1cm} (5.12)

\[ P_{\text{output}} = T \left( \frac{1}{V_3} \right) \]  \hspace{1cm} (5.13)

Further,

\[ e = 0.89 \left[ 1 - \exp \left( -\frac{\omega}{50} \right) \right] \]  \hspace{1cm} (5.14)
5.3.5 Results

*Highway cycle, charge sustaining*

For the sake of comparison, three kinds of runs were made: the first was a conventional run, the second was a run in which the SOC was not allowed to fall below its initial value, and in the third case it was allowed to fall up to .1 below the initial SOC. Figure 5.26 shows the optimal SOC profiles for run 2 (left) and run 3. In Run 0, the SOC remained constant or increased indefinitely depending on whether the em was allowed to regenerate.

![Graphs showing SOC profiles for Run 2 and Run 3](image)

**Figure 5.26** Comparison of optimal SOC profiles for Run 2 (left) and Run 3

A comparison of fuel economy between the runs is tabulated below

---

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<table>
<thead>
<tr>
<th>Run type</th>
<th>Mileage (mpg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional (Run 0)</td>
<td>37.51</td>
</tr>
<tr>
<td>Run 1</td>
<td>38.73</td>
</tr>
<tr>
<td>Run 2</td>
<td>40.09</td>
</tr>
</tbody>
</table>

Table 5.3 Fuel economy comparison between runs

The next figure shows a snapshot of the torque split for run 2.

![An Optimal Torque Split for the purely regenerative case](image)

Figure 5.27 A snapshot of the optimal torque split between the IC Engine and E machine for run 2
The next figure is a comparison of engine operating points.

Figure 5.28 Comparison of engine operating points in Runs 1 through 3 (CW from above)
Highway cycle, charge depleting

Some interesting results were obtained for the charge depleting case. In this case, the arc cost was formulated as follows:

\[
arc\ cos = \text{chemical energy consumed} + \alpha(\text{electrical energy consumed/recuperated})
\]

Two runs were made with \( \alpha=0 \) (Run 1) and \( \alpha=0.5 \) (Run 2). The optimal SOC profile for the runs is shown in Figure 5.29, and a comparison of engine operating points in .

Figure 5.29 Optimal SOC profiles for Run 1 (left) and Run 2
Figure 5.30 A comparison of engine operating points for Run 1 (left) and Run 2

In the charge depleting case, it is not enough to characterize the performance of the vehicle using mileage only. An additional parameter representing the electrical energy consumed must also be included. A comparison of these figures is given below.

<table>
<thead>
<tr>
<th>Run #</th>
<th>Mileage (mpg)</th>
<th>Mileage (L/100km)</th>
<th>Electric J/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>52.35</td>
<td>4.5</td>
<td>894.6</td>
</tr>
<tr>
<td>2</td>
<td>36.07</td>
<td>6.5</td>
<td>54.73</td>
</tr>
</tbody>
</table>

Table 5.4 Energy consumption comparison between runs
City cycle, charge sustaining

For the sake of comparison, four runs were made: the first was a conventional run, the second was a run in which the SOC was not allowed to fall below its initial value, and in the third case it was allowed to fall up to .1 below the initial SOC, and the fourth time it was allowed to fall up to .2 below its initial value. Figure 5.31 shows the optimal SOC profiles for run 2 (left) and run 3. In Run 1, the SOC remained constant or increased indefinitely depending on whether the em was allowed to regenerate.

Figure 5.31 Comparison of optimal SOC profiles for Runs 1 through 3, CW from top left

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A comparison of fuel economy between the runs is tabulated below. Needless to say, the fourth run yielded the best mileage, because it was the least constrained.

<table>
<thead>
<tr>
<th>Run type</th>
<th>Mileage (mpg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional (Run 0)</td>
<td>25.45</td>
</tr>
<tr>
<td>Run 1</td>
<td>30.32</td>
</tr>
<tr>
<td>Run 2</td>
<td>31.87</td>
</tr>
<tr>
<td>Run 3</td>
<td>33.43</td>
</tr>
</tbody>
</table>

Table 5.5 Fuel economy comparison between runs

The next figure shows a snapshot of the torque split for run 3.

Figure 5.32 A snapshot of the optimal torque split between the IC Engine and E machine for run2
The next figure is a comparison of engine operating points.

**Figure 5.33** Comparison of engine operating points in Runs 1 through 3 (Clockwise from above)
City cycle, charge depleting

Some interesting results were also obtained for the charge depleting case. In this case, the arc cost was formulated as follows:

\[ \text{arc cost} = \text{chemical energy consumed} + \alpha (\text{electrical energy consumed/recuperated}) \]

A comparison of energy consumption figures is given in Table 5.6.

<table>
<thead>
<tr>
<th>Run #</th>
<th>Mileage (mpg)</th>
<th>Mileage (L/100km)</th>
<th>Electric J/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34.49</td>
<td>6.83</td>
<td>118.8</td>
</tr>
<tr>
<td>2</td>
<td>34.32</td>
<td>6.86</td>
<td>-61.01</td>
</tr>
<tr>
<td>3</td>
<td>19.7</td>
<td>11.95</td>
<td>-2469</td>
</tr>
</tbody>
</table>

Table 5.6 Energy consumption comparison between runs

Three runs were made with \( \alpha = 0 \) (Run 1) and \( \alpha = 0.3 \) (Run 2) and \( \alpha = 0.5 \) (Run 3). The optimal SOC profile for the runs is shown in Figure 5.34, and a comparison of engine operating points in 5.35.
Figure 5.34 Optimal SOC profiles for Runs 1, 2 and 3, CW from top left
Figure 5.35 A comparison of engine operating points for Runs 1, 2 and 3, CW from top left
5.4 Further Work

In this thesis, the optimal energy management problem was cast in terms of power and efficiency as control variables and solved using dynamic programming. An advantage of this approach is the clear insight available into the physics of the problem, which is clouded when implementation level control variables are used. Implementation level control variables are lower level variables like speed, torque, voltage etc.. Work needs to be done in the detailed definition of the mapping between the power-efficiency and speed-load coordinates, for all types of components. Maps for major components like IC Engines and Electric Machines already exist. Maps for other components must be developed. In this respect, the approach of Rizzoni et. al. [18] can be very appropriate, because it presents an opportunity to represent component maps in terms of algebraic relations. The power of the methods presented in this thesis can be fully realized with further concurrent development in component maps.

Another advantage of the methods presented here was that it allowed different systems to be treated in a unified manner and it gives us a method for the unified representation of control policies. These ideas of unification and control policy parametrization initiated in this thesis need further thought and refinement. It is hoped that this thesis will serve as very good starting point for that purpose.

The methods presented here are very well suited to off-line optimization, i.e., known load profiles. Extension of these methods to apply to on-line optimization is another area where further work needs to be done. In the on-line context, load profiles are not known a priori. There are three ways by which the author feels that these methods could be used on-line.

The first is to develop reliable and robust load prediction techniques, and use the methods in exactly the same form as they are presented here on the predicted load profiles. The
second way is to modify these methods by incorporating learning into the dynamic programming models as is done in adaptive control theory. The third would be to find a way to apply these methods along with a hybrid control systems approach. This would probably involve the most labour, as the field of hybrid optimal control is still quite nascent.

Finally, as mentioned in Chapter 2, further work needs to be done in the hybrid powertrain design optimization field. It is hoped that this thesis will contribute towards the development of standard design optimization procedures for hybrid powertrains.
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