Stress-Strain Behavior for Actively Confined Concrete Using Shape Memory Alloy Wires

THESIS

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By

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Abstract

Within this work a new and innovative method using shape memory allow (SMA) wires to actively confine concrete members is discussed and further researched in great detail. This new confinement method utilizes a constant confining pressure that is a direct result to the SMA material being able to recover large inelastic strains with the application of heat. Due to the extraordinary behavior that these SMAs display, this confining method is classified as “active confinement”, which has been proven to be superior in almost every way to the ordinary passive confinement method that is almost exclusively used throughout the world.

In an attempt to fully understand this confinement method, great detail was spent on accurately formulating the stress-strain relationship of confined concrete members under certain varied parameters. To achieve this goal Mander et al. (1988)’s passive unified stress-strain model was modified for application with SMA wires, and multiple equations were incorporated within the model for comparison purposes to analytically correlate the stress-strain relationship of the actively confined concrete.

Once multiple analytical formulations are shown and discussed, a medium scale experiment was undertaken, where the compressive stress-strain relationship of 15 SMA confined concrete cylinders was obtained. With this experimental data, the confining effectiveness of the model was verified and the most accurate formulation was concluded to be from equations by the Xiao et al. (2010).

This research highlights that SMAs have an endless amount of possible applications, and due to the drastic reduction in price that has occurred in the past and which is expected to continue to occur in the future, structural engineers need to be aware of these alloys and continue to find innovative methods that utilizes their extraordinary behavior. With this in mind, the confining method described within this work could do
just that and eventually become the predominant structural confining method in the not so distant future, due to the superior stress-strain relationship shown.
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Fields of Study

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Chapter 1: Introduction & Background Information

1.1 Introduction

The potential high recovery stress associated with shape memory alloy (SMA) wires, can make them a very useful material for application of active confinement on concrete structural members. The active confining pressure that results from the SMA confinement has been proven superior to the conventional passive confinement method that is widely used throughout the world and much easier to apply than many other active confinement methods that often require large mechanical devices to apply post-tension forces. However, active confinement of concrete members using prestressed SMA wires is a relatively new concept within the structural engineering community and the amount of experimental research is extremely limited, severely restricting the possibility for real world applications. It should also be noted, that the research that does currently exist for this topic has generally been focused on validating the confining potential under optimum scenarios. While these early results have been very promising in proving the effectiveness of SMAs for confining applications, very little research if any, has been conducted on situations that would be representative of real world applications. These tests, representing the so called real world applications, will be the next step in the overall process and will be required to demonstrate the confinement potential under lower and more cost effective confinement ratios.

Within this work, multiple topics dealing with confinement of concrete members will be discussed, such as: the overall importance of structural confinement, the difference between active and passive confinement in both general application and the resulting stress-strain relationships, a detailed introduction for SMAs describing all necessary background information and their confining potential, and a detail description of Mander’s et al. (1988) confinement model along with some results from other
important models recently developed for active confinement situations. With this background information, the stress-strain relationship for SMA actively confined concrete specimens were analytically evaluated and used to show the possible variations that can occur depending on various parameters. These parameters include: the spacing of the confining material (confinement ratio), the recovery stress of the SMA wire used, the unconfined concrete strength, and the applicable equations that are used to describe the confined concrete stress-strain behavior.

With all of the analytical formulations characterized and analyzed, the actively confined stress-strain relationships were then experimentally tested in an attempt to verify, and/or readjust the characteristic relations (if needed) to be as accurate as possible. With the experimentally verified stress-strain relationships, the results were then used to describe the stress-strain behavior of full scale SMA actively confined reinforced concrete members. This was done in an attempt to further aid the understanding and development for real world applications of SMA confined concrete members.

1.2 Stress-Strain Curves for Concrete Members

The relationship between the stress and strain that a particular material displays is known as that material’s stress-strain curve. This curve is unique for each material and is found by recording the amount of strain or deformation at distinct intervals of stress, with respect to the original specimen’s cross-sectional area and length. These curves reveal important properties of the material such as the modulus of elasticity (E), yield stress (in metallic materials), ultimate strength, and ultimate strain, but more importantly is the fundamental information required for advanced engineering applications, such as moment-curvature analysis which gives an indication of the available flexural strength and ductility.

With concrete being a brittle material its ductility is severely lower than that of steel, however it still exhibits considerable deformation before failure. To obtain an approximation of the complete deformation potential, a displacement or strain controlled compressive test is preferred as opposed to a load control method. With the strain controlled method the concrete specimen is compressed at a constant displacement rate; meaning that the overall displacement will remain fairly constant but the stress applied
will vary throughout the loading procedure. If the compressive stress applied to the concrete member is done with a load control rate, the specimen will fail shortly after the peak stress value is reached and display only the ascending portion of the stress-strain relationship of the concrete specimen leaving out important information detailing the post-peak relationship.

A typical stress-strain relationship for an unconfined concrete member (4000 psi concrete) is shown in Figure 1.1 below. From this typical relationship it can be seen that with the application of compressive force, the stress will increase with an initial linear portion that exist up to about 30-40% of the ultimate load. This will be followed by a non-linear portion where large strains will occur for small increments of stress until the peak stress is reached. The non-linearity portion of this curve is primarily due to cracks forming within the concrete. These cracks start with the formation of microcracks at the paste-aggregate interface, followed by cracking within the cement paste matrix with increasing stresses, until the cracks eventually increase in size and severity. This causes a network of large cracks to develop, continually reducing the load carrying capacity until the peak stress of the member is reached.
Figure 1.1. Stress-Strain Relationship for Typical Unconfined Concrete (4000 psi Max Compressive Strength)

Due to the fact that concrete displays a behavior known as “strain-softening”, which indicates a reduction in stress beyond the peak value with an increase in the strain. Obtaining the complete stress-strain relationship is difficult, but it is often necessary because an understanding of the complete (ascending and descending) stress-strain behavior of concrete is essential for accurate constitutive modeling (approximation of the response of a certain material to external forces). It should also be noted that the ascending portion of the axial stress-strain curve provides key material parameters such as the Young’s modulus, while the descending or softening portion gives an indication of the ductility of the concrete (Attard and Setunge 1996). For unconfined concrete the descending portion of the stress-strain curve is often approximated simply with a decreasing linear line until the ultimate compressive strain value of 0.004 (inch/inch) or a larger value is reached and the unconfined specimen is assumed to be completely crushed and unable to support further loads.
1.3 Importance of Ductility

The ability of a member to deform under stress is known as ductility and it is a very important quality within structural engineering. Past research has shown that ductility is comparable to strength in the overall importance of structures and should be incorporated within both individual components and the entire structural configuration.

The main goal for most structural design is to achieve a strong yet ductile structure. A structure meeting the overall strength and ductility requirements assures that the capacity of the structure is greater than the overall demand and allows the members to deform plastically, yet still carry the load. This permits overloaded parts of the structure to yield (plastically deform) but still redistribute stress without experiencing failure. The concept of ductility is significant because it prevents progressive and disproportionate collapses by ensuring that moment redistribution can occur if localized failure arises. Achieving ductility is partly a matter of design and partly a matter of detailing. Therefore, over the years most structural codes have been designed to assure a level of useful ductility by strict detailing procedures.

The most important design consideration for ductility within reinforced concrete columns is the provision of sufficient transverse reinforcement in the form of spirals or circular hoops, or rectangular arrangements of steel; depending on the shape of the original columns. This transverse reinforcement is essential in order to confine the compressed concrete, prevent buckling of the longitudinal bars, and prevent shear failure of the structural member.

It is well known that ductile materials exhibit large strains and yielding (when dealing with most metals such as steel) before the specimen ruptures. On the contrary, brittle materials fail suddenly and without much warning at much lower strains. Thus, ductile structures are naturally more desirable because they provide considerable warning through visual damage before a structure would ultimately fail, as opposed to brittle structures whose sudden failure is the main reason most seismic related deaths occur.

Ductility is advantageous under static loading but is clearly crucial to the structural response under dynamic loading, because it is directly linked with energy absorption capability. The energy absorbed is simply the area under a force versus
displacement curve or the area under a stress-strain curve (per unit volume). Figure 1.2 shows a sample stress-strain curve with the same peak stress and the potential energy dissipated (hatched area) for either a brittle or ductile failure mode. By comparing the potential energy dissipated in this figure, it is easily observed how ductility can be comparable to overall strength, since ductile materials are capable of absorbing much larger quantities of energy before ultimate failure occurs.

![Figure 1.2. Typical Energy Dissipated for A.) Brittle Failure. B.) Ductile Failure](image)

The stress-strain curves of brittle and ductile specimens shown in Figure 1.2 A and B respectively, also display how these types of structures would behave during a dynamic event such as an earthquake. From the example shown in Figure 1.2A it can be easily observed that brittle structures only slightly deform before ultimate failure, therefore most of its energy dissipation potential is directly related to the specimen’s peak stress value. Thus, brittle structures will show either very little or no damage at all during a seismic event, or experience a complete collapse with little or no warning. However, from Figure 1.2B it is obvious that ductile structures can deform significantly before ultimate failure, with most of the specimens energy dissipation potential existing after peak stress has been reached. Thus, highly ductile structures will frequently display lots of damage in a seismic event but the structure will still remain standing, greatly reducing potential casualties. Therefore, it is essential to remember that during dynamic loading such as seismic events, a strong but brittle structure is usually less desirable than a slightly weaker structure with adequate ductility.
1.4 Structural Confinement

Structural Confinement is utilized for the enhancement of the strength and ductility of a member/structure giving added restraint against loads such as dead or live loads, seismic, explosive, impact, or severe weather loads. Besides the major benefits of increasing strength and ductility, confinement of concrete also increases the overall stiffness; while decreasing the extent of micro-cracking and therefore crack propagation. Figure 1.3 shows the drastic difference that confinement can make for a typical concrete specimen. In this example the confined specimen had a 15% and 310% increase in the compressive strength and ultimate strain, respectively compared to the unconfined specimen. While it is important to remember that the benefits of concrete confinement could either increase or decrease as they are directly related to the type and overall amount of confining material that is used, this example clearly shows the potential benefits that can be expected from concrete confinement.

![Figure 1.3. Stress versus Radial and Axial Strain for both Confined and Unconfined Concrete Cylinders. (Andrawes et al. 2010)](image)

With respect to concrete, this confinement is often obtained initially (during construction) from internal sources such as transverse hoops or spirals often made of steel, or externally after construction is completed, with the addition of materials such as
jackets or wraps to the outside of the member and are often made of steel or carbon fiber reinforced polymers (CFRP).

While the overall stress-strain behavior displayed for either internal or external reinforcement is very similar in nature, this work will focus more on the retrofitting of members or the addition of confining materials after construction has been completed. Additional confining material may be required due to multiple reasons. Some of the main reasons include: an increase in service load above initial design requirements, simple reduction in strength due to material deterioration over time, changes within structural codes making the confining material already internally applied insufficient and unsafe for the current updated code, or simply to repair structural damages that may have occurred.

It should be noted that recently many structures built prior to 1970 have required added confinement of members because they have been deemed to display inadequate lateral strength and stiffness, as well as inadequate ductility from simple deterioration and/or damage of the structure. Many reinforced concrete members designed in this period often have inadequate shear capacity due to lack of transverse steel and confinement, inadequately lapped longitudinal steel, and premature termination of longitudinal steel.

The revision of newer and stricter building codes from this time frame has often played a major role in the use of concrete confinement. It was determined that older codes lacked adequate seismic resistance; therefore, the ductility of the members was often unacceptable and needed upgraded. Since added confinement greatly increases the strength and ductility, many members in seismic areas have required added external confinement to either help repair seismic damage that has already occurred or to preemptively protect from the possibility of future damage.

The increased structural use of high-strength concrete (HSC) and high strength lightweight aggregate (HS-LWA) concrete throughout the last few decades has also greatly increased the need for structural confinement. It is widely accepted that the potential benefits of a structural member are dependent on the strength to weight ratio of a member. These stronger and lighter options offer many structural benefits such as decreasing the overall weight and dimensions of the members, resulting in increased
usable floor space along with the span and live load capacity of the structure. Recent cost analysis studies have also shown that the incurred savings due to the increased benefits are significantly greater than the added cost of the higher quality concrete, which has made the use of HSC and HS-LWA concrete very prominent in the civil engineering industry, presumably assuring an overall increasing trend in the foreseeable future.

However, with concrete and most other materials, an increase in overall strength is accompanied by a decrease in the materials ductility (increased brittleness). This often limits their use in certain structures and or location such as in potentially seismic areas. To overcome this decreased ductility and brittle failures of HSC and HSLWAC members, the aforementioned benefits from added structural confinement is often required.

1.5 Structural Confining Approaches

Every possible structural confining method that exists will fall under one of two fundamental confining approaches; either passive or active confinement. The distinction between these two broad confining approaches is directly related to how the confining pressure is applied to the structural member. Each approach, when applied correctly, will significantly and beneficially increase the overall behavior of structural members, but major differences within the stress-strain curves will exist depending on the approach used.

1.5.1 Passive Confinement

Passive confinement is the first possible approach used for structural confinement and this method is by far the more common of the two approaches used to enhance the strength and ductility capacity of vulnerable members. Passive confinement of concrete is often applied with external steel jackets, but can also be applied with the application of other materials such as wraps made of fiber reinforced polymer (FRP). With this conventional method, the confining pressure applied is directly dependent on the lateral expansion of the concrete due to the axial load applied (attributed to Poisson’s effect) and the stress strain relationship of the confining material that was used (Richart et al. 1929).

In other words, with the passively confined method higher values of lateral strain of the concrete will directly result in higher confining pressures from the externally
applied material until the ultimate strain of the material is reached or the concrete crushes. Due to the fact that the confinement pressure is directly related to the concrete expansion, it should be noted that if the applied axial load upon a member is nonexistent or relatively small, the confining pressure from the external confining material will initially be negligible, and therefore will not have any effect on the load deformation behavior of the member. Only once the applied axial load is sufficiently increased causing the member to decrease in length (in the direction of the applied axial force) and increase in length in the lateral direction due to lateral expansion of the concrete, will the confining material begin to apply any beneficial confinement for the member. As this lateral concrete expansion continues to increase due to greater axial loads, the confinement pressure applied by the confining material will also increase until the material begins to yield and the ultimate failure of the confining material is reached.

The major disadvantage with this method being a direct function of the concrete expansion would be that in order for this technique to be fully engaged and the confining material to be fully utilized, the concrete has to have already experienced at least some sort of damage (cracking). It should also be noted that lateral expansion of concrete under an axial compressive load has been known to decrease with an increase in concrete strength. Therefore, this dramatically reduces the effectiveness of passive confinement when higher strength concrete is used. This decrease in lateral expansion with higher strength concrete is another drawback for this approach, as it causes a direct increase in the amount of confining material that will be required for a particular level of ductility to be reached in a HSC member as opposed to a member consisting of lower strength concrete (Ahmad and Shah 1985).

1.5.1.1 Compressive Stress-Strain Behavior of Passively Confined Concrete

To fully understand how passive confinement is actually applied to structural members, it helps to look at the stress-strain curves that passive confinement typically produces. Figure 1.4 below has been provided for this purpose, as it shows the typical stress-strain representation for a concrete member passively confined with steel hoops with varying levels of confinement. It should be noted that this confinement example,
representing a concrete member being passively confined with steel is by far the most common real world confinement method used.

![Figure 1.4. Schematic of Stress-Strain Curves of Passively Confined Concrete Specimens (Moghaddam et al. 2010)](image)

From Figure 1.4, it can be seen that passively confined concrete shows a negligible stiffness reduction until a stress level of about 0.7-0.8 of the unconfined concrete strength ($f'_{co}$). This result is expected, due to the understanding that during this initial portion the confined member has not exhibited enough lateral deformation to interact with the confining material. If the load and therefore stress on the member is increased to the end of this initial portion, the location noted as the critical point ($\varepsilon_{cr}, f_{cr}$) will eventually be reached. This location denotes that the rate of propagation of major cracks has become significant, and most investigators have been known to describe the stress at this region as the critical stress ($f_{cr}$) (Chen 1994). Once the critical stress has been reached, the stiffness of the member will start to decrease due to the continued development of cracks within the member and will begin to rapidly decrease as the stress-strain curve continues towards the second key point known as the yield point ($\varepsilon_{cc}, f_{cc}$). At the so-called yield point, the start of significant irreversible dilation in the concrete (due to the combined effect of cracks and plasticity) finally activates the external passive
confinement. Up until this location, the amount of strain and corresponding deformations in the concrete member were too small for the confinement level to have any noticeable effect on the confined member. This left the steel confining material simply sitting on the outside of the member not contributing in any beneficial way. At this point, it’s important to clarify that when concrete fails (once suitable forces are applied), it actually crushes and crumbles and therefore doesn’t yield like ordinary metallic materials. With this in mind, the term yielding or yield point could be confusing when describing concrete, but was simply used as a general term to describe that permanent deformations have occurred.

It should be noted that this so called yield point, shown on Figure 1.4, would approximately correspond to the peak stress location of an unconfined concrete member, and the stress-strain curve would then be followed by a sharply decreasing linear line until ultimate failure eventually occurred (refer back to Figure 1.1 for a clearer representation). However, since we are now talking about confined concrete this yield point doesn’t mean that the concrete member has failed; it does however represent the fact that the confining material is now the only mechanism maintaining the concrete from failure. It should be noted, that only after this yield point has been reached, the effect of the passive confinement finally starts to become apparent. From Figure 1.4 it can be seen that once this yield point location has been reached, the stress-strain curve shows a linear trend with a slope that is strongly influenced by the confinement level applied. This slope will be negative for low levels of confinement, but can also become positive if the confinement level is increased sufficiently high. The linear trend continues until the rupture of the confinement material is reached at the ultimate point ($\varepsilon_{ult}$, $f_{ult}$), followed by the curve sharply decreasing with a negative slope equivalent to the post-peak slope of unconfined concrete. It should be mentioned that if the confining material was metallic, the material would eventually yield before the ultimate rupture location is reached; giving this linear line a decreasing nonlinear portion (this yielding behavior of the confining material is not shown for overall simplicity).
1.5.2 Active Confinement

Active confinement is the other possible approach used for structural confinement. While this method is far less commonly used, it does have some major advantages when compared to the passive confinement applications. With the active confinement method, the confining pressure is not dependent on the lateral dilation of the concrete like passive confinement, but applied independently by the confining material. Figure 1.5 shows the difference in conventional passive confinement and active confinement prior to lateral loads being applied. Multiple materials have been used to apply active confinement, including the same materials routinely used for passive confinement such as steel bands or FRP wraps, with the only difference being that these same materials now must be either post-tensioned or prestressed in some manner.

Some of the major benefits of active confinement are that the confining pressure is only dependent on the material used to apply the confining pressure and the amount of prestress or post-tensioning that is applied. Because of this independent relation with the confining pressure and the concrete, the confining pressure applied is constant throughout the stress-strain curve and is applied before any damage to the concrete has occurred (before concrete dilation). This independent relationship that active confinement displays is crucial because it ultimately results in a superior stress-strain behavior when compared to passive confinement.

Figure 1.5. Conventional Passive Confinement and Active Confinement before Axial Loads are Applied (Shin and Andrawes 2009)
1.5.2.1 Compressive Stress-Strain Behavior of Actively Confined Concrete

Figure 1.6 shows the typical stress-strain model for actively confined concrete with varying level of confinements. This stress-strain model for actively confined concrete is noticeably harder to predict than the stress-strain model for passively confined concrete because the level of confinement affects the model throughout the entire stress-strain curve unlike the passively confined method where the confinement level only greatly affects the model after the yield point has been reached.

Actively confined concrete, like passively confined concrete, starts from the origin with an initial slope equal to the modulus of elasticity of concrete ($E_c$) and doesn’t show a negligible stiffness reduction until the critical point ($\varepsilon_{cr}, f_{cr}$) is reached. However, unlike the passive confinement method the location of this point varies greatly. The critical stress ($f_{cr}$) of actively confined concrete is almost always estimated as a constant percent of the yield stress ($f_{cc}$), and can best be assumed to be approximately 85% of the yield stress (Moghaddam et al. 2010). This yield stress value for active confinement however can be significantly increased by increasing the amount of confinement applied, as opposed to the behavior of passively confined concrete whose critical stress location cannot be altered by any level of confinement.

After this point has been reached, the rate of propagation of major cracks again becomes significant within the member causing the stiffness to again be rapidly decreased until the location of the next key point, known as the yield point ($\varepsilon_{cc}, f_{cc}$), is reached. The locations of the yield point again signifies the start of significant irreversible dilation in the concrete (due to the combined effects of cracking and plasticity), however, with the active confinement application the location of this yield point can vary greatly depending on the level of confinement that was applied to the members.

After the yield point, the confining material is the only thing maintaining the yielded concrete, just like with the passively confined method. However, the post yielding slopes for actively confined concrete are almost always negative and at best cannot be much greater than zero (slightly positive). This expected negative post yielding slope is due to the fact that the confining material stiffness has now been
reduced to at most the strain hardening stiffness. This noticeable behavior is due to the fact that the confining material itself must yield at the exact time as the concrete member; opposed to the post-yield slope for passively confined concrete that can take large positive values due to the material still possibly behaving elastically after the concrete yield point ($\varepsilon_{cc}, f_{cc}$) is reached. The linear trend of the post yielding slope continues until the rupture of the confinement material is reached at the ultimate point ($\varepsilon_{ult}, f_{ult}$), and then the curve again sharply decreases with a negative slope equivalent to the post-peak slope of unconfined concrete.

With the knowledge displayed above, the complete stress-strain curve that passes through the origin, critical point, yield point, and has an initial slope at origin of $E_c$ and final slope at yield point equal to the post-yield slope can be obtained with the use of a forth order polynomial. However, for engineering purposes a second order polynomial can accurately approximate the pre-yield branch of the curve by neglecting the initial slope at origin of $E_c$ and final slope at yield point equal to the post-yield slope.

![stress-strain curve](image)

Figure 1.6. Schematic of Stress-Strain Curves of Actively Confined Concrete Specimens (Moghaddam et al. 2010)
1.5.3 Important Distinctions between Active and Passive Confinement

1.5.3.1 Stress-Strain Behavior

By comparing Figures 1.4 and 1.6, Moghadam et al (2010) discovered the following important distinctions between the stress-strain behavior of active and passive external confinement:

- The critical stress \( f_{cr} \) of actively confined concrete is almost always a constant percent of the yield stress \( f_{cc} \), and this value can be significantly increased by increasing the amount of confinement, as opposed to the behavior of passively confined concrete which does not alter the critical stress.

- The confining material of passively confined concrete yields after the concrete yield point \( (\varepsilon_{cc}, f_{cc}) \), while material of actively confined specimens always yield just at the yield point of the material, implying optimum utilization of the material in concrete confinement.

- The post yielding slopes for actively confined concrete can’t be much greater than zero, because the confining materials stiffness vanishes to (at most) the strain hardening stiffness, while the post-yield slope for passively confined concrete can take large positive values due to the material still possibly behaving elastically after the concrete yield point \( (\varepsilon_{cc}, f_{cc}) \) has been reached.

- For identical confinement layouts, the actively confined concrete specimens will show higher strength and less ultimate strain in comparison with passively confined concrete.

- The key factor attributing to the active confinement method superiority is the delay in the damage sustained by the concrete as a result of early application of the confinement pressure; in the case of passive confinement, the concrete would have to deform laterally (i.e., dilate) and be damaged to some extent for the confining pressure to be fully activated.

1.5.3.2 Stress-Volumetric Strain Behavior

To view the benefits of active confinement displayed in another way, Figure 1.7c shows the stress versus volumetric strain (ratio of change in volume due to deformation
to original volume) of unconfined, actively confined, and passively confined concrete. On this plot the stress is plotted on the y-axis and the volumetric strain is shown on the x-axis, with volumetric compaction being signified by being on the right side of the centerline and volumetric expansion being signified by being on the left side of the centerline.

From this plot it can be seen that with the continual increase in stress the unconfined concrete specimen first experiences volumetric compaction in the elastic region (right portion of the graph), until it eventually crosses the centerline and starts expanding rapidly in the plastic region (left portion of the graph) due to deformations and crack propagation until reaching the ultimate failure of the member. Similar to unconfined concrete, the volume of passively confined concrete members under axial stress begins by reducing in the elastic region, until it eventually starts expanding rapidly in the plastic region. However, the passive confinement and resulting confining pressure helps in delaying the point where the concrete starts expanding volumetrically, and therefore allows for increased stresses to be reached and also delays the ultimate failure of the member. In the active confinement case, the confining pressure which is applied to prestress the concrete element laterally, prior to loading, exerts an initial volumetric strain $\varepsilon_v^o$ due to compaction of the confined member. In order to overcome the effect of this initial volumetric strain, larger axial strains and stresses are required, thus resulting in the failure point of the concrete to become further delayed compared to the passively confined concrete (Shin and Andrawes 2010).
1.6 Objectives and Scope

With a general understanding of the superior stress-strain behavior displayed by active confinement now obtained, a new and innovative method to confine concrete specimens will be discussed and further investigated within this thesis. This confining method utilizes shape memory alloy (SMA) wires to provide the constant confining pressure that is essential for active confinement.

Within this thesis all relative information with regards to SMAs and the confining potential will be further investigated in great detail, and the following overall primary objectives have been selected:

1. Critical review of literature and assessment of analytical models
   a. A critical review of literature is discussed for assessing the suitability of various models toward their applicability to shape memory alloy (SMA) induced confinement of concrete cylinders.
   b. Computational procedures for a chosen SMA confined concrete cylindrical specimen are adopted to make the assessment meaningful for addressing primarily two required goals: enhancement of peak stress, and post-peak stress-strain response, that is necessary to estimate the increase in the ductility of concrete columns.
2. Development of an experimental program

This program deals with the following aspects:

a. Mechanical characterization of the selected SMA wire that was used as the confining mechanism.

b. Selecting concrete mix designs, to achieve a variation in the compressive strength of concrete.

c. Development of appropriate procedures required to wrap the SMA wire around the concrete cylinders.

d. Carrying out the appropriate loading tests for the concrete cylinders, (both unwrapped and wrapped) to determine their stress-strain response until ultimate failure occurred.

3. Experimental/analytical correlations

a. To assess the usefulness of the SMA confinement model toward achieving the expected increase in the ductility of the specimens beyond peak stress.

b. To make some correlations with the appropriate models with regard to the descending portion of the stress-strain response. These correlations will be limited toward the overall trends that are displayed rather than the numerical values obtained.

c. To carry out these correlations for assessment of the applicability of the best models toward analyzing the enhancement of peak stress and ductility, if any.

d. To assess the accuracy of the post-peak stress-strain response.


The scope of this work is limited to exclude:

- Steel reinforced concrete cylinders
- Dynamic loading effects
- Development of new models that may correlate better with the experimental data

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This work is also limited due to the relatively smaller number of specimens that could ultimately be tested. This was primarily due to the lack of resources available to buy the expensive SMA wires, but also due to the load testing machine and data acquisition system not being ideal.

1.7 Project Summary

This work entails detailed information describing the main differences between active and passive confinement, especially discussing how the confining pressure is ultimately applied and the resulting stress-strain behavior of concrete members confined under each method. With a full understanding of each approach, the overall superiority of the active confinement method becomes widely apparent, and it’s easily understood why the active confinement method appears to be the future of concrete confinement.

One method that appears realistically feasible to actively confine concrete members involves the use of shape memory allow wires and an innovative technique that utilizes the recovery stresses that these alloys can generate. These SMAs will therefore be discussed throughout this work in full detail, with a main focus existing on the potential beneficial use of SMAs for structural confinement.

Mander et al. (1988) unified passive stress-strain model is then described in detail and modified for the active confinement application that shape memory alloys wires will provide. Due to relatively difficult nature of formulating the stress-strain behavior that occurs with active confinement applications, multiple active confinement equations are then incorporated within this model to display the wide range of assessed behavior that may occur under certain varied parameters.

Once multiple analytical formulations are displayed and discussed, a medium scale experimental procedure was undertaken. The main objectives of this experiment were to verify the confining effectiveness that this SMA confining method provided under certain scenarios, and ultimately determine which author’s formulations appeared to be the most accurate when compared to the experimentally obtained data.

After a conclusion has been made on which authors model most accurately formulates the stress-strain behavior of SMA confined concrete, the results were then
used within a numerical example to predict the behavior of a full scale concrete column with ordinary longitudinal steel and transverse SMA wires.
Chapter 2: Shape Memory Alloys

2.1 Shape Memory Alloy Introduction

Shape memory alloys (SMAs) are a very unique group of metallic alloys that are capable of recovering apparent permanent strains when they are heated above a certain temperature, due to a reversible phase change that occurs within the atomic structure of the alloy. With these amazing alloys, a temporary deformed shape can be virtually held forever until the right stimulus (mainly temperature) is applied to trigger a shape recovery and return the alloy back to the original undeformed shape. Due to this reversible process, SMAs have an endless amount of possible applications and appear to be one of the main materials of the future.

These extraordinary alloys were first discovered in the early 1930s, although they didn’t start to attract real attention for engineering applications until 1971 when significant recoverable strain was observed in a Ni-Ti alloy at the Naval Ordnance Laboratories (USA). Currently, three major types of SMA compositions exist: nickel titanium based (Ni-Ti), copper based (Cu-Al-Ni and Cu-Zn-Al), and iron based (Fe-Mn-Si, Fe-Ni-C and Fe-Ni-Co-Ti). The nickel titanium based alloys are by far the most common due to their superior mechanical behavior, and if price is not a major concern, they are always the best choice. The copper based alloys are the second most common type, and have also seen some real world applications. These alloys, while being cheaper than the Ni-Ti based alloys, also display inferior mechanical properties and therefore the importance of cost and function must be weighed when choosing between the two. The iron based alloys have only been used seldom for actual applications, as they are still mostly in the research and development stage. These alloys are expected to be heavily used in the future as they are predicted to be even cheaper than the copper based alloys.
and only a fraction of the cost of current Ni-Ti based alloys that are used today, greatly reducing the overall cost for any application.

SMAs have already been used for multiple applications within the biomedical field (eyeglasses, dental wires, and reinforcement for arteries and veins), and aerospace industries (fixed wing and rotary-wing components) in the past. Recently, they have also started to attract a lot of attention in other engineering fields due to the price decrease that has occurred over the past 10 to 15 years. This price decrease which is mainly due to enhanced manufacturing processes over this time frame is expected to continue within the future and should drastically decrease due to the eventual increase in both copper and iron based alloys, which has made previously large scale applications now realistically possible. Like almost all other science and engineering fields, structural engineers have also taken notice to the potential benefits and now reasonable cost that SMAs may provide, which has led to multiple applications now being further researched, if not already implemented. One such potential structural application that SMAs may provide is for the active confinement of concrete members, which will be discussed in great detail within this chapter and throughout the paper.

2.1.1 Shape Memory Behavior

Shape memory alloys (SMAs) are a class of metals known for their unique thermomechanical characteristic which gives the alloy the ability to return to their predetermined undeformed shape, after undergoing large inelastic deformations. These alloys achieve this by either displaying a superelastic behavior, which allows the alloy to recover deformations with the removal of the applied stress (like a rubber band), or a shape-memory behavior, which enables the alloy to recover deformations with the application of heat.

Contrary to plastically deforming metals, the nonlinear deformation of SMAs is metallurgically reversible due to a realignment of the alloy’s structure that occurs at the atomic level. This atomic realignment that occurs within in SMAs is a two-way transformation (forward and reverse) between two stable atomic phases: a high temperature austenitic phase and a low temperature martensitic phase. In addition, the low temperature martensitic phase can be in one of two forms: twinned (undeformed) or
d detwinned (deformed) if sufficient force is applied. All three of the possible SMA phases along with the general path SMAs follow is shown in Figure 2.1. These phases are directly related to the molecular alignment of the alloy with the martensitic structure being the low energy molecular formation (natural atomic state) and the austenitic structure being the high energy formation. These phases or molecular formations of the alloy will therefore change with the application of energy and can be either stress or temperature induced.

As can be seen from this figure, the original twinned martensitic phase of the alloy is basically the natural (low energy) atomic structure of the alloy, and this structure will be preferably displayed, unless acted upon by either force or temperature. If a suitable force is applied, the alloy will start to transform from the twinned to detwinned martensitic phases, which has up to 24 martensitic variants until the completely detwinned martensitic phase is reached. The austenitic phase however, only has one single atomic structure, which is fundamental for SMAs to display the shape memory effect (SME) and recover any preliminary deformations that have occurred. Basically, no matter what phase the martensitic alloy is in (either twinned or any possible variation of the detwinned alloy), if the alloy is heated to a high enough temperature the alloy will always revert back to this predetermined austenitic crystalline structure. This basically allows the alloy to reset back to the twinned martensitic phase when the alloy’s temperatures are cooled low enough.
In other words, for the SME to be shown in SMAs, a quasi-plastic deformation is induced in the martensitic form, transferring the alloy from twinned to a detwinned molecular structure. The deformation can then be recovered by raising the temperature and changing the molecular structure again, this time from the detwinned (deformed) martensitic phase to austenitic phase. If the heat source is removed and the temperature is cooled low enough the alloy will change once again, this time from the austenitic form back to the original (twinned) martensitic structure, while in the process recovering all previous deformations and allowing the process to be restarted all over again if needed.

A great way to comprehend the physical behavior of SMAs, based on the alloys phase and temperature, is to visually depict a simple spring made of a typical SMA material. This representation is shown in the Figure 2.2. From this figure all possible phase transformations are portrayed, by either increasing or decreasing the temperature, or by deforming the SMA with the addition of an adequate mass. As can be seen from this representation when starting with a spring in the martensitic twinned phase (left hand side of the figure), the addition and eventual removal of heat, will transform the alloy to the austenite phase and then back into the martensitic twinned phase, with no physical movement occurring within the spring. If however a sufficient mass is applied, the
martensitic twinned spring will become detwinned, causing the spring to stretch out and increase in length. From this martensitic detwinned phase, if sufficient heat is applied the alloy will transform to the austenitic phase. During this transformation the alloy will recover all deformations within the spring, assuming that the mass of the object and the recovery stress of the alloy permit this to occur. If the heat is removed and the alloy is allowed to cool back down, the alloy will then transform back to the martensitic detwinned phase due to the fact that the mass is still attached to the spring. However, if this mass is removed and the spring is reheated, the alloy will transform back into the austenitic phase recovering all deformations due to the previously attached mass. With no mass attached, and with the removal of heat, the alloy will again transform back to the twinned martensitic form of the alloy.

Figure 2.2. Physical Representation of a SMA Spring, Showing All Possible Phase Changes Due to Temperature and Deformation.

2.1.2 SMA Transformation Temperatures

As noted above, the amount of martensite or the martensitic fraction of a SMA will change with respect to temperature. Knowing this martensitic fraction from 0-100% (where zero percent is a completely austenitic alloy) is crucial, because this fraction will
result in different stress-strain behavior and mechanical properties for the alloy. As depicted in Figure 2.3A, there are four transformation temperatures which locations vary based on material properties (especially the alloys composition) and are known as the martensite finish ($M_f$), martensite start ($M_s$), austenite start ($A_s$), and austenite finish ($A_f$).

If the material is in the low temperature martensite phase (100% martensite) and heat is applied, the material will begin the transformation to austenite once the temperature corresponding to the $A_s$ location has been reached, and will become entirely austenitic after reaching the $A_f$ temperature of that alloy (0% martensite). Likewise, if the material is in the high temperature austenite phase (0% martensite) and the material is cooled, the material will begin the transformation to martensite at the temperature corresponding to the $M_s$ location and will become entirely martensitic once the $M_f$ temperature has been reached (100% martensite).

The phase of the material (martensitic or austenitic) is very important because it will determine the behavior of the alloy. For example, when the alloy is in its low temperature martensitic phase, the SMA will behave plastically and quasi-permanent deformations will exist after a sufficient stress has been applied and removed (Figure 2.3B). However, if the alloy is in the high temperature austenite phase, the SMA will behave elastically (just like a rubber band) and deformations will be recovered with the removal of stress (superelasticity). The stress strain behavior is shown graphically in Figure 2.3B for both the martensitic and austenitic phases of the alloy, along with an approximation for the transformation region between phases.
While the most common method used to transform the phase in SMAs is with the application of heat, as explained above, it’s important to not overlook the fact that this phase transformation can also occur with pure mechanical loading if the material is in the austenitic phase. It is well known now that the transformation temperatures associated with SMAs are not constant values, but actually increase approximately linear with applications of stress. Figure 2.4 displays the typical plot of stress versus temperature for SMAs. From this plot it can be seen how the typical transformation temperature locations vary based upon the level of applied stress, and the linear relationship that ultimately results.
As an example, from this figure it can be seen that if the SMA is in the twinned martensitic form, the increased application of stress will only transform the alloy to the detwinned structure until it eventually begins to deform due to permanent (non-recoverable) plastic deformation. This result is fairly straightforward and besides the switch from twinned to detwinned, this behavior is just like any other metallic material under tension. However, major problems may arise if the SMA is in the austenitic phase and stress is applied while the temperature is held constant. As it can be seen from Figure 2.4, if a SMA wire is under little or no stress while in the austenitic form, special care needs to be taken to make sure that the alloy doesn’t transform back into the detwinned martensitic phase. Due to the estimated slopes of the transformation temperatures and the typical yield point of SMAs, this stress induced transformation will most likely only happen near the point of plastic deformation. However, it is important to note that if a SMA is in the austenitic phase and stressed in tension until ultimate rupture, the temperature will most likely need to be increased once yielding has occurred or the SMA will switch its phase back to the detwinned martensitic phase of the alloy.

This stress versus temperature relationship is key for actual SMA use, especially if the consumer has purchased a non-deformed SMA and is planning to prestrain the SMA material themselves, as the SMA can be guaranteed to have higher transformation temperatures then reported by the manufacturer.
2.1.3 SMA Mechanical Properties

The mechanical properties under tension, compression, shear, and torsion can vary widely with SMAs. This possible large variation in SMA properties is not only due to variations in chemical composition, which are practically endless, but also due to the thermomechanical processing and original heat treatment of the alloy from the manufacture, and of course the atomic arrangement of the alloy at the time of testing (either twinned or detwinned martensite, or austenite).

With these inherently large variations within SMAs, often only ranges or at best approximate values for the mechanical properties can be assumed. Experimental testing to obtain the mechanical properties can also be difficult, due to the sensitive nature of SMAs. Even samples from the same batch may slightly vary in properties such as; elastic modulus, yield stress, or recovery stress. To get some approximate values for the mechanical properties of SMAs, various SMAs in the form of wires and bars of varying diameters have been tested by a number of investigators over the years. A sample of some of these results and some of their results are summarized in Table 2.1. It should be noted that the table below is specifically for the Ni-Ti based alloys under tension only.

As explained earlier, the Ni-Ti based alloys have been known to display far superior mechanical behavior and therefore should be taken as the top end potential for SMAs.

<table>
<thead>
<tr>
<th>Property</th>
<th>Phase</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus, $E_y$ (GPa)</td>
<td>Austenite</td>
<td>30-98</td>
</tr>
<tr>
<td></td>
<td>Martensite</td>
<td>21-52</td>
</tr>
<tr>
<td>Yield Strength, $f_y$ (MPa)</td>
<td>Austenite</td>
<td>100-800</td>
</tr>
<tr>
<td></td>
<td>Martensite</td>
<td>50-300</td>
</tr>
<tr>
<td>Ultimate Strength, $f_u$ (MPa)</td>
<td>Austenite</td>
<td>800-1500</td>
</tr>
<tr>
<td></td>
<td>Martensite</td>
<td>700-2000</td>
</tr>
<tr>
<td>Elongation at failure, $\varepsilon_u$ (%)</td>
<td>Austenite</td>
<td>15-20</td>
</tr>
<tr>
<td></td>
<td>Martensite</td>
<td>20-60</td>
</tr>
</tbody>
</table>

Table 2.1. Mechanical Properties of Ni-Ti Alloys under Tension (Manach and Favier 1997; Otsuka and Wayman 1999; Rejzner et al. 2002; & DesRoches et al. 2004)
As seen from Table 2.1, the values for each mechanical property can vary widely for both phases. However, general conclusions can still be made off of the trends shown. From this data it appears that the austenitic phase shows superior mechanical behavior when compared to the martensitic phase. This however was expected based upon the knowledge for the stress-strain relationship of SMAs shown earlier within this chapter. For example, with the knowledge that the martensitic phase by nature will deform relatively easy and the austenitic phase will try to recover deformation or fight strain when stress is applied (like a contracting rubber band), it makes sense that the austenitic phase would have a higher range for Young’s modulus (stiffer) and have a lower elongation at failure range. The yield strength is also much higher for the austenitic phase since it yields like normal metals as opposed to the martensitic phase, which in a sense, is yielding from the initial switch from twinned to detwinned starts to occur. The values for the ultimate strength at each phase were slightly surprising as it appears the martensitic phase may have a slightly higher ultimate strength capacity; although the overall range of the two phases were still relatively similar overall.

2.1.4 Free or Constrained Recovery

Two different types of deformation recovery can occur with SMAs, which are classified as either free or constrained. With free recovery there is no physical restraint on the movement of the alloy as it transforms from the deformed martensite phase into the austenite phase, therefore the element can freely return to its original undeformed shape at zero stress. However, in the case of constrained recovery the change in the crystalline structure from martensitic to austenitic state still takes place by altering the temperature, although the physical movement of the alloy is prevented. Since the shape recovery is prevented (strained martensite to unstrained austenite), stress which is a function of final strain (that is, unresolved recovery or contact strain) is generated.

The SME phenomenon under constrained recovery is capable of large recovery stresses up to 900 MPa being realistically possible (Janke et al. 2005). This recovery stress does however depend on multiple factors; such as the material composition, manufacturing procedure, and the amount of prestrain applied. The max recovery stress (largest amount of stress the SMA is capable of recovering) occurs at temperatures above
the $A_f$ location and has been known to increase linearly with the amount of prestrain. This linear relationship generally holds true up until a strain of approximately 8%, which is usually agreed to be the max strain allowed for SMAs or the potential for nonrecoverable plastic deformation exists which will reduce the recoverable stress of the wire significantly (Tautzenberger 1990). Once the heating source is removed and the temperature is lowered back to room temperature (alloy still austenitic), the recovery stress of the SMA will drop slightly from the max recovery stress associated with the wire, to what is called the residual stress. This residual stress is the remaining recovery stress that occurs once the wire has been transformed into the austenitic phase, but cooled down to ambient temperatures. This residual stress is the actual stress that defines the potential of the SMA wire in real world applications. Just like the max recovery stress, the residual stress is dependent on multiple factors and will vary for each alloy but has been approximated as 80% of the maximum recovery stress (Shin and Andrawes 2009).

2.1.5 Prestrain Losses

With post-tensioned or prestressed materials, like the SMAs to be used within active confinement, the amount of prestrain loss is a major issue that needs to be accounted for accurately. The residual prestrain, after all losses have occurred is extremely important because it’s the value that will determine the effective amount of confining pressure that is actually applied to the specimens. When dealing with SMA confinement, prestrain losses can come from multiple sources. These include: geometric imperfections of the spiral (bends or kinks), wire slippage that could occur at the splicing connections or anchor points after the shape recovery of the wire has been activated, and most likely from the simplest form where slack develops between the wire and the confined specimen greatly reducing the confinement potential of the SMAs.

While it should be stressed that the amount of prestrain loss will vary with each application method, Shin and Andrawes (2009 and 2011) attempted to quantify this value for their active confinement procedure by accurately measuring the average value obtained from two extensometers that were attached to the SMA spiral on opposite sides of the concrete specimens. In their particular setups, the prestrain losses increased consistently after activation of the wire, until reaching a constant max value of 0.67%
(2009) and 0.97% (2011) prestrain loss. Obtaining these approximate losses is significant because of the knowledge that the recovery stress increases linear with increasing prestrain. Therefore, the predicted recovery stress to be applied to the concrete specimen can then be adjusted due to the estimated prestrain losses, based on the slope assuming at least two recovery stress values are known for that specific SMA wire at different prestrained value.

2.2 SMA Active Confinement Application Procedure for Concrete Specimens

For the application of active confinement using SMA wires on concrete specimens, the SMA wire is first prestrained usually within 5-8%. Any higher and the potential for nonrecoverable plastic deformation can reduce the recoverable stress of the wire significantly. Next the prestrained wire is wrapped around the concrete member often in the form of a continuous spiral, although individual hoops are also possible (Figure 2.5B). The SMA spiral is then heated above the alloys $A_f$ temperature to activate its shape recovery (Figure 2.5C). However, since the spirals are fully constrained at the ends often by clamps and throughout the length of the wire by the surface of the concrete specimen, a large recovery stress (hoop stress) is developed in the spiral creating a large confining pressure on the wrapped specimen (Figure 2.5A).
2.3 SMA Classifications

It should be noted that not every SMA type is suitable for this active confinement application. SMAs are classified as type 1, 2, or 3 depending on the relative position of the thermal hysteresis loop (distance from $M_s$ to $A_s$) and the location of ambient temperature (Figure 2.6). Type 1 SMAs and type 2 SMAs have the thermal hysteresis loop located completely below and above ambient temperature respectively, while type 3 SMAs are classified as having ambient temperature within their thermal hysteresis loop.

Type 3 SMAs (Figure 2.6C) are almost always required for civil application because the material can be deformed (prestrained) often by the manufacturer, stored virtually forever on the shelf, and applied when needed at ambient temperature in the martensitic phase without experiencing shape recovery. The material can then be triggered/activated by applying a heat source to the alloy (either by temperature or electricity), transforming the alloy into the austenitic phase. The heat source can then be removed after activation, allowing the SMA to cool and return to ambient temperature, while still remaining in the stressed austenitic phase resulting in usable residual stress for confinement.
Type 1 SMAs (Figure 2.6A) cannot be used for this application, due to the fact that the four transformation temperatures are too low relative to ambient temperatures, making it nearly impossible to apply the SMA wire to the concrete member in the martensitic prestrained state. With this type of SMA the material is in the austenitic phase at room temperature, causing them to display the superelastic effect, not the desired shape memory effect that is needed for the described application.

Type 2 SMAs (Figure 2.6B) are also not practical for real world applications of active confinement because the four transformation temperatures are too high relative to ambient temperatures. With this type of SMA, the wire can be safely deformed and applied to the concrete specimen in the martensitic phase without problems, but would require a unrealistic constant heat source to be applied throughout the life span of the alloy, to transform the wire into the austenitic phase and trigger the desired shape memory effect. With this type of SMA, once the heat source was removed and the wire was allowed to cool back down to ambient temperature the alloy would transform back into the martensitic phase, resulting in the loss of all recovery stresses, even with the alloy being physically restrained.
Figure 2.6 Location of the Hysteresis Loop Relative to Ambient Temperature for (A) Type 1 SMA (B) Type 2 SMA (C) Type 3 SMA

2.3.1 SMA Hysteresis Loop

While the type of SMA is very important, the size of the thermal hysteresis loop is also another obstacle that must be considered for correct active confinement application, as this determines the environment in which a SMA may be used in real engineering application.
It is well known that SMAs are very sensitive and even a 1% difference in the alloys composition can greatly alter the locations of the transformation temperatures and relative size of the hysteresis loop. For instance, the hysteresis gap can narrow or widen considerably, with just a small amount of copper (Cu) or niobium (Nb) respectively in the same nickel-titanium (Ni-Ti) based SMA. At first, the sensitive nature of SMAs seems to be a major disadvantage. However, this should be looked at positively as it gives the material tremendous flexibility allowing the user the ability to customize the materials to their exact specifications.

Most civil applications have utilized the NiTiNb alloy because it is classified as a type 3 SMA and has a very wide hysteresis range. This alloy’s favorable characteristics makes it suitable for the wide temperature variations possible in the industry (-5 to 110°F), without the threat of unwanted phase transformations due to seasonal temperature changes.

However, with that being said, the experimental investigation that will be further discussed within the upcoming chapters of this work incorporated a type 2 Ni-Ti SMA with an equal composition of both elements. This SMA was ultimately selected due to the fact that this composition is by far the most common of all SMAs and therefore was the one of the cheapest possible selections that could be adequately used within this work. Any and all other required mechanical properties or details about the SMA selected, will be further discussed in Chapter 5.

Chapter 3 of this experimental investigation discusses the pertinent literature reviews related to this topic. This chapter and literature review are separated in two main sections, with the first section detailing the most useful theoretical constitutive models of confined concrete cylinders, and the second section detailing previous research obtained on SMAs used for active confinement.
Chapter 3: Literature Review

3.1 Relevant Research Dealing with Stress-Strain Modeling of Confined Concrete

Mander et al. (1988) proposed a theoretical stress-strain model for concrete subjected to uniaxial compressive loading confined by transverse reinforcement. With this theoretical model a single equation is used for the stress-strain response. The model allows for cyclic loading and includes the effect of strain rate. The influence of various types of confinement is taken into account by defining an effective lateral confining stress, which is dependent on the configuration of the transverse and longitudinal reinforcement.

The model utilizes an energy balance approach to predict the longitudinal compressive strain in the concrete corresponding to first fracture of the transverse reinforcement by equating the strain energy capacity of the transverse reinforcement to the strain energy stored in the concrete as a result of the confinement.

The analytical stress-strain model for confined concrete suggested three important conclusions. First, for a particular transverse reinforcement configuration the effective confining stresses in the x and y directions can be calculated from the transverse reinforcement and a confinement effectiveness coefficient \( k_e \) which defines the effectively confined concrete core area by taking into account the arching action that occurs between the transverse hoops and between the longitudinal bars. Second, the form of the stress-strain curve for confined concrete can be expressed in terms of a simple uniaxial relation suggested by Popovics (1973) and only requires three control parameters (peak stress of confined concrete \( f'_{cc} \), strain at the peak concrete stress \( \varepsilon_{cc} \), and modulus of elasticity of concrete \( E_c \)). Third, with the energy balance approach, when the work done exceeds the available strain energy of the transverse steel, then hoop
fracture occurs and the section can be considered to have reached its ultimate
deformation.

Xiao et al. (2010) presented on their finding through modeling of the stress-strain
behavior of actively confined high-strength concrete (HSC) from a database with 51
actively confined concrete cylinder specimens from Attard and Setunge (1996), Imram
and Pantazopolou (1996), Lu and Hsu (2006), Candappa et al. (2001), and Tan and Sun
(2004). The unconfined concrete strengths of the database ranges from 51.8 to 126 MPa,
while the confinement ratio, which is defined as the ratio of the confining pressure to the
unconfined concrete strength, ranges from 0.01 to 0.84. The new active-confinement
model developed, worked well for both HSC and normal strength concrete (NSC) and
was shown to outperform past models that generally would underestimate the
stress-strain curves because of an underestimated peak axial stress.

Moghaddam et al. (2010) proposed an analytical model to predict the compressive
stress-strain curve of strapped concrete with post-tensioned yielding metal strips as a
function of the confinement level. The critical, yield, and ultimate points were
determined and the full stress-strain curve was then obtained by passing through these
key locations with a good degree of accuracy. The model was calibrated based on
experimental data of 72 cylindrical and prismatic specimens by the authors and 41
specimens used by Frangou and Pilakoutas (1996). The specimens covered several
parameters such as volumetric ratio, yield strength, and ultimate strain of the confining
material, and the strength, shape, and size of the concrete specimens.

Lokuge et al. (2005) presented an approach in predicting the stress-strain
relationship of high strength concrete (HSC) subjected to active lateral confinement. The
proposed model formulation was based on the experimental results reported by Candappa
(2000), which included 24 cylindrical specimens using four grades of concrete (40, 60,
75, and 100 MPa) and three confining pressures (4, 8, and 12 MPa) as test variables.
The analytical results from the proposed model indicated that when confined concrete is
stressed, initially it contracts (reducing in volume), after a certain stress, it starts to
expand (increasing in volume). At the time of peak axial stress, the volume of concrete is
back to its original unloaded volume. That is, the volumetric strain was observed to be
back at zero at peak axial stress. This suggests that the magnitude of lateral strain at peak axial stress is half of the corresponding axial strain magnitude.

3.2 Relevant Research Dealing with Shape Memory Alloy Confinement

Krstulovic-Opara and Thiedeman (2000) investigated the use of self-stressing composites for active confinement of concrete members with the goal of maximizing member strength and ductility for both normal strength and high-strength lightweight concrete (HS-LWC), with the main focus being on HS-LWC. Within this work the authors embedded SMAs as muscular fibers in cementitous composites to develop self-inducing prestressing composites. This work used a type 3 Ni-Ti SMA wire and focused on two different types of confinement: confinement with SMAs only; and confinement with a combination of SMAs and slurry infiltrated fiber mat concrete (SIMCON) with each being tested for retrofit and new construction applications.

The obtained data indicated that an increase in confinement, increased crack stability, material compaction, and corresponding stress throughout the test and at ultimate strength. Brittle behavior of NSC was prevented, while an already ductile response of HS-LWA fiber reinforced concrete (FRC) was further improved. Active confinement was more effective than passive confinement in restraining cracking and improving crack stability. Thus, active confinement resulted in a higher increase in the parameters previously listed. Furthermore, for the same level and type of confining stress, the increase in ultimate strength was typically higher for HS-LWA FRC than for NSC specimens. The investigation also indicated that a combination of adding fibers and using active confinement leads to a large increase in strength and ductility of HS-LWA concrete, and could thus permit their use in seismic-resistant design.

Choi et al. (2008) examined the ability of two 1.0 mm diameter SMA wires shape memory alloy wires to confine a total of 6 concrete cylinders. The first SMA (Ti-49.7Ni) remained in the martensitic phase at room temperature, and the second SMA wire (Ti-50.3Ni) remained in the austenitic form at room temperature. The martensitic wire utilized the shape memory effect and was prestrained before being wrapped around a concrete cylinder as opposed to the austenitic shape memory wire which utilized a
superelastic behavior, and was prestrained as they were wrapped around the concrete cylinder.

Based upon the results from this experimental work, the authors concluded that the two types of SMA wire jackets performed similar and both types of SMAs were capable of inducing active confinement and therefore can be used to increase the failure strain and the energy dissipation capacity of the concrete cylinder. The results obtained in this work were then compared with a constitutive model of concrete confined by steel jackets and lateral reinforcement by Li et al. (2005a, 2005b) to conduct a parametric study to understand how the amount of SMA wires can significantly improve the strength and the difference in using steel jackets compared to SMA jackets.

Shin and Andrawes (2009) completed a study that focused on investigating the feasibility of using spirals made of NiTiNb SMA to apply active confining pressure on concrete members. Three types of experimental tests were conducted. First, thermo-mechanical tests were performed on 2.0 mm diameter SMA wires to determine their recovery stresses. Second, several mechanical splicing connections were examined in tension to determine their adequacy in splicing the SMA wires to develop the total length of continuous wire needed. Third, uniaxial compression tests of SMA, SMA-glass fiber reinforced polymers (GFRP), and GFRP wrapped concrete cylinders were conducted and the results were compared.

Thermo-mechanical tests revealed that the recovery stress of the SMA wires varied linearly with the amount of prestrain, and the average residual recovery stress is approximately 80% of the maximum recovery stress. Splicing connections tests were conducted on three types of connections including sleeves, U-clamps, and welded connections. These tests revealed that using multiple U-clamps provided the best results as long as the average capacity of the number of U-clamps used was greater than the expected maximum recovery stress of the spiral.

Using the lateral confining pressure as the base for comparison, it was found that applying the same confining pressure from either passive or active confinement methods, produced significantly different concrete behavior (see Sections 1.5 through 1.5.3.2). For example, applying confining pressure of 1.0 MPa using SMA spirals and GFRP wraps resulted in an increase in the concrete strength and ultimate strain of approximately 1.2
and 10 times, respectively, when the SMA was compared to the GFRP application. Furthermore, the hybrid SMA-GFRP confinement technique exhibited a superior performance to the conventional passive confinement technique using GFRP. This increased performance is due to the small amount of SMAs used in the hybrid specimens that delayed the rupture of the GFRP sheets as well as the entire specimen, in addition to maintaining up to 60% of the concrete peak strength until failure.

This study suggested that SMA spirals could be used solely or as a supplementary confining technique along with FRP composite wraps. The addition of the SMAs could result in a significant reduction in the amount of FRP material needed to achieve a certain level of ductility, which could result in a dramatic reduction in the cost of future structural retrofitting and rehabilitation projects. The authors felt it was worth noting that due to the small number of specimens tested in this study, that the conclusions should not be generalized and that more research is needed.

Choi et al. (2010) assessed the confining effectiveness of shape memory alloy (SMA) wire jackets for concrete, and compared the performance to that of steel jackets. For this study 1.0 mm diameter shape memory alloy wires of Ni-Ti-Nb (Ni47.4-Ti37.86-Nb14.69 %wt.) and Ni-Ti (Ni53.85-Ti46.15 %wt.) at varying prestresses (up to 7%) were used to confine a total of four concrete specimens (two with each type of SMA). This study used data from both types of SMA wires, and suggested through regression analysis that the confining effectiveness \( k_1 \) of a SMA jacket for concrete equaled 4.33. This estimated value of confining effectiveness for SMAs was higher than the suggested 4.1 value for steel by Richart et al. (1928), and the suggested 2.0 value for FRP materials by Lam and Teng (2002). The authors noted that SMA properties vary with components and the manufacturing process and, thus, more data from various SMA wires should be used for a more general determination of the confining effectiveness of SMA wire jackets.

Andrawes et al. (2010) explored the feasibility of using Ni-Ti SMA spirals for seismic retrofitting of reinforced concrete bridge columns both experimentally and analytically. The experimental results of SMA spirals and CFRP wraps of varying but equal confinement levels were used to develop a model, displaying the method’s ability at improving the behavior of reinforced concrete columns under cyclic and seismic loadings.
The analytical results demonstrated a considerable advantage for the actively confined column using SMAs compared to the passively confined column using CFRP. Under seismic excitations, damage to the concrete core and longitudinal steel were decreased; while the strength, effective stiffness (indication of the amount of damage), and column residual drifts (governs functionality) were all increased with the use of SMA spirals compared to the CFRP wraps. The superiority of the proposed technique is primarily attributed to the early increase in concrete strength associated with using active confinement, which delays the damage experienced by both concrete and steel.

In the paper by Andrawes et al. (2010), the authors used Mander et al. model to compare formulated stress-strain relationships with some initial experimental results. However, in this paper the authors used the original equations suggested by Mander et al. that were designed for passive confinement, and did not investigate any other potential equations to accurately predict the stress-strain relationship of the SMA confined specimens. The use of Mander’s original equations was however deemed appropriate for these comparisons due to the large amount of SMA material (ρ_s) that was used, as the authors mainly researched the ultimate potential of SMA confinement, and not what would be representative in real world situations. As shown in Figure 1.6, this increased confining ratio (ρ_s) ultimately allowed the tested specimens to reduce the severity of the sharply decreasing post-peak relationship that typically occurs in actively confined concrete specimens.

Shin and Andrawes (2011) examined experimentally the effectiveness in enhancing the flexural ductility of vulnerable reinforced concrete columns by testing four reduced-scale (1/3-scale) reinforced concrete single-cantilever columns, representative of bridge columns. The quasi-static cyclic behavior of the as built column was compared with that of a column retrofitted by using actively confined NiTiNb 2.0 mm diameter SMA spirals, passively confined GFRP wraps, and a hybrid (passive plus active) confined approach using SMA spirals and GFRP jacket simultaneously; all designed to have the same confinement pressure. The hybrid approach was sought in this study as a more economical approach for applying active confinement because the amount of SMAs will be reduced significantly compared to the case with only SMA spirals.
To assess the overall performance of each retrofitting technique, the strength, displacement ductility (ratio of the drifts at the ultimate and yielding points), and hysteretic energy (area enclosed within the force-displacement curves until the ultimate point was reached) were compared for each method. The results showed that the SMA and SMA/GFRP columns exhibited a slight increase in strength and a significant increase in flexural ductility and ultimate drift capacity compared with the as-built column, whereas the GFRP column showed only a moderate enhancement in ductility and drift capacity.

Applying active confinement using SMA spirals significantly enhanced the ability of the columns to dissipate energy compared with the passive confinement applied by only using GFRP wraps. It was also noted by the authors that when assessing the damage of the four tested columns, during and after testing, revealed that the damage to the SMA and SMA/GFRP columns was far less than that sustained by the GFRP column.

3.2.1 SMA Confinement Essential Findings

- Active confinement was more effective than passive confinement in restraining cracking and improving crack stability. Thus, active confinement resulted in a higher increase in crack stability and corresponding stress throughout the test and at ultimate strength.
- Applying the same confining pressure from either passive or active methods, produced significantly different concrete behavior with regard to the location of the peak stress and the slope/curvature of the post-peak relationship.
- Studies suggested that SMA spirals could be used solely or as a supplementary confining technique.
- Analytical results demonstrated a considerable advantage for the actively confined columns using SMAs compared to the passively confined columns using CFRP. The superiority of the proposed technique is primarily attributed to the early increase in concrete strength associated with using active confinement, which delays the damage experienced by both concrete and the confining material.
Applying active confinement using SMA spirals significantly enhanced the ability of the columns to dissipate energy compared with the passive confinement applied by only using GFRP wraps.

It was also noted that when assessing the damage of the confined columns, during and after testing, revealed that the damage to the SMA confined columns was far less than that sustained by the passively confined GFRP column.

3.2.2 Critiques of Past Findings

The work done by Andrawes (2010 and 2011) and Choi (2008, 2009, and 2010) used very high amounts of SMA transverse confining material, in an attempt to prove the overall feasibility of the SMA confinement method. However, these authors did not investigate SMA confinement under lower and much cheaper confining ratios that would be more representative to real world confinement situations.

None of the previously mentioned research truly investigated which formulations actually provide the most accurate stress-strain representation of SMA confined concrete. Under the previously mentioned work, high levels confining ratios allowed the use of Mander’s passive equations to be incorporated into the formulation of the stress-strain representation, but when lower confining ratios are used the behavior is expected to change greatly and these equations are expected not to be adequate. Therefore, it is still critical to determine which of the many active confinement models can most accurately predict the stress-strain behavior of SMA confined concrete members.

The previously mentioned work was very exploratory in nature, and therefore the authors were only able to test a limited number of confined specimens (usually around 3 or 4). Due to the lower number of specimens tested, the data obtained appears to be incomplete and at least somewhat inconclusive overall. It also appears obvious the both the reliability and repeatability of this confinement method still needs to be further investigated on a much larger scale.
• While Andrawes (2010) did attempt to formulate the stress-strain behavior of SMA confined specimens with the use of Mander’s unified stress-strain model, Mander’s passive confinement equations were used within this portion as the results were only used for preliminary investigations. This formulation was only lightly discussed in general, and due to the low number of specimens tested the data obtained still needs to be verified and further investigated under different confining scenarios.

• The type of material used within the previous SMA confinement research has been very limited in general, and therefore more data with different SMA wires are still required in an attempt to accurately formulate the confining effectiveness of a wide range of SMA wires.
Chapter 4: Analytical Models and Their Assessment toward SMA Confined Concrete Columns

4.1 Mander’s Model Introduction

In 1988 Mander et al. proposed a unified stress-strain model (continuous function for both ascending and descending branches) based on equations from Popovics (1973). Mander’s model was designed for passively confined concrete members and is applicable to both circular and rectangular shaped transverse reinforcement. This model has been widely accepted throughout the structural engineering community for both its accuracy and relative simplicity in evaluating the stress-strain relationship of confined concrete members.

Due to the fact that Mander’s model will be used as a base for the stress-strain prediction of actively confined concrete members, the entire model will be described in detail within Appendix A, and the main equations will be briefly discussed within this chapter. It should be noted that only circular reinforcement will be described within this work and the reader should refer to Mander’s et al. (1988) work for further clarification on rectangular shaped transverse reinforcement.

4.2 Mander et al. (1988) Stress-Strain Model

Within Mander’s model, the main equation which determines the relationship between axial compressive stress \( f_c \) and axial strain \( \varepsilon_c \) is given by the following axial stress-strain equation which was originally proposed by Popovics (1973):

\[
\frac{f_c}{f_{cc}} = \frac{\varepsilon_c}{\varepsilon_{cc}} \left[ 1 + \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right)^{r} \right]^{-\frac{1}{r}}
\]

(4.2.1)
where \( f'_{cc} \) = compressive strength (peak stress) of confined concrete and will be defined later, \( \varepsilon_c \) = longitudinal compressive concrete strain, and \( \varepsilon_{cc} \) = the strain at maximum concrete stress (\( f'_{cc} \)) and can be found with the following equation from experimental work by Gerstle et al. (1979) which was based on a simple relationship proposed by Richart et al. (1928):

\[
\varepsilon_{cc} = \varepsilon_{co} \left[ 1 + 5 \left( \frac{f'_{cc}}{f'_{co}} - 1 \right) \right]
\]

(4.2.2)

where \( f'_{co} \) and \( \varepsilon_{co} \), equals the unconfined concrete strength and the corresponding strain, respectively. Generally, \( \varepsilon_{co} = 0.002 \) can be assumed based on the results from Richart et al. (1928), and is used within Mander’s et al. model for unconfined concrete.

The effective lateral confining pressure is defined with the following equation:

\[
f'_{l} = \frac{1}{2} \cdot k_e \cdot \rho_s \cdot f_{yh}
\]

(4.2.3)

where \( f_{yh} \) = yield strength of the transverse reinforcement, and \( \rho_s \) = the ratio of the volume of transverse confining steel to the volume of the confined concrete core

The confined compressive strength (\( f'_{cc} \)) can now be calculated using the effective lateral pressure (\( f'_{l} \)) and the unconfined concrete compressive strength (\( f'_{co} \)). To develop this equation Mander et al. used the “five-parameter” multiaxial failure surface equation given by William and Warnke failure criterion (1975):

\[
f'_{cc} = f'_{co} \left( -1.254 + 2.254 \sqrt{1 + \frac{7.94 f'_{l}}{f'_{co}} - 2 \times \frac{f'_{l}}{f'_{co}}} \right)
\]

(4.2.4)
With the value of the confined compressive strength ($f'_{cc}$) now known, the entire stress strain curve can be calculated using Equation 4.2.1 with varying longitudinal compressive concrete strain values up to the ultimate concrete compressive strain ($\varepsilon_{cu}$), which is calculated based on the amount and type of transverse confining material used with a strain energy balance approach.

By equating the ultimate strain energy capacity of the confining reinforcement per unit volume of concrete core ($U_{sh}$) to the difference in area between the confined ($U_{cc}$) and the unconfined ($U_{co}$) concrete stress-strain curves, plus additional energy required to maintain yield in the longitudinal steel in compression ($U_{sc}$), the longitudinal concrete compressive strain corresponding to hoop fracture can be calculated, with the following equation:

$$U_{sh} = U_{cc} + U_{sc} - U_{co}$$

(4.2.5)

4.2.1 Required Modifications of Mander’s Model for SMA Confinement

With Mander et al. (1988) model, the critical strain for the confined concrete ($\varepsilon_{cc}$) was already taken from experimental work on actively confined concrete specimens by Gerstle et al. (1979), and therefore already corresponds to a constant confinement pressure that exists within active confinement applications. As a result, the value of $\varepsilon_{cc}$ shown within Equation 4.2.2 does not require modification. However, it should be noted that multiple investigators have attempted to find ($\varepsilon_{cc}$) for active confinement with more precise equations and these results will be further discussed later within this chapter.

The confined compressive concrete strength ($f'_{cc}$), is determined through the use of the unconfined concrete compressive strength ($f'_{co}$) and the effective lateral confining pressure ($f'_{l}$) at the yielding of the transverse reinforcement which signifies a passively confined approach. The expression given by Mander et al. for the value of the effective lateral confining stress ($f'_{l}$) is given in Equation 4.2.3 and will require modification for active confinement applications with the use of prestressed SMA wires. It is also important to note that the left hand side of the simplified Equation 4.2.5 used for predicting the ultimate concrete strain ($\varepsilon_{cu}$) will also have to be adjusted, since that
portion of the equation was originally designed for steel transverse confinement as opposed to the SMA material that will be used.

4.2.2 Andrawes et al. (2010) SMA Adapted Model

The analytical stress-strain model developed by Mander et al. (1988) for concrete with lateral steel reinforcement was extended and used to describe the effect of active confinement provided by exterior SMA reinforcement by Andrawes et al. (2010). Within Mander’s model the effective lateral confining stress on concrete from transverse reinforcement is only used within Equation 4.2.4, while solving for the peak stress of the confined concrete (f"cc). At this location a constant confinement pressure was assumed to be equal to the yield strength of the transverse reinforcement because the corresponding strain level was assumed to be high enough to cause the steel lateral reinforcement to yield.

With the addition by Andrawes et al. (2010), the constant confinement pressure was taken as the lateral pressure exerted on the cylinder due to the recovery stress of the SMA wire. The lateral confining pressure (f_l) could then be calculated by considering the equilibrium of the resulting SMA forces, shape of the column, and the spacing of the transverse reinforcements with the following equation:

\[ f_l = \frac{2A_{sma} \times f_{sma}}{d \times s} \]  

(4.2.2.1)

where \( A_{sma} \) = cross-sectional area of the SMA wire, \( f_{sma} \) = recovery stress in the SMA wire, \( d \) = diameter of the specimen confined, and \( s \) = spiral pitch

The spacing between the SMA wires still results in a reduced confining pressure, which according to the model by Mander et al. requires the calculation of a modified (effective) lateral confining pressure (f’’i) which then equals Equation 4.2.2.2:
Once \( f'_{l} \) has been calculated for active confinement with prestressed SMA wires, the rest of Mander et al. (1988) model can remain unchanged until the value of the ultimate concrete compressive strain \( (\varepsilon_{cu}) \) has been reached.

The location of \( \varepsilon_{cu} \) on the stress-strain curve for actively confined concrete using SMA wire will be greatly influenced by the ultimate strength and the fracture strain of SMA material. As previously mentioned in Chapter 2, these critical values will vary greatly depending on the overall composition and the state of the SMA (Martensitic or Austenitic), and therefore will have to be evaluated on a case by case basis.

4.3 Analytical SMA Confined Stress-Strain Relationships

To get a better idea of the potential stress-strain relationship and overall benefit of SMA confined concrete members, Mander’s et al. (1988) proposed unified stress-strain model, that was modified for active confinement as described in detail within the previous sections, was implemented within the computer program Matlab. With the use of Matlab, multiple stress-strain relationships were evaluated for 4x8 inch actively confined concrete cylinder specimens. The stress-strain relationships were formulated based on a number of criteria, such as: the spacing of the transverse SMA wires, the recovery stress of the SMA wire, and the compressive strength on the unconfined concrete. It should be noted that changing the spacing of the wire changed the values for \( \rho_s \) (the ratio of the volume of transverse confining material to the volume of confined concrete core) and would essentially be the same as using different diameter SMA wire.

4.3.1 Stress-Strain Relationship Using Mander et al. Modified Model for SMA Confinement

The stress-strain relationships have been generated using Matlab, and the following geometrical and material parameters are used.
Unconfined Concrete Information

- Unconfined concrete strength ($f_{co}^') = 4000$ psi (unless given within plot)
- Strain at max stress $f_{co}^'$ of unconfined concrete ($\varepsilon_{co}^') = 0.002$
- Max strain of unconfined concrete ($\varepsilon_{cu}^'$ “unconfined”) = 0.04

Sample Column Dimensions

- Circular Column
  - Height = 8 inches
  - Diameter = 4 inches

Shape Memory Alloy Specifications

- Wire diameter = 0.02 inches (0.5 mm)
- Recovery stress of SMA = 116,000 psi (800 MPa)
- Ultimate tensile Strength = 150,000 psi (1034 MPa)
- Fracture Strain = 0.15

For these formulated plots the mechanical properties of the SMA material used would be considered conservative (refer back to Section 2.1.3), as the idea of this section was to show a realistic representation of SMAs confining behavior, not their ultimate potential. Therefore it should be noted that the values for the ultimate tensile strength, fracture strain, and especially the SMA diameter can be increased, which would give even better confinement benefits.

The calculated stress-strain relationship for various SMA transverse spacing configurations using Mander et al. (1988) modified model for active confinement is shown in Figure 4.1. From this plot one can clearly notice the expected improvements in both the compressive strength and overall ductility of the confined concrete members as the SMA spacing is decreased (increasing $\rho_s$). The peak compressive strength of the members increased from the unconfined concrete strength of 4000 psi to over 4500 psi, and the ductility of the members increased from approximately 0.004 to 0.0125 when the spacing was at the smallest value with a 1/5th inch pitch. While the increase in peak compressive strength from the SMA confinement is valuable, the main benefit from confinement is often the increase in ductility, which is very significant.
Figure 4.1. Stress-Strain Relationship Formulated for Various SMA Spacing Configurations Using Mander’s Original Model Modified for Active Confinement (4000 psi Confined Concrete with SMA Recovery Stress of 116,000 psi)

In Figure 4.2 the formulated stress-strain relationship for various SMA recovery stresses using Mander’s original model, modified for active confinement, are shown. From these curves, one can again see the expected improvements in the compressive strength of the concrete over the unconfined concrete strength from 4000 psi to nearly 4500 psi, as the SMA recovery stress was increased. However, from these plots an interesting and slightly counterintuitive behavior can be noticed with regard to the ultimate strain values. As one can see from the plots the overall ductility of the confined concrete members actually decreased as the recovery stress of the SMA was increased. For example the ultimate strain of the 25,000 psi SMA was nearly 0.014, while the ultimate strain of the 125,000 psi SMA was only around 0.0095, giving a decrease of approximately 32% for the ultimate strain.
This unexpected result occurred because increasing the SMA recovery stress didn’t affect the value $\rho_s$ (as it stayed constant throughout Figure 4.2), it did however, increase the overall strength of the confined concrete. As the increase in strength became more pronounced, the potential energy that the SMA confining material could dissipate (area under the curve) was simply reached earlier.

In Figure 4.3 the formulated stress-strain relationship for various concrete strengths using Mander’s original model modified for active confinement are shown. From these plots, any trend displaying the effect the SMA confinement had on the unconfined concrete strength is actually fairly hard to see. Therefore, the actual data itself had to be examined closer. From the individual data points it was noted that the evaluated increase of compressive strength at peak stress ($f'_{cc}$) was similar throughout all unconfined concrete strengths, ranging from an approximate increase of 360.5 psi in the 3000 psi concrete to a max increase of 368.6 psi in the 8000 psi concrete. The fact that these increases in compressive strength were so eerily similar seems to point to one of two conclusions. The first is that perhaps the unconfined concrete strength has little
effect on compressive strength at peak stress \((f'_{cc})\), and the second is, that perhaps a slight weakness exists in Mander’s et al. models ability to decipher differences between the evaluated compressive strength at peak stress \((f'_{cc})\) and the unconfined concrete strength used. Either way, it appears that this behavior should ultimately be investigated further, in an attempt to better understand these results.

While these values are surprisingly similar to one another, an overall expected trend does start to emerge if one looks at the increase in compressive strength as a percent difference to its corresponding unconfined concrete strength. These percent increases in compressive strength are shown in Figure 4.4. From this plot it can be seen that the 3000 psi unconfined concrete has an evaluated strength increase of approximately 12%, while the 8000 psi unconfined concrete only has an evaluated strength increase of around 4.6%. From this overall trend, it can be concluded that increasing the concrete strength decreases the overall confinement’s effectiveness when using Mander’s model, which makes sense due to the fact that the lateral confining stress remaining constant throughout.
Figure 4.3. Stress-Strain Relationship Formulated for Various Concrete Strengths Using Mander’s Original Model Modified for Active Confinement (SMA Recovery Stress of 116000 psi with 1/3\textsuperscript{rd} inch SMA Pitch)

Figure 4.4. Percent Compressive Strength Increase Using Mander’s Model for Various Unconfined Concrete Strengths Due to SMA Confinement (SMA Recovery Stress of 116000 psi with 1/3\textsuperscript{rd} inch SMA Pitch)
4.4 Recently Developed Analytical Equations for Active Confinement

The performance of an active confinement model greatly depends on its accuracy in evaluating the peak axial stress, the axial strain at peak stress, and the main axial stress-axial strain equation. The equations used to predict these values are often derived from actual experimentation, and can vary greatly. Within this chapter one of the main methods used to develop these predictive equations, the Mohr-Coulomb failure criterion, will be discussed in detail; along with the resulting equations that some investigators have previously developed with this method.

4.4.1 Peak Stress Equation Using Mohr-Coulomb Failure Criterion

The Mohr-Coulomb theory is a mathematical model describing the response of brittle materials, such as concrete, to shear stress as well as normal stress. Generally the theory applies to materials for which the compressive strength far exceeds the tensile strength, making it ideal for concrete.

The Mohr-Coulomb failure criterion is the classical failure criterion used in many applications to determine failure load as well as the angle of fracture of a displacement fracture. This theory assumes one of two different failure modes: the sliding failure, which occurs when a shear force produces a failure parallel to the direction of the force (either with or without an angle of fracture) causing the displacements to move parallel to the rupture surface; or the separation failure which causes the displacements to move perpendicular to the rupture surface. The separation failure method often occurs when tensile forces are applied. Therefore, the sliding failure will almost assuredly be the failure method expected when dealing with confined concrete and will be our major focus throughout this chapter.

This theory is still used extensively with most investigators, due to the overall simplistic nature and relative accuracy that exist with this method. The Mohr-Coulomb theory in its basic form for sliding failure is usually written as shown in Equation 4.4.1.1 that follows:

\[ \sigma_1 = f'_c + k_1 \sigma_3 \]

(4.4.1.1)
where: \( \sigma_1 = f'_{cc} \) = peak axial stress; \( f'_c = f'_{co} \) = uniaxial strength \((f'_c > 0)\);
\( \sigma_3 = f'_l \) = lateral confining pressure \((\sigma_3 > 0)\); and \( k_1 \) = a constant
determined experimentally that is a function of the concrete mix and the lateral pressure applied and can be found from Equation 4.4.1.2.

\[
k_1 = \frac{1 + \sin \phi}{1 - \sin \phi}
\]

(4.4.1.2)

Where: \( \phi \) = internal-friction angle of concrete defined in degrees (angle measured between the normal force and the resultant force that is attained when failure just occurs in response to shearing stress)

For confined concrete, Equation 4.4.1.1 is often normalized with respect to the uniaxial strength as:

\[
\frac{\sigma_1}{f'_c} = 1 + k_1 \frac{\sigma_3}{f'_c}
\]

(4.4.1.3)

Or similarly:

\[
\frac{f'_{cc}}{f'_co} = 1 + k_1 \frac{f'_l}{f'_co}
\]

(4.4.1.4)

The constant \( k_1 \) can only be found using triaxial testing and is often assumed to be equal to 4 because normal strength unconfined concrete will generally fail with an internal friction angle \( (\phi) \) of 37°. While assuming \( k_1 = 4 \) is suitable for unconfined concrete, it should be noted that this value will vary depending on the level of confinement within the experiment. The level of confinement is often described as the ratio of the lateral confining pressure and the compressive strength of unconfined concrete \((f'_l/f'_co)\), and is represented by the far right portion of Equation 4.4.1.4.
To get a better idea of just how much the confinement level can affect the value of this constant, multiple authors have investigated this topic. Some of the most notable results are listed below along with their general trends discovered for this constant. Dahl (1992) found that using the assumed value of \( k_1 = 4 \) often resulted in over predicting the peak stresses once the level of confinement \( (f'_l/f'_c) \) exceeded 0.5. Ansari and Li (1998), then studied the results of \( k_1 \) during high levels of confinement (up to 1.0) and from their research found the best fit value of \( k_1 \) could be as low as 2.6. Most recently Candappa et al. (2001) discovered a value of \( k_1 = 5.3 \) gave the best peak stress values for relatively low confinement levels \( (f'_l/f'_c < 0.2) \), when he studied past results from Dahl (1992), Xie et al. (1995) and Attard and Setunge (1996). The results from these authors are listed in Table 4.1 and the approximated trend is shown in Figure 4.5. From Figure 4.5, it’s easy to see that the constant \( k_1 \) clearly increases as the confinement ratio is decreased.

<table>
<thead>
<tr>
<th>Author</th>
<th>Confinement Level ( (f'/f'_c) )</th>
<th>( k_1 ) Value</th>
<th>Internal Friction Angle (( \phi ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candappa et al. (2001)</td>
<td>&lt;0.2</td>
<td>5.3</td>
<td>( \approx 47^\circ )</td>
</tr>
<tr>
<td>Dahl (1992b)</td>
<td>&gt;0.5</td>
<td>&lt;4.0</td>
<td></td>
</tr>
<tr>
<td>Ansari and Li (1998)</td>
<td>Up to 1.0</td>
<td>low as 2.6</td>
<td>( \approx 26^\circ )</td>
</tr>
</tbody>
</table>

Table 4.1. Values of the Experimentally Determined Constant \( k_1 \), for Varying Confinement Levels.
Figure 4.5. Approximate Values for the Constant $k_1$ Using Mohr-Coulomb Failure Criterion Based on the Confinement Level (Results from Dahl (1992b), Ansari and Li (1998), and Candappa et al. (2001))

With the inherent variability that exists for the experimentally verified constant $k_1$, it’s easy to understand that predicting the peak compressive strength of confined concrete may never be exact, and often the best case scenario is to simply find an accurate approximation that covers most situations. Listed below in Table 4.2, are some of the more notable equations for the compressive strength (peak stress) for actively confined concrete. Special attention should be paid to the values and variability of $k_1$ that was used within each model’s equations, as certain conclusions can potentially be made dealing with the confinement ratio used within their testing procedure. It should also be noted that it appears that Attard and Setunge (1996) and Moghaddam et al. (2010) did not use the Mohr-Coulomb Failure Criterion, but were still included for completeness.
<table>
<thead>
<tr>
<th>Investigators</th>
<th>Compressive Strength (peak stress) of Confined Concrete Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Richart et al. (1928)</td>
<td>( \frac{f'<em>{cc}}{f'</em>{co}} = 1 + 4.1 \frac{f'<em>{i}}{f'</em>{co}} )</td>
</tr>
<tr>
<td>Balmer (1949)</td>
<td>( \frac{f'<em>{cc}}{f'</em>{co}} = 1 + 5.6 \frac{f'<em>{i}}{f'</em>{co}} )</td>
</tr>
<tr>
<td>Attard and Setunge (1996)</td>
<td>( \frac{f'<em>{cc}}{f'</em>{co}} = \left( 1 + \left( \frac{f'<em>{i}}{0.288(f'</em>{co})^{0.67}} \right) \right)^k )</td>
</tr>
<tr>
<td></td>
<td>with ( k = 1.25 \left[ 1 + 0.062 \left( \frac{f'<em>{i}}{f'</em>{co}} \right) \right] (f'_{co})^{-0.21} )</td>
</tr>
<tr>
<td>Candappa et al. (2001)</td>
<td>( \frac{f'<em>{cc}}{f'</em>{co}} = 1 + 5.3 \frac{f'<em>{i}}{f'</em>{co}} )</td>
</tr>
<tr>
<td>Lu and Hsu (2006)</td>
<td>( \frac{f'<em>{cc}}{f'</em>{co}} = 1 + 4.0 \frac{f'<em>{i}}{f'</em>{co}} )</td>
</tr>
<tr>
<td>Jiang and Teng (2007)</td>
<td>( \frac{f'<em>{cc}}{f'</em>{co}} = 1 + 3.5 \frac{f'<em>{i}}{f'</em>{co}} )</td>
</tr>
<tr>
<td>Moghaddam et al. (2010)</td>
<td>( \frac{f'<em>{cc}}{f'</em>{co}} = 1 + 8 \frac{f'<em>{i}}{f'</em>{co}} - 4 \left( \frac{f'<em>{i}}{f'</em>{co}} \right)^{1.2} )</td>
</tr>
</tbody>
</table>

Table 4.2. Equations for Compressive Strength (Peak Stress) of Confined Concrete
4.4.2 Axial Strain at Peak Stress Equation

Now that the method and equations used to find the value for the peak compressive stress of actively confined concrete has been discussed, the next step involves developing an equation that will accurately locate the corresponding axial strain. Early investigators discovered that, just like the general Equation 4.4.1.1 used to predict the value for peak stress, a simple relationship can be used to determine axial strain at peak stress. This general relationship is given as follows:

\[ \varepsilon_1^u = \varepsilon_c^u (1 + k_2 \frac{\sigma_3}{f_c'}) \]  
(4.4.2.1)

where:  
\( f_c' = f_{co} = \) uniaxial strength \( (f_c' > 0) \);  
\( \sigma_3 = f_l = \) lateral confining pressure \( (\sigma_3 > 0) \);  
\( \varepsilon_c^u = \varepsilon_{co} = \) axial strain at peak stress in uniaxial compression;  
\( \varepsilon_1^u = \varepsilon_{cc} = \) axial strain at peak stress in triaxial compression and \( k_2 = \) coefficient that is a function of the concrete mix and the lateral pressure, which can be approximated from the slope when the ratio \( (\varepsilon_{cc}/\varepsilon_{co}) \) is plotted versus the ratio of \( (f_l/f_c') \).

Or, similarly:

\[ \varepsilon_{cc} = \varepsilon_{co} (1 + k_2 \frac{f_l}{f_{co}}) \]  
(4.4.2.2)

Just like with Equation 4.4.1.4 and the constant \( k_j \), the most important thing with this simple relationship is to determine an accurate value for the constant \( k_2 \). It’s important to note, that again, a fairly easy method does exist due to the fact that the axial strain at peak stress \( (\varepsilon_{cc}) \) has been discovered to increase linearly with the level of confinement, regardless of the uniaxial strength of the concrete. This is important because with this knowledge, the constant \( k_2 \) can be approximated by calculating the slope from plots of \( (\varepsilon_{cc}/\varepsilon_{co}) \) versus \( (f_l/f_c') \). An example of this linear relationship and the value derived from the plot is shown in Figure 4.6 (Candappa 2001).
From Figure 4.6 it can be seen that the value for the constant $k_2$ can easily be calculated by simply using the slope intercept equation ($y=m(x) + b$), once the data is plotted in the correct manner. With this method the $y$-intercept ($b$) is often set = 1, and then the slope ($m$ or $k_2$) can be determined from the plot of experimental data points. Or in other words, the equation takes the form as “$y=k_2(x) + 1$” and constant $k_2$ can be found with Equation 4.4.2.3.

$$k_2 = \frac{y - 1}{x}$$

(4.4.2.3)

Essentially, this constant ($k_2$) determines the amount the failure strain will increase, depending on the level of confinement applied. As a numerical representation, it can be seen that providing a level of confinement of 0.1 will result in the failure strain increasing by threefold, based upon $k_2 \approx 20$, as shown in Figure 4.6.

While this constant can be easily approximated due to the linear relationship that exists, it still can only be determined from experimentally obtained data, and therefore a certain amount of variability will exist. Listed below in Table 4.3 are some notable
equations to predict the axial strain at peak stress for active confinement, which include the values, derived for the constant $k_2$:

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Axial Strain at Peak Stress Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Richart et al. (1928)</td>
<td>$\varepsilon_{cc} = \varepsilon_{co} \left[ 1 + 20.5 \left( \frac{f^<em>}{f^</em>_{co}} \right) \right]$</td>
</tr>
<tr>
<td>Attard and Setunge (1996)</td>
<td>$\varepsilon_{cc} = \varepsilon_{co} \left[ 1 + (17 - 0.06 f^<em>_{co}) \left( \frac{f^</em>}{f^*_{co}} \right) \right]$</td>
</tr>
<tr>
<td>Candappa et al. (2001)</td>
<td>$\varepsilon_{cc} = \varepsilon_{co} \left[ 1 + 20 \left( \frac{f^<em>}{f^</em>_{co}} \right) \right]$</td>
</tr>
<tr>
<td>Lu and Hsu (2006)</td>
<td>$\varepsilon_{cc} = \varepsilon_{co} \left[ 1 + 19.21 \left( \frac{f^<em>}{f^</em>_{co}} \right) \right]$</td>
</tr>
<tr>
<td>Jiang and Teng (2007)</td>
<td>$\varepsilon_{cc} = \varepsilon_{co} \left[ 1 + 17.5 \left( \frac{f^<em>}{f^</em>_{co}} \right)^{1.2} \right]$</td>
</tr>
<tr>
<td>Moghaddam et al. (2010)</td>
<td>$\varepsilon_{cc} = \varepsilon_{co} \left( \frac{f^<em>}{f^</em>_{co}} \right)^{1.1}$</td>
</tr>
</tbody>
</table>

Table 4.3. Equations for Axial Strain at Peak Stress of Actively Confined Concrete

4.4.3 Active-Confinement Model for all Concrete Strengths

The equations listed above in Tables 4.2 and 4.3 are very useful in predicting the peak stress and the corresponding strain for concrete. However, each of these equations’ accuracy is limited by the strength of the unconfined concrete to be used. Basically, these equations were strictly intended for use with either normal strength concrete (NSC)
or high strength concrete (HSC), but cannot cover the full range of concrete strengths available without often decreasing the accuracy.

With this limitation in mind, Xiao et al. (2010) attempted to develop a unified active-confinement model applicable to both HSC (compressive strength above 50 MPa or 7250 psi) and NSC. To derive this model, the authors assembled a large database that included the results of 51 actively confined concrete cylinder specimens from Attard and Setunge (1996), Imram and Pantazopoulou (1996), Lu and Hsu (2006), Candappa et al. (2001), and Tan and Sun (2004). The unconfined concrete strength of the specimens ranged from 7500-18275 psi (51.8 to 126 MPa), while the confinement ratio ranged from 0.01 to 0.84.

From regression analysis of the specimen database, Xiao et al. (2010) developed the following peak axial stress equations for only HSC which is shown in Equation 4.4.3.1 and a unified equation for both NSC and HSC, which is shown in Equation 4.4.3.2:

\[
f'_{cc} = f'_{co} \left[ 1 + 3.34 \left( \frac{f'_c}{f'_{co}} \right)^{0.79} \right] \tag{4.4.3.1}
\]

\[
f'_{cc} = f'_{co} \left[ 1 + 3.24 \left( \frac{f'_c}{f'_{co}} \right)^{0.80} \right] \tag{4.4.3.2}
\]

Similar regression analysis of the test results of the actively confined specimens led to the following equations for axial strain at peak stress equations for only HSC (Equation 4.4.3.3) and a unified equation for both NSC and HSC (Equation 4.4.3.4):

\[
\varepsilon_{cc} = \varepsilon_{co} \left[ 1 + 18.8 \left( \frac{f'_c}{f'_{co}} \right)^{1.1} \right] \tag{4.4.3.3}
\]
The authors used a root-mean-square deviation (RMSD) to measure the differences between the values evaluated by their model and the values actually observed experimentally. From these results the authors found that the curves formulated by the proposed equations gave results that were in close agreement with the experimental curves, and shown to be considerably better than that of corresponding equations in existing models, including the one used in the model of Jiang and Teng (2007) (Equation 4.4.1.10 and 4.4.2.7). The results for the peak axial stresses and axial strain at peak stress are shown in Figures 4.7 and 4.8, respectively. From these figures, which display the results from Equations 4.4.3.1 through 4.4.3.4, Xiao et al. concluded that the behavior of actively confined NSC is very similar to that of actively confined HSC, which is a very important conclusion but will require further research to be verified.

\[
\varepsilon_{cc} = \varepsilon_{co} \left[ 1 + 17.4 \left( \frac{f_l'}{f_{co}'} \right)^{1.06} \right]
\]

(Eq. 4.4.3.4)

Figure 4.7. Peak Axial Stress of Actively Confined Concrete (Xiao et al. 2010)
Special notice should be paid to the equations developed from Xiao et al. (Equations 4.4.3.1 through 4.4.3.4) as the authors have introduced exponents to the confinement ratio, which can tell us information about both the peak stress and corresponding strain values, but also about the general trend in regards to the confining ratio overall.

From Equations 4.4.3.1 and 4.4.3.2, it can be seen that the authors have included an exponential number (0.79 or 0.8) to the confinement ratio when predicting the peak stress. This exponential value produces slightly counter intuitive results to the peak stress value as it actually gives better than previously expected values, due to the fact that the confining ratio is also a value smaller than 1. This additional exponential value also gives an intriguing trend when comparing the normalized peak stress versus the confinement ratio (Figure 4.6), as it predicts above average normalized peak stress values for low confinement ratios, and below average normalized peak stress values for relatively higher confinement ratios.

From Equations 4.4.3.3 and 4.4.3.4 it can be seen that the authors have also included an exponential number (1.1 or 1.06) to the confinement ratio when predicting
the strain at the peak stress. This exponential value produces lower than previously expected strain at peak stress results, again due to the fact that the confining ratio is smaller than 1. This additional exponential value also gives an intriguing trend when comparing the normalized axial strain at peak stress versus the confinement ratio (Figure 4.7), as it predicts slightly lower failure strains than previously expected during low confinement levels, and slightly higher failure strains during higher confinement levels.

4.5 Stress-Strain Relationship Formulated from Various Active Confinement Models

From the previous sections within this chapter, it is obvious that multiple researchers have attempted to accurately predict the values for the peak stress and the corresponding strain of actively confined concrete in attempts to ultimately determine the complete stress-strain relationships. Therefore, the question that now needs to be asked is: how much of a difference do these equations actually make?

To hopefully answer this question, within this section some of these equations will be used to predict the stress-strain relationship of some typical SMA confined concrete examples. The computer program Matlab was implemented to display these stress-strain curves for 4x8 inch actively confined concrete specimens with the geometrical and material parameters previously given in Section 4.3.1. It’s again important to note that the properties selected are fairly conservative as the idea of this portion was to again show a realistic representation of their confining behavior, not their ultimate potential.

The first stress-strain relationship is shown in Figure 4.9 and would be considered very lightly confined with a SMA pitch of 1/3 inch \((\rho_s = 0.000967)\). From even this lightly confined scenario, the overall differences within each models formulated relationship can be viewed. It appears, at this low confinement level, that the ascending branch from all of the models is just about identical up until a compressive stress of around 3500 psi or approximately 85% of the unconfined concrete strength, and only after this portion do any noticeable differences exist. Some variation does exist with the peak stress value as it varies from a low of 4196 psi by Jiang and Teng, to a high value of 4421 by Xiao et al. A general trend also appears to exist when comparing the peak stress
value and the date the equations were derived. As it appears that the older equations seem to predict lower peak stress values as opposed to the newer equations.

The real variation exists within the descending branch of the stress-strain curve, where even for this low confinement example a relatively large amount of variation can be seen. First it’s important to notice how Mander et al. curve is by far the highest when looking strictly at the descending branch. This response appears to be due to the fact that their equation is based on the passive confinement method, in which at the peak stress location the confining material may not have or only slightly began to yield, as opposed to active confinement where the confining material, by definition, must have already yielded for it to be in the post-peak region (refer to Sections 1.5.1 and 1.5.2).

![Figure 4.9. Lightly Confined Stress-Strain Relationship Formulate from Various Investigators (4000 psi Unconfined Concrete Strength, 116 ksi SMA Recovery Stress, 1/3\textsuperscript{rd} inch Pitch)](image)

The second stress-strain relationship is shown in Figure 4.10 and would also be considered lightly confined, although with a slightly higher value of $\rho_s = 0.0016$, due to the SMA spacing being reduced to 1/5\textsuperscript{th} an inch pitch. Again from this confined scenario overall differences within each models formulated relationship can be viewed, although

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this time they are even more pronounced than within Figure 4.9. From this plot, it can again be seen that all models are generally very similar, with their ascending branch up until the unconfined concrete stress level of 4000 psi. Some variation occurs when looking at the peak stress value, as it again appears that the newer equations generally predict a higher value relative to the old equations. When comparing the strain at peak stress, it’s important to notice the general shift towards the left as it varies from a low of 0.0023 by Moghaddam et al., and a high of 0.0036 by Mander et al. This shift towards the left is expected and important because it signifies a major difference between active and passive confinement.

The real variation, as expected, again exists within the descending branch of the stress-strain curve, where yet again a relatively large amount of variation can be seen. Mander’s curve is again by far the highest when looking strictly at the descending branch, and follows the same reasoning as was explained for Figure 4.9. The curve by Moghaddam et al. shows a very steep drop within its descending curve, which signifies a significant reduction in the confinement due to yielding of the SMA material. The ultimate strains also greatly vary within this plot from a low value of 0.00125 from Mander’s, to a high of approximately 0.03 from Moghaddam. This large discrepancy is due to the much higher post peak region shown by Mander’s curve, causing the energy dissipation capacity to be reached much sooner, as opposed to curve formulated by Moghaddam.
While the results are purely analytical at this point, the correlations by Xiao et al. (2010) and Moghaddam et al. (2010) appear to give the best overall results. These curves along with the one formulated by Mander et al., have been plotted again without the other models for viewing ease, and can be seen in Figures 4.11 and 4.12. With both of these models the location of the peak stress is shifted more towards the left when compared to Mander’s model, which is to be expected with active confinement due to the constant lateral pressure that is applied independent of axial stresses. These equations also predict lower stress values throughout the descending branch due to an expected yielding of the confining material. This lower trend allows these equations to also predict a higher ultimate strain value due to the fact that it takes longer to reach the SMA energy dissipating capacity as opposed to Mander’s.
Figure 4.11. Lightly Confined Stress-Strain Relationships Formulated by Mander, Xiao, and Moghaddam (4000 psi Unconfined Concrete Strength, 116 ksi SMA Recovery Stress, 1/3" inch Pitch)

Figure 4.12. Lightly Confined Stress-Strain Relationship Formulated for Mander, Xiao, and Moghaddam (4000 psi Unconfined Concrete Strength, 116 ksi SMA Recovery Stress, 1/5" inch Pitch)
4.5.1 Analytical Assessments

- The ascending branch of the stress-strain relationship evaluated using all the models is generally very similar, until approximately 85% of the unconfined concrete compressive strength is reached.
- When looking at the peak stress values, it appears that the equations by Xiao and Moghaddam generally assess a slightly larger increase in peak strength.
- When comparing the strain at peak stress, it’s important to notice the general shift toward the origin throughout the formulations. This shift towards the origin is expected and it is important because it signifies a major difference between active and passive confinement, where the peak stress occurs at a larger strain value for passive confinement leading to increased damage (cracking) within the concrete.
- When looking at the descending portion of the curves, it appears that the equations by Xiao, Jiang and Teng, and Moghaddam generally assess a much lower overall stress-strain behavior.
- This lower descending trend allows those equations to also assess a higher ultimate strain value, which is due to the fact that it then takes more deformations (strain) to reach the SMA energy dissipating capacity.

Based upon these analytical observations, it appears that a great deal of uncertainty still exists within the stress-strain relationship of actively confined concrete members, even under a relativity low amount of transverse confinement. It also appears that past models generally underestimated the benefits that active confinement may actually provide, due to the general trends that appear to exist within the estimation of both the value and location of the peak stress, and the general slope/curvature of the descending curve. With that being said, it does appear that the models by Xiao et al. and Moghaddam et al. may be the most accurate in formulating the stress-strain response of SMA actively confined concrete, although at this point in the investigation this is purely speculation.
With these analytical observations in mind, it appeared that further experimental investigation is required in an attempt to verify which model is the most accurate and should ultimately be used to formulate the stress-strain relationship of SMA confined concrete members. This experimental investigation will be greatly detailed and discussed in Chapters 5, 6, and Appendix C, and as mentioned earlier the models by Xiao, Moghaddam and Mander will be the main focus for experimental correlations.
Chapter 5: Experimental Testing Program for Correlations with the SMA Confined Models

5.1 Introduction

As one can very clearly see from Figures 4.9 through 4.12, the various models do significantly differ from each other in their predictions, especially in the domain of the descending stress-strain behavior. Not only were variations within the model clearly evident, but they also were found to be greatly dependent on the degree of confinement provided by the SMA, as they appear to be amplified with increased confining levels. With this in mind, the experimental study that follows becomes even more important, as it hopefully will provide more insight into the actual stress-strain behavior of SMA confined concrete under various levels of SMA confinement.

5.2 Experimental Objective

The main experimental objective was to obtain actual stress-strain relationships of SMA confined concrete specimens, and validate their overall confining potential. This information can then be used, in an attempt to determine which actively confined predictive equations (shown earlier within Chapter 4) most accurately predicts the descending stress-strain behavior of SMA confined concrete. Once the accuracy of the model was validated, the information could also then be used to verify the effectiveness of the newly modified active confinement model with regards to the increases in both ductility and peak stress.

The experimental portion of this research also incorporated four secondary goals, which included:

1. To assess how well the energy balance approach used within the model, correlates with the ultimate strain location of SMA confined concrete specimens.
2. To assess how well certain parameters, such as the amount of confining material (transverse confining ratio) and the unconfined concrete strength affect the stress-strain behavior.

3. To assess, which method is the most suitable to actually attach the SMA wire to the concrete specimens.

4. To investigate the overall accuracy of the stress-strain curves for SMA confined concrete, once the most appropriate method of attaching the SMA to the concrete specimen was selected.

With these secondary goals, it should be understood however, that due to the large amount of variability that exists with prestrained SMAs, it would be nearly impossible to test all confining possibilities, and therefore only certain instrumented specimens could be tested by keeping various variables manageable.

5.3 Experimental Procedure

To achieve the main experimental objective,

- Axial compression tests were conducted on multiple SMA confined concrete cylinders with varying degrees of concrete compressive strength and SMA confining material. From these axial compression tests, the stress versus strain relationship was calculated from the gathered force and displacement data, for each specimen tested.

- Once all data was obtained, the results were compared with the unconfined (control) specimens, to determine the approximate increase in overall strength and ductility that the SMA confinement scenario provided.

- To determine the confining potential of SMAs, it was deemed important to vary the confining ratio provided from the SMA confinement, and therefore cylinders were either tested with a SMA spacing of either 1/3\textsuperscript{rd} inch pitch ($\rho_s =0.000967$) or 1/5\textsuperscript{th} inch pitch ($\rho_s =0.0016$).

- It was also important to vary the concrete compressive strength used within the cylinders. To obtain this, the concrete used for the compressive tests were from one of three batches, each with a different compressive strength value.
These values were designed and anticipated to cover a fairly wide range of strengths.

- From each concrete batch, certain cylinders that were not to be confined would be tested as control specimens, and would be used to determine the average 28 day compressive strength, and the stress versus strain relationship of the unconfined concrete.
- The cylinders that were not designated as control cylinders would be confined with varying amounts of SMA material, and these stress-strain curves would ultimately be compared with the unconfined control cylinders, to determine the increase in both strength and ductility that the SMA confinement provided.

5.4 Experimental Testing Materials

Within this section the materials used for conducting the experiments are described in detail. The materials to be discussed include all relevant information obtained from experimental testing or directly from the manufacturer regarding the concrete and SMA material used, such as:

- Recovery stress - defines the amount of stress and overall active confinement capability that the SMA may provide to the concrete specimen.
- Transformation temperatures - defines when the SMA transforms to the Austenitic stage and then attempts to recover the previously applied strain deformations.
- Stress versus strain behavior under tension - determines the ultimate tensile strength and fracture strain of the SMA, which are directly required by the energy balance approach to calculate the predicted ultimate strain of the confined concrete specimen.
- SMA attachment method - required to tightly secure the SMA wire to the outside of the concrete cylinder and causing a fixed recovery boundary condition, so the SMA can apply a recovery stress to the concrete as it attempts to recover strain (deformation).
Once the SMA wire selected for experiments has been described in detail, general information about the concrete including specimen quantities and mix proportions used are discussed.

5.4.1 SMA Material (Flexinol) Introduction

The SMA wire that was used within the study to actively confine the concrete cylinders is known as Flexinol, and was partially donated by the company Dynalloy, Inc. Makers of Dynamic Alloys. These Flexinol wires were prestrained by the manufacture and are primarily used for actuator purposes, as they contract in length they create a force when they are activated. Due to this primary function, these SMA wires by design, work great for repetitive contractions, but they may also work perfectly for the constant constrained recovery that is required for active confinement applications.

These Flexinol SMA wires are round with a cross section diameter of 0.5 mm (0.020 inch) and are nickel-titanium based, with almost an exact 50/50 mixture. This Flexinol SMA wire is a type 2 SMA and therefore will require constant heating to maintain the wire in the Austenitic phase (Figure 2.6), allowing the SMAs to achieve the desired recovery stress necessary for confinement purposes. This wire was ultimately selected based on cost, as type 2 SMA are almost always the cheapest classification. As stated previously in Chapter 2, one would ideally want to use a type 3 SMA wire for this type of confinement application, as a type 2 SMA naturally has some slightly greater material limitations. The main issue with this type of wire will be due to the fact that the recovery stress displayed by the SMA, will greatly depend on the exact temperature during the time of testing, and therefore variations in the recovery stress are expected to occur in the range of 5-10% of the max possible recovery stress throughout. While this is obviously not an ideal behavior, this SMA can still be suitable for the relevant nature of this experiment.

With this in mind, the mechanical properties of the SMA in the high temperature austenitic phase are of the primary concern, as all active confinement will occur during this phase. It should be mentioned though, that since the SMA wires will be tested until the ultimate rupture point, the possibility does exist and is actually expected for the SMA wire to transform back into the martensitic phase towards the very end of the compressive
testing (Figure 2.4). While this transformation is important to understand, by the time it actually occurs the wires will have almost assuredly, plastically deformed so severely that the wire will be assumed to no longer be capable of still displaying a recovery stress, making the transformation back to martensite inconsequential.

Flexinol SMA wires comes with the possibility for two transformation temperatures, a low temperature alloy that will transform to austenite above 158°F (70°C) and a high temperature alloy that will transform to austenite above 194°F (90°C). The plots of temperature versus percent strain, for both the low and high temperature alloys are shown in Figure 5.1 and this data was taken directly from the Flexinol technical characteristic sheet off of the Dynalloy, Inc. website. For this experiment, the high temperature 194°F (90°C) alloy was used. From Figure 5.1, it can be seen that with the high temperature Flexinol wire, the atomic structure transforms from Martensite to Austenite causing the contraction in length that is expected to begin at a temperature of roughly 194°F (90°C) and should be completed at a temperature of approximately 248°F (120°C). This contraction force can be significant and when the SMA recovery is constrained, the Flexinol wire is capable of exerting the necessary large recovery stress that is required for active confinement.
As it can be seen from Figure 5.1, the Flexinol wires were already prestrained by the manufacturer (Dynalloy Inc.) and are designed to contract approximately 4-5% when heated into the high temperature Austenitic phase. This manufactured prestrain was a key factor in the selection of Flexinol, because it permitted that all of the SMA wires used within the experiment were uniformly prestrained throughout. This reasonable assumption will give the SMA equal recovery stresses and transformation temperatures that are given directly within the technical characteristic sheet from the company. If the prestraining was done manually, which was the other option, the possibility would have existed for non-uniform recovery stresses and the transformation temperatures would have to have been experimentally verified, which would have added unnecessary variability throughout the experiment.

5.4.2 Flexinol Recovery Stress

The most important characteristic of Flexinol that needed to be verified was the amount of recovery stress that the SMA, could actually provide during a constrained recovery situation. Since Flexinol was ultimately designed to be used for actuator
purposes and repeat small contractions, most of the recovery stress information from the manufacture was only given as conservative limits to be followed ensuring repetitive motion without damaging the SMA. Therefore, only general guidelines for the total recovery stress of Flexinol could be found from either the manufacturer or the technical characteristic sheet, making it apparent that the recovery stress value was going to have to be experimentally verified.

For this verification, simple plots of temperature (°F) versus force (lbs) were obtained for multiple small two inch samples of the as-shipped prestrained wire by conducting appropriate experiments. The temperature was increased by controlling the amount of current applied through the SMA with a portable power supply (Joule heating). The results from the first temperature versus force test are shown below in Figure 5.2, and by simply dividing the force by the area of the wire (0.02 inch or 0.5 mm diameter), the corresponding temperature versus stress (MPa) plot is shown in Figure 5.3. From Figure 5.2 it can be seen that the force generated by the wire reached a high of nearly 35.5 lbs, when the heat source was ultimately reduced after a temperature of roughly 270°F was reached. It should be noted that the maximum temperature reached, approximately 270°F, was selected because it corresponded to a value of 2.0 amps (1.4 volts) for the sample, and while reaching higher temperatures could have resulted in a slightly greater recovery force the test was stopped for fear of damaging the wire. The maximum contraction force recorded of 35.5 lbs. corresponds to a stress of roughly 779 MPa and is shown in Figure 5.3, along with the rest of the temperature versus stress values. On both of these plots the blue upward pointing arrows that are shown, designate the general heating trend that that occurred first, followed by the decrease temperature behavior which is shown by the red downward pointing arrow.
Figure 5.2. Flexinol Constrained Recovery Stress (Test #1), Plotting Temperature (°F) versus Force (Lbs).

Figure 5.3. Flexinol Constrained Recovery Stress (Test #1) Plotting Temperature (°F) versus Stress (MPa).
The recovery stress was tested again in the exact same method, because the Flexinol wire was delivered in two separate batches, and it was deemed critical to verify that both batches would give comparable recovery stress values during the experiment. The results from the second temperature versus force test are shown in Figure 5.4, and the corresponding temperature versus stress (MPa) plot is shown in Figure 5.5. From these plots it can be seen that the constrained SMA wire behaved very similar as the first test, although this time the max force recorded was slightly less reaching a value of 34.3 lbs. This maximum force value again corresponded to roughly 2.0 amps, causing a maximum temperature of 250°F when the heat source was again reduced.

![Figure 5.4. Flexinol Constrained Recovery Stress (Test #2), Plotting Temperature (°F) versus Force (Lbs.).](image-url)
Figure 5.5. Flexinol Constrained Recovery Stress (Test #2), Plotting Temperature (°F) versus Stress (MPa).

From both of the experimental tests (Figures 5.2 and 5.4) it was interesting to note that the Flexinol alloy didn’t have a very well defined transformation region from the low temperature martensite phase to the high temperature austenite phase. Before these tests were conducted the Flexinol SMA was expected to behave as is shown in Figure 5.1, where very little force would be generated until the austenite transformation temperature was ultimately reached, at a temperature greater than roughly 194°F (90°C). This idea was originally expected to be very similar to how water changes phases with the addition of heat, where the temperature will remain fairly constant during phase changes (solid → liquid → gas) until the phase has completely transformed. However this wasn’t the case, as can be seen from these experimental tests that the Flexinol alloy actually increased in recovery stress pretty much linearly throughout the tests.

After further examination of these plots, it seems that perhaps the Flexinol SMA is acting as small individual portions or particles instead of one uniformly mixed alloy, where certain areas can be transformed to austenite, while other areas can still be martensitic even at the same temperatures. This response may be due to poor material
characteristics or simply testing errors where the temperature was increased too fast for the transformations to be completed throughout the SMA. Either way this result should have very little effect on the active confinement application as long as the temperature of the SMA is assured to be high enough for the austenitic transformation to completely occur (≥ 250 °F). It’s also important to note, that while the results were not identical, it still appears to be reasonable to assume that both batches of Flexinol are very similar and any slight difference that may occur between the two are negligible, in regard to the expected amount of recovery stress.

5.4.3 Flexinol Recovery Stress at Varying Levels of Strain

The maximum recovery stress, which will be discussed in detail in the following section, is crucial for active confinement application, as this value determines the ultimate confining potential of SMA in question. However, obtaining the recovery stress for SMA wires at varying strain levels is also very important, because it’s the best way to predict the actual recovery stress after prestress losses have occurred (Section 2.1.5).

Typically, with a type 3 SMA, once the wire is sufficiently heated to activate the material, the recovery stress applied to the specimen would equal the peak recovery stress based on the amount of prestrain, minus a certain value due to prestrain losses. These prestrain losses can be measured with strain gauges or other precise equipment, but is often approximated to be in the range of less than one percent. The amount of prestrain can then be used to recalculate the current recovery stress, due to the fact that this stress has been ascertained to increase or decrease linearly with the amount of prestrain.

However, as stated earlier, with the experiment utilizing a type 2 SMA, the recovery stress of the wire is directly dependent on the temperature during the experiment, thus the prestrain loss becomes much harder to determine (Section 5.4.1). Due to this slight material limitation that this wire classification presents, no prestrain loss will be included into the expected recovery stress. While a zero prestress loss actually occurring during the proposed experiments is unlikely, this assumption is still justifiable because the focus is more on the behavioral trends instead of precise numerical results.
5.4.4 Flexinol Mechanical Properties

The mechanical properties of Flexinol are summarized in Table 5.1, and are labeled as either the expected values (from the manufactures) or experimentally obtained values for both the martensitic and austenitic phases.

<table>
<thead>
<tr>
<th>Property</th>
<th>Phase</th>
<th>Expected</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformation Temperature</td>
<td>Martensitic</td>
<td>158°F (70°C)</td>
<td>≈150-200°F</td>
</tr>
<tr>
<td></td>
<td>Austenitic</td>
<td>194°F (90°C)</td>
<td>≈ &gt;200°F</td>
</tr>
<tr>
<td>Recovery Stress</td>
<td>Martensitic</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td></td>
<td>Austenitic</td>
<td>N.A.</td>
<td>≈116ksi (800 MPa)</td>
</tr>
<tr>
<td>Ultimate Strength, $f_{u}$</td>
<td>Martensitic</td>
<td>170 ksi (1172 MPa)</td>
<td>N.A.</td>
</tr>
<tr>
<td></td>
<td>Austenitic</td>
<td>170 ksi (1172 MPa)</td>
<td>N.A.</td>
</tr>
<tr>
<td>Elongation at failure, $\varepsilon_a$</td>
<td>Martensitic</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td></td>
<td>Austenitic</td>
<td>17.5%</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

Table 5.1. Mechanical Properties of Flexinol SMA Wire Listing Both Expected Results from the Manufactures and Experimentally Verified Results.

5.4.5 Concrete Mix Proportions and the Cylindrical Specimens

The concrete specimens that were used for this experiment were cast in 4 inch diameter 8 inch tall single-use plastic molds. The material used for the concrete specimens included Cemex type 1 Portland cement, Quikrete mason sand (No. 1952-49), and Quikrete all-purpose gravel (No. 1151). The sand and gravel was oven dried at 104°C (219°F) for at least 24 hours to assure that the aggregate contained no water in the pores or on the surface. The maximum size of the gravel used for the coarse aggregate was 3/8 inch (10 mm). The specimens were removed from the plastic molds after 24 hours and put into room temperature water baths to be continuously and uniformly moist cured for 28 days.

For this experiment three different concrete strengths were created for the specimens, by varying the overall percent of the cement and fine aggregate of the mix designs. No additives were used in any of the concrete mixes. The three different batches were used to assess the effect of the plain concrete strength on the behavior of confined specimens, and were designed to give a slight range of different concrete strengths. The
concrete mix designs listed by overall percentage of each ingredient, water/cement ratio, and the measured slump from each batch are presented in Table 5.2. The average compressive strength for each batch is also shown within the table giving the average 28 day compressive strength after water curing.

The total number of specimens tested from each concrete batch along with the amount of SMA spacing applied for confinement is also shown in Table 5.3. It should be noted that the concrete specimens were not tested in the cured moist condition, but in an air dried state as it took a few days to apply the SMA confinement to each batch.

<table>
<thead>
<tr>
<th>Batch # (Average 28 Day Strength)</th>
<th>Cement</th>
<th>Coarse Aggregate</th>
<th>Fine Aggregate</th>
<th>Water</th>
<th>Water/Cement Ratio</th>
<th>Slump (inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batch #1 (3068 psi)</td>
<td>18.22%</td>
<td>37.23%</td>
<td>35.06%</td>
<td>9.50%</td>
<td>0.52</td>
<td>2.9</td>
</tr>
<tr>
<td>Batch #2 (4563 psi)</td>
<td>23.75%</td>
<td>36.67%</td>
<td>29.62%</td>
<td>9.97%</td>
<td>0.42</td>
<td>1.1</td>
</tr>
<tr>
<td>Batch #3 (3693 psi)</td>
<td>25.36%</td>
<td>35.11%</td>
<td>29.26%</td>
<td>10.27%</td>
<td>0.40</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Table 5.2. Concrete Mix Proportions Used, Displaying the Amounts in Percentages, Along with the Average Compressive Strength Values Obtained for Each of the Three Batches.
<table>
<thead>
<tr>
<th>Concrete Batch (Strength)</th>
<th>Control Specimens$^2$</th>
<th>Confined Specimens$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unconfined</td>
<td>Unconfined w/ Concrete Anchor</td>
</tr>
<tr>
<td>Batch #1 (3068 psi)$^1$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Batch #2 (4563 psi)$^1$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Batch #3 (3693 psi)$^1$</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

1. Average 28 day compressive strength values of control cylinders
2. All concrete specimens are 4 inch diameter 8 inch tall

Table 5.3. Concrete Specimen Test Matrix, Displaying Total Number of Specimens Tested and Corresponding Confining Level

5.5 SMA Wire Attachment Method

A key portion and a major obstacle of actively confining concrete members with SMA wire, is finding a suitable method to attach the SMA wire to the concrete specimen. The method that is ultimately selected must reduce or completely eliminate the potential of slack developing between the SMA wire and the outside of the cylinder which would reduce or completely negate the effect of the recovery stress, while still being reliable, repeatable, and nondestructive to the concrete. For this reason multiple methods and connections were researched and eventually tried, in an attempt to discover which method would work best for these specimens.

For each attachment method attempted and described below, the SMA wire was purposely stopped 0.75 inches short of the end of the concrete cylinders being confined. Therefore, each SMA confined cylinder roughly only confined 6.5 of the possible 8 inch height of concrete. This 0.75 inch buffer was intentionally included due to fear of the SMA wire attachment method getting in the way of the compressive load frame during the actual testing procedure. While this extra buffer did leave roughly 1.5 inches of unconfined concrete per each confined cylinder, its effect was assumed to be negligible to the overall stress-strain relationship, and is not included within the actual calculations.
The first method attempted to attach the wire to the cylinder, involved the use of U-clamps at the top and bottom of the cylinders in a nondestructive manner. With this method the idea was to basically attach the SMA wire to the cylinder by using the tension forces of the SMA wire, as it was tightly wrapped around the cylinders and eventually clamped at its ends. For this method to work the end of a continuous length of SMA wire must be tightly wrapped around the cylinder creating a complete loop of SMA wire. After this has been achieved, a u-clamp would then be used to attach the SMA wire from the completed loop to the rest of the SMA wire used to confine the length of the cylinder. With this method the tension force within the SMA wire is basically holding itself securely to the concrete cylinder, and therefore no damage is done to the actual cylinder. This method is visually depicted in Figure 5.6 and overall gave unfavorable results.

The problem with this method was that it was just too hard to tighten the nuts, locking the SMA tightly within the u-clamp, while still retaining enough tension within the wire to keep the SMA tightly wrapped around the concrete cylinder. It also appeared that some slippage was occurring within the SMA wire and the u-clamps, due to the small diameter size of the SMA wire, and the design of the u-clamps themselves.
The second method attempted, involved the use of simple nuts and bolts, to hold the SMA securely against the concrete cylinders. Again this method similar to the first, attempted to basically hold itself together from the tension in the SMA wire in a nondestructive manner. For this method to work the end of the SMA was first attached to the bolt, by wrapping the SMA around the bolt and tightly securing the wire with a nut. Once this was completed, the SMA was then wrapped completely around the cylinder and then again attached to the bolt in the same manner, by wrapping the SMA around the bolt and again securing the wire with another nut. This method is visually shown in Figure 5.7.
Overall, this method worked better than the u-clamps and was successful in attaching one section of the cylinder, either top or bottom (whichever side was attempted first). The problem with this method occurred when excess slack would often develop between the SMA and the concrete cylinder while trying to repeat the process at the opposite end of the cylinder. With this method the wire itself could be securely attached to the nut and bolt apparatus, but the wire would often contain too much slack at the opposite end and begin to slide down the concrete cylinder. This method worked better than the U-clamp attachments since the wire was held very tightly and did not slip within the nut and bolt apparatus, but it was eventually determined to be too difficult to accurately repeat.

After the first two nondestructive methods gave unfavorable results, and consistently allowed for too much slack, it was obvious that some type of physical anchor point needed to exist within the concrete cylinders themselves. Therefore, the third method incorporated the use of actual screws embedded within the concrete cylinders, giving the SMA wires a location to be physically attached. For these anchor points, a
3/16<sup>th</sup> x 1-1/4 inch Tapcon concrete anchor screw with a hex washer head was used. These anchor screws required predrilling 5/32 inch diameter holes 1-1/2 inches deep into the concrete at approximately 3/4 inches from both ends of the concrete cylinders. With this method the SMA wire was wrapped around the concrete anchor points as they were roughly 90% screwed into the concrete, and then securely tightened the rest of the way, holding the SMA wire firmly in place. Figure 5.8 displays this method along with general representation of the location of the anchor bolts.

The anchor screw attachment shown in Figure 5.8 was ultimately selected as the best method to secure the SMA wire to the concrete cylinders, and was used in all experimental testing. This method tightly held the SMA wire to the concrete cylinders, with little to no slack being visible while still being the most reliable, repeatable, and by far the easiest of the three described methods.

The only concern of using this method was of the potentially weakening or cracking the concrete, due to predrilling the holes or physically inserting the anchor screws into the ends of the cylinders. It should be noted however, that a similar anchor
screws attachment method was also used by Choi et al. (2008) who used anchor screws roughly the same size, with a diameter of 5 mm (0.2 inches) and a length of 20 mm (0.79 inches). From the experimental data obtained during Choi et al. (2008) research, comparing the peak stress and stress-strain relationship of both anchored and plain concrete, it was concluded that anchoring into the concrete with such small anchor screws did not influence the behavior of the concrete cylinders. Nevertheless, it was still viewed necessary to again experimentally verify that the anchor screw method used did not noticeably alter the behavior of plain concrete in a negative manner, potentially negating some of the benefits of the SMA confinement. To verify this result, some of the unconfined concrete control specimens, were also compressively tested with the concrete anchor screw embedded within. The results from these tests will be shown and further discussed later in this thesis.

5.6 Testing Equipment

Within this section the equipment used within the actual experimental testing is described in detail. The equipment to be discussed, includes all relevant information regarding heating the SMA wire and the compressive load frame used.

5.6.1 Compressive Load Frame

To experimentally test the concrete cylinders in compression and obtain the stress-strain relationship of the concrete specimens, a recently calibrated Durham Geo-Enterprises CM-415 model concrete load frame was used. Attached to the CM-415 compressive load frame were two Durham Geo E-405 smart digital indicators (single channel digital readouts), used to measure the compressive load applied by the machine (kips), and the overall displacement of the cylinders with an attached one inch linear variable displacement transducer (LVDT). The machine was located in the civil engineering materials laboratory at The Ohio State University, and had a maximum compressive load capacity of 500 kips. Figure 5.9 depicts the general layout and equipment used throughout this experiment. Figure 5.10 displays the LVDT and lever-arm used for the displacement measurements.
Figure 5.9. General Experimental Layout and Equipment Layout Used Within This Experiment
5.6.2 Flexinol Electrical Guidelines and Required Power Supply

Do to the nature of the compressive testing that needed to be completed; it was obvious that the easiest way to heat and activate the SMA was with joule heating through the use of an electrical power supply. Flexinol has a high resistance compared to copper and most other metallic materials, but is still conductive enough to easily carry current. The same conventional electrical rules apply for Flexinol, except that the resistance of the SMA actually goes down as it is heated and contracts. This result is partially due to the shortened wire, and partially due to the fact that the wire gets slightly thicker as it shortens. It’s also important to note that it makes no difference if either alternating or direct current is used for heating purposes.

With this information in mind, heating with the use of a power supply is fairly simple and only requires two key parameters to determine the strength of the power supply required. These required parameters are the resistance of the SMA in Ohms per unit length, which is based on the diameter of the wire, and the largest total length of wire that will require heating.
The required electrical guidelines for the Flexinol wire used with this experiment were taken directly from the technical characteristic sheet provided by the manufacturer and are shown in Table 5.4. From this table it can be seen that the resistance for a 0.02 inch (0.5 mm) Flexinol wire is roughly 0.11 ohms/inch, and approximately 4 amps will heat the wire sufficiently enough to contract the wire in 1 second. It should also be noted that based upon the SMA confinement spacing’s for the cylinders of 1/3\textsuperscript{rd} inch pitch and 1/5\textsuperscript{th} inch pitch, it was easily determined that the largest amount of SMA wire was approximately 36 ft, for the 1/5\textsuperscript{th} inch pitched SMA confined cylinders.

With both of these key parameters now known, it was possible to calculate that roughly 150 volts would be required from the power supply if a slightly lower current of 3000 mA was used. To obtain this required amount of voltage a Mastech 250 volt 12A variable transformer was used, which is shown in Figure 5.1.

<table>
<thead>
<tr>
<th>Diameter Size inches (mm)</th>
<th>Resistance ohms/inch (ohms/meter)</th>
<th>Approx. Current for 1 Sec Contraction mA</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.020 (0.51)</td>
<td>0.11 (4.3)</td>
<td>4000</td>
</tr>
</tbody>
</table>

*Data from Dynalloy, Inc. Makers of Dynamic Alloys

Table 5.4. Flexinol Electrical Wire Guidelines
5.7 Experimental Obstacles

This experimental investigation, like all experiments had its fair share of obstacles, some expected and some unforeseen, that needed to be resolved for this project to even be undertaken. While most of these obstacles were minor, they still required quick and creative resolutions, and that were ultimately both practical and inexpensive. Some of the obstacles that had to be overcome during this experimental investigation included:

1. During the compressive cylinder tests, metal end caps are required to be applied to the top and bottom of the concrete cylinders, to assure that the load applied is evenly distributed throughout the end of the cylinders during the tests. These metal end caps are designed to fit snugly on a certain size of concrete cylinders. Due to this snug fit the hexagonal tips of the concrete anchor screws would often stick out too far, causing the end caps to not fit correctly onto the ends of the cylinders.
To remedy this problem the ends of the concrete anchor screws were required to be shaved off with a metal file, until the end caps could fit correctly. The proximity of the metal end caps used and the concrete anchor screws after the end was filed down, is shown in Figure 5.12.

2. Due to the limited amount of space between the inside of the metal end cap and the outside of the concrete cylinders, extra care had to be taken to avoid direct contact of the metal end caps with either the concrete anchor screws or the Flexinol SMA wire during the heating of the SMA wire. This unwanted metal-metal contact was greatly increasing the resistance of the SMA confined cylinders and essentially causing the 250 volt power supply to be unable to sufficiently heat the Flexinol SMA wire.

   To remedy this problem, all metal surface of the end caps, except for the bottom inside portion, was covered with a layer of electrical tape. The bottom inside portion, was the only area in direct contact with the ends of the concrete cylinders, and therefore was specifically left uncovered for fear of altering the displacement measurements due to the layer of electrical tape. The metal end caps used in this experiment before and after altering can be viewed in Figure 5.13, and a typical SMA confined cylinder ready for testing can be viewed in Figure 5.14.

3. With the Flexinol SMA wire being classified as a type 2 SMA, it was required to heat the SMA material above 194°F (90°C) in order transform the SMA wire into the high temperature austenitic phase required for active confinement. To achieve the necessary increase in heat in the SMA wire during testing, an electrical power supply was deemed the best option, as this method was able to be easily measured and controlled throughout the tests, in a reliable and accurate method, while also assuring a uniform temperature throughout.

   During the compressive test though, it was observed that the Flexinol SMA wire could not handle the application of electrical current, and the physical act of yielding that occurred in the Flexinol wire due to the expansion of the concrete at the same time.
In almost every test, shortly after the peak stress of the concrete cylinder was reached, on the descending portion of the stress-strain curve as the concrete cylinder started to noticeably crush and forcing the SMA wire to severely yield, the connection between the ends of the SMA wire and power supply would violently spark and detach from each other, with a sudden blowout. This blowout and physical detaching of the connections was unexpected and an apparently undocumented occurrence of SMAs, and would often happen to both connections simultaneously (though not always). This sudden blowout would consistently leave a length of about 1 inch of SMA wire still attached to the power supply connection. Interestingly enough, this blowout always occurred at the very end of the SMA wire directly where the power supply was attached, at a location past the concrete anchor screws. Due to this blowout occurring after the location of the concrete anchor screws, this meant that the portion of Flexinol wire that eventually sparked and detached should have been under no stress, as this portion of wire was only included to easily connect the power supply. Therefore, this portion of the SMA wire should have been under no stress throughout the entire compressive test, and would appear to be unaffected by the rest of the Flexional wire yielding. Pictures displaying the extra length of Flexinol wire used to attach the power supply, and the resulting outcome from the power supply/Flexinol wire connection detachment, can be viewed in Figure 5.15 and 5.16.

In an attempt to remedy this problem, multiple electrical connections were tried throughout the experiment to assure that this behavior was due to a limitation in the Flexinol wire and not due to the connection used. However, the same blowout behavior appeared to occur in relatively the same location no matter what connection was used. Once, the electrical connections used in the experiment were determined to make no difference, the only conclusion possible at this time is that with this experimental application, the Flexinol wire simply could not handle the application of electrical current while simultaneously yielding, due to the limitations of the Flexinol wire material
alone. It is however, important to note that this sudden blowout in the connection is not fully understood, and will require further investigation to better understand this unexpected behavior, as it has not been reported in previous literature.

Due to this conclusion, the rate of loading during the compressive tests had to be increased to a relatively high value of 0.250 inch/min. Increasing the rate of loading was necessary due to the fact that once the power supply and SMA wire connection detached the temperature of the wire would begin to drop causing the Flexinol wire to transform back into the Martensitic phase if enough time was allowed to pass. Since, temperatures above 194°F are required for the austenitic behavior to be displayed, increasing the rate of loading allowed for the test to be completed fast enough, allowing most of the tests to reach ultimate failure or at least very close to it before the temperature in the wire dropped too low switching the alloy back into the martensitic phase.

While this behavior was unexpected, this result should only marginally affect the stress-strain behavior, due to the fact once the specimens tested reached their peak strength and began to display its post-peak curve, the Flexinol wire would have already begun to severely yield, causing the recovery stress of the SMA wire, to either be greatly reduced or completely eliminated.
Figure 5.12. Visual Depiction of Concrete Anchor Screws and Metal End Cap Interaction

Figure 5.13. Metal Ends Caps (Original and Altered) Used During the Experiments
Figure 5.14. Typical SMA Confined Concrete Cylinder Ready for Compressive Testing.
Figure 5.15. Concrete Anchor Screw and Typical Length of Extra Flexinol Wire Used for Connection Purposes

Figure 5.16. Power Supply/Flexinol SMA Wire Electrical Connections After Detaching Occurrence
Chapter 6: Experimental Results from SMA Confined Concrete

6.1 Introduction

To determine the confining potential of SMAs, the accuracy of the active confinement models discussed in Chapter 4, and to determine which models best described the post-peak behavior of SMA confined concrete specimens in Chapter 5, an experimental investigation was undertaken. As previously discussed, all experimental data obtained from this investigation will be susceptible to some natural limitations due to: (i) inherent concrete material non-homogeneity and variation of the mechanical properties (ii) sensitivity of testing equipment and testing procedures. It is well known that the behavior of concrete can greatly vary within similar sets of specimens primarily due to the progressive cracking that occurs during the loading process until ultimate failure eventually occurs. Therefore, the emphasis of this experimental investigation was placed on discovering the overall behavioral trends rather than on the numerical values obtained.

At this point it should be mentioned that all information regarding the accuracy of analytical model with respect to the experimentally obtained descending curve is contained in Appendix C and the actual (as measured) stress-strain relationship for each specimen tested can be viewed in Appendix E. Each of the stress-strain relationships shown from the experiment undertaken in this chapter, have been slightly adjusted due to unavoidable machine errors that occurred in the strain measurement at the start of each specimen tested. This initial strain error that occurred in each test was due to errors in the displacement measurements from non-uniform contact with the top and bottom platen of the compressive load frame and the concrete cylinder, at the start of each test. However, it’s important to note that this type of error is not unusual with this type of experimental work and specifically with the type of load frame that was used.
To eliminate this machine error, a linear trendline was approximated and pushed backward until a zero stress value was ultimately reached, based on the elastic deformation that was eventually shown after sufficient contact was made. With this newly approximated elastic relationship (with the initial machine error removed), the complete stress-strain representation was then able to be adjusted towards the y-axis, until the origin and the zero stress representation over lapped. A visual representation of this required adjustment is shown in Figures 6.1 and 6.2, which show a typical as measured stress-strain curve with the described initial error, and the stress-strain relationship after the plots were adjusted.

Figure 6.1. Recorded (As Measured) Stress versus Strain for the First Unconfined Anchored Cylinder in Concrete Batch #1.
6.2 Concrete Batch #1

The concrete mix proportions and specimen test matrix used within this experimental investigation were previously shown in Tables 5.2 and 5.3, respectively.

6.2.1 Unconfined Concrete Specimens

For each of the three concrete batches tested it was necessary to determine the unconfined peak strength of the concrete cylinders that were to be confined. This peak strength information was directly required for two main reasons: the first was for input into the active confined models that were discussed in Chapter 4, and the second, was for comparison purposes with the SMA confined cylinders in an attempt to discover the overall benefits that the SMA confinement may provide.

To determine the peak unconfined concrete strength of each concrete batch, two cylinders were tested under compression until ultimate failure occurred. During this initial testing phase, where the unconfined peak stress was being determined, it was also deemed important to verify that the addition of the concrete anchor screws didn’t alter the stress-strain behavior, or noticeably damage the concrete specimens. This verification about the concrete anchor screws was essential, because if the concrete anchor screws
used within the experiment were to damage or severely alter the stress-strain relationship of the unconfined cylinders, it would be nearly impossible to determine the actual benefits the SMA confinement actually provided.

The stress versus strain results from concrete batch #1, for two unconfined cylinders and two unconfined anchored cylinders are shown in Figure 6.3. Multiple cylinders were tested for each category due to the variation in the behavior that naturally exists within concrete. From the results shown in this plot, it can be seen that the concrete anchor screws appeared to have no noticeable negative effects, on either the peak concrete strength or the overall stress-strain relationship of the concrete specimens for the first concrete batch, as the two cylinders tested with the anchor screws embedded actually performed better than the unconfined control cylinders. However, it’s important to note that although in this test the cylinders with the concrete anchor screws performed better than the unaltered control cylinders, this performance is assumed to be due to simple variation of the material properties that exists within concrete and not directly related to the concrete anchor screws. Nevertheless, this behavior will also be monitored in the other two concrete batches to see if this overall trend continues.

With this newly obtained information about the addition of the concrete anchor screws having a negligible effect on the concrete specimens, it was then possible to determine from this data that the average peak compressive strength of the four cylinders (two unconfined and two unconfined with anchor screws) was approximately equal to 3068 psi, and this value would be taken as the expected compressive strength of the 1st concrete batch. The average stress-strain relationship of the four cylinders is also shown in Figure 6.3, and can now be taken as the most likely stress-strain representation for the unconfined concrete cylinders from this concrete batch.
6.2.2 Confined Concrete Specimens

For each SMA confined concrete cylinder the experimentally obtained stress-strain relationships after the required plot adjustment occurred are shown in this chapter. Alongside these actual stress-strain relationships are approximated polynomial trendlines that were required to accurately represent the stress-strain representations in equation form. These derived equations allowed the experimental stress or strain to be determined at any location throughout the curve which than can be compared adequately with the data from the appropriate model. These approximated trendline were often found from a 6th degree polynomial and obtained directly from Matlab. To make the approximated polynomial trendlines as accurate as possible to the actual recorded stress-strain relationship, each trendline was broken into two connecting sections, consisting of an ascending and descending (pre-peak and post-peak) portion of the stress-strain relationship.

In an attempt to numerically determine the accuracy of the approximated trendline with the experimental data, the coefficient of determination ($R^2$) value, was determined for each confined cylinder. The $R^2$ value is often used in statistics to describe how well a
regression line fits a set of data, and was deemed the best possible way to numerically prove the accuracy of the overall trendline representation. With this statistical method a $R^2$ value near 1.0 indicates that the regression line fits the data very well, while a $R^2$ value closer to 0.0 indicates that the regression line does not fit the data accurately. The $R^2$ values were determined from either Microsoft Excel or Matlab due to the large number of data points that existed.

6.2.2.1 SMA Confined 1/3rd Inch Pitch Specimens

The experimental results from the Flexinol SMA confined concrete specimens with 1/3rd inch SMA pitch and concrete batch #1 (unconfined compressive strength 3068 psi) are presented in this section. Figures 6.4 through 6.6, display the results from the three 1/3rd inch pitch SMA confined specimens.

As can be seen in Figure 6.4, the first specimen had a peak stress of approximately 3723 psi and a corresponding strain at peak stress equal to 0.0045. This specimen also had an ultimate stress of 1125 psi at failure, with an ultimate strain of 0.0228. This specimen showed a very sharply decreasing behavior directly after the peak location was reached and continued to decrease throughout the post peak region until the ultimate failure occurred.
Figure 6.4. Experimental and Approximated Stress versus Strain Relationship for 1/3\textsuperscript{rd} inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinder #1, from Concrete Batch #1 (Compressive Strength 3068 psi) (Plot Adjusted towards the Origin by 0.0016).

The stress-strain curve shown in Figure 6.5 is from the second 1/3\textsuperscript{rd} inch pitch SMA confined specimen from concrete batch #1. From this figure it can be seen that this specimen had a peak stress of approximately 3919 psi and a corresponding strain at peak stress equal to 0.0072. This specimen also had an ultimate stress at failure of 2292 psi, with an ultimate strain of 0.0243. This specimen again showed a very sharply decreasing behavior immediately after the peak stress was reached, but eventually leveled out and maintained the same stress level of approximately 2300 psi until ultimate failure was reached. The behavior shown in this test was slightly unexpected, as the cylinder was able to maintain its compressive strength so close to failure. This constant level of available compressive strength towards the end of the stress-stress relationship must be at least somewhat attributed to the concrete alone, as it has already been determined that the SMA confining material must have already yielded, greatly reducing its confining potential.
Figure 6.5. Experimental and Approximated Stress versus Strain Relationship for 1/3\textsuperscript{rd} inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinder #2, from Concrete Batch #1 (Compressive Strength 3068 psi) (Plot Adjusted towards the Origin by 0.00277).

Figure 6.6 shows the stress versus strain relationship for the third 1/3\textsuperscript{rd} inch pitch SMA confined specimen from concrete batch #1, and it can be seen that this specimen had a peak stress of approximately 3671 psi and a corresponding strain at peak stress equal to 0.0071. This specimen also had an ultimate stress at failure of 2846 psi, with an ultimate strain of 0.01994. This specimen showed an unexpected post peak behavior as the stress-strain relationship only dropped slightly, around 1000 psi, after the peak stress was reached. After this initial drop occurred the stress-strain relationship remained fairly constant with only minor fluctuations occurring until the ultimate failure was eventually reached. This behavior, with a fairly constant post-peak, is more representative to what would be expected from a typical passively confined specimen, and can really only be described by the inherent variability within concrete as it reaches ultimate failure.
Figure 6.6. Experimental and Approximated Stress versus Strain Relationship for 1/3<sup>rd</sup> inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinder #3, from Concrete Batch #1 (Compressive Strength 3068 psi) (Plot Adjusted towards the Origin by 0.0016).

The average stress-strain relationship from the three cylinders is shown in Figure 6.7 and the corresponding stress-strain values for the critical locations are shown in Table 6.1. From this average curve, it’s now possible to visualize the most likely stress-strain behavior for the 1/3<sup>rd</sup> inch pitch SMA confined concrete cylinders, which then can be compared with that of the unconfined cases in an attempt to understand the potential benefit that the SMA confining wire actually had on the concrete specimens.
Figure 6.7. Experimental and Approximated Average (Cylinders 1-3) Stress versus Strain Relationship for 1/3\textsuperscript{rd} inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders from Concrete Batch #1 (Compressive Strength 3068 psi).

<table>
<thead>
<tr>
<th>Cylinder</th>
<th>Peak Stress (psi)</th>
<th>Strain at Peak Stress (inch/inch)</th>
<th>Ultimate Stress (psi)</th>
<th>Ultimate Strain (inch/inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder 1</td>
<td>3723</td>
<td>0.0045</td>
<td>1125</td>
<td>0.0228</td>
</tr>
<tr>
<td>Cylinder 2</td>
<td>3919</td>
<td>0.0072</td>
<td>2292</td>
<td>0.0243</td>
</tr>
<tr>
<td>Cylinder 3</td>
<td>3671</td>
<td>0.0071</td>
<td>2846</td>
<td>0.01994</td>
</tr>
<tr>
<td>Average</td>
<td>3699</td>
<td>0.0062</td>
<td>2300</td>
<td>0.0217</td>
</tr>
</tbody>
</table>

Table 6.1. Stress-Strain Values from Concrete Batch #1 1/3\textsuperscript{rd} inch Pitch SMA Confined Cylinders

As can be seen from Figure 6.7, the average 1/3\textsuperscript{rd} inch pitch SMA curve from concrete batch #1 increased fairly linearly until approximately 3000 psi where a noticeable nonlinear curve develops until the peak stress was reached. Once the peak stress was reached, the stress-strain representation sharply decreased until a strain value of approximately 0.01 was obtained, which led to a much more subtle decreasing nature until the ultimate failure eventually occurred, at an approximate stress value of 2300 psi and ultimate strain value of 0.0217.
For comparison purposes the average stress-strain relationships of the unconfined concrete cylinders from Figure 6.3 and the average 1/3\textsuperscript{rd} inch pitch SMA confined cylinders from the figure above are shown in Figure 6.8. From this figure the large increase in ductility due to the SMA confinement can easily be noticed, along with a slight increase in compressive strength. From this representation, it appears that the addition of SMA confinement increased the ultimate strain from approximately 0.0052 that occurred in the unconfined cylinders to approximately 0.0217, or an increase of around 317\%. The peak stress also appears to have increased from approximately 3068 psi that occurred in the unconfined cylinders to approximately 3699 psi, or an increase of around 20.5\%. It is however important to keep in mind that due to the material nonhomogeneity of concrete, the peak stress and ultimate strain values can greatly vary throughout the tested specimens and therefore these increases should not be taken as definite value, only as a possible behavioral trend that will need to be further investigated before any definitive conclusions can be drawn.
Figure 6.8. Experimental Average Stress versus Strain Relationship for Unconfined Concrete and 1/3\textsuperscript{rd} inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders from Concrete Batch #1 (Compressive Strength 3068 psi).

6.2.2.2 SMA Confined 1/5\textsuperscript{th} Inch Pitch Specimens

Shown below are the experimental results from the Flexinol SMA confined concrete specimens with 1/5\textsuperscript{th} inch pitch SMA spacing for concrete batch #1 (unconfined compressive strength 3068 psi). The only difference within the 1/3\textsuperscript{rd} and 1/5\textsuperscript{th} inch pitch confined specimens is that due to restrictions in the amount of Flexinol SMA wire that was available, only two cylinders were able to be confined per concrete batch to determine the average representation. It is important to note that the exact same process that was undertaken in Section 6.2.2.1 for the 1/3\textsuperscript{rd} inch pitch SMA confined specimens was also carried out for the specimens within this classification (and all other remaining specimens tested throughout this experimental investigation), although the amount of data shown was slightly altered for viewing ease. Within this section the stress-strain relationship from the two 1/5\textsuperscript{th} inch pitch SMA confined concrete cylinders is shown in
Figure 6.9, and the corresponding stress-strain values for the critical locations are shown in Table 6.2.

When looking strictly at the behavior of the descending curves in Figure 6.9, it appears that the first specimen crushed in a slightly odd manner with only a relatively small decrease in stress after the peak location, and then remained mostly constant or slightly decreasing, until the ultimate failure eventually occurred. When looking at the descending portion of the second specimen tested, it’s important to notice the sharp decrease that occurred directly after the peak stress location was reached. After this sharp decrease, the stress-strain relationship slightly increased as the specimen completed its second stage of crushing. Once this second crushing stage was completed, the specimen remained constant until ultimate failure eventually occurred.

From this plot and corresponding values just discussed, it should be noted that the peak stress obtained of 3857 psi is very similar to the peak stress values recorded from the 1/3" inch pitch SMA confined cylinders. This result, while being hard to confirm due to only two confined specimens being tested, does seem to point to the fact that with this amount of SMA wire, the increased confinement level appears to have a small effect on the peak strength of the confined specimens. It’s also reassuring that the strain at ultimate failure is higher than each of the 1/3" inch pitch SMA confined cylinders described previously, as this behavior was expected due to the increase in confining material.

The average stress-strain relationship from cylinders 1 and 2 is also shown in Figure 6.9, and from this representation it’s now possible to visualize the most likely stress-strain behavior for the post peak region for 1/5" inch pitch SMA confined concrete cylinders with an unconfined concrete compressive strength of 3068 psi.
Figure 6.9. Experimental and Approximated Average (Cylinders 1 and 2) Stress versus Strain Relationship for 1/5\textsuperscript{th} inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders from Concrete Batch #1 (Compressive Strength 3068 psi).

<table>
<thead>
<tr>
<th></th>
<th>Peak Stress (psi)</th>
<th>Strain at Peak Stress (inch/inch)</th>
<th>Ultimate Stress (psi)</th>
<th>Ultimate Strain (inch/inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder 1</td>
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<td>2411</td>
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<td>Cylinder 2</td>
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<td>Average</td>
<td>4192</td>
<td>0.0062</td>
<td>2425</td>
<td>0.0322</td>
</tr>
</tbody>
</table>

Table 6.2. Stress-Strain Values from Concrete Batch #1 1/5\textsuperscript{th} inch Pitch SMA Confined Cylinders

The average stress-strain relationships of the unconfined concrete cylinders and the 1/5\textsuperscript{th} inch Pitch SMA confined cylinders are shown in Figure 6.10. From this figure the large increase in ductility due to the SMA confinement can again easily be viewed, along with the increase in compressive strength. From this representation, the addition of the SMA confinement increased the ultimate strain from approximately 0.0052 that occurred in the unconfined cylinders to approximately 0.0323, or an increase of around 521%. The peak stress also appears to have increased from approximately 3068 psi that
occurred in the unconfined cylinders to approximately 4173 psi, or an increase of around 36.0%.

Figure 6.10. Experimental Average Stress versus Strain Relationship for Unconfined Concrete and 1/5\textsuperscript{th} inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders from Concrete Batch #1 (Compressive Strength 3068 psi).

6.3 Concrete Batch #2

At this point, the exact same process discussed for the 1\textsuperscript{st} concrete batch, is discussed for concrete cylinders with a different concrete mix design and compressive strength. Table 5.2 provides the mix proportions used for this batch.

6.3.1 Unconfined Concrete Specimens

The stress versus strain results from concrete batch #2 for two unconfined cylinders and two unconfined anchored cylinders are shown in Figure 6.11. Multiple cylinders were again tested for each category due to the random behavior that naturally exists within concrete. From the results shown in this plot, it can be seen that the concrete
anchor screws appeared to again have no noticeable negative effects, on either the peak concrete strength or the overall stress-strain relationship of the concrete specimens for the second concrete batch. It’s important to note that oddly enough, the two unconfined anchored cylinders again had slightly higher peak strengths when compared to the unconfined control specimens. This behavior at this point in time is still assumed to be due to the inherent variability in concrete, as it would appear unlikely that the addition of the concrete anchor screws would be enough to increase the peak strength of the specimens.

With the same assumption about the addition of the concrete anchor screws having a negligible effect on the concrete specimens, it was then possible to determine from this data that the average peak compressive strength of the four cylinders (two unconfined and two unconfined with anchor screws) was approximately equal to 4563 psi. The average stress-strain representation of the four cylinders is also shown in Figure 6.11, and can now be taken as the most likely representation for the unconfined concrete cylinders from this concrete batch.
Figure 6.11. Stress versus Strain Results for Both Unconfined and Unconfined Anchored Concrete Cylinders and the Resulting Average Relationship, from Concrete Batch #2.

6.3.2 Confined Concrete Specimens

Shown below are the experimental results from the Flexinol SMA confined concrete specimens with 1/3\textsuperscript{rd} and 1/5\textsuperscript{th} inch SMA pitch for concrete batch #2 (unconfined compressive strength 4563 psi).

6.3.2.1 SMA Confined 1/3\textsuperscript{rd} inch Pitch Specimens

The stress-strain relationship from the three 1/3\textsuperscript{rd} inch pitch SMA confined concrete cylinders, and the corresponding average relationship is shown in Figure 6.12. The critical stress-strain values for each of the stress-strain representations are also summarized in Table 6.3.

It’s important to note that an error did occur during the testing of the first specimen within this batch and the data for the complete descending curve was unable to be recorded for this cylinder. As a result of this error the descending curve and ultimate strain recorded, was much lower than the results obtained for the other two specimens with the same 1/3\textsuperscript{rd} inch pitch SMA confinement, but would have been expected to be
similar if the data was able to be recorded throughout the tests entirety. Without the entire descending curve it’s difficult to determine any real behavior trends that occurred from this cylinder, besides noting the sharp decrease that occurred directly after the peak stress was reached.

The second specimen displayed a very narrow peak on its stress-strain curve as it somewhat oddly reached peak stress and drastically decreased in a very rapid manner. After this peak was reached, the stress-strain relationship then continued to decrease moderately until ultimate failure eventually occurred. While the third specimen showed a very sharply decreasing behavior directly after the peak stress was reached, but eventually leveled out as most of the cylinders tested have done, and eventually maintained the same stress level of approximately 2300 psi until ultimate failure occurred.

The average stress-strain relationship from cylinders 1 through 3 is also shown in Figure 6.12, and from this average curve it’s now possible to visualize the most likely stress-strain behavior for the post peak region for the 1/3rd inch pitch SMA confined concrete cylinders with an unconfined concrete compressive strength of 4563 psi. At this point it should be noted that each of the three 1/3rd inch pitch SMA confined cylinders in concrete batch #2, all had a slightly lower peak compressive strength than the average unconfined compressive strength of 4563 psi that was determined earlier. This result is most likely simply due to randomly selecting one or two unconfined cylinders with slightly above average peak strengths. With this possible statistical anomaly in mind, it’s important to note that the compressive strength of this batch for the predictive model (Appendix C) was still taken as 4563 psi, even though a slightly lower number near 4000 psi is probably more representative of the compressive strength of this concrete batch.
Figure 6.12. Experimental and Approximated Average (Cylinders 1-3) Stress versus Strain Relationship for 1/3\textsuperscript{rd} inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders from Concrete Batch #2 (Compressive Strength 4563 psi).

The average stress-strain relationship of the unconfined concrete cylinders and the 1/3\textsuperscript{rd} inch pitch SMA confined cylinders is shown in Figure 6.13. From this plot the unlikely scenario discussed before dealing with the average peak stress can be viewed. The unconfined cylinders actually had a slightly higher average peak stress of 4563 psi, than the average peak stress of the 1/3\textsuperscript{rd} inch pitch SMA confined cylinders, which only averaged 4039 psi, or a decrease in peak stress of 11.5%. This unexpected and unlikely result, again demonstrates the large variability that often exists within the compressive
stress of concrete, and can only be described as a statistical anomaly and not a behavioral trend that is expected to continue. What is important from this plot is to again notice the large increase in ductility that occurred due to the SMA confinement. From this representation the addition of SMA confinement increased the ultimate strain from approximately 0.0072 that occurred in the unconfined cylinders, to approximately 0.0247, or an increase of around 243%.

![Stress-Strain Relationship](image)

**Figure 6.13.** Experimental Average Stress versus Strain Relationship for Unconfined Concrete and 1/3rd inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders from Concrete Batch #2 (Compressive Strength 4563 psi).

6.3.2.2 SMA Confined 1/5th inch Pitch Specimens

Shown below are the experimental results from the Flexinol SMA confined concrete specimens with 1/5th inch SMA pitch for concrete batch #2 (unconfined compressive strength 4563 psi). The stress-strain relationship from the two 1/5th inch pitch confined cylinders is shown in Figure 6.14, and the corresponding stress-strain values for the critical locations are summarized in Table 6.4.
From Figure 6.14 it can be seen that the peak stress values obtained for both cylinders appears to be slightly lower than what would be expected due to the additional SMA confinement of concrete specimens with an unconfined compressive strength of 4563 psi. This result appears to further agree with the idea that the unconfined concrete strength used within the model of 4563 psi is at least slightly high.

When looking strictly at the behavior of the descending curves, a sharp decrease in available compressive strength occurred directly after the peak stress was reached for both specimens. After this sharp decrease occurs, it again appears that the concrete tries to maintain its available compressive strength although this time at a strongly reduced level now in the range of approximately 3000 psi until ultimate failure occurs due to the eventual rupture of the SMA confining material. It’s also important to not overlook the fact that the ultimate strain for this cylinder was very similar to the values obtained for the 1/3\textsuperscript{rd} inch pitch SMA confined specimens previously discussed for this batch. This result is noteworthy because it would seem to point towards a slightly earlier failure than would be expected, as the ultimate strain for this cylinder should be at least slightly larger due to the extra SMA confining material that was used.

Based upon these experimentally obtained plots for the two 1/5\textsuperscript{th} inch pitch SMA confined cylinders, the average stress-strain relationship was able to be determined, which is also shown in Figure 6.14. From this average curve, it was now again possible to visualize the most likely stress-strain behavior for the post peak region for 1/5\textsuperscript{th} inch pitch SMA confined concrete cylinders with an unconfined concrete compressive strength of 4563 psi.
Figure 6.14. Experimental and Approximated Average (Cylinders 1 & 2) Stress versus Strain Relationship for 1/5\(^{th}\) inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders from Concrete Batch #2 (Compressive Strength 4563 psi).

<table>
<thead>
<tr>
<th></th>
<th>Peak Stress (psi)</th>
<th>Strain at Peak Stress (inch/inch)</th>
<th>Ultimate Stress (psi)</th>
<th>Ultimate Strain (inch/inch)</th>
</tr>
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<td>2637</td>
<td>0.0255</td>
</tr>
<tr>
<td>Cylinder 2</td>
<td>4780</td>
<td>0.0044</td>
<td>2204</td>
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</tr>
<tr>
<td>Average</td>
<td>4835</td>
<td>0.0056</td>
<td>2324</td>
<td>0.0242</td>
</tr>
</tbody>
</table>

Table 6.4. Stress-Strain Values from Concrete Batch #2 1/5\(^{th}\) inch Pitch SMA Confined Cylinders

The average stress-strain relationship of the unconfined concrete cylinders and the 1/5\(^{th}\) inch Pitch SMA confined cylinders are shown in Figure 6.15. From this figure the large increase in ductility due to the SMA confinement can again easily be viewed, along with a slight increase in compressive strength. From this representation the addition of SMA confinement increased the ultimate strain from approximately 0.0072 that occurred in the unconfined cylinders to approximately 0.0242, or an increase of around 236.1%.

The peak stress also appears to have increased from approximately 4563 psi that occurred in the unconfined cylinders to approximately 4835 psi, or an increase of around 5.96%.

This increase in peak stress is much smaller than the increase that occurred when
comparing the specimens from concrete batch #1, and as previously discussed is most likely due to the apparently higher than expected peak stress values recorded for the unconfined cylinders in this concrete batch.

![Graph showing stress versus strain](image)

Figure 6.15. Experimental Average Stress versus Strain Relationship for Unconfined Concrete and 1/5th inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders from Concrete Batch #2 (Compressive Strength 4563 psi).

6.4 Concrete Batch #3
6.4.1 Unconfined Concrete Specimens

The stress versus strain results from concrete batch #3 (refer to Table 5.2 for mix proportions) for two unconfined cylinders and two unconfined anchored cylinders are shown below in Figure 6.16. From the results shown in this plot, it can again be seen that the concrete anchor screws appeared to again have no noticeable negative effects, on either the peak concrete strength or the overall stress-strain relationship of the concrete specimens for the third concrete batch. With the same assumption about the addition of the concrete anchor screws having a negligible effect on the concrete specimens, it was then possible to determine that the average peak compressive strength of the four cylinders (two unconfined and two unconfined with anchor screws) was approximately
equal to 3693 psi. The average stress-strain representation of the four cylinders is also shown in Figure 6.16, and can now be taken as the most likely representation for the unconfined concrete cylinders from this concrete batch.

![Figure 6.16. Stress versus Strain Results for Both Unconfined and Unconfined Anchored Concrete Cylinders and the Resulting Average Relationship, from Concrete Batch #3.](image)

6.4.2 Confined Concrete Specimens

Shown below are the experimental results from the Flexinol SMA confined concrete specimens with 1/3\textsuperscript{rd} and 1/5\textsuperscript{th} inch SMA pitch for concrete batch #3 (unconfined compressive strength 3693 psi). The stress-strain relationship from the three confined cylinders is shown in Figure 6.17, and the corresponding stress-strain values for the critical locations are shown in Table 6.5.

6.4.2.1 SMA Confined 1/3\textsuperscript{rd} inch Pitch Specimens

The first 1/3\textsuperscript{rd} inch pitch SMA confined specimen from concrete batch #3 clearly displayed the crushing and crumbling affect that has been shown multiple times before, as this cylinder broke in two easily defined phases. The first crushing phase occurred right after the peak stress was reached and the second crushing phase occurred at a strain
in the range of 0.015-0.023. During this second crushing phase the stress-strain relationship actually slightly regained some its available strength before its eventual ultimate failure occurred.

The second 1/3rd inch pitch SMA confined specimen from concrete batch #3 also again crushed in two distinct regions, but displayed a slightly odd behavior as the specimen was almost able to completely recover its peak available stress, by reaching a value of 3120 psi at a corresponding strain value of 0.0164. After this secondary peak was reached the stress-strain curve again sharply decreased until the ultimately failure occurred at a stress of 2031 psi and a corresponding strain value of 0.02338.

The third and final 1/3rd inch pitch SMA confined specimen from concrete batch #3, had a noticeable higher peak stress than the other cylinders tested in this group with a value of approximately 4735 psi. This specimen crushed almost ideally, as it crushed in very small sections giving the post-peak region a very smooth decreasing curvature throughout. Due to the crushing behavior that occurred during this test, this specimen sharply decreased after the peak was reached and eventually leveled out as most of the cylinders tested have done, until a stress value of approximately 2300 psi was reached and ultimate failure occurred.

Just like within the previous sections, the data obtained from these three stress-strain relationships were used to determine the average experimental stress-strain relationship for this batch. This average stress-strain relationship is also shown in Figure 6.17. From this average curve, it’s now possible to visualize the most likely stress-strain behavior for the post peak region for 1/3rd inch pitch SMA confined concrete cylinders with an unconfined concrete compressive strength of 3693 psi.
Figure 6.17. Experimental and Approximated Average (Cylinders 1-3) Stress versus Strain Relationship for 1/3rd inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders from Concrete Batch #3 (Compressive Strength 3693 psi).

<table>
<thead>
<tr>
<th></th>
<th>Peak Stress (psi)</th>
<th>Strain at Peak Stress (inch/inch)</th>
<th>Ultimate Stress (psi)</th>
<th>Ultimate Strain (inch/inch)</th>
</tr>
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<tr>
<td>Cylinder 2</td>
<td>3739</td>
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<td>2031</td>
<td>0.02338</td>
</tr>
<tr>
<td>Cylinder 3</td>
<td>4735</td>
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<td>2087</td>
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<tr>
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Table 6.5. Stress-Strain Values from Concrete Batch #3 1/3rd inch Pitch SMA Confined Cylinders

For comparison purposes the average stress-strain relationships of the unconfined concrete cylinders and the 1/3rd inch pitch SMA confined cylinders are shown in the following Figure 6.18. From this figure the large increase in ductility due to the SMA confinement can easily be viewed, along with a slight increase in compressive strength. From this representation the addition of SMA confinement increased the ultimate strain from approximately 0.009 that occurred in the unconfined cylinders to approximately 0.0257, or an increase of around 185.6% in ultimate strain. The peak stress also appears
to have increased from approximately 3693 psi that occurred in the unconfined cylinders to approximately 4109 psi, or an increase of 11.3%.

![Figure 6.18](image)

Figure 6.18. Experimental Average Stress versus Strain Relationship for Unconfined Concrete and 1/3\text{rd} inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders from Concrete Batch #3 (Compressive Strength 3693 psi).

6.4.2.2 SMA Confined 1/5\text{th} inch Pitch Specimens

Shown below are the experimental results from the Flexinol SMA confined concrete specimens with 1/5\text{th} inch SMA pitch for concrete batch #3 (unconfined compressive strength 3693 psi). The stress-strain relationships from two confined cylinders are shown in Figure 6.19, and the corresponding stress-strain values for the critical locations are shown in Table 6.6.

From the stress-strain relationship of the first 1/5\text{th} inch pitch SMA confined concrete cylinder it can be seen that this specimen had a peak stress of 4221 psi and a corresponding strain of 0.0043. This specimen also had a stress at ultimate failure of 2567 psi with a corresponding strain of 0.02043. From this plot, it can be seen that the peak stress value reached for this cylinder fits very closely to what would be expected with an unconfined compressive strength of 3693 psi that was determined previously.
It’s also important to not overlook the fact that the ultimate strain for this cylinder was lower than expected based upon the values obtained for the 1/3\textsuperscript{rd} inch pitch SMA confined specimens previously discussed for this batch. This result is noteworthy because it would seem to point towards a slightly earlier failure than would be expected, as the ultimate strain for this cylinder should be at least slightly larger than the 1/3\textsuperscript{rd} inch pitch SMA confined cylinders due to the extra SMA confining material that was used.

When looking strictly at the behavior of the descending curve, this specimen displayed a rather shallow descending slope throughout its post peak region. This result, which was fairly unexpected, is most likely due to the specimen being crushed in two distinct regions: a relatively minor initial failure that occurred at the peak location, and a secondary failure that occurred shortly thereafter. The relative weakness of the initial failure and the fact that the second crushing location occurred so close to the peak location, most likely directly resulted in the relatively shallow slope and curvature shown in this figure causing the specimen to not display the sharply decreasing behavior that has been expected.

From the stress-strain relationship for the second 1/5\textsuperscript{th} inch pitch SMA confined concrete cylinder it can be seen that this specimen had a peak stress of 5557 psi and a corresponding stress of 0.0062. This specimen also had a stress at ultimate failure of 2328 psi with a corresponding strain of 0.02658. When again looking strictly at the behavior of the descending curve, this cylinder crushed in an almost ideal manner with many small stages of crushing occurring that gave the descending stress-strain portion a relatively smooth curvature. The same general trend that has been discussed many times before also occurred, where the specimen showed a sharp decrease directly after the peak stress location was reached, followed by an eventual leveling out or slightly decreasing stress-strain relationship towards the end of the curve, until the ultimate failure of the confined cylinder was reached.

With the data obtained from the two specimens tested in this batch, the average experimental stress-strain results was also able to then calculated, and is shown below in fig 6.19. From this descending average curve, it’s now again possible to visualize the most likely stress-strain behavior for the post peak region for the 1/5\textsuperscript{th} inch pitch SMA
confined concrete cylinders with an unconfined concrete compressive strength of 3693 psi.

Figure 6.19. Experimental and Approximated Average (Cylinders 1 & 2) Stress versus Strain Relationship for 1/5\textsuperscript{th} inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders from Concrete Batch #3 (Compressive Strength 3693 psi).

<table>
<thead>
<tr>
<th></th>
<th>Peak Stress (psi)</th>
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<th>Ultimate Stress (psi)</th>
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</table>

Table 6.6. Stress-Strain Values from Concrete Batch #3 1/5\textsuperscript{th} inch Pitch SMA Confined Cylinders

The average stress-strain relationship of the unconfined concrete cylinders and the 1/5\textsuperscript{th} inch pitch SMA confined cylinders are shown in Figure 6.20. From this figure the large increase in ductility due to the SMA confinement can again easily be viewed, along with a fairly large increase in compressive strength. From this representation, the addition of SMA confinement increased the ultimate strain from approximately 0.009 that
occurred in the unconfined cylinders, to approximately 0.0235, or an increase of around 161.1%. The peak stress also appears to have increased from approximately 3693 psi that occurred in the unconfined cylinders to approximately 4894 psi, or an increase of around 32.5%. This increase in peak stress is fairly large when comparing the results from either of the two previously discussed concrete batches.

![Graph showing experimental average stress versus strain relationship for unconfined concrete and 1/5th inch pitch SMA (Recovery Stress 800MPa), confined cylinders from concrete batch #3 (Compressive Strength 3693 psi).](image)

Figure 6.20. Experimental Average Stress versus Strain Relationship for Unconfined Concrete and 1/5th inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders from Concrete Batch #3 (Compressive Strength 3693 psi).

6.5 Experimental Results Summary

6.5.1 Concrete Batch #1

Figure 6.21 shows the experimental average results of the unconfined, 1/3rd, and 1/5th inch pitch SMA confined cylinders tested for the first concrete batch. From this plot the benefits of SMA confinement for concrete members are easily visible, and it’s apparent that the SMA confinement strongly influences the level of ductility as the ultimate strain location increased as expected for each classification. For example, the average unconfined ultimate strain value was approximately 0.0052, and the 1/3rd and
1/5\textsuperscript{th} inch pitch SMA confined average cylinders had ultimate strains of 0.0217 and 0.0323, respectively. This figure also clearly displays the slight increase in the peak stress that the SMA confinement provided. For example, the average unconfined peak stress was equal to 3068 psi, and the 1/3\textsuperscript{rd} and 1/5\textsuperscript{th} inch pitch SMA confined average cylinders had a peak stress of 3699 psi and 4192 psi, respectively. These values are summarized in Table 6.7, along with the percent increase in both ultimate strain and peak stress that the confinement provided.

![Figure 6.21. Experimental Average Stress versus Strain Relationships for the Unconfined, 1/3\textsuperscript{rd} inch Pitch SMA Confined, and 1/5\textsuperscript{th} inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders for Concrete Batch #1.](image-url)
Table 6.7. Experimental Obtained Data from the Specimens of Concrete Batch #1

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<td>317.3%</td>
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<td>36.0%</td>
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<td>521.2%</td>
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</table>

6.5.2 Concrete Batch #2

Figure 6.22 shows the experimental average results of the unconfined, 1/3\(^{rd}\), and 1/5\(^{th}\) inch pitch SMA confined cylinders tested for the second concrete batch. From this plot the benefits of SMA confinement are again easily visible, and it’s apparent that the SMA confinement strongly influenced the level of ductility as the ultimate strain location greatly increased from both levels of confinement when compared to the unconfined average. With the specimens from this concrete batch it again appears that increasing the level of confinement also at least slightly increased the peak strength of the cylinders, even though the measured unconfined average value of 4563 psi seems to have been slightly higher than expected. The values obtained for both peak stress and ultimate strain for each confinement level are summarized in Table 6.8, along with the percent increases that the confinement provided.
Figure 6.22. Experimental Average Stress versus Strain Relationships for the Unconfined, 1/3\textsuperscript{rd} inch Pitch SMA Confined, and 1/5\textsuperscript{th} inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders for Concrete Batch #2.

<table>
<thead>
<tr>
<th>Confinement Level</th>
<th>Peak Stress (psi)</th>
<th>Peak Stress Increase</th>
<th>Ultimate Strain</th>
<th>Ultimate Strain Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconfined Avg.</td>
<td>4563</td>
<td>N.A.</td>
<td>0.0072</td>
<td>N.A.</td>
</tr>
<tr>
<td>1/3\textsuperscript{rd} inch Pitch</td>
<td>4039</td>
<td>-11.5%</td>
<td>0.0247</td>
<td>243.1%</td>
</tr>
<tr>
<td>SMA Avg.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/5\textsuperscript{th} inch Pitch</td>
<td>4835</td>
<td>5.9%</td>
<td>0.0242</td>
<td>236.1%</td>
</tr>
<tr>
<td>SMA Avg.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.8. Experimental Obtained Data from the Specimens of Concrete Batch #2

6.5.3 Concrete Batch #3

Figure 6.23 shows the experimental average results of the unconfined, 1/3\textsuperscript{rd}, and 1/5\textsuperscript{th} inch pitch SMA confined cylinders tested for the third concrete batch. From this plot, the benefits of SMA confinement are once again easily visible, as the SMA confinement again strongly influenced the level of ductility. The values obtained for both peak stress and ultimate strain for each confinement level are summarized in Table 6.9, along with the percent increases that the confinement provided.
Figure 6.23. Experimental Average Stress versus Strain Relationships for the Unconfined, 1/3\textsuperscript{rd} inch Pitch SMA Confined, and 1/5\textsuperscript{th} inch Pitch SMA (Recovery Stress 800MPa), Confined Cylinders for Concrete Batch #3.

<table>
<thead>
<tr>
<th>Confinement Level</th>
<th>Peak Stress (psi)</th>
<th>Peak Stress Increase</th>
<th>Ultimate Strain</th>
<th>Ultimate Strain Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconfined Avg.</td>
<td>3693</td>
<td>N.A.</td>
<td>0.009</td>
<td>N.A.</td>
</tr>
<tr>
<td>1/3\textsuperscript{rd} inch Pitch SMA Avg.</td>
<td>4109</td>
<td>11.3%</td>
<td>0.0257</td>
<td>185.6%</td>
</tr>
<tr>
<td>1/5\textsuperscript{th} inch Pitch SMA Avg.</td>
<td>4894</td>
<td>32.5%</td>
<td>0.0235</td>
<td>161.1%</td>
</tr>
</tbody>
</table>

Table 6.9. Experimental Obtained Data from the Specimens of Concrete Batch #3

6.6 Concluding Remarks

The following conclusions can be drawn from the data obtained directly from the experimental investigation. When reading these conclusions it’s important to remember that due to the material and experimental limitations, only some significant trends can be identified. With this in mind, these conclusions are generally based on the overall behavior that tends to be prevalent throughout the experimental work, and obviously more research is needed to arrive at some appropriate numerically driven assessments.
6.6.1 Experimental Correlations

1. It appears that the concrete anchor screws were very effective in attaching the SMA wire to the concrete cylinders as no noticeable slippage in the SMA confining wire appeared to occur during testing, and the ultimate failure of each SMA confined cylinder occurred within an acceptable range.

2. The addition of a relatively small amount of SMA wire ($\rho_s = 0.000967$ or 0.0016) appears to be very effective at increasing the ductility of the concrete specimens. It appears that an increase in the ultimate strain of approximately 150-500% can realistically be expected.

3. Increasing the SMA confining material, for the $1/5^{th}$ inch pitch cylinders resulted in increased average peak strength in all three concrete batches, when compared to the $1/3^{rd}$ inch pitch confined cylinders. However, the ultimate strain value was not affected in any significant way, due to the overall increased stress-strain relationship that resulted throughout the curve.

4. Any correlations between the unconfined concrete strength used and the resulting increase in ductility are inconclusive at this point.
Chapter 7: Numerical Example to Show the Effect of SMA Confinement

7.1 Numerical Example Introduction

A numerical example was chosen to determine the resulting stress versus strain and axial load versus displacement relationships, for a typical real world SMA actively confined reinforced concrete column. To solve this numerical example, the stress-strain relationship predicted for confined concrete by Xiao et al. (2010) will be used, as his model was concluded to provide the most accurate representation for SMA confined concrete (Appendix C).

7.2 Given Information

For this numerical example, consider a circular column that is 10ft high with a 1.5ft diameter. This column has been designed with eight ordinary No. 9 longitudinal steel bars, but is being actively confined with a 0.375 inch diameter continuous SMA spiral for the transverse reinforcement. This column has a clear cover of 1.5 inches and the unconfined concrete has a peak stress of $f'_{c}=4000$ psi and ultimate strain of $\varepsilon_{cu}=0.004$. The mechanical properties for the longitudinal (ordinary) steel and the SMA spiral used within this example are shown in Tables 7.1 and 7.2, respectively, and were chosen due to the fact that these mechanical properties would be considered typical as they closely represent what would most likely be used in a real world situation.
Table 7.1. Mechanical Properties of Longitudinal Steel (Ordinary) Used in Example

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity ($E$)</td>
<td>29000 ksi</td>
</tr>
<tr>
<td>Yield Stress ($f_y$)</td>
<td>60 ksi</td>
</tr>
<tr>
<td>Ultimate Stress ($f_u$)</td>
<td>100 ksi</td>
</tr>
<tr>
<td>Yield Strain ($\varepsilon_y$)</td>
<td>0.00207</td>
</tr>
<tr>
<td>Strain Hardening ($\varepsilon_{sh}$)</td>
<td>0.010</td>
</tr>
<tr>
<td>Yield Plateau ($SH%$)</td>
<td>0 (Flat)</td>
</tr>
<tr>
<td>Strain at Max Stress ($\varepsilon_{wu}$)</td>
<td>0.08</td>
</tr>
<tr>
<td>Rupture Strain ($\varepsilon_{rupt}$)</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 7.2. Mechanical Properties of Transvers SMA Spiral Used in Example

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>0.375 inches</td>
</tr>
<tr>
<td>Ultimate Strength ($f_{uSMA}$)</td>
<td>170 ksi</td>
</tr>
<tr>
<td>Fracture Strain ($\varepsilon_{ruptSMA}$)</td>
<td>0.0175</td>
</tr>
<tr>
<td>Recovery Stress ($f_{SMA}$)</td>
<td>800 MPa (116,000 psi)</td>
</tr>
</tbody>
</table>

Due to the fact, that varying the amount of transverse confining material by altering the pitch/spacing throughout the column will greatly influence the overall behavior, this numerical example will be solved for four different scenarios. These four individual scenarios include:

1. A control case with no ties or transverse material (for comparison purposes only)
2. Then three other scenarios by varying the pitch/spacing of the transverse SMA material throughout the column height with values of:
   a. 18 inches
   b. 12 inches
   c. 4 inches.

Based upon these four different scenarios the SMA material in these configurations will give the column $P_s$ (volume of SMA/volume of confined concrete) values of 0, 0.0013, 0.002, and 0.006, respectively. It’s also important to mention that the 18 and 12 inch spaced scenarios, would be representative to an ordinary non-seismic column design, while the 4 inch spaced scenario represents a column strictly designed for high seismic activity.
7.3 Numerical Results

Based upon the mechanical properties listed above, the stress-strain relationship for the longitudinal steel that will be used within each numerical configuration can be viewed in Figure 7.1.

![Stress-Strain Relationship](image.png)

**Figure 7.1.** Stress-Strain Relationship for the Longitudinal Steel Based on the Given Information from the Numerical Example.

The stress-strain relationships of the confined portion for each of the four configurations are shown in Figure 7.2. From these plots the importance of the transverse ties and the potential benefit from the SMA material are clearly evident, as drastic differences can easily be seen in the peak stress, ultimate strain, and the slope/curvature in the post-peak behavior, even though the same predictive stress-strain equation by Xiao et al. was used for each scenario (Equations 4.4.3.2 and 4.4.3.4). Based upon this plot, it can be seen that for typical levels of SMA transverse material (18 and 12 inches) the increase in peak stress is only marginal when compared to the unconfined scenario of 4000 psi, as each case predicted a peak stress of 4346 psi and 4596 psi respectively. However, even with this relatively small amount of transverse SMA, the large increase in ductility is still very apparent, as the 18 inch, and 12 inch spaced SMA gave ultimate
strain values of 0.0243 and 0.0365 respectively compared to the unconfined scenario of 0.004. For both of these two cases, the increase in ductility can still be directly attributed to the additional transverse SMA material, although it’s important to note that at these wider spaced scenarios (lower values of $P_s$), the amount of transverse confining material is still too small to avoid a very narrow peak, due to the sharply decreasing slope/curvature that occurs in the stress-strain relationship once the peak stress location has been reached.

When looking at the scenario with the 4 inch spaced SMA spiral, which would most likely occur for a column designed in a highly seismic area, the potential benefit that the SMA material provides becomes even more predominant. With this large amount of transverse SMA, the peak stress and ultimate strain reached values of 5790 psi, and 0.0542 respectively, but the post-peak stress-strain relationship also now decreased at a much slower rate. Due to this lower post-peak slope that occurs within this scenario, it’s important to also notice the much larger stress levels that remain until ultimate failure eventually occurs.

The values obtained from the stress-strain relationships are shown in Figure 7.2 for both the peak stress and ultimate strain, are also summarized in Table 7.3 along with the percent increase in both these values, when compared with the unconfined scenario with no transverse SMA material.
Figure 7.2. Stress versus Strain Relationship for Varying Degrees of Transverse Reinforcement

<table>
<thead>
<tr>
<th>Transverse Pitch/Spacing</th>
<th>Peak Stress</th>
<th>Percent Increase</th>
<th>Ultimate Strain</th>
<th>Percent Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconfined (No SMA)</td>
<td>4000 psi</td>
<td>N.A.</td>
<td>0.004</td>
<td>N.A.</td>
</tr>
<tr>
<td>18 inches</td>
<td>4346 psi</td>
<td>8.7%</td>
<td>0.0243</td>
<td>507.5%</td>
</tr>
<tr>
<td>12 inches</td>
<td>4596 psi</td>
<td>14.9%</td>
<td>0.0365</td>
<td>812.5%</td>
</tr>
<tr>
<td>4 inches</td>
<td>5790 psi</td>
<td>44.8%</td>
<td>0.0542</td>
<td>1255%</td>
</tr>
</tbody>
</table>

Table 7.3. Stress versus Strain Values and Percent Increases from Figure 7.2.

With the stress-strain relationships of both the longitudinal steel and the confined concrete both known; the axial load versus displacement of the entire column can now be determined. The load versus displacement plots for each scenario is shown below in Figure 7.3, and for viewing ease, an enhanced plot showing the locations of the yielding of the longitudinal steel, and the spalling off of the unconfined concrete is shown in Figure 7.4. From these figures, the benefits due to the SMA confinement can again be easily noticed, especially when compared to the unconfined column.
Figure 7.3. Load versus Displacement Relationship for Varying Degrees of Transverse Reinforcement

Figure 7.4. Enhanced Load versus Displacement Relationship for Varying Degrees of Transverse Reinforcement
The unconfined concrete column with no SMA transverse material has a similar axial load capacity to both the 18 and 12 inch pitch scenarios, although it was slightly smaller with a peak load value of 1.482e6 lbs., but differs after this location. As with this scenario, once the axial load capacity is surpassed the longitudinal steel within the column begins to yield and the response curve sharply decreases until a strain of 0.004, or a displacement of 0.48 inches is reached and the column completely fails as the concrete crushes. With this column having no columns ties and therefore no confined concrete, the response is extremely brittle and therefore very dangerous.

The 18 and 12 inch spaced response curves are fairly similar in both shape and axial capacity, with the later response curve having a slightly higher axial load capacity after the longitudinal steel begins to yield and a more ductile response due to the slightly closer spaced transverse SMA. The 18 inch spaced scenario had a peak axial load of 1.543e6lbs, while the 12 inch spaced scenario had a slightly higher peak axial load of 1.571e6lbs due to the slightly higher peak stress value that was shown in Figure 7.2. Once the longitudinal steel began to yield, both response curves decreased with regards to the axial load, with a constant slope until spalling of the cover concrete occurred at a strain of 0.004 and a displacement of 0.48 inches was reached. After the clear cover spalled off, the available axial load sharply dropped while the response curves for both scenarios continued to decrease until eventually leveling off and ultimately failing at values of 2.92 and 4.38 inches, respectively for each scenario.

As expected, the 4 inch pitch SMA column, gave the best results overall. With this response, the axial load capacity at the time of the longitudinal steel first yield is about 1.641e6 lbs, and even continued to increase slightly to a peak axial load value of 1.653e6 lbs even though the longitudinal steel had begun to yield. At an axial load of 1.587e6 lbs a strain of 0.004 corresponding to a displacement of 0.48 inches was reached and the concrete cover again spalled off, but due to the close spacing of the transverse SMA the axial load capacity only dropped down to 1.45e6 lbs. After this drop occurred, it’s important to notice that the column was then able to maintain most of its axial load capacity. As this response curve only slightly decreased until the ultimate failure of the column occurred at a value of 1.137e6 lbs, which corresponded to a displacement of 6.51 inches.
A summary of the axial load versus displacement data is given in Table 7.4, where the peak axial load, ultimate displacement, and the percent increases of both values can be viewed.

<table>
<thead>
<tr>
<th>Transverse Pitch/Spacing</th>
<th>Peak Load (Lbs.)</th>
<th>Percent Increase</th>
<th>Peak Displacement (inch)</th>
<th>Percent Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconfined (No SMA)</td>
<td>1.482e6</td>
<td>N.A.</td>
<td>0.48</td>
<td>N.A.</td>
</tr>
<tr>
<td>18 inches</td>
<td>1.543e6</td>
<td>4.1%</td>
<td>2.92</td>
<td>508.3%</td>
</tr>
<tr>
<td>12 inches</td>
<td>1.571e6</td>
<td>6.0%</td>
<td>4.38</td>
<td>812.5%</td>
</tr>
<tr>
<td>4 inches</td>
<td>1.653e6</td>
<td>11.5%</td>
<td>6.51</td>
<td>1256.3%</td>
</tr>
</tbody>
</table>

Table 7.4. Load versus Displacement Values and Percent Increases from Figure 7.3.
Chapter 8: Conclusions and Suggestions for Future Research

8.1 Overall Conclusions

Multiple analytically and experimentally conclusions have been included and discussed within this project (Sections 6.6.1 and C.5.1). With this in mind, the conclusions on the active confinement of concrete members with SMAs are as follows:

1. The stress-strain response obtained under active confinement is significantly better than the response obtained under conventional passive confinement. This superior response is due to the fact that the active confinement method applies confining pressure independent of the lateral expansion of concrete, and therefore optimally utilizes the transverse confining mechanism.
2. Under constrained recovery situations, SMAs are very capable of developing large recovery stresses that can effectively be utilized to actively confine concrete members.
3. The SMA active confinement method has shown the ability to significantly increase the ductility and energy dissipation capacity of concrete members.
4. Due to the large variability that may exist within the post-peak stress-strain relationship of SMA actively confined concrete, the energy balance approach was determined to be the most suitable method to predict the ultimate strain location.
5. Xiao et al. (2010) peak stress and axial strain at peak stress equations, when incorporated into Mander et al. unified stress-strain model, have been observed to provide the most accurate stress-strain relationship for SMA actively confined concrete members. These equations (4.4.3.2 and 4.4.3.4) are the most accurate in predicting the sharply decreasing slope/curvature that occurs in the post-peak stress-strain relationship, and the ultimate strain location.

8.2 Future Research

More extensive experimental research investigations are needed before accurate assessments of SMAs and their use in enhancing the required ductility of reinforced concrete columns can be made. These investigations must first continue the essential
steps of varying the main parameters required for compressive testing of SMA confined concrete specimens, such as:

- Number of specimens tested
- Strength and size of the cylinders
- Amount of confining material
- Level of recovery stress applied
- Overall composition of the SMA

However, multiple more detailed items for future research have also been listed below, but it’s important to note that with this topic still in its infancy, this is only a small portion of topics that will eventually need to be further investigated, and they are as follows:

1. Determine what effect, the unconfined concrete strength has on the overall stress-strain behavior, and verify that the equations by Xiao et al. (2010) can effectively predict the relationship for all unconfined concrete compressive strengths.
2. Understand and approximate the amount of prestrain loss that occurs during confinement applications.
3. Large scale tests on real world columns.
4. Better understanding of the recovery stress that the SMA may still provide throughout the post-peak descending portion of the stress-strain relationship.
5. The concrete anchor screw method will need to be researched in greater detail in an attempt to better understand the resulting connection for application purposes.
6. SMA confined concrete members will need to be tested under dynamic loads.
7. Further research will be required comparing the behavior of SMA confined concrete, and other confinement methods, such as fiber reinforced polymers (FRPs).
References


Ansari, F., and Q. Li. (1998)."High-strength Concrete Subjected to Triaxial Compression." ACI Materials Journal 95, no. 6


Appendix A: Mander’s Model

A.1 Mander’s et al. (1988) Stress-Strain Model

Within Mander’s model, the main equation which determines the relationship between axial compressive stress \( f_c \) and axial strain \( \varepsilon_c \) is given by the following axial stress-strain equation which was originally proposed by Popovics (1973):

\[
f_c = \frac{f_{cc}' \varepsilon_c}{\varepsilon_{cc}} \cdot r \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right)^r
\]

(A.1.1)

where \( f_{cc}' \) = compressive strength (peak stress) of confined concrete and will be defined later, \( \varepsilon_c \) = longitudinal compressive concrete strain, and \( \varepsilon_{cc} \) = the strain at maximum concrete stress \( f_{cc}' \) and can be found with the following equation from experimental work by Gerstle et al. (1979) which was based on a simple relationship proposed by Richart et al. (1928):

\[
\varepsilon_{cc} = \varepsilon_{co} \left[ 1 + 5 \left( \frac{f_{cc}'}{f_{co}} - 1 \right) \right]
\]

(A.1.2)

where \( f_{cc}' \) and \( \varepsilon_{co} \), equals the unconfined concrete strength and the corresponding strain, respectively. Generally, \( \varepsilon_{co} = 0.002 \) can be assumed based on the results from Richart et al. (1928), and is used within Mander’s et al. model for unconfined concrete.

The value of \( r \) in Equation A.1.1 can be calculated with Equation A.1.3 and the use of Equation A.1.4:
\[
  r = \frac{E_c}{E_c - E_{sec}}
\]

(A.1.3)

\[
  E_c = 57000 \sqrt{f_{co}} \text{ psi}
\]

(A.1.4)

where \( E_c \) = tangent modulus of elasticity of the concrete, and \( E_{sec} \) = secant modulus of confined concrete at peak stress, and can be found using the following equation:

\[
  E_{sec} = \frac{f'_{cc}}{\varepsilon_{cc}}
\]

(A.1.5)

An approach similar to the one used by Sheikh and Uzumeri (1980) was adopted to determine the effective lateral confining pressure on the concrete section. The maximum transverse pressure from the confining steel can only be exerted effectively on the part of the concrete core where the confining stress has fully developed due to arching action that is assumed to occur between the levels of transverse circular hoop reinforcement. The area of ineffectively confined concrete will be largest midway between the levels of the transverse reinforcement, and the area of effectively confined concrete core \( (A_e) \) will be the smallest. Figure A.1 below, visually shows this effective lateral confining pressure effect for a circular confined section.
The area of the confined concrete is the area of the concrete within the center lines of the perimeter spiral or hoop, $A_{cc}$. Since $A_e < A_{cc}$, the effective lateral confining pressure is defined with the following equation:

$$f'_l = \frac{1}{2} \cdot k_e \cdot \rho_s \cdot f_{yh}$$

(A.1.6)

where $f_{yh}$ = yield strength of the transverse reinforcement, and $\rho_s$ = the ratio of the volume of transverse confining steel to the volume of confined concrete core, which can be calculated from:
\[ \rho_s = \frac{4 \cdot A_{sp}}{d_s \cdot s} \]  
(A.1.7)

where \( A_{sp} \) = area of transverse reinforcement bar, \( d_s \) = diameter of spiral between bar centers, and \( s \) = center to center spacing of pitch of spiral or circular hoop. \( k_e \) = confinement effectiveness coefficient which in general equals:

\[ k_e = \frac{A_e}{A_{cc}} \]  
(A.1.8)

The coefficient \( k_e \) reflects the geometrical effectiveness of the longitudinal and transverse reinforcement in confining the concrete and depends on the shape of the section and arrangement of the transverse reinforcement. With this in mind, Equation A.1.8 simplifies for circular hoops (A.1.9), and circular spirals (A.1.10) respectively to become:

\[ k_e = \frac{1 - \left( \frac{s'}{2 \cdot d_s} \right)^2}{1 - \rho_{cc}} \]  
(A.1.9)

\[ k_e = \frac{1 - \frac{s'}{2d_s}}{1 - \rho_{cc}} \]  
(A.1.10)

where \( s' \) = the clear vertical spacing between spiral or hoop bars, and \( \rho_{cc} \) = ratio of area of longitudinal reinforcement to area of core of section
At this point the confined compressive strength \((f'_{cc})\) can now be calculated using the effective lateral pressure \((f'_l)\) and the unconfined concrete compressive strength \((f'_{co})\). To develop this equation Mander et al. used the “five-parameter” multiaxial failure surface equation given by William and Warnke failure criterion (1975):

\[
f'_{cc} = f'_{co} \left( -1.254 + 2.254 \sqrt{1 + \frac{7.94 f'_l}{f'_{co}} - 2 \frac{f'_l}{f'_{co}}} \right)
\]

(A.1.11)

A.2 Ultimate Concrete Compressive Strain

With the value of the confined compressive strength \((f'_{cc})\) now known, the entire stress strain curve can be calculated using Equation A.1.1 with varying longitudinal compressive concrete strain values up to the ultimate concrete compressive strain \((\varepsilon_{cu})\), which still needs to be evaluated. Scott et al. (1982) proposed that the ultimate concrete compressive strain be defined as the longitudinal strain at which the first hoop fracture occurs, since that strain can be regarded as the end of the useful region of the stress-strain curve for the confined concrete core.

Using Scott et al (1982) ultimate concrete compressive strain concept, which has been widely accepted by almost all confinement models, Mander et al. (1984) proposed a rational method for predicting the longitudinal concrete compressive strain at first hoop fracture based on an energy balance approach. With this approach, the additional ductility available from concrete confinement is due to the energy stored in the transverse reinforcement. As discussed earlier, the area under stress-strain curve represents the total strain energy per unit volume required to “fail” the concrete. Therefore, the increase in strain energy at failure resulting from the confinement (shown as the shaded area in Figure A.2) can only be provided by the strain energy capacity of the confining material as it yields in tension due to the lateral expansion of the concrete.
By equating the ultimate strain energy capacity of the confining reinforcement per unit volume of concrete core ($U_{sh}$) to the difference in area between the confined ($U_{cc}$) and the unconfined ($U_{co}$) concrete stress-strain curves, plus additional energy required to maintain yield in the longitudinal steel in compression ($U_{sc}$), the longitudinal concrete compressive strain corresponding to hoop fracture can be calculated, with the following equation:

$$U_{sh} = U_{cc} + U_{sc} - U_{co}$$

(A.1.12)

Equation A.1.12 can then be expanded further and written as:

$$\rho_s A_{cc} \int_0^{\varepsilon_{sf}} f_s d\varepsilon_s = A_{cc} \int_0^{\varepsilon_{cu}} f_c d\varepsilon_c + \rho_{cc} A_{cc} \int_0^{\varepsilon_{cu}} f_{sl} d\varepsilon_c - A_{cc} \int_0^{\varepsilon_{sp}} f_c d\varepsilon_c$$

(A.1.13)
where \( \rho_s \) = the ratio of the volume of transverse confining steel to the volume of confined concrete core; \( A_{cc} \) = area of concrete core; \( f_s \) and \( \varepsilon_s \) = stress and strain in transverse reinforcement; \( \varepsilon_{sf} \) = fracture strain of transverse reinforcement; \( f_c \) and \( \varepsilon_c \) = longitudinal compressive stress and strain in concrete; \( \varepsilon_{cu} \) = ultimate concrete compressive strain; \( P_{cc} \) = ratio of volume of longitudinal reinforcement to volume of concrete core; \( f_{sl} \) = stress in longitudinal reinforcement; and \( \varepsilon_{sp} \) = spalling (chipping or flaking) strain of unconfined concrete.

The Equation A.1.13 was then simplified with results from experimental data obtained from research by Mander et al. (1984) on varying diameters and grades of steel. The data from this research suggested that the first term on the left-hand side of Equation A.1.13 shown below as Equation A.1.14, which designates the total area under the stress-strain curve for the transverse reinforcement up to the fracture strain (\( \varepsilon_{sf} \)) is effectively independent of bar size or yield strength, and may be taken (within ±10%) as

\[
\int_{0}^{\varepsilon_{sf}} f_s \, d\varepsilon_s = U_{sf} = 110 \text{ MPa} \quad (1 \text{ MPa}=145 \text{ psi})
\]

where \( U_{sf} \) = the area beneath the stress-strain curve from zero load to fracture of the transverse confining material. It is important to note that for the steel tested by Mander et al. (1984) the fracture strain ranged between 0.24 and 0.29.

The last term in Equation A.1.13 represents the area under the stress strain curve for unconfined concrete. This required unconfined concrete area designated from the stress-strain curve has been greatly studied by multiple investigators who tested numerous plan concrete specimens throughout the past century, and may be accurately approximated as

\[
\int_{0}^{\varepsilon_{sp}} f_c \, d\varepsilon_c = 0.017 \sqrt{f_{co}} \text{ MPa} \quad (1 \text{ MPa}=145 \text{ psi})
\]
Therefore, Equation A.1.13 may be written as follows

\[ 110\rho_s = \int_0^{\varepsilon_{cu}} f_c d\varepsilon_c + \int_0^{\varepsilon_{cu}} f_{sl} d\varepsilon_c - 0.017\sqrt{f'_{co}} \text{ MPa} \]

(A.1.16)

The ultimate strain of confined concrete can now be evaluated using Equation A.1.16 along with the results for the longitudinal concrete stress \( f_c \) from Equation A.1.1 and the stress in the longitudinal steel reinforcement \( f_{sl} \) as a function of longitudinal strain.
Appendix B: Notations

B.1 Notations

The following symbols are use in this thesis:

\[ A_{cc} = \text{area of core within center lines of perimeter spiral or hoops excluding area of longitudinal steel;} \]
\[ A_e = \text{area of effectively confined core concrete;} \]
\[ A_{sma} = \text{cross-sectional area of SMA wire;} \]
\[ A_{sp} = \text{area of spiral bar;} \]
\[ d = \text{diameter of the cylinder;} \]
\[ d_s = \text{diameter of spiral;} \]
\[ E_c = \text{modulus of elasticity of concrete;} \]
\[ E_1 = \text{experimental value;} \]
\[ E_{sec} = \text{secant modulus of confined concrete at peak stress;} \]
\[ f_c = \text{longitudinal concrete stress;} \]
\[ f'_{cc} = \text{compressive strength (peak stress) of confined concrete;} \]
\[ f'_{co} = \text{compressive strength of unconfined concrete;} \]
\[ f_l = \text{lateral confining stress on concrete from transverse reinforcement;} \]
\[ f'_{l} = \text{effective lateral confining stress;} \]
\[ f_s = \text{stress in the transverse reinforcement;} \]
\[ f_{sl} = \text{stress in longitudinal reinforcement;} \]
\[ f_{sma} = \text{recovery stress of the SMA wire;} \]
\[ f_y = \text{yield stress;} \]
\[ f_{yh} = \text{yield strength of transverse reinforcement;} \]
\[ f_u = \text{ultimate rupture stress;} \]
\[ k_e = \text{confinement effectiveness coefficient;} \]
\[ n = \text{number of data points;} \]
\[ P_i = \text{predicted value;} \]
\[ s = \text{spiral spacing or pitch;} \]
\[ s' = \text{clear spacing between spiral or hoop bars;} \]
\[ U_{cc} = \text{strain energy stored by confined concrete per unit volume;} \]
\[ U_{co} = \text{strain energy stored by unconfined concrete per unit volume;} \]
\[ U_{sc} = \text{strain energy stored by longitudinal} \]
\[ U_{sf} = \text{area beneath stress-strain curve from zero load to fracture;} \]
\[ U_{sh} = \text{strain energy capacity of transverse confining material per unit volume of concrete core; } \]

\[ \mathcal{E}_c = \text{longitudinal concrete strain; } \]

\[ \mathcal{E}_{cc} = \text{strain at maximum concrete stress } f'_{cc}; \]

\[ \mathcal{E}_{co} = \text{strain at maximum stress } f'_{co} \text{ of unconfined concrete; } \]

\[ \mathcal{E}_{cu} = \text{ultimate concrete compressive strain, defined as strain at first hoop fracture; } \]

\[ \mathcal{E}_s = \text{strain in the transverse reinforcement; } \]

\[ \mathcal{E}_{sf} = \text{fracture strain of the transverse reinforcement; } \]

\[ \mathcal{E}_{sh} = \text{strain at the onset of strain hardening; } \]

\[ \mathcal{E}_{sp} = \text{spalling strain of unconfined concrete; } \]

\[ \mathcal{E}_{su} = \text{strain at maximum stress; } \]

\[ \mathcal{E}_{rupture} = \text{strain at rupture; } \]

\[ \rho_{cc} = \text{ratio of volume of longitudinal reinforcement to the volume of the concrete core of the section; and } \]

\[ \rho_{s} = \text{ratio of volume of transverse confining material to volume of confined concrete core.} \]
Appendix C: Experimental Correlations with SMA Confinement Models

C.1 Concrete Batch #1

The descending average curves that are displayed within this section were directly taken from the experimentally obtained data that was previously displayed in section 6.2.2.1 and 6.2.2.2.

C.1.1 SMA Confined 1/3rd inch Pitch Specimens

Using the peak stress information for the unconfined concrete cylinders of concrete batch #1 (3068 psi), the stress-strain relationship was then able to be determined for three selected models (Mander, Xiao, and Moghaddam) in an attempt to compare the assessed post-peak behavior, with the experimentally determined average results. The plot displaying the formulated results and the descending average curve is shown in Figure C.1.
From Figure C.1, it can be seen that based on the data obtained from this batch of cylinders the active confinement model was fairly accurate in predicting the increase in both peak strength and ductility due to the SMA confinement. First off, from this plot it can be seen that the peak strength of the descending average is fairly similar to the max stress assessed by the three models shown. While this result is encouraging, comparing the peak stress of the experimental results and the analytical model was never a key goal of this experiment, due to the fact that for any solid conclusions to be made in regards to the increase in peak strength would require more specimens to be tested, as this value can greatly vary throughout cylinders from even the same batch. For example, with the experimental results that were obtained it’s very possible that the average strength of the three SMA confined specimens was simply slightly higher than the 3068 psi compressive strength shown in the unconfined tested specimens discussed earlier, skewing the results.

In regards to increase to the overall length of the descending curve or increase in ductility, it also appears that the energy balanced approach used within the active confinement model was also fairly accurate, as the results again appear comfortably
within an acceptable range. However, it’s important to note that a direct comparison of
the experimentally obtained and analytically predicated ultimate strain values is very
difficult to attain, due to the energy balanced approach taking into account the area below
the stress-strain representation, to determine the ultimate failure location.

Due to the discrepancy in overall location between the formulated results from
the active confinement model and the average descending curve from the experimental
testing, reaching any solid conclusions about which models equations best formulated the
descending curve with the results as they are shown in Figure C.1 would be difficult at
best, and most likely ineffective in general. With that being said, important information
about the behavior of the descending curve can still be reached by simply readjusting the
peak location of the experimentally determined average descending branch to align with
the peak stress location of each of the three formulated stress-strain relationships. With
the experimentally determined average curve correctly lined up at the peak locations of
each of the three possible formulated stress-strain relationships selected, the overall
behavior of the descending branch could then easily be compared, and the differences can
be numerical quantified. At this point, it is very important to mention that physically
adjusting the peak location of the descending average will not alter in any way either the
slope or the curvature of the experimentally determined average stress-strain curve and
therefore will not alter any possible conclusion that may eventually be made from these
results.

Once the peak location of the experimentally determined descending average has
been adjusted to align with the analytically formulated peaks, it would then be possible to
visually compare and numerically quantify the stress-strain relations. To numerically
quantify the stress-strain relationships of the curves and compare the accuracy of the
proposed stress-strain models with the experimentally determined results, a root-mean-
square-deviation (RMSD) was applied to each curve. The RMSD method was used
because it is a good measure of accuracy and is frequently used to measure the difference
between predicted values by a model or an estimator and the values actually observed.
The RMSD method was used to get a single measure of predictive power for the
descending stress-strain relationship, and is defined by Equation C.1 that follows. With
the RMSD method a smaller value implies a more accurate representation, and each
RMSD value relating the two descending stress-strain relationships is shown within the following figures.

\[
RMSD = \sqrt{\frac{\sum_{i=1}^{n} (P_i - E_i)^2}{n}}
\]

(C.1)

Where \( E \) = experimental value; \( P \) = predicted value; and \( n \) = number of data points.

While this RMSD method will be helpful in determining the best representation for SMA confined concrete specimens, this method also has three major limitations that need to be kept in mind. These limitations include:

1. Due to the energy balance approach being used to determine the ultimate strain location, the length of the descending curve that is able to be compared will always be the smallest for Mander’s model. This behavior is due to the fact that Mander’s model always formulates the highest post peak stress-strain behavior of all the models and therefore the area under the stress-strain relationship (energy absorbed) will be greater for this model at any particular level of strain causing an earlier predicted ultimate failure. With this same reasoning in mind, the opposite will also be true for Moghaddam’s model, where this model will always predict the largest ultimate strain values due to the fact that it always formulates the lowest post peak stress-strain relationship, and therefore will take a higher ultimate strain value in order for the energy capacity of the SMA material to be exceeded. Due to these reoccurring behavior trends, the length of the descending curve (number of values) formulated by Mander’s model, will determine the amount of values that are used to determine the RMSD value for the other models. This is important to note and essentially required for comparison purposes, since naturally the formulations appear to have a much more difficult time assessing the stress-strain representation further away from the peak stress location.
2. This method doesn’t take into account whether the models being compared overestimates or underestimates the experimental results. Due to the conservative approach that is often required in structural engineering, ideally this method would penalize models that overestimate the stress-strain representation and possibly give a false sense of security for the structural response of the confined member.

3. This method computes the whole region that is being compared to a single value, and therefore doesn’t take into account where within the stress-strain curve the prediction is accurate. In an ideal situation the stress-strain representation at lower strain values, would be weighed stronger than the prediction at larger strain values, as the lower strain values are much more likely to be reached in real world situations.

As shown in Figure C.2, when Mander’s analytical model and the experimental descending curves are aligned so the peak stress locations occur at the same location, Mander et al. prediction was fairly accurate, although it’s important to note that this prediction overestimated the stress-strain representation throughout the curve. This result was expected as Mander et al. original model uses predictive equations designed for passive confinement and therefore should over predict the post peak region being compared for the actively confined SMA specimens due to not accounting for the yielding of the confining material directly after the peak location is reached. Mander’s equations also underestimated the ultimate strain of the SMA confined specimens by predicting a value of 0.0143 which was approximately 30% less than the 0.0186 value experimentally obtained. When numerically comparing Mander’s model with the experimental results, a RMSD value of 356 was obtained, which corresponded to the second most accurate prediction in this category.
Figure C.2. Stress versus Strain Relationships, Based on the Model by Mander et al. (1988) and the Experimentally Determined Adjusted Descending Average for 1/3\textsuperscript{rd} inch Pitch SMA (Recovery Stress 800MPa), Confined Batch #1 Concrete Cylinders.

In Figure C.3, the experimentally obtained descending average is compared with the Xiao’s analytical model. This predictive relationship from Xiao et al. was the most accurate in predicting the descending curve and had a RMSD value of the 320. From this figure it can easily be noticed that Xiao’s equations were fairly accurate in predicting the descending stress-strain relationship directly after the peak location was reached, although this prediction did underestimate the stress-strain curve as it got closer to the ultimate strain location, which did increase the RMSD value. When comparing the ultimate strain location and overall increase in ductility, Xiao prediction was very accurate as it formulated an ultimate strain value of approximately 0.0186, which resulted in only a 3\% overestimation of the experimental value of 0.018.
Figure C.3. Stress versus Strain Relationships, Based on the Model by Xiao et al. (2010) and the Experimentally Determined Adjusted Descending Average for 1/3rd inch Pitch SMA (Recovery Stress 800MPa), Confined Batch #1 Concrete Cylinders.

Shown in Figure C.4 are the experimentally obtained descending average curve and the relationship based on Moghaddam et al. analytical model. Moghaddam equations assessed the sharpest post-peak decrease and the lowest stress-strain relationship of the three models. When comparing the descending average and the predictive relationship, it can be seen that the equations used by Moghaddam are very accurate directly after the peak stress is reached as it almost matches the experimental results exactly. However, it’s important to note that this model severely underestimated the stress-strain representation towards the ultimate failure location once higher strain values were reached. Moghaddam’s model also greatly overestimated the location of the ultimate strain by formulating an ultimate strain value of approximately 0.0257, which resulted in a 31% overestimation of the experimental value of 0.0177. This large overestimation is due to the fact that the descending curve formulated by Moghaddam displays such a sharply decreasing behavior throughout the curve, and therefore takes longer to fail based upon the smaller amount of area below the stress-strain curve. This ultimate strain location formulated, appears to signify that perhaps for Moghaddam’s equations to be
useful, an ultimate strain location will need to be estimated by some other method as the energy balanced approach will almost always predict an unreasonably large ultimate strain value. Moghaddam et al. equations obtained a RSMD value of 526 which was highest of the three models.

Figure C.4. Stress versus Strain Relationships, Based on the Model by Moghaddam et al. (2010) and the Experimentally Determined Adjusted Descending Average for 1/3rd inch Pitch SMA (Recovery Stress 800MPa), Confined Batch #1 Concrete Cylinders.

C.1.2 SMA Confined 1/5th inch Pitch Specimens

Just like before using the peak stress information for the unconfined concrete cylinders of concrete batch #1 (3068 psi), the formulated stress-strain relationship was then able to be determined for three selected models (Mander, Xiao, and Moghaddam) in an attempt to compare the formulated post-peak behavior with the experimentally determined average results. The plot displaying the formulated results from the model and the descending average curve (once it was again adjusted to align with the peak theoretical results) is shown in Figure C.5.
As Figure C.5 shows, when the formulated relationship by Mander et al. and the experimental descending curves are aligned so the peak stress locations occur at the same location, it can again be seen that Mander’s et al. prediction overestimated the available strength throughout the descending portion of the stress-strain relationship. As stated before, this result will be expected as Mander’s et al. original model uses predictive equations designed for passive confinement and therefore should over predict the post peak region for SMA actively confined specimens, due to not accounting for the yielding of the confining material once the peak location is reached. Mander’s representation again severely underestimated the ultimate strain location as these equations with the use of the energy balance approach evaluated an ultimate strain value of 0.0197, which underestimated the experimental value of 0.0301 by approximately 52%. It is however important to mention, that when numerically comparing Mander’s model with the experimental results, a RMSD value of 488 was obtained, which signified the most accurate representation of the three formulations even with the constant overestimation that occurred throughout.
The predictive relationship from Xiao et al. was the second most accurate in predicting the descending curve and had a RMSD value of 534. From this figure, it can be seen that equations from Xiao do a fairly accurate job at predicting the sharp decrease directly after the peak stress is reached, although it does slightly overestimate this portion. It can also be seen, that just like with the 1/3\textsuperscript{rd} inch pitch SMA confined specimens this stress-strain relationship begins to overestimate the available strength as it approaches the ultimate failure location. When looking strictly at the increase in ductility and ultimate strain location, Xiao formulations with the energy balance approach was again accurate, as it evaluated an ultimate strain value of 0.0273, which only underestimated the experimental results of 0.029 by approximately 6%.

As typical, Moghaddam et al. assessed the steepest descending curve and the lowest post-peak stress-strain relationship of the three formulations. This relationship did a really good job predicting the initial decrease that occurs after the peak stress was reached. However, in the model by Moghaddam, the estimated relationship again basically follows this initial decrease until the ultimate strain value is reached, which differs greatly from the experimental descending average that tends to eventually level out as the concrete cylinder crushes. Due to this difference in relationships, it appears that Moghaddam’s prediction will continually underestimate the available compressive strength towards the last 2/3\textsuperscript{rd}s of the stress-strain curve. Moghaddam’s representation was stopped at a strain value of 0.035, as this prediction with the energy balance approach predicts unrealistically high ultimate strain values, and therefore no ultimate strain comparison will be made. Moghaddam et al. equations obtained a RMSD value of 900 even with the accurate early representation due to the fact that towards the end of the prediction this model was so far off from the experimental results.

C.2 Concrete Batch #2

The descending average curves that are displayed within this section were directly taken from the experimentally obtained data that was previously displayed in section 6.3.2.1 and 6.3.2.2.
C.2.1 SMA Confined 1/3\textsuperscript{rd} inch Pitch Specimens

Using the peak stress information for the unconfined concrete cylinders of concrete batch #2 (4563 psi), the formulated stress-strain relationship was then again able to be determined for the three selected models (Mander, Xiao, and Moghaddam) in an attempt to compare the formulated post-peak behavior with the experimentally determined average results. The plot displaying the formulated results and the experimentally determined descending average curve after it was adjusted to align with theoretical peaks is shown in Figure C.6.

![Figure C.6. Stress versus Strain Relationships, Formulated by Select Models and the Experimentally Determined Descending Average for 1/3\textsuperscript{rd} inch Pitch SMA (Recovery Stress 800MPa), Confined Batch #2 Concrete Cylinders.](image)

From the stress-strain plots shown in Figure C.6 it appears that all of the predictive relationships are much steeper and do not match when compared to the experimental average. This slightly inaccurate performance is most likely due to the odd occurrence of the unconfined cylinders having an average peak stress higher than the average 1/3\textsuperscript{rd} inch pitch SMA confined cylinder, which goes directly against all expectations and obtained results thus far. Like stated earlier from the three 1/3\textsuperscript{rd} inch
When comparing the relationship formulated by Mander et al. and the experimental descending curve, once it was aligned so the peak stresses occurred at the same location, it can again be seen that this time Mander’s et al. prediction was very accurate in predicting the descending curve. Although, it’s important to mention that the prediction by Mander usually has the shallowest decreasing slope of the three models, and the likely erroneous and slightly higher unconfined concrete strength inputted into the model, would appear to both work towards improving this prediction while decreasing the accuracy of the other two. Nevertheless, with this scenario, Mander’s prediction had a RMSD value of only 274, although it again slightly overestimated the relationship directly after the peak stress location and underestimated the curve towards the ultimate failure. While this prediction was very accurate in the portion of the prediction shown, it again did a fairly poor job of predicting the ultimate strain location, as the value of 0.0106 that was evaluated by the model, underestimated the experimental result of 0.0216 by approximately 104%.

For this scenario, the predictive relationship from Xiao et al. obtained a very large RMSD value of 1046. Xiao’s formulated relationship severely underestimated the experimentally obtained average, almost completely throughout the prediction. This much larger than normal RMSD value is again in all probability due to the apparently erroneous unconfined concrete strength used within the model. With that said, it again appears that Xiao prediction again did a fairly good job when visually comparing the length of the descending curve, although the ultimate strain value appears to be slightly off due to the sharply decreasing curve that was formulated. With this in mind, Xiao
assessed an ultimate strain value of 0.0162, which underestimated the experimentally obtained ultimate strain value of 0.0211 by 30.2%.

Moghaddam et al. again formulated the lowest post-peak stress-strain relationship of the three models. When comparing the experiential descending average with this prediction it can be seen the Moghaddam again strongly underestimates the stress-strain relationship of the entire region compared. While this underestimating behavior has been shown in all of his formulations, this behavior appears much more severe due to the perceived unconfined concrete strength error, giving this prediction another high RMSD value of 1332. Moghaddam’s model also again overestimated the location of the ultimate strain by assessing an ultimate strain value of approximately 0.0282, which resulted in a 26% overestimation of the experimental value of 0.0209.

C.2.2 SMA Confined 1/5th inch Pitch Specimens

Just like before using the peak stress information for the unconfined concrete cylinders of concrete batch #2 (4563 psi), the formulated stress-strain relationship was then able to be determined for three selected models (Mander, Xiao, and Moghaddam) in an attempt to compare the post-peak behavior with the experimentally determined average results. The plot displaying the formulated results and the adjusted descending average curve is shown in Figure C.7.
From the relationships shown in Figure C.7, it can again be seen that Mander et al. prediction overestimates the stress-strain relationship when compared to the experimentally obtained descending average directly after the peak location is reached, but was able to very accurately predict the relationship towards the end of the prediction. For this scenario the stress-strain relationship formulated by Mander et al. obtained a RMSD value of 511, due to the inaccurate ability to account for the confining material yielding directly at the peak location. This representation, once again also did a fairly poor job at predicting the ultimate failure location, as the value of 0.0151 that was assessed by the model, underestimated the experimental result of 0.0217 by approximately 44%.

The predictive relationship from Xiao for this scenario obtained a RMSD value of 1016. Xiao’s formulated relationship was fairly accurate initially in predicting the immediate decrease that occurred near the peak location, it did again however severely underestimated the stress-strain relationship towards the ultimate strain location. Xiao prediction of the ultimate failure location, which is often the best, was slightly off in this
case with an ultimate strain value of 0.0283 being assessed, or approximately 25% larger than the experimental value obtained of 0.0211.

Again Moghaddam’s equations accurately formulated the relationship directly after the peak stress was reached, but it again severely underestimates the available strength towards the end of the region being compared. Due to this severe underestimation that occurred within the last 2/3rds of this prediction, Moghaddam et al. equations again obtained the highest and therefore least accurate prediction with a RMSD value of 1518.

C.3 Concrete Batch #3

The descending average curves that are displayed within this section were directly taken from the experimentally obtained data that was previously displayed in section 6.4.2.1 and 6.4.2.2.

C.3.1 SMA Confined 1/3rd inch Pitch Specimens

Using the peak stress information for the unconfined concrete cylinders of concrete batch #3 (3693 psi), the formulated stress-strain relationship was then able to be determined for three selected models (Mander, Xiao, and Moghaddam) in an attempt to compare the formulated post-peak behavior with the experimentally determined average results. The plot displaying the formulated results and the adjusted descending average curve is shown in Figure C.8.
From Figure C.8 it can again be seen that Mander’s et al. prediction overestimates the stress-strain relationship when compared to the experimentally obtained descending average throughout the entire region being compared. However, for this scenario the stress-strain relationship formulated by Mander et al. obtained a RMSD value of 364, which corresponded to the most accurate prediction. This prediction by Mander, once again shows the severe underestimation of the ultimate strain location, as the Mander equations with the use of the energy balanced approach assessed an ultimate strain value of 0.0125, which underestimated the experimental result of 0.0246 by approximately 96.8%.

The predictive relationship by Xiao et al. was the second most accurate of the three models with a RMSD value of 398, which was only slightly higher than the RSMD value previously determined from Mander’s representation. While the RSMD value is higher in this comparison, it’s important to note that this equation was very accurate in predicting the initial post-peak relationship and almost perfectly matched the initial slope of the decreasing curvature.
While this prediction was initially extremely accurate, this method again underestimated the stress-strain relationship the closer it got to the ultimate failure location. While this method eventually became less accurate towards the end of the prediction, it should be noted that it again did a fairly accurate job in predicting the ultimate failure location by assessing a value of 0.0171, which underestimated the experimental result of 0.0241 by approximately 40.9%.

Moghaddam et al. model again did a decent job in predicting the relationship directly after the peak stress was reached, but it again severely underestimates the available strength towards the end of the region being compared. Due to the severe underestimation that occurs with this prediction towards the end of region being compared, Moghaddam’s model again obtained the highest and therefore least accurate prediction with a RMSD value of 632. This prediction was however very accurate in predicting the ultimate strain location with an assessed value of 0.0249, which only overestimated the experimentally obtained result of 0.0237, or by 4.8%.

C.3.2 SMA Confined 1/5th inch Pitch Specimens

Just like in the previous sections, using the peak stress information for the unconfined concrete cylinders of concrete batch #3 (3693 psi), the formulated stress-strain relationship was then able to be determined for three selected models (Mander, Xiao, and Moghaddam) in an attempt to compare the formulated post-peak behavior with the experimentally determined average results. The plot displaying the formulated results and the adjusted descending average curve is shown in Figure C.9.
From Figure C.9 it can again be seen that Mander’s et al. prediction overestimates the stress-strain relationship when compared to the experimentally obtained descending average throughout the entire region being compared. Mander’s equations, again does a poor job accounting for the sharp decrease in available strength that occurs directly after the peak location is reached. This prediction was however, fairly accurate at predicting the ultimate strain location with an assessed value of 0.0174, underestimating the experimental obtained value of 0.022 by only 26.4%. For this scenario the stress-strain relationship formulated by Mander et al. obtained a RMSD value of 587, which corresponded to the second lowest RMSD value.

The predictive relationship from Xiao et al. for this scenario was the most accurate of the three models with a RMSD value slightly better than the curve formulated by Mander et al. with a corresponding value of 310. Xiao’s model was accurate throughout the relationship, although towards the end of the region being compared, this relationship did slightly underestimate the available strength of the confined specimens. This prediction also did a suitable job in predicting the ultimate strain location by
overestimating the experimental value of 0.021 by 20.8% with a value of 0.0265 being assessed.

Moghaddam et al. model again severely underestimated the available strength towards the end of the region being compared, and again obtained the highest and therefore least accurate prediction, with a RMSD value of 833. This model with the use of the energy balanced approach, once again severely overestimated the ultimate strain location as well, and therefore was forced to be stopped at a strain value of 0.0035, with no ultimate strain comparison able to be made.

C.4 RMSD Values Obtained

As with any situation utilizing the RMSD statistical method, there is no real way to assign a level of accuracy to a single obtained value, and therefore the values obtained are best used to show accuracy when compared to other results from similar situations. With that being said, Table C.1 below summarized the RMSD values calculated after comparing the experimental averages with each of the three models formulated post peak relationship from each concrete batch/scenario tested within this experimental investigation.

From Table C.1, it’s easy to see that the models formulated by both Mander and Xiao were the most accurate, giving low and very similar RMSD values throughout, especially when remembering that an error is perceived to have occurred within concrete batch #2. However, it is important to restate, that the RMSD values obtained was just one of the multiple methods used to determine the most accurate stress-strain relationship for SMA confined concrete specimens, and this method does has some major limitations that were previously discussed in Section C.1.1. With this in mind, the RMSD values can really only be used to numerically display how inaccurate the model formulated by Moghaddam was, as this model received the highest values throughout this work.
<table>
<thead>
<tr>
<th>Model</th>
<th>Concrete Batch #1</th>
<th>Concrete Batch #2</th>
<th>Concrete Batch #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mander et al.</td>
<td>1/3rd 356 488</td>
<td>1/5th 274 511</td>
<td>1/5th 364 587</td>
</tr>
<tr>
<td>Xiao et al.</td>
<td>1/5th 320 534</td>
<td>1/5th 1046 1016</td>
<td>1/5th 398 310</td>
</tr>
<tr>
<td>Moghaddam et al.</td>
<td>1/5th 526 900</td>
<td>1/5th 1332 1518</td>
<td>1/5th 632 833</td>
</tr>
</tbody>
</table>

Table C.1. Summary of RMSD Values Obtained

C.5 Concluding Remarks

The following conclusions can be drawn from the experimental investigation and assessment of the analytical models with regard to their correlations with the experimental data. When reading these conclusions it’s again important to remember that due to the material and experimental limitations, only some significant trends can be identified. With this in mind, these conclusions are generally based on the overall behavior that tends to be prevalent throughout the experimental work, and obviously more research is needed to arrive at some appropriate numerically driven assessments.

C.5.1 Assessment of the Confinement Model and Experimental Correlations

1. The active confinement model used in this thesis appears to be reasonable in predicting the stress-strain relationship of SMA confined specimens, as long as four key parameters are known:
   a. Unconfined concrete strength.
   b. SMA ultimate tensile strength.
   c. SMA fracture strain.
   d. Ratio of the volume of transverse SMA to the volume of the confined concrete core.

2. The energy balanced approach used within the active confinement model does a fairly accurate job in predicting the ultimate strain location of the SMA confined concrete cylinders. However, due to the fact that this method only takes into account the area under the stress-strain relationship, the equation
that is ultimately used to predict this relationship will greatly affect this location.

3. Moghaddam’s Model:
   a. Often accurately matched the initial decrease in the stress-strain relationship that occurred directly after the peak location was reached, but severely underestimated the post-peak stress-strain behavior thereafter.
   b. This prediction could be very accurate if the concrete specimens would fail ideally, but due to the unpredictable crushing nature of concrete, the experimental stress-strain relationships were often drastically different from an ideal situation.
   c. The least accurate of the three models compared.

4. Mander’s Model:
   a) Consistently overestimated the stress-strain relationship throughout the post-peak region, due to the fact that this model does not take into account the yielding of the SMA confining material.
   b) The energy balance approach often severely underestimated the ultimate strain location of the SMA confined specimens.
   c) The descending portion of this stress-strain formulation was often less accurate towards the peak stress location, and more accurate towards the ultimate strain region due to the slope and curvature that this model assessed.
5. Xiao’s Model:
   a) Often accurately matched the initial descending curve in the stress-strain relationship that directly occurred after the peak location was reached.
   b) Although it did tend to underestimate the available strength towards the ultimate strain location this method was at least correctly on the conservative side.
   c) With the use of the energy balance approach this formulation was consistently the most accurate in predicting the ultimate strain location.

6. Based upon the assessment of the analytical models with the data obtained from the experimental investigation, the model by Xiao et al. was the most accurate model investigated, and may be used to accurately express the stress-strain relationship of SMA confined concrete.
Appendix D: Matlab Codes

D.1 Code for Unconfined Concrete Stress-Strain Relationship

```matlab
function [fc]=ConcreteCode(epc)

% fc'   =compressive strength of concrete            =4000psi
% epcu  =max usable compressive strain for concrete  =0.004
% epco  =strain at fc' (max concrete stress)         =0.002
%        for unconfined concrete
% epcc  =strain at fc' (max concrete stress)         =0.002
%        for confined concrete

fcprime=4000;
epcu=0.004;
epco=0.002;
epcc=0.002;

Ec=57000*((fcprime)^.5);
Esec=fcprime/epco;
r=Ec/(Ec-Esec);
ep50u=(3+0.002*fcprime)/(fcprime-1000);

x1=epco;
y1=fcprime;
x2=ep50u;
y2=0.5*fcprime;

Slope=(y2-y1)/(x2-x1);
yint=fcprime-Slope*epco;

% epc=-epc;

if epc <= epco && epc>0
    fc=((fcprime*r*(epc/epcc))/(r-1+((epc/epcc)^r)));
elseif epc<=epcu && epc>0
    fc=Slope*epc+yint;
else
    fc=0;
end
end
```
D.2 Code for Steel Stress-Strain Relationship

```matlab
function [fs]=SteelCode(ep)

% Es    =modulus of elasticity of Steel
% fy    =yielld strength of steel
% eps   =strain of steel
% epsy  =strain at the yield point
% epsrup =strain at rupture of steel

%Units are PSI
Es=29000000;
fy=60000;
fu=100000;
epsy=0.00207;
epsh=0.010;
epsu=0.08;
epsrup=0.12;
SHpercent=0;
NegStrain=0;

if ep<0
    NegStrain=1;
    ep=-ep;
    %A Marker being used to avoid errors if Negative Strain
    %values are inputed into the function
end
fsh=fy+(epsh-epsy)*SHpercent*Es;

if ep>epsrup;
    fs=0;
elseif ep<=epsy;
    fs=Es*ep;
elseif ep<=epsh && ep>=epsy
    fs=fy+SHpercent*Es*(ep-epsy);
else
    fs=fu-(fu-fsh)*(((epsu-ep)/(epsu-epsh))^2);
end

if NegStrain==1
    fs=-fs;      %Switches Negative stress value from Negative
    %Strain to Positive
end
end
```

D.3 Typical Code Displaying the Formulated Stress-Strain Relationship of SMA Confined Concrete for All Models

```matlab
close all
clear all
clc
```
tic

%Unconfined Concrete Stress in PSI
fc_prime=4000; %Max unconfined concrete stress
fco_prime=fc_prime;
epco=0.002; %Strain at max stress of unconfined concrete
epcu_unconfined=0.004; %Going to need to use a different variable for this value and the value of the confined concrete

%Column Information
h=8; %Height (inches)
d=4; %diameter (inches)
ColumnArea=pi*((d/2)^2); %Area of Column (Inches)^2

%SMA information
fSMA=116000; % 800MPa
diamSMA=0.02; %inch
areaSMA=pi*(diamSMA/2)^2; %[Inch]^2
ds=d+(diamSMA/2)+(diamSMA/2); %diameter of spiral between bar centers (inch)
tieDiam=0; %This portion has no column ties

%SMA information Used for calculating the Ultimate Strain Capacity
Ult_ten_Str_SMA=150000; %Ultimate Tensile Strength of SMA USed (Psi)
Fracture_strain_SMA=0.15; %Fracture Strain of The transverse SMA

%Unconfined Concrete
%This portion finds the stress-strain curve for unconfined concrete which is used for solving an energy balance equation, that is required to find the ultimate strain of the confined concrete
w=1;
for epc=0:0.0001:epcu_unconfined;
    allepc(w)=epc;
    fc_unconfined=ConcreteCode(epc);
    allfc_unconfined(w)=fc_unconfined;
    UnconfinedConcrete_load=fc_unconfined*ColumnArea;
    allUnConLoad(w)=UnconfinedConcrete_load;
    dispUncon=epc*h;
    alldispUnCon(w)=dispUncon;
    w=w+1;
end

Xint_Unconfined=allepc;
Yint_Unconfined=allfc_unconfined;

Unconfined_concrete_energy=trapz(Xint_Unconfined,Yint_Unconfined);
%The unconfined concrete energy capacity based on Manders et al. equations
for q=1:1:6;
    loops_per_inch=5;

    s=(1/loops_per_inch)-(diamSMA/2);  %s=center line spacing between ties (inches)

    s_prime=s-diamSMA;  %clear distance between ties (inch)

    roecc=0;  %roecc=ratio of area of longitudinal reinforcement to area of core of section
    %The core samples have no longitudinal reinforcement and therefore equals 0

    Acc=((pi/4)*((ds)^2)*(1-roecc));  %in^2
    ke=(1-((s_prime)/(2*ds)))/(1-roecc);

    roe_s=(4*areaSMA)/(ds*s);  %ratio of the volume of transverse confining steel to the volume of confined concrete core
    allroe_s(q)=roe_s;  %Unitless

    %epcu_confined=epcu_unconfined+0.14*(roe_s)*(fSMA/fc_prime);
    %The other possible equation for ultimate strain
    epcu_confined=0.04;  %experimental value used from solving manders equations

    fl=(2*areaSMA*fSMA)/(d*s);

    fl_prime=ke*fl;

    i=1;
    for ep=0:0.00025:epcu_confined;
        num(q)=i;

        allep(q,i)=ep;

        if q==1
            %Using Manders (1988) Equations
            fcc_prime=fco_prime*(-1.254+2.254*(1+((7.94*fl_prime)/(fco_prime)))^0.5)-

            2*(fl_prime/fco_prime));
            epcc=epco*(1+5*((fcc_prime(q)/fco_prime)-1));

        elseif q==2
            %Using Richarts (1928) Equations
            fcc_prime=fco_prime*(1+4.1*(fl_prime/fco_prime));
            epcc=(1+20.5*(fl_prime/fco_prime))*epco;

        elseif q==3
            %Using Candappa (2001) Equations

        end
    end

end
fcc_prime=fco_prime*(1+5.3*(fl_prime/fco_prime));
epcc=(1+20*(fl_prime/fco_prime))*epco;

elseif q==4
%Using Jiang and Teng (2007) Equations
fcc_prime=fco_prime*(1+3.5*(fl_prime/fco_prime));
epcc=(1+17.5*((fl_prime/fco_prime)^1.2))*epco;

elseif q==5
%Using Xiaos 2010 Equations NSC and HSC
fcc_prime=(1+3.24*((fl_prime/fc_prime)^0.8))*fco_prime;
epcc=(1+17.4*((fl_prime/fco_prime)^1.06))*epco;

else
%Using Moghaddam 2010 Equations
fcc_prime=fco_prime*(1+8*(fl_prime/fco_prime)-
4*((fl_prime/fco_prime)^1.2));
epcc=epco*((fcc_prime/fco_prime)^1.1);
end

x=ep/epcc;
allx(q,i)=x;
Ec=57000*((fco_prime)^(0.5));
Esec=fcc_prime/epcc;
r=Ec/(Ec-Esec);

fc=(fcc_prime*x*r)/(r-1+(x^r));
allfc(q,i)=fc;
i=i+1;
end

%This portion is separating the stress and strain values apart from
%each curve based upon the spacing value to be used in calculating
%the energy capacity (area under the curves)

%USH=Ultimate Strain Energy Capacity of the confining
%reinforcement per unit volume of concrete core
%Ush will vary for each SMA spacing category because it changes the
%value of roe_s for each

if q==1 %Manders
Xint_Confined_Mander=allep(1,:);
Yint_Confined_Mander=allfc(1,:);

Ush_Mander=Ult_ten_Str_SMA*Fracture_strain_SMA*roe_s;

elseif q==2 %Richart
Xint_Confined_Richart=allep(2,:);
Yint_Confined_Richart=allfc(2,:);

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Ush_Richart=Ult_ten_Str_SMA*Fracture_strain_SMA*roe_s;

elseif q==3 %Candappa
Xint_Confined_Candappa=allep(3,:);
Yint_Confined_Candappa=allfc(3,:);

Ush_Candappa=Ult_ten_Str_SMA*Fracture_strain_SMA*roe_s;

elseif q==4 %Jiang and Teng
Xint_Confined_Jiang_and_Teng=allep(4,:);
Yint_Confined_Jiang_and_Teng=allfc(4,:);

Ush_Jiang_and_Teng=Ult_ten_Str_SMA*Fracture_strain_SMA*roe_s;

elseif q==5 %Xiao
Xint_Confined_Xiao=allep(5,:);
Yint_Confined_Xiao=allfc(5,:);

Ush_Xiao=Ult_ten_Str_SMA*Fracture_strain_SMA*roe_s;

else %Moghaddam
Xint_Confined_Moghaddam=allep(6,:);
Yint_Confined_Moghaddam=allfc(6,:);

Ush_Moghaddam=Ult_ten_Str_SMA*Fracture_strain_SMA*roe_s;
end
end

%For all sections below
% Confined Concrete Portion
% For this setup there is no longitudinal steel or unconfined
% concrete therefore only the confined concrete portion is
% contributing to the stress-strain relationship

%P_column=ConfinedConcrete_load+UnconfinedConcrete_Load+Steel_Load
% However, for this setup there is no unconfined concrete or
% longitudinal steel therefore they both equal zero and all load is
% from the ConfinedConcrete only

%Manders 1988 Equations.
for z=2:1:numel(Xint_Confined_Mander);
    Confined_Concrete_Energy_Mander=trapz(Xint_Confined_Mander(1:z),Yint_Confined_Mander(1:z));
    %Approx Energy Capacity for a curve that is Extended deliberately long
    %pass a realistic ultimate strain value
    if Confined_Concrete_Energy_Mander <= Ush_Mander+Unconfined_concrete_energy
        %Finds when the Energy capacity of the linch SMA spaced
        stress-strain curve is greater than the combined
%energy from the transverse confining SMA and the energy from your typical unconfined concrete energy.

epcu_confined_Mander=Xint_Confined_Mander(z);
%Once the Ultimate Strain value has been reach this line
%assigns that value to the stress-strain curve
zz=z;

allep_Mander_until_Ult_Strain=Xint_Confined_Mander(1:zz);
allfc_Mander_Until_Ult_Strain=Yint_Confined_Mander(1:zz);
%stops the stress and strain values at the ultimate strain location

ConfinedConcrete_load_Mander=allfc_Mander_Until_Ult_Strain*ColumnArea;
P_column_Mander=ConfinedConcrete_load_Mander;
disp_Mander=allep_Mander_until_Ult_Strain*h;
end

end

%Richart 1928 Equations.
for v=2:1:numel(Xint_Confined_Richart);

Confined_Concrete_Energy_Richart=trapz(Xint_Confined_Richart(1:v),Yint_Confined_Richart(1:v));
%Approx Energy Capacity for a curve that is Extended deliberately long
%pass a realistic ultimate strain value

if Confined_Concrete_Energy_Richart <= Ush_Richart+Unconfined_concrete_energy
%Finds when the Energy capacity of 1/2inch stress-strain curve is greater than the combined
%energy from the transverse confining SMA and the energy from
%your typical unconfined concrete energy.

epcu_confined_Richart=Xint_Confined_Richart(v);
%Once the Ultimate Strain value has been reach this line
%assigns that value to the stress-strain curve
vv=v;

allep_Richart_Until_Ult_Strain=Xint_Confined_Richart(1:vv);
allfc_Richart_Until_Ult_Strain=Yint_Confined_Richart(1:vv);
%stops the stress and strain values at the ultimate strain location

ConfinedConcrete_load_Richart=allfc_Richart_Until_Ult_Strain*ColumnArea;

end
P_column_Richart=ConfinedConcrete_load_Richart;

disp_Richart=allep_Richart_Until_Ult_Strain*h;
end
end

%Candappa.
for b=2:1:numel(Xint_Confined_Candappa);
    Confined_Concrete_Energy_Candappa=trapz(Xint_Confined_Candappa(1:b),Yint_Confined_Candappa(1:b));
    %Approx Energy Capacity for a curve that is Extended delibertly long %pass a realistic ultimate strain value
    if Confined_Concrete_Energy_Candappa <= Ush_Candappa+Unconfined_concrete_energy
        %Finds when the Energy capacity of 1_3rdinch stress-strain curve is greater than the combined %energy from the transverse confining SMA and the energy from %your typical unconfined concrete energy.
        epcu_confined_Candappa=Xint_Confined_Candappa(b);
        %Once the Ultimate Strain value has been reach this line %assigns that value to the stress-strain curve
        bb=b;
    end

    allep_Candappa_Until_Ult_Strain=Xint_Confined_Candappa(1:bb);
    allfc_Candappa_Until_Ult_Strain=Yint_Confined_Candappa(1:bb);
    %stops the stress and strain values at the ultimate strain location
    ConfinedConcrete_load_Candappa=
    allfc_Candappa_Until_Ult_Strain*ColumnArea;
    P_column_Candappa=ConfinedConcrete_load_Candappa;
    disp_Candappa=allep_Candappa_Until_Ult_Strain*h;
end
end

%Jiang and Teng.
for e=2:1:numel(Xint_Confined_Jiang_and_Teng);

    Confined_Concrete_Energy_Jiang_and_Teng=trapz(Xint_Confined_Jiang_and_Teng(1:e),Yint_Confined_Jiang_and_Teng(1:e));
    %Approx Energy Capacity for a curve that is Extended delibertly long %pass a realistic ultimate strain value
if Confined_Concrete_Energy_Jiang_and_Teng <= 
Ush_Jiang_and_Teng+Unconfined_concrete_energy
    %Finds when the Energy capacity of 1_4thinch stress-strain 
curve is greater than the combined 
    %energy from the transverse confining SMA and the energy 
from 
    %your typical unconfined concrete energy.
    
epcu_confined_Jiang_and_Teng=Xint_Confined_Jiang_and_Teng(e);
    %Once the Ultimate Strain value has been reach this line 
    %assigns that value to the stress-strain curve
    ee=e;

allep_Jiang_and_Teng_Until_Ult_Strain=Xint_Confined_Jiang_and_Teng(1:ee);
allfc_Jiang_and_Teng_Until_Ult_Strain=Yint_Confined_Jiang_and_Teng(1:ee);
    %stops the stress and strain values at the ultimate strain 
location

ConfinedConcrete_load_Jiang_and_Teng=allfc_Jiang_and_Teng_Until_Ult_Strain*ColumnArea;
P_column_Jiang_and_Teng=ConfinedConcrete_load_Jiang_and_Teng;

    disp_Jiang_and_Teng=allep_Jiang_and_Teng_Until_Ult_Strain*h;
end

%Xiao.
for p=2:1:numel(Xint_Confined_Xiao);
    Confined_Concrete_Energy_Xiao=trapz(Xint_Confined_Xiao(1:p),Yint_Confined_Xiao(1:p));
    %AppFoi Energy Capacity for a curve that is Extended delibertly long 
    %pass a realistic ultimate strain value
    
    if Confined_Concrete_Energy_Xiao <= 
Ush_Xiao+Unconfined_concrete_energy
        %Finds when the Energy capacity of 1_5thinch stress-strain 
curve is greater than the combined 
        %energy from the transverse confining SMA and the energy 
from 
        %your typical unconfined concrete energy.
        epcu_confined_Xiao=Xint_Confined_Xiao(p);
        %Once the Ultimate Strain value has been reach this line 
        %assigns that value to the stress-strain curve
        pp=p;

end

allep_Xiao_Until_Ult_Strain=Xint_Confined_Xiao(1:pp);
allfc_Xiao_Until_Ult_Strain=Yint_Confined_Xiao(1:pp);
%stops the stress and strain values at the ultimate strain
location

ConfinedConcrete_load_Xiao=allfc_Xiao_Until_Ult_Strain*ColumnArea;
P_column_Xiao=ConfinedConcrete_load_Xiao;

disp_Xiao=allep_Xiao_Until_Ult_Strain*h;
end
end

%Moghaddam.
for u=2:1:numel(Xint_Confined_Moghaddam);

Confined_Concrete_Energy_Moghaddam=trapz(Xint_Confined_Moghaddam(1:u),Yint_Confined_Moghaddam(1:u));
%Approx Energy Capacity for a curve that is Extended delibertly long
%pass a realistic ultimate strain value

if Confined_Concrete_Energy_Moghaddam <= Ush_Moghaddam+Unconfined_concrete_energy
%Finds when the Energy capacity of 1_5thinch stress-strain
curve is greater than the combined
%energy from the transverse confining SMA and the energy
from
%your typical unconfined concrete energy.
epcu_confined_Moghaddam=Xint_Confined_Moghaddam(u);
%Once the Ultimate Strain value has been reach this line
%assigns that value to the stress-strain curve
uu=u;

allep_Moghaddam_Until_Ult_Strain=Xint_Confined_Moghaddam(1:uu);
allfc_Moghaddam_Until_Ult_Strain=Yint_Confined_Moghaddam(1:uu);
%stops the stress and strain values at the ultimate strain
location

ConfinedConcrete_load_Moghaddam=allfc_Moghaddam_Until_Ult_Strain*ColumnArea;
P_column_Moghaddam=ConfinedConcrete_load_Moghaddam;

disp_Moghaddam=allep_Moghaddam_Until_Ult_Strain*h;
end
end

plot (allep_Mander_until_Ult_Strain, allfc_Mander_Until_Ult_Strain,'k',
allep_Richart_Until_Ult_Strain, allfc_Richart_Until_Ult_Strain,'g--',
allep_Candappa_Until_Ult_Strain, allfc_Candappa_Until_Ult_Strain, 'm:',

D.4 Typical Code Displaying the Results from the Numerical Example Predicting the Stress-Strain and Load vs. Displacement for the 12 inch Spaced SMA Column

close all
clear all
clc
tic

% SMA information
fSMA=116000;   %(PSI) (800MPa)
diamSMA=.5 ;   %(inch)
areaSMA=pi*((diamSMA/2)^2);  %(Inch)^2

% SMA information Used for calculating the Ultimate Strain Capacity
Ult_ten_Str_SMA=170000;  %Ultimate Tensile Strength of SMA USed (Psi)
Fracture_strain_SMA=0.175;  %Fracture Strain of The transverse SMA

% Circular Column Information
h=120;        %Height (inches)
d=18; %diameter (inches)
ColumnArea=pi*((d/2)^2); %Area of Column (Inches)^2

ds=d+(diamSMA/2)+(diamSMA/2); %diameter of spiral between bar centers (inch)

cc=1.5; %clear cover inches
ConfinedColumnArea=pi*(((d-(2*cc)-diamSMA)/2)^2);

% 8 No. 9 longitudinal bars
NumLongBars=8; %Number of longitudinal bars
ast9=1.00; %Area of a number 9 bar
diam9=1.128; %Diam of a number 9 bar
astlong=NumLongBars*ast9; %Area of Longitudinal Steel

%Unconfined Concrete Stress in PSI
fc_prime=[4000]; %Max unconfined concrete stress
fco_prime=fc_prime;
epc=0.002; %Strain at max stress of unconfined concrete
epcu_unconfined=0.004; %Ultimate Strain for unconfined concrete

%Unconfined Concrete
%This portion finds the stress-strain curve for unconfined concrete which
%is used for solving an energy balance equation, that is required to find
%the ultimate strain of the confined concrete
w=1;
for epc=0:0.0001:epcu_unconfined;
    allepc(w)=epc;
    fc_unconfined=ConcreteCode(epc);
    allfc_unconfined(w)=fc_unconfined;
    w=w+1;
end

Xint_Unconfined=allepc;
Yint_Unconfined=allfc_unconfined;

Unconfined_concrete_energy=trapz(Xint_Unconfined ,Yint_Unconfined);
%The unconfined concrete energy capacity based on Manders et al. equations

%This portion is for SMA confined concrete Specimens that
%has only Transverse SMA confining material and incorporates information
%from the unconfiend concrete strength listed directly above
for q=2;
s_prime=s-diamSMA; %clear distance between ties (inch)
ac=pi*(((d-(2*cc)-diamSMA)/2)^2);
%roecc
roecc = astlong / (ac); % area of longitudinal reinforcement/area of core of

acc = ((pi/4)*((ds)^2)*(1-roecc)); % in^2
ke = (1-((s_prime)/(2*ds)))/(1-roecc);

roe_s = (4*areaSMA) / (ds*s); % ratio of the volume of transverse confining
% steel to the volume of confined concrete core (Unitless)

fl = (2*areaSMA*fSMA) / (d*s);
fl_prime = ke*fl;

i = 1;
epcu_confined = 0.3;
% unrealistically high value used to cover all relatisic examples
for ep = 0:0.0001:epcu_confined;
    num = i;
    allep(q, i) = ep;

    % Using Xiaos 2010 Equations NSC and HSC
    fcc_prime = (1+3.24*((fl_prime/fc_prime)^0.8))*fco_prime;
    epcc = (1+17.4*((fl_prime/fco_prime)^1.06))*epco;

    x = ep / epcc;
    allx(q, i) = x;
    Ec = 57000*((fco_prime)^0.5);
    Esec = fcc_prime / epcc;
    r = Ec / (Ec - Esec);

    fc = (fcc_prime * x * r) / (r - 1 + (x^r));
    allfc(q, i) = fc;

    i = i + 1;
end
% USH=Ultimate Strain Energy Capacity of the confining
% reinforcement per unit volume of concrete core

%Xiao
Xint_Confined_Xiao = allep(2, :);
Yint_Confined_Xiao = allfc(2, :);

Ush_Xiao = Ult_ten_Str_SMA * Fracture_strain_SMA * roe_s;
end

% This portion below finds the ultimate strain location based on the energy
% balance approach using the stress-strain raltionship predicted by Xiao's
% relationship
for p = 2:1: numel(Xint_Confined_Xiao);
    allp(p, 1);
Confined_Concrete_Energy_Xiao=trapz(Xint_Confined_Xiao(1:p),Yint_Confined_Xiao(1:p));
%Approx Energy Capacity for a curve that is Extended delibertly long
%pass a realistic ultimate strain value

if Confined_Concrete_Energy_Xiao <=
Ush_Xiao+Unconfined_concrete_energy

%Finds when the Energy capacity of the stress-strain curve
%is greater than the combined energy from the transverse confining SMA
%and the energy from your typical unconfined concrete energy.

epcu_confined_energy=Xint_Confined_Xiao(p);
%Once the Ultimate Strain value has been reach this line
%assigns that value to the stress-strain curve
pp=p;

allep_Xiao_Until_Ult_Strain_Energy=Xint_Confined_Xiao(1:pp);
allfc_Xiao_Until_Ult_Strain_Energy=Yint_Confined_Xiao(1:pp);
%stops the stress & strain values at ultimate strain location
end
%This portion finds the untilmate strain value using a conditional ultimate
%strain value corresponding to 20% of the peak stress value calculated,
as
%any value higher should be considered unrealistic
for m=2:1:numel(Xint_Confined_Xiao);
    allm=(1:m);
    Stop=fcc_prime*0.2;

    if Yint_Confined_Xiao(m) >= Stop && Xint_Confined_Xiao(m) >=
0.02
        mm=m;

        epcu_confined_percent=Xint_Confined_Xiao(mm);
%Once the Ultimate Strain Value has been reach this line
%assigns that value to the stress-strain curve

allep_Xiao_Until_Ult_Strain_Percent=Xint_Confined_Xiao(1:mm);
allfc_Xiao_Until_Ult_Strain_Percent=Yint_Confined_Xiao(1:mm);
%stops the stress &strain values at ultimate strain location
end

%This selects the smaller ultimate strain value from either the energy
%balance approach or based on the conditional percented of the peak
stress
if  epcu_confined_percent < epcu_confined_energy

    epcu_confined_Xiao = epcu_confined_percent;
    allep_Xiao_Until_Ult_Strain = allep_Xiao_Until_Ult_Strain_Percent;
    allfc_Xiao_Until_Ult_Strain = allfc_Xiao_Until_Ult_Strain_Percent;
else
    epcu_confined_Xiao = epcu_confined_energy;
    allep_Xiao_Until_Ult_Strain = allep_Xiao_Until_Ult_Strain_Energy;
    allfc_Xiao_Until_Ult_Strain = allfc_Xiao_Until_Ult_Strain_Energy;
end

%Now that the ultimate strain value has been obtained we can find the
load %vs displacement plot for the column with 1.5inch clear cover
(unconfined %concrete), longitudinal steel, and the confined SMA core

%Load carried by the Confined Concrete Portion
allConfinedConcrete_load=allfc_Xiao_Until_Ult_Strain*ConfinedColumnArea

ww=1;
for ep2=0:0.0001:epcu_confined_Xiao;
    allep2(ww)=ep2;
    %Steel Portion
    fsteel=SteelCode2(ep2);
    allfsteel(ww)=fsteel;
    Steel_load=fsteel*astlong;
    allSteel_load(ww)=Steel_load;
    %Unconfined Concrete portion
    clearcoverarea=(ColumnArea-ConfinedColumnArea);
    funcon=ConcreteCode2(ep2);
    allfuncon(ww)=funcon;
    Uncon_load=funcon*clearcoverarea;
    allUnConfinedConcrete_load(ww)=Uncon_load;
    ww=ww+1;
end
P_column_Xiao=allConfinedConcrete_load+allUnConfinedConcrete_load+allSteel_load;
disp=allep_Xiao_Until_Ult_Strain*h; %Total Displacement of the Column
plot (allep_Xiao_Until_Ult_Strain, allfc_Xiao_Until_Ult_Strain,'b--');
legend('12inch Spacing')
title('Confined Concrete Stress vs. Strain Relationship')
xlabel('Strain "inch/inch"')
ylabel('Compressive Stress "Psi"')
figure
plot(allelp_Xiao_Until_Ult_Strain, allfsteel)
title('Stress vs. Strain Relationship Typical Steel')
xlabel('Strain "inch/inch"')
ylabel('Compressive Stress "Psi"')
figure
plot(disp, P_column_Xiao)
legend('12inch Spacing')
title('Axial Load vs. Axial Displacement of Column')
xlabel('Displacement (Inch)')
ylabel('Axial Load (Lbs)')
toc
Appendix E: As Measured Stress-Strain Plots

Figure E.1. Recorded Stress versus Strain for the First Unconfined Cylinder in Concrete Batch #1.
Figure E.2. Recorded Stress versus Strain for the Second Unconfined Cylinder in Concrete Batch #1.

Figure E.3. Recorded Stress versus Strain for the First Unconfined Anchored Cylinder in Concrete Batch #1.
Figure E.4. Recorded Stress versus Strain for the Second Unconfined Anchored Cylinder in Concrete Batch #1.

Figure E.5. Recorded Stress versus Strain for the First 1/3\textsuperscript{rd} inch Pitch SMA Confined Cylinder in Concrete Batch #1.
Figure E.6. Recorded Stress versus Strain for the Second 1/3\textsuperscript{rd} inch Pitch SMA Confined Cylinder in Concrete Batch #1.

Figure E.7. Recorded Stress versus Strain for the Third 1/3\textsuperscript{rd} inch Pitch SMA Confined Cylinder in Concrete Batch #1.
Figure E.8. Recorded Stress versus Strain for the First 1/5th inch Pitch SMA Confined Cylinder in Concrete Batch #1.

Figure E.9. Recorded Stress versus Strain for the Second 1/5th inch Pitch SMA Confined Cylinder in Concrete Batch #1.
Figure E.10. Recorded Stress versus Strain for the First Unconfined Cylinder in Concrete Batch #2.

Figure E.11. Recorded Stress versus Strain for the Second Unconfined Cylinder in Concrete Batch #2.
Figure E.12. Recorded Stress versus Strain for the First Unconfined Anchored Cylinder in Concrete Batch #2.

Figure E.13. Recorded Stress versus Strain for the Second Unconfined Anchored Cylinder in Concrete Batch #2.
Figure E.14. Recorded Stress versus Strain for the First 1/3\textsuperscript{rd} inch Pitch SMA Confined Cylinder in Concrete Batch #2.

Figure E.15. Recorded Stress versus Strain for the Second 1/3\textsuperscript{rd} inch Pitch SMA Confined Cylinder in Concrete Batch #2.
Figure E.16. Recorded Stress versus Strain for the Third 1/3rd inch Pitch SMA Confined Cylinder in Concrete Batch #2.

Figure E.17. Recorded Stress versus Strain for the First 1/5th inch Pitch SMA Confined Cylinder in Concrete Batch #2.
Figure E.18. Recorded Stress versus Strain for the Second 1/5th inch Pitch SMA Confined Cylinder in Concrete Batch #2.

Figure E.19. Recorded Stress versus Strain for the First Unconfined Cylinder in Concrete Batch #3.
Figure E.20. Recorded Stress versus Strain for the Second Unconfined Cylinder in Concrete Batch #3.

Figure E.21. Recorded Stress versus Strain for the First Unconfined Anchored Cylinder in Concrete Batch #3.
Figure E.22. Recorded Stress versus Strain for the Second Unconfined Anchored Cylinder in Concrete Batch #3.

Figure E.23. Recorded Stress versus Strain for the First 1/3rd inch Pitch SMA Confined Cylinder in Concrete Batch #3.
Figure E.24. Recorded Stress versus Strain for the Second 1/3\textsuperscript{rd} inch Pitch SMA Confined Cylinder in Concrete Batch #3.

Figure E.25. Recorded Stress versus Strain for the Third 1/3\textsuperscript{rd} inch Pitch SMA Confined Cylinder in Concrete Batch #3.
Figure E.26. Recorded Stress versus Strain for the First 1/5th inch Pitch SMA Confined Cylinder in Concrete Batch #3.

Figure E.27. Recorded Stress versus Strain for the Second 1/5th inch Pitch SMA Confined Cylinder in Concrete Batch #3.