SELECTIVE DEUTERON ACCELERATION USING TARGET NORMAL SHEATH ACCELERATION

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By
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Abstract

It has been known for more than a decade that surface contaminants from a thin foil will be accelerated to multi-\(MeV\) energies after irradiation with an ultra-intense laser. The versatility of an ion beam for the generation of neutrons can be improved by tailoring which ions are accelerated. Nominally, the dominant species accelerated is the lightest present on a target surface typically contaminated with hydrocarbons and water: protons. This work elucidates a method of in-situ cryogenic coating of heavy ice on ultra-intense laser targets and experimental confirmation of the dominant acceleration of \(\sim MeV\) deuterons. 1D pseudo-Lagrangian calculations investigating the initial stages of ion acceleration with various levels of surface contamination are also presented.

The first successful demonstration of selective deuteron acceleration by target normal sheath-field acceleration, in which the normally dominant contaminant proton and carbon ion signals are suppressed by orders of magnitude relative to the deuteron signals is reported. Using a laser pulse with 0.5 \(J\), 120 \(fs\) duration, and \(5 \times 10^{18} W/cm^2\) mean intensity, the deuterons originating from a surface layer of heavy ice with energies up to 3.5 \(MeV\) comprised > 99% of the recorded ion signal.

The design, calibration, and implementation of a Thomson parabola spectrometer to measure the target normal ion spectra is presented. In addition to estimations of the target coating and contamination rates, the effect of contamination thickness is modeled presented. Analytic calculations predicting characteristics of various neutron sources utilizing this deuteron source are presented.
To my love, Alicia. Who gave me GT5, hasn’t bought a van, tolerates my vehicle, desire for garage space, and other idiosyncrasies that need not to be mentioned in a public forum.
ACKNOWLEDGMENTS

So research is painful. You know if you are reading this that everything is understated. I’m sure you been told that you learned something in elementary school that no one learns in elementary school. What you may not have realized is that when setting up experiments is discussed many elements are discussed as if they are trivial. Then there is Murphy’s Law, the Conservation of Errors (Stroud), Conservation of Luck (Me?), and Entropy are always in effect and can be mitigated by planning but also can render planning useless. It should also be mentioned that where there’s a will there’s a way, you just have to wring it out of mother nature (Rick), explain to the experiment it is either you or me (Linn) and threaten it with a wrench. These phrases exist for a reason.

So taking these together putting my name alone at the top of this work is somewhat disingenuous. Experimental research by its nature requires quite a collection of people. The truth of that statement varies with the subfield. Current experimental High Energy Density Physics involves large experimental apparatus. High peak power and high intensity laser systems which commonly fill multiple optical tables, rooms, and buildings. Extremely high current devices, Z-pinches, have been scaled up to enormous scales as have various magnetically confined plasma devices such as tokamaks. Buy their nature these apparatus are commonly the work of several individuals. The experiments due to their complexity and time constraints involve several to many individuals. Not to mention the clerical support staff, building staff, various shop and departmental staff.

From the beginning we assist, enrich, harm, and damage those near us intentionally or unintentionally. On average we climb in skill, resources, connections, and effectiveness. But your time is a hard limit. So you gather help. But help takes time. So one also slowly becomes an administrator, and often becomes a seeker of money. The structure seems similar to African Driver ants climbing a structure to become part of the structure.

Thanks to all, such as Sam Feldman for running the Glass Hybrid optical parametric chirped pulse amplification (OPCPA) System Testbed (GHOST) laser and assisting with experimental setup with Mike Storm. Special thanks to Dr. Enam Chowdhury for a wealth of knowledge, help, and ideas. The majority of my knowledge of optics, alignment, short pulse laser design can be attributed to hours in the lab working on Sisyphus with Tony
Link and “The Man.” Thanks to Chris Willis, with whom the analysis of the Thomson parabola spectrometer (TPS) spectrometer would have been much more painful and time consuming. Modeling of the magnetic fields in the TPS yoke and resulting section 4.3.3 are primarily the fruits of his labor. Andy Krygier contributed the time resolved electron spectra from large scale plasma (LSP) modeling enabling Chapter 5.

As for enabling me to complete this work thank you all for your patience and understanding. Thank you Tony Link, Andy Krygier, Rebecca Daskalova, and others for making this not only more tolerable, but enjoyable. My gratitude to my advisors Linn Van Woerkom for the opportunity and Richard Freeman for their continued support, knowledge, and patience.

Finally, this work was inspired and motivated by my coven\(^*\)/coterie/clan who’s care is my responsibly and pleasure. My thanks, love, and gratitude for their implicit sacrifice, and generous explicit love and support.

\(^*\)Terry Pratchett, not Shakespeare
VITA

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Publications


**Presentations**


**Fields of Study**

Major Field: Physics

Studies in:

- High Energy Density Physics
- Laser design and operation

Prof. R.R. Freeman & Prof. L. Van Woerkom

Dr. E.A. Chowdhury
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<tr>
<td>$A$</td>
<td>the mass in grams $g$ of a mol of material, approximately equal to the number of nucleons</td>
<td>atomic mass</td>
</tr>
<tr>
<td>$a_0$</td>
<td>pump strength of an oscillating field $\frac{eA}{mc^2}$</td>
<td>normalized vector potential</td>
</tr>
<tr>
<td>$a_{IS}$</td>
<td>radius greater than which an ion in a plasma is shielded in the ion-sphere model</td>
<td>ion-sphere radius</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>after reflection, the path traveled at which the transmission is $1/exp$</td>
<td>attenuation length</td>
</tr>
<tr>
<td>$B$</td>
<td>vector field describing the force on a moving charge by moving charges and time-varying electric fields</td>
<td>magnetic field</td>
</tr>
<tr>
<td>$C_s$</td>
<td>the speed of density/pressure waves or &quot;pressure communication&quot; in a medium $342 \text{ m/s}$ for dry air at STP</td>
<td>speed of sound</td>
</tr>
<tr>
<td>$\delta R$</td>
<td>radius of the pin hole image of the target on the detector in a TPS when used to detect laser accelerated ion, this is determined by geometry because the ion front expands laminarly as if from a point source</td>
<td>deflection ambiguity</td>
</tr>
<tr>
<td>$E$</td>
<td>vector field describing the force exerted on a charge by surrounding charges and time-varying magnetic fields</td>
<td>electric field</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Term</td>
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<td>--------</td>
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<tr>
<td>$E$</td>
<td>the energy of the lowest energy state in a system, $-13.6eV$ for an electron bound to a proton</td>
<td>energy</td>
</tr>
<tr>
<td>$E_0$</td>
<td>the energy of the lowest energy state in a system, $-13.6eV$ for an electron bound to a proton</td>
<td>ground state energy</td>
</tr>
<tr>
<td>$E_f$</td>
<td>relevant in systems where the particle spacing in on the order of the de Broglie thermal wavelength $\Lambda_d$, the energy of the state with 50% occupancy at absolute zero, all states with lower energy are filled as per the Pauli exclusion principle</td>
<td>Fermi energy</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>internal energy per unit mass, similar to specific heat but more inclusive</td>
<td>specific internal energy</td>
</tr>
<tr>
<td>$\eta$</td>
<td>resistance to electronic currents arising from electron scattering</td>
<td>resistivity</td>
</tr>
<tr>
<td>$F$</td>
<td>Helmholtz free energy or free energy $U - TS = -k_B T \ln Z$</td>
<td>Helmholtz free energy</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>definition, here</td>
<td>flux</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>an index such that $p \propto \rho^{\gamma}$, a fluid or gas described in this way is a polytropic gas</td>
<td>polytropic index</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>The relativistic gamma factor $\sqrt{1 - \frac{v^2}{c^2}}$</td>
<td>Lorentz factor</td>
</tr>
<tr>
<td>$I$</td>
<td>energy flux incident on a surface per unit time $Wcm^{-2}$</td>
<td>intensity</td>
</tr>
<tr>
<td>$J_C$</td>
<td>population of moving electrons characterized by short mean free path and local thermalization, often referred to as return current</td>
<td>cold current</td>
</tr>
<tr>
<td>$J_H$</td>
<td>population of moving electrons characterized by large mean free path and non-local thermalization, often referred to as hot or relativistic electron current</td>
<td>hot current</td>
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<td>Symbol</td>
<td>Description</td>
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<tr>
<td>$K_I$</td>
<td>coefficient, relating reactant concentrations with the reaction rate: for collisional ionization $\frac{d}{dt} n_m - 1 = n_m n_e$</td>
<td>ionization rate coefficient</td>
</tr>
<tr>
<td>$K_R$</td>
<td>coefficient, relating reactant concentrations with the reaction rate: for three-body recombination $\frac{d}{dt} n_m + 1 = n_m n_e^2$</td>
<td>recombination rate coefficient</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>for lasers, usually measured in $\mu m$</td>
<td>wavelength</td>
</tr>
<tr>
<td>$\lambda_D$</td>
<td>characteristic length of electronic shielding of the ion’s charge in a plasma or a dilute electrolyte</td>
<td>Debye length</td>
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<tr>
<td>$\Lambda_d$</td>
<td>the average de Broglie wavelength of an ideal gas $\Lambda_d = \frac{h}{\sqrt{2\pi mk_BT}}$</td>
<td>de Broglie wavelength</td>
</tr>
<tr>
<td>$m$</td>
<td>an object’s resistance to acceleration, famously proportional to an object’s total energy content.</td>
<td>mass</td>
</tr>
<tr>
<td>$\lambda_{mfp}$</td>
<td>the average distance traveled by a particle before an interaction, this is scaling term in Beer’s Law</td>
<td>mean free path</td>
</tr>
<tr>
<td>$m$</td>
<td>the angular momentum designation as in $2p$ or $n = 2, m = 1$, arising from the azimuthal solution for hydrogenic electron eigenstates</td>
<td>azimuthal quantum number</td>
</tr>
<tr>
<td>$N$</td>
<td>the total number of particles as in canonical or grand canonical ensemble</td>
<td>particle number</td>
</tr>
<tr>
<td>$n$</td>
<td>number of particles per unit volume</td>
<td>number density</td>
</tr>
<tr>
<td>$n_e$</td>
<td>number density of electrons where non-relativistic intensities of $\omega_0 \leq \omega_p$ light are reflected</td>
<td>critical density</td>
</tr>
<tr>
<td>$n$</td>
<td>the shell designation as in $1s$ or $n = 1, m$, arising from the radial solution for hydrogenic electron eigenstates</td>
<td>principal quantum number</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>central(ish) optical frequency of the spectrum of a laser pulse from</td>
<td>nominal laser frequency</td>
</tr>
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<td>Symbol</td>
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<td>-------------</td>
<td>-------------------------------------------</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>resonant frequency of electronic plasma oscillations or Langmuir waves</td>
<td>plasma frequency</td>
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<tr>
<td>$\Phi$</td>
<td>the electric potential energy scaled by the charge $e$ - commonly voltage</td>
<td>electric potential</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>electric potential normalized to the average electron energy such that $\varphi = \frac{\Phi}{k_B T_e}$</td>
<td>normalized electric potential</td>
</tr>
<tr>
<td>$d$</td>
<td>thickness of phosphor layer in imaging plates (IP)</td>
<td>phosphor thickness</td>
</tr>
<tr>
<td>$P$</td>
<td>a frequentist definition - the ratio of defined outcomes:total events</td>
<td>probability</td>
</tr>
<tr>
<td>$R$</td>
<td>the path length traveled by a particle before stopping in a given material</td>
<td>particle range</td>
</tr>
<tr>
<td>$r_{cy}$</td>
<td>radius of the circular motion of a moving charged particle in a uniform magnetic field $\mathbf{B}$ field.</td>
<td>cyclotron radius</td>
</tr>
<tr>
<td>$p$</td>
<td>force per unit area</td>
<td>pressure</td>
</tr>
<tr>
<td>$\rho$</td>
<td>mass per unit volume</td>
<td>density</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>the average distance traveled by a photon before a uniform scattering event</td>
<td>scattering length</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>the effective geometric cross section of a particle which describes the probability of an event for an incident particle, typically measured in barns $1\text{barn} = 10^{-24}\text{cm}^2$ as in the broad side of a . . .</td>
<td>cross section</td>
</tr>
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<td>$T$</td>
<td>bulk measurement of the average kinetic energy of particles</td>
<td>temperature</td>
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<td>$T_C$</td>
<td>temperature assigned/fitted to the less energetic component of a two temperature approximation of an electron energy distribution</td>
<td>cold $e^-$ temperature</td>
<td>33, 77, 78</td>
</tr>
<tr>
<td>$T_d$</td>
<td>temperature characterized by the average kinetic energy of electrons to be equal to the Fermi energy $E_f$</td>
<td>degeneracy temperature</td>
<td>110</td>
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<tr>
<td>$T_e$</td>
<td>characteristic temperature assigned/fitted to electrons thermalized with themselves but not other present particle species, i.e. ions</td>
<td>the $e^-$ temperature</td>
<td>xvii, 28–31, 105, 108</td>
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<tr>
<td>$T_i$</td>
<td>characteristic temperature assigned/fitted to ions thermalized with themselves but not other present particle species, i.e. $e^-$</td>
<td>the ion temperature</td>
<td>28</td>
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<td>$T_H$</td>
<td>temperature assigned/fitted to the more energetic component of a two temperature approximation of an electron energy distribution</td>
<td>hot $e^-$ temperature</td>
<td>33, 77, 78, 81</td>
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<td>$l$</td>
<td>target thickness</td>
<td>target thickness</td>
<td>48</td>
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<td>$t$</td>
<td>time</td>
<td>27, 30–32, 48, 67, 77</td>
<td></td>
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<tr>
<td>$U_p$</td>
<td>cycle averaged energy of an electron in a laser field $m_e c^2 \left( \sqrt{1 + \alpha_0^2} - 1 \right)$</td>
<td>ponderomotive potential</td>
<td>35</td>
</tr>
<tr>
<td>$V$</td>
<td>yea, volume, here for consistency</td>
<td>volume</td>
<td>48, 106</td>
</tr>
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<td>$\nu$</td>
<td>vector field of local velocities in a fluid</td>
<td>fluid velocity</td>
<td>27, 28, 31, 32, 48, 56–58, 67</td>
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<td>$x$</td>
<td>position</td>
<td>position</td>
<td>29–33, 37, 77</td>
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<td>$Z$</td>
<td>encodes the probability distribution of the microscopic states of a system</td>
<td>partition function</td>
<td>105, 106</td>
</tr>
<tr>
<td>$Z_e$</td>
<td>charge of an ion</td>
<td>ionization state</td>
<td>29, 31–33, 38–41, 56–58, 78, 106–109</td>
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<tr>
<td>$Z$</td>
<td>the partition function of a free electron</td>
<td>free electron partition $f^n$</td>
<td>105, 106</td>
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<tr>
<td>$g$</td>
<td>the partition function on an ion can be separated into that of the ground state times the sum of the probable occupancy of each electron configuration. The statistical weight is the degeneracy of each excited state.</td>
<td>statistical weight</td>
<td>106, 107</td>
</tr>
<tr>
<td>$Z_m$</td>
<td>the partition function for an $m$ times ionized atom contains both the kinetic states of the atom and the states of electrons bound to the nucleus</td>
<td>ion partition function</td>
<td>105, 106</td>
</tr>
<tr>
<td>$Z_{nuc}$</td>
<td>number of protons in a nucleus</td>
<td>nuclear charge</td>
<td>37, 39, 92, 110, 111</td>
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# List of Abbreviations

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<td>ADK</td>
<td>Ammosov, Delone, Krainov</td>
<td>110</td>
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<tr>
<td>ASE</td>
<td>amplified spontaneous emission</td>
<td>21</td>
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<tr>
<td>ATI</td>
<td>above threshold ionization</td>
<td>110, 112</td>
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<td>BEB</td>
<td>binary-encounter-Bethe</td>
<td>111, 112</td>
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<tr>
<td>BED</td>
<td>binary-encounter-dipole</td>
<td>111, 112</td>
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<tr>
<td>CCD</td>
<td>charge coupled device</td>
<td>43–45</td>
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<tr>
<td>CPA</td>
<td>chirped pulse amplification</td>
<td>11</td>
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<tr>
<td>CR-39</td>
<td>Columbia resin #39</td>
<td>xii, 68, 71, 72, 88</td>
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<tr>
<td>CSDA</td>
<td>continuously slowing down approximation</td>
<td>66</td>
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<tr>
<td>CVD</td>
<td>chemical vapor deposition</td>
<td>86</td>
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<tr>
<td>DEEP</td>
<td>Deposition of Energy due to Electrons and Protons</td>
<td>9</td>
</tr>
<tr>
<td>DLA</td>
<td>direct laser acceleration</td>
<td>35</td>
</tr>
<tr>
<td>DNA</td>
<td>deoxyribonucleic acid</td>
<td>8</td>
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<tr>
<td>DWA</td>
<td>dielectric wall accelerator</td>
<td>11</td>
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<tr>
<td>ECU</td>
<td>electronic control module</td>
<td>1</td>
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<tr>
<td>EMP</td>
<td>electro-motive pulse</td>
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<tr>
<td>ENDF</td>
<td>Evaluated Nuclear Data File</td>
<td>114, 115</td>
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<tr>
<td>EOS</td>
<td>equation of state</td>
<td>27, 40, 104</td>
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<tr>
<td>EXFOR</td>
<td>Experimental Nuclear Reaction Data</td>
<td>114, 115</td>
</tr>
<tr>
<td>FFAG</td>
<td>fixed field alternating gradient</td>
<td>11</td>
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<tr>
<td>FLYCHK</td>
<td>generalized population kinetics and spectral model for rapid spectroscopic analysis for all elements</td>
<td>104</td>
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<tr>
<td>FWHM</td>
<td>full width half max</td>
<td>xii, 43, 89</td>
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<td>GHOST</td>
<td>Glass Hybrid OPAP System Testbed</td>
<td>iv, xi, 43, 67, 72, 78–80, 87</td>
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<tr>
<td>HED</td>
<td>high energy density</td>
<td>25</td>
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<tr>
<td>HEDP</td>
<td>high energy density physics</td>
<td>25, 110</td>
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<tr>
<td>HV</td>
<td>high voltage</td>
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<tr>
<td>IAEA</td>
<td>International Atomic Energy Agency</td>
<td>11, 104, 114</td>
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<tr>
<td>ICE</td>
<td>internal combustion engines</td>
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<tr>
<td>ICF</td>
<td>Inertial Confinement Fusion</td>
<td>15, 23</td>
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<td>IP</td>
<td>imaging plate</td>
<td>xvi, 69–75, 91</td>
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<td>IR</td>
<td>infrared</td>
<td>48, 87</td>
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<tr>
<td>ITDB</td>
<td>Illicit Trafficking DataBase</td>
<td>11</td>
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<tr>
<td>ITS</td>
<td>Integrated Tiger Series</td>
<td>117</td>
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<tr>
<td>LINAC</td>
<td>LINear ACcellerators</td>
<td>35</td>
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<tr>
<td>LMD</td>
<td>Lee-More-Desjarlais resistivity</td>
<td>39</td>
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<tr>
<td>LPI</td>
<td>laser plasma interaction</td>
<td>11, 32, 34, 35, 38–40, 78, 80, 109, 112</td>
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<tr>
<td>LSP</td>
<td>large scale plasma</td>
<td>v, 33, 35, 78, 80</td>
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<tr>
<td>LTE</td>
<td>local thermal equilibrium</td>
<td>41, 104, 112</td>
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<tr>
<td>MCNP</td>
<td>Monte-Carlo N particle</td>
<td>9, 10, 74, 113, 117</td>
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<tr>
<td>MIRV</td>
<td>Multiple Independently-targetable Reentry Vehicle</td>
<td>14, 15</td>
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<tr>
<td>MOAB</td>
<td>Mother Of All Bombs</td>
<td>14</td>
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<tr>
<td>NIST</td>
<td>the National Institute of Standards and Technology</td>
<td>111</td>
</tr>
<tr>
<td>NOₓ</td>
<td>mono-nitrogen oxide</td>
<td>104</td>
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<tr>
<td>OAP</td>
<td>off axis parabola</td>
<td>43</td>
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<tr>
<td>OPCPA</td>
<td>optical parametric chirped pulse amplification</td>
<td>iv, xxi, 43</td>
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<tr>
<td>PET</td>
<td>polyethylene terephthalate</td>
<td>70</td>
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xxi
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<tr>
<td>PIC</td>
<td>particle-in-cell</td>
<td>33, 35, 41, 76–78, 92</td>
</tr>
<tr>
<td>PMT</td>
<td>photo-multiplier tube</td>
<td>73</td>
</tr>
<tr>
<td>PSL</td>
<td>photo-stimulated luminescence</td>
<td>70, 74, 89</td>
</tr>
<tr>
<td>SI</td>
<td>Systèmes des unités International</td>
<td>4, 29</td>
</tr>
<tr>
<td>SRIM</td>
<td>the Stopping-power and Range of Ions in Matter</td>
<td>10, 74, 75</td>
</tr>
<tr>
<td>TCC</td>
<td>target chamber center</td>
<td>44</td>
</tr>
<tr>
<td>TNSA</td>
<td>target normal sheath-field acceleration</td>
<td>ii, xi, 13, 16, 20, 22–24, 26–29, 34, 40, 41, 45, 46, 76–78, 86, 91, 92, 112</td>
</tr>
<tr>
<td>ToF</td>
<td>time of flight</td>
<td>67</td>
</tr>
<tr>
<td>TPS</td>
<td>Thomson parabola spectrometer</td>
<td>ii, v, xi–xiv, 45, 51, 54, 55, 57, 58, 67–69, 71, 74, 88, 89</td>
</tr>
<tr>
<td>UHV</td>
<td>ultra-high vacuum $\sim 10^{-8}$ Torr</td>
<td>23, 41</td>
</tr>
<tr>
<td>UV</td>
<td>ultraviolet</td>
<td>48, 87</td>
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## List of Constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Constant</th>
<th>Value</th>
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<tr>
<td>$a_B$</td>
<td>most probable distance between an electron and proton in a hydrogen atom $\frac{4\pi\varepsilon_0h^2}{m_ee^2}$</td>
<td>$1.67262 \times 10^{-11} \text{ m}$</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of light</td>
<td>$2.99792 \times 10^8 \text{ m/s}$</td>
</tr>
<tr>
<td>$e$</td>
<td>elementary charge (of an electron)</td>
<td>$1.60218 \times 10^{-19} \text{ C}$</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>the permittivity of free space</td>
<td>$8.85419 \times 10^{-12} \text{ C/Vm}$</td>
</tr>
<tr>
<td>$e$</td>
<td>number such that $\frac{d}{dx}(e^x) = e^x$</td>
<td>$2.7182818$</td>
</tr>
<tr>
<td>$h$</td>
<td>Planck’s constant $h = 1243eV nm$</td>
<td>$6.62607 \times 10^{-34} \text{ kg m}^2/\text{s}$</td>
</tr>
<tr>
<td>$\hbar$</td>
<td>reduced Planck’s constant $\frac{h}{2\pi}$</td>
<td>$1.05457 \times 10^{-34} \text{ kg m}^2/\text{s}$</td>
</tr>
<tr>
<td>$k_B$</td>
<td>physical constant relating probabilities of individual particles having specific energies $\propto \sqrt{E}e^{-E/k_BT}$ and the temperature observed at the bulk level $8.61733 \times 10^{-5} eV/K$</td>
<td>$1.38065 \times 10^{-23} \text{ J/K}$</td>
</tr>
<tr>
<td>$E_H$</td>
<td>the hydrogenic ionization energy (13.6 eV)</td>
<td>$2.18 \times 10^{-18} \text{ J}$</td>
</tr>
<tr>
<td>$m_e$</td>
<td>mass of an electron (0.511 MeV/c$^2$)</td>
<td>$9.10938 \times 10^{-31} \text{ kg}$</td>
</tr>
<tr>
<td>$m_u$</td>
<td>mass of 1/12 of a carbon-12 atom (931.50 MeV/c$^2$)</td>
<td>$1.66054 \times 10^{-27} \text{ kg}$</td>
</tr>
<tr>
<td>$N_A$</td>
<td>the number of particles in a mol, nearly the number of protons in a gram</td>
<td>$6.02214 \times 10^{23}$</td>
</tr>
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</table>
Generally speaking this work is aimed at the starting graduate student from the scientific or possibly engineering community as they are the only audience with a probability of reading this with the intent of learning something from it. Depending upon your proficiency with the field, my chosen level of complexity may be too low or insufficient n. I hope this is informative and somewhat complete if one includes the extensive body of work referenced and cited. For those that feel strongly that scientific communication must be strictly professional, free of sarcasm, and free of personification, ignore the rest of this preface and forgive many of the footnotes as this preface is it not part of the scientific content of this work.

Help From . . .

There is something I need to point out. Thank you to those who have gone before me. Oh, on the shoulders of giants and all that. Drake - Wonderful overview though my edition is somewhat error prone. Kruer - Loving examples of lasers into pre-plasma. Gibbon - Well done sir. Feynman - Dear lord what an example to try and follow. J.C.A. Wevers - bless you and your generosity, but a little more explanation would have helped. Also, the extended body of work found in citation.

To the executives of all stripes: a breech of etiquette

Some may label me a tree hugger, or possibly a hypocritical one as I’ve recently fooled the electronic control module (ECU) in an ’87 Toyota Supra to nearly double the fuel consumption under load. Power to weight ratio is a wonderful thing. Maybe the next hack will be lighter. That aside, we have a serious issue with the current energy economy. Right now things look quite good for positive motion in the energy industry. There seems to be an awareness of the problem. However, due to whatever reason you like to cite, the major players wish to be a big part of the problem. I believe if we want to help them I imagine the military, the oil & gas companies, etc. would all like to be a part of the set of solutions
by the time they are implemented.

The oil industry definitely needs to be part of the solution. The average life time of an oil field seems to be about thirty years to crest in production. We are finding fewer and fewer new fields and utilization of further fields such as the Tar sands or deep sea sources require and increasing fuel cost. Utilizing strategic reserves like ANWAR belittles their purpose but would generate revenue.

Worthy of note here is that demand for energy is looking to pace close to population unless significant economic costs/incentives and supply/demand and possibly social/media pressures. So captain obvious would like to have the floor. Yes, with the introduction of fracking (natural gas fracturing) we have begun to secure a large new energy source that is quite portable, with huge reserves. If we were to adopt this for transportation, it only delays the supply problem and exacerbates the consequences of a change in the second derivative of fuel supply without the proper social, economic, and industrial preparations.*

To the young activist

Some advice. Please understand that an appeal to "If only people could make the right choices . . ." is useless. It doesn’t matter. When one tries to use this as an agent of change, it is a sapling in the path of an elephant. For the majority of your elders, especially those with children, decisions are rarely made on the basis of right and wrong, they are mostly made based on damage control and the allocation of the most precious resources - time & money. To effectively try and alter the habits of those who control the wealth[1] appeals to right and wrong are not particularly effective.

To the mystics, priests

The lure of the darkside. Peace, it is always the leading argument it uses. Protection. The constant conflict of differentiation and the desire for love and progeny, and those that proceed us.

Fulfillment of these things and the rationalization thereof is the path. The desire for wealth is not the root cause I hope. The seven sins all sit opposite and mingle with the Commandments. The cleverness is in the presentation.

To my Daughters

Thunder Cats, Voltron, Iron Man, The Avatar, Star Wars and Star Trek are all better than Bratz. I’ll withhold judgment on the Looney toons.

*I think this argument can be made without the appeal to global warming. Which is such a hot topic that it can put people off instantaneously. I don’t think it needs to be used because the consequences of the energy crash are severe enough to convince people to begin changing.
Math and knowledge are powerful things. The world you live in is amazing in so many ways. I promise. In many ways we are trying to be the Federation somewhat successfully. But so unsuccessfully and for that I apologize. I hope in the future I can honestly say, "I've done all I can do. . ."

Majors

To those unfortunate/fortunate loving/dangerous talented/frightening individuals that may have reason to read this. The picture of knowledge as an expanding sphere into the unknown where current research endeavors are the shell works well. To push the analogy beyond its usefulness, remember that what can be found on the Internet, though phenomenal, is a smattering of the knowledge (correct and incorrect) and often poorly connected or explained. There is an elegance, depth, legitimacy, and scope to the old-school storage media. Also note as that shell expands the systems of interest are getting increasingly complicated and cross-disciplined. When faced with problems of grand scope, those giants that past before you may have not yet migrated to the web. Find the Library of Congress Catalog location of a book that discusses your topic/problem and physically walk to stacks* and much of your problem is discussed, solved, or elucidated in the works physically near your book. You can probably even pick the level of understanding/complication you wish.

The other side of this argument hardly needs to be stated. The entire patent database is on-line.† There are obviously several one stop shops for a first cut at a topic that don’t need to be mentioned.

How to. . .

Problem solving - Top down or bottom up? Our societal mechanisms for reward are a bit lopsided in the business world and broken in the corporate world. Business has made an art of optimizing resources to the maximal advantage of a subset of individuals. Instead of being a traditional useless activist, and elucidate problems and rail against status quo, I’ll try and enlist my readers to a different paradigm for resource allocation, motivation and reward.

Something the engineers do better than the physicists. Crowd sourcing. Possibly, the nature of problems makes giving genuine portions of the work to undergraduates and fresh graduate students insufficient. Crowd sourcing could and should be one of the most transformative tools brought to bear to improve social constructs/contracts. The beginnings of this are already growing. What they lack is direction and reward. I wish for a return to

*Provided society hasn’t decided they are useless yet, and you have access to a decent one.
†The patent database is an interesting terrain full of ridiculous ideas, clearly indefensible patents due to obviousness or are too broad, and yet still contains nearly all of the gems that are the genesis of modern technology.
“We the people. . .” but mean it. Terminology and legality can come later. First I will describe an entity that I wish to pit against the traditional corporate structure.

Individuals should be able to contribute where ever they are interested and capable. They should be rewarded for and by their contribution. What really drives society forward? Work. Real work. Ideas, designs, fabrication, resources, production, programing, education. This is what needs to be rewarded. Decisions, I think, a properly tempered and communicating mob has that in spades. Especially those that care. Those that care are usually motivated and self-selected, give them the opportunity and the incentive to contribute and they will work for you more than any carrot & stick you can find.

A corporation can be set up internally in nearly any way one chooses. Ownership of the company gives you access to the profit made off the backs of the employees. A model that has been tried before is employment being equivalent to ownership. I wish to make it more loose: investment, contribution. Unfortunately, the devil is in the details.

Scales

Of fundamental importance to problem solving is an understanding of scale. If you can get the scaling law, and a handle on one fixed scale, you have much of the problem solved and something to begin hanging your intuition on. For example, our Systèmes des unités International (SI) unit energy the joule, $J$ is not particularly useful for considering single particles. The old stand-by electron-volt, $eV$, is the energy gained or lost by an electron traversing a one volt, $V$, potential. This requires the charge of an electron $e^-$ for scale. For clarity the natural exponent $e \neq e$. The reduced Plank’s constant $\hbar$ scales the energy of a photon $E = h \omega$ and the speed of light $c = 2\pi \omega \lambda$ allows one to get a handle on how the photon’s angular frequency $\omega$ and wavelength $\lambda$ are related to its energy, but doesn’t really place any photons we are familiar with. A good place to start is with a firm reference point such as Plank’s constant $h$ in units one uses with lasers: $1243\ eV\ nm$ or $1.243\ eV\ \mu m$. A more colorful set of references is in Fig. 1.

When contemplating ionization $J$ is uninformative and again commonly favor $eV$ as it is comparable to ionization energies like the hydrogenic ionization energy $E_H = 13.6 eV$. Another fundamental constant, the Boltzmann constant $k_B$, relates the energy of individual particles to the temperature $T$ such that $k_B T \approx$ the average energy. Often working with the particle picture it is convenient to work with $k_B T$ as an energy scale in $eV$, $keV$, or $MeV$. Though for reminding oneself of just how hot these distributions are a good reference is room temperature which is $\sim \frac{1}{40} eV$ or equivalently, a one $eV$ temperature is $\sim 12,000 K$. Everything is a gas and may be partially ionized. A rich cherry red chunk of steel is certainly hot, or $\sim \frac{1}{10} eV$.

Speeds are also important to get a handle on what is going on. The $c \approx 3 \times 10^8\ m/s$ is
Figure 1: Scale references for the electromagnetic spectrum adapted from the web-comic xkcd[2].

not particularly informative. In terms of the lab, 30 \( cm/ns \) or about a foot per nanosecond helps visualize long pulse (ns) lasers as a few feet long. Many lasers are optical, so knowing how far light travels in an optical cycle is also useful: \( c \approx \frac{1}{3} \mu m/fs \).

Another critical scale is the natural frequency of a plasma plasma frequency \( \omega_p \). Bulk electron oscillations in plasma were first studied by Tonks and Langmuir in 1929, and it is not uncommon to see them referred to as Langmuir waves[3]. (Apparently Tonks drew the short straw.) Plasmas are generally quasi-neutral clouds of electrons and ions which, in the steady state, will have uniform density and virtually zero internal electric fields. For small motions of the electrons, the ions can be treated as fixed, due to their much greater mass. Introducing a perturbation in the electron density changes the local potential, which introduces a force on the electrons opposing the density gradient. These electrons will accelerate towards the exposed charge of the background ions. Their kinetic energy carries them past an equilibrium position, setting up another density gradient. This
motion repeats, thus setting up an oscillation. Given the scale governing the range of the
electrostatic force the permittivity of free space $\epsilon_0$, Tonks and Langmuir showed that for a
plasma with and electron number density $n_e$ small variations the characteristic frequency
or plasma frequency is $\omega_p = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}}$ in S.I. units and the group velocity is zero.

When the electron number density $n_e$ is high enough that the natural frequency of the
plasma matches the driving frequency of the laser, the electrons can re-radiate or reflect
the laser light. This is commonly referred to as critical density $n_c$. For 800 nm light
($\sim 60 \text{THz}$) $n_c = 1.6 \times 10^{21} \text{cm}^{-3}$ which helps explain why metals* like Aluminum† with
$n_{Al} = 1.8 \times 10^{23} \text{cm}^{-3}$ are opaque.

Ion acoustic waves similarly have an associated frequency and speed, both unsurprisingly
slower as each nucleon’s mass is $\sim 1800$ times larger than the mass of an electron $m_e$. The
convenient mass for ions is how many nucleons or atomic mass $A$. Variations in the binding
energy per nucleon‡ demands that a specific example is required to for a unit, and so $1/12$ of
$^{12}C$ is the atomic mass unit $m_u$.§ Though it does require the assumption that the ions
are much colder then the electrons, the ion plasma frequency $\omega_{pi} = \sqrt{\frac{e^2 n_e}{\epsilon_0 A m_u}}$.

Murphy’s Law

The simplest laser based ion acceleration experiment requires one to first build a laser.¶
This is by far the lion’s share of what one needs to do from an empty room. After that
all you need to do is put a foil target $\lesssim 10\mu\text{m}$ in front of it diagnose the laser, and set up
diagnostics.

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*One can argue that the inside of a metal, or any conductor, is an example of a high density low
temperature plasma. A more strict interpretation notes that the ions are not in-fact mobile.
†If one assumes the three valence electrons are free.
‡Iron has highest binding energies per nucleon which can explain why light nuclei tend to fuse (fusion)
and heavy nuclei tend to split (fission).
§So $A$ is the mass in g of a mol or Avogadro’s number $N_A$ of atoms and $Am_u$ is the mass of a single
atom.
¶Seriously this is broken. With the technology available today, these things should be bullet proof. They
are currently anything but. Part of the problem is that everyone is crazy. Let’s build a car and run it
continuously at full throttle or brake for 100,000 miles? No. That is how we run Semis, however.
Figure 2: Expect time and money to be lost to the expected unexpected. There’s a reason Murphy’s Law persists. You may lose time to an unnoticed sign error in your calculation, or... (a) is an example everyone who has spent some time in a laser lab is familiar with. Here, specifically are damages on the surface of an ND:YAG rod (gain medium) in one of the long pulse pump lasers. (b) Setting up diagnostics often requires shipping some components, which only requires some appropriate packaging. Your package will fall off the truck. (c) Other people have bigger problems. This is Cairo, Illinois from the air during the 2011 flooding of the Mississippi (darker) and Ohio rivers (lighter) while en route from my experiments in Texas. Cairo was possibly saved by breaching a near by levy and flooding vast nearby areas of less densely populated farmland.
Chapter 1
INTRODUCTION

1.1 Why do we care?

The potential benefits for fundamental research on laser-based ion acceleration include issues and problems of great importance to society. The potential exists for a controllable source of macroscopic quantities of relativistic particles which is considerably more compact than current sources. I begin with one description of a personally relevant application that we can all agree is worthy: cancer therapy. A second, more directly related potential application is the development of a neutron source for non-destructive evaluation of components or identification of nuclear materials.∗

1.1.1 Ion Beams: Cancer

Cancer is one of those things in hindsight I didn’t want to learn about.† Of all the things that frightened me as a child, things that movies have thought up and told me in adolescence and young adulthood, cancer holds a special place as both disturbing and probable. Simplistically, cancer is a direct result of cellular evolution. Some cell’s deoxyribonucleic acid (DNA) is altered which grants it with a growth advantage in its habitat. Most of the time, the collection of human cells and symbiotes that make our immune system and us purge these cells or they self destruct before the new strain becomes a problem. If it does not trigger a reaction from the habitat a tumor develops from most cell types. Both remarkable and disturbing is that the stain comprising the primary tumor acquires or evolves the ability to grow within other habitats within the body. This metastasis occurs frequently enough in


†This is my current understanding as a layman, through a not insignificant amount of reading, listening, and several conversations with oncologists.
cancer cases that the extent of the spread is medically classified or described as stages. Yet this commonly involves the cancer cells acquiring new abilities such as an acidic coating to penetrate dividing membranes within the body, or the ability to metabolize and survive in new environments; this is amazing, terrifying evidence of the mechanics of an evolutionary world more than 6000 years old.

The idea/reality of cancer affects me more on a visceral level than I would care to admit. At the same time, the path can be merciful in the opportunity for family and patient to cope with the nearing of the end of life. Recovery is uncertain, as put in an excellently blunt and sympathetic comparison to Beer’s law in xkcd. Exponential decay describes your chances of survival and the length of remission before a relapse. Increasing that half-life which commonly coincides with a lack of symptoms is a most noble of goals.

To this end a host of drugs/poisons and treatments have been devised to attack the types of cells that the primary tumor like evolved from with a hopefully limited effect on healthy tissue. This chemotherapy is often the first treatment even before surgery and has developed into an impressive array of treatments of varying effectiveness depending upon the type of cancer. After chemotherapy and surgery, radiation therapy is often employed and can increase the half-life of remission by a factor of two or better.*

The current radiation therapy technology employed is based on a linear accelerator, slinging nearly mono-energetic electrons into a bremsstrahlung target producing an x-ray spectrum. This source causes little collateral damage to surrounding tissue for surface tumors. For internal tumors a higher energy is optimal and the characteristic exponential decay of the depth-dose curve at greater depths means significant collateral damage is unavoidable. To the industry’s credit, the source can be swung around the patient, limiting the severity of the collateral damage but increasing damage extent[4]. High energy protons, due to the Bragg peak and narrow scattering divergence, deposit energy at a significantly more specific depth as shown in figure 1.1.†

With the hope of dramatically reducing the total dose of radiation and associated collateral internal burning (see fig. 1.1), hadron therapy has been receiving clinical interest.

*This is based on various collection of patient information pamphlets which report second-hand a reduction in the chance of relapse over a period of time by 40 – 70%.
†When using MCNP, one must be careful to be aware of how the code calculates values. A mistake/lack of foresight that wasted my time, and made it past two Phys. Rev. B reviewers[5] in a paper using the Deposition of Energy due to Electrons and Protons (DEEP) code. When calculating the energy deposition one may think that simply tracking the net particle/energy flux through a surface would yield the energy crossing the surface and by extension the energy deposited in a box of surfaces. Unfortunately, if a large portion of the particles are crossing a surface at a shallow angle they may not move normal to the surface enough to be counted. My Monte-Carlo N particle (MCNP) calculations yielded remarkably similar results to Udalagama[5] showing a hollowing of the electron beam as it scatters through the material. Upon review the MCNP manual explicitly warns against tracking flux across a surface when particles cross with a shallow angle (≲ 10° for MCNP) as they are not be counted properly. MCNP recommends using a specified routine (tally) to calculate the energy deposition which considers integrates both photon and electron energy deposition. This counts the particles entering or leaving a box, not crossing surfaces.
Figure 1.1: The al al distribution of the deposited energy or dose (log) and dose vs. depth \( dE/dz \) (linear) are shown for several particle types in water, which is a good approximation for human tissue. The proton deposition was calculated using SRIM. The photon, electron, and bremsstrahlung source (using a 1 cm converter and 1 mm diameter, 3 cm lead collimator) depositions were calculated using MCNP. At the time of writing, 30 MeV linear accelerators using either the electrons directly or photons from lead bremsstrahlung converter are commonly used in radiation therapy. Protons have the advantages reducing surface burning and depositing much of their energy at a limited depth, limiting collateral damage to surrounding tissue. To lowest order, both electrons and ions lose energy \( \propto v^{-4} \), the ions have dramatically lower scattering divergence.
Yet the larger cyclotrons or synchrotrons, more expensive devices require significantly more investment and as such, the number of these facilities remains in the single digits. Producing a usable, narrow energy band ion beam from a more cost effective source to reduce harrowing the patient, is a hallowed pursuit.

To this end new exotic accelerator types (fixed field alternating gradient (FFAG) accelerator[6], dielectric wall accelerator (DWA)[7], superconducting cyclotrons[8] like those at IBA are being developed. Among these, laser-based energetic ion sources enjoy a unique position. The laser has a variety of benefactors and since the development of chirped pulse amplification (CPA) in the early 80's[9], the technology of short pulse lasers has improved the power density significantly. The peak intensity has increased in an exponential fashion similar to Moore’s law of transistor count in processors. The implementation and problems of a laser-based system for cancer therapy based on current techniques have been considered[10].

**1.1.2 Neutron (and Gamma) Source: Non-Destructive Identification/ Active Interrogation of Special Nuclear Materials**

A more direct motivation for this work is the non-destructive identification of nuclear materials. The International Atomic Energy Agency (IAEA) reports from 20-40 confirmed incidents per year involving nuclear material to the Illicit Trafficking DataBase (ITDB). The ITDB also reports a number of incidents of trafficking small quantities of highly enriched uranium or plutonium[11]. Though well below quantities sufficient to make a device, these confirmed incidents are possibly indicative of a larger quantities in black market circulation or in the possession of groups with malicious intentions.

Current techniques* for detection rely on passive detection of ionizing radiation such as Geiger counters and scintillating detectors. A better choice is nuclear capture and subsequent decay in $^3$He tubes, of which the supply is famously waning. Regardless of the operation, sensitivity, and care taken in the read out of these detectors, they all run into a fundamental limit: background radiation.† Levels of background radiation vary depending on local concentrations of radioactive materials, such as Rn, and shielding from local architecture‡. As the flux of radiation from a source becomes similar to the flux of background radiation, it becomes difficult to impossible to confirm the presence of the source.

*To this authors knowledge limited to unclassified material.
†$^{13}$He tubes are significantly less sensitive to background photons as they can discriminate by only counting events with the characteristic 0.76 MeV decay.
‡Concrete and re-bar construction reduces exposure to cosmic radiation and other natural sources. While determining if shielding from radiation generated in laser plasma interaction (LPI) experiments was adequate we inadvertently discovered that the building provided enough shielding that a dosimeter placed near an exterior window received a higher dose than those stuck on or near the target chamber.

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For any relevant quantity of nuclear material being carried by a pedestrian, passive detection is likely to be sufficient as shielding of the nuclear material is difficult to carry. Currently, commercially available passive detection systems for vehicles improve sensitivity by forcing traffic through a portal where shielding can be installed to reduce background radiation from other directions. Useful applications are for scrap metal recycling to prevent contaminated material from being sold as scrap. Detection of cross border trafficking of nuclear material for weapons introduces a new set of difficulties if one assumes the trafficker to be moderately intelligent∗ and intends to defeat detection attempts.

At border crossings or ports involving vehicles and shipping a detailed inspection of each vehicle or container is not feasible, and the cavities are large enough to contain nuclear material and sufficient shielding in the form of steel, lead, and/or plastic to defeat passive detection by reducing the levels of emitted radiation to well below background levels. To this end, active interrogation of special nuclear materials is of interest.

Active interrogation can drastically improve signal discrimination by increasing the activity of special nuclear materials creating energetic short lived states. Temporal gating of the detectors, and the stimulated activity dramatically improves the sensitivity of a detection scheme. Though there are a variety schemes utilizing a variety of reactions[12–14], the vast majority of these utilize energetic γ-rays or neutrons due to their penetrating power. High-intensity laser-based sources have the potential to provide both in a single device. Specific to this work is interest in the demonstration of the feasibility of a laser-based neutron source. The basic scheme is to utilize laser accelerated energetic ions in fusion and other neutron producing nuclear reactions(figure 1.2). Utilizing deuterons in a \(^7\text{Li}(D,nx)\) reaction, has significant advantages over protons in absolute yield as discussed in detail in appendix B.

![Diagram](image)

Figure 1.2: Scheme for utilizing short pulse high-intensity lasers to generate a neutron source. A primary target foil is used to accelerate ions to produce neutron producing nuclear reactions in a secondary foil.

∗As some of these incidents have involved nuclear scientists this is not unreasonable
Neutron (and Gamma) Source: Non-Destructive Evaluation of Critical Components

A similar and related application is non-destructive evaluation of critical components. For the majority of engineered components, after determining the requirements one designs with a factor of safety\(^*\) sufficient that small defects such as those common in cast components will not effect the durability of the finished product. Other applications, such as any shear pin, require significantly greater tolerances on the material strength. Internal defects can weaken the finished component compromising its effectiveness. Turbines on commercial airliners are mounted to the wing with shear pins intended to fail before the wing is removed if a catastrophic vibration develops in the turbine.\(^†\) The presence of significant internal defects could cause such a failure when unwanted: during severe turbulence.

Non-destructive evaluation (NDE) of aerospace components, specifically turbine components is an important part of the quality control process. Inspection of the cooling passages in turbine blades has been accomplished with neutron radiography [15]. Blades are cast with a ceramic mold (such as Y\(_2\)O\(_3\)) which later is removed to form internal air passages. Removal of the core is not always complete, compromising the effectiveness of the cooling passage. Thermal neutrons pass through the blade material without much interaction but are readily absorbed by gadolinium, which can be a dopant in the mold material. Because of this, thermal neutron radiography has been the tool of choice for locating leftover core material inside the blade without destroying the blade. Neutron sources with various characteristics can also be utilized to detect corrosion, cracks, and other internal defects in a variety of aerospace components [15]. However these sources are difficult to produce and often require a research nuclear reactor. Though MeV neutrons can be produced, sources with large source sizes require collimation to achieve the image resolution required for many applications. Laser-based X-ray and particle sources provide some of the most advanced sources for non-destructive evaluation - owing to their relatively small size, potential to tailor source spectral characteristics, small particle source size, and therefore low divergence without collimation.

At a laser intensity of \(10^{19}\ Wcm^{-2}\), the TNSA ions have a thermal-like distribution with mean energies exceeding 1 MeV. These ions can initiate nuclear reactions. For example, the deuterium ion bombardment of a deuterium rich material generates 2.45 MeV neutrons through the D(d,n) reaction. Deuterium ion bombardment of Li leads to neutrons with a distribution of energies through the Li(d,n) reaction. Reaction choice is discussed in greater detail in appendix B. Since these neutrons are emitted from a volume whose dimensions are

\(^*\)from 2 upwards of 10 depending on the application. Pressure vessels, for example, are required by law to survive stress tests with pressures 3.5 times the rated pressure.

\(^†\)Nationwide Airlines’ Boeing 737 Flight CE723 landed safely after such an incident.
on the order of the laser focal spot (\(\sim \text{mm}\))∗ and on time scales on the order of laser pulse duration (\(\sim \text{psec}\)), they have the potential to be a source for noninvasive detection of fissile materials, with unprecedented spacial and temporal resolutions. Recent work discusses the optimization of these processes [16–18] and is summarized in appendix B.

1.2 An aside for my conscience

I feel the need to explain and defend my support for the acquisition of knowledge, documentation, and study of the fusion reaction in its various guises, though my conscience is clear for the above reasons. The application to destruction has been played out, as should be no surprise when one considers the scope of the resources that we have applied as a nation. To those who are uncomfortable with fusion as an avenue of research and to whom National Labs are only weapons labs, much of the knowledge required for the various applications overlap. Consider table 1.1: a list of applications as a function of reaction size in terms of number of particles \(N\) of neutrons and the energy \(E\) released.

The thermo-nuclear devices in table 1.1 are approximately accurate for the US, probably the UK, Germany, France, and certainly Russia. Scaling up the fusion reaction for eventual use as a weapon is absurd. Consider that the MIRV is preferred to one immense warhead. Decreasing in scale, the tactical nuclear warhead and the conventional Mother Of All Bombs (MOAB) already exist, and have advanced beyond new fundamental or even applied research undertaken by academic physics. Decisions pertaining producing and using these weapons are well into the hands of engineers, military officers, and policy makers. In contrast, fusion reactions of a useful size are unfortunately still in comparatively early stages when compared to use as a weapon.

1.3 Short pulse laser ion acceleration: A Summary Introduction

As mentioned above in motivating this work, high-intensity short-pulse lasers when incident upon a thin foil target, provided various conditions are met, will accelerate the target surface, producing an ion beam with a thermal-like energy distribution with \(MeV\) temperatures perpendicular to the target surface[20–22].† Without§ effort these ion sources

∗The many \(MeV\) neutrons may benefit from a virtual source size smaller than the interaction region if the center of momentum frame is significantly boosted and the virtual source size of the ions (\(\sim\)microns) becomes important.

†barring a Hollywood level redirection of national interests

‡I include Maksimchuk here because these are all experimental results. Though his proposed mechanism was later modeled[23] and subsequently shown[24] to not be the dominant mechanism, his paper still shows conclusive evidence for an ion beam and a suggested mechanism. Exactly what is expected from experimental science.

§and often despite
<table>
<thead>
<tr>
<th>Application</th>
<th>Type of Problem</th>
<th>Energy released</th>
<th># of Neutrons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cargo Scanning</td>
<td>Extensive Applied Research</td>
<td>$\sim 0.3 \text{ J}$</td>
<td>$\sim 10^{11}$</td>
</tr>
<tr>
<td>Remote Contraband detection</td>
<td>Fundamental Research &amp; Technological Advancements</td>
<td>$\sim 30 \text{ J}$</td>
<td>$\sim 10^{13}$</td>
</tr>
<tr>
<td>Inertial Confinement Fusion (ICF) demonstration</td>
<td>Science Experiment (We still don’t have one car across the bridge)</td>
<td>$\sim 30 \text{ MJ (14 lbs of TNT)}$</td>
<td>$\sim 10^{19}$</td>
</tr>
<tr>
<td>Fusion Energy</td>
<td>Still 30 years out after 40 years of effort†</td>
<td>$\sim 1 \text{ GJ (0.2 Tons)}$</td>
<td>$\sim 3 \times 10^{20}$</td>
</tr>
<tr>
<td>Hiroshima</td>
<td>Used (Fission)</td>
<td>$\sim 65 \text{ TJ (15 kTons)}$</td>
<td>$\sim 2 \times 10^{24}$</td>
</tr>
<tr>
<td>Multiple Independently-targetable Reentry Vehicle (MIRV)</td>
<td>Yes we made this.</td>
<td>$3 - 10 \times \sim 1600 \text{ TJ (390 kTons)}$</td>
<td>$\sim 1.5 - 5 \times 10^{26}$</td>
</tr>
<tr>
<td>Remove 1 km diameter nickel asteroid to save earth</td>
<td>Hollywood level redirection of national interests</td>
<td>$\sim 5 \times 10^{18} \text{ J (1.5 GTon)}$</td>
<td>$\sim 1.5 \times 10^{20}$</td>
</tr>
<tr>
<td>Remove Moon</td>
<td>Engineering problem</td>
<td>$\sim 10^{22} \text{ J (2500 GTon)}$</td>
<td>$\sim 3 \times 10^{32}$</td>
</tr>
<tr>
<td>Stars</td>
<td>Givers and takers of life (Solved-ish)</td>
<td>sustained burn many consumed</td>
<td></td>
</tr>
<tr>
<td>Supernovas</td>
<td>Nature actually does this (Solved-ish)</td>
<td>Type 1a</td>
<td>$\sim 5 \times 10^{55}$</td>
</tr>
</tbody>
</table>

Table 1.1: These are simplistic approximations for illustrative purposes only. The ICF yields follow Rosen [19]. Asteroid and moon removal based on 20% thermal coupling and raising the entire body to $1 \text{ eV}$. 

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are predominantly protons. The ion beam generated also retains features of the target surface in addition to being produced along the global target normal\cite{25, 26} and propagates in a laminar fashion as if it was produced from a source about the size of the laser focus (\(\sim 20\mu m\)), much smaller than the physical source size of 100 – 200 \(\mu m\) on the rear of the target\cite{27}.

The simplest model consistent with the basic characteristics of such a source is illustrated in figure 1.4. The laser generates a population of extremely energetic electrons which pull that target apart. Looking back into the literature, the fundamental mechanisms for the generation of these beams has been previously explained. Explosions, even some of the most violent, even ionizing, could be predicted reasonably well using shock dynamics, a subset of fluid dynamics, assuming that the electrons and ions remain in quasi-static thermal equilibrium\cite{28}. Specifically of interest is what happens when a shock wave reaches a dramatic drop in density.

1.3.1 Rarefactions

Rarefaction waves are what wasn’t mentioned in introductory mechanics or thermodynamics when one is asked to consider the expansion of a gas. The quasi-static assumption is made more often than it should, but continues to be made as the simplifications are immense and the results are often reasonably accurate even when the assumption is clearly false. Yea, that problem, work being done by an expanding gas on a piston. What happens when the speed of the piston is on the order of the speed of sound \(C_s\) in the gas?\footnote{Alternatively, when the length of the cavity is sufficient that a single pressure cannot describe the gas because gas closer to the piston has expanded more than gas further away. This case is handled in detail in Zel’Dovich and Raizer I.7\cite{28}.} Relevant to this work, is the extreme case when the conditions for expansion are generated on a much shorter time scale than the expansion itself, which is analogous to the piston disappearing instantaneously.\footnote{For atmospheric pressure, the piston need only exceed speed of sound \(C_s \approx 343m/s\) initially and accelerate to remain ahead of the expansion front. The conditions for the TNSA expansion are created by the first relativistic electrons reaching the rear of the target, effectively removing the piston at the speed of light.}

So what does one expect intuitively of the distribution after the piston disappears? The fluid initially in contact with the piston should expand under the influence of its own internal pressure and should be accelerated as the pressure exerted by the bulk medium is no longer balanced by contact with a rigid wall. As the surface begins to move and expand, material immediately behind also now has unbalanced pressures and it should begin to accelerate toward the missing piston, and begin to expand. For subsequent times, a drop in pressure moves into the medium including more and more material in expansion. The initial drop in the density, or rarefaction head, moves into the medium at whatever speed
pressure disturbances travel in the initial medium. The speed of sound \( C_s \) is this speed which communicates the lack of a piston to the rest of the medium.

The gas initially in contact with the piston has the most time to accelerate and expand. This surface has the lowest density and temperature, and the highest velocity. The conditions and velocities of the expanding gas should gradually approach that of the unchanged medium nearer to the rarefaction head. One may guess that the properties of the medium between the rarefaction head and the expansion front follow an exponential behavior, and in fact this is the solution.

This is the basic picture of a rarefaction, or free expansion. Rarefactions are most commonly complicated by the presence of a lower density medium to move into, and are often one component of a more complicated behavior. To connect the concept to reality some examples one may have encountered or thought of before are helpful. Some fun ones - catastrophic failure of a thin boundary between two very disparate density/pressure gases or plasmas. Say, a sub millimeter thick Beryllium window fails on a vacuum chamber. If you are or were more of a social deviant, the initial disassembly of a solid density explosive in air is another example. The initial behavior of fluid near a fast acting valve opening such as an intake or exhaust valve in an internal combustion engine is another example, though the flow through the valve is limited and may dominate the system depending on the conditions. A physically different system, but having analogous behaviors may be helpful for only intuitive reasons. The height, potential energy, and velocity of water near the complete failure of a tall dam can be thought of as the density, temperature, and fluid velocity of a gas/plasma in a rarefaction. This discussion is expanded upon for general application yet more useful detail in chapter 2, and with more considerations specific to laser-based applications added in chapter 3.

1.3.2 Rarefactions at (extremely) high temperature

Even assuming that the electron and ions remain in thermal equilibrium, at sufficiently high temperatures \( \gtrsim 1 \) the electron mean free path between collisions becomes comparable to the scale of the expansion, and a plasma begins to behave more like two fluids coupled by electromagnetism, rather than a single fluid following the ideal gas law. In an unperturbed plasma at constant density, the soup appears to be charge neutral provided one does not

* Shocks and rarefactions are often found together like bread and butter, jam and peanut butter, hammers and saws, Electric and magnetic fields.
† Fire cracker, M80, stick of dynamite, pipe filled with gunpowder, an inappropriately rated pressure vessel filled with liquid nitrogen, sealed and left to heat. For times much greater than the time it takes the rarefaction head to reach all the fluid/plasma/gas/fuel the behavior is described more accurately as a blast wave.
‡ As with most analogies I expect this to break down with sufficient scrutiny.
§ . . or low densities for astrophysical systems. For solid density material with a sharp interface, electron temperatures on the order of a few keV have mean free paths on the order of a \( \mu m \) which is approximately the length scales of the expanding laser plasmas at early times.
look too closely. The attraction between the ions and electrons conspire to keep the positive charge of the ions shielded effective for distances $\gtrsim$ the Debye length $\lambda_D$.

In the late 60’s and 70’s the problem of what happens in the early stages of a freely expanding plasma with hot enough electrons to be considered collision-less came of interest\cite{29–32}. This was motivated both by astrophysical interest in low density large expansions\footnote{Coronal ejections, portions of super nova expansions} and by the promise of extremely rapid heating of foil targets by lasers. These discussions focus on an initially charge neutral, constant density plasma slab with a steep interface to a low density (the back of a target in a near vacuum). One can either imagine something holding this density gradient in place being removed, or as in modeling a rapid expansion, after the target is instantaneously heated\footnote{In the case of a long pulse laser incident on a foil target, this concerns time after the front of heated electrons reaches the rear surface of the target.}. If one considers the density gradient in the absence of fields, the much lighter electrons will begin to expand much more rapidly than ions of the same temperature. This separates the charges which in turn generates a quasi-static electric field. This field is large enough to trap the majority of the electrons and to accelerate ions near the surface.

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Figure 1.3: Sketch of a collision-less two species rarefaction. Initial conditions (above) after the electron population has reached a quasi equilibrium density with a self-consistent electric field, but before any ion motion. After some time $t$ (below) the plasma expands, and beginning of the expansion (rarefaction head) moves into the bulk with a characteristic speed $C_s$. The shape of the expansion at different times is similar, with the scale of $x$ increasing with increasing time. Much of the expansion remains quasi neutral except near the rarefaction head and the ion expansion front.
As with a traditional expansion, the internal energy of the medium is converted into kinetic energy. Without additional heating, the electron population will cool, reducing the potential (pressure) accelerating the expanding ions. Analogous to the pressure and pressure gradient (force) in a traditional expansion, as the surface beings to expand the gradient of the potential weakens and the acceleration is reduced. Eventually, the remaining pressure gradient is weak enough that further acceleration is negligible.

1.3.3 (Ultra-) High intensity lasers

For the purposes of this work, a high\footnote{By all rights $10^{13} \text{ W/cm}^2$ at $\sim 1 \mu\text{m}$ wavelength (near infrared) is a fantastically intense source capable of stripping electrons from pretty much anything. We commonly achieve intensities a million times greater.} intensity laser converts approximately $15 - 50\%$ of its energy to a population of quasi-thermal\footnote{Though these ‘hot’ electrons have a quasi-thermal energy distribution, the distribution is not generated by a collection of collisions well described by statistical mechanics, but by a collection of conditions related to their interaction with the laser field in the pre-plasma.} relativistic electrons at the incident surface of a solid target. Though emitted in all directions, the more energetic electrons favor the laser axis in a cone with a half angle of $\sim 30 - 60^\circ$\textsuperscript{[33, 34]}. For further details see chapter 3.

Most laser systems operating in this regime have sufficient energy in pedestal and pre-pulses (figure 1.5) to ionize and modify the surface that the main pulse interacts with.\footnote{It has been shown that the divergence of the reflected light increases as one consequence of the pre-plasma which may be used to characterize the pre-plasma’s scale length\textsuperscript{[35]}.} A $1 \text{ J}$ pulse with a mean intensity of $\sim 10^{19} \text{ W/cm}^2$ will generate $\sim 1 \times 10^{13}$ hot electrons in a thermal-like distribution well described by a sum of Boltzmanns with $\sim 1 \text{ MeV}$ average energy. The majority of the electrons lose little energy traversing the few $\mu\text{m}$ thick targets often employed.\footnote{They do lose energy to scattering and resistive stopping\textsuperscript{[36]} especially after successive passes through the target.} As greater numbers of electrons leave and travel further from the initially neutral target, the resulting charge separation, associated electric field, and potential well strengthen. Climbing out of this well the electrons lose energy, ‘fall’ back into the well and are trapped, confined to within or near the surface of the target. The electrons spread into a distribution with each initial energy is associated with a distance from the target (height on potential hill) at which it is pulled (falls) back to the target. Since the hot electron spectra is quasi-thermal, having only a thermal distribution of energies and not having completely randomized directions, the resulting number of forward going electrons decays with distance from the target surface (fig. 1.3 Top). Only a small fraction escape to infinity\textsuperscript{[37]} to be detected. Within the conductive plasma slab, any charge separation is expelled to the surface, leaving only the uncovered positive charges near the surface to accelerate in the electric field\footnote{The positively charged target has an increased electric potential: a potential energy well trapping the electrons.} generated by the charge separation. As the trapped,
Figure 1.4: Simplistic schematic explanation of target normal sheath-field acceleration. A high-intensity $\gtrsim 10^{18} \text{W/cm}^2$, short pulse laser accelerates electrons to relativistic energies generally in the direction of the laser propagation. A small fraction of the electron kinetic energy is lost traversing the target. The earliest electrons escape to infinity charging the target. Subsequent electrons with sufficient energy escape, charging the target further. The majority of the electrons are trapped in the potential generated by the escaping electrons and reflux back and forth through the target. The same potential that traps the majority of relativistic electrons ionizes, and accelerates the target surface. The surface of a target is nominally covered with hydrocarbon and water contaminants, the lightest of which are accelerated the most: protons[21, 24]. More detailed descriptions follow in subsequent chapters.
Figure 1.5: Most real Ultra short pulse high intensity lasers have pedestals proceeding the main pulse by a few to $\sim 10$ ns created by ASE, in addition to pre-pulses, similar to the main pulse. Due to imperfect pulse compression there is often also a foot around the main pulse, not shown here, down $\sim 10^{-3}$ with ps durations. Though the specifics vary, these are common features which modify the surface of the target before the arrival of the main pulse. A common assumption is to assume the main pulse interacts with a pre-plasma, exponentially decaying from the surface with a few $\mu$m scale length.

Reflexing, electrons traverse the target multiple times they lose energy to accelerating the ions, scattering, and resistive stopping\[36\]. The number density decreases as the electron divergence and consecutive passes spread the refluxing population over a larger surface area.

To estimate then number density $n_e$ of the hot electrons, consider the volume occupied by a source with a diameter of the laser spot ($\sim 10$ $\mu$m diameter), $45^\circ$ angle divergence, whose length is on the duration of the laser pulse ($\sim 100$ fs or $\sim 30$ $\mu$m). Neglecting refluxing, and assuming half of the hot electrons are in this cone, the hot electron number density $n_{eH} \sim 2 \times 10^{20}$ cm$^{-3}$. This is significantly less than solid density $n \simeq 10^{23}$ cm$^{-3}$ and so one cannot expect behavior as indicated in the preceding section and illustrated in figure 1.3. There are two distinct electron populations. A thermal or cold electron population comprising the majority of the electrons which is heated by the significantly less numerous suprathermal or hot laser generated relativistic electrons.

Two temperature electron distributions and subsequent ion acceleration was considered in the early 80’s by True and Albritton\[40\] as a result of observations of double-peaked distributions of ions from target disassembly in the mid 70’s\[41, 42\]. As elucidated in section

\*\*A variety of experimental and modeling efforts have considered this problem, with a variety of results. The majority report significant divergence of the relativistic electrons ranging from $30^\circ - 90^\circ$ or more depending upon the target, intensity, and measurement\[33, 38, 39\].

\†\†or, equivalently, one electron population well characterized by two temperatures.
2.4, at late times, the behavior of the expansion is approximately that of two independent ion expansions, each maintaining quasi-neutrality with their respective electron populations, as illustrated in figure 1.6a.

![Image: Three Species Rarefaction](image)

![Image: Independent Rarefactions (Two Temperature Ion Spectra)](image)

Figure 1.6: (a) Sketch of a three species (two species, two temperature) rarefaction. At late times the expansion can be characterized by two independent quasi neutral expansions, each similar to figure 1.3, driven by two electron populations[40]. (b) Two temperature ion spectra if the hot population was 20 times more energetic and \( \frac{1}{70} \) as numerous. Actual values and rates vary dramatically depending on the experimental conditions. In TNSA generated by relativistically intense lasers, the more energetic population has a characteristic temperatures measured in MeV and is 100-1000 times more numerous than the keV expansions of the bulk. Experimentally, spectrometers used to characterize the more energetic populations commonly do not have enough dynamic range to detect ions from the bulk expansion(figures 6.1 and 6.2).

1.3.4 Ion acceleration experiments

Though the above description is extremely simplified, it does provide some useful insights. Current TNSA ion acceleration experiments rarely identify the two temperature nature of the expansion as the complete disassembly of the target has \( \sim \) keV energies* and are often outside the dynamic range of detectors intended to measure spectra from a few to 10’s

*It could be argued that this is not true for extremely thin targets where the entire column of the target is thought to be accelerated.
or even 100’s of MeV. This also implies that consideration of the dynamics of the target disassembly have little influence on the dynamics of the ion acceleration. As a consequence of the hot electrons comprising a small fraction of the total electron density, which decreases as they spread to fill the target, only the surface of the target is accelerated. Which explains why when the first MeV ion beams observed were primarily protons[20–22]. Neglecting cleaving materials in ultra-high vacuum \( \sim 10^{-8} \text{Torr} \) (UHV) every surface including the target will be coated with a thin layer of water and hydrocarbon contaminates, of which the lightest and most easily accelerated is the hydrogen \( p^+ \).

Describing the relaxation of the target as a single species of ions has yielded a variety of insights, but clearly falls far short of a complete description. The acceleration of protons and heavy ions to multi-MeV energies by TNSA has important applications in nuclear physics [43], and Inertial Confinement Fusion, specifically fast ignition [17, 44], in addition to medicine and non-destructive evaluation.

Previous work to accelerate ions via laser-solid interactions has been hindered by the presence of contaminants at the rear surface of the target[45]. Petrov et al. (2010) have recently reported on the suppression role played by the contaminants in ion acceleration [46]. In their description, electrostatic fields arising from the hot electron sheath in excess of 1 TV/m continuously “peel off” the contaminant ions on the rear surface. The rarefaction wave, associated with the expanding ion front, moves back into the solid-density material, successively exposing new material to the fields. Since the rear surface of the laser-irradiated target is invariably coated with a contamination layer of proton-rich hydrocarbons and water, the overwhelming majority of accelerated ions originate from this layer. Since the protons have the highest charge to mass ratio, the accelerated ion population is dominated by protons, independent of the native target material.

Several attempts have been reported to suppress the acceleration of the contaminant ions. Pulsed laser irradiation of the target rear surface to produce a long scale length plasma reduced the observed proton flux by two orders of magnitude[47]. However, this reduction was attributed to a reduction in the strength of the accelerating field at the target rear surface as opposed to a process of deselecting the protons for acceleration. Heating of the target also reduces the yield of contaminant protons, yet enhances the heavier bulk target ions[48, 49]. Observations show that the peak energy and total yield of protons from the rear surface of a 50 \( \mu \text{m} \) Al target heated to more than 600 °K is reduced by a factor of around 8 to 10[48]. In situ Ar ion beam sputtering of 15 \( \mu \text{m} \) thick Au targets demonstrate a two order of magnitude reduction in the proton yield with more than a factor of two reduction in the peak proton energy[24]. Hou et al. (2011) observe 3 to 5 times enhancement in the deuteron yield by placing 1 ml of D_2O in the target chamber to generate a partially deuterated contaminant layer but also observed large numbers of protons[50].
1.4 This Work

This thesis presents the first successful demonstration of deuteron acceleration in which the proton and carbon ions are almost entirely suppressed. The deuterium ions originate from a layer of heavy ice that was deposited onto the rear surface of a 500 \( nm \) thick membrane of Si\(_3\)N\(_4\) and Al under vacuum prior to firing the laser. This represents a new approach in dealing with the contamination issue; rather than being removed or replaced, the water and hydrocarbon layer is covered. Also presented in this thesis is a simple pseudo-Lagrangian 1D model following previous work[32, 51] of the ion expansion with the intension of aiding the intuition for future studies. These simple 1D calculations indicate the possibility of accelerating protons with a narrow spectral feature using TNSA.
Chapter 2

RAREFACTION WAVES - QUASI-STATIC PLASMA EXPANSION

The physics of Shocks and high temperature expansions has its own rich field and associated areas of research investigating properties of high energy density (HED) plasmas. Fundamentally this discussion is rooted in fluid dynamics but is directly related to knowledge of material properties that arise from the particle, atomic, and/or quantum behavior of the material. Drake’s book[52] is an excellent primer, first reference for explanations, and a source summarizing progress near the turn of the century.* Zel’dovich and Raizer[28] is a detailed, thorough, and heavily theoretical work, but an excellent reference. Crow[32] and later more completely Mora[51, 53] employ similar approaches generating expansion/acceleration models working with a non-interacting electron and single ion species. Much of the discussions in this chapter follows these works in part.

2.1 Overview

What follows is a more rigorous and detailed explanation of section 1.3 of the introduction. One of the simplest descriptions of material expanding from the surface of a target is to assume the laser deposits some amount of energy into the foil, instantaneously heating the foil, which then expands into vacuum. This casts ion acceleration into an adiabatic expansion, or rarefaction. Adiabatic expansions assume that mechanical work is done but that there is no heat flow within the fluid and hence the expansion is isentropic. Since more detailed theories share many of the characteristics of a gas rarefaction, this is a useful example and commonly used as a first cut description.

More relevant to high energy density physics (HEDP) experiments utilizing intense lasers is a two fluid model where the electrons are instantaneously heated and couple electrostatically to the ions. Two self-similar† solutions have been employed to inform different

*Particularly later editions.
†A self-similar solution has a specific mathematical definition (a solution of the form $u(x, y) =$
aspects of the expansion and relaxation of the target. Using a Gaussian density profile to approximate the target yields an adiabatic self-similar solution that can incorporate an electron population self-consistently losing energy to accelerating ions\cite{54}. This solution also lends itself to a cylindrical geometry to calculate the transverse expansion of the ion beam as it expands away from the target surface\cite{49, 55}.* Closely following the 1D solution, in '98 Dorozhkina\cite{56} published a versatile 3D solution which Bellie\cite{57} uses in an excellent synopsis including visualizations of both the ion and electron trajectories for a Gaussian disk. These solutions require quasi-neutrality for all space, do not represent the target evolution at early times, and do not lend themselves simply to multiple ion species.

In order to investigate how the ion species interact, the simplistic model discussed in this chapter is based on a semi-infinite collision-less isothermal plasma allowed to freely expand, similar to figure 1.3. Though the evolution from the initial conditions is unclear, an exponential decay moving into the plasma with a characteristic speed, $C_s$, has been shown to be a self-similar solution\cite{29}. This solution also requires the expansion region to be quasi neutral, and indicates that there is no maximum energy. The simplest modification is to cut off the high energy component. Moving to a pseudo-Lagrangian treatment of the ions\textsuperscript{†} (see section 5.1) and assuming the electron energy distribution is maintained by an external source or sufficiently large bulk yields a model close to the exponential self-similar solution. One advantage is a self-consistently generated high energy cutoff, which can be easily understood to simply have the position and velocity of the original surface layer of the target\cite{32}. The high energy cutoff can be analytically calculated by determining the electric field strength at the expansion front\cite{51}. This description also lends itself to a single explanation of both the ion acoustic expansion and the much hotter and less numerous expansion commonly referred to as TNSA\cite{40} (figure 1.6a).

The pseudo-Lagrangian approach following Crow\cite{32} and later Mora\cite{51} described in chapter 5 does have the advantage of being relatively easily manipulated to involve two fully ionized species, namely protons and deuterons, to investigate how they shield one another in various concentrations and initial conditions. Though it neglects a host of known effects and is ill suited to make accurate quantitative predictions of the spacial distribution and spectra of the accelerated ions, it does provide intuitive, qualitative understanding of how various ions species interact in the early stages of the expansion and provides scaling laws which match experimental results\cite{58}. The models are within reasonable agreement with

\[ y^a F(\zeta; \zeta = xy^b) \] for some constants $a, b$) which is useless for intuition. These solutions can be identical when the underlying variables are appropriately scaled. Exponential growth, when scaled appropriately looks the same.

\* These papers used an isothermal solution for the radial expansion.

\textsuperscript{†} Lagrangian in that the co-ordinate system is allowed to move, as one may when trying to find an optimal co-ordinate system for the Lagrangian formulations. Computationally, pseudo-Lagrangian co-ordinates are discretization of material with continuous space, as opposed to a continuous density and fixed spacial co-ordinate system, or discrete density on a fixed spacial grid.
2.2 Basic Fluid Mechanics

To begin the discussion I will briefly review the foundation of fluid mechanics starting with the Euler equations. The Euler equations, together with knowledge of the fluid properties, form a complete system of equations. The Euler equations are continuity equations which can be expanded for more general situations, and relate the fluid velocity $\mathbf{v}$, density $\rho$, pressure $p$, and the polytropic index $\gamma$ or the specific internal energy $\varepsilon$ as a function of time $t$. Equation 2.1 describes conservation of mass, 2.2 describes conservation of momentum, and the conservation of energy in conservative form 2.3 or for a polytropic fluid 2.4.*

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$  \hspace{1cm} (2.1)

$$\nabla p = \rho \frac{d\mathbf{v}}{dt} = \rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right)$$  \hspace{1cm} (2.2)

$$\frac{\partial}{\partial t} \left( \begin{array}{c} \rho \varepsilon \\ \frac{\rho v^2}{2} \end{array} \right) = -\nabla \cdot \mathbf{v} \left( \rho \varepsilon + \frac{\rho v^2}{2} \right) + \mathbf{v} \cdot \mathbf{F}$$  \hspace{1cm} (2.3)

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p = -\gamma p \nabla \cdot \mathbf{v}$$  \hspace{1cm} (2.4)

An important and relevant quantity for any acoustic (pressure) disturbance is the speed at which an acoustic wave travels. This value is commonly derived by linearizing $p = p_0 + p_1$; $p_0 \gg p_1$ and similarly $\rho$ in equations 2.1 and 2.2. Keeping only first order terms and substituting, yields the acoustic wave equation.

$$\frac{\partial^2 \rho}{\partial t^2} - \frac{\partial p}{\partial \rho} \nabla^2 \rho = 0$$  \hspace{1cm} (2.5)

Here $\frac{\partial p}{\partial \rho} = C_s^2$,† and if one expresses all of the changing energy, density, and ionization

*A polytropic fluid is one that is thermodynamically reversible. Gasoline mixed with O$_2$ in a box is not polytropic. Nitrogen is more so.

†For metals and solid density, low ionization state plasmas, $C_s \sim 10 \text{ km/s}$, which is much slower than the TNSA protons $\sim 10000 \text{ km/s}$. 

experiment, enough to reproduce experimental trends[58].
information within $\gamma$ then

$$\frac{\partial p}{\partial \rho} = C_s^2 = \gamma \frac{p}{\rho}$$

(2.6)

It is also important to note that the dispersion relationship for linear acoustic waves is

$$\omega^2 - C_s^2 k^2 = 0$$

indicating that the phase and group velocities are equal.*

### 2.3 Two Fluid Model of (Collision-less) Plasma Expansion

One of the first refinements made to adiabatic/isothermal rarefactions model is that electrons and not the ions are excited by the laser.† The electron temperature increases to the point where the mean free path $\lambda_{mfp}$ is greater than the scale of the interface and the subsequent expansion. This is implicit in the cartoon description of the TNSA mechanism and treating the plasma as a single thermalized expanding fluid is clearly a poor description. The laser accelerated electron population of interest to this work have characteristic temperatures on the order of an $MeV$ traversing a room temperature or cryogenically cooled target initially with ion temperatures of $\sim 1/40 - 1/80 eV$.‡ The electrons are freely mobile on the scale of the expansion and initially are at a different temperature than the ions. As such the electrons are better described as a second fluid which is coupled to the ion fluid electro-statically.§ For a constant electron temperature and stationary ion distribution, the net current will be approximately zero as electrons trapped near the initially net neutral bulk climb the potential well and are balanced by electrons falling back down returning to the bulk.

If the electrons are hot enough to assume that they are collision-less, and the electron temperature $T_e \gg$ the ion temperature $T_i$, then one is implicitly assuming that the pressure of the ions is effectively zero, and electrostatic coupling to the electrons is the only significant force acting on the ions. Conservation of stuff (mass), equation 2.1, still holds for the ions, but the force in the conservation of momentum equation (2.2) is replaced by the electrostatic force $-eE$ and Poisson’s equation, written in equation 2.7 in terms of electric potential $\Phi$.

*The useful definition of $\gamma$ changes depending on what one is investigating. The above definition is useful to ascertain the local speed of sound in a medium. Though one should not expect a constant $\gamma$ to be a sufficiency description when the temperature changes dramatically, creating a temporally and spatially varying $\gamma$.

†The ions can be directly excited by the laser through Brillouin scattering exciting phonons, but this a very small fraction of the absorbed laser energy.

‡Since the population of hot electrons is much less numerous than the bulk target, even if all the laser energy was coupled to bulk ions, the temperatures would still be on the order of 10’s of $eV$. Reduced mass targets have the potential to increase this to several $keV$, but this is still a slow expansion relative to TNSA.

§I would argue that this is still a quasi-static system with the constraint that the electric field $E$ still dominates the magnetic field $B$ and the force on the electrons $|e(\mathbf{v}_e \times \mathbf{B})| \ll |eE|$. 

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the \( n \) of the electrons and ions, the ionization state \( Z \), and in good old SI with \( \epsilon_0 \).

\[
\nabla^2 \Phi = -\frac{e}{\epsilon_0} (Zn_i - n_e) \tag{2.7}
\]

For the determination of \( n_e \), an argument similar to ones made when classically modeling free electrons trapped in the potential well of an ion or nucleus is employed. Given a thermal distribution of electrons with \( \frac{dN}{dE} = \frac{n_{e0}}{k_B T_e} e^{-E/k_B T_e} \) and nominal density \( n_{e0} \), then for a position \( x \) with electric potential \( \Phi \) the \( e^- \)'s present will be those with sufficient energy to climb the potential hill to get to \(-e\Phi\) or beyond, i.e. \( E \geq -e\Phi \). More explicitly,

\[
n_e(\Phi) = \int_{-e\Phi}^{\infty} \frac{dN}{dE} dE = n_{e0} e^{e\Phi/k_B T_e} \tag{2.8}
\]

Combining the electron density with Poisson’s equation 2.7, allows for calculation of \( \Phi(x) \) given \( T_e, n_i(x), \) and \( Z(x) \).

\[
\nabla^2 \Phi = \frac{e}{\epsilon_0} (n_{e0} e^{e\Phi/k_B T_e} - Zn_i) \tag{2.9}
\]

Notice that using the modified conservation of mass eq. 2.1, with equations 2.7, and 2.8, momentum and energy are not conserved. The exchange of momentum and energy from the electrons to the ions is not accounted for in the electrons. The temperature of the electrons should be altered to account for the loss of energy[59]. This significantly complicates the calculation, and it has been shown that experimental trends are reproduced without it[51, 58]. We also know from energy balance that only \( \sim 10\% \) of the energy in the hot electrons is transferred to the TNSA ions.

### 2.3.1 Initial Conditions

For simplicity, the model considered is an initially neutral, \( n_{e0} = Zn_{i0} \equiv n_0 \), half infinite plasma where the ions are cold and the electrons are described by a single temperature. Since plasma is a conductor, one demands that sufficiently far into the plasma \( E = 0 \), which we know from electromagnetism, i.e. conductors expel charge imbalance and fields to the surface. Since the target is initially charge neutral, and should remain so for macroscopic scales, then infinitely far from the plasma vacuum interface \( E = 0 \) also.

As always, choice of potential is specific to the problem at hand. If one is interested in the escaping electrons, then choosing the potential to be fixed at zero an infinite distance from the target and treating the target as a charging capacitor informs how much energy electrons lose climbing out of the potential well created by the target charging as electrons escape to infinity[37]. Link et al. also shows that only a small fraction, a few percent, of the initial electron distribution is required to charge the target to trap the majority of the electrons. If one is interested in the behavior of the ions, since less than 1% of the electrons
escape to infinity\cite{37}, one can assume that all the electrons are trapped and fix $\Phi = 0$ in the plasma and then $\Phi$ far from the plasma must be sufficient to trap most of the electrons which have a characteristic energy of $k_B T_e$, so $-e\Phi \gg k_B T_e$.

Another simplifying assumption is that the ion motion is slow enough that the electrons have time set up an initial spatial distribution and corresponding potential well before there is significant ion motion.\footnote{The lower the electron energy one considers, this becomes more true. For the more energetic portion of the electron spectra, if they are strongly relativistic electrons, the time taken to be accelerated back towards the target may be longer than the laser pulse and associated electron pulse. Since there is no continuous supply of electrons the assumption that the entire population equilibrates cannot be accurate. More handles this in more detail\cite{59}.}

Given a planar geometry and an initial step function in the ion density and that the plasma is singly ionized,

$$\nabla^2 \Phi = \frac{e n_0}{\varepsilon_0} \frac{e\Phi}{k_B T_e} - 1 \quad (2.10a)$$

$$\nabla^2 \Phi = \frac{e n_0}{\varepsilon_0} e\Phi/k_B T_e \quad (2.10b)$$

Following Crow et al.\cite{32} integration of eq. 2.10a can be carried out formally and solved numerically, but equation 2.10b can be integrated\footnote{...by taking the ln of both sides and using knowledge of $d\Phi/dx$} giving the initial potential to be\footnote{$e\Phi(x=0)/k_B T_e$ is 1 in the normalized coordinates in the following section.}

$$\begin{align*}
\text{for } x < 0 : & \quad x = -\sqrt{\frac{\varepsilon_0 k_B T_e}{2 n_0 e^2}} \int_{e\Phi(x=0)/k_B T_e}^{e\Phi/k_B T_e} \left( e\Phi/k_B T_e - \frac{e\Phi(x=0)}{k_B T_e} \right)^{-1/2} \frac{e}{k_B T_e} d\Phi \\
\text{for } x \geq 0 : & \quad \frac{e\Phi}{k_B T_e} = -2 \ln \left( 1 + x \sqrt{\frac{n_0 e^2}{2 \varepsilon_0 k_B T_e}} \right) - \frac{e\Phi(x=0)}{k_B T_e} \quad (2.11a) \end{align*}$$

**Boundary Conditions**

The outermost atomic layer, under the current simplifications, should always see the greatest electric field at any given point in time and remain the layer with the greatest velocity.\footnote{Notice that the boundary from ions to no ions causes an inflection in the derivative of $E$, and is therefore at least a local extrema in $E$. It can be deduced from inspection of the slopes in figure 2.1 that the extrema at the origin is the global extrema.} For $t > 0$ and $x > x_{\text{front}}$, $n_i = 0$. Therefore equation 2.7 simplifies as 2.10b, and can be solved by guessing a solution of the form $a e^{b\Phi/k_B T_e}$ where $a$ and $b$ are constants.

\cite{30}
This leads to the condition

$$-\frac{d\Phi}{dx} = E_{\text{front}}(x = x_{\text{front}}) = \sqrt{\frac{2n_e0k_BT_e}{\epsilon_0}} e^{\Phi/2k_BT_e}$$  \hspace{1cm} (2.12)

which can be integrated for a more general form of equation 2.11b

$$\text{for } x \geq x_{\text{front}}: \quad \frac{e\Phi}{k_BT_e} = \frac{e\Phi_{\text{front}}}{k_BT_e} - 2\ln \left(1 + \frac{(x - x_{\text{front}})}{e\epsilon_02k_BT_e}\right)$$  \hspace{1cm} (2.13)

**Scales and Normalization**

In order to compare results or calculations with the majority of the literature, in addition to the traditional reasons of identifying governing parameters and characteristic scales,* length, time, velocity†, and the electric potential are scaled in the following way.  

- $x$ is in $\lambda_D = \left(\frac{\epsilon_0k_BT_e}{n_0e^2}\right)^{1/2}$ electronic Debye length  \hspace{1cm} (2.14a)
- $t$ is in $\omega_{pi}^{-1} = \left(\frac{\epsilon_0Am_u}{n_0e^2}\right)^{1/2}$ inverse ionic plasma frequency  \hspace{1cm} (2.14b)
- $v$ is in $C_s = \lambda_D\omega_{pi} = \left(\frac{k_BT_e}{Am_u}\right)^{1/2}$ ion sound speed  \hspace{1cm} (2.14c)

$\Phi$ is scaled to the normalized electric potential $\varphi = \frac{\epsilon\Phi}{k_BT_e}$ average electron energy  \hspace{1cm} (2.14d)

**2.3.2 Two Fluid Isothermal Rarefaction**

As alluded to above, some assumption needs to be made about the electron temperature. The simplest is to assume that the electron temperature remains constant during the expansion. This constant temperature can be attributed to a source constantly supplying a fixed electron temperature, or by positing that the bulk of the target is heated before the expansion begins and is a sufficiently large thermal reservoir that a drop in temperature is negligible, hence nearly isothermal.

*Normalization: to reduce the clutter of carrying around too many constants, and finding scales that normal humans think well in, say numbers from $10^{-3}$ to $10^3$, was also a necessity for calculations as computers had much more significant restrictions on what numbers they could represent back in the 8 bit punch card days.

†the sound speed in a plasma is better given by $\sqrt{\gamma^2k_BT_e/Am_u}$ if one includes collisions etc, but this scale is close as $\gamma$ is $\sim 1 - 2$ and $1 < Z \lesssim 100$. 

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Figure 2.1: The ion (Black) and electron (Blue) normalized density assuming an initially sharp interface, and instantaneously heated, Maxwellian electron population. Also plotted is the normalized electro-(quasi)static trapping potential (Green) from equation 2.11. Shown here times $-1$.

This case has been extensively studied and reported on. The traditional self-similar* solution[28] can be found if one assumes that quasi-neutrality is maintained throughout the expansion. This solution maintains the rarefaction head as a characteristic where the density remains unchanged and the ion velocity $v_i = 0$ for $x \leq -C_s/t$ and is described for $x > -C_s/t$ by

\begin{align}
    n_e &= Z n_i = n_0 e^{-x/C_s t - 1} \quad (2.15a) \\
    v_i &= C_s + xt \quad (2.15b)
\end{align}

### 2.4 Three Fluid Model

Electrons generated in the LPI have energies high enough that they do not thermalize with the ions or the bulk electrons on the time scale of the expansion, as described in section 3.2.1.† The next increase in complexity is to add a second species of electrons to more accurately represent the electron spectra within the material. This is accomplished

* A solution that appears the same all times and/or all length scales. Apparently if one of the co-ordinates lacks a clearly defined origin this is the same thing. Here, the solution is unchanging when the co-ordinate considered is $x/C_s t$.

† The higher energies also improve the assumption that the electrons are collision-less until the cross-section for pair production increases significantly: $\sim 1 - 10 \, MeV$ depending on the material.
in detail by True et al. circa 1981[40] and more recently visited with finite target thickness by Mora[53]. The extension to the governing equations is fairly straight forward. Given the hot electron temperature $T_H$, the cold electron temperature $T_C$, and their associated densities: $n_H + n_C = n_e$ equation 2.9 becomes

$$\nabla^2 \Phi = \frac{e}{\epsilon_0} (n_H e^{e\Phi/k_BT_H} + n_C e^{e\Phi/k_BT_C} - Z n_i)$$  \hspace{1cm} (2.16)$$

The boundary condition at the ion front (equation 2.12) calculated as above becomes

$$E_{\text{front}}(x = x_{\text{front}}) = \left( \frac{2 n_H k_B T_H}{\epsilon_0} e^{e\Phi/k_BT_H} + \frac{2 n_C k_B T_C}{\epsilon_0} e^{e\Phi/k_BT_C} \right)^{1/2}$$  \hspace{1cm} (2.17)$$

and to calculate $\Phi$, $E$, and $n_e$ for $x \geq x_{\text{front}}$ a numerical integration similar to equation 2.11 is required.

Though it is not obvious that one can or could recover a self-similar solution, this can be implemented numerically with an arbitrary number of temperatures. This also has the potential advantage that one can apply different conditions to the electron populations to conserve energy.∗

The initial condition is very similar to the one described in section 2.3.1. However, due to their relative energies and the potential well that the electrons set up, the colder population is much more tightly confined to the bulk, and the hotter population, which energetically must be responsible for the majority of the potential well, comprises more of the sheath.

The presence of the colder population limits $\lambda_D$ at the rarefaction head and rate at which material is exposed to the accelerating potential of the hotter portion of the electron population. Restive stopping of the hot electrons and associated heating of the cold electrons, among other effects, will alter electron spectra driving the expansion as it proceeds. The resulting expansion proceeds as two relatively independent expansions; each with characteristic energies and relative number set by the initial electron spectra[40] as illustrated in figure 1.6a.†

∗Due diligence would require one to keep track of the electron distribution and calculate the changing distribution as a function of $E$. Modern codes particle-in-cell (PIC) and hybrid PIC codes such as LSP address this more directly by keeping track of scaled particles.

†The hot electron population heats the cold electron population, but by the time the majority of the ion acceleration has occurred, only a small fraction of the energy has been lost to the cold population for few micron targets and relativistic laser intensities[36].
Chapter 3
ION ACCELERATION UNDER EXPERIMENTAL CONDITIONS

*The models discussed in chapter 2 do provide insights into some of the properties of laser accelerated ions. These models do inform experimentally observable quantities such as the maximum ion energy† and the general form of the accelerated spectra to be Boltzmann-like. However, many of the assumptions made are not physical and these models neglect a host of mechanisms. Of interest would be an explanation of the conversion efficiency of laser light to ions, an explanation of the the relative spectra of different ion species, and these spectra as a function of angle.

The discussion will focus on, but not be limited to the regime of the experiment discussed in chapter 4. The state of the target at the arrival of the main pulse as a function of the pre-pulse duration and intensity will be discussed. The energy and intensity of the laser in relation to the target thickness can inform the type of ion acceleration mechanism [60]. Upon inspection of the parameters used in the experiment presented in this document, it will be shown that the TNSA mechanism, influenced by surface materials, is the most relevant.

3.1 Laser accelerated electron population

This is a field unto itself, richly populated with theory, simulation, and experimental studies. For the purposes of this discussion we need a description of the energetic elections produced in the LPI. For the purposes of this work I will elucidate the simplest approach, using the

†Or are artificially cropped to be consistent with experiment.
empirical and semi empirical scaling laws. I will use recent LSP simulation results and briefly discuss the capacitor model to infer the electron distribution in the target.

To begin to understand how relativistic electrons are generated from the LPI on the front of the target, looking at the motion of an isolated electron in a high intensity laser field is a good place to start. Gibbon[61] provides an analytical analysis, and a detailed quantum-electrodynamic analysis of the ionization and subsequent acceleration of electrons in strong laser fields can be found in Dr. Enam Chowdhury’s Thesis[62, 63].

### 3.1.1 Physical Scaling Law: Wilks Scaling

For laser intensities greater than $10^{18} W/cm^2$, Gibbon shows what is often refereed to as Wilks scaling best fits a series of experimental data from 1992-2001[61]. Wilks[64] finds that the energy of an electron oscillating in the transverse laser field is a good estimate of the characteristic temperature of the energetic electrons produced in 2D PIC simulations of LPI with a steep density gradient. The Lorentz factor $\gamma_r$, when in terms of the cycle averaged energy of an electron oscillating in a laser, is a function of the pump strength for electrons or normalized vector potential $a_0$ (shown here in terms of intensity $I$ and wavelength $\lambda$) and of the laser field is relativistic ponderomotive potential $U_p = (\gamma_r - 1) m_e c^2$. Using 800 nm light at $10^{18} W/cm^2$, $U_p$ and empirical characteristic temperature is 100 keV, and 700 keV at $10^{19} W/cm^2$.

$$k_B T \approx U_p = m_e c^2 \left( \sqrt{1 + a_0^2} - 1 \right) = m_e c^2 \left( \sqrt{1 + \frac{I \lambda^2}{1.37 \times 10^{18} W cm^{-2} \mu m^2}} - 1 \right)$$

(3.1)

Though this formulation does match the behavior of the temperature of the accelerated $e^-$'s, it does little to explain the experimentally observed energy distribution or maximum electron energies. Colloquially, the distribution has been explained in a variety of ways, but the most convincing and coherent explanation is currently being written by my colleague A. Krygier. Essentially, considering the average electron velocity neglects the effects of when the electron is relativistic moving with the EM wave. As the electrons become increasingly relativistic, the forward motion also approaches $c$ moving with the EM wave and can be accelerated for a period of time before the EM peak passes them and decelerates them. This effect is exploited by LINeAR ACcellerators (LINAC)s, has been previously noted in laser plasma channels[65], and is attempting to be exploited in direct laser acceleration (DLA)[66]. In the laser focus, electrons enter with or acquire a random momentum through

*A brief summary of Dr. Enam Chowdhury’s results for the electron momenta can be found in Drake[52] on page 459.

†This is a reasonable description when considering intensities where the electron velocity $|| \hat{k}$ is not strongly relativistic: $\sim 10^{18} W cm^{-2}$.
scattering. The alignment the electron’s momenta to the laser $\hat{k}$, the position, and the relation to the laser’s phase all need to be ideal to achieve maximum energies. The stochastic nature of these variables leads to the Boltzmann like energy distribution and is partially responsible for the hot electron divergence.

3.2 Other Considerations/Discussion

Though space constraints prevent a detailed discussion, the expansion and virtual point source/cathode nature of the ion beams should be mentioned. The angular spread of the beams can be attributed to a combination of the source $E$ field having a radial component in $r$ because either the differing rates of expansion as a function of radius on the rear surface and/or the internal collisional pressure of the electrons and ions the expansion proceeds.

3.2.1 Electron Transport

As previously alluded to, the relativistic electrons excited by the laser are considered as a hot population, whose energies are sufficiently high that the effects of scattering as they traverse the target has a negligible impact on their energies. At early times in the expansion some of these relativistic electrons escape the target which positively charges the target, creating a potential well for the electrons. In an insulator, this interface initially exists at the edge of the plasma and unionized target. The depth of the well and strength of the electric field increases until the insulator near the interface is field ionized (Appendix A.3.1) and the interface propagates quickly through the target[67, 68]. The leading relativistic electrons lose energy to ionizing an insulated target. Subsequent electrons escaping to a detector lose energy escaping this potential well, further increasing the depth of the potential well for subsequent electrons. Electrons with insufficient kinetic energy to escape the charging target will reflux, or circulate through and around the target. An approximation of the depth of the potential well caused by the target charging, and the effects on the escaping electron population has recently been considered in detail for small or reduced mass targets, a few microns thick and hundreds of microns in extent by my colleagues A. Link et al.[37].

Scattering

Possibly the most obvious energy loss and divergence increasing mechanism, scattering, has surprisingly little effect on relativistic electrons. Revisiting figure 1.1, note that 30 $MeV e^{-}$’s still have a significant amount of energy after traversing several $cm$’s of water. Through a few $\mu m$’s of water effectively no energy is lost. One can also note that the majority of the energy is carried in a relatively narrow beam in the first $cm$.*

*I suspect that the width in the first $cm$ is primarily due to secondary ‘knock-on’ electrons, scattered by the incident electrons, depositing energy.
A more relevant example: the stopping power \(-\frac{dE}{dx}\) for 1 MeV electrons is \(\approx 190 \text{ eV/\(\mu\)m}\) in water. \textit{EStar} is an excellent quick reference for approximations. More detailed discussions of how the relativistic stopping power changes with ionization and its relation to scattering and effects on electron divergence\[69, 70\] can be found in the literature. For the majority of the background electrons scattering losses are more appropriately interpreted in terms of their collective resistivity \(\eta\).

\textbf{Resistive Stopping}

The vast majority of the electrons in the target are considered as a cold or background electron population. This population has significantly lower mean energy, certainly not more than a few to tens of keV at most, are considered to be strongly collisional, and do not have sufficient energy to begin to climb the potential well created by the escaping electrons. The collisionality of the cold electrons give rise to their resistivity (Drude model). The resistivity of the cold electrons will both heat the cold electrons, and create an electric field (Ohm’s law), transferring energy from the hot electrons to the cold electrons as they \textit{nearly} neutralize the hot electron current.\(^\dagger\)

The cold electrons in the bulk target, which are unable to climb the potential hill at the edge of the target, leave \textit{only} the hot, refluxing electrons to shield the charge in the region of accelerating ions. Argued another way, since the target is a conductor one expects it to expel electric fields, and through Gauss’s law arguments and or by inspection of \(\nabla \cdot \mathbf{E} = \nabla^2 \Phi = \frac{\epsilon}{\epsilon_0} (n_e - n_i)\), one expects any \textit{net} charge to reside at the surface. Forgoing a more complex discussion of the dynamics of the refluxing electrons and attempting to integrate those dynamics to ascertain the local number density of the hot electrons, one simply considers of the population of electrons, how many have enough energy to reach the local potential. For an electron population with a Boltzmann distribution with energy \(k_B T\) and a number density of \(n_{eo}\), \(dN/dE = n_{eo} e^{E/k_B T}\), the local number density of electrons

\(^\star\)A few rules of thumb for weakly ionized plasmas and everything else. For relativistic electrons the collisional stopping power, primarily due to elastic collisions with other electrons, for any material is \(\sim 1 - 2 \text{ MeV/cm} \times \rho (g/cm^3)\) and more directly scales with \(n_e\). When considering \(e\)'s with higher energies, the radiative losses through bremsstrahlung equals the collisional stopping power for heavier atoms, such as \(\text{Pb}\) or \(\text{Au}\), around 10 MeV and for lighter atoms and compounds such as \(\text{H}_2\text{O}\), around 100 MeV. Collisional stopping power is nearly constant for relativistic energies, in contrast the nuclear charge \(Z_{\text{nuc}}\) scales bremsstrahlung losses \(\propto \gamma^6 Z_{\text{nuc}}^2\)

\(^\dagger\)Though every electron in the target would better be modeled by its wave function interacting with the the local electric field, it is not currently feasible to model every atomic field. Though this does bring an interesting (and likely useless) mental image to mind. One could imaging this as a field of wells (nuclei), partially filled with pockets of charge fuzz (bound electrons), but covered over and smoothed out by a sea of slow moving charge fuzz (free cold electrons). The moving and trapped fuzz collide with each other exchanging rolls and creating little ripples on the surface they are all sitting on. Through this sea a small number of fast charge packs move, causing a bulk creep of the slow fuzz in the opposite direction which swirl and slosh by the partially filled wells as they flow slowly down the shallow hill that is the floor of the sea. The fast fuzz slows down slowly climbing the shallow slope.
\( n_e(\Phi) \), provided the potential is monotonically increasing, will be

\[
n_e(\Phi) = n_{eo} e^{\Phi / k_B T}
\]  

(3.2)

Once the range of the electrons is greater than the range of the evanescent laser field, or skin depth, the laser field is certainly no longer sufficient to describe the fields or the source of the heating. The traditional approach is to consider just the electron transport through the material. It had long been supposed that there must be more numerous electrons traveling anti-parallel to the more energetic LPI produced electrons to maintain a nearly charge neutral plasma. The traditional approach is to just separate the electrons into a ‘hot’ population, or hot current \( J_H \), that is not locally thermalized, and a ‘cold’ population, or cold current \( J_C \), that is. The charge neutrality condition is then \( J_C \approx -J_H \). The designation ‘hot’ and ‘cold’ is somewhat ludicrous considering the densities and temperatures at which the electrons become ‘hot’, but this nomenclature is common. Then with an application of the superposition theorem, one can relate the \( \mathbf{E} \), the currents and resistivity \( \eta \) ohm’s law as \( \mathbf{E} = \eta C J_C + \eta H J_H \). For the relativistic electrons \( \eta_H J_H \) is a macroscopic description of the scattering discussed above. If science continues to be self consistent, then the scale of the resistive fields for \( \sim MeV \) e’s \( \approx 100 MV/m \) (for comparison only).

The resistivity of the cold current thus becomes a source of heating for the cold population of electrons and is responsible for the strength of the ohmic or resistive fields that slow the hot population of electrons. Since the resistivity is a function of the electron temperature, and temperature depends on the heat capacity of the electrons, this quickly requires a self-consistent solution.

The resistivity depends on the temperature of the bulk because the resistivity is a bulk expression of the effect of electron collisions in the material. The collision rate and therefore the resistivity is strongly dependent on both the energy of the electrons and the free electron density. Increasing the temperature initially increases the resistivity by increasing the number of excited states (which have a larger cross-section) and also by increasing the electron density through ionization. For sufficiently high electron temperatures, the electron collision cross section reduces, lowering the collision rate. The resistivity begins to decrease with increasing temperature above \( \sim 10 – 100 eV \) such that \( \eta \propto T^{-3/2} \) in what is commonly referred to as Spitzer regime[71]. Subsequent models, Lee - More[72] and later modified by

\[^*\]This is the same condition on the electron density that is used to derive the Debye length.

\[^1\]10keV electrons thermalize in \( \sim 1 \mu m \) of Al, and \( \sim 3 \mu m \) of H\(_2\)O. Barely hot enough to be ‘hot’ in this context, but if everything at solid densities is high \( Z \) plasma safely hot by any personal context.

\[^\dagger\]\( \eta_H \) is not an emergent property. Counter propagating relativistic electron currents do not suddenly start missing the background electrons simply because the net current is lower.

\[^\ddagger\]For sufficiently high energy electrons, \( \gtrsim keV \), the total electron density is more relevant.
Desjarlais[73] are applicable at lower temperatures.*

The electron density is directly tied to other thermodynamic properties of the fluid such as the heat capacity and pressure, which are of direct relevance to the shock dynamics of the fluid and the expansion properties. To determine these properties the average ionization state is sufficient (though not necessary, one could group this physics into a table and simply reference the $\gamma$).

There is a lot of physics to be considered if the initial stages of ionization is of interest. A few priming electrons may be required before significant ionization can occur. When the average kinetic energy is sufficient to remove the first electron from the ground state, and an ample supply of electrons streams through the target from the LPI region, of greater interest is how rapidly equilibrium is established and how quickly the ionization state responds to a changes in electron temperature. If these time scales are sufficiently short or long, determining the ionization state would be greatly simplified. Many of these issues are discussed in appendix A.

**Refluxing and Ohmic Stopping**

The ohmic stopping fields are primarily relevant for the first pass of the target. Once the electrons begin the reflux, the net hot current will be significantly less than the current of hot electrons expelled from the LPI. This proportionally reduces the cold electron current required to maintain charge neutrality and therefore the strength of the ohmic stopping fields. When neglecting the electron excursions into vacuum and considering the target surfaces as perfect mirrors, ohmic stopping has little effect on the electron energies and subsequent ion expansion[36].

### 3.2.2 Ions

The ionization state of the bulk target is required to calculate or approximate the density of the electrons and the resistivity of the bulk target for the hot electron population. Though somewhat less clear, the average charge state of the ions should be sufficient to describe the acceleration. For the models in chapter 2 the average ionization state is all that is required. Although the ion’s acceleration $ZeE/m_i$ depends on $Z$, and the population of ions exist in a distribution of charge states near $\overline{Z}$, any individual ion’s charge state is not static or slowly varying. Certainly in the initial phases of the acceleration, the ions are continuously changing state by recapturing electrons at the apex of their climb out of the potential well of the target, and being ionized by hot electrons still climbing the potential hill. The sheath electric fields from this potential are on the order of a few $MV/\mu m$ or $TV/m$ if you prefer. This field is sufficient to reduce the ionization potential by $\Delta E = e\Phi \sim 40 ev \sqrt{Z_{\text{nuc}}}$. One

*Sometimes referred to as the LMD resistivity model.
would not expect the individual charge states to acquire different kinetic energies until the electron density is significantly lower.∗

In the much slower hydrodynamic expansion, where the pressure of the plasma is the dominant driver, the separation of the different species within the expansion is not considered until the density has dropped significantly and collisions and Brownian motion stop mixing the species faster than any difference in their velocities can separate them. Extrapolating this behavior to TNSA is contradicted by experimental evidence. The majority of the energy in the ion spectra are carried by the lightest species available in the material accelerated, which one would expect to be partitioned by the concentration of the species if they remained mixed during the expansion.†

Though not so much a consideration as a useful estimate, the speed at which the ions respond to the disturbance of the accelerating fields can be found by exploiting the analogous nature of the electrostatic potential $Ze\Phi$ and the pressure $p$, the gradient of each is the force per unit stuff.‡

Using the fluid definition of $C_{si}^2 = \partial p / \partial \rho$ and what is effectively an EOS in equation 3.2, the ionic speed of sound will be determined by the hot electron temperature $T_h$ and the ion charge to mass $m$ ratio.

$$C_{si} = (Z_kB T_h/m)^{1/2} \quad (3.3)$$

### 3.2.3 Angular Dependence

First consider a planar target much larger than the ion source size. Ignoring the details of the LPI the electron source has a large divergence[38]. As a consequence, nominally the hot electron number density and temperature are greatest near the laser axis. After several reflux passes, the sheath field strength, associated maximum ion energy, and associated ion characteristic temperature, decreases radially. As the ions move away, normal to the rear surface, quasi-neutral expanding cloud of ions and electrons is free to expand in the radial direction and can be treated with an approach similar to those used to model the foil expansion[49, 57]. As a general rule, the hottest spectra is within $\sim 5^\circ$ of target normal, and the majority of the accelerated ions within $10 - 15^\circ$ for planar targets. For targets on the order of the ion source size or smaller, the divergence of the accelerated ions increases[74]. Various experimental and theoretical efforts exist to focus the ions with a hemisphere[49, 57, 75].

∗These lower densities, also imply later times at which the electrons will have lost most of there energy and the acceleration will be nearly complete.

†Though collisions between the ions may transfer some energy from the lighter species to the heavier ones, this effect is neglected as implied by the collision-less moniker. This is also equivalent the fluid having no viscosity - i.e. setting the stress tensor to zero in the Navier-Stokes equation

‡per electron and per unit volume
3.3 The Problem with Accelerating Light Ions: The Lightest Ions

As discussed in Chapter 2, fluid properties move with the fluid, so in an expansion where local thermal equilibrium (LTE) is maintained, and the largest force on the fluid is internal pressure, one would expect the concentrations of various species to remain consistent as the plasma expands. Similarly material interfaces are maintained, with mixing occurring at the interface. Though similar to the adiabatic plasma expansion\[52\], the force on TNSA ions is dominated by the Lorentz force, primarily from the electric field generated by escaping and refluxing MeV electrons accelerated by the laser. This sheath-field is strongest initially at the target surface and subsequently near the ion front. By the time the relativistic electrons reach the rear surface, the target is ionized* and the acceleration \( a = \frac{ZeE}{m_i} \) of each species is dependent upon their charge to mass ratio \( \frac{Ze}{m_i} \) and the local electric field. Of the ions close enough to the rear surface to be in the field generated by the hot electrons traversing the surface, the species with the highest charge to mass ratio will overtake the other ions and shield them from the strongest accelerating fields.

Using PIC simulations, Petrov \[46\] finds that a nm layer of proton contamination can substantially reduce the maximum deuteron energy and conversion efficiency of the laser to energetic deuterons. In addition to providing some rules of thumb exploiting energy balance arguments, these PIC simulations were used to show that if the laser fluence is sufficiently high or the contaminate layer sufficiently thin, the conversion efficiency and the maximum deuteron energy will not be significantly reduced.

3.3.1 Surface Contaminants

In vacuum chambers that are not UHV concentrations of water and hydrocarbons from oils and other contaminants form a few nm thick layer on the surface of everything in the chamber. Some surfaces such as polished copper or gold mitigate the thickness of the contaminant layer, but not enough to eliminate the layer.

Several attempts, all involving considerable effort, have been reported to suppress the acceleration of the contaminant ions. Pulsed laser irradiation of the target rear surface to produce a long scale length plasma reduced the observed proton flux by two orders of magnitude\[47\]. However, this reduction was attributed to a reduction in the strength of the accelerating field at the target rear surface as opposed to a process of deselecting the protons for acceleration. Heating of the target also reduces the yield of contaminant protons, yet

*Even if the target is not initially a conductor, in order for the large current of hot electrons to propagate, the hot and cold current must be nearly neutral (as discussed in section 3.2.1), as the hot electrons propagate into an insulator charge separation will generate an electric field strong enough to field ionize the insulator. This will slow the front of relativistic hot electrons as the leading electrons’ energy is consumed by this field. This ionization front was observed in gases at the rear of a target to be \( \sim \frac{e}{10} - \frac{e}{30} \)[67].
enhances the heavier substrate-derived ions[48, 49]. Observations show that the peak energy and total yield of protons from the rear surface of a 50 µm Al target heated to more than 600 °K is reduced by a factor of around 8 and 10 [48]. In situ Ar ion beam sputtering of 15 µm thick Au targets demonstrate a two order of magnitude reduction in the proton yield with more than a factor of two reduction in the peak proton energy[24].
Chapter 4

EXPERIMENTAL SETUP

4.1 GHOST

The experiments demonstrating preferential deuterium acceleration, the namesake of this document, were performed on the GHOST\cite{76} laser system at and in collaboration with The University of Texas at Austin. GHOST nominally has a 120 fs pulse width, 1053 nm wavelength, 2 J max energy pre-compressor, and an energy contrast of $\sim 10^6$\footnote{This was not measured at any time during our run and is suspect, but not particularly relevant as the energies are fairly low and extremely unlikely to destroy the target before the main pulse interacts with it.}. The system has an OPCPA front end which is used to pump ND:glass amplifiers and can be fired one every seven minuets. The laser is focused with an f/2 off axis parabola (OAP) to a 6 $\mu$m (FWHM)focal spot corresponding to a mean intensity of $2 \times 10^{19}$ W cm$^{-2}$. The target chamber is a cylinder 91.4 cm in diameter and 38 cm deep and is commonly completely reconfigured between experiments. An image and schematic of the experimental setup if found in Figure 4.1.

A common issue with high-intensity lasers are back reflections from the target interaction scattering back up the laser chain. The amplifiers, which still have a portion of their gain states\footnote{the longer life time excited state, often not directly stimulated by the pump} populated, amplify the back scattered light and may damage an optic further up the laser chain. Commonly, a minimum angle is chosen\footnote{Alternatively, Faraday rotators and/or Pockels cells may be used to isolate the amplifiers from back reflected light.} such that the specular reflection from the target has no possibility of being collected by the OAP. This was 22.5° on GHOST.

In order to determine the position of the focal plane and the surface of the target, a set of optical diagnostics was implemented. A low magnification (mag) imaging system to be able to view the target features consisted of a charge coupled device (CCD) camera, a lens\footnote{Which conveniently had a port for in-line illumination which could be used to illuminate the system or to quickly focus and sight the lens and camera by projecting the light to produce an in-focus reticle in the

43
Figure 4.1: The GHOST target chamber. (a) Schematic of the experimental setup. (b) An image of the experimental setup in the GHOST target chamber including the cryogenically cooled copper box surrounding TCC and its $\sim 25 \mu m$ Al foil (kitchen foil) insulated coolant lines.

and a mirror. An achromatic lens on an $xyz$ stage was placed inside the chamber which could be driven on the laser axis when the chamber was evacuated. Imaged onto another CCD with high magnification, the focal spot or focal plane monitor was used to align the parabola and position a target fiducial at the focal spot. The most useful fiducial was an edge of the etched window without a target as described in Section 4.2.1. The location of this edge could be transferred easily from the low magnification to high magnification systems. The edge of a few micron foil was also used to transfer the laser focal position and initially align the optical diagnostic used to co-locate the target surface and laser focus.

The low magnification system doesn’t have a short enough depth of field to accurately define the focal plane, and the focal spot monitor cannot be used with target in place. So a third system using specularly reflected light from the target was used to position the targets in the focal plane of the laser. The light from either the main laser, or a co-propagating $1 \mu m$ or HeNe alignment laser, specularly reflected from the target, was imaged with a high magnification on a CCD. Given that the target is a plane, so long as a portion of the light specularly reflected is collected by a lens, it will make an image on the CCD. If the target plane does not intersect the laser in the same position as was used for alignment, the image will translate. Due to the non ideal surface quality of target foils the specular image can be highly speckled and much larger than a magnified focal spot in the object plane. Placing a pin hole in the imaging system produces a small, well defined spot. Since one is only interested in a well defined geometry, and the CCD and the pinhole are fixed, this is sufficient to identify the position of the target. Provided there is a significant distance object plane.
between the pinhole and the CCD, changing position along the laser axis of the target will produce a translation of this spot on the CCD, which we were able to calibrate and detect a $\sim 2 \mu m$ translation along the laser axis. The higher the surface quality of the target the more reliable and accurate this diagnostic becomes.*

### 4.2 Target Preparation

This is arguably the key to being able to preferentially accelerate deuterons from the surface of a foil using TNSA. At issue is the composition and surface quality of the rear, and possibly front, surface(s) of the target. As alluded to above, high target surface quality also aids in consistent target alignment. Specifically, where the target plane intersects the laser focus needs to be well known for consistent laser intensity, and the target normal direction needs to be reliably aligned to the TPS for reliable measurement of the accelerated ions.

#### 4.2.1 Targets

With intention to take multiple shots in one vacuum pumping cycle and to measure the ions accelerated normal to the target, desire to have the target surface be globally flat, is implicit. In addition, in order to ensure reasonable ion acceleration, the target needs to be extremely thin, especially if one intends to add a layer of ice. Several attempts to use few micron foil for this experiment were less than successful, though instructive for comparison. These experiences led to coated and etched silicon wafer targets.

As alluded to above, preliminary experiments using copper foil yielded limited success accelerating significant quantities of deuterium†, but did inform us that for the strong laser focus $\sim f/3$ our few micron targets and alignment using shadowgraphy would not yield consistent results. This was probably due to several issues which I will describe, though no systematic study was performed to determine the contributions of the following.

Though care was taken to mount the foils little could be done to improve the surface quality of the Cu foils, which has several micron features easily observable with an optical microscope. In addition, the thickness of the shadowgraphs indicated the global flatness varied by $\sim 30 – 500 \mu m$ which made positioning of the target withing the Rayleigh range $\sim 10 \mu m$ of the focal plane extremely unlikely.

Aligning the target normal to the TPS was dubious at best. An alignment laser through the axis of the TPS reflected off of the rear surface of the target would be centered on the pinhole of the TPS only if the target normal falls on the axis of the TPS. The poor surface quality and associated specular reflection made this difficult. Though one generally expects

*We found this difficult to use with Cu foils, but worked well with the optical quality silicaon wafer targets.

†These experiments, though without prior knowledge of, yielded similar results to an article by Hou et al.[50]
a $\sim 5 - 10^\circ$ half angle beam of the most energetic ions$^{[21, 27]}$, a spatially modulated rear surface modulates the TNSA ion beam$^{[26]}$ and can even be used as a destructive technique to image the surface topography. Unfortunately, this is less than ideal if one is trying to infer the generated ion spectra from an extremely small solid angle.

To mitigate these problems, the decision was made to use optically flat targets, by etching and coating silicon wafers. Since the goal of this experiment was to demonstrate deuterium acceleration, the target thickness before ice coating was chosen to be 500 $nm$ so the targets could still be handled without the need for extreme care, yet still thin enough to effectively accelerate ions after being coated with $\sim 1 \mu m$ of heavy water. The targets are etched silicon wafers with 500 $nm$ remaining wall thickness consisting of a 200 $nm$ layer of Si$_3$N$_4$ coated with a 300 $nm$ layer of Al. The targets are 200 $\mu m$ in the transverse dimension. The Si$_3$N$_4$ is a stop etchant that remains after the silicon wafer is removed. The Al was added to increase the strength and aid in target alignment. Fig.4.4 illustrates the structure of a single membrane edge on. The entire target was a 3 by 3 array of the supported membranes to limit the number of vacuum cycles.

4.2.2 Deuterated Plastic Coating

Preliminary experiments, here primarily as control experiments, utilized Cu and Ti foil targets a few microns thick or the silicon wafer targets described above coated with deuterated plastic. The intention was to clean the surface to improve the deuterium ion signals from TNSA. The target rear surface is coated with a solution of deuterated plastic (Cambridge Isotopes CD 98%) dissolved in toluene in the ratio of 1:150. Upon evaporation of the toluene, a residue of CD constituted the deuteron source and target final layer.

This layer was transparent and using a combination of white light interference and optical microscopy the thickness of CD was approximately determined. The layer was thick enough not to make thin film interference patterns with optical frequencies. The implication being that the plastic layer to be greater than $\sim 0.7 \mu m$. Using an optical microscope, both the surface of the plastic layer and the foil could be brought into a sharp focus at different depths under high magnification. Due to limitations of the micrometer the difference in focal position could only be determined to be $\lesssim 2 \mu m$. Contamination of the solution or the residue layer could not be prevented.

4.2.3 Surface Contaminants

As discussed in sections 3.3 and 3.3.1, the presence of proton rich surface contaminants dramatically spoils the acceleration of other light ions. As was widely agreed in the literature, elimination of this layer is key to accelerate significant quantities of the

*This is roughly a phase difference of two $\lambda$ for red.
next lightest ions. Other more exotic and experimentally challenging mechanisms attempt to accelerate the entire target, requiring higher intensity lasers (\( \gtrsim 10^{20} \text{ W/cm}^2 \)), extremely thin targets (\( \sim 50 \text{ nm} \)), and therefore impressive pulse contrasts to ensure the target survives to interact with the main pulse of the laser.

With any gas solid interface, the gaseous atoms can be weakly bound to the surface through hydrogen bonds or Van der Waal’s forces. Imagine starting with a clean surface; some fraction of the incident gaseous particles will excite one or more phonons in the solid, and become weakly bound to the surface. Phonons from the solid or subsequent collisions from the gas may excite the bound particles to the continuum. After some time the coverage and thickness of the bound particles will reach an equilibrium. Atoms and molecules bound to the surface of a solid are referred to as contaminants or surface contaminants in the context of this document.

This is another field unto itself of considerable interest to surface chemistry\(^*\). There are a variety of approaches and components of the problem. Of concern for laser target interactions is how thick or how much the contaminants cover, the relative concentrations of various contaminants within this layer, and how one expects these values to change. Provided there is no phase or chemical change on the surface, then the layer will reach an equilibrium coverage of a mono-layer determined by the bond strengths and the temperature. The sticking coefficient\(^†\) is simply the ratio of particles that stick to a surface vs the number of incident particles. As a surface becomes covered, the sticking coefficient is the fractional sum of the coefficients for particles sticking on the clean surface and the particles to themselves. The sticking coefficient for both the gaseous particles incident on the surface and the gaseous particles sticking already stuck particles determines how rapidly a clean surface reaches equilibrium. This is on the order of \( 4 \times 10^{-4} \text{ Torr sec} \) for H\( _2 \)O on Aluminum[77] at room temperature. Generally, as temperature increases the sticking coefficient decreases, the coverage of the contaminants decreases, and the time to equilibrium increases.

Allen et al.[24] using x-ray photo-emission spectroscopy, found a 1.2\( \text{ nm} \) layer consisting of 27\% Au, 60.5\% hydrocarbons (CH\( _2 \)) and 12.2\% H\( _2 \)O on Au in a \( 2 \times 10^{-5} \text{ Torr} \) vacuum. One can imagine that this is a mono-layer of long carbon chains from various oils present, and then filled in with H\( _2 \)O.

If the target is cooled below the sublimation temperature of water (as for the deposition described in the next section) the temperature is likely near or below the sublimation temperature for most carbon chains long enough to be not particularly volatile.\(^‡\) As the

\(^*\)The catalytic converter in an automotive exhaust system relies on this mechanism to burn the remaining CH chains in the exhaust gas, and to reduce NO\( _X \).

\(^†\)There exists several related (I presume) definitions of the sticking coefficient in the literature. The definition used in the text is a common one.

\(^‡\)Diesel contains hydrocarbon chains C\( _8 \) to C\( _{21} \). C\( _{15} \) and up to C\( _{40} \) are used in lubricants and are less
temperature of a solid and the gas at the interface approaches the sublimation temperature, the sticking coefficient approaches one.\textsuperscript{78} Below the sublimation temperature the gaseous particles condense and freeze on the surface and the sticking coefficient remains near 1 even after the surface is coated.

The number of particles striking a surface per unit area is \( \frac{1}{2} n \langle |v_x| \rangle \) where \( \langle |v_x| \rangle = \sqrt{2k_B T/\pi m} \) is the mean of the magnitude of the thermal velocity perpendicular to the surface. The volume \( V \) of any one particle is proportional to volume of a mol of liquid or \( A/\rho \). Then of course the bridge from macro to micro \( N_A \) gives the particle \( V \sim A/N_A \rho \).

So, the contamination rate increasing the target thickness \( l \) is

\[
\frac{dl_c}{dt} \approx \frac{nA}{2N_A\rho} \sqrt{\frac{2k_B T}{\pi m}}
\]

(4.1)

The temperature of the particles in the chamber will be cooled by interactions with the cryogenic components in the vacuum chamber, and the sticking coefficient of warm gas on the cryo target will be less than 1. Using room temperature, and 1 for the sticking coefficient will set an upper bound on the contamination rate. For a chamber at 294 K, \( 1 \times 10^{-5} \text{Torr} \) this upper bound for a contamination rate on a cryo cooled surface is \( 15 \text{nm/s}, 10 \text{nm/s}, 3 \text{nm/s}, \) and \( 2.8 \text{nm/s} \) for C\textsubscript{15} and C\textsubscript{8} chains, H\textsubscript{2}O, and D\textsubscript{2}O respectively if each species is uniquely responsible for the pressure.

One expects an accurate description of the contaminants to be considerably more complicated as the target and gas temperatures are significantly different. Energetic gaseous particles are continuously heating the surface which is cooled by bulk thermal conductivity of the target, making the temperature of the surface contaminants likely much higher than that of the bulk target. Experimentally, we did not observe a \( 0.3 - 1.5 \text{ \mu m/100 sec} \) contaminant layer growth after the temperature sensor indicates a cooled target.

### 4.2.4 Cryogenic Heavy Water Deposition

In order to accelerate significant quantities of deuterons, the rear surface needed to be comprised of deuterons free of a hydrocarbon contaminant layer. Our initial attempts involved dissolving deuterated plastic in toluene. After attempting, with limited success, to use \( ns \) pulses of infrared (IR) and later ultraviolet (UV) to ablate and clean the rear surface of the target, I decided that cleaning was useless at worst and extremely difficult at best. Coating the target in vacuum with heavy water covers any contaminants present on the surface and, provided rate of contaminant deposition is slow enough, leaves D\textsubscript{2}O as a rear surface.

volatile than water at room temperature. Though not definitive, their sublimation temperatures are likely to be near or above that of water.

\textsuperscript{*}The experiment was hydrocarbons on platinum, but the general behavior was explained.
Initial investigations were performed using 10 $\mu m$ thick and $\sim 1 \, mm$ wide Cu foils in a setup similar to the one used in the deuterium acceleration experiment without the Cu shroud. The cooling and heavy water delivery system is shown schematically in figure 4.2. The initial goal was to determine for the nozzle used what trapped volume of water vapor was required to add a $1 \, \mu m$ coating to the target. The foil was imaged perpendicular to the surface normal and a $1.0 \, L$ volume of $18\, Torr$ vapor was deposited through the nozzle.

![Diagram of the cooling and heavy water delivery system](Image)

**Figure 4.2:** A cartoon of the cooling and heavy water delivery system pictured in figure 4.1b. A target mount is connected by a Cu braid to a LN$_2$ reservoir and its temperature is monitored with a thermocouple. The quantity of the heavy water delivered is determined by the temperature of the liquid D$_2$O and the volume of the vapor flask. The vapor nozzle was a 1/4" stainless line with a 4.4 $mm$ inner diameter and 3 $mm$ diameter opening 20 $mm$ from the rear surface. The ice layering was observed in real time using a microscope objective and the relationship between the ice thickness and the injected heavy water volume was established to be $1 \, \mu m/100 \, ml$.

We quickly discovered that simply cooling the Cu foil below the sublimation temperature to $\sim -130^\circ C$ and adding water vapor did not produce the desired surface coating. Instead, comparatively large ice crystals grew, and the surface density of the crystals grew with the addition of subsequent volumes of vapor. It was posited that when the water initially condensed, the surface tension would draw the water to bead, which would then freeze. Subsequent condensation would be drawn to the forming ice crystals. An example crystal is shown in figure 4.3a. I believe this is not dissimilar to how frost forms on an initially dry surface in winter.

After varying the temperature of the target and rate of deposition in addition to trying some traditionally hydrophilic materials such as titanium oxide growths, a solution was
Figure 4.3: (a) Images of the edge of a 10 μm thick Cu foil in the setup described in figure 4.2. (Left) An ice crystal formed from 0.2 L of water vapor. (Middle) A bare Cu foil, for reference the edge position is marked at the top. (Right) The same foil treated with Rain-X® Anti-Fog™ after 10 L of water vapor was deposited. (b) The morning frost on the antenna mount on the roof of a car after a dry just below freezing night.

found during a trip to the auto-parts store.∗ Rain-X®, in yellow, caught my attention as an interesting example of intentionally improving the effects of surface tension, presumably by decreasing the surface defects and/or making the surface more hydrophobic.† While wondering if there was a product to do exactly the opposite I noticed Rain-X® Anti-Fog™, in black, next to the Rain-X®, in yellow. Rain-X® Anti-Fog™ is intended to prevent condensation on the inside of one’s windshield beading up and forming a micro-lens array which we are all familiar with and mitigate with the defroster. It also works perfectly well to wick the water vapor condensate smoothly across the foil surface before it freezes, as shown in figure 4.3a.

It is possible that other surfactants would also perform as well, but contamination of the liquid is intended to be kept at a minimum. Though no study was performed I suspect that since Anti-Fog™ is intended to continue to perform after repeated applications of water, and is not easily removed with water that is not very water soluble. Thus I suspect that it is better choice than soaps.‡ The ice layering was observed in real time using a microscope objective and the relationship between the ice thickness and the injected heavy water volume

∗...which is required when one owns older vehicles and is unable to pay someone to fix them.
†For completeness: The intention is that rain beads up more and slides off ones windshield more readily to improve visibility in the rain. This works for a time, but without repeated application every few months or less the product hinders more than it helps.
‡Rain-X® Anti-Fog™ also remained bound to the Cu foil surface over three days at 5 × 10⁻⁶ Torr in sufficient quantities such that one could still cool the foil and re-coat the surface without large crystal growth.
was established to be 1 $\mu m/100\,ml$ for the nozzle parameters described in figure 4.2.

The trapped volume used to coat the silicon wafer targets was 100 $mL$. During the experiment, the ice growth was determined by standard thin film interference reflectometry using a helium-neon laser reflected off the target rear surface at an 11° incident angle. The reflectivity of the vacuum ice interface was on the order of a few percent, where the $D_2O$-Al interface was $\sim 90\%$ reflective. This disparity made discerning the changing interference from the DC signal impractical. Application of a Sharpie® before the Rain-X® Anti-Fog™ sufficiently reduced the reflectivity of the $D_2O$-Al interface so that the interference was observable.

A cycle of weak and strong reflection, corresponding to a $2\pi$ change in the relative phase difference of light reflected from the ice and aluminum, represented a 240 nm increase in the ice thickness. The injection volume of 100 $ml$ at 17.5 Torr (vapor pressure of water) caused five full fringe shifts corresponding to a total ice thickness of $1.2 \pm .2\,\mu m$.

The reflectometry diagnostic also provided feedback on the heavy ice deposition. For reasons not entirely clear to the author, upon injection of the heavy water the laser spot monitored for observing the thin film interference would speckle as if reflected from a rough surface. This corresponded to weak ion signals from TPS. I infer that either the surfactant was improperly applied or rendered ineffective by being covered. Upon review of the times of between beginning to cool the targets and attempting to coat, keeping the targets below the sublimation temperature for more than an hour resulted in a speckled reflection off of the rear surface. The best results were when the target was kept cool but not below the sublimation temperature ($\sim -80^\circ C$) until just prior to attempting the experiment. An explanation may be that the Rain-X® was covered with a contaminant layer sufficiently thick to render it ineffective.

Due to an increase in chamber pressure that accompanies the heavy water injection, the laser cannot be introduced until the chamber pressure recovers to prevent damage to the compressor gratings. In our setup the chamber pressure began to recover after $\sim 10\,sec$ and recovers after $\sim 20\,sec$.

### 4.2.5 Contaminant Control

During the delay after the water injection while the chamber pressure recovers, the newly formed heavy ice layer is susceptible to contamination by hydrocarbons and $H_2O$ as discussed above in section 4.2.3. A cryogenic shroud around the target was employed to prevent significant contamination of the heavy ice. The shroud consists of a copper box with appropriately drilled openings for the laser and diagnostic lines-of-sight and for target

*Developing the coating was performed on a different target chamber which had more pumping & also had a $1.2\,m$ cryo-cooled cold finger concentric to the beam line and another auxiliary chamber before the compressor. This setup recovered considerably faster. It is possible that for an even smaller volume of water vapor, the compressor may not have needed to be isolated."
manipulation. For pressures where gaseous $\lambda_{mfp}$ is the same order as the dimensions of the vacuum chamber, the shroud decreases the contamination rate of the target proportional to the solid angle that it covers by intercepting contaminants that would otherwise strike the target. Possibly more importantly, during the injection gaseous contaminants will be entrained in heavy water vapor flowing* out of the shroud.

The mean free paths of the $\text{H}_2\text{O}$ and $\text{C}_n\text{H}_{2n}$ are greater than the size of the vacuum chamber and will not behave as a fluid, but primarily follow ballistic trajectories. For a gas, the $\lambda_{mfp}$ can be expressed in terms of the collision cross section $\sigma$ as $k_B T / \sqrt{2} \sigma p$†

One can estimate $\sigma$ as the particle size, determined by the liquid density $\rho$, the $A$, and $N_A$: $\sigma \simeq 4(A/N_A \rho)^{2/3}$‡. This approximation neglects the packing fraction in liquid form, inter-particle interactions, and the differential cross-sections for different orientations, but is roughly the same order, sufficient for the current purpose.

$$\lambda_{mfp} \simeq \frac{k_B T}{4\sqrt{2} p} \left( \frac{N_A \rho}{A} \right)^{2/3} \tag{4.2}$$

If one assumes that all the particles in the chamber are water, then at $294 \, \text{K}$ and $p = 1 \times 10^{-5} \, \text{Torr}$, $\lambda_{mfp} \simeq 0.7 \, \text{m}$. If it was all $\text{C}_{15}$, $\lambda_{mfp} \simeq 0.2 \, \text{m}$. The chamber was $\sim 1 \, \text{m}$ in diameter so the assumption that the particles behave ballistically is not a bad one. Since the majority of the particles after a few collisions with the shroud will be stuck, the partial pressure inside the shroud should be significantly lower.

During the water injection, the chamber pressure was allowed to rise to $10^{-2} \, \text{Torr}$. Neglecting the cooling of the heavy water through expansion§, $\lambda_{mfp} \sim 200 - 700 \, \mu\text{m}$, significantly smaller than the dimensions of the shroud, or the holes in the shroud. Gaseous particles within the shroud prior to the injection are expected to be entrained in the heavy water vapor expanding and flowing out of the shroud. As the chamber pressure recovers the partial pressure of heavy water is initially significantly higher than any other species as it is responsible for increasing the pressure three orders of magnitude. It is higher still inside the shroud while heavy water vapor still flows out.

As the pressure recovers, $\lambda_{mfp}$ increases and gaseous particles behave more ballistically. As with the cryogenic target, the sticking coefficient will be very near unity and the shroud will reduce the contamination rate roughly by the solid angle from the rear of the target not subtended by the shroud to $2\pi$. Between the hole in the floor of the shroud to allow for target

*chamber pressure increases reducing $\lambda_{mfp}$ to mm’s.

†This is simply the number of particles within the interaction cross-section swept into a column along the particle trajectory. Additional path length due to collisions if all the particles are thermalized, the average closing velocity $\sqrt{2} v_{\text{el}}$. As the thermal velocity of the target particles becomes insignificant compared to the incident particle velocity, $v_{\text{el, closing}} \rightarrow v_{\text{el}}$. And of course the ideal gas law.

‡For two identical spheres colliding, the max separation of the centers is the diameter. The area swept out from the center of the moving sphere to the center of the targets is $\pi d^2$ or 4 times the area.

§Some of the energy will be lost to throttling, increasing entropy, through the valve, but much of the thermal energy will be converted to bulk velocity through the nozzle.
manipulation, the hole for the focal spot monitor, two for the rear surface interferometry, and one for the TPS, the covered to uncovered solid angle the rear surface views is 10:1, roughly decreasing the contamination rate by an order of magnitude after gaseous behavior becomes more ballistic.

Figure 4.4: A cryogenically cooled silicon wafer substrate supports a $200 \mu m \times 200 \mu m \times 0.5 \mu m$ thick membrane of Si$_3$N$_4$ and Al. The target rear surface is coated with heavy ice just prior to interaction with the laser. To minimize contamination the target is protected by a cryogenically cooled shroud. Figure not to scale. Reprinted with permission from J.T. Morrison, M. Storm, E. Chowdhury, K.U. Akli, S. Feldman, C. Willis, R.L. Daskalova, T. Growden, P. Berger, T. Ditmire, L. Van Woerkom, and R.R. Freeman, *Physics of Plasmas*, vol. 19, 300707 (2012). Copyright 2012, American Institute of Physics.

Though I have elucidated several reasons why the cryogenic shroud reduces the contaminant deposition rate, it is unclear that these reasons alone are sufficient. In the few seconds after the chamber pressure recovers before the laser shot, the the upper bound of apx. a mono-layer per second, even reduced an order of magnitude still would deposit a layer similar to one expected on the warm metal targets. I wouldn’t be going through all the trouble of working these things out if it didn’t work. So a couple more suggestions as to what may be happening.
4.3 Thomson Parabola Spectrometer

Thomson parabola spectrometers are used to characterize MeV ion beams produced in high intensity laser interactions. These spectrometers disperse multiple ion species according to their charge to mass ratio through the use of parallel electric and magnetic fields. All of these spectrometers have their roots in the work of J.J. Thomson, for which they are named[79]. Used in Thomson’s work on positive rays (accelerated charged atoms and molecules) and his studies of corpuscles† or electrons, he was able to produce some impressive images of traces various ions made on a phosphor screen he used to detect the ions[80].

Ions leaving the thin foil are collimated by passing through a pinhole in the spectrometer which is set a large distance (compared to the target size) from the foil. The transmitted ions pass through parallel electric and magnetic fields orientated perpendicular to the ions’ initial momentum. The Lorentz force disperses the ions perpendicular to both their initial momentum and the magnetic field according to their charge to mass ratio and energy. The result is a series of parabolic traces on a detector positioned perpendicular to the initial velocity. This separation of species into individual tracks permits the measurement of spectra when multiple ion species are present.

In order to get accurate spectra from a TPS, the electric and magnetic fields must be well characterized. Commonly, the geometry of the spectrometer is designed to meet assumptions of field uniformity and negligible fringe fields. This simplifies the analysis, giving a simple analytical solution for the ion positions at the detector[80, 81].

However, experimental setups often require that the spectrometer be made more compact and with high dynamic range. These restrictions often result in designs that violate the simplifying assumptions above. For these designs, a more thorough model of the fields is needed to accurately extract ion spectra from traces on the detector. Depending on design parameters, the error in the analytical assumption can be comparable to the energy resolution. Estimates are provided to approximate the error on the total ion deflection. A method for modeling ion trajectories including fringing effects is presented below using software freely available or in common use. The magnetostatic fields are modeled in 3D, including material properties of nearby magnetic materials using Radia[82]. Electrostatic fields are modeled in 2D for a spectrometer implementing angled plates using the partial differential equation toolbox in MATLAB®. Using these models to calculate the ion trajectory allows for analysis of a Thomson parabola spectrometer with an arbitrary field


† A term that generally refers to any minute particle. J.J. Thomson used the term for small particles of negative charge suspended within the positive pudding which supplied the size and mass of an atom. It still lives on as a free floating cell such as a blood cell. Isaac Newton used the same term for light particles in his particle model of light.
configuration.

4.3.1 Canonical Design and Data Reduction

Traditionally, a TPS will consist of parallel, uniform electric and magnetic fields. The uniform electric field is obtained with parallel field plates, and the uniform magnetic field is obtained either with a yoked pair of permanent magnets, or a yoked electromagnet[83]. The typical analysis of TPS systems involves assumptions that the magnetic field and electric field are uniform, and fringe fields are ignored. These assumptions require the effect of possible fringe fields to be minimized. Reducing the gap between the magnets, yoking the magnets, injecting the ions on the symmetry axis of the magnets, and extending the magnets all improve the validity of the analytical analysis.

The canonical analysis assumes the electric field $E$ and magnetic field $B$ are parallel to one another and are uniform in space. This permits the change in the ion’s momentum due to each field to be treated independently. A fit of the analytic solution presented below and experimental data for this design is shown in Figure 1. Designs that cannot be modeled to sufficient accuracy by these analytic solutions require a numerical model as presented in section 4.3.2.
TPS Dimensions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Old TPS</th>
<th>Compact</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{\text{pinhole}}$</td>
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<td>0.11</td>
</tr>
<tr>
<td>$x_{\text{total}}$</td>
<td>120</td>
<td>82</td>
</tr>
<tr>
<td>$y_{\text{total}}$</td>
<td>71</td>
<td>76</td>
</tr>
<tr>
<td>$z_{\text{total}}$</td>
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<td>220</td>
</tr>
<tr>
<td>$\phi$</td>
<td>deflection angle</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
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<td>4.8°</td>
</tr>
<tr>
<td>$x_B$</td>
<td>25.4</td>
<td>25.4</td>
</tr>
<tr>
<td>$y_1$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$y_2$</td>
<td>10.1</td>
<td>12.4</td>
</tr>
<tr>
<td>$z_0$</td>
<td>208</td>
<td>196</td>
</tr>
<tr>
<td>$z_B$</td>
<td>50.8</td>
<td>50.8</td>
</tr>
<tr>
<td>$z_E$</td>
<td>150</td>
<td>120</td>
</tr>
</tbody>
</table>

Table 4.2: As implemented dimensions of both the larger TPS based on the Bandyopadhyay and Neely design [84], and a smaller TPS referred to as the compact TPS, including the width of magnets $x_B$, external dimensions of the device $x_{\text{total}}$, $y_{\text{total}}$, $z_{\text{total}}$, and pinhole radius $R_{\text{pinhole}}$.

First consider the motion of an ion with initial velocity $\mathbf{v} = v_0 \hat{z}$ through a region with a uniform magnetic field $B$ with extent length of magnets $z_B$ which crosses a region with $B = 0$ before striking the detector distance to detector $z_0$. A non-relativistic ion with $m$ moving in a uniform magnetic field follows a circular orbit with cyclotron radius $r_{cy} = \frac{mv_0}{eZB}$. Assuming that the field outside the magnets is negligible, ions traversing the magnetic field region follow a section of the cyclotron orbit. Ions leave the region between the magnets tangential to their orbits; their velocity in the plane perpendicular to the $E$ and $B$ fields is then constant over the distance to the detector, $z_0 - z_B$. The length of magnets $z_B$ and the deflection angle $\phi$ are related in Figure 4.5 by $\sin \phi = z_B/r_{cy}$. Assuming $z_B \ll z_0$ and $z_B \ll r_{cy}$ then the displacement due to the magnetic field

Figure 4.5: Schematic of ion trajectory within TPS. Top: Side view of TPS showing opening angle of the field plates $\theta$, front spacing of the field plates $y_1$, length of electric field plates $z_E$, length of magnets $z_B$ and distance to detector $z_0$ as used in equations 4.3 and 4.6. Bottom: Top view of the TPS showing a typical ion path through the magnetic field, including the deflection angle $\phi$ and $r = r_{cy}$ as used in the derivation of equation 4.3. The magnets are mounted to a yoke described by Figure 4.12. Reprinted with permission from J.T. Morrison, C. Willis, R.R. Freeman and L. Van Woerkom, Review of Scientific Instruments, Vol. 82, 033506, (2011). Copyright 2011, American Institute of Physics.
is \( x(z_0) = z_0 \tan \phi \simeq z_0 z_B / r_{cy} = z_0 z_B eZB / mv_0 \). Without the preceding simplifying assumptions

\[
x(z_0, r_{cy}) = \frac{z_0 - z_B}{\sqrt{r_{cy}^2 / z_B^2 - 1}} + r_{cy} \left( 1 - \sqrt{1 - z_B^2 / r_{cy}^2} \right)
\]

(4.3)

Figure 4.6: Ion traces produced on the larger TPS designed to meet the assumptions necessary to derive the analytical model in equations 4.3 and 4.6. False color image of ion traces captured on BAS-TR 2025 image plates[85] from shots on 5 µm Ti, with 500 mJ at \( \sim 1 \times 10^{20} \) W cm\(^{-2}\) using 2500 V potential across the electric field plates. Fits of the analytical model are overlaid on the experimental data. The plotted energies are 5, 4, 3, 2.5, 2, 1.8, 1.6, 1.4, 1.2, 1, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, and 0.2 MeV. Minimum detected energies are 0.29, 1.0, 0.76, 0.49, and 0.29 MeV for p+, C4+, C3+, C2+, and C1+ respectively. Noise on either side of the proton traces is due to protons scattering through the foil near the pinhole. Reprinted with permission from J.T. Morrison, C. Willis, R.R. Freeman and L. Van Woerkom, Review of Scientific Instruments, Vol. 82, 033506, (2011). Copyright 2011, American Institute of Physics.

Assuming that the electric field plates are parallel, the ions will be uniformly accelerated and follow a trajectory \( y(z) = eZe^2 / 2mv_0^2 \), where \( z \) is the distance the ion travels in the electric field \( \mathbf{E} = -\hat{E}y \) and \( |\mathbf{E}| = E_y \). After traversing length of electric field plates \( z_E \), the velocity is constant. The total displacement from \( \mathbf{E} \) at the detector plane is given by[81]:

\[
y(z_0) = \frac{eZe}{mv_0^2} z_E \left( \frac{z_E}{2} + (z_0 - z_E) \right)
\]

(4.4)

A simple generalization to the parallel field plate configuration can greatly improve the spectrometer’s ability to record low energy ions[84]. In order to increase the dynamic range at a given electric field deflection, angled electric field plates can be employed. This decreases the energy at which ions collide with the negative field plate. If the opening angle of the field plates \( \theta \) and front spacing of the field plates \( y_1 \), the electric field is approximately \( \mathbf{E} = \Delta \Phi / d(z) \hat{z} \) for plates with a voltage difference \( \Delta \Phi \) and a separation distance \( d(z) = y_1 + z \sin \theta \)[84, 86]. Using the average separation distance
\[d_{avg} = y_1 + (z_E \sin \theta)/2\] in equation 4.4 approximates the ion deflection from electric field plates\(^\ast\). A more accurate solution is obtained by integrating Newton’s second law with the spatially varying field, yielding

\[y(z \leq z_E) = \frac{eZ \Delta \Phi}{mv_0^2 \sin \theta} \left[ \left( \frac{y_1}{\sin \theta} + z \right) \ln \left( 1 + \frac{\sin \theta}{y_1} z \right) - z \right]\] (4.5)

Neglecting fringe fields, the ion trajectory is unchanged beyond the field plates such that \[y(z \geq z_E) = y(z_E) + (z - z_E) \frac{v_y(z_E)}{v_0}\] yields the total deflection at the detector

\[y(z_0) = \frac{eZ \Delta \Phi}{mv_0^2 \sin \theta} \left[ \left( \frac{y_1}{\sin \theta} + z_0 \right) \ln \left( 1 + \frac{\sin \theta}{y_1} z_E \right) - z_E \right]\] (4.6)

Setting \(y = y_1 + z_E \sin \theta\) and \(z = z_E\) in equation 4.5 determines the energy at which ions collide with the negative plate, which is the low energy cutoff for the spectrometer.

The resolution of a given TPS design is limited by the uncertainty in ion positions on the detector. The radius of the image on the detector formed by the TPS pinhole \(\delta R\) is characteristic of the width of the traces formed by the TPS. As discussed in chapter 3, experimental observations of laser accelerated ions indicate a laminar expansion from a virtual point source\([26, 27]\). The expected ion track width on the detector and associated ambiguity in total deflection \(\delta R\) is the projection of the point source through the pinhole on the detector.\(^\dagger\) If the track width determines the ambiguity in the deflection, the resolution is a function of displacement from the trajectory of a neutral particle at the detector, or the neutral point. The distance from the neutral point is as a function of energy is \(R(E) = \sqrt{x(E)^2 + y(E)^2}\) using equations 4.3 and 4.6 evaluated at \(z = z_0\) and \(E = mv_0^2/2\). The energy uncertainty is \(\delta E = \delta R \frac{dE}{dR}\). The normalized uncertainty \(\delta E/E\) is plotted for the compact design in figure 4.7.

### 4.3.2 As Implemented Design

The analysis above gives satisfactory results provided the magnets are spaced closely enough to render the influence of fringe fields negligible, and the ions are inserted along the symmetry axis of the magnets, where the \(B\) field is normal to the magnet faces. Placing the electric field plates outside of the magnet region allows for reduction in magnet spacing, and permits the ions to travel in the plane of symmetry. However, experimental space constraints may not accommodate such a design. A more compact design places the electric field plates between the magnets. The result is that the magnets must be separated by a much larger distance. Minimizing magnet separation while using angled field plates requires ions to be

\(\ast\)Here it is assumed that \(\theta\) is small.

\(\dagger\)To limit noise on the detector as observed in figure 4.6, the pinhole should be thick enough to stop the highest energy protons expected. If not, ions scattered through the pinhole will show up as a background from the point source projected on the detector through the collimator.
Figure 4.7: Energy resolution plotted as a function of energy for protons. The blue curve results from the analytical model. Red points are calculated numerically from 3D field models. The uncertainty in energy $\delta E / E$ is roughly proportional to the uncertainty in deflection $\delta R / R$, where $\delta R$ is the ion track width from figures 4.6, 4.8, or 4.9. The numerical particle tracking program is more likely to under resolve the fringing regions for higher energy ions. In addition the numerical solution is more susceptible to noise at higher energies, where ions have a lower deflection distance R.
inserted off of the symmetry axis of the magnets, increasing the effects of the fringing B fields. Ion deflection from these fields is no longer parallel to the magnets and this effect can be observed primarily in the low energy ions without zero electric field, as shown in Figure 4.8.

![Figure 4.8](image.png)

**Figure 4.8:** The predicted location of ion traces for zero voltage on field plates is plotted in red over experimental data from two shots, one with 3 kV on the electric field plates and one without. Note that this zero voltage trace is curved, indicating that ions encounter magnetic fields not normal to the magnet surface. This indicates further analysis is required beyond the solutions presented in equations 4.3 and 4.6. Reprinted with permission from J.T. Morrison, C. Willis, R.R. Freeman and L. Van Woerkom, *Review of Scientific Instruments*, Vol. 82, 033506, (2011). Copyright 2011, American Institute of Physics.

To further reduce the overall size of the spectrometer, the length of the electric field plates can be reduced, provided the voltage across them is increased to maintain the total electric field deflection at the detector. However, this configuration causes the ions to deflect more sharply in the electric field, increasing the minimum ion energy that the spectrometer can record. To counteract this, the angle between the field plates is increased to reduce the energy at which ions collide with the negative field plate. Increasing the angle between the electric field plates while increasing the field causes low energy ions to encounter \( \mathbf{E} \) field components parallel to the ions’ initial velocity. This renders the simple kinetic expressions for displacement from the \( \mathbf{E} \) field invalid for large \( \theta \) and low energy ions.

One approach to improve the accuracy of the equations governing the displacement of the ions is to measure the magnetic field component normal to the magnets with a gauss-meter. The B field can then be interpolated and the magnetic deflection solved numerically to obtain predicted ion traces. While this approach aids in determining the spacial dependence of the magnetic field strength, it still ignores the variation in the direction of the magnetic field.
The following model determines the magnetic and electric fields (both magnitude and direction) in our spectrometer over all points in space using 3D magnetostatic and 2D electrostatic models. The magnetic field of the yoked NdFeB magnets is modeled in Radia\(^{82}\). Electrostatic fields are modeled using the Partial Differential Equation Toolbox in MATLAB\(^{®}\). Equations of motion are solved for ions at various energies using an adaptive ordinary differential algorithm in MATLAB\(^{®}\).

### 4.3.3 Magnetostatic Modeling

Radia builds a 3D geometry of the system, segmenting this geometry into \(n\) regions or elements. Based on the material properties applied to the model, Radia builds an \(n \times n\) interaction matrix defining how each element’s field magnetizes elements in the rest of the geometry.

To calculate the magnetization of a given geometric model, the interaction matrix is iteratively applied until the magnetization of the elements converges, thus permitting the calculation of the magnetic field at any point in space. When using Radia to solve a system, the choice of segmentation is particularly important. If the number of segments, \(n\) is too small, Radia does not accurately model the magnetization of the materials and hence the field.

In practice, the number of elements is increased in successive iterations until the
calculated field values at relevant positions (e.g., along the symmetry axis of the yoke) converge to some value. Increasing the segmentation of the model increases the size of the interaction matrix and the number of calculations. In addition to the number of segments, the type of segmentation is also important. Segmenting the model such that the magnetic fields are nearly normal or nearly parallel to the segment boundaries improves the accuracy for a given number of segments, allowing the use of fewer segments in the model. This reduces the number of calculations needed for convergence, and increases the accuracy of the final result.

A simplified model of the yoke and permanent magnets, without fasteners or fillets is constructed in Radia. In the corners of the yoke, where the fields bend through a sharp corner, a cylindrical segmentation rather than a rectangular one is employed. Material properties are applied according to Radia’s built in database for magnetic steel and NdFeB materials. The magnets are defined initially as regions of constant magnetization according to the manufacturer’s specifications. The properties of the magnets and yoke are applied during iterative application of the interaction matrix. After initial setup of geometry and material properties, the model converges in less than one minute on a 3GHz dual-core 64-bit computer. Our model converges with \( n \sim 1500 \) and \( \delta B \sim \pm 1 \text{ gauss} \).

The Radia prediction of the field near the symmetry center of one of the magnetic surfaces varies from a direct hall-probe measurement by a constant 5\%. Thus for further calculations of absolute deflection of the ions, the Radia code is normalized in all space by this comparison ratio.

The magnetic field \( B_x, B_y, \) and \( B_z \) between the magnets is exported to MATLAB® along a 5 mm grid at several different planes between the magnets and extending beyond the magnets by 15 mm.

4.3.4 Electrostatic Modeling

For the compact design described above, the fields are modeled in 2D using the Partial Differential Equation Toolbox in MATLAB®. This toolbox employs an adaptive finite-element method to find the solution of the Laplace equation \( \nabla^2 \Phi = 0 \) in the region near the field plates and grounded magnets. The geometry of the field plates is inserted using the toolbox’s graphical user interface∗.

Dirichlet boundary conditions are imposed at the edges of relevant geometry. The appropriate voltage is specified on each field plates. Other nearby conductors such as the magnets and yoke are grounded. In addition, the boundary condition is imposed that \( \Phi = 0 \) far away from the device. After defining the geometry, an adaptive triangular

∗When using this interface, geometry elements are limited to logical combinations of rectangles, polygons, or ellipses. More complicated shapes can be included by parametrically defining the geometry and boundary conditions in separate geometry files and boundary files.
mesh is created using the graphical user interface. This mesh is densely spaced near edges and corners of geometry, where potential varies rapidly, but is more sparsely spaced far away from any boundary conditions. The mesh can be easily refined in the user interface, permitting higher resolution in the potential calculation, and higher accuracy in calculating the electric field.

After the geometry and mesh are defined, the Laplace equation is solved on the vertices of the adaptive mesh, yielding the electrostatic potential as a function of position. Other built-in functions are then used to find the electric field at a given position, defined by \( \mathbf{E} = -\nabla \Phi \). This electric field vector is then passed to the adaptive differential algorithm in MATLAB®.

As this modeling is performed in 2D, fringing effects in the non-modeled dimension should be avoided. Therefore, the \( \mathbf{E} \) field plates should have sufficient extent in the direction of magnetic field deflection that ions do not approach the edges of the plates. For electric field plate designs that cannot be rendered in 2D, it may be necessary to model the 3D field in SIMION® or similar software.

For the Compact design there was a difference between the design and the fabricated spectrometer. The electric field plates were slightly warped, a difference of less than 1 \( \text{mm} \) across the front gap, but was < 0.1 \( \text{mm} \) at \( z = z_E \). This difference between the physical spectrometer and the modeled one was apparent in the higher energy protons. Instead of resorting to a 3D field solver such as SIMION®, two 2D models were used in different \( xz \) planes and as the plate separation varied slowly and linearly in \( x \). The 2.5D field values passed to the Newtonian solver are a linear interpolated between the two 2D models.

Using an adaptive ordinary differential equation algorithm in MATLAB® and the field models described above, trajectories for several species of ions are calculated across a log\( \text{10} \) array of energies. The positions of these ions at the detector plane is then output as a function of energy and ion species in order to make a mapping of the predicted traces at the detector. The resulting traces are shown overlaid on experimental data in Figure 4.9. The dimensions of our compact spectrometer are shown in Table 4.2.

### 4.3.5 Estimates of Analytical Error

In order to design a spectrometer such that the analytical solutions are adequate, the limits to the approximations inherent in the analytical solution need exploration. Two dimensional models of the electric field between the electric plates show that the analytical approximations described in Section 4.3.1 are reasonably accurate for positions a greater distance from field plate edges than the separation of the plates. At \( z = y_1 \) the model and analytical solution are nearly identical. The magnitude of the modeled field decreases to \( \sim 60\% \) the strength of the analytical solution at \( z = 0 \) and falls below \( \sim 10\% \) of the analytical solutions maximum at \( z = -2y_1 \). A symmetric argument can be made at the
other end of the electric field plates. If, as in both of above designs, the plate separation and therefore $\sim |E|^{-1}$ is $\sim$ ten times that of the edge near the pinhole, it may be neglected. The error in deflection due to the fringe field near the plates exit will also be greatly reduced as this is much closer to the detector. By considering only the errors from neglecting the fringe fields associated with the leading edge, the analytic solutions above under estimate the ion deflection introducing a relative error of $\sim y_1/z_E$ assuming $z_E \gg (z_0 - z_E)$.

![Electric Field Model](image)

Figure 4.10: Model of the electric field near $z = 0$ around the field plates using the 2D Partial Differential Equation Toolbox in MATLAB®. (a) A false color plot of the magnitude the electric field in the $x = 0$ plane. The black line is $y = 0$. (b) A line out of $E_y$ along $x = z = 0$. The black curve is the electric field from the The red curve is the simple analytical approximation described in Section 4.3.1 which is basically the parallel plate capacitor solution using the local plate separation.

The numerical solutions predicting the magnetic field are shown in Figure 4.11 for
geometries with and without a yoke. For magnets yoked as in Figure 4.12, the field is 95% of the peak value for separation of the magnets $y_2 < z < (z_B - y_2)$, where again $y_2$. Assumption of constant B field overestimates the magnetic field deflection introducing a relative error of $\sim y_2/(z_0 - z_B/2)$, assuming $z_B \ll r_{cy}$. This error increases if the ion trajectories also sample the fringe field in $x$.

Figure 4.11: In the case where permanent magnets are used without a yoke, assumptions of field uniformity and negligible fringing effects are particularly stressed. In the absence of a yoke, the field normal to the magnet is far less uniform than in the yoked case as shown above. (a) Division of yoke into cells for modeling in Radia[82]. (b) The Magnetic field of two permanent NdFeB magnets 2” × 1” × 1/4” in size, separated by 4 mm is plotted along the symmetry axis of the magnet pair. The magnets and yoke are symmetric in $z$ around 25.4 mm. The solid curves (left) are the magnitude perpendicular to the magnet face along the symmetry axis, and the dashed curves (right) are the magnitude parallel to the magnet face at 10% of the magnet separation, or gap. The blue (lower) curve is the field from just the permanent magnets. The red (higher) curve is the resulting field after the addition of a magnetic steel yoke. Note that the field is nearly twice as strong, and is more uniform after addition of a yoke. In addition to poor field uniformity, non-normal field components are a larger fraction of the overall field in the unyoked magnet pair. Figure (b) is reprinted with permission from J.T. Morrison, C. Willis, R.R. Freeman and L. Van Woerkom, Review of Scientific Instruments, Vol. 82, 033506, (2011). Copyright 2011, American Institute of Physics.
4.3.6 Additional Considerations

The maximum strength of the electric field is limited by spark breakdown between the plates. The Paschen curve describes the relation between pressure gap distance, and voltage at which a Townsend avalanche breakdown occurs.[87] For high vacuum, field emission of electrons from the cathode limits the maximum potential.[88] In these designs, a 14 kV potential cannot be maintained at 3 \times 10^{-5} \text{Torr} and 12 kV is considered a practical limit.

Appropriate selection of a pinhole is directly related to the distance to the source. Experimental observations of laser accelerated ions indicate a laminar expansion from a virtual point source[26, 27]. The expected ion track width on the detector and associated ambiguity in total deflection \( \delta R \) is the projection of the point source through the pinhole on the detector. To limit noise on the detector, the pinhole should be thick enough to stop the highest energy protons expected. If not, ions scattered through the pinhole will show up as a background from the point source projected on the detector through the collimator. The lightest ions, protons will generally have the highest energies, and longer particle range \( R \) for the same energies. Again, continuously slowing down approximation (CSDA) ranges* for protons are available at Pstar to determine the necessary thickness for a given design. Unfortunately, available machining techniques may be the limiting factor on how thick a pinhole can be for a given diameter.

*These ranges correspond to the most probable range. However ranges for various individual ions do not have a large standard deviation.
4.3.7 Ringing & EMP

High frequency noise can be induced in high voltage (HV) cabling during a shot, dramatically altering the electric field. This change in the electric field is exhibited as oscillations or kinks in the ion traces. A low-pass filter in the HV cabling\cite{89} in addition to shielding is recommended. Grounding the HV power supply connections and cable shielding independent of the target chamber can also reduce these effects.

As an electro-motive pulse (EMP) diagnostic the TPS is fairly poor. If one were to embark on a careful study of the exact source of the antenna that is coupling the EMP to the voltage across the field plates, then maybe one could begin to infer something about the magnitude of frequencies being coupled. One can infer which frequency components of the EMP are being coupled to the spectrometer\footnote{One could imagine that the ion trajectories themselves are being altered in a sufficiently strong EMP and the displacement at the detector is an artifact of these deflections projected over the distance to the detector. However, this would foil experiments that use the protons as a back-lighter for imaging other events using different energy windows to correlate the images to times.} in order to choose the appropriate low pass filter.

Since the time period in which the ions are accelerated is relatively short $\sim ns$, and the ions are not relativistic, any alteration of the trajectory of the ions some distance away from the ion acceleration target will have time encoded in the different ion energies, similar to a time of flight (ToF) detector. Noting the distance from the target to the TPS detector, a simple inspection of the ringing ion trace reveals the number of periods the oscillation undergoes from the time the highest energy ions recorded arrive to the time the low energy ions that just miss the negative plate arrive at the detector.\footnote{I suspect that this has something to do with either target size and/or some combination of intensity and energy. Using the setup at GHOST we observed at most one period, and though I’m really not sure if it was aligned at all, when they attempted to field the device on the Texas Petawatt the ringing appeared much more rapid, as with Dustin’s data\cite{90} and Higginson et al\cite{91}. Though this doesn’t discount the device or installation. They put the TPS in a different auxiliary chamber, possibly most importantly, with a different diameter window to the main chamber. A plausible explanation is that the EMP is strongest at lower frequencies, so the lowest frequency which the port (wave guide) will pass is likely the strongest driving frequency.}

For illustration, the data acquired at Ghost has a ring in the proton trace undergoing 1 – 7 periods depending on the shot and target. The maximum energies are $\sim 7 MeV$ and the low energy cutoff for the proton trace is $0.29 MeV$. Using intro physics $v = \sqrt{2mE/2}$ and the definition of non-relativistic velocity this implies $\delta t \approx 100 ns$ and the corresponding frequencies are $\sim 100 – 700 MHz$.
4.3.8 CR-39

CR-39\textregistered or allyl-diglycol carbonate\textsuperscript{*} is a plastic polymer resin which can be used to detect energetic neutrons and ions. Incident energetic particles with significant mass cause recoil or ionization damages as they scatter off of the atoms within the CR-39. After exposure the CR-39 is etched with NaOH which removes the damaged material faster than the surrounding undamaged resin causing a relief or pit in the surface of the CR-39 which can be subsequently view under a microscope as in Figure 4.13

Figure 4.13: CR-39 as a particle track detector. (a)Pits in CR-39 viewed under x40 magnification. These pits are $\sim 8 \mu m$. (b) The CR-39 was etched in a $6 M = 6 mol/L$ solution of NaOH for one hour at a temperature of 95$^\circ C$.

CR-39 is not significantly susceptible to damage from $\gamma$-rays and x-rays because they are unlikely to damage the surface of the CR-39, due to their comparatively low cross sections, which is crucial pit formation. This makes CR-39 an attractive ion detector. In addition, the quantum efficiency of CR-39 is well-defined by unity\textsuperscript{†} so it can be used as an absolutely calibrated detector. Work has been done on CR-39 to quantify and characterize the pit formation and depth as a function of ion energy and mass and the etching parameters. One can reveal substantially higher energy tracks from a saturated piece of CR-39 by etching the lower energy tracks completely away. More aggressive etching parameters can also be used to erase tracks to reuse the CR-39 if desired.

Unfortunately, if one does not posses a microscope or other device that can scan an exposed piece of CR-39, self focus, measure and log pit sizes and global positions, extracting data can be extraordinarily painful. When using a TPS one does not need to infer energy from the CR-39, so for this work CR-39 was used to calibrate the sensitivity of BAS-TR

\textsuperscript{*}The most common use of CR-39 is in the eyeglasses. According to PPG this was the 39\textsuperscript{th} resin formula from the Columbia thermalsetting resin project. It is currently a trade marked product of PPG Industries, Inc. I will drop the trademark from here on.

\textsuperscript{†}Provided the track density is low enough that one can detect all the pits\textsuperscript{[92]}. 68
2025 image plates described in the following section.

4.3.9 Image Plates

Up until now, I have only described how one correlates position in the detector plane of a TPS with the energy $E$ for a given charge to mass ratio, and a detector that would be useless for all practical purposes without the appropriate microscope and software which was not available for this work. A detector commonly used in the medical field as a reusable imaging material usually in place of x-ray film, an IP or a photo-stimulable phosphor (PSP) plate is phosphorescent* utilizing active layer of grains of BaFBr:Eu2+. BaFBr:Eu2+ is a phosphor with a long lifetime and the additional property that the excited states can relax by photo-stimulated emission or luminescence [93, 94]. I will neglect some of the detail discussion originally framed as x-ray imaging detectors.

Any ionizing radiation creates electron and electron holes in the storage phosphor proportional to the local energy deposited. Spatial variation in the energy deposition is preserved in the number density of the electron-hole pairs. The spacial variation in the energy deposition can be recovered subsequently by illumination with a HeNe laser which excites the hole pairs causing them to relax resulting in blue Eu2+ luminescence proportional to the intensity of the illuminating light and the number density of the hole pairs. Scanning an exposed image plate with focused HeNe light and measuring the characteristic blue luminescence with an appropriately filtered, characterized, and digitized photomultiplier tube produces a spacial data set of the uniquely related to the deposited energy within some error. IP’s may be reused after being erased by an extended exposure to stimulating light, as supplied by intense incandescent light†. IP’s have a tremendous dynamic range exceeding 5 orders of magnitude[95] with a linear response to x-ray dose‡. For a practical application of IP’s as a detector, measurements made in this work used BAS-TR 2025 IP’s , commercially available from Fujifilm Holdings Corporation, which are blue in appearance due as the phosphor layer is the outer surface§.

A Fujifilm FLA-7000 IP scanner was used to digitize the exposed image plates and record

*The difference between fluorescence and phosphorescence is much fuzzier than usually implied. Something fluorescent absorbs photons and re-emits the energy gained at a different frequency. The process for phosphorescence is essentially identical accept the relative life times of the states. Laser gain mediums have life times from $ns$ to $\mu s$ depending on the relevant time scale this could be a delayed emission or not. An example of this can be found in compact florescent bulbs in most homes. Though, named fluorescent for the coating which produces a more preferred and visible spectrum than the plasma inside the bulb, these bulbs (at least the ones in my house) have a 'warm up' time and continue to glow after they have been shut off. Definitely a phosphor.

†Fluorescent lights can activate the IP’s presumably from some of the more energetic photons in their spectra

‡I believe it is reasonable to expect, based on the mechanism, that this will also be true for any ionizing radiation but that is not certain

§Ions have a very short range and are extremely unlikely to penetrate the protective coating on other image plates.
the photo-stimulated luminescence (PSL) in an absolute unit PSL*. The electron hole pairs in the IP’s active layer have lifetimes which are temperature dependent. To use the imaging plates as an absolutely calibrated detector it is necessary to account for both the time before scanning and the temperature. As illustrated in Figure 4.14, if one intends to compare Image plates with different scan times and temperatures it can be an issue. Scanning the IPs takes \( \sim 30 - 60 \text{ sec} \) depending on the area scanned, so one usually waits \( 30 - 45 \text{ min} \) after exposure to scanning.

The implication of the different decay rates for different incident particles is more restrictive. Ohuchi’s work indicates that the different situations have a set of four or five characteristic states with decay rates that are effected by temperature and for the TR IPs, are activated in different proportions for different incident particles.\(^1\) This implies that applying a single decay rate to a data set with different energies will probably be progressively less accurate for increasingly different scan times, especially for shorter scan times. Also note that if the IPs are exposed to any bleaching or erasing radiation, such as room light, the activated phosphor with erase with a rate that is decreasing as of depth as a consequence of equation 4.7 discussed below.

With the above in mind, one can still effectively calibrate image plate sensitivity for a specific time, and temperature. Mančić et. al.\(^85\) calibrated BAS-TR 2025 IPs by placing both the IP and CR-39 as detectors in a magnetic spectrometer, splitting the trace so that protons\(^4\) with the same energy are detected on adjacent portions of the two detectors. This allows for the calculation of the image plate response \( \text{PSL}/N_p(E) \) to protons. The scan time reported was \( \sim 7 \text{ min} \).

Simplistically, due to the comparatively short ranges of ions, one can expect for most low energy ions to stop within the phosphor depositing all their energy in the active layer. Some proportion of this energy will active meta-stable states in the phosphor. The response of the IP we expect to be nearly linear for these ions. For sufficiently high energies of lighter ions, they will penetrate the active layer and not deposit all their energy there; for these energies, the response will be increasingly sub-linear. For even higher energies, where the Bragg peak is not in the phosphor, one expects the cross-section, the stopping power, and the IP response to decrease with increasing energy. This is qualitatively the behavior

\* General Electric has purchased the Scientific Imaging business from Fujifilm and as of when this was written no longer uses fixed sensitivity settings as the majority of their customers are interested in an image not a measurement. There exists a package for the image manipulation program ImageJ which would interpret the header in the output file from the Fujifilm scanner and supply an absolute unit PSL. As of this writing there is no such calibration for the GE Typhoon FLA-7000.

\(^1\) Ohuchi also finds that MS IPs, with a polyethylene terephthalate (PET) protective coating \( 9 \mu m \) thick, do not decay with significantly different rates for different particles. It seems that either the higher energy ion collisions or the activations sights near the surface are to blame because the \( 5.8 \text{ MeV} \) alpha has \( \sim 25 \mu m \) range.

\(^4\) As described in section 3.3 protons typically dominate the accelerated ion spectra. It is unclear how the contribution from other ions was inferred and therefore how much error was introduced, though a two micron mylar sheet would preferentially reduce the signal from heavier ions.
Figure 4.14: Image plate response decay as a function of time. The black lines are MS type image plates which have a protective plastic coating over the active layer. The red and blue lines are TR type image plates with no coating for use with low energy photons, low energy electrons, or most ions. These plates decay at different rates under exposure to 5.8 MeV α’s (red) or γ and β emitters ⁶⁰Co and ¹³⁷Cs (blue), and the rates increase for higher temperatures. Two ‘room’ temperatures are plotted: 19°C (solid) and 22°C (dashed). These plots use the decay characteristics as determined by Ohuchi[96]. Using 40 minute exposures removes the 20% in 20 min decays reported[85, 97].

I require a calibration, at minimum, for both protons and deuterons. To this end I developed both an experimental calibration, which is most useful as an absolute calibration, and a theoretical model to determine the functional form for different ion species. The experimental calibration is similar to the above calibration but uses the larger TPS described above and slotted pieces of CR-39 intersecting the traces as shown in Figure 4.15.

During a shot, I placed a piece of CR-39 on top of an IP which was machined with slots such that moving along a trace in the detector plane, ions would be detected on the CR-39 and IP alternately as in Figure 4.15. The 1.5 mm wide 1.5 mm separated slots are arranged perpendicular to the magnetic field deflection axis of the TPS such that successive slots intercept increasingly higher energy ions. Using a spatially calibrated optical microscope, images of the pits 100 μm wide were acquired. The edges of the CR-39 were used to correlate images with position on the IP. A counting routine plugin for ScopeTek®ScopePhoto™ was used to count the pits. To aid the routine, the images were acquired slightly defocused such that there were bright spots in the center of each pit*. The counting routine produces

*The pits are approximately spherical, the center of which form a negative lens. The pits created a virtual focus with light from the microscope back-lighter imaged by the microscope. Since all the pits are in the same plane, and nearly the same size for any image every pit in the image will produce a virtual focus.
Figure 4.15: (a) Schematic of method used to calibrate IPs with CR-39. A slotted piece of CR-39 is placed over an IP before the shot. After the etching the CR-39, an optical microscope was used to determine the position of images and take images of the pits. Feature finding software was used to count the pits, in addition to a manual inspection to correct errors in the counting. (b) Successful calibration shot IP data in PSL. This was a CD coated Si wafer target described in section 4.2.1 illuminated with 1.5 J of the GHOST laser.
an image with a + over each point it identifies. Each image was visually checked for counted defects and uncounted pits. Due to pits near the edges of each 100 µm trace, and questionable collections of pits, the counting error is 2% depending upon the individual image.

There has been an effort to calibrate IP x-ray sensitivity[95, 97, 98]. Though not directly applicable to the ion calibration, there are some notable advances in the understanding of the IP’s phosphor by Thoms[95] that enable modeling of how the energy deposition, stimulating light intensity, and luminosity change with depth. Thoms determined the optical scattering length $\lambda_s = 60 \mu m$ and attenuation length $\alpha$ is $0 \approx \alpha \gg \lambda_s$ in the ST type IPs for the stimulating HeNe light. As far as I can determine, the different imaging plates use the same phosphor material, and possibly grain size$^*$; so these values are taken as reasonable assumptions. Thoms derives the probability that stimulating light will be scattered back out of the phosphor before reaching a depth $z$, and the relative flux $\varphi$ of the stimulating light as a function of depth $z$ in an active phosphor thickness $d$, with the simplify assumption that the attenuation is negligible.

$$\varphi_{stm}(z) = \frac{6 \varphi_0 \lambda_s + d - z}{2 \lambda_s + z} \quad (4.7)$$

Note that the flux near the front of the image plate can be larger than the incident flux $\varphi_0$ due to multiple passes by scattered photons. A similar argument is made for the probability $P$ of detecting blue luminescence detected as signal on the photo-multiplier tube (PMT) or diode in the scanner$^\dagger$.

$$P_{det}(z) = P(0) \frac{\lambda_s + d - z}{2 \lambda_s + z} \quad (4.8)$$

The decay of the activation in the phosphor as a function of time, the stimulating photon flux 4.7 and the probability of detection as a functions of depth 4.8 are all defined to within some unknown constants related to specifics of the activation and the method of detection. Similarly, with the assumption that the activation of phosphor is proportional to the energy deposition $dE_{dep}(z)/dz$ by the ions, the image plate response can be calculated to within the same plane.

$^*$Though not definitive, and without a response from Fujifilm, using the same phosphor and grain size would streamline production, and the claims of the different types of image plates can be achieved by varying the thicknesses of the protective and phosphor layers.

$^\dagger$In principal $\lambda_s$ and $\alpha$ are a functions of wavelength, and are different for the blue stimulated luminescence. However the attenuation $\alpha$ can be assumed to be negligible as the band gap of the phosphor is much higher (8.3 eV vs. 3.2 eV)[98]. Also, if optical scattering length $\lambda_s$ is dominated by the grain size of the crystals and therefore by the density of interfaces and not the differences in index, then $\lambda_s$ will be similar in both cases.
an arbitrary constant.

\[
\frac{\text{PSL}}{N_p(E)} = \text{const} \int_0^d \frac{dE_{\text{dep}}(z)}{dz} \varphi_{\text{stm}}(z) P_{\text{det}}(z) dz \tag{4.9}
\]

What remains is a determination of the energy deposition as a function of depth. The simplest approach for protons would be similar to the discussion in Section B.2. A similar manipulation can yield \(dE/dz\) from the stopping power–\(dE/dR\). One could also use MCNPX which currently can handle protons. Ziegler’s SRIM\textsuperscript{99} is a freely available code which takes a primarily theoretical approach to ion stopping, providing calculations with various levels of complexity. Using the properties described in Table 4.3 and SRIM, a simple 1D model was used to calculate the theoretical energy deposition in TR image plates of a variety of ions and energies.

<table>
<thead>
<tr>
<th>Material</th>
<th>(d(1))</th>
<th>(d(2))</th>
<th>Phosphor*</th>
<th>PET</th>
<th>Ferrite</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR</td>
<td></td>
<td></td>
<td>Element</td>
<td>Stoich</td>
<td>Element</td>
</tr>
<tr>
<td>Phosphor*</td>
<td>50</td>
<td>52</td>
<td>C</td>
<td>0.05115</td>
<td>C</td>
</tr>
<tr>
<td>PET</td>
<td>250</td>
<td>11</td>
<td>H</td>
<td>0.05115</td>
<td>H</td>
</tr>
<tr>
<td>Ferrite</td>
<td>160</td>
<td>247</td>
<td>O</td>
<td>0.1023</td>
<td>O</td>
</tr>
<tr>
<td>MR</td>
<td></td>
<td></td>
<td>N</td>
<td>0.05115</td>
<td></td>
</tr>
<tr>
<td>PET</td>
<td>9</td>
<td>9</td>
<td>Ba</td>
<td>0.2481</td>
<td></td>
</tr>
<tr>
<td>Phosphor</td>
<td>115</td>
<td>120</td>
<td>F</td>
<td>0.2481</td>
<td></td>
</tr>
<tr>
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<td>188</td>
<td>202</td>
<td>Br</td>
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</tr>
<tr>
<td>Ferrite</td>
<td>160</td>
<td>160</td>
<td>Eu</td>
<td>0.03178</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3: Layer thicknesses and compositions for TR and MR IPs. The thicknesses reported are from two different sources \[90, 96\] that agree on the phosphor and protective covering thicknesses. The elemental composition and stoichiometry (Stoich) are derived from\[100\] who also gives densities for the TR phosphor, MS phosphor, PET, and Ferrite layers as 2.61 g/cm\(^3\), 3.3 g/cm\(^3\), 1.3 g/cm\(^3\), and 3.0 g/cm\(^3\) respectively. *The phosphor layers have different ratios of urethane: phosphor grains, the above phosphor stoichiometry is for the TR phosphor.

As discussed in the literature, ionization is believed to be responsible for activating the IPs. As shown in Figure 4.16, using the energy deposited by protons and deuterons ionizing the phosphor layer in equation 4.9 yields a calibration consistent with previously reported experimental data\[85\]. In addition, after accounting for the IP decay, the same scaling factor is consistent with the experimental calibration discussed above.

Unfortunately, this method cannot be extended to heavier ions. C.G. Freeman et al.\[101\] calibrate the same image plates using a TPS and an ion beam accelerator. They find similar results for protons and deuterons as in Figure 4.16, yet significantly less sensitivity to He\(^{2+}\).
Figure 4.16: BAS-TR response to protons and deuterons after 7 min decay. Mančić et al.[85] fit (black) to BAS-TR’s response to protons as a function of energy. SRIM calculated ionization as a function of depth, transformed into IP response, as in equation 4.9, for protons (blue) and deuterons (red). Accounting for different decay times as in Figure 4.14, a single scaling factor matches the SRIM IP response to both Mancic’s fit and experimental data in Figure 4.15b. Though also a function of decay time and temperature, \( \text{const} = 0.58 \text{MeV}^{-1} \) for this plot.

I also find that the experimental IP response to C\(^{4+}\) is significantly less than predicted by the method used for protons and deuterons. Ohuchi postulates that the differing decay characteristics for \( \alpha \)-particles is a result of the higher stopping power locally saturating the phosphor and activating different meta-stable states. SRIM reports the energy lost to ionization for a 1 MeV C\(^{4+}\) ion is \( \sim 10 \times \) that of a 1 MeV proton. If a single heavy ion does locally saturate the phosphor, and the phosphor exhibits differing decay characteristics when saturated, then attempting to model the image plate response to heavy ions will be nearly useless.
Chapter 5
Pseudo-Lagrangian 1D Simulations

To investigate the indirect interaction of the deuterons and protons in the expansion a 1D pseudo-Lagrangian code was utilized that is an extension of previous work\[32, 40, 51\]. The theoretical background for these calculations can be found in chapter 2. Though obviously limited these calculations provide insight into how various ion species within the sheath field caused by the refluxing electrons shield each other, preventing any single species from being optimally accelerated. The following computational results also support the descriptions previously offered explaining why surface contaminants, specifically protons, are most efficiently accelerated and disproportionately acquire the highest final energies.

The model and simulation results described in this chapter are relatively simplistic in nature compared to state of the art fluid and PIC plasma simulations. This method does not conserve energy, does not include the presence of any ions other than deuterons and protons, and cannot inform any estimates of angular divergence. However, as of the time of this writing, PIC is prohibitively computationally expensive if one wishes to both resolve the fields and surface with a few atom thick resolution, and have the simulation be large enough for the expansion to proceed. There also is little expectation that the following 1D pseudo-Lagrangian simulations should be in good quantitative agreement with experimental results. It is known from simple conservation that the hot electrons should lose less than 15% of their energy to the TNSA mechanism.*

The initial motivation was to explain the experimental spectra (fig. 6.2). Specifically, I hoped to explain how the maximum deuteron energies measured exceeded that of the protons and the limited spectral width, non-thermal proton spectra measured which is not expected from TNSA. Though these results do not fully match or explain the experimental data, they do provide valuable insight for future efforts, including the possibility of a narrow spectral feature generated by TNSA.

*Given an estimated total conversion efficiency of $\sim 2\%$ of laser light to ions, and $\sim 15\%$ coupling of laser to target\[102]\, then the hot electrons could lose up to $\sim 12\%$ of their initial energy to TNSA.
5.1 Overview

In essence, the calculation described in this chapter is based on conservation of mass (eq. 2.1) and conservation of momentum (eq. 2.2) using Poisson’s equation 2.7 (or specifically 2.9) to determine the force. Utilizing the initial conditions from section 2.3.1, in simplistic numerical calculation style, the ion position is updated by assuming constant acceleration for some small $\Delta t$. Then using the boundary conditions from section 2.3.1, Poisson’s equation is solved using a “finite difference code [bvp5c] that implements the 4-stage Labatto IIIa [...] implicit Runge-Kutta formula[103],” from MATLAB®.

The formalism in chapter 2 assumes that the ions move slowly enough to be considered static by the refluxing electrons and that the electron energy distribution is described by a sum of Boltzmanns. Then the local electron density $n_e(x)$ is a function of the local potential $\Phi$ as in equation 2.8. As such, electron fluid need only be described by the temperature and nominal density of each population. A two temperature model for the electrons need only track $n_{eH}, n_{eC}, k_B T_H, k_B T_C$ for each time.

Simply discretizing $n_i(x) \propto \rho$ onto a Cartesian grid can complicate solving equations 2.1 and 2.2 as both the fluid velocity and its divergence need to be known. PIC codes track particle positions. This simplifies conservation of mass to simple accounting, and turns conservation of momentum back into the familiar $F = ma$. The fields are calculated using a spacial grid where density is calculated by mapping the mass of the pseudo-particles to the corners of the cell in which they are located. These codes are extremely versatile and assuming the appropriate physics is included are extremely informative though extremely computationally expensive.

Pseudo-Lagrangian simulations discretize materials, here ions, and have continuous space similar to PIC codes. They do not keep track of a separate spacial grid, instead the density is determined by the particle spacing and fields/forces are calculated at the particle locations. As a result the local spacial resolution is determined by the local separation of the fluid segments/particles. These codes do not self-consistently handle temperature and pressure using particle scattering properties as in PIC, which is problematic at best when trying to model a parameter space were material properties (thermal/electrical conductivity etc.) are an active area of inquiry.

By the nature of the TNSA expansion, the vast majority of the particles involved originate from a few nm surface layer, which expands to several $\mu$m’s before accelerating fields relax. If a fixed Cartesian grid is used to calculate the densities and fields etc., the resolution required for the initial acceleration near the surface, and the scale of the subsequent expansion makes this an extremely expensive and difficult though not impossible PIC problem.∗

∗The drawbacks to the pseudo-Lagrangian approach are handled thusly: Material properties don’t matter
5.2 Multiple Ion Species

Extending governing equations of the model above simply requires replacing $Zn_i$ with $\sum_j Z_j n_{ij}$ in equation 2.7 or 2.16. In principle any arbitrary number of ion mass and charge states could be included. Ideally, one could include as much ionization physics as desired and update the ion charges states and number densities on every time step of the calculation. At minimum, one would like to include field ionization (A.3.1). For this work only protons and deuterons were included.

5.3 Time History of Electron Temperatures

A one dimensional model of the electron temperatures, both the relativistic and background portions, could be estimated by using a combination of geometric and ohmic arguments following D. Hey[36, 104]. For this work 2D3V PIC code LSP was utilized to generate a time history of the the electron population crossing the rear surface of a cryo target described in section 4.2.1. Due to computational constraints the resolution was insufficient to model both the LPI and the TNSA ion expansion. The LPI was modeled for $0.5J$ and $1.2J$ pulses from GHOST with a not unreasonable $1\mu m$ scale length pre-plasma as shown in figure 5.1. As such, the electron population used in model presented here was generated from a fixed ionization LPI LSP model[105].∗ This model includes subsequent losses due to ohmic stopping (sec. 3.2.1) and changes in the electron temperature and number due to geometric effects from the transport and refluxing. This model does not include modifications to the electron energies involved in accelerating the ions. As the ions accelerate the shape and position of the potential well trapping the majority of the electrons (fig. 2.1 and 5.3) changes as the ions move. For the refluxing electrons accelerating the ions, this means falling down a shallower hill than they climbed[36, 59].†

Since the number density of the hot electron population is not constant in time, there is no obvious way to implement a single temperature model as described in section 2.3. As previously noted (sec. 2.4) expansions from two distinct electron temperatures $T_H \gg T_C$, aside from the interface between the two, proceed relatively independently. For simplicity, an isothermal temperature of $50eV$ was assumed for colder electron population, a choice which will not influence the TNSA ions[40](see sec. 2.4).‡ The maximum number density of the hot electrons was chosen to be $1/100^{th}$ of solid density, by the crude approximation assuming half of the hot electrons are coupled to a forward directed $60^o$ half angle cone and I’m not trying to conserve energy. Hence I, nor the reader, should expect a particularly quantitative model.

∗The materials and thicknesses, and laser parameters corresponded those described in section 4.
†As previously noted this should $\sim 15\%$ of the electron energy, as so we expect the qualitative behavior of the model to be reasonable.
‡Initially, this is $\sim 1/100^{th} eV$ and depending on laser and target parameters could be significantly higher than a keV. Changing this from $5eV - 5keV$ has no noticeable effect on the TNSA ions.
Figure 5.1: Cross section of the the electron number density \( n_e \) used in 2D3V particle-in-cell simulations used to generate the hot electron distributions in figure 5.2. The target has a 1\( \mu \text{m} \) scale length pre-plasma on the front surface of the target. The laser is introduced by manipulating the boundary conditions on the edge of the simulation. The hot electrons moving in positive \( x \) are recorded at the target rear surface, here \( x = 0 \). Time \( t = 0 \) is defined as when the first recorded hot electrons cross \( x = 0 \).

A third of the laser energy is coupled into the hot electrons. This also roughly corresponds to \( n_e/n_{e0} \) for the peak laser intensity.\(^*\)

### 5.4 Deuteron Acceleration 1D results

Despite the noted caveats, these simulations were illuminating\(^†\) especially for informing the intuition about exactly how the protons ‘shield’ the heavier ions and the importance of a sharp interface. Though, as will be discussed (sec. 6.4), it is evident that the chosen physics is an incomplete description.

Though a variety of initial contaminant thicknesses were utilized, the results of three simulations are used to illustrate characteristic behaviors. The CD targets (sec. 4.2.2) were left to their own devices to acquire a 1 \( \sim \) 2 \( \text{nm} \) contaminant layer (sec. 4.2.3) before 1.2 \( J \) of GHOST was introduced. A 2 \( \text{nm} \) proton layer was added to the front of an semi-infinite slab composed of 99\% deuterons and 1\% protons to model the CD plastic and heavy water(fig. 5.4). The cryogenic heavy water targets (sec. 4.2.4) had little time to acquire a hydrocarbon contaminant layer (sec. 4.2.5) before 0.5 \( J \) of the GHOST laser was introduced. I infer from the experimental measurements (figures 6.1 and 6.2) that the contaminant layer was limited.

\(^*\) As if the laser field was throwing the entire surface of electrons it can reach through the target.

\(^†\) Any coding effort takes substantially more time than one expects or reasonably hopes.
Figure 5.2: The hot electron number density (black) and characteristic energy (red) vs. time utilized in the 1D pseudo-Lagrangian calculation in this chapter. These plots are generated by tabulating electrons leaving the rear surface of a 2D3V LPI simulation of the 0.5 J and 1.2 J, 120 fs FWHM, λ = 1064 nm GHOST pulses incident on a target in figure 5.1 with a 1.2 µm layer of D2O ice. Here t = 0 corresponds to the time when the first hot electrons reach the rear of the target surface in the LSP simulation as in figure 5.1. The initial drop in characteristic electron temperature is likely attributed to an initial steepening of the electron density, as shaper interfaces have been known to reduce the characteristic electron temperatures. The increase at late times is attributed to more energetic electrons completing one refluxing cycle and exiting the rear surface a second time as there was no significant numerical heating observed when calculating the energy balance of the simulations.
Figure 5.3: The initial conditions for a 1D simulation modeling a 2 nm thick contaminant layer on top of pure deuterons. $T_H = 0.52 \Phi$ and the initial hot electron number density is $4.8 \times 10^{-4} n_0$ from figure 5.2b and a background electron temperature of 5 keV. $\Phi$ and $\lambda_D$ are normalized to the peak characteristic hot electron temperature 0.93 MeV and initial electron $n$.

Possibly completely, as in figure 5.6, or possibly an incomplete layer of H$_2$O represented by a 0.2 nm layer comprised of 1/3 protons and 2/3 deuterons, as in figure 5.5, which are the other two initial conditions used in the following simulations.
Figure 5.4: Species number density (a) and ion spectra (b) from a 1D pseudo-Lagrangian simulation of the expansion of the rear surface of 99% deuton 1% proton semi-infinite slab with a 2 nm proton contaminant layer using the electron temperature profile in figure 5.2b after 320 fs. At this time the acceleration of the ions is $1/100^{th}$ of the peak acceleration and the ions have acquired the majority of their final velocity. Though the deuterons acquire significantly less energy than the simulations with fewer contaminants, there is not a significant reduction in number as observed experimentally (fig. 6.1 and 6.2).
Figure 5.5: Species number density (a) and ion spectra (b) from a 1D pseudo-Lagrangian simulation of the expansion of the rear surface of 99% deuteron 1% proton semi-infinite slab with a 0.2 nm, 33% proton contaminant layer using the electron temperature profile in figure 5.2a after 430 fs. At this time the acceleration of the ions is $1/100^{th}$ of the peak acceleration and the ions have acquired the majority of their final velocity. Though the deuterons acquire significantly more energy than the simulation with thicker contaminants, and the most numerous protons have a limited spectral width, the most energetic protons acquire more energy than the most energetic deuterons, which is again inconsistent with experimental results (fig. 6.1 and 6.2).
Figure 5.6: Species number density (a) and ion spectra (b) from a 1D pseudo-Lagrangian simulation of the expansion of the rear surface of 99% deuteron 1% proton semi-infinite slab no contaminant layer using the electron temperature profile in figure 5.2a after 430 fs. At this time the acceleration of the ions is $\frac{1}{100}$th of the peak acceleration and the ions have acquired the majority of their final velocity. Though the deuterons acquire significantly more energy than the simulation with thicker contaminants, and the most numerous protons have a limited spectral width, the most energetic protons acquire more energy than the most energetic deuterons, which is again inconsistent with experimental results (fig. 6.1 and 6.2).
Figure 5.7: For all initial density distributions, including no surface contaminants ((b) and fig. 5.6) the simulations show a leading proton feature with a limited spectral width, and similar yield which is inconsistent with experimental results (fig. 6.2 and 6.1) in which one species is significantly suppressed. (a) The plotted times are 0 fs, 6 fs, 19 fs, 49 fs, 118 fs, and 276 fs, $\lambda_D = 29$ nm, $\omega_0^{-1} = 4.4$ fs, and $n_0 = 6.0 \times 10^{22}$ cm$^{-3}$. (b) The plotted times are 0 fs, 6 fs, 21 fs, 59 fs, 150 fs, and 373 fs, $\lambda_D = 27$ nm, $\omega_0^{-1} = 4.4$ fs, and $n_0 = 6.0 \times 10^{22}$ cm$^{-3}$.
Chapter 6
RESULTS AND CONCLUSIONS

If we knew what we were doing, it wouldn't be called Research.
-A. Einstein

*Though the ultimate goal of producing and investigating a neutron source derived from a pitcher catcher arrangement was not attempted, excellent evidence of a deuterium dominated TNSA was acquired. This is a significant advancement of the state of the art and will hopefully enable significant improvements in the yield of neutron generation experiments in the near future. In investigating the potential causes of the measured spectra, calculations indicate the potential for a narrow spectra feature in protons generated from TNSA if they originate from a mono-layer contaminant surface which has been sough after in order to begin the development of a device for proton therapy.

6.1 Preliminary and Related Experiments

Several related attempts to selectively accelerate deuterium were performed prior to the in situ heavy ice deposition experiment. I will briefly discuss them partially in order to justify that unlike my method of in-situ heavy ice deposition, target cleaning efforts are to difficult and have limited effectiveness to be effective in practical neutron generation strategies. Admittedly, for substrates and targets resilient enough to withstand temperatures sufficiently high to remove the contaminant layer, various cleaning methods become viable. Broadly speaking this restricts such a technique to ceramics, certain crystals such as chemical vapor deposition (CVD) diamond, and a few metals, which excludes light ion acceleration such as deuterons.

Despite what is now apparent to this author, we attempted several experiments, that did not enter the literature, with the intension of applying and cleaning a surface mostly

comprised of deuterium. Data presented subsequently for comparison was from the simplest such experiment in which no cleaning was attempted. Fig. 6.1b shows data from a shot on GHOST (for target detail see section 4.2.2).

One of the earliest attempts involved ablating contamination from the rear surface using a \textit{ns IR} laser. For the combination of time delays and ablation laser energies chosen, there was little to no evidence of any suppression of the proton acceleration. Admittedly, the number of shots taken was limited and was far from a complete and conclusive study, but further study seemed a fruitless endeavor.\footnote{As Prof. Freeman is fond of reminding, “You can’t tweak zero.”}

A subsequent attempt was centered on the concept that \(4.6\, eV, 266\, nm\) photons have enough energy to break C-H and O-H bonds of \(\sim 4 - 5\, eV\).\footnote{These bonds can have higher dissociation energies at significant intensities as one needs to add enough energy to the electron such that the electro-magnetic wave does not deposit the electron back at the ion with zero velocity at the next node in the wave.} This was achieved using a \textit{ns IR} laser frequency doubled twice yielded similar results to using the fundamental. For ambient pressures of \(\sim 10^{-5}\, Torr\), the intensity required to overcome the recontamination rate seems to move the interaction into a regime where chemical disassociation is irrelevant. When the fluence of the \textit{UV} ablation laser was low enough not to destroy the target there was no measurable benefit. Fluences that did destroy the target served only to foil the ion acceleration. It is possible that with significant effort this method could become beneficial, however with no measurable improvement other methods were tried.

Back filling the chamber with heavy water vapor did show limited promise. The hope was that water was responsible for enough of the contamination in the vacuum chamber that replacing the water with heavy water would improve the number and energy of the accelerated deuterium spectra. As discussed above (section 4.2.3) there is evidence that water could be responsible for less than half of the contaminants\cite{24}. So one still expects protons be the predominantly accelerated species. Hou \textit{et al.} performed a contemporaneous and independent study exploring this technique\cite{50} with similar results. Hou \textit{et al.} showed that replacing H\textsubscript{2}O contamination with D\textsubscript{2}O improved the peak deuteron energy relative to the peak proton energy and increase the deuteron yield by a factor of 3 to 5, yet the protons significantly out number the deuterons in the measured ion spectra.

\textbf{6.2 Experiments on GHOST}

A set of experiments were performed on GHOST at the University of Texas at Austin comparing the simplistic approach of simply having a deuterated target or target rear surface, and the in-situ heavy ice coating of the target rear surface. Due to a few issues with the laser\footnote{Any contemporary ultra high-intensity laser system suffers the same ills including (especially) commercial systems.} the two targets were not compared under identical experimental conditions. The
pulse durations were nearly identical, as was the target incident angles. The successful shots acquiring TPS spectra for the CD coated wafer targets had pulse energies from $0.9 - 1.2 J$. Successful shots utilizing the heavy ice deposition were all acquired in succession and had energies from $0.4 - 0.5 J$.

To compare, a shot on a CD coated target analyzed was irradiated with $1.2 J$ of laser energy and the solid angle subtended by the TPS was $4 \times 10^{-7} \text{ sr}$. The spectrum of accelerated ions was dominated by protons with a maximum observed energy of $7 \text{ MeV}$ and is shown is figure 6.1b. Carbon ions form the next most abundant species with $C^{4+}$ ions producing the brightest line. The $C^{4+}$ line appears broader than other spectral lines due to unresolved oxygen lines that are immediately adjacent. Because deuterium ions and $C^{6+}$ ions have the same charge-to-mass ratio, their trajectory through the TPS is identical. Consequently deuterium ions cannot be distinguished from $C^{6+}$ ions directly. Using CR-39 as a detector, we found no evidence of $C^{6+}$ in this track. In figure 6.1a the deuteron line extends to around $2 \text{ MeV}$.

The ion spectra are recorded on imaging plates and/or CR-39 after passing through TPS, which is describe in detail in section 4.3. The TPS view is along the target normal subtending a solid angle of $5 \times 10^{-8} \text{ sr}$. Fig.6.1a shows spectroscopic data for one shot in which the target rear surface is coated with a $1.2 \pm 0.2 \text{ mm}$ layer of ice. The laser energy is $0.5 J$ with an estimated peak intensity of $10^{19} \text{ Wcm}^{-2}$, yielding a maximum observed deuterium ion energy of $3.5 \text{ MeV}$. Because the deuterium ions are so much lighter than the oxygen ions, the hot electron energy is preferentially used in deuteron acceleration. The very weak proton and carbon contaminant ion signals indicate these species are not present in significant quantities (for the heavy water case) in the vicinity of the target rear surface.

The ion flux and energy spectrum in Fig. 6.2 are extracted from the ion track data. A detailed analysis of the ion energy dispersion and charge-to-mass dispersion in the TPS is also presented in Ref.[106]. On selected shots during the experiment, slotted pieces of CR-39 plastic overlaid the imaging plates in order to measure the sensitivity of the imaging plates (section 4.3.9).

Fig. 6.2 shows the spectrum of protons and deuterons obtained from the tracks displayed in Fig. 6.1. For the plastic coated targets, the spectrum shows that most of the energy available for ion acceleration is transferred to the protons which acquire a maximum energy of around $7 \text{ MeV}$. The spectrum of deuterium ions from the plastic coated target extends only to around $2 \text{ MeV}$ and contains considerably less total energy. In contrast, the measured spectrum of ions that are accelerated from the heavy ice target is dominated by deuterium ions whose spectrum extends to around $3.5 \text{ MeV}$. The measured proton spectrum only extends to $0.9 \text{ MeV}$. The measured deuterium ion signal is greater than $99\%$ of the total ion spectra and is three orders of magnitude greater than the proton ion signal. **Note that the ratio of protons to deuterons in the measured spectra is consistent with the purity of**
Figure 6.1: Comparison of TPS ion spectra detected on calibrated image plates from heavy-ice and CD plastic coated targets irradiated with a 1 µm, 120 fs, laser with a 6 µm FWHM diameter focal spot. (a) Tracks from ions accelerated from the rear surface of a target coated with 1.2 ± 0.2 µm of heavy ice irradiated with 0.5 J at ∼ 5 × 10^{18} W cm^{-2}. The signal is dominated by deuterium ions with a maximum observed energy of 3.5 MeV. (b) Tracks of ions accelerated from the rear surface of a target that is coated with a 1 µm layer of deuterated plastic irradiated with 1.2 J at ∼ 1.2 × 10^{19} W cm^{-2}. The signal is dominated by protons and the maximum observed deuterium ion energy is around 2 MeV. Note the logarithmic PSL scale. Reprinted with permission from J.T. Morrison, M. Storm, E. Chowdhury, K.U. Akli, S. Feldman, C. Willis, R.L. Daskalova, T. Growden, P. Berger, T. Ditmire, L. Van Woerkom, and R.R. Freeman, *Physics of Plasmas*, vol. 19, 300707 (2012). Copyright 2012, American Institute of Physics.

The heavy water used in the experiment. An absolute comparison between the two sets of spectra cannot be made since the ice coated targets are irradiated with less than half the laser energy provided to the plastic coated targets. However, preliminary experiments performed using plastic coated targets with around 0.5 J of laser energy indicate a proton spectrum that is similar in magnitude and extent to the deuteron spectrum produced by the heavy ice targets reported here.*

*These were performed with Cu foils which were not flat making the viewing angle of the TPS ambiguous. Much of the data collected suffered from mis-alignment of the TPS making the location of the neutral point ambiguous and therefore the energy of the extracted spectra error prone.
Figure 6.2: Ion signal (Number/MeV/sr/J) accelerated from the target rear surface extracted from ion spectra described in Fig. 6.1. (a). For the ice-coated targets there is a dramatic decrease in both the observed proton number and the maximum observed energy (black). (b) The energy and measured number of deuterium ions accelerated from the rear surface of an ice-coated target is enhanced (black line). Reprinted with permission from J.T. Morrison, M. Storm, E. Chowdhury, K.U. Akli, S. Feldman, C. Willis, R.L. Daskalova, T. Growden, P. Berger, T. Ditmire, L. Van Woerkom, and R.R. Freeman, *Physics of Plasmas*, vol. 19, 300707 (2012). Copyright 2012, American Institute of Physics.
6.3 Extrapolation and Predictions

To estimate the total deuteron yield we assume the deuterium ions expand in a forward directed cone of half-angle $\sim 10^\circ$ from a 100 $\mu$m diameter region of the target rear surface. This is consistent with simulation[74] and previously reported data [26, 27] using protons with similar experimental parameters. The total number of forward directed deuterons is estimated to be $\sim 1 \times 10^{11}$, corresponding to a laser-to-deuteron-ion energy conversion efficiency of $\sim 2\%$. If the deuterium ion source is 100 $\mu$m in diameter then the $\sim 1.2 \mu$m thick heavy ice layer contains $\sim 1 \times 10^{16}$ deuterium atoms. We expect that the conversion efficiency will improve with increased laser energies, and optimally thick targets, thinner than those used here[60].

6.4 Conclusion and Final Remarks

Experimentally, this work shows that one can preferentially accelerate deuterons from a cryogenic heavy ice target using TNSA. The explanation as to exactly how is incomplete. Using a straight-forward extension of previous calculations essentially yielded the following prediction/explanation. If a surface contaminate layer of protons is simply added to the front surface, one expects a nearly continuous ion spectra switching from protons to the underlying deuterons when the surface layer was depleted(fig. 5.4). Though intuitively satisfying and supporting, the description of the fields peeling way surface layers sequentially does not model the experiment well.

If the protons are present at the surface, they will acquire the most energy. Though the protons measured in figures 6.1 and 6.2 have a limited spectra width, it is likely this feature is an artifact of the limited sensitivity of the IPs used. As to the limits of the proton spectral band having lower energies than the peak deuterons, I can only suggest that the initially the proton contaminant layer was sandwiched within a few nm’s of the surface of the D$_2$O ice. However, it remains unclear how this could have transpired. Alternatively, it is possible that in the absence of a contaminant layer, the expansion front of the deuterons and its associated E field could build a spectral feature not unlike shock acceleration. Protons passing the deuteron front could gain similar energies as they pass. Unfortunately, I would expect evidence of this within the simulations described in chapter 5 if this were the case. More detailed simulation work may provide more information.

Another feature of the experimentally observed spectra is the disparity between the relative numbers of protons and deuterons which is reversed in the heavy ice experiment. It has been noted that kinetic modeling of the electrons can preserve conservation of energy self-consistently[59]. This results in a hollowing of the electron spectra, as most of the energy gained by the ions in these simulations comes from the electron energies which circulate through the accelerating ions. This does not result in a limited spectral width of the ions
as one might expect if one takes the ambipolar diffusion/charge neutrality condition too seriously. It is possible that these features work them selves out at later times.

I suspect that more likely at fault is a combination of a lack of proper ionization dynamics and calculation that run until the acceleration ceases and the ionization states are frozen. In order to explain carbon and oxygen ion species concentrations it is seems that the E field and collisions may be sufficient to motivate an explanation and model. Though since the $\sim MeV/\mu m$ field are clearly strong enough to completely ionize $Z_{nuc} = 1$, this is not likely to explain the lack of deuterons in the observed spectra these simulations predict. My suspicion is that my assumptions predict D and I’m detecting only D+.

There was a time when it was interesting that expansions could be violent/rapid enough that the the ionization state could freeze out. The characteristic white glow of recombining plasma doesn’t reach all the way to the detector, and in slower expansions it was reasonable to assume that the majority of the ions recombined as the expanding plasma cooled. If the density drops quickly enough the recombination rates become negligible before the plasma cools enough to return to the gaseous phase, freezing the ionization state[28].

The tacit assumption has been that the TNSA and related thin foil expansions are violent/rapid enough that there is no recombination. Furthermore, that highest ionization states are frozen early in the expansion and are unchanged to the detector. Though this is likely true for the most energetic ions, I suspect this is not the case for the entire spectra, far from it. For future work the ionization mechanisms in appendix A should be included in a code with kinetic electrons, pseudo-Lagrangian or PIC, and run to significantly longer times where one can confirm that the ionization states have indeed frozen out.

In conclusion, this work successfully demonstrates deuterium ion acceleration from the rear surface of a laser-irradiated thin foil target coated with a $\sim 1.2 \pm .2 \mu m$ layer of heavy ice. The ice is formed by injecting 100 ml of heavy water, under vapor pressure, onto the surface of a cryogenically cooled, 500 nm thick, Si$_3$N$_4$ and Al membrane. The acceleration mechanism overwhelmingly favors the acceleration of deuterium ions with the proton and carbon ion signal being more than two orders of magnitude lower. The maximum observed deuterium ion energy and yield within $5 \times 10^{-8} \ sr$ as measured at our detector are 3.5 MeV and $1.2 \times 10^{12} \ sr^{-1}$ respectively. Application of this technique will significantly improve laser based neutron generation yields in the future. I’ve also predicted an exciting new avenue of investigation producing spectrally shaped protons of great potential use greatly improving access to proton cancer therapy in the future.
Bibliography


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of Plasmas, 18:040702, 2011. The authors observe 3 to 5 times enhancement in the deuteron yield by placing 1 ml of D$_2$O in the target chamber to generate a deuterated contaminant layer but also observed large numbers of protons.


[96] H Ohuchi and A Yamadera. Dependence of fading patterns of photo-stimulated luminescence from imaging plates on radiation, energy, and image reader. *Nuclear


[105] A. Krygier. The Ohio State University (private communication).


Appendix A
IONIZATION

A brief discussion of the processes involved and under what conditions they may dominate will help frame the following discussion. The simplest model states that the ionization state dynamics are a result of ionization rate and the recombination rate. If one attempts to solve the system in detail by utilization of rates and cross sections, one requires knowledge of the various populations of excited states of atoms and ions, the life times of all these states and their decay modes, and excitation channels. With some simplifications and averaging, this is the most general approach taken by generalized population kinetics and spectral model for rapid spectroscopic analysis for all elements (FLYCHK) [107] and other non-LTE ionization and spectroscopy codes. This is an excellent example where the heavy theoretical hammer of statistical physics can be applied: enter the Saha equation. The primary benefit is the reduction of required knowledge of the atomic levels. One only needs information on the various excited energy levels and their degeneracy. Unfortunately this approach can only be directly applied to equilibrium or quasi-equilibrium systems, yet can be used to inform rates for similar conditions[108].

For temperatures where few of the electrons have sufficient energies to remove a bound electron from the ground state, ionization from the excited states will be the most prevalent. As temperature increases, and the average electron temperature becomes comparable to the ionization potential of the first excited state, the contributions of excited states become less important as the electrons can ionize the ground state just as easily.

*The discussions on the EOS that appear in this chapter are analogous to the calculations undertaken in emissions reduction in internal combustion engines (ICE). For fuel efficiency and reducing the emission of soot, one desires a reduction of unburnt hydrocarbons. By reducing the available hydrocarbons, the reaction rates improve for oxidation of hydrocarbons, but also increases the combustion temperatures. Primarily produced at higher temperatures combustion temperatures, mono-nitrogen oxide (NOx) contributes to smog and acid rain. Predicting the stoichiometry of the exhaust gases follows two similar veins to the discussion of ionization presented in this chapter: minimize the Helmholtz free energy to ascertain the stoichiometry at LTE conditions or compute the reaction rates of all the various species for a given set of states which also involves minimizing the Helmholtz free energy.

†Many simple calculations can be done on the IAEA website.
‡the hammer needs to be used on either oneself or the theory for understanding
A.1 Saha Equation

The Saha equation[109, 110] is a statement that relates the relative densities of two species with maxwell-Boltzmann partition functions. In statistical mechanics, the Helmholtz free energy or free energy $F^*$ is a powerful tool used to describe a number of systems which move towards equilibrium by minimizing the free energy.† Zel’dovich and Raizer[28] use a purely statistical approach to deriving the Saha equation minimizes Helmholtz free energy for changing particle densities, similar to Plank’s 1924 derivation for the special case of hydrogen[111]. The same result can be achieved using the ionization and excitation transition probabilities for photo-ionization, excitation, recombination, and relaxation[112]. The following calculations have been employed for a variety of problems and continue to be an actively applied to specific situations, such as for a carbon oxygen mixed plasma[113].

A few clues to the derivation are here to illustrate the assumptions implicit in the Saha equation. Elucidating Zel’dovich, the Saha equation is a specific result of applying the concept of Helmholtz free energy or free energy $F$, which is proportional to the partition function $Z$. For an ideal Boltzmann gas of non-interacting particles the partition function is a product of the individual partition functions, where each type of particle is divided by the $N!$ to avoid over counting‡. Here it is useful to separate $Z$ into $Ze$, the partition function of a free electron, and the various ions and ionization levels $Z_m$, the partition function of an $m$-ion. Remembering that the the partition function of sub systems is the product of the sub systems, the partition function for a plasma

$$Z = \prod_i N_e \prod_m N_m Z_{m} = \frac{Ze^{N_e}}{N_e!} \left( \prod_m \frac{Z_m^{N_m}}{N_m!} \right)$$  \hspace{1cm} (A.1)

Notice that $Z_m$ includes the bound electron states. The reason for using Helmholtz free energy or free energy $F$ is that it relates $Z$ to $T$ by $k_B$, $F = -k_B T \ln Z$, and combined with the Sterling approximation $N! \approx N^N/e^N$ yields

$$F = -N_e k_B T_e \ln \frac{Ze_{e}}{N_e} - \sum_m N_m k_B T_m \ln \frac{Ze_{m}}{N_m}$$  \hspace{1cm} (A.2)

Now, any two ionization states $m$ and $n$ at equilibrium will minimize $F$. For constant

*For those who remember their chemistry of solutions this is similar to the Gibb’s free energy which accounts for energy required to add or remove particles from solution without explicit calculation of $pdV$ work which is of paramount importance here.

†In general systems tend to move towards the lowest energy state, Helmholtz’ free energy, Gibb’s free energy from thermodynamics, Le Chatlier’s principle from chemistry, Lenz’s Law from electrodynamics, and the principle of least action from mechanics are all explicit expressions thereof.

‡Generally, the possible states of a system are a permutation of all the possible states of each particle. At high temperatures, the identity of individual particles is irrelevant and the number of states is over counted by $N!$ and is more analogous to a combination (choose) arrangement function. There is some confusion on the use of the $\sum$ which, regrettably, is used interchangeably in some references with a sum of sums or as known to the rest of the world often as product $\Pi$. 

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and $N_m = -\delta N_n = -\delta N_e$, an expression of the ionization reaction, to obtain

$$\frac{N_n N_e}{N_m} = \frac{Z_n Z_e}{Z_m} \tag{A.3}$$

We have only neglected the contributions of photons. The free-free electron interactions, the free electron ion interactions, and the bound electron states are all correctly included in $Z_m$ and $Z_e$. To obtain the Saha equation one neglects the particle interactions, uses Boltzmann statistics, and assumes that the bound electron energies are not affected by plasma conditions. With these approximations, the kinetic or translational contributions of the ions cancel, leaving contributions from the free electrons and the bound electrons. Factoring the ground state energy $E_0$ of the bound electrons from their $E_i$,

$$Z_m = \sum_i e^{-E_i/k_BT} = e^{-E_0/k_BT} \sum_i e^{-(E_i-E_0)/k_BT}$$. The remaining part of $Z_m$ is the statistical weight $g_m$ of the ion or the sum of the statistical weights $g$ of the excited states. *

$$g_m = \sum_i g_0 + g_1 e^{-(E_1-E_0)/k_BT} + g_2 e^{-(E_2-E_0)/k_BT} + \cdots \tag{A.4}$$

$Z_e$ for a Boltzmann gas is handled by Statistical Mechanics texts and involves integrating over phase space using the Heisenberg position, momentum uncertainty to relate $dx$ to $dp$.

Here I will appeal to de Broglie thermal wavelength $\Lambda_d$ the average length between states for a given $T$ and $m_e$ therefore $Z_e = 2V/\Lambda_d^3$.† Using the above evaluations of $Z$ in equation A.3 yields the Saha equation for ionization state balance relating $n_m$ of ions with $T$ and the ionization potential, which can also be written in terms of Bohr radius $a_B$ and $E_H$ noting that $a_B^2 E_H = \hbar^2/8\pi^2 m_e$.

$$\frac{n_n n_e}{n_m} = 2 \left( \frac{2\pi m_e k_BT}{\hbar^2} \right)^{3/2} g_n e^{-(E_n-E_m)/k_BT} = 2 \left( \frac{k_BT}{4\pi a_B^2 E_H} \right)^{3/2} g_n e^{-(E_n-E_m)/k_BT} \tag{A.5}$$

Combined with particle conservation $\sum_m n_m = n_i$ and $\sum_m m n_m = n_e$, the Saha equation is a complete set of equations determining relative ionization states. Following Drake[52], a characteristic $Z_{bal} \approx \bar{Z}$ can be determined by finding two imaginary, non-integer ionization states $Z_{m-1/2}$ and $Z_{m+1/2}$ that are in equilibrium. Assuming the states

As an example, $g$ for Al$^{3+}$ is the number of different configurations of the electrons. Al$^{3+}$ is Ne like which has a full outer shell and no valance electrons. $g_0 = 1$, $g_1 = 2$ corresponding to the 3s orbital, and either $g_2 = 2$ & $g_3 = 4$ corresponding to 2p with azimuthal quantum number $m= 0$ and $m= -1,1$ respectively or $g_2 = 6$ depending on the level of detail one wishes to use.

The factor of 2 is because the electrons are either spin 1/2 or -1/2.
are hydrogenic, then for an isolated ion $E_n - E_m \simeq Z_n^2 E_H$

$$Z_{\text{bal}} = \sqrt{\frac{k_B T}{E_H}} \ln \left[ \frac{1}{Z_{\text{bal}} n_i} \frac{1}{2} \left( \frac{k_B T}{4 \pi a_B^2 E_H} \right)^{3/2} \frac{g_{\text{bal}+1/2}}{g_{\text{bal}-1/2}} \right] - \frac{1}{2}$$ \hspace{1cm} (A.6)

In principle the Saha equation A.5 forms a system of transcendental equations which define would uniquely define the ionization state. This system could be solved as a system of linear equations if the electron density was known. Using $Z_{\text{bal}}$ as an initial guess for the average ionization state, one could calculate the ionization states and recalculate the electron density until the calculation converges.

### A.2 Debye-Hückel Shielding and the Ion-Sphere Model

An implicit assumption in the evaluation of the ionization states, as described in section A.1, is the system’s $F$ is dominated by electrons and the photon contribution in negligible. This implies validity of the Saha equation requires sufficiently high densities, yet the particles have been treated as an ideal gas. Of obvious interest are the effects of high density on the electron distribution and the excited ion state populations and energies.\(^*\) Treating this problem generally one could apply many-body theories. However, much can be learned from two comparatively simple models that consider the electron density in the vicinity of an ion. If the free electrons are still approximately an ideal gas, then the primary effect is on the partition function of the ions through the energy levels, and Saha equation can still be utilized.

For sufficiently high densities, \(\sim\)solid or higher, the average spacing between ions is \(\sim 1\ \text{Å}\), or the size of an ion. Since the electron density must be on average constant throughout the plasma, the $Z$ free electrons must occupy the same volume as the ion. Said another way, the electrons freed from ions, on average don’t occupy any more volume than the electrons bound to the nucleus so the ion still is effectively neutral. So, for distances greater than ion-sphere radius $a_{IS}$, the ion is effectively an atom, where $Z = (4\pi/3)a_{IS}^3 n_e$ defines $a_{IS}$. The electron density is sufficiently high to shield the bound states from the nuclear charge, which can be described by a reduced effective potential for the bound electrons and excited states as described by Griem[108]. The net effect is to decrease the ionization potential and is often referred to as continuum lowering. For high densities this shift can exceed the vacuum ionization energy causing ionization even at low temperatures, referred to by the misleading term pressure ionization.

Determining charge distribution of the free electrons within the Ion-sphere has numerous approaches, the simplest is to assume a constant density for the electrons. Griem [108] notes that this approximation is most reasonable if the electrons are Fermi-degenerate,

\(^*\)Griem page 193 discusses the consequences[108].
which implies a modification to the derivation of the Saha equation \( A.5 \) where one uses the partition function of Fermions to describe the electrons. An application of Gauss’s Law gives the perturbing field of the free electrons on the atomic \( \Phi(r) \approx \frac{en_e}{6\epsilon_0} r^2 \). Once one grinds through first-order perturbation theory\[108\] or uses a semi-classical argument\[28\] the outer bound state with principal quantum number \( n \) will be up-shifted by

\[
\Delta E \approx \frac{n^4}{Z^2} n_e E_H a_B^3
\]  
(A.7)

If ionization energy in vacuum for excited ions with hydrogenic states is \( \sim \frac{Z^2 E_H}{n^2} \), then the highest bound state will be in the shell with

\[
n = \sqrt{\frac{Z a_{IS}}{a_B}}
\]  
(A.8)

The energy up-shift associated with this highest bound state in the simplistic depiction outlined above is effectively a reduction in the energy required to reach the continuum. Hence the term continuum lowering, which can be found by substituting the highest bound state \( A.8 \) into equation \( A.7 \).

\[
\delta E \approx Z E_H \left( \frac{a_B}{a_{IS}} \right)
\]  
(A.9)

Debye Hückel theory\[114\] was originally applied to the interaction of ions in electrolytes, yet the name persists for descriptions of the characteristic shielding distances, \( \lambda_D \), and the electron density increases near ions, Debye cloud.* Again, on average the plasma is charge neutral, so there must be some distance over which the Coulombic potential of the ion is shielded by the electrons as in the Ion sphere model. Here, the densities are lower and one considers the electrons to have a Boltzmann thermal distribution. Looking at the time-averaged spacially smooth potential near an ion, the Poisson equation can be solved to determine the equilibrium charge distribution around the ion. This charge distribution results in an exponential potential screening of the ions which has a characteristic scale, \( \lambda_D \).

\[
\lambda_D = \sqrt{\frac{\epsilon_0 k_B}{e^2 n_e T_e^{-1} + e^2 \sum_m Z_m^2 n_m T_m^{-1}}} \approx \sqrt{\frac{\epsilon_0 k_B T_e}{e^2 n_e}}
\]  
(A.10)

An important consideration that the reader and author are likely to forget about in a few years is that \( \lambda_D \) also depends on the ions. Also, local charge imbalances where the electrons separate from the ions, yet global charge neutrality is maintained can exist in plasmas when electron velocities\(^1\) exceed the thermal velocities of the thermal electrons and

*Hückel apparently drew the short stick, just as Tonks did in the naming of Langmuir (plasma) waves.

\(^1\)Commonly referred to as 'hot electrons,’ these superthermal electrons are involved in a host of interesting phenomena outside of the purview of this document. Coulomb explosions, wake fields, and the various forms of ion acceleration are all manifestations of the this double layer. The Debye Hückel theory may still be applied to these superthermal electrons to give a rough estimation of the extent of the double layer, or the
ions. This double layer supports density perturbations on the order of $< 20 \lambda_D$ depending on the ratio of the energies of the thermal and superthermal electrons.

Also, the above approximation is only valid on time scales where the ions are immobile. Certainly the case for LPI and certainly false for a thermalized plasma where the ions also conspire to reduce the shielding length further. The primary approximation implicitly used above is that the energy of the coulomb interaction between the particles considered to be mobile is small compared to their thermal energy $(Ze)^2/4\pi\varepsilon_0 r \ll k_B T$, or expressed in terms of particle density:

$$\sum_m n_m \ll \left(\frac{4\pi\varepsilon_0 k_B T}{Z^2 e^2}\right)^3 \text{ and } n_e \ll \left(\frac{4\pi\varepsilon_0 k_B T}{e^2}\right)^3$$  \hspace{1cm} (A.11)

The presence of the charge distribution around the ion serves to reduce the ionization potential, commonly described as due to the reduced energy of a free electron being due to the potential of the ion. This lowering of the continuum energy is $2ZE_H(a_B/\lambda_D)$. Even though it is clear that the regions of validity cannot overlap, one commonly approximates the net effect of density on the continuum energy of an electron as

$$\Delta E = ZE_H \times \text{the smaller of } \frac{2a_B}{\lambda_D}, \frac{a_B}{a_{IS}}$$  \hspace{1cm} (A.12)

The zero\textsuperscript{th} order approach is to just use continuum lowering to predict where shells spill out into the continuum as the energy required to be in the continuum is lowered. Unfortunately this predicts the non-physical model where the ionization state of the plasma has large discontinuities. For the ion-sphere regime with temperatures without significant excited states Drake pg. 77 gives a similar general approximation as Griem for the number of bound electrons. However, this only indicates ionization for high-Z materials at solid density and shocked low-Z materials. This approach also neglects important quantum-mechanical effects, things we are all familiar with, - metals and conduction bands, etc. Continuum lowering due to the Debye-Hückel and the ion-sphere theory are most useful as corrections to the Saha equation to extend the conditions over which it provides a useful approximation.\footnote{It seems to me that this is the explanation for why the work function of metal is not zero, nor is it the first ionization potential. Does one get the work function of a metal if one uses the proper number of conduction band electrons? This doesn’t have the complexity of the valence band theories, but it doesn’t require a periodic structure and applies just as easily to Mercury.}

scale at which the system appears nearly charge neutral.

\footnote{see Griem reference More 1986 for a 3/2 factor.}

\footnote{It seems to me that this is the explanation for why the work function of metal is not zero, nor is it the first ionization potential. Does one get the work function of a metal if one uses the proper number of conduction band electrons? This doesn’t have the complexity of the valence band theories, but it doesn’t require a periodic structure and applies just as easily to Mercury.}
A.3 Thomas-Fermi Model

For substantially condensed electron fluids one can model the free electrons as a Fermi sea after first determining the average ionization state. At high enough electron densities it is not Coulomb collisions, but quantum mechanical effects that create pressure. The Pauli exclusion principle prevents electrons from occupying the same quantum mechanical state.

In a Fermi degenerate system or Fermi sea, the Fermi energy \( E_f \)

\[
E_f = \frac{\hbar^2}{2m_e} \left( \frac{3n_e}{8\pi} \right)^{2/3}
\]

\( E_f(n_e = 10^{23} \text{ cm}^{-3}) = 7.9 \text{ eV} \) (A.13)

is the energy of a the state whose occupancy is 50% at absolute zero. Though typically applied to extremely cold, condensed systems, any system where the inter-particle spacing is on the order of \( \Lambda_d \) or is near the degeneracy temperature \( T_d \), defined by \( k_B T_d = E_f \). Other examples of systems where the quantum mechanical effects become important, and the Thomas-Fermi model is a good simple description, include massive Jovian planets, white dwarf stars, and HEDP implosion experiments.

A.3.1 Field Ionization

The most straight-forward description of field ionization is the semi-classical potential and the Bohr model of the atom. The potential well of the charge of the nucleus is 'tipped' by the addition of an electric field which for simplicity is assumed to be linear on the scale of the atom. For sufficiently large electric fields, the potential barrier is lowered anti parallel to \( E \) such that bound electrons can tunnel through the barrier and are free to accelerate in the electric field. Field Ionization occurs when the field is strong enough that the elecor is no longer bound and spill out over the saddle point in the potential. A more complete description involves calculations of the Ammosov, Delone, Krainov (ADK) tunneling probabilities to give an accurate accounting of the ionization rates and average ionization states. To predict the ionization state in the presence of a \( \sim \text{ ps} \) quasi-static field. In the presence of a strong laser field above threshold ionization (ATI) rates should be included. Of interest to this work is the quasi-static accelerating \( E \) fields generated on the surfaces of the target. For purposes of estimating the ionization states, the simplest description is provided.

In the presence of an external \( E\hat{z} \equiv E_z \), the atomic potential from \( Z_{nuc} \) along \( \hat{z} \) will be

\[
\Phi(z) = \frac{eZ_{nuc}}{4\pi\epsilon_0} z + zE_z
\]

and with simple calculus, the highest \( \Phi_{enc} \) enclosed for any \( r \) will be \( \Phi_{enc} = -2\sqrt{eZ_{nuc}E_z/4\pi\epsilon_0} \). This is the first term in a more general treatment considering states
with angular momentum and $m$ \cite{115}

$$
\Phi_{\text{enc}} = -2 \sqrt{\frac{eZ_{\text{nuc}}E_z}{4\pi\epsilon_0}} + |m| \left( \frac{eZ_{\text{nuc}}E_z}{4\pi\epsilon_0} \right)^{3/4} + \frac{3}{16} m^2 e Z_{\text{nuc}} E_z \frac{4\pi\epsilon_0}{4\pi\epsilon_0} \tag{A.15}
$$

This has been done quantum mechanically for an isolated atom in a relativistic laser field by Enam “The Man” Chowdhury\cite{62, 63}. Here it should be noted that in a laser field the electron will accelerate back towards the ion $\pi\omega_0$ later in the optical cycle. The electron can collide with the ion, causing further ionization, or recombine with the ion, emitting photons with integer $n$ multiples of the driving photon’s energy $n\hbar\omega_0$. These harmonics have interesting properties which have been utilized for the production of atto-second pulse trains.

Of prime relevance to the conditions in LPI regions and regions where the sheath fields are sufficiently strong, is how the global fields remap the higher energy states from bound to free. The ion and Debye sphere models both consider continuum lowering. The collective effect of the free electrons is to shield the nuclear fields, making states that would be bound in an isolated atom free. To use similar arguments to those used for continuum lowering, one would need to walk through perturbation theory to determine the energy levels with the addition of the external field.\* The most simplistic way to account for the effect of field ionization is similar to continuum lowering, using $\Phi_{\text{enc}}$ to determine continuum lowering $\Delta E = e\Phi_{\text{enc}}$.

### A.4 Collisional Ionization

Cross-Sections. Energetic ion - electron collisions can expel electrons. At base, just taking the weighted average of the cross section tables over a thermal distribution should get one what one wants. Unfortunately, Murphy tells us reality is not that kind if you would like any reasonable answer. The simplest things to add are the hydrogenic like excited states and their relative populations.

To determine the cross sections for electron-impact ionization one can turn to the National Institute of Standards and Technology (NIST) here. For the extrapolation and calculation of unknown cross sections one may use the binary-encounter-Bethe (BEB) model or the binary-encounter-dipole (BED) model\cite{116}. The BED model is more accurate, and relies on a detailed calculation of the cross section of a free electron with a bound electron. The Mott cross section is an expansion of the traditional Rutherford cross section and adds

\*Littman\cite{115} experimentally determined, in the case of the external electric field being applied by a laser, that the energy level of the unperturbed state being above or below the saddle point was not only sufficient to determine which states were ionized, it was a better description than the tunneling description. Though it is probable that Littman’s results are a result of the short time that the bound states are exposed to the peak field of the laser, the differences between the two approaches are probably small compared to other approximations made above.
spin exchange. Generalizing the Mott cross section by describing the incident particles with a velocity or momentum distribution yields binary-encounter cross section. So far these cross sections, describe with increasing accuracy, free-free electron collisions. The BED model considers a bound electron by considering it as an oscillator, and requires detailed knowledge of the power spectrum of each sub-shell.

The BEB model is uses a simplified model of the bound electrons which is hydrogenic[117]. This model loses the contribution of resonances that can be included in the BED model. Yet knowledge of the binding energies of the bound electrons is sufficient to calculate the total cross for electron impact ionization and not cross section differential in energy loss and angle. The BEB model fits experimental data for H, He, and H$_2$ well[116]. For large target atoms, $n \geq 3$, then the BEB requires a slight adjustment to the form and requires consideration of auto-ionization[118, 119]. Further digression into this field is not within the scope of this document. However it should be noted that this description does not currently account for changes in the target atoms structure due to the presence of an external electric field.*

**A.4.1 Electron Ion Recombination**

Dielectric recombination, or direct capture, and three-body recombination will reduce the ionization state of the expanding plasma. Three-body recombination is essentially an electron undergoing a collision that reduces its kinetic energy near an ion, increasing the two-body recombination rate. In the magnetically confined plasmas with low densities $\sim 10^{15} cm^{-3}$ and moderate temperatures $\sim 1eV$ studies have found that the recombination rate for deuterium plasma is $\sim 3 \times 10^{18} cm^{-3}s^{-1}$ and is dominated by three-body recombination[120]. Having three reactants (and no slow steps), three-body recombination scales as $n_e^2n_i$. This implies that for LPI relevant densities $10^{18} cm^{-3}$ the recombination rate for a $\sim 1 eV$ deuterium plasma is $\sim 10^{27} cm^{-3}s^{-1}$ or $\sim 1\%$ every 10 ps.

For conditions relevant to TNSA the recombination rate is dominated by three-body recombination rates, down to much lower temperatures and densities. The rate of recombination will be balance by ionization in LTE. The ionization recombination rate coefficient $K_R$, a reaction requiring an ion and two electrons, and the ionization rate coefficient $K_I$, with one ion and electron are then related by

$$K_R n_{m+1} n_e^2 = K_I n_m n_e.$$  \hspace{1cm} (A.16)

A common method used to determine $K_R$ is to calculate $K_I$ for the states in thermodynamic equilibrium and applying the Saha equation A.5 to the ionization balance

*Potentially combining collisional ionization by the BEB mode with ATI field ionization is possible, but combining either calculation with its more accurate and complicated versions looks terrible.
A.16. This reaction rate is in principle, valid for non-LTE conditions provided the same set of assumptions used to derive Saha are valid. This is not the complete story. At low densities electron capture or radiative capture rates will dominate, and other effects such as dielectric recombination should be considered. The another consideration is how to account for a significant portion of the electrons having a thermal distribution. One expand the distribution into thermal distributions probably. Most relevant to this work will be the laser accelerated MeV electrons streaming through the target. Just considering the disparity in the energies associated with bound electrons to the ‘hot’ electrons indicates that capture is unlikely *. The cross-section for capture is neglected by electron transport codes such as MCNP until the electrons have a few keV or less. A more relevant question is how rapidly these hot electrons become part of the thermal population.

*Fermi’s Golden I believe
Appendix B

Fuel: Deuterium Tritium, diesel, natural gas, gasoline. One plays the same game.

A fuel problem is always an energy balance problem with the goal of efficiently utilizing the energy in one form, and converting it to a desired energy or product. Another common element is the existence of some threshold which must be overcome to begin the reaction, and finally that the majority of the knowledge necessary to understand and predict a reaction is not required to use the fuel. Use and consequences can often be conveyed with one plot or a few functions.

B.1 Thermo Nuclear

For reaction cross sections the Evaluated Nuclear Data File (ENDF) found here is updated and maintained by IAEA. The ENDF can be compared to experimental cross sections archived in Experimental Nuclear Reaction Data (EXFOR).

B.2 As a Neutron Source

The motivation for much of the research and discussion in this work is the desire to produce a neutron source with high-intensity lasers. The method differs from a thermo-nuclear burn wave, but shares the calculation of the yield for energetic particles stopping in a material. The model utilized for a laser-based neutron source is the so-called 'pitcher catcher' set-up. The high intensity laser interacts with a 'pitcher' target which accelerates ions from the rear surface of the target into a second witness or 'catcher' target containing the ions through which the ions from the first surface of the first foil scatter, losing energy, and react with. A distinction should be made here about the quasi absolute nature of the reaction efficiency. In other reactions the reactions continue until the bulk conditions fail to be met (fusion
Figure B.1: Cross sections in Barns $= 10^{-23} \text{m}^2$ as a function of incident particle energy. The $\text{D}(D,n)^3\text{He}$ and $\text{T}(D,n)^4\text{He}$ reactions have been studied extensively and are the dominant reaction paths for incident deuterons. The $^7\text{Li}(p,n)^7\text{Be}$ also has reasonable amount of experimental data in EXFOR to back up the ENDF calculated cross sections. Deuterons incident on $^7\text{Li}$ does not have much experimental data in EXFOR $^7\text{Li}(D,nx)$ curve.

burn waves, fossil fuel fires etc.) Here the condition is on the incident particles and the probability of a reaction is dependent upon the initial energy of a reacting particle entering the material.

Ions accelerated from the 'pitcher' foil have much higher characteristic energies than those discussed elsewhere in this chapter and decisions on which reactions to pursue reflect this. In the second 'catcher' target, the following discussion ignores collective effects of the bombardment of ions and electrons. One typically assumes that the material is unchanged by the bombardment to first order.

The simplest calculation of the yield of a given reaction product from the bombardment of reactants is for the case when the witness foil is thin enough that the energy loss of the reactants to scattering is small compared to their initial energy. The total $\sigma$ for various neutron producing reactions is shown in figure B.1. The total cross-sections are simply the statistical chance of a reaction by an area. If the trajectory of an incident particle falls inside the area of the cross-section there will be a reaction, outside not.
Figure B.2: Particles with spacing $a$ and radius $r$ such that $\sigma = \pi r^2$ for a given reaction in a plane perpendicular to incident particles 1 and 2. Particle 1 has a reaction.

Given particles with a spectrum described by $dN(E)$ incident on a foil of thickness $R$ and $n$, the number of reactions for a given cross section $\sigma$ is

$$N = \int_0^{E_{\text{max}}} dN(E) \sigma(E) nR dE \quad (B.1)$$

For thicker targets, the incident particles will lose energy due to scattering losses described by the stopping power $-dE/dx$ which is a derivative of the energy as a function of depth, which is related to the particle range $R$.\footnote{Note that the stopping power is dependent upon the path not the displacement.} Inverting the range $R(E)$ is the range as a function of initial particle energy $E_R(R)$.\footnote{The particle range calculation can be lengthy, and is available for a variety of particles. Estar, Pstar, MCNPX etc. can calculate these things.} The inverted range $E_R(R)$ can be used to calculate the energy $E_s$ as a function of scattered path $R$ and initial particle energy $E_0$.

$$E_s(E_0, R) = E_R(R(E_0) - R) \quad (B.2)$$

Now given $E_s(E_0, R)$, one can calculate the volumetric cross section assuming that all the incident particles stop in the material.

$$\sigma_V(E_0) = \int_0^{R(E_0)} \sigma(E_s(E_0, R)) dR \quad (B.3)$$

The above description is a first cut and only considers the number of neutrons produced by the primary reaction. Provided that cross sections for secondary reactions are much less, they can be ignored. If this is not the case one can extend the above calculation to consider other reaction products given knowledge of all probable cross sections. If one
considers reactions that produce the incident particle species the calculations are no longer
decoupled. In addition if one is interested in the the spacial, angular, or spectral distribution
of particles the above description is woefully insufficient.

Though it is true that there are a variety of differential and integral techniques
to calculate resultant particle distributions, these calculations quickly require numerical
solutions as the complexity increases. A more common and currently supported approach
is to use Monte-Carlo techniques and codes such as MCNP and Integrated Tiger Series
(ITS).

B.3 Aside - Fossil

If only as an additional example that I expect people to understand,* fossil fuels at the
time of writing are a ubiquitous fuel in transportation and electricity. Though I’m certain
that everyone remembers or can infer that the primary reaction is some hydrocarbon chain
reacting with oxygen to make carbon dioxide, water, and heat. It doesn’t matter what fuel
one considers one will also produce some undesired products from incomplete combustion
(soot and carbon monoxide) and the acid NO\textsubscript{x} from achieving a high temperature using air
as the oxidizer.

Best efficiency and lack of acid are directly at odds, as power density is at odds with
soot and efficiency. Figure B.3 shows this explicitly for gasoline though these trends are
common for all fossil fuels oxidized with air. Modeling this is a fantastically complicated
endeavor with a plethora of intermediate products which all have temperature dependent
reaction rates which is not dissimilar to another problem which I gloss over in this work.

B.4 Laser Light - Also a heat source

The analogy to fuel is weak, but similar. The study of laser absorption has a significant
collection of literature and is dependent on a variety of parameters and a host of physics.
However, at the end of the day one is interested in the absorption of the light and the
temperature produced.

*Irrationally, admittedly.
Figure B.3: Emissions from and internal combustion engine as a function of octane air/fuel mixture (mass ratio). Soot (HC), and carbon monoxide production as a function of air-fuel mixture. Lean mixtures are characterized by problems with ignition. Rich mixtures are used when producing peak power to ensure ignition and to bring combustion temperatures down to prevent the pistons from melting. Original from Toyota Motor Sales[121] adapted by EndTuning[122].