Design and Comparison of Induction Motor and Synchronous Reluctance Motor for Variable Speed Applications: Design Aided by Differential Evolution and Finite Element Analysis

THESIS

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By

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Abstract

The advances in power electronics devices have opened the possibility of counting on Synchronous Reluctance Motors (SRM) in applications where variations of speed are required. Evidently, for many years the Induction Motors (IM) have successfully supplied this demand, and they have become in undeniable leaders in the field. The strategy preferred to control both motors is based upon vector control, which allows them to operate in a wide range of speed. The necessity of having motors with low-cost and robust construction opened up to discussion and comparison between these two types of motors.

In this study, a new approach based upon the Differential Evolution (DE) Algorithm was developed in order to design a 55Kw Induction Motor for variable speed applications, posteriorly, the performance of the final design was optimized by means of Finite Element Method (FEM) and evaluated through Indirect Field Oriented Current Control (IFOCC) strategy. Likewise, an equivalent Synchronous Reluctance Motor with inductance ratio in the range of 6-10 was also designed by using the same stator than the Induction Motor. A Vector Control (VC) strategy based upon the point of maximum power factor was implemented to test its performance.

Initial sizing and torque density optimization of an Inverter-driven Induction Motor were performed by using a novel algorithm based upon the Differential Evolution paradigm, the resultant geometry demonstrated containing the transient and steady state responses desired.
Finally, both motors optimized for variable speed applications were compared under the same variations of load, voltage and frequency in order to assess their consumption from an input apparent power point of view.

By assuming a limit for the input power through the same Variable Frequency Driver (VFD), the Induction Motor was able to bear a higher overload, whereas the Synchronous Reluctance Motor compared favorably and showed lower power consumption in average under intermittent loads.

It has been proved the feasibility of using Differential Evolution paradigm (DE) to lead the design of Inverter-driven Induction Motors. Furthermore, in being compared the aforementioned motors, the results suggest that there is no need to oversize the Variable Frequency Drive (VFD) in using the Synchronous Reluctance Motor (SRM) in spite of its modest power factor, the outcomes also demonstrate that the only disadvantage in the Synchronous Reluctance Motor is a lower overload capability of about 5%-10% with respect to the Inverter-driven Induction Motor designed through the Differential Evolution paradigm.
To my beloved daughter Sofia, my unconditional love Glenda, and my always supportive and selfless mother Lourdes . . .
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I am also very thankful for the freedom that my advisor, Professor Longya Xu, gave me in order to explore my interests and his guidance through this track.

This achievement would not be possible without the unconditional support of my family, Sofia, Glenda, Lourdes, Junior and Alfredo, who have accompanied me through a tough path full of uncertainties, and have always believed in me and my desire to succeed. Appreciative of my wife’s effort in wanting to face this challenge along with me.

My friends in the Ohio State University and for those who are living around Columbus, they deserve be mentioned since they have enriched my and my family’s experience during these years living in Columbus.

Because of all of you, this accomplishment have been reached.
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Abbreviations

DE          Differential Evolution
EA          Evolutionary Algorithm
FEA         Finite Element Analysis
IEC         International Electrotechnical Commission
IM          Induction Motor
IFOCC       Indirect Field Oriented Current Control
NEMA        National Electrical Manufacturers Association
SRM         Synchronous Reluctance Motor
VFD         Variable Frequency Drive
VC          Vector Control
Physical Constants

Copper Current Density \( \sigma_{cu} = 5 - 6 A/mm^2 \)

Copper Resistivity \( \rho = 2.1 \mu \Omega \text{cm} \)

Magnetic Air Permeability \( \mu_o = 4\pi 10^{-7} H/m \)

Pi \( \pi = 3.141592 \)

Silicon Steel Density \( \gamma_{iron} = 7872 Kg/m^3 \)
Symbols

\[ A_b \] Rotor Bar Cross Section Area \quad mm^2
\[ A_c \] Conductor Cross Section Area \quad mm^2
\[ A_{cr} \] Rotor End Ring Cross Section Area \quad mm^2
\[ A_t \] Specific Stator Current Loading \quad A/m
\[ A_{ss} \] Stator Slot Area \quad mm^2
\[ AT \] Magnetomotive Force \quad Amp – turn
\[ B_{cr} \] Rotor Yoke Flux Density \quad T
\[ B_{cs} \] Stator Yoke Flux Density \quad T
\[ B_g \] Air Gap Flux Density \quad T
\[ B_{tr} \] Rotor Teeth Flux Density \quad T
\[ B_{ts} \] Stator Teeth Flux Density \quad T
\[ b_{0s}, b_{0r} \] Stator/Rotor Slot Opening \quad m
\[ b_c \] End Winding Coil Width \quad m
\[ C \] Number of Circuits
\[ C_o \] Esson’s constant \quad J/m^3
\[ C_s \] Conductors per Slot
\[ c_k \] Constraint \( k \)
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<td>$v_{qd0,s}$</td>
<td>Voltage DQ0 in the Stator</td>
<td>volts</td>
</tr>
<tr>
<td>$W$</td>
<td>Coil Span</td>
<td></td>
</tr>
<tr>
<td>$W_{ts}$, $W_{tr}$</td>
<td>Stator/Rotor Teeth Width</td>
<td>$mm$</td>
</tr>
<tr>
<td>$W_{1r}$, $W_{2r}$</td>
<td>Rotor Slot Width</td>
<td>$mm$</td>
</tr>
<tr>
<td>$W_{1s}$, $W_{2s}$</td>
<td>Stator Slot Width</td>
<td>$mm$</td>
</tr>
<tr>
<td>$Z$</td>
<td>Arc Length Occupied by the Same Phase Belt</td>
<td></td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>Flux Density Shape Factor</td>
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</tr>
<tr>
<td>$\xi_{zz}$</td>
<td>Zig Zag Leakage Coefficient</td>
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<tr>
<td>$\eta$</td>
<td>Efficiency</td>
<td></td>
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<tr>
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<td>Magnetizing Current Angle Respect to D Axis</td>
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<tr>
<td>$\lambda$</td>
<td>Stack Aspect Ratio</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{qd0,r}$</td>
<td>Rotor Flux Linkage in DQ0 Axes</td>
<td>$Wb$ – $turns$</td>
</tr>
<tr>
<td>$\lambda_{qd0,s}$</td>
<td>Stator Flux Linkage in DQ0 Axes</td>
<td>$Wb$ – $turns$</td>
</tr>
<tr>
<td>$\lambda_m$</td>
<td>Magnetizing Flux Linkage</td>
<td>$Wb$ – $turns$</td>
</tr>
<tr>
<td>$\lambda_{md}$</td>
<td>Magnetizing Flux Linkage in D Axis</td>
<td>$Wb$ – $turns$</td>
</tr>
<tr>
<td>$\lambda_{mq}$</td>
<td>Magnetizing Flux Linkage in Q Axis</td>
<td>$Wb$ – $turns$</td>
</tr>
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<td>$\lambda_r$</td>
<td>Rotor Flux Linkage</td>
<td>$Wb$ – $turns$</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>Stator Flux Linkage</td>
<td>$Wb$ – $turns$</td>
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### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Magnetizing Flux Linkage Angle</td>
<td>degrees</td>
</tr>
<tr>
<td>$\mu_B$</td>
<td>Permeance at the Bottom of Slot</td>
<td>$H/m$</td>
</tr>
<tr>
<td>$\mu_T$</td>
<td>Permeance at the Top of Slot</td>
<td>$H/m$</td>
</tr>
<tr>
<td>$\mu_{TB}$</td>
<td>Permeance at the Middle of Slot</td>
<td>$H/m$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Resistivity, Angle Rotating Reference Frame</td>
<td>$\Omega m, rad/s$</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>Stator Pole Pitch</td>
<td>$m$</td>
</tr>
<tr>
<td>$\tau_{pm}$</td>
<td>Pole Pitch at Midpoint of Stator Slot</td>
<td>$m$</td>
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<td>Stator Slot Pitch</td>
<td>$m$</td>
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<td>$\tau_{sm}$</td>
<td>Slot Pitch at Midpoint of Stator Slot</td>
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<tr>
<td>$\theta_r$</td>
<td>Rotor Position Angle</td>
<td>rad/s</td>
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<tr>
<td>$\theta_2$</td>
<td>Displacement Due to Slip</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\Gamma_{(Bz,B)}$</td>
<td>Loss Due to Flux Density</td>
<td>watts/Kg$^3$</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Stator Current Angle Respect to D Axis</td>
<td>degrees</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Synchronous Speed</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Skew Angle</td>
<td>rad</td>
</tr>
</tbody>
</table>
Chapter 1: Design of Electric Motors Aided by Evolutionary Algorithms

1.1 Introduction

Evolutionary Computing or Evolutionary Algorithms is a concept that includes a wide group of techniques for the resolution of complex problems through emulating the natural processes of evolution. That is to say, these techniques use mechanisms of evolution theory for their design and implementation. The main contribution of the Evolutionary Computing to problem resolution methodology is the use of mechanisms to select potential solutions, and to create new candidates for recombining of characteristics of others already present, as it occurs in the evolution of the natural organisms [4, 5]. Research has been conducted in recent years to explore how the Artificial Intelligence techniques, particularly those known as Evolutionary Algorithms, are being used to adopt a new approach to optimization problems in designing electric motors, since this process is highly non-linear and quite complex from the material
properties and geometry points of view. Electric Motors, particularly Induction Motors, consume a considerable amount of electrical power worldwide, since they are the most attractive for industry due to robust construction, low cost and reduced maintenance. Nevertheless, there is little information available on the use of Evolutionary Algorithms in designing Induction Motors, specifically, for applications that require speed variations. There is an abundance of Induction Motors that are controlled by varying their supply voltage and frequency in order to change their speed without affecting their torque properties. For those reasons, the purpose of this investigation was to determine the effectiveness of including Evolutionary techniques into the designing stage of Induction Motors for applications where inverters are utilized to adjust speed and torque.

The results of this study may provide a useful approach that can be incorporated into existing designing tools in order to have a better exploration of optimum non-linear solutions in designing Induction Motors for variable speed applications.

1.2 Evolutionary Algorithms (EAs)

The purpose of the Evolutionary Algorithms is to guide a stochastic search causing a group of structures to evolve and selecting iteratively the most capable. During each iteration, best known as generation, there exists a set of solutions for a certain problem. This group is called the population; thus, the individuals or solutions must be encoded and decoded such that they are able to represent a potential solution.
Chapter 1. *Design of Electric Motors Aided by Evolutionary Algorithms*

To illustrate this, the pseudo-code of an Evolutionary Algorithm is depicted (See Algorithm 1).

**Algorithm 1 :** Evolutionary Algorithm Pseudo-code

```plaintext
Begin
    t ← 0
    P(t) ← InitializationPopulation()
    Evaluation(P(t))
    while t < generations do
        P'(t) ← SelectionParents(P(t))
        Crossover(P'(t))
        Mutation(P'(t))
        Evaluation(P'(t))
        P(t + 1) ← Survive(P(t), P'(t))
        t + +
    end while
End
```

As can be seen, the general structure of most of Evolutionary Algorithms includes the following stages: initialization, evaluation, selection, crossover, mutation and replacement. Initialization is the process of creating the starting population; this might be accomplished either by randomly generating or by including individuals who have been previously obtained -(for example, through analytical methods). Therefore, the Evolutionary Algorithms can benefit from other mechanisms[6]. Notice that there only one point of interaction exists between the individuals and the problem to solve; this point is the evaluation stage. Through the fitness function (normally ”cost function”), each individual of the population is measured in order to know how good the solution for the problem it is. Along with the fitness function and replacement stage, the selection is designed to score the individuals in order to get the fittest candidates
who would go to the reproduction. This way, they would spread their genes to a new
generation. In accordance to [6] the most popular techniques are fitness-proportionate
methods. In these methods, the probability of selecting an individual for breeding
is proportional to its fitness. The Evolutionary Algorithms have a set of operators
which changes the structure of an individual every generation. Two operators have
been classically used to this aim, in spite of other existing operators. The crossover
and mutation mechanisms allow Evolutionary Algorithms to exchange information
among individuals either by creating new potential solutions using the information
contained in the parents or by randomly substituting the chromosome at a certain
position by a different one. As the recombination is applied, the resulting individuals
are completely composed of information taken from their parents, thus, the genes will
be transmitted to the offspring[7] and the population will keep the good genes based
upon the performance obtained in computing the fitness function. However, the mu-
tation is slightly different since new material is randomly injected in the population,
the new individuals will have to compete in order to know if they become in potential
solution for the problem and then, they will be able to keep alive in the population.
Finally, the replacement stage keeps the population size fixed since it is usually de-
sired to have the same number of individuals in every generation. Therefore, the new
individuals created from recombination and mutation must substitute others who are
already present in the population. This can be done in several ways, according to
the literature consulted. The strategies more often used are replacement of the worst;
random; tournament and direct replacement[4–7]. At the beginning of their existence,
only three classical families were known but new variants have recently emerged, so the Evolutionary Algorithms are currently composed for:

- Evolutionary Programming (EP)
- Evolution Strategies (ESs)
- Genetic Algorithms (GAs)
- Evolution Programs (EPs)
- Genetic Programming (GP)
- Memetic Algorithms (MAs)

1.3 Evolutionary Algorithms Applied to Electric Motors

Among these techniques mentioned in the previous section, only the Genetic Algorithms have been successfully used to design motors [8–17], however two other techniques have been recently introduced. They are so-called Particle Swarm Optimization (PSO) [9, 12, 18] and Differential Evolution (DE) [19], which have shown a significant performance in being used as optimization mechanisms [20].

It is worth noticing that most of EAs have been specially applied to Permanent Magnet Motors [9–12, 14, 18, 19], and few towards Induction Motors [8, 13, 16], while other type of machines have not been widely studied or reported.
Genetic Algorithms (GAs), Particle Swarm Optimization (PSO) and Differential Evolution (DE) have gained ground to solve optimization problems because they have become in an efficient way to find solutions for nonlinear problems with constraints when searching in large-multidimensional spaces and multi-objective solutions is required. All of these methods are stochastic and they usually need more iterations but being meta-heuristics, they have a high probability to find an optimal solution unlike those gradient-based methods that are normally unable to escape from local optimum[7].

**The Genetic Algorithms** are the most known among Evolutionary Algorithms. In this approach each individual is usually encoded as a binary string although more complex individuals can be implemented. For example, if one single chromosome representing air gap length is to be formed by ten binary digits and its minimum and maximum limits are set between 1mm and 3mm, there will be 2¹⁰ potential solutions to explore in that range, that is to say, an accuracy of about $2\mu m$ (micro-meter). Then, as aforementioned, the chromosome will be improved by verifying all of the values in that range during the evolution by means of recombination and mutation until stopping point is reached.

**Particle Swarm Optimization** is a parallel evolutionary computation technique inspired by the social behavior of bird or bee flocking. Like Evolutionary Algorithms, Particle Swarm Optimization is a search method based on population. During bird flight, the position and velocity of each particle is set according to its own experience
as well as the whole population’s experience. However, this technique is not considered as an Evolutionary Algorithm since Particle Swarm Optimization does not use recombination and mutation to update its individuals. Otherwise, the particle updates itself by using a tracking memory to record its own previous best position and velocity.

**Differential Evolution** is another stochastic algorithm based upon generations. In the next section it will be explained further.

It is evident that there exists a clear influence to utilize Genetic Algorithms in designing motors since they are best known and have been widely used for many years. However, outstanding performance is also reported by authors who decided to optimize their designs through PSO or DE. Studies aforementioned have concluded that these other techniques are quite useful in electrical machines design. Similarly, an assessment was conducted by Vesterstrom and Thomsen[21] to determine which Evolutionary Algorithms were most suitable for optimization. They reported that Differential Evolution performed with outstanding results. Likewise, in 2011 Duan and Ionel[20] compared several techniques specifically for electrical machine design, the results showed superiority of DE. Those arguments serve as reasons to have chosen the Differential Evolution Algorithm as optimization technique for this study.
1.4 Differential Evolution (DE)

Differential Evolution is an heuristic approach proposed by Price and Storn in 1997 based upon the paradigm of Evolutionary Algorithms for optimizing continuous space functions. This technique was developed to use vectors of floating points as individuals, unlike other EAs aforementioned. The authors [1] state that this novel minimization method is able to outperform other techniques when the following characteristics are assessed:

- Handling of non-differentiable, non-linear and multi-modal cost functions.
- Parallelism in computing cost functions.
- Robustness and easiness due to its few control variables to tune.
- Consistent convergence.

The Differential Evolution utilizes a population size $N_P$ that does not change during the process; each vector is randomly initialized to cover the entire parameter space. Mutation is carried out by adding the weighted difference between at least two vectors to a third one, with the weight $F$ as a constant factor $\in [0,2]$ which multiplies the difference. After this new mutant vector is obtained, it is recombined to a population vector in order to generate a new "trial vector"; this stage is so-called Crossover. The Crossover constant $CR \in [0,1]$ is the likelihood to inherit parameter values from their parents, the target or the trial vector. Finally, the smaller function cost value among
the target vector and the trial one allows the algorithm to decide which vector should become part of next generation[1, 22].

![Differential Evolution Algorithm](image)

**Figure 1.1:** Differential Evolution Algorithm [1]

1.4.1 Variants of Differential Evolution

The strategies are classified on three categories and presented in [22], depending upon what vector is to be perturbed, the number of vectors differed to generate the mutant vector and also, the type of crossover used.

- **Vector to be mutated:** It can be randomly chosen or selected because it has the lowest population cost.

- **Number of difference vectors:** It can be single or double, for perturbation with a single vector difference, any two vectors are differed and added to the third
vector. As double vector difference is used, five vectors are chosen and the vector difference of each pair of any four is added to the fifth one.

• Type of crossover: Binomial or exponential crossover uses CR value as probability to take parameters from either the target vector or the trial vector.

1.5 Implementation of DE for Variable Speed Induction Motors

As was aforementioned, only few applications of DE for Induction Motors have been reported nevertheless no one has been found for variable speed specifically, so far. This study proposes a new approach based upon the DE algorithm in order to obtain an initial model of Induction Motors to be driven by an inverter. The resultant geometric model is carefully analyzed by using Finite Element Method and its performance studied to verify that it meets the requirements.

Inverter-driven Induction Motors require certain characteristics that allow the machine to operate in the torque-speed region for which a good trade-off among efficiency, power factor and peak torque is achievable.

It is evident that Inverter-driven Induction Motors have become a mature technology and their design is dependent upon how rotor bar skin effect and leakage inductance are adjusted, among other reasons [23]. Likewise, torque production in these types of machines is related to the rotor slip frequency and the rotor resistance as they are operated with constant rotor flux control [24]. For these reasons, the approach
presented in this study pays carefully attention to the machine geometry in order to enhance shapes and sizes, due to their relation to leakage inductance and resistance.

On figure 1.2 is depicted the geometry utilized in the algorithm developed, where each individual in the population is coded as an Induction Motor geometry –that is to say, a set of parameters, calculations and constraints that determine the main geometrical features of these machines.

Differential Evolution, as well as other Evolutionary Algorithms, is able to find the global minimum, instead of a local minimum, unlike other numerical optimization techniques widely used such as Non Linear Programming. The search space for each parameter selected is shown on table 1.1. It is worth noting that it is quite broad, taking into consideration that the design target for this study is 55Kw; thus, the minimum and maximum values are the boundaries, so that the algorithm can obtain
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Abbreviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator Inner Diameter</td>
<td>$D_{is}$</td>
<td>50mm</td>
<td>700mm</td>
</tr>
<tr>
<td>Stator Outer Diameter</td>
<td>$D_{os}$</td>
<td>50mm</td>
<td>1200mm</td>
</tr>
<tr>
<td>Stack Iron Length</td>
<td>$L$</td>
<td>50mm</td>
<td>800mm</td>
</tr>
<tr>
<td>Stator Tooth Width</td>
<td>$W_{ts}$</td>
<td>3mm</td>
<td>40mm</td>
</tr>
<tr>
<td>Stator Yoke Height</td>
<td>$H_{cs}$</td>
<td>10mm</td>
<td>100mm</td>
</tr>
<tr>
<td>Stator/Rotor Slots Combination</td>
<td>$S_1/S_2$</td>
<td>1</td>
<td>28</td>
</tr>
</tbody>
</table>

**Table 1.1:** List of Parameters and Their Search Space

a logical solution from geometrical point of view. It is also important to mention that DE does not require the use of derivatives which are not always easily obtainable or may not even exist, and its initial conditions do not need to be close to actual values [13, 25].

The algorithm is tuned in accordance to [22], where it has been reported that most of the optimization problems have been solved with DE by setting the weight $F$ in the range $[0.4 , 1]$, the crossover probability $CR$ close to 1.0 and a population size $NP$ about 5 to 10 times the parameters vector dimension. Taking into account those recommendations and after running the algorithm several times, the fastest convergence is reached with the following values:

- Population Size $NP = 60$
- Weight $F = 0.8$
- Crossover Constant $CR = 0.9$
- Strategy = DE/local-to-best/1
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Every generation the algorithm selects a set of values for each geometry, and those individuals are validated through the following relationships, which are based upon the simplified model depicted on figure 1.2.

\[ W_{1s} = \frac{\pi}{S_1} \left[ D_{is} \left( 1 - \frac{B_g}{B_{ts} K_{is}} \right) + 2H_{s01} \right] \]  
(1.5.0.1)

\[ W_{2s} = \frac{\pi}{S_1} \left[ D_{os} - D_{is} \left( \frac{B_g}{B_{ts} K_{is}} + \frac{2 B_g}{P_{ts} K_{is}} \right) \right] \]  
(1.5.0.2)

\[ H_{ss} = \left( W_{2s} - W_{1s} \right) \frac{S_1}{2\pi} \]  
(1.5.0.3)

\[ g = 5 \times 10^{-3} D_{is} \sqrt{\frac{\pi}{2P_1}} \]  
(1.5.0.4)

\[ W_{tr} = \frac{\pi}{S_2} (D_{or} - 2H_{sr}) - \frac{W_{1r} + W_{2r}}{2} \]  
(1.5.0.5)

\[ A_{ss} = \frac{K_{fill}(W_{1s} + W_{2s}) H_{ss}}{2} \]  
(1.5.0.6)
These relationships presented in [26–28] allow the algorithm to size and validate the dimensions; nevertheless, the flux densities at stator and rotor have to be defined first as follows:

\[ B_{ts} = \frac{B_g \pi D_{is}}{S_1 W_{ts} K_{is}} \]  \hspace{1cm} (1.5.0.7)

\[ B_{cs} = \frac{B_g D_{is}}{P_1 H_{cs} K_{is}} \]  \hspace{1cm} (1.5.0.8)

\[ B_{tr} = \frac{B_g \pi D_{or}}{S_2 W_{tr} K_{ir}} \]  \hspace{1cm} (1.5.0.9)

In designing an Induction Motor for variable speed applications, one is interested in the operating region between the point of breakdown torque and the point of synchronous speed. By adjusting the input voltage and the input frequency through an inverter, the motor keeps working in that area in spite of the speed it is driven. In addition, a small slip difference between these two points is essential in order to maintain the motor operating at high efficiency and power factor.

Therefore, the torque-speed curves A and B shown on 1.3, are the most suitable in selecting an Inverter-driven Squirrel Cage Induction Motor, while the NEMA Designs C or D are generally not good choices due to their lower starting torque, larger current rating and slip when driving by variable voltage and frequency [29].

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In order to achieve such behavior, the rotor slot shape must provide a wider section to the air gap, and inverted tear drop or trapezoidal shapes are recommended; that is to say, the slot can be wider and shorter than a direct-to-line starting Induction Motor. For instance, the rotor slot height should not be more than 150% of its width. Likewise, a large slot area is required to decrease the resistance, and avoid excessive losses. These characteristics also provide a small leakage reactance which has the advantage of reducing the slip frequency for the rated torque, keeping high the power factor and the breakdown torque as well [24, 30, 31]. On the other hand, the only limitation for increasing the slot area is the flux density in rotor teeth, resulting in higher iron losses if the rotor diameter is not increased.
Chapter 1. Design of Electric Motors Aided by Evolutionary Algorithms

\[ \frac{W_{1r} + W_{2r}}{2} = \sqrt{\frac{2}{3} \frac{A_b}{K_{ab}}} \]  \hspace{1cm} (1.5.0.10)

\[ H_{sr} = \frac{3}{2} \frac{W_{1r} + W_{2r}}{2} \]  \hspace{1cm} (1.5.0.11)

The equations (1.5.0.10) and (1.5.0.11) are proposed to lead the Differential Evolution Algorithm to search for solutions that fulfill the characteristics of rotor resistance and rotor leakage inductance aforementioned.

Being the cross section of the rotor bars, end rings and the design of the stator winding dependent upon current flowing through them, the set of equations below are used in sizing those elements. The material properties used by the algorithms are fixed since Copper has been initially selected as material for the conductors not only in the stator, but it also in the rotor; therefore, in deciding to use a different material like aluminum, its properties must be first included within the algorithm.

\[ N_s = \frac{\sqrt{2}}{4 \sqrt{3} K_w f_B g_p D_{is} L} \frac{P_1 V_L}{C_s} \]  \hspace{1cm} (1.5.0.12)

\[ C_s = \frac{2 m N_s}{S_1} \]  \hspace{1cm} (1.5.0.13)
\[ I_b = \frac{K W S_1 C_s I_s \cos \theta}{S_2} \]  \hspace{1cm} (1.5.0.14)

\[ I_{er} = \frac{I_b}{2 \sin \left(\frac{\pi P_1}{2 S_2}\right)} \]  \hspace{1cm} (1.5.0.15)

\[ A_b = \frac{I_b}{\sigma_{cu}} \]  \hspace{1cm} (1.5.0.16)

\[ A_{er} = \frac{I_{er}}{\sigma_{cu}} \]  \hspace{1cm} (1.5.0.17)

As has been already demonstrated, the main advantages of DE over other numerical optimization techniques are its simplicity, accuracy and fast convergence in minimizing a cost function. The capability of DE for multi-objective optimization is another superiority point which makes it worthy for the design of electric motors since a designer is often obligated to sacrifice some performance to enhance others.

Thus far, it has been explained how obtain the main dimensions for an Induction Motor along with some details about how to meet the requirements so that the motor is driven by adjustable voltage and frequency; however, the optimization problem for this study is focused on getting the geometry for a torque-density-improved motor.
Some software packages to aid the design of Induction Motors have been reported [24, 28], however, they are based upon the iterative increasing or decreasing of the machine dimension until meet the requirements initially specified. It is clear that from a numerical optimization point of view that those approaches are inefficient considering the computational cost required as well as the high non-linear relation among the parameters in the machine which defines a non-linear search space that cannot be totally covered by simple iterative techniques.

Hence, a multi-objective optimization technique is needed to find a global optimum when several cost functions are evidently contrasting among them, as happens in improving torque density, which involves a function to maximize torque while the volume function is minimized, as it is described by the equations (1.5.0.18) to (1.5.0.20).

\[ T_d = \frac{T_e}{V} \quad (1.5.0.18) \]

\[ V = \pi D_{os}^2 L \quad (1.5.0.19) \]

\[ T_e = \frac{P}{(1 - s)\omega} \quad (1.5.0.20) \]

Nevertheless, the actual relationship among the machine parameters is defined in [23] and described by equations (1.5.0.21) to (1.5.0.25), and they allow to express the
electromagnetic torque from the electric and magnetic loading, which are function of
the main motor geometry as well as the flux density paths throughout the iron and
air gap.

\[ A_l = \frac{2mN_sI_s}{\pi D_{is}} \]  

(1.5.0.21)

\[ C_o = K_F\alpha_iK_W\pi^2 A_lB_g \]  

(1.5.0.22)

\[ \lambda = \frac{P_1L}{\pi D_{is}} \]  

(1.5.0.23)

\[ F_o = \frac{1}{\left(1 + \frac{2A_lB_l}{\sigma_cuK_WB_gD_{is}} + \frac{\alpha_i\pi B_g}{P_1B_c}\right)^2} \]  

(1.5.0.24)

\[ P = \frac{F_oD_{os}^22\pi\lambda C_o f\eta\cos \theta D_{is}}{P_1^2K_E} \]  

(1.5.0.25)

Other authors who have applied similar techniques [25, 32, 33] have defined a variety
of objective functions such as production cost, rated torque, efficiency or ripple torque.

To obtain a fast convergence, it is useful add to the algorithm as much experience
as possible by taking into consideration the best practices gained over last years in
designing and building Inverter-driven Induction Motors. All of those details can be included as constraints and penalties in order to lead DE towards the optimum search space; furthermore, constraints and penalties play a vital role in avoiding that each individual takes illogical values occasioning this way, absurd solutions.

Some constraints, that have been tested and help the algorithm to find optimum values, are described in table 1.2, and are based upon the experience collected by [2, 23, 26, 27].

Due to the differences among magnitudes of the constraints aforementioned, it is mandatory to carry out a normalization among them; otherwise the candidates that fulfill the constraints with larger magnitude will be preferred, perturbing then the convergence to optimum solutions. Those drawbacks can be solved with no impact over the algorithm by applying the following normalization.

$$c_k = \begin{cases} 
\vartheta_k \left( \frac{c_k - c_{k\text{MAX}}}{c_{k\text{MAX}} - c_{k\text{MIN}}} \right) & \text{if } c_k > c_{k\text{MAX}} \\
\vartheta_k \left( \frac{c_{k\text{MIN}} - c_k}{c_{k\text{MAX}} - c_{k\text{MIN}}} \right) & \text{if } c_k < c_{k\text{MIN}} \\
0 & \text{otherwise}
\end{cases}$$

(1.5.0.26)

Where $\vartheta_k$ is a penalty coefficient that can be also adjusted to reduce the impact of a specific constraint on the evolution process. Likewise, the penalization strategy can not only be used on constraint functions, but it is also useful so that the objective function is weighted, for example, for multi-objective optimization, where the coefficient is applicable as the designer is interested in balancing the optimization towards a
### Table 1.2: List of Constraints that Incline DE Towards Inverter-driven Induction Motors Solutions

<table>
<thead>
<tr>
<th>Description</th>
<th>Definition</th>
<th>Minimum ( (c_{k_{MIN}}) )</th>
<th>Maximum ( (c_{k_{MAX}}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator Teeth Flux Density</td>
<td>( B_{ts} )</td>
<td>-</td>
<td>1.7 T</td>
</tr>
<tr>
<td>Stator Core Flux Density</td>
<td>( B_{cs} )</td>
<td>-</td>
<td>1.6 T</td>
</tr>
<tr>
<td>Rotor Teeth Flux Density</td>
<td>( B_{tr} )</td>
<td>-</td>
<td>1.8 T</td>
</tr>
<tr>
<td>Rotor Core Flux Density</td>
<td>( B_{cr} )</td>
<td>-</td>
<td>1.7 T</td>
</tr>
<tr>
<td>Stator Geometry Consistence</td>
<td>( D_{os} - D_{is} )</td>
<td>( 2(H_{ss} + H_{cs} + H_{s01}) )</td>
<td></td>
</tr>
<tr>
<td>Stator Geometry Consistence</td>
<td>( \pi(D_{is} + 2H_{s01}) )</td>
<td>( S_1(W_{ts} + W_{1s}) )</td>
<td></td>
</tr>
<tr>
<td>Stator Geometry Consistence</td>
<td>( \pi(D_{os} + 2H_{cs}) )</td>
<td>( S_1(W_{ts} + W_{2s}) )</td>
<td></td>
</tr>
<tr>
<td>Stack Aspect Ratio</td>
<td>( \lambda )</td>
<td>0.9</td>
<td>2.1</td>
</tr>
<tr>
<td>Inner to Outer Stator Diameter Ratio</td>
<td>( \frac{D_{is}}{D_{os}} )</td>
<td>0.4</td>
<td>0.7</td>
</tr>
<tr>
<td>Minimum Stator Slot Area</td>
<td>( A_{ss} )</td>
<td>( \frac{C_s \pi}{4} d_c^2 )</td>
<td>-</td>
</tr>
<tr>
<td>Stator Slot Aspect Ratio</td>
<td>( \frac{2H_{ss}}{W_{1s} + W_{2s}} )</td>
<td>1.5</td>
<td>3.5</td>
</tr>
<tr>
<td>Minimum Stator Slot Width</td>
<td>( W_{1s} )</td>
<td>( \frac{d_c}{K_{fult}} )</td>
<td>-</td>
</tr>
<tr>
<td>Rotor Slot Aspect Ratio</td>
<td>( \frac{2H_{sr}}{W_{1r} + W_{2r}} )</td>
<td>-</td>
<td>1.5</td>
</tr>
<tr>
<td>Rotor Slot Width Ratio</td>
<td>( \frac{W_{1r}}{W_{2r}} )</td>
<td>0.9</td>
<td>-</td>
</tr>
<tr>
<td>Output Power</td>
<td>( P )</td>
<td>55 kW</td>
<td>-</td>
</tr>
</tbody>
</table>
specific function more than the others. As far as single-objective optimization is concerned, those weights are determinant in the summation of each weighted functions into the final fitness function.

The next section will present the results obtained by using the approach explained above which is able to generate a geometry that meets the requirements for Induction Motors for variable speed applications, in addition to presenting reduced volume, being compared to frame sizes defined by NEMA and IEC standards.

1.6 Simulation Results

First of all, the algorithm convergence towards optimum solutions as well as the main parameters variability during the generations are presented. Secondly, the geometry is built and analyzed by utilizing a Finite Element Software. Subsequently, the main characteristics obtained are described; however, exhaustive analyses are performed and detailed later in the next chapter.

Figure 1.4 depicts the torque density trajectory over the optimization process. As is expected, after each generation either the torque is increased or the volume is reduced on the candidates in order to maximize the torque density which has been defined as the aim for this study. During the optimization the best individuals are going from the low torque density region (the blue/cyan region) through the high torque density region (the orange/red region).
When the objective functions are separated as is shown on figures 1.5 a and b, it is noticeable how the optimization is taking place. Since the algorithm is seeking a balance between high torque and low volume, the best solutions are not precisely the ones with largest torque or smallest volume, but a trade-off between them. In spite of the solutions converging to their optimums, the plot a shows that during the optimization some candidates with better torque appeared. However, the final goal, which is torque density, is improved after each generations as it is expected (see figure 1.5 c).

It is also important to note the constraints satisfaction on figure 1.5 d, and recall the equation 1.5.0.26, where the constraints applied to the algorithm are defined and normalized. The plot presents the result of the constraints satisfaction previously.
Figure 1.5: Objective Functions a) Torque Optimization b) Volume Optimization c) Torque Density Optimization d) Constraints Satisfaction

explained on table 1.2 for one of the best candidates. A zero value means that the condition has been totally met; likewise, a negative value means the parameter is below the reference, which in some cases is acceptable, while a positive value is intolerable by the DE definition. For example, the constraint one is the difference between the target output power (55Kw) and the output power calculated for each solution by using 1.5.0.25, which is about 65Kw for the graph shown. The second constraint that presents the largest difference is related to the rotor core flux density,
and it ended up considerably below (60%) of the reference applied (1.7T).

Use of the Pareto-dominance selection criteria is the most convenient approach in analyzing the results of Multi-Objective optimization problems. This technique typically groups the family of best-compromise solutions that provides the user with a clear view of various trade-offs among pareto-optimal solutions [32]. A pareto front for this study is depicted on figure 1.6. The blue line contains the best-compromise Induction Motor geometries. The algorithm based upon Differential Evolution paradigm is able to find solutions with volume less than 0.02 \( m^3 \) and torque above 350 \( Nm \) for a slip of 2%.

![Figure 1.6: Optimum Solutions Pareto Front](image)

Sizing an electric motor is the first step in designing, and this has been the intention of developing an algorithm based upon Differential Evolution. Defining a geometry
requires the compromise of several parameters that are generally correlated by non-linear relationships. There is not an analytical straightforward method that leads to optimum solutions which meet the whole set of design requirements; besides, the experience plays a vital role in wanting to obtain successful models. This all can be inferred from figures 1.7 and 1.8, where the $x$ axis depicts the number of generations or can also be interpreted as the convergence towards optimum values and the $y$ axis is the value chosen for each parameter. On those graphs, although the models mostly tend to have lower dimensions as the iterations pass, the internal geometry (for example, rotor size and slots shapes) is complexity-evaluated without any apparent pattern, demonstrating this way the algorithm’s capability to perform searching in a non-linear space of solutions.

A candidate from pareto front is selected and the respective model is built by using Finite Element Software in order to observe and enhance its behavior under different
values of voltage and frequency. Then, the motor is modeled and some of its dimensions are slightly varied taking advantage of magnetostatic and electrostatic analyses provided by Finite Element Method which give better accuracy in calculating the flux density in the iron. Authors [13, 15, 19, 28, 34] agree that advances in computer sciences have made possible the extensive use of FEA at various design stages; however it should be specially utilized to investigate the effectiveness of an already optimized motor obtained by means of other suitable design tools less expensive from...
As has been already mentioned, an Inverter-driven Induction Motor is designed to hold small leakage inductance as well as low rotor resistance; therefore, some of the major challenges faced by designers are the shapes and sizes of stator/rotor slots. To deal with those challenges, detailed flux density distribution in the core is suggested so that the influence of slot geometry on core losses and harmonics produced by the different types of leakage inductance can be studied.

After having selected a model generated by the Differential Evolution Algorithm, its dimensions are taken as the initial design. The final design is subsequently obtained by applying Finite Element Analyses in order to optimize the geometry taking advantage of its accurate flux and current density calculation. Table 1.3 shows the difference between initial and final designs proposed for this study. The largest modifications

![Figure 1.9: Induction Motor Dimensions a) Stator b) Squirrel Cage Rotor](image)
Table 1.3: Geometry Generated by DE and Optimized by FEA

are evidenced on slot dimensions; nevertheless, the reasons were mentioned before. In
general, the small deviation between their geometries comes to affirm the feasibility
of using the algorithm proposed in this chapter which demonstrates how powerful
tool it is in sizing Induction Motors for variable speed applications.

At this point, it is worth comparing results of the method proposed here to other
dimensions defined by either NEMA or IEC Standard motor frames (figures 1.10 and
1.11). First, it must be stated that dimensions reported by NEMA or IEC include
enclosure (housing), so the core dimensions are assumed slightly smaller; however,
they still serve as a reference. Secondly, the target for this study is 55Kw thus, the
frames selected for comparison are based upon this condition.

From the NEMA standard, the dimension $D$ and $2F$ are equivalent to stator outer
radius and stack length, respectively. As far as the IEC standard, the dimensions to compare are $H$ and $B$. Usually, $55KW$ motors come at $250S$ frames for IEC or $404U$ frames for NEMA, respectively.

Lower dimensions can be reached by Induction Motors for variable speed applications, and the results on table 1.4 demonstrate the algorithm’s efficiency in finding a geometry with reduced size that has a good trade-off among requirements such as torque, losses, leakage inductance, resistance, power output, power factor, etc.

Thus far, this section has exhibited the results of proposed motor from dimensions point of view; however, the performance has not been analyzed yet. The steady state response is presented on this chapter in order to show only the general behavior that
Chapter 1. *Design of Electric Motors Aided by Evolutionary Algorithms*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>① Stator Outer Radius Proposed Motor</td>
<td>182.5mm</td>
</tr>
<tr>
<td>② NEMA Frame 404U Dimension &quot;D&quot;</td>
<td>254.0mm</td>
</tr>
<tr>
<td>③ IEC Frame 250S Dimension &quot;H&quot;</td>
<td>250.0mm</td>
</tr>
<tr>
<td>Relative Dimension to ② and ③</td>
<td>(71.9-73)%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>① Stack Length Proposed Motor</td>
<td>165.0mm</td>
</tr>
<tr>
<td>② NEMA Frame 404U Dimension &quot;2F&quot;</td>
<td>311.2mm</td>
</tr>
<tr>
<td>③ IEC Frame 250S Dimension &quot;B&quot;</td>
<td>311.0mm</td>
</tr>
<tr>
<td>Relative Dimension to ② and ③</td>
<td>53 %</td>
</tr>
</tbody>
</table>

Table 1.4: Dimensions Comparison Between Proposed Motor and NEMA/IEC Standards

the Induction Motor is expected to have. A complete description of each machine’s parameter, in addition to a detailed analysis when being controlled by varying voltage and frequency will be given in the following chapter.

Under no-load conditions, the motor develops the flux density depicted on figure 1.12, and its slip is about 0.1%. As has been aforementioned, the lower the slip frequency, the higher the speed where the breakdown torque might occur. In this zone the inductive reactance becomes nearly equal to the rotor resistance. These are the reasons that low rotor resistance, low leakage inductance and small slip are preferred. The Inverter-driven Induction Motor is forced to operate at small slip despite the fact that supply voltage and frequency are changed. Likewise, there are other benefits associated with this type of operation, for example, at higher speed, the losses in an Induction Motor are smaller. Furthermore, rapid torque changes are
The speed-torque curve for the proposed motor on figure 1.13 confirms the performance desired. One of the most important characteristics in the steady state analysis of Inverter-driven Induction Motor is the region between the breakdown torque and the synchronous speed which should be as vertical and linear as possible, and even more optimized than NEMA A designs.

![Induction Motor Resultant Under No Load](image)

**Figure 1.12:** Induction Motor Resultant Under No Load a) Flux Density b) Flux Lines

In contrast to that operating region, the starting and pull-up torque shown at steady state response are no longer determinant on being driven by inverter, since the motor would be able to start with a torque as high as its breakdown torque when supply voltage and frequency are correctly managed. That is why high-starting-torque motors such as NEMA C and D designs are not desirables for variable frequency conditions.
Finally, current, efficiency and power factor graphs give an overview on main motor features even though Transient and Field Oriented response might slightly vary from these.
1.7 Discussion

This chapter attempted to assess the effectiveness of including an algorithm based upon the Differential Evolution (DE) paradigm as initial stage in designing Inverter-driven Induction Motors. Originally, it was assumed that the ability of Evolutionary Algorithms to find global optimums in non-linear search spaces could be incorporated on it; thus, a considerable reduction of designing time as well as optimum geometries for specific optimization problems could be achieved.

The findings based upon steady state simulations and Finite Element Analyses (FEA) imply that the algorithm developed has capability of leading the Induction Motor geometry so that the machine behaves as it expected when it is controlled by variable voltage and frequency.

It was proved that by appropriately defining the constraints as well as cost functions, satisfactory geometries were obtained, which demonstrated maximum values of torque density since that was initially the aim of this study.

Sizing and optimizing electrical machines by using this approach has been studied; however, this chapter expanded its applicability for Induction Motor with special requirements such as variable voltage and frequency. Furthermore, the technique chosen has provided evidence of accuracy, quickness and convergence.

This chapter has only focused on Induction Motors torque density optimization for a specific purpose since the next chapters will take those results to perform comparisons.
to Synchronous Reluctance Motors (SRM) –although it is likely that other types of optimization can be fulfilled with a similar formulation.

The algorithm outlined over this chapter should be replicated and compared to other design tools; moreover, better precision would be expected if the flux density calculation based upon Finite Elements is included. This was inferred from the possibility to optimize even more the slot geometries after the Differential Evolution Algorithm generated the solutions. On the other hand, even though incorporating Finite Element calculations (FEM) might attempt against algorithm quickness, it would still be far away from other time-consuming approaches aided purely on FEM.
Chapter 2 : Design and Field Oriented Current Control of Induction Motor

2.1 Introduction

The Squirrel Cage Induction Motors are generally considered as a mature technology which has permitted wide use in the industry for uncountable applications. Due to advances in power electronics technologies, the control of AC drives involving Induction Motors has been realizable giving the industry the possibility of counting on low-cost and robust machines able to perform tasks with high precision. Finding design methodologies for those machines has been motivation of research over last decades, nevertheless a novel approach for variable frequency Induction Motors has been proposed and set forth in the previous chapter.

In order to verify its effectiveness in leading the model’s geometry so that the motor finally shows the desired behavior, this chapter will dedicate to present successive analyses that put the model to test. First at all, the passage from the geometrical and electromagnetic data obtained from the lamination and winding design to the
equivalent circuit is detailed [2, 23, 26, 35–37]. This is no easy task, even though information enough on the equivalent circuit parameter can be found in literature, but the relationship between the models is not frequently available or discussed. After that, a Field-Oriented Vector Control (FOVC) is chosen since its actual applicability to these sort of machines has been proved and reported by several authors [23, 38–43].

Having validated the capabilities of the Induction Motor designed to be driven through inverters throughout this chapter, the following ones are going to be devoted to comparing this with a Synchronous Reluctance Motor (SRM) under similar circumstances. This study may be useful in giving a full sight to those researchers and manufacturers who are currently arguing that Synchronous Reluctance Motors have the ability of doing equivalent tasks to Induction Motors, with higher efficiency and lower material costs because of improvements in switching devices that provide the Synchronous Reluctance Motors with feasible control strategies (turning them into a new alternative for the market mainly headed by Induction Motors). It is worth emphasizing that research has been conducted to compare both machines; however, there is not information enough about their comparison from variable Volt/Hz point of view, particularly Vector Control, which is the primary intention of this work.

2.2 Design Considerations

Previous chapters described a geometrical approach that permits obtaining the desired behavior in designing Induction Motors for being driven by inverters. It is also
obvious that the machine shaping and sizing lead to modifying its internal parameters, which ultimately define the performance. The importance of reducing leakage inductances and resistances through adjusting dimensions carefully has been previously mentioned, in order to improve efficiency, power factor and peak torque. For those reasons, the relationship among geometries and parameters of Induction Motor’s equivalent circuit is presented as follows.

![Single-phase Equivalent Circuit for a Polyphase Induction Motor](image)

**Figure 2.1:** Single-phase Equivalent Circuit for a Polyphase Induction Motor

The parameters given by the equivalent circuit are widely used, and are frequently obtained by different means. For instance, several testing methods provide accurate values as the machine is physically available; other techniques that involve parameter identification and observers design are taken into consideration for control purposes. Nevertheless, from design of electric motor point of view, those previously mentioned are useless, theoretical approaches and Finite Element Methods are preferred, instead.

A straightforward method is presented in the next sections collected from the work of several authors over years [2, 23, 26–28, 36, 37, 43, 44]. By using the calculations as follows an effective balance between accuracy and promptness is reached, which
can also be utilized to iteratively model the Induction Motor performance since the equations are dependent upon machine geometry thoroughly set forth in the last chapter.

Finite Element Methods are also worthy in calculating machine parameters—most of all, when the geometries get complex. FEM offers high flexibility although its iterative calculation requires substantial computing power. Further details of FEM are beyond the scope of this work; however, the results generated by the following equations are quite similar to those obtained by FEM for shapes and sizes analyzed through this work.

2.2.1 Resistances and Leakage Inductances

The stator resistance per phase per circuit is given by the equation 2.2.1.1, where the conductor length $L_C$ covers not only the effective stator length, but also must include the coil segment located at both end windings as it is shown on figure 2.3.

\[ R_s = \rho \frac{N_s L_c}{C A_c} \]  

(2.2.1.1)

Resistances and inductances are not constant as the motor is operating, so magnetic saturation and skin effect should be considered. Nonetheless, the skin effect in the stator conductors is highly dependent upon the conductor height and the stator slot height and some techniques are utilized to reduce its effect [26]. Particularly, for
Inverter-driven Induction Motors, as it was aforementioned, decreasing the slot height in the stator and rotor with respect to its width is a common practice. Hence, a minimized skin effect can be neglected from resistance and leakage inductance calculations as long as this is taken into consideration in sizing the slots while the geometrical design is carried out.

Figure 2.2: Double-Layer Stator Slot and Semi-Closed Rotor Slot

The leakage flux can be divided into various components. The first one is the slot leakage flux, which consists of the amount of flux linking two teeth through one slot, in consequence, the material inside of slot and its placement define the permeance needed to calculate the leakage inductance due to slot leakage.
The relation 2.2.1.2 suggests the permeance calculation for two layer winding inserted into the slot, as it is depicted on the stator slot on figure 2.2. It must be mentioned that the total permeance is estimated as the summation of the permeance at the top, the bottom and the middle of tooth-slot height due to the self-inductances and the mutual inductance between the layers. Finally, the equation 2.2.1.3 turns them into slot leakage inductance.

\[
\mu_T = \mu_o \left( \frac{d_{3s}}{3b_{ss}} + \frac{d_{2s}}{b_{ss}} \right) \\
\mu_B = \mu_o \left( \frac{d_{5s}}{3b_{ss}} + \frac{d_{2s} + d_{3s} + d_{4s}}{b_{ss}} \right) \tag{2.2.1.2}
\]

\[
\mu_{TB} = \mu_o \left( \frac{d_{2s}}{b_{ss}} + \frac{d_{3s}}{2b_{ss}} \right)
\]

Another fundamental component of leakage flux is produced in the end winding. An exact calculation of this flux is probably unfeasible because of its nonuniform shape and proximity to other magnetic and conductor materials. Generally, an approximated semi circumference is assumed [37] to model the end winding, obtaining rough outcomes. Alternatively, the method of images has been set forth in [26], permitting this way to count on good-enough estimated values for more complex end winding geometries such as depicted on figure 2.3.
In order to have the leakage flux in the end winding computed, it is necessary to define some relationships (see figure 2.3), which are expressed in terms of winding pitch \( p \), pole pitch \( \tau_{pm} \) and slot pitch \( \tau_{sm} \) at midpoint of stator slots.

\[
p = \frac{W}{\tau_p}
\]
\[
\tau_{pm} = \frac{\pi}{P_1} \left( D_{is} + H_{ss} \right)
\]
\[
\tau_{sm} = P_1 \frac{\tau_{pm}}{S_1}
\]

Likewise, the projected sides from both machine ends are compounded by \( L_{ew1} \) and \( L_{ew2} \). Their derivation and the total end winding flux leakage was proposed by Liwschitz-Garik in 1964 and summarized in equations 2.2.1.4 and 2.2.1.5, which
include the pitch and distribution factor due to harmonic presence because of the
distributed winding.

\[ L_{ew1} = \frac{p\tau_{pm}(b_c + t_e)}{2\sqrt{\tau_{sm}^2 - (b_c + t_e)^2}} \]  

(2.2.1.4)

\[ L_{Lew} = 9.6\mu_o \frac{N_s^2}{P_1} \left[ \frac{1}{q} \sin \left( n \frac{\pi W}{2 \tau_p} \right) \frac{\sin \left( n \frac{\pi Z}{2 \tau_p} \right)}{\sin \left( n \frac{\pi Z}{2 q \tau_p} \right)} \right]^2 \left( L_{ew2} + \frac{L_{ew1}}{2} \right) \]  

(2.2.1.5)

There are also leakage fluxes crossing the air gap, but they contrast with air gap flux
since do not link the stator-rotor windings. Some authors divide them into two main
components, belt leakage and zig zag leakage [26] although others account for them
as a whole, (for instance, differential leakage [23] or simply air gap leakage [35]).

\[ L_{Lbt} = L_m \left[ \frac{1}{K_{W1}^2} \sum_{n=2}^{\infty} \left( \frac{K_{Wn}}{n} \right)^2 \right] \]  

(2.2.1.6)

\[ L_{Lzz} = \xi_{zz} \frac{N_s^2 L_c}{S_1} \mu_o \frac{W_{ts}W_{tr}(W_{ts}^2 + W_{tr}^2)}{6g_e\tau_s^3} \]  

(2.2.1.7)

The equation 2.2.1.6 defines the belt leakage inductance and it is evident that the
inclusion of all the space harmonics in the air gap except the fundamental component,
which is the magnetizing flux. On the other hand, the zig zag leakage (equation
2.2.1.7) flowing through stator and rotor teeth is not only dependent upon tooth geometry, particularly tooth width, but it is also tightly related to the ratio between coil span and pole pitch, such relationship is set by the coefficient $\xi_{zz}$, as it is shown on graph 2.4.

![Figure 2.4: Coefficient for Zig-Zag Leakage Inductance](image)

Finally, if the stator winding is skewed, the skew leakage inductance is obtained in 2.2.1.8 by using the skew angle $\chi$.

$$L_{Lsk} = L_m \left[ 1 - \left( \frac{\sin \left( \frac{\chi}{2} \right)}{\frac{\chi}{2}} \right)^2 \right]$$  \hspace{1cm} (2.2.1.8)

The total stator leakage inductance $L_{ls}$ is given by the summation of each individual leakage inductance above mentioned.

$$L_{ls} = L_{Lsl} + L_{Lew} + L_{Lbt} + L_{Lzz} + L_{Lsk}$$
Before continuing with rotor parameters calculation, the winding factor $K_W$ is introduced in order to have the whole parameters in the equivalent circuit (2.1) referred to the stator. The $n$–th harmonic of the winding factor is shown as follows:

$$K_{Wn} = \frac{1}{q} \sin \left( \frac{n \pi W}{2 \tau_p} \right) \frac{\sin \left( \frac{n \pi Z}{2 \tau_p} \right) \sin \left( \frac{n \pi b_o}{2 \tau_p} \right) \sin \left( \frac{n \pi Z}{\tau_p} \right)}{\sin \left( \frac{n \pi Z}{2 q \tau_p} \right) \sin \left( \frac{n \pi b_o}{2 \tau_p} \right) \sin \left( \frac{n \pi Z}{\tau_p} \right)}$$  \hspace{1cm} (2.2.1.9)

The overall winding factor consists of the pitch factor, distribution factor, slot opening factor and skew factor, respectively.

While the rotor resistance in a wound rotor Induction Machine is frequently calculated by following the same method as explained for stator resistance, a Squirrel Cage Induction Motor possess bars and end rings attached to its rotor where the current is induced by the air gap flux density.

By assuming the rotor circuit has the same quantity of phases than the stator circuit, the rotor resistance per phase is accounted for a portion of end ring and a number of rotor bars, separately.

$$L_{bsk} = \frac{L_b + \frac{2}{3} t_{be}}{\cos \left( \frac{2\pi}{S_2} \right)}$$ \hspace{1cm} (2.2.1.10)

$$R_b = \rho \frac{L_{bsk}}{A_b}$$
Chapter 2. Design and Field Oriented Current Control of Induction Motor

For better accuracy, the rotor bar resistance calculation must consider the bar extension length as well as the end ring resistance value has to take into account the rotor pole pitch at the middle of the end ring. The total rotor resistance referred to the stator and based upon the total rotor-cage joule losses [37] is presented in the equation 2.2.1.12.

\[
R_e = \rho \frac{\pi (D_{or} - h_{be})}{A_{cr} S_2} \quad (2.2.1.11)
\]

\[
R'_r = 12 \frac{K_w}{W_1} N_s^2 \left[ R_b + \frac{R_e}{2 \sin^2 \left( \frac{\pi P_1}{2 S_2} \right)} \right] \quad (2.2.1.12)
\]

Once again, the skin effect is not discussed here and left out of equations since it has been minimized in decreasing the slot rotor height respect to its width, by setting it a ratio below 1.5, as it was mentioned in table 1.2 and suggested by the authors of [24, 30, 31] in order to reduce the rotor resistance and rotor leakage inductance. This provides higher breakdown torque at lower slip where inductive reactance becomes equal to rotor resistance [29], among other advantages. Further details about the skin effect in the equivalent circuit parameters is proposed by the authors in [36].

The rotor leakage due to rotor bars can follow the same procedure used to derive the slot leakage inductance in the stator, being even simpler since one rotor bar fills the whole slot; as a result, it can be considered as a single layer unlike the stator
slot that has two (figure 2.2). The slot permeance factor for several types of slot geometries can be found on [26, 35], the relation 2.2.1.13, thus reflects the particular shape implemented by this work.

\[ L_{bar} = n_s^2 L e \mu_o \left[ \frac{d_{3r}}{3b_{sr}} + \frac{d_{1r}}{b_{sr} - b_{or}} \log e \left( \frac{b_{sr}}{b_{or}} \right) + \frac{d_{or}}{b_{or}} \right] \]  

(2.2.1.13)

For the end ring leakage inductance, Levi in 1984 introduced the equation 2.2.1.14, also used for damper winding in synchronous machines.

\[ L_{er} = \frac{4}{3m} \mu_o \frac{S_2}{P_1} \left[ 2g + 0.18 \pi \left( D_{or} - 2d_{sr} - d_{re} \right) \right] \]  

(2.2.1.14)

The author in [26] derives an additional component of leakage inductance due to harmonics presence in the rotor bars, which is added to the total rotor leakage inductance as well, and finally referred to the stator in the equation 2.2.1.16.

\[ L_{b(har)} = \mu_o \left( \frac{D_{or} L e}{g_e} \right) \left( \frac{S_2}{\pi P_1^2} \right) \sum_{n=1}^{\infty} \left( \frac{1}{2nS_2 P_1 + 1} \right)^2 \]  

(2.2.1.15)

\[ L'_{ir} = 12 \frac{K_{W1}^2 N_s^2}{S_2} \left[ L_{bar} + \frac{L_{er}}{2 \sin^2 \left( \frac{\pi P_1}{2S_2} \right)} + L_{b(har)} + L_{Lsk} \right] \]  

(2.2.1.16)
The magnetic properties of laminations in the stator and rotor as well as the air gap flux density define the magnetizing parameters such as magnetizing current and magnetizing inductance. The total magnetomotive force (ampere-turns) from the curve of magnetic materials is needed and generally provided by the laminations’ manufacturer.

It is important to mention that the dc magnetization curves are commonly expressed as a function of flux density; consequently, a complete flux density analysis is recommended so that the magnetomotive force in teeth and cores are obtained. In contrast,
the ampere-turns in the air gap only takes into consideration the air magnetic reluc-
tivity or reciprocal of air magnetic permeability ($\mu_0^{-1}$), in addition to the effective air
gap by using the Carter’s Coefficient.

The flattened flux density due to the saturation effect in teeth is represented on figure
2.6. This phenomenon is usually accounted for in stator teeth, rotor teeth and air gap
magnetomotive force calculation by carrying out a 30 degrees of displacement with
respect to the fundamental component, whereas the yoke in good designs needs no
longer such approximation, besides, its maximum is reached as the air gap flux cross
zero, as it is presented in the figure.

Taking into consideration all the effects previously mentioned, the magnetizing in-
ductance can be obtained by the following relation:
\[ L_m = 2.7 \frac{V_L K_w N_s \cos 30^\circ}{2 \pi f P_1 A T_{30}} \] (2.2.1.17)

### 2.2.2 Losses

After the equivalent circuit parameters have been obtained, the losses in the Induction Machine can be estimated by means of analytical methods. In Induction Machines the efficiency is mainly determined by the effect of copper losses, iron losses, stray load losses and mechanical losses over each different operating point that the machine is driven.

The analysis of mechanical losses is beyond the scope of this work; however, the authors in [45] go deeply into details by subdividing the mechanical losses into friction in bearings, windage losses of outside fan, friction air losses, windage losses due to end rings and friction losses of V-ring seals. Finally, they conclude that these effects contribute to the total losses in a range from 0.6% up to 2.7% of total input power, of which the lower range is to 6-pole medium size motors, while the higher losses are for 2-pole small motors.

Where the copper losses are concerned, it is worth recalling that the conductor resistance changes due to temperature. In [44], the authors report an increase in the conductor resistance of about 4% for temperature rise of 10 degrees based upon normal ambient temperature and up to an increment of 0.71% in efficiency as the motor
is operated at low temperature. Those resistance changes clearly affect the following equations utilized to find the copper losses in the stator as well as in the rotor.

\[ P_{cu,s} = 3R_sI_s^2 \] (2.2.2.1)

Similarly, the so-called rotor copper losses, in spite of the rotor conductors made of any other material different from copper, consist of the losses due to the rotor current flowing through bars and end rings. Hence, the rotor current estimation has to be made firstly in order to proceed with the calculation. Additionally, the current induced in the rotor is dependent upon the frequency and slip at which the motor is driven.

The equation 2.2.2.2 is introduced by the author in [46], which allows to derive the slip over each operating point in terms of terminal voltage for any condition.
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\[
a = 1 + \frac{PK_W R'_r}{mV^2_\phi} \left[ \left( \frac{L_m + L'_{lr}}{L_m} R_s \right)^2 + \left( 2\pi f \left( \frac{L_{ls} + L_m + L_{lr}}{L_m} \right) \right)^2 \right]
\]

\[
b = 1 - \frac{2R_s PK_W_1}{mV^2_\phi} \left[ \left( \frac{L_m + L_{ls}}{L_m} \right) \left( \frac{L_m + L'_{lr}}{L_m} \right) - \frac{L_{ls} + L_m + L_{ls} L'_{lr}}{L_m} \right]
\]

\[
c = \frac{PK_W R'_r}{mV^2_\phi} \left[ \left( \frac{L_m + L_{ls}}{L_m} \right)^2 + \left( \frac{R_s}{2\pi f L_m} \right)^2 \right]
\]

\[
s = \frac{c}{b} \left( 1 + \frac{ca}{b^2} + 2 \left( \frac{ca}{b^2} \right)^2 \right)
\]

Counting on slip for any condition, the induced current referred to the stator on bars and end rings is rather straightforward by following 2.2.2.3 for any point in the steady state response, based upon steady state analysis of equivalent circuit [2, 39–41, 43, 47].

\[
I''_R = \frac{V_L}{\sqrt{3}} \frac{ \left( R_s + \frac{R'_r}{s} \right) - j2\pi f(L_{ls} + L'_{lr}) }{ \left( R_s + \frac{R'_r}{s} \right)^2 + (2\pi f (L_{ls} + L'_{lr}))^2 } \tag{2.2.2.3}
\]

Upon obtaining the induced current for any part of the rotor and any frequency, the rotor copper losses can be estimated as follows;
$P_{cu,r} = \frac{R_b}{S_2} (2mN_sK_{W1}'I_R')^2 + 2R_e \left( \frac{mN_sK_{W1}'I_R'}{S_2 \sin \left( \frac{\pi P_1}{S_2} \right)} \right)^2 \tag{2.2.2.4}$

In general, the core or iron losses refer to the hysteresis and eddy current effect occurring in the lamination material due to fundamental flux component. Estimating iron losses is not an easy task since not only it depends upon material’s magnetic properties—which can be very changing in being subjected to different thermal, stress, mechanical or electromagnetic conditions—but it also the lack of an accurate measure of flux density distribution all over the stator/rotor teeth and core.

Nevertheless, to have an approximated idea of iron losses, a rough value for multiples operating points can be achieved by utilizing the air gap flux density $B_g$ for different current, voltage and load conditions. Posteriorly, the relations 1.5.0.7 to 1.5.0.9 are used to forecast the flux density average over teeth and cores. Up to this point, it is important to remember the saturation effect aforementioned, which flattens the air gap flux density due to the teeth saturation.

Making allowance for this effect, the flux density in stator and rotor teeth are calculated through the employment of 30 degrees of displacement in the fundamental component of air gap flux density, as shown on 2.6.

The plot 2.7 indicates the core losses ($watts/lb$) for each flux density value expressed in terms of Gausses, particularly, in non-oriented electrical silicon steel $M19$ with a
lamination thickness of 26Ga, which is the material used as core in this work for both motors designed. It is clear for this material as well as others commonly utilized to produce either stator or rotor laminations that the higher the frequency, the higher the iron losses.

Due to the variability in the input frequency as the motor is inverter driven, multiple material curves should be taken in order to get the best estimated. On the other hand, from a designer viewpoint, the worst condition appears when the motor is operated at maximum speed; that is to say, at the maximum design frequency the higher core losses can be accounted for.
As a result, the quantity of material subjected to such flux density is often divided into two main machine components; for instance, the losses occurring at the teeth and those occurring at the core.

\[
P_{\text{iron}(ts)} = S_1 \left[ W_{ts} H_{ss} + \left( \tau_s - \left( \frac{W_{1s} + b_{0s}}{2} \right) \right) d_{2s} + d_{0s} \left( \tau_s - b_{0s} \right) \right] L'_{\gamma_{\text{iron}}} \Gamma_{(Hz,B)}
\]  \hspace{1cm} (2.2.2.5)

\[
P_{\text{iron}(cs)} = \pi \left[ \left( \frac{D_{os}}{2} \right)^2 - \left( \frac{D_{is}}{2} + d_{0s} + d_{2s} + H_{ss} \right)^2 \right] L'_{\gamma_{\text{iron}}} \Gamma_{(Hz,B)}
\]  \hspace{1cm} (2.2.2.6)

Finally, the equations 2.2.2.5 and 2.2.2.6 summarize the above mentioned by including the material volume, its material density \( \gamma_{\text{iron}} \) and its losses due to flux density saturation \( \Gamma_{(Hz,B)} \) in order to obtain iron losses in the stator.

Similarly, the equations 2.2.2.7 and 2.2.2.8 are employed for the rotor even though careful attention must be paid to the fact that the frequency in the rotor is proportional to slip, for which the material curves tend to be those at lower frequency. To illustrate this, let us remember that an Induction Motor is operated at slips between the breakdown torque and nearly synchronous speed by the variable frequency converter. In that region, therefore, the rotor slip frequency is about 1 to 3Hz for NEMA Designs B when started at 60Hz across-the-line [29]. As a consequence, the rotor core losses are smaller than those at the stator regardless of having a magnitude of flux density in the rotor core generally higher than the one presents in the stator iron.
Chapter 2. *Design and Field Oriented Current Control of Induction Motor*

\[
P_{\text{iron(tr)}} = S_2 \left[ W_{tr} H_{sr} + \left( \tau_r - \left( \frac{W_{1r} + b_{0r}}{2} \right) \right) d_{1r} + d_{0r} (\tau_r - b_{0r}) \right] L_{\gamma_{\text{iron}}} \Gamma_{(Hz,B)} \tag{2.2.2.7}
\]

\[
P_{\text{iron(cr)}} = \pi \left[ \left( \frac{D_{or}}{2} - d_{0r} - d_{1r} - H_{sr} \right)^2 - \left( \frac{D_{in}}{2} \right)^2 \right] L_{\gamma_{\text{iron}}} \Gamma_{(Hz,B)} \tag{2.2.2.8}
\]

Accounting for load conditions along with material properties at different frequencies the efficiency can be estimated, where variations in any parameter can lead to changes in efficiency, for instance, the maximum efficiency does not necessarily occur neither at full load nor maximum speed [48].

### 2.3 Transient Response

The steady state response of the Inverter-driven Induction Motor proposed was addressed in previous chapter. This section shows the transient response which is worthy in analyzing the motor performance when a sudden change in the input parameters is applied to the machine, as it happens in being either connected or disconnected from the supply, or when a short-circuit appears at its terminals. Nonetheless, transient analysis is also useful, and it is the biggest interest for this work, to study the electromagnetic and mechanical transient when they are fed by inverters.
Through the use of equivalent circuit (figure 2.1) and phasor diagram of the Induction Machine (figure 2.8) is derived the equation for voltage, current and flux linkage in the stator and the rotor.

\[
\begin{align*}
v_{abc}^{s} &= i_{s}^{abc} R_{s} + \frac{d\lambda_{abc}^{s}}{dt} \\
v_{abc}^{r} &= i_{r}^{abc} R_{r}^{'} + \frac{d\lambda_{abc}^{r}}{dt} \\
\lambda_{abc}^{s} &= L_{ss} i_{s}^{abc} + L_{sr} i_{r}^{abc} \\
\lambda_{abc}^{r} &= L_{rs} i_{s}^{abc} + L_{rr} i_{r}^{abc}
\end{align*}
\]

(2.3.0.9)

The flux linkages are assumed to be linear function of inductances and currents, where the self-inductances in the stator are defined by 2.3.0.10 since they are independent.
of any motion.

\[
L_{ss}^{abc} = \begin{bmatrix}
L_{ls} + L_m & -\frac{1}{2}L_m & -\frac{1}{2}L_m \\
-\frac{1}{2}L_m & L_{ls} + L_m & -\frac{1}{2}L_m \\
-\frac{1}{2}L_m & -\frac{1}{2}L_m & L_{ls} + L_m
\end{bmatrix}
\]  

Likewise, an imaginary symmetrical winding is assumed in the rotor in order to define its self-inductances. Furthermore, a sinusoidally distributed winding is considered in both the stator and rotor in order to assume a displacement of 120 degrees among phases. Defining the inductance matrices \(L_{ss}^{abc}\) and \(L_{rr}^{abc}\) as follows.

\[
L_{rr}^{abc} = \begin{bmatrix}
L'_{lr} + L_m & -\frac{1}{2}L_m & -\frac{1}{2}L_m \\
-\frac{1}{2}L_m & L'_{lr} + L_m & -\frac{1}{2}L_m \\
-\frac{1}{2}L_m & -\frac{1}{2}L_m & L'_{lr} + L_m
\end{bmatrix}
\]

Whereas the self-inductance of each phase in the rotor and the stator as well as the mutual inductance between phases are independent of rotor motion, the mutual inductances among the stator phase windings with respect to the rotor phase windings must consider the rotor position at any point since the windings are in relative motion each other. So the magnetic coupling \(L_{sr}^{abc}\) and \(L_{rs}^{abc}\) are given by the rotor angle \(\theta_r\) with respect to the fixed stator axis.
The derivation of electromagnetic torque is detailed in [43, 47], where the magnetic force can be obtained through either energy or co-energy principles. The electromagnetic torque in rotating machines consists of energy or co-energy variation with respect to the motion, if a linear magnetic system is assumed, the torque will be proportional to the current as well as the variation of magnetic coupling with respect to the rotor motion. In summary, the equation 2.3.0.13 defines the electromagnetic torque for three-phase Induction Machines due to time varying mutual inductances.

\[ T_{e}^{abc} = - \frac{P_{1}}{2} L_{m} \left\{ i_{as} \left( i_{ar} - \frac{i_{br}}{2} - \frac{i_{cr}}{2} \right) + i_{bs} \left( i_{br} - \frac{i_{ar}}{2} - \frac{i_{cr}}{2} \right) \right. \\
\left. + i_{cs} \left( i_{cr} - \frac{i_{br}}{2} - \frac{i_{ar}}{2} \right) \right\} \sin \theta_{r} \\
+ \frac{\sqrt{3}}{2} \left\{ i_{as} (i_{br} - i_{cr}) + i_{bs} (i_{cr} - i_{ar}) + i_{cs} (i_{ar} - i_{br}) \right\} \cos \theta_{r} \] 

(2.3.0.13)

From now on a change of variable is introduced in the model in order to incorporate a set of fictitious winding that is not in relative motion. This approach finds for
the elimination of the variation in the mutual inductances \[42, 47\] through purely mathematical transformations.

The Induction Machine model is transferred to a new reference frame where the stator and rotor windings are converted to equivalent windings with no motion among them. Detailed bellow is the stationary reference frame for which the new rotor circuit must be stationary with respect to the new stator circuit. For this reference frame is evident that the stator circuit remains fixed but the rotor circuit is transformed by making use of \( \theta_r \).

\[
\begin{align*}
\bar{f}_{qd0}^s &= T_s f_{abc}^s \\
\bar{f}_{qd0}^r &= T_r f_{abc}^r
\end{align*}
\]  
(2.3.0.14)

The change towards DQ-reference frame from ABC-reference frame (2.3.0.14) is carried out by taking advantage of Clarke and Park’s transformation which allows definition of the transformation matrices for the stator \( T_s \) and the rotor \( T_r \).

\[
T_s = \frac{2}{3} \begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\
1 & 1 & 1 \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}, \quad
T_s^{-1} = \begin{bmatrix}
1 & 0 & 1 \\
-\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \\
\frac{1}{2} & \frac{\sqrt{3}}{2} & 1
\end{bmatrix}
\]  
(2.3.0.15)
Because the existence of its inverse, the transformation is bidirectional, and any
time-varying parameter in the ABC-reference frame might be turned into a constant
parameter in the DQ-reference frame and vice versa, such as voltages, currents, in-
ductances, flux linkages and torque.

By substituting the transformation matrices into the equations 2.3.0.14 for each pa-
rameter we are interested in, the DQ model in the stationary reference frame is finally
represented as follows;

\[
v_{qs} = R_s i_{qs} + \frac{d\lambda_{qs}}{dt}
\]
\[
v_{ds} = R_s i_{ds} + \frac{d\lambda_{ds}}{dt}
\]
\[
v_{0s} = R_s i_{0s} + \frac{d\lambda_{0s}}{dt}
\]

(2.3.0.18)
where the flux linkages aligned to the DQ axes do not depend any more on relative motion, unlike the original model. Likewise, the electromagnetic torque 2.3.0.13 is reduced to its equivalent 2.3.0.21.

\[
\begin{align*}
\lambda_{qs} &= \frac{3}{2}L_m (i_{qs} + i_{qr}) + L_{ls} i_{qs} \\
\lambda_{ds} &= \frac{3}{2}L_m (i_{ds} + i_{dr}) + L_{ts} i_{ds} \\
\lambda_{0s} &= L_{ls} i_{0s} \\
\lambda_{qr} &= \frac{3}{2}L_m (i_{qs} + i_{qr}) + L'_{lr} i_{qr} \\
\lambda_{dr} &= \frac{3}{2}L_m (i_{ds} + i_{dr}) + L'_{lr} i_{dr} \\
\lambda_{0r} &= L'_{lr} i_{0r}
\end{align*}
\] (2.3.0.20)

\[
T_e^{qd0} = \frac{3}{4}P_1 (\lambda_{qr} i_{dr} - \lambda_{dr} i_{qr})
\] (2.3.0.21)
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One of the main benefits of this transformation will be set forth later in the next section, which is the possibility to establish control strategies to independently command the fluxes and currents in the new reference frame.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Voltage Line-to-Line</td>
<td>480 Volt</td>
</tr>
<tr>
<td>Base Phase Current</td>
<td>84 Amp rms</td>
</tr>
<tr>
<td>Target Output Power</td>
<td>55 Kw</td>
</tr>
<tr>
<td>Stator Leakage Inductance</td>
<td>0.9734 mH</td>
</tr>
<tr>
<td>Stator Resistance</td>
<td>0.0623 Ω</td>
</tr>
<tr>
<td>Rotor Leakage Inductance</td>
<td>0.8865 mH</td>
</tr>
<tr>
<td>Rotor Resistance</td>
<td>0.0419 Ω</td>
</tr>
<tr>
<td>Magnetizing Inductance</td>
<td>24.6690 mH</td>
</tr>
<tr>
<td>Poles</td>
<td>4</td>
</tr>
<tr>
<td>Pole Pitch</td>
<td>9 slots</td>
</tr>
<tr>
<td>Coil Pitch</td>
<td>8 slots</td>
</tr>
<tr>
<td>Winding Factor</td>
<td>0.945</td>
</tr>
<tr>
<td>Turns per phase</td>
<td>84</td>
</tr>
<tr>
<td>Conductor per slot</td>
<td>14</td>
</tr>
<tr>
<td>Laminations</td>
<td>Silicon Steel M19 24Ga</td>
</tr>
<tr>
<td>Silicon Steel Density</td>
<td>7872 Kg/m³</td>
</tr>
<tr>
<td>Conductors, Bars and End Rings</td>
<td>Copper $\sigma_{cu}, \rho = 2.1\mu\Omega cm$</td>
</tr>
</tbody>
</table>

*Table 2.1: Induction Motor Parameters*

As was initially intended, the final part of this section will be dedicated to show the transient response for the Induction Motor for variable speed applications that has been detailed throughout this chapter and the previous one. To accomplish this, the list of parameters shown on table 2.1 was calculated by employing the methodology aforementioned; in addition, the values obtained by this approach were compared to those generated in modeling, analyzing and optimizing the motor through Finite
Element Method. It is worth mentioning that physical measurements have not been effectuated since the model has not been built by the moment this work has been submitted and published.

The transient response serves to assess the performance of electrical machines in being subjected to sudden changes in their input supply (figures 2.9, 2.10 and 2.11), are obtained from the Induction Motor proposed when it is to start directly connected to line under no-load conditions.

![Figure 2.9: Torque and Speed for Direct-to-Line Start for Different Volts/Hz Conditions](image)

In the figures are shown the behavior in the Induction Motor as it is started at a lower frequency than its rated value. For instance, the figure 2.9 demonstrates how the breakdown torque appears at lower speed, and then it moves along with the
frequency when increased, as it is expected. That behavior in varying the input voltage and frequency keeping constant its ratio, is the fundamental principle for variable speed applications in Induction Motors, and the starting torque therefore is not limited to the normal value generated by the motor; it can be as high as the breakdown value, instead [29].

Similarly, the currents in the stationary reference frame for stator and rotor are also presented when the Induction Motor is started at reduced voltage and frequency (11 Hz) as well as for rated conditions at 60 Hz. This demonstrates the feasibility of studying the motor performance by employing the DQ-Model as it could be done with the original reference frame.

![Figure 2.10: Stator Currents in Stationary Reference Frame for Different Volt/s/Hz Conditions](image)
2.4 Field Oriented Control

The Induction Machine is one of the most attractive and widely used electromechanical energy conversion devices. Extensive research has been done involving these machines becoming the Induction Machines as one of technologies more mature in the industry as far the electrical machines are concerned. This section will be focused upon the explanation of one control strategy for these sort of devices, which is broadly accepted and documented in [23, 38, 40, 42] in order to be applied to the Induction Motor model proposed.

Since the DQ-Model for Induction Machines has been introduced in the previous section, it will be then utilized for the derivation of Field Oriented Current Control.
strategy.

As observed in equation 2.3.0.21, the electromagnetic torque developed by an Induction Machine is proportional to the vectorial product between the rotor flux and the current. Hence, the magnitude is maximum when the flux and the current are orthogonal to one another.

The DQ-Model allows for decoupling the flux and the current into two perpendicular components so that they can be independently manageable. In orienting the rotor field towards one axis while the current is adjusted towards the other one, the motor torque can be maximized, in addition to be controlled by time-unvarying components.

Hence, under this strategy it is desired to orient the rotor flux toward D-axis, which is equivalent to saying that the Q component of the flux is equal to zero $\lambda_{qr} = 0$.

By eliminating rotor flux in the Q-axis in equations 2.3.0.20 and substituting for $i_{qr}$, the electromagnetic torque can be written in terms of flux linkage in D-axis and the stator current in Q-axis, as follows.

\[
\frac{3}{2} L_m i_{qs} + \left( \frac{3}{2} L_m + L'_{lr} \right) i_{qr} = 0
\]  
(2.4.0.22)

\[
\therefore \quad i_{qr} = \frac{-\frac{3}{2} L_m i_{qs}}{\frac{3}{2} L_m + L'_{lr}}
\]  
(2.4.0.23)
Evidently the relationship shown in 2.4.0.24 implies that the torque can be adjusted by means of maintaining the rotor flux constant in the D-axis as the stator current in Q-axis is varied, just to name one of its implications.

\[ T_e^* = -\frac{3}{4} P_1 \lambda_{dr} i_{qr} = \frac{3}{4} P_1 \frac{3}{2} \frac{L_m}{L_m + L'_r} \lambda_{dr} i_{qs} \]  
(2.4.0.24)

If the voltage equations 2.3.0.19 are rearranged by considering \( \lambda_{qr} \) equals zero and the rotor circuit is short-circuited, then the reference stator currents can be obtained as follows.

\[ R'_r i_{dr} + \frac{d\lambda_{dr}}{dt} = 0 \]

Where \( i_{dr} \) can be found from flux linkage equations:

\[ i_{dr} = \frac{\lambda_{dr} - \frac{3}{2} L_m i_{ds}}{\frac{3}{2} L_m + L'_r} \]

\[ R'_r \lambda_{dr} - \frac{3}{2} R'_r L_m i_{ds} + \left( \frac{3}{2} L_m + L'_r \right) \frac{d\lambda_{dr}}{dt} = 0 \]  
(2.4.0.25)
Ultimately, from equations 2.4.0.24 and 2.4.0.25 the reference is given by the following currents.

$$i^*_q = \frac{4}{3} P_1 \frac{\frac{3}{2} L_m + L'_{lr}}{3} \frac{T^*_e}{P_1 \lambda_{dr}}$$

$$i^*_{ds} = \frac{R'_{r} \lambda_{dr} + \left( \frac{3}{2} L_m + L'_{lr} \right) \frac{d\lambda_{dr}}{dt}}{\frac{3}{2} R'_{r} L_m}$$

Having defined the current controller, it is now necessary to establish the desired angle for field orientation. Direct field orientation control scheme mainly relies upon flux measurement in the air gap, which is somewhat problematic and inaccurate at low frequencies [38, 42]. On the contrary, the indirect scheme calculates the angle $\rho$ through the slip speed, as it is explained below; nonetheless, it is worth mentioning that this scheme is quite susceptible to variations in machine parameters.

In order to demonstrate this, it is assumed that flux rotor remains constant and it is oriented towards D-axis. As a result, it is also implied that there are no variations in $\lambda_{dr}$ and $\lambda_{qr}$ with respect to the time.

$$\lambda_{dr} = c \quad \therefore \quad \frac{d\lambda_{dr}}{dt} = 0$$

$$\lambda_{qr} = 0 \quad \therefore \quad \frac{\lambda_{qr}}{dt} = 0$$
Hence, we will obtain that $i_{dr} = 0$ since $R'_r i_{dr} = 0$ from 2.3.0.19, that yields

$$\lambda_{dr} = \frac{3}{2} L_m i_{ds} \quad (2.4.0.27)$$

Which shows nothing else but that the magnitude of rotor flux in the D-axis is proportional to the stator current in D-axis as well as can directly be controlled by this one. Furthermore, from Q-axis voltage 2.3.0.18 and current 2.4.0.23, the slip speed is defined by $\omega_2$ below.

$$v_{qr} = 0 = R'_r i_{qr} + \frac{d\lambda_{qr}}{dt} + \omega_2 \lambda_{dr}$$

$$\therefore$$

$$\omega_2 = -\frac{R'_r i_{qr}}{\lambda_{dr}} = \frac{R'_r}{\frac{3}{2} L_m + L'_{lr}} \frac{i_{qs}}{i_{ds}} \quad (2.4.0.28)$$

Finally, the angle between the stator a-phase axis and the rotating reference frame in the rotor is the summation of the rotor position and the displacement due to slip.

$$\theta_2 = \int \frac{R'_r}{\frac{3}{2} L_m + L'_{lr}} \frac{i_{qs}}{i_{ds}} dt$$

$$\rho = \theta_r + \theta_2$$

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Such as it was already mentioned, it is clear that the current controller as well as the angle estimation are quite dependent upon rotor parameters mainly, consequently, on-line parameter adaptive techniques are utilized to ensure the proper operation of Indirect Field Oriented scheme [42]. In spite of that, this strategy is widely preferred as control method, specially, as low-speed operations or control of position is required. The block diagram implemented by this work is presented in figure 2.12, and the simulations are detailed in next section.

![Diagram](image)

**Figure 2.12:** Indirect Field Oriented Current Control for Induction Motors

### 2.5 Simulation Results

The Induction Motor for variable speed application designed by Differential Evolution and set forth in chapter one, whose geometry and parameters were shown on tables 1.3 and 2.1, respectively, has been modeled and simulated under variable frequency conditions by making use of indirect Field Oriented Current Control.
After developing the model and the control strategy in MATLAB/Simulink, different load and speed conditions were recreated in order to evaluate the performance of the Induction Motor proposed. The figure 2.13 shows the voltage and the current required by the Induction Motor designed in order to handle efficiently the changes in the load, while variations in the speed are performed. It should be noted that the motor is operated in the four quadrants; that is to say, motoring and generating as well as accelerating and braking, being able to handle the load within nominal input supply even at starting at full-load.

Mostly the graphs are depicted in per unit system so that it is simpler the comparison to the rated values which can be seen on table 2.1. Additionally, the base values
are 1800rpm for speed, 300Nm for torque, 480Volt and 85Amp rms for line-to-line voltage and phase current, respectively.

Figure 2.14: Efficiency, Power Factor and Slip Under Different Speed and Load Conditions

Figure 2.14 presents the motor’s efficiency, which is calculated by taking into consideration the properties of Non-oriented electrical silicon steel M19 26Ga for the rotor and stator stacks, similarly, casting and wire copper for the conductors in the rotor and stator, respectively. It is worth mentioning that the losses in the rotor iron were obtained by using the curve at 50Hz for lacking of curves at lower frequencies (laminations manufacturers do not usually account for measuring the properties at
low frequencies); therefore, the efficiency should be somewhat higher than the one shown on figure. The results demonstrate that the rotor is subjected to low frequencies since they are proportional to the slip, also presented on figure, and being the slip nearly below 3% overall the operation, this can be inferred. Besides, one of the most important aims in designing the Induction Motor was to keep the slip as small as possible in order to make the motor operate in the pseudo-linear region between the breakdown torque and the no-load torque, both points close to the synchronous speed.

![Diagram](image)

**Figure 2.15:** Indirect Field Oriented Control Response Under Different Speed and Load Conditions
The values of efficiency and power factor achieved for the Induction Motor modeled are quite similar to those offered by renowned motor manufacturers [49–52] in comparison to motors at the same rated values. However, there is a clear advantage in the Induction Motor proposed by this work and previously stated on chapter one, which is the optimized or reduced size that can be understood as a decrement in the amount of material required to build the motor, finally leading to cost reduction.

As has been claimed from the beginning, the Induction Motor has been designed to be driven by an inverter, and the simulation of Vector Control through Indirect Field Oriented was the strategy chosen to assess the performance of the model proposed. On figure 2.15 the objective of the control scheme is clearly seen, and the prevailing parameters set forth on section 2.4 confirm that the rotor flux $\lambda_r$ is oriented toward D-axis since its component in the Q-axis remains fixed around zero. Furthermore, regardless of the nature of changing load, the stator current in D-axis $i_{ds}$ is maintained constant in order to avoid variations in the rotor flux $\lambda_{dr}$ since they are directly related, while the electromagnetic torque is independently controlled by the...
stator current in the Q-axis $i_{qs}$. As a result, the smooth transition among different conditions demonstrates the feasibility of the motor designed can be efficiently driven by changing Volt/Hz control schemes, as expected.

When the full load is applied, the flux density in the motor, as well as the current density in conductors, is presented on figure 2.16. Through Finite Element Method the distribution in the flux density reveals that the saturation only occurs on small portions of stator and rotor teeth; however, their values are considered on the acceptable range. On the other hand, the current density will depend upon the cooling method, and thermal analyses are recommended for such purpose since high values can lead to excessive copper losses overheating the motor and affecting the electromagnetic performance [28]. Nonetheless, the values reported are within the tolerance for totally-enclosed-fan-cooled TEFC machines [26].

2.6 Discussion

Over this chapter, exhaustive approaches were described to go from the geometrical model to the electromagnetic one in order to count on equivalent parameters that have been posteriorly used to conduct performance analyses.

First at all, the resultant parameters demonstrated that the geometrical design leaded by Differential Evolution initially, and then optimized by the Finite Element Method, contains the transient and steady state responses desired in an Induction Motor for variable speed applications. Among the findings are that the rotor impedance is small
enough, so that the breakdown torque occurs at low slip without affecting excessively its peak value. Furthermore, the losses and power factor are found comparable to other motors in the market, despite the motor designed presents smaller dimensions. Hence, the torque density optimization carried out has been also proved effective.

Secondly, one of the main goals of this work has been to confirm that the motor designed is able to perform well for different speed and load conditions; for that reason, the DQ model and Indirect Field Oriented Current Control was derived and simulated. The outcomes evidence that the motor is capable of starting under low frequency at full-load with no significant increase of starting current; moreover, the amount of reactive power consumed is within the acceptance limits.

After analyzing its transient response and performance under speed variations, it can be certainly concluded that the evolutionary design aided by DE and optimized by FEM behaves as expected in being driven by an inverter, this work particularly focused upon testing Indirect Field Oriented Current Control strategy.

It is important to emphasize that the skin effect was reduced through some assumptions taken over design stage, and thus it has been neglected in converting the model from geometrical to electromagnetic; however, at higher frequencies these assumptions are no longer valid. Likewise, saturations and parameters non-linearities have not been taken into account in the simulation. Otherwise, its effects were studied by the Finite Element Method.
The thermal performance is subject to future analysis in spite of counting on results within reasonable limits reported in literature.
Chapter 3 : Design and Vector Current Control of Synchronous Reluctance Motor

3.1 Introduction

The advances in power electronics devices have opened the possibility of counting on Synchronous Reluctance Motors (SRM) in applications where variations of speed are required, in addition to the necessity of having motors with low-cost and robust construction. Evidently, for many years the Induction Machines have successfully supplied this demand, and they have become in undeniable leaders in the field. On the other hand, there is a certain amount of benefits associated to Synchronous Reluctance Motors that allow them to overcome problems—for example, in applications with large torque and low speed where Induction Machines can present high losses with consequently thermal deformation problems. Additionally, the Cage-less Synchronous Reluctance Motors might mean a reduction in the manufacturing cost and an improvement in motor robustness.
The Synchronous Reluctance Machine employs the principle of reluctance torque which is the tendency of the rotor to line up along the minimum reluctance path. This imaginary path is synchronously rotating according to the air gap flux density produced by the distributed winding in the stator. Therefore, a SRM takes the principle of energy conversion through reluctance torque from Variable Reluctance Machines, popularly known as "Switched Reluctance" Motors, and merges it with the idea of sinusoidal flux density in the air gap from either Induction or Synchronous Machines.

In order to provide the fundamentals of Synchronous Reluctance Motor design, research has been conducted by several authors [53–58], where they explore different approaches to achieving high values of inductance ratio and inductance difference by means of changing specific parameters in the geometry and also proposing alternative configurations. In this chapter, those principles are validated and extended in designing and modeling an SRM that is to be compared to the Induction Motor in next chapter.

The motor proposed by this chapter is subject to an extensive analysis that includes core loss and cross magnetization effects, Finite Element Method as well as analytic models based upon equivalent circuit are presented. Finally, the model is simulated by using Vector Control and its performance is analyzed.

The results of this chapter will provide an equivalent motor to the Induction Motor
aforementioned, which will be compared in the next chapter. Due to the increasing interest in comparing these two machines, this work offers a useful perspective by driving them both through Vector Control and studying them from an energy conversion viewpoint.

3.2 Design Considerations

In order to perform a comparison as fairly as possible, the same stator design is utilized by both machines— the Induction Motor described in the previous chapters and the Synchronous Reluctance Motor that will be set forth below. Hence, over this chapter the attention will be only centered upon the rotor design.

![Figure 3.1: Rotor of Synchronous Reluctance Motor](image)

As far as Synchronous Reluctance Motor (SRM) is concerned, the most important parameters from a design and control viewpoint are both the ratio and the difference between the inductances in the D and Q axes, since they define machine characteristic
Chapter 3. Design and Vector Current Control of Synchronous Reluctance Motor

such as power factor, torque per ampere and constant-power speed range [59]. As will
be presented in the expressions below, the torque is dependent upon the difference
between inductances; nevertheless, special attention is given to the inductance ratio
$L_d/L_q$ due to its influence on power factor as well as the allowable tolerance on
parameter estimation for accurate motor control [53].

3.2.1 Rotor Design

Several approaches have been proposed in designing the rotor in order to attempt
obtaining the maximum saliency ratio achievable. Among the structures are: the
salient pole, the axially laminated, the transversally laminated and recently the solid-
rotor [53–57, 59–61]. A salient pole rotor is similar to the rotor used in Variable
Reluctance Machines (VRM) and made by removing iron from rotor in the transversal
region [60], in consequence, SRM has the same problems associated with those VRMs,
such as torque pulsations and noise.

The largest inductances ratios have been reported by Axially Laminated Anisotropic
(ALA) rotors [62]. In those, the laminations are stacked together through pole holders
connected and oriented towards the motor shaft. Despite its attractive ratio, the
disadvantages are related to higher amount of iron losses in the rotor due to eddy
currents induced by harmonic fields [63]; besides this, its manufacturing process in
practice is more challenging, which evidently might increase the costs.
On the other hand, laminations in the Transversally Laminated Anisotropic (TLA) rotors are punched and stacked conventionally where a multi-barriers configuration allows for achieving a modest inductance ratio capable of competing in terms of performance with Induction Motors.

A solid structure in the rotor has been also studied where the magnetic and non-magnetic material are bonded together in order to create a solidified composition that leads to stronger structural characteristics appropriated for high-speed applications where centrifugal forces are extremely high; however, this configuration presents no constraint to the flow of eddy currents occasioning the necessity of special attention in rotor iron losses [61].

Further details about every rotor structure can be found in literature referenced below, although it is the author’s opinion that the most feasible configuration to be used in the industry in order to provide an alternative in applications leaded by Induction Motors is the transversally laminated anisotropic. Indispensable prerequisites are not only to achieve a geometry able to offer high inductance ratio under saturation conditions, but the geometry must also be viable for building without raising manufacturing costs.

First, to have an idea of torque capability in SRM and its dependence upon DQ inductances, let us introduce the basic torque expression 3.2.1.1 and the figure 3.2, where the proportionality of inductance difference and current angle in the electromechanical energy conversion is clearly seen.
Chapter 3. Design and Vector Current Control of Synchronous Reluctance Motor

\[ T_{eq} = \frac{3}{4} P_1 (L_d - L_q) \frac{I_s^2 \sin 2\varphi}{2} \]  
(3.2.1.1)

When the iron losses are neglected, the torque production is presented on figure 3.2 for the SRM that will be discussed in this chapter.

![Figure 3.2: Electromagnetic Torque Variations with Respect to the Current Angle Neglecting Iron Losses](image)

The inductance ratio and difference are increased in maximizing the one in the D-axis, whereas the one in the Q-axis is kept as low as possible; in [53] was showed the highest realizable values, which consist of having a rotor made of solid magnetic material for a high \( L_d \) and if it is replaced by non-magnetic material a very low \( L_q \) is reached.
In general, the designer must make sure to provide a guide for flux lines to maximize the flow through D axis and a set of barriers to oppose the flow along with Q axis. This can be done by orienting nonmagnetic barriers perpendicularly to Q axis, and paths of magnetic material converging to D axis. For those reasons, authors in [53, 54] define a group of parameters that can be altered in designing a Synchronous Reluctance Motor, whose underlying purpose is increase the $L_d$ magnitude with respect to $L_q$. Those factors are the number of barriers punched in the lamination, the barrier width with respect to the lamination width and the stator slot configuration (number/design).

Likewise, Matsuo and Lipo in [54] and Staton et al. in [53] stated that the best results are accomplished with multi-barrier configurations even though a number higher than 10 does not present much benefit. Furthermore, the insulation/lamination width ratio should be between 0.5 and 1, out of that range the performance is degraded. Finally,
stator designs with 24 and 36 slots were analyzed, and the comparison seems to favor geometries with larger numbers of slots.

Since the same stator is used for the Induction Motor described in previous chapters, the slot configuration remained unaltered. Nevertheless, by using simulations based upon the Finite Element Method the maximum torque and ripple torque were plotted on figure 3.4 in order to observe the variations in the electromagnetic model as the number of barriers and its width are subject of modification.

Over this stage of design process, the stator current is fixed so that maximum torque and ripple torque are compared, and as can be seen, there are no patterns when the parameters are changed, which leads to select a geometry based upon a finite number of simulations within a non-linear space of solutions.

**Figure 3.4:** Torque and Torque Ripple Variability with Respect to the Rotor Geometry
Chapter 3. *Design and Vector Current Control of Synchronous Reluctance Motor*

The amount of harmonics and saturation level due to stator teeth effect in the ribs is of vital incidence in the performance of a Synchronous Reluctance Motor, which as a consequence, degrades the torque capability because its elevated ripple torque. By taking this into consideration, a saliency ratio of 0.62 and 9 barriers are selected, besides the importance of accounting for a realizable design due to machining tolerance errors by the moment it is built. With a rotor diameter of 233.8mm, 9 insulation barriers of 6.2mm wide and 10.0mm of iron width, a satisfactory inductance ratio under either unsaturated or saturated conditions is fulfilled, as depicted in figure 3.5.

![Figure 3.5: Inductances Saturation Due to Stator Current](image)

This combination of insulation and iron laminations provides a range for the inductance ratio between 5 and 10, while the inductance difference is between $20\,mH$ and $70\,mH$. It is worth noticing that the current angle is maintained fixed in each measurement in order to avoid errors due to effects of discrepancy in cross magnetizing current since the inductance in D axis is more sensible to suffer saturation as the current in Q axis is increased.
In Synchronous Reluctance Motors, the saturation shown on figure 3.5 and the cross magnetization have several effects on performance. One of them, which will be explained below, is that the maximum torque per ampere tends to occur at current angles greater than 45 degrees unlike the theoretical appreciation given by expression 3.2.1.1 and figure 3.2. Additionally, the power factor, which is the biggest concern in designing those machines, is degraded with decreasing the inductance ratio due to the saturation on D axis as consequence of stator current increment.

### 3.3 Parameters Computation

In general, a Synchronous Reluctance Motor can be modeled by following the expression given by 3.3.0.2, where the inductances defined in the \( abc \) reference frame are dependent upon rotor position, as shown on 3.3.0.3 and 3.3.0.4.

\[
v_s^{abc} = i_s^{abc} R_s + L(\theta_r)^{abc} \frac{di_s^{abc}}{dt}
\]  

(3.3.0.2)

\[
L(\theta_r)^{abc} = \begin{bmatrix}
L_{ls} + L_{m0} - L_{mR} \cos (2 \theta_r) & -\frac{L_{m0}}{2} - L_{mR} \cos \left( \frac{2 \theta_r - 2 \pi}{3} \right) & -\frac{L_{m0}}{2} - L_{mR} \cos \left( \frac{2 \theta_r + 2 \pi}{3} \right) \\
-\frac{L_{m0}}{2} - L_{mR} \cos \left( \frac{2 \theta_r - 2 \pi}{3} \right) & L_{ls} + L_{m0} - L_{mR} \cos \left( \frac{2 \theta_r - 4 \pi}{3} \right) & -\frac{L_{m0}}{2} - L_{mR} \cos \left( \frac{2 \theta_r + 2 \pi}{3} \right) \\
-\frac{1}{2} L_{m0} - L_{mR} \cos \left( \frac{2 \theta_r + 2 \pi}{3} \right) & -\frac{L_{m0}}{2} - L_{mR} \cos \left( 2 \theta_r + 2 \pi \right) & L_{ls} + L_{m0} - L_{mR} \cos \left( \frac{2 \theta_r + 4 \pi}{3} \right)
\end{bmatrix}
\]  

(3.3.0.3)
\[
L_{m0} = \frac{1}{3}(L_{mq} + L_{md}) \\
L_{mR} = \frac{1}{3}(L_{md} - L_{mq})
\]  

(3.3.0.4)

The inductance matrix is expressed in terms of minimum inductance factor \(L_{m0}\) that is independent upon motion and the term \(L_{mR} \cos 2\theta_r\) which represents the reluctance variation with respect to the rotor mechanical position. Nonetheless, both are conveniently defined in terms of magnetizing inductances in DQ axes settled on figure 3.3.

\[
T_{eabc} = \frac{P}{2} \frac{\partial W}{\partial \theta_r} = \frac{P}{4} [i_{as} \ i_{bs} \ i_{cs}] \begin{bmatrix}
2L_{mR} \sin (2\theta_r) & 2L_{mR} \sin \left(2\theta_r - \frac{2\pi}{3}\right) & 2L_{mR} \sin \left(2\theta_r + \frac{2\pi}{3}\right) \\
2L_{mR} \sin \left(2\theta_r - \frac{2\pi}{3}\right) & 2L_{mR} \sin \left(2\theta_r - \frac{4\pi}{3}\right) & 2L_{mR} \sin (2\theta_r) \\
2L_{mR} \sin \left(2\theta_r + \frac{2\pi}{3}\right) & 2L_{mR} \sin \left(2\theta_r + \frac{4\pi}{3}\right) & 2L_{mR} \sin (2\theta_r)
\end{bmatrix} \begin{bmatrix}
i_{as} \\
i_{bs} \\
i_{cs}
\end{bmatrix}
\]  

(3.3.0.5)

As a result, the derivation of torque electromagnetic shows the reluctance effect occasioned by the reluctance difference among D and Q axes, in addition to the proportionality to stator current applied in every rotor position.
Several simplifications initially assumed yield the torque expression 3.3.0.6, which provide a general idea of torque production in a Synchronous Reluctance Motor. First attempts at modeling the performance were made by means of introducing a simple equivalent circuit in DQ reference frame [53, 54, 57–59, 64–67] and the phasor diagram depicted on figure 3.6. Posteriorly, the iron losses and cross magnetization have been reason of researching and discussion owing to their important incidence in the behavior and control of Synchronous Reluctance Motors [61, 68–73].

\[
T_{e}^{abc} = \frac{P_1}{6} (L_{md} - L_{mq}) \left[ \frac{\sqrt{3}}{2} \left( i_{bs}^2 - i_{cs}^2 - 2i_{as}i_{bs} + 2i_{as}i_{cs} \right) \cos 2\theta_r 
+ \left[ i_{as}^2 - \frac{i_{bs}^2}{2} - \frac{i_{cs}^2}{2} - i_{as}i_{bs} - i_{as}i_{cs} + 2i_{bs}i_{cs} \right] \sin 2\theta_r \right]
\] (3.3.0.6)

Therefore, the equivalent circuit on figure 3.7 is utilized and the expressions in the
DQ reference frame are introduced in 3.3.0.7. If the aforementioned effects are con-
sidered, it is clear that the whole DQ current injected into the stator is no longer
producing torque; hence, the relationships have to be rearranged in function of DQ
magnetizing components. For instance, it is explicitly divided the direct and quadra-
ture inductances into the stator leakage component and the magnetizing component,
as is depicted in equation 3.3.0.10, in order to show how the DQ flux linkage (equation
3.3.0.11) is affected independently by stator and magnetizing currents, respectively.

\[ v_{qs} = R_s i_{qs} + L_{ls} \frac{di_{qs}}{dt} + \omega_r \lambda_{md} + \frac{d\lambda_{mq}}{dt} = R_s i_{qs} + L_{ls} \frac{di_{qs}}{dt} + R_m (i_{qs} - i_{mq}) \]  
\[ v_{ds} = R_s i_{ds} + L_{ls} \frac{di_{ds}}{dt} - \omega_r \lambda_{mq} + \frac{d\lambda_{md}}{dt} = R_s i_{ds} + L_{ls} \frac{di_{ds}}{dt} + R_m (i_{ds} - i_{md}) \]  

Figure 3.7: Equivalent Circuit of Synchronous Reluctance Motor a) Q axis b) D axis

In other words, when Synchronous Reluctance Motors are either simulated by using
Finite Element Methods or actual motors are subject of experimentation, the results
evidencing a high non-linearity which are very distant of results obtained if core losses
and cross coupling effect are neglected.
Chapter 3. Design and Vector Current Control of Synchronous Reluctance Motor

By operating with the resultant equations of analyzing the equivalent circuit 3.7, voltage in DQ axes can be expressed in two ways: either by the speed voltage or by the core losses 3.3.0.7.

\[
R_m(i_{qs} - i_{mq}) = \omega_r \lambda_{md} + \frac{d\lambda_{mq}}{dt} \tag{3.3.0.8}
\]

\[
R_m(i_{ds} - i_{md}) = -\omega_r \lambda_{mq} + \frac{d\lambda_{md}}{dt}
\]

\[
\lambda_{mq} = \int R_m(i_{qs} - i_{mq}) - \omega_r L_{md} i_{md} \, dt \tag{3.3.0.9}
\]

\[
\lambda_{md} = \int R_m(i_{ds} - i_{md}) + \omega_r L_{mq} i_{mq} \, dt
\]

When those two equations (3.3.0.7) are algebraically added, the magnetizing flux linkage across the air gap in DQ axis (3.3.0.9) is obtained and the influence of core losses and cross coupling is clearer. In [70], the cross magnetization effect in the inductances \( L_d \) and \( L_q \) is studied by means of Finite Element Method and actual measurement; the authors confirm the decrement of \( L_d \) occasioned by current \( i_q \) and the same effect but in lower proportion in \( L_q \) due to \( i_d \), besides the largest changes are attached to high current ratio \( i_d/i_q \). Thus, it is more appreciable the incidence on both inductances at a lower current.
\[
L_q = L_{ls} + L_{mq} \\
L_d = L_{ls} + L_{md}
\] (3.3.0.10)

However, at a higher current level the reduction in inductance values also appears, but in this case it is mainly due to iron saturation. Their results state that cross magnetization effect can lead to a reduction of up to 20% in the torque production and 6% in power factor.

\[
\lambda_q = L_{ls}i_{qs} + L_{mq}i_{mq} \\
\lambda_d = L_{ls}i_{ds} + L_{md}i_{md}
\] (3.3.0.11)

The simpler explanation for those reductions is the consequent degradation in the inductance ratio and difference, but further details could be found if the torque expression is rearranged in terms of magnetizing inductances and currents as described by equation 3.3.0.12, which could be considered as real torque production parameters.

\[
T_e = \frac{3}{4} P_1 \left( \lambda_{ds}i_{qs} - \lambda_{qs}i_{ds} \right) = \frac{3}{4} P_1 \left( L_{md}i_{md}i_{qs} - L_{mq}i_{mq}i_{ds} \right)
\] (3.3.0.12)

Now, let us make allowance for losses by including them into the torque equation.
In consequence, the torque expression derived in 3.3.0.13 clearly shows all those effects above mentioned: first, the core losses are taken into consideration through the term $i_{fe}$, the difference $(\lambda_{md} - \lambda_{mq})$ represents the cross coupling effect set forth by 3.3.0.9, and finally, the current angle where the maximum torque is expected.

The torque angle will be discussed again below; however, up to this point the torque expression permits the realization that the maximum torque occurs in the vicinity of magnetizing current angle $\epsilon$ equals to 45 degrees with respect to the D-axis. Therefore, if it is desired to relate it to the stator current angle, let us recall the phasor diagram 3.6, where it is seen that the magnetizing current angle is nearly 45 degrees as long as the stator current angle is bigger than that, and the dependence upon those angle is determined by cross magnetization and iron losses.

### 3.3.1 Inductances

As it has been discussed so far, the relationship between the inductances sets the behavior of a Synchronous Reluctance Machine; hence, the better the accuracy with those parameters are estimated, the more efficient will be the control strategy adopted.
The flux linkage in the air gap can be calculated by integrating the normal component of flux density over the air gap surface, but by making the assumption of magnetic flux density uniformity across the surface, it can be approximated as shown on equation 3.3.1.1.

\[
\lambda_m = N_s \int_B \mathbf{B} \cdot dS \approx \frac{2}{\pi} B_{g1} \tau_p L N_s K_{W1}
\]

_Figure 3.8: a) Air Gap Flux Density and b) Flux Linkages Variations Under Different Levels of Excitation_

Since the flux density in the air gap is flattened by the saturation effect, the fast fourier transform FFT allows for counting on the fundamental component so that magnetizing flux linkage is estimated as it is plotted on figure 3.8. The resultant flux linkage presents a slight displacement as the stator current is varied, although the ratio \( i_d/i_q \) remains constant over all the experiment.
The slight increment in the magnetizing angle $\delta$ while increasing the stator current is owing to a larger amount of current relatively ($I_s/I_m$) that is going to produce torque, in consequence, the efficient raises up to certain point that will be analyzed later.

If the stator leakage inductance is neglected in the meantime, and the assumption of being practically unaltered because no variations are applied to stator configuration leads to the conclusion that the inductances $L_d$ and $L_q$ can be straightforwardly obtained by means of expressions 3.3.1.2 and 3.3.0.10.

\[
\begin{align*}
L_{mq} &= \frac{\lambda_m \sin \delta}{i_s \sin \varphi} \\
L_{md} &= \frac{\lambda_m \cos \delta}{i_s \cos \varphi}
\end{align*}
\]  
(3.3.1.2)

Nevertheless, it is worth mentioning that the stator leakage inductance has minimum effect in the torque due to its cancellation in the inductances difference. On the contrary, from power factor and control point of view, this inductance significantly affects the ratio owing to its order of magnitude in being compared to $L_d$ and $L_q$ magnitudes; that is to say, the inductance ratio decreases considerably for high values of stator leakage inductance since it will be modified in bigger proportion to the value of $L_q$ than the one corresponds to $L_d$. 

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3.3.2 Power Factor

As has been already mentioned, the power factor is one of the parameters to which special attention has been drawn by researchers and manufacturers since modest values are achieved by Synchronous Reluctance Machines when they are not aided by permanent magnets. Despite this deficiency, the high efficiency in those machines still allows that the comparison to Induction Machines is up for discussion.

In order to keep expressing the performance parameters of Synchronous Reluctance Machines in terms of magnetizing components, let us introduce the internal power factor, which is the angle $\theta_i$ between the induced EMF in the air gap and the magnetizing current. It is closely linked to the real power factor and the proximity of stator current angle with magnetizing current angle is defined by the core losses, so the higher the efficiency, the closer the power factor to the internal power factor in the air gap.

Let us define the internal power factor as $\cos \theta_i = \cos(\pi/2 - \epsilon + \delta)$, where $\epsilon$ is the magnetizing current angle and $\delta$ is the magnetizing flux linkage angle, both with respect to the D axis. Therefore, the internal power factor can be written as follows.

$$\cos \theta_i = \frac{\tan \epsilon - \tan \delta}{1 + \tan \epsilon \tan \delta} \sqrt{1 + \left(\frac{\tan \delta - \tan \epsilon}{1 + \tan \epsilon \tan \delta}\right)^2} \quad (3.3.2.1)$$
If the expression 3.3.2.1 is rearranged and the magnetizing components are included into the equation, furthermore, for sake of simplicity the cross coupling effects are let out of the final result, 3.3.2.2.

\[
\cos \theta_i = \frac{\frac{L_{md}}{L_{mq}} - 1}{\sqrt{\left(\frac{L_{md}}{L_{mq}}\right)^2 \left(\frac{1}{\sin^2 \epsilon}\right) + \left(\frac{1}{\cos^2 \epsilon}\right)}}
\]  

(3.3.2.2)

Notice that not only the inductance ratio is determinant factor in the internal power factor calculation, but the magnetizing current ratio also is able to change the amount of reactive power consumed by the Synchronous Reluctance Machine.

By using the expression presented on 3.3.2.2 the curves on 3.9 are graphed, and describe how the power factor is enhanced at higher inductance ratios as it is expected; likewise, it is interesting to appreciate that a magnetizing current angle controlled to
remain approximately in the vicinity of 70 degree permits the machine to operate at its maximum internal power factor.

Consequently, this performance could be displaced toward the power factor at the machine terminals, where some implications have to be taken into account—for example, the inductance ratio is now dependent upon $L_d$ and $L_q$; low value of stator leakage inductance is therefore preferred to ensure better saliency since its order of magnitude affects mostly the Q axis inductance. On the other hand, the magnetizing current angle can be directly related to the DQ current as long as the core losses are small enough, which means that the stator current angle is slightly bigger than the magnetizing one; hence, by controlling the stator current angle, a desired magnetizing current angle can be fulfilled.

### 3.3.3 Losses

One of the reason for which researchers and manufacturers have recently shown interest in the Synchronous Reluctance Motors is the absence of rotor slip losses, besides advances in the implementation of control strategies that allow for providing better tangible solutions for problems historically faced by Induction Motors. For example, at low speed, the losses might be low enough to relieve the need for an external fan [59].
Like in other machines, the losses in Synchronous Reluctance Motors are classified in copper losses, iron losses, stray load losses and mechanical losses. As was aforementioned, the analysis of mechanical losses is beyond the scope of this work and further details can be found in [45].

In [56], the author proposes the expressions 3.3.3.1 and 3.3.3.2 - 3.3.3.3 for the calculation of copper and core losses in the stator, respectively. Since the stator remained unaltered over this work, a similar approach as presented for Induction Motors in the previous chapter can be conducted to estimate the stator losses by accounting for the material properties depicted on 2.7.

\[
P_{cu,s} = 3R_s (I_d^2 + I_q^2) \quad (3.3.3.1)
\]

\[
P_{iron(ts)} = S_1 \left[ W_{ts} H_{ss} + \left( \tau_s - \left( \frac{W_{1s} + b_{0s}}{2} \right) \right) d_{2s} + d_{0s} \left( \tau_s - b_{0s} \right) \right] L_{\gamma_{iron}} \Gamma_{(Hz,B)} \quad (3.3.3.2)
\]

\[
P_{iron(cs)} = \pi \left[ \left( \frac{D_{os}}{2} \right)^2 - \left( \frac{D_{ts}}{2} + d_{0s} + d_{2s} + H_{ss} \right)^2 \right] L_{\gamma_{iron}} \Gamma_{(Hz,B)} \quad (3.3.3.3)
\]

In generally, the complex rotor geometry for pole configuration bigger than two hinders the designer from estimating the rotor iron losses by means of using analytical methods with acceptable accuracy even though several attempts have been reported.
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by authors in [69, 73, 74]. For the Synchronous Reluctance Motor proposed by this work, the core losses were measured by utilizing Finite Element Method. The graph 3.10 shows the total core losses as the stator current is increased and the rotor is spinning at 1800rpm.

![Figure 3.10: Core Losses as Function of Stator Current at 60Hz](image)

If the stator iron losses are subtracted from those results approximating them through the approach above mentioned, the rotor iron losses are obtained which are in a range from 16% to 20% of total core losses for the Synchronous Reluctance Motor proposed.

Finally, both core losses are extremely smaller in being compared to the copper losses; in addition, as the speed increases, the flux linkage decrease is inversely proportional to the speed. For this reason, the core losses tend to remain at their value of base speed for the entire constant power region [56].
3.4 Vector Control

Generally, the control of a Synchronous Reluctance Motor is considered simpler than its equivalent in an Induction Motor. In addition, its inertia and rotor weight are also slightly lower, which can contribute significantly to speed control and time to reverse speed [59].

The close loop control of Synchronous Reluctance Motors can be implemented by means of several strategies and usually those techniques are referred as Flux Oriented or Field Oriented Control since they make use of Park and Clark’s transformation in decoupling the components. However, in those Vector Control strategies there is no flux orientation [40]. The control strategies are mainly classified as:

- Constant D axis current control
- Fast torque response control
- Maximum torque per ampere control
- Maximum power factor control

At this point it is worth mentioning that the strategy of maximum power factor control has been chosen to be implemented in this work, since the final goal is the comparison between the Induction Motor and the Synchronous Reluctance Motor designed from input rating point of view, so this strategy is the most suitable to
Chapter 3. Design and Vector Current Control of Synchronous Reluctance Motor

enhance the power factor deficiencies in the motor proposed. For further details in the other strategies above mentioned is suggested the following references [40, 56, 72].

In order to maximize the power factor, the denominator of 3.3.2.2 has to be minimized, so taking its derivative with respect to the magnetizing current angle $\epsilon$ the point of maximum power factor is found.

$$\tan \epsilon = \sqrt{\frac{L_{md}}{L_{mq}}} \quad (3.4.0.4)$$

Therefore, the maximum internal power factor is obtained in substituting 3.4.0.4 into 3.3.2.2 leading to the following expression.

$$\cos \theta_{i,max} = \frac{L_{md}}{L_{mq}} - 1 \quad (3.4.0.5)$$

As a result, the possibility of adjusting the power factor performance is evident not only through design parameters such as DQ inductances, but it is also feasible by means of a control strategy capable of managing the magnetizing current angle.

3.4.1 Current Angle

As for Direct Torque Control (DTC), the core losses influence on torque production should be accounted for; nevertheless, a more simplified Synchronous Reluctance
Motor model can be used for speed control.

In other words, by neglecting the core losses and cross coupling effect, the magnetizing current angle $\epsilon$ tend to be equals to the stator current angle $\varphi$ and the point of maximum power factor can be expressed as follows.

$$\tan \varphi = \sqrt{\frac{L_d}{L_q}} \quad (3.4.1.1)$$

In decoupling the stator current into the components in D and Q axes, the angle $\varphi$ can be better adjusted. Likewise, a trade-off between power factor and torque has to be carried out in controlling the angle. For that reason, the graph 3.11 and 3.12 are presented as result of simulations by employing Finite Element Method with the core losses effect.

**Figure 3.11:** Variations of Power Factor Due to Changes of Stator Current Angle
In those plots, the magnitude of stator current remains fixed at 78 Amp rms, but the angle is varied over a broad range and the best trade-offs are shown. As can be seen, the optimal range for the current angle is between 65 and 75 electrical degrees and beyond those ranges the performance is degraded by degeneration on either torque, power factor or both.

**Figure 3.12:** Variations of Torque Due to Changes of Stator Current Angle

Hence, the control strategy should be expected to drive the Synchronous Reluctance Motor proposed around 70 electrical degrees in order to count on an acceptable power factor without affecting the torque production.
3.4.2 Current Controller

Since the speed control will be utilized to compare the motors over next chapter, the simplified motor model set forth on 3.2.1.1 would be more convenient and straightforward to derive the current controller. Therefore, by substituting the maximum point of power factor into the torque expression as follows,

\[ T_e^* = \frac{3}{4} P_1 (L_d - L_q) \sqrt{\frac{L_d}{L_q}} i_d^2 \]  

(3.4.2.1)

and the reference currents are subsequently defined by 3.4.2.2.

\[ i_d^* = \frac{4T_e^*}{3P_1 (L_d - L_q) \sqrt{\frac{L_d}{L_q}}} \]

\[ i_q^* = \sqrt{\frac{L_d}{L_q}} i_d \text{ sign } T_e^* \]  

(3.4.2.2)

From those equations, a considerable dependence upon machine parameters such as stator leakage inductance and DQ magnetizing inductances is clearly appreciable in driving the Synchronous Reluctance Motor; thus, precise estimations in observer design and parameter identification would be determinant for control efficiency.
Finally, the block diagram for the whole control strategy is depicted on figure 3.13, where it has been included an inductance saturation model based upon the inductance measurements conducted and described previously in order to get a simulated behavior as close as possible to a real one in a Synchronous Reluctance Motor.

3.5 Simulation Results

The Synchronous Reluctance Motor for variable speed application designed and set forth over this chapter has been modeled and simulated under variable frequency conditions by making use of Vector Current Control. Machine parameters and rotor geometry are shown on table 3.1, and its stator is the same as used by the Induction Motor explained before.

After developing the model and the control strategy in MATLAB/Simulink different load and speed conditions were recreated in order to evaluate the performance of the
Synchronous Reluctance Motor proposed. The figure 3.14 shows the voltage and the current required by the motor designed in order to efficiently handle the changes in the load, while variations in the speed are performed. It should be noted that the motor is operated in the four quadrants: motoring and generating as well as accelerating and braking, and being able to handle the load within nominal input current although the input voltage is slightly above the rated value of 480 Volts.

The graphs are mostly depicted in per unit system so that is simpler to make the
comparison to the rated values which have been adjusted to the same values in accordance to Induction Motor for comparison effects in the next chapter. However, let us recall that the base values are 1800rpm for speed, 300Nm for torque, 480Volt and 85Amp rms for line-to-line voltage and phase current, respectively.

![Figure 3.14: Voltage and Current Under Different Speed and Load Conditions](image)

As it was emphasized on section 3.4.1, the control strategy so that the SRM presents the maximum power factor must guarantee a current angle around of vicinity of 70 degrees. This value also provides torque high enough to handle the load, and it differs from the theoretical maximum torque angle of 45 degrees due to saturation and cross...
magnetizing effects, as explained on section 3.3. For the purpose of confirming the effects of a current angle within the vicinity of 70 degrees, several simulations were performed by means of Finite Element Method and displayed on figures 3.11, 3.12 and summarized on table 3.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\varphi = 65$</th>
<th>$\varphi = 70$</th>
<th>$\varphi = 75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase Voltage (Volts rms)</td>
<td>277</td>
<td>265</td>
<td>255</td>
</tr>
<tr>
<td>Phase Current (Amp rms)</td>
<td>78</td>
<td>78</td>
<td>78</td>
</tr>
<tr>
<td>Input Rating (KVA)</td>
<td>65</td>
<td>62</td>
<td>60</td>
</tr>
<tr>
<td>Power Factor</td>
<td>0.82</td>
<td>0.77</td>
<td>0.71</td>
</tr>
<tr>
<td>Speed (rpm)</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
</tr>
<tr>
<td>Average Torque (Nm)</td>
<td>290</td>
<td>310</td>
<td>265</td>
</tr>
<tr>
<td>Ripple Torque (%)</td>
<td>7</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 3.2: Synchronous Reluctance Motor Performance in the Vicinity of 70 degrees

As the SRM is posteriorly simulated, the outcomes corroborate that the Vector Current Control implemented is certainly able to keep the stator current angle within the range desired so that the power factor is maximized, as it appears on figure 3.15. Despite the fact that the angle is effectively controlled, the power factor varies within the range of 0.7 and 0.8, which is determined by the effect of saturation in the inductances related to load changes.

As for the motor’s efficiency, the core losses are calculated by taking into consideration the properties of Non-oriented electrical silicon steel M19 26Ga for the rotor and stator stacks, while the resistivity $\rho = 2.1 \mu \Omega cm$ is used for the copper losses calculation. The efficiency in Synchronous Reluctance Motor is mainly determined by the copper
losses which are considerably higher than the core losses. In the graph is shown that
the motor proposed is able to achieve an efficiency of nearly 95% when it is loaded
under nominal torque, nevertheless, the major advantage is evident at lower loads
since the efficiency remains substantially high as well as the power factor.

In the next plots (figure 3.16) the parameters are compared that influence the motor
performance, and how the control strategy carries out the current regulation. The
ratio of the quadrature axis current to the direct axis current defines the current
angle which is desired to keep fixed at value equals to square root of inductance ratio.
Likewise, the saturation effect is modeled and simulated for which the inductance values are considered as function of stator current.

As a result, it can be inferred that having a power factor of nearly 0.8 requires not only a current angle of about 70 degrees, but it requires an inductance ratio above 8 approximately as well, and this is only feasible with small loads.

The model of saturation effect shows a decrement in the inductance ratio as full load is applied to the motor, that is to say, the ratio diminishes until taking values of nearly 6. Consequently, the maximum power factor achievable is also reduced to 0.7.
Further details are appreciable if the flux density within the Synchronous Reluctance Motor is examined when a current with an angle of 70 degrees is applied to the stator winding (as depicted on figure 3.17). The saturation in $L_q$ is desired and the width of the web section has been made large enough to minimize rotor saturation in $D$ axis; nonetheless, the saturation mainly occurs in the ribs near the rotor surface and the stator tooth, which implies that stator optimization could be made in order to count on a better flux density distribution in the stator–diminishing this way, the effect of stator leakage inductance in the inductance ratio.

![Vector Current Control for Synchronous Reluctance Motors](image)

**Figure 3.17:** Vector Current Control for Synchronous Reluctance Motors

### 3.6 Discussion

The design of an SRM has been proposed and exhaustive analyses have been carried out to assess its performance. Over the design process a multi-barrier configuration was chosen and its number of barriers–insulation width, iron width, barrier angle,
among other parameters– have been varied and compared in order to fulfill the desired behavior. The findings state that there is no apparent pattern when those parameters are changed. Hence, the selection of the best geometry is based upon the best trade-off observed among average torque, torque ripple, inductance ratio and inductance difference, within a finite number of simulations in the non-linear space of solutions.

The core loss and cross magnetization were discussed and their effect in the electromagnetic torque, current angle, inductance saturation and power factor were evaluated. The outcomes indicate that a magnetizing current angle of about 70 degrees and an inductance ratio over 8 can lead the SRM to achieve a power factor around 0.8 without affecting significantly the torque production. After calculating the total losses for the motor proposed, it is confirmed that iron losses are smaller than copper losses for which high efficiency is achievable, even at small loads and low speed.

Vector current control was utilized to regulate speed and torque in the SRM proposed, and the maximum power factor control strategy was implemented. Finally, it was proved through simulation results that the Synchronous Reluctance Motor responds effectively under the input variations applied for the controller, and the desired current angle is therefore obtained. On the other hand, the incidence of machine parameters in the controller calculation is clear, which exposes the necessity of a precise parameter estimation, most of all, due to the parameters variability over the whole operating range, so that the control not only is really efficient but the degradation of motor performance is also taken into consideration.
Chapter 4 : Comparing an Induction Motor to a Synchronous Reluctance Motor for Variable Speed Applications

4.1 Introduction

Owing to increasing interest in comparing these two types of motors, studies have been presented recently, and they attempt to prove the viability for Synchronous Reluctance Motor to face applications where Induction Motors are usually preferred because their reliability and low manufacturing cost.

In the mid-nineties, the authors in [75] and [57] compared analytically both types of machines. For example, Lipo et al. [57] state that a Synchronous Reluctance Motor produces theoretically 82% of the torque of the Induction Motor with a 50% of the losses. Therefore, by increasing the current in the SRM in order to match the copper losses and the flux in both machines a 142% more torque with respect to the equivalent Induction Motor could be expected, although if the cross magnetization effect is considered, that torque could be reduced in about 15-20%.
For its part, Miljavec and Jereb in [75] conducted a similar study and states that a Synchronous Reluctance Motor is able to provide higher torque by assuming that the ratios $L_d/L_m$ and $L_m/L_r$ are almost equal to one, where $L_m$ and $L_r$ are the magnetizing and the total rotor inductance in the equivalent Induction Motor, respectively. Additionally, they suppose that all the losses in both motors are the same but the copper losses. Therefore, the copper loss in the SRM at nominal working point can be doubled and the phase current can be increased by a factor of $\sqrt{2}$. In their study the saturation effect is neglected which led them to conclude that the larger overload reserve belongs to SRM and lower over-heating associated with it.

More recently, in [76] and [77], the authors compare the motors from the thermal point of view, and their results are based upon experimental verification of torque when both motors are running at the same winding temperature. They claim a clear advantage of torque production of Synchronous Reluctance Motor above the equivalent Induction Motor, which is about (10%-15%). They furthermore test both machines at same load conditions finding a SRM up to 23 Celsius Degrees cooler than its equivalent IM. For those reasons, they ultimately conclude that the larger the motor is, the larger is the expected torque advantage, when passing from an Induction Motor to the equivalent Synchronous Reluctance Motor.

Likewise, Germishuizen et al. conduct a comprehensive experimental analysis in [78] with both motors at 110kW power level. From the results of their experiments they conclude that in the high speed region the Synchronous Reluctance Motor compares
unfavorably with that of Induction Motor because of its limited constant flux weakening region, but they say that the stator in the SRM can be optimized in order to improve it. Furthermore, they claim that different PWM switching was used to drive each motor, so the efficiency could have been affected by that. Although the experiments were carried out by using inverters in order to drive both motors at different frequencies, the measurements were performed only around nominal torque value of 700Nm and measurement results at steady state were presented.

As can be seen, practical and theoretical comparison has been set forth in literature, however, it is the author’s opinion that that has not been fair enough for several reasons, and it is the intention of this work to provide a broader view to help strengthening the discussion. For example, in [57, 75–77], they do not analyze the apparent power needed to achieve such torque values; in addition to, they use general purpose Induction Motors instead of optimized Induction Motor to be driven by inverters. In [78], on the other hand, worthy results are shown although they are only taken around nominal torque, and as it is known, in the practice the motors are subject to different load conditions, not only the nominal.

It is worth mentioning that one of the biggest concerns regarding Synchronous Reluctance Motors is their modest power factor, and their main advantage is the high efficiency, so the apparent power needed to operate such motors should be the center of attention since of that depends upon the sizing process for the inverter. It is evident that the cost of USD/KVA is much more higher in the inverter than in the motor;
hence, the feasibility of using a Synchronous Reluctance Motor would be jeopardized if an over-sized Variable Frequency Drive (VFD) is required.

4.2 Results Under Different Load and Speed Conditions

In previous chapters both designs were independently analyzed, step loads were applied and input phase current and phase voltage were presented as part of the results, as well as their efficiency and power factor over different speed and load conditions. Before continuing, let us recall the base values that were previously used and will be also kept fixed during the results presented below for simplicity in the comparison. These values are 1800 rpm for speed, 300 Nm for torque, 480 Volt and 85 Amp rms for line-to-line voltage and phase current, respectively.

From now on, some assumptions are accounted for, which are based upon the practical implementation of the proposed analysis in this chapter. First at all, both the Synchronous Reluctance and the Induction Motor were designed to operate at an output power of 55kW, thus, an inverter able to manage such amount of energy should be chosen.

Secondly, A 55kW motor is a NEC standard power rating and the commercial Variable Frequency Drivers also come in standard KVA rating; hence, as part of this work, different inverters’ manufacturers were consulted in order to count on information about standard frames for VFD and their respective KVA rating.
Chapter 4. *Comparing an Induction Motor to a Synchronous Reluctance Motor for Variable Speed Applications*

In the industry, motors of 55kW are usually driven by inverters of at least 75KVA in order to provide the system with a gap large enough to beat sudden overloads or any momentary failure such as unbalanced phases or over-heating due to multiple starting. So an imaginary standard inverter of 75KVA will be used as reference for the purpose of comparison.

Likewise, commercial inverters are commonly designed and built to bear overloads of 50% for up to 60 seconds before triggering the protections; for that reason, two lines are drawn in the input power rating plot—one of them at nominal power of 75KVA and the other one at 112.5KVA, which corresponds with an overload of 50%.

The figure shown on 4.1 is the outcome of applying a ramp load from zero up to base torque while both motors are spinning at base speed, and then, the load remains constant while the motors are stopped and restarted.

In general, both motors meet the requirements since both provide output power of about 58kW while the input apparent power (KVA) remains below the limit that the VFD is able to manage underrated conditions.

Likewise, by taking a close look at the results, several conclusions can be drawn. For example, at full load, the power factor of 0.7 in the Synchronous Reluctance Motor punishes its performance; in consequence, the apparent power that the VFD has to provide is 70KVA, which is a 7% more in comparison to 65KVA in the Induction Motor in spite of the fact that efficiency in the Inverter-driven Induction Motor falls down about 2% at full load.
On the contrary, in applications where intermittent loads are driven by the motors and they spent most of the time operating under either no-load or low-load conditions, the Synchronous Reluctance Motor would be preferred over the Induction Motor due to its high efficiency in those operating regions as well as because the saturation effect would not reduce that much its power factor, as it is shown on the figure 4.1 before
1.5 seconds when there is no load. The drawback in the Induction Motors is evident as they run with no load because their high losses.

In what follows, the load conditions are slightly changed, which is to say that in the next simulation (see figure 4.2) the ramp load is applied while the motors are accelerating up to base speed, posteriorly, a sudden step overload has been simulated in order to analyze its effects in both motors.

From the graphs presented in this chapter and the previous ones, it can be inferred that no matter what type of load (ramp or step) or for what speed conditions are applied, both motors can be driven by the same 75KVA inverter as long as the load is within the nominal range (0 - 300Nm). Nevertheless, a different treatment is needed when the overload appears.

As aforementioned, a line of 112.5KVA, meaning a manageable overload of up to 50% for 60 seconds by means of using a 75KVA VFD, has been drawn in order to have the input KVA limited and to observe to which motor belongs the higher overload reserve. Making allowance for this, several overloads were applied until reaching the limit and the higher one is depicted in figure 4.2.

Some authors ensure that Synchronous Reluctance Motors owns a higher torque production than its equivalent Induction Motor; however, if the input energy is limited, efficiency and power factor play also vital role in the energy conversion. Based upon previous results explained, it would be predictable to think that the Induction Motor can manage a larger load with lower apparent power.
Chapter 4. Comparing an Induction Motor to a Synchronous Reluctance Motor for Variable Speed Applications

The simulation results indicate that the Inverter-driven Induction Motor can bear an increment of up to 60% in the load before reaching the KVA limit imposed by the inverter and the protections, while the Synchronous Reluctance Motor bears up to 50%. It is also important to notice that the main restriction for SRM to accomplish a better energy conversion as it is overloaded keeps being the power factor decrement.
Chapter 4. *Comparing an Induction Motor to a Synchronous Reluctance Motor for Variable Speed Applications*

due to saturation effect. Hence, it is the author’s opinion that the design and control of the Synchronous Reluctance Motor must be mainly focused upon power factor optimization in order to ratify its undoubted applicability as AC Drive.

### 4.3 Conclusions

In the first two chapters of this work, it was proved that by appropriately defining the constraints as well as cost functions in a novel algorithm based upon Differential Evolution, satisfactory geometries of Inverter-driven Induction Motors were obtained, which demonstrated maximum values of torque density.

The resultant parameters in the Induction Motor confirmed that the geometrical design leaded by a novel methodology based upon Differential Evolution initially, and then optimized by the Finite Element Method, contains the transient and steady state responses desired in an IM for variable speed applications. Furthermore, the losses and power factor are found comparable to other motors in the market, despite the motor designed presents smaller dimensions.

During the last chapter of this work, a worthy and comprehensive comparison has been presented; an Induction Motor optimized for being driven by inverter is compared to a Synchronous Reluctance Motor at output power of 55kW. The approach set forth allows for counting on transient and steady state response from the input power point of view.
This work contributes to enrich the discussion and to verify the feasibility of using SRM as alternative to Inverter-driven IM, since two equivalent motors particularly designed to be driven by Vector Current Control are subject to study under several frequency, voltage and load conditions. Hence, the approach presented provides a fair comparison to ensure results that have not been covered by studies before.

Based upon previous studies, it was anticipated that Synchronous Reluctance Motors have benefits enough to substitute Induction Motors in applications where variable speed is requested by taking advantage of their higher efficiency, rotor robustness, and lower manufacturing cost from materials perspective. In spite of their poor values of power factor, previous works point out the capability of producing larger torque than their Induction Motors equivalents.

The results obtained in this work provide evidence that there is no need to oversize the Variable Frequency Driver for the Synchronous Reluctance Motor when the load is within the nominal range. Certainly, it was proved that the total power apparent consumed by the SRM under full load is larger than that of the Inverter-driven Induction Motor; however, the consumption average would tend to be lower, if intermittent loads are applied instead.

On the other hand, the Induction Motor driven by inverter compares favorably over the SRM when they are overloaded, and the results suggest that it is able to bear between 5% and 10% more load without triggering the protections or disconnecting the VFD. Nevertheless, it could be inferred that the rotor thermal implications of
such overloads would tend to favor to the cooler rotor in the Synchronous Reluctance Machine.

For larger machines is likely that the modest power factor is to be an important constraint in Synchronous Reluctance Motors, but careful studies are recommended in order to verify their feasibility. As for small machines, the results of this work are valid and lend support to the assumption that SRM are functional in this type of applications.

It is worth recalling that the comparison has been done by employing the same stator for both motors, despite it was originally designed for the Inverter-driven Induction Motor, which lead us to believe that a stator design improvement could be carried out in order to diminish the saturation effect, and subsequently, enhance the power factor performance in the SRM.

Owing to the motors designed in this work have not yet been built at the time this material is being written, the approach outlined in this study is subject to experimental verification by implementing the control algorithms here described and building the geometries proposed in previous chapters. Additionally, the experimental study should include thermal data in order to complete the results. This way, several aspects not covered by previous research will be provided and verified.
Bibliography


Appendix A: MATLAB/Simulink Vector Control Models

Figure A.1: Indirect Field Oriented Current Control of Induction Motor
Figure A.2: Maximum Power Factor Vector Current Control of Synchronous Reluctance Motor
Appendix B: Single-Phase Equivalent Circuit of a Synchronous Reluctance Motor

\[ \ddot{V}_d + j\dot{V}_q = (R_s + j\omega_r L_{ls}) (\ddot{I}_d + j\dot{I}_q) - \omega_r L_{mq} \ddot{I}_{mq} + j\omega_r L_{md} \ddot{I}_{md} \]

\[ \ddot{V}_s = (R_s + j\omega_r L_{ls}) \ddot{I}_s + j\omega_r L_{md} \ddot{I}_m + \ddot{I}_{mq} \omega_r (L_{md} - L_{mq}) \]  \hspace{1cm} (B.0.0.1)

\[ \ddot{V}_s = (R_s + j\omega_r L_{ls}) \ddot{I}_s + R_m (\ddot{I}_s - \ddot{I}_m) \]

Figure B.1: Single-Phase Equivalent Circuit for a Synchronous Reluctance Motor