Isothermal Fatigue Life Prediction Techniques

DISSERTATION

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Abstract

Substantial progress has been made in advancing a pre-existing energy-based fatigue life prediction method into a powerful tool for real-world application through three distinct analyses, resulting in considerable improvements to the fidelity and capability of the existing model. First, a torsional fatigue life prediction method with consideration for the identification and incorporation of loading multiaxiality was developed and validated against experimental results from testing of Aluminum 6061-T6 specimens at room temperature. Second, a unique isothermal-mechanical fatigue life testing capability was constructed and utilized in the development of an isothermal-mechanical fatigue life prediction method. This method was validated against experimental data generated from testing of Aluminum 6061-T6 specimens at multiple operating temperatures. Third, alternative quasi-static and dynamic constitutive relationships were applied to the isothermal-mechanical fatigue life prediction method. The accuracy of each new relationship was verified against experimental data generated from testing of two material systems with dissimilar properties: Aluminum 6061-T6 at multiple operating temperatures and Titanium 6Al-4V at room temperature. Each investigation builds upon a previously-developed energy-based life prediction capability, which states: the total strain energy dissipated during both a quasi-static process and a dynamic process are equivalent and a fundamental property of the material. Through these three analyses, the energy-based life prediction framework has acquired the capability of assessing the fatigue life of
structures subjected to unplanned multiaxial loading and elevated isothermal operating temperatures; furthermore, alternative constitutive relationships have been successfully employed in improving the fidelity of the life prediction models. This work represents considerable advancements of the energy-based method, and provides a firm foundation for the growth of the energy-based life prediction framework into the thermo-mechanical fatigue regime. This future work will utilize many of the models developed for isothermal-mechanical fatigue; additionally, the isothermal-mechanical testing capability will be readily modified to perform thermo-mechanical fatigue.
Dedication

To my wife. I could not have done this without her.
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Nomenclature

\[ A \] = quasi-static curve-fitting parameter
\[ C \] = axial dynamic curve-fitting parameter
\[ C_f \] = full life parameter
\[ D \] = torsional shear dynamic curve-fitting parameter
\[ E \] = elastic modulus
\[ E_{st} \] = elastic modulus, steel
\[ F \] = generalized curve-fitting parameter
\[ G \] = shear modulus
\[ H \] = generalized curve-fitting parameter
\[ K_d \] = dynamic curve-fitting parameter
\[ K_{qs} \] = quasi-static curve-fitting parameter
\[ n \] = strain hardening exponent
\[ N_c \] = critical life prediction
\[ N_{c,H} \] = critical life prediction, modified Halford approach
\[ N_{c,RO} \] = critical life prediction, modified Ramberg-Osgood approach
\[ N_f \] = full life prediction
\[ T \] = temperature
\[ T_o \] = initial temperature
\[ W \] = strain energy density

xx
$W_d$ = dynamic strain energy density
$W_{qs}$ = quasi-static strain energy density

$\alpha$ = coefficient of thermal expansion
$\gamma_d$ = torsional shear dynamic strain
$\gamma_o$ = torsional shear quasi-static curve-fitting parameter
$\gamma_{pp}$ = torsional shear peak-to-peak strain
$\gamma_{qs}$ = torsional shear quasi-static strain
$\gamma_u$ = torsional shear failure ductility
$\varepsilon$ = axial strain
$\Delta\varepsilon_p$ = axial plastic strain range
$\varepsilon_d$ = axial dynamic strain
$\varepsilon_{mean}$ = mean strain
$\varepsilon_n$ = axial necking strain
$\varepsilon_o$ = axial quasi-static curve-fitting parameter
$\varepsilon_{pp}$ = axial peak-to-peak strain
$\varepsilon_{qs}$ = axial quasi-static strain
$\varepsilon_{thermal}$ = axial thermal strain
$\varepsilon_u$ = axial failure ductility
$\sigma$ = axial stress
$\text{d}\sigma$ = infinitesimal increment of axial stress
$\Delta\sigma_p$ = axial plastic stress range
\( \sigma_a \) = axial dynamic stress amplitude

\( \sigma_c \) = axial dynamic curve-fitting parameter

\( \sigma_{\text{mean}} \) = mean stress

\( \sigma_o \) = axial quasi-static material parameter

\( \sigma_p \) = principle stress amplitudes

\( \sigma_{pp} \) = peak-to-peak axial stress

\( \sigma_u \) = axial failure stress

\( \sigma_v \) = von Mises yield stress

\( \sigma_y \) = axial yield stress

\( \tau \) = torsional shear stress

\( d\tau \) = infinitesimal increment of torsional shear stress

\( \tau_c \) = torsional shear dynamic curve-fitting parameter

\( \tau_o \) = torsional shear quasi-static material parameter

\( \tau_{pp} \) = torsional shear peak-to-peak stress

\( \tau_u \) = torsional-shear failure stress

\( \tau_y \) = torsional-shear yield stress
Chapter 1: Introduction

1.1: A Brief History of Fatigue

Classical mechanics warns that engineered structures may fail in one of three prescribed ways: through overloading, buckling, or fatigue [1]. The phenomenon of fatigue can be described as diminished strength and ductility arising from alternating stresses below the ultimate tensile strength of the fatigued material [2]. Because even alternating stresses below the yield strength of the material will cause cumulative damage, classical mechanics cannot adequately describe the fatigue process; furthermore, early fatigue damage is often imperceptible to the naked eye and may initiate within the fatigued material [3]. As such, fatigue is perhaps the least understood and most insidious of the three possible failure mechanisms [4].

The danger of fatigue gained recognition in the early 19th century as early railroad systems were developed and employed. Incidents involving the sudden failure of locomotive and carriage axles alerted designers to the hazards of repeated structural loading. It was during this period that various researchers initiated investigations into fatigue failure [5-15]. A typical axle failure of the era is illustrated in Figure 1 [16]. Then in May of 1842, the lead locomotive of a passenger train travelling the Paris-Versailles railroad experienced an axle failure caused by fatigue cracking, incurring a high cost in lives and materiel [16].
The magnitude of the disaster accelerated fatigue research and served as a spur for the investigations of August Wöhler, widely regarded as a contemporary leader in the area of fatigue [5]. Wöhler summarized his conclusions in 1870 with this generalized statement:

“Rupture may be caused, not only by a steady load which exceeds the carrying strength, but also by repeated application of stresses, none of which are equal to the carrying strength. The differences of these stresses are measures of the disturbance of the continuity, in so far as by their increase the minimum stress which is still necessary for rupture diminishes [17, 18].”

Following Wöhler, research into fatigue progressed with greater rapidity. Scientists such as Bauschinger and Kirsch examined fundamental fatigue concepts including the Bauschinger effect and the effect of stress concentrators, respectively [5, 19-22]. Others like Ewing and Humfrey were the first to examine fatigue through a metallurgical lens – an approach that gained popularity and culminated in important metallurgical work regarding dislocation theory and its application to fatigue processes [5, 23-25].

Figure 1. Carriage axle fatigue failure, [16]
At the turn of the 20\textsuperscript{th} century, fatigue research struggled to keep pace with development as new technologies employed ever-increasing mechanical complexity [4]. Industrial tools, pressure vessels, and automobiles presented unique and pressing fatigue challenges. One of the most challenging technologies developed in this era was the aircraft, which relies on the flawless operation of multiple subsystems routinely subjected to alternating thermal and mechanical loading caused by aerodynamic forces, pressurization, and high temperature fluid flow. The most complex of these subsystems is the gas turbine engine, which provides the power and thrust utilized by many modern commercial and military aircraft. A typical gas turbine engine is routinely subjected to a high temperature operating environment coupled with combined thermal and mechanical dynamic loading. Under these conditions, engine components are expected to operate flawlessly for billions of cycles. Abrupt failures of engine structures entail dire consequences in terms of human life and financial resources. Thus, gas turbine engine technologies represent both the culmination of technological complexity with respect to design for fatigue and a vital area of basic research.

1.2: Motivation for Fatigue Research

Although widely considered a contemporary marvel, gas turbine engine development stems from the mid-seventeenth century. Examples include the use of steam jets to drive an impulse turbine in 1629 and a model carriage which utilized a steam jet to provide power in 1678. Other notable examples of gas turbine engine technology in antiquity include a patent granted to John Barber in 1791 for the first true gas turbine and the 1895 introduction of Parsons radial flow generators for power production in the
Cambridge Power Station. Development of gas turbine engines for application to aircraft grew largely from the work of Frank Whittle in the early 1930’s [26, 27]. This work eventually led to the adoption of jet propulsion for military aircraft in the later years of World War 2, followed by application to large commercial aircraft near the mid-century.

Engineering challenges escalated as designs of gas turbines for electricity generation and jet propulsion progressed. The pursuit of higher thrust and improved efficiency required ever lighter materials, higher operating temperatures, and thinner structures. Today, gas turbine engines are highly susceptible to fatigue failure upon an excess of loading cycles, wherein the quantity of excess is dependent upon both the nature of the thermo-mechanical loading and the attendant material processes activated under prescribed operating conditions. For these reasons, the reliability of gas turbine engine components from the perspective of fatigue failure is of great concern to the design engineer, as the failure by fatigue of such structures has been attributed to a variety of aircraft disasters [17, 28]. Unfortunately, the prediction of fatigue life is difficult even for circumstances of simple loading; as such, the juncture of intricate loading and hostile operating environment endemic to gas turbine hardware presents a considerable obstacle.

1.2.1: Gas Turbine Engine Basics

The basic design of a gas turbine engine is presented in Figure 2. The primary stages of the gas turbine engine are described as follows [29]:

a-1: Free-stream air is accelerated at the intake. The velocity of the air upstream from the engine is the flight velocity.
1-2: Air velocity is decreased as it passes through the inlet diffuser and ducting system to the compressor.

2-3: The air is compressed by a dynamic compressor. During compression, the pressure and temperature of the incoming airflow are increased. Compressor structures are subjected to considerable mechanical loading.

3-4: Fuel is injected into the compressed airflow within the combustor. The fuel/air combination is mixed and the mixture ignited. This further increases the pressure, temperature, and flow velocity of the air. Although the combustor utilizes no moving components, thermal loading on the walls of the combustor demand highly engineered materials.

4-5: The hot, high pressure air is expanded through a turbine, which supplies power to the compressor through a connecting shaft. Turbine structures are subjected to extreme thermal loading, radial centrifugal forces, and out-of-plane dynamic bending forces.

5-6: The expanded air is accelerated through a nozzle and exhausted to the outside environment. Acceleration of the air through the engine produces forward thrust.

The complex nature of loading on the turbine structures – thermal loading exceeding 1900 K, radial centrifugal loading, and dynamic bending loading – make them the primary area of concern for current fatigue life prediction technologies [30, 31].
1.2.2: Low-Cycle Fatigue Challenges

Low-cycle fatigue challenges arise from successive loading of structural components during engine acceleration through its operational range. This action generates stresses at, or exceeding, the elastic limit of the material and may cause failure in fewer than 10,000 operational cycles, where an operational cycle is defined as one acceleration through the operational range of the engine [30]. These conditions are worsened by other damage factors, such as creep mechanisms which initiate upon thermal loading above the creep activation threshold. Rare failure events such as rotor burst may also be attributed to low-cycle fatigue. Rotor burst is caused by transient stressing of the rotor disk resulting from over-speed conditions leading to catastrophic failure. Upon failure, dispersal of the disk fragments cascade damage throughout the engine. Figure 3 depicts one example of a burst rotor [32]. The dangers of low-cycle fatigue tend to be mitigated through a variety of measures taken by engine maintenance organizations. These measures include on-wing inspections, analysis of engine data, and the complete disassembly and inspection of an in-service engine. Early warning signs of
fatigue failure caused by most low-cycle fatigue mechanisms can be identified and the damaged part repaired or replaced [30].

Despite their utility and effectiveness, each of these maintenance practices requires considerable expenditure. Thus, a need exists for accurate, low-cost fatigue life prediction tools for low-cycle fatigue of gas turbine engines.

1.2.3: High-Cycle Fatigue Challenges

High-cycle fatigue challenges arise from repeated cycling of gas turbine engine components below the elastic limit. As loading occurs below the elastic limit, no gross structural deformations or obvious fatigue crack initiation patterns are developed during loading. Combined with the statistical variations in material properties and the uncertainty inherent to the microstructural processes governing fatigue, high-cycle fatigue failure tends to occur with little warning. The dangers of high-cycle fatigue are exacerbated by the same additive damage factors – such as the high temperature creep mechanism – prevalent in low-cycle fatigue.
As maintenance methods to identify impending high-cycle fatigue failures remain only moderately effective and require considerable expense, a great need exists for the development of fatigue life prediction tools for high-cycle fatigue of gas turbine engines.

1.3: Fatigue Life Prediction Methods

1.3.1: Classical Fatigue Evaluation Tools

Tools to predict the fatigue life of engineered structures originated in centuries past. The emergence of fatigue failure in the early 1800’s as a prominent damage mechanism was addressed by researchers in the latter half of the century with the introduction of the stress-versus-cycle (SN) curve and the development of various relationships between the alternating stress and mean stress [33-36].

The SN curve (Figure 4) is derived from empirical observation of fatigue testing across multiple stress amplitudes at a given stress reversal ratio, defined as the ratio of maximum stress to minimum stress. For each stress amplitude, the number of cycles to failure is recorded. The number of cycles to failure and stress amplitude are plotted logarithmically on the abscissa and linearly on the ordinate of the SN curve, respectively. This process is repeated for stress amplitudes at a variety of stress reversal ratios. SN curves permit the design engineer to estimate the fatigue life of a structure subjected to a given stress amplitude/stress reversal ratio combination within the bounds of existing SN data [33]. By convention, two primary regions (low-cycle and high-cycle) are typically denoted on the curve; yet, the ranges of these regions are debatable. This variability emerges from the active nature of fatigue research, where even simple definitions remain mutable.
Several relationships (i.e. the Goodman, Gerber, and Soderberg relations) express the connection between alternating stress and mean stress for a constant number of cycles to failure (Figure 5). Combinations of mean stress and alternating stress are plotted linearly on the abscissa and ordinate, respectively. These relationships permit the engineer to generate a safe design space for a given fatigue life across multiple stress reversal ratios [34].

Figure 4. Schematic of a typical SN curve, adapted from [35]

Figure 5. Schematic of a typical Haigh diagram, adapted from [36]
The classical methods outlined herein remain valuable, providing a conservative buffer against fatigue failure. However, these methods require ample data derived from destructive testing. For elaborate structures shaped in expensive materials and subjected to multiaxial loading, destructive testing is exorbitantly costly. Simplification of structural geometries or reduction of loading complexity may be employed to lessen costs; however, as testing conditions deviate from those applied to the actual component, the veracity of fatigue results for the original design quickly diminish. Additionally, these simplifications fail to reduce the time required to produce a complete SN curve or Haigh diagram. Servo-hydraulic mechanical testing at high stress amplitudes (low-cycle fatigue) may be completed in relatively short periods of time (~$10^2$-$10^4$s) with appropriate choice of operating frequency; however, testing at low stress amplitudes (high-cycle fatigue) may require long periods time (~$10^5$s) for operating frequencies within reasonable limits for servo-hydraulic testing systems. Thus, despite their obvious utility, both the SN curve and those methods found in the Haigh diagram are neither cost-effective nor true fatigue life prediction tools.

1.3.2: The Energy-Based Life Prediction Model

The foundation of a fatigue life prediction method lies in the early 20th century with scrutiny of the strain energy dissipated during dynamic loading [37]. Strain energy dissipation is a consequence of strain hardening, the plastic or irreversible straining of metal characterized by an increase in the stress necessary to produce a defined amount of plastic strain [38]. During cyclic loading a portion of the work applied to a structure
dissipates as strain energy, a process known as hysteresis and evidenced by the finite area formed during the conclusion of a fatigue cycle (Figure 6) [39, 40].

![Figure 6. Schematic of a hysteresis loop, [37]](image)

Subsequent work on the development of energy-based fatigue life prediction methods yielded poor results; for example, a promising model developed by Hanstock in the mid-1940’s was later invalidated under further scrutiny [38, 41]. Notable successes began in 1955, when Enomoto proposed that the summation of the strain energy dissipated during a cyclic process reached a critical value at fatigue failure [42]. This proposal was employed by Feltner and Morrow, who devised a logarithmic energy-based life prediction model built upon the foundation of three primary assumptions: first, that plastic strain energy is a measure of fatigue damage; second, that the damaging energy per cycle for a given stress amplitude is constant and equal to the area under the static stress-plastic strain curve; and third, that the total damaging energy required to cause fatigue fracture is constant and as a first approximation equal to the area under the static true stress-true strain curve [39]. A comparison of Feltner and Morrow’s theoretical
prediction versus experimental data for SAE 4340 steel specimens subjected to axial fatigue is presented in Figure 7.

![Graph](image-url)

Figure 7. Prediction versus axial SAE 4340 steel experimental fatigue data [39]

Although roughly accurate in the transition region between low-cycle and high-cycle fatigue, the fidelity of the theoretical prediction decreases in both the very low-cycle and very high-cycle regions. Moreover, the theoretical prediction within the low-cycle to high-cycle transition region is overly conservative, reducing the utility of the prediction.

The work of Feltner and Morrow was followed in 1966 by that of Stowell, which merged principles advanced by preceding researchers with a direct accounting of strain energy dissipation through evaluation of strain energy density integrals [43]. These efforts resulted in an energy-based life prediction method capable of predicting the fatigue life of engineered structures with some accuracy. An example of the fidelity of the prediction is presented in Figure 8 for Aluminum 7075-T6 specimens subjected to axial fatigue.
Stowell’s theory was expanded into a viable energy-based fatigue life prediction framework for various load paths by Scott-Emuakpor et al. [44-46]. Capabilities developed within this framework included models for axial fatigue, axial fatigue with mean stress, uniaxial bending fatigue, and transverse shear fatigue of Aluminum 6061-T6 and Titanium 6Al-4V. For simplicity, the energy-based life prediction framework is detailed through evaluation of room temperature axial loading. This approach applies the fundamental assumption of equality between the strain energy dissipated during both a quasi-static process and a dynamic process (Equation 1). Calculation of these strain energy densities requires two forms of a single constitutive relationship, presented in Equations 2 and 4, to model the strain induced in a quasi-static process (Equations 2 and 3) and a dynamic process (Equation 4).

\[ W_{qs} = N_c W_d \]  
\[ \varepsilon_{qs} = \sigma/E + \varepsilon_o \sinh(\sigma/\sigma_o) \]  
\[ \sigma_o = (\sigma_u - \sigma_y)/\ln(\varepsilon_n/0.002) \]
\[ \varepsilon_d = \sigma/E + (1/C)\sinh(\sigma/\sigma_c) \]  

(4)

The constitutive relationships employ a hyperbolic sine to model the physical mechanism of plasticity in both the quasi-static and dynamic stress-strain curves. Quasi-static curve-fit parameters are determined through minimization of the difference between the experimental quasi-static curve and the theoretical quasi-static curve established by the governing quasi-static equation. Dynamic curve-fit parameters are developed for a single stress amplitude and then utilized to model the dynamic strain energy dissipation at other stress amplitudes. These parameters are determined through minimization of the difference between the cyclic strain energy dissipated experimentally and the theoretical cyclic strain energy ascertained through application of the governing dynamic equation. As such, the use of the hyperbolic sine stems from both historical validation and unparalleled utility [43-46].

Strain energy density is generally formulated as a stress integral (Equation 5); however, it may be re-written with strain inhabiting the integrand, thereby permitting the use of the governing equations denoted in Equations 2 and 4. The strain energy dissipated during each process is presented in Equations 6 and 7 and as idealized schematics in Figures 9 and 10.

\[ W = \int_\varepsilon \sigma d\varepsilon \]  

(5)

\[ W_{qs} = \sigma_u \varepsilon_u - \int_0^{\sigma_u} \varepsilon_{qs} d\sigma \]  

(6)
\[ W_d = \sigma_{pp} \varepsilon_{pp} - 2 \int_{0}^{\sigma_{pp}} \varepsilon_d d\sigma \]  

(7)

Figure 9. Schematic of quasi-static strain energy density

Figure 10. Schematic of dynamic strain energy density

Replacing these formulations into Equation 1 and dividing \( W_{qs} \) by \( W_d \) produces the axial critical life equation (Equation 8) [45].

\[ N_c = \frac{\{\sigma_u [\varepsilon_a - (\sigma_u/2E)] - \sigma_o \varepsilon_o [\cosh(\sigma_u/\sigma_o) - 1]\}}{\{(2\sigma_c/C)[(\sigma_u/\sigma_c) \sinh(2\sigma_a/\sigma_c) - \cosh(2\sigma_a/\sigma_c) - 1]\}} \]  

(8)
Predictions resulting from Equation 8 are used to develop a theoretical SN curve. Validation of the predictive capabilities of the energy-based framework for axial, axial with mean stress, uniaxial bending, and transverse shear loading are presented versus Aluminum 6061-T6 experimental data in Figures 11, 12, 13, and 14, respectively.

Figure 11. Prediction versus axial Aluminum 6061-T6 experimental data [45]

Figure 12. Prediction versus mean stress Aluminum 6061-T6 experimental data [45]
Each of these models retained the assertion that the damaging energy per cycle for a given stress amplitude is constant [44-46]. Though sufficient for ideal fatigue processes, actual structural components experience measurable periods of material softening and hardening impacting the rate of strain energy dissipation. Figure 15 schematically depicts material hardening and softening during strain cycling. Because the energy-based life prediction framework depends heavily on an accurate accounting of dynamic strain
energy dissipation, a clear understanding of the evolution of the material properties during the dynamic process is necessary for an accurate prediction of fatigue life.

Figure 15. Schematic of material hardening and softening, [4]

Ozaltun et al. examined this issue for room temperature axial fatigue of Aluminum 6061-T6 structures. The effect of operating frequency on cyclic strain energy was quantified, indicating that cyclic strain energy dissipation tended to increase with operating frequency. Additionally, analysis of the strain energy dissipation history of a room temperature axially fatigued Aluminum 6061-T6 specimen not only demonstrated expected variations in strain energy dissipation rate but also exposed a rapid increase in the dissipation rate near the limit of the fatigue life (Figure 16) [47]. It was hypothesized that this final strain energy dissipation rate acceleration indicated substantial internal structural changes, serving as a secondary metric of the progression of fatigue damage. This idea was further pursued by Letcher et al. with the introduction of the critical life concept [48].
1.4: Areas of Improvement and Opportunity

Despite the palpable successes in past work, many opportunities exist for improvement and expansion of the energy-based life prediction approach. These opportunities can be expressed as a series of goals, which include:

1. To incorporate the effect of unplanned loading multiaxiality into the torsional-shear fatigue life prediction model through several measures, including crack-path examination and the utilization of principle stresses.

2. To extend the fatigue life prediction capability into the isothermal-mechanical regime, where a structure is subjected to mechanical loading at temperatures below the creep-activation limit.

3. To examine the applicability of alternative constitutive relationships with regards to the energy-based life prediction framework. Additionally, to re-examine the idea of the energy-based full life prediction.

Each goal answers a fundamental engineering challenge. The first goal handles unexpected loading multiaxiality, permitting the design engineer to quickly translate the
fatigue life of a structure subjected to a set of planned loading conditions to a set of unplanned loading conditions. The second goal handles several engineering challenges. First, it provides the design engineer the capability to design a structure for sustained thermal and mechanical loading; second, it serves as a stepping-stone to generalized thermo-mechanical fatigue, one of the most rigorous service conditions sustained by gas turbine structures; and third, the creation of a deterministic isothermal-mechanical fatigue life prediction method functions as the foundation of an ENSIP-favored probabilistic energy-based life prediction framework [49]. Finally, the third goal looks to improve upon the fidelity of the existing life prediction framework by incorporating alternative constitutive relationships. As such, the work put forth within this manuscript seeks to meet these three wide-reaching goals, thereby extending, expanding, and improving the existing life prediction framework.
Chapter 2: Experimental Setup and Theoretical Development

Each goal defined in Section 1.4: Areas of Improvement and Opportunity required the assembly of an experimental testing apparatus, the design of efficient and cost-effective experimentation plans, and the development of new energy-based life prediction theories.

2.1: Torsional Shear

2.1.1: Torsional Shear Test Setup

Torsional shear experimentation was conducted on an Instron model 1321 load frame equipped with MTS model 647 hydraulic wedge grips, MTS 647.02b 0.79-1.07 in. serrated V-style wedges, and an Instron model 6467-107 load transducer (Figure 17). Data was collected with various devices. VIC-3D, an optical measurement system developed by Correlated Solutions, obtained strain data during quasi-static testing via the stereoscopic recording of the displacement of a paint-based speckle pattern applied to the surface of each specimen. This process was performed at an acquisition rate of 19 frames per second. The VIC-3D software then interpreted those images to model the strain state on the surface of the specimen. Data was collected with an NI USB-6251 data acquisition system and processed on local hardware running the VIC-3D software suite. During dynamic testing, Vishay CEA-13-062UV-350 strain gauges connected to a Vishay model 2360 signal conditioner was substituted for the VIC-3D system as the fidelity of the
visual measurements was poor for small displacements. Remaining data – including angular displacement and torque – were recorded by the MTS FlexTest SE controller.

The test specimen (Figure 18) was designed in accordance with ASTM E2207 [50]. Specimens were lathed to a nominal roughness average surface finish of 0.813 μm from a single heat of Aluminum 6061-T6 tube stock. Further refinement was unnecessary due to the negligible effect of surface finish on the fatigue life of the material [51, 52].

Figure 17. Torsional shear fatigue setup

Figure 18. Torsional shear specimen geometry, dimensions in mm
Testing was performed at ambient temperature \( T_0 = 298 \text{ K} \) and humidity. Alignment of the load frame was attempted using a specimen fitted with two axial and two shear strain gauges placed in the center of the gauge section and separated circumferentially by 90 degrees. Alignment testing was conducted under zero-load conditions. Results of the initial test indicated the load frame was aligned within the 5% alignment band permitted by ASTM E466 at zero load; however, the absence of strain gauges located at either root of the radius leading to the gauge section limited the precision of the alignment measurements [53]. Testing of the alignment only at the zero-load condition further limited measurement quality. Subsequent calibration was attempted using dial indicators applied to the gauge section of the specimen; however, the precision of the dial indicator measurements was low. Regardless of the value of the various measurements, attempts to align the load frame were further frustrated by the lack of a dedicated alignment fixture.

An experimental procedure was used to eliminate inconsistencies between tests. This procedure detailed the method and location of strain gauge application, the position and orientation of each specimen within the hydraulic grips, and the grip pressure used to secure each specimen. Gauges were applied beginning with a three-step surface cleaning of the specimen followed by application of the adhesive to the gauge itself. The gauges were oriented such that the transverse and longitudinal centerlines of the gauge aligned with the transverse and longitudinal centerlines of the specimen, respectively. Specimens were placed within the grips with the strain gauge facing the test operator, ensuring that gauge wires fully cleared the load frame. The grip pressure applied to each specimen was
based on MTS specification. Design and execution of the various tests was accomplished through Multi-Purpose Testware, an MTS software suite coupled to the load frame controller.

2.1.2: Torsional Shear Testing Plans

Five experimental methods were used during torsional shear testing. The methods are detailed as follows:

1. *Dynamic testing* – Load-controlled torsional shear dynamic tests were conducted at multiple stress amplitudes for SN curve construction. Tests were operated at a frequency between 1 Hz and 5 Hz depending on the expected number of cycles to failure such that testing frequency increased with expected cycles. Data was recorded at a rate of 200 samples per cycle. Failure was defined as the cycle at which the angle of rotation exceeded by 10% the averaged maximum angle derived from mid-life data (Figure 19), ensuring fundamental agreement between the energy-based theory and experimental results by considering only those cycles to macro crack initiation.

![Torsional shear failure criterion](image)

Figure 19. Torsional shear failure criterion
2. **Quasi-static testing** – Displacement-controlled torsional shear quasi-static tests were conducted to generate the material constants and curve-fit parameters required by the life prediction model. Tests were operated at a displacement ramp rate simulating a strain rate of 137.2 µε/s. Data was recorded at a rate of 10 samples per second. Quasi-static failure was defined as the point of ultimate shear stress (Figure 20) [54]. Note that for torsional shear quasi-static testing, the engineering stress – engineering strain curve and the true stress – true strain curve are identical due to the lack of necking at the yield stress. Quasi-static material properties and curve-fit parameters were determined reduced using the procedure outlined in *Appendix A.1: Quasi-Static Data Reduction Procedure*.

![Quasi-static true stress – true strain](image)

**Figure 20.** Schematic of quasi-static true stress – true strain, adapted from [54]

3. **Dynamic frequency evaluation testing** – Load-controlled torsional shear dynamic tests were conducted across a range of operating frequencies at a specified stress amplitude. Data was collected at a rate of 200 samples per cycle regardless of operating frequency. The number of cycles captured at each frequency scaled with
the operating frequency to effectively utilize testing time. Dynamic frequency evaluation data was reduced using the procedure outlined in Appendix A.2: Dynamic Frequency Evaluation Data Reduction Procedure. Dynamic curve-fit parameters were determined using the procedure outlined in Appendix A.3: Dynamic Curve-Fit Parameter Procedure.

Dynamic frequency evaluation explored the inconsistency between the total strain energy dissipated during the fatigue process and the strain energy directly contributing to fatigue damage. This inconsistency develops from the nature of experimental hysteresis loops, which are a composite of anelastic (non-damaging, recoverable) and plastic (damaging, non-recoverable) behavior [4]. Ozaltun et al. proposed dynamic frequency evaluation testing as a method of minimizing the contribution of the anelastic strain energy through optimization of the operating frequency [47]. Data collection of dynamic testing performed at the optimized operating frequency (the optimal frequency) primarily captures plastic strain energy, improving the accuracy of the energy-based life prediction.

4. Combined quasi-static/dynamic testing – Load-controlled torsional shear dynamic tests were conducted to a specified number of cycles followed by displacement-controlled quasi-static testing to failure. Dynamic testing was performed at the optimal operating frequency discussed in Section 2.1.2, Dynamic frequency evaluation testing, and quasi-static testing was performed at the strain rate discussed in Section 2.1.2, Quasi-static testing. During the quasi-static phase 10 samples of data were recorded per second. During the dynamic phase 200 samples
of data were recorded per cycle. Dynamic/Quasi-static combined testing data was reduced using a combination of procedures conforming to those outlined in Appendix A.1: Quasi-Static Data Reduction Procedure and Appendix A.2: Dynamic Frequency Evaluation Data Reduction Procedure.

Combined quasi-static/dynamic testing examined the effect of service on a pristine structure through evaluation of the decreasing quasi-static strain energy dissipation with increasing dynamic strain energy dissipation. Ozaltun et al. theorized that reduction in the remaining quasi-static strain energy with increasing dynamic strain energy dissipation served as a metric of fatigue damage [47].

5. **Energy dissipation history testing** – Load-controlled torsional shear dynamic tests were conducted with continuous data acquisition. Testing was performed at the optimal frequency determined from *Section 2.1.2, Dynamic frequency evaluation testing* during periods of data acquisition and otherwise at a frequency of 1 Hz to minimize testing time. Data was recorded at a rate of 200 samples per cycle. The full life parameter was ascertained as the ratio between the number of cycles to complete separation of the specimen and the number of cycles at which the energy dissipated per cycle exceeded by 5% the average strain energy dissipated within the stable dissipation region. Energy dissipation history data was reduced using the procedure outlined in *A.4: Energy Dissipation History Testing Data Reduction Procedure*.

Energy dissipation history testing examined the effects of the evolution of material properties on total strain energy dissipation. In addition, this testing
highlighted the rapid changes in strain energy dissipation near the end of the fatigue life. Ozaltun et al. reasoned that changes in the strain energy dissipation rate near fatigue failure serve as a further metric of fatigue damage [47].

Results from these five methods were utilized in the development of the torsional shear energy-based fatigue life prediction with incorporation of the effects of multiaxiality.

2.1.3: Torsional Shear Energy-Based Theory

The equations for the torsional shear energy-based fatigue life prediction theory closely mirror those employed for the axial life-prediction method. As Equation 1 represents the foundation of the energy-based life prediction framework it remains wholly unchanged. Equations 2, 3, and 4 are re-written in terms of shear stress and shear strain as presented in Equations 9, 10, and 11, respectively.

\[ \gamma_{qs} = \tau/G + \gamma_o \sinh(\tau/\tau_o) \]  \hspace{1cm} (9)

\[ \tau_o = (\tau_u - \tau_y)/\ln(\gamma_u/0.002) \]  \hspace{1cm} (10)

\[ \gamma_d = \tau/G + (1/D)\sinh(\tau/\tau_c) \]  \hspace{1cm} (11)

Note how Equation 10 utilizes the ultimate strain compared to the use of necking strain in Equation 3. This is because a structure subjected to torsional shear does not experience necking during plastic deformation. The strain energy dissipated during each process is developed in Equations 12 and 13.
Replacing these formulations into Equation 1 and dividing $W_{qs}$ by $W_d$ produces the torsional shear critical life equation (Equation 14) [46].

\[
N_c = \frac{\{\tau_u (\gamma_u - \frac{\tau_u}{2G}) - \tau_o \varepsilon_o [\cosh(\frac{\tau_u}{\tau_o}) - 1]\}}{(2\tau_c/D)[(\frac{\tau_o}{\tau_c})\sinh(2\frac{\tau_o}{\tau_c}) - [\cosh(2\frac{\tau_o}{\tau_c}) - 1]]}
\]

2.2: Axial Isothermal-Mechanical

2.2.1: Axial Isothermal-Mechanical Design Requirements and Setup

Development of an isothermal-mechanical fatigue setup focused on four specific requirements which are detailed as follows:

1. The load frame must meet the mechanical and thermal demands of the testing procedure and produce repeatable data.

Experimentation was conducted on a custom load frame driven by an Instron A1267-3010 linear actuator (Figure 21). Two styles of grips were utilized: testing on Aluminum 6061-T6 specimens was performed with water-cooled MTS 651.02 buttonhead grips, while testing on Titanium 6Al-4V specimens was performed using MTS 647 hydraulic wedge grips equipped with water-cooled MTS 647.10 0.5 in. diameter surfalloy-coated circular wedges. Each grip style
offered unique advantages and disadvantages. The buttonhead grips (Figure 22A) were bi-material, constructed of stainless steel in the water-cooled main body and of a high temperature metal in the central grip shaft. This dual layer of protection effectively shielded components beyond the grips from heat transfer caused by thermal loading of the specimen.

The disadvantage of this setup stemmed from the necessity to use buttonhead inserts, which were hand-actuated and therefore increased the start-up and break-down time of each test. Additionally, the geometry of the buttonhead grips severely limited possible geometries of the induction coil.

The hydraulic grips (Figure 22B) were constructed of stainless steel and used stainless steel water-cooled wedge inserts. This setup provided the capability to rapidly switch specimens and permitted the use of a wide spectrum of specimen geometries. Furthermore, the hydraulic grips permitted a wider variety of induction coil designs. Disadvantages included a slight increase in heat transfer,
although water-cooling the wedges successfully mitigated the effect of thermal loading.

Figure 22. Grip styles, buttonhead (A) and hydraulic (B)

Alignment of the load frame was achieved with an MTS 609 alignment fixture in conjunction with an alignment specimen (Figure 23). Use of this system ensured alignment to within the 5% band permitted by ASTM E466 [53]. Positioned at the top of the load frame, the alignment fixture was isolated from thermal loading by the upper water-cooled grip.

The various sensors used by the load frame were calibrated prior to testing, and the calibration was checked at intervals throughout the testing process. The load cell was shunt calibrated to a simulated applied load of a known magnitude. Comparison of the known magnitude to the read value indicated the calibration state of the load cell. The extensometer was calibrated using an MTS 650.03 calibrator across two strain ranges (±4,000 µε and +200,000/-100,000 µε).
2. Thermal output must be generated through the use of a repeatable, reliable heating device that permits thermocouple and extensometer access to the specimen.

Several options for applying thermal loading were available for use, including high temperature furnaces, heat lamps, and induction systems. Furnaces are reliable and produce even thermal distributions; however, they also require significant space for mounting, are challenging to instrument, and need considerable time to both reach high temperature and return to room temperature. Heat lamps provide faster heating and cooling than furnaces and provide ample room for instrumentation. Yet, heat lamps rely primarily on radiation to conduct heat; as such, an array of heat lamps must be utilized to ensure total coverage. Induction systems are small, produce rapid heating and cooling rates, and provide ample space for instrumentation. However, induction systems only work on conductive materials, and the resulting thermal distribution is dependent on the design of the induction coil, the geometry of the test specimen, and the thermal boundary conditions.
An induction system (Figure 24) was chosen to provide thermal loading for both the aforementioned reasons and to provide the capability to expand into thermo-mechanical fatigue. This system was comprised of two components, including the induction oven and a water chiller for cooling. Manufactured by Ambrell, the 5060 6 kW induction oven utilized a transformer to generate high frequency electricity, which was then transmitted to an induction work head containing resonant tank capacitors. The work head conducted the electricity along a hollow copper coil. The alternating current within the copper coil induced eddy currents within the specimen which produced rapid heating. The transformer equipment and work head were water-cooled by an Electro Impulse RU-300 chiller.

![Ambrell 5060 6 kW induction system with original induction coil](image)

Figure 24. Ambrell 5060 6 kW induction system with original induction coil

The thermal distribution produced by the induction system was a function of the induction coil geometry, the geometry of the test specimen, and the thermal boundary condition imposed on the specimen by the water-cooled wedge grips.
As the geometry of the specimen was fixed, and the heat transfer to the water-cooled wedge grips was steady state, the thermal distribution was primarily affected by the induction coil geometry. Coil geometry was decided by five parameters: the number of turns of the coil, the spacing between the coils, the distance from the centerline of each coil to the centerline of the specimen, the diameter of the tubing used to construct the coil, and the material selected for the coil. Additional constraints on coil geometry stemmed from the geometry of the water-cooled grips and the necessity to provide access to the surface of the specimen for the thermocouples and extensometer.

For isothermal-mechanical testing ASTM standard required a thermal distribution varying by less than 1% of the set-point temperature [55]. Designing an induction coil capable of this exacting distribution was a challenging process requiring multiple coil iterations. Every design began with 99.99% pure extruded 0.25 in. hollow copper tubing chosen for its conductivity and workability. The induction coil crafting process evolved during the design process. Initial coils were hand-wrapped on wooden dowels marked with guide points for the chosen coil spacing and number of turns. This method elicited a fast turnaround from initial specification to finished coil but often resulted in slanted coils, coil spacing deviating from the desired dimensions, and uneven transitions between coil turns. Later coil iterations were hand-wrapped around molds designed in SolidWorks and constructed using a rapid prototype machine (Figure 25). These molds
required greater investments of time but produced finished induction coils with flat coils, exact spacing, and even transitions between coil turns.

![Figure 25. Example coil form](image)

The capability of each coil iteration was determined through experimentation. For each coil, a testing specimen fitted along the primary axis with seven K-type thermocouples was heated to three target temperatures. Temperature readings at each of the seven thermocouples were recorded for each set-point temperature. The temperature readings were then compared to the 1% deviation bands at the target temperature.

Examples of induction coil designs are presented in Figure 26. The first induction coil design iteration mimicked the dimensions of the coil provided with the induction system, featuring three turns at large diameters with loose spacing. Testing indicated that this design resulted in large thermal gradients between the coil turns and near the water-cooled grips. Several variations were attempted, including changes to the diameter of the coil turns and the coil spacing; however,
none of these efforts were effective in improving the thermal distribution. The next design iteration featured five turns at small turn diameters with variable spacing. This design – including variations of the number of turns and turn diameter – improved on the previous design, yet produced large, asymmetrical thermal gradients near the water-cooled wedge grips.

![Coil design iterations](image)

Figure 26. Coil design iterations

The final design iteration featured seven turns at small turn diameters with variable, asymmetric spacing. This design produced a thermal distribution meeting the ASTM requirements (Figure 27). The variable turn distribution provided excess heating near the grip section to mitigate heat transfer caused by the water-cooled wedges, while asymmetry of the distribution of the coil turns opposed a greater thermal gradient at the lower grip section than was found at the upper grip section.
To eliminate a radial thermal gradient, the induction system was activated twenty minutes prior to acquisition of temperature data. This warm-up period was applied to subsequent elevated temperature testing. It was assumed that the angular thermal gradient was negligible as both the induction coil geometry and the test specimen were symmetric about the primary axis.

During axial isothermal-mechanical quasi-static testing, temperature was controlled by a thermocouple tack welded then cemented to the center of the gauge section. As gross axial displacements lead to a reduction of the gauge area initially by the Poisson effect and then by necking, it was necessary to account for how these processes altered the coupling distance between the gauge section and the induction coil. Placement of the control thermocouple in the center of the gauge section – where most of the area reduction occurred – solved this issue, and was possible due to the low sensitivity of quasi-static results to the small stress concentrator caused by the tack welding process. The purpose of the two methods of fixing the thermocouple – tack welding followed by cementing – was to
provide a solid mechanical connection to the specimen (tack welding) and to reduce heat transfer from the thermocouple wires to the surrounding air by convection (cementing). Without cementing, convection from the thermocouple was a great concern, as transient disturbances to the airflow within the testing chamber would result in fluctuations of the recorded temperature.

During axial isothermal-mechanical dynamic testing, the temperature at the center of the gauge section was indirectly controlled by a thermocouple tack welded then cemented to the lower fillet. The control temperature correlated to the desired set-point value at the center of the gauge section. Unlike quasi-static testing, welding of the controlling thermocouple in the center of the gauge section would have produced a stress concentrator and fatigue crack initiation location detrimentally skewing the dynamic results. Furthermore, gross deflections giving rise to substantial area reduction did not occur within the gauge section at the dynamic stress amplitudes being considered. As such, the temperature profile correlated from a zero-load condition showed insignificant differences to that encountered under dynamic loading and made controlling from the lower fillet the only viable alternative.

During testing, the temperature was controlled with a feedback loop provided by a Chino DP1000C digital program controller, which modulated the power applied to the induction work head (Figure 28).
3. Measurement systems must be shielded from both the thermal loading and interference generated by the heating device.

The combination of high temperatures and the magnetic field generated by the induction coil created an environment hostile to measurement systems. Thus, in some cases specialized devices were employed to avoid damage and interference.

Three measurement devices were used during axial isothermal-mechanical fatigue testing. Load data was collected with an MTS 661 22,000 lbf load cell (Figure 29). Positioned above the water-cooled upper grip, the load cell was fully shielded from the high temperatures at the specimen. Voltage output from the load cell was conditioned with a Vishay 2360 signal conditioner and subsequently passed to an Instron Labtronic 880 controller. Aggressive grounding practices were utilized to isolate the load cell from electrical interference caused by the induction oven.
Strain was measured with an Epsilon 3648 high temperature capacitive extensometer (Figure 30), which employed long alumina rods to isolate the body of the extensometer from heat transfer. The alumina rods also ensured that accidental contact with the induction coil would not lead to erroneous signal transmission. Voltage output from the extensometer was conditioned with an Epsilon 3603 signal conditioner before transmission to the Instron controller.

Displacement information was acquired with a Linear Variable Differential Transformer (LVDT) located at the base of the load frame. The voltage signal from the LVDT was directly conditioned by the Instron controller. The LVDT was isolated from the high temperatures at the specimen by the lower water-cooled grip, actuator shaft, and lower machine deck.
4. Specimens must be developed to fit specialized grips and must promote rapid, even heating.

ASTM E606 provided guidelines for the development of the isothermal-mechanical specimens [55]. Two design iterations were necessary to accommodate the two styles of grips. The first iteration (Figure 31) featured grip sections designed for the buttonhead geometry.
To use this specimen, a two-piece stainless steel buttonhead insert (Figure 32) was first fitted around the grip section. Internally the buttonhead insert matched the specimen grip geometry, while externally the insert was threaded with a hexagonal collar. The specimen and insert were together threaded into the receiver of the buttonhead grip.

![Figure 32. Buttonhead insert](image)

With the buttonhead inserts tightened into the receiver, internal pistons were engaged against both ends of the specimen to lock it into place. The pistons were pressurized through actuation of a hydraulic pump connected to each grip.

The second specimen iteration (Figure 33) employed a simpler grip section design. This was possible due to the nature of the wedges fitted within the hydraulic grips, which featured a circular channel in which the specimen was placed. Once the specimen was placed within the channel, the wedges were forced together by actuation of the hydraulic grip. Pressure was applied to the grips by the hydraulic pump operating the load frame.
Each set of specimens was lathed to a nominal roughness average surface finish of 0.813 μm from single heats of Aluminum 6061-T6 or Titanium 6Al-4V rod stock in an effort to reduce inconsistencies in material properties. Further refinement of the surface finish on either the Aluminum 6061-T6 or the Titanium 6Al-4V specimens was not explored due to its negligible effect on the fatigue life within the fatigue regime being examined [51, 52].

2.2.2: Axial Isothermal-Mechanical Testing Plans

Four experimental methods were utilized during axial isothermal-mechanical testing. Those tests involving specimens crafted from Titanium 6Al-4V were performed exclusively at room temperature (ranging from $T_0=295$K to $T_0=298$K). Tests involving Aluminum 6061-T6 specimens were performed at four temperatures below the creep activation threshold of 423K (including $T_0=295$-298 K, $T_1=348$ K, $T_2=373$ K, and $T_3=398$ K) [56]. The experimental methods are described as follows:
1. Dynamic testing – Load-controlled axial dynamic tests were conducted at multiple stress amplitudes for construction of SN curves. Dynamic tests were performed at an operating frequency between 1 Hz and 10 Hz depending on the expected number of cycles to failure, such that the operating frequency increased with expected cycles. Data was acquired at a rate of 200 samples per cycle. Fatigue failure was defined as the approximate cycle at which the maximum axial displacement exceeded by 10% the averaged maximum displacement derived from data at the mid-life of the specimen. This failure criterion ensured fundamental agreement between the energy-based life prediction theory and the experimental results by considering only those cycles to macro crack initiation. Generally, the failure criterion was met just prior to separation of the specimen.

2. Quasi-static testing – Displacement-controlled axial quasi-static tests were conducted to generate the material constants and curve-fit parameters required by the life prediction model. Quasi-static tests were performed at a displacement ramp rate designed to simulate a strain rate of 1,333 με/s to maintain consistency with previous research and to conform to ASTM E8 [44-47, 57]. Data was acquired at a rate of 10 samples per second. Quasi-static failure was defined as the point of ultimate stress. The final area of each specimen was averaged from six measurements of the minimum diameter of the fully-separated specimen. Quasi-static material properties and curve-fit parameters were determined using the procedure outlined in Appendix A.1: Quasi-Static Data Reduction Procedure.
3. *Dynamic frequency evaluation testing* – Load-controlled axial dynamic tests were conducted across a range of operating frequencies at multiple stress amplitudes. Data was collected at a rate of 200 samples per cycle regardless of operating frequency. The number of cycles captured at each frequency increased with operating frequency to effectively utilize testing time. Dynamic frequency evaluation data was reduced using the procedure outlined in *Appendix A.2: Dynamic Frequency Evaluation Data Reduction Procedure*. Dynamic curve-fit parameters were determined using the procedure outlined in *Appendix A.3: Dynamic Curve-Fit Parameter Procedure*.

4. *Energy dissipation history testing* – Load-controlled axial dynamic tests were conducted with continuous data acquisition. Testing was performed at the optimal operating frequency discussed in *Section 2.2.2, Dynamic frequency evaluation testing*. Data was acquired at a rate of 200 samples per cycle. The full life parameter was ascertained as the ratio between the number of cycles to complete separation and the number of cycles at which the energy dissipated per cycle exceeded by 5% the average strain energy dissipated within the stable dissipation region. Energy dissipation history data was reduced using the procedure outlined in *A.4: Energy Dissipation History Testing Data Reduction Procedure*.

Results of these four methods were used in the development of the axial isothermal-mechanical energy-based fatigue life prediction method and in consideration of the capability of alternative constitutive relationships with regards to the energy-based fatigue life prediction method.
2.2.3: Axial Isothermal-Mechanical Energy-Based Theory

Development of the axial isothermal-mechanical energy-based fatigue life prediction theory was dependent on the choice of operational boundary conditions, which were defined primarily by the method of application of thermal loading. Upon initiation of thermal loading, the load frame control system was commanded to maintain zero load. As the specimen deformed due to thermal strain, the control system displaced the actuator to maintain the zero load condition; thus, thermal strain developed in the absence of thermal stress. This boundary condition was defined as a fixed-free condition, as one end of the specimen was restrained while the other was free to displace to maintain the desired load. Figures 34 and 35 denote the effect of the fixed-free boundary condition on the quasi-static and dynamic stress-strain curves, respectively.

Figure 34. Schematic, boundary condition effect on quasi-static stress-strain curve
The axial isothermal-mechanical energy-based fatigue life prediction model is a direct evolution of the axial room temperature fatigue life prediction method, which hinges on the equality between the strain energy density dissipated during both a quasi-static process and a dynamic process (Equation 15) [44]. Calculation of the two strain energy densities required two forms of a single constitutive relationship, presented in Equations 16 and 18, to model the strain induced in a quasi-static process (Equations 16 and 17) and a dynamic process (Equation 18).

\[
W_{qs}(T) = N_c(T)W_d(T) \tag{15}
\]

\[
\varepsilon_{qs}(T) = \frac{\sigma}{E(T)} + \varepsilon_o(T) \sinh(\frac{\sigma}{\sigma_o(T)}) \tag{16}
\]

\[
\sigma_o(T) = \frac{(\sigma_u(T) - \sigma_y(T))}{\ln(\sigma_u(T)/0.002)} \tag{17}
\]

\[
\varepsilon_d(T) = \frac{\sigma}{E(T)} + (1/C(T)) \sinh(\frac{\sigma}{\sigma_c(T)}) \tag{18}
\]
The strain energy densities dissipated during each process are presented in Equations 19 and 20 and as idealized schematics in Figures 36 and 37, respectively.

\[
W_{qs}(T) = \sigma_u(T)(\epsilon_u(T) - \epsilon_{thermal}) - \int_0^{\sigma_u(T)} (\epsilon_{qs}(T) - \epsilon_{thermal}) d\sigma
\]

\[
W_d(T) = \sigma_{pp}(\epsilon_{pp}(T) - \epsilon_{thermal}) - 2\int_0^{\sigma_{pp}} (\epsilon_d(T) - \frac{1}{2}\epsilon_{thermal}) d\sigma
\]

Figure 36. Schematic of quasi-static strain energy density at elevated temperature

Figure 37. Schematic of dynamic strain energy density at elevated temperature
By substituting Equation 21 into Equations 19 and 20 and dividing $W_{qs}(T)$ by $W_d(T)$ in Equation 15, the axial isothermal-mechanical fatigue life prediction equation was developed (Equation 22).

$$
\varepsilon_{\text{thermal}} = \alpha(T)(T - T_o)
$$

(21)

$$
N_c(T) = \frac{\left\{ \sigma_u(T)[\varepsilon_u(T) - \sigma_u(T)/2E(T)] - \sigma_y(T)\varepsilon_y(T)[\cosh(\sigma_u(T)/\sigma_o(T)) - 1]\right\}}{\left\{ [2\sigma_c(T)/C(T)][(\sigma_a/\sigma_c(T))\sinh(2\sigma_a/\sigma_c(T)) - (\cosh(2\sigma_a/\sigma_c(T)) - 1)]]\right\}}
$$

(22)

Equation 22 lacks the $\varepsilon_{\text{thermal}}$ term, the absence of which was a product of the boundary conditions employed during testing: as the thermal strain simply offsets the quasi-static and dynamic curves, it causes no additional plastic strain energy dissipation. Thus, the primary mechanism affecting the fatigue life of structures subjected to elevated isothermal loading with fixed-free boundary conditions is the interaction between temperature and material properties.

2.2.4: Alternative Constitutive Relationships

The current governing constitutive relationship forming the basis of the energy-based fatigue life prediction method has been employed successfully since the original work of Stowell; however, concurrent research manifested other, equally viable relationships ripe for incorporation into the life prediction framework [43, 58, 59]. Moreover, recent improvements in life prediction of the low-cycle to high-cycle transition region have not been realized in the high-cycle regime, where materials displaying an endurance limit necessarily dissipate damaging energy at a significantly
diminished rate. Accounting for this reduced rate is difficult, as hysteresis loops in this region tend to contain immeasurably small areas and be the result of primarily non-damaging mechanisms [59-62]. Auspiciously, an empirical method readily applicable to the current life prediction framework has been proposed to address this issue [59].

The primary component of the energy-based life prediction framework – the governing constitutive equation – connects the applied stress to the resulting strain and was employed in the development of the strain energy dissipated during both quasi-static and dynamic testing. The current constitutive relationship is presented in its generalized form in Equation 23.

\[
\varepsilon = \frac{\sigma}{E} + \left(\frac{1}{F}\right) \sinh\left(\frac{\sigma}{H}\right)
\]  

(23)

The first alternative constitutive relationship to be examined was developed by Ramberg and Osgood (Equations 24 and 25), where the plastic region of both the compressive-to-tensile half of the dynamic and quasi-static stress-strain curve are modeled with a power law relationship [58]. Note that these equations are specialized to the case of room temperature axial loading.

\[
\varepsilon_d = \frac{\sigma}{E} + \left(\frac{\sigma}{K_d}\right)^{(1/n)}
\]  

(24)

\[
\varepsilon_{qs} = \frac{\sigma}{E} + \left(\frac{\sigma}{K_{qs}}\right)^{(1/n)}
\]  

(25)
The second set of alternative equations were provided by the work of Halford (Equations 26 and 27). Halford paired a modified version of the Ramberg-Osgood dynamic stress-strain formulation with an empirically-based quasi-static strain energy formulation designed to address the endurance limit phenomenon by simulating the decrease in dynamic strain energy dissipation with increasing cycles as an increase in the quasi-static strain energy [59].

\[
\varepsilon_d = \frac{\sigma}{E} + \Delta \varepsilon_p \left(2 \frac{\sigma}{\Delta \sigma_p}\right)^{(1/n)}
\]

(26)

\[
W_{qs} = \left(\frac{E}{E_{qs}}\right)A(2N_c)^{(1/3)}
\]

(27)

Note that Equations 24, 26, and 27 utilize curve-fit parameters designed to be ascertained at individual stress amplitudes. Yet, application of these equations to a predictive life prediction methodology required the aforementioned parameters be optimized to accurately capture the elastic and plastic response across a range of stress amplitudes. As such, these factors were treated in a similar fashion as the curve-fit parameters \(C, D, \) or \(\sigma_c\) used by Stowell’s dynamic stress-strain relationship. Implementation of Equations 24-27 to the established methodology (Equations 1, 6, and 7) resulted in two new energy-based life prediction equations (Equations 28 and 29).

\[
N_{c,RO} = \frac{\{\sigma_u[\varepsilon_u - (\sigma_u/2E)] - K_{qs}^{-1}[n/(n+1)]\sigma_u^{[n+1]/n}\}}{\{[(1-n)/(1+n)]K_{d}^{-1}[2\sigma_d]^{[n+1]/n}\}}
\]

(28)
\[ N_{c,H} = \frac{\left[2^{1/3} AE (\Delta \sigma_p)^{1/3} (1 + n)\right]}{\left[E_s \Delta \varepsilon_p (2\sigma_a)^{(n+1)/n} (1 - n)\right]} \]  

To provide distinction, the approach developed by Stowell is defined as the “original method,” the approach applying the work of Ramberg-Osgood is defined as the “modified Ramberg-Osgood approach,” and the approach applying the work of Halford is defined as the “modified Halford approach.” The fitness of each method was considered as a function of the accuracy of the method, the number of material properties and curve-fit parameters employed by the method, and number of curve-fit parameters subjected to a numerical optimization process.

2.2.5: Critical Life Versus Full Life

The critical life prediction is theoretically valid during the region of stable strain energy dissipation forming the majority of the dynamic process; however, the onset of unstable strain energy dissipation signals the formation and propagation of macro cracks through the bulk material and foreshadows fatigue failure [47, 48, 62]. Within this region the governing energy-based theory loses validity as other mechanical processes not captured by the governing constitutive relationship are activated. Estimation of the total fatigue life of the structure requires the application of the full life parameter, which was measured as the ratio between the number of cycles to complete separation of the specimen and the number of cycles to the onset of the unstable region. Once defined, the full life parameter modified the critical life equation to predict the total life of the structure (Equation 30).
\[ N_f = C_f N_e \] 

(30)

The full life prediction represents an initial attempt to understand the material processes activated near failure and incorporate the action of those mechanisms into the fatigue life prediction methodology.
Chapter 3: Torsional Shear Fatigue Life Prediction with Multiaxiality Corrections

As detailed in Section 2.1.2: Torsional Shear Testing Plans, five experimental methods were used during torsional shear testing. These include quasi-static testing, dynamic testing, dynamic frequency evaluation, combined quasi-static/dynamic testing, and energy dissipation history testing.

3.1: Torsional Shear Experimental Results

3.1.1: Torsional Shear Quasi-Static Testing

Several torsional shear quasi-static stress strain curves were developed. Torsional shear stress was calculated as the ratio between the load vector and the initial area. The shear strain was directly measured with the VIC-3D visual strain system. Typical results are presented in Figure 38.

Figure 38. Typical torsional shear quasi-static testing results
As noted previously, necking does not occur during torsional shear loading; thus, the engineering-stress engineering-strain curve and the true-stress true-strain curve are identical. Relevant material properties and curve-fit parameters generated from these results are presented in Table 1. These properties and parameters were generated using Appendix A.1: Quasi-Static Data Reduction Procedure.

<table>
<thead>
<tr>
<th>$\tau_u$ [MPa]</th>
<th>192</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_u$ [mm/mm]</td>
<td>1.14E-01</td>
</tr>
<tr>
<td>$G$ [MPa]</td>
<td>23,000</td>
</tr>
<tr>
<td>$\tau_0$ [MPa]</td>
<td>11</td>
</tr>
<tr>
<td>$\gamma_0$ [mm/mm]</td>
<td>3.45E-9</td>
</tr>
</tbody>
</table>

Table 1. Torsional shear quasi-static material properties and curve-fit parameters

The relative standard deviation of the material properties did not exceed 7% across 25 quasi-static tests. Thus, the properties presented in Table 1 were assumed to be typical of the tested material.

3.1.2: Torsional Shear Dynamic Testing

Dynamic testing was performed at fully-reversed torsional shear stress amplitudes between 93 MPa and 119 MPa and operational frequencies ranging from 1 to 5 Hz to generate an SN curve. The results of this testing are presented in Figure 39.
3.1.3: Torsional Shear Dynamic Frequency Evaluation Testing

Dynamic frequency evaluation testing was performed at a fully-reversed alternating torsional shear stress amplitude of 106 MPa for seven operating frequencies between 0.01 and 5 Hz. The frequency range selected for testing represents the limits of feasibility: testing at frequencies below 0.01 Hz would require costly investments of time, while frequencies above 5 Hz would create considerable noise in the strain signal. The results of this testing are presented in Figure 40.
Results indicated a decrease in the dynamic strain energy dissipation between 0.01 Hz and 1 Hz followed by an increase between 1 Hz and 5 Hz. The trend present in Figure 41 – and the use of the dynamic frequency evaluation testing to determine the optimal testing frequency – is supported by the work of Dieter [4].

The frequency range chosen for testing fell within the first half of the trend outlined by Dieter, where the hysteresis energy generally rises with increasing frequency. Thus, 1 Hz was the frequency at which the minimum dynamic strain energy density was dissipated for the frequency range subset considered. At this frequency it was inferred that most, if not all, of the dissipated strain energy was due to plastic mechanisms. The results of this testing established the optimal operating frequency for subsequent torsional shear dynamic testing.

3.1.4: Torsional Shear Combined Quasi-Static/Dynamic Testing

Torsional shear combined quasi-static/dynamic testing was performed on several specimens. Each test was designed to dissipate strain energy equivalent to a dynamic test at a fully-reversed torsional shear stress amplitude of 106 MPa. Results from Section 3.1.2: Torsional Shear Dynamic Testing indicated that fatigue failure generally occurred at 22,000 cycles for the target torsional shear stress amplitude; thus, three specimens were dynamically loaded to 6,000, 18,000, and 21,000 cycles prior to application of quasi-static loading. Quasi-static results of this testing are compiled in Figure 41.
The progression of quasi-static curves for increasing amounts of dynamic loading indicated that strain energy dissipation remained low until near the vicinity of fatigue failure. These results also supported the assertion that the quasi-static strain energy remaining after dynamic loading could be utilized as a metric of fatigue damage [63].

3.1.5: Torsional Shear Energy Dissipation History Testing

Torsional shear energy dissipation history testing was conducted at a torsional shear stress amplitude of 106 MPa to maintain consistency with previous testing. Results for a single specimen are presented in Figure 42. Strain energy dissipation in the stable region is shown on the primary ordinate, while the total strain energy dissipation (including the final instability in dissipation rate) is shown on the secondary ordinate. For each set of data, the abscissa has been normalized by the total number of cycles in the data set.
Figure 42. Torsional shear energy dissipation history testing results

Results within the stable region showed a combination of material softening and hardening over the duration of the dynamic test. Substantial hardening near failure was followed by rapid energy dissipation caused by gross geometric changes in the gauge section. Thus, rapid material hardening followed by accelerated strain energy dissipation proved to indicate imminent fatigue failure. Further analysis of the data yielded a full life parameter value of 1.1, as per Section 2.1.2: Torsional Shear Testing Plans, Energy Dissipation History Testing, and Appendix A.4: Energy Dissipation History Testing Data Reduction Procedure. Data derived from energy dissipation history results was utilized to generate dynamic curve-fit parameters (Table 2). These parameters were found using Appendix A.3: Dynamic Curve-Fit Parameter Procedure.

<table>
<thead>
<tr>
<th>$D$ [mm/mm]</th>
<th>1.174E-03</th>
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</thead>
<tbody>
<tr>
<td>$\tau_c$ [MPa]</td>
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</tr>
</tbody>
</table>

Table 2. Torsional shear dynamic curve-fit parameters
3.2: Life Prediction with Multiaxiality Corrections

Using the material properties and curve-fit parameters developed from quasi-static testing and energy dissipation history testing (detailed and tabulated in Section 3.1.1: Torsional Shear Quasi-Static Testing and Section 3.1.5: Torsional Shear Energy Dissipation History Testing, respectively), the energy-based critical life prediction was calculated and the results are plotted in Figure 43.

![Figure 43. Torsional shear critical life prediction versus experimental data](image)

The poor correlation between the critical life prediction and the experimental data was construed as evidence of loading multiaxiality. This theory was supported by irregularities present in the failure patterns of specimens subjected to dynamic loading. Two features were prominent: first, specimens consistently exhibited crack initiation at the same location within the gauge length; and second, the path of the failure crack on each specimen was dissimilar to typical torsional shear crack path geometries. Figure 44 schematically illustrates a typical torsional shear crack path, while Figure 45 presents one example of the actual crack path found at fatigue failure.
Generally, fatigue failure initiates at the location of a stress concentrator either at the surface (due to a surface feature such as a small scratch) or within the material (due to an inclusion) [4]. Statistically, the random distribution of surface features and inclusions refuted the possibility of a consistent failure crack location. Thus, multiaxial loading conditions represent the only viable explanation for the preferential crack initiation location.

Figure 45. Actual torsional shear crack path
Multiaxial loading was caused by one of two sources: either the system was actively applying axial loading during dynamic testing, or misalignment of the load frame was applying bending loading to each specimen. If the control system was applying axial loading there would not exist a preferential crack initiation location, as the axial loading would be evenly distributed throughout the cross-section of the gauge section. Furthermore, the control system was commanded to zero the axial load throughout the duration of dynamic testing. Thus, the only possible source of multiaxial loading was through misalignment of the load frame. As load frame alignment was attempted at a control load of zero, it was likely that the magnitude of the misalignment loading varied in-phase with the torsional shear loading.

Load frame misalignment invalidated the use of the pure shear prediction model, rendering the results presented in Figure 43 questionable at best. To address this challenge, a fatigue life criterion incorporating loading multiaxiality was developed (Equation 31) [64].

$$N_c = \frac{\{\sigma_u[\epsilon_u - (\sigma_u/2E)] - \sigma_o[\cosh(\sigma_u/\sigma_o) - 1]\}}{\sum_{p=1}^{3}(2\sigma_c/C)[(\sigma_p/\sigma_c)\sinh(2\sigma_p/\sigma_c) - [\cosh(2\sigma_p/\sigma_c) - 1]]}$$  \hspace{1cm} (31)

The denominator of the fatigue life prediction equation (the dynamic strain energy dissipation) was modified to account for the three principle stresses, which can be derived from an assumed stress state. Decomposition of the multiaxial stress state into the three principle stresses was made possible by the underlying nature of the energy-based...
framework, which assumes that the contribution of damage from a complex loading state may be deconstructed to a group of superimposed simple load states.

Material properties and curve-fit parameters utilized in Equation 31 were developed as per the defined multiaxial life prediction procedure from fully-reversed axial dynamic testing and quasi-static testing of torsional shear specimens (Table 3) [64]. As the torsional shear to axial misalignment ratio was unknown, various ratios were applied to the multiaxial life prediction model. These ratios are listed in Table 4, with corresponding life predictions presented in Figure 46.

Note that misalignment of the load frame affects not only the theoretical fatigue life predictions but also the experimental data. During testing, the number of cycles to fatigue failure were recorded for the given torsional shear stress amplitude. However, as multiaxial loading was assumed to be occurring during testing, it was necessary to account for the dynamic misalignment loading in the previously-derived SN curves. This was done by adjustment of the experimental stress data from the recorded torsional shear alternating stress amplitudes to equivalent von Mises stress amplitudes for each of the assumed torsional shear to axial biaxial stress ratios using Equation 32. Henceforth, experimental data corrected in this manner is referred to as adjusted experimental data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
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<td>(\sigma_u) [MPa]</td>
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<td>(\epsilon_u) [mm/mm]</td>
<td>7.75E-02</td>
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<td>(E) [MPa]</td>
<td>68900</td>
</tr>
<tr>
<td>(\sigma_o) [MPa]</td>
<td>16</td>
</tr>
<tr>
<td>(\epsilon_o) [mm/mm]</td>
<td>3.17E-10</td>
</tr>
<tr>
<td>(\sigma_c) [MPa]</td>
<td>29.3</td>
</tr>
<tr>
<td>(D) [mm/mm]</td>
<td>3.401E9</td>
</tr>
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Table 3. Multiaxial material properties and curve-fit parameters
### Stress Ratio

<table>
<thead>
<tr>
<th>Stress Ratio</th>
<th>$\sigma_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 to 1</td>
<td>0.25$\tau_a$</td>
</tr>
<tr>
<td>3 to 1</td>
<td>0.33$\tau_a$</td>
</tr>
<tr>
<td>3 to 2</td>
<td>0.67$\tau_a$</td>
</tr>
<tr>
<td>4 to 3</td>
<td>0.75$\tau_a$</td>
</tr>
</tbody>
</table>

Table 4. Torsional shear to axial biaxial stress ratio

\[
\sigma_v = \sqrt{\frac{1}{2} \left[ 2\sigma_x^2 + 6\tau^2 \right]}
\]  

(32)

Figure 46. Multiaxial fatigue life prediction at various torsional shear to axial biaxial stress ratios

The assumption of a 3-to-2 torsional shear to axial biaxial stress state produced the closest correlation between the theoretical critical life prediction and the adjusted experimental data. This assumption was further validated by comparison of the actual crack path geometry of a typical torsional shear specimen to the crack path geometry of a fatigue specimen subjected to a 3-to-2 torsional shear to axial biaxial stress state presented in schematic form for various biaxial stress ratios in Figure 47 [65].
Figure 47. Crack path geometries for torsional shear stress, 3-to-2 torsional shear to axial biaxial stress, 3-to-5 torsional shear to axial biaxial stress, and axial stress [65]

Additionally, both the mismatch between the shear modulus and elastic modulus of Aluminum 6061-T6 (~3x difference) and the oversized geometry of the test specimens support the possibility of load frame misalignment causing a 3-to-2 torsional shear to axial biaxiality. Application of this assumption to both the critical life and full life predictions is presented versus adjusted experimental data in Figure 48. These results illustrated two primary conclusions. First, the assumed 3-to-2 torsional shear to axial biaxial stress state verified by crack path comparisons approximated the stress state present during testing. Second, the energy-based fatigue life prediction method was shown to be capable of determining the fatigue life not only of structures subjected to a dynamic torsional shear stress state, but also of structures subjected to a dynamic multiaxial stress state.
Figure 48. Multiaxial critical life and full life predictions versus experimental data
Chapter 4: Axial Isothermal-Mechanical Fatigue Life Prediction

4.1: Axial Isothermal-Mechanical Experimental Results

As detailed in Section 2.2.2: Axial Isothermal-Mechanical Testing Plans, four experimental methods were employed during axial isothermal-mechanical testing. These methods include quasi-static testing, dynamic testing, dynamic frequency evaluation, and energy dissipation history testing.

4.1.1: Axial Isothermal-Mechanical Quasi-Static Testing

Several axial isothermal-mechanical quasi-static curves were developed. Stress to necking was calculated as the ratio between the load vector and the initial area, while the ultimate stress was calculated as the ratio between the final load and the final area. Strain was directly measured with the high-temperature extensometer. Figure 49 details representative quasi-static curves at temperatures $T_0$, $T_1$, $T_2$, and $T_3$.

![Figure 49. Typical axial isothermal-mechanical quasi-static testing results](image-url)
The primary trend of quasi-static testing below the creep activation threshold was found as: at temperature $T_{n+1}$, the specimen was less capable of sustaining plastic loading but exhibited higher ductility than at temperature $T_n$. Samples of the total strain energy dissipated during quasi-static testing is presented in Table 5.

<table>
<thead>
<tr>
<th>Temperature [K]</th>
<th>Total Strain Energy [MJ/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>298</td>
<td>351</td>
</tr>
<tr>
<td>298</td>
<td>370</td>
</tr>
<tr>
<td>298</td>
<td>360</td>
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<td>327</td>
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<td>398</td>
<td>376</td>
</tr>
<tr>
<td>398</td>
<td>369</td>
</tr>
</tbody>
</table>

Table 5. Strain energy dissipated during quasi-static testing at temperature

The strain energies dissipated at temperatures $T_1$, $T_2$, and $T_3$ were found to be distributed within two standard deviations of the average strain energy dissipated at temperature $T_0$ (Figure 50).
As such, quasi-static strain energy density was considered to be temperature-independent for operating temperatures below the creep activation threshold. This constancy was attributed to the trend of increasing ductility offsetting decreasing ultimate stress. Thus, the single set of quasi-static material properties and curve-fit parameters presented in Table 6 were utilized for life prediction across all operating temperatures. These properties and parameters were generated using Appendix A.1: Quasi-Static Data Reduction Procedure.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\sigma_u$ [MPa]</td>
<td>469</td>
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<td>$\varepsilon_u$</td>
<td>8.53E-01</td>
</tr>
<tr>
<td>$E$ [MPa]</td>
<td>63639</td>
</tr>
<tr>
<td>$\sigma_0$ [MPa]</td>
<td>57</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>4.30E-04</td>
</tr>
</tbody>
</table>

Table 6. Axial isothermal-mechanical quasi-static material properties and curve fit parameters
4.1.2: Axial Isothermal-Mechanical Dynamic Testing

Axial isothermal-mechanical dynamic testing was performed at fully-reversed stress amplitudes between 165 MPa and 276 MPa for temperatures $T_0$, $T_1$, $T_2$, and $T_3$. Results are shown in Figure 51.

![Figure 51](image)

In general, results displayed a trend of decreasing stress amplitude with increasing operating temperature for a constant number of cycles to failure. Thus, greater damage occurred to specimens at higher temperature during dynamic loading.

4.1.3: Axial Isothermal-Mechanical Dynamic Frequency Evaluation Testing

Dynamic frequency evaluation testing was performed across multiple operating temperatures and fully-reversed alternating axial stresses at seven operating frequencies between 0.05 Hz and 4 Hz. Representative results of a test conducted at temperature $T_3$ and a fully-reversed axial stress of 221 MPa are presented in Figure 52.
The ideal operating frequency was found to be approximately 0.25 Hz regardless of operating temperature. This indicated that the dominant mechanism affecting anelastic behavior during isothermal testing was relaxation time governed by the operating frequency. The results of this testing established the optimal operating frequency for subsequent axial isothermal-mechanical dynamic testing. Furthermore, cyclic stress-strain data generated during dynamic frequency evaluation testing was utilized to ascertain dynamic curve-fit parameters for temperatures $T_0$, $T_1$, and $T_3$ (Table 7). These parameters were developed using Appendix A.3: Dynamic Curve-Fit Parameter Procedure.

<table>
<thead>
<tr>
<th>Temperature [K]</th>
<th>$\sigma_c$ [MPa]</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
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<td>70</td>
<td>6084200</td>
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<td>348</td>
<td>54</td>
<td>14756200</td>
</tr>
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<td>398</td>
<td>49</td>
<td>56630500</td>
</tr>
</tbody>
</table>

Table 7. Axial isothermal-mechanical dynamic curve-fit parameters
4.1.4: Axial Isothermal-Mechanical Energy Dissipation History Testing

Axial isothermal-mechanical energy dissipation history testing was conducted across the four operating temperatures at axial stress amplitudes matching those utilized during dynamic testing. Typical results at each operating temperature for the same stress amplitude are presented in Figure 53. To permit easy comparison, Figure 53 has been presented such that the ordinate of each data set was normalized by the maximum strain energy dissipation of that set, and the abscissa of each data set was normalized by the number of cycles to failure of that set.

![Graph showing normalized strain energy dissipation](image)

Figure 53. Axial isothermal-mechanical energy dissipation history testing results

At each operating temperature the value of the full life parameter was ascertained to be approximately 1.16 using Section 2.2.2: Axial Isothermal-Mechanical Testing Plans, Energy Dissipation History Testing and Appendix A.4: Energy Dissipation History Testing Data Reduction Procedure. As the value of this parameter nears unity, it was inferred that geometric changes were experienced late in the fatigue life of the specimen. This trend is characteristic of high-cycle fatigue, wherein the number of cycles
between crack formation and final failure are a small percentage of the total life of the structure.

4.2: Axial Isothermal-Mechanical Fatigue Life Prediction

Using the material properties and curve-fit parameters developed from quasi-static testing and dynamic frequency evaluation testing (detailed in Section 4.1.1: Axial Isothermal-Mechanical Quasi-Static Testing and Section 3.1.3: Axial Isothermal-Mechanical Dynamic Frequency Evaluation Testing), the critical life prediction was calculated for temperatures $T_0$, $T_1$, and $T_3$ (Figures 54, 55, and 56, respectively).

![Graph showing alternating axial stress (MPa) versus number of cycles for experimental data, theoretical critical life, and theoretical full life.]

Figure 54. Axial isothermal-mechanical critical life and full life predictions versus experimental data, $T_0$
Figure 55. Axial isothermal-mechanical critical life and full life predictions versus experimental data, $T_1$

Figure 56. Axial isothermal-mechanical critical life and full life predictions versus experimental data, $T_3$

An excellent correlation exists between the theoretical predictions and experimental data at each operating temperature. Thus, it has been shown that the isothermal-mechanical fatigue life prediction method can be applied separately at each temperature to successfully predict both the critical life and the full life of in-service Aluminum 6061-T6 structures subjected to fully-reversed axial mechanical loading and thermal loading below the creep activation threshold. These successes are further
extended through the development of a relationship between the dynamic curve-fit parameters and temperature (Figure 57).

![Figure 57. Axial isothermal-mechanical dynamic curve-fit parameters versus temperature](image)

Applying regression to the data presented in Figure 57, each dynamic curve-fit parameter was written as a function of temperature (Equations 33 and 34).

\[
\sigma_c(T) = 131.7 - 0.2068T \tag{33}
\]

\[
C(T) = 7308.4e^{0.022\nu} \tag{34}
\]

Equation 33, Equation 34, and the assumption of constant quasi-static strain energy dissipation were utilized to reformulate the original isothermal-mechanical life prediction equation (Equation 22), producing Equation 35.
\[
N_e(T) = \frac{\{\sigma_u [\varepsilon_u - \sigma_u / 2E] - \sigma_u \varepsilon_u [\cosh(\sigma_u / \sigma_c) - 1]\}}{\{2\sigma_c (T)/(C(T))\}[(\sigma_a / \sigma_c (T)) \sinh(2\sigma_a / \sigma_c (T)) - \cosh(2\sigma_a / \sigma_c (T)) - 1]\}}
\]

The full life remains as denoted in Equation 30. Utilizing the quasi-static material properties and curve-fit coefficients presented in Table 6 and setting \(T = T_2\), Equations 35 and 30 were solved for various fully-reversed stress ratios to produce theoretical critical life and full life estimates (Figure 58).

Figure 58. Axial isothermal-mechanical critical life and full life predictions versus experimental data, \(T_2\)

These results indicated that a strong correlation exists between the reformulated axial isothermal-mechanical life prediction method and experimental data. Thus, using the functions developed to describe the dynamic curve-fit parameters, the isothermal-life prediction method requires data only at temperature \(T_0\) to accurately assess the fatigue life of in-service Aluminum 6061-T6 structures subjected to fully-reversed axial mechanical loading and thermal loading below the creep threshold.
Chapter 5: Implementation of Alternative Constitutive Relationships

5.1: Axial Experimental Results

As detailed in Section 2.2.2: Axial Isothermal-Mechanical Testing Plans, four experimental methods were employed during alternative constitutive relationship testing. These methods include quasi-static testing, dynamic testing, dynamic frequency evaluation testing, and energy dissipation history testing.

5.1.1: Axial Quasi-Static Testing

Several quasi-static curves were developed for Titanium 6Al-4V. Stress to necking was calculated as the ratio between the load vector and the initial area, while the ultimate stress was calculated as the ratio between the final load and the final area. Strain was directly measured with the high-temperature extensometer. Figure 59 details a representative quasi-static curve.

![Figure 59. Typical Titanium 6Al-4V axial quasi-static testing results](image.png)
Relevant Titanium 6Al-4V material properties and curve-fit parameters for the original method, the modified Ramberg-Osgood approach, and the modified Halford approach generated from these results are presented in Table 8. Aluminum 6061-T6 material properties and curve-fit parameters for the modified Ramberg-Osgood approach and the modified Halford approach generated from previous axial isothermal-mechanical results are presented in Table 9. These properties and parameters were developed using Appendix A.1: Quasi-Static Data Reduction Procedure.

### Table 8. Titanium 6Al-4V axial quasi-static material properties and curve fit parameters, original method, modified Ramberg-Osgood approach, and modified Halford approach

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Approach</th>
<th>Original</th>
<th>modified Ramberg-Osgood</th>
<th>modified Halford</th>
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</tr>
<tr>
<td>$A$ [#]</td>
<td></td>
<td>-</td>
<td></td>
<td>4169</td>
</tr>
<tr>
<td>$\Delta\sigma_p$ [MPa]</td>
<td></td>
<td>-</td>
<td></td>
<td>1495</td>
</tr>
</tbody>
</table>

### Table 9. Aluminum 6061-T6 axial isothermal-mechanical quasi-static material properties and curve-fit parameters, modified Ramberg-Osgood approach, and modified Halford approach

<table>
<thead>
<tr>
<th>Temperature [K]</th>
<th>modified Ramberg-Osgood approach</th>
<th>modified Halford approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n$ [#] $K_{qs}$ [MPa]</td>
<td>$n$ [#] $A$ [#]</td>
</tr>
<tr>
<td>298</td>
<td>0.153 508</td>
<td>0.153 571</td>
</tr>
<tr>
<td>348</td>
<td>0.156 504</td>
<td>0.156 686</td>
</tr>
<tr>
<td>373</td>
<td>0.148 493</td>
<td>0.148 506</td>
</tr>
<tr>
<td>398</td>
<td>0.149 447</td>
<td>0.149 505</td>
</tr>
</tbody>
</table>

Table 9. Aluminum 6061-T6 axial isothermal-mechanical quasi-static material properties and curve-fit parameters, modified Ramberg-Osgood approach, and modified Halford approach
The relative standard deviation of the Titanium 6Al-4V material properties did not exceed 7% across four quasi-static tests. Thus, the properties presented in Table 8 were assumed to be representative of the material. Additionally, quasi-static material properties presented in Table 9 were assumed to be representative of Aluminum 6061-T6 at the specified temperatures.

5.1.2: Axial Dynamic Testing

Axial dynamic testing was performed on Titanium 6Al-4V specimens at fully-reversed stress amplitudes between 593 MPa and 910 MPa. Test results are shown in Figure 60.

![Figure 60. Titanium 6Al-4V axial dynamic testing results](image)

5.1.3: Axial Dynamic Frequency Evaluation Testing

Dynamic frequency evaluation testing was performed on Titanium 6Al-4V specimens across multiple fully-reversed alternating axial stresses at six operating frequencies between 0.005 Hz and 0.5 Hz. Representative results of a test conducted at a fully-reversed axial stress of 689 MPa are presented in Figure 61.
These results established the optimal operating frequency – 0.05 Hz – for subsequent axial testing. Data generated during dynamic frequency evaluation testing was also utilized to ascertain Titanium 6Al-4V dynamic curve-fit parameters for the original method, the modified Ramberg-Osgood approach, and the modified Halford approach (Table 10). Aluminum 6061-T6 material properties and curve-fit parameters for the modified Ramberg-Osgood approach and the modified Halford approach from previous axial isothermal-mechanical results are presented in Table 11. These parameters were found using Appendix A.3: Dynamic Curve Fit Parameter Procedure.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Original</th>
<th>modified Ramberg-Osgood</th>
<th>modified Halford</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_c$ [MPa]</td>
<td>266</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$C$ [mm/mm]</td>
<td>1125000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$n$ [#]</td>
<td>-</td>
<td>0.125</td>
<td>0.125</td>
</tr>
<tr>
<td>$K_d$ [MPa]</td>
<td>-</td>
<td>4626</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta \epsilon_p$ [mm/mm]</td>
<td>-</td>
<td>-</td>
<td>5.92E-05</td>
</tr>
</tbody>
</table>

Table 10. Titanium 6Al-4V axial dynamic curve fit parameters, original method, modified Ramberg-Osgood approach, and modified Halford approach
<table>
<thead>
<tr>
<th>Temperature [K]</th>
<th>modified Ramberg-Osgood approach</th>
<th>modified Halford approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( n )[#] ( K_d )[MPa]</td>
<td>( n )[#] ( \Delta \varepsilon_p )[mm/mm]</td>
</tr>
<tr>
<td>298</td>
<td>0.153</td>
<td>2049</td>
</tr>
<tr>
<td>348</td>
<td>0.156</td>
<td>1995</td>
</tr>
<tr>
<td>373</td>
<td>0.148</td>
<td>1814</td>
</tr>
<tr>
<td>398</td>
<td>0.149</td>
<td>1736</td>
</tr>
</tbody>
</table>

Table 11. Aluminum 6061-T6 axial isothermal-mechanical dynamic curve fit parameters, modified Ramberg-Osgood approach, and modified Halford approach

5.1.4: Axial Energy Dissipation History Testing

Axial energy dissipation history testing was conducted on Titanium 6Al-4V specimens at axial stress amplitudes matching those utilized during dynamic testing. Typical results are presented in Figure 62. For easy comparison, Figure 62 has been presented such that the ordinate of the data set was normalized by the maximum strain energy dissipation of that set, and the abscissa of the data set was normalized by the number of cycles to failure of that set.

![Normalized Number of Cycles](image)

Figure 62. Titanium 6Al-4V Axial energy dissipation history testing results
For Titanium 6Al-4V specimens, the full life coefficient was determined to be 1.01 using both Section 2.2.2: Axial Isothermal-Mechanical Testing Plans, Energy Dissipation History Testing and Appendix A.4: Energy Dissipation History Testing Data Reduction Procedure. As the value of this parameter nears unity, it was inferred that the structure experienced large geometric changes only in the final cycles. This trend is characteristic of high-cycle fatigue, wherein the number of cycles between crack formation and final failure are a small percentage of the total life of the structure.

5.2: Life Prediction with Alternative Constitutive Relationships

5.2.1: Life Prediction of Titanium 6Al-4V Specimens at Temperature $T_0$

Fatigue life predictions utilizing the original method, the modified Ramberg-Osgood approach, and the modified Halford approach are presented in Figure 63 for Titanium 6Al-4V specimens fatigued at temperature $T_0$. Figure 64 directly compares the accuracy of each prediction against the known number of cycles to failure at a given stress amplitude. Note that the line denoting a perfect prediction has a slope of unity and 30% bounds were included to clarify accuracy.

Figure 63. Titanium 6Al-4V axial critical life prediction versus experimental data, $T_0$
Examination of Figure 63 indicated that the modified Halford approach accurately predicted the fatigue life of the Titanium 6Al-4V specimens across the low-cycle and early high-cycle fatigue regimes; comparatively, both the original method and the modified Ramberg-Osgood approach failed to predict the fatigue life in the low-cycle or early high-cycle regions with any accuracy. This assessment was bolstered by inspection of Figure 64, which indicated most of the modified Halford approach predictions fell within the 30% bounds.

For Titanium 6Al-4V at temperature $T_0$, the predictive accuracy of the modified Halford approach far exceeded that of either the original method or the modified Ramberg-Osgood approach; however, the other factors contributing to the fitness of the method must still be considered. Between these methods, the modified Halford approach required the fewest material properties and curve-fit parameters: the elastic modulus, $E$, and the strain hardening exponent, $n$, were ascertained from quasi-static data; the plastic strain range at one stress amplitude, $\Delta\varepsilon_p$, was determined from dynamic data; and the values of $A$ and $\Delta\sigma_p$ were determined by minimization of the error between the
experimentally-determined dynamic strain energy derived from the stable region of a
dynamic test at a single stress amplitude and the theoretical dynamic strain energy
estimated by the application of Equation 24 to Equation 7 for the same stress amplitude.
Despite this advantage, the modified Halford approach did require optimization over two
variables, $A$ and $\Delta \sigma_p$. Overall, the unparalleled accuracy of the modified Halford approach
across the low-cycle and early high-cycle regions outweighed the challenges presented by
the necessity of optimizing over two variables.

5.2.2: Life Prediction of Aluminum 6061-T6 at Temperatures $T_0$, $T_1$, $T_2$, and $T_3$

Fatigue life predictions utilizing the original method, the modified Ramberg-
Osgood approach, and the modified Halford approach are presented in Figures 65, 66, 67,
and 68 for Aluminum 6061-T6 specimens fatigued at temperatures $T_0$, $T_1$, $T_2$, and $T_3$,
respectively. Figures 69, 70, 71, and 72 directly compare the accuracy of each prediction
against the known number of cycles to failure at a given stress amplitude for temperatures
$T_0$, $T_1$, $T_2$, and $T_3$, respectively. Note that the line denoting a perfect prediction has a slope
of unity and 30% bounds were included to clarify accuracy.
Figure 66. Aluminum 6061-T6 axial critical life prediction versus experimental data, $T_1$

Figure 67. Aluminum 6061-T6 axial critical life prediction versus experimental data, $T_2$

Figure 68. Aluminum 6061-T6 axial critical life prediction versus experimental data, $T_3$
Figure 69. Aluminum 6061-T6 life prediction accuracy comparison, $T_0$

Figure 70. Aluminum 6061-T6 life prediction accuracy comparison, $T_1$

Figure 71. Aluminum 6061-T6 life prediction accuracy comparison, $T_2$
Scrutiny of Figures 65-68 indicated the accuracy of both the original method and modified Ramberg-Osgood approach predictions; comparatively, the modified Halford approach predictions were consistently worse. This distinction was expected, as the Aluminum 6061-T6 material system does not display the endurance limit phenomenon evident in Titanium 6Al-4V. Examination of Figures 69-72 showed that the original method was slightly more accurate than the modified Ramberg-Osgood approach; however, this increased accuracy was offset by the increased difficulty of implementing the original method. The original method required the determination of eight material properties and curve-fit parameters prior to application: the elastic modulus, $E$, the ultimate stress, $\sigma_u$, the ultimate strain, $\varepsilon_u$, and the curve-fit constants $\varepsilon_o$ and $\sigma_o$ (a function of the ultimate stress, $\sigma_u$, the yield stress, $\sigma_y$, and the necking strain, $\varepsilon_n$) found from quasi-static data; and the values of the curve-fit parameters $C$ and $\sigma_c$ determined through the defined procedure. By comparison, the modified Ramberg-Osgood approach required six material properties and curve-fit parameters: the elastic modulus, $E$, the ultimate stress, $\sigma_u$, the ultimate strain, $\varepsilon_u$, and the strain hardening exponent, $n$, from quasi-static; the
curve-fit constant $K_{qs}$ from quasi-static data, and the curve-fit constant $K_d$ determined through the defined procedure. As the modified Ramberg-Osgood approach used fewer material properties and curve-fit parameters than the original method, it offered the best compromise between simplicity and predictive accuracy.
Chapter 6: Conclusions

The energy-based method envisioned by Jasper, realized by Stowell, and extended by Scott-Emuakpor et al. possesses immense utility to the design engineer. As the method stands, it provides the capability to predict the fatigue life of engineered structures subjected to room temperature dynamic axial, axial with mean stress, uniaxial bending, transverse shear, and torsional shear loading; however, the prior energy-based method provided no consideration of the effects of dynamic multiaxial loading, the effect of isothermal loading applied during dynamic mechanical loading, or the possibility of alternative constitutive equations to improve the accuracy of the prediction. It was the great contribution of this manuscript to address each of these pressing quandaries through multiple research efforts.

The first of these research efforts addressed the impact of multiaxial loading on experimental data and the critical life and full life predictions attempting to predict that data. Load frame misalignment during testing caused unanticipated dynamic bending loading in conjunction with the applied dynamic shear loading to act on the torsional shear specimen. In response, a multiaxial fatigue life prediction equation was developed and utilized with great success to accurately predict both the critical life and the full life. This effort was further aided by crack path propagation analyses, which determined the ratio of biaxial loading experienced by the specimen and showed that a design engineer could determine the degree of unexpected multiaxiality from experimental data.
The second of these research efforts addressed the method of determining the critical life and full life of structures subjected to combined mechanical and isothermal loading. To start, an isothermal-mechanical testing capability was developed from a given set of demanding testing requirements. Various methods of heating were explored, and considerations were made for operation and data collection under elevated temperature conditions. Specimen and induction coil geometries were optimized to provide even thermal loading over the duration of testing. With this task complete, an energy-based axial isothermal-mechanical fatigue life criterion was constructed for the Aluminum 6061-T6 system, using the previously-developed room temperature axial approach as a basis. This research comprised several advancements to the energy-based theory, including:

1. The discovery of constant quasi-static strain energy dissipation below the creep activation threshold.
2. The application of room temperature curve-fitting methods to elevated temperature data.
3. The development of dynamic curve-fit parameters as functions of temperature, thereby negating the need to perform elevated isothermal-mechanical testing below the creep threshold for the given material system.

Validation of the axial isothermal-mechanical fatigue life prediction method was accomplished by comparison of the critical life and full life predictions against experimental data. This comparison showed a strong correlation between the two life predictions and the acquired data.
The third of these research efforts was devoted to examining the predictive capability of alternative constitutive equations through the lens of the energy-based fatigue life prediction method. To address this issue, new quasi-static and dynamic constitutive equations were applied to the existing energy-based framework with good results. Compared to the original method, the modified Halford approach was shown to improve the accuracy of the life prediction across every fatigue regime for the Titanium 6Al-4V system due to the incorporation of a mechanism to simulate the decrease in dynamic strain energy dissipation with increasing cycles. As expected, the gains in predictive accuracy obtained through the modified Halford approach were not realized for the Aluminum 6061-T6 system, which does not exhibit an endurance limit. In that case, it was shown that both the original method and the modified Ramberg-Osgood approach were equally proficient in predicting the fatigue life of the Aluminum 6061-T6 system at operating temperatures ranging from room temperature to just below the creep activation threshold. However, it was shown that the modified Ramberg-Osgood prediction provided the best compromise between simplicity and predictive accuracy.

These three research efforts represent considerable contributions to the energy-based life prediction framework, greatly broadening the scope of the method and furthering the endeavor of reducing fatigue failures in real-world situations through the application of research-based engineering tools.
Chapter 7: Future Work

Despite the considerable progress documented within this manuscript, several methods of expanding or improving the energy-based fatigue life prediction framework exist. Three specific areas of improvement are identified: first, determining the effect of mean stress on fatigue life; second, expanding the energy-based method into the thermo-mechanical fatigue regime; and third, investigating the effect of creep on fatigue life.

One area of improvement for the energy-based life prediction framework stems from the effect of mean stress on fatigue life. Scott-Emuakpor et al. previously investigated this effect for stress reversal ratios greater than -1 (tensile mean stresses) \[45\]. The effect of mean stress on the energy dissipated per cycle was determined as presented in Equation 36.

\[
W_d = \sigma_{pp} (\varepsilon_{pp} + \varepsilon_{mean}) - \int_{\sigma_{mean}}^{\sigma_{pp} + \sigma_{mean}} \varepsilon_d d\sigma - \int_{0}^{\sigma_{pp}} \varepsilon_d d\sigma \quad (36)
\]

As shown by Equation 36, the effect of mean loading is to increase the strain energy dissipated per cycle, thereby decreasing the fatigue life. However, these initial investigations did not consider the effect of stress reversal ratios lesser than -1 (compressive mean stresses) on fatigue life. If the fatigue life of structures subjected to compressive mean stresses trended oppositely those subjected to tensile mean stresses,
then the compressively loaded structures should survive considerably longer. However, data from the Titanium 6Al-4V material system indicates that the fatigue life of structures subjected to dynamic compressive mean loading remains roughly the same as for fully-reversed dynamic loading [36]. Thus, some material process negates the advantage of compressive mean loading.

Complete understanding of the energy-based life prediction framework requires investigation of this situation. The best method of investigation involves the direct examination of the damage accrued during the dynamic process for compressive, zero, and tensile mean loading. How damage from compressive mean loading, zero mean loading, and tensile mean loading fundamentally differs, and how one phase of the dynamic process affects another phase of the dynamic process, would provide greater insight into the effects of mean loading. From this work, a new dynamic energy dissipation quantity will be proposed. Early results of the effect of sequential dynamic loading phases – from compressive loading, to zero loading, to tensile loading – are presented in Figure 7 with respect to energy dissipation. Results indicate that the dynamic compressive mean loading causes more damage than the lowest dynamic tensile mean loading. This result is unexpected, and indicates that this area merits further research.
Figure 73. Strain energy dissipation versus various cycles across five mean stress amplitudes

The second area for improvement concerns expansion of the energy-based life prediction framework into the thermo-mechanical fatigue regime. Thermo-mechanical fatigue poses dangers commensurate with those caused by isothermal-mechanical fatigue; yet, understanding of thermo-mechanical damage processes remains limited due to the challenging nature of the research. Fortunately, the isothermal-mechanical fatigue capability previously devised is readily converted to thermo-mechanical fatigue through a series of targeted upgrades. These upgrades include the addition of a dedicated temperature control axis, advanced software to facilitate adjustable-phase TMF loading waveforms, and an active cooling rake to enhance thermal control. Once completed, these upgrades will permit phase-locked thermo-mechanical cycling at frequencies of ~0.1 Hz.

With an experimental capability in place, it is necessary to devise the theory supporting an energy-based fatigue life prediction model. The initial conditions explored will mirror those found during isothermal-mechanical fatigue testing, including temperatures below creep activation threshold and fixed-free thermo-mechanical
boundary conditions. Thus, the preliminary theoretical model will draw heavily from the isothermal-mechanical fatigue life prediction equation (Equation 22). The assumption of temperature-independent quasi-static strain energy will again be employed, making the numerator of Equation 22 constant. However, the denominator will necessarily be dependent not only on the stress amplitude but also on the variation of temperature with mechanical cycling. Additional consideration must be given to the phase angle between the thermal and mechanical loading, which will affect the strain energy dissipated per cycle.

The third area of improvement relates to the effect of creep on the fatigue life of a structure. Creep is a temperature-dependent, time-dependent irreversible straining process found in metals stressed below the yield limit. This process is currently not addressed by the energy-based fatigue life prediction framework. Incorporation of the effect of creep would require time-dependency of the major material properties and curve-fit parameters in a similar fashion as the current dependency on temperature. However, unlike temperature the creep process may affect the quasi-static strain energy dissipation, requiring modifications to the numerator of the life prediction equation.
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Appendix A: Data Reduction Procedures

A.1: Quasi-Static Data Reduction Procedure

Quasi-static data reduction was performed using an established procedure developed as a spreadsheet in Microsoft Excel. The data to be reduced generally included a time vector \((t)\) in seconds, a displacement vector \((d)\) in inches, a load vector \((L)\) in pounds-force, and a strain vector \((\varepsilon)\) in microstrain. The following steps outline the procedure used to reduce this data: Note that for torsional shear data, the engineering stress vector did not need to be converted to a true stress vector.

1. The time vector \((t)\) and displacement vector \((d)\) were removed.

2. The strain vector \((\varepsilon)\) and load vector \((L)\) were smoothed using a moving average over five samples.

3. The initial area of the specimen \((A_i)\) was input in units of inches squared.

4. The strain vector was converted from microstrain to strain using Equation A1.

\[
\varepsilon_{\text{strain}} = \varepsilon_{\text{microstrain}} \times \left(\frac{1}{1000000}\right) \times \left(\frac{\text{strain}}{\text{microstrain}}\right) \quad (A1)
\]

5. The engineering stress vector \((\sigma_{\text{eng}})\) was determined from the load vector \((L)\) and initial area \((A_i)\) by Equation A2 in units of pound-force per inches squared.
\[ \sigma_{\text{eng}} = \frac{L}{A_i} \]  \hspace{1cm} (A2)

6. The true strain vector \((\varepsilon_{\text{true}})\) was determined from the engineering strain vector \((\varepsilon_{\text{eng}})\) by Equation A3.

\[ \varepsilon_{\text{true}} = \ln(1 + \varepsilon_{\text{eng}}) \]  \hspace{1cm} (A3)

7. The true stress vector \((\sigma_{\text{true}})\) was determined from the engineering stress vector \((\sigma_{\text{eng}})\) and the engineering strain vector \((\varepsilon_{\text{eng}})\) by Equation A4.

\[ \sigma_{\text{true}} = \sigma_{\text{eng}} (1 + \varepsilon_{\text{eng}}) \]  \hspace{1cm} (A4)

8. The true stress vector \((\sigma_{\text{true}})\) and true strain vector \((\varepsilon_{\text{true}})\) were truncated at the point of necking, represented by the maximum engineering stress \((\sigma_{\text{eng}})\). Beyond necking, the cross-sectional area of the specimen reduced in a nonlinear fashion, invalidating further recorded data.

9. The necking true stress \((\sigma_{n,\text{true}})\) and necking true strain \((\varepsilon_{n,\text{true}})\) were determined as the last true stress \((\sigma_{\text{true}})\) and true strain \((\varepsilon_{\text{true}})\) values, respectively.

10. The final area \((A_f)\) was input in units of inches squared.

11. The ultimate strain \((\varepsilon_u)\) was determined from the final area \((A_f)\) and the initial area \((A_i)\) by Equation A5 and added to the end of the true strain vector \((\varepsilon_{\text{true}})\).
\[ \varepsilon_u = LN(A_f/A_f) \] (A5)

12. The ultimate stress (\(\sigma_u\)) was determined from the load just prior to failure (\(L_f\)) and the final area (\(A_f\)) by Equation A6 and added to the end of the true stress vector (\(\sigma_{true}\)).

\[ \sigma_u = L_f/A_f \] (A6)

13. The modulus of elasticity (\(E\)) was determined as the slope of the linear region of the true stress (\(\sigma_{true}\)) – true strain (\(\varepsilon_{true}\)) vector.

14. The tensile yield stress (\(\sigma_y\)) was determined as the true stress (\(\sigma_{true}\)) at which the quantity in Equation A7 attained a negative value. Equation A7 represents the true strain vector (\(\varepsilon_{true}\)) subtracted from the 0.2% offset strain determined from the true stress vector (\(\sigma_{true}\)) and the modulus of elasticity (\(E\)).

\[ Difference = ((\sigma_{true}/E) + 0.002) - \varepsilon_{true} \] (A7)

15. The strain hardening exponent (\(n\)) was chosen to reduce the error between the theoretical and experimental quasi-static curves.
16. The material parameter ($\sigma_o$) was determined from the ultimate stress ($\sigma_u$), the tensile yield stress ($\sigma_y$), and the necking true strain ($\varepsilon_{n,\text{true}}$) by Equation A8.

$$\sigma_o = (\sigma_u - \sigma_y) / LN(\varepsilon_{n,\text{true}}/0.002)$$ (A8)

17. The curve-fit parameters ($\varepsilon_o$) and ($K_{qs}$) were chosen to reduce the error between the theoretical and experimental quasi-static curves.

A.2: Dynamic Frequency Evaluation Data Reduction Procedure

Dynamic frequency evaluation data reduction was performed using an established procedure developed in MATLAB. The data to be reduced generally included a load vector ($L$) in pounds-force and a strain vector ($\varepsilon$) in microstrain. The following m-file was used to reduce this data:

```matlab
clear all; close all; clc;

% Retrieve files to be processed
[filename,pathname] = uigetfile({'*.txt','Text Files (*.txt)';},...
   'MultiSelect','on'); clc;

if isequal(filename,0) || isequal(pathname,0)
    disp('No files were selected. Session terminated.')
    pause(2)
    clear all; clc;
    break
else
```
if ischar(filename) == 1
    length_filename = 1;
else
    length_filename = length(filename);
end
end

% Disable xlswrite warning.
warning off MATLAB:xlswrite:AddSheet

% Pre-allocate variables
avg_plastic_energy = zeros(length_filename,1);
rstd_plastic_energy = zeros(length_filename,1);

for file_number = 1:length_filename
    % Read file data using function read_data.
    if ischar(filename) == 1
        [~,loop_data] = read_data(strcat(pathname,filename));
    else
        [~,loop_data] = read_data([pathname filename{1,file_number}]);
    end
    if file_number == 1
        data_points = input('Data-points per cycle:'); clc;
        area = pi*input('Specimen diameter:')^2/4; clc;
        freqs = input('Frequencies [#;#;...]:'); clc;
    end
end
end

% Transform continuous data to cycle-separated data.
matrix_size = size(loop_data);
for i = 1:floor(matrix_size(1)/data_points)
    if i == 1
        loops = [loop_data(1:data_points,1)/1E6...
            loop_data(1:data_points,2)/area];
    else
        loops(:,:,i) = [loop_data(((i-1)*data_points)+...
            1:i*data_points,1)/1E6 loop_data(((i-1)*...
            data_points)+1:i*data_points,2)/area];
    end
end

% Determine the number of cycles in the retrieved data file.
number_of_cycles = size(loops);

% Pre-allocate variables.
plastic_energy = zeros(number_of_cycles(3),2);

% Determine cyclic energy with polyarea.
for cycle_number = 1:number_of_cycles(3)
    [loop] = zero_check(loops(:,:,cycle_number));
    loop = [smooth(loop(:,1),4) smooth(loop(:,2),4)];
    plastic_energy(cycle_number,:) = [cycle_number polyarea(loop(:,1),...
% Write each file to an excel sheet.
result_name = char(filename(file_number));
xlswrite(strcat(char(pathname),result_name(1:7)),plastic_energy,...
strcat(num2str(file_number)));

% Determine average properties based on frequency.
if length(freqs) > 1
    avg_plastic_energy(file_number,1) = freqs(file_number);
    avg_plastic_energy(file_number,2) =
    mean(plastic_energy(length(plastic_energy)/4:length(plastic_energy)-1,2));
    rstd_plastic_energy(file_number,1) = freqs(file_number);
    rstd_plastic_energy(file_number,2) =
    std(plastic_energy(length(plastic_energy)/4:length(plastic_energy)-1,2))/mean(plastic_energy(length(plastic_energy)/4:length(plastic_energy)-1,2))*100;
end

% Concatenate plastic_energy matrices into total energy matrix.
if file_number == 1
    total_energy = plastic_energy;
else
    plastic_energy(:,1) = total_energy(end,1)+plastic_energy(:,1);
    total_energy = cat(1,total_energy,plastic_energy);
% Plot total energy.
if input('Generate plots? (Y/N):','s') == 'Y';
    if length(freqs) == 1
        figure(1)
        plot(total_energy(:,1),total_energy(:,2),'bo'); grid minor;
        title('Plastic Energy vs. Cycles')
        xlabel('Cycles [#]')
        ylabel('Plastic Energy [lbf-in./in.^3]')
    else
        figure(1)
        plot(avg_plastic_energy(:,1),avg_plastic_energy(:,2),'bo'); grid minor;
        title('Average Plastic Energy vs. Frequency')
        xlabel('Frequency [Hz]')
        ylabel('Average Plastic Energy [lbf-in./in.^3]')
    end
    clc;
end
%Write data to file.
if length(freqs) == 1
    xlswrite(strcat(char(pathname),result_name(1:7)),total_energy,'Total');
else
    x1swrite(strcat(char(pathname),result_name(1:7)), [avg_plastic_energy...
            rstd_plastic_energy(:,2)], 'Average Plastic');
end

%Delete empty Excel sheets.
DeleteEmptyExcelSheets(strcat(char(pathname), result_name(1:7), '.xls'));
delete *.asv

A.3: Dynamic Curve-Fit Parameter Procedure

Determination of dynamic curve-fit parameters was performed using an established procedure developed in MATLAB. The data to be reduced generally included a load vector ($L$) in pounds-force and a strain vector ($\varepsilon$) in microstrain. The following m-file was used to reduce this data:

clear all; close all; clc;
stress = ; % Stress amplitude.
WD = ; % Measured dynamic strain energy.
cycles = ; % Measured cycles to failure.
b = ; % Halford constant.
n = ; % Cyclic strain hardening.
del_eps = ; % Plastic strain range.
E = ; % Elastic modulus.
Est = ; % Elastic modulus, Steel.
search_space = ; % [C;Sigma;D] search space.

% C/sigma_C, NS(WQS_S, C, sigma_c)
C = search_space(1,1);
for i = 1:search_space(1,3)
    sigma_c = search_space(2,1);
    for j = 1:search_space(2,3);
        ER(j,1) = sigma_c;
        ER(j,2) = abs(WD - ((2*sigma_c/C)*((stress/sigma_c)*sinh(2*stress/...
                      sigma_c)-(cosh(2*stress/sigma_c)-1))))/WD*100;
        sigma_c = sigma_c+search_space(2,2);
    end
    [~,J] = min(ER(:,2));
    min_in_ER(i,:) = [ER(J,1) C ER(J,2)];
    C = C+search_space(1,2);
end
[~,I] = min(min_in_ER(:,3));
min_ER = min_in_ER(I,:);
disp('Optimizing for C and Sigma_C produces:)
disp('Sigma_C=')
disp(min_ER(1))
disp('C=')
disp(min_ER(2))
disp('Percent Error=')

disp(min_ER(3))

% K_c, NRO(WQS_RO, n, K_c)

K_c = search_space(1,1);

for i = 1:search_space(1,3);
    ER(i,1) = K_c;
    ER(i,2) = abs(WD-(((1-n)/(1+n))*(((2*stress)^(n+1))/K_c)^(1/n))/WD*100;
    K_c  = K_c+search_space(1,2);
end

 [~,I] = min(ER(:,2));

min_ER = ER(I,:);

disp('Optimizing for K_c.')

disp('K_c=')

disp(min_ER(1))

disp('Error=')

disp(min_ER(2))

% A/K_c, NH-RO

A = search_space(1,1);

for i = 1:search_space(1,3)
    K_c = search_space(2,1);
    for j = 1:search_space(2,3);
        ER(j,1) = K_c;
    end
end
ER(j,2) = abs(cycles-((2^(1/3))*A*(E/Est))/(((1-n)/(1+n))^...((stress^(n+1))/K_c)^(1/n)))^(3/2))/cycles*100;

K_c  = K_c+search_space(2,2);

end

[~,J] = min(ER(:,2));

min_in_ER(i,:)= [ER(J,1) A ER(J,2)];

A = A+search_space(1,2);

end

[~,I] = min(min_in_ER(:,3));

min_ER = min_in_ER(I,:);

disp('Optimizing for A and K_c.')

disp('K_c=')

disp(min_ER(1))

disp('A=')

disp(min_ER(2))

disp('Error=')

disp(min_ER(3))

delete *.asv

---

A.4: Energy Dissipation History Data Reduction Procedure

Energy dissipation history data reduction was performed using a procedure developed in MATLAB. The data to be reduced generally included a load vector (L) in
pounds-force and a strain vector ($\varepsilon$) in microstrain. The following m-file was used to reduce this data:

```matlab
- clear all; close all; clc;

  % Retrieve files to be processed

  [filename,pathname] = uigetfile({'*.txt','Text Files (*.txt)';},...
    'MultiSelect','on'); clc;

  if isequal(filename,0) || isequal(pathname,0)
    clear all; clc;
    break
  else
    if ischar(filename) == 1
      length_filename = 1;
    else
      length_filename = length(filename);
    end
  end

  % Input

  data_points = ;
  area = ;
  ii=1;

  % Read file data using function read_data.

  [~,loop_data] = read_data(strcat(pathname,filename));
```

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% Transform continuous data to cycle-separated data.

matrix_size = size(loop_data);

for i = 1:floor(matrix_size(1)/data_points)
    if i == 1
        loops = [loop_data(1:data_points,1)/1E6...
                 loop_data(1:data_points,2)/area];
    else
        loops(:,:,i) = [loop_data(((i-1)*data_points)+...
                                  1:i*data_points,1)/1E6 loop_data(((i-1)*...
                                  data_points)+1:i*data_points,2)/area];
    end
end

% Determine the number of cycles in the retrieved data file.

number_of_cycles = size(loops);

% Determine cyclic energy with polyarea. This portion also sorts the
% loops from the minimum stress and determines the point where the loop
% crosses the x-axis.

for cycle_number = 1:number_of_cycles(3)
    [loop] = zero_check(loops(:,:,cycle_number));
    loop = [smooth(loop(:,1),5) smooth(loop(:,2),5)];
    plastic_energy(cycle_number,:) = [cycle_number polyarea(loop(:,1),loop(:,2))];
    if cycle_number == 50
        % Further processing...
    end
end
mid_cycle(:,ii) = loop(:,1);
mid_cycle(:,ii+1) = loop(:,2);

ii = ii+2;
end
end

% Write excel data

warning off MATLAB:xlswrite:AddSheet
xlswrite(strcat(char(pathname),'New'),plastic_energy);
xlswrite(strcat(char(pathname),'New'),mid_cycle,'Mid-Cycle');
DeleteEmptyExcelSheets(strcat(char(pathname),'New','.xls'));
delete *.asv

---

A.5: Supporting MATLAB m-files

These m-files were employed by many of the data reduction programs. Several of them were downloaded from the MATLAB m-file database. They are presented here for completeness.

function [header_mat,data_mat,j] = read_data(file)

if nargin < 1
    error('Function requires one input argument');
elseif ~ischar(file)
    error('Input argument must be a string representing a filename');
end

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% Open the file. If this returns a -1, we did not open the file successfully.

fid = fopen(file);
if fid=-1
    error('File not found or permission denied.');
end

% Initialize loop variables
no_lines = 0;
max_line = 0;
ncols = 0;
data = [];
j=1;

% Start processing
line = fgetl(fid);
if ~ischar(line)
    disp('Warning: file contains no header and no data')
end;

while line~=-1
    [data, ncols, errmsg, nxtindex] = sscanf(line, '%f');
    while isempty(data) | nxtindex==1
        no_lines = no_lines+1;
        max_line = max([max_line, length(line)]);
        eval(['line', num2str(no_lines), '=line;'])
line = fgetl(fid);
if ~ischar(line)
    disp('Warning: file contains no data')
    break
end;
[data, ncols, errmsg, nxtindex] = sscanf(line, '%f');
end

header_mat(1:no_lines,1:max_line,j) = char(' '*ones(no_lines, max_line));
for i = 1:no_lines
    if length(eval(['line' num2str(i)]))
        varname = eval(['line' num2str(i)]);
    else
        varname = ['line',num2str(i)];
    end
    header_mat(i,1:length(varname),j)=varname;
end
data = [data; fscanf(fid, '%f')];

% Resize output data
    eval('data_mat(1:length(data)/ncols,1:ncols,j) = reshape(data, ncols, length(data)/ncols);
    line = fgetl(fid);
    j=j+1;
no_lines = 0;
end
fclose(fid);

function [loops_modified] = zero_checker(loops)
    for i = 1:length(loops)
        if loops(i,1) ~= 0
            loops_modified(i,:) = loops(i,:);
        end
    end
end
return

function DeleteEmptyExcelSheets(fileName)
% Check whether the file exists
if ~exist(fileName,'file')
    error([fileName ' does not exist !']);
else
% Check whether it is an Excel file
    typ = xlsfinfo(fileName);
    if ~strcmp(typ,'Microsoft Excel Spreadsheet')
        error([fileName ' not an Excel sheet !']);
    end
    end
% If fileName does not contain a "\" the name of the current path is added
% to fileName. The reason for this is that the full path is required for
% the command "excelObj.workbooks.Open(fileName)" to work properly.

if isempty(strfind(fileName,'\'))
    fileName = [cd '\' fileName];
end

excelObj = actxserver('Excel.Application');
excelWorkbook = excelObj.workbooks.Open(fileName);
worksheets = excelObj.sheets;

sheetIdx = 1;

sheetIdx2 = 1;

numSheets = worksheets.Count;

% Prevent beeps from sounding if we try to delete a non-empty worksheet.
excelObj.EnableSound = false;

% Loop over all sheets
while sheetIdx2 <= numSheets

    % Saves the current number of sheets in the workbook
    temp = worksheets.count;

    % Check whether the current worksheet is the last one. As there always
    % need to be at least one worksheet in an xls-file the last sheet must
    % not be deleted.
    if or(sheetIdx>1,numSheets-sheetIdx2>0)

        % worksheets.Item(sheetIdx).UsedRange.Count is the number of used cells.

        %...
% This will be 1 for an empty sheet. It may also be one for certain other
% cases but in those cases, it will beep and not actually delete the sheet.
if worksheets.Item(sheetIdx).UsedRange.Count == 1
    worksheets.Item(sheetIdx).Delete;
end
end

% Check whether the number of sheets has changed. If this is not the
% case the counter "sheetIdx" is increased by one.
if temp == worksheets.count;
    sheetIdx = sheetIdx + 1;
end

sheetIdx2 = sheetIdx2 + 1; % prevent endless loop...
end

excelObj.EnableSound = true;
excelWorkbook.Save;
excelWorkbook.Close(false);
excelObj.Quit;
delete(excelObj);
return;