Seismic Evaluation of Reinforced Concrete Columns and Collapse of Buildings

DISSERTATION

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By

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ABSTRACT

There are a large number of reinforced concrete (RC) buildings in seismically active areas of the world that are not designed and constructed in accordance with modern seismic design provisions. These buildings are vulnerable to severe damage or collapse due to their low lateral displacement capacity and rapid degradation of shear strength during strong ground motions. Typically, columns in such buildings lack adequate strength and ductility in reverse cyclic loading and experience brittle shear failure and loss of axial load carrying capacity. To assess vulnerability to earthquake damage and decide on the required level of retrofit, expected behavior of the columns in terms of strength and deformation capacity must be evaluated. This can be achieved by estimating the load-deformation response considering all potential failure mechanisms associated with axial, flexure and shear behavior.

This study presents an analytical model for estimation of lateral load-displacement response of reinforced concrete columns. In the proposed model, flexural deformations are calculated through fiber section analysis employing cracked concrete behavior while shear behavior is modeled through Disturbed Stress Field Model (DSFM). The interaction between flexural and shear mechanisms is considered through axial strains and concrete compression softening. The proposed model also considers other critical aspects such as deformations due to reinforcement slip, buckling of compression
bars, enhancement in strength and ductility of the concrete due to confinement, concrete tension stiffening and tension softening, and concrete compression softening effects. The comparison of predicted responses with experimental data indicates that proposed model is a suitable displacement-based evaluation approach that can be employed to accurately estimate load-displacement relationships and failure modes.

During a seismic activity, local structural failure in the lower-story columns can initiate vertical or progressive collapse in the buildings with inadequate ductility if gravity loads cannot be transferred to undamaged columns. After one or more columns fail, an alternate load path is needed to transfer the loads carried by failed member (s) to other structural members. If adjoining elements cannot resist and redistribute the additional loads, a series of failures will occur until entire or substantial part of the structure collapses. In order to investigate redistribution of gravity loads resulting from column failure, a study is presented using a progressive collapse model and experimental data. During the experimental phase, an existing reinforced concrete building of regular structural configuration was tested by physically removing one first-story exterior column. The structural response of the test building was monitored by recording strains and displacements of selected frame members in the vicinity of removed columns. During the computational phase of the research, two- and three-dimensional models of the test building were generated in SAP-2000. Linear static and non-linear dynamic analyses were performed for both two- and three-dimensional building models. The experimental and predicted structural performances are compared and effectiveness of currently available analysis and design methodologies is discussed.
DEDICATION

This document is dedicated to my parents, M.N.U.K Lodhi and Hafeez Akhtar
and my wife, Tabassum Shadab
and my children, Muhammad Ibrahim Khan Lodhi, Aliza Khan Lodhi, and Muhammad
Wasay Khan Lodhi
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# TABLE OF CONTENTS

Abstract .......................................................................................................................... ii
Dedication ....................................................................................................................... v
Acknowledgments .......................................................................................................... vi
Vita .................................................................................................................................. vii
List of Tables .................................................................................................................. xiv
List of Figures ................................................................................................................ xv
Chapters

1.  **INTRODUCTION** ................................................................................................... 1
   1.1 General Background ............................................................................................... 1
   1.1.1 Desirable Seismic Performance ................................................................. 2
   1.1.2 Performance of RC Buildings in Past Earthquakes ............................... 3
   1.1.3 Seismic Performance and Progressive Collapse ............................... 5
   1.2 Research Significance and Impetus ................................................................. 6
   1.3 Research Objectives and Scope .................................................................... 9
   1.4 Organization ..................................................................................................... 11

2.  **LATERAL BEHAVIOR OF REINFORCED CONCRETE COLUMNS-**
    **MODELING OF INDIVIDUAL DISPLACEMENT COMPONENTS** .......... 16
   2.1 Introduction ......................................................................................................... 16
   2.2 Background and Overview ............................................................................. 17
   2.3 The Analytical Procedure of DCMI............................................................... 21
   2.3.1 Modeling of Flexural Behavior .............................................................. 22
   2.3.2 Reinforcement Slip Deformations .......................................................... 25
   2.3.3 Modeling of Shear Deformations ......................................................... 28
   2.4 Modified Compression Field Theory ......................................................... 30
4. PROPOSED MODEL FOR RESPONSE ESTIMATION OF RC COLUMN......71

4.1 Introduction ............................................................................................................................................71

4.2 Background Information on Displacement Component Model ..........................................................72

4.3 Modeling of Shear Behavior and Deficiencies in DCMI .......................................................................73

4.3.1 Pre-Peak Response ..........................................................................................................................73

4.3.2 Post-Peak Response ..........................................................................................................................76

4.4 Development of New Proposed Model ....................................................................................................77

4.5 Modeling of Shear Behavior ....................................................................................................................79

4.5.1 Disturbed Stress Field Model ...........................................................................................................79

4.5.2 Compatibility Relationships .............................................................................................................81

4.5.3 Equilibrium Relationships ..............................................................................................................84

4.5.4 Constitutive relationships ...................................................................................................................84

4.5.4.1 Concrete Compression Softening .................................................................................................85

4.5.4.2 Pre-Cracking Concrete Tensile Behavior .....................................................................................86

4.5.4.3 Post-Cracking Concrete Tensile Behavior ....................................................................................87

4.5.4.4 Reinforcement Stress-Strain Relationship ....................................................................................88

4.5.5 Local Cracks Conditions ....................................................................................................................89

4.5.6 Modeling of Crack Shear-Slip ............................................................................................................91

4.6 Calculation of Average Crack Spacing ...................................................................................................93

4.7 Material Stiffness Formulations ............................................................................................................94

4.8 Modeling of Flexural Behavior ............................................................................................................97

4.8.1 Flexural Section Analysis ................................................................................................................98

4.8.2 Flexural Deformations .....................................................................................................................98

4.8.3 Axial Strain Due to Flexure ..............................................................................................................99

4.9 Compatibility and Equilibrium Conditions .............................................................................................100

4.9.1 Compatibility Conditions .................................................................................................................100

4.9.2 Equilibrium Conditions ................................................................................................................102

4.10 Interaction between Axial-Flexure and Axial-Shear Model ....................................................................103

4.10.1 Axial Strain Interaction Methodology ..........................................................................................103

4.10.2 Concrete Compression Softening ..................................................................................................105
6.5 Summary of Assumptions ................................................................. 153

7. EXPERIMENTAL RESEARCH ON PROGRESSIVE COLLAPSE .......... 165

7.1 Introduction ...................................................................................... 165

7.2 Overview of the Experimental Work ............................................. 165

7.3 Experimental Procedure .................................................................. 166

7.3.1 Preparatory Stage ......................................................................... 166

7.3.2 Instrumentation ........................................................................... 167

7.3.3 Column Removal ......................................................................... 172

7.3.4 Data Recording and Reduction .................................................. 172

7.4 Test Results ..................................................................................... 173

8. PROGRESSIVE COLLAPSE SIMULATION OF TEST BUILDING .......... 185

8.1 Introduction ...................................................................................... 185

8.2 Design Standards and Approaches for Progressive Collapse .......... 185

8.2.1 Design Approaches for Progressive Collapse ................................. 186

8.2.2 Design Guidelines for Progressive Collapse .................................. 187

8.2.3 Analysis Procedures for Progressive Collapse ............................... 188

8.3 Analysis Procedure for Test Building ............................................. 189

8.3.1 Loading Conditions for Analysis .................................................. 190

8.3.2 Acceptance Criteria for Progressive Collapse ............................... 191

8.4 Modeling of Test Building .............................................................. 193

8.4.1 Material Properties and Modeling Details .................................... 194

8.4.2 Calculation of Loads .................................................................. 194

8.5 Analyses of the Test Building ......................................................... 196

8.6 Comparison of the Computational and Experimental Results ......... 198

8.6.1 Procedure for Computation of Strains ......................................... 199

8.6.2 Comparison of Computed and Experimental Strains ................... 200

9. CONCLUSIONS .................................................................................. 218

9.1 Summary .......................................................................................... 218

9.2 Conclusions ...................................................................................... 223

9.3 Recommendations for Future Work ............................................... 226
LIST OF TABLES

Table 5.1. Properties of test columns ................................................................. 136
Table 6.1. Details of the columns in Johnston Hall ........................................... 155
Table 6.2. Types of beams in Johnston Hall ...................................................... 156
Table 6.3. Characteristics of beams used in Johnston Hall .............................. 157
Table 6.4. Details of slabs in Johnston ................................................................. 159
Table 8.1. Calculated moment demand and capacity for frame members -2D linear static ................................................................. 202
Table 8.2. Calculated moment demand and capacity for frame members - 3D linear static ................................................................. 202
Table 8.3. Calculated frame member rotations - 2D nonlinear dynamic analysis .... 203
Table 8.4. Calculated frame member rotations -3D nonlinear dynamic analysis .... 203
LIST OF FIGURES

Figure 1.1. Olive View Hospital damaged in the 1971 San Fernando Earthquake
(Steinbrugge, K. V., NISEE) ........................................................................................................ 13
Figure 1.2. Failure of first story columns in the Olive View Hospital (Steinbrugge, K. V.,
NISEE) ........................................................................................................................................ 13
Figure 1.3. Imperial County Services Building damaged during the 1979 Imperial Valley
earthquake (Bertero, V. V., NISEE) ............................................................................................... 14
Figure 1.4. Column failures during 1999, Kocaeli (Turkey) earthquake (EQIIS image
database, http://nisee.berkeley.edu/eqiis.html) ........................................................................ 14
Figure 1.5. Structural failure due to soft story during 2005 Kashmir (Pakistan) earthquake
(MAE) Center, 2005 ........................................................................................................................ 14
Figure 1.6. Bottom two stories collapsed in 5-story reinforced concrete frame building
during 2010 Haiti earthquake (Photographs by E. Fierro) .............................................................. 15
Figure 1.7. Examples of damage observed in reinforced concrete buildings during 2011
off The Pacific Coast of Tohoku (Japan) earthquake (Aydan & Tano, 2011) .......................... 15
Figure 2.1. Components of the total lateral deformation in a fixed ended column .......... 40
Figure 2.2. Behavior of the cracked concrete in compression .................................................... 40
Figure 2.3. Plastic hinge model for calculating flexural displacements ................................. 40
Figure 2.4. Reinforcement slip model (Sezen and Setzler, 2008) ............................................ 41
Figure 2.5. Shear response model ................................................................................................. 41
Figure 2.6. Reinforced concrete element: (a) in-plane stresses conditions; (b) average
strains from in-plane loading; (c) average strains in cracked concrete; and (d) Mohr’s
circle for average strains ............................................................................................................... 42
Figure 2.7. MCFT constitutive relationships: (a) cracked concrete compression; (b) cracked concrete in tension; and c) reinforcing steel .................................................. 43

Figure 2.8. Average and local stresses in the element; (a) average stresses between cracks; and (b) local stresses at a crack ................................................................. 43

Figure 3.1. Spring representation of total response model ........................................ 65

Figure 3.2. Classification of columns into categories and rules governing combination of the deformation components (Setzler and Sezen, 2008) ................................. 65

Figure 3.3. Interaction of compression softening and axial strains (Mostafaei and Kabeyasawa, 2007; Mostafaei and Vecchio, 2008) ........................................... 66

Figure 3.4. Proposed compression bar buckling model ............................................ 66

Figure 3.5. Constitutive relationships of concrete in compression ............................ 67

Figure 3.6. Constitutive relationship of concrete in tension .................................... 67

Figure 3.7. Constitutive relationship for reinforcing steel ....................................... 68

Figure 3.8. Experimental and analytical results for (a) flexural response of Specimen 1; (b) flexural response of Specimen 4; (c) reinforcement slip response of Specimen 1; (d) reinforcement slip response of Specimen 2; (e) shear response of Specimen 1; and (f) shear response of Specimen 2 ................................................................. 69

Figure 3.9. Lateral load – displacement relationships (a) Specimen 1; (b) Specimen 2; and (c) Specimen 4 ................................................................. 70

Figure 4.1. Reinforced concrete element subjected to uniform loading (Vecchio, 2000) ................................................................. 109

Figure 4.2. Compatibility conditions in DSFM: (a) Deformations due to average constitutive response (net concrete strains); (b) deformations due to rigid body slip along crack; (c) combined deformations; (d) Mohr’s circle of net concrete strains; and (e) Mohr’s circle of total strains (Vecchio, 2000) ................................................................. 109

Figure 4.3. Equilibrium conditions in DSFM (Vecchio, 2000) ................................ 110

Figure 4.4. Constitutive relations in DSFM: (a) compression softening model; (b) tension softening model; (c) tension stiffening model; and (d) reinforcing steel response (Vecchio, 2000) ................................................................. 110

Figure 4.5. Parameters influencing crack spacing (Collins and Mitchell, 1991) ....... 111
Figure 4.6. Plastic hinge model for calculating flexural displacements .................. 111
Figure 4.7. Axial-Flexural model and determination of axial strain due to flexure .... 112
Figure 4.8. Interaction of compression softening and axial strains ...................... 113
Figure 5.1. Flow chart for implementation of the proposed model ...................... 137
Figure 5.2. Details of the modeled columns .................................................. 138
  (a) Typical test specimen and column details (Lynn, 2001; Sezen, 2002) .......... 138
  (b) Typical test specimen and column details (Saatcioglu and Ozcebe, 1989) .... 138
Figure 5.3. Constitutive relationships of concrete in compression ...................... 139
Figure 5.4. Constitutive relationship for reinforcing steel .................................. 139
Figure 5.5. Lateral load-displacement relationship for specimen-1 ..................... 140
Figure 5.6. Lateral load-displacement relationship for specimen-2 ..................... 140
Figure 5.7. Lateral load-displacement relationship for specimen-4 ..................... 141
Figure 5.8. Lateral load-displacement relationship for 2CLH18 ......................... 141
Figure 5.9. Lateral load-displacement relationship for 2CMH18 ......................... 142
Figure 5.10. Lateral load-displacement relationship for specimen 3CLH18 .......... 142
Figure 5.11. Lateral load-displacement relationship for specimen 3CMD12 .......... 143
Figure 5.12. Lateral load-displacement relationship for specimen 3CMH18 .......... 143
Figure 5.13. Lateral load-displacement relationship for specimen 3SLH18 .......... 144
Figure 5.14. Lateral load-displacement relationship for specimen 3SMD12 .......... 144
Figure 5.15. Lateral load-displacement relationship for specimen U4 ............... 145
Figure 5.16. Lateral load-displacement relationship for specimen U6 ............... 145
Figure 6.1. Exterior view of the Johnston Laboratory from southwest (KSA Digital
  Library-The Ohio State University) ......................................................... 160
Figure 6.2. Exterior view of the Johnston Laboratory from southeast showing building
  construction (KSA Digital Library-The Ohio State University) ..................... 160
Figure 6.3. Exterior views of Johnston Laboratory before demolition (KSA Digital
  Library-James Collier-by-nc) ................................................................. 161
Figure 6.4. Typical floor plan of the Johnston Laboratory .................................. 161
Figure 6.5. Plan and elevation view of the Johnston Laboratory ......................... 162
Figure 6.6. Details of actual and idealized spandrel beams for 2nd and 3rd floors .... 163
Figure 6.7. Details of actual and idealized spandrel beams for 4th floor and roof slab 164
Figure 7.1. Plan view of Johnston Laboratory showing structural members involved in testing ................................................................................................................................. 177
Figure 7.2. East side exterior view of the Johnston Laboratory before testing and after removal of exterior wall ................................................................................................................................. 177
Figure 7.3. Instruments used for testing of Johnston Laboratory (a) Strain gauge (CEA-06-250UW-120/P2) (b) Linear Displacement Sensor (Model HS-100 Micro-Measurements) (c) Data Acquisition System (System 5000) .............................................. 178
Figure 7.4. Location of strain gauges and displacement sensors on Johnston Laboratory. ............................................................................................................................................................................. 179
Figure 7.5. Strain gauges placement at typical section of columns and beams (a) Column bending about y-axis (b) Column bending about z-axis (c) Strain gauge placement on beams ........................................................................................................................................................................ 180
Figure 7.6. Concrete cover being removed with the help of hydraulic jack hammer (a) Concrete cover removal from column (b) Concrete cover removal from beam..... 181
Figure 7.7. Typical instrumentation of the Johnston Laboratory (a) Strain gauge installed on exposed and prepared surface of reinforcing bar (b) Linear displacement sensor installed under the beam ........................................................................................................................................................................ 181
Figure 7.8. Column removal process in the test building (a) Column A9 being removed (b) Column A9 removed ............................................................................................................................................................................. 182
Figure 7.9. Displacement measured with the help of LVDTs near column joint A9 .... 182
Figure 7.10. Strain gauge readings for column B9 ................................................................................................................................. 182
Figure 7.11. Strain gauge readings for column A8 (2/3 of height) ....................... 183
Figure 7.12. Strain gauge readings for column A8 (mid-height) ....................... 183
Figure 7.13. Strain gauge readings for column A10 ........................................... 183
Figure 7.14. Strain gauge readings for Beam-1 .................................................. 184
Figure 7.15. Strain gauge readings for Beam-2 .................................................. 184
Figure 7.16. Strain gauge readings for Beam-3 .................................................. 184
Figure 8.1. Calculation of beam rotation (Song, 2010) ............................................. 204
Figure 8.2. 2-D model of frame-A in SAP2000 .................................................... 204
Figure 8.3. 3-D model of test building in SAP2000 ................................................................. 205
Figure 8.4. Moment diagram before column removal ................................................................. 205
Figure 8.5. Moment diagram after column removal ................................................................. 206
Figure 8.6. Deflected shape of frame-A after column removal .................................................. 206
Figure 8.7. Moment-curvature relationship for spandrel beam in the second floor .... 207
Figure 8.8. Moment-curvature relationship for spandrel beam in the third floor ........ 207
Figure 8.9. Moment-curvature relationship for spandrel beam in the fourth floor ...... 208
Figure 8.10. Moment-curvature relationship for spandrel beam in roof........ 208
Figure 8.11. Moment diagram of Frame-A after column removal ................................. 209
Figure 8.12. Moment diagram of Frame-9 after column removal ........................................... 209
Figure 8.13. Moment diagram of Frame-A after column removal in 2-D nonlinear dynamic analysis ................................................................. 210
Figure 8.14. Moment diagram of Frame-A after column removal in 3-D nonlinear dynamic analysis ................................................................. 210
Figure 8.15. Moment diagram of Frame-A after column removal in 3-D nonlinear ... 211
Figure 8.16. Moment diagram of Frame-A after column removal in 3-D nonlinear.... 211
Figure 8.17. Comparison of the measured and computed strains for column A_8 at 2/3 of column height (2D LS analysis) ................................................................. 212
Figure 8.18. Comparison of the measured and computed strains for column A_8 at mid-height of the column (2D LS analysis) ................................................................. 212
Figure 8.19. Comparison of the measured and computed strains for column A_{10} (2D LS analysis) ................................................................................................................ 213
Figure 8.20. Comparison of the measured and computed strains for beam A_{9,10} (2D LS analysis) ................................................................................................................ 213
Figure 8.21. Comparison of the measured and computed strains for beam A_{8,9} (2D LS analysis) ................................................................................................................ 213
Figure 8.22. Comparison of the measured and computed displacements .......... 214
Figure 8.23. Comparison of the predicted and computed strains for beam 9_{A,B} (3D LS analysis) ................................................................................................................ 215
Figure 8.24. Comparison of the predicted and computed strains for column B₉ (3D LS analysis) .................................................................................................................................................. 215
Figure 8.25. Comparison of the predicted and computed strains for column A₈ at 2/3 of column height (NLD analysis) ................................................................................................................................................. 216
Figure 8.26. Comparison of the predicted and computed strains for column A₈ at mid-height (NLD analysis) .................................................................................................................................................. 216
Figure 8.27. Comparison of the predicted and computed strains for column A₁₀ (NLD analysis) .......................................................................................................................................................... 217
Figure 8.28. Comparison of the predicted and computed displacements (NLD) ........ 217
Figure A.1. Second floor framing plan of the Johnston Laboratory ........................................ 240
Figure A.2. Third and fourth floor framing plan of the Johnston Laboratory .................... 241
Figure A.3. Roof slab framing plan of the Johnston Laboratory ........................................ 242
Figure A.4. Remodeling drawings of the Johnston Laboratory, third floor equipment
    relocation and demolition plan .............................................................................................. 243
Figure A.5. Remodeling drawings of the Johnston Laboratory, fourth floor equipment
    relocation and demolition plan .............................................................................................. 244
Figure A.6. Remodeling drawings of the Johnston Laboratory, Existing first and second
    floor plans-new construction ............................................................................................... 245
Figure A.7. Remodeling drawings of the Johnston Laboratory, third floor plan – new
    construction .......................................................................................................................... 246
Figure A.8. Remodeling drawings of the Johnston Laboratory, fourth floor plan – new
    construction .......................................................................................................................... 247
Figure A.9. Remodeling drawings of the Johnston Laboratory, third and fourth floors-
    new equipment plan ............................................................................................................. 248
Figure A.10. Remodeling drawings of the Johnston Laboratory – Structural notes .... 249
Figure B.1. Recorded displacement readings from LVDT-1 ................................................. 251
Figure B.2. Recorded displacement readings from LVDT-2 ................................................. 251
Figure B.3. Recorded strains from strain gauge 1 ................................................................. 251
Figure B.4. Recorded strains from strain gauge 2 ................................................................. 252
Figure B.5. Recorded strains from strain gauge 3 ................................................................. 252
Figure B.6. Recorded strains from strain gauge 4 ........................................... 252
Figure B.7. Recorded strains from strain gauge 5 ........................................... 253
Figure B.8. Recorded strains from strain gauge 6 ........................................... 253
Figure B.9. Recorded strains from strain gauge 7 ........................................... 253
Figure B.10. Recorded strains from strain gauge 8 ....................................... 254
Figure B.11. Recorded strains from strain gauge 9 ....................................... 254
Figure B.12. Recorded strains from strain gauge 10 .................................... 254
Figure B.13. Recorded strains from strain gauge 11 .................................... 255
Figure B.14. Recorded strains from strain gauge 12 .................................... 255
Figure B.15. Recorded strains from strain gauge 13 .................................... 255
Figure B.16. Recorded strains from strain gauge 14 .................................... 256
Figure B.17. Recorded strains from strain gauge 15 .................................... 256
Figure B.18. Recorded strains from strain gauge 16 .................................... 256
Figure B.19. Recorded strains from strain gauge 17 .................................... 257
Figure B.20. Recorded strains from strain gauge 18 .................................... 257
CHAPTER 1

INTRODUCTION

1.1 General Background

There are a large number of reinforced concrete (RC) buildings in seismically active areas of the world that are not designed and constructed in accordance with modern seismic design provisions. Post-earthquake observations and research on performance of such buildings reveal structural deficiencies in their design, details and construction. These structures exhibit non-ductile behavior and possess very limited capacity to absorb and dissipate the energy of strong ground shaking beyond their limited elastic range. Typically, this class of RC structures often has low lateral displacement capacities and undergoes rapid degradation of shear strength and axial load carrying capacity during strong ground motions and hence is extremely vulnerable to excessive structural damage or collapse during future earthquakes.

In the United States and other parts of the developed world, these buildings were constructed between 1930s to mid-1970s prior to the inclusion of seismic design requirements in building codes. Even today, in low to moderate seismic regions and in some developing countries, reinforced concrete structures are being designed and built without essential seismic details which are vital to withstand large lateral loads.
Undoubtedly, existing stocks of such vulnerable buildings constitute most critical hazard risk in seismic regions of the world.

### 1.1.1 Desirable Seismic Performance

Current seismic codes provide standards for design, detailing and construction of reinforced concrete structures to ensure adequate load-bearing capacity, stiffness, deformability, and energy-dissipating capacity. At system level, seismic design follows strong column-weak beam design philosophy and mandates provision of ductility, adequate lateral-load resisting mechanism (e.g., special moment-resisting frames or shear walls) and redundancy (alternative load paths) in the structural system. It discourages irregularities in plan or elevation of the structures and does not allow presence of soft or weak stories and short columns. At element level, code requirements are mostly focused on proportioning and detailing of frame members (beams, columns, and joints) with the objective to achieve certain amount of ductility in addition to the required strength. Such detailing requirements include provision of adequate and closely spaced transverse reinforcement with 135 degree end hooks, and continuity of longitudinal reinforcement in the regions that experience the most inelastic deformations and stress reversals during earthquakes.

The seismic details ensure that the structural members have the capacity to withstand high seismic activity. For example, large amount of transverse steel provides sufficient shear strength to resist lateral earthquake loads while close spacing of the ties and 135 degree end hooks keep the core concrete confined at high displacements. This lateral confinement substantially increases compressive strength and ductility of concrete
(Mander et al., 1988); the key features required for desirable seismic performance of reinforced concrete building structures. Properly confined concrete can remain intact and withstand large displacements even if it has cracked under flexural tension. In an unconfined or poorly confined concrete, cycling of the shear force causes rapid crushing of the concrete and loss of strength as soon as cracks start to appear.

1.1.2 Performance of RC Buildings in Past Earthquakes

Reinforced concrete structures with insufficient seismic design and construction deficiencies have sustained widespread damage during past earthquakes. For example, during Kashmir (Pakistan) earthquake of 2005 and Haiti earthquake of 2010, extensive structural damage to residential, commercial and government buildings was observed. The damage was attributed largely to lack of earthquake-resistant design, poor standard of construction and inferior quality of building materials. In majority of the collapsed or damaged structures, structural types, member dimensions and detailing practices (insufficient lap length, improper lap location, lack of confinement in hinge, joint and splice regions etc.) were found inadequate to resist forces imposed by these earthquakes (EERI, 2005; MAE, 2005; USGS/EERI, 2010).

The 2011 Tohoku (Japan) earthquake is another example that signifies the need for proper seismic design and construction. Although, majority of causalities and large scale destruction of infrastructure during magnitude 9.0 megathrust earthquake was caused by ensuing tsunami, limited damage to the well-designed reinforced concrete buildings due to ground shaking was reported (PEER/EERI/GEER, 2011; Takewaki et al., 2011). However, extensive and severe structural damage was observed in older
residential and commercial buildings that were constructed prior to 1978 code revision of Japan. The modern structures built to withstand seismic demands did not sustain any substantial and widespread damage (Aydan and Tano, 2011).

Figures 1.1 through 1.7 present various examples of the structural damage and poor performance of reinforced concrete buildings in past earthquakes. Figure 1.1 shows Olive View Hospital building which was damaged in the 1971 San Fernando Earthquake. The building sustained significant damage in almost all columns in the first story. Close up views of the two columns in Figure 1.2 show severe shear damage to the columns. The concrete is entirely crushed and the columns have virtually lost their ability to carry lateral and axial loads. The Imperial County Services Building, as shown in Figure 1.3, also suffered significant damage to its first-story columns during the Imperial Valley earthquake of 1979. Transverse reinforcement did not provide sufficient confinement to core concrete, resulting in crushing of the concrete and buckling of the longitudinal reinforcement. Figures 1.4 through 1.7 show few other examples of the poor performance of reinforced concrete buildings during recent earthquakes around the world (Sezen et al., 2001; MAE, 2005).

The pattern of damage observed during past earthquakes indicates that in reinforced concrete frame buildings with poorly detailed components, most common cause of the significant structural damage or collapse under gravity and lateral earthquake loads is failure of the columns, beam-column joints or both (EERI, 2000; Ghannoum et al., 2006; Sezen et al., 2001). Beam damages and failures are less likely and less critical to the structure’s performance (Moehle and Mahin, 1991; Sezen et al., 2001). Beam-column joints with poor details and proportions might be susceptible to damage with
ensuing reduction in strength and ductility of joints or adjacent framing members. However, recognizing the fact that the non-ductile construction (older existing buildings and buildings with deficient seismic design) generally does not follow strong column-weak beam design philosophy and gravity loading often dictates member strengths, inelastic action and damage under earthquake loading commonly is limited to the columns. Typically, columns fail in shear with a subsequent loss of lateral and/or axial load carrying capacity. Such column behavior is primary focus of the research presented in this study.

1.1.3 Seismic Performance and Progressive Collapse

Post-earthquake studies show that primary cause of reinforced concrete building collapse during earthquakes is the loss of vertical-load-carrying capacity in critical building components leading to cascading vertical collapse, rather than loss of lateral-load capacity (Ghannoum et al., 2006; Moehle et al., 2002). Once axial failure occurs in one or more components, vertical loads from gravity and inertial effects are transferred to adjacent framing components. The ability of the frame to continue to support vertical loads depends on both the capacity of the framing system to transfer these loads to adjacent components and the capacity of the adjacent components to support the additional load. When any of these conditions is deficient, progressive failure of the structure may ensue.

Progressive collapse is generally defined as small or local structural failure resulting in collapse of the adjoining members and, in turn, causing total collapse of the building or a disproportionately large part of it. During a seismic activity, local structural
failure in the lower-story columns can initiate vertical or progressive collapse in the buildings with inadequate ductility if gravity loads cannot be transferred to undamaged columns (Gurley, 2008; Wibowo and Lau, 2009). After one or more columns fail, an alternate load path is needed to transfer the loads carried by failed member(s) to other structural members. If adjoining elements cannot resist and redistribute the additional loads, a series of failures will occur until entire or substantial part of the structure collapses.

Seismic design follows the “strong column-weak beam” philosophy, where the columns must have sufficient strength and ductility to push significant inelastic action and damage to the beams. Studies have shown that progressive collapse resistance of reinforced concrete frame structures improves substantially if well known seismic details are used in the design of the load carrying elements including columns and shear walls. However, in older and existing reinforced concrete buildings with inadequate seismic design, strength, ductility, continuity and/or redundancy may not be sufficient to redistribute the loads and prevent spread of damage. Hence, these buildings are extremely vulnerable to vertical or progressive collapse by not being able to carry gravity loads after failure caused by lateral loads. In the second part of the study, gravity load carrying capacity and behavior of reinforced concrete structures is studied through experimental and analytical assessments of the progressive collapse response of an existing building.

1.2 Research Significance and Impetus

Reinforced concrete structures with deficient seismic design are vulnerable to excessive structural damage or collapse during future earthquakes; therefore, it is vital
that these structures be retrofitted to sustain seismic loadings. Generally, seismic retrofit approaches consist of system behavior improvements (increasing overall capacity of the structural system by installing new concrete infill walls or steel bracings) and/or component strengthening (strengthening of the individual components such as beams, columns, and beam-column joints). Retrofitting vulnerable existing buildings is often economically more feasible than to completely replace them. However, to assess vulnerability of a structure to earthquake damage and suggest desired level of retrofit, it is imperative to first evaluate expected behavior of the structure in terms of strength and deformation capacities, progression of the damage, and collapse mechanisms.

In addition to the retrofitting requirements, there may be many other situations where structures are required to be analyzed accurately to evaluate their structural behaviors. For example, in performance- and displacement-based design philosophy (Priestley et al, 2007, SEAOC, 1995), important existing and planned structures may need to be evaluated to assess their maximum load capacity, ultimate deformation capacity, ductility, collapse resistance and failure mechanism. Seismic damage analysis of reinforced concrete structures using damage indices or indicators is another example towards this end. The seismic damage indices are the means to numerically quantify the damage sustained under an earthquake loading (Williams and Sexsmith, 1995) and generally refer to the deformations and/or energy absorbed during the damage (Park et al., 1988). These indicators are typically used for reliability studies of existing structures to decide on pre-earthquake strengthening, post-earthquake damage assessment to define required measures of repair and seismic performance predictions of the structures of great importance (Kappos, 1997). Thus, a realistic seismic damage analysis, in pre- or post-
earthquake scenario, requires development of analytical models to accurately predict non-linear structural behavior during the seismic event (Mergos & Kappos, 2010).

While much is known about the behavior of reinforced concrete components and systems, there are still areas that require further understanding. For a structure whose behavior is dominated by flexural mechanism, issues regarding its performance evaluations have been studied well and design procedures are relatively well established. However, for the structures whose behavior is affected by shear related mechanisms, accurate modeling remains elusive with many available approaches and theories. In a reinforced concrete structure, total lateral deformation is mainly caused by flexural and shear mechanisms. At element level, these mechanisms interact with each other and corresponding deformations do not occur independently. Therefore, any analytical procedure that aims to model overall lateral load behavior of the structure must take this aspect into account.

The expected behavior of the structure can be evaluated by determining load-deformation responses of the frame elements such as beams, columns and shear walls, considering all potential failure mechanisms associated with axial, flexure and shear behavior. As columns are the one of the critical vertical load carrying elements to prevent the collapse, understanding of their response to applied seismic loads is vital for overall assessment of the structural performance. Currently available studies for lateral response estimation of non-ductile reinforced concrete columns show that the approaches that can predict structural behavior with good accuracy employ complicated and computation-intensive procedures that may not be amenable and are difficult to implement. As a result, many approaches try to simplify the process by making simplifying assumptions but in
most cases this is done at the cost of accuracy. Therefore, it is aimed in this study to propose a suitable procedure to address critical modeling issues in lateral and gravity load behavior of the columns while predicting the response accurately and keeping overall computational process simple with easy implementation. It will be shown subsequently that the proposed model can effectively be employed to predict the structural response of reinforced concrete columns accurately with efficiently.

1.3 Research Objectives and Scope

Seismic Response Evaluation of Columns. This study presents an analytical model for estimating load-displacement response of reinforced concrete columns subjected to lateral loads and identifies critical structural and loading parameters which affect overall structural response in terms of strength and deformation capacity. The research on lateral load behavior is focused on columns commonly found in the older existing buildings. Traditionally, these columns have insufficient and widely spaced transverse reinforcement and non-seismic details such as 90-degree end hooks and splicing of the longitudinal bars in the regions experiencing largest inelastic deformations near column ends. Due to such deficiencies, columns may not have sufficient shear strength to develop plastic hinges at the ends. Also, wide spacing of column ties does not provide good confinement to the core concrete resulting in non-ductile behavior and sudden brittle failure. Although, this study focuses on modeling columns with poor seismic details, the proposed analytical procedure is equally applicable to predict structural response of the columns designed to meet the requirements of modern seismic codes.
Specifically, the objectives in the proposed research lateral load behavior of reinforced concrete columns are summarized as follows.

- Development of an analytical model to predict monotonic load-displacement relationship of the reinforced concrete column subjected to lateral loading.
- Analysis and identification of the loading and structural parameters influencing lateral response of the columns in terms of strength and deformation capacities, progression of the damage and failure mechanisms.
- Investigation and modeling of shear failure, post-peak strength degradation, and axial load failure in RC columns.

**Progressive Collapse Analysis.** Investigate and model progressive collapse response of reinforced concrete buildings while performing following tasks:

- Test an existing reinforced concrete building by physically removing one first-story column simulating failure or loss of the columns and response of the buildings with the help of measured strain and displacement data.
- Implementation and evaluation of current design methodologies, code standards and guidelines for collapse analysis.
- Computer simulations and analyses of the test building by performing linear static and nonlinear dynamic analyses of the two- and three-dimensional models of the test building.
- Comparison of the experimental and predicted performance, and accessing the effectiveness of currently available analysis and design methodologies.
1.4 Organization

This dissertation is organized into nine chapters. Chapter 2 and 3 present background and overview of the previous research on response estimation of reinforced concrete columns subjected to lateral loading. Various aspects of column behavior and their modeling are explained by presenting the details of an analytical procedure that was developed earlier by the author (Lodhi, 2010). The contents in these two chapters form the basis for learning and understanding complex modeling issues and new column research presented in this report. The details presented in chapter 2 and 3 shall be used for reference, discussion and development of a new procedure for modeling column behavior under lateral load. The components of total lateral displacement are discussed and procedures to model each component are explained in chapter 2. In chapter 3, the interaction between component displacement mechanisms and other aspects affecting total response are discussed and procedures are explained to capture these aspects in overall modeling approach. Chapter 4 discusses the weaknesses and limitations of previous research and presents details and development of a new analytical model proposed for estimation of lateral load-displacement response of reinforced concrete columns. The chapter explains the theoretical and procedural aspects of the proposed model. As Disturbed Stress Field Model (Vecchio, 2000) employed in proposed model is a complex approach, analytical procedure and relevant aspects are explained in detail. In chapter 5, implementation and verification of the proposed model are presented. Major analytical steps, calculation procedures and solution algorithm are explained. The procedures for calculation of strain components due to crack-shear-slip and local stresses at crack interfaces are outlined. Previously tested column specimens are modeled by
implementing proposed procedure and their load-displacement relationships and failure modes are calculated. The comparison between predicted and experimentally observed responses for test columns is finally presented to validate the proposed model.

Chapter 6 and 7 provide the details of the experimental work on progressive collapse research. Chapter 6 presents general description of the test building, characteristics of the structural components such as beams, columns and slabs, and material properties used in the research. Chapter 7 outlines overall experimental procedure and explains major stages of the building test. Detailed procedures for preparation and instrumentation of frame members, removal of the column, and recording of test data are presented. The time-history strain and displacement data recorded during the experiment are provided and responses of the instrumented elements are discussed.

Chapter 8 deals with the computational aspects of the research on progressive collapse. It presents the procedures and models of the test building used for the analyses. The analytical results from linear static and non-linear dynamic analyses for two-and three dimensional building models are presented and discussed. The predicted responses are compared with experimental data and evaluation of the existing analysis and design procedures for progressive collapse is discussed.

A summary of the research completed is included in Chapter 9. This chapter also discusses conclusions drawn from this study, and suggests several areas of future research.
Figure 1.1. Olive View Hospital damaged in the 1971 San Fernando Earthquake
(Steinbrugge, K. V., NISEE)

Figure 1.2. Failure of first story columns in the Olive View Hospital
(Steinbrugge, K. V., NISEE)
Figure 1.3. Imperial County Services Building damaged during the 1979 Imperial Valley earthquake (Bertero, V. V., NISEE)

Figure 1.4. Column failures during 1999, Kocaeli (Turkey) earthquake (EQIIS image database, http://nisee.berkeley.edu/eqiis.html)

Figure 1.5. Structural failure due to soft story during 2005 Kashmir (Pakistan) earthquake (MAE Center, 2005)
Figure 1.6. Bottom two stories collapsed in 5-story reinforced concrete frame building during 2010 Haiti earthquake (Photographs by E. Fierro)

Figure 1.7. Examples of damage observed in reinforced concrete buildings during 2011 off The Pacific Coast of Tohoku (Japan) earthquake (Aydan & Tano, 2011)
CHAPTER 2

LATERAL BEHAVIOR OF REINFORCED CONCRETE COLUMNS-
MODELING OF INDIVIDUAL DISPLACEMENT COMPONENTS

2.1 Introduction

This chapter presents background and overview of the previous research on response estimation of reinforced concrete columns subjected to lateral loading. Critical aspects of column behavior and their modeling are explained by presenting the details of a previously developed analytical model by the author (Lodhi, 2010). The model is entitled as Displacement Component Model with Interaction (DCMI). The contents of this chapter help in learning and understanding complex column behavior and modeling issues and form the basis for the research presented in this dissertation. Various aspects of DCMI explained in this chapter shall be used for reference, discussion and development of a new analytical model for evaluating column response under lateral loads.

The DCMI is a macro-model for estimating lateral load-displacement relationship of the reinforced concrete columns. It models each component of lateral displacement due to flexure, shear and longitudinal bar slip individually while considering interaction between shear, flexure and axial load mechanisms. Individual displacement components are combined together depending upon dominant failure mode (shear or flexure) to obtain
total response. The model also considers critical second order effects such as buckling of compression bars under excessive compressive strains, enhancement in concrete strength due to confinement, softening of the cracked concrete in compression, and concrete tension stiffening.

2.2 Background and Overview

When a typical fixed ended reinforced concrete column is subjected to the lateral loads at its ends, it undergoes total lateral deformation that is mainly comprised of three components due to flexure, reinforcement slip and shear mechanisms, as shown in Figure 2.1 (Setzler and Sezen, 2008; Sezen and Moehle, 2004). For a column whose behavior is dominated by flexure, issues regarding its performance evaluation have been studied well and current design procedures for flexural strength estimation are generally considered well established. Among available approaches, most of which either are based on lumped plasticity models or distributed nonlinearity models, fiber models are considered advanced analytical procedures that can conveniently be employed for evaluating structural response. It must however be noted that fiber models are appropriate tools for analyzing flexural performance only and behavior of the columns dominated by shear related mechanisms cannot be simulated.

For evaluating shear response of structural elements, such as beams and columns, many analytical models and theories have been presented in the past. Some of the most commonly used approaches are strut and tie models (Mörsh, 1902; Ritter, 1899; Schlaich et al., 1987) and empirical formulations/rational theories based on experimental observations such as Arakawa equation (Arakawa, 1970), Modified Compression Field
Theory (MCFT) (Vecchio and Collins, 1986) and Disturbed Stress Field Model (DSFM) (Vecchio, 2000). These approaches are fundamentally different in their theoretical modeling and conceptual development. Their applicability to structural members, computational demand and accuracy also vary in wide range from one approach to the other. Hence, accurate modeling of the shear behavior in beams and columns remains elusive.

MCFT is a powerful tool to model the response of reinforced concrete elements subjected to in-plane shear and normal stresses. However, in order to evaluate flexure-shear response of the reinforced concrete columns by MCFT, the member needs to be discretized into large number of biaxially stressed elements and analyzed using nonlinear finite element procedure (Vecchio, 1989). Vecchio and Collins (1988) extended concept of MCFT to fiber model approach for response estimation of reinforced concrete beams loaded in combined axial, shear and flexural forces. In this approach, concrete fibers are treated as biaxially stressed elements in the cross section, and analyzed for in-plane stress field based on MCFT. Later, this approach was improved for accurate determination of shear stress distribution on the cross section and advanced formulations were implemented successfully into a non-linear section analysis computer program called Response-2000 (Bentz, 2000). The application of the MCFT in finite element approach or sectional analysis approach yields reliable flexure-shear response, but results in fastidious computations, which are not simple for practical applications.

Total lateral deformation of a concrete column is mainly comprised of the flexure and shear components. These mechanisms interact with each other and corresponding deformations do not occur independently. For example, in the web of a reinforced
concrete column, axial strain due to flexural mechanism will increase principal tensile strain and width of the shear crack resulting in lower shear capacity of the element. On the other hand, experimental evidence has established that principal compressive stress in the concrete is a function of principal compressive strain as well as of principal tensile strain (Vecchio and Collins, 1986). Compressive strength and stiffness of concrete decrease as tensile strains increase. The concrete in the web of laterally loaded element is subjected to shear stresses in addition to the normal stresses due to axial load and flexure. As the shear stresses increase, principal tensile strains increase and concrete exhibits weaker and softer response in compression, resulting in lower flexural strength of the element.

Therefore, any numerical procedure that aims to model overall lateral load-displacement relationship must take the interaction of flexural and shear mechanisms into account. Recently, Mostafaei and Kabeyasawa (2006, 2007) presented Axial-Shear-Flexural Interaction (ASFI) approach for the displacement-based analysis of reinforced concrete elements such as beams, columns and shear walls by considering interaction between axial, shear and flexural mechanisms. This macro-model based approach consists of two models evaluating axial-flexural and axial-shear responses simultaneously to obtain total response of element subjected to axial, flexural and shear loads. In this approach, axial-flexural behavior is simulated by employing conventional section analysis or fiber model whereas axial-shear response is determined through MCFT by considering one integration point in the in-plane stress conditions. The axial-flexural and axial-shear mechanisms are coupled in average stress-strain field considering axial deformation interaction and softening of concrete compression strength while satisfying
compatibility and equilibrium conditions. Although, ASFI approach reduces computational demand considerably as compared to other models implementing MCFT into finite element analysis approach or sectional analysis approach, computational process is still intense and complicated due to coupling of the axial-flexure and axial-shear mechanisms, and requires a deliberate iterative scheme at each loading step. However, concepts from ASFI approach are utilized in the model proposed in this study.

Few studies in the recent past have also addressed the issues of stiffness degradation and strength deterioration in the reinforced concrete elements dominated by shear or shear-flexure behavior. These studies represent advanced formulations for fiber-based element (Ceresa et al., 2007, 2009; Chao and Loh, 2007; Mullapudi et al., 2008; Mullapudi and Ayoub, 2010; Xu and Zhang, 2011; Zhang et al., 2011) and Macro-element model (Mergos and Kappos, 2008, 2010) and consider interaction between inelastic shear and nonlinear flexural behaviors with different conceptual backgrounds, solution strategies, and implementation complexities. A state-of-art review is presented on fiber elements with focus on concentrated plastic-hinge type model that can be implemented in displacement-based finite element programs (Ceresa et al., 2007).

Currently available studies for response estimation of non-ductile reinforced concrete columns show that the approaches that can predict structural behavior with good accuracy employ complicated and computation-intensive procedures that may not be amenable and are difficult to implement. As a result, many approaches try to simplify the process by making simplifying assumptions but in most cases, this is done at the cost of accuracy. DCMI is a suitable procedure that addresses critical modeling issues while predicting the response accurately and keeping overall computational process simple with
easy implementation. It can effectively be employed to predict the strength and total lateral displacement capacity, considering the deformation components due to flexure, shear and reinforcement slip.

2.3 The Analytical Procedure of DCMI

Flexural and shear deformations in the DCMI are calculated independently while considering the interaction between these mechanisms and then combined together depending upon dominant failure mode. The flexural deformations are determined through fiber section model considering shear effects by employing compressive constitutive law for cracked concrete. Shear deformations are calculated by combination of MCFT (Vecchio and Collins, 1986) and shear response envelope by Sezen (2008), while considering effect of axial strains due to flexure on shear mechanism. Lateral deformation component due to reinforcement slip in beam-column joint regions is determined separately and added to the flexural and shear deformation components to obtain total response. The interaction between flexure and shear mechanisms allows for accurate response estimation while decoupled flexural analysis minimizes complexity of calculations and makes the analysis process relatively simple and easy. In addition, buckling of compression bars under large compressive strains is also incorporated in the analysis by employing separate stress-strain relationships for reinforcing steel in compression. The effects of concrete tension strength and softening of cracked concrete in compression are also considered.

The modeling approaches for component deformations and total deformations are presented in the following sections.
2.3.1 Modeling of Flexural Behavior

For reinforced concrete elements subjected to bending moment and axial load, such as beams or columns, flexural deformations can accurately be determined by performing section analysis on a fiber model in one-dimensional stress field. This is usually handy and accurate approach if realistic material constitutive relationships and actual stress distribution across the depth of the cross-section are considered in the analysis.

In fiber section analysis, a reinforced concrete cross section is discretized into finite number of concrete and steel fibers. Each of the fibers is idealized as a uniaxial element with its unique stress-strain relationship. Bernoulli’s principle, that plane section before bending remains plane after bending, is the main hypothesis in the analysis and implies that the longitudinal strain in concrete and steel at any point in the cross section is proportional to its distance from neutral axis resulting in linear strain distribution. Based upon the resulting strain profile, stress distribution for concrete and reinforcing steel can be determined in accordance with their respective stress strain relationships. By satisfying equilibrium equations at the cross section, the moment capacity of the section is determined. The process is repeated number of times by incrementing longitudinal strain until either the concrete or the steel fails as per defined failure criterion.

In conventional flexural section analysis, the concrete behavior is simulated by its response usually derived from standard cylinder test where it is subjected to uniaxial compression. The strain conditions for the concrete in the web of a laterally loaded reinforced concrete beam or column are significantly different from those in a cylinder test. The concrete in a cylinder test is subjected to only small tensile strains primarily due
to Poisson’s effect, whereas, the concrete in the web experience shear stresses in addition to the normal stresses due to axial load and flexure. Due to applied shear stresses, concrete in the web cracks diagonally in the direction normal to principal tensile strain. As mentioned earlier, experimental evidence has shown that that principal compressive stress in the concrete is not only the function of principal compressive strain but is also affected by the coexisting principal tensile strain in a way that compressive strength and stiffness of the concrete decrease as tensile strains increase (Vecchio and Collins, 1986). This implies that the concrete subjected to combined normal compressive and shear stresses is weaker in compression than the concrete subjected to normal compressive stresses only. Hence, the concrete in the web of a laterally loaded column must exhibit weaker and softer response as compared to the concrete subjected to uniaxial compression in cylinder test. The behavior of the cracked concrete in the manner explained above is called compression-softening and is illustrated in Figure 2.2. In this figure, $f'_c$ is compressive strength of the concrete, $f_{c2}$ is principal compressive stress in the concrete and $\varepsilon_{c2}$ is principal compressive strain in the concrete.

The effect of shear stress on degrading compressive strength of the concrete can be taken into account by considering compressive stress-strain relationships of diagonally cracked concrete in flexural section analysis instead of employing conventional constitutive relationship for uniaxially compressed concrete. This can be done by softening the response of concrete in uniaxial compression by a factor, which decreases as shear deformations increase. This factor, known as compression softening factor $\beta$, is function of principal tensile strain in the concrete and is defined as following (Vecchio and Collins, 1986).
\[ \beta = \frac{1}{0.8 - 0.34 \frac{\varepsilon_{c1}}{\varepsilon_{co}}} \leq 1.0 \]  

where \( \varepsilon_{co} \) is concrete strain corresponding maximum concrete cylinder strength and \( \varepsilon_{c1} \) is principal tensile strain in the concrete which can be determined through in-plane shear analysis of the flexural element. The procedure for determining principal tensile strain in the concrete and compression-softening factor is explained in the subsequent section.

In addition to considering cracked concrete behavior in the fiber model, enhancement in the strength and ductility of the concrete due to confinement and contribution of the concrete tensile properties to section moment capacity must also be considered in the analysis. For determining realistic moment capacity and analyzing the buckling of the longitudinal bars under excessive compressive strains, confined core concrete and unconfined cover concrete must be modeled separately with their respective stress-strain relationships.

Fiber model analysis results in a moment-curvature relationship for given geometric and material properties, reinforcement details and applied axial load for the cross-section being analyzed. From here, lateral load \( V \) corresponding to respective moment capacity, resulting average shear stress from flexural analysis \( \tau_f \) and maximum lateral force sustainable by the column can be calculated with the help of following equations.

\[ V = \frac{M}{a} \quad ; \quad \tau_f = \frac{V}{bd} \quad ; \quad V_p = \frac{M_p}{a} \]  

where, \( M \) is the flexural section moment capacity at any load level, \( b \) is width of the section, \( d \) is the effective depth of the section, \( a \) is the shear span equal to cantilever
column length and one half of the length a fixed ended column, and \( M_p \) is maximum moment capacity from flexural section analysis.

Flexure deformations are calculated with the help of plastic hinge model in which elastic and inelastic curvatures are idealized separately. In this model, a linear curvature distribution is assumed in the elastic range over the length of the column, and the inelastic curvatures are lumped at the column end over the plastic hinge length. The conceptual illustration of the plastic hinge model for a cantilever column is presented in Figure 2.3. Hence, lateral displacement due to flexure \( \Delta_f \) can be calculated by integrating curvature over the length of the column as per Equation 2.3,

\[
\Delta_f = \int_0^s \phi(x) x \, dx
\]

where \( \phi(x) \) is section curvature at distance \( x \) measured along column axis, and \( \phi_y \) is curvature at yield point. The plastic hinge length \( L_p \) is taken as one-half of the section depth \( h \).

### 2.3.2 Reinforcement Slip Deformations

When a reinforcing bar embedded in the concrete is subjected to a tensile force, strain accumulates in the embedded length of the bar which causes the bar to extend or slip relative to the concrete in which it is embedded. The same phenomenon is observed when a reinforced concrete column is subjected to bending moment. The concrete on tension face of the cross-section, being weak in tension, cracks at the early stage of loading and becomes ineffective in anchoring the column to the base. Resultantly, the
reinforcing bars carry tensile loads from column to the anchoring concrete. The bond stresses at concrete-steel interface in the anchoring concrete cause a tensile stress gradient in the bars over their embedded length. The steel stresses vary from zero at the dead end of the embedded bar to a maximum value at the face of beam-column or beam-footing joint. The length of the bar over which these stresses are distributed, and eventually transferred to concrete by the bond stresses, is called the development length. The accumulation of the strain over the development length causes the extension of the bars relative to the anchoring concrete. This extension is commonly known as reinforcement slip and it leads to rigid-body rotation of the column, as shown in figure 2.1. This results in lateral displacement that can be as large as 25 to 40 % of the total lateral deformations (Sezen, 2002).

The flexural deformations determined through conventional fiber section analysis (moment-curvature analysis) do not account for deformations caused by reinforcement slip at column ends. These deformations add another component of lateral displacement to overall column response. Therefore, reinforcement slip deformations must be calculated separately and resulting lateral displacement must be added to the other components of lateral displacement due to flexure and shear.

In this study, lateral displacement due to reinforcement slip is calculated using the model illustrated in Figure 2.4 (Sezen and Setzler, 2008). The model approximates the bond stress as bi-uniform function with different values for elastic and inelastic steel behaviors. The bond stress in the elastic and inelastic range is taken as \( u_e = 12\sqrt{f'_c} \text{ psi} \) (\( 1\sqrt{f'_c} \text{ MPa} \)) and \( u'_e = 6\sqrt{f'_c} \text{ psi} \) (\( 0.5\sqrt{f'_c} \text{ MPa} \)), respectively, where \( f'_c \) is concrete
compressive strength. Slip \( s \) at the loaded end of the reinforcing bar can be calculated by integrating bi-linear strain distribution \( \varepsilon_s(x) \) over the development length as follows,

\[
s = \int_0^{l_d} \varepsilon_s(x) \, dx
\]

(2.4)

where \( l_d = \frac{f_y d_b}{4u_b} \) and \( l'_d = \frac{(f_s - f_y)d_b}{4u'_b} \) are development lengths for the elastic and inelastic portion of the bar, respectively. Hence, integrating Equation 2.4, extension or slip of the reinforcing bars is,

\[
s = \begin{cases} 
\frac{\varepsilon_s l_d}{2} = \frac{\varepsilon_s f_y d_b}{8u_b} & \text{for } \varepsilon_s \leq \varepsilon_y \\
\frac{\varepsilon_s l_d + (\varepsilon_s + \varepsilon_y) l'_d}{2} = \frac{\varepsilon_s f_y d_b}{8u_b} + \frac{(\varepsilon_s + \varepsilon_y)(f_s - f_y)d_b}{8u'_b} & \text{for } \varepsilon_s > \varepsilon_y
\end{cases}
\]

(2.5)

where \( \varepsilon_s \) is the strain at loaded end of the bar, \( \varepsilon_y \) is steel yield strain, \( f_s \) is stress at loaded end of the bar, \( f_y \) is steel yield stress, and \( d_b \) is diameter of the longitudinal bar.

The reinforcement slip is assumed to occur in tension bars only and cause the rotation about the neutral axis as shown in Figure 2.4. Rotation caused due to reinforcement slip can be calculated as,

\[
\theta_s = \frac{s}{d - c}
\]

(2.6)

where \( d \) and \( c \) are the distances from the extreme compression fiber to the centroid of the tension steel and neutral axis, respectively. The lateral displacement due to slip at free end of a cantilever column can be calculated as the product of slip rotation \( \theta_s \) and length of the column \( L \) as,

\[
\Delta_s = \theta_s L
\]

(2.7)
2.3.3 Modeling of Shear Deformations

Shear deformations in reinforced concrete members have traditionally been ignored in design and research. This is mainly due to lack of their complete understanding and being difficult to measure, independent of other deformation components, in an experimental set up or a real structure. For a well-designed reinforced concrete column, shear deformations are small as compared to the flexural deformations and are often less than 10 percent of total deformations. Contrary, for a reinforced concrete column not designed according to stricter seismic design provisions, shear behavior could be the governing failure criterion. Shear deformations in such shear critical reinforced concrete column could contribute large percentage towards total deformations and hence cannot be ignored if an accurate analysis of deformation capacity is required.

Shear deformations in the DCMI are calculated using a combination of MCFT (Vecchio and Collins, 1986) and post-peak shear response envelope (Patwardhan, 2005; Sezen, 2008). In this model, pre-peak non-linear shear force-shear deformation response is obtained from in-plane analysis of the shear element based on MCFT while considering the interaction of the axial strain (Mostafaei and Kabeyasawa, 2007). Axial strain obtained from flexural section analysis is incorporated into the total axial strain of the shear element to include the effect of flexural behavior on shear response. After the peak strength has reached, shear strength is first assumed to remain constant at its peak value until the onset of the shear strength degradation and then declines linearly with increasing shear deformations to the point of axial load failure (Figure 2.5). At the point of axial load failure, lateral strength is assumed zero. The peak strength $V_{peak}$ in the
proposed shear response model refers to the point where response estimation by MCFT terminates due to either shear failure or reaching the load step corresponding to the peak flexural strength prior to experiencing shear failure. Hence, the peak strength $V_{\text{peak}}$ is the minimum of the shear strength of the column $V_n$ and shear force corresponding to the maximum moment that can be carried by the section $V_p$.

By modifying the equation proposed by Gerin and Adebar (2004), shear displacement at the onset of shear degradation $\Delta_{v,u}$ can be calculated as follows (Patwardhan, 2005; Sezen, 2008).

$$\Delta_{v,u} = \left(4 - 12 \frac{V_n}{f_c'}\right) \Delta_{v,n}$$

(2.8)

where $V_n = \frac{V_{\text{peak}}}{bd}$ is the shear stress at peak strength, $f_c'$ is the concrete compressive strength, and $\Delta_{v,n}$ is the shear displacement corresponding to the peak strength as determined from MCFT analysis. The shear displacement at axial load failure $\Delta_{v,f}$ is calculated as,

$$\Delta_{v,f} = \Delta_{ALF} - \Delta_{f,f} - \Delta_{s,f} \geq \Delta_{v,u}$$

(2.9)

where $\Delta_{ALF}$ is the total displacement at axial load failure, $\Delta_{f,f}$ and $\Delta_{s,f}$ are the flexural and slip displacement at the point of axial load failure, respectively. The total displacement at axial load failure is determined by the expression based on a shear friction model and an idealized shear failure plane (Elwood and Moehle, 2005).
\[ \Delta_{ALF} = \frac{0.04L(1 + \tan^2 \theta)}{\tan \theta + P \left( \frac{s_h}{A_{sv} f_{sv} d_c \tan \theta} \right)} \] (2.10)

where \( \theta \) is the angle of the shear crack, \( P \) is the axial load, \( A_{sv} \) is the area of transverse steel with yield strength \( f_{sv} \) at spacing \( s_h \), and \( d_c \) is the depth of the core concrete measured to the centerlines of the transverse reinforcement. In the derivation, \( \theta \) is assumed 65 degrees. The values of \( \Delta_{f,f} \) and \( \Delta_{s,f} \) in Equation 2.9 are determined according to the expected failure mode and classification of the column into categories as explained in subsequent subsection.

For modeling pre-peak shear response of the columns, MCFT is employed in this model. As MCFT is a complex approach, analytical procedure and relevant details are explained in a separate section below. These details will be frequently referred and discussed subsequently in development of new proposed model.

2.4 Modified Compression Field Theory

MCFT is a suitable approach for response estimation of reinforced concrete membrane elements subjected to normal and shear stresses. Vecchio and Collins (1986) developed the theory after modifying previously proposed Compression Field Theory (CFT) (Mitchell and Collins, 1974; Collins and Mitchell, 1980). Relative to CFT, MCFT includes compression-softening effects, contribution of tensile stresses in cracked concrete and local stress conditions at crack. It is essentially a smeared rotating crack model in which cracked concrete is treated as a new orthotropic material with its unique stress-strain characteristics. The theory consists of compatibility, equilibrium, and
constitutive relationships formulated in terms of average stresses and strains. The critical aspect of MCFT is the consideration of local stress-strain conditions at cracks ensuring that the tension in the concrete can be transmitted across the crack and shear stress on the surface of the crack does not exceed maximum shear provided by the aggregate interlock. Thus, load deformation response of the members loaded in shear can be estimated by considering compatibility of average strains for concrete and reinforcement, equilibrium relationships involving average stresses in concrete and reinforcement, and appropriate stress-strain relationships for reinforcement and diagonally cracked concrete.

2.4.1 Compatibility Conditions in MCFT

Consider an orthogonally reinforced concrete membrane element as shown in Figure 2.6(a). The element consists of smeared reinforcement in longitudinal (x) and transverse (y) directions, with the corresponding reinforcement ratios \( \rho_x \) and \( \rho_y \). The yield strengths of longitudinal and transverse reinforcement are \( f_{yx} \) and \( f_{yy} \), respectively. The concrete is characterized by a cylinder compressive strength \( f'_c \), a strain at peak \( \varepsilon_{co} \) and a tensile cracking stress \( f_{cr} \). The element’s edge planes are subjected to uniform normal stresses \( f_x \), \( f_y \) and shear stress \( \nu_{xy} \). The deformation of the element is assumed to occur such that the edges remain straight and parallel.

Under the applied loads, an equilibrium condition is attained resulting in unique strain condition defined by two normal strains \( \varepsilon_x \) and \( \varepsilon_y \) and the shear strain \( \gamma_{xy} \) as shown in Figure 3.4(b). From the Mohr’s circle of the average strains (Figure 3.4c and
3.4d), average concrete principal tensile strain $\varepsilon_{c1}$, average concrete principal compressive strain $\varepsilon_{c2}$ and orientation of principal strain field can be determined as,

$$\varepsilon_{c1} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \frac{1}{2}\sqrt{\left(\varepsilon_x - \varepsilon_y\right)^2 + \left(\gamma_{xy}\right)^2}$$  \hspace{1cm} (2.11)\\

$$\varepsilon_{c2} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) - \frac{1}{2}\sqrt{\left(\varepsilon_x - \varepsilon_y\right)^2 + \left(\gamma_{xy}\right)^2}$$  \hspace{1cm} (2.12)\\

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}\right)$$  \hspace{1cm} (2.13)\\

The inclination of the principal strain $\theta_p$ as given by Equation 2.13 can either be orientation of principal tensile strain $\theta_t$ or orientation of principal compressive strain (crack angle) $\theta_c$ depending upon the magnitude of normal strains $\varepsilon_x$ and $\varepsilon_y$. Hence,

$$\theta_p = \theta_t \quad \text{for} \quad \varepsilon_x > \varepsilon_y$$

$$\theta_p = \theta_c \quad \text{for} \quad \varepsilon_x < \varepsilon_y$$  \hspace{1cm} (2.14)\\

The principal tensile plan and principal compressive plane are 90° apart, i.e., $\theta_t = \theta_c \pm 90°$ or $\theta_t = \theta_c \pm 90°$. In addition, positive orientation of principal planes is considered to make counterclockwise angle with positive x-axis. Hence, following these rules, the relationship between $\theta_t$ and $\theta_c$ can be summarized as,

If $\theta_p = \theta_t > 0$ \quad $\Rightarrow$ \quad $\theta_c = \theta_t - 90°$

If $\theta_p = \theta_t < 0$ \quad $\Rightarrow$ \quad $\theta_c = \theta_t + 90°$

If $\theta_p = \theta_c > 0$ \quad $\Rightarrow$ \quad $\theta_t = \theta_c - 90°$

If $\theta_p = \theta_c < 0$ \quad $\Rightarrow$ \quad $\theta_t = \theta_c + 90°$  \hspace{1cm} (2.15)
2.4.2 MCFT Constitutive Relationships

In addition to relevant compatibility and equilibrium conditions, constitutive relationships are required to link average stresses to average strains for concrete and reinforcement in MCFT. Derivation of these laws, especially for the concrete, is one of the important aspects in development of the theory and simulates cracked concrete behavior in compression and tension.

2.4.2.1 Concrete in Compression

The constitutive laws for concrete in compression in MCFT have been derived by Vecchio and Collins (1986) after investigating experimental stress-strain behavior of cracked concrete. These laws consider reduction in compressive strength and stiffness of concrete due to shear after softening the uniaxial response by compression softening factor. The relationship for compressive concrete behavior in MCFT was suggested as, and is shown in Figure 2.7(a),

\[ f_{c2} = \beta f_{co} \left[ 2 \left( \frac{\varepsilon_{c2}}{\varepsilon_{co}} \right) - \left( \frac{\varepsilon_{c2}}{\varepsilon_{co}} \right)^2 \right] \]  

(2.16)

where \( \beta \) is compression softening factor defined as per Equation 2.1, \( \varepsilon_{c2} \) is concrete compressive strain and \( f_{c2} \) is corresponding concrete stress. The term multiplied with \( \beta \) in Equation 2.16 is the Hognested Parabola relationship for concrete in uniaxial compression, often used for normal strength concrete.
2.4.2.2 Concrete in Tension

The constitutive relationship for the concrete in tension was also developed by Vecchio and Collins (1986) based on their reinforced concrete panel tests. The concrete tensile stress-strain relationship, as illustrated in Figure 2.7(b), relates principal concrete tensile stress \( f_{ct} \) to principal concrete tensile strain \( \varepsilon_{ct} \). The relationship suggested prior to cracking i.e., \( 0 \leq \varepsilon_{ct} \leq \varepsilon_{cr} \) is linearly elastic and is given by the expression,

\[
f_{ct} = E_c \varepsilon_{ct}
\]

(2.17)

where, \( E_c \) is modulus of elasticity of the concrete, \( \varepsilon_{cr} \) is cracking strain corresponding to uniaxial cracking strength of concrete \( f_{cr} \), and \( f_{co} \) uniaxial compressive strength of concrete in MPa. These quantities are given by following expressions,

\[
E_c = 2 f_c / \varepsilon_{co} \quad ; \quad \varepsilon_{cr} = f_{cr} / E_c \quad ; \quad f_{cr} = 0.33 \sqrt{f_c}
\]

(2.18)

After cracking, the concrete tensile stresses continue to exist due to bond interaction between concrete and reinforcement and decrease as the principal concrete tensile strains increase. This phenomenon is known as tension stiffening. The relationship suggested after cracking, i.e., \( \varepsilon_{ct} > \varepsilon_{cr} \) is,

\[
f_{ct} = \frac{f_{cr}}{1 + \sqrt{200\varepsilon_{ct}}}
\]

(2.19)

For large reinforced concrete elements, this relationship is taken as (Vecchio and Collins, 1986),

\[
f_{ct} = \frac{f_{cr}}{1 + \sqrt{500\varepsilon_{ct}}}
\]

(2.20)
2.4.2.3 Reinforcement Stress-Strain Relationship

MCFT adopts bi-linear stress-strain relationship for the longitudinal and transverse reinforcement in both tension and compression that consists of initial ascending linear-elastic branch followed by a yield plateau. In the development of the theory, it is assumed that the axial stress in the reinforcement depends only on one strain parameter, the axial strain in the reinforcement. The shear stress resisted by the reinforcement on the plane normal to the reinforcement is zero. The relationship for average axial stress $f_x$ and average strain $\varepsilon_x$ used in MCFT is shown in Figure 3.5c and is described by the following equations,

$$
\begin{align*}
    f_{sx} &= E_s \varepsilon_{sx} \leq f_{sx,yield} \\
    f_{sy} &= E_s \varepsilon_{sy} \leq f_{sy,yield}
\end{align*}
$$

(2.21)

The subscripts $x$ and $y$ represent $x$-direction (longitudinal) and $y$-direction (transverse) reinforcement, respectively. $E_s$ is modulus of the elasticity of the steel, and $f_{sx,yield}$ and $f_{sy,yield}$ are yield stress of the reinforcement in $x$- and $y$-directions, respectively.

Compatibility of the average strains in MCFT requires that strain in reinforcement and concrete must match with the in plane strains in $x$ and $y$-directions i.e., $\varepsilon_{sx} = \varepsilon_x$ and $\varepsilon_{sy} = \varepsilon_y$. Hence, the Equation 3.14 takes the form as,

$$
\begin{align*}
    f_{sx} &= E_s \varepsilon_x \leq f_{sx,yield} \\
    f_{sy} &= E_s \varepsilon_y \leq f_{sy,yield}
\end{align*}
$$

(2.22)
2.4.3 Considerations of Local Cracks Conditions

Consideration of the local stress conditions at the cracks is another very important aspect in development of the MCFT. These considerations ensure that the stresses can be transmitted across the cracks. The formulations considering average stresses and strains do not capture local variations that may occur at the cracks. For example, tensile stresses in the reinforcement will be higher than the average at the cracks and lower than the average midway between the cracks. On the other hand, the concrete tensile stresses will be zero at the crack and higher than the average midway between the cracks. These local variations are important as the response of bi-axially loaded element may be governed by the reinforcement’s ability to transmit tension across the cracks or sliding shear failure along the cracks. To address these possibilities, MCFT limits the local stresses at the cracks and the average concrete tensile stress between the cracks.

Figure 2.8(a) shows average stresses at a section between the cracks perpendicular to the principal tensile stress direction and Figure 2.8(b) shows local stresses at the free surface of the crack. At the free surface of the crack, the average concrete tensile stresses reduce to zero. This causes reinforcement stresses to increase locally at the crack in order to transmit tensile stresses across the crack. Hence, average concrete tensile stresses must be limited to avoid failure of the reinforcement at the crack. Static equivalency of the average and local stresses in the direction normal to the crack surface results in the condition that limits the concrete average tensile stress to the following upper limit to insure its transmission across the crack.

\[
f_{c1} \leq \rho_x \left( f_{srx} - f_{sx} \right) \cos^2 \theta_{nx} + \rho_y \left( f_{sry} - f_{sy} \right) \cos^2 \theta_{ny}
\]  

(2.23)
where, $\rho_x$ and $\rho_y$ are reinforcement ratio in $x$ and $y$-directions respectively, $f_{srx}$ and $f_{sry}$ are local reinforcement stresses at crack, and $\theta_{nx}$ and $\theta_{ny}$ are the angles between the normal to the crack and reinforcement in $x$ and $y$-directions, respectively. The values for $\theta_{nx}$ and $\theta_{ny}$ are defined positive counterclockwise, as shown in Figure 2.8(b), and may be determined as,

$$\begin{align*}
\theta_{nx} &= -\theta_i \\
\theta_{ny} &= -\theta_c
\end{align*}$$

(2.24)

In MCFT, yielding of the reinforcement is the upper limit for local reinforcement stresses at the crack and hence, the average concrete tensile stresses are limited by reserve capacity of the reinforcements.

$$f_{ci} \leq \rho_x (f_{sx \text{yield}} - f_{sx}) \cos^2 \theta_{nx} + \rho_y (f_{sy \text{yield}} - f_{sy}) \cos^2 \theta_{ny}$$

(2.25)

The other consideration of local stresses at the crack deals with the shear stresses, which are present locally at the crack surface due to reinforcement crossing the cracks at skew angles (Figure 2.8(b)). This consideration limits the local shear stresses at the crack surface by the shear resistance provided by the aggregate interlock mechanism. Static equivalency of the average and local stresses in the direction tangential to the crack determines local shear stresses as follows,

$$v_{ci} = \rho_x (f_{srx} - f_{sx}) \cos \theta_{nx} \sin \theta_{nx} + \rho_y (f_{sry} - f_{sy}) \cos \theta_{ny} \sin \theta_{ny}$$

(2.26)

This local shear stress value at the crack must not exceed the shear strength provided by the interface between the cement paste and the aggregate particles due to aggregate interlock mechanism.

$$v_{ci} \leq 0.18v_{cimax}$$

(2.28)
where \( w \) is crack width in millimeters, \( a \) is maximum aggregate size in millimeters and \( f_{co} \) is concrete compressive strength in MPA. The crack width \( w \) in Equation 2.29 is average crack width over the crack surface and can be taken as product of the principal tensile strain \( \varepsilon_{c1} \) and the average spacing of diagonal cracks in the direction normal to the crack \( S_{cr} \).

\[
w = \varepsilon_{c1} S_{cr}
\]  

where,

\[
S_{cr} = \frac{1}{\sin \theta + \cos \theta} \left( \frac{S_{mx}}{S_{my}} \right)
\]

\( S_{mx} \) and \( S_{my} \) in the above equation are the crack spacing that indicate crack control characteristics of \( x \) and \( y \)-reinforcement respectively. These quantities can be approximately taken as \( S_{mx}=1.5 \times \) maximum distance from \( x \)-bars and \( S_{my}=1.5 \times \) maximum distance from \( y \)-bars. The term \( \theta \) in Equation 2.31 is inclination of the crack with respect to longitudinal axis as shown in Figure 2.8(a). Its value can be determined as \( \theta = |\theta_c| \) such that \( 0 < \theta \leq 90^\circ \).

If Equation 2.28 is not satisfied i.e., the local shear stress on the crack exceeds the shear resistance provided by the aggregate interlock mechanism, the average concrete tensile stress \( f_{c1} \) must be reduced as,

\[
f_{c1} = 0.18 \nu_{c1\text{max}} \tan \theta_c
\]
2.5 Summary

This chapter presented general background and overview of previous research on lateral response estimation of reinforced concrete columns. The components of total lateral displacements were explained and approaches for modeling each component were described. The modeling of individual displacement components form part of the analytical procedure called Displacement Component Model with Interaction (Lodhi, 2010). Various aspects of the model presented in this chapter will used in subsequent chapters. The flexure model incorporates concrete tensile behavior, interaction of compression softening and buckling of compression bars into the flexural analysis. The shear model includes the effect of flexural deformation on shear behavior. The pre-peak response is evaluated by employing MCFT and post-peak shear response modeled by a piecewise linear envelope by Sezen (2008). This chapter also provides a detailed description of Modified Compression Theory (MCFT). Deformation due to reinforcement slip was explained and model to capture resulting deformations was presented.
Figure 2.1: Components of the total lateral deformation in a fixed ended column

Figure 2.2: Behavior of the cracked concrete in compression

Figure 2.3: Plastic hinge model for calculating flexural displacements
Figure 2.4: Reinforcement slip model (Sezen and Setzler, 2008)

Figure 2.5: Shear response model
Figure 2.6. Reinforced concrete element: (a) in-plane stresses conditions; (b) average strains from in-plane loading; (c) average strains in cracked concrete; and (d) Mohr’s circle for average strains.
Figure 2.7. MCFT constitutive relationships: (a) cracked concrete compression; (b) cracked concrete in tension; and c) reinforcing steel

Figure 2.8. Average and local stresses in the element: (a) average stresses between cracks; and (b) local stresses at a crack
CHAPTER 3

LATERAL BEHAVIOR OF REINFORCED CONCRETE COLUMNS–MODELING OF TOTAL RESPONSE

3.1 Introduction

In Chapter 2, details of displacement components due to flexure, reinforcement slip and shear along with respective modeling approaches were presented. In this chapter, the procedure for combining individual displacement components and other critical aspects of column behavior, such as interaction between flexure and shear mechanism and buckling of compression bars, are explained. The implementation of DCMI is shown for four previously tested reinforced concrete column specimens and predicted responses are compared with experimentally observed responses. The capabilities of procedures employed to model displacement components, total displacement and other aspects considered in the analysis are highlighted.

3.2 Modeling of Total Lateral Response

In order to model the total lateral response of a column, three deformation components due to flexure, reinforcement slip and shear (explained in Chapter 2) should be combined together considering their interconnectedness. In DCMI, total column lateral response is modeled as a set of three springs in series; each spring representing lateral
displacement component due to flexure, bar slip and shear. Each spring is subjected to same force and total displacement is sum of responses of each spring. The pre-peak total response is obtained by simply adding deformation components due to flexure, bar slip and shear mechanism as described above. After reaching the peak, the mechanism limiting the peak strength (flexure or shear) will dominate the behavior. The procedure for combining deformation components for post peak is explained below and the model is illustrated in Figure 3.1.

For post-peak behavior, the column is classified into one of the five categories based on a comparison of its shear, yield and flexural strength and rules are specified for the combination of the deformation components for each category (Setzler and Sezen, 2008). The yield strength $V_y$ is defined as the lateral load corresponding to the first yielding of the tension bars in the column and flexural strength $V_p$ is the lateral load corresponding to the peak moment calculated from flexural analysis. Both of these loads are calculated from moment-curvature analysis of the fiber model as explained in Section 2.3.1. The shear strength $V_n$ for the columns failing in shear prior to the reaching flexural strength or failing close to flexural strength is determined from the shear model, where $V_n = V_{peak}$ discussed above. For other columns where peak strength by the shear model is close to the flexural strength, shear strength is calculated as a function of displacement ductility (Sezen and Moehle, 2004).

$$V_n = k(V_c + V_s) = k \left[ \frac{6\sqrt{f'_c}}{a/d} \sqrt{1 + \frac{P}{6f'_cA_g}} \right] 0.80A_g + \frac{A_{yw}f_{yw}d}{s}$$  \hspace{1cm} (3.1)
where $V_c$ is the concrete contribution to shear strength, $V_s$ is the transverse steel contribution to shear strength, $A_g$ is gross cross-sectional area, $a/d$ is the aspect ratio and $k$ is a factor related to the displacement ductility which is the ratio of the maximum displacement to the yield displacement. The value for $k$ varies from 0.7 to 1.0 for displacement ductility from less than 2 to greater than 6, respectively. In this study, the value for $k$ is taken as 1.0 as classification of the columns is based on initial or low-ductility shear and flexural strengths.

The classification system and rules governing the post peak response in each category are described below and are illustrated in Figure 3.2. The peak column response is limited by the smaller of the shear strength and flexural strength, and post-peak response is assumed to be governed by the limiting mechanism (i.e., flexure or shear).

3.2.1 Category – I ($V_n < V_y$)

In this category of the columns, shear strength is less than the yield strength and column fails in shear while the flexural behavior remains elastic. The deformation at peak strength (i.e., shear strength) is the sum of deformations in each spring at the peak strength. After the peak strength is reached, the shear behavior dominates the response. As the shear strength degrades, the flexure and slip springs unload along their initial responses. The post-peak deformation at any lateral load level is the sum of the post-peak shear deformation and the pre-peak flexural and slip deformations corresponding to that load.
3.2.2 Category – II \( (V_p \leq V_n < 0.95 V_p) \)

The shear strength is less than flexural strength and column fails in shear, however inelastic flexural deformation occurring prior to shear failure affects the post-peak behavior. The deformation at peak strength is the sum of the deformations in each spring at the peak strength. Shear deformations continue to increase after the peak shear strength is reached, but the flexure and shear springs are locked at their peak strength values. Hence, post-peak deformations at any lateral load level is the sum of flexural and slip deformations at the peak strength and post-peak shear deformation at that load.

3.2.3 Category – III \( (0.95 V_p \leq V_n \leq 1.05 V_p) \)

The shear and flexural strengths are nearly identical. Shear and flexural failure are assumed to occur “simultaneously,” and both mechanisms contribute to the post-peak behavior. The post-peak deformation at any lateral load level is the sum of the post-peak flexure, slip, and shear deformations corresponding to that load.

3.2.4 Category – IV \( (1.05 V_p < V_n \leq 1.4 V_p) \)

The shear strength is greater than the flexural strength and the column may potentially fail in flexure, however large shear deformations affect the post-peak behavior and shear failure may occur as the displacements increase. The deformation at peak strength is the sum of the deformations in each spring at the peak strength. After the peak strength is reached, flexural and slip deformations continue to increase according to their models, but the shear spring is locked at its value at peak strength. The post-peak
deformation at any lateral load level is the sum of the post peak flexural and slip deformations corresponding to that load and the shear deformation at peak strength.

### 3.2.5 Category – V ($V_n > 1.4V_p$)

The shear strength is much greater than the flexural strength and column fails in flexure while shear behavior remains elastic. The peak strength of the column is the flexural strength calculated from the flexure model. If the column strength degrades, flexural and slip deformations continue to increase according to their models, while the shear spring unloads with an unloading stiffness equal to its initial stiffness. The post-peak deformation at any lateral load level is the sum of the post-peak flexural and slip deformations and the pre-peak shear deformation corresponding to that load.

For category-I columns, $\Delta_{f,f}$ and $\Delta_{s,f}$ values to be used in Equation 2.9 are assumed zero. For the category-II columns, shear strength is less than flexural strength and these values are taken as the flexural and slip deformations at the load equal to the shear strength of the columns. For categories III, IV, and V specimens, $\Delta_{f,f}$ and $\Delta_{s,f}$ are the maximum calculated flexural and slip deformations.

### 3.3 Interaction Between Flexure and Shear Mechanisms

When a fixed-ended reinforced concrete column is subjected to lateral loading, such as during an earthquake, flexural and shear mechanisms interact with each other and affect overall response of the column. The interaction between flexural and shear deformations in the DCMI is based on the ASFI approach (Mostafaei and Kabeyasawa, 2007). Interaction methodology in ASFI approach couples axial-flexure and axial-shear
models with each other. Both mechanisms have to be evaluated simultaneously which makes ASFI approach relatively complicated and computationally intensive. The computational effort can be reduced significantly, if the analyses for flexural and shear behavior can be performed independently. Therefore, in the DCMI, the interaction of the shear deformations on flexural performance, and vice versa, are considered in a simplified manner that allows easy implementation and decoupled flexural and shear response evaluations.

3.3.1 Interaction of Concrete Compression Softening

Cracked concrete behavior is considered in flexural section analysis to represent degradation in compressive strength of the concrete due to applied shear stresses. This requires determination of the compression-softening factor $\beta$ to lower concrete stresses in uniaxial compression (Figure 2.2). The procedure for determining compression-softening factor can be adopted from Uniaxial-Shear-Flexure Model (USFM) by Mostafaei and Vecchio (2008). This is an approximate approach, which is derived after simplifying ASFI approach. USFM employs few fundamental equations of the MCFT and two assumptions on average principal compressive strain and average centroidal axial strain of the element to determine average principal tensile strain. The details of formulation, implementation and verification of USFM approach can be found in Mostafaei and Vecchio (2008).

Compression softening factor $\beta$, as defined in Equation 2.1, is a function of concrete principal tensile strain $\varepsilon_{ct}$ of the element being analyzed. The procedure to approximately determine principal tensile strain and subsequently compression softening
factor for a fixed ended column subjected to in-plane lateral load is illustrated in Figure 3.3. For an element considered between inflection point and one of the end sections of the column, $\varepsilon_{c1}$ can be determined from the following MCFT equation.

$$\varepsilon_{c1} = \varepsilon_x + \varepsilon_{yv} - \varepsilon_{c2}$$  \hspace{1cm} (3.2)

where, $\varepsilon_x$ is average axial strain at the centroid for the element and is obtained by averaging the values of centroidal axial stain at one of the end section $\varepsilon_o$ and axial strain of the inflection point $\varepsilon_{xa}$. Likewise $\varepsilon_{c2}$ is concrete principal compressive strain for the element. Its value, as per USFM assumption (Mostafaei and Vecchio, 2008), can be taken as the average of the uniaxial concrete compressive strain corresponding to resultant compressive force of the stress block at end section $\varepsilon_c$ and axial strain at the inflection point $\varepsilon_{xa}$. Hence,

$$\varepsilon_x = \frac{(\varepsilon_o + \varepsilon_{xa})}{2} \quad \varepsilon_{c2} = \frac{(\varepsilon_c + \varepsilon_{xa})}{2}$$  \hspace{1cm} (3.3)

The other unknown quantity in Equation 3.2, strain of the transverse reinforcement $\varepsilon_{yv}$, can be determined from the following MCFT based relationship.

$$\varepsilon_{yv} = \sqrt{b^2 + c - b}$$  \hspace{1cm} (3.4)

where,

$$b = \frac{f_{c1}}{2\rho_y E_{sy}} - \frac{\varepsilon_{c2}}{2}; \hspace{0.5cm} c = \frac{(\varepsilon_x - \varepsilon_{c2})(f_{c1} - f_{cx}) + f_{c1}\varepsilon_{c2}}{\rho_y E_{sy}}; \hspace{0.5cm} f_{cx} = f_x - \rho_y f_{sx}$$  \hspace{1cm} (3.5)

where, $\rho_y$ is transverse reinforcement ratio, $E_{sy}$ is modulus of elasticity of transverse reinforcement, $f_{cx}$ is concrete stress in longitudinal axis of the column, $f_x$ is applied
axial stress, $\rho_x$ is longitudinal reinforcement ratio, $f_{sx}$ is longitudinal steel stress obtained from section analysis based on average centroidal strain, $f_{c1} = 0.145\sqrt{f_c}$ is concrete principal tensile stress (in MPa), $\varepsilon_x$ is normal strain at the centroid and $\varepsilon_{c2}$ is average concrete principal compressive strains, both determined from Equation 3.3.

After calculating concrete principal tensile strain $\varepsilon_{c1}$ from Equation 3.2, compression-softening factor $\beta$ is determined from Equation 2.1 for a given curvature. This is the estimated value of $\beta$ which is employed in fiber model analysis to lower concrete stresses. In the DCMI, compression softening factor determined with the help of above mentioned procedure is employed till peak flexural strength and then a constant value equal to the last lowest is used for post-peak flexural analysis.

### 3.3.2 Interaction of Axial Strains

The effect of flexural deformations on shear behavior can considered by incorporating axial strain and shear stress due to flexure into in-plane analysis of the shear element based on axial strain interaction methodology of ASFI approach (Mostafaei and Kabeyasawa, 2007) and equilibrium of shear stresses in flexural and shear mechanisms. In this procedure, interaction of axial strain is taken into account by adding flexibility component of axial deformation due to flexure to the corresponding flexibility component of axial-shear model. By employing flexural shear stress to in-plane stress-strain relationship of the shear element, shear deformations are determined. The procedure for axial deformation interaction and determination of shear strain is described here for a fixed ended column subjected to lateral load.
The length of the column between inflection point and one of the end sections is considered as a shear element subjected to constant normal stress due to applied axial load and average shear stresses due to applied lateral load. Performing flexural analysis on fiber model of the end section and inflection point, average centroidal axial strain due to flexure $\varepsilon_{sf}$ (Figure 3.3) and corresponding flexibility component $f_{sf}$ can be determined with the help of following ASFI equations (Mostafaei and Kabeyasawa, 2007).

$$\varepsilon_{sf} = \frac{(\varepsilon_o - \varepsilon_{ml})}{2}, \quad f_{sf} = \frac{\varepsilon_{sf}}{\sigma_x}, \quad \sigma_x = \frac{P}{bd}$$

where, $\sigma_x$ is applied axial stress in longitudinal direction of the column and can be determined by dividing the applied axial load $P$ by the effective area of the cross section.

A stress-strain relationship in terms of flexibility matrix for an in plane shear element can be defined as,

$$
\begin{bmatrix}
    f_{11} & f_{12} & f_{13} \\
    f_{21} & f_{22} & f_{23} \\
    f_{31} & f_{32} & f_{33}
\end{bmatrix}
\begin{bmatrix}
    \sigma_x \\
    \sigma_y \\
    \tau_{xy}
\end{bmatrix}
=
\begin{bmatrix}
    \varepsilon_x \\
    \varepsilon_y \\
    \gamma_{xy}
\end{bmatrix}
$$

where $f_{ij}$ ($i, j = 1, 2, 3$) are flexibility components of in plane shear model, $\sigma_x$ is normal applied stresses in longitudinal direction, $\sigma_y$ is normal stress in transverse direction, $\tau_{xy}$ is shear stress, $\varepsilon_x$ is normal strain in axial direction, $\varepsilon_y$ is normal strain in transverse direction, and $\gamma_{xy}$ is shear strain.
Axial strain due to flexure $\varepsilon_{sf}$ can be taken into account in the axial-shear model by adding flexibility component obtained from Equation 3.6 into Equation 3.7.

\[
\begin{pmatrix}
(f_{11} + f_{sf}) & f_{12} & f_{13} \\
 f_{21} & f_{22} & f_{23} \\
 f_{31} & f_{32} & f_{33}
\end{pmatrix}
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix}
=
\begin{pmatrix}
\varepsilon_x + \varepsilon_{sf} \\
\varepsilon_y \\
\gamma_{xy}
\end{pmatrix}
\]  
(3.8)

In Equation 3.8, stresses in transverse direction (clamping stresses) are zero due to inexistence of lateral external force along the column, i.e., $\sigma_y = 0$. In addition, the applied shear stress $\tau_{xy}$ of the element is taken from flexural section analysis (Equation 2.2) as $\tau_{xy} = \tau_f$. In Equation 3.8, knowing the applied stresses, corresponding strains can be calculated. The flexibility matrix is obtained by inverting stiffness matrix of the shear element formulated using secant stiffness methodology of the MCFT approach (Vecchio and Collins, 1986).

### 3.4 Buckling of Compression Bars

Longitudinal reinforcing bars in columns may experience inelastic axial compression under severe loading and exhibit large lateral deformation known as buckling. The behavior in the compressive face of a concrete member at overload depends on a variety of factors such as, size and shape of the cross-section, the amount of longitudinal compression steel, the amount of transverse reinforcement providing confinement to the section, thickness of the cover concrete, and stress-strain properties for the steel and concrete (Potger et al., 2001).
The tendency for the compressively loaded steel bars to buckle and deflect outwards is initially resisted by the lateral restraint provided by the surrounding cover concrete as well as the transverse steel ties. As the compressive loads increase and approach the section capacity, the concrete surrounding the compressive bars carries large longitudinal compressive stress, and eventually becomes prone to longitudinal cracking, and spalling. After the cover concrete spalls off, ties restrain lateral movement and buckling.

In this study, compression steel stresses were reduced to account for buckling using the model shown in Figure 3.4. According to this model, compression stresses in longitudinal bars start to decrease when unconfined cover concrete starts to spall. When this happens, corresponding strain in the relevant steel layer can be calculated from flexural strain distribution across the cross section depth. This strain is $\varepsilon_{sp}$ as shown in the figure. This point can fall anywhere on typical stress-strain relationship for steel depending upon the level of flexural strain. Steel stresses follow their usual constitutive stress-strain relationship until strain reaches this limit. Then compression stresses in reinforcement follow new path defined by a line joining peak stress point to residual strength point having a slope $m$, which is calculated from following relationship (Inoue and Shimizu, 1988).

$$m = 100\epsilon_{sv}\left(\frac{1}{\sqrt{1 + 0.005\lambda^2}} - 1\right)E_{sv} ; \quad \lambda = \frac{\alpha s_h}{l_r}$$

(3.9)
where, $\varepsilon_{yx}$ is yield strain of longitudinal bars, $E_{ss}$ is modulus of elasticity for longitudinal steel, $\alpha$ is 1.0 for corner bars and 0.5 for intermediate bars, $s_h$ is stirrup or tie spacing, and $i_r$ is radius of gyration of longitudinal bar.

Diameter of the longitudinal bar and spacing of the transverse reinforcement is important parameters that affect the buckling of the compression bars (Monti and Nuti, 1992). Smaller diameter bars restrained by widely spaced ties are most likely to undergo lateral deformations and buckling much earlier during loading history than larger diameter bars confined by closely spaced transverse reinforcement. Therefore, in the DCMI, for tie spacing to bar diameter ratio $s_h/d_h$ of less than 5.0, no bar buckling is considered and compressive behavior of the reinforcement is similar to its tensile behavior. For $s_h/d_h$ ratio above 11.0, the bars are considered to buckle as soon as reinforcement yields. For $s_h/d_h$ ratios between 5.0 and 11.0, post-buckling softening is considered soon after spalling of the cover concrete in the proposed model.

### 3.5 Summary and Steps for Implementation of the DCMI Procedure

Total lateral displacement of a concrete column can be calculated under lateral loads. Note that the procedure can be used to model the response prior to peak strength (under increasing loads) and, also beyond peak response (possible decreasing lateral loads due to strength and stiffness degradation). The major steps of the procedure are to:

- Define uniaxial material properties for unconfined and confined concrete, and reinforcing steel. Include the effect of compression softening of concrete (Equation 2.1
and Figure 2.2). Consider the effect of compression bar buckling under large axial deformations (Equation 3.9 and Figure 3.4).

- Define fiber cross section for flexural analysis and perform moment-curvature analysis of the cross section.
- Calculate lateral load versus flexural displacement by integrating curvatures over the height of column (Equations 2.2 and 2.3).
- Calculate lateral load versus reinforcement slip displacement (Equations 2.5 through 2.7).
- Perform MCFT analysis to establish lateral load versus shear displacement relationship up to maximum shear strength (Figure 2.5) while considering the interaction of axial strains due to flexure (Mostafaei and Kabeyasawa, 2007). Alternatively, an approximate method (Sezen, 2008) can be used and this step can be skipped. Obtain lateral load versus shear displacement envelope (Figure 2.5).
- Classify the column and combine flexure, slip, and shear responses (Section 3.2, and Figure 3.2). During calculation of combined or total displacement, at each step, consider the interaction between axial-shear and flexure mechanisms (Section 3.4 and Figure 3.3).

3.6 Model Verification

The proposed procedure is implemented to evaluate response of previously tested reinforced concrete columns tested by Sezen (2002) and predicted responses are compared with experimental test data. These columns are very useful for modeling purpose as they provide experimental force-displacement data for each of the flexure, slip, and shear components individually in addition to the overall response. Hence, the
experimental data from these columns are used to validate the component and total deformation models proposed in this study (Lodhi, 2010; Sezen and Moehle 2006).

3.6.1 Test Specimens and Material Properties

The columns (Sezen, 2002) modeled in this study are lightly reinforced and have shear and flexural design strengths very close to each other. These are 18 in. (457 mm) square columns with fixed ends at top and bottom having height of 116 in. (2946 mm). The columns had eight No.9 bars and No. 3 column ties with 90-degree end hooks spaced at 12 in. (305 mm). Specimens-1 and -4 were tested with a constant axial load of 150 kips (667 KN), whereas, Specimen-2 was tested under a constant axial load of 600 kips (2670 KN). The columns were tested under unidirectional cyclic lateral loading, except for Specimen-4, which was tested under monotonically increasing load after few initial cycles of elastic loading. All of the test specimens are modeled with average concrete compressive strength of 3.08 psi (21.2 MPa). The yield strength of longitudinal and transverse reinforcement are taken to be 63 ksi (434 MPa) and 69 ksi (476 MPa), respectively. Other details of test specimens, material properties used for the development of reinforcing steel and unconfined and confined concrete models can be found in Setzler and Sezen (2008) and Sezen (2002 and 2008).

3.6.2 Material Constitutive Relationships

3.6.2.1 Concrete Behavior in Compression

Concrete behavior in compression for confined core concrete and unconfined cover concrete is modeled with the help of Mander et al. (1988) model, and shown in
Figure 3.5. According to this model, compressive stress-strain relationship of the confined concrete is defined as,

\[
f_c = \frac{f_{cc}r\left(\varepsilon_c/\varepsilon_{cc}\right)}{r-1+\left(\varepsilon_c/\varepsilon_{cc}\right)^2},\text{ where } \varepsilon_{cc} = \varepsilon_{co} \left[1+5\left(\frac{f_{cc}}{f_{co}}-1\right)\right], \quad r = \frac{E_c}{E_{sec}}, \quad E_{sec} = \frac{f_{cc}}{\varepsilon_{cc}} \quad (3.10)
\]

where \( f_{cc} \) is the peak confined concrete strength, \( \varepsilon_c \) is the concrete strain, \( \varepsilon_{cc} \) is the concrete strain at peak stress for confined concrete, \( \varepsilon_{co} \) is the concrete strain at peak stress in unconfined concrete (taken here as 0.002), \( f_{co} \) is the concrete compressive cylinder strength, \( E_c = 57,000 \sqrt{f_{co}} \text{ psi} \) \( (5000 \sqrt{f_{co}} \text{ Mpa}) \) is the modulus of elasticity of normal weight concrete, and \( E_{sec} \) is the secant modulus of the concrete.

In this study, ultimate compressive strain \( \varepsilon_{cu} \) for confined concrete is calculated from Equation 2.21, which is obtained by slightly modifying the maximum strain formula for spirally confined concrete in Priestley (1996).

\[
\varepsilon_{cu} = 0.004 + 0.14 (\rho_x + \rho_y) \frac{f_{yy}}{f_{cc}}
\]

\[
\rho_x = \rho_y = \frac{\pi d_i^2}{2 s_h d_c} \text{ (for square columns)}
\]

where \( f_{yy} \) is yield strength, \( d_i \) is diameter and \( s_h \) is center-to-center spacing of the transverse reinforcement, respectively, and \( d_c \) is depth of concrete core in the cross section.

For unconfined cover concrete, compressive stress-strain relationship is defined by following equation,

\[
f_{c,unconf} = \frac{f_{co} r_1\left(\varepsilon_c/\varepsilon_{co}\right)}{r_1-1+\left(\varepsilon_c/\varepsilon_{co}\right)^{r_1}} \text{ for } \varepsilon_c \leq 2\varepsilon_{co}, \text{ where } r_1 = \frac{E_c}{E_{sec1}}, \quad E_{sec1} = \frac{f_{co}}{\varepsilon_{co}} \quad (3.12)
\]
After reaching the strain of $2\varepsilon_{co}$, cover concrete is assumed to start spalling and part of falling branch in the region where $\varepsilon_c > 2\varepsilon_{co}$ is assumed to be a straight line which reaches zero stress at spalling strain $\varepsilon_{sp}$, taken equal to 0.006 in this study (Figure 3.5). In order to apply compression softening effect, compressive behavior for confined and unconfined concrete obtained from Equation 3.10 and 3.12 must be multiplied by compression softening factor $\beta$ from Equation 2.1.

### 3.6.2.2 Concrete Behavior in Tension

Concrete behavior in tension is simulated by the model developed by Vecchio and Collins (1986), as illustrated in Figure 3.6. As per this model, relationship suggested prior to cracking is linearly elastic and is given by the expression.

\[
f_{cl} = E_c \varepsilon_{cl}, \text{ for } 0 \leq \varepsilon_{cl} \leq \varepsilon_{cr}
\]  
(3.13)

where, $E_c \approx 2f_{co}/\varepsilon_{co}$ is modulus of elasticity of the concrete, $\varepsilon_{cr} = f_{cr}/E_c$ is cracking strain corresponding to uniaxial cracking strength of concrete $f_{cr} = 0.33\sqrt{f_{co}}$ MPa.

After cracking, the concrete tensile stresses continue to exist due to bond interaction between concrete and reinforcement and decrease as the principal concrete tensile strains increase. This phenomenon is known as tension stiffening. The relationship suggested after cracking is,

\[
f_{cl} = \frac{f_{cr}}{1 + \sqrt{200\varepsilon_{cl}}}, \text{ for } \varepsilon_{cl} > \varepsilon_{cr}
\]  
(3.14)

For large reinforced concrete elements, this relationship can be modified slightly to the following expression (Vecchio and Collins, 1986),

...
\[ f_{c1} = \frac{f_{ce}}{1+\sqrt{500\varepsilon_{c1}}} \] (3.15)

3.6.2.3 Reinforcing Steel Behavior

The reinforcing steel behavior (Figure 3.7) in this study is modeled considering a linear elastic behavior, a yield plateau, and a non-linear strain-hardening region, as per set of following equations,

\[
\begin{align*}
    f_s &= E_s \varepsilon_s \quad \text{for } \varepsilon_s \leq \varepsilon_y \\
    f_s &= f_y + (\varepsilon_s - \varepsilon_{sh}) \alpha E_s \quad \text{for } \varepsilon_y \leq \varepsilon_s \leq \varepsilon_{sh} \\
    f_s &= f_u - (f_u - f_{sh}) \left(\frac{\varepsilon_s - \varepsilon_{sh}}{\varepsilon_u - \varepsilon_{sh}}\right)^p \quad \text{for } \varepsilon_{sh} \leq \varepsilon_s \leq \varepsilon_u
\end{align*}
\] (3.16)

where \( E_s \) is the elastic modulus of steel, \( \varepsilon_s \) is the steel strain, and the subscripts \( y, sh, \) and \( u \) refer to the yield point, the onset of strain hardening, and the ultimate stress, respectively. The order of the curve \( p \) defines the strain-hardening region, and is often taken as 2 for a parabolic curve, and \( \alpha \) is a coefficient that defines the slope of the yield plateau. For the columns tested by Sezen (2002), these parameters were used to define the longitudinal steel stress-strain model as \( \varepsilon_{sh} = 0.016, \alpha = 0.02, f_u = 93.5 \text{ ksi (645 MPa)}, \) \( \varepsilon_u = 0.23, p = 6, E_s = 29,000 \text{ ksi (200,000 MPa)}. \) In this study, compressive steel behavior is not the same as tensile steel behavior, and is modified to consider effect of compression bar buckling. After reaching buckling strain, compressive steel stresses are reduced as per the bar buckling model explained in Section 3.4.
3.6.3 Comparison of the Predicted and Experimental Responses

Lateral load-flexural displacement relationships for Specimen-1 and 4 are presented in Figure 3.8(a) and 3.8(b), respectively. In this comparison, test specimens are also analyzed using another displacement component model (Setzler and Sezen, 2008). This model also treats deformations due to flexure, bar slip and shear individually, however, does not consider softening of concrete compression strength, concrete tensile behavior and buckling of compression bars in flexural analysis. It can be seen that both approaches predict identical pre-peak responses, which match very well with the experimental data. Peak load and deformation at peak load is also estimated very well by both approaches. For post peak behavior, however, predicted responses are quite different. After reaching the strains corresponding to the start of compression bar buckling, response predicted by the proposed procedure gradually drops and generally follows stiffness of the measured response. The diverging near-peak and post-peak predicted responses by both approaches highlight the need to consider concrete softening and bar buckling effect in the analysis. Figure 3.8(c) and 3.8(d) presents load-displacement relationships due to reinforcement slip for Specimen-1 and 2, respectively. The predicted responses by displacement component model and proposed method produce almost identical response up until peak load and then diverge in the post peak range. Again, this highlights the need for considering buckling of compression bars in the flexural analysis.

Lateral load-shear displacement relationships for Specimen-1 and 2 are presented in Figure 3.8(e) and 3.8(f), respectively. For comparison of the predicted shear responses, the columns are also analyzed with ASFI approach (Mostafaei and Kabeyasawa, 2007).
The predicted responses by ASFI approach and proposed procedure are identical until observed peak and follows experimental data generally well. Peak shear strength is generally captured well by both approaches. For post-peak behavior, proposed model shows strength degradation as deformations increase. In ASFI approach, after reaching peak load, shear deformations are calculated from secant stiffness at peak strength, which is kept constant for post peak behavior. As a result, post-peak predicted shear response in ASFI approach does not show shear strength degradation.

Figure 3.9 shows the comparison of predicted and experimental lateral load-total displacement relationships for Specimen-1, 2 and 4. Shear strength of Specimen-1 (Figure 3.9(a)) is calculated as 69.0 kips and flexural strength from moment-curvature analysis is 70.0 kips. Hence, this specimen is classified as category-III column, for which total displacement at any point in the response is sum of flexural, slip and shear displacement at that load step (Figure 3.5). With the proposed procedure, initial response is predicted very well up to the peak strength. Peak strength and deformation at peak load and the post peak response are captured well. The Specimen-2 (Figure 3.9(b)) has shear and flexural strengths of 92.0 and 72.0 kips, respectively, and is classified as category-IV column. For this column, shear deformation is frozen at its value at peak strength (flexural strength, 72.0 kips) and added to flexural and slip displacements for post-peak response. Predicted response by the proposed approach slightly overestimates the pre-peak stiffness and peak load in the positive direction and follows post peak experimental response fairly well in both directions. Specimen-4 (Figure 3.9(c)) is identical to Specimen-1 except that it was tested under monotonically increasing lateral load after few initial elastic cycles. Comparison of shear and flexural strength classifies this column
into category-III column. The predicted response by the proposed procedure follows the trend in experimental data but slightly overestimates the initial stiffness and peak strength.

3.7 Summary and Conclusions

The chapter presented the analytical procedure for response estimation of reinforced concrete columns subjected to lateral loads. The details on modeling of individual displacement components were presented in Chapter 2. In this chapter, the procedure to combine these components to obtain total lateral load-displacement response was described. In addition, the methodology for interaction of flexural and shear deformations, procedure to model buckling of compression bars, and major steps for implementation of the model are explained. The behavior of previously tested columns in terms of individual and total load-displacement relationships was modeled and predicted responses were compared with the experimentally observed responses.

The model (DCMI) determines flexure, bar slip and shear deformations individually considering interaction between these mechanisms and combines displacement components depending upon dominant failure mode. Through the described interaction methodology, flexural mechanism is decoupled from shear model that allows for relatively simpler analytical procedure. The flexural model considers concrete tensile behavior, interaction of compression softening and buckling of compression bars. The shear model includes the effect of flexural deformation on shear behavior. The pre-peak response is evaluated through MCFT, which cannot be used when the strength starts to
degrade beyond peak strength. Post-peak shear response is modeled by a piecewise linear envelope by Sezen (2008).

All deformation components, i.e., flexural, bar slip and shear, are combined together to determine total response of the column. The total/combined peak response is limited by the smaller of the shear and flexural strength of the column and limiting mechanism governs the post peak response. The presented procedure employs relatively simple calculations for the overall response estimation. The comparison of the predicted and observed responses indicates that the DCMI is a suitable displacement-based analytical procedure that performs well in predicting the individual displacement components and total response.
Figure 3.1. Spring representation of total response model

Figure 3.2. Classification of columns into categories and rules governing combination of the deformation components (Setzler and Sezen, 2008)
Figure 3.3. Interaction of compression softening and axial strains (Mostafaei and Kabeyasawa, 2007; Mostafaei and Vecchio, 2008).

Figure 3.4. Proposed compression bar buckling model
Figure 3.5. Constitutive relationships of concrete in compression

Figure 3.6. Constitutive relationship of concrete in tension
Figure 3.7. Constitutive relationship for reinforcing steel
Figure 3.8. Experimental and analytical results: (a) flexural response of Specimen 1; (b) flexural response of Specimen 4; (c) reinforcement slip response of Specimen 1; (d) reinforcement slip response of Specimen 2; (e) shear response of Specimen 1; and (f) shear response of Specimen 2
Figure 3.9. Lateral load – displacement relationships: (a) Specimen 1; (b) Specimen 2; and (c) Specimen 4
CHAPTER 4

PROPOSED MODEL FOR RESPONSE ESTIMATION OF RC COLUMNS

4.1 Introduction

This chapter presents the development and details of an analytical procedure for estimating load-displacement relationship of reinforced concrete (RC) columns subjected to lateral loading. This model is reformulation of a previously developed model (Lodhi and Sezen, 2012) and addresses few deficiencies found in earlier formulations. The major advancement is the modeling of the shear behavior, which explicitly employs Disturbed Stress Field Model (Vecchio, 2000) for estimating non-linear pre- and post-peak shear response. The new model refines modeling of shear behavior and thus improves prediction of overall column response including load-deformation relationships, ultimate load capacity and failure modes. Implementation of Disturbed Stress Field Model into proposed model eliminates systematic inaccuracies of MCFT and allows estimation of post-peak shear behavior based on the physical mechanics rather than empirical relations.

The proposed model consists of two sub-models, axial-flexure and axial-shear model, simultaneously representing the interaction of flexural and shear behaviors of the column element between inflection point and one of the end sections. At any load stage, both sub-models are coupled through axial deformation interaction and softening of
concrete compression response. Considering compatibility and equilibrium conditions in average stress-strain field, axial-flexure and axial-shear models are reduced to one-element model under combined axial, flexure and shear loading. The lateral load carried by the column at any given displacement is obtained by analyzing reduced one-element model for in-plane stress conditions while considering interaction between axial, flexural and shear mechanisms.

In this chapter, a brief summary of the previously developed model (Lodhi and Sezen, 2012) is presented and its deficiencies are highlighted. The major portion of the rest of the chapter is dedicated to the conceptual aspects of the proposed model and its development. This chapter only explains the procedural and theoretical aspects of the proposed model. Its implementation, solution strategy and validation shall be presented in the next Chapter 5.

4.2 Background Information on Displacement Component Model

In Chapter 2 and 3, detailed description of an analytical model (Lodhi, 2010; Lodhi and Sezen, 2012) for estimating load-displacement relationship of the reinforced concrete columns subjected to lateral loading was presented. This section briefly summarizes the model, referred as Displacement Components Model with Interaction (DCMI), and discusses few relevant aspects of it that lead to the development of new model proposed in this study.

In DCMI, each component of lateral displacement due to flexure, shear and longitudinal bar slip is modeled individually while considering interaction between shear, flexure and axial mechanisms. Individual displacement components are combined
together depending upon dominant failure mode (shear or flexure) to obtain total response. The flexural deformations are determined through fiber section analysis in one-dimensional stress field while employing compressive constitutive law for cracked concrete. Lateral displacement component due to longitudinal reinforcement slip is calculated separately using the model developed by Sezen and Setzler (2008). Shear deformations are calculated using a combination of MCFT (Vecchio and Collins, 1986) and a post-peak shear response envelope (Patwardhan, 2005; Sezen, 2008). The model also considers other second order effects such as buckling of compression bars under excessive compressive strains, enhancement in concrete strength due to confinement, softening of the cracked concrete in compression, and concrete tension stiffening.

The comparison of the predicted response with experimental test data indicates that the DCMI is a suitable displacement-based evaluation process that performs well in capturing overall behavior of laterally loaded RC columns (Lodhi and Sezen, 2012). Specifically, the procedure employed for modeling flexural and reinforcement slip deformations and other second order effects were found adequate and accurate (Lodhi, 2010). However, some deficiencies can be highlighted in modeling shear behavior of the column, as explained below.

4.3 Modeling of Shear Behavior and its Deficiencies in DCMI

4.3.1 Pre-Peak Response

In DCMI, pre-peak non-linear shear force-shear deformation response is obtained from in-plane analysis of the shear element based on MCFT (Vecchio and Collins, 1986) while considering the interaction of the axial strains. MCFT is a powerful tool to model
behavior of reinforced concrete elements subjected to in-plane shear and normal stresses. Since its formulation, MCFT has been applied to the analysis of a wide range of structural elements, details, and loading conditions. For response estimation of reinforced concrete frame members, the theory has successfully been implemented into number of analytical procedures (Mostafaei and Kabeyasawa, 2007; Vecchio, 1989; Vecchio, 2000; Vecchio and Collins, 1988) and computer programs (Bentz, 2000; Guner and Vecchio, 2008; Vecchio, 1987) following finite element modeling or fiber based modeling approaches.

Since its formulation, MCFT has been found to provide reliable shear response predictions of cracked reinforced concrete elements with an accuracy that is acceptable in most engineering contexts (Vecchio, 2000). However, some deficiencies have been revealed for certain structures under specific loading conditions. For example, MCFT generally overestimates strength and stiffness of the elements with no or light transverse reinforcement and elements in which significant orientation of stress-strain fields and crack directions occur. On the other hand, for heavily reinforced elements or elements where reinforcement and loading conditions are such that there is no or little rotation of principal stress and principal strain conditions, MCFT overestimates strength and stiffness. Reduced accuracy has also been observed for shear-critical beams or columns, where fully rotating crack modeling of MCFT may result in overestimated ductility and strength (Vecchio, 2000).

It has been found by experimental evidence that principal stress directions, in a reinforced concrete element loaded with combined normal and shear stresses, lags behind the principal strain directions (Vecchio, 2000). However, in MCFT, inclinations of
average principal strain directions are assumed to remain coincident with inclinations of average principal stress directions in the concrete. The enforced coaxiality of average principal stress and strain fields embedded in MCFT and its fully rotating crack modeling approach are the primary reasons believed for the inaccuracies noted above.

Another weakness of MCFT stems from its handling of shear stresses at the crack surface and difficulty in implementation of crack shear check. The cracks in concrete are assumed to align with average principal stress directions hence average shear stresses in directions normal to crack do not exist. However, at the crack surface, local stress conditions are significantly different than the average stress conditions in concrete continuum, and can generate shear stresses at the crack surface. As a result, local rigid body shear slip may occur along the cracks, which must be considered in addition to the average strains resulting from the constitutive response of the concrete to the average stresses. The compatibility relationships in MCFT, however, do not account for this aspect and completely ignore localized deformations caused by the shear slip. Alternatively, MCFT checks the magnitude of local stresses at the crack and ensures that these do not exceed limiting values and average stresses are transmitted across the cracks. The application of crack shear check in MCFT is complex and poses a significant computational challenge in its implementation. This is one aspect of the theory that has not been well understood, and often even ignored, by others in their application of the MCFT.

The MCFT is implemented in DCMI as shear-stress-based approach in force controlled environment. Therefore, it is only possible to obtain non-linear shear response
up to peak load. This is a serious limitation of MCFT that does not allow for the shear analysis in post-peak regime.

4.3.2 Post-Peak Response

For post-peak shear force-displacement behavior, a piecewise linear shear response envelope by Sezen (2008) is adopted in DCMI. According to this model, after the peak strength has reached, shear strength is first assumed to remain constant at its peak value until the onset of the shear strength degradation and then declines linearly with increasing shear deformations to the point of axial load failure (Figure 2.5). At the point of axial load failure, lateral strength is assumed zero. Shear displacements at the onset of shear degradation and at axial load failure are calculated as per procedure explained in Chapter 2.

The calculation of shear displacement at axial load failure requires knowledge of total displacement at axial load failure. In DCMI, total displacement at axial load failure is calculated from expression (Equation 2.10) based on a shear friction model and an idealized shear failure plane by Elwood and Moehle (2005). Though, the equation was derived from physical mechanics of shear-friction model, it was calibrated based on the data from a very limited number of test specimens (only twelve columns) with almost identical material and design parameters. Therefore, application of the Elwood-Moehle axial failure model may not be appropriate for columns which are not representative of the test specimens used for development of their model. Secondly, this model assumes a constant critical crack angle (i.e., inclination of shear failure plane) of 65 degrees. It further assumes that shear failure plane is continuous and distinct. However, complex
column behavior during shear failure may result in disjointed shear plane with sliding surfaces intercepted by multiple cracks at various angles. Also, fixed inclination of shear failure plane at 65 degrees may not accurately capture the behavior of the elements where crack directions are rotating and are influenced by reinforcement and loading conditions.

After displacement components due to flexure, reinforcement slip and shear are determined in DCMI, total lateral load-displacement response is obtained by combining individual responses as per the rules based on classification of the columns into various response categories. The classification is based on comparison of the shear strength, yield strength, and flexural strength of the column (Setzler and Sezen, 2008). DCMI uses a shear strength model developed by Sezen and Moehle (2004) to determine shear strength of the column for classification purpose. The shear strength model was developed from a database of 51 shear critical columns covering range of parameters commonly found in non-ductile reinforced concrete construction. Though, this equation generally yields reliable predictions of shear strength, its application to the columns falling outside the range of parameters considered for its development, may not be appropriate.

4.4 Development of New Proposed Model

Considering the limitations and weaknesses cited above, a new model is proposed to determine lateral load-displacement relationship of reinforced concrete columns. This model improves the prediction of shear behavior by eliminating inaccuracies of MCFT and allowing estimation of post-peak shear behavior based on the physical mechanics rather than empirical relations. The new proposed model retains the useful features of DCMI such as fiber model approach for flexural analysis, considerations of
reinforcement slip deformations, compression bar buckling mechanism, confinement effects, concrete compression softening, and concrete tension stiffening. However, unlike DCMI, flexural and shear deformations in the new model are coupled and calculated simultaneously at each load step while considering interaction between axial, flexural and shear mechanisms.

In the proposed model, flexural deformations are calculated through fiber section analysis considering cracked concrete behavior while shear response is evaluated based on the Disturbed Stress Field Model (DSFM) (Vecchio, 2000). The DSFM is an extension of MCFT that addresses the weaknesses, improves the reduced accuracy of MCFT, and provides an effective approach for describing behavior of cracked reinforced concrete elements. Modeling of column shear behavior through DSFM in the proposed model is a new advancement relative to DCMI, and constitutes a key feature of the analytical procedure described here. It allows for better representation of shear behavior and improves overall prediction of column strength, load-displacement relationship, and failure modes. In the proposed formulations, the DSFM is explicitly employed for modeling both pre- and post-peak shear behavior.

The proposed model consists of two sub-models, axial-flexure and axial-shear models, simultaneously evaluating flexural and shear behaviors of the column element between inflection point and one of the end sections. At any load stage, both sub-models are coupled through axial deformation interaction and softening of concrete compression response. Considering compatibility and equilibrium conditions in average stress-strain field, axial-flexure and axial-shear models are reduced to one-element model under combined axial, flexure and shear loading. The lateral load carried by the column at any
given displacement is obtained by analyzing reduced one-element model for in-plane stress conditions. The implementation of the proposed modeling procedure follows displacement controlled approach, as will be explained in the next chapter.

In this chapter, the development and details of the new proposed model are presented. Only the aspects which differ and/or have been improved relative to the previously explained DCMI are discussed here. The new developments are explained in detail and repeating details have been referred to previous sections/chapters and appropriate literature.

4.5 Modeling of Shear Behavior

The shear behavior of the reinforced concrete column is evaluated by considering axial-shear element of the column between inflection point and end-section, and modeling its response for in-plane stress conditions based on DSFM. The implementation of the DSFM for modeling shear response is one of the main features of the proposed analytical procedure, and is explained here by describing DSFM and its relevant aspects employed in new procedure.

4.5.1 Disturbed Stress Field Model

Disturbed Stress Field Model was proposed by Vecchio (2000) to address two main weaknesses of the MCFT, namely, enforced alignment of the principal stress and strain directions, and handling of the crack shear stresses. The theory was developed by reformulating and extending MCFT in several aspects. The key advancements include consideration of discrete crack shear slip in compatibility relations, decoupling of
principal stress and principal strain directions, revision of compression softening and tension stiffening mechanisms, and improvement of constitutive relations for concrete and reinforcement. The explicit inclusion of the crack slip deformations in DSFM allows for the deviation of the principal stress field from principal strain field (i.e., inclinations of principal stresses and principal strains are not coincident) and removes the complex crack shear check which is required by MCFT. The new formulations occupy middle ground between rotating-crack models and fixed-crack models combining strengths of each, thereby, giving an improved representation of crack mechanisms and resulting in increased accuracy.

The DSFM is essentially a smeared delayed-rotating-crack model in which cracked concrete is treated as a new orthotropic material with its unique stress-strain characteristics. The theory consists of compatibility, equilibrium and constitutive relationships formulated in terms of average stresses and strains with particular attention given to compression softening and tension stiffening mechanisms. The critical aspect of DSFM is the consideration of local stress-strain conditions at cracks ensuring that the tension in the concrete can be transmitted across the crack and shear stresses along the crack surfaces can appropriately be quantified. Complete details of DSFM including its formulation, implementation and validation can be found in Vecchio (2000, 2001) and Guner (2008). In this study, only the aspects relevant to its implementation in new proposed model are presented.
4.5.2 Compatibility Relationships

Consider an orthogonally reinforced concrete element as shown in Figure 4.1(a). The element has smeared and evenly distributed reinforcement in longitudinal (x) and transverse (y) directions, with the corresponding reinforcement ratios \( \rho_x \) and \( \rho_y \), and yield strengths of \( f_{yx} \) and \( f_{yy} \), respectively. The concrete is characterized by a cylinder compressive strength \( f_c' \), a strain at peak \( \varepsilon_{eo} \) and a tensile cracking stress \( f_{cr} \). The element’s edge planes are subjected to uniform normal stresses \( \sigma_x \) and \( \sigma_y \), and shear stress \( \tau_{xy} \).

Under the applied loads, an equilibrium condition is attained resulting in unique strain state defined by normal and shear strains, as shown in Figure 4.2. The element is experiencing the deformations (total strains) composed of both continuum straining and discontinuous slip along the crack surfaces. The continuum strain results from mechanical compliance to stresses and smearing of cracks width over finite area (Figure 4.2(a)) while slip component is the result of rigid body movement along the crack interface (Figure 4.2(b)).

The DSFM decomposes the total strains (\( \varepsilon_x, \varepsilon_y, \) and \( \gamma_{xy} \)) into net concrete strains (\( \varepsilon_{cx}, \varepsilon_{cy}, \) and \( \gamma_{cxy} \)) and strains due to crack shear-slip (\( \varepsilon_x^s, \varepsilon_y^s, \) and \( \gamma_{xy}^s \)). Therefore,

\[
\varepsilon_x = \varepsilon_{cx} + \varepsilon_x^s \tag{4.1}
\]

\[
\varepsilon_y = \varepsilon_{cy} + \varepsilon_y^s \tag{4.2}
\]

\[
\gamma_{xy} = \gamma_{cxy} + \gamma_{xy}^s \tag{4.3}
\]
The strain components due to crack shear-slip \((\varepsilon_x^s, \varepsilon_y^s, \text{ and } \gamma_{xy}^s)\) are calculated from the average crack slip shear strain \(\gamma_s\), defined as the crack shear slip \(\delta_s\) divided by the average crack spacing \(s\) as follows,

\[
\gamma_s = \frac{\delta_s}{s}
\]  

(4.4)

With the help of Mohr’s circle, average crack slip shear strain \(\gamma_s\) can be resolved into orthogonal components \(\varepsilon_x^s, \varepsilon_y^s, \text{ and } \gamma_{xy}^s\) relative to the reference \(x-y\) system. Hence,

\[
\varepsilon_x^s = -\frac{\gamma_s}{2} \cdot \sin(2\theta)
\]  

(4.5)

\[
\varepsilon_y^s = \frac{\gamma_s}{2} \cdot \sin(2\theta)
\]  

(4.6)

\[
\gamma_{xy}^s = \gamma_s \cdot \cos(2\theta)
\]  

(4.7)

The discrete slip along the crack \(\delta_s\) and calculation average crack spacing \(s\) shall be discussed subsequently in separate sections.

In DSFM, the net concrete strains are employed in appropriate constitutive relationships to determine average concrete stresses. For this purpose, concrete principal strains are determined from net strains, as opposed to MCFT that uses total strains. Hence, from the Mohr’s circle in Figure 4.2(d), net principal concrete tensile strain \(\varepsilon_{c1}\) and net principal concrete compressive strain \(\varepsilon_{c2}\) can be determined as,

\[
\varepsilon_{c1} = \left(\frac{\varepsilon_{cx} + \varepsilon_{cy}}{2}\right) + \frac{1}{2} \sqrt{(\varepsilon_{cx} - \varepsilon_{cy})^2 + (\gamma_{cy})^2}
\]  

(4.8)

\[
\varepsilon_{c2} = \left(\frac{\varepsilon_{cx} + \varepsilon_{cy}}{2}\right) - \frac{1}{2} \sqrt{(\varepsilon_{cx} - \varepsilon_{cy})^2 + (\gamma_{cy})^2}
\]  

(4.9)
The orientation of the net principal concrete strains $\theta$ and orientation of net principal concrete stresses $\theta_\sigma$ are assumed equal in DSFM, and determined with respect to the $x$-axis from Mohr's circle (Figure 4.2(d)) relationships involving the net concrete strains,

$$\theta = \theta_\sigma = \frac{1}{2} \tan^{-1} \left( \frac{\gamma_{cxy}}{\varepsilon_{cx} - \varepsilon_{cy}} \right)$$  \hspace{1cm} (4.10)

Likewise, the orientation of the total principal strain field $\theta_\varepsilon$ is determined from the total strains $\varepsilon_x$, $\varepsilon_y$, and $\gamma_{xy}$ (Figure 4.2(e)) as,

$$\theta_\varepsilon = \frac{1}{2} \tan^{-1} \left( \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \right)$$  \hspace{1cm} (4.11)

The difference between the orientation of total principal strains and the orientation of the principal concrete stresses defines the rotation lag as,

$$\Delta \theta = \theta_\varepsilon - \theta$$  \hspace{1cm} (4.12)

The reinforcement is assumed perfectly bonded to the concrete, hence, average strains in reinforcements are calculated from total strains as follows,

$$\varepsilon_{sx} = \varepsilon_x$$  \hspace{1cm} (4.13)

$$\varepsilon_{sy} = \varepsilon_y$$  \hspace{1cm} (4.14)

where $\varepsilon_x$ and $\varepsilon_y$ are element total strains, and $\varepsilon_{sx}$ and $\varepsilon_{sy}$ are the reinforcement total strains in the $x$- and $y$-directions, respectively.
4.5.3 Equilibrium Relationships

Under externally applied stresses, the concrete and reinforcement components of the element may experience stresses as shown in Figure 4.3. Equilibrium of forces requires that the resultant of the externally applied normal stresses in the x- and y-directions ($\sigma_x$ and $\sigma_y$) be resisted by average concrete stresses ($f_{cx}$ and $f_{cy}$) and average reinforcement stresses ($f_{sx}$ and $f_{sy}$) in the x- and y-directions. Equilibrium of moment requires that the externally applied shear stress ($\tau_{xy}$) be entirely balanced by an average shear stress ($\nu_{cxy}$) in the concrete. It must, however, be noted that average stress conditions do not represent conditions at any one point but rather represent average smeared conditions. These equilibrium conditions can be summarized as follows,

$$\sigma_x = f_{cx} + \rho_x f_{sx} \quad (4.15)$$

$$\sigma_y = f_{cy} + \rho_y f_{sy} \quad (4.16)$$

$$\tau_{xy} = \nu_{cxy} \quad (4.17)$$

The average concrete stresses can be determined from the Mohr’s circle of stresses shown in Figure 4.1(b) as,

$$f_{cx} = f_{c1} - \nu_{cxy} \times \cot(90 - \theta) \quad (4.18)$$

$$f_{cy} = f_{c1} - \nu_{cxy} \times \tan(90 - \theta) \quad (4.19)$$

4.5.4 Constitutive relationships

Constitutive relations are required to associate average stresses with average strains for both the concrete and the reinforcement. The constitutive relations of the
DSFM are a revised and refined version of MCFT constitutive relations. Derivation of these laws, especially for the concrete, is one of the important aspects in development of the theory and simulates cracked concrete behavior in compression and tension. These relations were derived from a comprehensive series of panel element tests by Vecchio and Collins (1986, 1993).

4.5.4.1 Concrete Compression Softening

The compression response of the cracked reinforced concrete is substantially different than the one obtained from uniaxial compression tests, and is characterized by significant degree of softening arising from the effects of the transverse cracking (Vecchio and Collins, 1986; Vecchio, 2000). Like MCFT, DSFM also acknowledges the fact that the principal compressive stress in the concrete is function of both principal compressive strain and co-existing principal tensile strain. The compressive strength and stiffness of the concrete reduce as the tensile strains increase, thus resulting in the weaker and softer response (Figure 4.4(a)).

The modeling of compression softening mechanism in the DSFM is similar to that of the MCFT, but degree of softening is substantially reduced and less attributed to the tensile strain effect, as crack shear-slip is explicitly taken into account. This modeling approach is more consistent with experimentally observed response. The compression softening in concrete is captured by DSFM through the reduction factor $\beta_d$, defined as,

$$\beta_d = \frac{1}{1 + C_s C_d} \leq 1.0$$  \hspace{1cm} (4.20)
The factor \( C_s \) accounts for the influence of slippage on the crack. Its value is taken as 0.55 if slip on the crack is explicitly taken into account in element compatibility relations. When crack slip is ignored, as in MCFT, its value must be taken as 1.0. The reduction factor \( \beta_d \) is used to soften both the concrete compressive strength \( f_c' \) and corresponding strain \( \varepsilon_{co} \) to define peak compressive stress \( f_p \) and the strain at the peak stress \( \varepsilon_p \) in compression response of the concrete. Hence,

\[
f_p = \beta_d \cdot f_c'
\]

\[
\varepsilon_p = \beta_d \cdot \varepsilon_{co}
\]

In the Figure 4.4(a), \( f_{c2} \) is principal compressive stress in the concrete, and \( \varepsilon_{c2} \) is concrete principal compressive strain.

### 4.5.4.2 Pre-Cracking Concrete Tensile Behavior

The constitutive relations for tensile behavior of concrete, as illustrated in Figures 4.4(b) and (c), relates principal concrete tensile stress \( f_{c1} \) to principal concrete tensile strain \( \varepsilon_{c1} \). The pre-cracking tensile stress-strain relationship of the concrete is modeled as linear elastic response.

\[
f_{c1} = E_c \varepsilon_{c1} \quad \text{for} \quad 0 \leq \varepsilon_{c1} \leq \varepsilon_{cr}
\]

\[
\varepsilon_{cr} = f_{cr} / E_c \quad ; \quad f_{cr} = 0.33 \sqrt{f_c'} \quad ; \quad E_c = 2 f_c' / \varepsilon_{co}
\]
where $E_c$ is modulus of elasticity of the concrete, $\varepsilon_{cr}$ is cracking strain corresponding to cracking stress (tensile strength) of concrete $f_{cr}$, and $f'_c$ is uniaxial compressive strength of concrete in MPa.

### 4.5.4.3 Post-Cracking Concrete Tensile Behavior

The post-cracking (i.e., $\varepsilon_{ci} > \varepsilon_{cr}$) tensile stresses in the concrete are modeled through two independent mechanisms, namely, tension stiffening (Figure 4.4(c)) and tension softening (Figure 4.4(b)).

#### (1) Tension Stiffening Mechanism

Tension stiffening refers to the presence of average tensile stresses in cracked concrete as a result of load transfer between concrete and reinforcement via bond stresses. Tension stiffening in the DSFM is based on the formulation of Bentz (2000) and modified by Vecchio (2000).

$$f_{c1}^a = \frac{f_{cr}}{1 + \sqrt{C_i \varepsilon_{ci}}}$$  \hspace{1cm} (4.26)

$$C_i = 3.6 \times t_d \times m \text{ where } t_d = 0.6$$  \hspace{1cm} (4.27)

$$\frac{1}{m} = \frac{4 \rho_x}{d_{bx}} |\cos \theta| + \frac{4 \rho_y}{d_{by}} |\sin \theta|$$  \hspace{1cm} (4.28)

where $d_{bx}$ and $d_{by}$ are the diameter of bars in x- and y- direction, respectively. It should be noted that $f_{c1}^a$ is limited to the amount which can be transmitted across cracks as defined by Equation 4.35.
(2) **Tension Softening Mechanism**

Tension softening refers to fracture-associated mechanisms. This phenomenon may be significant in lightly reinforced concrete members. The cracking tensile stress due to tension softening may be calculated with the help of following linear relationship,

\[
f_{c1}^b = f_{cr} \left(1 - \frac{\varepsilon_{c1} - \varepsilon_{cr}}{\varepsilon_{ts} - \varepsilon_{cr}}\right) \geq 0
\]

(4.29)

The terminal strain \( \varepsilon_{ts} \) (the strain at which tensile stresses in plain concrete reduce to zero) is determined from the fracture energy parameter \( G_f \) (the area under the stress-strain curve of plain concrete, assumed to be \( 75 \times 10^3 \) N/mm), and the characteristic length \( L_r \) (assumed to be half the average crack spacing \( s \) from Equation 4.50).

\[
\varepsilon_{ts} = 2.0 \frac{G_f}{L_r \times f_{cr}}
\]

(4.30)

The post-cracking average principal tensile stress in the concrete is finally taken as larger of the values predicted by tension stiffening and tension softening mechanisms.

\[
f_{c1} = \max(f_{c1}^a, f_{c1}^b)
\]

(4.31)

**4.5.4.4 Reinforcement Stress-Strain Relationship**

The reinforcing steel behavior in tension or compression modeled with tri-linear average stress-strain relationship shown in Figure 4.4(d).

\[
f_{si} = E_{si}\varepsilon_{si} \quad \text{for} \quad 0 \leq \varepsilon_{si} \leq \varepsilon_{yi}
\]

(4.32)

\[
f_{si} = f_{yi} \quad \text{for} \quad \varepsilon_{yi} \leq \varepsilon_{si} \leq \varepsilon_{shi}
\]

(4.33)

\[
f_{si} = f_{yi} + E_{shi} \times (\varepsilon_{si} - \varepsilon_{shi}) \quad \text{for} \quad \varepsilon_{shi} \leq \varepsilon_{si} \leq \varepsilon_{ui}
\]

(4.34)
where the subscript $i$ refers to reinforcement in either the $x$- or $y$-direction. In any component of reinforcement, $f_{si}$ is the stress, $E_{si}$ is the elastic modulus, $\varepsilon_{si}$ is the strain, $f_{yi}$ is yield stress, $\varepsilon_{yi}$ is yield strain, $\varepsilon_{shi}$ strain at the onset of strain hardening, $E_{shi}$ is slope of strain hardening response, $f_{ui}$ is ultimate stress, and $\varepsilon_{ui}$ is the ultimate strain.

### 4.5.5 Local Crack Conditions

The compatibility, equilibrium and constitutive relationships in DSFM are derived considering average stress-strain conditions. However, at the crack interfaces, local stress conditions are significantly different than average conditions. For example, tensile stresses in the reinforcement are higher than the average at the cracks and lower than the average midway between the cracks. On the other hand, the concrete tensile stresses are zero at the crack and higher than the average midway between the cracks.

The local stress variations are important as the response of bi-axially loaded element may be governed by the ability of the reinforcement to transmit tension across the cracks or sliding shear failure along the cracks. Hence, local stress conditions must be checked to ensure that average stresses are compatible with the condition of the cracked concrete and can be transmitted across the cracks.

Figure 4.3(c) shows average stresses at a section between the cracks perpendicular to the principal tensile stress direction and Figure 4.3(d) shows local stresses on a free surface of the crack. At the crack, average tensile stresses in the concrete reduce to zero. This causes reinforcement stresses to increase locally at the crack in order to transmit tensile stresses across the crack. Hence, average concrete tensile stresses must be limited...
to avoid failure of the reinforcement at the crack. Static equivalency of the average and local stresses in the direction normal to the crack surface results in the condition that limits the concrete average tensile stress $f_{cl}$ to the following upper limit to ensure its transmission across the crack.

$$f_{cl} \leq \rho_x \left(f_{xx} - f_{xx}\right)\cos^2 \theta_{nx} + \rho_y \left(f_{yy} - f_{xy}\right)\cos^2 \theta_{ny}$$  (4.35)

The local reinforcement stresses can be calculated by static equilibrium in the direction normal to the crack surface (Figure 4.3(d)). The resulting equation becomes

$$f_{cl} = \rho_x \left(f_{scrx} - f_{xx}\right)\cos^2 \theta_{nx} + \rho_y \left(f_{scry} - f_{xy}\right)\cos^2 \theta_{ny}$$  (4.36)

Unlike MCFT, there is no need to limit the interface shear stress because the crack slip deformations are explicitly incorporated into the DSFM formulations. Elimination of this complex calculation gives the DSFM an advantage when implementing it into finite element programs. Defined by following equation, the interface shear stress $\nu_{ci}$ is still calculated and used in the slip strain calculations.

$$\nu_{ci} = \rho_x \left(f_{scrx} - f_{xx}\right)\cos \theta_{nx} \sin \theta_{nx} + \rho_y \left(f_{scry} - f_{xy}\right)\cos \theta_{ny} \sin \theta_{ny}$$  (4.37)

In above equations, $f_{scrx}$ and $f_{scry}$ are local reinforcement stresses at crack, and $\theta_{nx}$ and $\theta_{ny}$ are the angles between the normal to the crack and reinforcement in x- and y-directions, respectively.

$$\theta_{nx} = \theta$$  (4.38)

$$\theta_{ny} = \theta - 90$$  (4.39)
4.5.6 Modeling of Crack Shear-Slip

As mentioned earlier, DSFM accounts for the deformations resulting from crack shear-slip in the formulations of its compatibility relations. In order to quantify crack slip, DSFM employs two approaches and then combines both into one hybrid model. One is a stress-based approach that relates the amount of shear slip to the magnitude of the shear stress acting on the crack. The other approach specifies a constant rotation lag between inclination of the stress field in the concrete $\theta_{\sigma}$ and inclination of total principal strain field $\theta_{\varepsilon}$. Both approaches are used to calculate crack slip shear strain $\gamma_s$, and then larger of the two values is utilized to determine strain components $\varepsilon^s_x, \varepsilon^s_y$ and $\gamma^s_{xy}$. Complete details of shear slip model can be found in Vecchio, (2000).

However, in the model proposed herewith, crack shear-slip is estimated using Lai-Veccio constitutive model (Vecchio and Lai, 2004), described below. This model combines the useful features of two existing stress-based models (Okamura and Maekawa, 1991; Walraven and Reinhardt, 1981) and is shown to provide accurate simulation of crack slip phenomenon with easy implementation and improved numerical stability (Vecchio and Lai, 2004).

Consider the element experiencing discrete rigid body slip along the crack surfaces in Figure 4.2(b). Assume that cracks are inclined in the direction of net principal tensile strain $\theta$, and cracks have average width and spacing of $w$ and $s$, respectively. The slip along the crack surface $\delta_s$ is found as,

$$\delta_s = \delta_s \sqrt{\frac{\psi}{1 - \psi}}$$

(4.40)
where

\[ \delta_2 = \frac{0.5 \nu_{c_{\text{max}}} + \nu_{co}}{1.8w^{-0.8} + (0.234w^{-0.707} - 0.20)f_{cc}} \] (Mpa, mm) \hspace{1cm} (4.41)\\

\[ \Psi = \frac{\nu_{ci}}{\nu_{c_{\text{max}}}} \] (Mpa) \hspace{1cm} (4.42)\\

\[ \nu_{co} = \frac{f_{cc}}{30} \] (Mpa) \hspace{1cm} (4.43)\\

\[ \nu_{c_{\text{max}}} = \frac{\sqrt{f_{c}}}{0.31 + \frac{24w}{a + 16}} \] (Mpa, mm) \hspace{1cm} (4.44)\\

\[ w = \varepsilon_{ci} \cdot s \] \hspace{1cm} (4.45)\\

In above equations, \( \nu_{c_{\text{max}}} \) is maximum shear stress that can be resisted on the crack (Vecchio and Collins, 1986), \( \nu_{co} \) is an initial offset in crack-shear slip, \( \nu_{ci} \) is shear stress on the crack surface calculated from Equation 4.37, \( w \) is average width of the crack, \( a \) is maximum aggregate size, \( f_{cc} \) is concrete cube strength in Mpa = 1.2 \( f_{c} \), \( \varepsilon_{ci} \) is net principal tensile strain, and \( s \) is average crack spacing.

Knowing the magnitude of crack shear-slip \( \delta_s \), average crack slip shear strain can be calculated as,

\[ \gamma_s = \frac{\delta_s}{s} \] \hspace{1cm} (4.46)\\

where \( s \) is the average crack spacing in the direction normal to the crack and can be estimated as,

\[ s = \frac{1}{\sin \theta \cdot \cos \theta} \left( \frac{1}{s_{mx}} + \frac{1}{s_{my}} \right) \] \hspace{1cm} (4.47)
It must be noted that when shear stress on the crack surface $\tau_{ci}$ approaches the the maximum theoretical shear stress $\tau_{c,max}$ that can be resisted on the crack through aggregate interlock, shear slip failure occurs (i.e. $\delta_s$ in Equation 4.40 becomes infinitely large).

### 4.6 Calculation of Average Crack Spacing

An estimate of average crack spacing $s$ is needed for crack width and crack slip calculations, as shown above. In the proposed model, variable crack spacing formulation is adopted from Collins and Mitchell (1991) based on the CEB-FIB Code (1978). Unlike constant crack spacing, the variable crack spacing model considers that the crack spacing becomes larger as the distance from the reinforcement increases. In Equation 4.47, $s_{mx}$ and $s_{my}$ are average crack spacing that indicate crack control characteristics of $x$- and $y$-reinforcement, respectively. These quantities depend on bond properties and layout of the reinforcement and can be estimated from CEB-FIP Code as,

$$s_{mx} = 2(c_x + \frac{S_s}{10}) + 0.25k_1 \frac{d_{bx}}{\rho_x}$$

$$s_{my} = 2(c_y + \frac{S_s}{10}) + 0.25k_1 \frac{d_{by}}{\rho_y}$$

(4.48)

where $c_x$, $c_y$, and $S_s$ are the parameters determined from the reinforcement layout as shown in Figure 4.5, $k_1$ is taken as 0.4 for the deformed bars and 0.8 for plain bars, $d_{bx}$ and $d_{by}$ are longitudinal and transverse the bar diameters, respectively, and $S$ is center-to-center spacing of the transverse reinforcement. The crack control characteristics of the
longitudinal and transverse reinforcement can, alternatively, be also taken as \( s_{mx} = 1.5 \times \) maximum distance from x-bars and \( s_{my} = 1.5 \times \) maximum distance from y-bars. After calculating \( s_{mx} \) and \( s_{my} \) values, Equation 4.47 is used to determine average crack spacing in the direction normal to cracks.

4.7 Material Stiffness Formulations

Vecchio (1989) presented a procedure that showed that nonlinear analysis of reinforced concrete membrane elements can be performed accurately by simply modifying existing linear elastic finite element routines based on secant stiffness formulation, provided realistic constitutive relations for concrete and reinforcement are employed. In constructing an individual element stiffness matrix \([K]\), a material stiffness matrix \([D]\) is required to relate stresses to the strains. The material stiffness matrix for a linear elastic isotropic material in state of plane stress can be modified in a form that depends on type of stiffness moduli used to reflect nonlinear behavior of reinforced concrete according to appropriate set of constitutive laws. Using the same concept, the material stiffness matrix \([D]\) for axial-shear element is determined by first defining stiffness matrices for concrete and reinforcements with respect to their respective principal material directions using secant moduli. The total stiffness matrix is then obtained by combining the component stiffness matrices, after appropriate transformations to take into account the directional dependence of the materials.

\[
[D] = [T]^T [D]_c [T] + \sum_i [T]^T [D]_{si} [T]
\] (4.49)
where $[D_c]$ is the concrete material stiffness matrix evaluated with respect to principal 1 and 2 axes system, corresponding to the direction of the principal tensile strain and principal compressive strain, respectively.

$$[D_c] = \begin{pmatrix}
E_{c2} & 0 & 0 \\
0 & E_{c1} & 0 \\
0 & 0 & G_c
\end{pmatrix} \quad (4.50)$$

where $E_{c1}$ and $E_{c2}$ are secant moduli for the concrete and relate to the stress-strain behavior in principal directions. At a particular stress-strain state these can be determined as,

$$E_{c1} = \frac{f_{c1}}{\varepsilon_{c1}}, \quad E_{c2} = \frac{f_{c2}}{\varepsilon_{c2}}, \quad G_c \approx \frac{E_{c1}E_{c2}}{E_{c1} + E_{c2}} \quad (4.51)$$

where $\varepsilon_{c1}$ and $\varepsilon_{c2}$ are average net principal strains in the concrete, and $f_{c1}$ and $f_{c2}$ are corresponding principal stresses.

For each reinforcement component, a reinforcement material stiffness matrix $[D_{si}]$ is evaluated as follows,

$$[D_{si}] = \begin{pmatrix}
\rho_iE_{si} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix} \quad i = x, y \quad (4.52)$$

where $E_{si}$ is the secant moduli for the reinforcing steel and relate to the stress-strain behavior in two orthogonal directions $x$ and $y$. For a particular stress/strain state,

$$E_{sx} = \frac{f_{sx}}{\varepsilon_x}, \quad E_{sy} = \frac{f_{sy}}{\varepsilon_y} \quad (4.53)$$
The transformation matrix \([T]\) in Equation 4.49 will differ for each component and is given by the following expression.

\[
[T] = \begin{pmatrix}
\cos^2 \psi & \sin^2 \psi & \cos \psi \sin \psi \\
\sin^2 \psi & \cos^2 \psi & -\cos \psi \sin \psi \\
-2 \cos \psi \sin \psi & 2 \cos \psi \sin \psi & \cos^2 \psi - \sin^2 \psi
\end{pmatrix}
\]  

(4.54)

where \(\psi = \theta\) for the concrete component, \(\psi = 0\) and \(90^\circ\) for longitudinal and transverse reinforcement, respectively.

Having determined the total/composite material stiffness matrix \([D]\), it can then be used to relate stresses \(\{\sigma\}\) to strains \(\{\varepsilon\}\),

\[
\{\sigma\} = [D]\{\varepsilon\} - \{\sigma^o\} \quad \text{or} \quad \{\varepsilon\} = [D]^{-1}\left(\{\sigma\} + \{\sigma^o\}\right)
\]  

(4.55)

\[
\{\varepsilon\} = \{\varepsilon_c\} + \{\varepsilon^s\}
\]  

(4.56)

\[
\{\sigma^o\} = [D_e]\{\varepsilon^s\}
\]  

(4.57)

where

\[
\{\sigma\} = \begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix}; \quad \{\varepsilon\} = \begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{pmatrix}; \quad \{\varepsilon_c\} = \begin{pmatrix}
\varepsilon_{cx} \\
\varepsilon_{cy} \\
\gamma_{cxy}
\end{pmatrix}; \quad \{\varepsilon^s\} = \begin{pmatrix}
\varepsilon^s_x \\
\varepsilon^s_y \\
\gamma^s_{xy}
\end{pmatrix}
\]  

(4.58)

It must be noted that in defining the secant stiffness values, net concrete strains (\(\varepsilon_c, \varepsilon_{cy}, \text{ and } \gamma_{cxy}\)) are used. Net concrete strains are total strains (\(\varepsilon, \varepsilon_y, \text{ and } \gamma_{xy}\)) less strains due to crack slip (\(\varepsilon^s, \varepsilon_{ys}^s, \text{ and } \gamma_{ys}^s\)).
4.8 Modeling of Flexural Behavior

In the proposed procedure, flexural behavior of the fixed-ended reinforced concrete column is modeled by considering an axial-flexure element between inflection point and one of the end sections, and evaluating its response under given material properties, section geometry and loading conditions through standard fiber section analysis techniques. The section analysis is performed in the usual way except that constitutive relationships of diagonally cracked concrete are employed to consider the effect of shear deformations on flexural performance of the element.

The modeling of flexural behavior and procedure to calculate flexural deformations in the proposed model are the exactly same as employed in DCMI except two aspects concerning with the handling of concrete compression softening response. In DCMI, compression softening effect is estimated prior to performing detailed shear analysis through an approximate approach based on USFM (Mostafaei and Vecchio, 2008). The estimated effect is then utilized to perform decoupled flexural analysis. In the proposed model, however, the effect of concrete compression softening is taken directly from parallel running axial-shear model and supplied to axial-flexure model for flexural analysis of the element. Another variation relative to DCMI is the level of softening employed in compression response of the cracked concrete, which is to a lesser degree in propose model being based on DSFM.

The complete description of flexural modeling including the details of fiber section analysis, the need and procedure to consider compression softening response of cracked concrete, and procedure to calculate flexural deformations are presented in
Chapter 2. However, a brief summary of the procedure is presented here for convenience in understanding and easy reference.

4.8.1 Flexural Section Analysis

Fiber section analysis is performed in the usual manner, as explained in Chapter 2, except that compressive stress-strain relationships of cracked concrete are employed to capture the influence of increasing shear deformations on flexural behavior during the analysis. This is done by softening standard uniaxial constitutive response of the concrete by compression softening factor $\beta_d$ obtained from in-plane shear analysis of the flexural element (Equation 4.20). In addition to concrete compression softening, enhancement in the strength and ductility of the concrete due to confinement, contribution of the concrete tensile properties to section moment capacity, and buckling of the longitudinal bars under excessive compressive strains are also considered in flexural analysis. For determining realistic moment capacity, confined core concrete and unconfined cover concrete are modeled separately with their respective stress-strain relationships.

4.8.2 Flexural Deformations

The fiber section analysis results in moment-curvature relationship for given geometric and material properties, reinforcement details and applied axial load for the cross-section being analyzed. Flexure deformations are then calculated with the help of plastic hinge model, shown in Figure 4.6, in which elastic and inelastic curvatures are idealized separately. In this model, a linear curvature distribution is assumed in the elastic range over the length of the column, and the inelastic curvatures are lumped at the
column end over the plastic hinge length. Hence, lateral displacement due to flexure $\Delta_f$ can be calculated by integrating curvature over the length of the column as per following equation,

$$\Delta_f = \int_0^a \phi(x) \, dx$$

(4.59)

where $\phi(x)$ is section curvature at distance $x$ measured along column axis, $a$ is the distance from column end section to inflection point, and $\phi_y$ is curvature at yield point. The plastic hinge length $L_p$ is taken as one-half of the section depth $h$.

The flexural drift ratio $\gamma_f$ is then calculated and used subsequently in the analysis for determining stiffness of the axial-flexure element.

$$\gamma_f = \frac{\Delta_f}{a}$$

(4.60)

4.8.3 Axial Strain Due to Flexure

Axial strain due to flexure is an interaction term in the proposed model that is used from axial-flexure analysis into axial shear analysis. Complete details on axial strain interaction methodology shall be presented subsequently, however, as axial strain due to flexure is determined from flexural section analysis, it is appropriate to describe the way it can be calculated.

Average axial strain due to flexure $\varepsilon_{xf}$ between two flexural sections can be determined, as illustrated in Figure 4.7, based on relative centroidal deformation between two sections assuming linear strain distribution (Mostafaei and Kabeyasawa, 2007).
\[
\varepsilon_{sf} = \frac{1}{l_{12}} \int_{0}^{l_{12}} (\varepsilon_{x2} - \varepsilon_{x1}) \, dx = \frac{\varepsilon_{x2} - \varepsilon_{x1}}{2}
\]  
(4.61)

where \( \varepsilon_{x1} \) and \( \varepsilon_{x2} \) are centroidal strains of two consecutive flexural sections and \( l_{12} \) is distance between the sections.

For a fixed ended column shown in Figure 4.8, if one section is at inflection point and other is at one of the end-sections, and assuming that \( \varepsilon_{o} \) and \( \varepsilon_{xa} \) are corresponding centroidal strains, Equation 4.61 will gives the average axial strain due to flexure for axial-flexure element as,

\[
\varepsilon_{sf} = \frac{(\varepsilon_{o} - \varepsilon_{xa})}{2}
\]  
(4.62)

### 4.9 Compatibility and Equilibrium Conditions

The compatibility and equilibrium conditions employed in the proposed model are based on the formulations by Mostafaei and Kabeyasawa (2007). Axial-flexure and axial-shear models must satisfy following compatibility and equilibrium conditions while considering interaction in terms of axial deformations and concrete compression strength softening. These conditions reduce two sub-models into one-element model subjected to in-plane axial and shear stresses due to applied axial, flexural and shear loadings.

#### 4.9.1 Compatibility Conditions

Figure 4.7 illustrates an element between two flexural sections of a reinforced concrete column subjected to axial load, bending moment and shear force. Total axial deformation in a reinforced concrete column can be considered as sum of axial
deformations due to axial, flexural and shear mechanisms. In an axial-flexural mechanism, total axial strain is combination of axial strains caused by axial mechanism $\varepsilon_{saf}$ and axial strain due to flexural mechanism $\varepsilon_{sf}$. Likewise, in an axial-shear mechanism, axial strain is combination of axial strain due to axial mechanism $\varepsilon_{sas}$ and axial strain caused by shear mechanism $\varepsilon_{ss}$. Hence, total axial strain $\varepsilon_x$ of the column between the two sections can be obtained by extracting $\varepsilon_{sf}$ from axial-flexural model and adding to axial deformations of axial-shear model.

$$\varepsilon_x = (\varepsilon_{sas} + \varepsilon_{ss}) + \varepsilon_{sf} \tag{4.63}$$

Compatibility of axial deformations is satisfied when axial deformations due to axial mechanisms in axial-shear and axial-flexural elements are equal to the axial strain due to only applied axial load $\varepsilon_{sa}$, that is,

$$\varepsilon_{sa} = \varepsilon_{saf} = \varepsilon_{sas} \tag{4.64}$$

Hence, Equation 4.63 becomes,

$$\varepsilon_x = \varepsilon_{sa} + \varepsilon_{ss} + \varepsilon_{sf} \tag{4.65}$$

Furthermore, the total lateral displacement $\Delta$ between two flexural sections can be taken as summation of shear displacement $\Delta_s$ and the flexural displacement $\Delta_f$.

$$\Delta = \Delta_s + \Delta_f \tag{4.66}$$
4.9.2 Equilibrium Conditions

Equilibrium conditions are satisfied in average stress-strain field. For equilibrium, axial stress in axial-flexural element $\sigma_{sf}$ should be equal to axial stress in axial-shear element $\sigma_{sx}$. Also, the equilibrium of shear stress in axial-flexural element $\tau_{f}$ and shear stress in axial-shear element $\tau_{s}$ must be satisfied simultaneously throughout the analysis,

$$
\begin{align*}
\sigma_{sf} &= \sigma_{sx} = \sigma_o \\
\tau_{f} &= \tau_{s} = \tau
\end{align*}
$$

(4.67)

where $\sigma_o$ is axial stress due to applied axial load and $\tau$ is resultant shear stress. Shear stress in axial-flexural and axial-shear mechanisms are calculated as,

$$
\begin{align*}
\tau_{f} &= \frac{1}{bd_f} \left( \frac{M_1 - M_2}{l_{12}} \right) \\
\tau_{s} &= \frac{V}{bd_s}
\end{align*}
$$

(4.68)

where $M_1$ is larger moment on one of the flexural section, $M_2$ is smaller moment on other flexural section, $b$ is width of the cross-section, $d_f$ is effective flexural depth of the section which can be taken as $d_f = h$ before concrete cracks in flexure and $d_f = d$ afterwards. $h$ is overall depth of the section, $d$ is effective depth of the section, $V$ is applied lateral load, $d_s$ is shear depth of the section and can be taken as $d_s = h$ until concrete tensile crack due to flexure and then $d_s = d$.

Normal stress $\sigma_o$ due to applied axial load $P$ in both axial-flexural and axial-shear models is calculated as,
\[ \sigma_o = \frac{P}{bh} \]  \hspace{1cm} (4.69)

Normal stress in the direction perpendicular to the longitudinal axis of the column, or the clamping stresses \( \sigma_y \) and \( \sigma_z \) are neglected due to inexistence of lateral external forces in these directions.

\[ \sigma_y = \sigma_z = 0 \]  \hspace{1cm} (4.70)

4.10 Interaction between Axial-Flexure and Axial-Shear Model

Interaction between axial-flexure and axial-shear models is based on the same concept employed in DCMI (ASFI approach), but implemented in the proposed model using a slightly different procedure. Axial deformation and concrete compression softening are two main interaction terms that couple axial-flexural and axial-shear mechanisms.

4.10.1 Axial Strain Interaction Methodology

The effect of flexural deformations on shear behavior is considered by incorporating axial strain due to flexure into in-plane analysis of the axial-shear element. Interaction of axial strain is taken into account by adding flexibility component of axial deformation due to flexure to the corresponding flexibility component of axial-shear model. The procedure for axial deformation interaction and determination of shear strain is described here for a fixed ended column subjected to lateral load.

The length of the column between inflection point and one of the end sections is considered as a shear element subjected to constant normal stress due to applied axial
load and average shear stresses due to applied lateral load. Performing flexural analysis on fiber model of the end section and inflection point, average centroidal axial strain due to flexure $\varepsilon_{sf}$ (Figure 4.8) and corresponding flexibility component $f_{sf}$ can be determined with the help of following ASFI equations (Mostafaei and Kabeyasawa, 2007),

$$
\varepsilon_{sf} = \frac{(\varepsilon_o - \varepsilon_{sa})}{2}, \quad f_{sf} = \frac{\varepsilon_{sf}}{\sigma_s}, \quad \sigma_s = \frac{P}{bd}
$$

where, $\sigma_s$ is applied axial stress in longitudinal direction of the column, $\varepsilon_o$ is centroidal strain in axial-flexure model and $\varepsilon_{sa}$ is axial strain at inflection point.

A stress-strain relationship in terms of flexibility matrix for an in-plane shear element (axial-shear model) can be defined as,

$$
\begin{pmatrix}
  f_{11} & f_{12} & f_{13} \\
  f_{21} & f_{22} & f_{23} \\
  f_{31} & f_{32} & f_{33}
\end{pmatrix}
\begin{pmatrix}
  \sigma_x \\
  \sigma_y \\
  \tau_y
\end{pmatrix}
= 
\begin{pmatrix}
  \varepsilon_x = (\varepsilon_{sx} + \varepsilon_{sx}) \\
  \varepsilon_y \\
  \gamma_{xy}
\end{pmatrix}
$$

where, $f_{ij}$ ($i, j = 1, 2, 3$) are flexibility components of in plane shear model, $\varepsilon_x = \varepsilon_{sx} + \varepsilon_{sx}$ is axial strain in axial-shear element and $\varepsilon_y$ is strain in transverse reinforcement. Axial strain due to flexure $\varepsilon_{sf}$ can be taken into account in the axial-shear model by adding flexibility component obtained from Equation 4.71 and Equation 4.72.

Considering $x$ axis as the main longitudinal axis of the column, stress in $x$-direction is assumed equal to the applied axial stress, i.e., $\sigma_x = \sigma_o$. Stresses in $y$-direction (clamping stresses) are zero due to inexistence of lateral external force along the column, i.e., $\sigma_y = 0$. Hence, flexibility matrix for axial-shear-flexure element takes the form,
where \( (\varepsilon_{xa} + \varepsilon_{xs}) \) is axial strain in axial-shear model considering the compatibility, \( \varepsilon_{xs} = \varepsilon_{xa} \). The flexibility matrix is obtained by inverting material stiffness matrix of the shear element formulated using secant stiffness methodology as explained in Section 4.7.

### 4.10.2 Concrete Compression Softening

Compression softening in cracked concrete is the reduction of compressive strength and stiffness, relative to the uniaxial compressive strength, due to coexisting transverse cracking and tensile straining. Concrete compression softening is another interaction term in the proposed model. However, unlike interaction of axial deformation from axial-flexural model and axial-shear model, compression softening of axial-shear model is taken into account in axial-flexural model. Hence, compression softening factor \( \beta_d \) determined from axial-shear mechanism (Equation 4.20) must be employed in flexural analysis to soften the uniaxial compressive behavior of the concrete in axial-flexure model.

### 4.11 Reinforcement Slip Deformations

Lateral displacement component due to longitudinal reinforcement slip \( \Delta_{pull} \) must be calculated separately and added to flexural and shear displacements to obtain total response.
\[ \Delta = \Delta_f + \Delta_s + \Delta_{pull} \] (4.74)

Any of the available pullout models such as Otani and Sozen (1972), Hawkins et al. (1982), Morita and Kaku (1984), Alsiwat and Saatcioglu (1992), Lehman and Moehle (2000) and Sezen and Setzler (2008) can be used to determine deformations due to reinforcement slip and include these effects in overall response. However, in the proposed model, lateral displacement due to longitudinal reinforcement slip is calculated using the model developed by Sezen and Setzler (2008). This model approximates the bond stress at concrete-steel interface in anchoring concrete as bi-uniform function with different values for elastic and inelastic steel behaviors, and calculates the slip at the loaded end of the reinforcing bar by integrating bi-linear strain distribution over the development length. Complete details of modeling flexural deformations are provided in Chapter 2.

### 4.12 Buckling of Compression Bars

The proposed model considers the effect of bar buckling under high axial strains in the flexural analysis of the element. The compressive stress-strain relationships of the longitudinal bars is modified according to the bar buckling model explained in Chapter 3. Complete details of bar buckling model can be found in Section 3.4.

### 4.13 Consideration of Confinement Effects

Confined concrete exhibits enhanced strength and ductility in compression. In proposed model, enhancement in concrete strength is taken into account by strength enhancement factor \( K \), which can be calculated by the any of appropriate confinement
strength models. The value of $K$ is used to modify the concrete compression response curves by increasing both the uniaxial compressive strength, $f'_c$ and corresponding strain $\varepsilon_{co}$ to determine the peak compressive strength $f_p$ and corresponding strain $\varepsilon_p$ as follows,

$$f_p = K \beta_d f'_c$$

$$\varepsilon_p = K \beta_d \varepsilon_{co}$$

The term $\beta_d$ is compression softening factor calculated from Equation 4.20 and is used to soften concrete response in compression due to presence of principal tensile strain diagonal cracking.

In order to satisfy compatibility and equilibrium conditions explained in Section 4.9, same material constitutive relationships must be employed in both axial-flexural and axial-shear models. In an axial-flexure model, if constitutive laws of confined core concrete and unconfined cover concrete are defined differently, constitutive law for the concrete in axial-shear model must also be modified. Equivalent peak concrete compression stress $f_{p-as}$ and corresponding strain $\varepsilon_{p-as}$ can approximately be calculated as,

$$f_{p-as} = \beta_d \left( \frac{KA_{con} + A_{unconf}}{A} \right) f'_c$$

$$\varepsilon_{p-as} = \beta_d \left( \frac{KA_{con} + A_{unconf}}{A} \right) \varepsilon_{co}$$
where $A$ is gross cross-sectional area, $A_{\text{con}}$ is area of confined core, $A_{\text{unconf}}$ is area of unconfined cover. The confinement strength enhancement factor $K$ in proposed model is calculated as,

$$K = \frac{f_{cc}}{f_c}$$  \hspace{1cm} (4.78)

where $f_{cc}$ is peak confined concrete strength calculated from Mander et al. (1988) model.

### 4.14 Conclusions

In this chapter, conceptual and procedural development of an analytical model is presented for response estimation of reinforced concrete columns subjected to lateral loads. The proposed model is a macro-model consisting of axial-flexure and axial-shear models evaluating flexural and shear responses simultaneously while considering interaction between axial strains and concrete compression softening. The compatibility and equilibrium relationships are satisfied in average stress-strain conditions. Flexural behavior is modeled following fiber section model in one dimensional stress field while shear behavior is modeled using DSFM. Modeling of shear behavior through DSFM allows for better representation of shear behavior based on the physical mechanics rather than empirical relations, and improves overall prediction of column strength, load-displacement relationship, and failure. The displacement due to reinforcement slip is added to flexural and shear displacements to obtain total lateral displacement at any load step. In addition, certain effects such as compression bar buckling, confinement effects, concrete compression softening, and concrete tensile behavior due to tension stiffening and tension softening, are considered.
(a) Loading conditions of the element  
(b) Mohr’s circle for average concrete stresses

Figure 4.1. Reinforced concrete element subjected to uniform loading (Vecchio, 2000)

Figure 4.2. Compatibility conditions in DSFM: (a) deformations due to average constitutive response (net concrete strains); (b) deformations due to rigid body slip along crack; (c) combined deformations; (d) Mohr’s circle of net concrete strains; and (e) Mohr’s circle of total strains (Vecchio, 2000)
Figure 4.3. Equilibrium conditions in DSFM (Vecchio, 2000)

Figure 4.4. Constitutive relations in DSFM: (a) compression softening model; (b) tension softening model; (c) tension stiffening model; and (d) reinforcing steel response (Vecchio, 2000)
Figure 4.5. Parameters influencing crack spacing (Collins and Mitchell, 1991)

Figure 4.6. Plastic hinge model for calculating flexural displacements
Figure 4.7. Axial-Flexural model and determination of axial strain due to flexure
Figure 4.8. Interaction of compression softening and axial strains
CHAPTER 5

IMPLEMENTATION AND VALIDATION OF PROPOSED MODEL

5.1 Introduction

The theoretical and procedural development of the proposed analytical model for response estimation of laterally loaded reinforced concrete columns were presented in the previous chapter. The proposed model is a displacement-based response evaluation approach based on compatibility of strains, equilibrium of average stresses, and interaction between axial, flexure and shear mechanisms. Flexural behavior is simulated through fiber section model in one-dimensional stress field whereas shear behavior is modeled based on Disturbed Stress Field Model (DSFM) considering one integration point in in-plane stress conditions. The model considers number of effects such as reinforcement slip deformations, buckling of compression bars under excessive compressive strains, enhancement in concrete strength due to confinement, softening of the cracked concrete in compression, modeling of concrete tensile behavior through tension stiffening and tension softening mechanisms, variable crack spacing, flexibility of material constitutive models, and detailed modeling and interaction of failure modes.

This chapter presents implementation and verification of the proposed model by describing major analytical steps, calculation procedures, and solution strategy. The
5.2 Implementation of the Proposed Model

In this section, analytical procedure is described for implementation of the proposed model to evaluate monotonic response of laterally loaded reinforced concrete columns including load-displacement relationships, load capacities, and failure modes.

5.2.1 Calculation of Crack Shear-Slip Strain Components

As explained in Section 4.5, a reinforced concrete element under external loads may experience deformations that are composed of both continuum straining and discontinuous slip along the crack surfaces. Hence, total strains in the element \((\varepsilon_x, \varepsilon_y, \text{ and } \gamma_{xy})\) are sum of the net concrete strains \((\varepsilon_{cx}, \varepsilon_{cy}, \text{ and } \gamma_{cxy})\) resulting from continuum straining and shear-slip strains \((\varepsilon_x^s, \varepsilon_y^s, \text{ and } \gamma_{xy}^s)\) arising from discrete rigid body movement along the crack interfaces. i.e.,

\[
\begin{align*}
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{pmatrix}
&= 
\begin{pmatrix}
\varepsilon_{cx} \\
\varepsilon_{cy} \\
\gamma_{cxy}
\end{pmatrix}
+ 
\begin{pmatrix}
\varepsilon_x^s \\
\varepsilon_y^s \\
\gamma_{xy}^s
\end{pmatrix} \\
&= \mathbf{E_c} \mathbf{u_c} + \mathbf{E_s} \mathbf{u_s}
\end{align*}
\] (5.1)

The continuum straining is the result of mechanical compliance to applied stresses and smearing of crack widths over finite area. Therefore, net strains in the continuum must be employed in appropriate constitutive relations to determine average stresses from...
average strains for the concrete. The implementation of the proposed analytical procedure is iterative and starts by assuming initial values for element total strains \((\varepsilon_x, \varepsilon_y, \gamma_{xy})\). In order to calculate principal strains and proceed with the analysis, net concrete strains \(\varepsilon_{cx}\), \(\varepsilon_{cy}\), and \(\gamma_{cxy}\) must be known. These strain components can be calculated from Equation 5.1 by subtracting crack shear-slip strains from the total strains. Crack shear-slip model presented in Section 4.5.6 cannot be utilized here to calculate crack slip strains as it requires knowledge of crack shear stress \(\nu_{ci}\) which can only be determined if net concrete strains are known. Hence, an alternate procedure must be adopted to calculate crack-slip strains at starting stage of the analytical process.

As explained earlier in Chapter 4, rotation of the principal stress field \(\theta_\sigma\) lags behind the rotation of principal strain field \(\theta_\varepsilon\) in reinforced concrete elements loaded with combined normal and shear stresses. As per the experimental evidence (Vecchio, 2000), this lag is established as soon as concrete cracks and remains relatively constant during the early stages of the loading until shear stresses on the crack surfaces start to become significant and govern the crack slip. In presenting the formulation of DSFM, Vecchio (2000) adopted a hybrid model for calculating average shear-slip strain \(\gamma_s\) considering two separate approaches based on constant rotation lag and crack shear stresses. In later refined Vecchio-Lai formulation (Vecchio and Lai, 2004), crack shear-slip was modeled with a forced-based approach considering aggregate interlock and shear stresses on crack surfaces. The initial crack slip, which occurs before contact areas develop between cracks, was explicitly considered by term \(\nu_{co}\) as explained in Section
4.5.6. This eliminated the need for calculating crack-slip shear strain based on constant rotation lag approach to account for the initial slip occurring during early stages of the loading. Despite its discontinued use in new formulation, constant rotation lag approach remains a simple and rational approach to model crack-slip strains during initial loading. Therefore, in the proposed procedure, this approach is used to estimate strain components caused by discrete crack shear-slip. After estimating shear-slip strains \((\varepsilon'_x, \varepsilon'_y, \gamma'_{xy})\) with the procedure explained below, net concrete strains can then be determined from Equation 5.1 for subsequent calculations.

The constant rotation lag model (Vecchio, 2000) relates the post-cracking rotation of the principal stress field \(\Delta\theta_\sigma\) to the post-cracking rotation of the principal strain field \(\Delta\theta_\varepsilon\) by a specified rotation lag \(\theta^l\). As mentioned earlier, the rotational lag establishes soon after cracking, and generally falls in the range of 5° to 10° until the yielding of a reinforcement component. To implement this approach, it is necessary to first define the post-cracking rotation of the principal total strain axis \(\Delta\theta_\varepsilon\) relative to initial crack direction \(\theta_{ic}\) (i.e., the inclination of the principal strains/stresses at first cracking).

\[
\Delta\theta_\varepsilon = \theta_\varepsilon - \theta_{ic}
\] (5.2)

The inclination of the total principal strain field \(\theta_\varepsilon\) is determined from the total strains \(\varepsilon_x, \varepsilon_y\), and \(\gamma_{xy}\) as explained in Section 4.5,

\[
\theta_\varepsilon = \frac{1}{2} \tan^{-1} \left( \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \right)
\] (4.11)
The post-cracking rotation of the principal stress field $\Delta \theta_\sigma$ is then related to $\Delta \theta_e$ by the constant rotation lag is follows,

$$\Delta \theta_\sigma = \left( \Delta \theta_e - \theta' \right) \text{ for } |\Delta \theta_e| > \theta'$$  \hspace{1cm} (5.3)

$$\Delta \theta_\sigma = \Delta \theta_e \text{ for } |\Delta \theta_e| \leq \theta'$$  \hspace{1cm} (5.4)

The constant lag $\theta'$ is taken as 5 degrees and initial crack direction $\theta_{ic}$ is assumed as 45 degrees. The inclination of the stress field at the current load stage $\theta_\sigma$ is then calculated as sum of orientation at initial cracking and its post-cracking rotation,

$$\theta_\sigma = \theta_{ic} + \Delta \theta_\sigma$$  \hspace{1cm} (5.5)

Then using the expression relating total strain condition to the orientation of stress/strain field in the continuum, average crack-slip shear strain $\gamma_s$ can be calculated,

$$\gamma_s = \gamma_{xy} \cdot \cos(2\theta_\sigma) + \left( \varepsilon_y - \varepsilon_x \right) \cdot \sin(2\theta_\sigma)$$  \hspace{1cm} (5.6)

Once the crack-slip shear strain $\gamma_s$ is known, it can then be resolved into orthogonal components $\varepsilon_x^s$, $\varepsilon_y^s$, and $\gamma_{xy}^s$ as,

$$\varepsilon_x^s = -\frac{\gamma_s}{2} \cdot \sin(2\theta)$$  \hspace{1cm} (5.7)

$$\varepsilon_y^s = \frac{\gamma_s}{2} \cdot \sin(2\theta)$$  \hspace{1cm} (5.8)

$$\gamma_{xy}^s = \gamma_s \cdot \cos(2\theta)$$  \hspace{1cm} (5.9)

At early stages of loading, the shear slip is generally governed by the constant rotation lag model as it captures the initial slip as the crack surfaces develop traction. However, at higher load intensities, this approach may not be accurate as rotation lag
ceases to be relatively constant and begins to escalate, and shear slip largely depends on the shear stresses that develop on the crack surfaces. Therefore, at advanced stages, stress-based models, such as the one presented in Section 4.5.6, will become critical in predicting crack-shear slip response of the element.

In the proposed model, to start the iteration process at any load step during initial stages of loading, strains due to crack shear-slip are calculated using the constant rotation lag approach presented above. Under large loads, e.g., after yielding of the reinforcement, initial estimate of the crack-slip strains can either be obtained from the previous load step or calculated based on constant rotation lag approach. During the analysis, crack-slip strains are re-calculated using a stress-based approach (Vecchio-Lai model as explained in Section 4.5.6) and new estimates are obtained by averaging initially estimated and re-calculated values to be used in next iteration.

5.2.2 Calculations for Local Crack Conditions

At the crack interfaces, local stress conditions are significantly different than average conditions found between the cracks. Therefore, it is necessary to consider and check local stresses at the crack to ensure that average stresses are compatible with the condition of the cracked concrete.

At the crack, it is assumed that local principal tensile stress in the concrete diminishes to zero. Hence, for average concrete tensile stress $f_{ct}$ to be transmitted across the crack, local stresses and strains in the reinforcement must increase. The local stress increase in the reinforcement is produced by an increment in average principal tensile
strain $\Delta \varepsilon_{\text{icr}}$. It is assumed that local incremental strain $\Delta \varepsilon_{\text{icr}}$ occurs in principal stress direction and satisfies equilibrium condition represented by Equation 4.36. Hence, local strain in the x- and y-reinforcement will be,

$$\varepsilon_{\text{scrx}} = \varepsilon_{sx} + \Delta \varepsilon_{\text{icr}} \cdot \cos^2 \theta_{sx}$$  \hspace{1cm} (5.10)

$$\varepsilon_{\text{scry}} = \varepsilon_{sy} + \Delta \varepsilon_{\text{icr}} \cdot \cos^2 \theta_{sy}$$  \hspace{1cm} (5.11)

The local incremental strain $\Delta \varepsilon_{\text{icr}}$ must satisfy following equilibrium condition as explained earlier in Section 4.5.5,

$$f_{ci} = \rho_x (f_{\text{scrx}} - f_{sx}) \cos^2 \theta_{sx} + \rho_y (f_{\text{scry}} - f_{sy}) \cos^2 \theta_{sy}$$  \hspace{1cm} (5.12)

The objective is to calculate reinforcement stresses and strains at the cracks and to make sure that the average concrete stresses can be transmitted across the cracks by the reserve capacity of the reinforcement. In addition, the shear stresses $\nu_{ci}$ developed at the crack interface are calculated and used for modeling crack shear-slip behavior.

The local crack calculations are performed using an iterative process within each cycle of iteration at each load step. The local crack calculations start by assuming a very small value for the incremental strain $\Delta \varepsilon_{\text{icr}}$. The strains in the reinforcements at the crack are then calculated by Equations 5.10 and 5.11. Local reinforcement stresses at the crack $f_{\text{scrx}}$ and $f_{\text{scry}}$ can be calculated according to appropriate constitutive relationships. The equilibrium of the Equation 5.12 is checked, and if not satisfied, the incremental strain value is adjusted and the same calculations are repeated until satisfactory convergence is achieved. Once local reinforcement stresses are known, the interface shear stress $\nu_{ci}$ can be calculated through the use of Equation 4.37.
5.2.3 Analytical Steps for Implementation of Proposed Model

This section summarizes the proposed procedure for estimating load-displacement relationships of reinforced concrete columns under monotonically increasing lateral loads. The procedure can be used to model the response prior to peak strength and beyond by capturing possible reduction in lateral load due to strength and stiffness degradation in post-peak regime. The major steps of the proposed procedure are to:

1. Consider an axial-flexure element of the column from its inflection point to one of the end sections. Define uniaxial material properties for unconfined and confined concrete, and reinforcing steel.

2. Define fiber cross-section for flexural analysis and perform moment-curvature analysis of the cross section at column end. Consider compression-softening factor $\beta_d$ to soften the uniaxial compressive stress-strain relationship of the concrete given by any appropriate constitutive material model. Include the effect of compression bar buckling, enhancement in compressive strength and ductility of the concrete, and contribution of concrete tensile stresses.

3. Analyze the cross-section at inflection point and determine axial strain $\varepsilon_{ax}$ due to applied axial load only considering same material constitutive laws employed for flexural section analysis of the end section. Assuming a linear distribution of average centroidal axial strain between end section and section at inflection point, axial strain due to flexure $\varepsilon_{xf}$ is determined as
\[
\varepsilon_{sf} = \frac{1}{l_{in}} \int_0^{l_{in}} (\varepsilon_o - \varepsilon_{sa}) \frac{x}{l_{in}} \, dx = 0.5(\varepsilon_o - \varepsilon_{sa})
\]  (5.13)

where, \( l_{in} \) is the length of the column between end section and inflection point, \( \varepsilon_o \) is total centroidal axial strain determined through flexural section analysis of the end section, and \( \varepsilon_{sa} \) is the axial strain due to applied axial load only at inflection point. It may be noted that \( \varepsilon_{sf} \) is the average value of axial strains due to flexure only between at end section and section at inflection point and considered constant over the entire length of the axial-flexure element.

4. Calculate flexural displacement by integrating curvatures over length of column element between end section and inflection point.

5. Calculate lateral displacement due to reinforcement slip by employing any of the appropriate models discussed in Section 4.11.

6. Consider an axial-shear element from inflection point to end section and apply DSFM as explained in Section 4.5 and determine required material stiffness matrices considering secant stiffness formulation and relevant material axis transformations, as explained in Section 4.7. Consider tension stiffening and tension softening effects while determining average concrete tensile stresses.

7. Apply axial strain interaction methodology while satisfying compatibility and equilibrium conditions as per Section 4.10 and integrate axial-flexure and axial-shear elements into one axial-shear-flexure element.

8. Analyze axial-shear-flexure element under in-plane stress conditions. Calculate the strains using Equation 5.14 and check the convergence of deformations and acquired
variables. Explicitly consider the effects due to crack shear-slip while relating total stresses and total strains.

\[
\{\sigma\} = [D]\{\varepsilon\} - \{\sigma^o\} \quad \text{or} \quad \{\varepsilon\} = [D]^{-1}\left(\{\sigma\} + \{\sigma^o\}\right)
\] (5.14)

### 5.2.4 Solution Technique for Implementing Analytical Procedure

In the previous section, major steps in the proposed analytical model were presented. This section summarizes the solution technique for implementing above-mentioned analytical steps and describes calculation process in step-by-step manner. The solution technique is iterative and calculation procedure starts by making assumptions on initial value of the strain parameters. Note that the procedure mentioned below is only for laterally loaded columns under compressive axial loads.

1. Define or input material properties and geometry of the cross-section. Decide on a sign convention for tensile and compressive stresses and strains. In the implementation of proposed analytical procedure for modeling test columns, later in this chapter, tension is taken as positive and compression is taken as negative. The modeling of column behavior under tensile axial loads is not considered.

2. Input axial load \( P \). If there is no applied axial load, then consider a negligibly small value for axial load for numerical stability of the analytical procedure.

3. Select a small value of total drift ratio \( \gamma \), such as \( \gamma = 0.000001 \), as a starting value.

4. Consider the variables of iteration such as \( \varepsilon_{ai}, \phi, \varepsilon_{si}, \varepsilon_{yi}, \gamma_{si} \) and assume some very small values for each of them for the first iteration. These variables are axial centroidal strain at the end section in the axial-flexure element \( (\varepsilon_{ai}) \), curvature of the end
section in axial-flexure element \( (\phi_i) \), average total normal strain in \( x \)-direction for axial-shear element \( (\varepsilon_{xi}) \), average total normal strain in \( y \)-direction for axial-shear element \( (\varepsilon_{yi}) \) and average total shear strain of axial-shear element \( (\gamma_{xyi}) \), respectively.

5. Calculate inclination of total principal strain field \( \theta_e \) by Equation 4.11 using assumed total strain values \( \varepsilon_{si}, \varepsilon_{yi}, \) and \( \gamma_{xyi} \).

6. Estimate strain components due to crack-shear slip \( \varepsilon_x^s, \varepsilon_y^s, \) and \( \gamma_{xy}^s \) following the procedure explained in Section 5.2.1.

7. Determine net concrete strains \( \varepsilon_{cx}, \varepsilon_{cy}, \) and \( \gamma_{cxy} \) using Equation 5.1.

8. Using net strains, determine net principal strains in the concrete \( \varepsilon_{c1} \) and \( \varepsilon_{c2} \) by Equations 4.8 and 4.9, respectively.

9. Calculate compression-softening factor \( \beta_d \) with the help of Equation 4.20.

10. Perform flexural section analysis at the end section of the axial-flexure element and determine nominal moment capacity \( M \) and corresponding axial centroidal strain \( \varepsilon_{oi+1} \).

11. Determine flexural-shear stress \( \tau_f \) by Equation 4.68 modified for axial-flexure element of the column as,

\[
\tau_f = \frac{M}{bd_f l_m}
\]  

(5.15)

12. Analyze section at inflection point and determine axial strain, \( \varepsilon_{xa} \) due to applied axial load only. Calculate axial strain due to flexure \( \varepsilon_{xf} \) with Equation 5.13 by replacing \( \varepsilon_o \) with \( \varepsilon_{oi+1} \) determined in step 10.
13. Determine flexural drift $\gamma_f$ and lateral drift corresponding to reinforcement slip rotation $\gamma_{pull}$ from flexural and reinforcement slip displacements, respectively.

$$\gamma_f = \frac{\Delta_f}{l_{in}} \quad \gamma_{pull} = \frac{\Delta_{pull}}{l_{in}}$$

(5.16)

14. Determine $\sigma_o, f_{xf}, K_f, and K_{pull}$ from Equations 4.69, 4.71, 5.15 and 5.16, respectively.

$$K_f = \frac{\tau_f}{\gamma_f}, \quad K_{pull} = \frac{\tau}{\gamma_{pull}}$$

(5.17)

15. Apply DSFM to the axial-shear element considered between column inflection point and end section, as explained in Section 4.5 and follow following steps:

a. Determine inclination of principal stress field $\theta$ from Equation 4.10 using net concrete strains and establish the values for $\theta_{nx}$ and $\theta_{ny}$ (Section 4.5.5).

b. Determine average steel stresses $f_{sx}$ and $f_{sy}$ corresponding to the strains $\varepsilon_x$ and $\varepsilon_y$ in accordance with employed constitutive laws for reinforcement in x- and y-directions, respectively (Section 4.5.4.4).

c. Determine average principal tensile and compressive stresses in concrete by considering same material constitutive laws employed in axial-flexural model (step 10). However, if in axial-flexure model cover and core concrete are modeled separately then concrete compressive strength and corresponding strain must be modified according to the Equation 4.77 and 4.78.
d. Check local stress-strain conditions at cracks. Calculate the reserve capacity of the reinforcement (Equation 4.35) to ensure that the tension in the concrete can be transmitted across the crack. Follow the iterative procedure explained in Section 5.2.2 and calculate local reinforcement strains and corresponding stresses. Also find the shear stress on the crack using Equation 4.37.

e. Following the procedure explained in Section 4.6, calculate average crack spacing $s$ and width of the crack $w$ using Equations 4.45 and 4.47.

f. Calculate the crack shear-slip $\delta_s$ following the procedure described in Section 4.5.6. Determine average crack slip shear strain $\gamma_s$ from Equation 4.46, and decompose it into orthogonal components to obtain shear-slip strains $\varepsilon_x^s$, $\varepsilon_y^s$, and $\gamma_{xy}^s$ using Equations 4.5 through 4.7. These strain values are used to obtain the new estimate of crack-slip strain values for the next iteration.

g. Determine secant moduli for concrete and reinforcements and assemble respective material stiffness matrices using Equations 4.50 through 4.53. Determine total material stiffness matrix $D$ by transforming component material stiffnesses into global x- and y-directions using Equation 4.54.

16. Invert total stiffness matrix $[D]$ to get flexibility matrix $[f]$ of Equation 4.72. Add flexibility component of axial deformation due to flexure $f_{sf}$ to $f_{11}$ component of $[f]$ to determine flexibility matrix of coupled axial-shear-flexure element of Equation 4.73.
17. Considering concrete material stiffness matrix $[D]$, and crack shear-slip strains determined in step 6, calculate pseudo-prestress vector $\sigma^o$ as per Equation 4.57.

18. Consider $\sigma_x = \sigma_o$ (Equation 4.71), $\sigma_y = 0$, and $\gamma_{xyi}$ from initial assumed value in current iteration, use Equation 5.14 and calculate $\tau_s$. Determine shear stiffness as,

$$K_x = \frac{\tau_s}{\gamma_{xyi}} \quad (5.18)$$

19. Determine total stiffness $K_y$ and total shear stress $\tau$ using following equations,

$$\frac{1}{K_y} = \frac{1}{K_f} + \frac{1}{K_s} + \frac{1}{K_{pull}} \quad (5.19)$$

$$K_y (\gamma_f + \gamma_{pull} + \gamma_s) = K_y (\gamma) = \tau \quad (5.20)$$

20. Consider $\sigma_x = \sigma_o$ (Equation 4.71), $\sigma_y = 0$, and $\tau$ from step 19, and compute $\varepsilon_{si+1}, \varepsilon_{yi+1},$ and $\gamma_{si+1}$ for the integration point using Equation 5.14. Determine $\gamma_{f1}$ and $\phi_{i+1}$ for flexural section in next iteration.

$$\gamma_{f1} = \frac{\tau}{K_f}; \quad \phi_{i+1} = \frac{\gamma_{f1}}{\gamma_f} \cdot \phi_i \quad (5.21)$$

21. Assess the convergence of the acquired variables $\varepsilon_{si+1}, \phi_{i+1}, \varepsilon_{yi+1}, \varepsilon_{yi+1},$ and $\gamma_{xyi+1}$. If satisfactory convergence is achieved, then go to the next step otherwise repeat steps 5 to 20 with improved estimate of the iteration variables.

22. Compute lateral load capacity of the column corresponding to the given total drift by Equation 4.58 replacing $\tau_s$ with $\tau$ obtained in step 19.

23. Increment total drift ratio $\gamma$ and repeat steps 4 to 20 until required response is evaluated.
These analytical steps are summarized in a flow chart as shown in Figure 5.1.

5.3 Validation of Proposed Model

The proposed model is verified by observing consistent correlation between estimated response and experimental results for a number of reinforced concrete column specimens. In order to implement the proposed analytical procedure, a computer program in MATLAB (MathWorks, 2009) is written and lateral load-deformation responses of previously tested reinforced concrete columns are estimated.

5.3.1 Details of Test specimens and Material Properties

Three reinforced concrete columns tested by Sezen (2002), eight columns tested by Lynn (2001) and four columns tested by Saatcioglu and Ozcebe (1989) are analyzed in this study. Details of these columns are presented below and summary of their material properties and cross-section details are provided in Table 5.1.

5.3.1.1 Sezen (2002) Columns

The columns tested by Sezen (2002) are lightly reinforced and have shear and flexural design strengths very close to each other. These are 18 in. (457 mm) square columns with fixed ends at top and bottom having clear height of 116 in. (2946 mm). The columns had eight No. 9 bars and No. 3 column ties with 90-degree end hooks spaced at 12 in. (305 mm). Specimens-1 and -4 were tested with a constant axial load of 150 kips (667 kN), whereas, Specimen-2 was tested under a constant axial load of 600 kip (2670 kN). The columns were tested under unidirectional cyclic lateral loading, except for
Specimen-4, which was tested under monotonically increasing load after few initial cycles of elastic loading. All of the test specimens are modeled with average concrete compressive strength of 3.08 psi (21.2 MPa). The yield strength of longitudinal and transverse reinforcement are taken to be 63 ksi (434 MPa) and 69 ksi (476 MPa), respectively. Typical details of these column specimens in given in Figure 5.2.

5.3.1.2 Lynn (2001) Columns

Lynn (2001) tested eight full-scale columns with constant axial loads and cyclic lateral loads. The loading, boundary conditions and geometric properties of the test specimens is very similar to the Sezen (2002) columns. All of these are considered lightly reinforced columns, and are subject to shear and flexural failures at nearly the same time. Figure 5.2 shows the typical details and overall dimensions of the columns. Grade 40 deformed longitudinal steel and concrete strengths ranging between 3.7 ksi (25.5 MPa) and 4.8 (33 MPa) were used. The columns were tested under constant axial loads of 113 kips (500 kN) or 340 kips (1512 kN).

5.5.1.3 Saatcioglu and Ozcebe (1989) Columns

Saatcioglu and Ozcebe (1989) specimens are cantilever columns and were tested under constant axial load of 135 kips (600 kN). The cross-section was 14 in. (350 mm) square, with nine No. 8 longitudinal bars. The column height was 39.5 in. (1000 mm). Key properties are listed in Table 5.1, and Figure 5.2 shows typical details for these columns. The transverse reinforcement was different on each specimen, but in all cases the columns were designed to develop flexural yielding prior to shear failure. Columns
were loaded cyclically in both one lateral direction and two perpendicular directions, although forces applied in one direction were limited to less than the force corresponding to the yield point. The authors reported that displacements in one direction do not significantly affect behavior in the perpendicular direction, as long as they are kept less than the yield displacement of the column.

5.3.2 Constitutive Material Models

The constitutive relationships used to model the test columns are presented in following section. Same constitutive laws for concrete and reinforcements are used for both axial-flexure and axial-shear models.

5.3.2.1 Concrete Behavior in Compression

Concrete behavior in compression for confined core concrete and unconfined cover concrete is modeled with the help of Mander et al. (1988) model, and shown in Figure 5.3. According to this model, compressive stress-strain relationship of the confined concrete is defined as,

\[
f_c = f_{cc} \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right) \left( \frac{\varepsilon_c}{\varepsilon_{co}} \right), \quad \varepsilon_{cc} = \varepsilon_{co} \left[ 1 + 5 \left( \frac{f_{cc}}{f_{co}} - 1 \right) \right], \quad r = \frac{E_c}{E_c - E_{sec}}, \quad E_{sec} = \frac{f_{cc}}{\varepsilon_{cc}} \quad (5.22)
\]

where \( f_{cc} \) is the peak confined concrete strength, \( \varepsilon_c \) is the concrete strain, \( \varepsilon_{cc} \) is the concrete strain at peak stress for confined concrete, \( \varepsilon_{co} \) is the concrete strain at peak stress in unconfined concrete (taken here as 0.002), \( f_{co} \) is the concrete compressive
cylinder strength, \( E_c = 57,000 \sqrt{f_{co}} \text{ psi} \ (5000 \sqrt{f_{co}} \text{ MPa}) \) is the modulus of elasticity of normal weight concrete, and \( E_{sec} \) is the secant modulus of the concrete.

In this study, ultimate compressive strain \( \varepsilon_{cu} \) for confined concrete is calculated from Equation 5.23, which is obtained by modifying the maximum strain formula for spirally confined concrete in Priestley (1996).

\[
\varepsilon_{cu} = 0.004 + 0.14 \left( \rho_x + \rho_y \right) \frac{f_{yy}}{f_{cc}}
\]

\( \rho_x, \rho_y = \frac{\pi d_t^2}{2s_h d_c} \) (for square columns)

where \( f_{yy} \) is yield strength, \( d_t \) is diameter and \( s_h \) is center-to-center spacing of the transverse reinforcement, respectively.

For unconfined cover concrete, compressive stress-strain relationship is defined by following equation,

\[
f_{c,unconf} = \frac{f_{co} r_1 \left( \varepsilon_c / \varepsilon_{co} \right)}{r_1 - 1 + \left( \varepsilon_c / \varepsilon_{co} \right)^{r_1}} \text{ for } \varepsilon_c \leq 2\varepsilon_{co}, \text{ where } r_1 = \frac{E_c}{E_c - E_{sec}}, \ E_{sec} = \frac{f_{co}}{\varepsilon_{co}} \tag{5.24}
\]

After reaching the strain of \( 2\varepsilon_{co} \), cover concrete is assumed to start spalling and part of falling branch in the region where \( \varepsilon_c > 2\varepsilon_{co} \) is assumed to be a straight line which reaches zero stress at spalling strain \( \varepsilon_{sp} \), taken equal to 0.006 in this study (Figure 5.3). In order to apply compression softening effect, compressive behavior for confined and unconfined concrete obtained from Equations 5.22 and 5.24 must be multiplied by compression softening factor \( \beta_d \) from Equation 4.20.
5.3.2.2 Concrete Behavior in Tension

For flexural modeling, a linear elastic relationship between concrete tensile stress-strain is employed until cracking, and then concrete tensile stresses are assumed to drop linearly to zero at tensile strain of 0.003. For axial-shear model, as concrete tensile behavior is critical to overall response, detailed concrete models considering the effects of tension stiffening and tension softening are employed. Details of these models are presented in Section 4.5.

5.3.2.3 Reinforcing Steel Behavior

The reinforcing steel behavior (Figure 5.4) in this study is modeled considering a linear elastic behavior, a yield plateau, and a non-linear strain-hardening region, as per set of following equations,

\[
\begin{align*}
  f_s &= E_s \varepsilon_s \quad ; \text{for } \varepsilon_s \leq \varepsilon_y \\
  f_s &= f_y + (\varepsilon_s - \varepsilon_{sh}) \alpha E_s \quad ; \text{for } \varepsilon_y \leq \varepsilon_s \leq \varepsilon_{sh} \\
  f_s &= f_u - (f_u - f_{sh}) \left( \frac{\varepsilon_s - \varepsilon_{sh}}{\varepsilon_u - \varepsilon_{sh}} \right)^p \quad ; \text{for } \varepsilon_{sh} \leq \varepsilon_s \leq \varepsilon_u
\end{align*}
\]

(5.25)

where \( E_s \) is the elastic modulus of steel, \( \varepsilon_s \) is the steel strain, and the subscripts \( y, sh, \) and \( u \) refer to the yield point, the onset of strain hardening, and the ultimate stress, respectively. The order of the curve \( p \) defines the strain-hardening region, and is often taken as 2 for a parabolic curve, and \( \alpha \) is a coefficient that defines the slope of the yield plateau. For the columns in this study, these parameters were used to define the longitudinal steel stress-strain model as \( \varepsilon_{sh} = 0.016 \), \( \alpha = 0.02 \), \( \varepsilon_u = 0.23 \), \( p = 6 \), \( E_s = 29,000 \) ksi (200,000 MPa).
5.3.3 Comparison of the Predicted and Experimental Response

Following the analytical procedures for implementation of the proposed model, lateral load-displacement relationships of column specimens are estimated. The predicted responses are compared with experimentally observed column responses in Figures 5.5 through 5.16. The responses are also predicted and compared using another model referred as Displacement Component Model (DCM) (Setzler and Sezen, 2008).

The DCM is a macro model that considers that total lateral deformation of a reinforced concrete column is comprised of three components; flexural deformations, reinforcement slip deformations and shear deformations. In this model, each of three individual deformation components is estimated separately and then simply added together to get total pre-peak response. For post-peak analysis, the column is classified into one of the five categories based on comparison of its predicted shear and flexural strength and then individual deformation components are combined together according to a set of rules specified for each category. This model does not consider buckling of the bars under large compressive strains and interaction between flexural and shear deformations.

Figure 5.5 through 5.7 present lateral load-displacement relationships for the columns tested by Sezen (2000). Pre-peak responses by both models is predicted almost identically, which matches very well with experimental response. However, post-peak predicted responses are significantly different. For all specimens, response predicted by
DCM is over predicted in terms of both strength and displacements. The response predicted by proposed model closely follow the experimental response with matching strength and displacement predictions. The post-peak stiffness matches well with experimental data.

Figure 5.8 through 5.16 present lateral load-displacement relationships for the columns tested by Lynn (2002) and Saatcioglu and Ozcebe (1989). Again, the pre-peak response by both models is predicted almost identically that matches well with pre-peak stiffness of experimentally observed response. However, DCM significantly underestimates the peak strength and corresponding displacements for almost all of the Lynn’s specimens. On the other hand, the proposed model captures peak strength, displacements and post-peak response very well. The response predicted by the proposed model compares well with the experimental test data. For columns tested by Saatcioglu and Ozcebe (1989), DCM, and proposed model predict identical pre-peak responses that match well with experimental responses. Pre-peak stiffness, peak strength and corresponding displacements are predicted well. However, in post-peak predicted responses, DCM predicts conservative strengths. Again, the proposed model predicts post-peak response with good accuracy and follows the experimental post-peak response.

5.4 Summary and Conclusions

Analytical procedure and solution technique for implementation of the proposed model for response estimation of reinforced concrete columns is presented. The procedure for calculation of the strain components due to crack-shear slip were presented
in details. These strain components are required to start the analytical procedure. In addition, the method to perform checks at crack surface and calculation of crack shear strength were also described. The procedure is incorporated into a computer program written in MATLAB (MathWorks, 2009) and response of 12 columns is estimated. Comparison of calculated total lateral load-total lateral displacement responses of test specimens with their experimental responses indicates that the proposed model is an effective tool for response estimation of columns subjected to lateral loads.
Table 5.1. Properties of test columns

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<th>Type</th>
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<th>f_c</th>
<th>f_y</th>
<th>f_yv</th>
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<th>( \rho_y )</th>
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C = Cantilever; DC = Double Curvature
Figure 5.1. Flow chart for implementation of the proposed model
(a) Typical test specimen and column details (Lynn, 2001; Sezen, 2002)

(b) Typical test specimen and column details (Saatcioglu and Ozcebe, 1989)

Figure 5.2. Details of the modeled columns
Figure 5.3. Constitutive relationships of concrete in compression

Figure 5.4. Constitutive relationship for reinforcing steel
Figure 5.5. Lateral load-displacement relationship for specimen-1

Figure 5.6. Lateral load-displacement relationship for specimen-2
Figure 5.7. Lateral load-displacement relationship for specimen-4

Figure 5.8. Lateral load-displacement relationship for 2CLH18
Figure 5.9. Lateral load-displacement relationship for 2CMH18

Figure 5.10. Lateral load-displacement relationship for specimen 3CLH18
Figure 5.11. Lateral load-displacement relationship for specimen 3CMD12

Figure 5.12. Lateral load-displacement relationship for specimen 3CMH18
Figure 5.13. Lateral load-displacement relationship for specimen 3SLH18

Figure 5.14. Lateral load-displacement relationship for specimen 3SMD12
Figure 5.15. Lateral load-displacement relationship for specimen U4

Figure 5.16. Lateral load-displacement relationship for specimen U6
CHAPTER 6

DESCRIPTION OF THE TEST BUILDING

6.1 Introduction

Post-earthquake studies show that primary cause of reinforced concrete building collapse during earthquakes is the loss of vertical-load-carrying capacity in critical building components leading to cascading vertical collapse, rather than loss of lateral-load capacity (Ghannoum et al., 2006; Moehle et al., 2002). Once column failure occurs, particularly in lower stories, vertical loads from gravity and inertial effects are transferred to adjacent framing components. The ability of the frame to continue to support vertical loads depends on both the capacity of the framing system to transfer these loads to adjacent components and the capacity of the adjacent components (beams, columns and joints) to support the additional load. When any of these conditions is deficient, structure may collapse in vertical direction.

In previous chapters, behavior of reinforced concrete columns under axial and lateral loads was studied and modeled. In subsequent chapters, behavior of reinforced concrete frame buildings is studied to investigate redistribution of gravity loads after failure of lower story column using a progressive collapse model and experimental data. In order to understand overall structural response of the building in the context being
studied, understanding the behavior of the individual frame members under given loading conditions is critical. The column research presented in previous chapters is very helpful towards this end, and is used for investigating structural response of the individual frame elements and the building itself. Specifically, the models presented for evaluation of flexural response are used to calculate strains in the beams and column of the test building and compare with the recorded experimental strains.

Progressive collapse is generally defined as small or local structural failure resulting in collapse of the adjoining members and, in turn, causing total collapse of the building or a disproportionately large part of it. During a seismic activity, local structural failure in the lower-story columns can initiate vertical or progressive collapse in the buildings with inadequate ductility if gravity loads cannot be transferred to undamaged columns (Gurley, 2008; Wibowo and Lau, 2009). After one or more columns fail, an alternate load path is needed to transfer the loads carried by failed member(s) to other structural members. If adjoining elements cannot resist and redistribute the additional loads, a series of failures will occur until entire or substantial part of the structure collapses.

As part of the experimental research, an existing reinforced concrete frame building was tested at the Ohio State University by removing one of the critical load carrying elements. Redistribution of the gravity loads was investigated using a progressive collapse model and experimental data. In this testing, one first-story exterior column was physically removed and structural response of the building was monitored by recording strains and displacements of selected frame members in the vicinity of the removed column. The recorded test data and experimental observations are used to
validate various levels of evaluation and modeling techniques employed for prediction of progressive collapse response of the buildings. The building was tested prior to its scheduled demolition in December 2011.

This chapter presents details of the test building which are required for explaining experimental procedure and subsequent computational research. These details include general description of the test building, characteristics of structural elements (columns, beams, and slabs), and material properties used in this research.

6.2 General Description of the Test Building

The test building, Johnston Laboratory, was located at the Ohio State University (OSU) campus in Columbus, Ohio. The building was designed in 1942 and completed in April 1943. Southwest exterior view of the building is shown in Figure 6.1. It was a four-story reinforced concrete frame building with regular structural configuration. Figure 6.2 shows an early view of the building during its construction in 1942. Johnston laboratory was constructed as War Research Laboratory and was initially used to house war projects that were assigned to the OSU. Later, it was remodeled in late 1980s and was used primarily as an academic research laboratory and office space until it was demolished in December 2011. Figure 6.3 shows some views of the building before its demolition.

A typical floor framing plan of the Johnston Laboratory is shown in Figure 6.4. In this study, only the structural bays with regular configuration are modeled and two bays at northern end of the building are not considered. As the building was tested by removing one column from eastern perimeter frame near the middle of the building, redistribution of the loads took place in the immediate vicinity of the removed column
and structural bays at the farthest end did not experience any additional loads. Hence, modeling the test building without two irregular end bays is not expected to affect predicted response and analyses results. Location of the removed column is highlighted with red circle in Figure 6.4.

Figure 6.5 shows plan and elevation of the Johnston Laboratory modeled in this research. The building model had an approximately 134 ft by 45 ft rectangular floor plan with twelve bays in the longitudinal direction and three bays in transverse direction. Along long side of the building, all bays measured 10 ft-8 in. except a single bay of 16 ft width located near middle of the structure. In transverse direction, outer bays were 18 ft-2 in. wide while middle bay, used as corridor of the building, had a width of 8 ft-6 in. The story heights for the first, second, third and fourth floors were 13 ft, 13 ft-3 in., 13 ft-3 in., and 12 ft-6 in., respectively.

6.3 Characteristics of Structural Frame Members

The test building was an ordinary reinforced concrete frame buildings of regular structural configuration primarily designed for gravity loads. The details of the columns, beams and slabs used in Johnston Laboratory are presented in Tables 6.1 through 6.4. These details are taken from original drawings and design notes, provided in Appendix A. Only the structural details such as the size of the cross-section and main features of reinforcement, needed to describe test building response, modeling procedure and computational research, are presented in this chapter. Other minute details about cross-section geometry, material properties and reinforcement detailing can be found in detailed building drawings in Appendix A.
6.3.1 Details of the Columns

Table 6.1 presents major details of the reinforced concrete columns used in the Johnston Laboratory. The columns are identified by a letter followed by a number (a subscript). The alphabetical letters are the frame in longitudinal direction (such as A, B, C or D) while numbers (such as 1, 2, 3 etc.) represent transverse frame. As shown in the table, eight categories of columns differing in size of the cross-section and reinforcement details were used in the test building. Both square and rectangular columns were used in the first story while only square columns were used in all upper stories. Also, relatively larger cross-sections with heavier reinforcements were used in lower stories and gradually reducing column sizes with lighter reinforcements were used in upper stories.

As per the original drawings, closely spaced (1.50 in. to 2.75 in.) spirals of #2 or #3 bars were used as transverse reinforcement in the first and second story columns whereas rectangular ties of #2 bars spaced at 10 in. were mostly used in the third and fourth story columns. However, after removing concrete cover from first-story columns (A8, A10, and B9) for instrumentation during preparatory phase, it was found that those columns actually had #3 ties spaced at 10 in. – 12 in. against specified transverse reinforcement of #2 bar spiral with 1.75 in. spacing/pitch per original drawings. In this study, details taken from the original structural drawings and notes were used for modeling of the test building and subsequent computational research; however, whenever different than specified information was found, actually observed details were used.
6.3.2 Details of the Beams

Tables 6.2 and 6.3 present details of the reinforced concrete beams used in the Johnston Laboratory. Like column’s nomenclature, beams are also identified by combination of letters and numbers. The letters represent longitudinal frames and numbers indicate transverse frames. For example, beam $A_{1.2}$ is a beam in longitudinal frame-$A$ running between transverse frames 1 and 2. Likewise, beam $1_{A-B}$ represents a beam in transverse frame-1 spanning between longitudinal frames $A$ and $B$. For each floor, beams are further identified by distinct numbers. Table 6.2 shows beam assignments by type while Table 6.3 presents characteristics of each type of the beam in terms of size of the cross-section and main features of the longitudinal and transverse reinforcement.

In the longitudinal perimeter frames (east and west side of the building), large sized spandrel beams were used in all floors. For the rest of the structure, rectangular beams of regular sizes were used. The actual details of the spandrel beams in the test building, as taken from original building drawings, are shown in Figures 6.6 and 6.7. Due to their irregular shapes, these spandrel beams are idealized by rectangular beams of roughly same size and reinforcement details. The idealized spandrel beams used for building modeling and analyses are also shown in Figure 6.6 and 6.7.

Minimal shear reinforcement was provided in all beams. Mostly, #3 rectangular stirrups with large spacing were used for beams in longitudinal frames whereas #4 bar ties with relatively smaller spacing were provided for beams in transverse frames. The stirrups were provided only at beam ends near supports. Detailed information on cross-
section and reinforcement detailing of the beams in the Johnston Laboratory can be found in “CONCRETE BEAM SCHEDULE” in Figures A.1 through A.3 in Appendix A.

6.3.3 Details of the Slabs

The details of the reinforced concrete slabs used in the Johnston Laboratory are presented in Table 6.4. Typically, 6 inch thick solid slabs were used in the second floor and 5 inch thick slabs were used in all upper floors. Slabs were supported by the beams in the transverse frame, resulting in one-way slab action. In the larger bay located near the middle of the long side of the building, ribbed slabs were used in all floors. In the second, third and fourth floors, 4.5 in. wide and 10 in. deep joists with 2.5 in. thick slabs were used. In the roof ribbed slab, joists were 8 in. deep. The joists were uniformly spaced at 20 inches and were tapered near both ends; however, for simplicity a uniform joist width of 4.5 inch is used for modeling and computational analyses in this study. Further details about the slab layout, size, and reinforcement can be found in “CONCRETE SLAB SCHEDULE” in the original drawings provided in Appendix A (Figures A.1 through A.3).

6.4 Material properties

Concrete and steel material properties could not be found from the original 1942 structural drawings of the Johnston Laboratory. However, structural notes and details from remodeling drawings of late 1980s (Figure A.10) specified some of the material properties used in the project. As per these drawings, the construction was governed by the Ohio Basic Building Code. The concrete, in general, was to comply with ACI 301-84,
“Specifications for structural concrete for buildings” with specified compressive strength of 3500 psi. Reinforcing bars used in the remodeling construction were ASTM A615, grade 60.

As material properties for the original construction were not readily available, these were assumed in line with common construction practices in early 1940s and details found in remodeling drawings, as following:

(1) Compressive strength of the concrete - 3500 psi
(2) Reinforcing rebar - ASTM A615, grade 40
(3) Concrete block masonry - ASTM C90 (Hollow)
    - ASTM C145 (Solid)

6.5 Summary of Assumptions

In order to simplify modeling and analysis of the test building, the following assumptions were made for building and material properties.

1. The tapering of the joists at their ends in the ribbed slab panels in the middle of the building plan was ignored, and a uniform joist width of 4.5 inch was assumed.

2. Material properties assumed are:-
   - Compressive strength of the concrete - 3500 psi
   - Yield strength of longitudinal reinforcing bars - 40000 psi
   - Yield strength of transverse reinforcing bars - 40000 psi

3. The data from the original structural drawings and observations in the field were different for type, size and spacing of ties for three columns that were instrumented in the experiment. For these columns, observed properties/specifications are used whereas specified properties in the drawings are used for all other frame members.
4. Two irregular end bays (right end of the building in Figure 6.4) are ignored in the modeling and analysis as these are not likely to affect the behavior of building near removed column.
<table>
<thead>
<tr>
<th>Column number</th>
<th>Property</th>
<th>1&lt;sup&gt;st&lt;/sup&gt; to 2&lt;sup&gt;nd&lt;/sup&gt; Floor</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; to 3&lt;sup&gt;rd&lt;/sup&gt; Floor</th>
<th>3&lt;sup&gt;rd&lt;/sup&gt; to 4&lt;sup&gt;th&lt;/sup&gt; Floor</th>
<th>4&lt;sup&gt;th&lt;/sup&gt; Floor to Roof</th>
</tr>
</thead>
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<td>Size</td>
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<td>16&quot; x 16&quot;</td>
<td>15&quot; x 15&quot;</td>
<td>14&quot; x 14&quot;</td>
</tr>
<tr>
<td></td>
<td>Long. steel</td>
<td>12#6 bars</td>
<td>8#6 bars</td>
<td>8#6 bars</td>
<td>4#5 bars</td>
</tr>
<tr>
<td></td>
<td>Trans. steel</td>
<td>#2@1.50&quot; (13&quot; spiral)</td>
<td>#2@2.00&quot; (12&quot; spiral)</td>
<td>#2@10.0&quot; (ties)</td>
<td>#2@10.0&quot; (ties)</td>
</tr>
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<td>16&quot; x 16&quot;</td>
<td>14&quot; x 14&quot;</td>
</tr>
<tr>
<td></td>
<td>Long. steel</td>
<td>11#7 bars</td>
<td>10#6 bars</td>
<td>8#6 bars</td>
<td>4#5 bars</td>
</tr>
<tr>
<td></td>
<td>Trans. steel</td>
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<td>#2@2.00&quot; (13&quot; spiral)</td>
<td>#2@10.0&quot; (ties)</td>
<td>#2@10.0&quot; (ties)</td>
</tr>
<tr>
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<td>16&quot; x 16&quot;</td>
<td>15&quot; x 15&quot;</td>
<td>14&quot; x 14&quot;</td>
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<td>10#7 bars</td>
<td>9#6 bars</td>
<td>8#5 bars</td>
<td>4#5 bars</td>
</tr>
<tr>
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<td>Trans. steel</td>
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<td>#2@2.00&quot; (13&quot; spiral)</td>
<td>#2@2.00&quot; (12&quot; spiral)</td>
<td>#2@10.0&quot; (ties)</td>
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<td>Long. steel</td>
<td>11#7 bars</td>
<td>8#7 bars</td>
<td>7#6 bars</td>
<td>4#5 bars</td>
</tr>
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<td></td>
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<td>#2@1.75&quot; (13&quot; spiral)</td>
<td>#2@2.00&quot; (12&quot; spiral)</td>
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<td>14&quot; x 14&quot;</td>
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<td>Long. steel</td>
<td>8#7 bars</td>
<td>8#6 bars</td>
<td>6#5 bars</td>
<td>4#5 bars</td>
</tr>
<tr>
<td></td>
<td>Trans. steel</td>
<td>#2@1.75&quot; (15&quot; spiral)</td>
<td>#2@2.00&quot; (12&quot; spiral)</td>
<td>#2@2.00&quot; (12&quot; spiral)</td>
<td>#2@10.0&quot; (ties)</td>
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<td>16&quot; x 16&quot;</td>
<td>16&quot; x 16&quot;</td>
<td>15&quot; x 15&quot;</td>
<td>14&quot; x 14&quot;</td>
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<td>Long. steel</td>
<td>11#6 bars</td>
<td>7#6 bars</td>
<td>4#6 bars</td>
<td>4#5 bars</td>
</tr>
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<td></td>
<td>Trans. steel</td>
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<td>#2@2.00&quot; (12&quot; spiral)</td>
<td>#2@2.00&quot; (12&quot; spiral)</td>
<td>#2@10.0&quot; (ties)</td>
</tr>
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<td>Size</td>
<td>16&quot; x 16&quot;</td>
<td>16&quot; x 16&quot;</td>
<td>15&quot; x 15&quot;</td>
<td>14&quot; x 14&quot;</td>
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<td>#2@2.00&quot; (12&quot; spiral)</td>
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</tr>
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<td>16&quot; x 16&quot;</td>
<td>15&quot; x 15&quot;</td>
<td>14&quot; x 14&quot;</td>
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<tr>
<td></td>
<td>Long. steel</td>
<td>8#7 bars</td>
<td>8#6 bars</td>
<td>6#5 bars</td>
<td>4#5 bars</td>
</tr>
<tr>
<td></td>
<td>Trans. steel</td>
<td>#2@1.75&quot; (13&quot; spiral)</td>
<td>#2@2.00&quot; (12&quot; spiral)</td>
<td>#2@2.00&quot; (12&quot; spiral)</td>
<td>#2@10.0&quot; (ties)</td>
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Long. Steel – Longitudinal steel  
Trans. Steel – Transverse steel
Table 6.2. Types of beams in Johnston Hall

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<tr>
<th>Beam number</th>
<th>2\textsuperscript{nd} Floor</th>
<th>3\textsuperscript{rd} Floor</th>
<th>4\textsuperscript{th} Floor</th>
<th>Roof</th>
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<td>Beams in longitudinal direction</td>
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<td></td>
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<td></td>
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<tr>
<td>A\textsubscript{1-2}</td>
<td>205</td>
<td>305A</td>
<td>405A</td>
<td>505A</td>
</tr>
<tr>
<td>A\textsubscript{2-3, A4-5, A5-6}</td>
<td>203A</td>
<td>303A</td>
<td>403A</td>
<td>503A</td>
</tr>
<tr>
<td>A\textsubscript{6-7, D6-7}</td>
<td>204</td>
<td>304</td>
<td>404</td>
<td>504</td>
</tr>
<tr>
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<td>303</td>
<td>403</td>
<td>503</td>
</tr>
<tr>
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<td>203</td>
<td>303</td>
<td>403</td>
<td>503</td>
</tr>
<tr>
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<td>R3</td>
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<td>505</td>
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<td>306</td>
<td>406</td>
<td>506</td>
</tr>
<tr>
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<td>306</td>
<td>406</td>
<td>506</td>
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<td>306</td>
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<td>506A</td>
</tr>
<tr>
<td>Beams in transverse direction</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>213</td>
<td>313</td>
<td>413</td>
<td>513</td>
</tr>
<tr>
<td>1\textsubscript{B-C}</td>
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<td>308</td>
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<td>514</td>
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<tr>
<td>3\textsubscript{C-D, 4C-D}</td>
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<td>312</td>
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<td>512</td>
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<td>311</td>
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<tr>
<td>6\textsubscript{A-B, 7A-B, 7C-D}</td>
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<td>309</td>
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<tr>
<td>6\textsubscript{B-C}</td>
<td>215</td>
<td>315</td>
<td>315</td>
<td>515A</td>
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<tr>
<td>7\textsubscript{B-C}</td>
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R\textsubscript{2}, R\textsubscript{3}, R\textsubscript{5} – Ribbed slabs at 2\textsuperscript{nd} floor, 3\textsuperscript{rd} floor and roof slab levels, respectively.
<table>
<thead>
<tr>
<th>Beam type</th>
<th>Size (b x h)</th>
<th>Longitudinal reinforcement</th>
<th>Stirrups</th>
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<tbody>
<tr>
<td>203</td>
<td>SP2</td>
<td>SP2</td>
<td>SP2</td>
</tr>
<tr>
<td>203A</td>
<td>SP2</td>
<td>SP2 except 2#5 bars (B); 1#6 bar (B)</td>
<td>SP2</td>
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<tr>
<td>204</td>
<td>SP2</td>
<td>SP2 except 3#7 bars (B)</td>
<td>SP2</td>
</tr>
<tr>
<td>205</td>
<td>16&quot; x 37&quot;</td>
<td>3#6 bars (B); 1#5 bar (T)*</td>
<td>#3@4&quot;,8&quot;,10&quot; (EE)</td>
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<tr>
<td>206</td>
<td>16&quot; x 37&quot;</td>
<td>2#6 bars (B)</td>
<td>#3@4&quot;,8&quot;,10&quot; (EE)</td>
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<tr>
<td>206A</td>
<td>12&quot; x 24&quot;</td>
<td>3#7 bars (B); 1#6 (T,EE)</td>
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<tr>
<td>208, 211</td>
<td>14&quot; x 24&quot;</td>
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Table 6.3 continued

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<td>SP5</td>
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<td>#3@5&quot;,6&quot; (EE)</td>
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R2, R3, R5 See the details in CONCRETE SLAB SCHEDULE in original drawings in Appendix A

b - Width of the beam
h - Height/depth of the beam (includes slab thickness)
B - Bottom; T – Top; * Over end column
EE - Each end
SP2 - 2nd Floor Spandrel beam (See details in Figure 6.6)
SP3 - 3rd Floor Spandrel beam (See details in Figure 6.6)
SP4 - 4th Floor Spandrel beam (See details in Figure 6.7)
SP5 - Roof slab Spandrel beam (See details in Figure 6.7)
SQ - Square Bars
OC - Outside Column
IC - Inside Column
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<th>Slab</th>
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<th>3&lt;sup&gt;rd&lt;/sup&gt; Floor</th>
<th>4&lt;sup&gt;th&lt;/sup&gt; Floor</th>
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Figure 6.1. Exterior view of the Johnston Laboratory from southwest, inside rectangle (KSA Digital Library-The Ohio State University)

Figure 6.2. Exterior view of the Johnston Laboratory from southeast showing building construction (KSA Digital Library-The Ohio State University)
Figure 6.3. Exterior views of Johnston Laboratory before demolition
(KSA Digital Library-James Collier-by-nc)

Figure 6.4. Typical floor plan of the Johnston Laboratory
Figure 6.5. Plan and elevation view of the Johnston Laboratory

(a) Typical structural floor plan

(b) East elevation of Frame-A
Figure 6.6. Details of actual and idealized spandrel beams for 2\textsuperscript{nd} and 3\textsuperscript{rd} floors
Figure 6.7. Details of actual and idealized spandrel beams for 4th floor and roof slab
CHAPTER 7

EXPERIMENTAL RESEARCH ON PROGRESSIVE COLLAPSE

7.1 Introduction

Johnston Laboratory was tested for progressive collapse research before it was demolished in December 2011. The building was tested by physically removing a first-story exterior column. Structural response of the building was monitored by recording strains and displacements of selected frame members in the vicinity of the removed column.

Test building along with properties of its structural members was described in the previous chapter. In this chapter, details of the completed experimental work are presented. These details include procedure and methodology for instrumentation, column removal and data recording. The test results are presented and briefly discussed.

7.2 Overview of the Experimental Work

The test building, Johnston laboratory was a reinforced concrete frame building of regular structural configuration located at The Ohio State University campus in Columbus, Ohio. The building was tested to investigate redistribution of the gravity loads and to model progressive collapse mechanism as one of the critical load carrying element
is lost. In the test, column A_9 was removed while columns A_8, A_{10}, and B_9 and beams A_8, A_{9-10}, and 9_{A-B} were instrumented, as highlighted on plan of the building in Figure 7.1. The names of the beams and columns were defined in Chapter 6. The column was removed by cutting it at its mid-height in a short period of time simulating instantaneous loss. The strains and displacements of selected beams and columns were measured with the help of strain gauges and linear displacement sensors.

7.3 Experimental Procedure

Though, the experimental testing basically involves physical removal of the column and recording of strain and displacement data, the entire experimental procedure can be described by four major stages: 1) preparation, 2) instrumentation, 3) column removal, and 4) data collection. These stages are explained in the following sections.

7.3.1 Preparatory Stage

In preparatory phase, all furniture and the majority of the large mechanical equipment were removed from all floors of the test building. Structural members directly involved in the testing were identified and marked on ground. The column A_9 (highlighted by circle in Figure 7.1) was earmarked for removal while columns A_8, A_{10}, B_9 and beams A_{8-9}, A_{9-10}, 9_{A-B} (highlighted by rectangles in Figure 7.1) were selected to be instrumented. Structural bays between columns A_7 and A_{11} in the longitudinal frame direction and between columns A_{10} and B_{10} in the transverse direction, containing these frame elements, were then exposed by removing brick façade and infill/partition walls in frame along axis-A. Figure 7.2 shows the east side views of the building just before the
experiment and after removal of the first-story brick exterior wall. As seen, bays have been cleared and structural frame members involved in the experiment were fully exposed and ready for instrumentation.

7.3.2 **Instrumentation**

After selected beams and columns to be instrumented were exposed, strain gauges and displacement sensors were installed at critical locations along length of these members. The critical locations were chosen to effectively record strain and displacement data for studying behavior of the structure after column removal and investigate loads redistribution.

A total of 18 strain gauges and two linear displacement sensors (LVDTs) were used to instrument columns A_{8}, A_{10}, and B_{9} and beams A_{8-9}, A_{9-10}, and 9_{A-B}. Out of 18 strain gauges, 12 were installed on columns and 6 were installed on beams. On columns, strain gauges were installed at one-third, mid, and two-third heights, while on beams, these were installed at the ends towards the column being removed. Universal general-purpose strain gauges of the type shown in Figure 7.3 with a resistance of 120.0±0.3\% ohms and strain range of ±5% were used in the testing (Vishay Precision group-Micro Measurements, 2010). These strain gauges can measure strains caused by compressive or tensile forces. Hence, change in member axial and bending forces and deflections can be monitored during redistribution of loads caused by sudden column removal.

Figure 7.4 shows the location of all strain gauges and displacement sensors on the test building. Strain gauges are shown by numbers 1 through 18. Three strain gauges (1, 2 and 3), were installed at one-third height of the column B_{9} on north, west and east face,
respectively. On column A_8, six strain gauges were installed at two different locations. Strain gauges 4, 5, and 6 were installed at two-third height while 7, 8, and 9 were installed at mid-height of the column. At both locations, north, south and west faces were instrumented. Gauges on North-South faces primarily measured major bending in longitudinal direction. Similarly, three strain gauges (10, 11, and 12) were installed on column A_{10} at mid-height on north, west and south faces, respectively. In addition to the instrumentation of columns, three beams between target column A_9 and neighboring columns A_8, A_{10} and B_9 were instrumented with a total of six strain gauges (13 through 18). On each of the beams A_{8-9}, A_{9-10}, and 9_{A-B}, a set of two strain gauges was installed at a distance of approximately 6-8 in. from the respective face of the column A_9.

A typical placement of strain gauges at any strain gauge location on columns and beams is shown in Figure 7.5. At every strain gauge location on columns, two strain gauges were placed on opposite faces about the axis of major bending and one was placed on either of the other two faces. The gauges were placed on middle longitudinal bars in all faces. Strain gauges installed in this pattern can give strain distribution across depth of the cross-section which is used to accurately estimate loads (axial and bending) carried by the columns. On beams, a set of two strain gauges was installed at every strain gauge location on middle reinforcing bars in the bottom face of the beams, as shown in Figure 7.5 (c).

A set procedure was followed for installation of the strain gauges. First, concrete cover was removed with the help of a Hydraulic Jack Hammer, as shown in Figure 7.6, to expose reinforcing bars for instrumentation. The surface of the reinforcing bars was then prepared for strain gauge bonding. The objective was to obtain a chemically clean smooth
surface having the alkalinity compatible with gauge adhesive and surface roughness appropriate for gauge adhesion. The surface preparation process consisted of five operations listed below in the order of execution. Each of the operations is briefly explained in this chapter. The detailed description of these operations can be found in Vishay-Micro Measurements (2011).

(1) Degreasing. Degreasing is the first operation in surface preparation and is performed to remove surface contaminants such as oil, grease, organic contaminants, and soluble chemical residues. This operation is critical in surface preparation as it prevents surface contaminants to be driven into the surface material by subsequent abrading operation. Degreasing can be accomplished by number of techniques such as use of a hot vapor degreaser, an ultrasonically agitated liquid bath, aerosol-type cleaning solvents, or wiping with GC-6 Isopropyl Alcohol (Vishay Precision group-Micro Measurements, 2010). In the experimental work being described, aerosol-type degreasing solvent (CSM-2 Degreaser) was used to degrease approximately 4-6 in. length of the exposed bar surface. Prior to applying degreasing solvent, soil, dust and other loose contaminants were removed from the surface by brushing with a clean, dry brush.

(2) Surface Abrading. In preparation for strain gauge installation, surface abrading is generally performed to remove loosely bonded adherents (such as scale, rust, paint, galvanized coatings, oxides, etc.), and to develop a surface texture that is suitable for bonding. In this experiment, after degreasing, rebar surface was descaled and smoothened over a length of approximately 3 in. using disc grinder with abrasive grinding discs and wire brush wheels. Finish abrading (wet abrading)
was then done with 220-grit wet-or-dry silicon-carbide paper (SCP-1) while keeping the surface wet with mildly acidic solution (M-prep Conditioner A). Wet abrading procedure was repeated with 320-grit (SCP-2) and 400-grit (SCP-3) silicon carbide papers.

(3) Gauge-Location Layout Lines. Gauge layout lines are a pair of crossed reference line drawn on the test surface to accurately locate and orientate a strain gage at a point where strain measurement is to be made. These lines are made perpendicular to one another, with one line oriented in the direction of strain measurement. The gauge is then installed so that the triangular index marks defining the longitudinal and transverse axes of the grid are aligned with the reference lines on the test surface.

(4) Surface Conditioning. After layout lines were marked, gauge installation area was scrubbed repeatedly with water based acidic surface cleaner (M-Prep Conditioner A) and cotton tipped applicator until surface is properly cleaned.

(5) Neutralizing. Neutralization is the final step in surface preparation that neutralizes all traces of conditioner applied in previous operations and brings the surface conditions back to an optimum alkalinity which is suitable for gauge adhesives. In our experiment, this operation was performed by liberally applying water based alkaline surface cleaner (M- Prep Neutralizer 5A) to cleaned area and scrubbing the surface with cotton applicator. After cleaning and drying, the surface achieved an alkalinity of 7.0 to 7.5pH, which was suitable for the adhesive being used for strain gauge bonding in this experiment.
After appropriately preparing the rebar surfaces, strain gauges were installed using a catalyst solution (200 Catalyst-C) and adhesive (M-Bond 200 Adhesive). Detailed procedure for strain gauge application is presented in Instruction Bulletin B-127-14 (Vishay Precision group-Micro Measurements, 2011). At any point of strain measurement, surface preparation and strain gauge placement were done in one go without any time gap between these two operations. A view of the strain gauge installed on rebar surface is shown in Figure 7.6(a). After securely bonding, strain gauges were covered with protective coating (M-Coat FB-2) to provide protection from environmental and mechanical damages. The complete procedure for application of protective coating can be found in Instruction Bulletin B-134-5 (Vishay Precision group-Micro Measurements, 2010). All strain gauges were connected to a data acquisition system using 3-conductor lead wires (326-DFV) for recording measurements.

In addition to 18 strain gauges, two linear displacement sensors, of the type shown in Figure 7.3 (Vishay Precision group-Micro Measurements, 2011), were also installed on the test building. These sensors have total displacement range of 4 in. and were used to measure vertical displacement of the joint A9. These were installed on either sides of the column being removed as shown in Figure 7.4. The displacement sensors were secured in place under bottom face of the beams A_{8,9} and A_{9,10} with the help of an independent mounting assembly that was detached from the test building. A close up view of the displacement sensor installed under the beam is presented in Figure 7.7(b). After installation, both displacement sensors were connected to data acquisition system with the help of 4-conductor cable for recording displacement measurements during the experiment.
7.3.3 Column Removal

For progressive collapse analysis of the framed structure, GSA (2003) requires structural analyses under several column loss scenarios, each with one column loss. In one of these scenarios, the structure must be analyzed for instantaneous loss of a first-story exterior column located at or near the middle of the long side of the building. Consistent with GSA analysis guidelines, first story column A₉ was physically removed in our experiment from the east side exterior frame of the test building. The column A₉ is marked with a red cross in Figure 7.4. The column was removed in a short period of time to simulate instantaneous loss as required by GSA (2003) guidelines. The column was cut at its mid-height with the help of heavy duty excavator fitted with hydraulic shear. The hydraulic shear is an attachment commonly used with excavators for demolition of the structures. It consists of two jaws that are hydraulically operated to close in and pinch/crush reinforced concrete and steel members. Figure 7.8 shows the column being removed with the help of this equipment. The Figure 7.8 (b) also shows a close-up view of the test building after column is completely cut. The column was removed after all instrumentation was completed and connection/cables were secured.

7.3.4 Data Recording and Reduction

Prior to the column removal, instrumentation of selected frame members was completed and all strain gauges and displacement sensors were connected to a 20-channel data acquisition system for collecting and recording test data. The data acquisition system (System 5000), used in this experiment consisted of a scanner (Model 5100B), software (StrainSmart, Version 3.10), and a laptop computer and is shown in Figure 7.3(c). In this
system, scanner is used to acquire data and digitize analog inputs while personal computer and system’s software are used for data recording, reduction, display and storage. The scanner can acquire test data within 1 millisecond from all channels at the scan interval as short as 0.02 seconds (Vishay Precision group-Micro Measurements, 2011).

A data recording station was established approximately 150 ft away from the test location. The wires and cables from 18 strain gauges and two LVDTs were run to this station and connected to channel input connectors (Nine-pin-D-sub-style) of strain gauge cards (Model 5110A) on the scanner. The scanner was then connected to laptop computer with 16-bit interface card. After the data recording setup was completed, column was removed and readings from all strain gauges and two displacement sensors were recorded during and after column removal process. The data was recorded at the scan interval of 0.1 second, i.e., 10 readings per second per sensor. The recorded test data was then converted in MS Excel format using StrainSmart (Vishay Micro-measurement) software for further interpretation and analyses.

7.4 Test Results

The strain and displacement data recorded by individual strain gauges and displacement sensors is provided in Appendix B. In this chapter, measured data for each frame members is presented and analyzed to explain response of the test building after column removal. Figures 7.9 shows displacement sensors readings while Figures 7.10 through 7.16 present strain gauge readings for various frame members. Displacements were measured in mm units (plotted in inches in Figure 7.10) and strains were recorded in
micro-strain values. A positive strain value indicates measured tensile strain (elongation) while negative reading indicates compressive strains (contraction). Prior to column removal, all sensors (except strain gauges 3 and 14) recorded zero readings as expected. From time $t = 61$ sec to $t = 66$ sec, unstable readings with abrupt strain changes were recorded consistently in almost all channels. During this time period, column was being removed and load redistribution was taking place. At $t = 66$ sec, the loads redistribution had completed and frame members were carrying additional loads due to column loss. This is shown by range of stabilized non-zero strain readings recorded from 66 sec to 82 sec. The second period of unstable/disturbed strain readings from $t = 82$ sec to $t = 88$ sec was obtained when the operator of the demolition equipment approached the column for the second time to make the cut cleaner and bigger. After, the shear jaws of the machine disengaged from the column, the readings stabilized once again at the same values recorded earlier after the first cut was made.

As it can be seen in Figures 7.10 and 7.14, strain gauges 3 and 14 did not record zero initial strains before column removal, however showed same pattern of readings as recorded from other strain gauges. It appears that these gauges did not initialize at zero strain value in the start, but recorded accurate data during the experiment. The strain readings from both gauges show an approximately constant offset throughout their range of recording, which if adjusted will result in the same strain variations recorded from other strain gauges. Hence, for the purpose of analyses and comparison between computational and experimental results, readings from strain gauges 3 and 14 will be adjusted by subtracting respective initial non-zero strain offsets from recorded strain values of both strain gauges.
Figure 7.9 shows recoded data from two LVDTs installed on either sides of the removed column under joint A9. A very small displacement of less than 0.04 in. was recorded by both displacement sensors. Data from strain gauges (1, 2 and 3) installed on column B9 is presented in Figure 7.10. After column removal, a small change in compressive strain is recorded and all strain gauges read roughly same strain values. It indicates that very small loads are transferred in the transverse direction and columns are not subjected to significant bending. The change in compressive strains is predominantly caused by axial loads, and not by column bending. Figure 7.11 shows the strain readings from strain gauges (4, 5 and 6) installed at two-third height on column A8. Strain gauges 4 and 6 were installed on opposite faces of the column in North-South directions while strain gauge 5 was attached on the interior face of the column in transverse frame direction. All strain gauges recorded negative strain values in the range of 64 – 94 microstrains after removal of the column A9. Strain gauge 6 recorded relatively larger compression as compared to strain gauge 4, indicating that column A8 experienced some bending in longitudinal frame direction. However, as the difference between recorded compressive strains on opposite faces of the column is small (approximately 30 microstrains), the bending moment that column A8 experienced after removal of column was not very significant. The recording of negative/compressive strains of notable magnitude by all strain gauges also indicates that column A8 was subjected to significant additional axial loads after removal of column A9. Same conclusions are drawn from the recorded strains at the mid-height of the column, as shown in Figure 7.12.

Figure 7.13 presents strain readings from strain gauges (10, 11 and 12) installed at mid-height of the column A10. The recorded strains for this column show very similar
pattern as observed for column A_8. The column behaves in identical manner as that of column A_8 and is subjected to significant additional axial load after loss of column A_9. The change in bending moment during redistribution of loads is small as shown by relatively small difference in recorded compressive strains by strain gages 10 and 12 installed on the opposite faces of the column in the direction of bending.

Figures 7.14 through 7.16 show the recorded strains for three beams (A_8-9, A_9-10, and 9_{A-B}) framing in joint A_9. The strain gauges on all beams were installed on bottom reinforcing bars at their ends approximately 6-8 in. away from respective faces of the column A_9. As expected, the strain readings for beams A_8-9 and A_9-10, as shown in Figures 7.14 and 7.16, show bottom face of the beams in tension. The large values of the recorded strains indicate that beams are carrying large loads after loss of intermediate support and reinforcing bars may have experienced flexural yielding. Figure 7.15 presents the strain readings for transverse beam 9_{A-B}. The recorded strain values for this beam are very small as compared to the recorded strains for beams A_8-9 and A_9-10, indicating again that load redistribution in the transverse direction is not very significant.
Figure 7.1. Plan view of Johnston Laboratory showing structural members involved in testing

(a) Before testing/demolition                        (b) After brick wall/façade removed

Figure 7.2. East side exterior view of the Johnston Laboratory before testing and after removal of exterior wall
(a) Strain gauge (CEA-06-250UW-120/P2)

(b) Linear Displacement Sensor (Model HS-100 Micro-Measurements)

(c) Data Acquisition System (System 5000)

Figure 7.3. Instruments used for testing of Johnston Laboratory
Figure 7.4. Location of strain gauges and displacement sensors on Johnston Laboratory
(a) Column bending about y-axis

(b) Column bending about z-axis

(c) Strain gauge placement on beams

Figure 7.5. Strain gauges placement at typical section of columns and beams
Figure 7.6. Concrete cover being removed with the help of hydraulic jack hammer

Figure 7.7. Typical instrumentation of the Johnston Laboratory
Figure 7.8. Column removal process in the test building

Figure 7.9. Displacement measured with the help of LVDTs near column joint A₉

Figure 7.10. Strain gauge readings for column B₉
Figure 7.11. Strain gauge readings for column A8 (2/3 of height)

Figure 7.12. Strain gauge readings for column A8 (mid-height)

Figure 7.13. Strain gauge readings for column A10
Figure 7.14. Strain gauge readings for Beam A9-10

Figure 7.15. Strain gauge readings for Beam 9A-B

Figure 7.16. Strain gauge readings for Beam A8-9
CHAPTER 8

PROGRESSIVE COLLAPSE SIMULATION OF TEST BUILDING

8.1 Introduction

Johnston Laboratory was tested to study redistribution of gravity loads and its progressive collapse response before it was demolished in December 2011. The building was tested by physically removing one first-story exterior column. Structural response of the building was monitored by recording strains and displacements of selected frame members in the vicinity of the removed column. Test building along with properties of its structural member and experimental procedure were described in Chapters 6 and 7, respectively. In this chapter, details of the computational analysis, general design and analysis procedures for progressive collapse, loading and modeling procedures of test building, and acceptance criteria are presented. The details of the 2-D and 3-D models of the test building and results of the linear static and nonlinear dynamic analyses are also provided.

8.2 Design Standards and Approaches for Progressive Collapse

Following the events of catastrophic structural failures in the recent past, progressive collapse has become a subject of increasing interest for structural engineers. 

185
As a result of escalating occurrences and vulnerabilities, the prevention of progressive collapse is becoming a requirement in building design and analysis. Current design guidelines recommend adequate structural redundancy, integrity, continuity, ductility and appropriate path for load redistribution to prevent or minimize risk progressive collapse.

### 8.2.1 Design Approaches for Progressive Collapse

American Society of Civil Engineers (ASCE 7-05, 2005) defines two general design methods for minimizing progressive collapse risk. These are indirect design method and direct design method. The indirect design approach attempts to prevent progressive collapse indirectly through provision of minimum levels of strength, continuity, and ductility. The direct design approach, on the other hand, explicitly considers resistance of a structure to progressive collapse during design process. The direct design approach is implemented through two methods; the specific local resistance method and the alternate load path method. The specific local resistance method seeks to provide strength to resist progressive collapse while the alternate load path method seeks to provide alternative load paths to redistribute loads and stop and contain localized damage.

In the alternate path method, the design allows local failure to occur, but seeks to prevent major collapse by providing alternate load transfer paths. After a structural member fails, loads are transferred to the adjoining members. If the adjacent members have sufficient capacity and ductility, the structural system develops alternate load paths. In this method, a building is analyzed by instantly removing one or more load bearing elements from the building, and by evaluating the capacity of the remaining structure to
prevent subsequent damage. This is a threat independent method and analysis is valid for any type of the hazard causing member loss.

8.2.2 Design Guidelines for Progressive Collapse

The notable building codes, standards and design guidelines on progressive collapse include U.S. General Services Administration (GSA, 2003) and the Department of Defense (DoD, 2005), National Institute of Standards and Technology (NIST, 2005), American Society of Civil Engineering (ASCE 7-05, 2005), and American Concrete Institute (ACI 318-08, 2011). The analysis and design guidelines issued by GSA (2003) and DoD (2005) constitute most comprehensive set of instructions on progressive collapse in the U.S. These guidelines explicitly address progressive collapse mitigation and provide quantifiable and enforceable procedures to resist progressive (Humay et al., 2006).

The DoD (2005) guideline describes the procedure to analyze and design building structures to resist progressive collapse. The guideline can be applied to reinforced concrete, steel, masonry, and wood structures. It follows a combination of direct and indirect design approaches depending upon the required level of protection for the structure. It uses indirect design for low levels of protection, and both indirect and direct design (alternate path) for medium and high levels of protection.

The GSA (2003) guideline was specifically prepared to ensure that the potential for progressive collapse is addressed in the design, planning, and construction of new buildings and major modernization projects. The guidelines are intended to prevent widespread collapse after a local failure has occurred in the structure. It follows threat
independent approach in which progressive collapse analysis is performed by implementing alternate path method of design. The GSA guidelines can be used for minimizing the progressive collapse risk during design of new buildings and evaluating the potential for progressive collapse in existing buildings. In this study, GSA (2003) guidelines are used to assess the progressive collapse potential of the test building.

8.2.3 Analysis Procedures for Progressive Collapse

Four analytical procedures can be used to investigate the progressive collapse behavior of the structures. These procedures are Linear Static (LS), Nonlinear Static (NLS), Linear Dynamic (LD), and Nonlinear Dynamic (NLD) (Marjanishvili, 2004; Marjanishvili and Agnew, 2006; McKay, 2008; Powell, 2005).

The primary method of analysis in the GSA guidelines is the linear static analysis. In general, it is the most simplified of the four procedures, and thus the analysis can be completed quickly and conveniently. However, linear static analysis does not capture dynamic effects and material nonlinearity in the structure caused by sudden loss of one or more members (Kaewkulchhai and Williamson, 2003). The analysis is run under the assumptions that the structure undergoes small deformations and that the materials respond in a linear elastic fashion. The procedure is limited to simple and low- to medium-rise structures (less than ten stories) with predictable behavior (GSA, 2003).

In a nonlinear static procedure, geometric and material nonlinear behavior are considered during the analysis. This procedure is a step above the linear static procedure because structural members are allowed to undergo nonlinear behavior during the analysis. However, using this analysis for progressive collapse potential might lead to
overly conservative results (Marjanishvili, 2004). Dynamic analysis accounts for inertia, and damping effects. Considering these dynamic parameters, dynamic or time history analysis can be much more complex and time-consuming than static analysis. However, the linear dynamic procedure provides more accurate results, compared with static analysis.

The nonlinear dynamic procedure is the most detailed and thorough method of progressive collapse analysis. This method includes both dynamic effects and potential nonlinear behavior of the progressive collapse phenomenon. Accurate and realistic results can be obtained from nonlinear dynamic analysis, however, it is typically time-consuming approach (Marjanishvili, 2004). The analysis is performed by instantaneously removing a load-bearing member from the already loaded structure and calculating time history of the structure response caused by the loss of that member.

In the research reported in this chapter, progressive collapse potential of an existing building, that was tested earlier, shall be evaluated following GSA (2003) guidelines and using linear static and nonlinear dynamic analysis approaches. Both two- and three-dimensional building models shall be analyzed. The predicted response shall be compared with experimental data and effectiveness of evaluation approaches shall be investigated.

8.3 Analysis Procedure for Test Building

The primary method of analysis in GSA (2003) guideline is the static linear elastic approach. Linear procedures are recommended to be used for low- to medium- rise structures with ten or less stories and typical structural configurations. For atypical
structures and buildings with more than ten stories, the GSA guideline allows the use of detailed nonlinear procedures. The guideline also recommends that three-dimensional analytic models be used to account for potential three-dimensional effects and avoid overly conservative solutions. Nevertheless, two-dimensional models may be used provided that the general response and three-dimensional effects can be adequately accounted for.

8.3.1 Loading Conditions for Analysis

For progressive collapse analysis, GSA recommends a general loading factor to be used for every structural member in the building being evaluated or analyzed. Different loading conditions are applied to different analysis procedures. For linear static analysis of a structure under gravity loading, following load combination is applied as per GSA (2003) guideline,

\[ \text{Load} = 2(DL + 0.25 \times LL) \]  

(8.1)

where DL is the self-weight of the structure (i.e., Dead Load) and LL is live load of the structure. A dynamic amplification factor of 2 is used to account for deceleration effects and simulate dynamic response when using static analysis procedures.

For nonlinear dynamic analysis, the following loading condition is recommended in the GSA guidelines,

\[ \text{Load} = DL + 0.25(LL) \]  

(8.2)

In this research, the live load is assumed zero because the test building was not occupied, and most of the partitions, furniture and other non-structural loads were removed from the building prior to testing and demolition. At the time of testing, the
building frames carried only dead loads due to the weight of various structural members, including walls, concrete slabs, beams, and columns, and some of the leftover non-structural loads.

### 8.3.2 Acceptance Criteria for Progressive Collapse

To evaluate the results of a linear static analysis, the magnitude and distribution of predicted demands are determined by Demand-Capacity-Ratio (DCR). DCR for a structural component is defined as the ratio of the maximum force demand \( D \) determined in the member to its expected ultimate, un-factored capacity \( C \),

\[
DCR = \frac{D}{C}
\]  

For flexure, \( DCR = \frac{M_{\text{max}}}{M_p} \) where \( M_{\text{max}} \) is maximum moment demand of the element calculated from linear static analysis, and \( M_p \) is moment capacity of the element. In this study, the moment capacity of the frame members is calculated through fiber section analysis under given loading conditions, geometric details, and material properties (Chapter 2). If a DCR value is greater than 1.0, theoretically the member has exceeded its ultimate capacity at that location. However, this alone does not signify failure of the structure as long as other members are capable of carrying the forces redistributed after the initial plastic hinge formation or failure.

In order to prevent collapse of the structure, the DCR values for each structural element in reinforced concrete buildings must be less than or equal to the following,

\[
DCR \leq 2.0 \text{ for typical structural configuration}
\]  

(8.4)
The structural elements that have DCR values exceeding the above limits, will not have additional capacity for effectively redistributing loads. Such members are considered failed and can therefore, result in collapse of the entire structure.

The performance evaluation criteria for nonlinear dynamic analysis procedures are based on plastic hinge rotation and displacement ductility. Figure 8.1 shows the measurement of plastic hinge rotation angle, after the formation of plastic hinges caused by column removal. Based on Figure 8.1, plastic hinge rotation angle for beam members on each side of the removed column can be measured between horizontal line and tangent to maximum deflected shape.

\[ \theta = \tan^{-1}\left( \frac{\delta_{\text{max}}}{L} \right) \]  

(8.5)

where \( \theta \) is maximum hinge rotation, \( \delta_{\text{max}} \) is maximum displacement of beam at the location where the column is removed, and \( L \) is beam length or column spacing in the longitudinal direction.

Displacement ductility ratio \( \mu \) is defined as the ratio of maximum displacement to elastic deflection limit, 

\[ \mu = \frac{\delta_{\text{max}}}{\delta_{e}} \]  

(8.6)

where, \( \delta_{\text{max}} \) is maximum displacement of columns or beams at a reference point, which can be calculated from analysis, \( \delta_{e} \) is the elastic deflection limit at that point, which is vertical displacement when the first plastic hinge forms.

Based on the GSA guidelines (GSA 2003, Table 2.1), the acceptance criteria of plastic rotation for reinforced concrete beams is 6 degrees. Displacement ductility for a
reinforced concrete column should not exceed a value of 1.0 at failure after the development of catenary action.

### 8.4 Modeling of Test Building

The test building, Johnston Laboratory, was a four story reinforced concrete frame building with regular structural configuration. The building was located at the Ohio State University (OSU) campus and was tested prior to its scheduled demolition in December 2011. The complete description of the test building is provided in Chapter 6.

Two-dimensional (2-D) and three-dimensional (3-D) models of the test building, Johnston Laboratory, were created using the SAP2000 computer program (SAP2000, 2009). The SAP2000 is a powerful and well known structural analysis and design software that is commonly used for various types of structures and loading conditions. Figures 8.2 and 8.3 show the 2-D and 3-D models of the test building, respectively, developed in SAP2000. Both building models shall be used for linear static and nonlinear dynamic analyses to evaluate progressive collapse response of the test building and comparison of the computational and experimental data. The 2-D models help to investigate the overall perimeter frame response while 3-D models can adequately account for 3-D effects and avoid overly conservative results. Both the DoD and GSA guidelines recommended the use of 3-D models in the progressive collapse analysis of buildings (DoD, 2005; GSA, 2003).
8.4.1 Material Properties and Modeling Details

The test building was an ordinary moment-resisting structure of the regular configuration (Figures 6.4 and 6.5). It consisted of typical reinforced concrete beams, columns, beam-column joints and slabs. The column sizes and reinforcement details changed in every story. The various sizes of the beams were used in all floors. The size of spandrel beams along the longitudinal perimeter frames was significantly larger than the intermediate beams in all floors. The solid slabs having thickness of 6 in. thickness (2nd floor) and 5 in. (3rd floor, 4th floor and roof) were used, except a single bay, in all floors, where ribbed slab with pan-joist construction was used. The complete details of the test building and structural components are provided in Chapter 6 and Appendix A.

For 2-D and 3-D models, the actual dimensions, details and properties of frame members were obtained from the original structural drawings and available design notes. If deviations from standard building drawings were found during the testing, the actually observed details were used for modeling. Special attention was paid in modeling beams, columns and slabs framing at joints. The compressive strength of the concrete, yield strength of the both longitudinal and transverse reinforcing bars, and modulus of elasticity of steel were assumed as 3500 psi, 40,000 psi, and 29,000 ksi, respectively.

8.4.2 Calculation of Loads

At the time of the testing, the building was not occupied and majority of the furniture and laboratory equipment had been removed from the building. No live loads were considered. Only dead loads were modeled that include dead load (self weights) of the individual structural elements (beams, columns and slabs), dead load of the infill
masonry walls, and non-structural loads from some of the leftover laboratory equipment and mechanical pipes/fixtures. A uniform load intensity of 10 psf was assumed to account for non-structural loads. The material densities required for calculating various loads were taken as following.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (lb/ft$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforced concrete</td>
<td>150</td>
</tr>
<tr>
<td>Masonry infill wall</td>
<td>120</td>
</tr>
<tr>
<td>Glass in window panels</td>
<td>160</td>
</tr>
</tbody>
</table>

The dead loads of the structural components (beams and columns) were generated by SAP2000 through material densities and member cross-sections. The dead loads from reinforced concrete slabs, masonry walls and non-structural elements were calculated manually and applied to the model as externally acting gravity loads. The slab was supported on its edges by the beams running in the transverse direction. Therefore, the dead load of the slab and non-structural load intensity was distributed to supporting beams according to tributary areas and one-way slab action. For 2-D model of the longitudinal frame-A (Figure 6.4), loads from slabs, partition walls, and non-structural elements were applied on the beams in the transverse frames. Individual linear static analyses were run for all transverse frames (frame-1 through 13 (Figure 6.4)) and end-actions from transverse frames were applied as point loads on respective joint locations on frame-A. In addition, frame-A carried the loads of window panels on all of its spandrel beams at all floor levels. The calculated dead loads were applied on all beams in both longitudinal and transverse directions at all floor levels of the 3-D model.
8.5 Analyses of the Test Building

Linear static and nonlinear dynamic analyses were performed on both 2-D and 3-D models of the test building following the GSA (2003) guidelines on progressive collapse. GSA load conditions defined by Equations 8.1 and 8.2 were applied for linear static and nonlinear dynamic analyses, respectively. As mentioned earlier, live loads were not considered in the analysis. For linear static analysis, as per guidelines, designated column was deleted from the model, structure was loaded with amplification factor of 2.0 and analysis was run. For nonlinear dynamic analysis, the column removal was simulated by a time history function in a very short interval of time as required by GSA guidelines. No load factors were applied to dead loads in this case.

For linear static analysis, DCR values were calculated (Equation 8.3) for the critical frame members in the vicinity of the removed column and compared with the acceptance criteria (Equation 8.4) specified by GSA (2003). For calculating DCR values, demand and capacity were considered in terms of moments. The load effect ($M_{\text{max}}$) was taken directly from the SAP2000 analyses and moment capacity ($M_p$) of the frame members were calculated through fiber model section analysis. The details of the fiber section analysis are provided in Chapter 2. In defining the fiber model of the cross-section, unconfined cover concrete and confined core concrete were configure separately with their respective stress-strain relationships. The enhancement in strength and ductility of the reinforced concrete due to lateral confinement was also taken into account while calculating ultimate moment capacity of the frame members. The uniaxial material constitutive relations were employed and no strength increase factors as defined by GSA (2003, Table 4.2) were considered.
Figures 8.4 through 8.6 present the results from 2-D linear static analysis. Figure 8.4 shows the bending moment diagram before column removal while Figure 8.5 shows moment diagram after column removal. As it can be seen, the column removal caused moment reversal in the beams intersecting at the removed support. The calculated DCR values for the beams directly affected by column removal are presented in Table 8.1. As seen, no beam DCR value is overshooting the acceptance criteria of 2.0. The beams framing into the removed column support are affected the most and almost reach their failure criteria. Figures 8.7 through 8.10 present the calculated moment-curvature responses for spandrel beams in frame-A at the load levels after removal of the column. These moment-curvature relationships are used for calculating $M_p$ (maximum moment) and DCR values of the critical beams for both 2-D and 3-D linear static analyses.

Figures 8.11 and 8.12, and Table 8.2 present the results from 3-D linear static analysis. Figure 8.11 shows the bending moment diagram of the frame-A after column removal while Figure 8.12 shows moment diagram of transverse frame-9 after column removal. As in 2-D linear static analysis, the column removal caused moment reversal in the second floor beams joining at the removed support. In other words, the beams had negative moments at their ends before the column removal, however the continuous beam experienced a large positive moment at that joint after the column is removed. The frame members in 3-D model are subjected to lesser loads (axial and bending moment) as compared to those of the 2-D model. The calculated DCR values for the beams directly affected by column removal are presented in Table 8.2. All beam DCR values are less than the acceptance criteria of 2.0.
For nonlinear dynamic analyses, element rotations were directly determined from SAP2000 analyses and were compared with GSA acceptance criteria of 0.1 radians (6 degrees). Tables 8.3 and 8.4 present the calculated plastic rotations of the critical frame members in the vicinity of the removed columns. As it can be seen, no member exceeds the allowable limit. The beams in the immediate vicinity of the removed support in the second floor are affected the most. Figures 8.13 and 8.14 show the bending moment diagrams after column removal for 2- and 3-D analyses, respectively. The removal of column causes stress reversal in the beam at the joint location above lost support. The calculated internal forces (moments and axial loads) in frame members in 2-D model are larger than those in 3-D model.

8.6 Comparison of the Computational and Experimental Results

Another important aspect of this study is the comparison of the calculated strain and displacement values with experimental test data. During the field testing of the Johnston Laboratory, strain values at various locations on columns and beams in the vicinity of the removed column were recorded (Section 7.3.2). The recorded data in terms of strains and displacements is very critical in evaluating existing progressive collapse analysis and design techniques and procedures. It must be noted that the data recorded during testing is result of the additional loads arising from the removal of column and do not represent total load effect. Therefore, separate analyses, only under applied loads, must be run prior to running analyses that simulate column removal. The strains must be calculated separately from both analyses and their difference must be compared with the test data.
8.6.1 Procedure for Computation of Strains

As mentioned above, the strains measured in the field are caused by the additional loads arising from redistribution of loads as column support in removed. Strain gauges and LVDTs can only measure the change in the strains (or net strains) between loaded state of members before and after column removal. Hence, in order to compare with recorded strains, the member strains need to be calculated under loading conditions before and after removal of the column.

In this study, member strains are calculated from fiber section analysis and SAP2000 analysis. The procedure is explained below.

1. Run SAP2000 analysis of the building model under applied loading before column removal.
2. At the strain gauge location of the interest, obtain member internal forces (axial load and bending moment) from SAP2000 results.
3. Define fiber model of the cross-section at the location of the strain gauges on the member and determine moment-curvature relationship under axial load obtained from SAP2000 analysis in step 2.
4. From moment-curvature relationship, extract rebar strains at the location that correspond to the actual location of the strain gauge. Plot moment-rebar strain relationship separately for the member being analyzed. An example of such relationship is shown in Figure 8.15.
5. From the moment-rebar strain relationship, obtain the rebar strain corresponding to the moment value obtained in step 2. This is the strain at the location of interest before column removal.
6. Delete the removed column from the building model, run SAP2000 analysis, and repeat step 2 to 5. The strain value calculated in this step corresponds to the strain at the location of interest after column removal.

7. Calculate the difference between strains calculated before and after column removal with due regards to their signs (i.e., compressive or tensile) and compare with the corresponding strain gauge readings.

   It was observed during computational analyses that the columns are subjected to insignificant bending during the experiment, even after removal of the column A9, hence bending effect in columns were ignored. Similarly, beams were found to have negligible axial loads during the analyses, therefore, axial loads were neglected for beams. As bending in the columns was ignored, strains corresponding to the strain locations in columns were determined through axial load-axial strain relationship instead of moment-curvature relationships. An example of such relationship is provided in Figure 8.16.

8.6.2 Comparison of Computed and Experimental Strains

   Figures 8.17 through 8.28 compare the predicted and experimental responses of the frame members after removal of the column. Figures 8.17 through 8.22 compare computed strains from 2-D linear static analyses with experimentally observed strains for members in Frame-A. Computed strains are compared with average of the strain values measured by all strain gauges at that particular location on that specific member. Figures 8.17 through 8.19 show that the predicted strains for columns A8 and A10 are close to the average measured strains in those columns. For beams A8-9 and A9-10, predicted strains are relatively different from average of measured strains for both beams, but still
are within the range of minimum and maximum recorded strains (Figures 8.20 and 8.21). The predicted displacements, however, are not predicted well and are severely underestimated. The predicted displacement are taken directly from SAP2000 analyses and plotted in Figure 8.22.

Figures 8.23 and 8.24 compare the predicted and experimental strains for beam 9A-B and column B9 in the transverse frame, respectively, using 3-D linear static analyses. As it can be seen, the strains are predicted with better accuracy for both frame members, as compared to the predictions using 2-D linear static analyses for members in longitudinal frame.

Figures 8.25 through 8.28 present comparisons between computed strains from nonlinear dynamic analyses with experimentally observed strains. These figures also compare the strains predicted from linear static analyses. The results are presented for columns A8 and A10 and joint displacements. The predicted strains with nonlinear dynamic analyses compare very well with the measured strains for both columns. The predicted displacements are also close to experimentally observed displacements as compared to the displacements predicted through linear static analysis.
Table 8.1. Calculated moment demand and capacity for frame members-2D linear static

<table>
<thead>
<tr>
<th>Elm</th>
<th>2nd Floor (kip-ft)</th>
<th>3rd Floor (kip-ft)</th>
<th>4th Floor (kip-ft)</th>
<th>Roof (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{max}$</td>
<td>$M_p$</td>
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<td>$M_{max}$</td>
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<tr>
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Table 8.2. Calculated moment demand and capacity for frame members-3D linear static

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<th>3rd Floor (kip-ft)</th>
<th>4th Floor (kip-ft)</th>
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<tbody>
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<tr>
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### Table 8.3. Calculated frame member rotations - 2D nonlinear dynamic analysis

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<th>4th Floor (rad)</th>
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</tbody>
</table>

### Table 8.4. Calculated frame member rotations - 3D nonlinear dynamic analysis

<table>
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<th>4th Floor (rad)</th>
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<tr>
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<td>-0.0037</td>
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</tr>
<tr>
<td>A₇₋₈</td>
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<td>-</td>
<td>-</td>
<td>-</td>
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<td>-0.0145</td>
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</table>
Figure 8.1. Calculation of beam rotation (Song, 2010)

Figure 8.2. 2-D model of frame-A in SAP2000
Figure 8.3. 3-D model of test building in SAP2000

Figure 8.4. Moment diagram before column removal
Figure 8.5. Moment diagram after column removal

Figure 8.6. Deflected shape of frame-A after column removal
Figure 8.7. Moment-curvature relationship for spandrel beam in the second floor

Figure 8.8. Moment-curvature relationship for spandrel beam in the third floor
Figure 8.9. Moment-curvature relationship for spandrel beam in the fourth floor

Figure 8.10. Moment-curvature relationship for spandrel beam in roof
Figure 8.11. Moment diagram of Frame-A after column removal

Figure 8.12. Moment diagram of Frame-9 after column removal
Figure 8.13. Moment diagram of Frame-A after column removal in 2-D nonlinear dynamic analysis

Figure 8.14. Moment diagram of Frame-A after column removal in 3-D nonlinear dynamic analysis
Figure 8.15. Moment-Rebar strain relationship for second floor spandrel beams

Figure 8.16. Axial load-Axial strain relationship for columns A_8 and A_{10}
Figure 8.17. Comparison of the measured and computed strains for column A₈ at 2/3 of column height (2D LS analysis)

Figure 8.18. Comparison of the measured and computed strains for column A₈ at mid-height of the column (2D LS analysis)
Figure 8.19. Comparison of the measured and computed strains for column $A_{10}$ (2D LS analysis)

Figure 8.20. Comparison of the measured and computed strains for beam $A_{9-10}$ (2D LS analysis)
Figure 8.21. Comparison of the measured and computed strains for beam $A_{8-9}$
(2D LS analysis)

Figure 8.22. Comparison of the measured and computed displacements
(2D LS analysis)
Figure 8.23. Comparison of the predicted and computed strains for beam $9_{A,B}$
(3D LS analysis)

Figure 8.24. Comparison of the predicted and computed strains for column $B_9$
(3D LS analysis)
Figure 8.25. Comparison of the predicted and computed strains for column A₈ at 2/3 of column height (NLD analysis)

Figure 8.26. Comparison of the predicted and computed strains for column A₈ at mid-height (NLD analysis)
Figure 8.27. Comparison of the predicted and computed strains for column $A_{10}$ (NLD analysis)

Figure 8.28. Comparison of the predicted and computed displacements (NLD analysis)
CHAPTER 9

CONCLUSIONS

9.1 Summary

A significantly large number of reinforced concrete buildings in seismic regions of the world possess structural deficiencies in their design, details and construction and are vulnerable to excessive damage or collapse during future earthquakes. These structures have very limited capacity beyond their limited elastic range to absorb and dissipate the energy of strong ground motions. They often fail in unwanted/undesirable non-ductile manner. In order to assess the vulnerability of an existing structure to earthquake damage and hence suggest desired level of retrofit, it is imperative to first evaluate expected behavior of the structure in terms of strength and deformation capacities, progression of the damage, and collapse mechanisms. This requires determination of load-deformation response of the individual structural elements, such as beams and columns, from initial loading to final collapse considering all potential failure mechanisms associated with axial, flexure and shear behavior.

Post-earthquake observations and research on performance of non-seismically designed buildings reveal that columns, especially those in the first story, are often the most critical members, and their damage is primary cause for significant structural
damage or collapse. Typically, columns in non-ductile buildings have low lateral displacement capacities and undergo rapid degradation of shear strength and axial load carrying capacity under earthquake loading. Such column behavior was primary focus of the research presented in this study.

The focus of research on modeling column behavior was to suggest a suitable analytical procedure that can accurately predict the monotonic lateral force-displacement relationship for reinforced concrete columns by overcoming limitations of available models. An analytical model was proposed to predict lateral load-displacement response of reinforced concrete columns. The proposed model is continuation of author’s previous work on modeling of column behavior (Lodhi, 2010) and was presented to better represent shear behavior and improve post-peak response estimation. It is a displacement-based, macro model evaluation approach developed considering compatibility of strains, equilibrium of average stresses, and interaction between flexure and shear mechanisms.

The proposed model consists of two sub-models, axial-flexure and axial-shear models, simultaneously evaluating flexural and shear behavior of the column element between inflection point and one of the end sections. The flexural deformations are calculated through fiber section analysis considering cracked concrete behavior while shear response is evaluated based on the Disturbed Stress Field Model (DSFM). At any load stage, both sub-models are coupled through axial deformation interaction and softening of concrete compression response. Considering compatibility and equilibrium conditions in average stress-strain field, axial-flexure and axial-shear models are reduced to one-element model under combined axial, flexure and shear loading. The lateral load carried by the column at any given displacement is obtained by analyzing reduced one-
element model for in-plane stress conditions while considering interaction between axial, flexural and shear mechanisms. Total displacement at any load level is sum of flexural, shear and reinforcement slip displacements.

The DSFM provides an effective approach for describing behavior of cracked reinforced concrete elements. In the proposed model, the DSFM is explicitly employed for modeling both pre- and post-peak shear behavior, and thus allows for investigating shear behavior based on the physical mechanics rather than empirical relations. Its implementation into proposed model refines modeling of the shear behavior and improves overall prediction of column strength, load-displacement relationship, and failure modes.

The proposed model also considers number of effects that improve overall response prediction. These include consideration of reinforcement slip deformations, buckling of compression bars under excessive compressive strains, enhancement in concrete strength due to confinement, softening of the cracked concrete in compression, modeling of concrete tensile behavior through tension stiffening and tension softening, variable crack spacing, flexibility of material constitutive models, and detailed modeling and interaction of failure modes.

The solution strategy follows iterative, secant stiffness formulations. The analysis procedure is displacement-controlled and starts with assumptions of initial values for displacement variables. These variables include strain at section centroid and curvature for axial-flexure element, and total normal and shear strains for axial-shear element. At any load step, lateral load at given displacement is calculated based on compatibility of axial strains, equilibrium of axial and shear stresses, and interaction between axial strain
due to flexure and concrete compression softening. At that load step, total displacement is summation of flexural, shear and reinforcement slip displacements. Within each iteration, when equilibrium conditions are satisfied, shear stress converted from moment in axial-flexure model and shear stress in axial-shear model must have the same value. This is the shear stress acting on the element at displacement level being considered and lateral load can be calculated by multiplying shear stress with cross-section area.

The analysis procedure generates number of parameters during the analysis that are used to monitor the type of failure. In addition to considering the possibility of concrete crushing at extreme compression fiber or rupture of steel, the model explicitly determines failure forms in shear. These failure conditions include shear failure at crack, failure due to loss of compression strength in axial-shear model due to high shear force, and compression-shear failure that occurs due principal compressive strains within the element reaching peak value of the strain in compression.

The proposed analytical procedure was implemented to model the behavior of previously tested column specimens and predicted responses were compared with test data. Consistent correlations were achieved for analytical and experimental results. The model was generally able to successfully capture pre-peak response with accurate predictions of stiffness and peak strength. For post-peak responses, the model accurately captured the observed failure modes and showed strength degradation after detecting shear failure, however, degree of strength degradation in some cases was found to be less than the observed behavior.

The study on progressive collapse of reinforced concrete frame building consisted of experimental and computational research. During the experimental phase of research,
an existing reinforced concrete building, located at the Ohio State University campus, was tested. The goal was to investigate redistribution of the gravity loads and model progressive collapse mechanism as one of the critical load carrying element is lost. In this testing, one first-story exterior column was physically removed. Structural response of the building was monitored by recording strains and displacements of selected frame members in the vicinity of removed column. The recorded test data and experimental observations add to the existing knowledge on progressive collapse and can be utilized to validate various levels of evaluation and modeling techniques employed for prediction of progressive collapse response of the buildings.

As part of the computational research, following the General Services Administration (GSA) guidelines, two- and three-dimensional computer models of the test building were generated using SAP-2000 computer program to analyze and compare the progressive collapse response. Two different analysis procedures, linear static and nonlinear dynamic, were performed and evaluated for their effectiveness in modeling progressive collapse scenarios. The demand capacity ratio (DCR) and plastic hinge rotation were determined from linear static and nonlinear dynamic analysis procedures, respectively. These parameters were used to check the performance acceptance criteria for each analysis. The results from computer models were compared with the strain and displacement values recorded during field testing.

Despite many previous studies on computational models and analysis to improve the building design against progressive collapse, there is very little actual field data available to evaluate and verify progressive collapse resistance of structures. Therefore, the field experiments and SAP2000 analyses performed in this research provide both
practical and fundamental information on the progressive collapse response of existing building

9.2 Conclusions

1. The proposed model is a suitable analytical procedure that can accurately predict the monotonic lateral force-displacement relationship for reinforced concrete columns. Although, this study focuses on modeling of non-ductile columns with poor seismic details, the proposed analytical procedure is equally applicable to predict structural response of the columns exhibiting different failure modes, e.g., flexural failure with very limited or no shear effects, flexure and/or shear failure following the flexural yielding, and brittle shear failure prior to flexural yielding etc.

2. Implementation of DSFM for modeling shear behavior of the column is one of the new contribution and major advancement relative to the previously developed model by the author. The newly proposed formulations successfully addresses the weaknesses and limitations found in earlier column modeling approach (Lodhi, 2010). Particularly, it eliminates reduced accuracy that might be encountered under certain loading and reinforcement conditions due inherent weaknesses of the model (i.e., Modified Compression Field Theory, MCFT) employed for modeling shear behavior.

3. The DSFM is an extension and improvement of MCFT that provides an effective approach for describing behavior of cracked reinforced concrete elements. Its implementation into proposed model refines modeling of shear behavior and improves overall prediction of column strength, load-displacement relationship, and failure modes.
4. The proposed model allows for the accurate representation of shear behavior into a procedure that can simultaneously evaluate column response under axial and flexural loadings as well, while considering interactions between these mechanisms. The model also enables investigation of post-peak behavior based on physical mechanics rather than empirical formulations or formulations requiring an estimate of column shear strength to determine expected behavior. This gives proposed model the ability to explicitly detect initiation of the shear failure and capture subsequent strength degradation as lateral deformations increase.

5. The proposed model presents a suitable analytical procedure that not only accurately estimates load-displacement relationship but also provides better representation of internal mechanisms responsible for shear failure and post-peak strength degradation. At every load stage, the proposed model quantifies number of parameters such as average stresses and strains in concrete, average stresses and strains in reinforcement, local stresses and strains in reinforcement at cracks interfaces, crack angles, average crack widths, and shear stresses at cracks etc. These parameters help to evaluate dominant mechanism (shear or flexure) and determine damage or failure mode as loading proceeds. These internal variables can also be very helpful in conducting further research on other aspects of the column behavior such as loss of axial load capacity and related deformations, or develop simplified procedures for column repose estimation.

6. Successful implementation of the proposed analytical procedure indicates that DSFM can be employed within framework of compatibility and equilibrium
conditions, and interaction methodology of ASFI approach (Mostafaei and Kabeyasawa, 2007).

7. The modeling of flexural behavior through fiber section model approach successfully integrates with DSFM approach within overall analytical procedure and does not cause any issues on convergence of variables and numerical stability of entire calculation procedure. Same is true for the model by Sezen and Setzler (2008), which was employed for calculating reinforcement slip deformations. This model represented the experimental slip behavior well and can be used to model the reinforcement slip response with least complexity and relatively good accuracy.

8. The interaction between flexure and shear deformations is an important consideration in determining overall column behavior under lateral load, and therefore must be considered in the analysis procedure. The interaction methodology employed in this study successfully works with formulations and solution strategy of DSFM.

9. Buckling of compression bars affect post peak behavior and hence must be considered in the analysis. The procedure adopted in this research for incorporating this phenomenon worked well and is recommended to be used for the analysis.

10. Moment-curvature analysis employing fiber model approach while considering concrete behavior in tension, compression-softening factor, and buckling of compression bars, and the plastic hinge method of calculating flexural deformations worked well in this research. These tools appear to be appropriate for use in both well reinforced and lightly reinforced concrete columns.

11. The implementation procedure is straightforward and calculation process is robust. The proposed procedure is not suitable for hand calculations as it is iterative
and requires loops of calculations, however, can easily be implemented through user defined routine with any appropriate computer programming language by following analytical steps and solution strategy presented in this study.

12. The comparison of the predicted responses of the test columns with observed experimental responses indicated that the proposed model is a suitable displacement-based evaluation approach that can predict lateral load-displacement relationships with reasonably good accuracy. The model also well captures other aspects of lateral response such as ultimate load, ultimate displacement, and failure modes.

### 9.3 Recommendations for Future Work

#### 9.3.1 Modeling of Column Behavior under Lateral Load

1. The research reported here presents a model for the monotonic lateral deformation of lightly reinforced concrete columns subjected to lateral loads. However, there remains a large amount of work to be completed before the behavior of non-ductile concrete structures in earthquakes can be fully modeled and understood. Several recommendations for continued research on this topic follow.

2. The proposed model was used to predict the behavior of limited number of test columns. While general good agreement was shown between the test data and model, further comparisons should be made to complete validation of the model over a wide range of column properties.

3. Further work is needed to model loss of axial load capacity of the column after column experiences shear failure. This can be done by extending the work presented in this research and studying internal mechanisms and responses in post-peak regime,
captured by the proposed model. Similarly, based on the detailed analytical procedure of the proposed model, simplified procedures, that can be implemented easily in hand calculations, can be formulated for modeling column behavior.

4. Several areas warrant more experimental research to improve the knowledge base. Particularly, more testing of reinforced concrete columns to axial load failure is necessary. Most test data available stops before loss of axial capacity. More data would help improve models for shear capacity and axial capacity; both of these are necessary components of the proposed model. Additionally, more test specimens should be instrumented to monitor the individual displacement components, particularly shear displacements. This would aid in improving the component models, which should lead to improvements in the total model as well.

5. The most significant area of research that remains is the modeling of the hysteretic behavior of columns. Many hysteretic models for flexural deformations exist, which should be applicable. Several shear models are available as well, however, these would need to be evaluated in the context of lightly reinforced columns to determine their applicability. These hysteretic models would need to be combined to predict the overall cyclic lateral behavior of lightly reinforced concrete columns.

6. The effects of strain offsets in concrete and reinforcement must be incorporated into the proposed model. These strain offsets can be elastic and/or plastic. The elastic strains offsets include effects of thermal expansion, poisson’s effect, and shrinkage. The plastic strain offsets arise from cyclic loading conditions or loading into post-peak levels. These strain offsets can easily be handled in proposed procedure based on similar procedure employed for handling of strains due to crack shear-slip. The crack-
shear slip strains are also treated as a strain offset in proposed procedure and do not contribute to the secant stiffness formulations. There effect is considered through pseudo stress vector while relating total strains to total stresses. However, work is required to quantify elastic and plastic strain offsets through available models before these can be implemented into the proposed model.

9.3.2 Progressive Collapse Research

1. Results of this research suggest that computational simulation of progressive collapse is relatively simple to perform through SAP2000. The computational simulations must also be performed with other available finite-element computer programs such as ELS and results must be compared.

2. More experimental data would be required to validate computational analysis for the progressive collapse, and better simulate the actual behavior of structural members.

3. Installation of additional strain gauges in the upper floors would be useful to monitor changes in strains of columns and beams as the building floor gets higher. Strain data measured at each floor level would be very valuable to determine the alternate load path caused by the loss of load-bearing columns.

4. This research only considered the removal of one exterior frame column to evaluate progressive collapse potential. More column removal scenarios must be tested as suggested by GSA (2003) and DoD (2005) guidelines. In addition, as removal of interior columns can also make a structure susceptible to progressive collapse, therefore, it would be important to assess the potential of progressive collapse when interior frame columns
of the building are removed. Both field tests and SAP2000 analysis can be performed to investigate the structure’s response resulting from the loss of interior column(s).

5. In this research, a reinforced concrete frame building with regular structural configuration was tested. It would be helpful to evaluate the progressive collapse potential of other buildings with different structural systems and configurations. The experimental and computational assessments on progressive collapse potential of various buildings would enable making more specific conclusions for a wide range of building structures.

6. In this research, linear static and nonlinear dynamic analyses were conducted using SAP2000 computer program. It would be interesting to examine nonlinear static and linear dynamic analyses, and compare the analysis results. The effect of nonlinearity or the dynamic effect on the progressive collapse analysis could be examined by comparing the results from the additional two analysis options as well. It is also expected that this comparison can prove to be the most accurate and suitable method to analyze the progressive collapse vulnerability of the buildings.
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APPENDIX A: STRUCTURAL DRAWINGS AND NOTES OF THE JOHNSTON LABORATORY
Figure A.1. Second floor framing plan of the Johnston Laboratory
Figure A.2. Third and fourth floor framing plan of the Johnston Laboratory.
Figure A.3. Roof slab framing plan of the Johnston Laboratory
Figure A.4. Remodeling drawings of the Johnston Laboratory, third floor equipment relocation and demolition plan.
Figure A.5. Remodeling drawings of the Johnston Laboratory, fourth floor equipment relocation and demolition plan.
Figure A.6. Remodeling drawings of the Johnston Laboratory. Existing first and second floor plans—new construction.
Figure A.7. Remodeling drawings of the Johnston Laboratory, third floor plan – new construction
Figure A.8. Remodeling drawings of the Johnston Laboratory, fourth floor plan – new construction.
Figure A.9. Remodeling drawings of the Johnston Laboratory, third and fourth floors -new equipment plan
Figure A.10. Remodeling drawings of the Johnston Laboratory — Structural notes
APPENDIX B: RECORDED TEST DATA
Figure B.1. Recorded displacement readings from LVDT-1

Figure B.2. Recorded displacement readings from LVDT-2

Figure B.3. Recorded strains from strain gauge 1
Figure B.4. Recorded strains from strain gauge 2

Figure B.5. Recorded strains from strain gauge 3

Figure B.6. Recorded strains from strain gauge 4
Figure B.7. Recorded strains from strain gauge 5

Figure B.8. Recorded strains from strain gauge 6

Figure B.9. Recorded strains from strain gauge 7
Figure B.10. Recorded strains from strain gauge 8

Figure B.11. Recorded strains from strain gauge 9

Figure B.12. Recorded strains from strain gauge 10
Figure B.13. Recorded strains from strain gauge 11

Figure B.14. Recorded strains from strain gauge 12

Figure B.15. Recorded strains from strain gauge 13
Figure B.16. Recorded strains from strain gauge 14

Figure B.17. Recorded strains from strain gauge 15

Figure B.18. Recorded strains from strain gauge 16
Figure B.19. Recorded strains from strain gauge 17

Figure B.20. Recorded strains from strain gauge 18