PROGRESSIVE COLLAPSE: SIMPLIFIED ANALYSIS
USING EXPERIMENTAL DATA

Thesis

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ABSTRACT

A structure experiences progressive collapse when the following conditions occur: a primary structural member fails due to manmade or natural causes, the loads from the lost member are transferred to adjoining members, adjoining members fail due to the redistributed loads, and the process repeats until a disproportionate amount of the structure is damaged or collapses. This study investigates methods of collecting data from a test structure, modeling a structure, performing analysis, and ultimately developing a process to create a simplified model for engineers to use for fast and easy progressive collapse analysis of a structure.

In this research, a reinforced concrete structure is tested and modeled for analysis. The building consisted of three above ground floors and a basement in a regular rectangular layout, thick reinforced concrete slabs with no beams on the lower floors, and circular columns with drop panels. 14 strain gauges were placed strategically on three of the first story columns near the same corner of the building prior to its demolition. Strain data was collected and monitored from the gauges during the demolition process where three external columns, including the corner column, were removed by a processor. This was done to simulate a multiple column loss triggering event for progressive collapse.

Using the SAP2000 program, a detailed model is developed to represent the building and its behavior as accurately as possible. Various factors such as the loading conditions, development of structural elements with proper dimensions, and end constraints are modeled according to the building plans. Analysis is performed on the model and compared to the test data to verify that the model accurately represents the
measured behavior of the structure. Various simplifications are then made to the detailed model including a reduction in the number of floors, a reduction in the number of bays, and replacing the thick slabs with equivalent beams. A series of sensitivity analyses are performed to investigate and justify all simplification steps made to the models. The test data is compared to the calculated results from the models after each major simplification is made to ensure relative accuracy is maintained. Ultimately, a procedure for creating a final simplified model consisting of only a few structural elements is developed. A spring model is also developed from the final simplified model which can be used in future research to perfect the proposed model.

The research results provide valuable information to the study of progressive collapse as the behavior and response of a reinforced concrete structure subjected to multiple column losses is investigated and discussed at length. The proposed procedure and model are suggested for use by engineers for the quick and simple check of a structure’s ability to resist progressive collapse. The final simplified model consists of only a few frame elements and is developed for general use of analyzing different types of building structures.
DEDICATION

Dedicated to my parents
ACKNOWLEDGEMENTS

First I would like to thank my parents, Dan Morone and Lisa Morone, for the constant support and guidance they have given me all of my life. Without them I may have never had the personal drive to do everything to the best of my abilities and reach for my highest goals. They have had a huge effect on the making me the person I am proud to be today and they mean everything to me. Thank you so much Mom and Dad!

Next I want to thank all of my family and friends who have also been there to support me in the toughest of times. In particular, my Grandpa has always been by my side to help whenever he can and I will always be thankful for everything he has done for me. It is difficult to imagine where I would be without the support that all of my family and friends have given me. From academia to personal matters, I have learned a lot from them and I will never forget how they have helped me get where I am.

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Greg Ullom deserves to be thanked for his involvement in this project as he discovered the test location and was able to get the demolition company to help us. He
also ran the computer data collection system and helped with setting up the sensors. He is taking a different direction with the same data for progressive collapse research and I wish him the best of luck. I would also like to thank Kevin Giriunas and Brian Song for their guidance and previous work with this area of research in effort with Dr. Sezen.

Finally I would like to thank my advisor, Dr. Halil Sezen, for guiding me through the process of conducting research and sharing his wisdom of structural engineering to make better engineers of all of us in his classes. He has spent many hours discussing my research with me and has truly played a major role in my education and has had a great effect on my college career.
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CHAPTER 1

INTRODUCTION

1.1 Problem Statement

The term ‘progressive collapse’ can be defined as total collapse, or proportionately large failure, of a structure due to the spread of a local failure from element to element throughout the structure. Progressive collapse can be triggered by manmade or natural hazards. Fire, blast, earthquake, or similar extreme loads can cause the failure of a structure’s vertical load carrying elements and can lead to progressive collapse. Progressive collapse is a complicated dynamic process where the collapsing system should be able to redistribute the loads in order to prevent the loss of critical structural members and total collapse. Ductility, redundancy, and continuity must be considered in design of beams, columns, and frame connections to allow for potential redistribution of large loads and to prevent collapse.

Some of the more famous examples of progressive collapse phenomena include the collapse of the World Trade Center (2001) towers due to terrorist attack, the bombing of the Murrah Federal Building (1995) in Oklahoma City, and the collapse of the Ronan Point (1968) building due to a gas explosion. Through research, such as this study, progressive collapse can be better understood, prepared for, and possibly prevented in the future.
There are complex computer programs and simulation tools that can be used to model progressive collapse response of buildings. Many researchers proposed and used detailed and simplified models in the past to predict the progressive collapse potential and behavior of buildings. However, almost all of these models are theoretical and have not been verified using experimental data from real structures. There are a very limited number of frame and component experiments conducted in the laboratory. Also, laboratory tests are typically less helpful in understanding the three-dimensional response of a full-scale building. This research is unique because it involves experimental testing of an actual reinforced concrete (RC) building in the field, and the experimental data is used to validate the detailed and simplified models developed in this thesis. Code methods and procedures need to be validated with physical test data through methods such as those proposed in this study.

1.2 Research Significance

This research is important because while progressive collapse is not a common event, its effects can be catastrophic if design methods and specifications are not good enough to slow or stop the spread of damage throughout the structure. Advancements in research on this subject can help prevent a large amount of damage from spreading after the initial triggering event has occurred. In addition to having better damage control, buildings designed against progressive collapse provide more safety to those who use and occupy the structure. Many lives can be saved if damage is contained to the area of initial structural element loss rather than allowing the damage and its effects to spread through the building and collapse large areas. With the possibility of attacks or terrorist acts at higher levels in current times, this study can provide significant information that may be used in future design methods to prevent progressive collapse from various events.

Simplified models can help save time for engineers especially during the preliminary design stage. Instead of developing a full-scale three-dimensional model of a high-rise building, only a small portion needs to be modeled and quickly analyzed to investigate progressive collapse potential after the loss of one or more load carrying
members. Modeling a complete building can be time consuming and difficult and causes the analysis to require extensive amounts of time to be completed. The model simplifications show what parameters are most critical for future researchers and practicing engineers to focus more attention upon in future applications of this type of analysis.

1.3 Objectives and Scope

In this research, the SAP2000 program is used to model and perform analysis on a reinforced concrete building. The tested and modeled building consisted of four floors, circular columns, and a regular rectangular plan of flat slabs. Three columns were removed from the building prior to its demolition and the building’s response was monitored during the column removal process. A detailed model of the building is developed which accounts for the end constraints, various structural elements with correct dimensions, accurate loading conditions, and proper overall size. Multiple sensitivity analyses are performed to investigate contributions of different modeling parameters. Simplifications of the building model are analyzed and justified until a very general and simple model is obtained for generalized use with typical building structures. Modeling simplifications included restraint conditions, number of floors considered, number of bays considered, and modeling of slabs as equivalent beams. In the final simplified model, a method for developing structural elements as equivalent springs is proposed and analyzed as a further simplification. This spring model can be used and modified to perfect the current simplified model.

The modeling simplifications are made by comparing the numerical results from several models with varying complexities to the experimental data obtained from the actual building test. Certain columns were selectively removed as a simulation of a progressive collapse triggering event and the redistribution of loads were monitored in numerical models. Ultimately a simplified model with very few elements is developed and the procedure for simplified model development is described. The research results provide insight into the behavior of a reinforced concrete flat slab structure with drop panels and no beams in the lower floors. This simple procedure is expected to be used by
practicing engineers to check the ability of a structure to resist progressive collapse. The proposed model allows for quick progressive collapse evaluation using a small frame model with few elements instead of modeling the entire structure, which can be a multi-story or high-rise building.

1.4 Organization

The thesis has two major components. First, the experimental procedure is described, and then the detailed and simplified model development is presented. The problem statement, significance of research, objectives and scope of the project, and organization of this thesis are described in Chapter 1. Chapter 2 includes a literature review of studies performed by other researchers and engineers on the topics related to progressive collapse of structures. The research studies discussed in this chapter has provided valuable information and ideas used in this project. Chapter 3 provides a procedure for properly preparing to collect test data. Chapter 4 presents and explains the results obtained from the experiment tests. Chapter 5 describes the detailed building model and includes sensitivity analyses used for assumptions in the model. Chapter 6 describes the procedure for validating the detailed model by performing a time history analysis. Chapter 7 discusses and justifies many steps used to create a simplified model from the detailed model using the experimental data. Chapter 8 provides the details of the final simplified model and shows the final results in comparison to the test data. Chapter 9 provides the summary and conclusions of this research along with ideas for future work and improvements to the models and procedures developed in this study. Finally, several appendices are included to provide a comprehensive set of all experimental data, photographs, and simulation results obtained during this research.
CHAPTER 2

BACKGROUND INFORMATION

Progressive collapse was a topic of limited focus within the world of structural engineering research prior to 2001. A few specific incidences such as the Ronan Point collapse in 1968 and the bombing of the Murrah Federal Building in Oklahoma City in 1995 sparked some interest in the topic of progressive collapse of structures but the attack on the World Trade Center towers in New York City and their collapse was the major event that triggered new research interests among structural engineering researchers. Since then a plethora of research papers on this topic have been written. In preparation for this current research, dozens of recently published papers were reviewed in order to evaluate the current level of progressive collapse research and to determine what direction this study should take. This chapter discusses the papers that benefitted this project most.

2.1 Previous Research on Codes and Standards

It is important to gain an understanding of the different codes and standards such as those developed by the Department of Defense (DoD) and General Services Administration (GSA). Menchel et al. (2009) compared different simulation techniques recommended by these organizations for reinforced concrete structures. They investigated the specific assumptions and procedures of linear static and nonlinear static
procedures recommended by both the DoD and GSA. Also described included the load history dependent procedure, generalized theory of plasticity, and use of beam elements with Kirchhoff Theory. A few numerical examples were performed to show how to perform the various types of analysis. Marjanishvili (2004) also described the GSA and DoD guidelines and included some insight into how and which analysis procedure should be chosen. Step by step procedures and explanations of linear elastic static analysis, nonlinear static analysis, linear elastic time history analysis, and nonlinear time history analysis were all included in this study. The advantages and limitations of each analysis type were discussed as well.

Mohamed (2006) reviewed 19 research papers on topics such as triggering events, assessment of loads, methods of analysis, and different design philosophies. He also reviewed the progressive collapse standards and codes of the United States, Britain, and Canada for determining loads, structural analysis requirements, and various design approaches. Explanations of the ACI 318, ASCE 7, alternate path direct design approach, specific local resistance direct design approach, GSA (2003), and DoD (2005) were also provided.

Tsai and Lin (2008) evaluated the analysis procedure recommended by the GSA for earthquake-resistant RC buildings subjected to progressive collapse. They determined that the dynamic amplification factor of 2.0 is conservative for nonlinear static analysis. They suggest using a different factor from 2.0 for situations where the building is loaded into a significantly yielding phase to account for the inelastic dynamic effect.

Wibowo and Lau (2009) described the various progressive collapse design codes and standards. They also discuss different types of analysis and analytical tools that may be used. Also discussed is the significance of seismic load effects on progressive collapse behavior of structures. They conclude that “seismic progressive collapse of structures can be analyzed by modifying current the current analysis procedures.”

2.2 Experimental Research and Analysis Procedures

2.2.1 Experimental Testing Studies
Much of the knowledge obtained for this study was obtained through progressive
collapse research projects recently conducted at The Ohio State University. Prior work
by Song (2010) and Giriunas (2009) have created the foundation for which this research
is based as well as several current research studies based on similar building experiments
at The Ohio State University. Collaborative studies from Song (2010) have provided
deep insight into the experimental procedure used by this study as well as the use of

Giriunas (2009) completed a study involving the comparison of results from field
testing of a real building to that of a computer model developed using the computer
program SAP2000. Giriunas placed strain gauges on various members in a building in
Northbrook, Illinois prior to its demolition in order to gather physical data of the
building’s response to the sequential loss of four columns. While his experiment dealt
with a steel frame structure, the information provided by his study gave great insight into
the steps used to gather experimental data and how to use it to determine the accuracy of
a specific analysis method.

Similarly, Song (2010) also performed the experiments on the Ohio Union
building on the Ohio State University campus grounds. This was also a steel frame
building, from which four first-story columns were removed. This study, as well as the
one by Giriunas, involved modeling the structure and calculating the demand-capacity-
ratio (DCR) in all beams and columns in the structures to determine whether the building
would collapse.

Sasani has performed a large amount of research on the topic of progressive
collapse. His research includes both topics of experimental testing as well as computer
analysis of structure models. Sasani and Kropelnicki (2008) studied the behavior of a
continuous beam located above a removed column in a reinforced concrete structure. A
3/8 scale model was test and compared along with the results from a detailed finite
element model of the beam integrated with a three-dimensional nonlinear model of the
rest of the structure. It gives insight into the process of performing hybrid analysis and
shows that when two adjacent columns are removed, the DCR method can be overly conservative.

Sasani and Sagiroglu (2010) performed and experimental test where an internal column was removed with explosives from a 20 story reinforced concrete hospital. Analysis was performed using the finite element method. The study showed that the lower floors, nearest the removed column, experienced larger changes in forces and deformations when compared to the higher floors. They claim that this effect combined with the higher design stiffness of the lower floors shows that slabs and beams in lower floors provide a larger role in redistributing loads after a column loss. They claim that larger structures are not more susceptible to progressive collapse because the structural system has a high reserved capacity for redistributed loads.

Sasani and Sagiroglu (2008) performed experimental testing on a 6 story reinforced concrete hotel. Two adjacent exterior columns, including a corner column, were simultaneously removed. The focus of their investigation was on the change in direction of the beam bending moments located near the removed columns and the possibility that these changes could cause brittle failure at the column face due to poorly anchored reinforcement. It was determined that bidirectional Vierendeel action played a large role in redistributing loads and that moments in the beams near the removed columns did change directions. Also noted was that despite not satisfying current integrity requirements, the building’s three-dimensional response and redundancy helped resist progressive collapse.

Matthews et al. (2007) performed explosive testing to remove a column from a two story reinforced concrete structure. Detailed descriptions of instrumentation and the test procedure were provided along with analysis used to show the effects of the blast loading on the physical test data.

Yi et al. (2008) performed experimental testing on a three story one third scale model of a reinforced concrete structure. A description of the mechanical behavior of the model as well as the load redistribution mechanisms is provided. It was determined that
the calculated capacity of the frame was only 70% of the tested failure capacity when
catenary effects are included.

2.2.2 Purely Analytical Studies

Marjanishvili and Agnew (2006) compared available procedures for progressive
collapse analysis using SAP2000. The different combinations of linear, nonlinear, static,
and dynamic analysis were analyzed following the GSA guidelines. This study included
building analysis examples using the SAP2000 program and provided “clear conceptual
step-by-step descriptions of various procedures for progressive collapse analysis.”

Bao et al. (2008) performed a macromodel-based simulation of progressive
collapse in a reinforced concrete structure. The two buildings used in the simulation
were designed for the lateral load requirements in a seismic and non-seismic region. The
study investigates large deformation responses and ultimately concluded that special RC
moment frames design for high seismic activity better resist progressive collapse than
frames designed for less seismic risk. Khandelwal et al. (2008) performed a very similar
study where a steel frame structure was designed for seismic risk and modeled. This
study also found that frames designed for high seismic risk were less vulnerable to
progressive collapse than those that were not.

Pujol and Smith-Pardo (2009) investigated the ability of floor systems to survive
abrupt removal of a column. The study proposes two ways in which a structure’s floor
system can be designed to survive a sudden removal of its supports. These include
proportioning the system using the results of a linear static analysis model with a load
factor exceeding 1.5 and providing detailing that can allow the system to reach
deformations of at least 1.5 larger than the deformation associated with loading that
develops the system’s full strength.

Ettouney et al. (2006) investigated the global system response of buildings
subjected to progressive collapse. This was done because the alternate path method is
commonly used for progressive collapse analysis but it does not consider the overall
stability of the system of a damaged structure. Simple methods of modeling and analysis
that are compatible with current engineering methods are proposed. The proposed methods are also applicable to seismic and direct blast design methods.

2.3 Previous Research on Simplified Progressive Collapse Modeling

One of the main goals of this research was to create a small simplified model (of the vulnerable portion of the structure) that engineers could use rather than creating a finite element model of the entire structure. First, simplified models reported in the literature are reviewed. Also, since the reinforced concrete building tested in this study was a flat plate system that had no beams, there was a need to find information on how a structure could be converted from one type to another, i.e., from slab-column system to beam-column frame system. The following research papers provided useful information.

Lee et al. (2008) investigates punching shear behavior of flat plate connections with concrete filled tube columns. The columns in this investigation were not exactly like the columns in the test structure of the current study however it gave some insight into the punching shear problems in a flat plate system. They developed a method for idealizing the deflection pattern of the plate above a lost column. They suggested the double spanned beam acts as if the outer third on each side is a cantilever beam with a simply supported beam resting in the middle with its supports at the ends of the cantilevers. This approximation was then converted into a two-spring system for nonlinear analysis. Similar ideas were used in development of simplified models in this research. King and Delatte (2004) studied punching shear failure in flat slab structures. They provided a few specific case studies involving failed structures. The problems discovered were mostly due to construction errors and not following the proper code procedures.

Mitchell and Cook (1984) analyzed slab section deformations and contribution of slabs to progressive collapse. The slab sections between columns were described as “hanging nets” when they act as two-way membranes. They idealized load-deflection response of the two-way slabs in interior panels. This study provided a better understanding of how slabs behave in a flat plate structure such as the one tested in this
research. Figure 2.1 shows the differences in the “hanging net” action of beam supported slabs opposed to slabs in flat plate structures.

![Figure 2.1: "Hanging net" slab membranes as introduced by Mitchell and Cook](image)

Mohamed (2009) addressed the issue of progressive collapse in the corner floor panels of reinforced concrete buildings. He also used SAP2000 to analyze a structure that had the first-story corner column removed. The study mostly focused on the bending moments and torsion in the beams as well as the effects of using lateral load resistant bracing. This research gave insight into how the corner of a structure behaves when the corner column is removed. One corner column was removed from the test structure in the current study, therefore the information from Mohamed (2009) was useful for flexural analysis of the corner of the structure.

Kim and Kim (2008) assessed the ability of steel moment frames to resist progressive collapse. The study compared linear and nonlinear analysis of structures based on the GSA and DoD guidelines. This study compared the differences of static loads with a load factor of 2.0 applied to the dynamic service loads with no load factor applied. It showed that the simplification factor of 2.0 seems to just adequately represent
the difference between the static strain and the maximum magnitude of the dynamic strain. This topic is discussed and investigated further in the current study.

Masoero et al. (2010) performed research on the different mechanisms of progressive collapse in brittle and ductile framed structures. They presented a simplified model scheme of beam elements using springs for supports, which proved to be useful in this research. The model was described as “approximate static scheme of elements where the starting damage propagation occurs.” The simplified model consisted of a few structural members attached to springs which would represent the structure as a whole.

Izzuddin et al. (2008) proposed a simplified framework for assessment of progressive collapse in a multi-story building. The suggested simplified structure model was very helpful to justify various simplifications to the SAP2000 model that is developed later in this thesis. It showed the steps of starting with a complete building and simplifying it into a section of the building, then just one floor, and then taking the slabs and transforming them into beam elements. The final model is comprised of a few beams connected with springs on the outer boundaries. This idea was partially used to create the simplified model proposed in this study. This paper also provided a method to calculate the stiffness of the springs to properly represent the structural members.
CHAPTER 3

EXPERIMENTAL TESTING AND BUILDING DESCRIPTION

3.1 Introduction

A large number of demolition companies were contacted by the author and another graduate student, G. Ullom, in order to locate a structure suitable for the testing purposes of this study. A copy of a standard email sent to companies is included in Appendix A. Many of the companies were not scheduled to demolish buildings within a reasonable distance or in the near future. Of the companies with building demolitions scheduled, in general, either the buildings scheduled for demolition were irregular or unsuitable for testing, or vast majority of demolition companies were unwilling to take risk due to potential damage and collapse during progressive collapse testing. The demolition company, Steve R. Rauch Inc., agreed to perform collapse testing prior to demolition of Swallen’s building in Middletown, Ohio (near Dayton, Ohio).

Swallen’s building was a large department store/mall that was connected to a parking garage in the middle of town. The concrete parking garage had been mostly torn down prior to testing and was no longer attached to the building during the experiment.

3.2 Test Building Description

The rectangular test structure was designed to be large and robust due to its use as a department warehouse. The overall building dimensions were 202 ft in the north-south
direction by 218 ft in the east-west direction. A grid of 130 circular columns had a uniform spacing of 20 ft in both directions except around the perimeter spans, as shown in the plan view of the first floor in Figure B.1. The structure had three floors above ground with heights of 12 ft 8 in., 11 ft, and 12 ft from top to bottom, respectively. A basement floor with a height of 11 ft was present under almost the entire structure, including part of the tested area. Figure B.2 shows the exterior elevation details of the structure. Appendix B contains all of the vital blueprints required for determining the dimensions and loading conditions within the structure. The first floor of northeast corner of the building was the location of the test area and is the portion of the structure designed in the computer models for analysis purposes. Figure 3.1 shows the internal view of the first story of the building.

Figure 3.1: The inside view of tested area of the Swallen's building
3.3 Instrumentation

The first of two days on site was entirely dedicated to preparing the columns for instrumentation and preparing the wires for testing the next day. The materials required included the generator, jackhammer, and grinder provided by the demolition company as well as hard hats and goggles for safety, wires, strain gauges, adhesive and catalyst, tape, extension cords, and gasoline for the generator which were self-supplied. The work was laborious and required several hours to complete for three separate columns. The individual steps for instrumentation are presented in Appendix C.

Since the building was large, it was decided to concentrate the testing and gauges for data collection in one corner of the building around the columns that were scheduled to be removed. Figure 3.2 shows the placement of the strain gauges as well as the columns that were removed during testing. The crossed out columns in the figure were removed and are numbered in chronological order of their removal (M9, M11, K11 respectively). The numbers located around columns L11, L10, and M10 show the strain gauge numbers. The extra strain gauges shown on column L10 were placed about 2 ft below the four strain gauges that were located approximately at mid-height. The basic dimensions of the structure are also included along the left and bottom sides of the figure. It should be noted that this diagram shows the slabs extended beyond the last column lines on the top and right sides. This is discussed at length when the model is developed and analysis is performed using the SAP2000 program.
3.4 Data Collection

The second day at the test site involved recording final notes on measurements as well as preparing a laptop computer and portable data acquisition system to collect the test data. Measurements of the exact placement of the strain gauges on the columns including their height and variation from the axis were recorded. The measurements and other field notes taken are included in Appendix D. The researchers and portable data acquisition system were located approximately 200 ft away from the testing area to be away from danger of potential collapse during testing. It was decided that it would be too dangerous to enter the test structure area after testing had been completed to retrieve the gauges and wires since multiple columns were to be removed. The Strain Smart program was used to monitor and record strain gauge data.

The processor shown in Figure F.1 first removed the wall along the side of the building, along axis M in Figure 3.2, to gain access to columns M9 and M11. Next, the
three columns that were marked for removal were severed by the processor’s large claw in chronological order of M9, M11, and K11 respectively. The claw was placed around each column and forced closed until the concrete core of the column was crushed and the reinforcement bars were ruptured. This process of crushing the column was completed in a short amount of time, approximately one second for most of the concrete crushing and a few more seconds of the claw moving to rupture the longitudinal bars. Finally, some small exterior columns that connected the building to the parking garage along with dividing walls that separated the two structures were removed before the testing and data collection was completed. Figure 3.3 shows the building at the end of testing. The figure shows all three columns that were removed as well as the three columns that were instrumented. Appendix E includes additional pictures related to the preparation and instrumentation of columns. Other pictures are included in Appendix F show the step by step demolition process.
CHAPTER 4

EXPERIMENTAL TEST RESULTS

4.1 Introduction

This chapter presents the recorded data by strain gauges attached to the three columns near the removed columns. Strains measured at different locations on the same column are compared and the physical meaning of strain variations and changes are discussed. Of particular interest were the sudden strain drops that occurred during each column removal. The data acquisition system recorded ten data points per second and ran for an uninterrupted 2134.3 seconds while the processor removed the columns and walls around the building. This resulted in a total of 21,343 data points collected from each of the 14 different strain gauges. Data point values were recorded as micro-strain (με) and thus need to be multiplied by $10^{-6}$ in order to have the actual dimensionless strain values. The strain values obtained from testing ranged from a maximum value of $47\cdot10^{-6}$ to a minimum value of $-225\cdot10^{-6}$. Positive values indicate tension strain while negative values indicate compression strain in this thesis.

4.2 Strain Gauge Data from Column M10

The recorded data from column M10 is shown below. All other raw data is shown in Appendix G. Figure 4.1 through 4.3 each show the strains measured by four strain gauges (7, 8, 9, and 10) on column M10 along with a bold black line representing
the average column strain value from the four gauges as shown in Figure 3.2. The rectangular markers show the peak maximum and minimum values of strain which correlate to either the dynamic motion of the structure after column removal, or the effect of the processor removing the column. The rectangular markers at the beginning and end of each figure show the steady state values of strain from which the change in strain or load before and after column removal can be determined. The vertical axis of the figures represents micro-strain, as previously mentioned, and the horizontal axis represents time in units of seconds from the start of data collection.

The data presented in Figures 4.1 through 4.3 show that all four strain gauges were experiencing the same patterns of increased compressive strain throughout the process of each column removal. The entire time history of each strain gauge is included with all of the data shown in Appendix F. Figures 4.1 through 4.3 show only a snapshot of testing during which dynamic action was taking place while still showing the static residual strains before and after column removal. In general, the distance between each individual strain gauge reading on the graphs remains about the same for each section of testing between separate column removals. However, it can be seen that immediately after the large increase in compression strains, which show the instant the column was removed, there tends to be a noticeable change in these distances. Figure 4.4 shows the same data as Figure 4.1 but with the values initialized to zero.

For example, the distance between the sensor readings before and after 1055 seconds in Figure 4.2 shows relative differences in changes in strain. This change in distance between strain curves apparently represents a change in the applied moment of the column where strains are measured. The direction of the moment being applied to the column from the redistribution of loads in the structure can be determined from the difference in measured residual strains. The axial strains are of most importance to this study and any analysis of this data relating to the moment values is used to validate assumptions used in the SAP2000 models as well as comparing the simulation results.
Figure 4.1: Column M10 strains during first (column M9) removal

Figure 4.2: Column M10 strains during second (column M11) removal
Figure 4.3: Column M10 strains during third (column K11) removal

Figure 4.4: Strain gauge values initialized to zero during first column removal
Table 4.1 summarizes the average measured strains for column M10 including static strains before column removal, residual static strains after column removal, as well as the maximum and minimum dynamic strains recorded during each column removal.

Table 4.1: Summary of measured strain results for column M10 ($10^{-6}$)

<table>
<thead>
<tr>
<th></th>
<th>First Column</th>
<th>Second Column</th>
<th>Third Column</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Static strain</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before column removal</td>
<td>-13</td>
<td>-43</td>
<td>-108</td>
</tr>
<tr>
<td>After column removal</td>
<td>-40</td>
<td>-105</td>
<td>-101</td>
</tr>
<tr>
<td>Change in strain</td>
<td>-27</td>
<td>-62</td>
<td>7</td>
</tr>
<tr>
<td><strong>Dynamic strain</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>-7</td>
<td>-27</td>
<td>-97</td>
</tr>
<tr>
<td>Minimum</td>
<td>-63</td>
<td>-140</td>
<td>-109</td>
</tr>
<tr>
<td>Difference</td>
<td>-56</td>
<td>-113</td>
<td>-12</td>
</tr>
</tbody>
</table>

The relative change in static or steady strain values were the most important experimental results obtained as they are used in comparison with the SAP2000 analysis results. The axial strain values directly related to the amount of axial load transferred to column M10 as each column removal occurred. The dynamic strain values are also used later in this thesis to evaluate the accuracy of the simulation models for dynamic time history analysis.

### 4.3 Dynamic versus Static Strains and GSA Amplification Factor

Table 4.1 shows that the magnitude of the difference between the maximum and minimum dynamic strains is relatively close to double the value of the change in static strains. The ratios of the difference between dynamic strains to the change in static strain are 2.07 for the first column removal, 1.82 for the second column removal, and 1.71 for the third column removal. This corresponds to the current GSA guidelines of using a load amplification factor of 2.0 for static analysis to account for the dynamic affects. These ratios determined from the experimental data support the proposal of the GSA similar to other studies such as those performed at The Ohio State University by Song.
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The GSA dictates that this load factor of 2.0 is applied when performing static analysis so as to account for the dynamic behavior of the structure. It should be noted that for the remainder of this study, unless otherwise stated, all load values do not have this load factor included for purposes of simplicity but that they should be included in real design. The reason this factor is not included in the loading conditions is because this study focuses on the real loads of the structure and the real experimental static equilibrium strain values. The major goal of this research is to better understand the actual response of the structure and not to design the structure as intended by the GSA guidelines. During static analysis, actual loads (without 2.0 factor) are applied because the predicted strain results are compared with measured residual static strains.

4.4 Experimental Data Chosen for Model Development

For the purpose of this research, primarily values obtained from strain gauges 7, 8, 9, and 10 are used for model development and simplification purposes. These gauges were attached to column M10, as shown in Figure 4.5, and were chosen for several reasons. The most important of these reasons was due to the column’s location in reference to the columns that were removed and the order in which they were removed. The data from columns L10 and L11 is still used for verification purposes of the final model but column M10 is used for model development and simplification due to the reasons described below.

Column M10 was located directly between two columns that were removed. Figures 4.1 and 4.2, as well as Table 4.1, show that large axial loads are transferred to column M10 during the removal of the first two columns. This was expected because column M10 was closest to the removed columns and was expected to experience the largest strain changes as each column was removed. Figure 4.3 shows a small strain change occurring. Particularly very little dynamic effect was observed due to the removal of the third column. This was expected since column M10 is further away from that column and would receive much less of the redistributed load after the removal of column K11. Also, by the time of the third column removal, it is much more difficult to
predict how the structure behaves and where the loads are transferred with two columns already missing without a very detailed analysis.

Figure 4.5: Numbering and layout of strain gauges (crossed out columns were removed in the order shown)

The same argument is also true for column L11. One reason the data from column M10 is more useful for model development is because the column M10 was located between the first and second columns that were removed, rather than the second and third columns. This is important because as soon as the first column was removed, loads were redistributed throughout the structure in a way that cannot easily be predicted. For this reason, it is best to study the effects of the earliest columns removed compared to that of the later columns removed. Had column L11 been chosen for the analysis, there
would be a lot more uncertainty in the modeling and analysis results. This would make the computer analysis more difficult than necessary when an option is available that can give immediate and more direct relationships.

Column L10 originally appeared to be in a location that would receive a large portion of the unsupported loads and would not be significantly affected by the column removal order since it is symmetrically located. However, the recorded data shows that column L10 experience very small variations in strain of average magnitudes generally less than $10 \cdot 10^{-6}$ (Figures G.1 through G.6) while column M10 experienced average variations of strain as high as $140 \cdot 10^{-6}$ (Figure 4.2). Since the strain gauges were limited to an accuracy of integer micro-strain values, the data from column M10 incurs less relative error due to the limits of the strain gauges accuracy since the magnitude of the data is much larger. Another problem with using column L10 data for model development involved additional uncertainty from the three-dimensional load redistribution in the structure. The load redistributions within this test building were particularly difficult to predict because the building did not have typical beam-column frames, but rather had thick slabs across the entire floor and no beams to transfer loads to columns. The three dimensional load redistribution effects within the slab are further discussed in Chapter 7.

The recorded strain history for strain gauge 1 is shown in Figure 4.6 and indicates that column L10 typically experienced tensile strain rather than compression. This probably means that rather than receiving just redistributed axial loads, it received redistributed moments of a magnitude that were large enough to overcome the compression of any additional axial forces on some sides of the column during various stages of testing between column removals. It is also possible that an axial tension force acted on the column to force some of the strain gauge data to enter the tension zone. However, this situation is not likely since not all of the strain gauges attached to column L10 experienced increased tension strains at all times. Strains measured by gauges 1, 2, 3, 4, 5, and 6 attached to column L10 are provided in Figures G.1 through G.6 in Appendix G. This data is very valuable to understand three dimensional thick slab
behavior in flat plate structures for progressive collapse. The intent of this study is to use the changes in axial loads of the columns to learn about progressive collapse while validating the assumptions and analysis tools by comparing the measured axial and moment results. For this reason the data from column L10 is less desirable for the development of detailed and simplified models but useful for the final validation of those models.

![Figure 4.6: Measured strain gauge data from strain gauge 1 on column L10](image)

Another minor consideration for why column M10 may be a better choice than column L10 is due to the fact that column M10 is an edge column. Edge columns are more vulnerable because they are not connected to as many members to allow for redistribution of loads. In this case, since there are no beams, loads are redistributed through the slabs. While some overhang of the slab exists, that part of the slab would transfer little or no loads because there are no columns or connections at the end to carry any load. Rather, this means that edge columns have half of the slab area of an internal
slab through which it can transfer extra loads. Thus, an external column is likely to fail before an identical internal column because it has less capability to transfer the additional stress to other areas of the structure. Also, in terms of a man-made related accident or attack, it may be more likely for a structure to lose an external column than an internal one. Multiple column loss is obviously the most likely case but when considering the loss of any individual column it is more likely that an accident or attack would be from outside of the building than inside due to security clearances and the fact that a vehicle collision is more probable from the outside of a building than the inside.

Data from columns L10 and L11 is used in the final steps of this study in order to appropriately show a full validation of the model with the test data. It would not be acceptable to validate the model using only a small portion of the data. Therefore, the data from column M10 is used for the initial development and simplification of the computer models while all of the recorded data will be used to validate of the final models developed.
CHAPTER 5

DEVELOPMENT OF DETAILED MODEL

5.1 Introduction

This chapter presents the procedure and assumptions used to develop an initial detailed model of the test structure for the purpose of performing computer analysis. Many topics including creation of models for the slabs, drop panels, and columns, determining the restraint and loading conditions, selection of the proper initial size of the model, and determination of the proper length of slab overhangs are discussed and justified using the recorded test data. The program chosen to create the model and perform analysis for this study was SAP2000 (2010) which is a commercial structural analysis software package used to analyze different types of structures under various static and dynamic loads.

The first major step in the analysis portion of this study is to create a detailed model to represent the real behavior of the tested structure as accurately as possible. This model is the most complicated model necessary to accurately display the behavior of the building while concentrating on the area known to be affected by removal of columns. A detailed model of the entire structure would produce very similar analysis results to this detailed model. However, the detailed model of the portion of the structure is more simple to create and much faster to analyze than a model of the entire structure. This
provides engineers and researchers with an advantageous starting point for testing simplified models as opposed to starting with a complete model of the entire structure being analyzed.

5.2 Development of SAP2000 Model/Elements

The SAP2000 (2010) program allows users to define different types of elements such as columns, beams, and shells. The test structure had a flat plate design consisting of thick slabs and no beams. The SAP2000 program provides default design details as long as the basic material and dimensions of the elements are provided. These design details, such as the placement and amount of reinforcing steel bars for example, were then modified in the program to match that of the plans provided for the test structure. The following sections describe the different elements used in the model.

5.2.1 Column Elements

The structure was comprised of multiple different column designs. Some columns had oddly shaped cross sections and were used around the perimeter of the building. The test area of the actual structure, and thus the modeled area, did not include any of these types of columns. A few of these columns existed on the northern side of the building but they were used to only support the parking garage that was connected to the building. The most common type of column was a circular 18 in. diameter reinforced concrete column. All columns in the test area of the basement as well as the first and second floors were of this specification. The third floor consisted of circular 12 in. diameter reinforced concrete columns. The detailed model is comprised accordingly with 18 in. and 12 in. diameter column elements in the lower and third floors, respectively. Many of the columns had unique steel reinforcing details which are listed in Figure B.7 in Appendix B. The steel reinforcement details were incorporated into the appropriate columns within the model during model development.

After defining the column sections in SAP2000, it is possible to view the section designer which shows the capacity information for each column section. The following discussion of bending capacities of the columns is only used for determining if bending moments are negligible or not in later analysis. The columns defined for the detailed
model had bending moment capacities ranging from 107 kip-ft to 267 kip-ft. The axial load-moment interaction surface of the typical 18 in. column and table including axial load and moment capacity in units of kips and inches are given in Figure 5.1. The design interaction curve from SAP2000, which is calculated based upon ACI-318 (2011), shows that the relationship between bending capacity and axial load is approximately linear from the region of maximum tension to the peak point of maximum bending capacity. This can be seen as the lower portion of the axial load-moment interaction surface in Figure 5.1. Therefore, linear interpolation can be used to determine the columns bending capacity at a zero axial-load condition. By using the linear interpolation, it was calculated that the bending capacity of the column with no axial load was 1286 kip-in. or approximately 107 kip-ft. The maximum bending capacity occurs near 363 kips of axial force at balanced condition and has a value of approximately 3204 kip-in. or 267 kip-ft.

![Figure 5.1: Axial load-moment interaction surface of an 18 in. diameter reinforced concrete column](image-url)
The assumption of zero axial-load in the table is very conservative to determine the bending capacity of columns since the test building model and hand calculations showed that all of the columns have axial loads between roughly 40 kips and 110 kips prior to column removal and no more than 300 kips after column removal. The interaction surface shows that the columns would have higher bending capacities with any load within this range than at the zero axial-load condition. This bending capacity is only used to determine whether bending moments in the structure are negligible or not as opposed to being used for design or analysis purposes. Thus, the use of the zero axial-load capacity of 107 kip-ft is acceptable and highly conservative.

5.2.2 Shell/Slab Elements

The typical design of the structure required an 8 in. thick reinforced concrete slab. Normally with beams, a slab would be thinner and transfer its loads to the beams. In this structure, the slabs were designed to be thicker and directly transfer loads to the columns. These were created as thick shell elements with an 8 in. thickness in the detailed model. There were also drop panels on top of all columns except for those connected to the roof. These drop panels were 2.375 in. thick and designed to be squares of 7 ft in length. The difficulty in modeling the drop panels are discussed in the next section. The drop panel elements were created as thick shell elements with a total thickness of 10.375 in. to account for the slab and drop panel combined.

There were different thick shell elements created to represent the roof structure. The roof details are included in Appendix B and show the complexity of its design. Ultimately it was decided that the roof could best be modeled as a thick shell of concrete slab with a constant 8 in. thickness. There was then a modification factor of 0.5 applied to the mass and weight of this slab element to lessen the weight of the shell. This would account for the weight of the rigid insulation as well as the lesser weight added by the small steel beams supporting the roof in terms of weight. The 8 in. thickness helped approximate the stiffness of the steel beams and roof support system. Other thicknesses were used and applied with appropriate weight and mass modification factors and there were insignificant differences between the transferred loads in the various lower
members of the structure. For this reason, as well as the fact that the roof would later be removed from the model as a simplification, it was found to be acceptable to use the 8 in. thick shell with a weight modification factor of 0.5 to represent the roof structure.

5.3 Restraint Conditions

In this detailed model, the restraint conditions on the bottom of the columns are assumed to be fixed. This is because they were attached to footings that were buried underground which prevented most movement and rotation. All other supports throughout the structure are assumed to be rigid connections, not pinned.

5.4 Loading Conditions

All loads in the models are considered to be unfactored dead loads. The building had essentially been completely emptied and stripped of most items that would create live loads with the exception of a few pipes and ceiling tiles. The tiles and pipes and other nonstructural components were considered to be negligible loads comparatively and were not calculated or modeled.

To accurately represent the loading conditions in the structure, the column and slab elements were given a self-weight factor of 1. This basically takes the density of reinforced concrete, $150\text{lb/ft}^3$, and distributes it where the elements exist. This would create a load of $100\text{ lb/ft}^2$ for the areas of the slab that are 8 in. thick and a load of about $130\text{ lb/ft}^2$ for the drop panel areas where the thickness was 10.375 in. thick. The roof load is estimated by the weight of the 8 in. thick shell multiplied by a modification factor of 0.5 which was discussed in the Section 5.2.2. This creates a load of $50\text{ lb/ft}^2$ to represent the rigid insulation and steel girders that comprise the roof. Other weight and thickness combinations were tested and no significant difference was encountered by the columns of concern to this study.

5.5 Selecting the Model Size

The change in strain that any given column experiences, due to the removal of another column, depends on the distance between the two columns. Columns surrounding a removed column experience much larger changes in strain due to
redistributed loading than columns that are located multiple spans in distance from that removed column. This means that after a column removal, a high percentage of loads carried by that column are redistributed locally to the nearest surrounding columns rather than equally to all other remaining columns in the structure. This further implies that a structure with a large base will have a high percentage of unaffected columns when any given column is lost. This is very important because it allows for a large reduction in the size of a model needed for analysis of a column loss without sacrificing the accuracy provided by a model of the entire structure. To develop a detailed model for this study, it is very beneficial to utilize this behavior of buildings to reduce the size of the model and amount of time required for analysis. This concept is highly investigated in the current study and explains how the simulation models behave as they are simplified by reducing model size. An example using actual data is provided next to validate this concept.

Figure 5.2 below shows the layout of the northeast corner of the building. The bold dark line represents the outer walls of the structure while the crosses show which columns were removed during testing. In the test building, column L11 experienced very little change in strain when column M9 was removed. Column L10 and column M10 experienced a much larger change in strain due to their closer proximity to column M9. Similarly, when column K11 was removed, column L11 and column L10 experienced large changes in strain while column M10 experienced very little change in strain.

The experimental measured data shows columns in neighboring spans to the removed column experience large changes in strain due to the redistributed loads. It also shows that columns at a distance of two spans are still affected by the column removal but at a much smaller magnitude. In order to effectively utilize this concept, a sensitivity analysis needs to be performed to determine the required minimum size of a model needed to maintain high accuracy in analysis.
5.5.1 Sensitivity Analysis of Transferred Loads

The concept that load transfers remain localized around a removed column is proven and investigated through the use of a test model developed in SAP2000. The test model for this sensitivity analysis is composed of the same exact dimensions, structural elements, loading conditions, and restraint conditions as the test building according to the technical drawings. This test model is only used for this specific sensitivity analysis and is completely separate from all other models developed in this study. With the test model created based upon all of the parameters provided in the plans, columns M9 and M11 were removed to simulate the first two column removals of the experimental procedure. Two columns were removed to cause larger changes in strain and loads in the columns than a single column would provide. This better illustrates how far the effects of redistribution spread throughout the structure along one axis. At the same time this
makes the sensitivity analysis more simple than removing all three of the columns as performed experimentally.

The analysis was performed by first creating the model structure, performing static analysis, then deleting the two columns and performing the static analysis again. The numerical results were then compared in order to observe the changes in moments that occurred after the columns were removed. Figures 5.3 and 5.4 show the bending moment results for the northeast corner of the test model after columns M9 and M11 were deleted. Prior to deleting the columns, all of the columns had bending moments of approximately 4 kip-ft or less. After deleting columns M9 and M11, the figures show most of the columns in the structure have similar moments with magnitudes of less than 4 kip-ft except for a few locations. The columns directly above the removed columns, as well as the columns in directly neighboring spans in the floor above the removed columns had magnitudes greater than 100 kip-ft and 50 kip-ft, respectively. The effects of removing the columns are shown to be essentially confined to that same span and the neighboring span according to the test model. The ovals in Figures 5.3 and 5.4 show where the columns were located prior to removal. The dashed rectangles show the areas where moments increased drastically. The light rectangles highlight the closest remaining columns which show little or no effect from the column removal process.

The sensitivity analysis and the test data provide similar conclusions. They show that for this building the redistribution of loads and strains are essentially contained within two spans in each direction from a given removed column. Since the tested area of the building was in a corner, the boundaries of the model should be determined by the column axes that are located between the corner and two spans past each column removed furthest from the corner. The columns removed furthest from the corner fall in axis J and axis 9 in their respective directions. For this reason, the selected size of the detailed model includes column lines 7 through 11 in the north-south direction and H through M in the east-west direction. This selection reduces the size of the structure to be modeled to less than 20% of the original structure, which greatly reduces the time needed to create the model as well as the time needed to perform analysis. Initially, in this
detailed model, all three floors and the basement are included in the model to best represent the true behavior of the structure. This leads to the detailed model defined as consisting of five rows of columns in each direction, with slab overhang, as well a total of four floors in height. The detailed model from the SAP2000 program is depicted in Figure 5.5.

Figure 5.3: Sensitivity analysis visualized parallel to the numbered axis
Figure 5.4: Sensitivity analysis visualized parallel to the lettered axis

Figure 5.5: SAP2000 detailed model
5.6 Modeling of Drop Panels

The most complicated and time consuming step of developing the detailed model was due to the nature of how shell elements are formed within the SAP2000 program. When a flat plate structure is created in SAP2000, the slab of each floor is segmented into various ‘area sections’. These ‘area sections’ are defined by default in SAP2000 to have corner points where the columns connect to the slab and can be defined to have a specific thickness. This introduced a problem since the corners of the drop panels were not in locations where the columns connected to the slab. Figure 5.6 shows a representation of how an ‘area section’ is defined by default with its corners at the columns. Figure 5.6 also shows the required boundaries for the placement of the 7 ft. square drop panels. It can be seen that the ‘area section’ boundaries conflict with the necessary boundaries of the drop panels. These boundaries need to match with each other in order to allow the drop panel area to be selected as an ‘area section’ which ultimately allows the extra thickness of the drop panels to be present within the SAP2000 model.

![Figure 5.6: Drop panel details](image)

Figure 5.6: Drop panel details
In order to correct this modeling problem and allow for the correct modifications of slab thickness where drop panels exist, nodes needed to be created in the corner locations of the drop panels. This would then allow the proper drop panel area to be selected as an ‘area section’ which allows for the thickness to be changed. Node points can manually be placed on the edges of the area sections but this would consume an immense amount of time to accurately place and define the coordinates of every individual drop panel corner in the model. The fastest used in this research was to create a new model with columns in the location of every the corners of every drop panel. The column elements would automatically create a node in the location desired for the drop panels. This was a very time consuming process but was much simpler than manually entering each individual node for every corner of a drop panel in the model.

Figure 5.7 shows how complicated the process became using this proposed method to create the drop panels. The figure shows groups of three lines running parallel in both directions. The center of the three lines represents the axis line for the actual columns that exist in the structure. The outer two lines represent the pseudo column axis lines that were created solely for the purpose of creating extra nodes for the drop panels. The intersections of the outer lines create the boundary of the drop panels so that they can be defined as their own ‘area sections’.

With all of the necessary nodes created, the pseudo columns were deleted. This left a model that was exactly like it was previously, except with the addition of nodes in a placement that created ‘area sections’ for the drop panels. Finally, the groups of four small squares that appear in groups at each column location in Figure 5.7 were selected together as they represent the entire drop panel area. With all of those areas selected, the thickness of 10.375 in. was entered to create the physical dimensions of the drop panels within the model while the rest of the slab had a uniform thickness of 8 in. Although complicated, this did allow for the exact modifications to the slab areas that would represent the real conditions within the structure. A sensitivity analysis on the necessity of including the drop panels in the model is presented in the next section.
5.6.1 Investigation of Structural Contribution of Drop Panels

During the modeling of the drop panels, the need for inclusion of drop panels in the model was investigated. Obviously the drop panels increase the dead load due to their thickness and increase the bending and shear stiffness of the thick slab around the column to prevent punching shear and to transfer moments to the column. However, in order to simplify the modeling process, this investigation was performed to determine how the structure behaves if the drop panels were removed with the dead loads of the additional thickness still accounted for.

A sensitivity analysis of the detailed model, developed to this point, was performed for the two extreme slab thicknesses. This was done to observe the response of
the model with the two most extreme values of stiffness within the slab. Two versions of the detailed model were created, one with all slabs having a constant uniform thickness of 8 in. and the other having a constant uniform thickness of 10.375 in. since those represent the thicknesses of the original slab and the drop panels respectively. Since the detailed model accounts for the dead loads through self-weight, a modification factor was developed in order to create equal dead loads between the model with 8 in. slabs and the model with 10.375 in. slabs. This was simply done by placing a weight modification factor of 8/10.375 to all of slabs in the model consisting of 10.375 in. thick slabs. This then gives them an identical weight to the 8 in. slabs. This leaves the remaining difference between the two models to be only due to the stiffness of the slabs which represent the two most extreme possible cases from the actual structure. Both models were then analyzed statically with the first two columns, M9 and M11, removed as performed in the actual test structure. The resulting moment magnitudes within the investigated area of the structure were very similar to those shown in Figures 5.3 and 5.4. To compare the numerical differences between the model with 8 in. slabs and the model with 10.375 in. slabs, Tables 5.1 and 5.2 includes the moment values for those columns determined to undergo a significant change in loads before and after the columns were removed from the models. Moment 2-2 and Moment 3-3 refer to the bending moments within the columns in the x-direction and y-direction of the models, respectively.

Table 5.1: Moment results from 8 in. slabs

<table>
<thead>
<tr>
<th>8&quot; slabs</th>
<th>Moment 2-2 (k*ft)</th>
<th>Moment 3-3 (k*ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd Floor</td>
<td>Top Column</td>
<td>Bottom Column</td>
</tr>
<tr>
<td>M11</td>
<td>-21.9</td>
<td>25.5</td>
</tr>
<tr>
<td>L11</td>
<td>-12.9</td>
<td>11.7</td>
</tr>
<tr>
<td>M9</td>
<td>-19.9</td>
<td>23.1</td>
</tr>
<tr>
<td>L9</td>
<td>-10.2</td>
<td>9.4</td>
</tr>
<tr>
<td>M8</td>
<td>3.2</td>
<td>-3.3</td>
</tr>
<tr>
<td>L8</td>
<td>4.2</td>
<td>-4.2</td>
</tr>
</tbody>
</table>
Table 5.2: Moment results from 10.375 in. slabs with adjusted weight

<table>
<thead>
<tr>
<th>10.375&quot; slabs</th>
<th>Moment 2-2 (kip*ft)</th>
<th>Moment 3-3 (kip*ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd Floor</td>
<td>Top Column</td>
<td>Bottom Column</td>
</tr>
<tr>
<td>M11</td>
<td>-24.7</td>
<td>27.2</td>
</tr>
<tr>
<td>L11</td>
<td>-13.6</td>
<td>12.8</td>
</tr>
<tr>
<td>M9</td>
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<td>24.2</td>
</tr>
<tr>
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<td>10.2</td>
</tr>
<tr>
<td>M8</td>
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<td>-3.2</td>
</tr>
<tr>
<td>L8</td>
<td>4.3</td>
<td>-4.2</td>
</tr>
</tbody>
</table>

The average difference in magnitude of the moments between the 8 in. slab case and the 10.375 in. slab case was less than 1 kip-ft which can be considered negligible in comparison to the assumed 107 kip-ft bending capacity of the columns which was determined in Section 5.2.1. The largest discrepancy between the two models came on the corner column M11 which was directly above one of the two removed columns. Even in that location, the top of column M11 in Table 5.1, the difference is less than 3 kip-ft which is comparatively small to the bending capacity of the columns.

This sensitivity analysis ultimately shows that very little difference occurs between the results of a model with 8 in. slabs and a model with 10.375 in. slabs once the difference in dead loads is accounted for. Since the test structure had 8 in. slabs and 10.375 in. drop panels, it can be assumed that the overall average stiffness of the slabs falls between that of the two extreme cases. For this reason, as long as the dead loads are appropriately accounted for, the existence of the drop panels in the detailed model is not necessary considering the type of situations and analysis this study is performing. This behavior is shown to be true for the large sized detailed model developed in this chapter. However, any smaller and more simplified models require the existence of the drop panels because they represent the local behaviors of members and the previous sensitivity analysis does not apply to those situations.
5.6.2 **Simplification of Drop Panels in the Detailed Model**

Since the drop panels were determined to be unnecessary to the detailed model, some simplifications are required to maintain the proper distribution of dead loads. In order to do this, the area of the floor plan consisting of drop panels was compared to the entire floor plan area of the modeled portion of the structure. The ratio of these areas was determined to be approximately 19.3% according to the dimensions of the model. This 19.3% was then used to determine how much additional thickness should be added to the 8 in. slab uniformly to account for the dead loads of all drop panels. The additional thickness of the drop panels was 3.375 in. 19.3% of this value was determined to be approximately 0.65 in., which is rounded to 0.75 in.

The slabs were then made a constant 8.75 in. thick throughout the model. This change accounts for all dead loads and creates a slab stiffness that is between the two extreme cases previously shown which would represent and accurate behavior of the structure. The difference in axial loads between the model including drop panels and the model with only 8.75 in. slabs were found to have approximately 3% error due to the rounding. Tables 5.3 and 5.4 show the first floor column axial loads of the detailed model with the drop panels and the detailed model with the 8.75 in. slabs, respectively. Removing the drop panels from the detailed model and modifying the slabs to have an 8.75 in. thickness ultimately preserves the accuracy of the loading conditions and results while simultaneously simplifying the time needed for modeling and performing analysis by drastic amounts.

<table>
<thead>
<tr>
<th>Axis</th>
<th>H</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
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<td>100.3</td>
<td>87.5</td>
<td>56.5</td>
</tr>
</tbody>
</table>

Table 5.3: Axial loads in first floor columns of detailed model with drop panels (kips)
When the detailed model was created, it was necessary to select a model size that was smaller than that of the entire original structure. This was done for simplification purposes explained in Section 5.5. Maintaining continuous behavior around the boundaries of a model as its size is decreased is a major challenge. Shrinking the size of the model creates two new boundaries (the two sides that are not located on the actual edges of the structure). The columns on the new edges of the model must have approximately balanced moments in order to behave as a continuous structure. Considering this idea, the concept of ‘cut off’ edges is now discussed in order to determine the proper placement of boundaries and the proper way to balance the moments on the columns located at the new edges. Figure 5.8 shows an example section of the structure and labels which edges are actual structural edges and which are ‘cut off’ edges, or new boundaries, created to reduce the size of the model. The arrows pointing inward from the dashed ‘cut off’ edges show that they are moved in to create smaller models.

Two of the four sides of the detailed model are composed of the original outer edges of the structure. The other two sides were determined to be five bays in distance from the actual edges in order to create a square model with five bays in size in both directions. The two sides that were not the original sides are considered to be the cut off edges. Table 5.4: Axial loads in first floor columns of detailed model with 8.75 in. slabs (kips)

<table>
<thead>
<tr>
<th>Axis</th>
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<th>J</th>
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<th>L</th>
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<tr>
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<td>114.0</td>
<td>113.5</td>
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<td>105.3</td>
<td>104.8</td>
<td>90.5</td>
<td>57.9</td>
</tr>
</tbody>
</table>
edges since they are where the model creates a line to cut off part of the original structure. In order to create this continuous effect within the model, it is necessary to determine exactly where the edge of the model needs to be cut off. Since making the new edge right where the columns exist would not work because of the unbalanced moments, it is then necessary to determine how much of the slab should remain to create balanced forces and thus simulate continuous behavior. Too much overhang produces large moments bending outward because of the lack of support on the far side of the slab from the columns that were cut off. Too little overhang produces large moments in the opposite direction, similar to not having any overhang at all.

Figure 5.8: Different types of edges discussed for models

In an initial attempt, 50% of the slab span length was used in the detailed model. This percentage was used because when considering the tributary areas assigned to any given column, half of a span length on each side of that column is assigned to that column and should produce roughly balanced bending forces. Figure 5.9 shows that a
50% overhang was too large and ultimately produced large unbalanced bending moments in column lines I and J. This was because an overhang of 50% was too large and caused a large outward bending moment in column line I which was partially transferred to column line J. The figure shows how the center and original edge of the structure have very small moments in comparison to the cut off edge. Bending moments in upwards of 10 kip-ft were found on the cut off edge while the rest of the structure has moments of approximately to 1 or 2 kip-ft. A continuous structure would show these same small moments in all areas of the structure. Since the figure shows much larger moments on the cut off edge, it is known that the assumed overhang of 50% of a span length is incorrect for simulating continuous behavior in the structure.

Figure 5.9: Moments in a frame with 50% overhangs on cut off edges
In order to determine the exact overhang ratio needed to produce balanced moments in column lines I and J, as well as the other ‘cut off’ side of the model, typical structural engineering equations are used along with the known parameters of the structure. Figure 5.10 shows the equations for the moments at the ends of cantilever and fixed-ended beams. Figure 5.10a represents the cantilever condition which is how the cut off edges of the model behave. Figure 5.10b represents the fixed-fixed condition which is how the interior bays of the real continuous structure behave. In order to balance moments near the ‘cut off’ edges of the model, the moments produced by the slab or beam on each side must be roughly the same. The outermost rows of columns on the cut off edges of the model have the cantilever condition on the outside and the fixed-fixed condition. These conditions must be approximately balanced in order to produce low moments as shown in Figure 5.11.

Figure 5.10: Fixed end moment equations for a) cantilever, and b) fixed-fixed conditions

The ratio of the length of the cantilever to the length of the continuous span may be determined by equating the maximum moment equations. $L_1$ represents the length of the cantilever while $L_2$ represents the length of the continuous span. The distributed loading conditions, $P$, are the same on both sides since they represent the same slab and
thus can be cancelled from Equation 5.1. Figure 5.11 shows where $L_1$ and $L_2$ exist in the structure.

$$M_{max} = \frac{PL_2^2}{12} = \frac{PL_2^2}{2} \quad Eq(5.1)$$

Then, by solving Equation 5.1, a relationship between $L_1$ and $L_2$ can be obtained.

$$\frac{L_1}{L_2} = \sqrt{\frac{1}{6}} = 0.408 = 40.8\% \quad Eq(5.2)$$

With the percentage of overhang determined to be 40.8% of the span length, the model was redesigned to have its slab overhangs with a length of 40.8% of 20 ft, or 8 ft 2 in. Figure 5.11 shows the results of this change and how the moments on the cut off edge of the model are smaller and similar to those internally in the structure. The moments are difficult to see because they are almost nonexistent after using the 40.8% overhang length in the model. This is reasonable because a regular structure would experience relatively small internal moments under gravity loads. The moments on the cut off edges with the new 40.8% overhangs are almost exactly identical to those on the actual edge of the structure with magnitudes of 2 kip-ft or less. Ultimately, using an overhang length of 40.8% of the original span length proves to eliminate erroneous moments in most of the structure and helps accurately predict its behavior as if it were continuous.
5.8 Hand Calculation of Axial Loads

With the detailed model fully developed and the self-weight applied, loading conditions on the critical columns that were tested could be found. In order to assure no errors were made in the loading conditions or other steps of creating the model, some basic hand calculations were performed in order to check accuracy of these final column loads prior to any column removal. To perform these calculations, it was assumed that the loads in the structure were appropriately distributed according to the tributary areas of each column. Figure 5.11 shows the dimensions used to calculate the tributary areas of the columns and ultimately find the loads. To find these dimensions, all spans between columns were divided in half and loads were appropriately designated to the column to which they would be transferred. Appendix G contains the tables used for calculating all of the intermediate details of the loads including the tributary areas, volume, density, and the differences in column sizes for each floor.
After collecting the information from the SAP2000 model, it was found that the model produced axial loads on the critical first story columns within approximately 5% of the simple hand calculations. These results can be seen in Tables 5.5 and 5.6. The first floor loads were calculated because all of the testing was done in that area. This verifies that the self-weight factor was working properly in the SAP2000 model and that the dimensions were modeled accurately.

Table 5.5: Hand calculated loads on each first story column (kips)

<table>
<thead>
<tr>
<th>Axis</th>
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Table 5.6: Total dead calculated loads on each first story column calculated from SAP2000

<table>
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</table>
6.1 Introduction

To show the accuracy of the detailed model, a time history analysis of the structure was performed in SAP2000 (2010). The dynamic time history analysis could be performed in multiple stages, one for each simulated column removal. Since it was shown previously that the third column removal had relatively no effect on the strain gauge readings of column M10, it was not simulated. Also, since the detailed model has significant amount of data, it requires immense amount of time to run this type of step-by-step analysis. For the purpose of showing the accuracy of the detailed model, only the first stage of time history analysis is performed in this chapter. The more simplified models were analyzed in two stages similar to the experimental procedure with column M9 removed first (stage I), followed by a period of time to settle statically, and then column M11 was removed (stage II). This simulation and analysis gives a direct comparison of the calculated average strain of column M10 in the detailed model with the corresponding experimental test data.

6.2 Step by Step Procedure

The basic procedure for performing a time history analysis in SAP2000 is to start with the detailed model and: 1) delete the column which will be removed, 2) replace the
column with an upward joint load of the same magnitude as the axial force in the column that was deleted, and 3) use a time history function which will remove the load over a user defined time period to simulate the removal of the column. A step by step set of instructions is included below.

First, the axial loads that were present in the removed columns needed to be determined. In this example only column M9 is removed, however, in more simplified models both the first and second columns were removed as in testing. The axial load on column M9 which was removed in this example was 64.249 kips. This was determined from a basic static analysis performed on the detailed model before removing any columns.

Next, the column to be removed, which is M9 for this example, was selected and deleted. A load pattern was defined to represent the axial load from column M9 pushing up on the joint where the column had existed. The load pattern for this example was called ‘Column1’ to represent it was the load taking the place of the first removed column. Figure 6.1 shows the screen in SAP2000 for creating this load pattern.

![Figure 6.1: Load pattern to represent axial load in removed column](image-url)
Then the top joint location was selected from where the column was deleted and assigned a joint load. This load should be in the positive Z-direction (upward), due to the compressive axial force in the column (downward), and have an equal magnitude to the axial force in the deleted column. Previous sensitivity analyses have shown that the moment present in the top of the removed column due to gravity loads was considered negligible, less than 5% of the bending capacity, and thus is not represented in this example. This scenario also applies to the shear force value in the removed column. Under gravity loads, small shear forces resulted with magnitudes of less than 5% of the shear capacity of the column and thus were neglected as well. If the moment or shear values were of a large enough magnitude that they could not be neglected, they could be applied to the joint along with the vertical force representing the axial load in the column. This makes the structure behave as if the column is still intact and allows for the time history analysis of this support load being removed. As mentioned before, this load was 64.249 kips. Figure 6.2 shows the SAP2000 screen for creating this joint load.

![SAP2000 screen for creating joint load](image)

Figure 6.2: Upward joint load to represent removed column
Next, a time history function was defined to simulate the physical removal of the column in the model. The function started with values of 1 for a period of time to represent the structure with the created load acting as if it were the column, i.e. the upward joint load of 64.249 kips is multiplied by 1.0. Then at a specified time, the load multiplier was reduced to 0.0 to simulate the desired process of column removal. For this example, the load remained for 1.4 seconds at full capacity (multiplier=1.0). It then decreased linearly within one tenth of a second until a total time of 1.5 seconds. Finally the zero value of load continues until a total time of 3 seconds so that the structure has time to show its reactions and final static state after it settles down.

The load could be removed almost instantly or over a longer period of time. During actual testing, the majority of the column was crushed in a short period of time and the rest of the variations in the strain gauge readings were due to impact from the processor that had cut the column. Different lengths of time were tested with the gradual decrease of the joint load. When multiple seconds were used, no dynamic response was calculated because the removal of the load had occurred too slowly. Shorter removal time lengths than one tenth of a second would produce larger dynamic strains but would be less realistic in comparison to the real testing methods used. Figure 6.3 shows the screen for creating the time history function named ‘remove’ for this example as well as a graphical representation of the function.
Next, a load case was defined where the analysis was changed from static to dynamic time history and the load pattern created for the column to be removed was linked with the created time history function. The ‘direct integration’ method was used in SAP2000 for all dynamic time history analysis. It was necessary to include the dead loads of the structures self-weight with a constant time history function. Not accounting for the self-weight of the structure would cause the results of the analysis to be incorrect. With both types of loads (column load and all dead loads) linked to their respective time history functions, SAP2000 could then calculate all the loads and remove the assigned column load according to the time history function that was created for column removal. It was necessary to ensure that the time step size and number of times steps create an equal amount of time for analysis as the length of the time history function. For this example, 300 steps of 0.01 second intervals were used to create a total analysis time of 3 seconds which matches the time history function length. Finally, a damping ratio needs
to be defined for dynamic analysis. The structure for this study was made of reinforced concrete and therefore a damping ratio of around 1%-2% can be appropriate to represent this building. A damping ratio of 1% was chosen for use for all remaining models. Figure 6.4 shows the SAP2000 screen for creating the load case named ‘remove column’ for this example with dynamic analysis with direct integration.

![SAP2000 screen for creating the load case named ‘remove column’](image)

Figure 6.4: Load case definition for time history analysis

### 6.3 Comparison of Analysis Results

Absolute vertical displacement of the joint on the top of column M10 is calculated and shown in Figure 6.5. In order to compare the vertical displacement in Figure 6.5 to those in the experimental data, the absolute displacement is divided by the length of the
column to obtain the axial strain in the column. The calculated displacement is divided by the length of the column, 144 in., to obtain the average axial strain. These values could then be multiplied by 1,000,000 to change them into units of strain (\( \cdot 10^6 \)) which are the units of the experimental test results. The nearly constant displacements between 0.30 and 1.37 seconds, and 1.8 and 3.0 seconds in Figure 6.5 correspond to static equilibrium states. Displacement cycles after t=0 and t=1.5 seconds are due to dynamic behavior. The dynamic event of the column removal is between 1.5 and 2.0 seconds. The dynamic results between 0.0 and 0.3 seconds on the plot are caused by the loads of the structure being applied during the first time step occurs since they cannot be placed at zero time. SAP2000 does not allow placement of loads at negative times or at t=0 and thus the loads can only be applied during the first time step of the time history analysis. This is verified in the plot since there is no strain calculated at t=0 and it rapidly increases during the first time step as the gravity loads are applied. SAP2000 treats this as a dynamic although the change in strain during that time period simply represents the pre-existing strains that would have been in the structure prior to testing.

**Figure 6.5:** Vertical joint displacement time history above column M10 in the detailed model

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Figure 6.6 compares the measured test data and the model results with both data sets initialized to a value of zero since the pre-existing strain in the test structure was unknown. It can be seen that the overall change in steady state strain values before and after column removal match with great accuracy. The dynamic results did not match as well. The test data shows three large changes in strain within the test data that occurs over multiple time periods and drops much further than the calculated results. It is assumed that this was caused by the processor’s claw as it was pushing and pulling on the remains of the column and caused the large strain fluctuations before finally settling to the steady state. There were also assumptions made for the damping ratio as well as the duration of time for the column’s removal which could greatly change the dynamic strain results. Since this study is mostly focused upon the changes in static or permanent deformations, dynamic results are still shown but not discussed further. To more easily compare the results of the model simulation with a simplified version of the test data, a plot was created which used the average values of the static strains over long periods of time in Figure 6.7.

![Figure 6.6: Comparison of measured and calculated strain history in column M10 in the detailed model](image)
While some differences exist, it can clearly be seen that the SAP2000 model accurately predicts the permanent deformations or strain difference measured during the experiment. The overall measured change in static strain was \(-27 \cdot 10^{-6}\) for the experimental results and an average of \(-27.78 \cdot 10^{-6}\) for the SAP2000 results which is less than a 3\% difference. A difference of less than \(1 \cdot 10^{-6}\) could be caused due to fluctuation in the accuracy of the strain gauges. This proves the great accuracy of the model, especially considering the many assumptions and changes made to create a model of just a portion of the original structure.

### 6.4 Conclusions of the Detailed Model

Through the results obtained in the time-history analysis of the detailed model, it is shown that the model very accurately represents the behavior of the entire structure and matches the test data obtained within less than 3\% error. With the individual assumptions already justified, this analysis has verified the ability to combine the assumptions into a large model that maintains accuracy in analysis while reducing time for modeling and analysis of an entire structure.
CHAPTER 7

SIMPLIFICATION OF DETAILED MODEL

7.1 Introduction

The main objective of this study is to provide a simplified generic model that can be used in practice to design or determine the ability of a structure to resist progressive collapse. It is envisioned that this goal can be achieved by modeling and analyzing only a small vulnerable portion of the structure instead of the entire structure. The previous chapters described how a detailed model of an actual test structure was created as in SAP2000 and analyzed through a time-history function. This chapter shows the process of simplifying the detailed model much further to its simplest form. Sensitivity analyses and other methods of justification are provided with the various steps of simplification including the changing of support conditions, reduction of number of floors, reduction of number of bays in both directions, and the use of equivalent beams to represent thick slabs. Also included is some intermediate time-history analysis from SAP2000, which shows that the evolving model remains relatively accurate compared to test data collected from the Swallen’s building considering the magnitude of the simplifications.

7.2 Simplification of Support Conditions and Effect of Basement

The basement of this structure provides multiple opportunities for simplification to the detailed model. The two main concerns to be addressed with this area are the
support conditions of the very bottom of the columns as well as how the first floor, the test area, is affected by having a basement floor below it. Figure 7.1 shows the elevation view of the test area along with the spacial relationship between the columns removed and the actual supports of the structure prior to the simplification proposed to the detailed model. All frames in the structure had the same elevation dimensions and thus Figure 7.1 represents all frames in the structure, not a single specific frame. Figure B.2 in Appendix B shows these dimensions in the actual blue prints of the structure.

![Figure 7.1: Elevation view of structure with fixed column supports](image)

### 7.2.1 Removal of the Basement Columns

In order to determine how the basement affects the behavior of the structure in the first floor level, it is necessary to perform analysis in SAP2000 with the model under gravity loads as well as with the loading conditions of when columns are removed.
Under pure gravity loads prior to removing any columns, the only effects the basement columns would have on the first floor would be through moments at the top and bottom of the columns. Figure 7.2 shows the basement columns as well as the first floor columns in the location where the highest moments existed. Even with fixed restraint conditions, the moments at the bottom of the basement columns are less than 0.4 kip-ft due to application of gravity loads to the detailed model. The tops of the basement columns and the bottoms of the first floor columns have moments with magnitudes between 2 and 3 kip-ft. Once again, compared to the bending capacity of these columns (on the order of 100-120 kip-ft minimum), this is negligible.

![Figure 7.2: Basement and first floor column moments under gravity loads](image)

The presence of very low moments at the bottom of the basement columns is beneficial for simplification purposes because it helps to show that the existence of this basement floor has almost no effect on the rest of the structure while under gravity loads.
The more unknown scenario is how the basement reacts when a first floor column is removed. In order to represent an extreme case for this study, columns M9 and M11 were removed from the model and the static loads are shown in Figure 7.3.

The static moments after removal of two columns, shown in Figure 7.3, are very similar to those in Figure 7.2 due to the gravity load case where the moments all remain insignificantly small in the basement columns and bottom of the first floor columns. The moments in the tops of the basement columns directly under the removed columns increased but only to between 5 kip-ft and 7 kip-ft. This is still very small comparatively and can be neglected in comparison to the capacity of the columns as well as the larger bending moments present in the same area of the structure. The moments at the bottoms of the basement columns are approximately 2 kip-ft or less. This further proves that the basement columns are not necessary to the model and thus are removed as a simplification when analyzing the models from this point forward.

Figure 7.3: Basement and first floor column moments with removed columns (M9 and M11 in the first story) and fixed restraints
7.2.2 Effect of Support Conditions

Another result to be noticed from the previous section is that the moments are so small in the basement columns, as well as at the bottom of the first story columns, that they could almost be assumed to have zero value. This would essentially represent the condition of having pinned restraints at the base of the structure rather than fixed conditions under gravity loads or column removals. This is true whether the basement or the first floor is used as the base of the structure since they both have very small moments at their base. Figure 7.4 shows how the pinned restraint conditions essentially produce the same results when applied to the detailed model as the fixed restraint condition in Figure 7.3.

The moments in the tops of the basement columns with pinned conditions are about 4 kip-ft to 5 kip-ft. The moment values and Figures 7.3 and 7.4 show that for such a large and detailed model, the use of fixed conditions or pinned conditions for the base supports make very little difference except for simplicity of performing the calculations and time needed for analysis.

Another research study mentioned in Chapter 2 have taken advantage of this effect and modeled their multistory buildings as if they have pinned restraint conditions at their base in order to make analysis simpler (Masero, 2010). This change in restraint conditions would have been another early step in simplifying the detailed model for this study, however, when a very small and simplified model is reached, pinned conditions would no longer accurately represent the behavior of the structure. This is because very simple models would lose their stability as their total number of degrees of freedom is reduced. In studies such as this where a structure is being represented by a simplified model, the change from fixed restraints to pinned restraints would become inappropriate. However, in studies where analysis is based on the macro level with large portions of the structure remaining in the model, pinned conditions would be an acceptable simplification to make. The detailed and simplified models in this research use fixed restraints at the base of the structure, however, pinned supports could have been used in the detailed model without compromising much accuracy.
7.3 Elimination of Upper Floors

A major step in simplifying the model is to reduce its total height to be represented by a single floor. In order to maintain accurate loading conditions, the gravity loads of the removed floors must still be applied to the structure. The previous section has shown how very small the moments throughout the structure were under gravity loads with magnitudes below 7 kip-ft. This suggests that the only major loading contribution to the first floor structure is through axial loads since the bending moments in lower floors are relatively small. For this reason, the loads from all above floors can be represented as point loads pushing down from above directly on each column. Since it was already determined that the simple hand calculations were very close to the results calculated from the SAP2000 program, it would be acceptable to use the hand calculations or SAP2000 analysis results to determine the magnitude of the loads from the upper floors. To do this, all areas within the tributary area of each individual column must be calculated and transferred downwards until the total vertical loads applied to the
first floor columns are found. These values are then used as the point load values described. Figure 7.5 below shows the model after removing the upper floors and replacing them with them with equivalent point load forces.

![Model](image)

**Figure 7.5: Single floor model showing column M9 removed**

This simplification is justifiable for a few reasons. The first is that there are essentially no moments from the gravity loads that need to be accounted for in the model. With the total axial load still accounted for and distributed properly according to tributary areas, all initially applied forces are represented. The second way to show this is acceptable is to run the analysis of the gravity loads on the structure and compare the axial loads in the bottom columns from the detailed model to those in the single floor model. Table 7.1 shows the resulting axial loads in the single floor model. Through comparison with the first floor axial loads of the detailed model, it was found that the results of this single floor model are within an average of 4% of the detailed model which is very close for such a large simplification. For example, the total axial load on column M11 was 49.1 kips in the detailed model. The axial load on column M11 in the simplified single floor model is 46.9 kips.
Finally, it should be noted that the big difference between the two models is that simplifying the structure down to one story removes some of its ability to transfer loads to further members. A general rule is that shorter structures are less resistant to progressive collapse than larger ones because there are more potential load paths and more possibilities for load redistribution within a larger structure. Sasani and Sagiroglu (2010) suggest that taller structures have a larger reserved capacity to handle redistributed loads with. It would seem that taller structures would be more susceptible to progressive collapse because of the larger weights but actually the abundance of structural elements, or redundancy, allows for more redistribution and transfer of loads that need to be accounted for after a column is lost. This basically means that the simplified model represents a more vulnerable structure than the actual structure which is a good conservative assumption to have within analysis models. This idea is shown and proven in the following section through the time history analysis of this single floor model.

### 7.4 Single Floor Model Time History Analysis

Using the same methodology developed in Chapter 6, the single floor model can be analyzed for its accuracy in the situation of column removal. All steps were duplicated from that process (Section 6.2) and it was determined that the axial load of the column to be removed, column M9, was 64.25 kips. Performing the analysis on this model provides the results in Figure 7.6 which show the vertical displacement of the top of column M10 with respect to time.
By performing the same unit conversions as before (Section 6.3), a comparison of the SAP2000 single floor model data and the actual test data is shown in Figure 7.7. The simplified steady state comparison is also shown in Figure 7.8 where the results can be seen to be similar to those of the detailed model. There is a slight difference in the amount of change in strain for single floor model. The experimental and computed strains are similar and the computed strains remain more conservative than the experimental strains. The average residual strain for the single floor model is at $-31.25 \cdot 10^{-6}$ as opposed to $-27 \cdot 10^{-6}$ for the experimental results and $-27.78 \cdot 10^{-6}$ for the detailed model. This model is still within an average of less than 16% difference of the test data which is reasonable considering how simplified the model is. This strain difference of only $4 \cdot 10^{-6}$ can once again be partially due to the experimental measurements or various modeling and analysis assumptions made.

Figure 7.6: Vertical joint displacement time history above column M10 in the single floor model
Figure 7.7: Comparison of measured and calculated strain history in column M10 in the single floor model

Figure 7.8: Simplified comparison of average measured strains and calculated strains in column M10 in the single floor model
This small increase in the amount of strain in column M10 is because a structure with a single floor is more vulnerable to effects of progressive collapse than a taller building due to having fewer members to transfer loads. Changing the model to a single floor results in approximately 16% higher calculated axial strain values than the test results. These strains are approximately 13% higher than the original detailed model which is an acceptable conservative difference. When used in practice, the resulting calculated strain values would be slightly higher than those that should be actually expected in the structure. This indicates slightly higher axial compressive forces on the column than those that exist according to the test data.

7.5 Reduction of Bays and Intermediate Model

With the entire model reduced to a single floor with vertical point loads acting on columns, the number of bays can be reduced to achieve a smaller and more simplified model. In order to do this, a similar assumption can be made as the one used to determine the model size where continuity between the spans is broken (Section 5.7). As long as the loading conditions along the assumed boundaries of the model appear to act as if the structure is continuous, the behavior of the structure can be accurately represented. This also correlates to the assumption from reducing the number of floors because having fewer members to transfer loads makes the model behave more conservatively than the real structure. Thus, reducing the model down to a much simpler form can help it becoming more conservative while maintaining as much accurate behavior at the same time as possible.

Figure 7.9 shows a model developed in the process of reducing the number of bays in analysis which is called the ‘intermediate model’ since it falls between the size of the detailed model and the simple model. It consists of a single floor with point loads acting on the columns to represent the forces coming from the above floors. The drop panels are brought back into the models at this point since the models are small enough that local member behavior could contribute more to the overall analysis of the model.
This model is developed with two rows of four columns rather than five rows of five columns which reduces the area to that of which the loads are mostly transferred within. This choice of size was made by viewing the slab stress diagram produced as a result of the single floor model. The maximum moments in the slabs before and after column removal can be seen in Figure 7.10.
The top half of Figure 7.10 shows the slab moments prior to column removal where loads are evenly spread and the highest moments can be seen to be concentrated within the drop panels near the columns. The lower half of the figure shows the slab moments after simulation of the column removal. Most of the model acts similar to how it did before column removal. The moments are mostly equal and concentrated around the locations of the columns. However, the effects of removing the column can be seen mostly between the edge of the structure and the second row of columns as well as being between the columns on each side of the removed column. Forces transferred through the bending of the thick slab are contained in this same area.

After adjusting the slab widths and point loads in order to creating loading conditions that match the previous models created, analysis was performed and the resulting vertical displacement diagram shown in Figure 7.11 was obtained. These adjustments were made to sustain the 40.8% overhang length while maintaining the total vertical loads in the structure.

![Figure 7.11: Vertical joint displacement time history above column M10 in the intermediate model](image)
Figure 7.12 shows the calculated results of the model compared to the test data. Compared to previous diagrams, a larger change in static strain occurs before and after the column removal. This was expected since a smaller model with fewer floors and now less bays offers less members for the forces to be transferred to. The average change in static strain for this model is about $-36.81 \cdot 10^{-6}$ as opposed to the $-27 \cdot 10^{-6}$ of the test data. This means that the intermediate model results are roughly 36% more conservative than the test data for the axial strain in column M10. Less error would be preferred although it is still an acceptable predictor of the structure’s behavior considering that the boundaries of the model are approaching directly into the area where forces are being redistributed as shown in Figure 7.10. It must also be noticed that while there is a 36% difference, the magnitude of difference is less than $10 \cdot 10^{-6}$. The difference is acceptable because the simplifications made provide increasingly conservative results in comparison to the test data. Figure 7.13 shows the simplified relation of the model results to the test data.

![Graph showing comparison of measured and calculated strain history in column M10 in the intermediate model](image)

Figure 7.12: Comparison of measured and calculated strain history in column M10 in the intermediate model

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7.6 Replacing Slabs with Equivalent Beam Elements

This section focuses on making the SAP2000 models more general to different types of building structures. For example, this study concentrates on a structure that consists of thick slabs and no beams. Since the majority of structures consist of beams and thin slabs, they would transfer moment and torsional loads differently between elements. A thick slab, such as that in this structure, has a much more complicated and less understood behavior. In order to make the most simplified model as general as possible, it would be beneficial to determine a method for creating an equivalent beam element that would act with similar properties to that of the thick slab. Changing the slab elements into equivalent beam elements will make the model more general and adaptable for use in many different types of structures while preserving much of the 3-dimensional behavior the slab.

Before creating any type of equivalent elements, it is necessary to know the existing bending and torsional stiffness of the thick slab. Following the equations from

Figure 7.13: Comparison of calculated and measured average strains in column M10 in the intermediate model
ACI 318 Code Section 9.5.2.3 shown in Appendix I, the effective moment of inertia of the slab was calculated to determine the most realistic stiffness attributes of the slab. The actual values from the plans of the test structure are used to show how to perform these calculations.

SAP2000 uses this moment of inertia value to determine the bending stiffness of the slab. This calculation is performed in both directions and for all slab sections remaining in the mid-size model. This is done because a beam is need in both directions to connect the columns and represent the different behaviors of the slab for each axis. It is also done because the dimensions in the different sections varied which changed the moment of inertia calculations. Another set of equations from ACI Code Section 13.7.5.1 in Appendix J show how to find the torsional stiffness of the slab between the columns. Once again, actual values from the plans of the test structure are used to show how to perform these calculations.

A comprehensive set of tables of the vital values for these calculations is included in Appendix K. After determining the values of effective moment of inertia for the thick slab, the process for creating an equivalent beam can begin. The main factors to achieve are matching the bending stiffness, torsional stiffness, and axial stiffness of the beam with that of the slab. The mass and weight can be accounted for using modification factors in the elements or by making the beams have no self-weight and applying a dead load across the elements appropriately.

The proposed method for achieving equivalent stiffness is to create a typically dimensioned beam section which results in a moment of inertia approximately the same as that calculated for the slab. The simplest way to do this is to make a beam section that has a depth of approximately double the depth as the thickness of the slab. Then the beam width should be between half of the beam depth and the full beam depth in order to give the beam a depth to width ratio of between one and two. This is considered typical for concrete beam design. After the values are chosen, it is necessary to check the moment of inertia of the section to see if it matches that of the slab. Increase or decrease
the dimensions while remaining within the depth to width ratio range until the moment of inertia value of the equivalent beam matches that of the slab approximately.

For example, the original slab could have a depth of 8 in. and an effective moment of inertia of 4200 \text{ in.}^4. To make an equivalent beam section, start with a beam that has double the depth of the slab, 16 in. The width of the beam should be between half of the depth of the beam and the full depth of the beam. With a width of 8 in., the moment of inertia is only 2730 \text{ in.}^4. Since the moment of inertia needs to be close to 4200 \text{ in.}^4 the width must be increased. Increasing the beam width to 12 in. results in a moment of inertia of approximately 4100 \text{ in.}^4. Increasing this value to the next nearest quarter of an inch, 12.25 in., results in a moment of inertia value of approximately 4200 \text{ in.}^4. Since the depth to width ratio of the beam falls between a value of one and two, the process of creating an equivalent beam is complete. Had the depth to width ratio not been between one and two, the depth would need to be accordingly increased or decreased. Then the process of selecting a beam width and calculating the moment of inertia would repeat until all stated requirements are met.

It is also necessary to match torsional moments of inertia. In order to do this, a modification factor can be applied to the beam element to force it to behave in torsion as if it were the slab. This modification factor will depend on the dimensions and geometry chosen for the equivalent beam element. The ratio of the torsional stiffness calculated for the slab to the torsional stiffness of the equivalent beam is the modification factor that should be used for the beam’s torsional constant. For example, if the slab has a torsional stiffness of 850 \text{ in.}^4 and the equivalent beam has a torsional stiffness of 400 \text{ in.}^4, then the modification factor applied to the torsional stiffness of the equivalent beam element in SAP2000 would be 2.125.

The axial stiffness can be modified by a similar process. The ratio of the effective area of the slab to the area of the equivalent beam should be used as the modification factor for the beam’s area. Finally the weight of the slab needs to be compensated for in the model. Since the slabs were two-way slabs they act as triangular loads along the
length of the beam with no load at the end points and the peak load occurring in the middle.

In order to test the effectiveness of this proposed method for creating equivalent beam elements, another model was developed using the intermediate model but with equivalent beam elements instead of the slabs. This model is called the ‘intermediate stick model’ and maintains the principles described in Section 5.7. As described in Section 5.7, the overhanging portions are necessary to maintain realistic moments and axial loads in the columns of the model. The model can be seen in Figure 7.14. Figure 7.15 shows an example of the loading conditions applied to the equivalent beams which represents that of the two-way slabs in the actual structure.

The results of the time history analysis for the vertical displacement of column M10 are shown in Figure 7.16 while the simplified comparison of the model results to the test data is shown in Figure 7.17. The simplified comparison of the results of this model with the experimental data is shown in Figure 7.18.

Figure 7.14: Intermediate stick model developed in SAP2000
Figure 7.15: Loading conditions on the intermediate stick model

Figure 7.16: Vertical joint displacement time history above column M10 in the intermediate stick model
The average difference between the final results of the model and the test data was that the model produced 36% larger strain values than those found experimentally. This is in high agreement with the intermediate model which also produced a difference.
of about 36% larger results in the model than the test data. This shows that the equivalent beam elements provided a model that differed by less than 1% from the model with the slab elements when compared to the test data. The mechanism for the three-dimensional behavior of a thick slab is very complicated and hard to predict and thus creating equivalent beams that provide behavior within less than 1% error is acceptable.

7.7 Extended Validation

As the error of this mid-size stick model has grown to 36% and the simplification process is almost complete, it is necessary to show some extra validation of the model to prove it can accurately demonstrate the behavior of the structure. In order to do this, the remaining data for columns L10 and L11 can be used to show that the calculated results agree with all of the data rather than only the first set of data. This helps to prove that the model and the numerous assumptions made thus far accurately represent the data of 14 strain gauges in three locations rather than only 4 strain gauges from a single location. Also to help validate the model, data from the second column removal will be compared with the model results. This can even better prove the model represents actual behavior of the structure because removing multiple columns causes complicated transfers of loads that are even more difficult to predict.

First, to remain consistent with previous analysis, the calculated results and data for column M10 is compared with a simulation of the first two columns removed in sequence. The procedure for this simulation is the same as for removing a single column except that there is an additional time function, load pattern, and load placed on the corner column (columns M11) which was the second column to be removed. Also, the load case needed to be modified to include the removal of the second column in the same manner as the first column removal. Figure 7.19 below shows the results obtained from the intermediate stick model for column M10.

It can be seen that the first change in displacement of the column closely represents that shown previously for column M10 because they are the same model removal. A small difference existed due to the force added in place of the corner column which was used for the second column removal. The second column removal is now
shown and is compared to that of a simplified representation of the test data in Figure 7.20.

Figure 7.19: Vertical joint displacement time history above column M10 in the intermediate stick model with 2 columns removed

Figure 7.20: Comparison of calculated and measured average strains in column M10 in the intermediate stick model with 2 columns removed
For column M10 in the intermediate stick model, the calculated results were 27% larger than the test data after the first column removal. The calculated results were 23% smaller than the test data after the second column removal. The magnitudes of the differences were approximately $8 \cdot 10^{-6}$ for the first column loss and $17 \cdot 10^{-6}$ for the second column loss. While this comparison shows that the model can predict the general behavior of two column losses, there is less accuracy in the change in strain for the second column loss than the first. This was expected due to the difficulties of predicting where loads are transferred within the structure. It should also be noted that after the first column loss, the calculated strain results are considered conservative because they give larger strains than the test data. However, after the second column loss, the calculated strains are smaller than those observed in the test data which is not conservative.

This same model was analyzed and then compared to the data collected for column L10. Column L10 was initially not chosen for use with model development due to the complicated nature of the three-dimensional bending and torsional behavior of the thick slab. Another reason for not initially using this data was that the magnitudes of the strains were much smaller than the other data sets. This means any errors in the accuracy of the strain gauges could more highly affect the results. Figure 7.21 shows the results obtained from the mid-size stick model for column L10.

Figure 7.22 shows the simplified comparison of the calculated results to the test data for column L10. Once again, even with very small strain magnitudes, the model can relatively accurately predict the general behavior of the structure through the loss of two columns. After the first column loss, the two results only differ by approximately $1 \cdot 10^{-6}$. After the second column loss, the two results differ by approximately $3 \cdot 10^{-6}$ on average. As discussed previously, differences on such a small magnitude could occur due to the accuracy limits of the strain gauges as well as error in observing the graphs. In this column, the results from the model provided higher strains for both column removals and thus are always conservative. This internal column is predicted with high accuracy from the model, especially considering that the model was not developed using this set of
data. This greatly helps to validate the model and all of the assumptions made so far because the data for column L10 was treated independently until this point.

Finally, the test data from column L11 is compared to the results of the model. The data set from this column was initially not chosen for model development due to its placement with respect to the order of column removals. Figure 7.23 shows the results obtained from the intermediate stick model for column M10.

Figure 7.24 shows the simplified comparison of the calculated results to the test data for column L11. The removal of the first column caused essentially no strain changes in this column. The second column removal showed similar results to that of column M10 since they are almost symmetrically located around the second column that was removed (columns M11). After the first column removal, the two results differ by approximately $1 \cdot 10^{-6}$. After the second column removal, the two results differ by approximately $10 \cdot 10^{-6}$ which is a 35% difference for this case. Similar to column M10, the model results were higher and conservative after the first column removal but lower than the test data after the second column removal. This again shows that the model can predict the first column removal with high accuracy. The strains in second column can be generally predicted but with less accuracy than the first column. This is due to the uncertainty in where loads were transferred during the first column removal compounded with the same uncertainty for a second column removal.
Figure 7.21: Vertical joint displacement time history above column L10 in the intermediate stick model with 2 columns removed

Figure 7.22: Comparison of calculated and measured average strains in column L10 in the intermediate stick model with 2 columns removed
Figure 7.23: Vertical joint displacement time history above column L11 in the intermediate stick model with 2 columns removed.

Figure 7.24: Comparison of calculated and measured average strains in column L11 in the intermediate stick model with 2 columns removed.
7.8 Conclusions

According to the extensive data validation and analysis shown, the assumptions used in the development of the computer models up to this point can easily predict the general trends in strain changes when the structure experiences multiple column losses. The strain changes due to the first column loss can be very accurately predicted and always gives almost exact or conservative results. The resulting strain changes for a second column loss can be predicted but with less accuracy. These changes can be predicted with results that are conservative for internal columns but too small for external columns. Since the final simplified model is used for the analysis of a single column loss, this chapter has validated the existing models and assumptions. Table 7.1 provides a comprehensive view of all major models developed so far, along with their accompanying assumptions and accuracies when compared to the test data for column M10 during the first column removal.

<table>
<thead>
<tr>
<th>Model Name</th>
<th>Error %</th>
<th>Assumptions</th>
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<tbody>
<tr>
<td>Detailed Model</td>
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8.1 Introduction

The final simplified model consists of approximately 10% of the area that of the detailed model, which is approximately 2% of the size of the actual entire structure. As discussed in Chapter 7, numerous simplification and conservative assumptions have been used to create this simplified model which still represents the behavior of the portion of the structure affected by column loss. Various assumptions from loading and restraint conditions to the number of floors and bays have been discussed at length. These assumptions used in the creation of the models have been shown to be justified. Also, a procedure for the development of equivalent beams to represent thick slabs has been provided and justified for making the simple models more general to various types of structures. Figure 8.1 shows a view of the final ‘simplified model’ of the building model investigated in this study. It consists of three columns surrounding a removed column whereas column M10 exists as one of the remaining columns in the model.

To reach this final state of the simplified model from the intermediate stick model described in Section 7.6, one last simplification was made. This simplification was to remove some of the columns which were not directly neighboring the removed column in the longitudinal or transverse directions. These remaining columns provide the majority
of additional support required in the portion of the structure where the column is lost. They are also the most likely to contribute to a progressive collapse phenomena by failing under large redistributed loads after column loss. For this reason, the columns neighboring the lost column are the most crucial columns that need to be analyzed for progressive collapse.

As in the intermediate stick model, the 40.8% length overhangs of the beams remain to simulate continuity and balance of the structural members so that the model behaves as if it were continuous within the structure. The negative moment created by a cantilever beam at a column support is almost the same as the negative moment at the end of the continuous beam (Section 5.7). Figure 8.2 shows the loads applied to the equivalent beams which represent the gravity loads from one-way and two-way slabs that existed in the test structure.

![Diagram](image)

Figure 8.1: Final simplified model developed in SAP2000
This simplified model consists of only the removed column area, the three remaining neighboring columns, and the equivalent beams directly connected to the remaining columns. With the model simplified to this magnitude, the time-history analysis was performed in less than 10 seconds as opposed more than an hour for the detailed model with drop panels included (Chapter 6). Since this model was developed in a way so that it could be used with different buildings, it can be used to save an immense amount of modeling and calculation time for engineers with different structures. For example, collapse vulnerability of a high-rise building due to loss of a first-story column can be quickly investigated using the simple model with three columns shown in Figure 8.1.

8.2 Time History Analysis of Simplified Model

Since the simplified model consists of only columns M8, M10, and L9, the only test data that can be used to verify the model is that from column M10. This was the same data that was determined to be best for developing the models and shown to justify
all of the previous assumptions made in Chapter 7 and thus is preferable for maintaining consistent comparisons with the test data. The intermediate stick model was verified using data for two column losses from all three instrumented columns (Section 7.7). This final simplified model is one simplification step away from that intermediate stick model and thus can be verified properly with only the experimental data from column M10 since all of the more detailed models were compared in detail with all of the experimental data. Figure 8.3 shows the vertical displacement above column M10, calculated from SAP2000 after performing the same time-history analysis on the simplified model as all previous models (Section 6.2). Figure 8.4 shows the calculated results from Figure 8.3 compared to the test data. The simplified comparison of the results from Figure 8.3 with the experimental data is shown in Figure 8.5 when the column is suddenly removed at a time of 1.5 seconds.

Figure 8.3: Vertical joint displacement time history above column M10 in final simplified model
Figure 8.4: Comparison of calculated and measured average strains in column M10 in the final simplified model

Figure 8.5: Comparison of calculated and measured average strains in column M10 in the final simplified model
The results from the simplified model were expected to follow a similar trend to that of the previous models where accuracy was lost as more simplifications were made. This was expected because generally as more assumptions are made to a model the error between calculated and measured results grows. The results from SAP2000 for the first column loss had been consistently growing more conservative as simplifications were made in those previous models in Chapter 7. The final simplification in developing the final simplified model provided the opposite trend in results. The results for the simplified model almost matched that of the test data. The final simplified model shows a residual strain of \(-28.47 \times 10^{-6}\) while the test data had a residual strain of \(-27 \times 10^{-6}\). This is a difference of only about 5% with a magnitude of less than \(2 \times 10^{-6}\). The results from the model were more conservative than the experimental results, which is consistent with all previous models for the first column removal.

These final simplified model results are very similar to those of the detailed model which consisted of many more bays in both directions, all four floors, as well as the thick slab shell elements rather than the equivalent beams (Figure 5.5). The magnitude of the initial and final static strain differed slightly, with higher strains in the simplified model, but the change in strain due to column M9 removal was nearly identical. This helps to show that the assumptions made to produce the proposed final simplified model seem to be acceptable.

### 8.3 Discussion of Results

As mentioned in Section 8.2, the results from SAP2000 for the simplified model were more accurate than expected. The trend from previous models had shown that as more simplifications were made, the results became less accurate and more conservative. The final simplification made to the model produced the opposite effect. This was determined to be caused by the removal of the last columns (those that were not M8, M10, or L9). The simplified model only contains the columns directly neighboring the removed column while the intermediate stick model had four additional columns in further remaining bays (Figure 8.6a). These extra columns provided continuity of the structure and maintained the bending behavior of continuous beams in the structure.
Since the final simplified model has fewer columns, this attribute was lost. The absence of continuous behavior allowed the extended portion of the beam M10-11 to rotate more freely and transferred less compressive force to act on the column where strain measurements were taken. The smaller compressive force acting on column M10 then causes smaller compressive strains to occur. This idea is illustrated in Figure 8.6. Figure 8.6a shows the intermediate stick model, where column M11 is holding the end of the beam back downward and causing compression on column M10. Figure 8.6b shows the simplified model, where column M11 does not exist which allows the beam to rise and rotate upward, causing less compressive force on the column M10.

This loss of continuous beam bending behavior explains why the calculated strains in column M10 were smaller than expected after the final simplification. Removing the extra columns from the intermediate stick model caused the column M10 to experience less axial compressive force, and thus smaller compressive strain. The magnitude of this loss in compressive force from the beam bending coincided with the gain in force experienced from the various simplifications made to the models in Chapter 7. It is assumed that the initial simplifications, mostly the reduction in size between the single floor model and intermediate model, caused the higher strain values in the results that existed prior to completing the final simplified model. The final simplification removed columns in the transverse direction of column M9 that thus had the opposite effect on the change in error between the calculated results and test data. This balance of gained and lost forces due to simplifications resulted in final calculations that were very similar to the test data.

Future research may show that different structures do not produce results as accurate as those in this study because the final simplification may not balance the gained and lost forces as well as the structure investigated in this study. This could be due to the use of different dimensions or material properties than those used in this research. In such a case, and for quickly modeling the progressive collapse response of a multi-story building, a general simplified method is proposed in the following section.
Addition of Springs for Continuity

As discussed in the previous section, it is likely that future studies following the proposed method of this research will find larger errors in the calculated results of the final simplified model. In such cases, this study proposes the addition of spring elements to the final simplified model. The final results of this study have a relatively small error. However, if future research shows a trend that the proposed method consistently provides larger or smaller results than the test data, this format for creating a spring model can be used to modify and correct the final simplified model. Izzuddin (2008) previously expressed the idea of using springs in simplified models of structures for progressive collapse analysis and it is shown in Figure 8.7.
Figure 8.7: Use of springs in simplified progressive collapse modeling and analysis as proposed by Izzuddin (2008)

Figure 8.8 shows the ‘spring model’ as proposed by the current study. The spring model is derived directly from the final simplified model. There are two simple changes made to the final simplified model in order to create the spring model. The first change is to calculate all forces and moments contributed by the loads existing on the overhang beam sections. As done in any other transfer of forces, the distributed loads can be calculated and changed into a point load and set of moments acting directly on the beam-column connection. In this model, the loads are all in the vertical direction. The vertical forces are added to the previous point load acting down on the column which represented the loads from the above floors that were removed. The centroid of the distributed loads is then calculated and used to determine the moments that act on the beam-column connection in both directions. A time-history analysis was performed after making this transfer of loads and the results were found to perfectly match those of the final simplified model.
The second change is the addition of springs to the beam-column joints of the three columns surrounding the removed column. These springs are to be designed to account for the loss of rotational and continuous behavior in the beams. SAP2000 allows for these springs to be designed in two different ways. The left side of Figure 8.9 shows the simple approach which allows for the inputs of rotational and translational stiffness in all three dimensions. Including springs in the SAP2000 model can provide better continuous beam bending behavior and more realistic results. The right side Figure 8.9 shows the advanced menu which allows for the entire stiffness matrix of the spring to be entered. This study proposes using the more simple approach of entering only the translation and rotational stiffness values. This is because these stiffness values will account for most continuous beam behavior while maintaining the simplicity of the model.
The spring elements are to be placed at the top of the three remaining neighboring columns in the model. The coordinate system should be set as global so that all springs can be based upon a constant coordinate system that is visible using the axis in SAP2000. Next, in order to properly calculate the translation and rotational stiffness values, the following equations can be used where $A$ is area of the beam cross section, $E$ is modulus of elasticity of normal weight concrete ($= 57,000 \sqrt{f_c}$), $L$ is length of the beam, and $I$ is moment of inertia of the beam cross section.

$$k_{translation} = \frac{AE}{L} \quad Eq(8.1)$$

$$k_{rotation} = \frac{EI}{L} \quad Eq(8.2)$$

The beams being used to calculate the stiffness values for these equations are those that were removed in the first step of creating the spring model (beginning of Section 8.4). The full original length of the beams is to be used, not the shortened length of the overhangs. A sample calculation for the springs on column M10 based upon
Figure 8.10 is provided below. If necessary, the calculations for the stiffness matrix can be performed by using the stiffness matrix equations given in Appendix L.

![Figure 8.10: Sample calculation for translation and rotation stiffness values](image)

Beam 8: \( A = 144 \text{in.}^2, \quad f'_c = 3750 \text{psi}, \quad L = 240 \text{in.}, \quad l = 3072 \text{in.}^4 \)

Beam 9: \( A = 170 \text{in.}^2, \quad f'_c = 3750 \text{psi}, \quad L = 168 \text{in.}, \quad l = 4094 \text{in.}^4 \)

The x-translation stiffness is calculated using the properties of beam 9. The y-translation stiffness is calculated using the properties of beam 8. There is no spring for the z-translation since column M10 exists in the model. The x-rotation stiffness is calculated using the properties of beam 8 because bending around the x-axis passes through beam 8. The y-rotation stiffness is calculated using the properties of beam 9. The z-rotation stiffness is not considered.

\[
E = 57,000\sqrt{f'_c} = 57,000\sqrt{3750 \text{psi}} = 3490 \text{ksi}
\]

\[
k_{x-translation} = \frac{170 \cdot 3490}{168} = 3532 \text{kip/in.}
\]
The current study is accurate enough, with only 5% error, that creating the spring model would not be necessary. However, to prove that the spring model can work with relative accuracy and provide different results, a time-history analysis was performed on the spring model using the data from the current study.

Figure 8.11 shows the vertical displacement above column M10, calculated from SAP2000 after performing the time-history analysis. The spring model shows a residual strain of -26.18·10^{-6} while the test data had a residual strain of -27·10^{-6}. This is a difference of only about 3% with a magnitude of less than 1·10^{-6}. This shows great accuracy although the calculated strain is slightly less than the measured strain. This analysis shows that the spring model can prove to be very accurate when compared to the test data. Future research may find it useful to modify this model in order to create the best possible fully comprehensive model available for fast and simple progressive collapse analysis of structures. Figure 8.12 shows the simplified comparison of the experimental data with the analysis results.

\[
k_{y\text{-translation}} = \frac{144 \cdot 3490}{240} = 2094 \text{ kip/in.}
\]

\[
k_{x\text{-rotation}} = \frac{3490 \cdot 3072}{240} = 44,672 \text{ kip \cdot in.}
\]

\[
k_{y\text{-rotation}} = \frac{3490 \cdot 4094}{168} = 85,048 \text{ kip \cdot in.}
\]
8.5 Generalized Method of Creating Simplified Models

The previous chapters have described and justified the development of simplified models. However, an engineer using the proposed method does not need to follow every
step performed thus far in this study to create a similar simplified model. With all major assumptions validated and models shown to be accurate in comparison to the measured test data, the following steps provide a generalized method for creating a simplified model for any building being analyzed for progressive collapse. Figure 8.12 shows the generalized layout of the final simplified model.

1.) Collect all existing data about the structure to be modeled.
   a. Determine the materials used and their properties.
   b. Determine the type of structural elements and their dimensions and locations.
   c. Determine all loading conditions including magnitudes and locations.
2.) Determine which perimeter column is to be removed, and which neighboring column is to be analyzed for changes in axial strain.
3.) Perform hand calculations to determine basic loading conditions.
   a. Calculate tributary areas for the column that will be removed as well as the neighboring columns.
   b. Calculate the total tributary area loads (from step 1.c) that each of these columns support.
4.) Sum all calculated tributary loads acting above the removed column and neighboring columns as single point loads acting down on each column (e.g., Figure 8.1).
5.) Account for three dimensional slab bending effects.
   a. For flat plate structures, transform slabs into equivalent beams.
   b. For conventional RC beam-column frame structures, include the effect of slabs in beam modeling (not investigated in this study).
6.) Calculate a 40.8% beam overhang length for all beams connected to the neighboring columns except those three connected above the removed column (e.g., Figure 8.12).
7.) Using a structural analysis program, create the simplified model (e.g., Figure 8.2).
   a. Model the removed column as well as the three neighboring columns.
b. Model the beams, or equivalent beams, that connect these columns to each other.

c. Model the remaining beam overhangs with a 40.8% span length.

d. Apply loads on beams and hand calculated point loads to the columns.

8.) Perform time-history analysis and obtain response of beams, columns, or joints when the middle perimeter column is removed.

![Diagram of the final simplified model](image)

Figure 8.13: Generalized layout of the final simplified model

In order to advance the final simplified model to the spring model, the following extra steps must be performed. Figure 8.13 shows the generalized layout of the spring model.

9.) Remove the beam overhangs and replace the loads with equivalent transferred point loads and moments directly to the columns.

10.) Calculate the proper stiffness of springs and add them to the remaining columns of the model.
11.) Perform time-history analysis where the proper column is removed and the strain response in the measured column is obtained.

Figure 8.14: Generalized layout of the spring model
CHAPTER 9

SUMMARY AND CONCLUSIONS

9.1 Introduction

This research has performed an investigation and discussed at length the progressive collapse behavior of a reinforced concrete building. The Swallen’s building in Middletown, Ohio was instrumented with 14 strain gauges on three columns prior to being demolished. Three edge columns were selectively removed from the structure and the response from the strain gauges was monitored. The data was then compiled and analyzed to determine the physical events occurring within the structure during the column removals.

In effort to provide useful information to practicing engineers, a computer model was developed in SAP2000 to test the ability to predict how strains changed as columns were removed from the structure. In order to do this, a detailed model was created which would represent the test area of the structure along with the proper loading and constraint conditions present in the experiment. In order to save time and effort for engineers, multiple simplifications were made to the detailed model. These simplifications included reducing the four-story building to be represented by a single floor, reducing the floor plan down to a size of three bays in width and a single bay in depth, and changing the thick slabs of the test structure to be represented by equivalent beam elements. The
various models and assumptions were verified multiple times using most of the test data available. As the number of assumptions increased, the error between the calculated strains and the test data increased from 3% up to a maximum of 36%. Ultimately, a final simplified model was developed and shown to provide results within 5% of the test data.

This study has provided a lot of useful data as well as many ideas and procedures for future uses with modeling structures to investigate progressive collapse behavior. This chapter further summarizes and explains the results of the research performed.

9.2 Limitations of Research

While the response behavior of structural members and frames in this study are carefully investigated through computer analysis, it is important to recognize that this is the first attempt at developing a simplified model for progressive collapse analysis for practicing engineers. Many assumptions were made in order to simplify the modeling and analysis processes and attain reasonably accurate results. All loads were considered in a single load combination where no load factors were applied. In static analyses, actual dead loads were used without GSA’s recommended load factor of 2.0 for progressive collapse analysis. The inclusion of these factors could cause the stress to leave the elastic range and exhibit nonlinear behavior. Differences between experimental and predicted data was typically considered negligible when less than 5%. It is possible for multiple errors that were considered negligible to compound and effect analysis results more significantly than assumed. As assumptions accumulated while models were simplified, the response was overestimated as high as 36%. The difference between measured and calculated response must be recognized and accounted for with the use of any portion of this research. While most of the assumptions were attempted to be justified and conservative, the final model simplification had a counteractive effect which balanced out the conservative assumptions to attain a response with high accuracy. Data from the experimental testing of the Swallen’s building has been provided and shows that the model for this structure has reasonable accuracy. However, different sizes and types of structures may provide less accurate results than those found in this study. More
buildings with existing test data should be modeled through the proposed procedure to determine if the assumptions are valid for various types and sizes of structures.

The results obtained in this study should still prove to be beneficial for saving time and accuracy while analyzing structures for progressive collapse. Efforts were made to widen the applicable use of the results by reducing the original structure into a simplified model using general assumptions that can be applied to other types of buildings with different designs.

9.3 Summary of Results

This study has gone into great depth and detail to obtain the results that have been presented and provide new insight into the topic of progressive collapse in structures. General formulations and assumptions were made whenever possible and the limitations of the procedures and assumptions made have been described. Many useful results and observations have been made through the efforts of this study. This section will summarize each major contribution to the research of progressive collapse that has been made by this research.

9.3.1 Step-by-Step Experimental Procedure

Appendix C includes a detailed step-by-step procedure for properly performing experimental testing on a structure to acquire useful data for progressive collapse research. These procedures are useful when strain gauge instrumentation is required for testing a reinforced concrete type of structure. The instrumentation process included removing the cover concrete, grinding the longitudinal steel rebar, cleaning the steel and applying an adhesive, applying the catalyst to the strain gauge, and finally attaching the strain gauges.

The steps for the physical removal of the columns during testing are shown in Appendix F. The number and location of removed columns, as well as the manner in which they were removed are all described.

9.3.2 Selection of Model Size
A detailed model was developed to capture the behavior of the building during removal of multiple columns. In the detailed model the basement, floor slabs, drop panels, and support conditions were modeled as accurately as possible. The computational demand was high due to large size of the model which took over an hour to analyze for a single column removal. Chapter 5 provides a method for determining the required size of a detailed model which can accurately represent the behavior of an entire structure without modeling that entire structure. The effects of a removed column from a structure are shown to be essentially contained within two bays in each direction of the removed column.

9.3.3 Modeling Drop Panels

A method for effectively modeling thick slabs with drop panels was developed in Section 5.6. This procedure is useful for analyzing a structure with slab elements as well as drop panels around the column locations. A sensitivity analysis was performed and showed that the drop panels generally had no effect on the behavior of the structure when analyzed with relatively thick slabs for progressive collapse with a removed column.

9.3.4 Slab Overhang Lengths

The proper length for modeling overhangs of slabs to model realistic and continuous behavior throughout a model was determined to be 40.8% of the span length through the use of known structural engineering principles in Section 5.7. This span length ratio is useful for balancing internal member moments, and axial loads when reducing the size of a model. Ultimately, the use of overhang slabs/beams allows for a smaller model while capturing continuous structural behavior.

9.3.5 Step-by-Step Time History Analysis Procedure

Chapter 6 provides a detailed procedure for performing dynamic time-history analysis of a structure for progressive collapse with a removed column using the SAP2000 program. The creation of load patterns, joint loads, time history functions, load cases, and joint displacement functions were described at length.

9.3.6 Restraint Conditions
It was determined in Chapter 7 that large models of a structure, at least five bays in width in each direction, could be modeled with pinned or fixed conditions at the base of the lowest columns with essentially no difference in analytical results after a column was removed from the structure. Large models could benefit from using pinned conditions to reduce analysis time. Smaller models require the fixed end condition because the reduction in the number of degrees of freedom would eventually become a noticeable factor as the model reduced in size.

9.3.7 Reduction of Number of Floors

Chapter 7 also shows that for progressive collapse analysis it is justifiable to reduce a multi-story model, including a basement, into a single floor model with axial loads acting on each column. Hand calculations suffice to find the loads of the upper floors by using proper tributary areas for each column. This simplification is acceptable because it maintains axial loads while most moment values within critical columns are negligible. This also provides a conservative safeguard since it removes elements from the structure to where loads could be potentially redistributed after column removal. Floor reduction makes a conservative model that is greatly simplified and is useful to perform simpler progressive collapse analyses on multi-story buildings.

9.3.8 Reduction of Number of Bays

Using the same assumption as the proper length for a slab overhang, overhanging slabs with a span length of 40.8% was used in Chapter 7 when reducing the number of bays in the model. This allowed the model to become more simplified while still capturing the behavior of the structure. It also provided a more vulnerable model since more structural elements that could have supported redistributed loads were removed. This is again useful to further reduce the model size of a structure for progressive collapse analysis.

9.3.9 Equivalent Beam Procedure

A procedure to replace thick slab shell elements with equivalent beams was developed in Chapter 7. This was done to simplify the analysis since most structures consist of beam-columns rather than thick slabs to transfer loads. The method details a
series of steps for creating a beam with equivalent bending, axial, and torsional stiffness as the thick slab. This preserved the slab behavior and transformed the model into a more general one and allows for the direct comparison of different models. This procedure is useful to model and analyze “flat slab” structures.

9.3.10 Final Simplified Model

Ultimately, all of the research was directed at creating a single simplified model to be used for progressive collapse analysis of most building structures by practicing engineers. This was successfully achieved and discussed in Chapter 8. The difference between the experimental and calculated axial strains was $1.5 \cdot 10^{-6}$, a 5% difference. This finally concluded the success of a procedure to develop a simplified general model for quick progressive collapse evaluation of a structure.

9.3.11 Spring Model

Chapter 8 also proposes a procedure to be used for correcting error in proposed models by adding spring elements to the columns of the simplified model in order to simulate better structural continuity. The springs are intended to replace the continuous beam bending behavior that was lost in the final simplification to the model. A time-history analysis showed that the spring model can achieve results that more closely represent the data obtained from experimentation. This is useful for future researchers who are validating and improving the simplified model proposed in this study by using data from new test structures.

9.3.12 Summary of Simplified Model Development

The procedure for creating the final simplified model included many modeling assumptions and observations discussed in Chapters 5 through 8. Figure 9.1 shows a flowchart representing the major steps taken in developing the simplified model in this study. The various model names appear on top and the simplifications used to create each following model are listed below within each circle.
With the development of each model, a time-history analysis was performed in order to compare and evaluate the accuracy of each model with the test data. Figure 9.2 compares the results of the different models developed in this study with the test data. The detailed model had an error of less than 3% due to few assumptions and included essentially all of the area of the structure that was affected by removing column M9. The single floor model had an error of less than 16%. The growth in error was mainly due to the removal of two upper floors and the basement and conversion of gravity loads in upper floors into point loads acting downward on a single floor model of the structure. The intermediate model had an error of about 36%. This growth in error was due to the reduction in the number of bays in the model. Some of the continuity of the structure was lost when the size was reduced to the bounds of the area of the structure that was affected by column removal. The intermediate stick model also had a 36% conservative error. This model incorporated the use of equivalent beams in order to remove the thick slabs and create a more generalized beam-column model. Since the error of this model
matched that of the mid-size model, it was concluded that the procedure for developing beams with equivalent behavior as thick slabs was accurate and effective. The final simplified model had an error of approximately 5%. This reduction in error was caused by the final exclusion of extra columns in the creation of the final simplified model. With only three remaining columns, one on each side of the removed column, the model sacrificed the continuous beam bending behavior that was incorporated in the larger models.

The resulting errors from all previous models had been calculated to be larger than the test results, meaning they were conservative. All assumptions made until the final simplification contributed to the conservative error. The final exclusion of columns in the simplified model caused an opposite effect on the accuracy of the results. As described in Section 8.4, the loss of continuous beam bending behavior resulted in lower compressive forces in the measured column and thus it experienced smaller compressive strains. The conservative errors counteracted with the effects of this loss to produce final results with a 5% conservative error which was determined to be very accurate with the amount of simplifications made to develop the model.

![Figure 9.2: Comparison of measured and computed average axial strains from all models](image)

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9.4 Summary and Future Research

Future developments are encouraged to be made on this model and the procedures used for creating it. More experimental data should be collected as in this study and compared with the proposed methods to further evaluate this method’s accuracy and improve the assumptions and simplifications. Various factors such as different structure geometry, materials, and loading conditions should be considered in the selection of new test structures as well as the comparison of analysis with this study. The effects of using various load factors on the applied loads of the model should also be investigated to determine how the simplified models behave with factored loads applied and if nonlinear analysis is necessary.

The ultimate goal of this project was to provide data that would give insight into ways to help prevent progressive collapse of structures. Through the information provided in this study, including experimental procedures and methods of creating and simplifying computer models for analysis, this goal has been achieved. While it seems that the assumptions and judgment used in this study are sound with the general ideas used in structural engineering, more physical tests must be conducted to confirm the accuracy of the methods, models, assumptions, and simplifications proposed in this study. Sources of error or problems in the development of the models may be evaluated in this process and lead to important changes or limitations. Only after more tests have been conducted and the results have been perfected can these models and procedures be considered for widespread use in practice but hopefully the steps taken in this study are the first critical steps to lead towards greater safety in design and prevention of progressive collapse failures in the future.
REFERENCES

ACI Committee 318. 2011. Building Code Requirements for Structural Concrete (ACI 318-11) American Concrete Institute, Farmington Hills, MI


APPENDIX A

EMAIL SENT TO DEMOLITION COMPANIES

Dear __________,

Hello, my name is Justin Morone and I am a civil engineering student at The Ohio State University. I am performing some research for a Master’s thesis under Dr. Halil Sezen. I received your contact information from another student, Kevin Giriliris, who had contacted you in the past and indicated you may be able to help us with further projects. I am performing a study on the topic of progressive collapse and am looking for a building mostly of a basic rectangular layout with members made of reinforced concrete or preferably steel. I was wondering if you would be demolishing any buildings in Ohio, possibly near Columbus, of this type in the next few months and if you would be willing to help out with this project. If you would be willing to help or would like more information on the project I would gladly give you the information you need for what I will be doing. My contact information is listed below. Your response would be extremely appreciated either way. Thank you very much for your time and I look forward to hearing from you soon.

D. Justin Morone

Figure A.1: Standard e-mail sent to demolition companies
APPENDIX B

BUILDING PLANS/BLEUEPRINTS

Figure B.1: Floor 1 plans

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Figure B.2: Exterior elevation plans
Figure B.3: Floor 1 slab details
Figure B.4: Wall sections and details
Figure B.5: Roofing plans
Figure B.6: Section details
Figure B.7: Miscellaneous details including beam reinforcement
APPENDIX C

INSTRUMENTATION PROCEDURE

Removal of Concrete Cover Material

Concrete removal over column rebar was the most laborious step of the preparation for testing. A total of 14 strain gauges were attached on 12 longitudinal bars in 3 columns near the mid-height of the columns. The strain gauges were mainly used to measure uni-axial strains in the longitudinal bars. The recorded strains could then be used to determine stresses in the bars during testing and variation of axial and bending stresses in the instrumented columns.

A jackhammer was used to remove concrete that was over the reinforcement. The locations for placing the strain gauges were previously spray-painted in the proper locations, generally around mid-height of the column. Mid-height of the column was preferable gauge placement to determine column axial loads (strains) assuming the bending moments would be the smallest in that location under possible lateral loads during testing. To more accurately measure the moments, the strain gauges could have been placed near the top and bottom of the column where the maximum strain differences would occur on opposite sides of the column, indicating a moment. Figure C.1 shows the author using the jackhammer to remove concrete cover along the four primary axis of the circular column.
This process of removing concrete cover was performed on the four sides of three separate columns which were chosen in locations to acquire the most useful results for research purposes. The location choices are explained later. Column L10 (Figure 3.2) was prepared for two extra strain gauges because the data ports and gauges were available and more data helped to verify the consistency of the results.

**Grinding the Steel Rebar**

Once the concrete cover was removed and the steel bars were visible, grinding could take place. This step involved using a steel grinder to create a smooth surface on the reinforcing bars so the strain gauges could adhere well. The grinding wheel would be placed against the steel bar and moved in an up and down motion until a smooth and flat surface of about one inch long and half an inch wide was created (Figure C.2). This provided adequate of space for the sensor to be placed in a desirable position.
Cleaning Surface and Applying Adhesive

Once a smooth area was obtained on the rebar, the steel needed to be cleaned to ensure the adhesive would work to hold the strain gauge in place. A cleaning chemical was sprayed on the smooth area then removed with a cloth. Next an adhesive was placed on the same area of the rebar.

Applying Catalyst

Once the steel was prepared, the next step was to attach the strain gauges to the bars. Prior to the day of preparation, Ullom soldered the strain gauges to the wires and labeled them appropriately for attachment to the columns and computer. The catalyst was brushed to the side of the sensor which would be placed against the steel (Figure C.3). It would then react with the adhesive to create a stronger bond between the steel and the sensor.
Attaching the Strain Gauge

The next step was to apply tape to the opposite side of the sensor as the catalyst and place it against the steel, using the tape to hold the strain gauge in place until the adhesive became strong enough to hold it in place. The gauge needed to be pressed firmly until the tape and adherent were strong enough to keep the sensor from moving. Figure C.4 shows the strain gauge properly attached to the steel rebar with tape and adhesive.
APPENDIX D

STRAIN GAUGE LOCATIONS AND FIELD NOTES

Table D.1: Strain gauge locations

<table>
<thead>
<tr>
<th>Strain Gage Number</th>
<th>Column</th>
<th>Location</th>
<th>Horiz. Dist. from Centerline</th>
<th>Side of Centerline*</th>
<th>Height</th>
<th>Concrete Cover</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>L-10</td>
<td>NW</td>
<td>1/2&quot;</td>
<td>L</td>
<td>6'-11/2&quot;</td>
<td>2 1/2&quot;</td>
</tr>
<tr>
<td>2</td>
<td>L-10</td>
<td>NE</td>
<td>0&quot;</td>
<td>R</td>
<td>6'-2&quot;</td>
<td>2&quot;</td>
</tr>
<tr>
<td>3</td>
<td>L-10</td>
<td>SE</td>
<td>1&quot;</td>
<td>R</td>
<td>6'-2&quot;</td>
<td>3&quot;</td>
</tr>
<tr>
<td>4</td>
<td>L-10</td>
<td>SW</td>
<td>1/2&quot;</td>
<td>L</td>
<td>6'-2&quot;</td>
<td>2 3/4&quot;</td>
</tr>
<tr>
<td>5</td>
<td>L-10</td>
<td>NW</td>
<td>1/2&quot;</td>
<td>L</td>
<td>2'-11&quot;</td>
<td>2 1/2&quot;</td>
</tr>
<tr>
<td>6</td>
<td>L-10</td>
<td>SE</td>
<td>1&quot;</td>
<td>R</td>
<td>2'-9 1/2&quot;</td>
<td>3&quot;</td>
</tr>
<tr>
<td>7</td>
<td>M-10</td>
<td>NW</td>
<td>3&quot;</td>
<td>L</td>
<td>4'-6&quot;</td>
<td>2 1/2&quot;</td>
</tr>
<tr>
<td>8</td>
<td>M-10</td>
<td>NE</td>
<td>2&quot;</td>
<td>L</td>
<td>4'-6 1/2&quot;</td>
<td>3 1/4&quot;</td>
</tr>
<tr>
<td>9</td>
<td>M-10</td>
<td>SE</td>
<td>1 1/2&quot;</td>
<td>R</td>
<td>4'-6 1/2&quot;</td>
<td>2 3/4&quot;</td>
</tr>
<tr>
<td>10</td>
<td>M-10</td>
<td>SW</td>
<td>1&quot;</td>
<td>R</td>
<td>4'-6&quot;</td>
<td>2&quot;</td>
</tr>
<tr>
<td>11</td>
<td>L-11</td>
<td>NW</td>
<td>1/2&quot;</td>
<td>R</td>
<td>4'-6&quot;</td>
<td>3&quot;</td>
</tr>
<tr>
<td>12</td>
<td>L-11</td>
<td>NE</td>
<td>1&quot;</td>
<td>R</td>
<td>4'-6&quot;</td>
<td>2 1/2&quot;</td>
</tr>
<tr>
<td>13</td>
<td>L-11</td>
<td>SE</td>
<td>1/2&quot;</td>
<td>R</td>
<td>4'-6&quot;</td>
<td>2&quot;</td>
</tr>
<tr>
<td>14</td>
<td>L-11</td>
<td>SW</td>
<td>1&quot;</td>
<td>R</td>
<td>4'-5 1/2&quot;</td>
<td>2 1/2&quot;</td>
</tr>
</tbody>
</table>

* Looking NW or NE
Table D.2: Test log of events during data collection

<table>
<thead>
<tr>
<th>Date:</th>
<th>7/27/2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location:</td>
<td>Swallen's Building - Middletown, Ohio</td>
</tr>
<tr>
<td>Test:</td>
<td>Strain Gage / Column Removal</td>
</tr>
<tr>
<td>By:</td>
<td>Greg Ullom</td>
</tr>
<tr>
<td>Assisted:</td>
<td>Justin Morone</td>
</tr>
<tr>
<td>Contractor:</td>
<td>Steve Rauch</td>
</tr>
<tr>
<td>Contact:</td>
<td>Scott Wells</td>
</tr>
<tr>
<td>Description:</td>
<td>instrumentation of 3 columns and removal or adjacent walls, columns, etc.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time:</th>
<th>Elapsed Time (min):</th>
<th>Occurrence:</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:19</td>
<td></td>
<td>Start Recording (Begin StrainSmart Readings)</td>
</tr>
<tr>
<td>10:20</td>
<td>0:01</td>
<td>Excavator begins &quot;munching&quot; on NW wall</td>
</tr>
<tr>
<td>10:28</td>
<td>0:09</td>
<td>Completion of wall demolition</td>
</tr>
<tr>
<td>10:29</td>
<td>0:10</td>
<td>Break (discussion)</td>
</tr>
<tr>
<td>10:33</td>
<td>0:14</td>
<td>First Column Removed (M-9)</td>
</tr>
<tr>
<td>10:36</td>
<td>0:17</td>
<td>Second Column Removed (M-11)</td>
</tr>
<tr>
<td>10:37</td>
<td>0:18</td>
<td>&quot;munching&quot; on west SW wall</td>
</tr>
<tr>
<td>10:39</td>
<td>0:20</td>
<td>Third Column Removed (K-11)</td>
</tr>
<tr>
<td>10:46</td>
<td>0:27</td>
<td>Square Column Removed (G.19)</td>
</tr>
<tr>
<td>10:48</td>
<td>0:29</td>
<td>Square Column Removed (G.20)</td>
</tr>
<tr>
<td>10:54</td>
<td>0:35</td>
<td>Stopped Recording</td>
</tr>
</tbody>
</table>
APPENDIX E

PREPARATION AND INSTRUMENTATION

Figure E.1: Outside view of test area
Figure E.2: Inside view of the building corner to be instrumented

Figure E.3: Depth of concrete removed
Figure E.4: All three instrumented columns

Figure E.5: Soldered wire connections
Figure E.6: Ullom running the data collection system
APPENDIX F

DEMOLITION PROCESS

Figure F.1: Excavator used for testing and demolition
Figure F.2: Tearing down the wall to gain access to the columns

Figure F.3: Cutting of first column
Figure F.4: Final state of first column after cut

Figure F.5: Cutting of second column
Figure F.6: Final state of second column after cut

Figure F.7: Cutting of third column
Figure F.8: Final state of third column after cut

Figure F.9: Back view of the structure
Figure F.10: View of slab and garage column connection

Figure F.11: Removal of garage column and wall
Figure F.12: Final back view of the structure

Figure F.13: Inside view after first and second columns removed
Figure F.14: Inside view after second and third columns removed
APPENDIX G

RAW TEST DATA FROM SENSORS

Figure G.1: Sensor 1 test data
Figure G.2: Sensor 2 test data
Figure G.3: Sensor 3 test data
Figure G.4: Sensor 4 test data
Figure G.5: Sensor 5 test data
Figure G.6: Sensor 6 test data
Figure G.7: Sensor 7 test data
Figure G.8: Sensor 8 test data
Figure G.9: Sensor 9 test data
Figure G.10: Sensor 10 test data
Figure G.11: Sensor 11 test data
Figure G.12: Sensor 12 test data
Figure G.13: Sensor 13 test data
Figure G.14: Sensor 14 test data
Figure G.15: Column M10 strain readings during first column removal
Figure G.16: Column M10 strain readings during second column removal
Figure G.17: Column M10 strain readings during third column removal
## APPENDIX H

### HAND CALCULATED LOADS

Table H.1: Column and drop panel weight calculations by hand

<table>
<thead>
<tr>
<th>Floor</th>
<th>Radius (ft)</th>
<th>Area (ft²)</th>
<th>Height (ft)</th>
<th>Volume (ft³)</th>
<th>Weight (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor 1</td>
<td>0.75</td>
<td>1.767</td>
<td>11.135</td>
<td>19.7</td>
<td>2.952</td>
</tr>
<tr>
<td>Floor 2</td>
<td>0.75</td>
<td>1.767</td>
<td>10.135</td>
<td>17.9</td>
<td>2.687</td>
</tr>
<tr>
<td>Floor 3</td>
<td>0.50</td>
<td>0.785</td>
<td>11.167</td>
<td>8.8</td>
<td>1.316</td>
</tr>
</tbody>
</table>

Floor 1, 2, 3

<table>
<thead>
<tr>
<th>Density (kips/ft³)</th>
<th>0.15</th>
</tr>
</thead>
</table>

Drop Panels

<table>
<thead>
<tr>
<th>Length (ft)</th>
<th>Width (ft)</th>
<th>Thickness (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>7</td>
<td>0.198</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Floor 1</th>
<th>Weight (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.455</td>
</tr>
<tr>
<td>Floor 2</td>
<td>1.455</td>
</tr>
<tr>
<td>Floor 3</td>
<td>0</td>
</tr>
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</table>
Table H.2: Slab and roof weight calculations by hand

<table>
<thead>
<tr>
<th>Column</th>
<th>H</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trib. Width (ft)</td>
<td>20</td>
<td>20</td>
<td>19.5</td>
<td>16.5</td>
<td>11.167</td>
</tr>
<tr>
<td>Trib. Length (ft)</td>
<td>Trib. Area (ft^2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>15.208</td>
<td>304.2</td>
<td>304.2</td>
<td>296.6</td>
<td>250.9</td>
</tr>
<tr>
<td>10</td>
<td>18</td>
<td>360</td>
<td>360</td>
<td>351</td>
<td>297</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>400</td>
<td>400</td>
<td>390</td>
<td>330</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>400</td>
<td>400</td>
<td>390</td>
<td>330</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>400</td>
<td>400</td>
<td>390</td>
<td>330</td>
</tr>
</tbody>
</table>

Floor 1, 2, 3

<table>
<thead>
<tr>
<th>Slab Density (kip/ft^3)</th>
<th>Thickness (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.667</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column</th>
<th>H</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trib. Weight (kips)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>30.4</td>
<td>30.4</td>
<td>29.7</td>
<td>25.1</td>
<td>17.0</td>
</tr>
<tr>
<td>10</td>
<td>36</td>
<td>36</td>
<td>35.1</td>
<td>29.7</td>
<td>20.1</td>
</tr>
<tr>
<td>9</td>
<td>40</td>
<td>40</td>
<td>39</td>
<td>33</td>
<td>22.3</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>40</td>
<td>39</td>
<td>33</td>
<td>22.3</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>40</td>
<td>39</td>
<td>33</td>
<td>22.3</td>
</tr>
</tbody>
</table>

Roof

<table>
<thead>
<tr>
<th>Slab Density (kip/ft^3)</th>
<th>Thickness (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.333</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column</th>
<th>H</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trib. Weight (kips)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>15.2</td>
<td>15.2</td>
<td>14.8</td>
<td>12.5</td>
<td>8.5</td>
</tr>
<tr>
<td>10</td>
<td>18</td>
<td>18</td>
<td>17.6</td>
<td>14.9</td>
<td>10.1</td>
</tr>
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<td>9</td>
<td>20</td>
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<td>19.5</td>
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<td>20</td>
<td>20</td>
<td>19.5</td>
<td>16.5</td>
<td>11.2</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>20</td>
<td>19.5</td>
<td>16.5</td>
<td>11.2</td>
</tr>
</tbody>
</table>
APPENDIX I

CALCULATION OF EFFECTIVE MOMENT OF INERTIA

\[ I_e = \left( \frac{M_{cr}}{M_a} \right)^3 I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \leq I_g \]

\[ I_g = \frac{bh^3}{12} \]

\[ I_{cr} = \frac{b\bar{y}^3}{3} + nA_s(d - \bar{y})^2 \]

\[ \bar{y} = \frac{nA_s \left[ \sqrt{1 + 2\frac{bd}{nA_s}} - 1 \right]}{b} \]

\[ M_{cr} = \frac{f_r l_g}{\gamma_t} \]

\[ \gamma_t = \frac{h}{2} \]

\[ f_r = 7.5\lambda\sqrt{f_c^i} \]

\[ M_a = \frac{wl^2}{12} \]

\[ n = \frac{E_s}{E_c} \]

\[ I_e = \text{effective moment of inertia} \]

\[ I_g = \text{gross moment of inertia} \]
\( I_{cr} \) = cracking moment of inertia

\( M_{cr} \) = cracking moment of cross section

\( M_a \) = actual moment of cross section

\( b \) = width of section

\( h \) = height of section

\( d \) = depth to center of tension steel

\( A_s \) = area of steel

\( w \) = distributed loading

\( \lambda = 1 \) for normal concrete or \( \lambda = 0.75 \) for lightweight concrete

\( f_c' \) = concrete compressive strength

The slabs in this study were comprised of various tributary widths, lengths, and reinforcing steel layouts. For the purpose of performing a sample calculation, a single slab section of the structure was chosen and the initial values and calculations are shown below.

\( n = 8, \ b = 98in., \ h = 8in., \ d = 7in., \ A_s = 2in.\^2, \ f_c' = 3750psi, \ w = 0.893 \frac{kip}{ft}, \ l = 20ft \)

\[
M_a = \frac{0.893 \times 20^2}{12} = 29.77k \times ft
\]

\[
y_t = \frac{8}{2} = 4in.
\]

\[
f_r = \frac{7.5 \times 1 \times \sqrt{3750}}{1000} = 0.459ksi
\]
Since the value of \( I_g \) must be less than or equal to the value of \( I_e \) it can be seen that the final value of \( I_e \) must be reduced to the value of \( I_g \). This could have been discovered by realizing that whenever the actual moment, \( M_a \), is smaller than the cracking moment, \( M_{cr} \), then the calculated \( I_e \) will be too high and always need to be reduced to the value of \( I_g \).
Table I.1: Effective moment of inertia from slab section BM in the plans

<table>
<thead>
<tr>
<th>fc'</th>
<th>3750</th>
<th>psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>As</td>
<td>2</td>
<td>in.^2</td>
</tr>
<tr>
<td>As'</td>
<td>0</td>
<td>in.^2</td>
</tr>
<tr>
<td>d</td>
<td>7</td>
<td>in.</td>
</tr>
<tr>
<td>h</td>
<td>8</td>
<td>in.</td>
</tr>
<tr>
<td>b</td>
<td>98</td>
<td>in.</td>
</tr>
<tr>
<td>n</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>w</td>
<td>0.89323</td>
<td>k/ft</td>
</tr>
<tr>
<td>L</td>
<td>20</td>
<td>ft</td>
</tr>
</tbody>
</table>

\[
I_g = b*(h^3)/12 = 4181.33 \text{ in.}^4
\]

\[
Y = \frac{1.35738}{12} \text{ in.}
\]

\[
cr = b*(Y^3)/3+n*As(d-Y)^2 = 591.124 \text{ in.}^4
\]

\[
fr = 7.5*\sqrt{fc'} = 0.45928 \text{ ksi}
\]

\[
yt = 4 \text{ in.}
\]

\[
M_{cr} = fr*I_g/yt = 40.0083 \text{ k*ft}
\]

\[
M_a = w*L^2/12 = 29.7743 \text{ k*ft}
\]

\[
I_e = 9301.69 \text{ in.}^4
\]

Table I.2: Effective moment of inertia from slab section BR in the plans

<table>
<thead>
<tr>
<th>fc'</th>
<th>3750</th>
<th>psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>As</td>
<td>2.2</td>
<td>in.^2</td>
</tr>
<tr>
<td>As'</td>
<td>0</td>
<td>in.^2</td>
</tr>
<tr>
<td>d</td>
<td>7</td>
<td>in.</td>
</tr>
<tr>
<td>h</td>
<td>8</td>
<td>in.</td>
</tr>
<tr>
<td>b</td>
<td>72</td>
<td>in.</td>
</tr>
<tr>
<td>n</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>w</td>
<td>0.65625</td>
<td>k/ft</td>
</tr>
<tr>
<td>L</td>
<td>20</td>
<td>ft</td>
</tr>
</tbody>
</table>

\[
I_g = b*(h^3)/12 = 3072 \text{ in.}^4
\]

\[
Y = \frac{1.62156}{12} \text{ in.}
\]

\[
cr = b*(Y^3)/3+n*As(d-Y)^2 = 611.458 \text{ in.}^4
\]

\[
fr = 7.5*\sqrt{fc'} = 0.45928 \text{ ksi}
\]

\[
yt = 4 \text{ in.}
\]

\[
M_{cr} = fr*I_g/yt = 29.3939 \text{ k*ft}
\]

\[
M_a = w*L^2/12 = 21.875 \text{ k*ft}
\]

\[
I_e = 6581.22 \text{ in.}^4
\]
Table I.3: Values for effective moment of inertia from slab section TA in the plans

<table>
<thead>
<tr>
<th>fc'</th>
<th>3750 psi</th>
<th>As</th>
<th>4.03 in.(^2)</th>
<th>As'</th>
<th>0 in.(^2)</th>
<th>d</th>
<th>7 in.</th>
<th>h</th>
<th>8 in.</th>
<th>b</th>
<th>98 in.</th>
<th>n</th>
<th>8</th>
<th>w</th>
<th>0.89323 k/ft</th>
<th>L</th>
<th>20 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ig = b*(h^3)/12</td>
<td>4181.33 in.(^4)</td>
<td>Y</td>
<td>1.84218 in.</td>
<td>cr = b*(Y^3/3+n*As(d-Y)^2)</td>
<td>1061.91 in.(^4)</td>
<td>fr</td>
<td>7.5*sqrt(fc')</td>
<td>0.45928 ksi</td>
<td>yt</td>
<td>4 in.</td>
<td>Mkr = fr*Ig/yt</td>
<td>40.0083 k*ft</td>
<td>Ma = w*L^2/12</td>
<td>29.7743 k*ft</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>le</td>
<td>8630.26 in.(^4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Table I.4: Values for effective moment of inertia from slab section TK in the plans

<table>
<thead>
<tr>
<th>fc'</th>
<th>3750 psi</th>
<th>As</th>
<th>1.4 in.(^2)</th>
<th>As'</th>
<th>0 in.(^2)</th>
<th>d</th>
<th>7 in.</th>
<th>h</th>
<th>8 in.</th>
<th>b</th>
<th>72 in.</th>
<th>n</th>
<th>8</th>
<th>w</th>
<th>0.65625 k/ft</th>
<th>L</th>
<th>20 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ig = b*(h^3)/12</td>
<td>3072 in.(^4)</td>
<td>Y</td>
<td>1.32835 in.</td>
<td>cr = b*(Y^3/3+n*As(d-Y)^2)</td>
<td>416.531 in.(^4)</td>
<td>fr</td>
<td>7.5*sqrt(fc')</td>
<td>0.45928 ksi</td>
<td>yt</td>
<td>4 in.</td>
<td>Mkr = fr*Ig/yt</td>
<td>29.3939 k*ft</td>
<td>Ma = w*L^2/12</td>
<td>21.875 k*ft</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>le</td>
<td>6859.23 in.(^4)</td>
<td></td>
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</tbody>
</table>
APPENDIX J

CALCULATION OF TORSIONAL MOMENT OF INERTIA

\[ K_t = \sum \frac{9E_{cs}C}{l_2(1 - c_2/l_2)^3} \]

\[ C = \sum \left[ \left(1 - 0.63 \frac{x}{y}\right) \frac{x^3y}{3} \right] \]

- \( K_t \) = torsional stiffness
- \( l_2 \) = transverse spans on each side of the column
- \( c_2 \) = size of equivalent rectangular column cross section
- \( x \) = shorter dimension of rectangular intersection of slab with column
- \( y \) = longer dimension of rectangular intersection of slab with column

The slabs in this study were comprised of various tributary widths, lengths, and reinforcing steel layouts. For the purpose of performing a sample calculation, a single slab section of the structure was chosen and the initial values and calculations are shown below.

\[ x = 10.375in., \quad y = 18in., \quad l_2 = 133in., \quad c_2 = 15.95in. \]

\[ C = \sum \left[ \left(1 - 0.63 \frac{10.375}{18}\right) \frac{10.375^3 \times 18}{3} \right] = 4267\text{in.}^4 \]
\[ K_t = \sum \frac{9 \times E_{cs} \times 4267}{133(1 - 15.95/133)^3} = 2 \left( \frac{9 \times 1 \times 4267}{133(1 - 15.95/133)^3} \right) = 847E_{cs}in.^4 \]

Table K.5: Torsional stiffness values in the 'x' and 'y' directions

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>10.375</td>
</tr>
<tr>
<td>y</td>
<td>18</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
</tr>
<tr>
<td>I2</td>
<td>133</td>
</tr>
<tr>
<td>c2</td>
<td>15.948</td>
</tr>
<tr>
<td>C</td>
<td>4267.463</td>
</tr>
<tr>
<td>Kt</td>
<td>847.245</td>
</tr>
</tbody>
</table>
APPENDIX K

SPRING STIFFNESS MATRIX

\[
k = \frac{EI}{L^3} \begin{bmatrix}
(\frac{AL^2}{I}) & 0 & 0 & (\frac{-AL^2}{I}) & 0 & 0 \\
0 & 12 & 6L & 0 & -12 & 6L \\
0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\
(\frac{-AL^2}{I}) & 0 & 0 & (\frac{AL^2}{I}) & 0 & 0 \\
0 & -12 & -6L & 0 & 12 & -6L \\
0 & 6L & 2L^2 & 0 & -6L & 4L^2 \\
\end{bmatrix}
\]

\( k = \text{stiffness} \)

\( E = \text{modulus of elasticity} \)

\( I = \text{moment of inertia} \)

\( L = \text{length of member} \)

\( A = \text{area of member} \)

The local stiffness matrix of a structural member is represented by the matrix shown. This matrix could be used if springs are added to the boundaries of the final simplified model in order to make the spring model proposed by this study. The values for moment of inertia, area, length, and modulus of elasticity are to be taken from the beams (or equivalent beams in this case) in the model that had been removed from columns M8, M10, and L9 and replaced with shorter overhangs. These beams would have provided additional stiffness to prevent motion if they had not been removed from the model. Thus, adding springs to these locations can provide additional stiffness and
restore more realistic continuous beam behavior to the model if needed. Using the proposed method of adding springs to the final simplified model is unnecessary in this specific study since the error of the final results is only 5%. However, a sample below shows how to properly calculate the stiffness matrix for a set of springs at one location given the following beam properties.

$L = 98\text{in.}, \quad A = 196\text{in.}^2, \quad I = 4200\text{in.}^4$

$$k = \frac{4200E}{98^3}$$

$$k = \frac{4200 \cdot 3490}{98^3}$$

$$\begin{bmatrix}
(196 \cdot 98^2 / 4200) & 0 & 0 & (-196 \cdot 98^2 / 4200) & 0 & 0 \\
0 & 12 & 6 \cdot 98 & 0 & -12 & 6 \cdot 98 \\
0 & 6 \cdot 98 & 4 \cdot 98^2 & 0 & -6 \cdot 98 & 2 \cdot 98^2 \\
(-196 \cdot 98^2 / 4200) & 0 & 0 & (196 \cdot 98^2 / 4200) & 0 & 0 \\
0 & -12 & -6 \cdot 98 & 0 & 12 & -6 \cdot 98 \\
0 & 6 \cdot 98 & 2 \cdot 98^2 & 0 & -6 \cdot 98 & 4 \cdot 98^2
\end{bmatrix}$$

$$\begin{bmatrix}
448.2 & 0 & 0 & -448.2 & 0 & 0 \\
0 & 12 & 588 & 0 & -12 & 588 \\
0 & 588 & 38416 & 0 & -588 & 19208 \\
-448.2 & 0 & 0 & 448.2 & 0 & 0 \\
0 & -12 & -588 & 0 & 12 & -588 \\
0 & 588 & 19208 & 0 & -588 & 38416
\end{bmatrix}$$