Novel Implementations of Ultrawideband Tightly Coupled Antenna Arrays

Dissertation

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By

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Abstract

Ultrawideband (UWB) phased arrays play an increasingly indispensable role in emerging communication and sensing systems. Multifunctional apertures, advanced radars, software-defined radios, electronic countermeasure systems, and radio telescopes all demand wideband beamforming front ends. Tightly Coupled Arrays (TCAs) have emerged as a very attractive option for these wideband systems, as they provide large bandwidths in low profile implementations. This work presents three novel TCAs which address the following challenges: 1) the need for increased bandwidth (> 10 : 1) in low-profile arrays; 2) the need for integrated baluns in these extremely wideband array designs; 3) the high component cost of large UWB arrays.

First, the Superstrate-Enhanced Substrate-Loaded Array (SESLA) is presented, which represents a novel scheme for bandwidth enhancement of UWB ground plane backed arrays. The SESLA employs resistive substrate loading to suppress destructive ground plane interference that limits array bandwidth. This translates to an improvement in bandwidth by a factor of two or more. Of course, if used alone, this loading can lead to severe degradation of radiation efficiency. However, a key innovation of the SESLA is that in addition to the loading, it uses a synergistically designed superstrate which dramatically mitigates the efficiency degradation. A very simple equivalent circuit is presented which is used to co-optimize the loading and
the superstrate. The SESLA approach is validated through measurements of a 4x4 prototype array.

As balanced feeding is a major challenge in UWB arrays, a version of the SESLA with an integrated feed is also presented. The design uses a stripline-based folded Marchand balun. At broadside scan, it is matched across a 13.9 : 1 bandwidth ($VSWR \leq 2.4 : 1$, infinite array) and shows good scanning capability out to $\theta = 45^\circ$. Extensive measurements of a prototype 8x8 array validating this design are presented.

Finally, an UWB beam-scanning horn antenna is presented. This design incorporates an UWB Tightly Coupled Dipole Array (TCDA) into a horn antenna to provide a high gain scanned beam with reduced element count, compared to a conventional phased array. For limited-scan applications, this translates to major savings in cost and complexity. The design is a truncated pyramidal horn fed by a TCDA, where the horn structure augments the gain of the array, but still allows some scanning. This allows for navigation of the design space between an UWB array and a fixed aperture. This design concept is validated with a measured prototype.
Dedicated to my family.
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Chapter 1: Introduction

1.1 Ultrawideband Phased Arrays: Applications and Existing Technologies

Ultrawideband\(^1\) (UWB) phased arrays have received considerable attention in recent years to meet the demands of emerging military and commercial communications systems [1–3]. Much of this attention has been driven by the development of multifunctional apertures. As the number of sensing and communications systems on mobile platforms continues to rise, there is a growing need to consolidate them into a single wideband aperture [4]. Since these systems may operate on widely separated frequency channels, these apertures demand scanning arrays with very wide bandwidths.

Advanced radar systems also demand very wideband phased arrays. Examples include foliage penetrating (FOPEN) radars [5] and through-wall imaging radars [6]. Other systems demanding wideband arrays include Software-Defined Radios (SDRs) [7,8], Electronic Warfare (EW) systems [1] and radio telescopes [9,10].

A number of antenna technologies have been developed in the last several decades to address the escalating demand for wideband arrays. The exponentially tapered slot

\(^1\)herein defined as \(f_{\text{max}}/f_{\text{min}} \geq 3\)
antenna (commonly referred to as the “Vivaldi” antenna) is, perhaps, the most widely used UWB array element [1–3]. The Vivaldi array element is an endfire radiator which essentially uses a tapered slotline to transform the impedance of a guided wave to that of free space over a wide bandwidth. First implemented in the 1970s [3], the Vivaldi array is a very mature technology that has been proven to provide bandwidth in excess of 10 : 1 and wide scanning capability. However, a major limitation of the Vivaldi antenna is its height, which is typically greater than λ/4 at the lowest operational frequency. For wideband applications operating in the lower microwave range (e.g., airborne FOPEN radars), Vivaldi arrays could be particularly obtrusive [1, 5]. Also, the Vivaldi array suffers from high cross-polarization levels when scanning [3] and phase center instability [11].

Connected arrays are growing in popularity as an alternative to the ubiquitous Vivaldi element [1, 12–14]. Although the concept was introduced in the 1970s [15], many practical implementations of these arrays for wideband systems have been developed in the last decade. These arrays typically employ electrically connected dipoles or slots. In the absence of a ground plane, these arrays could theoretically exhibit frequency-independent performance [16]. However, in the presence of a ground plane, these arrays are limited to about 4 : 1 bandwidth without material loading. Using ferrite-coated ground planes have been shown to improve the bandwidth to a decade or more [17,18] at the expense of efficiency and weight.
1.2 Tightly Coupled Arrays: an Emerging Paradigm in Ultrawideband Arrays

1.2.1 Overview and Operational Principles

Tightly Coupled Arrays (TCAs) are increasingly recognized as a transformational technology in the field of UWB phased arrays [2, 19–21]. These arrays are comprised of elements that are capacitively coupled to their neighbors. They provide wide bandwidths in low-profile implementations by using the inter-element capacitance to match out ground plane reactance. Thus, these arrays use mutual coupling as a design degree of freedom (rather than suppress it). As such, TCA elements are typically only analyzed in an array environment: since they depend strongly on array mutual coupling, their isolated element performance is generally irrelevant. This coupling also implies that truncation effects have a profound impact on the performance of small finite TCAs [22] (as is the case with other wideband array designs [23]).
As seen in the equivalent circuit in Fig. 1.1, a planar array placed over a ground plane is subject to a shunt reactance ($Z_{GP}$), which can be modeled as a shorted transmission line. In uncoupled low-profile arrays, this reactance hinders wideband matching [20]. However, in a TCA, the inter-element coupling counter-acts this reactance, matching the low-profile array over a wide bandwidth. To illustrate this, Fig. 1.2 gives the impedance of the parallel combination of $Z_{GP}$ and $\eta_0$ (as defined in Fig. 1.1). It is noted that, fundamentally, any ground plane backed array must match to this load. As seen, at lower frequencies, the load becomes highly inductive, while the real part declines. This presents an unfavorable matching condition, and generally implies poor low frequency/low profile performance for uncoupled ground plane backed arrays. However, as seen in Fig. 1.2, the addition of inter-element capacitance counter-acts the ground plane inductance, allowing the load to more easily be matched. As an example, Fig. 1.3 gives the infinite array VSWR of an uncoupled dipole array and that of a Tightly Coupled Dipole Array (TCDA). The two designs
are exactly the same, except that the latter has an appropriately chosen lumped capacitance between the tips of adjacent dipoles. As seen, the TCDA provides superior bandwidth, and is specifically improved at the low end of the frequency band.

### 1.2.2 Examples of Tightly Coupled Arrays

The Current Sheet Array (CSA) [19,20] is among the most well known examples of TCAs. It is simply an array of dipoles with interdigital capacitors at their tips, similar to the design depicted in Fig. 1.1 (right). It operates over a 4.5-5:1 bandwidth with a profile of roughly $\lambda_{low}/10$ [19,24,25] without material loading. The addition of dielectric superstrates (or radomes) can improve the infinite array bandwidth of the CSA to nearly a decade [19]. The CSA is part of a common subset of TCAs classified as Tightly Coupled Dipole Arrays (TCDAs). The simplicity and wideband, low-profile performance of TCDAs make them very attractive for a number of applications. Not surprisingly, a number of variations of this design exist [24–31]. The Double-Legged Dipole Array (DLDA) is a TCDA which uses diagonally-oriented coupled dipoles, and operates over a 7 : 1 infinite array bandwidth ($VSWR \leq 2$) and a profile of $\lambda_{low}/17$ [24]. The Planar Ultrawideband Modular Antenna (PUMA) array is another TCDA which cleverly addresses the issue of wideband array feeding. Through the use of strategically placed shorting pins on the coupled dipoles, the PUMA array can be fed with an unbalanced excitation over about a 5 : 1 bandwidth [27–29]. In [26], the issue of feeding was addressed through the use of an integrated Marchand balun, which was shown to provide an infinite array bandwidth of 6.3 : 1. TCDAs can also be optimized for very wide scan capability as shown in [30,31], where a TCDA with an integrated balun demonstrated scanning capability to $\theta = 75^\circ$. 
The TCA concept is not restricted to dipole-like elements. The Interwoven SPliral Array (ISPA, depicted in Fig. 1.4) is a circularly-polarized implementation of a TCA. As seen, it achieves inter-element capacitance by weaving the spiral element’s arms into neighboring unit cells. Without material loading, it is capable of a 10 : 1 infinite array bandwidth at a profile of \( \frac{\lambda_{low}}{18} \) [32,33].
1.2.3 Finite Tightly Coupled Arrays

It is well known that moderately-sized finite wideband arrays may behave far differently than their ideal infinite array models [22,23]. In particular, there is a size-bandwidth relationship that exists, with larger arrays providing larger bandwidths (eventually approaching infinite array performance). This is an important consideration in practical implementation of TCAs: realistically-sized TCAs are often of insufficient size to realize full infinite array bandwidth. This underscores the need for unit cell designs with even greater bandwidths than current designs can offer. Another important consideration in TCA implementation is that the bandwidth of a finite TCA also depends on the aperture shape and the element geometry.

To illustrate the size-bandwidth relation, the TCDA depicted in Fig. 1.5 is considered. This array realizes inter-element capacitance by overlapping the arms of adjacent dipoles. As a metric of finite array matching, the VSWR looking into an ideal Wilkinson power divider feeding the finite array is considered (see Fig. 1.6).
The network’s input reflection coefficient is calculated as outlined in Appendix A. As seen in Fig. 1.6, for a larger array, the low frequency cutoff shifts downward, and the fractional bandwidth increases. This is also clearly illustrated in Fig. 1.7, which plots the fractional bandwidth ($f_{\text{max}}/f_{\text{min}}$, $VSWR \leq 3$) of the square finite arrays. The more wideband Interwoven SPiral Array (ISPA [32], depicted in Fig. 1.4) is also considered in this plot. The unit cell size and ground plane spacing is the same for the two arrays. As seen, finite ISPAs produce higher bandwidths than finite TCDAs of the same size. This illustrates that finite TCA bandwidth depends not only on aperture size, but also element type.

In addition to size and element type, the shape of a finite TCA also influences its bandwidth. To illustrate this, the $M \times N$ TCDA depicted in Fig. 1.8 is considered. Fig. 1.9 gives the array bandwidth for rectangular TCDAs where $M$ (number of elements in H-plane direction) is varied. As seen, after some “critical value” for $M$,
adding additional rows of elements does not improve the array bandwidth. This is in contrast to the phenomena seen in Fig. 1.10, which gives the bandwidth of rectangular TCDAs where $N$ (number of elements in E-plane direction) is varied: adding rows in the $E-$plane appears to always improve the bandwidth (albeit, with diminishing returns).

1.3 Contemporary Challenges in Ultrawideband Arrays

While much work has been done in the realm of UWB arrays over the past few decades, major challenges remain in this area. Some of these challenges include:

1. **Ever-increasing demand for wider (> 10 : 1) bandwidths in low profile arrays:** in recent years, the number of sensing and communication systems placed on mobile platforms has exploded. At the same time, platforms sizes have decreased. For example, Unmanned Aerial Vehicles (UAVs) have replaced much larger piloted aircraft for many military applications. UWB multifunctional apertures are critical in reconciling the escalating demands for data throughput with shrinking platform sizes. As more functionality is added to these platforms, even more wideband apertures (with bandwidths in excess of a decade [1]) will be required (of course, while low profile is maintained). Emerging Software Defined Radios (SDRs) also demand extreme bandwidths. In [7], it is suggested that an SDR for commercial wireless communications might require 22:1 bandwidth.

2. **Feeding of extremely wideband phased arrays:** UWB arrays typically require balanced feeding, but are usually excited with an unbalanced interface
(e.g., a coaxial or microstrip line). A balun is required to transform the unbalanced excitation to the required balanced feeding. Such a balun is required to:

- Excite a differential mode (or equivalently, suppress the common mode),
- Maintain impedance matching,
- Maintain scanning capability.

Meeting these requirements over extremely wide bandwidths (in excess of a decade) is a major design problem. Integration of these baluns within the array unit cell is also a significant challenge.

3. **Demands of array electronics:** array electronics such as T/R modules, beamformers or amplifiers are major drivers in the high cost of UWB arrays. One way of alleviating this issue is reducing array element count. This also reduces power consumption and data processing requirements (for arrays employing digital beamforming). However, minimizing element count while maintaining acceptable array performance (specifically gain, bandwidth, and scanning capability) remains a challenge.

### 1.4 Contributions of Dissertation

The key contributions of this work in addressing the aforementioned challenges include:

1. **Development of the Superstrate-Enhanced Substrate-Loaded Array (SESLA)**- a novel paradigm for major bandwidth enhancement of
ground plane backed arrays: Like approaches discussed in [2, 19, 34], the SESLA [35] employs resistive substrate loading to dramatically enhance bandwidth (by a factor of 2 or more). This is accomplished by using the loading to suppress destructive ground plane interference. If the loading is used alone, it can severely reduce array efficiency [19]. However, the SESLA employs a synergistically designed superstrate in addition to the loading to maintain a high radiation efficiency. Indeed, the co-optimization of the loading and the superstrate to maximize bandwidth and radiation efficiency represents the key innovation of the SESLA. The approach taken in the SESLA is general: it can be applied to any UWB planar, ground plane backed array. It is also very simple: as discussed in Chapter 2, the substrate loading and the superstrate are easily co-optimized with basic equivalent circuits. The SESLA approach is empirically validated with a 4x4 prototype array.

2. Development of a SESLA with an integrated feed: as mentioned, providing balanced feeding for UWB arrays is a major challenge, particularly for the SESLA, which can produce designs with bandwidths well in excess of a decade. In this work, a SESLA unit cell with an integrated balun is presented. The design is matched across a 13.9 : 1 bandwidth ($VSWR \leq 2.4$, infinite array) and exhibits very good scanning capability out to $\theta = 45^0$. The design is experimentally verified with an 8x8 prototype array, which demonstrates very impressive wideband scanning performance.

3. Development of a an UWB beam-steering horn antenna: this design uses a TCDA to feed a horn antenna providing a high gain scanned beam with a
reduced element count, compared to a conventional UWB array. As mentioned, reducing array element count reduces cost, power consumption, and computational load. The design is essentially a truncated horn fed by a TCDA. The horn structure augments the gain of the array, but still allows some scanning. The concept allows a design trade between cost and scanning capability, and is validated with a measured prototype.

1.5 Outline of Dissertation

In the next chapter, the Superstrate-Enhanced Substrate-Loaded Array (SESLA) concept is introduced. The theory behind the design paradigm is developed, and measured results of a SESLA prototype are discussed. Chapter 3 presents the SESLA with an integrated feed. Development of the feed design is discussed, and a final unit cell design is presented. Chapter 4 presents extensive measurements of the 8x8 prototype SESLA with integrated feed, validating the design’s many advantages experimentally. Chapter 5 presents the UWB beam-scanning horn antenna, a measured prototype, and various aspects of its design. Finally, Chapter 6 summarizes the findings of this document, and presents suggestions for future research in this area.
Chapter 2: The Superstrate-Enhanced Substrate-Loaded Array (SESLA)

2.1 Introduction

The suitability of Tightly Coupled Arrays (TCAs) for low-profile wideband systems were extensively discussed in the previous chapter. However, in spite of their advantages, these arrays are not without limitations. In particular, the bandwidth of a TCA (or any planar array) is limited by the presence of the ground plane [36]. Ground-plane backed TCAs have demonstrated infinite array bandwidths of about 10:1 [32], but for applications demanding bandwidths well in excess of a decade (such as multifunctional apertures or software-defined systems), existing designs may not provide the required bandwidth (particularly when modestly-sized apertures are used). To address this issue, this chapter introduces the Superstrate-Enhanced Substrate-Loaded Array (SESLA). This design represents an innovative scheme for enhancing the bandwidth of a planar, ground plane backed array. As will be discussed, the SESLA employs resistive substrate loading to enhance array bandwidth (by a factor of more than 2), and a synergistically designed superstrate to maintain high radiation efficiency. The simple, circuit model based co-optimization
of the loading and the superstrate (for maximum bandwidth and efficiency) differentiates the SESLA from other array designs using resistive substrate loading [2,19,34].

In the next section, it is shown that the ground plane limits the bandwidth of a TCA, and the underlying physical phenomena are discussed. In section 2.3, it is shown that this bandwidth limit can be circumvented by employing resistive substrate loading, and ground-plane backed designs with bandwidths in excess of 20 : 1 can be realized. Of course, use of a resistive loading has a deleterious effect on the array’s radiation efficiency, and if the loading is used alone, this effect can be quite severe. However, in Section 2.4, it is shown that if a synergistically designed superstrate is used in conjunction with the loading, the efficiency losses can be greatly mitigated. These findings are validated empirically with a measured prototype array in Section 2.5. Variations of the SESLA design, including one producing a 36 : 1 bandwidth, are discussed in Section 2.6. Finally, concluding remarks on this topic are presented in Section 2.7.

2.2 Limitations of Planar Tightly Coupled Arrays

As previously stated, the bandwidth of a TCA (or any ground-plane backed array) is fundamentally limited by the ground plane [36]. The general equivalent circuit of an infinite ground plane backed array is given in Fig. 2.1. Equivalent circuits such as these are used frequently in analysis of infinite arrays and are based on Floquet modal analysis [19,37,38]. As seen in Fig. 2.1, the ground plane manifests as a shunt reactance \( Z_{GP} \), in parallel with the radiation load, given by:

\[
Z_{GP} = j\eta_0 \tan(\beta_0 h),
\]  

(2.1)
Figure 2.1: Generalized equivalent circuit for the unit cell of a planar array (with air substrate) of infinite extent.

Figure 2.2: Ground plane impedance $Z_{GP}$ in circuit depicted in Fig. 2.1.

which is plotted in Fig. 2.2. As seen, when $h = \lambda/2$, $Z_{GP} = 0$, and the array is short-circuited. Thus, the maximum operational frequency of a ground plane backed planar array is given by:

$$f_{max} \leq c/(2h). \tag{2.2}$$

where $c$ is the free space speed of light. As an example, we can consider the Tightly Coupled Bowtie Array (TCBA) whose unit cell is depicted in Fig. 2.3. Infinite array
simulations are also provided. These simulations were obtained with ANSYS HFSS, modeling a single unit cell with periodic boundary conditions, a widely accepted method for infinite array analysis [39]. It is noted that this array is fed by twin 50Ω coaxial lines excited 180° out of phase by an ideal hybrid coupler (this feeding configuration is discussed in [2]). As seen, in Fig. 2.3, at roughly 3.2 GHz, the array is severely mismatched. This is not surprising, as this frequency corresponds roughly to \( f_{\text{max}} \) given in Equation 2.2, indicating that the array is short-circuited. It is noted that this disruption in matching is narrowband: that is, the short circuit separates wide well-behaved spectral regions. Clearly, if this disruption could somehow be suppressed, the array bandwidth could be doubled and extended to well beyond a decade.

To suppress the narrowband matching disruption and enhance bandwidth, \( Z_{GP} \) must be modified in some way to prevent the short circuit that occurs at \( f_{\text{max}} \approx 3.2 \text{GHz} \). An important realization is that this cannot be accomplished by modifying \( Z_{GP} \) with lossless circuitry [19]. This is due to the fact that if \( Z_{GP} \) is lossless, it must
Waves Destructively Interfere: 
No Radiation
Phase Reversal
\( h = \lambda / 2 \)

Figure 2.4: Conceptual depiction of destructive interference that disrupts array performance when \( h = \lambda / 2 \).

obey Foster’s Reactance Theorem:

\[
\frac{\partial Z_{GP}}{\partial f} > 0 \quad \text{if } Z_{GP} \in \mathbb{R}.
\]  

(2.3)

In other words, the purely reactive \( Z_{GP} \) increases monotonically with frequency [40,41] and will inevitably cross zero. This is apparent in Fig. 2.2.

The only possible way, then, to circumvent this limitation on bandwidth imposed by the ground is to modify \( Z_{GP} \) with a resistive element. Indeed, in the next section, it is shown that the bandwidth of a TCA can be increased by a factor of more than two using a resistive substrate loading.

2.3 Bandwidth Enhancement with Substrate Loading

As mentioned, when the ground plane spacing approaches \( \lambda / 2 \), the array is short-circuited, causing the matching disruption seen in Fig. 2.3. One physical interpretation is that when \( h = \lambda / 2 \), the ground plane reflection returns to the aperture out of phase as seen in Fig. 2.4. The reflection destructively interferes with the outgoing wave preventing any radiation, and thus, the array reflects all incident power.
Figure 2.5: Plane wave reflectivity of simple ground plane.

Figure 2.6: Plane wave reflectivity of ground plane with resistive FSS.

To avoid this issue, a resistive substrate loading can be placed between the array and ground plane. For example, we can consider the use of a resistive Frequency Selective Surface (FSS). The resistive FSS (alternatively called a Circuit Analog Absorber [42]) is simply a ring of resistive material. The reflectivity plots of Figs. 2.5 and 2.6 give the ground plane reflection responses at the array aperture surface for cases when a regular ground plane is used and when a resistive FSS is placed between the array and ground plane. As seen, the resistive FSS attenuates the ground plane reflection response at mid-band, and hence, can be used to suppress the destructive ground plane interference.
Fig. 2.7 depicts the Superstrate-Enhanced Substrate-Loaded Array (SESLA): a TCBA with a resistive FSS placed between the array and ground plane and a superstrate. Its corresponding infinite array bandwidth curve is given in Fig. 2.8. As with the previous infinite array example (see Fig. 2.3), it is assumed that the array is fed with an ideal 180° hybrid coupler. As seen, when the FSS ($R_S = 50\Omega/sq.$, $W = 1.5\,mm$, $L = 22.75\,mm$, see Fig. 2.7) is used, the maximum operational frequency of the array more than doubles, and a 21:1 bandwidth is achieved ($VSWR \leq 3$ from 0.28-5.91 GHz). Clearly, the destructive ground plane interference is suppressed by the loading and the bandwidth limitation described in the previous section is circumvented.

Since the FSS geometry is polarization-insensitive, it can be used in the dual-polarized configuration as depicted in Fig. 2.9. Fig. 2.10 gives the simulated $S_{11}$ and $S_{12}$ curves for the dual-polarized array fed by two 180° hybrid couplers (see inset). As seen, the array is matched across a 21:1 bandwidth ($VSWR \leq 3$ from 0.285-5.92GHz). Since the two dipole elements are almost symmetric in construction, the two polarizations are virtually uncoupled ($S_{12} \leq -27dB$).
2.4 Efficiency Enhancement with Superstrates

The equivalent circuit for the SESLA depicted in Fig. 2.11 provides insight on how a superstrate alleviates loss from the substrate loading. Using this circuit, we
obtain the following expression for the radiation efficiency:

$$e_r = \frac{P_{\text{radiated}}}{P_{\text{accepted}}} = \frac{P_R}{P_{GP} + P_R} = \frac{1}{P_{GP}/P_R + 1} \quad (2.4)$$

where $P_R$ and $P_{GP}$ represent the real powers transferred to complex loads $Z_R$ and $Z_{GP}$, respectively. Introducing the values $R_R = \text{Re} \{Z_R\}$ and $R_{GP} = \text{Re} \{Z_{GP}\}$, we obtain the power ratio:

$$\frac{P_{GP}}{P_R} = \frac{\frac{1}{2}|I_{GP}|^2 R_{GP}}{\frac{1}{2}|I_R|^2 R_R} = \frac{R_{GP}}{|Z_{GP}|^2} \left[ \frac{R_R}{|Z_R|^2} \right]^{-1}. \quad (2.5)$$

The ratio can be further simplified by introducing the variables:

$$\xi_{GP} = \frac{R_{GP}}{|Z_{GP}|^2}; \quad \xi_R = \frac{R_R}{|Z_R|^2}. \quad (2.6)$$

Thus, the radiation efficiency simplifies to:

$$e_r = \frac{1}{\xi_{GP}/\xi_R + 1}. \quad (2.7)$$

Here, it is noted that that $\xi_{GP}$ depends only on substrate loading parameters, whereas $\xi_R$ is associated solely with the superstrate. Since the substrate loading
Figure 2.14: Radiation efficiency for SESLA depicted in Fig. 2.7 as determined through simulation and Equation 2.7. Electrical length of the superstrate is fixed (λ/4 at 3.05 GHz).

is resistive, ξ_{GP} > 0 (see Fig. 2.12), reducing \( e_r \) (per Equation 2.7). However, as seen in Fig. 2.13, the addition of a superstrate increases ξ_{R}, compensating for the rise in ξ_{GP} and improving efficiency. In effect, the superstrate improves efficiency by drawing power to the radiation load and away from the resistive FSS. To illustrate this, Fig. 2.14 gives the radiation efficiency of the coupled bowtie array with the FSS \( (R_S = 50Ω/\text{sq.}, W = 1.5mm, L = 22.75mm, \text{see Fig. 2.7}) \) and various superstrates.

In addition to full wave simulation results, efficiency was computed via Equation 2.7. The impedance \( Z_{GP} \) was extracted from the simulated complex FSS reflection coefficient, \( \Gamma \):

\[
Z_{GP} = \eta_0 \frac{1 + \Gamma}{1 - \Gamma}. \tag{2.8}
\]

Also, \( Z_R \) was calculated from superstrate parameters:

\[
Z_R = \frac{\eta_1 \eta_0 + j\eta_1 \tan \beta_1 h_s}{\eta_1 + j\eta_0 \tan \beta_1 h_s} \tag{2.9}
\]
Figure 2.15: Infinite array VSWR of SESLA depicted in Fig. 2.7 for various superstrate dielectric constants (FSS resistivity is $R_S = 50 \Omega/\text{sq}$). Electrical length of the superstrate is fixed ($\lambda/4$ at 3.05 GHz).

where $h_s$, $\eta_1$, and $\beta_1$ refer to the height, impedance, and propagation constant of the dielectric superstrate.

As seen in Fig. 2.14, without a superstrate, the efficiency is as low as 42% (-3.8dB). However, when an $\epsilon_r = 4$ superstrate is used (corresponding to results in Fig. 2.8), the efficiency increases to above 73% (-1.4dB) across the entire 21:1 band. With an $\epsilon_r = 6$ superstrate, the efficiency further increases (to above 77% (-1.1dB)). This comes, however, at the expense of bandwidth. Specifically, from Fig. 2.15, the computed bandwidth of the infinite array with an $\epsilon_r = 6$ superstrate is roughly 17:1 (VSWR$\leq 3$ from 0.28-4.89GHz).

To maximize efficiency, the FSS and superstrate must be designed synergistically. As previously mentioned, the purpose of the superstrate is to increase $\xi_R$ and compensate for the FSS-induced rise in $\xi_{GP}$. In doing this, the superstrate acts as a single section impedance transformer (or quarter-wave transformer), as seen in Fig. 2.11. As such, the superstrate’s capacity to increase $\xi_R$ is band-limited. Thus, for optimal
Figure 2.16: Radiation efficiency for SESLA depicted in Fig. 2.7 (with $\epsilon_r = 4$) for various FSS sheet resistances.

efficiency, the bands of the FSS and the superstrate (as a quarter-wave transformer) should overlap. This implies that:

- the superstrate thickness is $\lambda/4$ at the frequency where $\xi_{GP}$ peaks,

- the FSS bandwidth is commensurate with that of the superstrate.

As an example, let us consider the case of an array where $R_s = 25\Omega/$sq. for the FSS and $\epsilon_r = 4$ for the superstrate. As depicted in Fig. 2.12, the FSS response ($\xi_{GP}$) is much more wideband than the superstrate response ($\xi_R$) plotted in Fig. 2.13. As a result, the efficiency suffers outside the band of the superstrate, as seen in Fig. 2.16. However, if a more narrowband 50$\Omega/$sq. FSS is used, the superstrate adequately compensates for the FSS, improving the overall efficiency. On the other hand, if an even more narrowband 100$\Omega/$sq. FSS is used, then the corresponding peak in $\xi_{GP}$ is too high for compensation by the superstrate, and the efficiency suffers at $f \approx 3GHz$.

It is noted that decreasing $W$ (width of FSS trace) was found to have the same effect.
on $\xi_{GP}$ as increasing $R_S$. This is expected, as narrowing a resistive trace amounts to increasing its resistance.

In addition to $\epsilon_r$, and $R_S$, other critical design parameters include the FSS height above the ground plane and $L$, (length of the FSS, see Fig. 2.7). The impact of these parameters on efficiency are inter-related. Hence, a design process must be employed which considers all variables. Fig. 2.17 provides a set of contour plots which illustrate the influence of these parameters on radiation efficiency. These plots were created using the circuit model given by Equation 2.7, and they provide the
minimum radiation efficiency within the array operational band. For each data point, the superstrate height was optimized to maximize this value.

As seen in all plots, optimal FSS height occurs slightly above the midpoint between the ground plane and the array. Further, the top left plot shows that for the $\epsilon_r = 4$ superstrate, $R_S$ is optimal at around 50Ω. The bottom left plot shows that the optimal value for $R_S$ is slightly higher when the $\epsilon_r = 6$ superstrate is used. Finally, in the right two plots, it is shown that the optimal FSS length is slightly less than the unit cell width (23.25 mm).

Since the superstrate thickness is less than $\lambda/2$ at the highest operational frequency, attenuation in the dielectric is trivial, assuming a low loss material is used. In the preceding analysis, $\tan\delta = 0.005$ was assumed for all superstrates. It is noted that a variety of bulk dielectric materials with $\tan\delta < 0.005$ are commercially available [43,44]. In addition to improving efficiency, an $\epsilon_r = 4$ superstrate has the added advantage of lowering the element impedance to $\approx 100\Omega$. This allows for a relatively straightforward feed using twin 50Ω lines (see Fig. 2.7). Ideally, the superstrate
Resistive FSS: 75Ω/□
Polyethylene Superstrate 12.7mm
19.9mm 93mm
54.1mm

Figure 2.19: Fabricated 4x4 SESLA prototype (depicted without superstrate in photo).

Figure 2.20: Printed circuit boards for fabricated array prototype.

should be flush with the array aperture, but a minor air gap does not severely deteriorate performance. Fig 2.18 gives the array radiation efficiency for the unit cell with a gap between the array and superstrate. As seen, for a gap less than 0.5 mm, the effect on the radiation efficiency is minor.

2.5 Measured prototype

To validate the design of the proposed SESLA, the 4x4 prototype depicted in Fig. 2.19 was fabricated. The resistive FSS (W = 2.5 mm, L = 22.25 mm, dimensions refer to Fig. 2.7) and coupled bowtie array were printed on 20 mil Rogers RO4003
substrates ($\varepsilon_r = 3.55, \tan \delta = 0.0027$). Both boards are depicted (prior to array assembly) in Fig. 2.20. The FSS is made of a commercially available resistive Nickel Phosphorous (NiP) alloy produced by Ohmega Technologies, Inc. The sheet resistance of this material was measured to be $R_s \approx 75 \Omega/$sq. This was done using a linear four point probe, with geometric factors described in [45]. A 12.7mm polyethylene ($\varepsilon_r = 2.25, \tan \delta = 0.0007$ [46]) superstrate was placed on top of the array.

The 16 elements were fed with twin 50$\Omega$ coaxial lines (100$\Omega$ differential impedance) whose jackets were soldered together (as discussed in [2]). For broadside gain measurements, the array was excited uniformly with a network of commercially available components. Specifically, the feed network is comprised of a single wideband 180$^\circ$ hybrid coupler and two identical 16-way power splitters (see Fig. 2.21). Measurements were conducted in the Ohio State University ElectroScience Lab compact range, with the array mounted on a 610$mm \times 610$mm ground plane, as depicted in Fig. 2.22. For gain calibration, two standard gain horns covering 0.7-1GHz and 1-6GHz were used, employing the measurement technique outlined in [47].
Fig. 2.22: Prototype $4 \times 4$ array mounted for measurements in compact range.

Fig. 2.23 gives the simulated and measured broadside gain of the array, with and without the substrate loading. The feed network losses (2-7dB) were subtracted from the measured gain curve. As expected, the gain plummets at approximately 3.6GHz without the loading. When the loading is used, the array gain is maintained well beyond this frequency, indicating the suppression of ground plane interference. As seen, the simulated and measured gain curves are in very good agreement. It is noted that the lower operational limit of this small finite array is approximately 1.2GHz, much higher than that of the infinite array. As noted in Section 1.2, small or moderately-sized UWB arrays produce considerably lower bandwidths than their infinite counterparts, due to strong edge effects. The bandwidth would, of course, increase with increased finite array size (eventually approaching infinite array bandwidth).
Figure 2.23: Simulated (solid) and measured (dashed) broadside realized gain of array prototype.

![Simulated and measured broadside realized gain](image)

Figure 2.24: Measured broadside cross-polarization level of the array prototype.

![Cross-polarization levels](image)

Fig. 2.24 gives the measured broadside cross-polarization response. As seen, cross-polarization is quite low (about 30dB below co-polarized gain), and is not appreciably affected by the loading’s presence.

Fig. 2.25 provides the simulated and measured H-plane radiation patterns of the prototype array at 2, 3.6, and 5GHz. Once again, excellent agreement between
Figure 2.25: Simulated (solid) and measured (dashed) H-plane gain (in dBi) for 4x4 array prototype.

Simulation and measurement is observed. It is noted that when no loading is used, the 3.6GHz pattern experiences a null at broadside. However, when the loading is used, the null is suppressed.

2.6 Design Variations

In this section, two variations of the previously presented SESLA are discussed. The first uses a printed or meta-structured superstrate for efficiency enhancement. When used in lieu of a dielectric superstrate, a printed superstrate can dramatically reduce the weight of the array implementation. The second variation uses multiple layers of substrate-loading to achieve bandwidths as high as 36 : 1. Both of these design features (printed superstrates and multi-layer resistive substrate loading) are present in the Fragmented Aperture Array (FAA) [2,34]. However, a key distinction
between these designs and the FAA is simplicity: these designs can be optimized with basic equivalent circuits to realize their full potential.

2.6.1 Substrate-Loaded Array with Printed (Meta-Structured) Superstrates

As discussed in Section 2.4, a superstrate improves the radiation efficiency of the SESLA by drawing power away from the loaded ground plane. However, for some applications, particularly those operating at lower frequencies, a bulk dielectric superstrate can be unacceptably heavy. A low-mass alternative is a printed or “meta-structured” superstrate, as depicted in Fig. 2.26. As seen, the printed superstrate is comprised of capacitively-coupled (overlapping) sub-resonant dipoles. The equivalent circuit of the design is also depicted in Fig. 2.26. As discussed in Section 2.4, in order to alleviate losses from the loading, $Z_R$ (see Fig. 2.26) must be designed to increase $\xi_R$ across frequencies where the loading increases $\xi_{GP}$ (see Equations 2.6 and 2.7). Much like a bulk dielectric superstrate, a printed superstrate accomplishes this by transforming $Z_R$ to a lower value.
We can first consider a design using a resistive FSS ($R_S = 50\Omega/sq.$, $W = 2.5mm$, $L = 22.25mm$, dimensions refer to Fig. 2.7) placed halfway between the array and the ground plane ($h_R = 0.5h$). The resulting $\xi_{GP}$ curve is plotted in Fig. 2.27 (recall that $\xi_{GP}$ and $\xi_R$ dictate array efficiency, per Equation 2.7). Referring to Fig. 2.26, the design uses a printed superstrate where $C_E = 0.25mm$, $h_s = 8.8mm$, producing the $\xi_R$ curve also plotted in Fig. 2.27. While the superstrate does cause a peak
in $\xi_{GP}$ implying improved efficiency, $\xi_R$ actually drops below the value of $\xi_R$ when no superstrate is used ($\xi_{R0}$) after this peak. This means that while this printed superstrate improves efficiency in part of the band, it actually makes efficiency worse at the upper end of the band. This is in contrast to a bulk dielectric superstrate, which, as seen in Fig. 2.27, produces a $\xi_R$ curve that never drops below $\xi_{R0}$. Another problem with this design is that after the peak in $\xi_R$, the real part of $Z_R$ becomes very low, as the impedance of the capacitive surface becomes low. This wreaks havoc on matching at the upper end of the band.

To avoid these issues, the printed superstrate can be designed such that peak $\xi_R$ occurs above mid-band. This would imply that peak $\xi_{GP}$ would need to also occur above mid-band, to align the peaks for optimal efficiency. To accomplish the latter goal, a lower resistivity FSS is used, and its position is changed. Fig. 2.28 gives $\xi_{GP}$ for a resistive FSS at various positions, where $R_S = 30\Omega/\text{sq}$. As seen, by placing the FSS closer to the ground plane, the $\xi_{GP}$ curve is shifted up in frequency. A printed superstrate which invokes a complimentary peak in $\xi_R$, (such as in Fig. 2.29) can
then be used. The ill effects seen after peak $\xi_{GP}$ are pushed out of the target band, beyond the grating lobe frequency (6.5GHz).

Fig. 2.30 gives the radiation efficiency of a design ($C_E = 0.19\,mm$, $h_S = 4.2\,mm$, $h_R = 0.54h$, $R_S = 30\Omega/sq.$) that employs this concept. As seen, the printed superstrate improves efficiency across the entire band for this design. Furthermore, as seen in Fig. 2.31, the design is matched ($VSWR \leq 3$, $Z_0 = 188\Omega$) from 0.4-6.0GHz (15:1 bandwidth). For comparison, a design using a synergistically designed $\epsilon_r = 4$ dielectric superstrate is considered. As with the printed superstrate, the dielectric superstrate was designed by aligning the peaks of $\xi_{GP}$ and $\xi_R$ to maximize efficiency. This results in a superstrate thickness of $h_{\text{diesel}} = 7.75\,mm$, compared to the printed superstrate height of 4.2mm. Thus, the design with the printed superstrate produces a similar radiation efficiency characteristic to the design with a dielectric superstrate, in spite of being lower profile. The dielectric superstrate, does however, produce better low frequency matching ($Z_0 = 100\Omega$), as seen in Fig. 2.31.

2.6.2 SESLA with Multiple Loading Layers

As discussed in Section 2.3, resistive substrate loading enhances array bandwidth by suppressing destructive ground plane interference. This removes the short circuit seen in the general equivalent circuit for a planar array (see Fig. 2.1), as predicted by Equation 2.1. It is noted that Equation 2.1 is periodic: the untreated ground plane impedance crosses zero (short-circuiting the array) when $h = n\lambda/2$, where $n$ is an integer. Thus, to realize bandwidths even greater than those demonstrated in Section 2.3, higher order substrate loading networks can be used to suppress interference not only when $h = \lambda/2$, but also at subsequent short-circuit frequencies.
As an example, Fig. 2.32 depicts an SESLA with three layers of resistive loading, along with the design’s equivalent circuit. For simplicity, resistive sheets are used for ground plane interference suppression instead of the ring-type FSS. Referring to Fig. 2.32, an example is considered with the following parameters: \( \epsilon_r = 2.25 \), \( h_s = 32.8\,\text{mm} \), \( h = 93\,\text{mm} \), \( h_1 = h_2 = h_3 = 23.25\,\text{mm} \), \( R_{s1} = R_{s3} = 377\,\Omega/\text{sq.} \), \( R_{s2} = 225\,\Omega/\text{sq.} \), \( d = 23.25\,\text{mm} \). The design produces the infinite array VSWR curve seen in Fig. 2.33, in which \( VSWR \leq 2.5 \) from 0.17-6.1 GHz \( (Z_0 = 150\Omega) \), a 36 : 1 bandwidth. The array’s radiation efficiency is depicted in 2.34. As seen, the efficiency is well above 50% for most of the band.

### 2.7 Summary

In this chapter, the Superstrate-Enhanced Substrate Loaded Array (SESLA) was presented. This design represents a novel paradigm for enhancing the bandwidth of
UWB ground-plane backed arrays. As discussed in Section 2.2, the ground plane limits the achievable bandwidth of an UWB array as destructive ground plane interference places an upper bound on its operational frequency. This limitation can be circumvented through the use of resistive substrate loading, as demonstrated in Section 2.3. A single substrate loading enables Tightly Coupled Array (TCA) designs with bandwidths as high as 21 : 1, but of course, the loading degrades the array’s radiation efficiency. In fact, if the loading were used alone, the degradation is quite severe (possibly in excess of 3\,dB). However, as shown in Section 2.4, this degradation can be drastically mitigated through the use of a synergistically designed superstrate. For example, an \( \epsilon_r = 4 \) superstrate was shown to improve a loaded array’s efficiency from 42\% (−3.8\,dB) to 73\% (−1.4\,dB). The SESLA design concept was validated experimentally, as detailed in Section 2.5. Finally, some variations of the SESLA paradigm were discussed in Section 2.6. In particular, a design using a printed superstrate for low-weight implementation was presented, as well as a design that uses multiple layers of substrate loading to operate across a 36 : 1 bandwidth.
Chapter 3: Superstrate-Enhanced Substrate-Loaded Array with Integrated Feed: Theory and Design

3.1 Introduction

Feeding represents one of the major challenges in the practical implementation of ultrawideband (UWB) phased arrays. Dipole-like elements, such as the one used in the Superstrate-Enhanced Substrate-Loaded Array (SESLA) presented in the preceding chapter, require balanced feeding. In other words, the currents exciting the element must be equal and opposite (see Fig. 3.1). Since most systems require an unbalanced interface (such as a coaxial line), a balun transformer is required. As discussed in Section 1.3, such a balun must:

- Excite a differential mode (or equivalently, suppress the common mode as depicted in Fig. 3.1),
- Maintain impedance matching across the operational band,
- Maintain scanning capability.

Design of baluns operating across the very wide bandwidth of the proposed SESLA is a major technical challenge. Furthermore, Tightly Coupled Array (TCA) elements
Differential Mode: Currents are Equal and Opposite
No Return Currents on Ground

Common Mode: Currents are Equal and Co-Directional
Return Currents

Figure 3.1: Differential (balanced) mode and common (unbalanced) mode in two-wire lines.

50Ω Unbal. Input (e.g. Coax. Line)

Balanced Excitation

$I_b$, $V_b$

Figure 3.2: Concept of a feed structure integrated with a TCA.

typically have impedances of about $100 - 200\Omega$. In order to provide a $50\Omega$ interface, some sort of matching network may need to be employed.

In this chapter, feed structures for the SESLA are discussed. The purpose of a feed structure is to excite the array element with a $50\Omega$ unbalanced line, as seen in Fig. 3.2. It thus acts as a balun and an impedance transformer. As shown by Doane, et. al., these two devices can be combined and co-optimized to minimize size and complexity [26]. The designs presented in this chapter employ a folded Marchand balun [48], the operation of which is discussed in the next section. In Section 3.3 a version of the SESLA with an integrated feed is presented. This design can be modeled by a detailed equivalent circuit which provides physical insight on the operation of the system. In Section 3.4 an improved version of the design is presented, resulting in a
practically implementable array unit cell that is matched across a 13.9 : 1 bandwidth ($VSWR < 2.4$, infinite array at broadside scan) and shows good scanning capability out to $\theta = 45^0$.

### 3.2 The Marchand Balun

The Marchand balun is a wideband balun topology introduced in 1944 [49] which has been shown to provide bandwidths in excess of a decade [51–53]. The topology can be readily integrated with dipole-like elements [26, 54], and hence, is an ideal option for feeding the SESLA presented in the preceding chapter.

The original Marchand balun design is depicted in Fig. 3.3. The device is comprised of coaxial lines housed within a shielded conducting box. An equivalent circuit used in analysis of Marchand baluns [50, 51] is also depicted. As demonstrated in Fig. 3.4, the Marchand balun achieves wideband common mode suppression through symmetry. In this image, the desired balanced output currents are depicted, along with the return currents in the structure that would result from an undesired common
mode excitation. As noted, the unbalanced return currents destructively interfere in the center of the structure, and thus, no common mode can propagate on the balanced lines. This current cancellation is frequency independent, and thus the main practical challenge in implementation of the Marchand balun is wideband matching.

Fig. 3.5 depicts the folded Marchand balun [48], a variation of the design depicted in Fig. 3.3. Like the original Marchand balun, the folded version exploits symmetry to excite a purely differential mode. As seen in Fig. 3.6, the undesired common mode excitation is subject to current cancellation at the junction of the two lines.

Early Marchand balun designs were primarily realized with coaxial lines. More recent versions are realized in a variety of PCB-based [26,50,53,55–57] and MMIC-based [51] transmission lines. This allows seamless integration with antennas or other microwave devices. In the next section, a PCB stripline-based folded Marchand balun is integrated with the SESLA to provide its required balanced feeding.
3.3 SESLA with Integrated Feed

Fig. 3.7 presents a version of the SESLA with an integrated feed. As will be discussed, this design was realized by incorporating the aforementioned UWB folded Marchand balun with the SESLA, and optimizing the resulting structure for wideband matching. Aside from inclusion of this feed, the design has some notable differences compared to the SESLA presented in the previous chapter, including:

- The design uses a resistive sheet as loading, instead of the ring-type resistive FSS. This allows for simpler construction.
The unit cell’s E-plane length is half its H-plane length. As will be discussed, this facilitates wideband matching.

An exploded view of the feed is given in Fig. 3.8. As seen, the structure is realized in printed stripline with a via fence. In addition to providing nearly frequency independent impedance and propagation constant, this type of line is realized in inexpensive PCB process.

As mentioned, the purpose of the integrated feed is to allow the balanced-fed element to be excited by a 50Ω unbalanced line. As seen in Fig. 3.8, a split in the outer jacket of the stripline, at the top of the structure’s loop provides the feed’s balanced port terminals. The feed essentially has two operational components: a folded Marchand balun to provide differential feeding, and a tapered line for impedance matching (see Fig. 3.9). As described in the preceding section, the folded Marchand balun suppresses common mode excitation (ensuring balanced excitation) through symmetry. The balanced port lines form a junction at which undesired common mode excitation
currents destructively interfere, as seen in Fig. 3.10. An equivalent explanation of the feed operation (also depicted in Fig. 3.10) is that a common mode excitation would produce equal potential at the labeled nodes, and thus, no current could flow to ground.

As depicted in Fig. 3.7, the array unit cell has an unequal aspect ratio. In other words, the array element is half as long in the E-plane as in the H-plane. This concept of “splitting the array unit cell” was introduced by Cavallo, et. al. [58], and has many advantages, including:
Figure 3.11: Implementation of unit cell to prevent overpopulation of array.

- Prevention of onset of a certain type of common mode related to the length of a dipole-type element [59]

- Reduction of element impedance to aid aperture matching to 50Ω [26]

- Improvement of wideband matching performance of array-balun combination by increasing the ratio $\frac{Z_{out}}{Z_{ant}}$ (see Fig. 3.9).

To prevent oversampling the array aperture, two unit cells can be combined and fed in-phase with a Wilkinson power divider [58], as shown in Fig. 3.11.

Fig. 3.12 depicts multiple views of the array and integrated feed, annotated with dimensions. For rapid optimization of this design, a circuit model was developed which predicts the array/feed performance. The circuit was realized by first developing a model for the feed structure, then developing a circuit for the element. The final model simply combines these two circuits. In the following sub-sections, the development of this circuit model is discussed.
3.3.1 Circuit Model for Integrated Feed

Fig. 3.13 gives an equivalent circuit of the feed structure which includes parasitic reactances. Approximating the fenced striplines as rectangular coaxial lines, the impedance values $Z_{oc}$ and $Z_{oe}$ are readily approximated by [60]:

$$Z = \frac{1}{vC'}$$  \hspace{1cm} (3.1)

$$C' = \frac{2\epsilon_{r, sub}\epsilon_0 s}{h_{sub}} + 4\epsilon_{r, sub}\epsilon_0\left[\frac{2}{\pi}ln\left(1 + \coth\frac{\pi g}{2h_{sub}}\right)\right]$$  \hspace{1cm} (3.2)

where $v$ is the free space velocity of light, $s$ and $g$ are as defined in Fig. 3.12, $\epsilon_{r, sub}$ is the dielectric constant of the substrate material, and $h_{sub}$ is the thickness of each substrate in the stripline (total thickness is $2h_{sub}$). The values $Z_{ot}$, $l_{ot}$, $L$ and $C$ (from Fig. 3.13) cannot be readily obtained with closed-form solutions. Hence, to use the circuit model, a single simulation of the feed must be carried out to de-embed these parameters. Once they are obtained, the model can predict the variation in feed performance as many parameters including: $h_1$, $h_2$, $z_1$, $s_1$, $g_1$, $s_2$ and $g_2$ are changed.
To de-embed the circuit parameters, a version of the feed (with arbitrary dimensions) is analyzed with a full wave simulator. The circuit shown in Fig. 3.13 is then constructed in circuit simulator such as Agilent ADS, with the known parameters inserted into the model. An optimizer is then used to find the unknown values, such that the circuit response closely matches the response obtained by full wave simulation.

Using the de-embedded values of \( C_b \), \( Z_{ot} \), \( l_{ot} \) and \( L_b \), the circuit model response (plotted in Fig. 3.14) was obtained for two values of \( z_1 \) (as defined in Fig. 3.12). It is noted that neither value of \( z_1 \) is the same as in the design from which the de-embedded parameters were obtained. Excellent agreement is seen between the circuit model and full wave simulation results. Similarly, Fig. 3.15 gives results for varying values of \( h_2 \). Again, excellent agreement between the model and full-wave simulations is seen. It is noted that \( h_2 \) affects the de-embedded parameter \( l_{ot} \) (see Fig. 3.9). Thus, when varying \( h_2 \), \( l_{ot} \) must be changed from its de-embedded value \( l_{ot,0} \):

\[
l_{ot} = l_{ot,0} + \Delta h_2.
\]
Figure 3.14: S-parameters of feed structure as $z_1$ is varied.

Figure 3.15: S-parameters of feed structure as $h_2$ is varied.

It is noted that the circuit model cannot predict radiation losses in the feed. These losses are low for $x_1 \leq 4\text{mm}$ as seen in Fig. 3.16, but may be significant for large values of $x_1$.

### 3.3.2 Circuit Model for Element in Free Space

Fig. 3.17 depicts the coupled bowtie array element in free space with its equivalent circuit. The infinite array couples to a bi-directionally radiating plane wave, which is represented as an unbounded transmission line of impedance $\eta_0$ [19, 37, 38]. The superstrate is represented as a transmission line segment of impedance $\eta_0/\sqrt{\epsilon_r}$. Since the array’s unit cell is rectangular (not square), the impedance of the radiation load is transformed by a factor $d_e/d_h$ [61]. This is accomplished in the circuit by the $\sqrt{(d_e/d_h)} : 1$ transformer. The conducting part of the element is represented by a transmission line, a capacitor and an inductor. The length and effective dielectric constant of the line are approximated as:
Figure 3.16: Radiation loss of feed structure as a percentage of accepted power.

Figure 3.17: Equivalent circuit of array element in free space.

\[ l_E = \frac{d_E - w_{\text{feed}}}{2} - w_{\text{cap}}, \quad (3.4) \]

\[ \epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2}, \quad (3.5) \]

where element dimensions are defined in Fig. 3.17. The values \( Z_E, L_E \) and \( C_E \) are, in general, not known and must be de-embedded from a single simulation of the element. Once these values are found (using an optimizer), the circuit model can predict the element behavior as several other parameters are varied.

Fig. 3.18, gives the element response as superstrate height \( (h_s) \) and superstrate dielectric constant \( (\epsilon_r) \) are varied. To obtain the circuit model results for all plots, a single simulation run was used to obtain values \( Z_E, L_E \) and \( C_E \) as discussed above. Excellent agreement between the circuit model and full wave simulation is seen. To vary superstrate height, the length of the transmission line in Fig. 3.17 is varied accordingly. This line is similarly varied for a change in \( \epsilon_r \), but the value \( Z_E \) also varies. Given that:

\[ Z = \sqrt{\frac{L'}{C'}}, \quad (3.6) \]
Figure 3.18: Reflection coefficient of element for varying values of superstrate height (left) and superstrate dielectric constant (right).

\[ C' \propto \epsilon_{eff} \approx \frac{\epsilon_r + 1}{2}, \]  

(3.7)

the variation in \( Z_E \) with varying \( \epsilon_r \) is captured by:

\[ Z \approx Z_{E0} \sqrt{\frac{\epsilon_r + 1}{\epsilon_r + 1}} \]  

(3.8)

where \( Z_{E0} \) and \( \epsilon_{r0} \) refer to de-embedded values.

### 3.3.3 Full Circuit Model Combining Feed and Element

The full circuit of the antenna-feed combination is depicted in Fig. 3.19. The model is comprised of the previously presented feed circuit, the element circuit, and a circuit representing the lossy ground plane. The resistance \( R \) (in \( \Omega \)) is given by the
sheet resistance of the R-card (in Ω/sq.). Fig. 3.20 gives the VSWR of an antenna-feed combination with and without the resistive loading. As seen, the circuit model is in excellent agreement with full wave simulations.

3.3.4 Infinite Array Performance

The circuit for the array-feed combination was optimized for maximum bandwidth using the criteria $VSWR \leq 2$, $e_r > 70\%$. Using the circuit, radiation efficiency is
Table 3.1: Optimized parameters of circuit model.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_S$</td>
<td>225Ω/sq.</td>
<td>331Ω/sq.</td>
</tr>
<tr>
<td>$z_1$</td>
<td>7.6mm</td>
<td>9.4mm</td>
</tr>
<tr>
<td>$s_1$</td>
<td>6.4mil</td>
<td>8.0mil</td>
</tr>
<tr>
<td>$s_2$</td>
<td>65mil</td>
<td>74mil</td>
</tr>
<tr>
<td>$\epsilon_{r,super}$</td>
<td>2.0</td>
<td>3.25</td>
</tr>
<tr>
<td>$h_2$</td>
<td>19.5mm</td>
<td>19.5mm</td>
</tr>
<tr>
<td>$h_1$</td>
<td>25.2mm</td>
<td>25.2mm</td>
</tr>
<tr>
<td>$h_R$</td>
<td>23.25mm</td>
<td>23.25mm</td>
</tr>
<tr>
<td>$h_S$</td>
<td>16.5mm</td>
<td>14.2mm</td>
</tr>
<tr>
<td>$w$</td>
<td>12.0mm</td>
<td>14.0mm</td>
</tr>
<tr>
<td>$w_{cap}$</td>
<td>1.0mm</td>
<td>1.0mm</td>
</tr>
<tr>
<td>$x_1$</td>
<td>3.0mm</td>
<td>4.0mm</td>
</tr>
</tbody>
</table>

approximated as:

$$
e_r = \frac{|S_{12}|^2}{1 - |S_{11}|^2}. \tag{3.9}
$$

The optimized parameters are given in Table 3.1 (under optimization A) and the VSWR is plotted in Fig. 3.21. Once again, very good agreement between the circuit model and simulation is seen. The design provides matching ($VSWR \leq 2$) from 0.45-5.93GHz, a 13.2:1 bandwidth. The circuit was re-optimized for greater bandwidth under a relaxed criterion of $VSWR \leq 2.5$. The re-optimized parameters are given in Table 3.1 (under optimization B) and the VSWR is given in Fig. 3.21. As seen, the re-optimized design is matched from 0.41-6.05GHz (14.8:1 bandwidth). Both optimizations were performed using Agilent ADS software. Using the circuit model, these optimizations took only minutes, in contrast to optimizations with a full wave simulator, which would likely take days.
The radiation efficiency curves of the two optimized designs were obtained and plotted in Fig. 3.22. As seen, they are in good agreement with full wave simulation results. The slight deviation between the model and the full wave simulation is due to the fact that, as mentioned, the feed has some minor radiation loss.

Fig. 3.23 gives the simulated infinite array VSWR for Optimization A as the (rectangular) unit cell is scanned to $\theta = 45^\circ$ in the $E-$ and $H-$planes. For $E-$plane scanning, the design remains well matched, however, for $H-$plane scanning, the matching
deteriorates at the high end of the band. This is believed to be due to the presence of the feed structure, which is oriented such that its “loop” lies in the E-plane. Fig. 3.24 gives the radiation efficiency as the design is scanned. As seen, the radiation efficiency varies only slightly in the scanned cases. Fig. 3.25 gives the polarization purity of the infinite array (Optimization A) under $\theta = 45^\circ$ scan. The polarization purity is above 10 dB across most of the band, however, for these scanned cases, it does drop below this value at the band edges. This is likely due to the fact that the feed structure’s “loop” is oriented in the cross-polarized direction.

With a working design of a feed structure, the next logical step is the integration of a power divider with the unit cell, as depicted in Fig. 3.11. This is important to prevent overpopulation of the array. Preferably, the divider could be fabricated on the same circuit card as the feed structures as in [26, 58] to minimize costs and simplify integration. However, the power divider must join unit cells in the E-plane while the feeds are oriented along the H-plane. Thus, the feeds and the divider cannot be printed on the same circuit card with this design. Due to this issue, as well as high scanned cross-polarization levels, the improved design presented in the next section
3.4 Improved Design of SESLA with Integrated Feed and Divider

As mentioned, the design presented in the previous section demonstrates wideband performance, but has practical implementation issues and high scanned cross-polarization levels. Since the design uses a split unit cell, it is necessary to combine two elements with a power divider [26, 58] to avoid overpopulating the array (see Fig. 3.11). The orientation of the feed in design presented in the preceding section complicates integration of the divider, as it prevents the feeds and the divider from sharing the same PCB.
The design presented in this section (depicted in Fig. 3.26) is an evolution of the one presented in the preceding section. The design uses the same type of folded Marchand balun feed, but orients the feed such that its “loop” is in the same direction as the element polarization. As seen in Fig. 3.26 the improved design’s unit cell includes an integrated divider, which is printed on the same substrate as the feeds (as in [26, 58]). Other modifications of this design compared to the one presented in the previous section include:

- Modified element geometry, depicted in Fig. 3.27. This design allows for easier soldering to the vertically protruding feed.
- More sparsely populated via fence in the stripline balun. This was found to have little to no effect on the balun performance, but dramatically reduces fabrication costs.
- Scaled to lower frequency band (0.29-4.03 GHz) for easier construction of prototype.

### 3.4.1 Operation and Design

Like the design presented in [26], the one depicted in Fig. 3.26 employs two elements with Marchand baluns per unit cell, united by an integrated divider. Use of the SESLA scheme, however, along with a more wideband balun design, allows this implementation to operate over a much wider bandwidth. As seen in Fig. 3.27 the elements are comprised of coupled bowties printed on the top side of a PCB substrate, capacitively coupled to each other through a metal strip on the bottom of the substrate. Each element has an integrated folded Marchand balun. The basic circuit model for the balun is depicted in Fig. 3.28. As seen, the balun includes a
three section matching transformer. As with the previous design, it is realized in a via-shielded stripline. As seen in Fig. 3.28, the substrate is removed within the loop that forms the shorted line $Z_{ot}$. This increases the impedance $Z_{ot}$, and thus allows for more wideband matching. The unit cell includes substrate loading (in the form of a resistive sheet) and a synergistically designed polyethylene ($\epsilon_r = 2.25$, $\tan \delta = 0.0007$ [46]) superstrate to improve efficiency and Wide Angle Impedance Matching (WAIM). Both of the unit cell’s baluns are connected to an integrated Wilkinson power divider. Each arm of the divider is tapered to transform the impedance seen at the balun port ($\approx 65\Omega$) to $100\Omega$ at the divider junction. This leads to an input impedance of $50\Omega$ at the unit cell’s unbalanced port.

As seen in Fig. 3.26 the balun’s “loop” is oriented in the same direction as the element polarization. As such, the element has a significant electrical coupling to the balun structure. Rigorous equivalent circuits for the feed and element cannot
Figure 3.29: Dimensions of unit cell and balun for design depicted in Fig. 3.26.
Table 3.2: Optimized parameters of design depicted in Fig. 3.29.

be combined to form a full circuit, as with the previous design. Thus, optimization of this design requires a more holistic approach. As a starting point, balun and element dimensions were taken from an optimized version of the design presented in the preceding section. The resistive substrate loading and the superstrate were co-optimized for optimal radiation efficiency as discussed in Chapter 2. The balun-antenna combination (a single balun/element infinite array cell with no power divider) was then tuned using ANSYS HFSS. Following this, the Wilkinson power divider (depicted in Fig. 3.29) was designed. As mentioned, the divider design employs a tapered line for impedance transformation. The line profile was chosen to conform to
a Klopfenstein taper [62,63]. For a line of length \( L \), the impedance varies with length parameter \( z \) as:

\[
\ln Z(z) = \frac{1}{2} \ln Z_0Z_L + \frac{\Gamma_0}{\cosh A} A^2 \phi\left(\frac{2z}{L} - 1, A\right), \quad \text{for } 0 \leq z \leq L, \quad (3.10)
\]

where:

\[
\phi(x, A) = -\phi(-x, A) = \int_0^x \frac{I_1(A\sqrt{1-y^2})}{A\sqrt{1-y^2}} dy,
\]

with special values:

\[
\phi(0, A) = 0
\]
\[
\phi(x, 0) = \frac{x}{2}
\]
\[
\phi(1, A) = \frac{\cosh A - 1}{A^2}.
\]

Other constituent variables include:

\[
I_1(x) = \text{modified Bessel function}
\]
The divider design was then tuned with ANSYS HFSS using a built-in optimizer to minimize mismatch and maximize its port-to-port isolation.

The balun can then be tuned using the circuit depicted in Fig. 3.30. This circuit, which models a set of two radiators (each with a balun), separates the interior lines of the balun from the rest of the circuit. As seen, the transmission lines in the circuit correspond to lines inside the stripline balun, while \([S_U]\), is some unknown \(S\)-matrix representing the rest of the circuit. To de-embed \([S_U]\), one full-wave simulation of the unit cell depicted in Fig. 3.30 is carried out with \(Z_{oc1} = Z_{oc2} = Z_{oc3} = Z_{oc}\), where \(Z_{oc}\) is some arbitrary impedance. The two-port \(S\)-matrix from the simulation run is then transformed into an \(ABCD\)-matrix \(([M_{tot}]\)), as discussed in [64]. This matrix can be decomposed as:

\[
[M_{tot}] = [M_{oc}] [M_{oe}] [M_{U}] [M_{oc}] [M_{oc}],
\]  
(3.11)

where \(M_{oc}\) is given by:

\[
[M_{oc}] = \begin{bmatrix}
\cos(\beta_m l_{oc}) & jZ_{oc} \sin(\beta_m l_{oc}) \\
(jY_{oe} \sin(\beta_m l_{oc})) & \cos(\beta_m l_{oc})
\end{bmatrix}
\]  
(3.12)

where \(\beta_m\) is the stripline propagation constant and \(l_{oc}\) is depicted in Fig. 3.30. Fenced stripline impedances \(Z_{oc}\) and \(Z_{oe}\) can be approximated by Equation 3.1, which gives the impedance of a rectangular coaxial line. \(M_{oe}\), which represents the series open circuit is given by:

\[
[M_{oe}] = \begin{bmatrix}
1 & 0 \\
(jZ_{oc} \tan(\beta_m l_{oc})) & 1
\end{bmatrix}
\]  
(3.13)
Figure 3.31: Infinite array VSWR at broadside scan for optimized unit cell depicted in Fig. 3.26.

Figure 3.32: Infinite array radiation efficiency at broadside scan for optimized unit cell depicted in Fig. 3.26.

where $l_{oe}$ is the length of the open stub. The $ABCD$–matrix representing $[S_U]$ is found by:

$$[M_U] = [M_{oe}]^{-1} [M_{oc}]^{-1} [M_{tot}] [M_{oc}]^{-1} [M_{oe}]^{-1}.$$  \hspace{1cm} (3.14)

The $ABCD$–matrix $M_U$ is then transformed to an $S$–matrix, and the balun lines $Z_{oc3}$, $Z_{oc2}$, $Z_{oc1}$, and $Z_{oe}$ can be optimized for wideband matching in a circuit simulator such as Agilent ADS. After this optimization, the unit cell can be fine-tuned with a full wave simulator such as ANSYS HFSS.

3.4.2 Infinite Array Performance

The final infinite array design was formed with dimensions given in Table 3.2 (referring to Fig. 3.29). The design was simulated, and the broadside VSWR is plotted in Fig. 3.31. Using the matching criterion $VSWR < 2.4$, the design is matched from 0.29-4.03GHz, a 13.9 : 1 bandwidth. The radiation efficiency for broadside scan, plotted in Fig. 3.32 is above 70% for most of the operational band (dropping below this value at only the lower band edge). It is noted that this radiation efficiency
includes losses from the isolation resistors in the integrated power divider. Figs. 3.33 and 3.34 provide the VSWR and radiation efficiency for the unit cell under E-plane scan, out to $\theta = 45^0$. For $\theta = 45^0$ scan in the $E$-plane, the array is subject to some mismatch at the upper edge of the band, reaching $VSWR = 3$ at $3.87GHz$.

The ripple seen in the scanned radiation efficiency curve (see Fig. 3.34) is due to the isolation resistors in the power divider. Scanning in the E-plane causes unequal reflections to enter the divider leading to some loss in these resistors. However, as seen in Fig. 3.34, this loss is very low, manifesting in only a few percentage points in the efficiency. Figs. 3.35 and 3.36 give the VSWR and radiation efficiency under H-plane scan. Using a more relaxed matching criterion of $VSWR < 3$, matching is maintained across the band for a scan angle $\theta = 45^0$ and radiation efficiency does not deteriorate. VSWR and radiation efficiency under D-plane scanning are depicted in Figs. 3.37 and 3.38. Matching and efficiency are well-maintained across the band for scanning out to $\theta = 45^0$. Finally, the scanned ($\theta = 45^0$) polarization purity is depicted.
in Fig. 3.39. As expected, cross-polarization levels are very low when scanned in the $E$– and $H$–planes and they are highest when scanned in the $D$–plane. Nevertheless, for a $\theta = 45^\circ$ scan, $G_{co-pol}/G_{cross-pol}$ is higher than $10\,dB$ across the entire band, and is higher than $15\,dB$ for most of the band.

3.5 Summary

In this section, versions of the Superstrate-Enhanced Substrate Loaded Array (SESLA) with integrated feeds were presented. The challenges of feeding UWB arrays were reviewed, and the suitability of the Marchand balun for the SESLA was examined. The development of a stripline folded Marchand balun for the SESLA was discussed, and the final design was presented in the previous section. It was shown that the design is matched across a 13.9 : 1 bandwidth ($VSWR \leq 2.4$, infinite array at broadside) and exhibits good scanning capability out to $\theta = 45^\circ$. 
Figure 3.37: Infinite array VSWR for D-plane scanning of design depicted in Fig. 3.26.

Figure 3.38: Infinite array radiation efficiency for D-plane scanning of design depicted in Fig. 3.26.

Figure 3.39: Infinite array scanned polarization purity for design depicted in Fig. 3.26.
Chapter 4: Superstrate-Enhanced Substrate-Loaded Array with Integrated Feed: Measured Prototype

4.1 Introduction

In this chapter, the Superstrate-Enhanced Substrate Loaded Array (SESLA) with integrated feed discussed in Section 3.4 is experimentally validated through extensive measurement of an 8x8 prototype array. Fabrication of the prototype is discussed in the next section. In Section 4.3, the measurement setup is detailed, and measured performance is presented. It is shown that the 8x8 fabricated prototype array operates over an 8.2 : 1 bandwidth and scans to $\theta = \pm 45^0$ in $E-$, $H-$, and $D-$planes across most of the band.

4.2 Fabrication of Prototype Array

The unit cell for the fabricated design has slightly different optimized parameters than the one presented in Section 3.4. This is simply due to the fact that the infinite array presented in Section 3.4 was further tuned (for better bandwidth) after the fabricated prototype was built. All dimensions (referring to Fig. 3.29) that vary from those in Table 3.2 are listed in Table 4.1. The infinite array bandwidth of this design is 12.6 : 1 ($VSWR \leq 2.5 : 1$ 0.325-4.11GHz).
Fig. 4.1 depicts images of the prototype at six different stages of the fabrication. As seen in Fig. 4.1-1, each row of feeds and elements is printed on a separate Rogers RO4003 ($\epsilon_r = 3.55$, $\tan \delta = 0.0027$) Printed Circuit Board (PCB). Thus, the design is comprised sixteen total boards: eight identical boards for the feeds, and eight identical boards for the elements. Referring to the images in Fig. 4.1, the fabrication steps are as follows:

1. **Boards are prepared for assembly:** As discussed in the previous section, each unit cell contains a Wilkinson power divider with three resistors. These resistors are soldered to the feed boards, as depicted in Fig. 4.1-1. Eight female edge-mount SMA connectors are also attached to each board.

2. **Feed boards are placed in metal frame:** to align the eight feed boards, eight metal rods are routed through the circular holes in the boards. To ensure proper spacing between the boards, Styrofoam ($\epsilon_r = 1.03$, $\tan \delta = 0.0001$, [65])

---

Table 4.1: Dimensions for unit cell employed in prototype (refer to Fig. 3.29), that deviate from those in Table 3.2.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Value</th>
<th>Design Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_S$</td>
<td>375Ω/sq.</td>
<td>$h_s$</td>
<td>19.5mm</td>
</tr>
<tr>
<td>$g_s$</td>
<td>0mm</td>
<td>$z_e$</td>
<td>0mm</td>
</tr>
<tr>
<td>$z_t$</td>
<td>3.15mm</td>
<td>$x_1$</td>
<td>2.7mm</td>
</tr>
<tr>
<td>$h_2$</td>
<td>27.3mm</td>
<td>$z_1$</td>
<td>11.6mm</td>
</tr>
<tr>
<td>$s_s$</td>
<td>1.27mm</td>
<td>$s_1 = s_2 = s_3$</td>
<td>0.178mm</td>
</tr>
<tr>
<td>$m_1$</td>
<td>0.838mm</td>
<td>$m_2$</td>
<td>0.732mm</td>
</tr>
<tr>
<td>$m_3$</td>
<td>0.564mm</td>
<td>$m_4$</td>
<td>0.483mm</td>
</tr>
<tr>
<td>$m_5$</td>
<td>0.574mm</td>
<td>$m_6$</td>
<td>0.465mm</td>
</tr>
<tr>
<td>$m_7$</td>
<td>0.333mm</td>
<td>$g_{cap}$</td>
<td>0.7mm</td>
</tr>
<tr>
<td>$w_{cap}$</td>
<td>4mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
blocks are placed between the boards (below the frame), as depicted in Fig. 4.1-2.

3. **Ground plane is built around boards:** to form the ground plane around the boards, metal slats are placed between the boards, directly above the metal

Figure 4.1: 8x8 array prototype at six different stages in fabrication.
Figure 4.2: Coaxial line measurement setup for characterizing resistive sheets.

Figure 4.3: Simulated and measured $S_{12}$ response for configuration shown in Fig. 4.2.

rods (see Fig. 4.1-2). Copper tape is used to ensure electrical connection of the feed boards to the ground plane, as seen in Fig. 4.1-3.

4. **Resistive sheets are placed in the design:** 31mm Styrofoam blocks are placed above the ground plane, between the vertically-protruding feed boards. When the blocks are secured with masking tape, strips of the resistive sheet material (375Ω/sq.) are secured on top of the blocks, as depicted in Fig. 4.1-4.

5. **Aperture layer is placed:** another set of 31mm Styrofoam blocks is placed on top of the existing ones such that only the sixteen tabs at the top of each feed board remain protruding. The element boards are then placed on top of the stack of Styrofoam, aligning the protruding tabs such that they fit through the holes in the element boards. Once the element boards are secured on the Styrofoam stack (as seen in Fig. 4.1-5), the protruding tabs (which connect to the feed) are soldered to the elements.
6. **Superstrate is placed:** as the final step, the polyethylene ($\epsilon_r = 2.2$, $\tan \delta = 0.0007$ [46]) superstrate is placed on top of the array. First, thin polyethylene slats are placed on top of the aperture, between the protruding tabs. These slats are slightly taller than the protruding tabs. Then, a much thicker block of polyethylene is placed on top of the slats. Once the superstrate is secured, the array is complete, as depicted in Fig. 4.1-6.

Commercially available window tint films were used for resistive sheets in the presented design. To verify the films’ resistive properties in the band of interest, the measurement setup depicted in Fig. 4.2 was employed. In this configuration, the resistive film is placed within an oversized 50Ω coaxial line (as seen in the image, the sheet rests transverse to the line’s direction of propagation). The transmission response of the line is then measured with a network analyzer, and compared to simulated results of a 50Ω line with a single shunt resistive sheet. The particular test fixture used was the Electro-Metrics EM-2107A [66], a unit designed for evaluating the electromagnetic shielding properties of films.

The simulated and measured transmission responses of various films are given in Fig. 4.3. Good agreement is seen between predicted and measured results, verifying the films’ resistive properties.

### 4.3 Array Measurements

The prototype’s far field gain and scanned patterns were measured in the Ohio State University ElectroScience Lab compact range. As depicted in Fig. 4.4, a 2.4m x 2.4m ground plane was affixed to the array during the measurements. The simulated and measured broadside realized gain is plotted in Fig. 4.5. As seen, good agreement
between simulation and measurement was observed. To obtain the simulated realized gain, the 8x8 array was simulated with integrated baluns (without the dividers) using a symmetry plane to reduce the model size. Simulations were conducted using ANSYS HFSS. For the measurements, a wideband 64-way power divider was connected to the array, to achieve uniform excitation. The losses of the divider ($\approx 1 - 3.5\, dB$) were removed from the measured gain curve. A standard gain horn was used to perform the gain calibration as discussed in [47].

To provide an estimate of the prototype array’s operational bandwidth, Fig. 4.6 gives the simulated total loss in the array under uniform excitation. The curves in this plot were generated from the same model that produced the simulated gain curve of Fig. 4.5 (which agrees well with measurements). Simulations suggest that the 8x8
Figure 4.5: Measured broadside realized gain of prototype 8x8 array.

prototype array operates over an 8.2 : 1 bandwidth (total loss < 2dB). It is noted
that within this band, resistive loss never reaches 1dB. Using a more relaxed loss
criterion (total loss < 3dB), the array operates over a 9.2 : 1 bandwidth.

The measured array scan patterns were obtained using the unit-excitation active
element pattern method [67]. In this scheme, the Unit-Excitation Active Element
Pattern (UEAEP) of each element is taken. The UEAEP for element \( i \), \( \vec{g}_i(\theta, \phi) \) is
the complex-valued field radiation pattern resulting from unit excitation of element
\( i \), when all other elements are terminated (see Fig. 4.7). The field radiation pattern
for the fully-excited \( N \)-element array is then given by:

\[
\vec{E}_T(\theta, \phi) = \sum_{i=1}^{N} V_i \vec{g}_i(\theta, \phi)
\]  

(4.1)

where \( V_i \) is the complex-valued excitation amplitude coefficient for element \( i \). The
major advantage of this method is that once the set of UEAEPs is known for the
array, the full array pattern can be obtained for any complex excitation distribution
\( \nabla = \{ V_1, V_2, ... V_N \} \) (hence, any scan condition). Also, since this method considers
Figure 4.6: Simulated losses in prototype 8x8 array (under uniform excitation).

Figure 4.7: Concept of the array Unit Excitation Active Element Pattern (UEAEP).

element patterns for all elements, it accounts for spatial variation in mutual coupling unlike “classical” array pattern multiplication methods that consider only a single element pattern. This is particularly important for wideband arrays such as this one, where the UEAEP will vary considerably between a centrally-located element and an edge element.

In order to form the scanned array patterns, the co-polarized and cross-polarized UEAEP was taken for all 64 elements in the E-, H-, and D-planes (see Figs. 4.8 and 4.9). Co- and cross-polarization are defined by the common Ludwig’s 3rd polarization definition [68], in which a linearly-polarized antenna oriented in the $\hat{x}$-direction
produces fields:

\[ E_{co-pol} = E_\theta \cos \phi - E_\phi \sin \phi \]  \hspace{1cm} (4.2)

\[ E_{cross-pol} = E_\theta \sin \phi + E_\phi \cos \phi \]  \hspace{1cm} (4.3)
Figure 4.11: Measured H-plane patterns of 8x8 array for various scan angles.

Since the UEAEP method involves coherently summing the element responses, it is critical that the setup is not modified as the UEAEPs are measured. To validate the fidelity of the measurement, Fig. 4.10 provides two measured H-plane array patterns obtained by: 1) using a 64-way power splitter, and 2) using the UEAEP method with uniform excitations. Their excellent agreement serves as validation of the measurement setup.

Fig. 4.11 gives the simulated and measured scanned patterns in the H-plane. In these measurements, uniform excitation was used and all patterns are normalized to the broadside gain. Patterns at $1 - 4\,GHz$ are depicted, representing most of the
array’s band. A frequency-dependent phasing (corresponding to a fixed time delay) was applied for UWB scanning. Excellent agreement is seen between simulated and measured co-polarized patterns as the scanned look angle is varied from $-45^\circ$ to $45^\circ$. Further, the array beams are well maintained as the array is scanned. The measured cross-polarized patterns are also provided (normalized to the co-polarized gain at broadside). As seen, the measured cross-polarized levels are very low.

To provide simulated results for the scanned pattern measurements, finite by infinite simulations of the 8x8 array were conducted as seen in Figs. 4.12 and 4.13. The setup given in Fig. 4.12 was used to predict the normalized array patterns for scanning in the H-plane, while that of Fig. 4.13 was used to predict scanned patterns in the E-plane. The simulations were, again, conducted using ANSYS HFSS.

Fig. 4.14 gives the simulated and measured scanned patterns in the E-plane. Very good agreement between simulated and measured co-polarized gain is once again seen.
Figure 4.14: Measured E-plane patterns of 8x8 array for various scan angles.

For the 2, 3, and 4 GHz plots, the scanned beam is well maintained as the look angle is varied from $-45^\circ$ to $45^\circ$. Cross-polarization levels are, again, also very low.

Fig. 4.15 gives the scanned patterns in the D-plane. Since a finite x infinite simulation could not be carried out for D-plane measurements (and the full 8x8 array is too large and detailed for full simulation), only measured results are provided. As expected, cross-polarization levels are higher in the D-plane as the array is scanned, but nevertheless remain significantly below co-polarized gain levels.

These results validate the design of the SESLA with integrated feed presented in Section 3.4. They also show that a moderately sized implementation of the design
Figure 4.15: Measured D-plane patterns of 8x8 array for various scan angles.

provides wide bandwidth and scanning capability to $45^0$ in E-, H- and D-planes across most of the band.

4.4 Summary

In this section, the Superstrate-Enhanced Substrate Loaded Array (SESLA) with integrated feed design was empirically validated. First, fabrication of an 8x8 prototype array of the design was discussed, followed by details on the measurement setup. Measured results were presented, which showed excellent agreement with simulations.
A plethora of application-specific metrics can be used to evaluate an array’s performance, but using the common metrics of total efficiency (or by extension, gain) and scanning capability, the fabricated prototype is quite wideband. It was shown that the $8 \times 8$ array operates across an $8.2 : 1$ bandwidth (for total loss $< 2dB$, or $9.2 : 1$ for total loss $< 3dB$) and scans to $\theta = \pm 45^0$ in $E-$, $H-$, and $D-$planes across most of the band.
Chapter 5: Ultrawideband Beam-Scanning Horn Antenna

5.1 Introduction

Array electronics such as T/R modules are major drivers in the high cost of phased arrays. This is particularly true for UWB arrays which demand more wideband (hence more expensive) components. Furthermore, these components add weight, increase power consumption, and create additional processing demands. Thus, a high gain UWB scanning array with many elements can be very expensive, heavy, power hungry, and computationally taxing. Reducing array element count, of course, alleviates these issues. However, this generally requires that the array is made smaller or, for a fixed aperture size, that the array elements are enlarged. The former option is undesirable.

Figure 5.1: Conceptual overview of proposed TCDA-fed horn.
since it reduces gain and the latter may hinder wideband matching and/or introduce grating lobes.

In this chapter, an UWB beam-steering horn antenna is presented. The design produces a high gain scanned beam with a much lower element count than a conventional phased array. Thus, for wideband applications that demand only limited scan, it can serve as a low cost/complexity alternative to a full phased array. The design is essentially a truncated pyramidal horn fed by a Tightly Coupled Dipole Array (TCDA), as depicted in Fig. 5.1. The horn structure augments the gain of the array, but still allows limited scanning. As illustrated in Fig. 5.2, this allows navigation of the design space between a fixed aperture and a conventional phased array where the key trade-offs exist between scanning capability, size, and cost. Like designs presented in [69, 70], the antenna is a “hybrid solution” between a fixed aperture and a

Figure 5.2: Representation of the array-fed horn as a compromise between a fixed aperture and a conventional phased array.
scanning array. However, a key difference is that the use of a TCDA as a feed allows the design to operate over a roughly 3:1 bandwidth.

In the next section, the design of a scanning TCDA-fed horn is presented and a corresponding fabricated prototype is presented in Section 5.3. In Section 5.4, the simulated and measured performance of this design is discussed. In Section 5.5, design variations of the UWB scanning horn are examined. Finally, the chapter’s findings are summarized in Section 5.6.

5.2 Design of UWB Beam-Scanning Horn Antenna

The proposed beam-scanning horn depicted in Fig. 5.1 has two design components: the feeding array and the horn structure. The design of these two elements, and their combination to realize the final design, are discussed in this section.

The TCDA unit cell used in the design is depicted in Fig. 5.3. The array element is a dipole which uses overlapping arms to realize capacitive coupling between elements. It is fed by two 100Ω coaxial lines which are excited 180° out of phase, for a total
element impedance of 200Ω. In practice, balanced feeding of this array requires the use of an external 180° hybrid coupler. Fig. 5.4 gives the infinite array VSWR of the design, as fed by an ideal 180° hybrid coupler (with port impedances equal to 100Ω). As seen the infinite array is matched ($VSWR < 2$) across a 3.6 : 1 bandwidth.

The horn flare was designed by following the procedure outlined in [71] for realizing a waveguide-fed pyramidal horn with optimal aperture efficiency. This procedure provides the dimensions for the horn given:

- a desired approximate gain $G_0$,
- a design frequency (or wavelength $\lambda$),
- waveguide dimensions $a$ and $b$ (see Fig. 5.5).

To find the other horn dimensions, the following inequality is solved for $\chi$:

$$\left(\sqrt{2\chi} - \frac{b}{\lambda}\right)^2 (2\chi - 1) = \left(\frac{G_0}{2\pi} \sqrt{\frac{3}{2\pi}} \frac{1}{\sqrt{\chi}} - \frac{a}{\lambda}\right)^2 \left(\frac{G_0^2}{6\pi^3} \frac{1}{\chi} - 1\right). \quad (5.1)$$

From that result, the horn flare dimensions $\rho_e$ and $\rho_h$ (see Fig. 5.5) are found by:

$$\frac{\rho_e}{\lambda} = \chi, \quad (5.2)$$
Figure 5.6: Left: dimensions of a waveguide-fed pyramidal horn. Right: dimensions of the truncated version with a TCDA-feed.

\[
\frac{\rho_h}{\lambda} = \frac{G_0^2}{8\pi^3} \left(\frac{1}{\chi}\right).
\]

(5.3)

The horn aperture dimensions can then be found:

\[
a_1 \approx \frac{G_0}{2\pi} \sqrt{\frac{3}{2\pi\chi}} \lambda,
\]

(5.4)

\[
b_1 \approx \sqrt{2\chi\lambda}.
\]

(5.5)

Fig. 5.6 gives the dimensions of a pyramidal horn designed following this procedure for \(G_0 = 20\, dB\) at 3.95\(GHz\), \(a = 72\, mm\), and \(b = 34\, mm\) (WR-284 waveguide). To design the beam-scanning horn, the pyramidal horn was truncated such that a 4x4 implementation of the TCDA fits as a feed. This configuration (along with relevant dimensions) is depicted in Fig. 5.6. A prototype version of this particular design was built. The fabrication is described in the next section.

5.3 Fabricated Prototype

The 4x4 TCDA used in the prototype UWB beam-steering horn is depicted in Fig. 5.7. As previously mentioned, the array is fed with two 100\(\Omega\) lines which are
excited out of phase by $180^\circ$. The array employs microstrip impedance transformers to transform each element’s set of $100\Omega$ feed lines to $50\Omega$ each. This allows use of off-the-shelf $50\Omega$ hybrid couplers to provide balanced feeding. Fig. 5.8 depicts the array during fabrication and shows how the transformers are connected to the array. Fig. 5.9 depicts the microstrip transformers which employ a Klopfenstein taper [62, 63]. The design equations for this type of impedance transformer were presented in Section 3.4.

The truncated horn structure was constructed with aluminum sheets and affixed to the fabricated $4\times4$ TCDA. The resulting prototype is depicted in Fig. 5.10.

## 5.4 Simulated and Measured Performance

For testing of the design’s broadside scan performance, the array was fitted with a uniform feeding network (see Fig. 5.11). The network is comprised of two 16-way power dividers and a single $180^\circ$ hybrid coupler. The two output ports of the
Figure 5.9: Microstrip 50 – 100Ω transformers used in fabricated 4x4 TCDA.

Figure 5.10: Fabricated TCDA-fed horn.

Figure 5.11: TCDA-fed horn prototype equipped with feed network for broadside scan.

coupler are split 16 ways, and are used to excite the balanced-fed array uniformly (this configuration was also used to test the prototype presented in Section 2.5, and is depicted in Fig. 2.21).

Fig. 5.12 gives the simulated and measured broadside realized gain of the 4x4 TCDA-fed horn. As with most aperture antennas, this design produces a frequency-dependent gain curve. This implies that applications with wide spectrum signals
Figure 5.12: Broadside realized gain of the TCDA-fed horn, optimal-gain waveguide-fed horn of same aperture size, 4x4 TCDA, and 5x6 TCDA. Element size is the same for all arrays. The stand-alone arrays are on ground planes that are the same size as the horn aperture.

(e.g., systems using short pulses) may require equalization. For gain comparison, the broadside realized gain curves of a stand alone 4x4 TCDA and 5x6 TCDA are also given. For all three arrays, the array element size is the same. Also, the stand-alone arrays are placed on ground planes that are the same size as the horn aperture. As seen, the 4x4 TCDA-fed horn provides a gain curve that is up to $3.5\text{dB}$ higher than that of the 4x4 TCDA alone. In fact its gain is comparable to that of a 5x6 TCDA, in spite of having roughly half the number of elements. Thus, for certain applications demanding only limited scan, this implementation can serve as a low-cost, low-power alternative to the 5x6 TCDA. For the measured curve, the losses of the feed network ($\approx 2 - 4.5\text{dB}$) were subtracted out. Very good agreement is seen between simulation and measurements. All simulated results were obtained with ANSYS HFSS.
Figure 5.13: Equivalent H-Plane scanned patterns at 2.5GHz formed through use of a fixed feed network and using the UEAEP method.

For comparison, the gain of the pyramidal horn depicted in Fig. 5.6 is also plotted in Fig. 5.12. This horn has the same aperture size and flare angle as the TCDA-fed horn. It is noted that the TCDA-fed horn produces a comparable gain to the waveguide-fed horn.

In addition to augmenting the gain of the array, the horn structure allows limited scanning. To verify this, scanned array patterns were measured via the Unit-Excitation Active Element Pattern (UEAEP) method [67]. The details of this method were discussed in Section 4.3. As mentioned, stability of the measurement setup is critical in the UEAEP method, since it requires a coherent summation of all element responses. To validate the fidelity of the measurement setup, a scanned pattern formed with the UEAEP method was compared to one obtained using fixed delay lines. As seen in Fig. 5.13, these patterns agree very well, validating the measurement setup.
Fig. 5.14 gives the simulated and measured $H$-plane scanned pattern at 2, 3, and 4 GHz. Simulation agrees well with measurement in showing that the TCDA-fed horn scans to about $\theta = 15 - 20^\circ$ in the $H$-plane. Cross-polarization levels remain $\approx 20 - 30\,\text{dB}$ below main beam levels. Similarly, Fig. 5.15 gives the scanned $E$-plane patterns. These plots show that the TCDA-fed horn scans to about $\theta = 15^\circ$ in the $E$-plane. Cross-polarization levels are, once again, very low.

To check the antenna’s wideband matching, the VSWR of the system fitted with a uniform feeding network was measured (see Fig. 5.16). To remove the effects of the network from the matching characteristic, the following steps were taken:
Figure 5.15: Measured E-plane patterns of TCDA-fed horn various scan angles.

- Feed network reflections in $S_{11}$ response were removed via time-domain gating
- Measured $S_{11}$ response was compensated with round-trip network loss, viz.:

$$S_{11,\text{comp}} = S_{11,\text{meas}} + 2 \times \text{NetworkLoss} \ [dB].$$  \hspace{1cm} (5.6)

The resulting measured VSWR curve is plotted in Fig. 5.16. For comparison, the simulated curve is also included. The simulated curve was found by combining results of the simulated 4x4 TCDA-fed horn with the response of an ideal (perfectly matched and isolated) uniform feed network. To calculate the input VSWR, the procedure described in Appendix A was followed. Good agreement is seen between the simulated
and measured VSWR, showing that the TCDA-fed horn is matched over a roughly 3 : 1 bandwidth. It is noted that by contrast, a standard waveguide-fed horn operates over a 1.9 : 1 bandwidth.

Another benefit of the TCDA-fed horn is that unlike a conventional waveguide-fed horn, it is not plagued by high $E$-plane sidelobes. This is due to the fact that it uses a shorter horn flare, reducing wavefront curvature and the resultant aperture phase errors. Figs. 5.17 and 5.18 give the broadside $H$- and $E$-plane patterns of the TCDA-fed horn, and for comparison, also give the patterns for the waveguide-fed horn depicted in Fig. 5.6. It is noted that the TCDA-fed horn has the same aperture dimensions and flare angle as its waveguide-fed counterpart. As seen in Fig. 5.18, the waveguide-fed horn has much higher $E$-plane sidelobe levels than the TCDA-fed horn. The TCDA-fed horn does produce higher $H$-plane sidelobes, but their levels are far below the waveguide-fed horn’s $E$-plane sidelobe levels.
Figure 5.17: Simulated and measured H-plane broadside pattern for TCDA-fed horn. For comparison, simulated pattern for waveguide-fed horn with same flare angle and aperture size is included.

5.5 Design Variations

The TCDA-fed horn is electrically large and has many design parameters. As such, a global optimization of the structure is impractical. Observing performance trends as these parameters are varied provides guidelines on modifying the design for specific application requirements. In this section, several important design parameters are considered, namely, the horn’s $E$–plane flare angle, $H$–plane flare angle, and $E$–plane wall height.
Figure 5.18: Simulated and measured E-plane broadside pattern for TCDA-fed horn. For comparison, simulated pattern for waveguide-fed horn with same flare angle and aperture size is included.

5.5.1 Effects of Varying Horn Flare Angles

The initial TCDA-fed horn design used flare angles specified by formulas for realizing an optimal-efficiency pyramidal horn, as described in Section 5.2. In this section, the effects of varying the flare angles are examined. The horn flare angles are specified by two parameters, $\alpha_H$ and $\alpha_E$, as illustrated in Fig. 5.19. First, effects on the design’s bandwidth are considered. Specifically, the VSWR seen at the input of a uniform feed network connected to the array (see Fig. 5.16) is examined as a metric
Figure 5.19: Parameters defining horn flare for TCDA-fed horn. Dimensions in mm.

Figure 5.20: Bandwidth ratios (VSWR ≤ 2, see configuration in Fig. 5.16) of array-fed horns with varying flare angle.

of the system bandwidth. Fig. 5.20 gives the bandwidth of the TCDA-fed horn as the flare angle is varied. While $\alpha_H$ has almost no effect on the bandwidth, increasing $\alpha_E$ increases bandwidth slightly. It is noted that, as shown in Fig. 5.20, the array has greater bandwidth within the horn structure than it does outside of it. This is likely due to the image effect caused by the horn walls, which could alleviate array edge effects.

Next, the effects of the horn flare on the system scanning capability was examined. As seen, in Fig. 5.21, a design with a wider H-plane flare allows for wider H-plane scanning. To further show this trend, Fig. 5.22 gives the maximum scan angle as a function of frequency for various values of $\alpha_H$. It is noted the “maximum” scan angle is taken to be the widest angle where:

- the scan loss is less than 3dB,

- the 1st sidelobe level is more than 6dB below the main beam.
As seen in Fig. 5.22, $\alpha_H$ clearly affects $H$-plane scanning, but has virtually no effect on the E-plane scanning capability.

Fig. 5.23 gives scanned E-plane patterns for cases where $\alpha_E = 14^0$ and $\alpha_E = 32^0$. While the wider flare dramatically improves scanning at 4GHz, it does not at 2GHz. Indeed, Fig. 5.24 shows that while there is a general trend linking a wider E-plane flare with wider E-plane scanning, the improvement is frequency-dependent. Specifically, at lower frequencies ($\leq 2.2GHz$), increasing the flare does not improve the the scan angle. It is suspected that diffraction from the E-plane walls leads to higher E-plane sidelobes, limiting the effective scan range. As seen in Fig. 5.24, $\alpha_E$ does not affect the H-plane scanning.

While increasing the horn flare does improve the scanning capability, it tends to diminish the broadside realized gain of the antenna at higher frequencies. This is evident from Figs. 5.25 and 5.26 which depict the broadside realized gain of the

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**Figure 5.21**: Scanned H-plane patterns of array-fed horn. Red curve denotes maximum scan angle. $\alpha_E = 17^0$.

**Figure 5.22**: Maximum scan angle for various H-plane flares ($\alpha_E = 17^0$).
design as the flare angles $\alpha_H$ and $\alpha_E$ are varied. Nevertheless, even for wide horn flares, (i.e., $\alpha_H = 45^0$ or $\alpha_E = 32^0$), the realized gain remains well above that of the 4x4 TCDA alone.

5.5.2 Design with Receding E-Plane Walls

One means of improving the $H-$plane scanning capability is lowering the $E-$plane horn walls, as depicted in Fig. 5.27. The variable $h_R$, as defined in Fig. 5.27 represents the degree of $H-$plane wall recession. As seen in Fig. 5.28, lowering the walls improves the maximum scan angle. For example, lowering the walls by 90mm (about half the total horn length) nearly doubles the $H-$plane scanning ability. Furthermore, as seen in Fig. 5.29, the gain penalty for lowering the walls by this amount is minimal: at worst the gain curve drops by only about 1dB. Thus, lowering the $E-$plane walls
Figure 5.25: Broadside realized gain of TCDA-fed horn for various H-plane flares ($\alpha_E = 17^0$).

Figure 5.26: Broadside realized gain of TCDA-fed horn for various E-plane flares ($\alpha_H = 20^0$).

is a viable option for improving $H$–plane scanning ability, with little compromise in gain.

Figure 5.27: TCDA-fed horn with receding E-plane walls for improved $H$–plane scanning.
Figure 5.28: Maximum scan angle for various values of $h_R$ (see Fig. 5.27).

Figure 5.29: Broadside realized gain for various values of $h_R$ (see Fig. 5.27).

5.6 Summary

In this section a scanning horn antenna fed by a Tightly Coupled Dipole Array (TCDA) was presented. The design provides a high gain scanned beam with a reduced element count compared to a conventional phased array, for savings in cost and complexity. The horn structure augments the gain of the array, but still allows some limited scanning. The fabricated prototype design operates over a $3:1$ bandwidth and scans to about $\theta = 15^0 - 20^0$. Also, as an added benefit, the design was shown to provide a lower overall sidelobe level than a conventional pyramidal horn antenna. In addition to the prototype design, variations were examined where the horn flare and $E$–plane wall height were changed.
Chapter 6: Conclusions and Future Work

6.1 Summary of this Work

The first chapter of this document discussed the ever-increasing need for Ultra-wideband (UWB) phased arrays and the tremendous potential of Tightly Coupled Arrays (TCAs) in this area. This work presents three innovative TCA designs which address several contemporary challenges in the implementation of TCAs.

In Chapter 2, the Superstrate-Enhanced Substrate-Loaded Array (SESLA) was presented. This design employs resistive substrate loading and a synergistically designed superstrate for major array bandwidth enhancement. This innovation addresses the need for low profile arrays with bandwidths well in excess of a decade for multifunctional apertures and software-defined systems. As discussed, the loading suppresses the destructive ground plane interference that limits array bandwidth, while the superstrate maintains high radiation efficiency by drawing power away from the loading. The presented bandwidth enhancement scheme is general: it can be applied to any UWB, planar, ground plane backed array. In Chapter 2, it was applied to a Tightly Coupled Bowtie Array (TCBA) to realize an infinite array bandwidth of 21 : 1 ($VSWR \leq 3$). The SESLA design concept was validated empirically with a measured 4x4 prototype array.
A major challenge in the implementation of extremely wideband designs such as the SESLA is balanced feeding. To address this, a version of the SESLA with an integrated feed structure was presented in Chapter 3. The design employs a folded Marchand balun, and provides an infinite array bandwidth of $13.9:1$ ($VSWR \leq 2.4$). The design was also shown to provide good scanning capability to $\theta = 45^\circ$. In Chapter 4, extensive measurements of an 8x8 prototype array serve as validation of the array and integrated feed design.

Chapter 5 presents an UWB beam-scanning horn antenna, which is essentially a truncated pyramidal horn fed by a Tightly Coupled Dipole Array (TCDA). The horn structure augments the gain of the UWB array, while still allowing some scanning. For applications demanding limited scan, this design offers an alternative to a conventional phased array providing lower cost, lower power consumption, and lower computing overhead (for digitally processed arrays). This design was validated with a measured prototype.

6.2 Future Work

While this work addresses several technical challenges in implementation of TCAs, some areas in which it may be expanded are suggested. Modifying the presented designs for dual-polarized operation (items 1 and 2) is perhaps the highest priority, as many of the target applications demand this. Optimization of finite versions of the SESLA (item 3) to maintain wide bandwidth might be the next highest priority, since the SESLA may be deployed on small platforms (e.g. unmanned aircraft or spacecraft), requiring the use of moderately-sized arrays. Other important research topics include improving the scanning capability of the presented designs (items 4 and
5), and realizing higher-order (more wideband) versions of the SESLA for applications demanding extreme bandwidth (item 6).

The suggested directions for expansion of this work are:

1. **Dual-polarized implementation of SESLA with integrated feed:** many applications targeted by the SESLA with integrated feed presented in Section 3.4 require dual-polarized implementation. In order to realize such a design, it is suggested that using an element configuration with offset phase centers (similar to that presented in [27]) could be employed to accommodate the feed structures for both polarizations.

2. **Dual-polarized implementation of the TCDA-fed horn:** similarly, many target applications of the presented scanning horn antenna require dual polarization. Modification of the element in the presented design is straightforward: a dual-polarized element similar to that presented in Fig. 2.9 (of course, without loading) can be used. The horn flare, however would need to be re-optimized with equal flare angles in both lateral dimensions.

3. **Finite array optimization using integrated baluns:** it is well known that finite ultrawideband arrays exhibit strongly varying impedances at different element locations (e.g., at an edge element vs. an interior element). As a consequence, a finite array designed with an infinite array model may experience strong mismatch in the edge elements. One way to alleviate this issue might be to optimize the element geometry with varying position in the array. However, the computational resources required to simulate larger finite arrays might prohibit this option. A more efficient approach would be to optimize parts of
the baluns that can be separated from the rest of the array (e.g., interior lines of the balun, as discussed in Section 3.4.1) Referring to Fig. 6.1, the matrix \([S_{array}]\) could be de-embedded, and the lines within the baluns could be optimized. Using this technique, the finite array optimization may be carried out with a circuit model, rather than full wave simulation, considerably reducing the computational load.

4. **Expansion of the SESLA circuit model to include scan effects:** Chapters 2 and 3 presented SESLAs whose designs relied on equivalent circuit models. While these models only considered broadside scan, they could be expanded to include scanning effects. This would allow the loading and superstrate to be co-optimized for efficiency and matching as the array is scanned, to improve the design’s overall scanning functionality.

5. **Modification of the TCDA-fed horn antenna for wider scan:** the scanning horn antenna presented in Chapter 5 produces a high gain scanned beam
with a reduced element count (compared to a regular array), while operating over a 3 : 1 bandwidth. This is a tremendously powerful concept, as lowering the element count translates to a major reduction in cost and complexity. However, the design's limited scanning capability \((\theta = 15^0 - 20^0)\) limits the applications in which it can be used. Improving the design's scanning capability could make the concept relevant to more applications. It is suggested that, to this end, alternative structures comprised of metal and/or dielectric could be investigated, to replace the simple linear flare horn for gain augmentation.

6. **Detailed study of Substrate Loaded Arrays (SLAs) with multiple loading layers:** in Section 2.6.2, an SLA design with three resistive substrate loading layers was considered. The design uses the loading to suppress multiple ground plane “short-circuit” frequencies to realize an infinite array bandwidth of 36 : 1 \((VSWR \leq 2.5)\). This design concept was discussed briefly in Section 2.6.2, but a more thorough investigation of the scheme may be useful for applications demanding extreme bandwidth. Specifically, use of capacitively coupled resistive layers (e.g. the resistive FSS discussed in Section 2.3) in conjunction with multiple dielectric and/or printed layers to maximize bandwidth and efficiency would allow this concept to realize its full potential. Such an analysis could be efficiently carried out with a slight variation of the simple circuit model presented in Section 2.6.2 (this simplicity is a key distinction between this design and the Fragmented Aperture Array (FAA), [2,34]).
Appendix A: Input Reflection Coefficient of Array fed by Arbitrary Feed Network

We consider an $N$–port array with S-matrix $[\bar{S}]$ fed by an arbitrary $N + 1$–port feed network with S-matrix $[\tilde{S}]$ as depicted in Fig. A.1. The goal of the forthcoming analysis is to determine the reflection coefficient at the input port of the feed network, $\Gamma_{NET}$. It is assumed that the $i^{th}$ port of the array is connected to the $i^{th}$ port of the feed network and the $(N + 1)^{th}$ port of the feed network is its input. Further, both S-matrices are normalized to the same characteristic impedance.

We begin with the general relation for incident $[\bar{V}^+]$ and reflected $[\bar{V}^-]$ wave amplitudes:

$$[\bar{V}^-] = [\tilde{S}][\bar{V}^+]$$

(A.1)
\[ [\overline{V}^-] = [\overline{S}] [V^+]. \] (A.2)

Equation (A.1) is re-written as:

\[
\begin{bmatrix}
\overline{V}_N^- \\
\overline{V}_{N+1}^-
\end{bmatrix} =
\begin{bmatrix}
\overline{S}_N & \overline{S}_M \\
\overline{S}_L & \overline{S}_{N+1,N+1}
\end{bmatrix}
\begin{bmatrix}
\overline{V}_N^+ \\
\overline{V}_{N+1}^+
\end{bmatrix},
\] (A.3)

where \( \overline{V}_N = [\overline{V}_1^-, \overline{V}_2^-, ..., \overline{V}_N^-] \). It is noted that:

\[ [V^+] = [\overline{V}_N], \] (A.4)

\[ [V^-] = [\overline{V}_N], \] (A.5)

and, hence, (A.2) can be re-written as:

\[ \overline{V}_N^+ = [\overline{S}] [\overline{V}_N]. \] (A.6)

Placing (A.6) into (A.3) gives:

\[
\begin{bmatrix}
\overline{V}_N^- \\
\overline{V}_{N+1}^-
\end{bmatrix} =
\begin{bmatrix}
\overline{S}_N & \overline{S}_M \\
\overline{S}_L & \overline{S}_{N+1,N+1}
\end{bmatrix}
\begin{bmatrix}
\overline{S} \overline{V}_N^+ \\
\overline{V}_{N+1}^+
\end{bmatrix},
\] (A.7)

From (A.7) we obtain:

\[ [\overline{V}_N] = [\overline{S}_N][\overline{S}] \overline{V}_N^- + [\overline{S}_M] \overline{V}_{N+1}^+, \] (A.8)

which is simplified to:

\[ [\overline{V}_N] = \left( \overline{I} - [\overline{S}_N][\overline{S}] \right)^{-1} [\overline{S}_M] \overline{V}_{N+1}^+. \] (A.9)

Also from (A.7):

\[ \tilde{V}_{N+1}^- = [\tilde{S}_L][\overline{S}] [\overline{V}_N] + \tilde{S}_{N+1,N+1} \tilde{V}_{N+1}^+. \] (A.10)
Placing A.9 into A.10, we obtain:

\[
\tilde{V}^{-}_{N+1} = [\overline{S}_L][\overline{S}] \left\{ \left( \bar{I} - [\overline{S}_N][\overline{S}] \right)^{-1} [\overline{S}_M]\tilde{V}^{+}_{N+1} \right\} + \widetilde{S}_{N+1,N+1}\tilde{V}^{+}_{N+1}, \tag{A.11}
\]

and, finally:

\[
\Gamma_{NET} = \frac{\tilde{V}^{-}_{N+1}}{\tilde{V}^{+}_{N+1}} = [\overline{S}_L][\overline{S}] \left\{ \left( \bar{I} - [\overline{S}_N][\overline{S}] \right)^{-1} [\overline{S}_M] \right\} + \widetilde{S}_{N+1,N+1} \tag{A.12}
\]
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