Building Pre-Service Teacher’s Mathematical Knowledge for Teaching of High School Geometry

DISSERTATION

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By

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Abstract

There were two primary goals of the research conducted for this dissertation study. Firstly, to fill a gap in the research literature and begin a discussion around secondary pre-service teachers Mathematical Knowledge for Teaching (MKT) as it pertains to geometry. Although a multitude of studies exist for elementary teachers’ MKT, few exist at the secondary level and thus work is needed in this area of research. The current study is meant to help begin to fill that gap and also to examine the complexities of MKT for geometry. Secondly, I investigated the impact of two quarters of coursework, including two methods courses centered around the analysis of student work and thinking in geometry, on pre-service teachers’ mathematical knowledge.

The participants of the study were composed of eight pre-service teachers seeking licensure to teach 7th to 12th grade mathematics courses at a large Midwestern university. Pretest and posttest surveys were administered to the participants. Additionally, to clarify answers on the surveys, follow up interviews were conducted with all of the participants. Based on their responses, three participants were chosen for an in-depth case study. The selection of the candidates for the case study was carried out based on Cooney, Shealy, and Arvold’s (1998) classification of pre-service teachers.

Analysis of the survey data revealed that the pre-service teachers’ scores on the posttest surveys were significantly lower than their scores on the pretest surveys. A closer
look at the data revealed that the pre-service teachers’ scores on items pertaining to the analysis of student work and thinking, and decompressing were significantly lower in the posttest. However, there was no significant difference between their pre- and posttest scores on items pertaining to instructional strategies, questioning, trimming, and bridging. Additionally there was no significant difference in their scores for self-efficacy from pre- to posttest.

The case studies revealed that the pre-service teachers relied on their past experiences as learners of mathematics as well as their work experiences while making pedagogical decisions. The knowledge of content trajectories was crucial in their pedagogical decision making process. Pre-service teachers were unable to utilize learning based assessment models to aid in assessing student work and developing instructional tasks. Even though the pre-service teachers were not able to use the assessment models, their attention to student work increased. These results relate to previous findings in the area of pre-service teachers MKT (Elbaz, 1983; Fuller, & Brown, 1975; Kahan, et al., 2003; Levin, & Ye, 2008). Implication for future research and practice are discussed.
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Fields of Study

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Table of Contents

Abstract ........................................................................................................................................... ii

Fields of Study .................................................................................................................................. v

List of Tables ...................................................................................................................................... xiii

List of Figures ................................................................................................................................... xv

Chapter 1: Introduction ..................................................................................................................... 1

The Purpose of the Study and its Significance .................................................................................. 6

Research Questions ............................................................................................................................ 12

Overview of the Study ......................................................................................................................... 13

Chapter 2: Literature Review and Theoretical Framework ............................................................ 15

Begle’s Review of Literature ............................................................................................................. 16

What is mathematical knowledge for teaching? .............................................................................. 23

Shulman’s Framework for Knowledge of Teaching .......................................................................... 23

Mathematical Knowledge for Teaching Framework by Ball and Colleagues .............................. 30

Even’s Framework ............................................................................................................................ 36

Knowledge for Algebra Teaching (KAT) Framework (Theoretical Framework) .......................... 40

Explorations of Pre-service Teachers’ MKT ................................................................................... 53

Learning Progressions ...................................................................................................................... 58
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>The van Hiele Model of Development of Geometric Thought</td>
<td>58</td>
</tr>
<tr>
<td>Pirie and Kieren’s Recursive Theory of Mathematical Understanding</td>
<td>61</td>
</tr>
<tr>
<td>Conclusion</td>
<td>64</td>
</tr>
<tr>
<td>Theoretical Framework</td>
<td>65</td>
</tr>
<tr>
<td>Chapter 3: Methodology</td>
<td>71</td>
</tr>
<tr>
<td>Participants</td>
<td>74</td>
</tr>
<tr>
<td>Bran</td>
<td>76</td>
</tr>
<tr>
<td>Cersei</td>
<td>77</td>
</tr>
<tr>
<td>Nedd</td>
<td>77</td>
</tr>
<tr>
<td>Instruments for Data Collection</td>
<td>79</td>
</tr>
<tr>
<td>Pretest Survey</td>
<td>79</td>
</tr>
<tr>
<td>Pretest Interviews</td>
<td>80</td>
</tr>
<tr>
<td>Posttest Survey</td>
<td>81</td>
</tr>
<tr>
<td>Posttest Interviews</td>
<td>81</td>
</tr>
<tr>
<td>Instrument Development</td>
<td>82</td>
</tr>
<tr>
<td>Validity of the instrument</td>
<td>84</td>
</tr>
<tr>
<td>The Methods Courses</td>
<td>87</td>
</tr>
<tr>
<td>Data Analysis and Number Sense Methods Course</td>
<td>88</td>
</tr>
<tr>
<td>Geometry Methods Course</td>
<td>96</td>
</tr>
</tbody>
</table>
Researcher .................................................................................................................. 102

Teacher-Researcher ................................................................................................. 102

Participant observer ................................................................................................. 103

Data Collection .......................................................................................................... 104

Phase I ......................................................................................................................... 104

Phase 2 ......................................................................................................................... 104

Data Analysis ............................................................................................................. 105

Surveys ......................................................................................................................... 105

Interviews ..................................................................................................................... 108

Inter-grader Reliability ............................................................................................... 109

Chapter 4: Analysis and Results ............................................................................... 111

Findings from the Surveys ......................................................................................... 111

Analysis of Student Work ......................................................................................... 113

Instructional Strategies .............................................................................................. 114

Questions: .................................................................................................................... 115

Decompressing ........................................................................................................... 117

Trimming ....................................................................................................................... 118

Bridging ......................................................................................................................... 119

Self-Efficacy ................................................................................................................. 121
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Case of Cersei</td>
<td>Analyses of Cersei’s survey data</td>
<td>123</td>
</tr>
<tr>
<td></td>
<td>Pretest Survey and Interview</td>
<td>129</td>
</tr>
<tr>
<td></td>
<td>Posttest Survey and Interview</td>
<td>138</td>
</tr>
<tr>
<td></td>
<td>Shifts in Focus from Pre to Post</td>
<td>142</td>
</tr>
<tr>
<td></td>
<td>Findings</td>
<td>142</td>
</tr>
<tr>
<td></td>
<td>Summary</td>
<td>163</td>
</tr>
<tr>
<td>The Case of Bran</td>
<td>Analyses of Survey Data</td>
<td>165</td>
</tr>
<tr>
<td></td>
<td>Pretest Survey and Interview</td>
<td>166</td>
</tr>
<tr>
<td></td>
<td>Summary: Bran at the point of entry</td>
<td>170</td>
</tr>
<tr>
<td></td>
<td>Posttest Survey and Interview</td>
<td>179</td>
</tr>
<tr>
<td></td>
<td>Findings</td>
<td>179</td>
</tr>
<tr>
<td></td>
<td>Summary</td>
<td>184</td>
</tr>
<tr>
<td></td>
<td>Conclusion</td>
<td>203</td>
</tr>
<tr>
<td>The Case of Nedd</td>
<td>Analyses of Survey Data</td>
<td>205</td>
</tr>
<tr>
<td></td>
<td>Pretest Survey and Interview</td>
<td>207</td>
</tr>
<tr>
<td></td>
<td>Conclusion</td>
<td>211</td>
</tr>
<tr>
<td></td>
<td>Posttest Survey and Interview</td>
<td>217</td>
</tr>
</tbody>
</table>

ix
Discussion of Findings ........................................................................................................... 280

Teachers’ personal theorizing: Past experiences .............................................................. 281

Mathematical Content Knowledge ..................................................................................... 284

Teachers’ Beliefs .................................................................................................................. 286

Looking back, moving forward: Interpretations and Implications .................................... 287

Pre-service teachers’ prior experiences ........................................................................... 290

Balancing group and individual learning ........................................................................... 292

Was the program a weak link? A theoretical accounting of findings .............................. 294

Connecting theory and practice: An ongoing dilemma ....................................................... 296

Limitations ......................................................................................................................... 299

Recommendations for future research .............................................................................. 301

References ......................................................................................................................... 304

Appendix A: Pretest Survey ............................................................................................... 316

Appendix B: Pretest Interview Protocol ............................................................................ 325

Appendix C: Posttest Survey ............................................................................................. 331

Appendix D: Posttest Interview Protocol .......................................................................... 339

Appendix E: Rubric for assessing pre-service teachers’ analysis of student work and thinking ........................................................................................................................................... 346

Appendix F: Rubric for analyzing instructional strategies used by pre-teachers .......... 349
Appendix G: Rubric for analyzing the open-ended questions and interviews for quality of questions posed by the pre-service teachers ................................................................. 353

Appendix H: IRB Protocol ......................................................................................... 354

Appendix I: Consent Forms ...................................................................................... 368

Appendix J: Verbal Script for Recruitment of Participants ....................................... 372
List of Tables

Table 1. List of Data Collection Instruments ............................................................................. 79
Table 2. Correspondence between Pretest Items and the Research Questions .................. 84
Table 3. Correspondence between Posttest Items and the Research Questions .............. 84
Table 4. Classification of Survey Items .................................................................................. 87
Table 5. Scoring Rubric for Prompts Pertaining to Analysis of Student Work and
Questioning .............................................................................................................................. 107
Table 6. Scoring Rubric for Prompts Pertaining to Designing Instructional Strategies . 107
Table 7. Coding of Interview Data ......................................................................................... 109
Table 8. Pre-test and Posttest Comparison of Means: Total Score .................................... 112
Table 9. Pretest to Posttest Comparison of Means: Analysis of Student Work .......... 114
Table 10. Pretest to Posttest Comparison of Means: Instructional Strategies ............... 115
Table 11. Pretest to Posttest Comparison of Means: Questions .......................................... 116
Table 12. Pretest to Posttest Comparison of Means: Decompressing ............................... 118
Table 13. Pretest to Posttest Comparison of Means: Trimming ........................................ 119
Table 14. Pretest to Posttest Comparison of Means: Bridging ........................................ 120
Table 15. Pretest to Posttest Comparison of Means: Self-Efficacy ................................... 122
Table 16. Timeline of Data Collection .................................................................................. 123
Table 17. Scoring of Cersei’s Responses to the Surveys ..................................................... 126
Table 18. Content on Surveys by Question ................................................................. 128
Table 19. Timeline of Data Collection ...................................................................... 165
Table 20. Scoring of Bran’s Responses to the Surveys ................................................. 167
Table 21. Content in Surveys by Question ................................................................. 169
Table 22. Timeline of Data Collection for Nedd ......................................................... 206
Table 23. Scoring of Nedd’s Responses to the Surveys ............................................... 207
Table 24. Content on Surveys by Question ................................................................. 209
Table 25: Prominent patterns of discourse per participants per interview. Percentages
represent number of occurrences of factors while analyzing student work and thinking 243
Table 26: Prominent patterns of discourse about analysis of student work. Percentages
represent the breakdown of the factors while analyzing student work and thinking..... 244
Table 27: Prominent patterns of discourse per participants per interview. Percentages
represent number of occurrences of factors while commenting on instruction or
instructional decision making ....................................................................................... 245
Table 28: Prominent patterns of discourse about analysis of student work. Percentages
represent the breakdown of the factors while commenting on instruction or instructional
decision making ......................................................................................................... 246
List of Figures

Figure 1. Shulman’s Framework for Knowledge of Teaching .............................................. 24
Figure 2. Shulman’s Forms of Knowledge ........................................................................ 27
Figure 3. Mathematical Knowledge for Teaching (Ball et al., 2008) .............................. 31
Figure 4. Knowledge of Algebra for Teaching (McCrory et al., 2010) ......................... 42
Figure 5. Pirie- Kieren Model of Growth of Mathematical Understanding (Pirie, & Kieren, 1994) ........................................................................................................ 62
Figure 6. Factors Influencing Cersei’s Pedagogical Decision making in the Pretest Items ........................................................................................................................................ 130
Figure 7. Cersei’s Response to Pretest Item # 12 .............................................................. 136
Figure 8. Factors Influencing Cersei’s Pedagogical Decision Making in the Posttest Items .......................................................................................................................................... 138
Figure 9. Mapping of Cersei’s Sources for Analyzing Student Work and Thinking for Pretest ........................................................................................................................................ 144
Figure 10. Cersei’s Response to Pretest Survey item # 13 .................................................. 145
Figure 11. Mapping of Cersei’s Sources for Analyzing Student Work and Thinking for Posttest ........................................................................................................................................ 147
Figure 12. Cersei’s Response to Posttest Item #3 .............................................................. 148
Figure 13. Cersei’s Response to Pretest Item #15................................................................. 150
Figure 14. Mapping of Cersei’s Sources for Designing Instruction for Pretest.............. 155
Figure 15. Mapping of Cersei’s Sources for Designing Instruction for Posttest............ 156
Figure 16. Cersei’s Response to Posttest Item #16............................................................. 159
Figure 17. Cersei’s Response to Posttest Item #8............................................................... 162
Figure 18. Factors Influencing Bran’s Pedagogical Decision Making in the Pretest Items
........................................................................................................................................... 171
Figure 19. Bran’s Response to Pretest Item #18............................................................... 175
Figure 20. Factors Influencing Bran’s Pedagogical Decision Making in the Posttest Items
........................................................................................................................................... 180
Figure 21. Mapping of Bran’s Sources for Analyzing Student Work and Thinking for
Pretest..................................................................................................................................... 185
Figure 22. Bran’s Response to Pretest Item #19............................................................... 186
Figure 23. Mapping of Bran’s Sources for Analyzing Student Work and Thinking for
Posttest..................................................................................................................................... 186
Figure 24. Bran’s Response to Posttest Item #8............................................................... 190
Figure 25. Bran’s Response to Pretest Item #21............................................................... 192
Figure 26. Bran’s Response to Posttest Item #11............................................................... 193
Figure 27. Mapping of Bran’s Sources for Designing Instructional Strategies in the
Pretest..................................................................................................................................... 195
Figure 28. Mapping of Bran’s Sources for Designing Instructional Strategies in the
Posttest..................................................................................................................................... 196

xvi
Figure 29. Bran’s Responses to Posttest Items #12 and 13. ............................................. 198
Figure 30. Bran’s Response to Pretest Item #17................................................................. 201
Figure 31. Factors Influencing Nedd’s Pedagogical Decision Making in the Pretest Items
.................................................................................................................................................. 211
Figure 32. Factors Influencing Nedd’s Pedagogical Decision Making in the Posttest Items
.................................................................................................................................................. 218
Figure 33. Mapping of Nedd’s Sources for Analyzing Student Work and Thinking for
Pretest........................................................................................................................................... 224
Figure 34. Mapping of Nedd’s Sources for Analyzing Student Work and Thinking for
Posttest ........................................................................................................................................ 225
Figure 35. Nedd’s Response to Posttest Item #8 ................................................................. 227
Figure 36. Nedd’s Response to Pretest Item #20................................................................. 229
Figure 37. Nedd’s Response to Posttest Item #9 ................................................................. 230
Figure 38. Mapping of Nedd’s Sources for Designing Instructional Strategies in the
Pretest........................................................................................................................................... 231
Figure 39. Mapping of Nedd’s Sources for Designing Instructional Strategies in the
Posttest ........................................................................................................................................ 232
Figure 40. Nedd’s Response to Pretest Item #21................................................................. 233
Figure 41. Nedd’s Response to Posttest Item #11 ................................................................. 234
Figure 42. Nedd’s Response to Pretest Item #14................................................................. 235
Figure 43. Nedd’s Response to Posttest Item #9 ................................................................. 236
Figure 44. Nedd’s Response to Pretest Item #17................................................................. 239
Figure 45. Nedd’s Response to Posttest Item #8 ................................................................. 240

Figure 46. Common Forces Influencing Participants’ Pedagogical Decision Making... 248

Figure 47. Nedd’s Response to Pretest Item #13 ............................................................... 258

Figure 48. Bran’s Response to Pretest Item #13 ................................................................. 259

Figure 49. Common Forces Influencing Participants’ Pedagogical Decision Making... 281
Chapter 1: Introduction

International comparisons of mathematical achievement of school children document poor performance among American students as compared to their counterparts in other countries (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2008; OECD, 2010). Trends in International Mathematics and Science Study (TIMSS), in particular, revealed that American students performed significantly lower than their peers from six other countries in virtually all content areas. Analysis of factors that impact students’ mathematical performance has also established that while curriculum and its implementation is vital to establishing quality education, equally as important is the presence of “high quality” teachers in classrooms. Despite ongoing debates on the range of attributes and skills that a qualified teacher might need to possess, the subject matter knowledge of teachers has been recognized as a key ingredient to effective practice (Ahn & Choi, 2004). Leading professional organizations such as National Council of Teachers of Mathematics (NCTM) and National Research Council (NRC), in an attempt to establish benchmarks for quality of knowledge needed for teaching, have issued guidelines in which the number and type of mathematics courses to be taken by teachers at levels K-12 have been recommended (Tooke, 1993). These guidelines currently shape assessment of teacher education programs across the country (NCATE, n.d.), assuming them as indicators of high quality teaching (Palardy & Rumberger, 2008). Highly qualified teachers are defined in terms of background characteristics such as
teacher certification and a bachelor’s degree in the subject area (U. S. Department of Education, 2002; Palardy & Rumberger, 2008). Research suggests that such criteria may not be adequate in judging the quality of teaching taking place in the classroom (Hill, Rowan, & Ball, 2005; Palardy & Rumberger, 2008). However, for secondary teachers there is an additional requirement of subject area competence (Palardy & Rumberger, 2008). The No Child Left Behind Act (NCLB) requires that the teachers must demonstrate subject matter competency evidenced through degree completion or course taking patterns (Hill et al., 2005). Researchers have challenged the merit of such proxies and argued that simply having taken higher-level mathematics courses or securing a degree in mathematics does not ensure instructional success (Ball & Bass, 2000; Hill et al. 2005), suggesting that the characteristics of highly qualified teachers put forth by the Department of Education are not supported by scientifically based research (Darling-Hammond & Youngs, 2002).

Lack of satisfaction with mathematics education in the US is not a new phenomenon. Indeed, efforts to enhance the quality of mathematics teaching and learning date back to the early 20th century, but most prominently and politically visible since the 1960s (Even, 1990). While calls for deep conceptual understanding and meaningful learning have been widespread, consensus on the quality of knowledge needed for facilitating such learning among school children remains unachieved. Literature indicates that even though mathematical content knowledge is necessary for effective practice, it is not sufficient to nurture the kind of teaching demanded by the reform documents. Moch (2004) summed up the difference between knowing mathematics and knowing mathematics for teaching as follows:
The person who knows mathematics can substantiate, prove, confirm, authenticate, validate, and collaborate the facts on which this individual bases a claim of knowledge. A teacher must be able to do this in addition to being able to apply pedagogical knowledge, instructional strategies, classroom management, and student guidance. (p. 126).

Issues surrounding what teachers need to know and be able to do in order to be effective teachers remain unresolved (Ball, Thames, & Phelps, 2008). According to NCTM’s Principles and Standards for School Mathematics (2000), effective mathematics teaching must, “balance purposeful, planned classroom lessons with the ongoing decision making that inevitably occurs as teachers and students encounter unanticipated discoveries or difficulties that lead them to unchartered territory” (NCTM, 2000, p. 18). In navigating the demands of such practice teachers would have to possess, among others, a deep knowledge of children’s thinking (Ball, et al., 2008), their conceptions and development, a sound understanding of curriculum and its trajectory (McCrorry, Ferrini-Mundy, Floden, Reckase, & Senk, 2010), facility with representational modes and teaching tools that provide children’s access to robust mathematics (Ball, Hill & Bass, 2005; Even, 1993). These skills and knowledge domains have been characterized to be a part of Pedagogical Content Knowledge, as coined by Shulman (1986).

Shulman (1986, 1987) described Pedagogical Content Knowledge (PCK) as “knowing the ways of representing and formulating the subject matter that make it comprehensible as well as understanding what makes the learning of specific topics easy or difficult” (cited in Even, 1993, pg. 94). Drawing on Shulman’s PCK construct, Ball and colleagues developed a framework to capture the various types of knowledge that are
required by a mathematics teacher into a single construct referenced as mathematical knowledge for teaching (MKT) (Hill, 2007). MKT consists of not only knowledge of mathematics content but also of how it might be taught. Central to the domain of how mathematics content is taught is knowledge about the degree of difficulty of topics according to children’s development, ways of sequencing topics so that student learning is optimized, representing mathematical concepts so that they can be understood by students, bridging ideas and concepts from various areas of mathematics (Stevens, Harvey, Cuoco, Lee, & Baldassari, 2005).

Within the past decade several studies have been conducted to analyze the mathematical knowledge for teaching (Ball, 1990; Hill, 2007; Hill, Ball, & Schilling, 2008). These studies attempt to establish a link between teachers’ knowledge and the student achievement. For instance, Monk (1994) reported that teachers’ mathematics knowledge had a positive effect on students’ knowledge. Ball et al. (2005) also reported a positive relationship between mathematical knowledge for teaching and student achievement at the elementary grade levels. They also pronounced the particular significance of knowledge of children’s thinking on effective practice. Compatible studies that explore either the nature of high school mathematics teachers’ knowledge or its impact on teaching have been rare. The majority of research reports that concern secondary teachers focus on unpacking the connections between teachers’ beliefs and their practice as it is enacted through curriculum implementation. One of the few reported research studies on secondary teachers’ knowledge was conducted by Even (1990) in which the researcher investigated pre-service conceptualization of the topic of functions. Even (1990) reported an absence of knowledge about the importance of the
concept of function among teachers he studied. The researcher highlighted the importance of this domain of knowing and its relevance to school curriculum and instruction for teachers, arguing that assisting teachers to develop such knowledge base in content courses designed for teachers must become a priority. A review of the existing literature on secondary teachers’ knowledge and secondary school teaching reveal a focus on identifying gaps in knowledge (or ways to enhance it) as opposed to unpacking the nature of teachers’ understanding of mathematics for teaching. This gap is due, without the loss of generality, to two important factors. On the one hand, MKT secondary school is conceptually underdeveloped and discussions exist surrounding how it might be similar to or different from teaching elementary level mathematics. The range of mathematical topics addressed in secondary curriculum is far more diverse than those covered at the elementary grades with far less understanding of epistemological or ontological issues surrounding their development. These factors have made it difficult for scholars to develop measures that could adequately capture teachers’ own knowledge for teaching. Indeed, although Ball and her colleagues (Ball, 1990; Ball & Bass, 2000; Ball et al., 2005; Hill, 2007; Hill, et al., 2005) have made great strides in developing a measure of mathematical knowledge for teaching elementary and middle school (Learning Mathematics for Teaching), similar attempts at the secondary level are still in infancy. Two particular efforts include McCrory et al. (2010) work towards developing and testing a framework for measuring the Knowledge for Algebra Teaching (KAT), and ongoing effort at the Center for Research in Mathematics and Science Teacher Development, University of Louisville aimed at building an instrument to capture mathematical knowledge for teaching Geometry at the secondary school level. The
outcomes of these efforts, while promising, remain unclear as neither of the teams has yet released comprehensive reports on their progress or efficacy of their efforts. As such, the field of study, as it pertains to unpacking MKT at the secondary level or measuring it, remains under-developed.

The Purpose of the Study and its Significance

The goals of the present study were twofold. First, it was my aim to explore the nature of mathematical knowledge of a cohort of pre-service secondary mathematics teachers’ knowledge for teaching high school geometry by eliciting their reactions to episodes of children’s work-samples on geometric tasks. A second goal of the study was to investigate the impact of two quarters of coursework centered around the analysis of student work samples and thinking about geometry on pre-service teachers’ conceptualization of teaching actions. In this study I utilized the KAT framework developed by McCrory et al. (2010). This framework consists of a two dimensional matrix; the rows of the matrix stating the tasks to be performed by the teacher and the columns indicating the categories of knowledge for teaching. There are also three overarching categories, which form a third dimension. The tasks for teaching include analyzing students’ mathematical work and thinking, designing, modifying and selecting mathematical tasks, establishing and revising mathematical goals for students, accessing and using tools and resources for teaching, explaining mathematical ideas and solving mathematical problems, and building and supporting mathematical community and discourse (McCrory et al., 2010). The categories of knowledge include core content knowledge, knowledge of representations, knowledge of content trajectories, knowledge of application and contexts, knowledge of language and conventions and knowledge of
mathematical proofs and reasoning (McCrory et al., 2010). The three overarching categories are bridging, trimming and decomposing. Thus the framework provides us with 36 cells, each of which describes the importance of a particular category of knowledge for a particular task of teaching and how that knowledge should be utilized through the processes of bridging, trimming, and decomposition.

This model examines teaching actions pertaining to mathematical work explicitly, breaking down the tasks that teachers need to perform in a classroom and the type of knowledge required to perform those tasks. Even though the KAT framework is designed for the teaching of algebra, the tasks of teaching and the categories of knowledge listed are applicable to all branches of mathematics, including high school geometry. Thus, the KAT framework serves as a suitable choice for the proposed study. This framework guided the development of the data collection instrument. The items on the surveys were specifically designed to meet categories defined by the authors.

For the purpose of this dissertation I focused on the task of analysis of student work and thinking and the categories of knowledge associated with that task. Listening to and understanding student thinking has consistently been supported by educators as a major and vital influence on effective practice (Crespo, 2000). Analysis of student work is viewed as a resource for teachers to make informed decisions (Crespo, 2000). For a successful transition into a mathematics classroom, teachers need to have an extensive understanding of mathematical ideas, connections between different mathematical concepts, and knowledge of children’s cognition (Manouchehri, & Goodman, 2000).

According to the Principles and Standards for School Mathematics (NCTM, 2000), “Effective mathematics teaching requires understanding of what students know
and need to learn and then challenging and supporting them to learn it well.” (p. 11). Isiksal & Cakiroglu (2011) support this statement: “To improve mathematics instruction, teachers need to challenge and support students and have a sound understanding of the gap between what students know and what they need to learn” (p. 214). Effective teachers are also required to possess the knowledge of students as learners. Having the knowledge of what the students understand helps teachers to make better curricular and instructional judgments. The analysis of student work should also be the basis on which the teacher decides if the student is learning what was intended and also the nature of that learning. This knowledge allows the teacher to respond to the students’ questions appropriately so to advance their thinking. According to Rowland (2008), teachers can structure their ‘next move’ with an appropriate analysis of student work and thinking. As such it is essential that teachers know how children think and be familiar with their common understandings and misunderstandings so that they can help the children overcome their difficulties. Assessing student work can be useful to teachers conceptualizing ways to help them overcome these misunderstandings. The need to analyze student work is also essential as it gives teachers exposure to the non-traditional ways in which students might solve problems. The analysis of such work could lead into the investigations of whether the particular method used works all the time or only in a given specific situation within the given constraints. Teachers could also build their instruction around these methods. This knowledge will be useful for teachers, so that they can give constructive feedback to the students. However, teachers whose teaching is not centered on student thinking often tend to impose their own formalized conceptions onto the student, which are not always effective (Cobb, 1988).
According to the NCTM standards (2000), it is essential that the teachers give feedback to students. Feedback based on thoughtful analysis of student work can help students in understanding the concept and explain to them why their approach would or would not work. Another important quality highlighted by the NCTM standards (2000) is that the teachers must be able to determine what knowledge students bring to the classroom, since students often connect new ideas to the prior knowledge that they possess. The standards also suggest that in order to improve their instruction, teachers should be able to recognize what their students are doing.

Using variety of strategies teachers should monitor students’ capacity and inclination to analyze situations, frame and solve problems, and make sense of mathematical concepts and procedures. They can use this information to access their students’ progress and to appraise how well the mathematical tasks, student discourse, and classroom environment are interacting to foster students’ learning. Then they use these appraisals to adapt their instruction. (p. 19).

Thus there is a call for teachers to base their instruction on student understanding and use their assessment of student work to design tasks in order to advance learning. It is useful to study student learning in a more broad sense as well. It is important for teachers to be able to analyze and understand typical student work (i.e. what students normally do in a given situation for a given problem). However, it is also essential to study students’ atypical responses and conceptualizations about mathematical topics. Understanding these atypical approaches to problem solving might give the teacher an insight as to what the student is thinking and also where the student might be making a mistake. Such
analysis could bring out the misconceptions that students might have as well as the source of the misconception.

Research has shown that pre-service teachers lack the skills needed to understand and analyze student work and thinking. Many pre-service teachers attribute student errors to learning deficiencies (Cooper, 2009). Also, despite being able to detect errors, pre-service teachers are not able to come up with alternate instructional strategies that can enhance student learning (Cooper, 2009). Pre-service teachers often tend to re-teach the content without changing or modifying the instructional approach (Cooper, 2009). Many researchers have suggested that pre-service teachers need to be exposed to the task of analyzing student work to combat these issues (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Isiksal & Cakiroglu, 2011, Kazemi & Franke, 2004).

Our own research also indicated that pre-service teachers entering the teacher education program do not have a rich analysis of student work and thinking (Gilchrist & Somayajulu, 2010ab; Gilchrist & Somayajulu, 2011). Our investigation of pre-service teachers who were at the beginning of the Master of Education program revealed that despite their observation of student work, they did not consider it while designing their instructional strategy. The pre-service teachers leaving the program had a better sense of analysis of student work. It was, however, not advanced (Gilchrist & Somayajulu, 2010ab; Gilchrist & Somayajulu, 2011). Our findings indicate that teachers need to be exposed to episodes of student work. Teacher education programs need to include courses that are integrated with tasks concentrating on student work and thinking.

Another issue involved with analysis of student work and thinking is that of teacher self-efficacy. Teachers’ self-efficacy is defined as “their belief in their ability to
have a positive effect on student learning” (Ashton, 1985, p. 142). Pre-service teachers’ self-efficacy is high (Bush, 1986). However, Smith (1996) argued that teachers’ sense of self-efficacy is at risk every time they ask a student to voice their thoughts and ideas because the teacher may not be able to understand. Thus, often teachers do not ask students to voice their concerns or thoughts in the open. However, the relationship between teachers’ self-efficacy and their knowledge of student work and thinking has not been explained. I propose to investigate this relationship in this study.

There have been various calls for re-designing education courses so as to better prepare teachers to understand student work (Cooper, 2009; Manouchehri, 1997, 2008; McLeman & Cavell, 2009). One of the remedies for this is the use of written, video, or animated examples of student work (Manouchehri, 2008). Some of the suggestions to improve pre-service teachers’ knowledge on student thinking include group work and video technology that leads to discussions about the course content (Cooper, 2009). Jacobs and Philipps (2004) suggested the inclusion of examples of student work to facilitate discussion about student thinking of mathematics. The examination and discussion of various examples of authentic student work can be used to increase teachers’ knowledge of analysis of student work (Timmerman, 2004). Cooper (2009) has recommended that the following tasks be included in methods courses:

(i) Analysis of children’s written work and related thinking.

(ii) A deeper exploration of mathematical content to develop alternate instructional strategies.

(iii) Observing children engaged in performing mathematical tasks
There is little evidence that pre-service teachers learn mathematics for teaching from the content courses focused only around the mathematics devoid of connections to pedagogy (Superfine & Wagreich, 2009). Typically, courses in the methods of teaching mathematics for pre-service teachers have been designed without enough care given to the pedagogical as well as mathematical requirements for pre-service teachers’ mathematical knowledge for teaching (Superfine & Wagreich, 2009). Some questions remain unanswered: What are the roadblocks faced while designing a mathematics methods course? How should these courses be taught? This study will also attempt to answer questions as to what obstacles are faced in designing and implementing courses in the teaching of geometry and measurement.

Research Questions

The research questions for the proposed study are as follows:

1. What factors do pre-service teachers consider when judging students’ mathematical work and thinking?

2. What is the effect of two quarters of coursework on pre-service teachers’ assessment of students’ mathematical work and thinking of geometry and measurement?

3. What is the effect of two quarters of coursework on pre-service teachers’ ability to develop instructional strategies that aid student understanding of geometry and measurement?

4. What is the effect of two quarters of coursework on the quality of questions posed by the pre-service teachers to elicit student understanding of geometry and measurement?
5. What is the relationship between levels of teachers’ self-efficacy and their knowledge of students’ learning and thinking?

Overview of the Study

The study took place in a large Midwestern university. The participants of the study were the 2011-2012 cohort of pre-service mathematics teachers seeking licensure to teach 7th-12th grade mathematics courses. The proposed study consisting of two phases commenced in the summer quarter of 2011. During the methods course in number sense and data analysis, which was offered during the first term of summer 2011, the incoming cohort of pre-service teachers were asked to complete a pretest survey as a part of their coursework to elicit their knowledge at the beginning of the program. The pretest survey consisted of 25 questions out of which 15 questions were open ended. Pretest interviews were conducted near the beginning of the autumn quarter of 2011. The aim of the pretest interview was to gather in depth information about the pre-service teachers’ responses to the pretest surveys.

For the second phase of the data collection, the pre-service teachers were requested to complete the posttest surveys, which were given to them as a part of a course assignment. The posttest survey consisted of 21 questions out of which 14 questions were open ended. This survey was followed by a posttest interview. The posttest survey and interview was similar in structure to the pretest survey and the pretest interview.

The purpose of chapter 1 was to provide background, goals, significance, and research questions. Chapter 2 will present an overview of the literature on Mathematical Knowledge for Teaching (MKT) including the various frameworks that have been
proposed to capture MKT, explorations of pre-service teachers’ MKT, van Hiele and Pirie-Kieren models of assessment, and the theoretical framework.
Chapter 2: Literature Review and Theoretical Framework

It would be foolish to question the centrality of content knowledge for teaching of mathematics. However, the nature of knowledge of mathematics needed for quality mathematics teaching has been a source of debate for decades. Recently, the concept of mathematical knowledge required for teaching has lured many mathematics educators (Hill, et al., 2005). Until a few years ago, teachers’ knowledge was measured using quantitative constructs such as the number of mathematics courses taken or scores earned on standardized tests (Ball et al., 2008; Even, 1993). However, these measures have been shown to be unreliable, as they do not reveal the true nature of teacher knowledge.

In this chapter, I first summarize Begle’s (1979) review of literature and point out the weak points in his study. Begle’s study is the first attempt at understanding the influence of teachers on students’ mathematics learning. Begle (1979) characterized teachers’ knowledge in terms of variables such as number of courses taken, degrees obtained etc. Secondly, I will present on Shulman’s framework (1986, 1987) of knowledge for teaching followed by the Mathematical Knowledge of Teaching (MKT) framework developed by Ball and colleagues (2008). Shulman’s work is extremely important because he was the first to coin the term pedagogical content knowledge (PCK), now widely used, in trying to develop educational programs for teachers. Ball et al. (2008) were the first to develop a framework for explaining mathematical
knowledge for teaching of mathematics. Ball’s framework has been widely used to study elementary teachers’ MKT and also to investigate the correlation between MKT and student achievement (McCrorry, et al., 2010, Ball et al., 2008). Next I will talk briefly about the subject-matter knowledge framework developed by Ruhama Even (1990) and the strengths and weaknesses of each of these models. Lastly I will present the Knowledge for Algebra Teaching (KAT) framework (Floden, McCrorry, Reckase, & Senk, 2009; McCrorry et al., 2010) which serves as the theoretical framework for my study. Finally I will summarize the literature on pre-service teachers’ analysis of student’s work.

Begle’s Review of Literature

Begle (1979) in his review of literature identified teachers as one of the most important variables in mathematics education. Begle (1979) put forth an effort to try and understand the ways in which teachers influence mathematics learning. According to Begle (1979), studies of teacher characteristics fell into two categories namely, studies of teacher knowledge of mathematics and studies of teacher attitudes.

Begle (1979) criticized a number of studies that were devoted to finding the characteristics that identify an effective teacher because according to him effectiveness of a teacher was relative and that defining a perfectly effective teacher was a futile task in the absence of comparison samples; “We can only deal with the effectiveness of a teacher relative to the effectiveness of a pool of teachers on the learning of their students” (p. 29). However, according to Begle (1979), the only satisfactory way of measuring teacher effectiveness is through student learning but he also pointed that the very concept of teacher effectiveness may not be valid and that it was not a stable indication of a teacher characteristic.
Also mentioned in Begle’s review of literature were characteristics of teacher’s background variables and their relation to student achievement. The following were the variables included (Begle, 1979).

1. Number of years of teaching
2. Highest Academic degree
3. Academic credits beyond BA
4. Math Credits beginning with Calculus
5. Credits in math methods
6. In-service or extension courses
7. Other preparation in the last five years
8. Sex
9. Age
10. Current marital status
11. Children
12. Math as a major or minor
13. Major field

All of these variables had shown to have a positive main effect on student achievement. However, none of these characteristics were powerful indicators of teacher effectiveness. In addition to these 13 variables, 7 affective variables were also considered namely, theoretical orientation, concern for students, involvement in teaching, non-authoritarian orientation, like vs. dislike, creative vs. rote and need for approval. These affective variables had a positive main effect with student achievement but similar to the
characteristics of teacher background, they did not have a strong influence on teacher effectiveness.

Most of the studies that were reviewed by Begle (1979) were based on the notion that teachers need a thorough understanding of mathematics in order to teach the subject. However, none of these studies ever mentioned how thorough the understanding needs to be (Begle, 1979). There is a minimum amount of mathematical content knowledge that is required by high school teachers in order to maintain a positive correlation between their mathematical content knowledge and student achievement. One of the studies conducted by Begle (1972) comprised of 308, 9th grade teachers from around the country revealed that there was no significant correlation between teachers’ understanding of abstract algebra and student’s achievement scores in 9th grade algebra. However, there was a positive correlation between teachers’ knowledge of the real number algebra and student achievement scores (Begle, 1972). Although comprehensive, Begle considered a highly biased sample of teachers. All the teachers were NSF Institute participants who had volunteered to participate in the study and thus were highly motivated (Eisenberg, 1977). His results, however, are interesting. Mathematical content knowledge in algebra was necessary and was positively correlated with student achievement, but having taken abstract algebra had no relation to student achievement (Eisenberg, 1977).

Eisenberg (1977) replicated Begle’s study using an unbiased sample of 28 junior high school teachers in Columbus, Ohio and reached the same conclusion that there was very little effect of teachers’ content knowledge on student achievement. Eisenberg (1977) also stated that there was perhaps a lower bound of knowledge but it was too low to worry about. In other words, in addition to content knowledge, it is important that
teachers also require a bridge between the content knowledge and how they apply it in the classroom. Eisenberg’s (1977) choice of teachers was a better representative of the national population as compared to Begle’s (1972) sample. However, the sample was too small. Further, the sample was from only one state and was restricted to urban schools. Despite this, both these studies concluded that in order to teach high school algebra, algebra knowledge limited to real number systems was a sufficient predictor of students’ positive achievement.

A similar study was conducted by Tooke (1993) which aimed to observe the effects of student teachers’ mathematical backgrounds on student achievement. In this study, the researcher divided the mathematics courses into six different groups. The first group contained courses that do not normally require a calculus prerequisite, the second group contained the calculus sequence (Tooke, 1993). The third group contained an elementary abstract geometry course, and the fourth group contained an abstract algebra course and an introductory analysis course while the last group contained all post calculus mathematics courses (Tooke, 1993). Student achievement was measured on the basis of a performance-based objective test in pre-algebra and post-algebra courses (Tooke, 1993).

The results of this study revealed that there was a significant positive correlation between the student teachers’ grade in pre-calculus and their students’ achievements in pre-algebra classes (Tooke, 1993). There was no significant correlation between the student teachers’ grade in pre-calculus and their students’ achievement in algebra courses or courses with an algebra prerequisite (Tooke, 1993). However, the student teachers’ grade point ratio in post-algebra courses had a significant positive correlation with
students’ achievement in algebra courses and in courses with algebra as a prerequisite (Tooke, 1993). It was also noted that the student teachers’ grade point ratio in post-calculus courses had no significant correlation to student achievement in pre-algebra courses. The geometry courses and the post-calculus courses had a strong positive correlation to students’ algebra and post-algebra achievement.

Tooke’s (1993) study suffered some major drawbacks in the methodology, as described by the researcher. The instruments used in the study were not tested for validity and reliability (Tooke, 1993). Moreover, only 23 student teachers were included for this study with approximately 30 to 120 students per student teacher (p. 6). Student achievement was measured in terms of their performance on the objective tests in pre and post-algebra. This was a weak measure of student achievement. Another drawback was that the study did not specifically mention what courses fell under the post-calculus group. I suggest this because there are many different paths that mathematics students may take after calculus. Another drawback was that it did not mention whether any of the student teachers took graduate level classes. Despite the limitations, the study supported the claim that high school teaching required some amount of mathematical content beyond pre-calculus.

Monk (1994) found that although there was a positive relationship between the number of undergraduate mathematics courses taken by the teacher and their student’s mathematics achievement at the sophomore and the junior level, it did not have much significance when the number of undergraduate mathematics courses taken by the teacher exceeded five courses. The sample for this study consisted of 2,829 students from 51
schools selected from around the nation (Monk, 1994). Among these, 60 tenth grade students from each school were randomly selected for the study (Monk, 1994).

The study revealed a positive relationship, at the sophomore and junior grade levels, between the number of undergraduate mathematics courses taken by the teachers and their students’ achievement in mathematics (Monk, 1994). For the sophomores, graduate level mathematics courses taken by the teacher had a positive relationship with students’ mathematics achievement when the other teacher variables were controlled (Monk, 1994). These results were in agreement with the findings of Tooke (1993) and Eisenberg (1977) in that they too suggested that teachers’ mathematical knowledge was related to student achievement at the high school level. However, this study further revealed an interesting relationship between the number of mathematics courses taken and student achievement at the junior grade level. The study suggested that teachers who had taken five or fewer mathematics courses had a strong positive impact on student achievement (Monk, 1994). This relationship was not as strong at the sophomore level. A curvilinear relationship was observed between the number of undergraduate level courses taken and the mathematics achievement of students at the junior level (Monk, 1994). As stated, “This model served the best when the distinction was drawn at having five or fewer versus more than five undergraduate mathematics courses” (Monk, 1994, p. 130). It is curious to see how the effect of a sixth mathematics course is considerably smaller than that of the first five courses. This study suggested a possible upper bound for the number of advanced courses which might impact students’ mathematics achievement (Monk, 1994). Monk (1994) stated that although more than five undergraduate mathematics courses have a positive effect on the student achievement; this effect was
too small to be considered as educationally significant. Since the sample in Monk’s (1994) study was a good representative of the entire population of high school students in the United States the results are of importance. The study also revealed a positive correlation between teachers’ subject matter preparation and the course taking patterns among their students (Monk, 1994). However, the author admitted that one could interpret this correlation in two different ways. First, it was possible that this correlation was because of the fact that brighter students made better use of the teachers’ mathematics knowledge compared to less bright students (Monk, 1994). Second, it could also be possible that teachers who were teaching advanced classes may have possessed a stronger mathematical background than those teaching other classes.

Collectively, the studies reported above indicate that while teachers do need to have an understanding of mathematics in order to teach at the high school level, the nature of mathematical understanding remains unclear. Research shows that simply having taken a large number of mathematics courses is not sufficient to ensure instructional success. These findings have provided the view that some specialized knowledge of mathematics might be needed for practice (Ball et al., 2008). To better unpack this specialized knowledge for teaching of mathematics demands the establishment of a framework which takes into account the various domains of knowing pertaining to the practice of teaching. Such a framework would need to account for, and explain, how teaching of mathematics is different than teaching of other subjects such as languages or social studies.
In the next section I will elaborate on the frameworks for knowledge that have been extensively used in the field. From this elaboration, I will then choose a framework that is most relevant to my study.

What is mathematical knowledge for teaching?

*Shulman’s Framework for Knowledge of Teaching*

One of the major breakthroughs in categorizing professional knowledge for teaching was made by Lee Shulman. According to Shulman (1987) the knowledge base for teaching consists of skills, understanding, technology, codes and ethics of teaching and also the means for representing and communicating the knowledge to the students. Shulman (1987) suggested that teachers can “transform understanding, performance skills, or other desired attitudes or values into pedagogical representations and actions” (p. 7); something that professionals in other fields may or may not have. According to him, teaching begins with, “a teacher’s understanding of what is to be learnt and how it is to be taught” (p. 7).
Among these content knowledge, curriculum knowledge and pedagogical content knowledge have gained considerable attention as they have been extensively utilized to develop educational programs for teachers and are specific to the discipline of mathematics. The categories are described below:

a) Content knowledge:

Content knowledge includes the knowledge, skill, and understanding of the content that is to be taught to schoolchildren. This includes knowing the subject matter and identifying the important concepts to be taught. This type of knowledge requires an understanding of the structure of the subject matter (Shulman, 1986). According to Shulman, “the structures of a subject include both the substantive and the syntactic” (p. 9). The substantive structures are the different ways in which the concepts and principles of the domain are structured while the syntactic structures are those that help us determine the validity or invalidity of the material under study. Another important aspect
of this knowledge domain is that the teacher not only understands whether a given argument is true or false but should also have the ability to reason why it is correct or false. This type of an understanding may or may not be required in other professions.

Mathematical content knowledge also includes the ability to identify errors in student responses and in textbooks (Ball, et al., 2008). According to Shulman (1987), knowledge of mathematical content is accumulated through literature and the historical and philosophical scholarship in the field of study. This knowledge also includes recognizing the important skills and ideas in the domain, the knowledge of adding new ideas to the domain and excluding defective ideas from it (Shulman, 1987).

b) Curriculum knowledge:

The curriculum knowledge, according to Shulman (1986) is, “represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contradictions for the use of particular curriculum or program materials in particular circumstances” (p. 10). This knowledge domain is essential to instruction as teachers are expected to possess understanding of the curricular materials and tools available to them for instruction. In addition, teachers should be aware of the alternate sources of knowledge that are at their disposal.

Shulman (1986) further divided curriculum knowledge into two sub categories namely lateral curriculum knowledge and vertical curriculum knowledge. Lateral curriculum knowledge consists of the teacher’s ability to relate the topics taught in class to topics that are taught in other classes and disciplines. An illustrative example of ability
to connect the topic of derivatives and slope of tangent lines to physics’ concepts of speed and acceleration. This source of knowledge is not exclusive to teachers but it plays a vital role in helping students see and make connections among different disciplines.

Vertical curriculum knowledge is the knowledge of the topics that have been taught and will be taught in the same domain during preceding and later years in school. An example of vertical curriculum knowledge is the teachers’ ability to extend the topic of similarity as covered in elementary or middle school and also know how this topic will be further addressed in more advanced classes.

c) Pedagogical content knowledge (PCK):

According to Shulman (1986) pedagogical content knowledge consists of the frequently taught topics in the field along with the most useful forms of representations of that topic with useful examples, analogies and explanations. This domain of knowledge allows the teacher to make the subject matter comprehensible to others. As such, it includes, “An understanding of what makes the learning of topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons” (p. 9). PCK also includes scaffolding techniques useful for breaking down a complex topic into simpler parts so that students can understand them. At the same time care must be taken to ensure that the essence of the topic remains intact.

Another important aspect of pedagogical content knowledge is the ability to identify a student’s misconceptions and to use methods that would help learners overcome them. According to Shulman (1987), “pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of
the pedagogue” (p. 8). In other words, pedagogical content knowledge is the knowledge that is specific to teaching and is not necessarily needed by practitioners in other fields.

**Forms of knowledge:**

Shulman (1986) identified three “forms” of knowledge, in which each of previously discussed knowledge domains may be organized. The three forms of knowledge include: a) Propositional knowledge, b) Case knowledge and c) Strategic knowledge.

Figure 2. Shulman’s Forms of Knowledge

**Propositional knowledge:**

Much of the content that teachers learn is in the form of propositions. Shulman (1986) suggested that research on teaching has essentially explored teachers’ propositions. He further categorized the propositional knowledge into three parts – principles, maxims, and norms. A principle, according to Shulman (1986), comes from empirical research. For example principles for teaching can be found in the literature for
teaching and school effectiveness (Shulman, 1986). Maxims are practical claims rather than theoretical ones. Maxims consist of hypotheses that have not yet been verified by research but seem to hold true in everyday practice. According to Shulman (1986) the statement “Break a large piece of chalk before you use it for the first time, to prevent squeaking against the board” (p. 11) would be an example of a maxim. The final type of propositional knowledge is norms. Norms reflect the everyday values, of justice, fairness, and equity that we expect teachers to employ in their classrooms. Norms “are neither theoretical nor practical, but normative” (Shulman, 1986, p. 11). They guide the teacher’s work. These propositions however are not very useful on their own. To apply principles in the classrooms, the teacher should also have the understanding of case knowledge.

Case knowledge:

Case knowledge is the knowledge of specific cases that are well documented and offer pedagogically rich avenues. Cases can be examples of specific events or instructional practice or detailed descriptions of how certain events occurred. Shulman further decomposes cases into three categories- prototypes, precedents and parables. Prototypes are illustrations of theoretical principles whereas precedents are collections of maxims. Parables communicate norms. Cases can serve more than one purpose at a time. For example, they can serve as prototypes as well precedents simultaneously (Shulman, 1986).

Strategic Knowledge:

Strategic knowledge is required when the teacher is faced with a unique situation that is “out of the ordinary”. For example, when teachers are confronted with situations or problems, whether theoretical or practical, that have no straightforward solution they
need to make use of strategic knowledge in order to solve that problem or successfully deal with the situation. Strategic knowledge is required in order to go beyond the basic understanding of the subject or the topic and develop an advanced or a more sophisticated understanding of the subject. In case of mathematics teaching, strategic knowledge is required when teachers are called upon to analyze the alternate strategies children use in solving problems. Teachers are required to, not only, make sense of these strategies, but also verify if they are mathematically sound.

Lee Shulman and his colleagues were the first to recognize that teaching of different subjects requires different types of knowledge. Their framework for teacher knowledge can be applied to teaching of different subjects, though it doesn’t offer specific discipline based guides for practice. However, one limitation of this framework is that it does not specify the differences among teaching of different subjects. For example, what makes teaching of mathematics so different than teaching of, say English? In light of this, researchers have argued that Shulman’s framework “is not sufficiently developed to be operationalised in research on teacher knowledge and teacher education” (Petrou & Goulding, 2011). According to Ball et al. (2008), Shulman does not make a clear distinction between content knowledge and pedagogical content knowledge. Hashweh (2005) posited that the framework fails to account for interactions between the various categories of knowledge. Fennema & Franke (1992) characterized Shulman’s framework as static, not accounting for the dynamic nature of knowledge in use. Additional criticisms have included the framework’s lack of attention to the role of classroom interactions with students on how subject matter is taught (Fennema & Franke, 1992). Despite these, Shulman’s framework has served as the foundation for researchers
in various disciplines for their ongoing work towards establishing more comprehensive models accounting for teacher knowledge. In mathematics education extensive use of Shulman’s groundbreaking work has been made. Indeed, the concept of pedagogical content knowledge continues to frame much of the research activities in the area of mathematics teacher preparation and education.

*Mathematical Knowledge for Teaching Framework by Ball and Colleagues*

According to Ball et al. (2008), Shulman’s concept of pedagogical content knowledge created interest in the mathematics education community because it bridged the gap between content and pedagogy, suggesting that neither of the two domains can operate independently of one another. Ball et al. (2008) were the first to coin the term Mathematical Knowledge for Teaching (MKT), describing it as “mathematical knowledge needed to carry out the work of teaching mathematics” (p. 395). Their scholarly efforts towards defining the content were guided by two questions (p. 395):

1. What are the recurrent tasks and problems of teaching mathematics? What do teachers do as they teach mathematics?

2. What mathematical knowledge, skills, and sensibilities are required to manage these tasks?

Teaching here refers to the various tasks that the teacher needs to perform in order to enhance the learning of their students.

In order to better explain what knowledge is required by teachers to teach mathematics more effectively, Ball et al. (2008) divided the knowledge for teaching of mathematics into two categories namely Subject Matter Knowledge and Pedagogical Content Knowledge, which is commonly known as PCK (see Figure 2).
Subject Matter Knowledge is divided into several sub domains: Common Content Knowledge (CCK), Horizon Content Knowledge and Specialized Content Knowledge (SCK).

Figure 3. Mathematical Knowledge for Teaching (Ball et al., 2008)

CCK consists of knowledge of the mathematical domain used not only in schools but also in other professions. This includes knowledge of concepts and algorithms to be taught; the ability to recognize faulty answers and wrong conclusions, calculate answers and more specifically mathematical problem solving skills (Ball, et al., 2008). CCK also includes knowing mathematical symbols and language. This sort of knowledge is used by professionals in other fields such as engineers, computer scientists, and physicists.
The next sub category of Subject Matter Knowledge was Horizon Content Knowledge. Hill and Ball (2009) describe this domain as: “A view of the larger mathematical landscape that teaching requires.” (p. 70)

The final sub-category of subject matter knowledge is called Specialized Content Knowledge (SCK); one, which the authors argued, was unique to teaching (Ball et al., 2008). This knowledge includes the ability to identify patterns in student errors, examine various approaches used by students and determine whether each is mathematically sound when solving problems. According to Ball et al. (2008) this knowledge domain “involves an uncanny kind of unpacking of mathematics that is not needed- or even desirable- in settings other than teaching” (p. 400). Such specialized knowledge becomes instrumental to teaching when presenting new mathematical ideas that are complicated, and teachers must break the mathematical topic down in ways that the students can comprehend. Additionally teachers are frequently confronted with the task of answering students’ questions about why things work the way they do (Ball et al., 2008). When explaining concepts to learners, teachers need to make use of examples. Finding good examples can be a challenging job. Teachers need to choose examples that illustrate to the student what is being taught and yet be careful about the type of representations they utilize. They must be able to link the representations to different mathematical ideas and also to other representations (Ball, et al., 2008).

An important aspect of SCK is that of linking one mathematical topic or area to other mathematical topics in later classes. Teachers should be able to make those

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1 I argue that CCK is the same as content knowledge as defined by Shulman (1987). Here we see a similarity between the two frameworks. It is also evident that this framework is grounded in Shulman’s model.
connections and also enable their students to see and understand those connections. If this knowledge domain isn’t present, teachers may not be able to make these connections on the spot and that could lead to disruptions in the classroom or gaps in student understanding (Ball et al., 2008).

According to Ball et al. (2008), teaching involves the use of decompressed mathematical knowledge. However, when teaching, the aim is to convey mathematics to learners in compressed form. Students should be able to use this sophisticated knowledge of mathematics and be able to apply mathematical ideas and processes. Regardless of the end result, teachers need to have an unpacked mathematical knowledge because they need to make “visible and learnable” the contents of mathematics. Teachers need to be able to explain to students how mathematical language can be different from that used in day-to-day life. For example, mathematical definition of similarity is quite different from the definition of similarity used in day-to-day life. Teachers also need to be able to explain and justify not only their own ideas but also those of their students. These competencies and demands may not be required in other professions.

Pedagogical Content Knowledge, commonly known as PCK is further divided into three categories: Knowledge of Content and Students (KCS), Knowledge of Content and Teaching, and Knowledge of Curriculum. Knowledge of Content and Students is a combination of teachers’ knowledge of what students know and their knowledge of teaching (Ball et al., 2008). Teachers often need to anticipate concepts students find easy to understand and those difficult to understand. This also translates into finding proper examples for students. Teachers need to be able to find examples that students understand. These examples need to be motivating, precise and efficient in conveying the
ideas clearly. It is often the case that teachers choose examples that they believe are easy for students but they fail to make the idea clear to the students. On the other hand, examples can be very rich but if they are too complex the students might lose motivation and interest in learning them.

Teachers should also be able to interpret and analyze students’ incomplete or inaccurate mathematical ideas. At times children’s ideas may not be in correct mathematical terms making it hard for teachers to get a full grasp of their thinking. According to Ball et al. (2008),

Recognizing a wrong answer is common content knowledge (CCK), whereas sizing up the nature of the error, especially an unfamiliar error, typically requires nimbleness in thinking about numbers, attention to patterns, and flexible thinking about meaning in ways that are distinctive of specialized content knowledge (SCK). In contrast, familiarity with common errors and deciding which of several errors students are most likely to make are examples of content knowledge and students (KCS) (p. 401).

Knowledge of Content and Teaching (KCT) seems identical to that of Shulman’s curriculum knowledge. Effective teaching requires that the teacher have a good grasp of the instructional materials and tools that are at their disposal. Teachers need to sequence topics so that each topic builds on the previous one in order for the students to see connections. Such linkage enables learners to understand mathematics better. This also requires the knowledge about which examples to begin with so that the student can understand and get a firm grip on the basic principles and which examples to use so that the students can get deeper into the topic and understand it thoroughly. This sub category
also includes the type of knowledge that teachers need when they indulge in discussions with the students deciding when to and how to do so. That is, they must decide what information to provide and what to withhold so that the students can discover that knowledge on their own. Another major challenge that teachers often face is deciding which viable instructional model to use. They often need to make on-the-spot decisions about how they may tackle a mathematical topic. Each instructional model has its own advantages and disadvantages. It is up to the teacher to select among them, with clear understanding of how these different models work and how they affect student learning.²

Ball et al. (2005) have carried out several studies to investigate the relationship between teachers’ mathematical knowledge and student achievement. Their findings indicate that the teachers’ performance on their measure of knowledge instrument, which includes both common and specialized content knowledge, was a significant predictor of their students’ achievement (Ball, et al., 2005). Ball et al. (2005) conclude that even though mathematical knowledge is not sufficient for reducing the achievement gap, it is certainly necessary to prevent it from growing. Their conclusions have been subjected to criticism due to the nature and form of items used on the assessment (Petrou, & Goulding, 2010).

The MKT framework has also been scrutinized for several shortcomings it presents. First, it does not take into consideration the importance of teachers’ beliefs in their teaching (Petrou & Goulding, 2010). Second, the distinction between SCK and PCK is not well defined (Petrou & Goulding, 2010). Further, some of the categories are too

² The MKT framework is similar to the framework by Shulman but it gives a deeper understanding of the knowledge that teachers of mathematics need to know in order to teach effectively. It is more specific to the field of mathematics than Shulman’s model.
broad and need to be narrowed. This framework also does not acknowledge the tasks of teaching and how the knowledge base is related to the various tasks that are involved in teaching. Lastly, a possible limitation of this framework is that it has been primarily designed for the lower and the middle school level. Certainly, the framework might prove suitable for measuring teacher knowledge at the high school level but it has been tested only at the lower and the middle school levels. It also appears that teaching mathematics at the high school level is much more complex than at the lower school or the middle school. Teachers cover a wider spectrum of mathematical topics at the high school than at the lower levels of schooling. Although Ball and her colleagues (Ball, 1990; Ball & Bass, 2000; Ball, et al., 2005; Hill, 2007; Hill, et al., 2005) have made tremendous progress in measuring and analyzing mathematical knowledge for teaching at the elementary and middle school, there are no appropriate theoretical frameworks and instruments to describe or measure this knowledge at the high school level (McCorry et. al, 2010), though scholars have attempted to unpack what specific ways of knowing might be involved in the teaching of specific concepts (Even, 1993; Wilson, 1994). In the section to follow I present Even’s framework which is an essential part of the Knowledge for Algebra Teaching framework, which is my current theoretical model for the study.

Even’s Framework

Even (1990) proposed a framework for subject matter knowledge and for teaching as it applies to the “function.” Even (1993) applied this framework to investigate the relationship between teachers’ subject-matter knowledge and pedagogical knowledge. The results showed that teachers who used modern terms when defining a function did not use the same terms when they were attending to students who had difficulties. An
interview with a teacher revealed that she did not use the same terms as she thought students would not understand modern terms such as “mapping” (Even, 1993). Most of the teachers in the study were of the opinion that functions are equations (Even, 1993). The study indicated that teachers who have a limited concept image may not be able to understand current mathematics based on modern definition of function (Even, 1993).

This framework consists of seven aspects:

a) Essential Features
b) Different Representations
c) Alternate Ways of Approaching
d) Strength of a Concept
e) Basic Repertoire
f) Knowledge and Understanding of a Concept
g) Knowledge about Mathematics

These are described below.

Essential features

This particular aspect of the framework deals with the concept image (Even, 1990). According to Even (1990), a concept image is a mental picture of a concept that one creates in his mind prior to the internalization of a formal understanding. Different people may have different concept images for the same concept. However, “teachers should have a good match between their understanding of a specific mathematical concept they teach and the ‘correct’ mathematical concept” (p. 523). Even (1990) argued that it is a part of the teacher’s job to judge whether a particular instance belongs to a concept family. Teachers need to decide using analytical techniques and not by
prototypical judgment. The first type of judgment is based on the properties of the instance while the second type of judgment “uses a prototypical example as the frame of reference either by applying a visual judgment to other instances or by basing the judgment on the prototype’s self attribute and imposing them on other concept examples” (p. 524). Teachers need to do more than just to distinguish between concept examples and non-examples. They need to investigate and further question those conceptions.

Different representations

This category has been widely explained and elaborated by both Shulman (1987) and Ball et al. (2005, 2008). Mathematical concepts can be represented in a variety of ways. It is possible that students understand one representation of the concept but cannot comprehend a different representation of the same concept. Use of different representational forms fosters a better understanding (and a more thorough picture) of the concept (Even, 1990). Even (1990) suggests that when comparing different representations of the same concept, it is essential that teachers understand the common aspects of the representations while ignoring the irrelevant aspects. For example, while comparing the different representations of a function, teachers need to concentrate on the common central theme of dependent versus independent variable.

Alternate ways of approaching

Teachers often need to explain concepts in different ways so that they are made accessible to all students. They need to utilize alternate ways of introducing and elaborating concepts whilst remaining sensitive to the limitations of each approach since not all of them are applicable in every situation.

Strength of a concept
The success of a concept depends on the new opportunities it provides (Even, 1990). “Concepts become important and powerful because there is something special about them which is very unique and opens new possibilities” (Even, 1990, p. 525). This implies that teachers need to have a good grasp of the concept and its properties. They should also be familiar with topics that are closely related to this concept. Otherwise the teachers will be unable to provide these new possibilities for their students.

Basic repertoire

This category includes the knowledge of examples that bring out the important features of the concepts such as properties, axioms, theorems etc (Even, 1990). This knowledge allows for deeper understanding of the concept and also gives access to more complex knowledge. Even (1990) suggested that basic repertoire should be readily available to use, but it should not be memorized, at least not without understanding.

Knowledge and understanding of a concept

Conceptual knowledge is rich in relationships. According to Bell, Costello and Kuchemann (1983), as stated by Even (1990), conceptual knowledge is comprised of a rich set of connections between concepts and relationships among the concepts. Unlike procedural knowledge which can be learned with or without meaning, conceptual knowledge should be learned meaningfully (Even, 1990). In order to learn mathematics, both types of knowledge are essential and so are the relationships between them. If teachers do not help students to link concepts and procedures, they may not be able to apply them to solve problems despite having a grasp of the concept (Even, 1990).
Knowledge about mathematics

This type of knowledge involves an understanding of the nature of mathematics. According to Even (1990), “this is a more general knowledge about a discipline which guides the construction and use of conceptual and procedural knowledge.” (p. 527).

The major drawback of Even’s framework’s is that the categories defined are very broad. Similar to Shulman’s framework, Even did not acknowledge the effect of classroom interactions with students on the subject matter. The framework also does not acknowledge the interactions between various categories of knowledge.

As seen from the discussion above, the frameworks developed by Shulman and Even are not dynamic. Ball and colleagues highlight the dynamic nature of mathematical knowledge of teaching by taking into consideration the importance of the interactions between teachers and students (Fennema & Franke, 1992). The principles and standards put forth by NCTM (2000) also highlight the importance of classroom interactions between teachers and students. As a result of interactions, teachers are called upon to take up the task of analyzing students’ mathematical work and thinking.

Knowledge for Algebra Teaching (KAT) Framework (Theoretical Framework)

In this work, I used the Knowledge for Algebra Teaching (KAT) framework (McCrory et al., 2010) as a guiding lens for both the design of data collection instruments and interpreting the data. The KAT framework was developed for the subject area of Algebra but it is broad enough to be applied to other fields of mathematics, in particular to geometry. The KAT framework takes into consideration the theoretical and empirical research on teachers’ knowledge, students’ difficulties in high school algebra and research on high school algebra curriculum. Further, in developing this framework,
interviews with algebra teachers, analysis of instructional materials and classroom observations were also utilized (McCrory et al., 2010). The KAT framework draws on Ball and Colleagues’ contribution as well as Even’s model. KAT examines two main algebraic topics specifically, algebraic equations and expressions, and linear relationships (Floden et al., 2009; McCrory, et al., 2010). Yet, it is plausible that it might apply to other areas of mathematics, in this case geometry. I will make connections, as I explain the framework, and I will identify specific applications to geometry.

According to Floden et al. (2009), the Knowledge for Algebra Teaching (KAT) framework is organized as a two dimensional matrix with three overarching categories. The rows of the matrix are categories of knowledge of algebra for teaching. The columns consist of tasks of teaching during which teachers are expected to use mathematical knowledge. The three overarching categories are labeled as decompressing, trimming and bridging. This framework can be adapted for geometry by replacing the rows with categories of knowledge of geometry for teaching. The columns and the overarching categories remain intact. The framework is explained below.
Figure 4: Knowledge of Algebra for Teaching (McCrory et al., 2010, p. 58)

_Categories of knowledge of algebra for teaching_

In this section I will discuss the categories of knowledge of algebra that are represented in the columns of the matrix.

1. Core content knowledge (CCK)

CCK includes the knowledge of mathematics, and the main topics and ideas, procedures, structures and frameworks that are used in mathematics (McCrory et al., 2010). For example the core content knowledge for algebra includes knowledge of variables, linear equations, expressions, slopes and procedures such as solving equations, simplifying expressions, factoring and connections between two or more topics (McCrory et al., 2010). In the case of geometry it includes the knowledge of concepts such as
parallel lines, area and volume, similarity and congruence, shapes and their properties and making connections between different topics. This is similar to what Hill et al. (2005) refer to as common content knowledge. This is also similar to Shulman’s (1987) notion of content knowledge. Even’s framework also consists of a similar category of knowledge called *essential features* (Even, 1990).

2. Representation

This category consists of the different models and forms in which different concepts and procedures are presented (McCrory et al., 2010). Representations should be meaningful and should bear some kind of a relationship between the symbol and the referent (McCrory et al., 2010). In other words, representations help make connections between mathematical objects and other objects. In the case of algebra, this would involve the graphical representations of functions, use of algebra tiles, descriptions of relationship between variables etc. In the area of geometry representations mostly involve figures such as squares, rectangles, circles and other geometric shapes. However, it is very difficult to properly represent three-dimensional objects. Often we see that students struggle to get a grasp of a three-dimensional object drawn on paper or in books.

Shulman (1987) and Ball et al. (2008) also talk about knowledge of representations. According to Shulman (1987), the knowledge of representations resides within the content knowledge and it is a part of the subject. Ball et al. (2008) put the knowledge of representations under the category of SCK. According to them the knowledge of representations is unique to the field of teaching.
3. Content trajectories

This category includes more than just knowing the subject matter for the level taught. This knowledge extends beyond what the students are expected to know and understand and includes an appreciation and understanding of the origins and the extensions of the various mathematical concepts and procedures (Floden, et al., 2009; McCrory, et al., 2010). For example, knowledge of how the concept of ratio and proportion develops into similarity and congruence, which further leads into equivalence, would fall under the category of content trajectory. A content trajectory can be thought of as an ordering or sequencing of mathematical topics in a way that best supports student learning (McCrory, et al., 2010). Content Trajectories also include the knowledge that teachers need to identify the main ideas of the domain from the peripheral ones and be able to locate them within the trajectory (McCrory et al., 2010). This category is closely related to what Shulman (1987) calls vertical curriculum knowledge. Finally, included in this category are examples of mathematical work. According to McCrory et al. (2010), the reason for including examples in this category is that canonical examples are “closely related to knowing about different approaches to a topic, and to different sequencing” (p.28). This was accounted for by Even (1990) as she argued that teachers should know “powerful examples that illustrate principles, properties, theorems etc.” (p. 525).

4. Applications and contexts

This category consists of “knowledge of problems that arise from situations, contexts, or circumstances outside of algebra, or within a different part of algebra” (McCrory, et al., 2010, p. 29). This has been previously discussed by scholars, under the concept of real world applications. Many such situated problems involve the use of
algebraic procedures and concepts. “Such problems are used in the high school algebra curriculum as sites both for introducing new concepts and for applying concepts in various “real world” or contextualized situations” (McCrory, et al., 2010, p. 29). Often we find that algebra textbooks have a section reserved at the end of each chapter for applications or so called ‘real world problems’. NCTM’s Principles and Standards for School Mathematics (2000) define mathematical modeling as “identifying and selecting relevant features of a real-world situation, representing those features symbolically, analyzing and reasoning about the model and characteristics of the situation, and considering the accuracy and limitations of the model” (NCTM, 2000, p. 303). Hence the knowledge of applications of mathematical concepts to fields other than algebra or even mathematics is essential for teachers.

A lot of real world modeling is based on geometry as well. For example, we often see in geometry textbooks examples on finding area of a rectangular field, problems involving distance and height, angles and right triangles. The concept of knowledge of application and contexts is essential in geometry as well as students are expected to solve ‘real world problems’ involving common geometric shapes and concepts.

5. Language and conventions

This category includes the syntactic knowledge that teachers require to teach mathematics. Mathematical language includes the use of symbols and signs that are not used in day-to-day language. Moreover, the terms commonly used in algebra may be ambiguous for students and they may not be used consistently across the curriculum. They could also contradict with “students’ everyday meaning of the words” (McCrory, et al., 2010, p. 31). Along with the mathematical language, students are also exposed to
mathematical conventions such as algebraic expressions, rules such as FOIL and PEMDAS (parenthesis, exponents, multiplication, division, addition and subtraction). In mathematics classrooms, teachers often switch back and forth between informal talk and mathematical language, which is often confusing to students. According to McCrory et al. (2010), in the field of algebra, the notation $f(x)$ often used to denote a function is very confusing for some students. Students could misunderstand the symbol and take it for multiplication of $f$ and $x$, rather than the true meaning of the expression i.e. $f$ of $x$. Until they get this true meaning, they may fail to realize that this convention represents a function and not an operation (McCrory et al., 2010). There are other such algebraic expressions that may be confusing to students as well. This however is difficult to achieve since teachers need to be able to help students clearly distinguish between these notations.

The use of language in geometry is also ambiguous. Geometry uses its own set of symbols and notations and its own language. For example, the word “similar” has a different meaning in geometry than in day-to-day language usage. In geometry the word similar has a different meaning than in day-to-day language. Definitions can also be confusing to students. Also in geometry concepts can be defined in different ways. Some definitions are too general while the others too specific. Some definitions are not too clear. Notations like $\approx$ and $\sim$ can also be confusing to students. Hence this knowledge is very essential for teachers of geometry as well since they need to explain differences to students. They will need to know which definitions are most suitable to introduce in a classroom and also the consequences associated with those choices. In fact knowledge of
language and conventions is essential for all teachers of mathematics regardless of whether they are teaching arithmetic, algebra or geometry.

6. Mathematical reasoning and proof

McCror et al. (2010) maintain that:

in school algebra knowledge of reasoning and proof includes such things as knowledge of the specialized vocabulary of reasoning (e.g. terms such as contra positive), the ability to find examples and counterexamples of statements, the ability to use analogies or geometric arguments to justify statements, and the ability to use various proof techniques within an axiomatic system to make convincing arguments (p. 32).

This knowledge also includes knowing the importance of proofs and axioms, knowing if and how two axioms are equivalent such as the Well Ordering Principle and the Principle of Mathematical Induction (McCrory, et al., 2010). Knowledge of reasoning and proof also requires syntactic knowledge. For example, simplifying algebraic expressions requires the knowledge of properties of polynomial rings, and laws of multiplication and addition because they are applied to other number systems.

Proofs are the heart and soul of geometry. Students’ first exposure to synthetic proofs is in a high school geometry class. Often we see that students are not very comfortable with these proofs. Students have trouble with deductive proofs (Chazan, 1993). Furthermore, high school students cannot distinguish between empirical and deductive arguments (Chazan, 1993; Oner, 2009). Teachers need to know a great deal about geometric proofs and geometric reasoning. Another issue is that geometric proofs often depend on graphical representations of geometric objects. Terms and notations such
as ASA, SAS etc are used and it makes it very confusing for students. Lots of geometric proofs involve the use of theorems and axioms making them even more complicated. Teachers also need to be familiar with alternate ways of proving results. Another important part of mathematical proofs is coming up with conjectures. Often students cannot come up with conjectures that will ultimately lead them to the solution.

Tasks of teaching

In this section I will describe the constructs of the tasks of teaching that are depicted in the rows of the KAT framework.

1. Analyzing students’ mathematical work and thinking

Teachers are faced with the everyday task of listening to and analyzing students’ ideas and explanation of those ideas. They need to interpret and respond to these ideas. This is a way of knowing what students know. Teachers need to analyze the students’ thought process and determine whether the students’ strategy or solution is valid. In case of an incorrect solution the teachers also need to find the source of the error. It is often the case that students misunderstand something and it is the duty of the teacher to correct that misunderstanding. Students also have misconceptions about mathematics, which they bring into the classroom. Teachers are often faced with the task of learning about these misconceptions and correcting them. Research in the past few years has suggested that analyzing student work can lead to changes in teaching practice. According to Doerr (2006), “When teachers understand how students might approach a mathematical task and how their ideas might develop, this would seem to provide the basis for the teacher to support students in ways that would promote students’ learning” (p. 3).
It is essential that teachers know how children think and be familiar with their common understandings and misunderstandings so that they can help them overcome their difficulties. The ability to assess and interpret student work is vital to teachers’ success in designing ways to help students overcome these misunderstandings. The need to analyze student work is also essential as it gives teachers exposure to the non-traditional ways in which students might solve problems. The analysis of such work could lead into the investigations of whether the particular method used works all the time or only in a given specific situation within the given constraints. This knowledge is also useful for teachers so that they can give feedback to the students that will be constructive. However, teachers who do not center their teaching on student thinking or are not able to understand it often tend to impose their own formalized conceptions onto the student, which are not always effective (Cobb, 1988).

2. Designing, modifying and selecting mathematical tasks

One major instructional activity involves selecting and designing mathematical tasks that are accessible to the particular group of learners with whom they work (McCrory et al., 2010). These tasks should be modified so that their desired intentions are preserved but that they also meet the needs of children (McCrory et al., 2010). Tasks should not be too easy or too difficult and should encourage students to learn the intended objectives while keeping them interested. Additional considerations include maintaining the cognitive demands of the task, scaffolding student work, and deciding whether or not the tasks allow for discussion between the students and the teacher (McCrory, et al., 2010). According to McCrory et al. (2010), “Deciding where tasks fit in mathematical trajectories, determining if a particular task is likely to be accessible in mathematically
useful ways to a range of students, and planning for how to maintain cognitive complexity are also aspects of this category” (p. 35).

3. Establishing and revising mathematical goals for students

Determining what ideas are central to the mathematical domain to be taught, which ideas need to be emphasized, how to sequence these ideas and how to approach particular topics are issues teachers consider as they set goals for their students. Setting goals requires “mathematical judgment” (McCrory, et al., 2010, p. 35). After these goals are established, then the teachers indulge in the tasks of refining, rethinking and reflecting on these goals.

4. Accessing and using tools and resources for teaching

Educational tools and resources include textbooks, technology such as calculators and computers, concrete materials etc. Teachers are exposed to a variety of such tools and resources and it is their task to decide which of these tools are appropriate for specific mathematical goals (McCrory, et al., 2010). Teachers also need to create some correspondence between the instructional tools and the mathematical concept that is being taught (McCrory et al., 2010). Another important task for the teacher is to understand the limitations of specific tools by considering how and when the use of particular tools and it can hinder learners’ progress. Teachers need to assure that the tools are readily accessible to the students and that they fully understand how to use them.

5. Explaining mathematical ideas and solving mathematical problems

Often teachers find themselves solving problems that they had not anticipated in advance. These problems emerge from observation of students’ work or questions, examination of textbooks and more often than not, teachers have to address these
problems in the presence of their students. Thus, not only they need to be good problem solvers but they should also do it in a way that makes it easy for the student to understand and emulate. Solving problems and explaining mathematical ideas is central to any mathematics course. Teachers need to explain ideas in a way that students can understand. Problem solving has always been the focus of mathematics education for a very long time. Polya (1957) in his book *How to solve it* puts a lot of emphasis on good problem solving techniques and how it is important to pose questions while problems solving.

6. Building and supporting mathematical community and discourse

    McCrory et al. (2010) emphasize the importance of establishing a mathematical community when teaching. Doing so demands reliance on various aspects of mathematical knowledge for teaching. Since teaching is so situated in a classroom (Anderson, Reder, & Simon, 1996; Greeno, 1998), it is up to the teacher to decide the norms and rules for that particular situation. Teachers decide on what assumptions should be made, at what point it might be necessary to engage students in discussions and when students should work on their own. This could also include developing definitions as a group and helping students to learn to interact with each other, respond to each other’s queries and collectively produce mathematical knowledge in the classroom (Floden, et al., 2009; McCrory, et al., 2010).

*Overarching categories*

    In this section I will describe the overarching categories that include decompressing, trimming and bridging.
1. Decompressing

Decompressing involves working backwards from more sophisticated and compressed understanding of mathematical content to more unsophisticated and less polished form. Teachers need to do this without actually losing the meaning of the concept or the core idea. According to McCrory et al. (2010), decompressing may include the task of “attaching meaning to symbols and algorithms that are typically employed by sophisticated mathematics users in automatic, unconscious ways” (p. 38). McCrory et al. (2010) argue that it is possible that for secondary school mathematics teachers, “decompressing involves developing knowledge that is new for those with sophisticated mathematical training, and that takes something seemingly simple and routine and ‘complexifies it’” (p. 39). Teachers engage in the task of decompressing while trying to clear students’ misunderstandings and also while designing lessons. Decompressing is useful in analyzing students’ mathematical thinking and work, and also in designing tasks (McCrory et al., 2010).

2. Trimming

Trimming is the process in which teachers transform an advanced or sophisticated mathematical idea into a form that preserves the fundamental nature of the idea but is made available to students at the high school level. According to McCrory et al. (2010), “trimming involves scaling down, and intentionally and judiciously omitting detail and modifying levels of rigor, and also being able to judge when a student or a textbook presentation, is trimming, and if so, whether the trimming is appropriate.” (p. 42). For example, we often see that students are introduced to exponential functions before calculus where they learn about raising any positive number to an arbitrary power using
the definition of a logarithmic function and then learning about the definition of an exponential function (McCrory, et al., 2010). Lots of care needs to be taken while trimming. If contents are not trimmed properly, it could lead to misconceptions on the part of the student. According to Graeber, & Tirosh (1990), as stated by McCrory et al. (2010), in elementary schools students often learn that multiplying two numbers results in a bigger number. This causes great problems in later schooling when students find out that multiplying by a real number between 0 and 1 does not result in bigger numbers. On the contrary it results in smaller numbers.

Trimming and decompressing are indeed very similar. While decompressing is working backwards from more complex form to an unsophisticated form that is easily comprehensible by students, trimming involves a step more and that is it demands attention to maintain the integrity of the topic (that is, the essence of the topic that is being trimmed should not be lost).

3. Bridging

Bridging involves making connections between various things such as teachers’ goals for the students and students’ understanding, connecting ideas from abstract mathematics to high school mathematics or connecting ideas from one area of school mathematics to another (McCrory, et al., 2010). Some of these connections are easy to make, while some are not so obvious, and the teachers need to figure out how to make those connections possible.

Explorations of Pre-service Teachers’ MKT

Explorations of elementary teachers’ MKT have been well documented (Ma, 1999). At the secondary level, this effort has been less visible. In fact, the literature of
pre-service teachers’ MKT falls under four major categories: teachers’ knowledge of specific mathematical concepts, most prominently functions (Even, 1993; Wilson, 1994). The second category of scholarly work has focused on the impact of use of different tools including technology (Bowers, & Stephens, 2011) and case-based tasks on pre-service teachers’ development of MKT (Manouchehri, 2008; Manouchehri, & Enderson, 2003). The third category has examined the pre-service teachers’ problem posing skills in connection to instructional assessment and planning. The fourth category concerns the pre-service teachers’ assessment of children’s thinking.

Assuming that MKT is essential to effective mathematics teaching, a successful transition into a mathematics classroom demands that the teachers have an understanding of mathematical ideas, connections among different mathematical concepts, and knowledge of children’s cognition (Manouchehri, & Goodman, 2000). The Agenda for Action (NCTM, 1980) emphasized problem solving, re-examination of basic skills, and an incorporation of multimedia into the mathematics curriculum along with an increased load of mathematics for all students. According to Bush (1986), “Teacher education is the process of learning to teach that occurs in school classrooms and through direct experiences” (p. 21).

One of the six principles set forth for school mathematics is for pre-service teachers to be able to understand what students know and need to learn (NCTM, 2000). This requires that teachers be capable of understanding student reasoning so that they can develop effective instructional strategies in the future. However, research has shown that pre-service teachers lack the ability to determine the reasoning behind student work (Ball, 1990). A study conducted by Cooper (2009) indicated that pre-service teachers were able
to identify computational errors in students’ work but experienced difficulty when trying to come up with reasons that might have led to these misconceptions. Thus, pre-service teachers chose to simply re-teach the content without changing or modifying the instructional approach (Cooper, 2009). Bray (2011) suggests that it is important to understand how teacher beliefs and knowledge influence their capacity to deal with student errors. Many pre-service teachers are of the opinion that misunderstandings are a sign for a learning disability (Cooper, 2009). Manouchehri (1997) argues, “teachers translate their knowledge of mathematics and pedagogy into practice through the filter of their beliefs” (P. 198). Teachers who hold certain beliefs about their students tend to direct their instruction accordingly. For example, teachers who view children as possessing some mathematical knowledge and having the ability to gain more knowledge by engaging in problem solving tend to build their instruction around student thinking (Fennema et al., 1996). On the other hand, according to Spilane, teachers who view students as not capable of higher order thinking tend to build their instruction as teacher centered (Bray, 2011). In a recent study Bray (2011) found that teachers who believed that their students had limited capacity to support each other’s learning were hesitant to encourage student collaboration. Thus, not only are the instructional approaches affected by teachers’ beliefs about student knowledge but so are teachers’ error handling approaches (Bray, 2011). Teachers who do not understand their students’ thinking often neglect it and impose their own thinking onto the students (Cobb, 1988). In a study conducted by Cooper (2009), based on their observations of student work and thinking, pre-service teachers suggested strategies that fell mostly under three categories: The first category of instructional strategies focused on procedure, while another strategy
employed was to simplify the problem. The final suggested strategy was a teacher-directed strategy. Not many of the teachers suggested an alternate instructional strategy.

Isiksal & Cakiroglu (2011) posited that “to improve mathematics instruction, teachers need to challenge and support students and have a sound understanding of the gap between what students know and what they need to learn” (p. 214). Following this recommendation, effective teachers are also required to possess the knowledge of students as learners. Having the knowledge of what the students understand helps teachers to make better curricular judgments and to plan accordingly for instruction. The analysis of student work should also guide the teacher’s assessment of student learning. This knowledge also provides teachers with productive means to respond to the students’ questions. According to Rowland (2008), teachers can structure their ‘next move’ with an appropriate analysis of student work and thinking.

Clearly, there is a need to educate pre-service teachers in order to help them to better understand and analyze students’ mathematical work. One approach for meeting this need has encompassed the use of written, video or animated examples of student work (Manouchehri, 2008). In their study Manouchehri (2008) found that the use of case-based tasks successfully engaged pre-service teachers in the process of analyzing students’ work. The pre-service teachers made explicit connections among different solutions and also devised strategies to make these connections public during instruction (Manouchehri, 2008). In this study, teachers were also successful in identifying mathematical structures that could be used in order to enhance student understanding in specific cases.
Some of the suggestions to improve pre-service teachers’ knowledge on student thinking include group work and video technology that lead to discussions about the course content (Cooper, 2009). Jacobs and Philipps (2004) suggested that inclusion of examples of student work to facilitate discussion about student thinking of mathematics should be made. The examination and discussion of various authentic examples of student work can be used to increase teachers’ knowledge about how to analyze student work (Timmerman, 2004). Cooper (2009) has recommended that the following tasks be included in methods courses:

(iv) Analysis of children’s written work and related thinking.

(v) A deeper exploration of mathematical content to develop alternate instructional strategies.

(vi) Observing children engaged in performing mathematical tasks.

Another suggestion to improve teachers’ knowledge on student thinking, as proposed by Battista (2011) is the use of learning progressions. According to Battista (2011), learning progressions not only helps the teachers to understand students’ thinking but also expands their focus to the “pedagogically critical, psychological interpretations of students’ mathematical thinking “(p. 557). In order for learning progressions to be useful to teachers, they must be linked to both assessment tasks as well as instructional tasks that are designed to develop student thinking further (Battista 2011). Learning progressions should be used to develop formative and summative assessments as well as to make instructional decisions (Battista, 2011).
Learning Progressions

The use of learning progressions has been emphasized to aid in teaching and assessment of student work (Battista, 2011). The National Research Council (2007) defines learning progressions as “descriptions of the successively more sophisticated ways of thinking about a topic that can follow one another as children learn about and investigate a topic” (p. 214). Learning progressions, according to Battista (2011) focus on conceptual and procedural knowledge in addition to skills. In addition to providing descriptions of student learning, learning progressions are also tools for teaching since they can be used to “guide instructional decisions and moment-to-moment teaching” (Battista, 2011, p. 512).

The most commonly used learning progressions to assess student learning in geometry are the van Hiele levels. In the following section I will give a brief overview of the van Hiele levels along with Battista’s (2007) elaborations of the van Hiele levels. Following that I will also present the Pirie-Kieren (1989) model of assessment since it was one of the assessment models that the preservice teachers were expected to utilize in their analysis of student work and thinking. Pirie-Kieren model provided a global view of how growth in mathematical knowledge could be characterized. Van Hiele provided a local view regarding geometry and its progression.

The van Hiele Model of Development of Geometric Thought

The van Hiele model of development of geometric thought consists of five levels of understanding (Crowley, 1987) namely: visualization, analysis, informal deduction, formal deduction and rigor.
Level 0: Visualization

Students identify and reason about shapes according to their appearance as a whole (Battista, 2007; Crowley, 1987). Students are not yet able to reason using the parts or properties of the geometric shapes (Crowley, 1987). For example, students may reason that a square is not a rectangle because they don’t look alike (Battista, 2007). Student’s reasoning is also affected by the orientations of the figures (Battista, 2007). Battista added two more sublevels: Pre-recognition and recognition. Pre-recognition is when students cannot identify common shapes while recognition is when students can identify common shapes correctly (Battista, 2007).

Level 1: Analysis

In this level students are able to tell apart characteristics of geometric shapes based on their observations and experimentations with those shapes (Crowley, 1987). They then use these characteristics to “conceptualize classes of shapes” (Crowley, 1987, p. 2). Students at this level, however, are not able to establish relationships between the properties of shapes and are not able to understand definitions (Crowley, 1987). Battista (2007) suggests that students’ conceptualizations vary in their levels of sophistication. Based on this suggestion, he offers three sublevels under Level 1. The first sublevel is visual-informal componential reasoning. In this sublevel students’ descriptions of parts and properties of shapes are imprecise and improper and focus on the visualization of the shape, initially focusing on parts of the shape and then moving on to the interrelationships between the parts (Battista, 2007). The language used by students to describe the shape is also very imprecise (Battista, 2007). The second sublevel is informal and insufficient-formal componential reasoning. In this level students use a
mixture of informal and formal descriptions of the geometric shape (Battista, 2007). The formal descriptions are insufficient to describe the shape correctly and hence they need to make use of informal descriptions in order to give a complete description of the geometric shape (Battista, 2007). The final sublevel is sufficient formal property-based reasoning wherein students move from relying on visualization to whether a shape satisfies a given property (Battista, 2007). In Level 1: Analysis, students are capable of using and formulating precise definitions (Battista, 2007).

Level 2: Informal deduction

Students in this level are able to identify the “interrelationships of properties both within figures and among figures” (Crowley, 1987, p. 3). For example they can now reason that in a quadrilateral if opposite sides are parallel then the opposite angles must be equal or squares are rectangles since they possess all properties of a rectangle (Crowley, 1987). Students at this level are capable of understanding definitions and informal arguments (Crowley, 1987). However, students at this level do not completely understand the process of deduction, nor do they understand the role of axioms (Crowley, 1987). Battista further divided this level into 4 sublevels. The first sublevel is empirical relations in which students rely on empirical data to decide between the interrelatedness of properties (Battista, 2007). The second sublevel is componential analysis in which students conclude that the occurrence of one property is based on the occurrence of another one by analyzing how different shapes can be constructed using their components (Battista, 2007). The third sublevel is logical inference in which students now work with property statements and not visual images of the shape (Battista, 2007). Finally the fourth sublevel is hierarchical shape classification based on logical inference. In this sublevel,
students use logical reasoning to arrange their classification of shapes into a hierarchy (Battista, 2007).

**Level 3: Deduction**

Students at this level are able to use deductive reasoning. Students are capable of understanding the interrelationships and role of axioms, postulates, theorems and proofs (Crowley, 1987). Students can, not only understand formal proofs but construct their own proofs in more than one ways (Crowley, 1987).

**Level 4: Rigor**

Students at this stage can work with different axiomatic systems (Crowley, 1987). Hence at this level, students are capable of understanding and using non-Euclidean geometries.

Thus, the van Hiele model provides us with a means to identify students’ maturity in geometric thought (Crowley, 1987). The van Hiele model is linear and in order to attain a level, the student must have attained the previous levels. This is similar to Piaget’s theory of development. Although widely criticized, this model currently serves as the primary lens used in teacher education as a means to enculturate them into the norm of cognition based teaching.

**Pirie and Kieren’s Recursive Theory of Mathematical Understanding**

Pirie and Kieren (1989) offer a recursive model to describe mathematical understanding. They characterize mathematical understanding as a recursive phenomenon where “recursion is seen to occur when the thinking moves between levels of sophistication. Indeed each level of understanding is contained within succeeding levels” (Pirie, & Kieren, 1989, p. 8). The model views understanding as an unstable and
retrogressive organic element: layered but non-linear, never-ending process of growth. Dismissing the notion of growth as monotonous they suggest it as a dynamic organizing and reorganizing process as individuals move through eight embedded layers of understanding. Although the layers of understanding in the theoretical model build outward, growth in understanding occurs as students continue to work within and move back and forth through different layers of understanding (Martin, 2008).

![Diagram](image)

Figure 5. Pirie- Kieren Model of Growth of Mathematical Understanding (Pirie, & Kieren, 1994)

*Primitive knowing:*

It is what the teacher assumes the learner already knows, in other words, usable knowledge (Pirie, & Kieren, 1994). Although there is no way of fully knowing the
primitive knowledge that a learner possesses, one can observe aspects of in the context of the problem being worked on. Primitive knowledge doesn’t imply a low level of mathematics (Pirie, & Kieren, 1994).

*Image making:*

At this level of understanding the learner uses his or her prior knowledge in a new way, in the context of the presented problem. Through the image making process, the learner is involved in two necessary and complementary features of acting and expressing called *image doing and image reviewing* (Pirie, & Kieren, 1994).

*Image having:*

At this level, the learner can use a mental construct about the topic without actually doing the steps that are required to get the construct (Pirie, & Kieren, 1994). According to Pirie, & Kieren (1989), image having marks the first level of abstraction by “building on images based in action” (p. 8).

*Property noticing:*

Property noticing occurs when the learner can construct context specific properties by manipulating or combining images (Pirie, & Kieren, 1994).

*Formalizing:*

This level “entails thinking consciously about the noticed properties, abstracting common qualities and discarding the origins of one’s mental actions” (Pirie, & Kieren, 1989, p. 9). It is at this level that a student learns to appreciate a formal mathematical definition or algorithm.

*Observing:*

A person who is formalizing is able to reflect on and coordinate
formal activity of the previous level and recognize patterns (Pirie, & Kieren, 1994). This action is called observing. At this level students are able to present the coordinated activities in the form of theorems (Pirie, & Kieren, 1994).

Structuring:

Students at this level are able to sequence their thoughts and are aware of their interdependence (Pirie, & Kieren, 1989). In other words, structuring is “setting one’s thinking within an axiomatic structure” (Pirie, & Kieren, 1989, p. 9). The learner is capable of making conjectures and proving them, which Pirie and Kieren (1994) call theorem conjecturing and theorem proving.

Inventizing:

This is the outermost level in the PK model. At this level a person is able to extend the current topic by creating new questions (Pirie, & Kieren, 1994). The learner is able to answer questions as to why the consequences of thoughts are true (Pirie, & Kieren, 1989).

Unlike other models that might be specific to a particular subject matter, Pirie and Kieran offered a model for capturing the mathematical thinking processes of individuals.

Conclusion

Analysis of the various frameworks aimed at capturing mathematical knowledge for teaching, reveals the need for development of a more comprehensive model that both describes and provides a framework for measuring MKT. Current frameworks do not take into account the different tasks that teachers need to perform as engaged in practice and knowledge required to perform them. Another drawback of the frameworks prominent in the field is that some of the categories, as defined, are ambiguous and broad.
Ball et al. also voice the need to investigate and further refine the concept of curricular knowledge (Petrou & Goulding, 2010). There is much to learn about how teachers use curriculum materials in their classroom teaching (Petrou & Goulding, 2010). Proper frameworks for analyzing teachers’ knowledge of mathematics, including their understanding of learners’ thinking at the secondary school level are also absent (McCrory et al, 2010).

Another issue that needs to be addressed is the gap between what is learned and how knowledge is applied. Research shows that what pre-service teachers learn in their methods courses serves as the basis for their classroom behaviors (Bush, 1986). Various suggestions have been put forth to introduce reform in teacher education (Manouchehri, 1997; Cooper, 2009). The push toward student centered learning places great emphasis on the analysis of student work and thinking. However, little is known about how such capacity is developed by teachers or might be nurtured in teacher education.

The current study is aimed at understanding how pre-service teachers analyze student work and thinking in mathematics. This study also examined the effect of two quarters of a teacher preparation program on the development of teachers’ MKT as it pertains to analysis of student work and thinking. Not much is known about teachers’ MKT in the field of geometry. This study would be an attempt to contribute to this field of research and mapping out the pre-service teachers approaches to analyzing student work and thinking in geometry.

Theoretical Framework

There has been a call for utilizing the theory of situated cognition in order to design pre-service teacher education (Lin, Hsu, & Cheng, 2011; Paretti, 2008). According
to Green and Reid (2004) “teacher education – like educational research as well as schooling itself – should always be understood as a situated practice. As such, it is best conceived as always located somewhere, socially, spatially and historically, and as always speaking from somewhere” (p. 255).

The theory of situated cognition is based on the premise that all knowledge and learning is situated within the activity and the culture in which the activity takes place (Anderson et al., 1996; Brown, Collins, & Duguid, 1989; Greeno, 1998). The term, *situated learning*, is based on the notion that learning is situation specific and much of what is learned depends on the situation it is learned in (Anderson, et al., 1996). Situated learning emphasizes the disconnect that exists between learning that takes place in schools and the “real world” (Anderson, et al., 1996). It provides an explanation of why students who perform well in schools sometimes fail to transfer that knowledge into the real world or vice versa. For example, a study conducted on Orange county housewives revealed that they were very apt at calculations at the supermarket but performed poorly on school-like arithmetic tests (Lave, 1988). Another frequently cited example is the study conducted by Nunes, Schliemann, & Carraher (1993) in which Brazilian street children did arithmetic quite well while making sales on the street but who were not able to do so well on school based arithmetic tests. Stein (1998) listed four key elements:

1. Learning is grounded in the actions of everyday situations;

2. Knowledge is required situationally and transfers only to similar situations;

3. Learning is the result of a social process encompassing ways of thinking, perceiving, problem solving, and interacting in addition to declarative and procedural knowledge; and
4. Learning is not separated from the world but exists in robust, complex, social environments made up of actors, actions, and situations (as cited in Paretti, 2008, p. 492).

Situated cognition puts much emphasis on structuring activities that are authentic. Such activities are meaningless to students until they are situated in some domain culture. According to Brown et al. (1989) from a situated perspective, knowledge is always situated in an authentic context. Situated cognition provides us with a means of interpreting “how pre-service teachers nurture their competence in teacher education” (Lin et al., 2011).

For the purpose of the current study, the theory of situated cognition was utilized for the development of tasks for the two methods courses that were offered to the pre-service teachers. We did not use situated cognition as a way of enhancing learning but as a framework for task design. The use of those tasks grounds analysis in episodes of teaching and learning and motivates engagement.

According to Grossman, Hammerness, & McDonald (2009), methods courses mostly focus on the practice including teaching for subject matter, classroom management and assessment and helping students acquire tools and develop strategies for teaching. However, there is a gap between theory and practice in teacher education (Grossman et al., 2009). Pre-service teachers often lack teaching experiences even though they possess the required content and pedagogical knowledge (Lin et al., 2011). In order to bridge this gap we made use of the case-based methodology (Manouchehri, & Almohalwas, 2008; Manouchehri, & Enderson, 2003; Sykes, & Bird, 1992). Case-based tasks enhance teacher preparation because
1. They provide teachers with real contexts for learning about problems of practice (Sykes & Bird, 1992).

2. Teachers learn more pedagogy and subject matter (Walen, & Williams, 2000).

3. Teachers learn about reform based teaching and help them develop instructional strategies (Merseth, 2003).

Designing successful instruction requires “(1) analysis of students’ needs; (2) knowing the goals of teaching development; (3) making decisions about teaching content; (4) arranging teaching procedures; and (5) evaluating the effects of teaching” (Bennett, 1997, as cited in Lin, 2011, p. 101). Case study methodology offers pre-service teachers with episodes and samples of children’s work (Manouchehri, & Almohalwas, 2008). Case studies also offer teachers opportunities to “gain deeper mathematics insights” (Manouchehri, & Almohalwas, 2008, p. 243) by examining the mathematical work that students create. Case-based methodology also familiarizes teachers with real classroom problems as well as make them familiar with the setting in which these problems occur (Sykes, & Bird, 1992, as seen in Manouchehri, & Enderson, 2003).

While developing case-based tasks, the aim should be help the teachers realize the interactions between pedagogy and content (Manouchehri, & Almohalwas, 2008). Case-based tasks must “collectively, increase the teacher candidates’ understanding of mathematics, pedagogical content knowledge, and current research-based knowledge about effective mathematics teaching and learning” (Manouchehri, & Almohalwas, 2008, p. 244). Manouchehri and Almohalwas (2008) have proposed goals that these cases must meet. Among the mathematical goals, the case must

- Address important mathematical concepts (Wu, 2005).
- Allow for the development of representational knowledge (NCTM, 2000).
- Provide opportunities for problem solving and posing (Cooney, 1994).
- Provide opportunities to engage in a discourse about the mathematical content (NAP, 2001).

The pedagogical goals include

- Challenging teachers’ assumptions about teaching (Manouchehri, & Enderson, 2003).
- Helping teachers realize how instructional planning should be guided by children’s work (Ball, & Bass, 2003 as cited in Manouchehri, & Almohalwas, 2008).
- Increasing teachers’ listening and engagement skills (Merseth, 2003).
- Make teachers aware of the cognitive obstacles associated with the teaching of mathematical topics (Wilson, 1992).

However, according to Yin et al., (2011) one challenge that prevents the theory of situated cognition to be applied to teacher education is that there is a lack of social interactions within the teaching community. In order to address this issue the social constructivist perspective was utilized in the methods courses so as to include interactions between pre-service teachers.

The underlying belief of social constructivism is that knowledge is influenced by politics, ideologies, religion and various other social aspects (Vygotsky, 1978). Vygotsky (1978) argued that learning occurs in a social context and not in isolation. According to Vygotsky (1978) learning is influenced by historical aspect of the learners’ experiences. Also, in social constructivism, there is a possibility for the transfer of knowledge. When
working in groups or during interaction with more knowledgeable peers or with teachers, knowledge is transferred between individuals and thus learning is fostered.

According to Vygotsky, meanings are constructed socially and so a person can be understood only in relation to the social relations in which he or she exists (Wertsch, 1986). In order to succeed, an individual needs to be aware of the social norms and practices of the society in which he or she lives (Manouchehri, & Enderson, 2003). In case of teacher education, pre-service teachers learn about the practices and norms of their profession through interaction with peers (Manouchehri, & Enderson, 2003). These interactions between pres-service teachers and their peers “help them establish what is worthwhile knowledge and what is valued for practice” (Manouchehri, & Enderson, 2003, p. 116). Thus teacher education programs have called for interactive group work in order to address pedagogical and content related issues that arise from the case-based tasks (Cooper, 2009). Timmerman (2004) also suggests that the presentation of examples of student work should be followed by meaningful discussions about student thinking of mathematics. Manouchehri, & Enderson (2003) argue that by facilitating communication among pre-service teachers so that there is a meaningful exchange of ideas, they begin to develop new understandings of mathematics and teaching. There is a call to develop a “community of learners in which students rely on feedback from peers and work towards developing reasoning skills based on collective inquiry” (Manouchehri, & Enderson, 2003, p. 116).
Chapter 3: Methodology

The goals of the present study were twofold. First, it was my aim to explore the nature of mathematical knowledge of a cohort of pre-service secondary mathematics teachers’ knowledge for teaching high school geometry by eliciting their reactions to episodes of children’s work-samples on geometric tasks. A second goal of the study was to investigate the impact of two quarters of coursework centered around the analysis of student work samples and thinking about geometry on pre-service teachers’ conceptualization of teaching actions. An academic quarter consisted of 10 weeks of classes and one week of exams. The research questions guiding data collection and analysis included:

1. What factors do pre-service teachers consider when judging students’ mathematical work and thinking?

2. What is the effect of two quarters of coursework on pre-service teachers’ assessment of students’ mathematical work and thinking of geometry and measurement?

3. What is the effect of two quarters of coursework on pre-service teachers’ ability to develop instructional strategies that aid student understanding of geometry and measurement?
4. What is the effect of two quarters of coursework on the quality of questions posed by the pre-service teachers to elicit student understanding of geometry and measurement?

5. What is the relationship between levels of teachers’ self-efficacy and their knowledge of students’ learning and thinking?

In this chapter I present a description of the methods that were used in order to complete this study. Namely, this includes a description of the participants, a description of the location in which the study took place, procedures for data collection, and methods for data analysis.

This study utilizes a case study methodology involving both qualitative and quantitative data. The study involved interviewing participants to provide insight into the thought processes and their reasoning. Furthermore, according to Rossman and Rallis (1998), qualitative research has several positive qualities. First, it is situated in the real world and employs interactive and humanistic methods. Secondly, it is interpretative. Since I needed to obtain information about how teachers analyzed student work and thinking in geometry and factors they considered as they engaged in such tasks, qualitative research methods were best suited to the needs of my study.

Since I wanted to gather as much in-depth information about the pre-service teachers as possible I also used a case study approach (Tellis, 1997). Case studies, according to Schramm (1971), are commonly defined as:

The essence of a case study, the central tendency among all types of case study, is that it tries to illuminate a decision or set of decisions: why they were taken, how they were implemented, and with what result. (as reported by Yin, 2003, p. 12)
Case study is a suitable choice because the variables under study are inseparable from the context in which they exist (Yin, 2003). There are at least four applications of case studies as identified by Yin (2003):

1. Case studies “explain the presumed causal links in real-life interventions that are too complex for the survey or experimental strategies” (p. 15). In my study, the case study approach was used to explain the causal links between the observed changes in pre-service teachers’ approach to student work and thinking and the effect of two quarters of coursework.

2. Case studies “describe an intervention and the real-life context on which it occurred” (p. 15). For this study, the case study approach revealed the pre-service teachers’ experiences while going through two quarters of coursework.

3. Case studies “illustrate certain topics within an evaluation, again in a descriptive mode” (p. 15). In my study, the case study approach illustrated the factors that affected the pre-service teachers’ orientations to teaching namely their beliefs about teaching and learning and their experiences and what effect they had on the pre-service teachers’ decision making.

4. Case studies “explore those situations in which the intervention being evaluated has no clear, single set of outcomes” (p. 15). As in many qualitative studies, I anticipated that some of my outcomes might be surprising or unexpected. For example, one of the unexpected outcomes was that the pre-service teachers’ survey scores on items pertaining to analysis of student work decreased despite taking two quarters of coursework centered on the analysis of student work and thinking. In the event that such outcomes presented
themselves, the case studies would help me gain a better understanding of the situations to be able to explore and explain further.

Participants

The participants consisted of ten pre-service mathematics teachers (2011-2012 cohort) seeking licensure to teach grades seven through twelve, enrolled in the Master of Education program at a large Midwestern university during the 2011-2012 academic year. The participants were recruited for the study in their methods course (number sense and data analysis) in the summer quarter of 2011. The pre-service teachers were given a consent form during the first week of the second summer term and asked to decide if they wished to participate in the study. Six of the pre-service teachers were female while the remaining four were males. Eight of the enrolled pre-service teachers had previously obtained a bachelors degree in mathematics, and the remaining two teachers had a background in engineering (computer and industrial engineering).

Among the 10 pre-service teachers who had initially consented to participate, one withdrew after taking the posttest survey. Another pre-service teacher was not able to schedule a posttest interview. Since the interviews were a key part of the data collection, I chose not to include these two pre-service teachers (both females) in the study.

Since I was using a case study methodology, I needed a framework for narrowing down to three cases. Initially I had intended to use gender as a filter for selection of specific cases, for example two males and two females. However, I rejected this sampling technique because this variable had not proven to be notable in previous studies. Another possible method was to use the total scores on the pretest surveys as a way to catalogue participant selection. This was also rejected because based on the surveys alone, some
questions remained unanswered, some answers were unclear and in the end it made more sense to divide the student teachers based on Cooney, Shealy and Arvold’s (1998) classification of pre-service teachers. I chose this classification to select my cases because it gives a way to both validate and advance previous research and this selection was better suited to my study because it allowed me to link teachers’ beliefs and their analysis of student work. Even though I did not utilize known measures of teachers’ beliefs, I captured their claimed beliefs via the interviews and compared them to their enacted beliefs that were elicited in the surveys.

Among the remaining eight participants, three were selected for an in-depth case study phase of the study. The selection of the cases was done based on Cooney, Shealy & Arvold’s (1998) categorization of pre-service teachers’ orientations toward learning to teach including: isolationist, naïve idealist, naïve connectionist and reflective connectionist. An isolationist is one who rejects beliefs of others, while a naïve idealist uncritically accepts others ideas without questioning them (Cooney, Shealy, & Arvold, 1998). The connectionists “emphasize reflection and attention to the beliefs of others as compared to one’s own” (Cooney, 1999, p. 172). Lastly, the naïve connectionist acknowledges others’ beliefs but fails to resolve the conflict between one’s own beliefs and others’ beliefs whereas the reflective connectionist is able to assimilate others’ views into his own (Cooney et al., 1998). The cases were categorized into one of the above four categories based on the data from the surveys, interviews and classroom observations of the candidates. Based on the interview and survey data, I was able to find one participant who fit in the naïve connectionist and one participant in the naïve idealist categories. However, the remaining cases fell in the isolationist category. For the isolationist
category I picked Cersei because of her background in engineering because she had a background in engineering unlike the others whose backgrounds were in mathematics.

_Bran_

Bran completed his bachelor’s degree in mathematics and joined the M. Ed. program to obtain a licensure to teach 7th-12th grade mathematics. I had the opportunity to observe Bran in-class during the course of my study. I was the instructor for the methods course he took in number sense and data analysis in the summer. I also observed the pre-service teachers in their geometry methods course. During both the observations, it was clear that Bran’s mathematical ability was advanced. He had no difficulty solving any of the mathematical problems that were presented to the group. Bran also showed a lot of interest and enthusiasm in learning new material that was presented during the class.

Bran fell under the category of naïve connectionist. While holding onto his own beliefs about teaching of mathematics, Bran drew from several other sources, namely his mentor teacher, his instructors in mathematics and mathematics education courses, and lastly from the classroom episodes (Boaler & Humphreys, 2005) that were shown in the methods course. Despite speaking positively about many of the theories he learned about in his courses, there were clashes between his beliefs and these theories. He was unable to assimilate these theories into his viewpoints on teaching of mathematics. For example, Bran specifically acknowledged the importance of learning progressions when assessing individual student’s work and planning instruction for that student and showed evidence of what they were, but he did not reconcile them with his own beliefs about teaching that were based on experience as a teacher in the classroom.
Cersei

Cersei was one of the two pre-service teachers with a background in engineering among the cohort studied in this project. She worked at a car manufacturing company. She decided to pursue teaching when her plant closed and teaching at a high school seemed to her to be the best fit. Previous to joining the Master of Education program, she had enrolled in an alternative licensure program at a different university where she was only one course short of obtaining her certification. However, she expressed that she felt it necessary to go through the M. Ed. program in order to be an effective teacher. Cersei had completed all the required coursework in mathematics as she had also completed few STEM education courses while enrolled in her previous certification program.

Cersei fell under the isolationist category. The M. Ed. coursework had very little impact on Cersei’s beliefs on the teaching and learning of mathematics. She viewed the lesson plans and theories that she learned in the program as being only requirements for the coursework and as such saw minimal use for them in practice. Her views changed little from pre to post test and further, in the moment, she remained unable to alter her viewpoint even when confronted with a contradiction. For example, in the octagon problem, Cersei could only view the shape as a rectangle. When confronted about this belief and explicitly told that the shape was, according to the definition, an octagon, Cersei adamantly stuck to her viewpoint of the shape as a rectangle.

Nedd

Nedd was a computer engineer and an electrical engineer prior to joining the Master of Education program. He decided to become a teacher as he felt that it would be a good profession to retire. He had no previous teaching experience and so he felt that he
needed to go through the program in order to learn how to teach. Nedd was a very outspoken individual and was very active in class. Unlike the other pre-service teachers in the study who were on a one year track, Nedd opted to take the program over two years. He felt that he had a lot to learn and did not want to rush through it. Nedd was also taking mathematics classes in order to fulfill his requirements for the program. At the time of the pre-interview, Nedd was enrolled in the Introduction to Higher Mathematics course.

Based on his interview and survey data and on my observations of Nedd in the two methods courses, I placed him in the naïve idealist category. Nedd readily accepted the views and beliefs that he was introduced to without questioning them. Nedd was quick to accept the findings from the readings that were assigned in class. He felt as if he needed this knowledge and he trusted the teacher educators and his professors and never questioned them.
Instruments for Data Collection

<table>
<thead>
<tr>
<th>Data Source</th>
<th>Purpose</th>
<th>Sample</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest survey</td>
<td>Get baseline information about pre-service teachers’ beliefs, self-efficacy and assessment of student work</td>
<td>All</td>
<td>First week of summer quarter of 2011</td>
</tr>
<tr>
<td>Posttest survey</td>
<td>Get information about the change in pre-service teachers’ beliefs, self-efficacy and assessment of student work after the methods course</td>
<td>All</td>
<td>Last day of class December 2011</td>
</tr>
<tr>
<td>Pretest interview</td>
<td>Get more in depth information on the pretest survey</td>
<td>All</td>
<td>Followed the pretest survey. Second and third week of second term of summer 2011</td>
</tr>
<tr>
<td>Posttest interview</td>
<td>Get more in depth information about the posttest survey</td>
<td>All</td>
<td>Second and third week of winter quarter in January 2012</td>
</tr>
</tbody>
</table>

Table 1. List of Data Collection Instruments

Pretest Survey

The pretest survey was used to collect baseline data about all the participants. The pretest survey consisted of four parts and 25 questions. The first part consisted of seven questions about future teaching plans. The second part of the survey consisted of three questions about teachers’ self-efficacy, which plays an important part in the learning environments that they create for their students (Bandura, 1993). The third part of the pretest survey consisted of thirteen questions regarding the mathematical knowledge for teaching, as it pertained to the analysis of student work and thinking in geometry. The questions in this section were open-ended. The reason for including open-ended questions was to elicit detailed information from teachers. Finally, the fourth and final
part consisted of two questions regarding the mathematical knowledge for teaching, as it pertained to the analysis of student work and thinking in algebra and statistics. The questions about teachers’ self-efficacy were Likert-type questions and were completed using a nine point Likert scale. The survey was scored according to the developed rubrics (see Appendix E, F, & G). The pretest survey was about two hours long.

Pretest Interviews

The pretest interviews were semi-structured and helped provide additional insight on factors pre-service teachers’ considered when responding to the pretest items. The pretest interviews consisted of four parts. The first part of the interview comprised of questions about the pre-service teachers’ ideal classroom. I asked the pre-service teachers about what they viewed as their ideal classroom, the role of the students, teachers and the type of curriculum used in their ideal learning environment. I also asked them if they viewed their course work to be helpful in implementing their ideal classroom and what obstacles they might face in doing so. The reason for pursuing this line of questioning was to elicit what factors they considered important to teaching. The second part of the interview consisted questions regarding the teachers beliefs about the teaching and learning of mathematics. The third part consisted of questions regarding their self-efficacy, especially about efficacy for instructional strategies and classroom management. Finally the fourth and final part of the interview was dedicated to eliciting in-depth information regarding the pre-service teachers’ responses to the pretest surveys. The pretest interviews were approximately 100 minutes long.
Posttest Survey

The posttest survey was used to record data and observe the changes in the pre-service teachers’ self-efficacy and MKT at the end of the study. The posttest survey consisted of four parts and 21 questions. The comparison of the pre and posttest items is given in Table 4. The first part consisted of four questions about demographic data including future teaching plans. The second part of the survey consisted of three questions about teachers’ self-efficacy. The third part of the posttest survey contained thirteen questions regarding the mathematical knowledge for teaching, as it pertained to the analysis of student work and thinking in geometry. The questions in this section were open-ended. Finally, the fourth and final part of the posttest survey consisted of two questions regarding the mathematical knowledge for teaching, as it pertained to the analysis of student work and thinking in algebra and statistics.

Posttest Interviews

The posttest interviews were semi-structured similar, in structure, to that of the pretest interviews. However, the posttest interviews consisted of two more categories in addition to the four categories that were also in the pretest interviews. One of the categories of questions that I included was about the pre-service teachers’ reactions to the M. Ed. Program. I wanted to see if they viewed the program as beneficial to their teaching and what, according to them, was the most influential learning experience in the program. Additionally I asked the pre-service teachers’ about their knowledge about models of assessment (van Hiele and Pirie & Kieren). I chose to include these questions based on their responses to the posttest survey items 8, 11, and 14 which had specific
prompts requiring students to use either the van Hiele or the Pirie-Kieren models to assess student work. The posttest interviews were approximately 90 minutes in length.

Instrument Development

The design of the data collection instruments, namely the pre/post surveys and interview questions, was guided by the KAT framework. The items on the instrument were developed with the intent of eliciting the knowledge bases from which the teachers drew in order to analyze students’ mathematical work and thinking via the actions of decompressing, trimming and bridging.

The pretest and posttest surveys addressed concepts that were deemed important in Principles and Standards (2000) and the Academic Content Standards for K-12 Mathematics (Ohio Department of Education, 2004). I covered important topics and concepts that are taught in secondary school geometry.

The text provided me with numerous previously recorded episodes of children’s problem solving sessions and their work on geometric tasks, consisted of illustrations of students’ mathematical thinking, heuristic usage and reflexive actions (Manouchehri, 2012). The prompts for pre-service teachers were then developed using the KAT framework. Even though I was concentrating on the task of student work and thinking in the area of high school geometry and the knowledge bases associated with that task, the survey did draw from the other domains of the KAT framework. For example, some of the questions elicited teachers’ capacity in the area of task development, while other questions asked for knowledge related to the task of explaining mathematical ideas and solving mathematical problems. Two questions on the pretest (questions 11 and 12) and one question on the posttest (question 18) were developed by
the Center for Research in Mathematics and Science Teacher Development, University of Louisville. Finally, I also included one question on algebra (question 24 on the pretest and question 20 on the posttest (KAT released items) and one question on statistics (question 25 on the pretest and question 21 on the posttest). The reason for including items from algebra and statistics was to see whether the knowledge/skills that the pre-service teachers had developed for analyzing student work in geometry would also be transferrable to other areas of mathematics.

The interview structure was borrowed from Gilchrist and Somayajulu (2010a; 2010b). Gilchrist and Somayajulu (2010a) asked specific questions from pre-service teachers based on responses they had provided on a survey that contained items pertaining to the analysis of student work. In this work, I maintained a similar interview structure, in that, the interviews served the purpose of obtaining more information about the written responses provided to the surveys by the pre-service teachers. The interviews were semi-structured. Tables 1 and 2 summarize the correspondence between the research questions and the items on the pretest and posttest surveys.
Table 2. Correspondence between Pretest Items and the Research Questions

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Table 3. Correspondence between Posttest Items and the Research Questions

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<td>21</td>
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Validity of the instrument

The surveys consisted of 4 parts. The first part obtained background information on the participants. The second part was The Ohio State Teacher Efficacy Scale, which has been field-tested for validity and reliability (Tschannen-Moran & Hoy, 2001).
While preparing the survey, feedback was obtained from practicing mathematics teachers. These reviews were completed to determine the community’s common expectations of knowledge for pre-service teachers upon their completion of the program. I was careful to limit the questions to the range of skills that teachers were expected to develop, while in teacher preparation, in order to teach high school geometry. The surveys were also tested by three secondary pre-service teachers enrolled in the 2010-2011 STEM M.Ed. program and also by 2 Ph.D. students majoring in mathematics education.

Initial testing of the instrument began when I met with a few coaches and teachers participating in the Mathematics Coaching Project (MCP). I shared with them the items developed and received their feedback on the content and the format of the questions. Based on their feedback, I added more topics to the surveys. Initially, I had not included geometric constructions, transformations and definitions in my surveys. However, these topics emerged as important themes and so were addressed. After revising my surveys, I then arranged for a presentation of my data collection instruments with my advisor and other doctoral students in mathematics education. Following the presentation, I was given feedback on my work. The feedback mainly concentrated on the language I had used to frame the questions. Revisions were made to the surveys before they were administered to the pre-service teachers of the 2010-2011 cohort for further validation.

Based on the feedback I received, a few changes were made to the items. There was one major change to the posttest survey. Question 12 and 19 on the posttest survey were added after two of the three secondary pre-service teachers had tested the instrument. There were no other major changes made to the surveys in terms of content. One of the major problems faced was the wording on the pretest item on transformations. Since this question
involved specific terminology used in the Geometer’s Sketchpad (GSP) software, I decided to put this item on the posttest and replace it with the item on transformations from the posttest. The pre-service teachers from the 2010-2011 cohort who completed this survey during the pilot phase revealed that the questions on transformations were the hardest for them to follow and that a pictorial representation of what the student was doing would have been very helpful for them. However, I decided not to include any pictorial representations, as the goal was to see how teachers deciphered student reasoning and make sense of their written work. As I received input from the pre-service teachers from the 2010-2011 cohort as well as my colleagues, I realized that some questions needed to be reworded in order to make them clearer. The content of these questions did not change but the prompts were reworded to make them clearer and to elicit more information from them. Another goal of the pilot testing was to determine the amount of time it took for the pre-service teachers and the PhD students to complete the surveys. On an average, approximately 150 minutes were needed to complete the test. Interviews with the pre-service teachers of the 2010-2011 cohort revealed that the questions did elicit the kind of knowledge I had intended to capture. Table 4 illustrates a classification of items and the topics covered on the surveys.
<table>
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<th>Content</th>
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<td>Question 9</td>
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Table 4. Classification of Survey Items

The Methods Courses

The M. Ed. program was a 4 quarter long teaching licensure program. The teacher candidates complete coursework during three quarters (summer, autumn and winter). The spring quarter is devoted to student teaching. The pre-service teachers were required to observe their mentor teachers in the autumn and winter quarters. They taught eight
lessons in the autumn quarter and one class for thirty days in winter quarter. The pedagogical content specific coursework consisted of three methods courses: data analysis and number sense, geometry and algebra. The methods course in data analysis and number sense was offered in summer 2011. I taught this course. The geometry methods course was offered in autumn 2011. This course, which I observed, was taught by a graduate student. Finally the third methods course in algebra was offered in winter 2012. In the next section I will elaborate on the content of the summer methods course in data analysis and number sense, which I taught and the autumn methods course in geometry, which I observed. Data collection for this dissertation research occurred during the time frame of these two mathematics-specific methods courses.

**Data Analysis and Number Sense Methods Course**

The course had the following aims and objectives:

1. Increasing pre-service teacher pedagogical content knowledge of number and probability concepts as appropriate for secondary students.

2. Provide hands-on activities for interns to use with students in middle and high school.

3. Use varied representations of mathematical ideas to support and deepen students’ mathematical understanding.

4. Becoming experienced in designing lessons plans that build around fostering conceptual understanding of data analysis and number concepts.

5. Becoming experienced in analyzing and understanding student work and thinking about mathematical concepts.
The first of the three methods courses was offered in summer. This course was a methods course on data analysis and number sense. This was one of the first classes that the pre-service teachers took upon entering in the Master of Education program. The class met for three hours twice a week for five weeks. However, we met only 9 times as one of the classes was cancelled due to a national holiday. Classroom discussions between the pre-service teachers were highly encouraged to aid in problematizing learning issues and teaching strategies (Manouchehri, 2002). As a part of the class requirement, the students were expected to work one on one with a student at the mathematics clinic. The aim of this activity was to introduce the pre-service teachers to the process of analysis of student work and thinking in mathematics. Based on their work with the student, the pre-service teachers were required to develop three lesson plans to be used specifically with the child with whom they had worked and as a final project for the class, the pre-service teachers had to write a case-based analysis of their student. They were also required to conduct a brief (10-minute long) interview with the student.

Session 1:

The first session began with an introduction to the methods course and the expectations of the pre-service teachers. This was followed by an activity in which pre-service teachers were asked to define mathematics. After all the teachers had shared their definitions, they discussed the strengths and weaknesses of each one of them. Another issue that stemmed from this activity was the characterization of problem. Most of the pre-service teachers’ definition of mathematics involved problem solving, which led to a discussion of what a good problem is. What consists of a good problem? Following this
activity, the pre-service teachers were given the pretest survey and they used the rest of the class to complete it.

Session 2:

Session 2 began with a discussion of assigned readings. The discussion lasted for about an hour during which we discussed the current state of mathematics education in the United States of America. The articles also introduced the concept of reform-based teaching. Following this activity, the class watched the “border problem” video (Boaler & Humphreys, 2005). Before watching the video, the pre-service teachers were asked to solve the border problem on their own. In the border problem, students are presented with a 10-by-10 grid with a colored border. The students are asked to calculate the number of colored squares in the border. This video led to discussions about the way the teacher in the video taught the lesson. The pre-service teachers focused on the different ways in which the students came up with a solution to the border problem. They made connections between the different ways in which the solution was represented. Their reflective analyses of the video consisted of how they thought the video was helpful for them in trying to plan a lesson differently. The session concluded with pre-service teachers working on a pattern recognizing activity. Pre-service teachers worked in groups of 2-3 and came up with different patterns of numbers.

Session 3:

Session 3 also began with a discussion of the assigned readings. We spent about an hour discussing issues regarding the teaching approaches used at the Railside School (Boaler, 2008) and how these approaches were beneficial to the students of that school. The discussion also involved the obstacles faced while employing similar methods. The
students had a debate as to how these classroom conditions of the Railside school could be emulated. Again, here the students referenced the border problem video that they were shown in session 2 and tried to compare the two classroom scenarios. The aim was getting them to think about obstacles and issues that they would face while implementing these methods as well as the strengths and weaknesses of the types of teaching that were modeled in the video as well as in the Railside School (Boaler, 2008).

Following the above described activity, the pre-service teachers were presented with two or three mathematical problems and were asked to solve them in groups of two-three and then they came together as a whole class to share their findings. The pre-service teachers worked in their groups and came up with solutions to the problems. Once they had finished, they were then encouraged to solve the problem using different representations. They were then asked to share their solutions with the entire class. Through this activity, I wanted to have pre-service teachers work with different representations. I also had hands on manipulatives at their disposal to aid them in solving the problem. Secondly, I asked them to share with the class, the way present these solutions to a high school mathematics class.

Session 4:

Session 4 was similar in structure to session 3. We started by talking about the assigned readings. The topic of our discussion for this week was high-stakes testing. The discussion involved issues around high stakes testing and whether this form of testing is helpful or not. All the pre-service teachers argued against the current method of testing, stating that it was ineffective and was not a good measure of student understanding or achievement. There was also a brief discussion on how teachers often taught for
achievement on tests rather than for understanding. There was also a brief discussion on number sense versus computation where the pre-service teachers discussed whether being proficient in computation implied a good number sense and vice versa.

Following this discussion the pre-service teachers were given a problem to solve. This activity was very similar to what we did in session 3. The pre-service teachers worked in groups to figure out a solution. Once again they were encouraged to use multiple representations. Hands on manipulatives were made available to the pre-service teachers. After they had completed the task, the pre-service teachers shared their solutions and work with the rest of the class. We also talked about what problems or obstacles the students could face while solving the problem.

Session 5:

Starting with session 5, the concentration of the class shifted from number sense to data analysis. The session started with a question: What is data analysis? This got the pre-service teachers to come up with a suitable definition of data analysis and ways in which it differs from other forms of mathematics analysis. This proved particularly hard for the pre-service teachers. They struggled to come up with a suitable definition of data analysis. The main difference that they noticed between data analysis and mathematical analysis was that in data analysis, the data can be very random and not necessarily follow any particular pattern, whereas in other forms of mathematical analysis, the data normally follows an identifiable pattern.

The next topics of discussion were the measures of central tendency. Pre-service teachers were set the task of defining mean, median and mode in ways that were accessible to high school students. Following that, the pre-service teachers were asked to
come up with scenarios in which median or mode would be used as a measure of central tendency as compared to the mean. The importance of each of these was discussed. This led to the topic of outliers. The class then worked in the direction of how outliers could be an obstacle for students while engaging in data analysis.

The next activity presented to the pre-service teachers introduced them to the task of analyzing student work and thinking in mathematics. Pre-service teachers were given the following problem to solve (Wilson, 2012):

A pie graph has four unequal slices representing a sample of n people taste testing their favorite soft drink. If the Coca-Cola slice is exactly 36% of the total graph, how many people were initially in the sample? Suppose that 8 more people join the study and ¾ of them choose Coca-Cola. What percent of the graph would the Coca-Cola portion be?

The pre-service teachers were asked to solve this problem. After they finished, they were asked to try and come up with a way a student would solve it. The pre-service teachers worked in groups and came up with possible ways that students could solve the problem. After they had shared their ideas with the rest of the class, the pre-service teachers were shown a video recording of a student who was working on this problem.

The aims of the activity were two fold. First, it introduced the teachers to the task of analyzing and assessing student work and thinking in mathematics as they begin to understand how the student approached the problem and how it might have been different than how they approached it. The reason for using a video recording was to use illustrative examples of student work to increase teachers’ knowledge of analysis of student work (Cooper, 2009; Jacobs & Philipps, 2004; Timmerman, 2004). According to
Manouchehri (2008) case-based tasks are effective in engaging pre-service teachers in the process of analyzing student work.

In the discussion that followed the video, the pre-service teachers analyzed the student approach and also tried to decipher his choice of numbers and his conclusions. The pre-service teachers then commented on the questions that were asked by the interviewer in the video episode. They also discussed about how to aid the student in his understanding and what instructional strategy they would use in order to do that.

Session 6:

Session 6 began with a completely different agenda than the previous 5 sessions. The pre-service teachers viewed another video of a different student working on the Coca-Cola problem. After watching this episode, the pre-service teachers analyzed this student’s approach to the problem. Following this discussion, the pre-service teachers were again shown the episode from the previous session. It was followed by a comparison between the approaches taken by the two students to the same problem. The pre-service teachers discussed the similarities and differences between the two approaches. The pre-service teachers then focused their discussion on the strengths of the two approaches.

The pre-service teachers talked about the interviewer’s role in aiding the student. They critiqued the questions posed by the interviewer and also came up with questions of their own, which they thought could have been beneficial.

Session 7:

Session 7 had a similar agenda as session 5. The class began with a discussion of the assigned readings followed by a video of student work in data analysis. The initial
discussions revolved around the topic of cultural issues and problems faced by teachers in urban school settings. The second round of discussions began with the concept of probability and issues faced with teaching probability. Pre-service teachers discussed at length about what possible obstacles could the student face while learning probability. This gave rise to the topic of theoretical probability vs. experimental probability. Pre-service teachers talked about making connections between the two.

The second activity involved another episode of student work. First the pre-service teachers were faced with the following problem (Manouchehri, & Zhang, in press):

Consider the two pay options. $300 a week or $7.50 an hour. What factors will affect your choice of which option to take? Draw a graph that compares the two pay options allowing the reader to determine which one would be the best for them.

The pre-service teachers began working in groups and talked about different factors that would affect the choice of the pay option. They talked about number of days, number of working hours and other possible factors. Once they had finished working, they discussed their findings with the class. After that they were shown a video of a student working on that problem. Following the episode, the teachers considered the approach taken by the student and discussed the strengths and weaknesses of this approach. They also looked closely at the graphs drawn by the student and assessed them carefully. The discussion of the episode then focused on the line of questioning that the interviewer chose. They also talked about instructional strategies that they felt would best help the student.
Session 8:

Session 8 was similar to session 6. This session began by showing the pre-service teachers another video of a different student working on the pay-option problem. After watching this episode, the pre-service teachers analyzed this student’s approach to the problem. Following this discussion, the pre-service teachers were again shown the episode from the previous session. It was followed by a comparison between the approaches taken by the two students to the same problem. The pre-service teachers discussed the similarities and differences between the two approaches. The pre-service teachers then focused their discussion on the strengths of the two approaches.

The pre-service teachers talked about the interviewer’s role in aiding the student. They critiqued the questions posed by the interviewer and also came up with questions of their own, which they thought could have been beneficial.

Session 9:

The final session of the summer methods course in data analysis and number sense began with the pre-service teachers summarizing what they had learned so far from the course and how they thought it had affected them. The pre-service teachers were then asked to fill out an assessment that was required by the program.

Geometry Methods Course

The aims of the geometry methods courses were as follows:

1. Becoming familiar with the state of teaching and learning of geometry.
2. Becoming familiar with the K-12 geometry standards.
3. Becoming familiar with cognitive obstacles in learning geometry concepts.
4. Examining traditional and reform based geometry texts.
5. Exploring Euclidian constructions.

6. Exploring dynamic geometry software and becoming experienced in using them for constructions and conjectures.

7. Becoming familiar with connections between geometry and algebra.

8. Becoming familiar with connections between high school geometry and university geometry curricula.

9. Becoming experienced with assessing student thinking and creating lesson plans that foster students’ conceptual understanding of geometry.

10. Becoming experienced in posing and solving various types of geometry problems/explorations applicable to secondary curriculum.

The autumn methods course in geometry was an 11-week-long course during which the pre-service teachers met once a week for three hours. The class met for 10 weeks. Of the ten, class was cancelled one week. The eleventh week was exam week and so there were no regular classes. This course was taught by a graduate student under the supervision of a faculty member. During this course, the pre-service teachers were introduced to dynamic geometric software namely Geometer’s Sketchpad (GSP) as well as the van Hiele model of assessment of student work. As with the summer methods course, in this course the pre-service teachers were required to conduct a teaching experiment with two students. The pre-service teachers were required to meet with the students 5 times during the entire quarter. During their last meeting with the students, the pre-service teachers were required to interview them and write a case-based analysis of the students.
Session 1:

The session began with a discussion of the Pirie & Kieren (1989) model for growth in mathematical thinking. This was followed by allowing pre-service teachers to apply the Pirie & Kieren model to assess an episode involving student work in geometry. This was followed by an activity, where the pre-service teachers looked at various traditional and reform-based geometry texts. The aim of this activity was to compare and contrast the different texts and also to get an idea of which topics remain prevalent in different texts.

Session 2:

Session 2 began with a discussion of the van Hiele model of geometric reasoning along with Battista’s (2007) elaboration of the van Hiele levels. The pre-service teachers were then given an episode of student work in geometry and were asked to use the van Hiele model to assess the student’s understanding of the concept. They then repeated this exercise with the Pirie & Kieren model. This activity was followed by a comparison of the two models and knowing how to apply the two models in order to get a better understanding of the level at which the student is.

Toward the end of the class, the instructor took the students to a computer lab, where they were introduced to Geometer’s sketchpad (GSP). The pre-service teachers were allowed to explore the software. They were given some Euclidean constructions which they were required to construct with the help of the GSP software. The pre-service teachers spent the rest of the class working with GSP.
Session 3:

The third session concentrated on discussions about van Hiele model and the Pirie & Kieren (PK) model. The aim was to get students to see that while van Hiele offered a linear model of assessment, the PK model was more dynamic and allowed for reflection. Following that, there was a discussion to the pre-service teachers’ response to a question that was given to them at the end of the previous class. The pre-service teachers were asked to list three important constructions. The discussion involved why these constructions were useful and how would they use dynamic geometry software to help students with Euclidean constructions. Following this pre-service teachers were introduced to the NCTM Illuminations website to use as a resource for finding lessons regarding constructions.

The pre-service teachers were then given the following problem: Draw a pentagon of your choice. Select a point on a plane and then construct a perpendicular to each of the sides of the pentagon. Mark and label the feet of the perpendiculars and connect them to construct another pentagon. The pre-service teachers were asked to share with each other their drawings. Following this the instructor presented the class with another problem namely; consider the statement: The diagonals of a rectangle divide the area into four equal parts. After the pre-service teachers shared their solutions, they were asked to apply the van Hiele levels to their own thinking for both problems. The aim was to get a better understanding of the levels and also to make the pre-service teachers realize that van Hiele model relies on experience with the content and without that experience the highest level cannot be achieved. There was again a discussion about the limitations of the van Hiele model and the advantages of using the PK model. Finally the pre-service teachers
were shown a video recorded episode of a student solving a geometric problem comparing the areas of three different shapes. They were then asked to analyze the student based on the two models of assessment.

*Session 4:*

A major part of this session was devoted to talking with the pre-service teachers about their progress on the assignments. Toward the latter half of the session the pre-service teachers were grouped into pairs and asked to work on transformational geometry problems. Each group was given two sets of problems, one consisting of middle school and the other consisting of high school problems. They were instructed to use GSP or other manipulatives to solve these problems.

*Session 5:*

Session 5 began with a discussion of the curriculum followed by a discussion of the pre-service teachers’ progress on their assignments. A couple of the pre-service teachers showed the recording of their interviews with their students. This was followed by an activity where in the pre-service teachers analyzed the student work using the models of assessment. Finally the pre-service teachers shared their analyses with the class.

*Session 6:*

Session 6 was similar to session 5. The session began with a discussion of the assignments. Two of the pre-service teachers showed episodes of their interview with their students and it was followed by a group analysis of the students’ work and thinking using the van Hiele and the PK models.
Session 7:

Session 7 began with the pre-service teachers solving problems that appeared on the Ohio Graduation Tests (OGT). The pre-service teachers were then asked to assess each other’s response using a constructive response item rubric. Following that they were shown samples of student work on these problems and were asked to grade the work according to the instructions on the rubric. They discussed possible difficulties and applications to their own teaching.

The discussion then moved on to mathematical proofs. The pre-service teachers engaged in a discussion of what is a mathematical proof and what do students need to know about this topic. This further led into the investigation of what reasoning skills are required by students and also the notion of formal vs. informal proof. There was also a discussion about the various proving techniques and how they are different. They worked on proving a couple of conjectures together and talked about why the certain activity would be easy or hard for students and what might the students need to know in order to prove those conjectures.

Session 8:

Session 8 began with a discussion of what teachers need to know in order to teach mathematics, in particular geometry. What makes a good geometry teacher? Pre-service teachers also addressed what the differences are between conventions, definitions and postulates. They argued about what information is necessary for defining a triangle. For example, is it enough to say that a triangle is a three-sided polygon or would you need to add more information?
The discussion then moved toward the 5 postulates of Euclidean geometry and the importance of the 5th postulate. This provided a transition into talking about non-Euclidean geometry. The pre-service teachers talked about the importance of non-Euclidean geometries, especially hyperbolic and spherical geometry. Further discussion ensued as to whether non-Euclidean geometries should be taught in high schools or not or whether high school students are capable of understanding non-Euclidean geometries.

Session 9:

The pre-service teachers did a video presentation of their case studies. This was the final class for the geometry methods course.

Researcher

According to Glesne (2006), a researcher assumes various roles, as he gets involved in fieldwork such as reformer, advocate, and friend. During the course of my study, I as the researcher will assume dual roles. For a part of my study, I will assume the role of a teacher-researcher and for the remainder of my study, I will assume the role of a participant-observer.

Teacher-Researcher

During the first five weeks of the summer of 2011, I was the instructor for the summer methods course in data analysis and number sense. Since part of my data consists of assignments for that course, I assumed the role of a teacher-participant. During this phase, I interacted with the research participants. One ethical concern was a potential conflict of interest regarding the data collected during that period. The research plan was submitted for ethical review and received approval of the Institutional Review Board (IRB). In order to avoid any ethical conflicts, the data collected during this period
(i.e. pretest surveys and interviews) were not be analyzed until the official grades were submitted. In this way, their participation had no bearing on their grades and they were assured of their right to withdraw at anytime. Before I collected that data for my research, I obtained written consent from the participants whether they wished to participate in the study or not

*Participant observer*

During the autumn quarter of 2012, I also observed a methods course in geometry. During this observation, my role as a researcher was that of a participant observer. According to Glesne (2006), through participant observation one can learn “how the actions of research participants correspond to their words” (p. 49). As a participant observer, the researcher becomes immersed in the research, participants as well as the research questions (Glesne, 2006). One of the issues faced by a participant observer is how much a researcher can participate. Glesne (2006) offers a solution to this, “Participate, but in a way that does not get you inextricably incorporated in a setting’s ongoing affairs unless you are choosing to do more action- or participant oriented research” (p. 72). My role as a researcher in this case was that of an observer’s role. I observed the class and the interactions that occurred between pre-service teachers and between the pre-service teachers and the instructor and took field notes. However, the only time I intervened was to clarify a pre-service teacher’s response or to elicit more information from the pre-service teacher, given his/her response to a class discussion or a question posed by the instructor or other pre-service teachers. I intervened only when I, as the researcher, felt that it would benefit my study.
Data Collection

The data collection process consisted of two phases. The first phase took place in the summer and beginning of autumn quarter of 2011, and the second phase took place toward the end of autumn quarter of 2011 and beginning of winter quarter of 2012. A detailed description of the three phases and data collection during each phase is as follows.

Phase I

During the data analysis and number sense methods course, which was offered during summer 2011, the incoming cohort of pre-service teachers completed a pretest survey as a part of their coursework. Since this information was collected before I received the IRB approval and since I was the instructor for the methods course, I did not analyze the data until I had issued the final grades in order to avoid any conflict of interest. Since I did not receive the IRB approval until the week before autumn quarter of 2011, the pretest interviews were conducted only during the autumn quarter of 2011, approximately 3 months after the administration of the pretest interviews. The aim of the pretest interview was to gather in depth information about the pre-service teachers’ responses to the pretest surveys.

Phase 2

Phase 2 commenced at the end of the autumn quarter of 2011. The pre-service teachers were required to fill out the posttest surveys as a part of their coursework for the geometry methods course. The posttest interviews were conducted in the January 2012, approximately a month after the posttest surveys were administered to the cohort.
Data Analysis

Surveys

Pre-service teachers’ responses to the surveys were classified into two categories of mathematical analysis and pedagogical analysis utilizing previously identified performance indicators (Manouchehri, 2011). These indicators are described below:

Mathematical analysis

- Articulate the basis for the mathematical decisions that children made.
- Identify the strengths and weaknesses of children’s ideas from a mathematical point of view.
- Identify the content trajectory and using it to address children’s conceptions.
- Reasoning how certain misconceptions or errors could have resulted and identifying their sources.
- Designing mathematically sound instructional strategies

Pedagogical analysis

- Identify why certain pedagogical moves were appropriate to pursue with children based on their analysis of student work and thinking.
- Successfully identify areas in which children would or would not be able to perform adequately.
- Offer a rationale for why certain pedagogical choices should be implemented both, individually or in class.
- Identify the advantages and disadvantages of use of instructional tools in instruction.
Based on these indicators the pre-service teachers’ responses to the surveys (pre and post) were ranked along a continuum from mathematically mature to mathematically naive and pedagogically mature to pedagogically naive. The responses were also assigned numerical scores. Since there were three levels of maturity (naive=0, developing=1, mature=2) for both the mathematical and pedagogical analysis, the maximum possible score was 4 (Mathematically and Pedagogically Mature) and the minimum possible score was 0 (Mathematically and Pedagogically Naive). However, while scoring, I realized that the responses to prompts for designing instructional strategies contained a wide range of responses. Some of the responses contained pedagogical approach which was specific to the mathematical content involved while other contained general pedagogical approaches, which could be effective when applied to the situation under study but there were no details as to how those approaches could be successfully applied to the situation under study. I deemed the content specific pedagogical approaches more mature than the general pedagogical approaches (See Appendix E, F and G for the criteria determining maturity of a response). Hence, I added an extra level for the pedagogical analysis of the responses to prompts pertaining to designing instructional strategies.

Thus, the maximum possible score for a response is 5 (Mathematically and Pedagogically Mature) and the minimum possible score is 0 (Mathematically and Pedagogically Naive). Initially, I had only 3 levels of pedagogical maturity (naive, developing and mature). However, during the grading of the responses, it became apparent that the pedagogically developing level contained a wide range of responses. Some of the responses contained a general pedagogical approach. Table 5 and Table 6 explain the ranking of the responses to the surveys (Somayajulu, 2012):
### Table 5. Scoring Rubric for Prompts Pertaining to Analysis of Student Work and Questioning

<table>
<thead>
<tr>
<th>Score</th>
<th>Mathematical Analysis</th>
<th>Pedagogical Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Mathematically Naive</td>
<td>Pedagogically Naive</td>
</tr>
<tr>
<td>1</td>
<td>Mathematically Developing</td>
<td>Pedagogically Developing</td>
</tr>
<tr>
<td>2</td>
<td>Mathematically Mature</td>
<td>Pedagogically Mature</td>
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</tbody>
</table>

### Table 6. Scoring Rubric for Prompts Pertaining to Designing Instructional Strategies

<table>
<thead>
<tr>
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<th>Mathematical Analysis</th>
<th>Pedagogical Analysis</th>
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<td>Pedagogically Naive</td>
</tr>
<tr>
<td>1</td>
<td>Mathematically Developing</td>
<td>Pedagogically Developing</td>
</tr>
<tr>
<td></td>
<td>(general pedagogy)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Mathematically Mature</td>
<td>Pedagogically Developing</td>
</tr>
<tr>
<td></td>
<td>(Content specific pedagogy)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>Pedagogically Mature</td>
</tr>
</tbody>
</table>

After ranking the responses on pre and post surveys were ranked, I compared the scores of the pre and the posttest surveys. Even though the number of questions were the same in each survey, the number of prompts in each question were different for each question. Hence in order to compare the two surveys, I calculated the percent score for each participant. I then calculated the total scores for prompts pertaining to analysis of student work, instructional strategies and questioning and based on them found the percent scores for each of those categories. Finally I also calculated percent scores for prompts pertaining to decompressing, trimming and bridging. After I had the percent scores for all the above mentioned categories, I then conducted a paired samples t-test.
(α=.05) to determine whether there was a significant difference between the pre and post scores.

**Interviews**

The aim of both the interviews was to gain a deeper understanding of the participants’ thinking and to also highlight the important factors that they considered when they analyze student work and thinking since this information may not have been reflected on the written surveys. Each interview was videotaped and transcribed. The Atlas.ti qualitative analysis software was utilized for coding the interview transcripts. While coding for the interviews, I was looking for references that the pre-service teachers made as they talked about teaching and learning of mathematics. Also, instead of coding line by line, I opted to code the entire responses to a question as a hermeneutic unit. This approach allowed me to keep all the responses in context. It also helped to keep from segmenting participants’ thoughts on teaching and learning which were not expressible in a single line.

The coding consisted of two levels. During the first level, I coded the responses into 8 categories, which are listed in Table 7. As I reviewed the analysis phase, I realized that these categories were too broad. Therefore, I divided the categories into narrower specific subcategories. After I finished coding, I counted the frequency of each code. After I had done this I divided the number of occurrences of a particular code by the total number of occurrences to get a percentage of occurrences for each code.
After calculating the percentage of occurrences for each code, I used the percentages to prepare a map to illustrate the sources that the pre-service teachers drew from while analyzing student work and thinking. The percentage scores also helped me identify the patterns in the data. The final step was to triangulate my interview data with the survey data. For this purpose, I compared the survey responses to the interview responses. In the event that the response to the survey was vague or unclear, I compared it with the responses to the interviews to clarify and assure that my conclusions were correct.

**Inter-grader Reliability**

In order to maintain inter-grader reliability, the surveys were graded by three different researchers and the scores were compared. In case of a unanimous decision, the score was accepted immediately. However, in case of a dispute, the scoring rubric was revisited and if two out of three graders agreed upon a score, then that score was assigned to that response. If there was no majority, all the three researchers explained their scoring and discussed the problem and reached a unanimous consensus.
In case of the surveys, each of the three graders coded the transcripts and compared the coded transcripts. In case of a mismatch in grading, the researchers explained their reasoning behind the assigned code. This was followed by a group discussion so to reach a unanimous conclusion.
Chapter 4: Analysis and Results

In this chapter I shall first present the findings from the surveys for the entire cohort of pre-service teachers followed by the individual case studies. After reporting about the case studies, I will then carry out a cross comparison of the three cases highlighting common and unique patterns of work and thinking as evidenced among the data.

Findings from the Surveys

Recall that the surveys were scored according to the scoring rubrics that were developed (see Tables 5 & 6). Once all the items were scored, I found a total percentage score for each of the participant. I carried out a paired samples t-test to compare the means from the pre- and the post- tests (α=.05).

Pretest and posttest data was gathered from eight pre-service teachers, with an overall pretest mean percent score of 45.125 (SD=10.26) and a overall posttest mean percent score of 31 (SD=8.635). A paired samples t-test was conducted to determine statistical significance, and indicated that the overall pretest and posttest percent scores were statistically different (t= 3.248, df= 7, p= .014) as presented in Table 8.

A paired-samples t-test (table 8) on the percent scores for mathematical and pedagogical analysis was also conducted. The pretest mean percent score for the mathematical analysis was 43.125 (SD=9.538) and the posttest mean percent score for the mathematical analysis was 27.5 (SD=10.967). The paired-samples t test indicated that
there was a significant difference ($t=4.358$, $df=7$, $p=.003$) between the pretest and posttest percent scores in the mathematical analysis area.

The pretest mean percent score for the pedagogical analysis was 47.5 (SD=10.542) and the posttest mean percent score for the pedagogical analysis was 35 (SD=9.071). The paired-samples t test indicated that there was a significant difference ($t=2.884$, $df=7$, $p=.024$) between the pretest and posttest percent scores in the area of pedagogical analysis as presented in Table 8.

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<th>$\bar{X}_{pre}$(SD)</th>
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<th>T</th>
<th>Df</th>
<th>P</th>
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<td>.014</td>
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<td>7</td>
<td>.003</td>
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<tr>
<td>Pedagogical</td>
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<td>35 (9.071)</td>
<td>2.884</td>
<td>7</td>
<td>.024</td>
</tr>
</tbody>
</table>

Table 8. Pre-test and Posttest Comparison of Means: Total Score

The mean percent scores for the posttest surveys were significantly less when compared to mean percent scores of the pretest surveys. There was also a decline in the mean percent scores for the mathematical and pedagogical analysis from the pretest to the posttest.

I also compared the mean percent scores for the prompts pertaining to “Analysis of student work”, “Instructional strategies”, and “Questions posed” by the pre-service teachers. Once again, a paired samples t-test analysis of the mean percent scores for the mathematical and pedagogical analysis for these prompts was conducted.
Analysis of Student Work

Pretest and posttest data gathered from eight pre-service teachers for the prompts pertaining to analysis of student work revealed that pretest mean percent score was 51 (SD=11.326) and a overall posttest mean percent score of 29.625 (SD=6.865). A paired samples t-test was conducted to determine statistical significance, and indicated that the overall pretest and posttest percent scores were statistically different (t= 4.814, df= 7, p= .002) as presented in Table 9.

I also conducted paired-samples t-tests on the percent scores for mathematical and pedagogical analysis for the prompts associated with analysis of student work. The pretest mean percent score for the mathematical analysis was 46.75 (SD=10.124) and the posttest mean percent score for the mathematical analysis was 24.75 (SD=7.066). The paired-samples t test, as presented in Table 9, indicated that there was a significant difference (t= 5.925, df= 7, p= .001) between the pretest and posttest percent scores for the mathematical analysis associated with analysis of student work.

The pretest mean percent score for the pedagogical analysis was 53.875 (SD=11.861) and the posttest mean percent score for the pedagogical analysis was 34.625 (SD=7.539). The paired-samples t test indicated that there was a significant difference (t= 3.873, df= 7, p= .006) between the pretest and posttest percent scores for the pedagogical analysis associated with the analysis of student work, as presented in Table 9.
Instructional Strategies

Pretest and posttest data was gathered from eight pre-service teachers, with an overall pretest mean percent score of 42 (SD=15.09) and a overall posttest mean percent score of 34.25 (SD=16.507) for the prompts pertaining to instructional strategies designed by the pre-service teacher. A paired samples t-test was conducted to determine statistical significance (Table 10), and indicated that the overall pretest and posttest percent scores were not statistically different (t= 1.221, df= 7, p= .262).

I also conducted paired-samples t-tests on the percent scores for mathematical and pedagogical analysis for the prompts associated with instructional strategies designed by the pre-service teacher. The pretest mean percent score for the mathematical analysis was 39.325 (SD=16.378) and the posttest mean percent score for the mathematical analysis was 30.625 (SD=19.834). The paired-samples t test indicated that there was no significant difference (t= 1.228, df= 7, p= .259) between the pretest and posttest percent scores for the mathematical analysis for the prompts associated with instructional strategies designed by the pre-service teachers, as presented in Table 10.

The pretest mean percent score for the pedagogical analysis was 44.125 (SD=15.179) and the posttest mean percent score for the pedagogical analysis was 36.875 (SD=15.037) for the prompts pertaining to the instructional strategies designed by the

<table>
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<th></th>
<th>$\bar{x}_{\text{pre}}$(SD)</th>
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<th>T</th>
<th>Df</th>
<th>P</th>
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<tr>
<td>Overall</td>
<td>51 (11.326)</td>
<td>29.625 (6.865)</td>
<td>4.814</td>
<td>7</td>
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<td>Mathematical</td>
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<td>24.75 (7.066)</td>
<td>5.925</td>
<td>7</td>
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<td>34.625 (7.539)</td>
<td>3.873</td>
<td>7</td>
<td>.006</td>
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</tbody>
</table>

Table 9. Pretest to Posttest Comparison of Means: Analysis of Student Work
pre-service teacher. The paired-samples t test indicated that there was no significant difference ($t=1.135, df=7, p=.294$) between the pretest and posttest percent scores for the pedagogical analysis for the prompts pertaining to the instructional strategies designed by the pre-service teachers, as presented in Table 10.

<table>
<thead>
<tr>
<th></th>
<th>$\bar{X}_{pre}$ (SD)</th>
<th>$\bar{X}_{post}$ (SD)</th>
<th>T</th>
<th>df</th>
<th>P</th>
</tr>
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<tbody>
<tr>
<td>Overall</td>
<td>42 (15.09)</td>
<td>34.25 (16.507)</td>
<td>1.221</td>
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<td>.262</td>
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<td>Mathematical</td>
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<td>.259</td>
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<td>Pedagogical</td>
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<td>36.875 (15.037)</td>
<td>1.135</td>
<td>7</td>
<td>.294</td>
</tr>
</tbody>
</table>

Table 10. Pretest to Posttest Comparison of Means: Instructional Strategies

*Questions:

Pretest and posttest data gathered from eight pre-service teachers on prompts regarding questions posed by the pre-service teacher had an overall pretest mean percent score of 39.625 (SD=13.804) and a overall posttest mean percent score of 31.5 (SD=20.135). A paired samples t-test was conducted to determine statistical significance, and indicated that the overall pretest and posttest percent scores were not statistically different ($t=.805, df=7, p=.447$), as presented in table 11.

I also conducted paired-samples t-tests on the percent scores for mathematical and pedagogical analysis for the prompts pertaining to questions posed by the pre-service teachers. The pretest mean percent score for the mathematical analysis was 41.75 (SD=17.774) and the posttest mean percent score for the mathematical analysis was 28.125 (SD=20.863). The paired-samples t test, as presented in Table 11, indicated that there was no significant difference ($t=1.357, df=7, p=.217$) between the pretest and
posttest percent scores for the mathematical analysis for the prompts pertaining to questions posed by the pre-service teachers.

The pretest mean percent score for the pedagogical analysis was 37.375 (SD=11.77) and the posttest mean percent score for the pedagogical analysis was 34.375 (SD=26.516) for the prompts corresponding to questions posed by the pre-service teachers. The paired-samples t test indicated that there was no significant difference (t= .248, df= 7, p= .811) between the pretest and posttest percent scores for the pedagogical analysis, as presented in Table 11.

<table>
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<th>$\bar{X}_{\text{pre}}$(SD)</th>
<th>$\bar{X}_{\text{post}}$(SD)</th>
<th>T</th>
<th>Df</th>
<th>P</th>
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<td>Overall</td>
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<td>Pedagogical</td>
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<td>34.375 (26.516)</td>
<td>.248</td>
<td>7</td>
<td>.811</td>
</tr>
</tbody>
</table>

Table 11. Pretest to Posttest Comparison of Means: Questions

Thus, there was a significant decrease in the mean percent scores for the prompts corresponding to the analysis of student work, including the mean percent scores for mathematical and pedagogical analysis. However there was no significant difference in the mean percent scores for the prompts relating to instructional strategies and questions posed by the pre-service teachers.

The items on the surveys were guided by the KAT framework. The intent of the surveys was to elicit the knowledge bases from which the teachers drew in order to analyse students’ mathematical work and thinking via the actions of decompressing, trimming, and bridging. I compared pre- and post- mean percent scores for the prompts
eliciting decompressing, trimming and bridging to see if there was an effect on the pre-service teachers’ abilities to perform these actions.

**Decompressing**

The data for the prompts pertaining to decompressing revealed a pretest mean score of 53.875 (SD=10.19) and a posttest mean score of 29.75 (SD=6.584). A dependent t-test conducted to determine the statistical significance indicated that the pretest and posttest scores for the prompts pertaining to decompressing were statistically different (t=6.049, df= 7, p= .001), as presented in Table 12.

I also conducted paired-samples t-tests on the percent scores for mathematical and pedagogical analysis for the prompts pertaining decompressing. The pretest mean percent score for the mathematical analysis was 50 (SD=9.812) and the posttest mean percent score for the mathematical analysis was 26.125 (SD=7.809). The paired-samples t test, as presented in Table 12, indicated that there was a significant difference (t= 6.867, df= 7, p< .001) between the pretest and posttest percent scores for the mathematical analysis for the prompts pertaining to decompressing.

The pretest mean percent score for the pedagogical analysis was 57.875 (SD=11.382) and the posttest mean percent score for the pedagogical analysis was 34.25 (SD=8.345) for the prompts corresponding to decompressing. The paired-samples t test indicated that there was a significant difference (t= 4.812, df= 7, p= .002) between the pretest and posttest percent scores for the pedagogical analysis for prompts pertaining to decompressing, as presented in Table 12.
The data for the prompts pertaining to trimming revealed a pretest mean score of 37.125 (SD=16.685) and a post test mean score of 33.75 (SD=10.25). A dependent t-test was conducted to determine the statistical significance, and indicated that the overall pretest and posttest scores were not statistically different (t= .532, df= 7, p= .611), as presented in Table 13.

I conducted paired-samples t-tests on the percent scores for mathematical and pedagogical analysis for the prompts corresponding to trimming. The pretest mean percent score for the mathematical analysis was 35.125 (SD=16.4) and the posttest mean percent score for the mathematical analysis was 30.75 (SD=12.78). The paired-samples t test, as presented in Table 13, indicated that there was no significant difference (t= .756, df= 7, p= .475) between the pretest and posttest percent scores for the mathematical analysis for the prompts pertaining to trimming.

The pretest mean percent score for the pedagogical analysis was 38.625 (SD=17.344) and the posttest mean percent score for the pedagogical analysis was 35.75 (SD=11.877) for the prompts corresponding to trimming. The paired-samples t test indicated that there was no significant difference (t= .387, df= 7, p= .710) between the pretest and posttest percent scores for the pedagogical analysis, as presented in Table 13.
The data for the prompts pertaining to bridging revealed a pretest mean score of 44 (SD=11.071) and a post test mean score of 33.875 (SD=23.853). A dependent t-test conducted to determine the statistical significance indicated that the overall pretest and posttest scores were not statistically different (t= 1.292, df= 7, p= .237), as presented in Table 14.

I also conducted paired-samples t-tests on the percent scores for mathematical and pedagogical analysis for the prompts pertaining to bridging. The pretest mean percent score for the mathematical analysis was 43.625 (SD=12.176) and the posttest mean percent score for the mathematical analysis was 30 (SD=27.625). The paired-samples t test, as presented in Table 14, indicated that there was no significant difference (t= 1.360, df= 7, p= .216) between the pretest and posttest percent scores for the mathematical analysis for the prompts pertaining to bridging.

The pretest mean percent score for the pedagogical analysis was 44.25 (SD=11.622) and the posttest mean percent score for the pedagogical analysis was 36.75 (SD=22.301) for the prompts corresponding to bridging. The paired-samples t test indicated that there was no significant difference (t= 1.114, df= 7, p= .302) between the pretest and posttest percent scores for the pedagogical analysis, as presented in Table 14.
There was a significant positive correlation between the mathematical analysis scores and the pedagogical analysis scores for both the pre- and the post-tests. The scores for the mathematical analysis and pedagogical analysis for the pretest surveys were strongly correlated, $r(8) = .918$. Similarly the scores for the mathematical analysis and pedagogical analysis for the posttest surveys were also strongly correlated, $r(8) = .801$.

Thus we see that there was a significant decrease in the mathematical analysis scores for the overall means as well as the means for prompts pertaining to analysis of student work and thinking as well as for the prompts pertaining to decompressing. One explanation for the decrease in scores could be that the pre-service teachers were not able to solve the mathematics involved on the posttest. Another reason could be that the attention to the mathematical content of the student work decreased in the posttest. Also as you can see from the above correlations that the pedagogical analysis scores for both the pretest and the posttest surveys are highly correlated with the mathematical analysis scores. Hence, if the pedagogical approach was not content specific, then the scores dropped.
Self-Efficacy

From the table above we see that the pre-service teachers’ self efficacy scores did not change from pretest to posttest. A paired samples t-test revealed that the mean score of 5.813 (SD=.913) for the pretest was not significantly different from the mean score of 6.109 (SD=.75) (t= .986, df= 7, p= .357). Similarly, the mean scores for the efficacy for instructional strategies were also not significantly different (t= 1.272, df= 7, p= .44), as presented in Table 15.

The pretest mean score for efficacy for classroom management was 5.531 (SD=1.348) and the posttest mean score was 6.234 (SD=.786). The paired-samples t test indicated that there was no significant difference (t= 1.887, df= 7, p= .101) between the pretest and posttest percent scores for the efficacy for classroom management, as presented in Table 15.

The pretest mean score for efficacy for student engagement was 6.172 (SD=1.092) and the posttest mean score was 5.953 (SD=.914). The paired-samples t test indicated that there was no significant difference (t= .549, df= 7, p= .6) between the pretest and posttest percent scores for the efficacy for student engagement, as presented in Table 15.
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<th>$\bar{X}_{pre}$ (SD)</th>
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<th>Df</th>
<th>P</th>
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<td>6.234 (.786)</td>
<td>1.887</td>
<td>7</td>
<td>.101</td>
</tr>
<tr>
<td>Student Engagement</td>
<td>6.172 (1.092)</td>
<td>5.953 (.914)</td>
<td>.549</td>
<td>7</td>
<td>.6</td>
</tr>
</tbody>
</table>

Table 15. Pretest to Posttest Comparison of Means: Self-Efficacy

Thus we see that the pre-service teachers’ ability to analyze student work and to decompress knowledge had decreased. Thus the coursework had no effect on their ability to analyze work. In fact, it appears that the coursework had a negative influence on their ability to analyze student work. We also see that there was no significant difference in the scores for the pre-service teachers’ self-efficacy. This validates the results of the surveys since there was also no change in the pre-service teachers’ scores on the surveys.

However, it is not a fair judgment since we have not taken a look at the items on the surveys nor have we looked at the effect of the coursework on the pre-service teachers’ ability to analyze student work and thinking. Hence, even with the global view of the cohort provided above, it is still unclear what resources these pre-service teachers drew from in their analysis of student work. To clarify that point and also to provide a more coherent view of the thinking, three in-depth case studies were carried out. The case studies of Cersei, Bran, and Nedd will provide a glimpse into the teachers’ analysis of student work and the resources that they draw from as well as an overall view of how the program seemingly affected them.
Cersei was one of the two pre-service teachers with a background in Engineering among the cohort studied in this project. She had worked at a car manufacturing company.

When the plant she was working in closed, Cersei decided to pursue teaching because she thought that teaching at a high school was the next best choice. Cersei preferred teaching in a high school rather than a middle school. Prior to joining the Master of Education program, she had enrolled in an alternative licensure program at a different university where she was only one course shy of obtaining her certification. Despite this she felt it necessary to go through the M.Ed. program in order to be an effective teacher. Cersei had completed all the required coursework in mathematics and she had also completed a few STEM education courses while enrolled in her previous certification program.
My first introduction to Cersei was during the number sense and data analysis methods course that I taught in the summer quarter of 2011. I also observed the pre-service teachers in their geometry methods course in the autumn quarter of 2011. Cersei was a quiet student and did not participate in discussions very often. Classroom observations suggested that Cersei was comfortable with the mathematics involved at the high school level, as she easily solved any of the mathematical problems that were presented to the group. Cersei had also showed a great deal of enthusiasm for learning course materials presented. The summer methods course was a 5 week long course, during which the pre-service teachers met twice a week for 3 hours each. The course was designed so that all the activities/assignments in that course were centered on the task of analyzing student work and thinking. During the first two classes, pre-service teachers were assigned to read chapters from two books ‘What’s math got to do with it?’ (Boaler, 2008) and ‘Connecting mathematical ideas’ (Boaler & Humphreys, 2005). For the rest of the course, students were presented with written or visual episodes of children engaged in performing mathematical tasks. These tasks were followed by a whole class discussion involving assessment of the student’s mathematical work, including identifying the sources of misconceptions or alternate conceptions as well as dealing with those misconceptions and formulating instructional strategies to address them. The pre-service teachers also took a class on learning theories and a class on assessments. However, Cersei never made references to or used any of those theories during class discussions or when analyzing student work. Cersei had also volunteered an hour a week at a tutoring center during the summer quarter.
In the sections to follow, I will first offer an analysis of Cersei’s survey scores followed by background information on Cersei, drawing from her responses to the surveys and the interviews. Lastly I will answer the research questions based upon data from her case.

Analyses of Cersei’s survey data

Table 17 summarizes Cersei’s percent scores on pre and post surveys in terms of her mathematical and pedagogical analysis of student work. Each response for every category, except instruction, was scored on a scale of 0-2 for a maximum of 2 points for mathematical and 2 points for pedagogical analysis. For prompts relating to instructional strategy, maximum possible score for mathematical analysis was 2 while maximum possible score for pedagogical analysis was 3. Hence for prompts relating to instructional strategies, the maximum possible score was 5. For all other prompts the maximum possible score was 4. Prompts on the pre and post surveys were categorized as analysis of work, instruction, and questions, based on whether the participant was analyzing student work, planning instructional strategies, or posing questions. Since the surveys were designed to elicit pre-service teachers’ knowledge via the processes of decompressing, trimming and bridging, the table also includes Cersei’s percent scores for prompts pertaining to these three processes.
As seen in the table, there was an overall decrease in Cersei’s scores both in the areas of mathematical (pre: 52; post: 34) and pedagogical (pre: 59; post: 29) analysis. Although there was an increase in her total percent scores for mathematical and pedagogical analysis, Cersei’s percent scores lowered on prompts pertaining to analysis of student work and decompressing. For prompts pertaining to analysis of student work, Cersei’s percent scores for mathematical analysis decreased from 42 to 14, while her percent scores for pedagogical analysis decreased from 50 to 31. Similarly her percent

<table>
<thead>
<tr>
<th></th>
<th>Mathematical Analysis</th>
<th>Pedagogical Analysis</th>
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<tbody>
<tr>
<td></td>
<td>Pretest (%)</td>
<td>Posttest (%)</td>
</tr>
<tr>
<td>Analysis of work</td>
<td>42</td>
<td>14</td>
</tr>
<tr>
<td>Instruction</td>
<td>67</td>
<td>31</td>
</tr>
<tr>
<td>Questioning</td>
<td>33</td>
<td>25</td>
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<tr>
<td>Decompressing</td>
<td>46</td>
<td>13</td>
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<td>Trimming</td>
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<td>33</td>
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<tr>
<td>Bridging</td>
<td>56</td>
<td>41</td>
</tr>
<tr>
<td>Total Score</td>
<td>52</td>
<td>34</td>
</tr>
</tbody>
</table>

Table 17. Scoring of Cersei’s Responses to the Surveys
scores for mathematical analysis decreased from 46 to 13 and for pedagogical analysis decreased from 56 to 33 on prompts pertaining to the process of decompressing.

Cersei’s percent score on prompts pertaining to instruction decreased in both domains of mathematical (pre: 67; post: 31) and pedagogical analysis (pre: 69; post: 36). Her scores for mathematical (pre: 33; post: 25) and pedagogical analysis (pre: 33; post: 25) for prompts pertaining to questioning also decreased. However, there were only 3 prompts on the pretest and 2 prompts on the posttest that addressed the pre-service teachers’ ability to pose questions.

Finally, the percent scores for mathematical and pedagogical analysis for the prompts eliciting the processes of trimming and bridging also increased. For trimming, the scores decreased from 64 to 33 for the mathematical analysis, while for the pedagogical analysis, they decreased from 65 to 35. The scores for mathematical analysis decreased from 56 to 41 and the scores for the pedagogical analysis decreased from 62 to 44 for prompts pertaining to the process of bridging.

The following table represents the content of each of the questions from the pretest and posttest surveys.
<table>
<thead>
<tr>
<th>Content</th>
<th>Pretest</th>
<th>Posttest</th>
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<tbody>
<tr>
<td>Measurement</td>
<td>Question 12</td>
<td>Question 8</td>
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<tr>
<td>• Area</td>
<td>Question 15</td>
<td>Question 18</td>
</tr>
<tr>
<td>• Volume</td>
<td>Question 19</td>
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<tr>
<td>Similarity</td>
<td>Question 11</td>
<td>Question 12</td>
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<tr>
<td>Definitions</td>
<td>Question 14</td>
<td>Question 9</td>
</tr>
<tr>
<td>Reasoning and Proof</td>
<td>Question 20</td>
<td>Question 8</td>
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<tr>
<td></td>
<td>Question 18</td>
<td>Question 10</td>
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<td>Question 14</td>
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<td>Question 16</td>
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<td>Triangles</td>
<td>Question 21</td>
<td>Question 11</td>
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<td></td>
<td>Question 22</td>
<td>Question 12</td>
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<tr>
<td>Polygons</td>
<td>Question 13</td>
<td>Question 9</td>
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<td></td>
<td>Question 16</td>
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<td></td>
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<td>Question 14</td>
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<td>Constructions</td>
<td>Question 23</td>
<td>Question 19</td>
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<tr>
<td></td>
<td>Question 22</td>
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<tr>
<td>3-D Geometry</td>
<td>Question 12</td>
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<td></td>
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<td></td>
<td>Question 22</td>
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<td>Transformations</td>
<td>Question 20</td>
<td>Question 12</td>
</tr>
<tr>
<td></td>
<td>Question 16</td>
<td>Question 17</td>
</tr>
</tbody>
</table>

Table 18. Content on Surveys by Question

Cersei’s scores in all categories dropped for questions in the area of measurement, polygons, constructions and 3-D geometry. Cersei’s scores on prompts pertaining to analysis of student work and instructional strategies were higher particularly in the areas of reasoning and proof. Her scores for prompts relating to instructional strategies increased in the area of transformations. Cersei’s scores for instructional strategies
decreased in the areas of reasoning and proof, circles, similarity, polygons and transformations, constructions, and measurement and triangles. There was not much difference in her scores in the areas of triangle and similarity. Also note that Cersei scored a 0 on prompts pertaining to the use of learning progressions on the posttest surveys. Unfortunately the prompts for questioning on the pre and post- surveys were in different areas and so content comparison between the pre- and posttest surveys couldn’t be done.

Cersei’s mathematical analysis scores were positively correlated with her pedagogical analysis scores for both the pre- and posttests. The scores for the mathematical and pedagogical analysis for the pretest surveys were strongly correlated, \( r(28)= .848 \ p< .01 \). Similarly the correlation between the scores for the mathematical analysis and pedagogical analysis for the posttest surveys was moderate to strong, \( r(33)= .487, p< .01 \). This offers an explanation for the decrease in Cersei’s pedagogical analysis scores.

**Pretest Survey and Interview**

Although the pretest survey was administered on the first day of classes in the third week of June, 2011, obtaining the pretest interview was carried out three months later due to the pending IRB approval. In the following section I will present the factors that were most influential on Cersei’s pedagogical decision making. As seen in Figure 7, the most influential factors affecting her decision making in her pretest surveys and interviews were her past experiences, views on mathematics, her beliefs about teaching and learning and their influences on her teaching.
Beliefs about teaching and learning

Cersei’s orientation towards teaching was influenced by four major factors namely her beliefs in teaching and learning, her views about the teacher education program, her views about mathematics and her experiences as a teacher and as a student of mathematics. Note that in this section, I will be presenting Cersei’s claimed beliefs and not her enacted beliefs which were highlighted in her analysis of student work.

When asked if she saw parallels between the professions of teaching and engineering, Cersei said that she saw similarities between teaching and her previous job as an industrial engineer. I asked her, in her opinion, how was being an engineer different from being a teacher. To which she replied (Pretest Interview transcript):

Cersei: Good question and I think the one thing too that’s kind of, you know, in the back of my mind led me this direction all along perhaps, but without me realizing it is that one thing that I really liked with industrial engineering was training other people. I was always the person that trained the new industrial
engineers coming into the department. So I guess I’ve kind of been teaching all along without really recognizing it but really liked in that whole experience just to being kind of a mentor and helping so many get started into the department. (p. 5-6, lines 183-188)

Cersei believed that just knowing the content was not sufficient to be a good teacher. She was of the opinion that “well if I am gonna be teaching, I need to know more than just how to do it myself. I need to be able to explain the reasoning behind it.” (Pretest interview: p. 1, lines 26-27). However, she did not believe that the mathematics classes she took in college were helpful to her as a teacher (Pretest Interview transcript).

Cersei: Umm, I’ve taken umm, only up through the… what I guess here is called the algebra 1 and algebra 2. Those were the most recent classes that I’ve… I took and quite honestly I mean I just kinda fulfilled the check mark that I’m done with those classes but, umm, I mean personally I think I probably could have benefitted some more from different classes. I don’t feel that those classes really helped me with anything as far as being a teacher.” (p. 1-2, lines 32-36)

In contrast, she thought that college geometry was the only beneficial class because after taking it, she empathized with students who just couldn’t “get math” (Pretest survey: p. 1).

Cersei suggested that lecturing was not the best way of teaching. She acknowledged that student interaction and student participation were essential to foster good learning. She was a strong proponent of “working with the student” and students working in groups. This was also evident when she described her ideal classroom. Her
classroom set up was supportive of her beliefs about how instruction should be carried out (Pretest Interview transcript):

Cersei: To me, I feel like it’s really… my personal opinion is it feels kind of old when you walk into a classroom and all desks are facing the front of the room and they are all separated from one another and its pretty apparent that on a regular basis they are not working together. And so I think to me the ideal classroom is you know tables or at least groups of desks and my mentor’s classroom has tables of 2 that are pushed together sometimes to make groups of four or six and… umm, there are no assigned seats which, uh… he does assign seats if there is a need to and in one of his classes there was, umm, so you know I like that approach that they have the opportunity of getting not to have assigned seats but certainly reasonably or if you need to you can reserve that right to assign the seats. So… but I like the group work and I like them being able to sit next to each other. (p.3, lines 84-92)

She conveyed this same belief when she was asked about her views about teaching of geometry. While replying to a question on the pretest survey about how geometry should be taught, she claimed that geometry should be discovered by students rather than presented as direct teaching. This belief was also evident in the interview when she referenced the way her mentor teacher conducted his geometry instruction (Pretest Interview transcript):

Cersei: So it’s wonderful that this quarter my mentor teacher teaches two advanced geometry classes. So I love that because now I get to see… and I love the way that he teaches it, the way that he approaches it. He never gives them the
theorems. In fact, they don’t even use the word theorem. He uses the word conjecture and they develop their own conjectures (p.2, lines 44-47).

Cersei also viewed group work beneficial to student learning. In justifying this belief she highlighted the vital role of collaboration on advancement of knowledge and understanding (Pretest Interview transcript):

Cersei: Then it’s interesting to know, you know somebody didn’t get the right answer… the same answer then it’s kinda like a little internal debate going on between the group to see how did you get your answer and kinda helping each other to figure out where you were wrong along the way and then once the group is comfortable with their answer and they all agree upon what the answer should be then I like the share idea of sharing out with the rest of the class. First of all now does everybody have the same answer and you know if there is a disagreement of answers and that’s really interesting because I mean a group of students became convinced among themselves about this answer that’s wrong and that just makes the dynamics all the more interesting if everybody… if all the groups have the same answer (p. 4, lines 117-125).

Teacher education program

Cersei was enrolled in an alternate certification program at a midwestern university before joining the M.Ed. program. Cersei also acknowledged that the program was very effective and had a positive influence on her. She referred specific course experiences particularly those which confirmed her own views regarding effective teaching and learning (Pretest Interview transcript):
Cersei: And I personally see that there is some value in understanding other ways to solve the problem, umm, helps you kind of understand your way even better even more. Umm, one way to encourage that I think is just to learn from the teacher with the border problem (p.8, lines 264-266).

Cersei had pre-conceived notions about what she needed for her to be an effective teacher. As we saw earlier, she did not see connections between the mathematics content courses and the mathematics that she would be teaching. However, she believed that the coursework in the education area was helpful because they provided her with teaching strategies (Pretest Interview transcript):

Cersei: Specifically I think the classes that helped me the most were in the education area because now it’s more about learning how to be a teacher rather than just knowing the content and you know I think that that’s umm, really what sets apart, umm, you know a good teacher and bad teacher. . . . So, umm, I think it’s really the education classes that I liked the most more so than the content just because that’s where I think that I personally, umm, can use some more… yeah (p. 9, lines 317-323).

The effect of the mentor teacher on Cersei was an influential factor especially in the area of classroom management and her approach to instruction, as the mentor teacher provided imageries of practice that matched her beliefs. At the time of the interview, Cersei had just started observing her mentor teacher for about a month, however, she referred to her mentor teacher in talking about her ideal classroom, and his approach to teaching of geometry. While describing the classes that she had taken Cersei mentioned
that she did not like the way she was taught geometry. Instead she was really fond of the way her mentor teacher approached geometry (Pretest Interview transcript):

So I absolutely love this approach and he has incredible results with his students. So he’s definitely a great mentor. Perfect for me being in the area of geometry because I am actually looking through the text book and the way he teaches things with the aha moment… that oh yeah I remember memorizing that, I remember doing that but now it’s obvious what it means (p. 2, lines 44-51).

*View on mathematics*

Cersei’s view on mathematics, especially geometry, had been the most influential factor on her decision making process. This was evident both in her pretest survey and interview. Cersei viewed geometry to be very visual and so her instructional approach involved visual representations of the content. Figure 7 shows Cersei’s response to an item that was posed during the pretest survey (p. 6):
Cersei chose not to use the formulaic approach to this problem but rather use a visual representation of the cube. When asked about which two topics (out of 3) she felt most comfortable teaching, she chose geometric constructions and centers of triangles over 3-D geometry as those were easier to visualize for her.

Cersei took a similar approach in her interview, in which she preferred to give lots of visual examples to students in order to help them understand similarity and congruence (Pretest Interview transcript):

C: So I would like to include that in the list of examples just because, umm, I think it’s a good distinction. Not that at this point I think that they should
understand the idea of a congruence and similar but just introduce this… the concept that these two are similar and these are similar (points to pairs drawn) but I liked the definition number 1 because it’s really defining by shape which they can realize whether they’re… (p. 28, lines 945-949).

Experience:

Cersei’s view about the teaching and learning of geometry was influenced by her experience with the subject both as a student and a teacher. Cersei’s experiences with geometry as a student informed her view about how geometry should not be taught. Her experiences with geometry were prominently negative as she was forced to memorize geometrical facts. Although she did not offer explicit description of how she would teach a geometry class, capitalized on the importance of helping children make sense of the content rather than treating mathematics as a puzzle. Cersei argued (Pretest Interview transcript):

…and you know I really learned geometry from the… here are the theorems, memorize these and be able to use them and I’d sit there going ok, I was supposed to prove this, given this. I didn’t have any idea how to get from here to here (beginning to end), I didn’t know what these theorems mean and I’d sit there trying to piece them together and it was more just a matter of trying to fit a puzzle rather than actually thinking through the process. So I guess I’ve kinda seen how I don’t think geometry should be taught (p.2, lines 39-44).

In summary we see that Cersei believed students learned geometry by first discovering it themselves, followed by collaboration and meaning making and finally sharing with others. Cersei viewed teaching as any other profession in terms of the skill
set required. She viewed the teacher as someone who was not only competent in mathematics but also who could explain the mathematics. Despite this view of a teacher, she did not believe that teachers needed to take higher level mathematics courses to be effective teachers as she had pre-conceived notions of what she needed to be an effective teacher. Cersei was a proponent of student interaction and group work in the classroom.

Posttest Survey and Interview

The posttest survey was administered to Cersei in the second week of December at the end of the autumn quarter of 2011. Cersei was interviewed again in January almost a 45 days after the administration of the survey. The interview lasted around 90 minutes.

Figure 8 below describes the factors that influenced Cersei’s pedagogical decision making in her posttest surveys and interviews. Two main factors influenced Cersei’s decision making during the posttest surveys and interviews, her beliefs about teaching and learning and her mentor teacher.

Figure 8. Factors Influencing Cersei’s Pedagogical Decision Making in the Posttest Items
Mentor teacher

Just as we saw in the first set of interviews, a major influence on Cersei’s pedagogical decision making was her mentor teacher and her practices. Cersei referred to her mentor teacher’s work and ideas repeatedly but most prominently when she talked about setting goals for her students. While describing how she planned her lesson, Cersei said that her main focus was on the goals that she was setting for her students, which was aided by the task set forth for her by her mentor teacher (Posttest Interview transcript):

Cersei: Umm, I’m happy to say this quarter that I do like the lot of flexibility from my mentor teacher and her only objective… this is great. Very exciting. Her only objective for me in the six weeks that I am teaching is that I teach them how to solve quadratic equations (p.4, lines 118-120).

Cersei: … So I’m actually now defining goals for solving quadratic equations. What’s my goal for the unit? What’s my goal for each segment? And what’s my… what’s my goal for each lesson?

Researcher: Okay.

Cersei: So I do like that approach and that structure to… umm everything kinda tying back into that ultimate unit goal (p. 4-5, lines 131-136).

Observing her mentor teacher, Cersei now became more relaxed about the curriculum in the classroom. Cersei expressed that she now felt more comfortable teaching because she recognized she did not have to follow the textbook as she had previously assumed. Observing her mentor teacher’s work without using the prescribed textbook made her realize that it was acceptable to teach the students without a textbook (Posttest Interview transcript):
Cersei: I’m feeling a little bit more comfortable now that I don’t have to go by the exact textbook given and now I’ve seen both my mentor… mentor teachers from the fall and this winter. Neither of them use the assigned [textbooks](p.6, lines180-182).

Cersei’s opinion of the M. Ed. Program was mixed. On the one hand she felt it was helpful for her to think about teaching and assessing student work. On the other hand, she expressed concern that the content was too theoretical to have practical merit. She believed translating these theories into pragmatic instructional guides was less than feasible. For instance, when talking about writing lesson plans, she felt that they were more of a hindrance to her, rather than helpful. She wrote lesson plans because she was expected to write them as a course requirement and not because she thought they were useful or necessary to her work (Posttest Interview transcript).

Cersei: So a lot of that type of thing has to be at the end of the, umm, lesson plan. I’m almost just doing it now still… I’m really just doing it now just as ok this is part of the paper work I have to fill out. Okay, what did I do in here? So it’s almost an afterthought, me tracking how I … how I incorporated, umm, which I understand the purpose because you still need to show that you are considering all your students’ needs. Umm, like this extension…

Researcher: Right.

Cersei: (looking at her lesson plan) I don’t plan on extending it. So I don’t really, you know, when I’m typing something in here I’m just thinking whatever. How do I extend this if I had to? You know? The time or the students that I thought
would benefit from that but quite honestly it’s… you know a lot of this stuff is not something… that’s I would change if I were required to (p. 4, lines 103-113)

**Beliefs about teaching and learning**

Cersei’s claimed beliefs about how mathematics should be taught as expressed in the post-interview were consistent with those she had articulated at the beginning of the program. She opposed direct instruction and felt much of the mathematics taught in classroom is telling students what to do. Rather, the approach to teaching mathematics should be having students discover it (Posttest Interview transcript):

Cersei: Umm, definitely an open ended, you know, I … I don’t like the lecture style and that’s how I was taught was with lecture but, umm, you know I don’t think that it works for most students and I really don’t like the idea in math class of telling students how to do a process… how to do a procedure (p. 5, lines 151-153).

When asked if her views of teaching and learning of mathematics had changed over time, Cersei expressed a more mature view as she realized assessment of student learning, the nature and substance of what students learned in class, was a complex process and not at all straightforward. Additionally, she admitted that student engagement did not always ensure student understanding as she had previously assumed (Posttest Interview transcript).

Cersei: Umm, now, umm … now I’m paying more attention to the looks on their faces, umm, the reaction to things, whether they, umm, not so much whether they are engaged or not because I think it’s more than that. It’s more than them paying
attention and being engaged in what I am saying but them interacting (p. 1, lines 27-30).

Following from her previous interview, Cersei relied heavily on her mentor teacher in terms of planning for instruction. Her beliefs about the learning and teaching of mathematics had not changed since the last interview. She still mentioned that traditional classroom instruction is not the best way to teach mathematics and that mathematics should be discovered by the students.

*Shifts in Focus from Pre to Post*

There were no noticeable shifts in either Cersei’s claimed beliefs on teaching and learning or in her focus on the mentor teacher. However, notice that there were no references made to her work or teaching experiences in the posttest interviews. She also avoided the mathematical aspect of children’s work. There was an increased focus on assessment of student work in terms of the complexity of the task itself.

In the following sections I will present an analysis of the data from the surveys and the interviews and using those findings to answer my research questions for the case of Cersei.

*Findings*

*Research Question 1: What factors does Cersei consider when judging pre-service teachers’ mathematical work and thinking?*

I built a map to indicate the factors that Cersei draws from while performing the task of analysis of student work and thinking, to get a better idea about Cersei’s approach to analysis of student work and thinking. In order to build the map, I first coded the entire transcript. After coding, I examined the questions specifically about the analysis of
student work. I noted each of the codes listed under analysis of student work and kept track of the frequency of each. This was done for both the pre- and posttest transcripts. One caution when interpreting the data below is that one reason for the reference to mathematical content is high because the pre-service teachers were asked content specific questions and were asked to analyze students’ mathematical work.

From Figure 9 we see that Cersei referenced the content approximately 47% of the time, while she referenced teaching about 20% of the time. She also referenced students approximately 18% of the time. Of the 47% references made to the content, about 35.5% of references were to mathematics involved. While referring to teaching, 30% of the references made were regarding Cersei’s goals of instruction for that particular topic. Other references made were to the M. Ed. program (7%), experience (3%), self (3%) and learning (2%).
Figure 9. Mapping of Cersei’s Sources for Analyzing Student Work and Thinking for Pretest

In case of Cersei, as evident from her interviews, her views on mathematics and her beliefs about teaching and learning of geometry played a significant role when analyzing student work and thinking. Figure 10 shows Cersei’s response to an item on the pretest survey:
Cersei’s analysis was that the student was using the wrong definition of side and hence his answer was incorrect. During the pretest interview, I asked Cersei what she meant by the student had a misconception about the definition of a side to which she replied (Pretest Transcript):

Cersei: I think this misconception is that, umm, a side doesn’t need to have an angle.

Researcher: Ok.

Cersei: Like, umm, for example here it could just be a straight line and just because there is a point there means that it’s a side. So they are actually thinking
that these two (points out to the two adjacent sides formed by three collinear points in the diagram) are separate different sides. Like they probably… they seem to recognize the eight points and eight of what they think are sides but don’t necessarily understand that this is ju… really just one side. You know they put a point on it (p. 14, line 514).

So Cersei’s view of octagons was that no three points on the octagon could be collinear. When the interviewer told Cersei that mathematically it was an octagon, she replied “I personally don’t think so (pointing to the survey) just because I don’t think it could be a straight line there” (Pretest Transcript: p. 16, lines 564-565). Thus, Cersei’s conceptualization of an octagon was so strong that she rejected the claim made by the interviewers that the figure given was, mathematically, an octagon.

The figure below (Figure 11) shows the references that Cersei made for the posttest interview questions addressing the analysis of student work and thinking. Cersei’s total references to content were lower than in her pretest interview. She referenced content approximately 39% of the time. Her references to students increased to 27% while her references to teaching decreased to 13%. The biggest increase was seen in her reference to learning which rose from 2% in the pretest to 13% in the posttest interview. Other categories that were referenced were the M. Ed. Program (6%), experience and self (both 2% each).
Figure 11. Mapping of Cersei’s Sources for Analyzing Student Work and Thinking for Posttest

Cersei’s posttest interviews showed the influence of her beliefs about the teaching and learning of mathematics. Cersei believed that high school students struggle with geometric reasoning and proof. This was supported by her response to an item on the posttest survey (Figure 12):
1. Consider the following dialogue between Amy and Peter as they try to reason why a line segment has only one midpoint:

Amy: (Draws a line segment on GSP. Using the construct menu she marks the midpoint of the segment and begins changing the length of the segment by moving one of the endpoints.) See? The midpoint moves with the segment.

There is only one midpoint (points at the screen). No matter how long or how short the midpoint moves with it. Here, see if I try to make another midpoint it gives us the same point (goes to the construct menu and selects construct midpoint again). The points coincide.

Peter: I look at it this way... I say if we have a triangle and we construct the median from the vertex then we can have only one median. It means we have only one midpoint. See, let me show you (He draws a triangle and constructs a median. Pointing at the image he explains). See, a median is a segment that connects that point (points at the vertex) to the midpoint of this side (points to the opposite side). We have only one median. So, there is only one midpoint.

Amy nods in agreement. (Manouchehri, in progress)

a. What is your assessment of Amy’s and Peter’s arguments? Do you find these arguments common among high school students? What tools would you use to assess the level of the students’ thinking?

b. What issues would the teacher need to address with children regarding their arguments?

c. What could be contributing to the way that the children argue about the uniqueness of the midpoint?

d. If you were the teacher of these children how would you proceed with your lesson? Explain your reasoning.

   a) Both provide good arguments. I think these would actually be advanced for high school. I would expect students to refer to the definition of a midpoint.

b) Amy: How do you know, if you constructed midpoints all day that the points would always coincide?

Peter: Isn’t this just because of the definition of a median? How does this prove the median will always intersect the line segment at the same place?

c) They are more visual and comfortable with visualization and conjectures based on their experiences. They are ready to move to proving their conjecture. 

d) Begin to explore proofs formally. Also, this really just depends on the lesson objective.
This belief that reasoning and proof is difficult for high school students was also observed in her response to an item where I asked the pre-service teachers to list 3 or 4 topics in geometry that they thought would be hard for students. To this Cersei replied (Posttest Survey):

Cersei: Proofs! Students often believe repeated examples prove something is true. Also, proofs are taught to students beginning with concepts they cannot relate to (p. 11).

Thus in the responses above, we see that Cersei’s beliefs about the teaching and learning of mathematics, especially geometry in this case, have an immense effect on how she analyzes student work. When asked to pick the 3 out of 4 topics in geometry that would be hard for students, Cersei’s other pick was Surface Area and Volume because according to her, these topics were taught by making students memorize the formulae.

Both, the pretest and posttest data, Cersei’s orientation to student work is influenced by her beliefs. Her beliefs are so strong that she even rejects the interviewers’ opinion that the octagon pictured in the pretest (Figure 10) was mathematically correct. She refused to change her analysis of student work or her approach even though she acknowledged the view of the interviewers.

Research Question 2: What are the effects of two quarters of coursework on Cersei’s assessment of students’ mathematical work and thinking of geometry and measurement?

As seen above, Cersei’s mathematical analysis scores for items pertaining to the analysis of student work dropped from 42% to 14%, while her pedagogical analysis scores decreased from 50% to 31%. In this section I attempt to explain reasons why she scored less on her posttest surveys.
a) *Increased emphasis on the ways students think and decreased emphasis on the actual mathematical work.*

Cersei’s response to student work in the pretest surveys was making judgments on their mathematical work and detecting errors/misconceptions in their mathematical work. However, in her posttest surveys Cersei payed more attention to ways in which students think. She concentrated on the pedagogy, while deemphasizing the mathematics involved. For example, consider Cersei’s response to the following item on the pretest survey (Problem 15):

![Figure 13. Cersei’s Response to Pretest Item #15](image)

Now consider Cersei’s response to the item on the posttest survey (Figure 12). We see that Cersei attended to the specific mathematical details in her analysis of the pretest
item on student work. However, in her response to the posttest item, when asked what factors could be contributing to the way students argue about the uniqueness of a midpoint, she did not focus on the responses being mathematically correct or incorrect as was the case in the pretest survey. Her posttest focus seemed to have shifted instead to student thinking. Instead she was referring to the type of the learner the student might be (part c of the response). However, another factor that could have contributed to Cersei’s different approaches is the type of questions posed. The question on the Pretest survey included examples that could be figured out by carrying out simple calculations, whereas the example from the posttest survey was more about the process that the student was carrying out. There were no calculations involved.

These observations from the surveys can also be supported by the interview data. As observed from the two maps I created in regard to Cersei’s approach to analysis of student work, we observe that Cersei’s references to mathematics decreased while her references to students increased along with her references to learning. When I interviewed Cersei about Peter and Amy’s approach to determining the uniqueness of a midpoint, she replied (Posttest Interview transcript):

Cersei: They’re both relying on the… one thing I don’t like is that they’re both relying on having to draw and having to see it. They weren’t convinced already like we would be that there’s only one midpoint on a line just based on the definition of a midpoint (p. 20, lines 651-653).

b) Although Cersei acknowledged the importance of collaborative learning as emphasized by the coursework, there was a disparity which became more evident in the survey.
From the interview data, it is apparent that Cersei believed in collaborative student work and communication in the classroom. However, from her responses to the survey it seems evident that she demonstrates a teacher centered orientation. In talking about what a teacher needs to know in order to be a good teacher, Cersei emphasized that the teachers’ role is in being the authority on content. She was of the opinion that “well if I am gonna be teaching, I need to know more than just how to do it myself. I need to be able to explain the reasoning behind it (Pretest Interview transcript: p. 1, lines 26-27).” For example, Cersei is of the opinion that a midpoint implies uniqueness and when Amy and Peter (Figure 12) were arguing about the uniqueness of a midpoint, she does not see the merit in their efforts since she believes that uniqueness of the midpoint lies within the definition and it does not have to be proved. Though she acknowledges their approaches in trying to prove the uniqueness of a midpoint, she feels that the effort was unnecessary (Posttest Interview transcript):

Cersei: It’s really just based upon the definition of a midpoint.

Researcher: Okay.

Cersei: And there couldn’t possibly be more than one midpoint based upon the definition. Umm, I mean they reason through it well and they’ve, you know, they’ve observed it. It seems like, you know from the comments that they make like they feel comfortable with there is just one which is perfect. At least they didn’t say oh there could be more. Sometimes there isn’t one, there is something like that so … I’m comfortable with what I, you know, reading their interpretations that at least they feel comfortable that there is only one midpoint. So that’s good. Good start (p. 22, lines 720-727).
Thus the responses to the survey at times seem to differ from the responses to the interview questions. While one the one hand she seems to support group learning, on the other hand she deemphasizes their need to argue because she believed that there was only one correct answer.

c) Cersei did not make explicit connections to assessment models, thereby not utilizing them to analyze student work and thinking.

Many of the posttest survey items contained prompts that required pre-service teachers to analyze student work and thinking in geometry using theoretical models of assessment. As in the case of other pre-service teachers, Cersei too was unable to make connections between the theoretical models and practice. To Cersei, the theoretical models were an afterthought to reflect on her analysis but they did not help her during analysis (Posttest Interview transcript):

Cersei: Yeah. Right. Umm, when we’ve use PK model or van Hiele model in any of our, umm, classes here, I’ve only used them when I’ve had to.

Researcher: Okay.

Cersei: And it’s been more of an afterthought. More of a I designed this lesson plan because I knew that this was the best way the next best step for this student, in particular from working one on one with the student like what we have in some of our classes. Then you can exactly design what that student needs next. And I get to that without using those tools of van Hiele and PK model. Then I have to take a step back and figure out how does it relate to theory. So I think of this as more theory (p. 12, lines 380-387).
Research Question 3: What are the effects of two quarters of coursework on Cersei’s ability to develop instructional strategies to aid students’ understanding of geometry and measurement?

From the figure below (Figure 14) we see that while making instructional decisions during the pretest interview, Cersei referenced the content approximately 36% of the time, while she referenced teaching about 31% of the time. She also referenced students approximately 22% of the time. Of the 47%, about 32% of the references were to the mathematical content involved. While referring to teaching, 31% of the references made were regarding Cersei’s goals of instruction for that particular topic as well as planning and instruction. Other references made were to the M. Ed. program (5%), experience (4%), and learning (2%).

Figure 15 represents the factors that Cersei draws from while making instructional decisions during the posttest interview. Here references to content increased to 38% while her references to teaching decreased to 27%. She referenced students 22% of the time. While referring to content, Cersei referred to her familiarity with the content 21% of the time. Also while referring to teaching Cersei referred to the use of tools and technology about 29% of the time. Other than referring to content, teaching and students Cersei also referred to herself (8%), experience (3%) and the M. Ed. Program (2%) of the time.
Figure 14. Mapping of Cersei’s Sources for Designing Instruction for Pretest
As observed previously, Cersei’s scores for designing instruction decreased from the pretest to posttest. Cersei’s mathematical analysis score decreased from 67% to 31%, while her pedagogical analysis scores decreased from 69% to 36%. In this section I provide more in depth analysis of why her scores decreased, drawing from her interview data to support the findings from the surveys.
a) There is less emphasis on the mathematical content.

Consistent with her beliefs about the teaching and learning of geometry, Cersei relies on the use of visual aids in order to help the students visually in solving geometrical problems. However, one major noticeable shift has been in her attention to mathematical detail. While Cersei’s instructional strategies in the pretest survey addressed the precise mathematics involved, her instructional strategies in the posttest survey were very vague in the amount of mathematical detail. Consider the example in figure (Figure 13). We observe that Cersei precisely outlined the procedures that she would carry out in order to address the student misconception/error. Her instructional strategy is indeed guided by her belief that geometry is best learned with visual aids but she also pays attention to specific mathematical detail.

Now consider the following problem from the posttest survey (Figure 12). Notice that Cersei is very vague in her outline of how instruction should be structured and she does not attend to the specific mathematical content associated with the problem. So during the interview I asked Cersei how she would go about introducing proofs to Amy and Peter. To which she replied (Posttest Interview transcript):

Cersei: Oh I guess it just depends on here like the lesson objective. If the lesson objective is just to see why a line segment has only one midpoint then I think they are really pretty much done unless they’ve told me it’s just because of the definition of a midpoint. If we really want them to be able to learn to, umm… if we wanted them to be able to start really improving things then this one is a pretty simple one because it’s just because of the definition of a midpoint. So it’s not really a … not really a good application I don’t think for teaching students how to
logically progress to formalization of their observations. But, umm, so I guess in D, really depending on, you know if I really wanted them to progress from, umm, any further I can’t think of any extensions to build on to, to this (p. 22, lines 730-737).

Once again we see that Cersei is hesitant to get into the details of the instructional plan. This was one of the drawbacks of the surveys as well as the interviews. The pre-service teachers were required to come up with a strategy on the spot. It did not give them enough time to actually consider the problem carefully and plan their instruction accordingly. Rather, it was a hurried process.

b) Cersei became more conscious about her own difficulties with the mathematical content

There was no observed instance in the pretest survey as well as the interview in which Cersei had expressed or indicated that she was not familiar with the content under study. The posttest surveys also did not reveal such concerns. However, during the posttest interview, Cersei revealed that she was not comfortable with certain topics such as transformations and constructions. Consider the following question on the posttest transcript:
While interviewing Cersei to get more information about her responses to the question, Cersei revealed that she was not very confident about her understanding of rotations and reflections. Consider the following dialogue (Posttest Interview transcript):

Cersei: Oh okay. Okay. And so this is saying (referring back to the problem on the survey) why a reflection of a reflection with an intersecting mirror must be a rotation? I’m a little bit confused on that one, thinking about it, is I am thinking that wouldn’t it be the same because you are reflecting it again. So it’s not really a rotation, right. It’s just you are coming back to the exact same object you started with. Right? I’m thinking of rotate… rotating meaning that it might be actually like this (draws). You know like it might…

Researcher: Right. I see. Okay.
Cersei: Or am I thinking about rotation wrong here? (p. 25, lines 825-832)

Another episode of where Cersei revealed she was not sure about the content was when she was talking about constructions. Cersei was asked to rank 5 constructions in the order of most important to least important (Pretest Interview transcript).

Cersei: I really like that problem. (inaudible) because I remember back to your class. This was the only time I didn’t post anything on there… I don’t think I posted anything online because I wasn’t really convinced myself of what would be the most, umm, important constructions (p. 27, lines 896-898)

This data is consistent with the scores on the surveys. As observed earlier her scores on items relating to transformations and constructions decreased from pre- to post.

Research Question 4: What is the effect of two quarters of coursework on the quality of questions posed by the pre-service teachers to elicit student understanding of geometry and measurement?

Cersei’s mathematical and pedagogical analysis scores for the pretest were 33% each and 25% for the posttest. These scores were based on responses to 3 questions on the pretest and 2 questions on the posttest that contained prompts for quality of questions posed by pre-service teachers.

Cersei scored consistently low on the prompts pertaining to questioning. Cersei avoided posing actual questions. Rather she outlined a task that she would conduct in order to further student understanding. Unfortunately, we did not have data from the interviews to support this claim. However, looking at the responses to the survey, we observe that Cersei did not provide with a clear understanding as to how her questions would help the students with the understanding of the concept. The pretest surveys reveal
that Cersei’s approach to prompts relating to questioning was having the students discover a pattern by exposing them to multiple examples. For example, consider Figure 13. Cersei had the student work with cubes with different base areas, which goes along with her answer to prompt b, where she wants the student to find the volumes for each of the cubes in order to notice the pattern of increase. In another example, Cersei created a table with different sides of a triangle and the scale factors to enable students to see the pattern in which the ratio of areas change.

Data from the posttest reveals that Cersei gave an explanation of why she considered her approach to be important but her explanation was not specific. She presented a very general overview of her plan of questioning but did not really concentrate on the mathematical details and what she wanted to accomplish. Another change in the posttest was that she included the use of technology to gauge student understanding but she never mentioned how she would use it. For example, Cersei (Figure 17) mentioned she would use Geogebra to further student’s understanding on volume, but she doesn’t mention how she would go about that task nor does she explain how it would help the student.
Part 3: Analysis of student work in geometry
1. Consider the following question to be given to students:

*What can you say about the volumes of a cube with side $a$, a sphere with diameter $a$, and a cylinder with height and diameter of the base to be $a$. What can you say about the volumes of these shapes? What can you say about the surface areas?*

While working on this problem, a student concludes that the volume of the sphere is less than the volume of the cube since the sphere can fit inside a cube. However, the student is not sure about the volume of the cylinder.

a. Why do you think the students might conceive of such a relationship?

b. How do you respond to this student’s conception?

c. What are some questions you could ask the student to further his/her understanding on this topic?

d. What tools would you use to assess the level of this student’s thinking?

e. What levels are relevant here?

\begin{itemize}
\item[a)] The student is able to visualize the sphere inside the cube. I am curious why the student does not visualize the cylinder inside the cube. I think the student is able to reason visually.
\item[b)] Ask the student to sketch these shapes and label “a.” I especially want to see the cylinder.
\item[c)] First, I would ask how the cylinder drawn relates to the circle. The last step would be to combine the cylinder + sphere. I would use Geogebra for volume.
\item[d)] I would observe them with Geogebra for volume + the net shapes for surface area. This is a good application of Van Hiele. The student appears visual right away. Are they capable of generalizing? At what point do they use formulas? Based on the level of answer initially gives.
\end{itemize}

Figure 17. Cersei’s Response to Posttest Item #8

**Research Question 5: What is the relationship between levels of teachers’ self-efficacy and their knowledge of students’ learning and thinking?**

Comparison between the pre- and posttest self-efficacy scores revealed that the mean score for her pretest was 6.333 ($SD=1.307$) and the mean score for his posttest was 162
6.666 ($SD=1.129$). A paired samples t-test revealed that the means were not significantly different ($t=-1.399$, $df=23$, $p=.175$). Thus there was no significant difference in the Cersei’s self-efficacy scores despite that her scores for her ability to analyze student work and thinking decreased over the course of two quarters.

Teachers’ self-efficacy is defined as “their belief in their ability to have a positive effect on student learning” (Ashton, 1985, p. 142). We notice that Cersei’s beliefs about teaching and learning did not change. Her pedagogical decisions were influenced by the beliefs that she held. This could offer a possible explanation to why her self-efficacy scores did not differ from pre- to posttest.

Summary

Cersei’s orientation toward teaching is highly influenced by her beliefs about learning and teaching of mathematics. She believes that geometry is very visual and it is best learnt by visual representations. She also believes that authority in judging the validity of the mathematical content lies with the teacher. Cersei cited her mentor teacher as her main source of learning and growth. She wanted to model her classroom after her mentor teachers’ classrooms.

The decrease in the scores for mathematical analysis can be explained from our observation from her interviews where she now pays less attention to the mathematical content. This observation follows from her view that mathematical coursework at the college level is not necessary for an effective high school mathematics teacher. Consistent with this view, we do not see a sincere effort on Cersei’s part to really decompact, trim or bridge her mathematical content knowledge to unpack or explain the students’ mathematical work, in her response to the survey items.
Though Cersei’s mathematical and pedagogical analysis scores decreased from pre to post, we still see a change in her approach to the analysis of student work. Cersei’s shift in her approach to analyzing student work may be observed in her focus on the students’ learning preferences rather than only on content. She believes that especially for geometry, most students learn the material via visual aids such as physical objects or drawings of the objects. However, she does not place emphasis on the mathematical details of the student work. She also does not explicitly make judgments on the correctness of student work.

Another major change that I observed was that there was a disparity between her responses to the survey and her responses to the interview questions. While Cersei said that she believed in collaborative group learning, her responses to the surveys revealed that she was of the opinion that the teachers should be the authority on the content. There was no evidence of group work in any of her responses.

I argue that Cersei falls under the isolationist category in Conney, Shealy and Arvold’s (1998) classification of pre-service teachers. Cersei’s view of mathematics was so rigid that she rejected alternate views of the subject. This is evident in a couple of her responses where she refused to move beyond how she viewed a problem, for example the octagon problem and the problem in Figure 10. She also never questioned her own beliefs about teaching and learning and about mathematics, even when others directly pushed her towards questioning her own beliefs.
The Case of Bran

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Course</th>
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<td>Autumn 2011</td>
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<tr>
<td>Winter 2012</td>
<td>Algebra</td>
<td>Posttest Interviews</td>
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Table 19. Timeline of Data Collection

Bran came into this program with a view that experience was the best teacher and that he would only be able to be successful after gaining experience in the field. I had the opportunity to observe Bran during the entire course of data collection for the current study. I was the instructor for the methods course he took in number sense and data analysis in the summer 2011. I also observed Bran in his geometry methods course. It was clear that Bran’s mathematical ability was advanced. He had no trouble solving any of the mathematical problems that were presented to the group in either one of the two methods courses. Bran also showed interest and enthusiasm towards learning new material that was presented during both courses. He always participated in class discussions and never hesitated to present his work and ideas to the group. He referenced educational terms that he had learned such as “summative and formative assessments” but never offered concrete examples of those methods of assessment when teaching specific mathematical ideas. The pre-service teachers also took a course on learning
theories and a course on assessments. However, Bran never made references or used any of those theories in class or when the assignments required an analysis of student thinking. The pre-service teachers were also required to work with a student at a Learning Center and write an analysis of the student’s mathematical work and thinking.

Analyses of Survey Data

Table 20 summarizes Bran’s percent scores on pre and post surveys in terms of his mathematical and pedagogical analysis of student work. Each response for every category except instruction was scored for a maximum of 2 points for mathematical and 2 points for pedagogical analysis. For prompts relating to instructional strategy, maximum possible score for mathematical analysis was 2 while maximum possible score for pedagogical analysis was 3. Hence for prompts relating to instructional strategies, the maximum possible score was 5. For all other prompts the maximum possible score was 4. Prompts on the pre and post surveys were categorized as analysis of work, instruction, and questions, based on whether the participant was analyzing student work, planning instructional strategies, or posing questions. Since the surveys were designed to elicit pre-service teachers’ knowledge via the processes of decompressing, trimming and bridging, the table also includes Bran’s percent scores for prompts pertaining to these three processes.
<table>
<thead>
<tr>
<th></th>
<th>Mathematical Analysis</th>
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Table 20. Scoring of Bran’s Responses to the Surveys

As seen in the table, there was an overall increase in Bran’s scores both in the areas of mathematical (pre: 42; post: 44) and pedagogical (pre: 45; post: 53) analysis. Although there is an increase in his total percent scores for mathematical and pedagogical analysis, Bran’s percent scores lowered on prompts pertaining to analysis of student work and decompressing. For prompts pertaining to analysis of student work, Bran’s percent scores for mathematical analysis decreased from 54 to 39, while his percent scores for pedagogical analysis decreased from 57 to 47. Similarly, his percent scores for
mathematical analysis decreased from 58 to 40 and for pedagogical analysis decreased from 59 to 47 on prompts pertaining to the process of decompressing.

In terms of Bran’s percent score on prompts for instruction, there was an increase in both mathematical analysis (pre: 35; post: 58) and pedagogical analysis (pre: 41; post: 59). Bran’s scores for mathematical (pre: 17; post: 25) and pedagogical analysis (pre: 17; post: 50) for prompts pertaining to questioning also increased. However, there were only 3 prompts on the pretest and 2 prompts on the posttest that addressed the pre-service teachers’ ability to pose questions.

Finally, the percent scores for mathematical and pedagogical analysis for the process of trimming and bridging also increased. For trimming, the scores increased from 29 to 44 for the mathematical analysis, while for the pedagogical analysis, they increased from 30 to 46. The scores for mathematical analysis increased from 33 to 63 and the scores for the pedagogical analysis increased from 42 to 77 for prompts pertaining to the process of bridging.

There was a significant positive correlation between Bran’s mathematical analysis scores and the pedagogical analysis scores for both the pre- and the post- tests. The scores for the mathematical analysis and pedagogical analysis for the pretest surveys were strongly correlated, $r(28)= .675, p< .01$. Similarly the scores for the mathematical analysis and pedagogical analysis for the posttest surveys were also strongly correlated, $r(31)= .663, p< .01$

The following table represents the content of each of the questions from the pretest and posttest surveys.
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<th>Pretest</th>
<th>Posttest</th>
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<td>Question 8</td>
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<td>Constructions</td>
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<td>3-D Geometry</td>
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Table 21. Content in Surveys by Question

Bran’s scores in all categories dropped for questions in the area of measurement and he consistently scored lower on the items concerning constructions except on prompts pertaining to instructional strategies and bridging, where an increase is evident. Bran’s scores on prompts pertaining to analysis of student work and decompressing were
lower particularly in the content areas of triangles, polygons and 3-D geometry, while his scores for the same increased on the topic of circles and reasoning and proof. Bran’s scores for instructional strategies increased in the areas of reasoning and proof, circles, similarity, polygons and transformations while they decreased on questions dealing with measurement and triangles. His scores for bridging increased in all areas except for measurement. Also note that Bran either left prompts on the posttest surveys relating to the use of learning progressions blank or he avoided their use completely. Unfortunately the prompts for questioning on the pre and post- surveys were in different areas and so content comparison between the pre- and posttest surveys couldn’t be done.

In the sections to follow, background on Bran will be provided drawing from his responses to the surveys and the interviews. Analysis of his responses to the surveys and interviews will then follow.

*Pretest Survey and Interview*

Bran took the pretest survey on the first day of classes in the summer quarter of 2011. However, due to multiple reasons, I was only able to interview him mid October, almost 4 months after the administration of the pretest survey. Figure 18 illustrates the major forces shaping Bran’s thinking about teaching and learning geometry as revealed during the first interview.
Figure 18. Factors Influencing Bran’s Pedagogical Decision Making in the Pretest Items

Experiences:

Bran’s orientation towards teaching was highly influenced by his experiences as a student as well as a teacher of mathematics. During the pretest interviews, Bran made several references to his experiences when talking about his views on teaching and learning of geometry highlighting “experience” as the key vehicle towards learning to teach. According to him, experience “is the most important piece I think to being a teacher” (Pretest transcript: p. 5, line 159). While talking about his ideal classroom or making connections between his coursework and his teaching, Bran drew heavily from his experience as a student. He expressed several times that he believed he would get better at teaching after having spent a few years of experience in the classroom.

Bran was asked about his views on how high school students learned geometry. In replying to this question, he drew from his experience as a student of geometry. He
referred to how he learned geometry by proof, logic and exposure to actual objects. So he
felt that students learn geometry through axioms (Posttest Interview transcript).

Bran: I’m just going to think back to my high school experience because I’m in a
middle school right now with algebra. So I think from what I remember high
school students learn geometry by proof, by learning the logic, by exposure to the
actual objects themselves. I remember doing a lot of… of constructions on the
board, a lot of parallel lines, triangles, circles and regular figures and looking at
angles and similarity. So lot of… lot of axioms and learning those and then
looking at how they apply to, uh, pictures so that we can actually utilize them in
discovering new things about pictures (p. 1, lines 17-22).

As a follow-up to this response, I asked Bran how he would use this information
to make pedagogical decisions or to design lessons. At this point Bran referred to his lack
of teaching experience and expressed that he would fall back on his experiences as a
learner of the subject to be taught (Pretest Interview transcript).

Bran: I think… I think remembering that, that’s how I learned it and that’s how it
sank in for me. So I don’t know… I don’t know how I would draw up the lesson
plan right now. I think the first ones would be based on how I learned it and then
adjust based off of what I see in my students in my classroom, the questions that
they have (p. 1, lines 27-30).

When I asked Bran to describe the role of a teacher in his ideal classroom, he
replied that the teacher would be asking a lot of questions and act like a facilitator
(Pretest Interview transcript).
Bran: What my classroom would look like, I like a lot of space, I like a lot of movement and I ask a lot of questions to students and… sometimes too many just because that’s how I was trained was facilitation, never give an answer (p. 3, lines 75-77).

Bran, citing his lack of teaching experience, fell back on his experience as a student to make pedagogical decisions or even deciding what mathematical topics would be easy or difficult for children to learn. In another instance, Bran was asked specifically what concepts related to circles he considered to be difficult for students. He immediately suggested that chords would be hard for students because in his experience with geometry, he found them to be challenging (Pretest Interview transcript).

Bran: Topics about circles? I always found chords the most frustrating because tangents I could do… I knew so many different things that could happen with it. Chords were just frustrating because there weren’t just as many tricks to it (p. 19, lines 622-624).

Bran considered mathematics teaching experience as the most viable factor in learning to be a successful and effective teacher. When talking about developing lesson plans rather than relying on guides provided for him and his colleagues in multiple courses he was taking at the time of interview, Bran said he was not very confident about how he would structure his lessons as he needed more experience (Pretest Interview transcript).

Bran: So I … I don’t know how I’d structure it yet. I don’t think I’m comfortable at this point in the program, having a geometry class and… and knowing exactly how to do it. I think that’s going to come from experience (p. 1, lines 31-33).
When faced with the question of describing his ideal curriculum, Bran felt that he was unable to make that choice because of the lack of experience. However, he was of the opinion that with experience he would be able to select appropriate texts. Rather than drawing from his knowledge of mathematics or learning objectives according to the content, he argued he would trust the administrators to choose the curriculum for him since they have experience (Pretest Interview transcript).

Bran: Curriculum I would be using… umm I don’t know if I have an ideal… we’ve looked at a lot of standards and my thoughts and my attitude so far is wherever I get hired, the principal and the administration have already got their set of standards that they like. They have looked at it for a lot longer than I am… I have. So at least for that first two or three years, umm, it’ll be trusting them and going off of what they…. What they feel best. And so after a while I’ll develop my own thoughts and opinions that I don’t know if I have right now just because I haven’t taught the material, and be able to structure it based off of that (p. 2-3, lines 69-75).

Need to meet learners’ needs:

Another major factor influencing Bran’s orientation towards teaching was his consideration for learners. From the survey responses it was evident that Bran did not discard student work even if it was incomplete or erroneous. Rather he tried to work “with” the student’s approach. Figure 19 is illustrative shows Bran’s typical responses to items that asked him to respond to the students based on their work. In nearly all such cases, he acknowledged that the student was on the right track and that her approach was
indeed a good one and that she had the correct idea although her reasoning may have been flawed.

18. Consider the following explanation offered by Terri for why the sum of opposite angles in a cyclic quadrilateral is 180 degrees. (A cyclic quadrilateral is a quadrilateral for which a circle can be circumscribed so that it touches each polygon vertex from: http://mathworld.wolfram.com/):

"If you draw an angle inscribed in a circle to build a convex polygon the fourth vertex is limited to the angle which it can subtend by the given (first angle drawn. Suppose the given angle is 80 degrees in measure. Then it subtends 160 degrees of the circle. 200 degrees of the circle is left. Let n equal the measure of the last vertex angle of the supposed polygon. Its measure can't exceed half of the measure of the 200 degrees (the rest of the circle which is the measure of the angle it subtends). Therefore the measure of n is 200/2 = 100." (Manouchehri, in progress)

a. What factors do you think influenced this student's reasoning?
b. How would you respond to this student?

A) The student understands how to prove the conjecture and why it is true (inscribed angles & circumference)

B) I would encourage the student because she is on the right track, and challenge her to prove this with any set of angles, not just 90°.

Figure 19. Bran’s Response to Pretest Item #18.

He was then asked if presented with three lesson plans on the same topic, which one would he choose. I wanted to know what all factors he considers to be important while teaching. His main motivation behind this was a consideration for the students (Pretest Interview transcript).

Bran: First things that come to mind are what type of learner my student is? So if I know that they are visual learner, I wanna make sure that my lesson has a lot of, you know, pictures. A lot of… of construction in front of them. If they are a kinesthetic learner, I want to make sure that they are… they’re using
manipulatives and I wanna do that for all of them but I would focus the major pieces of the lesson towards what my students actually learn best (p. 2, lines 48-52).

Bran claimed that he believed he could help all of his students learn mathematics provided that he got to know them on a personal level. He would then utilize the knowledge he gained from learning about his students to find out about their backgrounds and their personal goals. He felt that this knowledge was essential in order to “move them forward” in mathematics because he could then show utility of mathematics in their life. When I asked him what knowledge would he need to possess in order to initiate and maintain his ideal mathematics classroom he referenced knowledge of children’s thinking as the key ingredient to his practice (Pretest Interview transcript):

Bran: I want knowledge of who my students are. What their background is? Is it a suburban neighborhood where most of the kids have older siblings, parents at home and have seen… seen graphing calculators or is it a really urban neighborhood where, you know, graphing calculator maybe a foreign concept. You know, coming to school is viewed as social time instead of work time because they don’t get social time any time else. So I want knowledge about my students (p. 3, lines 87-91).

Beliefs about teaching and learning:

Bran’s beliefs about teaching and learning had a strong impact on his orientation towards teaching. Note that by beliefs here I mean to address claimed beliefs. Bran believed that higher level college mathematics courses were essential for him to become a high school math teacher. When I asked if Bran saw connections between the
Bran: Yes. Because, an example I’ll use is proportions. A lot of the students, when I am teaching proportions over the past few days, have been asking why can’t I just put in the calculator? Why do I have to show my work? And having been in those classes, I know that there are situations where you’re not gonna know all the information except one piece. You’re going to need to know two or three or four different pieces of information that you don’t have. And so using those proportions, I can say that there’s going to be… come a time, you know, maybe not in a math context but in the real world where you’re trying to solve a problem, uh, even if it’s how do I get, you know, my kids to soccer practice and you know the dance recital and make dinner for them all in one night and using … using the logic in reasoning that you… you gain through math classes, that’s what’s actually going to help you, not necessarily being able to put into a calculator… can I do that? (p. 4, lines 123-132).

Even though Bran expressed he believed there existed connections between his higher mathematics courses and school mathematics, he did not explicitly reference any such connections. When probed further about the connections between his advanced geometry course and high school geometry he would be teaching, he offered a generic answer referring to the uses and advantages of spherical geometry over plane geometry. He was explicit that he viewed geometry as “a set of rules and similar to a game.”
According to him, the difference between spherical and Euclidean geometries lay in the different set of rules they used.

Bran also viewed teaching of geometry to be different from teaching algebra, focusing on the heavy emphasis on proving processes in Geometry. One of the questions on the survey had asked the participants to describe how the teaching of algebra or geometry were different or similar. To this Bran replied: “Teaching geometry is proving or disproving conjectures. Teaching algebra is focused on creating methods to solve problems.” He also claimed that in contrast to algebra, geometry was “detail oriented and a new style of thinking (logic with constraints) (Pretest Survey, p. 2).” Bran viewed geometry and algebra as separate entities. To him algebra was about problem solving but geometry was about verifying conjectures.

Bran was of the opinion that students were capable of solving problems on their own. However, according to him, the rigor associated with problem solving depended on the students’ background. He also assumed it was okay for the students to utilize procedures without always understanding them. He supported this view because according to him people may not always be aware of why they need to do things as long as they work.

*Teacher autonomy and power:*

Bran believed that the teacher exerted tremendous influence on students’ cognition and their attitudes. I asked Bran if it was possible to influence students’ attitudes about their mathematical ability and whether you could help learners believe in their own capacity to do mathematics. He articulated the view that such student disposition could be achieved with positive reinforcement. He felt that this approach was
widely used by coaches in sports. He attributed the students’ poor performance in mathematics to boredom, tedious structure of the class, the pressure placed on using rules and procedures in mathematics not accommodating their creative thinking (Pretest Interview transcript).

Bran: I think positive reinforcement has a lot to do with it, umm, especially if a student is... there is a particular student in mind who is just not confident or is searching for someone to say yes she is doing it right. Umm and so there are two angles: giving her positive reinforcement, telling her that she is right and then at a certain point to tell her stop asking me if you are right. You know you are right. Be confident in it because you’ve gone through all these steps that you know you can do it. And I don’t think it’s unique to math either because I think that’s what coaches do in sports. I think that’s what mentors do for students (p. 11, lines 355-361).

Summary: Bran at the point of entry

Bran made most of his judgments on his experiences and his beliefs about teaching and learning. Bran based his decision making mostly on his experience and his consideration for learners. He believed experience to be the greatest influence on a teacher and the decisions that he/she made. He also expressed that teachers have the power to influence students’ attitudes towards mathematics. While not convinced of his current abilities to do so, he saw himself set for gaining knowledge through practice.

Posttest Survey and Interview

Bran was administered the posttest survey during the second week of December 2011, at the end of the autumn quarter. Bran was interviewed in early February nearly 2
months after he had taken the posttest survey. At this time Bran was enrolled in the algebra methods course.

Figure 20. Factors Influencing Bran’s Pedagogical Decision Making in the Posttest Items

**M. Ed. program:**

The M. Ed. Program seemed to have a significant effect on Bran’s orientation towards teaching. Recall that from his pretest data, Bran believed that experience was the best teacher. He expressed that the M. Ed. Program gave him that experience and that was the major avenue for learning to teach. When asked what he liked most about the program, Bran said it was “the experience. Actually being able to get into the classroom and try everything (Posttest Interview transcript: p. 15, line 678).”

Bran believed that the M. Ed. Program was instrumental in helping him better understand student thinking. He referred to activities in which he was able to look at student work, learn about the misconceptions or errors or alternate ways of approaching the problem. He expressed that they were very useful for him and that he could connect
those episodes with his own experiences as a student teacher (Posttest Interview transcript).

Bran: I really enjoyed, I don't remember which class it is, I think a couple of them, looking at actual students' work, seeing what their misconceptions were, or some of the errors they were making, or when they created a new method somehow always worked. I mean you have to think about will this hurt the student in future, or will it help the student in future, what are the obstacles. I think to me, that's the most real, it's less based in theory, which you should probably pick on that, the theories, until I actually can get them connected, or I have experiences to relate the theory to, they don't stick with me … I think that's the real world stuff, it's most helpful to me (p. 15, lines 642-657).

Bran felt that the program had also helped him to think more about the type of assessments he could use and setting goals and objectives for doing so. He expressed that he was more conscious about what he wanted to assess and if his tasks actually helped him reveal the type of knowledge he wished to elicit (Posttest Interview transcript).

Bran: I think [the program] helped a lot in terms of really challenging me to think about what am I assessing this student on, and are these assessment items I'm using, questions or tasks, actually assessing what I want to, or are they assessing something totally different (p. 15, lines 667-670).

Another source of influence on Bran’s thinking was his mentor teacher. He said that his mentor teacher really helped him in terms of planning and preparing for teaching by modeling for him ways to consider long term learning goals when designing lessons (Posttest Interview transcript).
Bran: I always thought that I was ahead, I always thought that I was thinking ahead, and this guy thinks about 40 million steps ahead of me. So it's great because he's organized at this point that he's way ahead. He takes his notes, like he'll deliver his lesson and immediately afterwards he'll go through to them and he'll change things right in there so that next year he's already changed it, he's already implemented everything. So he does a lot of modification, and it's instinct, it's right away. That way he's ready to go a year ahead of time. I'm really glad, so... it makes it more valuable to have resources and save them and name them appropriately and really think about them ahead of time too (p. 1-2, lines 42-49).

**Beliefs about teaching and learning:**

Bran’s claimed beliefs elicited by the posttest interviews showed changes when compared to his claimed beliefs expressed during the pretest interviews. Bran was asked during the post interview what knowledge was needed for effective mathematics teaching, to which he identified the knowledge about the content. However, by knowledge about the content, Bran just didn’t mean the Core Content Knowledge (McCrory, et al., 2010) but also knowledge about the common misconceptions or errors that prevailed among students, knowledge about which topics might be hard or easy for students and different ways of presenting that content to the students (Posttest Interview transcript).

Bran: You have to know the material as far as it's math. You have to know a number of different methods to each thing, you have to know common mistakes, you have to realize based on each student what their common mistakes are… I'll have to be dissecting, understanding, and thinking ahead of time and know that
they'll probably run into this issue, how can I help them through it. So, the knowledge to be able to think ahead of time to know students' mistake would do it (p. 2, lines 81-90).

Consistent with the pretest interviews, Bran expressed that teachers needed to know higher level college mathematics but he felt that he never really thought about it since he was never presented with a situation where he had to recall facts from his college mathematics courses.

There was also a shift in his view about geometry. In the posttest survey, Bran was asked if teaching of geometry and algebra was similar or different. His response indicated that he considered them as similar as both the fields required using reasoning, problem solving and creating new knowledge. However, geometry was different because “first intuition is not always correct” (Posttest survey, p. 1) and that unlike algebra it included proof with rigor and working with the general case.

Need to meet learners’ needs:

As with the pretest, a major factor influencing Bran’s orientation to teaching was his consideration for learners. When asked what information he needed when assessing student work, Bran indicated that he would like to know as much information about the student including his background (Posttest Interview transcript).

Bran: Yeah. I would want as much information as possible. I’d love to know their background of... where are they coming from, how much time they can put into their homework every night, and if they're working, if they're on the IEP... whatever I can know I would like to know. What do I need to know? That's a great question. What do I need to know about student? (p. 3, lines 136-140).
Findings

Research Question 1: What factors does Bran consider when judging pre-service teachers’ mathematical work and thinking?

In order to get more information and to get a better idea about Bran’s approach to analysis of student work and thinking, I constructed a map to indicate the factors that Bran draws from while performing those tasks. In order to build the map, I first coded the entire transcript. After coding, I examined the questions specific to the analysis of student work. I noted each of the codes listed under analysis of student work and kept track of the frequency of each. This was done for both the pre- and posttest transcripts. One caution when interpreting the data below is that one reason for the reference to mathematical content is high is because the pre-service teachers were asked content specific questions and were asked to analyze students’ mathematical work. The two maps are shown in the figures below.
As evidenced in Figure 21, while analyzing student work Bran referenced the content approximately 43% of the time, while he made references to students about 29% of the time. Of the 40%, about 48% of the references were to the mathematical content involved. While referring to students, 50% of the references made were regarding student’s work and thought process itself, while 25% were regarding student comprehension. Other references made were to learning (7%), experience (12%), the M.Ed program (4%) and teaching (4%).

Bran paid most attention to the mathematical content of the student work while conducting his analysis. This is evident from his responses to the pretest survey items. Figure 22 below shows the detail in which Bran explains the attention he gives to mathematical detail. The procedure was thoroughly laid out and explained step by step.
19. Consider the question: What can you say about the areas of a square of side length \( a \), a circle of diameter \( a \), and an equilateral triangle of side of length \( a \)? What can you say about their perimeters?

After working on this problem, a group of students suggested that “the bigger the perimeter, the larger the area and vice versa.”

a. Why do you think the students might conceive of such a relationship?
b. How could a teacher address this conception?

Figure 22. Bran’s Response to Pretest Item # 19.

Figure 23. Mapping of Bran’s Sources for Analyzing Student Work and Thinking for Posttest
The coding of the posttest interviews revealed that 37% of the references made by Bran were to Students while there was a drop in the number of references made to the content (26%). The reference to teaching increased to 16% while the number of references made to learning increased to 15%. There were no direct references made to the M.Ed. program itself. One of the major differences in the pretest and posttest interviews was that while no references were made to learning progressions or theories of learning in the pretest interviews, these theories of learning were referred to in the posttest interviews.

*Research Question 2: What are the effects of two quarters of coursework on Bran’s assessment of students’ mathematical work and thinking of geometry and measurement?*

We saw earlier that Bran’s mathematical analysis scores for items pertaining to student work dropped from 54% to 39% and his pedagogical analysis scores for the same items decreased from 57% to 47%. I compared Bran’s survey data to the data obtained from the interviews in order to obtain more information on what changes might have occurred in Bran’s approach to analysis of student work and thinking.

1. *There was a greater focus on student work in the posttest interviews although he did not make explicit connections to the theoretical models of assessment.*

Figures 21 and 23 reveal a greater focus on students rather than on content in the posttest as compared to the pretest. One would assume, therefore, that either there should be no difference between the pre-post scores or that the posttest scores would go higher. However, Bran’s scores on the prompts pertaining to analysis of student work decreased from pretest to posttest (Refer to Table 20). A closer examination of the interviews offers an explanation of what might have caused these changes.

187
The first thing to note was that even though Bran referenced the van Hiele model, the Pirie-Kieren model or the Harel-Sowder proving schemes, he did so only because I had asked questions with specific directions that called for the use of these theories. On further probing about the relevance of these theories, Bran revealed that he did not make strong connections to them in his work. Consider the following dialogue between the researcher and Bran (Posttest Interview transcript):

Researcher: Ok. What about the assessment models you've learned about? Like Van Hiele, PK. Do you consider those models? Are they sources when you analyze student's work?

Bran: I don't know. No they're not.

Researcher: Do you feel like they should be?

Bran: I don't know. I guess... I don't know. I don't think I'm an expert in them. I think that's not how I picture student work. I think it could be, but I don't feel I'm at the level where I want to be to actually use them effectively. (p. 5, lines 191-197)

When asked why he did not find these theories helpful in analyzing student work and designing instruction Bran revealed that he viewed the models of assessment too theoretical and complex to have practical merit for classroom teaching.

(Posttest Interview transcript)

Researcher: So they're not helpful.

Bran: No.

Researcher: Why? I think this is the best question I can ask.
Bran: I think they're really detailed. I view them as complex. I just don't feel that I'm fast enough with them. I feel like looking at students' work, I can say here's where I think they are going wrong and talk to them individually about it. Or if I see an entire class' work I can craft practice and highlight certain aspect where I think students are struggling. I don't feel like, I know the PK model or the Van Hiele model at the back of my hand that I can go and just craft the entire assessment or lesson based off them. (p. 5, lines 204-212)

This could be one possible explanation for his lower scores on the posttest surveys on items pertaining to the analysis of student work and thinking. The survey forced Bran to utilize the theories with which he wasn’t comfortable and did not understand. However, in the interviews, Bran focused more on student work and thinking and used his assessments of the same in his instructional strategies.

This phenomenon is also supported by Bran’s responses to the posttest survey. When asked to use van Hiele or the PK model to analyze student work, Bran either left the question blank or did not make appropriate use the models of assessment while analyzing student work. This is evident in his response to the posttest survey item number 8 (Figure 24).
Part 3: Analysis of student work in geometry

1. Consider the following question to be given to students:
   *What can you say about the volumes of a cube with side a, a sphere with
diameter a, and a cylinder with height and diameter of the base to be a. What
can you say about the volumes of these shapes? What can you say about the
surface areas?*

   While working on this problem, a student concludes that the volume of the
sphere is less than the volume of the cube since the sphere can fit inside a
cube. However, the student is not sure about the volume of the cylinder.

   a. Why do you think the students might conceive of such a relationship?
   b. How do you respond to this student’s conception?
   c. What are some questions you could ask the student to further his/her
understanding on this topic?
   d. What tools would you use to assess the level of this student’s thinking?
   e. What levels are relevant here?

   a) *LIKE THIS:* ![Diagram]
   b) *I THINK IT IS GREAT, AND I WOULD ASK THE STUDENT TO TRY TO DRAW THE CYLINDER INSIDE THE CUBE, TOO*
   c) *HOW DO YOU KNOW? EXPLAIN YOUR THINKING TO ME.*
   d) *HOW WOULD THEY DESCRIBE A SQUARE PYRAMID WITH ALL SIDE LENGTHS “a”, TO SEE IF THEY CAN EXTEND THEIR KNOWLEDGE*
   e) *SPATIAL LEVELS
   GEOMETRIC THINKING
   VISUALIZATION
   GENERALIZATION*

Figure 24. Bran’s Response to Posttest Item #8.

   b) *Greater attempt at understanding student comprehension at a deeper level.*

   Bran made a significant leap in his understanding of student comprehension. He
defines student comprehension in the pretest interview as “I interpret it as being able to
take in the information and the knowledge, processes and the logic you use and apply it to another different situation.” (pretest transcript: p. 8, lines 273-274). His definition of student comprehension was much more detailed in his posttest interview. He viewed comprehension as a two level process (Posttest Interview transcript).

Bran: To reach comprehension, I think it comes down to the type of question or that similar reasoning. If a student can plug in a number and find an answer, that's one level of comprehension, they know how to use an algorithm, they know how to use a formula. It's a different level of comprehension if they understand why they're doing it and can explain it to another student or to myself. Cause that's the level for why it works and it sets the foundation for the actual math instead of just being able to accomplish a task that's given to them. (p. 4, lines 175-181)

The shift in understanding of student comprehension led also to his desire to get to a deeper level in his assessment of student work and thinking. Consider the following response by Bran to an item on the pretest transcript (Figure 25). As you see, Bran’s response to part A of the question addressed at a basic level what the misconception was without actually considering where the misconception stemmed from. His analysis only considered what the student could not do without actually talking about how instruction might have led to this misconception, or how this topic was introduced which led to this following misconception.
21. Consider the following letter written by a high school student: “Today our teacher talked about the area of a triangle being ½ base times height. I think the formula is helpful when we have all the information but how are we supposed to find the area of a triangle if we don’t know what its height is?” (Manouchehri, in progress)

a. What do you think the basis for the student’s question is?
b. Do you think this is an important question for students to consider? Explain.
c. How would you respond to this student’s question?

A) It seems like the proof was made with a right triangle but the student doesn’t know what to do with a different orientation, even of the same triangle.

B) Yes, because it helps them understand how to find height when it is not obvious.

C) Ask him to identify resources and skills/knowledge that can help him find an answer.

Figure 25. Bran’s Response to Pretest Item #21

Considering Bran’s response to the same question on the posttest survey (Figure 26), His response to part A indicates that he noted the way instruction was framed in regards to area of triangles and also that instruction might affect the student’s conception of area of triangles. It did not only identify the misconception but also addressed the cause of problem which would help the teacher provide better future instruction for the student. The analysis considered not only where the student went wrong but also where the misconception stemmed from.
His understanding of student comprehension appeared more sophisticated and therefore he proposed probing with richer tasks to get a deeper student understanding. Initially, Bran’s approach to determining the level of student comprehension involved asking the student for an explanation of her thought process and validating her work. However, the posttest interview he opted for asking the student to work on an analogous problem and see if she can reason through it. According to him, this could help reveal if the student has the understanding of why things work and how they apply it to different situations with a different set of restrictions. Consider the following excerpts from the pretest and posttest interviews:

Figure 26. Bran’s Response to Posttest Item #11
Pretest Interview transcript

Researcher: Ok. So let’s say I gave this problem to a middle school student: So two diagonals of a rectangle divide the area into four equal parts. Ok? Do you think your students… the middle school students could solve this problem or do they need to know properties of say congruence before they can attempt this problem? Or knowledge of say area?

Bran: I think that they’d be able to describe it without knowing the properties. I think to prove it they would need those definitions and need to be able to work with it a little bit. (p. 5, lines 179-184)

Posttest Interview transcript

Bran: Yes. So I want to see if, you know, if they just figured out these three shapes or did they figure out the method where they can actually compare more shapes. So, a square pyramid is another shape that they are most likely not familiar with, and can't probably just picture it on the top of their head. So have they created a model for understanding this, can they bring in any shape now to find the method to actually compare them. (p. 9, lines 401-405)

The aim of the methods courses was to introduce the teachers to the task of analyzing student work and thinking and building instructional strategies to help students overcome their difficulties or misunderstandings. These classes were also venues where the pre-service teachers were introduced to learning theories and learning progressions and were required to use them in order to assess the student’s work and build instruction based off of it. However, Bran did not relate to these theories and did not see them fit for designing lesson plans or even for planning instruction.
Research Question 3: What are the effects of two quarters of coursework on Bran’s ability to develop instructional strategies to aid students’ understanding of geometry and measurement?

In order to obtain more specific information about Bran’s instructional decision making process and sources from which he drew when doing so, I mapped out the various sources he refers to while talking about instruction (as shown in Figure 27).

Figure 27. Mapping of Bran’s Sources for Designing Instructional Strategies in the Pretest
Figure 28. Mapping of Bran’s Sources for Designing Instructional Strategies in the Posttest

- Focus changed from short term success to long term learning

In order to ascertain Bran’s goals for instruction, he was asked what he considered choosing among three different lesson plans on the same topic. To this Bran identified that the most important thing for him to consider would be the type of learner his student was (Pretest Interview transcript):

Bran: First things that come to mind are what type of learner my student is? So if I know that they are visual learner, I wanna make sure that my lesson has a lot of, you know, pictures. A lot of… of construction in front of them. If they are a kinesthetic learner, I want to make sure that they are… they’re using manipulatives and I wanna do that for all of them but I would focus the major
pieces of the lesson towards what my students actually learn best. (p. 2, lines 48-52)

Clearly his focus is on student learning but narrowed to the personal preferences of his students and ways of learning. The reason I characterize this as narrow is because his goals are limited to the particular context of the lesson and the students.

Comparing this response with that given in the posttest interview, he articulated how his goals for teaching have changed, suggesting a greater emphasis on long term goals for the student. (Posttest interview)

Bran: …And so I find myself really taking a lot more time and I find it harder to think about how would a student understand this, what is the easiest way for a student to grasp this, and how can I, how can I build on that later. So I'm much more forward thinking with everything that I'm teaching. It's not just teach this one thing and have them get that one day; it's more long term. (p.1, lines 21-25)

Here his focus on the long term goals of instruction, which is to help students to not only make connections with the subject but also be able to use their understanding in future work while enhancing their understanding. This was also evident in his responses to his posttest survey items where he was asked to pick any two of three topics and justify his selection. Later he was also asked to outline a sequence of lessons (Figure 29).

Bran considered connections between topics and the content trajectory when choosing the two topics that he wanted to teach. He did not choose the topic of similarity because he felt that it could be derived using transformations. He chose right triangle trigonometry because according to him it formed a base for calculus and exploring non-
polygon shapes and transformations over, similarity because similarity could be derived from transformations.

Figure 29. Bran’s Responses to Posttest Items # 12 and 13.
b) His confidence in his ability to teach remains high despite the fact that he views teaching as a more complicated endeavor.

During the pretest interview, in response to a question regarding whether he could design a lesson plan for instruction, Bran expressed confidence in his ability to do so despite his insufficient knowledge of teaching (Pretest Interview transcript)

Bran:…I don’t think I’m comfortable at this point in the program, having a geometry class and… and knowing exactly how to do it. I think that’s going to come from experience. (p. 1, lines 31-33)

Similar comments reveal further Bran’s high level of confidence. (Pretest Interview transcript)

Researcher: Could you give me an example where… where geometric constructions… or could you design a lesson plan for a … an activity not a lesson plan… an activity for geometric constructions?

Bran: Right now?

Researcher: I know I’m putting you on the spot.

Bran: Yeah. I probably can. Umm, I can’t do it right now but yes I feel comfortable and I can do it.

Researcher: You can do it?

Bran: Yes. (p. 21, lines 670-676)

At the beginning of the posttest interview, Bran talked about why he felt teaching mathematics was hard. (Posttest Interview transcript)
Bran: I thought, ok, I understand the math, I can explain this, I'll be ready to do this. But the way I think of it sometimes is not the easiest way to present to students.

In both his interviews, he acknowledges the fact that teaching is a complicated endeavor and he needs to learn more as he progresses through the course and through experience. However, he expresses a high level of confidence in his ability to design instruction.

The pre- and the post-test surveys also consisted of questions about the self-efficacy of pre-service teachers. There were three categories of questions namely: efficacy for instructional strategies, efficacy for classroom management, and efficacy for student engagement.

*Research Question 4: What is the effect of two quarters of coursework on the quality of questions posed by the pre-service teachers to elicit student understanding of geometry and measurement?*

As seen from the results above, Bran’s scores for the posttest (mathematical analysis 25%: pedagogical analysis 50%) were lower than his scores for the pretest (mathematical analysis 17%: pedagogical analysis 17%). Consider the following two responses from the pretest (Figure 30).
Figure 30. Bran’s Response to Pretest Item #17

From Figure 12 we see that in response to part C that Bran avoided framing the actual questions. Instead he offered plans that were too general and obscure to the context. These plans were not clear as to what student response was expected or how he would further enhance student understanding on the topic via that line of questioning.

We also saw a change in Bran’s understanding of what characterizes a good question. In the pretest interview, when I asked Bran, to describe what types of questions he thought qualified as good questions, he identified guiding as a key feature. “I think a good question guides students in the right direction toward an answer without giving them an explicit answer.” (p. 7, lines 239-240). His answer to the same question on the posttest interview focused on both the context and form of tasks allowed for reasoning (Posttest Interview Transcript):
I'm a personal fan of open-ended questions, most of them starting with how or why. Because I think I can gain more from having students explain their reasoning, or explain why this can work or how this can work, and reason through everything out loud, then I can hear how students picture or think of it. (p.4, lines 151-154).

This shift in focus revealed increased tendency towards getting a better understanding of learners’ thinking. His claims of what he thought were good questions were supported by the questions that he framed in the surveys. Bran’s increased focus toward student work was supported by his conceptualization of good questions.

*Research Question 5: What is the relationship between levels of teachers’ self-efficacy and their knowledge of students’ learning and thinking?*

Comparison between the pre- and posttest surveys revealed that the mean self-efficacy score for his pretest was 5.875 (SD=1.393) and the mean score for his posttest was 6.875 (SD=1.676). A paired samples t-test revealed that the means were significantly different (t= -2.731, df=23, p= .012). However, a closer look at the three categories revealed that the means for efficacy for instructional strategies and efficacy for student management were not significantly different. The mean scores for the efficacy for classroom management were significantly different (t= -3.55, df= 7, p= .009).

There was an increase in Bran’s self efficacy scores from pretest to posttest. This is not surprising as we saw earlier that Bran’s confidence to teach increased from pretest to posttest. However, a disparity is observed between the survey results and the self-efficacy results. We see that Bran’s survey scores have increased in his ability to design instruction as well as in his ability to pose good questions, however, there is no
significant difference in his self-efficacy scores for instructional strategies. This can be explained by a closer inspection of the questions on the self-efficacy survey. Bran showed an increase on his self efficacy scores for questions that pertain to designing instructional strategies and questioning. This is supported by the results of the survey in which Bran showed improvement in the areas of designing instructional strategies and posing good questions. However, we see that Bran’s score for the question about using a variety of assessment strategies is lower for the posttest than for the pretest. He ranked himself 7 on the pretest self efficacy survey whereas he ranked himself a 4 on the posttest. This is supported by our findings from the surveys and the interviews where, Bran experienced difficulty in using assessment models to aid in his analysis of student work.

Summary

In attending to student work and developing instructional strategies, Bran focused on the mathematical content involved. Bran believed that the mathematics he learned at the college level was useful for him to be a successful mathematics teacher at the high school level. Bran’s focus on mathematical detail seemed to be consistent throughout the surveys. However, his focus on students increased in responses to the posttest interviews. Also notice that Bran’s posttest interviews revealed that the M. Ed. Program had an impact on his analysis of student work and thinking as well as his ability to develop instructional strategies.

One of the things that Bran relied on during both his interviews was the importance of teaching experience to be a successful teacher. Note that in the pretest interviews, Bran kept referring to his lack of teaching experience. However, during the
posttest interviews, he referred a lot to his student-teaching experiences he had had so far. Also one of the major influences that the M. Ed. Program had on Bran was that it provided him with those experiences where he could practice his learnings from the courses.

Data from the interviews and the surveys confirm that Bran fell in the naïve connectionist category (Cooney, Shealy, & Arvold, 1998). Bran cited the impact of the M. Ed. Program on his ability to analyze student work and thinking saying that he now definitely paid more attention to student work and his idea of student comprehension had deepened. However, while analyzing work and even while designing instruction Bran was not able to utilize the assessment models to aid his work. Bran acknowledged the importance of the models but was not able to apply the theory to practice. To him, his beliefs that experience is the best teacher prevented him from really applying those theories. To him, looking at student work and figuring out where the student had gone wrong or what misconceptions prevailed was the best way of looking at student work and even though he saw how the assessment models could help in aiding that task, he believed that it was via experience that a teacher got better at analyzing student work and thinking. He viewed the assessment models as helpful in one to one instruction but not for designing instruction at for the entire class.

Despite having his own beliefs and goals about teaching and learning, Bran was open to accepting other viewpoints. He drew from a multitude of sources including his mentor teacher, his instructors in mathematics and mathematics education courses, and classroom episodes (Boaler & Humphreys, 2005), which were shown in the methods courses. For example, Bran stated that he believed the theories of assessment discussed
in his education courses could be helpful. However, because he believed so strongly that teaching experience itself was essential to his growth, he was less than inclined to integrate those theories into own practice. Even more specifically, Bran singled out learning progressions as important, especially when assessing an individual student’s work and then planning instruction for that student. Again despite agreeing that learning progressions can help to identify levels of students’ mathematical understanding, he could never reconcile the chasm in his own thinking regarding the prior impact of practical knowledge on learning to teach/knowing how to teach.

The Case of Nedd

I first met Nedd during the summer methods course in number sense and data analysis that I taught. Nedd was an extremely active participant in class activities, especially discussions. Nedd was never shy to speak his mind and express his views. He was of the opinion that he knew the mathematics that was required in high school but had no experience in teaching and so he felt that this program was most suited for him as he would learn how to teach.

Nedd was the other candidate with a background in computer engineering and electrical engineering. Also Nedd was on a two year track as opposed to the other participants who were on a one year track. Nedd needed to take some additional mathematics courses as he did not meet the licensure requirements. Nedd was required to take the abstract algebra series. At the time of the initial interview, he was enrolled in the Foundations of Higher Mathematics course which was a pre-requisite to the abstract algebra series. Nedd had previously done some work towards getting his doctoral degree but he could not follow through with the program due to a variety of reasons.
Nedd decided to become a teacher because he thought it would be a good way to spend the latter half of his career as a teacher. He valued teaching and believed that he brought a different perspective to the profession due to his background. He also said that if for any reason he could not become a teacher, he would continue being an engineer. When asked why he chose to teach mathematics instead of physics or engineering, he replied that in order to become a physics teacher, the list of pre-requisites was longer and it would take more time. Another factor that influenced Nedd’s decision to become a teacher was his volunteer work in a charter school for non-traditional learners that are disruptive. He felt that he would have been satisfied teaching technology classes but he chose mathematics because he felt like 9th grade mathematics had the highest dropout rate and he was determined to make sure that more students stayed in school.

In the sections to follow, I offer some background information on Nedd followed by an analysis of his responses to the surveys and interviews. In the last section I will attempt to answer the research questions for Nedd.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Course</th>
<th>Data Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer 2011</td>
<td>Number Sense and Data Analysis</td>
<td>Pretest Surveys</td>
</tr>
<tr>
<td>Autumn 2011</td>
<td>Geometry</td>
<td>Pretest Interviews</td>
</tr>
<tr>
<td>Autumn 2011</td>
<td>Geometry</td>
<td>Posttest Surveys</td>
</tr>
<tr>
<td>Winter 2012</td>
<td>Algebra</td>
<td>Posttest Interviews</td>
</tr>
</tbody>
</table>

Table 22. Timeline of Data Collection for Nedd
Analyses of Survey Data

Nedd’s percent scores on pre and post surveys in terms of his mathematical and pedagogical analysis of student work are presented in Table 23. Prompts on the pre and post surveys were categorized as analysis of work, instruction, and questions, based on whether the participant was analyzing student work, planning instructional strategies, or posing questions. Since the surveys were designed to elicit pre-service teachers’ knowledge via the processes of decompressing, trimming and bridging, the table also includes Nedd’s percent scores for prompts pertaining to these three processes.

<table>
<thead>
<tr>
<th></th>
<th>Mathematical Analysis</th>
<th>Pedagogical Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pretest (%)</td>
<td>Posttest (%)</td>
</tr>
<tr>
<td>Analysis of work</td>
<td>39</td>
<td>23</td>
</tr>
<tr>
<td>Instruction</td>
<td>35</td>
<td>29</td>
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<tr>
<td>Questioning</td>
<td>67</td>
<td>50</td>
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<td>Decompressing</td>
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<td>27</td>
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<tr>
<td>Trimming</td>
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<td>50</td>
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<tr>
<td>Bridging</td>
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<td>13</td>
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<tr>
<td>Total Score</td>
<td>40</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 23. Scoring of Nedd’s Responses to the Surveys
As seen in the table, there was an overall decrease in Nedd’s scores both in the areas of mathematical (pre: 40; post: 26) and pedagogical (pre: 40; post: 31) analysis. Nedd’s percent scores for mathematical and pedagogical analysis lowered on prompts pertaining to analysis of student work and decompressing. For prompts pertaining to analysis of student work, Nedd’s percent scores for mathematical analysis decreased from 39 to 23, while his percent scores for pedagogical analysis decreased from 43 to 35. Similarly his percent scores for mathematical analysis decreased from 42 to 27 and for pedagogical analysis decreased from 48 to 36 on prompts pertaining to the process of decompressing.

Nedd’s percent score on prompts pertaining to instruction decreased in the domain of mathematical (pre: 35; post: 29) analysis but increased in the domain of pedagogical analysis (pre: 31; post: 33). His scores for mathematical (pre: 67; post: 50) analysis for prompts pertaining to questioning decreased where as his scores for pedagogical analysis (pre: 33; post: 25) remained the same. However, there were only 3 prompts on the pretest and 2 prompts on the posttest that addressed the pre-service teachers’ ability to pose questions.

Finally, the percent scores for mathematical and pedagogical analysis for the prompts eliciting the processes of trimming and bridging also increased. For trimming, the scores increased from 36 to 50 for the mathematical analysis, while for the pedagogical analysis, they increased from 30 to 42. The scores for mathematical analysis decreased from 44 to 13 and the scores for the pedagogical analysis decreased from 33 to 09 for prompts pertaining to the process of bridging.
The following table represents the content of each of the questions from the pretest and posttest surveys.

<table>
<thead>
<tr>
<th>Content</th>
<th>Pretest</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement</td>
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<td></td>
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<tr>
<td>- Area</td>
<td>Question 12</td>
<td>Question 8</td>
</tr>
<tr>
<td>- Volume</td>
<td>Question 15</td>
<td>Question 18</td>
</tr>
<tr>
<td>Similarity</td>
<td>Question 11</td>
<td>Question 12</td>
</tr>
<tr>
<td></td>
<td>Question 17</td>
<td></td>
</tr>
<tr>
<td>Definitions</td>
<td>Question 14</td>
<td>Question 9</td>
</tr>
<tr>
<td>Reasoning and Proof</td>
<td>Question 20</td>
<td>Question 8</td>
</tr>
<tr>
<td></td>
<td>Question 18</td>
<td>Question 10</td>
</tr>
<tr>
<td></td>
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<td>Question 14</td>
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<tr>
<td></td>
<td></td>
<td>Question 16</td>
</tr>
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<td>Triangles</td>
<td>Question 21</td>
<td>Question 11</td>
</tr>
<tr>
<td></td>
<td>Question 22</td>
<td>Question 12</td>
</tr>
<tr>
<td>Polygons</td>
<td>Question 13</td>
<td>Question 9</td>
</tr>
<tr>
<td></td>
<td>Question 16</td>
<td>Question 15</td>
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<td></td>
<td>Question 18</td>
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<td>Circles</td>
<td>Question 14</td>
<td>Question 14</td>
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<tr>
<td>Constructions</td>
<td>Question 23</td>
<td>Question 19</td>
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<tr>
<td></td>
<td>Question 22</td>
<td></td>
</tr>
<tr>
<td>3-D Geometry</td>
<td>Question 12</td>
<td>Question 8</td>
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<tr>
<td></td>
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<tr>
<td></td>
<td>Question 22</td>
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<tr>
<td>Transformations</td>
<td>Question 20</td>
<td>Question 12</td>
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<td></td>
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<td>Question 16</td>
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<td></td>
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<td>Question 17</td>
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</table>

Table 24. Content on Surveys by Question

Nedd’s scores in all categories dropped for questions in the area of measurement, polygons, constructions, reasoning and proof, and 3-D geometry Nedd’s scores for
instructional strategies decreased in the areas of reasoning and proof, circles, similarity, polygons and transformations, constructions, and measurement and triangles. There was not much difference in his scores in the areas of triangle and similarity. Also note that Nedd left 3 questions completely blank and hence they were not scored. Nedd also did not answer all prompts on 5 questions and so that also affected his scoring. Unfortunately the prompts for questioning on the pre and post-surveys were in different areas and so content comparison between the pre- and posttest surveys couldn’t be done.

Nedd’s mathematical analysis scores were positively correlated with his pedagogical analysis scores for both the pre- and posttests. The scores for the mathematical and pedagogical analysis for the pretest surveys were strongly correlated, $r(28)= .847 \ p< .01$. Similarly the correlation between the scores for the mathematical analysis and pedagogical analysis for the posttest surveys was strong, $r(33)= .722, \ p< .01$. However, notice that for the prompts pertaining to instructional strategies, Nedd’s mathematical analysis scores decreased but his pedagogical analysis scores increased.
Experiences:

Nedd believed that experience was the key in being an effective teacher. This was very much apparent when Nedd talked about his views on the mathematics curriculum at the high school level. Nedd said that he trusted that the curriculum had gone through sufficient changes and revisions and as a result he believed that whatever curriculum was used in schools was good. He refused to make judgments on curriculum because of his lack of experience. However, he believed that his experience as an engineer was valuable. (Pretest Interview transcript).

Nedd: Ah. Umm, so some of that I guess would be, umm trusting, you know, that this curriculum as we have it today has gone through, you know, it’s improvement. Right? And so this is a set of topics that over time has been deemed, umm, a good curriculum. And I am not going to make any quick
judgments on that. I’ve...I’ve nothing to base my opinion on yet. Right? Umm I think that what I’ve seen and what I’ve studied, umm, I think that there are like little extra dimensions that engineering can… can take you (p. 3, lines 78-82).

When talking about his ideal mathematics classroom, Nedd referred back to his volunteering experience at the charter school as well as briefly to his experience as an engineer. Nedd claimed that over the course of his time volunteering at the charter school, he had come to the conclusion that lecturing was not the best form of teaching. Even though he thought the teachers were well versed with the subject they were teaching and were not confusing, he felt that in his ideal classroom, teaching would be somewhat the opposite with a minimal amount of lecturing (Pretest Interview transcript):

Nedd: Uh, because, again, back to my engineering, I am more of a understand what’s there and then look for improvement or change. Uh, so I would be making some of this up. Um, I have observed classes, like I said, when I was teaching at, or not teaching, when I was volunteering at Gahanna. I was almost the whole time in a classroom and I found that um, you know the lecture for for 90% of the class just is not, doesn’t look appealing, it doesn’t look uh, you know of course the teacher was obviously, they communicated wonderfully. I was sitting there listening and I go, wow they are just so on task. They are not stumbling on terms. They are not confusing. At least for me they weren’t confusing, but, umm, I would say that something opposite of that, uh, would be more, umm, more my ideal (p. 5, lines 147-154).

Another passage that describes the influence of Nedd’s work experience on his views about teaching and learning is when he talked about student comprehension, and he
drew parallels to his work. Nedd was asked what he meant by the term comprehension and what indicators would he use to suggest that a person had a deep understanding or a person had comprehended something (Pretest Interview transcript).

Nedd: Uhh, if someone was… so, uh, I worked at speech technologies for a long time and so I worked with people that know these pretty intense approaches to uh, modeling data and pattern matching, and if they could describe this to hmm, me or managers or a customer, right, they can take a pretty complicated thing and they can raise the level so that the story still holds and they can communicate that. Umm, I think that’s a good measure of being… of knowing your… you know this, very detailed thing but you can describe it so that somebody who is not, you know maybe knowledgeable in that field could understand it (p. 19, lines 593-599).

Beliefs about teaching and learning:

Nedd’s claimed beliefs about teaching and learning played a significant role in his orientation as a teacher. Nedd firmly believed that “a teacher should be taught how to teach (Pretest Interview transcript: p. 6, line 175).” Another claimed belief that Nedd projected throughout the interview was that in order for students to comprehend the material, they needed to struggle a bit with it. Consider the following dialogue between the interviewer (R) and Nedd (Pretest Interview transcript).

R: And what is important for understanding?

N: Uh, struggle. (p. 4, lines 125-126)

In order to facilitate that, the teacher needed to make the lesson a bit challenging so that they could experience this struggle. When asked what type of tasks he would
choose in order to promote student understanding, Nedd replied (Pretest Interview transcript):

N: Umm, a little bit of that has to be, umm, again challenging. It has to… I mean teaching is moving somebody somewhere. You know I always say that breathing hard is not bad. Umm, so that probably might take a little bit more convincing but, umm, I think that’s the goal. (p. 8, lines 231-236)

Nedd also claimed to believe that it is important for teachers to get to know how students think about mathematics. When asked why he thought it was important, he said (Pretest Interview transcript):

N: And I definitely believe and coming to learn… coming to believe that umm, you know you just don’t open up somebody and pour something in and they know it, right? And they… they do need to take things on in their own terms. Umm, and so knowing where they are at is I think important 1) to not drill something that’s… something they already know and you’ll lose em. They are bored or you try to stretch them too far and they are just… they have no place to connect that information. It’s just time… well in both case time is wasted. (p. 9, lines 268-273)

Nedd claimed that mathematics that was required for teaching high school was different from the mathematics that was required for engineering. In fact he goes on to say that he does not see a relation between what he learned in high school and the mathematics that he used while training to be an engineer. (Pretest Interview transcript)

N: Umm. So I think it’s different in the sense that, umm, you know being in computers… so there’s a lot of logic, a lot of (inaudible) theory is something that is not touched on. Umm, it’s something you can build off… (thinks aloud: can
you build off of it?) I can’t say, you know, I would say it’s new. I don’t think, I can’t… I can’t think back and say there is something I learned in high school math that took me there.

On the other hand, Nedd also believes that being a teacher is similar to being an engineer as in his experience the skill set required to be an engineer is similar to the skill set required to be a teacher. This skill set consists of problem solving techniques which, according to him, are finding a problem, looking for alternatives, and finding the best method that helps you solve the problem. (Pretest Interview transcript)

R: Ok. I see. I see. So how do you think … how do you think being an engineer is different from or similar to being a teacher?

N: Yeah. So I think the biggest reason, the biggest thing is, umm, mostly continuous improvement. You know problem solving, adaptability, looking at all the inputs. You know, defining a problem, alternatives, what gets you to that next improvement. I mean, I think those are very drilled into me and I think that’s a skill a teacher needs to have. (p. 3, lines 68-73)

Teacher preparation:

Nedd believed that he had a lot to learn from the program. He had quite a few expectations from the program. when asked if he felt it necessary to go through the program in order to be a successful teacher, he said that he felt that the program was necessary to train and prepare him for the profession of teaching (Pretest Interview transcript).

R: Ok. Do you really think you really need to go through this teacher ed program to be a teacher?
N: Yes.
R: Why?
N: I… I think I am comfortable enough or have experienced enough that I am willing… I would go… I would be willing to go out and fail miserably but, umm, or quickly but I didn’t… I didn’t like that approach. I am a, by nature, a person who trains for things and I really wanted to learn how to teach. (p. 3-4, lines 98-103)

At the time of the interview Nedd had completed one quarter of in the M. Ed. Program. I asked him the following question: What are the two or three things you feel you need to learn about how to teach? To this he replied that there were quite a few things he still needed to learn. (Pretest Interview transcript)

R: Ok. So let’s see. You, without putting words in your mouth, so you want to learn some techniques, new things about how to teach mathematics?
N: Umm.
R: Or not even that?
N: I think maybe the way you worded it sounded like micro things. Little tricks and schemes and strategies. Umm. And that’s all fine. I mean you probably need to build up your tool chest of those things. Right? But I think I’ve been exposed to much bigger things
R: Ok. What are some… What are a few of those bigger things that were important, that you considered to be important?
N: Yeah. So I think I’ve always understood, at least from a teacher’s perspective, that you know there is sort of a sequence of knowledge right? You can follow along or you can solve a problem on your own or you could teach that topic right?
And that’s… that takes just different levels of understanding knowledge and uh being able to, you know, reflect back. Umm, I think (sighs)… I think the bigger things were basically a much more crystallization of what’s important for understanding right? (p. 4, lines 111-124).

Thus we see that Nedd felt that he was not prepared yet for teaching and that he had a lot to learn from the program.

*View on mathematics:*

Nedd’s view of high school mathematics was that it was basically a re-creation of something that had been done in the past by mathematicians and so the only concern was how to make that interesting for the students. (Pretest Interview transcript)

N: Well this is actually something I am looking forward to trying to figure out how to balance is, you know, their constructionism approach, the problem approach. To an extreme you are asking somebody to re-create something but it’s already been created. You know there’s already people who have solved these things, that can show an interest in technique. So how do you, uh, show that? How do you make that interesting to somebody to say ok I can look at something that’s already been solved and gain something from it. (p. 7, lines 220-225)

*Conclusion*

Thus we see that Nedd sees himself as unprepared for the profession of teaching and that he needs to go through the program in order to be a successful teacher. He sees parallels between teaching and engineering and his experience as an engineer as well as a volunteer at a charter school have a big impact on his orientation toward teaching. He
also felt that mathematics required for engineering and mathematics required for teaching high school were different.

*Posttest Survey and Interview*

Beliefs about teaching and learning:

Nedd still believed that lecturing is not the best way of teaching. He feels that lecturing does not enable retention of knowledge. When asked about his idea classroom, Nedd described that he was not in favor of lecturing because his goal was that students retain the knowledge. (Posttest Interview transcript)

Nedd: And also you know maybe the teacher feels good about it. Maybe they think they’ve made a great presentation you know they’re up there feeling good and um but the retention level is just not there, so. So I think my-my ideal classroom would be anything that would just increase retention. (p. 3, lines 72-75).
Nedd was also of the opinion that lecturing was easy as compared to other modes of teaching where you have to reflect and react to students. He felt like once you had lectured a few times, it was just a performance art. (Posttest Interview transcript)

Nedd: Hm. Um, well I think I’ve already said I think it’s going to be much harder.

Researcher: Yeah.

Nedd: Because lecture. You prepare it. You go up. If you’ve done it a few times, you know you could be you know- I used to think of teaching as performance art. But I now think that’s not, that’s not right.

Researcher: So what do you think of it now?

Nedd: Well the teacher’s not the focus. There’s no way. (p. 4, lines 126-132)

Another claimed belief that Nedd portrayed was his beliefs about student comprehension. When I asked Nedd what he meant by student comprehension he was of the opinion that if a student could apply something that he had learned to another student, then he had comprehended the material (Posttest Interview transcript).

Nedd: Yeah, so I think, I think comprehension is, is ah, you know…I always keep going in my head about nuances of all these things. It sounds too simplistic whatever answer. Um. So I think it’s mostly applying something to a new situation. So if the, if something has been taught and then what you hope was taught as part of that, then being able to take that and then use that as a skill or a tool in another place another problem. I think that that’s a, a high mark of comprehension. (p. 10, lines 354-359)
Nedd also felt that another way for checking if a student had comprehended the material was if he could explain it to someone who is not knowledgeable in the subject. It was clear that Nedd was drawing from his work experience while making this claim (Posttest Interview transcript).

Nedd: You know if I was gonna try to explain speech recognition to somebody, right? So, um, I think that I could probably explain it to almost anyone. Because I could abstract it to a point where they would understand you know my description. Uh, so I wouldn’t have to talk about all the, you know, all the processing or the math that’s at the very bottom, right? But I could describe that the, the basic solution is to create a template, a model of how people speak and then, then how would you do that? Well, you make, you collect samples of how people speak and then you make a model that sort of represents them. I think you can go through that and I think, I don’t know if there’s, quite a good analogy here for students (p. 9, lines 319-327).

Experiences:

While talking about his ideal mathematics classroom, Nedd said that a key factor in choosing the curriculum would be, the level of student engagement. When I asked Nedd, how he would facilitate student engagement, he replied that he had no experience with curricular materials but he would trust what he reads and if it convinced him and sounded logical, he would use it (Posttest Interview transcript).

Nedd: I-so again as a new teacher, you’re, y-you read things and hear things and a lot of them sound good and I’m not good at saying them again because they’re not my own yet. You know I haven’t experienced them to say you know do I really
like it, do I really believe? But if you hear it, it sounds good, it sounds logical.

You know, ok, you could convince me. Um, so I’ve, you know I’ve heard of people setting up instruction where, uh, um, the reading, or the reviewing of video lecture is done as homework. (p. 4, lines 109-114)

When I asked Nedd, if he thought student comprehension could be measured, he immediately drew from his engineering experience and said that for him measurement was all about getting a single point answer which he believed was not true. Thus based on his experience, he believed that student comprehension could not be measured (Posttest Interview transcript).

Nedd: Oh, ouch. Ah, so in teaching engineering, it’s about measure. You know if you don’t have a model, then you create one. Ah, so, but I’m also I’ve, I’ve seen this and very much believe that it’s, it’s wrong to set a very focused goal, a very single point answer. Because people will get to that. You know if you say that you know the unemployment rate should be 7, under 7%, people will figure out how to do it and, and you know the world of unintended consequences kind of wanders in. Um, so I would, I would immediately have to say if I was measuring it, it couldn’t be, um, down to an individual number. I would have trouble with that. But in the broader sense, you know, yeah, I think I have to, I have to know how to do it (p.11, lines 373-381).

Thus we see that Nedd still drew from his experience as an engineer while making certain pedagogical decisions. He also felt that experience was an important factor to become efficient in teaching.
**M. Ed. program:**

Nedd came into the program with a view that he had the mathematics background but needed to be trained in teaching. From his pretest interviews we gathered that Nedd viewed teaching as a learned profession.

Nedd claimed that the program had had a huge effect on his decision making skills especially when it came to planning instruction for students. He believed that the program changed his outlook on how students learn. Before entering the program he believed that students learned the material immediately after it was taught but now after being in the program, he came to believe that that’s not always the case (Posttest Interview transcript).

Nedd: Yeah, ok. Um, so you know, I would say a huge impact. Right, because I think that I’ve always, I, I guess I have been, I’m quite content with the idea of not needing to see that a student learns something immediately. I’m quite content with planting seeds. Right? That maybe it takes days or weeks before I see something come back, so um, I think that. That’s sort of my own view of things and I think that that’s kinda reinforced with the you know constructivism um, having students struggle, ah, with things, ah, you know, maybe starting with a problem so that you create a hook and get them connected and use that as a place to then bring in little vignettes of lecture, so. I think it’s, maybe it’s sort of like almost 180 of how I would have thought of some of this (p. 48, lines 1672-1680).

Another influence that the program had on Nedd was on his disposition towards his expected learning. He became more aware of his teaching style and going through
some of the courses, he felt that his approach toward teaching had changed (Posttest Interview transcript).

Nedd: Yeah. So I think um, ah, one of ‘em is, ah, what I shared earlier was that I’ve always been a tough love kinda person. Uh, I think, you know, my children would say I always used to say, “Buck up.” I would never some-tell somebody to be careful. I would say, you can do it or you can’t do it. If you can do it and you hurt yourself, that’s fine. Um, struggling is good. Now, I say all those things more, but then I have that impulsive reaction of wanting to help and wanting to give them the answer and not let them struggle too long. Right? And I realized I gotta find that balance and I think I’ve moved away, a little bit more into the let them struggle. You know, reflect a day or two, that’s fine, you know. At the end of a session, you don’t have to have the answer. Um, kind of walking away to reflect could be better. (p. 18, lines 661-668).

Summary

Nedd’s scores on his survey for prompts pertaining to the analysis of student work and thinking decreased from pretest to posttest. From the interview data we see that Nedd’s attention to mathematical work decreased. However, his attention to mathematical content increased while designing instructional tasks. We also notice that Nedd’s experiences had a big influence on his orientation towards teaching.

Findings

Research Question 1: What factors does Nedd consider when judging pre-service teachers’ mathematical work and thinking?
Figure 33 shows that Nedd referenced the students approximately 48% of the time, content about 20% of the time and teaching approximately 13% of the time. Of the 48%, about 44% of the references were to the students work and thinking. While referring to content, 45% of the references made were made to the mathematical content of that particular topic. Other references made were to the learning (13%), experience (4%), and self (2%).

![Diagram of Analysis of Student Work](image)

Figure 33. Mapping of Nedd’s Sources for Analyzing Student Work and Thinking for Pretest

Figure 34 below shows the references that Nedd made for the posttest interview questions addressing the analysis of student work and thinking. He referenced student
work and thinking approximately 28% of the time while 21% of his references were made to content and teaching. There was increase in his references to teaching and learning in the posttest interview. Nedd referenced learning approximately 16% of the time. Other categories that were referenced were the experience (8%), and self (6%).

Figure 34. Mapping of Nedd’s Sources for Analyzing Student Work and Thinking for Posttest
Research Question 2: What are the effects of two quarters of coursework on Nedd’s assessment of students’ mathematical work and thinking of geometry and measurement?

As we saw earlier, Nedd’s scores on the surveys for the prompts pertaining to analysis of student work and thinking decreased from pretest to posttest. In this section I attempt to address the possible reasons of why his scores decreased.

a) Increase in self-reflection

The questions on the posttest survey and interviews provided Nedd avenues to reflect on his own knowledge and beliefs. Consider the following question, shown in figure 35 that was on the posttest survey:
3. Consider the following dialogue between Amy and Peter as they try to reason why a line segment has only one midpoint:

Amy: (Draws a line segment on GSP. Using the construct menu she marks the midpoint of the segment and begins changing the length of the segment by moving one of the endpoints) See? The midpoint moves with the segment. There is only one midpoint (points at the screen). No matter how long or how short the midpoint moves with it. Here, see if I try to make another midpoint it gives us the same point (goes to the construct menu and selects construct midpoint again). The points coincide.

Peter: I look at it this way... I say if we have a triangle and we construct the median from the vertex then we can have only one median. It means we have only one midpoint. See, let me show you (He draws a triangle and constructs a median. Pointing at the image he explains). See, a median is a segment that connects that point (points at the vertex) to the midpoint of this side (points to the opposite side). We have only one median. So, there is only one midpoint.

Amy nods in agreement. (Manouchehri, in progress)

a. What is your assessment of Amy’s and Peter’s arguments? Do you find these arguments common among high school students? What tools would you use to assess the level of the students’ thinking?
b. What issues would the teacher need to address with children regarding their arguments?
c. What could be contributing to the way that the children argue about uniqueness of the midpoint?
d. If you were the teacher of these children how would you proceed with your lesson? Explain your reasoning.

   a) Amy gets good intuitive feel for midpoint, but it is inductive reasoning.
   b) Peter provides a deductive argument.
   c) Teacher should explicitly point out above.
   d) Amy’s level of focus is only segment. Peter is introducing relations to other geometry (triangle).
   e) Look to add (b) and (c). Both are significant (deduction/relational).

Figure 35. Nedd’s Response to Posttest Item #8

While the responses to the survey did not give much detail about his analysis, the interviews shed light on what Nedd was accounting for while responding to the question.
During the interview, I asked Nedd why he thought Peter’s reasoning was more insightful to which he said that he was more involved in figuring out if there was a deductive argument for solving the problem. He recognized that Amy’s argument was inductive but he seemed to pay less attention to her argument while trying to figure out if this problem could be solved by deductive reasoning (Posttest Interview transcript).

Nedd: Yeah. I think at … at… again at the time when I was reacting to this I … I guess I tried to convince myself that there was a, you know deductive argument for it. And, you know, again I am kinda making that conclu… conclusion … of the specifics but later I think I was sort of retracting that… that the, umm, you know Amy’s view, while it does seem like it’s intuitive, she’s making lots of examples. I… I still wonder whether there is still a deductive way of argument that can be made. (p. 38, lines 1297- 1302)

b) There is no attempt at explaining the student’s mathematical work.

Comparisons of the pre- and the posttest surveys indicated that there was no sincere attempt to analyze the mathematical work of the student. While this is also true in the pretest, it is more obvious in the posttest survey. Figure 36 shows Nedd’s response to a pretest item where he was asked to identify the factors that could have influenced the student’s explanations. While these explanations were very vague, he at least made an attempt to identify what could have influenced the student’s work. In part b of the response, he also made a judgment as to whether a student was right or wrong. However, he also admits that he was not able to understand student S1’s explanation.
However, given a similar question in the posttest (figure 37), he did not even attempt to analyze the students’ work. Rather he dismissed it as saying the student made an observation and commented on it. There was no indication of whether he even agreed with a student or if he thought the students’ work was mathematically correct.
Research Question 3: What are the effects of two quarters of coursework on Nedd’s ability to develop instructional strategies to aid students’ understanding of geometry and measurement?
Figure 38 shows that while making instructional decisions during the pretest interview, Nedd referenced the content approximately 34% of the time, while he referenced teaching about 30% of the time. He made references to students approximately 20% of the time. Of the 34% of references made to content, about 45% of the references were to the mathematical content involved. While referring to teaching, 45% of the references made were regarding his’s goals of instruction for that particular topic. Other references made were to experience (8%), learning (3%), M. Ed. Program (5%) and self (2%).
Figure 39 represents the factors that Nedd drew from while making instructional decisions during the posttest interview. His references to content increased to 44% while his references to teaching decreased to 24%. Nedd referenced students 18% of the time. While referring to content, Nedd referred to the actual mathematical content about 45% of the time. Also while referring to teaching Nedd referred to his instructional goals for the particular topic about 62% of the time. Other than referring to content, teaching and students Nedd also referred to self (4%), experience (4%), learning (4%), and the M. Ed. Program (3%) of the time.
a) *Increased attention to the mathematical content*

There was an increase in the attention given to the mathematical content in the responses to the posttest survey items. In the pretest survey, Nedd’s responses were not content specific, but very general. The pedagogical decisions made were not content specific. Consider Nedd’s response to the following item on the pretest survey (Figure 40):

![Figure 40. Nedd’s Response to Pretest Item # 21](image)

In response to part c which required Nedd to outline a response to the students’ question, he did not make an attempt to address the particular mathematical content of the question. Rather his approach was very generic and it was unclear as to how it would help the student with his understanding of the problem. Now consider Nedd’s response to the same problem on the posttest survey (Figure 41).
Here we see that Nedd’s response was very specific and he tried to further student understanding by relating the area of the triangle to the area of a rectangle and getting students to observe the changes in the height of the triangle.

b) There is greater self-reflection which led to a greater focus on teaching/pedagogy and the fact that he viewed teaching as a more complicated endeavor.

One of the questions that I had asked the participants on both the questionnaires was regarding the ordering of definitions from easiest to hardest and how should these definitions be sequenced to assure student understanding. I noticed that during the pretest survey, Nedd only concentrated on the easiness of the language while sequencing these definitions. Figure 42 shows Nedd’s response to the question in the pretest:
During the interview, when I asked him what factors he considered while judging the level of difficulty of the definitions and for sequencing the definitions, Nedd replied saying that he liked definitions that were simple and did not use complex language (Pretest Interview transcript).

Nedd: Ok so maybe what does students have… where they’re at, what they’re thinking, terms they are used to, umm… I think simple is good, you know. So, you know, I guess the… I thought one of these was you have to from Euclid’s
definition… then maybe the… I’m trying to remember how to… I guess that doesn’t look as bad as I remembered. I remember the question as being rather archaic in the sentence structure and that, you know, unfamiliarity… it’s like trying to read from something from the 17… something from the 1700’s. You know I can get through it but it’s really hard to read. So I think it if it’s something that it…. (p. 26, lines 819-825).

Now consider Nedd’s response to the posttest item regarding the sequencing of definitions in order to assure student understanding (Figure 43).
We can see that there is an element of reflection that is present in his response. Nedd reflected on the content and questioned himself if he believed it was true. This was also evident in his interview where he refers to what factors he would have to consider in order to present a definition to the students (Posttest Interview transcript).

Nedd: Yeah. Okay. Oh Okay. Yeah well see I even questioned right because I had a… I didn’t have an argument. I had a … a knowledge share with Dr. X about polygons and she realized that our conversation was eventually she’s gotta break down and correct my understanding, what a true definition was. And so, uh, and then I think I even mentioned to you my… my thinking about this right? So if people, if mathematicians are… are happy and there’s good reasons which I don’t know about that you know segments or sides… segments of a polygon can’t be collinear, you know which is different than definition A and so now I’ve… there’s an inconsistency in my reading of definitions. Umm, so then I would have to decide, you know I have to like research this a little bit to say which definition do I believe. If people like the collinear definition then I’m gonna… I’m gonna go with the collinear definition, right? (p. 35, lines 1206-1216)

Research Question 4: What is the effect of two quarters of coursework on the quality of questions posed by Nedd to elicit student understanding of geometry and measurement?

As seen above, Nedd’s scores for the prompts pertaining to questioning did not change that much. The scores for mathematical analysis decreased from 67 to 50 while his scores on the pedagogical analysis remained the same (50 for both).

There was a change in Nedd’s claimed perception of what describes a good question. During the posttest interviews, when I asked Nedd what questions he
characterized as good questions, he replied that to him the difference between a question and a good question was analogous to the difference between a problem and an exercise (Pretest Interview transcript).

Nedd: ... I like the definition of problem versus an exercise that I’ve learned. Umm and so I think that those are good questions, umm, are to dominate those that are truly problems. They can’t just solve, you know, 1000, 2000, 3000 and done. (p. 15, lines 479-481)

However his description of a good question changed when I interviewed him again after seven months. Nedd claimed to believe that the description of a good question depended on whether the question was a test or exam question versus a question that a teacher asked in the classroom to her students. He also claimed that multiple choice questions or fill in the blank type of questions could be good questions (Posttest Interview transcript).

Nedd: I think that there are definitely you know fill in the blank, multiple choice questions that can be really good questions. You know, I think that, um, you know a multiple choice question that has a misconception in it as one of the choices, is a good idea. I think that you know, you know the answer shouldn’t be so easy, right? Ah, for multiple choice. But I guess, what’s a little difficult I guess, at least under the, is this, is this a test situation you think? Or it’s like in general?

Researcher: No, it doesn’t have to be a test. It could be in general. (p. 8, lines 274-281)

…
Nedd: Oh, ok, I see. Ok, I see what you’re saying. Um. I’ve actually, the simple you know request to ask them to summarize it or re-explain it to me. Ok, now we’ve done all that. You know, let’s take a moment, catch our breath, you did a bunch of work, tell me what you, tell me what we just did. That, that’s actually a very useful, again, like on a one-on-one situation. (p. 9, lines 302-306)

Thus we see a shift in Nedd’s characterization of questions into good questions. However, this view was not represented in his work. From his surveys, we see that when responding to prompts regarding framing good questions to further student understanding, Nedd used questions that would guide the student. The questions were often about comparing two situations. Figure 44 below shows a sample from the pretest survey in which Nedd’s question was leading the student to apply his method to other shapes and then compare and contrast.

Figure 44. Nedd’s Response to Pretest Item #17
Note that according to the description of comprehension that Nedd gave earlier, he is checking for student comprehension via this line of questioning and not to further student understanding.

Now consider the following item on the posttest survey (Figure 45). Here Nedd is suggesting a comparison between the 2-D and the 3-D views.

![Part 3: Analysis of student work in geometry](image)

1. Consider the following question to be given to students:

   *What can you say about the volumes of a cube with side a, a sphere with diameter a, and a cylinder with height and diameter of the base to be a. What can you say about the volumes of these shapes? What can you say about the surface areas?*

   While working on this problem, a student concludes that the volume of the sphere is less than the volume of the cube since the sphere can fit inside a cube. However, the student is not sure about the volume of the cylinder.

   a. Why do you think the students might conceive of such a relationship?
   b. How do you respond to this student’s conception?
   c. What are some questions you could ask the student to further his/her understanding on this topic?
   d. What tools would you use to assess the level of this student’s thinking?
   e. What levels are relevant here?

   (G) See via mental image, or drawing
   (b),
   (c). Suggest view 2-D from side and ends (approximate height cylinder).

Figure 45. Nedd’s Response to Posttest Item #8

**Research Question 5: What is the relationship between levels of teachers’ self-efficacy and their knowledge of students’ learning and thinking?**

Comparison between Nedd’s pre- and posttest surveys revealed that the mean self-efficacy score for his pretest was 5.208 ($SD=.977$) and the mean score for his posttest was 6.292 ($SD=.620$). A paired samples t-test revealed that the means were
significantly different ($t = -4.376, df = 23, p < .01$). However, a closer look at the three categories revealed that the means for efficacy for student management were not significantly different. Nedd’s mean score for efficacy of instructional strategies for the pretest was $4.375 (SD = 1.060)$ while his mean scores for the posttest were $6.375 (SD = .744)$. A paired samples t-test revealed that the two means were significantly different ($t = -4.320, df = 23, p < .01$). Finally Nedd’s mean scores for efficacy of classroom management for the pretest were $5.375 (SD = .518)$ while his mean scores for the posttest were $6.250 (SD = .707)$. A paired samples t-test showed that the two means were statistically different ($t = -2.497, df = 23, p <= .041$).

Although, Nedd’s overall score on the surveys decreased from pretest to posttest, his efficacy scores increased. Looking at the three factors for self efficacy, we see that his scores increased significantly for efficacy of instructional strategies and efficacy for classroom management. We see that Nedd’s scores for instructional strategies on the surveys also increased. This is not surprising as we see that in his interview, Nedd felt that he had learned a lot from the courses and especially with the assessment models even though he is not able to apply them to practice. However, his scores on the prompts pertaining to analysis of student work decreased. While Nedd felt that he had been exposed to the task of analyzing student work and thinking, he still had trouble applying his new knowledge to practice. Hence we see that there is a disparity between his self efficacy scores and his scores on the survey.

**Summary**

Based on the data presented above I claim that Nedd falls in the naïve idealist category (Cooney et al., 1998). Nedd came into the program with a belief that he needed
to be trained to be a teacher. He possessed the skills required for teaching, which he had acquired via his engineering profession. Nedd never really questioned the readings or the professors. Rather he viewed them as the authority. Nedd also believed that the coursework was helpful to him because it had changed his outlook toward teaching and learning. He believed that the theories that he learned had helped him understand students better. However, we see a disparity that Nedd actually did not understand how the van Hiele model could be applied to analyzing a group of students and designing tasks but when I interviewed him, he said he was comfortable with them and he recognized their importance in education. He never really questioned the findings or readings that were assigned to him. He was often observed quoting his readings and basing his decision making based on those readings. For example, Nedd quotes an article he read which said that in order to have comprehended the material, the student needed to know at least 40% of it. So he assimilated that into his understanding of what comprehension was and never really questioned it.

We see that the interviews led him to self reflect on his own teaching and understanding of mathematics. He often questioned himself about the validity of the content and whether he agreed with what the student wrote. He reflected back on his instructional plans to check whether they were the most suitable or not.

A Cross Examination of the Three Case Studies: The Findings

Table 25 gives the percent of the total references made by the participants to the different factors they cited when analyzing student work and thinking. There, column 1 represents the first level of coding which aimed to capture the factors that the pre-service
teachers referenced. Column 2 was a further break down of the codes in column 1. The percentages indicate the frequency of references to the codes in column 2.

Table 26 has a similar structure to that of table 25 and gives the percent of references to items in column 2 while referencing the factors in column 1. Scores for both the pre and the posttest are provided for each participant.

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<td>0</td>
<td>3%</td>
<td>1%</td>
<td>2%</td>
<td>0</td>
<td>2%</td>
<td>.3%</td>
</tr>
<tr>
<td></td>
<td>Real-life 0</td>
<td>0</td>
<td>6%</td>
<td>2%</td>
<td>0</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
</tr>
</tbody>
</table>

Table 25: Prominent patterns of discourse per participants per interview. Percentages represent number of occurrences of factors while analyzing student work and thinking.
Table 26: Prominent patterns of discourse about analysis of student work. Percentages represent the breakdown of the factors while analyzing student work and thinking.

The tables 25 and 26 show that there is a decrease in the total number of references to the mathematical content while referring to student work, especially in the case of Cersei. Cersei also showed an increase in the references made to curriculum. Bran on the other hand never referenced the curriculum while analyzing student work and thinking. All the three participants showed an increase in references made to student comprehension. Cersei showed an increase in her references to her expectations from students (what they should already know and be able to do) whereas Bran and Nedd’s references to student expectations decreased.

Table 27 gives the percent of the total references made by the participants to the different factors they cited when commenting on instruction and/or instructional decision making. There, column 1 represents the first level of coding which aimed to capture the factors that the pre-service teachers referenced. Column 2 was a further break down of
the codes in column 1. The percentages indicate the frequency of references to the codes in column 2.

Table 28 has a structure similar to table 27 and gives the percent of references to items in column 2 while referencing the factors in column 1. Scores for both the pre and the posttest are provided for each participant.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Cersei</th>
<th>Bran</th>
<th>Nedd</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre</td>
<td>Post</td>
<td>Pre</td>
<td>Post</td>
</tr>
<tr>
<td>Content</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematics</td>
<td>12%</td>
<td>8%</td>
<td>21%</td>
<td>15%</td>
</tr>
<tr>
<td>View of mathematics</td>
<td>5%</td>
<td>3%</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td>Language</td>
<td>6%</td>
<td>5%</td>
<td>1%</td>
<td>5%</td>
</tr>
<tr>
<td>Conceptual Knowledge</td>
<td>0</td>
<td>2%</td>
<td>0%</td>
<td>9%</td>
</tr>
<tr>
<td>Teaching</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Planning</td>
<td>10%</td>
<td>5%</td>
<td>0%</td>
<td>2%</td>
</tr>
<tr>
<td>Goals</td>
<td>10%</td>
<td>8%</td>
<td>4%</td>
<td>6%</td>
</tr>
<tr>
<td>Beliefs about Teaching</td>
<td>5%</td>
<td>3%</td>
<td>4%</td>
<td>2%</td>
</tr>
<tr>
<td>Learning</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beliefs about Learning</td>
<td>2%</td>
<td>0%</td>
<td>5%</td>
<td>3%</td>
</tr>
<tr>
<td>Students</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expectations</td>
<td>4%</td>
<td>0%</td>
<td>4%</td>
<td>5%</td>
</tr>
<tr>
<td>Work/Thinking</td>
<td>9%</td>
<td>10%</td>
<td>3%</td>
<td>7%</td>
</tr>
<tr>
<td>Background</td>
<td>4%</td>
<td>3%</td>
<td>7%</td>
<td>3%</td>
</tr>
<tr>
<td>Comprehension</td>
<td>4%</td>
<td>6%</td>
<td>3%</td>
<td>6%</td>
</tr>
<tr>
<td>Experiences</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teaching</td>
<td>4%</td>
<td>3%</td>
<td>3%</td>
<td>2%</td>
</tr>
</tbody>
</table>

Table 27: Prominent patterns of discourse per participants per interview. Percentages represent number of occurrences of factors while commenting on instruction or instructional decision making.
Tables 27 and 28 indicate that there was an increase in the number of references made to student comprehension while commenting on instruction or instructional decision making. The greatest increase was seen in the case of Bran and Nedd. There was also an increase in the participants references made to conceptual knowledge. This was most prominent in the case of Bran, whose references increased from 0% to 9% (Table 27) and 0% to 23% (Table 28). Cersei showed a decrease in the references she made to the mathematical content while commenting on instruction or instructional decision making.

Thus we see that despite the category in which the participant was classified, there was an overall increase in the references made to student comprehension while analyzing
student work and thinking as well as while commenting on instruction or instructional
decision making. There was a decrease in the references made by Cersei (isolationist) to
the mathematical content while analyzing student work or when proposing instructional
plans. The decline in references to mathematics indicates that the sources contributing to
her decision making were prominently her beliefs about what constituted as quality
learning and her experiences in the classroom. Bran (naïve connectionist), on the other
hand, stressed on conceptual knowledge. Bran’s increase in references to tending to
facilitating or assessing conceptual understanding of the content indicated that his
decision making was prominently motivated by a desire to consider mathematical content
of the child’s work. In the case of Nedd (navive idealist), there was an increase in his
references to work experience while analyzing student work and thinking which indicate
that his decision making was influenced mostly by his personal experiences.

Figure 46 illustrates an overview of common forces influencing the participants’
thinking when judging students’ work and thinking and sources they considered when
making pedagogical decisions. Note that while the degree of influence that each factor
exerted on teacher thinking varied according to the teacher’s particular orientation each
seemed to be a prominent part of the teachers’ discourse.
Figure 46. Common Forces Influencing Participants’ Pedagogical Decision Making

Similarities between cases

1. Pre-service teachers relied on their experiences while analyzing student work and making pedagogical decisions.

   All three pre-service teachers relied on their experiences in making pedagogical decisions. Many times these references to their experiences were not explicitly stated but were evident in their work. For example Cersei’s view of teaching was influenced by her experience as an engineer. When she was asked if being an engineer was different from being a teacher, she claimed that they were not different because while working as an engineer she was also teaching since a part of her job was to train the new engineers that came into her department (Pretest Interview transcript).

   Cersei: I think the one thing too that’s kind of, you know, in the back of my mind led me this direction all along perhaps, but without me realizing it is that one thing that I really liked with industrial engineering was training other people. I
was always the person that trained the new industrial engineers coming into the department. So I guess I’ve kind of been teaching all along without really recognizing it but really liked in that whole experience just to being kind of a mentor and helping so many get started into the department. (p. 6, lines 183-188)

Nedd shared a similar view regarding similarities between teaching and engineering. He responded to the same question by saying that a teacher and an engineer shared a set of common skills and so they were similar professions namely, a focus on continuous improvement of knowledge base needed for practice along with the need to transform that knowledge fluidly and flexibly depending on immediate needs. (Pretest Interview transcript)

Nedd: Yeah. So I think the biggest reason, the biggest thing is, umm, mostly continuous improvement. You know problem solving, adaptability, looking at all the inputs. You know, defining a problem, alternatives, what gets you to that next improvement. I mean, I think those are very drilled into me and I think that’s a skill a teacher needs to have. (p. 3, lines 70-73)

Bran also relied on his prior professional experiences when unpacking what he viewed as his ideal classroom. In outlining the role of the teacher he made close comparisons between what he had experienced and learned the job that he had held for two years where he was trained to professionally facilitate and lead (Pretest Interview transcript).

Bran: I had a … a job for two years where I was you know trained professionally to facilitate and to lead workshops and we always called it weaponizing what you have and so taking the experience you have and using it as a tool to, you know,
help students learn or reach an objective or …. So that I can become a better teacher, umm, in just being able to use all that I’ve already experienced. (p. 5, lines 160-166)

All three pre-service teachers compared teaching to other professions they knew or had experienced and drew parallels between the two situations. Cersei considered teaching to be similar to training other professionals on the job. Nedd was of the opinion that teaching and engineering required the same set of skills. Bran’s perspective was that the teacher’s role was that of a facilitator and a leader. Despite this, Bran and Nedd were the only two candidates who believed themselves to be inexperienced in their pedagogical knowledge. Since Cersei believed viewed teacher preparation as being similar to that of an engineer, it limited her understanding of what skill sets are required to be an effective teacher. In contrast, Bran and Nedd were more receptive to different views on content and pedagogical aspects of teaching.

All three pre-service teachers anchored their ideas about curriculum and instruction in their own experiences as learners of mathematics. Using these experiences as either examples or non-examples of what they felt necessary to do, they referenced them to suggest ideas for how an ideal learning environment could be organized so to optimize student learning. When asked to outline factors they considered while making decisions about what to do in class in response to either students’ needs or mathematical topics to be shared each one referenced their own learning experiences as the defining data sources when doing so. When Bran was asked about how he would structure his lesson plans, he replied that since he had no experience, he would structure the first few
lesson plans based on how he himself had learned geometry, drawing from his experience as a student. (Pretest Interview transcript)

Bran: I think… I think remembering that, that’s how I learned it and that’s how it sank in for me. So I don’t know… I don’t know how I would draw up the lesson plan right now. I think the first ones would be based on how I learned it. (p. 1, lines 27-29).

Cersei on the other hand, recalled her experiences as a student when she was talking about the mathematics classes that she had taken. Unlike Bran, however, she claimed her experience as a student of geometry taught her how not to structure a class. (Pretest Interview transcript)

Cersei: I hate the way I learned geometry and everybody I talk to here at Ohio State say that they love their advanced geometry class and you know I really learned geometry from the… here are the theorems, memorize these and be able to use them and I’d sit there going ok, I was supposed to prove this, given this. I didn’t have any idea how to get from here to here (beginning to end), I didn’t know what these theorems mean and I’d sit there trying to piece them together and it was more just a matter of trying to fit a puzzle rather than actually thinking through the process. So I guess I’ve kinda seen how I don’t think geometry should be taught. (p. 2, lines 38-44)

Nedd drew primarily upon his experience as an observer of a geometry course when he was asked to describe his ideal classroom and the role of a teacher in that ideal classroom. Nedd, like Michelle, described this experience as one that taught him how not to organize mathematics teaching. (Pretest Interview transcript)
Nedd: Um, I have observed classes, like I said, when I was teaching at, or not teaching, when I was volunteering at Gahanna. I was almost the whole time in a classroom and I found that um, you know the lecture for for 90% of the class just is not, doesn’t look appealing, it doesn’t look uh, you know of course the teacher was obviously, they communicated wonderfully. I was sitting there listening and I go, wow they are just so on task. They are not stumbling on terms. They are not confusing. At least for me they weren’t confusing, but, umm, I would say that something opposite of that, uh, would be more, umm, more my ideal. (p. 5, lines 148-154).

The three pre-service teachers did not view teaching as a unique profession with unique demands and tended to generalize actions based on their practical experiences.

2. Pre-service teachers were not able to use assessment models such as van Hiele and Pirie & Kieren to aid in assessing student work or designing instructional tasks.

The posttest surveys had specific prompts for the pre-service teachers enabling them to utilize the learning assessment models (van Hiele and Pirie-Kieren) when examining student’s work and understanding in geometry. However, neither of the pre-service teachers successfully utilized those models and referenced them only when asked explicitly to do so. Neither one of the candidates viewed them to be helpful when designing instruction to further student understanding. On the survey items, either the pre-service teachers left the prompts pertaining to the use of assessment models blank or offered general descriptions of them with no connection to the specific questions they were asked to answer.
During the interviews, all three pre-service teachers mentioned that they did not view such theoretical models helpful when analyzing student work due to their own lack of comfort with their content and structure. Bran expressed these views clearly. (Posttest Interview transcript):

R: Ok. What about the assessment models you've learned about? Like Van Hiele, PK. Do you consider those models helpful? Are they sources when you analyze student's work?
B: I don't know. No they're not.
R: Do you feel like they should be?
B: I don't know. I guess... I don't know. I don't think I'm an expert in them. I think that's not how I picture student work. I think it could be, but I don't feel I'm at the level where I want to be to actually use them effectively. (p. 5, lines 189-195).

Cersei had similar views regarding the use of van Hiele and the Pirie-Kieren models. She expressed that she did not understand them and considered them too theoretical with little practical merit. The only reason she used them in class was because she was required to do so. For her it was more of an afterthought. (Posttest Interview transcript):

Cersei: And it’s been more of an afterthought. More of a I designed this lesson plan because I knew that this was the best way the next best step for this student, in particular from working one on one with the student like what we have in some of our classes. Then you can exactly design what that student needs next. And I get to that without using those tools of van Hiele and PK model. Then I have to take a step back and figure out how does it relate to theory. So I think of this as more theory. (p. 12, lines 383-387).
Cersei viewed them as merely theoretical tools for assessing rather than developing learning and articulated that her preference was to not use them (Posttest Interview transcript):

Cersei: so the PK model, I can’t really even use as much as I, you know, probably, umm, I don’t wanna say would like to be able to or… umm, because I don’t really want to… I really don’t want to work with it anymore. But the reason is not just because I don’t want to learn the PK model but van Hiele and PK model, both of them, I feel like they are more tools for assessing rather than developing learning. (p. 12, lines 374-377)

Nedd was of the opinion that he somewhat understood these theories and them as applicable but could not recognize when they could be used. When I asked him if he thought the theories that were presented to the pre-service teachers in class were helpful, he too commented that he found them too theoretical, lacking lacking connection to practice (Posttest Interview transcript):

Nedd: Um. (pause) I had an, um, um, I’m trying to remember an analogy I had which is more like a barbell. Where um, a lot of theory, which I think is good, and then the coursework had a lot of you know like specifics, the individual case studies. And so I think that like the middle part that’s missing (p. 5, lines 169-172).

None of the three pre-service teachers were able to apply the theories to practice. They did not view these theories as helpful in practice.

3. Even though pre-service teachers did not use theoretical models when assessing student work and thinking, their attention to student thinking improved over time.
In the previous section we observed that the pre-service teachers did not make use of the assessment models to help them better analyze student work and design instructional tasks to further student understanding. However, it was observed that two of the three of the pre-service teachers paid more attention to student thinking and factors that might affect its enhancement.

Bran highlighted the connection between instruction and student’s ways of conceptualization of mathematical topics. Bran’s response to the triangle problem, as shown in figures 25 and 26, are illustrative of a more sophisticated evaluation of student work. In the first figure, which was from the pretest survey, Bran merely cited the difficulty the student had without actually considering what might be the source of this difficulty. However, in response to the same question on posttest survey, he not only noted the specific difficulty that the student was experiencing with finding the area of the triangle but also paid attention to factors that could have led to such a problem. It is also revealing that Bran became more sensitive to student work and thinking while designing instructional strategies. This is evident in his responses to the interview questions. When I asked Bran to elaborate on his response to one of the survey questions regarding how he would respond to particular instances of children’s work, he identified that understanding the content of student ideas as the starting place for planning subsequent instructional move (Posttest Interview transcript):

Bran: When they come up with an answer for which one is the largest, smallest, in between, I want to know how they decided on that. They just guessed? If they happen to guess right, then I can determine if they really don't understand it, and that we need to revisit it. And if they explain exactly why they went through, if
it's all just based on formula, then I know they just focused on the formula. If they can explain based on spatial representation, then I understand they have a different representation in their mind that they can explain it and understand it differently. So I feel like if they can articulate what is actually going on in their heads, then it gives me the clues for what to have them do next to further their understanding in different directions. (p. 8, lines 345-353).

Cersei’s attention to student work also increased as evidenced in a shift in her tendency towards understanding student thinking rather than just detecting errors/misconceptions in the work presented.

Differences between cases
1. Cersei and Nedd, both with backgrounds in engineering were less inclined to pay attention to the mathematical details of children’s work. In contrast Bran, who had completed his bachelor’s degree in mathematics focused on the mathematical details of children’s work.

Both the pre-service teachers from engineering backgrounds paid lesser attention to the mathematical content of children’s work on nearly all items on the posttest surveys. In case of Cersei, we saw in the previous section that her attention to student work had increased. However, her attention to the detail in mathematical content of the student work decreased. Consider the Cersei’s response to problem 15 on the pretest survey (Figure 26). We see that Cersei paid attention to the students’ work, there by detecting the errors and also making judgments regarding the mathematical content of the student’s work. However, on a similar item on the posttest survey (Figure 25), we see that Cersei
only concentrates on the pedagogy while completely deemphasizing the mathematics involved.

These observations were also supported by the interview data. Cersei’s reference to mathematical content decreased from pre- to post- test. When Cersei was pressed during the interview to comment on accuracy of Peter and Amy’s approach to determining the uniqueness of a midpoint, she avoided making a mathematical judgement and instead offered descriptions of what the children may have considered with building arguments (Posttest Interview transcript):

C: They’re both relying on the… one thing I don’t like is that they’re both relying on having to draw and having to see it. They weren’t convinced already like we would be that there’s only one midpoint on a line just based on the definition of a midpoint (p. 20, lines 651-653).

Notice that she did not make an attempt to verify the correctness of the approach taken by Amy and Peter. Rather she gave a vague description of how they might have come up with those interpretations.

As in the case with Cersei, Nedd also did not pay much attention to the mathematical content involved in the students’ work. Consider Nedd’s response to the pretest item number 13 (Figure 47)
Figure 47. Nedd’s Response to Pretest Item #13

Note that Nedd only believed the student was just being sarcastic or “over smart” rather than identifying the mathematics behind what the student did. He dismisses that the student might have had any logical or meaningful thought toward this problem. In contrast, when answering the same question (Figure 48), Bran decided that based on the mathematical definition given, the student was correct. Additionally, when Bran is explaining his answer in part B, he draws on mathematical definitions of polygons as figures with interior angles less than 180 degrees. He also considers that the figure may not be visually accurate and does not assume any measurements because they are not given, which is also a very mathematical approach to the problem.
Although the above representations are only from the pretest, similar patterns were found in the posttest as well.

2. Cersei and Nedd, both with engineering backgrounds, were unable to make connections between different mathematics topics, whereas Bran who was a mathematics major was able to make those connections at the end of two quarters of M. Ed. Coursework.

This phenomenon was most obvious when I asked the pre-service teachers to pick two out of three topics to teach based on their importance. On the posttest surveys, I asked the pre-service teachers to choose any two topics among transformations, similarity and right triangle trigonometry. Bran’s response (figure 29) was indicative of the fact that he saw connections among topics and articulated that he had chosen not to include similarity because it could be derived from transformation (scaling). He also chose right triangle trig because he felt it was the basis for calculus. He was also capable of
explaining how he would introduce students to similarity via the use of transformations (Posttest Interview transcript):

R: Let's go with... you said similarity can be described through transformations, and how would you go about that?

S: I think I'd start with a shape. So a transformation of this would be to make each side longer. So I could keep this here, so I can make everything, like move these points twice as far away from each other. And so I could have two shapes that after the transformation, the shape is still similar. So I can tell that's a special case of transformation when the relationships between the sides and the angles are the same (p. 13, lines 583-589).

Nedd and Cersei, on the other hand, were not able to identify the trajectory of concepts to aid them in choosing the topics they considered as important. Cersei selected the topics of transformations and similarity because she viewed them as related. She chose not to include right triangle trigonometry because she felt just teaching right triangle trigonometry was limiting their view of trigonometry. Nedd opted not to select any topics because he claimed that he didn’t know the connections among concepts sufficiently well to do so.

Viewing Survey Results Through the Three Cases

In this section, I will try to augment the survey results based on the findings from the case studies. As we saw earlier, the survey scores for the prompts pertaining to analysis of student work and thinking decreased significantly from pretest to posttest. The case studies reveal that while analyzing student work, all the three participants, Cersei, Bran and Nedd, paid less attention to the mathematical content of the student’s work,
which explains the decrease in the scores for mathematical analysis. Another observation was that the two pre-service teachers with engineering backgrounds were not able to identify the content trajectories while developing instructional tasks, which again led to the decrease in their scores for instructional strategies. However, in the case of Bran, he was able to identify the content trajectories of concepts and used them to develop his tasks. The interviews also revealed that the reason that they could not identify the content trajectories was because they did not see the usefulness of higher level college mathematics in order to teach school mathematics. Their practical knowledge/application of the subject prevented them from appreciating mathematics as a discipline in itself. They did not see the value of doing mathematics outside of its practical applicability.

The pedagogical analysis scores were correlated to the mathematical analysis scores and hence they also decreased. A factor that had a major influence was the experiences that the pre-service teachers had had. This became evident in their interviews. We now see that one of the reasons that the pre-service teachers failed to utilize the assessment models that they had learned in their methods courses is because they relied heavily on their work experiences as well as experiences as learners and they reverted to those experiences while making pedagogical decisions instead of using the theories.

During the interview with the participants, they revealed that while the coursework increased their awareness about student work and thinking and identifying the students’ misconceptions, it did not help them improve their ability to analyze student work. The survey results reflected this view as they revealed a decrease in the scores for prompts pertaining to analyzing student work and thinking.
Also, we noticed earlier in the chapter that the self efficacy scores validated the survey results. However, we do not see evidence of any direct influence of pre-service teacher’s self efficacy on their pedagogical decision making. These findings merely suggest that this relationship between pre-service teachers’ self efficacy and their pedagogical decision making needs to be explored in detail.
Chapter 5: Summery, Discussion, Implications

This chapter provides an overview of the purpose, procedures, findings, and a discussion of the results of the study. An interpretive analysis as well as a theoretical inspection of factors that could have contributed to the results will be offered. Following a discussion of the limitations of the study, suggestions for future investigations are then offered.

Purpose of the Study

The goals of the present study were twofold. First, it was my aim to explore the nature of mathematical knowledge of a cohort of pre-service secondary mathematics teachers’ knowledge for teaching high school geometry by eliciting their reactions to episodes of children’s work-samples on geometric tasks. A second goal of the study was to investigate the impact of two quarters of coursework centered around the analysis of student thinking about geometry on pre-service teachers’ conceptualization of teaching actions. Five research questions guided the data collection and analysis:

1. What factors do pre-service teachers consider when judging students’ mathematical work and thinking?

2. What is the effect of two quarters of coursework on pre-service teachers’ assessment of students’ mathematical work and thinking of geometry and measurement?
3. What is the effect of two quarters of coursework on pre-service teachers’ ability to develop instructional strategies that aid student understanding of geometry and measurement?

4. What is the effect of two quarters of coursework on the quality of questions posed by the pre-service teachers to elicit student understanding of geometry and measurement?

5. What is the relationship between levels of teachers’ self-efficacy and their knowledge of students’ learning and thinking?

This research project was different from studies of teachers’ knowledge conducted in the past both in its examination of the content and in methodologies used in conducting the research. First, the study considered the Mathematical Knowledge for Teaching (MKT) of secondary pre-service teachers for the teaching of geometry. Studies examining the MKT of secondary teachers are rare especially in the area of geometry. Secondly, data collection consisted of both surveys and interviews which examined the full complexity of MKT as it pertained to analysis of student work and thinking and spanned over a course of 7 months during which I observed the Pre-service teachers in courses they took while in teacher preparation program. Such longitudinal and close inspection of teachers’ educational experiences provided the ground for a deeper analysis of data, allowing me an explanatory capacity for findings of the study.

**Theoretical Framework**

This research was informed by two genres of scholarly perspectives on professional knowledge construction and ways in which such knowledge might be elicited. On the one hand, the theory of situated cognition was utilized while designing
tasks and developing methodologies for task implementation in the methods courses that the participants completed. As such, the tasks were designed so to ground the teachers’ analysis of teaching in the episodes of teaching and learning. In order to achieve this, I utilized the case-based methodology to offer the pre-service teachers with avenues to not only gain insight into dilemmas of practice but also to provide opportunities for deep mathematical analysis (Manouchehri, & Almohalwas, 2008). Following the principles of social cognition throughout their program the pre-service teachers were encouraged to interact with one another and to solve mathematical and pedagogical problems collaboratively and in discussions with peers. According to Manouchehri and Enderson (2003), pre-service teachers need to interact within the teaching community in order to learn the norms and practices of their profession.

On the other hand, the Knowledge for Algebra Teaching (KAT) framework (McCrory et al., 2010) was utilized as the vehicle for the design of data collection instrument as well as how the data obtained from the participants were analyzed. KAT distinguishes the tasks of teaching to include analyzing students’ mathematical work and thinking, designing, modifying and selecting mathematical tasks, establishing and revising mathematical goals for students, accessing and using tools and resources for teaching, explaining mathematical ideas and solving mathematical problems, and building and supporting mathematical community and discourse (McCrory et al., 2010). The categories of knowledge include core content knowledge, knowledge of representations, knowledge of content trajectories, knowledge of application and contexts, knowledge of language and conventions and knowledge of mathematical proofs and reasoning (McCrory et al., 2010). The three overarching categories are bridging,
trimming and decomposing. Thus the framework provides us with 36 cells, each of which describes the importance of a particular category of knowledge for a particular task of teaching and how that knowledge should be utilized through the processes of bridging, trimming, and decomposition. The participants instantaneous knowledge along specific dimensions of KAT were elicited so to be able to trace their growth and development overtime.

Methodology

Participants

The study was conducted in a large Midwestern university. The participants of the study were a cohort of 8 pre-service teachers. Two of the participants had backgrounds in engineering, while the other six participants had completed a bachelor’s degree in mathematics. The sample consisted of four males and four females. Of the eight participants, three participants were chosen for an in depth case study. The selection of the candidates was guided by Cooney et al., (1998) categories of pre-service teachers’ orientation towards learning to teach: isolationist, naïve idealist, naïve connectionist and reflective connectionist. This classification allowed me to link the pre-service teachers’ beliefs to their analysis of student work. A brief description of each of the three candidates is offered below.

Cersei

Cersei worked as an industrial engineer before she joined the M. Ed. Program. Prior to joining the program, Cersei was enrolled in another alternative certification program where she completed all required courses except one. Cersei fell under the isolationist category. The M. Ed. coursework had very little impact on Cersei’s beliefs on
the teaching and learning of mathematics. She viewed the assignments that she was required to complete in the program as being only requirements for the coursework and as such saw minimal use for them in practice. Cersei’s view of mathematics was so rigid that she rejected alternate views of the subject. This was also evident in a couple of her responses where she refused to move beyond how she viewed a problem. She also never questioned her own beliefs about teaching and learning and about mathematics even when others directly pushed her towards questioning to her own beliefs.

_Bran_

Bran completed his bachelor’s degree in mathematics. He joined the M. Ed. Program to obtain a licensure to teach 7<sup>th</sup> to 12<sup>th</sup> grade mathematics courses. Bran showed a lot of interest in and enthusiasm for learning new material. 

Bran exhibited characteristics of a naïve connectionist, as described by Cooney and colleagues (1998). In describing teaching ideas and outlining pedagogical plans Bran drew from his own personal experiences and interactions with more knowledgeable figures guiding his development, namely his mentor teacher, his professors of mathematics and mathematics education. However, despite speaking positively about the various theories he had learned about in his teacher education program, he was unable to resolve the clashes between his own beliefs about student learning and trajectories these theories outlined. He was unable to apply these theories in practice. Bran specifically acknowledged the importance of learning progressions when assessing individual student’s work and planning instruction for that student and showed evidence of knowledge about the content and structure of theory, but he did not integrate that knowledge meaningfully in pedagogical schemes he had developed. Instead, he relied
solely on his experience as a classroom teacher to contemplate student learning and thus, pedagogical decision making.

**Nedd**

Nedd had a background in computer science and electrical engineering before joining the M. Ed. Program. Unlike the other pre-service teachers in the cohort who were on a 1 year track, Nedd opted for an extra year to finish the program. Nedd was also taking mathematics classes in order to fulfill the requirements that the program demanded. Nedd was not assigned a student teaching appointment but had opportunities to observe teachers and interact with school children both formally and informally.

Nedd resembled Cooney et al.’s (1998) naïve idealist typology of orientation towards learning to teach. It was observed that Nedd readily accepted the views and beliefs to which he was introduced without questioning them. Nedd was quick to accept legitimacy of ideas shared in course readings. He felt as if he needed this knowledge and he trusted the teacher educators and his professors and never questioned them. Nedd also believed that the coursework was helpful to him because it had changed his outlook toward teaching and learning. He believed that the theories that he learned had helped him understand students better.

**Instrument Development**

Cohort data were collected via pretest and posttest surveys. The surveys were specifically designed to elicit the pre-service teachers’ Mathematical Knowledge for Teaching (MKT) through the processes of trimming, decompressing, and bridging. While some questions spanned multiple processes, in total fifteen questions aimed to capture decompressing, nine trimming, and seven bridging.
The KAT framework guided the design of all data collection instruments including the surveys and interviews. To aid in exemplifying decompressing, trimming, in bridging, the survey items were developed from the intention to draw out any knowledge bases from which the teachers drew when analyzing students’ mathematical work and thinking. Previously recorded episodes of children engaged in problem solving on geometric tasks from a longitudinal study were utilized. The episodes were also chosen to illustrate rich instances of children’s mathematical thinking, heuristic usage, and reflexive actions (Manouchehri, 2012). Additionally, the mathematical content topics included in the videos were common to the U.S. secondary school curriculum. Although there were differences between the pretest and posttest surveys, they were designed so that cross comparisons could be carried out.

To establish content validity of each of the instruments, feedback was obtained from a variety of experts including practicing middle and high school teachers, mathematics educators, and mathematics education graduate students. A pilot testing of the instruments was carried out on mathematics education graduate students and pre-service teachers from the 2011 cohort and subsequent necessary modifications were made. Care was taken to remove ambiguities in the language and questions to the best of my ability.

Data Collection

The data collection consisted of two phases and commenced in the summer quarter of 2011. During the methods course in number sense and data analysis, which was offered during the first term of summer 2011, the incoming cohort of pre-service teachers completed a pretest survey as a part of their coursework, which elicited their
knowledge at the start of the program. The pretest survey consisted of 25 questions out of which 15 questions were open ended. Pretest interviews were conducted during the start of the autumn quarter of 2011. The aim of the pretest interview was to gather in-depth information about the pre-service teachers’ responses to the pretest surveys.

The second phase of the data collection began at the end of the two quarters of course work in the M. Ed. Program required the pre-service teachers to fill out the posttest surveys, which were given to them as a part of a course assignment. The posttest survey consisted of 21 questions out of which 14 questions were open ended. This survey was followed by a posttest interview. The posttest survey and interview was similar in structure to the pretest survey and the pretest interview.

Data Analysis

The data analysis consisted of two phases. During the first phase the pre-service teachers’ survey responses were coded and scored along mathematical analysis and pedagogical analysis using indicators that were identified by Manouchehri (2011). As such, the survey responses were ranked on a continuum from Mathematically and Pedagogically Naïve to Mathematically and Pedagogically Mature. The responses were also assigned numerical scores. Finally percent scores for prompts pertaining to decompressing, trimming and bridging were then computed. A paired samples t-test (α=.05) was conducted to determine whether there was a significant difference between the pre and post scores.

Interview data: The interviews were video recorded and transcribed. Content analysis of each transcribed interview then followed. Instead of completing a line by line coding of the transcripts, I opted to code the entire responses to a question as a
hermeneutic unit in an attempt to identify the loci of concerns and attention of the participants as described by them, first in each of the responses they provided to questions asked and then throughout the interview. This content analysis focus allowed me to keep all the responses in context. It also helped to keep from segmenting participants’ ideas on teaching and learning that were not expressible in a single line. Once the coding was completed, I calculated the percentage of occurrences for each coded category. I then used the percentages to prepare a map to illustrate the sources that the pre-service teachers drew from while analyzing student work and thinking. I triangulated my interview data with the survey data. For this purpose, I compared the survey responses to the interview responses. In the event that the response to the survey was vague or unclear, I compared them with the responses to the interviews to clarify and assure that my conclusions were correct.

Results

*General findings from the Cohort data*

1. What factors do pre-service teachers consider when judging students’ mathematical work and thinking?

   The major factor affecting teacher’s pedagogical decision making while analyzing student work and thinking was their past experiences. With the exception of two, all the participants relied on their experiences as learners of mathematics while two of the participants who had previous careers in other fields, relied on their work experiences while making decisions.
2. What is the effect of two quarters of coursework on pre-service teachers’ assessment of students’ mathematical work and thinking of geometry and measurement?

Results from the surveys indicated that the pre-service teachers’ scores on items pertaining to analysis of student work and thinking decreased significantly from pretest to posttest. Pre-service teachers’ scores on prompts pertaining to decompressing were also significantly lower on the pretest surveys. However, there was no difference between the scores pertaining to trimming and bridging.

3. What is the effect of two quarters of coursework on pre-service teachers’ ability to develop instructional strategies that aid student understanding of geometry and measurement?

There was no significant difference between the scores between the pre-service teachers’ pre and posttest scores on prompts pertaining to developing instructional strategies.

4. What is the effect of two quarters of coursework on the quality of questions posed by the pre-service teachers to elicit student understanding of geometry and measurement?

There was no significant difference between the scores between the pre-service teachers’ pre and posttest scores on prompts pertaining to quality of questions posed by pre-service teachers in order to elicit student understanding.

5. What is the relationship between levels of teachers’ self-efficacy and their knowledge of students’ learning and thinking?
There was no significant difference between pre-service teachers’ self efficacy scores from pre- to posttest. No changes in the pre-service teachers beliefs in their own abilities to create instructional tasks that would promote further student understanding were detected. Lastly, the mathematical analysis scores were statistically correlated to the pedagogical analysis scores. Decrease in the mathematical analysis scores indicates that the pre-service teachers’ attention to the mathematical detail of the students’ work decreased and so did the quality of their instructional strategies. Individual case studies provided an outlook for understanding these results, as outlined below.

**General findings from the case study data**

In order to understand the cohort results better, an in depth case study of three of the participants’ thinking was completed. The finding of the study, according to each of the research questions guiding this work is offered in light of analysis of data from Cersei, Bran and Nedd.

1. **What factors do pre-service teachers consider when judging students’ mathematical work and thinking?**

   **Cersei**

   Analysis of the interview data revealed that Cersei’s reference to children’s mathematical work decreased from pre- to posttest. However, the number of references she made to students and their need increased. Her beliefs about teaching and learning of geometry dictated her orientation towards analysis of student work. Another prominent force that guided Cersei’s orientation to student work was her self-efficacy about classroom management. Cersei felt the need to be in control and her actions were guided accordingly. Her experiences as a learner also had an effect on her analysis of student
work and thinking. Cersei claimed that she learned mathematics via direct instruction and that it did not help her understand the subject and hence she was of the opinion that direct instruction was not useful and that instruction should be student centered.

*Bran*

The main factors influencing Bran’s orientation to the task of analyzing student work and thinking were his experiences as a learner as well as his knowledge of the subject matter. Bran referenced his experiences as a learner whenever he talked about developing lesson plans or describe his ideal classroom. Bran was the only participant who saw the connections between higher level college courses and secondary school curriculum and as a result he drew on his knowledge of content trajectories while selecting topics that he felt were important for students.

*Nedd*

Nedd drew parallels between his work experience and teaching. Nedd believed that he had the skills required to be a teacher since he viewed teaching to be very similar to his job as an engineer. Nedd avoided getting into the mathematical details of the students work. Nedd’s belief that it was necessary to struggle in order to learn something was consistently referred to in his work. Nedd made the maximum number of references to students in terms of his expectations from them, student behavior and student work.

2. *What is the effect of two quarters of coursework on pre-service teachers’ assessment of students’ mathematical work and thinking of geometry and measurement?*
**Cersei**

There was a significant decrease in Cersei’s scores for both mathematical and pedagogical analysis on the prompts pertaining to the analysis of student work and thinking. After augmenting her interview data with her survey data it was also noticed that despite an increased attention to student work, there was a decrease in her attention to mathematical content. There was an observed disparity between her claimed beliefs and her enacted beliefs. Cersei also could not make explicit connections to models of assessment and thus was not able to use them in her analysis of student work and thinking.

**Bran**

Bran was the only participant whose overall scores increased from pretest to posttest. However, his scores for prompts pertaining to analysis of student work and thinking decreased from pretest (mathematical analysis: 54, pedagogical analysis: 57) to posttest (mathematical analysis: 39, pedagogical analysis: 47). His scores for decompressing also decreased from pretest (mathematical analysis: 58, pedagogical analysis 59) to posttest (mathematical analysis: 40, pedagogical analysis: 47) indicating that his ability to understand student work and work backwards to identify the source of error in the work had increased. In the area of analyzing student work and thinking, it was noticed that although in the interviews Bran showed greater attention to the mathematics work of children as described in each task; he did not make explicit connections to models of assessment. He showed the tendency to try and understand the basis for student work.
Nedd

Nedd’s overall scores showed a decrease from pretest to posttest. His scores for analysis of student work and thinking and decompressing decreased from pretest to posttest. Furthermore, it was observed that on tasks pertaining to analysis of student work and thinking, there was an increase in the number of self-reflective statements he made. He appeared more conscious of his own mathematical knowledge as well as how his decisions affected children’s learning.

3. What is the effect of two quarters of coursework on pre-service teachers’ ability to develop instructional strategies that aid student understanding of geometry and measurement?

Cersei

Cersei’s scores on prompts pertaining to developing instructional strategies decreased from pre- to posttest. While designing instruction, her attention to the mathematical details of the lesson decreased. This stemmed from her increased consciousness to her own difficulties with the mathematical content.

Bran

Bran’s scores on prompts pertaining to developing instructional tasks decreased from pre- to posttest. However, while planning instructional strategies, Bran’s focus changed from short term mastery of skills to long term development of ideas. His confidence in his ability to teach remained high despite that he viewed teaching to be a more complex endeavour compared to what he had envisioned at the beginning of his program.
Nedd

Nedd’s scores for the pedagogical analysis of prompts pertaining to instructional strategies also increased from 31 to 33. His scores for trimming of mathematical content increased from pretest (mathematical analysis: 36, pedagogical analysis: 30) to posttest (mathematical analysis: 50, pedagogical analysis: 42). Although his attention to the students’ mathematical work decreased for the prompts pertaining to designing instructional strategies, greater focus on the mathematical content was revealed. At the end of the second quarter Nedd also admitted that he viewed teaching as a more complicated endeavor than previously envisioned.

4. What is the effect of two quarters of coursework on the quality of questions posed by the pre-service teachers to elicit student understanding of geometry and measurement?

Cersei

Cersei’s scores on prompts pertaining to good questions decreased from pre- to posttest. Cersei did not list questions that she thought were good questions on a given topic that would further student understanding and instead outlined tasks that she would conduct in response to student work. Additionally, she did not directly connect how her questions would help the students grow to understand the concept. As a general trend on the pretest items, Cersei did prefer allowing the students to discover patterns after exposing them to several examples. However in the posttest, she gave increasingly more vague and generalized approaches to answering questions including suggesting that use of technology was important without referencing how or why that technology might be used.
Bran

Bran’s survey scores on prompts pertaining to questioning decreased from pretest to posttest. However, there was a noticeable change in his perception of a good question. At the beginning of the program, Bran characterized a good question as one that guided the student to the answer without actually giving them the answer. However, during the posttest interviews, his characterization of a good question was one that furthered student understanding. There was a shift in focus on student comprehension.

Nedd

Nedd’s scores for the prompts pertaining to questioning did not change significantly. The scores for mathematical analysis decreased from pre- to posttest, while his scores on the pedagogical analysis remained unchanged. There was a change in Nedd’s characterization of a good question. At the start of the program Nedd viewed the difference between a question and a good question to be similar to the difference between a problem and an exercise. At the end of the two quarters, Nedd was of the opinion that whether or not a question is good depended on the situation: whether the question was an exam question or was it a question that the teacher asked in class.

5. What is the relationship between levels of teachers’ self-efficacy and their knowledge of students’ learning and thinking?

Cersei

There was no significant difference between Cersei’s self-efficacy scores. It was observed that Cersei’s beliefs about teaching and learning did not change overtime. Her pedagogical decisions were influenced by the beliefs she had developed previously in her career and were quite unaffected by the range of experiences she encountered while in the
program. It is plausible to attribute a lack of change in her self-efficacy scores to the strength of these beliefs.

_Bran_

Overall, Bran’s self efficacy scores increased from the pretest to the posttest, however there was no significant difference in his self efficacy in the category of instructional design. This lack of significant difference was surprising given Bran’s increased scores on the survey in the categories of ability to design instruction and pose good questions. However, after a closer examination, we see that Bran’s score for the question about using a variety of assessment strategies is lower for the posttest than for the pretest. He ranked himself 7 on the pretest self efficacy survey whereas he ranked himself a 4 on the posttest. This is supported by our findings from the surveys and the interviews where, Bran experienced difficulty in using assessment models to aid in his analysis of student work.

_Nedd_

There was a significant increase in Nedd’s overall self-efficacy score. His self-efficacy scores for instructional strategies and classroom management were significantly higher in the posttest. Nedd’s scores for instructional strategies on the surveys also increased. During the posttest interview Nedd revealed that he felt he the courses and especially the assessment models even though he is not able to apply them to practice had influenced his decision making.
Cross Analysis

A cross comparison between the three cases led to the following observations:

1. Pre-service teachers relied on their experiences while analyzing student work and making pedagogical decisions.
2. Pre-service teachers were not able to use learning based assessment models such as van Hiele and Pirie-Kieren to aid in evaluating student work and designing instructional tasks.
3. Knowledge of mathematics and its trajectory had an impact on how participants evaluated school children’s work and planned to organize instruction.
4. Even though pre-service teachers did not use assessment models to aid them in assessing student work and thinking, their attention to student thinking increased.
5. Cersei and Nedd, both with engineering backgrounds, were unable to make connections between different mathematics topics, whereas Bran who was a mathematics major was able to make those connections at the end of two quarters of M. Ed. Coursework.

Discussion of Findings

Figure 49 illustrates the factors affecting teachers’ pedagogical decision making. As the figure illustrates, teachers’ past experiences as learners of mathematics and their professional experiences served as key ingredients for how they examined issues of learning and teaching. Teachers’ knowledge of trajectory of the content, along with their ability to draw from that knowledge to make sense of mathematical practices of children played a critical role in their ability to offer relevant pedagogical plans when examining children’s mathematical ideas. Lastly, the need to control student learning and the desire
to quickly diagnose and fix what they perceived as gaps in children’s knowledge was pivotal to how they went about analyzing children’s practices and suggestions they offered for curriculum and instruction.

Less influential or less frequently considered factors when judging children’s mathematical thinking and work across the cohort and most prominently among the three cases examined included: knowledge of learning theories and references to how such knowledge could inform practice. These are discussed more fully below.

![Diagram of Common Forces Influencing Participants’ Pedagogical Decision Making](image)

Figure 49. Common Forces Influencing Participants’ Pedagogical Decision Making

*Teachers’ personal theorizing: Past experiences*

Nearly three decades ago, Elbaz (1983) coined the construct of teacher’s practical knowledge and defined five sources of such knowledge to include: situation, personal, social, experiential, and theoretical. Elbaz (1983) also described how the structure of teachers’ practical knowledge included rules of practice, practical principles, and images
that guide actions. Most prominently characterized as Teachers’ Personal Theorizing, Elbaz (1983) illustrated the dynamic interactions between teachers’ personal theories, beliefs, and practices. Watershed work by scholars such as Alba Gonzalez Thompson (1984, 1992) and Richardson (2003) have shown that teachers’ beliefs and personal theories about teaching are rarely changed by interventions imposed in teacher education and indeed what is learned in teacher education often does not carry beyond student teaching (Thompson, 1992; Lortie, 1975). There is also general agreement that teachers’ beliefs shape their judgments and actions in the classroom during and after student teaching.

Levin and Ye (2008) posited that teachers use their Personal Practical theories as their personal guiding theories in the pre-active (planning), interactive (teaching), and postactive (reflective) stages of their teaching (pg. 56). Following an extensive review of the literature, the authors suggested that beliefs of both pre-service and experienced teachers expressed as their personal (outside classroom) practical (inside the classroom) theories are the fundamental sources of their pedagogical decisions about teaching and learning (pg. 58). In studying the sources and content categories of teachers’ beliefs and practical personal theories, Levin and Ye (2008) reported that over 40% of their 200 participants’ sources of teachers’ decisions came from their own personal and educational background and 37% from their more recent observations and teaching experiences in their field placements. The authors argued that the teacher education program coursework seemed to have the most influence (37%) on their beliefs about instruction. In the Practical Personal Theorizing (PPT) content category of the classroom, the major source of beliefs expressed by the subjects also stemmed from personal background and
their own educational background (40%). In the category of beliefs about students and the nature of student learning, the major source of the participants’ PPTs came from recent observations and teaching experiences in their field placements (37%) required for their teacher education program. The authors concluded that lack of influence of teacher education programs on beliefs about the classroom as a learning environment supported Lortie’s (1975) claim regarding the power of prior beliefs based on the apprenticeship of observation. Similar results were observed among the participants in the current study. Content analytical maps of the in-depth interviews provided evidence that the pre-service teachers in the current project also relied on past experiences, classroom observations and mentor teachers’ actions as sources for making pedagogical decisions regarding how to organize and implement curriculum. The most influential factor affecting teachers’ pedagogical decision making was their past experiences both as learners of the content as well as their work experiences. Indeed, in case of those participants with the most substantial professional experiences pedagogical decision making was often made independent of an attention to children’s work and instructional implications that such work might have implied. In almost all cases, when references to children’s mathematical work were made analysis emerged from the individual’s own experiences as mathematics learners. Teachers also emulated the actions of mentor teachers, relying on models they had seen as ways to offer venues for pursuing pedagogy. These personal experiences provide a lens through which the participants examined learning issues or even assessed what they had learned in the methods courses. The participants’ past mathematical experiences also affected their views about the content and their focus on student thinking. This influence remained strong throughout the two quarters of
coursework. Their experiential knowledge had a strong influence on their view of mathematics. For example, Cersei and Nedd did not view higher level college mathematics courses as important thereby limiting their ability to identify the content trajectories, whereas Bran who recognized the importance of college level mathematics courses was able to not only identify the content trajectories but also utilized them while developing instructional tasks.

**Mathematical Content Knowledge**

Findings of the study suggest that knowledge of content trajectory was a far more influential factor on teacher thinking and decision making than exposure to theoretical models of children’s thinking. According to McCrory et al. (2010), a content trajectory can be conceptualized as a sequencing of mathematical topics such that it best supports student learning. This category of knowledge also includes alternate ways of approaching a problem and selecting canonical examples (McCrory et al., 2010). Knowing the trajectory of content includes knowing what ideas are central to the topic and which ideas are peripheral (McCrory et al., 2010). This category of knowledge has also been recognized by Even (1990), wherein she argued that teachers should be familiar with “powerful examples that illustrate principles, properties, theorems etc” (p. 525). In order to be efficient, teachers require “understanding connections between different mathematical concepts, understanding underlying mathematical concepts in a deeper way to be able to assess alternative approaches and solutions” (Falkenberg, 2011; p. 59).

Vale, McAndrew, and Krishnan (2011) found that pre-service teachers’ pedagogical decision making was entwined with their knowledge of the content trajectories. In an earlier study Aubrey (1996) found that teachers who had a strong hold
on the mathematical concepts were able to successfully design tasks whereas teachers who lacked the subject matter knowledge were not able to design effective tasks. The teachers who were weak in their subject matter knowledge were not able to connect the tasks to the children’s mathematical knowledge and thus were not effective in furthering student learning (Aubrey, 1996). Kahan, Cooper, and Bethea (2003) studied a pre-service teacher and observed that in areas where her mathematical content knowledge was strong, she was able to help students focus on the central idea, whereas in areas where her mathematical content knowledge was not strong, she was not able to do so. Her unfamiliarity with the mathematical content limited her ability to make pedagogical decisions (Kahan et al., 2003).

Similar findings were observed in the current study. Pre-service teachers, who were unable to see the trajectories in content, avoided the mathematics involved while addressing student work. The two Cersei and Nedd were uncomfortable with some of the content and thus did not see the connections between the higher level mathematics courses and also did not appreciate the mathematical structure and thus were not able to utilize mathematical content knowledge while making pedagogical decisions and designing tasks. Bran on the other hand was able to draw from his knowledge of mathematics and was able to structure his lessons in a way that would foster student understanding and learning. This was evident when the pre-service teachers were asked to select two out of three topics to teach. While Bran picked the topics based on the content trajectory and connections to future mathematical concepts, Cersei and Nedd offered no such explanation.
Teachers’ Beliefs

Manouchehri (1997) argued, “Teachers translate their knowledge of mathematics and pedagogy into practice through the filter of their beliefs” (p. 198). The findings of the study support this claim and further expand on the structure of those beliefs that influence teacher thinking, as described below.

The three case studies revealed a significant disparity between what teachers claimed they believed and the beliefs that were evident in their assessment of student work. For example, Cersei claimed to believe that collaborative work and communication in the classroom were imperative to their understanding of the subject; however, her work seemed to suggest that she believed in a teacher centered classroom where the teacher had the central role as the giver of knowledge. She considered the teacher to be the sole authority when judging correctness and appropriateness of children’s work.

It was also noted that many of the participants’ beliefs stemmed from their previous experiences and professional backgrounds. These professional experiences seemed to have provided them with a lens through which they defined teaching roles and viewed progress. For example, Nedd’s considered teaching and engineering to be similar, relying on the same techniques and skills. This view granted him the tendency to view student learning as a “problem” to be solved, oftentimes quickly through specific actions. Ironically, he was the most reluctant to define student learning in terms other than procedural progression towards mastery. In light of this view, Nedd considered teaching as a problem solving process, with the main goal of the problem solver being the identification of deficiency or deficient element in the system and resolving the dilemma.
accordingly. Cersei also viewed teaching to be similar to engineering because her job as an engineer was to train the new engineering recruits.

Both of the participants with engineering backgrounds believed that higher level college mathematics was not necessary for teaching. They believed the mathematics they had sufficient since they felt successful in having completed their past professional duties with that knowledge base. Since both equated engineering and teaching (as professions), they also espoused that the mathematics needed for engineering should suffice for teaching mathematics as well. Neither of these two participants acknowledged the importance of knowing or learning higher level abstract mathematics courses such as analysis and algebra and seemed unable to attach mathematical significance to the mathematical practices of children depicted in problems used. This, in turn, limited their ability to trim and bridge the mathematical content in ways to make the content accessible to children or to build around children’s ideas to advance their thinking. They were not able to design tasks that addressed the content trajectory. They either failed to see connections between what children had asked or showed to school curriculum the connections they made were superficial. It was observed that in places where the teachers had an understanding of these connections they were more inclined to contemplate accuracy of the children’s mathematical work or to design lessons that emphasized the applications of concepts in different areas.

Looking back, moving forward: Interpretations and Implications

The primary goal of the research project was to investigate factors that pre-service teachers considered when judging and assessing mathematical work of children and ways in which their pedagogical decision making might have been influenced by such an
assessment. An additional aim of the study was to explore the impact of two methods courses on teachers’ thinking relative to the first consideration of this research. Guided by the situated cognition perspective on teacher learning and complying with the social constructivist view on enhancing learning both the course experiences and measurement instruments were developed so to tap into the mathematical knowledge of teachers and enhancing that knowledge. A primary tool used for both enhancement of knowledge and eliciting it throughout the data collection phase include case-based analysis of problems of teaching and learning. In retrospect, and considering the results of the study, it appeared that such an approach was not particularly effective in anchoring the habit of theory-based analysis among teachers as depicted in no significant change in the quality of teachers’ assessment and their sensitivity to children’s mathematical cognition. These outcomes can be interpreted in a number of ways. On the one hand, one could suggest that instruments used in the study were not adequate in eliciting or capturing teacher knowledge. On the other hand, lack of change might be attributed to ineffective instructional approaches used in methods courses. Both these interpretations are legitimate and demand consideration.

The findings confirm previously reported observation that pre-service teachers tend to ignore mathematical issues vital to analyzing student work as well as to consider instructional strategies that could be used to address them (Manouchehri, & Enderson, 2003). Similar to Manouchehri and Enderson’s (2003) participants, the cohort studied here tended to focus more and be more concerned with teaching action and instruction instead of unpacking or understanding children’s mathematical work. Such a focus can provide an explanation for why the participants’ survey scores decreased from pre- to
posttest. Previously, Manouchehri, & Enderson (2003) alerted that “when using case-based tasks with teacher candidates, facilitators should deliberately stress content analysis as an explicit part of individual and group investigations by posing specific mathematical questions for students to consider and examine (p. 243).” In this work, the cases used, had included prompts that intended to motivate the pre-service teachers to reflect on the content of the cases, these prompts were not specific enough to offer them a platform for analysis. Many of the prompts were general and as a result the pre-service teachers refrained from addressing the mathematical content involved in the children’s work. A number of participants in this study avoided judging whether the child’s work was mathematically correct. Instead they concentrated on the general pedagogical aspects of the case, offering models of teaching that were too general to have specific applications to the issue under study. Perhaps, when designing such instruments for data collection, care should be taken to make sure that the reflective questions call for a rigorous mathematical analysis particularly if claims are intended to be made about teachers’ knowledge. The usefulness of such an approach certainly merits attention despite the fact that it can also funnel teachers’ responses as opposed to eliciting their authentic modes of thinking and reasoning which may be used by them while in real classrooms.

Two decades ago Sykes and Byrd (1992) highlighted the complex nature of selection and sequencing of cases with other elements of teacher education and called for the need to design and study, robustly, research on impact and implementation of cases so to further explore issues of teacher learning: what do teachers learn from different types of cases and how do they learn it.

In nearly all cases used in the program and items that teachers explored they were
asked to identify what the student understood or did not understand and how the teacher might respond to the student and which response will be the more efficient. Some of the situations deliberately provided opportunities to raise challenging mathematical issues for teachers. While the scope of the current project did not include a systemic study of what teachers learned from their participation in these case discussions the outcome of their use, based on responses provided on the surveys and during interviews, did not match the effort invested in both design and implementation. While the results of previous research on what teachers had gained from analysis of cases remains inconclusive, some claimed great mathematical gains (Smith et al, 2005a; 2005b; 2005c; Manouchehri, 2010), while others propose substantial learning in the area of pedagogical content knowledge among learners as the result of their use (Smith et. al, 2005a; 2005b; 2005c), additional research is needed to critically evaluate the scope and type of learning gained from such activities. Perhaps a combination of exemplars (Carter, 1999) could assist teachers in develop ways to how to operationalize theories of learning, providing concrete instances of how to ground abstract ideas about learning, teaching and mathematics in real classrooms.

Pre-service teachers’ prior experiences

The results of this study are consistent with the views expressed by Ponte, Oliveira, and Varandas (2002):

It is not enough for preservice mathematics teachers to have knowledge of mathematics, educational theories, and mathematics education. Experience with these matters established on a purely theoretical level, in terms of declarative knowledge, does not guarantee an effective acquisition of professional knowledge. The fact that this knowledge is deeply personal

290
and connected to action and to reflection upon experience implies that for its development pre-service teachers need imaginative and varied working environments as well as experience of situations as close as possible to real professional practice (pg. 96).

Findings of the present study suggested that the pre-service teachers relied on their past experiences as learners and work experiences when making pedagogical decisions. This is not a new phenomenon. Manouchehri (1997) observed that pre-service teachers rely heavily on their experiences as learners while teaching. For example, Cersei and Nedd relied heavily on their engineering backgrounds and work experiences. Since past experiences can facilitate assimilation of new knowledge, shaping our conceptualizations of new knowledge (Piaget, 1964), these engineering backgrounds shaped both Cersei and Nedd’s beliefs for teaching of mathematics. Bran, on the other hand, cited his limited work experience and thereby drew on the only experience he possessed, that of being a student. In other words, we acknowledge the need to consider the diverse experiences that pre-service teachers bring with them and need to value and address them while providing them with new teaching experiences.

As an example, an assignment that the pre-service teachers completed during both of their methods courses included tutoring one child at a local mathematics clinic and then creating lesson plans for future sessions. The teachers then had to write a case study of the child and his learning. However, the pre-service teachers’ tutorial experiences were not discussed in an intellectually robust manner during class sessions. Although teachers were encouraged to reflect on the children’s actions, drawing from theoretical knowledge base they were provided in courses, their work did not reflect sensitivity or
responsiveness to these guides. It appears essential that these experiences are highlighted and that the pre-service teachers receive coherent and thorough feedback. This kind of feedback could include detailed modeling of how to set up teaching goals, assessment tools that could be used along with clear description of what might be gained from using them—if and how the assessments target the intended knowledge, how the content might be structured so to maintain the integrity of the subject while navigating both short- and long-term content acquisition validity. During this feedback cycle, it is especially important to concentrate on content specific pedagogy rather than generalized pedagogy. In this way, the pre-service teachers can begin to make new experiences and connections, moving forward in building enactment strategies that help expand their visions of teaching (Schoenfeld, 2011).

*Balancing group and individual learning*

The findings revealed that none of the pre-service teachers exhibited the ability (or desire) to utilize the cognition-based models in assessing student work and thinking or to design instructional strategies that could foster student understanding. While examining case studies in the methods courses, the pre-service teachers were asked to analyze the students work using either the van Hiele model or the Pirie-Kieren model of assessment. The pre-service teachers seemed to struggle with applying these models during the first quarter of their M. Ed program. However, by the end of the second methods course in geometry, they were able to apply these models to evaluate the work of an individual student. The pre-service teachers, while referencing student thinking, did not focus on the mathematical content involved. Rather, they focused on factors such as student comprehension, student background, and learning styles. Although the
teachers’ references to the need to be more sensitive to student comprehension increased over the course of their involvement in two methods courses, their pedagogical decision making remained unaffected. That is, even though they shared a more sophisticated understanding of what student comprehension meant, their inability to address the important mathematical concepts became a hurdle in developing instructional tasks that would further the understanding of students. Such a gap has been observed by other researchers (Grossman et al., 2009). In fact, according to Kothragen and Wubbels (2001, as cited in Grossman et al., 2009) pre-service teachers do not utilize much of the theory that they are taught in their teacher preparation programs and that they are not well prepared to encounter problematic situations that arise in practice. This is often the case because many teacher education programs teach the conceptual tools in the coursework but the pre-service teachers do not get to enact them until they start student teaching (Grossman et al., 2009).

An important component of the methods courses was requiring the pre-service teachers to complete a case-based analysis of student work. In order to complete this task, the pre-service teachers were required to work with a student an hour a week and after the end of 5 weeks, write a case-based analysis regarding the student. This task was intended to be used as a bridge to close the gap between theory and practice. However, it did not fully serve its purpose. As mentioned earlier in this section, the pre-service teachers were able to apply the assessment models, at least partially, in order to assess student work and thinking but they did not utilize them for designing instruction to further enhance student understanding. Also, even in the presence of this experience, the pre-service teachers were still unable to identify the connections between what was learned in their
coursework to build pedagogy. Despite this, the M. Ed. Program was successful in raising the pre-service teachers’ attention to student thinking. At the end of the second methods course teachers considered factors that could have affected student work, or misconceptions that the student might have had. Such a shift, albeit small in magnitude may be attributed to the focus of the two methods courses on analysis of sources for children’s difficulties when learning mathematics.

Was the program a weak link? A theoretical accounting of findings

Fuller and Brown (1975) identified three stages of pre-service teacher concerns. The first stage is the concern for survival where the teachers pay more attention to class control, being observed and evaluated by their supervisors and their survival as teachers (Fuller, & Brown, 1975). The second stage of concern is that of teaching situation where the teachers attach greater importance to frustrations that arise in teaching situations such as working with too many students, ineffective curriculum and so on. Finally the third stage is concern for pupils where in the pre-service teachers are concerned about the needs of their students (Fuller, & Brown, 1975). Similar sequence of concerns was also observed by He and Cooper (2011) and was evident among the participants of this study. Survival concerns, such as classroom management and getting through the assigned curriculum, were dominant in their initial interviews when they talked about what obstacles they anticipated facing when organizing instruction. Another characteristic of the first stage of survival stage is that the pre-service teachers view the required coursework irrelevant (Fuller, & Brown, 1975), assuming a gap between theory and practice. This same reaction was observed among the sample studied in this research project relative to the capital they placed on theories of learning they had acquired while
in methods courses. They failed to see connections between the theories to which they were introduced and classroom practice. While the participants of this study expressed concerns about their students and their learning, they were unable to address them in their instruction (Fuller, & Brown, 1975).

Certainly, one plausible interpretation of the relative lack of influence of the methods courses on teachers could be that the content of the courses was designed without much sensitivity to helping teachers bridge theory and practice, according to the developmental sequence offered by Fuller and Brown (1975). While developing the methods courses, the instructors focused more on the third stage of teacher concern, whilst almost exclusively neglecting to address the first two stages. The findings of the current study suggest that teacher education programs should be cognizant of pre-service teachers’ prior experiences as learners and draw from those experiences to design activities that reveal both effective and less productive pedagogical moves and their impact on children’s conceptual learning of mathematics. Additionally, efforts need to be made to make explicit connections between the assessment models and developing instructional tasks for the whole class or group of students. This seems of vital importance since the shift from individual analysis of children’s cognition to navigating group learning is a complex and demanding process. The transition from the examination of a task to making generalized pedagogical moves is challenging even for experienced teachers and teacher educators. It is plausible to assume that extended exposure to theoretical models, concretized along various mathematical concepts can provide pre-service teachers with more powerful enactment strategies they could use in their own practice. According to Grossman (1992), one way to meet this challenge is to encourage
pre-service teachers to be reflective on their own teaching. Other researchers have also supported the use of reflection as a means to help teacher acquire professional knowledge. He, & Cooper (2011) argued, for instance, that “by providing strategies for thinking about teaching experiences beyond subject matter content and ethical and moral issues, teacher educators offer additional, more meaningful, and lasting preparation for professional life beyond the security of teacher education programs” (p. 98). This however, offers only a rudimentary explanation for the phenomena observed in this project, which speak to the existing gap between theory and practice. This point is described below.

Connecting theory and practice: An ongoing dilemma

A major dilemma for all the participants in the study was connecting personal and theoretical knowledge about teaching and learning. Although, some researchers have proposed the use of personal narratives (Olive, 2010) as useful tools to help bridge this gap the efficacy of such approach in the absence of a broader theoretical model on teacher learning remains questionable (Jaworski, 2006). The dilemma of bridging theory and practice has been an enduring dilemma in teacher education for decades. Several researchers, most notably Jaworski (2006), argued eloquently that despite substantial advances in increased theoretical maturity within the mathematics education community, the position of mathematics teaching has remained underdeveloped. This, according to her, contributes to the ongoing gap between what teachers do and the theoretical knowledge generated to explain learning.

While theory provides us with lenses for analyzing learning, the big theories do not seem to offer clear insights to teaching and ways in which teaching
addresses the promotion of mathematics learning (Jaworski, 2006, p. 188).

In the courses that the teachers completed, illustrations of children’s work and thinking were used as problem sources and tools to engage the participants in using theoretical models for both explaining the level of understanding exhibited by children and devising plans for moving the children portrayed in their understanding of mathematical topic under study. Analysis of participants’ responses revealed that although teachers were indeed capable of referencing learning theories as a platform for analysis of children’s thinking, such use was not natural to their thinking. Indeed, references were made only when they were asked to frame their analysis with such theories. Furthermore, even in the presence of such references when analyzing mathematical practices of children, none of the participants seemed able to propose instructional plans that would address the particular cognitive needs of the children. These findings certainly support Jaworski’s (2006) analysis regarding the limitations of theories in providing guides for teaching actions. As she explained,

Theories help us analyze, or explain, by they do not provide recipes for action; rarely do they provide direct guidance for practice. We can analyze and explain mathematics learning from theoretical perspectives, but it is naive to assume or postulate theoretically derivative models or methods through which learning is supposed to happen (p. 188).

This phenomenon can certainly explain the participants’ own claims regarding their lack of inclination to rely on theory with examining teaching and learning issues. A majority of the participants articulated that they viewed theories of learning introduced in the course too general to have practical merit for their classroom practice. Although they
considered them as viable diagnostic tools when interacting with individual children, in a one-on-one setting, they struggled to find means for theorizing about whole group teaching in their presence. This struggle is certainly legitimate and an issue closely linked to the absence of compatible theories of teaching that would link theory and practice more meaningfully. The mathematics education community has not been short in offering perspectives on characterizing effective teaching. For instance, in a recent work Schoenfeld and Kilpatrick (2008), in an effort to conceptualize proficiency for mathematics teaching, provided a provisional framework consisting of a set of seven dimensions: Knowing school mathematics in depth and breadth; knowing students as thinkers; Knowing students as learners; crafting and managing learning environments; developing classroom norms and supporting classroom discourse as part of “teaching for understanding”; building relationships that support learning and reflecting on one’s practice. Less clear is how such proficiency might be nurtured in teacher education (Tirosh & Wood, 2008). The absence of such a theory further complicates the issue of teacher education. On the one hand, the courses designed for teachers are often done so in light of general theoretical views on how to facilitate learning. This was certainly the case in the current study as both the design of the content of the courses as well as the instruments used for data collection were grounded in theories of learning mathematics, situated cognition and social inquiry. However, these deep theoretical views did not provide an anchor for how to operationalize teacher preparation. Although lack of experience of both course instructors (myself and the geometry methods course instructor) with teacher preparation may account for seemingly weak impact of the program on teacher decision making a review of current literature reveals that little
theoretical guide currently exist on how the teachers’ experiences might have been better organized so to have optimized the desired outcome. On the other hand, it is not quite clear how teachers’ development might be gauged against a research based platform. Again, in an extensive review of literature we noticed only one instance of published work, which attempts to examine teacher learning, not in presence of specific tools used in teacher education (Tirosh & Wood, 2008). Developing an understanding of the type of learning trajectories teachers exhibit when exposed to new mathematical and pedagogical knowledge is vital to our success in establishing a coherent theory of teacher education.

Limitations

In the previous section several shortcoming of the research project were discussed in detail. Additionally, in examining knowledge used in action, a major limitation of design was that I did not take into account the prominent discourses of the methods courses teachers completed. Analysis of the classrooms’ discourse could have increased the explanatory power of findings offering means for determining how and why certain changes were made possible or difficult to establish. To such end, this study only explained the changes that took place but it did not explain how the methods courses may have influenced the outcome.

Another drawback of the study was that I did not observe the pre-service teachers while in action and during their teaching phase. Observing the pre-service teachers in real classroom setting could have provided more substantial information about what (if any) and how they may have applied the learning from the program to practice. It would also have been a suitable setting to view teachers’ use of their MKT.
In documenting teachers’ knowledge, and its growth, for teaching geometry as it pertains to children’s thinking, in this work I relied on an instrument that proved to be valuable in identifying gaps in teachers’ knowledge but was limited in capturing what they knew. Absence of such a vital tool in mathematics education community poses a major challenge to the genre of research that aims to investigate teacher knowledge. Admittedly, the instrument developed and used was long, about 150 minutes. It was noticed that by the end of the 150 minutes, some of the participants did not make a serious commitment to completing the post-survey, as several of items were left blank. Quite naturally, this phenomenon could have impacted the results reported on the overall progress of the cohort. The interview data revealed that discussion of the survey items did provide substantial learning opportunities for the teachers. Once challenged to consider alternative ways of examining children’s mathematics the teachers did indeed engage in tasks and asked mathematical questions that pertained specifically to how the artefact or scenario could have been interpreted or evaluated. If such tasks are used consistently and coherently throughout the teachers’ educational backgrounds, augmented with specific ways in which reliance on theory could assist in explaining the children’s work and thinking, considerable learning might emerge. This point, however, does not diminish the need for development of measurement tools that allow the teachers to show what they know versus what they may not know. The first step taken in the current study provided a glimpse of possibilities for utilizing mechanisms for capturing teachers’ orientations to teaching. This first step was certainly not sufficient enough to provide a full profile of their knowledge or how they may respond to immediate needs of
classroom instruction, particularly when time for reflection or contemplation is not provided during the assessment.

Note that the surveys covered a very narrow range of content competence. Also, of all the possible data sources that could have helped in obtaining richer account of their thinking such as classroom observations, assignments that the pre-service teachers wrote for their classes or the case-based analysis that they conducted based on their work with students, I only utilized the surveys and the interviews. This provided me with a mere snapshot of their orientation toward teaching. The results of this study are not an anticipation of their competence or success as teachers. Rather this study merely presents an account of their thinking and work at the time of their transition into their professional identity.

Recommendations for future research

The instrument used in the study proved to have limitations in terms of the content that was addressed, length, and the type of responses elicited from pre-service teachers. For future work on this instrument, there is need for developing instruments to capture the various aspects of MKT that cover a broader range of geometric topics. While doing so, care needs to be taken to make sure that the instrument is not so long that the participants fail to finish or provide sufficient effort. One way to improve this is to have the instruments tested with different pre-service teachers and then create multiple choice questions with the most common answers given.

We realized during the interview process that the interviews themselves served as venues for learning and growth of pre-service teachers, as articulated by them. Examining the actual impact of interviews on teacher development was not the focus of
this work but this issue does merit careful consideration in the future. This is for at least two major reasons. On the one hand, such studies may help better define the type of scaffolding tools for facilitating teacher learning when using case-based tasks in teacher preparation. This knowledge can assist in operationalizing theory based methodologies in teacher education. On the other hand, they may provide more well-defined pathways towards building models of teacher thinking when practicing theories. Both of these domains of knowledge are currently lacking in teacher education literature.

As discussed above, during the course of this study, discourse analysis of methods courses was not included. Closer inspection of the classroom interactions will provide a clearer account of effective mathematical teacher educating, particularly since such knowledge is currently absent in the literature. Indeed, such studies can offer a perspective on how activities of teacher educators might be sequenced so to attend to teachers’ concerns, as prevalent in the quality of questions asked or elaborations they elicit from the teacher educators. More importantly, they can offer a perspective on what knowledge bases might be needed for those teaching methods courses. That is, how might a teacher educator understand learning theories pertaining to children’s own learning as well as teacher learning to effectively establish and navigate generative learning environments for teachers? This line of inquiry can also provide a guide for the type of instruments that may need to be developed so to better capture teacher knowledge and development.

The findings of this research suggested a close link between the teachers’ knowledge of content trajectory, their ability to more accurately assess children’s mathematical work and to devise pedagogical plans responsive to and aligned with such
assessment. Although, the study was not designed to examine the teachers’ knowledge of content trajectory pertaining to secondary mathematics curriculum, the results suggest careful study of the impact of various domains of mathematical knowledge, as a discipline, on teachers’ ability to carry out the tasks of teaching. Understanding the complex interplay between subject matter knowledge and teaching at the secondary level is only in very early stages of development. Future studies are needed to assist in better define such interplay.

Lastly, in the current study, I utilized Cooney, Shealy, and Arvold’s (1998) categorization of pre-service teachers. This categorization informs us about the belief systems of these teachers. What is largely unknown is how these pre-service teachers orient themselves to teaching? Further studies are needed to use these categories to determine what might be the learning trajectories for pre-service teachers in each of these categories. Also, are these categories of pre-service teachers exclusive or is it possible that pre-service teachers may not fall under any of these categories?
References


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Appendix A: Pretest Survey

Please answer the following questions:

Part 1: General Information

Name: ____________________________________

Gender: M   F
1. Teaching preference
   a. Preschool
   b. Elementary school
   c. Middle school
   d. High School
   e. College

2. Subject area preference
   a. Pre Algebra
   b. Algebra 1
   c. Algebra 2
   d. Geometry
   e. Pre-Calculus
   f. Calculus (College Prep)
   g. AP Calculus
   h. Statistics

3. List the mathematics courses you have taken and completed during your undergraduate studies
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________

4. In which mathematics course did you gain the largest amount of learning? Please explain.
5. In which course did you learn the content that is most helpful to you as a teacher? Please explain.
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________

6. How do you think teaching of geometry is similar to or different from teaching Algebra or other mathematical subjects? Please explain
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________

7. What are some common difficulties you anticipate students would encounter when learning geometry? How can a teacher address these difficulties? Please explain.
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________

Part 2: Self – Efficacy (Ohio State Teacher Efficacy Scale)
*Please circle the option that describes you the best. The scale is as follows: 1. Nothing, 3. Very little, 5. Some influence, 7. Quite a bit, 9. A great deal*

8. Factor 1: Efficacy for instructional strategies
   a. To what extent can you use a variety of assessment strategies?
   1 2 3 4 5 6 7 8 9
   b. To what extent can you provide an alternative explanation or example when students are confused?
   1 2 3 4 5 6 7 8 9
   c. To what extent can you craft good questions for your students?
   1 2 3 4 5 6 7 8 9
   d. How well can you implement alternative strategies in your classroom?
   1 2 3 4 5 6 7 8 9
   e. How well can you respond to difficult questions from your students?
   1 2 3 4 5 6 7 8 9
   f. How much can you do to adjust your lessons to the proper level for individual students?
   1 2 3 4 5 6 7 8 9
g. To what extent can you gauge student comprehension of what you have taught?  
   1  2  3  4  5  6  7  8  9

h. How well can you provide appropriate challenges for very capable students?  
   1  2  3  4  5  6  7  8  9

9. Factor 2: Efficacy for classroom management  
a. How much can you do to control disruptive behavior in the classroom?  
   1  2  3  4  5  6  7  8  9

b. How much can you do to get children to follow classroom rules?  
   1  2  3  4  5  6  7  8  9

c. How much can you do to calm a student who is disruptive or noisy?  
   1  2  3  4  5  6  7  8  9

d. How well can you establish a classroom management system with each group of students?  
   1  2  3  4  5  6  7  8  9

e. How well can you keep a few problem students from ruining an entire lesson?  
   1  2  3  4  5  6  7  8  9

f. How well can you respond to defiant students?  
   1  2  3  4  5  6  7  8  9

g. To what extent can you make your expectation clear about student behavior?  
   1  2  3  4  5  6  7  8  9

h. How well can you establish routines to keep activities running smoothly?  
   1  2  3  4  5  6  7  8  9

10. Factor 3: Efficacy for student engagement  
a. How much can you do to get students to believe they can do well in schoolwork?  
   1  2  3  4  5  6  7  8  9

b. How much can you do to help your students value learning?  
   1  2  3  4  5  6  7  8  9

c. How much can you do to motivate students who show low interest in schoolwork?  
   1  2  3  4  5  6  7  8  9
d. How much can you assist families in helping their children to do well in school?
   1  2  3  4  5  6  7  8  9

e. How much can you do to improve the understanding of a student who is failing?
   1  2  3  4  5  6  7  8  9

f. How much can you do to help your students think critically?
   1  2  3  4  5  6  7  8  9

g. How much can you do to foster student creativity?
   1  2  3  4  5  6  7  8  9

h. How much can you do to get through to the most difficult students?
   1  2  3  4  5  6  7  8  9

Part 3: Analysis of student work

11. Refer to the circle and ellipse below

A student developed a special project in which she began with circles of various radii (R) and created ellipses by stretching the circle in the horizontal direction (only) by a factor of two. The student estimated the areas and inferred that the areas (A) of the ellipses are approximated by the formula \( A = 6R^2 \). What would you tell the student to help her understand this result? (Bush, Ronau, McGatha, Thompson, 2002)

12. A cube 4 units on a side has each of its corners sliced off with each cut intersecting the original edges 1 unit from the original corners. Students are asked whether these cuts have increased or decreased the surface area. One student argues that the surface area has increased because the cuts produced new surfaces. Another student argues that the surface area has decreased because the new object is smaller than the original. What activity could a teacher use to help determine which student is correct? Include a rigorous analysis of the result of the activity as well as a description of the activity. (Bush, Ronau, McGatha, Thompson, 2002)
13. An octagon is an eight-sided polygon (Weisstein, n.d.a). When asked to draw an octagon a student sketched the following figure.

![Octagon drawing](image)

a. If you were a teacher of this child how would you assess this example?
b. Explain the basis for your assessment.
c. What conceptions could have contributed to this drawing?

14. Which of the following definitions of circles do you think are the easiest for students to understand? Which do you think are the most difficult? Please explain the basis for your choice and ranking. How should the presentation of these definitions be sequenced so as to assure student understanding. Justify your choice.

a. Given two points O and A. The set of all points P such that segment OP is congruent to segment OA is called a circle with O as center, and each of the segments OP is called a radius of the circle (Greenberg, 1993, p. 15).
b. A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another. The point is called the center of the circle (Euclid, 2006, p.31)
c. A circle is a set of points in a plane that are equidistant from a given point O. The distance from the center is called the radius and the point O is called the center. (Weisstein, E. W., n. d.b)
d. A circle is defined as the locus of all points at a fixed distance from a given point (Math Open Reference, 2009)

15. A student was given the following problem: Consider a cube whose base area is 4 cm². If the area of the base increases to 16 cm², how much does the volume increase?

a. The student replies by saying that the volume of the cube would increase by 4096 cm³. How do you think the child arrived at this answer?
b. What techniques or tools may be used to help the child understand the solution?
c. What are some questions you can ask the student to further his understanding on the topic?
16. The formula for finding the number of diagonals in a polygon is given by \( \frac{N(N-3)}{2} \). Suppose a student asks how might we justify, geometrically, the factor \( (N-3) \) in the formula. How would you explain the result? (Manouchehri, 2012, p.)

17. In response to the following question: Consider two similar triangles ABC and XYZ with a scale factor of 2. What do you think would be the ratio of their areas? 20 of the 25 students in class wrote 4 as the answer. In responding to an extension of the task, which had asked students to determine the ratio of areas of two similar triangles with a scale factor of 5, the students suggested 10 as the answer.
   a. What is your assessment of the students’ responses to the two problems?
   b. What issues would you need to address in class in response to these issues?
   c. What are some questions you could ask students to further their understanding on this topic?

18. Consider the following explanation offered by Terri for why the sum of opposite angles in a cyclic quadrilateral is 180 degrees. (A cyclic quadrilateral is a quadrilateral for which a circle can be circumscribed so that it touches each polygon vertex) (Weisstein, E. W., n. d.d): “If you draw an angle inscribed in a circle to build a convex polygon the fourth vertex is limited to the angle which it can subtend by the given (first angle drawn. Suppose the given angle is 80 degrees in measure. Then it subtends 160 degrees of the circle. 200 degrees of the circle is left. Let n equal the measure of the last vertex angle of the supposed polygon. Its measure can’t exceed half of the measure of the 200 degrees (the rest of the circle which is the measure of the angle it subtends). Therefore the measure of n is 200/2=100.” (Manouchehri, 2012)
   a. What factors do you think influenced this student’s reasoning?
   b. How would you respond to this student?

19. Consider the question: What can you say about the areas of a square of side of length \( a \), a circle of diameter \( a \), and an equilateral triangle of side of length \( a \)? What can you say about their perimeters?

   After working on this problem, a group of students suggested that “the bigger the perimeter, the larger the area and vice versa.”
   a. Why do you think the students might conceive of such a relationship?
   b. How could a teacher address this conception?
20. The following is a summary of responses offered by a couple of 8th graders to the following question: A rotation of a rotation can sometimes be a translation. Explain exactly when that happens.

S1: Yes, a rotation of a rotation can sometimes be a translation. It happens when it is rotated around a fixed point when rotates a number of times around that fixed point until there is no fixed point and it looks like the original rotation which is also an identity.

S2: If the second rotation has a different fixed point than the first rotation and the second rotation causes the triangle to become the identity triangle then it would be a translation (Manouchehri, 2012, p. 45)

a. What factors do you think influenced these explanations?
b. Decide whether you agree or disagree with these students. Please explain your reasons.
c. What tasks would you use to help these students understand the result?

21. Consider the following letter written by a high school student:
“Today our teacher talked about the area of a triangle being ½ base times height. I think the formula is helpful when we have all the information but how are we supposed to find the area of a triangle if we don’t know what its height is?” (Manouchehri, 2012, 93)

a. What do you think the basis for the student’s question is?
b. Do you think this is an important question for students to consider? Explain.
c. How would you respond to this student’s question?

22. If you had to choose between teaching two of the following three topics which would you choose and why? How do you think these topics would help students when extended to other concepts?

a. Geometric constructions
b. Centers of triangles
c. 3-D geometry
23. In trying to construct the angle bisector of $\angle ABC$, a student suggests the following method.
   a. Find a point X on $\overrightarrow{BA}$ and a point Y on $\overrightarrow{BC}$
   b. Find the midpoint O of $\overline{XY}$
   c. Then $\overrightarrow{BO}$ is the bisector of $\angle ABC$.

   (Schoenfeld, 1988)
   a. What factors do you think influenced this construction?
   b. What tasks would you use to help the student understand the result?
   c. What are some questions you could ask the students to further his/her understanding of this topic?

Part 4: Analysis of student work in algebra and probability

24. Hottubs and swimming pools are sometimes surrounded by borders of tiles. The drawing below shows a square hot tub with sides of length $s$ feet. This tub is surrounded by a border of 1 foot by 1 foot square tiles.

How many 1-foot square tiles will be needed for the border of this pool?

   a. Paul wrote the following expression: $2s + 2(s+2)$
      Explain how Paul might have come up with his expression.

   b. Bill found the following expression: $(s+2)^2 - s^2$
      Explain how Bill might have found his expression.

   c. How would you convince the students in your class that the two
25. In a lottery called Pick 4, a 4-digit number like 2798 is generated. To win, the participant must have chosen the same 4-digit number. Albert has chosen the number 2222 and Bill has chosen the number 2332. Compare their chances of winning.
   a. Albert has a better chance of winning
   b. Bill has a better chance of winning
   c. They both have the same chance of winning

What might be some of the typical student responses and why? Please explain in detail. (Fast, 1999, p. 237)
Appendix B: Pretest Interview Protocol

Part 1: Initial thoughts

1. Describe your ideal mathematics classroom; what would the teacher be doing? What would the students be doing? What would be their roles? What type of curriculum would be used and why?
2. What kinds of knowledge might be need for teachers in order for them to be able to maintain such a learning environment?
3. What obstacles do you think you would face in creating your ideal classroom?
4. To what extent has your coursework prepared you to model that type of teaching/foster the type of learning/or implement the kind of curriculum you outline in your description of an ideal classroom?

Part 2: Beliefs

5. Do you think children are capable of solving problems on their own or do you think they need to be taught certain procedures before they can solve problems? (For clarification: for example, do you think high school geometry students are capable of proving that the two diagonals of the rectangle divide the area into four equal parts? Do they need to know the properties of congruence before they can attempt this problem or do you think they can solve this problem without any formal knowledge of high school geometry?) If the response is “it depends”, ask for examples to illustrate.

6. Is it okay if students to follow certain procedures without understanding them as long as they can use them to find the right answers? If so could you give an example? (Probe further: Is it okay if the students do not understand the Pythagorean theorem but are able to successfully use it to find the sides of a right triangle? Under what circumstances is rote memorization useful?) Again, if the response is “it depends, ask for specific examples to illustrate).

7. Should teachers encourage students to use multiple ways of solving a problem? Could you give a specific example? Why it would be useful in that case to approach it using different ways?
8. Should teachers encourage students to use multiple ways of solving a problem? Could you give a specific example? Why it would be useful in that case to approach it using different ways?

9. Who should be responsible for correcting students’ responses? Can students solve their own discrepancies by collaborative work? (Ask for examples to clarify

Part 3: Self-Efficacy

10. What information would you need and/consider when assessing students’ work?
11. Item 8c on the survey concerned good questions. What questions do you qualify as good questions? Consider a topic and give an example of a good question.

12. If you are assisting a student in trying to solve the following problem: Prove that the diagonals of a square bisect each other. What would be some good questions to ask the student?

13. Item 8g on the survey is about student comprehension. What does the term comprehension mean to you? (Probe about mathematics in particular). How could student comprehension be measured?

14. What behavior do you consider to be disruptive behavior? Would a student repeatedly asking questions or shouting out answers qualify as disruptive behavior? If so how would you deal with that?

15. Why do you think students engage in disruptive behavior?

16. What obstacles do you feel you might face when dealing with classroom management?

17. Is it possible to have an instructional strategy in which all students are equally occupied with the task? Could you give an example?

18. Is it possible to change students’ beliefs about their mathematical abilities? If so how could you make them believe that they can do well in their schoolwork?
19. What are some of the ways you would try and motivate the students who show low interest in schoolwork? How would you motivate a student who has no interest in mathematics or a student who doesn’t see the usefulness of mathematics other than in school?

20. What do you think are some of the reasons students are failing mathematics? Do you think they are ‘bad’ at math or just uninterested? Is it possible for students who have been failing in mathematics (or performing badly) to improve?

Part 4: Analysis of student work

21. Question 11:
   a. Do you think this particular question would be hard for high school geometry students?
   b. Could you explain your response to this question in detail?
   c. Why did you choose the particular approach?
   d. What are the advantages of your approach?
   e. What are the limitations of your approach?
   f. What are some of the obstacles you would face while using this approach in trying to get the student to understand the result?
   g. What are some of the obstacles that the students would face while solving this problem?
   h. What do you think would be some of the common misconceptions about this topic?
   i. How would you incorporate technology in your instruction to help further student understanding?
   j. What are some other ways in which you could help the student understand this result?

22. Question 12:
   a. Please explain your answer in detail.
   b. What would be the advantages of your instructional strategy?
   c. What would be the disadvantages of your instructional strategy?
   d. How would you use technology to aid your instruction?
   e. What do you think the students’ misunderstandings are?
   f. How would your instructional approach help both the students?

23. Question 13:
   a. Is the student’s response right or wrong? Please explain in detail.
   b. What are some advantages of thinking of octagons in this way?
   c. What are some disadvantages of thinking of octagons in this way?
d. If you had to design a lesson that discusses the advantages of these types of octagons, what would you do?
e. What do you think is a source of that type of conceptualization?
f. What do you think would be some of the common misconceptions about this topic?

24. Question 14:
   a. Please describe your answer in detail.
   b. Why do you think that particular choice would be hardest for students to understand?
   c. What are some other topics about circles that students would find hard to understand?
   d. How would you use technology to further student understanding about circles?
   e. What are some of the obstacles you would face as a teacher while teaching the concept of circles to students?

25. Question 15:
   a. Why do you think the student would make that particular error?
   b. Is it possible that it is just a calculation error? Or is it more likely a conceptual error?
   c. What strategies would you use/could you use to help this student? Why would these approaches be effective?
   d. What are the disadvantages of your approach?
   e. What types of questions would you ask the student in order to elicit their thinking about this problem? Why would those questions be needed? What information do you have to obtain?
   f. What kind of manipulatives would you use to aid the student?

26. Question 16, 17 & 18:
   a. Do you think this particular question would be hard for high school geometry students?
   b. Could you explain your response to this question in detail?
   c. What instructional approach would you use to help this student?
   d. Why did you choose the particular approach?
   e. What are the advantages of your approach?
   f. What are the limitations of your approach?
   g. What are some of the obstacles you would face while using this approach in trying to get the student to understand the result?
   h. What are some of the obstacles that the students would face while solving this problem?
   i. What do you think would be some of the common misconceptions about this topic?
j. How would you incorporate technology in your instruction to help further student understanding?
k. What are some other ways in which you could help the student understand this result?

27. Question 19:
   a. Please explain your responses in detail.
   b. What are some of the common sources of those conceptions?
   c. Do you think such problems should be intuitive to the students?
   d. What tasks would you design in order to address these conceptions? How would you address misunderstandings?
   e. What other activities could you use to help the students with this problem?
   f. What other questions do you foresee arising related to reasoning and proofs in high school geometry?

28. Question 20:
   a. Please explain your responses in detail.
   b. What are some of the common sources of those conceptions?
   c. Do you think such problems should be intuitive to the students?
   d. What tasks would you design in order to address those alternate conceptions? How would you address misunderstandings?
   e. What other activities could you use to help the students with this problem?
   f. How would you incorporate technology in your task to aid this student?

29. Question 21:
   a. Do you think this particular question would be hard for high school geometry students?
   b. Please explain your response in detail.
   c. Why did you choose the particular approach?
   d. What are the advantages of your approach?
   e. What are the limitations of your approach?
   f. What are some of the obstacles you would face while using this approach in trying to get the student to understand the result?
   g. What are some of the obstacles that the students would face while solving this problem?
   h. What do you think would be some of the common misconceptions about this topic?
   i. How would you incorporate technology in your instruction to help further student understanding?
   j. What are some other ways in which you could help the student understand this result?
30. Question 22:
   a. Please explain your response in detail.
   b. Why did you choose the following topics?
   c. Please explain in detail why you think students will benefit more from your choice?
   d. What are the disadvantages of leaving out the third topic?
   e. What are some of the obstacles that the students face?
   f. What do you think would be some of the common misconceptions about these topics?

31. Question 23:
   k. Do you think this particular question would be hard for high school geometry students?
   l. Please explain your response in detail.
   m. Why did you choose the particular approach?
   n. What are the advantages of your approach?
   o. What are the limitations of your approach?
   p. What are some of the obstacles you would face while using this approach in trying to get the student to understand the result?
   q. What are some of the obstacles that the students would face while solving this problem?
   r. What do you think would be some of the common misconceptions about this topic?
   s. How would you incorporate technology in your instruction to help further student understanding?
   t. What are some other ways in which you could help the student understand this result?
Appendix C: Posttest Survey

Please answer the following questions:

Part 1: General Information
Name: ____________________________________

Gender: M    F

1. Teaching preference
   a. Preschool
   b. Elementary school
   c. Middle school
   d. High School
   e. College

2. Subject area preference
   a. Pre Algebra
   b. Algebra 1
   c. Algebra 2
   d. Geometry
   e. Pre-Calculus
   f. Calculus (College Prep)
   g. AP Calculus
   h. Statistics

3. How do you think teaching of geometry is similar to or different from teaching Algebra or other mathematical subjects? Please explain
   _______________________________________________________________________
   _______________________________________________________________________
   _______________________________________________________________________
   _______________________________________________________________________
   _______________________________________________________________________

4. What are some common difficulties you anticipate students would encounter when learning geometry? How can a teacher address these difficulties? Please explain.
   _______________________________________________________________________
   _______________________________________________________________________
   _______________________________________________________________________
   _______________________________________________________________________
   _______________________________________________________________________
Part 2: Self – Efficacy (Ohio State Teacher Efficacy Scale)

Please circle the option that describes you the best. The scale is as follows:

4. Factor 1: Efficacy for instructional strategies
   a. To what extent can you use a variety of assessment strategies?
      1 2 3 4 5 6 7 8 9
   b. To what extent can you provide an alternative explanation or example when students are confused?
      1 2 3 4 5 6 7 8 9
   c. To what extent can you craft good questions for your students?
      1 2 3 4 5 6 7 8 9
   d. How well can you implement alternative strategies in your classroom?
      1 2 3 4 5 6 7 8 9
   e. How well can you respond to difficult questions from your students?
      1 2 3 4 5 6 7 8 9
   f. How much can you do to adjust your lessons to the proper level for individual students?
      1 2 3 4 5 6 7 8 9
   g. To what extent can you gauge student comprehension of what you have taught?
      1 2 3 4 5 6 7 8 9
   h. How well can you provide appropriate challenges for very capable students?
      1 2 3 4 5 6 7 8 9

5. Factor 2: Efficacy for classroom management
   i. How much can you do to control disruptive behavior in the classroom?
      1 2 3 4 5 6 7 8 9
   j. How much can you do to get children to follow classroom rules?
      1 2 3 4 5 6 7 8 9
   k. How much can you do to calm a student who is disruptive or noisy?
      1 2 3 4 5 6 7 8 9
   l. How well can you establish a classroom management system with each group of students?
      1 2 3 4 5 6 7 8 9
m. How well can you keep a few problem students from ruining an entire lesson?
   1 2 3 4 5 6 7 8 9

n. How well can you respond to defiant students?
   1 2 3 4 5 6 7 8 9

o. To what extent can you make your expectation clear about student behavior?
   1 2 3 4 5 6 7 8 9

p. How well can you establish routines to keep activities running smoothly?
   1 2 3 4 5 6 7 8 9

6. Factor 3: Efficacy for student engagement
q. How much can you do to get students to believe they can do well in schoolwork?
   1 2 3 4 5 6 7 8 9

r. How much can you do to help your students value learning?
   1 2 3 4 5 6 7 8 9

s. How much can you do to motivate students who show low interest in schoolwork?
   1 2 3 4 5 6 7 8 9

t. How much can you assist families in helping their children to do well in school?
   1 2 3 4 5 6 7 8 9

u. How much can you do to improve the understanding of a student who is failing?
   1 2 3 4 5 6 7 8 9

v. How much can you do to help your students think critically?
   1 2 3 4 5 6 7 8 9

w. How much can you do to foster student creativity?
   1 2 3 4 5 6 7 8 9

x. How much can you do to get through to the most difficult students?
   1 2 3 4 5 6 7 8 9
Part 3: Analysis of student work in geometry

7. Consider the following question to be given to students:

What can you say about the volumes of a cube with side $a$, a sphere with diameter $a$, and a cylinder with height and diameter of the base to be $a$. What can you say about the volumes of these shapes? What can you say about the surface areas?

While working on this problem, a student concludes that the volume of the sphere is less than the volume of the cube since the sphere can fit inside a cube. However, the student is not sure about the volume of the cylinder.

a. Why do you think the students might conceive of such a relationship?
b. How do you respond to this student’s conception?
c. What are some questions you could ask the student to further his/her understanding on this topic?
d. What tools would you use to assess the level of this student’s thinking?
e. What levels are relevant here?

8. Which of the following definitions of a polygon do you think is the easiest for students to understand? Which do you think are the most difficult? Explain the basis for your choice and ranking. How should the presentation of these definitions be sequenced so as to assure student understanding. Justify your choice.

a. A figure is a polygon if and only if it meets the following conditions:
   a. It is formed by three or more coplanar segments called sides
   b. Sides that have a common endpoint are non collinear
   c. Each side intersects exactly two other sides, but only at their endpoints.
   (Foster, Cummins & Yunker, 1984, p. 558)
b. A polygon is composed of three or more coplanar segments that intersect only at the endpoints. Each endpoint is shared by exactly two of the segments.
c. A polygon can be defined as a geometric object "consisting of a number of points (called vertices) and an equal number of line segments (called sides), namely a cyclically ordered set of points in a plane, with no three successive points collinear, together with the line segments joining consecutive pairs of the points (Coxeter, & Greitzer, 1967 as cited by Weisstein, n. d.c).
d. A polygon is a “closed plane figure with straight edges” (Gellert et al. 1989, p. 162)
9. Consider the following dialogue between Amy and Peter as they try to reason why a line segment has only one midpoint:
Amy: (Draws a line segment on GSP. Using the construct menu she marks the midpoint of the segment and begins changing the length of the segment by moving one of the endpoints) See? The midpoint moves with the segment. There is only one midpoint (points at the screen). No matter how long or how short the midpoint moves with it. Here, see if I try to make another midpoint it gives us the same point (goes to the construct menu and selects construct midpoint again). The points coincide.

Peter: I look at it this way… I say if we have a triangle and we construct the median from the vertex then we can have only one median. It means we have only one midpoint. See, let me show you (He draws a triangle and constructs a median. Pointing at the image he explains). See, a median is a segment that connects that point (points at the vertex) to the midpoint of this side (points to the opposite side). We have only one median. So, there is only one midpoint.

Amy nods in agreement. (Manouchehri, 2012, p. 19)

a. What is your assessment of Amy’s and Peter’s arguments? Do you find these arguments common among high school students? What tools would you use to assess the level of the students’ thinking?

b. What issues would the teacher need to address with children regarding their arguments?

c. What could be contributing to the way that the children argue about uniqueness of the midpoint?

d. If you were the teacher of these children how would you proceed with your lesson? Explain your reasoning.

10. Consider the following letter written by a high school student:
“Today our teacher talked about the area of a triangle being ½ base times height. I think the formula is helpful when we have all the information but how are we supposed to find the area of a triangle if we don’t know what its height is?”
(Manouchehri, 2012, p. 93)

a. What do you think the basis for this student’s question is?

b. Do you think this is an important question for students to consider? Explain.

c. How would you respond to this student’s question?

d. What tools would you use to assess the level of the students’ thinking?

11. If you had to choose between teaching two of the following three topics which would you choose and why? How do you think these topics would help students when extended to other concepts?
d. Transformations  

e. Similarity  

f. Right triangle trigonometry  

12. Pick one of the topics you chose in question 12. What issues would you consider when sequencing your lessons. What challenges do you foresee associated with student learning? How do you plan on dealing with those challenges? Please explain in detail the basis of sequencing the lesson.  

13. Consider the student’s explanation for explaining why the conjecture “Inscribed angles that share a common chord have the same measure” is true.  

“I noticed the two angles take up the same amount of the circumference so their openings must be the same.” (Manouchehri, 2012, p. 19)  

a. What tools would you use to assess the level of the students’ thinking?  

b. Why do you think the students might conceive of such a relationship?  

c. How do you respond to this student’s conception?  

d. What are some questions you could ask the student to further his/her understanding of the topic?  

14. The formula for finding the measure of any interior angle in a regular polygon with N sides is given by \( \frac{(N-2) \times 180}{N} \). One of the ways in which this formula can be derived is by triangulating the figure from one vertex as follows:  

Suppose a student uses a different triangulation method and argues that the formula generated from the table does not work in his model since he gets more triangles in his pictures.  

a. How do you think the student came up with this method?  

b. Could this method be explained and modified to so to produce the same result as the GENERAL FORMULA?
c. What other challenges might arise in trying to explain the concept of triangulation technique to students so to assure understanding?

15. The following is a summary of responses offered by a couple of 8th graders to the following question: Give a quick explanation of why a reflection of a reflection, with intersecting mirror, must be a rotation?

S1: A reflection of a reflection must be a rotation since it moves and does not flip over a fixed point. I can move around the mirrors to make a rotation

S2: It must be a rotation because if you reflect an object over intersecting mirrors you can also rotate the object to get it into that same position. A reflection is a flipped or mirrored image, and when you rotate an object around a point/line, it changes positions and the image is mirrored no matter where you put the line. (Manouchehri, 2012, p. 43)

a. What factors do you think influenced these explanations?
b. Decide whether you agree or disagree with these students. Please explain your reasons.
c. What tasks would you use to help these students understand the result?

16. Consider the following reflections:
   a. Reflection about the X-axis
   b. Reflection about the Y-axis
   c. Symmetry about the origin

   a. Which of the above isometries might be difficult for students to visualize? Please explain in detail why you believe them to be more difficult.
   b. What techniques might be effective in helping children identify similarities or differences among these three types of isometries? How do you know these techniques are effective?

17. Students are asked to find the volume of a square pyramid. The side length of the base is 11 inches and slant height is 15 inches. One answer given by the students was 451 in³. What error do you think the student has made? (Bush, Ronau, McGatha, & Thompson, 2002)

18. Consider the following 5 geometric constructions using Euclidean methods:
   i) Constructing a perpendicular line to a given line
   ii) Constructing a parallel line to a given line
   iii) Constructing an angle bisector
   iv) Constructing a congruent line segment
   v) Constructing a tangent to a circle from a point on the circle.
a. Rank these constructions starting with the most important to least important for high school students to understand and do. (If you believe they all are of equal importance please state your reasoning)
b. Please explain the basis for your ranking and why you believe they are important for students? That is, how would such sequencing help their conceptual development of Euclidean constructions?

19. List 3 or 4 topics in Geometry that you think will be hard for students. Please explain why they would be hard.

Part 4: Analysis of student work in algebra and probability
20. A student solved the equation $3(n - 7) = 4 - n$ and obtained the solution $n = 2.75$. What might the student have done wrong? (KAT released items) ((Ferrini-Mundy, & Senk, 2006, p. 35)

21. Part 1: Which of the following is the most likely result of five flips of a fair coin?
   d. HHHTT
   e. THHTH
   f. THTTT
   g. HTHTH
   h. All four sequences are equally likely.

   Part 2: Which of the above sequences would be least likely to occur? (Konold et al., 1993, p. 397)

   What might be some of the typical student responses and why? Please explain in detail.
Appendix D: Posttest Interview Protocol

Part 1: Initial thoughts

1. Describe your ideal mathematics classroom; what would the teacher be doing? What would the students be doing? What would be their roles? What type of curriculum would be used and why?
2. What kinds of knowledge might be need for teachers in order for them to be able to maintain such a learning environment?
3. What obstacles do you think you would face in creating your ideal classroom?
4. To what extent has your coursework prepared you to model that type of teaching/foster the type of learning/or implement the kind of curriculum you outline in your description of an ideal classroom?

Part 2: Beliefs

5. Do you think children are capable of solving problems on their own or do you think they need to be taught certain procedures before they can solve problems? (For clarification: for example, do you think high school geometry students are capable of proving that the two diagonals of the rectangle divide the area into four equal parts? Do they need to know the properties of congruence before they can attempt this problem or do you think they can solve this problem without any formal knowledge of high school geometry?) If the response is “it depends”, ask for examples to illustrate.

6. Is it okay if students to follow certain procedures without understanding them as long as they can use them to find the right answers? If so could you give an example? (Probe further: Is it okay if the students do not understand the Pythagorean theorem but are able to successfully use it to find the sides of a right triangle? Under what circumstances is rote memorization useful?) Again, if the response is “it depends, ask for specific examples to illustrate).

7. Should teachers encourage students to use multiple ways of solving a problem? Could you give a specific example? Why it would be useful in that case to approach it using different ways?
8. Should teachers encourage students to use multiple ways of solving a problem? Could you give a specific example? Why it would be useful in that case to approach it using different ways?

9. Who should be responsible for correcting students’ responses? Can students solve their own discrepancies by collaborative work? (Ask for examples to clarify)

Part 3: Self-Efficacy

10. What information would you need and/consider when assessing students’ work?
11. Item 5c on the survey concerned good questions. What questions do you qualify as good questions? Consider a topic and give an example of a good question.

12. If you are assisting a student in trying to solve the following problem: Prove that the diagonals of a square bisect each other. What would be some good questions to ask the student?

13. Item 5g on the survey is about student comprehension. What does the term comprehension mean to you? (Probe about mathematics in particular). How could student comprehension be measured?

14. What behavior do you consider to be disruptive behavior? Would a student repeatedly asking questions or shouting out answers qualify as disruptive behavior? If so how would you deal with that?

15. Why do you think students engage in disruptive behavior?

16. What obstacles do you feel you might face when dealing with classroom management?

17. Is it possible to have an instructional strategy in which all students are equally occupied with the task? Could you give an example?

18. Is it possible to change students’ beliefs about their mathematical abilities? If so how could you make them believe that they can do well in their schoolwork?
19. What are some of the ways you would try and motivate the students who show low interest in schoolwork? How would you motivate a student who has no interest in mathematics or a student who doesn’t see the usefulness of mathematics other than in school?

20. What do you think are some of the reasons students are failing mathematics? Do you think they are ‘bad’ at math or just uninterested? Is it possible for students who have been failing in mathematics (or performing badly) to improve?

Part 4: Analysis of Student Work

21. Question 8:
   1. Do you think this particular question would be hard for high school geometry students?
   m. Could you explain your response to this question in detail?
   n. What instructional approach would you use to help this student?
   o. Why did you choose the particular approach?
   p. What are the advantages of your approach?
   q. What are the limitations of your approach?
   r. What are some of the obstacles you would face while using this approach in trying to get the student to understand the result?
   s. What are some of the obstacles that the students would face while solving this problem?
   t. What do you think would be some of the common misconceptions about this topic?
   u. How would you incorporate technology in your instruction to help further student understanding?
   v. What are some other ways in which you could help the student understand this result?

22. Question 9:
   a. Please explain your response in detail.
   b. Why did you choose the following approach?
   c. What are the advantages of your approach?
   d. What are the disadvantages of your approach?
   e. How would you incorporate technology in your instruction to help further student understanding?
   f. What are some other ways in which you could help the student understand this result?

23. Question 10:
   g. Please explain your responses in detail.
h. What are some of the common sources of those conceptions?
i. Do you think such problems should be intuitive to the students?
j. What tasks would you design in order to address those alternate conceptions?
How would you address misunderstandings?
k. What other activities could you use to help the students with this problem?
l. What other questions do you foresee arising related to reasoning and proofs in high school geometry?

24. Question 11:
a. Could you explain your response to this question in detail?
b. What instructional approach would you use to help this student?
c. Why did you choose the particular approach?
d. What are the advantages of your approach?
e. What are the limitations of your approach?
f. What are some of the obstacles you would face while using this approach in trying to get the student to understand the result?
g. What are some of the obstacles that the students would face while solving this problem?
h. What do you think would be some of the common misconceptions about this topic?
i. How would you incorporate technology in your instruction to help further student understanding?
j. What are some other ways in which you could help the student understand this result?

25. Question 12:
k. Do you think this particular question would be hard for high school geometry students?
l. Could you explain your response to this question in detail?
m. What instructional approach would you use to help this student?
n. Why did you choose the particular approach?
o. What are the advantages of your approach?
p. What are the limitations of your approach?
q. What are some of the obstacles you would face while using this approach in trying to get the student to understand the result?
r. What are some of the obstacles that the students would face while solving this problem?
s. What do you think would be some of the common misconceptions about this topic?
t. How would you incorporate technology in your instruction to help further student understanding?
u. What are some other ways in which you could help the student understand this result?
26. Question 13:
   a. Do you think this particular question would be hard for high school geometry students?
   b. Could you explain your response to this question in detail?
   c. What instructional approach would you use to help the group?
   d. Why did you choose the particular approach?
   e. What are the advantages of your approach?
   f. What are the limitations of your approach?
   g. Is group work useful in this case or would it be more beneficial to work alone?
   h. What can you say about the students?
   i. What do you think would be some of the common misconceptions about this topic?
   j. How would you incorporate technology in your instruction to help further student understanding?
   k. What are some other ways in which you could help the students understand this result?

27. Question 14:
   a. Do you think transformations would be hard for high school geometry students?
   b. Could you explain your response to this question in detail?
   c. What are some of the obstacles you would face while using this approach in trying to get the students to understand various transformations?
   d. What are some of the obstacles that the students would face while solving problems involving transformations?
   e. What do you think would be some of the common misconceptions about this topic?
   f. How would you incorporate technology in your instruction to help further student understanding?

28. Question 15:
   a. Please explain your response in detail.
   b. Why did you choose the following topics?
   c. Please explain in detail why you think students will benefit more from your choice?
   d. What are the disadvantages of leaving out the third topic?
   e. What are some of the obstacles that the students face?
   f. What do you think would be some of the common misconceptions about these topics

29. Question 16:
   a. Why did you choose this particular sequence?
b. What are the advantages of choosing this sequence?
c. How would this sequence help students better grasp the concept?
d. How would you trace student progress using this sequence?

30. Question 17:
a. Please explain your response in detail.
b. Why did you choose the following definitions?
c. Please explain in detail why you think students will benefit more from your choice?
d. What are the disadvantages of your choice?
e. What are some of the obstacles that the students face?
f. What do you think would be some of the common misconceptions about this topic?

31. Question 18:
a. Do you think this particular question would be hard for high school geometry students?
b. Please explain your response in detail.
c. What instructional approach would you use to help this student?
d. What are the advantages of your approach?
e. What are the limitations of your approach?
f. What are some of the obstacles you would face while using this approach in trying to get the student to understand the result?
g. What are some of the obstacles that the students would face while solving this problem?
h. What do you think would be some of the common misconceptions about this topic?
i. How would you incorporate technology in your instruction to help further student understanding?
j. What are some other ways in which you could help the student understand this result?

32. Question 19:
a. Please explain your response in detail.
b. On what basis did you rank them?
c. Please explain in detail why you think students will benefit more from your choice?
d. What are the disadvantages of leaving out other constructions?
e. What are some of the obstacles that the students face?

Part 5: Reactions to the course
33. Did the course help you to better understand students’ thinking?
34. What new information have you learned about students that you were not aware of before?
35. How has this course helped you in terms of assessing students?
36. Has this course had any impact on your decision-making skills?
37. What is the one thing you liked the most about this course?
38. What are the things you would like to change about this course?
39. Do you think your beliefs about students and mathematics in general have changed? If so why?
Appendix E: Rubric for assessing pre-service teachers’ analysis of student work and thinking

(Gilchrist & Somayajulu, 2010ab; Manouchehri, 2011)

<table>
<thead>
<tr>
<th></th>
<th>Mathematically</th>
<th>Pedagogically</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Proficient</strong></td>
<td>• Can articulate the basis for mathematical decisions children make.</td>
<td>• Identifying strengths and weaknesses of each approach drawing from a long-term perspective on content development.</td>
</tr>
<tr>
<td></td>
<td>• Offer other relevant mathematical concepts that could be pursued with them.</td>
<td>• Referenced the impact of curriculum and instruction on children’s behaviors and mathematical performance.</td>
</tr>
<tr>
<td></td>
<td>• Successfully identifies students’ errors or misuses of language.</td>
<td>• Successfully able to identify other instances where students may not be able to perform adequately.</td>
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<tr>
<td></td>
<td>• Offers explanations of why certain mathematical actions are sophisticated and can identify the trajectory of concepts.</td>
<td>• Identifies the strengths and relevance of children’s ideas from a mathematical standpoint.</td>
</tr>
<tr>
<td></td>
<td>• Identifies connections between the different representations that students have to offer.</td>
<td>• Able to successfully unpack the mathematical work of children or their connections to other concepts.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Can identify related concepts that address the mathematics presented in the case.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Successfully able to utilize theoretical models for assessment.</td>
</tr>
<tr>
<td>Developing</td>
<td>Under-developed</td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td>-------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>• Have difficulty in articulating the basis for mathematical decisions children make.</td>
<td>• Unable to articulate the basis for the mathematical decisions children make.</td>
<td></td>
</tr>
<tr>
<td>• Having difficulty offering other relevant mathematical concepts that could be pursued with them.</td>
<td>• Unable to offer other relevant mathematical concepts that might be pursued with them.</td>
<td></td>
</tr>
<tr>
<td>• Can offer explanations of why certain mathematical actions are sophisticated but are not able to adequately identify the trajectory of concepts.</td>
<td>• Avoids mathematics. Does not understand the mathematics used by the child.</td>
<td></td>
</tr>
<tr>
<td>• Are able to see connections between different representations that students have to offer.</td>
<td>• Unable to identify connections/differences among different representations that students have to offer.</td>
<td></td>
</tr>
<tr>
<td>• Having difficulty identifying students’ errors or misuses of language.</td>
<td>• Fail to offer explanations of why certain mathematical actions are sophisticated and are unable to identify the trajectory of concepts.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Unable to identify strengths and weaknesses of each approach drawing from a long-term perspective on content development.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Have difficulty in identifying the impact of school curriculum and instruction on children’s work and thinking.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Have difficulty in identifying other instances where students may not be able to perform adequately.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Able to identify a few strengths and weaknesses of children’s ideas from a mathematical standpoint.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Unable to adequately unpack the mathematical work of children or their connections to other concepts.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Can identify some related concepts that address the mathematics presented in the case.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Having difficulty in utilizing theoretical models for assessment.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Unable to identify students’ errors or misuses of language.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Unable to identify the impact of curriculum and instruction on children’s behaviors and mathematical performance.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Unable to identify other instances where students may not be able to perform adequately.</td>
<td></td>
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<tr>
<td></td>
<td>• Are not able to unpack the mathematical work of children or their relevance.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Do not see the utility of theoretical models for assessment.</td>
<td></td>
</tr>
<tr>
<td>errors or misuses of language.</td>
<td></td>
<td></td>
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<tr>
<td>--------------------------------</td>
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<td></td>
</tr>
</tbody>
</table>

348
Appendix F: Rubric for analyzing instructional strategies used by pre-teachers

(Gilchrist, & Somayajulu, 2010ab; Manouchehri, 2011)
<table>
<thead>
<tr>
<th>Mathematically</th>
<th>Pedagogically</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mature</strong></td>
<td>• Strains suggested are mathematically accurate and address the key concepts and ideas.</td>
</tr>
<tr>
<td></td>
<td>• Instructional strategy builds upon the strengths and relevance of children’s mathematical work and thinking.</td>
</tr>
<tr>
<td></td>
<td>• Strategies suggested address children’s misconceptions and errors.</td>
</tr>
<tr>
<td></td>
<td>• Able to draw from theoretical views on learning to offer perspectives on why certain pedagogical choices would be suitable.</td>
</tr>
<tr>
<td></td>
<td>• Offer specific guides for how to extend children’s intellectual workspace.</td>
</tr>
<tr>
<td></td>
<td>• Is aware of the obstacles faced by the teacher in trying to implement the suggested instructional strategy.</td>
</tr>
<tr>
<td></td>
<td>• Suggests multiple strategies for helping the student with difficulties or misconceptions.</td>
</tr>
<tr>
<td></td>
<td>• Instructional strategies suggested are well developed and well organized.</td>
</tr>
<tr>
<td></td>
<td>• Strategies suggested make provisions for not just a student or a group of students but the entire class.</td>
</tr>
<tr>
<td></td>
<td>• Are able to set precise mathematical goals for children.</td>
</tr>
<tr>
<td><strong>Developing (content specific pedagogy)</strong></td>
<td>• Strategies suggested are mathematically accurate but do not address some key concepts or areas.</td>
</tr>
<tr>
<td></td>
<td>• Instructional strategy addresses some strengths and relevance of children’s work</td>
</tr>
<tr>
<td></td>
<td>• Instructional strategies are aimed at addressing students’ misconceptions and errors.</td>
</tr>
<tr>
<td></td>
<td>• In some situations are able to draw from theoretical views on learning to offer perspectives on why certain pedagogical choices would be suitable.</td>
</tr>
<tr>
<td></td>
<td>• Instructional strategies are content specific and offer general guides for how to</td>
</tr>
<tr>
<td>Developing: General Pedagogy</td>
<td>Instructional strategies are mathematically accurate and address some key concepts and ideas.</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Able to identify some obstacles faced by the teacher in trying to implement the suggested instructional strategy.</td>
</tr>
<tr>
<td></td>
<td>There is some evidence of how the instructional strategies are structured and organized.</td>
</tr>
<tr>
<td></td>
<td>Are able to set some mathematical goals for children and instruction.</td>
</tr>
<tr>
<td>Naïve</td>
<td>Instructional strategy is not mathematically accurate and doesn’t address any of the key concepts and ideas.</td>
</tr>
<tr>
<td></td>
<td>No instructional strategies suggested.</td>
</tr>
<tr>
<td></td>
<td>Avoids mathematics or mathematics incorrect.</td>
</tr>
</tbody>
</table>

- Are able to set some mathematical goals for children and instruction.
<table>
<thead>
<tr>
<th>misconception.</th>
<th>Unable to set mathematical goals for children and instruction.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Chooses to re-teach the concept without modifying instruction.</td>
</tr>
<tr>
<td></td>
<td>Solves the problem for the student.</td>
</tr>
</tbody>
</table>
Appendix G: Rubric for analyzing the open-ended questions and interviews for quality of questions posed by the pre-service teachers

(Gilchrist, & Somayajulu, 2010ab)

Teachers’ responses will be categorized according to their level of mastery along a continuum from proficient, developing and under-developed

I. Proficient
   - Questions are thought provoking and are aimed at eliciting student understanding about the topic
   - Questions are mostly open ended and not yes/no type
   - Questions allow the student to reflect on his or her own work
   - Questions are aimed at eliciting student understanding of the formulae and procedures rather than recalling them
   - Extending the problem to see if the student has understood the concept. Allowing for checking the validity of a student’s response under different situations

II. Developing
   - Questions are thought provoking but do not necessarily elicit student understanding about the topic
   - Questions are aimed at eliciting detailed student responses
   - Questions are not aimed at memorization of formulae and procedures
   - Questions enable students to reflect on their work and thinking

III. Under-developed
   - Questions are aimed at guiding the students to the solution
   - Questions mostly involve recalling memorized formulae and procedures
   - Questions do not allow students to reflect on their own work and thinking
   - Most of the questions are poorly formed and do not elicit student response other than a “yes/no/I don’t know” type.
Appendix H: IRB Protocol
INITIAL REVIEW OF HUMAN SUBJECTS RESEARCH
The Ohio State University Institutional Review Boards
Office of Responsible Research Practices (ORRP)
300 Research Administration Building, 1960 Kenny Road, Columbus, OH 43210
Phone: (614) 688-8457  Fax: (614) 688-0366  www.orrp.osu.edu

<table>
<thead>
<tr>
<th>INDEX</th>
<th>DATE RECEIVED:</th>
<th>DATE VERIFIED COMPLETE:</th>
<th>OSU PROTOCOL NUMBER:</th>
</tr>
</thead>
</table>

1. PROJECT TITLE
Building pre-service teachers’ mathematical knowledge for teaching of high school geometry

2. INSTITUTIONAL REVIEW BOARD
Select the Board to review this research: 
☐ Behavioral and Social Sciences
☐ Biomedical Sciences
☐ Cancer
Final Board assignment is determined by ORRP.

3. PRINCIPAL INVESTIGATOR (or Advisor) - see Qualifications for service as a PI
Name (Last, First, MI): Manouchehr Azza
University Academic Title: Professor of Mathematics Education
Department Name (TIU): School of Teaching and Learning
Campus Mailing Address: 244 Arps Hall
1946 N. High Street
OSU
Columbus, OH 43210

E-mail: manouchehr.1@osu.edu
Phone: 614-688-4258

Degree(s): PhD
College (TIU): EME
Department # (TIU): T&L 12750
OSU ID Number: 07176090

4. CO-INVESTIGATOR(S)
Are there any OSU Co-Investigators on this protocol? 
☐ Yes → Complete Appendix A1
☐ No

Signatures of Co-Investigator(s) are required on Appendix A1.

5. KEY PERSONNEL
Are there any OSU key personnel on this protocol? 
☐ Yes → Complete Appendix A1
☐ No

Key personnel are defined as individuals who participate in the design, conduct, or reporting of human subjects research. At a minimum, include individuals who recruit or consent participants or who collect study data.

6. EXTERNAL CO-INVESTIGATOR(S) & KEY PERSONNEL
Are any external (non-OSU) Investigators or key personnel engaged in the OSU research? 
☐ Yes
☐ No → Go to Question #7

“Engaged” individuals are those who intervene or interact with participants in the context of the research or who will obtain individually identifiable private information for research funded, supervised, or coordinated by OSU. See OHRP Engagemt Guidance or contact ORRP for more information.
If Yes → Who will provide approval for these external personnel? □ OSU IRB → Complete Appendix A2
□ Non-OSU IRB → Provide a copy of the approval(s)

7. ADDITIONAL CONTACT(S)
If further information about this application is needed, specify the contact person(s) if other than the PI (e.g., study or regulatory coordinator, research assistant, etc.).
N/A

Name (Last, First, MI): Phone:
E-mail: Fax:

Name (Last, First, MI): Phone:
E-mail: Fax:

All OSU individuals listed on this protocol will have access to information about IRB actions and the completion status of each individual’s administrative and training requirements (CITI, COI disclosure). Note: Personal financial information provided in COI disclosures is not included.

8. EDUCATION
Have all OSU investigators and key personnel completed the required web-based course (CITI) in the protection of human research subjects? ☑ Yes □ No

Educational requirements (initial and continuing) must be satisfied prior to submitting the application for IRB review. See CITI Training or contact ORRP for more information.

9. FINANCIAL CONFLICT OF INTEREST
Does any OSU investigator (including principal or co-investigator), key personnel, or their immediate family members have a financial interest (including salary or other payments for services, equity interests, or intellectual property rights) that would reasonably appear to be affected by the research, or a financial interest in any entity whose financial interest would reasonably appear to be affected by the research?

☑ Yes □ No

All OSU investigators and key personnel must have a current COI disclosure form (updated as necessary for the proposed research) filed before IRB review. Examples of financial interests that must be disclosed include (but are not limited to) consulting fees or honoraria; stocks, stock options or other ownership interests; and patents, copyrights and royalties from such rights. For more information, see Office of Research Compliance COI Overview and COI Forms.

10. FUNDING OR OTHER SUPPORT
If the research is federally funded and involves a subcontract to or from another entity, an IRB Authorization Agreement may be required. Contact ORRP for more information.

a. Is the research funded or has funding been requested?
☑ Yes □ No

If Yes → Specify sponsor:

Provide a copy of the grant application or funding proposal. The University is required to verify that all funding proposals and grants (new or renewals) have been reviewed by the IRB before funds are awarded.

b. Is any support other than monetary (e.g., drugs, equipment, etc.) being provided for the study?
☑ Yes □ No

If Yes → Specify support and provider:

11. OTHER INSTITUTIONAL APPROVALS
Check all that apply and provide applicable documentation. See websites listed below for information on obtaining approvals. IRB review cannot be conducted until required institutional approvals or exemptions are obtained, except as noted.

- [ ] None
- [ ] Clinical Research Center (CRC) Scientific Advisory Committee (SAC) – Approval required for research sponsored by the CRC. Final IRB approval will be held pending receipt of SAC approval.
- [ ] Institutional Biosafety Committee (IBC) – Approval required for research involving biohazards (recombinant DNA, infectious or select agents, toxins), gene transfer, or xenotransplantation.
- [ ] Comprehensive Cancer Center (CCC) Clinical Scientific Review Committee (CSRC) – Approval or exemption required for cancer-related research.
- [ ] Maternal-Fetal Welfare Committee – Approval required for some research involving pregnant women and fetuses.
- [ ] Human Subject Radiation Committee (HSRC) – Approval required for research involving radiologic procedures for research purposes (e.g., non-clinical care X-rays, DEXA or CT scans, nuclear medicine procedures, etc.).

### 12. LOCATION OF THE RESEARCH

Research to be conducted at locations other than approved performance sites will minimally require a letter of support and may require another IRB's approval if personnel are engaged. See OHRP Engagement Guidance or contact ORRP for more information.

a. List the specific site(s) at which the OSU research will be conducted (include both domestic and international locations).

<table>
<thead>
<tr>
<th>Location Name (or description)</th>
<th>Address (street, city and state, or country)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arps Hall</td>
<td>1945 N. High Street&lt;br&gt;The Ohio State University&lt;br&gt;Columbus, OH 43210</td>
</tr>
</tbody>
</table>

b. Are all the sites named above on the OSU list of approved research performance sites?

- [ ] Yes → Go to Question #13
- [ ] No

If No →
- [ ] Domestic sites → Provide a letter of support, as applicable
- [ ] International sites → Complete Appendix U

c. For multi-site research, is the OSU PI the lead investigator or is OSU the lead site?

- [ ] Yes
- [ ] No → Go to Question #13
- [ ] Not multi-site → Go to Question #13

i. Describe the communication between sites that might be relevant to the protection of participants, such as unanticipated problems, interim results, and protocol modifications.

ii. Describe IRB oversight arrangements for each site (i.e., who is providing IRB review and approval). Provide copies of the non-OSU approvals, as applicable. Contact ORRP if requesting OSU be the IRB of record.
13. EXPEDITED REVIEW

Are you requesting Expedited Review?
☐ Yes → Complete Appendix B
☐ No

14. SUMMARY OF THE RESEARCH

Summarize the proposed research using non-technical language that can be readily understood by someone outside the discipline. Explain briefly the research design, procedures to be used, risks and anticipated benefits, and the importance of the knowledge that may reasonably be expected to result. Use complete sentences (limit 300 words).

The aim of the proposed study is to investigate the nature of the prospective secondary teachers' mathematical knowledge as it pertains to the analysis of student thinking in high school geometry and to explore the impact of two quarters of course work that would help further enhance this knowledge. Grounding our work in theoretical description of dimensions of Mathematical Knowledge of Algebra (KAT) developed by McCrory et al. (2010), we propose to study the development of an entire cohort of secondary mathematics teachers enrolled in the STEM M.Ed. Program at The Ohio State University seeking licensure to teach 7th - 12th grade mathematics. This framework will guide both the development of course materials as well as the assessment instrument used to document teachers' mathematical knowledge for teaching geometry.

The proposed study will contain three phases. The first phase will consist of collecting baseline information about the participants of the study. In this phase, pretest questionnaires will be administered to the pre-service teachers during the first week of the Summer quarter of 2011 as a part of their in class assignment in EDU T&L 621. Another course assignment for EDU T&L 621 requires teachers to write a case study report of a student they have during the field-based component of the course. Since one of the aims of the study is to investigate the impact of two quarters of course work on pre-service teachers’ mathematical knowledge, it is essential that we collect this information during the first quarter of their enrollment in the program. The first phase of the study will also include pretest interviews, which will be aimed at obtaining in depth information about the pre-service teachers' responses to the items on the pretest questionnaire.

During the second phase of the study the pre-service teachers will be observed in their geometry methods course (EDU T&L 749.02) during the Autumn quarter of 2011. One of the course assignments of EDU T&L 749.02 requires the pre-service teachers to conduct clinical interviews with high school students at The Ohio State University. This phase will conclude in December of 2011. The final phase of the study will consist of gathering information about the pre-service teachers after the geometry methods course. During the last two weeks of the Autumn quarter of 2012, the pre-service teachers will be given a posttest questionnaire as a part of their coursework for EDU T&L 749.02. These posttest questionnaires will be followed by posttest interviews. The aim of the posttest interviews would be to get more information about the pre-service teachers' responses to the posttest questionnaires. The final phase will conclude with the administration of the final questionnaire at the end of the Winter quarter of 2012. The aim of the final questionnaire is to measure the impact of the long term effects of the two quarter long coursework on pre-service teachers knowledge as it pertains to the analysis of student thinking and learning.

15. SCIENTIFIC BACKGROUND & LITERATURE REVIEW

Summarize existing knowledge and previous work that support the expectation of obtaining useful results without undue risk to human subjects. Use complete sentences (limit 300 words).

Bass (2005) coined the notion of Mathematical Knowledge for Teaching (MKT) as a means to distinguish a specialized body of knowledge of subject matter that is needed for teaching from how, for instance, an engineer might use mathematics. In recent years studies have been conducted to assess and document the level of elementary school teachers' mathematical knowledge for teaching. These studies have claimed that teachers' knowledge is correlated with student achievement. Similar explorations at the secondary school level have been rare. In explaining this phenomenon it has been argued that secondary school mathematics is very complex and consists of a wide range of topics and concepts (McCarty, Ferrini-Mundy, Floden, Reckase, & Senk, 2010). The large scope of content and concepts covered at the secondary level makes it difficult to capture teachers' knowledge entirely. Although Ball and her colleagues (D. L. Ball, 1990; D. L. Ball & Bass, 2000; D. L. Ball, Hill, & Bass, 2005; Hill, 2007; Hill, et al., 2005) have made tremendous progress in measuring and analyzing mathematical knowledge for teaching at the elementary levels, compatible instruments that could be used to measure high school teachers' knowledge are not currently present (McCrory, et al., 2010). Efforts are currently taking place towards developing and testing a framework to measure the mathematical knowledge of algebra teaching (KAT) (Floden, R. E., 2005; McCrory et al., 2010). In the area of geometry however, despite suggestions for aspects of knowledge for teaching that might be fostered in teacher preparation neither specific guides for how the content of course might be organized so to better align with proposed domains of mathematical knowledge for teaching, nor measurement tools that could capture teachers' growth and
development exist.

16. RESEARCH OBJECTIVES

List the specific scientific or scholarly aims of the research study.

The primary goal of this study is to explore the quality and content of pre-service teachers' Mathematical Knowledge for Teaching (MKT) as it pertains to geometry. Another goal of the study was to investigate the impact of a methods course centered around the analysis of student work and thinking of geometry on pre-service teachers' mathematical knowledge.

The following questions will guide the data collection and analysis:

1. What factors do pre-service teachers consider when judging students' mathematical work and thinking?
2. What is the effect of two quarters of coursework on pre-service teachers' assessment of students' mathematical work and thinking of geometry and measurement?
3. What is the effect of two quarters of coursework on pre-service teachers' ability to develop instructional strategies that aid student understanding of geometry and measurement?
4. What is the effect of two quarters of coursework on the quality of questions posed by the pre-service teachers to elicit student understanding of geometry and measurement?
5. What is the relationship between levels of teachers' self-efficacy and their knowledge of students' learning and thinking?
6. What obstacles are faced when developing and implementing a geometry methods course which utilizes analysis of student work and thinking?

17. RESEARCH METHODS & ACTIVITIES

a. Identify and describe all interventions and interactions that are to be performed solely for the research study. Distinguish research (i.e., experimental) activities from non-research activities. Provide data collection forms to be used. Note: Do not include case report forms for multi-site industry-sponsored or cooperative group studies.

All materials used during the methods courses EDU T&L 621 and EDU T&L 749.02 are a part of the data.

During the second term of the Summer quarter of 2011, the researchers will meet with the potential participants of the 2011-2012 cohort and explain the goals of the research. Consent forms will be distributed among the participants to be taken home and completed. Contact information for the researchers of the study will be provided if additional information might be needed. The participants will be asked to return the signed consent forms to class the following week.

The pretest questionnaire would have been administered to the 2011-2012 cohort during the previous summer term in their EDU T&L 621 class as a part of their coursework. Also as a part of the coursework for EDU T&L 621, pre-service teachers will also be required to conduct clinical interviews with high school students and write a case based analysis of their findings. The pre-service teachers will then be administered a pretest interview to get more in depth information about their responses to the pretest interviews.

The pre-service teachers will be observed in the geometry methods course (EDU T&L 749.02) during the Autumn quarter of 2011. The posttest questionnaire will be administered to the 2011-2012 cohort during the 11th week of the Autumn quarter of 2011 as a part of their coursework for EDU T&L 749.02. The interviews will be conducted during the 11th and 12th weeks of the Autumn quarter of 2011.

Also as a part of the coursework for EDU T&L 749.02, the pre-service teachers will be required to conduct a half hour clinical interview with a high school student participating in the Young Scholar program. These clinical interviews will take place during Autumn quarter of 2011. Following those interviews, the pre-service teachers will be subject to a short follow up interview which will last for about 30 minutes.

The final questionnaire will be administered during the last week of the Winter quarter of 2012 as a part of their coursework for EDU T&L 749.01.

b. Check all research activities that apply:

- Anesthesia (general or local) or sedation
- Audio, video, digital, or image recordings
- Magnetic Resonance Imaging (MRI)
- Materials that may be considered sensitive, offensive, threatening, or degrading
The Ohio State University Institutional Review Boards - INITIAL REVIEW OF HUMAN SUBJECTS RESEARCH

☐ Biohazards (e.g., rDNA, infectious agents, select agents, toxins)
☐ Biological sampling (other than blood)
☐ Blood drawing
☐ Coordinating Center
☐ Data, not publicly available
☐ Data, publicly available
☐ Data repositories ▸ Complete Appendix C (future unspecified use, including research databases)
☐ Deception ▸ Complete Appendix D & Appendix M1
☐ Devices ▸ Complete Appendix F
☐ Diet, exercise, or sleep modifications
☐ Drugs or biologics ▸ Complete Appendix F
☐ Emergency research
☐ Focus groups
☐ Food supplements
☐ Gene transfer
☐ Genetic testing ▸ Complete Appendix G
☐ Internet or e-mail data collection
☐ Non-invasive medical procedures (e.g., EKG, Doppler)
☐ Observation of participants (including field notes)
☐ Oral history (does not include medical history)
☐ Placebo
☐ Pregnancy testing
☐ Program Protocol (Umbrella Protocol)
☐ Radiation (e.g., CT or DEXA scans, X-rays, nuclear medicine procedures) ▸ Complete Appendix V
☐ Randomization
☐ Record review (which may include PHI)
☐ Specimen research
☐ Stem cell research
☐ Storage of biological materials ▸ Complete Appendix H (future unspecified use, including repositories)
☐ Surgical procedures (including biopsies)
☐ Surveys, questionnaires, or interviews (one-on-one)
☐ Surveys, questionnaires, or interviews (group)
☐ Other
☐ Specify :

18. DURATION

Estimate the time required from each participant, including individual interactions, total time commitment, and long-term follow-up, if any.

Consent form (1 week)
Completion of pretest questionnaire (part of coursework in EDU T&L 621) (2 hours)
Clinical interviews in which pre-service teachers will interview high school students (part of coursework in EDU T&L 621) (30 minutes)
Pretest interviews (1 interview per person) (1.5 hours)
Completion of posttest questionnaire (part of coursework in EDU T&L 749.02) (2 hours)
Posttest interviews (1 interview per person) (1.5 hours)
Clinical interviews in which pre-service teachers will interview high school students (part of coursework in EDU T&L 749.02) (30 minutes)
Follow up interviews held immediately after the clinical interviews with high school students (30 minutes)
Final questionnaire (part of coursework in EDU T&L 749.01) (2 hours)

19. NUMBER OF PARTICIPANTS

The number of participants is defined as the number of individuals who agree to participate (i.e., those who provide consent or whose records are accessed, etc.) even if all do not prove eligible or complete the study. The total number of research participants may be increased only with prior IRB approval.

a. Provide the total number of participants (or number of participant records, specimens, etc.) for whom you are seeking OSU IRB approval.
Approximately 20

b. Explain how this number was derived (e.g., statistical rationale, attrition rate, etc.).
The participants are pre-service teachers enrolled in the one year STEM M.Ed. program 2011-2012 cohort at The Ohio State University seeking licensure to teach 7th-12th grade mathematics. There are 25 students in the 2011-2012 cohort.

c. Is this a multi-site study? Yes → Indicate the total number of participants to be enrolled across all sites:
   No

20. PARTICIPANT POPULATION

a. Specify the age(s) of the individuals who may participate in the research:
   Age(s): Each over 20 years old

b. Specify the participant population(s) to be included (check all that apply):
   - Adults
   - Decisionally Impaired Adults → Complete Appendix W
   - Children (< 18 years) → Complete Appendix I
   - Healthy Volunteers
   - Neonates (uncertain viability/ nonviable) → Complete Appendix K
   - Non-English Speaking → Complete Appendix J
   - Pregnant Women/Fetuses → Complete Appendix K
   - Prisoners → Complete Appendix I
   - Students from Participant Pools (e.g., REP) Specify:
   - Unknown (e.g., research using secondary data/specimens, non-targeted surveys, program protocols)

b. Describe the characteristics of the population(s) and explain how the nature of the research requires/justifies inclusion of the proposed population(s).

   The population are pre-service teachers enrolled in the one year STEM M.Ed. program 2011-12 cohort at The Ohio State University seeking licensure to teach 7th-12th grade mathematics.

d. Are any of the participants likely to be vulnerable to coercion or undue influence?
   Yes
   No

If Yes → Describe additional safeguards to protect participants' rights and welfare.
Since the one of the investigators will be in charge of the grades for pre-service teachers in EDU T&L 621, the pre-service teachers might feel forced to participate. However, to safeguard against the possibility of coercion, we will not approach the pre-service teachers until after the grades for EDU T&L 621 have been submitted.

e. Will pregnant women be excluded from participation in the research?
   Yes
   No

If Yes → Explain how the nature of the research requires/justifies their exclusion. Address means of pregnancy screening.

21. PARTICIPANT IDENTIFICATION, RECRUITMENT, & SELECTION

a. Provide evidence that you will be able to recruit the necessary number of participants to complete the study.

The participants are pre-service teachers enrolled in the one year STEM M.Ed. program 2011-2012 cohort at The Ohio State University seeking licensure to teach 7th-12th grade mathematics, following a rigorous selection process. The Ohio State University will provide the researchers with the names of all the candidates enrolled in the program.
The participants' privacy will be respected by not announcing who is interested in participating or who agrees to participate. Additionally, the participants will never be referenced in any public venue using their real names.

b. Describe how potential participants will be identified (e.g., advertising, individuals known to investigator, record review, etc.). Explain how investigator(s) will gain access to this population, as applicable.

The only eligibility for participation is enrollment in the STEM M.Ed. program at The Ohio State University. Recruitment of the participants will take place in summer of 2011 during a required course (EDU T&L 721.03). All potential participants will be contacted by the researchers and will be given an opportunity to learn about the study and to ask questions. They will be asked to provide researchers with consent to participate in the study.

c. List the names of investigator(s) and/or key personnel who will recruit participants and what process will be used to determine participant eligibility.

Azita Manouchehri and Ravi Somayejaulu will recruit participants. Ravi Somayejaulu, Sarah Gilchrist, Yating Liu and Jennifer Czocher will be involved in conducting interviews with the pre-service teachers.

The only eligibility for participation is the enrollment in the STEM M. Ed. program at The Ohio State University.

d. Describe the recruitment process; including the setting in which recruitment will take place. Explain how the process respects potential participants’ privacy. Provide copies of proposed recruitment materials (e.g., ads, flyers, website postings, recruitment letters, and oral/verbal scripts).

Recruitment of the 2011-2012 cohort will take place during a required course (EDU T&L 721.03) in the summer of 2011. All potential participants will be contacted by the researchers and will be given an opportunity to learn about the study and ask questions. They will be asked to provide the researchers with consent to participate in the study.

<table>
<thead>
<tr>
<th>22. INCENTIVES TO PARTICIPATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Will participants receive compensation or other incentives (e.g., free services, cash payments, gift certificates, parking, classroom credit, travel reimbursement) to participate in the research study?</td>
</tr>
<tr>
<td>Compensation plans should be pro-rated (not contingent upon study completion) and should consider participant withdrawals, as applicable.</td>
</tr>
<tr>
<td>If Yes → Describe the incentive, including the amount and timing of all payments.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>23. ALTERNATIVES TO STUDY PARTICIPATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other than choosing not to participate, list any specific alternatives, including available procedures or treatments that may be advantageous to the subject.</td>
</tr>
<tr>
<td>None</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>24. INFORMED CONSENT PROCESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Indicate the consent process(es) and document(s) to be used in the study. Check all that apply. Provide copies of documents and/or complete relevant appendices, as needed. See Consent for Research for templates or contact ORRP for more information.</td>
</tr>
<tr>
<td>✅ Assent – Form</td>
</tr>
<tr>
<td>✗ Assent – Verbal Script</td>
</tr>
<tr>
<td>✅ Informed Consent – Form</td>
</tr>
<tr>
<td>✗ Informed Consent – Verbal Script → Complete Appendix M2</td>
</tr>
<tr>
<td>✗ Informed Consent – Addendum</td>
</tr>
<tr>
<td>✅ Parental Permission – Form</td>
</tr>
<tr>
<td>✗ Parental Permission – Verbal Script → Complete Appendix M2</td>
</tr>
<tr>
<td>✗ Parental Consent/Assent – Form(s) → Complete Appendix M1</td>
</tr>
<tr>
<td>✗ Waiver of Consent Documentation → Complete Appendix M2</td>
</tr>
<tr>
<td>✗ Waiver or Alteration of Consent Process → Complete Appendix M1</td>
</tr>
<tr>
<td>b. List the names of investigator(s) and/or key personnel who will obtain consent from participants or their legally authorized representatives.</td>
</tr>
<tr>
<td>N/A</td>
</tr>
</tbody>
</table>
Ravi B. Somayajulu

c. Who will provide consent or permission (i.e. participant, legally authorized representative, parent and/or guardian)?

N/A

d. Describe the consent process. Explain when and where consent will be obtained and how subjects and/or their legally authorized representatives will be provided sufficient opportunity (e.g., waiting period, if any) to consider participation.

N/A

During the second week of the second term of the Summer quarter of 2011, the researchers will meet with the potential participants of the 2011-2012 cohort and explain the goals of the research. Consent forms will be distributed among the participants to be taken home and considered. Contact information for the researchers of the study will be provided if additional information might be needed. The participants will be asked to return the signed consent forms to class the following week.

c. Explain how the possibility of coercion or undue influence will be minimized in the consent process.

N/A

Since the one of the investigators will be in charge of the grades for pre-service teachers in EDU T&L 621, the pre-service teachers might feel forced to participate. However, to safeguard against the possibility of coercion, we will not approach the pre-service teachers until after the grades for EDU T&L 621 have been submitted.

f. Will any other tools (e.g., quizzes, visual aids, information sheets) be used during the consent process to assist participant comprehension?

Yes → Provide copies of these tools

No

g. Will any other consent forms be used (e.g., for clinical procedures such as MRI, surgery, etc. and/or consent forms from other institutions)?

Yes → Provide copies of these forms

No

25. PRIVACY OF PARTICIPANTS

a. Describe the provisions to protect the privacy interests of the participants.

Participants’ names will be replaced with an alias and their private information such as questionnaire responses and interview tapes will be kept in a locked office and file cabinet in the PI’s office on campus.

b. Does the research require access to personally identifiable private information?

Yes

No

If Yes → Describe the personally identifiable private information involved in the research. List the information source(s) (e.g., educational records, surveys, medical records, etc.).

26. CONFIDENTIALITY OF DATA

a. Explain how information is handled, including storage, security measures (as necessary), and who will have access to the information. Include both electronic and hard copy records.

Each participant will be assigned an alias and this alias will be used for all written and spoken communication to protect his or her identity. The PI and the co-investigators of the project will be the only individuals with access to the raw data corpus collected for this research. All electronic data will be stored on an external storage device. All paper data and the external storage devices will be locked in the PI’s office on campus.
b. Explain if any personal or sensitive information that could be potentially damaging to participants (e.g., relating to illegal behaviors, alcohol or drug use, sexual attitudes, mental health, etc.) will be collected. ○ N/A

c. Will you be obtaining an NIH Certificate of Confidentiality? ○ Yes → Provide a copy before you begin the research ○ No

See OSU HRPP policy Privacy and Confidentiality for more information.

d. Explain any circumstances (ethical or legal) where it would be necessary to break confidentiality. ○ N/A

e. Indicate what will happen to identifiable data at the end of the study. Research-related records should be retained for a period of at least three years after the research has been discontinued (i.e., no further data collection, long term follow-up, re-contact, or analysis of identifiable/coded data.)

- Identifiers permanently removed from the data and destroyed (de-identified)
- Identifiable/coded (linked) data are retained
- Identifiable data not collected

27. HIPAA RESEARCH AUTHORIZATION

Will individually identifiable Protected Health Information (PHI) subject to the HIPAA Privacy Rule requirements be accessed, used, or disclosed in the research study?

- No
- Yes → Check all that apply:
  - Written Authorization → Provide a copy of the Authorization Form
  - Partial Waiver (recruitment purposes only) → Complete Appendix N
  - Full Waiver (entire research study) → Complete Appendix N
  - Alteration (written documentation) → Complete Appendix N

28. REASONABLY ANTICIPATED BENEFITS

a. List the potential benefits that participants may expect as a result of this research study. State if there are no direct benefits to individual participants. Compensation is not to be considered a benefit.

The participants will potentially benefit from increased learning as the result of reflecting on what they do as they solve different problems and analyze student work.

b. List the potential benefits that society and/or others may expect as a result of this research study.

This study will be a significant step toward identifying the mathematical knowledge for teaching of high school geometry. Although there has been a significant amount of research conducted on elementary teachers' mathematical knowledge for teaching of mathematics (Bail et al., 1990, 2005, 2006), at the high school level much is still unknown about teachers' mathematical knowledge for teaching.

The study will also be a major step in identifying and overcoming obstacles faced when developing and implementing a geometry methods course, which utilizes analysis of student work and thinking.

The study will also be a significant step toward developing an instrument for measuring mathematical knowledge of geometry as it pertains to the analysis of students' work and thinking in high school geometry.
29. RISKS, HARMS, & DISCOMFORTS
   a. Describe all reasonably expected risks, harms, and/or discomforts that may apply to the research. Consider the range of risks, including physical, psychological, social, legal, and economic. As applicable, discuss severity and likelihood of occurrence.

   Breach of confidentiality

   b. Describe how risks, harms, and/or discomforts will be minimized.

   The likelihood of breaching confidentiality is minimal because the participants will be referenced by their aliases in publications and presentations. Published papers or presentations represent the most likely situations where confidentiality breaches could possibly occur. The nature of the study is such that the risk or breach of confidentiality is minimal.

30. MONITORING

   Does the research involve greater than minimal risk (i.e., are the harms or discomforts described in Question #29 beyond what is ordinarily encountered in daily life or during the performance of routine physical or psychological tests)?

   Yes ☐ No ☑

   If Yes → Describe the plan to oversee and monitor data collected to ensure participant safety and data integrity. Include the following:
   - The information that will be evaluated (e.g., incidence and severity of actual harm compared to that expected);
   - Who will perform the monitoring (e.g., investigator, sponsor, or independent monitoring committee);
   - Timing of monitoring (e.g., at specific points in time, after a specific number of participants have been enrolled); and
   - Decisions to be made as a result of the monitoring process (e.g., provisions to stop the study early for unanticipated problems).

31. ASSESSMENT OF RISKS & BENEFITS

   Discuss how risks to participants are reasonable when compared to the anticipated benefits to participants (if any) and the importance of the knowledge that may reasonably be expected to result.

   Given the risks that are posed to pre-service teachers, given the breach of confidentiality, there are major benefits. If this research is shared with others via presentations and publications, then the derived benefits will outweigh the risks to the participants, which are minimal.

32. PARTICIPANT COSTS/REIMBURSEMENTS

   a. List any potential costs subjects (or their insurers) will incur as a result of study participation (e.g., parking, study drugs, diagnostic tests, etc.).

   N/A

   b. List any costs to participants that will be covered by the research study.

   N/A
33. APPLICATION CONTENTS

Indicate the documents being submitted for this research project. Check all appropriate boxes.

☑ Initial Review of Human Subjects Research Application

☑ Appendix A1: OSU Co-Investigators & Key Personnel (questions 4 & 5)
☐ Appendix A2: External (non-OSU) Co-Investigators & Key Personnel (question 6)
☐ Appendix B: Expedited Review – Initial Review (question 13)
☐ Appendix C: Data Repositories (question 17b)
☐ Appendix D: Deception (question 17b)
☐ Appendix E: Devices (question 17b)
☐ Appendix F: Drugs or Biologics (question 17b)
☐ Appendix G: Genetic Testing (question 17b)
☐ Appendix H: Storage of Biological Materials (question 17b)
☐ Appendix I: Children (question 20b)
☐ Appendix J: Non-English Speaking Participants (questions 20b and 24a)
☐ Appendix K: Pregnant Women/Fetuses/Neonates (question 20b)
☐ Appendix L: Prisoners (question 20b)
☐ Appendix M1: Waiver or Alteration of Consent Process (questions 17b & 24a)
☐ Appendix M2: Waiver of Consent Documentation (question 24a)
☐ Appendix N: Waiver of HIPAA Research Authorization (question 27)
☐ Appendix U: Research in International Settings (question 12)
☐ Appendix V: Radiation (question 17b)
☐ Appendix W: Decisionally Impaired Adults (question 20b)
☐ Consent form(s), Assent Form(s), Permission Form(s), and Verbal Script(s), including translated documents (question 24a)
☐ HIPAA Research Authorization Form(s) (question 27)
☐ Data Collection Form(s) (question 17a)
☐ Data Collection Form(s) involving protected health information (Appendix N)
☐ Recruitment Materials (e.g., ads, flyers, telephone or other oral script, radio/TV scripts, internet solicitations) (question 21d)
☐ Script(s) or Information Sheet(s), including Debriefing Materials (question 24)
☐ Instruments (e.g., questionnaires or surveys to be completed by participants) (question 17b)
☐ Other Committee Approvals/Letters of Support (questions 11 & 12)
☐ Research Protocol
☐ Complete Grant Application or Funding Proposal
☐ Drug Manufacturer’s Approved Labeling/Investigator’s Drug Brochure (Appendix F)
☐ Device Manufacturer’s Approved Labeling (Appendix E)
☐ Other supporting documentation and/or materials

For Multi-Site Clinical Trials supported by DHHS, the submission will also include:
☐ DHHS-approved Sample Informed Consent Document (if one exists)
☐ DHHS-approved Protocol (if one exists)
34. ASSURANCE

PRINCIPAL INVESTIGATOR (or Advisor)

I agree to follow all applicable policies and procedures of The Ohio State University and federal, state, and local laws and guidance regarding the protection of human subjects in research, as well as professional practice standards and generally accepted good research practice guidelines for investigators, including, but not limited to, the following:

- Perform the research as approved by the IRB under the direction of the Principal Investigator (or Advisor) by appropriately trained and qualified personnel with adequate resources;
- Initiate the research after written notification of IRB approval has been received;
- Obtain and document (unless waived) informed consent and HIPAA research authorization from human subjects (or their legally authorized representatives) prior to their involvement in the research using the currently IRB-approved consent form(s) and process;
- Promptly report to the IRB events that may represent unanticipated problems involving risks to subjects or others;
- Provide significant new findings that may relate to the subjects willingness to continue to participate;
- Inform the IRB of any proposed changes in the research or informed consent process before changes are implemented, and agree that no changes will be made until approved by the OSU IRB (except where necessary to eliminate apparent immediate hazards to participants);
- Complete and submit a Continuing Review of Human Subjects Research application before the deadline for review at intervals determined by the IRB to be appropriate to the degree of risk (but not less than once per year) to avoid expiration of IRB approval and cessation of all research activities;
- Maintain research-related records (and source documents) in a manner that documents the validity of the research and integrity of the data collected, while protecting the confidentiality of the data and privacy of participants;
- Retain research-related records for audit for a period of at least three years after the research has ended (or longer, according to sponsor or publication requirements) even if I leave the University;
- Contact the Office of Responsible Research Practices for assistance in amending (to request a change in Principal Investigator) or terminating the research if I leave the University or am unavailable to conduct or supervise the research personally (e.g., sabbatical or extended leave);
- Provide a Final Study Report to the IRB when all research activities have ended (including data analysis with individually identifiable or coded private information); and
- Inform all Co-Investigators, research staff, employees, and students assisting in the conduct of the research of their obligations in meeting the above commitments.

I verify that the information provided in this Initial Review of Human Subjects Research application is accurate and complete.

______________________________
Signature of Principal Investigator (or Advisor)

______________________________
Date

______________________________
Printed name of Principal Investigator (or Advisor)

DEPARTMENT CHAIR (or Signatory Official)

As Department Chair (or Signatory Official) for the Principal Investigator, I acknowledge that this research is in keeping with the standards set by our unit and that it has met all Departmental/College requirements for review.

If the PI or any Co-Investigator is also the Department Chair, the signature of the Dean or other appropriate Signatory Official, such as the Associate Dean for Research, must be obtained.

______________________________
Signature of Department Chair

______________________________
Date

______________________________
Printed name of Department Chair
Appendix I: Consent Forms

The Ohio State University Consent to Participate in Research

Study Title: Building pre-service teachers’ mathematical knowledge for teaching of high school geometry

Researcher: Azita Manouchehri

Sponsor: None

This is a consent form for research participation. It contains important information about this study and what to expect if you decide to participate.

Your participation is voluntary.

Please consider the information carefully. Feel free to ask questions before making your decision whether or not to participate. If you decide to participate, you will be asked to sign this form and will receive a copy of the form.

Purpose: The purpose of this research study is to investigate the nature of pre-service teachers’ mathematical knowledge as it pertains to the analysis of student thinking in high school geometry and to explore the impact of two quarters of coursework that would help further enhance this knowledge.

Procedures/Tasks: Should you wish to be involved in this study, you will be interviewed once during the second term of the summer quarter of 2011 and then a final interview at the end of the autumn quarter of 2011. Each of these interviews will be between 1 hour and 1.5 hours long. In addition to this, you will be observed during your EDU T&L 749.02 methods course in autumn quarter of 2011. Finally some of your course assignments will be collected and analyzed. However, be assured that all the data collected for the purpose of the study will have no impact whatsoever on your grades for the courses in which you are enrolled or your standing in the program. It will be solely for the purpose of the study. You may be subject to a brief interview during your EDU T&L 749.02 methods course in the autumn quarter of 2011. These interviews, if conducted will be between 15 -30 minutes long. During these interviews, you will not be asked for any private information.
Duration:
The study will end in April 2011 upon the completion of 749.01. The only time required of you is when you are interviewed. In addition you will also be observed during your EDU T&L 749.02 during the autumn quarter of 2011. Some of your course assignments will be analyzed for the purpose of the study. However, this will be done after your grades are issued by the course instructor.
You may leave the study at any time. If you decide to stop participating in the study, there will be no penalty to you, and you will not lose any benefits to which you are otherwise entitled. Your decision will not affect your future relationship with The Ohio State University.
Please mark which of the following assignments you permit to be used for data analysis:

☐ Pre-test Questionnaire (EDU T&L 621)
☐ 3 Lesson plans (EDU T&L 621)
☐ Case-based analysis of student work (EDU T&L 621)
☐ 2 Lesson plans (EDU T&L 749.02)
☐ Post-test Questionnaire (EDU T&L 749.02)
☐ Transcripts of interviews with high school students (EDU T&L 749.02)

Risks and Benefits:
Your participation in this study will have absolutely no negative consequences for you. You will benefit from increased learning as the result of reflecting on what you already do when you analyze student work and thinking in mathematics.

This study will be a significant step toward identifying the mathematical knowledge for teaching of high school geometry. Although there has been a significant amount of research conducted on elementary teachers’ mathematical knowledge for teaching of mathematics (Ball et al., 1990, 2005, 2008), at the high school level much is still unknown about teachers’ mathematical knowledge for teaching.
The study will also be a major step in identifying and overcoming obstacles faced when developing and implementing a geometry methods course, which utilizes analysis of student work and thinking.
The study will also be a significant step toward developing an instrument for measuring mathematical knowledge of geometry as it pertains to the analysis of students’ work and thinking in high school geometry

Confidentiality:
You will be assigned an alias and this alias will be used for all written and spoken communications to protect your identities.
Efforts will be made to keep your study-related information confidential. However, there may be circumstances where this information must be released. For example, personal information
regarding your participation in this study may be disclosed if required by state law. Also, 
your records may be reviewed by the following groups (as applicable to the research):
- Office for Human Research Protections or other federal, state, or international 
  regulatory agencies;
- The Ohio State University Institutional Review Board or Office of Responsible 
  Research Practices;
- The sponsor, if any, or agency (including the Food and Drug Administration for FDA-
  regulated research) supporting the study.

Incentives: You will not be paid to participate in this study. With the exception of 
interviews, all materials used will be from assignments you are expected to complete as 
part of course expectations.

Participant Rights:

You may refuse to participate in this study without penalty or loss of benefits to which you 
are otherwise entitled. If you are a student or employee at Ohio State, your decision will not 
affect your grades or employment status.

If you choose to participate in the study, you may discontinue participation at any time 
without penalty or loss of benefits. By signing this form, you do not give up any personal 
legal rights you may have as a participant in this study.

An Institutional Review Board responsible for human subjects research at The Ohio State 
University reviewed this research project and found it to be acceptable, according to 
applicable state and federal regulations and University policies designed to protect the rights 
and welfare of participants in research.

Contacts and Questions:
For questions, concerns, or complaints about the study you may contact Azita Manouchehri 
manouchehri.1@osu.edu.

For questions about your rights as a participant in this study or to discuss other study-related 
careers or complaints with someone who is not part of the research team, you may contact 
Ms. Sandra Meadows in the Office of Responsible Research Practices at 1-800-678-6251.

If you are injured as a result of participating in this study or for questions about a study-
related injury, you may contact Azita Manouchehri manouchehri.1@osu.edu.
Signing the consent form

I have read (or someone has read to me) this form and I am aware that I am being asked to participate in a research study. I have had the opportunity to ask questions and have had them answered to my satisfaction. I voluntarily agree to participate in this study.

I am not giving up any legal rights by signing this form. I will be given a copy of this form.

Printed name of subject

Signature of subject

Date and time

Printed name of person authorized to consent for subject (when applicable)

Signature of person authorized to consent for subject (when applicable)

Relationship to the subject

Date and time

Investigator/Research Staff

I have explained the research to the participant or his/her representative before requesting the signature(s) above. There are no blanks in this document. A copy of this form has been given to the participant or his/her representative.

Printed name of person obtaining consent

Signature of person obtaining consent

Date and time

Page 4 of 4
Form date: 12/15/05
Appendix J: Verbal Script for Recruitment of Participants

Building pre-service teachers’ mathematical knowledge for teaching of high school geometry

To the participants

Dr. Azita Manouchehri                  Ravi Somayajulu

We are conducting a research study is to investigate the nature of pre-service teachers’ mathematical knowledge as it pertains to the analysis of student thinking in high school geometry and to explore the impact of two quarters of coursework that would help further enhance this knowledge. As a member of the 2011 cohort of pre-service teachers enrolled in the STEM M.Ed. program at The Ohio State University, you are invited to get involved.

If you choose to get involved, you will be asked for your permission to gain access to your assignments you submitted for your methods courses EDU T&L 621 and EDU T&L 749.02. You will also be called upon to be interviewed once in the summer quarter and once in the autumn quarter of 2011. You also might be subject to a short interview during the autumn quarter of 2011. In addition you will be observed during your EDU T&L 749.02 course in the autumn quarter of 2011. No private information will be accessed for the research.

We would like to encourage you to participate in our study. Your participation in this study will have absolutely no negative consequences for you. You will benefit from increased learning as the result of reflecting on what you already do when you analyze student work and thinking in mathematics.

In addition, this study will benefit the field of mathematics education research. One goal of this study is to gain a deeper understanding of mathematical knowledge for teaching of geometry at the secondary level. This is an area that has been vastly unexplored. The study will be a major step in identifying and overcoming obstacles faced when developing and implementing a geometry methods course, which utilizes analysis of student work and thinking. The study will also be a significant step toward developing an instrument for measuring mathematical knowledge of geometry as it pertains to the analysis of students’ work and thinking in high school geometry.
All data collected is confidential. All the participants will be assigned aliases, which will be used for any communication about the study. Only the researchers will have access to the raw data corpus from this study. You may leave the study at any time. Leaving this study will have no impact whatsoever on your grades for the courses you are enrolled in or your standing in the program.

Please read the consent form carefully. Feel free to contact us with any questions or concerns you might have about the study. Notice that on the last page of the form, there is a place for you to sign if you wish to be involved.

We thank you for your consideration to be involved in this study.