SPECTRAL METHODS FOR THE ESTIMATION OF
ACOUSTIC INTENSITY, ENERGY DENSITY, AND SURFACE VELOCITY
USING A MULTIMICROPHONE PROBE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Glen C. Steyer, B.S.M.E., M.S.M.E.

*****

The Ohio State University
1984

Reading Committee:

Rajendra Singh
Donald R. Houser
Henry R. Busby
Lynn L. Faulkner

Approved By

Rajendra Singh
Co-Adviser
Department of
Mechanical Engineering

Donald R. Houser
Co-Adviser
Department of
Mechanical Engineering
ACKNOWLEDGEMENTS

I would like to express my deep felt gratitude to a number of individuals for their assistance, both material and moral. First, there are my co-advisors Professors Donald Houser and Rajendra Singh. The extended time period taken to complete this dissertation has resulted in undue labor on their part. I am grateful for their advise and encouragement. A further debt is owed to the typists, Ms. Denice Gutierrez and Ms. Leslie Brady, who did a very professional and timely job with difficult material and little assistance. Finally, I would like to thank my wife, Colleen, for her patience and encouragement. Without her support, this work would not have been possible.
VITA

June 1, 1952 ....................... Born — Fostoria, Ohio
1974 .............................. B.S.M.E., The Ohio State University,
                         Columbus, Ohio
1975 .............................. M.S.M.E., The Ohio State University,
                         Columbus, Ohio
1975—1976 ......................... Research Engineer, Warner & Swasey
                                 Research Center, Solon, Ohio
1976—1978 ......................... Teaching Assistant, Department of
                                 Mechanical Engineering, The Ohio
                                 State University, Columbus, Ohio
1978—1984 ......................... Consulting Engineer, Structural Dynamics
                                 Research Corporation, Milford, Ohio

PUBLICATIONS

"Errors Due to Scattering Effects in Multiple Microphone Intensity Probes," to be presented
at Inter-Noise 84, December, 1984.

"Computer Simulation of a Reciprocating Compressor — Compressor/Valves/Piping Inter-

"Eliminating Reciprocating Compressor Vibration and Performance Problems Using a New


FIELDS OF STUDY

Major Field: Mechanical Engineering, Acoustics and Structural Dynamics
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>VITA</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF TABLES.</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF FIGURES.</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>xiii</td>
</tr>
<tr>
<td><strong>CHAPTER</strong></td>
<td></td>
</tr>
<tr>
<td>1. INTRODUCTION.</td>
<td>1</td>
</tr>
<tr>
<td>1.0 General</td>
<td>1</td>
</tr>
<tr>
<td>1.1 General Literature Review</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Scope and Objectives of Work</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Organization</td>
<td>4</td>
</tr>
<tr>
<td>2. ACOUSTIC INTENSITY, ENERGY DENSITY, AND VELOCITY FORMULATIONS</td>
<td>5</td>
</tr>
<tr>
<td>2.0 General</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Theoretical Definitions</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Intensity Estimation Utilizing Pressure Sum and Difference</td>
<td>8</td>
</tr>
<tr>
<td>2.3 Two Microphone Cross-Spectral Intensity and Energy Density Estimation</td>
<td>10</td>
</tr>
<tr>
<td>2.4 Variations on Spectral Estimations of Intensity and Energy Density</td>
<td>13</td>
</tr>
<tr>
<td>2.5 Construction of a Three-Dimensional Intensity Probe</td>
<td>21</td>
</tr>
<tr>
<td>2.6 Estimator Implementation</td>
<td>23</td>
</tr>
<tr>
<td>3. ERROR ANALYSIS</td>
<td>27</td>
</tr>
<tr>
<td>3.0 General</td>
<td>27</td>
</tr>
<tr>
<td>3.1 Conceptual Error Analysis</td>
<td>27</td>
</tr>
<tr>
<td>3.1.1 Plane Wave</td>
<td>29</td>
</tr>
<tr>
<td>3.1.2 Point Monopole Source</td>
<td>32</td>
</tr>
<tr>
<td>3.1.3 Point Dipole Source</td>
<td>35</td>
</tr>
<tr>
<td>3.1.4 Infinite Plate</td>
<td>39</td>
</tr>
<tr>
<td>3.1.5 Symmetry Axis of a Baffled Circular Piston</td>
<td>50</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (Continued)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1.6 Summary of Finite Difference Error Results</td>
<td>57</td>
</tr>
<tr>
<td>3.2 Physical Error Analysis</td>
<td>58</td>
</tr>
<tr>
<td>3.2.1 Scattering Effects</td>
<td>59</td>
</tr>
<tr>
<td>3.2.1.1 Literature Review</td>
<td>59</td>
</tr>
<tr>
<td>3.2.1.2 Theoretical Estimates</td>
<td>60</td>
</tr>
<tr>
<td>3.2.1.3 Experimental Studies</td>
<td>65</td>
</tr>
<tr>
<td>3.2.1.4 Experimental Results</td>
<td>68</td>
</tr>
<tr>
<td>3.2.1.5 Summary of Scattering Errors</td>
<td>83</td>
</tr>
<tr>
<td>3.2.2 Evanescent Field Effects</td>
<td>84</td>
</tr>
<tr>
<td>3.2.3 Infinity Errors</td>
<td>87</td>
</tr>
<tr>
<td>3.3 Measurement Errors</td>
<td>99</td>
</tr>
<tr>
<td>3.3.1 Calibration Induced Bias Errors</td>
<td>99</td>
</tr>
<tr>
<td>3.3.2 Random Error</td>
<td>104</td>
</tr>
<tr>
<td>3.3.3 Bias Errors Due to Extraneous Noise</td>
<td>111</td>
</tr>
<tr>
<td>3.3.4 General Data Acquisition and Processing Errors</td>
<td>111</td>
</tr>
<tr>
<td>3.4 Summary of Error Analysis</td>
<td>113</td>
</tr>
<tr>
<td>4. EXPERIMENTAL VERIFICATION</td>
<td>115</td>
</tr>
<tr>
<td>4.0 General</td>
<td>115</td>
</tr>
<tr>
<td>4.1 Piston Tests</td>
<td>117</td>
</tr>
<tr>
<td>4.2 Periodically Stiffened Plate Structure</td>
<td>126</td>
</tr>
<tr>
<td>4.3 Summary of Experimental Results</td>
<td>136</td>
</tr>
<tr>
<td>5. CONCLUSION</td>
<td>138</td>
</tr>
<tr>
<td>5.1 Summary</td>
<td>138</td>
</tr>
<tr>
<td>5.2 Accomplishments</td>
<td>140</td>
</tr>
<tr>
<td>5.3 Conclusions</td>
<td>142</td>
</tr>
<tr>
<td>5.4 Recommendations for Further Work</td>
<td>143</td>
</tr>
<tr>
<td>LIST OF REFERENCES</td>
<td>144</td>
</tr>
<tr>
<td>APPENDIX – DERIVATION OF RESISTIVE AND REACTIVE SPECIFIC ESTIMATORS</td>
<td>149</td>
</tr>
</tbody>
</table>
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Compilation of Alternative Velocity Power Spectrum Estimators</td>
<td>20</td>
</tr>
<tr>
<td>3-1</td>
<td>Scattering Induced Phase Angle Errors (degrees) for Various Microphones and Spacings</td>
<td>77</td>
</tr>
<tr>
<td>3-2</td>
<td>Maximum Experimental Phase Errors (degrees) for a Four Microphone Probe Constructed from One Quarter Inch and One Half Inch Microphones. Also Shown is the Phase Angle Between the Microphone Signals for a Propagating Plane Wave</td>
<td>78</td>
</tr>
<tr>
<td>3-3</td>
<td>Minimum Number of Sampling Ensembles (N) for an Optimum Sampling Approach to Estimating the Radiated Power from a Rectangular Plate with (p) Half Wavelengths Vibration in One Direction and (q) in the Other</td>
<td>94</td>
</tr>
<tr>
<td>4-1</td>
<td>Comparison of Difference Between Measured Structural Velocity and Measured Acoustic Velocity for the Periodic Plate at Specific Frequencies</td>
<td>135</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2–1</td>
<td>Geometry in the Use of Two Microphones for Measurement of Acoustic Intensity</td>
<td>10</td>
</tr>
<tr>
<td>2–2</td>
<td>Measurement Process in the Presence of Noise</td>
<td>18</td>
</tr>
<tr>
<td>2–3</td>
<td>Placement of Microphones in a Cartesian Three-Dimensional Probe.</td>
<td>22</td>
</tr>
<tr>
<td>2–4</td>
<td>Placement of Microphones in a Quadrahedron Three-Dimensional Probe.</td>
<td>22</td>
</tr>
<tr>
<td>3–1</td>
<td>Geometry for Consideration of a Uni-Directional Plane Wave</td>
<td>29</td>
</tr>
<tr>
<td>3–2</td>
<td>Normalized Velocity Finite Difference Error for a Uni-Directional Plane Wave Source</td>
<td>31</td>
</tr>
<tr>
<td>3–3</td>
<td>Normalized Resistive Intensity Finite Difference Error for a Uni-Directional Plane Wave Source</td>
<td>31</td>
</tr>
<tr>
<td>3–4</td>
<td>Maximum Allowable Microphone Separation for a Specified Maximum Plane Wave Finite Difference Error in Intensity Estimation, as a Function of Frequency Range of Interest</td>
<td>33</td>
</tr>
<tr>
<td>3–5</td>
<td>Magnitude of the Normalized Velocity Finite Difference Error for a Point Monopole Source</td>
<td>37</td>
</tr>
<tr>
<td>3–6</td>
<td>Normalized Resistive Intensity Finite Difference Error for a Point Monopole Source</td>
<td>37</td>
</tr>
<tr>
<td>3–7</td>
<td>Normalized Reactive Intensity Finite Difference Error for a Point Monopole Source</td>
<td>38</td>
</tr>
<tr>
<td>3–8</td>
<td>Magnitude of the Normalized Velocity Finite Difference Error for a Point Dipole Source</td>
<td>42</td>
</tr>
<tr>
<td>3–9</td>
<td>Normalized Resistive Intensity Finite Difference Error for a Point Dipole Source</td>
<td>42</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES (Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3–10</td>
<td>Normalized Reactive Intensity Finite Difference Error for a Point Dipole Source</td>
<td>43</td>
</tr>
<tr>
<td>3–11</td>
<td>Geometry Used for Radiation from Bending Waves on an Infinite Plane.</td>
<td>44</td>
</tr>
<tr>
<td>3–12</td>
<td>Normalized In-Plane Velocity Finite Difference Error for an Infinite Plane Source</td>
<td>45</td>
</tr>
<tr>
<td>3–13</td>
<td>Normalized Normal Velocity Finite Difference Error for an Infinite Plane Source</td>
<td>45</td>
</tr>
<tr>
<td>3–14</td>
<td>Normalized Resistive Intensity Finite Difference Error for a 0 Degree Angle ($\gamma$) Relative to an Infinite Plane</td>
<td>47</td>
</tr>
<tr>
<td>3–15</td>
<td>Normalized Resistive Intensity Finite Difference Error for a 45 Degree Angle ($\gamma$) Relative to an Infinite Plane</td>
<td>47</td>
</tr>
<tr>
<td>3–16</td>
<td>Normalized Reactive Intensity Finite Difference Error for an Angle ($\gamma$) Normal to an Infinite Plane</td>
<td>48</td>
</tr>
<tr>
<td>3–17</td>
<td>Normalized Reactive Intensity Finite Difference Error for a 45 Degree Angle ($\gamma$) Relative to an Infinite Plane</td>
<td>48</td>
</tr>
<tr>
<td>3–18</td>
<td>Schematic of a Piston Radiator in a Rigid Baffle</td>
<td>51</td>
</tr>
<tr>
<td>3–19</td>
<td>Magnitude of the Normalized Velocity Finite Difference Error for the Axis of a Circular Piston with $\Delta x/x = 1.0$</td>
<td>53</td>
</tr>
<tr>
<td>3–20</td>
<td>Magnitude of the Normalized Velocity Finite Difference Error for the Axis of a Circular Piston with $\Delta x/x = 0.5$</td>
<td>53</td>
</tr>
<tr>
<td>3–21</td>
<td>Normalized Reactive Intensity Finite Difference Error for the Symmetry Axis of a Circular Piston with $\Delta x/x = 1.0$</td>
<td>54</td>
</tr>
<tr>
<td>3–22</td>
<td>Normalized Reactive Intensity Finite Difference Error for the Symmetry Axis of a Circular Piston with $\Delta x/x = 0.5$</td>
<td>54</td>
</tr>
<tr>
<td>3–23</td>
<td>Variation Along Symmetry Axis of Acoustic Pressure Amplitude $</td>
<td>p</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3-24</td>
<td>Normalized Resistive Intensity Finite Difference Error for a Circular Piston with Δx/x = 1.0</td>
<td>56</td>
</tr>
<tr>
<td>3-25</td>
<td>Exact and Estimated Intensity for a Circular Piston a/x = 10, and Δx/x = 1.0</td>
<td>56</td>
</tr>
<tr>
<td>3-26</td>
<td>Geometry Considered for Theoretical Estimation of Probe Scattering Effects, Semi-Infinite Cylinder</td>
<td>62</td>
</tr>
<tr>
<td>3-27</td>
<td>Schematic of Scattering Test Arrangement and Equipment.</td>
<td>67</td>
</tr>
<tr>
<td>3-28</td>
<td>Schematic of the Four Cylinder Probe Arrangement. The Cylinder Faces are at the Corners of a Tetrahedron</td>
<td>69</td>
</tr>
<tr>
<td>3-29</td>
<td>Imaginary Part of the Scattering Function Originally Measured on a Single Cylinder as Illustrated in Figure 3-27</td>
<td>70</td>
</tr>
<tr>
<td>3-30</td>
<td>Imaginary Part of the Scattering Function Measured on a Single Cylinder After Repositioning the Cylinder by One Foot. Comparison to Figure 3-29 Illustrates Error Induced by Scattering from Floor Grating</td>
<td>70</td>
</tr>
<tr>
<td>3-31</td>
<td>Imaginary Part of the Scattering Transfer Function for a Two Cylinder Probe with d/a = 2.0</td>
<td>72</td>
</tr>
<tr>
<td>3-32</td>
<td>Magnitude of the Scattering Transfer Function for a Two Cylinder Probe with d/a = 2.0</td>
<td>72</td>
</tr>
<tr>
<td>3-33</td>
<td>Imaginary Part of the Scattering Transfer Function for a Two Cylinder Probe with d/a = 4.0</td>
<td>73</td>
</tr>
<tr>
<td>3-34</td>
<td>Magnitude of the Scattering Transfer Function for a Two Cylinder Probe with d/a = 4.0</td>
<td>73</td>
</tr>
<tr>
<td>3-35</td>
<td>Imaginary Part of the Scattering Transfer Function for a Two Cylinder Probe with d/a = 5.0</td>
<td>74</td>
</tr>
<tr>
<td>3-36</td>
<td>Magnitude of the Scattering Transfer Function for a Two Cylinder Probe with d/a = 5.0</td>
<td>74</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3–37</td>
<td>Imaginary Part of the Scattering Transfer Function for a Two Cylinder Probe with $d/a = 6.0$</td>
<td>75</td>
</tr>
<tr>
<td>3–38</td>
<td>Magnitude of the Scattering Transfer Function for a Two Cylinder Probe with $d/a = 6.0$</td>
<td>75</td>
</tr>
<tr>
<td>3–39</td>
<td>Imaginary Part of the Scattering Transfer Function for a Two Cylinder Probe with $d/a = 8.0$</td>
<td>76</td>
</tr>
<tr>
<td>3–40</td>
<td>Magnitude of the Scattering Transfer Function for a Two Cylinder Probe with $d/a = 8.0$</td>
<td>76</td>
</tr>
<tr>
<td>3–41</td>
<td>Imaginary Part of the Scattering Transfer Function for the Four Cylinder Probe as Indicated</td>
<td>79</td>
</tr>
<tr>
<td>3–42</td>
<td>Magnitude of the Scattering Transfer Function for the Four Cylinder Probe as Indicated</td>
<td>79</td>
</tr>
<tr>
<td>3–43</td>
<td>Imaginary Part of the Scattering Transfer Function for the Four Cylinder Probe as Indicated</td>
<td>80</td>
</tr>
<tr>
<td>3–44</td>
<td>Magnitude of the Scattering Transfer Function for the Four Cylinder Probe as Indicated</td>
<td>80</td>
</tr>
<tr>
<td>3–45</td>
<td>Imaginary Part of the Scattering Transfer Function for the Four Cylinder Probe as Indicated</td>
<td>81</td>
</tr>
<tr>
<td>3–46</td>
<td>Magnitude of the Scattering Transfer Function for the Four Cylinder Probe as Indicated</td>
<td>81</td>
</tr>
<tr>
<td>3–47</td>
<td>Imaginary Part of the Scattering Transfer Function for the Four Cylinder Probe as Indicated</td>
<td>82</td>
</tr>
<tr>
<td>3–48</td>
<td>Magnitude of the Scattering Transfer Function for the Four Cylinder Probe as Indicated</td>
<td>82</td>
</tr>
<tr>
<td>3–49</td>
<td>Normalized Error in Extrapolation of Surface Velocity from Acoustic Velocity at a Distance $d$ from the Radiator Surface. Data is versus $d/a$ for the Pulsating Sphere and the Circular Piston and is versus $(2d/\lambda_p)$ for the Planar Source, where $\lambda_p = \text{Structural Wavelength}$</td>
<td>88</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>3–50</td>
<td>Required Number of Samplings to Assure Within 90 Percent Confidence Level that the Average Intensity is Accurate to Within 1 dB, for an Intensity Level of X over Y Proportion of the Surface, and Unity over the Remaining (1−Y) Proportion..</td>
<td>90</td>
</tr>
<tr>
<td>3–51</td>
<td>Minimum Probe Velocity Allowed to Maintain Maximum Number of Statistical Degrees of Freedom in Swept Average for a Plate of Thickness (h). Velocities At or Above this Value, Allow for a Minimum of Random Error in the Estimate of Sound Power.</td>
<td>97</td>
</tr>
<tr>
<td>3–52</td>
<td>Maximum Probe Traverse Velocity for a Plate of Thickness (h), In Order to Assure that the Measured Frequency Shift is Less than One-Half of One Spectral Line Resolution with a 400 Line Spectral Analysis to a Maximum Frequency of $F_{\text{max}}$.</td>
<td>97</td>
</tr>
<tr>
<td>3–53</td>
<td>Minimum Frequency to which Intensity may be Measured with Less than a 1 dB Error for Plane Wave Propagation. Frequency is Shown for Various Microphone Spacings as a Function of the Calibration Phase Mismatch Error.</td>
<td>103</td>
</tr>
<tr>
<td>3–54</td>
<td>Schematic of Two Channel Measurement Process in the Presence of Uncorrelated Noise.</td>
<td>112</td>
</tr>
<tr>
<td>4–1</td>
<td>Schematic of Experimental Equipment Used</td>
<td>116</td>
</tr>
<tr>
<td>4–2</td>
<td>Schematic of Piston Test Arrangement</td>
<td>118</td>
</tr>
<tr>
<td>4–3</td>
<td>Velocity Power Spectral Density at the Center and Outer Edge of the Piston</td>
<td>119</td>
</tr>
<tr>
<td>4–4</td>
<td>$G_{uuI}$, $G_{uuII}$, $G_{uuIII}$, and $G_{uuIV}$ Overlaid for Measurement Point 11 on the Piston Radiator</td>
<td>119</td>
</tr>
<tr>
<td>4–5</td>
<td>Piston Center Velocity PSD and the Measured Acoustic Velocity at Point 11, with 25 mm Microphone Spacing.</td>
<td>121</td>
</tr>
<tr>
<td>4–6</td>
<td>Piston Center Velocity PSD and the Measured Acoustic Velocity at Point 11, with 12 mm Microphone Spacing.</td>
<td>121</td>
</tr>
<tr>
<td>4–7</td>
<td>Measured Acoustic Velocity PSD at Piston Locations 11, 21, 31, and 41.</td>
<td>122</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4–8</td>
<td>Comparison of Experimental and Theoretical Values of Surface Averaged Impedance for a Circular Piston</td>
<td>122</td>
</tr>
<tr>
<td>4–9</td>
<td>Radial, Vertical, and Tangential Components of Acoustic Velocity PSD Measured at the Edge of the Piston</td>
<td>124</td>
</tr>
<tr>
<td>4–10</td>
<td>Schematic of Piston Test With Background Noise</td>
<td>125</td>
</tr>
<tr>
<td>4–11</td>
<td>Acoustic Pressure PSD Due to the Piston, and Due to the Background Source</td>
<td>127</td>
</tr>
<tr>
<td>4–12</td>
<td>Measured Resistive Intensity With and Without Background Noise Source</td>
<td>127</td>
</tr>
<tr>
<td>4–13</td>
<td>Measured Reactive Intensity With and Without Background Noise Source</td>
<td>128</td>
</tr>
<tr>
<td>4–14</td>
<td>Measured $G_{uuI}$ Velocity PSD With and Without Background Noise Source</td>
<td>128</td>
</tr>
<tr>
<td>4–15</td>
<td>Measured $G_{uuII}$ Velocity PSD With and Without Background Noise Source</td>
<td>129</td>
</tr>
<tr>
<td>4–16</td>
<td>Measured $G_{uuIII}$ Velocity PSD With and Without Background Noise Source</td>
<td>129</td>
</tr>
<tr>
<td>4–17</td>
<td>Measured $G_{uuIV}$ Velocity PSD With and Without Background Noise Source</td>
<td>130</td>
</tr>
<tr>
<td>4–18</td>
<td>Schematic of the Periodic Plate Structure</td>
<td>131</td>
</tr>
<tr>
<td>4–19</td>
<td>Measured Plate Velocity (circles) and $G_{uuI}$ Velocity (solid line) for the Periodic Plate</td>
<td>132</td>
</tr>
<tr>
<td>4–20</td>
<td>Measured Plate Velocity (circles) and $G_{uuII}$ Velocity (solid line) for the Periodic Plate</td>
<td>132</td>
</tr>
<tr>
<td>4–21</td>
<td>Measured Plate Velocity (circles) and $G_{uuIII}$ Velocity (solid line) for the Periodic Plate</td>
<td>133</td>
</tr>
<tr>
<td>4–22</td>
<td>Measured Plate Velocity (circles) and $G_{uuIV}$ Velocity (solid line) for the Periodic Plate</td>
<td>133</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

a  Piston radius
b  Speed of sound in air
d  Distance between radiating surface and probe center
f  Frequency (Hz)
h  Plate thickness
i  Square root (-1)
j  Indices
k, k_a  Acoustic wavenumber
k_p  Plate bending wavenumber
l  Microphone separation distance
n  Number of ensembles for FFT averaging
n  Unit normal
p, p_1, p_2  Acoustic pressure
p, q  Number half sine waves for plate vibration
r, r_1, r_2  Microphone locations
t  Time
u  Acoustic velocity
x, y, z  Cartesian coordinates
B  Dipole source strength
B  Effective frequency bandwidth
C_l  Material plane wave speed of sound
F  Acoustic flux
G_{pp}  Acoustic pressure power spectral density (single sided)
G_{uu}  Acoustic velocity power spectral density (single sided)
LIST OF SYMBOLS (Continued)

\( G_{p_1p_2}, G_{12} \)  Single sided cross spectrum between microphones 1 and 2

\( G_{11} \)  Power spectral density estimated from cross spectra

\( H_n \)  Hankel function

\( H_{ij}, H_O, H_s \)  Transfer functions

\( I \)  Resistive acoustic intensity

\( J \)  Reactive acoustic intensity

\( L \)  Number of time points in finite length time sample

\( L_{ij} \)  Real part of \( G_{ij} \)

\( L_X, L_Y \)  Plate length and width

\( M \)  Magnitude of a complex quantity

\( M_1, M_2 \)  Ratio of actual microphone sensitivity to assumed sensitivity

\( N \)  Number of samples

\( P(\omega) \)  Fourier transform of acoustic pressure

\( P, Q, R \)  General system noise signals

\( Q_{ij} \)  Imaginary part of \( G_{ij} \)

\( S \)  Monopole source strength

\( T \)  Acoustic kinetic energy density

\( U(\omega) \)  Fourier transform of acoustic velocity

\( V \)  Acoustic potential energy density

\( W \)  Acoustic energy density

\( X, Y, Z \)  General system input time signals

\( Z \)  General random variable

\( \alpha \)  Normalized microphone separation \( k\Delta r \)

\( \beta \)  Normalized microphone standoff distance \( \Delta r/r \)

\( \gamma \)  Coherence;
    Angle between microphone centers and plate horizon;
    Ratio of \( a/x \) for a piston radiator;
    Fourier transform variable

\textit{xiv}
LIST OF SYMBOLS (Continued)

\[ \Delta_{12} \] Calibration phase angle bias error

\[ \Delta r \] Microphone separation distance

\[ \delta \] \( \sqrt{x^2 + a^2} - x \)

\[ \varepsilon \] Normalized error

\[ \theta \] Angle in four microphone probe assembly;
Angle relative to a dipole source;
Angle between scattering angle and incident wave

\[ \kappa \] Ratio of plate to acoustic wavenumbers (\( k_p / k_a \))

\[ \lambda \] Wavelength

\[ \nu \] Poisson’s Ratio

\[ \xi \] Unit normal

\[ \Pi \] Complex acoustic intensity

\[ \pi \] Pi

\[ \rho \] Nominal air density

\[ \tau \] Time variable

\[ \chi \] Reactance

\[ \omega \] Circular frequency

Subscripts

\[ d \] Downstream

\[ f \] In the presence of flow

\[ o \] Nominal property

\[ s \] Surface average

\[ T \] Total

\[ u \] Upstream

\[ 1, 2 \] For microphones 1 or 2

\[ I, II \] Indices indicating alternative formulations for

\[ III, IV \] estimating acoustic velocity
LIST OF SYMBOLS (Continued)

Superscripts

• Derivative with respect to time

/ Derivative with respect to a spatial coordinate

^ Estimated quantity

~ Complex quantity (using the complex algebra)

* Complex conjugate

→ Vector quantity

Operators

∇ Del operator

Re [ ] Real part of a complex quantity

Im [ ] Imaginary part of a complex quantity

< >_s Spatial average

< >_t Temporal average

F( ) Fourier transform

∠ Angle of a complex quantity

E [ ] Expectation operator

VAR [ ] Variance

COV [ ] Covariance
CHAPTER 1 – INTRODUCTION

1.0 General

The advent of modern computer based frequency analyzers has made possible a variety of signal processing capabilities which in the past were either impossible or only available to a select few. Recent advances have sparked a considerable amount of activity in the use of two microphone probes for acoustic intensity measurement. These studies have identified the potential use of such probes for a wide variety of measurements ranging from sound power determination to material impedance measurement. This has resulted in acousticians returning to experimental methods in search of new approaches to characterizing sound fields and sources.

In the past, acoustic measurements have been almost entirely limited to acoustic pressure measurements via a microphone. Other equipment such as pressure gradient microphones [1] have been used, but only rarely. The inability of measuring anything other than acoustic pressure has led to the need of developing elaborate procedures for determination of such fundamental quantities as the total sound power radiated by a source. These procedures often require the use of such extremely expensive and special purpose facilities as anechoic chambers and reverberation rooms. This has been a severe impediment for many researchers. Even for the researcher with an anechoic chamber, the previous methods have been very time consuming and provide a limited amount of information regarding local intensity distributions over the surface of the source.

1.1 General Literature Review

As early as the 1940’s researchers [1, 2, 3] were investigating methods of measuring acoustic intensity directly. Some limited success was reported, however many practical problems existed with the equipment of that period. The techniques were pursued into the early 1950’s [4, 5], however they were never widely accepted. Beginning in the early 1970’s and carrying into the 1980’s the topic was again revisited. A number of approaches were considered. They all may be classified either as surface intensity methods [6, 7], or acoustic intensity methods [8, 9, 10, 11]. The surface intensity approaches utilize a motion transducer on the surface of the source in conjunction with an acoustic pressure measurement directly above the surface point. This has a number of obvious limitations in applicability. The acoustic intensity methods use a minimum of two transducers in the sound field. These are at least one pressure microphone and either an acoustic velocity transducer or another
pressure microphone. The most commonly used approach is to use two pressure microphones with signal conditioning in which the velocity is inferred from a finite difference approximation to the pressure gradient. For the latter approach the signal conditioning may either be done on the time domain signals directly or in the frequency domain.

This topic has received considerable attention over the past few years, and various intensity techniques are rapidly finding application in many specialized areas, each with unique considerations. This method has been used to determine radiation efficiencies [12], reactive intensity estimates [13], and material impedances [14]. It has been applied to problems in the nearfield of radiators [15], the measurement of energy in reactive environments [16], and in determining sources and sinks in interior problems [17]. The method has been applied to measurements in one-dimensional mean flow fields [18], and underwater acoustics [19]. Each of these applications has unique concerns which must be dealt with.

Recently an experimental approach has been given by Forssen and Crocker [12] for estimating surface velocity from an intensity probe placed in the very nearfield of a radiator. These studies however resulted in less than the expected accuracy. While a number of investigators [20, 21, 22] have identified and quantified potential sources of error in the estimation of acoustic intensity by this method, no published literature could be found which discussed the errors in the velocity estimation, nor any mention of various alternative estimation formulations for the acoustic velocity.

An exhaustive survey of the literature on error analysis for intensity probes showed that there were several fundamental questions which had not been addressed. One of these was the strength of the finite difference errors which may be expected for common types of radiators. A considerable amount of research had been published on these errors for simple acoustic monopoles, dipoles, and quadrupoles, but very little information has been published on errors in the nearfield of plates and pistons.

A further question is in the use of intensity probes for the measurement of radiated sound power. This can be done by either discrete sampling of intensity over the surface or by sweeping the probe along the surface. No mention could be found of the relative advantages of one approach over the other, nor of the number of samples required for accurate sound power estimation.

The available literature in this field is voluminous. As a result the detailed literature review for the various categories of investigation has been presented in each individual chapter.
1.2 Scope and Objectives of Work

The subject of this thesis is multi-microphone acoustic intensity measurement. Specifically, the use of two closely spaced microphones in conjunction with cross-spectral formulations for the estimation of acoustic intensity, both resistive and reactive, and the estimation of acoustic energy density. While this thesis will deal only with those techniques using frequency domain conditioning with multiple pressure microphones, many of the results will be applicable to the other methods of intensity measurement.

While some papers [12, 23] have been published regarding the use of intensity measurement in the presence of a mean fluid flow, this is a particularly special application which should be addressed separately from a general investigation of acoustic intensity methods. For this reason the methods presented herein will be limited to the zero mean flow case without dissipative effects.

The initial impetus for this study was some personal experience in experimental application of Forssen and Crocker's [12] approach for estimating surface velocity from an intensity probe in the very nearfield of a radiator. They presented results which show the approach to provide a very accurate estimation of the surface velocity for frequencies below coincidence. Attempts at the preliminary stage of this study in applying this approach to a different type of structure than that studied by Forssen and Crocker [12] resulted in much poorer results than those reported by Forssen and Crocker. Further investigation determined that there was an alternative spectral formulation for acoustic velocity which could be used. By applying this estimator to the very same data files the alternative formulation provided a much more accurate estimation of the surface velocity.

From the initial studies it was obvious that there was a need to address these issues, not only for the velocity estimator, but also for the reactive intensity estimation. The primary objective of this work then was to provide the necessary estimation error analysis for reactive intensity and acoustic velocity. Furthermore, alternative velocity estimators were developed and evaluated. The results of this study were then used to provide guidelines in the construction and use of a probe for the estimation of acoustic intensity and velocity, and the evaluation of this approach for radiation efficiency estimation.

The results of this study were used to perform experimental measurements of intensity and acoustic velocity. These experimental results were used to correlate with the theoretical predictions of measurement error. The experimental study was further extended to cases for which the measurement error could be readily determined from an analytical approach. This aided in further qualifying the magnitude of the measurement errors in practical applications.

In performing these investigations it was apparent that a considerable amount of research has previously been done in the use of two microphone cross-spectral formulations for the estimation of resistive intensity. However the work is widely scattered, in some
areas incomplete, and at times results have been reported which are contradictory. A secondary, but essential objective of this work was then to condense the major areas of research into one document and provide a degree of completeness to the field. This includes an investigation of what information may be extracted from a multi-microphone probe, clarification of the various error sources and their implications in implementation of the technique, and the extension of the approach to a four microphone probe for measurements in a three dimensional acoustic field.

1.3 Organization

This study covers both the theoretical and experimental aspects of acoustic measurements with a multi-microphone probe. The thesis will first consider the definition of acoustic intensity and energy density, then investigate the various methods in which these properties may be estimated from spectral formulations with such a probe (Chapter 2). These methods include estimators which may be found in the literature, and some which will be unique. The design of a probe for measurement of these properties in a three dimensional acoustic field will be considered. Application of the proposed estimator formulations to this probe will be reviewed.

Next the errors involved in estimating the acoustic field properties will be considered for various types of fields and all of the proposed estimators (Chapter 3). This study is used to identify how the estimation may be done in the most accurate sense. This involves determining which of the alternative estimators is most accurate, how a probe should be constructed, and the proper procedure for use of the probe. The result of these studies will be used to evaluate the feasibility of using a multi-microphone probe for the estimation of radiation efficiency and material impedance.

Finally this study includes experimental measurement of intensity and energy density with such a probe (Chapter 4). The results of these tests are compared with the error analysis to verify the theoretical results.
CHAPTER 2 – ACOUSTIC INTENSITY, ENERGY DENSITY, AND VELOCITY FORMULATIONS

2.0 General

This chapter will address the definition of acoustic intensity through the use of an acoustic energy equation. The results will first be provided for the general case including convective effects. These will then be reduced to the zero mean flow results which are typically used for radiation problems. The use of a two microphone pressure sum-difference approach to intensity and energy density estimation will then be reviewed. This will result in a statement of the spectral equations used for intensity and energy density estimation. Variations on this approach will then be presented, and the potential benefits in their use discussed. Finally the use of these methods for measurement of intensity in three dimensions, and energy density will be reviewed. The specifics of probe construction, and the application of the spectral estimators will be presented.

2.1 Theoretical Definitions

There are two common approaches to derivation of the acoustic energy equation. The first utilizes the energy equation as given by the First Law of Thermodynamics, which is then reduced by the acoustic assumptions. This approach has been discussed by Pierce [24] and Morfey [25]. A second approach [26], which is used here, begins with the linearized continuity and momentum equations, accounting for mean flow but ignoring dissipation terms.

\[
\frac{D\rho}{Dt} + \rho_0 \nabla \cdot \vec{u}_T = 0
\]

(2.1)

\[
\rho_0 \frac{D(\vec{u}_T)}{Dt} + \nabla p = 0
\]

where \( \vec{u}_T = \vec{u}_D + \vec{u} \) and \( p, u \) are functions of time

(2.2)

where \( \frac{D}{Dt} \) is the total or Stokes derivative

For an ideal gas with the isentropic process
\[
\frac{Dp}{Dt} = \frac{1}{c^2} \frac{Dp}{Dt} 
\]  
(2.3)

Thus we may rewrite Equations (2.1) and (2.2) as

\[
\frac{1}{c^2} \left( \frac{\partial p}{\partial t} + \mathbf{\hat{u}_o} \cdot \nabla p \right) + \nabla \cdot (\rho_o \mathbf{\hat{u}}) = 0 
\]  
(2.4)

\[
\rho_o \left( \frac{\partial \mathbf{\hat{u}}}{\partial t} + \mathbf{\hat{u}_o} \cdot \nabla \mathbf{\hat{u}} \right) + \nabla p = 0 
\]  
(2.5)

We now take the dot product of \( u \) with equation (2.5) which results in

\[
\rho_o \mathbf{\hat{u}} \cdot \frac{\partial \mathbf{\hat{u}}}{\partial t} + \rho_o \mathbf{\hat{u}_o} \cdot \nabla \left( \frac{|u|^2}{2} \right) + \nabla \cdot (p \mathbf{\hat{u}}) - p \nabla \cdot \mathbf{\hat{u}} = 0 
\]  
(2.6)

By assuming uniform nominal density, Equation (2.6) may be manipulated to yield

\[
\frac{\partial}{\partial t} \left( \frac{\rho_o |u|^2}{2} + \frac{p^2}{2\rho_0 c^2} \right) + \mathbf{\hat{u}_o} \cdot \nabla \left( \frac{\rho_o |u|^2}{2} + \frac{p^2}{2\rho_0 c^2} \right) + \nabla \cdot (p \mathbf{\hat{u}}) = 0 
\]  
(2.7)

by defining \( W = \left( \frac{\rho_o |u|^2}{2} + \frac{p^2}{2\rho_0 c^2} \right) \) Equation (2.7) may be rewritten as

\[
\frac{\partial (W)}{\partial t} + \nabla \cdot (\mathbf{\hat{u}_o} W) - W \nabla \cdot \mathbf{\hat{u}_o} + \nabla \cdot (p \mathbf{\hat{u}}) = 0 
\]  
(2.8)

Through use of the continuity equation on the mean flow field we have

\[
\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{\hat{F}} = 0 
\]  
(2.9)

where \( \mathbf{\hat{F}} \equiv p \mathbf{\hat{u}} + \mathbf{\hat{u}_o} W \)  
(2.10)
The term \( \hat{F} \) is the acoustic energy flux. By integrating this equation over an arbitrary volume, the interpretation as an energy conservation law is made apparent. The volume integral of \( \hat{F} \) is reexpressed as a surface integral through Gauss’ Theorem. In the absence of absorption within the volume or on the surface, we get

\[
\frac{d}{dt} \iiint_V W \, dV + \iiint_S \hat{F} \cdot \hat{n} \, dA = 0
\]

(2.11)

The analysis is made complete by time averaging the above equation.

\[
\left< \frac{d}{dt} \iiint_V W \, dV \right>_t + \iiint_S \hat{I}_f \cdot \hat{n} \, dA = 0
\]

(2.12)

where \( \hat{I}_f = \left< \hat{F} \right>_t \)

(2.13)

With this result we may now define \( \hat{I}_f \) as the acoustic intensity in the presence of a mean flow field. The definition of intensity may be broadened somewhat by using a complex notation to describe a harmonic response. This results in the following expressions for pressure and velocity

\[
p(t) = \text{Re} \left[ \hat{p}(\omega) e^{i\omega t} \right] ; \quad u(t) = \text{Re} \left[ \hat{u}(\omega) e^{i\omega t} \right]
\]

(2.14)

For the remainder of the analysis, all variables will be assumed to be functions of frequency \( \omega \), unless otherwise stated. Using the time averaging result for the harmonic analysis we get

\[
\hat{I}_f(\omega) = \frac{1}{4} \text{Re} \left[ \hat{p} \hat{u}^* \right] + \frac{\hat{u}_0}{2} \left[ \frac{\rho_0 \| \hat{u} \|^2}{2} + \frac{\| \hat{p} \|^2}{2\rho_0 c^2} \right]
\]

(2.15)

Where \( \sim \) represents a complex quantity.

This result may be generalized by defining the complex acoustic intensity of \( \hat{I}_f \) as
\[ \Pi_f(\omega) = I_f(\omega) + \bar{J}(\omega) = \frac{1}{2} \left( \bar{\rho} \cdot \bar{\tilde{u}}^* \right) + \frac{u_o}{2} \left( \frac{\rho_o |\bar{\tilde{u}}|^2}{2} + \frac{|\bar{\tilde{p}}|^2}{2\rho_o c^2} \right) \] (2.16)

With this definition it is necessary to further qualify \( I_f(\omega) \) as the resistive acoustic intensity, and \( \bar{J}(\omega) \) as the reactive acoustic intensity defined by,

\[ \bar{J}(\omega) = \frac{1}{2} \text{Re} \left( \bar{\rho} \cdot \bar{\tilde{u}}^* \right) \] (2.17)

Furthermore, the term \( W \) may be recognized as the acoustic energy density, which is composed of two parts. Defining \( (T) \) and \( (V) \) as the time averaged acoustic potential energy density and acoustic kinetic energy density respectively;

\[ T(\omega) = \left\langle \frac{\rho_o |\bar{\tilde{u}}|^2}{2} \right\rangle_t \] (2.18)

\[ V(\omega) = \left\langle \frac{|\bar{\tilde{p}}|^2}{2\rho_o c^2} \right\rangle_t \] (2.19)

For the case of zero mean flow \((u_o=0)\), the resistive acoustic intensity reduces to

\[ I = \left\langle p(t) \bar{u}(t) \right\rangle_t \] (2.20)

or

\[ I(\omega) = \frac{1}{2} \text{Re} \left[ \bar{\rho}(\omega) \cdot \bar{\tilde{u}}^* (\omega) \right] \] (2.21)

\[ \bar{J}(\omega) = \frac{1}{2} \text{Im} \left[ \bar{\rho}(\omega) \cdot \bar{\tilde{u}}^* (\omega) \right] \] (2.22)

### 2.2 Intensity Estimation Utilizing Pressure Sum and Difference

The cross-spectral density approach to intensity estimation is based on the use of a finite difference approximation of the pressure gradient at a point. The sum of the signals for two closely spaced microphones are used as the average acoustic pressure at the point, and the difference is used to estimate the pressure gradient. This sum-difference approach
was first proposed by Bolt and Petranskas [3] in 1943. However the approach at that time was to perform the signal conditioning on the time domain signals. This method was the subject of further study in the 1950's by Baker [4], and then by Schultz [5]. Little work was done in the area again until the 1970's. At this time there was a renewed interest in acoustic intensity measurement. The work at the time was progressing on two fronts. On one side, the approach was to measure the surface intensity by using an accelerometer on the surface of a radiator along with a pressure microphone in the vicinity of the accelerometer. This approach is described by Brito [6]. A similar approach which made use of a pressure microphone in conjunction with a velocity microphone was also investigated by Burger et al. [26]. This approach never received wide use, primarily due to the lack of an acceptable velocity microphone.

The pressure sum-difference approach was used successfully by several researchers. Stahel and Lambrich [8], Fahy [9], and Pavic [10] all reported on the approach. Again, in this work the approach was based on time-domain signal processing. The first suggestion of using a cross-spectral formulation for intensity measurement appears to be provided by Fahy [27]. This report was followed in short order by other work which reported successful application of the cross-spectral approach. The most notable of these are those of Chung [11, 15, 28]. These reports received considerable attention by others in the field and spurred a flurry of activity.

At the beginning of this work it was widely recognized that phase and magnitude mismatches between the microphones would have a serious adverse effect on the accuracy of intensity estimates. Various methods of correcting for calibration mismatches between the measurement channels were developed as reported by Chung [29], Seybert and Ross [30], and Krishnamma [31]. These authors documented the effect of finite difference errors at the upper frequency limit for a progressive plane wave. Further study of general errors in the estimations were reported by Thompson and Tree [20], Seybert [21], and Pascal [13].

An interesting point concerning these activities is that not until very recently has a rigorous derivation of the estimation formulation been published. Fahy's initial paper [27] merely provided the formula, while many successive papers simply referred to previous authors. One result of this situation was that a series of errors seems to have propagated through the literature. These include a sign error, spurious factors of 1/2 and a misunderstanding between the use of a double sided versus single sided spectrum. Lahti [32] provides an excellent discussion which traces the development and provides a rigorous derivation of the final result. Lahti also provides the formulation for the reactive intensity which received little mention previously. Mathur [33] has since published a derivation of the intensity estimator from a stochastic approach. Munro and Ingard [34], and Comparin, Rapp, and Sing [18] have presented spectral formulations for the estimation of resistive intensity in the presence of one dimensional mean flow.
2.3 Two Microphone Cross-Spectral Intensity and Energy Density Estimation

In section 2.2 it was shown that both the acoustic pressure and the acoustic velocity must be measured in order to determine the intensity and energy density. In order to measure these two quantities, two pressure microphones will be used which are oriented colinear with the intensity vector component to be determined. The geometry is illustrated in Figure 2.1.

![Diagram of two microphones](image)

**Figure 2—1**
Geometry in the Use of Two Microphones for Measurement of Acoustic Intensity

The acoustic pressure at the midpoint between the microphone locations is determined by averaging the pressure at the two microphones. The acoustic velocity is determined from a finite difference approximation of the pressure gradient, and an application of the momentum equation. For the case of zero mean flow the pressure and velocity may be stated as follows

\[
\hat{p}(r;t) = \frac{1}{2} [p_1(t) + p_2(t)]
\]

(2.23)

\[
\hat{u}_r(r;t) = \frac{1}{\rho_0} \int_0^1 \frac{p_2(\tau) - p_1(\tau)}{\Delta r} d\tau
\]

(2.24)
Expansion of the complex intensity in terms of the above quantities expressed in the frequency domain can be shown to result in (see Appendix A)

$$\hat{I}_r(\omega) = \frac{1}{\rho_0 \omega \Delta r} \text{Im} \left[ G_{p_1 p_2}(\omega) \right]$$  \hspace{1cm} (2.25)

Where $\hat{I}_r(\omega)$ is the estimate of the single sided power spectrum for the resistive acoustic intensity. $G_{p_1 p_2}(\omega)$ is the single sided power spectrum between the acoustic pressure at the two microphone locations. The $G_{p_1 p_2}(\omega)$ term may be expressed in terms of finite Fourier Transforms of the two pressures $P_1(\omega; T), P_2(\omega; T)$ as follows [35], where $T$ is the length of the time window over which the data is sampled.

$$G_{p_1 p_2}(\omega) = \lim_{T \to \infty} \frac{2}{T} \text{E} \left[ P_1^*(\omega; T) P_2(\omega; T) \right]$$  \hspace{1cm} (2.26)

The single sided power spectrum for the reactive intensity is similarly found to be; see Appendix A for derivation.

$$\hat{J}_r(\omega) = \frac{1}{2 \rho \omega \Delta r} \left[ G_{p_1} - G_{p_2} \right]$$  \hspace{1cm} (2.27)

The potential energy estimation may be derived by combining equation (2.19) with (2.23) to yield;

$$\hat{V}(r) = \frac{1}{4 \rho c^2} \left\langle \left( p_1(t) + p_2(t) \right) \left( p_1(t) + p_2(t) \right) \right\rangle_t$$  \hspace{1cm} (2.28)

$$\hat{V}(r) = \frac{1}{4 \rho c^2} \left[ \left\langle p_1^2(t) \right\rangle_t + \left\langle p_2^2(t) \right\rangle_t + 2 \left\langle p_1(t)p_2(t) \right\rangle_t \right]$$  \hspace{1cm} (2.29)

Now using the complex algebra, this may be expressed as;

$$\hat{V}(r; \omega) = \frac{1}{8 \rho c^2} \left[ \tilde{p}_1^*(\omega)\tilde{p}_2^*(\omega) + \tilde{p}_2^*(\omega)\tilde{p}_1^*(\omega) + 2 \text{Re} \left( \tilde{p}_1(\omega)\tilde{p}_2^*(\omega) \right) \right]$$  \hspace{1cm} (2.30)

We now recognize that for a harmonic signal;
\[ G_{p_1p_2}(\omega) = \hat{\mathbf{p}}_1^*(\omega)\hat{\mathbf{p}}_2(\omega) \] (2.31)

Thus
\[ \hat{V}(r,\omega) = \frac{1}{8\rho c^2} \left[ G_{p_1p_1}(\omega) + G_{p_2p_2}(\omega) + 2 \text{Re} \ G_{p_1p_2}(\omega) \right] \] (2.32)

By combining equations (2.18) and (2.24) a similar result for the kinetic energy density for the velocity in the direction collinear with the microphones is:
\[ \hat{T}(r,\omega) = \frac{1}{2\rho \omega^2 \Delta r^2} \left[ G_{p_1p_1}(\omega) + G_{p_2p_2}(\omega) - 2 \text{Re} \ G_{p_1p_2}(\omega) \right] \] (2.33)

It should be recalled that the kinetic energy density is defined with a term which is the dot product of the acoustic velocity vector with itself. Thus the total kinetic energy density requires measurement of all three acoustic velocity components. These three components of kinetic energy density summed together will comprise the total kinetic energy density. The potential and kinetic energy density terms may thus be expressed as:
\[ V(r,\omega) = \frac{1}{2\rho c^2} \ G_{pp}(\omega) \] (2.34)

\[ T(r,\omega) = \frac{\rho}{2} \left[ G_{uu_x}(\omega) + G_{uu_y}(\omega) + G_{uu_z}(\omega) \right] \] (2.35)

Using equations (2.34) and (2.35) with (2.32) and (2.33) the following expression for the acoustic pressure and velocity power spectra are derived:
\[ \hat{G}_{pp} = \frac{1}{4} \left[ G_{p_1p_1}(\omega) + G_{p_2p_2}(\omega) + 2 \text{Re} \ G_{p_1p_2}(\omega) \right] \] (2.36)

\[ \hat{G}_{uu}(\omega) = \frac{1}{(\rho \omega \Delta r)^2} \left[ G_{p_1p_1}(\omega) + G_{p_2p_2}(\omega) - 2 \text{Re} \ G_{p_1p_2}(\omega) \right] \] (2.37)

Equation (2.36) is the same estimator proposed by Fahy and Elliot [36] and Forssen and Crocker [12].
2.4 Variations on Spectral Estimates of Intensity and Energy Density

The previous discussion resulted in a spectral formulation for estimation of acoustic intensity and energy density. As with all estimation methods, these estimators are subject to error. Therefore it is necessary to consider alternative formulations which may minimize the most significant errors. In this section two variations on the estimators will be presented. Chapter 3 will then investigate the error sensitivities of the various estimators.

The estimation formulations developed for the resistive intensity and potential energy densities are relatively well behaved. However both the reactive intensity and the kinetic energy density estimation are the result of differencing quantities which are expected to be of approximately equal amplitude. Thus these two estimators may be very prone to error.

An alternative formulation for the kinetic energy density is developed here which may be less sensitive to error. The formulation results from starting with the following identity;

\[ |\widehat{\mathbf{p}}|^2 |\widehat{\mathbf{u}}|^2 = I^2 + J^2 \]  

(2.38)

from this we can derive

\[ \hat{G}_{uuII} = \dfrac{\hat{I}^2 + \hat{J}^2}{\hat{G}_{pp}} \]  

(2.39)

or

\[ \hat{G}_{uuII} = \dfrac{1}{(\rho \omega \Delta r)^2} \left\{ \frac{[G_{p_1}p_1 - G_{p_2}p_2]^2 + 4 \left[ \text{Im} \ G_{p_1}p_2 \right]^2}{G_{p_1}p_1 + G_{p_2}p_2 + 2 \text{Re} \ G_{p_1}p_2} \right\} \]  

(2.40)

Initial consideration of this estimator shows two potential advantages. Firstly, the kinetic energy density is computed as the sum of two terms. The first term results from the reactive field, and the second from the resistive. This is a very convenient and informative separation of terms. The second potential benefit of \( G_{uuII} \) is in the quantities which are differenced. Recall equation (2.37) where \( G_{uuI} \) is given in terms of \( \Delta I \) as

\[ \Delta I = G_{p_1}p_1 + G_{p_2}p_2 - 2 \text{Re} \ G_{p_1}p_2 \]  

(2.41)
This contains both auto and cross power spectra. Conversely, Equation (2.40) for $\hat{G}_{uuII}$ contains only the auto power spectra in the differenced quantities.

\[ \Delta_{II} = G_{p_1 p_1} - G_{p_2 p_2} \]  

(2.42)

If a random incoherent noise signal is present in the two measurement channels, it will show up in the auto power spectral estimates but not in the cross power spectrum. Thus equation (2.40) will be less sensitive to noise. Therefore, $\hat{G}_{uuII}$ is better formulated than $\hat{G}_{uuI}$.

By using the coherence between the two microphone signals ($p_1$ and $p_2$) we have the following identity:

\[ |G_{p_1 p_2}|^2 = \left[ G_{p_1 p_1} G_{p_2 p_2} \right] \gamma_{12}^2 \]  

(2.43)

Thus equation (2.39) may be expanded as;

\[ \hat{G}_{uuII} = \left\{ \frac{G_{p_1 p_1}^2 - 2 G_{p_1 p_1} G_{p_2 p_2} + G_{p_2 p_2}^2 + 4 \left| \text{Im} \ G_{p_1 p_2} \right|^2}{(\rho \omega \Delta r)^2 \left[ G_{p_1 p_1} + G_{p_2 p_2} + 2 \text{Re} \ G_{p_1 p_2} \right]} \right\} \]  

(2.44)

\[ \hat{G}_{uuII} = \left\{ \frac{\left( G_{p_1 p_1} + G_{p_2 p_2} \right)^2 - 4 \left| \text{Re} \ G_{p_1 p_2} \right|^2}{(\rho \omega \Delta r)^2 \left[ G_{p_1 p_1} + G_{p_2 p_2} + 2 \text{Re} \ G_{p_1 p_2} \right]} \right\} \]  

(2.45)

This may be regrouped as
\[ \hat{G}_{uu_{II}} = \frac{1}{(\rho \omega \Delta r)^2} \left\{ \left[ G_{p_1 p_1} + G_{p_2 p_1} - 2 \Re G_{p_1 p_2} \right] - 4 \left| G_{p_1 p_1} \right|^2 \left( \frac{1}{\gamma_{12}^2} - 1 \right) \right\} \]

which may be better expressed as;

\[ \hat{G}_{uu_{II}} = \hat{G}_{uu_{I}} - \frac{4 \left( 1 - \gamma_{12}^2 \right) G_{p_1 p_1} G_{p_2 p_2}}{(\rho \omega \Delta r)^2 \left( G_{p_1 p_1} + G_{p_2 p_2} + 2 \Re G_{p_1 p_2} \right)} \]

From this we see that for \( \gamma_{12}^2 = 1 \), the estimator \( \hat{G}_{uu_{II}} \) is identical to the estimator \( \hat{G}_{uu_{I}} \). However, for lower values of coherence, the \( \hat{G}_{uu_{II}} \) estimator will be lower.

\[ \hat{G}_{uu_{II}} \approx \hat{G}_{uu_{I}} - \frac{(1 - \gamma_{12}^2) G_{pp}}{(\rho \omega \Delta r)^2} \]

The lack of perfect coherence between the two signals may be due to either contaminating noise, or to separate incoherent acoustic sources. The additional acoustic source may either be a background noise source which is outside the measurement control volume, or it may be due to separate incoherent sources on the radiating structure. For example, consider a flat plate which is being driven by a random pressure distribution over the reverse surface. This random pressure distribution may be described by a temporal variance and a spatial variance with a corresponding spatial correlation length. This excitation will cause random plate vibration such that surface points very near to one another will exhibit coherent vibration, while points well separated will experience incoherent vibrations. Thus the radiator may be visualized as a collection of distributed incoherent sources. In this case, the pressure at two microphone locations will not be perfectly coherent.

The \( \hat{G}_{uu_{II}} \) estimator is only proposed for very nearfield velocity estimations. It is not acceptable as a general kinetic energy density estimator. This is made apparent when the general case of measurements in a fully reverberent field are considered where the net acoustic intensity is zero. In this case, \( \hat{G}_{uu_{II}} \) will estimate zero velocity. However the actual kinetic energy density is nonzero.

It is interesting to note that Munro and Ingard [34] recommended an approach similar to the \( \hat{G}_{uu_{II}} \) estimator by including a \( \left( \frac{\partial p}{\partial x} \right)^2 \) term in the intensity estimator in the
presence of a mean flow. In that work it was stated that the \( \left( \frac{\partial p}{\partial x} \right)^2 \) term "may be best expressed for numerical purposes as follows".

\[
\left| \frac{\partial p}{\partial x} \right|^2 = \left[ \left( |P_D|^2 - |P_u|^2 \right) \varphi^{-1} \right]^2 + 4 \left( \text{Im} \, P_D P_u^* \right) \varphi^{-1} \left( |P_D|^2 + |P_u|^2 + 2 \text{Re} \, P_D P_u^* \right)^2
\]

(2.49)

This expression is seen to be of essentially identical form as that in equation (2.40) for \( \hat{G}_{\text{uuI}} \). Unfortunately no elaboration was provided as to how, or why this form of expression was better. The advantages of this approach have been discussed in a limited sense in this chapter. Further elaboration on the relative benefits are provided in Chapters 3 and 4.

A second variation is considered which will affect the accuracy of both the reactive intensity estimation and the kinetic energy density. It is commonly recognized in signal processing [35] that if independent random noise is superimposed on two measured signals, both of the auto power spectra will experience bias error, while the averaged cross power spectrum will be unbiased. As a result, it is sometimes preferred to express estimators in terms of cross power spectra rather than auto power spectra. This approach was used by Bolleter [37] to eliminate the effect of self generated noise at a microphone in the presence of a mean flow, for the purpose of intensity estimation. This procedure is also commonly used in dynamic testing for mechanical systems [38]. Using this approach, the auto power spectrum is estimated as follows.

\[
\hat{G}_{11} = \begin{vmatrix} \hat{G}_{12}^* & \hat{G}_{13} \\ \hat{G}_{23} & \end{vmatrix}
\]

(2.50)

This approach requires the presence of a minimum of three independent signals. Thus the effect is to utilize only that portion of the signal which is fully coherent between the channels. Application of this approach would involve replacing the \( G_{p_1 p_1} \) and \( G_{p_2 p_2} \) terms in the reactive intensity and kinetic energy density expressions in equation (2.27) and (2.37) with terms as in equation (2.50).

Use of this approach allows the possibility of reducing the effect of background noise on measurement of the reactive intensity and acoustic velocity power spectrum. However if the source being measured is composed of several independent random sources, this method will also tend to separate these.
The benefit of this approach can be appreciated by considering the measurement problem as described below. With the measured signals 1, 2, and 3 it is desired to determine the input powers \( X, Y, \) and \( Z \) in the presence of uncorrelated random noise added to each of the signals. This is illustrated in Figure 2.2. From signal processing theory [35] we have the following for the measured spectral matrix;

\[
\begin{bmatrix}
G_{11} & G_{12} & G_{13} \\
G_{21} & G_{22} & G_{23} \\
G_{31} & G_{32} & G_{33}
\end{bmatrix}
= \begin{bmatrix}
G_{xx} + G_{pp} & G_{xy} & G_{xz} \\
G_{yx} & G_{yy} + G_{qq} & G_{yz} \\
G_{zx} & G_{xz} & G_{zz} + G_{rr}
\end{bmatrix}
\]  \hspace{1cm} (2.51)

Evaluating \( G'_{11} \) from equations (2.50) and (2.51) we have;

\[
\hat{G}'_{11} = \left| \frac{G_{yx} \cdot G_{xz}}{G_{yz}} \right|
\]  \hspace{1cm} (2.52)

This may be related to the true input power spectra by recognizing:

\[
\begin{align*}
G_{yx} &= H_{yx} \cdot G_{yy} \\
G_{xz} &= H_{xz} \cdot G_{xx} \\
G_{yz} &= H_{yz} \cdot G_{yy}
\end{align*}
\]  \hspace{1cm} (2.53)

Thus;

\[
\hat{G}'_{11} = \left| \frac{H_{yx} \cdot H_{xz}}{H_{yz}} \right| \cdot G_{xx}
\]  \hspace{1cm} (2.54)
Figure 2–2
Measurement Process in the Presence of Noise
We further use;

\[ |H_{yx}|^2 = \frac{G_{xx}}{G_{yy}} \quad ; \quad |H_{xz}|^2 = \frac{G_{zz}}{G_{xx}} \quad ; \quad |H_{yz}|^2 = \frac{G_{zz}}{G_{yy}} \]  \hspace{1cm} (2.55)

From which we obtain;

\[ (\hat{G}'_{11})^2 = \left| \begin{array}{ccc} G_{xx} & G_{zz} & G_{yy} \\ G_{yy} & G_{xx} & G_{zz} \\ G_{yy} & G_{xx} & G_{xx} \end{array} \right| \]  \hspace{1cm} (2.56)

or

\[ |\hat{G}'_{11}| = |G_{xx}| \]  \hspace{1cm} (2.57)

This shows that the modified estimator of the auto power spectra will not be sensitive to extraneous noise. This estimator has a potential problem if \((G_{zz})\) is identically zero. However, for the use in an intensity probe this is only expected to occur at those frequencies for which there is no acoustic pressure content. Thus it should not cause any serious problems in implementation.

Application of this approach to the \(\hat{G}_{uu}\) estimator results in a new estimator \(\hat{G}_{uu}^{III}\). Application to \(\hat{G}_{uu}^{II}\) results in \(\hat{G}_{uu}^{IV}\). All four velocity estimators are summarized in Table 2.1.
### Table 2–1
Compilation of Alternative Velocity Power Spectrum Estimators

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Right Hand Side of Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{G}_\text{uu}_I$</td>
<td>$\frac{1}{(\rho_0 \omega \Delta r)^2} \left[ G_{11} + G_{22} - 2 \text{ Re } G_{12} \right]$</td>
</tr>
<tr>
<td>$\hat{G}<em>\text{uu}</em>\text{II}$</td>
<td>$\frac{1}{(\rho_0 \omega \Delta r)^2} \left[ \frac{(G_{11} - G_{22})^2 + 4(\text{Im } G_{12})^2}{G_{11} + G_{22} + 2 \text{ Re } G_{12}} \right]$</td>
</tr>
<tr>
<td>$\hat{G}<em>\text{uu}</em>\text{III}$</td>
<td>$\frac{1}{(\rho_0 \omega \Delta r)^2} \left[ \left</td>
</tr>
<tr>
<td>$\hat{G}<em>\text{uu}</em>\text{IV}$</td>
<td>$\frac{1}{(\rho_0 \omega \Delta r)^2} \left[ \left( \left</td>
</tr>
</tbody>
</table>
2.5 Construction of a Three Dimensional Intensity Probe

This section will address the physical design of a microphone probe for the measurement of three dimensional intensity and acoustic energy density. The effect of alternative constructions will be discussed. While there are relatively few variations possible in the probe construction, there are a number of variations which may be employed in processing the signals within the constraints of the estimators provided in the previous sections.

In choosing a construction for a three dimensional probe it is necessary to place four pressure microphones in close proximity such that the pressure and the full pressure gradient at a point in space may be determined. In designing the probe it is desirable to minimize the effects of scattering from the probe and to have a reasonably compact probe.

The only limitation placed on the orientation of the four microphones is that they not be coplanar. There are two obvious choices for microphone placement. The first would be to place one of the microphones at the origin of a cartesian coordinate system, and then each of the remaining three microphones a unit distance away on each of the three respective axes. The other option would be to place the microphones at the four corners of a quadrahedron. The basis for choosing between these two designs is how well the intensity vector for each of the three directions can be estimated for a common point in space.

As an example, if the cartesian geometry is chosen with microphones oriented as shown in Figure 2.3, the use of microphones 1 and 2 for estimation of intensity in the X-direction will use a central finite difference approximation center on the point (0.5,0,0). The use of microphones 1 and 3 for estimation of the intensity in the Y-direction will be centered at the point (0,0.5,0). The estimation point for each of these intensity vectors can be brought closer together by using all the microphones for estimation of each intensity component. For example, the cross spectra $G_{12}$, $G_{23}$, and $G_{24}$ would all be used for estimation of the X-component of intensity. The alternative approach is to use the quadrahedron geometry. In this case the normalized microphone locations are provided in Figure 2.4. One advantage of the quadrahedron design is that all of the microphones are equally spaced from one another. This is not true in the cartesian probe. This symmetric probe design facilitates the estimation of the intensity components at a common acoustical center. For these reasons the quadrahedron design was used in this study.
Figure 2–3
Placement of Microphones in a Cartesian 3-Dimensional Probe

<table>
<thead>
<tr>
<th>Microphone</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 2–4
Placement of Microphones in a Quadrahedron 3-Dimensional Probe

<table>
<thead>
<tr>
<th>Microphone</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>1</td>
<td>$\sqrt{1/3}$</td>
</tr>
<tr>
<td>2</td>
<td>$-\sqrt{1/12}$</td>
</tr>
<tr>
<td>3</td>
<td>$-\sqrt{1/12}$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>
The selection of the quadrahedron spacing only specifies the location of the center of the microphone face. The other parameter which may be varied is the orientation (θ) of each microphone relative to microphone 4. This will have an effect on the amount of scattering from the probe and will also affect the ease of handling the probe. In order that the measurements can be made at locations as close as possible to the surface of a radiator it is necessary that all microphones generally point down toward the source. Furthermore, in order to provide a design in which the scattering induced error is not strongly sensitive to direction of the acoustic wavefront it would be desirable to have microphones 1, 2, and 3 all be oriented axisymmetrically relative to the Y-axis. With this type of pattern it would be logical to have microphone 4 oriented along the Y-axis. Thus the only parameter left to choice is the angle (θ) between the lower microphones and the Y-axis as shown in Figure 2.4. In order to reduce the scattering effects this angle should be as close as possible to 90 degrees. However the probe would be easier to use in tight areas if this angle were zero degrees. A logical compromise would be either 30 or 60 degrees. For this study, all probe measurements were conducted with a 60 degree angle. However the scattering experiments reported in Chapter 3 were conducted with a 30 degree angle in order to identify the worst case errors.

2.6 Estimator Implementation

With the probe construction fixed, there still remain certain issues in application of the estimators. As was discussed previously, it would be desirable to compute the cartesian components of intensity and velocity as a linear combination of the vector components from the six individual microphone pairs, such that the effect centers for each of the three cartesian components are as close as possible to one another. However, there is one factor which would call for a slight adjustment to this approach. In the nearfield of a radiating surface at sub-critical frequencies, the inplane components of intensity and velocity are known to decay rapidly with distance from the surface. Furthermore it is expected that the probe would be used with the plane defined by microphones 1, 2, and 3 being parallel to the radiator surface. Thus it would be reasonable to only use microphones 1, 2, and 3 to estimate the X and Z components. This will keep the acoustic center in the lower plane. In order to maintain the acoustic center close to the Y-axis, the vectors are summed as follows. The effective acoustic centers \((C_{ax}, C_{ay}, C_{az})\) are as indicated.
\[ V_x = \frac{1}{\sqrt{3}} \left[ -V_{12} - V_{13} \right] \quad ; \quad C_{Ax} = \left( \frac{\sqrt{3}}{12}, 0, 0 \right) \quad (2.58) \]

\[ V_z = \frac{1}{3} \left[ 2V_{13} - 2V_{12} + V_{23} \right] \quad ; \quad C_{Az} = (0, 0, 0) \quad (2.59) \]

\[ V_y = \frac{1}{\sqrt{6}} \left[ V_{14} + V_{24} + V_{34} \right] \quad ; \quad C_{Ay} = (0, \frac{1}{\sqrt{6}}, 0) \quad (2.60) \]

Where \( V_{ij} \) is the measured quantity vector component between microphone \( i \) and microphone \( j \).

\( C_{Ax}, C_{Ay}, C_{Az} \) are the coordinates normalized to the microphone spacing, of the point at which \( V_x, V_y, \) and \( V_z \) are estimated using a central finite difference approach.

This approach was used to develop the following formulae for intensity and velocity estimation.

\[ \hat{i}_x = \frac{1}{\rho \omega \Delta r \sqrt{3}} \left[ -\text{Im} \; G_{12} - \text{Im} \; G_{13} \right] \]

\[ \hat{i}_z = \frac{1}{\rho \omega \Delta r \ 3} \left[ 2 \text{Im} \; G_{13} - 2 \text{Im} \; G_{12} + \text{Im} \; G_{23} \right] \quad (2.61) \]

\[ \hat{i}_y = \frac{1}{\rho \omega \Delta r \sqrt{6}} \left[ \text{Im} \; G_{14} + \text{Im} \; G_{24} + \text{Im} \; G_{34} \right] \]

\[ \hat{j}_x = \frac{1}{\rho \omega \Delta r \ 2 \sqrt{3}} \left[ G_{22} + G_{33} - 2 \; G_{11} \right] \]

\[ \hat{j}_z = \frac{1}{\rho \omega \Delta r \ 2} \left[ G_{33} - G_{22} \right] \quad (2.62) \]

\[ \hat{j}_y = \frac{1}{\rho \omega \Delta r \ 2 \sqrt{6}} \left[ 3 \; G_{44} - G_{11} - G_{22} - G_{33} \right] \]
\[ \hat{G}_{pp} = \frac{1}{4} \left[ G_{11} + G_{22} + G_{33} + G_{44} \right] \]  

(2.63)

In implementing the velocity power spectra estimators it must be recognized that the velocity power spectrum is not a vector quantity, but rather it is the square of the magnitude of a vector. As such, it is not possible to extract the cartesian components in the same fashion as was done for the intensity estimators. Instead the approach taken was to first evaluate \( G_{uu,I} \) from just microphones 2 and 3. The velocity spectrum for the X direction was obtained by recognizing that summation of the velocity estimation from the microphone pairs (1, 2) and (1, 3) results in a linear combination of the \( G_{uu,X} \) and \( G_{uu,Z} \) spectra. By subtracting the Z-component from this, the X-component is left. The Y-component was similarly determined using the microphone pairs (1, 4), (2, 4), and (3, 4). The results are shown in equations (2.64):

\[ \hat{G}_{uu, z_I} = \frac{1}{(\rho \omega \Delta r)^2} \left[ G_{21} + G_{33} - 2 \text{ Re } G_{23} \right] \]
\[ \hat{G}_{uu, x_I} = \frac{1}{3(\rho \omega \Delta r)^2} \left\{ 2 \left[ 2G_{11} + G_{22} + G_{33} - 2 \text{ Re } G_{12} - 2 \text{ Re } G_{13} \right] - \hat{G}_{uu, z_I} \right\} \]  

(2.64)

\[ \hat{G}_{uu, y_I} = \frac{1}{4(\rho \omega \Delta r)^2} \left\{ 2 \left[ G_{11} + G_{22} + 3G_{44} - 2 \text{ Re } G_{24} - 2 \text{ Re } G_{34} \right] - \hat{G}_{uu, z_I} - \hat{G}_{uu, x_I} \right\} \]

The \( \hat{G}_{uu,II} \) estimator was defined in terms of the intensity vectors and the pressure power spectrum. As a result, it is possible to first separate the intensity vectors into the cartesian components and then compute the \( \hat{G}_{uu,II} \) estimations. These are shown in Equations (2.65):

\[ \hat{G}_{uu, x_{II}} = \left[ \hat{I}_x^2 + \hat{J}_x^2 \right] / \hat{G}_{pp} \]
\[ \hat{G}_{uu, y_{II}} = \left[ \hat{I}_y^2 + \hat{J}_y^2 \right] / \hat{G}_{pp} \]  

(2.65)

\[ \hat{G}_{uu, z_{II}} = \left[ \hat{I}_z^2 + \hat{J}_z^2 \right] / \hat{G}_{pp} \]
A further concern is the application of the cross spectrum approach to estimation of the auto power spectra. For this approach only the three lower microphones (1, 2, and 3) will be used for the estimation of auto spectra in connection with X and Z reactive intensity and velocity estimates. As such, the following are the formulae for the auto spectra which are to be substituted into equations (2.62), (2.63), and (2.64). This will result in the modified reactive intensity estimator, and the velocity estimators Guu_{III} and Guu_{IV}.

\[
\hat{G}_{11}' = \left| \begin{array}{cc} G_{13} & G_{12}^* \\ G_{23} & G_{22} \end{array} \right| \\
\hat{G}_{22}' = \left| \begin{array}{cc} G_{23} & G_{12} \\ G_{13} & G_{11} \end{array} \right| \\
\hat{G}_{33}' = \left| \begin{array}{cc} G_{13} & G_{23}^* \\ G_{12} & G_{22} \end{array} \right|
\]  

(2.66)  
(2.67)  
(2.68)

For application of the approach to estimation in the Y-direction a similar approach is used. However in this case, when any microphone pair are being considered there are two choices for the third microphone. The procedure used was to always select the microphone which is counter-clockwise relative to the microphone in the X-Z plane.
CHAPTER 3 — ERROR ANALYSIS

3.0 General

This chapter addresses the magnitude of error in the intensity and velocity estimates. These errors result from a variety of sources. For the purpose of organization it is convenient to separate them into three categories:

1. Conceptual
2. Physical
3. Measurement

The conceptual errors include all errors resulting from the concept of using pressure measurements at a finite number of locations to estimate the acoustic velocity and intensity. In this category the major source of error is the effect of the finite difference approximation which is used to estimate the pressure gradient or velocity. Also coming under this category are the effects of mean flow fields, acoustic finite amplitude effects, and viscosity effects; they are beyond the scope of this study.

The physical errors include all errors resulting from physical effects or limitations. The primary error sources are, the scattering effects of the microphones, the presence of an evanescent velocity field in the very near field, and finiteness errors resulting from the sampling of the acoustic field at a finite number of locations.

The measurement errors include all errors associated with the acquisition and processing of the signal from the microphones. Among these are; calibration induced bias errors, random errors, extraneous noise, microphone positioning errors, aliasing, and numerical truncation/roundoff in the digitization and processing of the data.

Each of these error sources are evaluated for practical acoustic fields. The conceptual and physical errors will be independent of which alternative formulations are used in the data processing to estimate the acoustic velocity. However the measurement errors will be sensitive to the formulation used. Thus, for these errors, the analysis was repeated for each of the alternative formulations.

3.1 Conceptual Error Analysis

Conceptual errors were defined as those errors resulting from the concept of using a finite number of pressure measurements to estimate the acoustic velocity and intensity. These errors are of two basic types. The first type of errors are those associated with defining the acoustic velocity and intensity as a function of the acoustic pressure and its gradient. The exact function would include the effect of fluid viscosity, the effect of finite amplitude acoustics, and the mean flow field. For the vast majority of applications of
multi-microphone probes, neglecting these effects will cause negligible error. The one major exception is when measurements are to be taken in the presence of a mean flow field. Munro and Ingard [34] show that accurate intensity measurements with a two microphone probe are possible only in the one dimensional flow case. Extensions of intensity methods to consider the mean fluid flow effect is a major task in itself, and beyond the scope of this study.

The second type of conceptual error is that resulting from the use of a finite difference approximation to the pressure gradient. In typical applications of the acoustic intensity estimator the finite difference error is one of the most significant sources of error.

The finite difference errors in the estimate of the resistive intensity have been reported by many authors [13, 20, 22, 38, 29, 40]. Thompson and Tree [20] provide a discussion of these errors in the nearfield of monopoles, dipoles, and quadrapoles, in addition to the case of the unidirectional plane wave. The vast majority of the literature on this subject has been limited to a discussion of only the resistive intensity. However Pascal [13] extended this topic to include the reactive intensity. Pascal offered further insight by presenting asymptotic expressions for the complex intensity errors in the nearfield of an infinite plane with travelling surface waves.

While considerable discussion may be found in the literature regarding finite difference errors in the intensity estimate, only one reference has been found which addresses finite difference errors in the velocity estimator. Elko [40] discusses this error in the velocity estimator, but only provides formulae for the simple acoustic sources. The following material is an attempt to consolidate the error evaluation for resistive intensity, reactive intensity, and velocity estimators. All three of these estimators are evaluated for the following cases:

1. Unidirectional plane wave
2. Point monopole source
3. Point dipole source
4. Planar radiation from an infinite panel
5. Symmetry axis of a baffled circular piston

The majority of discussion of finite difference errors in the literature has been limited to the simple ideal acoustic sources, even though one would be hard pressed to identify a mechanical device which will resemble a simple quadrapole in the very near field. Cases 4 and 5 above, represent more realistic source types which are found in typical machinery. Thus consideration of these sources provide a more practical basis upon which to estimate the finite difference errors in actual practice.
### 3.1.1 Plane Wave

The simplest case to consider is a plane acoustic wave incident parallel to the orientation of the two microphones. The geometry is as shown in Figure 3–1. In this case the reactive intensity will be identically zero, and there will be no error in the estimate of this component.

![Figure 3–1
Geometry for Consideration of a Unidirectional Plane Wave](image)

For a plane wave the pressure, velocity, and intensity may be determined from the following equations.

\[ \tilde{p}(x;\omega) = P_0 e^{-i k x} \]  
(3.1)

\[ \tilde{u}(x;\omega) = \frac{P_0 e^{-i k x}}{\rho c} \]  
(3.2)

\[ I = \frac{P_0^2}{2 \rho c} \]  
(3.3)

The velocity will be estimated from the acoustic pressure at two neighboring points as follows.

\[ \tilde{v} \approx \frac{i}{\rho \omega} \left[ \frac{\tilde{p}(x + \Delta r/2) - \tilde{p}(x - \Delta r/2)}{\Delta r} \right] \]  
(3.4)

where

\[ \Delta r \equiv \text{microphone spacing} \]
This results in;

\[ \tilde{u} = \frac{P_o}{\rho c} \frac{\sin(a/2)}{(a/2)} e^{-jkx} \]  

(3.5)

where

\[ a \equiv k \Delta r \]

For the normalized error in the velocity estimate, defined as;

\[ \epsilon_u = \frac{\hat{u} - \hat{\tilde{u}}}{\hat{u}} \]  

(3.6)

The resulting normalized velocity error is:

\[ \epsilon_u = 1 - \frac{\sin(a/2)}{a/2} \propto a^2/24 \]  

(3.7)

This error is shown graphically in Figure 3—2. The intensity estimate is obtained as follows;

\[ \frac{\hat{\tilde{u}}}{\Pi} = \frac{1}{2} \frac{\hat{P} \hat{u}^*}{2\rho \omega} \left[ \frac{\tilde{p}(x + \Delta r/2) - \tilde{p}(x - \Delta r/2)}{2} \right] \left[ \frac{\tilde{p}(x + \Delta r/2) - \tilde{p}(x - \Delta r/2)}{\Delta r} \right] \]  

(3.8)

The intensity estimate will reduce to;

\[ \frac{\hat{I}}{I} = \frac{1}{2} \frac{P_o^2}{\rho c} \frac{\sin a}{a} \quad ; \quad \hat{J} = 0 \]  

(3.9)
Figure 3–2
Normalized Velocity Finite Difference Error for a Unidirectional Plane Wave Source

Figure 3–3
Normalized Resistive Intensity Finite Difference Error for a Unidirectional Plane Wave Source
From which the normalized intensity estimate error will be found to be;

$$
\varepsilon_I = 1 - \frac{\sin a}{a} \approx \frac{a^2}{6}
$$

(3.10)

This error is shown graphically in Figure 3—3. These results show that for a plane traveling wave the intensity error is approximately four times greater than the velocity error. The intensity error will be less than 0.5 dB for \((k \Delta r)\) values of 0.892 or less. Figure 3—4 shows the maximum microphone spacing for various allowable intensity errors and frequencies.

3.1.2 Point Monopole Source

The acoustic monopole is the next source to be evaluated. With the monopole, there is now a reactive component of intensity which may be estimated. For a monopole the following equations may be used to describe the pressure, velocity, and intensity terms [41].

$$
\tilde{p}(r;\omega) = \frac{k\rho c S}{4\pi r} e^{-i(kr + \pi/2)}
$$

(3.11)

$$
\tilde{u}(r;\omega) = \frac{-k S}{4\pi r} \left( 1 - \frac{i}{kr} \right) e^{-i(kr + \pi/2)}
$$

(3.12)

$$
I = \frac{\rho c}{2} \left( \frac{kS}{4\pi} \right)^2
$$

(3.13)

$$
J = \frac{\rho c}{2 kr} \left( \frac{kS}{4\pi} \right)^2
$$

(3.14)

The velocity estimate results in;

$$
\Delta \tilde{u} = \frac{kS}{4\pi r} \left[ \frac{\sin (a/2)}{a^2} - \frac{i \cos (a/2)}{kr (1 - \beta^2/4)} \right]
$$

(3.15)

where

$$
\beta = \frac{\Delta r}{r}
$$
Figure 3-4
Maximum Allowable Microphone Separation for a Specified Maximum Plane Wave Finite Difference Error in Intensity Estimation, as a Function of Frequency Range of Interest
The normalized velocity error is then:

\[ |e_u|^2 = \left[ \frac{1 - \beta^2 / 4 - \frac{\sin (a/2)}{(a/2)}}{1 + (\beta/a)^2} \right]^2 + \left( \beta/a \right)^2 \left( \frac{1 - \beta^2 / 4 - \cos (a/2)}{1 - \beta^2 / 4} \right)^2 \]  

(3.16)

This function is graphically displayed in Figure 3–5. This plot shows the error for various distances from the source as determined by \( \Delta r/r \). This figure shows that at high frequencies, the error is relatively independent of distance from the source center. However, at lower frequencies it is apparent that the velocity is underestimated for field points close to the source. These curves show that even at zero frequency there is a bias error. It should be noted that the first term in the numerator of the above equation results from the resistive acoustic field, and the second term from the reactive field.

The value of \( \beta \) can only extend from (0, 2) when considering the source to be a solid object. \( \beta=2 \), corresponds to the near microphone being coincident with the source. This is the reason for the singularity in the error equation at \( \beta=2 \). In the low frequency limit, the error reduces to;

\[ e_u \approx \frac{\beta^2 / 4}{(1 - \beta^2 / 4)} \]  

(3.17)

This is seen to be a bias error resulting from the reactive field. In the higher frequency range \( \alpha > \beta \), the equation simplifies to;

\[ e_u \approx \frac{1 - \sin (a/2)/{(a/2)}}{(1 - \beta^2 / 4)} - \frac{\beta^2 / 4}{(1 - \beta^2 / 4)} \]  

(3.18)

This shows that for a monopole, the plane wave error is amplified by a factor of \((1-\beta^2/4)^{-1}\) and also has a negative bias error term. The increase in error due to the scale factor is somewhat over-compensated by the additional bias error, resulting in an error which is lower for the monopole than for the plane wave at high frequencies.

The breakpoint in the curves of Figure 3–5 is in the region where the microphones are between the acoustic nearfield and the acoustic far field.
The intensity estimate results in:

\[
\hat{\epsilon} = \frac{\rho c}{2} \left( \frac{kS}{4\pi r} \right)^2 \frac{\sin a}{a (1 - \beta^2/4)}
\]  
(3.19)

\[
\hat{\epsilon} = \frac{\rho c}{2kr} \left( \frac{kS}{4\pi r} \right)^2 \left( \frac{1}{1 - \beta^2/4} \right)^2
\]  
(3.20)

The normalized error would thus be:

\[
\epsilon_I = \frac{1 - \sin (a)/a}{(1 - \beta^2/4)} - \frac{\beta^2/4}{(1 - \beta^2/4)}
\]  
(3.21)

\[
\epsilon_J = 1 - \left[ \frac{1}{(1 - \beta^2/4)} \right]^2
\]  
(3.22)

These functions are shown in Figures 3–6 and 3–7. Since the reactive intensity error is only dependent on the ratio of microphone spacing to the field point radius (i.e. Δr/r), this function is plotted versus Δr/r in Figure 3–7.

The plot of resistive intensity error shows that at low frequencies the intensity is overestimated, while at high frequencies the intensity is underestimated. For β=0, it should be noted that the error for a monopole is equal to the error for a plane wave.

The nature of the intensity error is very similar to that of the velocity error which was discussed previously. When compared to a plane wave source, the error is seen to be scaled by \((1-\beta^2/4)^{-1}\) and shifted by the negative bias error. This causes the monopole error to be greater than the plane wave error at low frequencies, but less at high frequencies.

### 3.1.3 Point Dipole Source

The next step in source complexity is the point dipole. The pressure, velocity and intensity for a point dipole are described by the following equations [41]. It should be noted that the variables are now dependent on the distance from the source \(r\), and angular orientation \(\theta\) relative to the source.
\[ \tilde{p}(r, \theta; \omega) = \frac{B}{r} \cos \theta \left( 1 - \frac{i}{kr} \right) e^{-ikr} \]  
(3.23)

\[ \tilde{u}(r, \theta; \omega) = \frac{B}{2\rho r^2} \cos \theta \left[ 1 - \frac{2}{(kr)^2} - \frac{2i}{kr} \right] e^{-ikr} \]  
(3.24)

\[ I_r = \frac{B^2 \cos^2 \theta}{2\rho r^2} \]  
(3.25)

\[ J_r = \frac{B^2 \cos^2 \theta}{2\rho r^2 (kr)} \left[ 1 + \frac{2}{(kr)^2} \right] \]  
(3.26)

The velocity estimate results in:

\[ \frac{A}{u} = \frac{B \cos \theta}{\rho r^2} \frac{e^{-ikr}}{\rho r^2} \left\{ \frac{\sin (a/2)}{(a/2)(1 - \beta^2/4)} - \frac{2 \cos (a/2)}{(kr)^2 (1 - \beta^2/4)^2} \right\} \]
\[ - \left[ \frac{\cos (a/2)}{kr (1 - \beta^2/4)} + \frac{\sin (a/2)}{(a/2) kr (1 - \beta^2/4)^2} + \frac{\beta^2 \sin (a/2)}{2a^2 (1 - \beta^2/4)^2} \right] \]  
(3.27)

The resulting normalized velocity error is:

\[ |\varepsilon_u|^2 = \left\{ \left[ (1 - \beta^2/4)^2 (1 - 2 \beta^2/a^2) - (1 - \beta^2/4) \frac{\sin (a/2)}{(a/2)} + 2 (\beta/a)^2 \cos (a/2) \right]^2 \right\} \]
\[ + (\beta/a)^2 \left[ (1 + \beta^2/4) \frac{\sin (a/2)}{(a/2)} + (1 - \beta^2/4) \cos (a/2) - 2 (1 - \beta^2/4)^2 \right]^2 \]
\[ (1 - \beta^2/4)^2 \left[ 1 + 4 (\beta/a)^2 \right] \]  
(3.28)

In the low frequency range \((a \ll 1)\), the error reduces to:

\[ |\varepsilon_u| \approx \frac{\cos (a/2)}{(1 - \beta^2/4)^2} - 1 \]  
(3.29)
Figure 3-5
Magnitude of the Normalized Velocity Finite Difference Error for a Point Monopole Source

Figure 3-6
Normalized Resistive Intensity Finite Difference Error for a Point Monopole Source
Figure 3-7
Normalized Reactive Intensity Finite Difference Error for a Point Monopole Source
This again is the bias error resulting from the reactive field. By comparing Figure 3–8 with Figure 3–5 for the monopole, it can be seen that the low frequency error in the vicinity of the dipole is about double that for a monopole. Over the entire frequency range the error for the dipole is seen to be considerably worse than that for a monopole.

The radial intensity estimate for the dipole may be shown to be:

\[
\hat{I} = \frac{B^2 \cos^2 \theta}{2 \rho c r^2} \cdot \frac{1}{1 - \beta^2/4} \left\{ \sin \frac{a}{\alpha} \left[ 1 + \frac{1}{(kr)^2} \frac{1}{1 - \beta^2/4} \right] - \frac{\cos a}{(kr)^2 (1 - \beta^2/4)} \right\}
\]

(3.30)

\[
\hat{J} = \frac{B^2 \cos^2 \theta}{2 \rho c r^2 (kr)} \frac{1}{1 - \beta^2/4} \left[ 1 + \frac{2}{(kr)^2} \frac{1 + \beta^2/4}{1 - \beta^2/4} \right]
\]

(3.31)

Which results in the following normalized error:

\[
\epsilon_I = 1 - \frac{1}{(1 - \beta^2/4)^2} \left\{ \sin \frac{a}{\alpha} \left[ 1 + \frac{(\beta/a)^2}{(1 - \beta^2/4)} \right] - \frac{(\beta/a)^2 \cos a}{(1 - \beta^2/4)} \right\}
\]

(3.32)

\[
\epsilon_J = \left[ 1 - \frac{1}{(1 - \beta^2/4)^2} \right] + \frac{2}{(a/\beta)^2} \left[ 1 - \frac{1 + \beta^2/4}{(1 - \beta^2/4) \cos a} \right]
\]

\[
\left[ 1 + \frac{2}{(a/\beta)^2} \right]
\]

(3.33)

These functions are shown in Figures 3–9 and 3–10. These figures show similar trends to those observed for the dipole source. However once again, the greater source complexity causes very large errors for measurements in the very nearfield.

### 3.1.4 Infinite Plate

The results of the preceding sections are informative in providing a general characterization for the sensitivity of intensity and velocity estimates to finite difference errors for complex sources. However very little practical benefit is gained from those results. The previous results merely show that the error may range from very small to very large depending on the nature of the source. Common noise sources bear little resemblance to either the point monopole or dipole sources in the near field. In order to provide quantitative results for realistic types of radiators, the finite difference errors are evaluated in the near field of an infinite plate which has plane wave bending. This should be similar to the actual
conditions which would exist when the acoustic intensity technique is used to characterize
the radiation of plate-like structures.

The most interesting region for infinite plate radiation is in the subcritical frequency
range, that is, for frequencies at which the acoustic wavelength is greater than the structural
wavelength. For frequencies above this, the acoustic field will closely resemble the plane
wave field studies earlier. At subcritical frequencies no acoustic energy is radiated normal
to the plate. Real acoustic energy will only propagate parallel to the plate, and will de-
crease exponentially with distance from the plate. The geometry considered is shown in
Figure 3–11.

The equations describing the pressure and velocity are as follows [42] (for
frequencies at which \( k_p > k \)).

\[
\hat{\rho}(x, y; \omega) = \frac{i\rho c}{\sqrt{\kappa^2 - 1}} \quad V_o \quad e^{-ikp_x} e^{-k\sqrt{\kappa^2 - 1}} \quad y
\]  
(3.34)

\[
\hat{u}_x(x, y; \omega) = \frac{i \kappa}{\sqrt{\kappa^2 - 1}} \quad V_o \quad e^{-ikp_x} e^{-k\sqrt{\kappa^2 - 1}} \quad y
\]  
(3.35)

\[
\hat{u}_y(x, y; \omega) = V_o \quad e^{-ikp_x} e^{-ik\sqrt{\kappa^2 - 1}} \quad y
\]  
(3.36)

Furthermore, it can be shown that the intensities will be described as follows:

\[
I_x = \frac{\rho c}{(\kappa^2 - 1)} \quad \frac{V_o^2}{e^{-2k\sqrt{\kappa^2 - 1}}} \quad y \quad ; \quad J_x = 0
\]  
(3.37)

\[
I_y = 0 \quad ; \quad J_y = \frac{\rho c}{\sqrt{\kappa^2 - 1}} \quad V_o^2 \quad e^{-2k\sqrt{\kappa^2 - 1}} \quad y
\]  
(3.38)

The velocity estimates can be shown to be:

\[
\hat{\Delta} \quad u_x = \frac{i \kappa}{\sqrt{\kappa^2 - 1}} \quad V_o \quad e^{-ikp_x} e^{-k\sqrt{\kappa^2 - 1}} \quad y \left[ \frac{\sin (k_p \Delta x/2)}{(k_p \Delta x/2)} \right]
\]  
(3.39)
\[
\frac{\Delta u_y}{u_o} = V_o e^{i k p x} e^{-k \sqrt{\kappa^2 - 1}} y \left[ \frac{\sinh \frac{k \Delta y}{2} \sqrt{\kappa^2 - 1}}{k \Delta y \sqrt{\kappa^2 - 1}} \right] \] (3.40)

For this case, a convenient normalization is to express the microphone separation distance in terms of the structural wavelength \(a_x = k_p \Delta x, a_y = k_p \Delta y\). The normalized velocity errors then become:

\[
\epsilon_{u_x} = 1 - \frac{\sin \left( \frac{a_x}{2} \right)}{\left( \frac{a_x}{2} \right)} \] (3.41)

\[
\epsilon_{u_y} = 1 - \frac{\sinh \left( \frac{a_y}{2 \kappa} \right) \sqrt{\kappa^2 - 1}}{\left( \frac{a_y}{2 \kappa} \right) \sqrt{\kappa^2 - 1}} \] (3.42)

These functions are shown in Figure 3–12 and 3–13. These figures show that the inplane velocity is underestimated, and the perpendicular velocity is overestimated.

For the intensity estimate, a microphone pair will be considered which is oriented at an angle \(\gamma\) relative to the plate as shown in Figure 3–11. The exact intensity for this probe will then be:

\[
I = \frac{\rho c V_o^2 \kappa}{2(\kappa^2 - 1)} e^{-2k \sqrt{\kappa^2 - 1}} y \cos \gamma \] (3.43)

\[
J = \frac{\rho c V_o^2}{2 \sqrt{\kappa^2 - 1}} e^{-2k \sqrt{\kappa^2 - 1}} y \sin \gamma \] (3.44)

Using the normalized variables:

\[
a = k_p \Delta r \cos \gamma \] (3.45)

\[
\beta = k_p \Delta r \frac{\sqrt{\kappa^2 - 1}}{\kappa} \sin \gamma \] (3.46)
Figure 3–8
Magnitude of the Normalized Velocity Finite Difference Error for a Point Dipole Source

Figure 3–9
Normalized Resistive Intensity Finite Difference Error for a Point Dipole Source
Figure 3–10
Normalized Reactive Intensity Finite Difference Error for a Point Dipole Source
Figure 3–11
Geometry Used for Radiation from Bending Waves on an Infinite Plane

\[ u(x) = V_0 e^{i(\omega t - k_p x)} \]

\[ \kappa = \frac{k_p}{k} \]
Figure 3–12
Normalized Inplane Velocity Finite Difference Error for an Infinite Plane Source

Figure 3–13
Normalized Normal Velocity Finite Difference Error for an Infinite Plane Source
The estimated intensities can be shown to be:

\[
\hat{I} = \frac{\rho c V_0^2 \kappa}{2 (\kappa^2 - 1)} \sqrt{\kappa^2 - 1} \gamma \cos \gamma \left[ \sin \left( \frac{\alpha}{2} \right) \cosh \left( \frac{\beta}{2} \right) - \sinh^2 \left( \frac{\beta}{2} \right) \right]
\]

(3.47)

\[
\hat{J} = \frac{\rho c V_0^2 e^{-2k} \sqrt{\kappa^2 - 1}}{2 \sqrt{\kappa^2 - 1}} \sin \gamma \left[ \frac{\sinh \left( \frac{\beta}{2} \right)}{\left( \frac{\beta}{2} \right)} \cos \left( \frac{\alpha}{2} \right) \cosh \left( \frac{\beta}{2} \right) + \sin^2 \left( \frac{\alpha}{2} \right) \right]
\]

(3.48)

This will result in the following for the normalized error:

\[
\varepsilon_I = 1 - \frac{\sin \left( \frac{\alpha}{2} \right)}{\left( \frac{\alpha}{2} \right)} \cos \left( \frac{\alpha}{2} \right) \cosh \left( \frac{\beta}{2} \right) - \sinh^2 \left( \frac{\beta}{2} \right)
\]

(3.49)

\[
\varepsilon_J = 1 - \frac{\sinh \left( \frac{\beta}{2} \right)}{\left( \frac{\beta}{2} \right)} \cos \left( \frac{\alpha}{2} \right) \cosh \left( \frac{\beta}{2} \right) + \sin^2 \left( \frac{\alpha}{2} \right)
\]

(3.50)

The resistive intensity error is shown in Figures 3–14 and 3–15 for (\gamma = 0) degrees and (\gamma = 45) degrees. At (\gamma = 90) degrees, both the exact and the estimated intensity will be zero. The result for (\gamma = 0) degrees reduces to the same result as obtained for the plane wave case. The figures also show that over the majority of the range of (k_p \Delta r) studies, the error for the 45 degree angle arrangement was less than the error obtained from the plane wave case. The reactive intensity error is shown in Figures 3–16 and 3–17 for (\gamma = 90) degrees and (\gamma = 45) degrees.

The results for the infinite plate may best be compared to previous results by reducing the error expressions to the asymptotic results for small (k_p \Delta r). The worst case errors are obtained by letting \kappa \to \infty in the limit. This results in:

\[
\varepsilon_{u_x} \approx 1 - \frac{\sin \left( \frac{a_x}{2} \right)}{\left( \frac{a_x}{2} \right)} \approx \frac{1}{2} \left( \frac{a_x}{2} \right)^2
\]

(3.51)

\[
\varepsilon_{u_y} \approx 1 - \frac{\sinh \left( \frac{a_y}{2} \right)}{\left( \frac{a_y}{2} \right)} \approx \frac{1}{2} \left( \frac{a_y}{2} \right)^2
\]

(3.52)
Figure 3-14
Normalized Resistive Intensity Finite Difference Error for 0 Degree Angle ($\gamma$) Relative to an Infinite Plane

Figure 3-15
Normalized Resistive Intensity Finite Difference Error for a 45 Degree Angle ($\gamma$) Relative to an Infinite Plane
Figure 3-16
Normalized Reactive Intensity Finite Difference Error for an
Angle (γ) Normal to an Infinite Plane

Figure 3-17
Normalized Reactive Intensity Finite Difference Error for a
45 Degree Angle (γ) Relative to an Infinite Plane
This shows that the velocity error for the infinite plate is nearly identical to that for the plane wave. The only difference is in the y-component which is now overestimated rather than underestimated, and that the field wavenumber of interest is that due to the plate \(k_p\) rather than the characteristic acoustic wavenumber \(k\). The intensity errors are then; \((\lim \kappa \to \infty)\)

\[
\varepsilon_I \approx 1 - \frac{\sin (a/2) \cos (a/2)}{(a/2)} \quad \text{for } \gamma = 0^\circ
\]

(3.53)

Which reduces to;

\[
\varepsilon_I \approx 1 - \frac{\sin a}{a} \approx \frac{a^2}{2}
\]

(3.54)

and;

\[
\varepsilon_J \approx 1 - \frac{\sinh (a/2)}{(a/2)} \cosh (a/2) \quad \text{for } \gamma = 90^\circ
\]

(3.55)

Which simplifies to;

\[
\varepsilon_J \approx 1 - \frac{\sinh a}{a} \approx -\frac{a^2}{2}
\]

(3.56)

Again, the intensity result is seen to reduce to that of the plane wave. Here the exception is for the reactive intensity. Its error is now seen to be approximately equal to the negative of the error in the resistive intensity.

One noticeable difference between the infinite plate results and the dipole results is the lack of bias error at the low frequency limit. Thus, for the infinite plate, as long as the microphones are spaced sufficiently close together relative to the wavelength, the estimate will be essentially free of a finite difference error.

The infinite plate example was studied in order to investigate a class of radiator which might be encountered in typical radiators. However real machinery will be constructed of finite, rather than infinite plates. While there are many similarities between these two classes of radiators, there are also certain important differences. Below coincidence infinite plate does not radiate sound energy away from the plate, and will have a much stronger reactive field. It is reasonable to assume that the acoustic field for an
infinite plate will model a typical machine structure much better than a point dipole or point monopole.

3.1.5 Symmetry Axis of a Baffled Circular Piston

The final case to be considered is one which provides results typical of real noise sources. This is one of the few examples which may be considered to be realistic, exhibit a complex acoustic field, and possess an exact closed form solution. The geometry is shown in Figure 3–18. The intensity probe is considered to be on the axis of symmetry of a rigid circular piston set in a rigid baffle. Along the X-axis, the acoustic pressure and velocity are given by the following [23];

\[
\tilde{p}(x; \omega) = V_o \rho c e^{-ikx} \left( 1 - e^{-ik\delta} \right)
\]

(3.57)

where

\[
\delta = \sqrt{x^2 + a^2} - x
\]

\[
\tilde{u}(x; \omega) = V_o e^{-ikx} \left[ 1 - \frac{x}{\sqrt{x^2 + a^2}} e^{-ik\delta} \right]
\]

(3.58)

The intensity is found to be;

\[
\tilde{I}_x = \frac{V_o^2 \rho c}{2} \left[ 1 - e^{-ik\delta} \right] \left[ 1 - \frac{x}{\sqrt{x^2 + a^2}} e^{-ik\delta} \right]
\]

(3.59)

The finite difference errors are a function of frequency (\(\omega\)), microphone spacing (\(\Delta x\)), field location (\(x\)), and radius of the piston (\(a\)). The results have been determined as functions of the following nondimensional variables;

\[
\alpha = k\Delta x
\]

(3.60)

\[
\beta = \Delta x/x
\]

\[
\gamma = a/x
\]

This case is particularly interesting, both due to the practical nature of the case itself and due to the asymptotic limits over the values of \(a/x\). As \(a/x\) approaches infinity,
Figure 3-18
Schematic of a Piston Radiator in a Rigid Baffle
the results will approach the plane wave results. As a/x approaches zero, the results will approach those of the point monopole case.

The results for the velocity estimator error are shown in Figures 3–19 and 3–20. Figure 3–19 shows the results for Δx/x=1. Examination of these curves show that as a/x approaches zero, the results approach the monopole results in Figure 3–5. Furthermore, as a/x approaches infinity, the results approach the unidirectional plane wave results shown in Figure 3–3.

The results for the velocity estimator error for Δx/x=0.5 are shown in Figure 3–20. These results show the same characteristics as regards the limits of a/x.

An interesting result of this study is that the piston radius must be significantly less than the probe standoff distance for the low frequency monopole bias error to be of any significance. This points out that for nearfield measurements on a realistic radiator, the low frequency bias error is not likely to be a problem.

Figure 3–21 shows the normalized error in the reactive intensity for the case of Δx/x=1. The result for a/x=0.1 agrees with the result obtained for the point monopole case. This gives the error independent of a/x. The result for a/x=100 can be compared to the unidirectional plane wave case. However no estimate of error for reactive intensity was developed for that case due to the lack of a reactive intensity term. This result shows that the error term will asymptotically approach the same error as was obtained for the velocity error.

Figure 3–22 shows the normalized error in the reactive intensity for the case of Δx/x=0.5. Again, the error for a/x=0.1 is almost identical to the point monopole error. For a/x=2 the error terms shown in Figure 3–22 show a fluctuating nature, with discrete values of kΔx at which the error gets quite large. This is a result of the acoustic field fluctuating over the spatial coordinates. This fluctuation is a function of frequency ω. A typical spatial variation of p is shown in Figure 3–23 for a value of ka/2π=5.5. Those locations with zero pressure will also have zero intensity. Since the measured intensity at these locations will likely be something other than zero, the normalized error will be large.

Figure 3–24 shows the normalized resistive intensity error for Δx/x=1. The error obtained for a/x=0.1 is very similar to the point monopole error shown in Figure 3–6. Again, the error for a/x=10 shows the presence of discrete values of kΔr at which the error approaches very large positive values. The accuracy of the estimator may best be evaluated by referring to Figure 3–25. This figure shows the exact intensity and the estimated intensity superimposed on one plot for a/x=10 and Δx/x=1. The error is shown as a function of kΔx, which may also be thought of as a function of kx. This shows that the exact intensity is a harmonic function fluctuating between the values of 0.0 and 1.0. The estimated intensity is harmonic with the same period. However the magnitude of the oscillations are reduced for higher kΔx, and the oscillations tend toward a mean of zero.
Figure 3-19
Magnitude of the Normalized Velocity Finite Difference Error for the Axis of a Circular Piston with $\Delta X/X = 1.0$

Figure 3-20
Magnitude of the Normalized Velocity Finite Difference Error for the Axis of a Circular Piston with $\Delta X/X = 0.5$
Figure 3-21
Normalized Reactive Intensity Finite Difference Error for the Symmetry Axis of a Circular Piston with $\Delta X/X = 1.0$

Figure 3-22
Normalized Reactive Intensity Finite Difference Error for the Symmetry Axis of a Circular Piston with $\Delta X/X = 0.5$
Figure 3–23
Variation Along Symmetry Axis of Acoustic Pressure
Amplitude $|p|$ with Distance $X$ from
Center of Circular Piston of Radius $a$ for
$ka/2\pi = 5.5$ (Pierce [17])
Figure 3-24
Normalized Resistive Intensity Finite Difference Error for a Circular Piston with $\Delta X/X = 1$

Figure 3-25
Exact and Estimated Intensity for a Circular Piston $a/X = 10$, $\Delta X/X = 1.0$
This results in certain ranges where the estimated intensity would be negative, while the exact intensity would be a nearly zero positive value. This result shows the intensity as a harmonic function of radial distance from the piston. However, the intensity will also exhibit a harmonic nature over a surface parallel to the piston. One practical use of intensity measurement is for the determination of the radiated power by integrating the intensity over the surface area. Thus, the running average of the intensity functions in Figure 3–25 would appear to be a more relevant function. This would tend to average the error for neighboring points. From Figure 3–25 the average value of the exact intensity remains constant over the range of kΔr, while the average value of the estimated intensity is nearly zero at kΔr=3. This would correspond to a normalized intensity error of approximately unity for this value of kΔr. This is similar to the error shown in Figure 3–3 for the plane wave case.

3.1.6 Summary of Finite Difference Error Results

The results of this section have shown that for the plane acoustic wave field, the intensity and velocity errors are relatively small for low values of (kΔr). However, when higher order simple acoustic sources are considered, the errors become considerable, even at low frequencies. For these sources the measurement errors are a function of distance from the acoustic center. The finite difference error becomes large when the microphone separation is approximately equal to the distance of the measurement location from the source. These errors were seen to be on the order of fifty percent or more.

These results would raise serious questions about the accuracy of acoustic intensity probe measurements for most applications. The acoustic intensity probe is commonly used in the very nearfield of vibrating surfaces (0.2≤Δr/r ≤1.0). Use of the probe in this region has the advantages of better source resolution, and a reduction in the room reverberation effects. However, the evaluation of monopoles and dipoles for this region implies that measurement errors on the order of 3-6 dB could be expected in intensity measurements, and 3 dB errors in velocity measurements. These could be especially troublesome as the errors are present even in the very low frequency range.

Fortunately, the computed errors for simple acoustic point sources are more severe than the errors expected for real noise sources. The large errors for point monopoles and dipoles are a result of the steep pressure gradients in the vicinity of the acoustic singularity. For a typical noise source, such as a vibrating panel, the acoustic sources are distributed over a plane. This greatly reduces the pressure gradients, and the resulting finite difference error. The real intensity errors in the nearfield of an infinite plate were shown to be very similar to the errors for the plane wave case. Additionally, the finite difference error was shown to be independent of the distance from the plate. These results are dramatically
different from those of the monopole and dipole. These results provide a much more optimistic evaluation of the accuracy of acoustic intensity measurements in the very near field.

Evaluation of the finite difference errors on the axis of a baffled circular piston provide further evidence that errors in the presence of real sources are moderate. This case was very illuminating, in that the plane wave results and the point monopole results are subsets. Furthermore, the piston assumption can readily be applied to most applications of intensity measurements to plate-like structures. The piston example may be used to evaluate a plate-like structure by assuming the piston radius to be equal to the spatial correlation length of the surface vibrations. The results of this analysis showed that as long as the effective piston radius is approximately equal to the distance between the probe and the surface, the intensity errors are less than twenty percent and the velocity error would be less than ten percent. This is much less than the error which is indicated by the consideration of point monopoles and dipoles. This implies that measurements within 1 dB of accuracy are reasonable to expect in the very near field of real structural radiators.

3.2 Physical Error Analysis

Physical errors were defined as those estimation errors resulting from physical effects or limitations. The first error which was considered is the effect of scattering from the microphones and probe assembly. Scattering modifies the pressure field by adding an additional component referred to as the scattered pressure. As a result, the microphones measure slightly different pressures from those of the undisturbed acoustic field. Several authors have reported on this error source [43, 44, 45, 46, 47], however there have been conflicting conclusions on the severity of these errors.

The second physical error source which was addressed is the presence of an evanescent wave in the nearfield. This phenomenon has an effect on finite difference errors, and was thus considered in section 3.1. However, this section discusses the effect of the evanescent field on extrapolation of the indicated acoustic velocity to the surface of the radiator. This is of particular importance in the measurement of either radiation efficiency, or material impedances.

The final topic in this section is finity error. Finitity errors result from sampling the acoustic field at discrete points in space. For estimation of surface averaged properties, such as for sound power measurement, the spatial sampling may be done in either a discrete sense, or by sweeping the probe over the surface while continuously sampling data. For this reason, the concept of finity error was extended to a swept probe sampling procedure in order to evaluate the variance in the estimated parameters.
3.2.1 Scattering Effects

3.2.1.1 Literature Review

One of the physical effects which may introduce significant error in any acoustic measurement is the effect of acoustic scattering about the microphone and any associated hardware. The problem of self-induced scattering has been studied to a large degree as it is an inherent problem in the use of a single microphone. However, for the application of multiple microphone intensity probes, the scattering effects are even more severe. This problem has been investigated to some degree in the past decade as it applies to a two microphone intensity probe [43, 44, 45, 46]. Rasmussen [43] discusses this effect and presents experimental results for two microphones for first a parallel arrangement and then for a face to face arrangement. The data presented by Rasmussen shows the variation in the effective center to center distance between the microphone pair, and thus may be related to an effective phase error associated with the resulting delay time. This data indicates that the face to face arrangement results in reduced error from the scattering effect. While this face to face arrangement may be implemented for a two microphone probe, it is not possible for a four microphone probe.

Tichy [44] presents experimental results obtained for a parallel microphone arrangement. The data is presented as phase error for two different sizes of microphones and for four different microphone separation distances. Tichy’s results will also have the effects of scattering from the microphone holder. Unfortunately no description was provided for the holder and thus its’ effect on overall error, which could be significant, can not be determined. While measurement error will result from scattering from both the microphones and the probe holder it is helpful to separate out these two effects. The scattering from the microphones represents an inherent error which can not be eliminated, while the scattering from the probe holder is an error which can be reduced by using suitable probe geometries. The data presented by Tichy shows measured scattering induced phase errors in the range of +5°. The tests were conducted in a reverberent space which would not be conducive to performing tests of the required accuracy. While these results were not definitive, they are useful in estimating the expected error on an order of magnitude basis.

Krishnappa [45] presents the results of a study of phase errors for both parallel microphones and face to face microphones. In this work the phase error was mainly attributed to the presence of microphone holders. The results for extended microphones in the absence of a holder showed minimal error due to scattering. Furthermore Krishnappa found no significant difference in the results for parallel microphones compared to face to face microphones. This recent work seems to contradict the conclusions Rasmussen arrived at previously.
Kiteck [46] presents a thorough investigation of the scattering effects. Like other investigators he has studied the scattering about both quarter inch and half inch microphones. However his investigation included a study of the scattering influence for both single microphones and two closely spaced microphones. As with the other authors he has concluded that the largest source of error is due to scattering from the microphone holder rather than from the microphones themselves. Kiteck took a relatively rigorous experimental approach, but his tests were limited by the fact that these were conducted in a reverberant field, and were done in the nearfield of a speaker which was used as the sound source. This obscures the nature of the incident acoustic field. Additionally, the reverberant field would increase the measurement error. The results published by Kiteck show that when the probe holder effects were eliminated, the remaining measurement error attributed to scattering appears to be very small and within typical experimental error.

The published literature on this topic appears to present conflicting results and conclusions, and in several cases the validity of the experimental data is in doubt. The literature also suffers the weakness that very little analytical work has been done to support the experimental results. Therefore the scattering effects have been studied here in some detail, both experimentally and analytically for both two and four microphone probe arrangements.

3.2.1.2 Theoretical Estimates

The only method of evaluating the scattering effects for the complete problem is through the use of numerical methods such as the boundary integral method [47]. Unfortunately, such an approach provides little insight into the parametric sensitivities, and must be repeated for every change in probe construction. In order to estimate the effect of scattering the problem was simplified to one which could be addressed by a separation of variables approach. The most complicated geometry which could be evaluated with this approach and which would be representative of the problem is that of a single semi-infinite circular cylinder. In order to evaluate the scattering from two closely spaced cylinders, the solutions for two individual cylinders were superimposed. This approach does not account for the second order effects arising from the scattering by cylinder A of the scattered acoustic field from cylinder B. Additionally the approach will not properly evaluate the scattering from the surface at the end of the semi-infinite cylinder which is perpendicular to the cylinder axis. Due to these limitations in the analytical approach, the results may only be used to provide information as to the general trends which may be expected, and an order of magnitude evaluation of the error induced by scattering. In order to provide a more accurate quantitative evaluation of the scattering error, a series of experiments were
performed on simulated microphone probes. The results of these tests were compared to
the theoretical predictions in order to determine the accuracy of this approach.

The geometry modeled was a semi-infinite cylinder which was irradiated with a
plane wave acoustic field. The cylinder was assumed to be truncated at the origin. The
cylinder axis extended along the z-axis from zero to plus infinity. The scattering was
evaluated only for the cylindrical wall and not the end of the cylinder at the origin. The
geometry considered is illustrated in Figure 3–26.

The analysis will first address the scattering for an incident plane wave. Then the
reactive field will be addressed.

**Plane Wave Incident**

The incident plane wave may be broken down into two parts. One part will be
traveling parallel to the cylinder and the second part will be traveling perpendicular to the
cylinder axis. The incident acoustic velocity in the first term will always be parallel to the
cylindrical surface, thus resulting in no scattering. The only scattering would come from the
face of the cylinder, which has been ignored in this analysis.

Thus, the only term of interest is the plane wave which propagates normal to the
cylinder axis, and is characterized by a transverse wave number \((k_t)\). For this analysis
harmonic motion is assumed. Thus the incident plane wave may be determined as follows.

\[
\tilde{p}_1(r, \theta; \omega) = P_i e^{-ik_tr} \cos \theta
\]  

(3.61)

The incident pressure may be expanded in an infinite Fourier series in theta (\(\theta\)) as
follows;

\[
\tilde{p}_1(r, \theta; \omega) = P_i \sum_{n=0}^{\infty} \epsilon_n i^n J_n(k_tr) \cos n \theta
\]  

(3.62)

where

\(\epsilon_n = 1\) if \(n = 0\)

\(= 2\) if \(n > 0\)

The momentum equation may then be used to determine the acoustic acceleration
normal to the cylinder surface as follows;
Figure 3–26
Geometry Considered for Theoretical Estimation of Probe Scattering Effects, Semi-infinite Cylinder
\[
\rho \tilde{W}_s(a) = \frac{\partial \tilde{P}_i(a)}{\partial \xi_0} = i \xi_0 \quad k_t \tilde{P}_i(a)
\]  
(3.63)

where

\[\xi_0\] is the unit surface normal

We thus have:

\[
\tilde{W}_s(\theta) = \frac{k_t P_i}{\rho} \left[ \sum_{n=0}^{\infty} e_n i^n J_n'(k_t a) \cos n \theta \right] f(z)
\]  
(3.64)

Where \(f(z) = U(0);\), the unit step function.

The velocity at the surface will be broken up into two components, the first being a function independent of \(Z\), and the second being an odd step function.

\[
\tilde{W}_s(\theta) = \tilde{W}_s^1 + \tilde{W}_s^2
\]  
(3.65)

The scattering pressure will be determined separately for these two functions. Starting out with the first term, we find that the result is exactly one half of the result we would have for scattering from an infinite circular cylinder. The surface acceleration is given by:

\[
\tilde{W}_s^1 = \frac{k_t P_i}{2 \rho} \sum_{n=0}^{\infty} e_n i^n J_n'(k_t a) \cos n \theta
\]  
(3.66)

and the resulting scattered pressure is:

\[
\tilde{P}^i(r, \theta; \omega) = \frac{-p_i}{2} \sum_{n=0}^{\infty} e_n i^n J_n'(k_t a) \frac{H_n(k_t r)}{H_n'(k_t a)} \cos n \theta
\]  
(3.67)

Now turning to the second term which will account for the axial dependence of the scattered pressure, the surface acceleration will be defined as follows.
\[ \tilde{W}_s^2 = \frac{k_t b_i}{2 \rho} \sum_{n=0}^{\infty} \epsilon_n \tau G_n (k_t \alpha) \cos \theta f(z) \]  

(3.68)

where

\[ f(z) = \begin{cases} 
-1 & \text{if } z < 0 \\
1 & \text{if } z > 0
\end{cases} \]

In order to solve for the scattered pressure, a Fourier transform on the z-coordinate will be applied. The transform of the z-dependent function is as follows.

\[ \tilde{F}(\gamma) = \int_{-\infty}^{0} -e^{i\gamma z} dz + \int_{0}^{\infty} e^{i\gamma z} dz = \frac{2}{i\gamma} \]  

(3.69)

The scattered pressure will be determined by substituting the transformed acceleration into the surface harmonic expansion for scattered pressure, and inverse transforming the result to obtain the final solution. This yields:

\[ \tilde{p}^2(r, \theta, z; \omega) = \frac{-\rho}{2\pi} \sum_{n=0}^{\infty} \tilde{W}_n \cos \theta \int_{-\infty}^{\infty} 2 e^{i\gamma z} H_n \left[ \frac{(k_t^2 - \gamma^2)^{1/2}}{i\gamma (k_t^2 - \gamma^2)^{1/2} \alpha} \right] d\gamma \]  

(3.70)

Upon investigating the above result it is noticed that for the Z=0 plane the entire integral term vanishes. This results because all of the integrand is an odd function of the transform variable, and the integration is performed over the entire axis from minus infinity to plus infinity. This is especially convenient due to the fact that for a two microphone intensity probe only the results at the microphone centers are of interest.

**Reactive Field Evaluation**

There are an infinite variety of reactive fields which are possible. Furthermore the details of scattering effects are very dependent on the spatial variations of the incident
sound field. Thus it will be impossible to quantify the scattering error due to all types of reactive fields. However, in the very near field of plate type radiating structures, the reactive field may be modeled as one in which the acoustic velocity will vary harmonically in time and which will not be propagating. This means that as one traverses away from the surface all points will exhibit an acoustic velocity which is in phase. By further discounting any spatial variation over the surface of the radiating plate we arrive at a field which may be characterized as follows.

\[ u_y = u_z = 0 \ ; \ \tilde{u}_x(x;\omega) = V_0 \] \hspace{1cm} (3.71)

\[ \tilde{p}_x(x;\omega) = -i \chi V_0 \] \hspace{1cm} (3.72)

This will result in the following scattering surface acceleration:

\[ \tilde{\dot{W}}_s = -\omega V_0 \cos \theta \quad \text{for} \ z > 0 \] \hspace{1cm} (3.73)

\[ \tilde{\ddot{W}}_s = 0 \quad \text{for} \ z > 0 \]

Upon inspection it is seen that this is actually the case of sound radiation from a rigid circular cylinder translating harmonically. The resulting pressure is seen to be similar in form to that determined for an incident plane acoustic field, except that for the reactive field only the \( n=1 \) term is present.

### 3.2.1.3 Experimental Studies

In order to better quantify the effect of scattering, a series of tests were performed. The scattering was measured for plane wave radiation on a 31mm diameter cylinder. This would represent a scale model with a scale factor of 5 for a quarter inch microphone and 2.5 for a one half inch microphone. The tests were performed in the anechoic chamber at the Ohio State University Mechanical Engineering Department. The chamber has approximately a four by three by three meter working space. The chamber normally has a steel grating floor. However, this was removed after the initial tests showed the scattering from the grating to cause a significant distortion of the acoustic field over the frequency range of 500-2000 Hz. The acoustic pressure measurements were acquired with two phase matched quarter inch B&K microphones. The effect of scattering on the magnitude ratio and phase angle between the two probe locations was determined by measuring the transfer function between the two microphones using a 400 spectral line two channel analyzer.
Even though the chamber floor grating was removed there was still an appreciable amount of scattering from the remaining steel beams used to support the grating. This was evidenced by a waviness in the phase angle between two closely spaced microphones in the far field of a speaker. In order to minimize the error resulting from the remaining chamber scattering effects the following analysis procedure was settled upon. This procedure was found to provide repeatable results and was felt to give sufficient information on the probe scattering effects. Figure 3–27 is a sketch of the test arrangement and instrumentation.

The point of interest in the studies was the scattered pressure at a nearfield point, relative to the total acoustic pressure at the center of the cylinder face. As an example, for the case of measuring the scattering about a single cylinder, as shown in Figure 3–27, the reference microphone was placed at the center of the cylinder face. The second microphone was placed at the field point, which was typically within several inches of the reference microphone. The cylinder was removed and the transfer function between the two microphones measured. This function $H_o$ was stored within the analyzer as the reference transfer function. Next, the scattering object (cylinder) was replaced and the test repeated. This measured transfer function $H_s$ was then divided by the previously stored reference transfer function. In this manner the change in the relative phase and magnitude resulting from the presence of the scatterer was determined. The analysis steps are shown in the following equations.

Free Field: \[ \tilde{H}_o = \frac{\tilde{p}_{2,0}}{\tilde{p}_{1,0}} \] \hspace{1cm} (3.74)

Scattered Field: \[ \tilde{H}_s = \frac{\tilde{p}_{2s}}{\tilde{p}_{1s}} \] \hspace{1cm} (3.75)

Scattering Function: \[ TF = \frac{\tilde{H}_s}{\tilde{H}_o} = M \propto \phi \] \hspace{1cm} (3.76)

The above approach provided a very accurate determination of the change in the field point response relative to the center of the cylinder, however the scattered pressure at the center of the cylinder itself was not determined. In order to document the effect of scattering on the pressure magnitude at the cylinder, the power spectrum of the pressure at this location was measured both with and without the probe present. Since the pressure measurements are made relative to the pressure at a point in space rather than being relative to the incident field source, this approach does not easily separate out the scattered pressure. However, this experimental approach provided very accurate results for scattering.
Figure 3-27
Schematic of the Scattering Test Arrangement and Equipment
about a probe pair. In this case the prime concern is the indicated measurement at one probe relative to the other. If the pressure at both microphones is phase shifted the same amount, there will be no increased error in the intensity or velocity estimate. This approach allowed the measured results for the cross effect between two microphone probes to be determined very accurately.

The scattering tests as described above were conducted for various microphone separation distances $R$ and orientations $\theta$ about a single scattering cylinder. The tests were also conducted to determine the mutual scattering effect between two adjacent cylinders with various separation distances $d$. Finally the tests were than conducted on a scale model of a four microphone probe with the cylinder ends located at the corners of a quadrahedron. The simulated four microphone probe is shown in Figure 3–28.

### 3.2.1.4 Experimental Results

Initial tests were conducted to verify the experimental procedure and assure repeatability of the results. The first tests were performed with the metal floor grating still in the anechoic chamber. In these tests it was impossible to obtain repeatable results. It was discovered that small changes in the test article location relative to the sound source made dramatic changes in the results. Additionally the resulting functions were found to exhibit strong fluctuations in the frequency domain. Figures 3–29 and 3–30 show the imaginary part of the scattering function for a single cylinder for two successive measurements. The strong fluctuations at frequencies above 1000 Hz were found to be due to scattering from the floor grating. While the grating may have relatively little effect on most measurements performed in such a chamber, it had a disastrous effect on this study. The intent of the tests were to measure small deviations in the phase angle between the acoustic pressure at two locations as a result of scattering from the cylinders. The strength of the scattering from the floor was apparently as strong as that from the test articles.

It is interesting to note that a similar fluctuating nature can be observed in the results of probe scattering studies found in the literature. This phenomenon may be the source of some of the controversy over the amount of error induced by probe scattering effects. Given the difficulty encountered in performing accurate scattering studies in this anechoic chamber, it is difficult to believe that meaningful scattering results could be obtained in anything but a true anechoic space. For the remainder of the tests the floor grating was removed.

The results for the mutual scattering effects of two closely spaced cylinders will be presented first. The experimental data was obtained for a plane acoustic wave propagating in a direction coincident with the line of centers between the two cylinders. The data was acquired for $(d/a)$ ratios of 2, 4, 5, 6, 8 where $(d)$ is the distance between cylinder centers
Figure 3-28
Schematic of the Four Cylinder Probe Arrangement.
The Cylinder Faces are at the Corners of a Quadrahedron.
Imaginary Part of the Scattering Function Originally Measured on a Single Cylinder as Illustrated in Figure 3-27

Imaginary Part of the Scattering Function Measured on a Single Cylinder after Repositioning the Cylinder by One Foot. Comparison to Figure 3-29 Illustrates Error Induced by Scattering from Floor Grating.
and (a) is the radius of the cylinders. The results are presented in Figures 3-31 through 3-40. The results are presented for both the magnitude of the transfer function and the imaginary part of the transfer function. The figures show the theoretically estimated results compared with the experimental data.

In general, the correlation between the predictions and the test results are quite good. The poorest correlation is observed in the data for d/a of 2. This is expected, since the multiple scattering effects are greatest for this case. Such effects will first appear as the second cylinder interferes with the scattered wave from the first cylinder. When considering the field point to be the center of the face of the second cylinder, it is apparent that for the case when d/a is 2, the second cylinder completely masks the scattered pressure from the first cylinder. As the separation distance increases, the masking becomes less significant and consequently the multiple scattering effects should be of lesser significance. This is seen in the data where the correlation of the test data to analytical data for d/a of 2 is quite poor. In fact the scattering effect is opposite in sign to the prediction. However for a d/a of 5, 6, and 8 the correlation is quite good at high frequencies. For low frequencies at all values of d/a studied, both the theory and the test data indicate very little effect on the imaginary component, however the theoretical result appears to be of the opposite sign to the test results. In practice the minimum value of d/a which is generally used in an acoustic intensity probe is 4. Accordingly, the theoretical approach used to predict the scattering should be quite acceptable for determining the high frequency scattering error. For the lower frequencies the theory appears to account for the magnitude of the error, however due to the error in sign it is necessary to either use the experimental data or a more rigorous analytical approach to quantify the scattering induced error.

The scattering errors can be broken down into two different frequency ranges. For frequencies below 1000 Hz on the scaled probe, the error was a small, negative value. This error was largest for d/a = 2.0 and almost nonexistent for d/a = 8. For the intermediate d/a values, the error was nearly independent of d/a.

For frequencies above 1000 Hz on the scaled probe, the scattering phase error became very large. The largest measured error was 12 degrees. For this range, the errors are large enough to cause intensity measurements to be unacceptably inaccurate. However, the scattering errors are generally not the limiting factor in determining the allowable maximum frequency range for a probe. The finite difference errors discussed in section 3.1 will result in 1 dB error at frequencies lower than will the high frequency scattering behaviour. Only if the allowable measurement error is relaxed to a value greater than 1 dB will the scattering error limit the upper frequency range.

For most probe applications, the most important scattering errors are those in the lower frequency range. While the errors in this range are small, they can cause phase error up to one degree. Furthermore, the acoustic phase difference at the two microphones is
Figure 3-31
Imaginary Part of Scattering Transfer Function for a Two Cylinder Probe with d/a = 2.0

Figure 3-32
Magnitude of Scattering Transfer Function for a Two Cylinder Probe with d/a = 2.0
Figure 3–33
Imaginary Part of Scattering Transfer Function for a Two Cylinder Probe with d/a = 4.0

Figure 3–34
Magnitude of Scattering Transfer Function for a Two Cylinder Probe with d/a = 4.0
Figure 3–35
Imaginary Part of Scattering Transfer Function for a Two Cylinder Probe with $d/a = 5.0$

Figure 3–36
Imaginary Part of Scattering Transfer Function for a Two Cylinder Probe with $d/a = 5.0$
Figure 3-37
Imaginary Part of Scattering Transfer Function for a
Two Cylinder Probe with d/a = 6.0

Figure 3-38
Magnitude of Scattering Transfer Function for a
Two Cylinder Probe with d/a = 6.0
Figure 3-39
Imaginary Part of Scattering Transfer Function for a Two Cylinder Probe with d/a = 8.0

Figure 3-40
Magnitude of Scattering Transfer Function for a Two Cylinder Probe with d/a = 8.0
much smaller in this frequency range. Thus the measurements in the low frequencies may be more sensitive to error than those in the higher frequencies. The results of these tests show that a (d/a) value of 2.0 can cause significant error, while a (d/a) value of 8.0 will result in practically zero scattering induced phase error in the usable probe frequency range. Table 3–1 presents the scattering induced phase errors for scaled frequencies for 1/4 inch and 1/2 inch microphones.

The other aspect of microphone scattering is the effect on the measured pressure magnitudes. If the magnitude at both microphones is affected equally, the error in the estimators will be minimal. This only results in a maximum error of approximately one-half dB. However if the pressure magnitudes are unequally affected at the center of the two microphones, this causes an error in the measured reactive intensity and acoustic velocity. From the plots of the magnitude of the scattering functions, the maximum magnitude error for the mid-frequencies results in a magnitude ratio of 0.95 rather than 1.0. This ratio consistently goes to 1.0 for the very low frequencies, thus resulting in minimal low frequency error. Due to this magnitude mismatch error, the minimum reliable value of the term \(G_{22} - G_{11}\) in the velocity estimator, is approximately equal to \((0.1 \times G_{11})\) for the mid frequency range of the scattering tests. This will have a minimal effect on the velocity estimate, since for this frequency range the resistive intensity contribution to velocity should be larger than this false reactive indication. However, this error will affect the reactive intensity estimate.

<table>
<thead>
<tr>
<th>1/4&quot; Microphone Frequency</th>
<th>1/2&quot; Microphone Frequency</th>
<th>Error, degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>400</td>
<td>d/a = 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td>2000</td>
<td>1000</td>
<td>d/a = 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td>4000</td>
<td>2000</td>
<td>d/a = 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-4.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.0</td>
</tr>
<tr>
<td>6000</td>
<td>3000</td>
<td>d/a = 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-4.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-6.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-6.0</td>
</tr>
<tr>
<td>8000</td>
<td>4000</td>
<td>d/a = 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-12.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0</td>
</tr>
</tbody>
</table>
Four Cylinder Probe Results

The next step was to perform scattering studies for the four cylinder probe arrangement. Due to the geometric complexity, this configuration was studied entirely from an experimental approach. However, the analytical results from the two microphone probe provide a qualitative understanding of what would be expected for scattering from a four microphone probe.

The experimental studies were conducted in the same manner as described previously for the two cylinder arrangement. The tests were conducted for various orientations of the probe with the incident sound field arriving at right angles to the probe centerline. Additionally, the tests were conducted with the probe axis pointed directly at the sound source. A total of 15 probe orientations were tested. These test results are summarized in Figures 3–41 through 3–48. Table 3–2 shows the maximum phase and magnitude error which would result at various frequencies for probes constructed with quarter inch and with one-half inch microphones.

<table>
<thead>
<tr>
<th>1/4&quot; Microphone Frequency, Hz</th>
<th>1/2&quot; Microphone Frequency, Hz</th>
<th>Error, degrees</th>
<th>Plane Wave Phase Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>400</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>2000</td>
<td>1000</td>
<td>-0.5</td>
<td>0.0</td>
</tr>
<tr>
<td>4000</td>
<td>2000</td>
<td>0.0</td>
<td>3.5</td>
</tr>
<tr>
<td>6000</td>
<td>3000</td>
<td>3.0</td>
<td>13.0</td>
</tr>
<tr>
<td>8000</td>
<td>4000</td>
<td>0.0</td>
<td>18.0</td>
</tr>
</tbody>
</table>
Figure 3-41

Imaginary Part of Scattering Transfer Function Measured for the Four Cylinder Probe as Indicated

Figure 3-42

Magnitude of Scattering Transfer Function Measured for the Four Cylinder Probe as Indicated
Figure 3-43
Imaginary Part of Scattering Transfer Function Measured for the Four Cylinder Probe as Indicated

Figure 3-44
Magnitude of Scattering Transfer Function Measured for the Four Cylinder Probe as Indicated
Figure 3-45
Imaginary Part of Scattering Transfer Function Measured for the Four Cylinder Probe as Indicated

Figure 3-46
Magnitude of Scattering Transfer Function Measured for the Four Cylinder Probe as Indicated
Imaginary Part of Scattering Transfer Function Measured for the Four Cylinder Probe as Indicated

Magnitude of Scattering Transfer Function Measured for the Four Cylinder Probe as Indicated
The phase errors for the four microphone probe showed minimal low frequency error. For the four microphone probe tested, the \((d/a)\) value was approximately 8. Thus, these results are consistent with those of the two microphone probe. However, the high frequency errors in the four microphone probe are significantly higher than those observed in the two microphone test. The largest phase error is for the arrangement shown in Figure 3—43. The maximum phase errors are summarized in Table 3—2 for an axial incident sound field, and a laterally incident field. Also shown in this table is the phase angle which would result from plane wave propagation with the probe tested. This shows that the scattering angle is small compared to the plane wave angle. For a probe designed with the microphones closer together, for higher frequency testing, the scattering phase angle will be more significant, but will still not represent an overly large error.

### 3.2.1.5 Summary of Scattering Errors

The analytical and experimental results obtained in this study provided useful information regarding the scattering induced measurement error. The magnitude errors for the useful frequency range of an intensity probe were shown to be of comparable or lower magnitude than the errors resulting from the finite difference approximation. Thus, when this error is compared with the total intensity and velocity estimation errors, one finds that scattering effects are not dominant.

The phase errors also show a similar behavior. The phase error was found to be a complicated function of both the incident wave number and the microphone size and spacing. This error was typically a maximum of approximately 4 degrees for the four microphone probe at the upper frequency limit of use as determined by the finite difference errors. Again, this error is not dominant when compared to the finite difference error. From the experimental studies on pairs of semi-infinite cylinders it appears that this error can be minimized even further by proper selection of the microphone spacing. The studies showed that for \(d/a\) microphone spacings of approximately 6-8, the phase error was minimized over the normal operating frequency range. This compares with typical \(d/a\) values of 4-8 commonly used for many acoustic intensity applications. This result points out that one-quarter inch microphones are preferred over one-half inch microphones even for relatively low frequency intensity studies.

In this study only the effect of a purely resistive plane wave field on cylinders has been considered due to the difficulties in simulating an appropriate reactive field for the physical dimensions of the cylinders tested. The results that were presented in this section however did determine an order of magnitude for the scattering error resulting from a reactive field. This study shows that the error for this type of field would be small, or at worst be of comparable magnitude, when compared to the error in a resistive field.
While the comparison between the analytical and experimental results were poor in the lower frequency range for close microphone spacing, the comparison was very good for high frequencies. As a result, the analytical approach can not be totally relied upon to quantify the measurement error due to scattering. However, the analysis did provide very good support for the accuracy of the experimental data. The high frequency results indicate that the test methodology is valid, and the discrepancies between the analytical and experimental results are within the recognized limitations of the analysis approach.

The measurements on the four microphone probe showed the scattering errors to be only slightly greater than those in a comparable two microphone probe. As with the case of a two microphone probe, the largest source of scattering error is likely to be in the design of the microphone holders. This situation could be improved by the use of goose-neck extensions, however this is also likely to make the use of the probe more awkward.

One point which can be made relative to the conclusions of previous investigators, is that the results and conclusions of this work would in general be in agreement with those of Krishnappa [45]. For general use of a two microphone probe for the measurement of intensity, there would appear to be little to gain by utilizing a microphone arrangement other than the conventional side-by-side arrangement. Far more scattering error is induced by microphone holders than by the microphones themselves. Additionally, that error which may be directly attributable to scattering about the microphone appears to be of acceptable magnitude when compared to all other error sources.

3.2.2 Evanescent Field Effects

Forssen and Crocker [12] have reported on work in which the acoustic velocity estimator, as measured in the nearfield, has been used in conjunction with the measured intensity to estimate the radiation efficiency. If this approach is practical, it would provide a very convenient tool for such studies. One of the errors in such an approach is in equating the surface vibration velocity to the measured acoustic velocity. The presence of an evanescent field means that a portion of the acoustic pressure and velocity will rapidly decay with distance from the surface of the radiator. This evanescent field is reactive and results in no radiated (resistive) sound power. Under certain conditions this field may decay very rapidly in an exponential fashion. Since the acoustic velocity is estimated for a point midway between the two microphones, it is impossible to measure the surface velocity directly. This effect was studied for several ideal sound sources to determine the practicality of a microphone probe as a surface velocity or radiation impedance meter.
Planar Radiator

The first source evaluated was the infinite plate radiator which was considered for the finite difference errors. From that example the acoustic velocity normal to the plane was to be;

\[
\tilde{u}(x, y; \omega) = V_0 e^{-ikx} e^{-k(\kappa^2 - 1)^{1/2}y}
\]  

(3.77)

This shows that the velocity decays exponentially with distance from the plane \((y)\). The decay will be strongest for frequencies well below coincidence \((\kappa >> 1)\). In this region the above equation simplifies to;

\[
\tilde{u}(x, y) = V_0 e^{-ikx} e^{-kpY}
\]

(3.78)

From this, the normalized error in estimating the surface velocity from the acoustic velocity is;

\[
\epsilon_u = \frac{\tilde{u}(x, 0) - \tilde{u}(x, y)}{u(x, 0)} = 1 - e^{-2\pi y/\lambda p}
\]

(3.79)

Circular Piston

The rigid circular piston example used in the finite difference analysis provides further illustration of the evanescent error. From that case the acoustic velocity along the axis is:

\[
\tilde{u}(x; \omega) = V_0 e^{-ikx} \left\{ - \frac{e^{-ikx} \left( \sqrt{1 + (a/x)^2} - 1 \right)}{\sqrt{1 + (a/x)^2}} \right\}
\]

(3.80)

The magnitude of the normalized error in estimating the surface velocity from the acoustic velocity in this case is:

\[
|\epsilon_u|^2 = 2 (1 - \cos kx) + \left[1 + (r_0/x)^2\right]^{-1} + 2 \left[1 + (r_0/x)^2\right]^{-1/2} X \\
\left[ (1 - \cos kx) \cos (kx \sqrt{1 + (r_0/x)^2}) - \sin kx \sin (kx \sqrt{1 + (r_0/x)^2}) \right]
\]

(3.81)
Pulsating Sphere

The final case considered was that of a pulsating sphere of radius \( a \). For this case the acoustic velocity is described by:

\[
\tilde{u}(r;\omega) = \frac{V_0}{r} \left[ \frac{1 + i \frac{k r}{a}}{1 + i \frac{k a}{r}} \right]
\]  

(3.82)

This results in the following normalized velocity estimation error (where the field point is spaced from the sphere surface by \( d \)):

\[
| \varepsilon_u | = \frac{(d/a)^2 \left[ (2 + d/a) + (1 + d/a) (ka)^2 \right]^2 + (kd)^2}{(1 + d/a)^2 \left[ 1 + (ka)^2 \right]}^{\frac{1}{2}}
\]

(3.83)

These three cases have been evaluated and the results shown in Figure 3—49. The results for the pulsating sphere are shown for \( ka=1 \) and for the limit as \( ka \) approaches zero and infinity. The circular piston is shown for \( ka=1 \) and for the limit as \( ka \) approaches zero. The limit as \( ka \) approaches infinity provides an error of zero (the plane wave result). The infinite plate error is shown as a function of the structural wavenumber times separation distance.

This figure indicates that for the infinite plate example, the error can become very large. In order to estimate the surface velocity to within a 1 dB accuracy, the center of the probe must be within 0.018 \( \lambda \) where \( \lambda \) is the structural wavelengths of the surface. For a 1/2 inch microphone probe, the closest that the acoustic center can physically be is 13mm. Thus, if the structural wavelength \( \lambda \) is less than 720mm, the velocity estimation will be in error by more than 1 dB.

The infinite plate, however, is a rather severe example of the evanescent error. The pulsating sphere and rigid piston examples are probably more characteristic of common radiators. For the low \( ka \) region of a piston, the same 1 dB error criteria requires that the piston diameter be no less than 100mm. This is seen to be much less severe, but still indicates the potential for significant velocity errors for the shorter structural wavelengths. The largest errors would be expected for large unstiffened plate-like structures at frequencies just below coincidence. In this range, even though the structural wavelength is short, the field is still primarily reactive. For many machines structures typically encountered, the evanescent error would likely be less than 1 dB. This indicates some potential for construction of a radiation efficiency meter. However, for any potential
application, it is necessary to investigate this error source and assure that the error magnitudes are acceptable. This error will be studied further in Chapter 4.

3.2.3 Finitity Errors

The final type of physical error considered is finiteness error. This is the error introduced due to sampling a continuous function over a surface at a finite number of locations. In most acoustic measurements, a surface averaged quantity is generally of interest. For example sound power is computed from spatially averaged intensity measurements, and surface averaged velocities are used to determine radiation efficiency.

When the velocity is to be measured, the common approach is to measure acceleration at a finite number of discrete locations and find the spatial average, since there is no reliable transducer for measuring the surface averaged velocity \( \langle v^2 \rangle_s \) directly.

In the case of acoustic measurements there are two possible approaches to obtain the surface averaged values. The first is to use the discrete sampling approach, the second is to sweep the transducer over the measurement surface while averaging the result. As long as the sweeping assures that the probe uniformly weights the data over the total surface, and there is no correlation between the sweeping motion and the acoustic signal, then the latter approach should provide the surface averaged measurement quite easily.

There is relatively little direct mention of finiteness errors in the literature. References [48,15] indicate the required number of sampling locations per structural wavelength. However, no analysis of the variance of the estimate has been found in the published literature. Furthermore, no discussion has been found relative to proper sweeping methods. The following analysis fills this void, and provides a statement of the advantages of the swept approach relative to discrete sampling.

a. Discrete Sampling Approach

In evaluating the error in the discrete sampling approach it is necessary to evaluate the error as a sampling problem of a random variable. Of interest is the variance in the estimate of the mean value of the sampled variable as a function of the number of samples, be it intensity or velocity squared. The variance of this estimated mean can be found as follows, independent of the nature of the statistical distribution of the sampled variable;

\[
\text{VAR} \left[ \hat{Z} \right] = \frac{\text{VAR} \left[ Z \right]}{N}
\]  

(3.84)
Figure 3-49
Normalized Error in Extrapolation of Surface Velocity from Acoustic Velocity at a Distance \( d \) from the Radiator Surface. Data is Versus \( d/a \) for the Pulsating Sphere and the Circular Piston, and is Versus \( (2d/\lambda) \) for the Planar Source, Where \( \lambda = \) Structural Wavelength.
The variance of the estimated mean value is thus dependent upon the variance of the function over the measurement surface. For a uniform plate radiator one might expect that this variance would be characterized by the variance of a sine squared function. However in practice, typical radiators may not have a uniform velocity distribution over the surface. In fact, there may well be localized areas where the velocities are significantly higher compared to the rest of the structure. Here an exercise will be used to investigate the effect of finite errors. It is assumed that the function over the measurement surface is one of two values (unity or the value X). The function is equal to (X) over (Y) proportion of the surface, and is unity over (1-Y) proportion of the surface. The variance in the estimate of the mean of the function over the surface is then computed based on N discrete random samples over the surface. The variance in the mean, normalized by the mean value itself, is then expressed as:

$$\frac{\text{VAR} \left( \frac{A}{Z} \right)}{Z^2} = \frac{Y(1-Y)(X-1)^2}{N \left( XY - Y + 1 \right)^2} \quad (3.85)$$

Of importance is how many samples are needed to assure that the exact mean value is within the estimated confidence range. It is assumed that the criterion is to be 90% confident that there is no more than a 1.0 dB error in the estimated value. For this case Figure 3—50 shows the required number of samples for various values of (X) and (Y). This example shows that the size of the surface to be measured does not directly determine the number of sampling locations required. However it will have an indirect effect, in that as the surface gets larger there is usually an increase in the variance of the sampled function over the surface. In other words, at a given frequency a small plate will tend to vibrate in a simple half sine wave mode, while a large plate will possibly have local modes resulting in locally high vibrations. From the figure, a reasonable real noise source may have ten percent of the surface area which has velocity squared values ten times higher than the remainder of the surface. This would require approximately 120 discrete samples to estimate the radiated power with the given criterion. As the strength of the hot spot decreases, for example to only five times the level of the remainder of the surface, the required number of averages greatly reduces. In this case to only approximately 45. In a normal application of acoustic intensity finity error tends to require a significant number of measurement locations.

The previous example shows how many spatial sampings must be used when the sampling at each point is exact and the intensity is a simple bi-level function over the surface. A more general analysis includes the effect of a variance in the sampling at each spatial point, and also considers the intensity as a general function over the surface.
Figure 3–50
Required Number of Samplings to Assure within a 90% Confidence Level that the Average Intensity is Accurate to within ±1dB, for an Intensity Level of $X$, Over $Y$ Proportion of the Surface, and Unity Over the Remaining $(1-Y)$ Proportion.
In this analysis we begin with the expression for the estimate of surface averaged intensity.

\[
\hat{I} = \frac{1}{N} \sum_{s} \hat{I}_{s} \quad (\hat{I}_{s} \text{ sampled at N locations})
\]  

(3.86)

From the definition of variance we have;

\[
\text{VAR} [\hat{I}] = E \left[ \left( \frac{1}{N} \sum_{s} \hat{I}_{s} - \hat{I} \right)^{2} \right]
\]

(3.87)

\[
\text{VAR} [\hat{I}] = \text{E} \left[ \frac{1}{N} \sum_{s} \left( \hat{I}_{s} - \bar{I} \right)^{2} \right]
\]

(3.88)

\[
\text{VAR} [\hat{I}] = \frac{1}{N^{2}} \text{E} \left[ \left( \frac{N}{\Sigma} \left( \hat{I}_{s} - \bar{I} \right) \right)^{2} \right]
\]

(3.89)

This may be expanded as follows;

\[
\text{VAR} [\hat{I}] = \frac{1}{N^{2}} \text{E} \left[ \left\{ \frac{N}{\Sigma} \left( \hat{I}_{s} - \bar{I} \right) - \frac{N}{\Sigma} \left( \bar{I} - \hat{I}_{s} \right) \right\}^{2} \right]
\]

(3.90)

\[
\text{VAR} [\hat{I}] = \frac{1}{N^{2}} \text{E} \left[ \left( \frac{N}{\Sigma} \left( \hat{I}_{s} - \bar{I} \right) \right)^{2} + \left( \frac{N}{\Sigma} \left( \bar{I} - \hat{I}_{s} \right) \right)^{2} - 2 \left( \frac{N}{\Sigma} \left( \hat{I}_{s} - \bar{I} \right) \right) \left( \frac{N}{\Sigma} \left( \bar{I} - \hat{I}_{s} \right) \right) \right]
\]

(3.91)

The last term represents the covariance between two independent random variables, which is identically zero. Furthermore, for a sampled, zero mean random variable (A), the

\[
\text{E} \left[ \left( \frac{N}{\Sigma} A \right)^{2} \right] = N \text{E} \left[ A^{2} \right]
\]

(3.92)

Thus, the variance simplifies to;

\[
\text{VAR} [\hat{I}] = \frac{1}{N} \left\{ \text{E} \left[ \left( \hat{I}_{s} - I_{s} \right)^{2} \right] + \text{E} \left[ \left( \bar{I} - I_{s} \right)^{2} \right] \right\}
\]

(3.93)

or

\[
\text{VAR} [\hat{I}] = \frac{1}{N} \left\{ \text{VAR} [\hat{I}_{s}] + \text{VAR} [I_{s}] \right\}
\]

(3.94)
This can be interpreted by stating that the variance of the estimated surface averaged intensity is \((1/N)\) times the sum of the surface average of the variance in the sampling of each point intensity plus the variance of the point intensity over the surface. This result may now be applied to the sub-critical frequency radiation from a rectangular, simply supported plate. For this case, the surface intensity may be expressed as:

\[
I_s(x, y) = I_o \sin (k_x x) \sin (k_y y) \quad \text{for } 0 < x < L_x \\
0 < y < L_y
\]  

(3.95)

where

\[
k_x = \frac{p\pi}{L_x} \quad ; \quad k_y = \frac{q\pi}{L_y} \quad ; \quad p, q \text{ integers}
\]

and

\(I_o\) a normally distributed random variable sampled over time

For this model the surface averaged intensity is identically zero unless both \(p\) and \(q\) are odd. For this case, the mean and variance of the surface intensity are:

\[
\bar{I} = \frac{4 I_o}{pq \pi^2}
\]  

(3.96)

\[
\mathbb{E} \left[ I_s^2 \right] = \frac{I_o^2}{4}
\]  

(3.97)

\[
\text{VAR} \left[ I_s \right] = \mathbb{E} \left[ I_s^2 \right] - \bar{I}^2
\]  

(3.98)

From Pascal [13];

\[
\text{VAR} \left[ \hat{I}_s \right] = \frac{I_s^2}{2B} \left[ \frac{1}{\gamma_{21}^2} + \cot^2 \phi_{12} \left( \frac{1}{\gamma_{21}^2} - 1 \right) \right]
\]  

(3.99)

If we assume \((\gamma_{21} = 1)\), this reduces to;

\[
\text{VAR} \left[ \hat{I}_s \right] = \frac{I_s^2}{2B}
\]  

(3.100)
\[
\text{VAR} \left[ \frac{\hat{I}_s}{I} \right] = \frac{\text{E} \left[ I_s^2 \right]}{BT}
\]

(3.101)

Thus, we have;

\[
\text{VAR} \left[ \frac{\hat{I}}{I} \right] = \frac{1}{N} \left\{ \frac{\text{E} \left[ I_s^2 \right]}{BT} + \text{E} \left[ I_s^2 \right] - \bar{I}^2 \right\}
\]

(3.102)

\[
\frac{\text{VAR} \left[ \frac{\hat{I}}{I} \right]}{\bar{I}^2} = \frac{1}{N} \left\{ \left( 1 + \frac{1}{BT} \right) \frac{\text{E} \left[ I_s^2 \right]}{I^2} - 1 \right\}
\]

(3.103)

Which results in;

\[
\frac{\text{VAR} \left[ \frac{\hat{I}}{I} \right]}{\bar{I}^2} = \frac{1}{N} \left\{ \left( 1 + \frac{1}{BT} \right) \frac{(pq)^2 \pi^4}{64} - 1 \right\}
\]

(3.104)

From this result it is now possible to determine the optimum sampling scheme.

The optimum scheme is that combination of the number of sampling locations (\(N\)), and number of ensembles (\(n\)) averaged at each location which will provide the minimum variance in the estimated average intensity for a given total data acquisition time (\(\tau\)).

Starting with the identities;

\[
n = BT
\]

(3.105)

\[
Nn = \frac{\tau}{t_o} = C
\]

(3.106)

where

\(t_o\) is the length of each individual time window

The variance may be restated as;

\[
\frac{\text{VAR} \left[ \frac{\hat{I}}{I} \right]}{\bar{I}^2} = \frac{1}{N} \left\{ \left( 1 + \frac{1}{n} \right) \frac{(pq)^2 \pi^4}{64} - 1 \right\}
\]

(3.107)
For integral values of \((N)\) and \((n)\) such that their product is a constant, it is seen that the optimum choice is with \(n=1\). This is due to the fact that increasing the number of ensembles per point only serves to decrease the variance in each sampled value, whereas an increase in the number of measurement locations has the effect of addressing both the point sampling variance and the variance of the intensity over the surface.

This result shows that for a discrete sampling scheme, the optimum choice is a discretized form of the swept microphone approach. Performing this measurement in a truly discrete sense would cause an inordinate amount of wasted time in relocating the microphone between each test. A preferred approach is simply to sweep the microphone as the data is being sampled.

Returning to the plate example, if the optimum sampling approach is chosen then;

\[
\frac{\text{VAR} \left[ \frac{\Delta}{I} \right]}{I^2} = \frac{1}{N} \left[ \frac{(pq)^2 \pi^4}{32} - 1 \right]
\]  

(3.108)

For the criterion of 90 percent confidence of the error being within 1 dB, the maximum allowable value for the normalized variance is 0.017. Using this, Table 3–3 shows the required number of samples for various values of \(pq\).

<table>
<thead>
<tr>
<th>((pq))</th>
<th>(N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>122</td>
</tr>
<tr>
<td>3</td>
<td>1576</td>
</tr>
<tr>
<td>5</td>
<td>4484</td>
</tr>
</tbody>
</table>
This shows a very pessimistic view of the amount of data averaging time which must be used to make accurate estimates of radiated sound power. The very large ensemble numbers required is a result of the very unfavorable ratio of the intensity surface variance to the mean. This is due to the biharmonic dependence over the surface. This situation will exist only for homogeneous plates at frequencies well below the coincidence frequency. Even in this case, this biharmonic nature will be limited to the very nearfield (roughly within \( \lambda_p/2 \) of the surface). For higher frequencies, or greater distances from the surface the spatial variance in the intensity will greatly decrease, thus allowing much fewer ensemble numbers.

The above analysis assumes that the source is stationary (not changing with time). For those instances where very long sample times are required, the stationarity assumption may no longer be valid.

b. Swept Sampling Approach

The alternative method of performing an intensity survey is to sweep the probe over the surface while averaging the data. In this fashion the estimated value will tend to the average value of intensity over the swept area. A primary concern is that the sweeping motion provide a uniform weighting of the intensity over the full surface area, and not reside over a particular area for an inordinately long period. This is usually accomplished by defining a fixed sweeping pattern and repeating it several times. The pattern normally includes sweeping motions in the various directions. It is theoretically possible to achieve an appropriate average by using a completely random motion to the probe. However a human is much more adept at maintaining a defined regular pattern than at performing a truly random task.

It was shown in the previous discussion that the swept sampling approach may potentially result in less random error in a sound power measurement. The previous analysis may be extended in order to evaluate the variance in the sound power estimate with a swept probe. Equation (3.107) may be used directly by determining the number of ensembles (n) which are collected while the probe is in essentially the same location. In order to determine this, it is first necessary to define what it means for the probe to be in essentially the same location. For example, in the case of the plate vibrations excited by turbulent flow over the surface, this is determined by the spatial correlation length of the resulting vibrations. That is, any two points which are dislocated by a distance less than the correlation length experience essentially the same vibration. The correlation length is a function of the types of vibration and the excitation mechanism. For the purposes of this analysis it will be assumed that this correlation length is equal to one quarter of the structural wavelength (\( \lambda_p/4 \)).
Now, we investigate the criterion on probe velocity. From the above approach it is possible to determine a minimum desirable probe sweep velocity in order to minimize the variance of the estimated power. This is determined by requiring the probe to move $\lambda/4$ in each sampling ensemble. The time window length may be determined by;

$$t_o = \frac{L}{2 F_{\text{max}}}$$ \hfill (3.109)

where

$L$ = number of time domain points per ensemble
$F_{\text{max}}$ = Nyquist frequency

Furthermore, the plate wavelength may be stated as;

$$\lambda_p = \left[ \frac{1}{12 \left(1 - \nu^2 \right)} \right]^{\frac{1}{2}} \left( \frac{2\pi h C_L}{f} \right)^{\frac{1}{2}}$$ \hfill (3.110)

Defining the minimum velocity $u_{\text{min}}$ as;

$$u_{\text{min}} \cdot t_o = \frac{\lambda_p}{4}$$ \hfill (3.111)

or

$$u_{\text{min}} = \frac{1}{4} \left[ \frac{1}{12 \left(1 - \nu^2 \right)} \right]^{\frac{1}{2}} \left( \frac{2\pi h C_L}{f} \right)^{\frac{1}{2}} \frac{2 F_{\text{max}}}{L}$$ \hfill (3.112)

Note that the minimum velocity is greater at lower acoustic frequencies. It is reasonable to determine the required sweep velocity for $f=0.1 F_{\text{max}}$ as a lower frequency limit. For this case, and with $u = 0.3; L = 1024$;

$$u_{\text{min}} = 2.13 \times 10^{-3} \left( h C_L F_{\text{max}} \right)^{\frac{1}{2}}$$ \hfill (3.113)

This velocity has been determined for several plate thicknesses as a function of frequency, and is shown in Figure 3-51. Choice of sweep velocities lower than shown in this figure will result in an increased random error in the sound power estimate.

One adverse effect of the sweeping motion is a smearing effect in the frequency domain. This smearing effect is similar to a Doppler shift. In the normal treatment of Doppler shift the amount of the shift is a function of the translational velocity relative to the speed of sound in air. In the case at hand the Doppler shift is a result of moving
Figure 3–51
Minimum Probe Velocity Allowed to Maintain Maximum Number of Statistical Degrees of Freedom in Swept Average for a Plate of Thickness (h). Velocities At or Above this Value, Allow for a Minimum of Random Error in the Estimate of Sound Power.

Figure 3–52
Maximum Probe Traverse Velocity for a Plate of Thickness (h), In Order to Assure that the Measured Frequency Shift is Less than One Half of One Spectral Line Resolution with a 400 Line Spectral Analysis to Maximum Frequency $f_{\text{max}}$. 

the probe in a field which has a spatial variation. This spatial variation is determined by the wavenumber in the radiating structure along with the wavenumber in the acoustic field. At the surface the wavenumber is identically that of the structure. There is then a transition in which the effective wavenumber shifts to that characteristic of the acoustic field as the probe is moved away from the surface. Thus it is possible to achieve significant frequency shifts even with relatively moderate transverse velocity. If a probe is moved with a velocity \( u \) over a surface which exhibits a spatially varying pressure

\[
\tilde{p}(x;\omega) = A \sin(k_p x)
\]

(3.114)

The signal sensed by the microphone will be of the form:

\[
p(t) = \frac{A}{2} \left[ \cos(k_p ut - \omega t) - \cos(k_p ut + \omega t) \right]
\]

(3.115)

Thus the frequency is apparently shifted by the ratio \( k_p u / \omega \). This may be expressed as \( u / (f \lambda_p) \). This will generally only be a problem when narrow band frequency analysis is being performed. In the event that a 400 line spectral analysis is being performed, and it is required that the frequency components shift no more than one spectral line, the maximum sweep velocity \( u_{\text{max}} \) may be determined as a function of the plate bending wavelength. If the structural wavenumber is that of bending waves in a plate, the following equation describes the frequency shift (where \( h \) is the beam depth):

\[
\frac{\Delta f}{f} = \frac{u}{(2 \pi f h C_L)^{1/4}} \left[ \frac{12}{(1 - \nu^2)} \right]^{1/4}
\]

(3.116)

In the acoustic farfield the apparent wavenumber will be that of the acoustic medium. The amount of frequency shift in this case is that resulting from a Doppler shift effect. As such the resulting equation for the shift is:

\[
\frac{\Delta f}{f} = \frac{u}{c_0}
\]

(3.117)

These equations have been used to determine the maximum allowable sweep speeds \( u_{\text{max}} \) to maintain the 1 in 400 resolution required for narrow band analysis, as a function of maximum frequency analysis, for several plate thicknesses. The sweep velocity should be chosen as the lowest of that determined by the acoustic wavenumber or the structural...
wavenumber approach. The result is shown in Figure 3–52. This shows that for a 2mm thick plate for testing up to 1000 Hz, the maximum sweep velocity allowable would be approximately 0.5 m/s. Therefore if in sweeping, the probe is traversed over paths two centimeters apart, it would take approximately fifty seconds to sweep a 0.5 m². Sweeping at a faster velocity would cause noticeable smearing of the frequency spectrum.

By combining Figures 3–51 and 3–52, a band of preferred sweep velocities is determined. For certain combinations of plate thickness and maximum frequency, the two criteria are mutually exclusive. This requires that either the random error be increased or that a limited amount of frequency shift be accepted.

The velocity requirements determined by both the variance requirement and the frequency shift criteria are dependent upon the bandwidth used for frequency analysis. Figures 3–51 and 3–52 were computed for a typical 400 line narrowband spectral analysis. If octave, or 1/3 octave analysis is used, the requirements placed on probe sweep velocity are relaxed considerably. Furthermore, the increased analysis bandwidth will normally result in a dramatic reduction in the spatial variance of the intensity over the surface. This allows for a considerable reduction in the required number of ensembles.

3.3 Measurement Errors

Measurement errors were defined as all the errors associated with the acquisition and processing of the microphone signals. Among these errors are; calibration induced bias errors, random errors, extraneous noise, microphone positioning errors, aliasing, and numerical truncation/roundoff errors. Since some of these errors will be dependent on the specific estimator used the errors for each velocity estimator were examined.

3.3.1 Calibration Induced Bias Errors

One of the major errors in acoustic intensity measurement is due to phase mismatch in the individual microphones. The velocity and intensity estimate is dependent on the difference in pressure at two closely spaced microphones. In the low frequency range it is often necessary to discern to within one degree of phase accuracy to obtain an accurate measurement. Furthermore, in the presence of either a highly reactive field or a high background noise level, it may be necessary to maintain a similar order of magnitude resolution in phase even at the higher frequencies. No two microphones and data acquisition channels will be sufficiently matched to allow direct measurement of velocity or intensity without some type of compensation for the phase mismatch. There are basically two calibration approaches.
The first approach involves the use of microphone switching [29]. In this approach the measurement is first made with microphone A at location 1, and microphone B at location 2. The measurement is then repeated with microphone A at location 2 and microphone B at location 1. These two results are then combined through one of several alternative methods [21, 29] to determine the average of the two measurements. Through this approach the bias error in the first measurement will cancel out with the error in the second measurement. This method is capable of producing a very accurate result, however it has the obvious disadvantage of requiring twice as many measurements.

The alternative approach is to calibrate the probes at the beginning of each test by measuring the phase and magnitude error between the two microphones. This measurement is then stored and used to compensate each ensuing measurement. The compensation is applied directly to the cross power spectrum measurement. This approach is considerably faster than the switching method and provides acceptable accuracy as long as the phase mismatch in the instrumentation does not drift. This approach is the most commonly used [21, 32]. However, there are at least two methods used, as discussed below, for acquiring the original compensation transfer function.

The first method of measuring the compensation function [49] is to perform a measurement with the two microphones exposed to exactly the same signal. This is a relatively simple thing to do if it is only necessary to calibrate for the electronics, however a significant phase mismatch may exist between the microphone cartridges. This requires that the microphones be exposed to an identical sound field for calibration. This is a difficult task when the calibration must be done above 1000 Hz. Various approaches have been suggested for construction of anechoic tubes and pressure cavities for this purpose [49, 50].

An alternative approach [51] to acquiring the compensation transfer function is to use a variation of the microphone switching approach during calibration. If the microphones are exposed to a sound field for a fixed number of ensemble averages, then switched and exposed to the same sound field while the averaging is repeated for the same number of ensembles, the direct sum of these two measurements should indicate zero intensity. If the measurements are performed using a sufficiently long averaging time, then any indicated phase angle in the cross power spectrum is due to the instrumentation phase mismatch.

Now, an analysis of errors associated with phase mismatch will be presented. If \( (H_{11}) \) represents the calibration transfer function measured between microphone (i) and 1, then a cross power spectrum may be compensated for calibration mismatch as follows.
\[
\hat{G}_{ij}^c = \left( H_{i1} H_{j1}^* \right)^{-1} \hat{G}_{ij}
\]

(3.118)

where

\( \hat{G}_{ij}^c \) is the compensated cross spectrum

\( H_{i1}, H_{j1} \) are the calibration functions measured between microphones i, j and 1.

The effect of calibration induced bias error will now be presented for the resistive intensity, reactive intensity, and velocity estimate.

**Resistive Intensity**

The estimator for resistive intensity is:

\[
\hat{I}(\omega) = \frac{-\text{Im}(\hat{G}_{12})}{\rho \omega \Delta r}
\]

(3.119)

This may be rewritten as:

\[
\hat{I}(\omega) = \frac{|\hat{G}_{12}| \sin \phi_{12}}{\rho \omega \Delta r}
\]

(3.120)

If we assume:

\[
\hat{G}_{11} = M_1^2 G_{11}
\]

(3.121)

\[
\hat{G}_{22} = M_2^2 G_{22}
\]

(3.122)

\[
\hat{G}_{12} = |M_1 M_2 G_{12}| \approx (\phi_{12} + \Delta_{12})
\]

(3.123)

Then, assuming small angles for the error in the measured phase angle \( \Delta_{12} \):

\[
\epsilon_I = 1 - M_1 M_2 \left[ 1 + \frac{\Delta_{12}}{\tan \phi_{12}} \right]
\]

(3.124)

and for small \( \phi_{12} \):

\[
\epsilon_I \approx \left[ 1 - M_1 M_2 \right] - M_1 M_2 \left( \frac{\Delta_{12}}{\phi_{12}} \right)
\]

(3.125)
If we consider a uni-directional plane wave, then $\phi_{12} = k\Delta r$. By further assuming that the phase mismatch may only be corrected to within 0.5 degrees, there will be at least a 1 dB error in the intensity for all values of $k\Delta r < 0.034$. For a 10mm microphone spacing, this corresponds to 184 Hz. If the phase can be corrected to within 0.1 degrees, the low frequency limit is then 37 Hz.

For resistive intensity, the calibration error is due to the phase mismatch. However, the magnitude calibration error also carries over to intensity estimation. For plane wave propagation the lowest frequency to which the intensity may be measured to within 1 dB is given by:

$$f_{\text{low}} = \frac{\Delta_{12} C_0}{1.12(2\pi)\Delta r}$$  \hspace{1cm} (3.126)

This is summarized in Figure 3–53 for various microphone spacings and phase error.

**Reactive Intensity**

The estimator for reactive intensity is

$$\hat{J} = \frac{\hat{G}_{11} - \hat{G}_{22}}{2(\rho \omega \Delta r)}$$ \hspace{1cm} (3.127)

This results in the following normalized error:

$$\epsilon_{\hat{J}} = \frac{(1 - M_1^2) G_{11} - (1 - M_2^2) G_{22}}{G_{11} - G_{22}}$$ \hspace{1cm} (3.128)

By defining a parameter ($\alpha = M_2/M_1$), the error expression is reduced to:

$$\epsilon_{\hat{J}} = (1 - M_1^2) - M_1 \epsilon^2 \frac{G_{22}}{G_{11} - G_{22}}$$ \hspace{1cm} (3.129)

This points out that there are two error sources. The first term is a general level error due to a common magnitude calibration error between the two microphones. The second, and more serious error, is due to a mismatched magnitude calibration error ($1 - \alpha^2$).
Figure 3-53
Minimum Frequency to Which Intensity May Be Measured with
Less than a 1 dB Error for Plane Wave Propagation.
Frequency is Shown for Various Microphone Spacings as a
Function of Calibration Phase Mismatch Error.
This latter error determines the low frequency limit to which reactive intensity measurements may accurately be made.

**Acoustic Velocity**

The estimator for acoustic velocity is:

\[
\hat{G}_{\text{uu}} = \frac{1}{(\rho \omega \Delta r)^2} \left[ G_{11} + G_{22} - 2 \text{Re} \ G_{12} \right] \quad \text{(3.130)}
\]

By using the previously defined parameters, the calibration error reduces to:

\[
\epsilon_{\text{uu}} = (1 - M_1^2) + \frac{M_1^2 (1 - a)}{(1 - \Delta_1^2 \tan \phi_1)} \left[ \frac{(1 + a)(1 - \Delta_1^2 \tan \phi_1) \ G_{22} - 2 \text{Re} \ G_{12}}{G_{11} + G_{22} - 2 \text{Re} \ G_{12}} \right] \quad \text{(3.131)}
\]

This again shows two error sources. The first, \(1-M_1^2\) is due to the general magnitude error. The second, \((1-a)\) is due to a magnitute calibration mismatch. This is more easily recognized when the normalized error is simplified by removing second order error terms as follows:

\[
\epsilon_{\text{uu}} \approx (1 - M_1^2) + \frac{2 (1 - a) \left[ G_{22} - \text{Re} \ G_{12} \right]}{G_{11} + G_{22} - 2 \text{Re} \ G_{12}} \quad \text{(3.132)}
\]

**3.3.2 Random Error**

Any measurement performed on a random variable is subject to error in the estimation of the mean value of that variable. For many engineering measurements of sound and vibration, random processes may be modeled as Gaussian. Jenkins and Watts [52] evaluate the variance in estimation of cross and auto power spectra, when ensemble averaging estimates from a Digital Fourier Transform. This results in the following covariance matrix, where \(Q_{12} = \text{Im} \left[ G_{12} \right] \) and \(L_{12} = \text{Re} \left[ G_{12} \right] \).
\[
\text{COV} \left( \begin{pmatrix} \hat{G}_{11}, \hat{G}_{22}, \hat{L}_{12}, \hat{Q}_{12} \end{pmatrix}, \begin{pmatrix} \hat{G}_{11}, \hat{G}_{22}, \hat{L}_{12}, \hat{Q}_{12} \end{pmatrix} \right) = 
\begin{bmatrix}
G_{11}^2 & |G_{12}|^2 & G_{11}L_{12} & G_{11}Q_{12} \\
|G_{12}|^2 & G_{22}^2 & G_{22}L_{12} & G_{22}Q_{12} \\
G_{11}L_{12} & G_{22}L_{12} & \frac{1}{2} G_{11}G_{22} + L_{12}^2 - Q_{12}^2 & L_{12}Q_{12} \\
G_{11}Q_{12} & G_{22}Q_{12} & L_{12}Q_{12} & \frac{1}{2} \left( G_{11}G_{22} - L_{12}^2 + Q_{12}^2 \right)
\end{bmatrix}
\]

\[\text{VAR} \left[ \frac{1}{\hat{\mu}} \right] = \left( \frac{1}{BT} \right) \left[ \frac{G_{11}G_{22} - L_{12}^2 + Q_{12}^2}{2Q_{12}} \right] \]

\[\text{VAR} \left[ \frac{1}{\hat{\mu}} \right] = \left( \frac{1}{2BT} \right) \left[ \frac{L_{12}^2 + Q_{12}^2}{Q_{12}^2} - \frac{L_{12}^2}{Q_{12}^2} + 1 \right] \]

The effect of spatial variance on the accuracy of sound power estimates has been discussed in section 3.2.3. This section will present the variance in estimates of resistive intensity, reactive intensity, and velocity for single point measurements.

Resistive Intensity

Using the notation of this section, the estimator for resistive intensity is

\[
\hat{I} = -\frac{Q_{12}}{\rho \omega \Delta r}
\]

From the spectral covariance matrix, equation (3.133), we thus have;

\[
\text{VAR} \left[ \frac{1}{\hat{\mu}} \right] = \left( \frac{1}{BT} \right) \left[ \frac{G_{11}G_{22} - L_{12}^2 + Q_{12}^2}{2Q_{12}} \right]
\]

\[
\text{VAR} \left[ \frac{1}{\hat{\mu}} \right] = \left( \frac{1}{2BT} \right) \left[ \frac{L_{12}^2 + Q_{12}^2}{Q_{12}^2} - \frac{L_{12}^2}{Q_{12}^2} + 1 \right]
\]
Or finally;

$$ \frac{\text{VAR} \left[ \hat{J} \right]}{\hat{J}^2} = \left( \frac{1}{2BT} \right) \left[ \left( \frac{1}{\gamma_{12}} + 1 \right) + \text{cotg}^2 \phi_{12} \left( \frac{1}{\gamma_{12}} - 1 \right) \right] $$

(3.137)

Which is in agreement with Pascal's result [13] as presented in Section 3.2.3. This may also be expressed as;

$$ \frac{\text{VAR} \left[ \hat{J} \right]}{\hat{J}^2} = \frac{1}{BT} \left[ 1 + \frac{1}{2} \left( \frac{1}{\gamma_{12}} - 1 \right) (1 + \text{cotg}^2 \phi_{12}) \right] $$

(3.138)

Reactive Intensity

The reactive intensity estimate is stated as;

$$ \hat{J} = \frac{\hat{G}_{11} - \hat{G}_{22}}{2 \rho \omega \Delta r} $$

(3.139)

The variance of this estimator is found by;

$$ \frac{\text{VAR} \left[ \hat{J} \right]}{\hat{J}^2} = \frac{\text{VAR} \left[ \hat{G}_{11} \right] + \text{VAR} \left[ \hat{G}_{22} \right] - 2 \text{COV} \left[ \hat{G}_{11}, \hat{G}_{22} \right]}{\left[ \hat{G}_{11} - \hat{G}_{22} \right]^2} $$

(3.140)

From the spectral covariance matrix equation (3.133), this may be expressed as;

$$ \frac{\text{VAR} \left[ \hat{J} \right]}{\hat{J}^2} = \left( \frac{1}{BT} \right) \left[ \frac{\hat{G}_{11}^2 + \hat{G}_{22}^2 - 2 \left| \hat{G}_{12} \right|^2}{\left[ \hat{G}_{11} - \hat{G}_{22} \right]^2} \right] $$

(3.141)

By expressing the last term in the numerator as $ \left| \hat{G}_{12} \right|^2 = \gamma_{12}^2 \hat{G}_{11} \hat{G}_{22} $, the variance simplifies to;

$$ \frac{\text{VAR} \left[ \hat{J} \right]}{\hat{J}^2} = \frac{1}{BT} \left[ 1 + \frac{2 \hat{G}_{11} \hat{G}_{22}}{\left( \hat{G}_{11} - \hat{G}_{22} \right)^2} \left( 1 - \gamma_{12}^2 \right) \right] $$

(3.142)
Velocity Estimation

The first velocity auto spectrum estimator is expressed as;

\[
\hat{G}_{u u I} = \frac{\hat{G}_{11} + \hat{G}_{22} - 2 \hat{L}_{12}}{(\rho \omega \Delta t)^2}
\]

(3.143)

The variance of this estimator is;

\[
\text{VAR} \left( \hat{G}_{u u I} \right) = \frac{1}{(\rho \omega \Delta t)^4} \left[ \text{VAR} \left( \hat{G}_{11} \right) + \text{VAR} \left( \hat{G}_{22} \right) + 4 \text{VAR} \left( \hat{L}_{12} \right) \right. \\
+ 2 \text{COV} \left( \hat{G}_{11}, \hat{G}_{22} \right) - 4 \text{COV} \left( \hat{G}_{11}, \hat{L}_{12} \right) - 4 \text{COV} \left( \hat{G}_{22}, \hat{L}_{12} \right) \right]
\]

(3.144)

By referring to the spectral covariance matrix, equation (3.133), the variance is expressed as;

\[
\frac{\text{VAR} \left( \hat{G}_{u u I} \right)}{\hat{G}_{u u I}^2} = \frac{\left( \hat{G}_{11}^2 + \hat{G}_{22}^2 + 2 \left( \hat{G}_{11} \hat{G}_{22} + \hat{L}_{12}^2 - \hat{Q}_{12}^2 \right) + 2 \hat{G}_{12} \hat{L}_{12} - 4 \left( \hat{G}_{11} + \hat{G}_{22} \right) \hat{L}_{12} \right)}{\text{BT} \left( \hat{G}_{11} + \hat{G}_{22} - 2 \hat{L}_{12} \right)^2}
\]

(3.145)

\[
\frac{\text{VAR} \left( \hat{G}_{u u I} \right)}{\hat{G}_{u u I}^2} = \frac{\left( \left( \hat{G}_{11} + \hat{G}_{22} \right)^2 - 4 \left( \hat{G}_{11} + \hat{G}_{22} \right) \hat{L}_{12} + 4 \hat{L}_{12}^2 \right)}{\text{BT} \left( \hat{G}_{11} + \hat{G}_{22} - 2 \hat{L}_{12} \right)^2}
\]

(3.146)

From which is finally obtained;

\[
\frac{\text{VAR} \left( \hat{G}_{u u I} \right)}{\hat{G}_{u u I}^2} = \frac{1}{\text{BT}}
\]

(3.147)

The variance of the second velocity estimator may also be determined as follows.

We begin with the initial definition of the estimator, which is given as;

\[
\hat{G}_{u u II} = \frac{\hat{J}^2 + \hat{J}^2}{\hat{G}_{pp}}
\]

(3.148)

where \( \hat{G}_{pp} = G_{11} + G_{22} + 2 \hat{L}_{12} \)
The variance is then:

\[
\text{VAR} \left[ \hat{G}_{uuII} \right] = \left( \frac{2 \hat{I}}{\hat{G}_{pp}} \right)^2 \text{VAR} \left[ \hat{I} \right] + \left( \frac{2 \hat{J}}{\hat{G}_{pp}} \right)^2 \text{VAR} \left[ \hat{J} \right] + \left( \frac{\hat{I}^2 + \hat{J}^2}{\hat{G}_{pp}} \right)^2 \text{VAR} \left[ \hat{G}_{pp} \right]
\]

\[
+ 2 \left[ \frac{2 \hat{I}}{\hat{G}_{pp}} \right] \left[ \frac{2 \hat{J}}{\hat{G}_{pp}} \right] \text{COV} \left[ \hat{I}, \hat{J} \right] + 2 \left[ \frac{2 \hat{I}}{\hat{G}_{pp}} \right] \left[ \frac{- (\hat{I}^2 + \hat{J}^2)}{\hat{G}_{pp}} \right] \text{COV} \left[ \hat{I}, \hat{G}_{pp} \right]
\]

\[
+ 2 \left[ \frac{2 \hat{J}}{\hat{G}_{pp}} \right] \left[ \frac{- (\hat{I}^2 + \hat{J}^2)}{\hat{G}_{pp}} \right] \text{COV} \left[ \hat{J}, \hat{G}_{pp} \right]
\]

(3.149)

The covariance terms will be investigated first.

\[
\text{COV} \left[ \hat{I}, \hat{J} \right] = E \left[ \frac{- (\hat{Q}_{12} - Q_{12})}{\rho \omega \Delta r} \left( \frac{\hat{G}_{11} - G_{11} - \hat{G}_{22} + G_{22}}{2 \rho \omega \Delta r} \right) \right]
\]

(3.150)

\[
\text{COV} \left[ \hat{I}, \hat{J} \right] = \frac{-1}{(\rho \omega \Delta r)^2} \left\{ E \left( \hat{Q}_{12} - Q_{12} \right) \left( \hat{G}_{11} - G_{11} \right) \right. \\
- E \left( \hat{Q}_{12} - Q_{12} \right) \left( \hat{G}_{22} - G_{22} \right) \right\}
\]

(3.151)

\[
\text{COV} \left[ \hat{I}, \hat{J} \right] = \frac{-1}{(\rho \omega \Delta r)^2} \left\{ \text{COV} \left[ \hat{Q}_{12}, \hat{G}_{11} \right] - \text{COV} \left[ \hat{Q}_{12}, \hat{G}_{22} \right] \right\}
\]

(3.152)

\[
\text{COV} \left[ \hat{I}, \hat{J} \right] = \frac{- \hat{Q}_{12} \left( G_{11} - G_{22} \right)}{2 \rho \omega \Delta r (\rho \omega \Delta r)^2}
\]

(3.153)

And finally,

\[
\text{COV} \left[ \hat{I}, \hat{J} \right] = \frac{\hat{I} \hat{J}}{\rho \omega}
\]

(3.154)
A similar analysis for the other two covariance terms results in;

$$\text{COV} \left[ \hat{I}, \hat{G}_{pp} \right] = \frac{\hat{I} \hat{G}_{pp}}{BT} \tag{3.155}$$

$$\text{COV} \left[ \hat{J}, \hat{G}_{pp} \right] = \frac{\hat{J} \hat{G}_{pp}}{BT} \tag{3.156}$$

By using equations (3.149) and (3.154 – 3.156), we have;

$$\text{VAR} \left[ \hat{G}_{uu\Pi} \right] = \frac{4}{\hat{G}_{pp}^2} \left\{ \hat{I}^2 \text{VAR} \left[ \hat{I} \right] + \hat{J}^2 \text{VAR} \left[ \hat{J} \right] \right\} + \frac{\hat{G}_{uu\Pi}^2}{\hat{G}_{pp}} \text{VAR} \left[ \hat{G}_{pp} \right]$$

$$+ \frac{8 \hat{I}^2 \hat{J}^2}{BT \hat{G}_{pp}^2} - \frac{4 \hat{G}_{uu\Pi} \left[ \hat{I}^2 + \hat{J}^2 \right]}{BT \hat{G}_{pp}}$$

$$\tag{3.157}$$

We thus have;

$$\text{VAR} \left[ \hat{G}_{uu\Pi} \right] = \frac{4}{\hat{G}_{pp}^2 BT} \left\{ \hat{I}^4 \left[ 1 + \frac{1}{2} \left( \frac{1}{\gamma_{12}} - 1 \right) \left( 1 + \cotg^2 \theta_{12} \right) \right] \right\}$$

$$+ \hat{J}^4 \left[ 1 + \frac{2 G_{11} G_{22}}{(G_{11} - G_{22})^2} (1 - \gamma_{12}^2) \right] + 2 \hat{I}^2 \hat{J}^2 \right\} - \frac{3 \hat{G}_{uu\Pi}^2}{BT}$$

$$\tag{3.158}$$

$$\text{VAR} \left[ \hat{G}_{uu\Pi} \right] = \frac{4}{BT} \left\{ \frac{\hat{I}^2 + \hat{J}^2}{\hat{G}_{pp}^2} \right\}^2 - \frac{3 \hat{G}_{uu\Pi}^2}{BT} + \frac{4 (1 - \gamma_{12}^2)}{BT \hat{G}_{pp}^2} \times$$

$$\tag{3.159}$$

$$\left\{ \frac{\hat{I}^4}{2} \left( \frac{G_{11} G_{22}}{L_{12}^2 + Q_{12}^2} \right) \left( 1 + \frac{L_{12}^2}{Q_{12}^2} \right) + \frac{2 \hat{J}^4 G_{11} G_{22}}{(G_{11} - G_{22})^4} \right\}$$
This may be simplified to:

\[
\text{VAR} \left[ \hat{G}_{uu}^{II} \right] = \frac{\hat{G}_{uu}^{II}}{BT} + \frac{4 \left( 1 - \gamma_{12}^2 \right) G_{11} G_{22}}{BT \hat{G}_{pp}^2 (\rho \omega \Delta r)^4} \times \\
\left[ \frac{Q_{12}^2}{2} \frac{(L_{12}^2 + Q_{12}^2)}{(L_{12}^2 + Q_{12}^2)} + \frac{2 \left( G_{11} - G_{22} \right)^4}{16 \left( G_{11} - G_{22} \right)^2} \right] 
\]

(3.160)

\[
\text{VAR} \left[ \hat{G}_{uu}^{II} \right] = \frac{\hat{G}_{uu}^{II}}{BT} + \frac{2 \left( 1 - \gamma_{12}^2 \right) G_{11} G_{22}}{BT \hat{G}_{pp}^2 (\rho \omega \Delta r)^4} \left[ Q_{12}^2 + \frac{\left( G_{11} - G_{22} \right)^2}{4} \right] 
\]

(3.161)

Or;

\[
\text{VAR} \left[ \hat{G}_{uu}^{II} \right] = \frac{\hat{G}_{uu}^{II}}{BT} + \frac{2 \left( 1 - \gamma_{12}^2 \right) G_{11} G_{22} \hat{G}_{uu}^{II}}{BT \hat{G}_{pp} (\rho \omega \Delta r)^2} 
\]

(3.162)

And finally;

\[
\frac{\text{VAR} \left[ \hat{G}_{uu}^{II} \right]}{\hat{G}_{uu}^{II}} = \frac{1}{BT} \left\{ 1 + 2 \left( 1 - \gamma_{12}^2 \right) \frac{\left[ G_{11} G_{22} / G_{pp} (\rho \omega \Delta r)^2 \right]}{G_{uu}^{II}} \right\} 
\]

(3.163)

This shows that for unity coherence, the variance in the \( \hat{G}_{uu}^{II} \) estimator is identical to that in the \( \hat{G}_{uu}^{I} \) estimator in equation (3.147). For non-unity coherence, the variance in the \( \hat{G}_{uu}^{II} \) estimator will be higher than for the \( \hat{G}_{uu}^{I} \) estimator.

These results show that, with the exception of the \( \hat{G}_{uu}^{I} \) estimator, the variance of each of the estimators is a function of the coherence between the two microphone signals. The resistive intensity estimator will be very sensitive to coherence for fields in which the phase angle \( \phi_{12} \) is small. The reactive estimator will be very sensitive to coherence for fields in which \( G_{22} - G_{11} \) is small. Both of these effects are a result of the potential for a small fraction of the dominant intensity component to be misinterpreted as the complimentary component.
3.3.3 Bias Errors Due to Extraneous Noise

In signal processing, the sensitivity of an estimator to extraneous noise signals is of concern. The extraneous signal may be due to electrical noise in the instrumentation, or due to other mechanisms such as self-induced turbulence noise when a microphone is placed in a mean flow. To evaluate this effect, we consider two variables U and V, which we measure as signals X and Y. Prior to measurement, the variable U is contaminated by the noise signal m, and V is contaminated by n. This is shown schematically in Figure 3–54. In this example, both the power spectra \( G_{xx} \) and \( G_{yy} \) are biased by the amount \( G_{mm} \) and \( G_{nn} \), respectively. However, as long as the random signals m and n are independent processes, the cross power spectrum \( G_{xy} \) is unbiased, and thus a good measure of \( G_{uv} \).

For the above reason, the estimators \( \hat{G}_{uu_{III}} \) and \( \hat{G}_{uv_{IV}} \) would be better estimators than \( \hat{G}_{uu_{I}} \) and \( \hat{G}_{uu_{II}} \). Furthermore, \( \hat{G}_{uu_{I}} \) and \( \hat{G}_{uu_{II}} \) may be compared in light of this situation. The amount of error in \( \hat{G}_{uu_{I}} \) would be;

\[
(G_{uu} - \hat{G}_{uu_{I}}) = G_{mm} + G_{nn} \quad (3.164)
\]

While the error in \( G_{uu_{II}} \) is;

\[
(G_{uu} - \hat{G}_{uu_{II}}) = G_{mm} - G_{nn} \quad (3.165)
\]

Thus, the second estimator would be favored over the first.

3.3.4 General Data Acquisition and Processing Errors

The final source of errors considered is that set of errors normally associated with digital signal processing using the Fast Fourier Transform algorithm [35].

The first error encountered is that due to the digitization process. Most analog to digital convertors used for such processing possess between 8 and 14 bits of resolution. An 8 bit A/D will be capable of a dynamic range of only 48 dB, while a 14 bit A/D will have a dynamic range of 84 dB.

The next source of error is due to aliasing. Aliasing is a phenomenon associated with digital sampling of a signal. If the frequency of the signal being sampled is more than one-half the sampling frequency, the signal may be misinterpreted as a signal of lower frequency. For this reason it is necessary to filter out the higher frequency components prior to digitization. Proper selection of the filter cutoff frequency and slope of the filter skirts is necessary to minimize the effect of aliasing. This topic is covered in detail by Bendat and Piersol [35].
Figure 3–54
Schematic of Two Channel Measurement Process in the Presence of Uncorrelated Noise
The next error is leakage. This is an error associated with performing a Finite Fourier Transform. The input signals are digitized into ensembles containing a finite number of points, such as 1024. If the sampled function is not exactly periodic on this time window, then errors are introduced due to the signal truncation. These errors may allow some of the signal power at one spectral line to "leak" into neighboring spectral components. Various time windows such as Hanning or Bessel, are normally applied to the sampled time histories in order to reduce the effect of leakage. These time windows are designed to taper the signal at the ends of the time window in order to reduce the discontinuities at the ends. Bendat and Piersol [35] provide a detailed discussion of leakage.

3.4 Summary of Error Analysis

In this chapter a wide range of error sources have been considered. For each of these errors, various criteria have been developed to quantify the magnitudes of the errors.

The finite difference errors were seen to be dependent on the nature of the source, the spacing between the microphones, and the distance from the source. In considering various types of sources there are two non-dimensional parameters which are important. These are;

\[ k_a \Delta r \]

\[ d/\Delta r \]

where

\[ k_a = \text{acoustic wavenumber} \]

\[ d = \text{standoff distance of probe from radiator surface} \]

While the magnitude of any errors are dependent on the source type, it may generally be said that the error magnitudes will be reasonable (less than 1 dB) for situations where the following criteria are met;

\[ k_a \Delta r < 1 \text{ and } d/\Delta r > 1.5 \]  \hspace{1cm} (3.166)

These criteria are less stringent than those proposed by Thompson and Tree [20]. The difference is due to the consideration of the complexity of the acoustic nearfield of typical radiators. From a review of the infinite plane and circular rigid piston sources, it is concluded here that most practical sources do not experience as sharp pressure gradients as the quadrapole. The acoustic dipole is felt to be a more realistic source assumption.
Microphone probe scattering errors were evaluated for various microphone arrangements and spacings. These studies show that the optimum microphone spacing is approximately 3–4 times the microphone diameter. For probes of this spacing, the scattering error was negligible for the range of frequencies allowed by finity error considerations. Poor design of the microphone holders may cause additional scattering errors. Measurement of microphone scattering induced errors on a prototype three-dimensional intensity probe showed the errors to be slightly higher than for a two microphone probe.

While the strongest effect of microphone scattering is seen in the relative phase error, there is also a corresponding error in the relative magnitude. The reactive intensity and kinetic energy density estimators are functions of the pressure difference. Thus, in certain cases, the scattering induced magnitude errors may cause significant errors.

The study of errors in surface velocity estimation due to an evanescent field pointed out practical limitations in directly measuring radiation efficiencies or radiation impedances with an intensity probe. In these fields, it is important that the measurement be made as close as physically possible to the surface. In this event, there will be competing demands in regards to evanescent field errors and finite difference errors. The finite difference errors may become significant in the very nearfield. In this instance, though, it is necessary to accept increased finite difference errors in favor of reducing the evanescent field errors.

The study of finity errors is important when surface averaged properties are desired. This is the case for sound power studies or radiation efficiency measurement. Much confusion appears to exist in the literature regarding the relative advantages of discrete point sampling versus microphone sweeping methods. The analysis of this section conclusively proves that the sweeping approach is more favorable for measurement of average surface properties. A disadvantage, is the potential for frequency shifts. However, this may be controlled by maintaining sweep velocities below those shown in Figure 3–52. This study also resulted in minimum recommended velocities in order to minimize the sample variance, and a method of determining the required number of ensembles which should be averaged together.

The final error studies point out the need to use accepted good practice in signal processing. That is, to use anti-aliasing filters, use an analog to digital converter with acceptable dynamic range, and make use of data smoothing windows to reduce the effects of leakage. The evaluation of random error in each of the estimators provides further criteria for determining required total sample times for acceptable estimates. This resulted in a quantitative measure of the reliability of the estimates. This analysis showed that the \( \hat{G}_{uu1} \) estimator is somewhat less sensitive to random error than the \( \hat{G}_{uu2} \) estimator, however, the opposite is true in regards to sensitivity to extraneous noise.
CHAPTER 4 – EXPERIMENTAL VERIFICATION

4.0 General

The analyses of the previous two chapters have provided a considerable amount of insight into the use of a four microphone probe for the measurement of acoustic intensity and velocity. These analyses have provided some quantification of the expected error. However, the analytical study has been limited to several simple radiators for which an exact solution is available for the radiated field. In order to verify the feasibility of application of such a probe to realistic radiators, the probe was constructed and used to perform intensity and velocity measurements for several test cases. First, measurements were performed on a simple baffled circular piston. This provided experimental data for which the accuracy in the measurements could be compared with the predictions of Chapter 3. Off-axis measurements were then made on this piston in order to evaluate the measurement accuracy in a full three dimensional sound field. The measurements were first conducted in an ideal situation with no background noise. The tests were then repeated with a contaminating noise source in the vicinity. These tests were used to evaluate the relative merits of the various alternative estimators proposed in Chapter 2.

After successful application of the probe to the circular piston, the probe was used for measurements on a plate weldment. This structure was unbaffled, and designed to be a periodically stiffened structure, similar to what might be typically found in a machine structure. Again, the relative merits of the alternative estimators were evaluated based on measurements conducted on this structure.

For these tests, a three dimensional probe was constructed using four Realistic 1/2 inch condensor microphones. The probes were mounted in a holder which allowed the microphone center to center distances to be adjusted. The microphones were arranged in the quadahedron scheme, as discussed in Chapter 2, with an angle $\theta = 60$ degrees. This experimental system is depicted in Figure 4—1. It should be noted that a multi-channel frequency analyzer was used to acquire the data and generate the averaged cross-spectral matrix for all four microphones. These matrices were stored on a hard disk and then transferred to a minicomputer for final analysis.
Figure 4—1
Schematic of Experimental Equipment Used
4.1 Piston Tests

Initial tests were performed on a baffled circular piston, made of aluminum, 200 mm in diameter, and 35 mm thick. The baffle was 2.4 x 1.2 meters rectangular, and constructed from 20 mm thick plywood. The piston was driven in the center by a 50 lb* electromagnetic shaker with broadband random noise signal. The piston arrangement is shown in Figure 4–2. Data were acquired and analyzed over a frequency range of 100-3000 Hz. However, the system anti-aliasing filters were set at 2400 Hz. Thus the usable frequency range corresponded to: 0.19 < ka < 4.4.

During the tests, structural accelerations were measured at the center and the outer rim of the piston. The microphone probe was located at nine separate measurement locations, as shown in Figure 4–2. The radial location numbers 11, 21, 31, and 41 were selected to represent equal areas of the piston, assuming axisymmetric motion.

Power spectral densities (G_{uu_a}) of the velocity measured using an accelerometer at the center and outer portions of the piston are shown in Figure 4–3. This shows a prominent peak in the spectral content at 2150 Hz. This resonance peak was due to the natural frequency of the system consisting of the mass of the piston and the compliance of the stinger between the shaker and the piston. Also note that for frequencies above 1000 Hz, the outer portion of the piston has significantly higher motions than the center. This is due to the axisymmetric bending mode of the circular plate, at the resonance frequency of approximately 3000 Hz. While this resonance is above the frequency range of interest, the results shown in Figure 4–3 are influenced by it as the response is nearing the plate bending resonance.

The estimated acoustic velocity power spectral density from the probe measurements at location number 11 (piston center) is shown in Figure 4–4. For this figure the estimators G_{uu_I}, G_{uu_II}, G_{uu_III}, and G_{uu_IV} have all been superimposed on the same plot. All four estimators lay over each other with excellent repeatability. This is due to the fact that the coherence between the four microphones was essentially unity for this test. Figure 4–5 shows the estimated acoustic velocity overlaid with the measured structural velocity at the center of the piston. This shows that the acoustic velocity is approximately 2 dB lower than the piston velocity for frequencies below 1000 Hz. At frequencies above 1000 Hz, the measured acoustic velocity is, at times, higher than the piston center velocity. However, in this frequency range, this piston was not rigid. Thus the acoustic velocity is higher than the piston center velocity due to the increased piston rim velocity.
Figure 4-2
Schematic of Baffled Piston Test Arrangement
Figure 4-3
Velocity Power Spectral Density at the Center and Outer Edge of the Piston

Figure 4-4
$G_{uu_1}$, $G_{uu_II}$, $G_{uu_III}$, and $G_{uu_IV}$ overlaid for Measurement Point II on the Piston Radiator
The results of the evanescent velocity error analysis for a piston radiator from Section 3.2.2 may be compared to this experimental value from Figure 4–5. In this instance \( d/a = 0.19 \) and for the low frequencies we use the results for \( ka = 0 \). From Figure 3–49 the theoretical error is 1.1 dB, which agrees reasonably well with the 2 dB measured difference.

In order to verify that the difference in the velocity spectra in Figure 4–5 was due to the evanescent field error, the data were re-acquired with a field point closer to the piston face. For the point 11, the microphone nearest the piston was almost touching the piston, thus the only way to bring the effective measurement point closer to the piston face was to reduce the microphone spacing.

The microphone spacing was reduced to 12 mm, and the new estimate of acoustic velocity along with the piston velocity is shown in Figure 4–6. This shows that the difference between the piston velocity and the acoustic velocity is approximately 1 dB. This confirms the significance of the evanescent field induced velocity error.

The theoretical studies of Chapter 3 were conducted for points along the piston axis only, but here the experimental data have been used to infer the evanescent field effect for various radial locations off the piston axis. Figure 4–7 shows the measured acoustic velocity at locations 11, 21, 31, and 41. The first point of interest is the data for location 41. This point is over the baffle, and thus should indicate zero velocity at the surface. The measured acoustic velocity spectrum is approximately 15 dB lower than that measured at the piston center. This measured velocity would, however, be influenced by the evanescent field effect, and by baffle compliance. Taking these into consideration, the measured velocity is found to be reasonably low.

The results for location 21 are seen to be approximately 1–2 dB lower than those for the piston center. Since the piston velocity is uniform over the piston face for lower frequencies, it is assumed that this difference is entirely due to the effect of the evanescent field.

At location 31 the measured results are approximately 5 dB below the results for the piston center. This is believed to be due to the fact that location 31 is very close to the edge of the piston. For this location one of the microphones was over the baffle rather than the piston. Thus, this discrepancy is assumed to be the result of an edge effect.

The experimental study indicates that the evanescent field effect results in larger differences between the piston (structural) and acoustic surface averaged velocities than indicated by the on axis result given in Section 3.2.2. For this test, an additional 1–2 dB evanescent error was found.
Figure 4-5
Piston Center Velocity PSD, and the Measured Acoustic Velocity at Point II, with 25mm Microphone Spacing

Figure 4-6
Piston Center Velocity PSD, and the Measured Acoustic Velocity at Point II, with 12mm Microphone Spacing
Figure 4-7
Measured Acoustic Velocity PSD at Piston Locations 11, 21, 31 and 41

Figure 4-8
Comparison of Experimental and Theoretical Values of Surface Averaged Impedance for a Circular Piston
One of the proposed probe applications is for radiation impedance measurement. The resistive and reactive intensities over the piston were used in conjunction with the piston velocity to determine the surface averaged impedance function. For this purpose, the average piston velocity $\langle v^2 \rangle_s$ was estimated as the average of the velocity spectrum at the center of the piston, and at the edge of the piston. The surface impedance is determined as follows:

$$\chi = \frac{\langle J \rangle_s}{\rho c \langle v^2 \rangle_s} \quad (4.1)$$

$$R = \frac{\langle I \rangle_s}{\rho c \langle v^2 \rangle_s} \quad (4.2)$$

The surface averaged impedances ($I_s, J_s$) were evaluated by averaging the data for measurement locations 11, 12, and 13; see Figure 4--8. Superimposed in this figure are the theoretical impedances for a piston radiator [53]. This shows excellent agreement between experiment and theory.

The amount of random error in Figures 4--4, through 4--7 may be compared to the predictions of Section 3.3.2. For the case of perfect coherence, the variance in the intensity and velocity estimates is simply equal to the reciprocal of the number of ensembles. This would result in a normalized variance of 0.006 for these tests. This would correspond to 90 percent of the measured values being within 1.8 dB of the actual results. The random fluctuations in the measured data are seen to be within these bounds.

Figure 4--9 shows the measured acoustic velocity in the vertical, radial, and tangential directions at the edge of the piston. Due to symmetry, the tangential velocity should be zero. This figure shows that the indicated velocity spectrum is approximately two orders of magnitude below (or 20 dB less than) the maximum velocity.

In order to evaluate the sensitivity of the measured properties to background noise, the tests were repeated with an uncorrelated sound source, a speaker, in the vicinity of the piston. For this test the piston was unbaffled. The speaker was driven by a broad band random signal which was incoherent with the piston motion. The speaker level was adjusted such that the acoustic pressures at the center of the piston was approximately equal for either source, while the other source was switched off. Figure 4--10 shows a schematic of the test arrangement. Figure 4--11 shows the acoustic pressure power spectrum measured for the speaker and for the piston. The speaker was located on the $-Z$ axis of the intensity probe.
Figure 4—9
Radial, Vertical and Tangential Components of
Acoustic Velocity PSD Measured at the
Edge of the Piston
Figure 4-10
Schematic of Piston Radiator Test with Background Noise Simulated by a Speaker
Figure 4–12 shows the resistive intensity measured with only the piston, and with both the piston and speaker driven. The low frequency range shows a very large variance in the measured intensity. This is due to the very low radiation efficiency of an unbaflled piston in this frequency range. Even with the large variance, the two results are similar. In the range of 1500 Hz, the measured intensities were very similar even though the background pressure is 5 dB higher than the level from the piston. This illustrates the extreme insensitivity of the intensity estimator to background noise when the probe is located in the very nearfield.

Figure 4–13 shows the reactive intensity estimate with, and without background noise. Again, this shows extreme insensitivity to background noise. Figure 4–14 shows the vertical acoustic velocity power spectrum for the \( G_{uu} \) estimator with, and without background noise. Figure 4–15 shows the same result for the \( G_{uu} \) estimator. These figures illustrate that \( G_{uu} \) is insensitive to background noise at low frequencies, but \( G_{uu} \) is very sensitive. At higher frequencies, both estimators are sensitive. These figures illustrate that the background noise increased the indicated velocity for \( G_{uu} \), and decreased the indicated velocity for \( G_{uu} \).

Figures 4–16 and 4–17 show the results for \( G_{uu} \) and \( G_{uu} \). The effect of background noise on these two estimators is seen to be very similar to the effect seen in \( G_{uu} \).

4.2 Periodically Stiffened Plate Structure

The second example case for experimental study was a periodically stiffened plate. The structure is schematically shown in Figure 4–18. The plate was constructed of 4 mm thick steel plate of size 430 mm x 140 mm. Stiffeners were placed around the plate perimeter and at four (4) equally spaced locations across the plate width. These stiffeners were made from the same plate and were 30 mm deep. The plate was thus divided into five equal cells. Masses were then added in each of these cells to reduce the fundamental plate cell natural frequency to approximately 500 Hz.

This structure was excited on the second stiffener as shown in Figure 4–18. The plate was otherwise in a free-free, unbaflled state. The intensity probe was used to perform measurements at the center of cell number 3. An accelerometer was used to measure the plate velocity at the same location.

Figure 4–19 shows the plate velocity \( (G_{uu}) \) as measured by the accelerometer, along with the velocity estimate \( G_{uu} \). For this test the microphones were 14 mm apart, and the centroid of the microphones was approximately 20 mm above the plate. Figures 4–20, 4–21, and 4–22 show similar data for \( G_{uu} \), \( G_{uu} \), and \( G_{uu} \).
Acoustic Pressure PSD due to Piston, and due to Background Source. Piston, Circles (ooo); Background, Line (—)

Measured Resistive Intensity With and Without Background Noise Source. Without Background, Circles (ooo), with Background Line (—)
Figure 4-13

Measured Reactive Intensity With and Without Background Noise Source. Without Background, Circles (o); with Background, Line (—).

Figure 4-14

Measured $G_{uu}$ Velocity PSD With and Without Background Noise. Without Background, Circles (o); with Background, Line (—).
Figure 4–15
Measured $G_{uu_{\|}}$ Velocity PSD With and Without Background Noise Source.
Without Background, Circles (ooo); with Background, Line (—)
Figure 4-17

Measured $G_\text{w}_{14}$ Velocity PSD With and Without Background Noise Source.

Without Background, Circles (ooo), with Background, Line (---)
Expected Mode Shapes

- Overall plate mode
  \( f = 160 \) Hz

- Cell mode
  \( f = 600 \) Hz

- Cell mode
  \( f = 800 \) Hz

- Cell mode
  \( f = 1300 \) Hz

Figure 4-18
Schematic of the Periodically Stiffened Plate Structure and Vibration Mode Shapes
Figure 4-21
Measured Plate Velocity (Circles) and $G_{uu_{III}}$ Velocity (Solid Line) for Periodic Plate

Figure 4-22
Measured Plate Velocity (Circles) and $G_{uu_{IV}}$ Velocity (Solid Line) for Periodic Plate
In each of these figures, the acoustic velocity estimate for frequencies of 500 Hz and higher was significantly lower than the plate velocity measured by an accelerometer. In the velocity spectrum, four distinct plate resonances are evident. At 160 Hz a breathing mode of the overall plate is evident. The mode at 600 Hz corresponds to the mode where each cell moves out-of-phase with its neighbor. The resonances at 850 and 1300 Hz correspond to higher modes in which one or more of the neighboring cells are in-phase with each other. The normal progression of mode shapes for a periodic beam are shown in Figure 4–18 [54], along with the measured natural frequencies.

The measured evanescent error at 600 Hz may be compared to the theoretical result from Section 3.2.2 for an infinite plate. In this case the structural wavelength corresponds to the distance between two cells (172 mm). This corresponds to \(2d/\lambda = 0.23\). From Figure 3–49, the acoustic velocity should be 6.6 dB lower than the plate velocity. This is in excellent agreement with the experimental data shown in Figures 4–19, 4–20, 4–21, and 4–22.

At higher frequencies the plate cells will vibrate in-phase, and the effective plate wavelength would correspond to the cell spacing (86 mm). This results in \(2d/\lambda = 0.46\). From Figure 3–49 the acoustic velocity should be 14 dB lower than the plate velocity. Again, this is in excellent agreement with the experimental results.

For frequencies above 500 Hz, all four velocity estimators provide virtually identical results. However, at frequencies lower than 500 Hz there are significant differences. From Figure 4–19, the estimator \(G_{uu1}\) is seen to estimate higher velocities below 500 Hz than those measured at the plate with an accelerometer. Furthermore, the spectral definition of the first mode at 160 Hz is significantly lost in the \(G_{uu1}\) estimator. The other three estimators are seen to provide much more accurate velocity estimation for this mode. It should be recalled that for this mode the structural wavelength will be significantly longer than for the higher frequency modes. The longer wavelength in \(2d/\lambda = 0.05\). For this ratio, Figure 3–49 shows that the acoustic velocity should be within 1 dB of the plate velocity. The amount of experimentally measured difference between the various acoustic velocity estimators and the measured surface velocity is summarized in Table 4.1.
Table 4–1
Comparison of Difference Between Measured Structural Velocity and Measured Acoustic Velocity for the Periodic Plate at Specific Frequencies

<table>
<thead>
<tr>
<th></th>
<th>120 Hz</th>
<th>600 Hz</th>
<th>800 Hz</th>
<th>1300 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{uu_a}$ (dB re: 1(mm/sec)^2/Hz)</td>
<td>-28</td>
<td>-4</td>
<td>-9</td>
<td>-25</td>
</tr>
<tr>
<td>$G_{uu_a} - G_{uu_I}$</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>$G_{uu_a} - G_{uu_{II}}$</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{uu_a} - G_{uu_{III}}$</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{uu_a} - G_{uu_{IV}}$</td>
<td>-1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For these frequencies the four estimators gave essentially identical results.
4.3 Summary of Experimental Results

The test results on the piston and plate structure provided excellent agreement with the predicted behavior of Chapter 3. The tests on the piston resulted in surface averaged radiation impedance functions which agreed very well with theoretical results. This verified the accuracy of the resistive and reactive intensity spectral estimators. The random variations in the measured intensity spectra were within the limits predicted by an evaluation of the random error. Studies of the intensity estimators in the presence of a background noise field showed that both intensity estimators are relatively insensitive to background noise.

The velocity estimation results on the piston showed that in the absence of background noise, all four of the velocity estimators provided the same result. This is a result of the coherence between the various microphones being unity, in which case all four of the velocity estimators degenerate to the same result. Comparison of the measured acoustic velocity on the centerline of the piston showed the velocity to fall off with distance from the piston in the same fashion as was predicted in Section 3.2.2. For this case, it was possible to use the probe to accurately estimate the piston center velocity to within 1 dB accuracy. Measurements performed at various radial locations showed a further error of approximately 1–2 dB at the outer edge of the piston. This shows that with 1/2 inch microphones, the error in surface averaged velocity measurements using the probe would be approximately 2–3 dB. This error can only be reduced by using smaller microphones. For the purpose of surface velocity measurements, the finite difference errors were seen to be insignificant compared to the evanescent field effect. This indicates that the microphone probe may be used to estimate the surface velocity, but only with an appreciable error. A further complication is that, due to the evanescent effect, it is necessary to have the probe as close as possible to the radiating surface. With the probe being so close to the surface it may be impractical to use a swept probe sampling approach, unless a mechanized probe were employed.

Experimental tests performed on the peridically stiffened plate structure provided a better evaluation of the velocity estimation method as applied to typical machine structures. Again, the evanescent field effect was seen to correspond quite well to the theoretical predictions. In this case the surface velocity was underestimated by 6–15 dB. This clearly points out that extrapolation of the surface velocity from the measured acoustic velocity must be used with great caution. The velocity extrapolation error seems to be maximum at frequencies in the vicinity of coincidence.

Experimental tests with a simulated background noise source showed that each of the four velocity estimators provided somewhat different results. However, there was less overall error in \( G_{uu} \) than the other estimators. In general, the \( G_{uu} \) estimator was seen to
overestimate the velocity, and the other estimators underestimate the velocity. This situation allows the use of $G_{uuI}$ and one of the other three estimators as a means of identifying an upper and lower bound to the acoustic velocity. This is practical only for measurements in the very nearfield of a radiator. For the case where the acoustic energy may flow in both directions, the $G_{uuII}$ estimator was shown in Chapter 2 to be a poor estimator, since it relies on the vector sum of the intensity components.

Even though there was no contaminating background noise during the stiffened plate tests, the coherence between the microphone signals was found to not be exactly unity. Thus, the four velocity estimators provided different results. For this test case, the estimators $G_{uuII}$ and $G_{uuIII}$ provided better resolution for the low frequency, low amplitude data than did the $G_{uuI}$ estimator. This is the expected result from the considerations discussed in Chapters 2 and 3.
CHAPTER 5 – CONCLUSION

5.1 Summary

First, the general definition of acoustic intensity and energy density was reviewed. This analysis included a general definition of the complex valued acoustic intensity, for the general case including mean flow effects. These results were then simplified to the zero mean flow case. These studies showed that for the no-flow case, the intensity and energy density may be completely determined from knowledge of the acoustic pressure and the three components of acoustic velocity at a point. The acoustic velocity may be determined by using three closely spaced microphones to estimate the pressure gradient for a point through the use of a finite difference approximation. Next, it was shown how this finite difference in the time domain corresponds to spectral formulations. The spectral formulation for the resistive and reactive intensity was developed, along with the formulation for the acoustic pressure and velocity power spectra. The formulations for reactive intensity and acoustic velocity were then shown to be formulated in such a manner that they were sensitive to measurement error. Several alternative formulations were developed in order to reduce this error sensitivity. The relative merits of these alternatives were evaluated by subsequent theoretical and experimental studies.

The design of a four microphone probe for the measurement of intensity and energy density in three dimensions was then studied. The various spectral formulations were applied in such a fashion as to reduce the scattering error, reduce the effect of evanescent fields, and maintain the acoustic center for the estimations in the three directions as close as possible.

The sources of error in the resistive and reactive intensity estimators, along with the error in the acoustic velocity estimator were thoroughly studied. This included a study of the alternative estimators for acoustic velocity. The first error source studied was the finite difference error. This error was shown to be strongly related to the nature of the source and the distance from the radiator surface and the probe. This study considered the following acoustic sources: a progressive plane wave, a point monopole, a point dipole, an infinite plane with bending waves, and a baffled piston. This provided insight to the errors for simple point sources and for more complex physical sources. The conclusion from this study was that for \( k\Delta r > 1 \), and \( d/\Delta r > 1.5 \), the finite difference error will generally be less than 1 dB. Other researchers [20, 22] have studied this effect and provided more conservative recommendations. This is due to the assumption that typical radiators may contain source characteristics similar to point quadrapoles. But this study has shown it to be an overly
severe assumption. The finite difference error study provided very informative results from a consideration of measurements on the symmetry axis of a circular piston, placed in a baffle. This analysis provided graphic illustration of the transition from a plane wave radiation case to that of a point monopole. The major conclusion from these new results is that accurate intensity measurements may generally be made in the very nearfield of typical radiators with very little additional concern for finite difference errors.

As part of this study, an accurate method of measuring and predicting the effect of probe scattering was developed. This study determined that, in general for a two microphone probe, scattering induced errors are not significant for ratios of microphone separation to microphone diameters in the range of 2–4. Use of a ratio of 1 (microphones touching) was shown to introduce significant phase error in the lower frequencies. The scattering results indicate no benefit in microphone arrangements other than the conventional side-by-side probe. For a four microphone probe the high frequency scattering errors were shown to be only slightly worse. While it is possible that these errors would be the controlling factor limiting the upper frequency range of a probe, it was shown that in the majority of probe designs the finite difference errors are more severe.

For the determination of material impedances and source radiation efficiencies, Forssen and Crocker have proposed to use the acoustic velocity measured by an acoustic intensity probe to estimate surface velocities [12]. One of the errors in such an approach is due to the presence of an evanescent pressure field. At sub-critical frequencies, such a field causes the acoustic pressure and velocity to decay very rapidly with distance from the surface. In many cases this is an exponential decay. Since the effective center of the acoustic intensity probe physically cannot be on the surface, the acoustic velocity measured by the probe will be lower than the surface velocity. The error caused by extrapolating this acoustic velocity to the surface was shown to be very considerable in many cases. For plate vibrations, this was shown both theoretically and experimentally to result in as much as a 15 dB error. For other frequencies and plate constructions, the error may be less than 1 dB. This result requires that this method be applied with great caution. In practice, the amount of error might be estimated by resorting to the curves in Figure 3-49 or by performing measurements at two different elevations above the surface. If these two measurements provide drastically different results, then the method should not be applied.

Most of the practical applications of intensity probes are for measurement of surface averaged properties. It was shown in this study that the use of a probe sweeping technique is superior to a discrete sampling approach. With the swept probe method the surface averaged properties may be estimated with less variance and with greater ease. Criteria were developed in this analysis for minimum and maximum recommended sweep velocities. The maximum velocity is determined by spectra smearing criteria. The minimum velocity was determined by a requirement to maintain the largest possible number of statistical degrees
of freedom in the data, in order to minimize variance. These results are then summarized for vibrations of plates. In this study, it was shown that at frequencies well below coincidence, the variance in the surface average properties may require an extremely large number of data samples. In the event that only the surface averaged intensity is required, this variance may be greatly reduced by placing the probe at a greater distance from the surface.

Experimental evaluation of the sensitivity to background noise showed that the intensity formulation is very insensitive. However, the velocity estimators were more sensitive. In the presence of background noise the $G_{uu}$ was shown (both theoretically and experimentally) to be a better estimator than the alternatives. However, the $G_{uu}$ estimator was shown to over predict, while the other estimators underpredicted. Thus, two of these estimators could be used to establish upper and lower bounds on the acoustic velocity power spectrum. Comparison of the various velocity estimators for plate vibrations showed that in the low frequency range, the alternative formulations were superior to the standard $G_{uu}$ estimator, as they provided more dynamic range.

5.2 Accomplishments

This work has provided a considerable amount of information in the use of multiple microphone probes for the use of nearfield measurement of resistive and reactive intensity, along with the acoustic kinetic energy density. A major original contribution to the state-of-the-art has been the identification of alternative estimators for the reactive intensity and acoustic velocity power spectrum. These alternative estimators have been proven to provide a more accurate estimation in acoustic nearfields which possess more than one statistical degree of freedom. A further advantage of the alternative estimators is that they nominally will underestimate the acoustic velocity. Use of these estimators in conjunction with the original velocity estimator will thus provide an approximate upper and lower bound to the velocity estimate. An additional advantage of these estimators is the potential for use in non-zero mean flow situations where the microphone self-induced turbulence is significant. The advantages of such alternative estimators were identified both analytically and experimentally.

The second major contribution was in the area of error analysis for various estimators; this included a wide range of error sources which were studied. Many of the considerations of this study are original contributions. Primary among these is the evaluation of finity errors in the estimation of surface averaged parameters. This evaluation provided a detailed analysis of the differences between a discrete surface sampling approach and a swept probe approach. There has been considerable disagreement in the acoustic intensity field over the relative advantages of these two approaches, however, no literature is available which thoroughly investigates the errors in the two methods. This study resulted in identifying that the
swept probe approach is an unqualified superior method for determination of averaged surface properties. Furthermore, the analysis identified maximum and minimum probe sweep velocities in order to control both frequency shift errors and the variance in the estimated quantity.

The error analysis also included a study of the finite difference errors for a range of ideal acoustic sources. Previous reports by Thompson and Tree [20] provided the error results for resistive intensity for an ideal monopole, dipole, and lateral quadrupoles, they concluded that probe standoff distances should be determined by assuming the source to be quadrupole in nature. But the results of this study show that the finite difference errors in the very nearfield of the quadrupole are more severe than those in the nearfield of more realistic sources such as plates and pistons. From this, a realistic evaluation of finite difference errors was provided. These results were extended to include the reactive intensity, and the acoustic velocity power spectrum. The study of finite difference errors on the axis of a piston radiator was a very useful, and original contribution. This example contains the plane wave case in one limit, and the point monopole source in another. This provided a very clear evaluation of the transition from one to another, thus providing a much clearer understanding of the type of errors to expect in the very nearfield of a real radiator.

A further accomplishment in the error analysis was a detailed study of microphone scattering induced errors. Existing literature in this field has been limited to experimental measurements of the scattering error on actual sized microphones. In this method, the measurement errors have commonly been of equal magnitude as the scattering effect being measured. An improved approach using measurements on scaled up models of microphones has been implemented which provides more accurate experimental data. Furthermore, an analytical approach has been developed which provided excellent correlation to the experimental results. This provides a method of predicting the effect of microphone scattering, and also provides a means of checking the accuracy of the experimental results.

The analysis of errors in extrapolating the measured acoustic velocity to the radiating surface provided guidelines as to the limitations in using an acoustic intensity probe for the direct measurement of acoustic radiation efficiency. This result is directly applicable to measurement of material acoustic impedance. This study identified the limits beyond which a multi-microphone probe may not be used to accurately measure radiation efficiency, or surface impedance. It was shown that, for many common types of radiators, the error in surface velocity estimation could be unbearably large.

The error analysis performed on the random error term was extended to the acoustic velocity estimator. Previous publications have only provided this analysis for the resistive and reactive intensity.
Finally, the work presented herein identifies the various considerations in construction of a probe for measurement of intensity and acoustic velocity in a three-dimensional field. These results show the advantages of various probe constructions, and furthermore suggests approaches to implementing the estimators in order to minimize various errors.

5.3 Conclusions

This study has provided valuable results regarding proper procedures for the accurate use of multi-microphone probes for intensity and velocity estimation. The study has shown that in order to perform accurate measurements the following criteria must be satisfied:

1. The finite difference errors normally determine the usable upper frequency range for intensity measurement. In the farfield regions this error is determined by consideration of a plane acoustic wave. In this region a probe should not be used above the frequency range as determined in Figure 3–4. For use in the nearfield of a radiator, the analysis results from an infinite plate and baffled piston should be used to determine the possible errors. These results are summarized in Figures 3–11 through 3–25. These results indicate the possibility of estimating intensity levels to within 1 dB accuracy. However, under very complex nearfield patterns local measurement errors may exceed this.

2. The accuracy of surface velocity and radiation impedance measurements are largely determined by evanescent field effects. The errors for various types of radiators are summarized in Figure 3–49. This figure should be used to determine how close the microphone probe must be placed to the surface in order to provide sufficient estimation accuracy.

3. The use of a swept probe approach is preferred over fixed spatial sampling for the estimation of surface averaged properties. The limits on probe sweep velocity may be determined by referring to Figures 3–51 and 3–52. The required sampling time is determined by Equation (3.108).

4. The major error source differentiating the accuracy of the various velocity estimators is the sensitivity to bias errors from extraneous signal noise. This consideration showed the $G_{uu_{||}}$ estimator to be a superior estimator to $G_{uu_{\perp}}$. However, the $G_{uu_{\perp}}$ estimator was shown to be superior in the presence of multiple independent acoustic sources. Thus, it is recommended that both estimators be used to provide an upper and lower bound to the velocity estimate.
5.4 Recommendations for Further Work

The largest single deficiency in the use of an intensity/energy density probe, is in the extrapolation of radiator surface velocities from measured nearfield acoustic velocities. It was shown that the presence of an evanescent field can cause very large errors in this extrapolation. The ability to use a probe for such a measurement would greatly simplify many acoustic measurements and be invaluable to the experimentalist. One possible means of reducing this error would be to use a back propagation method, as discussed by Williams, Maynard, and Skudrzyk [55]. In a modified back propagation method, both the acoustic pressure and velocity over a planar grid could be used. This would appear to provide the potential for greatly reducing the projection error. In this approach a form of least squares estimation could be used by evaluating the inconsistencies between the pressure and the velocity back propagation results. The intensity probe could be used to provide the pressure and velocity data for such computations.

The consideration of complicating effects should be investigated. Included here are the effects of mean flows, the presence of acoustic dissipation, and the effect of finite acoustic amplitudes. Munro and Ingard [34], and Comparin et al [18] have provided preliminary discussion of intensity estimation in mean flow situations. However, no mention was made of the acoustic velocity estimation. Furthermore, no consideration has been found in the literature regarding the presence of acoustic dissipation, such as may result from the viscous shear layer in flow over a plate. High amplitude pulsations may be expected in the very nearfield of intense sources. These finite amplitude effects should be studied further.

Finally, it is recommended that experimental studies be performed to verify the theoretical error analyses of this work. The results regarding upper and lower limits on probe sweep velocity are of special interest here.
LIST OF REFERENCES


LIST OF REFERENCES (Continued)


LIST OF REFERENCES (Continued)


LIST OF REFERENCES (Continued)


APPENDIX — DERIVATION OF RESISTIVE AND REACTIVE SPECTRAL ESTIMATORS

This derivation will follow along the general lines of that presented by Pascal [13] and Lahti [32]. We start out with equations (2.21) and (2.22) repeated here:

\[ I(\omega) = \frac{1}{2} \text{Re} \left[ \tilde{p}(\omega) \cdot \tilde{u}^*(\omega) \right] \] (A.1)

\[ J(\omega) = \frac{1}{2} \text{Im} \left[ \tilde{p}(\omega) \cdot \tilde{u}^*(\omega) \right] \] (A.2)

These equations are expressed using the complex algebra. A more useful form is to express them in terms of a single sided cross power spectrum as follows:

\[ \tilde{\Pi}(\omega) = I(\omega) + iJ(\omega) = G_{up}(\omega) \] (A.3)

The pressure at the field point is estimated as the average of the pressure at microphone locations 1 and 2. We thus have:

\[ p(t) = \frac{1}{2} \left[ p_1(t) + p_2(t) \right] \] (A.4)

By taking the Fourier Transform of both sides of equation (A.4) we have:

\[ P(\omega) = \frac{1}{2} \left[ P_1(\omega) + P_2(\omega) \right] \] (A.5)

The acoustic velocity is estimated by starting with the acoustic momentum equation:

\[ \frac{\partial u_r(t)}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p(t)}{\partial r} \] (A.6)

The spatial derivative of the pressure is obtained by a central finite difference using the pressures at the two microphone locations;
\[
\frac{\partial u_i(t)}{\partial t} = - \frac{1}{\rho_0} \left[ \frac{p_2(t) - p_1(t)}{\Delta r} \right]
\]  
(A.7)

By taking the Fourier Transform of both sides of equation (A.7) we have;

\[
i \omega U(\omega) = \frac{1}{\rho_0} \left[ \frac{P_2(\omega) - P_1(\omega)}{\Delta r} \right]
\]  
(A.8)

Or

\[
U(\omega) = \frac{i}{(\rho_0 \omega \Delta r)} \left[ P_2(\omega) - P_1(\omega) \right]
\]  
(A.9)

We now use the definition of \(G_{up}\):

\[
G_{up}(\omega) = \lim_{T \to \infty} \frac{2}{T} \mathbb{E} \left[ U^*(\omega) P(\omega) \right]
\]  
(A.10)

By using equations (A.3), (A.5), (A.9), and (A.10) we obtain;

\[
\tilde{\Pi}(\omega) = \lim_{T \to \infty} \frac{2}{T} \mathbb{E} \left[ \frac{-i}{(\rho_0 \omega \Delta r)} \left( P_2^*(\omega) - P_1^*(\omega) \right) \left( \frac{1}{2} \left| P_1(\omega) + P_2(\omega) \right| \right) \right]
\]  
(A.11)

This is simplified to;

\[
\tilde{\Pi}(\omega) = \frac{-i}{2 \rho_0 \omega \Delta r} \lim_{T \to \infty} \frac{2}{T} \mathbb{E} \left[ P_2^*(\omega) P_1(\omega) - P_1^*(\omega) P_2(\omega) + P_2^*(\omega) P_2(\omega) - P_1^*(\omega) P_1(\omega) \right]
\]  
(A.12)

\[
\tilde{\Pi}(\omega) = \frac{-i}{2(\rho_0 \omega \Delta r)} \left[ G_{21} - G_{11} + G_{22} - G_{12} \right]
\]  
(A.13)

From this we obtain;

\[
I(\omega) = \frac{-1}{2\rho_0 \omega \Delta r} \left[ \text{Re} \left( i G_{21} \right) - \text{Re} \left( i G_{11} \right) + \text{Re} \left( i G_{22} \right) - \text{Re} \left( i G_{12} \right) \right]
\]  
(A.14)
\[ I(\omega) = -\frac{1}{2 \rho_0 \omega \Delta r} \left[ -\text{Im}(G_{12}^*) + \text{Im}(G_{12}) \right] \]

(A.15)

Finally:

\[ I(\omega) = \frac{-\text{Im}(G_{12})}{\rho_0 \omega \Delta r} \]

(A.16)

And the reactive intensity is:

\[ J(\omega) = \frac{-1}{2 \rho_0 \omega \Delta r} \left[ \text{Im}(i G_{21}) - \text{Im}(i G_{11}) + \text{Im}(i G_{22}) - \text{Im}(i G_{12}) \right] \]

(A.17)

\[ J(\omega) = \frac{-1}{2 \rho_0 \omega \Delta r} \left[ -\text{Re}(G_{12}^*) - \text{Re}(G_{11}) + \text{Re}(G_{22}) - \text{Re}(G_{12}) \right] \]

(A.18)

Which is finally reduced to:

\[ J(\omega) = \frac{(G_{11} - G_{22})}{2 \rho_0 \omega \Delta r} \]

(A.19)