AN EXPERIMENTAL STUDY OF PLANAR MODELS FOR HUMAN GAIT UTILIZING ON-LINE COMPUTER ANALYSIS OF TELEVISION AND FORCE PLATE DATA

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

by

Shahram Rahmani, M.S.E.E.

* * * * *

The Ohio State University

June, 1979

Reading Committee: Approved By
Prof. C. N. Burnett
Prof. S. H. Koozekanani
Prof. R. B. McGhee
Prof. F. C. Weimer

Frank Carlin Weimer
Adviser
Department of Electrical Engineering
To Homa

and

My Parents
ACKNOWLEDGMENTS

I would like to acknowledge with gratitude, the advice, assistance and encouragement which I received from my advisor, Professor F. C. Weimer. I am in particular grateful to Professors R. B. McGhee and S. H. Koozakanani not only for their guidance and support during my studies at Ohio State, but also for their enthusiastic supervision and cooperation during development of this dissertation. I am also grateful to Prof. C. Burnett for her encouragement and review of this work.

My sincere appreciation is due to my close friend, Mr. Hoover Chen for his help and for providing stimulating discussions and a pleasant working atmosphere. I would also like to thank Professors H. Hemami and K. Majidzadeh for their advice and support during by graduate work.

I am grateful to Mrs. Joanne Bilbrey for her excellent typing and preparation of this dissertation, and Mr. Henry Pagean for his photographic work. My thanks go to Dr. D. S. Shultheis for his programming assistance and Mr. J. Cocumelli for his help in collecting some of the data used in this dissertation.

I wish to thank my parents and all members of my family for their support and confidence in me. A special expression of my thanks to my wife, Homa, for her patience with me, encouragement, and drafting of this work.
This research was supported by the National Science Foundation under Grant No. ENG-7818957.
VITA

May 9, 1950 . . . . Born - Sanandaj, Iran
1973 . . . . . . . . M.S., Electrical Engineering,
University of Tehran, Tehran, Iran
1974 . . . . . . . . Graduate Teaching Associate, Mathematics
Department, The Ohio State University,
Columbus, Ohio
1975 . . . . . . . . Graduate Research Associate, Civil
Engineering Department, The Ohio State
University, Columbus, Ohio
1976 - 1979 . . . . Graduate Research Associate, Digital
Systems Laboratory, and Graduate
Teaching Associate, Department of
Electrical Engineering, The Ohio State
University, Columbus, Ohio

PUBLICATIONS

Control Conference, Denver, Colorado, June 16-21, 1979, (with

FIELDS OF STUDY

Major Field: Electrical Engineering

and K. J. Breeding.

and R. E. Fenton.

Studies in Computer and Information Science. Professors H. H. Mei
and D. S. Kerr.

Studies in Statistics. Professors K. R. Eberhardt and
M. A. Fligner.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEDICATION</td>
<td>ii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iii</td>
</tr>
<tr>
<td>VITA</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xii</td>
</tr>
</tbody>
</table>

## Chapter

1. **INTRODUCTION**                                                  | 1 |
   1.1 General Background                                           | 1 |
   1.2 Applications of Research in Human Locomotion                | 2 |
   1.3 Organization of the Dissertation                             | 4 |

2. **SURVEY OF PREVIOUS WORK**                                       | 7 |
   2.1 Introduction                                                  | 7 |
   2.2 General Theory of Locomotion                                 | 7 |
   2.3 Studies of Body Center of Gravity                            | 8 |
   2.4 Body Segment Parameters                                      | 9 |
   2.5 Kinematics of Gait                                            | 10 |
   2.6 Gait Kinetics                                                 | 13 |
   2.7 Mathematical Models and Simulation of Locomotion             | 17 |
   2.8 Summary                                                       | 19 |

3. **A COMPARISON AND EVALUATION OF ALTERNATIVE FIVE-MASS DYNAMIC MODELS FOR GAIT UTILIZING ONLY TELEVISION DATA** | 21 |
   3.1 Introduction                                                  | 21 |
   3.2 Basic Five-Mass Model                                         | 22 |
   3.3 Mathematical Formulations of Five-Mass Dynamic Models        | 27 |
   3.4 Reference Systems for Estimation of Translational Accelerations | 35 |
   3.5 Sensitivity Force Estimates to Body Segment Parameter Values | 69 |
   3.6 Summary                                                       | 72 |
TABLE OF CONTENTS (Contd)

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. A SEVEN-MASS DYNAMIC MODEL USING ONLY TELEVISION DATA</td>
<td>75</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>75</td>
</tr>
<tr>
<td>4.2 Description of the Model</td>
<td>76</td>
</tr>
<tr>
<td>4.3 Mathematical Formulation and Comparison with Direct Force Plate Measurement</td>
<td>80</td>
</tr>
<tr>
<td>4.3.1 Mathematical Modeling</td>
<td>80</td>
</tr>
<tr>
<td>4.3.2 Comparison with Direct Force Plate Measurements</td>
<td>105</td>
</tr>
<tr>
<td>4.4 Summary</td>
<td>113</td>
</tr>
<tr>
<td>5. A THREE-MASS DYNAMIC MODEL USING BOTH TELEVISION AND FORCE PLATE DATA</td>
<td>116</td>
</tr>
<tr>
<td>5.1 Introduction</td>
<td>116</td>
</tr>
<tr>
<td>5.2 Description of the Model</td>
<td>117</td>
</tr>
<tr>
<td>5.3 Mathematical Formulation and Typical Results</td>
<td>119</td>
</tr>
<tr>
<td>5.3.1 Mathematical Formulation</td>
<td>119</td>
</tr>
<tr>
<td>5.3.2 Typical Results</td>
<td>132</td>
</tr>
<tr>
<td>5.4 Summary</td>
<td>137</td>
</tr>
<tr>
<td>6. MEASUREMENT TECHNIQUES</td>
<td>139</td>
</tr>
<tr>
<td>6.1 Introduction</td>
<td>139</td>
</tr>
<tr>
<td>6.2 Description of Gait Laboratory Equipment</td>
<td>140</td>
</tr>
<tr>
<td>6.2.1 Gait Laboratory Equipment</td>
<td>140</td>
</tr>
<tr>
<td>6.2.2 Hardware/Software Development</td>
<td>151</td>
</tr>
<tr>
<td>6.3 Statistical Characterization of Signal and Noise</td>
<td>160</td>
</tr>
<tr>
<td>6.3.1 Television Data</td>
<td>161</td>
</tr>
<tr>
<td>6.3.2 Force Plate Data</td>
<td>170</td>
</tr>
<tr>
<td>6.4 Suboptimal and Optimal Filtering</td>
<td>173</td>
</tr>
<tr>
<td>6.4.1 Suboptimal Filtering</td>
<td>174</td>
</tr>
<tr>
<td>6.4.2 Optimal Filtering</td>
<td>178</td>
</tr>
<tr>
<td>6.5 Summary</td>
<td>190</td>
</tr>
<tr>
<td>7. EXPERIMENTAL RESULTS</td>
<td>194</td>
</tr>
<tr>
<td>7.1 Introduction</td>
<td>194</td>
</tr>
<tr>
<td>7.2 Kinematic Results</td>
<td>195</td>
</tr>
<tr>
<td>7.3 Kinetic Results</td>
<td>232</td>
</tr>
<tr>
<td>7.3.1 Five-Mass Model Results</td>
<td>236</td>
</tr>
<tr>
<td>7.3.2 Seven-Mass Model Results</td>
<td>248</td>
</tr>
<tr>
<td>7.3.3 Three-Mass Model Results</td>
<td>264</td>
</tr>
<tr>
<td>7.4 Comparison of Five-Mass and Seven-Mass Model</td>
<td>282</td>
</tr>
<tr>
<td>7.5 An Application to Experimental Gait Studies</td>
<td>290</td>
</tr>
<tr>
<td>7.6 Summary</td>
<td>301</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS (Contd)

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8. SUMMARY AND CONCLUSIONS</td>
<td>303</td>
</tr>
<tr>
<td>8.1 Summary</td>
<td>303</td>
</tr>
<tr>
<td>8.2 Recommendations for Further Work</td>
<td>306</td>
</tr>
<tr>
<td>8.3 Conclusions</td>
<td>309</td>
</tr>
</tbody>
</table>

**APPENDICES**

| A. COMPUTER PROGRAM FOR DATA ACQUISITION | 314 |
| B. COMPUTER PROGRAMS FOR DATA PROCESSING | 318 |
| C. COMPUTER PROGRAM FOR FIVE-MASS DYNAMIC MODEL | 340 |
| D. COMPUTER PROGRAM FOR SEVEN-MASS DYNAMIC MODEL | 348 |
| E. COMPUTER PROGRAM FOR THREE-MASS DYNAMIC MODEL | 362 |
| F. PLOTTING Routines | 375 |

**REFERENCES** | 379 |


LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1. Symbols Used for Basic Five-Mass Dynamic Model</td>
<td>28</td>
</tr>
<tr>
<td>3.2. Mean Square Error Between Computed Horizontal and Vertical Components of Ground Force Reaction and Direct Force Plate Measurement for Various Number of Harmonics Employed to Smooth Angles ($n$), Angular Rates ($n_1$) and Angular Accelerations ($n_2$) for the Method of Approach #2</td>
<td>47</td>
</tr>
<tr>
<td>3.3 Mean Square Error Between Computed Horizontal and Vertical Components of Ground Force Reaction and Direct Force Plate Measurements for Various Number of Harmonics Employed to Smooth Angles ($n$), Angular Rates ($n_1$) and Angular Accelerations ($n_2$) for the Method of Approach #3</td>
<td>51</td>
</tr>
<tr>
<td>3.4 Mean Square Error Between Computed Horizontal and Vertical Components of Ground Force Reaction and Direct Force Plate Measurements for Various Number of Harmonics Employed to Smooth Angles ($n$), Angular Rates ($n_1$) and Angular Accelerations ($n_2$) for the Method of Approach #4</td>
<td>54</td>
</tr>
<tr>
<td>3.5 Means Square Error Between Computed Horizontal and Vertical Components of Ground Force Reaction and Direct Force Plate Measurements for Various Number of Harmonics Employed to Smooth Angles ($n$), Angular Rates ($n_1$) and Angular Accelerations ($n_2$) for the Method of Approach #5 with Massless Feet Included</td>
<td>62</td>
</tr>
<tr>
<td>3.6 Mean Square Error Between Computed Horizontal and Vertical Components of Ground Force Reaction and Direct Force Plate Measurements for Various Number of Harmonics Employed to Smooth Angles ($n$), Angular Rates ($n_1$) and Angular Accelerations ($n_2$) for the Method of Approach #5 with No Feet</td>
<td>62</td>
</tr>
<tr>
<td>3.7 Mean Square Error Between Computed Horizontal and Vertical Components of Ground Force Reaction and Direct Force Plate Measurements for Various Number of Harmonics Employed to Smooth Angles ($n$), Angular Rates ($n_1$) and Angular Accelerations ($n_2$) for the Method of Approach #6</td>
<td>68</td>
</tr>
<tr>
<td>Table</td>
<td>Title</td>
</tr>
<tr>
<td>-------</td>
<td>-----------------------------------------------------------------------</td>
</tr>
<tr>
<td>3.8</td>
<td>Effects of Variation of d Parameters on Mean Square Error</td>
</tr>
<tr>
<td></td>
<td>Between Computed Components of the Ground Reaction Force and Direct</td>
</tr>
<tr>
<td></td>
<td>Force Plate Measurements</td>
</tr>
<tr>
<td>3.9</td>
<td>Effects of Variation of f Parameters on Mean Square Error</td>
</tr>
<tr>
<td></td>
<td>Between Computed Components of the Ground Reaction Force and Direct</td>
</tr>
<tr>
<td></td>
<td>Force Plate Measurements</td>
</tr>
<tr>
<td>3.10</td>
<td>Effects of Variation of m Parameters on Mean Square Error</td>
</tr>
<tr>
<td></td>
<td>Between Computed Components of the Ground Reaction Force and Direct</td>
</tr>
<tr>
<td></td>
<td>Force Plate Measurements</td>
</tr>
<tr>
<td>4.1</td>
<td>Table of Symbols Used in the Seven-Mass Model</td>
</tr>
<tr>
<td>4.2</td>
<td>Switching Assumption of Center of Pressure for One Gait Period</td>
</tr>
<tr>
<td>4.3</td>
<td>Numerical Values of Left Leg Partition Coefficients During First DSP</td>
</tr>
<tr>
<td></td>
<td>of the Gait Cycle Obtained by Linear Force Transfer</td>
</tr>
<tr>
<td></td>
<td>Hypothesis and by Direct Computation Using Force Plate Measurement</td>
</tr>
<tr>
<td>4.4</td>
<td>Numerical Values of Left Leg Partition Coefficients During Second DSP</td>
</tr>
<tr>
<td></td>
<td>of the Gait Cycle Obtained by Linear Force Transfer</td>
</tr>
<tr>
<td></td>
<td>Hypothesis and by Direct Computation Using Force Plate Measurement</td>
</tr>
<tr>
<td>5.1</td>
<td>Table of Symbols Used in Method I</td>
</tr>
<tr>
<td>5.2</td>
<td>Table of Symbols Used in Method II</td>
</tr>
<tr>
<td>6.1</td>
<td>Outputs from Force Plate</td>
</tr>
<tr>
<td>6.2</td>
<td>Standard Deviation and Expected Value of the Force Plate Measurements</td>
</tr>
<tr>
<td>6.3</td>
<td>Power Spectra for Noise and Input with Optimal Gain Values for Ankle</td>
</tr>
<tr>
<td></td>
<td>Angle Using Different Harmonics</td>
</tr>
<tr>
<td>6.4</td>
<td>Power Spectra for Noise and Input with Optimal Gain Values for Knee</td>
</tr>
<tr>
<td></td>
<td>Angle Using Different Harmonics</td>
</tr>
<tr>
<td>6.5</td>
<td>Power Spectra for Noise and Input with Optimal Gain Values for Hip</td>
</tr>
<tr>
<td></td>
<td>Angle Using Different Harmonics</td>
</tr>
<tr>
<td>7.1</td>
<td>Gait Parameters for the Male Subject</td>
</tr>
<tr>
<td>7.2</td>
<td>Gait Parameters for the Female Subject</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3</td>
<td>Physical Data About Subject of Five-Mass Model</td>
<td>236</td>
</tr>
<tr>
<td>7.4</td>
<td>Physical Data About the Subject of Seven-Mass Model</td>
<td>254</td>
</tr>
<tr>
<td>7.5</td>
<td>Physical Data About the Subject of Three-Mass Model</td>
<td>282</td>
</tr>
<tr>
<td>7.6</td>
<td>Horizontal Force Parameters Averaged on Ten Trials for Six Subjects Each with Three Different Foot-Wears</td>
<td>295</td>
</tr>
<tr>
<td>7.7</td>
<td>Vertical Force Parameters Averaged on Ten Trials for Six Subjects Each with Three Different Foot-Wears</td>
<td>296</td>
</tr>
<tr>
<td>7.8</td>
<td>Ankle and Knee Torques Averaged on Ten Trials for Six Subjects Each with Three Different Foot-Wears</td>
<td>299</td>
</tr>
<tr>
<td>7.9</td>
<td>Hip Torque Averaged on Ten Trails for Six Subjects Each with Three Different Foot-Wears</td>
<td>300</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Time Parameters of the Gait Cycle</td>
<td>23</td>
</tr>
<tr>
<td>3.2</td>
<td>Stick Figure Representation of Basic Five-Mass Model for Gait</td>
<td>24</td>
</tr>
<tr>
<td>3.3</td>
<td>Free Body Diagram for Upper Body</td>
<td>29</td>
</tr>
<tr>
<td>3.4</td>
<td>Free Body Diagram for Left Thigh</td>
<td>30</td>
</tr>
<tr>
<td>3.5</td>
<td>Free Body Diagram for Right Thigh</td>
<td>30</td>
</tr>
<tr>
<td>3.6</td>
<td>Free Body Diagram for Left Shank</td>
<td>31</td>
</tr>
<tr>
<td>3.7</td>
<td>Free Body Diagram for Right Shank</td>
<td>31</td>
</tr>
<tr>
<td>3.8</td>
<td>Matrix A for Force and Torque Estimation for Five-Mass Dynamic Model for Gait</td>
<td>36</td>
</tr>
<tr>
<td>3.9</td>
<td>Vector B for Force and Torque Estimation for Five-Mass Dynamic Model for Gait</td>
<td>37</td>
</tr>
<tr>
<td>3.10</td>
<td>Horizontal Ground Force Reactions Measured by Force Plate (*) and Calculated from Basic Five-Mass Dynamic Model</td>
<td>41</td>
</tr>
<tr>
<td>3.11</td>
<td>Vertical Ground Force Reactions Measured by Force Plate (*) and Calculated from Basic Five-Mass Dynamic Model</td>
<td>42</td>
</tr>
<tr>
<td>3.12</td>
<td>Foot Motion Configuration During the Stance Phase: (a) Toe Off, (b) Foot Flat, and (c) Heel Strike</td>
<td>48</td>
</tr>
<tr>
<td>3.13</td>
<td>Three-Link Model of a Leg Employed to Illustrate the Effect of Adding Massless Feet to the Model for Approach #4</td>
<td>52</td>
</tr>
<tr>
<td>3.14</td>
<td>Stick Figure Representation of Five-Mass Dynamic Model With Massless Pelvis Included</td>
<td>55</td>
</tr>
<tr>
<td>3.15</td>
<td>Free Body Diagram of Pelvis</td>
<td>60</td>
</tr>
<tr>
<td>3.16</td>
<td>Horizontal Ground Force Reactions Measured by Force Plate (*) and Calculated from Five-Mass Dynamic Model of Approach #6(+)</td>
<td>66</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.17</td>
<td>Vertical Ground Reactions Measured by Force Plate (*) and Calculated from Five-Mass Dynamic Model of Approach Approach #6</td>
<td>67</td>
</tr>
<tr>
<td>4.1</td>
<td>Stick Figure of the Seven-Mass Model</td>
<td>77</td>
</tr>
<tr>
<td>4.2</td>
<td>Alternative Models for the Foot: (a) Straight-Line Segments, (b) Triangular Configuration with Implicit Toes, (c) Triangular Configurations with Explicit Toes</td>
<td>79</td>
</tr>
<tr>
<td>4.3</td>
<td>Free Body Diagram for the Upper Body</td>
<td>83</td>
</tr>
<tr>
<td>4.4</td>
<td>Free Body Diagram for Left Thigh</td>
<td>83</td>
</tr>
<tr>
<td>4.5</td>
<td>Free Body Diagram for Right Thigh</td>
<td>83</td>
</tr>
<tr>
<td>4.6</td>
<td>Free Body Diagram for Left Shank</td>
<td>84</td>
</tr>
<tr>
<td>4.7</td>
<td>Free Body Diagram for Right Shank</td>
<td>84</td>
</tr>
<tr>
<td>4.8</td>
<td>Free Body Diagram for Left Foot</td>
<td>84</td>
</tr>
<tr>
<td>4.9</td>
<td>Free Body Diagram for Right Foot</td>
<td>84</td>
</tr>
<tr>
<td>4.10</td>
<td>Position of the Center of Gravity for a Uniform Triangle</td>
<td>85</td>
</tr>
<tr>
<td>4.11</td>
<td>Left Foot Configuration During Stance Phase: (a) Toe-Off, (b) Heel-Strike</td>
<td>90</td>
</tr>
<tr>
<td>4.12</td>
<td>Right Foot Configuration During Stance Phase: (a) Toe-Off, (b) Heel-Strike</td>
<td>92</td>
</tr>
<tr>
<td>4.13</td>
<td>Free Body Diagram for a Massless Pelvis</td>
<td>92</td>
</tr>
<tr>
<td>4.14</td>
<td>Assumed Numerical Values for $CL_1$ and $CR_1$ During Gait Cycle</td>
<td>97</td>
</tr>
<tr>
<td>4.15</td>
<td>Horizontal Ground Force Reaction Associated with Left Leg During One Gait Cycle with Direct Force Plate Measurement (*) and Computed One From Seven-Mass Model Under Linear Assumption for $q_x(\cdot)$</td>
<td>107</td>
</tr>
<tr>
<td>4.16</td>
<td>Vertical Ground Force Reaction Associated with Left Leg During One Gait Cycle with Direct Force Plate Measurement (*) and Computed One From Seven-Mass Model under Linear Assumption for $q_y(\cdot)$</td>
<td>108</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES (Contd)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.17</td>
<td>Position of Center of Pressure Measured by Force Plate (*) and Calculated from Seven-Mass Dynamic Model (+)</td>
<td>114</td>
</tr>
<tr>
<td>5.1</td>
<td>Three-Mass Model of a Single Leg</td>
<td>118</td>
</tr>
<tr>
<td>5.2</td>
<td>Three-Link Model for the Force Plate Method (Adopted from [2])</td>
<td>121</td>
</tr>
<tr>
<td>5.3</td>
<td>Free Body Diagram of Thigh (Method I)</td>
<td>123</td>
</tr>
<tr>
<td>5.4</td>
<td>Free Body Diagram of Shank (Method I)</td>
<td>123</td>
</tr>
<tr>
<td>5.5</td>
<td>Free Body Diagram of Foot (Method I)</td>
<td>123</td>
</tr>
<tr>
<td>5.6</td>
<td>Free Body Diagram of Thigh (Method II)</td>
<td>128</td>
</tr>
<tr>
<td>5.7</td>
<td>Free Body Diagram of Shank (Method II)</td>
<td>128</td>
</tr>
<tr>
<td>5.8</td>
<td>Free Body Diagram of Foot (Method II)</td>
<td>128</td>
</tr>
<tr>
<td>5.9</td>
<td>Vertical Components of Joint Forces During One Gait Cycle; (+) Ground Reaction, (+) Ankle, (n) Knee, ($) Hip</td>
<td>133</td>
</tr>
<tr>
<td>5.10</td>
<td>Horizontal Components of Joint Forces During One Gait Cycle; (+) Ground Reaction, (+) Ankle, (n) Knee, ($) Hip</td>
<td>134</td>
</tr>
<tr>
<td>5.11</td>
<td>Joint Torques During One Gait Cycle; (+) Ankle, (+) Knee, (n) Hip</td>
<td>136</td>
</tr>
<tr>
<td>6.1</td>
<td>A General Illustration of Force Plate, Television Camera, and Lights During Data Acquisition in Gait Laboratory</td>
<td>141</td>
</tr>
<tr>
<td>6.2</td>
<td>Force Plate with Its Associated Quantities</td>
<td>143</td>
</tr>
<tr>
<td>6.3</td>
<td>Block Diagram of the Camera-Computer Interface System (Adopted from [55])</td>
<td>154</td>
</tr>
<tr>
<td>6.4</td>
<td>Software Flow Chart for Force Plate and Television Data Acquisition</td>
<td>159</td>
</tr>
<tr>
<td>6.5</td>
<td>Power Spectral Density for Ankle Angle vs. Number of Harmonics</td>
<td>167</td>
</tr>
<tr>
<td>6.6</td>
<td>Power Spectral Density for Knee Angle vs. Number of Harmonics</td>
<td>168</td>
</tr>
<tr>
<td>6.7</td>
<td>Power Spectral Density for Hip Angle vs. Number of Harmonics</td>
<td>169</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES (Contd)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.8</td>
<td>Power Spectral Density of Noise Associated with Ankle Angle</td>
<td>171</td>
</tr>
<tr>
<td>6.9</td>
<td>Power Spectral Density of Noise Associated with Knee Angle</td>
<td>172</td>
</tr>
<tr>
<td>6.10</td>
<td>Smoothed Ankle Angle Using Three Harmonics</td>
<td>175</td>
</tr>
<tr>
<td>6.11</td>
<td>Smoothed Ankle Angle Using Six Harmonics</td>
<td>176</td>
</tr>
<tr>
<td>6.12</td>
<td>Smoothed Ankle Angle Using Nine Harmonics</td>
<td>177</td>
</tr>
<tr>
<td>6.13</td>
<td>Smoothed Knee Angle Using Three Harmonics</td>
<td>179</td>
</tr>
<tr>
<td>6.14</td>
<td>Smoothed Knee Angle Using Six Harmonics</td>
<td>180</td>
</tr>
<tr>
<td>6.15</td>
<td>Smoothed Knee Angle Using Nine Harmonics</td>
<td>181</td>
</tr>
<tr>
<td>6.16</td>
<td>Smoothed Hip Angle Using Three Harmonics</td>
<td>182</td>
</tr>
<tr>
<td>6.17</td>
<td>Smoothed Hip Angle Using Six Harmonics</td>
<td>183</td>
</tr>
<tr>
<td>6.18</td>
<td>Smoothed Hip Angle Using Nine Harmonics</td>
<td>184</td>
</tr>
<tr>
<td>6.19</td>
<td>Input-Output Configuration</td>
<td>185</td>
</tr>
<tr>
<td>6.20</td>
<td>Error Component Due to Signal</td>
<td>187</td>
</tr>
<tr>
<td>7.1</td>
<td>Inertial Angle of the Upper Body for One Gait Cycle</td>
<td>196</td>
</tr>
<tr>
<td>7.2</td>
<td>Inertial Angle of Left Thigh for One Gait Cycle</td>
<td>197</td>
</tr>
<tr>
<td>7.3</td>
<td>Inertial Angle of Left Shank for One Gait Cycle</td>
<td>198</td>
</tr>
<tr>
<td>7.4</td>
<td>Inertial Angle of Left Foot for One Gait Cycle</td>
<td>199</td>
</tr>
<tr>
<td>7.5</td>
<td>Angular Rate of Upper Body for One Gait Cycle</td>
<td>201</td>
</tr>
<tr>
<td>7.6</td>
<td>Angular Rate of Left Thigh for One Gait Cycle</td>
<td>202</td>
</tr>
<tr>
<td>7.7</td>
<td>Angular Rate of Left Shank for One Gait Cycle</td>
<td>203</td>
</tr>
<tr>
<td>7.8</td>
<td>Angular Rate of Left Foot for One Gait Cycle</td>
<td>204</td>
</tr>
<tr>
<td>7.9</td>
<td>Angular Acceleration of Upper Body for One Gait Cycle</td>
<td>205</td>
</tr>
<tr>
<td>7.10</td>
<td>Angular Acceleration of Left Thigh for One Gait Cycle</td>
<td>206</td>
</tr>
<tr>
<td>7.11</td>
<td>Angular Acceleration of Left Shank for One Gait Cycle</td>
<td>207</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>7.12</td>
<td>Angular Acceleration of Left Foot for One Gait Cycle</td>
<td>208</td>
</tr>
<tr>
<td>7.13</td>
<td>Three Anatomical Joint Angles</td>
<td>209</td>
</tr>
<tr>
<td>7.14</td>
<td>Average of Normalized Anatomical Ankle Angle for Male Subject with Corresponding Confidence Interval for Barefoot Case</td>
<td>213</td>
</tr>
<tr>
<td>7.15</td>
<td>Average of Normalized Anatomical Ankle Angle for Male Subject with Corresponding Confidence Interval for Positive Heel Shoes</td>
<td>214</td>
</tr>
<tr>
<td>7.16</td>
<td>Average of Normalized Anatomical Ankle Angle for Male Subject with Corresponding Confidence Interval for Negative Heel Shoes</td>
<td>215</td>
</tr>
<tr>
<td>7.17</td>
<td>Average of Normalized Anatomical Knee Angle for Male Subject with Corresponding Confidence Interval for Barefoot Case</td>
<td>216</td>
</tr>
<tr>
<td>7.18</td>
<td>Average of Normalized Anatomical Knee Angle for Male Subject with Corresponding Confidence Interval for Positive Heel Shoes</td>
<td>217</td>
</tr>
<tr>
<td>7.19</td>
<td>Average of Normalized Anatomical Knee Angle for Male Subject with Corresponding Confidence Interval for Negative Heel Shoes</td>
<td>218</td>
</tr>
<tr>
<td>7.20</td>
<td>Average of Normalized Anatomical Hip Angle for Male Subject with Corresponding Confidence Interval for Barefoot Case</td>
<td>219</td>
</tr>
<tr>
<td>7.21</td>
<td>Average of Normalized Anatomical Hip Angle for Male Subject with Corresponding Confidence Interval for Positive Heel Shoes</td>
<td>220</td>
</tr>
<tr>
<td>7.22</td>
<td>Average of Normalized Anatomical Hip Angle for Male Subject with Corresponding Confidence Interval for Negative Heel Shoes</td>
<td>221</td>
</tr>
<tr>
<td>7.23</td>
<td>Average of Normalized Anatomical Ankle Angle for Female Subject with Corresponding Confidence Interval for Barefoot Case</td>
<td>223</td>
</tr>
<tr>
<td>7.24</td>
<td>Average of Normalized Anatomical Ankle Angle for Female Subject with Corresponding Confidence Interval for Positive Heel Case</td>
<td>224</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>7.25</td>
<td>Average of Normalized Anatomical Ankle Angle for Female Subject with</td>
<td>225</td>
</tr>
<tr>
<td></td>
<td>Corresponding Confidence Interval for Negative Heel Shoes</td>
<td></td>
</tr>
<tr>
<td>7.26</td>
<td>Average of Normalized Anatomical Knee Angle for Female Subject with</td>
<td>226</td>
</tr>
<tr>
<td></td>
<td>Corresponding Confidence Interval for Barefoot Case</td>
<td></td>
</tr>
<tr>
<td>7.27</td>
<td>Average of Normalized Anatomical Knee Angle for Female Subject with</td>
<td>227</td>
</tr>
<tr>
<td></td>
<td>Corresponding Confidence Interval for Positive Heel Shoes</td>
<td></td>
</tr>
<tr>
<td>7.28</td>
<td>Average of Normalized Anatomical Knee Angle for Female Subject with</td>
<td>228</td>
</tr>
<tr>
<td></td>
<td>Corresponding Confidence Interval for Negative Heel Shoes</td>
<td></td>
</tr>
<tr>
<td>7.29</td>
<td>Average of Normalized Anatomical Hip Angle for Female Subject with</td>
<td>229</td>
</tr>
<tr>
<td></td>
<td>Corresponding Confidence Interval for Barefoot Case</td>
<td></td>
</tr>
<tr>
<td>7.30</td>
<td>Average of Normalized Anatomical Hip Angle for Female Subject with</td>
<td>230</td>
</tr>
<tr>
<td></td>
<td>Corresponding Confidence Interval for Positive Heel Shoes</td>
<td></td>
</tr>
<tr>
<td>7.31</td>
<td>Average of Normalized Anatomical Hip Angle for Female Subject with</td>
<td>231</td>
</tr>
<tr>
<td></td>
<td>Corresponding Confidence Interval for Negative Heel Shoes</td>
<td></td>
</tr>
<tr>
<td>7.32</td>
<td>Horizontal Ground Force Reaction for One Gait Cycle Measured by Force Plate</td>
<td>233</td>
</tr>
<tr>
<td>7.33</td>
<td>Vertical Ground Force Reaction for One Gait Cycle Measured by Force Plate</td>
<td>234</td>
</tr>
<tr>
<td>7.34</td>
<td>Position of Center of Pressure Measured by Force Plate vs. Percentage of</td>
<td>235</td>
</tr>
<tr>
<td></td>
<td>Gait Cycle</td>
<td></td>
</tr>
<tr>
<td>7.35</td>
<td>Horizontal Ground Force Reaction Associated with Left Leg, (+) Calculated</td>
<td>238</td>
</tr>
<tr>
<td></td>
<td>from Five-Mass Dynamic Model and (*) Force Plate Measurement</td>
<td></td>
</tr>
<tr>
<td>7.36</td>
<td>Horizontal Component of Force at Left Knee Calculated from Five-Mass Dynamic</td>
<td>239</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td></td>
</tr>
<tr>
<td>7.37</td>
<td>Horizontal Component of Force Associated with Trunk Obtained from Five-Mass</td>
<td>240</td>
</tr>
<tr>
<td></td>
<td>Dynamic Model</td>
<td></td>
</tr>
<tr>
<td>7.38</td>
<td>Partition Coefficient of Horizontal Force at Hip Joint</td>
<td>241</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>7.39</td>
<td>Total Horizontal Ground Force Reaction Obtained from Five-Mass Dynamic Model</td>
<td>242</td>
</tr>
<tr>
<td>7.40</td>
<td>Vertical Ground Reaction Forces Obtained from Five-Mass Dynamic Model (+) and Force Plate Measurement (*)</td>
<td>243</td>
</tr>
<tr>
<td>7.41</td>
<td>Vertical Component of Force at Left Knee Obtained from Five-Mass Dynamic Model</td>
<td>244</td>
</tr>
<tr>
<td>7.42</td>
<td>Vertical Component for Force Associated with Trunk Obtained from Five-Mass Dynamic Model</td>
<td>245</td>
</tr>
<tr>
<td>7.43</td>
<td>Partition Coefficient of Vertical Force at Hip Joint</td>
<td>246</td>
</tr>
<tr>
<td>7.44</td>
<td>Total Vertical Ground Force Reaction Obtained from Five-Mass Dynamic Model</td>
<td>247</td>
</tr>
<tr>
<td>7.45</td>
<td>Left Ankle Torque Calculated from Five-Mass Dynamic Model</td>
<td>249</td>
</tr>
<tr>
<td>7.46</td>
<td>Left Knee Torque Obtained from Five-Mass Dynamic Model</td>
<td>250</td>
</tr>
<tr>
<td>7.47</td>
<td>Trunk Torque Obtained from Five-Mass Dynamic Model</td>
<td>251</td>
</tr>
<tr>
<td>7.48</td>
<td>Partition Coefficient of Torque at Hip Joint</td>
<td>252</td>
</tr>
<tr>
<td>7.49</td>
<td>Sum of Ankle Torques Obtained from Five-Mass Dynamic Model</td>
<td>253</td>
</tr>
<tr>
<td>7.50</td>
<td>Horizontal Ground Reaction Force Associated with Left Foot Obtained from Seven-Mass Dynamic Model (+) and Force Plate Measurement (*)</td>
<td>255</td>
</tr>
<tr>
<td>7.51</td>
<td>Horizontal Component of Force at Left Ankle Joint Obtained from Seven-Mass Dynamic Model</td>
<td>256</td>
</tr>
<tr>
<td>7.52</td>
<td>Horizontal Component of Force at Left Knee Joint Obtained from Seven-Mass Dynamic Model</td>
<td>257</td>
</tr>
<tr>
<td>7.53</td>
<td>Horizontal Component of Force at Left Hip Joint Obtained from Seven-Mass Dynamic Model</td>
<td>258</td>
</tr>
<tr>
<td>7.54</td>
<td>Horizontal Component of Force at Right Hip Joint Obtained from Seven-Mass Dynamic Model</td>
<td>259</td>
</tr>
<tr>
<td>7.55</td>
<td>Horizontal Component of Force at Right Knee Joint Obtained from Seven-Mass Dynamic Model</td>
<td>260</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>7.56</td>
<td>Horizontal Component of Force at Right Ankle Joint Obtained from Seven-Mass Dynamic Model</td>
<td>261</td>
</tr>
<tr>
<td>7.57</td>
<td>Horizontal Component of Force Associated with Upper Body</td>
<td>262</td>
</tr>
<tr>
<td>7.58</td>
<td>Total Horizontal Ground Force Reaction Obtained from Seven-Mass Dynamic Model</td>
<td>263</td>
</tr>
<tr>
<td>7.59</td>
<td>Vertical Ground Reaction Force Associated with Left Foot Obtained from Seven-Mass Dynamic Model (+) and Force Plate Measurement (*)</td>
<td>265</td>
</tr>
<tr>
<td>7.60</td>
<td>Vertical Component of Force at Left Ankle Joint Obtained from Seven-Mass Dynamic Model</td>
<td>266</td>
</tr>
<tr>
<td>7.61</td>
<td>Vertical Component of Force at Left Knee Joint Obtained from Seven-Mass Dynamic Model</td>
<td>267</td>
</tr>
<tr>
<td>7.62</td>
<td>Vertical Component of Force at Left Hip Joint Obtained from Seven-Mass Dynamic Model</td>
<td>268</td>
</tr>
<tr>
<td>7.63</td>
<td>Vertical Component of Force at Right Hip Joint Obtained from Seven-Mass Dynamic Model</td>
<td>269</td>
</tr>
<tr>
<td>7.64</td>
<td>Vertical Component of Force at Right Knee Joint Obtained from Seven-Mass Dynamic Model</td>
<td>270</td>
</tr>
<tr>
<td>7.65</td>
<td>Vertical Component of Force at Right Ankle Joint Obtained from Seven-Mass Dynamic Model</td>
<td>271</td>
</tr>
<tr>
<td>7.66</td>
<td>Vertical Component of Force Associated with Upper Body</td>
<td>272</td>
</tr>
<tr>
<td>7.67</td>
<td>Total Vertical Ground Force Reaction Obtained from Seven-Mass Dynamic Model</td>
<td>273</td>
</tr>
<tr>
<td>7.68</td>
<td>Position of Center of Pressure Obtained from Seven-Mass Dynamic Model (+) and Measured by Force Plate</td>
<td>274</td>
</tr>
<tr>
<td>7.69</td>
<td>Left Ankle Joint Torque Calculated from Seven-Mass Dynamic Model</td>
<td>275</td>
</tr>
<tr>
<td>7.70</td>
<td>Left Knee Joint Torque Calculated from Seven-Mass Dynamic Model</td>
<td>276</td>
</tr>
<tr>
<td>7.71</td>
<td>Left Hip Joint Torque Calculated from Seven-Mass Dynamic Model</td>
<td>277</td>
</tr>
<tr>
<td>7.72</td>
<td>Right Hip Joint Torque Calculated from Seven-Mass Dynamic Model</td>
<td>278</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>7.73</td>
<td>Right Knee Joint Torque Calculated from Seven-Mass Dynamic Model</td>
<td>279</td>
</tr>
<tr>
<td>7.74</td>
<td>Right Ankle Joint Torque Calculated from Seven-Mass Dynamic Model</td>
<td>280</td>
</tr>
<tr>
<td>7.75</td>
<td>Torque Associated with Upper Body Calculated from Seven-Mass Dynamic Model</td>
<td>281</td>
</tr>
<tr>
<td>7.76</td>
<td>Horizontal Component of Joint Forces Obtained from Three-Mass Dynamic Model, Force Plate Measurement (+), Ankle (*), Knee (n), and Hip ($)</td>
<td>283</td>
</tr>
<tr>
<td>7.77</td>
<td>Vertical Components of Joint Forces Obtained from Three-Mass Dynamic Model, Force Plate (+), Ankle (*), Knee (n), and Hip ($)</td>
<td>284</td>
</tr>
<tr>
<td>7.78</td>
<td>Joint Torques Obtained from Three-Mass Dynamic Model, Ankle (+), Knee (*), and Hip ($)</td>
<td>285</td>
</tr>
<tr>
<td>7.79</td>
<td>Partition Coefficient Associated with Vertical Ground Reaction Force, q_y, for One Gait Cycle</td>
<td>289</td>
</tr>
<tr>
<td>7.80</td>
<td>Display of Horizontal Components of Joint Forces Obtained from Three-Mass Model for One Gait Cycle</td>
<td>291</td>
</tr>
<tr>
<td>7.81</td>
<td>Display of Vertical Components of Joint Forces Obtained from Three-Mass Model for One Gait Cycle</td>
<td>292</td>
</tr>
<tr>
<td>7.82</td>
<td>Display of Joint Torques Obtained from Three-Mass Model for One Gait Cycle</td>
<td>293</td>
</tr>
</tbody>
</table>
Chapter 1

INTRODUCTION

1.1 General Background

The study of human locomotion has been a concern for quite a long time. With the invention of the camera, an accurate investigation of human gait became possible, and the scientific study of this area was started by Fischer [1] in Germany in the late 19th century. This early work showed the sequence of motions as well as plots of angles of the various body segments with respect to time. The first main aspect of the human locomotion is that of kinematics which deals with the spatial behavior obtained from the observation of human subjects. Some kinematic parameters such as the displacement of several body joints can be recorded versus time. Other kinematic parameters such as the corresponding velocities and accelerations can be computed by successive differentiation of the position data. Similarly, angular data including angles between various body segments, angular velocities and angular accelerations can also be determined.

The second important aspect of human gait is dynamics, which deals with the torques and forces generated to make the walking process possible. It should be noted that, contrary to the kinematic case, the study of the dynamics of the human body involves more complex approaches rather than simple observations. In fact, determination of the internal
forces and torques in a human subject is not an easy task, and only external forces such as ground reaction components can be directly measured. In other words, the study of the dynamics of human gait involves many difficulties due to the fact that the human body is an extremely complex system. One solution to this problem is to consider a simplified model of the human body. This approach makes the analysis of human gait practically possible, and as the behavior of simpler models becomes understood, one can proceed to work with more complex models.

Because of the speed and flexibility of digital computers and modern instrumentation, in comparison with time-consuming photographic methods employing hand calculation, efficient quantitative measurement and analysis of human gait as well as simulation of biped locomotion have become possible. Moreover, mathematical models, as a simplification of the complex human body, can be effectively used together with the kinematic data to obtain dynamic features of human gait [2]. Such biped models are assumed to move similarly to the actual bipeds. Then, the dynamic behavior of this mathematical model can be evaluated to obtain the forces and torques required to generate the motion of the associated model. These dynamic quantities can be considered as estimates of actual internal forces and torques in the human body.

1.2 Applications of Research in Human Locomotion

There are several areas to which research in human locomotion can be applied including physical medicine and physical education. A good example can be found in the field of rehabilitation applied to handicapped persons who may be enabled to walk as closely as possible
to normal people. Another example is clinical evaluation of the vestibular system [3].

The research of this dissertation has been undertaken to make a contribution in the areas of diagnosing abnormal gait and evaluation of corrective measures. The principles explained in this dissertation can be applied to design orthotic or prosthetic devices. The former are assistive devices that can be worn by the handicapped person. Crutches can be considered as orthotic devices which are based on compensating functions. That is, a fault in the function of the body is counteracted by the device to produce the desired motion.

The latter are devices that deal with the problem at its origin. As an example, the loss of activity in a nerve may cause the associated muscle not to function properly. Prosthetic devices can be used to provide the required impulse in the nerve [4]. One possibility is the knee joint prosthesis which causes the knee forces to become generated properly. Obviously, one has to be extremely careful in the design of the above devices. Otherwise, these devices may hurt the patient rather than help him. One of the purposes of this research is to develop a procedure to estimate the forces and torques applied to various joints of the human body.

Studies of human locomotion can also contribute to the study of sports techniques [5] as well as to the development of new athletic training [6,7] and to investigations of various sports shoes [8]. Also, there have been some studies concerning the determination of the forces exerted on various parts of the body while heavy loads are lifted [9]. Furthermore, diagnosis of pathological gait, evaluation of surgical
operations, and gait training and the corresponding treatments are among several applications of human locomotion studies.

Unfortunately, the analysis of human gait has not been used clinically until recently. The main reason is that the model required to represent the human body for gait analysis has a complex structure even if many simplifying assumptions are made. Moreover, it is necessary to have a system with high accuracy as well as speed, both for data acquisition and data analysis and simulation. Since computers have become widely available, several steps have been taken toward an analysis of the human gait. The Gait Laboratory recently established in the Department of Physical Medicine at The Ohio State University is one of the few presently available for both research and clinical applications.

1.3 Organization of the Dissertation

This dissertation is divided into eight chapters. The first chapter introduces the general aspects of human locomotion. It also discusses the objective of this research as well as some of the associated applications. Chapter 2 presents a general survey of previous studies in the area of human locomotion to give a general idea to the reader. One of the main purposes of this dissertation is to concentrate on the dynamic modeling of the human body. Chapter 3 begins with describing a five-mass model similar to that used by Gupta [2]. The model employs only the kinematic data obtained from television systems to determine the corresponding joint forces and torques. However, several modifications such as adding a massless pelvis and feet are introduced to improve the results by using the
actual values determined from a force plate. Also, the sensitivities of the results with respect to various body segment parameters (mass, moment of inertia, etc.) are tested to have a better understanding of the system. Chapter 4 modifies the model used in Chapter 3 by adding massive triangular feet to it. It discusses the dependency of various dynamic quantities upon each other, decomposes the set of equations, and finally presents explicit equations for all joint forces and torques to be determined. It also discusses the validity of the assumptions made about the variation of ground reaction forces during the double support phase of the walking cycle. Chapter 5 describes a dynamic model corresponding to a single leg. An attempt is made to get a simpler form for the dynamic equations previously obtained [10]. This model employs force plate data directly to determine other joint forces and torques.

Chapter 6 deals with the different aspects of the measurement system. All equipment used in the current research is described briefly. The system software together with the algorithm used to collect both force-plate and television data are explained. This chapter also studies the statistical characteristics of the force plate data as well as the data obtained from television. Furthermore, two methods of smoothing the noisy signal are applied to the TV data. One is a suboptimal filter based on a Fourier series approach. The other is a non-realtime optimal filter (so-called nonphysically realizable Wiener Filter).

Chapter 7 deals with the experimental results. Typical kinematic results are presented. Sets of dynamic results associated with each of the
three models are shown, and the models in Chapters 3 and 4 are compared. Also, the validity of some associated assumptions is argued. Several experimental data sets are applied to the model in Chapter 5 to investigate gait differences caused by walking with positive- and negative-heeled shoes and bare feet. Finally, Chapter 8 gives a summary, conclusions, and recommendations for further study in the area of human locomotion. In the appendices, a complete listing of all computer programs for on-line data acquisition, processing, and corresponding analysis is presented.
Chapter 2
SURVEY OF PREVIOUS WORK

2.1 Introduction

Locomotion and gait parameters have been studied for quite a long time. Many researchers have investigated different aspects of locomotion, and their results have been published. In this chapter, an attempt is made to present a brief summary of studies in the above areas. The study involves kinematics, kinetics, measurement of locomotion parameters, modeling and simulation of locomotion, and other related work.

2.2 General Theory of Locomotion

Locomotion studies can be divided in several areas, one of which is robot locomotion. Such machines are either multi-legged or biped. Extensive explanations and reviews of different legged locomotion machines can be found in McGhee [11] and Buckett [12]. Camana [13] built a learning model of biped machine postural control. Kato and Tsuiki [14] constructed two biped walking machines one of which is powered hydraulically and is capable of climbing stairs and carrying fairly heavy objects. However, these machines are rather slow. A second category of study is animal locomotion. Extensive studies dealing with the locomotion of different animals, such as horses or cats have been carried out by many investigators [15]. One
of the earliest studies was done in 1872 by Muybridge [16] who obtained
a fast sequence of photographs by using a series of cameras to investigate
the motion of a galloping horse. Muybridge later used the above tech-
nique to study the motion of many other animals [17] and humans [18].

A third area of study is human locomotion which includes gait. Because of the fact that the research conducted by the author was
concerned only with the human gait, this chapter will concentrate on
the human aspects of locomotion.

2.3 Studies of Body Center of Gravity

There have been studies of the location of the center of gravity
of the body along with other locomotion studies. In fact, the basic
purpose of locomotion can be considered as translation of the center
of gravity of the human body. One of the earliest attempts to determine
the center of gravity in the human body was reported by Borelli [19].
Wilhelm and Weber [20] measured the vertical position of C.G.B. (center
of gravity of the body) by putting the body on a balanced plank and
sliding it to reach equilibrium. Meyer [21] and Reynolds [22] obtained
both the vertical position and also the antero-posterior position of
C.G.E. In 1895, frozen bodies were used by Braune and Fischer [23]
to determine the C.G. of individual parts of the body. The results
showed the C.G.B. varies from 55 to 58 percent of the body height.
There have been studies to determine the vertical oscillations of the
and Coreen [26], in 1942 studied the vertical projection of C.B.G.
and showed that no man can stand absolutely still.
2.4 Body Segment Parameters

In order to study human locomotion and have a model to represent the human body, it is necessary to have some information about the individual parts. Fischer and Braune [23] might be considered the pioneers in this area together with several other researchers who have tackled this problem. Harless in 1858 [27], Meeh in 1884 [28], Zook in 1930 [29], Dempster in 1952 [30], Ivantizkiy in 1956 [31], Salzberger in 1947 [32], and Bernstein in 1930 [33] are among them. Body segment parameters (BSP) include the volume, mass, density, center of gravity, and moment of inertia of various segments. In 1966, Drillis and Contini [34] did a comprehensive study to measure BSP. They used three techniques to determine the volume, including a photographic technique. A mathematical equation was used to compute the density. Once the volume and density are known, mass can be easily calculated. Two other methods were employed to measure the mass [34]. The moment of inertia was computed by a few methods one of which allows the segment to rotate about its joint, which is fixed. The length of each segment was obtained by multiplying the height of the body by a coefficient. The results obtained were not very different from the ones computed by Fischer and Braune [23] who employed a much simpler method. The procedure is based on the assumption that BSP have a fixed relationship with the height and weight of the body. More precisely, there are three coefficients for each segment which can be used to compute the corresponding mass, CG, and moment of inertia, respectively. Contini in 1970 [35] and in 1972 [36] did some further study in determination of BSP using statistical relations which only require some
basic measurements of the body. While Contini and Drillis did not deal with trunk and head, Chandler et al. in 1975 [37] determined the above quantities to give a better knowledge of BSP.

2.5 Kinematics of Gait

Studies of the nature of body motion, such as the horizontal and vertical displacements of various joints and limbs, velocities and accelerations as well as angles between the segments, angular rates and angular accelerations as functions of time, are the concerns of kinematic investigations. Such studies need a measurement system to determine the trajectories in time that give time samples with adequate frequencies. Early studies began with determination of various gait parameters. Between 1873 and 1895 Marey [38,39] made many contributions to gait studies and measurements. He managed to measure footfall and footrise in addition to the vertical pressure of the foot against the ground. The method of chronophotography, in which successive exposures are made on the same plate by interrupting the light, was employed by Braune and Fischer in 1985 [23] to study human gait. Demeny [24], in 1887, used motion pictures to study the vertical motion of the center of gravity. In 1943, Hartley [40] concluded that even high-speed motion pictures were not sufficiently good for gait study and introduced an improved photographic recording method. Finally, in 1945, West [41] described measurement of joint angles using a goniometer. In 1937, Glanville and Kreezer [42] investigated stride length, gait velocity, age, trunk angle, stance width, and support duration. Several other investigations on the gait
parameters and their importance have been published. Lamoreux in 1971 [43] used a film technique in analysis of the gait.

In order to make a general mathematical analysis of gait, McGhee et al. [44,45,46,47] have conducted extensive theoretical studies in which each leg is considered as a binary machine. This work applies to multi-legged locomotion as well as biped locomotion.

Kinematic measurement requires registration of motion in time. Methods available for this purpose can be classified according to the devices used as follows:

1. Cinephotography has high speed and accuracy. Traditionally, it has been used by many people. Sutherland and Hagy in 1972 [48] represents an example of this method. This method, unfortunately, is very time consuming to obtain displacement and angular data and thus is not very practical for clinical applications.

2. Stroboscopic devices employ flashing lights. Murray [49], in 1964, used this method which has disadvantages similar to the previous technique. Milner [50], in 1973, employed stroboscopic polaroid photography for clinical use.

4. Accelerometer devices are used to accomplish acceleration measurements. While the advantage of using this approach is that the accelerations are obtained directly, there are several disadvantages associated with this method including the difficulty of determining the orientation of the accelerometer in space and the calibration of the accelerometer. In 1964, Gate [54] employed this system to study and analyze human gait.

5. Computer-television systems are among the most interesting devices. An interface can be used to connect a television camera to the computer. The use of a computer makes this method very attractive in both research and clinical applications which require a great deal of data processing. Cheng [55], and Winter et al. [56] and Chen [8] have described such systems. These systems have the advantages of being very fast, rather simple and inexpensive, and fairly accurate. The disadvantages of these devices include measurement noise inherent in commercially available TV systems, provision of suitable light sources, and limited resolution. However, they can be efficiently used for gait studies. Such a system [8] has been used by the author for the current research. A brief description of this equipment is provided in Chapter 6 of this dissertation.

6. Opto-electronic systems (Selspot) have recently been developed which are based on a lateral photo effect of light detecting diodes. Light emitting diodes (LED) are attached to the joints of the subjects, and their locations are measured with special cameras. In the focal plane of the lens of each camera, lateral photo-detectors are mounted which generate the electrical signals from which the coordinates of the LED's
are obtained. The Selspot system is commercially available and has the advantages of 1) high resolution, 2) high signal to noise ratio, 3) sampling frequency of up to 300 Hz compared with 60 Hz for computer-TV systems. Gustafsson and Lanshammer, 1977 [10] have employed such a system for the study of gait.

2.6 Gait Kinetics

While gait kinematic studies are concerned with detailed analysis of the relative motions of various limb segments, determination of the joint forces and moments associated with such motion requires more elaborate mathematical models and more complex measurement apparatus. Such investigations are usually referred to as kinetic studies [71].

From 1895 to 1904, one of the pioneers of kinetic studies of human locomotion, Fischer [1], used stroboscopy for recording human motion. Earlier, in 1889, he and Braune [23] had studied body segment parameters as explained in Section 2.4. Fischer's study was limited to the single support case, where he calculated joint forces by using equations of motion. However, he did not have any device to measure the ground-force reaction. Herbert Elftman did a series of investigations in human locomotion which started in 1934 [58] with a study of the distribution of pressure in the human foot. This quantity is very important in order either to calculate the forces or do other kinetic studies. Before this time, several other researchers had tackled this problem. According to Elftman, they only measured and identified the shape of the foot and not the pressure. Elftman found out that the distribution of pressure changes from moment to moment. The total
pressure is not only due to gravitational force, but also due to the
downward foot reaction on the ground as the body is raised by the
muscles of the leg. In 1938, Elftman [59], managed to design and
develop a force plate which was able to measure the three components
of the ground force reaction and the center of pressure. As a result,
Elftman succeeded in calculating forces and torques acting at the joints.
Elftman also did some studies in the function of arms in walking [60],
the forces and energy changes in walking [61], and rotation of the
body [62]. Although Elftman had a limited amount of experimental data,
he made a valuable contribution to the analysis of locomotion problems.
In the studies done by Elftman, only planar motion was considered.
These results were later used and developed by Bresler and Frankel [63],
in 1950, to describe a three-dimensional investigation of leg moments
and forces during level walking. They showed that the external
components of forces are necessary for lateral stability in walking
and contribute to lateral hip moment very much. In this research,
nine cameras with 40 frames per second and a force plate were used with
the help of desk calculators. The calculations took 500 man hours
for the first subject. Subsequently, this time was reduced to 250
man hours per subject [63].

All of the above researchers have been motivated to help
orthopedic surgeons to evaluate abnormalities in lower limbs of patients
so they can provide more effective treatment. The development of
computers was extremely helpful to achieve this goal. Especially,
as computers became faster and cheaper, they were efficiently and
widely used to take care of enormous computations. In fact, the
development of minicomputers and microprocessors is so helpful that they have become one of the basic parts of any efficient system of gait analysis.

In 1973, Leo et al. [64] employed a photographic method, which takes 20 pictures per second, to record the movement of the subject. He also used a force plate to measure the ground reaction forces which were introduced as inputs to a dynamic model of the human leg. The study involved the investigation of the forces and torques acting on the lower limbs of a two-dimensional model both in the case of level walking and walking upstairs.

Modern kinetical systems have taken advantage of the new technology by using TV-systems or Selspot systems which can be interfaced to the desired computer. Winter et al. [65,66] is one of the investigators who has done some important kinetic studies in recent years. He used a television camera system which was interfaced to a large computer (CDC 1700). There were two cameras for collecting the data. One of these cameras was positioned on one side (positioned parallel to the walkway) while the other was in front of the walking subject in order to get a three-dimensional representation of the motion of one side of the body. The frequency of measurement was 60 Hz and a 96 x 96 binary matrix was used to represent the picture. This obviously yields a fairly low resolution. Some energy studies were also done by Winter [67], in 1973. Another TV system has been used by Jarrett et al. [68,69]. The latter system has a frequency of 50 Hz per camera, and is capable of using six cameras simultaneously giving a potential for a combined frequency of 300 Hz. The computer utilized in
in this system is a PDP-12. The marker system, made of retro-reflective tape, is passive in order to restrict the subject's movement as little as possible. There are 1,000 intervals counted per horizontal scanned line, and there are 292 horizontal lines per frame, which determines vertical resolution.

In 1974, Cheng [55,70] designed an interface circuit to connect the output of a TV-system to a PDP-11/10 minicomputer. Pin lights were connected to selected joints of the subject, and the displacement data were recorded at a rate of 60 frames per second. The system included only one TV camera, and therefore only planar models could be investigated. This system was designed and built for both kinematic and kinetic studies. The former was done mostly by Cheng [55,70], and the latter was carried out later by Gupta [2] and McGhee et al. [71]. The kinetic study includes the evaluation of the forces and moments at the specific joints of the human model without using any force plate. This computer-TV system has the disadvantage of not being able to detect two lights which are on the same TV line. This prevents the full marking of the pelvis in the frontal view, and also may present problems during certain phases of the gait cycle in obtaining the coordinates of foot and ankle lights.

Gustafsson and Lanshammar in 1977 [10] studied the planar motion of human subjects. They used an opto-electronic system (Selspot) to obtain the kinematic data. Two cameras were positioned on both sides of the walkway to record the coordinates of both legs simultaneously. An HP21-MX computer was employed to help the collection and storage of data. A force plate was also placed on the walkway to measure the
ground reaction forces on one leg. The Selspot measurement frequency was 315 Hz which is much faster than Cheng's system [55]. Light-emitting diodes were used as landmarks. One of the major drawbacks of this system is that the subject must carry a rather heavy power supply or be connected via a cable to the power source for the LED's. The dynamical model is a five-mass, eight-link one including a massless pelvis and feet.

2.7 Mathematical Models and Simulation of Locomotion

Mathematical modeling of various aspects of human behavior has attracted the attention of many investigators, and several have studied simulation of human locomotion. Some such studies have been based on filming the motion of the individual. As described above, one motivation for this work has been to calculate the forces and torques at body joints. Obviously, the development of a complete dynamic model of a human being is an extremely complicated and difficult task. Consequently, researchers have tried to simplify the system of the human body. As a very simple model, some aspects of the inverted pendulum problem for modeling of human locomotion have been studied [72,73]. McGhee and Kuhner [74] investigated the stabilization of a two-link inverted pendulum biped model. The upper link represents the upper body while the lower one consists of legs. For simplicity, the lower part (legs) was massless. The system was found to be stable in a small region about the vertical position when linear feedback of the angles and rates was used to generate the required torque. Beckett and Chang [75] limited the scope of their work by considering leg motion (a subsystem of the whole human skeleton) during the swing phase. The model consisted of three segments:
the thigh, the shank, and the foot. There were many approximations involved, such as the hip joint moving forward along a sinusoidal path. Lagrangian equations were written for the system and numerical methods were employed to get the solutions.

Mathematical modeling of the human body was also studied by Jacobson and Chow [76]. There were several constraints made to obtain the motion of several joints of the body. For example, during the foot flat to toe-off, the ankle was assumed to move in a circular arc about the toe. Also, the motion of the hip was constrained to follow a sinusoidal path together with a constant forward velocity. A performance function which was dependent on hip and knee torques was chosen, and the task was to minimize this function. The study yielded the resultant hip, knee and ankle angles as well as the joint forces and moments. Hartrum [77] did an extensive study of modeling and simulation of a three-dimensional kinematic model of the human body.

Feedback control has also been employed to simulate the motion of the human body [57,73,74,79]. In the general case, velocity and position feedback were used. Typically, in such studies, the joint moments are assumed to be proportional to the difference of the specified angular quantities and the ones obtained from the model. Golliday and Hemami [80,81] studied a two-link biped model extensively. Many aspects of the model, including controllability and observability were investigated. Jaswa [82] studied a three-link model including knee and hip joints. He used linear feedback together with an "inverse plant" for computing nominal joint torques to investigate the postural stabilization of the model during sitting down and standing.
up process. Farnsworth [83] continued the study with a more complete model which consists of five massive links. He used the single support case of walking for his simulation. Ceranowicz [84] did some more studies of the five-link control of the biped system including methods to decouple the motions of the model's segments. Camana [13] considered a two-link model and studied its posture. He used experiments with human subjects to test the model.

2.8 Summary

A survey of the available literature in the area of legged locomotion with emphasis on human gait is presented in this chapter. Studies of the body center of gravity and estimation of body segment parameters are quite important relative to gait analysis, and thus an overview of the available literature on these subjects is given.

Previous work in two areas of gait analayis, namely, kinematics and kinetics is reviewed. Various methods of recording kinematic and kinetic information are discussed and classified and their advantages and disadvantages are described. The available literature in the area of simulation of human locomotion and the associated mathematical models are also summarized in this chapter.

Considering the many previous studies in the area of human locomotion and gait analysis, the author believes that the following factors are of prime importance in the design of equipment and procedures for experimental studies in this area:

a) Type of equipment for measuring displacement and angular data.

b) Number of dimensions chosen for the models.

c) Type of markers on the human body.
d) The positioning of cameras.

e) The number of force plates used to measure ground force reactions.

f) Determination of the body segment parameters.

g) Measurement frequency and resolution.

h) Type of filter used to smooth noisy data.

i) Method of differentiation to obtain velocities and accelerations.

j) Mathematical modeling of the human subject.

k) Method of solving the kinetic equations.

l) Computation equipment and storage media.

m) Methods and facilities for presentation of results.

n) Capability of improving hardware and software.

All of these issues are addressed in the remainder of this dissertation.
Chapter 3

A COMPARISON AND EVALUATION OF ALTERNATIVE FIVE-MASS DYNAMIC MODELS FOR GAIT UTILIZING ONLY TELEVISION DATA

3.1 Introduction

The purpose of this chapter is to investigate various dynamic models for the human musculoskeletal system having the property that television data alone can be used to estimate quantities which may be useful in the analysis of gait and in locomotive rehabilitation. As in earlier work on this problem [2,71], in this chapter only five-mass planar models will be considered. However, in order to account for certain important features of gait, some of the models to be developed include additional massless links. As a first step in this chapter, a five-link model which is essentially the same as Gupta's model [2] is considered. This model includes a trunk, two thighs, and two shanks. The joints are assumed to be pin joints. No modeling of frictional losses is included. Rather, only the net forces and moments associated with observable motion are estimated. Successive elaborations of this model produce a total of six different methods for estimating joint forces and moments. These are compared and contrasted, and a recommendation concerning the best method is presented. The sensitivity of the preferred method to errors in estimation of body segment parameters is also investigated.
3.2 Basic Five-Mass Model

The method used here is the same as the so called "whole body method" which was used by Gupta [2]. A computer television system is used to obtain body motion data [8]. The subject is fitted with six lights normally wears athletic clothing. A photograph showing a typical arrangement of the lights is included in Chapter 6. In order to consider the upper body as a single part, the subject crosses his arms while walking. This also has the advantage of not blocking the hip light which could happen if the arms were moving. In this method, only the angular data are used during a gait cycle which contains two steps or one stride. Once the heel of the subject touches the force plate the cycle starts. The termination of this cycle is obtained by measuring the coordinates of the ankle light. The toe-off time of the same foot is also calculated by evaluation of the force-plate output which is sampled and collected, and stored by the computer. In the present research in which only one TV camera is involved, the subjects are all normal. This implies that one may assume symmetry between the right leg and the left leg. Under such an assumption, if the total period of one gait cycle (e.g., from the first left-heel strike (LHS) to the second LHS) is measured to be T, the right-heel strike (RHS) will happen at T/2. Therefore, the left toe off (LTO) occurs at \( t = T/2 + \delta \) where \( \delta \) indicates the period of time between one leg's heel strike and toe off of the other leg.

Fig. 3.1 illustrates toe off and heel strike during one gait cycle. Starting with LHS, experimental results show that under normal speed LTO happens somewhere between 62 and 70 percent of the gait cycle.
Figure 3.1. Time Parameters of the Gait Cycle.

The shaded areas in Fig. 3.1 indicate the times that both feet are on the ground, which is known as double support phase (DSP). On the other hand, single support phase (SSP) is the period that one leg is swinging in the air while the other is supporting the whole body. Generally, DSP and SSP take about 30 and 70 percent of the gait cycle (normal speed), respectively.

Gupta [2] used one of the legs which is in touch with the ground as the reference leg for half of the gait cycle. For example, in Fig. 3.1, the left leg is the reference one during the first half of the gait cycle, and the right leg is the reference during the second half. This is not quite correct since it results in discontinuities in forces and moments at switching times. Gupta's results show this effect [2].

Fig. 3.2 shows the stick figure of a five-link model walking from right to left. The horizontal line is always considered to be the reference. Using the coordinates of the pin lights attached to the body of the subject, it is possible to calculate the angles $\theta_1$, $\theta_2$, and $\theta_3$ during the whole gait cycle. The rest of the angles, $\theta_4$ and $\theta_5$, are constructed by using the symmetry assumption as follows:
Figure 3.2. Stick Figure Representation of Basic Five-Mass Dynamic Model for Gait.
For $0 \leq t < T/2$

\[
\theta_4(t) = \theta_2(t + T/2) \quad (3-1)
\]

\[
\theta_5(t) = \theta_1(t + T/2) \quad (3-2)
\]

For $T/2 \leq t < T$

\[
\theta_4(t) = \theta_2(t - T/2) \quad (3-3)
\]

\[
\theta_5(t) = \theta_1(t - T/2) \quad (3-4)
\]

Similarly, for the other thigh and other shank we have:

For $t = 0, T/2$

\[
\dot{\theta}_4(t) = \dot{\theta}_2(t + T/2) , \quad \ddot{\theta}_4(t) = \ddot{\theta}_2(t + T/2) \quad (3-5)
\]

\[
\dot{\theta}_5(t) = \dot{\theta}_1(t + T/2) , \quad \ddot{\theta}_5(t) = \ddot{\theta}_1(t + T/2) \quad (3-6)
\]

and for $t = T/2, T$

\[
\dot{\theta}_4(t) = \dot{\theta}_2(t - T/2) , \quad \ddot{\theta}_4(t) = \ddot{\theta}_2(t - T/2) \quad (3-7)
\]

\[
\dot{\theta}_5(t) = \dot{\theta}_1(t - T/2) , \quad \ddot{\theta}_5(t) = \ddot{\theta}_1(t - T/2) \quad (3-8)
\]

In this research, derivatives are obtained by digital filtering based on Fourier analysis methods. This aspect of the research is explained further in Chapter 6.
As previously stated, this research is involved only with planar motion, and the model has even been more simplified. It is obvious that when the entire body is considered, the resulting system is extremely complex [85]. That is why this model represents only planar motion and combines the upper body, arms, and head into a single equivalent rigid segment. Also, at this point, feet are ignored and no pelvic motion is introduced. Therefore, as can be seen in Fig. 3.2, the two thighs of the model are connected to each other at a single point. When one leg is in the stance phase, the lowest part of it, the ankle, is assumed to be pinned to the ground. This point is considered to be the reference point as explained before. Thus, this simple model does not include feet, and the ground force reactions are therefore taken to be equal to the forces at the ankle.

During the single support phase (SSP), the system of Fig. 3.2 is fully deterministic while this is not true during DSP. Thus an assumption has to be made on how the torque and forces transmitted by the upper body are divided between the two legs during DSP. This ratio is a variable to be calculated from the model equations. The partition coefficients [2], \( a, \beta, \) and \( \gamma \) are the three ratios which are considered for the transfer of force in \( x \) and \( y \) directions, and torque about \( z \) direction, respectively. Gupta assumed that the ground reactions are transferred linearly in time from one leg to the other leg during the DSP. Therefore, before LHS, the right leg is supporting the whole body. At LHS time, the left leg starts contributing to the support of the body. During DSP, LHS to RTO, the right leg transfers more and more of the net ground reaction to the left leg. Finally, at RTO, the left
leg has the support of the whole body while the right leg starts its swing phase. The same transfer happens in reverse during the second half of the gait cycle. As will be seen later, the total ground reaction forces and torque are still a variable to be solved for.

3.3 Mathematical Formulations of Five-Mass Dynamic Models

There are a number of alternatives available for obtaining the dynamical equations of motion for the model of Fig. 3.2 [86]. One possible approach is to consider the classical Lagrangian equations [64] which involves kinetic and potential energy obtained in terms of the angles, \( \theta_i \). This classical approach is rather tedious and the equations are quite complex [86]. A second approach is the so-called "free-body" approach [71,86] which is relatively simpler and easier to work with. With the free-body approach, for each segment in the model, three simple equations can be written

\[
\begin{align*}
\sum m_i \ddot{x}_i &= \sum F_{xi} \\
\sum m_i \ddot{y}_i &= \sum F_{yi} \\
\sum J_i \ddot{\theta}_i &= \sum T_i
\end{align*}
\]

(3-9)  (3-10)  (3-11)

Table 3.1 shows the definitions of all symbols used in this method. Eqs. (3-9) through (3-11) are D'Alembert's equations written for the free-body model. In fact, these are the results of effecting dynamic equilibrium in three directions.

In order to get the equations for all segments of the model, each is considered individually with the corresponding forces and
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>mass of the segment</td>
</tr>
<tr>
<td>L</td>
<td>length of the segment</td>
</tr>
<tr>
<td>J</td>
<td>moment of inertia of the segment</td>
</tr>
<tr>
<td>d</td>
<td>distance from the common joint with the proximal link to the C.G of the link under consideration angle</td>
</tr>
<tr>
<td>θ</td>
<td>angle with respect to the horizontal axis</td>
</tr>
<tr>
<td>x</td>
<td>horizontal displacement</td>
</tr>
<tr>
<td>y</td>
<td>vertical displacement</td>
</tr>
<tr>
<td>F_x</td>
<td>horizontal component of force</td>
</tr>
<tr>
<td>F_y</td>
<td>vertical component of force</td>
</tr>
<tr>
<td>T</td>
<td>torque</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>g</td>
<td>acceleration due to gravity</td>
</tr>
</tbody>
</table>

Torques. These are shown in Figs. 3.3, 3.4, 3.5, 3.6, and 3.7.

Effecting dynamic equilibrium in the x direction yields:

\[
m_1\ddot{x}_1 = F_{x1} - F_{x2} \tag{3-12}
\]
\[
m_2\ddot{x}_2 = F_{x2} - \alpha F_{x3} \tag{3-13}
\]
\[
m_3\ddot{x}_3 = F_{x3} \tag{3-14}
\]
\[
m_4\ddot{x}_4 = F_{x4} - (1-\alpha) F_{x3} \tag{3-15}
\]
\[
m_5\ddot{x}_5 = F_{x5} - F_{x4} \tag{3-16}
\]
Figure 3.3. Free Body Diagram for Upper Body.
Figure 3.4. Free Body Diagram for Left Thigh.

Figure 3.5. Free Body Diagram for Right Thigh.
Figure 3.6. Free Body Diagram for Left Shank.

Figure 3.7. Free Body Diagram for Right Shank.
The quantity $\alpha$ appearing in Eq. (3-15) is the partition coefficient for the horizontal forces acting between the upper body and the two thighs at the hip joint.

The total ground reaction force in x direction, $F_{x0}$ is the sum of two forces, $F_{x1}$ and $F_{x5}$, acting on the left and right leg at any time, respectively. Under the assumption of a linear transfer of the forces from one leg to the other leg during the DSP, the values of $F_{x1}$ and $F_{x5}$ are

$$F_{x1} = F_{x0} \cdot q$$

$$F_{x5} = F_{x0} \cdot (1-q)$$

(3-17)

(3-18)

where $q$ is defined as

$$q = \frac{t - t_{LHS}}{t_{RTO} - t_{LHS}} \quad LHS \leq t < t_{RTO}$$

$$q = 1.0 \quad t_{RTO} \leq t < t_{RHS}$$

$$q = \frac{t_{LTO} - t}{t_{LTO} - t_{RHS}} \quad t_{RHS} \leq t < t_{LTO}$$

$$q = 0.0 \quad t_{LTO} \leq t \leq T$$

(3-19)

Obviously, the value of 'q' equal to unity indicates that the left leg is supporting the whole body, while 'q' equal to zero expresses the same thing about the right leg.
Effecting dynamic equilibrium in y direction gives

\[ m_1 \ddot{y}_1 = F_{y1} - F_{y2} - m_1 g \]  \hspace{1cm} (3-20)

\[ m_2 \ddot{y}_2 = F_{y2} - \beta F_{y3} - m_2 g \]  \hspace{1cm} (3-21)

\[ m_3 \ddot{y}_3 = F_{y3} - m_3 g \]  \hspace{1cm} (3-22)

\[ m_4 \ddot{y}_4 = F_{y4} - (1-\beta) F_{y3} - m_4 g \]  \hspace{1cm} (3-23)

\[ m_5 \ddot{y}_5 = F_{y5} - F_{y4} - m_5 g \]  \hspace{1cm} (3-24)

Here, \( \beta \) is the partition coefficient of the forces in the y direction at the hip joint. Again, if the total ground reaction in the vertical direction is represented by \( F_{y0} \), the corresponding values for \( F_{y1} \) and \( F_{y5} \) are:

\[ F_{y1} = F_{y0} q \]  \hspace{1cm} (3-25)

\[ F_{y5} = F_{y0} (1-q) \]  \hspace{1cm} (3-26)

where \( q \) is calculated from Eq. (3-19). Finally, the equations for dynamic equilibrium for rotational motion are:

\[ J_1 \dddot{\theta}_1 = T_1 - T_2 + [F_{x1} \cdot d_1 + F_{x2} \cdot (\ell_1 - d_1)] \sin \theta_1 \]

\[ - [F_{y1} \cdot d_1 + F_{y2} \cdot (\ell_1 - d_1)] \cos \theta_1 \]  \hspace{1cm} (3-27)

\[ J_2 \dddot{\theta}_2 = T_2 - \gamma T_3 + [F_{x2} \cdot d_2 + \alpha F_{x3} \cdot (\ell_2 - d_2)] \sin \theta_2 \]

\[ - [F_{y2} \cdot d_2 + \beta F_{y3} \cdot (\ell_2 - d_2)] \cos \theta_2 \]  \hspace{1cm} (3-28)
\[ J_3 \ddot{\theta}_3 = T_3 + F_{x3} \cdot d_3 \sin \theta_3 - F_{y3} \cdot d_3 \cdot \cos \theta_3 \quad (3-29) \]

\[ J_4 \ddot{\theta}_4 = T_4 - (1-\gamma) \cdot T_3 + [F_{x4} \cdot d_4 + (1-\alpha) \cdot F_{x3} (\ell_4-d_4)] \]
\[ \cdot \sin \theta_4 - [F_{y4} \cdot d_4 + (1-\beta) \cdot F_{y3} (\ell_4-d_4)] \]
\[ \cdot \cos \theta_4 \quad (3-30) \]

\[ J_5 \ddot{\theta}_5 = T_5 - T_4 + [F_{x5} \cdot d_5 + F_{x4} (\ell_5-d_5)] \cdot \sin \theta_5 \]
\[ - [F_{y5} \cdot d_5 + F_{y4} \cdot (\ell_5-d_5)] \cdot \cos \theta_5 \quad (3-31) \]

where \( \gamma \) is the partition coefficient of the torque in z direction at the hip joint. If \( T_0 \) represents the total ground reaction torque, \( T_1 \) and \( T_5 \) can thus be calculated as

\[ T_1 = T_0 \cdot q \quad (3-32) \]

\[ T_5 = T_0 \cdot (1-q) \quad (3-33) \]

where \( q \) is again obtained from Eq. (3-19).

Eqs. (3-12) through (3-33) can be solved simultaneously to get the joint forces and torques provided that the displacement accelerations in x and y directions are known. Knowing the displacement accelerations, one can solve Eqs. (3-14), (3-22) and (3-29) independently to obtain the torque and forces associated with the upper body, that is, \( T_3, F_{x3}, \) and \( F_{y3} \).

Substituting the upper body components into the equations of the system, one can simplify them to get a twelfth order set of
equations which can be solved simultaneously by numerical means. These equations can be written in matrix form as

\[ \mathbf{A} \cdot \mathbf{x} = \mathbf{b} \]  \hspace{1cm} (3-34)

where \( \mathbf{x} \) is a 12 by 1 column vector as follows:

\[ \mathbf{x} = [F_{x0}' \ F_{x2}' \ \alpha \ F_{x4}' \ F_{y0}' \ F_{y2}' \ \beta \ F_{y4}' \ T_0 \ T_2 \ T_4 \ \gamma]^T \]  \hspace{1cm} (3-35)

The matrices \( \mathbf{A} \) and \( \mathbf{b} \) are presented in Figs. 3.8 and 3.9. In order to solve Eq. (3-34), the Gaussian Elimination Method with partial pivoting is chosen [87]. This is to guarantee the accuracy of the result even if matrix \( \mathbf{A} \) is ill-conditioned. Several iterations can be used to further reduce possible errors. Unfortunately, it turns out that the matrix \( \mathbf{A} \) is sometimes very nearly singular, and the iterations therefore do not always improve the results. A solution to this problem will be discussed in Chapter 4.

3.4 Reference Systems for Estimation of Translational Accelerations

So far, the free-body equations of the basic five-mass model have been discussed. The use of these equations requires that a reference system be defined to permit computation of the \( x \) and \( y \) coordinates of the center of mass for all five links and the first and second derivatives of these quantities. Most researchers, either those who were doing simulation studies [83, 84, 86] or those who were interested in kinetic studies [71], have chosen one of the lowest extremities which is in contact with the ground to be the reference point. Consequently, this will be the first approach to be taken in this section of this chapter.
\[
\begin{bmatrix}
q & -1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.0 & -F_{x3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & F_{x3} & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1-q & 0 & 0 & -1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -F_{y3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & q & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & F_{y3} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1-q & 0 & 0 & -1 & 0 & 0 \\
0 & d_2 \sin \theta_2 & \cdot F_{x3} \sin \theta_2 & 0 & 0 & -d_2 \cos \theta_2 & \cdot F_{y3} \cos \theta_2 & 0 & 0 & 1 & 0 & -T_3 \\
d_1 q \sin \theta_1 & (\ell_1-d_1) \sin \theta_1 & 0 & 0 & -q d_1 \cos \theta_1 & -(\ell_1-d_1) \cos \theta_1 & 0 & 0 & q & -1 & 0 & 0 \\
0 & 0 & -(\ell_4-d_4) & d_4 \sin \theta_4 & 0 & 0 & (\ell_4-d_4) \cdot F_{y3} & -d_4 \cos \theta_4 & 0 & 0 & 1 & T_3 \\
(1-q)d_5 & \cdot \sin \theta_5 & (\ell_5-d_5) & -(1-q)d_5 & (1-q) \cdot \sin \theta_5 & \cdot \cos \theta_5 & 0 & 0 & -(\ell_5-d_5) & 1-q & 0 & -1 & 0
\end{bmatrix}
\]

Figure 3.8. Matrix A for Force and Torque Estimation for Five-Mass Dynamic Model for Gait.
\begin{align*}
    m_1 \ddot{x}_1 \\
    m_2 \ddot{x}_2 \\
    m_4 \ddot{x}_4 + F_{x3} \\
    m_5 \ddot{x}_5 \\
    m_2 \ddot{y}_2 + m_2 g \\
    m_1 \ddot{y}_1 + m_1 g \\
    m_4 \ddot{y}_4 + m_4 g + F_{y3} \\
    m_5 \ddot{y}_5 + m_5 g \\
    \cdots \\
    J_2 \dot{\theta}_2 \\
    \cdots \\
    J_1 \dot{\theta}_1 \\
    J_4 \dot{\theta}_4 + T_3 - (l_4 - d_4) \cdot F_{x3} \cdot \sin \theta_4 + (l_4 - d_4) \cdot F_{y3} \cdot \cos \theta_4 \\
    J_5 \ddot{\theta}_5
\end{align*}

Figure 3.9. Vector B for Force and Torque Estimation for Five-Mass Dynamic Model for Gait.
Approach #1: Since the gait cycle starts with left-heal strike, the bottom of the left heel is considered to be the initial reference point. This point will then be regarded as fixed for the entire first half of the gait cycle. This is of course an approximation because in fact a human being raises his left heel before right-heal strike occurs. Therefore, this assumption will introduce some error. However, for the current footless model, this may be tolerable.

In the second half of the gait cycle, which starts with the heel strike of the right leg, the bottom of the right heel is considered to be the reference point. Therefore, the whole calculation must be switched at each heel strike from one reference point to the other one. Eqs. (3-36) through (3-45) show the x-y coordinates of the center of gravity of the five masses.

For $0 \leq t < T/2$:

\[ x_1 = d_1 \cos \theta_1 \]

\[ x_2 = \ell_1 \cos \theta_1 + d_2 \cos \theta_2 \]

\[ x_3 = \ell_1 \cos \theta_1 + \ell_2 \cos \theta_2 + d_3 \cos \theta_3 \]

\[ x_4 = \ell_1 \cos \theta_1 + \ell_2 \cos \theta_2 + (\ell_4 - d_4) \cos (\pi + \theta_4) \]

\[ x_5 = \ell_1 \cos \theta_1 + \ell_2 \cos \theta_2 + \ell_4 \cos (\theta_4 + \pi) + (\ell_5 - d_5) \cos (\pi + \theta_5) \]

and

\[ y_1 = d_1 \sin \theta_1 \]
\[ y_2 = l_1 \sin \Theta_1 + d_2 \sin \Theta_2 \]  \hspace{1cm} (3-42) \\
\[ y_3 = l_1 \sin \Theta_1 + l_2 \sin \Theta_2 + d_3 \sin \Theta_3 \]  \hspace{1cm} (3-43) \\
\[ y_4 = l_1 \sin \Theta_1 + l_2 \sin \Theta_2 + (l_4 - d_4) \sin (\pi + \Theta_4) \]  \hspace{1cm} (3-44) \\
\[ y_5 = l_1 \sin \Theta_1 + l_2 \sin \Theta_2 + l_4 \sin (\pi + \Theta_4) \]
\[ + (l_5 - d_5) \sin (\pi + \Theta_5) \]  \hspace{1cm} (3-45)

For \( T/2 \leq t \leq T \), the subscripts 1 and 2 are replaced by the subscripts 5 and 4, respectively, and vice versa. Evidently, one can differentiate the above equations to obtain the velocity and the acceleration of each point in \( x \) and \( y \) directions.

The previous approach using Eqs. (3-37) through (3-45) together with the equations of the system explained in Sec. 3.3 was used by Gupta [2] to calculate the joint forces and torques. Although he got some interesting results, there are some points to be noticed.

i) Gupta made a few mistakes in the equations he used which affect all results he got. The errors detected by the author are:

a) 2 errors in the equation for \( T_3 \).
b) 1 error in the equation for AA (9,3)
c) 1 error in the equation for AA (9,7)
d) 1 error in the equation for BB (7)

These errors have all been corrected in Figs. 3.8 and 3.9 and also in the programs appended to this dissertation.
ii) Although the general shape of the vertical ground reaction force at the ankle joint obtained by this method is more or less correct, its amplitude is too large. Experiments accomplished with the force plate show that the peak value of the vertical ground reaction force should not exceed the weight of the body by more that about 12 percent. However, in both sets of results obtained by Gupta, the vertical ground force reactions show more than 50 percent increase with respect to the body weight. Figs. 3.10 and 3.11 obtained in this research show that this effect persists after corrections of Gupta's equations.

iii) Because of the switching action which happens at each heel strike, there is a discontinuity which can be observed in Gupta's results and also in Figs. 3.10 and 3.11. This is an important problem which is due to the modeling simplification.

There are both physiological and mathematical explanations to the above problem. The physiological answer is related to the model for a human being which has been oversimplified. There is no pelvis included in the model which causes a great deal of approximation in the hip motion which will affect the rest of the system also. In other words, during DSP, the position of the hip computed once by using the left leg as the reference and then considering the right leg to be the reference do not give the identical results simply because of the length and the motion of the pelvis.

A mathematical review of the system coordinates, exactly before and exactly after the right-heel strike (which cases the switching) yields the following results:
Figure 3.10. Horizontal Ground Force Reactions Measured by Force Plate (*) and Calculated from Basic Five-Mass Dynamic Model (+).
Figure 3.11. Vertical Ground Force Reactions Measured by Force Plate (×), and Calculated from Basic Five-Mass Dynamic Model (+).
At \( t = T/2 - \varepsilon \), the left heel (ankle) is the reference point and the coordinates of the center of gravity of two arbitrary masses, for example, (left thigh and upper body) are as follows:

\[
x_2(T/2 - \varepsilon) = l_1 \cos[\theta_1 (T/2 - \varepsilon)] + d_2 \cos[\theta_2 (T/2 - \varepsilon)]
\]

\[
y_2(T/2 - \varepsilon) = l_1 \sin[\theta_1 (T/2 - \varepsilon)] + d_2 \sin[\theta_2 (T/2 - \varepsilon)]
\]

\[
x_3(T/2 - \varepsilon) = l_1 \cos[\theta_1 (T/2 - \varepsilon)] + l_2 \cos[\theta_2 (T/2 - \varepsilon)] + d_3 \cos[\theta_3 (T/2 - \varepsilon)]
\]

\[
x_4(T/2 - \varepsilon) = l_1 \sin[\theta_1 (T/2 - \varepsilon)] + l_2 \sin[\theta_2 (T/2 - \varepsilon)] + d_3 \sin[\theta_3 (T/2 - \varepsilon)]
\]

At \( t = T/2 + \varepsilon \), the reference is switched to the right leg which gives:

\[
x_2(T/2 + \varepsilon) = l_5 \cos[\theta_5 (T/2 + \varepsilon)] + l_4 \cos[\theta_4 (T/2 + \varepsilon)] - (l_2 - d_2) \cos[\theta_2 (T/2 + \varepsilon)]
\]

\[
y_2(T/2 + \varepsilon) = l_5 \sin[\theta_5 (T/2 + \varepsilon)] + l_4 \sin[\theta_4 (T/2 + \varepsilon)] - (l_2 - d_2) \sin[\theta_2 (T/2 + \varepsilon)]
\]

\[
x_3(T/2 + \varepsilon) = l_5 \cos[\theta_5 (T/2 + \varepsilon)] + l_4 \cos[\theta_4 (T/2 + \varepsilon)] + d_3 \cos[\theta_3 (T/2 + \varepsilon)]
\]
\[ y_3(T/2 + \varepsilon) = l_5 \sin[\theta_5 \ (T/2 + \varepsilon)] + l_4 \sin[\theta_4 \ (T/2 + \varepsilon)] \]
\[ + d_3 \sin[\theta_3 \ (T/2 + \varepsilon)] \]  

(3-53)

Using the symmetry of the body and Eq. (3-1) and (3-2), Eq. (3-50) through (3-53) become:

\[ x_2 \ (T/2 + \varepsilon) = l_1 \cos[\theta_1(\varepsilon)] + l_2 \cos[\theta_2(\varepsilon)] - l_2 \cos[\theta_2 \ (T/2 + \varepsilon)] \]
\[ + d_2 \cos[\theta_2 \ (T/2 + \varepsilon)] \]  

(3-54)

\[ y_2 \ (T/2 + \varepsilon) = l_1 \sin[\theta_1(\varepsilon)] + l_2 \sin[\theta_2(\varepsilon)] - l_2 \sin[\theta_2 \ (T/2 + \varepsilon)] \]
\[ + d_2 \sin[\theta_2 \ (T/2 + \varepsilon)] \]  

(3-55)

\[ x_3 \ (T/2 + \varepsilon) = l_1 \cos[\theta_1(\varepsilon)] + l_2 \cos[\theta_2(\varepsilon)] \]
\[ + d_3 \cos[\theta_3 \ (T/2 + \varepsilon)] \]  

(3-56)

\[ y_3 \ (T/2 + \varepsilon) = l_1 \sin[\theta_1(\varepsilon)] + l_2 \sin[\theta_2(\varepsilon)] \]
\[ + d_3 \sin[\theta_3 \ (T/2 + \varepsilon)] \]  

(3-57)

A comparison between Eqs. (3-46) through (3-49) and Eqs. (3-50) through (3-57) explains the discontinuity which occurs at \( t = T/2 \) (RHS).

In conclusion, approach #1, although has the advantage of being simple, yields very large errors in the estimation of joint forces and moments. The results obtained for this model show a mean-square error of at least \( 0.5 \times 10^5 \) (Newtons\(^2\)) for both horizontal and vertical ground
reaction components compared with the actual values measured by the force plate (Figs. 3.10 and 3.11).

**Approach #2:** Approach #2 is basically the same as the previous one except that an attempt is made to take care of the discontinuity at heel strike. A very simple method which eliminates this discontinuity is a linear interpolation of translational accelerations used during the DSP. This, although not a true assumption due to both logical and physiological facts, may be advantageous and useful to get an idea about the shape of the forces and torques and the amount of the mean-square error without the previous discontinuity. The interpolation is applied to the accelerations directly. Results obtained by this method show just a little improvement due to the elimination of the discontinuity. However, the mean-square error is still very large. Consequently, the model needs to be improved if better results are desirable. A comparison between the force plate results and the horizontal and vertical forces at the ankle joint indicates that the general shape of the vertical force is fairly acceptable. Nevertheless, this is not true for the horizontal component. In fact, $F_{x1}$ ($F_{x5}$) changes sign too many times which may imply a high frequency that needs to be eliminated.

In this research, in general, the raw angular data is expanded to get the equivalent Fourier series. Then, a number of components are selected (experimentally) to represent the smoothed data. Gupta [2] used 4 or 5 harmonics in most cases. On the other hand, the angular rate and the angular accelerations are directly obtained from the smoothed data. These angular derivatives contribute directly in the
calculation of displacement accelerations. Table 3.2 shows the experimental results obtained for various frequencies. The author found it sometimes helpful to smooth the angular rate and angular accelerations even further. This may give a better shape as far as the horizontal ground reaction force is concerned. Note that \( n \) represents the number of harmonics used to calculate the angular data and the corresponding derivates. Also \( n_1 \) and \( n_2 \) represent the number of terms used for further smoothing of angular rate and angular acceleration, respectively. This secondary smoothing is effective only if either \( n_1 \) or \( n_2 \) or both are less than the original number of terms, \( n \). The effect of the number of harmonics used \((n, n_1, n_2)\) on the mean square error (in comparison to force plate data) for a specific data set is presented in Table 3.2. Although the results may be different for any other data set, in general \( n = 4 \) and \( n = 3 \) seem to yield the best results (smallest MSE) in most cases. Still, there is no optimal value to be used generally.

It is known from previous work and from direct force plate measurements that numerical values for the vertical component of ground reaction are much larger that for the horizontal component. Therefore, one would expect \( \text{MSE}(F_y) \) to be larger than \( \text{MSE}(F_x) \) even if the quality of both components is comparable. From Table 3.2, the contrary condition is observed. This means that further efforts to improve Approach #2 should be concentrated mainly on factors affecting the horizontal component of ground reaction force.

**Approach #3:** In Sec. 3.2, it was mentioned that the mass of the lower limb of the model includes both masses of the corresponding
Table 3.2

Mean Square Error Between Computed Horizontal and Vertical Components of Ground Force Reaction and Direct Force Plate Measurement for Various Number of Harmonics Employed to Smooth Angles (n), Angular Rates (n₁), and Angular Accelerations (n₂) for the Method of Approach #2.

<table>
<thead>
<tr>
<th>n</th>
<th>n₁</th>
<th>n₂</th>
<th>MSE(Fₓ) x 10⁴ (Newtons²)</th>
<th>MSE(Fᵧ) x 10⁴ (Newtons²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>7</td>
<td>7</td>
<td>11.80</td>
<td>1.56</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>6</td>
<td>12.80</td>
<td>1.22</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>5</td>
<td>9.06</td>
<td>1.31</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>6</td>
<td>12.70</td>
<td>1.31</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>9.04</td>
<td>1.30</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>5</td>
<td>8.79</td>
<td>1.41</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3.91</td>
<td>1.40</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>6.17</td>
<td>0.659</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>3.68</td>
<td>1.38</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4</td>
<td>4.59</td>
<td>1.79</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>5.99</td>
<td>0.706</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>6.06</td>
<td>0.695</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>7.24</td>
<td>0.954</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4.66</td>
<td>1.94</td>
</tr>
</tbody>
</table>

shank and foot combined as a single component. Then, the lower end of this link is assumed to stay on the ground and to be the reference point for half of the gait cycle. However, this assumption introduces an error due to the heel-rise characteristic of the latter part of support phase in gait. That is, a human being typically raises the heel of a supporting leg while the ball of that foot is still on the ground, and this normally begins while the other leg is in its swing phase.
Consequently, the assumption that the reference point (the ankle of the supporting leg) in the model of Fig. 3.2 is fixed is not correct. Hence the calculation of the position of the center of gravity of all links must be corrected.

This correction can be easily accomplished by just adding the heel rise of the supporting leg to the length of the supporting link in the model. To do this, one needs to measure the angle of the foot with respect to the horizontal axis. Fig. 3.12 shows how heel rise happens. Fig. 3.12(c) shows heel strike. Fig. 3.12(b) shows the foot-flat phase and Fig. 3.12(a) shows the elevation of the heel while the toe is still on the ground. A triangular foot is considered to be a more accurate geometrical configuration. The ankle joint is assumed to be immobile in this model. Thus, support phase begins with zero heel rise which is maintained until foot flat occurs, after which heel rise follows immediately. Measurement of $\theta_F$ can be

![Figure 3.12. Foot Motion Configurations During the Stance Phase: (a) Toe Off, (b) Foot Flat, and (c) Heel Strike.](image)
accomplished by placing one light on the ankle joint and one on the ball of the foot. The angle $\phi_0$ is fixed and is measured when the foot is in a flat situation. This can be done by measuring $\theta_F$ when the toe off of the other leg occurs. For example, if the left heel strike starts the gait cycle, $\phi_0$ will be obtained as

$$\phi_0 = \theta_F (\text{RT0})$$

(3-58)

From Fig. 3.12(c), it follows that for $0 \leq t \leq T/2$

$$d_F = l_0 \cos \phi_0$$

(3-59)

$$\theta(t) = \theta_F(t) - \phi_0$$

(3-60)

$$\Delta y(t) = d_F \sin \theta(t)$$

(3-61)

$$\Delta x(t) = d_F (1 - \cos \theta(t))$$

(3-62)

Using these relationships, the heel rise for the supporting reference leg can be calculated as

$$H_r(t) = \sqrt{\Delta x^2(t) + \Delta y^2(t)}$$

(3-63)

For $T/2 \leq t \leq T$, from symmetry, the heel rise for the other leg is

$$H_r(t) = H_r(t - T/2)$$

(3-64)

The length of the lower limb of the supporting reference leg, thus is a function of time. If $l_1$, $d_1$, $l_5$, and $d_5$ represent the fixed lengths already defined, then for $0 \leq t \leq T/2$, then the modified lengths are:
\[ \lambda_1'(t) = \lambda_1 + Hr(t) \quad (3-65) \]

\[ d_1'(t) = d_1 + Hr(t) \quad (3-66) \]

For \( T/2 < t \leq T \)

\[ \lambda_5'(t) = \lambda_5 + Hr(t) \quad (3-67) \]

\[ d_5'(t) = d_5 + Hr(t) \quad (3-68) \]

Eqs. (3-65) through (3-68) show the only changes to be made in order to use the equations of the system obtained in the previous sections with the heel rise included. Experimental results obtained by this approach indicate that the heel rise represented by these equations may reach a value of up to four centimeters in normal human gait.

Table 3.3 shows that the mean square error of the ground force reaction shows a significant improvement as a result of the inclusion of this effect. However, the magnitudes of both ground reaction forces are still considerably larger than the true values obtained from force plate measurements. This problem is more severe for the horizontal component than for the vertical one. This indicates that Approach #3, although useful in reducing the mean square error in force plate estimation, is nevertheless not complete enough to yield acceptable results.

The idea of using different harmonics for angular rates and angular accelerations was also employed here to compare the results with the previous approach. Again it was noticed that depending on the data set, the use of a lower number of harmonics may be helpful
Table 3.3

Mean Square Error Between Computed Horizontal and Vertical Components of Ground Force Reaction and Direct Force Plate Measurement for Various Number of Harmonics Employed to Smooth Angles (n), Angular Rates (n₁), and Angular Accelerations (n₂) for the Method of Approach #3.

<table>
<thead>
<tr>
<th>n</th>
<th>n₁</th>
<th>n₂</th>
<th>MSE(Fₓ) x 10⁴ (Newtons²)</th>
<th>MSE(Fᵧ) x 10⁴ (Newtons²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>7.63</td>
<td>2.23</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>4</td>
<td>2.77</td>
<td>2.48</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>2.87</td>
<td>1.93</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>5.22</td>
<td>0.899</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>2.62</td>
<td>1.98</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>5.06</td>
<td>0.974</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5.23</td>
<td>1.11</td>
</tr>
</tbody>
</table>

in reducing the error. In general, as the previous case, the use of three or four Fourier components tends to produce the best results while smaller or larger values increase the mean square error.

Approach #4. Since Approach #3 still produced unacceptably large errors in ground reaction force estimation, some further elaboration of the basic five-link model is required. An immediate suggestion that one may make is that feet should be included in the model in a more realistic way than merely adding heel rise. Both Morecki [88] and Gustafsson et al. [10] have included feet in their modeling of the human being gait analysis. Since in this chapter a five-mass model with the equations shown in Sec. 3.2 is discussed, only massless feet are considered to contribute to the action of the other links.
A model with foot mass included will be developed in Chapter 4 of this dissertation.

With the addition of a foot with a movable ankle joint, the position of the center of gravity of each link can be easily calculated by using the relationships illustrated by Fig. 3.13. The foot angle is measured with respect to the top of the foot; therefore, the length of the supporting shank should be correspondingly reduced. That is, the coordinates of the center of gravity of the first and second links are ($0 \leq t < T/2$)

\[
\begin{align*}
    x_1 &= \ell_0 \cos \theta_F + (d_1 - d_0) \cos \theta_1 \\
    y_1 &= \ell_0 \sin \theta_F + (d_1 - d_0) \sin \theta_1
\end{align*}
\]  

(3-69)  

(3-70)
\[ x_2 = l_0 \cos \theta_F + (l_1 - d_0) \cos \theta_1 + d_2 \cos \theta_2 \]  
\[ y_2 = l_0 \sin \theta_F + (l_1 - d_0) \sin \theta_1 + d_2 \sin \theta_2 \]  
(3-71)

The rest of the coordinates are obtained in a similar way. In the second half of the gait cycle \((T/2 \leq t \leq T)\), only index changes are needed to get the required coordinates. For example, indexes 1 and 2 are replaced by 5 and 4, respectively. Note that the angle \(\theta_F\) in the second half is repeated according to its value in the first half of the gait cycle. That is,

\[ \theta_F(T/2+t) = \theta_F(t) \]  
(3-73)

Also, it should be mentioned that the foot related to the swinging leg does not have any role in the model simply because it is massless. In order to study the effect of introduction of massless feet to the model, the same set of data is again used. Table 3.4 shows the results obtained by using Approach \#4. Again, the case of different values for the harmonics is considered because it is more general. It is interesting to notice that in all cases the mean-square error for the horizontal component was reduced. This reduction, although again significant, shows that the horizontal force errors are still rather large and that further improvements to this model are desirable. The mean square error in vertical ground reaction force estimation is also generally reduced over that obtained in the previous models employed in this chapter, although this effect is not as significant as the reduction in horizontal force error.
Table 3.4

Mean Square Error Between Computed Horizontal and Vertical Components of Ground Force Reaction and Direct Force Plate Measurement for Various Number of Harmonics Employed to Smooth Angles (n), Angular Rates (n₁) and Angular Accelerations (n₂) for the Method of Approach #4.

<table>
<thead>
<tr>
<th>n</th>
<th>n₁</th>
<th>n₂</th>
<th>MSE(Fₓ) x 10⁴ (Newtons²)</th>
<th>MSE(Fᵧ) x 10⁴ (Newtons²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>3.12</td>
<td>0.609</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>2.60</td>
<td>1.34</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>1.15</td>
<td>1.20</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>2.46</td>
<td>1.19</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>1.03</td>
<td>1.09</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1.89</td>
<td>0.497</td>
</tr>
</tbody>
</table>

Approach #5. Up to this point in this chapter, the basic five-mass model used by previous investigators for the study of gait has been improved mainly by successively more realistic modeling of the kinematic function of the feet. The action of the pelvis in coupling the motion of the two legs to the upper body has not been dealt with explicitly. Rather, the discontinuities in limb segment positions, velocities, and accelerations resulting from the omission of pelvic function has been eliminated by the simple artifice of linearly interpolating the acceleration terms entering the force and moment equations during DSP. Therefore, it seems reasonable to expect that further improvements in gait modeling could result from explicitly including the kinematic function of the pelvis. Thus, the author was motivated to study and include the pelvic motion in the five-mass model. However,
to make the problem simpler, a massless pelvis with variable length is considered. The definition of the pelvis for the model shown in Fig. 3.2 is achieved by defining two points which represent the left hip (LH) and the right hip (RH). Fig. 3.14 shows the new modified model.

![Diagram](image)

**Figure 3.14. Stick Figure Representation of Five-Mass Dynamic Model With Massless Pelvis Included.**

The pelvic motion for the above planar model consists of two motions: 1) Pelvic tilt, which results in motion of the hip in the y-direction, and 2) Pelvic rotation, which produces hip motion in the x-direction. Inclusion of this motion evidently requires that the
coordinates of the left and right hip points be obtained. Obviously if one has such information, the coordinates of the center of gravity of all five links with non-zero mass can be easily computed from the angular data. The reference point is again the bottom of the shank (ankle) or the ball of the foot depending on whether or not feet are included in the model. With the gait cycle being initiated by the LHS, the coordinates of the left hip can be computed easily for the period between LHS and LTO as follows:

\[ x_{LH} = l_0 \cos \theta_F + l_1 \cos \theta_1 + l_2 \cos \theta_2 \]  

\[ y_{LH} = l_0 \sin \theta_F + l_1 \sin \theta_1 + l_2 \sin \theta_2 \]  

(3-74)  

(3-75)

The first term in the right-hand side of both Eqs. (3-74) and (3-75) must be omitted if feet are not included in the model. Determination of the right hip coordinates must be done for two conditions. For the DSP, the right hip coordinates can be calculated if the separation between legs (step length) is known. If 'S' represents the step length, specifically, for \( 0 \leq t < t_{RTO} \)

\[ x_{RH}(t) = x_{LH}(t + T/2) - S \]  

\[ y_{RH}(t) = y_{LH}(t + T/2) \]  

(3-76)  

(3-77)

For \( T/2 \leq t < t_{LTO} \)

\[ x_{RH}(t) = x_{LH}(t - T/2) + S \]  

\[ y_{RH}(t) = y_{LH}(t - T/2) \]  

(3-78)  

(3-79)
\[ y_{RH}(t) = y_{LH}(t - T/2) \]  \hspace{1cm} (3-79)

where \( T \) is the period of the gait cycle. The pelvic rotation displacement can be expressed as follows:

\[ x_p(t) = x_{RH}(t) - x_{LH}(t) \]  \hspace{1cm} (3-80)

\[ y_p(t) = y_{RH}(t) - y_{LH}(t) \]

The value of the step length can be estimated by measuring the displacement of the ankle light from TV data over the entire period of gait cycle. Also, the distance between the TV camera and the walking subject is required for the above computation. The result will be the stride length which is twice as large as the step length. This, of course, is based on the assumption made throughout this chapter that the motion of a subject is considered to be normal and symmetric which implies that left step and right step are equally long.

During the first SSP, there is no point of reference to determine the RH coordinates unless some kind of assumption is made about the pelvic motion. The simplest approach is a linear interpolation over the period of SSP. Knowing the values of left hip and right hip coordinates in DSP, one can write.

\[ x_p(t_{RTO}) = x_{RH}(t_{RTO}) - x_{LH}(t_{RTO}) \]

\[ y_p(t_{RTO}) = y_{RH}(t_{RTO}) - y_{LH}(t_{RTO}) \]  \hspace{1cm} (3-81)
\[ x_p(T/2) = x_{RH}(T/2) - x_{LH}(T/2) \]
\[ y_p(T/2) = x_{RH}(T/2) - x_{LH}(T/2) \] (3-81)

For \( t_{RTO} < t < T/2 \)

\[ x_{RH}(t) = x_{LH}(t) + x_p(t_{RTO}) \frac{T/2 - t}{T/2 - t_{RTO}} + x_p(T/2) \frac{t - t_{RTO}}{T/2 - t_{RTO}} \] (3-82)
\[ y_{RH}(t) = y_{LH}(t) + y_p(t_{RTO}) \frac{T/2 - t}{T/2 - t_{RTO}} + y_p(T/2) \frac{t - t_{RTO}}{T/2 - t_{RTO}} \] (3-83)

Eqs. (3-82) and (3-83) can be used to compute RH coordinates over the first SSP period. In order that the hip coordinates can be obtained over the second SSP, the symmetry assumption can be used which yields \( (t_{LTO} < t < T) \).

\[ x_{LH}(t) = x_{RH}(t - T/2) + S \] (3-84)
\[ y_{LH}(t) = y_{RH}(t - T/2) \] (3-85)
\[ x_{RH}(t) = x_{LH}(t - T/2) + S \] (3-86)
\[ y_{RH}(t) = y_{LH}(t - T/2) \] (3-87)

Once the hip coordinates are known over the entire period of the gait cycle, the desired coordinates of the center of mass of the five links are computed as follows:
\[ x_1 = x_{\text{LH}} - \ell_2 \cos \theta_2 - (\ell_1 - d_1 \cos \theta_1) \] (3-88)

\[ x_2 = x_{\text{LH}} - (\ell_2 - d_2) \cos \theta_2 \] (3-89)

\[ x_3 = \frac{1}{2} (x_{\text{LH}} + x_{\text{RH}}) + d_3 \cos \theta_3 \] (3-90)

\[ x_4 = x_{\text{RH}} - (\ell_4 - d_4) \cos \theta_4 \] (3-91)

\[ x_5 = x_{\text{RH}} - \ell_4 \cos \theta_4 - (\ell_5 - d_5) \cos \theta_5 \] (3-92)

\[ y_1 = y_{\text{LH}} - \ell_2 \sin \theta_2 - (\ell_1 - d_1 \sin \theta_1) \] (3-93)

\[ y_2 = y_{\text{LH}} - (\ell_2 - d_2) \sin \theta_2 \] (3-94)

\[ y_3 = \frac{1}{2} (y_{\text{LH}} + y_{\text{RH}}) + d_3 \sin \theta_3 \] (3-95)

\[ y_4 = y_{\text{RH}} - (\ell_4 - d_4) \cos \theta_4 \] (3-96)

\[ y_5 = y_{\text{RH}} - \ell_4 \cos \theta_4 - (\ell_5 - d_5) \cos \theta_5 \] (3-97)

The addition of a massless pelvis to the basic five-mass model introduces a new relationship among the acting moments. That is, instead of dealing with division of torques between the right and left hips by means of the partition coefficient \( \gamma \), a new equation must be developed. This can be easily done by considering a massless pelvis as shown in Fig. 3.15. The static rotational equilibrium equation for the above segment can be written as

\[ T_3 - \gamma T_3 - T' + \frac{v}{2} (1 - 2\alpha) F_{x3} + \frac{x}{2} (2 \beta - 1) F_{y3} = 0 \] (3-98)

59
Figure 3.15. Free Body Diagram of the Pelvis.
where $X_p$ and $Y_p$ are defined in Eq. (3-80). Eq. (3-98) can be rewritten as:

$$T^* = T_3 - \gamma T_3 + \frac{y_p}{2} (1 - 2\alpha) F_{x3} + \frac{x_p}{2} (2\beta - 1) F_{y3} \tag{3-99}$$

Thus, in order to include the effect of the massless pelvis into dynamic equations for the basic five-mass model, it is only necessary to modify the moment equation associated with the right thigh. By doing this, the matrix $A$ in Fig. 3.8 is modified by adding $y_p \cdot F_{x3}$ to $A(11,3)$ and adding $-x_p \cdot F_{y3}$ to $A(11,7)$. Also, the matrix $B$, shown in Fig. 3.9 must be modified by adding $y_p \cdot F_{x3} - x_p \cdot F_{y3}$ to $B(11)$ so that the effect of the massless pelvis is fully considered.

The above approach was studied for two cases: 1) massless feet included, 2) model does not have any feet. The results for both cases are shown in Tables 3.5 and 3.6, respectively. In both cases, different harmonic methods are employed, and the mean-square error for both vertical and horizontal ground force reactions are computed. From Table 3.5, it is important to notice that error is rather large, although the pelvis is included in the model.

It is interesting to observe that the values of error in Table 3.6, with no feet in the model, are much smaller than values of Table 3.5. This may possibly have two reasons: 1) adding new parts to the model does not necessarily improve the results because the complexity of the skeleton of a human being is much more than that of the model under consideration, and 2) the pelvic motion assumed in Approach #5 involves many approximations and may be over-simplified. What is most important, however, is that in both cases addition of the pelvis,
Table 3.5

Mean Square Error Between Computed Horizontal and Vertical Components of Ground Force Reaction and Direct Force Plate Measurement for Various Number of Harmonics Employed to Smooth Angles \( n \), Angular Rates \( n_1 \) and Angular Accelerations \( n_2 \) for the Method of Approach #5 with Massless Feet Included.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( \text{MSE}(F_x) \times 10^4 ) (Newtons²)</th>
<th>( \text{MSE}(F_y) \times 10^4 ) (Newtons²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>20.6</td>
<td>3.40</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>16.6</td>
<td>6.90</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>13.6</td>
<td>5.07</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>16.0</td>
<td>6.50</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>13.0</td>
<td>4.71</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>11.7</td>
<td>1.59</td>
</tr>
</tbody>
</table>

Table 3.6

Mean Square Error Between Computed Horizontal and Vertical Components of Ground Force Reaction and Direct Force Plate Measurement for Various Number of Harmonics Employed to Smooth Angles \( n \), Angular Rates \( n_1 \) and Angular Accelerations \( n_2 \) for the Method of Approach #5 with No Feet.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( \text{MSE}(F_x) \times 10^4 ) (Newtons²)</th>
<th>( \text{MSE}(F_y) \times 10^4 ) (Newtons²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>19.30</td>
<td>2.32</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>9.42</td>
<td>3.25</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>8.91</td>
<td>1.34</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>8.73</td>
<td>3.18</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>8.41</td>
<td>1.31</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>8.49</td>
<td>1.28</td>
</tr>
</tbody>
</table>
so far, has increased the mean-square error rather than decreasing it as was hoped. Evidently, improvements in the pelvic model are required.

**Approach #6.** The method used in Approach #5, based on linear interpolation for estimation of RH coordinates, Eq. (3-82) and (3-83), is not very accurate. Considering the negative results obtained with this method, it appears that it is necessary to estimate the shape of the pelvic motion curves by some other means. Hartrum [77] discusses this subject in detail and presents results obtained with a kinematic model using a sinusoidal approximation to pelvic motion. While Hartrum’s model, or an even more complex model could be used for this purpose, such alternatives have not been investigated. Rather, in light of the apparent importance of accurate representation of pelvic action, the author decided to assume no *a priori* form for pelvic tilt and rotation, but rather to make a direct measurement of hip motion throughout an entire locomotion cycle, thereby eliminating the modeling entirely. Such an approach implies shifting the reference point for leg motion from the foot to the hip. This can be done, as will be explained in the following, and has some advantages which will be discussed.

In order to measure the angular data for human gait, six lights were placed at the desired points, including the hip point (left hip). The data obtained by TV can be processed to get the motion of the left hip. In this analysis, the location of the left hip at the beginning of the gait cycle (LHS) is considered to be the reference point for all calculations. The procedure to estimate the left hip vertical displacement is the same as that used in previous
approaches for angular data. That is, the vertical displacement of the hip can be assumed to be a periodic function. Fourier series methods can be used both for smoothing and differentiation. To obtain the horizontal component, the procedure is slightly different. That is, horizontal motion involves a small oscillation superimposed on a steady forward progression. Thus

\[ x_{LH} = B_x t + \tilde{x}_{LH} \tag{3-98} \]

where \( \tilde{x}_{LH} \) is the periodic component of \( x_{LH} \). For any complete cycle of gait, \( B_x \) can be computed as

\[ B_x = \frac{x_{LH}(T) - x_{LH}(0)}{T} \tag{3-99} \]

where \( T \) is the gait cycle period. From Eqs. (3-98) and (3-99) \( \tilde{x}_{LH} \) can be computed, smoothed, and then be put back in Eq. (3-98) to get the desired estimate of horizontal position of the left hip. As before, smoothing is accomplished by appropriate truncation of the Fourier series representation of \( \tilde{x}_{LH} \). The derivatives of \( x_{LH} \) are computed by differentiation of the truncated series, yielding

\[ \dot{x}_{LH}(t) = k_x + \ddot{x}_{LH}(t) \tag{3-100} \]

\[ \ddot{x}_{LH}(t) = \dddot{x}_{LH}(t) \tag{3-101} \]

The displacement data for the right hip are obtained from the symmetry assumption. That is, for \( 0 \leq t \leq T/2 \)
\[ x_{RH}(t) = x_{LH}(t + T/2) - S \]  \hfill (3-102)

\[ y_{RH}(t) = y_{LH}(t + T/2) \]  \hfill (3-103)

where \( S \) is the step length. For \( T/2 \leq t \leq T \)

\[ x_{RH}(t) = x_{LH}(t - T/2) + S \]  \hfill (3-104)

\[ y_{RH}(t) = y_{LH}(t - T/2) \]  \hfill (3-105)

Once the hip coordinates are known, Eqs. (3-88) through (3-97) can be used to get the desired positions of center of gravity of all segments of the model. Then, the dynamic equations of the five masses will yield the joint forces and moments.

Application of the above equations to the data used throughout this chapter yields mean square ground reaction force estimation errors as listed in Table 3.7. Comparison of this table to the best results previously obtained, Table 3.4, shows a dramatic improvement in the horizontal MSE. The curves corresponding to Table 3.7, Figs. 3.16 and 3.17, likewise reveal a great improvement when compared to Figs. 3.10 and 3.11. It thus appears that using the hip coordinate as a reference point for calculation of limb segment motions provides some fundamental advantages and this approach will therefore be used throughout the remainder of this dissertation. In particular, besides reducing the MSE, referencing all motion to the hip eliminates entirely the need for modeling the kinematic function of massless feet, a great simplification both from a computation and analytic point of view. The same is of course true with regard to pelvic motion.
Figure 3.16. Horizontal Ground Force Reactions Measured by Force Plate (*) and Calculated from Five-Mass Dynamic Model of Approach #6 (+).
Figure 3.17. Vertical Ground Reactions Measured by Force Plate (*) and Calculated from Five-Mass Dynamic Model of Approach #6 (+).
Table 3.7

Mean Square Error Between Computed Horizontal and Vertical Components of Ground Force Reaction and Direct Force Plate Measurement for Various Number of Harmonics Employed to Smooth Angles (n), Angular Rates (n₁), and Angular Accelerations (n₂) for the Method of Approach #6.

<table>
<thead>
<tr>
<th>n</th>
<th>n₁</th>
<th>n₂</th>
<th>MSE(Fₓ) x 10⁴ (Newtons²)</th>
<th>MSE(Fᵧ) x 10⁴ (Newtons²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>0.203</td>
<td>1.51</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>0.278</td>
<td>1.43</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>0.386</td>
<td>0.568</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>0.256</td>
<td>0.141</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>0.210</td>
<td>0.561</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>0.211</td>
<td>0.564</td>
</tr>
</tbody>
</table>

Despite the fact that Approach #6 shows more than a ten-fold improvement over Approach #1 with regard to mean square estimation errors for both the horizontal and vertical components of ground reaction force, careful examination of Figs. 3.15 and 3.16 show that the estimated forces still differ considerably from the actual forces, and the author feels that still further improvements are needed. At a minimum, both the assumption that the mass of the foot can be lumped with that of the shank, and the assumption that ground reaction forces transfer linearly in time from one supporting let to the other during DSP, need to be examined further. These issues will be addressed in the following Chapter 4.
3.5 Sensitivity Force Estimates to Body Segment Parameter Values

Specifically, each segment of the model has four parameters which are: 1) mass of the segment (M); 2) length of the segment (l), 3) moment of inertia of the segment (J); and 4) distance between the common joint with the previous link and the center of gravity of the segment under consideration (d). These parameters have been estimated by many researchers and the results are well documented [23,89]. Specific parameter values for one of the subjects used in this research are listed in Chapter 7. Using these values and motion data for the this subject, Approach #6 as described above yields the following

\[
\text{MSE}(F_x) = 0.240 \times 10^4
\]  
(3-106)

\[
\text{MSE}(F_y) = 0.314 \times 10^4
\]  
(3-107)

To test the sensitivity of these estimation errors to body segment parameter values, the first parameter to be varied was moment of inertia. Three cases are investigated which change \( J_1(J_5), J_2(J_4) \) and \( J_3 \), respectively. For each case, in one instance 10 percent of the inertia was added, and in the other case 10 percent was subtracted. For all cases, the mean-square errors were computed, and none of them showed any changes at all for such variations in J. This implies that it should be possible to separate the force and torque calculations, thereby reducing the size of the matrix inversion problem implied by Eq. (3-34). This matter is treated in detail in Chapter 4.

The second parameter to be varied is 'd'. Table 3.8 shows the results obtained, and the percentage of the variation for each
Table 3.8

Effects of Variation of $d$ Parameters on Mean Square Error Between Computed Components of the Ground Reaction Force and Direct Force Plate Measurements.

<table>
<thead>
<tr>
<th>Distance to be Varied</th>
<th>MSE($F_x$)</th>
<th>Percent of Variation</th>
<th>MSE($F_y$)</th>
<th>Percent of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1(d_3) + 10%$</td>
<td>$0.248 \times 10^4$</td>
<td>3.33</td>
<td>$0.312 \times 10^4$</td>
<td>-0.64</td>
</tr>
<tr>
<td>$d_1(d_5) - 10%$</td>
<td>$0.235 \times 10^4$</td>
<td>-2.08</td>
<td>$0.318 \times 10^4$</td>
<td>+1.27</td>
</tr>
<tr>
<td>$d_2(d_4) + 10%$</td>
<td>$0.235 \times 10^4$</td>
<td>-2.08</td>
<td>$0.313 \times 10^4$</td>
<td>-0.32</td>
</tr>
<tr>
<td>$d_2(d_4) - 10%$</td>
<td>$0.246 \times 10^4$</td>
<td>2.50</td>
<td>$0.316 \times 10^4$</td>
<td>+0.64</td>
</tr>
<tr>
<td>$d_3 + 10%$</td>
<td>$0.242 \times 10^4$</td>
<td>0.83</td>
<td>$0.314 \times 10^4$</td>
<td>0.0</td>
</tr>
<tr>
<td>$d_3 - 10%$</td>
<td>$0.240 \times 10^4$</td>
<td>0.0</td>
<td>$0.315 \times 10^4$</td>
<td>+0.32</td>
</tr>
</tbody>
</table>

case is also indicated which is positive if the error is larger than the one with exact values. From Table 3.8 one can see that the errors do not change very much when $d_3$ varies. On the other hand, 10 percent change in either $d_1$ or $d_2$ causes about 2 to 3 percent change in the mean-square error (MSE) of the horizontal component.

The third parameter to be altered is length ($\ell$). Table 3.9 tabulates the results obtained by changing the length of each segment by 10 percent. Note that variation of $\ell_3$ does not introduce any difference because $\ell_3$ does not appear in any equations of the system. The MSE of the vertical component varies slightly as either $\ell_1(\ell_3)$ or $\ell_2(\ell_4)$ change, while the MSE of the horizontal component varies a few percent more. This indicates that the horizontal ground reaction force is more sensitive to the variations of the body segment lengths.
Table 3.9

Effects of Variation of \( \xi \) Parameters on Mean Square Error Between Computed Components of the Ground Reaction Force and Direct Force Plate Measurements.

<table>
<thead>
<tr>
<th>Length to be Varied</th>
<th>( \text{MSE}(F_x) )</th>
<th>Percent of Variation</th>
<th>( \text{MSE}(F_y) )</th>
<th>Percent of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_1 (\xi_5) + 10% )</td>
<td>( 0.231 \times 10^4 )</td>
<td>(-3.75)</td>
<td>( 0.322 \times 10^4 )</td>
<td>(+2.55)</td>
</tr>
<tr>
<td>( \xi_1 (\xi_5) - 10% )</td>
<td>( 0.256 \times 10^4 )</td>
<td>(+6.67)</td>
<td>( 0.310 \times 10^4 )</td>
<td>(-1.27)</td>
</tr>
<tr>
<td>( \xi_2 (\xi_4) + 10% )</td>
<td>( 0.257 \times 10^4 )</td>
<td>(+7.08)</td>
<td>( 0.319 \times 10^4 )</td>
<td>(+1.59)</td>
</tr>
<tr>
<td>( \xi_2 (\xi_4) - 10% )</td>
<td>( 0.226 \times 10^4 )</td>
<td>(-5.83)</td>
<td>( 0.311 \times 10^4 )</td>
<td>(-0.96)</td>
</tr>
<tr>
<td>( \xi_3 + 10% )</td>
<td>( 0.240 \times 10^4 )</td>
<td>(0.00)</td>
<td>( 0.314 \times 10^4 )</td>
<td>(0.00)</td>
</tr>
<tr>
<td>( \xi_2 - 10% )</td>
<td>( 0.245 \times 10^4 )</td>
<td>(0.00)</td>
<td>( 0.314 \times 10^4 )</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

The last parameter to be varied is the mass of body segments. The results are tabulated in Table 3.10. The effect of any change of mass, as one goes upward from shank to upper body, increases for both components. However, this variation for the horizontal component is much less than that for the vertical one. Indeed, the MSE of the vertical ground reaction force is extremely sensitive to the mass of the upper body (more than 60 percent increase when \( m_3 \) is reduced by 10 percent).

In conclusion, the horizontal ground reaction force is mostly sensitive to lengths of the lower limbs of the body and also to the mass of the upper body. The vertical component, on the other hand is rather sensitive to the masses of the body segments. The sensitivity is fairly larger when mass of the thighs change, and becomes much larger.
Table 3.10

Effects of Variation of m Parameters of Mean Square Error Between Computed Components of the Ground Reaction Force and Direct Force Plate Measurements.

<table>
<thead>
<tr>
<th>Mass to be Varied</th>
<th>MSE(F_x)</th>
<th>Percent of Variation</th>
<th>MSE(F_y)</th>
<th>Percent of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_1(m_5) + 10%</td>
<td>0.243 x 10^4</td>
<td>+1.25</td>
<td>0.304 x 10^4</td>
<td>-3.18</td>
</tr>
<tr>
<td>m_1(m_5) - 10%</td>
<td>0.236 x 10^4</td>
<td>-0.83</td>
<td>0.332 x 10^4</td>
<td>+5.73</td>
</tr>
<tr>
<td>m_2(m_4) + 10%</td>
<td>0.249 x 10^4</td>
<td>+3.75</td>
<td>0.293 x 10^4</td>
<td>-6.69</td>
</tr>
<tr>
<td>m_2(m_4) - 10%</td>
<td>0.232 x 10^4</td>
<td>-3.33</td>
<td>0.355 x 10^4</td>
<td>+13.06</td>
</tr>
<tr>
<td>m_3 + 10%</td>
<td>0.255 x 10^4</td>
<td>+6.25</td>
<td>0.302 x 10^4</td>
<td>-3.82</td>
</tr>
<tr>
<td>m_3 - 10%</td>
<td>0.227 x 10^4</td>
<td>-5.42</td>
<td>0.503 x 10^4</td>
<td>+60.19</td>
</tr>
</tbody>
</table>

in the case of the variations of the mass of the upper body. As a final overall observation, since the effect of changing body segment parameters is rather small (with the single exceptions of the upper body mass), additional improvements to estimation of joint forces and moments from TV data can be obtained only by a further critical analysis of the assumptions made in deriving the dynamic models of this chapter. Such an analysis is presented in Chapter 4.

3.6 Summary

In this chapter, an attempt is made to study the dynamics of human gait by means of several different mathematical models, each requiring only television data. The free body method is employed to obtain the dynamic equations for each model. Several assumptions
associated with this analysis such as the one concerning the symmetry of the gait and the linear transfer (with respect to time) of the ground reaction forces and torque are described. If the angular and translational kinematic information are known, kinetic quantities associated with the upper body can be directly computed. The remaining equations can be written in a matrix form to be solved to yield various joint forces and torques.

Special attention has been paid to some of the possible approaches used to obtain the translational accelerations associated with the center of mass of limb segments. As the first selection of the reference point, the ankle joint of the supporting leg is considered to be used during each half of the locomotion cycle. However, this approach, which was employed by Gupta [2], generates discontinuities at switching times \( t=0, T/2 \). Also, the ground force reactions computed using this method are quite different from the actual values obtained from direct force plate measurement. As a mathematical solution to this problem, a linear interpolation method is used to calculate the translational accelerations during DSP. Nevertheless, MSE values are still rather large.

The next two attempts which are made to improve the results consist of, i) inclusion of the heel-rise of the supporting leg during its stance phase and, ii) addition of massless feet to the model in a more realistic way. Results indicate that successive reductions of mean square errors are obtained through these two steps. A further modification in gait modeling is obtained by adding a massless pelvis to the model which is also a solution to the discontinuity problem.
already mentioned. As the first choice, the pelvic motion, although known during DSP, is assumed to be obtained by using a linear interpolation method during SSP. This approach fails to improve the results implying that a better method for the pelvic motion measurement is needed. Consequently, in order to overcome this problem, the pelvic motion is obtained from the trajectory of the light attached to the hip joint of the subject using a symmetric assumption. Moreover, the coordinates of the hip light are employed to determine the reference point required in calculation of the translational accelerations. This approach yields the least MSE values, and thus is the best for the basic five-mass dynamic model. Furthermore, a comparison between results obtained from the very first method and the one just mentioned indicates a dramatic improvement in terms of the reduction of the MSE values.

In this chapter, it is also attempted to test the sensitivity of results with respect to the variations of body segment parameters. The nominal value of each parameters is changed $\pm 10\%$, and the corresponding MSE values are obtained. Results show that variations of all moment of inertia parameters do not change the force quantities at all. Moreover, the horizontal ground force reaction is mostly sensitive to the lengths of the lower limbs as well as to the mass of the upper body. On the other hand, the vertical component of the force is rather sensitive to the masses of limb segments, which is expected.
Chapter 4

A SEVEN-MASS DYNAMIC MODEL USING ONLY TELEVISION DATA

4.1 Introduction

As it was pointed out in Chapter 3, one of the purposes of this research is to develop dynamic models of the human body for the estimation of the joint forces and moments. While in the previous chapter the number of segments of the model with nonzero mass was limited to five, this chapter deals with a more complex model which is obtained by adding triangular feet with the corresponding masses to the model under consideration. Needless to mention, the feet have great significance in human gait simply because they are the only segments in touch with the walking surface. In fact, the ground reaction forces are directly applied to the feet and due to the importance of the point of application (center of pressure), feet perform very significant roles in the dynamics of human gait.

This chapter describes a seven-mass, eight-link model. An attempt is made to show the complexities introduced by adding new segments to the previous model. Next, a method is developed to break down the dynamic equations of the system and a much simpler approach is presented. The method not only saves computer time and avoids dealing with a set of 18 equations and matrix inversion, but also explains the dependencies of various quantities upon the other ones and shows exactly how the equations can be decomposed. This
decomposition helps the author to tackle the hypothesis of linear transformation of the ground reaction forces and torques (with respect to time) during the double support phase.

4.2 Description of the Model

The dynamic model described in this chapter consists of seven segments of nonzero mass including two feet, two shanks, two thighs and the trunk which consists of the upper body, arms and head all lumped as a single segment. Moreover, a massless pelvis with variable length is added to the model which is similar to what was discussed and used in Chapter 3. A computer-television system [8] is used to get the data required for the model. As explained earlier, the subject is fitted with six lights, and wears athletic clothing. The details of the experimental methods are explained in Chapter 6. In this method, the displacement data of the hip joint, as well as the angular data corresponding to various segments of the body are used. Both the angular and displacement information are obtained for one period of the gait cycle. This period as well as the toe-off time are determined similarly to what was explained in Chapter 3. Furthermore, all the subjects are assumed to be normal which implies that a good symmetry exists between the right leg and the left leg.

Fig. 4.1 illustrates a stick figure of a seven-mass, eight-link model. The horizontal axis is considered to be the reference, and the positive direction of the angles is clockwise. The computer-television system can provide the coordinates of the left hip \((x_{LH}, y_{LH})\) as well as four angles \((\theta_1, \theta_2, \theta_3, \theta_4)\) for one complete gait cycle. As for the five-mass model, the rest of the data are constructed by using the
Figure 4.1. Stick Figure of Seven-Mass Model.
symmetry assumption as follows:

For $0 \leq t < T/2$

$$x_{RH}(t) = x_{LH}(t + T/2) - S \quad (4-1)$$

$$y_{RH}(t) = y_{LH}(t + T/2) \quad (4-2)$$

$$\theta_4(t) = \theta_3(t + T/2) \quad (4-3)$$

$$\theta_5(t) = \theta_2(t + T/2) \quad (4-4)$$

$$\theta_6(t) = \theta_1(t + T/2) \quad (4-5)$$

For the second half of the gait cycle $x_{RH}$, $\theta_4$, $\theta_5$, and $\theta_6$ are constructed by evaluating $x_{LH}$, $\theta_3$, $\theta_2$, and $\theta_1$ at $(t - T/2)$, respectively.

Next, Fourier analysis with a specific number of harmonics is applied to smooth the data. This also has the advantage of obtaining the first and second derivatives directly.

In Chapter 3, independent motion of feet with nonzero mass was not considered. Rather, the mass of the foot was lumped with that of the shank. Although this is rather far from reality, for gait studies, it can be considered as a first approximation to the human body. In fact, the mass of each foot is small compared with the mass of the corresponding shank. This is why the above assumption could be made. On the other hand, the motion of each foot is very significant and rather complex while it is in touch with the walking surface. In order to have a good model of the foot, it is not wise to represent it similar to other segments because 1) the length of the foot is not
much larger than its other dimension, and 2) it is important to have a good model of the foot because it is directly involved with the location of the center of pressure as well as the estimated ground reaction forces.

Fig. 4.2 shows three possible models for the foot all based on triangular forms. Among these three models, (a) is the simplest one, (b) is the one which is used by the author, and (c) is the most complex one. The mass of the foot is assumed to be uniformly distributed, which allows the corresponding center of mass to be determined. The top point of the triangle foot is connected to the associated shank, and the joint is assumed to be pin-like, similar to the others. Obviously, the center of pressure moves from heel to toe as the foot goes through a heel-strike to a toe-off process.

![Figure 4.2. Alternative Models for the Foot: (a) straight-line segments, (b) triangular configuration with implicit toes, (c) triangular configuration with explicit toes.](image)

Like the five-mass model, the seven-mass model is completely deterministic during the single support phase due to the absence of closed kinematic chains. However, during DSP, it is again necessary either to make some assumption regarding the manner in which the forces are distributed between the two legs or to make a direct
measurement of ground reaction forces. Both of these cases will be treated in this chapter.

4.3 Mathematical Formulation and Comparison with Direct Force Plate Measurement

This section is divided into two parts. The first part deals with mathematical aspects of the modeling of the human body and the equations of motion. In the second part, experimental data are applied to the model and the quality, acceptance, or rejection of the results are discussed.

4.3.1 Mathematical Modeling

Because of its simplicity and ease of application, the free-body approach to dynamic modeling of the musculoskeletal system will again be used in this chapter. With this approach, the equations for the upper body can be independently solved because it is assumed that the top of this segment is not in touch with any object. The rest of the equations, however, are more or less dependent on each other. The number of these equations was 12 for the five-mass model and is increased to 18 for the current system. According to the conventional methods, this set of equations are to be solved simultaneously. Consequently, one can write these equations in matrix form and use either a matrix inversion method or some equivalent procedure for numerical solution of linear simultaneous equations. It turns out, however, that analytic solution of this system of equations is possible and the numerical matrix inversion problem can be avoided entirely. The greater part of the balance of this chapter is devoted to the derivation of this analytic solution.
All the symbols which are used in conjunction with the system of Fig. 4.1 together with their corresponding definitions are listed in Table 4.1. In order to obtain dynamic equations of motion for each link, it is necessary to get the displacement accelerations as well as the angular acceleration. Therefore, as for the five-mass model, a coordinate system is required. The position of the left hip can be directly measured and therefore it is chosen as the reference point. The process of hip motion including smoothing for both its horizontal

Table 4.1

Table of Symbols Used in the Seven-Mass Model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>mass of the segment</td>
</tr>
<tr>
<td>ℓ</td>
<td>length of the segment</td>
</tr>
<tr>
<td>J</td>
<td>moment of inertia of the segment</td>
</tr>
<tr>
<td>d</td>
<td>distance from the common joint with the proximal limb to the C.G. of the link under consideration</td>
</tr>
<tr>
<td>e</td>
<td>difference between ℓ and d of the segment</td>
</tr>
<tr>
<td>θ</td>
<td>angle with respect to the horizontal axis</td>
</tr>
<tr>
<td>G</td>
<td>acceleration due to gravity</td>
</tr>
<tr>
<td>x</td>
<td>horizontal displacement</td>
</tr>
<tr>
<td>y</td>
<td>vertical displacement</td>
</tr>
<tr>
<td>F</td>
<td>force</td>
</tr>
<tr>
<td>T</td>
<td>torque</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>H</td>
<td>hip</td>
</tr>
<tr>
<td>R</td>
<td>right</td>
</tr>
<tr>
<td>L</td>
<td>left</td>
</tr>
<tr>
<td>q</td>
<td>partition coefficient of the ground reaction force</td>
</tr>
<tr>
<td>p</td>
<td>1 - q</td>
</tr>
</tbody>
</table>
and vertical coordinates, and construction of the right hip motion using the stride length have been carefully explained in the previous chapter, and therefore will not be discussed here. Once the positions of both left hip \((x_{LH}, y_{LH})\) and the right hip \((x_{RH}, y_{RH})\) are known, they can be used to determine the position of the center of gravity for other segments. Figs. 4.3 through 4.9 show the free body diagrams of the links with nonzero mass which can be used to obtain the desired coordinates as follows:

i) Upper Body

\[
x_7 = \frac{1}{2} (x_{LH} + x_{RH}) + d_7 \cos \theta_7
\]

\[
y_7 = \frac{1}{2} (y_{LH} + y_{RH}) + d_7 \sin \theta_7
\]

ii) Left Thigh and Right Thigh

\[
x_3 = x_{LH} - e_3 \cos \theta_3
\]

\[
y_3 = y_{LH} - e_3 \sin \theta_3
\]

\[
x_4 = x_{RH} - e_4 \cos \theta_4
\]

\[
y_4 = y_{RH} - e_4 \sin \theta_4
\]

iii) Left Shank and Right Shank

\[
x_2 = x_{LH} - l_3 \cos \theta_3 - e_2 \cos \theta_2
\]

\[
y_2 = y_{LH} - l_3 \sin \theta_3 - e_2 \sin \theta_2
\]
Figure 4.3. Free Body Diagram for Upper Body.

Figure 4.4. Free Body Diagram for Left Thigh.

Figure 4.5. Free Body Diagram for Right Thigh.
Figure 4.6. Free Body Diagram for Left Shank.

Figure 4.7. Free Body Diagram for Right Shank.

Figure 4.8. Free Body Diagram for Left Foot.

Figure 4.9. Free Body Diagram for Right Foot.
\[ x_5 = x_{RH} - l_4 \cos \theta_4 - e_5 \cos \theta_5 \]  
\[ y_5 = y_{RH} - l_4 \sin \theta_4 - e_5 \sin \theta_5 \]  

iv) Left Foot and Right Foot.

In determination of the center of gravity of each foot, one needs to be more careful. If one can assume that the mass of the triangular foot is uniformly distributed over its area, the position of C.G. is easily computed. Fig. 4.10 shows such a model. Knowing three parameters \( a, b, \) and \( h, \) one can show that

\[ \bar{X} = \frac{a + b}{3} \]  
\[ \bar{Y} = \frac{h}{3} \]

where \( \bar{X} \) and \( \bar{Y} \) are the desired coordinates. Consequently, the distance between C.G. of the foot and its ankle joint which is shown as "\( \rho_1 \)" (Figs. 4.8 and 4.9) as well as the angle "\( \omega_1 \)" can be easily measured, which leads to the following equations:

![Diagram](image)

Figure 4.10. Position of Center of Gravity for a Uniform Triangle.
\[ x_1 = x_{\text{LH}} - \ell_3 \cos \theta_3 - \ell_2 \cos \theta_2 - \rho_1 \cos(\theta_1 - \omega_1) \quad (4-17) \]

\[ y_1 = y_{\text{LH}} - \ell_3 \sin \theta_3 - \ell_2 \sin \theta_2 - \rho_1 \sin(\theta_1 - \omega_1) \quad (4-18) \]

\[ x_6 = x_{\text{RH}} - \ell_4 \cos \theta_4 - \ell_5 \cos \theta_5 - \rho_1 \cos(\theta_6 - \omega_1) \quad (4-19) \]

\[ y_6 = y_{\text{RH}} - \ell_4 \sin \theta_4 - \ell_5 \sin \theta_5 - \rho_1 \sin(\theta_6 - \omega_1) \quad (4-20) \]

Eqs. (4-5) through (4-14) and (4-17) through (4-20) can be differentiated to get the displacement accelerations which are required to solve the dynamic equations of the system. As an example, the displacement accelerations of center of gravity of the left foot will be expressed as follows:

\[ \ddot{x}_1 = \ddot{x}_{\text{LH}} + \ell_3 (\cos \theta_3 \cdot \dot{\theta}_3^2 + \sin \theta_3 \cdot \ddot{\theta}_3) + \ell_2 (\cos \theta_2 \cdot \dot{\theta}_2^2 + \sin \theta_2 \cdot \ddot{\theta}_2) 
+ \rho_1 (\cos(\theta_1 - \omega_1) \cdot \dot{\theta}_1^2 + \sin(\theta_1 - \omega_1) \cdot \ddot{\theta}_1) \quad (4-21) \]

\[ \ddot{y}_1 = \ddot{y}_{\text{LH}} + \ell_3 (\sin \theta_3 \cdot \dot{\theta}_3^2 - \cos \theta_3 \cdot \ddot{\theta}_3) + \ell_2 (\sin \theta_2 \cdot \dot{\theta}_2^2 - \cos \theta_2 \cdot \ddot{\theta}_2) 
+ \rho_1 (\sin(\theta_1 - \omega_1) \cdot \dot{\theta}_1^2 - \cos(\theta_1 - \omega_1) \cdot \ddot{\theta}_1) \quad (4-22) \]

The rest of the accelerations can be obtained in the same way. The corresponding equations for all other limb segments can be found in the computer programs listed in Appendix D.

The dynamic equations of the system can be presented in general form of D'Alembert's equations (Eqs. (3-9) through (3-11)). By applying these equations to each segment of the body, the following results will be obtained using the symbolic configurations shown in Figs. 4.3 through 4.9.
i) Upper Body:

\[ F_{x1} = m_2 \ddot{x}_2 \]  \hspace{1cm} (4-23)

\[ F_{y1} = m_2 \ddot{y}_2 \]  \hspace{1cm} (4-24)

\[ T_7 = J_7 \ddot{\theta}_7 + F_{y7} \cdot d_7 \cos \theta_7 - F_{x7} \cdot d_7 \sin \theta_7 \]  \hspace{1cm} (4-25)

ii) Left Thigh:

\[ F_{x2} - F_{x3} = m_3 \ddot{x}_3 \]  \hspace{1cm} (4-26)

\[ F_{y2} - F_{y3} = m_3 \ddot{y}_3 \]  \hspace{1cm} (4-27)

\[ T_2 - T_3 + (d_3 F_{x2} + e_3 F_{x3}) \sin \theta_3 \]

\[- (d_3 F_{y2} + e_3 F_{y3}) \cos \theta_3 = J_3 \ddot{\theta}_3 \]  \hspace{1cm} (4-28)

iii) Right Thigh:

\[ F_{x5} - F_{x4} = m_4 \ddot{x}_4 \]  \hspace{1cm} (4-29)

\[ F_{y5} - F_{y4} = m_4 \ddot{y}_4 \]  \hspace{1cm} (4-30)

\[ T_5 - T_4 + (d_4 F_{x5} + e_4 F_{x4}) \sin \theta_4 \]

\[- (d_4 F_{y5} + e_4 F_{y4}) \cos \theta_4 = J_4 \ddot{\theta}_4 \]  \hspace{1cm} (4-31)

iv) Left Shank:

\[ F_{x1} - F_{x2} = m_2 \ddot{x}_2 \]  \hspace{1cm} (4-32)
\[ F_{y1} - F_{y2} = m_2 \ddot{y}_2 \quad (4-33) \]

\[ T_1 - T_2 + (d_2 F_{x1} + e_2 F_{x2}) \sin \theta_2 \]
\[ - (d_2 F_{y1} + e_2 F_{y2}) \cos \theta_2 = J_2 \ddot{\theta}_2 \quad (4-34) \]

v) Right Shank:

\[ F_{x6} - F_{x5} = m_5 \ddot{x}_5 \quad (4-35) \]

\[ F_{y6} - F_{y5} = m_5 \ddot{y}_5 \quad (4-36) \]

\[ T_6 - T_5 + (d_5 F_{x6} + e_5 F_{x5}) \sin \theta_5 \]
\[ - (d_5 F_{y6} + e_5 F_{y5}) \cos \theta_5 = J_5 \ddot{\theta}_5 \quad (4-37) \]

vi) Left Foot:

\[ q_x F_{x0} - F_{x1} = m_1 \ddot{x}_1 \quad (4-38) \]

\[ q_y F_{y0} - F_{y1} = m_1 \ddot{y}_1 \quad (4-39) \]

\[ -T_1 + \rho_1 \sin(\alpha_1 - \omega_1) \cdot F_{x1} - \rho_1 \cdot F_{y1} \cdot \cos(\alpha_1 - \omega_1) \]
\[ + q_y F_{y0} \cdot C_1 + q_x \cdot F_{x0} \cdot C_2 = J_1 \ddot{\theta}_1 \quad (4-40) \]

Note that "\( \omega_1 \)" and "\( \rho_1 \)" are known parameters and are shown in Fig. 4.8. \( F_{x0} \) and \( F_{y0} \) are total ground reaction forces applied to the body in horizontal and vertical directions, respectively, \( q_x \) and \( q_y \) are the partition coefficients which correspond to \( F_{x0} \) and \( F_{y0} \), respectively.
and are associated with the left foot. These two coefficients are known during the single support phase (SSP) and unknown during the double support phase (DSP). For this reason, it has been necessary to make some sort of assumption about the values of \( q_x \) and \( q_y \) for DSP. In the case of the five-mass model, it was assumed that \( q_x = q_y = q \) where \( q \) is a linear function of time and is expressed in Eq. (3-19). To accomplish the current research, it is necessary to prove or disprove such an assumption. A quantitative expression of \( q_x \) and \( q_y \) will be presented later in this chapter.

There are two new quantities in Eq. (4-4), namely \( CL_1 \) and \( CL_2 \). The former is an unknown and represents the position of the center of pressure on the bottom of the foot. That is, it yields the instantaneous distance between vertical ground reaction force on the left foot and the vertical projection of its C.G. The latter is a quantity which indicates the perpendicular distance of the C.G. of the left foot from horizontal component of the ground force reaction. The variable, \( CL_2 \), can be computed by using two configurations of the triangular foot, as shown in Fig. 4.11. Note that all parameters indicated in this figure such as, \( \rho_2, \rho_3, \omega_2, \omega_3, \) and \( \beta \) can be measured and thus are known.

Since the process of walking has been chosen to begin with the heel-strike, it is appropriate to first consider Fig. 4.11(b).

\[
\delta(t) = \beta - (\pi - \theta_1(t)) \tag{4-41}
\]

\[
CL_2(t) = \rho_3 \sin(\omega_3 + \delta(t)) \tag{4-42}
\]

Note that the state of the left foot gradually changes to a foot-flat situation. That is, \( CL_2 \) approaches its minimum value \((\rho_3 \sin \omega_3)\).
Figure 4.11. Left Foot Configuration During Stance Phase: (a) Toe Off, (b) Heel Strike.

This situation can be easily determined by checking angle "$\delta$" whose value is positive at heel-strike time and decreases to zero at foot-flat situation. During this phase (I), $CL_2$ can be obtained as expressed in Eq. (4-42). After foot-flat, the foot is gradually lifted forward and angle "$\delta$" becomes negative. This phase (II) continues, as shown in Fig. 4.11(a), until toe off occurs. Thus,

$$CL_2(t) = \rho_2 \sin[\pi - \theta_1(t) - \omega_2] = \rho_2 \sin[\theta_1(t) + \omega_2]$$  \hspace{1cm} (4-43)

Obviously, constantly checking with angle "$\delta$" is necessary in order that one can recognize when Phase I ends and Phase II starts.

While $CL_2$ can be computed by either Eq. (4-42) or (4-43), determination of $CL_1$ is not as easy and requires that the set of equations of the system be solved. However, an intuitive observation of the model concludes that $CL_1$ has a value of '-$d_B$' at heel-strike (see Fig. 4.8). As the process of walking continues, it becomes zero and continues by taking positive values, and finally it reaches its maximum value '+$d_F$' at toe-off time.
vii) Right Foot:

\[ P_{x x0} - F_{x6} = m_6 \ddot{x}_6 \]  \hspace{1cm} (4-44)

\[ P_{y y0} - F_{y6} = m_6 \ddot{y}_6 \]  \hspace{1cm} (4-45)

\[-T_6 + \rho_1 \sin(\theta_6 - \omega_1) \cdot F_{x6} - \rho_1 \cdot F_{y6} \cdot \cos(\theta_6 - \omega_1) \]

\[ + p_y F_{y0 CR_1} + p_x F_{x0 CR_2} = J_6 \ddot{\theta}_6 \]  \hspace{1cm} (4-46)

where \( \rho_1 \) and \( \omega_1 \) are identical to the ones expressed in Eq. (4-40), while \( p_x \) and \( p_y \) are partition coefficients of horizontal and vertical components of ground reaction forces associated with the right foot, respectively. Note that:

\[ p_x + q_x = 1 \]  \hspace{1cm} (4-47)

and

\[ p_y + q_y = 1 \]

Like the left foot, \( CR_1 \) and \( CR_2 \) represent the perpendicular distance of C.G. of the right foot from vertical and horizontal components of the associated ground reaction forces, respectively.

Fig. 4.12 shows the right foot in two extreme cases. One can use the same argument applied to the left foot in order to get the expression for \( CR_2 \). Specifically, the right foot elevation angle, \( \gamma(t) \), is given by:

\[ \gamma(t) = \beta - (\pi - \theta_6(t)) \]  \hspace{1cm} (4-48)

For \( \gamma(t) \geq 0 \), it follows that

\[ CR_2(t) = \rho_3 \sin(\omega_3 + \gamma(t)) \]  \hspace{1cm} (4-49)
while for $\gamma(t) \leq 0$

$$CR_2 = \rho_2 \sin(\pi - \theta_6(t) - \omega_2) = \rho_2 \sin(\theta_6(t) + \omega_2)$$  \hspace{1cm} (4-5)

Like $CL_1$, $CR_1$ is an unknown quantity and can be obtained by solving the dynamic equations of the system.

In addition to the above equations for the limb segments with nonzero mass, there are three other equations describing the equilibrium of the forces and torques applied to the massless pelvis. These equations can be derived by considering Fig. 4.13.

---

**Figure 4.12.** Right Foot Configuration During Stance Phase: (a) Toe Off, (b) Heel Strike.

**Figure 4.13.** Free Body Diagram for Massless Pelvis.
\[ F_{x7} = F_{x3} + F_{x4} \quad (4-51) \]
\[ F_{y7} = F_{y3} + F_{y4} \quad (4-52) \]
\[ T_7 - T_3 - T_4 + \left( F_{x4} - F_{x3} \right) \frac{y_p}{2} + \left( F_{y3} - F_{y4} \right) \frac{x_p}{2} = 0 \quad (4-53) \]

where \( x_p \) and \( y_p \) can be defined as:
\[ x_p = x_{RH} - x_{LH} \quad (4-54) \]
\[ y_p = y_{RH} - y_{LH} \quad (4-55) \]

The following comments are important in solving the set of equations associated with the seven-mass dynamic model of the human body.

1. The upper body can be considered independently, which implies that the corresponding three equations (4-23, 4.24, and 4-25) can be directly solved. This yields \( F_{x7} \), \( F_{y7} \) and \( T_7 \).

2. Instead of defining partition coefficients related to the quantities at the hip joint, namely \( \alpha \), \( \beta \), and \( \gamma \), (as seen in Chapter 3) all forces and torques are explicitly defined here. This indeed makes a more uniform set of variables and directly yields the explicit values of forces and torques rather than implicit ones.

3. During single support phase (SSP), the set of equations of the system is fully deterministic. That is, number of unknowns is equal to the number of equations. However, this is not true for the double support phase (DSP). This requires some sort of assumptions which must be made if the system is to become fully determined. In
this chapter, $q_x$, $q_y$, and either $CL_1$ or $CR_1$ are the quantities whose values must somehow be estimated before one attempts to solve the equations. In all work until now it has been assumed that $q_x$ and $q_y$ are linear functions of time. Later in this chapter, force plate measurements are used to investigate the validity of such assumptions. Moreover, $CL_1$ and $CR_1$ are considered to be known and unknown alternately. The assumptions associated with these variables which will be discussed later, must be such that the system has a symmetric behavior.

4. Once all the assumptions are made, and the upper body components are determined, there will be twenty-one equations and twenty-one unknowns as follows:

$$\bar{x}^T = [F_{x0}, F_{x1}, F_{x2}, F_{x3}, F_{x4}, F_{x5}, F_{x6}, F_{y0}, F_{y1}, F_{y2}, F_{y3}, F_{y4}, F_{y5}, F_{y6}, C_1, T_1, T_2, T_3, T_4, T_5, T_6]$$ \hspace{1cm} (4-56)

where $C_1$ represents either $CL_1$ or $CR_1$ depending on which one is the unknown. In Chapter 3, the corresponding equations were of dimension twelve and were dealt with as a whole. This requires a large amount of both computer time and memory which is desirable to avoid even if one does not have to utilize matrix inversion. Fortunately, the author has found that the equations can be decomposed into three semi-independent sets of equations, each with seven unknowns, listed as follows:

$$\bar{x}_1^T = [F_{x0}, F_{x1}, F_{x2}, F_{x3}, F_{x4}, F_{x5}, F_{x6}]$$ \hspace{1cm} (4-57)
\[
\begin{align*}
\mathbf{x}_2^T &= [F_{y0}, F_{y1}, F_{y2}, F_{y3}, F_{y4}, F_{y5}, F_{y6}] \\
\mathbf{x}_3^T &= [C_1, T_1, T_2, T_3, T_4, T_5, T_6]
\end{align*}
\] (4-58) (4-59)

The term 'semi-independent' implies both the first and the second sets of equations can be solved completely independently. However, the third set requires the solutions from the first two to be substituted in its equations. This process does not raise any problems.

There are two advantages associated with the new system of equations: (1) a much smaller memory space is required, and three sets of equations can be solved consecutively, (2) it is interesting to note the dependencies of equations upon each other. Obviously, the sets of the horizontal force components and vertical force components are both independent of all torques and also each other. On the other hand, torque equations are more complex and do depend upon forces as expected. The matrix representations of these three sets of equations are presented as follows:

(a) Horizontal Forces:

\[
\begin{bmatrix}
q_x & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & +1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & +1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & +1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & +1 \\
p_x & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & +1 & +1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
F_{x0} \\
F_{x1} \\
F_{x2} \\
F_{x3} \\
F_{x4} \\
F_{x5} \\
F_{x6}
\end{bmatrix}
= \begin{bmatrix}
m_1\ddot{x}_1 \\
m_2\ddot{x}_2 \\
m_3\ddot{x}_3 \\
m_4\ddot{x}_4 \\
m_5\ddot{x}_5 \\
m_6\ddot{x}_6 \\
m_7\ddot{x}_7
\end{bmatrix}
\] (4-60)
(b) Vertical Forces

\[
\begin{bmatrix}
q_y & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & +1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & +1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & +1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & +1 \\
p_y & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & +1 & +1 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
F_{y0} \\
F_{y1} \\
F_{y2} \\
F_{y3} \\
F_{y4} \\
F_{y5} \\
F_{y6}
\end{bmatrix}
= \begin{bmatrix}
m_1(\ddot{y}_1 + g) \\
m_2(\ddot{y}_2 + g) \\
m_3(\ddot{y}_3 + g) \\
m_4(\ddot{y}_4 + g) \\
m_5(\ddot{y}_5 + g) \\
m_6(\ddot{y}_6 + g) \\
m_7(\ddot{y}_7 + g)
\end{bmatrix}
\tag{4-61}
\]

(c) Torques:

In order to write the matrix form of the torque equations, it is necessary to point out the changes which must be applied to them within a complete gait cycle. In other words, although there are seven unknowns at any time of the cycle, one of them and thus one of the equations changes at certain times. This is due to the problem concerning the determination of the center of pressure on both feet, which has already been mentioned. These two quantities, namely CL₁ and CR₁, are both unknown during DSP, and as it was pointed out before, there should be an assumption about the numerical value of one of them so that the other one can be computed. Furthermore, the symmetric feature of the system must be valid over the whole gait period.

One possible solution is based on the assumption that the center of pressure on each foot stays at the heel for a certain period of time, say τₜ, after heel-strike occurs and similarly, it goes and
stays at the toe a little before the corresponding toe-off (TO) happens. Fig. 4.14 illustrates the variations of $CL_1$ and $CR_1$ with respect to time based on the above assumptions. Note that $\tau_D$ represents the period of DSP in each half of the gait cycle and $\tau_S (0 \leq \tau_S \leq \tau_D)$ indicates when the switching must happen. Table 4.2 shows features of the above assumption. Note that $U$ stands for 'unknown' and indicates the variable for which the equations must be solved. Note also that $CL_1$ and $CR_1$ do keep the symmetry of the system, and only one of them is unknown at a time. That is, since there is exactly one unknown between $CL_1$ and $CR_1$, there is no need to introduce a torque partition coefficient as was done in Chapter 3. This is one of the advantages gained from the explicit foot model used in the present chapter.

![Diagram](image_url)

**Figure 4.14.** Assumed Numerical Values for $CL_1$ and $CR_1$ During Gait Cycle.
Table 4.2
Switching Assumption of Center of Pressure for One Gait Period

<table>
<thead>
<tr>
<th>Time</th>
<th>$C_L$</th>
<th>$C_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, $\tau_S$</td>
<td>$-d_B$</td>
<td>U</td>
</tr>
<tr>
<td>$\tau_S$, $\tau_D$</td>
<td>U</td>
<td>$+d_F$</td>
</tr>
<tr>
<td>$\tau_D$, T/2</td>
<td>U</td>
<td>0</td>
</tr>
<tr>
<td>T/2, T/2+$\tau_S$</td>
<td>U</td>
<td>$-d_B$</td>
</tr>
<tr>
<td>T/2+$\tau_S$, T/2+$\tau_D$</td>
<td>$+d_F$</td>
<td>U</td>
</tr>
<tr>
<td>T/2+$\tau_D$, T</td>
<td>0</td>
<td>U</td>
</tr>
</tbody>
</table>

In order to show the matrix representation of joint torques, one should pay attention to the fact that the set of seven equations and seven unknowns does not stay the same during one gait period and thus makes it a little difficult to express the system of equations in a single matrix form. However, one solution is to consider eight variables instead of seven in which the known variable can be identified according to Table 4.2. The result is shown on the following page. $U$ indicates the unknown quantity.

5. The matrix forms of the equations of the system can be even more simplified. This will yield explicit expressions for the unknown quantities in terms of the known parameters of the body and accelerations of various segments. The resulting equations associated with each individual variable are listed as follows:
\[
\begin{bmatrix}
+1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & +1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & +1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & +1 & 0 & 0 \\
0 & 0 & +1 & +1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & q_y F_y0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & p_y F_y0 \\
\end{bmatrix}
\]

\[
J_2 \ddot{\theta}_2 = (d_2 F_{x1} + e_2 F_{x2}) \sin \theta_2 + (d_2 F_{y1} + e_2 F_{y2}) \cos \theta_2
\]

\[
J_3 \ddot{\theta}_3 = (d_3 F_{x2} + e_3 F_{x3}) \sin \theta_3 + (d_3 F_{y2} + e_3 F_{y3}) \cos \theta_3
\]

\[
J_4 \ddot{\theta}_4 = (d_4 F_{x4} + e_4 F_{x4}) \sin \theta_4 + (d_4 F_{y5} + e_4 F_{y4}) \cos \theta_4
\]

\[
J_5 \ddot{\theta}_5 = (d_5 F_{x5} + e_5 F_{x5}) \sin \theta_5 + (d_5 F_{y5} + e_5 F_{y5}) \cos \theta_5
\]

\[
J_7 \ddot{\theta}_7 = d_7 (F_{y7} \cos \theta_7 - F_{x7} \sin \theta_7) + (F_{x4} - F_{x3}) \frac{y_p}{2} + (F_{y3} - F_{y4}) \frac{x_2}{2}
\]

\[
J_1 \ddot{\theta}_1 = \rho_1 (-F_{x1} \cdot \sin(\theta_1 - \omega_1) + F_{y1} \cdot \cos(\theta_1 - \omega_1) - q_x \cdot F_{x0} \cdot CL_2
\]

\[
J_6 \ddot{\theta}_6 = \rho_1 (-F_{x6} \cdot \sin(\theta_6 - \omega_1) + F_{y6} \cdot \cos(\theta_6 - \omega_1) - p_x \cdot F_{x0} \cdot CR_2
\]

\[
F_{x1} = q_x \sum_{j=1}^{7} m_j \ddot{x}_j - m_1 \ddot{x}_1
\]

\[
F_{x2} = q_x \sum_{j=1}^{7} m_j \ddot{x}_j - \sum_{j=1}^{2} m_j \ddot{x}_j
\]

\[
F_{x3} = q_x \sum_{j=1}^{7} m_j \ddot{x}_j - \sum_{j=1}^{3} m_j \ddot{x}_j
\]

\[\text{(4-62)}\]

\[\text{(4-63)}\]

\[\text{(4-64)}\]

\[\text{(4-65)}\]
\[ F_{x4} = p_x \sum_{j=1}^{7} m_j \dddot{x}_j - \sum_{j=4}^{6} m_j \dddot{x}_j \]  \hspace{1cm} (4-66)

\[ F_{x5} = p_x \sum_{j=1}^{7} m_j \dddot{x}_j - \sum_{j=5}^{6} m_j \dddot{x}_j \]  \hspace{1cm} (4-67)

\[ F_{x6} = p_x \sum_{j=1}^{7} m_j \dddot{x}_j - m_6 \dddot{x}_6 \]  \hspace{1cm} (4-68)

\[ F_{x0} = \sum_{j=1}^{7} m_j \dddot{x}_j \]  \hspace{1cm} (4-69)

Eq. (4-69) indicates the important fact that the total horizontal ground reaction force is equal to the sum of the horizontal forces due to accelerations of the individual segments. Similarly, for the vertical forces

\[ F_{y1} = q_y \sum_{j=1}^{7} m_j (\dddot{y}_j + g) - m_1 (\dddot{y}_1 + g) \]  \hspace{1cm} (4-70)

\[ F_{y2} = q_y \sum_{j=1}^{7} m_j (\dddot{y}_j + g) - \sum_{j=1}^{2} m_j (\dddot{y}_j + g) \]  \hspace{1cm} (4-71)

\[ F_{y3} = q_y \sum_{j=1}^{7} m_j (\dddot{y}_j + g) - \sum_{j=1}^{3} m_j (\dddot{y}_j + g) \]  \hspace{1cm} (4-72)

\[ F_{y4} = p_y \sum_{j=1}^{7} m_j (\dddot{y}_j + g) - \sum_{j=4}^{6} m_j (\dddot{y}_j + g) \]  \hspace{1cm} (4-73)
\[ F_{y5} = p_y \sum_{j=1}^{7} m_j (\ddot{y}_j + g) - \sum_{j=5}^{6} m_j (\ddot{y}_j + g) \quad (4-74) \]

\[ F_{y6} = p_y \sum_{j=1}^{7} m_j (\ddot{y}_j + g) - m_6 (\ddot{y}_j + g) \quad (4-75) \]

\[ F_{y0} = \sum_{j=1}^{7} m_j (\ddot{y}_j + g) \quad (4-76) \]

Likewise, Eq. (4-76) explains the fact that the total vertical ground reaction force is equal to the sum of all gravitational forces as well as forces due to accelerations of various segments.

It is possible to write the new explicit equations in a more general form because of the similarity which exists among them:

\[ F_{xi} = q_x \sum_{j=1}^{7} m_j \dddot{x}_j - \sum_{j=1}^{1} m_j \dddot{x}_j \quad i=1,2,3 \quad (4-77) \]

\[ F_{xi} = p_x \sum_{j=1}^{7} m_j \dddot{x}_j - \sum_{j=1}^{6} m_j \dddot{x}_j \quad i=4,5,6 \quad (4-78) \]

\[ F_{yi} = q_y \sum_{j=1}^{7} m_j (\ddot{y}_j + g) - \sum_{j=1}^{1} m_j (\ddot{y}_j + g) \quad i=1,2,3 \quad (4-79) \]

\[ F_{yi} = p_y \sum_{j=1}^{7} m_j (\ddot{y}_j + g) - \sum_{j=1}^{6} m_j (\ddot{y}_j + g) \quad i=4,5,6 \quad (4-80) \]
Eqs. (4-76) through (4-80) together with Eq. (4-69) provide very simple and interesting forms to express the entire joint forces associated with the seven-mass model of the human body. Note that horizontal and vertical components are completely independent of each other. Furthermore, the partition coefficients \( q_x(p_x) \) and \( q_y(p_y) \) appear in all equations. This emphasizes the importance of the assumptions made about the coefficients during DSP.

In order to investigate the distribution of the ground reaction forces and obtain the corresponding expressions for \( q_x \) and \( q_y \), it is necessary to measure the forces by some other methods. Fortunately, the author had access to a force plate which provides the required ground reaction forces. On the other hand, the simplified equations which have just been obtained can be used to get an expression for each of the partition coefficients. Let \( R_x \) and \( R_y \) represent the true values of horizontal and vertical ground reaction forces associated with the left foot. According to Eq. (4-69) and (4-76), the following relations can be written:

\[
R_x = q_x \sum_{j=1}^{7} m_j \ddot{x}_j
\]

\[
R_y = q_y \sum_{j=1}^{7} m_j (\ddot{y}_j + g)
\]

The next step is to solve the above equations to obtain \( q_x \) and \( q_y \), which yields:

\[
q_x = \frac{R_x}{\sum_{j=1}^{7} m_j \ddot{x}_j}
\]
\[ q_y = \frac{R_y}{\sum_{j=1}^{7} m_j (\ddot{y}_j + g)} \]  \hspace{1cm} (4-84)

These equations are very useful. Not only can one employ them to investigate the validation of the previous linear assumptions, but also they may be used to check the goodness of the model during SSP. Quantitative results of such evaluations will be presented later.

6. Eq. (4-62) indicates that torque equations have rather more complex configurations compared to the ones associated with forces. A very important variable, however, is the location of the center of pressure \( CL_1(CR_1) \). This quantity can be computed from the model and be compared with the actual values obtained from the force plate. In order to have an explicit expression for \( CL_1(CR_1) \), one may manipulate the associated equations (Eq. (4-62)) to get the desired form. This has been done by the author and the result includes both \( CL_1 \) and \( CR_1 \) as follows:

\[ q_y CL_1 + p_y CR_1 = SB \]  \hspace{1cm} (4-85)

where \( SB \) can be determined from the expression

\[
SB = \frac{1}{F_{y0}} \left[ \sum_{j=1}^{7} J_j \ddot{\theta}_j - \rho_1 x_1 \sin(\theta_1 - \omega_1) + \rho_1 y_1 \cos(\theta_1 - \omega_1) \right. \\
\left. - \rho_1 x_6 \sin(\theta_6 - \omega_1) + \rho_1 y_6 \cos(\theta_6 - \omega_1) \\
- F_{x0}(q_x CL_2 + p_x CR_2) - BZ \right] 
\]  \hspace{1cm} (4-86)

103
\[ BZ = \sum_{j=2}^{5} \left[ \left( d_j F_{xj} + e_j F_{xj} \right) \sin \theta_j - \left( d_j F_{yj} + e_j F_{yj} \right) \cos \theta_j \right] \]
\[ - d_7 (F_{y7} \cos \theta_7 + F_{x7} \sin \theta_7) \]
\[ - \frac{y_p}{2} (F_{x4} - F_{x3}) - \frac{x_p}{2} (F_{y3} - F_{y4}) \]  
\[ (4-87) \]

and

\[ \lambda = -1 \text{ if } j = 2, 3 \]
\[ \lambda = +1 \text{ if } j = 4, 5 \]

Obviously, one needs to compute the numerical values of \( BZ \) and \( SB \) in order to get the location of the center of pressure. According to Table 4.2, only one of the two variables \( CL_1 \) and \( CR_1 \) is to be determined at any time and the value of the other one is known. For example, \( CL_1 \) can be expressed as:

\[ CL_1 = \frac{SB - p_y CR_1}{q_y} \]  
\[ (4-88) \]

From Eq. (4-85) through (4-88), it is quite obvious that the center of pressure does depend on all quantities involved in the model, and the expression is much more complex than the ones for forces. Once the values of \( CL_1 \) and \( CR_1 \) are known, they can be substituted into torque equations to obtain the corresponding torques in the correct order. The result will be:
\[ T_1 = q_y F_{x0} CL_1 + q_x F_{x0} CL_2 + p_1 (F_{x1} \sin(\theta_1 - \omega_1) - F_{y1} \cos(\theta_1 - \omega_1)) - J_1 \ddot{\theta}_1 \]  

(4-89)

\[ T_2 = T_1 + (d_2 F_{x1} + e_2 F_{x2}) \sin\theta_2 - (d_2 F_{y1} + e_2 F_{y2}) \cos\theta_2 - J_2 \ddot{\theta}_2 \]  

(4-90)

\[ T_3 = T_2 + (d_2 F_{x3} + e_3 F_{x3}) \sin\theta_3 - (d_3 F_{y2} + e_3 F_{y3}) \cos\theta_3 - J_3 \ddot{\theta}_3 \]  

(4-91)

\[ T_6 = p_y CR_F y_0 + p_x CR_F x_0 + p_1 (F_{x6} \sin(\theta_6 - \omega_1) - F_{y6} \cos(\theta_6 - \omega_1)) - J_6 \ddot{\theta}_6 \]  

(4-92)

\[ T_5 = T_6 + (d_5 F_{x6} + e_5 F_{x5}) \sin\theta_5 - (d_5 F_{y6} + e_5 F_{y5}) \cos\theta_5 - J_5 \ddot{\theta}_5 \]  

(4-93)

\[ T_4 = T_5 + (d_4 F_{x5} + e_4 F_{x4}) \sin\theta_4 - (d_4 F_{y5} + e_4 F_{y4}) \cos\theta_4 - J_4 \ddot{\theta}_4 \]  

(4-94)

Providing that values for the partition coefficients \( q_x \) and \( q_y \) are assumed or known, the above torque equations together with the corresponding force relationships, Eqs. (4-77) through (4-80), permit a complete kinetic analysis of the model of Fig. 4.1 from television data only.

4.3.2 Comparison with Direct Force Plate Measurements

The displacement and angular data obtained from human subjects can be used for the seven-mass model in order to investigate the behavior of the system. Obviously the best way to test the quality of results is to compare them with the corresponding force plate...
values as was done in Chapter 3. Figures 4-15 and 4-16 show the horizontal and vertical ground reaction forces, respectively. The force plate measurements are also included so that the comparison becomes possible. The partition coefficients for both horizontal and vertical forces are assumed to vary linearly with respect to time. The curves plotted with '×' sign are those obtained from the seven-mass model, and the one shown by the '×' sign represent force plate measurement. Both Figs. 4.15 and 4.16 show the ground reaction forces associated with the left foot. Obviously, they are zero once the swing phase starts. The amount of force indicated on the vertical axis is in Newtons and each time unit on the horizontal line is 1/60 of a second. In order to get a better idea about the quality of results, the mean square errors corresponding to each case have been computed and the results are as follows:

$\text{MSE}(F_x) = 0.144 \times 10^4 \text{ (Newtons)}^2$

$\text{MSE}(F_y) = 0.146 \times 10^4 \text{ (Newtons)}^2 \quad (4-95)$

Although these two values are nearly equal, a comparison of Figs. 4.15 and 4.16 indicates that since the actual values for vertical force are much larger than those for the horizontal force, on a percentage basis, the vertical force estimate is actually much more accurate than the horizontal force estimate. This suggests that there is still a significant modeling error in the representation of the ground reaction forces in the $x$ direction. In particular, in order to investigate the quality of the results fully, it is important to check the validity of the numerical values used for $q_x$ and $q_y$. These
Figure 4.15. Horizontal Ground Force Reaction Associated with Left Leg During One Gait Cycle with Direct Force Plate Measurement (*) and Computed from Seven-Mass Model under Linea: Assumption for $q_x$ (+).
Figure 4.16. Vertical Ground Force Reaction Associated with Left Leg During One Gait Cycle with Direct Force Plate Measurement (*) and Computed One From Seven-Mass Model Under Linear Assumption for $q_y$ (+).
quantities can be actually measured for the model by Eqs. (4-82) and (4-83) by using the force plate outputs. Tables 4.3 and 4.4 list numerical values of \( q_x \) and \( q_y \) during first and second DSP of gait cycle, respectively. Tables include both values obtained from the model of linear variation with respect to time and those computed from the corresponding formulas.

Table 4.3

Numerical Values of Left Leg Partition Coefficients During First DSP of the Gait Cycle Obtained by Linear Force Transfer Hypothesis and by Direct Computation Using Force Plate Measurement

<table>
<thead>
<tr>
<th>Field Number</th>
<th>Linear Values</th>
<th>Computed Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( q_x )</td>
<td>( q_y )</td>
</tr>
<tr>
<td>1</td>
<td>0.083</td>
<td>0.083</td>
</tr>
<tr>
<td>2</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>5</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>6</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>7</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>8</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td>9</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>10</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>11</td>
<td>0.92</td>
<td>0.92</td>
</tr>
</tbody>
</table>
Table 4.4
Numerical Values of Left Leg Partition Coefficients During Second DSP of the Gait Cycle Obtained by Linear Force Transfer Hypothesis and by Direct Computation Using Force Plate Measurement

<table>
<thead>
<tr>
<th>Field Number</th>
<th>Linear Values</th>
<th>Computed Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$q_x$</td>
<td>$q_y$</td>
</tr>
<tr>
<td>1</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>2</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>3</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>4</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td>5</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>6</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>7</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>8</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>9</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>10</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>11</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

A comparison between the values of $q_x$ in two cases reveals that the linear assumption is completely contrary to the values obtained by force plate measurement. In fact, most of the time, neither the magnitudes nor the signs match with each other. Note that $q_x$ changes sign during DSP while linear assumption does not deal with such a feature. This can be better understood if the motion of
both legs is considered. During double support phase, there are times that the front leg is pushed forward because of the inertia of the body, and on the other hand, the back leg is pushed backward so that the body can move forward. Obviously, the horizontal forces acting on the two feet at such a time are in opposite directions. Furthermore, it does happen that at certain times these forces have identical magnitude which means that the total horizontal ground reaction force is zero. This in fact can be easily checked by plotting $F_{x0}$ for one period of the gait. The results indicate that $F_{x0}$ becomes zero during each DSP. In such periods of time, when $F_{x0}$ has a very small magnitude, the situation becomes critical because the two components of the horizontal ground reaction force namely $q_{x}F_{x0}$ and $p_{x}F_{x0}$ may be much larger than $F_{x0}$ which implies that $q_{x}(p_{x})$ can have a large amplitude. Under these circumstances, the horizontal partition coefficient will have a sudden jump and discontinuity can be infinitely large depending on the sampling instant. Consequently, the prediction of $p_{x}(q_{x})$ is practically not possible. A comparison between Table 4.3 and Table 4.4 also indicates that the computed values of $q_{x}$ are quite different for two DSP's simply because the corresponding ground reaction forces are not the same.

The partition coefficient associated with the vertical ground reaction force, $q_{y}$, is also listed in Tables 4.3 and 4.4. The computed values and the ones obtained from linear assumption are fairly close to each other in both tables. This implies that the linear assumption, although not perfect, is an acceptable one in the case of vertical force. However, in order to get a good estimate of the horizontal component,
it is necessary to have some information about $q_x$ during DSP. Obviously, one solution is access to a force plate. However, the whole point of this chapter is to develop a method which does not require force plate data. Examination of Fig. 4.15 shows that by far the greater part of the error in horizontal force estimation is associated with the two double support phases. This observation taken together with the results of Tables 4.3 and 4.4 implies that it is necessary to reject the linear transfer hypothesis for the horizontal forces. This matter is discussed further in Chapter 7 of this dissertation.

So far, determination of forces has been discussed in detail without saying anything about torques. In order to evaluate the amount of torques at various joints of the model, it is first necessary to compute all the forces which have effects on the torques. Moreover, the position of the center of pressure (CP) will also be involved. As a matter of fact, once the CP is known, Eq. (4-83) through (4-94) can be used to obtain all joint torques. During the walking cycle, the CP starts at the heel and then moves forward up to the toe-off time. This motion can be measured by the force plate, and results show that the position of CP varies approximately as a linear function of time. In this research, an attempt has been made to compute the position of the CP by using the equations developed for the model, namely Eqs. (4-86) through (4-88). The result is shown in Fig. 4.17 with the assumptions in Table 4.2 included. For this example, $\tau_B$ and $\tau_S$ (see Fig. 4.14) are 12 and 7 time units, respectively, while each time unit is 1/60 second. The values of $d_B$ and $d_F$ are 0.096 and 0.164 meters, respectively.
Examination of Fig. 4.17 shows that, as expected, the actual forward motion of the CP as measured by the force plate is very close to a linear function of time. On the other hand, while the location of the CP estimated by the model of this chapter deviates from this line by only a few centimeters, the shape of this letter curve is not quite correct. In fact, there is a swing in the middle of §5 which implies that the subject pushes his CP backwards. Obviously, this is not the kind of motion employed by a normal person during the walking process. Nevertheless, one should not expect that a seven-mass model behaves exactly like the body of a human being which has many more degrees of freedom. Further, consideration of Eq. (4-86) through (4-88) explains that the position of CP depends upon all parameters of the body as well as all the horizontal and vertical forces. Therefore any error in the above quantities will appear and influence the results obtained for CP. In other words, the position of center of pressure is dependent on many variables and with such a simple model, a difference of a few centimeters from the actual value should be expected.

4.4 Summary

In this chapter a further step toward dynamic modeling of the human body for gait analysis has been taken. Feets with nonzero mass have been added to the model to permit better estimates of joint forces and torques to be obtained. Modeling of the foot is, of course, a complicated process, and it has been necessary to make assumptions to simplify the system. It is important that the modeling of the foot has an especially significant role in computations and numerical
Figure 4.17. Position of Center of Pressure Measured by Force Plate (*) and Calculated from Seven-Mass Dynamic Model (+).
results regarding the horizontal forces as well as the position of the center of pressure. Also, in this chapter an attempt was made to present a simpler form of the dynamic equations. First the set of equations was decomposed into three semi-independent smaller sets, and then explicit, simple, and general forms of equations were introduced which yield the unknowns directly. This has the great advantage of showing the dependencies for each unknown. Next, sample results were presented to help in evaluation of the model. The assumptions made about the partition coefficients were analyzed in detail and analytical expressions were obtained to help the acceptance or rejection of the previous linear transfer hypothesis. It was found that this assumption is acceptable for $q_y$, while this is not the case for $q_x$. Finally, the position of the center of pressure was computed and plotted. Although the result does not match exactly with reality, it is surprisingly close considering the fact that the model of this chapter is a greatly simplified version of the body of a human being, and thus some mismatch or error should be expected.
Chapter 5

A THREE MASS DYNAMIC MODEL USING BOTH TELEVISION AND FORCE PLATE DATA

5.1 Introduction

The dynamic models which have so far been investigated in this dissertation have utilized only data obtained from television. Force plate measurements were used only in order to test the quality of the model. In fact, all the efforts were made to improve the model so that a smaller mean square error between forces obtained from the model and measured by force plate can be achieved. This chapter, however, deals directly with the information collected from the force plate. In other words, the force plate provides the inputs to the dynamic system such as the ground reaction forces and position of center of pressure. The rest of the required information including the trajectory of the hip joint as well as angles, angular rates, and angular accelerations are taken from television data as discussed in previous chapters. The goal is to determine the forces and torques at the joints of the model. This method was employed by Leo [64] in 1973 and Gupta [2] in 1975. Although the idea presented in this chapter is the same, a different algorithm to derive equations of the system is introduced. The technique is more or less similar to those seen in Chapters 3 and 4; it is simple to understand and easy to apply.
5.2 Description of the Model

The dynamic model considered in this chapter is the same as the one used in Chapter 4 from one point of view and is different from that one from another point of view. That is, model does include a foot, shank, thigh, and in a sense the upper body and another leg, all with nonzero mass. However, it is desired to concentrate only on one leg. Therefore, contrary to previous models which used to deal with the body as a whole, this model specifically investigates the dynamics of the leg which steps on the force plate up to the hip joint, and does not get involved with the interaction between this leg and the rest of the body. In fact, this separation is possible because the force plate information is directly used by the system. Once the leg starts its swing phase, the system is still fully deterministic because there are no forces at the bottom of the foot.

The model is shown in Fig. 5.1. It consists of three segments, foot, shank, and thigh, linked together with pin joints. Motion is again limited to the sagittal plane which is the plane of major significance in locomotion. The computer-television system [8] together with a force plate are employed to provide the data required for the system. Similar to previous cases, the subject is fitted with a suitable number of lights and walks in front of a television camera while stepping on the force plate at the same time. The details of experimental techniques are explained in Chapter 6. The information obtained from television is the coordinates of the hip \((x_H, y_H)\), foot angle \((\phi_F)\), shank angle \((\phi_S)\), and thigh angle \((\phi_T)\) over the entire period of the gait cycle. On the other hand, force plate provides the horizontal and vertical ground reaction forces as well as the position
Figure 5.1. Three Mass Model of a Single Leg.
of center of pressure as long as the foot is in contact with the force plate, which starts with its heel-strike and ends with its toe-off.

Since the force plate has reliable and accurate outputs, the simple model of the foot shown in Fig. 5.1 was considered to be accurate enough for the model [2,64]. Therefore, the author also follows the same approach of modeling the foot. The body segment parameters can be determined from the ratios established by Braune and Fischer which give quite accurate results. This includes the location of the center of mass of the foot. The displacement of the hip joint as well as the angles $\phi_F$, $\phi_S$, and $\phi_T$ are smoothed by the Fourier Series approach with a specific number of harmonics. Since this model only deals with one leg, the subject does not have to be normal while this was a requirement for a five-mass and a seven-mass model.

5.3 Mathematical Formulation and Typical Results

This section is divided into two parts. This first one discusses the dynamic equations of motion. The second part presents some typical kinetic results obtained by applying the model to the data taken from a normal subject.

5.3.1 Mathematical Formulation

In Sec. 5.1, it was mentioned that Leo et al. [64] and Gupta [2] studied dynamic behavior of the three-mass model shown in Fig. 5.1. The mathematical approach used by them is slightly different from the one that is proposed by the author although both methods use the same principles. In order to study two different algorithms applied to the same dynamic model, the equations associated with each method are obtained and compared with each other as follows:
Method I:

Figure 5.2 illustrates the three-mass model with the corresponding specifications. The angles $\theta_1$, $\theta_2$, and $\theta_3$ are measured with respect to the vertical line. Since the amount of forces are known at the bottom of the foot, the system can be solved from bottom to top. The vertical axis is named $z$ instead of $y$ because of the terminology used in the force plate system. Table 5.1 lists all the symbols used in this method. Each segment is considered individually. D'Alembert's principles, which are expressed in Eqs. (3-9) through (3-11) are used so that the equations of system can be obtained. There are three kinds of forces involved with the model, namely: i) reaction forces, ii) gravitational forces, iii) inertial forces. Note also that the moments acting at each joint are due to the muscles acting on two bone segments. Free body diagrams of the three segments are shown in Figs. 5.3, 5.4 and 5.5. For each link, it is necessary to obtain the coordinates of the corresponding center of gravity from which the inertial accelerations can be computed. The position of the hip is considered to be the reference, and the results are as follows [2]:

(a) Foot

\[
\begin{align*}
\mathbf{r}_{AG} &= (x_H + l_t \sin \theta_1 - l_s \sin \theta_2 + d_F \sin \theta_3) \mathbf{i} \\
&\quad + (z_H - l_t \cos \theta_1 - l_s \cos \theta_2 - d_F \cos \theta_3) \mathbf{k} \\
\mathbf{v}_{AG} &= (x_H \dot{\theta}_1 + l_t \cos \theta_1 \ddot{\theta}_1 - l_s \cos \theta_2 \ddot{\theta}_2 + d_F \cos \theta_3 \ddot{\theta}_3) \mathbf{i} \\
&\quad + (z_H \dot{\theta}_1 + l_t \sin \theta_1 \ddot{\theta}_1 + l_s \sin \theta_2 \ddot{\theta}_2 + d_F \sin \theta_3 \ddot{\theta}_3) \mathbf{k}
\end{align*}
\]  

\text{(5-1)}

\text{(5-2)}
Figure 5.2. Three-Link Model for the Force Plate Method. (Adopted from [2]).
Table 5.1

Table of Symbols Used in Method I

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>Displacement vector</td>
</tr>
<tr>
<td>v</td>
<td>Velocity vector</td>
</tr>
<tr>
<td>a</td>
<td>Acceleration vector</td>
</tr>
<tr>
<td>F</td>
<td>Force vector</td>
</tr>
<tr>
<td>M</td>
<td>Moment vector</td>
</tr>
<tr>
<td>A</td>
<td>Ankle</td>
</tr>
<tr>
<td>K</td>
<td>Knee</td>
</tr>
<tr>
<td>H</td>
<td>Hip</td>
</tr>
<tr>
<td>X</td>
<td>Component in X direction</td>
</tr>
<tr>
<td>Z</td>
<td>Component in Z direction</td>
</tr>
<tr>
<td>x</td>
<td>x coordinate</td>
</tr>
<tr>
<td>z</td>
<td>z coordinate</td>
</tr>
<tr>
<td>AG</td>
<td>Center of gravity of foot</td>
</tr>
<tr>
<td>KG</td>
<td>Center of gravity of shank</td>
</tr>
<tr>
<td>HG</td>
<td>Center of gravity of thigh</td>
</tr>
<tr>
<td>T</td>
<td>Thigh</td>
</tr>
<tr>
<td>S</td>
<td>Shank</td>
</tr>
<tr>
<td>F</td>
<td>Foot</td>
</tr>
<tr>
<td>l</td>
<td>Length</td>
</tr>
<tr>
<td>m</td>
<td>Mass</td>
</tr>
<tr>
<td>d</td>
<td>Distance from the proximal point to center of gravity of the limb under consideration</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>i</td>
<td>Unit vector in x direction</td>
</tr>
<tr>
<td>k</td>
<td>Unit vector in z direction</td>
</tr>
</tbody>
</table>
Figure 5.3. Free Body Diagram of Thigh (Method I).

Figure 5.4. Free Body Diagram of Shank (Method I).

Figure 5.5. Free Body Diagram of Foot (Method I).
\[ a_{AG} = (\ddot{x}_H + l_T(\dot{\theta}_1 \cos \theta_1 - \dot{\theta}_2 \sin \theta_1) - l_S(\dot{\theta}_2 \cos \theta_2 - \dot{\theta}_2 \sin \theta_2) \\
+ d_F(\dot{\theta}_3 \cos \theta_3 - \dot{\theta}_3 \sin \theta_3) \hat{i} + (\ddot{z}_H + l_T(\dot{\theta}_1 \cos \theta_1 + \sin \theta_1 \hat{v}_1) \\
+ l_S(\dot{\theta}_2 \cos \theta_2 + \sin \theta_2 \hat{v}_2) + d_F(\dot{\theta}_3 \cos \theta_3 - \dot{\theta}_3 \cos \theta_3)) \hat{k} \]  

(5-3)

and the dynamic equations are

\[ F_{AX} = \ddot{x}_X - m_F(\ddot{x}_H + \dot{\theta}_1 \dot{l}_T \cos \theta_1 - \dot{\theta}_2 \dot{l}_T \sin \theta_1 - \dot{\theta}_2 \dot{l}_S \cos \theta_2 \\
+ \dot{\theta}_2 \dot{l}_S \sin \theta_2 + \dot{\theta}_3 d_F \cos \theta_3 - \dot{\theta}_3 d_F \sin \theta_3) \]  

(5-4)

\[ F_{AZ} = -m_F g + R_Z - m_F(\ddot{z}_H + \dot{\theta}_1 \dot{l}_T \sin \theta_1 + \dot{\theta}_2 \dot{l}_T \sin \theta_1 - \dot{\theta}_2 \dot{l}_S \cos \theta_2 \\
+ \dot{\theta}_2 \dot{l}_S \cos \theta_2 + \dot{\theta}_3 d_F \sin \theta_3 + \dot{\theta}_3 d_F \cos \theta_3) \]  

(5-5)

\[ M_A = J_T \ddot{\theta}_3 + m_F \dot{x}_H \dot{d}_F \cos \theta_3 + m_F \dot{z}_H \dot{d}_F \sin \theta_3 + m_F \ddot{l}_T \dot{d}_F \cos(\theta_1 - \theta_3) \\
- m_F \dot{\theta}_2 \dot{l}_S \dot{d}_F \cos(\theta_2 + \theta_3) + m_F \ddot{\theta}_2 \dot{d}_F^2 + m_F \ddot{\theta}_2 \dot{l}_S \dot{d}_F \sin(\theta_2 + \theta_3) \\
+ m_F \dot{\theta}_3 \dot{l}_T \dot{d}_F \sin(\theta_3 - \theta_1) + m_F g \dot{d}_F \sin \theta_3 \\
- R_Z (x_H - \dot{x}_H - \dot{l}_T \sin \theta_1 + \dot{l}_S \sin \theta_2) \\
- R_X (z_H - \dot{z}_H - \dot{l}_T \cos \theta_1 - \dot{l}_S \cos \theta_2) \]  

(5-6)

(b) Shank

\[ r_{KG} = r_K - d_S \sin \theta_2 \hat{i} - d_S \cos \theta_2 \hat{k} \]

\[ = (x_H + l_T \sin \theta_1 - d_S \sin \theta_2) \hat{i} + (z_H - l_T \cos \theta_1 - d_S \cos \theta_2) \hat{k} \]  

(5-7)
\[ v_{KG} = (\dot{x}_H + l_T \cos \theta_1 \dot{\theta}_1 - d_s \cos \theta_2 \dot{\theta}_2)i + (\dot{z}_H + l_T \sin \theta_1 \dot{\theta}_1 + d_s \sin \theta_2 \dot{\theta}_2)k \]  

(5-8)

\[ a_{KG} = (\ddot{x}_H + l_T (-\dot{\theta}_1^2 \sin \theta_1 + \cos \theta_1 \ddot{\theta}_1) - d_s (-\dot{\theta}_2^2 \sin \theta_2 + \cos \theta_2 \ddot{\theta}_2)i \]

\[ + (\ddot{z}_H + l_T (\dot{\theta}_1^2 \cos \theta_1 + \sin \theta_1 \ddot{\theta}_1) + d_s (\dot{\theta}_2^2 \cos \theta_2 + \sin \theta_2 \ddot{\theta}_2))k \]  

(5-9)

and the corresponding dynamic equations are:

\[ F_{KX} = F_{AX} - m_s (\ddot{x}_H + \dot{\theta}_1 l_T \cos \theta_1 - \ddot{\theta}_1^2 l_T \sin \theta_1 - \dot{\theta}_2 d_s \cos \theta_2 + \dot{\theta}_2^2 d_s \sin \theta_2) \]  

(5-10)

\[ F_{KZ} = F_{AZ} - m_s g - m_s (\ddot{z}_H + \dot{\theta}_1 l_T \sin \theta_1 + \ddot{\theta}_1^2 l_T \cos \theta_1 + \dot{\theta}_2 d_s \sin \theta_2 \]  

\[ + \dot{\theta}_2^2 d_s \cos \theta_2) \]  

(5-11)

\[ M_K = -J_s \ddot{\theta}_2 - m_s \dot{\theta}_2 d_s \sin \theta_2 + F_{AZ} l_s \sin \theta_2 - F_{AX} l_s \cos \theta_2 + M_A \]  

(5-12)

(c) Thigh

\[ r_{HG} = r_H + d_t \sin \theta_1 \hat{i} - d_t \cos \theta_1 \hat{k} \]

(5-13)

\[ v_{HG} = (\dot{x}_H + d_t \cos \theta_1 \dot{\theta}_1)\hat{i} + (\dot{z}_H + d_t \sin \theta_1 \dot{\theta}_1)\hat{k} \]  

(5-14)

\[ a_{HG} = (\ddot{x}_H + d_t (-\sin \theta_1 \dot{\theta}_1^2 + \cos \theta_1 \ddot{\theta}_1))\hat{i} \]

\[ + (\ddot{z}_H + d_t (\sin \theta_1 \dot{\theta}_1^2 + \dot{\theta}_1^2 \cos \theta_1))\hat{k} \]  

(5-15)

and the dynamic equations are

\[ F_{HX} = F_{KX} - m_s (\ddot{x}_H + \dot{\theta}_1 l_T \cos \theta_1 - \dot{\theta}_1^2 d_t \sin \theta_1) \]  

(5-16)

125
\[ F_{HZ} = F_{KZ} - m_T g - m_T (\ddot{z}_H + \dddot{\theta}_1 + \ddot{d}_T \sin \theta_1 + \dot{\theta}_1^2 \cos \theta_1) \] (5-17)

\[ M_H = -J_T \ddot{\theta}_3 - m_T g d_S \sin \theta_1 + F_{KZ} l T \sin \theta_1 - F_{KX} l T \cos \theta_1 + M_k \] (5-18)

Note that the above equations can be solved consecutively and there is no need to solve them simultaneously.

Method II:

This method is more or less similar to Method I with a few differences which have the advantage of making the equations easier to understand. The model is shown in Fig. 5.1, and the angles are shown by \( \phi_F \), \( \phi_S \), and \( \phi_T \), which are different from previous notations in order not to be mistaken with the ones in Method I. However, these angles as well as the hip coordinate notations are similar to the ones used in previous chapters. Table 5.2 shows all the symbols used in Method II. The free body diagram of the three segments of the model are shown in Figs. 5.6, 5.7 and 5.8. In order to make the model compatible with the ones in Chapters 3 and 4, an identical coordinate system is chosen with the angles measured with respect to the horizontal axis. Again, the hip joint is chosen to be the reference point and the coordinates of the center of gravity of all segments are computed as follows:

(a) Thigh

\[ x_T = x_H - e_T \cos \phi_T \] (5-19)

\[ y_T = y_H - e_T \sin \phi_T \] (5-20)
### Table 5.2

Table of Symbols Used in Method II

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>Mass of the segment</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Length of the segment</td>
</tr>
<tr>
<td>J</td>
<td>Moment of inertia of the segment</td>
</tr>
<tr>
<td>d</td>
<td>Distance from the common joint with the proximal link to the center of gravity of the link under consideration</td>
</tr>
<tr>
<td>e</td>
<td>Difference between ( \lambda ) and ( d ) of the segment</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Angles with respect to the horizontal axis</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>X</td>
<td>Horizontal axis</td>
</tr>
<tr>
<td>Y</td>
<td>Vertical axis</td>
</tr>
<tr>
<td>F</td>
<td>Force</td>
</tr>
<tr>
<td>T</td>
<td>Torque</td>
</tr>
<tr>
<td>H</td>
<td>Hip</td>
</tr>
<tr>
<td>K</td>
<td>Knee</td>
</tr>
<tr>
<td>A</td>
<td>Ankle</td>
</tr>
<tr>
<td>R</td>
<td>Ground reaction force</td>
</tr>
<tr>
<td>( a_Y )</td>
<td>Location of the center of pressure</td>
</tr>
</tbody>
</table>
Figure 5.6. Free Body Diagram of Thigh (Method II).

Figure 5.7. Free Body Diagram of Shank (Method II).

Figure 5.8. Free Body Diagram of Foot (Method II).
The corresponding accelerations are:

\[
\ddot{x}_T = \ddot{x}_H + e_T (\sin \phi_T \ddot{\phi}_T + \cos \phi_T \dddot{\phi}_T) \tag{5-21}
\]
\[
\ddot{y}_T = \ddot{y}_H + e_T (\sin \phi_T \ddot{\phi}_T - \cos \phi_T \dddot{\phi}_T) \tag{5-22}
\]

(b) Shank

\[
x_S = x_H - \ell_T \cos \phi_T - e_S \cos \phi_S \tag{5-23}
\]
\[
y_S = y_H - \ell_T \sin \phi_T - e_S \sin \phi_S \tag{5-24}
\]

and after two times differentiation:

\[
\dddot{x}_S = \dddot{x}_H + \ell_T (\sin \phi_T \ddot{\phi}_T + \cos \phi_T \dddot{\phi}_T) + e_S (\sin \phi_S \ddot{\phi}_S + \cos \phi_S \dddot{\phi}_S) \tag{5-25}
\]
\[
\dddot{y}_S = \dddot{y}_H + \ell_T (\sin \phi_T \ddot{\phi}_T - \cos \phi_T \dddot{\phi}_T) + e_S (\sin \phi_S \ddot{\phi}_S - \cos \phi_S \dddot{\phi}_S) \tag{5-26}
\]

(c) Foot

\[
x_F = x_H - \ell_T \cos \phi_T - \ell_S \cos \phi_S - e_F \cos \phi_F \tag{5-27}
\]
\[
y_F = y_H - \ell_T \sin \phi_T - \ell_S \sin \phi_S - e_F \sin \phi_F \tag{5-28}
\]

which yields the following accelerations

\[
\dddot{x}_F = \dddot{x}_H + \ell_T (\sin \phi_T \ddot{\phi}_T + \cos \phi_T \dddot{\phi}_T) + \ell_S (\sin \phi_S \ddot{\phi}_S + \cos \phi_S \dddot{\phi}_S)
\]
\[
+ e_F (\sin \phi_F \ddot{\phi}_F + \cos \phi_F \dddot{\phi}_F) \tag{5-29}
\]
\[
\dddot{y}_F = \dddot{y}_H + \ell_T (\sin \phi_T \ddot{\phi}_T - \cos \phi_T \dddot{\phi}_T) + \ell_S (\sin \phi_S \ddot{\phi}_S - \cos \phi_S \dddot{\phi}_S)
\]
\[
+ e_F (\sin \phi_F \ddot{\phi}_F - \cos \phi_F \dddot{\phi}_F) \tag{5-30}
\]
Once the accelerations of the three segments of the model are determined, the dynamic equations can be easily written and solved. These equations are written as follows:

(i) Horizontal Forces:

\[ F_{\text{AX}} = R_x - m_{\text{F}} x_{\text{F}} \]  
\[ (5-31) \]

\[ F_{\text{KX}} = F_{\text{AX}} - m_{\text{S}} x_{\text{S}} \]  
\[ (5-32) \]

\[ F_{\text{HX}} = F_{\text{KX}} - m_{\text{T}} x_{\text{T}} \]  
\[ (5-33) \]

(ii) Vertical Forces:

\[ F_{\text{AY}} = R_y - m_{\text{F}} (y_{\text{F}} + g) \]  
\[ (5-34) \]

\[ F_{\text{KY}} = F_{\text{AY}} - m_{\text{S}} (y_{\text{S}} + g) \]  
\[ (5-35) \]

\[ F_{\text{HY}} = F_{\text{KY}} - m_{\text{T}} (y_{\text{T}} + g) \]  
\[ (5-36) \]

(iii) Torques

\[ T_{\text{A}} = J_{\text{F}} \dot{\phi}_{\text{F}} + (F_{\text{AY}} \cos \phi_{\text{F}} - F_{\text{AX}} \sin \phi_{\text{F}}) e_{\text{F}} \]
\[ + (a_{\text{F}} e_{\text{F}})(-R_{\text{F}} \sin \phi_{\text{F}} + R_{\text{y}}) \]  
\[ (5-37) \]

\[ T_{\text{K}} = J_{\text{S}} \dot{\phi}_{\text{S}} + T_{\text{A}} + (F_{\text{AY}} d_{\text{S}} + F_{\text{KY}} e_{\text{S}}) \cos \phi_{\text{S}} \]
\[ - (F_{\text{AX}} d_{\text{S}} + F_{\text{KX}} e_{\text{S}}) \sin \phi_{\text{S}} \]  
\[ (5-38) \]

\[ T_{\text{H}} = J_{\text{T}} \dot{\phi}_{\text{T}} + T_{\text{K}} + (F_{\text{KY}} d_{\text{T}} + F_{\text{HY}} e_{\text{T}}) \cos \phi_{\text{T}} \]
\[ - (F_{\text{KX}} d_{\text{T}} + F_{\text{HX}} e_{\text{T}}) \sin \phi_{\text{T}} \]  
\[ (5-39) \]
Eqs. (5-31) through (5-39) give all the quantities that are of our interest. Note that there is no need to solve the equations simultaneously. In fact, the only requirement is that the equations must be solved in the right order, such as the one already presented. In other words, all the unknown quantities in the right-hand side of each equation have already been obtained from previous ones.

A comparison between Method I and Method II will help us reach the following conclusions:

(a) The coordinate systems used in order to get the acceleration of each segment are very much the same.

(b) In Method I, the torque equations are written with respect to the upper joint of the segment. This is why, for example, the torque equations associated with the foot has too many terms. Method II, however, uses the center of gravity of each segment as the reference point to get the torque equations.

(c) The features of Method II explained in part (b) not only causes a more uniform algorithm be presented to study dynamic behaviors of various links, but also it is compatible with the techniques applied to the models in previous chapters. Thus, they can all fit in a rather general approach which can be used to investigate dynamic models.

(d) The force equations given by Method II are presented so that they are very easy to understand. However, this is not quite the case in Method I although both forms express the same thing.

Based on the above argument, the author has decided to use Method II to study the three-mass model of the human body.
5.3.2 Typical Results

The kinematic data collected from TV-computer system together with data obtained from force plate are used to investigate the model already developed. The results can be expressed in terms of six forces and three torques. Fig. 5.9 shows four vertical forces included in the model. The vertical axis shows the amount of forces in Newtons and the horizontal axis gives percentage of the gait cycle (1.3 sec) normalized to 100 units. The characteristic of the plots are such that +, *, ⊙, and $\$, represent $R_y$, $F_{AY}$, $F_{KY}$, and $F_{HY}$, respectively. Note that $R_y$ is directly measured by the force plate and has the largest magnitude during the stance phase of the corresponding foot. Then the ankle force, knee force, and hip force are in the order that vertical force decreases. In other words, the vertical force at the bottom of the foot is largest during stance phase. Moving upward from the ankle to the hip, the force is reduced. This feature of the vertical forces can be called the 'cushion effect' because it looks as if the upper curve were pushed down to yield the other ones. Obviously the cushion effect is due to both gravitational and inertial accelerations.

During the swing phase which occurs after toe-off, the ground reaction force goes to zero, and the other forces become negative. This indicates that forces are generated so that they can hold the leg while it is swinging. In this period of time, the hip force has the largest magnitude at the joints and ankle has the smallest one.

The horizontal components of the forces are shown in Fig. 5.10. Similar to the vertical forces, graphs plotted by +, *, ⊙, and $\$, show
Figure 5.9. Vertical Components of Joint Forces During One Gait Cycle; (+) Ground Reaction, (*) Ankle, (n) Knee, ($) Hip.
Figure 5.10. Horizontal Components of Joint Forces During One Gait Cycle; (+) Ground Reaction, (a) Ankle, (n) Knee, ($) Hip.
$R_x$, $F_{AX}$, $F_{KX}$, and $F_{HX}$, respectively. The vertical axis shows the amount of forces in Newtons while the horizontal axis is normalized to percentage of the gait cycle which is about 1.3 sec. During the stance phase of the leg, first $R_x$ becomes negative, and after foot-flat takes positive values. The other horizontal forces, namely $F_{AX}$, $F_{KX}$, and $F_{HX}$ more or less have the same form with the magnitude decreasing because of the inertial forces. During the swing phase of the leg, $R_x$ becomes zero but the other horizontal forces take considerable values due to accelerations of three segments and cause the leg to swing forward so that the process of walking becomes possible.

Finally, the three torques associated with the model are shown in Fig. 5.11. $T_A$, $T_K$, and $T_H$ correspond to graphs plotted with +, *, and $\bigcirc$, respectively. The vertical axis indicates the amount of torques in Newton-meters and the horizontal axis again represents the percent of gait cycle. The ankle torque starts with very small magnitude at heel-strike time. It takes negative values and becomes larger and larger as the body continues in stance phase. This has the effect of rotating the ankle joint forward while the other leg is in its swing phase. Before toe-off time, $T_A$ reaches its largest value because of the tremendous amount of torque needed to lift the heel and make the body ready for toe-off. As the stance phase gets close to its end, the magnitude of $T_A$ reduces fairly rapidly. During the swing phase of the leg, the ankle torque has very small values because there is little torque action associated with the ankle.

Before heel-strike the knee and hip torques have negative values and try to stretch the leg and make it ready for its contact with the
Figure 5.11. Joint Torques During One Gait Cycle; (+) Ankle, (*) Knee, (n) Hip.
ground. As the stance phase starts, $T_K$ and $T_M$ have fairly large negative values. As walking proceeds, $T_K$ faster and $T_H$ more slowly approach zero and then take positive values. At about toe-off time, $T_K$ and $T_H$ take more and more positive values so that the leg can be bent and thus the swing phase becomes possible. As the leg proceeds to be stretched, torques at knee and hip joints take negative values and the action continues.

5.4 Summary

The three-mass model discussed in this chapter is much simpler than the previous ones. Furthermore, it utilizes force plate data directly to determine the other joint forces and torques. Consequently, the simplicity and accuracy involved in the analysis of this dynamic model leads to reliable results. In other words, this model does not involve any kind of hypothesis, and the procedure to obtain various quantities is very simple and straightforward. Method II which was developed in Sec. 5.3.1 indicates this simplicity, and the corresponding equations are in a form compatible with the ones obtained in Chaps. 3 and 4.

The simplicity of the model allows it to be applied to a larger amount of data. This can be related to many areas such as clinical applications, study of sport shoes, study of various ordinary footwear and so on. To take advantage of this attractive feature of the model, a study involving determination of joint forces and torques with two different types of footwear, namely positive heel shoes and negative heel shoes (or so-called earth shoes) as well as barefoot walking has been done. This represents one of the several applications
of the dynamic modeling of the human body, and some of the results will be presented and discussed in Chapter 7.
Chapter 6

MEASUREMENT TECHNIQUES

6.1 Introduction

The study of human locomotion has always been of great interest to many investigators, and many theories have been developed to explain various features of kinematic and dynamic behavior of human beings. This chapter includes a description of a system by which the measurement of ground reaction forces, displacements of various points of the body and some important angles can be obtained.

The first part of this chapter discusses the development of the Gait Laboratory where the current research is taking place. The purpose of the establishment of such a laboratory together with the equipment which are used for various research objectives are also discussed. The system from hardware and software points of view is investigated and methods of data acquisition are explained. Then, the quality of the data as well as the improvement techniques are studied. Section 6.3 involves characterization of signal and noise using a power spectral density approach. Next, a suboptimal filtering (smoothing) approach, which is indeed the same as the Fourier series analysis, is applied to the data. Also an optimal filter, based on spectral densities of noise and signal, is discussed and proposed for smoothing the raw data. Finally, some experimental results on the number of harmonics used for smoothing the data are presented.

139
6.2 Description of Gait Laboratory Equipment

The Gait Laboratory was established in the Physical Medicine Department at the Ohio State University in early 1977. The above title was chosen for the laboratory because it has been primarily operated for study of human gait. The current research is mainly about dynamic modeling of human gait and is one of the ongoing projects of this young laboratory. It will be further developed for clinical applications. This section about the system used in the Gait Laboratory is divided into two parts. The first part describes the equipment purchased and brought to the laboratory while the second part explains the part of the system developed specifically for the Gait Laboratory applications including both hardware and software.

6.2.1 Gait Laboratory Equipment

The equipment used in the Gait Laboratory for the collection of kinematic and force date can be listed as: (a) force plate, (b) charge amplifier, (c) television camera (d) PDP-11/34 minicomputer, (e) graphic display system, and (f) landmarks (lights). An attempt is made to describe each of these components briefly and to explain how they contribute to the whole system.

a) Force Plate

This system is called the multicomponent measuring platform or more commonly just "force plate." Fig. 6.1 shows a subject stepping on such a plate which is placed inside the floor so that it is level with the walking surface. These plates were originally designed by Cunningham [90]. The one being used in the Gait Laboratory is made by Kistler Company [91] and the model number is 9261A. This system is
Figure 6.1. A General Illustration of Force Plate, Television Camera, and Lights During Data Acquisition in Gait Laboratory.
actually a piezoelectric transducer with which several quantities can be measured as follows:

i) The three components of the applied force \( F \), namely \( F_x \), \( F_y \), and \( F_z \).

ii) The two coordinates of force application (zero moment point or so-called center of pressure), namely \( a_x \) and \( a_y \).

iii) The free moment \( M_z \) about an axis normal to the surface of the plate.

There are four three-component quartz transducers which are fitted inside the platform so that high rigidity and thus minimal measuring deflection and wide range of frequency can be achieved. The operation of the system is based on the electric charges generated by the quartz transducers. The amount of electric charges are directly proportional to the components of the applied force. The next step is the use of charge amplifiers to convert the charges into analog dc voltages. The analog signals can be sampled and recorded later for analysis. This system costs about $6,000 and is used mostly for investigations in biomechanics and automotive engineering.

The measuring surface of this device is \( 440 \times 264 \text{ mm}^2 \). The weight of the platform is about 26 kg. In order to determine various quantities of interest, a positive Cartesian coordinate system is used as reference. This is shown in Fig. 6.2 together with the quantities associated with the force plate.

This positive direction of the \( z \)-axis is normal to the platform pointing into its surface. The positive direction of the \( y \)-axis points towards the end of the plate which has the connections. There are eight connections (outputs) on the force plate which supply the inputs to the charge amplifier. According to Fig. 6.2 each individual
Figure 6.2 Force Plate with Its Associated Quantities.
transducer which is positioned in one of the four corners of the platform generates three-component charges corresponding to the three components of the force. In other words, there are 12 individual forces to start with:

\[(F_{xi}, F_{yi}, F_{zi}) \quad i=1,2,3,4\]

However, some of the individual transducer outputs are paralleled together permanently and are designated as measuring platform outputs. This is why there are only 8 outputs for 12 quantities. Table 6.1 gives all the outputs as well as the associated components.

**Table 6.1**

Outputs From Force Plate

<table>
<thead>
<tr>
<th>Output Number</th>
<th>Notation</th>
<th>Corresponding Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(x_{12})</td>
<td>(F_{x12} = F_{x1} + F_{x2})</td>
</tr>
<tr>
<td>2</td>
<td>(x_{34})</td>
<td>(F_{x34} = F_{x3} + F_{x4})</td>
</tr>
<tr>
<td>3</td>
<td>(y_{14})</td>
<td>(F_{y14} = F_{y1} + F_{y4})</td>
</tr>
<tr>
<td>4</td>
<td>(y_{23})</td>
<td>(F_{y23} = F_{y2} + F_{y3})</td>
</tr>
<tr>
<td>5</td>
<td>(z_1)</td>
<td>(F_{z1})</td>
</tr>
<tr>
<td>6</td>
<td>(z_2)</td>
<td>(F_{z2})</td>
</tr>
<tr>
<td>7</td>
<td>(z_3)</td>
<td>(F_{z3})</td>
</tr>
<tr>
<td>8</td>
<td>(z_4)</td>
<td>(F_{z4})</td>
</tr>
</tbody>
</table>
The component of the resultant forces are obtained as follows:

\[
F_x = F_{x12} + F_{x34} \quad (6-1)
\]

\[
F_y = F_{y14} + F_{y23} \quad (6-2)
\]

\[
F_z = F_{z1} + F_{z2} + F_{z3} + F_{z4} \quad (6-3)
\]

The magnitude of the total force can be easily obtained:

\[
F = (F_x^2 + F_y^2 + F_z^2)^{1/2} \quad (6-4)
\]

One can also compute the corresponding angles to determine the position of vector \( \vec{F} \) in the coordinate system.

As it was pointed out before, in addition to three components of the force applied to the force plate, there are three other quantities, namely \( a_x \), \( a_y \), and \( M_z' \) to be determined. They can be computed by using the eight quantities obtained at the output of the systems as follows:

\[
a_x = \frac{F \cdot a_z - M_y}{F_z} \quad (6-5)
\]

\[
a_y = \frac{F \cdot a_z + M_z}{F_z} \quad (6-6)
\]

\[
M_z' = M_z - \frac{F \cdot a_x}{x \cdot y} + \frac{F \cdot a_y}{y \cdot z} \quad (6-7)
\]

where \( M_x' \), \( M_y' \), and \( M_z' \) are given by:

\[
M_x = b(F_{z1} + F_{z2} - F_{z3} - F_{z4}) \quad (6-8)
\]

\[
M_y = a(-F_{z1} + F_{z2} + F_{z3} - F_{z4}) \quad (6-9)
\]
\[ M_z = b(-F_{x12} + F_{x34}) + a(F_{y14} - F_{y23}) \]  \hspace{1cm} (6-10)

The numerical values of the platform parameters are

\[ a = 0.132 \text{ meters} \]
\[ b = 0.220 \text{ meters} \]
\[ a_z = -0.037 \text{ meters} \]

The details of derivation of Eqs. (6-5) through (6-10) are discussed in [91]. Note that the final results include three components of the force, and the position where it is applied (center of pressure), and the free moment about an axis parallel to the z-axis.

b) Multi-Channel Charge Amplifier

The multi-channel amplifier provides eight analog dc signals corresponding to eight force components listed in Table 6.1. The model used in the Gait Laboratory, which costs about $6,000, is a multi-channel amplifier assembly consisting of eight Type 5001 charge amplifiers, one Type 5671 central control unit, and three Type 5721 blank panel (optional) in a Type 5719A rack case.

The eight inputs to multi-channel charge amplifier system come from the outputs of the platform (Type 9216A) already discussed. Each input centers one of the charge amplifier units (Type 5001). Each unit consists of a dc amplifier of very high input impedance with capacitive negative feedback. The system is designed to convert the electric charge from a piezoelectric transducer into a proportional voltage at the low impedance amplifier output [92]. Its frequency range is from 0 to 180 KHz. There is a range dial through which twelve ranges are available, and it is calibrated in mechanical units per volt. The
calibration factor is set by the transducer sensitivity potentiometer. The system has the advantage of being remotely controlled. There is a switch which determines the time constant (long, medium, and short). For quasistatic measurements, the switch must be set to the 'long' position. For measurement of dynamic processes, 'medium' or 'short' positions can be used.

In the present research, the motion is limited to the walking process. The time constant switch is set to the 'long' position. The transducer sensitivity range is set to 10 with the potentiometer set to the value obtained from the force plate manual tables. The range for the mechanical units V is set to 1000 for vertical components and 500 for the horizontal components. If the output from the charge amplifier is between +2.5V and -2.5V, it corresponds to 1024 units obtained from the A/D converter. Thus, in computation of total vertical forces, each unit obtained from A/D converter corresponds to $\frac{1000 \times 2.5}{512} = 4.88$ Newtons. For the total horizontal forces this coefficient is 2.44 Newtons/FP unit. The operation is remotely controlled using a Type 5651 remote control device. It is important to know that the multi-channel charge amplifier system must be on for about 45 minutes prior to the beginning of data collection so that it can be warmed up. The outputs from charge amplifiers will then go to A/D converters associated with the PDP-11/34 minicomputer to be sampled and stored.

It is important to notice that the force plate system operates independently of the television system which will be discussed later; and although the data from both sources are collected simultaneously, they come from two different origins.
c) Television Camera

At the current time, the Gait Laboratory employs one TV camera, but a three TV camera system is already planned. A Panasonic Model WV-341F standard black and white television camera is being used. It has the standard scanning system (525 horizontal lines/frame, 60 fields/sec, and 30 frames/sec). The camera's dimensions are length - 14 1/2", height - 6 1/3" and width - 5 1/2". It weighs about 14 lbs., and it consists of a video pickup head, a control unit, and sync pulse generators. The camera system converts a video scene into composite electrical signals which include the video signal, horizontal blanking pulses, and vertical blanking pulses. The information existing in the video signal will be obtained and used via an interface to the minicomputer.

d) PDP-11/34 Minicomputer

This is a 16-bit, general purpose minicomputer which is made by Digital Equipment Corporation (DEC). It can be considered as the heart of the Gait Laboratory because of the important role it performs in many respects. It samples, collects and stores the data obtained from the force plate; it is interfaced to the television camera through which the kinematic data are provided and stored, and it processes all the data and presents the results either through its associated display system or on hard copy.

The PDP-11/34 minicomputer has eight general purpose registers and a bus system structure. The bus (unibus), on which all communications are done consist of 56 signal lines, 51 of them bidirectional. All the connections to the unibus are in parallel. The instructions may take one, two or three words. The addressing techniques of the
PDP-11 are very powerful and convenient to the user. It has eight modes and all registers can be involved. The average execution time for an instruction in the PDP-11/34 is about 4.7 to 6.7μs [94]. The computer storage media used in the Gait laboratory are diskettes or so-called floppy disks which look like phonograph records. Each floppy disk has 77 tracks and every track consists of 26 sectors. There are 64 words in each sector where one word has 16 bits of information. Therefore, each floppy disk has the capacity of storing about 250,000 bytes (half words) or 2 million bits of information.

The software used by the Gait laboratory system is the RT-11 which is a very powerful and attractive system. It is a single-user programming and operating system. It provides two operating environments: single-job operations, and foreground/background (FB) capability.

Single job operation allows only one program to be in the memory at any time. The program is executed until either it is completed or it is physically interrupted by the user. However, in foreground/background situation, two independent programs may be in memory. The foreground program is given the priority and is executed until it passes the control to the background program which is executed until again the foreground program requires the control, and so on. This sharing of the system increases the efficiency of the processor usage. The F/B feature of the RT-11 system has been successfully employed in the Gait Laboratory.

The RT-11 can be extensively used for program development. There are many system files such as text editor, assembler, peripheral interchange, linker, librarian (to generate user's own library), on-line debugging, etc. It also has the capability of handling high-level
languages such as BASIC or FORTRAN IV which has been extensively used in the Gait Laboratory. The RT-11 Fortran System Subroutine Library (SYSLIB) is a collection of Fortran callable routines that make the program requests and many utility functions available to the Fortran programmer.

e) Display System

There is a display system, Model GT40, which is one of the PDP-11 graphic systems. It mainly includes the VT11 Graphic System with the associated display processor that is interfaced with the PDP-11 unibus. The VT11 controls a cathode ray tube display and a light pen. The display screen provides a basic 9 1/4" (20.7 centimeter) by 9 1/4" viewing area. This area can be considered as a coordinate grid (x,y coordinates) with 1024 logical units on each axis. That is, there are 1,048,576 individually addressable positions on the screen. The character capacity of the screen is 73 per line and a total of 31 lines per screen. The display screen is refreshed at a rate of 30 Hz.

The display processor is controlled by the RT-11 VT11 handler. There is a variety of graphics control subroutines which can be implemented to support the VT11 display processor. By calling these routines from Fortran programs under RT-11, the user can take advantage of several associated attractive and powerful features.

The display processor is a direct-memory access device. The graphic system has a refresh CRT capability and supports a solid-state light pen by which one may interact with the display processor to select options, move pictures, and do other dynamic alterations.
f) Lights

There are six small incandescent lights used as landmarks in the experiments done in the Gait Laboratory. Each light is mounted on a piece of wood so that it can be attached by some means to the important anatomical joints of the body. A power supply is used to generate the required power which includes 14 volts and 0.08 amperes per light. There is a trailing wire, through which the power is supplied, and it is connected so that it does not interfere with the walking process.

6.2.2 Hardware/Software Development

Before studying how the system development is done, it is necessary to describe the experimental procedure. The main purpose is collection of some important information about displacement of anatomical joints of the body. These data can then be used to compute the corresponding angles. There are six small lights (incandescent lights) connected to the body of the subject who walks in front of the television camera. Fig. 6.1 shows the positions of the lights on the left side of the body. From bottom to top they are positioned on (1) ball of the foot, (2) ankle joint, (3) below knee joint, (4) above knee joint, (5) hip joint, and (6) top of the shoulder. The system is designed so that the displacement of each light with respect to time can be obtained by using the television-computer system. During each television field, the TV screen is scanned by an electron beam and one set of information, including all coordinates of the six lights, is obtained. Thus, the rate of the video information is 60 per second. The room is required to be dark, and the subject wears sports clothing and walks from right to left from the television point of view.
He also steps on the force plate so that the force data can be collected simultaneously. The walking floor is usually covered by a black cloth so that there is no reflection of the lights. The subject crosses his arms in front of himself so that the hip light is not covered. Also, this has the advantage of allowing the arms and hands to be included in the upper body in the dynamic models. There are three anatomical and four inertial angles which can be calculated by knowing the position of the lights. The first three are the anatomical angles corresponding to the ankle joint, knee joint, and hip joint; while the second four are inertial angles of the upper body, thigh, shank, and foot with respect to the horizontal axis.

There are a couple of problems with the camera system used in the Gait Laboratory. The first one is due to the blooming effect of the videcon tube of the TV camera. This causes the lights to appear a little larger than they should be. Therefore, an error will be introduced in the identification of the positions of the lights. This problem can be taken care of by using 'array cameras' [8]. The second problem is that while the subject is moving, not only the lights attached to his body are seen, but also the corresponding tails are observed on the TV screen. The reason for this is that the videcon integrates over the entire field time. As a result, the intensity of the lights appears much smaller during the fast phases of the motion. This problem can be cured if the lights are strobed so that they are on only for a short time within each field. Note that incandescent lamps cannot be strobed, and light emitter diodes (LED) are needed for such a purpose. For the current system, the subject is asked to walk in the opposite direction of the TV scanning so that the light itself and not its tail can be detected.
In order to understand the system completely, it will be appropriate to have a description of its hardware and software. Note that this discussion is associated with the single TV camera system.

6.2.2.1 Hardware

The system hardware consists mainly of the TV interface which receives the television signal and takes the useful information out of it and finally prepares the data corresponding to the light coordinates. The system was originally designed by Cheng [55] and Chen [8] built a similar one for the Gait Laboratory. The block diagram of the system is shown in Fig. 6.3. The function of the whole system is controlled by a program which handles the entire data acquisition process. This includes the data obtained from the force plate. The interface has a register called the status register which contains two useful bits. The first is bit 7 which represents the ready flag. It is set when a light is detected and reset by the program. The second bit is 15 which is the field-done flag. It is reset at the beginning of each field by vertical blanking pulses, and set when the number of horizontal lines is counted to 232, which is close to the end of the field.

The input to the interface is the composite video signal. It is then amplified and separated into three parts, namely the video signal, blanking pulses, and vertical blanking pulses. The video signal goes to a voltage comparator which has an adjustable potentiometer, and if the input amplitude is larger than a certain amount (this corresponds to the cases that the scan beam reaches the light), there is a high output at the voltage comparator. The positions of the lights are determined by two 8-bit counters. The first one is the
Figure 6.3. Block Diagram of the Camera-Computer Interface System. (Adopted from [55]).
x-counter, and its input comes from a 4.45 MHz clock which generates about 241 pulses during the 53μs duration of a horizontal line. As soon as a light is seen, the output of the voltage comparator gates the contents of the x-counter into a buffer as the horizontal coordinate of the light. The blanking pulses are used as inputs to the y-counter. In each field, there are about 246 horizontal lines. Similarly, the content of the y-counter gates to a buffer when a light is seen. The x-counter is reset by blanking pulses while the y-counter is reset by the vertical blanking pulses. At the time the contents of the counters are gated, the ready flag is set also to let the programmer know that the data are in buffers. Once the user receives this information, he resets the ready flag and waits for the next light. One set of lights are obtained during each field and the field-done flag indicates its termination. The details of the interface components can be obtained for [8,55,93]. Note that since the width and height of the picture frame are not the same, there is a scale factor of 0.78 involved. That is, the y-coordinates must be multiplied by 0.78 to obtain the correct values.

6.2.2.2 Software

It was mentioned before that the TV-computer interface is a programmed interface. That is, the whole function is controlled by an assembly language program. Furthermore, this program must supervise the sampling process of the force plate data at the same time. All the data collected will then be processed and used for gait study.

Because of the attractive foreground/background option of the RT-11 system, it is possible to have a completely realtime system.
In other words, the foreground program can be in assembly language to do the data acquisition job. On the other hand, the background program may be in Fortran to take care of the data processing. This method has been successfully employed by the author with both programs residing in the memory of the PDP-11/34. However, the procedure is only good for small amounts of processing and analysis. As the current research was further developed, needs for the following tasks were obvious: editing and sorting the raw data, computation of angular data and their derivatives, smoothing the raw data, extraction of all useful parameters, applying the data to a complex dynamic model, and finally, displaying the results on the screen or obtaining hard copy graphs. Although the memory capacity of the system has been increased from a total of 16K to a total of 32K (semiconductor memory), still it does not provide sufficient space for all the processing needs. On the other hand, the task of data collection together with system setup is time consuming and not quite convenient. Consequently, the current procedure is based on storing the results on the floppy disks, and this is done in several steps. First, the assembly language program collects the data from the TV and force plate simultaneously. Because the time required for writing (storing) the data onto the disk is larger than the rate of data acquisition, the memory stores all data until the experiment terminates. Then, results are all written on the disk. Second, a Fortran program reads, edits and sorts the data and again writes them back on the disk. Next, another Fortran program reads the results, computes the angular values, smoothes both displacement and angular quantities and their derivatives and obtains all the gait parameters (heel-strike time, toe-off time, etc.) and writes them all
back so that the third Fortran program can use them later to get the kinetic results and store them for a fourth program to either display them or to plot them on the paper.

Since the assembly program controls the function of TV interface and sampling of the force plate data, it requires a special attention. In order to study the process of data acquisition, it is important to know about the AR11 system, which is one module realtime analog sub-system that interfaces with the PDP-11. The AR11 includes a 16-channel, 10-bit A/D converter with sample-and-hold and a programmable realtime clock with one external output. The analog inputs can be programmed for unipolar (0 - 5V) or bipolar (+2.5V) operation, which corresponds to the force plate output. There are several ways to start an A/D conversion including the overflow of the realtime clock. The user can choose any of the 16 single-ended channels of analog input under control of A/D Status Register (ADSR). There is also an A/D Buffer Register (ABDR) that takes the 10-bit converted values.

The realtime clock includes a Clock Status Register (CKSR), Clock Buffer/Preset (CKBR), and clock counter. The clock counter cannot be loaded directly from the processor, but can be read directly. When bit 0 of the CKSR is cleared, loading the CKBR also loads the clock counter register.

The clock counter register can be loaded with a negative number, say -17. Then the frequency of the clock can be set to some specific value, say 1 KHz. The counter will then start counting up with the above frequency until an overflow happens which causes the A/D conversion. For the above case, every 0.017 second one sample is obtained.
The flow chart for the software of the system is shown in Fig. 6.4. It uses several macro-instructions. The period of time for data collection and number of lights as well as several register and address specifications are parts of the program initializations. Next, the program is written such a way that it waits until the subject's heel touches the force plate. The procedure is as follows: One of the four components of the vertical force is sampled at a rate of 200 Hz, and its value is checked. If it is more than a threshold value, action starts.

The collection of data starts with sampling all eight channels at a rate of 2 KHz/channel. Once one set of eight values of force data is stored in the memory, the collection of TV data can be started. The status register of the interface has the address 164100, and the associated buffer whose lower and upper eight bits give the x and y coordinates, respectively, has the addresses 164102. Program checks the ready flag of the status register. If a light is detected, its coordinates are tested. The vertical coordinate is checked to ascertain that it is not due to the overshoot. Also, it is possible that one light may be detected twice because of the fact that it is not a 'point light.' Therefore, if the differences of both the horizontal and vertical components of the current light and previous light are at least five units, the data are accepted to be valid and considered as the x and y of a light. The valid data are stored in the memory and the ready flag is reset so that another light can be detected. Due to the fact that the testing process takes some time, the system is not able to detect two lights on the same horizontal scan line. The action for light detection is continued until the field-done flag is set, which
Figure 6.4. Software Flow Chart for Force Plate and Television Data Acquisition.
implies the termination of the TV field. A specific number of memory locations is reserved for each field which is enough for number of lights plus two data. This allows all lights to be detected in the presence of up to two invalid data points. Experimentally, in more than 99 percent of the cases, the number of invalid data points does not exceed to two. If the number of data points is more than the number of lights plus two, the extra ones will be ignored. If it is less, the unused space in memory is filled with zeros. In any case, a marker is written at the end of the field to indicate whether the number of data points is the same as the number of lights. The process continues with sampling of the force plate outputs until the required number of data points are taken. Note that the A/D conversion starts during retrace of the vertical blanking pulses.

Once all data is obtained and stored in the memory, they are all written onto the user's floppy disk under a specific data name. For ordinary gait study, the data is collected for about 2.5 seconds, which is far more than one normal gait cycle.

6.3 Statistical Characterization of Signal and Noise

In this section, an attempt is made to investigate some statistical characteristics of both television and force plate data. While the force plate signal is basically very steady and reliable, the TV signal does have some variation which requires more attention.

It must be mentioned that particularly there are some further preprocessing and editing associated with the raw television data which must be done in addition to the original redundancy test during data acquisition. In other words, it is necessary to have an algorithm
to take care of any missing information as well as the extra pieces of information. The method employed for such an editing or sorting purpose is based on a linear interpolation approach to fill in for the missing data points. On the other hand, the order of lights as well as the test of redundancy of extra data points are done by checking and comparing their coordinates. Thus, before any smoothing process, translational and angular information are in complete form without any redundancy.

6.3.1 Television Data

The data obtained from the TV cameras is employed to calculate both angular and translational information. Since human gait is a periodic process, any of the above quantities, say $f(t)$, can be expressed in the form of a Fourier series. That is, one can write:

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos n \omega_0 t + B_n \sin n \omega_0 t$$  \hspace{1cm} (6-11)

where $\omega_0 = \frac{2\pi}{Q}$ is the fundamental angular frequency and $Q$ is the period of the quantity. The above coefficients can be calculated as follows:

$$A_0 = \frac{1}{Q} \int_{0}^{Q} f(t) \, dt$$  \hspace{1cm} (6-12)

$$A_n = \frac{2}{Q} \int_{0}^{Q} f(t) \cos n \omega_0 t \, dt$$  \hspace{1cm} (6-13)

$$B_n = \frac{2}{Q} \int_{0}^{Q} f(t) \sin n \omega_0 t \, dt$$  \hspace{1cm} (6-14)

In general, a reasonable approximation can be obtained by retaining a finite number of harmonics and neglecting the higher ones.
For the case under consideration, the TV information consists of a finite number of data points, \( N \), in each period. Then, Eqs. (6-12) through (6-14) can be approximated in the form of summations instead of integrals as:

\[
A_0 = \frac{1}{N} \sum_{K=1}^{N} f(KT) \quad (6-15)
\]

\[
A_n = \frac{2}{N} \sum_{K=1}^{N} f(KT) \cos \left( \frac{2\pi n K}{N} \right) \quad (6-16)
\]

\[
B_n = \frac{2}{N} \sum_{K=1}^{N} f(KT) \sin \left( \frac{2\pi n K}{N} \right) \quad (6-17)
\]

where \( T \) is the sampling period, and \( \omega_0 \) is replaced by its equivalent, \( \frac{2\pi}{NT} \).

In the analysis of a random single (noise), its Fourier series representative, which is valid for a particular time interval of \( Q \) sec, and in practice is very limited, does not give very useful information. The reason is that if another period of \( Q \) sec is taken, the results are not the same. Thus, it is necessary to develop a spectral analysis of the noise that 'settles down' to a fixed frequency spectrum. This can be done by providing a measure of power density in the random signal (actually its mean-squared) at different frequencies. In other words, the difference in spectral analysis between deterministic and random signals can become clear. In the former case one can expand the time function in its Fourier series or transform, resulting in an amplitude (and phase) spectrum. However, in the latter case, it is the power
density at each frequency that plays the equivalent role. Note that throughout this dissertation, only stationary and ergodic processes [95] are assumed.

In order to explain an algorithm to obtain the 'power spectral density' or just the 'power spectrum', it is important to note that we may be dealing with two different cases. In the first case, we have a periodic function \( f(t) \). If \( Q \) is the corresponding period, one can use the complex Fourier series for the simplicity of the method and write

\[
f(t) = \frac{Q}{2} \sum_{n=-\infty}^{\infty} C_n e^{-j \frac{2\pi n t}{Q}}
\]

(6-18)

where \( C_n \) is the complex Fourier coefficient which can be expressed as:

\[
C_0 = A_0
\]

(6-19)

\[
C_n = A_n - j B_n \quad n \neq 0
\]

(6-20)

The explicit formula for \( C_n \) can be written as follows:

\[
C_n = \frac{2}{Q} \int_{-Q/2}^{Q/2} f(t) e^{-j \frac{2\pi n t}{Q}} \, dt
\]

(6-21)

In some cases, it is more convenient to get the expression by using the average value over \( M \) periods:

\[
C_n = \frac{2}{MQ} \int_{-MQ/2}^{MQ/2} f(t) e^{-j \frac{2\pi n t}{Q}} \, dt
\]

(6-22)
or

$$C_n = \frac{1}{M} \sum_{k=1}^{M} \left[ \int_{\frac{(k-1)Q}{2}}^{\frac{KQ}{2}} f(t) e^{-\frac{2\pi n t}{Q}} dt \right]$$

(6-23)

The power spectrum can then be obtained [96] as:

$$\phi_{FF}(n) = \frac{Q}{4} |C_n|^2$$

(6-24)

Note that for a periodic function, the average of M evaluations of $C_n$ on the whole interval of MQ is the same as the value obtained on one period Q.

As the second case, consider a function, g(t), which is not periodic. This corresponds to a random signal. Each interval of duration Q gives a different value for $C'_k$. If a period of MQ is taken, one can write:

$$C'_n(M) = \frac{2}{MQ} \int_{\frac{-MQ}{2}}^{\frac{MQ}{2}} g(t) e^{-j\frac{2\pi n t}{Q}} dt$$

(6-25)

and the power spectrum will be

$$\phi_{GG}(n) = \lim_{M \to \infty} \frac{MQ}{4} |C'_n(M)|^2$$

(6-26)

Since only the discrete type of information is available, it is necessary to work with the appropriate formulas. This can be done by taking advantage of the Discrete Fourier series method already explained. More specifically, Eqs. (6-15) through (6-17) can be used to obtain a smooth representation of the data, x(KT). To study the
statistical characteristics of the signal in the presence of noise, the corresponding data can be considered to be periodic simply because human gait is a periodic process, and the power of noise is much less than the signal's. The result can be expressed as

\[ \Phi_I \left( \frac{2\pi n}{NT} \right) = \frac{N}{4} |C_n|^2 \]

\[ = \frac{N}{4} (A_n^2 + B_n^2) \quad (6-27) \]

where \( N \) is the number of data points for one gait cycle, \( T \) is the sampling period (1/60 sec) and subscript \( I \) represents input.

While in the presence of informative data (signal), it can be assumed that one is dealing with a periodic function, this is not the case that the input consists of noise only. Experimentally, the noise data can be obtained by placing the lights on a solid and fixed object so that they are located as they would be if attached to a human body. Any variation of the angles will be due to the noise associated with the system.

The problem that is raised here is how to obtain the power spectrum of noise which is a random signal and not periodic. Therefore, associating any period to noise is not very logical. However, it is necessary to compute the power spectrum of noise for the frequencies equal to multiples of the fundamental frequency of the signal. In order to take care of this difficulty, it is suggested that the power spectrum can be estimated by [96]

\[ \hat{\Phi}_{NN} \left( \frac{2\pi n}{NT} \right) = \frac{N}{4} (A_n^2 + B_n^2) \quad (6-28) \]
where \( N \) is the number of data points, \( T \) is the sampling time, \( n \) is the harmonic number, \( A_n^' \) and \( B_n^' \) are obtained from expressions similar to Eqs. (6-15) and (6-17). Nevertheless, there is a possible difficulty which needs attention. It turns out that the variance of the random variable \( \hat{\phi}_{NN}(n) \) does not decrease with increasing \( N \) [96]. Thus, \( \hat{\phi}_{NN}(n) \) does approach \( \phi_{NN}(n) \) only in a statistical sense. Practically, \( \hat{\phi}_{NN}(n) \), given by Eq. (6-22) continues to oscillate randomly about \( \phi_{NN}(n) \) and does not settle down to the desired value no matter how many points are taken. One solution to avoid this undesirable feature is to take several, say \( M \), set of \( N \) data points of noise. Next, \( \hat{\phi}_{NN}(n) \) can be computed for each set, and then average the resultant values of estimated power spectrums to obtain a good approximation of \( \phi_{NN}(n) \). In fact, if each of these computed values are statistically independent of each other, the variance of the average value will be reduced by a factor of \( \frac{1}{M} \).

The power spectra of the television data (signal plus noise) have been obtained for three different anatomical angles corresponding to ankle, knee and hip joints, and are shown in Figs. 6.5, 6.6, and 6.7, respectively. In each case an average of five data sets are obtained. That is, in order to obtain a more reliable results, the power spectra for five different gait cycles are obtained and then averaged. In all cases, \( N=75 \) data points and \( T=1/60 \) sec. The horizontal axis in all figures represents the harmonic number and it varies from one (fundamental value) to 12. Figure 6.5 shows that for the ankle angle the second harmonic has the most power. The function then decreases so that its value for \( n \geq 6 \) is rather small. Figure 6.6 illustrates the power spectrum associated with the knee angle. The
Figure 6.5. Power Spectral Density for Ankle Angle vs. Number of Harmonics.
Figure 6.6. Power Spectral Density for Knee Angle vs. Number of Harmonics.
Figure 6.7. Power Spectral Density for Hip Angle vs. Number of Harmonics.
term corresponding to the fundamental harmonic is the greatest. In fact, most of the power exists for the first three harmonics, and for \( n \geq 4 \) it becomes quite small. Finally, the hip angle, shown in Fig. 6.7, has also the most of the power in the first two harmonics. For \( n \geq 3 \), it has some sort of oscillation. For \( n \geq 7 \) it takes fairly small values.

The power spectrum of the noise can be obtained by taking an approach similar to what has already been discussed. In order to obtain a better estimate of the power spectrum, 24 sets of noise data are taken. For each set, \( N=75 \) and \( T=1/60 \) sec. Moreover, the power spectrum for each set is computed, and the results are then averaged. Two angles are considered for this purpose. Figure 6.8 shows the average of 24 sets for the ankle angle. It appears that for \( n=4 \), the maximum noise power is obtained. Furthermore, for \( n=3, 6, \) and 12 the values are greater than the rest. Likewise, Fig. 6-9 shows the power spectrum of the noise corresponding to the knee angle. Again, for \( n=4 \), the maximum noise power is obtained. Next are \( n=3, 6, \) and 12 that when compared with the rest yield higher noise power.

It is important to note that the sampling time of the system is fixed (1/60 sec). This limits the accuracy for the computation of the values of power spectrum associated with the higher harmonics. Also, the rectangular integration rule which is employed to calculate the Fourier coefficients, may generate some undesired error. Therefore, the number of harmonics considered is limited to 12 or less.

6.3.2 Force Plate Data

The force plate system is one of the sources that provides information for the system. Due to the fact that it is a rather
Figure 6.8. Power Spectral Density of Noise Associated with Ankle Angle.
Figure 6.9. Power Spectral Density of Noise Associated with Knee Angle.
sophisticated system, the data obtained from such a system is considered to be quite reliable. Furthermore, the force plate outputs have been used so that the quality of the dynamic models can be evaluated. In other words, one does not expect to receive any undesired or noisy information from the force plate. Nevertheless, it will be helpful to check the quality of the force plate output. For this purpose, a solid weight of 300 lbs (1328 Newtons) is put on the platform and the force plate outputs are measured. Next, the six quantities associated with the force plate are calculated, and for each of them, the corresponding standard deviation and expected value are obtained. If \( x_1(i=1,N) \) represents the quantity which has been measured, the standard deviation is obtained by

\[
S = \left( \frac{\sum_{i=1}^{N} (x_i - x)^2}{N} \right)^{1/2}
\]

(6-29)

Table 6.2 shows the results of the experiments for the six quantities. Note that the maximum value of the standard deviation for any of the forces is 1.06 Newton (0.238 lbs) which is not very large. The position of the center of pressure is obtained very accurately and the associated standard deviations are very small. The free moment component has also a small standard deviation.

6.4 Suboptimal and Optimal Filtering

This section is divided into two parts. The first part is to obtain a suboptimal filter to be applied to the television data. The second part takes advantage of the statistical characteristics of
Table 6.2
Standard Deviation and Expected Value of the Force Plate Measurements

<table>
<thead>
<tr>
<th></th>
<th>(F_Z) (Newton)</th>
<th>(F_X) (Newton)</th>
<th>(F_Y) (Newton)</th>
<th>(a_X) (Meters)</th>
<th>(a_Y) (Meters)</th>
<th>(M'_Z) (Newton Meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>1.060</td>
<td>0.8250</td>
<td>0.6645</td>
<td>(\leq 10^{-7})</td>
<td>(\leq 10^{-7})</td>
<td>0.0463</td>
</tr>
<tr>
<td>Expected Value</td>
<td>1329.1</td>
<td>0.65</td>
<td>0.24</td>
<td>-0.034</td>
<td>-0.002</td>
<td>2.17</td>
</tr>
</tbody>
</table>

signal and noise which were discussed in the last section to obtain an optimal filter.

6.4.1 Suboptimal Filtering

This method is a smoothing techniques based on the minimization of the least square error (conventional Fourier series approach) shown by Eq. (6-17). The translational or the angular data obtained for one gait cycle are taken and the Fourier coefficients are computed by Eqs. (6-13) through (6-20). The final results can be determined by substituting the coefficients into Eq. (6.16). The cut-off harmonic is chosen by the user and is selected experimentally. In order to show the effect of the number of harmonics used to get the smoothed data, Figs. 6.10 through 6.12 illustrate the ankle angle obtained with 3, 6, and 9 harmonics, respectively. The vertical axis represents the amount of angle in degrees, and the horizontal axis indicates the time with each unit equal to 1/60 sec, and the total time corresponds
Figure 6.10. Smoothed Ankle Angle Using Three Harmonics.
Figure 6.11. Smoothed Ankle Angle Using Six Harmonics.
to one complete gait cycle. It is obvious that as the number of harmonics increases, there appears to be too many ups and downs which are neither expected in the motion nor desirable. Therefore, an attempt should be made to keep the cut-off harmonic rather small. Figures 6.13 through 6.15 show the knee angle obtained with 3, 6, and 9 harmonics \( m \), respectively. It can be seen that the increase of \( m \) does not change the shape of the graph very much. This conclusion is expected because we already have observed that the powers associated with the first two or three harmonics of the knee angle are much larger than the others. Thus, the change is rather small if one adds higher frequencies. Finally, Figs. 6.16 through 6.18 illustrate the hip angle for three cases \( m = 3, 6, \) and 9, respectively. Note that the effect of increasing the number of harmonics is less than that associated with the ankle angle and more than that corresponding to the knee angle. In other words, \( m=9 \) is too large, but \( m=5 \) or 6 can be acceptable.

In order to have a general algorithm to smooth the television data, many data sets from several subjects have been tested. From the experimental results, it has turned out that the best value for \( m \) varies between 3 and 6, depending on the data set and the type of the quantity and application.

6.4.2 Optimal Filtering

One can now investigate the problem of how to filter linearly an observed, discrete-time, random process \( y(KT) \) to obtain the best estimate by minimizing the mean square error, which is determined by comparing \( y(KT) \) with the desired discrete random process, \( x(KT) \). The problem can be tackled by investigating the corresponding
Figure 6.13. Smoothed Knee Angle Using Three Harmonics.
Figure 6.15. Smoothed Knee Angle Using Nine Harmonics.
Figure 6.16. Smoothed Hip Angle Using Three Harmonics.
Figure 6.17. Smoothed Hip Angle Using Six Harmonics.
Figure 6.18. Smoothed Hip Angle Using Nine Harmonics.
continuous-time version [98] and then converting the results to the
discrete case [99].

Consider a continuous case where the input to the system is
\( f_1(t) \) which consists of an informative signal \( s(t) \), and the noise
component \( n(t) \). Fig. 6.19 shows such a system which has transfer
function \( G(s) \). The output is represented by \( f_0(t) \). Furthermore,
the desired output can also be shown by \( f_\alpha(t) \). The goal is to design
a linear system described by the impulse response \( g(t) \), such that
the mean-square error between the actual output and the desired output
is minimized. Since one is working with a random process, it makes
sense to perform the calculations in terms of the frequency domain
functions (power spectra and transfer functions).

\[
\begin{array}{ccc}
\text{Input} & \phantom{\text{g}(t)} & \text{Output} \\
\varepsilon_1(t) = s(t) + n(t) & G(j\omega) & f_0(t) \\
\end{array}
\]

Figure 6.19. Input-Output Configuration.

Under the assumption that signal and noise are uncorrelated,
there are two components involved with the determination of the mean-
square error. Since linear systems are considered, these two can be
evaluated separately. The first part is the error due to the trans-
mission of noise through the system. This is corresponding to the case
that the input to the system consists of noise only. If \( \phi_{NN}(\omega) \)
represents the power spectrum of the noise, the associated output is
\[ \phi_{\text{ON-ON}}(\omega) = \left| G(j\omega) \right|^2 \phi_{\text{NN}} \]  

(6-30)

Obviously this output is undesirable and generates the error whose mean square is

\[ \overline{e_n^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{\text{ON-ON}}(\omega) \, d\omega \]

\[ = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(j\omega)| \phi_{\text{NN}}(\omega) \, d\omega \]  

(6-31)

The second components of the error is resulted from the difference between the desired output and the actual output caused by the signal \( s(t) \). This is described in Fig. 6.20. This error exists because the linear system does not operate on \( s(t) \) properly to obtain the desired output. \( G_d(j\omega) \) is the desired transfer function which can yield the desired output. \( \phi_{ss}(\omega) \) and \( e_s(t) \) represent the power spectrum of the signal and the component of the error due to the signal, respectively. Like Eq. (6-31), system has a transfer function of \( G_d(j\omega) - G(j\omega) \), and one can write

\[ \overline{e_s^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| G_d(j\omega) - G(j\omega) \right|^2 \phi_{ss}(\omega) \, d\omega \]  

(6-32)

Throughout this research it is assumed that the signal and noise are obtained from independent sources and therefore uncorrelated. Under this assumption, the mean square value of the total error is simply the sum of the two components already expressed. The problem is now to minimize
\[
e^{-2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ |G(j\omega)|^2 \phi_{NN}(\omega) + |G(j\omega) - G_d(j\omega)|^2 \phi_{SS}(\omega) \right] d\omega
\] (6-33)

Note that a stationary random process is also assumed. The minimization algorithm without regard to physical realizability of the system can be done by considering magnitude-phase form for \(G(j\omega)\) and \(G_d(j\omega)\) as

\[
G(j\omega) = A(\omega) e^{j\theta(\omega)}
\] (6-34)

\[
G_d(j\omega) = A_d(\omega) e^{j\theta_d(\omega)}
\] (6-35)

Now \(G(j\omega)\) and \(G_d(j\omega)\) can be written in terms of their real and imaginary parts. By doing this and taking the corresponding magnitude, we will have:

\[
e^{-2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| A^2 \phi_{NN} + \left[A_d^2 + A^2 - 2AA_s \cos(\theta - \theta_d)\right] \phi_{SS}\right| d\omega
\] (6-36)
where the arguments ω and jω are omitted for convenience. Since A, A_d, and φ_{ss} are all non-negative, the minimum value of the integral is achieved by setting θ = θ_d. Thus error will become

\[
\mathcal{E}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ A^2 (\phi_{SS} + \phi_{NN}) - 2AA_d \phi_{SS} + A^2_d \phi_{SS} \right] d\omega \quad (6-37)
\]

The above equation may be rearranged as

\[
\mathcal{E}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ \left( A \sqrt{\phi_{II}} - \frac{A_d \phi_{SS}}{\sqrt{\phi_{II}}} \right)^2 + \frac{A^2_d \phi_{SS} \phi_{NN}}{\phi_{II}} \right] d\omega \quad (6-38)
\]

where \( \phi_{II}(\omega) = \phi_{II} = \phi_{NN} + \phi_{SS} \) is the power spectrum of the whole input. It is obvious that the minimum error occurs when the square term in Eq. (6-38) is zero. Thus

\[
A = A_d \frac{\phi_{SS}}{\phi_{II}} \quad (6-39)
\]

Since we are dealing with pure filtering, \( A_d = 1 \). Also, \( \phi_{SS} \) is the difference between \( \phi_{II} \) and \( \phi_{NN} \) which are measurable quantities. Consequently, we can write

\[
A_{opt}(\omega) = \frac{\phi_{II}(\omega) - \phi_{NN}(\omega)}{\phi_{II}(\omega)} \quad (6-40)
\]

Eq. (6-40) is equivalent to the so-called Wiener Filter. Subscript opt is used to indicate the optimum. Note that this filter is not physically realizable in realtime.

In the case of discrete systems, a similar expression can be used. That is
\[ A_{opt} \left( \frac{2\pi m}{NT} \right) = \frac{\phi_{II} \left( \frac{2\pi n}{NT} \right) - \phi_{NN} \left( \frac{2\pi n}{NT} \right)}{\phi_{II} \left( \frac{2\pi n}{NT} \right)} \]  \hspace{1cm} (6-41)

where \( m \) is a positive integer representing the harmonic number, \( N \) is the number of data points and \( T \) is the sampling time (1/60 sec). In Sec. 6.3, a method by which the power spectrum can be computed has already been studied. The results have been presented for both noise and signal in the presence of noise. Therefore the associated values can be employed to obtain the corresponding optimal gains.

Table 6.3 shows the power spectra of the inputs and noise as well as the corresponding optimal gains for the ankle angle, and \( n \) represents the harmonic number. \( \phi_{NN} \) is obtained by the averaging method explained in Sec. 6.3. It is interesting to see that the gain values stay more or less in the neighborhood of unity. In other words, it looks that the amount of noise power is much less than the input power.

The same approach is taken for the knee angle and the results are shown in Table 6.4. Like those obtained for the ankle angle, the gain values for the knee angle are fairly close to unity and they do not vanish as the frequency increases.

Table 6.5 shows the set of optimal gains for the hip angle. The approach is more or less similar to the previous ones. It is again obvious that the values of the gain are fairly close to unity and do not vanish as frequency increases.

An overall study of the optimal filter applied to various angular data reveals that the amount of noise power is rather small.
Table 6.3

Power Spectra for Noise and Input With Optimal Gain Values for Ankle Angle Using Different Harmonics

<table>
<thead>
<tr>
<th>m</th>
<th>$\phi_{NN}$</th>
<th>$\phi_{II}$</th>
<th>$A_{opt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.14</td>
<td>628.14</td>
<td>0.998</td>
</tr>
<tr>
<td>2</td>
<td>1.52</td>
<td>1051.65</td>
<td>1.000</td>
</tr>
<tr>
<td>3</td>
<td>4.94</td>
<td>297.54</td>
<td>0.983</td>
</tr>
<tr>
<td>4</td>
<td>11.21</td>
<td>183.92</td>
<td>0.939</td>
</tr>
<tr>
<td>5</td>
<td>2.09</td>
<td>62.89</td>
<td>0.967</td>
</tr>
<tr>
<td>6</td>
<td>6.65</td>
<td>54.53</td>
<td>0.878</td>
</tr>
<tr>
<td>7</td>
<td>2.28</td>
<td>41.80</td>
<td>0.945</td>
</tr>
<tr>
<td>8</td>
<td>3.42</td>
<td>62.89</td>
<td>0.946</td>
</tr>
<tr>
<td>9</td>
<td>3.80</td>
<td>80.75</td>
<td>0.593</td>
</tr>
<tr>
<td>10</td>
<td>1.90</td>
<td>59.66</td>
<td>0.968</td>
</tr>
<tr>
<td>11</td>
<td>2.09</td>
<td>34.77</td>
<td>0.940</td>
</tr>
<tr>
<td>12</td>
<td>6.08</td>
<td>35.15</td>
<td>0.827</td>
</tr>
</tbody>
</table>

and in most cases the optimal gain values are very close to 1. As the harmonic number increases, the error associated with the integration method used to obtain power spectra can get large which is why the study has been done for up to $m=12$. Note that the noise in this case is limited to the electrical system and is not all the noise received during the collection of gait data.

6.5 Summary

This chapter has discussed the factors which are involved with obtaining both kinematic and kinetic measurements. All of the equipment currently used in the Gait Laboratory is briefly described.
Table 6.4
Power Spectra for Noise and Input
With Optimal Gain Values for Knee
Angle Using Different Harmonics

<table>
<thead>
<tr>
<th>m</th>
<th>$\phi_{NN}$</th>
<th>$\phi_{II}$</th>
<th>$A_{opt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.641</td>
<td>7913.70</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>0.931</td>
<td>6608.70</td>
<td>1.000</td>
</tr>
<tr>
<td>3</td>
<td>2.500</td>
<td>972.60</td>
<td>0.997</td>
</tr>
<tr>
<td>4</td>
<td>5.456</td>
<td>68.90</td>
<td>0.921</td>
</tr>
<tr>
<td>5</td>
<td>1.449</td>
<td>44.02</td>
<td>0.967</td>
</tr>
<tr>
<td>6</td>
<td>2.655</td>
<td>47.12</td>
<td>0.944</td>
</tr>
<tr>
<td>7</td>
<td>0.529</td>
<td>13.81</td>
<td>0.962</td>
</tr>
<tr>
<td>8</td>
<td>1.224</td>
<td>9.19</td>
<td>0.867</td>
</tr>
<tr>
<td>9</td>
<td>1.096</td>
<td>19.91</td>
<td>0.945</td>
</tr>
<tr>
<td>10</td>
<td>0.466</td>
<td>21.14</td>
<td>0.978</td>
</tr>
<tr>
<td>11</td>
<td>1.044</td>
<td>6.65</td>
<td>0.843</td>
</tr>
<tr>
<td>12</td>
<td>2.657</td>
<td>4.16</td>
<td>0.360</td>
</tr>
</tbody>
</table>

This gives a good insight about the functions of the whole system to the reader as well as some information which can be helpful in employing similar equipment with the part numbers given as references. Also, a description of the experimental environment was presented together with the hardware-software requirements. Moreover, all the data acquisition techniques were discussed. It was obvious that this system is fairly simple and easy to employ. The television components, however, have some drawbacks such as the sampling rate, the failure to detect two lights on the same horizontal line, the blooming feature of the videocon tubes, and the tailing effect of the lights. On the
Table 6.5

Power Spectra for Noise and Input With Optimal Gain Values for Hip Angle Using Different Harmonics

<table>
<thead>
<tr>
<th>m</th>
<th>(\phi_{NN} )</th>
<th>(\phi_{II} )</th>
<th>(A_{opt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.176</td>
<td>1963.10</td>
<td>0.998</td>
</tr>
<tr>
<td>2</td>
<td>4.506</td>
<td>645.70</td>
<td>0.993</td>
</tr>
<tr>
<td>3</td>
<td>18.678</td>
<td>205.60</td>
<td>0.909</td>
</tr>
<tr>
<td>4</td>
<td>18.574</td>
<td>275.98</td>
<td>0.933</td>
</tr>
<tr>
<td>5</td>
<td>11.434</td>
<td>110.58</td>
<td>0.897</td>
</tr>
<tr>
<td>6</td>
<td>23.962</td>
<td>159.53</td>
<td>0.850</td>
</tr>
<tr>
<td>7</td>
<td>6.450</td>
<td>62.64</td>
<td>0.897</td>
</tr>
<tr>
<td>8</td>
<td>9.957</td>
<td>79.63</td>
<td>0.875</td>
</tr>
<tr>
<td>9</td>
<td>6.085</td>
<td>122.66</td>
<td>0.950</td>
</tr>
<tr>
<td>10</td>
<td>4.112</td>
<td>130.85</td>
<td>0.969</td>
</tr>
<tr>
<td>11</td>
<td>5.161</td>
<td>93.41</td>
<td>0.945</td>
</tr>
<tr>
<td>12</td>
<td>20.374</td>
<td>57.61</td>
<td>0.646</td>
</tr>
</tbody>
</table>

whole, one may conclude that the system is just good for the gait study. More modifications are needed in order that a higher accuracy can be obtained or other aspects of human locomotion can be studied.

The quality of both force plate and television data were also studied. The standard deviations corresponding to the force plate data were rather small. A Fourier series approach was employed to get the power spectrum of data. The analysis of the input signal pointed out that most of the power is in the first three or four harmonics. The study of the noise characteristics were based on taking several data sets and averaging the associated power spectra. Results showed that the highest noise power belonged to the fourth
harmonic \( (n = 4) \). However, the noise power spectrum which is mainly due to the undesired electrical sources was much smaller than the signal power.

Suboptimal smoothing methods were applied to the TV data. The experimental results on the number of harmonics showed that the knee angle is not very sensitive to higher harmonics. On the other hand, hip angle does include some higher harmonics which are not quite desirable. The worst case is concerned with the ankle angle which requires fairly small number of harmonics \( (n = 3, 4) \) to avoid an improper form.

The study of optimal filters for various angles results in high gain values for most of the cases. In fact, if a small number of harmonics is used, the gains are practically all ones. It is important to know that there are other sources of noise in addition to the electrical noise which has been considered so far. For example, the lights are not quite fixed on the subject's body, and actually they move a little no matter how tightly they are attached. Moreover, there are also vibrations during gait due to the process of stepping. This may also generate some undesirable high frequency components.

Experimentally, the value of \( m \) is chosen to be between 3 and 6 which can be applied to translational and angular data as well as their derivatives. However, the experimental suboptimal value of \( n \) for kinetic studies turned out to be \( n = 3, 4 \). The difference can be explained by the fact that the joint forces and torques are not linear functions of the angular data, and thus do not necessarily require the same number of harmonics.
Chapter 7

EXPERIMENTAL RESULTS

7.1 Introduction

This chapter presents a variety of results obtained for experiments that have been conducted in the Gait Laboratory. The reader, then will have a better insight about what has been discussed so far, both quantitatively and qualitatively. Also, these results will help him to make his own judgments and conclusions.

Kinematic results will be presented in Sec. 7.2. This includes various joint angles as well as their velocities and accelerations. Next, in order to study the quality of the measurements, several experiments are done on the same individual and the results are statistically presented. Moreover, the effect of the type of shoe is studied by observing results of three different cases: positive heel shoes, bare-foot, and negative heel shoes (so-called earth shoes). Sec. 7.3 presents several kinetic results obtained from the dynamic models developed in Chapters 3 and 4. Various joint forces and torques are shown associated with each model. Sec. 7.4 makes an argument about the results seen in Sec. 7.3 and compares the quality of models with each other. It also discusses the acceptance or rejection of different assumptions made in each case. Sec. 7.5 presents an example application to the experimental gait studies by applying the model explained in Chapter 5 to data sets obtained from several male or female
individuals wearing positive or negative heel shoes or just barefoot. Finally, a summary of the results will be presented in Sec. 7.6.

7.2 Kinematic Results

There are two kinds of angular measurements. In one case the angles are measured with respect to the horizontal axis, which are called inertial angles. These are the same as the angles used in all three dynamic models explained in Chapters 3 through 5. The second kind considers the anatomical angles, that is, the angles between various segments of the body.

Since the inertial angular data have been extensively used to study the human gait, the corresponding results are presented first. There are four angles associated with the segments of the body. These angles are (1) the angle of the upper body; (2) thigh angle; (3) shank angle; (4) foot angle. They are shown in Figs. 7.1 through 7.4, respectively. These correspond to four angles shown in Fig. 4.1 as $\theta_7$, $\theta_3$, $\theta_2$, and $\theta_1$, respectively. In any of the four figures, the vertical axis indicates the angle in radians and the horizontal axis represents the time with each unit equivalent to 1/60 sec. Each figure shows the variation of the associated angle for one complete gait cycle which starts with the heel-strike of the left foot and concludes with the next heel-strike of the same foot. The period of the gait is 1.27 sec., and toe-off time is 0.83 sec., which is about 65 percent of the gait cycle. Note that the sub-optimal approach (conventional Fourier series) explained in Secs. 6.3 and 6.4 has been used with the first five harmonics included. Note also that all four angles decrease as the body goes toward the toe-off situation. However, angles increase during swing phase. This is the sort of result that one expects.
Figure 7.1. Inertial Angle of the Upper Body for One Gait Cycle.
Figure 7.2 Inertial Angle of Left Thigh for One Gait Cycle.
Figure 7.3. Inertial Angle of Left Shank for One Gait Cycle.
Since the angular rates and angular accelerations are also important and directly are involved in dynamic equations of the body, it is worthwhile to present one sample of each quantity. Figs. 7.5 through 7.8 show four angular rates corresponding to upper body, thigh, shank, and foot, respectively. Also, Figs. 7.9 through 7.12 illustrate the associated angular accelerations, respectively. Note that the upper body has rather small angular rates and accelerations. This is expected because of its limited motion during the walking process.

On the other hand, as one moves from top to bottom, the magnitudes of angular rates and accelerations corresponding to lower segments become larger. This increase is valid for both positive and negative values. It can be seen that angular accelerations exhibit larger values during the swing phase which is quite natural.

A second set of kinematic data is presented so that the statistical properties can be observed. Moreover, it includes the effects of wearing different kinds of shoes. There are three anatomical angles illustrated in each case. These angles are associated with ankle, knee, and hip joints as shown in Fig. 7.13, and correspond to the angles between the body links. Because of the fact that a statistical analysis of results is desirable, ten data sets, each representing one gait cycle of the same person, are averaged together. Obviously, the average (mean) of the ten gait cycles is a much better indication of the kinematic behavior of the subject. Furthermore, in order to determine the variation of the average gait, the upper and lower bounds of its 95 percent confidence interval using a t-distribution method are illustrated.

The data obtained from two subjects, a
Figure 7.5. Angular Rate of Upper Body for One Gait Cycle.
Figure 7.6. Angular Rate of Left Thigh for One Gait Cycle.
Figure 7.7. Angular rate of left shank for one gait cycle.
Figure 7.8. Angular Rate of Left Foot for One Gait Cycle.
Figure 7.10. Angular Acceleration of Left Thigh for One Gait Cycle.
Figure 7.11. Angular Acceleration of Left Shank for One Gait Cycle.
Figure 7.12. Angular Acceleration of Left Foot for One Gait Cycle.
Figure 7.13. Three Anatomical Joint Angles.
male and a female, are presented here. There are two kinds of shoes (positive heel and negative heel) as well as the barefoot case per subject. A more complete qualitative investigation involving a total of twenty subjects can be found in [100]. In addition to three angles obtained in each case, there is some other information such as stride time, stance (toe-off) time, stride length, and vertical heel-rise that can be obtained and presented. Tables 7.1 and 7.2 show the results (average of ten cycles) of the gait parameters associated with the male and female subjects, respectively.

Table 7.1

Gait Parameters for the Male Subject

<table>
<thead>
<tr>
<th></th>
<th>Height: 69 1/2&quot;</th>
<th>Weight: 187 lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barefoot</td>
<td>1.29</td>
<td>1.31</td>
</tr>
<tr>
<td>Positive Heel</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>Negative Heel</td>
<td>1.04</td>
<td>1.09</td>
</tr>
<tr>
<td>Stride Time (sec)</td>
<td></td>
<td>1.08</td>
</tr>
<tr>
<td>Stride Length (m)</td>
<td></td>
<td>1.08</td>
</tr>
<tr>
<td>Stance Time (sec)</td>
<td>0.87</td>
<td>0.94</td>
</tr>
<tr>
<td>Heel-Rise (m)</td>
<td>0.23</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Although any general judgment based on only two subjects will not be very accurate, one may make some conclusions by observing the above numerical values. The first things to be pointed out are both the stride time and the stance time which have the smallest values in
Table 7.2

Gait Parameters for the Female Subject

<table>
<thead>
<tr>
<th></th>
<th>Height: 67&quot;</th>
<th>Weight: 125 lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Barefoot</td>
<td>Positive Heel</td>
</tr>
<tr>
<td>Stride Time (sec)</td>
<td>1.14</td>
<td>1.25</td>
</tr>
<tr>
<td>Stride Length (m)</td>
<td>1.03</td>
<td>1.00</td>
</tr>
<tr>
<td>Stance Time (sec)</td>
<td>0.74</td>
<td>0.82</td>
</tr>
<tr>
<td>Heel-Rise (m)</td>
<td>0.23</td>
<td>0.20</td>
</tr>
</tbody>
</table>

the 'barefoot' case. The next conclusion to be made is about the heel-rise with the least value obtained from the positive heel case. This may be due to the reason that subjects raised their feet more and less up to a certain maximum level during the swing phase. Since in the case of positive heel, the heel is higher compared with the other two cases, its difference with the maximum is least.

The angular data for each subject is divided into three groups. Each group consists of one of the anatomical angles for the two shoe types and barefoot cases. Note that in order to be able to get the average of ten gait periods, it was necessary to normalize each cycle.

The normalization procedure is based on the expansion of angular functions by using a constant value equal to $N/101$, where $N$ is the number of data points for the complete gait cycle. Values of the normalized function for the new sampling times are obtained by
employing a linear interpolation method. Let \( x(i) \) represent an array of angular information with \( N \) elements for one walking cycle. The elements of the normalized function \( X(J) \) can be calculated as follows:

For \( J=1 \) through 101

\[
X(J) = x(K)(K-B+1) + x(K+1)(B-K) \tag{7-1}
\]

where

\[
B = \frac{J \times N}{101} \tag{7-2}
\]

\[
K = \text{Integer part of } B \tag{7-3}
\]

One must be careful about points close to the two ends. More specifically, it is assumed that for \( K=0 \), the normalized value is equal to \( x(1) \). Also, for \( K=N \), the normalized value will be \( x(N) \).

It should be noted that the new function has 101 points. One can shift the index so that it can represent the numerical values corresponding from 0 to 100 percent of the gait cycle.

Consequently, each data set has been converted to its equivalent by using percentage of the gait cycle instead of the conventional time unit. In all graphs, the average values of the angles are shown by '+' sign while the upper and lower bounds of the 95 percent confidence intervals are shown by '*' and 'o' signs, respectively. Figs. 7.14 through 7.16 show the ankle angle graphs for the male subject. Similarly, Figs. 7.17 through 7.19 show the knee angle graphs and Figs. 7.20 through 7.22 illustrate the graphs associated with the hip angle for the same subject.
Figure 7.14. Average of Normalized Anatomical Ankle Angle for Male Subject with Corresponding Confidence Interval for Barefoot Case.
Figure 7.16. Average of Normalized Anatomical Ankle Angle for Male Subject with Corresponding Confidence Interval for Negative Heel Shoes.
Figure 7.17. Average of Normalized Anatomical Knee Angle for Male Subject with Corresponding Confidence Interval for Barefoot Case.
Figure 7.18. Average of Normalized Anatomical Knee Angle for Male Subject with Corresponding Confidence Interval for Positive Heel Shoes.
Figure 7.19. Average of Normalized Anatomical Knee Angle for Male Subject with Corresponding Confidence Interval for Negative Heel Shoes.
Figure 7.20. Average of Normalized Anatomical Hip Angle for Male Subject with Corresponding Confidence Interval for Barefoot Case.
Figure 7.21. Average of Normalized Anatomical Hip Angle for Male Subject with Corresponding Confidence Interval for Positive Heel Shoes.
Figure 7.22. Average of Normalized Anatomical Hip Angle for Male Subject with Corresponding Confidence Interval for Negative Heel Shoes.
A comparison of the ankle angles with each other indicates that the magnitude of the angle with positive heel is the most and with barefoot is the least. Also, it appears that the difference between maximum and minimum points for the negative heel (NH) shoe is greater than the other two cases, and the peaks for NH are a little sharper than the others.

The knee angles for the three cases show no obvious difference and the graphs are very similar to each other. The same comparison about the hip angles indicates that in the case of positive heel (PH) shoes, at the beginning and at the end, the angle is a little smaller than the others. Also, before the end of the gait cycle, angles for the barefoot (BF) and NH cases are almost constant, but it is not true about PH.

The kinematic data obtained from the female subject are also divided into three groups with the same format. Figs. 7.23 through 7.31 show the results of ankle, knee, and hip angles, respectively. A comparison about the ankle angles indicates again that the PH one is generally larger than the others. Also, the peaks and valleys of the graphs for PH and NH cases are sharper than BF. The knee angles do not show any obvious differences and more or less look the same. However, the hip angles indicate that again for the PH case, at the beginning and at the end of the gait cycle the corresponding values are smaller. Actually, the hip angle values toward the end of the swing phase are decreasing for PH while for other cases they either increase (BF) or stay the same (NH).

A general conclusion about the results obtained from two subjects can be made by emphasizing that for each individual the
Figure 7.23. Average of Normalized Anatomical Ankle Angle for Female Subject with Corresponding Confidence Interval for Barefoot Case.
Figure 7.24. Average of Normalized Anatomical Ankle Angle for Female Subject with Corresponding Confidence Interval for Positive Heel Shoes.
Figure 7.25. Average of Normalized Anatomical Ankle Angle for Female Subject with Corresponding Confidence Interval for Negative Heel Shoes.
Figure 7.26. Average of Normalized Anatomical Knee Angle for Female Subject with Corresponding Confidence Interval for Barefoot Case.
Figure 7.27. Average of Normalized Anatomical Knee Angle for Female Subject with Corresponding Confidence Interval for Positive Heel Shoes.
Figure 7.28. Average of Normalized Anatomical Knee Angle for Female Subject with Corresponding Confidence Interval for Negative Heel Shoes.
Figure 7.29. Average of Normalized Anatomical Hip Angle for Female Subject with Corresponding Confidence Interval for Barefoot Case.
Figure 7.30. Average of Normalized Anatomical Hip Angle for Female Subject with Corresponding Confidence Interval for Positive Heel Shoes.
Figure 7.31. Average of Normalized Anatomical Hip Angle for Female Subject with Corresponding Confidence Interval for Negative Heel Shoes.
general shapes of the graphs do not vary much for BF, PH, and NH cases. However, it appears that the corresponding graphs obtained from two persons are quite different; and in other words, every individual has his own pattern.

7.3 Kinetic Results

As it was discussed in Chapter 6, the force plate system is able to measure six quantities from which only three are used in this research. These are namely two components of the ground reaction forces and the location of center of pressure (zero moment point), and all three of them are in the sagittal plane. An example of these measurements are shown in Figs. 7.32 and 7.34. In the experiment, the subject only steps with one foot on the platform, and thus there is no measurement during swing phase of the corresponding leg. The first two figures show the forces, and the subject weighs 149 lbs. or 663 Newtons. The vertical axis in Figs. 7.32 and 7.33 are in 'Newtons' where the horizontal axis represents the time, and each unit is equivalent to 1/60 sec. Fig. 7.34 also shows the variation of center of pressure (in meters) with respect to time. In all three figures, the process starts with heel-strike, and the total time is one complete gait cycle. Note that the vertical ground force reaction exceeds the weight of the body twice and drops below this value once due to the inertial forces. Also, the horizontal force changes in direction during the stance phase. The position of the center of pressure is shown with the center of mass as the reference point.
Figure 7.32. Horizontal Ground Force Reaction for One Gait Cycle Measured by Force Plate.
Figure 7.33. Vertical Ground Force Reaction for One Gait Cycle Measured by Force Plate.
Figure 7.34. Position of Center of Pressure Measured by Force Plate as Percentage of Gait Cycle.
In previous chapters, three dynamic models of human body were developed and discussed. While the first two did not employ force plate measurements explicitly, the third one directly applied them to the model. In this section, the results associated with each model are presented extensively.

7.3.1 Five-Mass Model Results

This model is explained in Chapter 3, and all of the modifications to improve its quality have already been discussed in detail. The model assumes that the ground reaction forces and torques (applied at the ankle joint) transfer linearly with respect to time from one leg to the other during double support phase of the gait cycle. Results include several joint forces and torques as well as the hip partition coefficients, α, β, and γ. The body segment parameters are obtained by using Braune and Fischer's coefficients. Table 7.3 shows these parameters as well as other physical information associated with the subject.

Table 7.3
Physical Data About Subject of Five-Mass Model

<table>
<thead>
<tr>
<th></th>
<th>Trunk</th>
<th>Thigh</th>
<th>Shank &amp; Foot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex: Male</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height: 1.8m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight: 663N</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>l (meters)</td>
<td>0.846</td>
<td>0.441</td>
<td>0.513</td>
</tr>
<tr>
<td>m (kgs)</td>
<td>45.1</td>
<td>7.02</td>
<td>4.19</td>
</tr>
<tr>
<td>J(kg.ms)</td>
<td>2.23</td>
<td>0.084</td>
<td>0.100</td>
</tr>
<tr>
<td>d (meters)</td>
<td>0.286</td>
<td>0.253</td>
<td>0.273</td>
</tr>
</tbody>
</table>
The various routines which were used to obtain joint forces and
torques are shown in the Appendices and the function of each of
them is also described. In order to test the quality of the results,
the ground reaction forces are compared with the force plate output.
In this particular experiment, the period of gait cycle is 1.27 sec.,
or 76 time units (each time unit 1/60 sec.). Action starts with left
heel-strike (LSH). The other parameters are: RTO=12, RHS=38,
LTO=50 time units. The horizontal forces are shown in Figs. 7.35
through 7.37, representing the horizontal ground reaction forces
together with the corresponding force plate measurement (shown by 'x'),
the component at the knee, and the associated trunk force, respectively.
Fig. 7.38 shows the partition coefficient $\alpha$, and Fig. 7.39 illustrates
the total horizontal ground reaction force, $F_{x0}$. The corresponding
mean-square error for the horizontal force is $0.212 \times 10^4$ (Newtons)$^2$.
All forces are expressed in terms of Newtons and each graph is plotted
for one complete gait cycle with one time unit equal to 1/60 sec.
The rest of the horizontal joint forces can be obtained similarly. In
fact, since motion is assumed to be symmetric, there is only a phase shift
difference between the corresponding components (example, $F_{x2}$ and $F_{x4}$).

Similarly, the vertical force components are shown in Figs. 7.40
through 7.42, representing the ground force reaction together with force
plate measurement (shown with 'x'), the associated knee force and the
vertical force corresponding to the upper body. The corresponding
partition coefficient $\beta$, and the total vertical ground force reaction
are shown in Figs. 7.43 and 7.44, respectively. Like the previous
case, all forces are expressed in Newtons. Again, a linear assumption
Figure 7.35. Horizontal Ground Force Reaction Associated with Left Leg, (+) Calculated from Five-Mass Dynamic Model and (*) Force Plate Measurement,
Figure 7.36: Horizontal Component of Force at Left Knee Calculated from Five-Mass Dynamic Model.
Figure 7.38. Partition Coefficient of Horizontal Force at Hip Joint.
Figure 7.39. Total Horizontal Ground Force Reaction Obtained from Five-Mass Dynamic Model.
Figure 7.40. Vertical Ground Reaction Forces Obtained from Five-Mass Dynamic Model (+) and Force Plate Measurement (*).
Figure 7.41. Vertical Component of Force at Left Knee Obtained from Five-Mass Dynamic Model.
Figure 7.42. Vertical Component for Force Associated with Trunk Obtained from Five-Mass Dynamic Model.
Figure 7.43. Partition Coefficient of Vertical Force at Hip Joint.
Figure 7.44. Total Vertical Ground Force Reaction Obtained from Five-Mass Dynamic Model.
is considered during the double support phase. The mean-square error related to the vertical ground reaction forces is \( 0.304 \times 10^4 \) (Newtons)^2.

Finally, the joint torques are illustrated in Figs. 7.45 through 7.47 which include the ankle torque, knee torque, and torque associated with the upper body, respectively. The partition coefficient, \( \gamma \), is shown in Fig. 7.48, and finally Fig. 7.49 represents the total torque at the two ankles. All torques are expressed in Newton meters.

7.3.2 Seven-Mass Model

The seven-mass dynamic model is fully discussed in Chapter 4. It is more or less similar to the five-mass model with the feet added to get a better representation of the human body. This model, similar to the previous one, assumes that linear transfer of ground reaction forces takes place. The corresponding assumption about the torque does not exist here because of the model of the foot and the variation of the center of pressure. Instead, an assumption has been made about the location of the zero moment point. Without such assumptions, the system of equations is not fully determined.

The physical data about the subject for this model are shown in Table 7.4. The routines which are used to obtain the results are listed in the appendices, and their functions are explained. The quality of the results are again tested by comparing them with the platform measurements. The partition coefficients \( \alpha \), \( \beta \), and \( \gamma \) are no longer involved. Instead, forces are explicitly considered.
Figure 7.46. Left Knee Torque Obtained from Five-Mass Dynamic Model.
Figure 7.47. Trunk Torque Obtained from Five-Mass Dynamic Model.
Figure 7.48. Partition Coefficient of Torque at Hip Joint.
Figure 7.49. Sum of Ankle Torques Obtained from Five-Mass Dynamic Model.
Table 7.4

Physical Data About the Subject of the Seven-Mass Model

<table>
<thead>
<tr>
<th></th>
<th>Trunk</th>
<th>Thigh</th>
<th>Shank</th>
<th>Foot</th>
</tr>
</thead>
<tbody>
<tr>
<td>m (kg)</td>
<td>45.10</td>
<td>7.020</td>
<td>3.370</td>
<td>1.670</td>
</tr>
<tr>
<td>J (kg-m^2)</td>
<td>2.23</td>
<td>0.084</td>
<td>0.051</td>
<td>0.001</td>
</tr>
<tr>
<td>d (meters)</td>
<td>0.286</td>
<td>0.253</td>
<td>0.186</td>
<td>---</td>
</tr>
<tr>
<td>L (meters)</td>
<td>0.846</td>
<td>0.441</td>
<td>0.473</td>
<td>---</td>
</tr>
</tbody>
</table>

Foot specifications:

- \( \rho_1 = 0.085 \text{ m} \)
- \( \rho_2 = 0.151 \text{ m} \)
- \( \rho_3 = 0.111 \text{ m} \)
- \( \omega_1 = 0.52 \text{ rad} \)
- \( \omega_2 = 0.245 \text{ rad} \)
- \( \omega_3 = 0.34 \text{ rad} \)
- \( d_F = 0.154 \text{ m} \)
- \( d_B = 0.106 \text{ m} \)
- \( \beta_1 = 1.56 \text{ rad} \)
- \( \beta_2 = 0.54 \text{ rad} \)
- \( \beta_3 = 1.03 \text{ rad} \)
- \( \text{RTO} = 12 \)
- \( \text{RHS} = 38 \)
- \( \text{LTO} = 50 \)

The horizontal components of joint forces are shown in Figs. 7.50 through 7.58. These quantities, according to their definitions in Chapter 4, are the component of the ground reaction force on the left foot, \( q_{Fx_0} \) together with \( Fx_1, Fx_2, Fx_3, Fx_4, Fx_5, Fx_6, Fx_7, \) and \( Fx_0 \) (the total ground reaction force due to both legs), respectively. Fig. 7.50 also includes the force plate measurement shown with '*' signs. The forces are all in terms of Newtons, and the horizontal axis shows the time corresponding to one gait cycle with each unit equal to 1/60 sec. The mean-square error obtained from comparing the
Figure 7.50. Horizontal Ground Reaction Force Associated with Left Foot Obtained from Seven-Mass Dynamic Model (+) and Force Plate Measurement (*).
Figure 7.51. Horizontal Component of Force at Left Ankle Joint Obtained from Seven-Mass Dynamic Model.
Figure 7.52. Horizontal Component of Force at Left Knee Joint Obtained from Seven-Mass Dynamic Model.
Figure 7.54. Horizontal Component of Force at Right Hip Joint Obtained from Seven-Mass Dynamic Model.
Figure 7.55. Horizontal Component of Force at Right Knee Joint Obtained from Seven-Mass Dynamic Model.
Figure 7.56. Horizontal Component of Force at Right Ankle Joint Obtained from Seven-Mass Dynamic Model.
Figure 7.58. Total Horizontal Ground Force Reaction Obtained from Seven-Mass Dynamic Model.
platform output and the one obtained from the model is $0.171 \times 10^4$ (Newtons)$^2$ under a linear assumption for $q_x$.

The vertical components of the joint forces can be independently computed. These, according to the notations in Chapter 4, are $q_y$, $F_{y0}$, $F_{y1}$, $F_{y2}$, $F_{y3}$, $F_{y4}$, $F_{y5}$, $F_{y6}$, $F_{y7}$, and $F_{y0}$ (the total ground reaction) shown in Figs. 7.59 through 7.67, respectively. The vertical ground reaction force obtained from the force plate is also shown in Fig. 7.59 by '●' signs, and the corresponding mean-square error is $0.134 \times 10^4$ (Newtons)$^2$. All forces are in Newtons, and $q_y$ is a linear function of time during DSP. The graphs are plotted for one complete gait cycle.

As discussed in Chapter 4, the joint torques of the seven-mass model as well as the position of center of pressure can be computed. Results include $C_{l1}$, $T_1$, $T_2$, $T_3$, $T_4$, $T_5$, $T_6$, and $T_7$ which are shown in Figs. 7.68 through 7.75, respectively. All torques are in Newton-meters, and the location of the zero moment point is in meters. All graphs are plotted for one walking cycle. The quantity $t_s$ is assumed to be seven time units ($7/60$ sec).

7.3.3 Three-Mass Model Results

This model employs the force plate outputs to compute the joint forces and torques associated with the leg stepping on the platform. The details have been discussed in Chapter 5. Table 7.5 shows the physical data of the subject of the experiment.

Similar to what was done in Chapter 5, each group of graphs is plotted together so that the cushioning effect can be seen. The time is normalized and shown in terms of percentage of gait cycle.
Figure 7.59. Vertical Ground Reaction Force Associated with Left Foot Obtained from Seven-Mass Dynamic Model (+) and Force Plate Measurement (*).
Figure 7.61. Vertical Component of Force at Left Knee Joint Obtained from Seven-Mass Dynamic Model.
Figure 7.62. Vertical Component of Force at Left Hip Joint Obtained from Seven-Mass Dynamic Model.
Figure 7.63. Vertical Component of Force at Right Hip Joint Obtained from Seven-Mass Dynamic Model.
Figure 7.64. Vertical Component of Force at Right Knee Joint Obtained from Seven-Mass Dynamic Model.
Figure 7.65. Vertical Component of Force at Right Ankle Joint Obtained from Seven-Mass Dynamic Model.
Figure 7.66. Vertical Component of Force Associated with Upper Body.
Figure 7.67. Total Vertical Ground Force Reaction Obtained from Seven-Mass Dynamic Model.
Figure 7.68. Position of Center of Pressure Obtained from Seven-Mass Dynamic Model (+) and Measured by Force Plate.
Figure 7.70. Left Knee Joint Torque Calculated from Seven-Mass Dynamic Model.
Figure 7.71. Left Hip Joint Torque Calculated from Seven-Mass Dynamic Model.
Figure 7.72. Right Hip Joint Torque Calculated from Seven-Mass Dynamic Model.
Figure 7.74. Right Ankle Joint Torque Calculated from Seven-Mass Dynamic Model.
Table 7.5

Physical Data About the Subject of a Three-Mass Model

<table>
<thead>
<tr>
<th></th>
<th>Thigh</th>
<th>Shank</th>
<th>Foot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex: Male</td>
<td>Height: 180m</td>
<td>Weight: 746N</td>
<td></td>
</tr>
<tr>
<td>m (kg)</td>
<td>7.900</td>
<td>3.790</td>
<td>1.880</td>
</tr>
<tr>
<td>J (kg\cdot m^2)</td>
<td>0.106</td>
<td>0.065</td>
<td>0.001</td>
</tr>
<tr>
<td>d (meters)</td>
<td>0.253</td>
<td>0.186</td>
<td>0.068</td>
</tr>
<tr>
<td>L (meters)</td>
<td>0.441</td>
<td>0.473</td>
<td>0.250</td>
</tr>
</tbody>
</table>

Figs. 7.76 and 7.77 illustrate the horizontal and vertical forces, respectively. The force plate measurements, ankle forces, knee forces, and hip forces are plotted by '+' , '*' , 'n' , and 'S' , respectively. Fig. 7.78 shows the corresponding joint torques. The symbols '+' , '*' , and 'n' represent ankle, knee, and hip torques. All figures correspond to one complete gait cycle.

7.4 Comparison of Five-Mass and Seven-Mass Results

Development of dynamic models of human gait is one of the goals of this dissertation. Also, an attempt is made to obtain the joint forces and torques by using only kinematic data collected by the television system. The first approach starts with a five-mass (five-link) model which had been considered by Gupta [2]. However, the mean-square errors which have been computed by using force plate measurements were rather large. After several modifications including the addition of a massless pelvis, the errors were considerably reduced.
Figure 7.76. Horizontal Components of Joint Forces Obtained from Three-Mass Dynamic Model, Force Plate Measurement (+), Ankle (*) Knee (n), and Hip ($).
Figure 7.77. Vertical Components of Joint Forces Obtained from Three-Mass Dynamic Model, Force Plate (+), Ankle (*), Knee (\(\wedge\)), and Hip ($).
Figure 7.78. Joint Torques Obtained from Three-Mass Dynamic Model, Ankle (+), Knee (*), and Hip (△).
All the algorithms used the linear variation assumption for the ground force reactions. That is, the ground force reactions transfer from one leg to the other one vary linearly with respect to time. The associated partition coefficients, $q_x$ and $q_y$, are linear functions and are equal to each other with the common notation 'q' throughout the analysis of the five-mass model. Under this situation, typical results presented in the previous section indicate that mean-square errors are

$$\text{MSE}(F_x) = 0.212 \times 10^4 \text{ (Newtons)}^2$$

$$\text{MSE}(F_y) = 0.304 \times 10^4 \text{ (Newtons)}^2$$

Since the amount of errors are not very small, it was decided to add more segments to the model. Obviously, feet have very important roles in human gait. One must be very careful with the modeling of the feet because of its different form. With the triangular shape chosen for the model, the same data set is used in order to compare the results with a five-mass model. Under the assumption that $q_x$ and $q_y$ vary linearly with respect to time the mean-square error results were

$$\text{MSE}(F_x) = 0.144 \times 10^4 \text{ (Newtons)}^2$$

$$\text{MSE}(F_y) = 0.146 \times 10^4 \text{ (Newtons)}^2$$

Evidently, there has been some improvement in the mean-square errors associated with both horizontal and vertical forces. In fact, it appears that there has been a greater reduction of error related to vertical forces. More precisely, by comparing the errors computed
for two models, it can be seen that $\text{MSE}(F_x)$ for the seven-mass model is 68 percent of that obtained for the five-mass model, which implies a 32 percent improvement. On the other hand, $\text{MSE}(F_y)$ for a seven-mass model has been reduced to about one half of that for the five-mass model, and there is a 52 percent improvement. In other words, the mean-square error corresponding to the horizontal forces did not decrease as much as expected.

By observing the shape of the ground force reactions obtained from the model, it is apparent that the maximum deviation from the force plate measurement happens during the double support phase (DSP) of the gait cycle. Therefore, one may conclude that the linear variation assumption may not be correct. According to the algorithms developed and discussed in Chapter 4, the partition coefficients, $q_x$ and $q_y$, can be computed, and the results have already shown that, although $q_y$ may be approximated with a linear function of time, such an assumption is absolutely false about $q_x$. The numerical values presented in Chapter 4 show that $q_x$ changes sign during each DSP with the corresponding discontinuity.

According to the expression for $q_x$, it is $\frac{R_x}{F_{x0}}$ where $R_x$ is the real value of the horizontal ground reaction force due the the left foot (measured by the force plate) and $F_{x0}$ is the total ground reaction force in the horizontal direction. It is important to notice that $F_{x0}$ becomes zero four times during the gait cycle, twice during two DSP's and twice during two SSP's. The reason can be explained as follows: there is a time during each DSP when the ground reaction forces associated with two legs are equal (non-zero) but have opposite
directions, which causes $F_{x0}$ to become zero. Also, during each SSP, there is an instant when the horizontal ground reaction force for the supporting leg changes its direction and thus becomes zero. At this time, the corresponding force for the swinging leg is obviously zero also. Therefore, $F_{x0}$ becomes zero. Evidently, at the above four moments, it is impossible to predict $q_x$. While this is not a severe problem for SSP cases, it is an undesirable situation for the DSP cases. In fact, $q_x$ may get infinitely large during DSP's depending on the time of the sampling. This makes the prediction of numerical values for $q_x$ practically impossible; and therefore, Gupta's hypothesis concerning linear variation for $q_x$ is definitely rejected. As a solution to this problem, it is necessary to obtain accurate values of $q_x$, and thus the horizontal ground force reaction, by some other means such as a force plate. Note that this is mainly required for DSP of walking cycle.

While Gupta's hypothesis was incorrect about $q_x$, it can be accepted as far as $q_y$ is concerned. In order to show the whole idea, the value of $q_y$ is computed and plotted vs. time in Fig. 7.79. Note that ideally, $q_y$ is one during single support phase (SSP) of the cycle is zero during the swing phase. Under such a circumstance, one would expect numerical values of forces exactly equal to force plate measurements. However, since the model is not perfect and there are some errors involved, $q_y$ is not one during SSP. Still, Fig. 7.79 illustrates that a linear variation for $q_y$ during DSP can be acceptable.
Figure 7.79. Partition Coefficient Associated with Vertical Ground Reaction Force, q_y, for One Gait Cycle.
7.5 An Application to Experimental Gait Studies

So far the experimental results corresponding to one particular subject have been studied so that the behavior of various models can be investigated. This section considers one of these models, namely the three-mass model described in Chapter 5, to study the effects of wearing positive-heeled and negative-heeled shoes as well as the barefoot case on various joint forces and torques associated with the model. Furthermore, an attempt is made to compare male and female subjects by testing corresponding joint forces and torques. For this experiment, three male and three female persons who are assumed to be normal have been chosen. The subjects wearing one of the two footwear or just barefoot have walked in front of the television camera 10 times and thus the data for 30 gait cycles were collected. The stored information includes both force plate and television data. A larger set of these data has been analyzed from a kinematic point of view which are only corresponding to TV data [100].

In order to be able to deal with such a huge amount of data, all ten similar gait cycles have been normalized to obtain both TV and force plate data in terms of the percentage of the cycle. Next, the average of force plate and TV data have been computed. This yields three averaged sets of data per person associated with the positive heel (P), negative heel (N), and barefoot (B) cases. For each data set, the ground reaction forces and the computed forces at various joints together with the associated torques are determined. All results can be displayed on the CRT, and the examples of them are shown in Figs. 7.80 through 7.82.
Figure 7.80. Display of Horizontal Components of Joint Forces Obtained from Three-Mass Model for One Gait Cycle.
Figure 7.81. Display of Vertical Components of Joint Forces Obtained from Three-Mass Model for One Gait Cycle.
Figure 7.82. Display of Joint Torques Obtained from Three-Mass Model for One Gait Cycle.
The results are presented in three groups: horizontal forces, vertical forces, and torques. Table 7.6 shows various quantities related to the horizontal forces. The first column indicates the subjects with the first three subjects being males and the last three females. The second column shows the type of shoes or barefoot case. The third column represents the time when, roughly at the middle of the stance phase, the horizontal ground force reaction, \( R_x \), becomes zero (changes direction). The next three columns show the time at which \( R_x \) takes its minimum value, the relative value of \( R_x \) with respect to the weight of the body, and the difference between \( R_x \) and the horizontal component of force at hip joint (\( F_{Hx} \)) at the point of \( \min R_x \) (relative with respect to weight of the body), \( d_x (\min) \), respectively. The last three columns show the time, the relative maximum value of \( R_x \), and the relative value of difference between \( R_x \) and \( F_{Hx} \) at that point, respectively. It can be seen that \( t_{\min R_x} \) and \( t_{\max R_x} \) have the least value for barefoot case (B) and mostly the largest value for P case. Note that all forces are shown in terms of their relative values with respect to the weight of the body.

By comparing several force values, it is obvious that there are no consistent differences as far as various types of shoes are concerned. Every person has his own force properties, regardless of being male or female. Moreover, the magnitude of the maximum and minimum values of \( R_x \) are not necessarily the same for each individual.

A similar kind of analysis can be done about the vertical forces. Table 7.7 shows the results with the first two columns being identical to the ones in Table 7.6. The next three columns
Table 7.6

Horizontal Force Parameters Averaged on Ten Trials for Six Subjects Each with Three Different Foot-Wears

<table>
<thead>
<tr>
<th>Subject</th>
<th>Shoe</th>
<th>$T_{Rx}=0$</th>
<th>$t_{Min , R_x}$</th>
<th>Min $R_x$</th>
<th>$d_x$ (Min)</th>
<th>$t_{Max , R_x}$</th>
<th>Max $R_x$</th>
<th>$d_x$ (Max)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P</td>
<td>37</td>
<td>16</td>
<td>-0.18</td>
<td>0.03</td>
<td>57</td>
<td>0.13</td>
<td>0.10</td>
</tr>
<tr>
<td>1</td>
<td>N</td>
<td>38</td>
<td>14</td>
<td>-0.20</td>
<td>0.06</td>
<td>58</td>
<td>0.15</td>
<td>0.10</td>
</tr>
<tr>
<td>1</td>
<td>B</td>
<td>38</td>
<td>14</td>
<td>-0.17</td>
<td>0.04</td>
<td>55</td>
<td>0.14</td>
<td>0.11</td>
</tr>
<tr>
<td>2</td>
<td>P</td>
<td>36</td>
<td>14</td>
<td>-0.14</td>
<td>0.03</td>
<td>57</td>
<td>0.18</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>N</td>
<td>38</td>
<td>13</td>
<td>-0.13</td>
<td>0.02</td>
<td>58</td>
<td>0.17</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>35</td>
<td>11</td>
<td>-0.11</td>
<td>0.03</td>
<td>56</td>
<td>0.17</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
<td>P</td>
<td>37</td>
<td>14</td>
<td>-0.12</td>
<td>0.03</td>
<td>54</td>
<td>0.24</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>N</td>
<td>37</td>
<td>11</td>
<td>-0.13</td>
<td>0.10</td>
<td>51</td>
<td>0.21</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>33</td>
<td>13</td>
<td>-0.13</td>
<td>0.06</td>
<td>52</td>
<td>0.22</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
<td>P</td>
<td>33</td>
<td>14</td>
<td>-0.11</td>
<td>0.03</td>
<td>54</td>
<td>0.16</td>
<td>0.06</td>
</tr>
<tr>
<td>4</td>
<td>N</td>
<td>35</td>
<td>12</td>
<td>-0.19</td>
<td>0.03</td>
<td>53</td>
<td>0.16</td>
<td>0.06</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>34</td>
<td>9</td>
<td>-0.09</td>
<td>0.05</td>
<td>52</td>
<td>0.17</td>
<td>0.06</td>
</tr>
<tr>
<td>5</td>
<td>P</td>
<td>34</td>
<td>16</td>
<td>-0.12</td>
<td>0.03</td>
<td>57</td>
<td>0.15</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>N</td>
<td>37</td>
<td>12</td>
<td>-0.10</td>
<td>0.02</td>
<td>57</td>
<td>0.16</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>32</td>
<td>12</td>
<td>-0.14</td>
<td>0.03</td>
<td>55</td>
<td>0.16</td>
<td>0.04</td>
</tr>
<tr>
<td>6</td>
<td>P</td>
<td>34</td>
<td>16</td>
<td>-0.08</td>
<td>0.03</td>
<td>55</td>
<td>0.20</td>
<td>0.06</td>
</tr>
<tr>
<td>6</td>
<td>N</td>
<td>35</td>
<td>12</td>
<td>-0.16</td>
<td>0.02</td>
<td>54</td>
<td>0.20</td>
<td>0.07</td>
</tr>
<tr>
<td>6</td>
<td>B</td>
<td>33</td>
<td>9</td>
<td>-0.09</td>
<td>0.04</td>
<td>51</td>
<td>0.18</td>
<td>0.06</td>
</tr>
</tbody>
</table>
Table 7.7
Vertical Force Parameters Averaged on Ten Trials
for Six Subjects Each with Three Different Foot-Wears

<table>
<thead>
<tr>
<th>Subject</th>
<th>Shoe</th>
<th>( t_{\text{Max}1 , R_y} )</th>
<th>Max1 ( R_y )</th>
<th>CE ( \text{Max1} )</th>
<th>( t_{\text{Min} , R_y} )</th>
<th>Min ( R_y )</th>
<th>CE ( \text{Min} )</th>
<th>( t_{\text{Max}2 , R_y} )</th>
<th>Max2 ( R_y )</th>
<th>CE ( \text{Max2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P</td>
<td>.19</td>
<td>1.17</td>
<td>.22</td>
<td>32</td>
<td>0.84</td>
<td>0.16</td>
<td>51</td>
<td>1.28</td>
<td>0.20</td>
</tr>
<tr>
<td>1</td>
<td>N</td>
<td>19</td>
<td>1.08</td>
<td>.23</td>
<td>30</td>
<td>0.87</td>
<td>0.13</td>
<td>52</td>
<td>1.26</td>
<td>0.21</td>
</tr>
<tr>
<td>1</td>
<td>B</td>
<td>18</td>
<td>1.04</td>
<td>.25</td>
<td>31</td>
<td>0.84</td>
<td>0.14</td>
<td>49</td>
<td>1.27</td>
<td>0.17</td>
</tr>
<tr>
<td>2</td>
<td>P</td>
<td>20</td>
<td>1.03</td>
<td>.21</td>
<td>33</td>
<td>0.95</td>
<td>0.19</td>
<td>53</td>
<td>1.09</td>
<td>0.21</td>
</tr>
<tr>
<td>2</td>
<td>N</td>
<td>23</td>
<td>1.04</td>
<td>.17</td>
<td>31</td>
<td>0.96</td>
<td>0.18</td>
<td>52</td>
<td>1.12</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>20</td>
<td>1.04</td>
<td>.21</td>
<td>30</td>
<td>0.90</td>
<td>0.16</td>
<td>49</td>
<td>1.07</td>
<td>0.19</td>
</tr>
<tr>
<td>3</td>
<td>P</td>
<td>19</td>
<td>1.05</td>
<td>.27</td>
<td>30</td>
<td>0.94</td>
<td>0.16</td>
<td>49</td>
<td>1.17</td>
<td>0.21</td>
</tr>
<tr>
<td>3</td>
<td>N</td>
<td>16</td>
<td>1.08</td>
<td>.29</td>
<td>28</td>
<td>0.90</td>
<td>0.13</td>
<td>45</td>
<td>1.11</td>
<td>0.19</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>14</td>
<td>1.05</td>
<td>.26</td>
<td>28</td>
<td>0.90</td>
<td>0.13</td>
<td>45</td>
<td>1.05</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>P</td>
<td>20</td>
<td>1.06</td>
<td>.17</td>
<td>33</td>
<td>0.87</td>
<td>0.18</td>
<td>51</td>
<td>1.15</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>N</td>
<td>17</td>
<td>1.18</td>
<td>.21</td>
<td>30</td>
<td>0.94</td>
<td>0.15</td>
<td>46</td>
<td>1.19</td>
<td>0.18</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>18</td>
<td>1.09</td>
<td>.20</td>
<td>30</td>
<td>0.95</td>
<td>0.16</td>
<td>47</td>
<td>1.29</td>
<td>0.19</td>
</tr>
<tr>
<td>5</td>
<td>P</td>
<td>22</td>
<td>1.10</td>
<td>.16</td>
<td>35</td>
<td>0.94</td>
<td>0.19</td>
<td>46</td>
<td>1.04</td>
<td>0.16</td>
</tr>
<tr>
<td>5</td>
<td>N</td>
<td>26</td>
<td>1.06</td>
<td>.15</td>
<td>35</td>
<td>0.95</td>
<td>0.13</td>
<td>51</td>
<td>1.10</td>
<td>0.19</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>19</td>
<td>1.09</td>
<td>.22</td>
<td>31</td>
<td>0.96</td>
<td>0.15</td>
<td>49</td>
<td>1.06</td>
<td>0.18</td>
</tr>
<tr>
<td>6</td>
<td>P</td>
<td>16</td>
<td>1.05</td>
<td>.22</td>
<td>28</td>
<td>0.89</td>
<td>0.14</td>
<td>52</td>
<td>1.14</td>
<td>0.24</td>
</tr>
<tr>
<td>6</td>
<td>N</td>
<td>20</td>
<td>1.12</td>
<td>.28</td>
<td>28</td>
<td>0.95</td>
<td>0.14</td>
<td>47</td>
<td>1.22</td>
<td>0.18</td>
</tr>
<tr>
<td>6</td>
<td>B</td>
<td>18</td>
<td>1.06</td>
<td>.20</td>
<td>25</td>
<td>0.94</td>
<td>0.13</td>
<td>45</td>
<td>1.11</td>
<td>0.18</td>
</tr>
</tbody>
</table>
indicate the time when \( R_y \) reaches its first maximum, the associated relative value of \( R_y \), and the relative value of the difference between \( R_y \) and the corresponding hip component at this point which can be called the Cushioning Effect (CE), respectively. The next three columns show the time, the relative value, and the relative CE value at the minimum point of \( R_y \), respectively. Finally, the last three columns are the time, the relative value, and the relative CE value at the second maximum point of \( R_y \).

It is important to note that the difference between the components of the ground reaction forces and the corresponding hip forces can be described analytically by the use of the equations developed in Chapter 5 as follows:

\[
R_x - F_{Hx} = m_F \ddot{x}_F + m_S \ddot{x}_S + m_T \ddot{x}_T \tag{7-1}
\]

\[
R_y - F_{Hy} = m_F (\dot{y}_F + g) + m_S (\dot{y}_S + g) + m_T (\dot{y}_T + g) \tag{7-2}
\]

Eq. (7-2), which is equal to the Cushioning Effect (CE), implies that this quantity only depends on the kinematic data and not the force measurements.

By observing various quantities shown in Table 7.7, it can be concluded that there is no significant difference among the results. In other words, wearing positive or negative heeled shoes or being barefoot does not have an obvious effect on the form of the forces obtained from the three-mass model. Although the results vary as shoe type changes, this variation is not consistent.
The last group of the quantities to be considered here are the joint torques. Table 7.8 shows the values of the ankle and knee torques all expressed in Newton-meters. The first two columns are identical to the ones in the previous tables. The third column indicates the time where the ankle torque ($T_A$) takes its minimum value and the next column shows the corresponding amount of torque. The first maximum of the knee torque happens in about 20 percent of the gait cycle. Then, it is followed by a local minimum and another maximum. The times of these three points followed by their values are also shown in Table 7.8. The last two columns indicate the integral of magnitudes of $T_A$ and $T_K$ over the entire gait cycle, respectively. In most cases, it appears that the minimum of the ankle torques has the least magnitude with the bare feet, and it happens usually sooner than the times associated with the other cases (P and N). The integral of the magnitude of $T_A$ has the least value for case B.

The time required to reach the first maximum point of $T_K$ is the least in case of N. The value of this point is usually the least with positive heel shoes and the most for barefoot. It can also be seen that the values of $T_K$ at the minimum and the second maximum points are usually the least for the positive heel shoes and most for the barefoot.

Finally, Table 7.9 describes several values related to the hip torque. These correspond to the minimum point at the beginning and the maximum point where each is specified by the time of the occurrence and its value. The integral of the torque magnitude is also presented in the last column. The minimum value indicates that
| Subject | Shoe | $t_{TA,min}$ | $\text{Min}(T_A)$ | $t_{1,\text{Max}T_K}$ | $\text{Max}1(T_K)$ | $t_{\text{Min}}(T_K)$ | $\text{Min}(T_K)$ | $t_{2,\text{Max}T_K}$ | $\text{Max}2(T_K)$ | $\int_T T_A \, dt$ | $\int_T |T_K| \, dt$ |
|---------|------|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------------|----------------|
| 1       | P    | 49           | -166.2          | 22              | 13.5            | 42              | -26.6           | 56              | 99.8            | 60.6           | 31.0           |
| 1       | N    | 49           | -137.4          | 18              | 47.3            | 36              | 16.6            | 56              | 122.2           | 46.3           | 35.0           |
| 1       | B    | 47           | -145.4          | 18              | 51.7            | 40              | 12.3            | 55              | 118.6           | 42.8           | 32.4           |
| 2       | P    | 53           | -117.9          | 24              | -2.8            | 43              | -40.3           | 63              | 40.8            | 57.3           | 27.6           |
| 2       | N    | 52           | -105.7          | 24              | 16.4            | 43              | -23.9           | 62              | 53.6            | 52.9           | 20.8           |
| 2       | B    | 50           | -71.1           | 24              | 38.7            | 41              | 1.3             | 60              | 55.0            | 29.8           | 23.1           |
| 3       | P    | 48           | -74.2           | 21              | 3.4             | 34              | -8.7            | 58              | 73.6            | 25.7           | 22.6           |
| 3       | N    | 44           | -74.6           | 19              | 18.3            | 34              | 2.8             | 54              | 98.8            | 23.8           | 24.2           |
| 3       | B    | 43           | -53.5           | 19              | 35.7            | 32              | 10.8            | 54              | 69.6            | 20.1           | 24.1           |
| 4       | P    | 50           | -42.7           | 22              | 8.5             | 43              | -4.6            | 59              | 29.6            | 13.9           | 9.9            |
| 4       | N    | 47           | -45.2           | 18              | 29.5            | 34              | 9.7             | 56              | 48.4            | 14.1           | 17.7           |
| 4       | B    | 46           | -47.1           | 23              | 29.0            | 36              | 14.5            | 54              | 46.2            | 11.4           | 18.1           |
| 5       | P    | 53           | -50.9           | 30              | 20.8            | 42              | 3.8             | 61              | 21.2            | 21.6           | 15.7           |
| 5       | N    | 53           | -81.1           | 16              | 8.0             | 42              | -6.6            | 62              | 33.0            | 34.6           | 11.5           |
| 5       | B    | 55           | -42.9           | 17              | 38.0            | 41              | 25.8            | 58              | 45.6            | 12.6           | 27.2           |
| 6       | P    | 49           | -96.6           | 28              | -10.2           | 43              | -20.0           | 58              | 59.0            | 33.9           | 22.8           |
| 6       | N    | 47           | -80.6           | 17              | 12.0            | 31              | -1.4            | 57              | 69.0            | 32.5           | 15.8           |
| 6       | B    | 44           | -54.3           | 22              | 20.3            | 31              | 12.9            | 53              | 78.2            | 18.8           | 21.7           |
Table 7.9

Hip Torque Averaged on Ten Trials for Six Subjects Each with Three Different Foot-Wears

<table>
<thead>
<tr>
<th>Subject</th>
<th>Shoe</th>
<th>tMin T_H</th>
<th>Min T_H</th>
<th>tMax T_H</th>
<th>Max T_H</th>
<th>(\int T_H , dt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P</td>
<td>3</td>
<td>-180.6</td>
<td>55</td>
<td>192.1</td>
<td>63.7</td>
</tr>
<tr>
<td>1</td>
<td>N</td>
<td>3</td>
<td>-154.5</td>
<td>55</td>
<td>209.7</td>
<td>60.0</td>
</tr>
<tr>
<td>1</td>
<td>B</td>
<td>3</td>
<td>-166.0</td>
<td>53</td>
<td>204.1</td>
<td>58.9</td>
</tr>
<tr>
<td>2</td>
<td>P</td>
<td>7</td>
<td>-116.7</td>
<td>61</td>
<td>71.9</td>
<td>40.2</td>
</tr>
<tr>
<td>2</td>
<td>N</td>
<td>4</td>
<td>-100.6</td>
<td>60</td>
<td>86.8</td>
<td>39.0</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>3</td>
<td>-98.1</td>
<td>57</td>
<td>96.0</td>
<td>45.5</td>
</tr>
<tr>
<td>3</td>
<td>P</td>
<td>3</td>
<td>-138.5</td>
<td>57</td>
<td>107.7</td>
<td>48.2</td>
</tr>
<tr>
<td>3</td>
<td>N</td>
<td>6</td>
<td>-162.9</td>
<td>53</td>
<td>142.5</td>
<td>47.1</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>4</td>
<td>-132.4</td>
<td>54</td>
<td>110.8</td>
<td>47.5</td>
</tr>
<tr>
<td>4</td>
<td>P</td>
<td>4</td>
<td>-46.0</td>
<td>56</td>
<td>57.4</td>
<td>21.5</td>
</tr>
<tr>
<td>4</td>
<td>N</td>
<td>4</td>
<td>-47.8</td>
<td>55</td>
<td>78.2</td>
<td>29.5</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>3</td>
<td>-59.7</td>
<td>51</td>
<td>84.8</td>
<td>32.7</td>
</tr>
<tr>
<td>5</td>
<td>P</td>
<td>4</td>
<td>-67.3</td>
<td>54</td>
<td>43.0</td>
<td>23.8</td>
</tr>
<tr>
<td>5</td>
<td>N</td>
<td>5</td>
<td>-58.0</td>
<td>61</td>
<td>44.0</td>
<td>21.9</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>3</td>
<td>-83.5</td>
<td>57</td>
<td>59.1</td>
<td>29.8</td>
</tr>
<tr>
<td>6</td>
<td>P</td>
<td>10</td>
<td>-108.5</td>
<td>58</td>
<td>76.3</td>
<td>36.1</td>
</tr>
<tr>
<td>6</td>
<td>N</td>
<td>5</td>
<td>-67.4</td>
<td>56</td>
<td>87.8</td>
<td>29.4</td>
</tr>
<tr>
<td>6</td>
<td>B</td>
<td>5</td>
<td>-91.7</td>
<td>53</td>
<td>95.7</td>
<td>35.5</td>
</tr>
</tbody>
</table>
it has the least value for male subjects and the largest value for females when subjects are barefoot. The maximum point, however, has usually the least magnitude in the case of positive heel shoes (P).

As a general conclusion, it is apparent that most of the differences can be seen in the joint torques rather than the forces. Nevertheless, for this small number of subjects, the differences are neither very obvious nor quite large. In some cases when the differences exist, they are not always consistent.

7.6 Summary

This chapter presents both kinematic and kinetic results. All the experimental results are obtained basically by using the Gait Laboratory's equipment. A sample of angular measurements (angles, angular rates, and angular accelerations) which are directly used for dynamic modeling of the human body are illustrated. Moreover, a variety of anatomical angles between various limbs of the body are presented by averaging several cycles of motion and normalizing them in terms of the percentage of the gait period. By using t-distribution approach, the 95 percent confidence interval region for each graph has been shown. Results indicate only a small amount of variation with the standard deviation most of the time being less than 1.5 degrees. This implies that the quality of the system is good and the results are reliable for gait studies. A brief conclusion about the effect of shoe-heel has been made, and the difference in terms of gait angles or gait parameters are discussed. It appears that every individual has his own gait pattern.
The kinetic results have been presented in three groups associated with three dynamic models. Also, some force plate measurements are shown. By testing the mean-square error of the ground reaction forces from the model and from the platform, it is apparent that the seven-mass model does improve the quality of the estimated quantities. However, the error associated with the horizontal component of the force has less change mainly because the linear assumption about $q_x$ is not correct. Therefore, it is necessary to use a system (such as a force plate) to estimate the values of $q_x$ during DSP or else to choose some more suitable analytic form for $q_x$. The investigation of this question is not part of this dissertation. On the other hand, the linear hypothesis regarding $q_y$ can be acceptable, and the seven-mass dynamic model yields rather accurate results.

As an example application to experimental gait studies, both force plate and TV data obtained from six subjects, three males and three females, have been used to study the effects of positive and negative heel shoes in addition to barefoot case. The general view of the results indicates that the joint forces are rather close to each other and there are not consistent differences associated with them. On the other hand, there are some small differences corresponding to the joint torques which can be observed. In most cases, the largest difference can be seen by comparing positive heel shoes and barefoot results with the negative heel shoes often being in between.
Chapter 8

SUMMARY AND CONCLUSIONS

8.1 Summary

The analysis of the human gait has not been used on a large scale for either clinical or other kinds of applications. One of the main goals of this dissertation is to make a contribution to this field so that some quantitative information related to human gait becomes available.

The Gait Laboratory was established at the Ohio State University in April, 1977. Both people in electrical engineering and physical medicine have been involved in the ongoing projects. This author has been primarily responsible for development of a software based system to study human gait. There are various measurement devices which are interfaced to the minicomputer being used at the laboratory. A description of the equipment can be found in Chapter 6. This equipment forms the starting point for the research of this dissertation. The experimental procedure is based on attaching some proper lights sources to anatomical landmarks (important joints) in order to obtain their trajectories. From the coordinates of each marker in successive motion the gait parameters of the human locomotion can be determined and analyzed in real time. These parameters include step length, step frequency, velocity, stride length, and time or relative time for the gait phases. Furthermore, kinematic information such as displacement of
various joints of the body as well as the various segments centers of masses are measured. Angular information such as relative ankle, knee and hip angles and the absolute angles (with respect to horizontal axis) for feet, shanks, thighs and the trunk can also be obtained.

Once the kinematic data is available, the next requirements are preprocessing and editing. That is, some points may be missing or may need correction. A linear interpolation method has been employed to obtain these values. Since the process of walking is periodic, a least square error technique (conventional Fourier series) has been used to smooth the noisy data. This is considered to be a suboptimal filtering method. Chapter 6 also explains an optimal filter (so-called Wiener filter) which was studied and applied to raw data. Throughout this study, stationary processes as well as no correlation between signal as noise have been assumed.

One of the main objectives of this research is to investigate several dynamic models of the human body so that the forces and torques associated with various joints of the model can be obtained. All models discussed in this dissertation are planar. Furthermore, since the motion of only one of the legs is recorded, a symmetric gait has been assumed whenever both legs must be included in the model. The kinetic study of the human body, which results in knowledge of the joint forces and torques, is very helpful in locomotion rehabilitation and essential for a quantitative and objective analysis of gait.

There are three dynamic models investigated in this dissertation. The force plate system which directly measures all three components of the ground reaction forces as well as the position of center of pressure is explicitly involved in one of the models discussed in
Chapter 5. This model is analyzed from bottom to top and it only includes the leg which steps on the force plate. The two more complex models involve both legs as well as the upper body which includes two arms and the head. These models use only television data, and the force plate results are merely employed to test the quality of the models. Chapter 3 discusses a five-mass model together with several attempts to improve the results such as adding a massless pelvis with a variable length. For every change, quantitative results are presented. Moreover, a sensitivity test of the various body segment parameters has been done. These parameters (mass, moment of inertia, etc.) are determined based on Braune and Fischer's coefficients. A more complete model with seven massive segments is considered in Chapter 4 which includes an extensive argument about the dynamic equations of the model and dependencies of various kinetic quantities upon each other. It also discusses the assumptions made about the transfer of the ground force reactions from one leg to the other one during double support phase of the gait cycle. Such an assumption is necessary to make the system of associated dynamic equations completely deterministic.

Extensive experimental results are presented in Chapter 7. Kinematic results include various anatomical as well as inertial angles, angular rates and angular accelerations. A few statistical evaluations of the results with 95 percent confidence interval are also shown. Kinetic results include one set of several joint forces and torques corresponding to each model. Furthermore, results obtained from five-mass and seven-mass models are compared and the associated assumptions are discussed. Finally, as an application to experimental gait studies, the three-mass model in Chapter 5 is employed to study
the effects of positive heel and negative heel shoes (so-called earth shoes) as well as barefoot cases on the corresponding joint forces and torques.

8.2 Recommendations for Further Work

The Gait Laboratory has been primarily established and developed in order to obtain both kinematic and kinetic evaluations of human gait which have many applications such as diagnosis of the abnormalities and physical therapy. The system has a fairly good speed and accuracy. Nevertheless, it has a few drawbacks as follows:

1. One television camera is barely sufficient for the proper study of human gait. In fact, in all current cases a normal (symmetric) walking process has been assumed so that the construction of all required data becomes possible. This problem will be solved by having a three television camera system, which is currently being developed at the laboratory.

2. Some of the gait parameters such as the stride time or the toe-off time are estimated by software (using TV and force plate data). This method may not be quite accurate. Use of either a foot switch or a conducting walking surface can give more reliable information.

3. The current television system has a few disadvantages such as the blooming effect of CRT or the trailing effect caused by incandescent lights. Thus, the quality of data is reduced, and this makes the editing and preprocessing actions quite necessary. This introduces some error in the results. Use of light emitting diodes (LED) which can be strobed as well as employing Selspot or array camera systems [8] will help to solve the current problem.
4. During the time required to detect a light, check the redundancy, and get its coordinates, the system is unable to detect any other light. More precisely, if two lights are positioned on the same horizontal TV scan line, only the first one can be detected. Therefore, it is desirable to have a faster system.

5. In order to study the kinetics of the human gait more efficiently, it is necessary to have a system which consists of at least two force plates so that the individual measurements associated with both legs can be obtained. Obviously, four force plates are even more desirable.

This dissertation has concentrated on dynamic models of planar models. There are two options to increase the complexity of the model: a) by adding more segments to obtain a more complex planar model and b) by considering a three-dimensional model. The latter is obviously a more difficult task to do while the former is fairly straightforward because the algorithm described in Chapter 4 can be easily expanded for this purpose. In fact, for an eleven-mass model with arms included, the complexity of the closed loop of the lower limbs, which is the more important part, stays the same, and similar explicit form of equations can be obtained for the whole model. On the other hand, the study of the three-dimensional model will not be as easy and does require a television camera system with at least two cameras to collect the proper kinematic information.

One of the most complex dynamic models which has already been studied is a fifteen-segment model in three dimensions [101]. In this research, bilateral stereo photogrammetric films are used to collect

307
kinematic data. Reference points are fixed to the subject, and there are two force plates which operate independently. This model measures the positions, velocities, and accelerations, both translational and rotational, independently for each limb segment. That is, it is a true free body model without kinematic constraints. One result of this approach is that body parts appear to separate when the results are presented on a CRT display. The author feels that this is a fairly serious defect which may limit the usefulness of any kinetic analysis method which ignores kinematic constraints. This question requires considerable additional research.

Another area of research is the study of the energetics of the human gait. This dissertation does not become involved in the activities or functions of the muscles which are sources of the energy required to generate the human motion. As a matter of fact, such a study will be very helpful in interpretation of the results obtained by the author. A recent study in the area of energetics of human locomotion has been accomplished by Hardt [102]. He studied a five-mass three-dimensional model to investigate muscle activities based on minimum force or minimum energy hypothesis. A Selspot system is used to track multiple free bodies in three-dimensional space. There are six light emitting diodes attached to each segment to obtain the associated orientation and position, one of which is approximately positioned at the corresponding location of center of mass. Since there is no force plate involved, a linear force and torque transfer hypothesis is employed by Hardt.

By improving the current system existing in the Gait Laboratory and increasing its speed (data collection rate) and accuracy, many other
aspects of human locomotion can be investigated. Sitting down, standing up, running and any other kind of motions such as those used in performing certain types of sports can be studied and analyzed in the laboratory.

Finally, clinical applications are among many areas of research which can take advantage of the current study. The principles and algorithms used in this dissertation can be applied to locomotive rehabilitation. Design of various orthotic or prosthetic devices are among many possible applications. A proper example of this type of work is a study recently done by Winarski [103] concerning an optimal below knee prosthesis alignment and its relationship to cumulative energy expenditure. This study includes a seventeen-segment three-dimensional true free body type of biped model. Response of the amputee was photographed on 16mm reversal films at a rate of 32/sec. The experiment was to perform flexion-extension adjustment on the prosthesis, and to document the corresponding response to each perturbation. There were two cameras used in this research (sagittal and frontal views). Force measurements were provided by strain gauges mounted on the pylon of the artificial limb.

8.3 Conclusions

There have been basically two kinds of quantities which have been analyzed throughout this research, namely those related to kinematics and dynamics of the human being. Most of the kinematic quantities can be either measured directly or computed easily. Due to the speed, resolution and other factors associated with the current system, it is fairly accurate and efficient for gait analysis. The
system operates in real time with a high speed of data collection. For example, during the study of effects of wearing positive heel shoes, negative heel shoes, and the barefoot case in the human gait, it took about 45 minutes to collect 33 data sets (11 for each case) per subject. This excludes the set-up and calibration time (10-15 minutes in normal situations) which must be done only once for all experiments. The statistical analysis of the angular data yields a standard deviation of less than 1.5 degrees for the average of 10 data sets per subject. The power spectral analysis of the television data pointed out that the most power is in the first four harmonics. A similar study regarding the associated noise showed that the maximum noise power belongs to the fourth harmonic. Nevertheless, the noise power is much smaller than the signal power. The standard deviations of the force plate data have been rather small. For the force quantitities, the highest noise standard deviation value as been about 1 Newton (in about 1300 Newtons). The corresponding values for the position of center of pressure and the free moment component are less than $10^{-4}$ millimeters and 0.05 Newton meters, respectively.

The study of optimal filters for different angles results in rather high (close to 1) gain values for most of the situations. It must be mentioned that this filter corresponds to only electrical noise and the others sources such as the one due to the motions of the lights on the skin or vibration during stepping process are not included. For the suboptimal filtering which includes all possible noises, a number of three harmonics is chosen. In fact, for kinematic studies, five harmonics are usually used. However, in the dynamic analysis of
the gait, normally three or four harmonics are employed. Note that forces and torques are not linear functions of the angular quantities, and thus do not necessarily require the same number of harmonics.

Among the three dynamic models which have been employed in this research, the three-mass model described in Chapter 5 is rather simple and easy to apply because it only deals with one leg and uses the force plate measurements directly. On the other hand, the other two models which consider the model as a whole require the kinematic information concerning both legs. Consequently, the data associated with one leg is constructed by assuming a symmetric gait.

In the evaluation of all dynamic models, the reference system from which the coordinates and thus accelerations of the centers of mass of various segments are obtained is very important. The position of the hip light was chosen to be the reference point consistently. Evidently, this choice is a rather appropriate one especially since it helps to consider the pelvis as well.

Throughout the study of the five-mass model, the linear assumption of transfer of forces and torques during the double support phase was kept. The mean square error of the ground reaction forces obtained from the model and direct measurements from the force plate have been employed to test the quality of the models. The sensitivities of the mean-square error values have been studied by changing the body segment parameters ±10 percent. Since the upper body mass consists of a good portion of the whole mass of the body, it generates the greatest sensitivities. Moreover, the error corresponding to horizontal component of the force is mostly sensitive to lengths of the lower
limbs and the error associated with the vertical component is rather sensitive to the masses of the body segments.

The dynamic equations of the human body become increasingly complicated when more complex models are considered. The method used previous to this dissertation research was to solve the equations simultaneously. The number of equations were 12 for a five-mass model and 18 for a seven-mass model. However, this dissertation shows that not only can the set of equations be partitioned, but also explicit forms can be obtained for all joint forces and torques. This has been described in Chapter 4 with dependencies shown.

A comparison between the results of the five-mass model and the seven-mass model indicates that there is some improvement in the mean-square errors corresponding to vertical and horizontal ground force reactions when the number of masses is increased. For example, a particular data set shows about 32 and 52 percent reduction of mean-square error associated with the horizontal and vertical components, respectively.

The ground force reactions have been assumed to transfer linearly with respect to time from one foot to the other during the double support phase of the gait cycle. Force plate measurements have been used to obtain both algebraic expression and numerical values for the above partition coefficients. Results have pointed out that the linear assumption is fairly acceptable for the vertical component and is absolutely rejected for the horizontal component of the ground force reaction. In fact, the partition coefficient associated with the horizontal force may get infinitely large and practically impossible
to predict. Consequently, while the television data are rather sufficient to estimate the vertical component on the entire gait cycle, the horizontal component can only be estimated during single support phases. Additional information is needed to determine the horizontal force during double support phases. Obviously, a force plate can provide such information.

The position of the center of pressure has also been estimated by using the seven-mass model. Although the values do not match with reality exactly, the results are surprisingly accurate, and the observed errors merely reflect the fact that this model is a very simplified version of the body of a human being. The author hopes that this dissertation will encourage others to develop still more accurate mathematical models to further improve the correspondence between the model and physical reality with respect to human gait.
APPENDIX A

COMPUTER PROGRAM FOR DATA ACQUISITION

Program STVF6.MAC is the routine which is employed to collect the information from both TV-camera and force plate simultaneously. It then writes the data onto the disk under the file name FKTV6.DAT. The file structure (21 element file) is so that for each field, the first eight numbers are force plate outputs, the next twelve numbers are the coordinates of the six lights and the 21 number is -1 if all lights are detected completely. The author has also used another program call GO [8] for data acquisition, alternatively (25 element file).
* THIS PROGRAM TAKES DATA FROM FORCE PLATE
* AND TURNS 6 LIGHTS ON, AND WRITES THEM ONTO DISK.
* S. RAHMANI 5-APR-78.

.MCAll ...V2..RESDEF..EXIT..FETCH..ENTER..WRITE..CLOSE

.REGDEF

.CKRM: 170404  I/O CLOCK STATUS REGISTER

.CKSR: 170404  I/O CLOCK STATUS REGISTER

.ADRM: 170400  I/A D STATUS REG.

.ADRR: 170802  I/A D BUFFER REG.

STR=8

.START

MIV #SR+R2

MOV #BUFF+R3

CLR (R2)

MOV $32,RCKSR

SET IO -5 TO GET DATA AFTER 5 CNTS.

MOV $400+BADR

ENABLE OVERFLOW TO START A/D CURT

A11

MOV $411,RCKSR

MODE=1, FREQ.=1KHZ

A21

TSTB RCKSR

B1:

TSTB BADR

BPL B1

CMP $374BADR

REX A1

IF START TAKING DATA WHEN F.P. IS TOUCHED

LOOP2

MOV #0A1+BADR

CLR R3

MOV $0+TMP1

GET DATA FROM F.P.

CLR TF+5

A2:

MOV $407,RCKSR

MODE=1, FREQ.=2KHZ

CMP $400+SNR

CALL 8 SAMPLES DONE?

BET MORE1

SUB $4000+BADR

GET READY FOR NEXT SET

CLR TF+5

BPL MORE1

BR LOOP

MORE1

TSTB RCKSR

CHECK FOR O.F.

BPL MORE1

CNVT1

TSTB BADR

BPL CNVT

JSR CLR.READ

SET GET THE DATA

INC TF+5

BR B2

LOOP1

TST (R2)

TEST DONE FLAG

BMP WOT

IF DONE FLAG SET, GO TO WAIT

TSTB (R2)

TEST READY FLAG

BPL LOOP

IF NOT READY, GO BACK

*
STVF6.MAC

NEXT: MOV TMP2,TMP1
CLRB (R2)
BR LOOP

WAIT: TST (R2)
SMI WAIT
CMP #LTS,R1
BEQ VALID
MOV #LTS,R1
MOV $0,(R3)+
SBR R1,CLEAR
MOV $-2,(R3)+
SET BOUNDARY FOR INVALID DATA (-2)
BR S2

VALID: MOV SDADEP+R4
BRING POINTER BACK
MOV #LTS,R1
MOV (R4)+TMP4
BIC $177400,(R4)
MOV (R4)+1,(R3)+
SWAB TMP4
BIC $177400, TMP4
MOV TMP4,(R3)+
SBR R1,S1
MOV $-1,(R3)+
SET THE BOUNDARY -1 FOR VALID DATA
S2: MOV $0,TMP1
INC COUNT
INC HEADERBACK
INC LODB2
INC COUNT AND TEST COUNT FOR ZERO

G1: MOV $4444,(R3)+
MOV $99,R1
MOV $0,(R3)+
SBR R1,S3
THE REST OF THE BLOCK FILLED WITH ZERO
CLR R1
S3: MOV #FREE+R2
BRING PIONIER BACK

S4: MOV #FREE+R2
FETCH R2 NAME
MOV $AREA+R5
CLR R4
ITER R5,R4,NAME
WRITE R5,R4,R3+256,R1
INC R1
ADD $512,R3
UPDATE PIONTER
CMP $255,R1
IF MORE THAN 25 NEEDED BLOCKS
BNE S4
CLOSE S4
EXIT S4
READ: TSTR @ADDR
BPL READ
MOV @ADDR+(R3)+
ADD $400,ADDR
RTS FC

; EVEN

COUNT2: WORD -300,
TMP21: WORD 0
TMP22: WORD 0
TMP23: WORD 0
DATADDR: BLKW 24.
AREA1 BLKW 10
BUFF: BLKW 6400.
NAME: @ADDR /DK/
RADIO /VR/
RADIO /DAT/
FREE: +2
END START

317
APPENDIX B

COMPUTER PROGRAMS FOR DATA PROCESSING

Program PT6.FOR lists the content of the raw data file and the number of valid fields. This program is mainly used to check the quality of the data. Program PAPPL6.FOR reads the raw data from the disk and after processing them, plots three anatomical angles on the paper. Program PMI1.FOR does the same thing for four inertial angles and angular rates and accelerations. Program ST9 works with 25-word format data fields. It processes the data and normalizes them. Then, the corresponding average values and standard deviations are calculated. The output consists of the average and the 95 percent confidence interval graphs which are plotted. Program P4.FORM does the optimal filtering. It reads the raw data from the disk and after some preprocessing, plots power spectra for both noise and data in the presence of noise and lists the corresponding numerical values and the optimal gains.

Program RAWC63.FOR is the master program for data processing which calculates angular quantities and hip coordinates. It provides all necessary information for five-mass and seven-mass dynamic models. Results are written back into the disk under the name FILE6.DAT. Subroutine ANGL.FOR calculates the angle between two lines obtained by connecting four points. Subroutine AVE.FOR does some preprocessing on the raw data to take care of the missing

318
points. Subroutine FOOR.DAT calculates the power spectral density of a given function for various frequencies. Subroutine FOUR.FOR has two versions. It basically calculates the Fourier coefficients to smooth the raw data. Version I only gives the smoothed function. Version II calculates both the smoothed values and the first and second derivatives.
C PROGRAM TO PRINT DATA FROM F.P. & 6 LIGHTS
C S. RAHMASHI 5-APR-78
DIMENSION KRD(21)
INTEGER KRD
CALL ASSIGN(2,'DK:JOHN6.DAT';2,'OLD','NC',1)
DEFINE FILE 2(120,21,1,116)
ISS=0
50 10 K=1,120
READ(2,'K') KRD
IF(KRD(21).EQ.-1) ISS=ISS+1
TYPE 30*K=(KRD(I),I=1,8)
TYPE 30* (KRD(I),I=9,21)
30 FORMAT(1X,13I6)
10 CONTINUE
TYPE 20-ISS
20 FORMAT(1X,'NO. OF VALID DATA=',I3)
STOP
END
GO TO 7
TOEOFF=TOEOFF+1.
GO TO 28

7 IF(KF.EQ.1) GO TO 19
   COR=KRD(17)
   DF=ABS(COR-CORF)
   CORF=KRD(17)
   KH=TOEOFF+23.
   IF(K.LT.KH.OR.DE.GT.2.) GO TO 19
   HS=FLOAT(K)
   INS=K
   KF=1

19 IF(IAN.NE.4) GO TO 5
   TYPE 8*K*(KRD(J)),J=1,8
   FORMAT(2X,I3,O16)
   TYPE 9*(KRD(J)),J=9,21
   FORMAT(1X,I3,O16)
   GO TO 10

5 DO 50 J=9,20
   J1=J-8
   F1D(11)=FLOAT(KRD(J))
   GO TO 50

50 CONTINUE
   F1D(12)=F1D(12)*SFR
   F1D(13)=F1D(13)*SFR
   F1D(16)=F1D(16)*SFR
   F1D(17)=F1D(17)*SFR
   IF(KRD(21).EQ.-1) GO TO 40
   TKN=0.

40 GO TO 10
   ISO=ISO+1
   IF(IAN.NE.3) GO TO 44
   CALL ANGL(F1D(1),F1D(2),F1D(3),F1D(4),F1D(5),F1D(6),F1D(7),
     F1D(8),F1D(9),F1D(10),THET(K))
   IF(IAN.NE.2) GO TO 46
   CALL ANGL(F1D(1),F1D(2),F1D(3),F1D(4),F1D(5),F1D(6),F1D(7),
     F1D(8),F1D(9),F1D(10),THET(K))
   IF(IAN.NE.1) GO TO 13
   CALL ANGL(F1D(7),F1D(8),F1D(9),F1D(10),F1D(11),F1D(12),THET(K))

13 IF(I1.EQ.0) GO TO 10
   TYPE 12*K,F1D(12),THET(K)
   FORMAT(1X,I3,O13,F8.0)
   CONTINUE

10 CONTINUE
   IEND=NPOINT
   GO TO 17

11 IEND=I-2

17 END=IEND
   TIME=END/60.
   TTOE=TOEOFF/60.
   TNS=HS/60.
   IF(I1.NE.0) GO TO 210
   IF(IAN.GE.1.AND.IAN.LE.3) GO TO 33
   IF(IAN.EQ.4) GO TO 1
   TYPE 30=I11

30 FORMAT(1X,'TOTAL NO. OF VALID DATA=',I3)
   GO TO 350

33 CONTINUE

C29 FORMAT(1X,'TYPE11 FOR LOOK AHEAD 2 FOR 3 POINT AVE.
C 1,3 FOR BOTH AND 4 FOR NONE.'
C ACCEPT 3,ME
MC=3
   TYPE 55=TOEOFF,INS,IEND,TTOE,TNS,TIME
   FORMAT(1X,F4.2,2X,I3,2X,I3,2X,F4.2,3X,
     1X,TNS,'=','F4.2,3X','TOTAL TIME=','F4.2,3X','SEC')
   T2=TOE-2.
   NPOINT=IEND
   CALL AVE(THET,ME,NPOINT,STHET)
   CALL FOUR(STHET,INTERMS,NPOINT,FTHET)
IF(NF.EQ.1) KP=IEND
IF(IEND.LT.KP) KP=IEND
KP=IEND
IF(NF.EQ.1) NPOINT

DO 350 I=1,NPOINT
   XY(I,J)=FTHET(I)
350
   CONTINUE

CALL CLOSE(2)
CONTINUE

ACCEPT 3,MZ
IF(MZ.NE.1) GO TO 92
IO 91 J=1,KP

TYPE 90*I*XY(I,J)*J,=1,6

90  FORMAT(1X,I3,3X,6F8.2)
91  CONTINUE
92  CALL VEL01(I*XY,J*KE,256,NPLOT,0,0,0,0,0,0)

GO TO 1
STOP

END
* THIS PROGRAM FLOTS ALL THE RESULTS ON THE PAPERS
* IT FLOTS PHI,PHID,PHIFOR 4 ANGLES WRT HON. AXIS USING RAWC63

DIMENSION RT(100+13), JXY(24), A(18)
CALL ASSIGN(2, 'HIFILE6, PAT, 12', 'GLD', 'NC', 1)
DEFINE FILE 2(100+13+6, U, NRES=5)
FI=1.1.1.1
CONTINUE
DO 5 J=2, 13
RT(I,J)=0.0
5 CONTINUE

TYPE 7
FORMAT(1X,'TYPE THE INDEX NO. ')
ACCEPT &L

FORMAT(12)
JXY(1)=1
IF(L.NE.1) GO TO 21
JXY(2)=2
21 IF(L.EQ.2) JXY(2)=3
   IF(L.EQ.3) JXY(2)=4
   IF(L.EQ.4) JXY(2)=5
   IF(L.EQ.5) JXY(2)=6
   IF(L.EQ.6) JXY(2)=7
   IF(L.EQ.7) JXY(2)=8
   IF(L.EQ.8) JXY(2)=9
   IF(L.EQ.9) JXY(2)=10
   IF(L.EQ.10) JXY(2)=11
   JXY(2)=L+1
READ(2'1, (A(1), J=1, 18))
TERMS=A(1)
WPHASE=A(2)/2.
NPOINTS=NPHASE.
IRT0=A(3)
IL0=A(4)

TYPE 200, NTERMS, NPOINT, IRT0, IL0
FORMAT(1X, 418)
DO 10 L=1, 100
   IP=IP+1
READ(21, IF) (A(J), J=1, 18)
B0. 14 J=1, 12
J(J)=I(J)
RT(I,J)=A(J)
14 CONTINUE
RT(I+11)=PI-RT(I+11)
RT(I+12)=RT(I+12)
RT(I+13)=RT(I+13)
IF(L.NE.0) GO TO 12
12 CONTINUE

6 FORMAT(1X, 418)
DO 10 L=1, 100
   CALL WPLT.R(T, JXY, NPOINT, 100, 1, 0, 0, 0, 0)
GO TO 98
100 SDE
END
C PROGRAM TO NORMALIZE & GET THE AVE. & STAN. DEV.
C USING INTERPOLATION (6 LIGHTS).
DIMENSION KRD(25), FRD(20), THETA(T), THEK(T), THE0(T), THE0(T)
1: FTHETA(100), SM(101, 15, 3), ANY(100)
2: TMTA(8), SF(101, 12), AVERAGE(101), ST(101), JXY(10)
INTEGER KRD
C
TYPE 215
C
FORMAT(1X, 'TYPE IN THE COEFFICIENT OF TV DISTANCE ')
ACCEPT 216, DI
C
TYPE 216
C
FORMAT(F4.2)
NPOINT = 90
C
TYPE 219
C
CONTINUE
C
TYPE 220
C
FORMAT(1X, 'TYPE # OF HARMONICS NEEDED')
ACCEPT 220, NIERMS
C
TYPE 220
C
FORMAT(I1)
SFR = 0.78
C
FORMAT(1X, 'TYPE 1 FOR ANKLE, 2 FOR KNEE, 3 FOR HIP')
1 ANGLES, 4 FOR ALL RESPECTIVELY, ELSE TYPE 6
2 TO TERMINATE.')
C
TYPE 220
C
FORMAT(I1)
DO 6 I = 1, 101
DO 6 J = 1, 10
DO 6 K = 1, 3
SM(I, J, K) = 0.
6 CONTINUE
C
TYPE 300
C
FORMAT(1X, 'TYPE NO. OF DATA SETS')
ACCEPT 4, NSETS
C
TYPE 300
C
FORMAT(I2)
NSR = I
C
IF (NSETS.EQ.4) NSR = 3.11
IF (NSETS.EQ.5) NSR = 2.77
IF (NSETS.EQ.6) NSR = 2.57
IF (NSETS.EQ.7) NSR = 2.45
IF (NSETS.EQ.8) NSR = 2.37
IF (NSETS.EQ.9) NSR = 2.30
IF (NSETS.EQ.10) NSR = 2.26
IF (NSETS.EQ.11) NSR = 2.23
JXY(1) = 1
JXY(2) = 2
JXY(3) = 1
JXY(4) = 3
JXY(5) = 1
JXY(6) = 4
JXY(7) = 1
JXY(8) = 5
JXY(9) = 1
JXY(10) = 6
C
**************************************
C
TYPE 310
C
FORMAT(1X, 'T.O.TIME(S)', '2X', 'STRP TIME(S)', '2X',
1: 'STRP LENGTH(M)', '2X', 'HEELRIST(M)', '2X', 'HIP K.S.(DEG)',
2: 'KNEE M.S.(DEG)', '2X', 'ANK.M.S.(DEG)', '2X', 'DIFF(ANKLE)(DEG)')/
3: X)
C
ASTR = 0.
STIR = 0.
AMH = 0.
AMR = 0.
AMDIF = 0.
BTR = 0.
ATD = 0.
AMH = 0.
ASTR = 0.
C
325
DO 400 NPE=1,NSETS
CALL ASSIGN(2,'DK:XXXXX.DAT',-12,'GLD','NC',1)
DEFINE FILE 2(100,25),1,NSRTT)
IFLG=0
TOEDF=1.
NMSE=0.
STRIDE=0.
XF=0
IA=0
C
DO 10,KE=1,90
READ(2,K) KRD
IF(K.EQ.1) AM=KRD(10)
6
NM(N)=KRD(N)
IF(IFLG.LE.1) GO TO 7
IF(K.GE.2) GO TO 22
CPR=KRD(18)
26
DO 27 JI=1,8
TMJ(JI)=KRD(JI)
27
CONTINUE
22
GO TO 7
MA=0
21
DO 20 JI=1,4
THM=THM+TMJ(JI)
20
CONTINUE
THM=THM/4.
TB=ABS(THM-512.)
IF(TH.NE.10.,0.,OR,K.LE.40.,OR,KRD(4),GT,600.) GO TO 24
IFLG=1
GO TO 7
24
TOEDF=TOEDF+1.
GO TO 28
7
IF(KF.EQ.1) GO TO 19
CPR=KRD(18)
EF=ABS(COR-CPR)
COR=KRD(18)
KR=TOEDF+23.
IF(K.LT.KR OR,DEF,GT,2.) GO TO 19
HS=FLOAT(K)
IHS=K
KF=1.
19
AMEND=KRD(18)
CONTINUE
5
DO 50 JI=10,21
FRD(JI)=FLOAT(KRD(JI))
50
CONTINUE
FRD(2)=FRD(2)*SFR
FRD(4)=FRD(4)*SFR
FRD(6)=FRD(6)*SFR
FRD(8)=FRD(8)*SFR
FRD(10)=FRD(10)*SFR
FRD(12)=FRD(12)*SFR
40
CONTINUE
CALL ANUL(FRD(1),FRD(2),FRD(3),FRD(4),FRD(5),FRD(6),THET(K))
CALL ANUL(FRD(3),FRD(4),FRD(5),FRD(6),FRD(7),FRD(8),FRD(9),THET(K))
CALL ANUL(FRD(12),FRD(10),FRD(9),FRD(10),FRD(11),FRD(12),THETA(K))
10
CONTINUE
END=END+1.
50
CONTINUE
17
TICE=TOEDF/60.
IJO=TOEDF
THS=HS/60.
326
ST9. FOR

SIRIDE = (DI) * ABS(ANI - AWEND)
ANYMIN = ANY(1)
ANYMAX = ANY(1)
30. 34. K = 1, IHS
IF (ANY(K), LT, ANYMIN) ANYMIN = ANY(K)
IF (ANY(K), GT, ANYMAX) ANYMAX = ANY(K)
34 CONTINUE
HRISE = (DI) * ABS(ANYMAX - ANYMIN)
C IF (HS, GT, 20.) FC = IODEFF * 100. / HS
IFC = PC
C
AMR = AMR + HRISE
ASTRD = ASTRD * STRIDE
BTO = BTO + PC
MIDS = IODEFF / 2.0
SFA = 101. / HS
NMIDS = SFA * (1., 0, MIDS)
SIM = SIMHHS
IHS = HS
NPDP = IHS
C DO 402 K = 1, 3
IF (K, NE, 1) GO TO 404
CALL FOUR (THETA, ITERM, IHS, FTHET)
404 IF (K, NE, 2) GO TO 406
CALL FOUR (THETA, ITERM, NPDP, FTHET)
406 IF (K, NE, 3) GO TO 408
CALL FOUR (THETA, ITERM, NPDP, FTHET)
C
DO 320 L = 1, 101
AL = L
LK = L
LRB = L - 1
IF (LK, NE, 0) GO TO 318
SM(L, NP, K) = FTHET(1)
GO TO 320
318 SM(L, NP, K) = FTHET(LRB) + (FTHET(LRB) - FTHET(LRB1)) * (B1 - FR1)
320 CONTINUE
402 CONTINUE
C
AMAX = SM(NMIDS, NP, 1)
DG. 409 L = NMIDS / 85
IF (SM(L, NP, 1), GT, AMAX) AMAX = L
IF (SM(L, NP, 1), GT, AMAX) AMAX = SM(L, NP, 1)
409 CONTINUE
AMIN = SM(1, NP, 1)
DG. 410 L = 1, HMAX
IF (SM(L, NP, 1) - LT, AMIN), AMINA = SM(L, NP, 1)
410 CONTINUE
ADIF1 = AMAX - AMIN
AADIF = AADIF + ADIF1
AMK = AMAX + SM(NMIDS, NP, 1)
ANK = AMK + SM(NMIDS, NP, 2)
AMIN = AMIN + SM(NMIDS, NP, 3)
ATOK = TOT + HG
ASTK = AST + HG
IYPE = 30. NF = IKOE, THS, SIRIDE, HRIZE, SM(NMIDS, NP, 3) SM(NMIDS, NP, 2)
1. SM(NMIDS, NP, 1) ADIF1
30 FORMAT (X, 13, BX, 4, 2, 9, X, F4.2, 10X, F4.2, 10X, F5.2, 10X, F5.1, 10X, F5.1)
1. 10X, F5.1, 10X, F5.1, 10X, F5.1
C IF (NP, NE, NETS) GO TO 399
ATOMAD/SETS
RT0 = RT0/SETS
AMK = AMR/SETS
ASIRD=ASTRD/SETS
ASTI=ASTT/SETS
AMH=AMH/SETS
AMK=AMK/SETS
AMA=AMA/SETS
AARIF=AARIF/SETS

320 CALL CLOSE(2)
400 CONTINUE
C
96 TYPE 33
33 FORMAT(1X,40X,'***** THE AVERAGE VALUES *****'//,+1X)
TYPE 30+SETS+ATD+ASTI+ASTRD+AH+AMH+AMK+AMA+AARIF
TYPE 2

ACCEPT 3+IANG
IF(IANG.EQ.6) GO TO 99
IF(IANG.NE.4) GO TO 451

451 DO 453 I2=1,101
453 DO 455 I1=1,101
455 ST(I2)=0.
456 CONTINUE
DO 457 I1=1,101
DO 458 J=1,NSETS
ST(I2)=ST(I2)+SM(I,J,IANG)
459 CONTINUE
C
C GET THE MEAN
DO 321 I1=1,NSETS
321 DO 322 L=1,101
322 AMEAN(L)=AMEAN(L)+ST(L)*SETS
IF(L.EQ.NSETS) AMEAN(L)=AMEAN(L)/SETS
322 CONTINUE
321 CONTINUE
C
C GET THE STAND.DEV.
DO 332 L=1,101
331 CONTINUE
332 CONTINUE
C
C GET THE UPPER & THE LOWER PART
DO 350 J=1,101
350 CONTINUE
IF(IANG.NE.1) GO TO 461
350 CONTINUE
350 CONTINUE
IF(IANG.NE.2) GO TO 462
350 CONTINUE
461 IF(IANG.NE.1) GO TO 462
462 UTD=160.
463 VBD=190.
GO TO 464.
461 VBD=190.
GO TO 465.
462 UTD=160.
GO TO 464.
463 CALL UPL1T(SIP,JXY,101,101,1,0,100,1,UBD1,UBD2)
455 CONTINUE
GO TO 96
99 STOP

END
PROGRAM TO USE WEINER FILTER TO SMOOTH THE DATA
IT USES FOURIER SERIES TO GET THE POWER SPEC.
TAKES THE AVE. OF POWER SPEC. OF DATA SETS.

DIMENSION JXY(4), PNN(30), PI(30), TPI(30), TPIII(30), TPIV(30), TA(8)
DIMENSION KARD(25), FRD(12), THET(200), TPII(30), TPFN(30)
PI=3.141592

IC=0
NTERMS=30
NFRD=79
DO 36 I=1,30
TFII(I)=0.
TPNN(I)=0.
36 CONTINUE
DO 130 NTIM=1,2
130 TYPE 60

IF(NSET, 'TYPE NO. OF DATA SETS')
ACET 1
NSET=1

TYPE 4
ACET 1
INDEX
LDIF=0.
DO 33 J=1,200
THET(J)=0.
CONTINUE
DO 50 NN=1,NSET
ACET 52
NP
ANF(NP)=
CALL ASSIGN(2, 'OK1:DATA,FIL!', -12, 'OLD', 'NC', 1)
DEFINE FILE 2:600, 25:1+NXRDC
IF (NTIM.EQ.2) IC=0
IF (NTIM.EQ.12) IC=1
IF (IC.LT.1) IC=IC+1
READ (2, K) (KARD(J), J=1,25)
K=K-IC
37 FORMAT (I8, 2I8)
DO 30 J=10+2,
J=J-9
FRD(J)=KARD(J)
30 CONTINUE
DO 34 J=2,12,
FRD(J)=FRD(J)+SFR
CONTINUE
IF (KARD(9).NE.6) GO TO 51
IF (INDEX.NE.5) GO TO 31
CALL ANQL(FRD(1), FRD(2), FRD(3), FRD(4),
FRD(5), FRD(6), THET(KK))
GO TO 3
IF (INDEX.NE.2) GO TO 32
CALL ANQL(FRD(3), FRD(4), FRD(5), FRD(6),
FRD(7), FRD(8), FRD(9), FRD(10), THET(KK))
GO TO 3
32 IF (INDEX.NE.1) GO TO 3
CALL ANQL(FRD(7), FRD(8), FRD(9), FRD(10),
FRD(11), FRD(12), THET(KK))
3 CONTINUE
51 CONTINUE
52 FORMAT (1I4)
CALL CLOSE(2)
IC=IC+NP
IF (NTIM.NE.11) GO TO 63
CALL FORD (THET, NTERMS, NP, PNN)
DO 55 L=1,30
TPNN(L)=TPNN(L)+PNN(L)
55 IF (NP.EQ.NSET) TPNN(L)=TPNN(L)/NSET
CONTINUE
GO TO 50
56 CALL FORD (THET, NTERMS, NP, PI)

349$ 
#34L$
P4.FOR

DO 56 L=1,30
   TPIL(l)=TPII(l)+PII(l)
   IF(NN,66,58) TPIL(l)=TPII(l)/SETS
56   CONTINUE
50   CONTINUE
130  CONTINUE
C
DO 66 I=1,30
   AXY(I,1)=1
   AXY(I,2)=AMP/4.*TPWN(I)
   AXY(I,3)=(AMP/4.*PII(I)
   AXY(I,4)=(TPII(I)-TPWN(I))/TPII(I)
   AA(I)=AXY(I,4)
66   CONTINUE
   DO 67 JJ=1,3
   II=2*II-1
67   CONTINUE
   TYPE 41
   FORMAT(1X,'TYPE 1 FOR NOISE, 2 FOR INFL, AND 3 FOR RATIO:')
   ACCEPT 1,J3
   IF(J3.EQ.1) JA(2)=2
   IF(J3.EQ.2) JA(2)=1
   IF(J3.EQ.3) JA(2)=4
   CALL UPLT(AXY,JA,12,30,1*0,0,0,0,0,0)
   TYPE 42
   FORMAT(1X,'/',1X)
   DO 68 I=1,30
   TYPE 69,1(X,AXY(I,NN),NN=2,4)
68   CONTINUE
69   FORMAT(1X,1X,19,F10.3)
C
1   FORMAT(I3)
4   FORMAT(1X,'TYPE 1 FOR ANGLE, 2 FOR KNEE, 3 FOR HIP')
7   FORMAT(1X,'THE MEAN = ',F12.5)
23  FORMAT(1X,'TYPE IN 1 DIGIT FOR .5 OF PRIN TO BE PLOTTED')
24  FORMAT(1X,'TYPE 1 TO CONTINUE, 0 TO STOP')
STOP
END
RAWC63.FOR

C THIS_PROGRAM.READS_DATA_BOTH_FROM_FILE.
C AND DOES SOME MODIFICATIONS. ANGULAR COMPUTATIONS
C AND WRITES ANGULAR DATA BACK ONTO THE DISK.
C DIMENSION KRD(21),KRD(19),KRD(130),KRD(130),KRD(130),KRD(130),KRD(130),KRD(130)
C 1:TM(10),THETA(130),DTHE1(130),DTHET1(130),TM(130),TM(130),TM(130)
C 2:DDTHETA(130),DDTHT1(130),DDTHT1(130),DTHE1(130),DTHET1(130),DTHET1(130)
C 3:ALX(130),ALZX1301,AL3(130),DDTHEF(130)
C 4:DXH(130),DXH(130),DXH(130),DXH(130),DXH(130),DXH(130),DDYH(130),DDYH(130)
C INTDEF=2 KRD
C CALL ASSION(2,'DX11DGN6A.DAT','-12,'DLR','NC',1)
C CALL ASSION(3,'DKFILES.DAT','-12,'NEW','NC',1)
C DEFINE FILE 2(100,2,10,0,21,0,0)
C DEFINE FILE 3(131,36,33,NNNTT)
C TYPE 2
C FORMAT(1X,'TYPE II')
C ACCEPT 3 III
C FORMAT(1I)
C SFR=0.78
C ISR=0
C TOEOFF=1.0
C IFLG=0
C KE=0
C NP=130
C IA=0
C DO 10 K=1,130
C READ(2,K) KRD
C CHECK THE END OF THE VALID DATA
C IF(KRD(21),EQ,-1) IA=0
C IF(KRD(21),EQ,-2) IA=IA+1
C IF(KRD(100)AND,IA,GE,3) GO TO 11
C GET THE T.O. TIME
C IF(IFLG,EQ,1) GO TO 7
C IF(KRD(2),EQ,2) GO TO 22
C CORR=KRD(12)
C YMIN=KRD(7)+KRD(8))/2.
C DO 27 J1=1,8
C TM(J1)=KRD(J1)
C CONTINUE
C DO 27 J1=1,8
C TM(J1)=KRD(J1)
C CONTINUE
C TM=TM/4.
C TB=ABS(TM-512.)
C IF(TB,GE,10.0,OR,K.RD(4),GE,600) GO TO 24
C CONTINUE
C IFLG=1
C GO TO 7
C TOOEFF=TOOEFF+1.
C GO TO 28
C GET H.S.
C IF(KRD(2),EQ,1) GO TO 19
C CORR=KRD(12)
C DE=ABS(CORR-CORR)
C CORR=KRD(17)
C KM=TOOEFF+23.
C IF(KM,LE,50.0,OR,K,DE,1.1) GO TO 19
C IHS=K
C KE=1
C CONTINUE
C DO 50 J=9,20
C J=J-8
C 331
FRED(J) = FLOAT(KRB(J))
CONTINUE
FRED(2) = FRED(2) * SFR
FRED(4) = FRED(4) * SFR
FRED(6) = FRED(6) * SFR
FRED(8) = FRED(8) * SFR
FRED(10) = FRED(10) * SFR
FRED(12) = FRED(12) * SFR

C

XH(K) = FRED(3)
YH(K) = FRED(4)

C

IF (KRD(21), EQ, -1) GO TO 40
THETA(K) = 0
THETS(K) = 0
THEH(K) = 0

GO TO 10

40
ISS = ISS + 1

DU = FRED(3) - 10.

CALL ANGL(FRED(1), FRED(2), FRED(3), FRED(4), FRED(5), FRED(4),
1DU+FRED(4), THET(K))
DM = FRED(5) - 16.

CALL ANGL(FRED(3), FRED(4), FRED(5), FRED(6), FRED(5),
1DUF+FRED(6), DU+FRED(6), THETS(K))
DM = ERD(9) - 10.

CALL ANGL(FRED(7), FRED(8), FRED(9), FRED(10), FRED(9), FRED(10),
1DU+FRED(10), THEH(K))
DM = ERD(11) - 10.

CALL ANGL(FRED(9), FRED(10), FRED(11), FRED(12), FRED(11), FRED(12),
1DU+FRED(12), THET(K))

CONTINUE

IEND = 136

GO TO 12.

11

IEND = I-A

17

END = END

TIME = END/60.
AAM = A

IF (AAM, LT, TOEFOF) TOEFOF = AAM

KDE = TOEFOF - TIME

THS = HS/60.

C

TYPE 30, ISS

C30

FORMAT(1X, 'TOTAL NO. OF VALID DATA=' , I3)
    ME = 5

TYPE 55, KTOE, JHS, KEND, ITDE, ITMO, TIME

55

FORMAT(1X, 'TOTAL TIME=' , F4.2, ' SEC')

T2 = TOEFOF - 2.

TYPE 6A

66

FORMAT(1X, 'TOTAL NO. OF POINTS' , R, T, D, & L, D, T, O, )

C

ACCEPT 47, NERMS, NT1, NT2, NY, ITO, ITO

FORMAT(11, I1, I11, I11, I11, I11, I11, I11, I11, I11)

NPOINT = NN.

CALL AVE(THET, ME, NPOINT, THET)

DO 68 I = 1, 130

THETH(I) = 0.

68

CONTINUE

CALL FUC(THET, NTERMS, NPOINT, THET, AL1, AL2)

CALL FUR(AL1, NT1, NPOINT, THET, AL3, THET)

CALL FUR(AL2, NT2, NPOINT, THET, AL1, THET)

CALL AVE(THETS, ME, NPOINT, THET)

DO 71 I = 1, 130

THETS(I) = 0.

71

BTHTS(I) = 0.

CONTINUE

CALL FUG(THET, WTERMS, NPOINT, THET, AL1, AL2)

CALL FUG(AL1, NT1, WPOINT, THET, AL3, THET)

CALL AVE(THETW, ME, WPOINT, THET)

DO 71 I = 1, 130

THETW(I) = 0.

71

BTHTW(I) = 0.

CONTINUE

CALL F6H(THET, NTTERMS, NPOINT, THET, AL1, AL2)

CALL F6H(AL1, NT1, NPOINT, THET, AL3, THET)

CALL F6H(AL2, NT2, NPOINT, THET, AL1, THET)

CALL AVE(THET6, ME, NPOINT, THET)

DO 71 I = 1, 130

THET6(I) = 0.

71

BTHT6(I) = 0.
DDTHES(I)=0.0
CONTINUE
CALL FOUR(THE1, NT1, NPOINT, THE1, AL1, AL2)
CALL FOUR(AL1, AL2, NPOINT, DTHES, AL1, AL3)
CALL FOUR(THE1, NT1, NPOINT, DTHE1, AL1, AL2)
CALL FOUR(THE1, NT1, NPOINT, DTHE1, AL1, AL3)
CALL AVE(THE1, NT1, NPOINT, DTHE1)
BEGIN
DO 73 I=1, NPOINT
IF(I.GT.10) GO TO 73
IF(XH(I).EQ.0.) LB=LB+1
73 CONTINUE
DO 76 I=1, LB-1
XH(I)=XH(LB+1)
76 CONTINUE
CALL AVE(XH+ME*NPOINT, AL1)
CALL AVE(YH+ME*NPOINT, AL2)
C
DO 51 I=1, NPOINT
AL1(I)=300.-AL1(I)
AL2(I)=300.-AL2(I)
51 CONTINUE
XC=AL1(I)
YC=AL2(I)
DO 53 I=1, NPOINT
AL1(I)=AL1(I)-XC
AL2(I)=AL2(I)-YC
53 CONTINUE
PN=NPOINT-1
SKX=AL1(NPOINT)/PN
DO 79 I=1, PN
AI=I-1
AL1(I)=AL1(I)-SKX*AI
79 CONTINUE
BEGIN
CALL FOUR(AL1, NT1, NPOINT, XH, DXH, DXH)
CALL FOUR(AL2, NT2, NPOINT, YH, DYH, DYH)
C
TAME THE AVE OFF
C
UX=0.
DO 72 I=1, NPOINT
UX=UX+XH(I)
72 CONTINUE
POINTS=NPOINT
UX=UX/POINTS
DO 78 I=1, NPOINT
AI=I-1
XH(I)=SKX*AI+XH(I)-UX
DXH(I)=SKX+DXH(I)
78 CONTINUE
C
F1=UX(1)
F2=UX(1)
TYPE 41
FORMAT(1X, 'TYPE THE COEFF')
ACCEPT 42, MCFF
42 FORMAT(1X)
F2=MCFF
F1=F1/MCFF
DO 77 I=1, NPOINT
XH(I)=(XH(I)-F1)*F2
YH(I)=$(YH(I)-F2)*F2
RAWC63.FOR

DXH(I)=DXH(I)+#FAF
DYH(I)=DYH(I)+#FAF

TYPE B3.1, X(I), DXH(I), DDH(I), YH(I), DYH(I), DDYH(I)

83 FORMAT (1X, I3, 6F10.5)
77 CONTINUE
RT(1)=INTERMS
RT(2)=NPOINT
RT(3)=TRYD
RT(4)=LTO
DO 74 I=1, 18
RT(I)=0, 0
74 CONTINUE
WRITE (3, 1) (RT(I), I=1, 18)
DO 80 K=1, 110
IF (K .LE. NPOINT) GO TO 85
THETH(K)=0.0
THETS(K)=0.0
THETA(K)=0.0
DTHET(K)=0.0
DTHET2(K)=0.0
DTHETA(K)=0.0
DTHETH(K)=0.0
DTHETE(K)=0.0
DTHETF(K)=0.0
XH(K)=0
YH(K)=0
DXH(K)=0
DYH(K)=0
85 IF (RT(1) .GE. 0) THEN
RT(2)=THETS(K)
RT(3)=THETH(K)
RT(4)=DTHETE(K)
RT(5)=DTHETF(K)
RT(6)=DTHETE(K)
RT(7)=DTHETA(K)
RT(8)=DTHETH(K)
RT(9)=DTHETE(K)
RT(10)=DTHETF(K)
RT(11)=DTHETE(K)
RT(12)=DTHETF(K)
RT(13)=XH(K)
RT(14)=YH(K)
RT(15)=DYH(K)
RT(16)=DXH(K)
RT(17)=DDYH(K)
RT(18)=DDXH(K)
WRITE (3, 'K1) (RT(I), I=1, 18)
IF (RT(1) .GE. 0) GO TO 80
TYPE B2.1, (RT(I), I=1, 18)
TYPE B2.1, (RT(J), J=1, 18)
82 FORMAT (1X, I3, 1X, I3, 1X, I3, 1X, 12F8.2)
80 CONTINUE
STOP
END
ANG.FOR

SUBROUTINE ANG(LAX,AY,BIX,BIY,B2X,B2Y,CX,CY,ANG)

PI=3.141592
E1X=AX-B1X
E1Y=AY-B1Y
E2X=AX-B2X
E2Y=AY-B2Y
E1L=SQRT(E1X**2+E1Y**2)
E2L=SQRT(E2X**2+E2Y**2)
UE1X=E1X/E1L
UE1Y=E1Y/E1L
UE2X=E2X/E2L
UE2Y=E2Y/E2L

D0TP=UE1X*UE2X+UE1Y*UE2Y

IF(ABS(D0TP).LE.0.01) GO TO 100
TANG=SQRT(1.-D0TP**2)/D0TP
IF(D0TP.GT.0.0) GO TO 80
ANG=PI-ATAN(-TANG)
GO TO 120

80 ANG=ATAN(TANG)
GO TO 120

100 ANG=PI/2.

120 CONTINUE

RETURN

END
Subroutine AVE (THET, ME, NPOINT, STHEI)

Dimension THET(300), STHEI(300)

DO 20 I=1,NPOINT
   IF (ABS (THET(I)) .LE. 0.001) GO TO 20
   ISTART=1
   GO TO 40
20 CONTINUE

IF (ISTART .EQ. 1) GO TO 25
   ISM1=1, ISTART
   GO TO 10
10 CONTINUE

50 CONTINUE
   ISP1=ISTART+1
   GO TO 40
   IF (ABS (THET(I1)) .GT. 0.001) GO TO 50
   IF (ABS (THET(I2)) .LE. 0.001) GO TO 40
   THE1(I1)=THE1(I1)+THE1(I2))/2.0
   GO TO 50
   THET(I1)=THE1(I1)
   CONTINUE
   IF (THE1(NPOINT) .EQ. 0.0) THET(NPOINT)=THE1(NPOINT)
   GO TO 100
50 CONTINUE
   CONTINUE

*** Check the method ***
   IF (ME.NE.1 .OR. ME.NE.3) GO TO 80
   IFLAG=0
   NPM2=NPOINT-2
   GO TO 70
60 I=1,NPM2
   IP1=I+1
   IF (IFLAG .EQ. 0) AND (STHEI(I,LT STHEI(IP1)) .AND. STHEI(IP1)
   GT STHEI(IP2)) GO TO 70
   IF (IFLAG .EQ. 1) GO TO 77
   IF (STHEI(I,LT STHEI(IP1)) .AND. STHEI(IP1,LT STHEI(IP2))
   IFLAG=1
   GO TO 70
70 STHEI(IP1)=(STHEI(IP1)+STHEI(IP2))/2.0
   GO TO 70
77 IF (STHEI(I) .GT. STHEI(IP1)) AND (STHEI(IP1) .GT. STHEI(IP2))
   I=I+1
   IF (STHEI(I) .GT. STHEI(IP1) .AND. STHEI(IP1) .GT. STHEI(IP2))
   IFLAG=0
   GO TO 70
70 CONTINUE

80 IF (ME.NE.2 .OR. ME.NE.3) GO TO 90
   *** Weighted 3 point AVE, method ***
   GO TO 85
80 CONTINUE
   NPM2
   IP2=I+2
   STHEI(IP1)=(STHEI(IP1)+2.0*STHEI(IP1)+STHEI(IP2))/4.0

85 CONTINUE
90 RETURN

END
C THIS SUB. IS USED TO GET THE POWER SPECTRUM
C
SUBROUTINE FDOOR(Y, NTERMS, NPOINT, FY)
DIMENSION FY(30), Y(1200), A(30), X(30)
N=0.0
N=1.041592
DO 20 I=1, NPOINT
20 CONTINUE
N=AG/NPOINT
DO 50 I=1, NTERMS
A(I)=0.0
END
50 CONTINUE
DO 100 J=1, NTERMS
XI=I
DO 100 I=1, NPOINT
XI=I
A(J)=A(J)*Y(I)*SIN(2.0*XJ*XJ*(XI-1.0)/NPOINTS)
B(J)=B(J)*Y(I)*COS(2.0*XJ*XJ*(XI-1.0)/NPOINTS)
CONTINUE
DO 50 I=1, NTERMS
A(J)=2.0*A(J)/NPOINTS
B(J)=2.0*B(J)/NPOINTS
FY(I)=A(I)**2+B(I)**2
150 CONTINUE
RETURN
END
FOUR.FOR(I)

C THIS SUBROUTINE CALCULATES THE FOURIER COEFF.
C THAT MINIMIZE THE MEAN SQUARE ERROR
C
SUBROUTINE FOUR(Y, NTERMS, NPOINT, FY)
DIMENSION T(300), FY(300), Y(300), A(30), B(30)
A=0.0
FI=3.141592
DO 20 I=1,NPOINT
A=AO+Y(I)
20 CONTINUE
POINTS=NPOINT
AO=AO/POINTS
DO 50 I=1, NTERMS
A(I)=0.0
B(I)=0.0
50 CONTINUE
DO 100 J=1, NTERMS
XI=J
DO 150 I=1, NPOINT
A(I)=A(I)+Y(I)*SIN(2.0*XJ*FI*(XI-1.0)/POINTS)
B(I)=B(I)+Y(I)*COS(2.0*XJ*FI*(XI-1.0)/POINTS)
100 CONTINUE
DO 150 J=1, NTERMS
A(I)=2.0*A(I)/POINTS
B(I)=2.0*B(I)/POINTS
150 CONTINUE
I(I)=1.0
DO 180 I=2, 300
I(I)=1.0
180 CONTINUE
DO 200 I=1, NPOINT
FY(I)=AO
200 CONTINUE
XJ=J
FY(I)=FY(I)+A(J)*SIN(2.0*XJ*FI*(I-1.0)/POINTS)
B(J)*COS(2.0*XJ*FI*(I-1.0)/POINTS)
200 CONTINUE
RETURN
END

338
FOUR.FOR(II)

C THIS SUBROUTINE CALCULATES THE FOURIER COEFF.
C THAT MINIMIZE THE MEAN SQUARE ERROR
C
SUBROUTINE FOUR(Y,NTERMS,NPOINT,FY,DFY,DDFY)
DIMENSION Y(200),FY(200),X(200),A(30),B(30),DFY(200),DDFY(200)
A(0)=0.0
PI=3.141592
DO 10 I=1,NPOINT
   AO=A0+Y(I)
   CONTINUE

10 CONTINUE
C TYPE 10,NPOINT,POINTS,NTERMS
C
DO 20 I=1,NTERMS
   AI=0.0
   BI=0.0
   CONTINUE

20 CONTINUE
DO 100 J=1,NTERMS
   XI=J
   XI=X(1)+Y(I)*SIN(2.0*XJ*PI/(X1+1.0)/POINTS)
   XJ=K(J)+Y(I)*COS(2.0*XJ*PI/(X1+1.0)/POINTS)
   CONTINUE

100 CONTINUE
DO 200 I=1,NTERMS
   FY(I)=AO
   CONTINUE

200 CONTINUE
DO 300 I=1,NTERMS
   FY(I)=FY(I)+A(I)*SIN(2.0*XJ*PI/I(POINTS))
   DFY(I)=DFY(I)+2.0*XJ*PI/I(POINTS)*A(I)*COS(2.0*XJ
   END
   CONTINUE
   DO 300 I=1,NTERMS
   DDFY(I)=DFY(I)*60.0
   CONTINUE
   RETURN
   END
APPENDIX C

COMPUTER PROGRAM FOR FIVE-MASS DYNAMIC MODEL

Program WHPHM.FOR works with RAWC63.FOR expressed in Appendix B. It reads the information from the disk and applies them to the model (Approach #6) to calculate all joint forces and torques. Results are written into the disk under the name RESULT.DAT to be used later.

Subroutine SOLN.FOR solves a set of linear equations.
WHFM.FOR

C PROGRAM TO USE 5-MASS MODEL
C
C HIP JOINT IS THE REF. FGINIT.
C
DIMENSION SM(6),SL(6),SJ(6),D(6),RT(18),PHI(130,7)
1,DFH(130,7),DDFH(130,7),XX(12),RF(21),DXLH(130),DXXH(130)
2,DXLY(130),DXYH(130),DXXH(130),DXXH(130),DLYH(130),DXYH(130)
3,XX(130),YYH(130),XYH(130),XYYH(130),YYH(130)
4,XX(130),YY(130),XY(130),XYY(130),YY(130)
5,XX(130),YY(130),XY(130),XYY(130),YY(130)
6,XX(130),YY(130),XY(130),XYY(130),YY(130)
7,XX(130),YY(130),XY(130),XYY(130),YY(130)
8,XX(130),YY(130),XY(130),XYY(130),YY(130)
9,XX(130),YY(130),XY(130),XYY(130),YY(130)

CALL ASSIGN(3,'DK:FILE6.DAT',12,'OLD','NC',1)
CALL ASSIGN(2,'DK:RESULT.DAT',13,'NEW','NC',1)
DEFINE FILE 3(131,36,'LNXXR)
DEFINE FILE 2(130,42,'LNXRST)

TYPE 113

1 FORMAT(1X,'TYPE II IF CHANGE X10) & STEP LENGTHS IN CM')
2 ACCEPT 1,11
3 FORMAT(I)
4 ACCEPT 3,1Y1,Y12
5 FORMAT(12=1X,12,1X,13)
6 STI=1Y1
7 STJ=STJ/100.
8 ST2=1Y2
9 ST2=ST2/100.
10 SM(1)=4.55
11 SM(2)=7.05
12 SM(3)=49.0
13 SM(4)=SM(2)
14 SM(5)=SM(1)
15 SLO=.14
16 SL(1)=0.502
17 SL(2)=0.431
18 SL(3)=0.827
19 SL(4)=SL(2)
20 SL(5)=SL(1)
21 SJ(1)=0.105
22 SJ(2)=0.099
23 SJ(3)=2.35
24 SJ(4)=SJ(2)
25 SJ(5)=SJ(4)
26 D(1)=-0.267
27 D(2)=-0.247
28 D(3)=-0.280
29 D(4)=D(2)
30 D(5)=D(1)
31 G=9.81
32 PI=3.1416
33
34 SM(6)=0.
35 SL(6)=0.
36 SJ(6)=0.
37 D(6)=0.

IF(J,NC,J) GO TO 5

TYPE 116

116 FORMAT(1X,'TYPE THE LENGTH-WEIGHT FACTORS IN F4.2')
117 ACCEPT 117,WM

118 CONTINUE

118 CONTINUE

111 FORMAT(1X,'TYPE M+L,J,ID,IS(FOR +,-X10)')
112 ACCEPT 112,M+L,J,ID,IS

C
WHPHM.FOR

D(4)=D(2)
D(5)=D(1)
CONTINUE
C
READ(3,1) (RT(I),I=1,10)
NTMAX=RT(1)\nNPOINT=RT(2)
NLHNRT=RT(2)/2.0
NPOINT=NLHNRT*2
ITG=RT(3)
ILT=RT(4)

TYPE 2;RT0,NPHASE,ILT0,NPOINT

FORMAT(1X,‘RT0’,I3,1X,‘NLHNRT’,I3,3X,‘END’,I6)
C
NOW READ ANGULAR DATA
DO 4 K=1,NPOINT
KP1=K+1
READ(3,‘KP1’) (RT(J),J=1,18)
C
DATA IS IN PROPER UNIT
PHI(I)=RT(I)
PHI(I+2)=RT(2)
PHI(I+3)=RT(3)
PHI(I+4)=RT(4)
PHI(I+5)=RT(5)
PHI(I+6)=RT(6)
PHI(I+7)=RT(7)
PHI(I+8)=RT(8)
PHI(I+9)=RT(9)

C
TAKE FEET INTO ACCOUNT
TEF=PI-RT(10)
PHI(I+10)=TEF
PHI(I+11)=RT(11)
PHI(I+12)=RT(12)

C
XLH(I)=RT(13)
YLH(I)=RT(14)
DYLH(I)=RT(15)
DDYLH(I)=RT(16)
DAXLH(I)=RT(17)
DDAXLH(I)=RT(18)

CONTINUE
C
PHI,DFHI,AND DDFHI ARE ANGLES, VELOCITIES, AND ACC, RESPECTIVELY.
C
CONSTRUCT THE DATA FOR THE OTHER THIGH AND SHANK
DO 10 I=1,NPHASE
N2=I+NPHASE

PHI(N2+1)=PHI(I+1)
PHI(N2+2)=PHI(I+2)
PHI(N2+3)=PHI(N2+1)
PHI(I+1)=PHI(N2+2)
PHI(I+2)=PHI(N2+3)
PHI(I+3)=PHI(N2+4)
PHI(N2+4)=PHI(I+4)
PHI(N2+5)=PHI(I+5)
PHI(I+5)=PHI(N2+6)
PHI(N2+6)=PHI(I+6)
PHI(N2+7)=PHI(I+7)
DFHI(N2+5)=DFHI(I+1)
DFHI(I+1)=DFHI(N2+5)
DFHI(N2+6)=DFHI(I+2)
DFHI(I+2)=DFHI(N2+6)
DFHI(N2+7)=DFHI(I+3)
DFHI(I+3)=DFHI(N2+7)
DDFH(I)=DDFH(I+1)
DDFH(I+1)=DDFH(I)
DDFH(N2+5)=DDFH(I+2)
DDFH(I+2)=DDFH(N2+5)
DDFH(N2+6)=DDFH(I+3)
DDFH(I+3)=DDFH(N2+6)

C
XRH(I)=XLH(N2)–ST1
YRH(I)=YLH(N2)

342
WHPHM.FOR

XRH(N2)=XLH(I)+ST2
YRH(N2)=YLH(I)
DXRH(I)=DXLH(N2)
DXH(N2)=DXLH(I)
DYRH(I)=DYLH(N2)
DYH(N2)=DYLH(I)
DDYRH(I)=DDYLH(N2)
DDYH(N2)=DDYLH(I)
CONTINUE

DO 204 I=1,NPOINT
XF(I)=XRH(I)-XLH(I)
YF(I)=YRH(I)-YLH(I)
CONTINUE

204 FORMAT(1x,13,2x,10F7.3)

C

C ****************************************************************************

C

C  DO 12 I=1,NPOINT
I.F(I,GE,IRTO) GO TO 15
Q=1
Q=O/(IRTO)
GO TO 30

15 IF(I,GT,NPHASE) GO TO 18
O=1.0
GO TO 30

18 IF(I,GE,ILTO) GO TO 22
O=(ILTO-I)
GO TO 30

22  CONTINUE

C

C DO 32 K=1,5
R(K)=COS(PHI(I,K))+(DPHI(I,K)**2)+SIN(PHI(I,K))+(DPHI(I,K)**2)+SIN(PHI(I,K))
S(K)=SIN(PHI(I,K))+(DPHI(I,K)**2)+COS(PHI(I,K))+(DPHI(I,K)**2)
CONTINUE

C

C DDX(1)=DDXH(I)+SL(2)*R(2)+(SL(1)-D(1))*R(1)
DDX(2)=DDXH(I)+(SL(2)-D(2))*R(2)
DDX(3)=(DDXH(I)+DXRH(I))/2.-D(3)*R(3)
DDX(4)=DXRH(I)+(SL(4)-D(4))*R(4)
DDX(5)=DXXH(I)+SL(4)*R(4)+(SL(5)-D(5))*R(5)
DDY(1)=DYLH(I)-SL(2)*S(2)-(SL(1)-D(1))*S(1)
DDY(2)=DDYLH(I)-(SL(2)-D(2))*S(2)
DDY(3)=(DDYLH(I)+DDYRH(I))/2.+D(3)*S(3)
DDY(4)=DDYRH(I)-(SL(4)-D(4))*S(4)
DDY(5)=DDYRH(I)-SL(4)*S(4)-(SL(5)-D(5))*S(5)

C

C TYPE 37,1:XLH(I),XRH(I),XF(I),DXLH(I),DXRH(I)
TYPE 37,1:YLH(I),YRH(I),YF(I),DYLH(I),DYRH(I)

38 TYPE 37,1:(DXH(N)+H=1.5),:DDYH(N)+H=1.5

37 FORMAT(1x,13,2x,10F10.2)

C

C NOW GET THE UPPER BODY COMPONENTS DIRECTLY
C

C FX=SM(3)*DDX(3)
FK=SM(3)*DDY(3)+
TF=SF(3)*DDYH(I)+FX3*3.3*SIN(PHI(I,3))+FY3*D(3)*
1COS(PHI(I,3))
RE(1)=FX3
RE(2)=FY3
RE(3)=TF
C

C CONSTRUCT THE MATRIX FOR EQ. OF ORDER 12
DO 44 I=1,12

44  CONTINUE
44 4 4  d 1 = 1, 12  
AA(1+1)=0  
CONTINUE  
AA(1,2)=1.0  
AA(2,2)=1.0  
aa(2,3)=FX3  
aa(3,3)=FX3  
aa(3,4)=1.0  
aa(4,1)=1.0  
aa(4,4)=1.0  
aa(5,6)=1.0  
aa(5,7)=FY3  
aa(6,5)=0  
aa(6,6)=1.0  
aa(7,7)=FY3  
aa(7,9)=1.0  
AA(8,5)=1.0  
AA(8,8)=1.0  
aa(9,2)=D(2)*SIN(PHI(I,2))  
aa(9,3)=(SL(2)-D(2))*SIN(PHI(I,2))*FX3  
aa(9,6)=D(2)*COS(PHI(I,2))  
aa(9,7)=FY3*(SL(2)-D(2))*COS(PHI(I,2))  
aa(9,10)=1.0  
aa(9,12)=T3  
aa(10,1)=D(1)*SIN(PHI(I,1))  
aa(10,2)=(SL(1)-D(1))*SIN(PHI(I,1))  
aa(10,5)=D(1)*COS(PHI(I,1))  
aa(10,6)=-(SL(1)-D(1))*COS(PHI(I,1))  
aa(10,9)=0  
aa(10,10)=1.0  
aa(11,3)=FY(I)*FX3*FX3*(SL(4)-D(4))*SIN(PHI(I,4))  
aa(11,4)=D(4)*SIN(PHI(I,4))  
aa(11,7)=-FY3*FY3*(SL(4)-D(4))*COS(PHI(I,4))  
aa(11,8)=D(4)*COS(PHI(I,4))  
aa(11,11)=1.0  
aa(11,12)=T3  
aa(12,1)=1.0  
aa(12,2)=D(5)*SIN(PHI(I,5))  
aa(12,4)=SL(5)-D(5)*SIN(PHI(I,5))  
aa(12,5)=1.0  
aa(12,8)=-(SL(5)-D(5))*COS(PHI(I,5))  
aa(12,9)=1.0  
aa(12,11)=1.0  
C  
NOW GET BB MATRIX  
BB(1)=SH(1)*DX(1)  
BB(2)=SH(2)*DX(2)  
BB(3)=SH(4)*DX(4)*FX3  
BB(4)=SH(5)*DX(5)  
BB(5)=SH(2)*DY(2)*SM(2)*G  
BB(6)=SM(1)*DDY(1)*SM(1)*G  
BB(7)=SM(4)*DDY(4)*FY3*SM(4)*G  
BB(8)=SM(5)*DDY(5)*SM(5)*G  
BB(9)=SJ(2)*DDPHI(I,2)  
BB(10)=SJ(1)*DDPHI(I,1)  
BB(11)=SJ(4)*DDPHI(I,4)+T3-FX3*(SL(4)-D(4))*SIN(PHI(I,4))  
FY3*(SL(4)-D(4))*COS(PHI(I,4))  
BB(12)=SJ(5)*DDPHI(I,5)  
CALL SOLN(12,AA,BB,XX)  
FX0=XX(1)  
FX1=FX0  
FX5=FX0+(1.0-0)  
FX2=XX(2)  
ALPHA=XX(3)  
FX4=XX(4)  
FY0=XX(5)
WHPHM.FOR

FY1=FY0&D
FY5=FY0*(1.0-D)
FY2=XX(6)
BETA=XX(7)
FY4=XX(8)
T0=XX(9)
T1=T0&D
T5=T0*(1.0-D)
T2=XX(10)
T4=XX(11)
GAMMA=XX(12)
RE(4)=FX1
RE(5)=FX5
RE(6)=FY1
RE(7)=FY5
RE(8)=T1
RE(9)=T5
DO 48 J=1,12
IF (J.EQ.9) GO TO 12
RE(JF9)=XX(J)
CONTINUE
WRITE(2'I) (RE(LE),LE=1,21)
48 CONTINUE
IF (J.EQ.1) GO TO 12
WRITE(1X,16,4X,FB,3)
CONTINUE
DO 49 I=1,21
DA(I)=6.0
CONTINUE
WRITE(2'NPL1) (DA(I),I=1,21)
STOP
END
C SUBROUTINE TO SOLVE A SET OF LINEAR EQUATIONS
SUBROUTINE SOLN(NN, A, B, X)
DIMENSION A(12,12), B(12), X(12), UL(12,12)
COMMON IFS(12)
CALL DECOMP(NN, A, UL)
CALL SOLVE(NN, UL, B, X)
RETURN
END

C*************************************************************************
C*************************************************************************
C SUBROUTINE DECOMP(NN, A, UL)
DIMENSION A(12,12), UL(12,12), SCALES(12), IFS(12)
COMMON IFS
N=NN

C INITIALIZE IFS, UL & SCALES
DO 5 I=1,N
IFS(I)=1
ROWNM=0.0
DO 2 J=1,N
UL(I,J)=A(I,J)
AI=ABS(UL(I,J))

1 IF(ROWNM<AI) GO TO 2
2 CONTINUE
3 SCALES(I)=1.0/ROWNM
GO TO 5

4 SCALES(I)=0.0
CONTINUE

C GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING
NM1=N-1

DO 17 K=1,NM1
BIG=0.0
DO 11 I=K,N
IP=IFS(I)
SIZE=ABS(UL(IP,K))*SCALES(IP)

10 IF(Size>BIG) GO TO 11
11 CONTINUE
12 BIG=SIZE
IF(IP=K) GO TO 13
13 I=IP
CONTINUE
GO TO 17
14 J=IFS(K)
IFS(K)=IFS(IP)
IFS(IP)=J
KPI=IFS(K)

15 FIVOT=UL(KP,K)
KP1=K+1
DO 16 I=KP1,N
IFS(I)=E
16 EM=-UL(IP,K)/FIVOT
UL(IP,K)=EM
DO 17 J=KP1,N
UL(IP,J)=UL(IP,J)+EM*UL(KP,J)
17 CONTINUE
CONTINUE
KP=IFS(N)
IF(UL(KP,N))=19
18 CALL SING
19 RETURN
END

346
SUBROUTINE SOLUC(NU,UL,R,X)
DIMENSION UL(12,12),B(12),X(12),IFS(12)
COMMON IF6
NU=NN
IFS(1)=IFS
DO 2 I=2,NU
2 SUM=SUM+UL(I)*X(I)
C 1 J=1,IFS(I)
1 SUM=SUM+UL(IF,J)*X(J)
C 2 CONTINUE
2 X(J)=B(IF,J)/SUM
C 3 I=IFS(I)
3 X(N)=X(N)/UL(IF,N)
C 4 CONTINUE
4 X(I)=(X(I)-SUM)/UL(IF,I)
C RETURN
END

SUBROUTINE SING(IWHY)
11 FORMAT(10X,'MATRIX WITH ZERO ROW IN DECOMPOSE.')
12 FORMAT(10X,'SINGULAR MATRIX IN DECOMPOSE. ZERO DIVIDE IN SOLVE.')
13 FORMAT(10X,'NO CONVERGENCE IN IMPREV. MATRIX IS NEARLY SING.')
NOUT=3
GO TO (1,2,3),IWHY
1 TYPE 11
GO TO 10
2 TYPE 12
GO TO 10
3 TYPE 13
RETURN
END
APPENDIX D

COMPUTER PROGRAM FOR SEVEN-MASS DYNAMIC MODEL

There are two routines used for the seven-mass model, both work with RAWC63.FOR. Program B8M.FOR employs the information obtained from the disk and calculates all joint forces and writes the results into the disk under the name RESULT.DAT. Program C86M.FOR is similar to the previous one except that after calculation of joint forces, it uses them to compute all joint torques and the location of the center of pressure. These new quantities are now written into the disk as RESULT.DAT.
PROGRAM TO USE WHOLE BODY METHOD
IT USES RAWC63+HTP MOTION IS INCLUDED.
IT IS THE ORIGINAL PROGRAM FOR 7-MASS,8-LINK MODEL.
DIMENSION SM(6),SL(6),SJ(6),X(18),RT(18),PHI(130),7
1=DPHI(130),7,DPHI(130),7,XX(18),RE(21),DXLH(130),DXRH(130)
2=RYLH(130),DXYLH(130),DXXLH(130),DXYRH(130),DYRH(130)
3=XLH(130),XH(130),XH(130),YRH(130)
4=R(18),S(5),DXK(4),DDY(7)
S=R(18),R(6),X(6),D(15)
CALL ASSIGN(3,DKIFILE+DAT+12,'OLD','NC',1)
CALL ASSIGN(2,DKIRESULT.DAT+13,'NEW','NC',1)
DEFINE FILE 3(131,36),UNNXTR
DEFINE FILE 2(130,42),UNNEST

TYPE II
FORMAT(1X,'TYPE II(1 IF CHANGE XI0) & STEP LENGTHS IN CM')
ACCEPT 1:II
FORMAT(II)
ACCEPT 3+II+1Y2
FORMAT(II,1X,12,1X,13)
ST1=II1
ST2=II2
ST3=II3
FORMAT INFORMATION
SL0=0.190
THET1=1.56
THET2=0.54
THET3=1.03
W1=0.52
W2=0.245
W3=0.34
BK=0.100
BF=0.154
SL.F=0.20
SMF=1.67
SJS=0.001
RO1=0.085
RO2=0.151
RO3=0.111

SM(1)=3.37
SM(2)=7.85
SM(3)=49.0
SM(4)=SM(2)
SM(5)=SM(1)
SL(1)=0.46
SL(2)=0.431
SL(3)=0.827
SL(4)=SL(2)
SL(5)=SL(1)
SJ(1)=0.052
SJ(2)=0.089
SJ(3)=2.35
SJ(4)=SJ(2)
SJ(5)=SJ(1)
D(1)=0.181
D(2)=0.247
D(3)=0.280
D(4)=D(2)
D(5)=D(1)
D=9.81
P=3.1416

C
SM(6)=0.
SL(6)=0.
SJ(6)=0.
D(6)=0.
IF(II.EQ.0) GO TO 5

TYPE 116

116 FORMAT(1X,'TYPE THE LENGTH,WEIGHT FACTORS IN F4.2')
ACCEPT 117, WL, WM

117 FORMAT(F4.2,1X,F4.2)
SMF=SMF#WM
RW1=RW1#WL
RW2=RW2#WL
RW3=RW3#WL
BK=BK#WL
BF=BF#WM
DO 118 I=1,5
SM(I)=SM(I)#WM
SL(I)=SL(I)#WL
PJ(I)=PJ(I)#WL
SJ(I)=SJ(I)#WM#WL
118 CONTINUE

119 CONTINUE

TYPE 111

111 FORMAT(1X,'TYPE M,L,J,ID,IS FOR +,-X10')
ACCEPT 112, M, L, J, ID, IS

112 FORMAT(5(I1,I1))
IF(IS.EQ.3) GO TO 5
IF(IS.EQ.1) AS=1.1
IF(IS.EQ.2) AS=0.9
IF(M.EQ.7) SMF=SMF#AS
IF(L.EQ.7) RO1=RO1#AS
SH(M)=SH(M)#AS
SL(L)=SL(L)#AS
SJ(J)=SJ(J)#AS
D(ID)=D(ID)#AS
SM(4)=SM(4)#AS
SM(5)=SM(5)
SL(4)=SL(2)
SL(5)=SL(1)
SJ(4)=SJ(2)
SJ(5)=SJ(1)
D(4)=D(2)
B8M.FOR

D(5)=D(1)
GO TO 119
C
5 CONTINUE
C
READ(3',1) (RT(I),I=1,10)
NTERM=RT(1)
NPOINT=RT(2)
NPHASE=RT(2)/2.0
NPOINT=NPHASE#2
IRT0=RT(3)
ILT0=RT(4)
TYPE 2,IRT0,NPHASE,ILT0,NPOINT
2 FORMAT(1X,'RT0=',13.6,1X,'3X,'LT0=',13.3X,'END=',16)
C
NOW READ ANGULAR DATA
DO 4 K=1,NPOINT
KP=K+1
READ(3',KP1) (RT(J),J=1,10)
C
DATA IS IN PROPER UNIT
PHI(K,1)=RT(1)
PHI(K,2)=RT(2)
PHI(K,3)=RT(3)
DPHI(K,1)=RT(4)
DPHI(K,2)=RT(5)
DPHI(K,3)=RT(6)
DDPHI(K,1)=RT(7)
DDPHI(K,2)=RT(8)
DDPHI(K,3)=RT(9)
C
4 TAKE FEET INTO ACCOUNT
TEF=RT(10)
PHI(K,6)=TEF
DPHI(K,6)=RT(11)
DDPHI(K,6)=RT(12)
C
XLH(K)=RT(13)
Ylh(K)=1.94RT(14)
DYLH(K)=RT(15)
DDYLH(K)=RT(16)
DXLH(K)=RT(17)
DDXLH(K)=RT(18)
CONTINUE
C
PHI,DPHI AND DDPHI ARE ANGLES, VELOCITIES, AND ACC, RESPECTIVELY.
C
CONSTRUCT THE DATA FOR THE OTHER THIGH AND SHANK & RIGHT HIP.
DO 10 I=1,NPHASE
N2=I+NPHASE
PHI(N2,5)=PHI(I,1)
PHI(N2,6)=PHI(I,2)
PHI(N2,7)=PHI(I,3)
PHI(N2,8)=PHI(I,4)
PHI(N2,9)=PHI(I,5)
DPHI(N2,5)=DPHI(I,1)
DPHI(N2,6)=DPHI(I,2)
DPHI(N2,7)=DPHI(I,3)
DPHI(N2,8)=DPHI(I,4)
DPHI(N2,9)=DPHI(I,5)
C
XRH(I)=XLH(N2)-ST1
YRH(I)=Ylh(N2)
C
351
B8M.FOR

1+DCOS(U6)DPHI(I,6)
DDY(7)=DDY#F(I)-SL(4)SL(5)S(5)R01*(SIN(U7)DPHI(I,7))
1+DCOS(U7)DPHI(I,7))
FY=0.
DO 30 MA=1.5
30 FY=FYN+SMF*(DDY(MA)+G)
CONTINUE
FY=FYN+SMF*(DDY(6)+G+DDY(7)+G)
C
FORMAT(1X,13,2X,F6.2,5X,F6.2)
C
NOW GET THE UPPER BODY COMPONENTS DIRECTLY
FX3=SMF#DDX(3)
FY3=SMF#DDY(3)+G
T3=SMF#DDPHI(I,3)+FX3#D(3)*SIN(Phi(I,3))+FY3*D(3)*
ICOS(Phi(I,3))
RE(1)=FX3
RE(2)=FY3
RE(3)=T3
C
CONSTRUCT THE MATRIX FOR EQ. OF ORDER 18
DO 44 IJ=1,6
44 DO 44 JJ=1,6
AA(IJ)=0.0
AA(I1,J1)=0.0
CONTINUE
AA(I1)=0.3
AA(I2)=1.0
AA(I3)=1.0
AA(I4)=1.0
AA(I5)=1.0
AA(I6)=1.0
AA(I7)=0.04
AA(I8)=1.0
BB(I)=SMF*DDX(I)
RH(I)=SMF*DDX(I)
C
DO 81 IJ=1,6
81 AA(I1,J1)=0.0
CONTINUE
AA(I1)=0.3
AA(I2)=1.0
AA(I3)=1.0
AA(I4)=1.0
AA(I5)=1.0
AA(I6)=0.04
AA(I7)=1.0
AA(I8)=1.0
BB(I)=SMF*DDX(I)
RH(I)=SMF*DDX(I)
C
CALL SOLN(d*AA*BB+XX)
C
DO 81 IJ=1,6
81 AA(I1,J1)=0.0
CONTINUE
AA(I1)=0.3
AA(I2)=1.0
AA(I3)=1.0
AA(I4)=1.0
AA(I5)=1.0
AA(I6)=0.04
AA(I7)=1.0
AA(I8)=1.0
BB(I)=SMF*(DDY(I)+G)
RH(I)=SMF*(DDY(I)+G)
C
CALL SOLN(d*AA*BB+X6)
DO 82 I1=1,6
XX(I1:6)=XO(I1)
82 CONTINUE
84 CONTINUE
C
FX7=RE(1)
FY7=RE(2)
T7=RE(3)
C
THE NOTATION ACCORDING TO 7-MASS MODEL IS
C
1=FXY7,2=FY7,3=I7,4=FXO7,5=FYO7,6=FYO7,7=FYO,8=FX1,9=FX2
C
10=FX3,11=FX4,12=FX5,13=FX6, 14=FY1,15=FY2,16=FY3,17=FY4
C
18=FX5,19=FY6,20=FX0,21=FY0
C
FL=FXO#03
RE(4)=XX(1)#03
RE(5)=XX(1)
RE(6)=XX(7)#0
RE(7)=XX(7)
RE(8)=XX(2)
RE(9)=XX(3)
RE(10)=RE(1)#XX(6)
RE(11)=RE(1)#(1,-XX(6))
RE(12)=XX(4)
RE(13)=XX(5)
RE(14)=XX(8)
RE(15)=XX(9)
RE(16)=RE(2)#XX(12)
XX(17)=RE(2)#(1,-XX(12))
RE(18)=XX(10)
RE(19)=XX(11)
RE(20)=FXM
RE(21)=FYH
IF(II,NE,2) GO TO 14
ACCEPT 1;ID
IF(10,EJ,1) GO TO 17
14 WRITE(2,'(I15,2,LE=4,1,21)')
12 CONTINUE
DO 49 I1=1,21
RE(A(I1))=6,0
49 CONTINUE
WRITE(2,'(M10)') (BA(I),I=1,21)
STOP
END
C86M.FOR

C  C86M.FOR
C PROGRAM TO USE WHOLE BODY METHOD
C IT IS THE SAME AS THE ORIGINAL PROGRAM FOR 7-MASS,B-LINK MODEL.
C HOWEVER, IT CALCULATES TORQUES & C.P. TO BE PLOTTED.
DIMENSION SM(6),SL(6),SJ(6),D(6),RT(18),PHI(130+7)
1,E(5),PHI(130+7),DDPHI(130+7),XX(18),RE(21),DXLH(130),DXRH(130)
)
2,DYLH(130),DYRH(130),DXLH(130),DXRH(130),DDYLH(130),DDXRH(130)
)
3,XLH(130),YLH(130),XRH(130),YRH(130)
4,R(5),S(5),DX(7),DDY(7)
5,BA(18,18),BB(6),X(6),BA(15)
CALL ASSIGN(3,'FILE1.DAT',12,'OLD','NC',1)
CALL ASSIGN(2,'RESULT.DAT',13,'NEW','NC',1)
DEFINE FILE 3(131,36,0,NXTR)
DEFINE FILE 2(130,42,0,NREST)

C TYPE 113
113 FORMAT(1X,'TYPE II(1 IF CHANGE X10) & STEP LENGTHS IN CM')
ACCEPT 1=II
1 FORMAT(I1)
ACCEPT 3=I1Y1,IY2
3 FORMAT(I2,1X,I2,1X,I3)
STI=IY1
ST1=STI/100.
ST2=IY2
ST2=ST2/100.

C FOOT INFORMATION
SL0=0.190
THET1=1.56
THET2=0.54
THET3=1.03
W1=0.52
W2=0.545
W3=0.34
BK=0.100
BF=0.154
SLF=0.20
SMF=1.67
SJF=0.001
R01=0.085
R02=0.151
R03=0.111

C SM(1)=3.37
SM(2)=7.85
SM(3)=49.0
SM(4)=SM(2)
SM(5)=SM(1)
SL(1)=0.46
SL(2)=0.431
SL(3)=0.827
SL(4)=SL(2)
SL(5)=SL(1)
SJ(1)=0.052
SJ(2)=0.089
SJ(3)=2.35
SJ(4)=SJ(2)
SJ(5)=SJ(1)
D(1)=0.181
D(2)=0.247
D(3)=0.280

355
C86M.FOR

C
SM(6)=0.
SL(6)=0.
SJ(6)=0.
D(6)=0.
IF(I2.EQ.0) GO TO 5
TYPE 116
FORMAT(1x,'TYPE THE LENGTH,WEIGHT FACTORS IN F4.2')
ACCEPT 117,WL,WM
117 FORMAT(F4.2,1x,F4.2)
SMF=SMF*WM
R01=R01*WL
R02=R02*WL
R03=R03*WL
IF=BF*WM
DO 118 I=1,5
SM(I)=SM(I)*WM
SL(I)=SL(I)*WL
D(I)=D(I)*WL
SJ(I)=SJ(I)*WM*WL
118 CONTINUE
CONTINUE
119 TYPE 111
111 FORMAT(1x,'TYPE M+L,J,ID,IS(FOR +/-X10)')
ACCEPT 112,M+L,J,ID,IS
112 FORMAT(5(I1,1x))
IF(IS.EQ.0) GO TO 5
IF(IS.EQ.1) AS=1.1
IF(IS.EQ.2) AS=0.9
SM(M)=SM(M)*AS
SL(L)=SL(L)*AS
SJ(J)=SJ(J)*AS
D(ID)=D(ID)*AS
SM(4)=SM(4)
SM(5)=SM(5)
SL(4)=SL(4)
SL(5)=SL(5)
SJ(4)=SJ(4)
SJ(5)=SJ(5)
D(4)=D(4)
D(5)=D(5)
GO TO 119
5 CONTINUE
DO 6 IB=1,5
F(IB)=SL(IB)-D(IB)
6 CONTINUE
C
READ(3,1) (RT(I),I=1,18)
INTERMS=RT(1)
NPOINT=RT(2)
NPHASE=RT(2)/2.0
NPHASE=NPHASE#2
IRTO=RT(3)
ILTO=RT(4)
ACCEPT 114,JS
114 FORMAT(13)
TYPE 2,IRTO,NPHASE,ILTO,NPOINT
2 FORMAT(1x,'RTD=','3I6','JTO=','3I6','END=','I6')
C
NOU READ ANGULAR DATA
DO 4 K=1,NPOINT
KP1=K+1
4
C
READ(3,KF1) (RT(J),J=1,18)
C
DATA IS IN PROPER UNIT
PHI(K,1)=RT(1)
PHI(K,2)=RT(2)
PHI(K,3)=RT(3)
DFPHI(K,1)=RT(4)
DFPHI(K,2)=RT(5)
DFPHI(K,3)=RT(6)
DFPHI(K,1)=RT(7)
DFPHI(K,2)=RT(8)
DFPHI(K,3)=RT(9)
C
IF RT(10) = TEF
PHI(K,6)=TEF
DFPHI(K,6)=-RT(11)
C
IF RT(12) = 0
XLY(N)(K)=RT(13)
YLY(N)(K)=RT(14)
DLXK(K)=RT(15)
DYLXK(K)=RT(16)
DYLXK(K)=RT(17)
DYLXK(K)=RT(18)
C
CONTINUE
C
PHI, DPHI, AND DDPHI ARE ANGLES, VELOCITIES, AND ACC, RESPECTIVELY.
C
WHICH ARE NEEDED
C
CONSTRUCT THE DATA FOR THE OTHER THIGH AND SHANK
DO 10 I=1,NPHASE
N2=I+NPHASE
PHI(N2,5)=PHI(I,1)
PHI(N2,4)=PHI(I,2)
PHI(N2,3)=PHI(I,3)
PHI(N2,2)=PHI(I,4)
PHI(N2,1)=PHI(I,5)
DFPHI(N2,5)=DFPHI(I,1)
DFPHI(N2,4)=DFPHI(I,2)
DFPHI(N2,3)=DFPHI(I,3)
DFPHI(N2,2)=DFPHI(I,4)
DFPHI(N2,1)=DFPHI(I,5)
C
XRH(I)=XLYH(N2)-ST1
XRH(N2)=XLYH(I)+ST2
YRH(I)=YLYH(N2)
YRH(N2)=YLYH(I)
IXRH(I)=DXLH(N2)
IXRH(N2)=DXLH(I)
DYRH(I)=DYLH(N2)
DYRH(N2)=DYLH(I)
DDXRH(I)=DDYLH(N2)
DDXRH(N2)=DDYLH(I)
DDYRH(N2)=DDYLH(N2)
C
CONTINUE
C
**********************************************************************
FS12=FI-PHI(I(1,6)-W2
FS13=THE2+PHI(I,6)-PI+W3
FS14=THE2+PHI(NPHASE+I,6)-PI+W3

357
C56M.20R

DO 12 I=I+NPOINT
12 IF(I.GE.IRTO) GO TO 15
   G=1
   GO TO 30
IF(I.GT.NPHASE) GO TO 19
   G=1.0
   GO TO 30
IF(I.GE.ILTO) GO TO 22
   G=(ILTO-I)
   Q=3/(ILTO-NPHASE)
   GO TO 30
   Q=0.0
   Q=Q+0.0
   D=(Q-1.0)*D
   DOX(I)=DXLH(I)+SL(2)*R(2)+(SL(1)-D(1))*R(1)
   DOX(I)=DOXH(I)+SL(2)*R(2)
   DOX(I)=DOXH(I)+DOXRH(I)/2-D(3)*R(3)
   DOX(I)=DOXH(I)+(SL(4)-D(4))*R(4)
   DOX(I)=DOXH(I)+SL(4)*R(4)+(SL(5)-D(5))*R(5)
   U6=P1-FH(I+6)/W1
   U7=P1-FH(I+7)/W1
   TYPE=37*I*6+U7
   DOX(I)=DOXH(I)+SL(2)*R(2)+SL(1)*R(1)+RO1*(-COS(U6))*6FH(I+6)*2
   +SIN(U6)*6FH(I+6))
   DOX(I)=DOXH(I)+SL(4)*R(4)+SL(5)*R(5)+RO1*(-COS(U7))*6FH(I+7)*2
   +SIN(U7)*6DHFI(I+7))
   FX3=SM(3)*DOX(3)
   FXM=FX3+5SF*(DOX(6)+DOX(7))+SL(1)*(DOX(1)+DOX(5))+
   1SM(5)*(DOX(2)+DOX(4))
   DDY(1)=DNYLH(I)-SL(2)*S(2)-(SL(1)-D(1))*S(1)
   DDY(2)=DNYLH(I)-(SL(2)-D(2))*S(2)
   DDY(3)=DNYLH(I)+(SL(4)-D(4))*S(4)
   DDY(4)=DNYLH(I)-(SL(4)-D(4))*S(4)
   DDY(5)=DNYLH(I)-SL(4)*S(4)-(SL(5)-D(5))*S(5)
   DDY(6)=DNYLH(I)-SL(2)*S(2)-(SL-1)*S(1)+RO1*(SIN(U6))*6FH(I+6)*2
   +COS(U6)*6DHFI(I+6))
   DDY(7)=DNYLH(I)-SL(4)*S(4)-(SL(5)-S(5)+RO1*(SIN(U7))*6DHFI(I+7)*2
   +COS(U7)*6DHFI(I+7))
   FORMAT(1X,13,2X,7F8.2,5X,7F8.2)
   CL2=0.
   CR2=0.
   FP2=FI-PHI(I+1)-W2
   FP3=THE2-PHI(I+1)-W3
   FP4=FI-PHI(I+2)-W2
   FP5=THE2-PHI(I+2)-W3
   IF(FP3.LE.W3) GO TO 56
   CL2=RO1*SZ(FP3)
   GO TO 57
57 CONTINUE
   IF(FP3.LE.W3) GO TO 56
   CR2=RO1*SZ(FP3)
   ND TO 59

358
C86M.FOR

58  CR2=RO2*SIN(FF4)
59  CONTINUE
60  IF(I.EQ.IRT0.AND.I.LE.MPHASE) CR2=0.
61  IF(I.EQ.ILT2) CL2=0.

41  C
42  NOW GET THE UPPER BODY COMPONENTS DIRECTLY
43  C
44  FX3=SM(3)*DDX(3)
45  FY3=SM(3)*(DDY(3)+G)
46  T3=SJ(3)*DOPHI(I,3)-FX3*D(3)*SIN(PHI(I,3))+FY3*D(3)*
47  COS(PHI(I,3))
48  TFI=(YRH(I)-YLC(I))/YRH(I)-XLC(I))
49  FI=ATAN(TFI)
50  SLB=(YRH(I)-YLC(I))*S2+(XRH(I)-XLC(I))*S2
51  SB=(SLB/2)**((FX3*XX(6)-FX3*(1.-XX(6)))SIN(FI)+
52  (FY3*XX(12)+FY3*(1.-XX(12)))*COS(FI))
53  RE(I)=FX3
54  RE(2)=FY3
55  RE(3)=T3

C

CONSTRUCT THE MATRIX FOR EQ. 9F ORDER 18

44  DO 44 II=I,6
45  DO 44 JJ=I,6
46  AA(I,J)=0.0
47  CONTINUE
48  AA(1,1)=03
49  AA(1,2)=-1.0
50  AA(2,2)=1.0
51  AA(2,3)=-1.0
52  AA(3,3)=1.0
53  AA(3,6)=-FX3
54  AA(4,4)=1.0
55  AA(4,6)=FX3
56  AA(5,4)=-1.0
57  AA(5,5)=1.0
58  AA(6,1)=0.4
59  AA(6,5)=-1.0
60  BB(1)=SMF*DDX(6)
61  BB(2)=SM(1)*DDX(1)
62  BB(3)=SM(2)*DDX(2)
63  BB(4)=SM(4)*DDX(4)+FX3
64  BB(5)=SM(5)*DDX(5)
65  BB(6)=SMF*DDX(7)
66  CALL SOLN(6,AA,BB,XX)

C

81  DO 81 II=I,6
82  DO 81 JJ=I,6
83  AA(I,J)=0.0
84  CONTINUE
85  AA(1,1)=0
86  AA(1,2)=-1.0
87  AA(2,2)=1.0
88  AA(2,3)=-1.0
89  AA(3,3)=1.0
90  AA(3,6)=-FY3
91  AA(4,4)=1.0
92  AA(4,6)=FY3
93  AA(5,4)=-1.0
94  AA(5,5)=1.0
95  AA(6,1)=1.0
96  AA(6,5)=-1.0
97  BB(1)=SMF*DDY(6)+G)
98  BB(2)=SM(1)*(DDY(1)+G)
99  BB(3)=SM(2)*(DDY(2)+G)
100  BB(4)=SM(4)*(DDY(4)+G)-FY3
C86M.FOR

BB(5)=SM(S(5)+1(DY(5)+6)
BF(5)=SM*(DDY(7)+6)
CALL SOLN(6,AA,EB+X6)
DO 82 I=1,6
XX(I+6)-X(I+6)
82 CONTINUE

C

DLX=XRH(I)-XLY(I)
DLY=YRH(I)-YLY(I)
TF=0.5*(DLX(XXX(I2)-1)*FY3+DLY(1-XX(6))*FX3)

C

BZ=(D(1)*XX(2)+E(1)*XX(3)+SIN(PHI(I+1))-(D(1)+XX(8)
I+1)*XX(9)+COS(PHI(I+1))+(D(2)+XX(3)+E(2)*XX(6)*FX3)*
2SIN(PHI(I+1)))-(D(2)+XX(3)+E(2)*XX(6)*FY3)+COS(PHI(I+1))
BZ=BZ/(D(4)+XX(4)+E(4)*(1-XX(6))*FX3)*SIN(PHI(I+1))
1-(D(4)*XX(10)+E(4)*(1-XX(12))^3)*FY3*COS(PHI(I+1))+(D(1)+XX(5)
I+1)*XX(4)+SIN(PHI(I+1))-(D(5)+XX(11)+E(5)*XX(10))
3COS(PHI(I+1))
BZ=BZ

C

SR1=SB*(DDPHI(I+6)+DDPHI(I+1)+DDPHI(I+5))
1DDPHI(I+6)=DDPHI(I+1)+DDPHI(I+5)
1DDPHI(I+6)=DDPHI(I+1)+DDPHI(I+5)
BZ=RB*(XX(2)+SIN(U6)+XX(8)+COS(U6)+XX(5)+SIN(U7)+XX(11)+
1COS(U7)-XX(1)+C3*(CL2+(1+03)*CR2)

C

SR=SB+2+SB
SR=SB+2+SB
123 FORMAT(1X,13.4X,4F11.3,7X,3Fa,1)

C

P=1.0
IF(I.GT.JS) GO TO 61
CL1=0
CR1=(SB-Q*CL1)/P
GO TO 66
61 IF(I.GE.IRTD) GO TO 62
CR1=0
IF(I.GT.NPHASE) GO TO 63
CL1=0
CR1=0
GO TO 66
62 IF(I.GT.NPHASE) GO TO 63
63 IF(I.GT.(NPHASE+JS)) GO TO 64
64 IF(I.GE.ILTO) GO TO 65
65 CL1=0
CR1=0
GO TO 66
66 CONTINUE

C

TYPE 39*I=CL1
FORMAT(1X,13.3F10.3,"*** ','+2F10.3+4X,2F10.3)

C

RE(4)=CL1
RC(5)=CR1
RE(6)=CL2
RE(7)=CR2
TT1=XXX(7)+CL1+63*XX(I/4+CL2+
1D0(I)*XX(2)+SIN(PHI(I)+W1)-XX(8)+COS(PHI(I)-W1))-SJF*
2DDPHI(I+6)
TT2=TT1+(D(I)+XXX(2)+SL(I)-D(I)+XXX(3)+SIN(PHI(I+1))
I-(D(1)+XX(8)+SL(I)-D(1)+XXX(9)))*COS(PHI(I+1))-

360
C86M.FOR

C

500    CONTINUE

C

14    WRITE(2,'(I)') (RE(MG),MG=1,21)

12    CONTINUE

DO 49 I=1,21

49    CONTINUE

END
APPENDIX E

COMPUTER PROGRAM FOR THREE-MASS DYNAMIC MODEL

The routines used for this model work with 25-word format data files. KL.FOR reads several data files and after processing, normalizes them and calculates their average. It then writes the results into the disk. Routine K22M.FOR reads the results from the disk and displays joint forces, torques, and the location of the center of pressure on the screen of the display system. K23M.FOR does the same job except that it provides a hard copy of the plots associated with those quantities.
C PROGRAM TO NORMALIZE & GET THE AVERAGE OF SEVERAL DATA SETS.

DIMENSION KRD(25), FRD(20), THETA(90), THEIR(20), THEIF(20)

LH0(90), LH4(90), LH9(40), LH10(90), H(90), H(90), H(90), H(90), H(90)

LTHETA(90), KRD

TYPE 211

211 FORMAT(1X, 'TYPE VER. & HOR. SCALE FACTORS')

216 FORMAT(F4.2, 1X, F4.2)

SCALE=400.

2 FORMAT (1X, 2F2, 31)

NPOINT=90

CONTINUE

TYPE 220

220 FORMAT (1X, 'TYPE # OF HARMONICS INCLUDED')

ACCEPT 220+ TERMS

220 FORMAT (I1)

WFR=0.70

3 FORMAT (1X)

C

DO 6 I=1, 101
DO 217 K=1, 118

SM(I, K)=0.0

217 CONTINUE

C

TYPE 300

300 FORMAT (1X, 'TYPE NO. OF DATA SETS')

ACCEPT 300+NSETS

4 FORMAT (I1)

NSETS=SSETS

C

DO 10 I=1, NSETS

CALL ASIGMO(2, 'DK', XXXX, OLD, 'NC', J1)

DEFINE FILE 210, 12, LDNS(10)

IF (LDNS(10)) THEN 10

CONTINUE

1 FORMAT (1X, 'TIME(S)')

CALL SM(I, J1)

C

DO 10 I=1, 108

READ (2, K) KRD

IF (KRD > 6) GO TO 98

IF (FRD> 20) GO TO 7

C

20 CONTINUE

27 CONTINUE

C

22 ME=0

MP=0.

DO 21 J1=1, 4

INM=ABS(INM-512)

CONTINUE

TM=INM/4.

INM=ABS(INM-512).

21 continue

344 # 344

363
IF TB.GE.10.0 OR KL.EQ.40 OR KRD4.GT.600 GO TO 24
IFLG=1
GO TO 7

24 TOE=TOEFF+1.
GO TO 20

7 IF(KF.EQ.1) GO TO 19
COR=KRD(18)
LFR=KRD(18)
CORF=KRD(18)
KH=TOEFF+23.
KE=KRD(18) OR LE.GT.21 GO TO 19
HS=FLOAT(K)
IH=K
KF=1

19 CONTINUE
GO 50 J=10,21

50 FFK(J)=FLOAT(KRD(J))
CONTINUE

FRD(21)=FRD(21)*SFR
FRD(4)=FRD(4)*SFR
FRD(6)=FRD(6)*SFR
FRD(8)=FRD(8)*SFR
FRD(10)=FRD(10)*SFR
FRD(12)=FRD(12)*SFR

C DO 331 IE=1,8
K0(18)=KRD(18)-KHO(18)

331 CONTINUE

C TYPE 3,X

C LY=0.22 M., LX=0.132 M., THICKNESS OF FE=0.037 M.
C 512 UNITS=2.5 V, 1 V=400 NEWTONS
C C1=2.5V/512(UNIT)#400(NEWTONS)/1V
C C1=1.553125
C RY=K0(1)+K0(2)+K0(3)+K0(4)
C XMM=0.22(1.0+K0(1)/K0(1)+K0(2)+K0(3)+K0(4)
C IF (ABS(FR(K,K)) GT .01) GO TO 91
A(K,K)=(-0.037*FR(K,K)+XMM)/FR(K,K)
GO TO 92

91 A(K,K)=0.
92 CONTINUE

C DU=FRD(5)-10.
CALL ANGLE(FR(3),FR(3),FRD(3),FRD(3),FRD(3),FRD(3),
1FRD(12),DU,55D6,THETA(K))
DU=FRD(9)-10.
CALL ANGLE(FR(K(3),FRD(8),FRD(9),FRD(9),FRD(9),FRD(9),
1FRD(8),THETA(K))
DU=FRD(11)-10.
CALL ANGLE(FR(9),FRD(10),FRD(10),FRD(10),FRD(10),FRD(10),
1FRD(12),DU,FRD(12),THETA(K))

10 CONTINUE
C IEND=POINT
C TDOE=TOEFF
C 1T=TOEFF
C TNS=HS
C IF(HS.GT.20.) PC=TOEFF*100./HS
C IPC=PC

C 34444$
C 34444$
K1.FOR

BTO=BT0+FC
SEA=101./HS
IHS=HS
NPOINT=IHS

C
DO 402 K=1,N
TYPE I=K,K
IF(K.NE.1) GO TO 404
CALL FOUR(THETF,NTERMS,IHS,FT,DFT,DDFT)
K=1
K2=2

404 IF(K.NE.2) GO TO 406
CALL FOUR(THEIA,NTERMS,IHS,FI,DFT,DDFT)
K=4
K2=5
K3=6

406 IF(K.NE.3) GO TO 408
CALL FOUR(THEK,NTERMS,IHS,FT,DFT,DDFT)
K=7
K2=8
K3=9

408 IF(K.NE.4) GO TO 410

DO 51 I=1,NPOINT
XH(I)=XH(I)-XH(I)
YH(I)=YH(I)-YH(I)
CONTINUE

51 XG=XH(I)
YC=YH(I)
DO 53 I=1,NPOINT
XH(I)=XH(I)-XG
YH(I)=YH(I)-YC
CONTINUE
PN=NPOINT-1
XH(I+PN)=XH(I+PN)
DO 79 I=1,NPOINT
AI=I-1
XH(I)=XH(I)-SKX*AI
CONTINUE
CALL FOUR(XH,NTERMS,NPOINT,FT,DFT,DDFT)
C TAKE THE AVE. OF

C VX=0.
DO 77 I=1,NPOINT
VX=VX+FT(I)
CONTINUE
POINT=POINT+1
VX=VX/POINT
DO 78 I=1,NPOINT
AI=I-1
FT(I)=SKX*AI*FT(I)-VX
DFT(I)=SKX*DFT(I)
CONTINUE
C
TYPE 110
FORMAT(1X,'TYPE DISG COEFF')
AC=111,FAF
111 FORMAT(F6.4)
FAF=FT(I)
DO 77 I=1,NPOINT
FT(I)=FT(I)-FAF
DFT(I)=DFT(I)-FAF
CONTINUE

K=10
K2=11

365
410 IF(K.NE.5) GO TO 412
      FAF=4.1/368
      CALL FOUR(YH,TERMS,NPOINT,F1,DFT,DDFT)
      FAF=FT(1)
      DO 75 I=1,NPOINT
           FT(I)=FT(I)-FAF
           DFT(I)=DFT(I)+FAF
      75      CONTINUE
      K1=13
      K2=34
      K3=15

412 IF(K.NE.6) GO TO 414
      DO 55 I=1,NPOINT
           FT(I)=FT(I)+DFT(I)
           DDF(I)=DDF(I)/N
      55      CONTINUE
      K1=14
      K2=12
      K3=18

414 CONTINUE

50 320 L=1+101
   AL=1
   BL=AL*HS/101
   LB1=BL1
   LB2=LB1+1
   IF(LB1.NE.0) GO TO 310
   SM(L,K1)=FT(I)
   SM(L,K2)=DFT(I)
      GO TO 320
310 SM(L,K1)=FT(LB1)+(FT(LB2)-FT(LB1))*(BL1-FB1)
   SM(L,K2)=DFT(LB1)+(DFT(LB2)-DFT(LB1))*(BL1-FB1)
   SM(L,K3)=DDFT(LB1)+(DDFT(LB2)-DDFT(LB1))*(BL1-FB1)
   GO TO 320
320 CONTINUE
402 CONTINUE

C ATO=ITOE+ATO
   ASTI=ASTI+THS
   TYPE...NP*ITOE,THS
   FORMAT(1X,13.2,F9.2,/,1X)
60 30 IF(NP.NE.NSETS) GO TO 399
   ATO=ATO/SETS
   ASTI=ASTI/SETS
   TYPE...NP*SETS
   399 CALL CLOSE(2)
   NO 397 J=1,101
   NO 327 M=1,18
   ZM(J+M)=ZM(J,M)+SM(J,M)
   IF(NP.NE.NSETS) ZM(J+M)=ZM(J,M)/SETS
   392 CONTINUE
400 CONTINUE
50 33 FORMAT(1X,14X,*** THE AVERAGE VALUES ***,1X)
C .
   TYPE...NP*SETS
   396 FORMAT(1X,TYPE NAME OF THE AVE..1LL **** THE AVERAGE VALUES ****,1X)
   CALL ASSIGN(3,'DKAVE.DAT','12',NEW,'NC',1)
   DEFINE FILE 3(102+36+U.NAVR)
   ATO=ATO+101/ASTI
   AN(2)=TERMS
```
TYPE 33
TYPE 62, NSETS*ATO*ASTI*AM(1)
FORMAL (1X, 15, JFF, 2)
DO 54 I=1, 18
AM(I)=0.0
54 CONTINUE
WRITE(3, 'AM(I), I=1,18')
C: AM HAS ONE SET OF ALL QUANTITIES
DO 60 I=1, 101
DO 61 K=1, 18
AM(K)=2M(I, K)
61 CONTINUE
IP1=IP1+1
WRITE(3, 'AM(II), II=1,18')
60 CONTINUE
GO TO 99
98 TYPE 99
FORMAT (1X, 'DATA SET DOES NOT HAVE ENOUGH FRAMES')
99 STOP
END
```
PROGRAM TO USE 3-MASS MODEL (FORCE PLATE METHOD) TO CALCULATE
FORCES & TORQUE. IT DISPLAYS FORCES & TORQUES.

COMMON/FILE/INWC(3000)
DIMENSION RT(18), FMH(3), DFMH(3), DDFMH(3)
2X12(3), S(3)
CALL ASSIGN(3, 'DK:FILEA,DAL=13, 'OLD X 'NC', 1)
DEFINE FILE 3 (102:16, U=MTRX)

C

TYPE 65

A5 FORMAT('1X,' 'TYPE W & H FACTORS')
ACCEPT 66-AW, AH

66 FORMAT('F4.0,JX,F4.1')
AW=AW#4,44982/718.
AH=AH#2,54/175.
WEI=AW#2,54/175.
ACCEPT JX, JI
FORMAT(13)
SHF=AW#1,67
SHS=AH#3,37
SHM=AW#6,65
SLF=AH#0,14
SLS=AH#4,46
SLT=AH#4,41
SJE=0,001
SJS=0,052
SJL=0,68
DFH=AH#(C,14,-038)
DS=AH#(A6,-181)
DT=AH#(A6,-186)
D=9,81
F1=3,1416

C

READ(3,'1') (RT(I),J=1:18)
INTERM=RT(2)
NPOINT=100
NPHASE=50
JLJO=RT(1)
INTO=ITLO=50

TYPE 2 INTO, NPHASE, ITLO, NPOINT
FORMAT('1X,' 'RT0=,' 'I3,' '16,' '3X,' 'LTD=,' 'I3,' '3X,' 'END=,' 'I6')
C
NEW.READ THE DATA
READ(3,'2') (RT(J), J=1:18)
Y1=RT(18)
READ(3,'101') (RTJ,J=1:18)
YN=RT(18)
SLF=ABS(YN-Y1)
DFH=AH#SLF#0,102/0,14
EF=SLF-DF

C

CONTINUE

TYPE 2

FORMAT('1X,' 'TYPE 1 FOR FX, 2 FOR FY, 3 FOR TORQUE. 4 FOR AY,
AND 6 TO 90 CF RESPECTIVELY')
ACCEPT I1P
ACCEPT I1 NPHASE
IF(NPHASE.EQ.0, NPHASE=50)
IF(1F.EQ.1) GO TO 99
CALL INIT(3000)
IF(1P,NE.1) GO TO 20
CALL SCL0(0,0.-150,100,150.)
CALL APNT(0,0.-150.)
CALL VECC0(0,300.)
CALL APNT0(0,0.)
CALL VECC(100,0,0.)
CALL APNT(115,145,115,145,1)
CALL TEXT('HORIZONTAL FORCES')
CALL APNT1(1,116,145,115,1)
CALL TEXT1(115,115,115,115,1)

\*
CALL APNT(1,-146,.,-4)
CALL TEXT(’-150 N’)
DO 35 L=1,11
A(L)=30.*L
A(L)+0(L)=180.
35 CONTINUE
MF=11
20 IF(IF_.NE.2) GO TO 21
CALL SCAL(0.,-200.,100.,1000.)
CALL APNT(0.,-200.,)
CALL VECT(0.,1200.,)
CALL APNT(0.,0.,)
CALL VECT(100.,0.,)
CALL APNT(45.,920.,-4)
CALL TEXT(’VERTICAL FORCES’)
CALL APNT(1.,795.,-4)
CALL TEXT(’1000 N’)
C CALL APNT(0.,-200.,-4)
C CALL VECT(100.,0.,0.,3)
C CALL APNT(1.,-195.,-4,4,1)
CALL TEXT(’-200 N’)
DO 36 L=1,13
A(L)=100.*L-300.
36 CONTINUE
MF=15
21 IF(IF_.NE.3) GO TO 22
CALL SCAL(0.,-200.,100.,-200.,)
CALL APNT(0.,-200.,)
CALL VECT(0.,400.,)
CALL APNT(0.,0.,)
CALL VECT(100.,0.,)
CALL APNT(45.,195.,-4)
CALL TEXT(’JOINT TORQUES’)
CALL APNT(1.,196.,-4)
CALL TEXT(’200 NM’)
CALL APNT(1.,-196.,-4)
CALL TEXT(’-200 NM’)
DO 38 L=1,11
A(L)=40.*L-240.
38 CONTINUE
MF=11
22 IF(IF_.NE.4) GO TO 23
CALL SCAL(0.,0.,100.,0.,30)
CALL APNT(0.,0.,)
CALL VECT(0.,0.,3)
CALL APNT(0.,0.,)
CALL VECT(100.,0.,)
CALL APNT(45.,0.29,.,-4)
CALL TEXT(’CENTER OF PRESSURE’)
CALL APNT(1.,0.29,.-4)
CALL TEXT(’-30 CM’)
DO 39 L=1,7
A(L)=0.05*(L-1)
39 CONTINUE
MF=7
23 CONTINUE
TYPE=110
110 FORMAT(1X,’TYPE HOR & VER COEFFS’)
ACCEPT II, HF, VFE
111 FORMAT(F4.2,1X,E4.2)
DO 4 KA=1,101
II=1.0*(KA-1)
IF=II*L
READ(3,KF1) (RT(J),J=1,18)
RX=HF*RT(17)
RY=UF*RT(16)
C DATA IS IN PROPER UNIT
PH(1)=RT(1)
K22M.FOR

```
PH(2)=RT(4)
PH(3)=RT(7)
DPH(1)=RT(2)
DPH(2)=RT(5)
DPH(3)=RT(8)
DDPH(1)=RT(3)
DDPH(2)=RT(6)
DDPH(3)=RT(9)
C
PH, DPH AND DDPH ARE ANGLES, VELOCITIES AND ACC, RESPECTI()
C
C WHICH ARE NEEDED
C
GET THE L. HIP COORDS.
DDXY=RT(15)
DDYH=RT(10)-Y1
C
203 DO 203 K=1,1
       (X(K)=COS(PH(K))*DPH(K)**2+SIN(PH(K))*DDPH(K))
       S(K)=-SIN(PH(K))*DPH(K)**2+COS(PH(K))*DDPH(K)
   203 CONTINUE
C
       DDXF=DDXY+SLT(R(1)+SLS*R(2)/(SLF-DF)*R(1))
       DDXS=DDXY+SLT(R(3)+(SLS-DS)*R(2))
       DDTX=DDXT=SLT(DT)*R(3)
       DDYF=DDYH+SLT(R(3)-SLS*R(2))-(SLF-DF)*S(1)
       DDYS=DDYH+SLT(R(3)-SLS-DS)*S(2)
       DDTY=DDTY=SLT(DT)*S(3)
C
37 FORMAT(1X,13.2X,10F10.3)
C
       FX=RX-SMF*DDXF
       FAY=RY-SMF*(G+DDYF)
       TA=SLF*(DPH(1))+(AY-EF)*(-FX*SN(PH(1)))*RY
       TH=SLF-DF)(&-FAX*SN(PH(1)))*FAX*CS(PH(1))
       FX=FAX-SMF*DDXS
       FKY=FAY-SMF*(G+DDYS)
       TK=SLF*(DPH(3))+TAXS*(FAX*CS(PH(2))-FAX*
       15IN(PH(2)))+(SLS-DS)**(FAX*CS(PH(3))-FAX*SN(PH(2)))
C
       FHX=FKY-SMF*DDXH
       FHY=FKY-SMF*(G+DDYT)
       TH=SLF*(DPH(3)+TAYT*(FHX*CS(PH(3))-FHX*SN(PH(3))
       1)*(SLS-DS)*FHX*CS(PH(3))-FHX*SN(PH(3)))
C
IF(II,NE.1) GO TO 5
   TYPE 37,KA,A,AY,RX,FAX,FKY,RY,FAY,FKY,TA,TK,TH
C
5 CONTINUE
C
IF(IF,NE.1) GO TO 10
   CALL SCAL(0.0,-150.0,100.0,150.0)
   CALL AENT(JX,FX)
   CALL APNT(T1,FAX)
   CALL APNT(T1,FKY)
   CALL APNT(T1,FHX)
   IF(KA,NE,NPHASE) GO TO 10
   CALL APNT(T1,FX)
   CALL TEXT(" RX")
   CALL TEXT(" A")
   CALL APNT(T1,FKX)
   CALL TEXT(" H")
   CALL APNT(T1,FHX)
   CALL TEXT(" H")
C
10 IF(IF,NE.2) GO TO 11
   CALL APNT(T1,RX)
   CALL APNT(T1,FAX)
   CALL APNT(T1,FKY)
   CALL APNT(T1,FHX)
   IF(KA,NE,NPHASE) GO TO 11
```

34A*34L*
CALL APNT(T1*RY)
CALL APNT(T1*EAY)
CALL APNT(T1*EAY)
CALL APNT(T1*FHY)
CALL APNT(T1*FHY)
CALL APNT(T1*RT)
CALL APNT(T1*RT)
CALL APNT(T1*RT)
IF(KA,NE.3) GO TO 12
CALL APNT(T1*TA)
CALL APNT(T1*TA)
CALL APNT(T1*TH)
CALL APNT(T1*TH)
IF(KA,NE.3) GO TO 12
CALL APNT(T1*TH)
CALL APNT(T1*TH)
CALL APNT(T1*TH)
12 IF(IP,NE.4) GO TO 4
IF(KA,NE.10),AY=0.
CALL APNT(T1*AY)
CONTINUE
DO 15, L=10,100,10
AL=L
CALL APNT(AL*0.4+B1)
15 CONTINUE
CALL APNT(VS,1,-4*4-1)
CALL APNT(1,-1*4-4)
CALL APNT(0,0*4-4)
DO 16 L=1,MP
CALL APNT(0,0*4+L*8+1)
16 CONTINUE
GO TO 7
99 STOP
END
C PROGRAM TO USE 3-MASS MODEL (FORCE PLATE METHOD) TO CALCULATE
FORCES & TORQUE. IT PLOTS FORCES & TORQUES ON THE PAPER.
DIMENSION RI(10), PH(2), DFH(3), DDPH(3)
REAL XJ(12), Z(101:13)+R(3)+S(3)
CALL ASSIGN(3, 'DK: FILEA, DAI', -13, 'OLD', 'NC', 1)
DEFINE FILE .3102X36, U, NXXTRI
C
ACCEPT 1, I1

1 FORMAT (11)
   SMF=1.67
   SNS=1.37
   SMTP=2.65
   SLF=0.14
   SLS=4.6
   SLT=4.11
   SJS=0.01
   SJ=0.52
   DJ=0.08
   DF=0.14-.038
   NS=.46-.18!
   DI=41-.168
   G=9.81
   P=3.1416

2 READ(3,'1') (RT(I), I=1,10)
   ITERM=RT(2)
   NEPRINT=100
   NP_PHASE=50
   ILTO=RT(1)
   IRTO=RLTO=50
   TYPE 2+RTO+NP_PHASE+ILTO+NPOINT
   FORMAT(1X,'RTO',13, 16.3X,'LTO=', 13, 3X, 'END:', 16)
   C
   NOW READ THE DATA
   READ(3,'2') (RT(J), J=1,10)
   Y1=RT(I)
   READ(3,'3L4F11') (RT(J), J=1,10)
   YN=RT(1)
   X=ABS(YN-Y1)
   DF=SLF-DF
   TYPE 37+ILTO+Y1+YN+SLF+DF+EF
   C
   FORMAT(1X,'TYPE 1 FOR FX, 2 FOR FY, 3 FOR TORQUE, 4 FOR M,,
   1 AND 6 TO STOP, RESPECTIVELY')
   C
   TYPE 2I
   C
   FORMAT(1X,'TYPE H & V COEFF')
   ACCEPT 72+CH, CV
   C
   FORMAT(4, 2X, 1X, 4, 2)
   DO 4 # KAI=1, 101
   4 IF(KAI=KAI)
      READ(4,'1P1') (RT(J), J=1,10)
      Rx=RT(17)*CH
      Ry=RT(16)*CV
   C
   DATA 15 IN PROPER UNIT
   PH(1)=RT(1)
   PH(2)=RT(1)
   PH(3)=RT(7)
   PH(4)=RT(2)
   PH(5)=RT(5)
   DFH(3)=RT(8)
   DDPH(1)=RT(3)
   DDPH(2)=RT(6)
   DDPH(3)=RT(9)
   C
   PH, DFH, AND DDPH ARE ANGLES, VELOCITIES, AND ACCELERATIONS,
   RESPECTIVELY,
   C
   WHICH ARE NEEDED
   C
   GET THE H, V COORDS.
   ECH=RT(12)
   ERY=RT(15)
   C
   344434.4
AT=ABS(RI(1B)-Y1)

C DO 203 K=1,3
D(0K)=COS(FH(K))#DPH(K)#2+SIN(EH(K))#DNEH(K)
S(K)=SIN(FH(K))#DPH(K)#2+COS(EH(K))#DUPH(K)

203 CONTINUE

C DDXF=DDXY+SLR*(3)+SLS#R(2)+(SLF-DF)#R(1)
DDXY=DDX+SLR*(3)+(SLS#DS)#R(2)
DDXT=DDXY#(SLT-AT)#R(3)
DYYF=DYYH-SLT*S(3)-SLS*S(2)-(SLF-DF)#S(1)
DYS=DDYH-SLT*S(3)-(SLS#DS)#S(2)
DYYT=DDYH#(SLT-AT)#S(3)

C FORMAT(1X,13,2X,10F10.3)

C FAX=RX-SMF#DDXF
FAY=RY-SMF#DDYF
TA=SJF#DDPF(1)+AY-EE#RX#SIN(FH(1))#RY;
1+SLF-DF#(-FAX#SIN(FH(1)))+FAY*COS(FH(1)).

C FKY=FAY-SMF*(G+DDYS)
TK=SJF#DDPF(2)+TA#DS*(FAY*COS(FH(2))-FAX#
1+SIN(FH(2)))+(SLS#DS)*(EY*COS(FH(2)))+FXX#SIN(FH(2)).

C FXY=FXK-SMT#DDXM
FXY=FXK-SMT*(G+DDY)
TH=SJT#DDPF(3)+TK#DT*(FXY*COS(FH(3))-FXX#SIN(FH(3))
1+(SLT-AT)*(FMT#COS(FH(3))-FXX#SIN(FH(3)))).

C IF(II,NE.1) GO TO 5
TYPE 37,FA#RFX,FX#RY#FAY,FL,TA,TH,TH
CONTINUE
Z(KA=1)=RX
Z(KA=2)=FAX
Z(KA=3)=FXK
Z(KA=4)=FX
Z(KA=5)=RY
Z(KA=6)=FAY
Z(KA=7)=FXK
Z(KA=8)=FL
Z(KA=9)=TA
Z(KA=10)=TH
Z(KA=11)=TH
Z(KA=12)=0.
IF(KA.6,0.90#(KA=1)

7 CONTINUE

C DO 7 J=1,5
J1=2*J-1
JX(J1)=1

C CONTINUE

50 TYPE 8
ACCEPT 1:IP
IF(IP.EQ.6) GO TO 99
IF(IP.NE.1) GO TO 10
JX(2)=1
JX(4)=2
JX(6)=3
JX(8)=4
NC=4

10 IF(IP.NE.2) GO TO 11
JX(2)=5
JX(4)=6
JX(6)=7
JX(8)=8

34#34L*
NC=4
11 IF(IP.NE.3) GO TO 12
   JX(2)=9
   JX(4)=10
   JX(6)=11
   NC=3
12 IF(IP.NE.4) GO TO 13
   JX(2)=12
   NC=1
13 CALL UPLLOT(Z,JX,101,101,NC,0.,0.,0.,0.,0.)
   TYPE 1A.
14 FORMAT(1X,////1X)
   GO TO 50
99 STOP
   END
APPENDIX F

PLOTTING ROUTINES

Program QIM.FOR reads the results stored in the file RESULT.DAT and plots various quantities on the paper. It also reads the force plate data and plots the corresponding forces simultaneously. The associated mean square values are also calculated. Subroutine VPLOT.FOR is used to plot various quantities.
QIM.FOR

0  THIS PROGRAM PLOTS ALL THE RESULTS ON THE PAPER
1  IT ALSO PLOTS THE FF MEASUREMENTS.
2  DIMENSION Ri(150,24),KRD(21),JXY(21),A(21)
3  IND(2),AL(150,4),AL2(150,2)
4  CONTINUE
5  CALL ASSIGN(2,'PK:RESULT.DAT','13','OLB','NC','1')
6  DEFINE FILE 2(150:42,1:1,NRESXY)
7  DO 6 I=1,150
8  R(i,j)=1-1
9  DO 6 J=2,24
10  R(i,j)=0.0
11  CONTINUE
12  TYPE 40
13  FORMAT('IX','TYPE 1 IF FF DATA NEEDED 0 O.W.')
14  ACCEPT B,UF
15  TYPE 41
16  FORMAT('IX','TYPE IN # OF CURVES TO BE PLOTTED')
17  ACCEPT B,NCR
18  DO 42 I=1,NCR
19  FORMAT('7'
20  ACCEPT B,L
21  CONTINUE
22  IND(I)=L
23  CONTINUE
24  IF(L.EQ.0) GO TO 100
25  JXY(1)=I
26  IF(L.NE.1) GO TO 21
27  JXY(2)=I
28  21  IF(L.EQ.2) JXY(2)=3
29  IF(L.EQ.3) JXY(2)=4
30  IF(L.EQ.4) JXY(2)=5
31  IF(L.EQ.5) JXY(2)=6
32  IF(L.EQ.6) JXY(2)=7
33  IF(L.EQ.7) JXY(2)=8
34  IF(L.EQ.8) JXY(2)=9
35  IF(L.EQ.9) JXY(2)=10
36  IF(L.EQ.10) JXY(2)=11
37  JXY(2)=L+1
38  JXY(3)=1
39  IF(L.EQ.0) JXY(4)=24
40  100  JXY=JXY+150
41  READ(211,(A(I,J),J=1,21)) IF(A(2).EQ.-6.0.AND.A(11).EQ.-6.0) GO TO 20
42  20  J=1
43  IF(I.EQ.J) RT(I-J)=A(J)
44  14  CONTINUE
45  CONTINUE
46  NPOINT=I-1
47  CALL CLOSE(2)
48  IF(NP.NE.1) GO TO 60
49  60  CALL ASSIGN(3,'XXXXL.DAT','12','OLB','NC','1')
50  DEFINE FILE 3(150:21,1:1,OLXX)
51  DO 50 I=1,150
52  READ(211,(KRD(J,J),J=1,21))
53  AL(I+1)=KRD(1)
54  AL(I+2)=KRD(2)
55  AL(I+3)=KRD(3)
56  AL(I+4)=KRD(4)
57  AL(I+5)=KRD(5)
58  AL(I+6)=KRD(6)
59  50  CONTINUE
60  CALL CLOSE(3)
61  TYPE 86
62  FORMAT('IX','TYPE HF,UF')
63  86  CONTINUE
64  34A34L*
ACCEPT 02, HF, VF  

FORMAT(F4.2,1X,F4.2)  
DO 51 I=1,NPOINT  
E1=0.  
DO 52 J=1,N  
E1=E1+(AL1(I,J)-AL1(I+100,J))  
CONTINUE  
E1=E1/4.  
RT(I+2)=E1*2.5*400./(512.*VF)  
E2=AL2(I+1)-AL2(I+100)+AL2(I+2)-AL2(I+2)  
52  
E2=E2/2.  
RT(I+24)=E2*2.5*400./(512.*HF)  
CONTINUE  
POINTS=NPOINT  
XMSE=0.  
YMSE=0.  
DO 53 I=1,NPOINT  
XMSE=XMSE+(RT(I,5)-RT(I+24))**2  
YMSE=YMSE+(RT(I,7)-RT(I+23))**2  
51  
CONTINUE  
XMSE=XMSE/POINTS  
YMSE=YMSE/POINTS  
TYPE 54,XMSE,YMSE  
FORMAT('1x','MSE(FX)=','E10.3','2x','MSE(FY)=','E10.3')  
54  
CALL UPLOILRT(JX1,NPOINT,150,H2C,0,0,0,0,0,0)  
60  
GO TO 5  
STOP  
END  

377
REFERENCES


379


80. Golliday, C. L., Jr., Toward Development of Biped Locomotion Controls: Planar Motion Control of the Kneelless Biped Standing and Walking Gaits, Ph.D. dissertation, The Ohio State University, Columbus, OH 43210, June, 1975.


