A GENERALIZED ARCHITECTURE FOR THE
FREQUENCY-SELECTIVE DIGITAL PREDISTORTION
LINEARIZATION TECHNIQUE

Dissertation

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of The Ohio State University

By

Jiwoo Kim, M.S., B.S.

Graduate Program in Electrical and Computer Engineering

The Ohio State University

2012

Dissertation Committee:

Prof. Patrick Roblin, Advisor
  Prof. Furrukh Khan
  Prof. Joanne DeGroat
Abstract

The Linearization of power amplifiers is a research area of growing importance. Among the many approaches which have been tried, digital predistortion (DPD) remain, one of the most promising techniques. In this thesis, a new generalized frequency-selective DPD technique is investigated. The proposed linearization algorithm is signal independent and any band limited signal can be linearized. 3rd and 5th order linearizations are demonstrated using various multi-tone signals. For 3rd order linearization, more than 15 dB cancellation for the inband distortion, 14 dB cancellation on 3rd order interband intermodulation distortion (IMD) were achieved. For the 5th order linearization, more than 15 dB IMD cancellation on inband, 16 dB cancellation on the 3rd order interband IMD, and 6 dB on 5th order interband IMD cancellation were achieved. To demonstrate the applicability of the algorithm to multiple bands, the two-band theory was modified to a three-band theory with up to 3rd order compensation. In the three-band case, the interband linearization played an important role in the overall performance. Without the interband linearization, only 3 – 4 dB IMD cancellation was observed. However, with interband compensation, more than 10 dB IMD cancellation was achieved. For the three-band case, to investigate the robustness of the DPD system, the middle channel was turned off and the same coefficients worked well for two bands.
In recent DPD applications, the linearization of largely spaced two-band signals have generated a lot of interest. In this work, the same algorithm was applied to 250 MHz spaced signal and more than 15 dB IMD cancellation was achieved. For the signal separation, a digital IF technique was proposed which uses only a single local oscillator (LO) for synthesizing the band separation. Previous works used two different LOs for the signal generation using expensive commercial synthesizers. In this work, a low cost testbed consisting of a field programmable gate arrays (FPGA) and two commercial upconverters is used.

For higher peak to average power ratio (PAPR) signal, a two-band crest factor reduction (CFR) block is proposed. MATLAB simulations and an subsequent FPGA implementation yield an excellent agreement on the performance with a 20 dB adjacent channel power ratio (ACPR) improvement achieved with 2.5 dB PAPR reduction.
This is dedicated to my family.
Acknowledgments

I would like to truly thank my advisor, Professor Patrick Roblin, for all of his time, effort, advice, and encouragement with my research and entire studies. Without his support, I would not be able to complete this work. His insights and guidance have made this work to be done successfully.

Together with my advisor, Professor Furrukh Khan, and Professor Joanne De-Groat guided me through the dissertation phase of this study by forming the committee. I would like to express thank to them for the dedication to many hours of their time to review this dissertation.

I also would like to express my gratitude to my colleagues in our Non-linear RF Lab. In particular, Dr. Seokjoo Doo, Dr. Inwon Suh, Mr. Shashank Mutha, Dr. Xi Yang, Mr. Young-Seo Ko, Mr. Haedong Jang, Dr. Christophe Quindroit, and Mr. Naveen Naraharisetti. I thanks to all for sharing good times which make a small home in U.S. away from home in Korea.

Many thanks go to Dr. Kwangjoo Kwak who helped me a lot like an older brother and Ms. Sehye Kim who spent fun time with me in Seoul as well as Columbus.

I have to say thank to Mr. Hyunchae Kim at Solid for discussing with me regarding technical issues which I was not familiar with at the very early stage in the study, Mr. Sung-Jin Kim at Broadcom for helping me in professional as well as private life.
My special thanks also go to my two sisters and their families who have encouraged me all the time.

Finally, I would like to take the opportunity to thank my parents who sacrificed so much for me to be here. With their love, belief, and encouragement through my entire life, I have been able to overcome many hardships and make this accomplishment.
Vita

April 16, 1975 ......................... Born - Kyunggi, Republic of Korea

1998 ................................. B. S. Electronic Engineering,
Sungkyunkwan University,
Suwon, Republic of Korea

2000 ................................. M. S. Electronic and Electrical Engineering,
Pohang University of Science and Technology (POSTECH),
Pohang, Republic of Korea

2000 - 2006 .............................. Engineer,
Solid Technologies, Inc.,
Seoul, Republic of Korea

2006 - present ......................... Graduate Student,
Electrical and Computer Engineering,
The Ohio State University

Jan. 2011 - Aug. 2011 ................. Intern,
Broadcom Corp.,
Irvine, CA

Publications

Journal Publications


Fields of Study

Major Field: Electrical and Computer Engineering

Studies in Microwave and Circuit Systems: Prof. Patrick Roblin
Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Dedication</td>
<td>iv</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>v</td>
</tr>
<tr>
<td>Vita</td>
<td>vii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>xii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>xiii</td>
</tr>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1.1 Motivation</td>
<td>1</td>
</tr>
<tr>
<td>1.1.2 Review of the field</td>
<td>5</td>
</tr>
<tr>
<td>1.2 Overview of thesis</td>
<td>6</td>
</tr>
<tr>
<td>1.3 Contributions</td>
<td>7</td>
</tr>
<tr>
<td>2. Theory of Frequency Selective Digital Predistortion</td>
<td>8</td>
</tr>
<tr>
<td>2.1 Higher Order Intermodulation Terms Generation</td>
<td>9</td>
</tr>
<tr>
<td>2.2 Frequency-Selective Digital Predistortion Linearization: Memoryless Case</td>
<td>10</td>
</tr>
<tr>
<td>2.3 Frequency-Selective Digital Predistortion Linearization: Memory Case</td>
<td>21</td>
</tr>
<tr>
<td>2.4 Theory Extension to Three-Band Case</td>
<td>28</td>
</tr>
<tr>
<td>2.5 Conclusion</td>
<td>30</td>
</tr>
</tbody>
</table>
B. Proof of the higher order IMD polynomial equivalence .............. 88

Bibliography ................................................................. 92


List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Resource usage of the implemented WiMax signal generator and CFR</td>
<td>40</td>
</tr>
<tr>
<td>4.1 Signal order for using the same 3rd order interband generation block.</td>
<td>49</td>
</tr>
<tr>
<td>4.2 Signal order for using the same 5th order interband generation block.</td>
<td>50</td>
</tr>
<tr>
<td>5.1 Band pass filter specification for largely spaced DPD</td>
<td>77</td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Evolution of wireless standards with spectral efficiencies [1].</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Simplified concept of a digital predistortion linearization technique [1].</td>
<td>4</td>
</tr>
<tr>
<td>2.1 Proposed two-band frequency-selective digital predistortion architecture</td>
<td>13</td>
</tr>
<tr>
<td>2.2 Simulation of the proposed algorithm using two bands WiMax signals.</td>
<td>22</td>
</tr>
<tr>
<td>2.3 AM/AM and AM/PM plot of before and after the proposed DPD algorithm.</td>
<td>23</td>
</tr>
<tr>
<td>2.4 Constellation plot for lower sideband.</td>
<td>24</td>
</tr>
<tr>
<td>2.5 Constellation plot for upper sideband.</td>
<td>25</td>
</tr>
<tr>
<td>2.6 3 Bands with associated linearization coefficients.</td>
<td>31</td>
</tr>
<tr>
<td>3.1 Two-band CFR block.</td>
<td>33</td>
</tr>
<tr>
<td>3.2 Polar clipping diagram</td>
<td>34</td>
</tr>
<tr>
<td>3.3 Digital-IF setup</td>
<td>35</td>
</tr>
<tr>
<td>3.4 Block level design of the proposed CFR algorithm</td>
<td>36</td>
</tr>
<tr>
<td>3.5 MATLAB simulation results of hard and soft clipping in spectrum domain</td>
<td>37</td>
</tr>
<tr>
<td>3.6 VSA capture of the implemented hard and soft clipping.</td>
<td>37</td>
</tr>
</tbody>
</table>
3.7 CCDF simulation result of the original, hard, and soft clipping. About 8.7 dB PAPR at 0.01 % CCDF is obtained. 

3.8 VSA CCDF capture of the implemented soft clipping with Gaussian reference. About 8.75 dB PAPR at 0.01 % CCDF is obtained.

4.1 A simplified non-real time testbed setup [27]

4.2 Top block of the proposed frequency-selective digital predistortion linearization

4.3 Envelope Generator Block

4.4 In-band correction block

4.5 Block diagram of the higher order generator.

4.6 Block diagram of the 3rd order IMD generator.

4.7 Block diagram of the 5th order IMD generator.

4.8 Block diagram of the interband correction block.

4.9 Screen capture of functional simulation result.

4.10 A spectrum capture of the output of the 3rd order IMD generator with 4 MHz fundamental single tone.

4.11 A spectrum capture of the output of the 5th order IMD generator with 4 MHz fundamental single tone.

4.12 CCDF plot of 16-, 96- and 128-tone multisine signal.

4.13 Testbed setup for the proposed frequency-selective predistortion

4.14 Original spectrum of 16-tone two bands signal with up to 3rd order IMDs

4.15 Inband LSB linearization of up to 3rd order IMD

4.16 Inband USB linearization of up to 3rd order IMD
<table>
<thead>
<tr>
<th>Section Number</th>
<th>Description of Content</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.17</td>
<td>3rd order interband linearization</td>
<td>61</td>
</tr>
<tr>
<td>4.18</td>
<td>+3rd order interband linearization</td>
<td>62</td>
</tr>
<tr>
<td>4.19</td>
<td>Performance of the proposed frequency-selective DPD algorithm of up to 3rd order IMD input</td>
<td>63</td>
</tr>
<tr>
<td>4.20</td>
<td>Original spectrum of 16-tone two bands signal with up to 5th order IMDs</td>
<td>64</td>
</tr>
<tr>
<td>4.21</td>
<td>Inband LSB linearization of up to 5th order IMD input</td>
<td>64</td>
</tr>
<tr>
<td>4.22</td>
<td>Inband USB linearization of up to 5th order IMD input</td>
<td>65</td>
</tr>
<tr>
<td>4.23</td>
<td>-3rd order interband linearization of up to 5th order IMD input</td>
<td>65</td>
</tr>
<tr>
<td>4.24</td>
<td>+3rd order interband linearization of up to 5th order IMD input</td>
<td>66</td>
</tr>
<tr>
<td>4.25</td>
<td>-5th order interband linearization of up to 5th order IMD input</td>
<td>66</td>
</tr>
<tr>
<td>4.26</td>
<td>+5th order interband linearization of up to 5th order IMD input</td>
<td>67</td>
</tr>
<tr>
<td>4.27</td>
<td>Performance of the proposed frequency-selective DPD algorithm of up to 5th order IMD input</td>
<td>68</td>
</tr>
<tr>
<td>4.28</td>
<td>CCDF of the test signal with 96-tone per three bands</td>
<td>69</td>
</tr>
<tr>
<td>4.29</td>
<td>Original spectrum of the 3-band signal</td>
<td>69</td>
</tr>
<tr>
<td>4.30</td>
<td>3-band linearization without interband linearization</td>
<td>70</td>
</tr>
<tr>
<td>4.31</td>
<td>3-band linearization output with interband linearization</td>
<td>71</td>
</tr>
<tr>
<td>4.32</td>
<td>Performance comparison with and without interband linearization</td>
<td>72</td>
</tr>
<tr>
<td>4.33</td>
<td>Mid-channel off case after all 3-band linearization</td>
<td>73</td>
</tr>
<tr>
<td>4.34</td>
<td>Performance comparison between before and after turning off the mid-channel off after all 3-band linearization</td>
<td>74</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>5.1 Largely spaced digital predistortion architecture using digital IF</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>5.2 Photo of the testbed</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>5.3 CCDF plot for the proposed largely spaced DPD using digital IF</td>
<td>79</td>
<td></td>
</tr>
<tr>
<td>5.4 LSB (upper plot) and USB (lower plot) linearization performance of the proposed largely spaced DPD using digital IF</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>5.5 Both bands linearization performance of the proposed largely spaced DPD using digital IF</td>
<td>81</td>
<td></td>
</tr>
</tbody>
</table>
ACRONYMS

ACPR: Adjacent Channel Power Ratio
ADC: Analog to Digital Converter
AM: Amplitude Modulation
BPF: Band Pass Filter
CCDF: Complementary Cumulative Distribution Function
CDMA2000: Code Division Multiple Access 2000
CFR: Crest Factor Reduction
DAC: Digital to Analog Converter
DAC EVM: DAC5682z EVAluation Module
DC: Direct Current
DPD: Digital Pre-Distortion DSP: Digital Signal Processing
DUT: Device Under Test
EER: Envelope Elimination and Restoration
ESG: Electronic Signal Generator
EVM: Error Vector Magnitude
FPGA: Field Programmable Gate Array
GPIB: General Purpose Interface Bus
GUI: Graphic User Interface
IF: Intermediate Frequency
IMD: InterModulation Distortion
LINC: Linear amplification with Nonlinear Components
LO: Local Oscillator
LPF: Low Pass Filter
LSB: Lower Side-Band
LSNA: Large Signal Network Analyzer
LTE: Long Term Evolution
LUT: Look-Up Table
LVDS: Low Voltage Differential Signaling
MSB: Middle Side-Band
OFDM: Orthogonal Frequency Division Modulation
PA: Power Amplifier
PAE: Power Added Efficiency
PAPR: Peak to Average Power Ratio
PC: Personal Computer
PD: Pre-Distortion
PM: Phase Modulation
PSG: Performance Signal Generator
QAM: Quadrature Amplitude Modulation
RF: Radio Frequency
TI EVM: DAC5682z EVM from Texas Instruments
USB: Upper Side-Band
VSA: Vector Signal Analyzer
WCDMA: Wideband Code Division Multiple Access
WiMAX: Worldwide Interoperability for Microwave Access
Chapter 1: INTRODUCTION

1.1 Introduction

1.1.1 Motivation

Recent portable devices have wireless Internet access capability and users can explore Internet services in almost everywhere. As the number of mobile device users increases, an higher data rate is thus required. The way of raising the data rate is by increasing the bandwidth or the power. Since the RF spectrum is a limited resource, the spectral efficiency was increased in more advanced standards, i.e., CDMA2000, W-CDMA, WiMAX, and LTE. Fig. 1.1 shows the evolution of wireless standards with spectral efficiencies.

PA used in communication devices and systems, i.e., cell phone and base station. Indeed the PA is one of the most nonlinear components in communication system. In base station, linearity is an essential characteristic to avoid the degradation of the symbol error rate. By using highly linear amplifiers, the communication link has a higher quality for the transmitted signal which leads to a lower error rate. As a result, the wireless carrier can design its network with a more economical cell planning.

In base station, the operating class of the power amplifier is either A or AB to remain in the linear response region, and to avoid a strong nonlinear behavior of the
Rapid growth in both subscribers and digital content is stressing wireless infrastructures as more data traffic demand is placed on a limited amount of the wireless spectrum. Meeting this demand has escalated procurement costs for base station systems and their operating expenses, which are driven by high energy consumption. Base station power amplifiers (PAs) – the devices that drive wireless signals outward from a base station – can account for as much as 30 percent of a base station’s cost.

Implementing crest factor reduction (CFR) and digital pre-distortion (DPD) techniques before wireless signals reach the base station PA can improve the quality and coverage of the base station’s signal while reducing the system’s procurement and operating costs.

**Optimizing Performance and Efficiency of PAs in Wireless Base Stations:**

Digital pre-distortion reduces signal distortion at high power levels.

### Figure 1: Evolution of wireless standards with spectral efficiencies [1].

Power amplifier. Theoretical maximum PAEs of class A and AB are 50% and 78.5%, respectively [2]. In consequence, significant amount of supply power is dissipated and not used for actual amplification. In the service provider perspective, using a larger output power amplifier is beneficial because they pay much for installing and maintaining wireless base stations. Therefore, larger wireless cell overage of a base station reduces the number of stations and this means single base station is able to support a larger number of wireless subscribers resulting in a smaller operating expense for the provider.

Over more than two decades huge research and development have been done in designing higher power amplifiers [3]. Designing a highly linear power amplifier is challenging as well as expensive. For those reasons, modern cell planning incorporates
a sector concept. In conventional cell planning, one base station provides a certain radius of area coverage using one power amplifier with a single dipole antenna. In the cell planning with sectorization, the original cell is divided into multiple sectors and different amplifiers and directional antennas are used for each of individual sector. This technique makes possible to support an increased number of subscribers without increasing the number of base station sites. However, under the paradigm shift from voice to data centric communication, the base station sectorization is not a sufficient solution and there still is a demand for higher power amplifier with higher linearity for the new standards.

There are many types of linearization techniques [4], i.e., feedforward [5] [9], EER [10] [11], LINC [6], predistortion [7] [8]. In feedforward linearization, the input signal is going into the main amplifier. The main amplifier generates the original signal with gain and IMD as well. The output of the main amplifier is divided into two paths. One path is going to a delay line and the other path is going to an error amplifier with proper attenuation, i.e., the inverse of the main amplifier’s gain, with opposite phase. At the end those two paths are combined together and the IMDs from the two different paths cancel each other because they have opposite phases. Feedforward technique shows the best performance yet they use two amplifiers and this makes for a higher system cost. In EER, the input signal is divided into PM and AM signals. The PM signal is going into a switching mode power amplifier for which the supply power is controlled by the AM signal. In this technique, phase matching between the two paths is critically important [4]. Also restoring the envelope efficiently is a challenging issue. In LINC, two constant envelopes with phase modulated signals can be synthesized from any non-constant envelope signal. Then the two separated signals
go into two different PAs and the output of the PAs are summed together. However, precise signal separation at RF frequencies is very challenging. In predistortion, the signal is pre-distorted before injecting it at the input of a nonlinear power amplifier so that the nonlinear response of the power amplifier could be compensated and the output of the amplifier yields a linear response. Fig. 1.2 shows a conceptual diagram of a digital predistortion linearization. The original idea was developed using analog techniques which does not have many degrees of freedom for changing the response of the nonlinear power amplifier. Recent advance in DSP technology is enabling this linearization technique.
Modern wireless systems use complex modulation schemes i.e., 64 QAM for W-CDMA, and OFDM for WiMax and LTE, to achieve higher spectral efficiency, and it generally leads to higher PAPR of the signal.

To handle with a signal with higher PAPR, a conventional way of using PAs is backing off the signal power such that the signal power at the PA input is small enough so that the PA cannot be saturated even for the peak of the signal. However, the higher power back off, the lower the PAE of the PA as well. Since the PA is the most expensive component in a base station, it leads to a higher system cost. The other way is to limit the higher signal power via PAPR reduction or crest factor reduction [12]. In the signal processing point of view, limiting a signal generates spectral regrowth in the frequency domain and increases the symbol error rate. Therefore a careful manipulation on the original signal is required.

1.1.2 Review of the field

In this thesis, we concentrate on the linearization and crest factor reduction techniques, especially for the frequency-selective digital predistortion linearization. Digital predistortion linearization is well known in the community and many previous works have been reported so far [13] [14] [15]. In [20], the first frequency-selective digital predistortion technique was introduced. In this paper, the differential memory effect had been characterized using a LSNA. Differential memory effect is a memory effect dependent on frequency tone/band spacing exhibiting a response and behavior different for each sideband. Therefore each band needs to be taken care of independently, especially for signals with widely separated bands. However, it was hard to reach the optimum performance because the inband and the interband were not fully
orthogonal. In [21], an orthogonal block was introduced to make the inband and the interband orthogonal. The previous achievements in [20] [21] were limited up to 3rd order IMD compensation. However, for high power applications, a high order IMD compensation is needed. Also a more compact and generalized mathematical expression is needed for the higher order IMD analysis.

Although there have been since many emerging research reports on digital pre-distortion for widely spaced two-band signals [30] [31] [32], most of those works had been done using two different LOs which makes for a more expensive system implementation. Also in [30] [31], they included memory effects but the work was limited to 2 bands and the demonstration was done for a limited band separation using two vector signal generators.

In all the previous work, 2-band linearization was proposed, analyzed, and tested as an example of multiband linearization. In the 2-band linearization case, only the inband linearization is needed. However, for the linearization of a larger number of bands, i.e., 3 bands or larger, interband linearization plays an important role and no prior work addresses this issue.

1.2 Overview of thesis

The organization of this thesis is the following:

Chapter 2 presents the frequency-selective digital predistortion theory. Beginning with memoryless two bands case, the memory case of frequency-selective theory is presented. Also a three-band theory is presented.
Chapter 3 presents a two-band CFR technique. In this chapter, a block level design, MATLAB simulation, testbed setup, and measurement results are presented.

Chapter 4 presents the hardware implementation of the frequency-selective algorithm. Detail block level design, testbed setup, and measurement results are presented.

Chapter 5 presents the digital predistortion of largely spaced multiband signals.

Chapter 6 presents the conclusion of this thesis and a discussion on future works.

1.3 Contributions

This work generalizes the two-band frequency-selective digital predistortion algorithm and demonstrates up to 5th order linearization using a low cost FPGA testbed [16]. The two-band theory can be extended to a higher number of bands case and in this work, we demonstrate three bands linearization. In three bands linearization, interband linearization plays a key role and no paper or report has dealt with that. Also the linearization of 250 MHz spaced signals using digital IF technique in an FPGA implementation was demonstrated based on the frequency-selective algorithm [17].
Chapter 2: THEORY OF FREQUENCY SELECTIVE DIGITAL PREDISTORTION

The linearization of RF transmitters is one of the most important and challenging issues in modern wireless communication systems since higher linearity is necessary to deliver signals with low error probability. Linearization is also a critical requirement for increasing the capacity of base-stations enabling a more economic cell planning.

Memory polynomials have been successfully introduced [18] to compensate for memory effects and several subsequent works including [19] adopted them to develop powerful linearization algorithms. However it is very challenging to apply them to linearize the amplification of two communication bands separated by more than 80 MHz. In the other hand, a frequency-selective digital predistortion linearization technique has been proposed and demonstrated for the addressing the large differential memory effects associated with arbitrary band spacing [20]. One major advantage of the frequency-selective linearization approach is that if an amplifier is to amplify two widely separated band (say for example 500 MHz) with 10 MHz bandwidth each, each band can be upconverted by different modulators before being combined and amplified, thus reducing drastically the bandwidth requirement on the predistorter. Yet the frequency-selective predistortion of each band accounts for PA distortion induced by the combined bands. Further the channel bandwidth requirement on the receiver
for the feedback path of the adaptation is similarly reduced since each band can be linearized independently while accounting for the other. Given the receiver ADC bandwidth capabilities are even more constrained than those of DACs in transmitters, this is another important advantage of the frequency-selective scheme.

In [21], an incremental progress was reported, in which an orthogonal condition was added to the algorithm. With this feature the inband and interband linearization could be successfully tuned independently. However, this architecture was limited to 3rd order did not fully utilized the advantage of the reduced feedback bandwidth requirement and did not incorporate the time-selective dimension of memory polynomials [22].

This chapter presents a theory of the frequency-selective digital predistortion algorithm which extends the original frequency-selective algorithm up to 5th order. Also the architecture can be modified using memory polynomials to address memory effects in each of the individual bands.

2.1 Higher Order Intermodulation Terms Generation

Consider an RF amplifier excited by a modulated RF signal:

\[ x_{in} = I(t) \cos(\omega t) - Q(t) \sin(\omega t) \]

The baseband signal can be separated in terms of its LSB and USB signal:

\[ I = I_L + I_U \quad \text{and} \quad Q = Q_L + Q_U \]

Let us assume the two tones are amplitude and phase modulated such that we can write:

\[ I_L(t) = E_L(t) \cos \left[ \frac{1}{2} \Delta \omega t + \phi_L(t) \right] \]
\[ Q_L(t) = -E_L(t) \sin \left[ \frac{1}{2} \Delta \omega t + \phi_L(t) \right] \]
\[ I_U(t) = E_U(t) \cos \left[ \frac{1}{2} \Delta \omega t + \phi_U(t) \right] \]
\[ Q_U(t) = E_U(t) \sin \left[ \frac{1}{2} \Delta \omega t + \phi_U(t) \right] \]

(2.1)

The intermodulation terms generated will be of the form:

\[ I_p = E_L^{(p-1)/2} E_U^{(p+1)/2} \cos \left[ \Delta \omega_p t + \frac{p+1}{2} \phi_U - \frac{p-1}{2} \phi_L \right] \]
\[ Q_p = E_L^{(p-1)/2} E_U^{(p+1)/2} \sin \left[ \Delta \omega_p t + \frac{p+1}{2} \phi_U - \frac{p-1}{2} \phi_L \right] \]
\[ I_{-p} = E_U^{(p-1)/2} E_L^{(p+1)/2} \cos \left[ \Delta \omega_{-p} t + \frac{p+1}{2} \phi_L - \frac{p-1}{2} \phi_U \right] \]
\[ Q_{-p} = E_U^{(p-1)/2} E_L^{(p+1)/2} \sin \left[ \Delta \omega_{-p} t + \frac{p+1}{2} \phi_L - \frac{p-1}{2} \phi_U \right] \]

(2.2)

with \( \Delta \omega_{\pm p} = \pm \left( \frac{p}{2} \right) \Delta \omega \).

It is quite unpractical (costly and less inaccurate) in practice in a DSP implementation to calculate the instantaneous phases \( \phi_U \) and \( \phi_L \). An alternative approach is to calculate these intermodulation terms directly from \((I_L, Q_L)\) and \((I_U, Q_U)\). See Appendix A for an example of such expansions for the \( I_k \) and \( Q_k \) functions.

2.2 Frequency-Selective Digital Predistortion Linearization: Memoryless Case

Let us consider two separate frequency bands represented each by a pair of \( I \) and \( Q \) baseband signals: \( I_L \) and \( Q_L = \hat{I}_L \) for the lower band and \( I_U \) and \( Q_U = \hat{Q}_L \) for the upper band. The hat notation is used to signify an Hilbert transform. Assuming the bands are located at frequency \( f_1 \) and \( f_2 \) respectively, the 3rd order intermodulation introduces spurious bands at \( 2f_1 - f_2 \) and \( 2f_2 - f_1 \). The 5th order intermodulation introduces additional spurious bands at \( 3f_1 - 2f_2 \) and \( 3f_2 - 2f_1 \).
In [20] [21] an orthogonal two-band frequency-selective linearization scheme was introduced and demonstrated. In that algorithm the linearization of an amplifier with 3rd order nonlinearities relied on six complex coefficients to compensate for the inband and interband distortion independently.

Let us first consider the generation of the fundamental 3rd and 5th order IMDs. Given the \((I_L, Q_L)\) and \((I_U, Q_U)\) lower and upper baseband signals, we shall call \((I_{-3}, Q_{-3})\) and \((I_3, Q_3)\) the associated lower and upper baseband signals introduced by the 3rd order intermodulation at the spurious bands \(2f_1 - f_2\) and \(2f_2 - f_1\) respectively. Similarly we shall call \((I_{-5}, Q_{-5})\) and \((I_5, Q_5)\) the associated lower and upper baseband signals introduced by the 5th order intermodulation at the spurious bands \(3f_1 - 2f_2\) and \(3f_2 - 2f_1\) respectively.

For the 3rd and 5th order these IMD terms are readily found to be given by:

\[
I_{-5} = I_L^3 I_U^2 - I_L^3 Q_U^2 + 6I_L^2 Q_L I_U Q_U - 3I_L Q_L^2 I_U^2 + 3I_L Q_L^2 Q_U^2 - 2Q_L^3 I_U^2 Q_U,
\]

\[
Q_{-5} = Q_L^3 Q_U^2 - Q_L^3 I_U^2 + 6Q_L^2 Q_L I_U Q_U - 3Q_L I_U^2 Q_U^2 + 3Q_L I_U^2 Q_U^2 - 2I_L^3 Q_U I_U,
\]

\[
I_{-3} = (I_L^2 - Q_L^2) I_U + 2I_L Q_U Q_L,
\]

\[
Q_{-3} = -(I_L^2 - Q_L^2) Q_U + 2I_U I_L Q_L,
\]

\[
I_3 = (I_U^2 - Q_U^2) I_L + 2I_U Q_U Q_L,
\]

\[
Q_3 = -(I_U^2 - Q_U^2) Q_L + 2I_U I_L Q_U,
\]

\[
I_5 = I_U^3 I_L^2 - I_U^3 Q_L^2 + 6I_U^2 Q_U I_L Q_L - 3I_U Q_U^2 I_L^2 + 3I_U Q_U^2 Q_L^2 - 2Q_U^3 I_L Q_L,
\]

\[
Q_5 = Q_U^3 Q_L^2 - Q_U^3 I_L^2 + 6Q_U^2 Q_U Q_L I_L - 3Q_U I_U^2 Q_L^2.
\]
beside \( I_{-1} = I_L, Q_{-1} = Q_L, I_1 = I_U \) and \( Q_1 = Q_U \).

Note that the equations reported above recast the 3rd order frequency-selective theory in a more compact way than previously reported in [20] while adding the targeted new 5th order corrections. For example, 3rd order IMDs are given by,

\[
I_L(3) = (I_L^2 - Q_L^2)I_U + 2I_LQ_UQ_L,
\]

\[
Q_L(3) = -(I_L^2 - Q_L^2)Q_U + 2I_UI_LQ_L,
\]

\[
I_U(3) = (I_U^2 - Q_U^2)I_L + 2I_UQ_UQ_L,
\]

\[
Q_U(3) = -(I_U^2 - Q_U^2)Q_L + 2I_UI_LQ_U,
\]

where \( I_L(3), Q_L(3), I_U(3), \) and \( Q_U(3) \) are equivalent to \( E^2I + \tilde{E}^2Q, \tilde{E}^2I - E^2Q, E^2I - \tilde{E}^2Q, \) and \( \tilde{E}^2I + E^2Q \) terms in the previous work, respectively. Therefore, for \( p = \pm 3 \) case, the equations are equivalent to the topology in the previous work. The equivalence has been demonstrated in the Appendix B.

To take full advantage of the differential memory capability of the frequency-selective linearization architecture, the different inband and interband components \((I_p, Q_p)\) must be amplitude rescaled and phase shifted independently using an IQ modulator. The input and output relationship of the IQ modulator for a \( p \)-th order component is of the form:

\[
\begin{bmatrix}
I'_p \\
Q'_p
\end{bmatrix} = \begin{bmatrix}
\alpha_p(E_U^2, E_L^2) & -\beta_p(E_U^2, E_L^2) \\
\beta_p(E_U^2, E_L^2) & \alpha_p(E_U^2, E_L^2)
\end{bmatrix}
\begin{bmatrix}
I_p \\
Q_p
\end{bmatrix},
\]

where \( I_p, Q_p, I'_p, Q'_p \) are the inputs and outputs of the IQ modulator, respectively, and \( \alpha_p \) and \( \beta_p \) are the complex coefficients used by the modulator. \( E_U^2 \) and \( E_L^2 \) are the envelope of lower and upper side band signals which are given by \( E_L^2 = I_{-1}^2 + Q_{-1}^2 \)
Figure 2.1: Proposed two-band frequency-selective digital predistortion architecture

and $E_U^2 = I_1^2 + Q_1^2$, respectively. The same matrix equation holds also for the inband components $p = \pm 1$ beside the interband components $|p| > 1$. Note that additional envelopes $E_p^2 = I_p^2 + Q_p^2$ may be introduced for intermodulation linearization as the intermodulation corrections injected in the PA effectively introduces new bands beside the original input bands.

The functional dependence of the $\alpha_p$ and $\beta_p$ on the envelopes can be expressed using a Taylor expansion. For the case of a two envelope dependence $E_U^2$ and $E_L^2$ and a 5th order expansion this gives:

$$\alpha_p \simeq \alpha_{p,0,0} + \alpha_{p,0,1}E_U^2 + \alpha_{p,1,0}E_L^2 + \alpha_{p,0,2}E_U^4 + \alpha_{p,2,0}E_L^4 + \alpha_{p,1,1}E_U^2E_L^2, \quad (2.3)$$

$$\beta_p \simeq \beta_{p,0,0} + \beta_{p,0,1}E_U^2 + \beta_{p,1,0}E_L^2 + \beta_{p,0,2}E_U^4 + \beta_{p,2,0}E_L^4 + \beta_{p,1,1}E_U^2E_L^2. \quad (2.4)$$

The final output, $I_{out}$ and $Q_{out}$ are then given by:

$$I_{out}(t) = \sum_{p=-P}^{P} P \sum_{p \text{ is odd}} I_p'(t), \quad \text{and} \quad Q_{out}(t) = \sum_{p=-P}^{P} P \sum_{p \text{ is odd}} Q_p'(t)$$
The resulting proposed algorithm structure is shown in Fig. 2.1. Note that it is also beneficial while pursuing intermodulation linearization to perform the inband linearization first before performing the interband linearization [20].

To investigate the algorithm in mathematically, let us assume that the output of a power amplifier excited by the above two band signal can be given by the following quasi-memoryless Volterra expansion:

$$b_{out}^{QML}(t) = \sum_{p(ODD)=-P}^P [I_{out}(t) \cos(\omega_0 t) - Q_{out}(t) \sin(\omega_0 t)] \quad (2.5)$$

In a matrix notation we can write the output as:

$$V_{out}(t) = G[V_{in}(t)] \cdot X_{PA}$$

where we define

$$V_{out}(t) = \begin{bmatrix} I_{U,out}(t) \\ Q_{U,out}(t) \\ I_{L,out}(t) \\ Q_{L,out}(t) \end{bmatrix}, \quad V_{in} = \begin{bmatrix} I_U(t) \\ Q_U(t) \\ I_L(t) \\ Q_L(t) \end{bmatrix} \quad \text{and} \quad X_{PA} = \begin{bmatrix} \alpha_{PA,U} \\ \beta_{PA,U} \\ \alpha_{PA,L} \\ \beta_{PA,L} \end{bmatrix},$$

with

$$\alpha_{PA,U} = \begin{bmatrix} \alpha_1 \\ \alpha_3 \\ \vdots \\ \alpha_P \end{bmatrix}, \quad \beta_{PA,U} = \begin{bmatrix} \beta_1 \\ \beta_3 \\ \vdots \\ \beta_P \end{bmatrix}, \quad \alpha_{PA,L} = \begin{bmatrix} \alpha_{-1} \\ \alpha_{-3} \\ \vdots \\ \alpha_{-P} \end{bmatrix} \quad \text{and} \quad \beta_{PA,L} = \begin{bmatrix} \beta_{-1} \\ \beta_{-3} \\ \vdots \\ \beta_{-P} \end{bmatrix}.$$
The PA is now to be linearized by predistortion using a function of the same form as the PA model:

\[ V_{PD}(t) = G[\alpha V_{in}(t)] \cdot X_{PD} \] (2.12)

\[ V_{out}(t) = G[V_{PD}(t)] \cdot X_{PA} \] (2.13)

with \( X_{PD} \) the predistortion coefficients and with \( \alpha \leq 1 \) an attenuation factor. Let us assume that the maximum output power (envelope \( I_{out}^2 + Q_{out}^2 \)) occurs for the maximum input power (envelope \( I^2 + Q^2 \)) (PA with monotonous gain and with weak enough memory effects). Then if we have scaled the input signal such that the maximum input power occurs at the P1dB compression point, we need to select the attenuation \( \alpha \leq 10^{-1\text{dB}/20} = 0.8913 \) to allow for the linearized PA to have a linear gain up to P1dB.

Let us introduce the perturbation \( \delta V \) such that \( V_{PD} \) can be written:

\[ V_{PD} = \alpha V_{in} + \delta V \]

with the small perturbation \( \delta V \) defined as:

\[ \delta V = \begin{bmatrix} \delta I_U(t) \\ \delta Q_U(t) \\ \delta I_L(t) \\ \delta Q_L(t) \end{bmatrix}. \]
For any function \( f(x) \), we can write \( f(x_0 + \delta x) \approx f(x_0) + f'(x_0)\delta x \) by the Taylor series expansion. Therefore, for \( I_U[\alpha V_{in}] \) vector, \( I_U[\alpha V_{in} + \delta V] \) is given by,

\[
I_U[\alpha V_{in} + \delta V] = \left[ I_1(\alpha V_{in} + \delta V) \ I_3(\alpha V_{in} + \delta V) \ \cdots \ I_P(\alpha V_{in} + \delta V) \right] \\
\simeq I_U[\alpha V_{in}] + \left[ J_{I_1}(\alpha \delta V_{in})\delta V \ J_{I_3}(\alpha \delta V_{in})\delta V \ \cdots \ J_{I_P}(\alpha \delta V_{in})\delta V \right],
\]

where \( J_{I_P}(\alpha \delta V_{in}) \) is the Jacobian of the vector \( I_P \) at \( \alpha \delta V_{in} \).

Similarly, we can derive \( Q_U[\alpha V_{in} + \delta V] \), \( I_L[\alpha V_{in} + \delta V] \), and \( Q_L[\alpha V_{in} + \delta V] \) as followings:

\[
Q_U[\alpha V_{in} + \delta V] \approx Q_U[\alpha V_{in}] + \left[ J_{Q_1}(\alpha \delta V_{in})\delta V \ J_{Q_3}(\alpha \delta V_{in})\delta V \ \cdots \ J_{Q_P}(\alpha \delta V_{in})\delta V \right], \\
I_L[\alpha V_{in} + \delta V] \approx I_L[\alpha V_{in}] + \left[ J_{I_{-1}}(\alpha \delta V_{in})\delta V \ J_{I_{-3}}(\alpha \delta V_{in})\delta V \ \cdots \ J_{I_{-P}}(\alpha \delta V_{in})\delta V \right], \\
Q_L[\alpha V_{in} + \delta V] \approx Q_L[\alpha V_{in}] + \left[ J_{Q_{-1}}(\alpha \delta V_{in})\delta V \ J_{Q_{-3}}(\alpha \delta V_{in})\delta V \ \cdots \ J_{Q_{-P}}(\alpha \delta V_{in})\delta V \right].
\]

Since for any \( k(1 \leq k \leq P) \), \( J_{I_k}(\alpha V_{in})\delta V = \frac{\partial I_k}{\partial I_U} \delta I_U + \frac{\partial I_k}{\partial Q_U} \delta Q_U + \frac{\partial I_k}{\partial I_L} \delta I_L + \frac{\partial I_k}{\partial Q_L} \delta Q_L \)

and \( J_{Q_k}(\alpha V_{in})\delta V = \frac{\partial Q_k}{\partial I_U} \delta I_U + \frac{\partial Q_k}{\partial Q_U} \delta Q_U + \frac{\partial Q_k}{\partial I_L} \delta I_L + \frac{\partial Q_k}{\partial Q_L} \delta Q_L \), we can rewrite the matrix \( G_U \) and \( G_L \) as followings:

\[
G_{U/L}[\alpha V_{in} + \delta V] \approx G_{U/L}[\alpha V_{in}] + \left[ \frac{\partial I_{+1}}{\partial I_U} \frac{\partial I_{+3}}{\partial I_U} \ \cdots \ \frac{\partial I_{+P}}{\partial I_U} \ \\
\frac{\partial I_{+1}}{\partial Q_U} \frac{\partial I_{+3}}{\partial Q_U} \ \cdots \ \frac{\partial I_{+P}}{\partial Q_U} \ \\
\frac{\partial I_{+1}}{\partial I_L} \frac{\partial I_{+3}}{\partial I_L} \ \cdots \ \frac{\partial I_{+P}}{\partial I_L} \ \\
\frac{\partial I_{+1}}{\partial Q_L} \frac{\partial I_{+3}}{\partial Q_L} \ \cdots \ \frac{\partial I_{+P}}{\partial Q_L} \ \\
\frac{\partial I_{-1}}{\partial I_U} \frac{\partial I_{-3}}{\partial I_U} \ \cdots \ \frac{\partial I_{-P}}{\partial I_U} \ \\
\frac{\partial I_{-1}}{\partial Q_U} \frac{\partial I_{-3}}{\partial Q_U} \ \cdots \ \frac{\partial I_{-P}}{\partial Q_U} \ \\
\frac{\partial I_{-1}}{\partial I_L} \frac{\partial I_{-3}}{\partial I_L} \ \cdots \ \frac{\partial I_{-P}}{\partial I_L} \ \\
\frac{\partial I_{-1}}{\partial Q_L} \frac{\partial I_{-3}}{\partial Q_L} \ \cdots \ \frac{\partial I_{-P}}{\partial Q_L} \ \\
\frac{\partial Q_{+1}}{\partial I_U} \frac{\partial Q_{+3}}{\partial I_U} \ \cdots \ \frac{\partial Q_{+P}}{\partial I_U} \ \\
\frac{\partial Q_{+1}}{\partial Q_U} \frac{\partial Q_{+3}}{\partial Q_U} \ \cdots \ \frac{\partial Q_{+P}}{\partial Q_U} \ \\
\frac{\partial Q_{-1}}{\partial I_U} \frac{\partial Q_{-3}}{\partial I_U} \ \cdots \ \frac{\partial Q_{-P}}{\partial I_U} \ \\
\frac{\partial Q_{-1}}{\partial Q_U} \frac{\partial Q_{-3}}{\partial Q_U} \ \cdots \ \frac{\partial Q_{-P}}{\partial Q_U} \ \\
\frac{\partial Q_{+1}}{\partial I_L} \frac{\partial Q_{+3}}{\partial I_L} \ \cdots \ \frac{\partial Q_{+P}}{\partial I_L} \ \\
\frac{\partial Q_{+1}}{\partial Q_L} \frac{\partial Q_{+3}}{\partial Q_L} \ \cdots \ \frac{\partial Q_{+P}}{\partial Q_L} \ \\
\frac{\partial Q_{-1}}{\partial I_L} \frac{\partial Q_{-3}}{\partial I_L} \ \cdots \ \frac{\partial Q_{-P}}{\partial I_L} \ \\
\frac{\partial Q_{-1}}{\partial Q_L} \frac{\partial Q_{-3}}{\partial Q_L} \ \cdots \ \frac{\partial Q_{-P}}{\partial Q_L} \ \\
\delta I_U \ \\
\delta Q_U \ \\
\delta I_L \ \\
\delta Q_L \right].
\]
where every derivatives are evaluated at $\alpha V_{in}$. Therefore,

$$
\begin{align*}
\frac{\partial G}{\partial I_U} & \triangleq \begin{bmatrix}
\frac{\partial I_1}{\partial I_U} & \cdots & \frac{\partial I_P}{\partial I_U} & -\frac{\partial Q_1}{\partial I_U} & \cdots & -\frac{\partial Q_P}{\partial I_U} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial I_{L-1}}{\partial I_U} & \cdots & \frac{\partial I_{L-P}}{\partial I_U} & -\frac{\partial Q_{L-1}}{\partial I_U} & \cdots & -\frac{\partial Q_{L-P}}{\partial I_U} \\
0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial Q_1}{\partial I_U} & \cdots & \frac{\partial Q_P}{\partial I_U} & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial I_{L-1}}{\partial I_U} & \cdots & \frac{\partial I_{L-P}}{\partial I_U} \\
0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial Q_1}{\partial I_U} & \cdots & \frac{\partial Q_P}{\partial I_U} & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial I_{L-1}}{\partial I_U} & \cdots & \frac{\partial I_{L-P}}{\partial I_U} \\
\end{bmatrix},
\end{align*}
$$

$$
\begin{align*}
\frac{\partial G}{\partial Q_U} & \triangleq \begin{bmatrix}
\frac{\partial I_1}{\partial Q_U} & \cdots & \frac{\partial I_P}{\partial Q_U} & -\frac{\partial Q_1}{\partial Q_U} & \cdots & -\frac{\partial Q_P}{\partial Q_U} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial I_{L-1}}{\partial Q_U} & \cdots & \frac{\partial I_{L-P}}{\partial Q_U} & -\frac{\partial Q_{L-1}}{\partial Q_U} & \cdots & -\frac{\partial Q_{L-P}}{\partial Q_U} \\
0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial Q_1}{\partial Q_U} & \cdots & \frac{\partial Q_P}{\partial Q_U} & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial I_{L-1}}{\partial Q_U} & \cdots & \frac{\partial I_{L-P}}{\partial Q_U} \\
0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial Q_1}{\partial Q_U} & \cdots & \frac{\partial Q_P}{\partial Q_U} & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial I_{L-1}}{\partial Q_U} & \cdots & \frac{\partial I_{L-P}}{\partial Q_U} \\
\end{bmatrix},
\end{align*}
$$

$$
\begin{align*}
\frac{\partial G}{\partial I_L} & \triangleq \begin{bmatrix}
\frac{\partial I_1}{\partial I_L} & \cdots & \frac{\partial I_P}{\partial I_L} & -\frac{\partial Q_1}{\partial I_L} & \cdots & -\frac{\partial Q_P}{\partial I_L} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial I_{L-1}}{\partial I_L} & \cdots & \frac{\partial I_{L-P}}{\partial I_L} & -\frac{\partial Q_{L-1}}{\partial I_L} & \cdots & -\frac{\partial Q_{L-P}}{\partial I_L} \\
0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial Q_1}{\partial I_L} & \cdots & \frac{\partial Q_P}{\partial I_L} & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial I_{L-1}}{\partial I_L} & \cdots & \frac{\partial I_{L-P}}{\partial I_L} \\
0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial Q_1}{\partial I_L} & \cdots & \frac{\partial Q_P}{\partial I_L} & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial I_{L-1}}{\partial I_L} & \cdots & \frac{\partial I_{L-P}}{\partial I_L} \\
\end{bmatrix},
\end{align*}
$$

$$
\begin{align*}
\frac{\partial G}{\partial Q_L} & \triangleq \begin{bmatrix}
\frac{\partial I_1}{\partial Q_L} & \cdots & \frac{\partial I_P}{\partial Q_L} & -\frac{\partial Q_1}{\partial Q_L} & \cdots & -\frac{\partial Q_P}{\partial Q_L} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial I_{L-1}}{\partial Q_L} & \cdots & \frac{\partial I_{L-P}}{\partial Q_L} & -\frac{\partial Q_{L-1}}{\partial Q_L} & \cdots & -\frac{\partial Q_{L-P}}{\partial Q_L} \\
0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial Q_1}{\partial Q_L} & \cdots & \frac{\partial Q_P}{\partial Q_L} & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial I_{L-1}}{\partial Q_L} & \cdots & \frac{\partial I_{L-P}}{\partial Q_L} \\
0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial Q_1}{\partial Q_L} & \cdots & \frac{\partial Q_P}{\partial Q_L} & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \frac{\partial I_{L-1}}{\partial Q_L} & \cdots & \frac{\partial I_{L-P}}{\partial Q_L} \\
\end{bmatrix}.
\end{align*}
$$

Therefore, in the limit of small perturbation $\delta V$ we can write in the form of

$$
G[\alpha V_{in}(t) + \delta V(t)] \simeq G[\alpha V_{in}(t)] + \frac{\partial G}{\partial I_U}[\alpha V_{in}(t)] \delta I_U(t) + \frac{\partial G}{\partial Q_U}[\alpha V_{in}(t)] \delta Q_U(t) \\
+ \frac{\partial G}{\partial I_L}[\alpha V_{in}(t)] \delta I_L(t) + \frac{\partial G}{\partial Q_L}[\alpha V_{in}(t)] \delta Q_L(t) \tag{2.14}
$$

Let us assume for the time being that for small input powers the PA is linear with gain $G_{PA}^{lin,0}$ and memoryless (no dispersion requiring equalization) such that we have:

$$
V_{out}(t) = G[V_{in}(t)] \cdot X_{PA} \simeq G_{PA}^{lin,0} V_{in}(t)
$$

The matrix $G_{PA}^{lin,0}$ is defined as:

$$
G_{PA}^{lin,0} = \begin{bmatrix}
G_{PA,U}^{lin,0} & 0 \\
0 & G_{PA,L}^{lin,0}
\end{bmatrix}
$$
with
\[
G_{PA,U}^{lin,k} = \begin{bmatrix} \alpha_{1,k} & -\beta_{1,k} \\ \beta_{1,k} & \alpha_{1,k} \end{bmatrix} \tag{2.15}
\]
\[
G_{PA,L}^{lin,k} = \begin{bmatrix} \alpha_{-1,k} & -\beta_{-1,k} \\ \beta_{-1,k} & \alpha_{-1,k} \end{bmatrix} \tag{2.16}
\]

Note that \( k = 0 \) corresponds to a single zero delay as is appropriate for the dispersion less model presently considered.

Our goal is to find the predistortion coefficient \( X_{PD} \) so that the output of the PA is linear for all input \( \alpha V_{in}(t) \) on the predistorter such that we have:
\[
V_{out}(t) = G[V_{PD}(t)] \cdot X_{PA} = \alpha G_{PA}^{lin,0} V_{in}(t) \tag{2.17}
\]

We assume that for small input powers the PD system is linear and memoryless with unity gain such that we have:
\[
V_{PD}(t) = G[\alpha V_{in}(t)] \cdot X_{PD} \simeq \alpha V_{in}(t)
\]

Given that we have:
\[
\delta V(t) = V_{PD}(t) - \alpha V_{in}(t)
\]
\[
= G[\alpha V_{in}(t)] \cdot X_{PD} - \alpha V_{in}(t)
\]
\[
= G[\alpha V_{in}(t)] \cdot X_{PD}.
\tag{2.18}
\]

Itemizing each component of \( \delta V(t) \) we have:
\[
\delta I_U(t) = I_U[\alpha V_{in}(t)] \cdot \alpha_{PD,U} - Q_U[\alpha V_{in}(t)] \cdot \beta_{PD,U} - \alpha I_U(t)
\]
\[
= I_U[\alpha V_{in}(t)] \cdot \alpha_{PD,U} - Q_U[\alpha V_{in}(t)] \cdot \beta_{PD,U}
\]
\[
\delta Q_U(t) = Q_U[\alpha V_{in}(t)] \cdot \alpha_{PD,U} + I_U[\alpha V_{in}(t)] \cdot \beta_{PD,U} - \alpha Q_U(t)
\]
\[
= Q_U[\alpha V_{in}(t)] \cdot \alpha_{PD,U} + I_U[\alpha V_{in}(t)] \cdot \beta_{PD,U}
\]
\[ \delta I_L(t) = I_L[\alpha V_{in}(t)] \cdot \bar{\alpha}_{PD,L} - Q_L[\alpha V_{in}(t)] \cdot \bar{\beta}_{PD,L} - \alpha I_L(t) \]
\[ = I_L[\alpha V_{in}(t)] \cdot \bar{\alpha}_{PD,L} - Q_L[\alpha V_{in}(t)] \cdot \bar{\beta}_{PD,L} \]
\[ \delta I_U(t) = Q_L[\alpha V_{in}(t)] \cdot \bar{\alpha}_{PD,L} + I_L[\alpha V_{in}(t)] \cdot \bar{\beta}_{PD,L} - \alpha Q_L(t) \]
\[ = Q_L[\alpha V_{in}(t)] \cdot \bar{\alpha}_{PD,L} + I_L[\alpha V_{in}(t)] \cdot \bar{\beta}_{PD,L} \]

with
\[ \bar{\alpha}_{PD,U} = \begin{bmatrix} 0 \\ a_3 \\ a_5 \\ \vdots \\ a_P \end{bmatrix}, \quad \bar{\beta}_{PD,U} = \begin{bmatrix} 0 \\ b_3 \\ b_5 \\ \vdots \\ b_P \end{bmatrix}, \quad \bar{\alpha}_{PD,L} = \begin{bmatrix} 0 \\ a_{-3} \\ a_{-5} \\ \vdots \\ a_{-P} \end{bmatrix} \quad \text{and} \quad \bar{\beta}_{PD,L} = \begin{bmatrix} 0 \\ b_{-3} \\ b_{-5} \\ \vdots \\ b_{-P} \end{bmatrix} \]

and with:
\[ X'_{PD} = \begin{bmatrix} \bar{\alpha}_{PD,U} \\ \bar{\beta}_{PD,U} \\ \bar{\alpha}_{PD,L} \\ \bar{\beta}_{PD,L} \end{bmatrix} \]

The small perturbation term, \( V_{PD}(t) \triangleq \alpha V_{in}(t) + \delta V(t) \) is coming from the predistortion block. From the equation (13) and (16),
\[ G[\alpha V_{in}(t) + \delta V(t)]X_{PA} \approx \left[ G[\alpha V_{in}(t)] + \frac{\partial G}{\partial I_U} \delta I_U + \frac{\partial G}{\partial Q_U} \delta Q_U + \frac{\partial G}{\partial I_L} \delta I_L + \frac{\partial G}{\partial Q_L} \delta Q_L \right] X_{PA} \]
\[ \approx \alpha G_{PA}^{lin.0} V_{in}(t) \]

Inserting each component of \( \delta V \) gives,
\[ \alpha G_{PA}^{lin.0} V(t) = G[\alpha V_{in}(t)] \]
\[ \approx \frac{\partial G}{\partial I_U} X_{PA} \delta I_U + \frac{\partial G}{\partial Q_U} X_{PA} \delta Q_U + \frac{\partial G}{\partial I_L} X_{PA} \delta I_L + \frac{\partial G}{\partial Q_L} X_{PA} \delta Q_L \]
\[ = \frac{\partial G}{\partial I_U} X_{PA}(I_U \cdot \bar{\alpha}'_{PD,U} - Q_U \cdot \bar{\beta}'_{PD,U}) + \frac{\partial G}{\partial Q_U} X_{PA}(Q_U \cdot \bar{\alpha}'_{PD,U} + I_U \cdot \bar{\beta}'_{PD,U}) \]
\[ + \frac{\partial G}{\partial I_L} X_{PA}(I_L \cdot \bar{\alpha}'_{PD,L} - Q_L \cdot \bar{\beta}'_{PD,L}) + \frac{\partial G}{\partial Q_L} X_{PA}(Q_L \cdot \bar{\alpha}'_{PD,L} + I_L \cdot \bar{\beta}'_{PD,L}) \]
\[
\begin{bmatrix}
\frac{\partial G}{\partial I} X_{PAI_U} + \frac{\partial G}{\partial Q_U} X_{PAQ_U} \\
-\frac{\partial G}{\partial I} X_{PAQ_U} + \frac{\partial G}{\partial Q_U} X_{PAI_U} \\
\frac{\partial G}{\partial I} X_{PAI_L} + \frac{\partial G}{\partial Q_L} X_{PAQ_L} \\
-\frac{\partial G}{\partial I} X_{PAQ_L} + \frac{\partial G}{\partial Q_L} X_{PAI_L}
\end{bmatrix}
\begin{bmatrix}
\alpha'_{PD,U} \\
\beta'_{PD,U} \\
\alpha'_{PD,L} \\
\beta'_{PD,L}
\end{bmatrix}
= Y(t) \cdot X_{PD}.
\]

Therefore,
\[
Y(t) \triangleq \begin{bmatrix}
\frac{\partial G}{\partial I} X_{PAI_U} + \frac{\partial G}{\partial Q_U} X_{PAQ_U} \\
-\frac{\partial G}{\partial I} X_{PAQ_U} + \frac{\partial G}{\partial Q_U} X_{PAI_U} \\
\frac{\partial G}{\partial I} X_{PAI_L} + \frac{\partial G}{\partial Q_L} X_{PAQ_L} \\
-\frac{\partial G}{\partial I} X_{PAQ_L} + \frac{\partial G}{\partial Q_L} X_{PAI_L}
\end{bmatrix}
\end{equation}

Note that for simplicity, we use \(\frac{\partial G}{\partial I_U}, \frac{\partial G}{\partial Q_U}, \frac{\partial G}{\partial I_L}, \) and \(\frac{\partial G}{\partial Q_L}\) instead of \(\frac{\partial G}{\partial I_U}[\alpha V_{m}(t)],\frac{\partial G}{\partial Q_U}[\alpha V_{m}(t)],\) and \(\frac{\partial G}{\partial Q_L}[\alpha V_{m}(t)],\) respectively.

For the given matrix \(Y\), we rewrite the equation for mathematical convenience as follow:
\[
Y = \begin{bmatrix}
\frac{\partial G}{\partial I_U} X_{PAI_U} + \frac{\partial G}{\partial Q_U} X_{PAQ_U} \\
-\frac{\partial G}{\partial I_U} X_{PAQ_U} + \frac{\partial G}{\partial Q_U} X_{PAI_U} \\
\frac{\partial G}{\partial I_U} X_{PAI_L} + \frac{\partial G}{\partial Q_L} X_{PAQ_L} \\
-\frac{\partial G}{\partial I_U} X_{PAQ_L} + \frac{\partial G}{\partial Q_L} X_{PAI_L}
\end{bmatrix}
\]

where

\[
Y_1 = AI_U + BQ_U,
Y_2 = -AQ_U + BI_U,
Y_3 = CI_L + DQ_L,
Y_4 = -CQ_L + DL_L.
\]

\[
A = \begin{bmatrix}
\frac{\partial I_U}{\partial I_U} \cdot \alpha_{PA,U} + \frac{\partial I_U}{\partial Q_U} \cdot \beta_{PA,U} \\
\frac{\partial Q_U}{\partial I_U} \cdot \alpha_{PA,U} + \frac{\partial Q_U}{\partial Q_U} \cdot \beta_{PA,U} \\
\frac{\partial I_L}{\partial I_U} \cdot \alpha_{PA,L} + \frac{\partial I_L}{\partial Q_U} \cdot \beta_{PA,L} \\
\frac{\partial Q_L}{\partial I_U} \cdot \alpha_{PA,L} + \frac{\partial Q_L}{\partial Q_U} \cdot \beta_{PA,L}
\end{bmatrix},
\]

\[\text{Note that for simplicity, we use } \frac{\partial G}{\partial I_U}, \frac{\partial G}{\partial Q_U}, \frac{\partial G}{\partial I_L}, \text{ and } \frac{\partial G}{\partial Q_L} \text{ instead of } \frac{\partial G}{\partial I_U}[\alpha V_{m}(t)], \frac{\partial G}{\partial Q_U}[\alpha V_{m}(t)], \frac{\partial G}{\partial Q_L}[\alpha V_{m}(t)], \text{ respectively.}\]
The linearization has been done for each side band, separately. Fig. 2.2 shows the simulation of the proposed algorithm using two bands WiMax signal. From the simulation, 10 – 20 dB IMD cancellation is expecting. Fig. 2.3 shows AM/AM and AM/PM plots before and after the proposed algorithm. Red dots are the responses of the real PA output and yellow dots are the responses of the PD output. Blue dots are the responses of the sum of PA and PD responses which shows a linear line. This means that nonlinear PA response is compensated with PD and the total PA and PD systems behaves like a linear system. Figs. 2.4 and 2.5 show the constellation diagrams for both LSB and USB.

2.3 Frequency-Selective Digital Predistortion Linearization: Memory Case

We wish now to generalize the linearization theory for the case where the PA is exhibiting memory effects (dispersions) in each of the band $p$.

$$V_{out}(t) = \sum_{k=1}^{K} G[V_{in}(t - k\tau)] \cdot X_{PA}^{(k)}$$

with $\tau$ a delay (typically the sampling time interval).
Figure 2.2: Simulation of the proposed algorithm using two bands WiMax signals.
Figure 2.3: AM/AM and AM/PM plot of before and after the proposed DPD algorithm.
Figure 2.4: Constellation plot for lower sideband.
Figure 2.5: Constellation plot for upper sideband.
Now the PD has no memory compensation while the PA is exhibiting memory effects.

\[ G[\alpha V_{in} + \delta V] \simeq G[\alpha V] + \frac{\partial G}{\partial I_U} \delta I_U + \frac{\partial G}{\partial Q_U} \delta Q_U + \frac{\partial G}{\partial I_L} \delta I_L + \frac{\partial G}{\partial Q_L} \delta Q_L, \]

\[ V_{PD} = G[\alpha V] \cdot X_{PD} \simeq \alpha V_{in}, \]

\[ \delta V = V_{PD} - \alpha V_{in} = G[\alpha V_{in}] \cdot X'_{PD}. \]

From the problem statement,

\[ \mathbf{V}_{out}(t) = \sum_{k=0}^{K} G[V_{PD}(t - k\tau)] \cdot \mathbf{X}^{(k)}_{PA} \simeq \sum_{k=0}^{K} G'_{in(k)} \cdot \mathbf{V}_{in}(t - k\tau), \]

and

\[ G[V_{PD}(t - k\tau)] \simeq G[\alpha V_{in}(t - k\tau) + \delta V(t - k\tau)] \]

\[ = G[\alpha V_{in}(t - k\tau)] \]

\[ + \frac{\partial G}{\partial I_U} [\alpha V_{in}(t - k\tau)] \delta I_U(t - k\tau) + \frac{\partial G}{\partial Q_U} [\alpha V_{in}(t - k\tau)] \delta Q_U(t - k\tau) \]

\[ + \frac{\partial G}{\partial I_L} [\alpha V_{in}(t - k\tau)] \delta I_L(t - k\tau) + \frac{\partial G}{\partial Q_L} [\alpha V_{in}(t - k\tau)] \delta Q_L(t - k\tau). \]

Therefore,

\[ \sum_{k=0}^{K} \left\{ G'_{PA} \cdot \mathbf{V}_{in}(t - k\tau) - G[\alpha V_{in}(t - k\tau)] \cdot \mathbf{X}^{(k)}_{PA} \right\} \]

\[ \simeq \sum_{k=0}^{K} \left\{ \frac{\partial G}{\partial I_U} [\alpha V_{in}(t - k\tau)] \delta I_U(t - k\tau) + \frac{\partial G}{\partial Q_U} [\alpha V_{in}(t - k\tau)] \delta Q_U(t - k\tau) \right. \]

\[ + \frac{\partial G}{\partial I_L} [\alpha V_{in}(t - k\tau)] \delta I_L(t - k\tau) + \left. \frac{\partial G}{\partial Q_L} [\alpha V_{in}(t - k\tau)] \delta Q_L(t - k\tau) \right\} \cdot \mathbf{X}^{(k)}_{PA} \]

\[ \triangleq \sum_{k=0}^{K} \left\{ \frac{\partial G^{(k)}}{\partial I_U} \delta I_U^{(k)} + \frac{\partial G^{(k)}}{\partial Q_U} \delta Q_U^{(k)} + \frac{\partial G^{(k)}}{\partial I_L} \delta I_L^{(k)} + \frac{\partial G^{(k)}}{\partial Q_L} \delta Q_L^{(k)} \right\} \cdot \mathbf{X}^{(k)}_{PA}, \]

where

\[ \delta I_U^{(k)} = I_{U}^{(k)} \cdot \alpha'_{PD,U} - Q_{U}^{(k)} \cdot \beta'_{PD,U}, \]
\[
\delta Q_U^{(k)} = Q_U^{(k)} \cdot \tilde{\alpha}_{PD,U}^{(k)} + I_U^{(k)} \cdot \tilde{\beta}_{PD,U}^{(k)},
\]
\[
\delta I_L^{(k)} = I_L^{(k)} \cdot \tilde{\alpha}_{PD,L}^{(k)} - Q_L^{(k)} \cdot \tilde{\beta}_{PD,L}^{(k)},
\]
\[
\delta Q_L^{(k)} = Q_L^{(k)} \cdot \tilde{\alpha}_{PD,L}^{(k)} - I_L^{(k)} \cdot \tilde{\beta}_{PD,L}^{(k)}.
\]

After some algebra, we have the new equation:
\[
\sum_{k=0}^{K} Y^{(k)}(t) \cdot X_{PD}^{(k)} = \sum_{k=0}^{K} \left\{ G_{PA}^{lin,k} \cdot V_{in}^{(k)} - G^{(k)} \cdot X_{PA}^{(k)} \right\},
\]
where
\[
Y^{(k)} = \begin{bmatrix}
\frac{\partial G^{(k)}}{\partial I_U} \cdot X_{PA}^{(k)} \cdot I_U^{(k)} + \frac{\partial G^{(k)}}{\partial Q_U} \cdot X_{PA}^{(k)} \cdot Q_U^{(k)} \\
-\frac{\partial G^{(k)}}{\partial I_U} \cdot X_{PA}^{(k)} \cdot Q_U^{(k)} + \frac{\partial G^{(k)}}{\partial Q_U} \cdot X_{PA}^{(k)} \cdot I_U^{(k)} \\
-\frac{\partial G^{(k)}}{\partial I_L} \cdot X_{PA}^{(k)} \cdot I_L^{(k)} + \frac{\partial G^{(k)}}{\partial Q_L} \cdot X_{PA}^{(k)} \cdot Q_L^{(k)} \\
-\frac{\partial G^{(k)}}{\partial I_L} \cdot X_{PA}^{(k)} \cdot Q_L^{(k)} + \frac{\partial G^{(k)}}{\partial Q_L} \cdot X_{PA}^{(k)} \cdot I_L^{(k)}
\end{bmatrix}
\]

We wish now to generalize the linearization theory given in question (a) for the case where the predistortion accounts for memory effects (dispersions) in each of the intermodulation band \( p \).

\[
V_{PD}(t) = \sum_{k=0}^{K} G[V_{in}(t - k\tau)] \cdot X_{PD}^{(k)}
\]

Now the PD block can compensate memory effects. For small input power and its perturbation,

\[
V_{PD} = \sum_{m=0}^{M} G[\alpha_m V_{in}(t - m\tau) + \delta V^{(m)}] \cdot X_{PD}^{(m)} \simeq \sum_{m=0}^{M} \left\{ \alpha_m V_{in}^{(m)}(t) + \delta V^{(m)} \right\}.
\]

Since the PA is also exhibiting memory effects,

\[
V_{out} = \sum_{k=0}^{K} G[V_{PD}] \cdot X_{PA}^{(k)}
\]

\[
\simeq \sum_{k=0}^{K} G \left\{ \sum_{m=0}^{M} (\alpha_m V_{in}^{(m)} + \delta V^{(m)}) \right\} \cdot X_{PA}^{(k)}
\]

\[
\simeq \sum_{k=0}^{K} \sum_{m=0}^{M} \left\{ G[\alpha_m V_{in}^{(m)}] + \frac{\partial G^{(m)}}{\partial I_U} \delta I_U^{(m)} + \frac{\partial G^{(m)}}{\partial Q_U} \delta Q_U^{(m)} \right\}.
\]
\[ \sum_{k=0}^{K} G_{PA}^{\text{lin},k} \cdot X_{in}(t - k\tau) \triangleq \sum_{k=0}^{K} G_{PA}^{\text{lin},k} \cdot V_{in}^{(k)} \]

Therefore, for any \( 0 \leq k \leq K \),

\[ G_{PA}^{\text{lin},k} \cdot V_{in}^{(k)} - \sum_{m=0}^{M} G[\alpha_m V_{in}^{(m)}] = \sum_{m=0}^{M} \left\{ \frac{\partial G^{(m)}}{\partial I_U} \delta I_U^{(m)} + \frac{\partial G^{(m)}}{\partial Q_U} \delta Q_U^{(m)} \right. \\
+ \left. \frac{\partial G^{(m)}}{\partial I_L} \delta I_L^{(m)} + \frac{\partial G^{(m)}}{\partial Q_L} \delta Q_L^{(m)} \right\} \cdot X_{PA}^{(k)} \]

After some algebra, we have the new equation:

\[
Y^{(k)} = \begin{bmatrix}
\sum_{m=0}^{M} \left\{ \frac{\partial G^{(m)}}{\partial I_U} \cdot X_{PA}^{(k)} \cdot I_U^{(m)} + \frac{\partial G^{(m)}}{\partial Q_U} \cdot X_{PA}^{(k)} \cdot Q_U^{(m)} \right\} \\
\sum_{m=0}^{M} \left\{ -\frac{\partial G^{(m)}}{\partial I_U} \cdot X_{PA}^{(k)} \cdot Q_U^{(m)} + \frac{\partial G^{(m)}}{\partial Q_U} \cdot X_{PA}^{(k)} \cdot I_U^{(m)} \right\} \\
\sum_{m=0}^{M} \left\{ -\frac{\partial G^{(m)}}{\partial I_L} \cdot X_{PA}^{(k)} \cdot Q_L^{(m)} + \frac{\partial G^{(m)}}{\partial Q_L} \cdot X_{PA}^{(k)} \cdot I_L^{(m)} \right\} \\
\sum_{m=0}^{M} \left\{ -\frac{\partial G^{(m)}}{\partial I_L} \cdot X_{PA}^{(k)} \cdot Q_L^{(m)} + \frac{\partial G^{(m)}}{\partial Q_L} \cdot X_{PA}^{(k)} \cdot I_L^{(m)} \right\} 
\end{bmatrix}^T \tag{2.20}
\]

Compared with the memoryless case in Eq. (2.19), Eq. (2.20) has the summations over \( m \) to take care of memory effect.

### 2.4 Theory Extension to Three-Band Case

In the previous sections, we developed the theory of frequency-selective digital predistortion for the two-band case. The theory can be easily extended to a larger number of bands where interband linearization plays an important role. In this section, the three-band case is presented for a 3rd order linearization.

Fig. 2.6 shows three bands situation with its associate coefficients. For each band, we have

\[ X_k = I_k + jQ_k, (k = L, M, U) \]

\[ E_k = |X_k|, \]

28
\[
C_k = f(E_L^2, E_M^2, E_U^2) \\
\simeq c_{k,0} + c_{k,L} E_L^2 + c_{k,M} E_M^2 + c_{k,U} E_U^2,
\]

where, \( k \) is a band index, \( X_k \) is the complex signal of the band \( k \), \( E_k \) is the envelope of the complex signal \( X_k \), and \( C_k \) is the complex linearization coefficients to be found. For mathematical simplicity, the derivation has been done up to 3rd order Taylor expansion.

The important thing in the three bands case is the interaction of the LSB and MSB which generates additional interband modulation terms. One of these terms is located exactly in the position of the USB if all the three bands are equally spaced. Similarly, the interaction of MSB and the USB generates an interband term which falls into the location of the LSB. Even if they are not equally spaced, those interband modulation terms are in the band of interest and need to be removed. Furthermore if each band is getting closer, the band rejection requirement of the RF filter is getting sharper and the cost would be increased steeply. Therefore, interband linearization is an essential block for multi-band linearization.

For the interband linearization, the same formulas could be used as in the two bands case. For example, the interaction of MSB and USB generates the -3rd IMD term in the location of the LSB, and the interaction of LSB and MSB generates the +3rd IMD term in the position of the USB. From the equations in the two bands case, the following IMD terms are easily obtained as:

\[
I_{MU,-3} = (I_M^2 - Q_M^2)I_U + 2I_M Q_U Q_M, \\
Q_{MU,-3} = -(I_M^2 - Q_M^2)Q_U + 2I_U I_M Q_M, \\
I_{LM,+3} = (I_M^2 - Q_M^2)I_L + 2I_M Q_L Q_M,
\]
\[ Q_{LM,3} = -(I_M^2 - Q_M^2)Q_L + 2I_MI_LQ_M, \]
\[ I_{LM^*U} = I_LI_MI_U + I_LQ_MI_U \]
\[ - Q_LI_MI_U + Q_LQ_MI_U, \]
\[ Q_{LM^*U} = I_LI_MQ_U - I_LQ_MI_U \]
\[ + Q_LI_MI_U + Q_LQ_MI_U. \]

Associated with the above 3rd order IMD terms are the complex linearization coefficients \( c_{MU,-3}, c_{LM,3} \) and \( c_{LM^*U} \). Note that two other three band products \( X_L^*X_MI_U \) and \( X_LX_MI_U^* \) can be identified which also generate interband terms. However those two IMD terms are all located out of the band of interest (inband is limited to L, M and U) and can thus be readily removed using conventional filtering for sufficiently large band-spacing.

### 2.5 Conclusion

This chapter presented the theory of the frequency-selective digital predistortion algorithm. Starting from the original two-band memoryless case, a generalized architecture of the algorithm was proposed. MATLAB simulation showed 10 – 20 dB IMD improvement. Also mathematical expressions for the two-band memory case was developed. The proposed algorithm can be extended to a larger number of bands and a three-band case was developed where interband took an important place.
Figure 2.6: 3 Bands with associated linearization coefficients.
Chapter 3: CREST FACTOR REDUCTION (CFR) TECHNIQUE

3.1 Background

Modern communication systems use complex modulation schemes such as WCDMA or OFDM, which causes higher PAPRs in transmitted signals [20]. Since higher PAPRs call for large power inefficient PAs, maintaining low a PAPR while keeping a low signal distortion is one of the key system design goal. Therefore, PAPR reduction techniques have been extensively studied [23] [24]. However, schemes have not been developed for multi-band signals linearized with a frequency selective characteristics. In this chapter, a two-band crest factor reduction technique has been designed and implemented which is applicable to frequency selective DPD systems.

3.2 Two-band Crest Factor Reduction Technique

To address the differential memory nonlinear effects, separating the upper sideband and the lower sideband is needed. In Figure 3.1, a two-band CFR block is proposed. In the correction factor calculation block, the instantaneous signal amplitude and angle between I and Q have been estimated, then the signal amplitude
which exceeds a predefined value has been clipped while keeping the signal angle (polar clipping). Figure 3.2 shows this algorithm. Each correction signal for $I_L$, $I_U$, $Q_L$, and $Q_U$ has been generated using following equations:

\[
I_{CU} = \frac{I_C + \hat{Q}_C}{2}; I_{CL} = \frac{I_C - \hat{Q}_C}{2},
\]
\[
Q_{CU} = \frac{Q_C - I_C}{2}; Q_{CL} = \frac{Q_C + I_C}{2}.
\]

A simple clipping process (hard clipping) causes a strong spectral regrowth which contaminates adjacent channels. To prevent this effect, a Root-Raised-Cosine filter has been used for spectral shaping (soft clipping).

### 3.3 Simulation and FPGA Implementation of Two-band Signal

The algorithm has been implemented on a Terasic DE3 board which has an Altera Stratix-III FPGA [25]. Fig. 3.4 shows a block level design of the proposed CFR

![Figure 3.1: Two-band CFR block.](image-url)
Figure 3.2: Polar clipping diagram
algorithm. To prevent IQ-imbalancing and LO-leakage problems, a digital IF scheme has been adopted. Figure 3.3 shows the testbed setup for the digital IF scheme. In the setup, every signal source has been synchronized to the master clock (Agilent MXG Analog Signal Generator).

Figures 3.5 and 3.6 show MATLAB simulation results and experimental measured data for a two-band CFR in frequency domain. The measured data captures on a spectrum analyzer are close to the simulation plot and exhibit about 20 dBc ACPR improvement for 8.7 dB clipping level. Figures 3.7 and 3.8 show CCDF curves from both MATLAB simulations and experimental measurement, respectively.

Table 3.1 shows the FPGA (Stratis-3 SL150 Device) resource usage of the implemented CFR algorithm and WiMax signal generator.
Figure 3.4: Block level design of the proposed CFR algorithm
Figure 3.5: MATLAB simulation results of hard and soft clipping in spectrum domain

Figure 3.6: VSA capture of the implemented hard and soft clipping.
Figure 3.7: CCDF simulation result of the original, hard, and soft clipping. About 8.7 dB PAPR at 0.01% CCDF is obtained.
Figure 3.8: VSA CCDF capture of the implemented soft clipping with Gaussian reference. About 8.75 dB PAPR at 0.01 % CCDF is obtained.
Table 3.1: Resource usage of the implemented WiMax signal generator and CFR

<table>
<thead>
<tr>
<th></th>
<th>Logic Utilization</th>
<th>Total Block memory</th>
<th>DSP Block (18-bit)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Combinational ALUTs</td>
<td>Memory ALUTs</td>
<td>Dedicated Logic Registers</td>
</tr>
<tr>
<td>Resource Available</td>
<td>113,600</td>
<td>56,800</td>
<td>113,600</td>
</tr>
<tr>
<td>WiMax Generator</td>
<td>6,889</td>
<td>16,808</td>
<td>6,316</td>
</tr>
<tr>
<td>CFR</td>
<td>23,371</td>
<td>937</td>
<td>39,077</td>
</tr>
</tbody>
</table>

3.4 Conclusion

This chapter presented a 20 MHz two-band crest factor reduction technique to reduce high PAPR for WiMax application. Soft and polar clipping method were used for the implementation. In both MATLAB simulation and FPGA implementation, 20 dB ACPR improvement was achieved while reducing PAPR to 8.7 dB from 11.2 dB original PAPR.
Chapter 4: HARDWARE IMPLEMENTATION AND RESULTS

Many linearization works, i.e., [27] [26], have been demonstrated in non real-time based system. Fig. 4.1 shows a simplified schematic diagram of this non real-time based system testbed. A PC running MATLAB and VSA software [28] is connected to an ESG over a GPIB bus. Then a signal data is downloaded from the PC to the ESG and the ESG plays the digital data to generate the RF frequency signal. The RF output is connected to a RF power amplifier which is the DUT. The RF power amplifier output is connected to a spectrum analyzer through an attenuator if necessary. This spectrum analyzer is also communicating with the PC over the GPIB bus. Inside the VSA, the RF signal is down converted to I and Q baseband signals. These I and Q data are sent to the PC. Then the VSA software on the PC does post processing on the baseband I and Q data so that the software measures various RF parameters such as EVM, CCDF, etc. This approach is the most common way to do linearization in the literatures.

However, in a real implementation, one cannot rely on these high performance testing facilities not only because of their cost but also because they do not provide for the needed real time linearization of the unknown RF signal. Indeed in a realistic
implementation, all the processing must be performed in real time in a compact integrated linearization system.

In this chapter, an hardware implementation of multiband DPD based on a on a FPGA testbed is presented. This FPGA testbed will be used for all the experiments reported through the remained part of work this thesis.

### 4.1 Hardware block overview

Fig. 4.2 shows the top schematic of the proposed 5th order digital predistortion system. Two band signals, $I_U$, $Q_U$, $I_L$, and $Q_L$ are coming to the in-band correction block as well as envelope generator block. The envelope generator block generates signal envelopes from the original two bands signals. In the in-band correction block,
linearization on in-band has been performed. The block takes 10 complex coefficients (5 for each of sidebands) from MATLAB GUI on PC. The output of in-band correction are connected to the higher order generator block which generates the higher order IMD terms up to 5th order. Each of IMD terms generated from the higher order block is going into the inter-band correction block and IMD compensation has been performed independently for each of IMD terms. IQ balancing is also included in the inter-band correction block.
In the following part of this chapter, more detail explanation on each of block is provided.

4.2 Envelope Generator Block

Fig. 4.3 shows the detail envelope generator block diagram in which the envelope signals are generated from the original input. In [22], a frequency-selective model is given by,

\[ y(m) = x(m) \cdot \sum_{k=1}^{K} a_k \cdot |x(m)|^{k-1}, \]

with \( x(m) = I(m) + jQ(m) \) is the input signal, \( y(m) = I'(m) + jQ'(m) \) is the output signal, \( a_k \) are complex coefficients, and \( K \) is the polynomial order which is an odd number.

For two band signals, an envelope signal can be defined for each of the upper sideband signal, \( x_U(m) \), and lower sideband signal, \( x_L(m) \). They are given by \( E^2_U = |x_U(m)|^2 \) and \( E^2_L = |x_L(m)|^2 \). In 4.3, the generated envelopes are \( EU2 = E^2_U = I^2_U + Q^2_U \), \( EL2 = E^2_L = I^2_L + Q^2_L \), \( EU4 = E^4_U = I^4_U + Q^4_U \), \( EL4 = E^4_L = I^4_L + Q^4_L \), and \( EU2EL2 = E^2_U \cdot E^2_L = (I^2_U + Q^2_U) \cdot (I^2_L + Q^2_L) \).

The original input signals are going into the squarers to calculate square values of each of the input. The squarers have been implemented using memory based multipliers. The output bit-width of the block is 24-bit for \( E^2_U \) and \( E^2_L \). For \( E^4_U \), \( E^4_L \), and \( E^2_U \cdot E^2_L \) terms, 24-bit width is truncated from the original 48-bit. The validity of truncation is that higher order terms generate smaller values for small perturbation. The output of the squarers are also going to another squarer block which calculates four squarers for \( E^4_U \) and \( E^4_L \).
Figure 4.3: Envelope Generator Block
4.3 In-Band Correction Block

Fig. 4.4 shows the USB in-band correction block which takes the original input signals, i.e., $I_U$ and $Q_U$ as well as the envelope signals. The LSB in-band correction block is identical to the USB block with different signal input, i.e., $I_L$ and $Q_L$. Each of 12 bit input signals are modulated by each of 24-bit envelope signals.

The modulated 36-bit signals are amplitude scaled and phase shifted using a complex multiplier. In the schematic diagram, the complex multiplier is denoted by a square box with $\text{CX}$ notation, while a plain real multiplier is denoted by a square
box with $X$. $\alpha$ and $\beta$ have following relationship: $\alpha = M \cos \theta$, $\beta = M \sin \theta$, where $M$ is amplitude scaling factor which is 12-bit in the digital hardware and $\theta$ is phase shifting factor in radian which is 12-bit as well.

Each of the modulated, amplitude scaled, and phase shifted paths is summed together with the original signal.

The final summed outputs are in-band outputs which are $I(1)$, $Q(1)$, $I(-1)$, and $Q(-1)$, respectively.

4.4 Higher-Order IMD Generator Block

Higher-order IMD generator block is a key block in this work. To compensate 3rd and 5th order IMD terms, the block generates those terms in each sideband based on the input which are the output of the in-band linearization. The mathematical formulas for the 3rd and 5th order terms are presented in chapter 2 and repeated here:

$$I_{-5} = I_L^3 I_U^2 - I_U^3 Q_L^2 + 6I_U^2 Q_L I_U Q_U - 3I_L Q_L^3 Q_U - 3I_L Q_L^2 Q_U^2 - 2Q_L^3 I_U Q_U,$$

$$Q_{-5} = Q_L^3 Q_U^2 - Q_U^3 I_L^2 + 6Q_L I_U Q_U I_U - 3Q_L I_U Q_L Q_U^2 + 3Q_L I_U^2 Q_U^2 - 2I_L Q_U Q_U I_U,$$

$$I_{-3} = (I_L^2 - Q_L^2) I_U + 2I_U Q_U Q_L,$$

$$Q_{-3} = -(I_L^2 - Q_L^2) Q_U + 2I_U I_L Q_L,$$

$$I_3 = (I_U^2 - Q_U^2) I_L + 2I_U Q_L Q_U,$$

$$Q_3 = -(I_U^2 - Q_U^2) Q_L + 2I_U I_L Q_U,$$

$$I_5 = I_U^3 I_L^2 - I_U^3 Q_L^2 + 6I_U^2 Q_L I_U Q_L - 3I_U Q_L^2 Q_U - 3I_U Q_L^2 Q_L^2 - 2Q_U^3 I_U Q_U L,$$

$$Q_5 = Q_U^3 Q_L^2 - Q_U^3 I_L^2 + 6Q_U I_U Q_L Q_U - 3Q_U I_U Q_L Q_L^2 + 3Q_U I_U^2 Q_L^2 - 2I_U^3 Q_L I_L.$$
4.4.1 Cube Generator Block

Each IMD term includes square and cube terms in each of the equations. For efficient calculation, a block which calculates those higher order products has been implemented separately. In Fig. 4.5 shows the higher order generator block. The block can be used for all paths and the input is denoted by $X$ which takes either $I_L$, $Q_L$, $I_U$, or $Q_U$. The associated output are denoted by $X^2$ and $X^3$. For example, for the input $I_L$, the block generates $I_L^2$ and $I_L^3$. The original input $X$ is going through the delay elements to match the total delay with other higher products. The original input is 12 bit and the square and cube output is 24 bit and 36 bit, respectively.

4.4.2 Higher-Order IMD Generator Block

Fig. 4.6 shows the 3rd order IMD generator block for $I(+3)$. In the notation of $I(+3)$, ‘I’ stands for the I component, ‘+’ sign stands for the upper sideband, ‘3’ stands for the 3rd order IMD. Therefore, $Q(-3)$ stands for 3rd order IMD of Q in the lower sideband, in the same manner. Since the 3rd IMD formulas are symmetric, the block can be used for all four cases with different input signal sets. Table 4.1 shows the signal order for using the same 3rd order block.

Fig. 4.7 shows a 5th order IMD generator block. Similar with the 3rd IMD block, the 5th order can be used for all four cases with different input signal sets. Table 4.2 shows the signal order to use the same 5th order block.

4.5 Inter-Band Correction Block

Fig. 4.8 shows inter-band linearization block. Each path has been linearized using amplitude scaling and phase shifting which are done by a complex modulator.
Figure 4.5: Block diagram of the higher order generator.

Table 4.1: Signal order for using the same 3rd order interband generation block.
Figure 4.6: Block diagram of the 3rd order IMD generator.

Table 4.2: Signal order for using the same 5th order interband generation block.
Figure 4.7: Block diagram of the 5th order IMD generator.
Fundamental, 3rd IMD, and 5th IMD are summed separately. To address analog impairment in DAC, mixer, modulator, etc., IQ-balancing blocks are introduced for each band. The balanced output are summed together at the final stage.

4.6 Hardware Simulation and Verification

All hardware has been implemented on Altera Stratix III FPGA. Fig. 4.9 shows a functional simulation of the implemented hardware.
Figure 4.9: Screen capture of functional simulation result.
To verify the 3rd and 5th order IMD generation blocks, 4 MHz single tone has been generated and applied to 3rd and 5th order blocks. Fig. 4.10 shows a spectrum capture of the output of the 3rd order IMD generation block. A clear single tone at 12 MHz has been observed. There is another tone at 88 MHz and this is the image from the DAC. Note that 100 MHz FPGA clock has been used for the test and 88 MHz = 100 MHz - 12 MHz. Similarly Fig. 4.11 shows a spectrum capture of the output of the 5th order IMD generation block. A clear single tone at 20 MHz has been observed.
Figure 4.11: A spectrum capture of the output of the 5th order IMD generator with 4 MHz fundamental single tone.
4.7 Testbed setup

4.7.1 Test Signals

For the system excitation, the input signal could be any band limited signals. In this work, two-band multisine signals with a constant amplitude and specific phases have been developed as follows:

\[ I_U(t) + jQ_U = \left\{ A \sum_{k=1}^{N} e^{j(k\Delta\omega t + \phi_{U,k})} \right\} e^{j2\pi F_{modU} t} \] (4.1)

\[ I_L(t) + jQ_L = \left\{ A \sum_{k=1}^{N} e^{-j(k\Delta\omega t + \phi_{L,k})} \right\} e^{-j2\pi F_{modL} t} \] (4.2)

Multisine signal could have higher a PAPR when each of the individual tones is superimposed in phase. In this study, each of the phases, \( \phi_{U,k} \) and \( \phi_{L,k} \), has been chosen carefully, so that total signals can have realistic PAPR. In this work, three signal sets have been developed as follows: 1) 16-tone per two bands (5MHz bandwidth for each band) with 7.5 dB PAPR, 2) 96-tone per three bands (5MHz bandwidth for each band) with 8.7 dB PAPR, and 3) 128-tone per two bands (10MHz bandwidth for each band) for largely spaced signal linearization with 8.3 dB PAPR. Fig. 4.12 shows the CCDF of three developed signals.

In this case, total \( 2N \)-tone signals are generated and implemented using LUT on the FPGA device.

4.7.2 Two- and Three-Band Case

The overall experimental testbed used in this work is shown in Fig 4.13. Multitone signals for each lower and upper side band signal with specific phases are generated by MATLAB and implemented by a LUT on the Stratix-III 3SE260 FPGA on Altera DE3 board. The FPGA system clock is 125 MHz. The predistorted baseband signal
Figure 4.12: CCDF plot of 16-, 96- and 128-tone multisine signal.
from the FPGA is connected to a TI DAC5682z DAC EVM over LVDS. The TI EVM has dual 16-bit D/A converters with up to 1.0 GSPS, a clock distribution chip, CDCM7005, and analog quadrature modulator, TRF3703, to upconvert the baseband DAC output to RF. Interpolation filter with ×4 rate and the LC LPF with 300 MHz 3 dB corner frequency are used between the DAC and the IQ modulator to remove the Nyquist images from the DAC. DC offset and IQ-imbalancing have been carefully removed. 890 MHz RF frequency was chosen to investigate the digital cellular band response and the external LO fed into the DAC EVM. The RF output of the EVM is connected via a pre-amplifier to the Mini-Circuit amplifier ZX60-43-S+ to be linearized.
4.8 Results

4.8.1 Two-Band Case

In the measurement, first, the power amplifier output power was set so that only the 3rd IMD came out so that it could be linearized. Second, the power amplifier output power was raised so that both the 3rd and 5th IMDS came out to be linearized for both 3rd and 5th IMDS.

Fig. 4.14 shows the original output signal with 16-tone for two bands whose CCDF is 7.5 dB. To take full advantage of the frequency-selective nature, linearization had been done using the following steps: 1) Inband LSB linearization, 2) Inband USB linearization, 3) -3rd order IMD linearization, and finally 4) +3rd order IMD linearization.

Fig. 4.15, Fig. 4.16, Fig. 4.17 and Fig. 4.18 show the step by step linearizations.

Fig. 4.19 shows the linearization before and after the proposed frequency-selective DPD algorithm has been applied to the 3rd order IMD outputs. From the measurement, more than 15 dB IMD cancellation in both inbands and more than 14 dB IMD cancellation on 3rd order IMD were achieved.

For higher output power input, the 5th order IMD came out at the output, the same multisine test signal was used. Fig. 4.20 shows the original output. Similarly to the 3rd order linearization case, the following linearization steps had been done: 1) Inband LSB linearization, 2) Inband USB linearization, 3) -3rd order IMD linearization, 4) +3rd order IMD linearization, 5) -5th order IMD linearization, and finally 6) +5th order IMD linearization.

Fig. 4.21, Fig. 4.22, Fig. 4.23, Fig. 4.24, Fig. 4.25, and Fig. 4.26 show the step by step linearizations.
Figure 4.14: Original spectrum of 16-tone two bands signal with up to 3rd order IMDs

Figure 4.15: Inband LSB linearization of up to 3rd order IMD
Figure 4.16: Inband USB linearization of up to 3rd order IMD

Figure 4.17: -3rd order interband linearization

61
Fig. 4.27 shows the linearization before and after the proposed frequency-selective DPD algorithm has been applied to 5th order IMD output. From the measurement, more than 15 dB IMD cancellation in both inbands, more than 16 dB IMD cancellation on 3rd order IMD, and more than 6 dB IMD cancellation on 5th order IMD were achieved.

### 4.8.2 Three-Band Case

In the 3 bands measurement, only 3rd order IMD linearization had been done. 32-tone multisine signal per band was used for the test. Fig. 4.28 shows the CCDF of the test signal for this measurements whose original PAPR is 8.7 dB (red line).
Figure 4.19: Performance of the proposed frequency-selective DPD algorithm of up to 3rd order IMD input
Figure 4.20: Original spectrum of 16-tone two bands signal with up to 5th order IMDs

Figure 4.21: Inband LSB linearization of up to 5th order IMD input
Figure 4.22: Inband USB linearization of up to 5th order IMD input

Figure 4.23: -3rd order interband linearization of up to 5th order IMD input
Figure 4.24: +3rd order interband linearization of up to 5th order IMD input

Figure 4.25: -5th order interband linearization of up to 5th order IMD input
Figure 4.26: +5th order interband linearization of up to 5th order IMD input

Fig. 4.29 shows the original spectrum of the power amplifier. For the three-band algorithm, a manual linearization method similar to the two-band method was used: 1) turning off MSB and USB while turning on LSB with increased power by $\sqrt{3}$ to maintain the same output power, then finding self inband linearization coefficient, i.e., $c_{LL}$, 2) Repeat the step 1) for MSB and USB, i.e., $c_{MM}$ and $c_{UU}$, 3) turning off USB while turning on LSB and MSB with increased power by $\sqrt{3}/2$ so that the final output power is the same as the normal 3 bands. Then find two inband linearization coefficients and +3rd interband linearization coefficient, i.e., $c_{LM}$, $c_{ML}$, and $c_{LM,3}$, 4) Repeat the step, 3) for MSB and USB, i.e., $c_{MU}$, $c_{UM}$, and $c_{MU,3}$, and finally 5) turning on all three bands with normal power and find $c_{LU}$ and $c_{UL}$.
Figure 4.27: Performance of the proposed frequency-selective DPD algorithm of up to 5th order IMD input
Figure 4.28: CCDF of the test signal with 96-tone per three bands

Figure 4.29: Original spectrum of the 3-band signal
Without the interband linearization, three bands would not able to be linearized fully and Fig. 4.30 shows the spectrum capture of it. With the interband linearization, a clean spectrum was achieved and Fig. 4.31 shows the final DPD output. Fig. 4.32 shows the performance comparison with and without interband linearization.

To investigate the robustness of the DPD system, the mid-channel was turned off. The associated CCDF for the resulting two-band signal is 6.5 dB and shown in Fig. 4.28 with blue line. Fig. 4.33 shows the spectrum capture of the mid-channel off from the three bands input.

Fig. 4.34 shows the comparison before and after turning off the mid-channel, and there is no performance degradation between two. Therefore, the DPD system was well extracted using the three-band signal.
With interband linearization, about 10 dB IMD cancellations for all three bands were achieved. Without interband linearization, only 3 – 4 dB IMD cancellations for LSB and USB were observed.

4.9 Conclusion

In this chapter, we have presented the implementation of the frequency-selective digital predistortion algorithm on FPGA device, and demonstrated two- and three-band signal cases. In the two-band signal test, a 7.8 dB PAPR test signal has been used and input power levels had been chosen for demonstrating the performance of 3rd and 5th IMD linearization of the proposed algorithm. For the 3rd order IMD signal, 15 dB inband IMD cancellation and 14 dB on the 3rd interband IMD cancellation
Figure 4.32: Performance comparison with and without interband linearization
were achieved. For the 5th order IMD linearization test, more than 15 dB on inband IMD cancellation, 16 dB on the 3rd order interband IMD cancellation, and 6 dB on the 5th order interband IMD cancellation were achieved. For the three-band test, 8.7 dB PAPR test signal was used for the three-band linearization. Without interband linearization, only 3 – 4 dB IMD cancellation were observed on LSB and USB. With interband linearization, more than 10 dB IMD cancellation were achieved. Therefore, interband play an important role in multi-band linearization. To investigate how the linearization coefficients are well characterized, turning the mid-channel off had been done and no performance degradation was observed.
Figure 4.34: Performance comparison between before and after turning off the mid-channel off after all 3-band linearization.
Chapter 5: LARGELY SPACED DPD

5.1 Introduction

Linearization of largely spaced signals is an emerging topic in linearization research [29] [30] [31] [32]. This technique has advantage of economical system cost for wireless service providers because they could use the same linearization block for different frequency bands and share a same broadband PA. Previously, service providers had to use different PAs for different frequency bands. In [30] [31], two identical processing stages were used for two different bands using two different LOs and summed together before applying to the PA. Also they assumed those two input bands were far enough so that the intermodulation terms could be removed easily using an external filter. In their work, they used 100 MHz of band spacing and achieved 10 – 16 dB ACPR improvement depending on the signal scenarios. In [32], 60 MHz band separation with two carrier per band was used.

In this chapter, a new scheme of largely spaced signals DPD using digital IF is proposed, in which a single LO is required and 250 MHz band separation is achieved using digital coarse mixer.
5.2 Largely Spaced Signals DPD

The DAC5682 [33] supports coarse mixing mode which is capable of shifting the input signal spectrum by the fixed mixing frequencies $f_s/2$ or $\pm f_s/4$. For the complex signal, $I(t) + jQ(t)$, the output of the coarse mixer block, $I_{out}(t)$ and $Q_{out}(t)$ are given by following:

\[
I_{out}(t) = I(t) \cos(2\pi f_{IF} t) - Q(t) \sin(2\pi f_{IF} t),
\]
\[
Q_{out}(t) = I(t) \cos(2\pi f_{IF} t) + Q(t) \sin(2\pi f_{IF} t),
\]

where $f_{IF}$ is the fixed mixing frequency. For example, $f_{IF} = +f_s/4$ case is

\[
\cos(2\pi \cdot f_{IF} \cdot t)|_{t=nT_s} = \cos\left(2\pi \cdot \frac{f_s}{4} \cdot t\right)|_{t=nT_s} = \cos\left(\frac{\pi}{2} \cdot \frac{1}{T_s} \cdot nT_s\right) = \cos\left(\frac{\pi}{2} n\right) = 1, 0, -1, 0, \ldots
\]

\[
\sin(2\pi \cdot f_{IF} \cdot t)|_{t=nT_s} = \sin\left(2\pi \cdot \frac{f_s}{4} \cdot t\right)|_{t=nT_s} = \sin\left(\frac{\pi}{2} \cdot \frac{1}{T_s} \cdot nT_s\right) = \sin\left(\frac{\pi}{2} n\right) = 0, 1, 0, -1, \ldots
\]

Therefore, the output sequences for $f_{IF} = +f_s/4$ case are $I_{out} = \{+I, -Q, -I, +Q\}$ and $Q_{out} = \{+Q, +I, -Q, -I\}$. Similarly, for $f_{IF} = -f_s/4$ case, the output sequences are $I_{out} = \{+I, +Q, -I, -Q\}$ and $Q_{out} = \{+Q, -I, +Q, -I\}$.

By connecting additional DAC5682z EVM to the DE3 FPGA board, and choosing $f_{IF} = +f_s/4$ for the USB signal and $f_{IF} = -f_s/4$ for the LSB signal, the baseband...
Table 5.1: Band pass filter specification for largely spaced DPD

<table>
<thead>
<tr>
<th>BPF</th>
<th>Passband (MHz)</th>
<th>Stopband Rejection</th>
<th>Insertion Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPF1 for LSB</td>
<td>690 - 790</td>
<td>40 dBC at 850 MHz</td>
<td>0.23</td>
</tr>
<tr>
<td>BPF2 for USB</td>
<td>930 - 970</td>
<td>55 dBC at 750 MHz</td>
<td>1.08</td>
</tr>
</tbody>
</table>

signals are spaced with $2f_{IF}$ each other. In this test, 125 MHz sampling frequency has been used and in consequence, a two bands signal with band separated by 250 MHz has been generated.

The advantages of this architecture is that only a single LO is required for the signal separation and no physical multiplier is needed for the spectrum shifting because the output sequences are just bypassing the original or sign-bit flipping of the signals.

## 5.3 Testbed and result

Fig. 5.1 shows the proposed largely spaced DPD architecture. Since the two bands are largely separated, the interband modulation terms are located far enough and easily removed using filters. Hence, only the inband linearization part is needed for this test. To remove the images and other unwanted signals, two BPFs from K&L Microwave have been used. Table. 5.1 shows the brief specification of two BPFs. Fig. 5.2 shows the photo of the testbed.

64-tone per each 10 MHz bandwidth has been generated to simulate WiMax or LTE signals. Fig. 5.3 shows the CCDF of the test signal. CCDF of the 2 bands test signal (128-tone) has 8.3 dB PAPR.
Figure 5.1: Largely spaced digital predistortion architecture using digital IF

Figure 5.2: Photo of the testbed
Fig. 5.3: CCDF plot for the proposed largely spaced DPD using digital IF

Fig. 5.4 shows the LSB (upper plot) and USB (lower plot) performance of the proposed architecture. Fig. 5.5 shows the combined linearization results for both of the bands linearization result. Note that asymmetric output level of the two bands is due to the gain dependence of the amplifier.

From the measurement, more than 15 dB IMD cancellation for both bands has been achieved.

5.4 Conclusion

This chapter proposed a new architecture of largely spaced signals DPD using digital IF technology. More than 15 dB IMD cancellation was achieved for both sidebands. Using the proposed architecture, only a single LO is needed for the signal separation. Therefore, compared to the previously proposed architecture in [30] [31],
Figure 5.4: LSB (upper plot) and USB (lower plot) linearization performance of the proposed largely spaced DPD using digital IF.
Figure 5.5: Both bands linearization performance of the proposed largely spaced DPD using digital IF
this approach is cost effective while yielding the same performance. However, there is
a band separation limitation due to digital clock. This disadvantage could be resolved
using higher speed interpolation supported DAC.
Chapter 6: CONCLUSION AND FUTURE WORK

Linearization of power amplifiers is most emerging research area. Many approaches have been tried and digital predistortion is one of promising techniques. In this thesis, generalized frequency-selective digital predistortion technique was investigated. Beginning with memoryless two-band frequency-selective DPD technique, simulation and real hardware has a good agreement on the performance. The proposed linearization algorithm is signal independent and any band limited signal could be used for the linearization. 3rd and 5th order linearization were demonstrated using various multi-tone signals. For 3rd order linearization, more than 15 dB IMD cancellation on inband, 14 dB cancellation on 3rd order interband IMD. For 5th order linearization, more than 15 dB IMD cancellation on inband, 16 dB cancellation on 3rd order interband IMD, and 6 dB on 5th order interband IMD cancellation were achieved. To demonstrate an extendability of the algorithm to larger multi-band, two-band theory was modified to three-band with up to 3rd order compensation. In three-band case, interband linearization played an important role in overall performance. Without interband linearization, only 3 – 4 dB IMD cancellation was observed. However, with inband compensation, more than 10 dB IMD cancellation was achieved. For three-band case, to investigate robustness of the DPD system, middle channel was turned off and the same coefficients worked well for two bands.
In another point of view of recent DPD application, linearization on largely spaced signals has been paid attention. In this work, the same algorithm was applied to 250 MHz spaced signal and more than 15 dB IMD cancellation was achieved. For the signal separation, digital IF technique was proposed which uses only single LO for the separation. Other previous works used two different LOs for the separation.

For higher PAPR signal, two-band CFR block was proposed. MATLAB simulation and FPGA implementation had an excellent agreement on the performance; 20 dB ACPR improvement was achieved with 2.5 dB PAPR reduction.

In this work, the following contributions were made:

1) A new frequency-selective DPD theory and generalized architecture were developed and proposed, respectively. The theory was developed for two-band memoryless case. However, the architecture can easily incorporate a memory polynomial model.

2) The theory can be extendable to higher multi-band case where interband take a key place. In this work, the three-band case was demonstrated and to our best knowledge, this is the first report showing importance of the interband linearization.

3) Two-band CFR block was demonstrated. Previous works focused on only a single band CFR.

As for future work, higher order, i.e., 5th or even higher, linearization on higher multi-band, i.e., 3-band or even higher, should be considered. Also the integration
of both DPD and CFR blocks should be considered carefully. Each of block has been developed and tested yet optimization of both blocks was not considered in this work. Finally, an adaptive scheme should be developed so that the DPD trains itself automatically.
Appendix A: HIGHER ORDER INTERMODULATION TERMS GENERATION

The following identities can be established:

\[ I_1 = I_U \]
\[ Q_1 = Q_U \]
\[ I_{-1} = I_L \]
\[ Q_{-1} = Q_L \]
\[ I_3 = I_U^2 I_L + 2I_U Q_U Q_L - Q_U^2 I_L \]
\[ Q_3 = -I_U^2 Q_L + 2I_U Q_U I_L + Q_U^2 Q_L \]
\[ I_{-3} = I_U I_L^2 - I_U Q_L^2 + 2Q_U I_L Q_L \]
\[ Q_{-3} = 2I_U I_L Q_L - Q_U I_L^2 + Q_U Q_L^2 \]
\[ I_5 = I_U^3 I_L^2 - I_U^3 Q_L^2 + 6I_U^2 Q_U I_L Q_L - 3I_U Q_U^2 I_L^2 + 3I_U Q_U Q_L^2 - 2Q_U^3 I_L Q_L \]
\[ Q_5 = -2I_U^3 I_L Q_L + 3I_U^2 Q_U I_L^2 - 3I_U Q_U Q_L^2 + 6I_U Q_U^2 I_L Q_L - Q_U^3 I_L^2 + Q_U^3 Q_L^2 \]
\[ I_{-5} = I_U^3 I_L^3 - 3I_U^2 Q_U Q_L^2 + 6I_U Q_U I_L^2 Q_L - 2I_U Q_U Q_L^3 - Q_U^3 I_L^3 + 3Q_U^3 I_L Q_L^2 \]
\[ Q_{-5} = 3I_U^3 I_L^2 Q_L - I_U^3 Q_L^3 - 2I_U Q_U I_L^3 + 6I_U Q_U I_L Q_L^2 - 3Q_U^2 I_L^2 Q_L + Q_U^2 Q_L^3 \]
\[ I_7 = I_U^4 I_L^3 - 3I_U^4 I_L Q_L^2 + 12I_U^3 Q_U I_L^2 Q_L - 4I_U^2 Q_U^2 I_L^2 Q_L - 6I_U^2 Q_U Q_L^3 - 12I_U Q_U^3 I_L^2 Q_L + 18I_U Q_U^2 I_L^2 Q_L + 4I_U Q_U^3 Q_L^3 + Q_U^4 I_L^3 - 3Q_U^4 I_L Q_L^2 \]
Q_7 = -3I_U^3I_L^2Q_L + I_U^4Q_L^2 + 4I_U^2Q_UI_L^2 - 12I_U^3Q_UI_LQ_L + 18I_U^2Q_U^2I_L^2Q_L

-6I_U^2Q_U^3Q_L^3 - 4I_UQ_U^4I_L^2 + 12I_UQ_U^3I_LQ_L^2 - 3Q_U^4I_L^2Q_L + Q_U^4Q_L^3

I_7 = I_U^3I_L^2 - 6I_U^2I_L^2Q_L^2 + I_U^5Q_L^4 + 12I_U^4Q_UI_L^4Q_L - 12I_U^3Q_UI_LQ_L^3 - 3I_UQ_U^2I_L^4

+18I_UQ_U^2I_L^2Q_L^2 - 3I_UQ_U^3I_LQ_L - 4Q_U^3I_L^3Q_L + 4Q_U^3I_LQ_L^3

Q_{-7} = 4I_U^3I_L^2Q_L - 4I_U^3I_LQ_L^3 - 3I_UQ_UI_L^4 + 18I_U^2Q_UI_L^2Q_L^3 - 3I_UQ_UQ_L^4

-12I_UQ_U^2I_L^3Q_L + 12I_UQ_U^3I_LQ_L^3 + Q_U^3I_L^4 - 6Q_U^3I_L^2Q_L^2 + Q_U^3Q_L^4

I_9 = I_U^3I_L^2 - 6I_U^2I_L^2Q_L^2 + I_U^5Q_L^4 + 20I_U^4Q_UI_L^4Q_L - 20I_U^3Q_UI_LQ_L^3 - 10I_U^2Q_U^2I_L^5

+60I_U^2Q_U^2I_L^4Q_L - 10I_U^2Q_U^3I_LQ_L^3 - 40I_U^2Q_U^3I_L^2Q_L + 40I_UQ_U^3I_LQ_L^3

+5I_UQ_U^4I_L^4 - 30I_UQ_U^4I_L^2Q_L^2 + 5I_UQ_U^4Q_L^4 + 4Q_U^5I_L^2Q_L - 4Q_U^5I_LQ_L^3

Q_9 = -4I_U^3I_L^2Q_L + 4I_U^3I_LQ_L^3 + 5I_U^4Q_UI_L^4 - 30I_U^3Q_UI_L^2Q_L^3 + 5I_U^4Q_UQ_L^4 +

40I_U^3Q_U^2I_L^2Q_L - 40I_U^2Q_U^3I_LQ_L^3 - 10I_U^2Q_U^3I_LQ_L^3 + 60I_U^2Q_U^3I_L^2Q_L - 10I_U^2Q_U^3Q_L^4

-20I_UQ_U^4I_L^2Q_L + 20I_UQ_U^4Q_L^3 + Q_U^5I_L^4 - 6Q_U^5I_L^2Q_L^2 + Q_U^5Q_L^4

I_{-9} = I_U^4I_L^5 - 10I_U^3I_L^3Q_L^2 + 5I_U^4I_LQ_L^4 + 20I_U^3Q_UI_L^5Q_L - 40I_U^3Q_UI_L^2Q_L^3

+4I_U^3Q_U^2I_L^3Q_L^2 - 6I_U^2Q_U^3I_L^3Q_L^2 + 60I_U^2Q_U^3I_L^2Q_L - 30I_U^2Q_U^3I_LQ_L^4 - 20I_UQ_U^3I_L^4Q_L

+40I_UQ_U^3I_L^2Q_L^3 - 4I_UQ_U^3Q_L^5 + Q_U^4I_L^5 - 10Q_U^4I_L^3Q_L^2 + 5Q_U^4I_LQ_L^4

Q_{-9} = 5I_U^4I_L^3Q_L - 10I_U^3I_L^3Q_L^2 + I_U^4Q_L^5 - 4I_U^3Q_UI_L^5 + 40I_U^3Q_UI_L^2Q_L^3

-20I_U^2Q_U^3I_L^3Q_L^2 - 30I_U^2Q_U^3I_L^2Q_L + 60I_U^2Q_U^3I_LQ_L^4 - 6I_U^2Q_U^3Q_L^5 + 4I_UQ_U^3I_L^5

-40I_UQ_U^3I_L^2Q_L^3 + 20I_UQ_U^3I_LQ_L^4 + 5Q_U^4I_L^4Q_L - 10Q_U^4I_L^2Q_L^2 + Q_U^4Q_L^5

87
Appendix B: PROOF OF THE HIGHER ORDER IMD POLYNOMIAL EQUIVALENCE

In [20], only 3rd order IMD terms had been compensated. To extend to higher order IMDs (i.e. up to 5th, 7th, or even higher order), MATLAB code has been developed which returns higher order IMD terms. In this section, the equivalence between the previous scheme and the new one is verified.

Let us assume \( I' \) and \( Q' \) are the output of the IQ-modulator of the inter-band stage, then

\[
I' = \alpha I - \beta Q \tag{B.1}
\]

\[
= \{\text{Re}(\alpha_3)E^2 - \text{Im}(\alpha_3)\hat{E}^2\}I - \{\text{Re}(\beta_3)E^2 - \text{Im}(\beta_3)\hat{E}^2\}Q,
\]

\[
Q' = \beta I + \alpha Q \tag{B.2}
\]

\[
= \{\text{Re}(\beta_3)E^2 - \text{Im}(\beta_3)\hat{E}^2\}I - \{\text{Re}(\alpha_3)E^2 - \text{Im}(\alpha_3)\hat{E}^2\}Q.
\]

In the general case, we have:

\[
\alpha = \frac{Z_1 - jZ_2}{2},
\]

\[
\beta = \frac{Z_2 + jZ_1}{2}.
\]

For the lower sideband comparison, let us assume \( \alpha_3 = Z_1 \) and \( \beta_3 = jZ_1 \) then,

\[
\alpha_3 = \text{Re}(Z_1) + j\text{Im}(Z_1),
\]

88
\[ \beta_3 = -\text{Im}(Z_1) + j\text{Re}(Z_1). \]

Therefore, we have:

\[ \text{Re}(\alpha_3) = \text{Im}(\beta_3); \text{Im}(\alpha_3) = -\text{Re}(\beta_3). \]  

(B.3)

Inserting Eq.B.3 in (1) and (2) gives,

\[ I' = \text{Re}(\alpha_3)E^2I - \text{Im}(\alpha_3)\hat{E}^2I - \text{Re}(\beta_3)E^2Q + \text{Re}(\beta_3)\hat{E}^2Q \]

\[ = \text{Re}(\alpha_3)E^2I - \text{Im}(\alpha_3)\hat{E}^2I - \text{Im}(\alpha_3)E^2Q + \text{Re}(\alpha_3)\hat{E}^2Q \]

\[ Q' = \text{Re}(\alpha_3)E^2Q - \text{Im}(\alpha_3)\hat{E}^2Q + \text{Re}(\beta_3)E^2I - \text{Im}(\beta_3)\hat{E}^2I \]

After a few algebra steps, we obtain

\[ I' = \text{Re}(\alpha_3)(E^2I + \hat{E}^2Q) - \text{Im}(\alpha_3)(\hat{E}^2I - E^2Q) \]  

(B.4)

\[ Q' = -\text{Im}(\alpha_3)(E^2I + \hat{E}^2Q) - \text{Re}(\alpha_3)(\hat{E}^2I - E^2Q) \]  

(B.5)

\[ E^2I + \hat{E}^2Q \]

\[ = 2(I_U I_L - \hat{I}_U \hat{I}_L)(I_U + I_L) + 2(I_U \hat{I}_L + \hat{I}_U I_L)(Q_U + Q_L) \]

\[ = 2(I_U^2 I_L - I_U \hat{I}_L I_L + I_U I_L^2 - I_L \hat{I}_U \hat{I}_L) \]

\[ + 2(I_U \hat{I}_U \hat{I}_L + I_L \hat{I}_U^2 - I_U \hat{I}_L^2 - I_L \hat{I}_U \hat{I}_L) \]

\[ = 2(I_U^2 + I_U^2)I_L + 2(I_L^2 - \hat{I}_L^2)I_U - 4I_L \hat{I}_U \hat{I}_L \]

\[ = 2\{E_U^2 I_L + (I_L^2 - Q_L^2)I_U + 2I_L Q_U Q_L\} \]

\[ = 2\{E_U^2 I_L + I_L(3)\} \]  

(B.6)
Similarly, we obtain

\[
\hat{E}^2 I - E^2 Q
= 2(I_U \hat{I}_L + \hat{I}_U I_L)(I_U + I_L) - 2(I_U I_L - \hat{I}_U \hat{I}_L)(Q_U + Q_L)
= 2(-I_U Q_L + Q_U I_L)(I_U + I_L) - (I_U I_L + Q_U Q_L)(Q_U + Q_L)
= 2\{-I_U^2 Q_L + Q_U I_U I_L - I_U I_L Q_L + Q_U I_L^2
- I_U I_L Q_U - Q_U^2 Q_L - I_U I_L Q_L - Q_U Q_L^2}\}
= -2\{E_U Q_L + 2I_U I_L Q_L - (I_L^2 - Q_L^2)Q_U\}
= -2\{E_U^2 Q_L + Q_L(3)\} \quad \text{(B.7)}
\]

For the upper sideband, let us assume \(\alpha_3 = Z_2\) and \(\beta_3 = -jZ_2\) then,

\[
\alpha_3 = \text{Re}(Z_2) + j\text{Im}(Z_2),
\beta_3 = \text{Im}(Z_2) - j\text{Re}(Z_2).
\]

After same simple algebra, we obtain

\[
I' = \text{Re}(\alpha_3)(E^2 I - \hat{E}^2 Q) - \text{Im}(\alpha_3)(\hat{E}^2 I + E^2 Q) \quad \text{(B.8)}
Q' = \text{Im}(\alpha_3)(E^2 I - \hat{E}^2 Q) + \text{Re}(\alpha_3)(\hat{E}^2 I + E^2 Q) \quad \text{(B.9)}
\]

The terms, \(E^2 I - \hat{E}^2 Q\) and \(\hat{E}^2 I + E^2 Q\), are given by the following equations:

\[
E^2 I - \hat{E}^2 Q = 2\{E_L^2 I_U + I_U(3)\} \quad \text{(B.10)}
\]

\[
\hat{E}^2 I + E^2 Q = 2\{E_U^2 Q_U - Q_U(3)\} \quad \text{(B.11)}
\]

The terms, \(E_L^2 I_U, E_U^2 Q_L, E_U^2 I_U,\) and \(E_L^2 Q_U,\) in equations (B.6), (B.7), (B.10), and (B.11), respectively are from the non-orthogonal structure and can be eliminated by adopting an orthogonal scheme.
Higher order terms, i.e. 5th, 7th, and even higher terms, can be calculated numerically from the MATLAB code.
Bibliography


