Developmental Analysis and Design of a Scaled-down Test Facility for a VHTR Air-ingress Accident

THESIS

Presented in Partial Fulfillment of the Requirements for the Degree Master of Science in the Graduate School of The Ohio State University

By

David J. Arcilesi Jr.

Graduate Program in Nuclear Engineering

The Ohio State University

2012

Master's Examination Committee:

Dr. Richard N. Christensen, Advisor

Dr. Xiaodong Sun, Co-advisor
Abstract

A critical event in the safety analysis of the Very High-temperature Gas-cooled Reactor (VHTR) is a loss-of-coolant accident (LOCA). This accident is initiated, in its worst case scenario, by a double-ended guillotine break of the hot duct, which leads to a rapid reactor depressurization. In a VHTR, the reactor vessel is located within a reactor cavity that is filled with air during normal operating conditions. During a LOCA, an air-helium mixture may enter the reactor vessel following a reactor vessel depressurization. Since air chemically reacts with high-temperature graphite, this could lead to damage of core-bottom and in-core graphite structures as well as core heat-up, toxic gas release, and failure of the structural integrity of the system unless mitigating actions are taken. Therefore, it is imperative to understand the dominant mechanism(s) in the air-ingress process so that mitigating measures can be considered for VHTR designs.

Early studies postulated that the dominant mechanism of air ingress is molecular diffusion. In general, however, molecular diffusion is a slow process, and recent studies show that the air-ingress process could be initially controlled by density-driven stratified flow of hot helium and a relatively cool air-helium mixture in the hot duct. If density-driven stratified flow initially dominates, earlier onset of natural circulation within the core would occur. This would lead to an earlier onset of oxidation of internal graphite structures and, most likely, at a more rapid rate. Thus, it is important to understand both of these air ingress mechanisms in a VHTR. These mechanisms may be important at
different times for different scenarios, specifically breaks of varying size, orientation, shape, and location.

Since no experimental data are readily available to understand the phenomena and determine which mechanism will dominate for various break conditions, there’s a need to design and construct a scaled-down experimental test facility to generate data. In this thesis, the scaling analysis, the developmental analysis and design of a scaled-down air-ingress accident test facility will be given. As part of the developmental analysis, the non-dimensional Froude number was preserved in establishing hydraulic similarity. On average, the non-dimensional resistance number of the scaled-down facility deviates 2.81% in terms of relative accuracy from the non-dimensional resistance number of the prototypic design. A 1/8th geometric scale is utilized for the entire geometry except for the hot duct length, support column pitch and support column diameter. The exceptions to the 1/8th geometric scale are to avoid large distortion of the loop pressure loss distribution (modified hot duct length) and to preserve the non-dimensional Froude number (modified support column diameter and pitch).

A heat transfer characterization of the lower plenum of the prototypic and scaled-down system was performed. The characterization focused on the support columns which are the principal heat source in the lower plenum during an air-ingress accident scenario. This analysis shows that a lumped capacitance approximation for the support columns is valid. Also, the analysis determines an operational heater power ($\dot{Q} = 125$ W) for shell/heater rods in the scaled-down system so that the rod surface temperature and the rod average radial heat flux ($\dot{Q} = 0$ W) can be preserved from the prototypic case.
In addition, a containment free volume \((V = 1 \text{ m}^3)\) was determined to house the scaled-down facility. With the containment free volume known, initial vessel pressures to preserve the air-to-helium mole ratio \((P = 40 \text{ psig})\) and mixed mean temperature \((P = 34.2 \text{ psig})\) of the prototype case were calculated. Finally, vessel design drawings and instrumentation are given.
Acknowledgements

This research is being performed using funding received from the DOE Office of Nuclear Energy’s Nuclear Energy University Programs and The Ohio State University. The author would also like to acknowledge the assistance and support of his advisors - Dr. Richard N. Christensen and Dr. Xiaodong Sun.
Vita

June 2000 .......................................................Skyline High School, Salt Lake City, UT
2008.............................................................B.S. Mathematics, University of Utah
2008.............................................................B.S. Physics, University of Utah
2012.............................................................Graduate Fellow, Department of Mechanical
                                          Engineering, Nuclear Engineering Graduate
                                          Program, The Ohio State University

Publications

   Analysis for Air Ingress Experiments for a VHTR,” Transactions of the American

   Ingress and Hot Plenum Natural Circulation for a VHTR,” Transactions of the American

   Characterization for a Scaled-down Air-ingress Accident Test Facility,” Submitted and
   Accepted for ANS 2012 Annual Meeting in Chicago, IL

Fields of Study

Major Field: Nuclear Engineering
Table of Contents

Abstract ............................................................................................................................... ii

Acknowledgements ............................................................................................................. v

Vita ..................................................................................................................................... vi

List of Tables ..................................................................................................................... xi

List of Figures .................................................................................................................. xiii

Chapter 1: Background and Literature Review ............................................................... 18

Introduction ................................................................................................................... 18

Air-ingress Accident Phenomenology .......................................................................... 21

Geometry ....................................................................................................................... 23

Scaling Analysis ............................................................................................................ 26

Thermo-physical Properties .......................................................................................... 27

Chapter 2: Theoretical Derivations for the Physical Setup of the System ................. 29

Geometric Scaling Analysis on Reactor System ........................................................... 29

Hydraulic Similarity in the Hot Duct-Hot Plenum System ........................................... 32

Heat Transfer Characterization on Reactor System ...................................................... 43

Power Scaling Analysis for Scaled-down Facility ........................................................ 51
Support Column One-dimensional Heat Diffusion Transient Analysis ...................... 56

Support Column Two-dimensional Heat Diffusion Transient Analysis .............. 62

Design Analysis of Containment ........................................................................... 71

Transient Blowdown Analysis ............................................................................. 75

Chapter 3: Physical Description of the Scaled-down Facility .................................. 87

Introduction .......................................................................................................... 87

Design Drawings .................................................................................................... 88

Instrumentation of the Physical Setup ................................................................... 103

Pressure ................................................................................................................. 103

Temperature ............................................................................................................. 104

Oxygen Sensors ....................................................................................................... 105

Local Velocity .......................................................................................................... 108

Conclusion ............................................................................................................... 109

Appendix A: The Complete Results from One-dimensional Support Column Fin

Analysis .................................................................................................................... 116

Prototype Geometry and Specifications ............................................................... 117

Scaled-down Geometry and Specifications ......................................................... 122

Appendix B: MATLAB Codes .................................................................................. 127
MATLAB code for heat transfer characteristics of a support column using a one-dimensional fin analysis with fixed temperature (Dirichlet) boundary conditions..... 127

MATLAB code for heat transfer characteristics of a support column using a one-dimensional fin analysis with mixed boundary conditions ........................................... 131

MATLAB code for heat transfer characteristics of a support column using a one-dimensional fin analysis with constant wall temperature................................. 135

MATLAB code for one-dimensional transient temperature distribution of prototypic and scaled-down support columns ......................................................... 139

MATLAB code for one-dimensional transient temperature distribution of shell/heater system for scaled-down test section ......................................................... 142

MATLAB code for two-dimensional transient temperature distribution of prototypic support column ....................................................................................... 145

MATLAB code for two-dimensional transient temperature distribution of support column in scaled-down test section ......................................................... 150

MATLAB code for two-dimensional transient temperature distribution of shell/heater system in scaled-down test section ......................................................... 155

MATLAB code for mixed mean temperature in reactor/containment system without air mass reduction ......................................................................................... 160

MATLAB code for mixed mean temperature in reactor/containment system with air mass reduction ......................................................................................... 162
MATLAB code for blowdown transient analysis of prototypic GT-MHR................. 165

MATLAB code for blowdown transient analysis of scaled-down test section.......... 168
List of Tables

Table 1. List of key dimensions for VHTR pressure vessel ............................................ 25
Table 2. Species composition and temperatures for each segment for single-species calculations ....................................................................................................................... 37
Table 3. Species composition and temperatures for each segment for mixed species calculations ....................................................................................................................... 40
Table 4. Resultant support column pitches and diameters ........................................... 41
Table 5. Relative accuracy of resistance for each case ................................................. 42
Table 6. Summary of one-dimensional fin analysis ...................................................... 45
Table 7. Summary of air-ingress phenomenon time scales .......................................... 56
Table 8. Boundary conditions and initial condition for transient one-dimensional analysis ................................................................................................................. 57
Table 9. Mesh size and time step for one-dimensional transient analysis ................. 58
Table 10. Boundary conditions for two-dimensional analysis .................................... 63
Table 11. Total time scale, convective heat transfer coefficient, and far-field temperature for case (1), (2), and (3) ................................................................................................................. 64
Table 12. Mesh size and time step for two-dimensional transient analysis ............... 65
Table 13. Temperature gradient at the vertical mid-plane of the support column ......... 69
Table 14. Scaled-down containment free volume for different initial vessel pressures .. 72
Table 15. Initial conditions for final temperature analysis ........................................... 73
Table 16. Results of transient control volume analysis following depressurization........ 74

Table 17. Critical dimensions for the scaled-down test facility................................. 88

Table 18. Pressure transducer specifications ............................................................. 103

Table 19. List of oxygen sensor candidates considered for scaled-down facility ....... 106
List of Figures

Figure 1. Schematic of 600 MWth GT-MHR pressure vessel (Reference design for the VHTR) [10]............................................................................................................................................. 19

Figure 2. Progression of air-ingress scenario [10].................................................................................................................... 20

Figure 3. Schematic of the 600 MWth GT-MHR [10] ................................................................................................................. 24

Figure 4. Ingress to hot exit plenum (blue) and enlarged view of hot duct-hot plenum system ................................................................................................................................................................................. 30

Figure 5. Hot duct-hot exit plenum system with control volumes outlined............................................................................................................ 34

Figure 6. Average heat transfer coefficient v. far-field temperature for prototype geometry ........................................................................................................................................................................... 46

Figure 7. Average heat transfer coefficient v. air mole fraction for prototype geometry. 46

Figure 8. Average Biot number v. far-field temperature for prototype geometry .......... 47

Figure 9. Average Biot number v. air mole fraction for prototype geometry.............. 47

Figure 10. Effect of thermo-physical properties of air-helium mixtures on heat transfer coefficient ........................................................................................................................................................................ 50

Figure 11. Flow depth of the heavy current [10] ..................................................................................................................... 52

Figure 12. Thermal time constant v. far-field temperature for prototype geometry....... 55

Figure 13. Thermal time constant v. far-field temperature for scaled-down geometry... 55

Figure 14. Radial temperature profile for prototype geometry at different times (t/\tau_{total}) 59
Figure 15. Radial temperature profile for scaled-down geometry at different times ($t/\tau_{total}$) ........................................................................................................................................ 59

Figure 16. Radial temperature profile for shell/heater system at heater power of 150 W 60

Figure 17. Radial temperature profile for shell/heater system at heater power of 125 W 60

Figure 18. Radial temperature profile for shell/heater system at heater power of 100 W 61

Figure 19. Radial temperature profile for shell/heater system at heater power of 0 W ... 61

Figure 20. Temperature contour plot for prototype geometry (Case 1) at $t=3\tau_{total}$ ........ 66

Figure 21. Temperature contour plot for scaled-down geometry (Case 2) at $t = 3\tau_{total}$ ... 67

Figure 22. Temperature contour plot for shell/heater system for 150 W (Case 3a) at $t=3\tau_{total}$ ........................................................................................................................................ 67

Figure 23. Temperature contour plot for shell/heater system for 125 W (Case 3b) at $t=3\tau_{total}$ ........................................................................................................................................ 68

Figure 24. Temperature contour plot for shell/heater system for 100 W (Case 3c) at $t=3\tau_{total}$ ........................................................................................................................................ 68

Figure 25. Temperature contour plot for shell/heater system for 0 W (Case 3d) at $t=3\tau_{total}$ ........................................................................................................................................ 69

Figure 26. Pressure versus time during depressurization of prototypic vessel ............... 81

Figure 27. Helium temperature versus time during depressurization of prototypic vessel ........................................................................................................................................ 81

Figure 28. “Wall” temperature versus time during depressurization of prototypic vessel ........................................................................................................................................ 82

Figure 29. Pressure versus time during depressurization of scaled-down test section .... 84
Figure 30. Helium temperature versus time during depressurization of scaled-down test section ........................................................................................................................................... 84

Figure 31. “Wall” temperature versus time during depressurization of scaled-down test section ........................................................................................................................................... 85

Figure 32. Hot-exit plenum of the GT-MHR 600 MWth by General Atomics, Inc. 87

Figure 33. Side view of the bottom semi-hemispherical shell ................................................................. 89

Figure 34. Conax® gland at the end of the pipe on the bottom semi-hemispherical shell (Units: in.) ........................................................................................................................................... 91

Figure 35. Bottom view of the bottom semi-hemispherical shell ................................................................. 92

Figure 36. Bottom view of the bottom semi-hemispherical shell with support column positions projected ........................................................................................................................................... 92

Figure 37. Top view of bottom plate (top) and side view of bottom plate (bottom) ........... 93

Figure 38. Bottom view of the bottom plate with (left) and without (right) support column positions projected ........................................................................................................................................... 94

Figure 39. Different views of the middle shell or test section ................................................................. 96

Figure 40. Bottom view (left) and side view (right) of middle shell ................................................................. 96

Figure 41. Low pressure sight glass on the side of the middle shell (B = 2.500 in., C = 1.875 in., D = 0.760 in., E = 0.290 in.) [30]. ........................................................................................................................................... 97

Figure 42. Top view of the top plate with (left) and without (right) support column positions projected ........................................................................................................................................... 97

Figure 43. Bottom view (top) and side view (bottom) of the top plate ......................................................... 98

Figure 44. Side view of the top semi-hemispherical shell ........................................................................ 100
Figure 45. Top view of the top semi-hemispherical shell .............................................. 101
Figure 46. Top view of top semi-hemispherical shell with support column positions projected .......................................................................................................................... 101
Figure 47. Side view of the assembled vessel ................................................................. 102
Figure 48. Side cut view of the assembled vessel .......................................................... 102
Figure 49. Schematic of Honeywell STG944 Pressure Transducer [31] ....................... 104
Figure 50. Schematic of Teledyne Series 9060 Zirconium Oxide Oxygen Sensor [32] 107
Figure 51. Average Biot number v. far-field temperature for prototype geometry ...... 117
Figure 52. Average Biot number v. air mole fraction for prototype geometry .......... 117
Figure 53. Average thermal time constant v. far-field temperature for prototype geometry ......................................................................................................................... 118
Figure 54. Average thermal time constant v. air mole fraction for prototype geometry .... 118
Figure 55. Average boundary layer thickness v. far-field temperature for prototype geometry ......................................................................................................................... 119
Figure 56. Average boundary layer thickness v. far-field temperature for prototype geometry ......................................................................................................................... 119
Figure 57. Average heat transfer coefficient v. far-field temperature for prototype geometry ......................................................................................................................... 120
Figure 58. Average heat transfer coefficient v. far-field temperature for prototype geometry ......................................................................................................................... 120
Figure 59. Average Rayleigh number v. far-field temperature for prototype geometry 121
Figure 60. Average Rayleigh number v. air mole fraction for prototype geometry ...... 121
Figure 61. Average Biot number v. far-field temperature for scaled-down geometry .. 122
Figure 62. Average Biot number v. air mole fraction for scaled-down geometry ....... 122
Figure 63. Average thermal time constant v. far-field temperature for scaled-down
gridometry ......................................................................................................................... 123
Figure 64. Average thermal time constant v. air mole fraction for scaled-down geometry ......................................................................................................................... 123
Figure 65. Average boundary layer thickness v. far-field temperature for scaled-down
gridometry ......................................................................................................................... 124
Figure 66. Average boundary layer thickness v. air mole fraction for scaled-down
gridometry ......................................................................................................................... 124
Figure 67. Average heat transfer coefficient v. far-field temperature for scaled-down
gridometry ......................................................................................................................... 125
Figure 68. Average heat transfer coefficient v air mole fraction for scaled-down
gridometry ......................................................................................................................... 125
Figure 69. Average Rayleigh number v. far-field temperature for scaled-down geometry ......................................................................................................................... 126
Figure 70. Average Rayleigh number v. air mole fraction for scaled-down geometry . 126
Chapter 1: Background and Literature Review

Introduction

The potential for an air-ingress accident stems from consideration of a postulated loss-of-coolant accident (LOCA) in a very high temperature gas-cooled reactor (VHTR). It is considered to be one of the most important safety issues facing the VHTR and, therefore, needs to be examined carefully for the reactor safety analysis [1]. This accident is initiated by a break in the hot duct of the reactor cross vessel. In the worst case scenario, the hot duct is completely severed and the reactor pressure vessel is disjoined from the energy conversion unit in what is referred to as a double-ended guillotine break. Following the break, a rapid depressurization of the vessel occurs. This leads to the ingress of an air-helium mixture into the reactor vessel. Since air chemically reacts with high-temperature graphite, this can lead to damage of in-core graphite structures and fuel, release of carbon monoxide, core heat up, failure of the structural integrity of the system, and release of radionuclides to the environment. Therefore, the air-ingress accident scenario and phenomena are very important in VHTR safety analyses.

Initially, the air-ingress mechanism, as shown in the literature, was considered to be controlled by molecular diffusion when a one-dimensional numerical tool was used [1] - [5]. According to these studies, when the air-ingress speed is controlled by molecular diffusion, governed by Fick’s law, the onset of natural circulation takes on the order of
100 hours to begin. This type of scenario gives the reactor operators sufficient time to take necessary actions to mitigate the accident scenario.

![Schematic of 600 MWth GT-MHR pressure vessel (Reference design for the VHTR)](image)

Figure 1. Schematic of 600 MWth GT-MHR pressure vessel (Reference design for the VHTR) [10]

However, recent studies have shown that the previous works in which the air-ingress mechanism is dominated by molecular diffusion might mislead what will actually occur during an accident scenario [7] - [10]. These recent studies show that the air
ingress process might not initially be controlled by molecular diffusion but rather by a density driven-stratified flow of hot helium and a relatively cold air-helium mixture. This generally happens when a heavy fluid (relatively cold air-helium mixture) in the reactor cavity intrudes into a light fluid (hot helium) in the lower plenum. Calculations using multi-dimensional computational fluid dynamic (CFD) models have shown that the time for the onset of natural circulation is significantly reduced to a couple of minutes when density-driven stratified flow dominates. This certainly changes the accident scenario and the scale for mitigating procedures. Moreover, these simulations also demonstrate a new air-ingress scenario that consists of four steps: (1) depressurization, (2) stratified flow (stage 1), (3) stratified flow (stage 2), and (4) natural circulation [10].

Figure 2. Progression of air-ingress scenario [10]
The aforementioned air-ingress mechanisms – molecular diffusion and density-driven stratified flow – may be important for different scenarios, specifically for different break sizes, shape, and locations. However, there is no experimental data readily available to verify under which break conditions each mechanism dominates. Therefore, it would be useful to have experimental data on the temperature, pressure, velocity and species concentration of fluid flowing through the hot duct and lower plenum of the pressure vessel. From the experimental data, the air-ingress phenomena will be better understood so that potential mitigating strategies can be identified, it will provide an understanding for which air-ingress mechanism dominates under a given set of break conditions and it will also provide a means to validate CFD analyses. This report will discuss the analysis and design for a scaled-down experimental facility which will investigate the different phenomena surrounding a VHTR air-ingress accident stemming from a break in the hot duct of the cross vessel.

Air-ingress Accident Phenomenology

After a hot duct break in the cross vessel of the VHTR, the coolant (helium) inside the reactor is discharged out of the reactor vessel into a vented, air-filled containment. Following the depressurization, the air-helium mixture in the containment enters into the hot exit plenum of the reactor vessel through the bottom part of the hot duct. There are two mechanisms by which the air-helium mixture in the containment ingresses into the hot exit plenum: density-driven stratified flow and molecular diffusion. Density-driven stratified flow, a countercurrent flow, is driven by a density difference between the relatively cool air-helium mixture (outside the vessel) and hot helium (inside
the vessel). It should be noted that the density difference between the two fluids is due to
the difference in effective molecular weight of each fluid as well as the difference in each
fluid’s temperature. On the other hand, molecular diffusion is driven by the difference in
air concentrations inside and outside the reactor. According to the literature [10], the
time scale of density driven stratified flow is three orders of magnitude less than the time
scale of molecular diffusion. A small time scale means the process is fast and a large
time scale means the process is slow. Therefore, the density-driven stratified flow is the
dominant mechanism as the air-helium mixture is filling the hot exit plenum and
molecular diffusion can be neglected.

After the air-helium mixture fills the hot exit plenum, another type of
countercurrent flow occurs driven by the temperature differences between the inside and
outside of the vessel. The physical mechanism that drives this phenomenon is similar to
that discussed in the density-driven stratified flow case except this buoyancy flow is due
only to the temperature difference; not the temperature difference and the effective
molecular weight of the fluid. Therefore, this flow does not stop during the air-ingress
process since the temperature gradient on the inside and the outside of the reactor vessel
always exist. This natural convective flow can continue into the reactor core when the
core heats up sufficiently. By the core heating up, this creates a density difference
between the fluid in the core and the fluid in the upcomer. When this density difference
is large enough to overcome the pressure losses in the circulation loop, this initiates the
onset of global natural circulation. The scaled-down facility is designed to examine the
density-driven stratified flow and hot plenum natural circulation, the local natural convective flow through the hot plenum.

**Geometry**

Fundamental to the design and construction of a scaled-down experimental facility is an understanding of the design specifications and geometry of the prototype facility. An extensive literature review was performed to find all available information on the geometry of the VHTR.

From Kodochigov [11], average core specific power, number of fuel compacts per fuel block, number of reactivity control rods (in/out core), number of reserve shutdown systems, allocated operative reactivity margin on control rods, number of hexagonal blocks in the outer reflector and its height, number of hexagonal blocks in the active core region, number of hexagonal blocks in the inner reflector and its height were found.

From the NGNP Point Design [12], the coolant pressure, the reactor pressure vessel height and its material composition, the cross vessel material, the core barrel material, the percentage of bypass flow between fuel elements, the maximum allowable temperature of the vessel wall, and the material composition of control rods were found.

In reference [13], the hot duct inner radius, the hot duct pipe thickness, the hot duct material, the permanent side reflector material, the inner diameter of the core barrel, the core barrel thickness, and its material composition were given. Also, figures showing a cross-section of the VHTR shutdown cooling system as well as the metallic internal structures, core, and control rod guide tubes were found.
In Neylan et al [14], the vessel inner radius is disclosed. Tak et al. give a descriptive figure of the hexagonal fuel block [15]. Condie lists the vessel height, the number of coolant channels in a standard fuel element, and the number of coolant channels in a control fuel element [16].
Reza gives the active core height, the hot exit plenum height, the support column height, the support column thickness, the distance between support columns, and the gap between plenum heads [17].

General Atomics, Inc. gives the core power, the core inlet and outlet temperatures, the helium mass flow rate, the vessel dimensions, the shutdown cooling system dimensions, the control rod assembly housing dimensions, and the hot duct and cold duct dimensions [18].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prototype (m)</th>
<th>1/8th Scale (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vessel Height</td>
<td>23.7</td>
<td>2.963</td>
</tr>
<tr>
<td>Vessel Inner Diameter</td>
<td>7.8</td>
<td>0.975</td>
</tr>
<tr>
<td>Vessel Outer Diameter</td>
<td>8.4</td>
<td>1.050</td>
</tr>
<tr>
<td>Core Height</td>
<td>11</td>
<td>1.375</td>
</tr>
<tr>
<td>Active Core Height</td>
<td>7.8</td>
<td>0.975</td>
</tr>
<tr>
<td>Support Column Height</td>
<td>2.84</td>
<td>0.355</td>
</tr>
<tr>
<td>Cold Duct Inner Diameter</td>
<td>2.29</td>
<td>0.286</td>
</tr>
<tr>
<td>Hot Duct Inner Diameter</td>
<td>1.43</td>
<td>0.179</td>
</tr>
<tr>
<td>Support Column Diameter*</td>
<td>0.212</td>
<td>0.027</td>
</tr>
<tr>
<td>Support Column Pitch*</td>
<td>0.36</td>
<td>0.045</td>
</tr>
</tbody>
</table>

Table 1. List of key dimensions for VHTR pressure vessel

Table 1 is a list of the key dimensions for the prototype and those for a 1/8th geometric scale VHTR pressure vessel. The 1/8th geometric dimensions are shown in Table 1 for illustrative purposes and can be used for the scaled-down facility if the 1/8th scale proves to be an acceptable choice. Since this study is focused on the phenomena occurring within the hot duct and lower plenum geometries, some of the dimensions will not be directly used in this study. However, it is beneficial and instructive to include them for completeness of the overall basic geometry. Building a full scale model is not
practically feasible for financial reasons as well as not having sufficient space to house a facility that large. Therefore, under the current parameters, a scaled-down model is the only solution in order to collect experimental data for the early stages of the air-ingress scenario. A scaled-down factor is determined by performing a parametric analysis of the Rayleigh number. In the prototypic geometry, the Rayleigh number is on the order of $10^9$ where the support column height is taken to be the characteristic length. Therefore, the goal is to find a scale-down factor that is small enough that it maintains the Rayleigh number in the laminar flow regime ($10^4 \leq \text{Ra} \leq 10^9$) while still being large enough to allow the scaled-down facility to be economical and practical to work with. Utilizing a 1/8th geometric scale, the Rayleigh number is on the order of $10^6$. This Rayleigh number preserves the natural circulation phenomenology of the prototype geometry. Moreover, with a 1/8th geometric scale, the overall height of the facility is approximately 7 feet. This is a height that is practical to work with and economical. Justification for the actual scale used for the scaled-down facility is the topic of this thesis.

Scaling Analysis

Several scaling analyses were procured in the available literature [19] - [23], [27]. These analyses ([19] - [23]) start with the fundamental equations of transport phenomena; namely, the continuity equation, the momentum equation and, in most cases, the energy equation. These fundamental equations are non-dimensionalized in various forms (depending on the paper), yielding different non-dimensional scaling parameters. Since most these scaling analyses focused mainly on natural circulation and do not consider density-driven stratified flow, they are not used as models for this scaling analysis to the
scaled-down test facility. Reyes et al. [27] derived a scaling analysis which considers many of the same phenomena on a similar type of facility that will be examined on the scaled-down test facility. Therefore, this analysis is used, in part, as a motivation for the scaling analysis of the scaled-down test facility. The derivation of this analysis is presented at the beginning of Chapter 2.

**Thermo-physical Properties**


An understanding of the air-ingress phenomenology, the prototypic geometry, the scaling analysis as presented in the beginning of Chapter 2, and thermo-physical properties will be used to demonstrate hydraulic and heat transfer similarity between the prototype and scaled-down hot duct-lower plenum system in Chapter 2. Furthermore, a design for the scaled-down facility will be given in Chapter 3.
Chapter 2: Theoretical Derivations for the Physical Setup of the System

Geometric Scaling Analysis on Reactor System

In this analysis, prototypic fluids at prototypic pressure and temperature have been assumed [27]. Therefore, no fluid-to-fluid scaling was performed. From the continuity equation, the mass flow rate at every cross-section for the $i^{th}$ segment along the loop is constant in Figure 4. Mathematically, this can be expressed as seen in equation [1].

$$\dot{m} = \dot{m}_i$$  \[1\]

where $\dot{m}_i$ is the mass flow rate and $\dot{m}_i$ is the mass flow rate in the $i^{th}$ segment. The integrated loop momentum equation is written as follows:

$$\frac{d\dot{m}}{dt} \sum_l I_i = (\rho_c - \rho_h)gH - \frac{\dot{m}_i^2}{\rho_ra_i^2} \sum_r \left( \frac{f_l}{d_h} + K_i \right) \left( \frac{a_r}{a_i} \right)^2$$ \[2\]

where $l_i$ and $a_i$ are the length and the cross-sectional area of the $i^{th}$ segment, respectively. $\rho_c$, $\rho_h$, and $g$ are the cold-side density, the hot-side density and the acceleration due to gravity, respectively. $H$, $\rho_c$, and $a_c$ are the vertical distance between the thermal centers of the hot and cold side, the density of the reference segment and the cross-sectional area, respectively.
of the reference segment, respectively. \( f, d, \) and \( K \) are the Darcy friction factor, the hydraulic diameter and the minor loss coefficient of the \( i^{th} \) segment, respectively.

The loop momentum equation can be made dimensionless by normalizing the terms relative to their initial conditions or boundary conditions. This is denoted by the subscript "o". That is,

\[
\dot{m}^* = \frac{\dot{m}}{m_o} = \frac{\dot{m}}{\rho_r a_r w_o} \tag{3}
\]

\[
\rho_{H}^* = \frac{\rho_{H}}{\rho_c - \rho_{H_o}} \tag{4}
\]
Substituting these ratios into the governing equation yields the following dimensionless equation:

\[
\frac{dm^*}{dt^*} = \Pi_f \left[ \left( \frac{\rho^*_c - \rho^*_h}{\rho^*_f} \right) \frac{H^*}{\Pi_{fr}} - (m^*)^2 \Pi_f \left[ \sum_i \frac{1}{2} \left( \frac{fl}{d_h} + K \right) \left( \frac{a_e}{a_i} \right)^2 \right] \right]
\]

with

\[
H^* = \frac{H}{H_o}
\]

The non-dimensional time scale is as follows:

\[
t^* = \frac{t}{\tau} = \frac{lw_o}{H_o}
\]

where \( w_o \) and \( H_o \) are the velocity of the reference section at the onset of natural circulation and the geometric height of the reference section, respectively. The non-dimensional groups are defined wherein the length or geometric scale, the non-dimensional Froude number and the friction number are given as follows:
\[ \Pi_L = \frac{H_o}{a_r \sum \frac{l_i}{a_i}} \]  

[10]  

\[ \Pi_{Fr} = \frac{\rho_r w_o^2}{(\rho_c - \rho_h)gH_o} \]  

[11]  

\[ \Pi_F = \left[ \sum_{i} \frac{1}{2} \left( \frac{f_l}{d_h} + K_i \left( \frac{a_c}{a_i} \right)^2 \right) \right] \]  

[12]  

Hydraulic Similarity in the Hot Duct-Hot Plenum System  

Utilizing the geometric scaling analysis, a model was created to find the best support column diameter and pitch for the test facility to mimic the pressure loss distribution of a hot duct-hot exit plenum system. This will be accomplished by calculating the pressure loss distribution and the Froude Number in the prototype system and then modifying the pitch and diameter of the support columns until the pressure loss distribution and Froude number in the experimental system matches that of the prototype.  

The model assumes that the test facility's height, support column height, hot duct diameter and plenum diameter are a 1/8th scale of the prototype dimensions. Furthermore, the hot duct length is assumed to be 0.1 m. In the event of a hot duct break at the edge of the power conversion unit, the scaled-down length is 0.3575 m (which is equal to 1/8 of 2.86 m). Therefore, in this model, the hot duct length is approximately 28% of the scaled-down hot duct length. Other dimensions such as the support column diameter and pitch are scaled by different factors to be determined in the current analysis.
The model, which is used to find the support column diameter and pitch in the scaled-down facility, divides the circulation flow path in the hot duct-hot exit plenum system into five segments or control volumes. The first segment is the bottom half of the hot duct. The second segment is the bottom half of the hot exit plenum whose constant width is equal to the vessel diameter (6.8 m in the prototype geometry; 0.85 m in the scaled-down geometry). This means that for the purpose of this analysis the shape of the second segment is a right rectangular prism or right cuboid – not a right circular cylinder which is the prototypic geometry. This type of geometry is prescribed for the analysis because the pressure loss correlation for lateral flow resistance across bare rod arrays is dependent on the number of tube rows in the direction of flow [30]. Therefore, it gives a value to the pressure loss for a rectangular array of tubes. The third segment is the gas rising from the bottom to the top of the hot exit plenum. In this analysis, the flow area of this segment is circular-shaped which is expected based on the prototype geometry. The fourth segment is the top half of the hot exit plenum whose constant width is again equal to the vessel diameter as explained for the second segment. The fifth segment is top half of the hot duct so its flow area is a semicircle. The hot duct-hot exit plenum system with its corresponding five control volumes is shown in Figure 5.
Figure 5. Hot duct-hot exit plenum system with control volumes outlined

The first and fifth control volumes correspond to the bottom and top half of the hot duct, respectively. In these two segments, the friction loss is the only pressure loss taken into consideration. The interface of segment (1) and segment (5) is treated as an additional boundary due to a quasi no-slip boundary condition. This subtle detail becomes important when calculating the hydraulic diameter of these segments. Mathematically, the friction loss coefficient is computed as follows:

\[ K_{friction} = f \frac{I}{D_e} \]  \[ \text{[13]} \]

where \( f = \begin{cases} \frac{64}{\Re}, & 0 < \Re < 2308.1487 \\ \frac{3.03 \times 10^{-12} \Re^4 - 3.67 \times 10^{-8} \Re^2 + 1.46 \times 10^{-4} \Re - 0.151}{\Re^{0.3164}}, & 2308.1487 \leq \Re < 4210.0770 \\ \frac{0.3164}{\Re^{0.82}}, & 4210.0770 \leq \Re < 51094.3686 \\ \frac{0.184}{\Re^{0.2}}, & \Re \geq 51094.3686 \end{cases} \]

The piecewise function for the friction factor is continuous. The friction factor correlation for the laminar-to-turbulent transition regime can be found in J.P Abraham et al [28].
As the fluid passes from segment (1) to segment (2), the pressure loss due to expansion is taken into account. The calculation of the expansion loss coefficient utilizes the ratio of the cross-sectional area of segment (1) and the maximum cross-sectional area of segment (2). This results in a larger expansion loss coefficient than the real geometry would produce. The calculation done here assumes that the fluid empties directly into the largest cross-sectional area of the plenum which is obviously not the case. In reality, the fluid empties more gradually from the bottom half of the duct as it approaches the maximum cross-sectional area of the plenum. This is due to the cylindrical geometry of the hot exit plenum. The expansion loss coefficient is based on the equations and correlation given by Idelchik [29] (pp. 160) and vary with the Reynolds number.

The second and fourth control volumes correspond to the bottom and top half of the hot exit plenum, respectively. In these segments, there is a staggered array of bare rods that fill the entire control volume. Therefore, the only form of pressure loss that is accounted for in these segments is the friction pressure loss due to the flow normal to the triangular array of bare rods. It should be noted that this model assumes that all the fluid passes along the entire length of the plenum. This results in an overestimate of the friction pressure loss. In reality, only a fraction of the fluid will pass along the entire length of the plenum. Moreover, due to the nature of the friction correlations available, this model assumes that bottom of the plenum is a square or rectangular shape as opposed to a circle which is the actual geometry of the prototype. This assumption also leads to an overestimate of the friction pressure loss. Mathematically, the friction loss coefficient is expressed as follows:
where \( f \) is the friction factor based on a correlation given in Todreas [30]; \( N \) is the number of tube rows in the direction of flow; \( Z \) is a correction factor depending on the array arrangement [30].

As the fluid passes from segment (4) to segment (5), the pressure loss due to sudden contraction is taken into account. The calculation of the contraction loss coefficient utilizes the ratio of the cross-sectional area of segment (5) and the maximum cross-sectional area of segment (4). This results in a larger contraction loss coefficient than the real geometry would produce. By similar reasoning given previously, the calculation performed for this model assumes that the fluid empties directly from the largest cross-sectional area of the plenum into the top half of the hot duct which is not the case. In reality, the fluid is funnelled gradually into the top half of the hot duct from the maximum cross-sectional area of the top half of the hot exit plenum. However, since the same model is applied to both the prototype geometry as well as the scaled-down geometry, the overestimate in pressure loss can be neglected due to the consistent nature of the analysis for both geometry types. It is essentially the loop pressure loss distribution through which the hydraulic similarity is established. The contraction loss coefficient is based on the equations and correlation given by Idelchik (pp. 168) and vary with respect to the Reynolds number.
The third control volume corresponds to the vertical motion of the fluid from the bottom of the hot exit plenum to its top. In this segment, the frictional pressure loss is the only pressure loss taken into consideration. This friction loss is due to the fluid flow along the support columns.

To find the support column diameter and pitch, nine cases are considered – six of which are single-species calculations (either air or helium) and the other three are binary mixed-species calculations (both air and helium mixed to prescribed mole fractions). Table 2 shows the six single-species cases that are considered.

<table>
<thead>
<tr>
<th>Case</th>
<th>Species Composition</th>
<th>Hot Temperature (°C)</th>
<th>Cold Temperature (°C)</th>
<th>Average Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100% Helium</td>
<td>850</td>
<td>25</td>
<td>437.5</td>
</tr>
<tr>
<td>2</td>
<td>100% Helium</td>
<td>850</td>
<td>170</td>
<td>510</td>
</tr>
<tr>
<td>3</td>
<td>100% Helium</td>
<td>850</td>
<td>500</td>
<td>675</td>
</tr>
<tr>
<td>4</td>
<td>100% Air</td>
<td>850</td>
<td>25</td>
<td>437.5</td>
</tr>
<tr>
<td>5</td>
<td>100% Air</td>
<td>850</td>
<td>170</td>
<td>510</td>
</tr>
<tr>
<td>6</td>
<td>100% Air</td>
<td>850</td>
<td>500</td>
<td>675</td>
</tr>
</tbody>
</table>

Table 2. Species composition and temperatures for each segment for single-species calculations

For each case, segments (1) and (2) are at the cold temperature. Segments (4) and (5) are at the hot temperature and segment (3) is at the average temperature. The hot and cold temperatures find their origins from the transient blowdown analysis (For details, see Transient Blowdown Analysis in Chapter 2). The pressure for all five segments is atmospheric pressure. Since there is a temperature difference within the system, there exists a density difference or a driving force, $\Delta P_d$, for natural circulation.

Mathematically, the driving force is expressed as
\[ \Delta P_d = (\rho_c - \rho_h)gh \]  

where \( \rho_c \) and \( \rho_h \) are the densities for the cold and hot temperature, respectively; \( g \) is the acceleration of gravity and \( h \) is the height of the plenum. Therefore, for a given case, there is a set driving force for natural circulation in the system.

Having established the natural circulation driving force for the system, the velocity is iterated until the resistance pressure drop, \( \Delta P_{res} \), is essentially equal to the driving force, \( \Delta P_d \); i.e. \( |\Delta P_d - \Delta P_{res}| < 10^{-6} \). Mathematically, the resistance pressure drop is expressed as

\[ \Delta P_{res} = \frac{1}{2} \sum_{i=1}^{5} K_{T,i} \rho_i v_i^2 \]  

where \( K_{T,i} \) is the total loss coefficient for the \( i^{th} \) segment; \( \rho_i, v_i \) is the density of the fluid and the fluid velocity for the \( i^{th} \) segment, respectively. It should be noted that the model assumes that the amount of mass within the entire system does not change with time. Therefore, the mass flow rate remains constant from segment to segment. Using this fact along with the density and flow area of each segment, the velocity for a given control volume can be found.

Once this procedure has been followed for the prototypical geometry for a given set of temperatures, there is a unique non-dimensional Froude number that is recorded.
This is the same non-dimensional Froude number from the geometric scaling analysis and is defined as

\[ \Pi_{Fr} = \frac{\rho_r v_r^2}{(\rho_c - \rho_h) gh} = \frac{\rho_3 v_3^2}{(\rho_c - \rho_h) gh} \]  

[17]

Now, with the scaled-down geometry; which is to say, the plenum diameter, the plenum height, the support column height and hot duct diameter at 1/8th scale and the hot duct length reduced to 0.1 m, the velocity is adjusted to preserve the non-dimensional Froude number for a given case. This means that the velocity is reduced by a factor of \( \sqrt{8} \) in order to preserve the non-dimensional Froude number. Since an adjustment of both the fluid velocity as well as the test facility geometry has taken place, the driving force, in general, is no longer equal to the resistance pressure drop. The two quantities can be equated by scaling down the support column pitch and diameter by the same factor. The scaling factor is iterated until \( |\Delta P_f - \Delta P_{res}| < 10^{-6} \).

These simulations were completed for nine different cases. Cases (1) – (6), given in the previous section (Table 2), are single species simulations (either Air or He). Cases (7) – (9) are mixed species simulations. Again, all simulations were performed at atmospheric pressure for all control volumes. Table 3 shows the species composition and temperature by control volume.
Fluid density and dynamic viscosity for the mixed species compositions are calculated according to relations that take into account the combined effects of air and helium. The mixture density is determined from the ideal gas laws as a linear combination of the mole fraction of the components (1: air; 2: helium).

\[ \rho_{mix} = \rho_1 x_1 + \rho_2 x_2 \]  \[18\]

where \( x_1 \) and \( x_2 \) are the mole fractions of the individual components in the mixture. The dynamic viscosity of the mixture is determined by the following set of equations.

\[ \mu_{mix} = \mu_1 \left( 1 + \frac{x_2}{x_1} \Phi_{12} \right)^{-1} + \mu_2 \left( 1 + \frac{x_1}{x_2} \Phi_{21} \right)^{-1} \]  \[19\]

where

\[ \Phi_{ij} = \left[ 1 + \left( \frac{\mu_i}{\mu_j} \right)^{1/2} \left( \frac{M_j}{M_i} \right)^{1/4} \right]^2 \left[ 8 \left( 1 + \frac{M_i}{M_j} \right) \right]^{1/2} \]  \[i \neq j \]  \[20\]
These relations for density and dynamic viscosity can be found in Banerjee and Andrews [31].

By maintaining the non-dimensional Froude number similarity and adjusting the support column pitch and diameter to balance the natural circulation driving force with the pressure drop, a set of support column pitches and diameters is collected. Each case has a unique pitch and diameter that ensures that these conditions are satisfied. These values are tabulated in Table 4 along with the arithmetic average for the nine values.

<table>
<thead>
<tr>
<th>Case</th>
<th>Support Column Pitch (m)</th>
<th>Support Column Diameter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.118</td>
<td>0.069</td>
</tr>
<tr>
<td>2</td>
<td>0.152</td>
<td>0.089</td>
</tr>
<tr>
<td>3</td>
<td>0.234</td>
<td>0.138</td>
</tr>
<tr>
<td>4</td>
<td>0.072</td>
<td>0.042</td>
</tr>
<tr>
<td>5</td>
<td>0.071</td>
<td>0.042</td>
</tr>
<tr>
<td>6</td>
<td>0.100</td>
<td>0.059</td>
</tr>
<tr>
<td>7</td>
<td>0.066</td>
<td>0.039</td>
</tr>
<tr>
<td>8</td>
<td>0.105</td>
<td>0.062</td>
</tr>
<tr>
<td>9</td>
<td>0.065</td>
<td>0.038</td>
</tr>
<tr>
<td>Average</td>
<td>0.109</td>
<td>0.064</td>
</tr>
<tr>
<td>Adjusted Avg.</td>
<td>0.094</td>
<td>0.055</td>
</tr>
</tbody>
</table>

Table 4. Resultant support column pitches and diameters

Therefore, from the tabulated values, the support column pitch and diameter for the scaled down test facility to ensure hydraulic similarity is 10.9 cm and 6.4 cm, respectively. Moreover, if Case 3 is removed from consideration, **the averages drop to 9.4 cm for the pitch and 5.5 cm for the diameter**. This is justified since it’s not physically realizable to have 100% helium at a low temperature of 500°C and at a high temperature of 850°C circulating through the hot duct-hot exit plenum system within the scope of the accident. It should be noted that the pitch and diameter were scaled by the
same factor. Hence, the pitch-to-diameter ratio in the scaled-down test facility is equal to the pitch-to-diameter ratio in the prototype.

From the geometric scaling analysis, another key non-dimensional Pi term is the resistance number. The resistance number is defined as follows:

\[
\Pi_r = \sum_i \left( \frac{f_i}{D_i} + K \right) \left( \frac{a_i}{a_i} \right)^2
\]  

For the nine cases given in Table 5, the relative accuracy with respect to the resistance number of the prototype is given; i.e. 

\[
\frac{\Pi_{F_{prot}} - \Pi_{F_{Scaled}}}{\Pi_{F_{prot}}} \times 100\%.
\]

<table>
<thead>
<tr>
<th>Case</th>
<th>Relative Accuracy of the Resistance Number (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
</tr>
<tr>
<td>2</td>
<td>0.16</td>
</tr>
<tr>
<td>3</td>
<td>4.65</td>
</tr>
<tr>
<td>4</td>
<td>3.08</td>
</tr>
<tr>
<td>5</td>
<td>1.99</td>
</tr>
<tr>
<td>6</td>
<td>1.71</td>
</tr>
<tr>
<td>7</td>
<td>5.52</td>
</tr>
<tr>
<td>8</td>
<td>8.14</td>
</tr>
<tr>
<td>9</td>
<td>0.83</td>
</tr>
<tr>
<td>Average</td>
<td>3.01</td>
</tr>
<tr>
<td>Adjusted Average</td>
<td>2.81</td>
</tr>
</tbody>
</table>

Table 5. Relative accuracy of resistance for each case

This shows that there is good agreement between the resistance number of the prototype and the resistance number of the scaled-down facility.
Heat Transfer Characterization on Reactor System

Understanding the heat transfer properties of the system is pivotal in characterizing how quickly the air ingress phenomenon will transition from the first stage of density driven air ingress to the second stage of hot plenum natural circulation. To understand the time scale of this transition, one needs to understand the primary heat source during the course of the accident which is mainly the graphite support columns. The graphite support columns receive most of their heat energy via conduction from the core.

Using a one-dimensional, steady-state fin analysis, a temperature profile was derived for a single IG-110 graphite support column for three limiting cases. In case 1, Dirichlet boundary conditions were prescribed where $T_{\text{top}} = 850^\circ C$ and $T_{\text{bottom}} = 490^\circ C$ as demonstrated in Figure 4. These two temperatures were chosen because they are the outlet and inlet temperatures of the core during normal operation [18]. In case 2, mixed boundary conditions were employed where $T_{\text{top}} = 850^\circ C$ and $q_{\text{bottom}} = 0$. In case 3, the wall temperature for the entire column is assumed to be constant; that is, $T(z) = 850^\circ C$ for $z \in [0, H]$ where $H$ is the support column height. For case 1, the excess temperature distribution is described by the following equation:

$$\theta(z) = \frac{\theta_{\text{bottom}} \sinh (mz) + \theta_{\text{top}} \sinh (m(H-z))}{\sinh (mH)}$$

[22]

where $\theta(z) = T(z) - T_\infty$, $\theta_{\text{bottom}} = T_{\text{bottom}} - T_\infty$, $\theta_{\text{top}} = T_{\text{top}} - T_\infty$, $m^2 = \frac{hP}{kA_c}$.
\( h \): convection heat transfer coefficient, \( P \): wetted perimeter of support column,

\( k \): support column thermal conductivity, \( A_c \): support column cross-sectional area

For case 2, the excess temperature distribution is described by the following equation:

\[
\theta(z) = \theta_{top} \frac{\cosh(m(H-z))}{\cosh(mH)}
\]  \[23\]

By integrating \( \theta(z) \) over the interval from 0 to \( H \) and dividing by \( H \), the average excess temperature, \( \bar{\theta} \), is found. For case 1, the average excess temperature is described by the following equation:

\[
\bar{\theta} = \frac{\theta_{top}}{mH \sinh(mH)} \left[ \left( \frac{\theta_{bottom}}{\theta_{top}} + 1 \right) \left( \cosh(mH) - 1 \right) \right]
\]  \[24\]

For case 2, the average excess temperature is described by the following equation:

\[
\bar{\theta} = \frac{\theta_{top}}{mH} \tanh(mH)
\]  \[25\]

For case 3, the average excess temperature is described by the following equation:
\[ \bar{\theta} = \theta(z) = T(z) - T_{\infty} \] [26]

A summary of the three cases is given in Table 6.

<table>
<thead>
<tr>
<th>Case</th>
<th>Excess Temperature Distribution, ( \theta(z)) (°C)</th>
<th>Average Excess Temperature, ( \bar{\theta} ) (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[ \theta(z) = \theta_{\text{bottom}} \frac{\sinh(mz) + \theta_{\text{top}} \sinh(m(H-z))}{\sinh(mH)} ]</td>
<td>[ \bar{\theta} = \frac{\theta_{\text{top}}}{mH} \frac{\theta_{\text{bottom}} + 1}{\sinh(mH)} (\cosh(mH) - 1) ]</td>
</tr>
<tr>
<td>2</td>
<td>[ \theta(z) = \theta_{\text{top}} \frac{\cosh(m(H-z))}{\cosh(mH)} ]</td>
<td>[ \bar{\theta} = \frac{\theta_{\text{top}}}{mH} \tanh(mH) ]</td>
</tr>
<tr>
<td>3</td>
<td>[ \theta(z) = 850 - T_{\infty} ]</td>
<td>[ \bar{\theta} = \theta(z) = 850 - T_{\infty} ]</td>
</tr>
</tbody>
</table>

Table 6. Summary of one-dimensional fin analysis

Using the three different cases, a parametric study was performed to calculate the convection heat transfer coefficient, the support column Biot number, the transient conduction thermal time constant, the boundary layer thickness at the top of the support column, and the Rayleigh number. The MATLAB code is shown in Appendix B. The average value of the three cases is calculated for the five different heat transfer characteristics. They are calculated for different far-field temperatures (from 25–500 °C in increments of 50°C) and for different air/helium species compositions (from 0–100% air mole fraction in 10% mole fraction increments). These calculations were performed at prototype dimensions at prototype temperatures (850°C) as well as at scaled-down dimensions at 750°C.
Figure 6. Average heat transfer coefficient v. far-field temperature for prototype geometry

Figure 7. Average heat transfer coefficient v. air mole fraction for prototype geometry
Figure 8. Average Biot number v. far-field temperature for prototype geometry

Figure 9. Average Biot number v. air mole fraction for prototype geometry
Figure 6 and Figure 7 show how the heat transfer coefficient varies with far-field temperature and species composition. Figure 8 and Figure 9 show how the Biot number varies with the far-field temperature and species composition. Appendix A contains the remaining figures of the heat transfer characteristics from the fin analysis for the prototype geometry at prototype temperature (850°C) and the scaled-down geometry at 750°C. The heat transfer coefficient varies from about 3-12 W/(m²·K) for the prototype geometry. It varies from 5-15 W/(m²·K) for the scaled-down geometry. These results are encouraging for a couple of reasons. First, despite imposing three different types of boundary conditions and varying both the far-field temperature and the species composition over a wide range of values, the heat transfer coefficient remains in a small enough interval that the natural circulation phenomenology will not change significantly. This statement applies to both types of geometry - prototype and scaled-down. Second, the range of heat transfer coefficients for the prototype and scaled-down geometries are very similar - 3-12 W/(m²·K) for the prototype geometry and 5-15 W/(m²·K) for the scaled-down geometry. Therefore, there is considerable overlap between the two ranges. This means that the heat transfer coefficient in the prototype system and the scaled-down system will be close enough to each other during the course of an event that the basic natural circulation phenomenology will be preserved. This ensures that a high degree of heat transfer similarity will be maintained.

Other observations from the one-dimensional fin analysis include that for a given species composition the heat transfer coefficient and the Biot number decrease monotonically as the far-field temperature increases. Also, it can be observed that for a
given far-field temperature the heat transfer coefficient and the Biot number increase as
the air mole fraction increases from 0 to 0.5 and decrease as the air mole fraction
increases from 0.5 to 1. The non-monotonic behavior of the heat transfer coefficient as a
function of the air-mole fraction is due to the parabolic nature of the dynamic viscosity
and thermal conductivity of an air-helium mixture as the air-mole fraction varies for a
given far-field temperature. These relations can be found in Banerjee and Andrews [31]
and are given in equations [27] and [28].

Equation [27] is the relation describing the dynamic viscosity of the air-helium
mixture. The following indices describe the components of each mixture (1: air; 2:

\[
\mu_{\text{mix}} = \mu_1 \left(1 + \frac{x_2}{x_1} \Phi_{12} \right)^{-1} + \mu_2 \left(1 + \frac{x_1}{x_2} \Phi_{21} \right)^{-1}
\]

where

\[
\Phi_{ij} = \left[1 + \left(\frac{\mu_i}{\mu_j}\right)^{1/2} \left(\frac{M_j}{M_i}\right)^{1/4} \right]^{-2} \left[8 \left(1 + \frac{M_i}{M_j}\right)^{1/2}\right]^{-1}
\]

[27]

[28] is the relation describing the thermal conductivity of the air-helium mixture.

\[
k_{\text{mix}} = k_1 \left(1 + \frac{x_2}{x_1} \Phi_{12} \right)^{-1} + k_2 \left(1 + \frac{x_1}{x_2} \Phi_{21} \right)^{-1}
\]

where
\[
\Phi_y = 1.065 \left[ 1 + \left( \frac{k_i P_j}{k_j P_i} \right)^{1/2} \left( \frac{M_j}{M_i} \right)^{1/4} \right]^2 \left[ 8 \left( 1 + \frac{M_i}{M_j} \right) \right]^{-1/2} \quad i \neq j
\]

and

\[
P_i = 0.115 + 0.354 \left( \frac{c_{pi}}{R_i} \right)
\]

[28]

\(x_1\) and \(x_2\) are the mole fractions of the individual components in the mixture. \(M_i\) and \(M_2\) are the molecular weights of the pure gases. \(c_{pi}\) and \(R_i\) are the specific heat capacity and specific gas constant for the \(i^{th}\) component, respectively.

Assuming that there is a linear variation of helium concentration, \(C_{he}(z)\) in the hot exit plenum where the helium concentration is zero at the bottom and one at the top of the plenum. The resultant heat transfer coefficient, \(h(z)\), is shown in Figure 10. This also assumes that the film temperature is constant along the height of the support column.

Figure 10. Effect of thermo-physical properties of air-helium mixtures on heat transfer coefficient
Power Scaling Analysis for Scaled-down Facility

With the small Biot numbers \((10^{-3} \text{–} 10^{-4})\) calculated from a one-dimensional, steady-state fin analysis, the lumped capacitance approximation seems valid. However, to confirm this approximation and to also establish the heater power in the scaled-down facility so that the natural circulation phenomenology is preserved, a MATLAB code (see Appendix B) was written to produce radial temperature profiles for (1) a prototypic support column, (2) a scaled-down support column, and (3) a scaled-down shell/heater system. A scaled-down shell/heater system is a practical and viable design in that a high shell outer wall temperature (750°C) can be achieved without exceeding the maximum heater sheath temperature (1120°C) of a Watlow® MULTICELL™ heater. A one-dimensional, steady-state calculation shows that a shell outer wall temperature of 750°C can be achieved while the heater is exerting 1000W. Under these conditions, the calculation also shows that the surface temperature of the heater is approximately 900°C. This surface temperature is well below the aforementioned maximum heater sheath temperature.

On each figure (Figure 14 - Figure 19), there are multiple distributions which correspond to different non-dimensional times \((t/\tau_{\text{total}})\) in the transient. The non-dimensional times are 0, 0.05, 0.10, 0.25, 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0. This allows one to see how the temperature profile progresses over time for each geometry. The total time scale, \(\tau_{\text{total}}\), for the prototype is 16.06 s and the total time scale for the scaled-down facility is 5.67 s. The total time scale is derived using the following method. First, the time scale for the density-driven air ingress phenomenon, \(\tau_{\text{DD}}\), is calculated. This
calculation is similar to one found in Oh and Kim [10]. The speed of the gravity current for $0 < \gamma < 0.281$ is calculated using the following relation.

$$
U = \sqrt{(1-\gamma)gH \left[ \frac{1}{\gamma} \frac{h}{H} \left( 2 - \frac{h}{H} \right)^{1-\frac{h}{H}} \right]^{1/2}}
$$  \[29\]

where $g$ is the acceleration due to gravity, $H$ is the channel depth, $h$ is the current depth, and $\gamma$ is the density ratio ($\rho_1/\rho_2$). For this calculation, the density ratio of helium (at 850°C, 1 atm) and air (at 25°C, 1 atm) is 0.03666. Using Figure 11, the depth of the current is calculated to be $h = 0.06$ m in the prototype geometry and $h = 0.0075$ m in the scaled-down geometry. From equation [29], the current velocity is $U = 5.287$ m/s for the prototype geometry and $U = 1.869$ m/s for the scaled-down geometry.

Figure 11. Flow depth of the heavy current [10]
To determine the density-driven air ingress time scale, a length scale needs to be determined. Since the exact breaking point can change depending on the event, a minimum time scale can be determined by taking the shortest possible distance. Therefore, the length scale was determined to be one-half of hot exit plenum total length. This is equal to \( L = 3.4 \) m for the prototype geometry and \( L = 0.425 \) m for the scaled-down geometry. Originally, the density-driven time scale was compared to the air diffusion time scale which assumes that the air is uniformly spread throughout the whole duct cross section [10]. In order to achieve a more equitable time scale comparison, a superficial air velocity was computed for the density-driven air ingress phenomenon. The superficial velocities will be calculated for both geometry types even though the current analysis does not consider the molecular diffusion time scale. The superficial velocity is estimated by

\[
U_s = \frac{U_h}{H} \quad [30]
\]

This is calculated to be 0.211 m/s for the prototype geometry and 0.075 m/s for the scaled-down geometry. Therefore, the minimum density-driven air ingress time scale, \( \tau_{DD} \), is calculated by

\[
\tau_{DD} = \frac{L}{U_s} \quad [31]
\]
This equals 16.08 s for the prototype geometry and 5.68 s for the scaled-down geometry.

Since the Biot number is much less than 0.1, the error associated with using the lumped capacitance method is negligible. Therefore, the thermal time constant, which describes how quickly the temperature of the support column approaches the temperature of the surroundings, is given by

\[ \tau_{\text{thermal}} = \frac{\rho V c_p}{h A_s} \]  

[32]

where \( h, \rho, V, c_p \) and \( A_s \) are the convective heat transfer coefficient, support column density, support column volume, support column specific heat and support column surface area, respectively. The thermal time scale for a wide range of parameters is shown in Figure 12 and Figure 13. These two figures correspond to the prototypic and scaled-down geometries, respectively. Since the exact value of the thermal time constant is not nearly as important as its order of magnitude, an average of the 11 cases (each value corresponding to a different air-helium mole fraction at 25°C) is the thermal time constant used for this analysis. The thermal time constants for these 11 cases correspond to the minimum thermal time constants for a given species composition. Therefore, the average thermal time constant calculated in this analysis is a minimum average thermal time constant. For the prototype geometry, the minimum average thermal time constant is 14,008 s. For the scaled-down geometry, the minimum average thermal time constant is 3,299 s.
Figure 12. Thermal time constant v. far-field temperature for prototype geometry

Figure 13. Thermal time constant v. far-field temperature for scaled-down geometry
With the minimum density-driven air ingress time scale, $\tau_{DD}$, and minimum thermal time constant, $\tau_{thermal}$, known, the minimum total time scale, $\tau_{total}$, can be calculated. The total time scale is expressed as

$$\frac{1}{\tau_{total}} = \frac{1}{\tau_{DD}} + \frac{1}{\tau_{thermal}}$$  \[33\]

The minimum density-driven air ingress time scale is three orders of magnitude smaller than the thermal time constant for both types of geometry. This means that the minimum total time scale is dominated by the density-driven air ingress time scale. Table 7 summarizes the time scales considered.

<table>
<thead>
<tr>
<th>Geometry Type</th>
<th>Density-driven Time Scale (s)</th>
<th>Thermal Time Constant (s)</th>
<th>Total Time Scale (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prototype</td>
<td>16.08</td>
<td>14,008</td>
<td>16.06</td>
</tr>
<tr>
<td>Scaled-down</td>
<td>5.68</td>
<td>3,299</td>
<td>5.67</td>
</tr>
</tbody>
</table>

Table 7. Summary of air-ingress phenomenon time scales

**Support Column One-dimensional Heat Diffusion Transient Analysis**

The one-dimensional fin analysis provided an axial temperature distribution of a support column rod for a given set of boundary conditions. Calculating an average temperature of the rod, heat transfer characteristics of the rod can be calculated such as the Biot number, the heat transfer coefficient, and the transient conduction time scale. The calculated Biot numbers and transient conduction time scales suggest that a lumped capacitance approximation of the support column is valid. In order to confirm this
approximation and to establish an operational heater power for the shell/heater system, a transient one dimensional analysis was performed on MATLAB.

The code (See Appendix B) utilizes a finite-difference discretization and an explicit time-marching method. The discretization is derived using a uniform mesh. Constant thermo-physical properties are assumed for this calculation. Density is taken to be 1770 kg/m³. Specific heat capacity is taken to be 1720 J/(kg·K). Thermal conductivity is taken to be 85 W/(m·K). These thermo-physical properties correspond to a IG-110 graphite temperature of 750°C [32]. The governing equation is a single spatial dimension (radial variable) transient heat diffusion equation for cylindrical coordinates.

Mathematically, this is expressed as follows:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right); r_1 \leq r \leq r_2; t \geq 0$$

[34]

where \(r_1, r_2\) is the inner and outer radius of the column/annulus, respectively. The radial variable is normalized on Figure 14 - Figure 19 for easier comparison among the different geometries. Table 8 lists the boundary and initial conditions for cases (1), (2) and (3).

<table>
<thead>
<tr>
<th>Case</th>
<th>Left Boundary Condition</th>
<th>Right Boundary Condition</th>
<th>Initial Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{\partial T}{\partial r} \bigg</td>
<td>_{r=0} = 0 )</td>
<td>(-k_s \frac{\partial T}{\partial r} \bigg</td>
</tr>
<tr>
<td>2</td>
<td>(-k_s \frac{\partial T}{\partial r} \bigg</td>
<td><em>{r=r_2} = q</em>{\text{heater}} )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8. Boundary conditions and initial condition for transient one-dimensional analysis
Sufficiently small time steps need to be taken in order to ensure the stability of the explicit time-marching scheme. The maximum time step for this scheme is determined by the following equation:

$$\Delta t_{\text{max}} = \frac{\Delta r^2}{2\alpha}$$  \[35\]

where $\Delta r$ (m) is the nodal distance and $\alpha$ (m$^2$/s) is the thermal diffusivity of IG-110 graphite. The actual time step is 0.2 times the maximum time step. This increases confidence that the scheme will converge to a solution.

<table>
<thead>
<tr>
<th>Case</th>
<th>$r_1$ (m)</th>
<th>$r_2$ (m)</th>
<th>Number of Nodes</th>
<th>$\Delta r$ (m)</th>
<th>$\Delta t_{\text{max}}$ (s)</th>
<th>$\Delta t$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.106</td>
<td>1001</td>
<td>1.06e-4</td>
<td>2.0122e-4</td>
<td>4.0243e-5</td>
</tr>
<tr>
<td>2</td>
<td>0.0275</td>
<td>0.0127</td>
<td>301</td>
<td>9.1667e-4</td>
<td>1.5048e-4</td>
<td>3.0096e-5</td>
</tr>
<tr>
<td>3</td>
<td>0.0127</td>
<td>0.0275</td>
<td>251</td>
<td>5.9200e-5</td>
<td>6.2762e-5</td>
<td>1.2552e-5</td>
</tr>
</tbody>
</table>

Table 9. Mesh size and time step for one-dimensional transient analysis

Figure 14-Figure 19 are temperature profiles at different non-dimensional times for the prototype, scaled-down, and the scaled-down heater-shell system geometries.
Figure 14. Radial temperature profile for prototype geometry at different times ($t/\tau_{\text{total}}$)

Figure 15. Radial temperature profile for scaled-down geometry at different times ($t/\tau_{\text{total}}$)
Figure 16. Radial temperature profile for shell/heater system at heater power of 150 W

Figure 17. Radial temperature profile for shell/heater system at heater power of 125 W
Figure 18. Radial temperature profile for shell/heater system at heater power of 100 W

Figure 19. Radial temperature profile for shell/heater system at heater power of 0 W
There is good agreement among Figure 14-Figure 19. The temperature profile varies negligibly over the radius of the rod/annulus. Therefore, the lumped capacitance method is a good approximation. Also, a heater power of 125 W creates a temperature profile through the annular portion of the scaled-down rod-heater system (Figure 17) similar to that shown in the scaled-down support column (Figure 15) and the prototypic support column (Figure 14). It should be noted, however, that even if 0 W is pushed through the heater there is a very good similarity between the temperature profiles of the shell/heater system, scaled-down rod and prototype rod. Therefore, there exists a heater power that can be utilized during the course of an experiment such that similar temperature profiles can be achieved among the three different support column geometries. This preserves the natural circulation phenomenology of the prototype during the course of an experiment on the scaled-down facility.

Support Column Two-dimensional Heat Diffusion Transient Analysis

In addition to the one-dimensional transient analysis, a two-dimensional transient analysis was performed. The major difference between the current analysis and the 1-D transient analysis is that the current analysis takes into account a heat source. More specifically, the current analysis takes into account the heat conducted at the top and the bottom boundaries of the support column. Mathematically, this is demonstrated by imposing fixed temperature (Dirichlet) boundary conditions at the top and bottom boundaries of the support column. The governing equation is a two-dimensional heat diffusion transient equation for cylindrical coordinates.
Mathematically,

\[ \frac{\rho c_v}{\rho} \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right); \quad r_1 \leq r \leq r_2; \quad t \geq 0 \]  \[36\]

For cases (1) - (3) as defined in the 1-D transient analysis, the boundary conditions are summarized in Table 10.

<table>
<thead>
<tr>
<th>Case</th>
<th>Top Boundary Condition</th>
<th>Bottom Boundary Condition</th>
<th>Left Boundary Condition</th>
<th>Right Boundary Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fixed Temperature (750°C)</td>
<td>Fixed Temperature (750°C)</td>
<td>( \frac{\partial T}{\partial r} \bigg</td>
<td>_{r=r_0} = 0 )</td>
</tr>
<tr>
<td>2</td>
<td>Fixed Temperature (750°C)</td>
<td>Fixed Temperature (750°C)</td>
<td>(-k_1 \frac{\partial T}{\partial r} \bigg</td>
<td><em>{r=r_1} = q</em>{heater} )</td>
</tr>
<tr>
<td>3</td>
<td>Fixed Temperature (750°C)</td>
<td>Fixed Temperature (750°C)</td>
<td>(-k_1 \frac{\partial T}{\partial r} \bigg</td>
<td><em>{r=r_1} = q</em>{heater} )</td>
</tr>
</tbody>
</table>

Table 10. Boundary conditions for two-dimensional analysis

The same initial condition is imposed on the rod/annulus that is imposed in the 1-D transient analysis. Namely,

\[ T(r,z,t=0) = 750 \text{ °C}; \quad r_1 \leq r \leq r_2; \quad 0 \leq z \leq H \] \[37\]

The total time scale, heat transfer coefficient and far-field temperature for the 2-D analysis are given in Table 11. The total time scales are taken from Table 7. The heat
transfer coefficients are the largest values calculated from the 1-D fin analysis. The far-field temperature is the lowest value considered in the 1-D fin analysis.

<table>
<thead>
<tr>
<th>Case</th>
<th>Total Time Scale (s)</th>
<th>Heat Transfer Coefficient (W/(m²K))</th>
<th>Far-field Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.06</td>
<td>12.688</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>5.67</td>
<td>15.571</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 11. Total time scale, convective heat transfer coefficient, and far-field temperature for case (1), (2), and (3)

Under these conditions, this allows for the maximum possible heat removal from a support column. So, if a temperature profile is relatively flat under these conditions, it is certain that the lumped capacitance approximation is valid under all other possible conditions within the scope of the experiment. Furthermore, it determines the maximum required heater power to maintain temperature profile similarity in the shell/heater system.

The governing equation is discretized using a finite volume discretization and explicit time-marching scheme. The stability is ensured by taking Δt to be 0.2 of Δt_max, where

\[
\Delta t_{\text{max}} = \frac{1}{2\alpha \left( \frac{1}{\Delta r^2} + \frac{1}{\Delta z^2} \right)} \quad [38]
\]
\( \alpha \) is the largest thermal diffusivity of IG-110 graphite over the range of 20–800°C, \( \Delta r \) is the horizontal mesh spacing and \( \Delta z \) is the vertical mesh spacing. This yields the smallest \( \Delta t_{\text{max}} \). The largest value occurs at 20°C and equals 1e-4 m²/s. The mesh size and corresponding \( \Delta t_{\text{max}} \) are shown for each case in Table 12.

<table>
<thead>
<tr>
<th>Case</th>
<th>Support Column Inner Radius, ( r_1 ) (m)</th>
<th>Support Column Outer Radius, ( r_2 ) (m)</th>
<th>Support Column Height, ( H ) (m)</th>
<th>Number of Cells in Radial Direction</th>
<th>Number of Cells in Axial Direction</th>
<th>( \Delta r ) (m)</th>
<th>( \Delta z ) (m)</th>
<th>( \Delta t_{\text{max}} ) (s)</th>
<th>( \Delta t ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.106</td>
<td>2.84</td>
<td>40</td>
<td>1000</td>
<td>0.0027</td>
<td>0.0028</td>
<td>0.0188</td>
<td>0.0038</td>
</tr>
<tr>
<td>2</td>
<td>0.0275</td>
<td>0.356</td>
<td>40</td>
<td>200</td>
<td>6.88e-4</td>
<td>0.0018</td>
<td>0.0021</td>
<td>4.11e-4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0148</td>
<td>0.356</td>
<td>20</td>
<td>200</td>
<td>7.40e-4</td>
<td>0.0018</td>
<td>0.0023</td>
<td>4.67e-4</td>
<td></td>
</tr>
</tbody>
</table>

Table 12. Mesh size and time step for two-dimensional transient analysis

Using this method, temperature contour plots have been generated for cases (1)–(3) at time \( t = 3 \tau_{\text{total}} \). To simplify comparison, the radial and axial direction (or x- and y-axis, respectively) have been normalized with respect to the case’s column geometry. Also, the thermal conductivity and specific heat capacity are not constant, but vary with temperature. Correlations were derived from data in the available literature [32]. Density is taken to be 1770 kg/m³. Both the thermal conductivity and specific heat capacity for the IG-110 graphite are expressed in the following relations:

\[
k_s(T) = 3 \times 10^8 T^3 - 1 \times 10^{-5} T^2 - 0.0612T + 125.56 \quad [39]
\]

\[
c_p(T) = 9 \times 10^{-7} T^3 - 0.0027T^2 + 3.0203T + 643.24 \quad [40]
\]
where $T$ is in units of °C. Equations [39] and [40] are valid for the temperature range of 20–800°C. Figure 20 - Figure 25 are the resulting temperature contour plots of the 2-D analysis at $t = 3 \tau_{total}$. Figure 20 is the temperature contour plot for the prototype geometry. Figure 21 is the temperature contour plot for the scaled-down geometry. Figure 22-Figure 25 are temperature contour plots for the shell/heater system at $\dot{Q} = 150$ W, 125 W, 100 W, and 0 W, respectively.

![Temperature contour plot for prototype geometry (Case 1) at $t=3\tau_{total}$](image-url)

Figure 20. Temperature contour plot for prototype geometry (Case 1) at $t=3\tau_{total}$
Figure 21. Temperature contour plot for scaled-down geometry (Case 2) at $t = 3\tau_{total}$

Figure 22. Temperature contour plot for shell/heater system for 150 W (Case 3a) at $t=3\tau_{total}$
Figure 23. Temperature contour plot for shell/heater system for 125 W (Case 3b) at $t = 3\tau_{total}$

Figure 24. Temperature contour plot for shell/heater system for 100 W (Case 3c) at $t = 3\tau_{total}$
Figure 25. Temperature contour plot for shell/heater system for 0 W (Case 3d) at $t=3\tau_{\text{total}}$

The results of the 2-D calculations are promising and, in many respects, give similar conclusions to those found in the 1-D transient analysis. At first glance, the average temperature gradient in the radial direction at the mid-plane for all six cases is similar. Table 13 quantifies the average radial temperature gradient for Figure 20 - Figure 25. Please note that the non-dimensional radial thickness is defined as $\frac{r_2 - r_1}{r_2}$.

<table>
<thead>
<tr>
<th>Case</th>
<th>Heater Power (W)</th>
<th>Non-dimensional Radial Thickness</th>
<th>Radial Thickness (m)</th>
<th>Temperature Difference ($^\circ$C)</th>
<th>$\frac{\Delta T}{\Delta r}$ ($^\circ$C/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>N/A</td>
<td>1</td>
<td>0.106</td>
<td>3.5</td>
<td>33.02</td>
</tr>
<tr>
<td>2</td>
<td>N/A</td>
<td></td>
<td>0.0275</td>
<td>0.75</td>
<td>27.28</td>
</tr>
<tr>
<td>3a</td>
<td>150</td>
<td></td>
<td>0.5382</td>
<td>0.75</td>
<td>50.68</td>
</tr>
<tr>
<td>3b</td>
<td>125</td>
<td></td>
<td>0.1480</td>
<td>0.75</td>
<td>50.68</td>
</tr>
<tr>
<td>3c</td>
<td>100</td>
<td></td>
<td></td>
<td>0.625</td>
<td>42.23</td>
</tr>
<tr>
<td>3d</td>
<td>0</td>
<td></td>
<td></td>
<td>0.5</td>
<td>33.78</td>
</tr>
</tbody>
</table>

Table 13. Temperature gradient at the vertical mid-plane of the support column
From Table 13, Case 3d is the best case for simulating the average radial thermal gradient at the midplane. This suggests that case 3d, whose heater power is 0 W, has the closest average radial heat flux to that of the scaled-down and prototypic support columns.

The surface temperature for a majority of the support column for all six cases is within one degree celsius. Outside this region, the surface temperature does not increase more than five degrees celsius for any of the geometries. Case 3b ($Q = 125$ W), however, appears to have the most similar surface temperature distribution to those found in the prototypical and scaled-down geometries. It’s interesting to note that even if no power is pushed through the heater in the shell/heater system, there’s still good similarity of the surface temperature distribution at $t = 3\tau_{\text{total}}$ among all three geometry types.

Beyond $t = 3\tau_{\text{total}}$, the change in the temperature contour plots would be more distinct among the three different geometries given the mass difference of each geometry type. If it’s necessary to run the experiments past this time limit ($t = 3\tau_{\text{total}}$), additional calculations would need to be performed to see the maximum required heater power to simulate the temperature profiles in the prototypic and scaled-down geometries.

However, up to $t = 3\tau_{\text{total}}$, the surface temperature distribution of the shell/heater system is most similar to the surface temperature distribution of the prototype and scaled-down support columns when the heater is emitting 125 W (Case 3b).

In conclusion, if the heater power is 0 W (Case 3d), this creates the closest average radial heat flux at the mid-plane of the shell/heater system to those average radial heat fluxes found at the mid-plane of the prototypic and scaled-down support columns (Case 1 & Case 2), as seen in Table 13. On the other hand, if the heater were emitting
125 W of power, a surface temperature distribution along the shell/heater system would be created that is the most similar to the surface temperature distribution of the prototypic and scaled-down support columns.

**Design Analysis of Containment**

Following a depressurization of the pressure vessel, hot helium from the reactor vessel mixes in the vessel’s containment with relatively cool air from the reactor cavity. The mixing process creates a mixed species temperature as well as a mixed species concentration which have a significant effect on the following stages of an air-ingress accident especially the density-driven stratified flow. In this analysis, considerations are made to calculate the free volume of the scaled-down containment and the initial pressure of the scaled-down vessel. By calculating the values, the final mixing temperature and air-to-helium mole ratio in the prototypic case will be preserved in the scaled-down case. Furthermore, with an understanding of the initial pressure of the scaled-down vessel, the initial operating conditions for a given experiment are now determined.

Utilizing the prototypic geometry and normal operating conditions of the GT-MHR, it was found that the mole ratio of air in the containment to helium in the pressure vessel is 5.15. In order to maintain this mole ratio, the containment volume of the scaled-down geometry was calculated for different initial vessel pressures. In these calculations, the volume of the prototypic containment is taken to be 25,000 m³, the free volume of the pressure vessel is taken to be 265 m³. The initial pressure and temperature of the vessel is 7 MPa and 850°C, respectively. The initial pressure and temperature of the containment is 0.101325 MPa and 25°C, respectively. The free volume of the scaled-
down vessel is 0.210 m³. The initial helium temperature is 750°C in the scaled-down pressure vessel. In these calculations, the temperature and pressure of the scaled-down containment are the same as those for the prototype containment. The ideal gas law was used to calculate the free volume of the scaled-down containment so that the air to helium mole ratio is the same that is found in the prototypic case. The results are summarized in Table 14 for different initial vessel pressures.

<table>
<thead>
<tr>
<th>Initial Scaled-down Vessel Pressure (psig)</th>
<th>Initial Vessel Temperature (°C)</th>
<th>Scaled-down Containment Volume (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>750</td>
<td>0.315</td>
</tr>
<tr>
<td>30</td>
<td>750</td>
<td>0.963</td>
</tr>
<tr>
<td>60</td>
<td>750</td>
<td>1.605</td>
</tr>
</tbody>
</table>

Table 14. Scaled-down containment free volume for different initial vessel pressures

In addition to the above analysis, the mixed mean temperature of the pressure vessel containment system is calculated by using the first law of thermodynamics,. This analysis was performed to give the final temperature of the air-helium mixture after the helium is emptied into the containment. This temperature occurs after the contents are well mixed. It’s also assumed that that no heat is lost or gained from the confinement and that no work is done by the control volume which makes sense since it’s a rigid volume. The governing equation is given by

\[ U_f - U_i = 0 \]  

[41]

72
where $U_f$, $U_i$ are the internal energy of the system at the final and initial stages, respectively. By manipulating equation [41], the final temperature of the system can be solved as seen in equation [42].

$$T_f = \frac{(m_{he}c_{v,i,he}T_{i,he} + m_a c_{v,i,a} T_{i,a})}{(m_{he}c_{v,f,he} + m_a c_{v,f,a})} \quad [42]$$

Since the specific heat of air is a function of temperature, equation [42] is solved iteratively. Using the initial conditions listed in Table 15, the final temperature is 382.6 K. Using the ideal gas law, the final pressure is 0.1538 MPa.

<table>
<thead>
<tr>
<th></th>
<th>Confinement</th>
<th>Pressure Vessel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure (MPa)</td>
<td>0.101325</td>
<td>7</td>
</tr>
<tr>
<td>Volume (m$^3$)</td>
<td>25,000</td>
<td>265</td>
</tr>
<tr>
<td>Temperature (K)</td>
<td>300</td>
<td>1123</td>
</tr>
<tr>
<td>Species</td>
<td>100% Air</td>
<td>100% Helium</td>
</tr>
</tbody>
</table>

Table 15. Initial conditions for final temperature analysis

In the GT-MHR containment, there are vents located on the wall. Therefore, the global pressure within the containment is maintained at or very near atmospheric pressure. Since the final pressure in the first law analysis is above atmospheric pressure, a transient control volume analysis was performed. The governing equation is given in equation [43]. It is assumed that only air escapes out of the containment and it does so at its initial temperature.
\[ U_f - U_i = -m_e h_e \]  \[43\]

Rearranging equation [43], the final temperature can be solved for as seen in equation [44].

\[
T_f = \frac{\left( -m_e c_{p,i,a} T_{i,a} + m_{he} c_{v,i,he} T_{i,he} + m_{ai,a} c_{v,i,a} T_{i,a} \right)}{m_{he} c_{v,i,he} + m_{ai,a} c_{v,i,a}} \]  \[44\]

where \( m_e = m_{a,i} - m_{a,f} \). Iterating over equation [44] and using the ideal gas law, a final temperature can be found. All final pressures are atmospheric pressure. A summary of the results can be found in Table 16. Different cases are based on the initial vessel pressure in the scaled-down facility.

<table>
<thead>
<tr>
<th>Geometry Type (Initial Vessel Pressure)</th>
<th>Final Temperature (K)</th>
<th>Percent of Initial Air Mass Lost</th>
<th>Final Air/He Mole Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prototype (7 MPa)</td>
<td>366</td>
<td>37.2%</td>
<td>3.22</td>
</tr>
<tr>
<td>Scaled-down (60 psig)</td>
<td>406.2</td>
<td>42.4%</td>
<td>1.85</td>
</tr>
<tr>
<td>Scaled-down (53.4 psig)</td>
<td>394.7</td>
<td>37.1%</td>
<td>2.21</td>
</tr>
<tr>
<td>Scaled-down (40 psig)</td>
<td>374.0</td>
<td>26.5%</td>
<td>3.21</td>
</tr>
<tr>
<td>Scaled-down (34.2 psig)</td>
<td>366.0</td>
<td>21.9%</td>
<td>3.81</td>
</tr>
<tr>
<td>Scaled-down (30 psig)</td>
<td>360.5</td>
<td>18.7%</td>
<td>4.34</td>
</tr>
<tr>
<td>Scaled-down (0 psig)</td>
<td>327.1</td>
<td>-4.0%</td>
<td>16.67</td>
</tr>
</tbody>
</table>

Table 16. Results of transient control volume analysis following depressurization

From Table 16, it can be seen that the final temperature of the prototype geometry can be attained when the initial vessel pressure of the scaled-down vessel is 34.2 psig and
a containment free volume of 1 m$^3$. The final air-to-helium mole ratio of the prototype
gеometry can be attained in the scaled-down geometry when the initial vessel pressure is
40 psig. The air-to-helium mole ratio is also an important aspect of this analysis since the
density, viscosity, thermal conductivity and specific heat of the air/helium mixture are
dependent on the mole fractions of these two species. Less importantly, the percentage of
air mass lost from the containment can be simulated if the initial vessel pressure in the
scaled-down system is 53.4 psig.

**Transient Blowdown Analysis**

In the event of a LOCA of a GT-MHR, the pressure boundary of the reactor is
compromised. Under such circumstances, the reactor initially undergoes a
depressurization or blowdown in which helium in the reactor system empties into the
surroundings, namely, a containment that houses the reactor vessel. In this analysis,
helium is released from the reactor vessel through a break in the cross vessel of the
reactor. It will be shown that the blowdown is a rapid process whose time scale is much
shorter than the time scale of the density-driven stratified flow and hot plenum natural
circulation.

During the release of helium, the temperature of the fluid initially decreases with
time due to the expansion of the helium inside the vessel. As the pressure of the helium
inside the vessel approaches the ambient pressure, the heat transferred from the reactor
internals and vessel wall to the fluid cause the temperature of the fluid to increase. The
interaction of these two effects leads to a dynamic behavior of the fluid temperature
during the depressurization process.
In the present analysis, a simple model for describing the depressurization process of a helium-filled reactor vessel with internal structures has been employed. This simple method does not incorporate local details such as flow patterns or temperature fields, but, instead, uses a lumped capacitance model to reveal the integral changes taking place during the depressurization. Also, this simplified model assumes constant thermo-physical properties except for density and a constant heat transfer coefficient between the solid structures and the helium. Since natural convection is the dominant mode of heat transfer during a depressurization, the heat transfer coefficient is taken to be 2 W/(m²·K).

The energy conservation equations for helium and the internal solid structures are given in equation [45] and [46], respectively.

\[
\frac{dU(t)}{dt} = \dot{Q}_{\text{conv}}(t) - m_i(t) \Rightarrow \frac{d(m(t)c_vT(t))}{dt} = hA_s(T_w(t) - T(t)) - \dot{m}(t)c_pT(t) \quad [45]
\]

where \( i \) is the helium enthalpy, and \( h \) is the heat transfer coefficient [33]. For all solid structures (vessel wall and internals), the energy equation is

\[
\frac{dH_w(t)}{dt} = \dot{Q}_{\text{gen}}(t) - \dot{Q}_{\text{conv}}(t) \Rightarrow \rho_w V_w c_{p,w} \frac{dT_w}{dt} = 0.066P_0t^{-0.2} - hA_s(T_w(t) - T(t)) \quad [46]
\]

where \( P_o \) is the constant operating power level, and \( T_w \) is the wall temperature. The mass balance equation is simply
\[ m(t) = m_o - \int \dot{m}(t) \cdot dt = m_o - \dot{m}_{\text{exit}} \Delta t \]  \hspace{1cm} [47]

The density as a function of time is derived by dividing equation [47] by the free volume of the vessel. It is shown in equation [48].

\[ \rho(t) = \rho_o - \frac{1}{V} \int \dot{m}(t) \cdot dt = \rho_o - \frac{\dot{m}_{\text{exit}}}{V} \Delta t \]  \hspace{1cm} [48]

By taking \( \delta t \) to be sufficiently small, \( \dot{m}(t) \) can approximated as a constant. This way, the use of integrals can be avoided in the MATLAB code (See Appendix B). From the momentum equation for steady incompressible flow,

\[ \frac{p_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_L \]  \hspace{1cm} [49]

the velocity as it leaves the hot duct can be derived where \( h_L \) is the head loss for the flow from the vessel to the containment. It takes into account the sudden contraction from the vessel to the hot duct and the friction along the duct. The velocity out of the hot duct can be expressed as

\[ v(t) = \sqrt{\frac{P(t) - P_{\text{amb}}}{\rho_{\text{avg}}} \left( \frac{\alpha}{2} + \frac{1}{2} \frac{K_c + f \cdot l}{D_h} \right)} \]  \hspace{1cm} [50]
where $\alpha$ is the kinetic energy coefficient and $K_s$ is the pressure loss coefficient for a sudden contraction. The kinetic energy coefficient is taken to be one which corresponds to a flat velocity profile. This is a fair approximation since the flow through the duct is turbulent.

The state equation is the ideal gas law shown in equation in [51].

$$P(t) = \rho(t) \cdot R \cdot T(t)$$  \[51\]

The initial conditions are as follows:

$$m = m_i, P = P_i, T = T_i, T_w = T_i \text{ at } t = 0$$  \[52\]

Equations [45]–[48], [50] and [51] can be rewritten in their discretized form as shown in equations [53]-[58].

**Helium Temperature**

$$T(i+1) = \frac{hA_s \Delta t}{c_s m(i)} T_w(i) + \left( 1 - \frac{(hA_s + \dot{m}(i) R) \Delta t}{c_s m(i)} \right) T(i)$$  \[53\]
**Temperature of Vessel Wall and Internals**

\[
T_w(i+1) = \frac{0.066 \cdot \Delta t \cdot P_0 \cdot ((i-1) \cdot \Delta t + \Delta t^2)^{0.2}}{\rho_w V_c p_{p,w}} + \frac{h A_t \Delta t}{\rho_w V_c p_{p,w}} T(i) + \left(1 - \frac{h A_t \Delta t}{\rho_w V_c p_{p,w}}\right) T_w(i) \tag{54}
\]

**Helium Mass in Vessel**

\[
m(i+1) = m(i) - \dot{m}(i) \cdot \Delta t \tag{55}
\]

**Helium Density in Vessel**

\[
\rho(i+1) = \rho(i) - \frac{\dot{m}(i)}{V} \cdot \Delta t \tag{56}
\]

**Helium Exit Velocity**

\[
v(i) = \sqrt{\frac{P(i) - P_{amb}}{\rho_{avg} \left(\frac{\alpha}{2} + 1 + \frac{1}{2} \left(K_c + \frac{f \cdot l}{D_h}\right)\right)}} \tag{57}
\]

**Helium Pressure**

\[
p(i+1) = \rho(i+1) \cdot R \cdot T(i+1) \tag{58}
\]

From initial conditions in equation [52], the velocity as seen in equation [57] can be calculated. If the velocity is greater than or equal to the speed of sound than the mass flow is calculated based on the choked flow velocity; otherwise, the mass flow rate is based the calculated velocity. With the mass flow rate and other given or calculated
parameters, the helium temperature and “wall” temperature can be calculated from equations [53] and [54]. Furthermore, the helium mass and density in the vessel can be calculated based on equations [55] and [56], respectively. Finally, a new helium pressure is calculated based on the new helium density and temperature. The process is repeated until the helium pressure is equal to the ambient pressure.

Figure 26 through Figure 28 show the helium pressure in the vessel, the helium temperature in the vessel and temperature of the vessel wall and internal structures as a function of time during a blowdown event following a double-ended guillotine break. There are three distinct curves on each figure corresponding to different heat transfer coefficients. This is for the prototypic geometry and conditions. Initial temperature of the helium and the vessel internal structures is 850°C (1123.15 K) and the initial vessel pressure is 7 MPa.
Figure 26. Pressure versus time during depressurization of prototypic vessel.

Figure 27. Helium temperature versus time during depressurization of prototypic vessel.
From Figure 26, it can be observed that the blowdown is essentially over in 0.3 s for all three heat transfer coefficients. The duration of the blowdown is two orders of magnitude smaller than the density-driven air ingress time scale and five orders of magnitude smaller than the hot plenum natural circulation time scale. This means that the duration of the blowdown is two orders of magnitude smaller than the total time scale, $\tau_{\text{total}}$. Therefore, the duration of the blowdown is negligible when compared to the total time scale of the air-ingress accident (See Table 7) in the prototype geometry. However, as seen in Figure 27, there’s a significant decrease in the helium temperature during the blowdown process due to the net loss in thermal energy from the helium due to its expansion. It can also be seen that temperature of the helium in the vessel is dependent on the heat transfer coefficient. With higher heat transfer coefficients, more thermal energy is transferred to the solid wall, leading to a decrease in the helium temperature.
energy is released from the vessel internal structures into the helium. This causes the local minimum of the helium temperature to increase as the heat transfer coefficients increases. Also, the time derivative of the helium temperature increases as the heat transfer coefficient increases. This means that with a higher heat transfer coefficient the density difference between helium within the vessel and the air-helium mixture in the containment will be greater, causing the plume in the density-driven stratified flow to ingress more quickly. Figure 28 shows that the change in temperature of the vessel’s internal structures is minimal over the duration of the blowdown despite the heat transfer coefficient. These results indicate the importance of knowing the helium temperature at the end of the blowdown phase and the temperature of the air-helium mixture at the end of the blowdown. It is these two temperatures and their corresponding densities that will determine how quickly the density-driven stratified flow will occur.

Figure 29 through Figure 31 show the helium pressure in the vessel, the helium temperature in the vessel and temperature of the vessel wall and internal structures as a function of time during a blowdown event following a double-ended guillotine break. There are three distinct curves on each figure corresponding to different heat transfer coefficients. This is for the scaled-down geometry and conditions. Initial temperature of the helium and the vessel internal structures is 750°C (1023.15 K) and the initial vessel pressure is 75 psia or 517106 Pa.
Figure 29. Pressure versus time during depressurization of scaled-down test section

Figure 30. Helium temperature versus time during depressurization of scaled-down test section
Figure 31. “Wall” temperature versus time during depressurization of scaled-down test section

Figure 29 shows that the blowdown in the scaled-down test facility terminates in approximately 0.008 s for all three heat transfer coefficients. The duration of the blowdown is three orders of magnitude smaller than the density-driven stratified flow and six orders of magnitude smaller than the hot plenum natural circulation in the scaled-down geometry. This means that the duration of the blowdown is three orders of magnitude smaller than the total time scale, $\tau_{total}$. Therefore, the duration of the blowdown is negligible when compared to the total time scale of the air-ingress accident in the scaled-down test section (See Table 7). Comparing Figure 27 and Figure 30, it can be seen that there’s a significant difference between the helium temperature in the prototype and the scaled-down geometry. This issue has been resolved by preserving the
non-dimensional Froude number over that temperature range. This is discussed in more
detail in the section Hydraulic Similarity in the Hot Duct-Hot Plenum System of Chapter
2. From Figure 31, the “wall” temperature varies negligibly during the duration of the
transient for all three heat transfer coefficients.
Chapter 3: Physical Description of the Scaled-down Facility

Introduction

With the critical dimensions of the scaled-down facility established, a design of the experimental facility can be presented. The facility is essentially a one-eighth geometric scaled-down model of the hot-exit plenum of the GT-MHR 600 MWth constructed by General Atomics, Inc. Figure 32 shows clearly which region of the reactor is modeled in the scaled-down facility.

Figure 32. Hot-exit plenum of the GT-MHR 600 MWth by General Atomics, Inc.
All dimensions of the scaled-down facility are a one-eighth geometric scale except for the hot duct length, the support column diameter, and the support column pitch. The support column pitch and diameter are approximately one-fourth of the prototype dimensions. The method for determining this factor was explained in Chapter 2. The hot duct length is 0.10 m or about 28% of the one-eighth scale which is 0.35 m. This is done to maintain similarity of the loop pressure loss distribution between the prototype and the scaled-down geometry.

<table>
<thead>
<tr>
<th>Component</th>
<th>Dimension (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot Duct Length</td>
<td>10.00</td>
</tr>
<tr>
<td>Hot Duct Inner Diameter</td>
<td>18.75</td>
</tr>
<tr>
<td>Support Column Diameter</td>
<td>5.5</td>
</tr>
<tr>
<td>Support Column Pitch</td>
<td>9.4</td>
</tr>
<tr>
<td>Test Section Inner Diameter</td>
<td>85.00</td>
</tr>
<tr>
<td>Test Section Height</td>
<td>35.50</td>
</tr>
</tbody>
</table>

Table 17. Critical dimensions for the scaled-down test facility

**Design Drawings**

Computer drawings are annotated to show the dimensions of each part of the vessel. All parts to the vessel that are shown in Figure 33-Figure 48 are constructed of Alloy 800H. Figure 33 is a side view of the bottom semi-hemispherical (2:1 elliptical head) shell of the air-ingress experimental facility. The shell has an outside diameter of 34” and a height of 8.5”. The shell thickness is ¼”. Seventeen - ¾”, schedule 40 pipes are welded to the bottom of the shell. These pipes are fabricated from Alloy 800H and vary in length – the shortest being 8.5” long. All pipes, however, extend to the same vertical level which is 17” below the top of the bottom semi-hemispherical shell. This
allows clearance for insulation to be placed along the bottom of the vessel without interfering with the entrance of the instrumentation ports. Also, the additional distance away from the vessel reduces the temperature requirements for the sealant of the Conax® glands which will attach the thermocouples, pressure transducer tubing, and oxygen sensors to the vessel while maintaining the pressure boundary of the entire system. The reduced temperature requirement is also beneficial for the instrumentation by reducing the possibility of an electrical failure.

Figure 33. Side view of the bottom semi-hemispherical shell
To determine an appropriate length of pipe, a 1-D fin calculation was performed on the shortest pipe protruding from the bottom head 8.5”. This calculation shows that with a base temperature of 750°C and a heat transfer coefficient of 2 W/(m²·K), the temperature at the end of the pipe is about 650°C. The same calculation shows that with a heat transfer coefficient of 5 W/(m²·K), the pipe-end temperature drops to 535°C. These calculations suppose that one-half of the pipe length is wrapped in perfect insulation. Although these temperatures are high and, in some situations, are too high for the sealant of a Conax® gland (See Figure 34), this analysis does not take thermal radiation into account. A Fluent calculation shows that when radiation is accounted for, this greatly reduces the pipe-end temperature to 280°C. This corresponds to an effective heat transfer coefficient 25 W/(m²·K). It is preferred to keep the facility as short as possible for convenience in performing experiments, conducting maintenance, and generally working around the facility. In the current design, if a three-foot clearance – the distance from the bottom of the pipes to the floor - is taken into consideration, then the overall height of the facility is 85 in. (2.159 m). A minimum three-foot clearance is necessary in order to insert and extract instrumentation and instrumentation piping that will need to be able to span a maximum distance of 31 in. (0.787 m). This is the distance from the bottom of the pipe to the bottom of the top plate. The bottom semi-hemispherical is welded to the bottom of the middle shell or test section shown in Figure 39 and Figure 40.

Figure 34 is a schematic of the Conax® gland used in the facility for sealing pressure transducer tubing, oxygen sensors and thermocouples into the vessel. The
pressure transducer gland will seal two pressure transducer tubes per gland while the thermocouple gland will seal four thermocouples per gland. An oxygen sensor gland will seal a single oxygen sensor. The metal housing is constructed of 303SST. The gland sealant is made of Grafoil which operates up to 495°C in oxidizing environments. Grafoil is a sealant which is made out of 99% pure graphite. 303SST has an annealing temperature of 1010°C; this is well above the 280°C calculated pipe-end temperature.

Figure 34. Conax® gland at the end of the pipe on the bottom semi-hemispherical shell\textsuperscript{34} (Units: in.)

Figure 35 and Figure 36 are bottom views of the bottom semi-hemispherical shell of the air-ingress experimental facility. Figure 36 shows the relative position of the support columns to the instrumentation ports. The perforated lines represent the support column position. Furthermore, in these figures, the type, position and spacing of the instrumentation ports are shown.
Figure 35. Bottom view of the bottom semi-hemispherical shell

![Diagram of Bottom View of the Bottom Semi-Hemispherical Shell](image1)

<table>
<thead>
<tr>
<th>Ballon No.</th>
<th>Hole for Instrumentation</th>
<th>Inner Diameter (in)</th>
<th>Outer Diameter (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pressure Transmitter</td>
<td>0.034</td>
<td>1.050</td>
</tr>
<tr>
<td>2</td>
<td>Oxygen Sensor</td>
<td>0.034</td>
<td>1.000</td>
</tr>
<tr>
<td>3</td>
<td>Temp. Sensor</td>
<td>0.024</td>
<td>1.060</td>
</tr>
</tbody>
</table>

Figure 36. Bottom view of the bottom semi-hemispherical shell with support column positions projected

![Diagram of Bottom View of the Bottom Semi-Hemispherical Shell with Support Column](image2)
Figure 37 is a top view (top-left) and a side view (bottom-left) of the bottom plate which separates the bottom semi-hemispherical shell from the middle shell or test section. It rests on a 0.5 in. (12.7 mm) lip on the bottom of the middle shell. The support columns located in the middle shell will be positioned in the hexagonal array of 55 circles. The support columns are made out of IG-11 graphite. Each circle has a diameter of 55 mm (2.17 in.) and is 0.5 in. (12.7 mm) deep. The center-to-center pitch is 94 mm (3.70 in.). This drawing is oriented in such a way that the outlet duct points to the right.

Figure 37. Top view of bottom plate (top) and side view of bottom plate (bottom)
Figure 38 is the bottom view of the bottom plate with and without the support column positions projected onto the plate. The holes for the various types of instrumentation are shown. The “heater” holes are in place to allow the unheated length of the heater to protrude into the bottom semi-hemispherical shell. This permits the heated length of the heater to coincide with the length of the support column. The “miscellaneous” holes are in place to allow one to fill the bottom semi-hemispherical shell with \( \frac{1}{4} \)” ceramic balls to take up the free volume in the lower shell. A chamfer is used for each hole and its dimensions are outlined accordingly. The reason for the chamfer is to facilitate the successful installation of the instrumentation into the test section as it’s drawn up from the bottom. Note that the bottom plate is smaller in diameter than the top plate so that both fit.

![Figure 38. Bottom view of the bottom plate with (left) and without (right) support column positions projected](image)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pressure Transducer</td>
<td>0.761</td>
<td>1.031</td>
</tr>
<tr>
<td>2</td>
<td>Oxygen Sensor</td>
<td>0.750</td>
<td>1.000</td>
</tr>
<tr>
<td>3</td>
<td>Bump, Sensor</td>
<td>0.781</td>
<td>1.031</td>
</tr>
<tr>
<td>4</td>
<td>Heater</td>
<td>1.000</td>
<td>N/A</td>
</tr>
<tr>
<td>5</td>
<td>Electric controller</td>
<td>1.000</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Figure 39 and Figure 40 show the middle shell or test section of the air-ingress experimental facility. On the bottom of the shell, a lip is located that will support the bottom plate (Figure 37 & Figure 38). A flange is welded around the top of the shell. The top semi-hemispherical shell will be bolted to this flange. The duct with a flanged end is located at the front. Extensions can be connected to the duct to simulate different break types. An air piston will be used to secure the duct cover and initiate the accident sequence. There are two 2-inch pipe extensions of approximately 7 in. in length on the opposite side of the duct that are capped off with low pressure sight glasses whose windows are constructed of quartz and whose metal fittings are constructed of 303SST (See Figure 41). The pipe extensions are in place to allow insulation to be placed around the vessel without interfering with the laser’s line of sight. The windows allow the laser of the PIV/PLIF system to pass through to the test section while being able to maintain a pressure boundary up to 1000 psi. This is more than sufficient for the experimental needs of this project since the vessel will only be pressurized up to 50 psig. There’s also a ¾” NPT pipe protruding from the bottom of the hot duct. This is an instrumentation port where up to four thermocouples can be inserted into the hot duct.
Figure 39. Different views of the middle shell or test section

Figure 40. Bottom view (left) and side view (right) of middle shell
Figure 41. Low pressure sight glass on the side of the middle shell (B = 2.500 in., C = 1.875 in., D = 0.760 in., E = 0.290 in.) [35]

Figure 42. Top view of the top plate with (left) and without (right) support column positions projected

<table>
<thead>
<tr>
<th>Ballion No.</th>
<th>Holes For Instrumentation</th>
<th>Inner Diameter [in]</th>
<th>Outer Diameter [in]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Oxygen Sensor</td>
<td>0.750</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>Camera Ports (Top)</td>
<td>2.067</td>
<td>N/A</td>
</tr>
<tr>
<td>3</td>
<td>Heater</td>
<td>1.000</td>
<td>1.259</td>
</tr>
<tr>
<td>4</td>
<td>Screw</td>
<td>0.250</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Support column Alignment</td>
<td>1.000</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Figure 42 is the top view of the top plate with (left) and without (right) the support column positions projected. The top plate separates the middle shell from the top semi-hemispherical shell which is shown in Figure 44 through Figure 46. The top side of the plate has chamfers on the holes that are designed to allow the oxygen sensors and heaters to pass through more easily to the test section. The two larger holes are for the camera to take images with the PIV/PLIF system. Here, velocity and concentration measurements are taken. The orientation of this drawing is such that the outlet duct is pointed to the right.

Figure 43 shows a bottom view and a side view of the top plate. The hexagonal array of holes is designed to secure the support columns. Each hole has a chamfer to make it easier to secure the support column within the hole. This alignment is critical to preserve the flow phenomenology. The 1” holes, in each support column anchor, are in place to ensure that the support columns are correctly inserted to their proper position.

Figure 43. Bottom view (top) and side view (bottom) of the top plate
Figure 44 is a side view of the top semi-hemispherical shell. This piece is bolted down to the middle shell as seen in Figure 39 and Figure 40. There are 11 openings on the top of the head – 1 for insertion of ceramic balls, 2 for camera ports, 2 for oxygen sensors, and 6 for heaters. All pipes extending from the shell end at the same vertical level (17 inches above the shell bottom). This allows clearance for insulation to be installed and relaxes the temperature requirement for the Conax® glands. It also reduces the possibility of electrical failure in the instrumentation heads. A low pressure sight glass as discussed for the laser ports (Figure 41) is used in the camera ports to maintain the pressure boundary and still allow images to be taken by the camera. To fill the free volume of the upper head, ceramic balls will be inserted from the top of the semi-hemispherical shell. The ceramic balls (which are primarily aluminum oxide and silicon dioxide) add a load of approximately 270 kg to the top plate. Even at the high operational temperatures of 750°C, the added weight on the top plate produces a minimal deflection of 0.0025”.
Figure 44. Side view of the top semi-hemispherical shell

Figure 45 and Figure 46 are a top view of the top semi-hemispherical shell without and with the support column positions projected, respectively. These figures show the type, position and relative spacing of the ports with respect to each other and, in the case of Figure 46, the support columns. For both figures, they are oriented such that the outlet duct is pointing right.
Figure 45. Top view of the top semi-hemispherical shell

Figure 46. Top view of top semi-hemispherical shell with support column positions projected
Figure 47 and Figure 48 show a side view and a side cut view of the assembled vessel, respectively.
Instrumentation of the Physical Setup

Instrumentation is essential to any experiment. In the following paragraphs, the different types of instrumentation will be discussed. The location of the instrumentation within the scaled-down test facility has been shown in Figure 33 through Figure 48. There are four major parameters that will be measured in the facility – pressure, temperature, oxygen concentration and local gas velocity.

**Pressure**

The most straight-forward instrumentation for the facility are the pressure and temperature sensors. To measure pressure in the vessel, Honeywell STG944 Pressure (Gage) Transducers are used. The transducers are used to measure pressure inside the middle shell (or test section) as seen in Figure 39 and Figure 40. During density-driven air ingress and hot plenum natural circulation, large pressure variations from atmospheric pressure are not expected occur. However, that instrumentation will be in place to measure the gage pressure at different locations throughout the vessel which consists mainly of the lower plenum of the reactor vessel. The pressure tubing can be moved vertically if measurements need to be made at different vertical positions. The transducer is also used to measure the pressure dynamics during the initial depressurization phase of the experiments.

<table>
<thead>
<tr>
<th>Pressure Range</th>
<th>Temperature Range</th>
<th>Output Range</th>
<th>Frequency Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 500 psi</td>
<td>-40 - 110°C</td>
<td>4 - 20 mA</td>
<td>350 ms (Analog mode)</td>
</tr>
</tbody>
</table>

Table 18. Pressure transducer specifications
In analog mode, the accuracy of the pressure transducer is ±0.075% of the calibrated span or upper range value, whichever is greater. In digital mode, the accuracy of the pressure transducer is ±0.0625% of the calibrated span or upper range value, whichever is greater.

![Figure 49. Schematic of Honeywell STG944 Pressure Transducer [36]](image)

**Temperature**

For temperature measurements, a grounded Type N thermocouple constructed by OMEGA is used. The thermocouples have a quick response time (0.55 s) and can handle temperatures up to 1335°C. The accuracy of the thermocouple is ±1.1°C or ±0.40% of
the reading, whichever is greater. Their location is shown clearly on Figure 33 and Figure 36. A row of thermocouples is placed along the midline extended from the hot duct to capture the plume as it moves across the bottom of the test section. Another row of thermocouples is perpendicular to the previous row. This arrangement is to capture temperature information with respect to the plume’s width as the front moves across the bottom of the lower plenum. As in the case of the pressure tubing, the thermocouples can be moved vertically if measurements need to be made at different vertical positions.

**Oxygen Sensors**

Oxygen concentration instrumentation suitable for the current application is a challenging technical issue. An extensive search was performed to find the best candidates for the experimental facility. In-situ oxygen sensors seemed to be the best type of oxygen sensor since they generally had the lowest response times. However, the formal criteria for the oxygen sensors are as follows: (i) small probe diameter to avoid large flow disturbances or so that it can fit inside of a hollowed-out support column, (ii) quick response time due to the small time scale associated with the density-driven stratified flow in the scaled-down facility [38], (iii) allowable sample temperature range of the sensor (ambient to 900°C) contains the temperature range of interest (200-800°C) for the experiments, (iv) sturdy sensor construction allows sensor to structurally withstand elevated pressures (up to 50 psig) without cracking, (v) high accuracy, and (vi) small minimum insertion length so that measurements close to the bottom of the test section can be made. A total of 22 different oxygen sensors were researched. The oxygen sensor models are listed in Table 19.
No. | Oxygen Sensor Make/Model
---|------------------------
1 | Econox/Carboprobe HT
2 | Econox/Carboprobe DS
3 | Econox/LT Probe
4 | Bosch/Automotive Sensor
5 | Ametek, Inc./WDG-INSITU
6 | General Electric/FGA-311
7 | Bhoomi/BI 2000
8 | Datatest/Model DT 3000
9 | Yokogawa/ZR202G
10 | Yokogawa/ZR402G
11 | United Process Controls/CS 87
12 | United Process Controls/Oxyfire
13 | Rosemount/In Situ Oxymitter
14 | Preferred Instruments, Inc./Model ZP
15 | Land Instruments/WDG1200
16 | Land Instruments/WDG1210
17 | Honeywell/MF020
18 | Honeywell/GMS-10 RVS Series
19 | Honeywell/KGZ-10 Series
20 | Forney/ZR-22
21 | Air Instruments & Measurements, LLC/Model 3600
22 | Teledyne Analytical Instruments 9060H Zirconium Oxide Oxygen Sensor

Table 19. List of oxygen sensor candidates considered for scaled-down facility

Given the criteria listed above, the Teledyne 9060H Zirconium Oxide sensor was chosen as the best suitable sensor for the scaled-down facility. The sensor probe has a probe diameter of 0.75” as shown in Figure 50. The response time of the sensor is 90% in less than 4 seconds. The sensor can operate in temperatures up to 900°C. It can structurally withstand pressures up to 50 psig without cracking. The Teledyne 9060H has an accuracy of ±1% of the actual measured oxygen value with a repeatability of ±0.5% of the measured value. The 9060H also has a very small minimum insertion length. It only
requires that the sensing element which is located at the tip of the probe be exposed to the sample gas to be analyzed.

Figure 50. Schematic of Teledyne Series 9060 Zirconium Oxide Oxygen Sensor [37]

Non-intrusive methods for measuring oxygen concentration were also explored and will be used in the current design. This can be done through planar laser-induced
fluorescence. With the current window design that allows one to look into the vessel, oxygen concentration can be measured using a Nd:Yag Pulsed Laser set to a wavelength of 266 nm while seeding the air with acetone. The images are then captured on a high speed electronic camera and processed on the computer. Measurements of this type have been up to temperatures of 1000 K.

Local Velocity

To make local velocity measurements, traditional methods were considered such as pitot tubes and thermal anemometers. However, due to the low-flow conditions, the resulting dynamics pressure will not be sufficiently large that velocity can be accurately measured. Furthermore, due to the high temperature and transient concentration, thermal anemometry becomes very difficult, if not impractical, to implement. Therefore, the best option to measure local velocity conditions for high temperature, low-flow conditions is particle image velocimetry.

Particle image velocimetry (PIV) tracks the motion of seeding particles that are embedded in the flow with a laser and camera system. This is similar to what is done in planar laser-induced fluorescence (PLIF) except with PIV the camera is tracking the reflection of the laser beam from the seeding particles. During PLIF, there’s actually a molecular excitation so the wavelength of light that is observed by the camera is different than the wavelength of the laser. The seeding particles that are used for PIV are titanium dioxide. These have an exceptionally high melting point approximately 1830°C so they are suitable for this application.
Conclusion

The concept for a modified 1/8th geometric scaled-down facility has been demonstrated. The scaled-down test facility is similar to the prototypic geometry in terms of hydraulic resistance. In fact, on average, the non-dimensional resistance number of the scaled-down facility deviates 2.81% in terms of relative accuracy from the non-dimensional resistance number of the prototypic design while the non-dimensional Froude number is matched perfectly.

A heat transfer characterization of the scaled-down facility is performed and compared against the prototypic case. Utilizing different boundary conditions of a one-dimensional fin analysis, the mole fraction of air was varied as well its far-field temperature. The results demonstrate that the heat transfer coefficient varies from about 3-12 W/(m²·K) for the prototype geometry and from 5-15 W/(m²·K) for the scaled-down geometry. Since the heat transfer coefficients vary over a small interval, despite different boundary conditions and parameter values, the natural circulation phenomenology of each geometric case will be preserved. Moreover, the range of heat transfer coefficients for the prototype and scaled-down geometries are very similar. This means that the heat transfer coefficient in the prototype system and the scaled-down system will be close enough to each other during the course of an event that the basic natural circulation phenomenology will be preserved. This ensures that a high degree of heat transfer similarity will be maintained.

In addition to a heat transfer characterization of the scaled-down facility, calculations were performed to determine the operational heating power of shell/heater
system and to validate the lumped capacitance approximation. The calculations show that with a heater power of 125 W the surface temperature distribution of the rod is closely matched to the surface temperature distribution of a scaled-down solid rod and a prototypic support column. Moreover, the average radial heat flux of the shell/heater system is closely matched to the prototypic and scaled-down rods when the heater power is 0 W. In the course of this analysis, an overall time scale was provided which is dependent on the density-driven stratified flow and hot plenum natural circulation phenomena. The overall time scale is 16.07 s for the prototypic case and 5.67 s for the scaled-down case.

A containment free volume \((V = 1 \text{ m}^3)\) was determined to house the scaled-down facility. With the containment free volume known, initial vessel pressures to preserve the air-to-helium mole ratio \((P = 40 \text{ psig})\) and mixed mean temperature \((P = 34.2 \text{ psig})\) of the prototype case were calculated. The initial stages of the blowdown were studied. It was found that in both types of geometry that the duration of the blowdown is negligible when compared to the total time scale of the air-ingress accident, but that helium temperature dropped appreciably. This issue was resolved by maintaining similarity of the non-dimensional Froude number over the temperature range of interest.

Vessel design drawings have been provided as well as the rationale for the current design. The type of instrumentation (temperature, pressure, velocity, and concentration) and the sensors’ locations were provided. The location of these sensors was determined, in large part, by the computational fluid dynamic studies available in the literature as well as those being performed at the author’s institution.
References


Appendix A: The Complete Results from One-dimensional Support Column Fin Analysis

These figures represent the complete set of results for the one-dimensional support column heat transfer fin analysis that was performed in Chapter 2. The first set of figures is for the prototype geometry (Figure 51 - Figure 60) while the second set of figures is for the scaled-down geometry (Figure 61 - Figure 70). The Biot number, the thermal time constant, the boundary later thickness, the average heat transfer coefficient, and average Rayleigh number are plotted against the far-field temperature and the air mole fraction while being parameterized by the other in a given figure.
Prototype Geometry and Specifications

Figure 51. Average Biot number v. far-field temperature for prototype geometry

Figure 52. Average Biot number v. air mole fraction for prototype geometry
Figure 53. Average thermal time constant v. far-field temperature for prototype geometry

Figure 54. Average thermal time constant v. air mole fraction for prototype geometry
Figure 55. Average boundary layer thickness v. far-field temperature for prototype geometry

Figure 56. Average boundary layer thickness v. far-field temperature for prototype geometry
Figure 57. Average heat transfer coefficient v. far-field temperature for prototype geometry

Figure 58. Average heat transfer coefficient v. far-field temperature for prototype geometry
Figure 59. Average Rayleigh number v. far-field temperature for prototype geometry

Figure 60. Average Rayleigh number v. air mole fraction for prototype geometry
Scaled-down Geometry and Specifications

Figure 61. Average Biot number v. far-field temperature for scaled-down geometry

Figure 62. Average Biot number v. air mole fraction for scaled-down geometry
Figure 63. Average thermal time constant v. far-field temperature for scaled-down geometry

Figure 64. Average thermal time constant v. air mole fraction for scaled-down geometry
Figure 65. Average boundary layer thickness v. far-field temperature for scaled-down geometry

Figure 66. Average boundary layer thickness v. air mole fraction for scaled-down geometry
Figure 67. Average heat transfer coefficient v. far-field temperature for scaled-down geometry

Figure 68. Average heat transfer coefficient v. air mole fraction for scaled-down geometry

125
Figure 69. Average Rayleigh number v. far-field temperature for scaled-down geometry

Figure 70. Average Rayleigh number v. air mole fraction for scaled-down geometry
MATLAB code for heat transfer characteristics of a support column using a one-dimensional fin analysis with fixed temperature (Dirichlet) boundary conditions

```matlab
clc; clear all; close all
format long

% Input
T_inf = 50; % units = [C]; far-field temperature
y_a = 1; % unitless; air mole fraction
y_he = 1 - y_a; % unitless; helium mole fraction
H = 2.84; % units = [m]; prototype rod height
D = 0.212; % units = [m]; prototype rod diameter
T_o = 750; % units = [C]; upper-boundary temperature - top of the rod
T_H = 490; % units = [C]; lower-boundary temperature - bottom of the rod

% Initial Temperature and Pressure
Tw = (T_o + T_H)/2; % initial average wall temp; units = [C]
P = 1.01325; % units = [bar]

% Constants
g = 9.81; % units = [m/s^2]; acceleration of gravity

M_he = 4.002602; % units = [g/mol]; Helium molecular weight
M_a = 28.9644; % units = [g/mol]; Air molecular weight
```
mf_he = (y_he*M_he)/(y_he*M_he + y_a*M_a); % unitless; Helium mass fraction
mf_a = 1 - mf_he; % unitless; Air mass fraction

R = 8.314462175; % units = [J/mol/K]; Universal Gas Constant
R_a = 1000*R/M_a; % units = [J/kg/K]; Specific Gas Constant for Air
R_he = 1000*R/M_he; % units = [J/kg/K]; Specific Gas Constant for He

tol = 1e-12; % error tolerance
res = 0.1; % error residual
iter = 0; % iteration count

while res > tol
    T = (T_w + T_inf)/2;

    % Helium Thermo-physical Properties
    mu_he = 3.674e-7*(T + 273.15)^0.7; % Dynamic Viscosity; units = [kg/(m*s)]
    rho_he = 48.14*P/(T + 273.15)*(1 + 0.4446*P/(T + 273.15)^1.2)^-1; % Density; units = [kg/m^3]
    nu_he = mu_he/rho_he; % kinematic viscosity; momentum diffusivity; units = [m^2/s]
    kf_he = 2.682e-3*(1 + 1.123e-3*P)*(T + 273.15)^(0.71*(1 - 2e-4*P)); % Fluid thermal conductivity; units = [W/(m*K)]
    cp_he = 5193; % units = [J/kg/K]; Specific Heat Capacity at Constant Pressure

    % Air Thermo-physical Properties
    mu_a = 2.5914e-15*(T + 273.15)^3 - 1.4346e-11*(T + 273.15)^2 +... - 5.0523e-8*(T + 273.15) + 4.1130e-6; % Dynamic Viscosity; units = [Pa*s]
    rho_a = 345.57*((T + 273.15) - 2.6884)^-1; % Density; units = [kg/m^3]
    nu_a = mu_a/rho_a; % kinematic viscosity; momentum diffusivity; units = [m^2/s]
    kf_a = 3.42253617e-4 + 9.8497252e-5*(T + 273.15) - 4.88240e-8*(T + 273.15)^2 +... + 1.7542e-11*(T + 273.15)^3; % Air thermal conductivity; units = [W/(m*K)]
    cp_a = 1000*(1.3864e-13*(T + 273.15)^4 - 6.4747e-10*(T + 273.15)^3 +... + 1.0234e-6*(T + 273.15)^2 - 4.3282e-4*(T +273.15) + 1.0613); % units = [J/kg/K]; % Specific Heat Capacity at Constant Pressure

    % Air-Helium Mixture Thermo-physical Properties
    rho = y_he*rho_he + y_a*rho_a; % units = [kg/m^3]; Mixture Density
    phi_a = (1 + (mu_a/mu_he)^(1/2)*(M_he/M_a)^(1/4))^2*(8*(1 + M_a/M_he))^(1/2);
\[ \phi_{he} = (1 + \left( \frac{\mu_{he}}{\mu_a} \right)^{1/2} \left( \frac{M_a}{M_{he}} \right)^{1/4})^2 \left( 8 \left( 1 + \frac{M_{he}}{M_a} \right) \right)^{1/2}; \]  
% units = [Pa*s]; Mixture Dynamic Viscosity

\[ \mu = \mu_a \left( 1 + \frac{y_{he}}{y_a} \phi_a \right)^{-1} + \mu_{he} \left( 1 + \frac{y_a}{y_{he}} \phi_{he} \right)^{-1}; \]  
% units = [Pa*s]; Kinematic Viscosity

\[ \nu = \frac{\mu}{\rho}; \]  
% units = [m^2/s]; Kinematic Viscosity

\[ \rho_{a} = 0.115 + 0.354 \left( \frac{cp_a}{R_a} \right); \]  
\[ \rho_{he} = 0.115 + 0.354 \left( \frac{cp_{he}}{R_{he}} \right); \]

\[ \phi_{a} = 1.065 \left( 1 + \left( \frac{p_{he}}{p_{a}} \right)^{1/2} \left( \frac{M_{he}}{M_a} \right)^{1/4} \right)^2 \left( 8 \left( 1 + \frac{M_a}{M_{he}} \right) \right)^{-1/2}; \]  
% units = [W/m/K]; Mixture Thermal Conductivity

\[ \phi_{he} = 1.065 \left( 1 + \left( \frac{p_{a}}{p_{he}} \right)^{1/2} \left( \frac{M_{a}}{M_{he}} \right)^{1/4} \right)^2 \left( 8 \left( 1 + \frac{M_{he}}{M_a} \right) \right)^{-1/2}; \]  
% units = [W/m/K]; Mixture Thermal Conductivity

\[ cp_{a} = mf_{a} \times cp_{a} + mf_{he} \times cp_{he}; \]  
% units = [J/kg/K]; Mixture Specific Heat at Constant Pressure

% Non-Dimensional Numbers

\[ Pr = \frac{\mu \times cp}{kf}; \]  
% Prandtl Number; dimensionless

\[ Gr = \frac{g}{(T + 273.15)} \left( T_w - T_{inf} \right) H^3 \nu^2; \]  
% Grashof Number; dimensionless

\[ Ra = Pr \times Gr; \]  
% Rayleigh Number; dimensionless

\[ if \ Ra > 1e9 \]  
\[ \quad Nu = 0.10 \times Ra^{1/3}; \]  
\[ else \]  
\[ \quad Nu = 0.59 \times Ra^{1/4}; \]  
\[ end \]

\[ h = Nu \times kf / H; \]  
% heat transfer coefficient; units = [W/m^2/K]

% Rod Thermo-physical Properties

\[ ks = 3e-8 \times T_{w}^3 - 1e-5 \times T_{w}^2 - 0.0612 \times T_{w} + 125.56; \]  
% unirradiated rod thermal conductivity; units = [W/m/K]

\[ cps = 9e-7 \times T_{w}^3 - 0.0027 \times T_{w}^2 + 3.0203 \times T_{w} + 643.24; \]  
% rod specific heat capacity; units = [J/(kg*K)]

\[ rhos = 1770; \]  
% rod density; units = [kg/m^3]

\[ m = \left( 4 \times h / (D \times ks) \right)^{1/2}; \]  
% units = [m^{-1}]

\[ Tw_{new} = \frac{T_H + T_o - 2 \times T_{inf} \times \cosh(m \times H) - 1}{(m \times H \times \sinh(m \times H))} + T_{inf}; \]  
% new average rod temp; units = [C]

\[ res = abs(Tw_{new} - Tw); \]  
\[ Tw = Tw_{new}; \]
\[ iter = iter + 1; \]
end

h
Bi = h*D/(4*ks)
Ra
tau = rhos*D*cps/(4*h)
BL = 3.93*Pr^(-1/2)*(0.952 + Pr)^(1/4)*Gr^(-1/4)*H % Boundary Layer Thickness; units = [m]
MATLAB code for heat transfer characteristics of a support column using a one-dimensional fin analysis with mixed boundary conditions

```matlab
clc; clear all; close all
format long

% Input
T_inf = 50; % units = [°C]; far-field temperature
y_a = 1; % unitless; helium mole fraction
y_he = 1 - y_a; % unitless; air mole fraction

H = 2.84; % units = [m]; prototype rod height
D = 0.212; % units = [m]; prototype rod diameter

%H = 0.3556; % units = [m]; scaled-down rod height
%D = 0.055; % units = [m]; scaled-down rod diameter

T_o = 750; % units = [°C]; upper-boundary temperature - top of the rod
T_H = 490; % units = [°C]; lower-boundary temperature - bottom of the rod

% Initial Temperature and Pressure
Tw = (T_o + T_H)/2; % initial average wall temp; units = [°C]
P = 1.01325; % units = [bar]

% Constants
g = 9.81; % units = [m/s²]; acceleration of gravity

M_he = 4.002602; % units = [g/mol]; Helium molecular weight
M_a = 28.9644; % units = [g/mol]; Air molecular weight

mf_he = (y_he*M_he)/(y_he*M_he + y_a*M_a); % unitless; Helium mass fraction
mf_a = 1 - mf_he; % unitless; Air mass fraction

R = 8.314462175; % units = [J/mol/K]; Universal Gas Constant
R_a = 1000*R/M_a; % units = [J/kg/K]; Specific Gas Constant for Air
R_he = 1000*R/M_he; % units = [J/kg/K]; Specific Gas Constant for He
```
tol = 1e-12; % error tolerance
res = 0.1; % error residual
iter = 0; % iteration count

while res > tol
T = (Tw + T_inf)/2;

% Helium Thermo-physical Properties
mu_he = 3.674e-7*(T + 273.15)^0.7; % Dynamic Viscosity; units = [kg/(m*s)]
rho_he = 48.14*P/(T + 273.15)*(1 + 0.4446*P/(T + 273.15)^1.2)^-1; % Density; units = [kg/m^3]
nu_he = mu_he/rho_he; % kinematic viscosity; momentum diffusivity; units = [m^2/s]
kf_he = 2.682e-3*(1 + 1.123e-3*P)*T + 273.15)^(0.71*(1 - 2e-4*P)); % Fluid thermal conductivity; units = [W/(m*K)]
cp_he = 5193; % units = [J/kg/K]; Specific Heat Capacity at Constant Pressure

% Air Thermo-physical Properties
mu_a = 2.5914e-15*(T + 273.15)^3 - 1.4346e-11*(T + 273.15)^2 + 5.0523e-8*(T + 273.15) + 4.1130e-6; % Dynamic Viscosity; units = [Pa*s]
rho_a = 345.57*((T + 273.15) - 2.6884)^-1; % Density; units = [kg/m^3]
nu_a = mu_a/rho_a; % kinematic viscosity; momentum diffusivity; units = [m^2/s]
kf_a = 3.42253617e-4 + 9.8497252e-5*(T + 273.15) - 4.88240e-8*(T + 273.15)^2 + 1.7542e-11*(T + 273.15)^3; % Air thermal conductivity; units = [W/(m*K)]
cp_a = 1000*(1.3864e-13*(T + 273.15)^4 - 6.4747e-10*(T + 273.15)^3 + 1.0234e-6*(T + 273.15)^2 - 4.3282e-4*(T + 273.15) + 1.0613); % units = [J/kg/K]; Specific Heat Capacity at Constant Pressure

% Air-Helium Mixture Thermo-physical Properties
rho = y_he*rho_he + y_a*rho_a; % units = [kg/m^3]; Mixture Density
phi_a = (1 + (mu_a/mu_he)^(1/2)*(M_he/M_a)^(1/4))^(1/2);
phi_he = (1 + (mu_he/mu_a)^(1/2)*(M_a/M_he)^(1/4))^(1/2);
mu = mu_a*(1 + y_he/y_a*phi_a)^(-1) + mu_he*(1 + y_a/y_he*phi_he)^(-1); % units = [Pa*s]; Mixture Dynamic Viscosity
nu = mu/rho; % units = [m^2/s]; Kinematic Viscosity
\[ P_a = 0.115 + 0.354 \left( \frac{cp_a}{R_a} \right); \]
\[ P_{he} = 0.115 + 0.354 \left( \frac{cp_{he}}{R_{he}} \right); \]
\[ \phi_{phip_a} = 1.065 \left( 1 + \left( \frac{kf_a P_{he}}{kf_{he} P_a} \right)^{\frac{1}{2}} \left( \frac{M_{he}}{M_a} \right)^{\frac{1}{4}} \right)^2 \times \left( 8 \left( 1 + \frac{M_a}{M_{he}} \right) \right)^{-\frac{1}{2}}; \]
\[ \phi_{phip_{he}} = 1.065 \left( 1 + \left( \frac{kf_{he} P_a}{kf_a P_{he}} \right)^{\frac{1}{2}} \left( \frac{M_a}{M_{he}} \right)^{\frac{1}{4}} \right)^2 \times \left( 8 \left( 1 + \frac{M_{he}}{M_a} \right) \right)^{-\frac{1}{2}}; \]
\[ kf = kf_a \left( 1 + \frac{y_{he}}{y_a} \phi_{phip_a} \right)^{-1} + kf_{he} \left( 1 + \frac{y_a}{y_{he}} \phi_{phip_{he}} \right)^{-1}; \% \text{ units } = [W/m/K]; \text{ Mixture Thermal Conductivity} \]
\[ cp = mf_a cp_a + mf_{he} cp_{he}; \% \text{ units } = [J/kg/K]; \text{ Mixture Specific Heat at Constant Pressure} \]

\% Non-Dimensional Numbers

\[ Pr = \mu cp/kf; \% \text{ Prandtl Number; dimensionless} \]
\[ Gr = g \times 1/(T + 273.15) \times (T_w - T_{inf}) \times H^3/\nu^2; \% \text{ Grashof Number; dimensionless} \]
\[ Ra = Pr \times Gr; \% \text{ Rayleigh Number; dimensionless} \]

\% Rod Thermo-physical Properties

\[ ks = 3e-8 \times T_w^3 - 1e-5 \times T_w^2 - 0.0612 \times T_w + 125.56; \% \text{ unirradiated rod thermal conductivity; units } = [W/m/K] \]
\[ cps = 9e-7 \times T_w^3 - 0.0027 \times T_w^2 + 3.0203 \times T_w + 643.24; \% \text{ rod specific heat capacity; units } = [J/(kg*K)] \]
\[ rhos = 1770; \% \text{ rod density; units } = [kg/m^3] \]
\[ m = (4h/(D \times ks))^{\frac{1}{2}}; \% \text{ units } = [m^{-1}] \]
\[ T_{w\_new} = (T_o - T_{inf})/(m \times H) \times tanh(m \times H) + T_{inf}; \% \text{ new average rod temp; units } = [C] \]

\% heat transfer coefficient; units = [W/m^2/K]
BL = 3.93*Pr^(-1/2)*(0.952 + Pr)^(-1/4)*Gr^(-1/4)*H \ % Boundary Layer Thickness; units = [m]
MATLAB code for heat transfer characteristics of a support column using a one-dimensional fin analysis with constant wall temperature

```matlab
%% Heat Transfer Coefficients, Biot Numbers, and Transient Conduction Time
%% Scales for Prototype/Scaled-Down VHTR Support Columns: 1-D Calculation
%% Constant Wall Temperature
%% Binary Mixture: Air/Helium
clc; clear all; close all
format long

% Input
T_inf = 50;

y_a = 1; % unitless; air mole fraction
y_he = 1 - y_a; % unitless; helium mole fraction

H = 2.84; % units = [m]; prototype rod height
D = 0.212; % units = [m]; prototype rod diameter

%H = 0.3556; % units = [m]; scaled-down rod height
%D = 0.055; % units = [m]; scaled-down rod diameter

% Initial Temperature and Pressure
Tw = 750; % initial average wall temp; units = [C]
P = 1.01325; % units = [bar]

% Constants

g = 9.81; % units = [m/s^2]; acceleration of gravity
M_he = 4.002602; % units = [g/mol]; Helium molecular weight
M_a = 28.9644; % units = [g/mol]; Air molecular weight

mf_he = (y_he*M_he)/(y_he*M_he + y_a*M_a); % unitless; Helium mass fraction
mf_a = 1 - mf_he; % unitless; Air mass fraction

R = 8.314462175; % units = [J/mol/K]; Universal Gas Constant
R_a = 1000*R/M_a; % units = [J/kg/K]; Specific Gas Constant for Air
R_he = 1000*R/M_he; % units = [J/kg/K]; Specific Gas Constant for He

iter = 0; % iteration count
```
T = (Tw + T_inf)/2;

% Helium Thermo-physical Properties
mu_he = 3.674e-7*(T + 273.15)^0.7; % Dynamic Viscosity; units = [kg/(m*s)]
rho_he = 48.14*P/(T + 273.15)*(1 + 0.4446*P/(T + 273.15)^1.2)^-1; % Density; units = [kg/m^3]
u_he = mu_he/rho_he; % kinematic viscosity; momentum diffusivity; units = [m^2/s]
kf_he = 2.682e-3*(1 + 1.123e-3*P)*(T + 273.15)^0.71*(1 - 2e-4*P); % Fluid thermal conductivity; units = [W/(m*K)]
cp_he = 5193; % units = [J/kg/K]; Specific Heat Capacity at Constant Pressure

% Air Thermo-physical Properties
mu_a = 2.5914e-15*(T + 273.15)^3 - 1.4346e-11*(T + 273.15)^2 + 5.0523e-8*(T + 273.15) + 4.1130e-6; % Dynamic Viscosity; units = [Pa*s]
rho_a = 345.57*((T + 273.15) - 2.6884)^-1; % Density; units = [kg/m^3]
u_a = mu_a/rho_a; % kinematic viscosity; momentum diffusivity; units = [m^2/s]
kf_a = 3.42253617e-4 + 9.8497252e-5*(T + 273.15) - 4.88240e-8*(T + 273.15)^2 + ...
1.7542e-11*(T + 273.15)^3; % Air thermal conductivity; units = [W/(m*K)]
cp_a = 1000*(1.3864e-13*(T + 273.15)^4 - 6.4747e-10*(T + 273.15)^3 + ...
1.0234e-6*(T + 273.15)^2 - 4.3282e-4*(T +273.15) + 1.0613); % Specific Heat Capacity at Constant Pressure

% Air-Helium Mixture Thermo-physical Properties
rho = y_he*rho_he + y_a*rho_a; % units = [kg/m^3]; Mixture Density
phi_a = (1 + (mu_a/mu_he)^(1/2)*(M_he/M_a)^(1/4))^2*(8*(1 + M_a/M_he))^(-1/2);
phi_he = (1 + (mu_he/mu_a)^(1/2)*(M_a/M_he)^(1/4))^2*(8*(1 + M_he/M_a))^(-1/2);
mu = mu_a*(1 + y_he/y_a*phi_a)^(-1) + mu_he*(1 + y_a/y_he*phi_he)^(-1); % units = [Pa*s]; Mixture Dynamic Viscosity
nu = mu/rho; % units = [m^2/s]; Kinematic Viscosity

P_a = 0.115 + 0.354*(cp_a/R_a);
P_he = 0.115 + 0.354*(cp_he/R_he);
phip_a = 1.065*(1 + ((kf_a*P_he)/(kf_he*P_a))^(1/2)*(M_he/M_a)^(-1/4))^2*... (8*(1 + M_a/M_he))^(-1/2);
\[
\text{phip}\_\text{he} = 1.065*(1 + \\
((\text{kf}\_\text{he}\*P\_a)/(\text{kf}\_\text{a}\*P\_\text{he}))^{(1/2)}*(M\_\text{a}/M\_\text{he})^{(1/4)})^{2*} \\
(8*(1 + M\_\text{he}/M\_\text{a}))^{(-1/2)}; \\
\text{kf} = \text{kf}\_\text{a}*(1 + y\_\text{he}/y\_\text{a}*\text{phip}\_\text{a})^{(-1)} + \text{kf}\_\text{he}*(1 + y\_\text{a}/y\_\text{he}*\text{phip}\_\text{he})^{(-1)}; \\
% \text{units} = [W/m/K]; \text{Mixture Thermal Conductivity}
\]

\[
\text{cp} = \text{mf}\_\text{a}\*\text{cp}\_\text{a} + \text{mf}\_\text{he}\*\text{cp}\_\text{he}; \\
% \text{units} = [J/kg/K]; \text{Mixture Specific Heat at Constant Pressure}
\]

% Non-Dimensional Numbers

\[
\text{Pr} = \mu*\text{cp}/\text{kf}; \quad \% \text{Prandtl Number}; \quad \text{dimensionless}
\]

\[
\text{Gr} = g*1/(T + 273.15)*(T\_\text{w} - T\_\text{inf})*H^3/\nu^2; \quad \% \text{Grashof Number}; \quad \text{dimensionless}
\]

\[
\text{Ra} = \text{Pr}\*\text{Gr}; \quad \% \text{Rayleigh Number}; \quad \text{dimensionless}
\]

\[
\text{if} \text{ Ra} > 1e9
\quad \text{Nu} = 0.10*\text{Ra}^{(1/3)};
\text{else}
\quad \text{Nu} = 0.59*\text{Ra}^{(1/4)};
\text{end}
\]

\[
\text{h} = \text{Nu}\*\text{kf}/H; \quad \% \text{heat transfer coefficient}; \quad \text{units} = [W/m^2/K]
\]

% Rod Thermo-physical Properties

\[
\text{ks} = 3e-8*T\_\text{w}^3 - 1e-5*T\_\text{w}^2 - 0.0612*Tw + 125.56; \quad \% \text{unirradiated rod thermal conductivity}; \quad \text{units} = [W/m/K]
\]

\[
\text{cps} = 9e-7*T\_\text{w}^3 - 0.0027*T\_\text{w}^2 + 3.0203*Tw + 643.24; \quad \% \text{rod specific heat capacity}; \quad \text{units} = [J/(kg*K)]
\]

\[
\text{rhos} = 1770; \quad \% \text{rod density}; \quad \text{units} = [kg/m^3]
\]

% hold on
% plot(T\_inf,h,**)
% xlabel('Far-field Temperature (C)')
% ylabel('Heat Transfer Coefficient (W/(m^2*K))')
% title('Heat Transfer Coefficient v. Far-field Temperature (C)')
% legend('100% Helium','10% Air/90% He','20% Air/80% He','30% Air/70% He',...
% '40% Air/60% He','50% Air/50% He','60% Air/40% He','70% Air/30% He',...
% '80% Air/20% He','90% Air/10% He','100% Air','-1)

\[
\text{h}
\]

\[
\text{Bi} = \text{h*D}/(4*\text{ks})
\text{Ra}
\text{tau} = \text{rhos*D*cp}/(4*\text{h})
\]
BL = 3.93*Pr^(-1/2)*(0.952 + Pr)^{(1/4)}*Gr^(-1/4)*H % Boundary Layer Thickness; units = [m]
MATLAB code for one-dimensional transient temperature distribution of prototypic and scaled-down support columns

```matlab
%% One-dimensional Transient Temperature Profile for prototypic and scaled-down VHTR Support Columns in Cylindrical Coordinates
%% Finite Difference Discretization; Explicit Time Marching Scheme
clc; clear all; close all
format long

% Prototype
% R = 0.106; % units = [m]; Prototype Support Column Radius
R = 0.0275; % units = [m]; Scaled-Down Support Column Radius
N = 301; % Number of Discretization Nodes
dr = R/(N-1);

% Prototype
% h = 6.1996496537452; % units = [W/m^2/K]; Lower Bound 750C; Constant Tw BC; 100% Air; T_inf = 25C
% h = 12.68797285887; % units = [W/m^2/K]; Upper Bound 750C; Constant Tw BC; 50% He/50% Air; T_inf = 25C
% tau = 16.06; % units = [s]; prototype time scale

% Scaled-down Facility
% h = 7.9490104123302; % units = [W/m^2/K]; Lower Bound 750C; Constant Tw BC; 100% Air; T_inf = 25C
% h = 15.571321316995; % units = [W/m^2/K]; Upper Bound 750C; Constant Tw BC; 100% He; T_inf = 25C
% tau = 5.67; %SD facility time scale

% Thermo-physical Properties of Support Column
k = 85; % units = [W/m/K]; Support Column Thermal Conductivity
rho = 1770; % units = [kg/m^3]; SC Density
cp = 1720; % units = [J/kg/K]; SC Specific Heat Capacity
a = k/(rho*cp); % units = [m^2/s]; SC Thermal Diffusivity
dtmax = dr^2/(2*a); % units = [s]; maximum time step
dt = 0.2*dtmax; % units = [s]; time step

Fo = a*dt/dr^2; % Fourier Number - Non-dimensional time
Bi = h*dr/k; % Biot Number; dimensionless number

T_i = 750; % units = [C]; Initial Temperature Condition
T_inf = 25; % units = [C]; Far-field temperature

% Initialization
```

for i = 1:N
    T_old(i) = T_i;
    r(i) = (i-1)*dr;
    rn(i) = r(i)/R;
end

iter = 0;
gamma = [0 0.05 0.1 0.25 0.5 1 1.5 2 2.5 3];
for j = 1:length(gamma)
    while iter <= nearest(gamma(j)*(tau/dt))
        % Left Node
        i = 1;
        T(i) = (1 - 4*Fo)*T_old(i) + 4*Fo*T_old(i+1);

        % Interior Nodes
        for i = 2:N-1
            T(i) = (Fo - (a*dt)/(2*dr*r(i)))*T_old(i-1) + (1 - 2*Fo)*T_old(i) +... ((a*dt)/(2*dr*r(i)) + Fo)*T_old(i+1);
        end

        % Right Node
        i = N;
        T(i) = 2*Fo*T_old(i-1) + (1 - (a*h*dt)/(k*r(i)) - 2*Fo*(1 + Bi))*T_old(i)+... ((a*h*dt)/(k*r(i)) + 2*Fo*Bi)*T_inf;
        for i = 1:N
            T_old(i) = T(i);
        end
        iter = iter + 1;
    end

    T_g(:,j) = T;
end

plot(rn,T_g(:,1),'b',rn,T_g(:,2),'--r',rn,T_g(:,3),'-.
k',rn,T_g(:,4),'m',rn,T_g(:,5),'.--b',rn,T_g(:,6),'.--r',... rn,T_g(:,7),'.k',rn,T_g(:,8),'.--m',rn,T_g(:,9),'.--b',rn,T_g(:,10),'r')
legend('t/\tau = 0','t/\tau=0.05','t/\tau=0.10','t/\tau=0.25','t/\tau=0.50',... 't/\tau=1.00','t/\tau=1.50','t/\tau=2.00','t/\tau=2.50','t/\tau=3.00',0)

title('Temperature v. Normalized Radial Position (Scaled-down Geometry)')
xlabel('Normalized Radial Distance from Support Column Centerline')
ylabel('Temperature (°C)')
grid on
MATLAB code for one-dimensional transient temperature distribution of shell/heater system for scaled-down test section

```matlab
%% One-dimensional Transient Temperature Profile for shell/heater system
%% in cylindrical coordinates
%% Finite Difference Discretization; Explicit Time Marching Scheme
clc; clear all; close all
format long

Ro = 0.0275; % units = [m]; Scaled-Down Support Column Outer Radius
Ri = 0.0127; % units = [m]; Scaled-Down Support Column Inner Radius
N = 251; % Number of Discretization Nodes
dr = (Ro - Ri)/(N-1);

% Scaled-down Facility
h = 7.9490104123302; % units = [W/m^2/K]; Lower Bound 750C; Constant Tw BC; 100% Air; T_inf = 25C
h = 15.5713213816995; % units = [W/m^2/K]; Upper Bound 750C; Constant Tw BC; 100% He; T_inf = 25C
tau = 5.67; % units = [s]; SD facility time scale

% Heater Geometry and Output
Q_dot = 0; % units = [W]
D = 0.935*2.54/100; % heater diameter; units = [m]
H = 14*2.54/100; % heater length; units = [m]
A_s = 2*pi*Ri*H; % heater surface area; units = [m^2]
q = Q_dot/A_s; % heater heat flux; units = [W/m^2]

% Thermophysical Properties of Support Column
k = 85; % units = [W/m/K]; Support Column Thermal Conductivity
rho = 1770; % units = [kg/m^3]; SC Density
cp = 1720; % units = [J/kg/K]; SC Specific Heat Capacity
a = k/(rho*cp); % units = [m^2/s]; SC Thermal Diffusivity
dtmax = dr^2/(2*a); % units = s; maximum time step
dt = 0.2*dtmax; % units = [s]; time step

Fo = a*dt/dr^2; % Fourier Number - Non-dimensional time
Bi = h*dr/k; % Biot Number; dimensionless number

T_i = 750; % units = [C]; Initial Temperature Condition
T_inf = 25; % units = [C]; Far-field temperature
```

142
% Initialization
for i = 1:N
    T_old(i) = T_i;
    r(i) = (i-1)*dr + 0.0125;
    rn(i) = r(i)/Ro;
end

iter = 0;
gamma = [0 0.05 0.1 0.25 0.5 1 1.5 2 2.5 3];
for j = 1:length(gamma)
    while iter <= nearest(gamma(j)*(tau/dt))
        % Left Node
        i = 1;
        T(i) = (1 - 2*Fo)*T_old(i) + 2*Fo*T_old(i+1) + (a*dt*q)/k*(2/dr - 1/r(i));

        % Interior Nodes
        for i = 2:N-1
            T(i) = (Fo - (a*dt)/(2*dr*r(i)))*T_old(i-1) + (1 - 2*Fo)*T_old(i) +... 
                  ((a*dt)/(2*dr*r(i)) + Fo)*T_old(i+1);
        end

        % Right Node
        i = N;
        T(i) = 2*Fo*T_old(i-1) + (1 - (a*dt)/(k*r(i)) - 2*Fo*(1 + Bi))*T_old(i) +... 
               ((a*dt)/(k*r(i)) + 2*Fo*Bi)*T_inf;

        for i = 1:N
            T_old(i) = T(i);
        end

        iter = iter + 1;
    end

T_g(:,j) = T;
end

plot(rn,T_g(:,1),'b',rn,T_g(:,2),'--r',rn,T_g(:,3),'-k',rn,T_g(:,4),'
.m',rn,T_g(:,5),'--b',rn,T_g(:,6),'--r',rn,T_g(:,7),'
.k',rn,T_g(:,8),'--m',rn,T_g(:,9),'
.b',rn,T_g(:,10),'r')
legend(['t/\tau = 0','t/\tau=0.05','t/\tau=0.10','t/\tau=0.25','t/\tau=0.50','...
't/\tau=1.00','t/\tau=1.50','t/\tau=2.00','t/\tau=2.50','t/\tau=3.00',0]
title('Temperature v. Normalized Radial Position (Shell/Heater System)')
xlabel('Normalized Radial Distance from Support Column Centerline')
ylabel('Temperature (^oC)')
grid on
MATLAB code for two-dimensional transient temperature distribution of prototypic support column

%% 2-D (Cylindrical Coordinates) Transient Heat Equation for VHTR Support Column
%% Rod (IG-110 Graphite) Using Explicit Time-Marching Method; Finite Volume Discretization;
%% Uniform Mesh; Non-Constant Thermophysical Properties
%% Prototype Geometry

%% Dirichlet BCs for top and bottom
clc; clear all; close all
format long
g
% Geometry
Lr = 0.106; % units = [m]
Lz = 2.84; % units = [m]
N = 40; % Number of Cells in Horizontal (r) Direction
M = 1000; % Number of Cells in Vertical (z) Direction
dr = Lr/N; % Cell Width - Horizontal Direction
dz = Lz/M; % Cell Height - Vertical Direction
P = 1/dr^2;
Q = 1/dz^2;
R = (dr/2)/Lr:dr/Lr:Lr/Lr;
Z = (dz/2)/Lz:dz/Lz:Lz/Lz;

% Temperatures
T_t = 750; % units = [C]; Upper Boundary Condition
T_b = 750; % units = [C]; Lower Boundary Condition
T_inf = 25; % units = [C]; Far-field Temperature
T_i = 750; % units = [C]; Initial Temperature Condition

% Heat Transfer Coefficients
%h = 6.15799399387808; % units = [W/m^2/K]; Lower Bound 850C; Constant Tw BC; 100% Air; T_inf = 25C
%h = 12.5616471739268; % units = [W/m^2/K]; Upper Bound 850C; Constant Tw BC; 50% He/50% Air; T_inf = 25C
%h = 6.19996496537452; % units = [W/m^2/K]; Lower Bound 750C; Constant Tw BC; 50% He/50% Air; T_inf = 25C
%h = 12.6879727285887; % units = [W/m^2/K]; Upper Bound 750C; Constant Tw BC; 50% He/50% Air; T_inf = 25C
%h = 3.81636529726588; % units = [W/m^2/K]; Lower Bound 850C; Constant Tw BC; 100% Air; T_inf = 500C
%h = 7.666060738358; % units = [W/m^2/K]; Upper Bound 850C; Constant Tw BC; 50% He/50% Air; T_inf = 500C

%h = 3.53442419860516; % units = [W/m^2/K]; Lower Bound 750C; Constant Tw BC; 100% Air; T_inf = 500C

%h = 7.1226789075961; % units = [W/m^2/K]; Upper Bound 750C; Constant Tw BC; 50% He/50% Air; T_inf = 500C

% Time

alpha = 1e-4; % thermal diffusion; units = [m^2/s]
dtmax = 1/(2*alpha*(P+Q));
dt = 0.2*dtmax;
tau = 16.06; % overall time scale; units = [s]
gamma = 3;

% Initialization

for i = 1:N
    for j = 1:M
        T_old(i,j) = T_i;
    end
end

% Position Vector

for i = 1:N
    r_o(i) = 0.5*(2*i - 1)*dr;
    r_e(i) = r_o(i) + dr/2;
    r_w(i) = r_o(i) - dr/2;
end
iter = 0;

while iter < nearest(gamma*tau/dt)
    iter = iter + 1;
end

% Support Column Thermophysical Properties

for i = 1:N
    for j = 1:M
        k(i,j) = 3e-8*T_old(i,j)^3 - 1e-5*T_old(i,j)^2 - 0.0612*T_old(i,j) + 125.56; % T[C]; units = [W/m/K]
        c_p(i,j) = 9e-7*T_old(i,j)^3 - 0.0027*T_old(i,j)^2 + 3.0203*T_old(i,j) + 643.24; % T[C]; units = [J/kg/K]
    end
end
rho = 1770; % density; units = [kg/m^3]
% Interior Cells

for i = 2:N-1
    for j = 2:M-1
        k_e = (k(i+1,j) + k(i,j))/2;
        k_w = (k(i-1,j) + k(i,j))/2;
        k_n = (k(i,j+1) + k(i,j))/2;
        k_s = (k(i,j-1) + k(i,j))/2;
        T(i,j) =
          dt/(rho*c_p(i,j))*(1/(r_o(i)*dr^2)*(k_w*r_w(i)*T_old(i-1,j)-
            (k_w*r_w(i)+k_e*r_e(i))*T_old(i,j)+k_e*r_e(i)*T_old(i+1,j))+
            1/dz^2*(k_s*T_old(i,j-1)-
            (k_s+k_n)*T_old(i,j)+k_n*T_old(i,j+1)))+
          T_old(i,j);
    end
end

% Top Boundary

for i = 2:N-1
    j = M;
    k_e = (k(i+1,j) + k(i,j))/2;
    k_w = (k(i-1,j) + k(i,j))/2;
    k_o = k(i,j);
    k_s = (k(i,j-1) + k(i,j))/2;
    T(i,j) =
      dt/(rho*c_p(i,j))*(1/(r_o(i)*dr^2)*(k_w*r_w(i)*T_old(i-1,j)-
        (k_w*r_w(i)+k_e*r_e(i))*T_old(i,j)+k_e*r_e(i)*T_old(i+1,j))+
        1/dz^2*((k_o/3+k_s)*T_old(i,j-1)-
        (3*k_o+k_s)*T_old(i,j)+(8/3)*k_o*T_t))+
      T_old(i,j);
end

% Bottom Boundary

for i = 2:N-1
    j = 1;
    k_e = (k(i+1,j) + k(i,j))/2;
    k_w = (k(i-1,j) + k(i,j))/2;
    k_n = (k(i,j+1) + k(i,j))/2;
    k_o = k(i,j);
    T(i,j) =
      dt/(rho*c_p(i,j))*(1/(r_o(i)*dr^2)*(k_w*r_w(i)*T_old(i-1,j)-
        (k_e*r_e(i)+k_w*r_w(i))*T_old(i,j)+k_e*r_e(i)*T_old(i+1,j))+
        1/dz^2*((8/3)*k_o*T_b-(k_n+3*k_o)*T_old(i,j)+k_n+k_o/3)*T_old(i,j+1)))+
      T_old(i,j);
end
% Right Boundary

for j = 2:M-1
  i = N;
  k_o = k(i,j);
  k_w = (k(i-1,j) + k(i,j))/2;
  k_n = (k(i,j+1) + k(i,j))/2;
  k_s = (k(i,j-1) + k(i,j))/2;
  T(i,j) = dt/(rho*c_p(i,j))*(1/(r_o(i)*dr)*(-1*h*r_e(i)*...
          ((3*T_old(i,j)-T_old(i-1,j))/2-T_inf) -
          k_w*r_w(i)/dr*(T_old(i,j)-T_old(i-1,j)))+...
          1/dz^2*(k_s*T_old(i,j-1) -
          (k_n+k_s)*T_old(i,j)+k_n*T_old(i,j+1)))+T_old(i,j);
end

% Left Boundary

for j = 2:M-1
  i = 1;
  k_e = (k(i+1,j) + k(i,j))/2;
  k_o = k(i,j);
  k_n = (k(i,j+1) + k(i,j))/2;
  k_s = (k(i,j-1) + k(i,j))/2;
  T(i,j) = dt/(rho*c_p(i,j))*(1/(r_o(i)*dr^2)*(k_e*r_e(i)*T_old(i+1,j)-...
          k_e*r_e(i)*T_old(i,j))+1/dz^2*(k_s*T_old(i,j-1)-...
          (k_s+k_n)*T_old(i,j)+k_n*T_old(i,j+1)))+T_old(i,j);
end

% Top-Right Cell

i = N;
j = M;
k_o = k(i,j);
k_w = (k(i-1,j) + k(i,j))/2;
k_s = (k(i,j-1) + k(i,j))/2;
T(i,j) = dt/(rho*c_p(i,j))*(1/(r_o(i)*dr)*(-1*h*r_e(i)*...
          ((3*T_old(i,j)-T_old(i-1,j))/2-T_inf) -
          k_w*r_w(i)/dr*(T_old(i,j)-T_old(i-1,j)))+...
          1/dz^2*((8/3)*k_o*T_t- (3*k_o+k_s)*T_old(i,j)+(k_o/3+k_s)*T_old(i,j-1)))+T_old(i,j);

% Bottom-Right Cell

i = N;
j = 1;
k_o = k(i,j);
k_w = (k(i-1,j) + k(i,j))/2;
k_n = (k(i,j+1) + k(i,j))/2;
T(i,j) = dt/(rho*c_p(i,j))*(1/(r_o(i)*dr)*(-1*h*r_e(i)*...
\((3T_{\text{old}}(i,j)-T_{\text{old}}(i-1,j))/2-T_{\text{inf}}) - k_w*r_w(i)/dr*(T_{\text{old}}(i,j)-T_{\text{old}}(i-1,j))+... \\
(1/dz^2)*)((8/3)*k_o*T_b - (k_n+3*k_o)*T_{\text{old}}(i,j)+(k_n+k_o/3)*T_{\text{old}}(i,j+1))+T_{\text{old}}(i,j+1);\)

% Bottom-Left Corner

\begin{verbatim}
i = 1; 
j = 1; 
k_o = k(i,j); 
k_e = (k(i+1,j) + k(i,j))/2; 
k_n = (k(i,j+1) + k(i,j))/2; 
T(i,j) = dt/(rho*c_p(i,j))*(1/(r_o(i)*dr^2)*(k_e*r_e(i)*T_{\text{old}}(i+1,j)-... 
k_e*r_e(i)*T_{\text{old}}(i,j))+(1/dz^2)*((8/3)*k_o*T_b - (3*k_o+k_n)*T_{\text{old}}(i,j)+... 
(k_n+k_o/3)*T_{\text{old}}(i,j+1))+T_{\text{old}}(i,j);\end{verbatim}

% Top-Left Corner

\begin{verbatim}
i = 1; 
j = M; 
k_e = (k(i+1,j) + k(i,j))/2; 
k_s = (k(i,j-1) + k(i,j))/2; 
k_o = k(i,j); 
T(i,j) = dt/(rho*c_p(i,j))*(1/(r_o(i)*dr^2)*(k_e*r_e(i)*T_{\text{old}}(i+1,j)-... 
k_e*r_e(i)*T_{\text{old}}(i,j))+(1/dz^2)*((8/3)*k_o*T_t - (3*k_o+k_s)*T_{\text{old}}(i,j)+... 
(k_o/3+k_s)*T_{\text{old}}(i,j-1))+T_{\text{old}}(i,j);\end{verbatim}

for i = 1:N 
    for j = 1:M 
        T_{\text{old}}(i,j) = T(i,j); 
    end 
end 

contourf(R,Z,T',10)
MATLAB code for two-dimensional transient temperature distribution of support column in scaled-down test section

```matlab
%% 2-D (Cylindrical Coordinates) Transient Heat Equation for VHTR Support Column
%% Rod (IG-110 Graphite) Using Explicit Time-Marching Method; Finite Volume Discretization;
%% Uniform Mesh; Non-Constant Thermophysical Properties
%% Scaled-down Geometry

%% Dirichlet BCs for top and bottom
clc; clear all; close all
format long

% Geometry
Lr = 0.0275; % units = [m]
Lz = 0.3556; % units = [m]
N = 40; % Number of Cells in Horizontal (r) Direction
M = 200; % Number of Cells in Vertical (z) Direction
dr = Lr/N; % Cell Width - Horizontal Direction
dz = Lz/M; % Cell Height - Vertical Direction
P = 1/dr^2;
Q = 1/dz^2;

R = (dr/2)/Lr:dr/Lr:Lr/Lr;
Z = (dz/2)/Lz:dz/Lz:Lz/Lz;

% Temperatures
T_t = 750; % units = [C]; Upper Boundary Condition
T_b = 750; % units = [C]; Lower Boundary Condition
T_inf = 25; % units = [C]; Far-field Temperature
T_i = 750; % units = [C]; Initial Temperature Condition

% Heat Transfer Coefficients
h = 7.9490104123302; % units = [W/m^2/K]; Lower Bound 750C; Constant Tw BC; 100% Air; T_inf = 25C
h = 15.5713213816995; % units = [W/m^2/K]; Upper Bound 750C; Constant Tw BC; 100% He; T_inf = 25C
h = 5.52941690843639; % units = [W/m^2/K]; Lower Bound 750C; Constant Tw BC; 100% Air; T_inf = 500C
h = 10.577265041911; % units = [W/m^2/K]; Upper Bound 750C; Constant Tw BC; 100% He; T_inf = 500C
```
alpha = 1e-4; % thermal diffusion; units = [m^2/s]
dtmax = 1/(2*alpha*(P+Q));
dt = 0.2*dtmax;
tau = 5.67; % overall time scale; units = [s]
gamma = 3;

% Initialization
for i = 1:N
    for j = 1:M
        T_old(i,j) = T_i;
    end
end

% Position Vector
for i = 1:N
    r_o(i) = 0.5*(2*i - 1)*dr;
    r_e(i) = r_o(i) + dr/2;
    r_w(i) = r_o(i) - dr/2;
end

iter = 0;

while iter < nearest(gamma*tau/dt)
    iter = iter + 1;

    % Support Column Thermophysical Properties
    for i = 1:N
        for j = 1:M
            k(i,j) = 3e-8*T_old(i,j)^3 - 1e-5*T_old(i,j)^2 -
                     0.0612*T_old(i,j) + 125.56; % T[C]; units = [W/m/K]
            c_p(i,j) = 9e-7*T_old(i,j)^3 - 0.0027*T_old(i,j)^2 +
                         3.0203*T_old(i,j) + 643.24; % T[C]; units = [J/kg/K]
        end
    end

    rho = 1770; % density; units = [kg/m^3]

    % Interior Cells
    for i = 2:N-1
        for j = 2:M-1
            k_e = (k(i+1,j) + k(i,j))/2;
            k_w = (k(i-1,j) + k(i,j))/2;
            k_n = (k(i,j+1) + k(i,j))/2;
            k_s = (k(i,j-1) + k(i,j))/2;
        end
    end
\[
T(i,j) = \\
\frac{dt}{\rho c_p(i,j)} \left(1 \left(\frac{1}{\rho d o(i) d r^2} \left(\frac{k_w r_w(i) T_{old}(i-1,j)}{r_o(i)} - \frac{(k_w r_w(i) + k_e r_e(i)) T_{old}(i,j)}{r_o(i)} + \frac{k_e r_e(i) T_{old}(i+1,j)}{r_o(i)}\right) + \frac{1}{dz^2} \left(\frac{k_s T_{old}(i,j-1)}{3} - \frac{(k_s + k_n) T_{old}(i,j)}{3} + \frac{k_n T_{old}(i,j+1)}{3}\right)\right)\right) + T_{old}(i,j); \\
\end{align*}
\]

end
end

% Top Boundary
\[
\text{for } i = 2:N-1 \text{ \hspace{1cm} } j = M; \\
\begin{align*}
& k_e = (k(i+1,j) + k(i,j))/2; \\
& k_w = (k(i-1,j) + k(i,j))/2; \\
& k_o = k(i,j); \\
& k_s = (k(i,j-1) + k(i,j))/2; \\
& T(i,j) = \\
\frac{dt}{\rho c_p(i,j)} \left(1 \left(\frac{1}{\rho d o(i) d r^2} \left(\frac{k_w r_w(i) T_{old}(i-1,j)}{r_o(i)} - \frac{(k_e r_e(i) + k_w r_w(i)) T_{old}(i,j)}{r_o(i)} + \frac{k_e r_e(i) T_{old}(i+1,j)}{r_o(i)}\right) + \frac{1}{dz^2} \left(\frac{8/3 k_o T_t}{3} - \frac{(k_n + 3 k_o) T_{old}(i,j)}{3} + \frac{(k_n + k_o/3) T_{old}(i,j+1)}{3}\right)\right)\right) + T_{old}(i,j); \\
\end{align*}
\]

end

% Bottom Boundary
\[
\text{for } i = 2:N-1 \text{ \hspace{1cm} } j = 1; \\
\begin{align*}
& k_e = (k(i+1,j) + k(i,j))/2; \\
& k_w = (k(i-1,j) + k(i,j))/2; \\
& k_n = (k(i,j+1) + k(i,j))/2; \\
& k_o = k(i,j); \\
& T(i,j) = \\
\frac{dt}{\rho c_p(i,j)} \left(1 \left(\frac{1}{\rho d o(i) d r^2} \left(\frac{k_w r_w(i) T_{old}(i-1,j)}{r_o(i)} - \frac{(k_e r_e(i) + k_w r_w(i)) T_{old}(i,j)}{r_o(i)} + \frac{k_e r_e(i) T_{old}(i+1,j)}{r_o(i)}\right) + \frac{1}{dz^2} \left(\frac{8/3 k_o T_b}{3} - \frac{(k_n + 3 k_o) T_{old}(i,j)}{3} + \frac{(k_n + k_o/3) T_{old}(i,j+1)}{3}\right)\right)\right) + T_{old}(i,j); \\
\end{align*}
\]

end

% Right Boundary
\[
\text{for } j = 2:M-1 \text{ \hspace{1cm} } i = N; \\
\begin{align*}
& k_o = k(i,j); \\
& k_w = (k(i-1,j) + k(i,j))/2; \\
& k_n = (k(i,j+1) + k(i,j))/2; \\
& k_s = (k(i,j-1) + k(i,j))/2; \\
\end{align*}
\]
\[
T(i,j) = \frac{dt}{(\rho c_p(i,j))} \left( \frac{1}{r_o(i) \cdot dr} \right) \left( -1 \cdot h \cdot r_e(i) \right) \cdot \left( 3 \cdot T_{old}(i,j) - T_{old}(i-1,j) \right) - \frac{1}{dz^2} \left( k_s \cdot T_{old}(i,j-1) - (k_n + k_s) \cdot T_{old}(i,j) + k_n \cdot T_{old}(i,j+1) \right) + T_{old}(i,j);
\]

\% Left Boundary

\begin{verbatim}
for j = 2:M-1
    i = 1;
    k_e = (k(i+1,j) + k(i,j))/2;
    k_o = k(i,j);
    k_n = (k(i,j+1) + k(i,j))/2;
    k_s = (k(i,j-1) + k(i,j))/2;
    T(i,j) =
        \frac{dt}{(\rho c_p(i,j))} \left( \frac{1}{r_o(i) \cdot dr^2} \right) \left( k_e \cdot r_e(i) \cdot T_{old}(i+1,j) - k_e \cdot r_e(i) \cdot T_{old}(i,j) \right) + \frac{1}{dz^2} \left( k_s \cdot T_{old}(i,j-1) - (k_s + k_n) \cdot T_{old}(i,j) + k_n \cdot T_{old}(i,j+1) \right) + T_{old}(i,j);
\end{verbatim}

\% Top-Right Cell

\begin{verbatim}
i = N;
j = M;
    k_o = k(i,j);
    k_w = (k(i-1,j) + k(i,j))/2;
    k_s = (k(i,j-1) + k(i,j))/2;
    T(i,j) =
        \frac{dt}{(\rho c_p(i,j))} \left( \frac{1}{r_o(i) \cdot dr} \right) \left( -1 \cdot h \cdot r_e(i) \right) \cdot \left( 3 \cdot T_{old}(i,j) - T_{old}(i-1,j) \right) - k_w \cdot r_w(i)/dr \cdot \left( T_{old}(i,j) - T_{old}(i-1,j) \right) + \frac{1}{dz^2} \left( \frac{8}{3} \cdot k_o \cdot T_t - (3 \cdot k_o + k_s) \cdot T_{old}(i,j) + (k_o/3 + k_s) \cdot T_{old}(i,j-1) \right) + T_{old}(i,j);
\end{verbatim}

\% Bottom-Right Cell

\begin{verbatim}
i = N;
j = 1;
    k_o = k(i,j);
    k_w = (k(i-1,j) + k(i,j))/2;
    k_n = (k(i,j+1) + k(i,j))/2;
    T(i,j) =
        \frac{dt}{(\rho c_p(i,j))} \left( \frac{1}{r_o(i) \cdot dr} \right) \left( -1 \cdot h \cdot r_e(i) \right) \cdot \left( 3 \cdot T_{old}(i,j) - T_{old}(i-1,j) \right) - k_w \cdot r_w(i)/dr \cdot \left( T_{old}(i,j) - T_{old}(i-1,j) \right) + \frac{1}{dz^2} \left( \frac{8}{3} \cdot k_o \cdot T_b - (k_n + 3 \cdot k_o) \cdot T_{old}(i,j) + (k_n + k_o/3) \cdot T_{old}(i,j+1) \right) + T_{old}(i,j+1);
\end{verbatim}

\% Bottom-Left Corner

\begin{verbatim}
i = 1;
j = 1;
\end{verbatim}
\begin{verbatim}
k_o = k(i,j);
k_e = (k(i+1,j) + k(i,j))/2;
k_n = (k(i,j+1) + k(i,j))/2;
T(i,j) = 
dt/(rho*c_p(i,j))*(1/(r_o(i)*dr^2)*(k_e*r_e(i)*T_old(i+1,j)-...
  k_e*r_e(i)*T_old(i,j))+(1/dz^2)*((8/3)*k_o*T_b-
(3*k_o+k_n)*T_old(i,j)+...
  (k_n+k_o/3)*T_old(i,j+1))+T_old(i,j));

% Top-Left Corner

i = 1;
j = M;
k_e = (k(i+1,j) + k(i,j))/2;
k_s = (k(i,j-1) + k(i,j))/2;
k_o = k(i,j);
T(i,j) = 
dt/(rho*c_p(i,j))*(1/(r_o(i)*dr^2)*(k_e*r_e(i)*T_old(i+1,j)-...
  k_e*r_e(i)*T_old(i,j))+(1/dz^2)*((8/3)*k_o*T_t-
(3*k_o+k_s)*T_old(i,j)+...
  (k_o/3+k_s)*T_old(i,j-1))+T_old(i,j));

for i = 1:N
  for j = 1:M
    T_old(i,j) = T(i,j);
  end
end
end
contourf(R,Z,T',10)
\end{verbatim}
MATLAB code for two-dimensional transient temperature distribution of shell/heater system in scaled-down test section

%%% 2-D Transient Heat Equation for VHTR/Scaled-down Model
Heater/Support Column
%%% Rod (IG-110 Graphite) Using Explicit Time-Marching Method; Finite Volume
%%% Discretization; Uniform Mesh; Non-Constant Thermophysical Properties

%%% Dirichlet BCs for top and bottom
clc; clear all; close all

% Rod Shell Geometry
Lr = 0.0148; % units = [m]
Lz = 14*0.0254; % units = [m]
N = 20; % Number of Cells in Horizontal (x) Direction
M = 200; % Number of Cells in Vertical (y) Direction
dr = Lr/N; % Cell Width - Horizontal Direction
dz = Lz/M; % Cell Height - Vertical Direction
P = 1/dr^2;
Q = 1/dz^2;

R = (0.0127+dr/2)/0.0275:dr/0.0275:0.0275/0.0275;
Z = (dz/2)/Lz:dz/Lz:Lz/Lz;

% Heater Geometry and Output
Q_dot = 125; % units = [W]
D = 0.935*2.54/100; % heater diameter; units = [m]
H = 14*2.54/100; % heater length; units = [m]
A_s = pi*D*H; % heater surface area; units = [m^2]
q1 = Q_dot/A_s; % heater heat flux; units = [W/m^2]

A2 = pi*0.0254*Lz;
q = q1*A_s/A2;

% Temperatures
T_t = 750; % units = [C]
T_b = 750; % units = [C]
T_inf = 25; % units = [C]
T_i = 750; % units = [C]

% Heat Transfer Coefficients
%h = 7.9490104123302; % units = [W/m^2/K]; Lower Bound 750C; Constant
Tw BC; 100% Air; T_inf = 25C
\[ h = 15.5713213816995; \text{units} = [W/m^2/K]; \text{Upper Bound 750C; Constant Tw BC; 100% He; T_{inf} = 25C} \]

\[ h = 5.52941690843639; \text{units} = [W/m^2/K]; \text{Lower Bound 750C; Constant Tw BC; 100% Air; T_{inf} = 500C} \]

\[ h = 10.577265041911; \text{units} = [W/m^2/K]; \text{Upper Bound 750C; Constant Tw BC; 100% He; T_{inf} = 500C} \]

% Time

\[ \alpha = 1e-4; \text{thermal diffusion; units = [m^2/s]} \]
\[ \text{dtmax} = 1/(2*\alpha*(P+Q)); \]
\[ \text{dt} = 0.2*\text{dtmax}; \]
\[ \text{tau} = 5.67; \text{overall time scale; units = [s]} \]
\[ \gamma = 3; \]

% Initialization

\[ \text{for } i = 1:N \]
\[ \quad \text{for } j = 1:M \]
\[ \quad \quad T_{\text{old}}(i,j) = T_i; \]
\[ \quad \text{end} \]
\[ \text{end} \]

% Position Vector

\[ \text{for } i = 1:N \]
\[ \quad r_o(i) = 0.5*(2*i - 1)*dr + 0.0127; \]
\[ \quad r_e(i) = r_o(i) + dr/2; \]
\[ \quad r_w(i) = r_o(i) - dr/2; \]
\[ \text{end} \]

\[ \text{iter} = 0; \]

\[ \text{while iter < nearest(\gamma*\text{tau}/\text{dt})} \]
\[ \quad \text{iter} = \text{iter} + 1; \]

% Support Column Thermophysical Properties

\[ \text{for } i = 1:N \]
\[ \quad \text{for } j = 1:M \]
\[ \quad \quad k(i,j) = 3e-8*T_{\text{old}}(i,j)^{3} - 1e-5*T_{\text{old}}(i,j)^{2} - 0.0612*T_{\text{old}}(i,j) + 125.56; \text{units = [W/m/K]} \]
\[ \quad \quad c_p(i,j) = 9e-7*T_{\text{old}}(i,j)^{3} - 0.0027*T_{\text{old}}(i,j)^{2} + 3.0203*T_{\text{old}}(i,j) + 643.24; \text{units = [J/kg/K]} \]
\[ \text{end} \]
\[ \text{end} \]

\[ \rho = 1770; \text{density; units = [kg/m^3]} \]

% Interior Cells
for i = 2:N-1
  for j = 2:M-1
    k_e = (k(i+1,j) + k(i,j))/2;
    k_w = (k(i-1,j) + k(i,j))/2;
    k_n = (k(i,j+1) + k(i,j))/2;
    k_s = (k(i,j-1) + k(i,j))/2;
    T(i,j) =
      dt/(rho*c_p(i,j))*(1/(r_o(i)*dr^2)*(k_w*r_w(i)*T_old(i-1,j)-
        (k_w*r_w(i)+k_e*r_e(i))*T_old(i,j)+k_e*r_e(i)*T_old(i+1,j))+...
        1/dz^2*(_k_s*T_old(i,j-1)_-
        (k_s+k_n)*T_old(i,j)+k_n*T_old(i,j+1)))+...
        T_old(i,j));
  end
end

% Top Boundary
for i = 2:N-1
  j = M;
  k_e = (k(i+1,j) + k(i,j))/2;
  k_w = (k(i-1,j) + k(i,j))/2;
  k_o = k(i,j);
  k_s = (k(i,j-1) + k(i,j))/2;
  T(i,j) =
    dt/(rho*c_p(i,j))*(1/(r_o(i)*dr^2)*(k_w*r_w(i)*T_old(i-1,j)-
      (k_w*r_w(i)+k_e*r_e(i))*T_old(i,j)+k_e*r_e(i)*T_old(i+1,j))+...
      1/dz^2*((k_o/3+k_s)*T_old(i,j-1)-
        (3*k_o+k_s)*T_old(i,j)+(8/3)*k_o*T_t))+...
      T_old(i,j));
end

% Bottom Boundary
for i = 2:N-1
  j = 1;
  k_e = (k(i+1,j) + k(i,j))/2;
  k_w = (k(i-1,j) + k(i,j))/2;
  k_n = (k(i,j+1) + k(i,j))/2;
  k_o = k(i,j);
  T(i,j) =
    dt/(rho*c_p(i,j))*(1/(r_o(i)*dr^2)*(k_w*r_w(i)*T_old(i-1,j)-
      (k_e*r_e(i)+k_w*r_w(i))*T_old(i,j)+k_e*r_e(i)*T_old(i+1,j))+...
      1/dz^2*((8/3)*k_o*T_b-
        (k_n+3*k_o)*T_old(i,j)+(k_n+k_o/3)*T_old(i,j+1)))+...
      T_old(i,j));
end
% Right Boundary
for j = 2:M-1
  i = N;
  k_o = k(i,j);
  k_w = (k(i-1,j) + k(i,j))/2;
  k_n = (k(i,j+1) + k(i,j))/2;
  k_s = (k(i,j-1) + k(i,j))/2;
  T(i,j) = dt/(rho*c_p(i,j))*(1/(r_o(i)*dr)*(-1*h*r_e(i)*
    ((3*T_old(i,j)-T_old(i-1,j))/2-T_inf)-
    k_w*r_w(i)/dr*(T_old(i,j)-T_old(i-1,j)))+
    1/dz^2*(k_s*T_old(i,j-1)-
    (k_n+k_s)*T_old(i,j)+k_n*T_old(i,j+1)))+T_old(i,j);
end

% Left Boundary
for j = 2:M-1
  i = 1;
  k_e = (k(i+1,j) + k(i,j))/2;
  k_o = k(i,j);
  k_n = (k(i,j+1) + k(i,j))/2;
  k_s = (k(i,j-1) + k(i,j))/2;
  T(i,j) = dt/(rho*c_p(i,j))*(1/(r_o(i)*dr)*(k_e*r_e(i)/dr*(T_old(i+1,j)-
    T_old(i,j))+
    q*r_w(i))+1/dz^2*(k_s*T_old(i,j-1)-
    (k_s+k_n)*T_old(i,j)+k_n*T_old(i,j+1)))+T_old(i,j);
end

% Top-Right Cell
i = N;
j = M;
  k_o = k(i,j);
  k_w = (k(i-1,j) + k(i,j))/2;
  k_s = (k(i,j-1) + k(i,j))/2;
  T(i,j) = dt/(rho*c_p(i,j))*(1/(r_o(i)*dr)*(-1*h*r_e(i)*
    ((3*T_old(i,j)-T_old(i-1,j))/2-T_inf)-
    k_w*r_w(i)/dr*(T_old(i,j)-T_old(i-1,j)))+
    (1/dz^2)*(8/3)*k_o*T_t-
    (3*k_o+k_s)*T_old(i,j)+(k_o/3+k_s)*T_old(i,j-1)))+T_old(i,j);

% Bottom-Right Cell
i = N;
j = 1;
  k_o = k(i,j);
  k_w = (k(i-1,j) + k(i,j))/2;
  k_n = (k(i,j+1) + k(i,j))/2;
  T(i,j) = dt/(rho*c_p(i,j))*(1/(r_o(i)*dr)*(-1*h*r_e(i)*...
\[
\frac{(3T_{\text{old}}(i,j)-T_{\text{old}}(i-1,j))/2-T_{\text{inf}}-}
k_w*r_w(i)/dr*(T_{\text{old}}(i,j)-T_{\text{old}}(i-1,j)))}{(1/dz^2)*((8/3)*k_o*T_b-
(k_n+3*k_o)*T_{\text{old}}(i,j)+(k_n+k_o/3)*T_{\text{old}}(i,j+1)))+T_{\text{old}}(i,j+1);
\]

% Bottom-Left Corner

\[
i = 1;
j = 1;
k_o = k(i,j);
k_e = (k(i+1,j) + k(i,j))/2;
k_n = (k(i,j+1) + k(i,j))/2;
T(i,j) = 
\frac{dt/(\rho*c_p(i,j))*(1/(r_o(i)*dr)*(k_e*r_e(i)/dr*T_{\text{old}}(i+1,j)-...
k_e*r_e(i)/dr*T_{\text{old}}(i,j)+q*r_w(i))+(1/dz^2)*((8/3)*k_o*T_b-
(3*k_o+k_n)*T_{\text{old}}(i,j)+...}
(k_n+k_o/3)*T_{\text{old}}(i,j+1)))+T_{\text{old}}(i,j);
\]

% Top-Left Corner

\[
i = 1;
j = M;
k_e = (k(i+1,j) + k(i,j))/2;
k_s = (k(i,j-1) + k(i,j))/2;
k_o = k(i,j);
T(i,j) = 
\frac{dt/(\rho*c_p(i,j))*(1/(r_o(i)*dr)*(k_e*r_e(i)/dr*T_{\text{old}}(i+1,j)-...
k_e*r_e(i)/dr*T_{\text{old}}(i,j)+q*r_w(i))+(1/dz^2)*((8/3)*k_o*T_t-
(3*k_o+k_s)*T_{\text{old}}(i,j)+...}
(k_o/3+k_s)*T_{\text{old}}(i,j-1)))+T_{\text{old}}(i,j);
\]

for i = 1:N
for j = 1:M
T_{\text{old}}(i,j) = T(i,j);
end
end

contourf(R,Z,T',10)
MATLAB code for mixed mean temperature in reactor/containment system without air mass reduction

```matlab
%%% Mixed Mean Temperature After Depressurization
clc; clear all; close all

%%% Initial Conditions and Parameters
Ph = 7e6; %% Absolute Pressure of Vessel; units = [Pa]
Vh = 265.9; %% Volume of Pressure Vessel; units = [m^3]
Th = 1123; %% Helium Temperature inside Vessel; units = [K]
Pa = 101325; %% Absolute Pressure of Containment; units = [Pa]
Va = 25000; %% Volume of Containment; units = [m^3]
Ta = 298; %% Air Temperature in Containment; units = [K]

%%% Constants and Thermo-physical properties
R = 8.314462175; %% Universal Gas Constant; units = [J/mol/K]
Mh = 0.004002602; %% Molar mass of helium; units = [kg/mol]
Ma = 0.0289656; %% Molar mass of air; units = [kg/mol]

nh = Ph*Vh/(R*Th); %% number of helium moles; units = [mol]
nai = Pa*Va/(R*Ta); %% number of air moles; units = [mol]

mh = Mh*nh; %% mass of helium in the system; units = [kg]
mai = Ma*nai; %% initial mass of air in the system; units = [kg]

chi = 3116; %% specific heat capacity for helium; units = [J/kg/K]
cni = (1000/0.239005736/28)*(1/1.4)*(9.3355-122.56*(Ta/100)^-1.5+...
     256.38*(Ta/100)^-2 - 196.08*(Ta/100)^-3); %% specific heat for nitrogen; units = [J/kg/K]
coi = (1000/0.239005736/32)*(1/1.4)*(8.9425 + 4.8044e-3*(Ta/100)^1.5 -...
     42.679*(Ta/100)^-1.5 + 56.615*(Ta/100)^-2); %% specific heat for oxygen; units = [J/kg/K]
cai = 0.8*cni + 0.2*coi; %% specific heat for air; units = [J/kg/K]

iter = 0;
tol = 1e-6;
Pf = (Ph + Pa)/2;
Tf = (Th + Ta)/2;
Pf_new = 0;

while abs(Pf_new - Pf) > tol
    Pf = Pf_new;
    cnf = (1000/0.239005736/28)*(1/1.4)*(9.3355-122.56*(Tf/100)^-1.5+...
```
256.38*(Tf/100)^{-2} - 196.08*(Tf/100)^{-3});
cof = (1000/0.239005736/32)*(1/1.4)*(8.9425 + 4.8044e^{-3}*(Tf/100)^{1.5}
-...
42.679*(Tf/100)^{-1.5} + 56.615*(Tf/100)^{-2});
caf = 0.8*cnf + 0.2*cof;
chf = 3116;
Tf = (mh*chi*Th + mai*cai*Ta)/(mh*chf + mai*caf);
Pf_new = (nh+nai)*R*Tf/(Va+Vh);
iter = iter +1;
end
MATLAB code for mixed mean temperature in reactor/containment system with air mass reduction

%%% Mixed Mean Temperature and Air-to-Helium Mole Ratio After Depressurization
clc; clear all; close all

%%% Initial Conditions and Parameters
% Scaled-down Parameters for Vessel
%Ph = 75*6894.759;  %% Absolute Pressure of Pressure Vessel; units = [Pa]
%Vh = 0.209835;  %% Volume of Pressure Vessel; units = [m^3]
%Th = 1023;  %% Helium Temperature inside Vessel; units = [K]

% Prototypic Parameters for Vessel
Th = 1123;  %% Helium Temperature inside Vessel; units = [K]
Ph = 7e6; %% Absolute Pressure of Vessel; units = [Pa]
Vh = 265.9;  %% Volume of Pressure Vessel; units = [m^3]

% Scaled-down Parameters for Containment
%Pa = 101325;  %% Absolute Pressure of Containment; units = [Pa]
%Va = 1;  %% Free Volume of Containment; units = [m^3]
%Ta = 298;  %% Air Temperature in Containment; units = [K]

% Prototypic Parameters for Containment
Pa = 101325;  %% Absolute Pressure of Containment; units = [Pa]
Va = 25000;  %% Volume of Containment; units = [m^3]
Ta = 298;  %% Air Temperature in Containment; units = [K]

%%% Constants and Thermo-physical properties
R = 8.314462175;  %% Universal Gas Constant; units = [J/mol/K]
Mh = 0.004002602;  %% Molar mass of helium; units = [kg/mol]

 nhi = Ph*Vh/(R*Th);  %% number of helium moles; units = [mol]
 nhf = nhi;

 mhi = Mh*nhi;  %% mass of helium in the system; units = [kg]
mhf = Mh*nhi;  %% mass of helium in the system; units = [kg]

 chi = 3116;  %% specific heat capacity for helium; units = [J/kg/K]

162
Ma = 0.0289656;  \% Molar mass of air; units = [kg/mol]

nai = Pa*Va/(R*Ta);  \% numbrt of air moles; units = [mol]

naf = nai;

mai = Ma*nai;  \% initial mass of air in the system; units = [kg]

maf = Ma*naf;  \% initial mass of air in the system; units = [kg]

cni = (1000/0.239005736/28)*(1/1.4)*(9.3355-122.56*(Ta/100)^-1.5+...
     256.38*(Ta/100)^-2 - 196.08*(Ta/100)^-3);  \% specific heat for nitrogen; units = [J/kg/K]

coi = (1000/0.239005736/32)*(1/1.4)*(8.9425 + 4.8044e-3*(Ta/100)^1.5 -...
     42.679*(Ta/100)^-1.5 + 56.615*(Ta/100)^-2);  \% specific heat for oxygen; units = [J/kg/K]

cai = 0.8*cni + 0.2*coi;  \% specific heat for air; units = [J/kg/K]

%%% Parameters for Outflow

T_esc = Ta;

cpn = (1000/0.239005736/28)*(9.3355-122.56*(T_esc/100)^-1.5+...
     256.38*(T_esc/100)^-2 - 196.08*(T_esc/100)^-3);

cpo = (1000/0.239005736/32)*(8.9425 + 4.8044e-3*(T_esc/100)^1.5 -...
     42.679*(T_esc/100)^-1.5 + 56.615*(T_esc/100)^-2);

cp_esc = 0.8*cpn + 0.2*cpo;

%%% Iterative Loop

iter = 0;
tol = 1e-8;
Pf = (Ph + Pa)/2;
Tf = (Th + Ta)/2;

while abs(Pf - Pa) > tol
    cnf = (1000/0.239005736/28)*(1/1.4)*(9.3355-122.56*(Tf/100)^-1.5+...
        256.38*(Tf/100)^-2 - 196.08*(Tf/100)^-3);
    cof = (1000/0.239005736/32)*(1/1.4)*(8.9425 + 4.8044e-3*(Tf/100)^1.5 -...
        42.679*(Tf/100)^-1.5 + 56.615*(Tf/100)^-2);
    caf = 0.8*cnf + 0.2*cof;
    chf = 3116;
    Tf = (-1*ma_esc*cp_esc*T_esc + mhi*chi*Th + mai*cai*Ta)/(mhf*chf + maf*caf);

    Pf_new = (nhf+naf)*R*Tf/(Va+Vh);
    if abs(Pf_new - Pa) > tol
        naf = Pa*(Va+Vh)/(R*Tf) - nhf;
        maf = naf*Ma;
        ma_esc = mai - maf;
    end

end
Pf = Pf_new;
iter = Iter + 1;
end

Tf
naf/nhf
ma_esc/mai*100
MATLAB code for blowdown transient analysis of prototypic GT-MHR

```matlab
%% Blowdown Transient Analysis for Prototypic GT-MHR
clc; clear all; close all

% Givens

R = 2076.9; % helium gas constant; units = [J/kg/K] 
c_p = 5190.1; % specific heat capacity under constant pressure for helium; units = [J/kg/K] 
c_v = 3122; % specific heat capacity under constant volume for helium; units = [J/kg/K] 
c_w = 1071.71; % specific heat capacity for vessel and internals; units = [J/kg/K] 
V_w = 799.05; % volume of vessel structure and internals; units = [m^3] 
rho_w = 4307.79; % density of vessel structure and internals; units = [kg/m^3] 
As = 10432; % total surface area; units = [m^2] 
V = 265.9; % vessel free volume; units = [m^3] 
Po = 6e8; % steady-state reactor power; units = [W] 

gamma = 5/3; % ratio of specific heats for helium; unitless 
T_amb = 365; % ambient temperature (confinement temperature); units = [K] 
rho_amb = 0.13359; % ambient helium density; units = [kg/m^3] 
mu_amb = 2.2804e-5; % ambient helium viscosity; units = [kg/m/s] or [Pa*s] 
a = sqrt(gamma*R*T_amb); % speed of sound at exit of pipe; units = [m/s] 
p_amb = 101325; % ambient pressure; units = [Pa] 

mu_i = 5.0180e-5; % initial helium viscosity; units = [Pa*s] 
rho_i = 2.9796; % intial helium density; units = [kg/m^3] 
p_i = 7000000; % initial vessel pressure; units = [Pa] 
m_i = rho_i*V; % intial helium mass in vessel; units = [kg] 
T_i = 1123.15; % initial helium temperature; units = [K] 
T_o = 273.15; % abs. temperature; units = [K] 
Twi = 1123.15; % initial wall temperature; units = [K] 
h = 2; % heat transfer coefficient; units = [W/m^2/K] 

eps = 4.5e-5; % pipe equivalent roughness for a new commercial steel pipe 
L = 2.86; % length of pipe; units = [m] 
d1 = 1.4238; % inner diameter of hot duct; units = [m] 
d2 = 1.5; % outer diameter of hot duct; units = [m] 
d3 = 2.29; % inner diameter of cold duct; units = [m]
```

165
\[ \text{Ac} = \frac{\pi}{4} (d_3^2 - d_2^2 + d_1^2) \] % flow area of duct; units = [m^2]
\[ \text{Pw} = \pi (d_3 + d_2 + d_1) \] % wetted perimeter of duct; units = [m]
\[ D = 4 \times \text{Ac}/\text{Pw}; \] % hydraulic diameter; units = [m]
\[ K_c = 0.5; \] % Loss coefficient for a sudden contraction; unitless
\[ f_g = 0.002; \] % friction factor guess; unitless
\[ \alpha_2 = 1; \] % kinetic energy coefficients; unitless
\[ \text{tol} = 1e-6; \] % tolerance
\[ \text{t_max} = 60; \] % duration of blowdown transient; units = [s]
\[ \text{delta}_t = 0.001; \] % temporal step; units = [s]
\[ n = \text{floor}(\text{t_max}/\text{delta}_t); \] % number of time steps; unitless

\[ \text{err} = 1; \]
\[ \text{err}1 = 1; \]
\[ T(1) = T_i; \]
\[ \text{Tw}(1) = \text{Twi}; \]
\[ p(1) = p_i; \]
\[ m(1) = m_i; \]
\[ \rho(1) = \rho_i; \]
\[ \mu(1) = \mu_i; \]
\[ \text{for } i = 1:n \]
\[ \text{t}(i) = (i-1) \times \text{delta}_t; \]
\[ \rho_{\text{avg}}(i) = (\rho(i) + \rho_{\text{amb}})/2; \]
\[ \mu_{\text{avg}}(i) = (\mu(i) + \mu_{\text{amb}})/2; \]
\[ \text{while abs(err) >= tol} \]
\[ v_g(i) = \sqrt{\frac{p(i) - p_{\text{amb}}}{\rho_{\text{avg}}(i) \times (\alpha_2/2 + 1/2 \times (K_c + f_g \times \text{L}/D))}}; \]
\[ \text{if } v_g(i) >= a \]
\[ R_{\text{g}}(i) = \rho_{\text{avg}}(i) \times a \times D/\mu_{\text{avg}}(i); \]
\[ \text{else} \]
\[ R_{\text{g}}(i) = \rho_{\text{avg}}(i) \times v_g(i) \times D/\mu_{\text{avg}}(i); \]
\[ \text{end} \]
\[ \text{if } R_{\text{g}}(i) >= 4000 \]
\[ f_1(i) = 0.184 \times R_{\text{g}}(i)^{-0.2}; \]
\[ \text{while abs(err1) >= tol} \]
\[ f(i) = (1/(\log_{10}((\epsilon/D)/3.7 + 2.5/(R_{\text{g}}(i) \times \sqrt(f_1(i))))))^2; \] % Colbrook Equation
\[ \text{err1} = f_1(i) - f(i); \]
\[ f_1(i) = f(i); \]
\[ \text{end} \]
\[ \text{else} \]
\[ f(i) = 64/R_{\text{g}}(i); \]
\[ \text{end} \]
\[ v(i) = \sqrt{\frac{p(i) - p_{\text{amb}}}{\rho_{\text{avg}}(i) \times (\alpha_2/2 + 1/2 \times (K_c + f(i) \times \text{L}/D))}}; \]
\[ \text{err} = v_g(i) - v(i); \]
\[ f_g = f(i); \]
\[ \text{end} \]
\[ \text{if } v(i) >= a \]
\[ m_{dot}(i) = \rho_{avg}(i) \cdot A_c \cdot a; \]
\[ Re(i) = \rho_{avg}(i) \cdot a \cdot D / \mu_{avg}(i); \]
\[ \text{else} \]
\[ m_{dot}(i) = \rho_{avg}(i) \cdot A_c \cdot v(i); \]
\[ Re(i) = \rho_{avg}(i) \cdot v(i) \cdot D / \mu_{avg}(i); \]
\[ \text{end} \]

\[ T(i+1) = \delta_t \cdot h \cdot A_s / (c_v \cdot m(i)) \cdot T_w(i) + (1 - \delta_t \cdot (h \cdot A_s + m_{dot}(i) \cdot R) / (c_v \cdot m(i))) \cdot T(i); \]

\[ T_w(i+1) = \delta_t \cdot 0.066 \cdot P_0 \cdot ((i-1) \cdot \delta_t + (\delta_t)^2)^{-0.2} \cdot (\rho_w \cdot V_w \cdot c_w) \cdots \]
\[ +\delta_t \cdot h \cdot A_s / (\rho_w \cdot V_w \cdot c_w) \cdot T(i) + (1 - \delta_t \cdot h \cdot A_s / (\rho_w \cdot V_w \cdot c_w)) \cdot T_w(i); \]

\[ t(i+1) = t(i) + \delta_t; \]
\[ m(i+1) = m(i) - m_{dot}(i) \cdot \delta_t; \]
\[ \mu(i+1) = 1.865 \times 10^{-5} \cdot (T(i+1) / T_0)^{0.7}; \]
\[ \rho(i+1) = \rho(i) - m_{dot}(i) \cdot \delta_t / V; \]
\[ p(i+1) = \rho(i+1) \cdot R \cdot T(i+1); \]
\[ \text{if } p(i+1) \leq p_{amb} \]
\[ \text{break} \]
\[ \text{end} \]
\[ \text{err} = \text{err} + 1; \]
\[ \text{err1} = \text{err1} + 1; \]
\[ \text{end} \]

plot(t,p)
grid on
xlabel('Time (s)');
ylabel('Pressure (Pa)');
title('Prototype Geometry - Pressure v. Time');
figure
plot(t,T)
grid on
xlabel('Time (s)');
ylabel('Helium Temperature (K)');
title('Prototype Geometry - Helium Temperature v. Time');
figure
plot(t,T_w)
grid on
xlabel('Time (s)');
ylabel('Solid Temperature (K)');
title('Prototype Geometry - Solid Temperature v. Time');
MATLAB code for blowdown transient analysis of scaled-down test section

```matlab
%% Blowdown Transient Analysis for Scaled-down Test Section
clc; clear all; close all

% Given
R = 2076.9; % helium gas constant; units = [J/kg/K]
c_p = 5190.1; % specific heat capacity under constant pressure for helium; units = [J/kg/K]
c_v = 3122; % specific heat capacity under constant volume for helium; units = [J/kg/K]
c_w = 1071.71; % specific heat capacity for vessel and internals; units = [J/kg/K]
V_w = 1.56; % volume of vessel structure and internals; units = [m^3]
rho_w = 4307.79; % density of vessel structure and internals; units = [kg/m^3]
As = 5.316822; % total surface area; units = [m^2]
V = 0.160848; % vessel free volume; units = [m^3]
Po = 6*125; % steady-state reactor power; units = [W]
gamma = 5/3; % ratio of specific heats for helium; unitless
T_amb = 365; % ambient temperature (confinement temperature); units = [K]
rho_amb = 0.13359; % ambient helium density; units = [kg/m^3]
mu_amb = 2.2804e-5; % ambient helium viscosity; units = [kg/m/s] or [Pa*s]
a = sqrt(gamma*R*T_amb); % speed of sound at exit of pipe; units = [m/s]
p_amb = 101325; % ambient pressure; units = [Pa]
mu_i = 4.6918e-5; % initial helium viscosity; units = [Pa*s]
rho_i = 0.24317; % initial helium density; units = [kg/m^3]
p_i = 517106; % initial vessel pressure; units = [Pa]
m_i = rho_i*V; % initial helium mass in vessel; units = [kg]
T_i = 1023.15; % initial helium temperature; units = [K]
To = 273.15; % abs. temperature; units = [K]
Twi = 1023.15; % initial wall temperature; units = [K]
h = 2; % heat transfer coefficient; units = [W/m^2/K]
eps = 4.5e-5; % pipe equivalent roughness for a new commercial steel pipe
    % units = [m]
L = 8*.025; % length of pipe; units = [m]
d1 = 7.5*0.025; % inner diameter of hot duct; units = [m]
Ac = pi/4*d1^2; % flow area of duct; units = [m^2]
Pw = pi*d1; % wetted perimeter of duct; units = [m]
```

168
\[ D = 4 \cdot \frac{A_c}{P_w}; \quad \text{hydraulic diameter; units} = [m] \]
\[ K_c = 0.5; \quad \text{Loss coefficient for a sudden contraction; unitless} \]
\[ f_g = 0.002; \quad \text{friction factor guess; unitless} \]
\[ \alpha_2 = 1; \quad \text{kinetic energy coefficients; unitless} \]
\[ \text{tol} = 1e-6; \quad \text{tolerance} \]
\[ t_{\text{max}} = 60; \quad \text{duration of blowdown transient; units} = [s] \]
\[ \text{delta}_t = 0.0001; \quad \text{temporal step; units} = [s] \]
\[ n = \text{floor}(t_{\text{max}}/\text{delta}_t); \quad \text{number of time steps; unitless} \]
\[ \text{err} = 1; \]
\[ \text{err1} = 1; \]

\[ T(1) = T_i; \]
\[ Tw(1) = Tw_i; \]
\[ p(1) = p_i; \]
\[ m(1) = m_i; \]
\[ \rho(1) = \rho_i; \]
\[ \mu(1) = \mu_i; \]

\[
\text{for } i = 1:n \\
\quad \text{% i = 1;} \\
\quad t(i) = (i-1) \cdot \text{delta}_t; \\
\quad \rho_\text{avg}(i) = (\rho(i) + \rho_\text{amb})/2; \\
\quad \mu_\text{avg}(i) = (\mu(i) + \mu_\text{amb})/2; \\
\quad \text{while abs(err) } \geq \text{ tol} \\
\quad \quad v_g(i) = \sqrt{\frac{(p(i)-p_\text{amb})}{\rho_\text{avg}(i) \cdot (\alpha_2/2 + 1/2 \cdot (K_c + f_g \cdot L/D))}}; \\
\quad \quad \text{if } v_g(i) \geq a \\
\quad \quad \quad \text{Re}_g(i) = \rho_\text{avg}(i) \cdot a \cdot D/\mu_\text{avg}(i); \\
\quad \quad \text{else} \\
\quad \quad \quad \text{Re}_g(i) = \rho_\text{avg}(i) \cdot v_g(i) \cdot D/\mu_\text{avg}(i); \\
\quad \quad \text{end} \\
\quad \quad \text{if } \text{Re}_g(i) \geq 4000 \\
\quad \quad \quad f_1(i) = 0.184 \cdot \text{Re}_g(i)^{-0.2}; \\
\quad \quad \quad \text{while abs(err1) } \geq \text{ tol } \\
\quad \quad \quad \quad f(i) = \frac{1}{(-2 \cdot \log_{10}((\varepsilon/D)/3.7 + 2.51/(\text{Re}_g(i) \cdot \sqrt{f_1(i)})))^2}; \quad \text{Colbrook Equation} \\
\quad \quad \quad \quad \text{err1} = f_1(i) - f(i); \\
\quad \quad \quad \quad f_1(i) = f(i); \\
\quad \quad \text{end} \\
\quad \text{else} \\
\quad \quad f(i) = 64/\text{Re}_g(i); \\
\quad \text{end} \\
\quad v(i) = \sqrt{\frac{(p(i)-p_\text{amb})}{\rho_\text{avg}(i) \cdot (\alpha_2/2 + 1/2 \cdot (K_c + f(i) \cdot L/D))}}; \\
\quad \text{err} = v_g(i) - v(i); \\
\quad f_g = f(i); \\
\quad \text{end} \\
\quad \text{if } v(i) \geq a \\
\quad \quad m_\text{dot}(i) = \rho_\text{avg}(i) \cdot A_c \cdot a; \\
\quad \quad \text{Re}(i) = \rho_\text{avg}(i) \cdot a \cdot D/\mu_\text{avg}(i); \\
\]
else
    m_dot(i) = rho_avg(i)*Ac*v(i);
    Re(i) = rho_avg(i)*v(i)*D/mu_avg(i);
end

T(i+1) = delta_t*h*As/(c_v*m(i))*Tw(i) + (1 - delta_t*(h*As + m_dot(i)*R)...
    /(c_v*m(i)))*T(i);

Tw(i+1) = delta_t*0.066*Po*((i-1)*delta_t + (delta_t)^2)^-0.2/(rho_w*V_w*c_w)...
    + delta_t*h*As/(rho_w*V_w*c_w)*T(i) + (1 - delta_t*h*As/(rho_w*V_w*c_w))*Tw(i);

    t(i+1) = t(i) + delta_t;
    m(i+1) = m(i) - m_dot(i)*delta_t;
    mu(i+1) = 1.865e-5*(T(i+1)/To)^0.7;
    rho(i+1) = rho(i) - m_dot(i)*delta_t/V;
    p(i+1) = rho(i+1)*R*T(i+1);
    if p(i+1) <= p_amb
        break
    end
    err = err + 1;
    err1 = err1 + 1;
end

plot(t,p)
grid on
xlabel('Time (s)');
ylabel('Pressure (Pa)');
title('Scaled-down Test Section - Pressure v. Time');
figure
plot(t,T)
grid on
xlabel('Time (s)');
ylabel('Helium Temperature (K)');
title('Scaled-down Test Section - Helium Temperature v. Time');
figure
plot(t,Tw)
grid on
xlabel('Time (s)');
ylabel('Solid Temperature (K)');
title('Scaled-down Test Section - Solid Temperature v. Time');