MACROECONOMIC IMPLICATIONS OF FRICTIONS IN HETEROGENEOUS AGENT ECONOMIES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of the Ohio State University

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In my first chapter, "Unemployment Benefits in a Three State Model of Employment Flows," I evaluate the welfare effects of unemployment insurance in an incomplete insurance framework where individuals face both employment and earnings risk. I extend the aggregate labor market model of Krusell et al (2011) to include an unemployment insurance (UI) program. The model UI program is similar to UI in the United States and includes a 50 percent replacement rate of income up to a benefit cap, a finite duration of benefits, limited eligibility, and an imperfectly monitored job search requirement. I calibrate the model to match the aggregate labor market moments and flows as well as the size and scope of UI in the United States. The model UI features, especially limited eligibility, limit the steady state welfare effects of UI. As a result, removing UI from the model leads to only a 0.1 percent consumption-equivalent increase in average welfare. The welfare effects are eight times larger when all job losers are eligible for UI. I also find that the moral hazard created by the imperfectly monitored job search requirement and the finite duration of unemployment benefits lead to a spike in the employment hazard of benefit recipients at their benefit expiration, which is consistent with the empirical findings. In contrast, if benefits are uncapped, the
overall rate at which benefit recipients search for work is halved in the first two months after a layoff as compared to the baseline model. Thus, modeling the effects of UI in the United States on welfare and individuals’ labor supply decisions requires careful modeling of the structure of UI benefits.

Recent empirical evidence suggests that the real response of the economy to a monetary shock is seasonally dependent. In my second chapter, “Seasonality in a Menu Cost Model,” I introduce a seasonal fluctuation into a menu cost model to examine the model’s ability to produce an empirically consistent seasonal cycle and a seasonally dependent response to a monetary shock. The model generates an equilibrium seasonal fluctuation in output of about 8 percent from peak to trough, which is consistent with the real seasonal cycle that I find in the data. The response of the economy to a monetary shock is seasonally dependent. An expansionary monetary shock generates stronger real effects when prices are near the trough of the seasonal cycle since firms prefer to delay their response until prices further increase in the seasonal cycle. The effect is asymmetric across seasons, so the initial and cumulative real effects of monetary shocks increase by up to 20 percent compared to the equivalent nonseasonal model.
I dedicate this dissertation to my wife, Fang Zhang, and my parents.
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CHAPTER 1
UNEMPLOYMENT BENEFITS IN A THREE STATE MODEL OF EMPLOYMENT FLOWS

1.1 Introduction

Following the official end of the recession in June 2009, the labor market has recovered at an anemic rate, and the average duration of unemployment remains near an all-time high of 40 weeks. Unemployment insurance (UI), specifically the extension of maximum UI benefit eligibility to 99 weeks in many states, has borne part of the blame for the historical labor market weakness. A broader issue is the long-term costs and benefits of the UI system in the United States. However, estimates of the aggregate welfare effects of the current UI system relative to a no insurance environment vary widely depending on the model environment and the features of the UI system. The objective of this paper is to study the welfare and labor supply effects of UI and identify how these effects depend on the exact features of the UI system.

---

1See Rothstein (2011) and Barro (2010) for contrasting views.
Evaluating the welfare and labor supply effects of UI in a model where individuals’ wages vary requires careful consideration of the structure of UI benefits in the United States. In practice, the amount of UI benefits that individuals receive depends on their recent earnings, and high earners are eligible for a maximum benefit that varies state to state. The heterogeneity in wages and benefits may provide individuals with high potential wages and a capped benefit a stronger incentive to return to work after a layoff than an individual with a low potential wage but high replacement rate of benefits, which create nontrivial differences in welfare effects. The distributional welfare effects of UI and related policy changes depend on how the government funds UI benefits. If the government decides to fund them with higher taxes, then high earners disproportionately suffer. Alternatively, individuals who do not work and have low potential wages suffer greatly from their inability to smooth their consumption if the government holds taxes fixed and reduces other transfer programs.

I introduce unemployment benefits into the model of aggregate employment developed in Krusell et al (2008, 2010, 2011a). Individuals in the model are heterogeneous in their productivity and thus their potential wages, and they face an endogenous labor supply decision and an exogenous employment risk in the form of a separation shock. Employed individuals who survive the separation shock may quit. Everyone else must decide whether they want to search for work. Job seekers incur a simple search cost and succeed in finding work with a fixed probability. The main departures of the model from Krusell et al are that individuals must actively search for work to receive an employment opportunity
and that some individuals who suffer the separation shock become eligible for the UI program.

The structure of the model UI program matches four important features of the UI program in the United States: 1) unemployment insurance benefits are available only to a fraction of individuals who suffer an involuntary job loss; 2) benefit recipients collect a 50% pretax replacement rate of income up to a maximum benefit; 3) benefit recipients can collect UI benefits for a finite duration during a given jobless spell; and 4) benefit recipients face an imperfectly monitored job search requirement. If the government discovers that they did not search for work, then they lose unemployment benefits for the current unemployment spell but regain eligibility if they are laid off in the future. The imperfect nature of the monitoring provides individuals an incentive to delay their job search and collect UI benefits, which is the source of moral hazard in the model. I calibrate the model to fit the aggregate size and scope of UI in the United States, in addition to matching other labor market and macroeconomic flows and moments. The model reasonably fits other, untargeted effects of UI, such as the average duration of UI receipt and the size of the empirically observed fall in consumption after a layoff.

The features of the UI program, especially limited eligibility, limit the welfare effects of UI. Removing UI from the baseline model increases output. Aggregate capital increases slightly because of increased precautionary savings and demand for capital after removing UI from the model, but the employment to population ratio of the economy falls by 1%. The positive effect of UI on labor supply comes
from low wage workers for whom UI added enough to the value of work to induce them to enter the labor market. The labor supply effect is strongest when the government reduces the income of low income individuals by reducing the transfers that they receive in order to fund UI. The cap on unemployment benefits limits the value of UI for individuals with a high potential wage, so UI has limited negative labor supply effects on productive individuals. Removing UI from the baseline model increases average welfare by less than 0.1%, compared to about a 1% reduction in welfare if I allow all job losers to receive benefits. Simply, UI has too small of a budget and scope to have large welfare effects in the model.

The distributional welfare effects of UI depend on how the model government funds changes in the UI budget. Workers, especially productive ones, bear the brunt of the negative welfare effects of more generous UI benefits if the government decides to balance its budget by increasing its income tax. Alternatively, the government could balance its budget by fixing the tax rate and reducing other spending. Such a policy will reduce the lump sum transfer that all individuals receive from the government in the model, which will lead to large welfare losses for individuals who are not working and have low potential wages.

The features of the UI program affect individuals’ job search incentives relative to simpler models of UI. A UI program with a fixed replacement rate of benefits and no benefit cap will overstate the moral hazard effects of UI on productive individuals compared to my model. The maximum benefit replaces a smaller percentage of potential income the more productive workers are. Hence, individuals with high earnings ability will tend to seek employment quickly after a
layoff. Without the benefit cap, such individuals will delay reentry into the job market, and the aggregate employment hazard of UI recipients is less than half of what it is with the benefit cap for the first two months after a layoff. On the other hand, a model with a fixed UI benefit will overstate the effects of UI for individuals with low potential wages since the UI benefit may exceed their potential working wage.

A model with an indefinite duration of benefits and with perfect monitoring to eliminate the moral hazard behavior will fail to create a spike in the exit rate from unemployment at benefit expiration, as found empirically in Moffitt (1985) and Katz and Meyer (1990a,b). First, stricter monitoring reduces the moral hazard of individuals to delay their job search, which reduces the spike. Strict government monitoring reduces the relative value of avoiding a job search and forces more individuals who do not search out of UI\(^2\). Second, a lower success rate of job finding lengthens the expected job search, which causes more benefit recipients to search earlier and reduces the size of the spike. Other model features affect the size of the spike. A higher benefit cap causes productive benefit recipients to delay their job search, which causes a larger spike. These factors, especially the imperfect monitoring, provide alternative explanations, in addition to administrative and data collection issues, to recent empirical evidence that casts doubt about the size of the spike in the employment hazard at benefit expiration, as in Card, Chetty, and Weber (2007).

\(^2\)Klepinger et al (2002) and Borland and Tseng (2007) provide empirical support for the strong connection between government monitoring of the job search and an increased job finding rate.
Unemployment insurance reduces the precautionary savings of individuals who self insure against the risk of involuntary job loss, but UI does not crowd out much savings in the baseline model since the savings asset pays interest. Under restrictive assumptions, an optimal unemployment insurance program will eliminate precautionary savings, as in Hansen and İmrohoroğlu (1992) when the benefit level is optimal and UI provides no moral hazard. Engen and Gruber (2001) find a larger role for precautionary savings after considering a finite duration, capped benefit. Both results assume that all individuals are eligible for UI benefits upon job loss and that the any assets that individuals accumulate are not interest bearing, which maximizes the insurance value of UI. The effects of UI policy changes on savings are also small in my model, but the model would be more sensitive to policy changes if the model did not restrict the scope of the UI program. A 50% decrease in the replacement rate of benefits reduces aggregate savings by four times more when all individuals who experience an involuntary job loss receive UI than in the baseline model.

The model contributes to the study of welfare effects of UI in dynamic economies by estimating the welfare effects of UI in a model that reasonably matches the labor market flows and the size and scope of the UI program in the United States. Young (2004) finds that UI reduces consumption equivalent steady state welfare by up to 1.1% as compared to a no UI environment. His model UI program allows all individuals who lose their jobs to be eligible for UI, which causes more individuals to receive benefits than in this model and thus larger welfare effects. Wang and Williamson (2002) find that the upper bound on the positive welfare
effects of UI is 1.5% of consumption. They emphasize the limitations on the welfare effects imposed by the limited eligibility for benefits, but individuals in the model can only save in a non-interest bearing asset. The distribution of wealth in their model is negatively skewed unlike the distribution of wealth in the United States, which distorts the welfare effects. Also, neither of these models feature heterogeneity in individual wages. Mukoyama (2011) focuses on the decomposition of the welfare effects of several comparison models in general equilibrium, but the UI program in his model is a simple fixed benefit available to all individuals who do not work. Pollak (2007) estimates the welfare effects of UI in a model where individuals are heterogeneous in their wages. His focus is on the UI system in Germany in the 1990s, and his model does not allow quits, which reduces the importance of the labor supply margin as compared to this model.

The paper also contributes to the study of the labor supply effects of UI by providing an example of a model where a UI program increases the net employment of an economy. Krueger and Meyer (2002) provide a good survey of the labor supply effects of social insurance. One of the most influential models in this literature is Mortensen (1977), who adds a finite duration UI benefit program in a simple search model. He finds that benefit recipients are more willing to search for work just before their benefits expire and will maintain a high search effort after their benefits expire. Krueger and Mueller (2009) test some of the predictions of the Mortensen model in the American Time Use Survey data and found that search effort decreases after UI benefits expire, unlike the predictions of the Mortensen model. In my model, selection causes the average search intensity of
individuals to decrease. After benefits expire, most individuals who immediately want a job have found work, which leaves a pool of long term unemployed who do not search.

Section 2 presents the model and a more detailed description of the model UI program. Section 3 presents the calibration of the model. The results of the model and the implications of each part of the realistic UI program are explored in section 4, and section 5 examines the welfare effects of UI. Section 6 concludes.

1.2 Model

The foundation of the model is Krusell et al (2010). A period is one month. A continuum of individuals with total mass 1 populate the economy. Individuals make labor supply decisions in an environment where they face three labor market frictions: 1) workers may be laid off or fired from their jobs; 2) job searchers suffer a fixed cost in terms of utility to search for work; and 3) they succeed in their job search with a fixed probability. Markets are incomplete, so individuals insure themselves against the employment risk by accumulating savings in the form of capital, though UI provides partial insurance. The model features a more realistic unemployment benefit program than much of the UI literature. Unemployment benefits are only available to workers who are laid off, not workers who are fired or who quit, and benefits recipients are eligible for only 6 months of benefits during a given unemployment spell. The government imperfectly monitors the job search of UI recipients and disqualifies individuals who do not search for work from receiving benefits for the rest of their unemployment spell. Benefit
recipients receive a pretax 50% replacement rate of their potential income up to a maximum benefit cap.

All individuals have the same preferences over consumption, time devoted to work, and time devoted to seeking work:

$$\sum_{t=0}^{\infty} \beta^t \left[ \log(c_t) - \psi_1 e_{1t} - \psi_2 e_{2t} \right]$$

Consumption in the model is $c_t \geq 0$, $0 < \beta < 1$ is the discount factor, $\psi_1 > 0$ is the disutility of work, and $\psi_2 > 0$ is the disutility of seeking work. Labor is indivisible, so $e_{1t} \in \{0, 1\}$ where $e_{1t} = 1$ if an individual works and $e_{1t} = 0$ if an individual does not work. I assume also that individuals differ neither in their search intensity nor disutility of searching for a job, so $e_{2t} \in \{0, 1\}$ where $e_{2t} = 1$ if an individual searches and $e_{2t} = 0$ if an individual does not search.

Entering a period, an individual’s state consists of her wealth level in terms of capital $k$, her productivity draw from last period $s_{-1}$, and her labor market status. Her labor market status could be employed ($e$), not employed and not receiving benefits ($n$), or not employed and has received employment benefits for $b - 1$ periods ($u_b$) where $b$ is the number of months that a benefit recipient would receive if she receives benefits in the current period. First, all individuals learn their new, idiosyncratic productivity draw $s$ and individuals employed entering the period learn if they face an exogenous separation shock. The productivity
shock follows the following $AR(1)$ process with normally distributed shock $\epsilon$:

$$\log(s_{t+1}) = \rho \log(s_t) + \epsilon_{t+1}$$

$$\epsilon \sim N(0, \sigma_\epsilon)$$

$$0 \leq \rho < 1$$

At the same time, employed individuals learn their idiosyncratic separation shock. With probability $\sigma$, they can continue to work. Individuals are laid off and are thus eligible for unemployment benefits with probability $\gamma_u$. Otherwise with probability $\gamma_n$, individuals are fired and ineligible for unemployment benefits.

Second, individuals make their labor supply and search decisions. Employed individuals who do not suffer a separation shock must decide whether or not to quit their job. Individuals without a job, including the employed who suffered the separation shock, choose whether or not to seek work. Third, all individuals make their consumption and savings decisions for the current period based on their finalized current employment status.

Individuals who enter the period employed and survive the separation shock must choose between working and quitting. On the job search is irrelevant in the model since all jobs offer the same wage to a person given their productivity draw. Individuals who quit their jobs are ineligible for unemployment benefits, so they choose the maximum value of remaining employed and not working without UI eligibility:

$$V(k, s, E) = \max\{V(k, s, e), V(k, s, n)\}.$$ 

Here, $V(\cdot, \cdot, e)$ is the expected discounted lifetime utility of their consumption and
savings decisions if they are employed, and \( V(\cdot,\cdot,n) \) is the expected discounted lifetime utility of not working and not receiving unemployment benefits. The upper case employment state indicates that the individuals are making their current labor supply decisions, and the lower case employment state indicates that the employment status of the individuals are finalized for the current period.

Individuals who enter the period not employed and not eligible for unemployment benefits and individuals who are fired choose whether or not to seek employment. They choose the maximum of the expected value of seeking work and the value of the status quo:

\[
V(k,s,N) = \max \{ \lambda V(k,s,e) + (1 - \lambda) V(k,s,n) - \psi_2, V(k,s,n) \}.
\]

The probability that the job seeker succeeds in finding work is \( \lambda \). Note that job seekers must pay the utility cost of seeking work \( \psi_2 \) regardless of the outcome of the job search.

Individuals who are eligible for UI also choose whether or not to seek employment. They choose between the maximum of the expected value of seeking work and the expected value of the status quo for individuals who, if they remain eligible for unemployment benefits, will be in their \( b \)-th period of receiving unemployment benefits.

\[
V(k,s,U_b) = \max \{ \lambda V(k,s,e) + (1 - \lambda) V(k,s,u_b) - \psi_2, (1 - m) V(k,s,u_b) + m V(k,s,n) \}
\]

Here, \( V(\cdot,\cdot,u_b) \) is the expected discounted lifetime utility of their consumption and savings decisions if they are in the \( b \)-th period of benefit recipiency. Similar to Abdulkadiroğlu et al (2002), the government imperfectly monitors benefit
recipients’ job seeking behavior and terminates UI eligibility if it discovers that
the benefit recipient did not actively seek work in the current period. The gov-
ernment catches individuals who do not actively seek work with probability $m$
$\left(0 \leq m \leq 1\right)$. Individuals whose benefits are terminated cannot receive unem-
ployment benefits for their current jobless spell. A smaller value of $m$ increases
the amount of moral hazard that the unemployment benefit program produces.
When $m$ is 1, the government perfectly monitors and disqualifies all individuals
who do not seek work from receiving unemployment benefits. As $m$ approaches
0, the government increasingly has difficulty monitoring individuals’ job seeking
behavior, which increases the incentive for individuals to collect benefits and not
seek work.

Conditional on working in the current period, individuals make their con-
sumption and savings decisions based on the following Bellman equation:

$$
V(k, s, e) = \max_{c, k'} \left[ \log(c) - \psi_1 + \beta E_s \left( \sigma V(k', s', E') + \gamma_u V(k', s', U_1') + \gamma_n V(k', s', N_1') \right) \right] \\
c + k' = (1 + r - \delta) k + (1 - \tau) ws + \Phi \\
k' \geq 0 \\
c \geq 0
$$

Individuals who work earn $ws$ pretax. A constant rate income tax $\tau$ funds the
unemployment benefits and a lump sum transfer $\Phi$ that is the same for all indi-
viduals in the economy. Individuals enter the period with capital $k$ and earn a
real interest rate of $r - \delta$ net of depreciation, where $\delta$ is the depreciation rate of
capital. Borrowing is not allowed in the economy.
Individuals who are not employed and do not receive unemployment benefits face the following Bellman equation:

\[
V(k, s, n) = \max_{c, k'} \left[ \log(c) + \beta E_s \left( V(k', s', N') \right) \right]
\]

\[
c + k' = (1 + r - \delta) k + \Phi
\]

\[
k' \geq 0
\]

\[
c \geq 0
\]

They do not work, so they do not receive labor earnings. They still receive income from capital holdings and the government transfer.

Unemployment benefit recipients face the following Bellman equation:

\[
V(k, s, u_b) = \max_{c, k'} \left[ \log(c) + \beta E_s \left( V(k', s', U_{b+1}) \right) \right]
\]

\[
c + k' = (1 + r - \delta) k + (1 - \tau) uw \min(s, \bar{s}) + \Phi
\]

\[
k' \geq 0
\]

\[
c \geq 0
\]

\[
b = 1, ..., B
\]

The pretax unemployment benefit \(uw \min(s, \bar{s})\) features a constant replacement rate of income \(u\) up to a maximum benefit level \(uw \bar{s}\). Unemployment benefits have been taxable income in the United States since 1987, so they are taxed at rate \(\tau\). Individuals who are in their final period of benefits \(B\) are not eligible for benefits in the next period, so \(V(k', s', U_{B+1}) = V(k', s', N')\).

A Cobb-Douglas production function in the aggregate capital and aggregate
labor inputs is the production technology of the economy. Aggregate capital, $K$, and aggregate labor, $L$, are as follows:

$$K = \int_0^1 k_idi$$

$$L = \int_0^1 s_ie_idi$$

The production function for the final good, $Y$, is as follows:

$$Y = K^\alpha L^{1-\alpha}$$

Individuals can use the final good interchangeably for consumption and investment. The wage and rental rate of capital are the marginal products of their respective aggregate inputs.

The recursive competitive equilibrium of this economy follows the standard definition. It consists of value functions $V$; consumption, savings, and labor supply decisions; wage $w$; interest rate $r$; government transfer $\Phi$; individuals’ decision rules $\Gamma^*$; and an invariant distribution $\Omega^*$ such that

1. $V(k,s,e), V(k,s,u_b)$, and $V(k,s,n)$, and the consumption, savings, and labor supply decisions solve the individuals’ problem;
2. the capital and labor markets clear;
3. the government budget constraint holds;
4. $\Omega^*$ is invariant to the individuals’ decision rules $\Gamma^*$.

The discrete labor supply decision requires a numerical solution to the problem. I discretize the shock process and capital state using a grid. I approximate the AR(1) process for the productivity shock using the Rouwenhorst (1995)
method, which provides a more reliable solution for a highly persistent process than the standard Tauchen (1986) method according to Kopecky and Suen (2010). For a given set of parameters, I guess a government transfer $\Phi$, an interest rate $r$, and a wage $w$ and then solve the individuals’ value functions. Cubic spline interpolation provides the values of the value functions between the grid points. I iterate on the firms decision rules to obtain the invariant distribution $\Gamma$ for the given $\Phi$, $r$, and $w$ and then update them and repeat the solution procedure until the labor and capital markets clear and the government budget constraint holds exactly.

The mechanics of the model differ from Krusell et al in two main ways. First and most importantly, my model features a realistic UI benefit program. Krusell et al focuses on fitting aggregate labor market and labor flow data for which UI was a relatively minor consideration. They implement UI in an extension as a lump sum transfer available to all individuals who do not work. They were able to calibrate the model to US data for a relatively low average replacement rate of benefits, but a higher replacement rate caused too many individuals to never seek work. Even with a disutility of work near 0, the model could not generate enough employment to match the US employment to population ratio. Mechanically, I split up the not working employment state into a no benefit state and a set of UI receipt states to account for the fact that only some individuals receive UI benefits.

Second, Krusell et al does not have costly job search citing evidence from Jones and Riddell (2006) about marginally attached workers and the extent to which
family connections are used to obtain jobs. Rather, they assume that job opportunities arrive with an exogenous probability, and individuals choose to accept or reject the offers. Hence, there is no formal search, and unemployed individuals in their model are simply individuals who did not receive a job opportunity but would take it if it were offered to them. I add search costs to add marginally attached workers to the model, which helps fit the model to the empirical labor market flows.

1.2.1 Evaluation of UI Benefit Features

There are a few abstractions from completely realistic UI in the US in the model formulation. I choose to model UI as a single aggregate program in a single aggregate labor market rather than modeling the quirks of each state-administered UI program local labor market. In practice, there are structural differences in UI programs across the US. For example, 35 states force UI recipients to wait a week before becoming eligible for UI, but I assume away this waiting period of benefits since the model is monthly.

I make two assumptions to remove employment history from the state space of the model. First, I assume that individuals become eligible for unemployment benefits after working for one month and that the system does not require a minimum amount of earnings to qualify for benefits. In many states, individuals must earn a minimum amount of money or work for a minimum number of weeks in the previous quarter before becoming eligible for UI. Since individuals who quit are ineligible for UI benefits, this assumption does not cause UI recipients to quit
and immediately collect benefits. Second, I assume that the value of unemployment payments depends on the UI recipients’ current productivity draw if they are below the benefit cap level $\bar{s}$. Typically, the size of the benefit depends on the earnings from the recipients’ last job. I allow the dependence between the benefit and current productivity level since the productivity distribution is highly persistent in the calibration, which would minimize the effect on average of changing productivity.

The imperfect monitoring of the job search requirement is plausible since the Department of Labor acknowledges that there are overpayments in the UI program through their Benefit Accuracy Measurement (BAM) program, many of which are neither reasonably detectable nor collectible. The BAM operation rate, which includes overpayments that states should reasonable detect and collect, averaged 5.62% for the 2002-2007 period, compared to the BAM annual rate of 9.5%, which also includes nonrecoverable or undetectable overpayments and payments deemed to be technically proper. The BAM program estimated the fraud overpayment rate at 2.5% over the same period. Common methods that state UI programs use to detect overpayments and fraud include cross-referencing UI recipients with employer new hire, layoff, and other government benefit data; investigating public tips of potential fraud; and validating recipients’ evidence of job search.

The penalties for unemployment fraud, benefit recipients who do not seek work and are caught in the model, are simplified relative to state practice. State unemployment agencies assess a variety of penalties for accidental overpayment
and fraud. At the very least, individuals will have to pay back any overpayments with interest, and the penalties for outright fraud are more severe, ranging from fines to criminal prosecution and potential imprisonment. I assume away overpayments of benefits and their resulting interest and instead assume that individuals are disqualified from receiving benefits for the current UI spell. The penalties in the model are most similar to “penalty weeks,” which are weeks of benefits that recipients must claim but not collect.

Given the simplified role of the firm in the model, the funding mechanism for UI does not depend on the aggregating firm’s layoff decisions. In reality, federal and state governments fund permanent UI through a couple of taxes. First, the Federal Unemployment Tax Act established a payroll tax at a 6% rate on the first $7,000 of gross earnings of each worker per year. Second, employers pay, on behalf of individuals, a marginal tax on income up to a state-determined maximum income level. The cap on this tax is again relatively low compared to average per capita income in most states. The tax employers pay depends on how many of their employees are laid off and claim UI benefits. This “experience rating” of firms discourages firms from laying off many workers by imposing additional taxes on the firm to partially fund the UI used by its laid off workers. Such a tax system would require a richer firm structure than the model here. Wang and

3The effective tax rate was 6.2% between 1988 and June of 2011 because of a temporary increase in the tax credit. The effective tax that employers pay, however, is much less in many cases since they are allowed to take a tax credit of up to 5.4% on the FUTA tax, depending on state unemployment taxes. See USC 3301 and 3302(b) for further details.
Williamson (2001) found little evidence for large aggregate effects caused by experience ratings, though the distributional effects were significant.

1.3 Calibration

The calibration of the model matches key facts about the US macroeconomy and labor market while maintaining a realistic size and scope of UI. The calibration focuses on the 1994-2007 period for which comparable labor market information is available. Table 1.1 summarizes the calibration results.

I calibrate twelve parameters and fix the remaining three. I fix the marginal tax rate, $\tau$, at 0.3 for consistency with Krusell et al, who choose the tax rate as representative of the literature that estimates average effective tax rates on labor income across countries stemming from Mendoza et al (1994), Prescott (2004), and McDaniel (2007). To match the mode of the pretax replacement rate of benefits across the various state UI programs, I set $u = 0.5$. I set the maximum duration of benefits to $B = 6$ months to match the 26 standard weeks of permanent UI that are available to eligible individuals regardless of the current unemployment rate and approved emergency benefits.

I next target standard macroeconomic moments. I choose $\beta$ to target a real interest rate of 4% and $\alpha$ to target a labor share of 70%. The depreciation rate, $\delta$, matches an investment to output ratio of 0.2. Since the wealth distribution of the model will affect the welfare criteria that I use, I match the wealth holdings of the top 60% of the population, who hold 99% of wealth according to Rodriguez et al (2002).
The aggregate labor market moments are drawn from Current Population Survey labor market flow data from Krusell et al (2011a). They compute the average monthly labor market flows from employment, unemployment, and out the labor force for the 1994 to 2007 period using the empirical procedure developed by Fujita and Ramey (2009). I choose the disutility of work, $\psi_1$, to match a 63.3% employment to population ratio. The job finding probability $\lambda$ is set to ensure a 5.1% standard unemployment rate. I also match a “generalized” definition of the unemployment rate that includes individuals who unsuccessfully searched for work during the period (standard definition of unemployment) and marginally attached workers who would search in the absence of the search cost, which pins down the disutility of search, $\psi_2$. Within the labor market flows, I choose to match the flow from employed to other labor market states to provide a realistic potential flow of individuals into UI. Formally, I match the 96.1% of the employed who remain employed and the 2.1% of the employed who become unemployed month to month on average.

To maintain a reasonable UI program, I match the model to aggregate UI program data for the standard, permanent UI program. The Department of Labor Wage and Hour Division maintains information about aggregate benefits paid out, the insured unemployment rate (UI recipients as a percentage of the labor force), and the exhaustion rate of benefits (the percentage of initial UI claimants who use all available periods of benefits). UI benefits from the main program averaged 0.273% of GDP between 1994 and 2007, which pins down the maximum benefit cap $\bar{s}$. The insured unemployment rate averaged 2.2%, which defines the
scope of the UI program and thus pins down the fraction of the employed who are laid off, $\theta_u$. The effectiveness of government monitoring determines how long individuals can delay their reentry into the labor force on average, so I set $m = 0.05$ to match the 35.6% exhaustion rate of benefits.

A couple of issues stand out in the calibration of the UI program. The rate of layoffs in the model seems relatively small at 19% of separations compared to the 43% recipiency rate of UI benefits, but the model matches the recipiency rate by construction. The common definition of the recipiency rate is the percentage of unemployed workers who collect UI benefits, so matching both the IUR at 2.2% and the standard definition of unemployment at 5.1% ensures that the model matches the recipiency rate by definition. UI recipients remain not employed for a longer period of time than individuals who are laid off or who lose UI benefits, so the rate of layoffs should be lower than the recipiency rate. Other institutional factors reduce the fraction of individuals who claim UI benefits in practice. Not all workers are eligible for UI in practice even if they are laid off. On average, 12.3% of the labor force was ineligible for UI between 1998 and 2007 (DOL), which includes the self-employed, some small farms, and other legally uncovered jobs. Card and Blank (1991) finds that roughly half of individuals ineligible for unemployment have not worked enough to receive UI. Not all workers accept UI even if they are eligible, particularly if the individual does not expect to be unemployed for long. Wang and Williamson (2002) allow 35% of laid off workers to receive unemployment benefits to match a 30.4% recipiency rate between 1977 and 1987, but their model features a weaker moral hazard incentive than this model.
The limited government monitoring in the model implies that 95% of benefit recipients who do not search for work continue to receive unemployment benefits each month. In the baseline model, the government pays 75% of total benefits to individuals who do not search for work, which is a higher overpayment rate than estimated by the Department of Labor. The Benefit Accuracy Measurement (BAM) program, which audits state UI programs to proper monitoring of benefit recipients, estimates that the state programs overpay UI benefits by about 10% overall. While the low value of \( m \) implies highly ineffective government monitoring, recall that the penalties match those of unemployment fraud, not those caused by accidental overpayment or other administrative issues. One could interpret a large subset of the individuals who did not search for work in the model as individuals who searched for work that did not match their skills in practice. This subset of individuals in practice would be eligible for unemployment benefits but would not likely find work. The enforcement rate in the model indeed matches the enforcement rate for UI fraud empirically. Gray (2003) notes that the monthly sanction rate of UI claims for a lack of job search evidence was 2.78%, compared to 3.2% in the baseline model. Similarly, the BAM’s estimate of overpayments due to fraud averaged 2.5% over the 2002-2007 period over which relevant data is available (DOL).
1.4 Results

The optimal employment and job search decisions of individuals provide insight about the model’s ability to fit the employment flows and the aggregate labor market moments. The employment and job searching choices of individuals follow a simple rule. For each employment state and each productivity draw \( s \), there is a cutoff level of capital holdings \( k \) under which individuals will want to remain employed or will search for work. Above this capital cutoff level, individuals would rather not work and live off of their savings and the government transfer. Figure 1 shows the cutoff capital levels for the baseline model. The bold line in Figure 1 shows the cutoff levels for employed individuals. Individuals in the choice stage who are still employed after the separation shock with capital and productivity combinations above the cutoff line will choose to quit their job, and otherwise they will choose to remain employed. Any individuals who are not employed and not receiving benefits with capital and productivity combinations between the employed cutoff line and the cutoff line for individuals who are not employed and not receiving benefits (solid, not bolded) are marginally attached workers.

The cutoff values in Figure 1 imply that individuals with relatively low productivity draws will not work and that the fraction of time individuals spend in the labor force increases with their productivity level. Figure 1.2 plots the employment to population ratio conditional on productivity, which confirms that the 10% of individuals with a productivity draw of 0.32 or below will not work
until their productivity rises and will subsist on the transfer benefit from the government. At the other extreme, individuals with high productivity draws will maximize their time in the workforce. This feature of the baseline model is important for the allocative impacts of UI funding. Individuals who do not work and subsist on the government transfer tend to have low productivity draws and do not benefit from increased UI benefits in the short run, and they are vulnerable to policy changes funded with the government transfer, especially since they likely have little to no savings. Workers with high wages receive some benefit from the unemployment insurance and are more vulnerable to the income tax.

The baseline model reasonably matches the flows out of employment and out of the labor force but does not perfectly fit the flows out of unemployment, as shown in Table 1.2. The calibration ensures that the model fits the labor market flows from employment to other states. This labor market flow match is similar to Krusell et al (2011a), who also have difficulty fitting the flows out of unemployment. They argue that a recalibration of their model to fit the flows of prime age males provides a better match. Alternatively, they can match the flow from unemployment to employment and ignore the unemployment rate. They also confirm that a transitory preference shock to the disutility of work can cause the unemployed to exit the work force temporarily, which will drive up the flow from unemployed out of the labor force. Relative to their baseline specification, my baseline model improves the fit of the flow from unemployment to employment at the expense of the flow of individuals who persist out of the labor force.

The search cost creates a pool of marginally attached workers, which allows
the model to better match the flow from unemployment to employment. The 
"generalized unemployment" definition in Krusell et al includes all individuals 
who would accept a job if offered. Since all job offers arrive exogenously in 
the Krusell et al models, individuals do not actively search, so all of the unem-
ployed would like a job but did not receive an employment opportunity. The 
generalized unemployed in my model include job seekers who did not succeed 
and marginally attached workers, who would search for a job if they could avoid 
the fixed cost of job search. Marginally attached workers are persistently unem-
ployed, so relatively fewer unemployed find jobs in the next period. The effect on 
the flows are large since marginally attached workers are 3.3% of the labor force 
in the calibration.

One side effect of the search cost is to slow the flow of individuals back into 
employment. Job losers or quitters will typically draw upon their savings to 
smooth their consumption until they find their next job. The search cost reduces 
the cutoff capital level for the job search, so job losers and job quitters will draw 
down their savings for a longer time. This effect is strongest among job quitters 
who, ceteris paribus, have the highest wealth holdings of individuals who exit 
employment. Some job losers will delay their job market reentry as well, particu-
larly if they held a relatively high amount of wealth. Figure 1.3 shows the effect 
of the search cost on the employment hazard of job quitters and fired individuals. 
The search cost reduces the employment hazard of job quitters by up to half for the 
first 18 months after quitting. The effect on fired individuals is at most 15%, and
the effect on UI recipients (not pictured) is smaller because of the dominating effects of the incentives created by UI. Table 1.3 compares the baseline labor market flows and the flows from the model where the search cost is reduced. The search cost improves the fit of the unemployment to employment flow at the expense of the persistence of the out of the labor force flow. While the search cost helps fit the flow from unemployment to employment, the side effect of this assumption makes the flow of individuals who persist out of the labor force too high.

Unemployment benefits have relatively minor effects on the job flows since only 2.2% of the labor force receives them at any given point of time. Table 1.4 compares the baseline and no unemployment labor market flows. As long as monitoring of the job search requirement is lax, benefit recipients will delay their job search, which will increase the flow of individuals who are technically out of the labor force. In practice, they drop out of the labor force for the job flow calculations, particularly when they first become eligible for benefits. Hence, a model with unemployment benefits will cause more individuals who have lost their jobs to drop out of the labor force.

Beyond the labor market flows, the baseline results fit reasonably well to other, untargeted aggregate metrics of UI. The average duration for which individuals received standard UI benefits was 15 weeks in the 1994-2007 period according to the Department of Labor, and the equivalent baseline model average duration is 16.8 weeks (DOL). Given that I match the exhaustion rate of benefits, this result suggests that slightly too few individuals exit UI at the beginning of UI recipiency, which could be attributed to a model benefit cap that is slightly too high.
The average replacement rate of pre-layoff wages suggests that slightly too many UI recipients collected the maximum benefit. The replacement rate from 1998-2007 averaged 35.7%, compared to a 39.5% average replacement rate in the model (DOL). Since I do not explicitly include part time workers in the model, a high replacement rate is expected. Finally, UI is partial insurance, and benefit recipients do not fully smooth their consumption in the face of the employment risk. Gruber (1997) estimates that the average worker experiences a 6.8% drop in food consumption after becoming unemployed in a Panel Survey on Income Dynamics sample. He also estimates that individuals who do not receive UI benefits would reduce their consumption by 22.2% on average. Average consumption of individuals who did not immediately find a job after suffering from a separation shock falls by 7.3% in the baseline model, but the consumption of the subset of individuals who are not eligible for unemployment benefits falls by only 8%. Highly productive individuals with significant savings easily smooth their consumption, so the consumption effects of firing among below average productivity individuals are more consistent with the evidence from Gruber. The average consumption drop for a fired individual with a below average productivity draw was 22.3% in the baseline model.

The model also fits other untargeted labor market metrics relatively well. Krusell et al find that the average duration of an employment spell is 22.5 months in the labor flow data, the average duration of an unemployment spell is 2 months, and the average duration of a out of labor force spell is 14.3 months. The average employment spell lasts 18.3 months, the average unemployment spell lasts 1.96
months, and the average out of labor force spell lasts 17.6 months in the baseline model. Blank and Card (1991) find in Current Population Survey data between 1977 and 1987 that 69.8% of the unemployed had been unemployed for 13 or fewer weeks, 15.5% had been unemployed for between 14 to 26 weeks, and 14.7% had been unemployed for more than 26 weeks. The respective fractions are 69.2%, 21.2%, and 9.4% in the baseline model. The match of the employment flows between states helps the ability of the model to reasonably match these other labor market metrics.

1.4.1 Moral Hazard and the Limited Duration of Benefits

The limited duration of benefits creates a declining value of job search avoidance as a benefit recipient lasts longer in the program, which may give recipients a stronger incentive to work as benefit expiration approaches. This stronger incentive would lead to a spike in the employment hazard of individuals when their benefits are about to expire. Limited duration unemployment benefits are not, however, sufficient to generate a spike in the reemployment hazard at benefit expiration in this environment, since effective government monitoring can prevent benefit recipients from delaying their reentry into the workforce. The government must either allow workers to receive benefits without searching for work or be unable to closely monitor and enforce a job search requirement for UI to create a moral hazard problem.

The spike in the employment hazard is one of the famous results of empirical studies of social insurance, such as in Moffitt (1985) and Katz and Meyer (1990)
A common interpretation of the spike suggests that many UI recipients delay their job search until just before their UI benefits expire because of the moral hazard that UI creates without a strictly monitored job search requirement. The size of the spike and the attribution of the spike to an increase in the employment hazard have been the subject of a recent empirical debate. Card, Chetty and Weber (2007) found that the spike in the employment hazard of individuals near benefit expiration was relatively small compared to the exit from registered unemployment in Austrian data. Since individuals must formally register as unemployed to receive UI benefits, many individuals simply stopped registering as unemployed after their benefits expired.

Figure 1.4 shows that the employment hazard of UI recipients decreases until just before benefits expire, when the employment hazard sharply rises. The employment hazard in the figure is defined as the fraction of remaining individuals within a defined employment status who find work in each specified month. The heterogeneity of workers helps to create this initially declining employment hazard with the benefit spike at the end. For the first several periods after a layoff, highly productive individuals strongly desire to return to work because of the benefit cap, so a relatively large fraction of UI recipients search for and find work. However, after three months, relatively few highly motivated individuals remain on benefits. Many of the individuals who remain on benefits do not actively seek work until their benefits are about to expire, which is the cause of the initial decline in the employment hazard. Benefit recipients delay their job market entry until they are about to exhaust their unemployment benefits, particularly if the
recipients’ benefits are less than the benefit cap, which is an implication of the capital cutoffs in Figure 1.1. The bottommost cutoff line in Figure 1.1 is that of the first-period benefit recipients. Few initial benefit recipients who have an average or below average productivity draw will seek work. The next cutoff line up in Figure 1.1 is for individuals in their third month of receiving benefits. The upward shift in the cutoff line relative to the first month indicates an increased desire to return to the workforce. Still, the disincentive to work for individuals is strong relative to the individuals who do not receive benefits. The next cutoff line up in Figure 1.1 is for individuals in their sixth and final month of collecting benefits. The future value of UI receipt decreases as the recipient collects UI for a longer period of time with finite duration of benefits.

Imperfect monitoring of benefit recipients allows individuals to collect benefits without seeking work and is thus the source of the moral hazard created by UI in the model. Stricter monitoring reduces the value of not seeking work and thus encourages more individuals to seek work before their benefits expire. Figure 1.5 shows the fraction of benefit recipients who seek work in each benefit period, varying \( m \) from the baseline value of 0.05. When the government cannot monitor individuals, the case where \( m = 0 \), fewer initial benefit recipients choose to seek work than in the baseline case. Instead, they wait for benefits to expire or nearly expire before seeking work, which also causes a more pronounced spike in the fraction of individuals who are seeking work at benefit expiration. As monitoring becomes more strict, benefit recipients are more likely to seek work initially after an unemployment spell. A consequence of stricter monitoring is that the exit
hazard from benefits as well as the reemployment hazard become flatter to the point where the spike entirely disappears when \( m = 0.5 \).

Towards the extreme case where the government can perfectly monitor whether benefit recipients have sought work, continued unemployment benefits begin to reward job search and reentry into the workforce, and practically all benefit recipients search for work in all periods, as shown in Figure 1.5 when \( m = 0.5 \). While this result would suggest that a policymaker who seeks to encourage reentry into the workforce, one must keep in mind that monitoring is costly and that the model penalties for UI are higher than in most sanction situations, which may not be the case if benefit recipients make a misdirected effort at searching for a job.

Mechanically, imperfect monitoring also limits the length of time that benefit recipients expect to receive benefits, which makes it essential for matching the exhaustion rate of benefits in the data. The baseline calibration ensures that 49.5% of job seekers find work and exit UI, and imperfect monitoring forces 5% of the rest off of benefits. Stricter monitoring of unemployment benefits encourages more recipients to search for work and forces the rest out of UI at a faster rate. As a result, the exhaustion rate of benefits and the expected value of benefits that individuals receive fall.

1.4.2 Cap on Benefits

The cap on benefits, \( \bar{s} \), directs unemployment benefits and their associated effects towards individuals who have relatively low productivity draws. In practice, models of UI without a benefit cap seek to match the mean replacement rate
of benefit recipients’ income, usually with a fixed benefit if no heterogeneity in wages exists in the model. While this approach makes sense in a model without heterogeneity in wages, a fixed benefit distorts the potential effect of the UI system on individuals’ labor supply decisions relative to a unemployment benefit that depends on wages. In my model, a fixed benefit set at the mean replacement rate of benefits understates the benefits received by individuals whose benefit would otherwise be at or below the benefit cap and overstate the benefits received by relatively more productive individuals if model benefits are heterogeneous. In my calibration, the benefit cap allows a 50% pretax replacement rate of benefits for UI recipients while still matching the benefits paid out by UI.

Productive benefit recipients face increased moral hazard without the benefit cap, ceteris paribus. For many productive individuals, the maximum unemployment benefit is a pittance relative to their possible working wage and is not enough of an incentive to keep them from returning to the workforce. Thus, the departure of productive individuals from UI in the first few periods after their layoff causes an initially decreasing aggregate hazard of employment for UI recipients. Figure 1.6 compares the employment hazard for UI recipients for the baseline model and the model where there is no cap on UI benefits and shows that the employment rate for benefit recipients in the uncapped UI model is less than half of the baseline model for the first two months after a layoff. Hence, a model without a benefit cap and a 50% replacement rate will overstate the effects of UI on relatively productive individuals and overstate the moral hazard they face.
1.4.3 Limited UI Eligibility

Limited eligibility plays a critical role in the model’s ability to match the labor market flows in the data, particularly from employment to other states, while maintaining a realistic UI scope. The model without limited eligibility, which I term the "separation eligibility" model, will not be able to match the labor market flows from employment to other states while matching the insured unemployment rate (IUR). In this model, all individuals who face the separation shock are eligible for the same UI benefit as in the baseline model. Most individuals who quit in the model exit the labor force because of the search cost and their savings, so the 1.8% of the employed who transition from employed to unemployed would need to be laid off in the separation eligibility model. The duration of UI benefit recipiency would have to be less than 2 months on average to match the IUR of 2.2%, which would be too short and imply essentially perfect monitoring of UI job search. Implementing the separation eligibility model and ensuring that markets clear and the government budget constraint binds implies an IUR of just over 10%. A potential workaround to the matching problem would be to impose a large transitory preference shock on individuals’ disutility to work, so the employed may decide to quit their jobs for a period but immediately seek work when they reenter the workforce. The problem with this approach is that it limits the importance of the labor supply decision and would reduce the average employment spell, which is already a little too low, and it would overestimate the expected insurance that individuals receive from UI.

Only 19% of the employed who face the separation shock earn UI benefits in
the baseline model, so the separation eligibility model provides over five times the expected benefits than the baseline model. This increase in the expected benefits of UI decreases the amount of precautionary savings that individuals hold in the separation eligibility model against the employment risk caused by the separation shock. Relatively less productive individuals who hold less capital will respond more strongly to the insurance difference than individuals with higher productivity draws and more wealth holdings because of their vulnerability to the separation shock. The top 60% of the wealth distribution, most of whom are highly productive and earn above the UI benefit cap, holds 97.6% of the wealth in the model, so the aggregate effect of UI on savings is quite small.

As Table 1.5 shows, the aggregate steady state effects of unemployment benefits on savings are minimal regardless of the assumptions about the program. Unemployment benefits tend not to affect aggregate savings much because of the general equilibrium effects of an increase in the availability of benefits on taxes and especially on government transfers and the relative unimportance of the lower part of the wealth distribution, whose precautionary savings would adjust the most given a different eligibility policy. Also, aggregate labor supply does not change much in response to the alternative assumptions. The experiment in Table 1.5 reduces the recipiency rate of UI benefits by half in the baseline and separation benefit model. The effects on savings in the policy experiment are several

4I do not recalibrate the model here. Rather, I set $\gamma_u = 1 - \sigma$, hold all other parameters fixed, and find the new market clearing wage, interest rate, and level of government transfer. Thus, the separation eligibility model no longer matches the size of the UI budget in the United States.
times larger in the separation eligibility case than in the baseline case. Assuming that the tax adjusts to the UI benefit change yields slightly stronger aggregate effects.

1.5 Welfare Effects

In this section, I compare the steady state welfare of the baseline UI model to a model without UI and models that do not incorporate the features of the UI program in the United States. The baseline model suggests that there are small welfare differences between the model economy with and without UI. Alternative simple UI plans generate much larger welfare effects.

I study the welfare effects of UI using the expected lifetime utility of an individual, as in Aiyagari and McGrattan (1998):

$$W = \int V(k, s, \xi) d\Omega^*(k, s, \xi)$$

The welfare criterion is $W$. Similar to the welfare criteria studied in Young (2004), I transform the units of the welfare criterion into a percentage of consumption by solving for $\phi$ that satisfies:

$$W_1 = W_0 + \frac{1}{1 - \beta} \log (1 + \phi)$$

Here, $W_1$ is the welfare criterion under a policy change, and $W_0$ is the welfare criterion in the baseline case. As a general rule, $\phi > 0$ indicates that the new policy leads to an average welfare gain. To determine the winners and losers of
policy changes, I consider the individuals’ expected lifetime utility conditional on their current productivity draw, $\kappa(s)$:

$$\kappa(s) = \int V(k, s, \xi) d\Omega^s (k, \xi)$$

The equilibrium distribution conditional on a productivity draw of $s$ is $\Gamma^s (k, \xi)$. The welfare measure has a similar transformation as the aggregate criterion to obtain a meaningful welfare comparison in terms of consumption:

$$\kappa_1(s) = \kappa_0(s) + \frac{1}{(1 - \beta)} * \log (1 + \mu(s))$$

Here, $\mu(s)$ is the disaggregated welfare criterion. Since the exogenous productivity shock process does not change when comparing two policies, the same measure of individuals will exist at each discretized shock process grid point, and the measurement will compare the average welfare of the same sets of individuals. For now, I ignore the transition costs of UI.

To compare the steady state equilibrium of the baseline to the policy change, I fix all of the calibrated parameters in the model except for the parameters that change in the alternative policy and solve for the new wage, interest rate, and transfer or tax that clears the labor and capital markets and allows a balanced government budget. The new policies will thus not match the moments in the calibration exercise, but they do not deviate much from the calibrated moments, unless otherwise specified.

To get a better understanding of the nature of the welfare costs, consider the welfare implications of removing the search cost from the model. Individuals
benefit from removing the labor market friction, which causes a suboptimal delay in labor market reentry after a job separation. In the aggregate, $\phi = 0.0027$, which means that individuals would need about 0.27% less consumption in the no friction case to equate average welfare in the baseline model. For comparison, Lucas (1987) estimates that the welfare benefit of removing consumption fluctuations caused by business cycles is about 0.1% in terms of consumption, but recent evidence by Krusell et al (2009) suggests the welfare gains of eliminating business cycles are larger—about 1% in terms of consumption. The disaggregated welfare gains compared to the baseline model, plotted in Figure 1.7, are hump-shaped with a maximum welfare benefit of 0.35% near the median productivity worker ($s = 1$). Removing the search costs benefits high productivity individuals less than the median productivity individuals. The concave welfare function de-emphasizes the effect of the fixed search cost for individuals with relatively high utility, and more productive individuals have fewer job searches since they rarely quit, so the welfare benefit for more productive individuals is smaller than for an individual with the median productivity draw. Individuals with low productivity draws have lower labor force participation rates and thus fewer job searches and a lower welfare effect. The aggregate welfare benefit is relatively high since the aggregation naturally puts more weight where there is a higher density of individuals on the productivity distribution.

Average welfare is higher after removing UI from the baseline model. UI reduces economic activity by reducing the capital input. Removing UI increases individuals’ demand for capital to self insure against the employment risk much as a
decrease in employment risk causes an increase in interest rates and a lower equilibrium capital in the incomplete markets model of Aiyagari (1994). At the same time, removing UI decreases the labor input by 0.2% and reduces the employment-population ratio by about 1 percentage point. While this result might seem counterintuitive, the negative labor supply response is concentrated among low productivity workers for whom UI provides an incentive to enter the labor force. UI for low productivity individuals adds to the potential value of employment since they can collect UI if they are laid off, and the marginal labor supply effect of additional compensation is high since their wealth and combined wage and transfer income are relatively low. The labor supply response of relatively productive individuals to removing benefits is muted because of the relative unimportance of UI in their income, especially for individuals who earn the maximum UI benefit. Even if individuals spend less time working overall because of the moral hazard effect of UI, the limited eligibility and finite duration of UI limits the average time that the person spends on UI. Overall, the effect on capital outweighs the aggregate labor supply effect after removing UI, and consumption-equivalent welfare improves by 0.04% if individuals receive a higher transfer from the government and by 0.08% if the government instead lowers the distortionary income tax. The employment to population ratio falls by 0.7%, and the aggregate labor input falls by 0.02% in that case for the same reasons as the transfer case.

The negative effects of UI mirror the results in Young (2004), who finds that labor and capital both fall because of unemployment insurance. The negative welfare effects of unemployment insurance are much larger in his model, 1.1%
or 1.7% consumption equivalent depending on the preferred specification, since his model does not match the recipiency rate of UI and thus the UI budget. The key difference between his model and my model, aside from the heterogeneity in wages, is his lack of a limited eligibility feature in his UI system. He matches the unemployment rate in the United States but does not focus on individuals out of the labor force, so his model is most similar to the separation eligibility model in this paper, which has quantitatively similar welfare effects (see below). Wang and Williamson (2002) and Mukoyama (2011) both find that UI increases welfare. The welfare gains in Wang and Williamson are at most 1.5% of consumption, and the welfare gains from switching to the optimal system are less than 0.1% of consumption since the endogenous labor supply margin allows individuals to adjust to the remaining employment risk that UI does not cover. Their model features limited eligibility for UI, which allows them to match the recipiency rate of unemployment benefits, but the household savings instrument is not interest-bearing in their model, which means that all savings insures against employment risk. Hence, their distribution of wealth is negatively skewed, which is the opposite of wealth distributions of economies in practice. Since the accuracy of the welfare criterion depends on the distribution of income and wealth in the economy, my model’s better match of the wealth distribution improves the accuracy of the welfare results. The welfare gain of UI in the exogenous employment model in Mukoyama (2011) is 0.03%, but it appears to be larger with an endogenous labor supply decision and moral hazard. The key difference between his models and
my model is the structure of the unemployment insurance program; all individuals who do not work receive unemployment benefits in his models, and he does not construct the programs to match the features of the UI system in the United States. Likewise, Launov et al (2009) studied the effects of the Hartz IV welfare reforms in German that reduced UI benefits and found that welfare fell by 0.5% in an extension of a Diamond-Mortensen-Pissarides model that did not include savings. One would expect that the welfare importance of UI would be much less in a model with savings.

The relative gain of productive vs. less productive individuals depends on whether the transfer or taxes adjust in the model when UI is removed. Figure 1.8 compares the disaggregated welfare criterion $\mu(s)$ for tax adjustment and transfer adjustment cases. Low productivity individuals have few assets in equilibrium, and UI induced them to enter the labor force. Removing UI causes them to have more difficulty smoothing consumption over time, particularly around the work/no work cutoff productivity value around 0.4. Removing UI hurts their welfare under the tax adjustment case since they do not earn much back from the reduced taxes, but they see welfare gains with the increased income from the higher transfer. Here, the lowest productivity individuals who do not work are slightly better off receiving the additional transfer since they see no immediate increase in income from reduced income taxes. Individuals with higher productivity draws tend to work up to 2% more on average with a maximum effect near the median worker after removing UI. These productive individuals benefit from the decrease in moral hazard from eliminating UI benefits and either lower taxes
or a higher government transfer. Productive individuals are better off with a tax adjustment than a transfer adjustment, because the income tax is distortionary. The equilibrium interest rate in the model decreases slightly after removing UI, which causes the welfare effect of the transfer case becomes slightly negative for highly productive individuals ($s > 4$). The welfare effect for the tax case remains positive since the additional income they receive in the no UI case is increasing in their productivity draw, unlike in the transfer case.

The disaggregated welfare measures are useful for comparing the baseline to the model without a benefit cap to show that the welfare comparisons of the steady states depend on the differences in UI funding between the models. In the aggregate, individuals are better off in the baseline case since output is higher. Productive individuals face greater moral hazard in the absence of a benefit cap, which reduces labor supply and labor input relative to the baseline model. Average welfare decreases less if an increase in taxes accounts for the change in UI funding ($\phi = -0.2\%$) versus a transfer decrease ($\phi = -0.9\%$). The distortionary income tax rises 1.1% to account for the differences in UI budgets between the models, which will again increasingly affect high productivity individuals. The transfer decrease significantly harms the less productive and poorer individuals, however, and individuals are relatively better off under the tax adjustment case for most productivity draws ($s > 4$). For less productive individuals, particularly those who do not work, the transfer is a significant portion of their income, and a large lump sum tax will affect them more than an increased income tax. Individuals who do not earn enough to receive the maximum unemployment benefit
do not receive any benefit after removing the cap, but they pay part of the bill for additional UI benefits. These two facts cause a large disparity in the welfare comparisons in Figure 1.9. Individuals who earned less than the benefit cap do not suffer much as the effect of the additional income tax is minor, but the welfare losses under the tax adjustment case exceed 2% of their average consumption for the least productive individuals in the economy. For the most productive individuals, a lump sum payment to fund the uncapped benefits has a decreasing effect on their welfare, whereas the tax is more distortionary.

The welfare costs of unemployment are relatively small because of the structure of the UI program, not because of the underlying labor market structure or the fit to the wealth distribution. I now consider a model with complete eligibility for UI, in which all individuals who are not working receive a 50% replacement rate of their potential income. One cannot fit the facts about UI and the labor market at the same time in this model since matching the employment-population ratio would force over 30% of individuals into UI each period. To fit the UI benefit budget, the replacement rate of UI benefits would have to be less than 1%. The model generates large aggregate welfare effects regardless of how the model accounts for the differences in the government budget. The welfare effects relative to the baseline case are more negative if the tax adjusts \((\phi = -3\%)\) than if the transfer adjusts \((\phi = -2.5\%)\), and the relative transfers that the new policy provides determine the disaggregated welfare effect. In the tax increase case, highly productive individuals suffer a 5% equivalent consumption decrease, but lower productivity individuals suffer smaller welfare losses or even welfare gains.
if they otherwise would not work, since they would also receive the UI benefit but don’t have to work to establish eligibility. The transfer case leads to up to 10% welfare losses for low productivity individuals since the UI budget is much larger, economic activity is much lower, and thus the transfer falls by about 20% compared to the baseline case, excluding the positive effect from additional UI benefits. The welfare loss is decreasing in productivity since individuals receive higher UI payments with a higher productivity draw in the absence of the distortionary tax.

The separation eligibility model, which provides UI to all individuals who face the separation shock, reduces welfare by 0.8% compared to the baseline case if the transfer adjusts and 1% if the distortionary tax adjusts. The latter number is roughly in line with the welfare results of Young (2004). The size of the welfare effects are not surprising given that this case is an intermediate case between the baseline and full eligibility cases. The distributional welfare effects are quite similar qualitatively to the full eligibility as well.

1.6 Conclusion

The design of a model unemployment insurance program matters for many common exercises in the UI literature. I add a UI program that has features consistent with the UI program in the United States to a general equilibrium model with labor market frictions and a labor supply decision to match not only key facts about the aggregate labor market but also the size and scope of UI in the US. Each of the UI features has nontrivial effects on individual incentives and in the
aggregate. The limited eligibility of individuals for UI limits the size of the UI program, limits the negative welfare effects of UI, and limits UI’s effect on other aggregate variables, such as savings. The welfare of the baseline model is much closer to the no UI equilibrium than alternative, broader UI policies, so a more robust welfare comparison for an optimal UI benefit is the no welfare case. The finite duration of UI benefits and the imperfect monitoring of job search leads to an empirically consistent spike in the employment hazard of benefit recipients at benefit expiration. The benefit cap limits the moral hazard that UI provides to highly productive individuals with high potential wages and allows the model to feature a 50% pretax replacement rate up to the cap, which helps focus the effects of UI on individuals who receive it in reality.

The welfare effects of UI are negative relative to a no UI environment since individuals reduce their capital holdings in the presence of additional insurance, which reduces output. The presence of UI increases labor supply slightly in the baseline model since the prospect of UI encourages low productivity workers to enter the labor force, and the cap on UI benefits limits the moral hazard of more productive workers. However, the increase in labor supply does not offset the decrease in the capital input on output. The size of the welfare effects are small, less than 0.1% of consumption, because of the features of the UI program. This welfare effect increases to 0.8% of consumption if UI eligibility expands to include all individuals who face an employment separation shock and to 2.5% or more if all individuals who do not work are eligible for UI. From the perspective of determining the relative welfare benefits of an optimal UI policy, it would be more
accurate to compare the optimal UI policy to a no UI benchmark than to com-
pare the optimal policy to one modeled after the US without a limited eligibility
requirement, though the optimal UI policy in this economy may be a 0% replace-
ment rate as in Young (2004).

The next natural step is to extend the model to evaluate the transition costs
of removing UI from the model. In general, the transition costs of UI depend
on how much the production inputs of the economy differ between the steady
states. Transition costs will decrease the welfare benefit of removing UI. The
small differences in the aggregate inputs should keep the transition costs of UI
small. The transition would also allow a proper analysis of the effects of extend-
ing UI benefits since UI benefit extensions typically coincide with a tight labor
market. Krusell et al (2011b) find that this class of model can account for business
cycle fluctuations while accounting for labor market flows.

Another potentially valuable extension would be to make UI benefits depend
on past income, which would require the model to explicitly track employment
and income history. The replacement rate of benefits in many states exceeds 50%
for low income individuals who just qualify for a state’s minimum UI benefit,
which could amplify the positive labor supply effect and extend the duration of
employment for low wage individuals. At the same time, individuals may not
qualify for UI if they do not work for a long enough time without a layoff, which
may reduce the value of UI for individuals with extremely low potential wages
who would have to work for too long to feasibly earn UI benefits.
The minimum earnings requirement to receive UI may affect individuals’ employment incentives in unemployment insurance programs where job quitters receive benefits. Individuals seeking short term employment may adjust their desired quitting date to when they first become eligible for unemployment benefits, which could cause a spike in quits upon benefit eligibility. Baker and Ham (1987) and Baker and Rea Jr. (1998) found spikes in the employment hazard of individuals that corresponded to when they became eligible for unemployment benefits in late 1980s and 1990 Canadian data. Unemployment benefits at the time were relatively easy to obtain—Canada provided unemployment benefits to job quitters at the time—so a model unemployment benefit program where eligibility depends on individuals’ employment history should be able to capture the spikes in the employment hazard.
Table 1.1: Model Calibration

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r - \delta$</td>
<td>4%</td>
<td>4%</td>
<td>$\beta$</td>
<td>0.9946</td>
</tr>
<tr>
<td>$wL/Y$</td>
<td>70%</td>
<td>70%</td>
<td>$\alpha$</td>
<td>0.3</td>
</tr>
<tr>
<td>$I/Y$</td>
<td>20%</td>
<td>20.3%</td>
<td>$\delta$</td>
<td>0.0067</td>
</tr>
<tr>
<td>Top 60% Wealth</td>
<td>0.99</td>
<td>0.976</td>
<td>$\sigma_\varepsilon$</td>
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</tr>
<tr>
<td>Tax Rate</td>
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<td>$\tau$</td>
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<tr>
<td>$E/P$</td>
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<td>63.4%</td>
<td>$\psi_1$</td>
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</tr>
<tr>
<td>$U/(E + U)$</td>
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<td>5.1%</td>
<td>$\lambda$</td>
<td>0.495</td>
</tr>
<tr>
<td>$U^G/(E + U^G)$</td>
<td>8.4%</td>
<td>8.5%</td>
<td>$\psi_2$</td>
<td>0.026</td>
</tr>
<tr>
<td>$Em \rightarrow Em$</td>
<td>0.961</td>
<td>0.962</td>
<td>$\sigma$</td>
<td>0.957</td>
</tr>
<tr>
<td>$Em \rightarrow Un$</td>
<td>0.021</td>
<td>0.020</td>
<td>$\rho$</td>
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</tr>
<tr>
<td>IUR</td>
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<td>2.2%</td>
<td>$\gamma_u$</td>
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<td>0.29%</td>
<td>$\bar{s}$</td>
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<td>35.6%</td>
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</tr>
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<td>replacement rate</td>
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<td>$u$</td>
<td>0.5</td>
</tr>
<tr>
<td>benefit length</td>
<td>Fixed</td>
<td></td>
<td>$B$</td>
<td>6</td>
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</tbody>
</table>
Table 1.2: Baseline Employment Flows

Data (Krusell et al) / Model

<table>
<thead>
<tr>
<th></th>
<th>Employed</th>
<th>Unemployed</th>
<th>Not in LF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employed</td>
<td>0.961</td>
<td>0.021</td>
<td>0.018</td>
</tr>
<tr>
<td>0.962</td>
<td>0.020</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>From Unemployed</td>
<td>0.251</td>
<td>0.507</td>
<td></td>
</tr>
<tr>
<td>0.289</td>
<td>0.675</td>
<td>0.242</td>
<td></td>
</tr>
<tr>
<td>0.036</td>
<td>0.919</td>
<td>0.951</td>
<td></td>
</tr>
</tbody>
</table>

| Not in LF | 0.034 | 0.017 | 0.047 | 0.032 | 0.919 | 0.951 |
Table 1.3: Employment Flows: Baseline vs. No Search Cost

No Search Cost / Baseline

<table>
<thead>
<tr>
<th></th>
<th>Employed</th>
<th>Unemployed</th>
<th>Not in LF</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>To</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>0.959 0.962</td>
<td>0.019 0.020</td>
<td>0.021 0.018</td>
</tr>
<tr>
<td><strong>From</strong></td>
<td>Unemployed</td>
<td>0.482 0.289</td>
<td>0.492 0.675</td>
</tr>
<tr>
<td>Not in LF</td>
<td>0.033 0.017</td>
<td>0.042 0.032</td>
<td>0.925 0.951</td>
</tr>
</tbody>
</table>
Table 1.4: Employment Flows: Baseline vs. No Unemployment

<table>
<thead>
<tr>
<th></th>
<th>Employed</th>
<th>Unemployed</th>
<th>Not in LF</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>To</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>0.963</td>
<td>0.022</td>
<td>0.015</td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.289</td>
<td>0.675</td>
<td>0.036</td>
</tr>
<tr>
<td>Not in LF</td>
<td>0.018</td>
<td>0.026</td>
<td>0.956</td>
</tr>
<tr>
<td><strong>From</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>0.962</td>
<td>0.020</td>
<td>0.018</td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.289</td>
<td>0.675</td>
<td>0.036</td>
</tr>
<tr>
<td>Not in LF</td>
<td>0.017</td>
<td>0.032</td>
<td>0.951</td>
</tr>
</tbody>
</table>

No Unemployment / Baseline
Table 1.5: Precautionary Savings Changes Conditional on Eligibility

Precautionary Savings \( \left( \frac{K}{L} \right) \) : Transfer Adjusts

<table>
<thead>
<tr>
<th>Replacement Rate</th>
<th>Eligibility</th>
<th>Baseline</th>
<th>Layoff or Fired</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u = 0.5 )</td>
<td></td>
<td>130.92</td>
<td>129.93</td>
</tr>
<tr>
<td>( u = 0.25 )</td>
<td></td>
<td>131.07</td>
<td>130.57</td>
</tr>
<tr>
<td>No Benefits</td>
<td></td>
<td>131.17</td>
<td></td>
</tr>
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</table>
Figure 1.1: Baseline Cutoff Capital Levels
Figure 1.2: Employment-Population Ratio
Figure 1.3: Reentry Rate to Workforce: No Search Cost vs. Baseline
Figure 1.4: Baseline Employment Hazards
Figure 1.5: The Effect of Monitoring on the Incentives of the Unemployed
Figure 1.6: Employment Hazard after Removing Benefit Cap
Figure 1.7: Steady State Welfare Effect of Removing the Search Cost
Figure 1.8: Steady State Welfare Effect of Removing Unemployment Insurance
Figure 1.9: Steady State Welfare Effect of Removing Benefit Cap
CHAPTER 2
SEASONALITY IN A MENU COST MODEL

2.1 Introduction

Seasonal fluctuations in the economy have long been seen as little more than an annoyance to be discarded in macroeconomic analysis\(^1\). However, that has begun to change. Empirical vector autoregression (VAR) evidence from Olivei and Tenreyro (2007) suggests that the response of the economy to monetary shocks is seasonally dependent, and earlier regression analysis by Barsky and Miron (1989) suggests that the seasonal cycle and the business cycle share certain qualitative features, which suggest at least some common causes. Both suggest a nontrivial connection between seasonal fluctuations and the business cycle. I explore the implications of seasonality for the potential connection between firms’ price-setting behavior and their resulting response to monetary shocks in a menu cost model. The connection between the composition of firms in a menu cost model and the potential response to monetary shocks, emphasized by Midrigan (2009), provides the potential for seasonal fluctuations to have substantial real effects.

\(^{1}\)See, for example, Sims (1974).
I develop a menu cost model with seasonal fluctuations in the form of a fluctuating labor market rigidity. The model shows that a relatively small fluctuation in the real rigidity of the economy can generate a sizable seasonal cycle. Output fluctuates by 7.8 percent from peak to trough in my baseline specification, which is the upper bound of the amplitude of the seasonal cycle that I find in GDP data. There are several competing explanations for the existence of the seasonal cycle and for the seasonal dependence in the effects of shocks. Olivei and Tenreyro (2007) point to time varying labor market fluctuations, such as staggered labor contracts. Nakamura and Steinsson (2008) suggest that seasonal fluctuations in demand could be responsible for the seasonal fluctuations in the micro price data, but neither study is conclusive. My model is more similar to Olivei and Tenreyro’s concept; the source of seasonality in my model is a generic, time-varying labor market friction. Seasonal demand shocks are not necessary to obtain sizable seasonal fluctuations in the economy.

Firms respond to the seasonal fluctuations by seeking to set their price nearer to the prices of the average firm in the economy, which causes firms to set approximately the same optimal prices across the seasonal cycle. Since firms cluster their optimal prices, differences in when firms decide to set their prices cause much of the seasonal fluctuations in the baseline model. Firms are relatively intolerant of having low prices when aggregate prices are rising in the seasonal cycle and vice versa.

The model features a seasonally-dependent response to monetary policy shocks and produces a greater average output response to seasonal fluctuations than the
equivalent nonseasonal model. The baseline model results produce a monetary re-
response pattern that is inconsistent with the VAR evidence of Olivei and Tenreyro–
the output response to a monetary shock in the model is larger is the latter half of
the year than in the first half of the year. Since prices are relatively high and out-
put is low at the beginning of the year, firms are much more likely to raise their
prices in the first few months of the year. Inflation causes many of these price
changes to be permanent, which causes prices to respond quickly to the monetary
shock, which limits its effects on output. Shocks that occur in the second half of
the year have a stronger output effect since firms delay responding to the shock
until the beginning of the next year.

Several potential fixes to the model could improve the model’s fit to the evi-
dence of Olivei and Tenreyro. The responses are empirically consistent if defla-
tion prevails in the economy. Alternatively, an imperfect information mechanism
could delay firms’ response to the shock long enough for the firms to respond to
the shock in the following year if the shock occurs early in the year, which would
increase the output response of shocks early in the year.

A methodological contribution of my paper is how I handle the solution of
the menu cost model. My model is the first dynamic general equilibrium model
with menu costs, heterogeneous agents, and seasonal fluctuations. Others have
solved models featuring a seasonally fluctuating economy (see Braun and Evans
(1995), Liu (2000), and Olivei and Tenreyro (2007)), but those models do not fea-
ture extensive firm heterogeneity. To solve for the steady state of a menu cost
model, one must solve for the stationary distribution of the heterogeneous firms
across states. Seasonality complicates the problem by introducing dependence between the value functions and decision rules across the seasons, which forces me to solve for the distribution of firms across each season in order to determine the steady state path of aggregate, prices, output, and wages. Ignoring seasonal fluctuations, I solve for the stationary distribution using a method similar to solving for the stationary distribution of a Markov chain. I then generalize the method to accommodate the differing pricing rules and distributions of firms across the seasons. This cycling distribution of firms across time is termed a cyclostationary distribution, a concept that has been rarely applied in economics.

Section 1 continues with a review of the related literature. I introduce the model in Section 2. Section 3 presents the planner equilibrium of the economy. Section 4 compares the seasonal and nonseasonal models, and Section 5 shows the effects of a monetary shock. Section 6 concludes.

2.2 Literature Review

Other Models with Seasonality

Seasonal fluctuations in the macroeconomy are larger than business cycle fluctuations in the short run. Barsky and Miron (1989) develop some basic facts about the seasonal cycle of the economy, which allow them to establish the existence of the seasonal cycle in the economy. Seasonal fluctuations dominate in the short run in the economy. Regressing on detrended data, Barsky and Miron find that output on average fluctuates by about 8% over the course of the year from the peak in the 4th quarter of the year to the trough in the first half of the year. The price
level moves in the opposite direction as output, but the fluctuations in the overall price level are much smaller, on the order of less than half of a percent. Also, the seasonal cycle and the business cycle share qualitative characteristics, which may imply that the fluctuations have some common causes. Their results are empirical, and economic models that incorporate seasonality are left for future research.

Braun and Evans construct a real business cycle model with seasonal fluctuations in preferences, technology, and government purchases to attempt to explain the seasonal cycle as laid out by Barsky and Miron. To explain the size of the seasonal cycle, their model requires technology to rise at a 24% annual rate in the fourth quarter of the year and to fall at a 28% annual rate in the first quarter of the year, which they consider evidence that the typical production technology in RBC models is misspecified.

While Braun and Evans (1995) create a model that can explain seasonal fluctuations in real variables, it could not explain fluctuations in nominal variables. Liu (2000) creates a two sector monetary model with seasonal fluctuations to determine whether and how the Federal Reserve should respond to the fluctuations in the seasonal cycle versus the business cycle. Historically, the Federal Reserve smoothed nominal interest rates over the seasonal cycle but not the business cycle. The real-bills doctrine suggests that a central bank should smooth fluctuations in nominal interest rates over both cycles while the quantity theory of money suggests that a central bank should smooth fluctuations in the money supply over the cycles. Liu finds, comparing the historical policy, the real-bills policy, and the quantity-theory of money policy, that the historical policy is closest to the optimal
policy prescribed by Friedman’s Rule. Notably, his model is the first to combine both business cycle fluctuations and seasonal fluctuations.

Olivei and Tenreyro (2007) also combine business cycle and seasonal fluctuations in a DSGE model to explain apparent seasonal fluctuations in the response of the economy to monetary shocks. Using a seasonal VAR, they show that the response of output to monetary policy shocks varies depending on the quarter in which the shock occurs. In the first half of the year, prices respond little initially and output responds quickly to a monetary shock. In the second half of the year, prices respond quickly to the shock, and output does not respond much at all. Olivei and Tenreyro suggest that seasonally varying labor market frictions are the source of the seasonal fluctuations in the response of the economy to monetary shocks. They find evidence that wage contracts are typically signed in the later half of the year, especially during the fourth quarter. If a monetary shock occurs in the latter half of the year, then the firms can price the shock into their wages. Wages respond quickly to the shock, and thus most firms’ marginal costs respond quickly to the shock. Prices respond quickly to the shock, which limits the real effects of the shock. In the beginning the year, many firms will have just renegotiated their wages, so marginal costs are roughly constant for those firms. If the monetary shock occurs in the beginning of the year, firms are less willing to respond to the shock since their marginal costs respond sluggishly. They are more willing to change their production, so output responds quickly and robustly to the shock. They create a model of staggered wage contracts in which firms face Calvo timing in wage setting. Every period, firms have a seasonally dependent
probability of being able to negotiate their wages. They set the probabilities of
adjustment so that firms are much more likely to renegotiate their wages at the
end of the year. Their model fits the general pattern that they observe in the VAR
analysis.

While seasonality in economics is the subject of a large literature, there are few
empirical applications of cyclostationarity in economics, and what applications
there are tend to be for time series analysis of macroeconomic or financial data.
Broszkiewicz-Suwaj et al (2004) apply cyclostationarity to financial data to find
look at the concept of cyclostationarity in time series data and various time series
models. Leskow (2001) applies an econometric test featuring cyclostationarity to
asset volatility data as an alternative to the ARCH and GARCH approaches to
modeling the variance of asset returns. Serpedin et al (2005) is an extensive bib-
liography on cyclostationarity across disciplines and a good reference for other
uses of cyclostationarity.

Menu Cost Models
State-dependent pricing models allow economists to evaluate the assumptions
underlying time-dependent models, and they allow economists to better model
firm behavior based on the ability to fit the micro data on prices. Menu cost mod-
els are a subset of state-dependent pricing models in which firms must pay a fixed
cost to change their prices. Some economists do not think that menu cost models
can generate sufficient monetary nonneutrality in response to a monetary shock
to be useful. These nonneutrality results, including Caplin and Spulber (1987) and Golosov and Lucas (2007), challenge the view that small nominal rigidities could be realistically amplified to explain large monetary nonneutrality in the macroeconomy. If money is neutral in a menu cost model, then either something fundamental about the model is misspecified or the underlying assumptions about firms’ price changes in time-dependent pricing models are incorrect. If the latter is true, then the result challenges the validity of a large literature of work that builds upon models that assume time-dependent pricing. On the other hand, others find that there are mechanisms by which small menu costs have substantial real effects. It is possible to develop these models further as a potential replacement for time-dependent models.

Mehrez (1998) explores the implications of the menu cost mechanism in an economy with seasonal fluctuations and hence is the paper most similar in spirit to my paper. Mehrez explores the adjustment incentives of firms, without the now typical DSGE framework, by assuming that firms desire to maximize the flow of profits with a simple penalty from deviating much from a seasonally varying target price. His framework is a sufficient framework to analyze firms’ adjustment incentives, but it is not sufficient to pin down the aggregate implications of seasonality. He finds that the observed seasonal fluctuations in the economy are less than the true seasonal fluctuations, because firms cannot adjust completely to the seasonal fluctuations. Also, as inflation rises in his economy, the amplitude of seasonal fluctuations rises as well. Matching Israeli price data from the 1980s, he finds
that the adjustment behavior of the firms is roughly consistent with his model of menu costs coupled with seasonal fluctuations in the firms’ desired price.

Caplin and Spulber (1987) create a menu cost model in which money is completely neutral. While firms face a nontrivial menu cost, firms are distributed uniformly across prices by assumption. To generate monetary nonneutrality, the distribution of firms across relative prices must change in response to a shock. If the distribution of firms across relative prices does not change, then the only reaction of firms to the shock will be for the firms with the lowest relative prices to have large nominal price changes. The distribution of firms across nominal prices would simply shift upwards by the amount of the monetary shock, and aggregate prices would fully accommodate the shock. Individual firms will not fully and immediately adjust to the shock, but the firms that do adjust will adjust enough to force monetary neutrality.

Dotsey, King, and Wolman (1999) create a menu cost model in which firms are subject to a random menu cost, which creates heterogeneity in the firm’s price adjustment decision. The technical problems of the model increase with this assumption since the steady state distribution of the firms across prices is unknown initially. The problem is tractable, however, and they find that the model produces significant monetary nonneutrality. Their nonstandard distribution of the menu cost shocks drives their results.

Golosov and Lucas (2007) create a continuous time menu cost model that builds upon the previous menu cost literature by matching price change moments implied by the model to micro data. Specifically, Golosov and Lucas match the mean
and variance of nonsale price changes and inflation from Klenow and Kryvtsov (2008). Previous models did not match the micro data about prices explicitly. In Dotsey, King, and Wolman (1999), all firms adjust their prices to a common price since the random component of the model is the menu cost, which does not affect the optimal chosen price of the firm. In Golosov and Lucas, firms face an idiosyncratic productivity shock that affects whether and how the firms adjust their prices, which allows them to match the micro data on prices. With a model that matches the micro facts about price changes, they then evaluate how a monetary shock affects the path of output and prices in the model and compare the model to a baseline Calvo model. They find that the model produces little monetary nonneutrality as a result of the monetary shock, consistent with the findings of Caplin and Spulber, which is not surprising since the only difference between Golosov and Lucas’ problem and Caplin and Spulber’s problem, to a log linear approximation, is the existence of idiosyncratic shocks.

Midrigan (2009) views the Golosov and Lucas and Caplin and Spulber nonneutrality result as a problem of the underlying assumption that the idiosyncratic shocks are normally distributed, the effect of which he calls the selection effect. The selection effect occurs when the firms that change their prices after a monetary shock are the firms that have large price changes. One way to minimize the selection effect is to minimize the number of firms that have large price changes by forcing as many firms as possible to have small price changes. Since large idiosyncratic shocks give firms an incentive to change their prices by a large amount, one way to mitigate the selection effect is to choose an idiosyncratic distribution for
the shock that groups most firms around the mean shock value. A leptokurtotic
distribution of firms across prices, one in which the tails of the distribution are
fat but a large mass of firms is clustered near the mean of the distribution, gives
the desired grouping of firms around the median price. Midrigan successfully ap-
plies a leptokurtotic distribution of cost shocks to create a menu cost model that
can generate substantial real rigidity in response to a monetary shock. The con-
sequence of the leptokurtotic distribution of shocks is a leptokurtotic distribution
of desired and actual price changes, which Midrigan finds is consistent with firm
level price scanner data.

Another contribution of Midrigan (2009) is a method by which to account for
small price changes in the data. Midrigan introduces economies of scope in price
setting in firms. Firms produce two related goods, and when the firm chooses to
change the price of one of its goods, it gets to change the other price for free. Since
the firms cluster around the median price and the firms will most often change
the price of one of its goods when it receives a large idiosyncratic shock in that
good, the free price change for the other good is often quite small. Combined with
the leptokurtotic shocks, Midrigan’s model is able to account for 80% as much
monetary nonneutrality as a time dependent model.

Nakamura and Steinsson (2009) create the Calvo-Plus model, a hybrid state
and time dependent pricing model, to fit both the micro data and provide an al-
ternative solution to the neutrality result of Golosov and Lucas. The model is a
hybrid in the sense that firms face a certain probability of having the either a high
or nearly zero menu cost, which is a state dependent pricing mechanism that has
elements of (time dependent) Calvo timing. By introducing sector heterogeneity and intermediate inputs into the model, they are able to generate roughly nine times the amount of monetary nonneutrality as a baseline menu cost model. The intuition behind the importance of sector heterogeneity in the model is the idea that the first price change by a firm after a monetary shock contributes the most towards the economy’s adjustment to a shock. The second time a firm adjusts its price after a monetary shock, it contributes little to the adjustment, because it has already priced in most of the shock. They find evidence that the frequency and size of price changes varies by sector in micro level pricing data. If a significant number of price changes occur within relatively few firms in the economy, then the firms that frequently change their prices will account for many of the price changes but contribute little to adjusting to the shock in the aggregate. Firms that are in sectors in which prices change infrequently will contribute few price changes, and they will change their prices more sluggishly to the monetary shock but account for much of the monetary nonneutrality of the model. Overall, firms tend to change their prices less in response to exogenous aggregate shocks, which boosts the monetary nonneutrality produced by the model. Forcing some firms in the model to account for a large number of price changes triples the monetary nonneutrality produced by their standard menu cost model.
Intermediate inputs are particularly important for my paper as it is one possible labor market friction that can justify my generic source of real rigidity. The intermediate inputs mechanism of Nakamura and Steinsson is based on the roundabout production method of Basu (1995) where all goods are final goods and inputs for all other goods in the market. A larger share of intermediate inputs in production slows down the rate at which firms change their prices since firms’ marginal costs respond more sluggishly to a monetary shock. Firms do not respond to the monetary shock per se; they respond to the changes in their marginal costs. In a standard menu cost model, aggregate wages will respond immediately and completely in response to a shock, which means that firms have a more immediate incentive to change their prices. With intermediate inputs, wages depend on the other firms’ chosen prices, because the prices of firms are essentially the marginal costs of other firms. The more incomplete the adjustment of marginal costs to a monetary shock, the more likely firms will not respond to the shock, and even if they adjust, adjustment will be incomplete. Hence, monetary nonneutrality increases significantly with intermediate inputs in the model.

Burstein and Hellwig (2007) investigate the size and importance of aggregate and firm level sources of real rigidity in a standard menu cost model. The aggregate real rigidity in their model takes the form of a generic labor market friction. Essentially, the wage that firms face in the model is the geometric weighted average of the money supply, which is the monetary shock and thus adjusts immediately to a monetary shock, and the price level, which responds sluggishly. The aggregate real rigidity is controlled by the weighted average parameter; as
more weight is put on the price level, the slower firms will respond (in terms of changing their prices) to the monetary shock so long as other firms have an incentive to respond sluggishly to the shock as well. The decreasing returns to scale production function in labor is the firm level rigidity in the model. Burstein and Hellwig find that the firm level rigidity cannot generate much monetary nonneutrality with reasonable implications on the micro facts about prices. Basically, as decreasing returns to scale become stronger, firms cannot adjust by changing their production as much in response to a monetary shock, so they will be more likely to change their prices. This additional desire to change prices limits the additional rigidity that the model can produce. The bulk of the monetary nonneutrality produced by the model is from the aggregate real rigidity, which is consistent with the findings of Nakamura and Steinsson.

Gorodnichenko (2009) develops a menu cost model with imperfect information about a nominal demand shock but perfect information about the aggregate price level. Firms must choose whether or not to change their prices and whether or not to purchase a better signal about the nominal demand shock. Firms are hesitant to change their prices in response to a monetary shock because their price change contains their information about the demand shock and the state of the economy. The aggregate price level acts as a public signal about the state of the shock, and when enough firms change their prices, enough information is released about the shocks into the public signal to reach a tipping point, and many more firms change their prices. The hesitation on the part of firms to change their
prices in response to any shock provides a novel source of real rigidity. Gorodnichenko’s model, while highly stylized, can generate not only a large amount of monetary nonneutrality in response to a monetary shock, but a delayed and smoothed hump-shaped response of prices (and thus a hump-shaped response of output) as well.

Kehoe and Midrigan (2008) create a menu cost model in which firms can elect to make not only a typical, permanent price change but a temporary price change as well. The temporary price change accommodates sales and other markdowns, and can be thought of as a price rental. Firms have both a regular price and a desired sales price. If the firm wants to change its regular price, it must pay the full menu cost, but it maintains that new price permanently. If the firm instead wants to use a sales price, then it will “rent” as price change at a lower cost than a regular price change. It will change its price for a period, and the price will revert to its regular price in the next period. Their model is able to account for a reasonably large amount of monetary nonneutrality. They also show that models calibrated to micro data without sales overstate the response of the economy to the shocks, and models calibrated to price data with sales cannot account for much monetary nonneutrality. Instead, a reasonable, imperfect compromise for a menu cost model without temporary price changes is to try to match the percentage of time prices are at their annual mode.
Studies of Price Setting and Seasonality in Price Setting

Seasonality in price setting has been of interest to economists for some time, and there has been a debate about the motives that firms have to adjust their prices. Most papers that look at the micro price data tend to look at price changes in the aggregate rather than season by season, because most practical uses for this data require seasonally adjusted data. Researchers in this literature also tend to remove sales from the data using various algorithms to smooth from the data what they think is unimportant noise for models that focus on sources of aggregate fluctuations. Also, the frequency of price changes, especially sales, in high frequency data would imply that prices change very frequently in menu cost models, which are typically specified as either monthly or quarterly models. In the menu cost literature, other than Kehoe and Midrigan (2008), the models do not have a way to accommodate sales, and my model is the first DSGE models to accommodate explicitly seasonal price changes. I will eventually need to expand upon the literature presented below, especially the work of Nakamura and Steinsson (2008), to find additional facts about the seasonal aspect of price changes.

Bils and Klenow (2004) examine BLS data used to compute the CPI to find some basic facts about micro price changes. The BLS collects monthly data for 70,000-80,000 goods from 22,000 stores in 88 geographic regions across the United States. Bils and Klenow find considerable heterogeneity in price changes across goods, which implies that standard time dependent sticky price models do not do a good job modeling prices at the good level, particularly for goods that have less frequent price changes. Also, they find that the average time per price change is
quite low; for half of the goods in the sample, the average duration of a price is less than 5.5 months. The median duration of nonsale price changes is about 4.3 months. They do not mention much about seasonality in price changes in their paper other to say that seasonally adjusted inflation features the lowest persistence and highest volatility, so only seasonal sales would not explain their findings.

Klenow and Kryvtsov (2008) find similar facts as Bils and Klenow in the same data. With sales, prices change on average every four months in the median category, and without sales, prices in the median category change on average every seven months. Price changes are typically large in absolute terms, about 10%, but there are many small price changes on the order of 5% or less. They also find that variance of aggregate inflation is mostly from changes in the intensive margin (the size) of price changes, rather than the extensive margin (the fraction of items) of price changes. They find that the 2004 vintage (and previous) of state dependent and time dependent models have a difficult time fitting all of the facts that they have at the same time. The Golosov and Lucas model does not generate enough small price changes, and the model of Dotsey, King, and Wolman does not generate enough large price changes. Time dependent models predict that older prices will feature larger changes when they do change, and they suggest an incorrect hazard of price changes. They do acknowledge that some of the newer models, such as Midrigan’s model, can generate reasonable price changes, however.

Nakamura and Steinsson (2008) work with the same data as Bils and Klenow and Klenow and Kryvtsov and establish what they term are the five facts about
prices. Notably for my model, the fourth fact is that price changes are highly sea-
sonal. In particular, they found a monotonically decreasing trend of price changes
over the four quarters of the year, after removing sales from the data. Within a
quarter, the frequency of price adjustment decreases monotonically as well. They
do not provide information about the size of price changes over different seasons,
however. Disaggregating the price adjustments into price increases and price de-
creases, they find that the frequency of price decreases stays steady throughout the
year while the frequency of price increases tends to fall monotonically through the
year. Also, the hazard of price adjustment for some goods, which gives the prob-
ability that a firm will adjust its price given the length of time since its last price
change, has a spike for some goods at 12 months, which implies that some prices
change annually.

They also find other facts that are important for calibrating a menu cost model.
The median frequency of non-sale price changes is about 9%-12% a month, half
of what it is leaving in sales. Sales comprise a large percentage of price changes.
Also, not all price changes, excluding sales, are price decreases. About one third of
nonsale price changes are price decreases. Finally, the slope of the hazard function
for individual prices tends to be downward sloping, which means that firms are
more likely to adjust in consecutive periods than in any other pair of periods.

Looking at scanner data from Dominick’s, a grocery store, in the Chicago area,
Kehoe and Midrigan (2008) find that the price changes in that data, including
sales, differ considerably from the BLS data that Bils and Klenow and Nakamura
and Steinsson use. First, they find that prices change rapidly. In their sample,
about 1/3 of prices change every week. While the size of relative price changes is high but volatile, firms spend most of the year (about 60%) at the mode, or regular, price. If a product deviates from its regular price, it is typically for a sale; 30% of the time, a good’s price is below its regular price. Hence, sales account for most (83%) price changes in their data, and sales are highly transient; a product’s price returns to its regular price the next week with about a 50% chance when the product is currently on sale.

One of the key pieces of the puzzle of creating a menu cost model with seasonality is to fit the high monetary nonneutrality, the high frequency of price changes in the first quarter of the year, and the tendency for seasonal sales to occur in some goods in the fourth quarter of the year (which also produces a slight decline in the CPI as documented by Barsky and Miron (1989)). If seasonality is indeed a motive for firms to adjust their prices, it would have to give firms a strong motive to change prices in the first quarter of the year, and the price changes in the fourth quarter of the year would have to be disproportionately price decreases. Seasonally varying demand shocks could be a culprit, or some other aggregate shock that boosts demand in the second half of the year could cause the fluctuations as well. But why do firms want to lower prices in periods in which demand for their goods is high? There are several competing theories.

Warner and Barsky (1995) evaluate the statistical properties of daily pricing data for eight goods from different stores around Ann Arbor, Michigan between November 1987 and February 1988. Their theory about why prices decrease when demand for the good is high has to do with economies of scale in search. For
consumers, the cost of researching goods and traveling between sites is roughly a fixed cost, so when demand is high, consumers optimally search and travel more. Hence, consumers are more sensitive to prices during periods of high demand; they find and purchase goods at lower prices, which lowers the average price for the good.

Chevalier, Kashyap, and Rossi (2003) provide evidence that firms lower prices on certain goods in periods of peak seasonal demand to attract customers into their stores. They provide support for the loss leader model of price setting and advertising, formalized by Lal and Matutes (1994), in which the firms’ optimal pricing decision is to lower prices on goods that are in the most demand. Consumers do not know all of the prices at a store before they travel there, but they do know the prices of goods that the firms’ advertise. The firms then compete on the basis of a subset of advertised goods, in particular, goods for which there is a high demand. While firms do not earn high margins on the advertised goods in high demand, firms make up for the losses by charging relatively high prices on unadvertised goods. Consumers acquiesce to the high prices because of the inconvenience and uncertainty of traveling to a different store. The loss leader theory does not rely on aggregate demand fluctuations to generate sales; rather, idiosyncratic demand fluctuations for goods are sufficient to generate sales in that good. Chevalier, Kashyap, and Rossi document that prices for certain goods that face idiosyncratic seasonal demand shocks, such as tuna during Lent, decrease during the periods of higher demand. Prices decrease for other goods during periods of high aggregate demand as well, such as beer, cheese, soup, and crackers.
at Christmas, but the loss leader theory produces seasonal fluctuations regardless of their source.

Nevo and Hatzitaskos (2006) provide an alternative theory of seasonal price fluctuations that can account for sales during holidays and other periods of high demand. During peak periods of seasonal demand, regardless of whether it is an idiosyncratic or aggregate fluctuation, more consumers enter the market for certain goods. These consumers may differ from the consumers who typically purchase the good, and they may be more sensitive to the price of a good. For example, during Lent, the additional consumers who buy tuna may not care as much about the quality of tuna purchased as tuna connoisseurs who purchase tuna throughout the year. Thus, the new entrants would tend to purchase cheaper tuna than the regular tuna purchasers. Alternatively, during holidays, people would tend to enjoy beer at parties or other events, boosting the demand for beer. As Nevo and Hatzitaskos put it (p. 2-3), “after a few beers, it is hard to distinguish between different brands,” so consumers of beer may decide to buy cheaper varieties of beer. As the frugal consumers enter the market, the average price of beer sold in the market may decline regardless of any sales in the market. Even if there are sales, they may not account for as much of the decline in average prices as the product substitution in markets. Nevo and Hatzitaskos reestimate the results of Chevalier, Kashyap, and Rossi using fixed weights price indices for the goods, which a change in the market share of brands will not affect. They find that the seasonal fluctuations in prices resulting from sales rather than substitutions are much less than implied by Chevalier, Kashyap, and Rossi. Also, for tuna
specifically, demand for higher quality white tuna does not increase much during Lent; the additional demand for tuna is predominantly for the cheaper, light tuna, and the prices of the two varieties of tuna that gain the most market share do not decrease. Both of these facts are inconsistent with the loss leader model and are consistent with the idea that product substitutions account for a significant portion of fluctuations in prices.

The loss leader theory and the substitution theory can account for seasonality in prices from both idiosyncratic and aggregate demand sources, but the search theory and the countercyclical markup theory can only account for seasonality in prices from fluctuations in aggregate demand. Hence, an important issue is the primary source of seasonality in prices. If idiosyncratic sources of seasonality only affect a small subset of goods, then the debate in the theory is not necessarily particularly important for the menu cost literature. Aggregate demand fluctuations would sufficiently account for the seasonality motive in price setting for firms. Bryan and Cecchetti (1995) examine seasonality in CPI components from 1982 to 1993 and find that seasonality from idiosyncratic sources dominates seasonality from a common aggregate source. They perform a linear decomposition of individual price movements into an average seasonally adjusted price movement, an average aggregate seasonal price movement, an idiosyncratic seasonal price movement, and measurement error. The ratio of the idiosyncratic seasonal variance to the aggregate seasonal variance across the goods provides some insight about the relative importance of the two sources of fluctuations. They find that only two out of the thirty-six goods in their sample, auto repair and food away
from home, have a variance ratio of less than one, which implies that few goods have small idiosyncratic contributions to overall seasonal price fluctuations. Furthermore, several goods feature extremely large variance contributions from the idiosyncratic seasonal component. The variance ratio for motor fuel, fruits, gas and electricity, fuel oil, and women’s apparel all exceed 100. Their analysis ignores the fact that the elasticity of demand and supply for goods is not necessarily the same, however, which could understate the importance of aggregate price changes on goods with a high elasticity of demand or inelasticity of supply. Another fact that they point out is that the source of seasonal fluctuations in the CPI goods does not necessarily come from a single source. Regressing the prices of goods on deterministic seasonal trends, they find that the months in which seasonality in price setting appears varies between the goods and may identify the source of the price fluctuations. For example, cereal and fruit prices tend to decline in autumn when supply is abundant, but they tend to increase in January when fresh supply is scarce. Public transportation, natural gas, and electricity prices have on average large January price increases, which could be an effect of regulation. Fluctuations from demand may not be the only reason for fluctuations in prices.
2.3 Model

At time $t$, firms, indexed by $i$ on $[0, 1]$, hire labor from households to produce goods in a monopolistically competitive market. The production function is constant returns to scale in labor:

$$y_t(i) = a_t(i) l_t(i)$$

Firms operate along their demand curves, so consumption of good $i$, $c_t(i)$, is equal to production, $y_t(i)$. Firm $i$ hires labor, $l_t(i)$, and faces a idiosyncratic and persistent productivity shock $a_t(i)$ that takes the following form:

$$\log a_t(i) = \rho_a \log a_{t-1}(i) + \theta_t$$

$$\theta_t \sim iid N \left(0, \sigma^2_a\right)$$

The firms’ current period nominal profit is:

$$\pi_t(i) = p_t(i) c_t(i) - w_t l_t(i)$$

Identical consumers provide labor $l_t$ to the firms at a nominal wage $W_t$ and earn dividends from equal ownership shares of the firms in the economy. The representative consumer seeks to maximize the lifetime discounted sum of utility defined over the aggregated consumption good and labor. The representative
consumer’s problem is:

$$\max E_t \sum_{t=0}^{\infty} \beta^t U(C_t, l_t)$$

subject to

$$\int_0^1 p_t(i)c_t(i)di \leq W_t l_t + \pi_t$$

The aggregate consumption good is $C_t$, the aggregate price level is $P_t$, the nominal wage is $W_t$, and the share of profits that the consumer receives from ownership of the firms is $\pi_t$. The consumer discounts the future at rate $\beta$. Aggregate output is equal to aggregate consumption in the model. Aggregate consumption and the aggregate price level come from the standard constant elasticity of substitution aggregators:

$$P_t = \left[\int_0^1 p_t(i)^{1-\epsilon}di\right]^{1/\epsilon}$$

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\epsilon-1}{\epsilon}}di\right]^{\frac{\epsilon}{\epsilon-1}}$$

The consumer discounts the future at a rate of $\beta$. Utility in the model is:

$$U(C_t, l_t) = \frac{C_t^{\gamma_s}}{\gamma_s} - \chi l_t$$

The labor-leisure condition from the consumer’s problem is:

$$w_t = \chi C_t^{1-\gamma_s}$$
$w_t$ is the real wage rate, defined as the nominal wage divided by the aggregate price level. Consumer demand is determined by the cost minimization motive of the consumer across all consumption bundles:

$$\min \int_0^1 p_t(i)c_t(i)di$$

subject to

$$U^* \leq \frac{C_t^{fs}}{\gamma_s} - \chi l_t$$

Consumer demand is then simply:

$$\frac{c_t(i)}{C_t} = \left( \frac{p_t(i)}{P_t} \right)^{-\varepsilon}$$

Note that the form for consumer demand is independent of the curvature of the utility function for the simple power/logarithmic forms of utility, so the curvature of the utility function only affects the representative consumer’s labor-leisure condition.

I introduce money, $M_t$, by setting it equal to nominal demand.

$$M_t = P_tC_t$$

Effectively, money in the model is the sum of labor income and profits of the firm as suggested by the binding budget constraint of the consumers. Money grows exogenously in the model following the process:

$$M_t = \mu M_{t-1}$$
The firms’ intertemporal problem is to maximize its discounted stream of profits. The Bellman equations that define the firms’ problem is:

\[
V_a(\eta(i), s) = \max_{\tilde{p}^*(i)} \left( \pi(\tilde{p}^*(i), \eta(i), s) + \hat{\beta} \int_0^1 V \left( \frac{\tilde{p}^*(i)}{\mu}, \eta(i'), s' \right) dF(\eta) \right)
\]

\[
V_n(\tilde{p}_{-1}(i), \eta(i), s) = \pi(\tilde{p}_{-1}(i), \eta(i), s) + \hat{\beta} \int_0^1 V \left( \frac{\tilde{p}_{-1}(i)}{\mu}, \eta(i'), s' \right) dF(\eta)
\]

\[
V(\tilde{p}_{-1}(i), \eta(i), s) = \max \left( V_a(\eta(i), s) - \tilde{w} \xi, V_n(\tilde{p}_{-1}(i), \eta(i), s) \right)
\]

Variables with a tilde have been normalized by the current period money supply. Firms pay a fixed menu cost in terms of labor, \( \tilde{w} \xi \), in the model, so firms must decide whether to change their price or to keep their current nominal price. The value of adjusting their price, \( V_a \), depends on a firm’s current productivity and the current season \( s \). If a firm adjusts, it chooses the current optimal normalized price, \( \tilde{p}^*(i) \), that maximizes the sum of current and discounted future profits. The value of keeping its current price, \( V_n \), depends on the profitability of the firm at its current price, conditional on the season and the firm’s current productivity. The firm decides to adjust its price if the value of adjustment exceeds the value of nonadjustment by more than the menu cost, which forms the basis for the overall value function for the firm, \( V \).

The model presented here is similar to other menu cost models in the literature, most notably Burstein and Hellwig (2007). Relative to Burstein and Hellwig, I omit a demand shock, omit the decreasing returns to scale production function, include seasonal fluctuations (described momentarily), and explicitly derive the representative consumer’s problem. Recall from the representative consumer’s labor-leisure condition that the real wage is related to aggregate consumption, the
disutility of labor, and the curvature of the utility function. Substituting out $C_t$ from the labor-leisure equation using the definition of money:

\[
\begin{align*}
    w_t &= \chi \left( \frac{M_t}{P_t} \right)^{1-\gamma_s} \\
    W_t &= \chi M_t^{1-\gamma_s} P_t^{\gamma_s}
\end{align*}
\]

Let $\chi = 1$, and thus:

\[
W_t = M_t^{1-\gamma_s} P_t^{\gamma_s} \tag{2.2}
\]

The model, with the restriction that $0 \leq \gamma_s < 1$, implies that the nominal wage is the geometric average of the money stock and the aggregate price level, which is a fundamental assumption in Burstein and Hellwig (2007). They assume this form for the law of motion for the nominal wage noting that it can be derived a result of increased curvature in the utility function relative to standard logarithmic preferences or the result of including intermediate inputs into the model.

The size of the labor market friction, and thus the real rigidity, in the model depends largely on the choice of $\gamma_s$. When $\gamma_s$ is low, the nominal wage will respond more quickly to a monetary shock because of the greater weight placed on the money supply in (2.2). When $\gamma_s$ is high, a greater weight is placed on the aggregate price level, which adjusts more slowly to a monetary shock. The aggregate price level adjusts slowly because of the fluctuating real rigidity as well.
Consider the first order condition of the firm’s problem without the adjustment cost:

\[
\log (\tilde{p}^*(i)) = \log \left( \frac{\epsilon}{\epsilon - 1} \right) + \gamma_s \log (\bar{P}) - \log (a(i))
\]

(2.3)

Note that as \(\gamma_s\) increases, the firm is more pressed to keep its price close to the prices of other firms (the aggregate price level). Firms will be hesitant to make large price adjustments if the aggregate price level does not respond strongly to the shock. Furthermore, the size of price adjustment should be lower on average since the firm will also want to keep its relative price low.

I implement seasonality in the model by allowing \(\gamma_s\) to vary depending on the season. Since \(\gamma_s\) affects the degree of real rigidity in the model, the source of the seasonal fluctuations is similar to the staggered labor market contracts in Olivei and Tenreyro. Seasonal fluctuations in preferences are also one potential source of seasonal fluctuations, as explored by Beaulieu and Miron (1990).

### 2.3.1 Planner Equilibrium

First, I seek the steady state equilibrium of the economy, which simplifies to finding the steady state of the equivalent planner economy of the model. The planner economy is the economy in which there is no aggregate uncertainty from monetary shocks, which holds so long as the money supply growth rate is exogenous and known. A more detailed summary of my solution method is in Appendix A and Appendix B, but a summary of the equilibrium is below.
The planner equilibrium of the nonseasonal economy follows the standard definition. It consists of a value function $V$, firm price setting decisions, wage $w$, aggregate price $P$ and output $C$, and an invariant distribution $\Gamma^*$ such that:

1. $V$ and the firms’ price setting decisions solve the firms’ problem;
2. $w$ clears the labor and goods markets;
3. $\Gamma^*$ is invariant to the firms’ price setting decisions across states; and
4. aggregate price $P$ and consumption $C$, aggregated from $\Gamma^*$, are consistent with (2.1).

The cyclostationary equilibrium of the economy is the planner equilibrium that is consistent across all seasons. There is interdependence between the firms’ problems across seasons: the current season’s pricing decision rules affect the initial distribution of firms next season, and the next season’s value function enters into the firm’s problem in the current season. More formally, the cyclostationary equilibrium consists of a sequence of value functions $V_s$, firm price setting decisions across seasons, a sequence of wages $w_s$, a sequence of aggregate prices $P_s$ and output $C_s$, and an invariant distribution $\Gamma_s^*$ such that:

1. $V_s$ and the firms’ price setting decisions solve the firms’ problem for each season;
2. $w_s$ clears the labor and goods markets for each season;
3. $\Gamma_s^*$ is invariant to the firms’ price setting decisions across states and seasons; and
4. aggregate prices $P_s$ and consumption $C_s$, aggregated from $\Gamma_s^*$, are consistent with (2.1).
I solve the cyclostationary equilibrium by guessing a sequence of wages $w_s$, solving the implied value functions, decision rules, and invariant distributions in each season. I then update the sequence of wage guesses using the implied wages from the previous guess and iterate until the sequence of wages converges.

Because of the menu costs and the seasonal fluctuations, there is no analytical solution to the problem, so I must solve the model numerically. I discretize the productivity shock process using the Tauchen (1986) method on a grid of 20 points. In addition, I discretize prices in the model. I solve for the value function $V$ on the grid of productivity and price points using value function iteration and cubic splines. I choose 20 points on the price grid when I solve the value function and 100 when I solve for the distribution of prices.

### 2.3.2 Parameterization

I calibrate some parameters in the model to micro price moments. The model parameters that I do not calibrate are largely from Burstein and Hellwig (2007). Since the models are similar, the parameters that best fit Burstein and Hellwig’s model without the demand shock should provide a good first approximation of the parameters necessary for my model to fit the data. The baseline parameters are in Table 2.1.

In a baseline case without seasonality, I calibrate $\rho_a$, $\gamma_s$, $\xi$, and $\sigma_a$ by minimizing, using a Nelder-Mead algorithm, the sum of weighted squared deviations of the nonseasonal model’s moments away from moments obtained from the Dominick’s scanner price data set by Midrigan (2009). I choose to match the mean
price change (0.1%), the mean absolute price change (12%), the fraction of positive price changes (65%), and the frequency of price changes (24%). In the cyclostationary equilibrium, I keep the calibration from the model without seasonality and introduce seasonal fluctuations by varying $\gamma_s$ to fit the real seasonal fluctuations in GDP in the US economy.

The baseline calibrated parameters are quite consistent with the existing literature. The key parameters in the model are $\gamma_s$ and $\rho_s$ because they will control the magnitude of the response to seasonal fluctuations. The calibrated value for $\gamma_s$ fits nicely into the existing literature about labor market rigidities. Burstein and Hellwig (2007) estimate a value for a firm specific rigidity and an aggregate (labor market) rigidity. The true real rigidity of the model is a function of both parameters. Their estimates of the true real rigidity and the aggregate real rigidity are between 0.75 and 0.85. Nakamura and Steinsson's (2008) materials share of intermediate inputs is similar to the real rigidity in this model. They chose a materials share of 0.7. Rotemberg and Woodford (1997) estimate a firm specific rigidity and an aggregate rigidity of around 0.8. My calibrated technology shock persistence is slightly lower than what other authors find but is still reasonable. For example, Burstein and Hellwig assume a persistence of 0.5, Midrigan finds a persistence of 0.483 by calibrating his model to similar moments, and Nakamura and Steinsson choose a persistence of 0.7.

In order to fit the model to the size of the real seasonal cycle in the U.S. economy, I first need to determine the size and features of the seasonal cycle. Thus, I must update the empirical evidence about the seasonal cycle obtained in Barsky.
and Miron (1989) to include more recent data. I focus on nonseasonally adjusted real GDP data from the Bureau of Economic Analysis, dated from Q1 1947 to Q4 2004. Following Barsky and Miron, I detrend the natural logarithm of the GDP series using the Hodrick-Prescott (HP) filter with a smoothing parameter of 1600, consistent with quarterly data. The HP filter decomposes the data into a deterministic trend component and a cyclical component. I regress the cyclical component on seasonal dummies as follows:

$$y_t^c = \sum_{s=1}^{4} \beta_s d_s + \epsilon_t$$  

(2.4)

The cyclical component of log GDP is $y_t^c$, $d_s$ are the seasonal dummies, $\beta_s$ are the regression coefficients, and $\epsilon_t$ are the regression errors. The regression coefficients can be interpreted as being percentage deviations from trend real GDP. The regression results are in Table 2.2.

Table 2.2 shows that the real size of the seasonal cycle is approximately 7% from peak to trough. The cycle starts at its lowest point, approximately 3.5% below trend GDP in the first quarter of the year. GDP rises to approximately trend GDP in the second quarter of the year, and rises slightly above trend in the third quarter of the year. GDP surges to approximately 3.5% above trend in the fourth quarter of the year, implying that GDP falls 7% between the fourth quarter and the first quarter of the following year on average, relative to trend. The qualitative features of the seasonal fluctuation are consistent with the results of Barsky and Miron, though the estimated size of the seasonal cycle was higher (about 8%) in their study.

The smaller estimate of the seasonal cycle implies that the size of the seasonal
cycle may have shrank in recent years, consistent with the idea that the volatility of U.S. output has decreased since the early 1980s\(^2\). To test the hypothesis that the size of the seasonal cycle has shrunk in recent years, I break the data into two parts: the early sample between 1947 and 1983, and a later sample between 1984 and 2004. I then estimate the regression equation (3.7) for each sample. The results of the regressions are in Table 2.3 and Table 2.4.

The results from the early sample imply that the seasonal cycle was larger, about 7.7% from peak to trough, while the later sample has a smaller cycle—closer to 5.7% peak to trough. Otherwise, the qualitative features of the trend are similar in both samples. The results of a Chow test yielded a F-test statistic of 2.997, which rejects the hypothesis that the coefficients of the regressions are the same at the 5% level. Thus, the seasonal cycle has likely shrunk since the last regression.

I choose to test the menu cost model’s ability to match the larger seasonal fluctuation in GDP in the 1947-1983 subsample in order to assess the upper bound of the effects of seasonality on the real economy. A high value for the real rigidity parameter leads to higher output and lower prices. Thus, an empirical consistent seasonal fluctuation would feature high \(\gamma_s\) in the beginning of the year, with \(\gamma_s\) slowly falling for the rest of the year. \(\gamma_s\) should also feature a large jump at the end of the year to account for the large differences in output between Q1 and Q4. The baseline \(\gamma_s\) that I choose are given in Table 2.5.

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\(^2\)Stock and Watson (2002) suggest that the reduction may be purely exogenous and coincidental, while Kahn, McConnell, and Perez-Quiros (2001) suggest that improving technology may be driving the reduction.
2.4 Results

Firms’ price adjustment behavior depends on their current price relative to the price of other firms, the menu cost, and their current productivity. Figure 2.1 shows the price adjustment behavior of firms for a month in which there is no seasonal cycle. Firms that are more productive produce more and set lower prices. Optimally, firms would like to set their price at the solid line in Figure 2.1. Since firms have to pay the menu cost in order to change their prices, firms are willing to tolerate having a price that differs from the optimal price so long as the losses from not setting their price at the optimal price do not exceed the menu cost. Losses from having a suboptimal price are increasing away from the optimal price, so for every productivity draw, there is a unique price below which firms will always increase their price and a unique price above which firms will always decrease their price. The dashed line on Figure 2.1 indicates the highest nonadjustment price for different productivity draws, and the dotted lines indicate the same for the lowest nonadjustment price. If a firm has a price and productivity combination that is between the dashed and dotted lines on Figure 2.1, then they will not adjust their price. Otherwise, they will adjust. Since prices on the graph are expressed in terms of the optimal normalized price ($\tilde{p}$), as time passes, the growth of the money supply will slowly erode firms’ prices. One could picture firms’ prices falling slowly over time on the graph if the firms’ productivity remains constant. Firms thus increase their prices more often than they decrease their prices. Firms decrease their prices if they face a positive productivity shock in the steady state.
with a high enough price, and also potentially if seasons change once seasonal fluctuations are introduced.

The seasonal fluctuations in the model cause firms to strategically cluster their prices close to each other through the seasonal cycle—the cyclostationary equilibrium is not simply a collection of the individual steady states strung together. Figure 2.2 shows the aggregated output values from the individual steady states that make up the baseline cyclostationary equilibrium and from the baseline cyclostationary equilibrium. While consumption fluctuates about 10% trough to peak in the cyclostationary equilibrium (7.8% if one takes the quarterly average output), the fluctuation between the individual steady states is larger at about 14%. In addition, firms anticipate and adjust to future changes in the seasonal cycle. For the first three months of the seasonal cycle, \( \gamma_s \) remains the same, yet prices gradually rise ahead of the expected higher prices in the fourth and fifth months of the baseline cycle. Firms anticipate and adjust to future increases in prices in the cycle as well as evidenced by the decrease in consumption (and thus increase in prices) in the eleventh and twelfth months of the cycle.

The clustering motive of firms in the seasonal cycle shows most strongly in the firms’ optimal price. Figure 2.3 shows the optimal normalized price that the firms set in each month of a simplified seasonal cycle\(^3\) if they choose to adjust, and

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\(^3\)I shorten the seasonal cycle to be 4 months instead of a year for exhibitional purposes. I keep the parameters from the initial calibration except for the seasonally varying real rigidity, which I will vary to assess the role of seasonality in the propagation of monetary shocks. The shortened cyclostationary equilibrium has the same qualitative price and consumption fluctuation over the cycle if one thinks of the months in the shortened cycle as quarters. Prices are relatively high and output is relatively low in the first month of the cycle. Prices and output are near the annual average during the second and third months of the cycle, and prices are relatively low and...
Figure 2.4 shows the optimal normalized price that firms set in the nonseasonal model. The prices in the seasonal model do not differ much for firms with relatively low productivity draws, while the prices in the nonseasonal model differ much more. For the last three months of the seasonal cycle, the optimal prices of firms with relatively low productivity draws in the seasonal model are practically the same. The optimal prices in the first month, when the aggregate normalized price level is at its peak in the seasonal cycle, are slightly higher than the prices in the other three months. Relatively productive and profitable firms respond more strongly to current and expected future seasonal fluctuations, however.

Seasonal fluctuations occur in aggregate prices because of differences in which firms adjust their prices in a given season. Figures 2.5 and 2.6 show the highest nonadjustment prices and lowest nonadjustment prices for firms through the simplified seasonal cycle. The highest nonadjustment prices in Figure 2.5 show that firms would not likely decrease their prices during the first month of the seasonal cycle, when the nonadjustment prices are higher than in the other three months of the cycle, particularly for relatively unproductive firms. This behavior makes sense since aggregate prices are highest in the first month of the cycle. On the other hand, firms are more likely to decrease their prices in the second month of the cycle because of the slightly higher optimal prices in the first month of the cycle.

For this section, I set month 1: \( \gamma_1 = 1.04 \times 0.7482 \); month 2: \( \gamma_2 = 0.7482 \); month 3: \( \gamma_3 = 0.7482 \); month 4: \( \gamma_4 = 0.96 \times 0.7482 \). This choice of parameters leads to a 2.0% fluctuation in output over the course of the seasonal cycle.
cycle and the lower threshold for price decreases. Also, inflation will encourage aggregate prices to fall if few firms wish to increase their prices during a particular month.

The lowest nonadjustment prices in Figure 2.6 show that firms are more interested in increasing their prices during the first month of the cycle than during the other three months. A higher lowest nonadjustment price indicates that fewer firms are willing to tolerate having relatively low prices during that period, so the upward movement from the fourth month nonadjustment line to the first month nonadjustment line represents firms’ desire to increase their prices when they know that the aggregate price level is relatively high. In this simplified model, firms are twice as likely to change their prices in the first month of the cycle, and over 90% of price changes are price increases in that month, compared to an average of 60% price increases in the other three months in the cycle.

The persistence of the firms’ idiosyncratic shock process affects the firms’ desire to respond to the seasonal fluctuations. Figure 2.2 also shows the path of output through the cyclostationary equilibrium for various values of the persistence of the productivity shock. Note that aggregate consumption does not vary as much in the seasonal cycle when the idiosyncratic shock is less persistent. Firms are more willing to adjust their prices away from the aggregate price level if their individual circumstances become relatively more important as suggested by the firms’ optimal pricing equation in the absence of menu costs (2.3). In the extreme case when the persistence is 0, firms always expect that their productivity draw will return to the median level, so only a large seasonal fluctuation in \( \gamma_s \) will cause
enough firms to change their prices to generate a substantial seasonal cycle. Mechanically, firms also will experience more extreme productivity draws when the productivity persistence is higher, which increases their potential response to the seasonal fluctuations, particularly for productive firms. The price adjustment behavior of relatively productive firms in Figure 2.6 in particular suggests that the differences in firms’ behavior over the seasonal cycle can become substantial for more extreme productivity draws, which leads to the substantial differences in price changing behavior that define the baseline cyclostationary equilibrium.

2.5 Monetary Shock

In this section, I evaluate the response of the model economy to a monetary shock and determine that seasonal fluctuations potentially amplify the real response of the economy to a shock. The shock that I consider is an unexpected shock to the money supply growth rate. Since the shock shifts the distribution of firms on normalized prices and causes no aggregate uncertainty as long as I assume that the future path of wages is known, I use a perfect foresight transition method to compute the effects of the shocks. Further details of the solution method are in Appendix C. To dissect the effects of the seasonal cycle, I focus on the simplified seasonal model with 4 months.

The degree to which the monetary shock affects output depends on the real rigidity in the nonseasonal model. A larger amount of real rigidity, which is higher value of $\gamma_s$ in the baseline model, will lead to a larger output response. Figure 2.7 shows the output response of a 0.5% monetary shock over the course
of a year following the shock for the baseline model and for values of $\gamma_s$ three percent higher or lower than the baseline $\gamma_s$. As the previous analysis predicts, the stronger clustering motive for firms dampens their price response. Initially in the baseline model, consumption rises 0.225% above steady state levels and falls slowly, eventually dropping below the steady state value 9 months after the shock. Compared to VAR evidence about the effects of a monetary shock, the output response is too small, too fast, and not persistent enough. Compared to the simple menu cost model of Golosov and Lucas (2007), however, the output response of the model is much larger. When $\gamma_s$ is smaller (0.7258), the initial output response following the shocks is smaller—about 0.2%, and consumption again falls more slowly after the shock. When $\gamma_s$ is larger (0.7706), the initial output response is about 0.25%, and the response is more persistent than the baseline case.

Ignoring the potential dynamic influence of the seasonal cycle, the $\gamma_s$ that define the cyclostationary equilibrium should allow an output response that is roughly consistent with the quarterly VAR evidence of Olivei and Tenreyro (2005). During the first quarter of the year, the real rigidity is high, which should lead to a larger output response than in the fourth quarter of the year when the real rigidity is low. The response should be more moderate in the second and third quarters of the year.

In practice, however, the changing aggregate price level through the seasonal cycle causes firms to react to the shock in a way that overwhelms the effect from the changing real rigidity. Firms know that they are more likely to increase their
prices at the beginning of the cycle, so when a shock occurs in the months just preceding a price increase, firms will avoid raising their prices until they were going to raise their prices anyway. Their strategy saves them from adjusting their prices twice. A stronger output response then follows in the months preceding the increase in prices in the seasonal cycle. Figure 2.8 shows the output response of the simplified seasonal model when the shortened real seasonal cycle is about 0.8%, which corresponds to the same parameter fluctuation as the baseline model. The lowest initial output response to the shock (0.18%) occurs when the shock occurs just before the first month of the cycle, when the real rigidity is relatively high. The response remains low for the months following the shock as well. The highest initial output response (0.38%) occurs during the fourth month of the cycle when the real rigidity is at its lowest, but the response dies off quickly. Several spikes in the response occur as the response in subsequent periods in the fourth month of the cycle, just before prices rise. When the shock occurs just before the third month in the cycle, the largest response occurs in the next month after the shock. When the shock occurs just before the second month in the cycle, the largest response to the shock occurs two months after the shock. In each case, the largest output response occurs in the fourth month of the seasonal cycle.

The larger output response to a monetary shock in the months preceding the upswing in prices in the seasonal cycle will also hold true when the monetary shock is negative. When the monetary shock is negative, firms have an incentive

\[ \begin{align*}
    \text{Month 1: } & \quad \gamma_1 = 1.02 \times 0.7482; \\
    \text{month 2: } & \quad \gamma_2 = 0.7482; \\
    \text{month 3: } & \quad \gamma_3 = 0.7482; \\
    \text{month 4: } & \quad \gamma_1 = 0.98 \times 0.7482.
\end{align*} \]
to reduce their prices, ignoring the seasonal fluctuation and inflation. Figure 2.9 shows the output response to a negative 0.5% monetary shock when the $\gamma$s are set to produce a 0.8% real seasonal cycle. As before, the largest output response occurs when the shock hits just before the third and fourth months of the seasonal cycle, and the smallest response occurs when the shock hits just before the first month of the cycle. A positive monetary shock is not the driving force behind the larger output responses in the second half of the cycle in the simplified model.

Why do firms delay their response to the shock late in the year regardless of whether the monetary shock is positive or negative? The reason is that inflation is positive, and firms are unlikely to need to correct the upward adjustment in future periods. Inflation, driven by the exogenous money growth, decreases firms’ relative price over time. Firms that have high relative prices can decrease their prices and pay the menu cost or do nothing and let inflation reduce their prices to make them more competitive. If firms have relatively low prices, their situation will never self-correct. With deflation, the model is able to generate a larger output response in the first quarter relative to the fourth quarter of the year. Figure 2.10 shows the effects of a 0.5% negative monetary shock when prices are falling at a 2.5% annual rate, and the $\gamma$s are set to produce a 2% real seasonal cycle. The initial response of output response is larger in absolute value when the shock occurs late in the year.

---

5 Month 1: $\gamma_1 = 1.02 * 0.7482$; month 2: $\gamma_2 = 0.7482$; month 3: $\gamma_3 = 0.7482$; month 4: $\gamma_1 = 0.98 * 0.7482$.

6 Month 1: $\gamma_1 = 1.04 * 0.7482$; month 2: $\gamma_2 = 0.7482$; month 3: $\gamma_3 = 0.7482$; month 4: $\gamma_1 = 0.96 * 0.7482$. 
just before the first month of the cycle (0.435%) than when the shock occurs just before the other months in the cycle. While the output response is not at its lowest in the fourth quarter of the year, the asymmetric nature of the price decreases in the seasonal cycle allows firms to decrease their prices. Also, in the fourth quarter of the year, firms would not necessarily respond strongly knowing that a temporary upswing in the seasonal cycle will occur in the following month. Firms will delay decreasing their prices following the shock until they would decrease them anyway after the first month of the cycle, which amplifies the effect of the shock on output.

Firms’ optimal pricing behavior following the shock reflects the differences in the initial partial adjustment to the shocks. Figure 2.11 compares the optimal prices for the first month in the cyclostationary equilibrium with the optimal prices for the economy where a 0.5% positive monetary shock occurs just before the first month in the seasonal cycle. The normalized price targets are slightly lower when the firms react to the monetary shock. Since $P_tC_t = M_t$, and $\tilde{P}_t = C_t$, nominal prices and output increase following the positive monetary shock, but an incomplete price response to the shock would lead to a decrease in the normalized aggregate price level and a decrease in the target normalized price that firms seek to set if they adjust. A decrease in the normalized aggregate price level is equivalent to an increase in output. The shock has larger effects on firms’ price-setting behavior if the shock occurs just before the fourth month in the seasonal cycle. Figure 2.12 compares the reaction of firms facing the first month and fourth month shocks, in terms of the percentage deviation of the firms’ optimal price from what
it would be if the shock had not occurred. Recall that the shock before the fourth month leads to a larger output response since many firms adjust to the shock in the following period when they increase their prices. For reference, if the deviation of the optimal price after the shock was 0%, then firms are fully adjusting for the shock in their prices, which would lead to no output response if prices were flexible. If the deviation of the optimal price was -0.5%, then firms are not expecting the aggregate normalized price level to change in response to the shock, and output will respond fully (0.5%) to the shock, so long as firms’ do not change when they choose to change their prices. The -0.05% price deviation for relatively unproductive firms for both shocks indicates that they are setting their prices as if the shock has little real effects. By contrast, a significant difference opens up between the price setting of the firms that are relatively productive. Firms reacting to the shock just before the fourth month set their prices much lower after the shock than they otherwise would in the absence of the shock. Comparing the firms’ decisions to increase or decrease their prices leads to similar conclusions.

Increasing the amplitude of the seasonal fluctuation also increases the mean output response, so long as the seasonal fluctuation is moderate\textsuperscript{7}. Figure 2.13 compares the mean output response to a 0.5% monetary shock for different amplitudes of seasonal fluctuations and to the nonseasonal shock equivalent. As the

\textsuperscript{7}In an extreme case, consider a seasonal fluctuation where all firms change their prices every period. When the monetary shock hits the economy, all firms will still change their prices, and the shock does not affect output. Essentially, the economy behaves as if prices are flexible. A 20% fluctuation in parameters in the simplified cycle (10% plus and minus the baseline $\gamma_s$) is sufficient to generate this phenomenon.
amplitude of the seasonal fluctuation increases, the initial mean response of output increases. The mean initial response of the nonseasonal model is 0.228%, compared to 0.258% for the seasonal model with a 0.8% real fluctuation and 0.277% for the seasonal model with a 2.0% real fluctuation. Measuring the cumulative response of output over the first twelve months after the shock leads to similar conclusions about the effects of seasonality. The cumulative response of output for the nonseasonal model using this measure is 0.583%, compared to 0.737% for the seasonal model with a 0.8% real fluctuation and 0.761% for the seasonal model with a 2.0% real fluctuation.

Three promising mechanisms exist that could potentially fix the models’ ability to fit the seasonal VAR evidence of Olivei and Tenreyro. First, an imperfect information mechanism, as in Gorodnichenko (2009), could delay firms’ ability to recognize that a shock has occurred, causing them to delay their response to the shock by several months. Firms reacting to an expansionary monetary shock that occurs at the end of the year recognize the shocks in the early part of the following year, which would give them an incentive to adjust for the shock when they would increase their prices early in the following year. The output response of the shock could be less than the response to a shock that occurs at the beginning of the year. Firms recognize that the shock occurs in the middle of the year, which gives them an incentive to wait to change their prices until they would otherwise increase their prices at the beginning of the following year. The longer delay to respond to the shock with a price adjustment would lead to a higher output response overall.
Second, allowing firms to index their prices to the current inflation rate would remove much of the firms’ incentive to delay their price increase until they would otherwise adjust their prices. No price increase would necessarily self-correct as time passes. This adjustment would increase the relative importance of the seasonally varying real rigidity, which supports a better fit to the seasonal VAR evidence.

Third, allowing some time-dependence of price change opportunities, as in Nakamura and Steinsson (2009), would allow firms better opportunities to change their prices at empirically consistent and convenient times. Nakamura and Steinsson’s Calvo-Plus model randomly provides firms with either a low or high menu cost. Firms have an incentive to change their prices during months when their menu costs are low. In other months, when menu costs are extremely high, firms have a strong incentive not to change their prices. If firms have a higher probability of receiving a low menu cost in later months, then they may choose to react to the shock earlier, producing more consistent empirical output responses to shocks.

2.6 Conclusion

I construct a menu cost model with a seasonally fluctuating labor market rigidity that matches the seasonal fluctuations in output in the US economy and produces seasonal fluctuations. The model can readily explain a 7.5\% gap between the peak and trough of the seasonal cycle in quarterly GDP with relatively small changes in the labor market rigidity.
I find that seasonal fluctuations in a menu cost model increase the model’s output response to a monetary policy shock. As long as inflation is positive, firms have a strategic incentive to respond to shocks when they are more likely to increase their prices. This incentive causes prices to respond more sluggishly to a monetary policy shock in the months before prices increase in the seasonal cycle. While the output responses of shocks that occur early in the year have a slightly smaller effect on output than the nonseasonal model, the average output effect of shocks across different months is still up to 20% larger initially in the nonseasonal model.

The key determinants of the effects of seasonality in the cyclostationary equilibrium are the size of the labor market rigidity and the persistence of the technology shock. As the size of the labor market (real) rigidity increases in the model, the profit penalty for firms that deviate far from the aggregate price level increases. Since seasonality moves the aggregate price level, firms increasingly cluster their optimal prices to avoid lost profit from fluctuations. The aggregate price level is endogenously determined by the firms’ behavior, so limitations in the firms’ response to fluctuations limit the fluctuation of nominal aggregate variables. The persistence of the technology shock demonstrates the tradeoff that the firms face between adjusting for the idiosyncratic technology shock and the seasonal fluctuations in the economy. As the persistence of the technology shock increases, any extreme idiosyncratic shock that the firm receives is less transient and has more of an effect on firm behavior. In response, aggregate variables and firms’ prices respond more to the seasonal fluctuation. Simply, the idiosyncratic component
becomes more important to the firms, which offsets the seasonal clustering motive.

While the model produces seasonal output responses that are inconsistent with the empirical evidence of Olivei and Tenreyro, the dynamic effect of the seasonal fluctuation could be brought into line with a couple of adjustments to the simple menu cost model. First, imperfect information may stall firms’ ability to respond to the shock. Firms immediately know that the monetary shock has occurred in the model, whereas firms may need time to determine if a shock has occurred and how large it is in reality. Firms may still be more likely to increase their prices during the beginning of a year, but it may take them until the following year to begin to respond fully. Firms that experience shocks later in the year may be able to identify and respond to them in time for the price increase at the beginning of the year. Second, the time dependence of price changes in Nakamura and Steinsson (2009) is a particularly appealing feature, particularly if it forces firms to consider increasing their prices more evenly through the seasonal cycle. Third, indexing firm prices to inflation or the money supply growth rate would eliminate much of the effect that inflation has on firms’ price-setting behavior, which would allow a greater role for the fluctuating real rigidity.

The methodology developed in this paper has other potential applications. For example, Alessandria, Kaboski, and Midrigan (2009) show that transportation costs and delivery lags produce a pattern of lumpy trade where firms import large quantities of goods and hold substantial inventories. If demand for certain goods is seasonal, then the lumpiness could be amplified by seasonal fluctuations,
because firms would optimally stock up on a good in preparation when people would purchase it. Kaneda and Mehrez (1998) argue that seasonal fluctuations are nontrivial when modeling international trade and that there are large seasonal fluctuations in the trade of some disaggregated goods and aggregate measures of trade. A general equilibrium model of international trade with heterogeneous goods with seasonally fluctuating demand for an importing country would be able to explore the connection between lumpiness and seasonality.
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*: calibrated
Table 2.2: Seasonal Fluctuations in US GDP Data (1947-2006)

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Table 2.3: Seasonal Fluctuations in US GDP Data (1947-1983)

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Table 2.4: Seasonal Fluctuations in US GDP Data (1984-2006)

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Figure 2.1: Firm Adjustment Behavior in the Steady State
Figure 2.2: Cyclostationary Equilibrium Consumption
Figure 2.3: Optimal Price Setting for Adjusters in the Cyclostationary Equilibrium

Optimal Price Setting for Adjusters, Simplified Seasonal Cycle

- **Month 1**
- **Month 2**
- **Month 3**
- **Month 4**

Normalized Price Level vs. Productivity Shock Value
Figure 2.4: Optimal Normalized Prices, Nonseasonal Models
Figure 2.5: Seasonal Price Decrease Decision Behavior

Behavior of Highest Nonadjustment Price, Simplified Seasonal Cycle

- Month 1
- Month 2
- Month 3
- Month 4

Normalized Price vs. Productivity
Figure 2.6: Seasonal Price Increase Decision Behavior

Behavior of Lowest Nonadjustment Price, Simplified Seasonal Cycle

- Solid line: Month 1
- Dashed line: Month 2
- Dotted line: Month 3
- Dashed-dotted line: Month 4

Normalized Price vs. Productivity
Figure 2.7: Nonseasonal Response to Monetary Shock

Response to 0.5% Monetary Shock, Nonseasonal Model, Varying Real Rigidity

- Dotted line: Low Rigidity ($\gamma=0.7258$)
- Black line: Baseline ($\gamma=0.7482$)
- Dashed line: High Rigidity ($\gamma=0.7706$)

% Deviation in Output Compared to Steady State vs. Time after Shock (Months)
Figure 2.8: Seasonal Response to Monetary Shock

Response to 0.5% Monetary Shock, Model with Seasonality, 0.8% Real Fluctuation

- **Mean Response**
- **Shock Occurs Before 1st Month**
- **Shock Occurs Before 2nd Month**
- **Shock Occurs Before 3rd Month**
- **Shock Occurs Before 4th Month**

% Deviation in Output Compared to Steady State

Time after Shock (months)
Figure 2.9: Seasonal Response to Negative Monetary Shock

Response to 0.5% Negative Monetary Shock,
Model with Seasonality, 0.9% Real Fluctuation

% Deviation in Output Relative to SS

Time after Shock (months)
Figure 2.10: Seasonal Response to Negative Monetary Shock under Deflation

Response to 0.5% Negative Monetary Shock
2.5% Annual Deflation, 2% Real Fluctuation

% Deviation in Output Relative to SS

Time after Shock (months)
Figure 2.11: Optimal Price Response to Monetary Shock

Comparison of Optimal Adjustment Price for Firms, 1st Month of Cycle

- **Cyclostationary Equilibrium**
- **Initial Shock Response**

**Normalized Price**

**Productivity**
Figure 2.12: Difference in Optimal Normalized Prices Compared to Cyclostationary Trend, Initial Month of Monetary Shock
Figure 2.13: Comparison of Mean Seasonal and Nonseasonal Responses to a Monetary Shock

Comparison of Mean Output Responses to 0.5% Monetary Shock, Different Sized Fluctuations

Time after Shock (Months)

% Deviation in Output, Compared to Steady State

Legend:
- Nonseasonal
- 0.6% Real Fluctuation
- 0.8% Real Fluctuation
- 2.0% Real Fluctuation
Appendix A: Computational Details for Planner Equilibrium

Figure A1 outlines the computational algorithm that I employ to solve for the steady state of the model without seasonality. The objective of the algorithm is to solve for (2.2).

The computational algorithm is similar to the method used in Khan and Thomas (2003). First, I must guess an initial distribution of firms across prices, an initial value function, and initial bounds on the possible equilibrium wage. Also, I must discretize the productivity shock and the normalized prices. The algorithm solves for the steady state wage using a bisection algorithm starting from the initial wage bounds. The wage bisection algorithm provides a guess for the steady state wage in the economy. Using the model equations, all other endogenous variables, except for the firm’s optimal price, can be expressed in terms of this wage guess, so for each state, I compute the firms optimal price using a golden section search algorithm on (2.2). To approximate the expected value function, I employ a cubic interpolation spline. Once I know the optimal price for each state, I know the value function implied by the wage and those prices. However, this value function is not necessarily the same as the one used for the spline approximate, so I perform value function iteration, repeatedly solving for optimal prices across states given the current approximation of the value function and for the implied value function until the value function converges. Then, I have the value function and optimal pricing decision for firms across all states. I must aggregate the economy by finding the stationary equilibrium of firms across the states, however, to
find the wage implied by the value function and decision rules. To find this distribution, I employ a technique similar to solving for the steady state of a Markov chain, which is described in more detail in Appendix B. With the distribution of firms across states in hand, I aggregate the firms’ individual prices to compute the aggregate price level using the CES aggregators. The aggregate normalized price level and aggregate consumption are inversely related as implied by (2.1), and the consumer’s labor-leisure condition then provides the wage implied by firms’ optimization. The implied wage informs of which bound in the bisection algorithm should be changed, and the algorithm continues with new guesses for the steady state wage until convergence. Once the wage converges, I have found the steady state.

Adding seasonality to the problem means that I will need to find the cyclostationary equilibrium and the cyclostationary distribution of firms across prices. To find the cyclostationary equilibrium of the model, I use an algorithm similar to the one above, except I solve for the cyclostationary normalized wages across seasons in a cycle until the aggregate variables in the model converge to some tolerable level across iterations. The solution to the above algorithm for each value of $\gamma_s$ is a good initial guess for initial values needed for the algorithm. Instead of using the current season’s value function as the approximate for the next period’s value function (as in the steady state), I use the next period’s value function in the cycle. When solving for the stationary distribution of firms across prices in the current period, I use the stationary distribution of firms across states from the other periods to pin down the distribution of firms across states after a complete
seasonal cycle (see Appendix B). The information necessary to solve the current period problem from the other periods’ problems is made available to the algorithm in the current cycle. Otherwise, the logic of the algorithm, essentially to find the market clearing wages, remains the same.
Figure A.1: Computational Process for Steady State

1. Wage bisection
2. Find optimal price
3. Value function iteration
4. Stationary firm distribution
5. Aggregate economy

Until convergence
Appendix B: Computational Method for Cyclostationary Distribution

This appendix provides the computational algorithm for the stationary or cyclostationary distribution of firms across states. The spirit of the solution method follows Tauchen (1986) in that I need to find the transition matrix between states in the model across periods. The following method finds the stationary distribution across firms of non-iid states $\omega_t$ in the case without seasonality:

Step 1: Sort the states into the iid shocks and the non-iid state variables (including persistent shocks). The states in $\omega_t$ are only the non-iid (discretized) states, and the overall distribution of firms across states is the joint distribution of the distribution of firms across the non-iid states and the independent distribution of firms across the iid shocks. If there are multiple non-iid state variables, the variables in $\omega_t$ are the (vectorized) Cartesian power of the states.

Step 2: Initialize a mass 1 of firms at every state in $\omega_t$ and then find the implied mass of firms across all states, including the iid shocks. The mass of firms at any state will then be equal to the mass of firms at the state implied by the joint distribution of the iid shocks. If there are no iid shocks, then the mass of firms at all states is simply 1.

Step 3: Apply the decision rules for the firms across all states once and track the mass of firms from each non-iid state in the present period to each non-iid state after the effect of inflation in the next period. Tracking these firms gives the elements of the transition matrix $A$ in the relationship $\omega_{t+1} = A\omega_t$. The element $A_{i,j}$ is the mass of firms that began at time $t$ at state $j$ and ended up at the beginning of time $t+1$ at state $i$. 132
Step 4: Create an initial distribution of firms across the states \( \omega^0 \).

Step 5: Note that \( \omega_{t+1} = \omega_t \) in equilibrium, and iterate \( \omega^{i+1} = A \omega^i \) until convergence.

The rationale for this approach follows from the solution method of the menu cost model and the problem of finding the stationary distribution of a Markov chain. For computational purposes in the menu cost model, one must discretize the states. It would be impossible to pin down the firms’ optimal price choices numerically given an infinite number of states. Hence, the distributions of the firms across the states must be discretized as well. The stationary distribution of continuous random variables is approximated by the stationary distribution of discrete random variables. If the states have the Markov property—that given the present state, future and past states are independent—then the problem of finding the stationary distribution simplifies to the problem of finding the transition matrix between states and then solving for the steady state of the Markov chain.

The first step to find is to find the transition matrix between the firms’ state now and their state in the next period. In a textbook Markov chain problem, the transition matrix is known, but in a computational setting, one must find the transition matrix implied by the firms’ decision rules at different states.

To solve for the cyclostationary distribution in the case with seasonality, one would need the transition matrices \( A_1, \ldots, A_s \) where \( s \) is the number of seasons in a year and where \( \omega_{t+q} = A_q \omega_{t+q-1} \) for \( 1 \leq q \leq s \). The cyclostationary distribution satisfies the properties \( \omega_{t+s} = A^* \omega_t \) and \( \omega_{t+s} = \omega_t \) for all \( t \). Substituting in the relationship \( \omega_{t+q} = A_q \omega_{t+q-1} \) simplifies the problem of pinning down \( A^* \):
Thus, the transition matrix $A^*$ for a given period $t$ is a function of the period by period transition matrices:

$$A^* = A_s A_{s-1} \ldots A_1$$

Solving for the cyclostationary distribution then uses the same methods as solving for the simple stationary distribution case given the transition matrix (from step 4 on).

Step 1: Sort the states into the iid shocks and the non-iid state variables (including persistent shocks). The states in $\omega_t$ are only the non-iid (discretized) states, and the overall distribution of firms across states is the joint distribution of the distribution of firms across the non-iid states and the independent distribution of firms across the iid shocks. If there are multiple non-iid state variables, the variables in $\omega_t$ are the (vectorized) Cartesian product of states.

Step 2: Initialize a mass 1 of firms at every state in $\omega_t$ and then find the implied mass of firms across all states, including the iid shocks. The mass of firms at any state will then be equal to the mass of firms at the state implied by the joint distribution of the iid shocks. If there are no iid shocks, then the mass of firms at all states is simply 1.
Step 3: Apply the decision rules for the firms across all states once and track the mass of firms from each non-iid state in the present period to each non-iid state after the effect of inflation in the next period. Tracking these firms gives the elements of the transition matrix $A_1$ in the relationship $\omega_{t+1} = A_1 \omega_t$. The element $A_1(i,j)$ is the mass of firms that began at time $t$ at state $j$ and ended up at the beginning of time $t+1$ at state $i$.

Step 4: Repeat steps 1-3 to find $A_2, ..., A_s$. Then compute $A^* = A_s A_{s-1} ... A_1$.

Step 5: Create an initial distribution of firms across the states $\omega^0$.

Step 6: Note that $\omega_{t+s} = \omega_t$ in equilibrium, and iterate $\omega^{i+1} = A^* \omega^i$ until convergence.
Appendix C: Computational Method for Monetary Shock

In this exercise, I compute the response of my menu cost model to an unexpected, one time increase to the money growth rate $\mu$. I denote the increase $\epsilon$. I use the rational expectations transition path method developed in Buera and Shin (2008) to solve for the effect of the shock.

A simple way to conceptualize the timing of the model and shock is to consider a simple decision and production process that takes place over the course of a day. The first action of the day is taken by the monetary authority, which increases the money supply at some growth rate $\mu$. Then, firm managers wake up in the morning and set their prices based on available information, including their initial state (current normalized price, productivity) and information about the future (the money supply growth rate in the case without any sort of aggregate uncertainty). Production takes place in the afternoon, and purchasing and consumption take place in the evening given the firms’ pricing rules set earlier in the day.

After the unexpected, one time positive shock to the money growth rate, the firms’ managers wake up the morning after the shock and realize that there has been $\epsilon\%$ more inflation than expected, so the entire distribution of firms across normalized prices has been shifted down relative to the steady state distribution. Since the one time shock has no effect on the future path of the money growth rate, the shock does not introduce uncertainty about aggregate conditions. Assuming perfect foresight about the transition path of $\mu$. However, not all firms adjust their
prices in the period after the shock, so the effect of the shock on other aggregate variables is persistent and nontrivial.

I must begin with initial conditions and starting parameters, including the steady state solution to the menu cost economy and the total number of periods over which I allow the economy to adjust to the shock, which I denote $T$. Here, I choose $T = 120$ for a quarterly specification. I assume that I have the steady state solution to the menu cost economy for the algorithm below.

There are four key objects that I need to compute every period when computing the effect of the shock: the value function, $V(p_t, a_t)$, the transition matrix for firms between states in period $t$ and $t + 1$, $A_t$, the distribution of firms across states, $\pi_t$, and the wage that clears the labor market, $w_t$. Optimization to find the market clearing wage, the transition matrix, and the distribution of firms across states in a given period relies on the one period ahead value function. Likewise, the value function depends on the current wage. Since the steady state value function is known, one can solve for the sequence of value functions over time backwards given a sequence of guesses for the market clearing wage. Given the value functions, one can solve forwards for the market clearing wages, update the wage guesses accordingly, and then repeat until convergence. The detailed solution algorithm is as follows:

1. Determine the transition matrix $A_0$ that would result from the steady state pricing decisions if inflation is $\mu + \varepsilon$ instead of $\mu$, the steady state rate of inflation for the steady state equilibrium. The initial distribution of firms across states $\pi_1$
is then simply $\pi_1 = A_0 \pi$ where $\pi$ is the steady state distribution of firms across states.

(2) Guess an initial sequence of market clearing wages $w^0_t, t = 1, ..., T$.

(3) Using backward induction, compute the value functions $V(p_t, a_t)$ for $t = 1, ..., T - 1$ while holding the wage fixed each period at the guess $w^i_t$. Note that the time $T$ value function is simply the steady state value function.

(4) Using the value functions computed in step 3 as guesses for the future value functions, solve forward for the implied wages $w^*_t$ for $t = 1, ..., T - 1$. Note that the initial distribution of firms across states depends on the results from the previous period as the initial distribution of firms across states at time $t$ is $\pi_t = A_{t-1} \pi_{t-1}$.

(5) Update the wage guesses using $w^{i+1}_t = (1 - \lambda) w^i_t + \lambda w^*_t, \lambda \in (0, 1)$. Here, I choose $\lambda = 0.98$.

(6) Repeat steps 3-5 until convergence of the wage guesses.

Note that one difference between the Buera and Shin method and this method is that the distribution of firms across states is computed using the discretized distribution of firms across states instead of simulations.

A different type of shock that would be easy to implement in this framework is a one period ahead expected shock where firms learn that there will be a one time shock to inflation one period in advance and are allowed to adjust their prices accordingly. However, firms that adjust ahead of the shock would price much of the
shock into their price changes, and without the distributional effects of the shock, the aggregate price level may actually rise ahead of the shock, which would cause a (temporary) negative response to consumption. In addition, an expected persistent shock to the money supply growth rate would be easy to implement since the deterministic path of the growth rate does not introduce aggregate uncertainty.

An unexpected, stochastic shock introduces aggregate uncertainty, so the Buera and Shin method would fail, and I would need to use the Krusell and Smith (1998) algorithm instead.
BIBLIOGRAPHY


