Two Essays on Business Cycle Models

Dissertation

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Abstract

Canonical business cycle theories are incapable of explaining some recent patterns in the data. My research explores generalizations of them towards a resolution of these empirical anomalies.

The first chapter, “An Empirical Evaluation of Sticky Information in a Two-Sector Model with Durable Goods,” examines the empirical plausibility of the model developed by Kitamura and Takamura (2008), which presents a solution to the negative comovement problem discussed by Barsky, House and Kimball (BHK, 2007). Using the theoretical model presented by Kitamura and Takamura (2008), I compare impulse response functions in the model with those obtained from a structural VAR. Model-implied impulse response functions for sectoral output and inflation fit data well, for both durable and nondurable goods, over the period 1980Q1-2007Q4. Moreover, the minimum distance estimate of the degree of sticky information is 0.70, which is lower than the value used by Mankiw and Reis (2002) in their one-sector economy. This suggests that sticky information and firm-specific factors offer both a simple method to correct for the BHK puzzle and a useful strategy to generate plausible impulse responses in a two-sector economy.

In the second chapter, “A General Equilibrium Model with Banks and Default on Loans,” I develop a model to explain aggregate business cycle patterns in the U.S. after 1990. A particular interest of this chapter is to analyze the interaction between
banks and the real sector. During the recent financial crisis, banks reduced new business lending amidst concerns about borrowers’ ability to repay. At the same time, firms facing higher borrowing costs alongside a worsening economic outlook reduced investment. My model explicitly introduces banks into a business cycle model. I assume that a banks’ ability to raise deposits is constrained by a limited commitment problem and that, furthermore, loans to firms involve default risk. In this environment, changes in loan rates affect the size of the business sector. I explore how banks influence the behavior of households and firms and find that both productivity and financial shocks lead to counter-cyclical default and interest rate spreads.

I examine the implications of a government capital injection designed to mitigate the effect of negative productivity and financial shocks in the spirit of the Troubled Asset Relief Program (TARP). I find that the stabilizing effect of such policy interventions hinges on the source of the shock. In particular, a capital injection is less effective against aggregate productivity shocks because easing banks’ lending stance only weakly stimulates firms’ demand for loans when aggregate productivity falls. In contrast, a capital injection can counteract the adverse effect of financial shocks on the supply of loans.

Finally, I measure aggregate productivity and financial shocks to evaluate the role of each in the business cycle. I find that the contribution of aggregate productivity shocks in aggregate output and investment is large until mid-2008. Financial shocks explains 65% of the fall in investment and 55% of the fall in output in the first quarter of 2009.
This document is dedicated to my wife, Misa.
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Chapter 1: An Empirical Evaluation of Sticky Information in a Two-Sector Model with Durable Goods

1.1 Introduction

Despite the fact that approximately 15% of real GDP in the U.S. are durable goods and residential investment, which are known to be interest-rate-sensitive components, recent business cycle literature has not paid a fair amount of attention to the role of durable goods in the macroeconomic dynamics. In an insightful paper, Barsky, House and Kimball (2007) investigate the propagation mechanism of a monetary shock in a two-sector dynamic stochastic general equilibrium (DSGE) model with time-dependent sticky prices à la Calvo and long-lived durable goods. Surprisingly, if durable goods are flexibly-priced and nondurable-goods have sticky prices, the output of durable goods contracts sharply following monetary easing. While the micro evidence of price stickiness in durable goods is scarce compared to non-durable goods, the BHK model implies that, for example, the production of houses whose prices are generally considered to be flexible through negotiation should decrease when aggregate demand expands. This is clearly at odds with the macroeconomic evidence: Structural VAR impulse response functions in Figure 1.1, which I created following the methods used by Erceg and Levin (2005, 2006), reveals that residential investment, durable goods and other components of GDP respond to a monetary-policy shock in
the same direction.\footnote{See the footnote of Figure 1.1 for details. Our sample size (1959Q1-2007Q4) is larger than that used by Erceg and Levine (2005, 2006). In addition, for our purpose, we present the impulse response functions for inflation that are not available in their paper.} Moreover, the peak response of residential investment (durable goods) is on average about 10 (4) times larger than that of other GDP components. In the BHK model, the negative comovement problem is prevalent even if durable goods have sticky prices. Figure 1.2 indicates cases where sectoral output exhibits a “positive comovement” after a monetary-policy shock in the BHK model.\footnote{See the footnote of Figure 1.2 for the exact definition of “positive comovement” that is employed to create the figure. Based on the observation identified in Figure 1.1, we regard that the model generates “positive comovement” if the peak impulse response of durable production relative to that of nondurable production is between 1 and 20. Shapes of impulse response functions, which I will discuss below, are not taken into account.} For a given level of price stickiness in nondurable goods, a positive comovement occurs only when the price stickiness of durable goods falls into the colored region. Suppose, for example, that the price stickiness parameter for nondurable-goods sector is 0.5, as suggested by Bils and Klenow (2004). Then, positive comovement requires the price stickiness parameter for the durable-goods sector to be between 0.4525 and 0.4625. Since there is no guarantee that the price stickiness of durable goods falls into this region, this implies that the model requires additional frictions to generate reasonable output impulse responses to a monetary-policy shock.

Kitamura and Takamura (2008) present a simple method to resolve the negative comovement puzzle in a two-sector model with durable goods. We introduce sticky information discussed by Mankiw and Reis (2002, 2006) to both types of goods in a model with flexibly-priced durable goods and sticky-price nondurable goods.\footnote{Based on the case study of Zbaracki et al. (2004), Mankiw and Reis (2002) argue that sticky information may be interpreted as capturing firms’ “managerial and customer costs which include the costs of information-gathering, decision-making, negotiation, and communication.” Recent studies pursuing the micro foundation of sticky-information models include Reis (2006a,2006b) and Gorodnichenko (2008).} We
focus on this particular combination of price stickiness because the negative comovement problem is especially prominent when the degree of price stickiness for durable goods approaches zero. Moreover, BHK (2007) point out that residential prices can be very flexible. The main findings of Kitamura and Takamura (2008) are summarized as follows. First, a sufficiently high degree of sticky information ensures positive comovement of sectoral output even when the factors of production are perfectly mobile. A delayed response of price setters to an expansionary monetary-policy shock under sticky information limits the excessive change in the relative price of durable goods, and induces consumers to demand both types of goods. Second, when the factors of production are firm-specific, even a low degree of informational friction results in the positive comovement of both goods. The firm-specificity of factors significantly weakens the cost pressure that transmits from one sector to the other and increases the degree of strategic complementarity in firms’ price-setting decisions within the same sector as explained by Woodford (2003). As opposed to the view presented in BHK (2003, 2007) that the mobility of factors will not qualitatively change the negative comovement results, Kitamura and Takamura’s (2008) result suggests that it makes a significant difference when other types of frictions coexist in the model.

Admittedly, this is not the only method to correct for the negative comovement problem. Following the ideas discussed by BHK (2003, 2007), some papers have already presented other frictions by which a two-sector model exhibits a positive comovement of durable and nondurable production. For example, Monacelli (2009) introduces a borrowing constraint on households, Carlstrom and Fuerst (2006) add sticky nominal wages to sticky prices and Sudo (2008) considers the input-output
structure of economy. In addition, Tsai (2009) shows a resolution based on the assumptions that firms must borrow to pay their wage bills, consumption has habit persistence and labor cannot move across sectors. In contrast to the papers mentioned above, however, the dual stickiness model (sticky information and sticky prices) presented by Kitamura and Takamura (2008) has an advantage that it can generate hump-shaped impulse responses, as in Figure 1.1, when production factors are firm specific.

In this chapter, I quantitatively examine to what extent sectoral output and inflation responses generated by the dual stickiness model of Kitamura and Takamura (2008) are comparable to those obtained from data, based on the recognition that our model generates realistic patterns in these variables when production factors are firm specific. Specifically, I compute model impulse response functions based on standard parameter values, an estimated value of sticky information using the minimum-distance estimation method, and an estimated standard deviation of monetary policy shock from SVAR. Then, I examine whether the model responses fall into their 95% confidence intervals constructed from data. From this exercise, I find that the impulse response functions of nondurable output, nondurable inflation and residential price inflation generated by the model fit reasonably well into the 95% confidence intervals. The model impulse response of residential investment fits into the 95% confidence interval during 1980Q1-2007Q4. However, the model peak response in residential investment is small and early relative to the mean of its empirical counterpart. I also find that the minimum distance estimate of the degree of sticky information during 1980Q1-2007Q4 is 0.70. This value is lower than that used by Mankiw and Reis (2002) in their one-sector economy. These results suggest that sticky information
with firm-specific factors is a simple method to correct for the BHK puzzle and a useful strategy to generate plausible impulse responses in a two-sector economy.

This chapter proceeds as follows. Section 1.2 explains the dual stickiness model with firm-specific factors developed by Kitamura and Takamura (2008) and elaborate on the synergy between sticky information and the firm-specificity of production factors. Section 1.3 conducts an empirical evaluation of the model presented in Section 1.2, and Section 1.4 concludes.

1.2 Two-Sector Dual Stickiness Models with Firm-Specific Factors

In this section, I introduce the benchmark dual stickiness model with firm-specific factors that will be examined against data in the next section. Kitamura and Takamura (2008) show that an introduction of sticky information to the BHK model is sufficient to generates a positive comovement of output in two sectors. I present the model with firm-specific factors for the following reasons. First, the negative comovement problem is eliminated with small sticky information. Second, the model with firm-specific factors generates hump-shaped impulse responses in the output and inflation of two sectors as in Figure 1.1.

1.2.1 Households

The model with firm-specific factors is essentially equivalent to the yeoman farmer model of Ball and Romer (1990), where each household operates a firm using its own work force. Alternatively, if markets are complete, households’ problems reduce to that of a representative household inhabited by heterogeneous workers who individually decide hours worked for a particular intermediate goods producer. In this case,
the lifetime utility of the representative household is
\[
E_t \left[ \sum_{i=0}^{\infty} \beta^i \left\{ u(C_{t+i}, D_{t+i}) - \int_0^1 v(N_{c,t+i}(z)) \, dz - \int_0^1 v(N_{x,t+i}(z)) \, dz \right\} \right],
\]
where \( C_t \) is nondurable consumption, \( D_t \) is the stock of durable goods, and \( N_{j,t}(z) \) is type-\((j,z)\) labor supplied in period \( t \) \((j = c, x, z \in [0,1])\). The representative household maximizes the lifetime utility by choosing \( C_t, D_t, X_t, S_t \) and \( N_{j,t}(z) \) subject to the budget constraint and the law of motion of durable goods:

\[
P_{c,t} C_t + P_{x,t} X_t + M_t \leq \int_0^1 W_{c,t}(z) N_{c,t}(z) \, dz + \int_0^1 W_{x,t}(z) N_{x,t}(z) \, dz + \Pi_t + T_t + (1 + i_{t-1}) S_{t-1} - S_t + M_{t-1},
\]

\[
D_t = X_t + D_{t-1} (1 - \delta), \quad (1.1)
\]

where \( W_{j,t}(z) \) \((j = c, x)\) is nominal wages for type-\((j,z)\) labor, \( P_{c,t} \) and \( P_{x,t} \) are nominal prices of nondurable and durable goods, \( X_t \) is investment on durable goods, \( M_t \) is nominal money balance carried over to period \( t \), \( \Pi_t \) is dividends from firms, \( T_t \) is lump-sum nominal transfers from the government, and \( i_t \) is the nominal rate of interest. Denoting \( MU_t^C \equiv \partial u(C_t, D_t) / \partial C_t \) and \( MU_t^D \equiv \partial u(C_t, D_t) / \partial D_t \), first order conditions for the household’s problem are

\[
v'(N_{j,t}(z)) = \frac{W_{j,t}(z)}{P_{c,t}} MU_t^C, \quad (1.2)
\]

\[
MU_t^C \frac{P_{x,t}}{P_{c,t}} = MU_t^D + \beta (1 - \delta) E_t \left\{ MU_{t+1}^C \frac{P_{x,t+1}}{P_{c,t+1}} \right\}, \quad (1.3)
\]

\[
MU_t^C = \beta (1 + i_t) E_t MU_{t+1}^C \frac{P_{c,t+1}}{P_{c,t}}. \quad (1.4)
\]

The first equation, (1.2), is a contemporaneous relationship that equates the marginal disutility of labor and the marginal utility of consuming \( W_{j,t}(z) / P_{ct} \) units of nondurable goods by working an additional hour. The second equation, (1.3), states that
consumers equate the opportunity cost of purchasing a unit of durable goods and the marginal utility that comes from an additional consumption of durable goods plus the expected capital gain from holding the additional stock of durable goods for one period. Finally, the third equation, (1.4), is a standard consumption Euler equation.

1.2.2 Money Demand and Supply

Following BHK (2007), money demand is given by the quantity equation,

\[ M_t = P_{c,t}C_t + P_{x,t}X_t. \] (1.5)

A central bank injects money into the economy, which follows a money growth rule of the form \( \Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t \), where \( m_t \) is the logarithm of money supply, \( \varepsilon_t \) is an i.i.d. mean-zero monetary-policy shock.

1.2.3 Firms

Final Goods Producers

Final goods producers purchase intermediate goods in a competitive market and amalgamate them into final goods through the Dixit-Stiglitz aggregator:

\[ X_t = \left[ \int_0^1 X_t(z)^{\varepsilon+1} \, dz \right]^{\frac{\varepsilon}{\varepsilon+1}} \quad \text{and} \quad C_t = \left[ \int_0^1 C_t(z)^{\varepsilon+1} \, dz \right]^{\frac{\varepsilon}{\varepsilon+1}}, \]

where \( \varepsilon > 1 \) is the elasticity of demand. From the optimality conditions of final goods firms, prices of final goods are given by

\[ P_{j,t} = \left[ \int_0^1 P_{j,t}(z)^{1-\varepsilon} \, dz \right]^{\frac{1}{1-\varepsilon}}, \quad \text{for} \quad j = c, x, \]

where \( P_{j,t}(z) \) is the price of an intermediate input indexed by \( z \in [0, 1] \) in sector \( j = c, x \). In addition, the demand for each intermediate good is given by

\[ X_t(z) = \left( \frac{P_{x,t}(z)}{P_{x,t}} \right)^{-\varepsilon} X_t \quad \text{and} \quad C_t(z) = \left( \frac{P_{c,t}(z)}{P_{c,t}} \right)^{-\varepsilon} C_t. \] (1.6)
Intermediate Goods Producers

Intermediate goods are produced by monopolistically competitive suppliers in two sectors. Following Dupor, Kitamura and Tsuruga (2010), Kitamura and Takamura (2008) assume that intermediate-goods producers are subject to two types of nominal rigidity: The price stickiness à la Calvo and the information stickiness à la Mankiw and Reis (2002). Importantly, producers in different sectors face different degrees of price stickiness. The Calvo probability is $\theta_x = 0$ for durable goods and $\theta_c \in [0, 1]$ for nondurable goods. This means that durable-goods firms are allowed to reoptimize their prices every period according to a mark-up pricing rule whereas nondurable-goods firms reset their prices, whenever possible, equal to the weighted average of expected future nominal marginal costs. On the other hand, sticky information allows firms to update their information sporadically. This implies that intermediate-goods firms may have to determine their optimal prices based on vintage information when they receive a signal to adjust their prices. In this environment, a firm that is unconstrained by the nominal frictions in this period chooses the optimal price that applies until it reoptimizes again, based on updated information. In addition, the firm determines pricing plans for future states in which the firm is allowed to adjust its price based on the information acquired in the current period. More formally, if a firm $z$ in sector $j$ with $k$-period-old information is able to reoptimize its price, it chooses $P^*_{j,t,k}(z)$ to maximize

$$E_{t-k} \sum_{i=0}^{\infty} (\theta_j \beta)^i MU_{t+i}^c \left[ \frac{P^*_{j,t+k}(z)}{P_{c,t+i}} Y_{j,t+i}(z) - \frac{W_{j,t+i}(z)}{P_{c,t+i}} N_{j,t+i}(z) \right]$$

subject to equation (1.6) and production technology

$$Y_{j,t}(z) = F(N_{j,t}(z), K), \quad (1.7)$$
where $Y_{j,t}(z)$ is the level of firm production and $K$ and $N_{j,t}(z)$ are capital and labor demand. Recall that labor and capital are not substitutable across firms. For simplicity, I assume that every intermediate-goods producer owns a fixed level of capital, $K$. By taking first-order conditions, we obtain the optimal price level as follows:

$$P^*_{j,t,k}(z) = \frac{\sum_{i=0}^{\infty} (\theta_j \beta)^i E_{t-k} \left[ MU_{t+i}^c P_{j,t+i}^c P_{c,t+i}^{-1} Y_{j,t+i} MC_{j,t+i}(z) \right]}{\sum_{i=0}^{\infty} (\theta_j \beta)^i E_{t-k} \left[ MU_{t+i}^c P_{j,t+i}^c P_{c,t+i}^{-1} Y_{j,t+i} \right]}$$

(1.8)

for $j = c, x,$ where $\mu \equiv \varepsilon / (\varepsilon - 1)$ is a markup rate,

$$Y_{j,t} = \begin{cases} C_t & \text{if } j = c \\ X_t & \text{if } j = x \end{cases},$$

and

$$MC_{j,t}(z) = \frac{W_{j,t}(z)}{\frac{dF}{dN}(N_{j,t}(z), K)}.$$  

(1.9)

The marginal cost of production, $MC_{j,t}(z)$, is firm-specific because wages are heterogeneous and the production function shows decreasing returns to scale with respect to labor for a fixed level of capital. In the Appendix to this chapter, I show that this heterogeneity is tractable in a log-linearized system. Moreover, it is shown that all firms in sector $j = c, x$ facing vintage $k$ information choose the same level of optimal price if they are not constrained by price stickiness. That is, $P^*_{j,t,k}(z) = P^*_{j,t,k}$.

1.2.4 Price Indices

The probability of price stickiness and information stickiness are independent, and each firm in sector $j = c, x$ will be able to update its information and price with probability, $1 - \phi$ and $1 - \theta_j$, respectively. This implies that, in every period, only a fraction $(1 - \theta_j) (1 - \phi)$ of firms in sector $j$ can adjust their prices based on the latest information, while a fraction $(1 - \theta_j) \phi$ of firms do so using vintage information. Notice that the fraction of reoptimizing firms with vintage $k$ information in the total
number of reoptimizing firms is \((1 - \phi) \phi^k\). Hence, the indices for optimal prices in period \(t\) can be written as

\[
Q_{j,t} = \left(1 - \phi\right) \sum_{k=0}^{\infty} \phi^k \left(P_{j,t,k}^* \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad \text{for } j = c, x. \tag{1.10}
\]

Moreover, the aggregate price index can be written in the following recursive form:

\[
P_{j,t} = \left[\theta_j P_{j,t-1}^{1-\varepsilon} + (1 - \theta_j) (Q_{j,t})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad \text{for } j = c, x. \tag{1.11}
\]

1.2.5 Positive Comovement and Hump-Shaped Impulse Responses

In the BHK model with common factor markets, the difference in price adjustment speeds leads to an abrupt change in the price of durable goods relative to nondurable goods. For example, suppose that there is an unexpected increase in money supply. While the prices of nondurable-goods gradually approach a long-run level of nominal marginal costs due to infrequent price adjustments, firms in the durable-goods sector immediately raise their prices above the long-run level since they surcharge for a short-run increase in factor prices associated with higher demand for nondurable goods. Changes in the relative price of durable goods, in turn, cause a huge substitution effect through (1.3), which dominates the income effect of monetary easing. Intuitively, the longevity of durable goods ensures that consumers do not suffer from reducing the investment on expensive durable goods until their prices decline.

The frictions in the model developed by Kitamura and Takamura (2008) completely eliminates the negative comovement problem. Figure 1.3 shows that, even if \(\phi\) is significantly smaller than 0.75,\(^4\) the output of goods in two sectors never exhibits

\(^4\)Mankiw and Reis (2002) and Carroll (2003) support \(\phi = 0.75\).
a negative comovement. The firm-specificity of factors is central to this outcome. It generates real rigidity between the durable-goods and nondurable-goods sectors. In other words, nominal marginal costs in the durable-goods sector becomes insensitive to changes in the production of nondurable goods. To see this, it is useful to present the pricing equations, (1.8) and (1.9) for \( j = c, x \), in log-linearized forms:

\[
p_{c,t,0}^* = (1 - \theta_c \beta) \sum_{i=0}^{\infty} (\theta_c \beta)^i E_t m_{c,t+i},
\]

\[
m_{c,t}^f = p_{c,t} + \frac{1 + \omega}{1 + \varepsilon \omega} c_t,
\]

\[
p_{x,t,0}^* = m_{x,t}^f,
\]

\[
m_{x,t}^f = -\frac{1}{1 + \varepsilon \omega} \gamma_t + p_{x,t} + \frac{\omega}{1 + \varepsilon \omega} x_t,
\]

where \( \gamma_t \) denotes the marginal value of durable goods, and \( m_{c,t}^f \) represents the nominal marginal cost of production that a firm in sector \( j = c, x \) would face if there were no nominal frictions, and lower case letters represent the percentage deviation of a variable from its steady-state level. As in BHK (2007), \( \gamma_t \simeq 0 \) in (1.13) since the stock of durable goods is large relative to their flows and their depreciation rate is low. This gives us a significant implication that only the price or the level of final goods in the same sector has a direct impact on nominal marginal costs in each sector. BHK (2007) argue that the negative comovement puzzle persists when labor is the only immobile factor across sectors. Moreover, BHK (2003) point out that even if all factors of production are immobile across sectors, the best we can expect is an acyclical durable-goods production following a monetary-policy shock (dashed lines in Figure

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\footnote{The model is solved after log-linearizing the optimality conditions around the steady state. The derivation of the log-linearized system is given in the Appendix to this chapter. See Table 1.1 for parameter values.}
1.3). Although their argument is correct when price stickiness on nondurable goods is the only nominal friction in the economy, Figure 1.3 reveals that factor mobility changes the negative comovement result significantly when sticky information exists in the economy.

Of course, along with firm-specific factors, sticky information is a necessary ingredient to generate strictly positive output responses in durable-goods for any value of $\phi$. Through (1.10), the presence of inattentive firms in both sectors causes a delayed reaction of firms to a monetary-policy shock. As a result, the relative price of durable goods changes smoothly so as to create positive demand for durable goods after a monetary expansion.

The synergy between sticky information and firm-specific factors also creates hump-shaped patterns in Figure 1.3. As is explained by Woodford (2003) in the context of one-sector sticky price models, firm-specific factors bring about real rigidity among firms within each sector. In short, reoptimizing firms have a strategic incentive to change their prices by less to protect their profit in a monopolistically-competitive environment, observing that some firms in the same sector are constrained from adjusting their prices; in this model, this is represented by the fact that coefficients on $c_t$ and $x_t$ in (1.12) and (1.13) are divided by $1 + \varepsilon \omega$. It is a well-known fact that sticky price models without real rigidity only weakly amplify the effect of nominal shocks. (Chari, Kehoe, and McGrattan 2000) With sticky information, the real rigidity within a sector acts as a catalyst to generate hump-shaped inflation responses. Within a group of firms that are free to adjust their prices, a majority of them use vintage information to determine their optimal prices right after a shock occurs. Given that inattentive firms have not adjusted their prices, reoptimizing firms facing the
latest information mute their reaction to the shock due to strategic complementarity. As time elapses, however, increasingly more firms will realize the shock, which accelerates the pace of price adjustment by reoptimizing firms with updated information. Consequently, the price path in each sector shows convexity in initial periods and concavity later on. This, in turn, leads to hump-shaped inflation responses in Figure 1.3, which is consistent with the empirical findings in Figure 1.1.

1.3 Empirical Evaluation of the Sticky-Information Model

Having shown that our model with firm-specific factors can resolve the negative comovement problem and generate hump-shaped responses in inflation and output of two sectors, I examine the ability of the model to replicate the empirical impulse response functions in these variables. As we mentioned earlier, Figure 1.1 shows the empirical impulse responses of sectoral output and inflation to a monetary-policy shock. It shows that (a) they are hump-shaped, (b) the peak of residential price inflation is earlier and larger than those of other inflation series, (c) residential investment and the production of durable-goods are more sensitive to a monetary-policy shock than other GDP components. Recall that we assumed that durable-goods firms in our theoretical model can price flexibly following the BHK’s (2007) argument that residential investment can be considered as flexibly-priced durable goods. In this section, I compare impulse response functions obtained from the model and those obtained from SVAR applied to the U.S. data. I examine responses in residential investment, residential price inflation, other GDP (GDP excluding durable goods
and residential investment) and its price inflation. We assume that other GDP corresponds to nondurable goods sector in the model.

To compare the empirical and model impulse response functions, I estimate the short-run interest semi-elasticity of money. For SVAR identifies impulse responses to an interest-rate shock while the model counterparts are responses to a shock on money supply. As Christiano et al. (1999, 2005) argue, estimates of the elasticity with respect to an annualized federal funds rate ranges from $-0.1\%$ to $-1\%$ depending on the type of monetary aggregate used for estimation. They estimate the elasticity from an implied contemporaneous relationship between the federal funds rate and monetary aggregate series, which is obtained by running SVAR including money. Following this approach, I included M2 in the SVAR and found that an initial one-standard deviation (64 basis points in an annualized rate) decline in the federal funds rate translates into a $0.2\%$ point increase in the growth rate of M2. This will be the size of the shock that feeds into the theoretical model. In terms of elasticity, the implied short-run interest semi-elasticity of money is approximately $-0.3\%$.

To compute model-implied impulse responses, I use the parameter values in Table 1.1. However, the value of sticky information, $\phi$, is yet to be determined. I estimate the value of $\phi$ using the minimum-distance estimation method following Christiano

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6Empirical evaluation methods are not limited to the one I adopted in this paper. I could have generated artificial data from the model using a random number generator and run SVAR on them using the recursive identification scheme to judge whether the resulting impulse response functions are close to the empirical ones. The merit of this approach is that we can impose the same SVAR restrictions on the actual and model-generated data. I did not use this method because the current version of the model does not have a mechanism to generate a liquidity effect. As is common to many New Keynesian models, an increase in money supply leads to a contemporaneous increase in the nominal interest rate. As a result, impulse response functions generated from simulated data will show a wrong sign.

7The result will be robust even if we use nondurable and services consumption instead of other GDP. However, empirical impulse response functions have wider confidence intervals.
et al. (2005). Namely, we choose the value of $\phi$ that minimizes the weighted sum of squared distance between the model and empirical impulse response functions from period 1 to 24. Period 0 is excluded because the empirical response is restricted to be zero by the recursive identification scheme in SVAR. Using the sample of 1959Q1-2007Q4, the estimated value of $\phi$ is 0.74, which is close to the value supported by Mankiw and Reis (2002) and Carroll (2003). In Figure 1.4, the model impulse response functions were plotted over the empirical counterpart with 95% confidence intervals. The figure shows that the model impulse response functions for other GDP, its price inflation and residential price inflation fit reasonably well into their confidence intervals. In contrast, the peaks response of residential investment in the model is less than a half of that in SVAR, and, moreover, the model-implied residential investment responses are outside the confidence interval in most periods.

To examine whether the timing of decisions in the model affects these results, I assume that certain economic decisions are predetermined one period in advance in the spirit of Christiano et al. (2005). Specifically, I assume that (a) the representative household determines the consumption level of nondurable goods one period in advance, (b) money demand is determined by a cash-in-advance constraint and (c) intermediate-goods producers must make their decisions one period earlier than in the model presented in Section 1.2. As in Figure 1.4, the model-implied responses show no movement in period 0 under these additional assumptions. Since the new timing of decisions adds delays in agents’ decisions, the estimated value of $\phi$ drops to 0.73. In this new setting, the production of other GDP fits better to the 95% confidence interval. In addition, the peak response of residential investment is closer

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8The details of this alternative model is explained in the Appendix to this chapter.
to its empirical counterpart than it is with the original timing assumptions. However, the size of peak response in residential investment is essentially unaffected by the additional timing assumptions.

I conducted the same exercises in a sample of 1980Q1-2007Q4 to separate the periods of different monetary policy regimes. Indeed, one standard deviation of the federal funds rate falls to approximately 36 basis points in an annualized rate and the short-run interest semi-elasticity of money declines to $-0.24\%$. In this sample, the minimum distance estimates of $\phi$ falls to 0.72 in the baseline model with firm-specific factors and 0.70 in the model with the additional timing assumptions. Figure 1.5 shows empirical and model impulse response functions in the sample of 1980Q1-2007Q4. As in the case of 1959Q1-2007Q4, the fit of model-implied responses in other GDP, price inflation in other GDP, and residential price inflation into their 95% confidence intervals is good. In addition, the model-implied response in residential investment fits better into the 95% confidence interval relative to the case in 1959Q1-2007Q4. Especially, the model-implied response is very close to the mean of the empirical counterpart during period 1 to 3. However, its peak response is still smaller and earlier than the empirical average.

The model developed by Kitamura and Takamura (2008) is simple. Nevertheless, it can explain three of the four impulse response functions reasonably well regardless of the sample periods I considered in this chapter. Accounting for residential investment is more challenging than others. In this model, a higher degree of sticky information is required to delay and increase the peak response of residential investment. When the relative price of residences changes more gradually due to stickier information, capital gains from residential investment also changes gradually over time. As a result, the
demand for residences will increase more gradually. In addition, more nominal rigidity
increases the real effect of a nominal expansion. But, of course, increasing the value of
\( \phi \) will worsen the overall fit of other variables. Other researchers also find it difficult to
generate an empirically-plausible impulse response pattern in residential investment.
For example, Carlstrom and Fuerst (2006) introduces sticky wages and an interest-
rate rule in the BHK model, where households make intertemporal decisions without
a cash-holding constraint. They find that residential investment responds too much
to a monetary-policy shock unless an adjustment cost of residential investment exists.
The development of a more elaborate economic model to improve the responses of
residential investment is left for future research.

1.4 Conclusion

In this chapter, I showed impulse responses, following a monetary-policy shock,
generated by the dual stickiness model developed by Kitamura and Takamura (2008),
and examined whether they are comparable in size and patterns to those obtained
from data. I found that model-implied impulse responses are reasonably similar to
their empirical counterparts in residential price inflation, other GDP (GDP compo-
nents excluding residential investment and durable goods), and price inflation in other
GDP. The response of residential investment reponse fits into the 95\% confidence in-

Overall, the results presented in this chapter bolters the conclusion of Kitamura
and Takamura (2008) that sticky information is a useful friction for solving the neg-
ative comovement problem discussed by BHK (2007).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_c$</td>
<td>Price stickiness for nondurables</td>
<td>0.5</td>
</tr>
<tr>
<td>$\theta_x$</td>
<td>Price stickiness for durables</td>
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</tr>
<tr>
<td>$\phi$</td>
<td>Information stickiness</td>
<td>various values</td>
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<td>$\alpha$</td>
<td>Capital share</td>
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<tr>
<td>$\beta$</td>
<td>Discount factor</td>
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</tr>
<tr>
<td>$\eta$</td>
<td>Frisch elasticity of labor supply</td>
<td>1</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Elasticity of demand for intermediate goods</td>
<td>11</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate of durable stock</td>
<td>0.0125</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Persistence of money-growth rule (M2)</td>
<td>0.69 in Figures 1.3 and 1.4</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Elasticity of real marginal cost</td>
<td>$(\alpha + \eta^{-1}) / (1 - \alpha)$</td>
</tr>
</tbody>
</table>

Notes: $\theta_c = 0.5$ is based on Bils and Klenow (2004). The value of $\rho_M$ is estimated from M2 series for the relevant sample periods. The sticky information parameter will be estimated by the minimum-distance method in Figure 1.4 and 1.5. All the other parameter values are identical to BHK (2007).

Table 1.1: Parameters Values
NOTES: The figure represents impulse response functions to an unexpected one-standard-deviation (16-basis-point) decline in the federal funds rate. Data issues are discussed in data appendix. We estimated a 9-variable VAR with 4 lags and constant terms on the U.S. data (1959Q1-2007Q4). The included variables are durable goods, residential investment, other GDP components, the price indices of these variables, a commodity price index, the federal funds rate and M2. We took the logarithm of each variable except for the federal funds rate. We followed the methodology used by Erceg and Levin (2005, 2006): The monetary-policy shock is identified by the block-recursive assumption with the ordering of variables mentioned above; dotted lines in the figure represent 95% confidence intervals generated by Monte Carlo method with 500 replications.

Figure 1.1: SVAR Impulse Responses (1959Q1-2007Q4)
NOTES: The definition of “positive comovement” in this figure is as follows: (a) the output responses of durables and nondurables to an expansionary monetary shock are strictly positive at least for the first three periods and (b) the ratio of the peak response of durable production to the peak of nondurable production within the first 30 periods falls in the range of $[1, 20]$. We searched for the combinations of parameters over the grids of $[0, 0.9] \times [0, 0.9]$ with 0.0025 intervals.

Figure 1.2: Combinations of Sticky-Price Probabilities Generating "Positive Comovement" in the Baseline BHK Model
NOTES: Model impulse responses are expressed as a percentage deviation from the steady state levels following an unexpected one percentage increase in money growth.

Figure 1.3: Impulse Responses in a Model with Firm-Specific Factors
NOTES: The estimated degree of sticky information is $\phi = 0.74$ in the benchmark model and 0.73 in the model with the alternative timing assumptions.

Figure 1.4: Data Versus Model Impulse Response Functions: 1959Q1-2007Q4
NOTES: The estimated degree of sticky information is $\phi = 0.72$ in the benchmark model and 0.70 in the model with the alternative timing assumptions.

Figure 1.5: Data Versus Model Impulse Response Functions: 1980Q1-2007Q4
Chapter 2: A General Equilibrium Model with Banks and Default on Loans

2.1 Introduction

Since the collapse of the U.S. housing market in 2007, growth in the United States slowed significantly alongside worsening bank solvency, damaged investor confidence and a poor outlook for corporate profits. Although it is generally recognized that the crisis itself ended by 2009, the economy is still overshadowed by a weak economic recovery, especially in employment, and shaken by a series of aftershocks such as the downgrading of U.S. bonds and sovereign debt problems in the Euro zone. The adverse effect of this financial crisis was not limited to over-leveraged investment banks; it also affected traditional commercial banks.\footnote{Ivashina and Scharfstein (2010) document that the total amount of U.S. corporate loans issued by large commercial banks fell sharply after mid-2007. Moreover, Koepke and Thomson (2011) explain that credit channels declined sharply in the banking sector in 2008 and 2009, followed by sluggish recovery in 2010.} The resulting contraction in the provision of new loans was associated with increases in interest-rate spreads and default rates.

These disruptions in financial markets, culminating with the failure of Lehman Brothers in the fall of 2008, led policy makers to deploy an unprecedented policy of purchasing private securities and bonds, bailing out financial institutions, viewed as
too-big-to-fail, and recapitalizing large commercial banks through the Troubled Asset Relief Program (TARP). Although these efforts to increase liquidity in the market have seen some success, default rates are still higher than pre-crisis levels. Moreover, many have argued that these policies have had limited effect on credit creation either due to the reluctance of banks to lend or that of firms to borrow.

In an effort to understand how a disruption in new lending can propagate through the economy, I introduce a financial sector into a dynamic stochastic general equilibrium (DSGE) model. More specifically, I build a “triple-decker”\textsuperscript{10} model of households, banks and firms where banks raise deposits from households and provide firms with loans that involve a risk of default. In this economy, I assume two financial frictions. First, a lack of commitment on the part of banks to repay depositors, as in Gertler and Karadi (2011), requires banks hold net worth against deposits. Since bank net worth is scarce, this deposit friction makes the volume of bank loans to firms inefficiently low and drives loan rates above deposit rates. Second, loans to firms involve the risk of default. This introduces a second channel through which loan rates exceed deposit rates. In the presence of these frictions, the accumulation of net worth allows banks to mitigate the capital requirement implied by the deposit friction \textit{and} to buffer against future default risk.

I consider two types of aggregate shocks that affect these financial frictions. First, financial shocks reduce the collateral value of bank net worth. Households become more concerned about banks’ solvency and banks are required to hold more net worth against deposits. These shocks originate in the financial market and directly impact the deposit friction. Second, aggregate productivity shocks change the ability of firms

\textsuperscript{10}This description is due to John Moore.
to repay their debt. When the aggregate productivity falls, default rates rise and bank net worth falls. These shocks allow me to investigate the interaction between the real and the financial sectors and to evaluate the quantitative importance of financial and real shocks in driving business cycles.

Using this framework, I conduct the following analyses. First, I derive responses that follow an aggregate productivity shock and a financial shock, which endogenously changes banks’ capital requirements, to examine whether they are consistent with empirical regularities. In particular, interest rate spreads and default rates are counter-cyclical in the data while aggregate output, investment, consumption, hours worked and new business loans are procyclical. Second, I analyze the effectiveness of a capital injection in mitigating the adverse effects of real and financial shocks. As I mentioned above, the U.S. government reacted to the financial crisis by injecting capital into large commercial banks through TARP. The objective of this exercise is to provide a quantitative assessment of whether such capital injections are effective against each type of shock. Third, I estimate aggregate productivity and financial shocks from the U.S. data using a Bayesian estimation method.

The main findings of this chapter are as follows. First, my model is consistent with the following empirical regularities: Interest rate spreads and default rates are counter-cyclical while aggregate output, investment, consumption, hours worked and new business loans are procyclical. While this holds for both aggregate productivity and financial shocks, there are important differences in how the economy responds to each. The effects of a financial shock are short-lived and interest rate spreads rise through a tightening of the deposit friction. In contrast, the effects of an aggregate
productivity shock are long-lasting and interest rate spreads rise even though the deposit friction is initially relaxed.

The presence of default is central to understanding the long-lasting effects of an aggregate productivity shock. When aggregate productivity falls, net worth of banks falls because of an unexpected increase in loan default. Bank net worth continues to fall for a while in subsequent periods as the effect of falling loan demand keeps the loan rate from rising sufficiently high to cover losses from default. Later, as the demand for loans recovers, banks are constrained in their ability to provide new loans by their net worth. On the other hand, the mechanism behind a financial shock is simpler. First, interest rate spreads rise as depositors become more concerned about the safety of their savings. With a higher loan rate, more firms default and less new loans are made. But the effect of the shock is short-lived because banks’ net worth is left intact and the productivity of firms is unaffected. As a result, banks quickly accumulate net worth to ease the higher capital requirements implied by a larger deposit friction.

Second, even though a negative financial shock and a negative aggregate productivity shock both lead to increases in interest rate spreads, capital injections are more effective against financial shocks than against aggregate productivity shocks. After a financial shock, a net worth injection helps reduce banks’ need to accumulate financial capital. In contrast, after an aggregate productivity shock, the benefit of a capital injection is limited because easing bank lending is not sufficient to stimulate firms’ demand for loans when aggregate productivity is falling. These results show that the

\[11\text{Nevertheless, it is important to note that interest rate spreads still increase due to higher default rates.}\]
effectiveness of a capital injection depends on the source of shocks and policy makers must seek more information beyond interest rate spreads to understand financial market conditions in real time.

Third, through an estimation of real and financial shocks, I find that aggregate productivity shocks are an important driving force for aggregate output and investment until mid-2008. However, financial shocks play an important role for explaining the sharp decrease in aggregate output and investment in 2009. In particular, the financial shock contributes to approximately 65% of the drop in investment and 55% of the fall in real GDP in 2009Q1.

This chapter is related to existing studies stressing the importance of financial frictions. Among others, Kiyotaki and Moore (1997) develop a model where changes in collateral values propagate shocks to the economy. The financial accelerator model of Bernanke, Gertler and Gilchrist (1999) introduces standard debt contracts based on the costly-state verification model of Townsend (1979). More recently, Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) develop a model of financial intermediaries holding net worth given deposit frictions, and analyze unconventional monetary policy.\textsuperscript{12} Jermann and Quadrini (2012) construct a model where production of firms are constrained by the availability of working capital borrowed from financial institutions. However, to the best of my knowledge, no existing research has developed a model with households, banks and firms with deposit frictions, bank net worth and banks’ loans to firms involving a risk of default. Moreover, while Gertler and Kiyotaki (2010) provide a qualitative analysis of capital injections to financial intermediaries in parallel to unconventional monetary policies, mine is the first quantitative analysis of financial intermediaries in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) invest in state-contingent claims on firms.

\textsuperscript{12}Financial intermediaries in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) invest in state-contingent claims on firms.
capital injections to commercial banks, that provide risky loans to firms, in a general equilibrium framework.

My model yields results that are distinct from those obtained in the above papers. For example, my model distinguishes itself from Gertler and Karadi (2011) in that the loan rate involves a risk premium. This drives interest rate spreads even when deposit frictions are relaxed following a negative aggregate productivity shock. In contrast, in Gertler and Karadi (2011), interest rate spreads and the tightness of deposit frictions have a one-to-one relationship.

More importantly, the result that the effect of aggregate productivity shocks is propagated through financial frictions is in contrast to existing findings in the literature. Particularly, Jermann and Quadrini (2012) find that financial shocks affecting the incentive constraint that limits loans to a representative firm account for a large fraction of business cycle fluctuations in aggregate output and hours worked whereas aggregate productivity shocks explain little.\textsuperscript{13} Even though both they and I find that aggregate productivity shocks relax the incentive constraints (the deposit friction in my model) on impact, the effect of a decline in productivity is dampened by the financial friction in their model whereas, in my model, it has an additional channel of propagation through loan default. Kocherlacota (2000) also supports the view that financial frictions discussed in Kiyotaki and Moore (1997) only weakly propagate aggregate productivity shocks. Khan and Thomas (2011) who generalize Kiyotaki and Moore (1997) to quantitatively examine the amplification of a large collateral (financial) shock also find the same result. An exception is Carlstrom and Fuerst (1997)

\textsuperscript{13}The choice of a financial shock in my model is motivated by their work.
who show that an aggregate productivity shock is propagated through financial frictions on capital production. However, in their model, the default rate is procyclical and any amplification is through entrepreneur net worth.

Other related studies include Meh and Moran (2010) who develop a model with bank net worth. Their paper embeds the double moral hazard problem developed by Holmstrom and Tirole (1997) in a New Keynesian DSGE framework and analyzes the propagation of a shock to net worth, productivity and monetary policy. My model differs in its underlying frictions and, crucially, in the role of endogenous default in driving results. Lastly, Gomes and Schmidt (2009) obtain counter-cyclical default in the absence of financial intermediaries and focuses on credit spreads for long-term bonds.

This chapter is organized as follows. Section 2.2 constructs the model. Section 2.3 presents calibration and steady state results. Section 2.4 discusses results in system dynamics. Section 2.5 extends the analysis to capital injections. Section 2.6 explains the measurement of shocks and evaluates the relative importance of aggregate productivity and financial shocks. Finally, Section 2.7 concludes.

2.2 Model

2.2.1 Overview of the Model

There are three types of private agents in the economy, households, firms, and banks. Households earn wages from firms, rental income from capital, interest income from bank deposits and dividends from both banks and firms. They purchase goods for consumption from firms and save through bank deposits or by holding physical capital. Firms operate one-period projects in different locations by renting capital,
$k_f$, and labor, $l_f$, to produce output, $y_f$, in competitive factor and output markets. The mass of potential projects has fixed measure, $M$. All projects shut down after production and are replaced by new projects. Firms at each project location decide whether to implement their projects in the next period. Project implementation requires paying a fixed cost, $\kappa$, at each location and banks are assumed to be the only entity that can finance this cost. In addition, new projects must pay a random administrative labor cost that is funded by households. Banks intermediate between households and firms, and they finance loans using their net worth and bank deposits made by households. There are two types of financial frictions in the model. First, banks have limited commitment to repay households’ deposits. This constrains the extent to which banks are leveraged. Second, bank loans have a risk of default and they cannot seize the profit from a project in full when a firm fails to repay its debt.

Firms hold a continuum of ex-ante identical potential project locations. Let $b$ denote the gross debt payment to a bank for a project that is funded. Given the state of the economy, a firm anticipates the profit of a project next period net of debt and weigh it against the administrative labor cost, $w \xi$, where $w$ is the wage rate and $\xi$ is a random variable. Only projects with $\xi$ lower than an endogenously determined threshold level, $\overline{\xi}$, will be implemented. Firm projects are heterogeneous, ex-post, in terms of their productivity levels. Let $\varepsilon$ be an idiosyncratic productivity level and $z$ represent the aggregate productivity level. Given $\varepsilon$ and $z$, firms in each project location produce output with a technology, $y_f = \varepsilon z F (k_f, l_f)$. Because of decreasing returns to scale, projects make profits after wages and capital rental costs are paid. But since debt is predetermined, projects with low idiosyncratic productivity levels
will default on loans. Insolvent projects will have zero value after banks confiscate any gross profits.

Banks start each period with a number of loans made in the previous period, \( \chi_s \), and the volume of deposit, \( s \). After agents receive signals on aggregate and idiosyncratic productivity levels, financial transactions on existing loan contracts are settled and bank net worth, \( n \), is determined. During this process, solvent projects repay \( b \) while banks liquidate insolvent projects and seize a fraction \( \lambda \in (0, 1) \) of their profits, where \( 1 - \lambda \) represents the costs of liquidation. Before banks make new loans, some die (fraction \( 1 - \theta \)) and are replaced by new banks. The start-up funds for these \( \theta \) new banks are provided by households. This assumption ensures that banks do not over-accumulate net worth to self-finance new loans.

Although individual banks collect deposits, \( s' \), from households, banking requires net worth due to a limited commitment to repay depositors. Following Gertler and Karadi (2011), I assume that banks may abscond with their funds, \( s' + n \), if the amount of borrowing is very large relative to their net worth. This implies that banks must possess a sufficiently large stake in their assets so as to convince depositors that banks’ cost of foregoing the value of implementing their business is large. Gertler and Karadi’s (2011) financial friction represents the banking sector’s capital requirement in a convenient way. Given the amount of funds in hand, banks choose the volume of new loans, \( \chi' \). Thereafter, \( b' \) balances the supply and demand for loans.

A unit measure of households derive utility from consumption and leisure and discount future utility by \( \beta \in (0, 1) \). They own firms and banks and have access to a complete set of state-contingent claims. The representative household’s expected
discounted lifetime utility is $\sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t)$, where $C$ and $L$ denote consumption and hours worked, respectively. Given an aggregate state of the economy, it chooses consumption, hours worked and savings through deposit and capital. The representative household’s individual state variables are deposit, $s$, and capital, $k$.

### 2.2.2 Firm Projects

Each project operated involves renting capital and labor in competitive factor markets to produce final goods. Given productivity levels, $\varepsilon$ and $z$, wage rate, $w$, and rental price, $r_k$, a firm project maximizes profit subject to the decreasing returns to scale production function, $y_f = \varepsilon z F(k_f, l_f)$. Here, $\varepsilon$ is assumed to be an i.i.d. random variable and $\log(z)$ follows an AR(1) process. Since every project is one-period lived, firms solve a static optimization problem: $\max_{k_f, l_f} \{y_f - r_k k_f - wl_f\}$. Let $f(\varepsilon; x)$ be the profit function before debt repayment and $x$ be a vector of aggregate state variables. Idiosyncratic shocks cause some firms to default on their debt. More specifically, a project involves default if $f(\varepsilon; x) < b$, where $b$ is the debt service for borrowing the start-up cost, $\kappa$. A threshold level of default, $\underline{\varepsilon}$, is the level of idiosyncratic productivity at which projects break even after repaying their loans:

$$f(\underline{\varepsilon}; x) = b.$$  \hfill (2.1)

After all financial transactions are made, solvent projects pay their net profit to households while insolvent projects surrender $f(\varepsilon; x)$ to banks, leaving no value to shareholders. Since profits are distributed to households only when projects are solvent, the final profit of a project is expressed as $1_{[\varepsilon \geq \underline{\varepsilon}]} (f(\varepsilon; x) - b)$, where $1_{[\varepsilon \geq \underline{\varepsilon}]}$ is an indicator function that takes the value of 1 if $\varepsilon \geq \underline{\varepsilon}$ and 0 otherwise.
The current generation of projects ends with production. Thereafter, firms will have new potential projects of measure $M$ and decide whether or not to produce next period. In doing so, they compare the value of implementing a project with a stochastic administrative cost, $w\xi$. A project will be implemented if the former is greater than or equal to the latter. Notice that the value of implementation involves the debt repayment, $b$, for a startup loan, $\kappa$, and the interest cost of borrowing. Since the value of implementation is the expectation of discounted final profit of a project, a threshold level, $\bar{\xi}$, is defined as

$$w\bar{\xi} = E \left[ \frac{\beta P'}{P'} \int_{\xi > \zeta'} (f(\xi'; x') - b') d\Pi(\xi) \right],$$

(2.2)

where $\Pi(\xi)$ is the probability distribution of $\xi$, $P$ is the household’s marginal utility of consumption, and $E$ is an expectation over aggregate states conditional on $x$.$^{14}$

The right-hand-side of this equation is integrated over idiosyncratic shocks above the threshold ($\varepsilon > \varepsilon$) because insolvent projects have no value to their owners. The condition, (2.2), implies a demand for loans. Let $J(\xi)$ be the probability distribution of $\xi$. As projects with $\xi < \bar{\xi}$ will be implemented, it follows that the demand for new loans (equivalently, the measure of projects) is $\chi' = J(\bar{\xi}) M$. Moreover, the total amount of labor hired for administration is $\chi' \int_{\xi < \bar{\xi}} dJ(\xi) / J(\bar{\xi})$.

### 2.2.3 Banks

The characterization of banks in this model builds upon Gertler and Karadi (2011). The main difference is that, in this chapter, banks make loans subject to default risk while, in the model of Gertler and Karadi (2011), banks hold claims on the state-contingent returns to capital. As we will see, the default channel generates the result

$^{14}$ $\beta P'/P$ is the stochastic discount factor.
that higher default risk raises interest rate spreads even when banks are willing to make more loans.

The timing of events in Figure 2.1 is useful for understanding bank’s problem. Every period begins with realizations of aggregate and idiosyncratic productivity levels. The ability of firm projects to repay debt depends crucially on these levels. Since \( \varepsilon \) is i.i.d., the average revenue from a loan is
\[
V(x) = [1 - \Pi(\varepsilon)] b + \lambda \int_{\varepsilon \leq \varepsilon'} f(\varepsilon, x) d\Pi(\varepsilon).
\]
Then, a bank’s net worth in this period is simply the gross interest revenue minus gross interest payment to depositors and any dividend payout to the bank owner:
\[
n = V(x) \chi_s - R(x) s - d_B,
\]  
where \( R \) and \( d_B \) are gross deposit rate and dividend payout, respectively.\(^{15}\) Banks that survive finance new loans \( \kappa \chi'_s \) through net worth and borrowings from depositors, \( s' \). That is, the balance sheet identity of a bank is
\[
s' + n = \kappa \chi'_s. \tag{2.4}
\]
Equation (2.4) implies that information on \( n_{-1} \) and \( \chi_s \) is sufficient to know \( s \). Substitution of (2.4) into (2.3) simplifies the individual law of motion of net worth as follows:
\[
n = \rho(x) \kappa \chi_s + R(x) n_{-1} - d_B, \tag{2.5}
\]
where \( \rho \equiv V/\kappa - R \) is the risk-adjusted excess return on a loan.

As I discussed in Section 2.2.1, banks are not able to borrow as much as they wish because of the capital requirement, \( (s' + n) \leq \psi^{-1} B(n, \chi'; x) \), where \( B \) represents the

\(^{15}\)Technically, it is possible to consider default of banks when their net worth drops to a negative value. As we will see later, banks differ only in their size in this model and returns to their assets are common. Thus, when bank default occurs, all banks must fail at the same time. I exclude this possibility by focusing on the local dynamics around steady state.
end-of-period value of a bank while the left-hand-side of the inequality is equivalent to the value of loans from (2.4). This capital requirement can be expressed as

\[ \kappa \chi'_s \leq \psi^{-1} B(n, \chi'; x). \]  

(2.6)

This capital requirement states that banks must hold sufficient net worth relative to their assets to guarantee that deposits are risk-free in equilibrium. Here, \( \psi \) is a stochastic variable that affects the financial capital required by depositors. When \( \psi \) increases, banks are required to hold more net worth against loans. In this chapter, \( \psi \) is regarded as a financial shock and I examine how such a shock, hitting the banking sector, affects business cycle fluctuations.\(^{16}\)

Given the law of motion of net worth and the capital requirement, I describe the Bellman equation of a bank’s problem as follows.

\[
B(n_{-1}, \chi'_s; x_{-1}) = E_{-1} \beta \frac{P_{-1}}{P} \max_{d_B, n} \left\{ d_B + (1 - \theta) n + \max_{\chi'_s} \theta B(n, \chi'_s; x) \right\}
\]  

(2.7)

subject to (2.5), (2.6), \( \chi'_s \geq 0, \ d_B \geq 0 \) and the law of motion of aggregate states, \( x = \Xi(x_{-1}) \). Below, I briefly characterize the solution of a bank’s problem; the derivation of these results is in the appendix.

Consider a bank that made \( \chi_s \) loans with \( n_{-1} \) units of net worth in the previous period. The bank chooses its level of dividend, paid to shareholders, \( d_B \). Any retained earnings, in the form of net worth, is paid to shareholders if the bank does not survive. When the bank does continue, which occurs with probability \( \theta \), it will choose the quantity of new loans, \( \chi'_s \). As shown in appendix, as long as (2.6) binds, banks do

\(^{16}\)Jermann and Quadrini (2012) consider a similar type of incentive constraint between households and firms and conclude that financial shocks play more important roles than aggregate productivity shocks in accounting for business cycles in the U.S. economy.
not pay out dividends: \( d_B = 0 \). Intuitively, a bank owner expects to obtain returns in excess of the deposit rate when the capital requirement limits the supply of loans below the efficient level. Because I focus on dynamics in the neighborhood of steady state where \((2.6)\) binds, this result always holds.

As the capital requirement binds in equilibrium, I can exploit the linearity of the problem to guess the solution to the value function as \( B(n, \chi'; \mathbf{x}) = g_n(\mathbf{x}) n + g_\chi(\mathbf{x}) \chi'_s \). Substituting this expression into \((2.6)\) proves that the total value of loans is proportional to the bank’s net worth as shown below (see the Appendix for a formal proof).

\[
\kappa \chi'_s = \phi(\mathbf{x}) n, \tag{2.8}
\]

where \( \phi \) is the leverage ratio (assets to net worth ratio) that is defined as \( \phi \equiv g_n/ (\psi - g_\chi / \kappa) \). Since the number of loans is linear in net worth, it is convenient to define \( G(\mathbf{x}) \equiv g_n(\mathbf{x}) + g_\chi(\mathbf{x}) \phi(\mathbf{x}) / \kappa \) to represent the real price of bank net worth.\(^{17}\) Then, we have

\[
\phi(\mathbf{x}) = \psi^{-1} G(\mathbf{x}). \tag{2.9}
\]

Substituting \((2.5)\) and \((2.8)\) into \((2.7)\), it is straightforward to show that \( G \) has a recursive representation.

\[
G(\mathbf{x}) = E \beta^{P'} \frac{P'}{P} (1 - \theta G(\mathbf{x}')) (\rho(\mathbf{x}') \phi(\mathbf{x}) + R(\mathbf{x}')) \tag{2.10}
\]

If there is no capital requirement, banks break even in expectation and \( G \) is always equal to one. That is, bank net worth is no more valuable than a unit of consumption goods. In this economy, however, the price of bank net worth is greater than one in the neighborhood of the steady state.

\(^{17}\)Note that \( B(n, \chi'; \mathbf{x}) = [g_n(\mathbf{x}) + g_\chi(\mathbf{x}) \phi(\mathbf{x})] n \) using \((2.8)\).
Finally, the aggregate law of motion of banks’ net worth must be derived. Let $N_{-1}$ and $\chi$ denote the aggregate quantities of net worth and loans, respectively, at the beginning of the current period. The stochastic death of banks forces a measure $1 - \theta$ of banks to be replaced by new banks. Following Gertler and Karadi (2011), the aggregate start-up fund is a fraction of existing aggregate bank assets, $\omega \kappa \chi$, where $\omega > 0$ is a constant. Because individual bank net worth is $\rho \kappa \chi_s + R n_{-1}$ from (2.5) with $d_B = 0$ and the probability of survival from stochastic death is $\theta$, the total volume of net worth held by continuing banks is $\theta [\rho \kappa \chi + R N_{-1}]$. Adding the aggregate start-up funds provided to new banks established in the same period, the aggregate law of motion of net worth is

$$N = \theta [\rho \kappa \chi + R N_{-1}] + \omega \kappa \chi.$$  \hfill (2.11)

### 2.2.4 Households

A household holds a non-negative amount of deposit, $s$, and capital, $k$, and receives gross returns of $(r_k + 1 - \delta)$ on capital and $R$ on deposits. Here, $\delta$ is the depreciation rate of capital. The household’s additional sources of income are wages and dividends from firms and banks. Household expenditure involves consumption and savings through deposits and capital. The utility maximization problem of the household is:

$$H (s, k; x) = \max_{c, L, s', k'} \{ u (C, 1 - L) + \beta E [H (s', k'; x')] \}$$
subject to
\[ c + s' + k' \leq w(x) L + Rs + (r_k + 1 - \delta) k + \pi, \]
\[ k', s' \geq 0, \]
\[ x' = \Xi(x), \]

where \( \pi \) includes profits from firms, dividend payments from banks and a net transfer from the government. Taking the first order condition, we obtain a standard consumption Euler equation.
\[ 1 = E\beta \frac{P'}{P} R', \]
where \( P = D_1 u(C, 1 - L) \). Since bank deposits and capital are perfect substitutes for households, an arbitrage condition must hold: \( R = r_k + 1 - \delta \).

### 2.2.5 Market Clearing Conditions

Market clearing condition in the final goods market is
\[ Y = C + I + \chi(1 - \lambda) \int_{\varepsilon < \bar{\varepsilon}} f(\varepsilon, x) d\Pi(\varepsilon), \tag{2.12} \]
where \( I = K' - (1 - \delta) K + \kappa \chi' \) is aggregate investment and \( Y = \chi \int y_f(\varepsilon, x) d\Pi(\varepsilon) \) is aggregate output. The supply of output is equal to consumption, investment and the total cost of default. In the labor market, household labor supply must match labor demand across projects, \( L_f \), and the total number of hours worked in preparation of new projects.
\[ L = L_f + \chi' \int_{\xi < \bar{\xi}} \xi \frac{dJ(\xi)}{J(\xi)}, \]
where \( L_f = \chi \int l_f(\varepsilon, x) d\Pi(\varepsilon) \). Finally, the capital rental market must clear:
\[ K = \chi \int k_f(\varepsilon, x) d\Pi(\varepsilon). \tag{2.13} \]
2.2.6 Recursive Competitive Equilibrium

A recursive competitive equilibrium is a set of functions, \((k_f, l_f, \xi, \overline{\xi}, G, \chi'_s, n, H, c, L, s', k', b, R, w)\), satisfying the following conditions.

1. Economic agents solve their problems:
   
   (a) Firms solve their respective problems and \((k_f, l_f, \xi, \overline{\xi})\) describes the associated decision rules for firms;
   
   (b) Banks solve their respective problems and \((\chi'_s, n)\) describes the associated decision rules for banks;
   
   (c) Households solve their respective problems and \((c, L, s', k')\) describes the associated decision rules for households;

2. Markets for final goods, labor and capital clear;

3. Laws of motion for aggregate state variables are consistent with individual decisions:
   
   (a) \(K' = k' (K, S; x), S' = s' (K, S; x)\);
   
   (b) \(N = \theta \int n (n_{-1}, \chi_s; x) d\mu (n_{-1}, \chi_s) + \omega \kappa \chi_s\), where \(\mu (n_{-1}, \chi_s)\) is the distribution of banks with net worth \(n_{-1}\) and the number of loans \(\chi_s\);
   
   (c) \(\chi' = \int \chi'_s (n_{-1}, \chi_s; x) d\mu (n_{-1}, \chi_s)\).

2.3 Calibration and Steady State

I assume that the representative household’s instantaneous utility stems from indivisible labor (Hansen (1985), Rogerson (1988)), \(u(c, L) = \log c + \eta_v (1 - L)\), and
the production function is Cobb–Douglas, \(\varepsilon z F(k, l) = \varepsilon z k^\alpha l^\nu\), where \(\alpha + \nu < 1\). I also assume that idiosyncratic productivity, \(\varepsilon\), follows Pareto distribution with the probability distribution function, \(\Pi(\varepsilon) = 1 - \varepsilon^{-\kappa}\), and that administrative employment, \(\xi\), has a uniform distribution with support \([0, A]\). I choose a quarterly time frequency and set the parameters of my model to match calibration targets in the steady state.

The subjective discount factor, \(\beta\), is chosen to generate a 4% real interest rate per year. Following Khan and Thomas (2007), the depreciation rate of capital, \(\delta\), is equal to the average growth-adjusted ratio of investment to capital between 1953 to 2002. The capital share parameter, \(\alpha\), is set to reproduce the annualized capital output ratio of 2.5.

The parameter on production function, \(\nu\), and the exponent on Pareto distribution, \(k_\varepsilon\), are determined to ensure that the labor income to output ratio is 0.64 and the gross charge-off rate is 1.27% per year. The gross charge-off rate, the amount of assets written-off as a percentage of average loan balances, is taken from the Federal Deposit and Insurance Corporation and used as a measure of default probability. The uniform distribution parameter, \(A\), is chosen to set the entry rate to 90% each quarter and the measure of potential projects, \(M\), is selected to normalize the measure of operating projects, \(\chi_{ss}\), to one. The parameter on labor disutility, \(\eta_\nu\), is adjusted so that average hours worked are one-third of normalized available hours of one.

I set interest rate spread, \(b/\kappa - R\), to 1.98%, from the difference between the U.S. bank prime loan rate and three-year Treasury constant maturity rate to pin down \(\kappa\). Following Gertler and Karadi (2011), the probability of bank’s survival is \(\theta = 0.972\) to imply banks’ average life of 10 years and bank’s leverage ratio, \(\phi_{ss}\), is set to 4, by which \(\omega\) as well as steady state financial shock, \(\psi_{ss}\), is determined. Finally, the
seizable fraction of insolvent project’s profit, $\lambda$, must be pinned down. The recovery rate of bank loans are usually high because they are secured by collateral. However, loans in my model are unsecured and I choose to match the 38% recovery rate of defaulted senior unsecured bonds reported in Moody’s (2007). Table 2.1 summarizes the parameter values.

Steady state results are summarized in Table 2.2. Total fixed cost as a share of output, $\kappa \lambda_{ss}/Y_{ss}$, is approximately 6.2%, of which net worth of banks accounts for one quarter. The liquidation cost of defaults is a very small portion of aggregate output. The excess return on net worth, $\rho$, is 117 basis points in annualized term.

I solve the model using a log-linear approximation around its steady state. Since only the mean of the bank distribution is required to aggregate bank net worth and the number of loans provided by banks, this method delivers a convenient and accurate approximation to local dynamics in the neighborhood of the steady state. The associated equilibrium conditions are shown in the appendix.

### 2.4 Results

In this section, I show that aggregate productivity and financial shocks are able to reproduce counter-cyclicality in default rates and interest rate spreads as well as procyclicality in GDP, investment and consumption. Towards this goal, I examine impulse response functions to an unexpected aggregate productivity shock and a financial shock that independently give rise to a recession.

The dynamic response of the economy is affected by financial market frictions and aggregate quantities exhibit a hump-shaped response to a monotone aggregate productivity shock. This is in sharp contrast to Khan and Thomas (2011) who argue...
that financial frictions do not propagate real shocks. Moreover, Jermann and Quadrini (2012) argue that real shocks are relatively weak. In contrast, in my model, they have a nontrivial role, and financial frictions interact with real shocks to reproduce empirically plausible aggregate responses.

**Aggregate Productivity Shock**

Figure 2.2 shows impulse response functions to an unexpected aggregate productivity decline. In period 0, aggregate productivity falls by 0.1 percent from steady state and gradually returns to steady state; its persistence is 0.9455.\(^{18}\) The figure shows that the models is successful in producing counter-cyclicality of both default rates and interest rate spreads as well as the procyclicality of number of projects (loans),\(^{19}\) output, investment and consumption. Importantly, these movements are consistent with what we observe in data. The mechanism behind the counter-cyclical result is as follows. First, the fall in aggregate productivity worsens the overall profit levels of firms. Then, since debt repayment, \(b\), is predetermined, default rates increase and, as a result, bank net worth falls.

Importantly, despite the damage to bank net worth, the Lagrange multiplier on the capital requirement is lower than its steady state level in period 0 and the interest rate spread rises in period 1. If there were no default as in Gertler and Karadi (2011), a lower multiplier would imply a smaller interest rate spread because (2.6) is the only financial friction at work. In contrast, in this chapter, default interacts with the friction between households and banks. On the one hand, the demand for loans

\(^{18}\)This persistence is the estimated value in section 2.6.

\(^{19}\)The number of loans and the number of projects are equivalent. I use these terms interchangeably.
falls as the expected value of entry declines with lower productivity. In addition, an increase in the price of bank net worth, $G$, partially offsets the tightening effect of falling bank net worth on the capital requirement. The price of net worth, whose response is identical to that of leverage when $\psi$ is held constant,\footnote{Recall that $\phi = \psi^{-1}G$ in (2.9).} increases since bank net worth is scarcer. Altogether, the capital requirement is relaxed, which puts a downward pressure on $b$ in period 1. On the other hand, a higher default rate due to persistently lower productivity puts an upward pressure on $b$ to compensate for the expected loss from default. The results in Figure 2.2 show that the latter effect is larger than the former so that interest rate spread rises in period 1.

Consequently, the rise in $b$, along with lower productivity, raises default rate on loans in period 1. That the rise in $b$ is partially offset by the demand-side effect further damages the net worth of banks by reducing the risk-adjusted excess return, $\rho$. The capital requirement tightens in period 5, after which default and the incentive constraint start to reinforce each other. This is evident in the persistence of the interest rate spread, default rates and number of loans, all of which are far more persistent than aggregate productivity. Even though the half life of aggregate productivity shock is approximately 13 periods, it takes 24 periods for aggregate output to reach their half lives. This is the propagation from the financial sector that constrains the pace of economic recovery.

To further highlight the propagation of real shocks through financial frictions, Figure 2.3 compares responses in aggregate output, investment, consumption and the number of firm projects for three different cases: (a) Blue lines are responses with financial frictions; (b) red dashed lines are responses without financial frictions.
(κ = 0); (c) green dashed lines are responses without financial frictions (κ = 0) or administrative employment costs (A = 0). The figure shows that financial frictions in this model adds persistence to the effect of real shocks and also generates non-monotonicity of responses in output and investment by dampening the initial response of the economy. These results stem from a persistent decrease in the number of loans.

**Financial Shock**

Shocks to aggregate productivity affect the profit of firms and the likelihood of repaying their debt. While there is no doubt that increased uncertainty on the performance of underlying assets is the fundamental problem for banks during recessions, the fear of systemic risk can make even relatively sound banks suffer by making it hard for them to raise funds. Although the fire sale of assets to deleverage balance sheets was particularly prominent in investment banks (Adrian and Shin (2009)), Ivashina and Scharfstein (2010) point out that commercial banks have also cut new business loans during the recent financial crisis. In the light of this evidence, I try to capture the effect of exogenous variation in bank creditworthiness through changes in ψ.\(^{21}\)

Figure 2.4 shows impulse response functions to an unexpected 0.1 percentage point increase in ψ that gradually returns to steady state with a persistence of 0.737.\(^{22}\) Similar to the aggregate productivity shock, default rates and interest rate spreads exhibit a counter-cyclical pattern while output, investment, consumption and the number of loans respond procyclically. The major difference with a productivity

\(^{21}\)Gertler and Kiyotaki (2010) consider an inter-bank loan market where lending banks limit the amount of loans to borrowing banks.

\(^{22}\)Again, this persistence parameter is taken from estimates in section 2.6.
shock is that the initial impact date responses are negligible. In particular, bank net worth does not decrease. This initial response follows from productivity being initially unaffected; the financial shock influences the decisions of firms implementing projects in future periods. The financial shock tightens banks’ capital requirement until period 4. The increase in \( \psi \) requires more net worth for banks to raise the same level of deposits, holding other things constant. Since the productivity of firms has not been changed, this supply-side effect overwhelms any demand-side effect that might affect the capital requirement, (2.6). As a result, interest rate spreads rise sharply in period 1 and thus discourage the implementation of new projects. In contrast to the aggregate productivity shock, bank net worth increases after the financial shock because the supply-side driven increase in \( b \) is sufficiently large to cover the subsequent rise in default rates. As bank net worth peaks in period 4, the effect of the financial shock becomes weaker and the capital requirement is relaxed relative to the steady state. Thereafter, interest rate spreads stay low and the number of new loans begins to rise.

Responses in output and investment reflect the higher debt levels associated with a financial shock and the resulting decisions of firms to implement their projects. It is worth pointing out that the responses for the financial shock do not persist longer than the underlying shock. This is primarily because the financial shock does not change the average productivity of firms. This suggests that financial shocks may be able to pick up short-lived variations in macroeconomic variables that cannot be explained by aggregate productivity shocks.
Given the results in this section, I can proceed further to examine the effectiveness of capital injection and the contribution of the two types of shocks in explaining business cycle fluctuations in the U.S.

2.5 Capital Injection to Banks

In the previous section, aggregate productivity and financial shocks generated macroeconomic fluctuations through conventional channels, banks’ capital requirement, \((2.6)\), and their effect on loan default. In this section, I examine the effectiveness of capital injections to banks during periods of financial distress. After the failure of Lehman Brothers in 2008, a series of policy actions were taken to assist the financial sector, which includes, among others, capital injections to large banks through the Troubled Assets Relief Program. This rescue program was large-scale but short-lived to minimize both the opposition to bailing out financial institutions and any possible moral hazard problems. To capture this aspect of the policy, I characterize a capital injection as a transitory shock to net worth in a recursive competitive equilibrium. I assume that before banks die stochastically, each bank receives \(\tau n_{-1}\) from the government, where \(\tau\) is a mean zero i.i.d. policy shock. This is assumed be a pure transfer from the government in the sense that banks have no obligation to pay any dividend to the government. Under this policy, the law of motion of individual bank net worth is expressed as

\[
n = \rho \kappa \chi + Rn_{-1} + \tau n_{-1}.
\]

The cost of policy is financed by lump-sum taxes on households, \(T\), every period. That is,

\[
T = \tau N_{-1}.
\]
By the introduction of this policy, (2.10) should be expressed as

\[ G(x) = E\beta \frac{P'}{P} (1 - \theta + \theta G'(x')) (\rho' \phi + R' + \tau') \]  

(2.14)

and (2.11) is replaced by

\[ N = \theta [\rho \kappa \chi + (R + \tau) N_{-1}] + \omega \kappa \chi, \]  

(2.15)

where \( x \) includes the current \( \tau \) in addition to the existing state variables.

Figure 2.5 shows the impulse responses following a capital injection in period 0 to counteract an unexpected aggregate productivity shock and a financial shock. The size of aggregate productivity shock is 0.1% as before while that of financial shock is adjusted to generate the same interest rate spreads for both shocks in period 1. The size of capital injection, \( \tau \), is set to 0.068 to roughly offset the initial fall in bank net worth after an aggregate productivity shock. Figure 2.5 indicates that even though the initial fall in bank net worth was fully offset by the capital injection, its effectiveness against an aggregate productivity shock is limited. Even though interest rate spreads fall sharply in period 1, default rates remain high and continue to rise until period 7. Thereafter, interest rate spreads start to catch up with default rates, deterring new borrowing by firms. As a result, recovery in output and investment is modest and temporary as we can see from the fact that red lines join blue lines rapidly.

In contrast, a capital injection is more effective in stabilizing fluctuations following a financial shock. Figure 2.6 indicates that peak responses are diminished by easing banks’ need for collateral. Table 2.3 summarizes the reductions in the peak responses for the case characterized by a capital injection relative to the case without policy intervention. These decrease at least 27% for interest rate spreads, default rates and
the number of loans and 22% for output and investment. The corresponding numbers for the aggregate productivity shock are significantly lower, especially in output and investment.

Why does a capital injection work more effectively against financial shocks? When a financial shock arrives, a rise in the interest rate spreads results from a worsening of the friction between depositors and banks. A capital injection is a direct solution to this problem as banks’ credibility improves with the level of their net worth. The results in this section show that a one-time capital injection works reasonably well against a persistent financial shock. In contrast, an increase in interest rate spreads immediately after an aggregate productivity shock is not attributable to a more severe tension between depositors and banks. Instead, it is associated with an increase in default as I discussed in Section 2.4. Recall that a lower demand for loans actually relaxes the capital requirement of banks. Since capital injections cannot increase the productivity of firms, they are at best an indirect measure that gives a buffer against future shocks.

The results of this policy exercise imply that the effect of capital injections may be limited depending on the source of shocks driving a recession. Although interest rate spreads are believed to be a good indicator of an economic downturn, it is difficult, in practice, to identify the underlying shocks in real time. If a worsening productivity of firms is the fundamental problem, the government will learn of it later through a lingering higher rate of default.
2.6 Measurement of Shocks

In this section, I measure the cyclical components of financial and aggregate productivity shocks to evaluate their relative importance in the business cycle. While there is no unique method to measure latent variables from data, especially financial shocks, Jermann and Quadrini (2012) use an enforcement constraint that corresponds to (6) to measure financial shocks, given the Solow residual series for aggregate productivity shocks. Then, the shock processes are estimated from these recovered observations and used to simulate their model. They find that financial shocks are the leading force in business cycle fluctuations in the United States. I do not follow this approach, however, because the standard Solow residual method is not consistent with aggregate supply in the model of this chapter. Instead, in an effort to match the model to data, I use a Bayesian estimation method to estimate the persistence and standard deviation of the underlying shocks given the calibrated structural parameters.\(^{23}\)

I implement a Bayesian estimation of shocks by including three types of data: (a) real GDP, (b) private investment and (c) bank net worth. For identification, at least three types of shocks are necessary to match three data series. To satisfy this requirement, I include i.i.d. measurement errors in private investment along with aggregate productivity shocks and financial shocks. I assume an AR(1) structure for aggregate productivity shocks, \(z\), and financial shocks, \(\psi\), and estimate their persistence parameters as well as the standard deviation of i.i.d. normal innovations.

\(^{23}\text{I do not estimate structural parameters because of a technical reason. Each time parameters are drawn to compute the log density, its steady state of the model must be recomputed to derive a state space representation. However, for some parameter values, the steady state default threshold of idiosyncratic productivity, }\xi, \text{ hits its lowest value, which prevents the use of log-linearization to solve the model. I leave the estimation of structural parameters using a non-linear solution method for my future research.}\)
to these shocks. For the i.i.d. normal measurement errors in investment, I estimate their standard deviation.

Data on real GDP and private investment are taken from the National Income and Product Accounts (NIPA). Tier 1 leverage capital of financial institutions affiliated to the Federal Deposit Insurance Corporation (FDIC) is used to represent bank net worth. The data start from 1990Q1 and end in 2010Q4 as determined by the availability of tier 1 capital series. All series are detrended using Hodrick-Prescott filter with the smoothing parameter of 1,600 for quarterly data.

Estimation is in two steps. First, I combine information from prior distributions of estimated parameters and the log-likelihood implied by the data to find the mode of the log posterior density. Next, I use this information to propose draws for simulating the posterior distribution. The Metropolis-Hastings Algorithm is used to implement the simulation step. I used 200,000 Monte Carlo Markov Chain draws for simulating the posterior distribution with this algorithm, which resulted in an acceptance rate of 23 percent.

The prior distributions on estimated parameters are chosen as follows. The persistence of aggregate productivity shocks, $\rho_z$, is beta distributed with mean, 0.9, and standard deviation, 0.05, while that of financial shocks, $\rho_\psi$, has the same distribution with mean, 0.8, and standard deviation, 0.05. The standard deviation of the innovations to aggregate productivity shocks, $\sigma_z$, is assumed to follow an inverse-gamma distribution with a mean of 0.01 and a standard deviation of infinity while that of innovations to financial shocks, $\sigma_\psi$, has the same distribution with a mean of 0.1 and a standard deviation of infinity. Finally, the standard deviation of measurement
errors in private investment, $\sigma_i$, has an inverse-gamma prior with a mean of 2 and a standard deviation of infinity.

Table 2.4 summarizes the estimates of shock parameters and Figure 2.7 shows their prior and posterior distributions. The persistence and standard deviation of aggregate productivity shocks are approximately 0.95 and 0.2 percent, respectively. Financial shocks have a lower persistence of 0.74 and a larger standard deviation of 0.66 percent. The standard deviation of measurement errors in investment has a posterior mean of approximately 2.2 percent.

Based on the posterior means, shocks are computed using Kalman smoother and are shown in Figure 2.8. Notice that there are sharp increases in financial shocks from 2008Q3 to 2009Q2. This captures liquidity problems in the banking sector around the time when Lehman Brothers failed. Prior to the financial crisis, such liquidity issues were mild from the late 1990s to 2000 in association with the dot-com boom, followed by a worsening of confidence with the collapse of IT stock prices. These movements in financial shocks are broadly consistent with our prior knowledge. The fluctuation in aggregate productivity shocks are also in line with the boom and bust of economic conditions. Specifically, there is a sharp fall in 2009 following the financial crisis in late 2008.

Given the shock series for aggregate productivity and financial shocks, I generate model predictions by feeding in the recovered aggregate productivity and financial shocks. Dashed lines in Figure 2.9 shows business cycle fluctuations of real GDP, real private investment and hours worked from the data. Plain solid lines, solid lines with

\[ \text{Dashed lines in Figure 2.9 shows business cycle fluctuations of real GDP, real private investment and hours worked from the data. Plain solid lines, solid lines with} \]

\[ \text{\textsuperscript{24}I assume that state variables in 1990Q1 are at their steady-state levels. Then, endogenous state variables are determined within the model over time. Moreover, even though I have the whole sequence of shocks, agents hold rational expectations of future state of the economy given the current state of the economy.} \]
circles, and solid lines with crosses indicate model predictions driven by both shocks, aggregate productivity shocks only and financial shocks only, respectively. Notice that differences between the dashed line and the plain solid line in private investment indicate measurement errors, or components unexplained by the underlying shocks of my model.

There are two observations that arise from this exercise. First, financial shocks play an important role during the financial crisis starting in 2008Q4. In 2009Q1, for example, approximately 65% of the drop in private investment and 55% of the drop in real GDP come from financial shocks. In 2009Q2, the contribution of financial shocks is 46% in private investment and 34% in real GDP. This role of financial shocks also holds for hours worked. Although the data lag slightly behind model predictions and the trough is larger than what the model predicts, significant decreases in hours worked are in part due to financial shocks.

Second, financial shocks have a relatively minor role in explaining variations in these variables prior to the financial crisis. In contrast, aggregate productivity shocks drive most of variations in real GDP, private investment and hours worked. This is prominent in real GDP and also in private investment and hours worked at least until 2002.

This historical importance of aggregate productivity is in contrast to Jermann and Quadrini (2012) who find that their model’s prediction worsens with aggregate productivity shocks using the sample period of 1984Q1-2009Q4. Of course, in this chapter, real GDP is perfectly matched by construction, however hours worked is not used for estimation. Aggregate productivity shocks work poorly in Jermann and Quadrini (2012) because the financial constraint in their paper relaxes in recessions,
making it easier for firms to borrow working capital for production. In this chapter, the deposit friction, (2.6), also relaxes in recessions. However, because of the mechanics of default, financial frictions amplify the procyclicality of output and hours worked.

In sum, through the exercises in this section, I have found that aggregate productivity shocks may be an important driving force of output, investment and hours worked. However, aggregate productivity shocks alone cannot explain the movements of these variables, especially in periods immediately after the financial crisis in late 2008. Financial shocks help explain real GDP and investment in early 2009.

2.7 Conclusion

In this chapter, I have shown that an environment wherein a friction between banks and depositors, alongside interactions among the deposit friction, bank net worth and loans subject to default generates counter-cyclicality in default rates and interest rate spreads and procyclicality in aggregate output, investment, consumption and hours worked. These results are consistent with data and hold for both aggregate productivity and financial shocks. Even though interest rate spreads rise in both cases, however, the magnitude and mechanisms of propagation are different, and the effectiveness of capital injection depends on the source of business cycle. This implies that policy makers should be careful about interpreting a higher interest rate spread when they design policies. I also find that aggregate productivity shocks are important in explaining business cycle fluctuations in real GDP and investment and hours worked through mid-2008 but cannot, by itself, explain movements in these series after the financial crisis. The financial shock I considered in this chapter fills
this gap in 2009. The finding for the role of aggregate productivity shocks is in sharp contrast to Jermann and Quadrini (2012) and serves to buttress their importance.

The model has one-period-lived projects and loans for simplicity, but introducing multi-period loans is an interesting direction for future work. This may allow us to address the maturity mismatch problem of banks. Introducing firm net worth to add a richer interaction between firms and banks may allow a better comparison to existing models. Finally, introducing nominal frictions will allow for meaningful insights on the relationship between default and monetary policy.
### Table 2.1: Baseline Parameter Values

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \delta )</th>
<th>( \alpha )</th>
<th>( \nu )</th>
<th>( k_e )</th>
<th>( A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.017</td>
<td>0.27</td>
<td>0.61</td>
<td>16.0</td>
<td>0.036</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( M )</th>
<th>( \eta )</th>
<th>( \kappa )</th>
<th>( \theta )</th>
<th>( \omega )</th>
<th>( \psi_{ss} )</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.11</td>
<td>2.49</td>
<td>0.062</td>
<td>0.972</td>
<td>0.0018</td>
<td>0.42</td>
<td>0.38</td>
</tr>
</tbody>
</table>

### Table 2.2: Steady State Results

<table>
<thead>
<tr>
<th>Asset</th>
<th>Interest rate spread</th>
<th>Default rate</th>
<th>Number of loans</th>
<th>Output</th>
<th>Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>C/Y</td>
<td>12%</td>
<td>12%</td>
<td>17%</td>
<td>7%</td>
<td>12%</td>
</tr>
<tr>
<td>I/Y</td>
<td>29%</td>
<td>28%</td>
<td>27%</td>
<td>22%</td>
<td>22%</td>
</tr>
<tr>
<td>N/Y</td>
<td>0.015</td>
<td>0.015</td>
<td>0.046</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>S/Y</td>
<td>0.046</td>
<td>0.046</td>
<td>0.17</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>( \rho )</td>
<td>117bp</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 2.3: Reductions in Peak Responses After Capital Injection

<table>
<thead>
<tr>
<th>Asset</th>
<th>( \rho_z )</th>
<th>( \rho_{\psi} )</th>
<th>( \sigma_z )</th>
<th>( \sigma_{\psi} )</th>
<th>( \sigma_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>prior mean</td>
<td>0.9</td>
<td>0.8</td>
<td>0.01</td>
<td>0.1</td>
<td>2.00</td>
</tr>
<tr>
<td>posterior mean</td>
<td>0.946</td>
<td>0.737</td>
<td>0.206</td>
<td>0.665</td>
<td>2.179</td>
</tr>
<tr>
<td>confidence interval (90%)</td>
<td>0.916, 0.975</td>
<td>0.657, 0.824</td>
<td>0.163, 0.249</td>
<td>0.542, 0.780</td>
<td>1.891, 2.445</td>
</tr>
</tbody>
</table>

### Table 2.4: Estimation Results
Figure 2.1: Timing of Events

State variables
- $K_t$: capital
- $N_{t-1}$: bank net worth
- $\chi_t$: number of loans
- $b_t$: firm debt
- $z_{t-1}$: aggregate productivity
- $\psi_{t-1}$: financial shock

Innovations realize and $z_t$ & $\psi_t$ are determined

Projects produce $Y_t$ by renting $K_t$ & $L_t$

Households determine $C_t, S_{t+1}, K_{t+1}$ & $L_t$

Projects default if $\epsilon_t \leq \epsilon^*_t$; otherwise repay $b_t$

Fraction $\theta$ banks exit and are replaced by new banks with start-up fund $N_t$ is determined

Potential projects draw $\xi_t$ and $\chi_{t+1}$ of them decide to produce next period

Banks lend $\kappa \chi_{t+1}$ to new projects using deposits $S_{t+1}$ and net worth $N_t$

Each project pays set-up costs $\kappa + w_t \xi_t$, where $w_t \xi_t$ is paid by households

Each project agrees to pay $b_{t+1}$ next period
Figure 2.2: Impulse Responses to Aggregate Productivity Shock
NOTES: (a) Solid lines are responses with financial frictions; (b) dotted lines are responses without financial frictions ($\kappa = 0$); (c) dashed lines are responses without financial frictions ($\kappa = 0$) and administrative employment costs ($A = 0$).

Figure 2.3: The Role of Financial Frictions in Propagating an Aggregate Productivity Shock
Figure 2.4: Impulse Responses to Financial Shock
NOTES: Dashed lines are impulse responses following an aggregate productivity shock when the capital injection policy is implemented. Solid lines show responses to the same shock in the absence of the policy.

Figure 2.5: Capital Injection Against a Productivity Shock
Figure 2.6: Capital Injection Against a Financial Shock

NOTES: Dashed lines are impulse responses following a financial shock when the capital injection policy is implemented. Solid lines show responses to the same shock in the absence of the policy.
NOTES: Dashed and solid lines show prior and posterior distributions, respectively.

Figure 2.7: Prior and Posterior Distributions

NOTES: The panels indicate percentage or percentage point deviation of shocks from the steady state.

Figure 2.8: Aggregate Productivity and Financial Shocks Measured by Bayesian Estimation
NOTES: Dashed lines represent business cycle fluctuations in data; solid lines show model-implied dynamics when feeding measured aggregate productivity and financial shocks into the model; solid lines with circles indicate model-implied dynamics when feeding only measured aggregate productivity shocks; solid lines with crosses represent model-implied dynamics when feeding only measured financial shocks into the model.

Figure 2.9: Model Predictions and Data
Bibliography


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Appendix A: Appendix to Chapter 1

A.1 Log-Linearized System

The system consists of (1.1), (1.2), (1.3), (1.5), (1.6), (1.7), (1.8), (1.9), (1.10), (1.11), the definition of GDP, \( Y_t = P_{c,t}C_t + P_{x,t}X_t \), and the definition of aggregate price level, \( P_t = \bar{P}_cC_t + \bar{P}_xX_t \). The unknowns are \( \{D_t, X_t, P_{j,t}, Q_{j,t}, P^*_j, MC_{j,t}, C_t, Y_t, P_t\} \) and \( M_t \) is exogenously determined by the money-growth rule. As in BHK (2007), the utility and production functions take the following functional forms.

\[
    u(C, D) = \psi_c \log C + \psi_d \log D ,
\]

\[
    v(N) = \psi_p \frac{\eta}{\eta + 1} N^{\frac{\eta + 1}{\eta}} ,
\]

\[
    F(K, N) = AK^\alpha N^{1-\alpha} .
\]

Letting the lower-case letters denote the percentage deviations from the steady-state values, the system of log-linearized equations can be derived as below.

\[
    d_t = \delta x_t + (1 - \delta) d_{t-1} ,
\]

\[
    \eta^{-1}n_{j,t}(z) = w_{j,t}(z) - c_t - p_{c,t} ,
\]

\[
    c_t - p_{x,t} + p_{c,t} = [1 - \beta (1 - \delta)] d_t + \beta (1 - \delta) E_t [c_{t+1} - p_{x,t+1} + p_{c,t+1}] ,
\]

\[
    m_t = p_t + y_t ,
\]

\[
    25 \bar{P}_c \text{ and } \bar{P}_x \text{ are steady-state values of sectoral prices.}
\]
\[y_{j,t}(z) = -\varepsilon (p_{j,t}(z) - p_{j,t}) + y_{j,t},\]  
(A.2)

\[y_{j,t}(z) = (1 - \alpha) n_{j,t}(z),\]  
(A.3)

\[p_{j,t}^*(z) = \sum_{i=0}^{\infty} (\theta_j \beta)^i E_t [mc_{j,t+i}(z)],\]  
(A.4)

\[mc_{j,t+i}(z) = w_{j,t}(z) + \alpha n_{j,t}(z),\]  
(A.5)

\[q_{j,t} = (1 - \phi) \sum_{k=0}^{\infty} \phi^k E_{t-k} p_{j,t}^* \text{ for } j = c, x,\]  
(A.6)

\[p_{j,t} = \theta_j p_{j,t-1} + (1 - \theta_j) q_{j,t} \text{ for } j = c, x,\]

\[y_t = \kappa_c c_t + (1 - \kappa_c) x_t.\]

\[p_t = \kappa_c p_{c,t} + (1 - \kappa_c) p_{x,t}\]

where \(\kappa_c \equiv \bar{P}_c \bar{C} / (\bar{P}_c \bar{C} + \bar{P}_x \bar{X}).\) In equation (A.6), \(p_{j,t}^*\) is defined as \(p_{j,t,0}^*\) and I used the fact that \(p_{j,t,k}^* = E_{t-k} p_{j,t}^*\).

In this model, it is easy to deal with firm heterogeneity. First, (A.1), (A.3) and (A.5) yields

\[mc_{j,t}(z) = c_t + p_{c,t} + \omega y_{j,t}(z),\]

where \(\omega \equiv (\alpha + \eta^{-1}) / (1 - \alpha).\) By substituting this condition into (1.8), I obtain

\[p_{j,t}^*(z) = \sum_{i=0}^{\infty} (\theta_j \beta)^i E_t [\omega y_{j,t+i}(z) + c_{t+i} + p_{c,t+i}]\]

\[= \sum_{i=0}^{\infty} (\theta_j \beta)^i E_t [-\varepsilon \omega (p_{j,t}^*(z) - p_{j,t+i}) + \omega y_{j,t+i} + c_{t+i} + p_{c,t+i}],\]

where (1.6) is used to get the result in the second line. After shifting the \(p_{j,t}^*(z)\) term from the right-hand-side to the left-hand-side, it is straightforward to derive

\[p_{j,t}^* = (1 - \theta_j \beta) mc_{j,t} + \theta_j \beta E_t p_{j,t+1}^* \text{ for } j = c, x,\]
where
\[ mc_{j,t}^n = \frac{1}{1 + \varepsilon \omega} \gamma_{j,t+1} + \frac{\varepsilon \omega}{1 + \varepsilon \omega} p_{j,t+1} + \frac{1}{1 + \varepsilon \omega} c_{t+1} + \frac{\omega}{1 + \varepsilon \omega} y_{j,t+1}. \] (A.7)

Note that the household’s first-order conditions with respect to \( C_t \) and \( X_t \) yields
\[ \Gamma_t = \frac{P_{x,t}}{P_{c,t}} M U_{c,t}, \] (A.8)

where \( \Gamma_t \) is the Lagrange multiplier on (1.1). By log-linearizing (A.8) and combining it with (A.7), we obtain (1.13) in Section 1.2.5.

\[ mc_{x,t}^n = -\frac{1}{1 + \varepsilon \omega} \gamma_t + p_{x,t} + \frac{\omega}{1 + \varepsilon \omega} x_t. \]

The system of log-linearized equations with lagged expectations can be solved by algorithms developed by Wang and Wei (2006) and Meyer-Gohde (2010). The sticky information model has an infinite lags of expectations. These methods truncates the lags at sufficiently large finite value and solves a finite-order lagged expectation model.

### A.2 The Model with Alternative Timing Assumptions

In Section 1.3, I introduced additional assumptions into the model to ensure that all the variables do not respond to a monetary-policy shock in period 0 and to further delay the responses. The first assumption forces the representative household to determine the amount of nondurable consumption one period in advance. The second assumption is a cash-in-advance constraint through which the representative household chooses the optimal level of money used in the next period to purchase nondurable and durable goods. These two assumptions are sufficient to suppress a durable investment response in period 0 as long as producers do not respond to shocks in the same period. The third assumption forces producers of intermediate goods to
make their decisions one period in advance. In other words, these producers determine period-\(t\) prices based on period \(t - 1\) information. The fraction of firms holding vintage \(k \geq 1\) information is \((1 - \phi) \phi^{k-1}\). Under these assumptions, the optimality conditions for the representative household are as follows:

\[
\begin{align*}
\Lambda_t &= \beta E_t (\Lambda_{t+1} + \Xi_{t+1}), \\
C_t^{-1} &= E_{t-1} \left[ P_{c,t} (\Lambda_t + \Xi_t) \right], \\
\Gamma_t &= P_{x,t} (\Lambda_t + \Xi_t), \\
D_t^{-1} &= \Gamma_t - \beta (1 - \delta) E_t \Gamma_{t+1}, \\
\Lambda_t &= \beta (1 + i_t) E_t \Lambda_{t+1}, \\
W_{j,t} (i) \Lambda_t &= \psi_n N_{j,t} (i)^{1/\eta}, \\
M_{t-1} &= P_{c,t} C_t + P_{x,t} X_t, \\
\end{align*}
\]

(A.9)

where \(\Lambda_t\), and \(\Xi_t\) are the Lagrange multipliers on the budget constraint and the CIA constraint, (A.9). Log-linearized optimality conditions for producers are

\[
p^{*}_{j,t} = (1 - \theta_j \beta) E_{t-1} m c_{j,t}^{\alpha} + \theta_j \beta E_{t-1} p^{*}_{j,t+1}
\]

and \(p^{*}_{j,t,k} = E_{t-k-1} p^{*}_{j,t}\). 

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A.3 Data

A.3.1 Sources

The following data series were obtained from the National Economic Accounts Data of the Bureau of Economic Analysis (http://www.bea.gov/).

<table>
<thead>
<tr>
<th>Series</th>
<th>Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP, quantity index</td>
<td>1.1.3</td>
</tr>
<tr>
<td>Gross Domestic Product</td>
<td>1.1.5</td>
</tr>
<tr>
<td>Price Indexes for Gross Domestic Product</td>
<td>1.1.4</td>
</tr>
</tbody>
</table>

Monthly federal funds rates and seasonally adjusted M2 series were downloaded from the Federal Reserve Economic Data (FRED) and converted to a quarterly series by taking the arithmetic means over three months. For commodity price index, we use the index for agricultural raw materials (00176BXDZF) in the International Financial Statistics (IFS) of the International Monetary Fund (IMF).

A.3.2 Construction of GDP and Price Data Series

Following Erceg and Levin (2005, 2006), we use the Tornqvist approximation explained by Whelan (2002), to construct the series of real durable goods plus real residential investment and the real GDP excluding these components. The formula given in Whelan (2002) is

$$\frac{\Delta Y_t}{Y_{t-1}} = \theta_t \frac{\Delta X_t}{X_{t-1}} + (1 - \theta_t) \frac{\Delta Z_t}{Z_{t-1}},$$

where $X$ is a component of $Y$, $Z$ is a series excluding $X$ from $Y$, and $\theta_t$ is the average of nominal expenditure shares of $X$ in period $t$ and $t - 1$. From any of the two series among $X$, $Y$ and $Z$, it is possible to construct the other one by applying this procedure sequentially starting from the base year. This procedure was also used to construct price series.
Appendix B: Appendix to Chapter 2

B.1 Banks Pay Out No Dividend

Consider the bank’s problem in Section 2.2.3. The first-order conditions with respect to $d_B$, $\chi'_s$ and $n$ are as follows.

$[d_B]$

$1 - \eta + \lambda = 0,$  \hspace{1cm} (B.1)

$[\chi']$

$\theta D_2 B (n, \chi'_s; x) + (\psi^{-1} D_2 B (n, \chi'_s; x) - \kappa) \bar{\mu} + \kappa \mu = 0,$  \hspace{1cm} (B.2)

$[n]$

$1 - \theta + \theta D_1 B (n, \chi'_s; x) + \psi^{-1} D_1 B (n, \chi'_s; x) \bar{\mu} = \eta,$  \hspace{1cm} (B.3)

where $\eta$, $\bar{\mu}$, $\mu$ and $\lambda$ are the Lagrange multipliers associated with (2.5), (2.6), $\chi'_s \geq 0$ and $d_B \geq 0$, respectively. Benveniste-Scheinkman conditions are

$D_1 B (n_{-1}, \chi; x_{-1}) = E_{-1} \beta \frac{P}{P_{-1}} R \eta,$  \hspace{1cm} (B.4)

$D_2 B (n_{-1}, \chi; x_{-1}) = E_{-1} \beta \frac{P}{P_{-1}} \rho \kappa \eta.$  \hspace{1cm} (B.5)

If $\bar{\mu} = 0$ for all periods, (B.3) together with (B.4) implies that $\eta = (1 - \theta) (1 + \theta + \theta^2 + \cdots) = 1$. But if (2.6) is binding or binds in the future, $\eta > 1$. Then, $\lambda > 0$ from (B.1). In this chapter, I consider dynamics around steady state in which
(2.6) binds. Also, notice that \( \chi'_s = 0 \) leads to no production in the next period, which cannot be an equilibrium. Hence, \( \mu = 0 \), and (B.2) and (B.5) imply that \( E (\beta P'/P) \rho' q' \geq 0 \) in equilibrium.

### B.2 Leverage Ratio and the Value of Banks

Because (2.6) is binding in the neighborhood of steady state, substitute \( B (n, \chi'_s; x) = g_n (x) n + g_\chi (x) \chi'_s \) into (2.6) to obtain \( \kappa \chi'_s = \psi^{-1} (g_n (x) n + g_\chi (x) \chi') \), or equivalently,

\[
\kappa \chi'_s = \frac{g_n (x)}{\psi - g_\chi (x)/\kappa} n
\]

\[
= \phi (x) n,
\]

where \( \phi \equiv g_n / (\psi - g_\chi / \kappa) \). Using the definition of \( G \), the following expression is derived.

\[
\phi (x) = \psi^{-1} \left( g_n (x) + \frac{g_\chi (x)}{\kappa} \phi (x) \right)
\]

\[
\equiv \psi^{-1} G (x).
\]

Next, substitute \( B (n, \chi'_s; x) = g_n (x) n + g_\chi (x) \chi'_s = G (x) n \) into (2.7):

\[
G (x_{-1}) n_{-1} = E_{-1} \beta \frac{P}{P_{-1}} \left( 1 - \theta + \theta G (x) \right) n
\]

\[
= \left[ E_{-1} \beta \frac{P}{P_{-1}} \left( 1 - \theta + \theta G (x) \right) (\rho (x) \phi (x_{-1}) + R (x)) \right] n_{-1},
\]

where the last equality uses (2.5) with \( d_B = 0 \) and (2.8). This gives us the expression in (2.10).
B.3 Lagrange Multiplier on Incentive Constraint

Notice that \( D_1 B(n, \chi'; x) = g_n(x) \) and \( D_2 B(n, \chi'; x) = g_\chi(x) \). From (B.2) and (B.5),

\[
\bar{\mu} = \frac{\theta g_\chi / \kappa}{1 - \psi^{-1} g_\chi / \kappa}.
\] (B.6)

Then, from (B.3), (B.6) and the definition of \( \phi \),

\[
\eta = 1 - \phi + \phi g + \psi^{-1} \bar{\mu} g
\]

\[
= 1 - \phi + \phi \left( g + g_\chi \frac{\psi^{-1} g_n}{\kappa (1 - \psi^{-1} g_\chi / \kappa)} \right)
\]

\[
= 1 - \phi + \phi G
\]

Since \( g_\chi / \kappa = E(\beta P'/P) \rho \eta \) from (B.5),

\[
\bar{\mu} = \frac{\theta E(\beta P'/P) \rho' \eta'}{1 - \psi^{-1} E(\beta P'/P) \rho' \eta'}
\]

\[
= \frac{\theta E(\beta P'/P) \rho' (1 - \theta + \theta G')}{1 - \psi^{-1} E(\beta P'/P) \rho' (1 - \theta + \theta G')}.
\]

B.4 Equilibrium Conditions

A set of conditions below constitutes the recursive competitive equilibrium. I log-linearize these conditions around steady state to compute impulse response functions.

Households:

\[
P_t = C_t^{-1}
\]

\[
w_t = \eta_t C_t
\]

\[
C_t^{-1} = \beta E_t C_t^{-1} R_{t+1}
\]

Firms:

\[
\tilde{z}_{it}^{1/(1-\alpha-\nu)} h_t = b_t
\]
\[ h_t = (1 - \alpha - \nu) z_t^{1/(1 - \alpha - \nu)} \Gamma_t^{\alpha/(1 - \alpha - \nu)} \Omega_t^{\nu/(1 - \alpha - \nu)} \]

\[ \Gamma_t = \frac{\alpha}{R_t - 1 + \delta} \]

\[ \Omega_t = \frac{\nu}{w_t} \]

\[ Y_t = \frac{k_{z,t}}{k_{z,t} - 1/(1 - \alpha - \nu)} \chi_t z_t^{1/(1 - \alpha - \nu)} \Gamma_t^{\alpha/(1 - \alpha - \nu)} \Omega_t^{\nu/(1 - \alpha - \nu)} \]

\[ w_t \xi_t = E_t \beta \frac{P_t}{P_{t-1}} \left[ \mu h_t(\xi_{t+1})^{-(k_{z,t} - 1)}} - b_{t+1} \xi_{t+1} \right] \]

\[ \chi_{t+1} = \frac{\kappa}{\bar{k}} M \]

Banks:

\[ G_t = E_t \beta \frac{P_{t+1}}{P_t} \left( \rho_{t+1} \phi_t + R_{t+1} \right) \{ (1 - \theta) + \theta G_{t+1} \} \]

\[ \phi_t = \psi_t^{-1} G_t \]

\[ \kappa \frac{\xi_t}{\bar{k}} M = \phi_t N_t \]

\[ N_t = \theta [ \rho_t \kappa \chi_t + R_t N_{t-1} ] + \omega \kappa \chi_t \]

\[ \rho_t \equiv V_t / \kappa - R_t \]

\[ V_t = [ 1 - \Pi^e (\xi_t) ] b_t + \lambda_t F_t \]

\[ F_t = \frac{k_{z,t}}{k_{z,t} - 1/(1 - \alpha - \nu)} h_t \left( 1 - \xi_t^{-(k_{z,t} - 1)}} \right) \]

Market-clearing conditions:

\[ Y_t = C_t + \kappa \chi_{t+1} + K_{t+1} - (1 - \delta) K_t + \chi_t (1 - \lambda) F_t \]

\[ K_t = \frac{k_{z,t}}{k_{z,t} - 1/(1 - \alpha - \nu)} \chi_t z_t^{1/(1 - \alpha - \nu)} \Gamma_t^{(1-\nu)/(1-\alpha-\nu)} \Omega_t^{\nu/(1-\alpha-\nu)} \]

\[ L_t = \chi_t \Gamma_t^{\alpha/(1-\alpha-\nu)} \Omega_t^{1/(1-\alpha-\nu)} z_t^{1/(1-\alpha-\nu)} \mu + \chi_{t+1} \xi_t / 2 \]

LOMs for exogenous variables:

\[
\begin{bmatrix}
\log z_{t+1} \\
\psi_{t+1} - \psi_{ss}
\end{bmatrix} =
\begin{bmatrix}
\rho_z & 0 \\
0 & \rho_\psi
\end{bmatrix}
\begin{bmatrix}
\log z_t \\
\psi_t - \psi_{ss}
\end{bmatrix}
+ \begin{bmatrix}
\epsilon_{z,t+1} \\
\epsilon_{\psi,t+1}
\end{bmatrix}
\]

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