AN EXPERIMENTAL AND THEORETICAL INVESTIGATION OF THE EFFICIENCY OF
PLANETARY GEAR SETS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy
in the Graduate School of The Ohio State University

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ABSTRACT

Planetary gear sets are preferred in many power transmissions for their advantages such as higher power density, lower radial support loads, lower noise levels, greater kinematic flexibility, and manufacturing error insensitivity compared to counter-shaft gearing. One potential disadvantage of planetary gear sets is power losses due to multiple planet branches, resulting in increased numbers of gear meshes and bearings. The main goal of this study is to theoretically and experimentally investigate the power losses of planetary gear sets.

An experimental power loss database for planetary gear sets is first developed through tightly-controlled experiments. Dependence of power loss on operating conditions is quantified. The influences of number of planets and planet surface roughnesses on planetary power loss are also included in the experimental test matrix.

Sources of planetary gear set power loss are grouped in two categories as load-dependent and load-independent losses. Major components of load-dependent power loss are gear mesh and planet bearing mechanical losses while load-independent power losses are formed by gear and carrier drag, planet bearing viscous losses, and gear mesh pocketing losses. Experimental data is analyzed to separate the load-dependent and load-independent effects, and separate modeling studies are performed corresponding to each. For modeling of the gear mesh mechanical losses, gear and carrier drag losses, and planet bearing viscous losses, well-established modeling
methodologies from recent literature are employed. In order to bridge the gaps in the literature, novel models for planet bearing mechanical power loss, and internal and external helical gear mesh pocketing power loss are developed.

For prediction of planet bearing mechanical power losses, a planet bearing load distribution model is first proposed to predict load intensities along the roller contacts due to combined radial force and overturning moment caused by helical gear mesh forces. This model takes into account planet bearing macro-geometry as well as micro-modifications to the roller and race surfaces. Predicted load distributions, bearing kinematic relationships and an elastohydrodynamic rolling power loss model are combined to predict load-dependent power loss of a planet bearing.

A new fluid dynamics model is proposed to predict pocketing power losses at both external (sun-planet), and internal (ring-planet) meshes of a planetary gear set. A numerical procedure and companion discretization scheme are proposed to quantify the pocket volumes, escape areas, and area and volume centroids from the transverse involute geometry of helical gears. Conservation laws of mass, momentum, and energy are applied to the governing multi-degree-of-freedom fluid dynamics system to predict power losses due to squeezing of air, oil, or an air-oil mixture from gear meshes.

Models for key components of planetary power loss are brought together in a single methodology to predict both load-dependent and load-independent power losses of a planetary gear set, including mechanical gear mesh (internal and external) losses, mechanical and viscous planet bearing losses, gear mesh pocketing (internal and external) losses, and gear and carrier drag losses. This methodology is used to simulate the planetary power loss experiments to
demonstrate its accuracy within wide ranges of operating, lubrication, surface, and design conditions.
To my loving and supportive fiancée
ACKNOWLEDGMENTS

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FIELDS OF STUDY

Major Field: Mechanical Engineering
   Focus of Gearing, Dynamic Modeling, and Power Loss Modeling
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<tr>
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<tr>
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<td>$\rho_{e(i,j)}^{(n)}$</td>
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CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

In a planetary gear set, also known as an epicyclic gear set, three or more power branches, each formed by a planet gear, are used to transmit power, allowing high power density (power to volume ratio) and compact designs. As shown in Figure 1.1, a simple planetary gear set consists of a central external gear (called a sun), a number (typically 3 to 7) of identical external gears (planets or pinions, also called stars in case of the carrier being non-rotating) in mesh with the sun gear as well as an internal gear (called a ring). All of the planets are held by a rigid structure (called a carrier) through planet pins and cylindrical roller bearings. The typical type of planet bearings are needle bearings in case of automotive applications while cylindrical roller bearings are also used in other planetary applications.

The axi-symmetric arrangement of a planetary gear set allows cancellation of radial gear mesh forces acting on the central members (sun, ring and carrier) leaving zero nominal radial loads, easing the support structure and bearing requirements significantly. This canceling of
Figure 1.1 Mechanical components of a planetary gear set.
radial loads allows for the use of floating central members to reduce sensitivity of the gear set to various manufacturing errors [1.1]. These advantages make planetary gearing useful in many power transmission applications including automotive, rotorcraft, aerospace, and wind turbines. As they are the primary power transmission components of such systems, power losses caused by them become critical to the overall power transmission efficiency. The research presented in this thesis focuses on prediction and measurement of planetary gear set power losses.

Power losses in any geared transmission can be broken into two main categories, load-dependent (mechanical) power losses and load-independent (spin) power losses as shown in Figure 1.2:

\[ P = P_{LD} + P_{LI}. \]  
\[ (1.1) \]

Load-dependent power losses can be attributed to loaded, lubricated contacts of the mating surfaces at the gear meshes and rolling element bearings. They originate from the relative sliding and rolling action of the elastohydrodynamic lubricant film at the contact interface that can be loosely defined as friction [1.2-1.4]. The mechanical loss of a gear system can be further split into its two main gear mesh \( P_{mg} \) and bearing \( P_{mb} \) components as

\[ P_{LD} = P_{mg} + P_{mb}. \]  
\[ (1.2) \]

Mechanisms for spin power losses are entirely different from those of the mechanical losses. As contact friction is negligible under no-load conditions, interactions of the gear set components with the surrounding medium (oil, air or any mixture of oil and air) are responsible for such losses. They have been often referred to as churning (in case of oil) and windage (in case of air or air/oil mix) losses. Regardless of the type of the fluid around the gear set, main
Figure 1.2 Main components of power losses of a planetary gear set.
components of spin power losses of a gear set are (i) the drag losses of the rotating components \( P_d \), (ii) losses due to pocketing of the fluid at the gear mesh interfaces \( P_p \) and (iii) viscous losses of rolling element bearings \( P_{vb} \), in addition to other secondary sources such as synchronizers and oil seals [1.5]. Accordingly,

\[
P_{LI} = P_d + P_p + P_{vb}.
\]  

The main goal of this research is to develop analytical models to predict mechanical and spin power loss components of planetary gear sets and validate them through comparisons to data collected through tightly-controlled planetary power loss experiments.

1.2 Literature Review

1.2.1 Gear Pair Mechanical Loss Studies

As reviewed extensively in References [1.6-1.8], studies on mechanical gear mesh losses can be grouped in three categories (i) experimental investigations, (ii) prediction models using a constant or empirically estimated coefficient of friction, and (iii) physics-based model employing elastohydrodynamic lubrication (EHL) formulations of the contacting surfaces. Experimental studies were mostly on spur and helical gear pairs focusing on heat generation and power losses [1.9-1.13]. These studies used data collected under loaded and unloaded conditions to separate spin losses from the total power loss in order to isolate mechanical gear mesh losses provided the gearbox bearing losses can be estimated accurately. Basic gear design parameters such as module, face width, pressure angle, helix angle, surface roughness type (shaved, ground, etc.) as well as surface roughness amplitudes (machined, chemically polished, etc.) were used as variable
parameters in these experiments to quantify their impact on mechanical power losses. Among them, smaller modules and smoother surfaces were reported to reduce power losses.

Recognizing that the instantaneous contact of a gear at any certain point can be represented by a two cylindrical rollers in relative sliding, earlier modeling efforts on gear mesh mechanical power losses [1.14-1.17] used a sliding friction coefficient that is either estimated as a constant parameter or determined empirically via twin-disk experiments. These models were limited in their accuracy and ability to handle wider ranges of lubricants and operating conditions while effects such as surface roughness were not included methodically.

The state-of-the-art in modeling of gear mesh mechanical power losses exists in a number of recent studies where EHL theory was employed to predict friction at gear contacts, not using a user defined or empirical friction description [1.2-1.4]. These studies either relied on a real-time transient analysis of EHL conditions of gear contacts as they move along the tooth surface or used friction models based on regression analysis of EHL results covering wide ranges of key contact parameters. Some of these models included only the sliding power losses under full-film lubrication conditions [1.2] while others captured both sliding and rolling losses that take place at gear contacts [1.4]. The latter models used mixed (or boundary) EHL formulations such that actual asperity (metal-to-metal) contacts are analyzed. This was shown to be absolutely necessary for prediction of power losses of heavily loaded and lower speed gear sets with typical surface roughnesses. This EHL based method was first used for spur gears where load along the face width can be assumed uniform. It was also applied to helical gears by using a more complex load distribution solution algorithm that takes into account the compliance of the gear tooth [1.18-1.19]. This method was also shown to compare well with the experiments of Petry-Johnson et al. [1.9].
This study will employ the methodology proposed by Li and Kahraman [1.4] to predict the mechanical power losses of external (sun-planet) meshes of the planetary gear sets. This model will be modified to incorporate new regression friction formulae of lubricants of interest. An internal gear pair mechanical power loss model will also be developed using the same methodology to handle ring-planet meshes of the planetary gear set.

1.2.2 Bearing Mechanical and Viscous Power Loss Studies

Empirical analysis of bearing mechanical power loss is the most commonly used method that is often attributed to Palmgren [1.20] and furthered by Harris [1.21]. A few bearing analysis packages have been developed such as ADORE [1.22], SHABERTH [1.23], and CYBEAN [1.24] for commercial purpose with little published information available in regards to the theory behind them. These packages have the capability to analyze in great detail roller-cage interactions, load distributions and bearing slip conditions. They lack the ability to predict power losses as they are not equipped with any EHL analysis of rolling friction mentioned previously. Nelias et al [1.25] and Chang et al [1.26] provide EHL studies on cylindrical roller bearings with simple radial loads and load distributions.

When used with helical gears, planet bearings, shown in Figure 1.1, are subjected to overturning moments in addition to a radial force. Literature lacks combined load distribution and mechanical power loss models for needle bearings under such loading conditions. The influence of other needle bearing features such as needle crowning on rolling power losses have also not been investigated in these previous studies. The proposed research aims at developing such a mechanical power loss model for planet bearings. For load-independent, viscous power
losses of bearings, empirical formulae of Harris [1.21] have been used extensively. The same
formulae will be adapted here for bearing spin losses.

1.2.3 Gear Spin Power Loss Studies

There are various published experimental studies on the power loss of a gear or a disk
rotating fully, or partially submerged in a lubricant [1.27-1.29]. Empirical relations for
dimensionless churning torque were provided from experiments for disks [1.30] and for gears in
mesh [1.31-1.32]. Ariura et al [1.33] presented experimental load independent power loss
measurements on jet lubricated spur gears. The experiments by Petry-Johnson et al [1.9]
provided data on the influence of face width and module of spur gears on spin losses under jet
lubricated (windage) conditions. More recently, Petry-Johnson [1.34] and Moorhead [1.35]
performed spur gear pair oil churning experiments with static oil levels as a test parameter.

A theoretical approach to load independent power loss analyzing oil fling-off is provided
by Akin et al [1.36-1.37]. Models on windage losses of gears rotating only in air have been
limited to empirical studies [1.38-1.39] or computational fluid dynamics models [1.40-1.41]. As
stated by the review paper of Eastwick and Johnson [1.42], most of these studies excluded losses
associated with fluid flow at the gear mesh interface. Pechersky and Wittbrodt [1.43] proposed a
theoretical analysis to compute the pressure and velocities of oil trapped in the meshing zone of
spur gears without computing the resultant power loss. Diab et al [1.39] introduced a model for
air trapping in the meshing zone. Most recently, Seetharaman and Kahraman [1.5] proposed a
fluid-mechanics based formulation to compute drag power losses along the sides and periphery of
rotating spur gears as well as pocketing losses at the gear mesh interface under dip lubrication
conditions. This incompressible flow formulation for oil churning losses was later replaced by a
compressible flow formulation to predict the same for windage losses under jet lubrication conditions [1.44]. The same investigators [1.45] provided comparisons between their predictions and experiments to validate their approach.

While the drag loss formulations of Seetharaman and Kahraman [1.5] are suitable for rotating components of a planetary gear set, a new pocketing loss model must be developed to handle internal and external helical gear meshes in order to model an automotive planetary gear set.

1.2.4 Planetary Gear Train Power Loss Studies

As the focus of the above studies has been on losses associated with a single gear or at most a gear pair, very little published work is available for planetary gear power losses. Limited experiments [1.46-1.49] provided some data on power loss of planetary gear sets within very limited parameter ranges. There is no study on accurate EHL-based prediction of power losses of the planetary gear sets. The only studies that are available on planetary gear train efficiency are limited to the kinematic analyses of the gear train, combined with either constant or empirically obtained efficiency values assigned to each gear mesh [1.48-1.52]. Chen and Angeles [1.53] provided the same analysis making a virtual-power flow argument. Fanghella [1.54] proposed a computational method independent of power flow analysis to predict the planetary gear train efficiency given the individual gear mesh efficiencies. Due to this apparent lack of experimental data and actual physics-based methods to predict losses due to individual gear and bearing contacts, a fundamental understanding of power losses of planetary gear sets does not exist.
1.3 Thesis Objectives and Scope

The overall aim of this study is to provide a complete investigation of planetary gear set power losses including both spin and mechanical components through comprehensive modeling as well as extensive laboratory experiments. The ultimate goal is to arrive at a validated power loss model of planetary gear sets. In addition to making complete use of verified external gear mesh mechanical loss formulations of Li and Kahraman [1.4] and drag spin loss formulations of Seetharaman and Kahraman [1.5], various new component-level formulations will be developed here to devise a complete power loss prediction model of planetary gear sets. Individual objectives of this dissertation research are as follows:

- Develop an experimental methodology for measurement of spin and mechanical power losses of a planetary gear set having \( n \) planet branches \((n \in [3,6])\), and conduct tightly-controlled experiments using this methodology to generate an extensive experimental database that includes various design, surface, lubricant and operating parameters including number of planets, torque, speed, lubricant temperature, and surface roughness amplitudes.

- Develop a new load distribution model for caged needle bearings to predict mechanical power loss using rolling friction formulations of Li and Kahraman [1.4].

- Develop a new pocketing power loss model for external and internal helical gear meshes considering leakage effects, as well as interactions between cavities occurring simultaneously at the gear mesh. Provide windage (compressible flow) and churning (incompressible flow) power loss formulations for the pocketing power losses of helical gear mesh interfaces.
• Devise a methodology to predict overall planetary gear set power loss (both spin and mechanical components) of a planetary gear set incorporating the above new and existing models according to Figure 1.2 and Eq. (1.1)-(1.3).

• Compare the methodology’s predictions to the experimental results to assess the accuracy of the methodology. Employ models to perform parametric studies towards arriving at guidelines on how to design more efficient planetary gear sets.

As the targeted application is the automotive planetary gear set in automatic transmissions, CVT transmissions and transfer cases, only needle bearings with cages will be considered as planet bearings. The full-complement (cageless) needle bearings will be kept outside the scope of this study. While the methodology will be general, specific friction models will be kept specific to automotive transmission lubricants. The operating speed, torque and temperature ranges considered in this study will also be representative of automotive conditions, as well as the gear surface roughness and accuracy conditions.

1.4 Thesis Outline

In line with the specific objectives listed above, first, the experimental study will be presented in Chapter 2. Design and procurement of an experimental rig and its instrumentation for measurement of planetary gear set torque loss will be described in this chapter. Results of the power loss tests on this experimental rig will be presented to quantify the impact of operating conditions (input torque, input speed, and inlet lubricant temperature), gear set design parameters (number of planets, \( n \)), and tooth surface roughness amplitudes. Torque loss measurements will be separated into load dependent and load independent components in order to establish mechanical and spin components of power losses.
Chapter 3 will provide a mechanical power loss model for caged needle bearings of planets. A load distribution model for caged cylindrical roller bearings will be presented, followed by a mechanical power loss model that combines the predicted load distribution with bearing kinematics and EHL rolling power loss predictions [1.4].

Chapter 4 will be dedicated to the development of a pocketing power loss model for helical gear meshes. A numerical approach for calculating volume of lubricant squeezed by the gear mesh will be presented. With numerically calculated, time-varying volumes, exit areas, and other geometric parameters, pocketing power losses will be predicted for both compressible and incompressible flow conditions.

All power loss components of a planetary gear set will be combined in Chapter 5 to predict overall power loss (both spin and mechanical components) of a planetary gear set, incorporating the new planet bearing mechanical power loss model, the new helical gear pocketing model, the existing gear mesh mechanical loss model, the existing drag loss formulation, and the existing bearing viscous power loss formulation. The model will be used to simulate the experiments presented in Chapter 2 to demonstrate the accuracy of the proposed methodology.

Chapter 6 summarizes this study and lists its major conclusions and contributions, ending with a number of recommendations for future research in this field.
References for Chapter 1


[1.19] LDP, Gear Load Distribution Program, Gear Dynamics and Gear Noise Research Laboratory, The Ohio State University, 2011.


CHAPTER 2

EXPERIMENTAL INVESTIGATION OF THE EFFICIENCY OF

PLANETARY GEAR SETS

2.1 Introduction

In this chapter, results from an experimental study on power losses of planetary gear sets are presented. The experimental set-up includes instrumentation for an accurate measurement of power loss, and a specialized test apparatus to operate a planetary gear set under tightly-controlled speed, load and oil temperature conditions. The test matrix consisted of gear sets having 3 to 6 planets operating under loaded and unloaded conditions in order to separate load independent (spin) and load dependent (mechanical) power losses. The test matrix also included tests with planet gears having two levels of tooth surface roughness amplitudes as well as tests at varying oil inlet temperature.

The overall goal of this chapter is to provide an experimental investigation of planetary gear set power loss including both spin and mechanical components through extensive tests. In accordance with this goal, specific objectives of this chapter are as follows:
• Develop an experimental methodology for measurement of spin and mechanical power losses of a planetary gear set having \( n \) planet branches \((n \in [3,6])\).

• Perform tightly-controlled tests using this methodology to generate an experimental database that quantifies the contributions of spin and mechanical losses to the total loss.

• Quantify the impact of various design, surface, lubricant and operating parameters including number of planets, torque, speed, lubricant temperature, and surface roughness amplitudes on the components of planetary gear set power loss.

2.2 Experimental Set-up

2.2.1 Test Gearbox and Instrumentation

An experimental set-up previously designed for planet load sharing and rim deflection studies [2.1-2.3] was adapted with several modifications for the measurement of torque loss. The gear box cross-section shown in Figure 2.1 reveals two planetary gear sets of the same basic design (one test set and one reaction set) connected to each other in a back-to-back arrangement. The sun gears and carriers of both gear sets are connected to each other, respectively, to form a power circulation loop. The ring gear of the test gear set is fixed (splined to the housing) while the reaction gear set ring gear is connected to a loading flange to allow application of a controlled magnitude of reaction torque via a torque arm. In this arrangement, the gear sets are loaded through this external loading mechanism while a relatively small DC motor attached to the sun gear shaft is sufficient to provide a torque input equal to the loss of the entire gearbox. This allows operation at the desired speed conditions. This arrangement provides circulation of power
Figure 2.1 Cross-sectional view of the test gearbox.
up to ~420 kW with the use of only a 25 kW DC motor. The cross-section shown in Figure 2.1 also shows that the sun gear shaft is permitted to float radially. This eases conditions that may cause planet load sharing issues [2.1].

Fixtures to connect a torque-meter between the DC motor and the gearbox were designed and procured and are shown in Figure 2.2. The torque sensor (Himmelstein MCRT 48703V(2-3)) has a torque range of 225 Nm and resolution of 0.05% of the maximum torque. Two flexible couplings (one single-stage and one two-stage) and a support bearing were selected to prevent any undesirable loading on the torque sensor. The two stage coupling is used to reduce the effects of the floating sun shaft, limiting radial forces on the torque-meter, while the support bearing eliminated radial forces on the torque-meter from the universal joint attaching the DC motor.

The gearbox is lubricated via a rotary union on the sun shaft. Lubricant is centrifugally moved to the gear meshes and bearings from the center of the sun shaft. A 6 kW lubricant heater is implemented in the external lube system in order to achieve and maintain the desired lubricant inlet temperature within a range of 40 to 100°C. A thermocouple on the inlet side is used to measure the oil inlet temperature. In order to avoid any accumulation of oil at the bottom of the gearbox housing which would cause churning losses, large openings were machined along the lower section of the housing allowing the oil to drain effectively. An oversized oil pan attached to the gearbox housing allows pooling of the oil to be pumped back to the main tank of the external lube system.
Figure 2.2 A view of the test set-up and instrumentation.
2.2.2 Test Matrix and Specimens

The test matrix includes ranges of speed, torque and temperature as operating condition parameters. It also includes number of planets as a design parameter as well as planet tooth roughness amplitude as a surface parameter. An input (sun) torque range of 0 to 1000 Nm (increment of 250 Nm) and an input (sun) speed range of 1000 to 4000 rpm (increment of 1000 rpm) were considered in the test matrix. Values of 40, 60, and 90°C were chosen as discrete inlet oil temperature values. With the automatic transmission fluid used in these tests, these temperature values represented viscosity values of 29.77, 15.39, and 7.32 cSt, respectively. Three, four, five and six-planet carriers were employed to perform tests with number of planets, \( n \) (\( n \in [3,6] \)), as a test variable. Figure 2.3 shows the 4, 5 and 6-planet carriers used in this study. Here, the 6-planet carrier was used for the 3-planet experiments as well. Table 2.1 lists the parameters of the test gears, while Table 2.2 lists the parameters of the test planet needle bearings.

Two sets of planet gears were procured in order to investigate the effect of surface roughness. A set of planets with shaved surfaces with root-mean-square (rms) roughness of about 0.4 µm and another set with chemically polished surfaces (rms roughness of about 0.1 µm) were both tested as part of this study. With these parameters, the test matrix included 300 individual tests to constitute a database for this gear set and lubricant. Baseline measurements of power loss for the 6-planet arrangement with shaved planet surfaces on both the test and reaction sides were performed first to establish the power loss of the reaction gear set. For the remaining tests, the reaction side of the gearbox was kept the same configuration, while the number of planets and roughness variations were later applied only to the test gear set.
Figure 2.3 Four, five and six-planet test carriers.
Table 2.1 Parameters of the test gear sets

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Table 2.2 Parameters of the test planet needle bearings

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<td>Planet bore diameter [mm]</td>
<td>34.025</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Unloaded tests (i.e. tests with no external load applied to the reaction ring gear) were performed in order to quantify the spin loss of the planetary gear set at given speed values. These spin loss values were subtracted from the loaded test results to isolate the load dependent losses. In other words, mechanical losses were calculated as the difference between the loaded power loss at a given speed and the spin power loss measured at the same speed.

2.3 Measured Planetary Gear Set Power Loss Results

2.3.1 Influence of Number of Planets

Figure 2.4(a) shows the variation of the total power loss, \( P \), of the 3-planet gear set (with shaved planet surfaces) with sun gear speed (\( \Omega_{sun} \)) at various sun gear torque (\( T_{sun} \)) values, and an inlet oil temperature of 90°C. The \( P \) values in this figure pertaining to zero torque represent the gear set spin losses, \( P_{LI} \). The gear set mechanical losses, defined as \( P_{LD} = P - P_{LI} \), are shown in Figure 2.4(b). The corresponding total and mechanical efficiencies are defined respectively as

\[
\eta = \frac{P_{in} - P}{P_{in}},
\]

\[
\eta_{LD} = \frac{P_{in} - P_{LD}}{P_{in}},
\]

where \( P_{in} = (\pi / 30)T_{sun}\Omega_{sun} \), is the input power. They are presented in Figures 2.4(c) and 2.4(d), respectively. Here, it is observed that the \( \eta \) values range from 98.85% to 99.1% while the \( \eta_{LD} \) values are within 98.9% and 99.4%. Spin power losses constitute a larger percentage of the total losses at high-speed and low-load conditions while the dominance of \( P_{LD} \) is evident under high-
Figure 2.4 Measured (a) total power loss, (b) mechanical power loss, (c) total efficiency, and (d) mechanical efficiency of a 3-planet shaved gear set at 90°C.
Figure 2.4 (continued).

(c) 

\[ \eta \quad \% \]

\[ \Omega_{sun} \quad \text{[RPM]} \]

(d) 

\[ \eta_{LD} \quad \% \]
load conditions. For instance, at $\Omega_{\text{sun}} = 4000$ rpm, $P = 3.78$ kW at $T_{\text{sun}} = 1000$ Nm of which about 85% is $P_{LD}$ and the remaining 15% is $P_{LI}$. At the same speed value and $T_{\text{sun}} = 250$ Nm about half of $P$ is $P_{LI}$. This is the reason why the total efficiency values dip significantly in Figure 2.4(c) at lower $T_{\text{sun}}$ values. It is also observed in Figures 2.4(a) and (b) that $P_{LD}$ varies almost linearly with speed confirming trends in earlier gear pair experiments [2.4] while $P_{LI}$ varies somewhat exponentially with speed, again in agreement with gear spin loss experiments [2.5].

Figures 2.5 through 2.7 show the measured $P$ and $P_{LD}$ values for 4, 5 and 6-planet gear set configurations, respectively, at the same operating conditions as in Figure 2.4. It is noted from Figures 2.4 to 2.7 that the mechanical power loss results for 3 to 6-planet gear sets are rather close to one another. The 4-planet gear set losses correspond to a mechanical efficiency range of 99.0–99.4% while $99.1 < \eta_{LD} < 99.4\%$ for both the 5-planet and 6-planet gear sets. As shown in Figure 2.8, the direct comparison of $P$ values of each gear set at 0 (spin loss), 500 and 1000 Nm confirms this observation, with the exception of Figure 2.8(a) that shows some separation of $P_{LI}$ values with the 6-planet gear set having up to 0.2 kW more spin loss than the 3-planet gear set.

The measured $P_{LD}$ values for gear sets having different number of planets are close to one another. This suggests that the mechanical power losses of a gear set operating at a certain speed, load, and temperature condition remains similar regardless of its number of planets. It also indicates that the portion of the input torque transmitted by each planet branch $T_{PB} = T_{\text{sun}}/n$ and speed are the two parameters that should be used to assess and present the mechanical power losses of planetary gear sets. With this, all the data points from Figures 2.4 to 2.7 can be rearranged by using the $T_{PB}$ and $\Omega_{\text{sun}}$ as the test parameters and the mechanical power loss per
Figure 2.5 Measured (a) total power loss, and (b) mechanical power loss of a 4-planet shaved gear set at 90°C.
Figure 2.6 Measured (a) total power loss, and (b) mechanical power loss of a 5-planet shaved gear set at 90°C.
Figure 2.7 Measured (a) total power loss, and (b) mechanical power loss of a 6-planet shaved gear set at 90°C.
Figure 2.8  Comparison of $P$ for 3 to 6-planet shaved gear sets at a) 0 Nm, b) 500 Nm, and c) 1000 Nm for 90°C.
Figure 2.8 (continued).

\[ P \quad [\text{kW}] \]

\[ \Omega_{sun} \quad [\text{RPM}] \]
planet branch \( (P_{MBL} = P_{LD}/n) \) as the output parameter. This reduces Figures 2.4 to 2.7 into a single plot as shown in Figure 2.9. This figure indicates that the \( P_{LD} \) value of a 6-planet gear set at a given \( T_{sun} \) and \( \Omega_{sun} \) is approximately equal to the \( P_{LD} \) value of its 3-planet equivalent set under the same conditions, while the \( P_{MBL} \) value for the 6-planet system is approximately half of that in the 3-planet system. This also indicates clearly that changing the number of planets of a planetary gear set is not a viable way of significantly changing the gear set efficiency.

2.3.2 Influence of Oil Inlet Temperature

Figure 2.10 compares the measured \( P_{LD} \) values of the 3-planet gear set at \( T_{sun} = 1000 \) Nm and the corresponding \( P_{LI} \) values at three oil inlet temperature levels of 40, 60 and 90°C. Here, an increase in temperature from 40 to 90°C results in a decrease in viscosity from 29.77 to 7.32 cSt for this particular lubricant. In line with the gear pair power loss predictions [2.6-2.8] and measurements [2.4], this sizable reduction in viscosity, results in sufficient reductions in the lubricant film thickness that causes the \( P_{LD} \) increases of 0.21 kW observed in Figure 2.10(a). Both drag and bearing viscous components of gear pair spin losses were reported to increase with increased viscosity [2.5,2.9]. Figure 2.10(b) points to the same effect for a planetary gear set seeing a 0.19 kW increase in power loss accompanying a decrease of lubricant inlet temperature from 90°C to 40°C.

Figures 2.11 to 2.14 present the corresponding temperature comparisons for the 4, 5 and 6-planet shaved gear sets, as well as the 6-planet chemically polished gear set. Overall trends in these figures are the similar to Figure 2.10. Figures 2.10(a) to 2.13(a) show little variance in
Figure 2.9  Branch mechanical power loss as a function of branch torque and sun speed at 90°C, 3 to 6-planet shaved gear sets.
Figure 2.10  Comparison of the a) mechanical power losses at 1000 Nm, and b) spin power losses of a 3-planet shaved gear set at 40, 60, and 90° C.
Figure 2.11 Comparison of the a) mechanical power losses at 1000 Nm, and b) spin power losses of a 4-planet shaved gear set at 40, 60, and 90°C.
Figure 2.12  Comparison of the a) mechanical power losses at 1000 Nm, and b) spin power losses of a 5-planet shaved gear set at 40, 60, and 90° C.

\[ P_{LD} \text{ [kW]} \]

\[ P_{LI} \text{ [kW]} \]

\[ \Omega_{sun} \text{ [RPM]} \]
Figure 2.13 Comparison of the a) mechanical power losses at 1000 Nm, and b) spin power losses of a 6-planet shaved gear set at 40, 60, and 90° C.

Figure 2.13 Comparison of the a) mechanical power losses at 1000 Nm, and b) spin power losses of a 6-planet shaved gear set at 40, 60, and 90° C.
Figure 2.14  Comparison of the a) mechanical power losses at 1000 Nm, and b) spin power losses of a 6-planet chemically polished gear set at 40, 60, and 90° C.
mechanical power loss with temperature. This implies that the lubricant film is small with respect to the surface roughness and all inlet temperatures result in a large amount of asperity contact. Figure 2.14(a), on the other hand, shows an appreciable increase in mechanical power loss with an increase in temperature, implying the increase in temperature is accompanied by an increase in asperity contact.

The tests at lower temperatures presented in Figure 2.10 to 2.13 should be interpreted with caution. While the oil inlet temperatures were maintained at the specified levels (40, 60 and 90° C), this does not directly represent the actual bulk temperatures of the gear contact surfaces or the temperature of the oil at the gear mesh and bearing surface interfaces. Three thermocouples were implemented in the gearbox to measure (i) the oil inlet temperature, (ii) oil exit temperature and (iii) the bulk temperature of the ring gear. A set of these three measurements are shown for, a spin \( T_{sun} = 0 \) Nm test in Figure 2.15, as well as a heavily loaded \( T_{sun} = 1000 \) Nm test in Figure 2.16, for the 6-planet gear set at all three oil inlet temperatures considered. Here, large temperature increases of the lubricant and the ring gear accompany large power losses in the system.

### 2.3.3 Influence of Planet Surface Roughness

Figure 2.17 shows the power loss measurements for a 6-planet gear set with chemically polished planets at the same operating conditions as Figure 2.7. A qualitative comparison of Figures 2.7 and 2.17 highlights the impact of roughness of contacting gear surfaces on the resultant power losses. As expected, the spin losses \( P \) at \( T_{sun} = 0 \) do not vary because the surface roughnesses influence only the mechanical losses. The \( P_{LD} \) values, meanwhile, are
Figure 2.15 Inlet lubricant, outlet lubricant, and ring gear temperatures at 0 Nm for a) 40°C, b) 60°C, and c) 90°C.
Figure 2.15 (continued).

![Graph](image-url)

- **Temp** [deg C]
- **Duration** [sec]
- **Input Speed**
- **Ring**
- **Outlet**
- **Inlet**
- **$\Omega_{sun}$** [rpm]
Figure 2.16  Inlet lubricant, outlet lubricant, and ring gear temperatures at 1000 Nm for a) 40°C, b) 60°C, and c) 90°C.

Figure 2.16  Inlet lubricant, outlet lubricant, and ring gear temperatures at 1000 Nm for a) 40°C, b) 60°C, and c) 90°C.
Figure 2.16 (continued).

![Graph showing temperature (Temp [deg C]) and duration (Duration [sec]) with marked points for Ring, Outlet, Inlet, and Input Speed with Omega symbol (Ωsun [rpm]).]
Figure 2.17 Measured (a) total power loss, and (b) mechanical power loss of a 6-planet chemically polished gear set at 90°C.
reduced in Figure 2.17 with smoother planet surfaces compared to Figure 2.7 with rougher (shaved) surfaces.

The direct differences between the $P$ values of Figures 2.7 and 2.17 ($P_{shaved} - P_{polished}$) are plotted in Figure 2.18. Here, the maximum reduction in $P$ value by making planet tooth surfaces smoother is about 0.65 kW at $T_{sun} = 1000$ Nm and $\Omega_{sun} = 4000$ rpm. At this test condition with $P_{in} = 419$ kW, this improvement amounts to a 0.15% improvement in the mechanical efficiency of the gear set. This might appear only a slight improvement. However, in these tests the sun and ring gears had similar roughnesses (about 0.4 $\mu$m rms). Accordingly, the composite gear contact roughnesses in Figure 2.7 were $S_{co} = [(S_{rms})^2_1 + (S_{rms})^2_2]^{1/2} = [(0.4)^2 + (0.4)^2]^{1/2} = 0.56$ $\mu$m compared to $S_{co} = [(0.4)^2 + (0.1)^2]^{1/2} = 0.41$ $\mu$m in Figure 2.10. This 25% reduction in composite gear contact roughness resulted in a 0.15% efficiency increase. If one were to chemically polish the sun and ring gear surfaces as well to the same levels, one would obtain $S_{co} = [(0.1)^2 + (0.1)^2]^{1/2} = 0.14$ $\mu$m with larger mechanical efficiency improvements over a broader range of load and speed.

Figures 2.19 and 2.20 show the difference in the $P$ values for inlet oil temperatures of 60, and 40°C, respectively. Here larger differences in mechanical power loss, $P_{LD}$, are seen with decreasing temperature. At 60°C, with $T_{sun} = 1000$ Nm and $\Omega_{sun} = 4000$ rpm, a power loss improvement of 0.82 kW is seen when chemically polishing the planet gear surfaces, which corresponds to a 0.20% improvement in mechanical efficiency. A larger improvement of 1.01 kW, corresponding to a 0.24% mechanical efficiency improvement, is seen at an oil inlet temperature of 40°C. The fact that mechanical efficiency improves when chemically polishing
Figure 2.18 The difference between the mechanical power loss values for a 6-planet gear set with shaved and chemically polished planets at 90°C.

Figure 2.19 The difference between the mechanical power loss values for a 6-planet gear set with shaved and chemically polished planets at 60°C.
Figure 2.20 The difference between the mechanical power loss values for a 6-planet gear set with shaved and chemically polished planets at 40°C.
the planet surfaces is expected; however, a larger improvement is expected when the lubricant film is the thinnest (viscosity is the lowest, temperature is the highest). Figures 2.18 through 2.20 show the opposite trend. This inconsistency with theory can be explained by the lubricant temperature increase presented in Figure 2.16. Here the rougher shaved surfaces cause more power loss, causing a larger temperature increase than that of the chemically polished surfaces. The comparisons shown in Figures 2.18 through 2.20 are actually comparing cases at different lubricant temperature conditions.

2.4 Summary

The experimental study presented in this chapter provides a database of both mechanical and spin power losses of a typical planetary gear set and lubricant. The database includes the change in power loss with varying number of planets, inlet oil temperature, surface roughness, operating speeds and operating torques. These operating configurations and conditions cover a wide range of gear and bearing lubrication conditions experienced by a planetary gear set. The results of this experimental study show that spin power loss increases slightly with increasing number of planets in the gear set while very little difference is observed amongst the mechanical power loss results for differing number of planets. They indicate that the portion of the input torque transmitted by each planet $T_{PB} = T_{sun} / n$ and speed should be used to assess and present the mechanical power losses of planetary gear sets. Changing the number of planets of a planetary gear set is not a viable way of changing the gear set mechanical efficiency. Next, gear sets having planet gears with much smoother tooth surfaces were shown to provide reduced mechanical power losses. This improvement is provided by both the reduction in sliding and rolling friction afforded by the smoother surfaces. Sliding friction is reduced due to the fact that
asperity contacts can be reduced with smoother surfaces, and rolling power loss is decreased due to the smoothening of the pressure distribution within the lubricant film [2.4]. Finally, increasing the oil inlet temperature was shown to reduce spin power losses while increasing the mechanical losses consistently.

In the following two chapters, the gaps in modeling planetary gear set power losses will be bridged by developing a planet bearing power loss model and a helical gear pocketing power loss model. In Chapter 5, an overall planetary gear set power loss methodology will be proposed. In the same chapter, the tests presented here will be simulated and compared directly to the measurements for the validation of the planetary gear set power loss prediction methodology.

References for Chapter 2


CHAPTER 3

A MECHANICAL POWER LOSS MODEL OF PLANET BEARINGS

3.1 Introduction

One necessary consequence of implementing planetary gearing in a power transmission application is introducing planet bearings for every branch. This constitutes an additional source of power loss, contributing to the overall power loss of the system in the form of mechanical and viscous bearing losses. Mechanical power losses of cylindrical roller bearings (with planet needle bearings being a special case) have been traditionally estimated by using empirical formulae \[3.1\] given as a simple linear function of radial load, \( F_r \), the bearing angular velocity, \( \omega_b \), and the bearing pitch diameter, \( d_m \), as

\[ P_{mb} = f_i F_r d_m \omega_b \]  

(3.1)

where \( f_i = 0.0002-0.0004 \) for caged bearings and 0.00055 for full compliment bearings (loose rollers with no cage), both originating from empirical studies. While this simple formula is useful in roughly estimating the bearing mechanical losses, it is not suitable for a more precise prediction where other critical parameters including surface roughness, lubricant properties, geometry, and temperature must be included.
In this chapter, a model to predict mechanical power loss of a cylindrical roller bearing is presented. While complete cylindrical roller bearings (with the outer and inner race as a unit) are used for larger-scale planetary gearboxes used in applications such as helicopter transmissions and wind turbine gearboxes, planet needle bearings used in automotive applications consist of a set of needles held by a cage with the planet pin and the planet bore acting as the inner and outer races, respectively. In either case, the same mathematical formulation applies.

The formulation proposed here consists of a model to predict the distribution of load on rolling elements (needles) of a caged cylindrical roller bearing under any arbitrary combination of radial and moment loading. This load distribution prediction is used with an elastohydrodynamic (EHL) lubrication based rolling friction model to predict the mechanical power loss of a cylindrical roller bearing.

The overall goal of this chapter is a theoretical investigation of planet bearing mechanical power losses. The specific objectives of this chapter are as follows:

- Develop a contact analysis model in order to predict cylindrical roller bearing load distribution due to arbitrary radial and moment loads.
- Combine the developed load distribution model with bearing kinematic relationships and EHL based formulations [3.2] to predict rolling power loss in cylindrical roller bearings.
- Perform a parametric study to quantify the impact of key planet needle bearing design parameters on the planet needle bearing mechanical power loss. Based on the parametric study results, arrive at guidelines in regards to design of high-efficiency planet bearings.
3.2 Cylindrical Roller Bearing Load Distribution Model

In the case of caged cylindrical roller bearings, several assumptions are made in the contact analysis. Any losses taking place along the contact between the cage and the rollers is assumed to be negligible. It is also assumed that the rollers (needles) remain equally spaced and parallel to the nominal rotational axis of the planet, as well as to each other as illustrated in Figure 3.1. Furthermore, in the case of double-row needle bearings, the right and left needles are assumed to line up perfectly with one another.

The rollers are divided into a prescribed number of $K$ axial slices of equal width. The radial force balance equation and overturning moment balance equation for the entire bearing, as well as the equation of contact for each roller are given, respectively, as [3.1]:

\[
\frac{1.24(10)^{-5} F_r}{w^{0.89}} - \sum_{z=1}^{Z} \frac{\cos \psi_z}{k_z^{0.11}} \sum_{\lambda=1}^{k_z} \left[ \Delta_z \pm \frac{1}{2} \theta(\lambda - \frac{1}{2})w \cos \psi_z - c_\lambda \right]_{1.11} = 0 , \quad (3.2a)
\]

\[
\frac{1.24(10)^{-5} M}{w^{0.89}} - \sum_{z=1}^{Z} \frac{\cos \psi_z}{k_z^{0.11}} \left\{ \sum_{\lambda=1}^{k_z} \left[ \Delta_z \pm \frac{1}{2} \theta(\lambda - \frac{1}{2})w \cos \psi_z - c_\lambda \right]_{1.11} (\lambda - \frac{1}{2})w \right. \\
\left. - \sum_{\lambda=1}^{k_z} \frac{1}{2} \ell [\Delta_z \pm \frac{1}{2} \theta(\lambda - \frac{1}{2})w \cos \psi_z - c_\lambda ]_{1.11} \right\} = 0 , \quad (3.2b)
\]

\[
\left[ \delta_r \pm \frac{1}{2} \ell \theta \right] - \frac{C_d}{2} - 2 \left[ \Delta_z \pm \frac{1}{2} \theta(\lambda - \frac{1}{2})w \cos \psi_z - c_\lambda \right]_{\text{max}} = 0 . \quad (3.2c)
\]

Referring to Figure 3.1, $\delta_r$ and $\theta$ are the radial and angular deflections of the bearing, $\ell$ is the roller length, $C_d$ is the bearing diametral clearance, $\Delta_z$ is the mean deflection of roller $z (z \in [1, Z])$, $Z$ is the number of rollers in the bearing, $\lambda$ is the index of the axial roller slices,
Figure 3.1 Definition of the needle bearing parameters used in the load distribution model.
\( w \) is the width of each slice, \( \psi_z \) is the azimuth angle of roller \( z \), \( c_{\lambda} \) is the magnitude of the roller crown drop, \( F_r \) is the bearing radial load, \( M \) is the bearing moment load, and \( k_z \) is the number of slices of roller \( z \) that are in contact.

This formulation is extended into two arbitrary orthonormal radial directions \( x \) and \( y \) and the equations are solved using Newton’s method. The load distribution (i.e. load intensity on each slice \( \lambda \) of each roller \( z \)) and the total roller load, respectively, are given by

\[
q_{\lambda,z} = \frac{[\Delta_z \pm \frac{1}{2} \theta(\lambda - \frac{1}{2})w\cos \psi_z - c_\lambda]}{1.24(10)^{-5} (k_z w)^{0.11}}, \tag{3.3a}
\]

\[
Q_z = \frac{w^{0.89}}{1.24(10)^{-5} k_z^{0.11}} \sum_{\lambda=1}^{k_z} [\Delta_z \pm \frac{1}{2} \theta(\lambda - \frac{1}{2})w\cos \psi_z - c_\lambda]^{1.11}. \tag{3.3b}
\]

Here \( q_{\lambda,z} \) is the load intensity distribution across each roller, and \( Q_z \) is the total load carried by each roller.

### 3.3 Cylindrical Roller Bearing Power Loss Model

Assuming that the relative sliding at the contacts of each roller with the inner race (planet pin) and outer race (planet gear bore) are negligible, the velocity of the contact can be considered to represent a pure rolling condition. Here, the rolling velocity is given by the rolling element angular velocity

\[
u_r = \frac{1}{4} d_m \omega_p \left(1 - \frac{d_r^2}{d_m^2}\right) \tag{3.4}
\]
where $d_r$ is the roller diameter.

With the rolling velocity, normal loads, and contact radii of each slice of each roller known, the EHL-based methodology proposed earlier by Li and Kahraman [3.2] can be used to predict the rolling power loss of a needle bearing considering the surface roughness, lubricant type, and lubricant temperature. A set of lubricated cylindrical contact parameters together with their typical ranges representative of automotive applications listed in Table 3.1 were considered here. A set of 2,880 combinations of these parameters (lubricant inlet temperature, equivalent radius of curvature of the surfaces, race RMS surface roughness, roller RMS surface roughness, hertzian contact pressure, and rolling velocity) covering possible contact conditions within these parameters ranges were defined and analyzed using the EHL model of Li and Kahraman [3.2]. The results of these 2,880 analyses were then regressed using the methodology proposed recently by Li and Kahraman [3.3] to obtain the following expression of the rolling friction power loss per unit roller width:

$$P_{WD} = \left[ (10)^{\frac{5}{m}} \frac{W}{m} \right] \exp \left[ \hat{a}_0 + \bar{G} (\hat{a}_1 \theta + \hat{a}_2 \hat{S}_{co}) \right] \left[ \hat{b}_1 \theta + \hat{b}_2 \ln \theta + \hat{b}_3 \ln \hat{U} + \hat{b}_4 \hat{S}_{co} \right] \cdot \bar{G} \left[ \hat{b}_5 \ln \hat{U} + \hat{b}_6 \hat{p} \right] \left[ \hat{b}_7 \ln \hat{R}_{eq} + \hat{b}_8 \ln \hat{S}_{co} \right]. \quad (3.5)$$

Here dimensionless parameters are defined as

$$\theta = \frac{r \sqrt{d}}{p h S_{co}}, \quad \hat{U} = \frac{u_r \sqrt{d}}{E_{eq} \ell_{ref}}, \quad \bar{G} = \alpha E_{eq},$$

$$\hat{R}_{eq} = \frac{R_{eq}}{R_{ref}}, \quad \hat{p} = \frac{p h}{E_{eq}}, \quad \hat{S}_{co} = \frac{S_{co}}{\ell_{ref}},$$

$$\hat{a}_0 = \frac{a_0}{a_{ref}}, \quad \hat{a}_1 = \frac{a_1}{a_{ref}}, \quad \hat{a}_2 = \frac{a_2}{a_{ref}}, \quad \hat{b}_1 = \frac{b_1}{b_{ref}}, \quad \hat{b}_2 = \frac{b_2}{b_{ref}}, \quad \hat{b}_3 = \frac{b_3}{b_{ref}}, \quad \hat{b}_4 = \frac{b_4}{b_{ref}}, \quad \hat{b}_5 = \frac{b_5}{b_{ref}}, \quad \hat{b}_6 = \frac{b_6}{b_{ref}}, \quad \hat{b}_7 = \frac{b_7}{b_{ref}}, \quad \hat{b}_8 = \frac{b_8}{b_{ref}}.$$
Table 3.1 EHL contact parameters used for bearing regression analysis.

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<th>Parameter</th>
<th>Symbol</th>
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<td>$R_{eq}$</td>
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</tr>
<tr>
<td>Rolling Velocity [m/s]</td>
<td>$u_r$</td>
<td>1, 5, 10, 15, 20</td>
</tr>
</tbody>
</table>
\[ E_{eq} = 2 \left[ \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right]^{-1}, \quad R_{eq} = \frac{r_1 r_2}{\eta + r_2}, \]

\[ S_{co} = \sqrt{(S_{rms})_1^2 + (S_{rms})_2^2}. \]

In these dimensionless quantities, \( \ell_{ref} = (10)^{-6} \) m, \( R_{ref} = 5(10)^{-3} \) m, \( \nu_d \) is the dynamic viscosity of the lubricant, \( \alpha \) is the pressure-viscosity coefficient of the lubricant, \( p_h \) is the Hertzian contact pressure, \( E_{1,2} \) and \( \nu_{1,2} \) are the Young’s moduli and Poisson’s ratios of the materials of contacting surfaces, \( r_{1,2} \) are the contact radii of the cylinders, and \( (S_{rms})_{1,2} \) are the RMS surface roughnesses of the cylinders. The Hertzian pressure \( p_h \) is calculated from the Hertzian cylindrical contact formula using the contact dimensions as well as the normal load intensity on the slice, \( q_{h\lambda z} \) (eq. (3.3a)). Coefficients \( \hat{a}_0 - \hat{a}_2 \), \( \hat{b}_1 \) to \( \hat{b}_8 \) are lubricant dependent regression coefficients listed in Table 3.2 for a common transmission lubricant (an automatic transmission fluid).

Using the regressed equation (3.5), rolling power loss density for each loaded slice \( \lambda \) of each needle \( z \) is found as \( PWD_{\lambda z}[u_r, p_h, r_{1,2}, E_{1,2}, \nu_{1,2}, \nu_d, \alpha, (S_{rms})_{1,2}] \). These power loss densities of individual loaded slices are multiplied by the contact width and summed to find the mechanical power loss of an entire bearing as

\[ P_{mb} = w \sum_{j=z}^{Z} \sum_{\lambda=1}^{k_z} PWD_{\lambda z}. \]  

\[ (3.6) \]
Table 3.2 Numerical values of the regression coefficients [3.2, 3.3] of Eq. (3.5).

<p>| | |</p>
<table>
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<tr>
<td>(\hat{b}_8)</td>
<td>0.233836</td>
</tr>
</tbody>
</table>
3.4 Parametric Studies on Planet Bearing Mechanical Power Loss

A parametric study of caged planet needle bearings is presented in this section. The double-row needle bearings of the experimental gear set described in Chapter 2 with sixteen rollers per row ($Z=16$) are simulated as the baseline example case. Table 3.3 lists the parameters of this bearing. Variations to the bearing geometry, lubricant conditions, loading conditions, and surface conditions are applied to quantify their influence on planet bearing mechanical power loss, $P_{mb}$, within a range of branch input torque of the planetary gear set ($T_{PB} = T_{sun}/n$ where $T_{sun}$ is the total sun gear torque and $n$ is the number of planets) and various input (sun) speed $\Omega_{sun}$ values.

Figure 3.2 shows the planet gear base cylinder with sun and ring gear mesh forces $W_s^{(n)}$ and $W_r^{(n)}$ applied along their respective planes of action. The sun gear mesh force is decomposed to its tangential, axial and radial components as $W_s^{(t)}$, $W_s^{(a)}$ and $W_s^{(r)}$. Likewise, the components of $W_r^{(n)}$ are shown in Figure 3.2 as $W_r^{(t)}$, $W_r^{(a)}$ and $W_r^{(r)}$. The resultant forces on the planet bearing are $W_s^{(t)} + W_r^{(t)}$ in the tangential direction, $W_s^{(r)} + W_r^{(r)}$ in the radial direction, and $W_s^{(a)} + W_r^{(a)}$ in the axial direction, whose vector sum equals $F_r$. Likewise the bearing moment should balance the axial mesh force components such that $M = r_{pp}W_s^{(a)} + r_{pp}W_r^{(a)}$. If the assumption that the pressure angles of the sun-planet and ring-planet meshes are equal and the pitch radii of the planet with the sun gear and the ring gear are equal, then $W_s^{(t)} = W_r^{(t)} = W_m^{(t)}$, $W_s^{(a)} = -W_r^{(a)} = W_m^{(a)}$ and $W_s^{(r)} = -W_r^{(r)}$. Under these typical conditions, radial bearing force $F_r$ and the bearing moment $M$ reduce to
Table 3.3 Parameters of the baseline planet needle bearings used in the parametric study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of rows</td>
<td>--</td>
<td>2</td>
</tr>
<tr>
<td>Number of rollers per row</td>
<td>--</td>
<td>16</td>
</tr>
<tr>
<td>Diametral clearance [mm]</td>
<td>$C_d$</td>
<td>0.004</td>
</tr>
<tr>
<td>Roller length [mm]</td>
<td>$\ell$</td>
<td>9.52</td>
</tr>
<tr>
<td>Roller diameter [mm]</td>
<td>$d_r$</td>
<td>4.015</td>
</tr>
<tr>
<td>Pin diameter [mm]</td>
<td>--</td>
<td>25.991</td>
</tr>
<tr>
<td>Planet bore diameter [mm]</td>
<td>--</td>
<td>34.025</td>
</tr>
</tbody>
</table>
Figure 3.2 Normal \((n)\) gear mesh forces and their radial \((r)\), tangential \((t)\) and axial \((a)\) components acting on a helical planet. They are balanced by the radial bearing force \(F_r\) and the bearing overturning moment \(M\).
\[ F_r = 2W_m^{(t)} = \frac{2T_{PB}}{r_{ps}}, \]  
\[ M = 2W_m^{(a)} r_p = \frac{2T_{PB} r_{pp} \tan \beta}{r_{ps}}, \]

where \( \beta \) is the helix angle, \( r_{pp} \) is the planet pitch radius, and \( r_{ps} \) is the sun pitch radius. While the formulation proposed in Sections 3.2 and 3.3 apply to any arbitrary combination of \( F_r \) and \( M \), Figure 3.2 and Eq. (3.7) and (3.8) show that \( F_r \) and \( M \) are defined by parameters \( \beta \), \( r_{pp} \), and \( r_{ps} \).

In Figure 3.3, resultant bearing load distributions are shown at load levels, \( T_{PB} = 42 \), 100, and 333 Nm. Here, the zero degree position represents the direction the radial force applied as shown in Figure 3.1. Roller #1 is placed at this position such that \( \psi_1 = 0 \). Rollers on the opposite side of the bearing from this zero degree position remain unloaded. Figure 3.3 shows equal and uniform load distributions along the right and left rows of the roller at this zero degree position, while the load distributions of other rollers get more skewed with an increase in the needle position angle, \( \psi_2 \). This shows the effect of \( M \) on the bearing. Since roller #1 at \( \psi_1 = 0 \) is positioned orthogonal to \( M \), its load distribution is not influenced (skewed) by \( M \). Figure 3.3(a-c) also shows the influence of crown on the rollers or raceways. As the load increases from Figure 3.3(a) to Figure 3.3(c) more of the initial separation from crowning is negated by contact deflection, and the contact length increases. Increase in load also causes two additional rollers at \( \psi_2 = 67.5^\circ \) and \( 292.5^\circ \) come into contact.
Figure 3.3 Roller load distributions at a) $T_{PB} = 42$ Nm, b) $T_{PB} = 100$ Nm, and c) $T_{PB} = 333$ Nm.
Figure 3.4 shows the mechanical power loss, $P_{mb}$, of the same example planet bearing as a function of $T_{PB}$ at oil inlet temperature values of 40, 60 and 90°C (corresponding to kinematic viscosity values of 29.77, 15.39, and 7.32 cSt, respectively) at input (sun gear) speed values of $\Omega_{sun} = 3000$ and 4000 rpm. Here, decreasing the oil temperature (or increasing lubricant viscosity) results in an increase in $P_{mb}$. For instance, at $\Omega_s = 4000$ rpm and $T_{PB} = 333$ Nm, $P_{mb} = 236, 195$ and 181 W for inlet oil temperature levels of 40, 60 and 90°C.

The increase of viscosity via reduced temperature causes an increase in rolling power loss, agreeing well with the EHL theory as well as empirical relations available in the literature on rolling power loss [3.4]. When Figures 3.4(a) and (b) are compared, it is observed that the temperature effects are more prominent at higher speeds. For $T_{PB} = 333$ Nm, decreasing oil temperature from 90°C to 40°C results in a 19% increase in $P_{mb}$ at $\Omega_{sun} = 3000$ rpm, while it results in 23% at $\Omega_{sun} = 4000$ rpm. Even larger reductions in $P_{mb}$ at higher oil temperatures are obtained at lighter loads. For instance, at $\Omega_{sun} = 4000$ rpm and $T_{PB} = 42$ Nm, a 35% reduction in $P_{mb}$ is achieved when the oil temperature is elevated from 40°C to 90°C compared to 23% at $T_{PB} = 333$ Nm. This may be due to the large thickness of lubricant film seen at all temperatures for very light loading.

In Figure 3.5, the $P_{mb}$ values corresponding to different combinations of roller and raceway surface roughness values are compared within the same $T_{PB}$ range at $\Omega_{sun} = 1000$ to 4000 rpm at 90°C oil inlet temperature. The combinations consisted of roller/raceway surface roughnesses $(S_{rms})_1 / (S_{rms})_2$ of 0.06/0.02, 0.06/0.11, 0.10/0.11, and 0.10/0.15 µm. These roughness values pertain to composite RMS surface roughnesses of $S_{co} = 0.063, 0.125, 0.149,$
Figure 3.4 Effect of oil inlet temperature on $P_{mb}$ at (a) $\Omega_{sun} = 3000$ rpm and (b) $\Omega_{sun} = 4000$ rpm. $S_{co} = 0.141$ μm.
Figure 3.5  Effect of composite surface roughness $S_{co}$ on $P_{mb}$ at (a) $\Omega_{sun} = 1000$ rpm, (b) $\Omega_{sun} = 2000$ rpm, (c) $\Omega_{sun} = 3000$ rpm, and (d) $\Omega_{sun} = 4000$ rpm. Oil inlet temperature is 90°C.
Figure 3.5 (continued).

(c) 

\[ P_{mb} \quad [W] \]

0.063 µm

0.125 µm

0.149 µm

0.180 µm

(d) 

\[ P_{mb} \quad [W] \]

0.063 µm

0.125 µm

0.149 µm

0.180 µm

\[ T_{PB} \quad [Nm] \]
and 0.180 µm, respectively. Figure 3.5 shows a large increase in $P_{mb}$ with an increase in $S_{co}$.

For instance, at $T_{PB} = 333$ Nm and $\Omega_{sun} = 4000$ rpm, $P_{mb} = 89$ W for the smoothest case of $S_{co} = 0.063$ µm while it is nearly three times higher for the roughest case of $S_{co} = 0.180$ µm.

Here, the rougher surfaces cause larger pressure ripples along the fluid film. This increases the magnitude of the film pressure gradient [3.3] to increase $P_{mb}$. The same figure also indicates that the increases in $P_{mb}$ with increasing surface roughnesses are more significant at lower speeds. In Figure 3.5(a), at $T_{PB} = 333.3$ Nm and $\Omega_{sun} = 1000$ rpm, $P_{mb} = 14$ W for the smoothest case of $S_{co} = 0.063$ µm, while $P_{mb} = 71$ W for the roughest case of $S_{co} = 0.180$ µm.

At slower speeds the lubricant film is thinner, magnifying any differences in pressure gradient within the film amplifying the effect of surface roughness on $P_{mb}$.

Figure 3.6 shows the effect of roller (needle) diameter $d_r$ on $P_{mb}$ within the same ranges of $T_{PB}$ and $\Omega_{sun}$ at 90°C. Here the planet pin (inner race) diameter, and number of rollers are kept the same while the planet bore (outer race) diameter is varied to accommodate for the rollers of different $d_r$. Roller diameter values of $d_r = 2.015, 3.015, 4.015$, and 5.015 mm are considered in Figure 3.6. Here it seems that $P_{mb}$ increases with increasing $d_r$ as the difference between the cases of $d_r = 2.015$ and 5.015 mm at $T_{PB} = 333$ Nm and $\Omega_{sun} = 1000$ rpm is about 13%, and at $T_{PB} = 333$ Nm and $\Omega_{sun} = 4000$ rpm is about 32%. Figure 3.7 shows that the differences amongst the predicted load distributions for different roller diameters are negligible, suggesting that the increase in power loss due to an increase in $d_r$ is caused primarily by an increase in rolling velocity, $u_r$, in Eq. (3.4).
Figure 3.6 Effect of roller diameter $d_r$ on $P_{mb}$ at (a) $\Omega_{sun} = 1000$ rpm, (b) $\Omega_{sun} = 2000$ rpm, (c) $\Omega_{sun} = 3000$ rpm, and (d) $\Omega_{sun} = 4000$ rpm. Oil inlet temperature is 90°C and $S_{co} = 0.141$ μm.
Figure 3.7 Roller load distributions at $T_{PB} = 333$ Nm for (a) $d_r = 2.015$ mm, (b) $d_r = 3.015$ mm, and (c) $d_r = 5.015$ mm.
Figure 3.8 shows the effect of increasing bearing diametral clearance, $C_d$, under the same operating conditions as Figure 3.6. Diametral clearance values of $C_d = 1, 4, 9, \text{ and } 14 \, \mu \text{m}$ are simulated here by adjusting planet bore diameter, in order to show the variation of $P_{mb}$ with $C_d$. Figure 3.8(d) shows that a modest maximum reduction of about 15% is obtained by increasing $C_d$ from 1 $\mu \text{m}$ to 14 $\mu \text{m}$. This increase is a direct result of the impact of $C_d$ on load distribution through Eq. (3.2c). As shown in the load distributions of Figure 3.9 for each $C_d$ variation, the overall contact length (total length of the loaded needle segments) is shown to increase with decreasing $C_d$, while the maximum contact stress is reduced at the same time. This indicates that reducing the total length of contact while increasing contact pressure reduces planet bearing rolling power loss.

Figures 3.10 illustrates the influence of the roller length, $\ell$, on $P_{mb}$ for the same example bearing. Here, the same baseline double-row bearing design with needle lengths of $\ell = 6, 8, 10, \text{ and } 12 \, \text{mm (in one row)}$ is used. In each case, needles are crowned along their ends for the final 20% of the roller length. Power loss is plotted in Figure 3.10 against $T_{PB}$ at the same four speed levels. Here, an increase in $P_{mb}$ accompanies an increase in roller length showing a 24% increase at $\Omega_{sun} = 1000 \, \text{rpm}$, and a 40% increase at $\Omega_{sun} = 4000 \, \text{rpm}$ when doubling roller length from 6 to 12 mm. The increase in $P_{mb}$ with decreasing maximum contact stress shows a similar trend as with the effect observed in Figure 3.8 and 3.9 for decreasing $C_d$. Here, again reducing the total contact length shows a modest reduction in planet bearing rolling power loss levels.
Figure 3.8  Effect of diametral clearance \( C_d \) on \( P_{mb} \) at (a) \( \Omega_{sun} = 1000 \) rpm, (b) \( \Omega_{sun} = 2000 \) rpm, (c) \( \Omega_{sun} = 3000 \) rpm, and (d) \( \Omega_{sun} = 4000 \) rpm. Oil inlet temperature is 90°C and \( S_{co} = 0.141 \mu m \).
Figure 3.8 (continued).

(c) and (d) show the relationship between $P_{mb}$ [W] and $T_{PB}$ [Nm] for different hole sizes (1 µm, 4 µm, 9 µm, 14 µm).
Figure 3.9  Roller load distributions at $T_{PB} = 333$ Nm for (a) $C_d = 1$ µm, (b) $C_d = 9$ µm, and (c) $C_d = 14$ µm.
Figure 3.10  Effect of single roller length $\ell$ on $P_{mb}$ at (a) $\Omega_{sun} = 1000$ rpm, (b) $\Omega_{sun} = 2000$ rpm, (c) $\Omega_{sun} = 3000$ rpm, and (d) $\Omega_{sun} = 4000$ rpm. Oil inlet temperature is 90°C and $S_{co} = 0.141 \mu m$. 

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Figure 3.10 (continued).

(c) 

\[ P_{mb} \text{ [W]} \]

(d) 

\[ P_{mb} \text{ [W]} \]

\[ T_{PB} \text{ [Nm]} \]
Figure 3.11 shows the change of $P_{mb}$ due to varying planet gear helix angle. In order to investigate the effect of changing the ratio of the radial force, $F_r$, to overturning moment, $M$, on the same example planet bearing, analyses are performed at planet gear helix angles of $\beta = 0$ (spur), 13.124 (baseline), 20, and 30 degrees. Figure 3.11 shows between a 45% and 60% increase in $P_{mb}$ for $TPB = 333$ Nm when $\beta$ is increased from 0 to 30 degrees, at various input speeds. Figure 3.12 shows the corresponding load distributions at $\beta = 0$, 20, and 30 degrees. Here, the additional moment loading is seen to cause an increase not only in roller loads but also in the overall loaded contact length, both of which contribute to the increased rolling power loss.

### 3.5 Summary

A load distribution model for caged cylindrical roller bearings under arbitrary radial and moment loading conditions has been developed in order to analyze planet bearing load distribution. This load distribution model has been combined with an EHL rolling friction model in order to predict planet bearing mechanical power loss. This combined model provides the ability to analyze the effects of surface roughness, lubricant properties, geometry, and temperature.

A parametric study has been performed in order to identify operating and geometrical bearing parameters that cause changes in planet bearing power loss. The study included variance of oil inlet temperature, composite surface roughness, roller diameter, bearing diametral clearance, helix angle of the planet gear, and individual roller length using the bearing design used in the experiments presented in Chapter 2 as a baseline. The results of the parametric study are summarized in Table 3.4. In Table 3.4, increasing bearing speed, planet torque, bearing
Figure 3.11 Effect of planet gear helix angle $\beta$, on $P_{mb}$ at (a) $\Omega_{sun} = 1000$ rpm, (b) $\Omega_{sun} = 2000$ rpm, (c) $\Omega_{sun} = 3000$ rpm, and (d) $\Omega_{sun} = 4000$ rpm. Oil inlet temperature is 90°C and $S_{co} = 0.141$ μm.
Figure 3.11 (continued).

(c) 

$P_{mb}$ [W] 

$T_{PB}$ [Nm] 

(d) 

$P_{mb}$ [W] 

$T_{PB}$ [Nm]
Figure 3.12  Roller load distribution at $T_{PB} = 333.3$ Nm for (a) $\beta = 0^\circ$, (b) $\beta = 20^\circ$, and (c) $\beta = 30^\circ$. 
Table 3.4 Influence of increasing various operating, surface and design parameters on $P_{mb}$.

($\uparrow\uparrow$) increases significantly, (+) increases modestly, (0) not influenced, (−) reduces modestly and (↓↓) reduces significantly.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Change in $P_{mb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil Temperature</td>
<td></td>
<td>↓↓</td>
</tr>
<tr>
<td>Bearing Speed</td>
<td>$\omega_b$</td>
<td>↑↑</td>
</tr>
<tr>
<td>Planet Torque</td>
<td>$T_{PB}$</td>
<td>↑↑</td>
</tr>
<tr>
<td>Surface Roughness</td>
<td>$(S_{rms})<em>1,(S</em>{rms})_2$</td>
<td>↑↑</td>
</tr>
<tr>
<td>Roller Diameter</td>
<td>$d_r$</td>
<td>+</td>
</tr>
<tr>
<td>Diametral Clearance</td>
<td>$C_d$</td>
<td>−</td>
</tr>
<tr>
<td>Roller Length</td>
<td>$\ell$</td>
<td>+</td>
</tr>
<tr>
<td>Helix Angle</td>
<td>$\beta$</td>
<td>↑↑</td>
</tr>
</tbody>
</table>
surface roughness, and planet gear helix angle are indicated to increase $P_{mb}$ significantly, increasing temperature is indicated to decrease $P_{mb}$ significantly, increasing roller diameter or roller length is indicated to increase $P_{mb}$ modestly, and increasing bearing diametral clearance is indicated to modestly decrease $P_{mb}$.

References for Chapter 3


4.1 Introduction

Replacing standard parallel axis gearing with planetary gearing in an application introduces more gear meshes to the system in the form of one sun-planet (external) and one planet-ring (internal) mesh per planet branch. In other words, a single gear mesh is replaced by $2n$ gear meshes ($n$ external meshes and $n$ internal meshes). This has the potential to cause additional load-independent (spin) power losses to the system primarily due to the pocketing power losses at the gear meshes. As stated in Section 1.2.3, the current literature on gear efficiency does not have a model to predict helical gear pocketing power losses of external and internal helical gear meshes in an air-oil mixture.

In this chapter, a fluid dynamics model to predict power loss caused by pocketing of fluid at a helical gear mesh interface is proposed. The geometry of external and internal helical gearing is analyzed numerically in order to provide inputs for the fluid dynamics model in the form of volumes and areas as a function of time. The proposed formulation relies on the prediction of fluid pressure and velocity distributions within the gear mesh interface in order to predict pocketing power loss. While the fluid dynamics formulation proposed here will be
presented in a form that is specific to helical gears (also to spur gears as they are a special case of helical gears), the fluid dynamics formulations will be generic such that it may also be used to predict pocketing losses of other types of gearing such as spiral bevel and hypoid gears.

From the view of the overall objectives of this dissertation focusing on power losses of planetary gear sets, the main goal of this chapter is to provide a theoretical modeling framework to predict pocketing power losses in both the sun-planet (external) and ring-planet (internal) meshes of a planetary gear set. However, the proposed model is not specific to planetary gear configurations and applies to any helical or spur gear mesh in counter-shaft arrangements as well.

The specific objectives of this chapter are as follows:

- Numerically calculate key geometrical parameters for external and internal helical gear meshes with varying mesh position (time), including (i) pocket volumes, (ii) exit areas, and (iii) centroids of each volume and area.

- Develop a general fluid dynamics model in order to predict pocketing power losses due to squeezing of lubricant, air, or a mixture of each, from the gear mesh.

- Perform a parametric study in order to identify key external and internal helical gear design parameters that impact pocketing power loss the most. Based on the parametric study results, arrive at guidelines in regards to design of higher efficiency helical gears.
4.2 Geometrical Computations of External and Internal Helical Gear Mesh Volumes and Areas

The mesh of a helical gear pair contains a number of pockets that are formed, compressed and expanded as the gears rotate in mesh. In order to illustrate this, consider the external spur gear mesh of Figure 4.1 that shows the teeth of gears in different sequential incremental mesh positions to indicate that there are multiple pockets formed (i) between the teeth of gear 1 and the root fillets of gear 2, and (ii) between the teeth of gear 2 and the root fillets of gear 1. It is also noted that there is a periodicity to these pockets, i.e. a pocket at a given time instant will have the same shape as the proceeding pocket one base pitch ahead when the gears are rotated by one mesh cycle.

The cross-sectional shapes shown in Figure 4.1 along the transverse plane of the gears remain the same along the face width direction only when the gears are spur type [4.1]. In that case, the volume of a pocket is the product of the side area, \( A_{n(i,j)}^{(n)} \), shown in Figure 4.1 and the active face width of the gears. In case of helical gears, however, the cross-sectional shape of the pocket at a given mesh position changes along the face width direction according to a rotation transformation defined by the helix angle. Figure 4.2 illustrates a pocket of an external helical gear pair at a given instantaneous position. Here, the three-dimensional shape of the pocket formed between a gear tooth and the space between two teeth of the mating gear is evident, as shown Figure 4.2(b), where the shape of each transverse slice of the pocket along the face width direction varies as well as the tooth contact point along the instantaneous contact line. Therefore, a form of discretization is required in the face width direction using the transverse involute geometry. For \( J \) discrete control volumes (slices) in the face width direction and a total of \( I \) pockets in the circumferential direction (see Figure 4.1), dimensions of interest are (i) \( I(J + 1) \)
Figure 4.1 Transverse pocket geometry at different mesh positions.
Figure 4.2 Discretization of helical gear pocket across face width.
end exit areas (to the next control volume or to outside if the slice is the first or the last slice along the face width of the gears) whose normal is in the face width direction, (ii) \(I(J + 1)\) centroids of these end areas, (iii) \(IJ\) volumes of discrete control volumes, (iv) \(IJ\) centroids of these volumes, (v) \((I + 1)J\) circumferential exit areas (including flow through the mesh backlash, and the gap before and after the gears are in contact), and (vi) \((I + 1)J\) centroids of these circumferential areas.

Figure 4.3(a) illustrates the numerical procedure devised to calculate the end area of a face width slice. The minimum distance between the two gear surfaces on both the contacting side and the backlash side are found using a search algorithm in order to define the points on both gears that define the pocket. Grid points are added to either surface in order to have \(K\) nodes on both surfaces. This method is based on numerically integrating the area bounded by two mating gear tooth surfaces approximately by using \(K-1\) quadrilaterals. Figure 4.3(a) shows a very coarse discretization using four quadrilaterals (\(K = 5\)). Here the \(j\)-th face width slice of the \(i\)-th pocket formed by the mating gears is shown in a rotational increment \(n\). This discretization and the width of the control volume in the face width direction, yield all required pocket information. Among them, the end area \(A_{e(i,j)}^{(n)}\) of a face width slice of a pocket is given by summing the areas \(A_{e(i,j,k)}^{(n)}\) of each quadrilateral. The area of each quadrilateral is given as one half of the magnitude of the cross product of its diagonals as shown in Figure 4.3(b), the end area is given as

\[
A_{e(i,j)}^{(n)} = \frac{1}{2} \sum_{k=2}^{K} \left\| \mathbf{x}_{(i,j,1,k)} - \mathbf{x}_{(i,j,2,k-1)} \times (\mathbf{x}_{(i,j,2,k)} - \mathbf{x}_{(i,j,1,k-1)}) \right\| 
\]  

(4.1)

where subscript \(e\) denotes an end area, subscript \(k\) refers to the index counting the number of discrete points along the surfaces of the pinion (gear index 1) and the gear (gear index 2),

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Figure 4.3 (a) A coarse discretization of a transverse slice of the mesh pocket, and (b) a single quadrilateral for the area analysis.
subscript \( i \) refers to the meshing pocket in the circumferential direction (see Figure 4.1), subscript \( j \) refers to the face width slice being analyzed, and superscript \( n \) denotes the time step. Vectors \( \mathbf{x} \) define the vertices of the \( k \)-th quadrilateral as defined in Figure 4.3. With this process, end exit areas \( A_{e(i,j)}^{(n)} \) and \( A_{e(i,j+1)}^{(n)} \) of the \( j \)-th discrete control volume of the \( i \)-th pocket at time step \( n \) are determined.

According to Figure 4.3(a), coordinates of the centroids \( \mathbf{x}_{c(i,j,k)}^{(n)} \) of each of the \( K-1 \) quadrilateral are found using any two triangles that form the quadrilateral as

\[
\mathbf{x}_{ec(i,j,k)}^{(n)} = \frac{A_{e1} \mathbf{x}_{e1} + A_{e2} \mathbf{x}_{e2}}{A_{e(i,j,k)}^{(n)}}.
\]  

(4.2)

where

\[
\mathbf{x}_{e1} = \frac{1}{3} \left[ \mathbf{x}_{(i,j,1,k-1)}^{(n)} + \mathbf{x}_{(i,j,2,k)}^{(n)} + \mathbf{x}_{(i,j,2,k-1)}^{(n)} \right],
\]

\[
\mathbf{x}_{e2} = \frac{1}{3} \left[ \mathbf{x}_{(i,j,1,k-1)}^{(n)} + \mathbf{x}_{(i,j,1,k)}^{(n)} + \mathbf{x}_{(i,j,2,k)}^{(n)} \right],
\]

\[
A_{e1} = \frac{1}{2} \left\| \left( \mathbf{x}_{(i,j,2,k)}^{(n)} - \mathbf{x}_{(i,j,1,k-1)}^{(n)} \right) \times \left( \mathbf{x}_{(i,j,2,k-1)}^{(n)} - \mathbf{x}_{(i,j,1,k-1)}^{(n)} \right) \right\|
\]

\[
A_{e2} = \frac{1}{2} \left\| \left( \mathbf{x}_{(i,j,2,k)}^{(n)} - \mathbf{x}_{(i,j,1,k-1)}^{(n)} \right) \times \left( \mathbf{x}_{(i,j,1,k)}^{(n)} - \mathbf{x}_{(i,j,1,k-1)}^{(n)} \right) \right\|
\]

With the centroids of the quadrilaterals forming the end area known, the overall centroid coordinates, \( \mathbf{x}_{ec(i,j)}^{(n)} \), of the exit area are calculated as a weighted summation of the quadrilateral centroids:
\[
\begin{align*}
\mathbf{x}_{\text{ec}(i,j)}^{(n)} &= \frac{\sum_{k=2}^{K} \mathbf{x}_{\text{ec}(i,j,k)}^{(n)} A_{(i,j,k)}^{(n)}}{A_{(i,j)}^{(n)}}. \\
\end{align*}
\] (4.3)

The control volume of the pocket defined between two end areas \( A_{e(i,j)}^{(n)} \) and \( A_{e(i,j+1)}^{(n)} \) is given by

\[
V_{(i,j)}^{(n)} = \frac{1}{2} b \left[ A_{(i,j)}^{(n)} + A_{(i,j+1)}^{(n)} \right] 
\] (4.4)

where \( b \) is the face width of the \( j \)-th control volume of the \( i \)-th pocket being analyzed. Likewise, the centroid \( \mathbf{x}_{\text{ec}(i,j)}^{(n)} \) of this control volume is the average of the centroids of its end exit areas

\[
\mathbf{x}_{\text{ec}(i,j)}^{(n)} = \frac{1}{2} \left[ \mathbf{x}_{\text{ec}(i,j)}^{(n)} + \mathbf{x}_{\text{ec}(i,j+1)}^{(n)} \right]. 
\] (4.5)

The circumferential exit areas, \( A_{r(i,j)}^{(n)} \) and \( A_{r(i+1,j)}^{(n)} \), are calculated from the backlash distance,

\[
d_{r(i,j)}^{(n)} = \left\| \mathbf{x}_{(i,j,2,k=1)}^{(n)} - \mathbf{x}_{(i,j,1,k=1)}^{(n)} \right\|,
\]

and the contact gap distance,

\[
d_{r(i+1,j)}^{(n)} = \left\| \mathbf{x}_{(i+1,j,2,k=K)}^{(n)} - \mathbf{x}_{(i+1,j,1,k=K)}^{(n)} \right\|,
\]

of the transverse slice as shown in Figure 4.3(a). They are given as

\[
A_{r(i,j)}^{(n)} = \frac{b}{2} \left[ d_{r(i,j)}^{(n)} + d_{r(i,j+1)}^{(n)} \right]. 
\] (4.6)

The centroid coordinates \( \mathbf{x}_{r_{\text{c}(i,j)}}^{(n)} \) of the circumferential exit area are calculated in the same manner as the end exit area individual quadrilateral centroids in Eq. (4.2) with \( k = 1 \) or \( k = K \) depending whether the calculation is on the contact or backlash side as
\[ x_{rc(i,j)}^{(n)} = \frac{x_{rc1}^{(n)} + x_{rc2}^{(n)}}{A_{r(i,j)}^{(n)}}. \]  (4.7)

where

\[ x_{rc1} = \frac{1}{3} \left[ x_{(i,j,1,k)}^{(n)} + x_{(i,j+1,k)}^{(n)} + x_{(i,j+1,2,k)}^{(n)} \right], \]

\[ x_{rc2} = \frac{1}{3} \left[ x_{(i,j,1,k)}^{(n)} + x_{(i,j,2,k)}^{(n)} + x_{(i,j+1,2,k)}^{(n)} \right], \]

\[ A_{r1} = \frac{1}{2} \left\| \left( x_{(i,j+1,1,k)}^{(n)} - x_{(i,j,1,k)}^{(n)} \right) \times \left( x_{(i,j+1,2,k)}^{(n)} - x_{(i,j,1,k)}^{(n)} \right) \right\|, \]

\[ A_{r2} = \frac{1}{2} \left\| \left( x_{(i,j,2,k)}^{(n)} - x_{(i,j,1,k)}^{(n)} \right) \times \left( x_{(i,j+1,2,k)}^{(n)} - x_{(i,j,1,k)}^{(n)} \right) \right\|. \]

Figure 4.4 shows an actual application of this numerical integration process for transverse pocket slices at two different time steps (or face width slices for helical gears).

This process is applied to the transverse slices of each tooth, beginning when a tooth first impinges upon a pocket (cavity between the two teeth of the mating gear) and ending when the tooth leaves that pocket completely. Repeating this for a discrete number of time steps through the mesh cycle, time variations of the volume, escape areas, the centroid of the volume, and centroids of the escape areas of each control volume are determined. A sample calculation is presented in Figure 4.5 to illustrate the time variation of an end area \( A_{e(i,j)}^{(n)} \), backlash-side area \( A_{r(i,j)}^{(n)} \), and contact-side area \( A_{r(i+1,j)}^{(n)} \) of a control volume through the meshing cycle. The areas here are plotted as a function of rotation angle. This angle can be used to find the area variation in
Figure 4.4  Examples of the discretization of transverse slices of a mesh pocket.
Figure 4.5 An example of the variation of (a) the end area, (b) the backlash area, and (c) the contact side area with the rotation of gears.
time (pinion gear rotation), or in the face width direction of a helical gear (helical lead angle). It is noted in Figure 4.5(a) that $A_{e(i,j)}^{(n)}$ reduces almost quadratically up to a certain rotational position where it reaches its minimum before expanding again quadratically. This indicates that the control volume associated with this end area first shrinks and then expands. A similar variation is observed for the backlash-side area in Figure 4.5(b) where a sizable portion of the meshing cycle is represented by a small but constant $A_{r(i,j)}^{(n)}$ defined by the nominal backlash of the gear pair. The contact-side escape area also varies in a similar manner, except there is a segment of the cycle with zero area, indicating that this opening has been blocked completely by the tooth contact.

With the pocket volumes and escape areas established, as well as, the variations of these quantities with time (rotational position), the problem in hand reduces to a multi-degree-of-freedom discrete fluid dynamics problem shown in Figure 4.6. Here, multiple time-varying control volumes are connected to each other in the circumferential direction through backlash and contact gap areas, and in the face width direction through end areas. In the vertical direction, a total of $I$ pockets exist along the circumferential direction with or without connections to the adjacent pockets. A blocked connection (denoted by a cross in Figure 4.6) indicates that these two pockets are separated by the tooth contact. An open connection represents a passage along the backlash (or gap between the mating teeth) allowing transport of the media between the pockets. The rows of pockets in Figure 4.6 indicate $J$ axial control volumes of each helical pocket that are connected to each other by the end areas defined in Eq. (4.1). All of the $IJ$ slices shown in Figure 4.6 change their volumes and areas of their openings to ambient or to other control volumes as the gears roll.
Figure 4.6  Multi-degree-of-freedom fluid dynamics problem governing helical gear pocketing.
4.3 External and Internal Helical Gear Mesh Pocketing Power Loss Model

In Figure 4.6, consider the fluid flow into and from any given discrete control volume \( j \) of pocket \( i \). The continuity equation for this control volume and the conservation of momentum at the exits of the control volume are given, respectively, as

\[
\frac{\partial}{\partial t} \int_{CV} \rho \, dV + \int_{CS} \rho u \, dA = 0 \tag{4.8}
\]

\[
\frac{\partial}{\partial t} (\rho u) + \frac{\partial \rho}{\partial x} = 0 \tag{4.9}
\]

where \( \rho \) is the fluid density, \( V \) is the pocket volume, \( A \) is the exit area, \( u \) is the exit velocity and \( \frac{\partial \rho}{\partial x} \) is the pressure gradient across an exit. \( CV \) denotes integration over the control volume, and \( CS \) denotes integration over the exit surfaces of the control volume.

Considering the geometrical inputs calculated, and the index notation presented in Section 4.2 for a discrete number of \( IJ \) control volumes shown in Figure 4.6, while also assuming pressure and velocity gradients between pockets to be linear, these fluid dynamics equations, (4.8) and (4.9), can be applied in discretized form for both space and time.

Consider the \( j \)-th control volume of the \( i \)-th circumferential pocket with pressure \( p_{i(j)}^{(n)} \) and density \( \rho_{i(j)}^{(n)} \). Circumferential fluid velocity \( u_{r(i,j)}^{(n)} \) is the fluid velocity between axial control volume \( j \) of pockets \( i \) and \( i-1 \). Axial fluid velocity \( u_{e(i,j)}^{(n)} \) is the fluid velocity between pocket \( i \) of axial control volumes \( j \) and \( j-1 \). These definitions follow the same index notation as
Section 4.2. Assuming a linear pressure gradient between control volumes, the pressure \( p_{e(i,j)}^{(n)} \) at axial connections and the pressure \( p_{r(i,j)}^{(n)} \) at circumferential connections are defined as

\[
p_{e(i,j)}^{(n)} = p_{(i,j-1)}^{(n)} + \left( \frac{p_{(i,j)}^{(n)} - p_{(i,j-1)}^{(n)}}{\| \mathbf{x}_{ec(i,j)}^{(n)} - \mathbf{x}_{c(i,j)}^{(n)} \| + \| \mathbf{x}_{ec(i,j)}^{(n)} - \mathbf{x}_{c(i,j-1)}^{(n)} \|} \right) \mathbf{x}_{ec(i,j)}^{(n)} - \mathbf{x}_{c(i,j-1)}^{(n)} \right) \|
\]

(4.10)

\[
p_{r(i,j)}^{(n)} = p_{(i-1,j)}^{(n)} + \left( \frac{p_{(i,j)}^{(n)} - p_{(i-1,j)}^{(n)}}{\| \mathbf{x}_{rc(i,j)}^{(n)} - \mathbf{x}_{c(i,j)}^{(n)} \| + \| \mathbf{x}_{rc(i,j)}^{(n)} - \mathbf{x}_{c(i-1,j)}^{(n)} \|} \right) \mathbf{x}_{rc(i,j)}^{(n)} - \mathbf{x}_{c(i-1,j)}^{(n)} \right) \|
\]

(4.11)

Density at the axial connections, \( \rho_{e(i,j)}^{(n)} \), and at circumferential connections, \( \rho_{r(i,j)}^{(n)} \), can also be defined. With these, the discretized form of the continuity equation becomes

\[
\rho_{e(i,j)}^{(n)} \left( \frac{V_{e(i,j)}^{(n)} - V_{e(i,j)}^{(n-1)}}{\Delta t} \right) + \rho_{e(i,j)}^{(n)} \left( \frac{p_{(i,j)}^{(n)} - p_{(i-1,j)}^{(n)}}{\Delta t} \right) + \sum_{k=0}^{1} z_k \rho_{e(i+k,j)}^{(n)} A_{e(i+k,j)} u_{e(i+k,j)} + \sum_{k=0}^{1} z_k \rho_{r(i,j+k)}^{(n)} A_{r(i,j+k)} u_{r(i,j+k)} = 0
\]

(4.12)

where \( \Delta t \) is the time step that is dependent on the rotational speed and the number of teeth of the driving gear (gear 1). Here, \( z_{k=0} = -1 \) and \( z_{k=1} = 1 \). Conservation of momentum in the circumferential direction is given by

\[
p_{r(i,j)}^{(n)} \left( \frac{u_{r(i,j)}^{(n)} - u_{r(i,j)}^{(n-1)}}{\Delta t} \right) + u_{r(i,j)}^{(n)} \left( \frac{p_{r(i,j)}^{(n)} - p_{r(i,j)}^{(n-1)}}{\Delta t} \right) + p_{r(i,j)}^{(n)} u_{r(i,j)}^{(n)} \left( \frac{u_{r(i+1,j)}^{(n)} - u_{r(i-1,j)}^{(n)}}{\| \mathbf{x}_{r(i+1,j)}^{(n)} - \mathbf{x}_{r(i-1,j)}^{(n)} \|} \right) = 0
\]
\[
\frac{u^{(n)}_{e(i,j)} - u^{(n-1)}_{e(i,j)}}{\Delta t} + \frac{p^{(n)}_{e(i,j)} - p^{(n-1)}_{e(i,j)}}{\Delta t}
\]
\[+ p^{(n)}_{e(i,j)}\frac{u^{(n)}_{e(i,j+1)} - u^{(n)}_{e(i,j-1)}}{\Delta t}
\]
\[+ \left(\frac{p^{(n)}_{p(i,j)} - p^{(n)}_{p(i,j-1)}}{\Delta t}\right) = 0.
\] (4.14)

Conservation of momentum in the axial direction is given by

\[
\frac{u^{(n)}_{e(i,j)} - u^{(n-1)}_{e(i,j)}}{\Delta t} + \frac{p^{(n)}_{e(i,j)} - p^{(n-1)}_{e(i,j)}}{\Delta t}
\]
\[+ \left(\frac{p^{(n)}_{p(i,j)} - p^{(n)}_{p(i-1,j)}}{\Delta t}\right) = 0.
\] (4.13)

Here, exits that are considered to be at ambient pressure include circumferential exits of index 1 or \(I + 1\), axial end exits of index 1 or \(J + 1\), and any exits connecting a control volume being analyzed to a control volume left out of the analysis (control volumes with no tooth impinging on the cavity). For these cases, the pressure gradient term in the momentum equations are modified accordingly.

If there are \(I\) circumferential pockets, and \(J\) control volumes in the face width direction, this set provides \(IJ + I(J + 1) + J(I + 1)\) equations in order to solve for \(IJ\) pocket pressures, \(J(I + 1)\) circumferential velocities, and \(I(J + 1)\) axial velocities for any time step \(n\), given initial condition \(p^{(n-1)}\) and \(u^{(n-1)}\). Newton’s method is used to solve the set of nonlinear equations for each time step individually until one mesh cycle is completed, and the final mesh position solution is used as an initial condition for the beginning of the mesh cycle. This process continues until the solution approaches a steady state solution within an acceptable range. Using
this solution for the pocket pressures and exit velocities, conservation of energy for the control volumes (ignoring gravitational effects) yields

\[
\frac{\partial}{\partial t} \int_{CV} (\rho E) dV + \int_{CS} \rho u \left( \frac{p}{\rho} + E \right) dA = \dot{W}
\] (4.15)

where \( E = c_v T + \frac{1}{2} u^2 \) is the internal energy of the fluid, \( T \) is the temperature, \( c_v \) is the constant volume specific heat, and \( \dot{W} \) is the work done on the control volume (equal to the pocketing power loss).

Assuming uniform flow at entrances from ambient conditions \( (\alpha_k = 1) \), fully developed flow elsewhere \( (\alpha_k = 2) \), and negligible kinetic energy stored in the control volume, the same discretization scheme used earlier is applied to Eq. (4.15) as well to find the power loss associated with pocketing of the \( j \)-th control volume of the \( i \)-th circumferential pocket as

\[
\dot{W}^{(n)}_{(i,j)} = c_v \left[ \frac{\rho^{(n)}_{(i,j)} v^{(n)}_{(i,j)} T^{(n)}_{(i,j)} - \rho^{(n-1)}_{(i,j)} v^{(n-1)}_{(i,j)} T^{(n-1)}_{(i,j)}}{\Delta t} \right]
\]

\[
+ \frac{1}{2} \sum_{k=0}^{1} A^{(n)}_{(i+k,j)} P^{(n)}_{(i+k,j)} \left[ c_v \rho^{(n)}_{(i+k,j)} \left( T^{(n)}_{(i-k,j)} + T^{(n)}_{(i-k+1,j)} \right) \right]
\]

\[
+ p^{(n)}_{(i-k,j)} + p^{(n)}_{(i-k+1,j)} + \alpha_k \rho^{(n)}_{(i+k,j)} \left( u^{(n)}_{(i+k,j)} \right)^2
\]

\[
+ \frac{1}{2} \sum_{k=0}^{1} A^{(n)}_{(i,j+k)} P^{(n)}_{(i,j+k)} \left[ c_v \rho^{(n)}_{(i,j+k)} \left( T^{(n)}_{(i,j-k)} + T^{(n)}_{(i,j-k+1)} \right) \right]
\]

\[
+ p^{(n)}_{(i,j-k)} + p^{(n)}_{(i,j-k+1)} + \alpha_k \rho^{(n)}_{(i,j+k)} \left( u^{(n)}_{(i,j+k)} \right)^2
\].

(4.16)

Summing up the individual control volume power losses at a given position \( n \), the total pocketing loss of the helical gear mesh in this instantaneous position is found as
With this formulation, the pocketing power loss associated with (internal or external) meshing of two helical gears can be calculated.

To this point, the modeling has been done assuming that the density of the fluid in the mesh is defined. These formulations simplify significantly for a gear mesh filled completely with an incompressible lubricant. As the other extreme, for a mesh filled completely with air, the ideal gas law,

$$\rho_a = \frac{p}{RT},$$  \hspace{1cm} (4.18)

can be employed for the density-pressure relationship where \( R \) is the specific gas constant for air. For the practical (and the most common) case of an air-oil mixture, an equivalent density of the air-oil mixture is defined by using the volumetric lubricant-to-air ratio, \( \xi \), at ambient pressure. Furthermore, the density of the lubricant in the mixture is assumed to remain constant (i.e. act as an incompressible fluid) while the air in the mixture is allowed to expand (be compressible) according to the ideal gas law, yielding an equivalent density

$$\rho_{eq} = \frac{\xi \rho_o + (1 - \xi) \left( \frac{p_a}{RT_o} \right)}{\xi + (1 - \xi) \left( \frac{p_a}{p} \right) \left( \frac{T}{T_a} \right)}$$  \hspace{1cm} (4.19)
where $\rho_o$ is the lubricant density, $p_a$ is ambient pressure, and $T_a$ is ambient temperature. Here the temperature of the air is also needed to define the equivalent density, and $IJ$ more solutions are needed for unknowns, $T^{(n)}_{(i,j)}$. Assuming isentropic expansion of the air,

$$
\frac{T^{(n)}_{(i,j)}}{T^{(n-1)}_{(i,j)}} = \left( \frac{p^{(n)}_{(i,j)}}{p^{(n-1)}_{(i,j)}} \right)^{\left( \frac{\gamma-1}{\gamma} \right)}
$$

(4.20)

where $\gamma$ is the specific heat ratio of air, $IJ$ equations can be added to the solution scheme for a complete solution.

4.4 A Parametric Study on Pocketing Losses of Helical Gear Meshes

A brief parametric study of pocketing power losses in helical gears is presented in this section. In this parametric study, a unity-ratio external helical gear pair is considered as an example baseline system. Table 4.1 lists the geometric parameters of the external gear pair, while the corresponding transverse geometry is shown in Figure 4.7. Sensitivity of the pocketing power loss $P_p$ of the example gear pair to various basic gear design parameters, namely helix angle, $\beta$, effective face width, $b$, nominal backlash, $\delta$, and transverse module, $m_t$, are quantified by using the model proposed in Sections 4.2 and 4.3. Various oil-to-air ratio, $\xi$, values are also considered to investigate their effect on $P_p$. Here, the lubricant is a standard automatic transmission fluid.

First, the influence of rotational speed on pocketing loss of the example external helical gear pair is demonstrated in Figure 4.8 for $\beta = 0^\circ$, $15^\circ$ and $30^\circ$ (the same transverse geometry...
Table 4.1 Parameters of the parametric study baseline external helical gear set

<table>
<thead>
<tr>
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<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth</td>
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<td>25</td>
</tr>
<tr>
<td>Normal module [mm]</td>
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<td></td>
</tr>
<tr>
<td>Helix angle [deg]</td>
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<td></td>
</tr>
<tr>
<td>Normal pressure angle [deg]</td>
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<td></td>
</tr>
<tr>
<td>Outside diameter [mm]</td>
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<td>54.00</td>
</tr>
<tr>
<td>Root diameter [mm]</td>
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<td>46.00</td>
</tr>
<tr>
<td>Center distance [mm]</td>
<td>50.00</td>
<td></td>
</tr>
<tr>
<td>Face width [mm]</td>
<td>15.00</td>
<td>15.00</td>
</tr>
<tr>
<td>Transverse tooth thickness [mm]</td>
<td>3.0916</td>
<td>3.0916</td>
</tr>
<tr>
<td>Diameter of thickness meas. [mm]</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>Transverse Backlash [mm]</td>
<td>0.10</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.7  Transverse geometry of the example baseline external helical gear set.
Figure 4.8 Variations of $P_p$ with pinion speed $\Omega$ for gear pairs having $\beta = 0^\circ$, $15^\circ$ and $30^\circ$. $\bar{b} = 15$ mm and $\xi = 0.05$. 
as Figure 4.7) each having $\bar{b} = 15$ mm and $\xi = 0.05$. An exponential increase in $P_p$ is observed in Figure 4.8 with the pinion speed $\Omega$. The speed sensitivity is $P_p \propto \Omega^{3.06}$ for $\beta = 0^\circ$ while it is $P_p \propto \Omega^{2.95}$ for $\beta = 30^\circ$.

The influence of the helix angle, $\beta$, on $P_p$ of the external gear pair is shown in Figure 4.9 for $\xi = 0.05$. Here, several helical gear pairs within $\beta \in [5^\circ, 35^\circ]$ (the same transverse geometry as Figure 4.7) and face width values of $\bar{b} = 15$, 25 and 35 mm were analyzed at pinion speeds of $\Omega = 5000$, 7500 and 10000 rpm to form Figure 4.9. Here, regardless of the value of $\bar{b}$, $P_p$ is reduced significantly with $\beta$. For instance, in Figure 4.9(a) for $\bar{b} = 15$ mm, $P_p = 120$ W at $\beta = 10^\circ$ while it is reduced to about half ($P_p = 57$ W) at $\beta = 35^\circ$. Likewise, for $\bar{b} = 25$ mm, $P_p$ is reduced from 291 W at $\beta = 10^\circ$ to only 94 W at $\beta = 35^\circ$, a three times reduction.

It is clear from Figure 4.9 that a helical gear pair experiences significantly less $P_p$ than its spur counterpart having the same transverse geometry. The spur gear pair with $\beta = 0^\circ$ was predicted to have nearly 2.3, 4.0, and 5.1 times higher $P_p$ than its helical counterpart having $\beta = 35^\circ$ at face width values of $\bar{b} = 15$, 25 and 35 mm, respectively.

Figure 4.10 shows the effect face width $\bar{b}$ on $P_p$. Here, helix angles of $\beta = 0^\circ, 10^\circ, 20^\circ$ and $30^\circ$ were considered at $\xi = 0.05$ and the same three speed levels as before. $P_p$ is seen to increase monotonically (in most cases, almost linearly) with $\bar{b}$.

Doubling the face width from $\bar{b} = 15$ mm to 30 mm at $\Omega = 10000$ rpm causes 4.8, 2.6, 2.3 and 1.9 times increases in $P_p$ for
Figure 4.9 Effect of helix angle $\beta$ on $P_p$ for (a) $\bar{b} = 15$ mm, (b) $\bar{b} = 25$ mm, and (c) $\bar{b} = 35$ mm. Oil inlet temperature is 90°C and $\zeta = 0.05$. 
Figure 4.9 (continued).
Figure 4.10  Effect of face width $b$ on $P_p$ for (a) $\beta = 0^\circ$, (b) $\beta = 10^\circ$, (c) $\beta = 20^\circ$, and (d) $\beta = 30^\circ$. Oil inlet temperature is 90°C and $\xi_i = 0.05$. 
Figure 4.10 (continued).

(c) \( P_p \) [W] vs. \( \bar{b} \) [mm] for different RPM:
- 5000 rpm
- 7500 rpm
- 10000 rpm

(d) \( P_p \) [W] vs. \( \bar{b} \) [mm] for different RPM:
- 5000 rpm
- 7500 rpm
- 10000 rpm
\[ \beta = 0^\circ, \ \beta = 10^\circ, \ \beta = 20^\circ, \ \text{and} \ \beta = 30^\circ, \ \text{respectively.} \]  This nearly linear increase in \( P_p \) with increasing \( \bar{b} \) is to be expected as the volume of each pocket (and therefore how much fluid present to be expelled), increases linearly with \( \bar{b} \), as seen in Eq. (4.4). The larger increases in the spur gear case can be attributed to the fact that, for larger \( \bar{b} \), addition of \( \beta \) has more effect in increasing the exit areas of the pocket.

The influence of increasing ambient oil-to-air volumetric ratio, \( \xi \), on \( P_p \) is shown in Figure 4.11. Here, four of the external helical gear designs previously investigated, namely (a) \( \beta = 0^\circ, \ \bar{b} = 15 \text{ mm} \), (b) \( \beta = 0^\circ, \ \bar{b} = 25 \text{ mm} \), (c) \( \beta = 30^\circ, \ \bar{b} = 15 \text{ mm} \), and (d) \( \beta = 30^\circ, \ b = 25 \text{ mm} \), are analyzed for \( \xi \in [0.01, 0.60] \). Here, the increase in \( P_p \) is linear with increasing \( \xi \). This is directly related to the kinetic energy term of Eq. (4.16). The equivalent density of the air-oil mixture to be expelled from the pocket is a linear function of \( \xi \), and the kinetic energy imparted to the fluid is linearly related to this density. The larger slope seen for higher speeds in Figure 4.11 shows the cubic relationship to the fluid velocity of this kinetic energy term.

An investigation as to whether or not an increase in nominal backlash, \( \delta \), can provide relief in order to decrease \( P_p \) is shown in Figure 4.12. The same geometrical cases presented in Figure 4.11 are analyzed here at \( \xi = 0.05 \). With a sizable increase in backlash from \( \delta = 0.05 \text{ mm} \) to 0.225 mm in spur gear cases, Figures 4.12(a,b) exhibit a modest reductions of 17\% and 20\% in \( P_p \) for \( \bar{b} =15 \text{ and} \ 25 \text{ mm at } \Omega =10000 \text{ rpm} \). Figure 4.12(c,d) shows the effect of increasing nominal backlash for gear pairs having \( \beta = 30^\circ \). Here, the trends are not as easily recognized as in the spur gear case. The helical gear pairs involve more circumferential communication between adjacent pockets, through backlash areas, and the openings before and after contact.
Figure 4.11 Effect of oil-to-air ratio $\xi$ on $P_p$ for (a) $\beta = 0^\circ$, $\bar{b} = 15$ mm, (b) $\beta = 0^\circ$, $\bar{b} = 25$ mm, (c) $\beta = 30^\circ$, $\bar{b} = 15$ mm, and (d) $\beta = 30^\circ$, $\bar{b} = 25$ mm. Oil inlet temperature is 90°C.
Figure 4.11 (continued).
Figure 4.12  Effect of nominal backlash $\delta$ on $P_P$ for (a) $\beta = 0^\circ$, $\bar{b} = 15$ mm, (b) $\beta = 0^\circ$, $\bar{b} = 25$ mm, (c) $\beta = 30^\circ$, $\bar{b} = 15$ mm, and (d) $\beta = 30^\circ$, $\bar{b} = 25$ mm. Oil inlet temperature is 90°C and $\xi = 0.05$. 
Figure 4.12 (continued).
occurs due to the fact that the contact lines are not parallel to the rotational axes. These interactions between pockets are more complex in the helical gear meshes and they do not necessarily provide relief for the exiting fluid.

As the final sensitivity study, Figure 4.13 shows the influence of the transverse gear module, \( m_t \), on \( P_p \). Here, the same four design cases are used and keeping the gear pitch diameter constant the number of teeth on the gear is varied from 23 to 28 in increments of one. The transverse geometries of each gear set analyzed in Figure 4.13 are shown in Figure 4.14 for \( m_t = 1.786, 1.852, 1.923, 2.0, 2.083 \) and \( 2.174 \) mm. Figure 4.13 shows little variance of \( P_p \) with \( m_t \), indicating that \( P_p \) is not very sensitive to \( m_t \).

4.5 Summary

A fluid dynamics model was proposed for predicting power losses due to pocketing of air, oil, or an air-oil mixture in helical gear meshes. Geometric inputs to the fluid dynamics model were defined through a numerical procedure that uses the transverse involute properties of a helical gear pair. The model takes into account geometric parameters, speed, and lubricant parameters to predict the instantaneous pocketing power losses.

The proposed model was employed to study the sensitivity of pocketing power losses to basic helical gear parameters including helix angle, face width, transverse module, nominal backlash, as well as the oil-air ratio. The results of the parametric study indicate the following sensitivities:
Figure 4.13  Effect of transverse module $m_t$ on $P_p$ for (a) $\beta = 0^\circ$, $b = 15$ mm, (b) $\beta = 0^\circ$, $b = 25$ mm, (c) $\beta = 30^\circ$, $b = 15$ mm, and (d) $\beta = 30^\circ$, $b = 25$ mm. Oil inlet temperature is 90°C and $\xi = 0.05$. 

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Figure 4.13 (continued).
Figure 4.14 Transverse geometries of the gear sets analyzed in the effect of module study, a) $m_t = 1.786$ mm, b) $m_t = 1.852$ mm, c) $m_t = 1.923$ mm, d) $m_t = 2.0$ mm, e) $m_t = 2.083$ mm, and f) $m_t = 2.174$ mm.
- Increasing helix angle in relatively large face width gears causes a significant reduction in pocketing power loss due to the increase in overall exit area provided in helical gears.

- Increasing the face width of a gear set causes a significant increase in pocketing power loss due to the increase of fluid present in the mesh that must be expelled in the meshing process.

- An increase in oil-to-air ratio causes an increase in pocketing power loss.

- Increasing nominal backlash in spur gears, for especially larger face width values, causes a modest decrease in pocketing power loss, while less effect is seen in helical gears.

- Pocketing power loss is not very sensitive to the transverse module of the gear set.

References for Chapter 4

5.1 Introduction

It was stated in Chapter 1 that the power losses of a planetary gear set can be classified in two groups based on their dependence on load:

\[ P = P_{LD} + P_{LI} . \]  \hspace{1cm} (5.1)

Referring to Figure 1.2, the load dependent (mechanical) power loss, \( P_{LD} \), includes all friction induced losses at the external and internal gear meshes (\( P_{mg} \)) as well as at the planet bearings (\( P_{mb} \)) such that

\[ P_{LD} = P_{mg} + P_{mb} . \]  \hspace{1cm} (5.2)

Meanwhile, three main components of \( P_{LI} \) were identified as the carrier and gear drag, \( P_d \), bearing viscous loss, \( P_{vb} \), and gear mesh pocketing loss, \( P_p \), such that
\[ P_{LI} = P_d + P_{vb} + P_p. \]  

(5.3)

It was stated in Chapter 1 that models are available to predict some of these power loss components. For the others, novel models were proposed in chapters 3 and 4 (planet bearing mechanical loss and helical gear mesh pocketing loss). This chapter has two major objectives:

- Develop a methodology that combines all the component-level power loss models to predict the overall power loss of a planetary gear set.
- Simulate the experiments of Chapter 2 using this methodology such that the accuracy of the power loss predictions can be accessed through direct comparisons to planetary gear set power loss measurements.

With respect to the first main objective, the following approaches will be employed:

- An existing drag formulation of Seetharaman and Kahraman [5.1] will be adapted to compute the components of the drag power loss, \( P_d \), associated with the rotating central gears (sun and/or ring) and the carrier.
- The model proposed in Chapter 4 will be employed to predict the pocketing power losses, \( P_p \), taking place at both sun-planet (external) and ring-planet (internal) gear meshes.
- Widely used empirical formulae of Harris [5.2] will be used to predict bearing viscous power losses, \( P_{vb} \).
A recently developed methodology of Li and Kahraman [5.3 – 5.5] will be applied to predict mechanical power losses, $P_{mg}$, of the sun-planet, and ring-planet gear meshes.

The model proposed in Chapter 3 will be employed to predict bearing mechanical power losses, $P_{mb}$.

In regards to the second main objective, the planetary gear set power loss experiments presented in Chapter 2 will be simulated. Predicted power loss components, $P_{Ld}$ and $P_{LD}$, will be compared to measurements. These comparisons will be used to determine the fidelity of the methods used in this prediction methodology within typical ranges of operating conditions and design variations.

5.2 Planetary Gear Set Power Loss Prediction Methodology

5.2.1 Prediction of the Load Independent Power Loss

Drag on the rotating gears and carrier is predicted using the method proposed by Seetharaman and Kahraman [5.1], which treats a gear as a rotating disk in a fluid (air-oil mixture with a certain equivalent density and viscosity). Here, the drag power loss for a member $i$, $P_{d(i)}$, is given as the sum of the drag on its periphery, $P_{dp(i)}$, and on its faces (side surfaces), $P_{df(i)}$, such that $P_{d(i)} = P_{dp(i)} + P_{df(i)}$ where [5.1]

$$P_{dp(i)} = 4\pi v_d b_i r_{oi}^2 \omega_i^2,$$  (5.4)
Here $\rho$, $v_d$, and $v_k$ are the density, the dynamic viscosity, and the kinematic viscosity of the air-oil mixture surrounding the gear set, respectively. $b_i$ and $r_{oi}$ are the face width and the outside radius of the rotating member $i$. The angular velocity of the member is denoted by $\omega_i$. Equation (5.5) provides two separate formulae for $P_{df}^{(i)}$ based on whether the flow regime is laminar (Reynolds number $Re < 5e5$) or turbulent (Re $> 5e5$). Equations (5.4) and (5.5) are applied to every rotating gear and carrier to determine the overall $P_d$ of the planetary gear set.

Bearing viscous losses are predicted from empirical relationships presented by Harris [5.2]. Here, bearing viscous loss of a planet bearing is given (in Watts) by

$$P_{vb} = \frac{\pi}{30} (10)^{-10} f_0 v_k^{2/3} \Omega_b^{5/3} d_m^3$$  \hspace{1cm} (5.6)$$

where $v_k$ is the lubricant kinematic viscosity (in centistokes), $\Omega_b$ is the bearing speed relative to the carrier (in rpm), $d_m$ is the bearing pitch diameter (in mm), and $f_0$ is a constant dependent on bearing type and lubrication condition. For a cylindrical roller bearing with a cage, $f_0 = 2.2 - 4.0$. Harris [5.2] recommends $2f_0$ for double-row caged roller bearings as was the case for the planet bearings of experimental gear sets of Chapter 2. Equation (5.6) applies to thrust bearings as well with $f_0 = 3.5$. 

\[ P_{df}^{(i)} = \begin{cases} 0.41\pi \rho v_k^{0.5} \omega_i^{2.5} r_{oi}^4, & Re < 5(10)^5 \\ 0.025\pi \rho v_k^{0.14} \omega_i^{2.86} r_{oi}^{4.72}, & Re > 5(10)^5 \end{cases} \]  \hspace{1cm} (5.5)
With the pocketing loss of an internal (ring-planet) mesh, $P^{(r)}_p$, and the pocketing loss of an external (sun-planet) mesh, $P^{(s)}_p$, defined using the model proposed in Chapter 4, the overall spin power loss of an $n$-planet gear set is defined as

$$ P_{LI} = n \left[ P^{(s)}_p + P^{(r)}_p + P_{vb} \right] + P^{(s)}_d + P^{(r)}_d + P^{(c)}_d. $$

(5.7)

### 5.2.2 Prediction of the Load Dependent Power Loss

Mechanical (sliding and rolling) power losses at the sun-planet and ring-planet meshes, $P^{(i)}_{mg}$ and $P^{(i)}_{mgr}$ ($P^{(i)}_{mg} = P^{(i)}_{mgu} + P^{(i)}_{mgr}$), are predicted in a manner analogous to the approach used for bearing mechanical power losses in Chapter 3. A gear load distribution model [5.6, 5.7] is used in predict the load intensities (force per length) along the helical gear tooth contact lines. The total contact lines are discretized into $K$ ($k \in [1, K]$) segments of uniform width. Contact of each segment is represented by cylindrical contacts with rolling and sliding velocities of $u_r = \frac{1}{2}(u_{1k} + u_{2k})$ and $u_s = u_{1k} - u_{2k}$, where the tooth surface velocities of mating gears at the contact with respect to the carrier are $u_{1k} = n_k (\omega_1 - \omega_c)$ and $u_{2k} = n_k (\omega_2 - \omega_c)$. Here, $n_k$ and $\omega_i$ are the radius of curvature of the contact point and angular velocity of the contact surface of gear $i$, and $\omega_c$ is the carrier angular velocity.

For a given contact segment $k$, a mixed elastohydrodynamic lubrication (EHL) model can be used to compute rolling and sliding friction power losses. Such a model, e.g. the mixed EHL model of Li and Kahraman [5.8], requires the following contact parameters:
(i) Geometric and kinematic parameters consisting of $u_r$, the slide-to-roll ratio 
\[ \hat{s} = \left| \frac{u_s}{u_r} \right| , \]
and the equivalent radius of curvature 
\[ R_{eq} = \eta r_2 / (\eta + r_2) . \]

(ii) Measured roughness profiles of tooth surfaces in the direction of rolling (involute direction).

(iii) Lubricant parameters including density-pressure, viscosity-pressure and viscosity temperature relations.

(iv) The normal contact load given in the form of load intensity $W'$ (load per unit contact length).

EHL simulations must be performed for all contacts segments $k \in [1, K]$ of the gear mesh at a given position. Given typical values of $K$, this amounts to 50 to 100 EHL simulations. As helical gear contacts move along the tooth surfaces, in the process changing the geometric, kinematic and loading conditions of the contacts, these simulations would also be repeated for at least 10 discrete gear mesh positions, bringing the total number of EHL simulations for a given gear mesh to 500 to 1000. In addition, planetary gear sets have two different types of gear meshes (internal and external) further doubling the number of required EHL analyses. This represents significant computational burden. In order to circumvent this problem, Xu et al [5.9] first proposed performing a set of (a large number of) up-front EHL analyses representing typical ranges of gear contact parameters for a given application with a given lubricant type. The results of these analyses would then be processed using a regression analysis to arrive at a closed-form, EHL-based friction coefficient formula. They used such formula in their gear mechanical power loss models to show a good agreement with measurements [5.10]. Later work of Li and
Kahraman [5.3-5.5, 5.8] enhanced this methodology by providing the capability to model excessive asperity interactions as well as rolling loss effects.

The methodology proposed by Li and Kahraman [5.8] was used to define an EHL-based regression formula for sliding friction coefficient $\mu$. With an automatic transmission fluid as the example lubricant and automotive transmission gearing as the targeted application, typical ranges of lubricated cylindrical contact parameters were defined in Table 5.1, together with the increments within each parameter range. A large number of combinations (33,600) of these parameters were defined to cover possible contact conditions within these parameter ranges, and an EHL simulation of each combination was carried out using the model of Li and Kahraman [5.5]. The results of these simulations were then processed by using linear regression according to the methodology proposed recently by Li and Kahraman [5.8] to obtain the following expression for the coefficient of sliding friction $\mu$ for a discrete contact point:

For $0 < \Lambda \leq 0.7$:

$$
\mu = \exp\left[ a_0 + \hat{s}(a_1\hat{G} + a_2\hat{S}_{eq}) + \Lambda(a_3\hat{p} + a_4\hat{G}) + \hat{R}_{eq}(a_5\hat{G} + a_6\hat{S}_{co}) \right]
\cdot \hat{s}^{\left[ h_1\hat{R}_{eq} + h_2\ln(\hat{S}_{co}) \right]}(\hat{R}_{eq})^{h_3}\hat{S}_{co}^{h_4\Lambda + h_5\ln(\hat{U})},
$$

(5.8a)

For $0.7 < \Lambda \leq 3.0$:

$$
\mu = \exp\left[ a_0 + \hat{R}_{eq}(a_1\hat{G} + a_2\hat{G} + a_3\hat{U}) \right]
\cdot \hat{s}^{\left[ h_1\hat{G} + h_2\ln(\hat{U}) + h_3\hat{R}_{eq} + h_4\ln(\hat{R}_{eq}) \right]}(h_5\hat{G} + h_6\hat{G})^{h_7\hat{R}_{eq} + h_8\ln(\hat{R}_{eq})},
$$

(5.8b)

For $\Lambda > 3.0$:

$$
\mu = \exp\left[ a_0 + a_1\hat{R}_{eq} + \hat{G}(a_2\hat{p} + a_3\hat{R}_{eq}) \right]
\cdot \hat{s}^{\left[ h_1\hat{G} + h_2\hat{p} + h_3\hat{R}_{eq} \right]}(h_5 + h_6\hat{R}_{eq})^{h_7\hat{R}_{eq} + h_8\ln(\hat{R}_{eq})},
$$

(5.8c)
Table 5.1 EHL contact parameters used for gear regression analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil Temperature [°C]</td>
<td>--</td>
<td>25, 50, 75, 100</td>
</tr>
<tr>
<td>Eq. Radius of Curvature [mm]</td>
<td>$R_{eq}$</td>
<td>5, 20, 40</td>
</tr>
<tr>
<td>Gear Surface Roughness [μm]</td>
<td>$(S_{rms})_2$</td>
<td>0.1, 0.3, 0.5, 0.7</td>
</tr>
<tr>
<td>Pinion Surface Roughness [μm]</td>
<td>$(S_{rms})_1$</td>
<td>0.1, 0.3, 0.5, 0.7</td>
</tr>
<tr>
<td>Hertzian Pressure [GPa]</td>
<td>$p_h$</td>
<td>0.5, 1.0, 1.5, 2.0, 2.5</td>
</tr>
<tr>
<td>Rolling Velocity [m/s]</td>
<td>$u_r$</td>
<td>1, 5, 10, 15, 20</td>
</tr>
<tr>
<td>Slide-to-Roll Ratio</td>
<td>$\hat{s}$</td>
<td>0.025, 0.05, 0.10, 0.25, 0.50, 0.75, 1.00</td>
</tr>
</tbody>
</table>
In these equations, dimensionless parameters are defined as

\[
\bar{U} = \frac{u_r v_d}{E_{eq} R_{eq}}, \quad \bar{G} = \alpha E_{eq}, \quad \bar{W} = \frac{W'}{E_{eq} R_{eq}}, \quad \bar{p} = \frac{p_h}{E_{eq}},
\]

\[
S_{co} = \sqrt{(S_{rms})_1^2 + (S_{rms})_2^2}, \quad \bar{S}_{co} = S_{co} / R_{eq},
\]

\[
\bar{R}_{eq} = \frac{R_{eq}}{b_h}, \quad b_h = \sqrt{\frac{8W'R_{eq}}{\pi E_{eq}}}, \quad E_{eq} = 2 \left[ \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right]^{-1}
\]

where \(v_d\) and \(\alpha\) are the dynamic viscosity and the pressure-viscosity coefficient of the lubricant, \(p_h\) is the Hertzian contact pressure, \(b_h\) is the Hertzian contact width, and \(E_{1,2}\) and \(\nu_{1,2}\) are the Young’s moduli and Poisson’s ratios of the materials of contacting surfaces 1 and 2. It is noted that Eq. (5.8) is defined within three ranges of parameter \(\Lambda = h_{\text{min}} / S_{co}\) where \(h_{\text{min}} = 1.714U^{-0.694}G^{0.568}W^{-0.128}R_{eq}\) [5.11]. This parameter, known as the lambda ratio (ratio of the smooth surface minimum film thickness \(h_{\text{min}}\) to the composite surface roughness \(S_{co}\)), is used commonly as a rough estimate of the type of the lubrication regime (boundary, mixed or full-film). Coefficients \(a_0 - a_6\), and \(b_1 - b_9\) are lubricant dependent regression coefficients listed in Table 5.2(a) for the automatic transmission fluid considered.

Using the regressed equation (5.8), the coefficient of friction at each contact segment \(k\) in rotational mesh position \(m\) (\(m \in [1, \Gamma]\)) is found as \(\mu_{km}\). These friction coefficient predictions, along with known normal load intensity \(W'_{km}\) (from load distribution prediction) and sliding velocity \(u_{s km}\) at each of the \(K\) contact segments define the sliding mechanical power loss of
Table 5.2 Numerical values of the regression coefficients [5.3, 5.8] of
(a) Eq. (5.8) and (b) Eq. (5.10).

(a)

<table>
<thead>
<tr>
<th>(0 &lt; \Lambda \leq 0.7)</th>
<th>(0.7 &lt; \Lambda \leq 3.0)</th>
<th>(\Lambda &gt; 3.0)</th>
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</thead>
<tbody>
<tr>
<td>(a_0)</td>
<td>(-0.92139)</td>
<td>(-0.92139)</td>
</tr>
<tr>
<td>(a_1)</td>
<td>(-0.00008711)</td>
<td>(-0.00008711)</td>
</tr>
<tr>
<td>(a_2)</td>
<td>(-440.2)</td>
<td>(-440.2)</td>
</tr>
<tr>
<td>(a_3)</td>
<td>(-208.356)</td>
<td>(-208.356)</td>
</tr>
<tr>
<td>(a_4)</td>
<td>(0.00034889)</td>
<td>(0.00034889)</td>
</tr>
<tr>
<td>(b_1)</td>
<td>(0.00201603)</td>
<td>(0.00201603)</td>
</tr>
<tr>
<td>(b_2)</td>
<td>(-0.0059483)</td>
<td>(-0.0059483)</td>
</tr>
<tr>
<td>(b_3)</td>
<td>(-1.90169)</td>
<td>(-1.90169)</td>
</tr>
<tr>
<td>(b_4)</td>
<td>(-0.579273)</td>
<td>(-0.579273)</td>
</tr>
<tr>
<td>(b_5)</td>
<td>(18.9479)</td>
<td>(18.9479)</td>
</tr>
<tr>
<td>(b_6)</td>
<td>(-0.00711570)</td>
<td>(-0.00711570)</td>
</tr>
</tbody>
</table>

(b)

<table>
<thead>
<tr>
<th>(0 &lt; \Lambda \leq 1.0)</th>
<th>(\Lambda &gt; 1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{a}_0)</td>
<td>(-55.5228)</td>
</tr>
<tr>
<td>(\bar{a}_1)</td>
<td>(-4049775804)</td>
</tr>
<tr>
<td>(\bar{a}_2)</td>
<td>(0.00036893)</td>
</tr>
<tr>
<td>(\bar{a}_3)</td>
<td>(91748360)</td>
</tr>
<tr>
<td>(\bar{b}_1)</td>
<td>(0.0035756)</td>
</tr>
<tr>
<td>(\bar{b}_2)</td>
<td>(-2.81418)</td>
</tr>
<tr>
<td>(\bar{b}_3)</td>
<td>(0.79315)</td>
</tr>
<tr>
<td>(\bar{b}_4)</td>
<td>(-1.57573)</td>
</tr>
<tr>
<td>(\bar{b}_5)</td>
<td>(0.71772)</td>
</tr>
<tr>
<td>(\bar{b}_6)</td>
<td>(-4.20259)</td>
</tr>
<tr>
<td>(\bar{b}_7)</td>
<td>(-0.162010)</td>
</tr>
</tbody>
</table>
gear mesh $i$ ($i = s, r$ where a superscript $s$ represents a sun-planet mesh and $r$ represents a ring-planet mesh)

$$P_{mg\mu}^{(i)} = \frac{1}{\Gamma} \sum_{m=1}^{K} \sum_{k=1}^{K} \mu_{km} w_{km}^{(i)} \mu_{km} w_{km}^{(s)_{km}}$$ \hspace{1cm} (5.9)

where $w_{km}$ is width of the $k$-th contact segment at mesh position $m$.

Li and Kahraman [5.8] also provided a companion procedure to compute the rolling mechanical power loss density $\Re$ (in W/m) as a part of the same regression analysis. For the same EHL simulations representing 33,600 distinct combinations of contact parameters listed in Table 5.1, the following expressions for the rolling power loss are obtained:

For $0 < \Lambda \leq 1.0$ :

$$\Re = (10)^{-5} \exp \left[ \bar{a}_0 + \bar{a}_1 \bar{U} + \bar{a}_2 \bar{G} + \bar{a}_3 \bar{S}_{eq} \bar{S}_{eq} \right]$$

$$\cdot \Lambda \left( \bar{b}_0 \bar{R}_{eq} \bar{b}_1 \bar{S}_{eq} \bar{S}_{eq} \bar{S}_{eq} \right) \left[ \bar{b}_0 + \bar{b}_1 \ln(\bar{U}) \right]$$ \hspace{1cm} (5.10a)

For $\Lambda > 1.0$ :

$$\Re = (10)^{-5} e^{\bar{a}_0} \Lambda \left( \bar{S}_{eq} \right)^{\bar{b}_0} \left[ \bar{b}_0 + \bar{b}_1 \ln(\bar{p}) \right] \left( \bar{S}_{eq} \right)$$

$$\left( \bar{R}_{eq} \right)^{\bar{b}_0} \left( \bar{S}_{eq} \right) \left[ \bar{b}_0 + \bar{b}_1 \ln(\bar{G}) + \bar{b}_2 \ln(\bar{U}) \right]$$ \hspace{1cm} (5.10b)

Additional dimensionless parameters defined to facilitate this regression are

$$S_{eq} = \frac{(S_{rms})(S_{rms})_2}{(S_{rms})_1 + (S_{rms})_2} \hspace{1cm} \bar{S}_{eq} = \frac{S_{eq}}{R_{eq}}.$$
In Eq. (5.10), $a_0 - a_3$, and $b_1 - b_{10}$ are lubricant dependent regression coefficients listed in Table 5.2(b) for the same automatic transmission fluid.

Using Eq. (5.10), rolling power loss density for contact segment $k$ at a given position $m$ is found as $\gamma_{km}$. Summing up over all $K$ contact segments and averaging over the $\Gamma$ mesh positions, the rolling mechanical power loss of the gear mesh $i$ is defined as

$$P_{mgr}^{(i)} = \frac{1}{\Gamma} \sum_{m=1}^{\Gamma} \sum_{k=1}^{K} \gamma_{km} w_{km}$$

(5.11)

where $w_{km}$ is width of the $k$-th contact segment at mesh position $m$.

With the mechanical loss of a planet bearing found using the model presented in Chapter 3 as $P_{mb}$, and the gear mesh mechanical losses defined above, the total mechanical power loss of an $n$-planet gear set is found to be

$$P_{LD} = n \left[ P^{(s)}_{mgr} + P^{(r)}_{mgr} + P^{(s)}_{mgm} + P^{(r)}_{mgm} + P_{mb} \right].$$

(5.12)

5.3 Comparisons Between the Predictions and Measurements

5.3.1 Spin Power Loss Comparisons

In this section, measured load independent power losses of planetary gear sets presented in Chapter 2 are compared to the predictions of the methodology presented in Section 5.2.1. With the ring gear stationary ($P_d^{(r)} = 0$), Eq. (5.7) reduces to $P_{LL} = n \left[ P_p^{(s)} + P_p^{(r)} + P_{vb} \right] + P_d^{(s)} + P_d^{(c)}$.

The bearing viscous loss at an additional thrust bearing (shown in the cross-section of Figure 2.1) is also added to the total $P_{LL}$. The drag calculations were performed with an equivalent fluid
density and viscosity corresponding to a lubricant-to-air ratio of $\xi = 0.05$ (5%) at ambient pressure. The pocketing calculations were also performed using the same $\xi$ value. The oil-air mixture densities for $\xi = 0.05$ are 43.84, 43.15 and 42.13 kg/m$^3$, and the dynamic viscosities are, 24.84, 12.67, and 5.90 centipoise at 40°C, 60°C, and 90°C, respectively.

Figure 5.1 shows the comparison between measured and predicted $P_{LL}$ for oil inlet temperatures of 40°C, 60°C, and 90°C. This comparison shows good qualitative agreement between the measurements and the prediction of $P_{LL}$. Defining percent difference between the measurement and the prediction as

$$\% \text{Diff} = 100 \times \left| \frac{P_{\text{measured}} - P_{\text{predicted}}}{P_{\text{measured}}} \right|,$$  \hspace{1cm} (5.13)

Figure 5.1(a), 5.1(b), and 5.1(c), at $\Omega_{\text{sun}} = 1000$ rpm, show maximum percent differences of 45.6%, 39.7%, and 50.4% while the absolute differences are only 49.4, 35.4 and 41.4 W. However at $\Omega_{\text{sun}} = 4000$ rpm, the predictions improve, showing maximum percent differences of 5.0%, 6.3%, and 18.8%. Here the prediction tends to deviate more from the measurements at the highest temperature.

In order to provide a simpler and clearer comparison, the spin power loss of a single planet branch (sun and ring pocketing losses of a planet and viscous loss of its bearing),

$$P_{SBL} = P_{p}^{(s)} + P_{p}^{(r)} + P_{vb},$$ \hspace{1cm} (5.14a)

is defined as one third the difference between the 6-planet, and the 3-planet spin power loss measurements, i.e.
Figure 5.1 Measured and predicted load-independent power losses $P_{LI}$ at (a) $40^\circ C$, (b) $60^\circ C$, and (c) $90^\circ C$. 
Figure 5.1  (continued).
\[
P_{SBL} = \frac{1}{3}[(P_LI)_{n=6} - (P_LI)_{n=3}]. \tag{5.14b}
\]

Since the drag power losses of the 3 and 6-planet gear sets should be nearly identical, one-third of the difference in their \( P_{LI} \) values should correspond to \( P_{SBL} \). Figure 5.2 shows measured and predicted \( P_{SBL} \) at oil inlet temperatures of 40°C, 60°C, and 90°C. Good qualitative and quantitative agreement between the model and the measurements is observed here. When the gear set is operated at \( \Omega_{sun} = 4000 \) rpm, the prediction differed from the measurement by 11.0%, 21.9%, and 23.5% (7.4, 17.4 and 16.4 W) at 40°C, 60°C, and 90°C, respectively. Figure 5.2 shows an exponential speed relationship of \( \Omega_{sun}^{2.16}, \Omega_{sun}^{2.29}, \) and \( \Omega_{sun}^{2.45} \) at 40°C, 60°C, and 90°C, for the predictions respectively. This implies that the influence of planet bearing viscous loss on \( P_{SBL} \) is less significant at higher temperature than those of pocketing losses since \( P_{vb} \propto \Omega^{5/3} \) and \( P_p \propto \Omega^3 \).

The comparisons presented in Figures 5.1 and 5.2 are subject to various uncertainties associated with the oil-to-air ratio \( \xi \) (\( \xi = 0.05 \) used in the comparisons as a reasonable value for the experiments of Chapter 2) and the bearing viscous loss constant \( f_0 \). Harris [5.2] specified a range of \( f_0 = 2.2 - 4.0 \) for this constant for single-row cylindrical roller bearings and stated that it should be doubled for double-row bearings. In this comparison to experiments using double-row bearings, \( f_0 = 6 \) was used. While predictions that are closer to the measurements could be obtained by varying the values of \( \xi \) and \( f_0 \), this was found unnecessary here as the differences are already within the measurement accuracy of the torque sensor. It should be noted that the experimental setup shown in Figure 2.1 includes a rotary union (a fluid slip ring to transfer oil.
Figure 5.2 Measured and predicted branch spin power losses $P_{SBL}$ at (a) 40°C, (b) 60°C, and (c) 90°C.
Figure 5.2 (continued).
from a stationary frame to a rotating component) in order to provide lubricant to the input shaft. This union contains seals that contribute to the spin power loss measurements. Rotary union spin losses can be attributed to some of the differences in Figure 5.1 while it should not affect the branch spin power loss comparisons presented in Figure 5.2.

5.3.2 Mechanical Power Loss Comparisons

In this section, predicted mechanical power losses, $P_{LD}$, are compared to those measured in Chapter 2 for 3 to 6-planet gear sets ($n \in [3,6]$) with shaved and chemically polished planet gear surfaces. According to Eq. (5.12), the mechanical power loss model of an $n$-planet gear set consists of sliding and rolling losses of $n$ sun-planet and ring-planet meshes, and rolling mechanical losses of $n$ planet bearings. Both gear and bearing mechanical power loss predictions were carried out at oil inlet temperatures of 40°C, 60°C, and 90°C. The kinematic viscosity values at these temperatures are $\nu_k = 29.77$, 15.39, and 7.32 centistokes, respectively.

Figure 5.3 shows the comparison between measured and predicted $P_{LD}$ for 3-planet, 4-planet, 5-planet, and 6-planet shaved gear sets at 40°C. The predicted and measured trends agree well qualitatively, while the predicted $P_{LD}$ are consistently lower that the measured ones, especially at higher torque values. Figures 5.4 and 5.5 show the same comparisons at 60°C and 90°C. It is noted here that the prediction becomes increasingly more accurate with increasing temperature. For instance At $\Omega_{sun} = 4000$ rpm and $T_{sun} = 1000$ Nm, Figure 5.3 for 40°C shows 27.4%, 15.8%, 20.2%, and 16.5% difference between predictions and measurements for $n = 3, 4, 5,$ and 6, respectively. Figure 5.4 for 60°C exhibits differences of 21.3%, 11.2%, 17.8%
Figure 5.3 Measured and predicted mechanical power losses $P_{LD}$ of (a) 3-planet, (b) 4-planet, (c) 5-planet, and (d) 6-planet gear sets having shaved planets at 40°C.
Figure 5.4 Measured and predicted mechanical power losses $P_{LD}$ of (a) 3-planet, (b) 4-planet, (c) 5-planet, and (d) 6-planet gear sets having shaved planets at 60°C.
Figure 5.4 (continued).

(c) $P_{LD} \text{ [kW]}$

(d) $P_{LD} \text{ [kW]}$

$\Omega_{sun} \text{ [RPM]}$
Figure 5.5 Measured and predicted mechanical power losses $P_{LD}$ of (a) 3-planet, (b) 4-planet, (c) 5-planet, and (d) 6-planet gear sets having shaved planets at 90°C.
Figure 5.5 (continued).
and 13.3% at the same speed and load condition while they are reduced to 7.6%, 3.3%, 6.6% and 1.8% in Figure 5.5 for 90°C. This apparent improvement of agreement between the model and the experiments can be attributed to the temperature increases of the gearbox during testing as in Figures 2.15 and 2.16. The viscosity of the lubricant used is much more sensitive to temperature changes around 40°C than around 90°C. Therefore temperature rises of the lubricant before reaching the gear meshes and planet bearings greatly influence the actual viscosity of the oil provided to the gear meshes in the lower temperature tests. This effect is more pronounced in tests subject to higher power losses, because of the increase in heat generation, as well as the high branch torque tests where the normal tooth forces are increased.

Figure 5.6 shows the comparison between measured and predicted $P_{LD}$ of a 6-planet chemically polished gear set at 40°C, 60°C, and 90°C. Again, the agreement between the measurement and prediction is observed to be good qualitatively. Here, the difference between the measurement and prediction at $\Omega_{sun} = 4000$ rpm and $T_{sun} = 1000$ Nm is 29.2%, 11.8%, and 17.3% for 40°C, 60°C, and 90°C, respectively. While this cannot be viewed as insignificant, it is noted that the model is capable of predicting the influence of planet surface roughness effects on power losses reasonably well.

As recommended in Chapter 2, Figures 5.7 to 5.9 present the load dependent power loss in terms of planet branch power loss, $P_{MBL} = P_{LD}/n$. Here $P_{MBL}$ is plotted against branch input power, $P_{PB} = P_{in}/n$. The measurements in Figures 5.7(a) to 5.9(a) and the predictions in Figures 5.7(b) to 5.9(b) both show an envelope formed by the low-speed tests on the high power loss side, and the high-speed tests on the low power loss side. This occurs due to the increased lube film thickness reducing power loss at higher speeds. Linear fits are provided for the measurements.
Figure 5.6 Measured and predicted mechanical power losses of a 6-planet gear set having chemically polished planets at (a) 40°C, (b) 60°C, and (c) 90°C.
Figure 5.6  (continued).

\( P_{LD} \) [kW] vs \( \Omega_{sun} \) [RPM]

- 250 Nm Exp
- 500 Nm Exp
- 750 Nm Exp
- 1000 Nm Exp
- 250 Nm Pred
- 500 Nm Pred
- 750 Nm Pred
- 1000 Nm Pred
Figure 5.7  (a) Measured and (b) predicted planet branch mechanical power losses $P_{MBL}$ of a gear set having shaved planets at 40°C.
Figure 5.8 (a) Measured and (b) predicted planet branch mechanical power losses $P_{MBL}$ of a gear set having shaved planets at 60°C.
Figure 5.9  (a) Measured and (b) predicted planet branch mechanical power losses $P_{MBL}$ of a gear set having shaved planets at 90°C.
and predictions at each temperature. The slope of these fits represent the compliment of the mechanical efficiency (i.e. $\eta_{LD} = 100\% - \text{Slope}$). The fits presented in Figures 5.7 to 5.9 show measured $\eta_{LD}$ values of 99.30%, 99.29%, and 99.25% compared to the predicted values of 99.44%, 99.38%, and 99.28% for 40°C, 60°C, and 90°C, respectively. The decrease in the difference between the prediction and measurement with increased oil inlet temperature is again due to increases in gearbox operating bulk temperatures.

Figures 5.10 to 5.12 compare the measurements and predictions for the 6-planet gear set having chemically polished planets in the same manner as before. The measurements in Figures 5.10(a) to 5.12(a) and the predictions in Figures 5.10(b) to 5.12(b) show the same type of envelope formed by the low- and high-speed tests. This envelope for the chemically polished gear set is somewhat narrower than that of the rougher shaved gear set (Figure 5.7) for the lowest temperature and tends to expand with increasing temperature. This shows the reduction in asperity interactions along the EHL contact due to reduced surface roughnesses of planet gears, an effect that is minimized by decreasing lubricant viscosity (increase in temperature). Linear fits are provided for the measurements and predictions at each temperature. The fits presented in Figures 5.10-5.12 yield measured $\eta_{LD}$ values of 99.56%, 99.52%, and 99.46% compared to predicted values of 99.44%, 99.46%, and 99.37% for 40°C, 60°C, and 90°C, respectively. The similar slopes of Figures 5.7(b) and 5.12(b) show that the model predicts similar behavior for the shaved and chemically polished planets at 40°C. This is the effect of the high viscosity of the lubricant providing adequate lubrication film thickness in both cases. At 90°C, the predicted improvement in mechanical efficiencies by chemically polished planets is 0.09% while the measurements show a 0.18% improvement.
Figure 5.10  (a) Measured and (b) predicted planet branch mechanical power losses $P_{MBL}$ of a gear set having chemically polished planets at 40°C.
Figure 5.11 (a) Measured and (b) predicted planet branch mechanical power losses $P_{MBL}$ of a gear set having chemically polished planets at 60°C.
Figure 5.12  (a) Measured and (b) predicted planet branch mechanical power losses $P_{MBL}$ of a gear set having chemically polished planets at 90°C.
5.4 Summary

In this chapter, a methodology was outlined to compute both load-dependent and load-independent gear and bearing components of power loss of planetary gear sets. The load-dependent components included the mechanical (sliding and rolling friction) losses of external and internal gear meshes as well as rolling friction losses of planet bearings. Load-independent losses included drag losses of rotating components, pocketing losses at external and internal gear meshes as well as bearing viscous losses.

The planetary gear set power loss experiments presented in Chapter 2 were simulated by using the planetary gear set power loss prediction methodology. The load-independent and load-dependent components of predicted power losses were compared to measurements to show a reasonably good match. The level of agreement between the predicted and measured load-independent power loss are highly dependent on the estimation of the oil-to-air ratio as well as the value of the bearing viscous power loss constant. The predicted load-dependent power losses match the measurements well, especially at high temperature where bulk temperatures of components of the experimental set-up are close to the oil inlet temperatures, and the lubricant viscosity is less sensitive to temperature change.

References for Chapter 5


[5.7] LDP, Gear Load Distribution Program, Gear Dynamics and Gear Noise Research Laboratory, The Ohio State University, 2011.


CHAPTER 6

CONCLUSION

6.1 Summary and Conclusions

This study experimentally and theoretically examined planetary gear set power losses. Sources of planetary gear set power loss were categorized as either load-dependent or load-independent losses. Major sources of load-dependent power losses were identified as gear mesh and planet bearing mechanical losses while gear and carrier drag losses, planet bearing viscous losses, and gear mesh pocketing losses constitute the components of load-independent power losses.

In Chapter 2, experiments designed to investigate power loss of a typical planetary gear set were presented. Changes in power loss with number of planets, inlet oil temperature, planet surface roughness, operating speed, and operating torque were investigated. These experimental results showed that increasing number of planets in the design causes an increase load-independent power loss while showing little effect on load-dependent power loss. Increasing oil inlet temperature was observed to reduce load-independent power loss, at the same time increasing the load-dependent power loss. It was also shown clearly that reducing planet surface
roughness levels causes a significant reduction in load-dependent power loss of a planetary gear set.

The theoretical study focused on developing a comprehensive methodology for predicting planetary gear set power loss. This methodology relied on previous work in order to predict gear mesh mechanical power loss [6.1-6.3], gear and carrier drag loss [6.4], and bearing viscous drag loss [6.5], while new models needed to be developed for predicting bearing mechanical loss and gear mesh pocketing loss components of planetary power loss.

Chapter 3 proposed a mechanical power loss model for caged cylindrical roller bearings including planet needle bearings as a subset. This model predicted the distribution of load amongst, and across rolling elements given bearing macro-geometry, and micro-modifications to the roller and race surfaces. With the load distribution and bearing kinematic relationships as inputs, an elastohydrodynamic rolling power loss model was used to quantify planet bearing mechanical power loss. A parametric study accompanied the model showing that the planet bearing mechanical power loss increases significantly with (i) decreasing lubricant temperature, (ii) increasing planet speed and/or torque, and (iv) increasing planet gear helix angle.

Chapter 4 presented a novel pocketing power loss model for internal and external helical gears that can be used to model sun-planet, and planet-ring meshes of a planetary gear set. The model consisted of a geometric numerical procedure and discretization scheme, and a multi-degree-of-freedom fluid dynamics model. The numerical procedure and discretization scheme provided the variation of pocket volumes and areas to define the squeezing of fluid from the gear mesh. With this input, the fluid dynamics model used principles of conservation of mass, momentum and energy to predict pocketing power loss. A parametric study on the influence of design and operating parameters on pocketing power loss was also performed in this chapter. It
showed an increase in pocketing power loss with (i) a decrease of helix angle in relatively large face width gears, (ii) an increase in face width, (iii) an increase in the ratio of lubricant to air, and (iv) a decrease in gear backlash especially for large face width spur gears.

The modeling methodology presented was compared in Chapter 5 to the experimental database of Chapter 2. Reasonably good correlation between the predictions and experiments was demonstrated. The accuracy of the load-independent power loss predictions was shown to be sensitive to the estimated values of oil-to-air ratio, and chosen proportionality constants for bearing viscous loss. The load-dependent power loss predictions and experimental measurements, on the other hand, matched very well, especially where the bulk temperature of the gear set and the inlet temperature are close to one another.

6.2 Major Contributions

This study provides an experimental database and a comprehensive modeling methodology, which provide insight into the sources of planetary gear set power loss that was previously unavailable. Some of the major contributions of this study are listed as follows:

- The experimental database quantifying planetary gear set power loss provided in this study represents the most complete database of its type, including various operating speeds, operating torques, inlet temperatures, number of planets, and planet surface finishes. This database will be critical for future planetary modeling investigations.

- The bearing mechanical power loss model presented constitutes a marked improvement over frequently used closed-form power loss equations that represented the current state of art [6.5]. It is a completely physics-based model that takes into
account both planet loading conditions (including moment loading) and performance of the lubricant film without relying on any empirical parameters.

- The pocketing power loss model presented represents the first comprehensive attempt to quantify power loss associated with squeezing an air-oil mixture from an inter-tooth space using fluid dynamics conservation laws. While it is applied only to parallel-axis gearing in this work, the model is generic so that it can be applied to other forms of gearing as well.

- The modeling methodology in this study matches well with the experimental results, making it the first validated methodology of the overall power loss of an entire planetary gear set.

6.3 Recommendations for Future Research

The following lists research that may enhance or build upon the work presented in this dissertation:

- The model can be extended to predict the power loss of an entire planetary transmission (automotive, wind turbine gearbox or rotorcraft gearbox), provided power loss associated with clutches, seals, and synchronizers can be assessed.

- Theoretical investigation of the mechanical power loss of full-compliment needle roller bearings (with no cage) appears to be a natural extension of the proposed bearing model, as such bearings are used commonly for low-speed and high-torque planetary applications of automatic transmissions. The load distribution prediction
presented in this study can be extended to analyze full-compliment bearings including the effect of roller separation and skew introduced by removing the retaining cage. The skew of the rollers and the contact between rollers in full compliment bearings necessitates analysis of sliding power loss in addition to the rolling component investigated in this study. An experimental investigation of planetary gear sets similar to the one presented in this study employing full-compliment needle bearings would provide experimental means to quantify the penalty of using full-compliment needle bearings in terms of overall power loss.

- The fluid dynamics model used to calculate helical gear pocketing power loss can be used in order to predict pocketing power loss of other types of gearing, including but not limited to spiral bevel, hypoid, worm, and face gearing. Here, the geometric calculation of pocket volumes, exit areas, and centroids would need to be replaced for the specific type of gearing.

References for Chapter 6:


REFERENCES


[1.19] LDP, Gear Load Distribution Program, Gear Dynamics and Gear Noise Research Laboratory, The Ohio State University, 2011.


[5.7] LDP, Gear Load Distribution Program, Gear Dynamics and Gear Noise Research Laboratory, The Ohio State University, 2011.


