GPS Antenna and Receiver for Small Cylindrical Platforms

Dissertation

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ABSTRACT

In the past few decades, GPS has revolutionized navigation positioning and timing with numerous civilian and military applications. Recently, there is increased interest in GPS navigation for small cylindrical platforms which can have a potentially high rotation rate (up to 350 Hz). The purpose of this work is to extend the state-of-the-art of GPS receiver and antenna technology for this specific application of small cylindrical platforms. This presents a set of design challenges for engineers, and this work will make contributions to three aspects of the problem: antenna design, satellite coverage, and receiver design. First, a novel dual-band antenna that provides right-hand circular polarization (RHCP) coverage at the GPS L1/L2 bands for reception of C/A-, P(Y)-, and M-coded GPS signals is designed. The availability of GPS measurements at two bands allows one to remove the biases due to the ionsphere and reception of P(Y) and M-coded signals improves navigation accuracy. Importantly, the antenna size is only $4\text{cm} \times 4\text{cm} \times 5.08\text{mm}$ $(\lambda/6 \times \lambda/6 \times \lambda/50)$. Second, this antenna is specifically designed to have a robust tuning such that it can be mounted on metal cylinders of various diameters (60-160mm) and still function properly. For these cylinders, the antenna has broad RHCP coverage and good gain bandwidth performance. Third, the satellite coverage provided by the antenna is investigated. As expected, a single element cannot provide the full spherical coverage which is needed for continuous satellite tracking as the platform rotates. It is shown that the
maximum gain method (i.e. choosing the element with the highest gain) is able to obtain full spherical coverage even with only two elements. However, it is a challenge to implement this method because the time-varying platform attitude is unknown. Therefore, a novel receiver tracking algorithm that implements the maximum gain method is designed by modifying the receiver itself, specifically the delay lock loop. Example results shown that the proposed approach is able to provide continuous satellite tracking as the platform rotates, minimize the number of elements, and eliminate the need for knowledge of the platform attitude.
Dedicated to Rachael
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<td>GPS</td>
<td>Global Positioning System</td>
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<tr>
<td>CDMA</td>
<td>Code division multiple access</td>
</tr>
<tr>
<td>RF</td>
<td>Radio frequency</td>
</tr>
<tr>
<td>PVT</td>
<td>Position, velocity and time</td>
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<tr>
<td>RHCP</td>
<td>Right-hand circular polarization</td>
</tr>
<tr>
<td>PEC</td>
<td>Perfect electrical conductor</td>
</tr>
<tr>
<td>HPBW</td>
<td>Half-power beamwidth</td>
</tr>
<tr>
<td>BW</td>
<td>Bandwidth</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-noise-ratio</td>
</tr>
<tr>
<td>CNR</td>
<td>Carrier-to-noise-ratio</td>
</tr>
<tr>
<td>BF</td>
<td>Beamforming</td>
</tr>
<tr>
<td>Max</td>
<td>Maximum gain</td>
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<td>PDW</td>
<td>Post-discriminator weighting</td>
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CHAPTER 1

Introduction

1.1 Background

Navigation has been one of the most important concerns for mankind throughout history [1]. Up to the nineteenth century, one’s position on the globe (latitude and longitude) were found using stars, a sextant, a compass, and an accurate clock. In the twentieth century, radio and aviation were developed that led to an outburst of inventiveness which was destined to revolutionize life even more completely than had the “introduction of the horse in the third millenium B.C.E” [2]. As wireless technology became more sophisticated with micro-electronics and aviation was extended to space with rocket-propelled satellites, it became possible to build a space-based navigation system. In the 1960s and 1970s, the Global Positioning System (GPS) was developed, and one could now use artificial stars and accurate clocks to estimate position, velocity, and time (PVT).

GPS uses trilateration as the navigation method to find user PVT. Trilateration is the method whereby ranging signals to known locations are used to solve the navigation problem. It is practical because code division multiple access (CDMA) allows one to obtain instantaneous ranging signals from various satellites, the satellite
orbits are known, and accurate receiver clocks are inexpensive. With these important
developments and methods, GPS has become ubiquitous to modern civilization.

There are numerous GPS applications spanning a wide variety of civilian and
military activities. Civil uses include agriculture, construction, geology, aviation,
transportation, banking, hiking, telecommunications, power generation, and the Inter-
ternet. Military uses include GPS receivers on aircraft, ships, tanks, trucks, soldiers,
and missiles. Since the 1970s, receiver, electronic, and antenna technology has im-
proved such that GPS has revolutionized navigation positioning and timing. GPS
related research is still ongoing because of its impact on many vital civil and military
applications.

The purpose of this work is to extend the state-of-the-art of GPS receiver and an-
tenna technology for the specific application of small cylindrical platforms. Recently,
there is increased interest in GPS navigation for small cylindrical platforms [3–7].
These platforms can have potentially high rotation rates of up to 350 Hz and short
flight times. Before launch, the system is relayed pertinent initialization data which
is essential for a fast acquisition. After launch, the system needs to quickly acquire
satellites, track them, and provide accurate navigation data. Furthermore, the system
should have low power consumption, low cost, and be small volume. This presents
a set of design challenges for engineers, and this work will focus on three aspects of
the problem: antenna design, satellite coverage, and receiver design. The design of
the RF front end, navigation processor (which uses the receiver measurements), and
guidance system proper (i.e. autopilot) will not be covered in this work. Note that
thin metal cylinders of diameter 60-160mm will be used in the study as the platforms
of interest and a radio frequency interference (RFI) free environment is assumed.
1.2 Previous Work

Previous work related to antenna design, satellite coverage, and receiver design for this application is discussed below. It will be shown that a good solution does not exist in the literature.

The main antenna design challenges are dual-band reception, enough bandwidth at each band, and antenna tuning that is not sensitive to geometrical changes on the platform. First, the antenna needs to provide reception at the GPS L1 and L2 frequencies (1575 and 1227 MHz). This enables the GPS receiver to make measurements (such as code delay and carrier phase) at each band and then remove the biases due to the ionosphere. Second, it is desired to have enough bandwidth to receive the C/A (civil) as well as the P(Y) and M (both military) codes. Although the C/A code has narrow bandwidth (2 MHz), the other codes occupy about 25 MHz around the L1 and L2 center frequencies. Receiving these signals is beneficial because the larger bandwidth means a smaller chip width, and hence a finer and more accurate code delay measurement leading to increased navigation accuracy. Third, it is desired that a single antenna design be applicable to the multiple diameter cylinders of interest. This means that the antenna tuning should not vary that much as the diameter of the cylinder varies.

There are also a number of electrical requirements that are standard for a GPS antenna. Since GPS transmitters are right-hand circular polarized (RHCP) antennas, the receiving antenna should be RHCP as well, so that there is enough desired signal power for the receiver. Also, the pattern of the antenna should be broad to receive the satellite signals spread throughout the sky. Finally, there are the physical requirements of the antenna. The antenna mounting needs to be conformal to maintain
aerodynamics and a small volume antenna is desired because of the limited available size for antenna mounting. Also, easy fabrication of the antenna is very important and the antenna must be made with material that can survive launch shock. To some it might seem surprising that a GPS antenna design with the above characteristics does not already exist. As discussed below, this is primarily because the antenna volume (particularly the height) needs to be so small.

There are a couple of options for antenna element selection. In [8], a monopole is placed inside the nose of a platform using the body to provide the ground plane. Although the monopole could be modified for dual-band by increasing its bandwidth, it has pattern nulls on the cylinder axis, is not circularly polarized, and is a large antenna. A spiral would provide circular polarization and a wide bandwidth, but the smallest spirals at this band are still too large in terms of aperture and profile [9]. The same antenna size conditions eliminate the dielectric resonator [10] or broadband micro-strip patch [11, 12] from contention. On the other hand, narrow band patch antennas can provide CP, small profile, small aperture, dual-band capabilities, and are practical for this application [13].

Dual-band GPS antennas have been studied in the literature for various applications [14–17]. Often a stacked patch forms the basic element and research has been carried out on reducing size [14], improving impedance and axial ratio bandwidth [15], and integrating the feed with the antenna [16,18]. But, practical stacked patches have a total height of about 10-16mm which is too high for this application. (In [19], a stacked patch is presented with a height of 7mm, but the simulated performance was unable to be replicated by measurements.) Another way to generate a second resonant frequency is to cut a slot in a single-layer patch. Examples include a rectangular
slot [20], triangular slot [21], circular slot [22], U-slot [23], and S-slot [17] patch. But, many of these are not designed for CP at both bands [20–23], have a frequency ratio between the higher and lower modes that is too high for GPS [24,25], have poor axial ratio bandwidth [26], or are too large for this application [17]. Therefore, a dual-band RHCP GPS antenna with small height is needed for this application.

Next, obtaining satellite coverage is a challenge because the platform of interest has unknown time-varying attitude due to pitch and roll during flight. Work has been done on attitude estimation for spinning vehicles using magnetic sensors [27,28], gyros [29], accelerometers [30], inertial measurement units [31], and strong interference sources [32]. But, the highly accurate sensors needed are too large and smaller, less accurate sensors can take up to 15 seconds to roughly estimate attitude. This is too long because of the short flight time for this application. It is clearly unwise to rely on these sensors for GPS satellite reception. (Note that GPS itself can be used for attitude estimation [33–37]).

Before a GPS navigation solution is obtained, the receiver must acquire and track at least four satellites. In order to do this, spherical antenna coverage is needed because the platform is rotating. But, spherical coverage cannot be provided by a single antenna as the platform introduces shadow regions of very poor gain. Therefore, to overcome this drawback, antenna diversity (i.e. using multiple elements) is often proposed. For this application, the elements can be spaced around the cylinder circumference. The challenge becomes how to combine the elements to achieve the desired spherical coverage given that the time-varying platform attitude is unknown.
In [38], a four element cylindrical microstrip GPS array is designed (single-frequency band). The cylinder circumference is $1.84\lambda$. To obtain the coverage, the authors simply sum the elements at radio frequency (RF). This gives adequate coverage in the roll plane (i.e. perpendicular to the cylinder axis), but as the observation point moves towards the cylinder axis the coverage becomes very poor. The same results are observed in [39]. In [40], it is concluded that one should simply place an element in each direction needed for coverage and then sum them. In [41], a GPS receiver was investigated with two antennas placed on a cylinder with $16.5\lambda$ circumference ($8.25\lambda$ per element). The GPS receiver lost satellite tracking after a spin rate of 2 Hz was exceeded. It is clear that summing the elements can lead to loss of coverage/tracking.

On the other hand, it is well known that digital beamforming [42] will provide very good coverage given that one has knowledge of the satellite direction-of-arrival (DOA) and time-varying platform attitude. But, as related above, knowledge of the attitude is not available. Even if it is available, one still needs the antenna manifold (complex pattern) and a separate receiver channel for each satellite. This additional complexity may be too great for such a small platform of interest.

Another approach to coverage is to use an antenna switching method prior to the receiver. In [43], a switchable antenna system is proposed, but it assumes the attitude and desired signal DOA are known. In [44], a similar switching approach is suggested with four antennas but this still needs accurate attitude and enough space for all of the antennas. In [45], two antennas are placed on a spinning cylinder and turned on/off based on the attitude towards zenith in order to receive signals from GPS satellites. Again, the obvious drawback is that it assumes the attitude is known. The other drawback, not obviously apparent, is that as the cylinder spins
the antenna begins to point towards the horizon in one direction. Yet, at the horizon in the opposite direction, there are satellites that will be in the shadow region of the “on” antenna. Even with accurate attitude, these satellites will not be continuously tracked by the receiver and maybe up to 30 – 45% of the sky could be periodically lost.

A different way to maintain good coverage/tracking is to modify the GPS receiver in some fashion (whether using multiple elements or a single antenna). In [46], the outputs of two antenna elements are sent to a two channel receiver where both channels are combined in-phase after each channel undergoes the standard cross-correlation. This works well assuming one knows the carrier phase of each channel which is tracked by a phase lock loop (PLL). But, a working PLL relies on the delay lock loop (DLL) to be tracking the satellites (i.e. the carrier phase cannot be tracked until the GPS signal codes are being tracked). But, this is the problem we are trying to solve in the first place.

In [47–49], the satellite tracking problem is tackled by introducing a rotational tracking loop and using a single antenna. The rotational tracking loop estimates the roll rate of the cylinder and demodulates the rotation of the incoming signal. Then, it is sent to the traditional code tracking (DLL) and carrier tracking loops (PLL). When one correlates the demodulated received signal with the reference signal, the post-correlation signal-to-noise-ratio (SNR) improves as the phase from both signals are roughly the same. There are two drawbacks to this approach. First, it increases the complexity of the receiver as one needs a numerically controlled oscillator (NCO), modulation controller, and three demodulator correlators in addition to the standard
loops. (In [49], the three demodulator correlators are eliminated.) The second drawback is that it assumes the received GPS signal is strong enough to be tracked. But, as pointed out above, the antenna may be facing away from the satellite leading to a large decrease in signal amplitude. The key point to emphasize is the amount of time the antenna is facing away from the satellite versus the correlation integration time. If it is short (high spin rate), then the above approach could work, but not if it is long (low spin rate). Therefore, it is only a partial solution.

Another way that receivers could maintain tracking with a single antenna and weak satellite signals is to deeply/tightly integrate the signal tracking and navigation processor (vector tracking loops) [50–56]. In this case, every satellite is jointly tracked with a Kalman filter that has the receiver position, velocity, and time (PVT) estimates as its states. The output of the Kalman filter is sent back to the signal correlators to maintain the joint satellite tracking. Note that the code phase and carrier phase of each satellite are not directly tracked, but they (or their derivative) are input to the Kalman filter and used to track the receiver PVT. If a satellite drops in signal amplitude, a traditional scalar tracking loop would lose that satellite. However, a vector tracking loop could still track the satellite because a good estimate of PVT (and, of course, satellite location) is available. Therefore, in theory, vector tracking seems to provide better performance. There are still questions to be answered, though. For example, the robustness of the loop as multiple satellites lose significant signal amplitude is difficult to estimate [57]. Also, if one properly compares scalar and vector loop tracking, recent results seem to indicate that the improvement in performance might not be as great as first thought [58]. Nevertheless, this area of research is still very active. For our application, though, vector tracking is not the
best choice because of the increase in receiver complexity. Therefore, a method is needed that will maintain continuous satellite tracking without drastically increasing system complexity or needing to know the time-varying platform attitude.

1.3 Contributions and Organization of This Document

The purpose of this work is to extend the state-of-the-art of GPS receiver and antenna technology for the specific application of small cylindrical platforms. There are a number of goals to accomplish this end. First, a RHCP, dual-band GPS antenna with adequate bandwidth at each band and robust tuning is needed that can be easily integrated onto a thin metal cylinder. Second, spherical satellite coverage is needed because the platform rotates. Assuming that multiple antenna elements are used, the design goal becomes how to combine the elements and how many are needed for the desired coverage. This leads to examining and modifying the receiver while also trying to minimize receiver complexity.

Chapter 2 presents a novel dual-band GPS antenna that meets the requirements for this application. The antenna design is presented in a step-by-step way and the method to obtain two resonant modes in a single-layer patch is discussed. The effect of various antenna parameters on tuning and bandwidth are studied. A bandwidth optimized final design that can receive C/A, P(Y), and M-coded signals is presented with small height and aperture. In this chapter, the antenna is placed on a finite ground plane. Measurements are done and confirm the simulation results which is important because previous dual-band designs have been shown to be sensitive to fabrication techniques.
Chapter 3 studies the performance of the novel antenna when it is placed on various diameter metal cylinders (60-160mm). The antenna tuning is shown to be robust when placed on the multiple diameter cylinders even though the antenna is small. It has been specifically designed to minimize the effect of the cylinder diameter on its tuning performance. Note that the cylinders do affect the antenna patterns (e.g. decreasing directivity), but good gain bandwidth performance is still obtained. Measurements of the antenna on a cylinder are carried out and demonstrate an excellent agreement to simulation performance. Therefore, Chapters 2 and 3 present the design and performance of a novel dual-band GPS antenna for use on thin cylinders.

Chapter 4 investigates the satellite coverage obtained when multiple antenna elements are placed on a thin, static cylinder. The signal-to-noise-ratio (SNR) performance of summing the elements at RF and beamforming are compared for different number of elements. It is concluded that both methods are not practical, because the RF sum method does not have spherical SNR coverage and beamforming needs to know platform attitude. The maximum gain method (i.e. choosing the element with the highest gain) is presented and studied because, in theory, it does not need to know platform attitude. Its SNR performance is compared with multiple elements and multiple diameter cylinders, and it is concluded that only two elements are needed for spherical SNR coverage. The question remains, though, about how one should practically implement this method. This is a challenge because the GPS signals prior to the receiver are below the noise floor and because the time-varying platform attitude is unknown.
Chapter 5 builds on the results from Chapter 4 by designing a novel receiver tracking algorithm that implements the maximum gain method and allows for continuous tracking of the satellites signals. To this end, the time-varying platform effects are included in the conventional receiver tracking loop model (delay lock loop). Therefore, the thin, dynamic cylinder is studied. Using the model, a novel receiver tracking algorithm that implements the maximum gain method is designed. Note that the proposed tracking algorithm can be used with an arbitrary number of elements. Example results show that the tracking algorithm provides continuous tracking of satellite signals without knowledge of the time-varying platform attitude.

Chapter 6 contains the summary and conclusion of this research as well as potential future work.
CHAPTER 2

A Novel Dual-Band GPS Receiver Antenna

It is well known that GPS receivers use multiple GPS bands for removal of the ionospheric delay bias as well as other techniques to improve navigation performance [1]. To this end, this chapter focuses on a novel dual-band GPS receiver antenna design. The antenna design needs to provide right-hand circular polarization (RHCP) coverage at both the L1 (1.575 GHz) and L2 (1.227 GHz) bands. The second challenge is to have enough bandwidth at each band for reception of C/A-, P(Y)-, and M-coded GPS signals. These signals have about 25 MHz bandwidth. The third challenge is robust antenna tuning so that it is not very sensitive to small changes in the platform. In addition, a constraint is placed on the antenna design in that it is desired to minimize the antenna height. This is necessary because the antenna will be conformally mounted on thin cylinders to maintain aerodynamics. The outline of the chapter is as follows. Section 2.1 presents and discusses the dual-band antenna design with its geometry, electric field distribution, and method of operation. The simulated performance of a preliminary design is investigated and it is determined that the bandwidth is sub-optimal. To remedy this, Section 2.2 studies the effect of the various antenna parameters on bandwidth. Section 2.3 optimizes the antenna bandwidth using the previous study and presents the simulated performance of the
final antenna design. Section 2.4 compares the measured and simulated performance of the final antenna design. Note that in this chapter the antenna performance is studied while on a flat ground plane. In the next chapter, the antenna will be placed on various diameter cylinders.

2.1 Antenna Design

First, the antenna design challenges, concepts, and features are discussed. In Section 1.2, it was determined that a single-layer micro-strip patch antenna is the best fit to be placed on a thin cylinder because it naturally reduces the antenna height. As a reminder, a patch antenna consists of a ground plane, substrate, metallic patch layer, and some type of feeding mechanism such as a probe or aperture.

The first antenna design challenge is to realize a single-layer, dual-band patch that radiates RHCP at both the GPS L1 (1.575 GHz) and L2 (1.227 GHz) bands. The first step is to achieve RHCP at both radiating bands (resonant modes). To generate two linearly polarized resonant modes, a slot can be cut into the metallic patch layer and an example of this approach has been known for over a decade (see Fig. 2.1(a)) [20]. The main challenge is then to generate RHCP at both bands. The two approaches to the problem are either a single-feed or dual-feed patch (or quad-feed). The single-feed RHCP patch would place either a single probe or aperture somewhere in/on the patch and then introduce asymmetry into the patch layer or substrate. But, if one does this, the 3 dB axial ratio bandwidth is narrower than the impedance bandwidth ($S_{11} < -10 \text{ dB}$) [17]. If one has a very large volume (aperture and height) available, then one could increase the impedance bandwidth enough to have good axial ratio bandwidth. But, in this application, this is not a solution.
Another approach is to use a dual-feed patch where there is a 90° phase difference between the two feeds. Now, the axial ratio bandwidth is greatly increased and the impedance bandwidth becomes the limiting factor for the patch performance. But in order to have good axial ratio for both modes, the physical symmetry of the patch needs to be maintained. Fig. 2.1(b) shows how to maintain the symmetry when introducing a second feed. The patch itself generates one mode and the slots generate a second higher mode. Note that both modes are excited by a single coaxial probe and the second probe is present for RHCP purposes.

The second step is to obtain the proper resonant frequency ratio between the two modes (1.227 GHz and 1.575 GHz). The antenna in Fig. 2.1(b) has a very large ratio between the two modes because the slots are not very long. One approach could be to place a very thin superstrate on top of the patch layer such that the antenna height is only slightly increased. But, there are serious fabrication issues with this because air/glue gaps would cause the resonant modes to shift. A better approach is shown in Figs. 2.1(c) and 2.1(d) where meandering is introduced to the slots to lower their resonant frequency. Since the antenna will be constructed with printed circuit board technology, these precise cuts do not lead to fabrication issues. The details of tuning the antenna will be presented after its other characteristics are discussed.

The two additional features of the antenna design are substrate selection and the introduction of perfect electric conductor (PEC) side walls (copper tape) on the substrate. To provide miniaturization and easy fabrication, Rogers TMM10i ($\epsilon_r = 9.8$, $\delta_d = 0.002$) is selected as the substrate with a thickness of 5.08mm (0.2in). Furthermore, PEC walls are placed on the sides of the substrate in order to provide tuning robustness [59]. The tuning robustness advantage of the PEC walls will become
apparent in Chapter 3 when the antenna is placed on different types of platforms. Note that the PEC walls also provide some antenna miniaturization (i.e. cavity loading).

Fig. 2.2 shows the top and side views of the proposed antenna design including the various physical parameters. In summary, the antenna consists of a micro-strip patch with meandered slots, a substrate, side PEC walls, two feed probes, and a ground plane. Initially, the antenna is placed on top of a 25.4cm × 25.4cm ground plane. To simulate the antenna performance, ANSYS HFSS v.13 is used.

The electric fields generated by the antenna are used to provide a qualitative description of how the antenna operates. Fig. 2.3 shows the electric field amplitude for the two modes in the plane of the micro-strip patch with one probe excited. The locations of red color indicate strong electric fields whereas blue color indicates weak electric fields. For the lower mode, the electric field is concentrated along the edge of the patch and in the slots. The slots affect its resonant frequency as expected [19]. Also, the PEC walls decrease the resonant frequency by increasing capacitance between the ground and patch edge [59]. Therefore, the lower mode is a slot and cavity loaded patch mode. For the higher mode, the electric field is mainly concentrated along the slots and its resonant frequency is only slightly dependent on the patch length. Thus, the higher mode is a slot mode.

Next, a quantitative discussion of the antenna resonant modes is presented. For the tuning study, the antenna size is set to 3cm × 3cm. The nominal design parameters are $S = 30\text{mm}$, $L = W = 28.8\text{mm}$, $a = 0.4\text{mm}$, $W_s = 0.4\text{mm}$, $L_s = 21\text{mm}$, $g_1 = 6\text{mm}$, $g_2 = 2.1\text{mm}$, $g_3 = 3\text{mm}$, $g_4 = 4.5\text{mm}$, and $d = 2\text{mm}$. The patch length ($L$) and slot meandering ($L_s$, $g_1$, and $g_4$) are varied independently while keeping the
Figure 2.1: Evolution of antenna design.
Figure 2.2: Top and side views of the proposed dual-band GPS antenna.
Figure 2.3: Electric field amplitude for the lower and higher modes in the \( xy \) plane with \( z = 5.08 \text{mm} \) with a single probe excited. Locations of red color indicate strong electric fields whereas blue color indicates weak electric fields.

other parameters at their nominal values. A single probe is used in the study to simplify the simulations.

Fig. 2.4 shows \( S_{11} \) versus frequency as the patch length and slot length are varied. As the patch length (\( L \)) increases, the lower mode resonant frequency decreases. Also, as the slot length increases, the lower mode resonant frequency decreases. This verifies that the lower mode is a slot loaded patch mode. (It is also cavity loaded because of the presence of the PEC walls, but this is not demonstrated as it is expected).

Next, as the slot length increases, the higher mode resonant frequency decreases. This occurs whether one varies \( L_s, g_1, \) or \( g_4 \). In addition, the higher mode resonant frequency only slightly varies as the patch length changes. These results confirm that the higher mode is a slot mode. Therefore, the quantitative tuning study verifies the qualitative understanding of how the two modes resonate.
Figure 2.4: $S_{11}$ versus frequency as the patch length ($L$) and slot length ($L_s$, $g_1$, and $g_4$) are varied.
The antenna is tuned to the GPS L1/L2 frequencies. For the preliminary antenna design, the size is set to 3cm × 3cm. The patch length \( L \) and slot length \( g_4 \) are \textit{jointly} varied to tune the antenna because the resonant modes are not independent. The preliminary design parameters are \( S = 3 \text{cm} \), \( L = W = 28.83 \text{mm} \), \( a = 0.4 \text{mm} \), \( W_s = 0.4 \text{mm} \), \( L_s = 21 \text{mm} \), \( g_1 = 9 \text{mm} \), \( g_2 = 1.7 \text{mm} \), \( g_3 = 3 \text{mm} \), \( g_4 = 5.14 \text{mm} \), and \( d = 2.7 \text{mm} \). Two probes are now present in the design and the antenna has been placed on a 25.4cm × 25.4cm (10in × 10in) ground plane.

Fig. 2.5 shows \( S_{11} \) and \( S_{12} \) versus frequency for the 3cm × 3cm antenna. It can be seen there are three \( S_{11} \) nulls instead of the expected two nulls. This occurs because of port coupling. At 1.4 GHz, \( S_{12} \) is very high and therefore the ports are strongly coupled. Fig. 2.6 shows the electric fields at 1.4 GHz in the plane of the microstrip patch with \textit{only} one probe excited. A single probe excites energy in both orthogonal modes (i.e. left/right and top/bottom slots) at this frequency and this causes energy from the first probe to be strongly coupled to the second probe. Hence, there is no radiation at 1.4 GHz. However, at GPS L2 (1.227 GHz) and L1 (1.575 GHz), \( S_{11} \) and \( S_{12} \) are both below -10 dB and power is radiated. This means that the antenna is operating at the desired bands of interest. But note that the port coupling is stronger at the L2 band than at the L1 band.

Fig. 2.7 shows the boresight RHCP realized gain versus frequency for the 3cm × 3cm antenna on a 25.4cm × 25.4cm (10in × 10in) ground plane. It can clearly be seen that there are only two radiating modes and no energy is being radiated at 1.4 GHz. This confirms the conclusions from the \( S \)-parameter results. The peak gain is 3.5 dBi at the L2 and L1 bands. Also, the antenna has poor co-pol to cross-pol ratio.
Figure 2.5: $S_{11}$ and $S_{12}$ versus frequency for the 3cm $\times$ 3cm antenna.

Figure 2.6: Electric field amplitude at 1.4 GHz in the $xy$ plane with $z = 5.08mm$ for the 3cm $\times$ 3cm antenna and with a single probe excited. Locations of red color indicate strong electric fields whereas blue color indicates weak electric fields. Note that this is a non-radiating mode.
Figure 2.7: Boresight realized gain versus frequency for the 3cm × 3cm antenna on a 25.4cm × 25.4cm ground plane.

at the L2 band. In sum, the antenna has RHCP radiation at the GPS L1 and L2 bands and the first antenna design challenge has been achieved.

Yet, the 3cm × 3cm antenna design has some major drawbacks. First, there is strong port coupling ($S_{12}$) at the L2 band which causes the co-pol to cross-pol ratio to decrease to 9.9 dB as seen in Fig. 2.7. This is unacceptable for an RHCP radiator. The probe distance ($d$) can be increased to reduce port coupling (and hence increase co-pol to cross-pol ratio), but the antenna matching will degrade as well. For example, if one selects a probe distance of 4.2mm (original 2.7mm), then the co-pol to cross-pol ratio is 23 dB at the L2 band which is good. But, $S_{11}$ degrades and is -8 dB at L2 and -4.5 dB at L1 which is clearly not good. The antenna loses matching efficiency to gain a better co-pol to cross-pol ratio.

Second, the bandwidth is small at both bands and particularly at the L2 band. The antenna 3 dB bandwidth from the peak boresight gain is only 11 MHz at the
Figure 2.8: Power spectral density versus frequency for the C/A-, P(Y)-, and M-coded GPS signals.

L2 band and 20 MHz at the L1 band. Fig. 2.8 shows the power spectral density versus frequency for the three desired GPS signals. It can be seen that a bandwidth of about 25 MHz is needed to receive the signals. In addition, the C/A- and P(Y)-codes have centered power spectral density main lobes, but the M-code has significant power on the band edges. Therefore, even if one solves the problem of obtaining good matching efficiency and co-pol to cross-pol ratio, the antenna may not have enough bandwidth to receive the desired signals. Achieving enough bandwidth at each band is of great importance for this application as it was outlined as the second antenna design challenge. To improve the antenna bandwidth, a parametric study has been carried out.

2.2 Bandwidth Study

To improve the antenna bandwidth, a parametric study is carried out. This is done by varying the parameter of interest while retuning the antenna to L1/L2 at each step using $L$ and $g_4$. To compare each parameter’s effect on impedance bandwidth, $S_{11}$
at the particular band is used. This is not meant to convey that this metric will be used to compare the antenna bandwidth and desired signal bandwidth. For that comparison, the 3 dB bandwidth from the peak gain is used as seen in the previous and next sections. (Of course, this assumes the peak gain is “good” which it is with values around 4-5 dBi.) Once the effect of each parameter on the impedance bandwidth at each band is known, a strategy for improving the antenna bandwidth is developed.

The parameters to vary are the antenna height, aperture \((S)\), slot width \((W_s)\), slot meander length \((L_s, g_1, g_4)\), slot meander width \((g_2)\), slot meander location \((g_3)\), and slot location \((a)\). Note that the feed location \((d)\) is varied to keep a good match when the parameter of interest is studied (i.e. eliminate the detuning effect on bandwidth). Also, the antenna aperture \((S)\) is 3cm unless otherwise stated.

The antenna height is varied first. Fig. 2.9 shows \(S_{11}\) versus frequency at the L1/L2 bands. As the height increases, the lower band has better bandwidth. This is expected as the lower mode is a patch mode. The slot mode (higher band) bandwidth does not vary much with a change in the antenna height.

Fig. 2.10 shows \(S_{11}\) versus frequency at the L1/L2 bands as the antenna aperture \((S)\) is varied. As the aperture increases, the lower band has better bandwidth. This is reasonable because the gap between the patch edge and substrate edge (PEC wall) is increasing as the aperture increases (see Table 2.1 and note that \(S\) as well as \(L\) are changing because the antenna is being retuned for each aperture). The bandwidth of the slot mode is not affected by change in the antenna aperture.

Next, the antenna volume and parameters related to the slot are varied. Fig. 2.11 shows \(S_{11}\) versus frequency at the L1/L2 bands as the slot width \((W_s)\) is varied.
Figure 2.9: $S_{11}$ versus frequency for the L1 and L2 bands as the antenna height is varied.

Table 2.1: Antenna aperture versus gap width

<table>
<thead>
<tr>
<th>Aperture ($S$)</th>
<th>Gap ($\frac{s-L}{2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30mm</td>
<td>0.6mm</td>
</tr>
<tr>
<td>32.5mm</td>
<td>1mm</td>
</tr>
<tr>
<td>35mm</td>
<td>1.5mm</td>
</tr>
<tr>
<td>37.5mm</td>
<td>2.2mm</td>
</tr>
</tbody>
</table>
The slot mode bandwidth does not change as the slot width varies. At first, this is very unexpected because it is well known that a thicker slot (or dipole) has more bandwidth than a thinner slot. But, in this study, the slot mode is retuned to the L1 band (i.e. $g_4$ increased) after the slot width is increased. So, on second thought, the results are reasonable in that the effect of slot width on bandwidth is not entirely isolated from the slot meander length ($g_4$). Also, note that the lower mode bandwidth has minimal change as the slot width is varied.

Now, the effect of slot meander length on bandwidth is directly studied. Fig. 2.12 shows $S_{11}$ versus frequency at the L1/L2 bands as the slot meander length is varied. “Less meandering” refers to a larger $L_s$ value and smaller $g_1/g_4$ value and “more meandering” vice versa. The slot mode bandwidth is inversely proportional to the meander length which is expected. The result is similar to a meandered dipole and demonstrates why increasing slot width ($W_s$) does not necessarily increase bandwidth.
The lower mode bandwidth is hardly affected by an increase in the slot meander length.

Fig. 2.13 shows $S_{11}$ versus frequency at the L1/L2 bands as the slot meander width ($g_2$) is varied. The higher mode bandwidth increases with slot meander width. This makes sense as increasing slot width can be thought of as decreasing the slot meandering. However, the lower mode bandwidth is inversely proportional to the slot meander width. This is somewhat surprising as the lower mode bandwidth was not affected much by slot meander length (see Fig. 2.12). Between the slot meander length and width parameters, $g_2$ affects the patch mode bandwidth more than $L_s$, $g_1$, or $g_4$. It is not known exactly why this is the case.

The final two parameters are slot meander location ($g_3$) and slot location ($a$). Fig. 2.14 shows $S_{11}$ versus frequency at the L1/L2 bands as the slot meander location is varied. The patch mode bandwidth is not affected by a change in $g_3$. The slot mode bandwidth has a slightly proportional relation with slot meander location. This is
Figure 2.12: $S_{11}$ versus frequency for the L1 and L2 bands as the slot meander length ($L_s$ vs. $g_1/g_4$) is varied.

Figure 2.13: $S_{11}$ versus frequency for the L1 and L2 bands as the slot meander width ($g_2$) is varied.
reasonable because if $g_3$ is too small it increases the slot meandering. Fig. 2.15 shows $S_{11}$ versus frequency at the L1/L2 bands as the slot location is varied. Note that for this study the substrate size ($S$) is 3.25cm. It can be seen that for both resonant modes the bandwidth is hardly affected by the slot location.

Table 2.2 displays the conclusions of the bandwidth study. In summary, the only way to increase the patch mode (L2) bandwidth is to increase the antenna volume through either its height or aperture. The only way to increase the slot mode (L1) bandwidth is to decrease the slot meandering (length or width). Furthermore, increasing the slot meander width increases the slot mode bandwidth but decreases the patch mode bandwidth. Therefore, volume, slot meander length, and slot meander width can be used to optimize the bandwidth of the two resonant modes.

A strategy to improve the antenna bandwidth was developed. In order to increase the lower mode bandwidth (L2), the antenna aperture or height has to be increased. It was decided to keep the height fixed because of the antenna application (see the
Figure 2.15: $S_{11}$ versus frequency for the L1 and L2 bands as the slot location (a) is varied.

Table 2.2: Summary of parametric bandwidth study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>L1 Bandwidth</th>
<th>L2 Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasing height</td>
<td>No change</td>
<td>Increases</td>
</tr>
<tr>
<td>Increasing aperture ($S$)</td>
<td>No change</td>
<td>Increases</td>
</tr>
<tr>
<td>Increasing slot width ($W_s$)</td>
<td>No change</td>
<td>No change</td>
</tr>
<tr>
<td>Increasing meander length ($g_1, g_4, L_s$)</td>
<td>Decreases</td>
<td>No change</td>
</tr>
<tr>
<td>Increasing meander width ($g_2$)</td>
<td>Increases</td>
<td>Decreases</td>
</tr>
<tr>
<td>Increasing meander location ($g_3$)</td>
<td>No change</td>
<td>No change</td>
</tr>
<tr>
<td>Increasing slot location (a)</td>
<td>No change</td>
<td>No change</td>
</tr>
</tbody>
</table>
beginning of Chapter 2). Therefore, the antenna aperture was increased. As outlined above, this improves the L2 bandwidth directly. In addition, there is now more space to fit the slots into such that slot meandering can be reduced. This increases the L1 bandwidth. Furthermore, the meander width \( g_2 \) can be varied to balance the bandwidth for the two modes. Using this strategy a final antenna design with improved bandwidth is presented next.

### 2.3 Final Design with Improved Bandwidth

Using the strategy developed at the end of the last section, a final antenna design with improved bandwidth is presented. A range of aperture sizes was investigated and a final design size of \( 4\text{cm} \times 4\text{cm} \times 5.08\text{mm} \) was selected. This is a size of \( \lambda/6 \times \lambda/6 \times \lambda/50 \) where \( \lambda \) is the free space wavelength at the lower band of interest. Also, note that the material was kept fixed to Rogers TMM10i. The final design parameters are \( S = 40\text{mm}, \ L = W = 33.4\text{mm}, \ a = 0.4\text{mm}, \ W_s = 0.4\text{mm}, \ L_s = 26\text{mm}, \ g_1 = 5.2\text{mm}, \ g_2 = 2.1\text{mm}, \ g_3 = 3\text{mm}, \ g_4 = 4.5\text{mm}, \) and \( d = 4.2\text{mm} \). In the following, simulated performance of this novel antenna is presented. The antenna is mounted on a \( 25.4\text{cm} \times 25.4\text{cm} \) ground plane and an ideal feed is used to excite the antenna.

Fig. 2.16 shows \( S_{11} \) and \( S_{12} \) versus frequency for the final antenna design. As seen in the \( 3\text{cm} \times 3\text{cm} \) antenna, the \( S_{11} \) null around 1.4 GHz corresponds to port coupling where power is not radiated. But, now \( S_{12} \) has decreased in the bands of interest because the probes are further apart. This improves the co-pol to cross-pol ratio as seen below. It should be noted, though, that the new probe location \( (d = 4.2\text{mm} \) and greater than the previous design of \( d = 2.7\text{mm} \)) also causes a slight detuning at 1.227
and 1.575 GHz. This was considered a necessary tradeoff in the design to achieve port isolation. Because this is a GPS receiver antenna, the fact that $S_{11}$ is not below -10 dB is not a major concern. (But, of course, we would like $S_{11}$ to be as low as possible.) Primarily, though, the increased aperture allows for larger bandwidth as compared to the 3cm × 3cm design which is highlighted in the next discussion.

Fig. 2.17 shows the boresight RHCP realized gain versus frequency of the final antenna design on a 25.4cm × 25.4cm ground plane. The peak RHCP realized gain is 5 dBi at the L2 band and 4 dBi at the L1 band. Table 2.3 compares the 3 dB bandwidth of the 3cm × 3cm and 4cm × 4cm antennas. The larger design has better 3 dB bandwidth. Now the bandwidth is at least 25 MHz for both bands. This is good performance considering the antenna height is only λ/50 and allows for reception of the desired signals. Also, good co-pol to cross-pol ratio of 26 dB at the L2 band and
Table 2.3: Comparing antenna bandwidth

<table>
<thead>
<tr>
<th>Antenna</th>
<th>L1 3 dB Bandwidth</th>
<th>L2 3 dB Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>3cm × 3cm</td>
<td>20 MHz</td>
<td>11 MHz</td>
</tr>
<tr>
<td>4cm × 4cm</td>
<td>31 MHz</td>
<td>25 MHz</td>
</tr>
</tbody>
</table>

Figure 2.17: Boresight realized gain versus frequency for the 4cm × 4cm antenna on a 25.4cm × 25.4cm ground plane.

36 dB at the L1 band is obtained and is much better than single-feed designs [17]. The antenna is an efficient RHCP radiator.

Fig. 2.18 shows the realized gain RHCP and LHCP patterns in the xz plane at L1 and L2 center frequency for the final antenna design. Note that, because of physical symmetry, the yz pattern will be identical. A broad pattern is obtained at both bands with a 3 dB beamwidth of 84° at L2 and 108° at L1. This is good because it allows for
reception of multiple satellite signals spread throughout the sky. A directive antenna is not desired for this GPS application. Furthermore, the co-pol to cross-pol ratio is higher than 15 dB until the observation point moves away from boresight to $\theta = \pm 51^\circ$ at L2 and $\theta = \pm 28^\circ$ at L1. This is reasonable considering that the ground plane is not modified (i.e. corrugations or RF chokes) [60]. Also, note that this antenna is not designed for precision applications which generally desire a co-pol to cross-pol ratio of greater than 15 dB up to and even below the horizon to reject multipath (usually LHCP signals).

Therefore, the antenna design achieves the first two challenges of RHCP radiation at the GPS L1 and L2 bands and having enough bandwidth for reception of C/A-, P(Y)-, and M-coded signals. In addition, it has reduced height and a final design size of 4cm × 4cm × 5mm ($\lambda/6 \times \lambda/6 \times \lambda/50$). This allows it to be mounted on a thin
cylinder. Chapter 3 will study the antenna on these platforms, but, first, Chapter 2 continues with confirming the simulation results with measurements.

### 2.4 Antenna Measurements

Next, the antenna performance is measured. The antenna was fabricated by Bay Area Circuits, Inc., using the Rogers TMM10i board. (In Chapter 3, the antenna fabrication and resonant frequency errors will be fully discussed. In this section, though, the antenna from Chapter 3 which was specifically tuned for a cylinder is retuned for use on a ground plane. The manual retuning is done by placing small pieces of copper tape over the ends of the slots, but the antenna size is not changed.) Fig. 2.19 shows the antenna when placed on the 25.4cm × 25.4cm ground plane and ready for measurement. Measurements were performed in the anechoic chamber of the ElectroScience Laboratory.

A surface mount 90° hybrid feed was purchased from MiniCircuits (part number QCN-19) to be used for the antenna measurements. Fig. 2.21 shows the hybrid when placed on a test board. The size of the hybrid is only 3mm x 2mm x 0.89mm allowing it to be integrated either around or under the antenna or in the RF front end. For the measurements, though, the test board is connected by coaxial cables to the ports of the antenna. The hybrid has excellent measured performance (results not shown in figures) with the reflection from the input port below -20 dB from 1.1 to 1.8 GHz. The output ports measured performance is also very good with less than 0.8 dB amplitude difference and 90° ± 1° phase difference from 1.1 to 1.8 GHz.

Fig. 2.22 shows simulated $S_{11}$ and $S_{12}$ and measured $S_{11}$, $S_{22}$, and $S_{12}$ versus frequency for the 4cm × 4cm antenna on a 25.4cm × 25.4cm ground plane. Important,
Figure 2.19: 4cm × 4cm antenna placed on a ground plane and ready for measurement.

Figure 2.20: Surface mount 90° hybrid on test board. The hybrid itself is only 3mm x 2mm x 0.89mm and is the white rectangular box in the center of the board.
the measured radiating modes at 1.227/1.575 GHz are aligned with the simulated performance. Also, as mentioned above, only the antenna resonating modes were retuned to the GPS L1/L2 frequencies. This explains why the simulated and measured $S_{11}$ nulls at approximately 1.4-1.5 GHz are not exactly lined up. In addition, the measured $S_{12}$ (port coupling) is slightly higher than the simulated performance but still good port isolation is obtained at the bands of interest.

Fig. 2.23 shows the simulated and measured realized gain versus frequency for the 4cm × 4cm antenna on a 25.4cm × 25.4cm ground plane. The simulated and measured RHCP gain have similar performance at 1.227 GHz (L2). At 1.575 GHz (L1), there is a 2.3 dBi difference in peak gain. Since the simulated and measured $S_{11}$ performance was similar and the patterns are both broad (see below), part of the reason for the realized gain drop is most likely due to a difference in radiation efficiency. But it is difficult to say for certain because radiation efficiency is hard to measure. In addition, the measured 3 dB bandwidth is 24 MHz for both bands. Also, the measured LHCP gain is higher than the simulated gain with a greater difference.
at the L1 band. The measured co-pol to cross-pol ratio is 18 dB at L2 and 16 dB at L1. This has increased (simulated 26/36 dB at L2/L1) partly because the measured antenna has greater port coupling than the simulated antenna. Nevertheless, the comparison between simulated and measured realized gain performance is reasonable.

Fig. 2.24 shows the simulated and measured realized gain patterns at the center frequencies of the GPS L2 and L1 bands for the 4cm × 4cm antenna on a 25.4cm × 25.4cm ground plane. At the L2 band, the simulated and measured RHCP gain is very similar with the main difference being in LHCP gain. At the L1 band, there is a drop off in gain between the simulated and measured performance as expected from the boresight gain versus frequency plot. But, the simulated and measured patterns are both broad with a measured 3 dB beamwidth of 81° at L2 and 117° at L1.
Figure 2.23: Simulated and measured boresight realized gain versus frequency for the 4cm × 4cm antenna on a 25.4cm × 25.4cm ground plane.

Figure 2.24: Simulated and measured realized gain patterns at the center frequencies of the GPS L2 and L1 bands for the 4cm × 4cm antenna on a 25.4cm × 25.4cm ground plane.
2.5 Summary & Conclusions

A novel dual-band GPS receiver antenna was designed, optimized, simulated, fabricated, and measured. The primary contribution of this antenna is that RHCP coverage at the L1/L2 bands for reception of C/A-, P(Y)-, and M-coded GPS signals is provided while the antenna height was kept to a minimum. The final antenna size is 4cm × 4cm × 5.08mm (λ/6 × λ/6 × λ/50). Table 2.4 summarizes the antenna simulated and measured performance. The important conclusions are that the antenna has measured 3 dB bandwidth of 24 MHz, co-pol to cross-pol ratio of at least 16 dB, and a broad pattern. This combined with its size make it an excellent candidate for small-size and reduced height dual-band GPS applications. In the next chapter, the antenna is placed on thin cylinders and its performance discussed.
Table 2.4: Summary of simulated and measured antenna performance on 25.4cm × 25.4cm ground plane

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>L1 Sim.</th>
<th>L1 Meas.</th>
<th>L2 Sim.</th>
<th>L2 Meas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak RHCP gain</td>
<td>4 dBi</td>
<td>1.7 dBi</td>
<td>5 dBi</td>
<td>5 dBi</td>
</tr>
<tr>
<td>3 dB BW</td>
<td>31 MHz</td>
<td>24 MHz</td>
<td>25 MHz</td>
<td>24 MHz</td>
</tr>
<tr>
<td>Co/cross-pol ratio at boresight</td>
<td>36 dB</td>
<td>16 dB</td>
<td>26 dB</td>
<td>18 dB</td>
</tr>
<tr>
<td>3 dB beamwidth</td>
<td>108°</td>
<td>117°</td>
<td>84°</td>
<td>81°</td>
</tr>
</tbody>
</table>
CHAPTER 3

Dual-Band GPS Receiver Antenna on Thin Cylinders

In the last chapter, a novel dual-band GPS receiver antenna was designed and its performance analyzed when placed on top of a ground plane. In this chapter, the antenna performance will be investigated when placed on various diameter cylinders (60-160mm). For placement, the antenna is cavity mounted on the cylinders. But, before comparing its performance on the cylinders, the effect of the cavity must be isolated from the effect of the cylinders. Once this is done, a proper investigation of the antenna performance on the cylinders can be made. Furthermore, it is desired that the antenna tuning be maintained even if the cylinder diameter changes. This is a challenge because the antenna is close to the cylinder curvature. The chapter outline is as follows. Section 3.1 compares the cavity mounted and top mounted antenna performance when placed on a ground plane. Next, Section 3.2 presents the antenna performance when placed on a 117mm diameter cylinder. Section 3.3 investigates the antenna performance on multiple diameter cylinders including its tuning performance. In Section 3.4, the fabricated antenna is placed on a 117mm diameter cylinder and its simulated and measured performance are compared.
3.1 Flush Mounted Antenna Performance

In Chapter 2, a dual-band GPS receiver antenna was designed and simulated when placed on top of a ground plane. Now, the antenna is placed flush mounted (or in a cavity) on a ground plane (see Fig. 3.1). Note that the substrate size of the antenna is the same size as the cavity itself and that the ground plane is circular and not a square as in Chapter 2. As before, the antenna size is 4cm × 4cm × 5.08mm (\(\lambda/6 \times \lambda/6 \times \lambda/50\)). Since the original design has PEC walls on the sides of the substrate, the antenna tuning does not change that much when it is flushed mounted. The flush mounted antenna has resonances at 1.234 and 1.578 GHz. This is slightly different than the desired 1.227 and 1.575 GHz. Therefore, the antenna is retuned when placed flush in the ground plane.

The flush mounted antenna is retuned with final design parameters of \(S = 40\text{mm},\) \(L = W = 33.68\text{mm},\) \(a = 0.4\text{mm},\) \(W_s = 0.4\text{mm},\) \(L_s = 26\text{mm},\) \(g_1 = 5.2\text{mm},\) \(g_2 = 2.1\text{mm},\) \(g_3 = 3\text{mm},\) \(g_4 = 4.48\text{mm},\) and \(d = 4.2\text{mm}.\) As compared to the antenna on top of the ground plane, the only differences in the antenna parameters are \(L = W = 33.4/33.68\text{mm} \text{ (top/flush)}\) and \(g_4 = 4.5/4.48\text{mm} \text{ (top/flush)}\). The antenna does not have to heavily modified when placed flush in a ground plane. The performance of the cavity antenna and top mounted antenna will be compared to isolate the effect of the cavity on antenna \(S\)-parameter, gain, bandwidth, and pattern performance. In the following, simulated performance is presented, the ground plane has diameter 22.86cm, and an ideal feed is used to excite the antenna.

Fig. 3.2 shows \(S_{11}\) and \(S_{12}\) versus frequency for the antenna design on top and flush to the 22.86cm diameter ground plane. One can see that there is a only a very
slight difference between the two antennas. Note that the port coupling difference is also minute.

Fig. 3.3 shows the boresight RHCP realized gain versus frequency for the antenna design on top and flush to the 22.86cm diameter ground plane. Again, the antennas are very similar with the flush mounted antenna having a slight degradation in bandwidth. The peak RHCP gain is 5.6 dBi at the L2 band and 5.2 dBi at the L1 band for the top mounted antenna and 5.2 dBi at the L2 band and 4.9 dBi at the L1 band for the flush mounted antenna. The 3 dB bandwidth is 24 MHz at the L2 band and 30 MHz at the L1 band for the top mounted antenna and 23 MHz at the L2 band and 29 MHz at the L1 band for the flush mounted antenna. Finally, the co-pol to cross-pol ratio is 24 dB at the L2 band and 29 dB at the L1 band for the top mounted antenna and 24 dB at the L2 band and 31 dB at the L1 band for the flush mounted antenna. It is clear that the antenna mounting does not degrade the boresight gain performance that much.

Fig. 3.4 shows the RHCP and LHCP realized gain patterns at the L1 and L2 center frequencies for the antenna design on top and flush to the 22.86cm diameter ground plane.
Figure 3.2: $S_{11}$ and $S_{12}$ versus frequency when the antenna is placed on top and flush to a 22.86cm diameter ground plane.

Figure 3.3: Boresight realized gain versus frequency when the antenna is placed on top and flush to the 22.86cm diameter ground plane.
Figure 3.4: Realized gain patterns at the center frequencies of the L1 and L2 bands in the $xz$ plane when the antenna is placed on top and flush to a 22.86cm diameter ground plane.

ground plane. At both bands, the antennas have very similar patterns. The 3 dB beamwidth is $79^\circ$ at 1.227 GHz (L2) and $72^\circ$ at 1.575 GHz (L1) for both the top mounted and flush mounted antennas.

Table 3.1 summarizes the performance of the top mounted and flush mounted antenna. The introduction of the cavity only slightly reduces the peak gain and 3 dB bandwidth. The co-pol to cross-pol ratio and 3 dB beamwidth are either similar or exactly the same for both antennas. Therefore, the antenna design can be cavity mounted on the cylindrical platforms of interest. Any changes in performance will be due to the cylinder and not the cavity.
Table 3.1: Antenna performance when mounted on top and flush to a 22.86cm diameter ground plane

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>L1 Top</th>
<th>L1 Flush</th>
<th>L2 Top</th>
<th>L2 Flush</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak RHCP gain</td>
<td>5.2 dBi</td>
<td>4.9 dBi</td>
<td>5.6 dBi</td>
<td>5.2 dBi</td>
</tr>
<tr>
<td>3 dB BW</td>
<td>30 MHz</td>
<td>29 MHz</td>
<td>24 MHz</td>
<td>23 MHz</td>
</tr>
<tr>
<td>Co/cross-pol ratio at boresight</td>
<td>29 dB</td>
<td>31 dB</td>
<td>24 dB</td>
<td>25 dB</td>
</tr>
<tr>
<td>3 dB beamwidth</td>
<td>72°</td>
<td>72°</td>
<td>79°</td>
<td>79°</td>
</tr>
</tbody>
</table>

3.2 Antenna Performance on 117mm Diameter Cylinder

Next, the 4cm × 4cm antenna is cavity mounted on a 117mm diameter cylinder (0.62λ$_{L1}$, 0.49λ$_{L2}$). Fig. 3.5 shows the antenna placed on the cylinder. The length of the cylinder is 61.6cm (3.24λ$_{L1}$, 2.57λ$_{L2}$). The length of the cavity is 5.87cm and the depth of the cavity is 0.95cm. Note that the substrate and cavity are no longer the same size. This is done to improve antenna tuning robustness (which is discussed below). Once it is placed on the cylinder, the antenna has to be slightly retuned because the platform changes the resonant frequencies. In this case, though, the pitch plane (xz) and roll plane (yz) modes have different PEC structures very close to the antenna which causes different tunings for each orthogonal mode. To account for this, the parameter $b_4$ is introduced (see Fig. 3.6). $W$, $L$, $g_4$, and $b_4$ are varied independently to tune the antenna. The final design tuning parameters are $L = 33.42$mm, $W = 33.52$mm, $a = 0.4$mm, $W_s = 0.4$mm, $L_s = 24.5$mm, $g_1 = 6$mm, $g_2 = 2.1$mm, $g_3 = 3$mm, $g_4 = 4.54$mm, $b_4 = 4.53$mm, and $d = 4$mm. Note that the
flush mounted antenna on a ground plane had $L = W = 33.68\text{ mm}$, $g_4 = b_4 = 4.48\text{ mm}$, and $d = 4.2\text{ mm}$. In the following, simulated performance is presented when an ideal feed is used to excite the antenna.

Fig. 3.7 shows $S_{11}$ (pitch mode), $S_{12}$, and $S_{22}$ (roll mode) versus frequency for the antenna placed on the 117mm diameter cylinder. First, there is good port isolation (low $S_{12}$) at both bands of interest. Due to the differences in the platform in the pitch and roll planes, the roll mode has lower bandwidth than the pitch mode. Therefore, there is a platform effect not only on the resonant frequencies but even on $S_{11}$ and $S_{22}$ performance.

Fig. 3.8 shows the boresight RHCP and LHCP realized gain versus frequency for the antenna on the 117mm diameter cylinder. The peak RHCP gain is 2.3 dBi at the L2 band and 2.6 dBi at the L1 band. The 3 dB bandwidth is 24 MHz and 27 MHz at the L2 and L1 bands, respectively, which is good for reception of the signals of interest. The co-pol to cross-pol ratio is 11 dB at the L2 band and 12 dB at the L1 band. It can be seen that the boresight RHCP gain has decreased when the antenna is placed on the cylinder as compared to a ground plane. This is reasonable because...
Figure 3.6: The 4cm × 4cm antenna design and its parameters when placed on a cylinder.

Figure 3.7: $S_{11}$ (pitch mode), $S_{12}$, and $S_{22}$ (roll mode) versus frequency for the antenna placed on the 117mm diameter cylinder.
the antenna becomes less directive (as compared to the ground plane) due to the curvature of the cylinder. Fig. 3.9 shows the electric field amplitude in the roll plane at the L2 and L1 center frequencies for the antenna placed on the cylinder. It can be seen that there are strong electric fields being guided by the cylinder curvature below the horizon. Furthermore, the LHCP component increases. This is partly because of diffraction off the ends of the cylinder (see Fig. 3.10). The main contribution to the increase in the LHCP component, though, is that the two orthogonal modes are no longer 90° out-of-phase. This happens because of the platform asymmetry close to the antenna. If one had complete control over the phase between the ports, then the LHCP component could be decreased. For example, if the ports had a 120° phase between them, then the co-pol to cross-pol ratio becomes 21 dB at the L2 band (2.6 dBi peak RHCP gain) and 28 dB at the L1 band (2.8 dBi peak RHCP gain). But, this type of control over the port phasing is difficult to achieve, particularly when it is needed at both bands. Therefore, the platform asymmetry close to the antenna causes an increase in the co-pol to cross-pol ratio and the curvature of the platform causes a decrease in the antenna directivity.

Fig. 3.11 shows the pitch and roll plane realized gain patterns at the center frequencies of the L1 and L2 bands for the antenna on the 117mm diameter cylinder. The antenna is less directive when placed on the platform with a pitch plane 3 dB beamwidth of 137° at L2 and 124° at L1 and roll plane 3 dB beamwidth of 126° at L2 and 114° at L1. Also, there is significant radiation in the lower hemisphere due to the cylinder curvature. Finally, the LHCP gain has ripple in the pitch plane upper hemisphere. This is due to diffraction off the ends of the cylinder. A study was done on the effect of cylinder length on LHCP ripple. It was found that different lengths cause
the ripple to increase/decrease or to change where the peaks and troughs are located (results not shown). But, the length of the cylinder is not an engineering parameter in this application. The only way to decrease the diffraction off the ends is to modify the cylinder in some fashion (smoothing, corrugation, etc.). Again, modifications to the cylinder are not considered engineering parameters for this application.

Table 3.2 summarizes the antenna performance on a 117mm diameter cylinder. The 3 dB bandwidth is at least 24 MHz for both bands which means that the antenna can still receive C/A-, P(Y)-, and M-coded GPS signals. (The slight decrease in bandwidth from 25 to 24 MHz will not cause a problem.) The antenna becomes less directive on the cylinder as compared to the ground plane performance with a lower peak gain and broader beam. This is not necessarily a negative because the antenna is trying to receive signals from satellites spread out across the sky. Furthermore, the antenna has a higher co-pol to cross-pol ratio as compared to the ground
Figure 3.9: Electric field amplitude in the roll plane for the antenna placed on the cylinder of diameter 117mm and length 61.6cm.
Figure 3.10: Electric field amplitude in the pitch plane for the antenna placed on the cylinder of diameter 117mm and length 61.6cm.
plane performance. Yet, this will not lead to degradation in navigation accuracy due to multipath GPS signals originating away from the cylinder. This is because the platform is considered to be in free space and only subject to long delay multipath which is automatically rejected by the receiver [1]. Note that the antenna/platform still has direction dependent code delay and carrier phase biases (see Chapter 5 for more details). Calculating these biases includes all multipath signals originating from the cylinder (platform effect) as well as the antenna effect itself on the biases.
Table 3.2: Summary of antenna performance on a 117mm diameter cylinder

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>L1</th>
<th>L2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak RHCP gain</td>
<td>2.6 dBi</td>
<td>2.3 dBi</td>
</tr>
<tr>
<td>3 dB BW</td>
<td>27 MHz</td>
<td>24 MHz</td>
</tr>
<tr>
<td>Co/cross-pol ratio at boresight</td>
<td>12 dB</td>
<td>11 dB</td>
</tr>
<tr>
<td>Pitch 3 dB beamwidth</td>
<td>124°</td>
<td>137°</td>
</tr>
<tr>
<td>Roll 3 dB beamwidth</td>
<td>114°</td>
<td>126°</td>
</tr>
</tbody>
</table>

3.3 Antenna Performance on Additional Diameter Cylinders

The performance of the 4cm × 4cm antenna when it is placed on additional diameter cylinders is now studied (see Fig. 3.12). The cylinders have diameters of 60mm (0.32\(\lambda_{L1}\), 0.25\(\lambda_{L2}\)), 75mm (0.39\(\lambda_{L1}\), 0.31\(\lambda_{L2}\)), 95mm (0.5\(\lambda_{L1}\), 0.4\(\lambda_{L2}\)), 135mm (0.71\(\lambda_{L1}\), 0.56\(\lambda_{L2}\)), and 160mm (0.84\(\lambda_{L1}\), 0.67\(\lambda_{L2}\)). The lengths of the cylinders are kept fixed to 61.6cm. In addition, the depth of the cavity on the cylinders has to be varied slightly in order to keep the antenna conformal. For the 95mm through 160mm cylinders, the depth is 0.95cm. For the 75mm cylinder, the depth is 1.11cm and, for the 60mm cylinder, the depth is 1.27cm. The length of the cavity is kept fixed to 5.87cm.

An important consideration in this study is whether or not one needs to retune the antenna design for each cylinder. The antenna tuned for the 117mm cylinder is placed on multiple diameter cylinders and the \(S_{11}\), \(S_{12}\), and \(S_{22}\) performance is compared (see Fig. 3.13). Note that for both modes and at both bands, the change
Figure 3.12: Antenna placed on various diameter cylinders each with length 61.6cm.
Table 3.3: Antenna resonant frequency (in MHz) on multiple diameter cylinders

<table>
<thead>
<tr>
<th>Res. freq</th>
<th>60mm</th>
<th>75mm</th>
<th>95mm</th>
<th>117mm</th>
<th>135mm</th>
<th>160mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 S11</td>
<td>1574</td>
<td>1575</td>
<td>1575</td>
<td>1575</td>
<td>1575</td>
<td>1575</td>
</tr>
<tr>
<td>L1 S22</td>
<td>1575</td>
<td>1575.5</td>
<td>1575</td>
<td>1575</td>
<td>1575</td>
<td>1575</td>
</tr>
<tr>
<td>L2 S11</td>
<td>1226</td>
<td>1227</td>
<td>1227</td>
<td>1227.5</td>
<td>1227.5</td>
<td>1227.5</td>
</tr>
<tr>
<td>L2 S22</td>
<td>1228.5</td>
<td>1228.5</td>
<td>1227.5</td>
<td>1227.5</td>
<td>1227.5</td>
<td>1227.5</td>
</tr>
</tbody>
</table>

in diameter causes very little shift in the resonant frequencies. The maximum shift is only 1.6 MHz which is considered to be within fabrication error (see Table 3.3 and note that the precise resonant frequency for L2 is 1.2276 GHz). Therefore, one can use the same antenna on a wide range of cylinders without need for retuning. The reason for this is the PEC walls placed on the substrate sides and the substrate length (4cm) being shorter than the cavity length (5.86cm). This combination helps to mitigate the change in tuning by isolating the antenna from the platform. The tuning robustness of the antenna is an important contribution and constitutes the third main antenna design challenge as outlined in Chapter 2.

Fig. 3.14 shows boresight realized gain versus frequency for the L1 and L2 bands when the antenna is placed on various diameter cylinders. All of the antennas have at least 23 MHz 3 dB bandwidth at the L1 and L2 bands (see Table 3.4). This is important for reception of the desired signals of interest. As the diameter decreases, the peak RHCP gain decreases as well. Since the antenna is matched, the antenna is less directive with increased curvature which is a reasonable result. It can be
Figure 3.13: $S_{11}$ (pitch mode), $S_{22}$ (roll mode) and $S_{12}$ versus frequency for the 4cm $\times$ 4cm antenna placed on multiple diameter cylinders.
Figure 3.14: Boresight realized gain versus frequency at the L1 and L2 bands for the antenna placed on multiple diameter cylinders. The top set of curves are RHCP and the bottom set are LHCP.

seen that the larger cylinders have better peak gain performance. In addition, the LHCP component increases for the smaller diameter cylinders. This is because the orthogonal modes are no longer 90° out-of-phase as mentioned above. The platform asymmetry that is close to the antenna causes this to occur. For the smaller diameter cylinders, the asymmetry (difference between flat PEC structure in pitch plane and curved PEC structure in roll plane) becomes greater. To decrease the LHCP gain, each cylinder would need a different phase between the antenna ports (e.g. 105° or 80°). But, this amount of control is not possible for this application as was discussed above. Note also that the large diameter cylinders, 95mm to 160mm, have similar performance.

Figs. 3.15 and 3.16 show the pitch and roll plane realized gain patterns at the L1 and L2 center frequencies for the antenna on various diameter cylinders. As the diameter decreases, the RHCP gain decreases. This occurs because the antenna
becomes less directive with increased curvature. In general, the larger cylinders have a narrower beam which is quite reasonable. The platform also causes the LHCP gain to increase and the larger diameter cylinders have better LHCP performance. This is because of the platform asymmetry close to the antenna (orthogonal modes are not 90° out-of-phase) and diffraction off the ends of the cylinder. Nevertheless, all of the cylinders provide broad RHCP coverage (see Table 3.5). Furthermore, the antenna on various cylinders has a higher co-pol to cross-pol ratio as compared to the ground plane performance. As discussed above, this will not lead to degradation in navigation accuracy due to multipath GPS signals originating apart from the cylinder. This is because the platform is considered to be in free space and only subject to long delay multipath which is automatically rejected by the receiver [1]. Of course, the antenna/platform still has direction dependent code delay and carrier phase biases.
Table 3.5: Antenna half-power beamwidth (HPBW) on multiple diameter cylinders

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>60mm</th>
<th>75mm</th>
<th>95mm</th>
<th>117mm</th>
<th>135mm</th>
<th>160mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 pitch HPBW</td>
<td>146°</td>
<td>143°</td>
<td>135°</td>
<td>124°</td>
<td>124°</td>
<td>121°</td>
</tr>
<tr>
<td>L2 pitch HPBW</td>
<td>143°</td>
<td>140°</td>
<td>138°</td>
<td>137°</td>
<td>130°</td>
<td>125°</td>
</tr>
<tr>
<td>L1 roll HPBW</td>
<td>120°</td>
<td>120°</td>
<td>120°</td>
<td>114°</td>
<td>115°</td>
<td>117°</td>
</tr>
<tr>
<td>L2 roll HPBW</td>
<td>128°</td>
<td>125°</td>
<td>129°</td>
<td>126°</td>
<td>116°</td>
<td>111°</td>
</tr>
</tbody>
</table>

(see Chapter 5). Calculating these biases includes all multipath signals originating from the cylinder (platform effect) as well as the antenna effect itself on the biases.
Figure 3.15: RHCP and LHCP realized gain patterns in the pitch plane at the L1 and L2 center frequencies for the antenna on various diameter cylinders.
Figure 3.16: RHCP and LHCP realized gain patterns in the roll plane at the L1 and L2 center frequencies for the antenna on various diameter cylinders.
3.4 Antenna Measurements on 117mm Diameter Cylinder

The antenna performance is measured when it is placed on a cylinder of diameter 117mm and length 61.6cm. The antenna was fabricated by Bay Area Circuits, Inc. As mentioned in Chapter 2, a surface mount 90° hybrid feed was used for the antenna measurements (see section 2.4). Fig. 3.17 shows the antenna configuration for measurement. All measurements were done in the RF anechoic chamber of the ElectroScience Laboratory.

Fig. 3.18 shows the antenna placed on a cylinder of diameter 117mm and length 61.6cm. The first parameter measured is the resonant frequencies of each port of the slotted patch. The measured resonant frequencies were 1.210 and 1.570 GHz for both the pitch and roll modes. This gives a 1.3% and 0.3% error from the desired frequencies of 1.227 and 1.575 GHz. This error is probably due to the material not having a precise dielectric constant of 9.8 because both measured resonant frequencies are slightly lower than the desired frequencies. Therefore, we retuned the antenna to 1.227 and 1.575 GHz. This is done by adding two small pieces of metal tape to two of the slots as shown in Fig. 3.18 and also by slightly lowering the metal tape on the sides of the antenna (although this cannot be seen in the photograph). To lower the percent error between the measured and desired frequencies in a more practical way, it is recommended that precise measurement of the dielectric material be made. All of the following results are obtained with the retuned antenna.

Fig. 3.19 compares simulated and measured $S_{11}$ and $S_{22}$ versus frequency for the antenna on the cylinder. As mentioned above, the $S_{11}$ and $S_{22}$ nulls around 1.4 GHz are due to port coupling and not due to radiation. At the GPS L1 and L2 bands, the
Figure 3.17: Antenna placed on 117mm diameter cylinder and ready for measurement.
Figure 3.18: Zoomed in view of antenna placed on 117mm diameter cylinder. Notice the small pieces of metal tape in the top left and bottom left corners of the patch to precisely tune the antenna to the GPS L1 and L2 frequencies.
simulated and measured resonant frequencies are very close together once the antenna is retuned. Also, the pitch and roll modes both had the desired resonant frequency.

Fig. 3.20 compares simulated and measured boresight realized gain versus frequency for the antenna placed on the 117mm cylinder. At the L2 band, the simulated and measured RHCP realized gain is similar to the measured peak RHCP gain of 1.9 dBi. At the L1 band, the measured peak RHCP gain is 1.7 dBi which is 0.9 dBi lower than simulation. The 3 dB bandwidth is 25 MHz at the L2 band and 27 MHz at the L2 band which is about the same bandwidth as simulation. Also, for LHCP and at both bands, the simulated gain is higher than the measured gain, which is unexpected. The measured co-pol to cross-pol ratio is 14 dB at the L2 band and 16 dB at the L1 band. After measurement, additional HFSS simulations were run at a very fine mesh size to check the results, but major changes in simulated RHCP or LHCP gain were not observed.
Figure 3.20: Simulated and measured boresight realized gain versus frequency for the antenna placed on the 117mm cylinder.

Fig. 3.21 compares simulated and measured RHCP and LHCP realized gain patterns in the pitch and roll planes at the L1 and L2 center frequencies for the antenna on the 117mm cylinder. In both planes, the simulated and measured RHCP patterns are similar at both frequencies. From $-90^\circ$ to $90^\circ$, for both frequencies and patterns cuts, the difference between simulated and measured RHCP gain is always less than a dB. The only exception to this is for the GPS L1 pitch plane, where the difference can reach a few dB in certain directions. Note the patterns being compared are not normalized. When comparing the LHCP patterns, one sees that the measured gain is almost always lower than the simulated gain which is expected because of the boresight gain results. The measured pitch plane 3 dB beamwidth is $131^\circ$ at the L2 band and $114^\circ$ at the L1 band and the measured roll plane 3 dB beamwidth is $122^\circ$ at the L2 band and $133^\circ$ at the L1 band. When comparing the measured beamwidths to the simulated beamwidths the greatest difference ($19^\circ$) occurs at the L1 band and in
Figure 3.21: Comparing simulated and measured RHCP and LHCP realized gain patterns in the pitch and roll planes at the L1 and L2 center frequencies for the antenna on the 117mm cylinder.

Table 3.6 summarizes the simulated and measured antenna performance on the 117mm diameter cylinder. The 3 dB bandwidth is about the same for the simulated and measured performance. Also, as compared to the measured antenna performance on a ground plane (see section 2.4), the antenna peak gain performance on the cylinder
Table 3.6: Summary of simulated and measured antenna performance on 117mm diameter cylinder

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>L1 Sim.</th>
<th>L1 Meas.</th>
<th>L2 Sim.</th>
<th>L2 Meas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak RHCP gain</td>
<td>2.6 dBi</td>
<td>1.7 dBi</td>
<td>2.3 dBi</td>
<td>1.9 dBi</td>
</tr>
<tr>
<td>3 dB BW</td>
<td>27 MHz</td>
<td>27 MHz</td>
<td>24 MHz</td>
<td>25 MHz</td>
</tr>
<tr>
<td>Co/cross-pol ratio at boresight</td>
<td>12 dB</td>
<td>16 dB</td>
<td>11 dB</td>
<td>14 dB</td>
</tr>
<tr>
<td>Pitch HPBW</td>
<td>124°</td>
<td>114°</td>
<td>137°</td>
<td>131°</td>
</tr>
<tr>
<td>Roll HPBW</td>
<td>114°</td>
<td>133°</td>
<td>126°</td>
<td>122°</td>
</tr>
</tbody>
</table>

has a better agreement with simulation. In addition, the co-pol to cross-pol ratio is better for the measured antenna. Finally, the 3 dB beamwidths has relatively good agreement with the largest difference of 19° at the L1 band in the roll plane.

3.5 Summary & Conclusions

The antenna designed for a dual-band GPS receiver was placed on various diameter cylinders and its performance was investigated. To isolate the cylinder curvature effect on performance from the cavity effect, the antenna was first flush mounted in a finite ground plane. The flush mounted and top mounted antenna results were compared and it was concluded that the cavity only slightly degrades the bandwidth. The antenna was then placed on a 117mm diameter cylinder. This caused a decrease in directivity as compared to the ground plane performance which is quite reasonable because of the curvature. On the 117mm diameter cylinder and at both bands, the antenna has at least 2.3 dBi peak RHCP gain, 24 MHz 3 dB bandwidth, 114° 3 dB
beamwidth, and a boresight co-pol to cross-pol ratio of 11 dB. The antenna tuned on the 117mm cylinder was placed on various diameter cylinders. The maximum resonant frequency shift observed was 1.6 MHz. Therefore, the antenna does not need to be retuned when placed on different diameter cylinders. This is an important contribution and constitutes the completion of the third major antenna design challenge as outlined in Chapter 2. Table 3.7 summarizes the simulated antenna performance when it is placed on multiple diameter cylinders. For all cylinders, the antenna has broad RHCP coverage (111° HPBW) and good gain bandwidth performance (1-2.8 dBi peak RHCP gain with at least 23 MHz 3 dB bandwidth). This is adequate to receive C/A-, P(Y)-, and M-coded GPS satellite signals. Finally, the fabricated antenna was placed on the 117mm diameter cylinder and its simulated and measured performance was compared. The resonant frequency error was only 0.3% and 1.3% at the GPS L1/L2 bands. This is an excellent result for an experimental resonant type antenna of small volume. Once the antenna was retuned, the simulated and measured performance had very good agreement. In the next chapter, the satellite coverage of the antenna on thin cylinders is investigated.
Table 3.7: Summary of antenna performance on multiple diameter cylinders

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>60mm</th>
<th>75mm</th>
<th>95mm</th>
<th>117mm</th>
<th>135mm</th>
<th>160mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 peak gain</td>
<td>1 dBi</td>
<td>1.6 dBi</td>
<td>2.1 dBi</td>
<td>2.6 dBi</td>
<td>2.6 dBi</td>
<td>2.7 dBi</td>
</tr>
<tr>
<td>L2 peak gain</td>
<td>1.1 dBi</td>
<td>1.8 dBi</td>
<td>2.1 dBi</td>
<td>2.3 dBi</td>
<td>2.6 dBi</td>
<td>2.8 dBi</td>
</tr>
<tr>
<td>L1 3 dB BW</td>
<td>28 MHz</td>
<td>28 MHz</td>
<td>27 MHz</td>
<td>27 MHz</td>
<td>27 MHz</td>
<td>27 MHz</td>
</tr>
<tr>
<td>L2 3 dB BW</td>
<td>26 MHz</td>
<td>25 MHz</td>
<td>24 MHz</td>
<td>24 MHz</td>
<td>24 MHz</td>
<td>23 MHz</td>
</tr>
<tr>
<td>L1 co/cross-pol at boresight</td>
<td>7 dB</td>
<td>10 dB</td>
<td>11 dB</td>
<td>12 dB</td>
<td>12 dB</td>
<td>13 dB</td>
</tr>
<tr>
<td>L2 co/cross-pol at boresight</td>
<td>6 dB</td>
<td>8 dB</td>
<td>10 dB</td>
<td>11 dB</td>
<td>11 dB</td>
<td>13 dB</td>
</tr>
<tr>
<td>L1 pitch HPBW</td>
<td>146°</td>
<td>143°</td>
<td>135°</td>
<td>124°</td>
<td>124°</td>
<td>121°</td>
</tr>
<tr>
<td>L2 pitch HPBW</td>
<td>143°</td>
<td>140°</td>
<td>138°</td>
<td>137°</td>
<td>130°</td>
<td>125°</td>
</tr>
<tr>
<td>L1 roll HPBW</td>
<td>120°</td>
<td>120°</td>
<td>120°</td>
<td>114°</td>
<td>115°</td>
<td>117°</td>
</tr>
<tr>
<td>L2 roll HPBW</td>
<td>128°</td>
<td>125°</td>
<td>129°</td>
<td>126°</td>
<td>116°</td>
<td>111°</td>
</tr>
</tbody>
</table>


CHAPTER 4

Satellite Coverage of Antenna on Thin Cylinders

In this chapter, the satellite coverage provided by the dual-band GPS receiver antenna on thin cylinders is investigated. As the platform rotates, spherical satellite coverage is needed for the GPS receiver because it allows for continuous satellite tracking. The main challenge is to provide a practical method for spherical satellite coverage without having knowledge of the time-varying platform attitude. Note that, in this chapter, the platform is not rotating and a static satellite coverage study is carried out. As expected, a single antenna element cannot provide the desired spherical coverage. Multiple elements are placed around the circumference of the cylinder and different methods to combine them are presented. The methods investigated are summing the elements directly (RF sum), the optimal beamforming method, and choosing the element with the highest gain (maximum gain method). To compare the methods, the signal-to-noise ratio (SNR) after combining the elements but prior to the receiver is used as the metric. The cylinders studied have diameters of 75mm and 160mm. Furthermore, the number of antenna elements is varied for each method. It is important to minimize the number of elements needed for good coverage because the metallic cylinder’s structural integrity should be maintained. The outline of this chapter is as follows. Section 4.1 presents a system model of the antenna and RF front
end for the various methods. It then derives the SNR equations for each method from the system models. Section 4.2 compares and discusses the satellite coverage results of the various methods when the antenna is placed on the 75mm cylinder. Both GPS L1 and L2 band results are presented. Section 4.3 extends the results and discussion to include the 160mm cylinder. Section 4.4 provides the summary and conclusions for the chapter.

4.1 Antenna and RF Front End Model

Multiple antenna elements (as designed in Chapters 2 and 3) are placed around the circumference of the small cylindrical platforms of interest. The elements are combined in various ways and their satellite coverage is compared. The combination methods are summing the elements (RF sum method), beamforming, and choosing the element with the highest gain (maximum gain method). Furthermore, the number of antenna elements is varied for each method. It is important to minimize the number of elements needed for good coverage because the metallic cylinder’s structural integrity should be maintained. The metric used to generate and compare results is SNR after the front end and combination but prior to the receiver. To this end, SNR equations for each of the methods are derived starting with the SNR of a single antenna element.

4.1.1 Single Antenna Element

The block diagram and system model of a single antenna element is shown in Fig. 4.1. The signal is received by the antenna and sent through the RF front end. The front end consists of a low noise amplifier (LNA), bandpass filter, mixer, and A/D converter. After preprocessing, the signal is sent to the receiver where the usual acquisition and tracking occur. The antenna is modeled as a filter with
The frequency response $A(f, \theta, \phi)$ where $f$ is the frequency, $\theta$ is the angle measured from zenith (complement of elevation angle), and $\phi$ is the azimuth angle. The antenna is considered to add noise $\hat{n}(t)$ to the system. This noise comes from external sources. The RF front end is modeled as an ideal bandpass filter with system bandwidth $B$. The front end adds internal noise (thermal), $n(t)$, to the signal prior to being input to the receiver. This is considered to come primarily from the LNA. The reason for splitting the noise into two separate sources will become apparent in the next section. Both noise sources are assumed to be white and Gaussian. Note that if one wanted to incorporate a non-ideal front end, $F(f)$, then the filter block in Fig. 4.1 would be $A(f, \theta, \phi)F(f)$. Also, band limited signals are assumed for this study.

The SNR of the single antenna element is

$$\text{SNR} = \frac{P_d}{P_n} = \frac{\int S_d(f)df}{\int (S_n(f) + S\hat{n}(f))df}$$  \hspace{1cm} (4.1)$$

where $P_d$ is the desired signal power, $P_n$ is the total noise power, $S_d(f)$ is the desired signal power spectral density after being filtered, $S_n(f)$ is the internal noise power spectral density, and $S\hat{n}(f)$ is the external noise power spectral density. For convenience sake, the integration $\int$ is assumed to mean $\int_{-B/2}^{B/2}$ where $B$ is the above mentioned system bandwidth. The desired signal power spectral density is given by

$$S_d(f) = C_d |H(f, \theta, \phi)|^2 G_d(f)$$  \hspace{1cm} (4.2)$$

where $H(f, \theta, \phi)$ is the antenna system response ($H = A$ for one element), $G_d(f)$ is the normalized desired signal power spectral density and $C_d$ is the desired signal power prior to filtering. A normalized power spectral density means that
\[ \int G_d(f) \, df = 1. \]  \hspace{1cm} (4.3)

\( S_n(f) \) is given by

\[ S_n(f) = \mathcal{F}\{E\{n(t)n^*(t - \tau')\}\} \]  \hspace{1cm} (4.4)

where * is the complex conjugate, \( E\{\cdot\} \) is the expectation operator, \( \mathcal{F} \) is the Fourier transform, and \( \tau' \) (delay between the random wide-sense stationary signals) is used instead of the usual \( \tau \) because \( \tau \) is used to denote a different concept in Chapter 5.

The Fourier transform kernel is the noise autocorrelation function. 4.4 simplifies to

\[ S_n(f) = \mathcal{F}\{N_0 \delta(\tau')\} \]  \hspace{1cm} (4.5)

\[ = N_0 \]  \hspace{1cm} (4.6)

where \( N_0 \) is the thermal noise variance with units of Watts/Hertz and the fact that the noise is white was used. In the same way, the external power spectral density is

\[ S_\hat{n}(f) = \hat{N}_0 \]  \hspace{1cm} (4.7)

where \( \hat{N}_0 \) is the external noise variance. Let the ratio of external to internal noise variance be \( \alpha = \hat{N}_0/N_0 \). For now, the ratio is left as a variable. Its possible values will be discussed later. will be discussed below. The total noise power \( P_n \) becomes

\[ P_n = \int (N_0 + \hat{N}_0) \, df = (N_0 + \hat{N}_0) B = (1 + \alpha) N_0 B. \]  \hspace{1cm} (4.8)

A new expression can be defined as \( C_n \triangleq (1 + \alpha)N_0 B \) where \( C_n \) is the noise power in a single channel. For the single element case, \( P_n = C_n \), but this in general is not
true as will be seen in the next sections. Simplifying the above equations leads to a final SNR expression of

\[
\text{SNR}(\theta, \phi) = \frac{C_d}{C_n} \int G_d(f) |A(f, \theta, \phi)|^2 df. \tag{4.9}
\]

Note that this equation depends on \( \theta \) and \( \phi \) which are considered to be the satellite direction with the cylinder being fixed. The ratio \( C_d/C_n \) is considered to be the SNR upon an isotropic antenna element (i.e. unity gain). The above equation simplifies to this if \( A(f, \theta, \phi) = 1 \). In this study, the ratio \( C_d/C_n \) will be fixed when comparing the various combination methods.

### 4.1.2 RF Sum Method

Fig. 4.2 shows a block diagram and system model for the RF sum method. In this method, the output from various antenna elements (total of \( K \)) are summed directly...
Table 4.1: Insertion loss for typical RF power combiners at the L band

<table>
<thead>
<tr>
<th>N</th>
<th>Insertion Loss</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.4 dB</td>
<td>0.91</td>
</tr>
<tr>
<td>3</td>
<td>0.5 dB</td>
<td>0.89</td>
</tr>
<tr>
<td>4</td>
<td>0.7 dB</td>
<td>0.85</td>
</tr>
<tr>
<td>5</td>
<td>1 dB</td>
<td>0.79</td>
</tr>
<tr>
<td>6</td>
<td>1 dB</td>
<td>0.79</td>
</tr>
</tbody>
</table>

together at RF and sent through the RF front end and on to the receiver. Note that there is not an LNA behind each element because the elements are considered very close together. This method is simple to implement but can lead to loss in performance because the antenna elements can be out-of-phase. In the system model, there is a noise source prior to the summation. These noise sources represent the external noise received by each element, and they are assumed to be uncorrelated. The RF sum is split into an ideal summation and a gain $L$ which is less than one and represents the insertion loss. The insertion loss comes from using a RF power combiner and depends on the total number of elements to be summed. Table 4.1 shows the values used for $L$ in this study. These values are taken from typical RF power combiners. The second noise source $n(t)$ represents noise added by the front end (mostly by the LNA).

Now, the system response is not from only one element but comes from the summation of the elements. It can be written as
Figure 4.2: Block diagram and system model for the RF sum method.
\[ H(f, \theta, \phi) = \sum_{k=1}^{K} A_k(f, \theta, \phi) \] (4.10)

and therefore \( S_d \) becomes

\[ S_d(f) = C_d L \left| \sum_{k=1}^{K} A_k(f, \theta, \phi) \right|^2 G_d(f). \] (4.11)

Similar to the single element case, \( S_n \) is written as

\[ S_n(f) = N_0 \] (4.12)

The total external noise power spectral density is

\[ S_{\hat{n}}(f) = \mathcal{F} \left\{ L E \left\{ \left( \sum_{k=1}^{K} \hat{n}_k(t) \right) \left( \sum_{k=1}^{K} \hat{n}_k(t - \tau') \right)^* \right\} \right\} \] (4.13)

\[ = \mathcal{F} \left\{ L \sum_{k=1}^{K} \hat{N}_0 \delta(\tau') \right\} \] (4.14)

\[ = \mathcal{F} \left\{ LK \hat{N}_0 \delta(\tau') \right\} \] (4.15)

\[ = LKN_0 \] (4.16)

where it has been assumed that the external noise from each channel is uncorrelated (i.e. \( E\{n_i(t)n_j(t - \tau')\} = 0 \) where \( i \neq j \)). The total noise power \( P_n \) is
\[ P_n = \int (N_0 + LK\hat{N}_0) \, df \]  
\[ = (N_0 + LK\hat{N}_0) \, B \]  
\[ = B \, (N_0 + \alpha LKN_0) \]  
\[ = B \, N_0 \, (1 + \alpha LK) \]  
\[ = C_n \left( \frac{1 + \alpha LK}{1 + \alpha} \right). \]

Using 4.1, 4.11, and 4.17 the SNR can be written as

\[ \text{SNR}(\theta, \phi) = \frac{C_d}{C_n} \left( \frac{L(1 + \alpha)}{1 + \alpha LK} \right) \int G_d(f) \left| \sum_{k=1}^{K} A_k(f, \theta, \phi) \right|^2 \, df. \]

In the above equation, the antenna responses are summed before their absolute value is taken. This can lead to an increase or decrease in SNR when the elements are in- or out-of-phase. Also, as compared to the single element case, there is a new factor in front of the integration. This factor depends on the insertion loss, number of elements, and ratio of external to internal noise variance.

### 4.1.3 Beamforming Method

Fig. 4.3 shows a block diagram and system model for the beamforming method. This is the most complicated of the methods. The output from each antenna element is sent through an RF front end and then combined in a digital beamformer. The beamformer has \( M \) outputs corresponding to the number of satellites in view. Each beamformer output is then input to a separate channel of a multi-channel receiver. The beamforming method gives good SNR performance in that the outputs of the
elements are combined using the known antenna response. This increase in performance comes at a cost because the weights depend on the satellite direction-of-arrival (DOA). To find the correct weights, the attitude of the platform must be known. As pointed out in Section 1.2, this assumption cannot be made for this application. The other costs are that each satellite needs a different set of weights and that the antenna manifold (i.e. response) needs to be known. Therefore, beamforming is not practical for this application, but its performance is included to compare with and give perspective about the other methods.

In the system model, each element has an RF front end which leads to external and internal noise sources for each channel. All of the noise sources are considered uncorrelated. Usually, the noise sources for each channel is represented by a single noise source. But, to be consistent with the RF sum method, the noise sources are kept separate. Also, the summation is modeled without insertion loss because the signals have been down converted and digitized and RF power combiners are not being used.

The weights for beamforming are [61]

$$w_k(\theta, \phi) = A_k^*(f_o, \theta, \phi)$$  \hspace{1cm} (4.23)

where $f_o$ is the center frequency of the band and the weight dependence on satellite direction is explicitly shown. The system response becomes

$$H(f, \theta, \phi) = \sum_{k=1}^{K} w_k A_k(f, \theta, \phi)$$  \hspace{1cm} (4.24)

$$= \sum_{k=1}^{K} A_k^*(f_o, \theta, \phi) A_k(f, \theta, \phi).$$  \hspace{1cm} (4.25)
Figure 4.3: Block diagram and system model for the $m^{th}$ satellite for the beamforming method.
$S_d$ becomes

$$S_d(f) = C_d \left| \sum_{k=1}^{K} A_k^*(f_o, \theta, \phi) A_k(f, \theta, \phi) \right|^2 G_d(f). \quad (4.26)$$

$S_n$ is now

$$S_n(f) = \mathcal{F} \left\{ E \left\{ \left( \sum_{k=1}^{K} w_k n_k(t) \right) \left( \sum_{k=1}^{K} w_k n_k(t - \tau') \right)^* \right\} \right\} \quad (4.27)$$

$$= \mathcal{F} \left\{ \sum_{k=1}^{K} \left| w_k \right|^2 N_0 \delta(\tau') \right\} \quad (4.28)$$

$$= N_0 \sum_{k=1}^{K} \left| w_k \right|^2 \quad (4.29)$$

$$= N_0 \sum_{k=1}^{K} \left| A_k(f_o, \theta, \phi) \right|^2 \quad (4.30)$$

where again the noise in different channels is uncorrelated. In the same way $S_{\hat{n}}(f)$ is

$$S_{\hat{n}}(f) = \hat{N}_0 \sum_{k=1}^{K} \left| A_k(f_o, \theta, \phi) \right|^2. \quad (4.31)$$

The total noise power becomes

$$P_n = \int (N_0 + \hat{N}_0) \sum_{k=1}^{K} \left| A_k(f_o, \theta, \phi) \right|^2 df \quad (4.32)$$

$$= (N_0 + \hat{N}_0) B \sum_{k=1}^{K} \left| A_k(f_o, \theta, \phi) \right|^2 \quad (4.33)$$

$$= C_n \sum_{k=1}^{K} \left| A_k(f_o, \theta, \phi) \right|^2 \quad (4.34)$$

Using the above equations, the final SNR expression is
\[
\text{SNR}(\theta, \phi) = \frac{C_d}{C_n} \sum_{k=1}^{K} |A_k(f_o, \theta, \phi)|^2 \int G_d(f) \left| \sum_{k=1}^{K} A_k^*(f_o, \theta, \phi)A_k(f, \theta, \phi) \right|^2 df.
\]

(4.35)

Again, the angles \(\theta\) and \(\phi\) are the satellite DOA which have to be known in order to find the weights. Also, the absolute value in the integration now occurs after summing the weighted response from each element. It is clear that the beamforming SNR will not decrease if the phase response from different elements is different. Note that there is also a factor in front of the integration similar to the RF sum method. In this case, the factor depends on the summation of the magnitude squared weights.

### 4.1.4 Maximum Gain Method

Finally, the maximum gain method is presented where the element with the highest gain is chosen. Fig. 4.4 shows a block diagram and system model for the maximum gain method. The output from each element is sent through the RF front end and to a specially designed receiver. The receiver design is the focus of Chapter 5. For the purpose of this chapter, the special receiver can be thought of as choosing the element with the highest gain. The system model consists of the antenna filters, external noise, internal noise, and a set of weights. Again, it is assumed that the noise sources are uncorrelated. But now the weights are

\[
w_k = \begin{cases} 
0 & \text{for } k \neq q, \ q^{th} \text{ element chosen} \\
1 & \text{for } k = q
\end{cases}
\]

(4.36)

where the \(q^{th}\) element chosen is the one with the highest gain. The main idea is that different elements are chosen as the satellite DOA changes. As mentioned above, Chapter 5 presents a method about how to achieve this result.
Figure 4.4: Block diagram and system model for the maximum gain method.
To derive the SNR equation, the system response becomes

\[ H(f, \theta, \phi) = A_q(f, \theta, \phi) \] (4.37)

and the desired signal power spectral density is

\[ S_d(f) = C_d |A_q(f, \theta, \phi)|^2 G_d(f). \] (4.38)

Similar to the single antenna element case, the noise power is simply

\[ P_n = C_n. \] (4.39)

Using the above equations the final expression for the SNR is

\[ \text{SNR}(\theta, \phi) = \frac{C_d}{C_n} \int G_d(f) |A_q(f, \theta, \phi)|^2 df \] (4.40)

where \( q \) depends on \( \theta \) and \( \phi \).

### 4.1.5 Noise Considerations

In this section, the relationship between the external and internal noise is investigated. As related above, this relationship is only important to the RF sum method because some of the noise occurs before the RF summation (i.e. external noise sources) and some occurs after the RF summation (i.e internal noise sources). For the other methods, the total noise is all that matters and not the ratio of external to internal. The noise power density can be thought of as a equivalent temperature multiplied by Boltzmann’s constant \((1.38 \times 10^{-23} \text{J/K})\) [1]. That is
\[ N_0 = T_i k' \]  \hspace{1cm} (4.41) \\
\[ \hat{N}_0 = T_e k' \]  \hspace{1cm} (4.42)

where \( T_i \) is the internal noise equivalent temperature, \( T_e \) is the external noise equivalent temperature, and \( k' \) is Boltzmann’s constant. Following [1], the total equivalent temperature is

\[ T_{eq} = T_i + T_e \]  \hspace{1cm} (4.43)

\[ T_{eq} = T_A + \left( \frac{1}{G} - 1 \right) 290 + \left( \frac{F - 1}{G} \right) 290 \]  \hspace{1cm} (4.44)

where the first term \( T_A \) is the antenna equivalent temperature (assumed to be 100 K). The second term comes from the cable and bandpass filter which are prior to the LNA. \( G \) is the gain of the cable/filter (less than one). The third term comes from the LNA with \( F \) as the noise figure. The number 290 comes from assuming that the cable, filter, and LNA have an operating temperature of 290 K. The terms after the LNA are omitted because of its high gain. Also, it is assumed that \( G = 0.8 \) and \( F = 2 \) [1]. The main issue is which noise terms should be prior to the sum junction in the RF sum method and which terms after the sum junction. There are two cases.

The first case is to assume that the external noise comes only from the antenna, \( T_e = T_A \). Then,

\[ T_i = 290 \left( \frac{1}{G} - 1 + \frac{F}{G} - \frac{1}{G} \right) \]  \hspace{1cm} (4.45)

\[ = 290 \left( \frac{F}{G} - 1 \right) = 435 \text{ K} \]  \hspace{1cm} (4.46)
and the ratio of external to internal noise is \( \alpha = 100 \text{K}/435 \text{K} = 0.23 \). The second case is to assume that the external noise should also include the cable and filter because these could occur before the sum junction of the RF sum method. Now,

\[
T_e = T_A + \left( \frac{1}{G} - 1 \right) 290 = 172.5 \text{K} \quad (4.47)
\]

and

\[
T_i = \left( \frac{F - 1}{G} \right) 290 = 362.5 \text{K}. \quad (4.48)
\]

The ratio becomes \( \alpha = 0.48 \). It can be seen that the ratio could be different based on the exact way the RF sum method is implemented. Also, the assumed values for \( G \) and \( F \) could be slightly different which would change the ratio as well. Therefore, the average of the ratio for the two cases will be used. Namely,

\[
\alpha = 0.36. \quad (4.49)
\]

Table 4.2 shows the RF sum method factor for different numbers of elements with the assumed ratio of external to internal noise. These numbers are used in the study.
Table 4.2: RF sum method factor versus number of elements

<table>
<thead>
<tr>
<th>N</th>
<th>( L_{\frac{1+\alpha}{1+\alpha LN}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.75</td>
</tr>
<tr>
<td>3</td>
<td>0.62</td>
</tr>
<tr>
<td>4</td>
<td>0.52</td>
</tr>
<tr>
<td>5</td>
<td>0.44</td>
</tr>
<tr>
<td>6</td>
<td>0.4</td>
</tr>
</tbody>
</table>

4.2 Satellite Coverage on 75mm Diameter Cylinder

The satellite coverage on the 75mm diameter cylinder is now investigated. One through four elements are placed on the cylinder. Fig. 4.5 shows the multiple antenna elements spaced around the circumference of the cylinder. The elements themselves were not retuned once placed on the cylinder because, as mentioned in Chapter 3, the antenna has excellent tuning robustness.

The scenario setup is as follows. The normalized desired signal power spectral density is

\[
G_d(f) = \frac{\text{sinc}^2(f/f_c)}{\int \text{sinc}^2(f/f_c) \, df} \tag{4.50}
\]

where \( f_c = 10.23 \) MHz and the null-to-null bandwidth is \( 2f_c \). This power spectral density closely approximates the GPS P-code. The SNR upon an isotropic element is set to \( C_d/C_n = -30 \) dB which is a typical value for the weak satellite signals [1].
Figure 4.5: Multiple antenna elements placed on the 75mm diameter cylinder.
Fig. 4.6 shows the SNR over the upper and lower hemisphere at the L1 and L2 bands for a single element on the 75mm diameter cylinder. For these types of charts, the center of the plot corresponds to zenith for the upper hemisphere and nadir for the lower hemisphere. The edge of the plot corresponds to the horizon and moving counterclockwise represents an increase in azimuth angle. Also, for this study, a SNR of -35 dB is used as the cutoff for good satellite signal reception. This represents a 5 dB SNR loss as compared to an isotropic signal. It can be seen that in the upper hemisphere for both bands good SNR coverage is obtained. But the lower hemisphere coverage is very poor and there are large regions with SNR equal to -45 dB or lower. For this application, the cylinder is considered to have an unknown time-varying attitude and, therefore, a single element is not able to ensure satellite signal reception or tracking.

Figs. 4.7 and 4.8 show the SNR over the upper and lower hemisphere for the RF sum method on the 75mm diameter cylinder for the L1 and L2 bands, respectively. The single element case is included for comparison. For both bands, the SNR coverage increases with the number of elements. Good coverage is obtained once at least three elements are present. But, even with at least three elements, there are angular regions with low SNR. These regions coincide with the ends of the cylinder. Therefore, the RF sum method provides good coverage but has two drawbacks. First, more than two elements are needed for good coverage. Second, there are two angular regions that will cause problems for tracking as the platform has time-varying pitch and roll angles. As these low SNR angular regions move across the sky, certain satellites will be lost. But, there may not be enough time to reacquire the lost satellites (once the
Figure 4.6: Single element upper and lower hemisphere SNR at L1 and L2 on the 75mm diameter cylinder.
low SNR angular regions have moved to a different part of the sky) given that the
application has a short flight time.

Figs. 4.9 and 4.10 show the SNR over the upper and lower hemisphere for the
beamforming method on the 75mm diameter cylinder for the L1 and L2 bands, re-
spectively. Again, SNR increases as the number of elements increases. Also, since
beamforming properly phases the elements prior to the summation, complete spher-
ical coverage is obtained as long as at least two elements are present. This is an
advantage as compared to the RF sum method.

Figs. 4.11 and 4.12 show the SNR over the upper and lower hemisphere for the
maximum gain method on the 75mm diameter cylinder for the L1 and L2 bands,
respectively. Again, the SNR increases with the number of elements. This increase is
not as much as the beamforming method, though. Importantly, complete spherical
coverage is obtained as long as at least two elements are present. Therefore, the
receiver will be able to continuously track as the platform rotates.

The previous plots give a good qualitative description of the SNR over the whole
sphere for the three methods. A more quantitative way of comparing is presented
next. The metric used is percent angular region over which a minimum SNR value is
reached. This converts the upper and lower hemisphere charts into a single curve of
percent angular region versus minimum SNR. This makes it easier to directly compare
the different number of elements and methods.

Fig. 4.13 and 4.14 show the percent angular region versus minimum SNR on the
75mm cylinder for various numbers of elements and for each of the three methods at
the L1 and L2 bands, respectively. For the single element case, the coverage is quite
poor. The percent angular region is less than 60% at -35 dB minimum SNR. The single
Figure 4.7: SNR over the upper and lower hemisphere at the L1 band for the RF sum method on the 75mm diameter cylinder.
Figure 4.8: SNR over the upper and lower hemisphere at the L2 band for the RF sum method on the 75mm diameter cylinder.
Figure 4.9: SNR over the upper and lower hemisphere at the L1 band for the beam-forming method on the 75mm diameter cylinder.
Figure 4.10: SNR over the upper and lower hemisphere at the L2 band for the beam-forming method on the 75mm diameter cylinder.
Figure 4.11: SNR over the upper and lower hemisphere at the L1 band for the maximum gain method and the 75mm diameter cylinder.
Figure 4.12: SNR over the upper and lower hemisphere at the L2 band for the maximum gain method and the 75mm diameter cylinder.
element is included in all the plots for reference. For the RF sum method, the percent angular region at -35 dB minimum SNR increases with the number of elements. Good coverage of at least 95% is obtained with a minimum of three elements. In addition, it can be seen that full spherical coverage is not obtained regardless of the number of elements. This occurs because there are deep nulls that coincide with the ends of the cylinder as mentioned above. For the beamforming case, excellent SNR is obtained with two or more elements which is expected. The maximum gain method also has good SNR when at least two elements are present. It can be seen that its coverage does not improve that much once at least three elements are present. Therefore, the maximum gain method with two elements is a good choice because it provides spherical coverage and minimizes the number of elements needed.

Table 4.3 summarizes the percent angular region above -35 dB minimum SNR for the three methods and at both bands. It can be seen that the two element maximum gain case is an excellent tradeoff between minimizing the number of elements and still obtaining good coverage (at least 95%). For the sum method, one could only achieve this amount of coverage with three elements.

Table 4.4 shows the minimum SNR obtained for at least 99% coverage for the three methods. The SNR results when two elements are present is discussed. The beamforming method has a 6.9 dB increase in minimum SNR over the sum method at the L1 band and a 7.6 dB increase in minimum SNR at the L2 band. This shows the loss in performance obtained when using the RF sum method and trying to minimize the number of elements. The maximum gain method recovers much of this lost minimum SNR. The minimum SNR improvement upon the sum method is 4.4 dB at the L1 band and 5.4 dB at the L2 band. This is a significant increase and
Figure 4.13: Percent angular region versus minimum SNR at the L1 band for the three methods on the 75mm diameter cylinder.
Figure 4.14: Percent angular region versus minimum SNR at the L2 band for the three methods on the 75mm diameter cylinder.
Table 4.3: Percent angular region above -35 dB minimum SNR for 75mm cylinder

<table>
<thead>
<tr>
<th>Method</th>
<th>1E</th>
<th>2E</th>
<th>3E</th>
<th>4E</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 Sum</td>
<td>52.6%</td>
<td>90%</td>
<td>98.2%</td>
<td>98.9%</td>
</tr>
<tr>
<td>L1 BF</td>
<td>52.6%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>L1 Max</td>
<td>52.6%</td>
<td>95.3%</td>
<td>99.2%</td>
<td>99.3%</td>
</tr>
<tr>
<td>L2 Sum</td>
<td>52.9%</td>
<td>96.1%</td>
<td>97.6%</td>
<td>98.3%</td>
</tr>
<tr>
<td>L2 BF</td>
<td>52.9%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>L2 Max</td>
<td>52.9%</td>
<td>96.5%</td>
<td>99.2%</td>
<td>99.3%</td>
</tr>
</tbody>
</table>

shows the advantage of the maximum gain method over the sum method when trying to obtain full spherical coverage and minimize the number of elements.

Therefore, the maximum gain method with two elements seems to be the best choice to obtain good satellite coverage on the 75mm cylinder. Although the beam-forming method has excellent performance, it needs to know the platform attitude and, therefore, is not practical. The sum method is simple and easy to implement, but, in order to obtain good coverage, it needs at least three elements. The maximum gain method provides a happy medium between knowledge needed (platform attitude) and number of elements needed on the one hand and SNR performance on the other. In the next section, the satellite coverage study is repeated for the 160mm diameter cylinder.
Table 4.4: Minimum SNR at 99% percent angular region for 75mm cylinder

<table>
<thead>
<tr>
<th>Method</th>
<th>1E</th>
<th>2E</th>
<th>3E</th>
<th>4E</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 Sum</td>
<td>-47.5 dB</td>
<td>-40.4 dB</td>
<td>-37.1 dB</td>
<td>-35.3 dB</td>
</tr>
<tr>
<td>L1 BF</td>
<td>-47.5 dB</td>
<td>-33.5 dB</td>
<td>-31.8 dB</td>
<td>-30.4 dB</td>
</tr>
<tr>
<td>L1 Max</td>
<td>-47.5 dB</td>
<td>-36 dB</td>
<td>-34.8 dB</td>
<td>-34.6 dB</td>
</tr>
<tr>
<td>L2 Sum</td>
<td>-46.3 dB</td>
<td>-41.1 dB</td>
<td>-38.6 dB</td>
<td>-36.5 dB</td>
</tr>
<tr>
<td>L2 BF</td>
<td>-46.3 dB</td>
<td>-33.5 dB</td>
<td>-31.7 dB</td>
<td>-30.5 dB</td>
</tr>
<tr>
<td>L2 Max</td>
<td>-46.3 dB</td>
<td>-35.7 dB</td>
<td>-34.7 dB</td>
<td>-34.6 dB</td>
</tr>
</tbody>
</table>

4.3 Satellite Coverage on 160mm Diameter Cylinder

The satellite coverage on the 160mm diameter cylinder is now investigated. One through six elements are placed on the cylinder. Fig. 4.5 shows the multiple antenna elements spaced around the circumference of the cylinder. The elements themselves were not retuned once placed on the cylinder because, as mentioned in Chapter 3, the antenna has excellent tuning robustness. The scenario setup is the same as described in Section 4.2. The desired signal power spectral density approximates the GPS P-code with a 20.46 MHz null-to-null bandwidth. The signal-to-noise-ratio incident upon an isotropic element \( \frac{C_d}{C_n} \) is -30 dB.

Figs. 4.16 and 4.17 show the SNR over the upper and lower hemisphere for the RF sum method on the 160mm diameter cylinder for the L1 and L2 bands, respectively. First, the single element case shows that the required spherical coverage
Figure 4.15: Multiple antenna elements placed on the 160mm diameter cylinder.
is not obtained because of the large decrease in SNR in the lower hemisphere. This is expected. For the RF sum method, the SNR for two and three elements at both bands is poor with large variations. Once at least four elements are present, the SNR is good and continues to improve as more elements are added. But, the RF sum method on the 160mm cylinder has similar drawbacks to the RF sum method on the 75mm cylinder. The method does not minimize the number of elements needed for full spherical coverage. In addition, there are still two angular regions with very poor SNR that coincide with the ends of the cylinder.

Figs. 4.18 and 4.19 show the SNR over the upper and lower hemisphere for the beamforming method on the 160mm diameter cylinder for the L1 and L2 bands, respectively. As the number of elements increase, the SNR increases. Also, as expected, complete spherical coverage is obtained as long as at least two elements are present. This is an advantage as compared to the RF sum method. Therefore, the beamforming method on the 160mm cylinder has similar conclusions as the beamforming method on the 75mm cylinder.

Figs. 4.20 and 4.21 show the SNR over the upper and lower hemisphere for the maximum gain method on the 160mm diameter cylinder for the L1 and L2 bands, respectively. The SNR increases with the number of elements. As expected, this increase is not as much as the beamforming method. Also, similar to the 75mm cylinder, complete spherical coverage is obtained as long as at least two elements are present.

Now, the quantitative percent angular region versus SNR plots are used to compare the different methods. Figs. 4.22 and 4.23 show the percent angular region versus minimum SNR on the 160mm cylinder for various numbers of elements and
Figure 4.16: SNR over the upper and lower hemisphere at the L1 band for the RF sum method on the 160mm diameter cylinder.
Figure 4.17: SNR over the upper and lower hemisphere at the L2 band for the RF sum method on the 160mm diameter cylinder.
Figure 4.18: SNR over the upper and lower hemisphere at the L1 band for the beam-forming method on the 160mm diameter cylinder.
Figure 4.19: SNR over the upper and lower hemisphere at the L2 band for the beam-forming method on the 160mm diameter cylinder.
Figure 4.20: SNR over the upper and lower hemisphere at the L1 band for the maximum gain method on the 160mm diameter cylinder.
Figure 4.21: SNR over the upper and lower hemisphere at the L2 band for the maximum gain method on the 160mm diameter cylinder.
each of the three methods for the L1 and L2 bands, respectively. Similar to the 75mm cylinder, the single element case provides poor SNR. For the RF sum method, the percent angular region at -35 dB minimum SNR increases with the number of elements at the L1 band. At the L2 band, this is not true as the three element case dips below the two element case. Good minimum SNR is obtained once four elements are present. But, there are still angular regions with very poor SNR that coincide with the ends of the cylinder regardless of the number of elements. For the beamforming method, excellent minimum SNR is obtained once at least two elements are present. The maximum gain method also has good coverage when at least two elements are present. It can be seen that its minimum SNR does not improve that much once there are at least three elements. Therefore, the maximum gain method with two elements is a good choice because it provides full spherical coverage and minimizes the number of elements needed. The same conclusion is drawn for the 75mm cylinder.

Table 4.5 summarizes the percent angular region above -35 dB minimum SNR for the three methods and at both bands. It can be seen that the two element maximum gain case is an excellent tradeoff between minimizing the number of elements and still obtaining good coverage (at least 97.8%). For the sum method, one could achieve this amount of coverage only with four elements.

Table 4.6 shows the minimum SNR obtained for at least 99% coverage for the three methods. The SNR results when two elements are present is discussed. The beamforming method has a 8.4 dB increase in minimum SNR over the sum method at the L1 band and a 9.8 dB increase in minimum SNR at the L2 band. This minimum SNR drop is larger for the 160mm cylinder as compared to the 75mm cylinder. It shows the loss in performance obtained with using the RF sum method and trying
Figure 4.22: Percent angular region versus minimum SNR at the L1 band for the three methods on the 160mm diameter cylinder.
Figure 4.23: Percent angular region versus minimum SNR at the L2 band for the three methods on the 160mm diameter cylinder.
to minimize the number of elements. The maximum gain method recovers much of
this lost minimum SNR. The minimum SNR improvement upon the sum method is
5.7 dB at the L1 band and 7.5 dB at the L2 band. This is a significant increase and
shows the advantage of the maximum gain method over the sum method when trying
to obtain full spherical coverage and minimize the number of elements needed.

Therefore, similar to the 75mm cylinder, the maximum gain method with two
elements seems to be the best choice to obtain good satellite coverage on the 160mm
cylinder. The reasons are the same as mentioned above. Namely, the beamforming
method has excellent performance, but it requires knowledge of the platform attitude
which is unknown and, therefore, beamforming is impractical. The sum method is
simple and easy to implement, but, in order to obtain full spherical coverage, it
needs at least four elements. As before, the maximum gain method provides a happy
medium between the knowledge needed (platform attitude) and number of elements
needed on the one hand and SNR performance on the other.
Table 4.5: Percent angular region over -35 dB minimum SNR for 160mm cylinder

<table>
<thead>
<tr>
<th>Method</th>
<th>1E</th>
<th>2E</th>
<th>3E</th>
<th>4E</th>
<th>5E</th>
<th>6E</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 Sum</td>
<td>53.3%</td>
<td>80.9%</td>
<td>84.7%</td>
<td>99%</td>
<td>99%</td>
<td>99.5%</td>
</tr>
<tr>
<td>L1 BF</td>
<td>53.3%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>L1 Max</td>
<td>53.3%</td>
<td>97.8%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
</tr>
<tr>
<td>L2 Sum</td>
<td>57%</td>
<td>92.5%</td>
<td>79.1%</td>
<td>98.7%</td>
<td>99%</td>
<td>99.2%</td>
</tr>
<tr>
<td>L2 BF</td>
<td>57%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>L2 Max</td>
<td>57%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 4.6: Minimum SNR at 99% percent angular region for 160mm cylinder

<table>
<thead>
<tr>
<th>Method</th>
<th>1E</th>
<th>2E</th>
<th>3E</th>
<th>4E</th>
<th>5E</th>
<th>6E</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 Sum</td>
<td>-48.1 dB</td>
<td>-41 dB</td>
<td>-45.3 dB</td>
<td>-35 dB</td>
<td>-33.8 dB</td>
<td>-32.8 dB</td>
</tr>
<tr>
<td>L1 BF</td>
<td>-48.1 dB</td>
<td>-32.6 dB</td>
<td>-30.5 dB</td>
<td>-29.5 dB</td>
<td>-28.4 dB</td>
<td>-27.7 dB</td>
</tr>
<tr>
<td>L1 Max</td>
<td>-48.1 dB</td>
<td>-35.3 dB</td>
<td>-34 dB</td>
<td>-34 dB</td>
<td>-33.9 dB</td>
<td>-33.9 dB</td>
</tr>
<tr>
<td>L2 Sum</td>
<td>-50.6 dB</td>
<td>-41.7 dB</td>
<td>-40.3 dB</td>
<td>-35.8 dB</td>
<td>-34.6 dB</td>
<td>-33.4 dB</td>
</tr>
<tr>
<td>L2 BF</td>
<td>-50.6 dB</td>
<td>-31.9 dB</td>
<td>-30.1 dB</td>
<td>-28.9 dB</td>
<td>-27.9 dB</td>
<td>-27.6 dB</td>
</tr>
<tr>
<td>L2 Max</td>
<td>-50.6 dB</td>
<td>-34.2 dB</td>
<td>-33.6 dB</td>
<td>-33.5 dB</td>
<td>-33.5 dB</td>
<td>-33.4 dB</td>
</tr>
</tbody>
</table>
4.4 Summary & Conclusions

The satellite coverage obtained using multiple dual-band GPS receiver antennas on thin cylinders has been investigated. A single element could not provide the necessary spherical coverage. For multiple elements, the RF sum, beamforming, and maximum gain methods were used to combine the elements. SNR was used as the metric to compare the various methods. To this end, SNR equations for each of the methods were derived. The cylinders investigated have diameters of 75mm (1-4 elements) and 160mm (1-6 elements). The GPS P-code power spectral density was used as the desired signal and the incident SNR upon an isotropic element was -30 dB. The RF sum method is the simplest method but cannot obtain full spherical coverage. The beamforming method has excellent SNR performance. But, it requires knowledge of the cylinder attitude which is not available. The maximum gain method does not need to know the attitude and has SNR performance in between the RF sum and beamforming methods. Very importantly, it provides complete spherical coverage even with only two elements which is true for both the small and large cylinders. Therefore, the maximum gain method with two elements is the best choice to obtain full spherical coverage, minimize the number of elements, and eliminate the need for knowledge of platform attitude. But, a practical way to implement this method prior to the receiver is unknown because the GPS signals are below the noise floor. In the next chapter, the receiver itself is modified to implement the maximum gain method by designing a novel tracking algorithm.
CHAPTER 5

A Novel GPS Receiver Algorithm for Continuous Satellite Tracking

In the previous chapter, it was demonstrated that the maximum gain approach (i.e., choosing the best element) with two antenna elements would provide full satellite coverage and hence satellite tracking. But the way to implement this approach was not discussed. This is a challenge because the GPS signals prior to the receiver are below the noise floor and the time-varying platform attitude is unknown. In this chapter, the maximum gain method from Chapter 4 is implemented in the GPS receiver itself. To this end, a GPS receiver tracking model (delay lock loop) is developed that accounts for the time-varying rotation of the small cylindrical platforms of interest. Using the model, it is shown that for a single antenna element continuous tracking is not maintained as the cylinder spins. Therefore, the receiver tracking loop is modified to implement the maximum gain method from Chapter 4 which will improve the tracking loop performance. For this chapter, results are presented when two antenna elements are placed on the 75mm diameter cylinder, and the integration time and spin rate are varied. It is shown that the proposed approach gives excellent tracking loop performance. The chapter outline is as follows. Section 5.1 reviews the
basics about a GPS receiver and presents the assumptions made about the GPS receiver acquisition process for this application. Section 5.2 investigates a GPS receiver tracking loop model for a single antenna element and derives equations necessary to generate analytic results. It is shown that tracking loop performance is poor for a single antenna on the spinning platform. Section 5.3 presents a novel GPS receiver algorithm that implements the maximum gain method from Chapter 4. It is shown that the new method has good tracking loop performance as the platform spins. Section 5.4 demonstrates an approach that even improves upon the maximum gain method tracking loop performance. Section 5.5 presents the summary and conclusions of the chapter.

5.1 GPS Receiver Review and Acquisition Assumptions

In section 1.1, the basics of GPS were described. A GPS receiver tracks the code delay (pseudorange) from various satellites. Each satellite signal has its own code that the receiver searches for. This is done by cross-correlating the received signal with a locally generated reference signal. The delay at which the cross-correlation has a peak is the code delay. Since the delay changes with time, it is tracked by updating the locally generated reference signal. This is known as a delay lock loop (DLL).

Before satellite tracking is feasible, though, the receiver must acquire the satellite signals. This is known as the acquisition phase. In this phase, the receiver roughly estimates the doppler frequency and pseudorange of the received satellite signal. This is done by searching over a fixed set of doppler bins and code delays which can take a few minutes. For this application, it is assumed that the receiver has \textit{a priori} estimates of the doppler frequency and code delay. Using the initial estimates, the search
space is reduced considerably and only takes a few seconds. The initial estimates are uploaded to the receiver by an outside source and this is termed a “hot” start [1]. Other assumptions made from the “hot” start are that the available satellites and their locations (ephemeris data) are known. This is necessary because decoding the ephemeris data from the received signal can take a few minutes. Once the acquisition process estimates the rough doppler frequency and code delay, the receiver transitions to tracking mode. As will be seen below, the main problem becomes maintaining good tracking loop performance as the platform rotates.

5.2 GPS Receiver Tracking

In this section, a model for receiver tracking is developed. The model incorporates the effect of the antenna response as well as the time-varying platform effect. This is first done for a single antenna element. Then, it is also done for the beamforming method that was described in Chapter 4. The beamforming method is studied because it provides optimal results and therefore a good benchmark to compare against the other algorithms. This section ends with a few example results of the single element and beamforming methods. It will be seen that the tracking loop performance is very poor with only a single antenna element.

5.2.1 Tracking Model for Single Antenna Element

Fig. 5.1(a) shows a block diagram for the delay lock loop. The signal is received by the antenna and sent through the RF front end. The front end consists of a low noise amplifier (LNA), bandpass filter, mixer, and A/D converter. The digitized signal is cross-correlated with the output of the code generator. The discriminator estimates the code delay and sends the estimate to the loop filter. The purpose of
the loop filter is to reduce the variance of the noisy code delay estimate. This is done by lowering the loop filter bandwidth. But there is a trade-off. If the bandwidth is lowered too much then the dynamic performance of the DLL suffers. The output of the loop filter is sent to the navigation processor where position, velocity, and time (PVT) estimation occurs. To close the loop, the output is also sent to a numerically controlled oscillator (NCO) which generates the new code delay to be used in the correlation. In this way, the code delay is tracked.

Fig. 5.1(b) shows the system model for the delay lock loop. The antenna on platform is time-varying because of the rotation. Therefore, the antenna is modeled as a linear time-varying (LTV) filter. (In the derivation, [62] is being followed and modified when necessary). The LTV filter is characterized by the function $h(t_1, t_2)$ which represents the response at observation time $t_1$ due to an impulse at time $t_2$. The internal noise (from the LNA in the RF front end) and external noise are represented by $n(t)$ and $\hat{n}(t)$, respectively. This noise is the same as discussed in Chapter 4 and the split between internal and external noise sources is continued in this chapter for ease of notation. The internal and external noise are white and Gaussian with zero mean and variance $N_0$ for $n(t)$ and variance $\hat{N}_0$ for $\hat{n}(t)$. The cross-correlation is a multiplication and integration of the received signal with the locally generated reference signal $x(t)$ with time shift $\tau$. The integration time is $T$. After going through the integration, the output is $\hat{R}$ where the hat here denotes an estimate corrupted by noise. Then, the discriminator generates a noisy estimate for the code delay $\hat{\tau}$. The noise at this stage is called measurement noise and its variance is not the same as the internal or external noise variance. The loop filter smoothes out the noisy code delay estimate and sends it to the navigation processor and NCO/code generator. The measurement noise after
the discriminator is very important in the model. If the noise is too high then the loop will not track the code delay. Also, excess measurement noise can degrade the quality of the PVT estimate. The measurement noise variance depends on $\hat{R}$. For this model, analytic expressions will be developed for $\hat{R}$. Therefore, sampled signals do not need to be used in the model which reduces simulation time. Also, note that PVT estimation is not covered in this work.
The measurement noise variance is inversely proportional to the carrier-to-noise-ratio (CNR) [1]. CNR is a very important parameter in a GPS receiver and is equivalent to the post-correlation SNR (i.e. after the integration) divided by the integration time. Therefore, it is a ratio of signal power to noise power density and has units of Hz. For the delay lock loop, it essentially demonstrates the quality of the code delay estimate. CNR is given by [62]

\[
\text{CNR} = \frac{1}{T} \frac{|\mathbb{E}\{\hat{R}(t_0, T, \tau_{peak})\}|^2}{\text{var}\{R(t_0, T, \tau_{peak})\}}
\]

(5.1)

where \(\tau_{peak}\) is the location of the cross-correlation peak. This equation shows that CNR depends on the mean and variance of the noisy cross-correlation function \(\hat{R}\). Specifically, it is the variance about the squared magnitude of the mean. Therefore, expressions for the mean and variance need to be derived that include the time varying antenna/platform effect.

As mentioned above, the LTV filter is characterized by the response \(h(t_1, t_2)\) at observation time \(t_1\) due to an impulse at time \(t_2\). The output of the LTV filter is

\[
y(t) = \int_{-\infty}^{\infty} h(t, t_2) r(t_2) \, dt_2.
\]

(5.2)

By definition, the response \(h(t_1, t_2)\) is related to the instantaneous antenna response by

\[
h(t_1, t_2) \triangleq \dot{a}(t_1, t_1 - t_2)
\]

(5.3)

where \(\dot{a}\) is the time domain antenna response including the time-varying platform effect. This antenna response will be discussed below. The input \(r(t)\) is considered to be an exact replica of the locally generated reference signal.
\[ r(t) = \sqrt{C_d} x(t) \]  

(5.4)

where \( C_d \) is the power of the received signal and \( x(t) \) has unit power. This input is considered an exact replica because only the antenna and platform rotation effects are of interest. Platform translation motion, tropospheric effects, ionspheric effects, navigation data bits, multipath, and other complications are not included in the model.

Now, the derivation of the mean and variance of \( \hat{R} \) is continued. The cross-correlation estimate is given by

\[ \hat{R}(t_0, T, \tau) = \frac{1}{T} \int_{t_0}^{t_0+T} (y(t) + n(t) + \hat{n}(t)) x(t - \tau) \, dt. \]  

(5.5)

The mean of 5.5 is

\[
R(t_0, T, \tau) = E\{ \hat{R}(t_0, T, \tau) \} \\
= E\left\{ \frac{1}{T} \int_{t_0}^{t_0+T} (y(t) + n(t) + \hat{n}(t)) x(t - \tau) \, dt \right\} \\
= \frac{1}{T} \int_{t_0}^{t_0+T} E\{ y(t) x(t - \tau) \} \, dt
\]

(5.6)

(5.7)

(5.8)

where it is assumed that the noise and the reference signal are uncorrelated. The integrand is written as

\[
E\{ y(t) x(t - \tau) \} = E\left\{ x(t - \tau) \int_{-\infty}^{\infty} h(t, \beta) r(\beta) \, d\beta \right\} \\
= \sqrt{C_d} \int_{-\infty}^{\infty} h(t, \beta) E\{ x(t - \tau) x(\beta) \} \, d\beta \\
= \sqrt{C_d} \int_{-\infty}^{\infty} h(t, \beta) R_{XX}(t - \tau - \beta) \, d\beta
\]

(5.9)

(5.10)

(5.11)
where $R_{XX}$ is the auto-correlation of $x(t)$. (Note that $x(t)$ is real-valued.) Let

$\alpha = t - \tau - \beta$ and the integrand becomes

$$E\{y(t)x(t - \tau)\} = \sqrt{C_d} \int_{-\infty}^{\infty} h(t, t - \tau - \alpha) R_{XX}(\alpha) d\alpha.$$  (5.12)

Substituting 5.12 into 5.6 gives

$$R(t_0, T, \tau) = \sqrt{C_d} \frac{T}{T} \int_{-\infty}^{\infty} \int_{t_0}^{t_0+T} h(t, t - \tau - \alpha) R_{XX}(\alpha) dt \, d\alpha.$$  (5.13)

Now, using the time domain antenna response definition of 5.3 in 5.14 leads to

$$R(t_0, T, \tau) = \sqrt{C_d} \frac{T}{T} \int_{-\infty}^{\infty} \int_{t_0}^{t_0+T} \hat{a}(t + \tau, \alpha + \tau) R_{XX}(\alpha) dt \, d\alpha.$$  (5.14)

$$= \sqrt{C_d} \int_{-\infty}^{\infty} \left( \frac{1}{T} \int_{t_0}^{t_0+T} \hat{a}(t + \tau, \alpha + \tau) dt \right) R_{XX}(\alpha) d\alpha.$$  (5.15)

The antenna response $\hat{a}$ can be approximated as

$$\hat{a}(t + \tau, \alpha + \tau) \approx \hat{a}(t, \alpha + \tau)$$  (5.16)

which assumes that the antenna response does not vary that much over the cross-correlation main lobe width. The C/A-code has the maximum main lobe width of about $0.24\mu s$. If the platform is spinning at a maximum rate of 350 Hz, then the maximum change in satellite DOA over the main lobe width is $360 \times (350 Hz) \times 0.24\mu s = 0.03^\circ$. This type of rapid change in the antenna pattern was not seen in Chapter 3. Therefore, the assumption is valid. Now, a new function is defined as

$$\hat{a}(t_0, T, \alpha + \tau) = \frac{1}{T} \int_{t_0}^{t_0+T} \hat{a}(t, \alpha + \tau) dt.$$  (5.17)
Substituting 5.17 into 5.14 (and using the approximation) gives

\[ R(t_0, T, \tau) = \sqrt{C_d} \int_{-\infty}^{\infty} \hat{a}(t_0, T, \alpha + \tau) R_{XX}(\alpha) d\alpha. \]  

(5.18)

Using a Fourier transform on 5.18 results in

\[ \mathcal{F}\{R\}(f) = \sqrt{C_d} \int_{-\infty}^{\infty} R_{XX}(\alpha) \hat{a}(t_0, T, \alpha + \tau) e^{-j2\pi f \tau} d\alpha d\tau. \]  

(5.19)

Using the substitution \( \tau'' = \alpha + \tau \) gives

\[
\mathcal{F}\{R\}(f) = \sqrt{C_d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{XX}(\alpha) \hat{a}(t_0, T, \tau''') e^{-j2\pi f \tau''} d\alpha d\tau''
\]

(5.20)

\[
\mathcal{F}\{R\}(f) = \sqrt{C_d} \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} R_{XX}(\alpha) e^{j2\pi f \alpha} d\alpha \right) \hat{a}(t_0, T, \tau''') e^{-j2\pi f \tau''} d\tau''
\]

(5.21)

\[
\mathcal{F}\{R\}(f) = \sqrt{C_d} \int_{-\infty}^{\infty} G_d(f) \hat{a}(t_0, T, \tau''') e^{-j2\pi f \tau''} d\tau''
\]

(5.22)

\[
\mathcal{F}\{R\}(f) = \sqrt{C_d} G_d(f) \hat{A}(t_0, T, f).
\]

(5.23)

where \( G_d(f) \) is the power spectral density of the reference signal (desired signal) as defined in Chapter 4. Note that, in the above derivation, the fact that \( R_{XX} \) is symmetric was used. Now, applying an inverse Fourier transform gives

\[ R(t_0, T, \tau) = \sqrt{C_d} \int_{-B/2}^{B/2} G_d(f) \hat{A}(t_0, T, f) e^{-j2\pi f \tau} df \]

(5.24)

where \( \hat{A} \) is

\[
\hat{A}(t_0, T, f) = \frac{1}{T} \int_{t_0}^{t_0+T} \hat{A}(t, f) dt
\]

(5.25)
Figure 5.2: Phase compensation $\psi$ for time-varying rotation $\mathbf{M}(t)$.

with bandlimited signals being used (hence the integration limits $\pm B/2$). The above equation is the mean of the noisy estimate $\hat{R}$. Before moving to the variance, the antenna response $\hat{A}$ is explicitly described. The antenna response is

$$\hat{A}(t, f) = A(f, \theta(t), \phi(t)) e^{j\psi(t)}$$

(5.26)

where $A$ is the frequency domain antenna response (same as Chapter 4), $\theta(t)$ and $\phi(t)$ are the time-varying satellite angles, and $\psi(t)$ is the time-varying phase compensation. First, the incident angles are discussed and then the phase compensation. The time-varying incident angles depend on the rotation of the platform. Let $\hat{r}(\theta, \phi)$ be the unit vector in the initial satellite direction $\theta, \phi$ and $\hat{r}'$ the unit vector after the rotation is applied (see Fig. 5.2). That is

$$\hat{r}' = \mathbf{M}(t)\hat{r}$$

(5.27)

where $\mathbf{M}$ is the standard rotation matrix ($3 \times 3$ and depending on roll, pitch, and yaw) and
\[ \mathbf{r} = \begin{bmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{bmatrix} \]  

(5.28)

with \( \theta, \phi \) being the initial satellites angles. The basis vectors for \( \mathbf{r} \) are the global (i.e. reference) co-ordinate system of \( \mathbf{x}, \mathbf{y}, \mathbf{z} \).

The phase compensation term \( \psi(t) \) is now discussed (see Fig. 5.2). This term comes from the fact that the received satellite signal was originally broadcast with a particular reference co-ordinate system. As the antenna rotates on the platform, the phase of \( A \) needs to be “rotated” as well starting from the initial satellite co-ordinate system. In [33, 63], the phase compensation term is mentioned but the antenna models are too simple. Let unit vector \( \hat{\theta} \) and unit vector \( \hat{\phi} \) be the \( \mathbf{x}_0 \) axis and \( \mathbf{y}_0 \) axis in the initial satellite co-ordinate system (this system is chosen because the satellite is transmitting towards the origin). \( \hat{\theta} \) and \( \hat{\phi} \) are the standard spherical unit vectors and are given by

\[ \hat{\theta} = \begin{bmatrix} \cos \theta \cos \phi \\ \cos \theta \sin \phi \\ -\sin \theta \end{bmatrix} \]  

(5.29)

and

\[ \hat{\phi} = \begin{bmatrix} -\sin \phi \\ \cos \phi \\ 0 \end{bmatrix}. \]  

(5.30)

Now, let \( \mathbf{M}(t) \) be the vectors \( \hat{\theta} \) and \( \hat{\phi} \) each rotated by \( \mathbf{M}(t) \). Also, let \( \theta' \) and \( \phi' \) be the new spherical angles after the rotation is applied (i.e. the spherical angles for \( \mathbf{r}' \)). Then, the phase compensation can be written as (see Fig. 5.2(b))
\[ \tan \psi(t) = \frac{\hat{\theta}' \cdot \hat{\phi}(\theta', \phi')}{\hat{\theta}' \cdot \hat{\theta}(\theta', \phi')} \]  
\[ = \frac{M(t)\hat{\theta}(\theta, \phi) \cdot \hat{\phi}(\theta', \phi')}{M(t)\hat{\theta}(\theta, \phi) \cdot \hat{\theta}(\theta', \phi')} \]  
\[ \psi(t) = \tan^{-1} \left( \frac{M(t)\hat{\theta}(\theta, \phi) \cdot \hat{\phi}(\theta', \phi')}{M(t)\hat{\theta}(\theta, \phi) \cdot \hat{\theta}(\theta', \phi')} \right) \]  
\[ (5.31) \]
\[ (5.32) \]
\[ (5.33) \]

where \( \cdot \) is the dot product. Note that the angles \( \theta' \) and \( \phi' \) also depend on \( M(t) \) but this has not been explicitly shown. The above equation shows that the phase compensation is essentially measuring the angle between two reference co-ordinate systems. The first being the initial system evaluated at \( \theta', \phi' \) and the second being the initial system rotated by \( M(t) \). In addition, the above equation is not unique in that it could also be written in terms of \( \hat{\phi}' \) instead of \( \hat{\theta}' \).

An example may provide a good illustration for what the phase compensation term does. Consider the scenario where the satellite incident unit vector is \( \hat{x} \) and that the rotation matrix only depends on a constant positive roll about \( \hat{x} \). In this case, the satellite angles will not change \( (\theta = \theta', \phi = \phi') \) and the antenna response \( A \) will be constant with time. The phase compensation \( \psi(t) \) will simply be the constant positive roll. This can be thought of as a roll-induced positive doppler frequency shift. The shift would be negative if the platform was spinning in the opposite direction.

Next, the variance of \( \hat{R} \) is found. This is given by

\[ \text{var}\{\hat{R}\} = E\{|\hat{R}|^2\} - |E\{\hat{R}\}|^2. \]

(5.34)

Using 5.5, the first term can be written as
\[
E[|\hat{R}|^2] = \frac{1}{T^2} \int \int E\{x(t - \tau)x^*(s - \tau)[y(t) + n(t) + \hat{n}(t)]
\]
\[
[y^*(s) + n^*(s) + \hat{n}^*(s)]\} dt ds.
\] (5.35)

The integrand can be written as

\[
E\{y(t)y^*(s)x(t - \tau)x^*(s - \tau) + n(t)n^*(s)x(t - \tau)x^*(s - \tau) + \hat{n}(t)\hat{n}^*(s)x(t - \tau)x^*(s - \tau)\}
\] (5.36)

where the fact that the noise sources and reference signal are uncorrelated has been used. Now, the second expression on the right-hand side of 5.34 is written as

\[
|E\{\hat{R}\}|^2 = \frac{1}{T^2} \int \int E\{y(t)y^*(s)x(t - \tau)x^*(s - \tau)\} dt ds.
\] (5.37)

Using the above equations, the variance simplifies to

\[
\text{var}\{\hat{R}\} = \frac{1}{T^2} \int \int E\{n(t)n^*(s)x(t - \tau)x^*(s - \tau) + \hat{n}(t)\hat{n}^*(s)x(t - \tau)x^*(s - \tau)\} dt ds.
\] (5.38)

Eq. 5.40 can be split into two pieces and each evaluated separately. The first piece is simplified as
\[
\frac{1}{T^2} \iiint E\{n(t)n^*(s)x(t-\tau)x^*(s-\tau)\}dt\,ds
\]  
(5.41)

\[
= \frac{1}{T^2} \iiint E\{n(t)n^*(s)\}E\{x(t-\tau)x^*(s-\tau)\}dt\,ds
\]  
(5.42)

\[
= \frac{1}{T^2} \iiint N_0 \delta(t-s) R_{XX}(t-s)dt\,ds
\]  
(5.43)

\[
= \frac{1}{T^2} \int_{t_0}^{t_0+T} N_0 R_{XX}(0)dt
\]  
(5.44)

\[
= \frac{N_0}{T}
\]  
(5.45)

where the fact that \(x(t)\) has unit power has been used. The second piece of 5.40 is simplified in the same way except the final answer is \(\hat{N}_0/T\). The variance then becomes

\[
\text{var}\{\hat{R}\} = \frac{N_0 + \hat{N}_0}{T} = \frac{(1 + \alpha) N_0}{T}
\]  
(5.46)

where \(\alpha\) is the ratio of external to internal noise variance as defined in Chapter 4. Now, using 5.1, 5.24, and 5.46, the analytic expression for CNR is

\[
\text{CNR} = \frac{C_d}{(1 + \alpha)N_0} \left| \int_{-B/2}^{B/2} G_d(f) \hat{A}(f)e^{-j2\pi f\tau_{peak}} df \right|^2.
\]  
(5.47)

CNR is a function of the time-varying platform, integration time \(T\), and the initial satellite incident angles. All of these effects are incorporated in \(\hat{A}\). The factor in front of the integration will be defined below as part of the scenario setup.

### 5.2.2 Tracking Model for Beamforming Method

The tracking model for the beamforming method is described next. Fig. 5.3 shows the receiver tracking loop model for the beamforming method. Similar to Chapter 4, the beamforming method sums the weighted responses of each element (total of
Figure 5.3: The receiver tracking loop model for the beamforming method.

$K$ and then sends the sum to the receiver tracking model. Now, each antenna is a time-varying filter $\hat{A}_n(t, f)$. The entire derivation for CNR is not given in this section. Rather, by inspection, the pertinent equations are modified. Eq. 5.24 becomes

$$R(t_0, T, \tau) = \sqrt{C_d} \int_{-B/2}^{B/2} G_d(f) \hat{H}(f)e^{-j2\pi f \tau} df$$

where $\hat{H}(f)$ is the beamforming method equivalent of $\hat{A}(f)$. It is given by

$$\hat{H}(f) = \frac{1}{T} \int_{t_0}^{t_0+T} \hat{H}(t, f) dt$$

and $\hat{H}(t, f)$ is the beamforming method equivalent of $\hat{A}(t, f)$. By inspection, $\hat{H}(t, f)$ is given by

$$\hat{H}(t, f) = \sum_{k=1}^{K} w_k(t) \hat{A}_k(t, f).$$
The weights \( w_k(t) \) are considered to be perfectly updated with the time-varying antenna response. Therefore, they are given by

\[
w_k(t) = A_k^*(t, f_o)
\]  

(5.51)

where \( f_o \) is the center frequency of the band of interest. The above equations give the mean of \( \hat{R}(t_0, T, \tau) \).

Next, the variance is derived. One can start with the first term in 5.40 and modify it to include the weights. This gives

\[
\frac{1}{T^2} \int \int E\left\{ \sum w_k(t) n_k(t) \left( \sum w_k(s) n_k(s) \right)^* \right\}
\]

(5.52)

\[
E\{x(t - \tau)x^*(s - \tau)\} \, dt \, ds
\]

(5.53)

\[
= \frac{1}{T^2} \int \int N_0 \delta(t - s) R_{XX}(t - s) \sum w_k(t) w_k^*(s) \, dt \, ds
\]

(5.54)

\[
= \frac{N_0}{T^2} \int_{t_0}^{t_0 + T} R_{XX}(0) \sum |w_k(t)|^2 \, dt
\]

(5.55)

\[
= \frac{N_0}{T^2} \int_{t_0}^{t_0 + T} \sum |w_k(t)|^2 \, dt
\]

(5.56)

where again the noise from different channels is uncorrelated. The variance of \( \hat{R} \) is therefore

\[
\text{var}\{\hat{R}\} = \frac{N_0}{T^2} \int_{t_0}^{t_0 + T} \sum |w_k(t)|^2 \, dt + \frac{\tilde{N}_0}{T^2} \int_{t_0}^{t_0 + T} \sum |w_k(t)|^2 \, dt
\]

(5.57)

\[
= \frac{(1 + \alpha) N_0}{T^2} \int_{t_0}^{t_0 + T} \sum |w_k(t)|^2 \, dt
\]

(5.58)

\[
= \frac{(1 + \alpha) N_0}{T} V(t_0, T).
\]

(5.59)

where \( V(t_0, T) \) is defined as
\[ V(t_0, T) = \frac{1}{T} \int_{t_0}^{t_0+T} \sum |w_k(t)|^2 dt \]  
\[ = \frac{1}{T} \int_{t_0}^{t_0+T} \sum |\dot{A}_k(t, f_o)|^2 dt. \] (5.61)

This function represents the contribution of the weights to the total noise power density. Using 5.1, 5.48, and 5.57 the beamforming method CNR becomes

\[
\text{CNR} = \frac{1}{(1 + \alpha)N_0 V(t_0, T)} |R(t_0, T, \tau_0)|^2  
\]
\[
= \frac{C_d}{(1 + \alpha)N_0 V(t_0, T)} \left| \int_{-B/2}^{B/2} G_d(f) \hat{H}(f)e^{-j2\pi f \tau_{\text{peak}}} df \right|^2. \] (5.63)

CNR depends on the time-varying antenna responses, the integration time \(T\), the incident satellite angles, and the time-varying weights. Note that these weights perfectly follow the time-varying antenna response. This represents a very fast weight update rate as the platform rotates. Therefore, for a given scenario, the beamforming method will provide the upper limit for CNR.

### 5.2.3 Example Results

Some example results are now presented for the single element and beamforming methods. The purpose of this section is to show how the CNR varies with platform rotation. A discussion regarding the relationship between CNR and tracking loop performance is also given. The scenario setup is as follows. Two dual-band GPS receiver antennas are placed on the 75mm diameter cylinder (see Fig. 5.4). The cylinder lies on the \( \hat{x} \) axis. The pitch and yaw of the platform do not vary with time and are set to zero. The roll angle \( \beta(t) \) varies with time and is considered to be a positive rotation around \( \hat{x} \). It is given by
Figure 5.4: Reference co-ordinate system and roll angle $\beta$ for two antenna elements placed on 75mm diameter cylinder.

$$\beta(t) = 360f_r t \text{ degrees}$$

(5.64)

where $f_r$ is the roll angle frequency. In this case, the rotation matrix becomes

$$M(t) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(\beta(t)) & -\sin(\beta(t)) \\
0 & \sin(\beta(t)) & \cos(\beta(t))
\end{bmatrix}. \quad (5.65)$$

The normalized desired signal power spectral density is

$$G_d(f) = \frac{\text{sinc}^2(f/f_c)}{\int \text{sinc}^2(f/f_c) \, df} \quad (5.66)$$

where $f_c = 10.23 \text{ MHz}$ and the null-to-null bandwidth is $2f_c$. This power spectral density closely approximates the GPS P-code. The desired signal power is set to $C_d = -156 \text{ dBW}$ [1]. The total noise power density is $(1 + \alpha)N_0 = -201 \text{ dBW / Hz}$. For a system bandwidth of 30 MHz, the incident SNR would then be $C_d/[(1 + \alpha)N_0 B] = -30 \text{ dB}$ which was the incident SNR used in Chapter 4. Therefore, the CNR on an isotropic element ($\hat{A}(f) = 1, \tau_{\text{peak}} = 0$) would be 45 dB-Hz.
Fig. 5.5 shows the CNR versus time at the L1 and L2 bands for three initial incident angles for two elements on the 75mm diameter cylinder. The integration time is 10 ms and the roll frequency is 5 Hz. First, the CNR for the single element case is discussed. It can be seen that the CNR varies significantly versus time. The variation also depends on the initial satellite incident angle. The variation can be as little as 3 dB-Hz or greater than 25 dB-Hz depending on the initial satellite incident angle. The actual CNR degrades to 20 dB-Hz. For the beamforming method, good CNR is obtained versus time. This is expected because the weights perfectly follow the time-varying antenna response. When $\theta = 0^\circ, \phi = 0^\circ$, the CNR varies about 3 dB-Hz. For the other incident angles, the CNR is relatively flat versus time. Therefore, the beamforming method provides excellent CNR performance.

In the above paragraph, the CNR was presented. It is also of interest to discuss the relationship of CNR to tracking loop performance. The two issues are loss of lock and increased measurement noise degrading the PVT estimate. There is not a one-to-one correspondence between a particular CNR value and tracking loop loss of lock [64]. Rather, the measurement noise may or may not cause loss of lock depending on its particular value. Now, if a particular measurement noise value is very strong, the location of the early and late correlators will move away from the true correlation peak. If they move too far, then the tracking loop will lose lock. As an example, consider the discriminator function shown in Fig. 5.6. Note that one chip spacing between correlators has been assumed. If the location of the correlators moves 1.5 chips (i.e. the measurement noise value is $\pm 1.5$ chips) then the tracking loop will lose lock [65]. This occurs because the input error is positive but the estimated input error is negative. The loop filter does reduce the discriminator noise but only to a certain
Figure 5.5: CNR versus time at the L1 and L2 bands for three initial incident angles for two elements on the 75mm diameter cylinder. The integration time is 10 ms and the roll frequency is 5 Hz.
extent. This extent is determined by the well known tradeoff between the dynamic stress error and noise error. A larger loop filter bandwidth reduces the dynamic error but increases the noise error and *vice versa*. For this application, a very small loop filter bandwidth is not practical.

The other issue is the effect of strong measurement noise on PVT estimation. Even if the tracking loop does not lose lock because of poor CNR, its measurements are still very noisy. When estimating position, velocity, and time, the measurement noise can increase the position, velocity, or timing error [1]. Of course, PVT errors also depend on the number of satellites, geometry of satellites, and the PVT implementation algorithm. Therefore, CNR is very important to maintain continuous tracking and obtain a good PVT estimate.

It would be advantageous for the reader to quantify the relationship between CNR and measurement noise variance. The measurement noise variance is the variance of the noise after the discriminator and it depends on CNR. Of course, this relationship also depends on the type of discriminator, but an example would still provide some
benefits. The early-power-minus-late-power discriminator with a correlator spacing of one chip is used as the example. In this case, the measurement noise standard deviation prior to the loop filter is

\[ \sigma = c T_c \sqrt{\frac{2}{T^2(CNR)^2} + \frac{1}{2T(CNR)}} \text{ meters} \]  

(5.67)

where \( c \) is the speed of light and \( T_c = 1/f_c \) is the chip width in seconds [1]. Now, there is also a loop filter in the delay lock loop that reduces the measurement noise variance. Using a linear model for the delay lock loop and a single order loop filter, the measurement noise standard deviation after the loop filter can be written as

\[ \sigma_L = \sigma \sqrt{2B_\tau T} \text{ meters} \]  

(5.68)

where \( B_\tau \) is the loop filter bandwidth in Hz. Note that \( \sigma \) is the measurement noise standard deviation prior to the loop filter and \( \sigma_L \) is the measurement noise standard deviation after the loop filter.

Table 5.1 shows the standard deviation in meters and chips versus CNR before and after the loop filter. A P-code chip width (97.8 ns), an integration time of 10 ms, and a loop filter bandwidth of 20 Hz [1] is used. It can be seen that for a CNR of 20 dB-Hz the tracking loop has a very good chance of losing lock because the standard deviation is one chip. For a CNR of 30 dB-Hz, the chance of losing lock is significantly reduced. But, \( \sigma_L \) is still at an unacceptable level of 4.9 m. At a CNR of 35 dB-Hz, \( \sigma_L \) is reduced to 2.5 m. At a CNR of 40 dB-Hz, the standard deviation after the loop filter is 1.3 m. Finally, at a CNR of 45 dB-Hz, the standard deviation after the loop filter is 0.7 m. For this example, it is considered that a CNR of around 30 dB-Hz is close to the lower limit to maintain tracking loop lock. It is also considered that a
Table 5.1: Measurement noise standard deviation for different values of CNR

<table>
<thead>
<tr>
<th>CNR</th>
<th>$\sigma$ (in chips)</th>
<th>$\sigma_L$ (in chips)</th>
<th>$\sigma$ (in chips)</th>
<th>$\sigma_L$ (in chips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>46 m</td>
<td>29 m</td>
<td>1.58</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>7.8 m</td>
<td>4.9 m</td>
<td>0.26</td>
<td>0.16</td>
</tr>
<tr>
<td>35</td>
<td>3.9 m</td>
<td>2.5 m</td>
<td>0.13</td>
<td>0.08</td>
</tr>
<tr>
<td>40</td>
<td>2.1 m</td>
<td>1.3 m</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td>45</td>
<td>1.2 m</td>
<td>0.7 m</td>
<td>0.04</td>
<td>0.025</td>
</tr>
</tbody>
</table>

CNR of 35 dB-Hz is the lower limit for good PVT estimation. Of course, the higher the CNR the better the PVT estimate. Note that this is a 10 dB-Hz drop from the isotropic antenna case.

The measurement noise is not the only corrupting influence on the true code delay. The total estimated delay can be written as

$$\hat{\tau} = \tau_0 + b + z \quad (5.69)$$

where $\tau_0$ is the true delay, $b$ is the code delay bias, and $z$ is the measurement noise. Since the antenna, platform, and front end filter the received signal, they will introduce a shift in the cross-correlation peak from the true value and this is called code delay bias. The code delay bias can be split into direction independent and dependent components. Direction independent biases are primarily associated with the front end and connecting cables (although there is a direction independent bias from the antenna/platform.) If they are not accounted for they will only cause a timing error.
in the PVT estimation. In a real system, these will be removed by calibration and therefore direction independent biases are ignored in the results presented below. On the other hand, the direction dependent bias occurs because the antenna/platform is a direction dependent filter. These biases will cause a position error in the PVT estimation because they depend on the location of the satellite. In addition, for this application, this bias will also be time-varying. Therefore, the direction dependent code delay bias on the moving platform is presented. Again, a P-code chip width of 97.8 ns and integration time of 10 ms is used.

Fig. 5.7 shows the code delay bias versus time at the L1 and L2 bands for three initial incident angles for two elements on the 75mm diameter cylinder and with a roll frequency of 5 Hz. The bias has a mean and variation. For both the single element and beamforming cases, the mean of the time-varying bias is not necessarily zero. As mentioned above, this causes position errors in the PVT estimation. For the different scenarios and at a given band, the difference between the bias means is only about 50 cm. This is considered good antenna-induced bias performance. This result does not change regardless of the method used and therefore its discussion is omitted from the rest of this chapter.

Now, the bias variation for a given scenario is discussed. For the single element cases, the bias variation depends on the initial incident satellite angle. The $\theta = 0^\circ, \phi = 0^\circ$ incident angle scenario has variation of about 50 cm. The $\theta = 60^\circ, \phi = 0^\circ$ incident angle scenario has a larger variation of over 100 cm. The $\theta = 87^\circ, \phi = 0^\circ$ incident angle scenario has variation of about 15 cm. It can be seen that the larger the CNR variation the larger the bias variation which is reasonable. This bias variation is more than the measurement noise standard deviation of 70 cm for a CNR of 45
dB-Hz. The code delay estimation will be adversely affected by the bias variation. For the beamforming method, the bias variation does not exceed 15 cm. This bias variation is less than the measurement noise standard deviation, and consequently, the bias variation will not cause a large error in the code delay estimation. Therefore, the beamforming method has much better bias variation performance than the single element cases.

In this section, the variation of CNR and code delay bias has been presented as the platform rotates. It was seen that a single antenna element has poor CNR performance and that the bias variation may exceed 100 cm. This will most likely lead to loss of lock and will definitely increase the error in the PVT solution. For the beamforming method, the CNR had excellent performance and the code delay bias variation is about 15 cm which is less than the measurement noise standard deviation. But, as discussed in Chapter 4, the beamforming method is impractical for this application. Also, in Chapter 4, the maximum gain method was presented and shown to have good satellite coverage. It was implied that this would lead to good tracking performance. Yet, the full discussion about how the maximum gain method is implemented in a GPS receiver was omitted. In the next section, a novel approach to implement the maximum gain method is presented.
Figure 5.7: Code delay bias versus time at the L1 and L2 bands for three initial incident angles for two elements on the 75mm diameter cylinder. The integration time is 10 ms and the roll frequency is 5 Hz.
5.3 Novel Receiver Tracking Algorithm

A novel approach to implement the maximum gain method in a delay lock loop is presented. This approach will demonstrate that good tracking performance can be maintained even as the platform undergoes rotation and without knowledge of the time-varying platform attitude.

5.3.1 Tracking Model for Maximum Gain Method

Fig. 5.8(a) shows the block diagram for the conventional approach to the maximum gain method implementation of the delay lock loop. The conventional approach represents how to implement the maximum gain method implementation in multiple completely separate element channels. After the loop filters, the element selection chooses the best element. The details of how this is done will be discussed below. The important consideration for the conventional approach is that the multiple element channel loops are not linked together in any fashion. This is a major problem. Suppose that there are only two element channels and that both channels start with enough CNR to estimate code delay. When the cylinder rotates, one element channel will still have good CNR, but the other element will have poor CNR as was shown above. The element with poor CNR will lose lock because its code delay estimate is extremely noisy. As the platform continues to rotate, the first element will then have poor CNR and the second element will then have good CNR. But, the second element has already lost lock, and, without a code delay estimate from the first element, it cannot re-obtain a good code delay estimate. After a few cylinder rotations, both
element channels would lose lock and have to reacquire the satellite signal. Therefore, for this application, the conventional approach will not continuously track the satellite signals.

Fig. 5.8(b) shows the block diagram for the proposed approach to the maximum gain method implementation of the delay lock loop. Now, the multiple element channels have a common NCO/code generator. The element selection block is done after the discriminators and its output is sent to the loop filter. The loop filter output updates the common NCO/code generator. This ensures that a good estimate of the code delay is available to all of the element channels so that each channel will continuously track the satellite signal. Even if only one of the channels has good CNR, then its estimate will be used to track the satellite on the other channels. It is assumed that at least one of the channels has a high enough CNR to obtain an estimate of the code delay. This is a reasonable assumption because the antennas on the cylindrical platform are pointed in various directions. Also, it is assumed that the true code delay for each of the elements is essentially the same because they are placed very close together (i.e. the code phase difference between elements is negligible).

Next, the element selection block details are discussed. In theory, this block chooses the discriminator output from the element channel with the highest CNR. This can be done because the signal power after the cross-correlation is higher than the noise floor. In practice, one compares the estimated post-correlation signal power

\[ \hat{C}_d = |\hat{R}(t_0, T, \tau_p)|^2, \]

where \( \tau_p \) is the delay for the prompt correlator, and chooses the element channel with the greatest \( \hat{C}_d \). Again, it is assumed that at least one of the element channels has a high enough CNR such that the estimated post-correlation
Figure 5.8: Block diagrams for the conventional and proposed approaches to the maximum gain method implementation of the delay lock loop.
signal power is higher than the noise floor. (The reason the element selection is done after the discriminators will become apparent in Section 5.4.)

Another important concern for the maximum gain method implementation is how the tracking loop is triggered. Above, the assumption was made that the various antenna elements are pointed in multiple directions. Therefore, at least one of the element channels will have high enough CNR to allow for satellite acquisition. Once the acquisition threshold is exceeded, the entire tracking loop is triggered. Note that acquisition does not need to occur for all of the elements. Therefore, since acquisition depends on code delay and doppler frequency, the code delay and doppler frequency are considered common to each of the channels. Implicit is this conclusion is that each channel has a common clock. This means that the doppler frequency from clock bias is common to each channel.

This maximum gain method implementation can be thought of as a type of vector delay lock loop (VDLL). In a standard VDLL, a single antenna is used but the correlation outputs from different satellites are used as multiple channels [50–56]. The advantage of a VDLL is that the channels with strong CNR can help maintain tracking for the channels with weaker CNR. Hence, this implementation of the maximum gain method is similar in concept to a VDLL. But, it is different in that the multiple channels derive from multiple antenna elements tracking the same satellite whereas, in the VDLL, the multiple channels derive from a single antenna tracking multiple satellites.

Fig. 5.9 shows the system model for the maximum gain method implementation of the delay lock loop. The antenna and platform effect is modeled as a time-varying filter. The model has internal noise (from the RF front end) and external noise which
Figure 5.9: System model for the proposed maximum gain method implementation of the delay lock loop.

are represented by noise sources $n(t)$ and $\hat{n}(t)$, respectively. All noise sources are Gaussian and white and uncorrelated between the various element channels. The correlation is done by multiplying by the local reference signal and integrating the result. The discriminator estimates the code delay which is represented by $\hat{\tau}_k$ where the hat denotes an estimate corrupted by measurement noise. The element selection in the block diagram is replaced by a weighted sum of the discriminator output from each channel. Note that after the discriminator the code delay estimate is real-valued. The weighted sum output $\hat{\gamma}$ is sent to the loop filter which is used to smooth out the noisy estimate. The output of the filter is sent to the navigation processor and fed back to the NCO/code generator. This creates the local reference signal with the common delay $\tau$. 

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The weights for the maximum gain method are

\[ w_k = \begin{cases} 
0 & \text{for } k \neq q, \ q^{th} \text{ element chosen} \\
1 & \text{for } k = q
\end{cases} \tag{5.70} \]

where the \( q^{th} \) element is chosen as the one with the highest \( \hat{C}_d \). With these weights, the CNR of the maximum gain method is simply

\[
\text{CNR} = \frac{C_d}{(1 + \alpha)N_0} \left| \int_{-B/2}^{B/2} G_d(f) \hat{A}_q(f) e^{-j2\pi f \tau_{\text{peak}}} df \right|^2. \tag{5.71}
\]

where

\[
\hat{A}_q(t_0, T, f) = \frac{1}{T} \int_{t_0}^{t_0+T} \hat{A}_q(t, f) \, dt \tag{5.72}
\]

and

\[
\hat{A}_q(t, f) = A_q(f, \theta_q(t), \phi_q(t)) e^{j\psi_q(t)}. \tag{5.73}
\]

The time-varying satellite incident angles \( \theta(t), \phi(t) \) and phase compensation \( \psi(t) \) are the same as discussed in section 5.2.1.

5.3.2 Example Results

In this section, the performance of the beamforming and maximum gain methods are compared. The scenario setup is the same as described in section 5.2.3.

Fig. 5.10 shows CNR versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. The integration times is 10 ms and the roll frequency is 5 Hz. For the beamforming method, it can be seen that the highest CNR performance is obtained. This is as expected. For the maximum gain method, it can be seen where the output is switched between the two
channels. As compared to the single element case, the maximum gain method provides good continuous CNR. The CNR does not have any large swings as the platform rotates. The tracking loop will not lose lock and the measurement noise will not be very strong. Therefore, for low spin rates, good CNR performance has been achieved using the maximum gain method.

Fig. 5.11 shows the code delay bias versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. The integration time is 10 ms and the roll frequency is 5 Hz. For the beamforming method, the bias variation does not exceed 15 cm. This is expected performance. For the maximum gain method, the bias variation is higher than the beamforming method and is on the order of 25 cm. Still, this is lower than the measurement noise standard deviation of 70 cm for a CNR of 45 dB-Hz, integration time of 10 ms, and loop filter bandwidth of 20 Hz. Therefore, the maximum gain method achieves good code delay bias performance.

Next, the integration time is reduced to 5 ms. Fig. 5.12 shows CNR versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles with a roll frequency of 5 Hz. For the two methods, it can be seen that the CNR performance is very similar to the above CNR performance for an integration time of 10 ms. Therefore, for slow spin rates, the integration time is not a major factor in terms of CNR performance. This reasonable because, in the limit of a static cylinder (i.e. no rotation), the CNR will not vary with integration time.

Fig. 5.13 shows the code delay bias versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. The integration time is 5 ms and the roll frequency is 5 Hz. For the beamforming method, the bias variation does not exceed 15 cm which is expected. For the maximum gain method,
Figure 5.10: CNR versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 10 ms and roll frequency of 5 Hz.
Figure 5.11: Code delay bias versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 10 ms and roll frequency of 5 Hz.
Figure 5.12: CNR versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 5 ms and roll frequency of 5 Hz.
the bias variation is higher than the beamforming method and is on the order of 25 cm. This is lower than the measurement noise standard deviation of 70 cm for a CNR of 45 dB-Hz, integration time of 10 ms, and loop filter bandwidth of 20 Hz. Therefore, for slow spin rates, the integration time is not a major factor in terms of code delay bias performance.

Now, the spin rate is increased to 175 Hz and the CNR performance and code delay bias compared for the beamforming and maximum gain methods. Fig. 5.14 shows CNR versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. The integration times is 10 ms and note that the time scale has changed to 0-60 ms. For the beamforming method, the CNR performance is very good. But, for the maximum gain method, the CNR is very low for two of the three initial incident angles. The reason for these results is that the integration time is longer than the spin rate. The integration time is 10 ms and the period for the spin rate is 5.7 ms (1/175). The spin creates a change in phase versus time (i.e. doppler frequency) and consequently a long integration time hurts CNR performance. The beamforming method is unaffected by the change in phase over the integration because the weights account for the phase. Note that the amount of CNR degradation in the maximum gain method is dependent on the initial incident satellite angle. Therefore, for fast spin rates, the integration time is very important to CNR performance.

Fig. 5.15 shows the code delay bias versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. The integration time is 10 ms and the roll frequency is 175 Hz. For the beamforming method, the bias variation is very small which is expected. For the maximum gain method, the
Figure 5.13: Code delay bias versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 5 ms and roll frequency of 5 Hz.
Figure 5.14: CNR versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 10 ms and roll frequency of 175 Hz.
bias variation is about 40 cm. Therefore, high spin rates cause a larger bias variation for this particular integration time. But, since the measurement noise is very strong due to the poor CNR of around 23-33 dB-Hz, this bias variation is still lower than the noise standard deviation $\sigma_L$ (see Table 5.1).

Now, the integration time is reduced to 1 ms. Fig. 5.16 shows CNR versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles and for a roll frequency of 175 Hz (roll period of 5.7 ms). Now, there are 5-6 integrations for every rotation. For the beamforming method, the CNR performance is still good for the given initial incident angles. This is expected, even though it can be seen that the CNR has some small variation versus time. For the maximum gain method, CNR performance is much higher than the 10 ms integration time CNR performance. This is because the phase over the integration time does not change as much as compared to the longer integration time. Therefore, for a fast spin rate, reducing the integration time increases the CNR.

Before discussing the code delay bias, the integration time is reduced even further to 0.25 ms. Fig. 5.17 shows CNR versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles with a roll frequency of 175 Hz (roll period of 5.7 ms). Now, there are 22-23 integrations for every rotation. For the beamforming method, the CNR performance is good with the modulation of the spinning platform easily seen. This is because the number of sampled points in the 60 ms range has increased due to the smaller integration time. For the maximum gain method, good CNR performance is obtained. Now, when comparing the results for an integration time of 1 ms to 0.25 ms, the shorter integration time has higher CNR for the maximum gain method. The initial incident angles of $\theta = 60^\circ, \phi = 0^\circ$
Figure 5.15: Code delay bias versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 10 ms and roll frequency of 175 Hz.
have the most improvement from a longer to shorter integration time. Therefore, the main conclusion is that a shorter integration time can be used to improve CNR performance when using the maximum gain method and the cylinder has a fast spin rate.

Fig. 5.18 shows code delay bias versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. The integration time is 0.25 ms and the roll frequency is 175 Hz (roll period of 5.7 ms). For the beamforming method, the code delay bias variation is less than 15 cm. For the maximum gain method, the bias variation is less than 25 cm. For this integration time, a CNR of 44 dB-Hz (average best result from maximum gain method), and 20 Hz loop filter bandwidth, the measurement noise standard deviation is $\sigma_L = 106$ cm from 5.67 and 5.68. Therefore, the bias variation from a shorter integration time is much less than the minimum measurement noise standard deviation.

Finally, the spin rate is increased to a very fast 325 Hz (roll period of 3.07 ms). Fig. 5.19 shows CNR versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles with an integration time of 0.25 ms. Now there are 12-13 integrations for every rotation. For the beamforming method, the CNR results are good. For the maximum gain method, the CNR has variation with a minimum of 38 dB-Hz for the different scenarios. Therefore, the maximum gain method has good CNR performance provided that at least 12 integrations are done for a single rotation of the cylinder.

Fig. 5.20 shows code delay bias versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. The integration time is 0.25 ms and the roll frequency is 325 Hz (roll period of 3.07 ms). For the
Figure 5.16: CNR versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 1 ms and roll frequency of 175 Hz.
Figure 5.17: CNR versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 0.25 ms and roll frequency of 175 Hz.
Figure 5.18: Code delay bias versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 0.25 ms and roll frequency of 175 Hz.
Figure 5.19: CNR versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 0.25 ms and roll frequency of 325 Hz.
beamforming method, the code delay bias variation is less than 10 cm. For the maximum gain method, the bias variation is less than 25 cm. Again, for this integration time, a CNR of 44 dB-Hz (average best result from maximum gain method), and 20 Hz loop filter bandwidth, the measurement noise standard deviation is $\sigma_L = 106$ cm. Therefore, the bias variation from a shorter integration time is much less than the minimum measurement noise standard deviation.

Therefore, it has been shown that the maximum gain method has a minimum CNR of 38 dB-Hz across the different scenarios. This assumes that a proper integration time is chosen based on whether the cylinder has a slow or fast spin rate. This minimum CNR ensures that the tracking loop will not lose lock and that good PVT estimation is obtained. The maximum gain method CNR performance is much better than the single antenna element cases. Also, it was demonstrated that the antenna- and platform-induced code delay bias was always less than the measurement noise standard deviation and hence measurement noise is the primary corrupting influence on the estimated delay. The maximum gain method does an excellent job of reducing the noise variance as the platform spins and rotates to ensure continuous satellite tracking.
Figure 5.20: Code delay bias versus time for the beamforming and maximum gain methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 0.25 ms and roll frequency of 325 Hz.
5.4 Further Performance Improvement

Now, when one observes the maximum gain method system model, the weights chosen for the channels are simply one or zero. The maximum gain method relies on good CNR from an individual channel to lower the measurement noise variance. The question arises of whether there is a better way to choose the weights to lower the measurement noise variance. For example, Fig. 5.5 showed that the CNR from the two elements is of comparable value when the satellite initial incident angles are $\theta = 87^\circ, \phi = 0^\circ$. In this case, it would be very advantageous to choose weights that use both channels such that the measurement noise variance after the sum junction is minimized. It is very important to note that the weights are after the discriminators and, therefore, the carrier phase of the channels is not assumed to be known. In this section, a procedure to choose the weights is presented.

5.4.1 Post-Discriminator Weighting Method

Optimal weights for minimizing the noise variance after the sum junction are derived in this section. For convenience, this weighting method will be called the post-discriminator weighting method (PDW). To this end, the code delay estimate for each channel $\hat{\tau}_k$ can be written as

$$\hat{\tau}_k = \tau_0 + b_k + z_k$$

(5.74)

where $\tau_0$ is the true delay, $b_k$ is the antenna/platform induced code delay bias for the $k^{th}$ element, and $z_k$ is the measurement noise for the $k^{th}$ element. The noise has zero mean, variance of $\sigma_k^2$ for the $k^{th}$ element, and is uncorrelated between channels. The code delay estimates for each channel can be arranged as an $K \times 1$ column vector $\vec{\tau}$.
\[ \vec{\tau} = \vec{\tau}_0 + \mathbf{b} + \mathbf{z} \] (5.75)

where

\[ \vec{\tau}_0 = \begin{bmatrix} \tau_0 \\ \vdots \\ \tau_0 \end{bmatrix}, \] (5.76)

\[ \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_K \end{bmatrix}, \] (5.77)

and

\[ \mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_K \end{bmatrix}. \] (5.78)

Using the above equations, the output after the sum junction is written as

\[ \hat{\gamma} = \mathbf{w}^T \vec{\tau} \] (5.79)

where \( \mathbf{w} \)

\[ \mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_K \end{bmatrix} \] (5.80)

and here the superscript \( T \) represents the transpose operator. Note that all variables are real-valued. Now, the variance of the output is
\[
\begin{align*}
\text{var}\{\hat{\gamma}\} &= E\{(\hat{\gamma})^2\} - (E\{\hat{\gamma}\})^2 \\
&= E\{w^T \bar{\tau}^T \bar{\tau} w\} - (E\{w^T \bar{\tau}\})^2 \\
&= w^T E\{((\bar{\tau}_0 + b + z)(\bar{\tau}_0 + b + z)^T)\} w - (w^T E\{(\bar{\tau}_0 + b + z)\})^2 \\
&= w^T((\bar{\tau}_0 \bar{\tau}^T + bb^T + \bar{\tau}_0 b^T + b \bar{\tau}_0^T + E\{zz^T\})w \\
&- (w^T \bar{\tau}_0 + w^T b)^2 \\
&= w^T((\bar{\tau}_0 \bar{\tau}^T + bb^T + \bar{\tau}_0 b^T + b \bar{\tau}_0^T + E\{zz^T\})w \\
&- w^T((\bar{\tau}_0 \bar{\tau}^T + bb^T + \bar{\tau}_0 b^T + b \bar{\tau}_0^T)w \\
&= w^T E\{zz^T\} w \\
&= w^T \Phi w
\end{align*}
\]

where \(\Phi = E\{zz^T\}\) is the correlation matrix. The matrix elements are zero everywhere except for the diagonal. On the diagonal, the values are \(\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2\).

To minimize the variance, the trivial solution is to set the weights to zero. Therefore, a constraint is introduced that prevents the weights from going to zero as one minimizes the variance. The constraint is

\[w^T u = 1\]  

(5.90)

where the constraint vector \(u\) is

\[
u = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}.
\]

Therefore, the final problem setup becomes
This is a standard linear algebra problem with a well-known solution. Hence, the solution will not be re-derived here. The answer is given by [42]

\[
\mathbf{w} = \frac{\Phi^{-1}\mathbf{u}}{\mathbf{u}^T\Phi^{-1}\mathbf{u}}
\]

(5.93)

where \( \Phi^{-1} \) is the inverse of \( \Phi \).

The previous results are generalized for any number of channels. In the following, two channels will be used to generate and compare results to the maximum gain method and beamforming method. In this case, the weights become

\[
\mathbf{w} = \frac{1}{\sigma_1^2 + \sigma_2^2} \begin{bmatrix} \sigma_2^2 \\ \sigma_1^2 \end{bmatrix}.
\]

(5.94)

Using these weights, the variance of the output is

\[
\text{var}\{\hat{\gamma}\} = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2}.
\]

(5.95)

For the maximum gain method, the variance of the output would be \( \sigma_q^2 \) where the \( q^{th} \) element is chosen. It can be seen that the variance of the optimal weighting method will always be either equal to or less than the variance of the maximum gain method.

Finally, an exact way to compare the beamforming, maximum gain, and PDW methods is discussed. For the beamforming and maximum gain methods, the CNR is explicitly given by the available equations. But, for the PDW method, the CNR is not explicitly defined because one is working with the noise variance. To compare the various methods, an equivalent CNR is defined for the PDW method. The equivalent
CNR is the CNR in a single channel that would generate a given measurement noise variance. In order to do this, a particular discriminator has to be assumed because the exact relationship between CNR and noise variance depends on the type of discriminator. The early-power-minus-late-power discriminator will be used to find the equivalent CNR. Note that this type of discriminator is not assumed for the beamforming or maximum gain methods. Using 5.67, the equivalent CNR can be found by

\[
\frac{\text{var}\{\hat{\gamma}\}}{T^2_c} \text{CNR}_{eq}^2 - \frac{1}{2T} \text{CNR}_{eq} - \frac{2}{T^2} = 0
\]

with solution

\[
\text{CNR}_{eq} = \frac{T^2_c}{T\text{var}\{\hat{\gamma}\}} \left[ 1 \pm 2 \sqrt{\frac{1}{4} + \frac{8\text{var}\{\hat{\gamma}\}}{T^2_c}} \right].
\]

The above equation gives two solutions (one positive and one negative) for the equivalent CNR of the PDW method. The negative solution is rejected because it does not have any meaning.

5.4.2 Example Results

In this section, the performance of the beamforming, maximum gain, and post-discriminator weighting (PDW) methods are compared. The scenario setup is the same as described in section 5.2.3. Also, for the PDW method, an early-power-minus-late-power discriminator is assumed to generate its equivalent CNR on a single channel. For more details, see the end of section 5.4.1. Note, that since the beamforming and maximum gain method results have already been presented, the PDW method results will be focused on.
Fig. 5.21 shows CNR versus time for the beamforming, maximum gain, and PDW methods at the L1 and L2 bands for three initial incident angles. The integration times is 10 ms and the roll frequency is 5 Hz. For the beamforming method, it can be seen that the highest CNR performance is obtained which is expected. For the maximum gain method, good continuous CNR is provided as seen above. Now, for the PDW method, the CNR is higher than the maximum gain method. In fact, the CNR approaches the optimal beamforming method and is within a small fraction of a dB-Hz of it. The PDW method does not have as many requirements as the beamforming method. The PDW method does not need to know the platform attitude. Also, its weights are updated every $T$ seconds as opposed to a continuous (ideal) or very fast (practical) rate for the beamforming method. Therefore, for low spin rates, the PDW method increases the CNR as compared to the maximum gain method while avoiding the impracticality of the beamforming method.

Fig. 5.22 shows code delay bias versus time for the beamforming, maximum gain, and PDW methods at the L1 and L2 bands for three initial incident angles. The integration time is 10 ms and the roll frequency is 5 Hz. For the beamforming method, the bias variation does not exceed 15 cm. For the maximum gain method, the bias variation is higher than the beamforming method and is on the order of 25 cm. For the PDW method, the bias variation is about 20 cm. Both the maximum gain and PDW method bias variations are lower than the measurement noise standard deviation of 70 cm for a CNR of 45 dB-Hz, integration time of 10 ms, and loop filter bandwidth of 20 Hz. Therefore, the maximum gain and PDW methods achieve good code delay bias performance.
Figure 5.21: CNR versus time for the beamforming, maximum gain, and PDW methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 10 ms and roll frequency of 5 Hz.
Figure 5.22: Code delay bias versus time for the beamforming, maximum gain, and PDW methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 10 ms and roll frequency of 5 Hz.
The spin rate is changed to 175 Hz (period of 5.7 ms). Fig. 5.23 shows CNR versus time for the beamforming, maximum gain, and PDW method at the L1 and L2 bands for three initial incident angles with an integration time of 0.25 ms. There are 22-23 integrations for every rotation. For the beamforming method, the CNR performance is good with the modulation of the spinning platform easily seen. For the maximum gain method, good CNR performance is obtained. For the PDW method, the CNR performance is higher than the maximum gain method and lower than the beamforming method. With the higher spin rate, the amount of improvement upon the maximum gain method depends on the initial incident angle. For example, the scenario with initial incident angles of $\theta = 87^\circ, \phi = 0^\circ$ has a PDW method CNR of 40.5 dB-Hz and a maximum gain method CNR of about 38 dB-Hz. But, in the other scenarios, the CNR for the maximum gain and PDW methods is closer. The higher spin rate also can cause the CNR of the PDW method to be less than the CNR of the beamforming method. This was not seen in the scenarios with a spin rate of 5 Hz. Therefore, a higher spin rate reduces the CNR performance of the PDW method as compared to beamforming, but it still has a higher CNR than the maximum gain method.

Fig. 5.24 shows code delay bias versus time for the beamforming, maximum gain, and PDW methods at the L1 and L2 bands for three initial incident angles. The integration time is 0.25 ms and the roll frequency is 175 Hz (roll period of 5.7 ms). For the beamforming method, the code delay bias variation is less than 15 cm. For the maximum gain method, the bias variation is less than 25 cm. For the PDW method, the variation is about 20 cm. As stated above, the minimum measurement noise standard deviation is about 100 cm using 5.67 and 5.68 with an integration time
Figure 5.23: CNR versus time for the beamforming, maximum gain, and PDW methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 0.25 ms and roll frequency of 175 Hz.
of 0.25 ms, CNR of 44 dB-Hz, and loop filter bandwidth of 20 Hz. Therefore, the bias variation is less than the minimum measurement noise standard deviation for all of the methods.

Finally, the spin rate is increased to a very fast 325 Hz (roll period of 3.07 ms). Fig. 5.25 shows CNR versus time for the beamforming, maximum gain, and PDW methods at the L1 and L2 bands for three initial incident angles with an integration time of 0.25 ms. Now there are 12-13 integrations for every rotation. For the beamforming method, the CNR results are good. For the maximum gain method, the CNR performance is also good with a minimum of about 38 dB-Hz. For the PDW method, the CNR performance is higher than the maximum gain method with a minimum of 40 dB-Hz. Therefore, the PDW method performs very well provided that at least 12 integrations are done for a single rotation of the cylinder.

Fig. 5.26 shows code delay bias versus time for the beamforming, maximum gain, and PDW methods at the L1 and L2 bands for three initial incident angles. The integration time is 0.25 ms and the roll frequency is 325 Hz (roll period of 3.07 ms). For the beamforming method, the code delay bias variation is less than 15 cm. For the maximum gain method, the bias variation is less than 25 cm. For the PDW method, the variation is about 20 cm. Similar to the results when the spin rate was 175 Hz, the minimum measurement noise standard deviation is larger than the bias variation for all of the methods.

Therefore, the PDW method provides an improvement in CNR performance as compared to the maximum gain method. For all scenarios, the PDW method minimum CNR was 40 dB-Hz and the maximum gain method minimum CNR was 38 dB-Hz. It is recommended to use the PDW method because of its higher CNR. The
Figure 5.24: Code delay bias versus time for the beamforming, maximum gain, and PDW methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 0.25 ms and roll frequency of 175 Hz.
Figure 5.25: CNR versus time for the beamforming, maximum gain, and PDW methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 0.25 ms and roll frequency of 325 Hz.
Figure 5.26: Code delay bias versus time for the beamforming, maximum gain, and PDW methods at the L1 and L2 bands for three initial incident angles. There are two antenna elements on the 75mm diameter cylinder with an integration time of 0.25 ms and roll frequency of 325 Hz.
extra cost of calculating the weights is considered minimal when compared to the performance increased of on average a few dB-Hz of CNR. In addition, for slow spin rates, the CNR performance of the PDW method approaches the CNR performance of the beamforming method. Finally, the bias variation of the PDW method (about 20 cm) is less than the maximum gain method (about 25 cm) and both are less than the measurement noise standard deviation (smallest result about 70 cm).

5.5 Summary & Conclusions

In this chapter, a novel GPS receiver algorithm for continuous satellite tracking was presented. It builds on the results from Chapter 4 where it was shown that the maximum gain method gives full spherical coverage (and hence continuous tracking when the platform rotates). But, it was not shown how one should implement the maximum gain method. This is a challenge because the GPS signals prior to the receiver are below the noise floor and because the time-varying platform attitude is unknown. Therefore, the maximum gain method is implemented in the receiver itself by modifying the standard GPS receiver tracking loop (DLL). To this end, analytic expressions for CNR with a single antenna were presented that included the effect of the time-varying platform and the antenna response. The expressions were then modified to include the beamforming method. Example results were presented and it was shown that a single antenna element cannot provide good continuous CNR. The CNR has large variations versus time with minimums of 20 dB-Hz. The beamforming method does provide good CNR performance with a minimum of 42 dB-Hz. But, the beamforming method is not practical for this application. Also, a discussion about the effect of CNR on tracking loop lock and PVT estimation was presented. It was
concluded that CNR below 30 dB-Hz will probably lead to loss of lock. In addition, CNR below 35 dB-Hz will lead to large error in the PVT estimate. The higher the CNR the better the PVT estimation.

A novel receiver tracking algorithm was presented that implements the maximum gain method by making two modifications to a standard tracking loop. The first consists of using a common NCO/code generator for all of the element channels (i.e. the same code delay estimate sent to all channels). The use of a common NCO/code generator allows the tracking loop to continuously track the satellite signals as long as one channel has good CNR even if the other channels have poor CNR. If a common code delay estimate is not used, then the tracking loop will lose lock. The second modification consists of selecting the element with the highest estimated post-correlation signal power. This is practical because the post-correlation signal power is higher than the noise floor. It was set up as a weighted sum of the discriminator outputs from the various elements. With the maximum gain method, it was demonstrated that good CNR is achieved as the platform rotates with a minimum of 38 dB-Hz. This conclusion assumes that at least 12 integrations during a rotation were carried out. Therefore, the implied conclusion of Chapter 4 that the maximum gain method provides an excellent way to provide continuous satellite tracking, minimize the number of elements, and eliminate the need for knowledge of the platform attitude was confirmed in the present chapter.

Furthermore, an additional method was presented that improves upon the CNR performance of the maximum gain method. This was the post-discriminator weighting method (PDW). In this method, the discriminator outputs of the various element channels were weighted and combined to minimize the measurement noise variance.
It does not have an analog method in Chapter 4 because its weights are integrated in the receiver. In the PDW method, an equivalent CNR was found using the minimized variance and assuming a particular type of discriminator. Example results showed that the PDW method always has the same or higher CNR than the maximum gain method. Also, for slow spin rates, the CNR performance of the PDW method approaches the CNR performance of the beamforming method. For faster spin rates, the PDW method CNR performance is not quite as high as the beamforming method but still higher than the maximum gain method. Similar to the maximum gain method, this conclusion assumes that at least 12 integrations during a rotation were carried out. With this assumption, the minimum CNR for the PDW method was about 40 dB-Hz for the different scenarios. Therefore, the maximum gain method has the lowest minimum CNR of 38 dB-Hz, the PDW method has minimum CNR of 40 dB-Hz, and the beamforming method has the highest minimum CNR of 42 dB-Hz. This leads one to the conclusion that the PDW method with two antenna elements is the best choice for the small cylindrical platform of interest given the above mentioned assumptions, goals, and expectations.
CHAPTER 6

Summary & Future Work

The purpose of this work was to extend the state-of-the-art of GPS receiver and antenna technology for the specific application of small cylindrical platforms. These platforms can have potentially high rotation rates of up to 350 Hz and short flight times. Before launch, the system is relayed pertinent initialization data which is essential for a fast acquisition. After launch, the system needs to quickly acquire satellites, track them, and provide accurate navigation data. Furthermore, the system should have low power consumption, low cost, and be small volume. This presents a set of design challenges for engineers and this work focused on three aspects of the problem: antenna design, satellite coverage, and receiver design. The design of the RF front end, navigation processor (which uses the receiver measurements), and guidance system proper (i.e. autopilot) was not covered in this work. Thin metal cylinders of diameter 60-160mm were used in the study as the platforms of interest and an RFI free environment was assumed.

The goals were as follows. First, a dual-band GPS antenna with adequate bandwidth to receive C/A-, P(Y)-, and M-coded signals and robust tuning was needed that can be easily integrated onto a thin metal cylinder. Second, spherical satellite coverage was needed because the platform rotates. Assuming that multiple antenna
elements are used, the design goal becomes how to combine the elements and how many are needed for the desired coverage. This leads to examining and modifying the GPS receiver in order to implement the proposed approach for spherical satellite coverage.

Chapter 2 presented a novel dual-band GPS antenna that meets the requirements for this application. The antenna provides RHCP coverage at the GPS L1/L2 bands for reception of C/A-, P(Y)-, and M-coded GPS signals. The antenna is also quite thin and the final antenna size is $4\text{cm} \times 4\text{cm} \times 5.08\text{mm}$ ($\lambda/6 \times \lambda/6 \times \lambda/50$). Simulation results showed that it has at least 4 dBi peak RHCP gain and 25 MHz 3 dB bandwidth at both bands. Measurements were carried out that generally confirmed the simulation results (there was a difference in peak RHCP gain at the L1 band). Therefore, the antenna is an excellent candidate for small-size and reduced height dual-band GPS applications.

Chapter 3 studied the performance of the novel antenna when it is placed on various diameter metal cylinders (60-160mm). A very important contribution from this chapter was that one does not need to retune the antenna when it is placed on the multiple diameter cylinders even though the antenna is small. The antenna has robust tuning performance. The cylinders do affect the antenna patterns but good coverage is still obtained. For example, on the 117mm diameter cylinder and at both bands, the simulated antenna has at least 2.3 dBi peak RHCP gain, 24 MHz 3 dB bandwidth, 114° 3 dB beamwidth, and a boresight co-pol to cross-pol ratio of 11 dB. For all cylinders, the antenna has broad RHCP coverage ($111^\circ$ HPBW) and good gain bandwidth performance (1-2.8 dBi peak RHCP gain with at least 23 MHz 3 dB bandwidth). This is adequate to receive C/A-, P(Y)-, and M-coded
GPS satellite signals. The fabricated antenna was placed on the 117mm diameter cylinder and its simulated and measured performance was compared. The resonant frequency error was only 0.3% and 1.3% at the GPS L1/L2 bands. This is an excellent result for an experimental resonant type antenna of small volume. Once the antenna was retuned, the simulated and measured performance had very good agreement. Therefore, Chapters 2 and 3 presented the design, simulation, and measurement of a novel dual-band GPS antenna for use on thin cylinders.

Chapter 4 investigated the satellite coverage when a single antenna or multiple antenna elements are placed on a thin cylinder. A single element could not provide the necessary spherical coverage. After placing multiple elements on the cylinder, the RF sum, beamforming, and maximum gain methods were used to combine the elements. Signal-to-noise-ratio (SNR) was used as the metric to compare the methods. To this end, SNR equations for each of the methods were derived. The cylinders investigated had diameters of 75mm (1-4 elements) and 160mm (1-6 elements). The GPS P-code power spectral density was used for the desired signal. The RF sum method is the simplest method but cannot obtain full spherical coverage. The beamforming method has excellent SNR performance. But, it requires knowledge of the time-varying platform attitude which is not available. On the other hand, the maximum gain method does not need to know the attitude and has SNR performance in between the RF sum and beamforming methods. Very importantly, it provides complete spherical coverage even with only two elements. This is true for both the small and large cylinders. The maximum gain method with two elements is the best choice to obtain full spherical coverage, minimize the number of elements, and eliminate the need for knowledge of platform attitude. But, how to implement it in a practical way
was not presented. This is a challenge because the GPS signals prior to the receiver are below the noise floor and because the time-varying platform attitude is unknown.

Chapter 5 built on the results from Chapter 4 by designing a receiver tracking loop that implements the proposed maximum gain method and allows for continuous tracking of satellites. To this end, the antenna and time-varying platform effects are modeled as a linear time-variant filter to be used in the conventional receiver tracking loop model. Using the model, it was shown that a single antenna element cannot provide good continuous carrier-to-noise-ratio (CNR). The CNR has large variations and minimums of 20 dB-Hz. The beamforming method does provide good CNR performance with a minimum of 42 dB-Hz, but it is not practical for this application. The relationship between CNR and tracking loop lock and position, velocity, and time (PVT) estimation was also presented. It was concluded that CNR below 30 dB-Hz will probably lead to loss of lock. In addition, CNR below 35 dB-Hz will lead to large error in the PVT estimate. The higher the CNR the better the PVT estimation.

A novel receiver tracking algorithm that implements the maximum gain method was designed by making two modifications to a standard tracking loop. The first consists of using a common NCO/code generator for all of the element channels (i.e. the same code delay estimate sent to all channels). This is a very important step. The second consists of selecting the element with the highest estimated post-correlation signal power which is practical because the post-correlation signal power is higher than the noise floor. With the maximum gain method, it was demonstrated that good CNR is achieved as the platform rotates with a CNR minimum of 38 dB-Hz. This conclusion assumes that at least 12 integrations during a rotation were carried out (regardless of a fast or slow spin rate). The maximum gain method is an excellent
approach to provide continuous satellite tracking, minimize the number of elements, and eliminate the need for knowledge of the platform attitude.

Furthermore, an additional method was presented that improves upon the CNR performance of the maximum gain method. This was the post-discriminator weighting method (PDW). In this method, the discriminator outputs of the various channels were weighted and combined to minimize the measurement noise variance. It does not have an analog method in Chapter 4 because its weights are integrated in the receiver. Example results showed that the minimum CNR for the PDW method was about 40 dB-Hz. Therefore, the maximum gain method has the lowest minimum CNR of 38 dB-Hz, the PDW method has minimum CNR of 40 dB-Hz, and the beamforming method has the highest minimum CNR of 42 dB-Hz. This leads one to the conclusion that the PDW method with two antenna elements is the best choice for the small cylindrical platform of interest given the above mentioned assumptions, goals, and expectations.

In this work, the antenna design, satellite coverage, and receiver design have been studied for the application of interest. Some future work could still be done on the receiver design (which is not surprising). The presented receiver design overcame the problem of providing continuous satellite tracking for the delay lock loop (DLL). The DLL is the basic tracking loop in the receiver but the phase lock loop (PLL) is also important. The PLL is used to estimate precise carrier phase and doppler frequency. In turn, these can be used to estimate platform attitude and pseudorange rate which will improve the DLL performance and PVT solution. Note that the PLL is dependent on the DLL.
Future work could investigate the effect of the time-varying platform attitude on the PLL. In this case, the carrier phase should be tracked separately for the different elements because the true carrier phase is different for each element (unlike the true code delay). The potential challenge is tracking the carrier phase under a very fast spin rate. In addition, fast spin rates will also cause direction-dependent doppler frequency biases for the different satellites which could be investigated. Finally, experimental results for the receiver maximum gain method implementation could be carried out to verify the conclusions of Chapter 5.
BIBLIOGRAPHY


