The Noise Signature and Production Mechanisms of Excited High Speed Jets

Dissertation

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By

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Abstract

Following on previous works showing that jet noise has significant intermittent aspects, the present work assumes that these intermittent events are the dominant feature of jet noise. A definition and method of detection for intermittent noise events are devised and implemented. Using a large experimental database of acoustically subsonic jets with different acoustic Mach numbers ($M_a = 0.5 – 0.9$), nozzle exit diameters ($D = 2.54, 5.08, & 7.62$ cm), and jet exit temperature to ambient temperature ratios (ETR = 0.84 – 2.70), these events are extracted from the noise signals measured in the anechoic chamber of the NASA Glenn AeroAcoustic Propulsion Laboratory. It is shown that a signal containing only these events retains all of the important aspects of the acoustic spectrum for jet noise radiating to shallow angles relative to the jet downstream axis, validating the assumption that intermittent events are the essential feature of the peak noise radiation direction. The characteristics of these noise events are analyzed showing that these events can be statistically described in terms of three parameters (the variance of the original signal, the mean width of the events, and the mean time between events) and two universal statistical distribution curves. The variation of these parameters with radiation direction, nozzle diameter, exit velocity, and temperature are discussed.

A second experimental database from the Ohio State University Gas Dynamics and Turbulence Laboratory of far-field acoustic data from an excited subsonic jet with hydrodynamic Mach number of 0.9 ($M_j = 0.9$) at various total temperature ratios (TTR =
1.0-2.5) is analyzed using the same process to determine what other characteristics of these noise events and their production can be identified. In addition to the experimental acoustic database, conclusions and observations from previous works using Localized Arc Filament Plasma Actuators (LAFPAs) are leveraged to inform discussion of the statistical results and their relationship to the jet flow dynamics. Analysis of the excited jet reveals the existence of a resonance condition. When excited at the resonance condition, large amounts of noise amplification can occur – this is associated with each large-scale structure producing a noise event. Conversely, noise reduction occurs when only one noise event occurs per several large-scale structures. One of the important conclusions from these results is that there seems to be a competition for flow energy among neighboring structures that dictates if and how their dynamics will produce noise that radiates to the far-field. The interaction of large-scale structures is explored and these results are related to the acoustic results.

Utilizing the results from both databases, several models for noise sources addressing different aspects of the results are discussed. A simple model for this kind of noise signal is used to derive a relationship between the characteristics of the noise events and the fluctuations in the integrated noise source volume. Based on the known flow-field dynamics and the acoustic results from the excited jet, a hypothetical model of the competition process is described. A wave-packet model on a cylindrical surface is used to analyze the impact of the azimuthal extent of a source. It is found that an axisymmetric
source is the most efficient radiator within the scope of the model. These various models speculate on the dynamics relating the noise sources to the signal in the far-field and, as such, the present work cannot provide a definitive description of jet noise sources, but can serve as a guide to future exploration.
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Table of Contents

Abstract ........................................................................................................................................ ii
Acknowledgments ..................................................................................................................... v
Vita .......................................................................................................................................... vi
Table of Contents .................................................................................................................... ix
List of Tables ........................................................................................................................... xii
List of Figures .......................................................................................................................... xiii
Nomenclature .......................................................................................................................... xvi
Chapter 1 : Introduction .......................................................................................................... 1
Chapter 2 : Background .......................................................................................................... 5
  2.1 The Acoustic Analogy ........................................................................................................ 6
  2.2 Vortex Sound .................................................................................................................... 9
  2.3 Flow Control .................................................................................................................... 12
  2.4 Traditional Analysis of Jet Noise .................................................................................... 15
  2.5 Supercritical Radiation ................................................................................................... 19
  2.6 Temporally Localized Signal Analysis ......................................................................... 20
Chapter 3 : Noise Event Definition and Signal Extraction .................................................... 24
Chapter 4 : Experimental Databases ....................................................................................... 28
  4.1 AAPL Database ............................................................................................................... 28
  4.2 GDTL Database ............................................................................................................... 30
List of Tables

Table 4.1: AAPL experimental operating conditions .................................................................29
Table 4.2: GDTL experimental operating conditions ...............................................................35
Table 4.3: GDTL excitation parameters ..................................................................................36
Table 6.1: Various calculated quantities for the event width distributions at $\phi = 30^\circ$. .........57
Table 6.2: Mean event width for all cases at $\phi = 90^\circ$. .......................................................61
Table 6.3: Various calculated quantities for the event intermittence distributions at $\phi = 30^\circ$. 69
Table 9.1: Radiated power versus azimuthal extent ($\delta \theta$) for $St_D = 0.2$, $M_a = 1$, & $\phi = 30^\circ$. 148
List of Figures

Figure 2.1: Jet noise sources cartoon reproduced from Tam et al. (2008) ........................................ 16
Figure 2.2: Tam’s similarity spectra ............................................................................................... 17
Figure 3.1: Cartoon of data extraction process ............................................................................. 26
Figure 4.1: Jet facility and anechoic chamber .............................................................................. 31
Figure 4.2: Heating system diagram ............................................................................................. 32
Figure 4.3: LAFPA circuit schematic .............................................................................................. 33
Figure 5.1: Schlieren images of jets demonstrating the behavior of actuators ............................ 39
Figure 5.2: FWHM and centerline TKE for various excited cases in a Mach 0.9 unheated jet. .... 42
Figure 6.1: Time-domain of a portion of data for $D = 2.54$ cm, TTR = 1.0, and $Ma = 0.9$ at three polar angles ($\phi = 30^\circ$, $90^\circ$, & $130^\circ$) showing the raw data (black) and the reconstructed signal (pink). ........................................................ .......................................................... 46
Figure 6.2: Example spectra for signal reconstruction ................................................................. 47
Figure 6.3: PDF of the amplitude of unaltered signals normalized by $p_{RMS}$ for a given signal. ................................................................................................................................. 52
Figure 6.4: PDF of peak amplitudes for all cases at $\phi = 30^\circ$ ................................................. 54
Figure 6.5: Distribution of event widths for all cases at $\phi = 30^\circ$ .......................................... 55
Figure 6.6: Distribution of event widths normalized by their respective means for data at $\phi = 30^\circ$. .............................................................................................................................................. 58
Figure 6.7: Distribution of event widths for all cases at $\phi = 90^\circ$ ........................................ 60
Figure 6.8: Joint PDF for event width ($\ell = 6$, $\xi = 0.17$) and amplitude ($\sigma = 1.2$) assuming the two quantities are independent random variables. ................................................................. 62
Figure 6.9: Joint probability density functions of event amplitude and width for several operating conditions at $\phi = 30^\circ$ .................................................................................................................. 64
Figure 6.10: Distribution of events intermittence for all cases at $\phi = 30^\circ$ ......................... 68
Figure 6.11: Intermittence distributions normalized by their respective means for data at $\phi = 30^\circ$. ................................................................. 70

Figure 6.12: Spectra at $\phi = 30^\circ$ showing the predictive capability of the mean intermittence. ................................................................. 72

Figure 6.13: Directivity patterns for mean event width................................................................. 74

Figure 6.14: Directivity pattern of mean width and intermittence for Case 5 ($D = 2.54$ cm, $M_a = 0.9$, & TTR = 1.0)................................................................. 75

Figure 6.15: Averaged event waveform for several polar angles in Case 5 ($D = 2.54$ cm, $M_a = 0.9$, TTR = 1.0). ................................................................. 77

Figure 6.16: Cartoon of one-and-one waveform (A or B) and the impact of averaging.............. 78

Figure 6.17: $\phi = 30^\circ$ events normalized by width and amplitude before averaging for Case 5 ($D = 2.54$ cm, $M_a = 0.9$, TTR = 1.0)................................................................. 80

Figure 6.18: Cartoon of low-angle jet noise signal based on the characteristics extracted from the analysis. ................................................................. 85

Figure 7.1: SPL (dB) map for the unexcited jet data from GDTL at various temperatures. ... 87

Figure 7.2: Baseline 1/3 octave spectra at 90° and 30° for multiple temperatures (similarity spectra are for the unheated case). ................................................................. 88

Figure 7.3: Narrowband spectra at 90° and 30° for $St_{DF} = 0.35$. .............................................. 90

Figure 7.4: Narrowband spectra at 90° and 30° for maximum reduction frequency at 30°... 91

Figure 7.5: $\Delta$OASPL (dB) for $m = 0$. ...................................................................................... 93

Figure 7.6: $\Delta$OASPL (dB) for $m = 1$. ...................................................................................... 94

Figure 7.7: $\Delta$OASPL (dB) for $m = 3$. ...................................................................................... 95

Figure 7.8: $\Delta$OASPL plots for $\phi = 30^\circ$. .................................................................................. 98

Figure 7.9: Example spectra for signal reconstruction. ............................................................. 99

Figure 7.10: Additional example spectra showing lack of over-prediction. ......................... 101
Figure 7.11: Time-domain data for TTR = 2.0, m = 0, and St_{DP} = 0.26 at \( \phi = 30^\circ \). ............... 102

Figure 7.12: PDF of peak amplitudes for all excitation frequencies at \( \phi = 30^\circ \). Higher frequencies are in red and lower frequencies in blue. ............................................................ 105

Figure 7.13: Gamma deviations for temperature ratios of TTR = 1.0 & 2.0 and \( \phi = 30^\circ, 45^\circ, 60^\circ, & 90^\circ \) ............................................................. 107

Figure 7.14: Intermittence distributions normalized by their respective means for the excited jet. .................................................................................................................... 108

Figure 7.15: Mean width and intermittence for the unexcited jet data from GDTL. .......... 111

Figure 7.16: Mean width and intermittence for various temperatures at \( \phi = 30^\circ \). T_F is the excitation period. ................................................................................................................... 113

Figure 7.17: Mean width and intermittence for two representative temperatures at \( \phi = 90^\circ \). 116

Figure 7.18: Unexcited joint-PDFs of amplitude and width from the GDTL data at various temperature ratios at \( \phi = 30^\circ \). ..................................................................................................................... 118

Figure 7.19: Joint-PDFs for several excited cases at TTR = 2.0 and \( \phi = 30^\circ \) ............... 120

Figure 8.1: Picture of microphone array setup. .................................................................... 127

Figure 8.2: The single pulse behavior of the jet and the actuator acoustic wave. ............. 128

Figure 8.3: Mean square pressure of the phase-averaged pressure signal for axisymmetric excitation at various frequencies. ...................................................................................................... 130

Figure 8.4: Examples of the large-scale structure signature for various excitation frequencies. ........................................................................................................................................... 132

Figure 9.1: Cartoon of low-angle jet noise signal based on the characteristics extracted from the analysis. ................................................................. 135

Figure 11.1: Wavelet amplitudes (|\( \hat{p} \)|^2) for a \( M_j = 0.9 \), TTR = 2.0 jet at \( \phi = 30^\circ \). .............. 160

Figure 11.2: Conceptual rendering of jet nozzle with screens in flow path. ....................... 161
Nomenclature

$A_i$ = $i^{th}$ event amplitude (Pa)
$a_\infty$ = ambient speed of sound (m/s)
$C_n$ = azimuthal Fourier coefficient
$D$ = jet exit diameter (cm)
$ETR$ = jet Exit Temperature Ratio
$G(\chi; \ell, \xi)$ = gamma distribution
$He$ = Helmholtz number ($f D/a_\infty$)
$J_\nu(x)$ = Bessel’s function of the first kind
$k$ = wavenumber (1/m)
$\ell$ = shape parameter of the gamma distribution
$M_a$ = jet acoustic Mach number ($U_j/a_\infty$)
$M_f$ = jet hydrodynamic Mach number ($U_j/a_j$)
$m$ = excitation azimuthal mode
$N(x; \sigma, \mu)$ = normal distribution
$OASPL$ = OverAll Sound Pressure Level (dB)
$p$ = acoustic pressure (Pa)
$PDF$ = Probability Density Function
$p_{RMS}$ = root mean square pressure (Pa)
$R$ = jet radius (m)
$Re_D$ = jet Reynolds number based on diameter
$r$ = radial coordinate (m)
$SPL$ = Sound Pressure Level (dB)
$St_D$ = Strouhal number ($f D/U_j$)
$St_{DF}$ = excitation Strouhal number ($f_F D/U_j$)
$T$ = temporal period (s)
$t$ = temporal coordinate (s)
$T_F$ = excitation period (s)
$T_i$ = $i^{th}$ event temporal coordinate (s)
$T_{ij}$ = Lighthill stress tensor (kg m$^{-1}$ s$^{-2}$)
$TTR$ = jet Total Temperature Ratio
$U_j$ = jet exit velocity (m/s)
$x$ = far-field observer coordinates (m)
$y$ = source-field coordinates (m)
$W_p$ = non-dimensionalized acoustic power
$\alpha$ = model event width parameter ($\mu$s)
$\beta$ = azimuthal Gaussian extent parameter (rad)
$\Delta_T$ = mean event intermittence (msec)
$\delta(x)$ = Dirac delta function

xvi
$\bar{\delta t}$ = mean event width (μs)
$\delta_{ij}$ = Kronecker delta function
$\delta t_i$ = $i^{th}$ event width (μs)
$\delta \theta$ = percentage of azimuthal Gaussian extent above $1/2$
$\theta$ = jet azimuthal angle (degrees)
$\Theta_{ni}$ = integrated azimuthal factor
$\lambda$ = wave-packet axial envelope parameter (m)
$\zeta$ = scale parameter of the gamma distribution (μs)
$\rho$ = fluid density (kg/m$^3$)
$\sigma$ = standard deviation parameter of normal distribution (Pa)
$\sigma_{ij}$ = viscous stress tensor (kg m$^{-1}$ s$^{-2}$)
$\tau_\infty$ = inverse Helmholtz number
$\tau_j$ = inverse Strouhal number
$\phi$ = jet polar angle relative to downstream axis (degrees)
$\Psi_{ij}$ = integrated noise source tensor (kg m$^2$ s$^{-2}$)
$\omega$ = temporal frequency (rad/s)
Chapter 1: INTRODUCTION

Jet noise is a problem that has plagued the use of jet engines since their inception. Despite six decades of research since the seminal work of Lighthill (1952), a clear picture of jet noise sources has not yet emerged (Jordan and Gervais 2008). Part of the problem is the sheer number of parameters that can be varied in a jet that have been shown to impact jet noise production (e.g. temperature, pressure, density, hydrodynamic Mach number, acoustic Mach number, nozzle geometry, nozzle exit boundary layer turbulence, etc.). Many of these parameters are interrelated and no unified standard exists for reducing them to a minimum set of independent parameters – this leads to additional confusion about overlapping experimental regimes. While there have been advances in empirically based models (e.g. Tam et al. 1996; Viswanathan 2006) and theoretical analysis (e.g. Cabana et al. 2007) of jet noise, the essential features of jet noise are still debated. Without a complete description of the essential features of jet noise, understanding of their sources is clearly impeded.

Through six decades of research, there have been two dominant methods of experimental data analysis in jet aeroacoustics: 1) Fourier spectrum analysis and 2) correlation analysis. Spectral analysis is the fundamental tool used by the aeroacoustics community, and for good reason. These two tools provide researchers with a wealth of information and insight, but with certain restrictions. Spectral analysis discards temporal information making it impossible to link particular aspects of the frequency domain back
to segments of the signal in time. Correlation analysis provides links between two signals in time, but only if their trends are sufficiently similar – it can indicate how similar the trends of two signals are and how the similarity is displaced in time.

The problem with the reliance on these two tools, particularly spectral analysis, is that they are likely to overlook some of the fundamental aspects of jet noise. Relatively recently, some researchers have started utilizing tools like wavelet transforms to obtain a more complete picture of the noise signal. The basic theme of these works is the supposition that acoustically subsonic jet noise (at least in the radiation to low angles) is made up of intermittent bursts as opposed to continuous variations. Understanding this kind of signal requires a different analysis methodology than has been prevalent in the literature. In a previous work from the Gas Dynamics and Turbulence Laboratory (GDTL) of the Ohio State University, Hileman et al. (2005) showed that, if the amplitude of intermittent bursts (in this context defined as portions of the signal that exceeded twice the root mean square far-field pressure fluctuations, $2p_{\text{RMS}}$) were reduced by 50%, the peak region of the spectrum was reduced by about 4 dB. Hileman et al. used the assumption that the bursts are a significant constituent of the noise signal, but not necessarily the primary feature. The purpose of the present work is to take this previous assumption to its extreme limit: assume that these bursts (hereafter referred to as noise “events”) are the dominant feature of jet noise. This work is divided as follows.

- Chapter 2 – The relevant aspects of the current understanding of jet noise and data analysis are discussed.
- Chapter 3 – A noise event as well as a method of its identification and extraction is defined and discussed.
• Chapter 4 – The experimental facilities and techniques used to collect the data are described. The experimental parameters are also described.

• Chapter 5 – The characteristics of the plasma actuators and relevant previous results are discussed.

• §6.1 – The spectrum of the event-only signal is computed and compared to the total signal.

• §6.2 – Statistical analysis of these events is performed for an extensive experimental data set of unexcited jets to reveal what these events can say about the nature of jet noise.

• Chapter 7 – The statistical analysis is repeated on a data set of excited jets to correlate changes in the far-field sound induced by excitation with changes in the statistical parameters.

• Chapter 8 – The impulse response of the jet flow-field and the interaction of large-scale structures are explored by measuring the near-field hydrodynamic pressure.

• Chapter 9 – The implications of the results on jet noise sources are discussed and a model is developed.

The work presented in this dissertation is part of a larger collection of work on excited heated jets that has been conducted over the last four years. The publications on this body of work include:

• (Kearney-Fischer et al. 2009, 2011b) – Characterization studies of the response of a Mach 0.9 jet to excitation at various temperatures using acoustic and Particle
Image Velocimetry (PIV) data. The purpose of these works was to develop a basic understanding of how the jet response changes with temperature, both in the flow-field and the acoustic far-field.

- (Kearney-Fischer et al. 2010; Kearney-Fischer and Samimy 2010) – Acoustic and PIV characterization studies of the response of a Mach 1.3 jet to excitation at various temperatures. The purpose of these works was to determine what if any changes occur in excitation response when the jet is supersonic.

- (Kearney-Fischer et al. 2011a) – A study combining acoustics, PIV, schlieren, and excitation to explore the onset and behavior of Mach wave radiation (one of the components of jet noise).

These works are largely peripherally related to the topics under discussion in the present work. Data and conclusions from these works will be incorporated into the present work where they are relevant so they are not summarized here in any more detail.
Chapter 2: BACKGROUND

The general summary of the present understanding of jet noise is as follows. The noise from a turbulent jet consists of several major components. It is well established that there exist flow structures (vortices) ranging in size from dissipation-scale to the order of the nominal exit dimension of the jet nozzle (i.e. large-scale structures). These structures are created and governed by instabilities that exist in the flow. It is generally accepted that jet noise is produced by the interaction and development, as well as the disintegration of these structures. Beyond this point, the description of noise production processes gets quite complicated. In subsonic jets, the evolution, interaction, and disintegration of flow structures produce what is known as mixing noise. A supersonic jet could also include screech and broad-band shock-associated noise. Depending on the velocity of the jet and the acoustic properties of the fluid into which the jet is exhausting, there is yet another noise component known as Mach wave radiation. Some of these noise production mechanisms are fairly well understood while others are not; the interdependence of the various mechanisms is not easily identified. While the basic governing dynamics behind screech and Mach wave radiation have been known for decades, the basic production mechanisms for mixing noise remain elusive. Theoretical approaches to the problem have provided insights, but an elegant description of a source and its relationship to the noise produced has not been obtained. Theoretical analyses, beginning with the pioneering
work of Lighthill, struggle to untangle the non-linear nature of the governing equations into a form that clearly identifies a source.

2.1 The Acoustic Analogy

Beginning with a paper published in 1952, Sir Michael James Lighthill revolutionized the study of aeroacoustics by developing a theory of sound derived from the fundamental governing equations of fluid mechanics (Lighthill 1952). This was the first of what have become known as acoustic analogies – a process by which the governing equations of the flow are transformed to describe propagating pressure waves and their relationship to some sort of source field. Before that time, studies of sound generated aerodynamically were primarily concerned with frequency. Experiments focused on showing that the frequency content of the flows was directly related to the sound produced. The theory developed in association with these experiments was concerned with explaining the production of such frequencies through flow instability. Instability theory based experiments included observing that oscillations introduced acoustically at the orifice of a jet or flame are rapidly amplified under certain conditions.

Since 1952, a number of researchers have formulated acoustic analogies, or different approaches pursuing the same goal, that are alternatives to Lighthill’s (e.g. Howe 1975; Goldstein 1984; Lilley 2003).

There are a number of assumptions and simplifications which went into Lighthill’s derivation that must be remembered when interpreting the results.

1. The evidence available from experiments of the time indicated that the sound produced is sufficiently weak to ignore any reverse process effects on the
flow unless a resonator was present in the flow to amplify the sound. It is assumed in this derivation that no reverse processes are present.

2. The effects of solid boundaries such as reflection, diffraction, absorption, and scattering are ignored. Furthermore, it is assumed that fluctuations are caused only by instabilities within the flow.

3. This derivation is restricted to subsonic flows and does not attempt to describe the additional complications created by shock structures in the flow.

Starting from the continuity and momentum equations of fluid mechanics, the equations are manipulated without any other simplifying assumptions into what is known as Lighthill’s equation:

$$\frac{\partial^2 P}{\partial t^2} - a_v^2 \nabla^2 \rho = \frac{\partial^2}{\partial x_i \partial x_j} \left\{ \rho u_i u_j - \sigma_{ij} + \left[ (p - p_0) - a_v^2 (\rho - \rho_0) \right] \delta_{ij} \right\}. \quad (2.1)$$

where $a_v$ is the speed of sound in the ambient, the subscript 0 denotes the mean quantity, $\sigma_{ij}$ is the viscous stress tensor, $\delta_{ij}$ is the Kronecker delta, and the repeated subscripts denote summation (i.e. Einstein notation). The right hand side of this equation describes the sources that give rise to the acoustic waves. This source term is often written as

$$F(\chi, t) = \frac{\partial^2}{\partial x_i \partial x_j} T_{ij}, \quad (2.2)$$

where $T_{ij}$ is known as the Lighthill stress tensor with dimensions of $\text{kg m}^{-1} \text{s}^{-2}$. Written for the pressure, the wave equation appears as

$$\frac{1}{a_v^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial^2}{\partial x_i \partial x_j} \left( \rho u_i u_j - \sigma_{ij} \right) - \frac{\partial^2 \rho_e}{\partial t^2}, \quad (2.3)$$

where $\rho_e$ is an excess density generated by changes in the entropy of the local flow field.
Using Green’s functions, it is possible to construct a solution to Lighthill’s equation. Green’s function solutions utilize the idea that the source term in a differential equation—the right hand side of (2.3)—can be thought of as a distribution of point sources (i.e. Dirac delta functions). If the differential equation is solved for one generic delta function source, that solution (known as the Green’s function solution to the differential equation) can be used to find the complete solution by integrating the product of the Green’s function solution and the source field. Green’s function solutions are a general class of solutions for boundary value problems and have specific constraints on when they can be applied. Since this derivation process is documented in previous works (Crighton et al. 1992), it is not repeated here. The solution to the acoustic pressure field in three dimensions is then

\[ p(\mathbf{x},t) = \frac{1}{4\pi} \int F \left[ y, t - \frac{|\mathbf{x} - \mathbf{y}|}{a_*} \right] d^3 y, \quad (2.4) \]

where \( F \) is the source term of (2.3), \( \mathbf{x} \) represents the observer location coordinates, and \( \mathbf{y} \) is the coordinates of the source field. It should be noted that there are a couple minor technical errors in the derivation documented in Crighton et al. (1992) resulting in a slightly different expression from the one in (2.4), but the solutions are functionally identical. One important characteristic of this solution is the property of retarded time. The source term contains dependencies on both space and time, but the time is evaluated while taking into account the time required for the energy to propagate from the source to the observer.
One additional assumption that is usually applied to the solution to Lighthill’s equation is the far-field approximation (|x| ≫ |y|). Using this assumption, there are several simplifications that can be made:

\[ |x - y| \approx |x| - \frac{x \cdot y}{|x|} \quad |x - y|^i \approx |x|^i \quad \frac{\partial}{\partial x_i} \approx \frac{-x_i}{|x|} \frac{\partial}{\partial t}. \]  

While Lighthill’s result is monumental, the source dynamics remain buried in an integro-differential equation. This has made it very difficult to determine what types of flow-field fluctuations are important to noise production. The solution to Lighthill’s equation will be invoked in Chapter 9 to discuss how the experimental results impact the understanding of jet noise sources.

### 2.2 Vortex Sound

As already stated, it is generally believed that the dynamics of vortices present in the flow are responsible for sound production. A more specific flow-noise relation is the belief that the dynamics of large vortices (i.e. large-scale structures) are responsible for the dominant sound radiated to low angles relative to the jet downstream axis. It has been speculated that dynamically significant events, as opposed to the mere existence of the structures, (e.g. the collision/interaction of two vortices or breakup of a vortex) are the mechanism that produces the noise radiated to the low angles. Thus the research on vortex sound has a close relationship to research on jet noise. Vortex sound theory, originally developed by Powell (1964), and refined by Möhring (1978) and others, was first experimentally tested by Kambe et al. (1983; 1986; 2010). The experimental work of Kambe et al. shows that the theory does a reasonably good job of predicting the noise generated by the head-on collision of two vortex rings. The sound pressure signal \( p \)
produced by these collisions appears as a primary peak accompanied by two lower amplitude side-lobes. Their work has also shown that these collisions produce a primarily quadrupole sound field whose amplitude scales as

\[ p \propto \frac{U^4}{r}, \]

where \( U \) is the collision speed of the vortices and \( r \) is the radial distance to the observation point. In terms of intensity, these collision noise events follow the same \( U^8 \) scaling as derived by Lighthill for jet noise. Additionally, they revealed that these events have a non-dimensional characteristic lifetime that scales as \( R_o/U \), where \( R_o \) is the initial radius of the vortex ring.

Another important example of experimental work on vortex sound is the research of Schram et al. (2004; 2005). This work focuses on the sound generated by ring vortex pairing events in low-speed jets (\( U_j \approx 5 \) & 34 m/s). Using a combination of excitation, axisymmetric geometry assumptions, and PIV, they were able to calculate the integrated source term from the velocity field and, therefore, calculate the acoustic pressure fluctuation produced by the pairing. By examining two different jet velocities, they observed that the structure of the acoustic pulse became more complex with increasing velocity. In the 5 m/s jet, vortex pairing produced an acoustic pulse that was essentially a single prominent peak with weak side-lobes of opposite sign. In the 34 m/s jet, pairing resulted in an acoustic pulse with two prominent peaks, one positive and one negative, with an additional weaker lobe on each side of the primary pulse.

Although much of the experimental work has focused solely on the head-on collision of two vortex rings, the sound produced by this well-studied type of vortex collision is a
passable model for the kinds of sound events created by various unsteady vortex processes. There have also been theoretical and simulation-based works looking at a wider variety of vortex phenomena and their sound production (e.g. Ko et al. 1999; Fedorchenko 2000; Tang and Ko 2000; Ran and Colonius 2009). This is an ongoing and active area of research and its precise relevance to jet noise is still unclear.

If it is accepted, however, that dynamics of large-scale structures such as interaction and disintegration (or vortex collision/disintegration) are the dominant mechanism of sound production in a jet, then the theory of vortex sound dictates several aspects of the acoustic signal. The most important aspect is intermittence. In this model, the sound source is not continuous and this means that spectral analysis is likely masking important characteristics. Additionally, while the rate of vortex creation in the jet can be thought of as being approximately periodic (at the jet column natural Strouhal number of $St_D \approx 0.3$), the timing of vortex collision/disintegration processes would be only loosely dictated by the creation timing. This is another area where spectral analysis may have trouble since by design it has a preference for periodic signals. The last area of relevance is the shape of the radiated sound wave and its relationship to the sources; specifically the implications for wave-packet models of sound sources. A good discussion of the history and development of wave-packet models is given by Obrist (2009). Almost all existing wave-packet models are based on simple travelling waves that produce far-field sound signals that are highly periodic and whose characteristic lifetimes (the reader can think of time between zero crossings as a metric of the lifetime for the present purposes) are on the same order as the period of the travelling source. These source periods are often modeled as being driven by the large-scale structure periodicity (i.e. dictated by $St_D \approx$
0.3. Within the framework of vortex sound describing jet noise, the characteristic shape and lifetime of the noise events would have little relation to the period between large-scale structures and a completely new source model would be needed.

2.3 Flow Control

Experiments on controlling the development of the jet plume have been going on for almost as long as jets have been in use. Applications for flow control cover a wide range of topics, but the most heavily studied are noise reduction and mixing enhancement. Previous studies of jet flow control include both passive (geometrical modifications of the nozzle such as chevrons, lobed nozzles, etc.) (e.g. Samimy et al. 1993; Zaman et al. 1994; Kim and Samimy 1999; Saiyed et al. 2003; Callender et al. 2004; Viswanathan 2005) and active (can be turned off to eliminate performance penalties when unneeded) control techniques (e.g. Crow and Champagne 1971; Kibens 1980; Zaman and Hussain 1980, 1981; Gutmark and Ho 1983; Ho and Huerre 1984; Cohen and Wygnanski 1987).

Jets have several instabilities which have been well researched in low-speed and low Reynolds number jets (e.g. Crow and Champagne 1971; Kibens 1980; Zaman and Hussain 1980, 1981; Gutmark and Ho 1983; Ho and Huerre 1984; Michalke 1984; Cohen and Wygnanski 1987; Monkewitz et al. 1990; Jendoubi and Sturkowskii 1994; Lesshafft et al. 2010). These instabilities are: the jet initial shear layer instability, the jet column or jet preferred mode instability, instability related to significant density gradients in the jet, and, in the case of an axisymmetric jet, the azimuthal component of instability. The initial shear layer instability is a Kelvin-Helmholtz type instability created by the presence of an inflection point in the velocity profile of the jet. As is well known, this type of instability
amplifies perturbations over a range of frequencies based on the relevant velocities ($U_j$ in jets) and the cross-sectional length scale of the shear layer (e.g. Greitzer et al. 2004 pp. 279-347). This mechanism causes small perturbations in the flow to roll up into large structures (i.e. vortices), even in high speed flows (e.g. Crow and Champagne 1971). As the shear layer around the jet grows, it eventually intersects itself (the region contained within the shear layer is known as the jet potential core). This point of intersection is unstable and is known as the jet column instability. The jet column instability amplifies a range of frequencies depending on the same velocities as the initial shear layer, but the spatial scale is the diameter of the jet. This instability has a preferred frequency, known as the jet column natural frequency, of $St_D \approx 0.3$. In an axisymmetric jet, these instabilities have an additional degree of freedom for their development. The azimuthal development, therefore, is another component in the instability processes in the jet. In shear layers with disparate densities on the two sides of the shear layer, additional instability phenomena can occur (e.g. Monkewitz et al. 1990). This instability, which is distinct from Rayleigh-Taylor instability, depends on the density change across the shear layer and has characteristics that are qualitatively similar to the initial shear layer instability. More information on these instabilities and how they are relevant to the control of jets can be found in previous works (Samimy et al. 2007a; Samimy et al. 2007b; Kearney-Fischer et al. 2009; Samimy et al. 2010).

In experimental work simulating practical conditions, as the Reynolds number of the jet increases for a given jet diameter and dynamic viscosity, so does the background noise, the instability frequencies, and the flow momentum. To operate in this environment, active control devices (actuators) must provide excitation signals of
increasingly large amplitude and frequency. As a result, the limited works in the active control of high-speed and high Reynolds number jets used acoustic excitation to control high subsonic jets around their preferred mode (Jubelin 1980; Lu 1983; Ahuja and Blakney 1985). There have been a few experiments which used active control in supersonic jets (e.g. Troutt and McLaughlin 1982; Morrison 1983; Ahuja and Blakney 1985; Ahuja and Whiffin 1985; Lepicovsky et al. 1985a; Lepicovsky et al. 1985b). The most relevant conclusion of these works is that both subsonic and supersonic jets support the existence of large turbulence structures; even in the presence of shocks. As described in more detail in the works cited (as well as the sources cited within those works), a large turbulent structure is a vortex whose size is on the order of the exit dimensions of the jet. Additionally, exciting supersonic jets alters their acoustic signatures in more complex ways than in subsonic jets. Unfortunately, acoustic drivers lose control authority in these highly energetic jets at Reynolds numbers (typically control authority wanes for Reynolds numbers greater than 100,000) which are too low to explore the dynamics of high speed and high Reynolds number jets used most commonly in application (i.e. jet engines). The loss of control authority occurs because a more energetic flow requires a greater input of energy to perturb the flow. Consequently, a new type of actuator is required, which can operate in this environment. For additional information on flow control mechanisms, see the following previous works from GDTL (Samimy et al. 2007a; Samimy et al. 2007b; Samimy et al. 2010).

The Gas Dynamics and Turbulence Laboratory (GDTL) has developed a class of plasma actuators, called localized arc filament plasma actuators (LAFPAs) that can provide excitation signals of high amplitude and high frequency for high-speed and high
Reynolds number flow control (Samimy et al. 2004; Kim et al. 2010). These actuators work by exciting the instabilities that exist in the jet. While plasmas have become a popular means of active flow control in recent years, LAFPAs utilize a different plasma formation process from those used in the past. For example, (McLaughlin et al. 1975; Benard et al. 2008) used glow discharge and dielectric barrier discharge, respectively. These previous plasma types suffer from many of the same control limitations as acoustic drivers. GDTL has used these actuators for noise and flow control studies in both subsonic and supersonic unheated jets (e.g. Samimy et al. 2007a; Samimy et al. 2007b; Kim et al. 2009; Samimy et al. 2010). More recently, GDTL started exploring the impact of jet temperature on the effectiveness of LAFPAs. The effect of heating on LAFPA performance for both mixing enhancement and noise mitigation is currently under investigation in both subsonic and supersonic jets (e.g. Kearney-Fischer et al. 2009, 2010; Kearney-Fischer and Samimy 2010; Kearney-Fischer et al. 2011b). The work documented in these publications is discussed as it is relevant in the following chapters.

2.4 Traditional Analysis of Jet Noise

A cartoon of the current understanding of jet mixing noise is shown in Figure 2.1. This figure is reproduced from Tam et al. (2008). The explanation of this cartoon is as follows. While there are a whole range of turbulence scales ranging from the diameter of the jet down to dissipation, they can be grouped into two categories: small and large. The large-scale structures produce noise that radiates preferentially downstream in fairly well organized wavefronts – presumably because their significant physical size gives those structures preferred orientations and directions of motion. The small-scale structures,
being much more isotropic in nature and distribution, produce noise that radiates to the sideline and upstream angles. In the present work, a large-scale structure is defined as the complete set of dynamical behaviors associated with a vortex that may or may not produce noise.

Fourier spectrum analysis (known as Fourier analysis, spectral analysis, frequency analysis, and many other names in the literature) is a powerful tool used throughout the scientific world for signal analysis of all kinds. This approach has contributed to many of the currently understood aspects of jet noise including, but not limited to: directivity, scaling with size and velocity, and identifying different types of noise sources. Consequently, this tool is a staple of the aeroacoustics community and a benchmark for any new theory or analysis technique. One example utilizing spectral analysis is Viswanathan’s work on scaling (Viswanathan 2006) that incorporates temperature and
directivity effects into an empirical model for noise scaling. Another example of the power of spectral analysis is Tam’s development of the two noise-source model (Tam et al. 1996; Viswanathan 2002) which shows that the mixing noise in jets can be described by two types of source spectra that are superposed with different weights at different polar angles.

While there is still debate about the implications of this empirical model, it is generally accepted that it does do a good job of representing the shapes of the spectra in mixing noise dominated jets. It is, therefore, a good basis for describing the current understanding of jet noise produced by spectral analysis. The two types of spectra, known as the Large-Scale Similarity (LSS) spectrum and the Fine-Scale Similarity (FSS) spectrum, are shown in Figure 2.2. The LSS is representative of the noise radiated to low-angles while the FSS is representative of noise radiated to sideline or upstream angles.

Figure 2.2: Tam’s similarity spectra.
For comparison purposes, the spectral peaks are aligned, but the LSS generally has a higher amplitude peak at a lower frequency than the FSS (see §6.1 or §7.1 for quantitative examples). It can be seen that the primary distinction between the LSS and FSS is the degree of “peakyness.” The FSS is a generally flatter spectrum, while the LSS has a relatively well defined peak.

Correlation analysis is another powerful tool that has contributed to many of the same areas of understanding of jet noise as spectral analysis (e.g. Hileman et al. 2004; Panda et al. 2005; Bogey and Bailly 2007; Ukeiley et al. 2007; Tam et al. 2008). Studies utilizing this tool are looking for relationships between different variables (e.g. velocity and density) or the same variables but at different regions of interest in a flow (e.g. near-field to far-field or flow-field to far-field). Typically, signals from different points in space are correlated to look at the relationships between those locations in terms of propagation or directivity. Many of these studies use correlation analysis to locate noise sources in space and/or time in an attempt to link the result (i.e. acoustic radiation) back to the cause (i.e. the turbulent dynamics that produce the noise).

The papers cited above are but a few examples of the enormous body of work that has been generated in the decades of research on jet noise. These examples were chosen simply to provide concrete instances of different utilizations of these tools. For many hundreds more examples of works utilizing these tools in the pursuit of understanding jet noise, the reader is referred to any of the several review papers that have been written on the subject (Ffowcs Williams 1977; Crighton 1981b, a; Tam 1998; Jordan and Gervais 2008).
2.5 Superdirective Radiation

The concept of superdirective radiation in jet noise, originally discussed by Crighton and Huerre (1990), describes the radiation pattern from a noise source that can’t be decomposed into a finite series of multipoles. This is currently a popular area of research for explaining the noise radiated to low angles (e.g. Obrist 2009) and is intimately related to wave-packet models for noise sources. The basic construction of these studies is to describe the noise source as a wave-packet whose oscillations and spatial extent are determined by the large-scale structures in the flow and it has been shown mathematically that such a source can produce noise that is radiated exclusively to the low angles (e.g. Cavalieri et al. 2011a).

A simple instance of this noise source model is

\[
T_{11}(y, \tau) \propto D^2 \delta(y_2) \delta(y_3) \exp\left[i(\omega \tau - k y_1)\right]\exp\left[-y_1^2/\lambda^2\right].
\]  

(2.7)

In this case, only one component of the Lighthill stress tensor is non-trivial and this term is compact in two of the three spatial dimensions. In these simple source models of low-angle jet noise, it is acceptable to retain only the \(T_{11}\) component because it can be shown—see §9.1 or Crighton (1975)—that the radiation to the far-field comes from the component of the Lighthill stress tensor aligned with the radiation direction. The coordinate \(y_1\) is the axial coordinate aligned with the jet downstream axis. The factor of \(D^2\) counters the dimensions of the delta functions so that (2.7) is dimensionless. Since the Lighthill stress tensor has dimensions (§2.1), a numerical analysis would scale the expression in (2.7) by a characteristic density and two characteristic velocities to arrive at
the appropriate units as might be inferred from (2.1). Using the far-field approximation, the solution to Lighthill’s equation is

\[ p(x, t) = \frac{x_j}{4\pi|x|^3} \frac{\partial^2}{\partial t^2} \int_{-\infty}^{\infty} T_j \left[ y, t - (|x| - |y| \cos \phi) / a_x \right] d^3y. \quad (2.8) \]

Plugging in the source model (2.7) and solving for the pressure yields

\[ p(x, t) \propto -\frac{\lambda D^2 \omega^2 \cos^2 \phi}{4\sqrt{\pi} \lambda^2} \exp \left[ -\frac{k^2 \lambda^2}{4} \left( 1 - \frac{\omega}{a_x k} \cos \phi \right)^2 \right] \exp \left[ i\omega(t - |x|/a_x) \right]. \quad (2.9) \]

In this kind of model, it is assumed that only the real part of (2.9) is representative of the noise – the imaginary part is extraneous and only appears because it is convenient to work in complex numbers. This result has an exponential directivity factor that can’t be described by a multipole expansion and that concentrates the radiated energy into the low angles. One important observation made by Crighton and Huerre (1990) is that the integral scale of the acoustic source is of order \( \lambda^2 \) which allows very small envelopes, relative to the wavelengths being emitted, to support superdirective properties. In subsequent chapters, the wave-packet and resulting far-field pressure discussed here are considered to be canonical examples of the use of wave-packets for describing jet noise.

2.6 Temporally Localized Signal Analysis

Fourier analysis utilizes oscillating signals with infinite extent and describes the examined signal in terms of those oscillations. Therefore it is not particularly useful for characterizing localized events.

Even without reference to the preceding discussion on the efficacy of vortex sound as framework for understanding jet noise, a few researchers have chosen to look at the
acoustic far-field with the assumption that at least some of the sources of noise are intermittent (e.g. Juve et al. 1980; Guj et al. 2003; Hileman et al. 2005; Kastner et al. 2009; Cavalieri et al. 2010; Grassucci et al. 2010; Kœnig et al. 2010; Cavalieri et al. 2011a; Low et al. 2011). A common theme that unifies these works is the use of wavelet analysis – the underlying principle of which is that the signals under examination cannot be adequately described by a set of periodic waves. This assumption of intermittence has produced some interesting results.

In the case of the previous works at GDTL (Hileman et al. 2005; Kastner et al. 2009), the assumption of intermittence provides a basis for a source localization method. A wavelet transform of the far-field signal showed that there were spikes in the signal. Based on this observation, noise events were defined as spikes rising above a specified threshold in the time domain. The localization method used a microphone array, and the times of arrival at the various microphones of these spikes to locate the source of an event in space-time. As discussed in those works, the calculated region of noise sources agrees with other research indicating that the noise radiated at low angles (~30°) relative to the jet downstream axis comes from an area near the end of the potential core. Simultaneous flow-visualizations using a MHz-rate imaging system showed that these events are associated with dynamically significant behavior of the large-scale structures. It was also shown that when the amplitude of the events was artificially reduced via signal post-processing, the amplitude of the spectral peak could be reduced by several decibels. Guj et al. (2003) used a similar kind of conditional averaging of the flow-field to determine that bursts of noise were related to dynamically significant fluctuations of the large-scale
structures. They also called attention to the limitations of Fourier analysis to illuminate this kind of phenomenon.

Cavalieri et al. (2010) looked at the direct numerical simulations (DNS) of an uncontrolled and an optimally noise-controlled two-dimensional mixing layer of Wei and Freund (2005). They showed that the optimally controlled case accomplishes noise reduction by suppressing certain intermittent peaks in the signal – highlighting the need to include intermittency in sound prediction schemes. Noise suppression in this simulation was related to preventing the merger of three vortices (a triple-merger) that was shown to produce a large spike in the acoustic signal. Cavalieri et al. (2011a) discuss a wave-packet model in which the envelope function varies in both space and time. This analysis, which follows the idea originally suggested by Kastner et al. (2006), shows that a high amplitude event (i.e. a pressure spike) can be produced when the wave-packet is truncated by fluctuations in the envelope. Grassucci et al. (2010) used a wavelet domain filter to separate near-field pressure fluctuations into intermittent and non-intermittent signals. They then related the intermittent signal to velocity fluctuations in the jet using Linear Stochastic Estimation (LSE). While this work is preliminary, their initial results are promising. Kœnig et al. (2010; 2011a; 2011b) have started using wavelet transforms and filtering in the wavelet domain to isolate these intermittent events for study. This analysis uses a 4\textsuperscript{th} order Paul wavelet to decompose the signals with a continuous wavelet transform. Results to date have mainly focused on the relationship of the resulting directivity patterns to wave packet models for jet noise. One important observation in Kœnig et al. (2011a) is that using Helmholtz number can achieve better spectral collapse for varying acoustic Mach number than Strouhal number scaling in unheated jets. This
scaling with Helmholtz number has also been seen by other researchers (e.g. Tanna 1977) and is further discussed in §6.2. Low et al. (2011) uses wavelet filtering and correlation on both near and far-field to determine how the near-field events are related to the far-field events. Their work is still preliminary, but warrants mentioning as a significant attempt to trace the intermittent aspects of jet noise back toward their sources.

These works show that jet noise does indeed contain intermittent events and that these events play a significant role in the overall acoustic picture of the jet. The results to date, however, are quite limited in their description of these intermittent events. Issues such as the importance of these events to the total signal spectra, many aspects of the nature of these events (lifetimes, frequency of occurrence, etc.), and exact relationship of these events to the flow-field dynamics remain to be determined.
Chapter 3: Noise Event Definition and Signal Extraction

The hypothesis of this analysis is as follows.

**Hypothesis:** The primary noise sources in a mixing noise dominated jet (i.e. acoustically subsonic), at least those that radiate to shallow angles relative to the jet downstream axis, are intermittent “events” with periods of relative silence in between.

Therefore, a post-processing routine that highlights these events while suppressing other signal components can shed light on the relevant dynamics of jet noise. While some of the previous work of other researchers lends support to this premise, only the work of Koenig et al. (see 2011a and others) has made the assertion that these events constitute the dominant feature of jet mixing noise. The validity of this premise will be investigated once the analysis process has been explained.

In the absence of a theoretical basis, this analysis requires an ad hoc definition of an event. In this analysis, an event is defined as a portion of the signal whose peak exceeds $\pm 1.5p_{RMS}$ where the RMS pressure is a unique value for every microphone (dictated by its position in the acoustic field) and the jet operating condition. This definition is chosen because it is consistent with the previous work at GDTL (Hileman et al. 2005; Kastner et al. 2009). While the definition of Koenig et al. (2011a) uses an arbitrary threshold in the wavelet domain, the current approach applies the thresholding in the time domain. The
complementary nature of these two approaches is beneficial, primarily because it should be useful for comparison of the two results for both contrast and confirmation. Other multiples of $p_{RMS}$ were explored and it was found that 1.5 was the largest threshold that sufficiently reproduced the important spectral characteristics (see §6.1). As one might expect, the distribution of amplitudes in a far-field noise signal—once normalized by $p_{RMS}$ for that signal—is the unit normal distribution (this is demonstrated in §6.2.1). The proportion of a signal contained within $z$ standard deviations is given by

$$proportion(z) = \text{erf}\left[\frac{z}{\sqrt{2}}\right],$$

(3.1)

where erf is the error function. Evaluated at $z = 1.5$, $proportion(1.5) = 0.866$ so this definition ignores about 87% of the signal in the time domain. It is important to take note of the fact that this threshold is unique for every signal. This selection criterion means that noise events are outliers with respect to the total signal in which they exist and this definition is self-consistent across any signal examined. A selection criterion based on a fixed threshold (either for the entire range of polar angles or based on a particular operating condition) would impose a definition that obviously would be incapable of accounting for scaling. Since these events might be expected to scale with properties like jet velocity and diameter, a definition that inherently scales is a superior choice. Another reason for this definition is that it is simple. Part of the motivation of this analysis is to see if the acoustic signals can be represented by a simplistic set of parameters.

A cartoon of the data extraction and fitting process for the basic types of encountered events is shown in Figure 3.1. As discussed in more detail in §6.1, the events in the figure are fitted with a Mexican hat wavelet. The wavelet reconstruction of each event is shown
as a separate curve and the final events only waveform is also plotted. Information on the events is extracted as follows:

1. Any contiguous set of points that exceeds the threshold ($\pm 1.5 p_{RMS}$) is identified as an event – this identifies five regions in the cartoon (1, 2, 3, 4a, & 4b).
2. For every event $i$, the peak amplitude ($A_i$) and temporal location ($T_i$) are identified.
3. The width of an event (i.e. its extent in time – $\delta t_i$), defined as the width at half of the peak amplitude—i.e. the Full Width at Half Maximum (FWHM), is
found by scanning outward in both directions from a given peak for the first occurrence of that criterion.

4. A check is performed to look for event overlap – events 4a and 4b in the cartoon. If the temporal extent of two or more peaks of the same sign overlaps, these events are merged into a single peak with the following properties.

   a. Width – Determined as the time between the left edge of the earliest peak and the right edge of the latest peak (time increasing from left to right) using the half-maximum criterion from the individual peaks.

   b. Peak Location – Determined as half way between the newly determined beginning and end of the merged peak.

   c. Peak Amplitude – the greatest amplitude in the merged event.

It should be noted that event location and width are determined to single sample accuracy of the discretely sampled acoustic signal and are not interpolated to a higher precision. One consequence of this data extraction method is that the minimum event width allowed is three samples. Any subsequent analysis of these quantities will also be quantized at single sample accuracy. With this information in hand, any number of analyses can be performed. In §6.2 these quantities are used to develop a statistical picture of the typical event characteristics.
Chapter 4: EXPERIMENTAL DATABASES

The experimental data for this work comes from two different databases generated in two different research facilities. The data comes from either the Aero-Acoustic Propulsion Laboratory (AAPL) at NASA Glenn or from the GDTL at Ohio State University. Analyzing data from both facilities eliminates facility biased conclusions by comparing points of overlap. Data from AAPL is used to determine scaling properties for the unexcited jet since that facility supports larger nozzle diameters than GDTL and has an archival database of a wide range of operating conditions. The GDTL facility’s ability to excite jets allows for certain jet dynamics to be exaggerated or isolated for study.

4.1 AAPL Database

The experimental database for the unexcited jet analysis is taken from the Small Hot Jet Acoustic Rig (SHJAR) at the NASA AeroAcoustic Propulsion Laboratory (AAPL). Since the data acquisition process was not directly part of the present work, the description of the experimental setup is more limited. This database, taken from a large facility validation database study (Bridges and Brown 2005; Brown and Bridges 2006), is designed to efficiently and effectively explore the various parameters that can affect subsonic jet noise. There are a total of 21 cases covering three jet diameters, five acoustic Mach numbers, and unheated as well as three elevated temperatures with Reynolds numbers (based on the jet diameter) ranging from $Re_D = 1 \times 10^5$ to $2 \times 10^6$. All three of the
converging nozzles in these experiments are axisymmetric. The exact values of these parameters are enumerated in Table 4.1. The “Case Number” will be used later in this work to refer to the different combinations of nozzle and operating conditions. Total Temperature Ratio (TTR) is the ratio of the jet stagnation temperature to the ambient temperature and the Exit Temperature Ratio (ETR) is the ratio the jet exit temperature to the ambient temperature. Unless stated otherwise, the Mach number referred to when discussing the data is the acoustic Mach number ($M_a = U_j/a_\infty$).

The SHJAR is housed in a fully anechoic geodesic dome (60-foot, 18.3 m, radius) that uses 24 inch (61 cm) long fiberglass wedges to eliminate reflections at all frequencies above 200 Hz. Compressed air (up to 150 psia, 1.03 MPa) is routed through a hydrogen gas combustor, a muffler, settling chamber, and then through a reducer and nozzle where it is exhausted through a large door to the ambient environment. The combustor produces tones as well as broadband combustion noise that complicate data analysis because they are not fully eliminated by the muffler. The facility can support flow rates up to 6 lbm/s (2.72 kg/s) with stagnation temperatures up to 1300 °F (704 °C). This facility has 24 microphones on a constant radius arc ranging from 15° to 130° relative to the jet downstream axis spaced every 5°. The arc radius is 100 inches (2.54 m).

<table>
<thead>
<tr>
<th>$D$ (cm)</th>
<th>TTR</th>
<th>$M_a$</th>
<th>ETR</th>
<th>Case Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.54</td>
<td>1.00</td>
<td>0.5, 0.6, 0.7, 0.8, 0.9</td>
<td>0.95, 0.93, 0.90, 0.87, 0.84</td>
<td>1-5</td>
</tr>
<tr>
<td>5.08</td>
<td>1.00</td>
<td>0.5, 0.6, 0.7, 0.8, 0.9</td>
<td>0.95, 0.93, 0.90, 0.87, 0.84</td>
<td>6-10</td>
</tr>
<tr>
<td></td>
<td>1.81, 1.92</td>
<td>0.5, 0.9</td>
<td>1.76</td>
<td>16-17</td>
</tr>
<tr>
<td></td>
<td>2.31, 2.43</td>
<td>0.5, 0.9</td>
<td>2.27</td>
<td>18-19</td>
</tr>
<tr>
<td></td>
<td>2.75, 2.84</td>
<td>0.5, 0.9</td>
<td>2.70</td>
<td>20-21</td>
</tr>
<tr>
<td>7.62</td>
<td>1.00</td>
<td>0.5, 0.6, 0.7, 0.8, 0.9</td>
<td>0.95, 0.93, 0.90, 0.87, 0.84</td>
<td>11-15</td>
</tr>
</tbody>
</table>

Table 4.1: AAPL experimental operating conditions.
from the nozzle exit for the 1” (2.54 cm) and 2” (5.08 cm) nozzles and 150 inches (3.81 m) for the 3” (7.62 cm) nozzle to ensure that microphones are in the acoustic far-field. All microphones are arranged for normal incidence on stands designed to minimize reflections. Data at this facility are acquired using Bruel & Kjaer model 4939 microphones and Nexus 2690 amplifiers connected using 100m cables. The output of the amplifiers are acquired at 200 kHz and low-pass filtered at 90 kHz by a DataMAX Instrumentation Recorder from RC Electronics Inc. About 8 seconds of data are acquired at each set point. Ambient conditions are monitored in real time to ensure that the properties like acoustic Mach number and ETR can be maintained. Many details such as the nozzle design and validation of the facility are available in (Bridges and Brown 2005; Brown and Bridges 2006).

4.2 GDTL Database

All of the experiments on excited jets are conducted in the anechoic chamber at GDTL within the Aeronautical and Astronautical Research Laboratories (AARL) at Ohio State University.

4.2.1 Test Facility

The jet simulation facility at GDTL is a blow-down type facility. The compressed air, supplied with three 5-stage reciprocating compressors, is filtered, dried, and stored in two cylindrical tanks with a volume of 43 m³ and pressure up to 16 MPa. The compressed air is passed through a storage heater at a set temperature to heat up the air to the desired temperature and supplied to the stagnation chamber of the jet facility with an axisymmetric nozzle. The air is discharged horizontally through the nozzle into an
The anechoic chamber and then through an exhaust system to the outdoors (Figure 4.1). The anechoic chamber in these experiments has an open volume of about 25 m³ and is rendered anechoic down to about 250 Hz using fiberglass wedges. The chamber validation is documented in Kerechanin et al. (2001). The nozzles for the experiments reported in this work are all stainless steel, axisymmetric with an exit diameter of $D = 2.54$ cm (1 in.).

The heating system is composed of a Watlow 15 kW electric heater and a heat storage tank. A diagram of this storage-based heating system is shown in Figure 4.2. This setup is referred to as an offline heating system because the electric heater is not in the path of the compressed air. It was determined that the electrical heating elements could add noise to the acoustic signature of the jet so a passive heat exchanger is used to
separate the heater. The heat storage tank is a 3.5 m (138 in) tall by 0.3 m (12 in) diameter cylinder packed with four sets of vertically aligned rows of stainless steel plates. An electric blower takes room air, passes it through the electric heater, through the heat storage tank, and discharges it outdoors. The electric heater has a maximum output temperature of 866 K (1100 °F) which produces a maximum jet stagnation temperature of \(~775\) K due to heat loss in the storage system. During experiments, pressurized air is forced through the heat storage tank to be heated before entering the jet stagnation chamber. The Mach 0.9 jet experiments can be run continuously for approximately 40 minutes with minimal temperature variation \(~0.2\) K/min. This system is limited, not by a maximum flow rate dictated by choking the flow through the heater, but by how long the storage tank can maintain a stable temperature.

4.2.2 Plasma Actuators

As discussed in §2.3, GDTL has developed a type of flow control actuator known as Localized Arc Filament Plasma Actuator (LAFPA). Each actuator consists of a pair of pin electrodes held in place using a nozzle extension. The electrodes are distributed around the nozzle perimeter, approximately 1 mm upstream of the nozzle extension exit plane. A ring groove of 0.5 mm deep and 1 mm wide is used to house the electrodes and to shield the plasma. The nozzle extension is made of boron nitride and tungsten wires of
1 mm diameter are used for electrodes. Measured center-to-center, the spacing between a pair of electrodes for each actuator is 4 mm, and the distance between the neighboring electrodes of two adjacent actuators is 6 mm. With this arrangement, eight actuators are uniformly distributed around the nozzle extension so that the azimuthal spacing between two adjacent actuators is 45°.

A multi-channel high-voltage plasma generator, designed and built in-house at the Ohio State University, is used to drive the actuators. The plasma generator enables simultaneous powering of up to eight LAFPAs distributed around the perimeter of the ceramic nozzle extension, with independent frequency, duty cycle, and phase control of individual actuators. A schematic of the LAFPA circuitry is shown in Figure 4.3. Each actuator is connected in series with a fast response, high repetition rate, high-voltage transistor switch, two approximately 15 kΩ high power solid body ceramic ballast resistors, and a high-voltage, high-current (10 kV, 1A) Glassman DC power supply. Two of these power supplies are used to energize eight actuators. If all eight actuators are
powered at the same time, the single actuator average current is limited to 0.25A. The switches are controlled using a multi-channel Transistor-Transistor Logic (TTL) output Peripheral Component Interconnect (PCI) card manufactured by the Measurement Computing Corporation and controlled by LabView software. The software generates separate pulse trains for each channel providing independent frequency, duty cycle, and phase control. The switches are capable of producing high voltage pulses (up to 10 kV) at repetition rates from 0 to 200 kHz, with a very short pulse rise/fall time (~0.1 μsec) and a variable duty cycle (from 0 to 100%). Every switch is liquid cooled to allow continuous operation at high frequencies, high voltages and high currents.

4.2.3 Diagnostic Tools

GDTL has several diagnostic systems that are used to study the jet including: two-component & three-component Particle Image Velocimetry (PIV), schlieren photography, hot-wire anemometry, near-field pressure measurement, and far-field pressure measurement.

The primary diagnostic tool used in the work discussed here is the far-field acoustic system. The far-field sound pressure level (SPL) is measured using ¼ inch Brüel & Kjaer 4939 microphones. Acoustic data are collected using a linear microphone array with ten microphones measuring angles of $\phi = 25^\circ, 30^\circ, 35^\circ, 40^\circ, 45^\circ, 50^\circ, 60^\circ, 70^\circ, 80^\circ,$ and $90^\circ$ relative to the jet downstream axis. The array axis is parallel to the jet axis and the microphones are mounted normal to the array axis. Testing confirmed that the only observable changes in spectra acquired with the microphones mounted as described, as opposed to radial (also referred to as normal) incidence, are due to the sensitivity of the
The resulting radial distances from the nozzle exit to the microphones range from \(49D\) (at 90°) to \(116D\) (at 25°). The acoustic signal from each microphone is band-pass filtered from 20 Hz to 100 kHz, amplified by one of three four-channel Bruel & Kjaer Nexus 2690 conditioning amplifiers, and acquired using National Instruments PXI-6133 A/D boards and LabView software. The microphones are calibrated using a 114 dB, 1 kHz sine wave, and the frequency response of the microphones is flat up to 80 kHz with the microphone grid cover removed. Sample signals are collected at 200 kHz with 8192 data points per sample producing a spectral resolution of 24.4 Hz. An average sound pressure level (SPL) spectrum is obtained from the root mean square of 100 short-time spectra. More information on the microphone hardware is available in (Samimy et al. 2007b).

### Table 4.2: GDTL experimental operating conditions.

<table>
<thead>
<tr>
<th>(D) (cm)</th>
<th>(M_j)</th>
<th>TTR</th>
<th>(M_a)</th>
<th>ETR</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.54</td>
<td>0.9</td>
<td>1.0</td>
<td>0.83</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>1.02</td>
<td>1.29</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1.18</td>
<td>1.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>1.32</td>
<td>2.15</td>
<td></td>
</tr>
</tbody>
</table>

**4.2.4 Experimental Parameters**

Since the goal of this work is to explore mixing noise from subsonic jets, a jet with a hydrodynamic Mach number of 0.9 at different total temperature ratios was chosen for study (Table 4.2). The Reynolds numbers of these jets range from \(Re_D = 2 \times 10^5\) to \(6 \times 10^5\). While some of these cases aren’t acoustically subsonic in the strictest sense \((M_a < 1)\), a previous work has shown that these jets do not display any strong supersonic noise characteristics (Kearney-Fischer et al. 2011a). Based on that result, it was determined
that the data from these experiments was acceptable for the analysis undertaken in the present work.

The excitation parameters used on these jets are shown in Table 4.3 – the subscript “F” signifies forced and is used to denote an excitation quantity for several variables. Previous works (e.g. Kearney-Fischer *et al.* 2011b) showed that the changes in the acoustic field of the jet were minimal outside this frequency range. The axisymmetric mode \((m = 0)\) and the 1\textsuperscript{st} order helical mode \((m = 1)\) are canonical choices for excitation based on the literature discussed in §2.3. Additionally, Suzuki and Colonius (2006) have shown that azimuthal \(m = 0\) mode growth in an unexcited jet is significantly more enhanced by heating of the jet than that of modes 1 and 2. This same phenomenon has also been reported by Hall and Glauser (2009) and previous work by GDTL (Kearney-Fischer *et al.* 2009) discusses some experimental results of this behavior with respect to LAFPAs. Previous work showed that \(m = 3\) (the highest mode accessible by the facility) produced the greatest reductions in the far-field noise levels (e.g. Kearney-Fischer *et al.* 2011b). The duty cycle of the excitation is determined from the following frequency dependent equation:

\[
\text{duty cycle(\%)} = \begin{cases} 
0.6(f_F/1000) + 2 & \text{if } f_F \leq 30,000 \\
0.29(f_F/1000) + 11.42 & \text{if } f_F > 30,000
\end{cases}
\]  

(4.1)

where \(f_F\) is the excitation frequency. The excitation parameter space (Table 4.3) is based on these observations. A spectral analysis of this data has been published in a previous
work (Kearney-Fischer et al. 2011b) and additional analysis not directly relevant to the current discussion can be found there.

4.3 Acoustic Spectral Post-Processing

In spectral analysis, it is common and often necessary to correct the spectra to remove facility and experimental dependencies to be able to compare spectra across time and among different research groups. Several corrections are applied to the raw spectra to remove facility and atmospheric dependencies. The spectra are scaled to a uniform radius of $80 \frac{r}{D}$ using $r^{-2}$ scaling. Linear propagation scaling is valid because, as enumerated in the preceding paragraph, the microphone distances place all of the microphones in the acoustic far-field (i.e. in the linear propagation domain). Due to the alignment of the microphone array, it is necessary to correct the spectra for the directional sensitivity of the microphones. Calibration data, along with the known geometry of the microphone array relative to the jet is used to correct all microphones to normal incidence. The spectra are also corrected to standard day atmospheric absorption properties as prescribed by ANSI Standard S1.26-1995 (ANSI 1995). These corrections assume that the vast majority of the noise is emitted near the jet exit compared to the propagation distance to the microphones. In the subsequent sections that utilize spectral analysis, these corrections are applied to the data unless otherwise noted.
Chapter 5: LAFPA Characteristics

This chapter discusses the behavior of the plasma actuators and the previous experimental results. The response of the jet flow-field to excitation by LAFPAs has been documented in previous works and the salient aspects are discussed here – the far-field acoustic response will be discussed directly in the sections where it is relevant.

Due to the physics of these actuators, their output has a square wave nature containing only a “positive” swing rather than the sinusoidal output of acoustic drivers. Additionally, the finite number of actuators limits the azimuthal modes available for excitation. The azimuthal modes of excitation possible for an 8 actuator configuration are $m = 0-3$, $\pm 1$, $\pm 2$, and $\pm 4$. One consequence of these arrangements is that the excitation leaks a fraction of the energy into azimuthal modes other than the one intended – this is true for any discrete actuation method (e.g. Bechert and Pfizenmaier 1977) – and also into higher frequencies. In the vast majority of cases, however, this leakage (pertaining to the development of large-scale structures) is not significant as evidenced by the flow-field results in publications on LAFPAs. For all of the azimuthal modes discussed in this work\(^1\), each actuator fires once per excitation period and each actuator imparts the same amount of energy to the flow as any other actuator. Thus, the energy imparted to the flow per period is the same regardless of azimuthal mode. The only difference is that, in the

\(^1\) In $m = \pm 1$, two of the eight actuators do not fire, but since this mode is only briefly included, it is omitted from the discussion on this page.
case of \( m = 0 \), the peak power is eight times higher since all eight actuators fire together. More information on the discharge power characteristics of the actuators can be found in (Utkin et al. 2007; Adamovich et al. 2009).

The energy imparted into the flow by these actuators manifests itself in the form of a compression wave. When an actuator is fired, the voltage between the two electrodes builds until it is sufficient to ionize the air between the electrodes. This ionization process is very rapid (on the order of 10 ns) and deposits large amounts of energy into a small volume very quickly (on the order of megawatts). This very fast deposition of energy heats the air between the electrodes, rapidly increasing the local pressure. This pocket of elevated pressure pushes out in an attempt to reestablish equilibrium – the result is a wave (referred to as a compression wave) that travels outward from the actuator. A schlieren image demonstrating the existence of these compression waves is in Figure 5.1a. In this figure, several actuators are arranged alongside a rectangular jet on a line whose axis points out of the page. Compression waves can be seen inside the jet as well.
as above and below it. The waves that pass through the jet are stretched by convection – giving them a non-circular shape. It is these compression waves that interact with the jet instabilities; seeding the jet with perturbations that can roll up into large-scale structures as discussed in §2.3. As an example, a phase-averaged schlieren image of a Mach 0.9 unheated jet excited with \( m = 1 \) and \( St_{DF} = 0.3 \) is shown in Figure 5.1b. The helical nature of the structures induced by excitation is readily apparent. It should be noted that the structures induced by this active control method have a primarily span-wise extent – in this context, span-wise means that the vortex axis is aligned with the azimuthal axis of the jet. Previous works (Samimy et al. 2010) have shown that excitation with these actuators produces regular structures over a wide range of frequencies and that the induced structure spacing is dictated by the excitation frequency.

Several previous works from GDTL have characterized the behavior of the jet flow-field response to excitation at various Mach numbers and temperatures (Samimy et al. 2007a; Kearney-Fischer et al. 2009; Kim et al. 2009; Kearney-Fischer et al. 2010; Samimy et al. 2010). To further characterize the response of the flow-field to the actuators, several figures from previous publications are reproduced and discussed here. The data in these figures are obtained from PIV data sets. The two quantities discussed here are the jet width (defined as the full width at half-maximum – FWHM) and the centerline Turbulent Kinetic Energy (TKE). The TKE is obtained from PIV data sets by calculating the mean square fluctuation magnitude of the velocity vectors relative to the mean flow-field and is normalized by the square of the jet exit velocity. These two quantities are basic metrics for the impact of excitation on the flow-field, but they also contain information about the structures induced by excitation. The FWHM and
centerline TKE, for excitation parameters covering the same range of azimuthal modes and excitation Strouhal numbers used in the present work (§4.2.4), in a $M_j = 0.9$ unheated jet are reproduced from Kim et al. (2009) in Figure 5.2. These profiles show that exciting the jet near the jet column natural frequency ($St_{DF} \approx 0.3$) increases the jet spreading (as determined by the FWHM at the end of the domain), increases the peak TKE levels, and moves the TKE saturation point closer to the nozzle exit – this is consistent with the literature. Exciting the jet at higher Strouhal numbers (roughly 0.6-1.5 depending on azimuthal mode) results in rapid early growth (as determined from the FWHM) followed by a region (about 3 jet diameters) of little or no growth and then a secondary growth period. While not directly shown here, the secondary growth onset location is associated with the end of the potential core. This implies that the structures are growing rapidly at first, and then convecting for a few jet diameters without much evolution (referred to as stable convection) before decaying or disintegrating at the end of the potential core. It should be noted that the flow-field trends observed here are consistent in the results for a supersonic jet as well as heated jets. One noteworthy difference is that, in a heated jet, this stable convection for excitation Strouhal numbers near 0.7 becomes more pronounced – particularly for $m = 0$ (Kearney-Fischer et al. 2010).
Figure 5.2: FWHM and centerline TKE for various excited cases in a Mach 0.9 unheated jet.
While not discussed here in detail, various data reduction methods in addition to the FHWM and TKE such as phase locking, proper orthogonal decomposition, and vortex identification techniques have been used to characterize the behavior of the excited large-scale structures. Based on these analyses, several conclusions were reached about the excited large-scale structure characteristics.

1. The largest structures are created by exciting the jet near the jet column natural frequency ($St_{DF} \approx 0.3$).

2. The largest structures are created by exciting mode ±1, mode 1, and then mode 0 in decreasing order of size.

3. Individual structures created by excitation are detectable from $St_{DF} \approx 0.1$ (the lowest Strouhal number ever examined) to $St_{DF} \approx 1.5$. Each firing of an actuator within this frequency range generates a structure.

4. Depending on the azimuthal mode of the excitation, the structures created by the individual actuators merge into different shapes (e.g. mode 0 merges into a ring, mode 1 into a helix, etc.). The rate at which this merger occurs depends on the azimuthal mode and also the excitation frequency.

5. Structures excited near $St_{DF} \approx 0.7$ initially grow more rapidly than those at $St_{DF} \approx 0.3$ and then convect for a few jet diameters (about 2-4) without significant development. This is particularly true of mode 0 and this behavior is accentuated at elevated temperatures.
Chapter 6: Unexcited Jet Analysis

In this chapter, the AAPL experimental database (§4.1) is analyzed using the noise event definition detailed in Chapter 3. The purpose of this exploration is to develop an understanding of the validity of the noise event definition and its characteristic dependencies with changes in jet size and operating conditions. Once this understanding is in place, it is easier to interpret the impact of excitation on the characteristics of the noise events.

6.1 Spectral Analysis

The first step in the process is to determine if the proposed definition of an event has any merit (i.e. does it capture the important aspects of the signal?). Given that spectral analysis is the standard tool of research on jet noise, it would be useful to compare the spectrum of the original signal to one that contains only the events. One method to accomplish this kind of decomposition is filtering in the wavelet domain as already discussed in the context of the work of Kœnig et al. (2010). This approach is quite involved, but results in a decomposed signal containing events with intricate shapes. If, alternatively, the raw signal was simply truncated (i.e. \( p(|p| < 1.5p_{\text{RMS}}) \) were set to zero), it would introduce a lot of high frequency content from the sharp corners at the edges of an event. Given that the event locations, amplitudes, and widths are known, it is relatively
easy to reconstruct an event decomposed signal with the appropriate choice of a model function. The model function chosen for this reconstruction is a Mexican hat wavelet

\[ \phi_j(t) = A_j \left( 1 - \frac{(t-T)^2}{\delta t^2} \right) \exp \left[ -\frac{(t-T)^2}{\delta t^2} \right], \]  

(6.1)

where \( A_j \) is the peak amplitude, \( \delta t \) is the event width (i.e. FWHM), \( T \) is the temporal location of the event peak, and \( \epsilon \) is an empirical adjustment factor to skew the width of the wavelet based on the width as determined from the data. By trial and error, it was found that a correction of 10% was needed to adjust the wavelets to satisfactorily fit the data (i.e. \( \delta t_{\text{wavelet}} = 0.9 \delta t = \epsilon^{3/2} \delta t_i \)). Therefore, \( \epsilon = 0.81 \) was set as a constant for all data processing. It should be noted that (6.1) is not the standard Mexican hat wavelet. This wavelet has weaker side-lobes resulting in a non-zero mean. The cartoon in Figure 3.1 demonstrates the fitting process. This model function was chosen because of its flexibility. If the true nature of a noise event is a single peak in isolation (event 1 in the cartoon), this function will probably fit it well unless it is highly asymmetric. If, however, the true nature of a noise event is more complex (i.e. involving multiple positive and negative swings such as the combined structure of events 2 and 3 in the cartoon), guessing an appropriate universal model function at this point is essentially impossible. Under the current definition, each positive and negative peak in a multiple swing event will be identified as a separate event and their parameters stored independently. When the events-only signal is reconstructed, the different pieces will be modeled as independent instances of the Mexican hat wavelet. Since the Mexican hat wavelet is amenable to superposition (hence its use as a wavelet), it should do a good job of representing a more complex shape (see cartoon events 2 and 3). Once the reconstructed signal is calculated,
it can be put through the same post-processing steps as the raw data for calculating the spectrum.

An example of the signal reconstruction is shown in Figure 6.1 for three polar angles ($\phi = 30^\circ$, 90°, & 130°). The abscissa is a non-dimensionalized time, $\tau_j = t \frac{D}{U_j} = 1/\text{St}_D$ (inverse Strouhal number), where $U_j$ is the jet exit velocity. The reconstruction is doing a good job, especially at the low angles, of reproducing the major aspects of the signal. At low angles, the time-domain is characterized by large slowly oscillating shapes. In contrast, the sideline and upstream angles are quite jittery (i.e. characterized by rapid
oscillations on the order of the sampling resolution). The implications of these different characteristics are discussed in more detail using the statistical analysis in §6.2.

The Sound Pressure Level (SPL) spectra for the raw data and reconstructed data at two polar angles ($\phi = 30^\circ$ & $90^\circ$) are shown in Figure 6.2 for two disparate jets and operating conditions. It is clear that, especially in the case of $30^\circ$, the reconstructed signal is doing a very good job of reproducing the important features of the spectrum (i.e. the peak location and amplitude and the shape of the spectral peak). The decreased high frequency content is expected since the reconstruction ignores the high-frequency content of the signal except for that which is imposed by the width of events. There is also an increase in the low frequencies, probably due to the basic spectral behavior of the model function which requires all frequencies, below a characteristic frequency, to represent the localized pulse.

The higher angles ($90^\circ$ being the representative sample) are reasonably well reproduced, but the spectral amplitudes are slightly over-predicted. It is worth noting that

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6_2}
\caption{Example spectra for signal reconstruction.}
\end{figure}
no corrections have been applied to these spectra. In typical spectral analysis, a microphone free-field correction, distance scaling correction, and an atmospheric absorption correction are applied to the calculated spectrum. It was found that the frequency dependent corrections, which have significant effect only at high frequencies, do not meaningfully alter the characteristics of the signal relevant to this analysis. Additionally, since the definition and subsequent analysis are in terms of $p_{RMS}$, it is not necessary to scale the data to a uniform distance.

Some additional information can be gleaned at this point by looking at the energy of the reconstructed signal ($E_R = \langle (p - p_{mean})^2 \rangle$) compared to the original ($E$). The ratio of $E_R/E$ is found to be essentially unity. Examining the various polar angles, it is found that this ratio has a range of $1 \pm 0.025$. This unity or slightly over unity reconstruction occurs because the smooth function used to model the events ends up adding some energy at low frequencies (as already discussed) which makes up for the energy eliminated from the high frequencies.

While there are many cases and polar angles that are not shown, these results are typical for all the cases studied. The low polar angles are well predicted by the event-only signal. The sideline and upstream angles are decently represented, but with the spectral amplitudes being a bit over-predicted. Without any additional analysis, these observations provide us some important insights.

1. This combination of event definition, data extraction, and signal reconstruction is capturing the vital aspects of jet noise using a simple set of equations with three parameters and a model function. It can therefore be concluded that acoustically
subsonic jet noise (at least for the low polar angles) is indeed well described by a fundamentally intermittent signal populated by localized events.

2. The noise radiated to the sideline and upstream angles has a distinct nature that is detectable in this analysis. As will be further quantified in §6.2, the educed noise events in these directions seem to have widths dictated by the sampling rate of the signal (i.e. the oscillations are too fast to be reasonably well resolved in the raw data). This kind of rapid oscillation can cause the determined event widths to be larger (typically by one sample) than the actual oscillation that they try to capture. These small amounts of added energy are likely the source of the over-predicted spectral amplitude since there are many events in the signal (in comparison to the shallow angles).

3. Based on the two preceding observations, it is concluded that the hypothesis is correct for low angles, but that sideline and upstream angles have a different nature. The exact nature of these signals will be quantified by further analysis in §6.2. The reconstruction technique is still reasonably reproducing the spectrum, but the nature of sideline and upstream noise is clearly different.

6.2 Statistical Analysis

Using the large number of events captured in a given data set for each polar angle (between 10,000 and 100,000 events depending on polar angle), it is possible to construct statistical distributions for the various quantities extracted from the data. The purpose of this section is to explore the nature of these extracted quantities through statistical analysis to determine how these quantities scale with jet diameter and operating
conditions. This is accomplished by examining the distributions of the three parameters (amplitude, location in time, and width) and determining scaling quantities for these parameters. In most of this section, analysis will be focused on 30° and 90° as representative of the two characteristic types of noise signals found in the subsonic jet.

At this point it is prudent to briefly describe two statistical distribution functions that will be used several times in the subsequent analysis. The normal (or Gaussian) distribution is very common and describes random quantities that fall symmetrically about some mean. The expression for the normal distribution is

\[
N(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi \sigma^2}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right],
\]

where \(\mu\) is the mean (and also the mode) and \(\sigma^2\) is the variance. If \(\mu = 0\) and \(\sigma = 1\), the distribution is called the standard normal or unit normal.

The other distribution of note is the gamma distribution. The gamma distribution is a statistical distribution that arises from Poisson processes. In a Poisson process, the occurrences of events of interest are independent from one another and so these occurrences (e.g. wait times) are distributed randomly about some mean. The gamma distribution is described by the following function

\[
G(x; \ell, \xi) = \frac{x^{\ell-1} \exp[-x/\xi]}{\Gamma(\ell)\xi^\ell},
\]

where \(\ell\) and \(\xi\) are known as the shape and scale parameters, respectively, \(\Gamma\) is the gamma function, and \(x\) is the quantity of interest (e.g. the wait time before the occurrence of the next event). A few of the relevant properties of the gamma distribution are: 1) the
mean occurs at $x = \ell \zeta$, 2) the mode occurs at $x = (\ell - 1)\zeta$, and 3) the variance of the distribution is $\ell \zeta^2$.

In order to look at the scaling properties of these statistics as well as their relationship to the spectra, there are a few quantities that will need to be defined. The first is a non-dimensionalized time ($\tau$). As seen in (6.4), time is non-dimensionalized using the jet diameter and either the jet exit velocity (resulting in a quantity that can be thought of as an inverse Strouhal number as already defined in §6.1) or the ambient speed of sound (in which case the quantity is an inverse Helmholtz number).

$$\tau_j = t \frac{U_j}{D} = \frac{1}{St_D} \quad \tau_\infty = t \frac{a_\infty}{D} = \frac{1}{He} \quad (6.4)$$

6.2.1 Amplitude Distributions

The first quantity to explore is the amplitude distribution of the peaks ($A_i$). In order to better understand the distribution of the peaks, it is prudent to first examine the distribution of amplitudes present in the original signal. The signal is quantized into 1000 amplitudes, normalized by the $p_{RMS}$ of that particular signal, and the Probability Density Function (PDF) is computed (note that the area under the curve of a PDF should always be one). Several examples of the resulting distributions are shown in Figure 6.3. In Figure 6.3a, the PDF of the signal from the 30° microphone is shown for all cases in Table 4.1—there are 21 curves of different colors in the figure. It is clear that, once normalized by $p_{RMS}$, the distribution of amplitudes collapses onto a single curve regardless of the jet size and operating conditions. The unit normal distribution curve is shown in Figure 6.3a as a black line. In Figure 6.3b, the PDF of the signal from all 24 microphones is shown for one jet with one operating condition along with the unit normal distribution curve.
Again, total collapse is achieved. While it isn’t possible to observe this from Figure 6.3a as rendered, it is found that lower velocity jets have greater scatter about the unit normal curve while higher velocity jets, such as the one shown Figure 6.3b, very closely hug the unit normal distribution. This change in scatter is true regardless of the jet diameter and temperature. Based on these results it is concluded, as might be expected, that the distribution of amplitudes in any acoustic signal examined for this study obeys the unit normal distribution and the only controlling parameter is $p_{RMS}$.

Looking at the event amplitudes (as defined in Chapter 3), the analysis becomes a bit more complicated. The PDF of the peak amplitudes for all 21 cases at $30^\circ$ (all the cases used in Figure 6.3a) is shown in Figure 6.4. While good collapse is once again achieved, the distribution is not quite the unit normal. As should be expected, the distribution is sharply cut off at $1.5p_{RMS}$ as a consequence of the event definition. The best fit curve, however is a normal distribution with a standard deviation of $\sigma = 1.2$. If the PDF was
determined from a signal that was simply truncated below $1.5p_{RMS}$, it would be equivalent to truncating the distribution in Figure 6.3a at 1.5 and renormalizing for unit area. The departure from the unit normal can be explained by two factors. First, this distribution examines only the peak amplitudes. Points that would contribute to the distribution (i.e. all points in the signal above $1.5p_{RMS}$) are discarded unless they happen to be a peak. This is effectively like turning the gradual rise and fall of a peak into a delta function. The result of this process is that the distribution becomes slightly skewed toward values that are more likely to be peaks (i.e. larger values). Second, when two initially distinct peaks are determined to be overlapping, the two peaks are classified as a single event and only the larger peak is kept (see data extraction step 4 in Chapter 3). This also results in a preferential selection of larger amplitudes. The final result is that the distribution of peak amplitudes for all signals examined in this study (i.e. every polar angle for all cases) is described by the normal distribution with a $\sigma$ of 1.2 and the value of $p_{RMS}$ for a given signal.
6.2.2 Width Distributions

As discussed in Chapter 3, the event width ($\delta t_i$) is defined as the Full Width at Half Maximum (FWHM). The distribution of event widths provides information about the characteristic time-scale of the flow-field phenomenon that produced the acoustic event.

6.2.2.1 Distribution at $30^\circ$

The distributions of event widths for the various cases at the $30^\circ$ microphone are shown in Figure 6.5. In the unheated jet cases, the various velocities (acoustic Mach numbers) group very tightly according to jet diameter (Figure 6.5a), indicating that the jet diameter is a representative length scale but the acoustic Mach number is not a representative velocity scale. However, upon close inspection (see Table 6.1) a trend with acoustic Mach number is detectable, but it is a weak dependence when compared to diameter and temperature variation. Given the very weak dependence on acoustic Mach
number and thus jet velocity, it can also be concluded that the convective velocity would not be a proper scaling. In the case of heated jets, the acoustic Mach number dependence becomes significant. In the elevated temperature cases, however, there is a clear trend toward larger values with increasing temperature – especially for the lower velocity. This suggests that something in the noise producing dynamics changes at elevated temperatures and this new behavior is much more sensitive to the jet velocity. Applying the obvious first choice for scaling, inverse Strouhal number ($\tau_j$), makes the distributions more disparate (see Figure 6.5b), so it is not a good choice, as it was already determined that the jet velocity is not an appropriate velocity scale.

Figure 6.5: Distribution of event widths for all cases at $\phi = 30^\circ$. 

a) PDF with time abscissa  
b) PDF with scaled time, $\tau_j$
Kœnig et al. (2011a) observe these same trends by looking at spectral collapse. They note that Helmholtz number scaling does the best job of collapsing the spectra at 30° for unheated jets of a single diameter while Strouhal number scaling does the best job for collapsing the hot jet cases. Their work to date does not include a parametric evaluation of jet diameter effects. It can be seen from Figure 6.5 that Helmholtz number scaling (lacking $U_j$ dependence) would be a superior choice to Strouhal number scaling for the event widths in unheated jets, and that Strouhal number scaling is also inappropriate for the hot jets. In a related work Cavalieri et al. (2011b) propose that the superiority of Helmholtz number scaling in the unheated jets is related to the non-compactness of the source associated with the azimuthally symmetric portion of the acoustic signal radiating to the low angles. They extend this idea to suggest that the source becomes more compact at elevated temperatures due to the disparity in the speed of sound between the jet core and the ambient. Cavalieri et al. (2011b) also note that, while the axisymmetric mode scales with Helmholtz number, the first helical mode scales with Strouhal number and observe that the axisymmetric mode is only dominant at the low angles. While the explanations proposed by Kœnig et al. are interesting and warrant further investigation, they are still speculative at this time. An alternate possibility is that the hot jet introduces additional compressibility and/or viscosity considerations. The work of Kambe et al. on vortex collisions showed that the viscous dominated aspects of the collision process are significant to the noise produced in the collision (e.g. Kambe 1984). Given the significant changes in viscosity that occur as the jet is heated, it is possible that the interaction dynamics are changing.
To a reader familiar with the gamma distribution, it should be apparent that these distributions are behaving at least somewhat like gamma distributions. Therefore, the distributions of event widths are fit with the gamma distribution and it is found that it is indeed a good fit (see Figure 6.6) – the fit parameters are enumerated in Table 6.1. Additionally in Table 6.1, the mean event width ($\delta_t$) for the various cases is enumerated and the scaled values for both $\tau_j$ and $\tau_\infty$ are calculated. While not tabulated, the product of $\ell$ and $\zeta$, the mean of the gamma distribution, is a pretty good match to the mean of the data (the maximum discrepancy is 10%). This agreement supports the statement that the gamma distribution is a good fit to the data. Detailed inspection of the distribution and best fit curves (not shown) confirms that these distributions are well described by the

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Table 6.1: Various calculated quantities for the event width distributions at $\phi = 30^\circ$.  

To a reader familiar with the gamma distribution, it should be apparent that these distributions are behaving at least somewhat like gamma distributions. Therefore, the distributions of event widths are fit with the gamma distribution and it is found that it is indeed a good fit (see Figure 6.6) – the fit parameters are enumerated in Table 6.1. Additionally in Table 6.1, the mean event width ($\delta_t$) for the various cases is enumerated and the scaled values for both $\tau_j$ and $\tau_\infty$ are calculated. While not tabulated, the product of $\ell$ and $\zeta$, the mean of the gamma distribution, is a pretty good match to the mean of the data (the maximum discrepancy is 10%). This agreement supports the statement that the gamma distribution is a good fit to the data. Detailed inspection of the distribution and best fit curves (not shown) confirms that these distributions are well described by the
gamma distribution. It can be seen from the values in Table 6.1 that neither a Strouhal number nor a Helmholtz type scaling properly collapses the data. Looking at the general trend in the average width versus jet diameter or temperature for a given acoustic Mach number, the average event width appears to be scaling in a uniform manner, but conclusions should not be drawn until the polar angle dependence is examined (§6.2.5).

To examine the relationship between the distributions and the trends in the mean width, the distribution widths were scaled by their respective means and the PDFs for all cases are shown in Figure 6.6. Additionally, a gamma distribution curve based on the averaged gamma parameters ($\ell = 5.72$ & $\xi = 0.17$) is shown as a black dashed line. It should be noted that $\xi$ must be non-dimensionalized by the mean event width for a given case before averaging. While the collapse is not quite as good as the amplitude distributions, it is still sufficient to say that the mean event width is the controlling parameter in the distribution of event widths. Once the mean value is known, the entire
distribution can be reasonably well predicted based on universal values for $\ell \approx 5.72$ and $\xi \approx 0.17$.

There are several conclusions to draw from these results.

1. The mechanism that produces these noise events is not particularly sensitive to jet velocity, but it is sensitive to diameter and temperature. Furthermore, the changes in the distributions seem to have a linear relationship to the jet diameter.

2. The agreement of the data with a gamma distribution indicates that the lifetimes or time-scales of these events are uncorrelated (i.e. independent from one another). This implies that the source mechanism of these events is sufficiently independent from one event to the next making it unique. This makes sense since the source flow-field is a highly turbulent flow.

3. There exists only one controlling parameter in the distribution (in this case the mean width). Once that parameter is known, the entire distribution is known – along with the universal values of $\ell$ and $\xi$.

If these results are interpreted within the context of vortex sound as discussed in §2.2, some interesting correlations are found. In Kambe’s work on vortex ring collisions, it was found that the characteristic lifetime of the noise generated by the collision grew as $R_o/U$, where $R_o$ is the initial radius of the vortex ring. The present results show that the characteristic lifetime (i.e. width) of the noise events in the jet are similarly scaling with jet size (or the nozzle exit diameter). Since Kambe’s work uses the collision velocity $U$, it is difficult to discuss the relationship between his experiments and present trends except to speculate that it might indicate that the collision (loosely interpreted as the interaction
of structures) speed doesn’t change significantly with increasing jet velocity in the unheated jets.

6.2.2.2 Distribution at 90°

The distributions of event widths for the various cases at the 90° microphone are shown in Figure 6.7. The marked grid-points on the abscissa are spaced at the resolution of the analysis (5 μs) – there is one data point per grid marker for these distributions. Per the definition of the event width, the minimum allowed event width is 10 μs. Regardless of operating conditions, the peak of the distribution occurs at a very short time and it is

Figure 6.7: Distribution of event widths for all cases at \( \phi = 90^\circ \).
also fairly constant. The peaks in all of the cases are separated by no more than four data points with the largest peak width being only 40 μs (eight data points). The mean event widths are enumerated in Table 6.2. The mean event widths have a slightly larger range (7 data points), but the result is quite clear. The events at the sideline and upstream angles based on the definition chosen in this work (Chapter 3) are rapid oscillations whose characteristics are dictated, not by the flow, but primarily by the sampling rate of the data.

The result here confirms and quantifies the conclusion reached in §6.1 and also agrees with the assessment of spectral analysis that the sound radiated in sideline and upstream directions has at least some of the characteristics of white noise. These events are fundamentally different from those at low angles and are very nearly delta functions with respect to the sampling rate. Furthermore, the sampling properties of the signals make a competent analysis of these events impossible – to determine if there is any significant structure in these signals would likely require increasing the sampling rates by at least a factor of 5. One last observation is that the event width distributions at 90° can also be described adequately by a gamma distribution, but this is not of any real use given the preceding discussion. Since it has been established that the event definition and

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Table 6.2: Mean event width for all cases at \( \phi = 90^\circ \).
analysis process cannot provide an informative description of sideline and upstream noise, further analysis of these radiation directions will be more restricted.

6.2.3 Joint PDF—Amplitude and Width – 30°

One might expect there to be a relationship between the amplitude and width of an event. To explore this possibility the joint PDF of the two variables is calculated for the 30° data. Based on §6.2.2.2 it is concluded that it is not useful to analyze the sideline and upstream angles. The distributions are normalized so that there is unit volume under the surface. Based on the scaling characteristics determined in §6.2.1 and §6.2.2, the distributions are presented with the event widths (\(\delta t\)) normalized by the mean event width and binned at the resolution of the data (5 μs). The peak amplitude (\(p_{\text{peak}}\)) is normalized by the RMS pressure and the data are divided into 70 bins between 1.4\(p_{\text{RMS}}\) and 3.5\(p_{\text{RMS}}\).
If the two variables are statistically independent, the combination of a given amplitude and width would simply be the product of the distributions for the two variables. Since universal distributions for these variables have already been established, it is easy to calculate what the independent joint PDF would look like. The independent joint PDF is shown in Figure 6.8 using $\ell = 6$, $\xi = 0.17$, and $\sigma = 1.2$. As expected, the most probable event width lies on a horizontal line at a value of 0.85. Another possibility is that the width and amplitude are related by some geometric relationship. A simple example of such a relationship is that of the width and height of an isosceles triangle with a constant spreading angle. In this simple example, fractional changes in width are equal to fractional changes in height (i.e. if the height doubles, the width doubles).

The joint PDFs for several cases are shown in Figure 6.9. Subfigures a-c show the variation with diameter, d-f show the variation with acoustic Mach number, g-i show the variation with ETR at $M_a = 0.5$, and j-l shown the variation with ETR at $M_a = 0.9$. There is very little change in the distribution with diameter – the peak of the distribution decreases slightly with increasing diameter. There is essentially no change in the distribution width acoustic Mach number; consistent with the conclusions from §6.2.2. Heating does produce noticeable changes. In both acoustic Mach numbers, increasing the temperature causes the distribution to elongate in the $p_{peak}$ direction (i.e. for a given width, larger amplitude events become more probable with increasing temperature) while the width dimension is relatively fixed. The elongation isn’t extreme, but it is significant.
Figure 6.9: Joint probability density functions of event amplitude and width for several operating conditions at $\phi = 30^\circ$. 

**a)** $M_a = 0.9, D = 2.54$ cm  
Case 5

**b)** $M_a = 0.9, D = 5.08$ cm  
Case 10

**c)** $M_a = 0.9, D = 7.62$ cm  
Case 15

**d)** $D = 5.08$ cm, $M_a = 0.5$  
Case 6

**e)** $D = 5.08$ cm, $M_a = 0.7$  
Case 8

**f)** $D = 5.08$ cm, $M_a = 0.9$  
Case 10

**g)** ETR = 0.85, $M_a = 0.5$  
Case 6

**h)** ETR = 1.76, $M_a = 0.5$  
Case 16

**i)** ETR = 2.70, $M_a = 0.5$  
Case 20

**j)** ETR = 0.85, $M_a = 0.9$  
Case 10

**k)** ETR = 1.76, $M_a = 0.9$  
Case 17

**l)** ETR = 2.70, $M_a = 0.9$  
Case 21
It is immediately clear that the width and the amplitude are not independent, but determining the nature of the dependence is considerably more difficult. The dashed line is the doubling line (i.e. the width doubles when the amplitude doubles) that passes through the peak of the distribution. For any given amplitude, the peak width follows the doubling line quite well across all of the cases. If the width and amplitude were deterministically linked and all events had the same spreading angle, then the distribution should tightly follow the dashed line on the figures. Since, however, there is no reasonable expectation (as seen from the broad variation in widths for any given amplitude) for all events (even of a single amplitude) to have the same spreading angle, it is concluded that the two quantities are correlated, but not deterministically. The amplitude seems to dictate a minimum width, but allows a range of widths above this. Also, the consistent match between the doubling line and the peak of the distribution suggests that there is a characteristic shape to the events that scales depending on the amount of energy in the event. One other conclusion is that the elongation of the distribution in the $p_{\text{peak}}$ direction with heating indicates that the amplitude and width are becoming less correlated.

6.2.4 Intermittence Distributions – 30°

The last characteristic of the noise event needed to complete the picture is the time between the defined noise events (the intermittence). It should be noted that this intermittence is not related to the fluid dynamics concept of turbulence “intermittency” (Pope 2000, pp.167-173). Based on the preceding discussion, only 30° results are
analyzed as the representative sample of low-angle noise. Generally, the $N^{th}$ order intermittence can be written as

$$\Delta T_{i}^{(N)} = T_{i} - T_{i-N}, \quad (6.5)$$

where $T_{i}$ is the temporal location of the $i^{th}$ noise event peak as discussed in Chapter 3. The distribution of the $N^{th}$ order intermittence is then found by calculating the histogram of the set $\{\Delta T_{i}^{(N)}\}_{i}$. As can be seen in the sample signals plotted in Figure 6.1, the true nature of these noise events often includes at least one positive and one negative swing beyond the $1.5p_{RMS}$ threshold. Since these are identified as distinct events, an indiscriminate calculation of the intermittence distribution would likely be skewed by the statistics regarding the spacing of these peak-to-peak swings. This peak-to-peak interval information is characterized by the event widths, so it is redundant to include it here. The simplest solution to this issue is to look at peaks of one sign only and this is the method that will be used. The results from looking at only positive peaks and then only negative peaks can be averaged together to conserve the statistical population size of the analysis.

Since at least some events consist of multiple positive and negative swings, it is also prudent to average over several orders of intermittence. Based on the assumption that the $N = 2$ distribution should have a characteristic value approximately twice that of the $N = 1$ distribution, the average of $M$ orders of the $N^{th}$ order distribution ($H_{j}^{(N)}$) can be calculated as

$$Y_{j} = \frac{1}{M} \sum_{N=1}^{M} NH_{jN}^{(N)}, \quad (6.6)$$
where \( j \) denotes the \( j^{\text{th}} \) bin of the distribution. The \( N^{\text{th}} \) order distribution is down-sampled by a factor of \( N \) (indicated by the subscript \( jN \)) and scaled by a factor of \( N \) to conserve the area under the curve. The down-sampled and scaled distributions can then be point-by-point averaged.

The distributions of event intermittence are shown for the 30° data in Figure 6.10. It is clear that, apart from the time scale, the distributions of event intermittence behave very similarly to the event width. The small secondary peak close to the origin is evidence of the peak-to-peak swing time scale already discussed. A relatively weak dependence on jet velocity for the unheated jets is observed, as are strong dependencies on jet diameter and temperature. In the hot jet cases, a velocity dependence is also present. This same behavior was observed in the wavelet transform analysis of Kœnig et al. (2011a). They indirectly infer a change in the intermittence through changes in the fraction of energy retained when filtering at the same threshold in the wavelet domain. Once again, the data lends itself to description by the gamma distribution.
The best fit gamma parameters as well as the mean event intermittence ($\overline{\Delta T}$) are listed in Table 6.3. The mean of the best fit gamma distribution matches the empirical mean pretty well (maximum discrepancy is 4%) and a detailed examination of the best fit curves (not shown) shows that the data are indeed well described by the gamma distribution. The degree of similarity in the trends between event width and intermittence is actually very high. If the ratio of the mean event width to the mean intermittence is calculated for all cases, the average and standard deviation of that set of ratios is $\overline{t/\Delta T} = 0.128 \pm 0.002$. This suggests that the periodicity of events and the lifetime of those events...
are strongly related. Additionally, the order of magnitude disparity in these quantities is in contravention to the sinusoidal nature of typical wave-packet models.

Following the same approach as in §6.2.2.1, the intermittence distributions are scaled by their respective means and plotted in Figure 6.11 along with a gamma distribution curve given by the average gamma parameters ($\ell = 6.0$ & $\xi = 0.17$). With the mean quantity scaled out, the distribution of the event intermittence is also seen to be universal and to be well predicted by a gamma distribution. The times between noise events are independent; that is, the occurrence of one event does not influence the occurrence of another. This conclusion was also reached by Guj et al. (2003). Again, these results are in
stark contrast to typical wave-packet models that are based on a nearly sinusoidal ansatz. Lastly, there is only one controlling parameter on the distribution (the mean) and, once known, the entire distribution may be predicted. It is also worth noting that, in addition to the link between their mean values as already discussed, the distribution of event intermittence has very similar gamma parameters to the event width distribution.

The reader may have already noticed that the mean intermittence has something important to say in relation to the spectra. As mentioned in §2.6, Koenig et al. (2011a) noted that scaling the frequency axis into Helmholtz number can achieve better spectral collapse for varying acoustic Mach number than Strouhal number scaling in an unheated jet. This implies that the spectral characteristics depend, at most, weakly on the jet velocity. Inspection of the 30° spectra on a Helmholtz number axis in Figure 6.12 reveals a fairly constant value of the peak Helmholtz numbers irrespective of acoustic Mach number for the unheated jets. This is similar to the trends observed in the event

![Figure 6.11: Intermittence distributions normalized by their respective means for data at $\phi = 30^\circ$.](image)
intermittence as well as event widths. The amplitudes of the spectral peaks in Figure 6.12 are artificially aligned. The collapse is quite similar to that observed in Kœnig et al. (2011a). The value of the mean intermittence, converted into Helmholtz number, is marked with triangles. The mean intermittence does a pretty good job of predicting the peak in the spectrum. The prediction is not quite as good for the low-speed 7.62 cm cases. This may be due to the properties of the anechoic chamber combined with the frequencies of the jet. As discussed in §4.1, the anechoic chamber is effective down to about 200 Hz. The peak frequencies of the 7.62 cm jet are close to the limits of the chamber so it is possible that the results are being skewed by reflections. In the elevated temperature cases, prediction is pretty good for the low-speed cases, but is a bit low for the high-speed cases. It is possible that combustor noise is skewing these results. Given the additional complexities of the hot jet cases, the agreement is decent. The consistency of the mean intermittence with the spectral peak indicates that the quasi-periodicity of these events is what creates the spectral peak.
6.2.5 Directivity of Mean Event Width

Analysis of the 30° and 90° microphone signals has shown that there are a few important variables in these noise events. With that information, more polar angles are examined. The mean width (\( \overline{\delta t} \)) directivity for the five acoustic Mach numbers in the unheated jet is shown in Figure 6.13a. The first thing to note is that there is very little variation in this directivity for the different acoustic Mach numbers. Apart from some

\[ a) \ D = 2.54 \text{ cm, TTR} = 1.0 \]
\[ b) \ D = 5.08 \text{ cm, TTR} = 1.0 \]
\[ c) \ D = 7.62 \text{ cm, TTR} = 1.0 \]
\[ d) \ D = 5.08 \text{ cm, elevated temperatures} \]

Figure 6.12: Spectra at \( \phi = 30^\circ \) showing the predictive capability of the mean intermittence.
subtle trends with velocity that have already been discussed in more detail in §6.2.2, these directivity patterns are essentially identical. There is some apparent variation in the upstream angles, but as already discussed, the information from these directions is of limited use due to the nature of the signals and the limitations of the data acquisition system. There is also more variation in the 15° microphone, but that might be expected given the close proximity to the jet plume. Another important observation is that beyond 55° the mean width becomes constant. This indicates (as can be confirmed by examination of the spectra) that the noise radiated to angles of 60° and higher has a similar nature. It is also found that the gamma parameters that describe the distributions follow similar trends with polar angle. The shape parameter ($\ell$) changes much less than the rate parameter ($\zeta$). The shape of the gamma distribution changes gradually with polar angle as the noise signal transitions from an intermittent nature to a more white noise type nature.

The directivity of the mean width for the three jet diameters is shown for the unheated jet and the middle acoustic Mach number ($M_a = 0.7$) in Figure 6.13b. While the change in the sideline and upstream angles is very little, it is clear that the mean width has a coupled dependence on the jet diameter and polar angle for the low-angle noise. The dependence on the jet temperature is also complex (Figure 6.13 c & d). Again there is very little change in the sideline and upstream angles. The mean width in the transition angles ($\phi = 40°$ to $60°$) changes more rapidly as the jet gets hotter. The low angles ($\phi = 15°$ to $35°$), however, have a fairly constant slope with their absolute values being dictated by the changes in the transition angles. Lastly, it is seen that a larger jet velocity (i.e. greater acoustic Mach number) significantly suppresses these trends with
temperature. It is not clear at this time how to construct an appropriate scaling scheme to account for these trends. It is likely that a much more extensive data set would be required to understand it fully.

The mean width ($\overline{\omega}$) and intermittence ($\overline{\Delta T}$) are shown vs. polar angle for case number 5 ($D = 2.54 \text{ cm}, M_a = 0.9, \text{TTR} = 1.0$) in Figure 6.14 where the mean intermittence has been scaled by 0.128 based on the conclusion reached in §6.2.4. Case 5 was chosen as a representative sample. This shows that there is a strong and consistent

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**Figure 6.13:** Directivity patterns for mean event width.
relationship between the mean event width and intermittence regardless of polar angle – further supporting the idea that the two are dynamically linked in some way. One possible explanation for this link can be found by looking at vortex sound. Vortex sound, as discussed in §2.2, has shown that the width of a noise event created by colliding vortices scales with the size of the vortices. The size of the typical vortex in the jet is dictated partly by the frequency with which they occur. Thus the frequency of occurrence (the intermittence) could be linked to the width of the noise event through the size of the typical vortex if it is found that vortex collision/disintegration processes are indeed responsible for these noise events.

Returning to the comparison of the mean intermittence to the spectral peak (Figure 6.12), it is found that, in terms of Helmholtz number, the spectral peak frequency at the low angles is fairly constant for a given jet diameter and polar angle in the unheated jet.
In contrast, the spectral peak Strouhal number for angles above about 50° is pretty constant. While there aren’t any additional figures (in the interest of brevity) showing direct comparisons for polar angles other than 30° (see Figure 6.12), the mean intermittence is a pretty good match to the spectral peak for the low angles. On the range $\phi = 15°$ to $40°$, the prediction is generally quite good. For higher polar angles, the peak frequency is consistently over-predicted, but as already discussed, this analysis does not expect to correctly predict the properties of radiation to the sideline and upstream angles.

### 6.2.6 Averaged Event Waveform

Up to this point, the data have revealed several important things about the nature of jet noise. One feature that has been notably absent is the shape of the typical noise event. While the instantaneous waveform changes, the shape obtained by averaging many events can provide some useful information. Averaging is performed on the raw data by extracting a portion of raw data around each identified event that is ten times the calculated width of that event. Note that this averaging does not utilize any wavelet fitting as discussed in §6.1. Events with negative peak amplitude are inverted. All events are aligned in time for a coincident peak and then averaged. A representative sample of this result is shown in Figure 6.15 for Case 5 ($D = 2.54$ cm, $M_a = 0.9$, TTR = 1.0). While only one example is shown, the subsequent discussion and analysis have been confirmed to be valid across all cases. This figure shows several features already identified in the previous sections (e.g. the typical event width at low angles is larger than sideline angles, angles greater than about $\phi = 50°$-$60°$ have consistent characteristics, etc.). The main piece of additional information provided by the averaged waveform is that the low angles
contain at least one statistically significant negative swing associated with the positive swing, and vice versa. Conversely, the sideline averaged waveforms have a much smaller negative swing. The shape of the average event waveform is very similar to autocorrelations of the signals (e.g. Tam et al. 2008) as might be expected.

It is important to note that this averaged waveform is not a good representation of the instantaneous signal. If the instantaneous waveform consisted of three alternating lobes, as the averaged waveform suggests, the intermittence analysis should have strongly detected a characteristic time $\Delta T$ on the order of $\delta t$. One possible waveform is an event containing one positive and one negative swing, but not necessarily in that order (referred to as a one-and-one waveform) – shown as item A or B in the cartoon in Figure 6.16. This waveform can explain many of the observed behaviors.

Figure 6.15: Averaged event waveform for several polar angles in Case 5 ($D = 2.54 \, \text{cm}, M_a = 0.9, \text{TTR} = 1.0$).
1. The one-and-one waveform is consistent with the averaged waveform. There is no way to separate the cases where the negative swing precedes the positive or vice versa without assuming something about the governing dynamics that produced the noise event (see cartoon items A and B). If, during averaging, the positive peaks of A and B are aligned and averaged, the result would be a symmetric waveform like that shown in the cartoon which matches the shape of the average waveform in Figure 6.15.

2. The existence of negative lobes in the average waveform indicates that there must be some swing of opposite sign associated with the primary peak. If the typical instantaneous event had no negative swing (e.g. Gaussian), the average waveform would not have negative lobes.

3. In the cartoon, the amplitudes of the negative lobes on the average waveform are half the amplitude of the positive lobe. In Figure 6.15, however, the secondary swing is of considerably reduced amplitude. This is an artifact of the averaging. There are events of all different widths averaged together with one enforced point of coincidence (time equals zero). The averaged waveform, therefore, becomes

Figure 6.16: Cartoon of one-and-one waveform (A or B) and the impact of averaging.
increasingly smeared with increasing distance from the enforced point of coincidence. For any given event, it is therefore possible that the negative swing would have amplitude comparable to the positive swing. In an effort to explore the impact of this averaging, the events used to create the average waveform for 30° in Figure 6.15 were scaled to remove the effect of varying amplitude and width before averaging. The time axis of each event was normalized by the event width ($\delta t$) and the peak amplitude of the event was scaled to one. The normalized events were averaged and the result is shown in Figure 6.17 along with the waveform that would result from applying the same averaging procedure to the proposed one-and-one instantaneous waveform. It is clear that, while there is a basic consistency, many of the details are different. This suggests that either the shape of the typical event is more complex than the simple model used in this section or that multiple types of events are present in the signal. The present model, however, is still acceptable for the analysis discussed here – a more complex waveform isn’t needed to discuss the basic nature of the event dynamics and, if there are multiple event types present, the presence of the negative lobes indicates that the event type under discussion must be prevalent.

4. From Table 6.1 the average event width at 30° is $\bar{\delta t} = 84 \mu s$ (which agrees well with the averaged waveform). The minimum in the averaged waveform occurs at 170 $\mu s$. If it is assumed that the instantaneous waveform is a one-and-one event, a crude model of this waveform is two adjacent isosceles triangles of equal dimensions with one pointed up and the other pointed down. In this case, the
geometry says that the distance between the two peaks is $2 \delta t$ which is a very good match to the location of the averaged waveform minimum.

Based on these findings, it is reasonable to conclude that the one-and-one waveform may be a reasonable representation of the instantaneous noise event. This could also explain portions of the signal with more than one positive and negative swing as being sequential occurrences of this basic signature. It is important, however, to stress that this model is not physically motivated – it is the simplest waveform that satisfies the statistics. The work of people like Schram et al. (2005) and Kambe et al. (1983) advocates for more complex waveforms based on the framework of vortex sound theory. Using Schram’s work (discussed more in §2.2) as an example, the inconsistencies between the one-and-one waveform and the vortex sound waveforms can be discussed. The typical acoustic pulse generated by vortex pairing in their work has two primary peaks of opposite sign accompanied by weaker lobes on either side of the primary

Figure 6.17: $\phi = 30^\circ$ events normalized by width and amplitude before averaging for Case 5 ($D = 2.54$ cm, $M_a = 0.9$, TTR = 1.0)
structure. The primary structure of their acoustic pulse is very similar to the one-and-one pulse. Between the additional complexities that arise from extrapolating Schram’s low-speed results to the high-speed jets studied here and the smearing effects of statistical averaging, it is very possible that the additional features in their characteristic acoustic pulse exist in the jet noise signal, but are not detectable in the present analysis. A definitive description of the typical instantaneous acoustic event from the jet simply not possible from the present work, but the basic characteristics gleaned from the statistics are still quite useful.

6.3 Summary of Results

Before proceeding further, it is sensible to summarize what has been learned to this point.

1. The acoustic signal radiating to the low polar angles can be well represented by intermittent bursts of noise. In this study, these bursts are defined as regions of the signal with amplitudes larger than $1.5p_{RMS}$.

2. The fundamentally distinct nature of the low-angle noise compared to sideline and upstream radiated noise is, once more, clearly revealed in this analysis. The primary conclusion is that the noise in the sideline and upstream directions is not dominated by an intermittent nature. This conclusion should be tempered, however, by the fact that the analysis was limited by the sampling rate of the signal and it is therefore possible that noise events are present in the sideline and upstream angles but are very short-lived events.
Using a model with three important parameters in the acoustic signal (peak amplitude, event width, and the time between events), the following are now known.

a. The distribution of peak amplitudes obeys a universal normal distribution across all polar angles and operating conditions once scaled by $p_{RMS}$ of the original signal. The normal distribution ($\sigma = 1.2$) is close to the unit normal but slightly skewed toward larger amplitudes as already discussed.

b. The distribution of event widths (looking only at the low polar angles) obeys a universal gamma distribution for all cases at a given polar angle once scaled by the mean event width ($\bar{d_t}$). There are small variations in the gamma parameters with polar angle, but the distributions are pretty well described by a single universal distribution for all angles below about $\phi = 50^\circ$. The fact that a gamma distribution accurately describes the event widths implies that the width of one event has no correlation to the width of other events – this result also implies that the noise sources are also independent.

c. The joint probability density functions of event width and amplitude show that there is a link between event amplitude and width that can be partially described by a simple geometric scaling.

d. The distribution of event intermittence has the same characteristics as the event width once scaled by the mean intermittence ($\bar{\Delta T}$). This
implies that the occurrence of one event is not correlated to the occurrence of preceding or subsequent events. It is also found that the universal distribution that describes the event intermittence is the same as the distribution of the event width (gamma parameters $\ell \approx 6.0$ & $\xi = 0.17$).

e. There is a consistent relationship between the mean event width and the mean intermittence $\bar{\delta}t/\bar{\Delta}T = 0.128$. This implies some sort of link between the governing dynamics controlling these two quantities.

f. The mean event intermittence is a good predictor of the spectral peak frequencies for the low angles.

g. The directional dependence of the mean quantities interacts with the dependencies on diameter and temperature. These coupled dependencies make developing a scaling law for the data quite challenging.

4. Based on these observations, the entire signal for noise radiated to the low angles can be predicted in a statistical sense with knowledge of three quantities ($p_{RMS}$, $\bar{\delta}t$, and $\bar{\Delta}T$) along with the $\sigma = 1.2$ normal distribution and the gamma distribution ($\ell \approx 6.0$ & $\xi = 0.17$). It is actually possible to eliminate one or other of the mean quantities if the relationship in point 3.e ($\bar{\delta}t/\bar{\Delta}T = 0.128$) is used.
5. The mean width and intermittence are fairly insensitive to changes in the jet velocity in an unheated jet, but are significantly dependent on diameter and jet temperature. In the hot jet, there is also velocity dependence.

6. Directivity analysis of the mean quantities confirms the dependencies already discussed, but also shows that the polar angle dependence is intertwined with the diameter and temperature dependencies.

7. The averaged event waveform suggests that the typical instantaneous noise event consists of one positive and one negative swing of comparable amplitudes and widths, but not necessarily in that order. It must also be remembered that this model for a typical instantaneous noise event is statistically, not physically motivated.

Points 1 and 2 are consistent with the observations of Kö nig et al. (2011a). Point 5 is also consistent with the work of Kö nig et al., though their discussion utilizes an indirect inference from filtering in the wavelet domain as opposed to these event quantities. It is important to note that using the mean value of the distributions is a convenient scaling – it would work equally well to use the peak of the distribution or any other descriptive quantity of the distribution. If the appropriate shape is indeed one positive swing followed by one negative swing (or vice versa) of comparable amplitude, then the characteristic lifetime of that event would be two times the event widths discussed. Based on this analysis, the conceptual picture of the low-angle jet noise signal appears as Figure 6.18.
Figure 6.18: Cartoon of low-angle jet noise signal based on the characteristics extracted from the analysis.
Chapter 7: EXCITED JET ANALYSIS

In this chapter, the impact of excitation on the noise events is analyzed using the GDTL database (§4.2). This is accomplished by first performing a spectral analysis of the data to determine how the jet responds to excitation using the traditional tool. The reason for this is to identify excitation cases that produce the greatest changes in the Sound Pressure Level (SPL) and/or the Overall Sound Pressure Level (OASPL) and then to relate those changes to changes in the noise event characteristics.

7.1 Spectral Analysis

In the previous works from GDTL on the acoustic field of an excited jet, it has been common to remove narrowband tones associated with excitation to focus on the changes in the broadband spectral shape (e.g. Kastner 2007; Kearney-Fischer et al. 2011b). This was done for two reasons: 1) there is potentially a component of the signal acquired by the DAQ system that is electronic noise associated with the plasma based excitation, and 2) the actuators emit a compression wave that is detectable at the microphones (referred to as actuator self-noise). It has been determined in previous works that these contaminants were relatively weak, but it will always be difficult to tell what portions of the narrowband tones originate in the flow-field. Given the obvious complications associated with attempting to extract the time domain signature of the electronic noise and compression waves associated with excitation without impacting the noise produced
by the jet, it will be assumed that the signal as acquired is valid and no attempt to condition the data will be made.

7.1.1 Unexcited Spectra

The first issue to examine is how the energy is distributed across the polar angles as a function of Strouhal number. This is done on the GDTL database to provide a basis for interpreting the changes in the acoustic field created by excitation. Figure 7.1 shows plots of SPL (in dB) for the baseline (no control) jet for two of the four temperature ratios experimented – the others are omitted due to their general similarity. The important observations from Figure 7.1 are:

1. The Strouhal number of the peak SPL is fairly constant - decreasing by a small amount as the temperature rises (from 0.18 at TTR = 1.0 to 0.15 at TTR = 2.5).
2. The ridgeline formed by the peak SPL through the range of polar angles has a similar shape as the temperature changes.
3. The peak noise amplitude increases significantly with the temperature, as expected for a constant hydrodynamic Mach number jet.

A better sense for the variations in spectral shape can be obtained by examining the 90° and 30° data. The spectra for these two polar angles are shown in Figure 7.2. For visual clarity the narrowband data were integrated into 1/3 octave bands for comparison to Tam’s similarity spectra (Tam et al. 1996). Additionally, the amplitudes of the spectral peaks are artificially aligned to focus on shape changes.

It is apparent from the results presented in Figure 7.2a that shape of the spectrum at a 90° radiation angle does not change significantly over the range of temperatures experimented here as demonstrated by the high degree of similarity in the data. The lack of the characteristic hump in the 90° spectra, as observed in Viswanathan (2004), indicates no observable low Reynolds number effects even though the Reynolds number drops from about $6 \times 10^5$ at ambient temperature to about $2 \times 10^5$ at a temperature ratio of
2.5. Viswanathan (2004) noted that jets with a Reynolds number above about $4 \times 10^5$ have similar characteristics, but that going below this Reynolds number could result in significant disparities in the spectral characteristics. The reason for the lack of Reynolds number effect even with this relatively low Reynolds number is thought to be the existence of a 20 mm straight section in the nozzle geometry (for holding the actuators) following the contraction section that provides relaxation to the accelerating boundary layer within the nozzle. This relaxation allows the boundary layer to reacquire a turbulent behavior following the favorable pressure gradient in the nozzle as discussed in Kearney-Fischer et al. (2009). Additionally, as observed by Viswanathan (2002), the fine-scale similarity spectrum fits the data at this radiation angle well (within about 2-3 dB) over all the temperatures examined.

The consistency of the baseline spectra with the observations of other researchers is further supported by the 30° spectra shown in Figure 7.2b. To avoid clutter, the large-scale similarity spectrum is shown only for the unheated case, but it is clear that the data are a good fit to the similarity spectra. While it is immediately apparent that heated jets have a faster high-frequency roll-off than the unheated jet, it may also be observed that this roll-off rate is continually increasing as a function of temperature, but that the rate of increase slows. A more subtle change is the slight increase in the range of frequencies over which peak radiation occurs.

7.1.2 Excited Narrowband Spectra

Sample narrowband spectra are presented to provide a sense for the influence of excitation on various parts of the spectrum. The excitation parameters used in the next
two figures were chosen to reinforce the interpretation of results which are discussed later.

The narrowband spectra for azimuthal modes 0 and 3, temperature ratios of 1.0 and 2.5, and polar angles 30° and 90° are shown in Figure 7.3 for an excitation Strouhal number of $St_{DF} = 0.35$. Mode 1 is omitted because the far-field response to excitation is qualitatively similar regardless of azimuthal mode. This $St_{DF}$ is close to the jet column natural frequency. Excitation at this frequency generates very large coherent structures in the unheated jet (Kim et al. 2009). The excitation Strouhal number is marked with triangles at the top and bottom of the graph. There are several things apparent in Figure 7.3.

1. The reduction/amplification characteristics depend on temperature, azimuthal mode, and polar angle.

2. The excitation tone amplitude of $m = 0$ at 30° increases dramatically at elevated temperatures while the increase at 90° is much less substantial. The
changes in the $m = 3$ excitation tone amplitude are similar in nature, but diminished in magnitude compared to $m = 0$.

At a temperature ratio of 1.0, there is a very slight reduction in low frequencies for $m = 3$ accompanied by amplification at high frequencies. At the same temperature, $m = 0$ has high frequency amplification and lacks low frequency reduction. At the elevated temperature, $m = 3$ has more pronounced reduction at $30^\circ$ (as compared to $m = 0$) and some reduction at low frequencies coupled with amplification in the high frequencies at $90^\circ$. In contrast $m = 0$ has only amplification at $90^\circ$.

Narrowband spectra are shown in Figure 7.4 for the excitation frequency that achieves the largest decrease in OASPL at $30^\circ$. At this $St_{DF}$, no observable coherent structures are generated in the unheated jet (Kim et al. 2009). One important thing to note is that the excitation frequency of maximum reduction is different for the two azimuthal modes at elevated temperatures. Thus there are three excitation frequencies in Figure 7.4 denoted by the triangular markers at the top and bottom of the figure: $St_{DF} = 2.00$ for both $\text{a)} 90^\circ$ narrowband spectra and $\text{b)} 30^\circ$ narrowband spectra.

Figure 7.4: Narrowband spectra at $90^\circ$ and $30^\circ$ for maximum reduction frequency at $30^\circ$. 

Narrowband spectra are shown in Figure 7.4 for the excitation frequency that achieves the largest decrease in OASPL at $30^\circ$. At this $St_{DF}$, no observable coherent structures are generated in the unheated jet (Kim et al. 2009). One important thing to note is that the excitation frequency of maximum reduction is different for the two azimuthal modes at elevated temperatures. Thus there are three excitation frequencies in Figure 7.4 denoted by the triangular markers at the top and bottom of the figure: $St_{DF} = 2.00$ for both $\text{a)} 90^\circ$ narrowband spectra and $\text{b)} 30^\circ$ narrowband spectra.
modes at TTR = 1.0, $St_{DF} = 1.32$ for $m = 0$ at TTR = 2.5, and $St_{DF} = 0.53$ for $m = 3$ at TTR = 2.5. The levels of reduction at $30^\circ$ in $m = 0$ excitation are comparable in both temperature ratios and the high frequency amplification at $90^\circ$ in the elevated temperature case is notably larger. For $m = 3$, the reduction at $30^\circ$ is dramatically improved by the elevated temperature, but at $90^\circ$, while there is some increase in low frequency noise reduction, the high frequency amplification is much more pronounced. Lastly, the excitation frequency which achieves maximum reduction decreases for both modes as the temperature rises, but the change is much more pronounced in mode 3 excitation.

These results clearly show that there are several parameters involved in changes in the acoustic characteristics induced by excitation: azimuthal mode, radiation direction, excitation frequency, and temperature ratio. At the elevated temperature, excitation adds or removes more than 5 dB from large portions of the broadband spectrum. At this point, many aspects of the excitation effect including the contribution of the narrowband excitation tones on the total energy of the spectrum remain to be discussed. Due to the large size of the parameter space, the following subsections will analyze the effects of various parameters using methodologies which allow for the efficient presentation and interpretation of the experimental results.

### 7.1.3 Changes in OASPL

The changes in OASPL for the angular domain as a function of excitation Strouhal number are presented and discussed to determine how the excitation parameters impact the noise. The $\Delta$OASPL (in dB) maps ($OASPL - OASPL_{BASELINE}$) for modes 0, 1, and 3
at the various temperatures are shown in Figure 7.5, Figure 7.6, and Figure 7.7 respectively. The discussion of these results will focus on one temperature at a time.

In the unheated cases, there is some consistency in the reductions observed at the small angles and the general amplification over most of the angles and excitation Strouhal numbers, but the generally erratic behavior indicates that the energy of the narrowband tones is playing a significant role in the measured OASPL. Furthermore, the variation in the energy of the narrowband tones as a function of the excitation frequency makes it difficult to discern any trends in the data. The narrowband spectra in Figure 7.4
showed that the broadband jet noise is being altered, but the complex nature of these \( \Delta OASPL \) maps makes interpretation almost impossible.

The results at the elevated temperatures show that the data become much more organized as the temperature increases. In mode 0, there is a strong amplification region at the low excitation frequencies and low angles. In contrast, modes 1 and 3 have no such strong amplification region. These results show that the tones observed in the narrowband spectra of Figure 7.4 can have a significant impact on the integrated result and that the tones can have a significant contribution that comes from the flow-field. These results...
clearly show that by proper selection of $St_{DF}$ and excitation azimuthal mode, one can obtain noise reduction across all measured radiation angles for temperature ratios $\geq 2.0$.

A more detailed examination of the $\Delta OASPL$ maps yields the following observations – most of the conclusions are withheld until the time-domain statistics have been analyzed (§7.2).

1. The far-field noise signature of the unheated jet seems to contain a significant amount of actuator self-noise that obscures trends in the OASPL. This diminishes as the jet is heated because the jet gets louder.
2. In the elevated temperatures, $m = 0$ develops a strong increase in OASPL at the low angles ($\phi < 50^\circ$) for excitation Strouhal numbers near jet column natural frequency ($St_D \approx 0.3$). Close inspection reveals that the greatest increase occurs at the excitation frequency of $St_{DF} = 0.18$ – near the spectral peak frequency. This suggests, based on the unexcited results (§6.2.4) that link the mean intermittence to the spectral peak frequency, that exciting the jet with mode 0 at that frequency reinforces the naturally occurring events. Examination of the narrowband spectra (Figure 7.4) indicates that this additional energy is contained in the narrowband tones. Since this behavior is not observed in the other modes, it is concluded that this is a flow-field response as opposed to actuator electronic noise or self-noise. The cause for this is likely the confluence of two separate ideas: a disproportionate increase in $m = 0$ energy content in unexcited heated jets and superdirective radiation.

3. While difficult to determine from the lower temperatures alone, the complete picture indicates that $m = 3$ produces the largest decreases in the OASPL and that these decreases occur at the lowest angles measured ($\phi = 25^\circ$ & $30^\circ$).

4. The excitation Strouhal number that removes the most energy at the low angles decreases with increasing temperature and is different for the different azimuthal modes. Mode 0 settles near $St_{DF} \approx 1.3$, while modes 1 and 3 settle at $St_{DF} \approx 0.5$.

5. Reduction at the low angles is accompanied by amplification at the higher angles.

It has been observed that the amount of energy found in the $m = 0$ flow-field mode of unexcited jets grows disproportionately faster than the other modes with increasing jet temperature (Suzuki and Colonius 2006; Hall and Glauser 2009). Reinforcing this natural
mechanism with excitation would likely produce axisymmetric flow-field structures that have significantly increased spatial and temporal coherence — a conclusion that is supported by a previous work using PIV (Kearney-Fischer et al. 2009). Another way to say this would be that the flow becomes more receptive to $m = 0$ development. The concept of superdirective radiation in jet noise, originally discussed by Crighton and Huerre (1990), describes the radiation pattern from a noise source that can’t be decomposed into a finite series of multipoles (§2.5). In an unexcited jet, there are a wide range of oscillation frequencies with randomized phases that would result in energy being distributed in time and frequency space. If one frequency becomes dominant (say through the combination of disproportionate increases in mode receptivity and excitation), the energy of the radiation should be concentrated in both time and frequency space. The directive aspects of the ΔOASPL map and the narrowband spectra support this assertion. The observation of this phenomenon only in the axisymmetric mode suggests that the spatial coherence of the structures (and consequently the wave-packets) is an important factor in the radiating efficiency of this mechanism.

To facilitate discussion in subsequent sections, the ΔOASPL plots for $\phi = 30^\circ$ are shown in Figure 7.8.
7.1.4 Events Only Comparison

Using the same process described for the unexcited jet data in §6.1, the spectra of the events only signal are constructed and plotted in Figure 7.9 for unexcited cases as well as two representative sets of excitation parameters at two of the four temperature ratios. The unexcited results are very similar to the AAPL, which confirms the generality of the results as well as the procedure, so they are not discussed in detail.

Figure 7.8: ΔOASPL plots for $\phi = 30^\circ$. 

(a) TTR = 1.0  
(b) TTR = 1.5  
(c) TTR = 2.0  
(d) TTR = 2.5
Figure 7.9: Example spectra for signal reconstruction.
The spectral behaviors in the excited jet are qualitatively similar to the unexcited jet results (§6.1) in most cases and the spectral reconstruction characteristics for temperatures and excitation parameters not shown are also qualitatively similar to those presented. For the unheated jet, the low-angle data are well represented and the sideline angle data have some problems. At high frequencies, the sideline spectra are highly distorted and elevated (taking on an almost completely flat profile—white noise). The reasons for this reconstruction deficiency at the sideline angles have already been discussed. In the elevated temperatures, the sideline spectra results are similar to the cold jet, but something more complicated is happening with the low angles. For the case shown in Figure 7.9d, the 30° spectral peak location is reasonably good, but the spectral amplitude is significantly over-predicted. Additional spectra for the same operating condition and excitation frequency are shown in Figure 7.10. These additional spectra show that this behavior is only present in the axisymmetric mode. Examining spectra at nearby frequencies and polar angles (not shown) reveals that this behavior occurs in the region of the ΔOASPL maps where strong amplification was observed. This over-prediction is another indicator that something different is happening in the acoustic field when the jet is excited axisymmetrically near the jet column natural frequency ($St_D = 0.3$) in these heated cases.
To understand how the spectral levels are being over-predicted in some of these cases, a portion of the reconstructed time-domain signal for one such case is shown in Figure 7.11 – the data is plotted versus inverse Strouhal number. It should be noted that most excitation parameters produce time-domain signals and reconstructions that look similar to those shown in Figure 6.1. The change in signal characteristics responsible for the changes observed in the ΔOASPL maps and narrowband spectra is immediately apparent. When excited with these parameters in the elevated temperatures, the noise radiated to low angles becomes highly periodic (with a period matching the excitation frequency) containing long chains of high-amplitude events. The behavior of the time-domain signal supports the discussion of superdirective radiation discussed in §7.1.3. While it is possible to conceptualize this noise signature being created by highly periodic vortex interactions, the characteristics match those of superdirective radiation very well.

Over-prediction of the spectral levels occurs because of the nature of the model function used to reconstruct the data. The widths of the events in relation to the period are
such that the negative lobe of the Mexican hat wavelet lies near the peak of the subsequent negative swing. Since the positive and negative swings are treated independently, the final signal is a superposition that results in excess amplitude being created at the peaks of the events (a few of these occurrences are circled in the figure). This excess energy gets distributed into the spectrum due to the properties of the Mexican hat wavelet resulting in the over-predicted spectral amplitudes. While it is possible to eliminate this over-prediction by using a different model function, it would complicate the analysis by requiring a method of determining when the alternate model function was needed for very little benefit. The over-prediction doesn’t impact any of the statistics discussed in §7.2 and it is also a convenient indicator for the occurrence of this phenomenon.

7.2 Statistical Analysis

Based on the spectral analysis of the excited jet (§7.1), it is concluded that the excited jet dynamics are fundamentally similar to the unexcited jet (Chapter 6) with the few exceptions already discussed. The most relevant conclusion is that the statistical
analysis of the events can focus on the low angles (30° is used) without missing important information. Additionally, the unexcited cases within the GDTL database do not need to be explored in great detail except for comparison purposes to excited cases. Since the unexcited jet results show that the event width and intermittence have very similar behaviors, only the intermittence is discussed in detail and the mean width is discussed in relation to the mean intermittence.

7.2.1 Amplitude Distributions

The amplitude distributions for the excited jet are examined by focusing on 30°. The spectral analysis results (§7.1) will inform the discussion to further reduce the number of points that need examining. The probability density functions for the peak amplitudes are shown in Figure 7.12 for all excitation frequencies at modes 0 & 1 and temperature ratios of 1.0 and 2.0. The distributions, for the most part, are very similar to the unexcited jet (Figure 6.4). There are two notable differences in the excited jet results. In the unheated jet, it can be seen that the higher frequencies (the redder curves) are skewing toward the unit normal distribution for both modes. This variation is relatively slight and, given the likelihood that the data has a large amount of actuator self-noise as already discussed (§7.1.3), it is not likely that it is a meaningful variation. At the elevated temperature cases (TTR = 2.0) shown in Figure 7.12, the departure from the unexcited distribution is associated with the strong increases in the OASPL. In mode 0, the low excitation frequencies (blues) have an amplitude distribution close to the unit normal while mode 1 has no such trend. This change is likely associated with a basic change in the nature of the typical event created by exciting with these parameters. As already discussed, the
unexcited event amplitude distributions skew away from the unit normal because the peaks only contain part of the information about points above the $1.5p_{RMS}$ threshold (§6.2.1). The typical event in these highly excited cases, however, is different from the typical event in most other cases. As seen in Figure 7.11, these cases are characterized by periodic high-amplitude oscillations that roll rapidly on and off. The peaky characteristics of these events mean that the peak amplitude is more representative of the data above the threshold so it recovers a distribution close to the unit normal—the distribution of the total signal (Figure 6.3). While not shown, the other polar angles, temperatures, and azimuthal modes have amplitude distribution characteristics consistent with a combined interpretation of the unexcited results and the range of excitation frequencies and polar angles where the strong amplification occurs.
7.2.2 Intermittence Distributions

The amplitude distributions indicate that, in most cases, the excited jet statistics are likely to behave similarly to the unexcited jet. Therefore, to get an overall picture of the intermittence distribution behaviors without presenting an unwieldy number of plots, the following metric is used to describe how much a particular case deviates from the unexcited distribution. The distributions are normalized by their respective means as done in Figure 6.11. The best fit gamma distribution for the unexcited data (baseline) is

Figure 7.12: PDF of peak amplitudes for all excitation frequencies at $\phi = 30^\circ$. Higher frequencies are in red and lower frequencies in blue.
determined for a given operating condition and polar angle. The Root Mean Square (RMS) error of a particular distribution with respect to the baseline best fit gamma distribution is computed. Finally, this RMS error is normalized by the RMS error of the baseline distribution with respect to its own best fit gamma distribution. In this way, quantities significantly greater than one indicate a distribution that is a meaningful departure from the baseline while removing the expected changes in the distribution that occur due to a changing mean. This quantity is referred to as the “gamma deviation.”

The gamma deviations for two temperature ratios at four polar angles ($\phi = 30^\circ, 45^\circ, 60^\circ, \& 90^\circ$) are shown in Figure 7.13. Looking at the unheated case, it is clear that all of the azimuthal modes show substantial deviation from a gamma distribution over a wide range of frequencies. The deviation has generally similar levels regardless of azimuthal mode, but there is some directional dependence. At the low angles where the jet is loudest, the deviation is generally weaker. The deviation becomes more prominent as the angle increases presumably because the jet noise is lower at the larger angles. This kind of indiscriminant deviation supports the previous conclusion that actuator self-noise is a prominent feature at this operating condition. The TTR = 2.0 jet is an entirely different story. Modes 1 and 3 have very little deviation except for a small amount at the sideline angles for high excitation frequencies. Mode 0, in contrast, shows a very strong localized deviation that correlates very well with the excitation frequency and directivity characteristics of the large OASPL increase already discussed (§7.1.3). The other polar angles and temperatures (not shown) have behaviors and trends consistent with the results shown. The TTR = 1.5 case behaves like the TTR = 2.0 case implying that the diminution of the self-noise problem occurs somewhere between TTR = 1.0 and TTR = 1.5.
Figure 7.13: Gamma deviations for temperature ratios of $TTR = 1.0$ & 2.0 and $\phi = 30^\circ$, 45°, 60°, & 90°.
Several example distributions are shown in Figure 7.14 to illustrate the different characteristics indicated by the gamma deviation. Note that these distributions are normalized by their respective means consistent with previous analysis (§6.2.4). In each of these figures, the unexcited intermittence distribution (Baseline) is shown as a black line and the excitation periods are indicated by downward pointing triangles.

In the cold jet at $\phi = 30^\circ$ excited with mode 0 (Figure 7.14a), the deviation at low excitation frequencies is due to narrowband spikes that are multiples of the excitation

Figure 7.14: Intermittence distributions normalized by their respective means for the excited jet.
period. The distribution at these conditions retains the same basic shape as the baseline so the gamma deviation values are near one. At the higher excitation frequencies, the intermittence distribution is being significantly altered. Very strong narrowband spikes are visible (multiple of the excitation period) and these cause the shape of the distribution to flatten resulting in large gamma deviations.

Looking at the same polar angle and excitation mode but at an elevated temperature (Figure 7.14b) reveals just how radically the distribution is being altered by excitation. For low excitation frequencies near the jet column natural frequency \((St_D \approx 0.3)\), the distribution is dominated by a spike at the excitation period with strong but weaker spikes at multiples of the fundamental period. The occurrence of events with these few periodicities is so high that the remaining portions of the distribution have negligibly small probabilities. For the same conditions at large excitation frequencies, however, the distribution is indistinguishable from baseline. To look at directivity, the intermittence distributions at \(\phi = 60^\circ\) are shown (Figure 7.14c) for the same temperature and excitation parameters as (Figure 7.14b). It is clear that the distribution deviates only slightly from the baseline due to excitation with the strongest deviation being a narrowband spike associated with the excitation period when that period is near the natural mean period. Finally, to examine the excitation mode response, the intermittence distributions for mode 3 (Figure 7.14d) are shown for the same polar angle and temperature as (Figure 7.14b) with one exception. In (Figure 7.14d), the distributions are plotted for all the excitation frequencies. It is clear that exciting this mode does not create strong deviations from the baseline distribution. There are a few narrowband spikes at the fundamental
periods for a few of the lower excitation frequencies, but the distribution shape is essentially unchanged.

These distributions show that the gamma deviation is indeed a good descriptor of changes in the distribution shape. Additionally, this result adds support to the conclusion that the jet at the elevated temperatures is generating strong superdirective radiation when excited with the axisymmetric mode near the jet column natural frequency.

7.2.3 Mean Width and Intermittence

Based on the unexcited jet analysis (§6.2), the mean width and intermittence are parameters governing the changes in the jet noise signal characteristics. Additionally, a strong link between these two quantities was observed in the unexcited jet \( \frac{\delta t}{\Delta T} = 0.128 \). That same scaling is applied to the data in this section to see if the behavior is preserved under excitation and to minimize the number of needed figures.

The unexcited (Baseline) mean quantities for the GDTL data are shown in Figure 7.15. Comparing these data to the unexcited database from AAPL (§6.2.5), there are similarities and some differences. It should first be noted that the scaling factor of 0.128 between the mean event width and intermittence is consistently a good scaling in the GDTL data. Looking at the unheated case (TTR = 1.0)—for which a nearly direct comparison exists in the AAPL database Case 4—the distributions are very similar. The mean width at \( \phi = 30^\circ \) is nearly identical between the two databases; as is the shape of the trend to the sideline angles. The mean width at \( \phi = 90^\circ \) is slightly lower in the GDTL facility, but it is unlikely that the discrepancy is significant. The previous discussion of the sideline angles (§6.2.2.2) showed that the noise event analysis is of very limited use at
the sideline and upstream angles. Additionally, the idea that noise radiated to the sideline comes from fine-scale turbulence dictates that differences in things like nozzle geometry, boundary layer turbulence, etc. should result in differences in the noise production. The only discrepancy of note in the unheated case is the behavior at $\phi = 25^\circ$. In the AAPL data, the event width continues increasing with decreasing polar angle. The GDTL data, however, changes direction below $\phi = 30^\circ$. This is most likely a facility dependence in the GDTL created by the location of the $\phi = 25^\circ$ microphone within the anechoic chamber. The $\phi = 25^\circ$ microphone is located in a corner of the anechoic chamber in relatively close proximity to the walls and the collector – AAPL, in contrast, has no such proximity issue. It is therefore likely that noise reaching this microphone is being altered by this proximity and so discussion of the data at $\phi = 25^\circ$ should be minimized.

The trend with increasing jet temperature in the GDTL database is a much more complex comparison. The GDTL database holds a fixed hydrodynamic Mach number
while varying the stagnation temperature ratio while the AAPL database holds the
coustic Mach number constant while varying the exit temperature ratio. The result is that
the AAPL approach isolates temperature as a variable while holding the jet velocity
constant whereas the GDTL approach results in a coupled change in the jet velocity and
temperature. The AAPL approach is therefore superior for hydrodynamically subsonic
jets, but is not practical for supersonic jets due to the additional complexities of a
supersonic flow. At the time of collection, this best practice was not known to GDTL –
otherwise the data would have been collected following the best practice. One
consequence of this is that the GDTL jet becomes acoustically supersonic (as seen in
Table 4.2) and previous work has shown that Mach wave radiation is emerging as a
significant noise source in the higher temperatures of the jet currently under discussion
(Kearney-Fischer et al. 2011a). The onset of an additional noise source complicates the
picture, but the previous work indicates that it should be a relatively weak contribution in
terms of the acoustic spectra. Looking at the transition \(\phi = 40^\circ\) to \(60^\circ\) and sideline
angles, the trend with increasing temperature in the GDTL data is similar to the AAPL
data. The low angles, however, have a trend that is similar to the AAPL data for the lower
temperatures, but that reverses direction for the two highest temperature ratios. Unless
compressibility effects, created by the combined increase in temperature and velocity, are
producing changes in the source mechanisms of the mixing noise sources, it is likely that
this behavior is indicative of the presence of Mach wave radiation, but that the Mach
wave radiation source is too weak to produce any significant changes in the spectral
shape. This result dictates that any significant changes in excitation response between
The mean width and intermittence for \( \phi = 30^\circ \) are shown in Figure 7.16 for the four temperature ratios. The excitation period \( (T_F) \) is also shown in these figures for context.

Looking at the cold jet first, it can be seen that the scaling of 0.128 is held quite well regardless of excitation parameters at this temperature. Again there is evidence that the data has significant actuator self-noise: the azimuthal modes are indistinguishable and
trend consistently downward to smaller and smaller intervals with increasing excitation frequency. It is difficult to say from the data, but it appears that the mean event width is asymptotically approaching the excitation period. Comparing these data to the ΔOASPL data (Figure 7.8a) offers very little insight except to confirm the indistinguishable nature of the different azimuthal modes at this temperature.

At a temperature ratio of 1.5, the behavior of the changes in the mean width and intermittence are starkly different from those of the unheated jet. The different azimuthal modes trend down to some minimum and then trend back toward the baseline levels with increasing excitation frequency. This reinforces the idea that the jet is being excited without overwhelming the acoustic field with actuator self-noise. It is also clear in this case that the different azimuthal modes are affecting the mean quantities in different ways.

In mode 0, the mean intermittence closely follows the excitation period for excitation frequencies less than \(St_{DF} \approx 0.3\) as might be expected from the previous discussions. Even at the lowest excitation frequencies measured, the noise source dynamics are still so strongly controlled by the excitation that the intermittence increases above the baseline. It should be remembered, however, that the relationship between the intermittence distribution and the mean is strained in these strong response cases. When the baseline distribution is obliterated by a strong periodicity, the mode of the distribution will be the best descriptor. As already discussed (§7.2.2), the mode of the distribution in these strong response cases is exactly the excitation frequency. This explains why the mean intermittence in these cases follows the excitation frequency while not matching it. Since these strong response cases constitute only a small portion of the parameter-space, the
mean of the distribution is still better than the mode as a general descriptor of the changes in the distribution in response to excitation. The amplification of the OASPL peaks when the excited mean intermittence matches the baseline (at an excitation frequency of $St_{DF} \approx 0.2$). This frequency is close to the spectral peak frequency of the low-angle noise. This strongly suggests the idea that the excitation frequency is matching a naturally occurring resonance. Another observation of mode 0 excitation is that, for the low frequencies ($St_{DF} < 0.6$), the changes in mean width are smaller than the changes in the intermittence. This suggests that, while the dynamics governing the width and intermittence are strongly related, the behavior of one does not necessarily determine the exact behavior of the other. There are two local minima in the excitation intermittence response, $St_{DF} \approx 0.4 \& 0.8$, at TTR = 1.5 & 2.0. As the temperature increases, the minimum at 0.8 vanishes. The locations of these minima are, roughly speaking, harmonics of the spectral peak frequency. Given that the jet responds very strongly to exciting at the spectral peak frequency with mode 0, it makes sense that strong responses might also be observed at harmonics of that fundamental.

The changes induced by exciting modes 1 and 3 aren’t as dramatic as mode 0. In these modes, there is always good agreement between the width and intermittence. Mode 1 has a gradual trend with a large flat bottom – the peak noise reduction occurs somewhere in this bottom. Mode 3 has a more pronounced minimum with noise reduction peaking at excitation frequencies just larger than the intermittence minimum. Apart from the change in behavior between the unheated and TTR = 1.5 cases that is attributable to the dominance of actuator self-noise, there aren’t any readily identifiable significant changes in the excitation response with increasing temperature. It is therefore
concluded that the additional noise source of Mach wave radiation is not a significant factor in the jet noise (at least with respect to excitation response) at these conditions.

For completeness, the mean width and intermittence data at $\phi = 90^\circ$ are shown for two representative temperature ratios in Figure 7.17. In the unheated jet, the issue of actuator self-noise is readily apparent – the mean width approaches the limits of the signal resolution (10 μs) and doesn’t trend back toward the baseline at the high excitation frequencies (Note that only the scaled intermittence values ever dip below 10 μs). The elevated temperature case does trend back toward the baseline. The event width and intermittence are always well matched by the 0.128 scaling and there is no detectable difference between the azimuthal modes. If the data at this polar angle has anything meaningful to say, it is that exciting the jet makes the oscillations in the sideline noise even more rapid.

The behavior of the jet in response to excitation frequency and azimuthal mode suggest a process of competition, both in the temporal axis (which is highly correlated...
with the axial direction) and the azimuthal axis. One of the general results is that significant noise reduction occurs when the excitation period is smaller than the induced mean intermittence, and also larger than the event width. The maximum noise reduction occurs when the induced mean intermittence frequency is roughly five times less than excitation frequency. Studies of the flow-field (Chapter 5) have shown that excitation at these frequencies generates a single structure for each pulse of the actuator. If the mean intermittence doesn’t match the excitation period, the implication is that only some of the structures are producing noise events. It appears that the noise sources radiating to the low angles (i.e. large-scale structures) are competing for flow energy. Noise reduction occurs when excitation produces an environment in which this competition limits the amount of energy that any one structure can consume. Conversely, noise amplification occurs when excitation tunes the jet to allow each structure to consume as much energy as possible. This idea will be discussed more in Chapter 9.

7.2.4 Joint PDF—Amplitude and Width – 30°

Before examining the response to excitation, it is prudent to look at the distributions of the unexcited cases from the GDTL database to see if there are differences relative to the AAPL database results discussed in §6.2.3. The joint PDFs for the 30° microphone are shown in Figure 7.18 for the four temperature ratios. On the whole, the distributions from the GDTL database are quite similar to the AAPL database results. The main difference is that the elongation with heating is more pronounced in the GDTL facility. It is difficult to say if this difference is a facility dependence or is due to heating while holding the hydrodynamic Mach number fixed. Given the high degree of similarity
between the most closely matched unheated cases (Figure 7.18a and Figure 6.9a) however, it is more likely that this difference is the result of holding the hydrodynamic Mach number fixed. As discussed in §6.2.3, the elongation with increasing temperature indicates that the width and amplitude are becoming less correlated.

In the interest of keeping the number of figures to a reasonable level, a few select excited distributions at a temperature ratio of TTR = 2.0 are shown in Figure 7.19. In addition to the doubling line, the excitation period is marked with a black triangle on the

Figure 7.18: Unexcited joint-PDFs of amplitude and width from the GDTL data at various temperature ratios at $\phi = 30^\circ$. 
right-hand edge of the figures. As already discussed, the results at a temperature ratio of TTR = 1.0 seem to have significant levels of actuator self-noise. It is decided that it is sufficient to present one temperature ratio as representative of the joint PDF characteristics given the likelihood of a small benefit for a large number of additional figures required to present multiple temperature ratios. Excitation azimuthal modes 0 and 3 are shown because they elucidate the characteristic behaviors and the frequencies chosen highlight significant points in the trends. While not shown, the trends and behaviors discussed here are also seen in the temperature ratio 1.5 and 2.5 data.

Looking at the distributions, it is clear that $m = 0$ is creating much more significant changes. As expected at this point from the other results, exciting the jet with $m = 0$ near $St_{DF} = 0.2$ radically reshapes the distributions. What is interesting about this distribution is that width is apparently independent of the amplitude when excited in this resonant regime. Additionally the width distribution is almost symmetric about the mean width. The distribution is also much more compact. These characteristics are all consistent with the results already discussed. As the excitation frequency increases (Figure 7.19b) the distribution trends back toward a shape similar to the unexcited jet. As the excitation period passes through the mean width (Figure 7.19 c & d), the distribution becomes bimodal in the event width; the second lobe is a harmonic of the primary lobe. By the time $St_{DF} = 3.0$ is reached, the distribution has returned to the baseline.
Azimuthal mode 3 is much less complicated than mode 0. The distribution characteristics are essentially unchanged by excitation. This is consistent with the results from preceding sections. The information from these joint-PDFs agrees with the other analyses and additionally shows a link between amplitude and width. Given the complexity of the link between the two quantities, however, it would be difficult to reduce to an analytical expression without over-simplifying the dynamics.
7.3 Summary of Results

As already established in previous publications (Kearney-Fischer et al. 2011b) and discussed in §7.1.2 and §7.1.3, the plasma actuators are capable of manipulating jet noise. The spectral analysis shows that:

1. The reduction/amplification characteristics depend on jet temperature, excitation azimuthal mode, and polar angle.

2. The quantitative amount of far-field acoustic energy that can be removed through excitation increases with increasing temperature.

3. The excitation tone amplitude of $m = 0$ at the low angles ($\phi < 50^\circ$) increases at elevated temperatures for excitation Strouhal numbers near the jet column natural frequency ($St_D \approx 0.3$). Close inspection reveals that the greatest increase occurs at the excitation frequency of $St_{DF} = 0.18$ – near the spectral peak frequency. This suggests, based on the unexcited results (§ 6.2.4) that link the mean intermittence to the spectral peak frequency, that exciting the jet with mode 0 at that frequency reinforces the naturally occurring events. These changes in the tone amplitude create large changes in the OASPL that occur only in mode 0. This is highly suggestive of strong superdirective radiation and its onset at elevated temperatures is likely related to the disproportionate growth of mode 0 energy in the jet when the temperature is elevated.

4. Actuator self-noise appears to be a significant contribution to the noise signature in the unheated jet, but becomes inconsequential at elevated temperatures because the jet gets louder.
5. While difficult to determine from the lower temperatures alone, the complete picture indicates that \( m = 3 \) produces the largest decreases in the OASPL and that these decreases occur at the lowest angles measured (\( \phi = 25^\circ \) & \( 30^\circ \)).

6. The excitation Strouhal number that removes the most energy at the low angles decreases with increasing temperature and is different for the different azimuthal modes. Mode 0 settles near \( St_{DF} \approx 1.3 \), while modes 1 and 3 settle at \( St_{DF} \approx 0.5 \).

7. Strong reduction at the low angles is accompanied by amplification at the higher angles. Uniform reduction is achievable when exciting high frequencies, but the reduction is quantitatively smaller than peak reduction levels. Also, there isn’t any well-defined optimum excitation frequency or azimuthal mode for reduction at the high frequencies.

The statistical analysis provides the following additional insights.

1. Actuator self-noise is an apparent problem in the statistical metrics of the unheated jet – reinforcing the conclusion from the spectral analysis.

2. In most cases, the statistical description of the jet noise (i.e. the shape of the distributions) is not being significantly changed by excitation.

3. When excited in the strongly resonant regime (i.e. \( m = 0 \) and \( St_{DF} \approx 0.1-0.5 \) at elevated temperatures), the naturally occurring (i.e. unexcited) statistical distributions of the event intermittence are totally obliterated by a single intermittence interval. This suggests that the excitation is achieving a resonance with the natural frequencies of the jet – supported by the fact that the strongest resonance occurs when the induced mean intermittence matches the baseline
mean intermittence and that this matching occurs when the jet is excited with frequencies roughly matching the spectral peak frequency \((St_{DF} \approx 0.2)\).

4. The relationship between the mean intermittence \((\overline{\Delta T})\) and width \((\overline{\delta t})\) as determined from the unexcited jet \((\overline{\delta t}/\overline{\Delta T} = 0.128)\) is still valid in the excited jet with one exception. In the strongly resonant regime, the mean width is not as strongly affected compared to the mean intermittence. This divergence in the relationship between these two quantities suggests that the mechanisms responsible for these characteristics are linked, but that one does not always dictate the exact nature of the other.

5. The nature of the jet response, both in terms of frequency and azimuthal mode, suggests a process in which noise sources are competing for flow energy. Noise reduction occurs when excitation produces a competitive environment that limits the amount of energy that any one structure can consume – limiting a structure’s ability to produce a noise event. Conversely, noise amplification occurs when excitation tunes the jet to allow each structure to consume as much energy as possible – resulting in large noise events.

The most probable cause for the increasing amounts of noise reduction with increasing temperature is the rapid growth in the energy of the \(m = 0\) mode as the temperature is elevated (Suzuki and Colonius 2006; Hall and Glauser 2009). Results supporting the observations about the behavior of mode 0 with regard to LAFPAs are presented in Kearney-Fischer et al. (2009). It is generally believed that the axisymmetric mode is the most efficient radiator of sound (Hall et al. 2006) – this idea is strongly supported by the present work. If the competition process is indeed present, the highly
organized perturbations seeded by exciting higher azimuthal modes (e.g. \( m = 3 \)) grow and consume as much of the available flow energy as possible. Consequently, there is less energy available for the \( m = 0 \) mode. Thus, the unexcited heated jet converts more energy into sound than the unheated jet, but when excited, the energy conversion is severely suppressed by diverting that energy into modes which are less efficient radiators. However, reduction achieved with mode 0 excitation indicates that there must also be a contribution from the highly organized nature of the excited structures. Given the apparent resonance conditions in which mode 0 excitation can produce large increases in noise, it makes sense that some noise reduction could be achieved in mode 0 by exciting the jet away from the resonance condition (i.e. frequencies considerably larger than \( \text{St}_{DF} = 0.2 \)) as long as the frequency isn’t so high that the jet is unresponsive.

The strong superdirective radiation created by exciting the axisymmetric mode suggests that the spatial coherence of the structures plays an important role. A possible explanation for the intermittent nature of low-angle noise would then be that temporal fluctuations in the spatial coherence of the large-scale structures produce periods of increased radiation efficiency. The characteristic frequency of the wave-packet would then be associated with the event width and frequency of the events would be related to the frequency of the coherence fluctuations. This idea is further explored in §9.2.2.
Chapter 8: Measurements of Structure Interaction

The results discussed in Chapter 7 highlighted questions about the behavior and interaction of the large-scale structures in the jet (both in terms of the excited jet behavior and also what that infers about the unexcited jet). The two most important observations are the discussion of a resonance condition and the idea that structures may be competing for flow energy. While there are many questions that could be explored regarding the nature of the structure interaction, the two most critical are:

1. What is the impulse response of the jet? This question is important because it looks at the behavior of a structure in isolation and can answer other questions such as ‘can a large-scale structure exist in isolation or does it require adjacent structures to support its growth?’ The impulse response of the jet is something that has not been well studied in the literature. The explorations of jet or shear layer excitation in the literature usually focus on the response to single frequency excitation and then sweep that frequency (e.g. Ho and Huerre 1984).

2. Can evidence of competition for flow energy be found and what is the behavior of that competition?

In an effort to address these questions, GDTL has begun experiments using excitation and a linear microphone array placed just outside the jet. The microphones detect a pressure signature associated with the fluctuations within the jet (including the
fluctuations due to passage of the large-scale structures) because they are in what is known as the near-field hydrodynamic region. In this region, the air-speed induced by the entrainment is quite low, but the pressure-field is dominated by the dynamics of the jet (e.g. Tinney et al. 2008). The jet is excited with frequencies ranging from very low (250 Hz or $St_{DF} = 0.02$) to moderate Strouhal numbers ($St_{DF} = 1.4$). Exciting the jet at very low frequencies is equivalent to exciting the jet with a single pulse because all of the timescales of the jet are very small compared to the excitation period. As discussed in Chapter 5, the perturbation produced by LAFPAs is much more similar to a delta function than a sinusoid. A single pulse from these actuators is then a very good tool for studying the impulse response of the jet. Studying the impulse response of the jet, as well as the signature of the large-scale structures over a wide range of frequencies, can provide support for the speculation on structure behavior in the preceding chapters. To date, only the axisymmetric mode has been explored so conclusions will be somewhat limited. These experiments are ongoing, but the results to date have some very relevant conclusions. These preliminary results are therefore presented and discussed in the context of jet noise. The jet examined here is the same Mach 0.9, $D = 2.54$ cm, unheated jet (TTR = 1.0) studied in Chapter 7. While it was shown that actuator self-noise was a problem in the acoustic far-field, the amplitude of the hydrodynamic pressure fluctuations is much higher so actuator self-noise is not a significant problem.

The linear microphone array consists of five Bruel & Kjaer microphones (model 4939) arranged along the jet axis as shown in Figure 8.1. The axial locations of the microphones are from $2D$ to $4D$ and the radial distances are roughly $1.5D$ – the exact radial distance increases with axial distance to compensate for jet spreading. Data are
acquired with the same system used to collect the far-field noise data (§4.2.3) with the same sampling rate and signal conditioning. The excitation control signal is also acquired to provide a well-defined time base for the data. Multiple microphones are used to track the velocity of the large-scale structures as well as changes in the structure energy (which can be inferred from the pressure-field) with axial location. For the purposes of the current discussion, data from only one microphone (located at $x/D = 3$ and $r/D = 1.5$) will be examined. The acquired data are phase-averaged to focus on the signature of the induced structures. While the data collected in these experiments has many facets and can be examined in a number of ways, the discussion here will focus exclusively on addressing the two questions listed above.

The impulse response of the jet to axisymmetric excitation is shown in Figure 8.2a. Technically the jet is excited with $m = 0$ at 250 Hz ($St_{DF} = 0.02$), but this frequency is low enough to be considered synonymous with the impulse response as will be confirmed. In this figure, time equals zero corresponds to the rising edge of the actuator control signal.
The impulse response signal shown in Figure 8.2a is the average of about 1000 pulses. Examining the impulse response, several features are apparent.

1. The acoustic wave from the actuators (i.e. the actuator self-noise) arrives at about 0.22 milliseconds which is consistent with the time taken to traverse the three jet diameters at the speed of sound in the ambient air. It is clear that the actuator self-noise is quite weak in comparison to the signature of the large-scale structure. The acoustic wave from a single actuator firing in quiescent air is shown in Figure 8.2b. The microphone is far enough from the actuators that the compression wave, which initially steepens into a shock wave (Utkin et al. 2007), has relaxed into an acoustic wave. The relaxation of the wave explains what might, at first, be a counter-intuitive feature – the leading edge of the compression wave is actually a pressure drop.
2. The large-scale structure signature consists of a low-pressure lobe (associated with a vortex) followed by a high-pressure lobe which must also be present since the pressure field of the jet must be isobaric in a net sense.

3. The pressure minimum arrives at about 0.5 milliseconds and the entire event lasts about 0.6 milliseconds.

While the width and depth of the negative lobe can be associated with the size and energy of the large-scale structure, these quantities change with axial location so they can’t, as such, be directly useful in a discussion of the jet noise dynamics. These characteristics will be further discussed in the context of interaction. This clearly indicates that large-scale structures do not require the existence of adjacent structures to grow. While the vast body of previous work relating jet noise to instability waves of a given frequency (e.g. typical wave packet models) can be and is still useful, this result supports the previous conclusion cautioning against the use of such an approach when the basic constituents of the dynamics are not of a fundamentally periodic nature.

One consequence of the impulse response nature of the jet is that, for excitation frequencies low enough to keep the structures separated, the energy per unit time should add independently and be directly related to the excitation frequency. Based on the impulse response duration of about 0.6 milliseconds, this would correspond to a frequency of about 1.67 kHz \((St_{DF} = 0.15)\). Relating \(St_{DF} = 0.15\) to a spatial distance using \(U_c = 0.6U_j\) results in \(\Delta x = 0.6 D/St_{DF} \rightarrow \Delta x = 4D\). For excitation frequencies higher than this, it is expected that interaction and competition should take place. To get an overview sense of this, the mean square pressure of the phase-averaged signals at various excitation frequencies is calculated and normalized by the jet exit kinetic energy density.
(Figure 8.3). The unexcited mean square pressure (Baseline) is marked for reference. The line associated with the independent addition of energy, using the impulse response energy as the reference point, is also shown. Just as expected, the excitation frequencies below about $St_{DF} = 0.15$ very closely follow the independent addition line. This confirms that the signature associated with exciting at $St_{DF} = 0.02$ is indeed representative of the impulse response. Beyond this frequency, the energy per unit time continues to increase, but starts to fall away from the independent addition line. The energy per unit time peaks at $St_{DF} = 0.25$ and then decays rapidly – falling below the baseline levels by a Strouhal number of about $St_{DF} = 0.45$.

To get a better sense for how the structures are being affected, several examples are shown in Figure 8.4. For each example, the phase-averaged signal is shown along with the impulse response signal and the excitation period is marked by a black bar. The first
example ($St_{DF} = 0.13$) is the highest frequency for which the adjacent impulse responses (including the actuator acoustic wave) don’t appreciably interact. It can be seen that, at this and any lower excitation frequency, the structures do not interact in an appreciable way. The second example ($St_{DF} = 0.26$) shows how the structures start interacting when the energy per unit time is maximized. The amplitude of the signal is unchanged from the impulse response, but the adjacent structures are compressing one another slightly resulting in an almost perfectly sinusoidal shape. The next example ($St_{DF} = 0.35$) shows a case where the interaction of the structures has started to inhibit structure growth. The amplitude is reduced and the structures are increasingly compressed (remember that the time axis of these data are directly related to spatial extent by way of the convective velocity of the structures). In the last example ($St_{DF} = 0.49$), the inhibition of structure growth by competition has become quite severe. The expected periodicity is still present, but the amplitude and shape of the structure signature is significantly altered.

These results reveal several important aspects about large-scale structure behavior and interaction in the jet. As seen in these data, as well as previous experiments using LAFPAs (Chapter 5), excitation produces one structure per excitation period over a wide range of frequencies – at this point it could be said that this statement is true over the range $St_{DF} = 0$ to $St_{DF} \approx 1.5$. The size and energy of a structure is only dependent on the frequency for frequencies greater than $St_{DF} \approx 0.15$. For frequencies greater than this threshold, the structures interact, first in a way that increases the energy density (i.e. energy per unit time), but then in a competitive manner that inhibits the ability of the structures to extract energy from the flow. The frequency that results in the greatest energy density ($St_{DF} = 0.25$) is in the same region as the postulated resonance...
phenomenon discussed in Chapter 7. While the numbers don’t line up exactly, they are quite close and factors such as temperature have not been taken into account at this point.

Turning these conclusions toward the unexcited jet reveals the following. Structures that are separated by at least a sufficient distance in space ($\Delta x \approx 4D$) have similar energy per structure, but greater separation equates to less energy per unit time. As the structure spacing approaches resonance ($St_{DF} \approx 0.2 \rightarrow \Delta x \approx 3D$), the maximum energy per unit time is reached. Beyond resonance, the structures compete for energy and drag down the energy per unit time. As long as the energy radiated to the far-field is even loosely related to the energy of the structures (a conclusion supported by all of the other results), this

Figure 8.4: Examples of the large-scale structure signature for various excitation frequencies.
distribution of energy provides an explanation for why the most energy in the acoustic far-field is associated with the resonance frequency.

While not discussed in detail here, the preliminary results to date have shown that the structure characteristics scale in time as Strouhal number and in amplitude with the jet exit kinetic energy density. These scaling parameters were used in the preceding discussion and are validated by results not presented. Since it is not known how the impulse response changes with temperature, excitation azimuthal mode, etc., the conclusions discussed here cannot be considered finalized, but they are significant nonetheless.
Chapter 9: THE IMPLICATIONS FOR NOISE SOURCES

Using the picture of jet noise radiated to the low angles, the implications of this description on the noise sources are now discussed.

9.1 From the unexcited jet

The well-known solution to Lighthill’s equation (1952) for the acoustic pressure with the far-field approximation is

\[ p(x, t) = \frac{x_j x_i}{4\pi |x|} \frac{\partial^2}{\partial t^2} \int T_{ij} \left[ y, t - (|x| - |y|\cos\phi) / a_\infty \right] d^3 y. \]  

(9.1)

See §2.1 & §2.5 for details on the origin of this equation. At a fixed radial distance, the evaluation in retarded time in the far-field is the same for all directions if the sources are acoustically compact (i.e. $|y|\cos\phi$ may be neglected) so the spatial integral over the source volume can be written as

\[ \int T_{ij} \left[ y, t - (|x| - |y|\cos\phi) / a_\infty \right] d^3 y = \Psi_{ij}(t), \]  

(9.2)

where the source region has been evaluated at the retarded time. If the sources are not acoustically compact, $\Psi_{ij}$ will have angular dependence. It has been shown that the essential feature of low-angle jet noise is a fundamentally intermittent signal. Double integration in time of this signal implies that the components of the integrated source tensor ($\Psi_{ij}$) responsible for low-angle noise are changing in time from one quasi-steady value to another. The second time derivative of this quasi-steady signal could produce the...
one-and-one signal proposed in §6.3. A cartoon showing the proposed behavior of $\Psi_{ij}$ associated with the low-angle noise cartoon is shown in Figure 9.1b (note that Figure 6.18 is reproduced here as Figure 9.1a).

An example of such a source can be shown as follows – for simplicity, this initial derivation will be done for a single point in the far-field. A simple model for the one-and-one pulse is the first derivative of a Gaussian curve with an appropriate normalization. The noise event model ($E$) is a representation of the far-field pressure $p(x,t)$ at some fixed location for a small period of time:

$$E(t; T, \alpha, A) = A \frac{T - t}{\alpha} \exp \left[ \frac{1}{2} - \frac{(t - T)^2}{2\alpha^2} \right]. \quad (9.3)$$

The noise event $E$ has parameters governing its location in time ($T$), its duration in time ($\alpha$) – i.e. width, and its amplitude ($A$). A positive value of $A$ produces an event with the positive swing preceding the negative swing. This model has the following properties:

- The distance between the positive and negative peaks is $2\alpha$. 

![Figure 9.1](image_url)  
**a)** Far-field pressure fluctuations  
**b)** Integrated source fluctuations  
Figure 9.1: Cartoon of low-angle jet noise signal based on the characteristics extracted from the analysis.
Using the same adjacent isosceles triangle approximation of §6.2.6 gives a rough approximation of the FWHM ($\delta t$) of one half of the one-and-one pulse as $\alpha$. In reality, the transcendental equation can be numerically solved to show that the FWHM = 1.60252$\alpha$.

Integrating $E$ twice in time produces the following

$$\int \int E(t; T, \alpha, A) dt \, dt = A \alpha^2 \sqrt{\frac{e \pi}{2}} \text{erf} \left[ \frac{t - T}{\sqrt{2} \alpha} \right],$$

(9.4)

where erf is the error function and a constant offset is omitted for simplicity. In addition to supporting the previous argument, this result provides information about the behavior of the integrated source because

$$\int \int E(t; t_0, \alpha, A) dt \, dt = \frac{1}{4 \pi |x| a_0^2} \frac{x_i x_j}{|x|^2} \Psi_{ij},$$

(9.5)

for some fixed $x$ remembering that $\Psi_{ij}$ has been evaluated at the retarded time. The change in the magnitude ($M$) of the integrated source for a noise event at that particular location (i.e. $\frac{x_i x_j}{|x|^2} \Psi_{ij}$) is then

$$M = 4\pi^{3/2} \sqrt{2e} A |x| \alpha^2 a_0^2,$$

(9.6)

where an additional factor of two arises from the fact that the range of the error function is [-1,1]. The time interval over which this change in the integrated source occurs can be estimated as $\Delta t \approx 4\sqrt{2\alpha}$ from the known properties of the error function. Taking this back to the original definition of the event width ($\delta t$) as the FWHM of only one half of the one-and-one pulse, the time-scale of the change in the integrated source is $\Delta t \approx 3.53 \delta t$. If typical values are inserted for the width ($\delta t = 100$ μs) and amplitude ($A = 315$).
1.75p_{RMS}) for sound measured at $\phi = 30^\circ$ and $100D$ from the jet where $a_\infty = 340$ m/s, then the typical amplitude and time-scale of the changes in the source field are $M = 4.09$ $p_{RMS}$ $D$ kg m$^2$ s$^{-2}$ and $\Delta t = 353$ $\mu$s.

Bringing directivity into this picture, the first thing to note is that the far-field coordinate is an implicit variable in the parameters $A$, $a$, and $T$. The limitation of the present analysis is that it only allows for interpretation in two dimensions so the third dimension will not be discussed (i.e. $x_1$ is defined as the jet downstream axis and $x_3$ is discarded). This choice makes the discussion simpler and is not an unacceptable simplification. As has been discussed by many researchers previously (e.g. Crighton 1975), the integrated source at some far-field coordinate takes on the form

$$
\frac{x_i x_j}{|x|^2} \Psi_{ij} = \cos^2 \phi \Psi_{11} + \cos \phi \sin \phi (\Psi_{12} + \Psi_{21}) + \sin^2 \phi \Psi_{22}.
$$

(9.7)

This provides a map for ascribing the changes in the parameters of the typical noise event onto the components of the integrated source tensor ($\Psi_{ij}$). Evaluating the integrated source at $\phi \approx 0$, the combined information in §6.2.5 and (9.7) says that the dominant characteristic of $\Psi_{11}$ should be the intermittent events. $\Psi_{22}$ on the other hand (i.e. $\phi \approx \pi/2$) should behave much more like a white noise source. Looking at intermediate angles, the exact nature of the transition between the behavior of $\Psi_{11}$ and $\Psi_{22}$ is much more difficult to speculate on except to say that $\Psi_{12}$ and $\Psi_{21}$ must be contributing components that both decrease the width and increase the frequency of noise events in a fashion that limits to the behavior of $\Psi_{22}$. One obvious problem that arises in this interpretation is that the experimental results show that upstream angles do not recover the low-angle intermittent
behavior, contrary to the model prediction. This discrepancy can likely be accounted for by including source models not discussed in this simple analysis – e.g. non-compact source models (i.e. superdirectivity §2.5) such as those discussed by Cavalieri et al. (2011a) in which the components of the integrated source tensor have directivity.

While this result cannot directly point out the sources responsible for generating the noise events, it indicates the amplitude and temporal scales of the fluctuations in the flow-field that produce the noise events. Additionally, the intermittence information indicates how often these fluctuations occur. Armed with this information, future experiments and simulations can narrow their focus accordingly in the search for jet noise sources.

9.2 From the excited jet

Previous works (Chapter 5) showed that there are separate structures for each excitation pulse from each actuator and that, under the right conditions (exciting near the jet column natural frequency \( St_{DF} \approx 0.3 \)), neighboring structures will merge into larger structures and grow to large energies/sizes before disintegrating. Under other conditions (\( St_{DF} \approx 0.6-1.5 \) depending on azimuthal mode), excitation creates structures that initially grow rapidly, but then stop growing and convect in a stable manner before breaking up.

The analysis of noise events shows that a resonance exists in the jet – evidenced by the large noise amplification that occurs when the jet is excited at a frequency matching the mean intermittence of the unexcited jet. In this case, at least in a statistically averaged sense, every large-scale structure produces a noise event. Additionally, it was found that very strong amplification occurred when the jet was excited axisymmetrically (i.e. \( m = \))
This analysis also showed that significant noise reduction occurs when only a fraction of the large-scale structures produce noise events. These results suggest a process in which noise sources are competing for flow energy and that these noise sources are closely related to the large-scale structures. These results divide into two areas: a general behavior of competition, and the specific impact of azimuthal extent.

### 9.2.1 The Competition Model

Based on the experimental results, a descriptive model encompassing the observed behaviors is now discussed. There are two aspects to this model that dictate noise production: the interaction of structures, and the intrinsic energy within a structure. Within a range of excitation frequencies, the amount of interaction between neighboring structures is controlled by the excitation. Obviously, the azimuthal mode of excitation dictates the interaction in the azimuthal dimension. If the frequency is too low, the jet naturally produces structures in between those generated by excitation. When the frequency is too high, the jet statistics trend back toward the baseline (see Chapter 5 and §7.1.3). It may be that the actuator amplitude is diminishing at these high frequencies or that the jet doesn’t convert the perturbations into individual structures, but excitation does seem to alter the jet in small ways (implying that these high frequencies are at least subtly altering the naturally occurring structures).

The energy contained within a structure is controlled by both the excitation and the Initial Shear Layer Instability (ISLI). The ISLI (as discussed in §2.3) preferentially amplifies a range of frequencies. At low excitation frequencies, the ISLI amplifies the perturbation generated by each pulse as a totally independent event (i.e. the impulse
response) and will amplify the perturbation according to the amplification rates inherent in the instability. When the excitation frequency is sufficiently high, the ISLI won’t amplify the perturbations as independent events and this will change the way in which the initial perturbations are amplified. The details of this behavior were discussed in Chapter 8.

The size and the energy of a structure are related – as should be expected. As shown in the work of Kambe et al. (e.g. 1983), the size of a structure impacts the time-scale of the noise event (i.e. width) produced by a vortex collision. While the dynamical link isn’t clear at this point, it is important to acknowledge that the energy of a structure beyond its size is also likely to be a parameter on the generation of noise events.

The idea of structure competition has been discussed in a previous work from GDTL (Kim and Samimy 2009) on non-ideally expanded supersonic jets. In that work, it was observed that the excited structures had to compete with the naturally occurring structures for flow energy. In a non-ideally expanded supersonic jet, a strong feedback loop is established by the interaction of the large-scale structures and the shock-train that naturally excites structures in a periodic fashion at frequencies near the jet column natural frequency. This feedback process generates a tonal noise called screech noise. This work showed that the range of excitation frequencies that produced robust structures was reduced compared to an ideally expanded jet and that the structure passage frequency in the jet would vacillate between the screech frequency and the excitation frequency. This is strong evidence that structures can and do compete for flow energy and provides precedent for the ideas discussed here.
Putting the results and the preceding discussion into an enumerated form, the model of the noise sources is as follows.

1. Premises

   1.a. The dynamics of the large-scale structures are at least causally linked to the noise events.

   1.b. A structure must have certain characteristics to generate a noise event (most probably, it may need to have absorbed enough energy from the flow – this would be manifest in the structure’s size, rotational kinetic energy, etc.).

2. The process by which a noise event is generated.

   2.a. A perturbation is seeded (naturally or artificially), rolls up into the beginnings of a structure, and the structure starts to develop.

   2.b. Depending on its proximity to other structures, the structure in question will develop in different ways. To facilitate this description, the axial and azimuthal separations are divided into a few categories: Axial separation (FAR $\rightarrow S_{t_{DF}} < 0.1$, RESONANT $\rightarrow S_{t_{DF}} \approx 0.2$, CLOSE $\rightarrow 0.5 < S_{t_{DF}} < 1.5$), Azimuthal separation (FAR $\rightarrow m = 3$, CLOSE $\rightarrow m = 0$ or 1).

   2.b.i. Axial – FAR, Azimuthal – FAR: Each structure develops with characteristics dictated by the impulse response of the ISLI. This case results in a relatively benign structure that convects and dies without producing a strong noise event.

   2.b.ii. Axial – FAR, Azimuthal – CLOSE: The structures azimuthally merge into a single structure whose characteristics are still dictated by ISLI, but that has greater ability to pull energy out of the flow (especially if the structure
is azimuthally complete – e.g. \( m = 0 \). This case has structures with more energy than 2.b.i, but the structures occur infrequently so they don’t result in significant energy per unit time reaching the far-field.

2.b.iii. Axial – RESONANT, Azimuthal – FAR: Neighboring structures reinforce each other and can grow to energies larger than those seen in cases 2.b.i or 2.b.ii, but are still relatively restrained by the confined azimuthal extent. This case produces well organized structures, but the confined azimuthal extent of the structures still largely limits the potential to produce strong noise events.

2.b.iv. Axial – RESONANT, Azimuthal – CLOSE: Neighboring structures merge into an azimuthally cohesive structure and axially neighboring structures reinforce each other. This case produces well organized structures with the largest energies and the strongest noise events.

2.b.v. Axial – CLOSE, Azimuthal – FAR: Axially neighboring structures compete with one another for flow energy; limiting their ability to grow. One consequence of these competing structures is that they prevent more energetic structures from developing. This case produces well organized, but reduced energy structures compared to 2.b.iv. These structures don’t produce particularly strong noise events, and they also prevent more energetic structures (that would produce strong noise events) from forming.

2.b.vi. Axial – CLOSE, Azimuthal – CLOSE: Axially neighboring structures compete with each other (as in 2.b.v), but the azimuthal merger process
still allows them to pull out more energy from the flow in comparison to 2.b.v. This case is not substantially different from 2.b.v, but its noise events will be somewhat stronger.

2.b.vii. Final note: If the excitation frequency is too high ($St_{DF} > 1.5$), the instability dynamics simply stop generating structures one-to-one with excitation. This places a limit on how closely structures can be packed and also says that this limit is imposed by the flow dynamics.

2.c. The structure decays due to viscous forces further downstream or disintegrates due to collisions with itself or other structures near the end of the jet potential core.

There are several important observations to be made about this model. In the unexcited jet, a mix of all these cases occur, but since those of 2.b.iv produce the strongest noise events, the far-field spectrum has its peak associated with this periodicity. In the reality of case 2.b.i, the jet will produce other structures in between the ones in question whereas 2.b.v doesn’t allow other structure patterns to occur – this is why exciting the jet into the 2.b.v configuration results in noise reduction while low frequency excitation doesn’t have any effect on the statistical noise picture. Exciting in the 2.b.v regime eliminates the 2.b.iv events that sometimes occur in the unexcited jet, but the rough shape of the spectrum isn’t radically changed because the 2.b.iv events only occur sometimes in the unexcited jet and the broadband shape of the spectrum is dictated, in large part, by the width of the noise events. Lastly, there is a balance that must be struck between the amplitude and the frequency of noise events. This model suggests that, as the structure frequency increases, the number of noise events per unit time will increase.
proportionally. The model also says that the strength of those noise events changes with frequency. As expected, noise reduction is achieved when these two factors are tuned so that the energy per unit time reaching the far-field is less than that of an uncontrolled jet.

9.2.2 The Relationship Between Azimuthal Extent and Radiation Power

As discussed in §7.3, the excited jet results showed that the azimuthal extent of the structures plays a significant role in the low-angle noise. One method for exploring this issue is through a superdirective noise model (§2.5). In this case, instead of using the one-dimensional model (2.7), the noise source is modeled as lying on a cylinder whose radius is the same as nozzle radius (R). Using the same model for the axial fluctuations as in §2.5, the noise source term can be written as

\[ T_{11}(\theta, r) \propto R \delta(r - R) \exp \left[ i(\omega \tau - k \gamma) - \frac{y_i^2}{2 \lambda^2} \right] C_\phi e^{i \theta}, \quad (9.8) \]

where \( \theta \) is the azimuthal coordinate, \( R \) is the jet radius, and there is an implied summation over all integers \( n \) (i.e. Einstein notation). Since the azimuthal content is described as a Fourier series, any azimuthal distribution can be represented.

The solution to Lighthill’s equation using only the \( T_{11} \) source term with the far-field assumption in cylindrical coordinates is

\[ p(x,t) = \frac{\cos^2 \phi}{4\pi |x| a_e^2} \frac{\partial^2}{\partial t^2} \int_{-\infty}^{\infty} \int_{0}^{\infty} T_{11} \left[ y_i, t - \left| x - y_i \right| / a_e \right] r dr \partial \gamma, \quad (9.9) \]

It can be assumed without loss of generality that the azimuthal coordinate of the observer is zero. Using this assumption, the distance between the source and observer in the far-field in cylindrical coordinates is approximately

\[ \left| x - y_i \right| \approx |x| - y_i \cos \phi - R \cos \theta \sin \phi. \quad (9.10) \]
Inserting the source term and evaluating the trivial radial integration, the far-field pressure is

\[
p(\chi, t) \propto \frac{R^2 \cos^2 \phi}{4\pi |x| a_e} \frac{\partial^2}{\partial t^2} \exp\left[i\omega(t - |x|/a_e)\right] \times \\
\int_{-\infty}^{\infty} \exp\left[i(\omega \cos \phi / a_e - k)y_i - y_i^2 / \lambda^2\right] C_n dy_i \int_0^{2\pi} \exp[i\omega R \cos \theta \sin \phi / a_e] e^{in\theta} d\theta.
\]

Equation (9.11)

Inspection reveals that the primary difference between (9.11) and its one-dimensional model equivalent in §2.5 (not shown) is the appearance of a factor containing the azimuthal integral – as should be expected. This integral is of the form of Bessel’s first integral so it can be evaluated as

\[
\Theta_n = \int_0^{2\pi} \exp[i\omega R \cos \theta \sin \phi / a_e] e^{in\theta} d\theta = 2\pi i^n J_n[\omega R \sin \phi / a_e] = 2\pi i^n J_n[\pi St, M_a \sin \phi],
\]

where \(J_n\) is the \(n\)th order Bessel function of the first kind. If \((\pi St, M_a \sin \phi) \approx 0\), all \(J_n \approx 0\) except for \(J_0\) so only the axisymmetric sources can radiate when the argument of the Bessel function is small unless this factor is offset by the Fourier coefficients – a result noted by Cavalieri et al. (2011a). This is mathematical support for the idea that the axisymmetric mode is the most efficient radiator of low-angle jet noise – at least within the scope of these simple models. Equation (9.12) also indicates that the axisymmetric mode is the most efficient radiator for low Strouhal numbers regardless of polar angle, but it must be remembered that the polar angle directivity is dictated by other factors in the model. This result provides a possible explanation for the experimental results showing that exciting the axisymmetric mode with low Strouhal numbers results in strong
amplification. Since it is possible for the azimuthal Fourier coefficients \((C_n)\) to be functions of \(t\) or \(y_1\), the coefficient factor must be left inside the axial integral and time derivatives of (9.11). If \(C_n\) are not functions of \(t\) or \(y_1\), the far-field pressure is the product of the result from the one-dimensional model (2.9) and the azimuthal factor \(C_n \Theta_n\):

\[
p(x,t) \propto A[x] \exp\left[i\omega(t-|x|/a_\infty)\right] C_n \Theta_n,
\]

where all of the various purely real valued factors have been aggregated into \(A\) for compactness.

In order to make a meaningful assessment of the contribution of varying azimuthal extent, it is necessary to assume a model of the azimuthal extent. While a time and space-varying model would be the most representative, that level of complexity rapidly becomes analytically intractable. Therefore, a model that is independent of both \(t\) and \(y_1\) will be used. A simple model for the axial extent is a Gaussian with the scale parameter \((\beta)\). As \(\beta \to \infty\), the Gaussian models an axisymmetric (i.e. \(m = 0\)) source while small values for \(\beta\) model azimuthally confined sources. Without loss of generality, the peak of the Gaussian can be placed at \(\theta = 0\). The Fourier coefficients of the Gaussian are

\[
C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-\beta^2/4} e^{-in\theta} d\theta
\]

\[
= \frac{\beta}{4\sqrt{\pi}} \left( \text{erf}\left[ \frac{\pi}{\beta} + \frac{in\beta}{2} \right] + \text{erf}\left[ \frac{\pi}{\beta} - \frac{in\beta}{2} \right] \right) e^{-\pi^2 \beta^2/4}.
\]

(9.14)

It can be shown that all of the coefficients \((C_n)\) are real and that \(C_n = C_{-n}\), as should be expected from the transform of a real even function. In order to put the width of the Gaussian on a more physical footing, \(\beta\) is defined as
\[ \beta \equiv \frac{\pi \delta \theta}{\sqrt{\ln[2]}}, \tag{9.15} \]

so that \( \delta \theta \) is the percentage of the azimuth with an amplitude greater than \( \frac{1}{2} \).

The properties of the Fourier coefficients of the Gaussian allow for additional simplification of the far-field pressure (9.13). The summation over the Fourier components can be simplified by the following sequence of operations:

\[
C_n \Theta_n = 2\pi C_0 J_0 + 2\pi \sum_{n=1}^{\infty} \left( i^n C_n J_n + i^{-n} C_{-n} J_{-n} \right) \\
= 2\pi C_0 J_0 + 2\pi \sum_{n=1}^{\infty} \left( i^n C_n J_n + i^{-n} C_n (-1)^n J_n \right) \\
= 2\pi \left( C_0 J_0 + 2\sum_{n=1}^{\infty} i^n C_n J_n \right) \\
= 2\pi \left( C_0 J_0 + 2\sum_{n=1}^{\infty} (-1)^{-n} C_{2n-1} J_{2n-1} + 2\sum_{n=1}^{\infty} (-1)^n C_{2n} J_{2n} \right) \\
= M_{\text{even}} + iM_{\text{odd}}, \tag{9.16}
\]

where the second step requires that \( C_n = C_{-n} \) and \( M_{\text{even}} \) & \( M_{\text{odd}} \) are the summations over the even and odd terms respectively. If \( C_n \) are all real (as is the case with the Gaussian), then the final step of (9.16) has decomposed the azimuthal factor into real and imaginary parts. Incorporating this result into (9.13) and retaining only the real portion, the far-field pressure is

\[
p(x, t) \propto A[x] \left\{ \cos \left[ \omega \left( t - \frac{|x|}{a_{\infty}} \right) \right] M_{\text{even}} + \sin \left[ \omega \left( t - \frac{|x|}{a_{\infty}} \right) \right] M_{\text{odd}} \right\}. \tag{9.17}
\]

Only the real part is retained because, as discussed in §2.5, only the real part represents the physical solution. The imaginary part is retained through the preceding calculation steps because of the mathematical convenience of complex numbers. From (9.17) it can be seen that the acoustic power in the far-field is
\[ W_p = \frac{1}{A^2} \int_0^T p^2 \, dt = \frac{1}{2} \left[ M_{\text{even}}^2 + M_{\text{odd}}^2 \right], \]  

(9.18)

where \( T \) is the period of the far-field signal and the power has been normalized by \( A^2 \) so that the quantity \( W_p \) is a dimensionless representation of the energy contribution from only the azimuthal factor. Strictly speaking, the factor of \( \frac{1}{2} \) in (9.18) comes from the integration of the trig functions. In order to numerically evaluate this result, values for \( St_D, M_a, \) and \( \phi \) are required. Using values relevant to the experimental basis for this discussion \( (St_D = 0.2, M_a = 1, \phi = 30^\circ) \), Table 9.1 is computed for several azimuthal extents \( (\delta \theta) \). In addition to the acoustic power \( (W_p) \), Table 9.1 also contains the energy of the Gaussian \( (E) \) and the acoustic power normalized by the Gaussian energy. Since the Gaussian always has unit amplitude, its energy changes as the extent \( (\delta \theta) \) changes. It is therefore possible that the acoustic power could be simply scaling with the energy of the Gaussian. Normalizing the acoustic power by the Gaussian energy scales eliminates this variable.

The first thing to notice is that the normalized power increases with increasing \( \delta \theta \). This shows that the radiating efficiency of a source increases with increasing azimuthal extent. As \( \delta \theta \rightarrow \infty \) (i.e. the extent limits to an axisymmetric source), the acoustic power

<table>
<thead>
<tr>
<th>( \delta \theta )</th>
<th>( \beta )</th>
<th>( E = C_n C_n )</th>
<th>( W_p )</th>
<th>( W_p/E )</th>
</tr>
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<td>0.01</td>
<td>0.038</td>
<td>( 7.53 \times 10^{-3} )</td>
<td>( 2.24 \times 10^{-3} )</td>
<td>0.297</td>
</tr>
<tr>
<td>0.1</td>
<td>0.377</td>
<td>( 7.53 \times 10^{-2} )</td>
<td>0.223</td>
<td>2.97</td>
</tr>
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<td>0.892</td>
<td>5.93</td>
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<tr>
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<td>0.376</td>
<td>5.22</td>
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<td>37.7</td>
<td>0.995</td>
<td>18.7</td>
<td>18.8</td>
</tr>
</tbody>
</table>

Table 9.1: Radiated power versus azimuthal extent \((\delta \theta)\) for \( St_D = 0.2, M_a = 1, \& \phi = 30^\circ \).
saturates indicating that sources which are approximately axisymmetric can be treated as axisymmetric when considering their radiation behavior.

While not tabulated, there are additional behaviors of note with regard to varying the Strouhal number. For \( \delta \theta \ll 1 \), the acoustic power is independent of the Strouhal number. When \( \delta \theta \geq 1 \), only the zeroth order Fourier coefficient is significantly non-zero so the acoustic power behaves like a zeroth order Bessel function (i.e. \( J_0 \)) when varying the Strouhal number. Under the simplification that only \( J_0 \) is significant, it is tempting to think that the oscillatory behavior of the Bessel function indicates that the solution is unphysical. One must remember, however, that the acoustic power (\( W_p \)) calculated here is only the azimuthal contribution – everything else was scaled out. Additional Strouhal number dependence can be (and is) present in the other factors. The combined effect of the various factors can be used to model appropriate Strouhal number dependence for the whole system. Obviously, the complete far-field pressure (9.17) has many dependencies resulting in a complex behavior that cannot be easily described in words—and this is a very simplistic model. The preceding highlights a few of the relevant parameters for the purpose of the present discussion.

The result of this azimuthal extent model provides a basis for some of the suppositions made in the model description (§9.2.1) and at the end of §7.3. Exciting the jet with higher order azimuthal modes restricts the azimuthal extent of the structures, which should create a corresponding restriction of the sources. Within the scope of this model, this result shows that these confined sources are less efficient radiators. If one imagines exciting azimuthal mode three as three independent sources each with an extent of \( \delta \theta = 0.1 \), the combined power would be \( 3 \times 2.97 = 8.91 \) (where 2.97 came from the
appropriate entry in Table 9.1). Thus, several azimuthally smaller sources are still weaker radiators than one axisymmetric source. As an alternative example, the acoustic power for a purely axisymmetric source using the same parameters as those used to generate Table 9.1 ($St_D = 0.2$, $M_a = 1$, $\phi = 30^\circ$) is $W_p = 18.8$ while the acoustic power for a purely mode three source with the same parameters is $W_p = 8.14 \times 10^{-6}$. The disparity in this case is very much larger because of the use of a single simple harmonic as the model of the azimuthal extent. In reality, the decrease in radiating efficiency created by exciting mode three is probably somewhere between the two examples just discussed since a simple harmonic is an over-simplification and the neighboring structures in an $m = 3$ excited jet aren’t actually independent.

If the azimuthal extent of a structure were to vary over the lifetime of a structure (as should be expected given the highly turbulent nature of the jet), the above provides a possible model for the inductive argument made at the end of §7.3. While it is possible to numerically model a source with time dependent Fourier coefficients, it is analytically intractable and the model already presented provides a sufficient basis for a discussion. When a structure is first created (i.e. has small azimuthal extent), this model indicates that it is a relatively inefficient radiator. Under the right circumstances (see §9.2.1) this structure will experience a period of time during which it has a large azimuthal extent and high noise source energy that would result in a period of increased radiating efficiency and radiating power. This period of increased radiation would be a noise event. This uncouples the axial frequency (which should be directly related to the intermittence) from the event width (which the data show is a fraction of the intermittence) while still
maintaining a close relationship between the two quantities through the governing
dynamics of vortex evolution.

9.3 Noise Event Point of Origin

It is tempting to think of the oscillations of the wave-packet models as being
synonymous with the large-scale structures. While they are obviously related, linking
them so strongly is, at best, a premature conclusion and, at worst, a mistake that can
create significant misunderstanding. The shape and placement of the envelope function
plays an important role in this interpretation. For the sake of a discussion, it is assumed
that the oscillating factor and the envelope are tuned to mimic the growth and decay
characteristics of the large scale structures. If linked in this way, the conclusion is as
follows. Once a structure is of sufficient energy, it radiates continually as it propagates
downstream until it is destroyed by viscous forces. Given that the structures in the jet can
be of significant size for several jet diameters preceding the end of the potential core
(Chapter 5), this would imply that the noise source is highly distributed (i.e. highly non-
compact). It is worth noting that this kind of radiation behavior has been known for many
years to exist in a different jet noise generation mechanism, Mach wave radiation (Tam
1971). In these flows, supersonically convecting structures produce this kind of highly
non-compact noise source. This noise source has been studied at GDTL and schlieren
images clearly showing the link between the structures and the Mach wave radiation are
found in Kearney-Fischer et al. (2011a). For further discussion of this noise generation
mechanism, the reader should refer to the cited GDTL publication.
While it is possible that the preceding interpretation is correct, there is some evidence that may contradict that interpretation. As discussed in §2.6, previous work at GDTL has shown that these noise events have an apparent origin in a region (about five jet diameters axially) around the end of the potential core (Hileman et al. 2005; Kastner et al. 2009). This was determined through the use of a beam-forming array utilizing cross-correlation between several microphones and linear acoustic propagation to determine an event origin in space-time. If these events are generated only near the end of the potential core, then the oscillations in a wave-packet model cannot be synonymous with the large-scale structures. At a minimum, the envelope function must describe some significant change in the flow-field that makes the structures radiation capable.

One potential problem with the determination of the space-time origin performed by Kastner and Hileman is that they used a very simple model and did not incorporate the directional dependency of the noise event characteristics (see §6.2.5 and §7.2.3). These directional characteristics can create errors in the calculation of the space-time origin. Additionally, if the noise sources do behave according to the highly non-compact model, they would likely have an apparent origin near the end of the potential core. The apparent location would arise because the shape of the outgoing wave created by the convecting source would be more elliptical rather than spherical. In calculating the origin of noise events, Kastner and Hileman used a fairly simple triangulation procedure that does not account for refraction effects let alone a convecting source. This does not make their work invalid (in fact their work was shown to closely match much more complex beam forming techniques); it just means that there is additional complexity buried in their results that has not been taken into account.
Chapter 10: SUMMARY AND CONCLUSIONS

Following on the previous works showing that jet noise has significant intermittent aspects, the present work hypothesized that these intermittent events are the dominant feature of jet noise. A definition and method of detection for noise events was devised and implemented. Starting with a large experimental database from the NASA AeroAcoustics Propulsion Laboratory (AAPL) of acoustically subsonic jets with different velocities, diameters, and temperatures, these events were extracted from the far-field noise signals. It was shown that a signal containing only these events retains all of the important aspects of the acoustic spectrum for jet noise radiating to shallow angles relative to the jet downstream axis. It is therefore concluded that these intermittent events are the essential feature of low-angle jet noise.

The characteristics of these noise events were statistically analyzed. It was shown that these events are uncorrelated and that they can be statistically described in terms of three parameters ($p_{RMS}$ of the original signal, the mean width of the events, and the mean time between events) and two universal statistical distribution curves. It was found that this intermittent nature occurred most prominently in the low angles and was not detectable for polar angles greater than about $\phi = 60^\circ$ – possibly due to the limitations of the acquisition. These parameters have strong dependencies on jet diameter and temperature. The parameters have a very weak dependence on jet velocity for unheated jets, but have significant velocity dependence in hot jets. The mean frequency of
occurrence of the events was shown to be a good predictor of the spectral peak frequency. It was also found that there exists a strong correlation between the mean width and the mean intermittence (occurrence of the events). The ratio of these two quantities is consistently 0.128. This disagrees with the sinusoidal behavior of typical wave-packet models. A relationship was also established between the amplitude and width that can be partially described by a geometric scaling, but whose details are much more complex than the relationship between the mean width and mean intermittence. The profile obtained by averaging all of the events indicates that the typical event may consist of one positive and one negative swing, but not necessarily in that order. This typical event model is not physically motivated, but is predicated on being the simplest waveform that matches the statistics.

A second experimental database from the Ohio State University Gas Dynamics and Turbulence Laboratory (GDTL) of far-field acoustic data from an excited subsonic \((M_j = 0.9)\) jet at various temperatures \((TTR = 1.0-2.5)\) was then analyzed using the same process to determine what other characteristics of these noise events and their production could be identified. The jet was excited using an array of eight Localized Arc Filament Plasma Actuators (LAFPAs) spaced azimuthally around the nozzle exit. The excitation azimuthal modes and Strouhal number examined were \(m = 0, 1, \& 3\) and \(St_{DF} = 0.09-3.0\) respectively. The basic characteristics of the signal analysis from this database matched those from the unexcited database; indicating that the dynamics being examined are not somehow unique to one facility. In addition to the experimental acoustic database, conclusions and observations from previous works using LAFPAs were leveraged to
inform discussion of the statistical results and their relationship to the flow-field dynamics.

The results from the excited jet provided several more important characteristics that wouldn’t have been discovered without the use of excitation. Analysis of the noise events revealed the existence of a resonance condition in the jet. When the jet is excited at a frequency matching the mean intermittence of the unexcited jet, large noise amplification can occur. This phenomenon is particularly pronounced when exciting the axisymmetric mode. When this occurs, every large-scale structure is producing a noise event. Conversely, noise reduction occurs when only a fraction of the large-scale structures produce noise events. This occurred most substantially when the jet was excited with $m = 3$ and frequencies on the range $St_{DF} = 0.5-1.0$ depending on jet temperature. The other modes also produce noise reduction, but $m = 3$ (the largest simple azimuthal mode that could be excited by 8 LAFPAs), was the most pronounced. These results suggest a process in which noise sources are competing for flow energy and that these noise sources are closely related to the large-scale structures.

A preliminary experiment to explore the behavior and interaction of large-scale structures was conducted. This experiment used a 2.54 cm diameter unheated jet operated at $M_j = 0.9$ excited with the axisymmetric mode. The signature of the jet structures was measured by microphones placed in the near-field hydrodynamic region. By exciting the jet with very low frequencies, the impulse response of the jet was determined. The frequency range of constructive and destructive interaction between large-scale structures was determined. It was found that the peak in the energy density of the large-scale structures corresponds to the resonance condition found in the acoustic results.
A simple model for this kind of temporal noise signal was used to derive a relationship between the characteristics of the noise events and the fluctuations in the integrated noise source volume. While this relationship cannot say a lot about the source dynamics, it represents the limit of how far it is practical to extrapolate the acoustic results without knowledge of the flow-field dynamics occurring at the time of the event generation.

Leveraging the known dynamics of the excited jet flow-field, it was possible to extrapolate further. Based on the known flow-field dynamics and the acoustic results from the excited jet, a hypothetical model of the competition process was described. A brief synopsis of the model is as follows. Depending on the axial and azimuthal proximity of adjacent structures, initially separate structures can combine, work collaboratively to extract energy from the flow, or stifle each other’s ability to extract energy. When the structures are able to consistently extract large amounts of flow energy, they produce strong noise events. When energy extraction is inhibited, noise production is reduced. Noise reduction through excitation is achieved by exciting the jet into a configuration that has reduced noise production while also inhibiting the naturally occurring structures.

Using a wave-packet model on a cylindrical surface, the impact of the azimuthal extent of a source was examined. By modeling the azimuthal extent as a Gaussian, it was possible to write down an analytical expression for the far-field acoustic power. Using this expression it was shown that an axisymmetric structure is the most efficient radiator of sound (within the scope of the model). A structure covering only a fraction of the azimuth was less efficient and the efficiency increases monotonically with increasing
azimuthal extent. It was proposed that a time varying azimuthal extent could explain the noise events by letting the lifetime of an event (i.e. the event width) be dictated by the time during which a structure has large azimuthal extent while the time between events (i.e. the intermittence) is dictated by the time between the large-scale structure occurrences.

In closing, it is important to acknowledge that the question of jet noise sources has not been answered. The present work provides insight on the characteristics of the noise radiated to the low angles (in the form of noise events) and speculates on the dynamics responsible for those events. It does not state the precise dynamics responsible for the noise events, let alone address the sources of noise that radiate to the sideline and upstream angles. While the results found in this work cannot pinpoint noise sources, this new information should help narrow the focus of future work in the pursuit of understanding jet noise.
Chapter 11: Future Work

There is yet more information to be extracted from the acoustic signals through the use of the wavelet transform. As discussed in §2.6, Koenig et al. (2011a) have started using wavelet filtering to extract noise events. The methodology of the present work was partially motivated by previous work at GDTL. In the interest of confirmation, the analysis in this work should be repeated using wavelet transforms. Obtaining the same trends with a noise event definition based on energy concentration, as opposed to simple amplitude, would be a good confirmation that these events are the important signal characteristic.

As an example of the potential power of wavelet transform analysis, a few examples are shown in Figure 11.1. The wavelet transform used is a 4th order Paul wavelet. These signals are taken from the $\phi = 30^\circ$ microphone in a $M_j = 0.9$ jet at TTR = 2.0: a) unexcited, b) an example of exciting the resonance, and c) an example of significant noise reduction. Each signal is normalized by its own $p_{RMS}$ before calculating the transform so that plots represent equivalent comparisons of energy concentration. The quantity plotted in Figure 11.1 is the square modulus of the wavelet transform. For reference, the mean width and intermittence for the unexcited data are on the order of 0.1 and 0.7 milliseconds respectively. It is clear that there are blobs of concentrated energy of varying intensity in the data. While it isn’t practical to examine the time axis in Figure 11.1a at 0.1 millisecond spacing, the unaided eye can pick out the 0.7 millisecond
characteristics. Additionally, the Scale axis shows that 0.1 and 0.7 milliseconds essentially bound a region containing the vast majority of the energy. In the resonance excited case (Figure 11.1b), it is clear that the signal is highly regular and that the energy is more concentrated. Lastly, the noise reduction case (Figure 11.1c) has reduced concentrations of energy as well as reduced occurrences of the very large events.

Additional reasons to do a wavelet transform based study include:

1. Investigating the possibility of algorithmic biases. It is possible, and some instances are discussed, for the definition and algorithm used in the present work to introduce biases. Utilizing a different approach with the same objectives in mind can help identify and eliminate those biases from analysis.

2. Signal reconstruction without adding energy to some portions of the spectrum. As discussed in §6.1, the signal reconstruction used in the present work adds energy to the low frequencies. An appropriately implemented wavelet filter could reconstruct the events without introducing significant amounts of additional energy.

3. Identify peak characteristics while leveraging the power of the wavelet transform. Local maxima in the square modulus wavelet space instantly provide the peak location in time, the dominant scale of the event (i.e. its width), and a metric for the energy concentration in the event. Using a 4th order Paul wavelet, the real and imaginary parts of the wavelet transform indicate if the event is symmetric or antisymmetric respectively. In the case of the antisymmetric events, the sign of the imaginary part indicates whether the event is a positive swing followed by a negative one or vice versa.
Another possibility for future work is to focus on the presence and behavior of the large-scale structures. It has been argued for a long time, and not without reason, that the large-scale structures are the source of low-angle jet noise as well as controlling jet mixing characteristics. It would be ideal if the large-scale structures could be studied...
without the mess of the small-scale turbulence, but this isn’t possible for several reasons. One possibility is to do the exact opposite – study a jet that has no large-scale structures, at least in the initial development of the jet through the end of the potential core.

It may be possible to produce a jet that has no large-scale structures by placing screens in the flow as shown in Figure 11.2. In this conceptual rendering, two screens are supported downstream of the nozzle exit: 1) is just barely beyond the nozzle exit (0.2 diameters) and 2) is 1.25 jet diameters downstream from the nozzle exit. The screens would be much finer that what is shown in Figure 11.2, but would have a high percentage opening so that they don’t present a significant obstruction to the flow. Screens at these locations should break up any large-scale structures before they have a chance to significantly develop. The absence of large-scale structures can be confirmed by exciting the jet in a regime that normally generates robust structures. If structures can’t be

Figure 11.2: Conceptual rendering of jet nozzle with screens in flow path.
detected under excitation, then it is likely that structures would be largely eliminated in
the unexcited jet. By removing a center-portion of the screens so that they don’t interfere
with the jet, it is possible to characterize the impact of the supporting bracketry on the
flow-field and acoustic field. The flow-field and acoustic fields of a jet lacking large-
scale structures could then be studied. Knowing how the jet behaves in the absence of
large-scale structures can shed light on many aspects of jet dynamics including separating
the component of low-angle jet noise that comes from phenomena other than the large-
scale structures.

The last possibility for future research based on the present work is relating structure
energy to noise. With the ability to excite the jet, it is possible to produce structures with
much more consistent characteristics than in the unexcited jet. Using techniques like
high-resolution PIV and hydrodynamic near-field pressure measurements, it is possible to
characterize the energy (size, etc.) of a typical structure and relate those characteristics to
the noise events. The present work has done this qualitatively, but it would be beneficial
to make a quantitative assessment of this relationship.
REFERENCES


