Spins in heterogeneous landscapes: Consequences for transport and imaging

Dissertation

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Spintronics is a burgeoning field that endeavors to use the spin of an electron for information processing. Remarkable progress has been made in understanding injection of spin-polarized electron populations/currents into semiconductors, which are typically non-magnetic, as well as manipulation and detection of said spins.

This dissertation presents a combination of numerical analysis and experiments aimed at understanding and developing imaging tools that utilize the direct coupling of spins with the spatially varying magnetic field of a micromagnetic probe. This coupling produces a spin detection technique which relies on the resulting magnetic force on a cantilever to which the magnetic probe is attached. While extensively used in materials research, such an imaging tool has lagged as a characterization tool for spintronics. This is because of the challenges associated with measuring extremely small forces from these samples, and a lack of understanding of the effects of the inhomogeneous vector field from the probe on the sample spin density.

I have developed a numerical framework, based on solving the spin-transport equation, for understanding behavior of spins in the presence of spatially varying parameters. Based on analysis of the analytically solvable case of spin in a uniform magnetic field, but with multiple vector components, I have identified a crucial quantity, $\theta_B$. This captures the competition between parallel and perpendicular field components dictating the collective spin precession behavior in these spin ensembles.

The numerical simulations show that in the proximity of the micromagnet the
Hanle response is broadening due to the field component parallel to the injected spin direction. This result also sheds light on the effect of magnetic injectors used in spintronics devices. Our simulations show that the Hanle response will also be artificially broadened in this case due to the magnetic field from the injector. Such broadening is the most likely cause for the larger than expected Hanle halfwidths measured in recent experiments, and should be taken into account while extracting spin lifetime from such data.

Insights from numerical analysis have led us to propose scanned magnetic perturbation imaging for spatially mapping spin properties. This technique enhances the imaging capabilities of existing detection methods by encoding local information in the field gradients from a micromagnet. The spin density is governed by a convolution of local spin properties and the local magnetic field that perturbs the spins. By scanning this local perturbation a contrast in the globally detected signal, resulting from sample inhomogeneity, can be seen.

On the experimental side I have measured the broadening of the Hanle response due to a micromagnetic tip, using a globally integrated spin photoluminescence signal from optically excited spins in n-GaAs epitaxial membranes. I have also conducted sensitive force detection experiments to measure the magnetic forces from the spins in these samples. I have measured small forces of the order femtonewtons, similar to that predicted from simulations. The Hanle response of this force signal behaves as expected. This may be the first force detection of non-equilibrium itinerant spin-polarized electrons in paramagnetic semiconductors.
Dedicated to my parents

&

Bhallamudi Seetaram Murty

I wish we had time for another game of chess...
A dissertation is the outcome of a team effort. While I may have been a central player in this one, it is the result of the efforts of numerous people and collaborations. I have been fortunate to have interacted with many who have taught me a lot about scientific enquiry and research.

First and foremost among these is my advisor Chris Hammel. I thank him for his remarkable patience with me during my long road to completion, especially with my kid-in-a-candy-store syndrome in a world with many interesting scientific problems. He is a true scientist who enjoys tackling hard experimental problems, something that I respect him a lot for and attracted me to work with him.

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Chapter 1

INTRODUCTION

Imaging spin-related properties is central to advancing the field of spintronics: the paradigm of utilizing the spin degree of freedom of electrons, in addition to their charge, for carrying information. Spin is related to magnetism and thus, in its simplest form, a spintronic device is one whose response is settable by manipulating the spin, for e.g., by using a magnetic field in addition to electric fields. However, in experiments and devices the magnetic environment experienced by the spins can be complex. The purpose of this dissertation is to theoretically and experimentally explore the behavior of spin-polarized itinerant electron populations in such complex real-world situations, which have not been studied well before. Such understanding has bearings both for the scientific and technological communities.

Research in spin-based devices has been gaining momentum since the early works of D’yakonov and Perel, Tedrow and Meservey, Johnson and Silsbee and others [1, 2, 3, 4, 5, 6]. Those early results heralded the works of Albert Fert, Peter Grünberg, and Stuart Parkin among others, which led to the discovery and eventual commercialization of Giant Magneto-Resistive (GMR) technology [7, 8, 9]. It has also given impetus for some important fundamental scientific exploration of the behavior of electronic spins in condensed matter systems [10]. Commercial hard disk read heads based on GMR technology were introduced within a decade of its discovery in 1988, and have
resulted in an increase of hard disk information storage densities by many orders of magnitude. Such spectacular early success has led to recognition such as the Nobel prize for physics in 2007 (Fert and Grünberg), and also to an explosion of research work.

The GMR devices mentioned in the previous paragraph are all-metallic structures. Interesting and remarkable research continues to focus on such all metallic structures, where the main thrust is towards making a viable commercial memory, popularly known as the Magnetic Random Access Memory (MRAM) and spin-torque based oscillators. MRAM devices are in early stages of commercialization and hold the potential to revolutionize the computing world. They integrate the read/write and the memory elements into the same device making them as fast as the conventional RAM, while maintaining the non-volatility of magnetic memory elements like those in hard drives. They can also be energy efficient, given the energy scales for magnetic processes. As such, MRAM promises to be a panacean memory which could replace all the various kinds of memory currently on the market.

While work on MRAM has also lead to some interesting scientific and technical innovations, such as spin-torque effect and current driven domain walls, many researchers have been focusing on the next step—combining logic and memory elements. The following sentence from one of the most cited reviews of this field [11] captures the long term vision of this branch of research: “It is envisioned that the merging of electronics, photonics, and magnetics will ultimately lead to new spin-based multifunctional devices such as spin-FET (field effect transistor), spin-LED (light-emitting diode), spin-RTD (resonant tunneling device), optical switches operating at terahertz frequency, modulators, encoders, decoders, and quantum bits for quantum computation and communication”. Spin-FET refers to a field-effect transistor like device where the source-drain current can be controlled via a magnetic knob, in addition
to the usual charge-coupled gate. Spin-LED provides the additional functionality of being able to control the polarization of the emitted light. Other devices also offer similar additional tunability via a magnetic knob.

1.1 Motivation for this research and organization of the chapters

However, good things usually come with caveats. Implementing logic typically requires integrating semiconducting elements with sources of spins, typically ferromagnetic contacts. This brings along with itself a host of issues relating to ferromagnet-semiconductor interfaces and their effect on injection and transport of spins. These issues and the control of spin transport within the semiconductor have been the crux of the problem in this field thus far. Several tools have been employed successfully to study these issues. Prominent among them are optical methods: spin-polarized electro-/photo-luminescence and Faraday/Kerr microscopy; and magneto-transport measurements. While these have resulted in significant progress in the field, they have limitations. Optical methods, for example, work well only for the subset of optically active materials such as the direct band gap III-V semiconductors. Electrical measurements have to deal with the compounded problem of multiple interface.

The primary motivation for this work was to investigate the potential of using a scanned magnetic force probe as an imaging tool [12, 13, 14, 15] for spintronic applications. Such a tool offers the crucial benefit of direct magnetic coupling to the electronic magnetic moment, making it largely material and structure insensitive. This makes it appealing for technologically important semiconductors like silicon, which is not optically active and still has significant issues with electrical methods. However, the development of such a tool is challenging. One of the reasons being the extremely small number of non-equilibrium spins “injected” in these paramagnetic structures
which results in extremely small forces. To establish the feasibility of force detection, the experiments in this research were done on GaAs using optical injection, which has been well studied by other detection methods. I used spin-photoluminescence (spin-PL) [16, 17] to understand spin physics in these sample before conducting experiments on the effects of/and on a magnetic probe. In **Chapter 2** I will provide the background for optical injection of spins and spin-PL detection. I will also provide the background for sensitive force detection.

Another important reason for the paucity of research on force detection is the lack of understanding of the effects of the spatial inhomogeneity of the magnetic field produced by the tip. Most of the experimental work in the field of spintronics relies on the assumption that spins largely see a uniform environment, especially with regards to magnetic fields. Only fairly recently [18, 19] have the consequences of inhomogeneities been acknowledged. However, a clear understanding is far from available. **Chapter 3** presents a numerical analysis for the behavior of spins in spatially varying environments. I show that, not only can the impact of the inhomogeneous fields be understood, one can also benefit from them. I will present in this chapter, one of the important results in this work, a proposal for a novel imaging technique for spin properties. This technique uses the spatially confined field of a scanned micromagnetic probe, in conjunction with existing electrical or *global* optical spin detection schemes. It is thus applicable to all materials systems susceptible to either of those approaches and may be widely applicable. The simulations also provide estimates for the forces that can be expected from spins injected into paramagnetic semiconductors.

In **Chapter 4** I will cover the experimental aspects of this research. I will first present detailed spin-PL measurements which establish the successful injection of spins and characterize the spin behavior in the samples used in my research. Then I will present the experiments measuring the effects of local micromagnetic probe on
the spin-PL signal, verifying some of the simulation results. I will then discuss experiments on force detection. While feasible according to simulations, these experiments remain quite challenging. Though many issues remain, I will present what may be the first measurement of magnetic forces from a non-equilibrium itinerant spin-polarized electron population.
The intent of this chapter is to provide sufficient background for understanding the research presented in this dissertation and its place in the general development of the field. The poster child of semiconductor spintronics is a device proposed in 1990 by Datta and Das [20], now popularly known as the spin-FET (fig. 2.1), in analogy to regular field-effect transistor (FET). However unlike regular FET devices which have normal metal contacts, the source for a spin-FET is a ferromagnet which injects spins into a 2D electron gas (2DEG). While all electrons possess spin, the term spins here refers to the notion of a spin-polarized electron population with a net number of spins of a particular orientation. The spins are manipulated in the 2-DEG by a gate voltage. The gate voltage is transformed into a magnetic field, $B_g$, due to the relativistic motion of the electron in an electric field. This is one kind of spin-orbit (SO)\textsuperscript{1} type field [21, 16, 22, 23] and $B_g \propto \nabla V_g \times \mathbf{p}$, where $V_g$ is the gate voltage and $\mathbf{p}$ the linear momentum of the electron. The spins precess around this perpendicular field at the Larmor frequency, $\Omega_L = g\mu_B B_g/\hbar$, where $g$ is the gyromagnetic ratio of the electron and $\mu_B$ is the Bohr magneton. The resistance to the current flow depends on the orientation of the ferromagnetic detector relative to the spins reaching it. This

\textsuperscript{1}See section 2.3.1 for more details.
will depend on the precession frequency, and thus in turn on the gate voltage. Spin information will be lost over a time $\tau_s$, called the spin lifetime, which is an important parameter. The spin diffusion/drift length, $L_s$, is given by $\sqrt{D_s\tau_s}$ or $v\tau_s$ for diffusion and drift regimes respectively, where $D_s$ is the diffusion constant and $v$ the drift velocity. $L_s$ sets the length scale for spin based devices.

While still unrealized, the spin-FET is a canonical device which has the three key components of a semiconductor spintronic device: a. injection of spins into a semiconductor, b. transport and manipulation/control of spins within the semiconductor, and c. detection of spins.

### 2.1 Spin dynamics in a non-magnetic material

The above mentioned concepts can be described in a more mathematical way by the Spin Diffusion/Transport Equation which governs the dynamics and transport of spins in non-magnetic materials, such as the paramagnetic semiconductors I am interested in for this research. The equation of motion for the spin density $S(r,t)$ is
given by \[24\]
\[
\frac{\partial S}{\partial t} = G + D_s \nabla^2 S + \zeta (E \cdot \nabla) S + \gamma \mathbf{B} \times \mathbf{S} - \frac{S}{\tau_s},
\]
(2.1)
where \( \mathbf{r} = (x, y) \) is the spatial coordinate within the 2D sample and \( t \) is time. The first term, \( G \), represents the spin generation or injection. The second and third terms dictate the diffusion and drift of the spins, where \( D_s \) is the diffusion constant, \( E \) is the electric field and \( \zeta \) the mobility. The fourth term represents a convenient way to manipulate spins, the precession of the spins around a net vector magnetic field \( \mathbf{B} \), with \( \gamma = g \mu_B / \hbar \) being the gyromagnetic ratio. The final term represents spin relaxation. From a device point of view one is interested in detecting \( S \) after spins have has been injected and then manipulated by various intrinsic and extrinsic factors such as diffusion, relaxation, and electric and magnetic fields.

In principle all the variables, \( \mathbf{B}, G, \tau, D_s, \) and \( \gamma \), can be functions of time and space, however I will only consider time-independent parameters. Furthermore, \( D_s \) and \( \gamma \) will also be independent of space in this dissertation. I will also assume \( E = 0 \), and unless stated otherwise I am interested in the steady state solution, \( \partial S / \partial t = 0 \).

This formalism can be made more comprehensive by including all the effects of spin-orbit coupling and hyperfine interactions. Even though I restrict myself to 2D samples in this dissertation, the formalism can also extend to the third spatial dimension. The simpler model helps focus on the problem I wish to study: inhomogeneous magnetic fields from a magnetic probe.

In the rest of this chapter, I will discuss the some background for understanding injection and detection of spins, and their relaxation and precession, as applicable to my research. In this chapter I will restrict myself to spatially uniform magnetic fields. In my experiments spins were created in a n-GaAs sample using optical injection. They were then detected using spin photoluminescence and magnetic force detection. I will discuss these concepts in more detail in the following section.
2.2 Optical Injection and detection

Optical injection was first studied, theoretically and experimentally, in the Soviet Union in the 1970s [1]. A net electron spin polarization can be created in the conduction band of direct bandgap optically active materials on excitation by circularly polarized light. This is made possible in some materials, e.g. III-V compounds with zinc blende structure such as GaAs, due to a combination of optical selection rules and band structure details.

2.2.1 Band structure of III-V optically active semiconductors

Gallium Arsenide is a typical example of a material with zinc blende structure. It has a face-centered cubic (FCC) lattice with a basis consisting of two different atoms at (0,0,0) and (1/4, 1/4, 1/4). In contrast, silicon and germanium have a diamond structure which has an FCC lattice with a basis at (0,0,0) only. The valence electrons in the Ga: 4s^2 3p^1 and As: 4s^2 3p^3 form sp^3 hybrid orbitals. Due to electronegativity considerations, As 3p orbitals contribute more to the valence band, while the conduction band have a more s-like character from the 4s band of Ga atoms. However, neither band is purely p or s in character.

The energy levels at the zone center for GaAs are shown in Fig. 2.2. I am interested in computing the wavefunctions of these bands in order to understand the optical transition probabilities on excitation with circularly polarized light. This can be done relatively easily for the s-like conduction bands due to the lack of orbital angular momentum (L = 0). The electron wavefunction in the conduction band is described by a Bloch function [16]

$$\Psi_{m,k}^{c} = e^{i\mathbf{k}\cdot\mathbf{r}}u_{m,j}(\mathbf{r})$$

Here $m$ represents the electron spin orientation, $\mathbf{k}$ is the crystal momentum and...
Figure 2.3: Energy levels near the Γ point in zinc-blende crystals.

Figure 2.2: Electronic band structure of GaAs near the zone center showing the s-like conduction band, heavy-hole and light hole valence sub-bands, and split-off sub-band.[25]

Bloch amplitude $u_{m_j}$ is invariant under the symmetry transformation of the crystal ($u_{m_j}(r) = u_{m_j}(r + T)$). Considering the angular part of the Bloch amplitude, it can easily be separated into orbital and spin parts since $L = 0$, and be written in the basis of $|l, m_l, s, m_s\rangle$

$$u_{1/2}(r) = S \uparrow \propto |0, 0, \frac{1}{2}, \frac{1}{2}\rangle, \quad u_{-1/2}(r) = S \downarrow \propto |0, 0, \frac{1}{2}, -\frac{1}{2}\rangle \quad (2.3)$$

where $|l, m_l\rangle = |0, 0\rangle = Y_0^0 = 1/\sqrt{4\pi}$, and $Y_0^0$ is the first spherical harmonic.

However, the situation in the valence band is more complicated, especially for heavier atom like As. There is significant spin-orbit coupling due to the crystal field of the nuclei, which has a Hamiltonian of the form $H_{so} = \lambda \mathbf{L} \cdot \mathbf{S}$. Due to this interaction,
and $S$ are no longer good quantum numbers and total angular momentum $J$ and $J_z$, derived from $\hat{J}^2 = (\hat{L} + \hat{S})^2$, should be considered. The spin-orbit interaction energy results a loss of degeneracy of the valance bands. At the zone center, sub-bands with $(J, J_z) = (3/2, \pm 3/2), (3/2, \pm 1/2)$ form a 4-fold degenerate band while $(1/2, \pm 1/2)$ form a 2-fold degenerate band as shown in Fig. 2.2. Away from the band center, $(J, J_z) = (3/2, \pm 3/2)$ and $(3/2, \pm 1/2)$ further split into two sub-bands with different curvatures, and are known as the heavy-hole (HH) and light-hole (LH) bands based on their effective masses. At the zone center, the irreducible representation is $\Gamma_6$ for the conduction band, $\Gamma_7$ for the split-off band of the valence band and $\Gamma_8$ for the heavy and light holes valence band.

Heavy hole and light hole wavefunctions in the valence band are given by

$$\Psi^{(hh, lh)}_{MK} = e^{i \mathbf{k} \cdot \mathbf{r}} \sum_{m_j} \chi_{Mm_j}^{(3/2)} u^{(3/2)}_{m_j}$$

where $M = \pm 3/2$ for HH and $\pm 1/2$ for LH; $\chi_{Mm_j}(\mathbf{k})$ is the coefficient that depends only on the direction of $\mathbf{k}$ and are related to the finite rotation matrices $D^{(3/2)}_{\mu M}(\phi, \theta, \psi)$, where $\theta$ and $\phi$ are the polar angles of the vector $\mathbf{k}$ with respect to some fixed coordinate frame; $\psi$ is an arbitrary angle defining the phase of the wavefunction. The index $M$ denotes the value of the component of the electron angular momentum in the valence band along the direction of the wave vector $\mathbf{k}$.

Likewise the electron wavefunction in the split-off band is given by

$$\Psi^{(so)}_{MK} = e^{i \mathbf{k} \cdot \mathbf{r}} \sum_{m_j} D^{(1/2)}_{m_j M}(\phi, \theta, \psi) u^{(1/2)}_{m_j}$$

The projection $M$ of the angular momentum in the direction of $\mathbf{k}$ assumes the values $M = \pm 1/2$. The index $m_j$ also assumes the value $\pm 1/2$. 
The Bloch amplitude $u^{l}_{m_{j}} = |j, m_{j}\rangle$ can be expressed in the basis of $|l, m_{l}, s, m_{s}\rangle$ by using the Clebsch-Gordon coefficients

$$u^{l}_{m_{j}} = |j, m_{j}\rangle = \begin{cases} 
|\frac{3}{2}, \frac{3}{2}\rangle = |1, 1, \frac{1}{2}, \frac{1}{2}\rangle & \text{hh and lh bands} \\
|\frac{3}{2}, \frac{1}{2}\rangle = \frac{1}{\sqrt{3}}(|1, 1, \frac{1}{2}, -\frac{1}{2}\rangle + \sqrt{2}|1, 0, \frac{1}{2}, \frac{1}{2}\rangle) \\
|\frac{3}{2}, -\frac{1}{2}\rangle = \frac{1}{\sqrt{3}}(|1, -1, \frac{1}{2}, \frac{1}{2}\rangle + \sqrt{2}|1, 0, \frac{1}{2}, -\frac{1}{2}\rangle) \\
|\frac{3}{2}, -\frac{3}{2}\rangle = |1, -1, \frac{1}{2}, -\frac{1}{2}\rangle & \text{split-off bands} \\
|\frac{1}{2}, \frac{1}{2}\rangle = \frac{1}{\sqrt{3}}(|\sqrt{2}|1, 1, \frac{1}{2}, -\frac{1}{2}\rangle - |1, 0, \frac{1}{2}, \frac{1}{2}\rangle) \\
|\frac{1}{2}, -\frac{1}{2}\rangle = -\frac{1}{\sqrt{3}}(|\sqrt{2}|1, -1, \frac{1}{2}, \frac{1}{2}\rangle + |1, 0, \frac{1}{2}, -\frac{1}{2}\rangle) & 
\end{cases}$$

(2.6)

### 2.2.2 Spin-polarized excitation with circularly polarized light

Let us now consider optical excitation by light of energy $E_{g}$, the bandgap of the semiconductor discussed above. By Fermi’s golden rule, the probability of transition between two bands on excitation by light of frequency $\omega$ is proportional to

$$R_{vb\rightarrow cb} = \frac{2\pi}{\hbar} \left| \langle \Psi_{cb} | e\mathbf{r} \cdot \mathbf{E} | \Psi_{vb}\rangle \right|^{2} \delta(E_{g} - \hbar\omega)$$

(2.7)

where $e$ is the electron charge, $\mathbf{r}$ the position operator, and $\mathbf{E}$ is the electric field of the light radiation. For right light of energy $(\sigma^{+})$ and left $(\sigma^{-})$ circularly polarized light, $\hat{r} = -(\hat{x} + i\hat{y})/\sqrt{2}$ and $(\hat{x} - i\hat{y})/\sqrt{2}$ respectively. Also, the light propagation direction is assumed to be normal to the sample plane, $\mathbf{k} = k\hat{z}$.

The second rule that governs the optical excitation of electrons by light is the quantum-mechanical selection rule that only allows transitions with $\Delta m_{j} = \pm 1$ only. Using these rules, the probability of excitation from various valance band states into the various conduction band states can be computed. Following are the results for
RCP light ((2.8a),(2.8b)) and for LCP light ((2.8c),(2.8d))

\[
\left| \langle \frac{1}{2}, \frac{1}{2} | \mathcal{Y}^{-1} \frac{3}{2}, -\frac{1}{2} \rangle \right|^2 = \left| \langle 0, 0, \frac{1}{2}, \frac{1}{2} | \mathcal{Y}^{-1} \left( \frac{1}{\sqrt{3}} |1, -1, \frac{1}{2}, \frac{1}{2}\rangle + \sqrt{2} |1, 0, \frac{1}{2}, -\frac{1}{2}\rangle \right) \right|^2 = \frac{A_0}{3}
\]

(2.8a)

\[
\left| \langle \frac{1}{2}, -\frac{1}{2} | \mathcal{Y}^1 \frac{3}{2}, -\frac{3}{2} \rangle \right|^2 = \left| \langle 0, 0, \frac{1}{2}, -\frac{1}{2} | \mathcal{Y}^1 |1, -1, \frac{1}{2}, -\frac{1}{2}\rangle \right|^2 = A_0
\]

(2.8b)

\[
\left| \langle \frac{1}{2}, \frac{1}{2} | \mathcal{Y}^{-1} \frac{3}{2}, \frac{3}{2} \rangle \right|^2 = \left| \langle 0, 0, \frac{1}{2}, \frac{1}{2} | \mathcal{Y}^{-1} |0, 1, \frac{1}{2}, \frac{1}{2}\rangle \right|^2 = A_0
\]

(2.8c)

\[
\left| \langle \frac{1}{2}, -\frac{1}{2} | \mathcal{Y}^{-1} \frac{3}{2}, \frac{3}{2} \rangle \right|^2 = \left| \langle 0, 0, \frac{1}{2}, -\frac{1}{2} | \mathcal{Y}^{-1} \left( \frac{1}{\sqrt{3}} |1, 1, \frac{1}{2}, \frac{1}{2}\rangle + \sqrt{2} |0, 1, \frac{1}{2}, \frac{1}{2}\rangle \right) \right|^2 = \frac{A_0}{3}
\]

(2.8d)

where \( Y^\pm l_1 \) is the spherical harmonic representing the operators for \( \sigma^\pm \) lights, and \( A_0 \) is a positive real constant. These probabilities are compactly shown in Fig. 2.3. As can be seen from the figure, excitation by purely circularly polarized light will produce \( \frac{A_0}{3}(1 - 1/3) = 50\% \) spin-polarized electrons.

While this may be considered as transferring the angular momentum of the light to the spin of an electron, the direct coupling between an electron spin and light is negligible. The probability of transitions from the split off bands can be computed for the cases where they are energetically allowed,

\[
\left| \langle \frac{1}{2}, \frac{1}{2} | \mathcal{Y}^1 \frac{1}{2}, -\frac{1}{2} \rangle \right|^2 = \left| \langle 0, 0, \frac{1}{2}, \frac{1}{2} | \mathcal{Y}^1 \left(-\sqrt{2/3} |1, -1, \frac{1}{2}, \frac{1}{2}\rangle + |1, 0, \frac{1}{2}, -\frac{1}{2}\rangle \right) \right|^2 = \frac{2A_0}{3}
\]

(2.9a)

\[
\left| \langle \frac{1}{2}, -\frac{1}{2} | \mathcal{Y}^{-1} \frac{1}{2}, \frac{1}{2} \rangle \right|^2 = \left| \langle 0, 0, \frac{1}{2}, -\frac{1}{2} | \mathcal{Y}^{-1} \left(\sqrt{2/3} |1, 1, \frac{1}{2}, \frac{1}{2}\rangle - |1, 0, \frac{1}{2}, \frac{1}{2}\rangle \right) \right|^2 = 2A_0/3.
\]

(2.9b)

As can be seen when the split-off bands are excited, along with the LL and HH
Figure 2.3: The probabilities of various transitions on excitation with circularly polarized light are shown in the figure. More of $m_j = -1/2$ ($+1/2$) will be excited by $\sigma^-$ ($\sigma^-$) light (of energy smaller than the split-off to conduction band transition) resulting in spin polarized excitation. However, if the split-off band transitions are also allowed, then the spin polarization is lost.

bands, the overall probability probability of transition to both the spin states would be equal and no spin polarization would be obtained. However, the orbital motion couples to the light through an electric dipolar coupling, and to the spin through spin-orbit coupling. It is only due to the spin-orbit coupling that spin-polarization is created in the conduction band on optical excitation.

2.2.3 Optical spin detection using luminescence

Spin-polarized electron populations recombine following the same quantum-mechanical selection rules and transition probabilities as discussed in the previous subsection and thus emit circularly polarized light. Spin polarized luminescence is thus the inverse of optical excitation discussed in the previous section.

The device shown in the Fig. 2.4 was used for one of the first successful demonstrations of electrical spin injection, and used BeZnMnSe as the spin injector [26] in a light
emitting diode structure. Injected spin-polarized electrons recombine with holes in a quantum well and emit spin polarized luminescence. The polarization of electrons, $P_n$, can be extracted by measuring the relative amounts of $\sigma^+$ and $\sigma^-$, right and left circularly polarized components respectively, using polarization sensitive optics. The advantage of this method is a simple and standard LED structure for fabrication. Since then the technique has also been used to demonstrate spin injection into GaAs and silicon from Fe via tunnel contacts [17, 27]. Circular polarization measured in Fe/Al$_2$O$_3$/Si/n-AlGaAs/i-GaAs/p-AlGaAs, n-i-p quantum well structures is shown in fig. 2.5. While spin-polarized EL is experimentally not as challenging as some of the other techniques, the LED structure is not easily compatible with different device structures or materials. Also, it does not have imaging capabilities.

Figure 2.4: Device schematic of a spin LED device for spin-EL detection. (From [26])

2.3 Spin relaxation and precession

Spin information of the electrons can be both manipulated by and lost due to magnetic fields through magnetic coupling of the electronic moment with the field. Spins (magnetic moments) precess around the net magnetic field they experience (Fig. 2.6), in a cone
Figure 2.5: Measured circular polarization spectrum of EL for spin injection into Si from Fe via Al₂O₃ tunnel barrier. (From [27])

whose half angle is the angle between the spin and the magnetic field, until damping processes cause the spins to align with the field. The equation of motion for the precession is given by \( \gamma B \times S \). The two main sources of fields which can govern spin behavior in a paramagnetic material are spin-orbit fields and externally applied magnetic fields.

### 2.3.1 Spin relaxation and spin-orbit coupling

The relativistic motion of an electron relative to an electric potential, \( V_{so} \), results in a Lorentzian transformation of the electric potential into a magnetic field [16, 22, 23]

\[
B_{so} = \frac{\hbar[\nabla V_{so} \times \mathbf{p}]}{4m_e^2c^2},
\]

where \( m_e \) is the effective mass of the electron, \( c \) is the speed of light, and \( e \) the electronic charge. There are different spin-orbit interactions for conduction electrons, depending on the origin of electric potential, \( V_{so} \). This can
be due to: crystal fields from the nuclei, that create spin-orbit split bands and allow for optical spin injection; gate voltages; lack of inversion symmetry in the crystal structure which results in potentials with non-zero SO field; built-in potentials in a semiconductor heterostructure resulting from band offsets; any strain in the system that can change the local potential in the lattice.

Spin orbit fields can be used for manipulating spins [20, 28, 29]. There are various degrees of freedom to modify the spin orbit coupling, such as material, device structure, strain, and external electric fields. Spin-orbit fields can be quite significant and of the order of tesla. While this field can be used to manipulate the spins via a gate voltage, it is also related to many of the spin-relaxation processes. Thus, being able to optimize SO-coupling is important to obtaining a controlled manipulation of spins.

Non-equilibrium spin information is lost due to two classes of mechanisms: 1.
**Spin-lattice relaxation**: spins loose energy to the surrounding thermal bath (the lattice) and relax to thermal equilibrium. The spin-polarization decays exponentially over a characteristic time scale called $T_1$; 2. **Spin-spin dephasing**: The decay of magnetization in the transverse direction to an externally applied field, and occurs due to the precession of spins in local magnetic fields. Different spins in an ensemble experience different local fields in space and time, and *dephase* relatively to each other. Polarization typically decays exponentially over a characteristic time called $T_2$. Both these time scales can be measured in different ways, and most commonly using magnetic resonance techniques. Many spintronic experiments also provide a way to measure these quantities.

The main mechanisms for relaxation/dephasing of conduction electrons result from local magnetic fields whose orientation relative to the spins can change as they undergo momentum scattering. These fields can cause the electrons to precess around them and dephase the spins relative to one another. In the presence of spin-orbit interaction, the eigen states of the electronic wavefunction are an admixture of the two spin states. This admixture is sensitive to the local magnetic fields resulting from spin-orbit interactions and can result in loss of spin information. Following are the important mechanisms for spin relaxation/dephasing of conduction electrons in semiconductors:

**Elliott-Yafet** [30] Electric potential from the lattice(phonons) and impurities can result in spin-orbit fields which cause spin relaxation. This is usually more important for metals, and small band-gap semiconductors with inversion symmetric structure and large $\mathbf{L} \cdot \mathbf{S}$ coupling.

**D'yakonov-Perel’** [2] This occurs in structures lacking inversion symmetry, where the consequent potentials can result in appreciable spin-orbit fields. Inversion asymmetry can be inherent to the material(for e.g., GaAs), or due to the device
being a heterostructure, or even strain.

**Bir-Aronov-Pikus** This is important for p-doped semiconductors where the exchange interaction between the holes and the electrons results in an effective magnetic field. The Hamiltonian for the interaction is given by \( AS \cdot J \delta(r) \), where \( A \) is the exchange integral between the valence band holes and conduction band electrons, \( J \) is the total angular momentum operator for holes, \( S \) the electron spin operator, and \( r \) is the relative position of electrons and holes.

Further details of spin relaxation mechanisms can be found in the review by Zutic [23]. For semiconductor conduction electrons spin lifetimes can reach up to microseconds at low temperatures (< 10K). However, these time scales are temperature dependent and at room temperature these are typically less than a nanosecond. Also, spin lifetime may have a non-monotonic dependence on the doping level. For GaAs spin lifetime tends to peak around a doping level of \( 3 \times 10^{16} \) cm\(^{-3} \) for n-GaAs, where it undergoes a metal-insulator transition. For this doping level D’yakonov-Perel’ is the dominant spin relaxation mechanism and spin lifetimes of \( \sim 100 \) ns are typical.

### 2.3.2 Spin precession and Hanle effect in an external field

If an external magnetic field transverse to the direction of injected spin is applied, the spins will precess around it resulting in a signature behavior called the Hanle curve (or response) [16]. This is a plot of the spin component along the original spin direction (by convention along the \( \hat{z} \)) versus the transverse Hanle field strength (by convention \( B = B_h \hat{y} \)). In the simple case of the continuous wave (CW) injection and globally integrated detection, as is the case in many optical injection and PL detection experiments, the integrated \( S_z \) is described by a Lorentzian lineshape. This behavior results from a combination of the spin precession and spin relaxation. The continuous injection process creates spins of different ages. Their precession angle,
and thus their projection along the $S_z$, will depend on their age and the magnetic field strength (Fig. 2.7). Spin relaxation acts as a weighting function to these projections, and mathematically this behavior can be expressed in the following form

\[ \mathcal{H}_a = \int_{A_d} S_z \propto \int_0^\infty \exp(t_a/\tau_s) \cos(\gamma t_a B_{th}) \, dt_a \]

\[ = \frac{1}{1 + (\gamma \tau_s B_{th})^2} \]

where $A_d$ is the area of the integral and in this case extends over the entire region where spins exist, $t_a$ is the age of the spin: the time from injection till detection, and $B_{th}$ is the transverse Hanle field.

The Hanle response is one of the acid measurements for successfully demonstrating spin injection and spin dynamics. As seen from Sec. 2.10b it can help in measuring the spin lifetime. However, it need not always follow the Lorentzian behavior seen here or be a Lorentzian with its width determined by the spin lifetime. One case where the width is set carrier recombination rates in presented in the next section. There will be a more detailed discussion of the Hanle behavior under different conditions in Chapter 3.

### 2.3.3 Spin relaxation vs. carrier recombination

In the discussion thus far, I have ignored the carrier recombination process, which can also limit the life of a spin in optical injection and detection experiments. Two extreme cases can be considered to illustrate the effect:

**Electron polarization in a p-doped material with $\tau_s \gg \tau_r$:** All the carriers in the conduction band are created by the optical excitation process only. Thus, no spins can survive beyond the carrier lifetime, $\tau_r$, given by electron-hole recombination process. Thus the effective spin lifetime is $\tau_r$. 

20
Figure 2.7: The different arrows show the evolution of spin in time. Spins precess around a transverse magnetic field at a rate proportional to the magnetic field strength. The reduction in the magnitude of the spin is due to spin relaxation over the time of precession. This precession along with relaxation determines the net projection of the spin ensemble along the injected spin direction, and results in the Hanle response.

**Samples with** $\tau_r \gg \tau_s$: Once excited into the conduction band, spin-polarized electrons will survive as long the spin relaxation processes allow them. Thus the effective lifetime is the intrinsic spin lifetime.

In general the effective spin lifetime for optical experiments are governed by a combination of carrier recombination and spin relaxation. Recombination also affects the net polarization that can be achieved in the optical pumping process.
Let us denote the recombination rate by \( r \), \( n \) and \( p \) as the electron and hole densities on excitation, \( n_0 \) and \( p_0 \) are the respective densities without excitation (either given by doping or the intrinsic densities of the semiconductor), the difference in electron densities of the two spin states by \( \Delta_n \), and the polarization of the electrons created by optical pumping process is \( P_i \) (0.5 for GaAs). The charge and spin rate balance equations can be now written as \[23\]

\[
\begin{align*}
  r(np - n_0p_0) &= G \\
  r\Delta_n np + \Delta_n / \tau_s &= P_i G
\end{align*}
\]  

These equations are written under the assumption of steady-state and that the hole spin lifetime is much shorter than the electron spin lifetime, and holes are essentially unpolarized on the time scale of interest here. From \[2.11\], the polarization of the electrons achieved in steady state, \( P \) can be written as

\[
P = P_i \frac{1 - \frac{n_0p_0}{np}}{1 + (\tau_s rp)^{-1}} \tag{2.12a}
\]

For a p-doped material, \( p \approx p_0 \) and \( n \gg n_0 \) and \( 1/rp_0 = \tau_r \), thus

\[
P = P_i \frac{1}{1 + \frac{\tau_s}{\tau_r}} \tag{2.12b}
\]

and for n-doped material where \( n \approx n_0 \) and \( p \gg p_0 \)

\[
P = P_i \frac{1}{1 + \frac{\tau_s}{\tau_f}} \tag{2.12c}
\]

Recombination lifetime is not defined for electrons in n-type materials and is replaced by \( \tau_j = n_0/G = rp_0 \), which accounts for the recombination dynamics. It is the time over which the fermi sea is replaced by photo-excited electrons. The effective spin lifetime, \( \tau_{s,eff} \), is given by an addition of the recombination and relaxation rates and
for a n-type material [16, 23],

\[
\frac{1}{\tau_{s,\text{eff}}} = \frac{1}{\tau_s} + \frac{G}{n_0}
\]  

(2.13)

2.4 Fundamentals of high sensitivity magnetic force detection

Having discussed the basics of creating and detecting spins, and their behavior within a semiconductor, I now turn my attention to discuss basics of another important tool that I used in my research. Scanned probe force microscopy is a very sensitive characterization tool for studying magnetic phenomena on a micro- or nano- scale. One of the main motivations for this research is to develop magnetic force microscopy as a tool for spintronics. Injected spin polarized populations in paramagnetic semiconductors are orders of magnitude smaller than those in ferromagnets, which are usually studied using this tool. Thus the forces are also expected to be orders of magnitude smaller than those for ferromagnetic samples, which are typically in the pico- to nano-newton range. Central to my discussion is the concept of sensitivity and how high sensitivity force detection may be achieved. Sensitivity is usually understood, quantitatively, in terms of signal-to-noise Ratio (SNR), which should be greater than one for the experiment to work, i.e., the signal should be greater than noise. Thus, the approach is two-fold: reduce noise and increase signal. I will discuss these two aspects in this section. A good understanding of the sources and types of noise allows one to take steps to mitigate it, either by reducing its amplitude or by using techniques, such as modulation, which allows for decoupling from the noise in the frequency space.
2.4.1 What is a magnetic force microscope

The schematic of a typical magnetic force microscope (MFM) set-up is shown in Fig. 2.8. The MFM consists of a transducer that converts magnetic force into a measurable voltage. This is achieved by using a micro-mechanical cantilever with a micro-magnetic tip at its end. The cantilever is a simple harmonic oscillator (SHO). However, its properties such as the spring constant \(k\), resonant frequency \(\omega_0\), and oscillation amplitude \(z_{osc}\) can change due to interactions between the tip and the sample. The imaging capability comes from mounting the cantilever (or the sample) on a scanning element, such as a piezo tube.

Note that the interactions between the tip and the sample need not be of magnetic origin only, and in fact other interactions can result in significant experimental challenges in measuring magnetic forces. The change of the cantilever properties, and thus the force between the tip and the sample, can be measured using some suitable system. It is important to understand how these properties depend on the tip-sample interaction and how these changes can be sensitively measured.

Commercially available cantilevers are usually 1-2 \(\mu\)m thick, 100-500 \(\mu\)m long, and 20-40 \(\mu\)m wide. \(k\) and \(\omega_0\) usually range from 0.010-10 N/m and 10 kHz to 400 kHz respectively. The cantilevers can be rectangular or triangular. The magnetic tip for commercial cantilever is typically a magnetic coating on a sharp integrated tip, formed during cantilever fabrication. As will be seen later, it is desirable to have magnetic tips which can provide high magnetic gradients at the sample. Also, it is desirable to have magnets with high coercivity whose magnetization remains stable when external fields are applied. Such probes can be made by gluing micromagnetic particles of hard-magnetic materials like neodymium iron boron (NdFeB) or samarium-cobalt (SmCo). These can be further shaped and/or reduced in size by using a focused ion beam (fig. 2.9).
2.4.2 What is the measured signal: force gradient and force detection

The cantilever is a simple harmonic oscillator (SHO) whose motion is described by

\[ m_c \ddot{z} + \frac{m_c \omega_0}{Q} \dot{z} + k z = F \] (2.14)

where \( k \) is the effective spring constant of the cantilever, \( m_c \) its effective mass, \( z \) is the direction of cantilever motion, and \( Q \) is the quality factor of the oscillator and is inversely related to its dissipation/damping. \( F \) is the net force acting on the cantilever.

The frequency response of the cantilever (and any SHO) may be obtained by solving the above equation for \( z \) and taking its Fourier transform,

\[ Z_\omega = \frac{F_\omega/m_c}{(\omega^2 - \omega_0^2) + i\omega \omega_0/Q} \] (2.15)

where \( Z_\omega \) and \( F_\omega \) are the Fourier transform of the cantilever displacement and the
force respectively. The frequency response is a Lorentzian, as shown in fig. 2.10. The resonant frequency occurs at $\omega_0 = \sqrt{\frac{k}{m_c}}$ and the width of the resonance is given by $\Delta \omega = \frac{\omega_0}{Q}$. Also, the amplitude is enhanced by $Q$ at resonance.

Forces between the tip and the sample change this frequency. This change is proportional to either the force or force gradient depending on the type of force.

### 2.4.3 Effect of position dependent force: Force Gradient detection

If the magnetic force depends on the tip-sample separation, then the cantilever experiences a varying force as it oscillates. This difference of force is an extra restoring force, due to the gradient of the force $\frac{dF(z)}{dz}z$. It acts as a correction to the spring constant, and can be seen by taking a Taylor series expansion about the cantilever...
neutral position, \( z_p \) (and some simple algebra)

\[
m_c \ddot{z} + \frac{m_c \omega_0}{Q} \dot{z} + \left( k - \frac{dF}{dz} \right) z = F(z_p) + \left( \frac{dF}{dz} \bigg|_{z=z_p} \right) z_p
\]

(2.16)

This is the equation a SHO with position independent force, but now with a modified spring constant. Since \( \omega_0 = \sqrt{k/m_c} \), the change in the effective spring constant shifts the resonant frequency. For small \( \delta k/k \), the change in resonant frequency is given by

\[
\delta f = \frac{1}{2} \frac{\delta k}{k} f_0 = -\frac{f_0}{2k} \frac{dF}{dz} \bigg|_{z=z_p}
\]

(2.17)

where I have replaced angular frequency \( \omega \) with ordinary frequency \( f \) (\( \omega = 2\pi f \)).

Figure 2.10: Frequency response of an ideal simple harmonic oscillator with a Q of 100. The frequency axis is normalized by the resonant frequency. (From www.wikipedia.com)
2.4.4 Effect of time varying force: Force detection

However when a time-varying force, \( F \), is applied in-phase with the cantilever position, the change in spring constant is proportional to the force and not the force gradient. This is shown in the following equations,

\[
m_c \ddot{z} + \frac{m_c \omega_0}{Q} \dot{z} + kz = F_0 e^{i \omega t} + \frac{F}{z_{osc}} z \\
= F_0 e^{i \omega t}
\]

\[
m_c \ddot{z} + \frac{m_c \omega_0}{Q} \dot{z} + (k - \frac{F}{z_{osc}})z = F_0 e^{i \omega t}
\]

where \( F_0 \) is some general force applied to oscillate the cantilever at some frequency \( \omega \). The change of resonant frequency is again given by eqn. 2.17.

Magnetic force detection using time dependent forces is common in Magnetic Force Resonance Microscopy (MRFM), which uses force detection to study magnetic resonance. Many electron polarization modulation schemes/protocols have been developed for MRFM that one can benefit from. In my experiments the modulation of the magnetization/polarization of electrons is achieved by modulating the polarization of light.

2.4.5 Cantilever-sample interaction

The magnetic force, \( F \), between the cantilever magnetic tip and the magnetic moments in the sample is given by

\[
F = \int_V dF = \int_V d (m_s \cdot \nabla B_t)
\]

\( V \) indicates volume integral over entire space, subscripts \( s \) and \( t \) refer to sample and tip respectively, \( m \) is the magnetic moment, and \( B \) is the magnetic dipolar field. This force can be equivalently expressed in terms of the moment of the tip and the gradient.
from the sample, and for a point dipole approximation for the tip

\[ F = m_t \cdot \int_V d (\nabla B_s) \]  \hspace{1cm} (2.20)

Both the expressions for the force are valid and can be used depending on convenience. While this is the total force between the tip and the sample, only forces in the direction of motion of the cantilever (z-axis) change its resonant frequency. Hence only the z-component, \( F_z \), needs to be considered

\[ F_z = \int_V d \left( m_{s,x} \frac{\partial B_{t,z}}{\partial x} + m_{s,y} \frac{\partial B_{t,z}}{\partial y} + m_{s,z} \frac{\partial B_{t,z}}{\partial z} \right) \]  \hspace{1cm} (2.21)

### 2.4.6 Measuring cantilever properties

As mentioned earlier, tip-sample interaction changes cantilever properties. I monitor these properties and extract information regarding the sample magnetization. The system for keeping track of cantilever properties has two main components.

**Tracking cantilever motion: Fiber Interferometry**

In my experiments I use optical fiber interferometry [31, 32] for detecting the motion of the cantilever. This technique is capable of achieving resolutions of 0.1 pm/\( \sqrt{\text{Hz}} \), and is the most widely used technique for high sensitivity cantilever motion detection, especially at cryogenic temperatures. Fig. 2.11 shows a schematic of a typical setup. The technique involves shining a laser beam on the cantilever through an optical fiber. The end of this optical fiber is typically fixed 30-50 \( \mu \text{m} \) away from the cantilever. Reflected light is collected by using an optical coupler, and is fed to a photo-diode (plus a transimpedance amplifier to convert into a voltage). Reflected light has two components— one from the end of the fiber and another from the cantilever. These two reflections interfere with each other, and their path difference is modulated by the cantilever motion. Thus, the reflected intensity is modulated at the instantaneous...
cantilever resonant frequency. This reflected light signal is then demodulated to extract the instantaneous cantilever frequency. Further details of fiber interometry can be found in the following references [33].

Figure 2.11: Schematic of an optical interferometry set-up. The two red arrows indicate two reflections: one from the end of the fiber and one from the cantilever. These reflections interfere and modulate the reflection intensity at the cantilever frequency.

**Frequency Shift detection**

Forces or force gradients can be measured from their effects on the cantilever resonant frequency. The sensitivity of the frequency detection scheme [34] is paramount in determining the force sensitivity of the overall MFM technique. Typically a frequency sensitivity of 1 part per million or better is required for measuring the small femtonewton forces that are expected in my experiments. Development of an in-house cost-effective yet sensitive frequency detector was one of early focuses of this project. This frequency shift detector has two key components:
**Excitation circuit**  It is a positive feedback circuit that adjusts the drive frequency naturally to the instantaneous cantilever resonant frequency.

**Demodulator**  A sensitive demodulator which can measure frequency with a high bandwidth is crucial for these experiments. I have developed such a demodulator which it is implemented on a PC using data acquisition cards (National Instruments PCI-6229) and an algorithm which allows for sensitive, yet high bandwidth, measurement of frequency. The algorithm is based on the fact for any sine wave, $y = A_0 \sin \omega_0 t + \phi$, has the following linear relationship with its second derivative with respect to time

$$\ddot{y} = -\omega_0^2 y$$  \hspace{1cm} (2.22)

Thus, by fitting a signal to its second derivative, the frequency of the signal may be extracted. The advantage of this method over other methods, such as those based on zer-crossings, is that is uses the entire time record of the signal to improve the measurement accuracy. Further details of the algorithm can be found in ref. [35]. The algorithm was implemented using Labview software.

### 2.4.7 Noise in MFM measurement

As mentioned earlier, techniques for countering noise can include reducing its amplitude or decoupling from it in frequency space using modulation. A useful way to analyze noise is to measure its power spectral density (PSD), which provides information on how the energy of a given variable is distributed in frequency. For any time series of a variable, $\lambda(t)$, the PSD is defined as

$$S_{\lambda}(\omega) = \lim_{t \to \infty} \frac{|\Lambda(\omega)|^2}{t},$$  \hspace{1cm} (2.23)
where $t$ is the time and $\Lambda(\omega)$ is the fourier transform of the variable. It should be noted that PSD is normalized to unity bandwidth and for frequency measurement will have units of $\text{Hz}^2/\text{Hz}$. Fig. 2.12 shows the power spectral density of the frequency measurement noise (which is related to force noise) typically seen in force microscopy measurements using frequency shift detection (data from K.C.Fong [33]). The bottom axis is presented as demodulated frequency, which refers to a frequency relative to the cantilever’s resonant frequency. In the absence of any modulation of the cantilever frequency, in response to any modulation of the force from the spins, this plot will have contributions only from the noise sources. The contributions of important noise sources are identified in the figure, and I will discuss them next.

Figure 2.12: The power spectral density of frequency measurement noise typically seen in frequency shift detection. Various sources of noise are identified. For the signal to be measurable, it has to be larger than the noise present at that frequency. (From ref. [33])
Thermal Noise of the cantilever

The single most important figure of merit for a force sensing cantilever is its thermal force noise [36]. It arises from the random motion of electrons in the cantilever at any finite temperature, and is analogous to Johnson noise in a resistor. *Cantilever thermal noise limited performance is the best that can be achieved in force microscopy.* The thermal force noise is given by

\[ F_{N,th} = \sqrt{\frac{4\kappa k_B T B}{\omega_0 Q}} \]

where \( k_B \) is the Boltzman constant, \( T \) is the temperature, and \( B \) is the bandwidth of measurement. The corresponding force gradient noise is given by \( F'_{N,th} = F_{N,th}/z_{osc} \).

The above expression emphasizes the need to work at cryogenic temperatures. The Q is also significantly affected by environmental parameters. Q can change from 100 in air to 100,000 in vacuum and at low temperatures. Contaminants also play an important role in determining the Q, and maintaining cleanliness is very important. By carefully choosing cantilevers, thermal noise figures of less than \( 10^{-16}/\sqrt{\text{Hz}} \) are achievable in vacuum at low temperatures, even with commercial cantilevers. The power spectral density frequency noise corresponding to the thermal noise is independent of demodulated frequency and is given by

\[ S_{\omega,th} = \frac{\omega_0 k_B T}{Q k z_{osc}^2} \]

Other noise sources

*Shot noise of interferometry* The laser used for interferometric detection has an associated shot noise. The SNR due to shot noise is proportional to \( \sqrt{P} \), where \( P \) is the power of the light. Thus increasing the light power used for interferometry can help. However, other considerations, such as heating of the cantilever or increasing
the cantilever oscillation amplitude due to light pressure, limit the power that can be used. The spectral density of shot noise is inversely proportional to the demodulated frequency [35].

1/f noise 1/f noise can be of varied origin and is very challenging to reduce. It is usually countered by modulating the signal at a frequency where 1/f noise is not substantial.

Mechanical noise Mechanical noise in a system is reduced by careful apparatus design which avoids strong mechanical resonances and vibrations. This is especially challenging for apparatus designed for low temperatures, due to differential shrinkage of various parts. Also, good vibration isolation techniques need to be in place.

Electrical noise Random electrical noise may be picked up through ground loops or from long leads acting as antennas. As such care needs to be taken in avoiding these.

Fig. 2.13 presents a typical force noise spectrum showing thermal noise limited performance, with the equivalent thermal force noise being $7 \times 10^{-17} \text{N/}\sqrt{\text{Hz}}$ at 4.2K. This data was collected using the force detection system used in the present research.

Modulation techniques to improve performance

When reducing the noise amplitude is not sufficient, modulation techniques need to be used to move the signal to a frequency where noise power is minimized. In the proposed experiments, the polarization of light can be modulated using an electro-optic modulator.
Figure 2.13: Figure shows power spectral density (PSD) of frequency measurement noise in my experimental set-up. As can be seen thermal noise limited performance has been achieved. Data collected in vacuum at 4.2K
Chapter 3

Numerical analysis of Spin dynamics in spatially inhomogeneous environments

In this chapter I will present one of the core ideas of this dissertation: numerically analyzing the behavior of non-equilibrium itinerant spins in paramagnetic semiconductors, under spatially varying conditions. Numerical analysis is necessitated for inhomogeneous conditions, since analytical solutions do not exist for the spin transport equation (eqn. 2.1) that governs the behavior of spins.

Top panel of Fig. 3.1 is a snapshot of a simulation which I will be describing in this chapter. It combines the schematic of typical simulation geometry with the spatial map of the spin density, obtained from the simulations described in this chapter. The bottom panel shows the orientation that I will assume, unless stated otherwise, for the important vectors involved in these simulations.

However, the first four sections of this chapter will not involve a magnetic tip or any other spatially varying parameters. Such analysis is valuable in understanding the more complex inhomogeneous cases.
Figure 3.1: a) A schematic of the main geometry I am interested in simulating. This figure assumes spins are uniformly injected through the sample. The color map and the arrows show the key results obtained from these simulations: the spatial map of spin density after their interaction with the tip. The arrows show the direction and magnitude of the spins, while the color map shows their component along the injection direction. Panel b) The 2D sample (light blue rectangle) is assumed to be in the \( xy \)-plane. Spins (\( S \)) are injected into it oriented along the \( z \)-axis. The dipolar micromagnet is also assumed to be oriented along the \( z \)-axis, but located at a height \( z_p \) above the sample plane. The Hanle field \( B_h \) is assumed to be along the \( y \)-axis.
3.1 Spin transport in a magnetic field

Eqn. 2.1 can be written in a scaled form with unitless parameters

$$\frac{\partial S^*}{\partial t^*} = D_s^* \nabla^2 S^* + B^*(r) \times S^* - \frac{S}{\tau_s^*(r)} + G^*(r)$$

(3.1)

where, the following transformations have been applied: $r^* = r/L_{sc}$, $t^* = t/t_{sc}$, $D_s^* = D_s t_{sc}/L_{sc}^2$, $B^* = \gamma B t_{sc}$, $G^* = G/G_{sc}$, and $S^* = S/G_{sc} \tau_s$; $L_{sc}, t_{sc}$, and $G_{sc}$ are appropriately chosen scaling quantities. Throughout this dissertation starred variables will denote unitless scaled quantities. All equations in this dissertation can be written using scaled or unscaled parameters. Solving the equation in its scaled form highlights the length and time scales that control the collective behavior of the spins. It gives insight into the interplay between the various terms, the various regimes of the problem, and the associated physics. The spin diffusion length, $L_s$, can provide a natural scaling for length, however it is not unique and in principle choose any length may be chosen. For example, in the case of inhomogeneous magnetic fields, it may be more appropriate to choose a length that captures the variation of the magnetic field. If the spin lifetime is uniform over the sample, $t_{sc} = \tau_s$ is the best choice.

While any component of the spin can be computed, I will mainly focus on $S_{||}$, the component parallel to the injected spin direction. Usually, this is the experimentally measured parameter. Along with the spatial map of $S_{||}$, I will also be focusing on the Hanle measurement. Typically in such experiments $S_{||}$ is measured, averaged over some region of space denoted by $A_d$, as a function of a transverse magnetic field $B_h$.

Mathematically,

$$H^*(B_h^*) = \int_{A_d} S_{||}^*(x^*, y^*, B_h^*) \, dr^*$$

(3.2)

The Hanle curve, $H$, measures the dephasing of the spins, as they precess in some
net magnetic field, due to a distribution of *ages* (time between injection and detection) within the spin ensemble being measured. This distribution results from two factors: 1) continuous injection and 2) diffusion, which causes spins to take variable amount of time to traverse from the injector to the detector. A Hanle curve can be measured *globally* or *locally*. For global Hanle, $H_g$, $A_d$ extends over entire space. Local Hanle, $H_l$, detects spins typically in a region $A_d \leq L_s^2$.

Finally, I consider the form of $G$. The experimentally most relevant mechanism of spin injection for this dissertation is optical injection and has been discussed in 2.2. In this case, I will assume a pump (circularly polarized light) beam propagating normal to the sample plane creates spins along the direction of propagation. Unless specified otherwise, I will assume the light propagation direction, and thus injected spin direction to be along $\hat{z}$. Optical injection typically uses gaussian laser beams, thus

$$G(r) = G_0 \exp\left(-2r^2/r_{pu}^2\right) \hat{z},$$

where $G_0$ is the peak intensity of the Gaussian beam and $r_{pu}$ its radius. The pump is assumed to be centered at $(0, 0)$, in my co-ordinate system. In Sec. 3.5.2, I will consider electrical injection, where spins are injected from a ferromagnetic injector into the sample. I will assume the dimensions of the injector to be infinitely large ($\gg L_s$) and its magnetization oriented along $\hat{z}$. These simulations can be extended to other forms of $G(r)$. 
3.1.1 Case of \( D = 0 \)

Eqn. 3.1 is easy to solve in the absence of diffusion, i.e. \( D = 0 \), even when \( B \) varies spatially. I introduce the \( 3 \times 3 \) matrix \( \mathbf{B}^* \)

\[
\mathbf{B}^* = \begin{pmatrix}
0 & B_z^* & -B_y^* \\
-B_z^* & 0 & B_x^* \\
B_y^* & -B_x^* & 0
\end{pmatrix}.
\]

The spin density is then given by

\[
\begin{align*}
S^*_x(r^*) &= [I - \mathbf{B}^*(r^*)]^{-1} \mathbf{G}^*(r^*) \quad (3.3a) \\
S^*_x(r^*) &= \frac{B_y^*B_z^* + B_y^*}{1 + B_x^2 + B_y^2 + B_z^2} \mathbf{G}^*_z \\
S^*_y(r^*) &= \frac{B_y^*B_z^* - B_x^*}{1 + B_x^2 + B_y^2 + B_z^2} \mathbf{G}^*_z \\
S^*_z(r^*) &= \frac{1 + B_z^2}{1 + B_x^2 + B_y^2 + B_z^2} \mathbf{G}^*_z. \quad (3.3d)
\end{align*}
\]

It should be noted however that \( \mathbf{B}^* \) and \( \mathbf{G}^* \) are the magnetic fields and generation functions at the position \( r^* \) only. This is expected, since in the absence of diffusion and other electric fields injected spins at any point in space are independent of spins at any other point in space.

3.2 Solving spin diffusion equation

3.2.1 Uniform magnetic field and k-space analysis

I begin by considering a spatially uniform magnetic field. In this case, the steady-state spin density can be calculated in the Fourier domain. The 2-D Fourier transformed spin density by

\[
S^*(k^*) = \frac{1}{2\pi} \int S^*(r^*) \exp(-i2\pi k^* \cdot r^*) \, dr^*, \quad (3.4)
\]
where the integral is over entire space. A similar expression for \( G^*(k^*) \). Then the spin diffusion equation can be written in the Fourier space as

\[
-k^*2D_s^*S^*(k^*) + B^* \times S^*(k^*) - S^*(k^*) + G^*(k^*) = 0
\]

(3.5)

\[
\Rightarrow S^*(k^*) = [((k^*2D_s^* + 1)I - B^*)^{-1}G^*(k^*)].
\]

(3.6)

Here \( I \) is the 3 \( \times \) 3 unit matrix, \( G(k^*) \) and \( S(k^*) \) are to be understood as 3 \( \times \) 1 column vectors, and \( [...]^{-1} \) is a matrix inverse. The real-space spin density is then given by the inverse Fourier transform of \( S^*(k^*) \)

\[
S^*(r^*) = \frac{1}{2\pi} \int S^*(k^*) \exp(i2\pi k^* \cdot r^*) dk^*.
\]

(3.7)

The case of \( k = 0 \): Global Hanle

From eqn. 3.4 the globally integrated spin density corresponds to the case of \( k = 0 \). Using eqn. 3.6, I can extract \( S_z(k = 0) \) as

\[
\mathcal{H}_g \propto S_z^*(k = 0) = \frac{G_z^*(k = 0)}{1 + (B^*)^2},
\]

(3.8)

This is identical to the form presented in Sec 2.3.2. As can be seen from eqns. 3.6 and 3.8, the global Hanle does not depend on the diffusion constant. The effect of a finite \( D_s \) is to cause the spins to move around. However, in a spatially uniform magnetic field spins undergo the same precession and relaxation as with \( D_s = 0 \), and diffusion will not matter.

3.2.2 Numerical solution using Euler’s iterative method

Eqn. 3.1 cannot be solved analytically either in the real space or Fourier space for spatially varying parameters and \( D_s \neq 0 \). My collaborators and I have found that the simplest yet sufficiently accurate approach is to numerically solve eqn. 3.1 in
time-dependent form. This approach involves starting from some initial condition $S(r, t = 0)$ and then iterating in time, using time step $\Delta t$, as follows

$$S^*(r^*, t^* + \Delta t^*) = S^*(r^*, t^*) + \left[ \nabla^2 S^*(r, t^*) + B^*(r^*, t) \times S^*(r^*, t) - S^*(r^*) + G^*(r^*) \right].$$

(3.9)

The Laplacian can be evaluated numerically by discretizing the two-dimensional sample into a grid of points separated by a distance $\Delta x$ in each direction. I can then write

$$\nabla^2 S(r, t) \sim \frac{1}{(\Delta x)^2} \sum_\delta [S(r + \Delta x \hat{\delta}, t) - S(r, t)],$$

(3.10)

where $\hat{\delta}$ represents the unit vectors along the spatial grid. This can be iterated until steady state is achieved, i.e. $S$ does not change significantly with time (Fig 3.2).

Figure 3.2: Algorithm for numerically solving spin diffusion equation using Euler’s method.
Figure 3.3: The spatial map of $S_z$ for spins diffusing in a uniform magnetic field, assuming point injection, $r_{pu} \ll L_s$

### 3.3 Stochastic view of spin diffusion and local Hanle behavior

Let us consider the steady state spin density in the presence of a spatially uniform Hanle field and a Gaussian injection beam, where $r_{pu} \ll L_s$. Fig. 3.3 shows the spatial profile of $S_z$ obtained using the numerical method described in the previous section.

An alternate and common way of deriving eqns. 2.10b and 3.8 is to solve the integral form of eqn. 2.1 [37]

$$\mathcal{H}(B^*_n) \propto \int_{-\infty}^{0} \mathcal{P}(t^*_a) \exp(-t^*_a) \cos(B^*_n t^*_a) \, dt^*_a,$$

(3.11)

where $t^*_a$ is the age (time between injection and detection, in scaled time units) of the spin and $\mathcal{P}$ is the probability density function for the age of spins being detected.
In the case of global Hanle with time-independent generation function, it is simply a constant independent of age, and eqn. 3.11 reduces to eqn. 2.10b.

I will now focus on the case of local Hanle response in various regions of a point injected sample. For finite $D_s$, the local Hanle curves are not Lorentzian. Further insight into the local Hanle curves can be obtained by understanding the distribution of the phase of the spins in that region. This depends on the subset of spins being detected and the form of $\mathcal{P}(t_a)$. I will consider the simple case of point injection and point detection as an illustration. For diffusive transport, which arises from a random walk of the spins, the probability of a spin of certain age being detected (per unit area per time) at a point distance $L$ away from the injection spot is given by

$$
\mathcal{P}_{rw}(t_a) = \frac{1}{4\pi D t_a} \exp\left(\frac{-L^2}{D t_a}\right).
$$

(3.12)

For $\mathcal{P} = \mathcal{P}_{rw}$, corresponding to a diffusive random walk model, the integral in eqn. 3.11 can be analytically solved, and

$$
\mathcal{H}_l(B^*_h, L^*) \propto \mathcal{R}e\left(K_0(L^* \sqrt{1 + iB^*_h})\right)
$$

(3.13)

Local Hanle curves at various distances away from the injection are shown in Fig. 3.4. As can be seen, the precession behavior of a local ensemble can be very different from the global signal.

At a given field $\mathbf{B} = B_h \hat{y}$, the phase of the precession vector is given by, $\theta = [B_h t_a^* \mod 2\pi]$. $\mathcal{P}(t_a)$ can be converted into a probability for the phase of the precession phase

$$
\mathcal{P}_\theta(\theta) = \sum_{t_a^* : [B_h t_a^* \mod 2\pi] = \theta} \mathcal{P}(t_a) \exp(-t_a^*)
$$

(3.14)

Figure 3.5 shows $\mathcal{P}_\theta(B_h^* = 2)$ for the positions corresponding to the local Hanle curves. Two features of the $\mathcal{P}_\theta$ curves are important: their width and the position of the peak. For a narrower distribution of angles, as seen at position A, the resulting Hanle curve
Figure 3.4: The top panel shows the dependence of $H_l$ as a function of the Hanle field (bottom axis) and distance away from the injection point (left axis). The decay of $H_l$ away from the injection point is due to spin relaxation. The bottom panel shows line cuts taken at various points along the $y$-axis of the top panel. These represent the local Hanle curves at various distances away from the injection spot: $L/L_s = 1/4$ (red), $L/L_s = 1$ (green), $L/L_s = 1$ (orange), $L/L_s = 1$ (black). The curves have been normalized. The solid blue line shows the global Hale response described by the Lorentzian in eqn. 3.8 for comparison. The local Hanle response is governed by the probability of a spins of certain age being present within the ensemble being collected in that local region.
is broadened due to a relative coherence amongst the spins. The peak position gives an idea of the average phase of the spins for a given field. At $L/L_s = 3$, majority of the spins have rotated by more than $\pi/2$. Thus, the negative lobe in the local Hanle curve is seen at this position.

![Figure 3.5: The probability of precession angles within the spin ensemble detected at various distances away from the injection point: $L/L_s = 1/4$ (red), $L/L_s = 1$ (green), $L/L_s = 1$ (orange), $L/L_s = 1/4$ (black). Note that these curves have been multiplied by 1, 20, 500 and 20000 respectively.](image)

3.4 Effect of fields with multiple vector components

I will now analyze the case of the presence of a spatially uniform field, $B_u = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$, being present in addition to the Hanle field. Thus, the total field around which the spins will precess (see Fig. 3.6) is

$$B = B_u + B_h = B_x \hat{x} + (B_y + B_h) \hat{y} + B_z \hat{z}.$$  \hspace{1cm} (3.15)
Figure 3.6: Schematic of spin precession in the presence of a spatially uniform field \( \mathbf{B}_u = B_x \hat{x} + B_y \hat{y} + B_z \hat{z} \) and \( B_h \). The competition between \( B^*_\parallel = B^*_z \) (component parallel to injection) and \( B^*_\perp = \sqrt{(B^*_x)^2 + (B^*_y + B^*_h)^2} \) (component perpendicular to injection) dictates spin dynamics.

The diffusion for non-zero \( D_s \) can again be solved by either by Euler’s iterative method or the Fourier transform approach. Matrix inversion provides an analytical form for \( D_s = 0 \), as given in eqn. 3.3. Rearranging those terms and defining some new terms results in the following expressions.

\[
S^*_z = [1 + \theta_B^2]^{-1} G^*
\]

(3.16)

Where,

\[
\theta_B = \frac{(B^*_\perp)^2}{1 + (B^*_\parallel)^2},
\]

(3.17)
and \( B^* = B^*_z \) and \( B^*_\perp = \sqrt{(B^*_x)^2 + (B^*_y + B^*_h)^2} \). I will call \( \theta_B \) the *dephasing angle*, and as will be seen in the following subsection, it plays a very important role in determining the behavior of spins in a spatially inhomogeneous field.

Since the fields are spatially uniform, the global Hanle curve will be the same for \( D_s = 0 \) and \( D_s \neq 0 \) and is given by the equations above. The results are shown in Fig. 3.7. The key point to note is that the global Hanle curves are still Lorentzian. However, \( B^*_1/2 \) is not set solely by the lifetime of the spins, but is rather enhanced by \( B^*_2 \) and \( B^*_z \),

\[
B^*_1/2 = \sqrt{1 + (B^*_2)^2 + (B^*_z)^2}.
\]  

(3.18)

Experimentally, this may result in the observance of a significantly enhanced Hanle width, and proper care should be taken in extracting the spin lifetime from Hanle measurements in the presence of a general field. The other key feature is the shift of the Hanle curve by an amount \( B^*_y \).

An understanding can again be gained by looking at the spin precession about the total magnetic field the spins experience (3.6). The equation of motion of \( S_z \) in the case of a general field is not a simple cosine term, but is given by

\[
S_z^*(t^*) = \left( \frac{B^*_\parallel}{B^*} \right)^2 + \left( \frac{B^*_\perp}{B^*} \right)^2 \cos B^* t^*
\]  

(3.19)

\( B^* \parallel \) inhibits precession. To illustrate the effect of \( B^* \parallel \), I will examine the limiting case of \( \mathbf{B} = B_h \mathbf{\hat{y}} + B_z \mathbf{\hat{z}} \), where \( B_z \gg 1/\gamma \tau_s \). When \( B_z \gg B_h \) the effective magnetic field vector is essentially along the \( \mathbf{\hat{z}} \) direction, the precession vector of the spins barely tips away from it, irrespective of the age of the spin. Thus it takes a Hanle field of the order of the parallel field before enough distribution in the precession angles can be generated to reduce \( S_z \) by half. As for \( B_x \), it tips the effective magnetic field away the injected spin direction and reduces \( S_z \) even at \( B_h = 0 \). In order to see any further reduction in \( S_z \), a transverse field at least comparable with the \( B_x \) needs to
Figure 3.7: Global Hanle curves for various cases of spatially uniform magnetic fields, $B_u$, applied in addition to the swept Hanle field, $B_h$. These curves are valid for all values of $D_s$ since globally averaged spin density is insensitive to diffusion when the sample dimensions are much greater than $L_s$.

Using eqns. 3.19 and 3.11 the global and local Hanles for the case of point injection and point detection can be obtained,

$$\mathcal{H}(B_h^*) \propto \int_0^\infty \mathcal{P}(t_a^*) \exp(-t_a^*)S_{zp}(t_a^*) \, dt_a^*$$  \hspace{1cm} (3.20)

$$\mathcal{H}_l(B_{h,L}^*) \propto \left(\frac{B_{||}^*}{B^*}\right)^2 K_0(L^*) + \left(\frac{B_{\perp}^*}{B^*}\right)^2 K_0(L^* \sqrt{1 + iB_h^*})$$  \hspace{1cm} (3.21)

where, $\mathcal{P} = \text{constant for global Hanle and } \mathcal{P} = \mathcal{P}_{rw}$ for local Hanle.

### 3.5 Spatially Varying Magnetic Field

I will now consider solving eqn. 3.1 for spatially varying magnetic fields. This equation is difficult to solve analytically for a general $B$ and needs to be solved numerically using the Euler’s method described in Sec. 3.2.
3.5.1 Effect of Dipolar Field on the Hanle response

I will use the insights from the uniform case to shed light on one of the key aspects of this paper: spin transport in spatially varying fields. Specifically, I will solve for the spin transport equation in the presence of a dipolar field. Such a field is of interest because the magnetic field from a micromagnetic tip, such as those used in magnetic force detection experiments, can be reasonably approximated as a dipolar field. Furthermore, complex magnetic fields can be modeled as a collection of dipole fields. Such complex fields might be important in understanding the effects of spatially rough magnetic injectors used in spintronics devices.

I consider a point dipole having a magnetic moment \( \mathbf{m} = m_z \hat{z} \) above the center of the injection spot at a height \( z_p \) above the sample plane (see Fig. 3.1). The 2D sample is located within the \( xy \)-plane. The spatially varying field from such a tip is given by

\[
B_{\text{dip}}(x, y) = \frac{\mu_0}{4\pi} \frac{3(\mathbf{m} \cdot \mathbf{r})\mathbf{r} - \mathbf{m}\mathbf{r}^2}{r^{5/2}},
\]

(3.22)

where \( \mathbf{r} = x\hat{x} + y\hat{y} + z_p\hat{z} \), and \( \mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1} \) is the permeability of free space. At any point \((x, y)\), the magnetic field experienced by the spins is the vector sum of \( B_{\text{tip}} \) and \( B_h \),

\[
\mathbf{B} = B_h + B_{\text{tip}}.
\]

(3.23)

The spin density, \( S(x, y) \), can again be computed by matrix inversion for the case of \( D_s = 0 \) and analyzing this case will help in understanding some salient features of the behavior of spins in spatially inhomogeneous fields which will be relevant even with diffusion.
Analysis without diffusion, $D_s = 0$

In the absence of diffusion every spin remains fixed in space over time and experiences the same field. Thus, effectively, every spin sees a uniform magnetic field like those discussed in previous sections. However, spins in different positions experience different magnetic field.

The top panel of Fig. 3.8 shows the $\theta_B$ (left axis) as a function of position along the $x$-axis, while the right hand axis shows the corresponding $B_\parallel$ component of the field for a magnetic moment $m_z \hat{z}$, where $(\mu_0/4\pi)(m_z/z_p^3) = 10$. Note that the scaling for the length is $z_p$. For the dipole field, $z_p$ provides a more natural length scale which is applicable for both $D_s = 0$ and $D_s \neq 0$ cases. The middle panel shows three different Gaussian generation functions with $r_{pu} \gg z_p \implies r_{pu}^* \gg 1$, $r_{pu} \sim z_p \implies r_{pu}^* \sim 1$, and $r_{pu} \ll z_p (r_{pu}^* \ll 1)$; reflecting the fact that there is now an additional length scale of importance, the tip-sample separation, which dictates how far the magnetic field extends and how rapidly it varies. The bottom panel shows the spatial variation of $S_z$ along the $x$-axis. For $B_H = 0$.

For uniform injection, $r_{pu} \gg z_p$, it can be clearly seen that $S_z(x,0)$ is inversely related to $\theta_B(x,0)$, as dictated by eqn. 3.16. Thus the lobed spatial structure of $\theta_B(x,0)$ is transferred to the spatial distribution of spins. As I will discuss in the next section, this could have important implications for imaging local spin properties of the sample on a length scale set by the magnetic field variations. This length scale can be smaller than the detector sizes used in many experiments. For $D_s = 0$ case, pumps of smaller diameter provide a simple weighting function to the uniform pump case.

Fig. 3.9 presents the Hanle curves for the $D_s = 0$ case. I will start with the local Hanle responses, since they can be understood more readily based on the discussion in the previous subsections. Panel a) presents the local Hanle curves for various positions
Figure 3.8: a) A line cut of $\theta_B$ (left axis) and $B_\parallel$ (right axis) along the $x$-axis, in the presence of a dipole moment oriented along $\hat{z}$. b) Three different Gaussian generation functions such that $r_{6u}^* \gg 1, r_{6u}^* \sim 1$ and $r_{6u}^* \ll 1$. c) Spatial spin density along the $x$-axis corresponding to the generation functions shown above.
Figure 3.9: Local and global Hanle curves in the presence of a dipole tip and with \( D_s = 0 \). a) Local Hanle curves along various points along the \( x \)-axis. b) Local Hanle curves for various points along the \( y \)-axis. c) Global Hanle response for the three different generation functions shown in Fig. 3.8b.
along the $x-$axis. These positions are marked in Fig. 3.8a. Note that $B_y(x,0) = 0$ along the $x-$axis. As discussed in Sec. 3.4, three quantities will determine the Hanle curves, $\theta_B$, $B_x$, and $B_z$. $\theta_B(x,0)$ dictates how much the $S_z(x,0)$ is lowered at zero Hanle field. As such, positions x1 and x5 (and also x2 and x4) have identical $S_z(B_h = 0)$, even though they experience very different magnetic field. On the other hand $B_x$ and $B_z$ determine the width of the Hanle curves (eqn. 3.18). As such the Hanle curve widths at positions x2 and x4 are very different even though they have identical $\theta_B$.

The middle panel b) shows the local Hanle curves for various positions along the $y$-axis. Along this axis, $B_x = 0$, but $B_y$ is non-zero. This results in the the peak of the Hanle curve get shifted by an amount $-B_y(r)$. The width is given is $\sqrt{1+B_z^2}$, while the $S_z(B_h = 0) \propto (1 + \theta_B)^{-1}$.

Finally, the global Hanle, which is the integral of the local Hanle curves over the entire space, can be calculated. These are shown in the bottom panel. I show the global Hanle curves for the three pump generation functions shown in Fig. 3.8b. The most obvious feature of these curves is that, as $r_{pu}$ becomes smaller, the Hanle curves become much broader. The reason for this behavior is that, for smaller $r_{pu}$, at smaller pump radius, most of the spins experience a large parallel tip field, which broadens the Hanle according to eqn. 3.18.

$D_s \neq 0$ case:

While $D_s = 0$ is an interesting test case, it is not relevant for conduction electron spins which are needed in spin injection devices. I will now put together all the insight gained in the previous sections of this chapter to tackle the difficult case of spins diffusing in a spatially varying field.

The spin density and Hanle curves will now depend on all three length scales: $r_{pu}$, $L_s$, and $z_p$. Fig. 3.10 shows the spatial map of the spin density for the case of
uniform spin injection, and in the presence of the same dipole moment as used in the $D_s = 0$ case. These are calculated using the numerical method described before. The graph also includes the spin density for the $D_s = 0$ case for comparison. Also plotted is the crucial quantity $\theta_B$ on the right hand axis. The most noticeable difference from the diffusionless case is the collapse of the central maxima and the formation of a hole in spin density.

Diffusion causes spins to move from areas of high spin density to low spin density. The areas of lowest spin densities are given by the maxima in $\theta_B$. These areas of high $\theta_B$ essentially act as sinks for the spin. Spins flow into this region from the central peak and the outer edges. However, there is a significant difference in the central peak region and the outer regions. For simulation presented here, the central minimum in $\theta_B$ is only an $L_s/2$ from the minima. Strong spatial spin density gradients

Figure 3.10: (a) Steady state spin density $S_z(x/z_p, y = 0)$ in the presence of a dipole field for $D_s = 0$ (red) and $D_s\tau_s/z_p^2 = 1$ (blue). The dipole $m_z\hat{z}$ (where $\mu_0 m_z z_p = 10$) is located at $r/z_p = (0, 0, 1)$ and uniform injection is assumed. The green curve shows the spin dephasing factor $\theta_B(x)$; note that $\theta_B = 1 \implies S_z = 0.5$ for $D_s = 0$. 
Figure 3.11: Global Hanle curves in the presence of a dipole tip, with and without diffusion, and for various generation functions: $r_{pu} \gg L_s$ (red), $r_{pu} \sim L_s$ (blue), $r_{pu} \ll L_s$ (green). The solid lines are assuming $D = z_p^2/\tau_s$, while the dashed lines represent $D = 0$ case.
cannot exist within such length scales. As the spin flows into the sink and the central peak region will reduce gradually to decrease spin density gradients. However, on the other side of the spin density minima, there are large regions with low $\theta_B$ which are more than an $L_s$ away from the minima. These regions act as spin sources that can continuously feed spins into the sink regions. Mathematically, there is more space to allow for lower spin density gradients.

Fig. 3.11 shows the global Hanle curves for the three generations function shown in Fig. 3.8b. I also plot the corresponding Hanle curves from the $D_s = 0$ case. As can be seen, for uniform generation, there is almost no difference between the two cases. This is expected, since most of the spins experience no tip field, and it reduces to the case of just the uniform Hanle field. From previous subsections it is known that there is no difference for global Hanle curves for the $D_s = 0$ and $D_s \neq 0$ cases. The most clear difference, as expected, is for the spatially narrowest generation function.

3.5.2 Effect of random injectors on spin injection devices

Understanding the impact of microscopic inhomogeneities on spin transport is a centrally important issue that calls for spatially resolved studies. Particularly important are inhomogeneities due to the stray fields arising from imperfect ferromagnetic injectors used for electrical injection [18, 19, 38]. Spin lifetime is typically obtained from the width of Hanle curves, but such global measures of spin polarization are sensitive to inhomogeneities, so microscopic approaches are needed to discern the various mechanisms that can affect the shape of a Hanle curve. Fig. 3.12 shows the schematic of a three-terminal electrical injection/detection device. Non-uniformity of the interface and complex domain structures can cause the injected spin population to experience significantly different magnetic fields, and lose spin information on time scales much shorter than spin lifetime due to dephasing. The numerical method developed in this
research is capable of simulating the impact of magnetic fields with random spatial variation. Fig. 3.13 shows calculated Hanle curves in the presence of random magnetic fields. I consider a field \( \mathbf{B}_r = \mathcal{N}_x(0, B_v)\hat{x} + \mathcal{N}_y(0, B_v)\hat{y} + \mathcal{N}_z(0, B_v)\hat{z} \) where I sample normal distributions of mean zero and variance \( B_v \) for each field component at every spatial point. Fig. 3.13 shows results for three values of \( B_v : \ll (\gamma \tau_s)^{-1}, = (\gamma \tau_s)^{-1}, \) and \( > (\gamma \tau_s)^{-1} \). As can be seen, the Hanle curves can be broadened significantly and the spin signal (proportional to spin polarization of the electrons) can decrease substantially. Furthermore, the Hanle peak can be shifted.

The results of Fig. 3.13 can be considered a superposition of the shifting and broadening seen in the case of the dipole field and in Sec. 3.4. Clearly these effects can confound the determination of spin lifetimes from Hanle widths. The detailed features of the stray field generated by a rough ferromagnetic injector will depend
Figure 3.13: Global Hanle curves for a sample with randomly spatially varying magnetic field, \( \mathbf{B}_r = \mathcal{N}_x(0, B_v) \mathbf{x} + \mathcal{N}_y(0, B_v) \mathbf{y} + \mathcal{N}_z(0, B_v) \mathbf{z} \). Such fields may be expected in electrical injection devices with rough injectors. The injected spins are assumed to be along the \( x \)-axis in this simulation.

on several characteristics including roughness, saturation magnetization, domain size, and thickness. Regardless of these details, stray fields of the order kilogauss are readily achievable a few nanometers away from a rough injector and these can substantially affect the spin polarization.

### 3.6 Scanned Magnetic perturbation Imaging: spatially varying fields and global detection

A technique which could spatially resolve the local variations of spin density and other key parameters, such as spin lifetime, in multicomponent spintronic devices would have enormous impact. Here I will show that local information about the spin properties can be encoded in the variation of the global spin polarization. The global signal is modified in response to a spatially scanned magnetic field from a micromagnetic probe. In analogy with electrostatic tip-induced scattering in a two-dimensional
electron gas [39] the local magnetization is modified to an extent determined by both the vector field of the micromagnetic probe and the local spin properties. This local perturbation is detected by established global spin polarization measurement techniques (such as spin photoluminescence or non-local voltage) which, in themselves, need not provide spatially resolved information. Hence this proposed method is applicable to any materials system (whether optically active or inactive). A schematic of this proposed imaging technique is shown in Fig. 3.14(a). As a magnetic dipole is scanned above the sample the dependence of a globally detected spin signal on position of the probe provides a map of the sample’s spin properties.

The spatially dependent response of the spin polarization to the magnetic field of the probe tip can be calculated by means of the numerical technique that I have described in the previous section. Vector precession is an added complexity which is not present in the case of (scalar) charge deflection by a scanned gate.

### 3.6.1 Image formation

I consider spins injected uniformly into the sample by some means (uniformity is not a necessary condition). The injected spins and the scanned dipole are assumed to be oriented along \( \hat{z} \). This sample has a small localized region in which the spin lifetime is five times longer than the rest of the sample. Panels (b) and (c) of Fig. 3.14 show spatial maps of the component of the spin density parallel to the injected spin direction, \( S_z \), for two positions of the probe relative to the lifetime inhomogeneity. Panel (d) presents the image of \( S_{z,g} \), the integral of \( S_z \) over the entire sample, as a function of the position of dipole over the sample. The region of slower relaxation rate is clearly distinguished from the rest of the sample.

I have already described the formation of a hole in the spin density due to the field dipolar field (Fig. 3.10). This local suppression of the spin polarization by the probe
Figure 3.14: Demonstration of the ability to image local properties using global measurements: (a) Measurement geometry of the proposed imaging experiment. Embedded in a uniform plane is a small region (dotted line) in which $\tau_s$ is five times larger than throughout the remaining area. The 2D sample is uniformly pumped with spins. A probe is positioned above the sample at a distance $z_p$ and its $xy$-coordinates are scanned to obtain the image presented in panel (d). (b,c) Spatial map of $S_z(x, y)$ when the enhanced $\tau_s$ region is found at $(2L_s, 2L_s)$ and $(6L_s, 6L_s)$ relative to the tip. (d) $S_{z,g}$, the globally integrated $S_z$ over the entire sample as a function of probe co-ordinates $(x_p, y_p)$.
tip field provides the basis for scanned probe imaging of a spintronic sample: the impact of this tip field is determined by local sample properties. The image contrast in Fig. 3.14(d) results from the fact that long-lived spins are more strongly dephased because they experience the tip field for a longer time before being replaced by fresh, well-oriented spins. Thus, the globally averaged spin density of the sample will be more strongly suppressed when the dipole is over the region of longer $\tau_s$. The size of this effect depends on the "global" size over which the spin polarization is integrated.

### 3.6.2 Imaging resolution

To estimate the spatial resolution of the imaging technique, I consider the effect of a linear magnetic field gradient: $\mathbf{B} = \alpha(y\hat{y} - z\hat{z})$ (see Fig. 3.15). The injected spins are uniformly oriented along $\hat{z}$. The resulting variation of $S_\parallel$ along the $y$-axis, for $\alpha = (\gamma\tau_s L_s)^{-1}$, where $(\gamma\tau_s)^{-1}$ is the halfwidth of the conventional Lorentzian Hanle curve, is presented in the inset of Fig. 3.15. *In the absence of diffusion*, spins at different positions along the $y$-axis will sample different fields, resulting in a mapping of the conventional Hanle curve onto the spatial domain (Fig. 3.15a). Thus, spins will exist primarily in the region where $|B| \leq (\gamma\tau_s)^{-1}$, and the resolution $L_{r,nD}$ is given by $L_{r,nD} \alpha \leq (\gamma\tau_s)^{-1}$. The location of the surviving spin magnetization can be spatially shifted by adding a uniform offset to the gradient field.

In the presence of diffusion, a measure of the resolution may be obtained from the width of a Gaussian fit, $L_r$, to the sample spin density (inset of Fig. 3.15b). This width as a function of the applied gradient is shown in Fig. 3.15. Please note that $L_{r,nD}$ that is used in the $x$-axis is inversely proportional to $\alpha$. The spatial resolution $L_r$ decreases in inverse proportion to the gradient until $L_r \lesssim L_s$ below which its rate of decrease slows. This situation is reminiscent of magnetic resonance imaging (MRI) where spatial information is encoded into field or frequency shifts through magnetic
Figure 3.15: a) Hanle curve is mapped onto the spatial domain (right axis) in the presence of a linearly varying field (left axis) of the form $B = \alpha (y\hat{y} - z\hat{z})$. The resolution in the absence of diffusion, $L_{r,nD}$, is given by $L_{r,nD} \alpha \leq (\gamma \tau_s)^{-1}$. b) The resolution length scale $L_r$ in the presence of diffusion as a function of resolution length scale $L_{r,ND} \propto \alpha^{-1}$, the magnetic field gradient $\alpha$. Note that the lengths are scaled by the diffusion length $L_s$, and field is in the units of $(\gamma \tau_s)^{-1}$. Inset) Variation of $S_z$ along the $y$-axis in the presence of a linearly varying field. A Gaussian fit to the numerically simulated data $S_z$ is shown. The width of the Gaussian may be considered to be the resolution for a given gradient.
field gradients [40]. As in MRI, diffusion degrades resolution. Though in the current approach it can be finer than the spin diffusion length if strong enough gradients are used. This fact is seen in numerical simulations shown in Fig. 3.15. Furthermore, it should be noted that diffusion lengths for many spintronic systems are of the order of a micron, especially at room temperatures. Field gradients exceeding 10 G/nm are achievable with rare-earth micromagnetic tips [15] such as are used in magnetic force resonance microscopy [41, 42, 43] indicating that high spatial resolution is possible with this technique. In the case of imaging with a dipole tip, the gradients (and the regions of suppressed magnetization shown in Fig. 3.10) cannot be smaller than the probe-sample separation, $z_p$, so for large $z_p$ this will set the resolution.

### 3.6.3 Feasibility and signal-to-noise

In addition to a mechanism for defining the resolved volume, imaging also requires that this suppression of signal due to the tip be detectable with adequate signal-to-noise ratio. Optical spin detection is highly sensitive, so this will not prevent high resolution imaging. For example, in optical imaging experiments in GaAs [24], signals detected from an area $A_i \sim 16 \mu m^2$ exhibited SNR $\zeta \sim 25$. Hence a perturbation that completely suppresses the spin magnetization throughout an area $A_i/\zeta \sim 600 \, \text{nm}^2$ would be detectable with unity SNR with no modification to the signal detection. Optical spectroscopy on few electron spin states in individual 100 nm GaAs quantum dots shows that much higher sensitivity is achievable [44, 45]. In the present context, this sensitivity would allow substantial further improvement of the resolution.

A similar balance of spatial resolution and effect size arises in electrical detection schemes (for which high SNR measurements have also been reported [46, 47, 48]). Although the analysis of the relationship between resolved volume and signal size is more complex, any spatial resolution afforded by the scanned probe in optically
inaccessible samples is an important advance since current electrical spin detection offers no imaging capabilities. Using our numerical approach we have calculated a signal suppression of more than 10% for spins collected in a narrow channel of width $w_c = L_s$ by a ferromagnetic electrode a distance $\ell_c = L_s$ from the injection site. For either electrical or optical spin detection experiments, the size of the effect can be varied by adjusting the parameters of the magnetic particle (e.g., size and saturation magnetization).

**3.7 Force on a cantilever with magnetic tip**

The spin density generated by the pump beam will exert a force on the magnetic tip. Mathematically this may be described in the following manner

$$F_{\text{tip}} = \int \mathbf{m}_s \cdot \nabla \mathbf{B}_{\text{tip}}$$  \hspace{1cm} (3.24a)

Denoting the spatial part of magnetic dipole function by $f_D(r, z_p)$ for the sake of brevity, the force may be written as

$$F = \int \mu_B \mathbf{S} \cdot \nabla \left[ \frac{\mu_0}{4\pi} \mathbf{m} f_D \right] \, dr^2$$  \hspace{1cm} (3.24b)

For a 2D sample with optical excitation this may be written in scaled parameters in the following manner

$$F = \int \left( \mu_B \mathbf{S}^* \frac{G t_{\text{sc}}}{\alpha} \right) \cdot \left[ \frac{\nabla^*}{L_{\text{sc}}} \left( \frac{\mu_0}{4\pi} \mathbf{m} \frac{1}{L_{\text{sc}}^3} f_D^* \right) \right] \left( L_{\text{sc}}^2 \, dr^* \right)$$  \hspace{1cm} (3.24c)

where, $\alpha$ is the absorption coefficient of the material. The individual vector components, $k = \{x, y, z\}$, of the force may thus be written as

$$F_k = \mu_B \frac{G t_{\text{sc}} \mu_0}{\alpha} \frac{1}{4\pi} m^* \sum_{i=x,y,z} \int S_i \frac{\partial f_D^*}{\partial i} \, dr^*$$  \hspace{1cm} (3.24d)
It should be noted that $G = \eta I_0 \alpha / E_p$, where the incident pump intensity is related to the incident power: $I_0 \approx P_0 / \pi r_{pu}^2$. $E_p$ is the energy of the exciting photon. An efficiency factor $\eta$ has been included since not every photon will result in an excited electron. Thus

$$F_k = c_0 F^*_k,$$  \hspace{1cm} (3.24e)

$$F^*_k = \sum_{i=x,y,z} \int {S^*_i} \frac{\partial \mathcal{R}^*_{D,j}}{\partial i} dr^*.$$  \hspace{1cm} (3.24f)

$$c_0 = \mu_B \left( \frac{P_0 t_{sc}}{\pi r_{pu}^2 E_p} \right) \left( \frac{\mu_0 m_k}{4\pi L_{sc}^2} \right)$$  \hspace{1cm} (3.24g)

It should be noted that $m_j$ is the tip-perturbed moment that I calculate using the numerical method described in Sec. 3.2. The above mentioned formalism was added to the numerical simulations to enable calculation of forces that may be expected from the GaAs optical injection samples. It should be pointed out that on the scale of the variation of magnetic field from the magnetic tip, the sample is not two dimensional. This may result in some non-ideality of these calculations. However, they can still serve as reasonable estimates for the kind of forces expected in my experiments.

Since my numerical code computes the spin-polarization of the sample, it allows for a comparison of the spin-PL (directly proportional to spin-polarization) and force signal. Simulations were done to calculate the force as a function of pump radius and distance of the tip from the sample. Simulations were done assuming constant incident power (0.1mW) which does not change as the pump size is varied, $\eta$ was assumed to be 0.1, and $\tau_s = 3$ ns which is close to the effective spin lifetimes I measured in my experiments. Fig. 3.16 provides the results of the simulations. The top row of the figure shows the force and spin-PL signals as a function of Hanle field and pump radius when the tip is 0.75 $\mu$m away. The bottom row show the same when it is 3 $\mu$m away.
Figure 3.16: Numerically calculated force (left hand panels) on a cantilever due to spin polarized carriers, as a function of pump radius, $r_{pu}$ and Hanle field, $B_h$. Also presented are calculations for spin-PL signal (directly proportional to the spin polarization) for comparison (right hand panels). The top row correspond to a tip-sample separation of 0.75 $\mu$m, while those in the bottom row are for 3 $\mu$m separation.
The salient points to note in these results are:

- The forces that can be expected from these samples are quite small of the order femtonewtons or smaller. The force decreases very rapidly with tip-sample separation, reflecting the fact that gradient of the dipole magnetic field decreases as the fourth power of distance.

- Higher forces are seen with smaller pump sizes. This may be understood by thinking of the equivalent expression for force $F_{\text{tip}} = \int m_{\text{tip}} \cdot \nabla B_{\text{sample}}$. A smaller pump size effectively increases the gradient of the magnetic field from the injected spins, thus enhancing the force. In the limit of an infinitely large pump, which will create uniform magnetization, the force will tend to zero.

- The force signal is more sensitive to the tip-sample distance and the pump size than the spin-PL signal. This may be because of the force detector interacts with fewer nearby spins than the spin-PL signal. The gradient from the tip is localized where as the spin-PL signal is global.

- Another remarkable feature revealed by these calculations is the fact that Hanle response of the force signal seems to much broader than the spin-PL. This, again, is most likely due to the fact that the spins being detected by a force signal most likely experience a strong magnetic field from the probe-tip, resulting in a broader Hanle response.
I will turn my attention to the experimental aspects of this research. Three kinds of experiments were performed: 1) establish spin injection and Hanle precession in the absence of a micromagnet, and measure spin lifetime of the samples; 2) study the effect of local magnetic probe on the spin-PL signal, following the ideas described in the last chapter; 3) study the force exerted by the injected spins on the micromagnet attached to a mechanical cantilever. Since the probe perturbs the spins, the force of the spins on the magnet is the newtonian back-action. Fig. 4.1 shows a schematic of the experiments that were performed.

All the experiments were performed on a GaAs membrane sample in vacuum at cryogenic temperatures in the range of 10 K – 20 K. Cryogenic environment was chosen to both increase the PL signal from the GaAs sample, and reduce the thermal noise of the force detector. An optical flow cryostat (ST-300, Janis Research), was used for these experiments. A turbo-molecular pump (V70, Varian Inc.) was used for the vacuum. An iron-core room temperature magnet with Helmholtz design (V-4005, Varian Associates), was used for providing the Hanle field and was powered using a bi-polar power supply (BM 15-20 M, Kepco).
Figure 4.1: A cartoon representation of the various experiments presented in this chapter. The first set of experiments verified the spin injection in GaAs and carefully characterized the Hanle response in the absence of the tip. Then the effects of perturbation due to the tip on the PL signal were studied. Finally force detection experiments were conducted.

4.1 Sample, micromagnetic probe, and the microscope

4.1.1 Sample

GaAs was chosen for these experiments since it is a well studied spin system, and should help understand the various issues with the new detection techniques I wish to implement. The 2 µm thick n-GaAs (100) layer with a nominal doping level of $3 \times 10^{16}$ cm$^{-3}$
cm$^{-3}$ was epitaxially grown in an MBE chamber. The growth structure is shown in Fig. 4.2. A sacrificial layer of AlGaAs was grown to facilitate back-etching of the wafers. Samples were fabricated using standard optical lithography and wet etching techniques. The wafers were back-etched to create large membranes (0.3 mm $\times$ 1 mm). The wafers were etched using a mixture of citric acid, hydrogen peroxide and water, to allow selective etch between GaAs and AlGaAs. Fig. 4.3 shows the image of a back-lit etched membrane. The shape of the membrane is due to the anisotropic nature of the etch along various crystalline directions and is not important for these experiments. A “bull’s eye” pattern was also etched from the top using HF, to assist positioning the cantilever precisely relative to the sample in a non-optical cryostat. However, since an optical cryostat was used, these patterns do not play any role in the sample. They were useful for focusing the optical elements on the sample plane.

4.1.2 Micromagnetic probe

Neodymium iron boron, NdFeB, spheres were used as micromagnetic probes. The spherical shape is expected to approximate the dipole moment better and NdFeB is a hard-magnet with high coercivity, and as such can maintain its magnetization even as the Hanle field is applied. It also has high magnetization and can thus provide higher fields. These particles are carefully glued onto the tip of a micromechanical cantilever under a microscope. The particles were typically glued on silicon nitride triangular cantilevers (spring constant, $k_0 \sim 0.01-0.03$ N/m). Stycast 1266 (Emerson and Cuming) was used to glue the particles because of its well known low temperature properties.
Cantilever Magnetometry

Cantilever magnetometry [49] is a very sensitive technique that can be used to measure the magnetic behavior of small micromagnetic particles, which have small moments \(10^{-10} \text{ J/T} - 10^{-12} \text{ J/T}\) and cannot be measured easily by other techniques. It is used to measure the magnetic moment and hysteretic switching behavior in external magnetic fields. Fig. 4.4 shows typical data obtained from such a measurement. Shown in the figure is a plot of the frequency of the cantilever as a function of an applied external magnetic field. Details on measuring a cantilever’s frequency are provided in Sec. 2.4. The inset shows the relative direction of the magnetization of the micromagnet, the direction of motion of the cantilever and the external field.
Figure 4.3: Optical micrograph of a typical membrane sample formed by back etching. The membranes are typically \( \sim 0.3 \text{ mm} \times 1 \text{ mm} \). The pattern on the top is not crucial to the experiments, and just helps in locating it.

direction. The cantilever’s motion (either due to thermal motion or driven using a piezo-element) causes misalignment with the applied external field. This results in a restoring torque, which for small motion of the cantilever is given by

\[
T_r = \frac{m \beta BH_c}{(B + H_c)} \tag{4.1}
\]

Where, \( m \) is the magnetic moment of the particle; \( \beta \) is the angle the cantilever makes with the field during its motion, \( B \) is the applied external field, and \( H_c \) is the coercive field of the magnetic particle. This torque effectively changes the cantilever spring constant and thus its frequency. The change of spring constant, \( k_0 \), is given by \( \Delta k = T_r/\beta L_e^2 \), where \( L_e \) is the effective length of the cantilever. Thus, the change in the resonant frequency, \( \delta f_0 \), for \( B \ll H_c \) is given by

\[
\frac{\Delta f}{f_0} = \frac{\Delta k}{2k_0} = \frac{m}{2k_0L_e^2} \Delta B \tag{4.2}
\]

where, \( f_0 \) is the resonant frequency at zero applied field and \( \Delta B \) is the change in applied field resulting in \( \Delta f \). Due to the linear relationship between \( \Delta f \) and \( \Delta B \), the moment of the particle can be calculated from the slope of the frequency vs. \( B \).
Figure 4.4: A plot of the cantilever response to an applied magnetic field in the geometry shown in the inset. The movement of the cantilever relative to the field results in a restoring torque that changes the cantilever frequency. The second inset shows an optical micrograph of the NdFeB particle glued on a Si$_3$N$_4$ triangular cantilever.

4.1.3 Microscope design

Fig. 4.5 shows the schematics for the microscope, which consists of a copper cage housing the stages on which the sample and the cantilever are mounted. The copper cage was screwed directly to the cold finger of the cryostat.

The sample was glued to a copper holder mounted on a cryogenic piezo-positioner from Attocube (model ANZ-100), which can move the sample vertically in the lab frame. The cantilever was also mounted on copper holder, but on top of two Attocube
Figure 4.5: a) The copper cage, housing the microscope, inside the optical flow cryostat. The copper age is bolted to the cold finger of the cryostat. b) Zoomed in view of the copper cage without the cryostat. c) Close up of the microscope, showing the sample stage (copper piece on the left) mounted on top an attocube that moves in the vertical direction. On the right hand side the the cantilever is mounted on another copper piece, which itself sits on top of two attocube stages that allow for motion in the horizontal plane (in the lab frame). d) Further zoomed in view of the sample (left hand side) and the cantilever assembly (right hand side). The green part is the cantilever chip, the thin red line indicates a the optical fiber used for interferometry and the yellow rectangular piece is the piezo disc used for driving the cantilever at its resonance frequency. Not shown here are the copper braids used for cooling the various stages. Pump beam shines on the sample from the side opposite to the cantilever.
stages (both model ANX-100), that moved in the other $xy$-plane. It should be noted that the lab frame’s $x$ corresponds to the sample $\hat{z}$-direction, since the sample is mounted vertically.

A Cernox temperature sensor was mounted on the copper piece to which the sample was glued. The temperature was regulated by a heater attached to a PID controller and by adjusting the flow rate of the liquid helium.

Pressed indium contacts were made from the back of the sample to the copper holder to improve thermal contact. Also, soft copper braids, soldered directly to the sample’s copper holder, were screwed into the cage to ensure thermal conduction to the main cold finger. Similarly, a thick copper braid was used to ground the piezo plate (1 mm thick). The cantilever sits on top of the piezo plate, with a high thermal conductivity sapphire plate (1 mm thick) sandwiched between them. The cantilever is heat sunk mainly through the copper braid grounding the piezo plate. Soft copper braids were also used to heat sink the copper piece holding the cantilever assembly, and also the a plate mounted between the attocubes.

Electrical feedthroughs carried the wires for the Attocube stages, peizo-drive for exciting the cantilever motion, and wires for the temperature sensor. A home-built optical feedthrough was used to feed an optical fiber, which was used to perform interferometry for measuring the cantilever position, into the vacuum space.

\section*{4.2 Spin photoluminescence experiments to study local field effects}

A combination of optical selection rule and spin-orbit coupling result in a spin-polarized luminescence from the recombination of a spin-polarized electron population with holes. The physics of spin-PL has been discussed in greater detail in Sec. 2.2. Three different geometries were used for obtaining the spin-PL shown in
4.2.1 Establishing spin physics on unetched epitaxial layers

The first set-up (Fig. 4.6) was primarily used for conducting the initial studies on unetched GaAs samples. These experiments were done in collaboration with Y. Jung [50]. A pump laser which excites the carriers in the semiconductor. The laser energy needs to greater than the band-gap of GaAs (1.52 eV), but smaller than the energy of split-off band to conduction band transition. In fact, due to faster spin-relaxation processes for electrons pumped higher into the conduction band [16], it is advisable to have the pump energy close to the bandgap. For these experiments a
Ti:Sapphire laser (Mira, Coherent Inc.) tuned to 780 nm was used. A Glan-Thompson polarizer was used to polarize the light along the vertical direction (in the lab frame of reference). An Electro-Optic Modulator (EOM, model EO-PM-NR-C1, Thorlabs), which is a voltage controlled waveplate, allows us to produce circularly polarized light, which was incident on the sample through a achromatic-doublet lens (focal length 25.4 mm).

One of the circular components of light was selectively collected at the detector by a combination of a quarter wave-plate and a polarizer. The fast-axis of the quarter waveplate was rotated using a rotation mount to select one or the other circular component. For these initial experiments, a spectrometer with liquid nitrogen cooled CCD (Spec-10:256 / ST133B, Princeton instruments) was used as a detector to measure wavelength-resolved PL (Fig. 4.7). The PL spectrum\(^2\) is similar to those seen in the literature at low temperatures, as is the spin polarization \([?]\).

A Hanle response needs to be measured to verify that the circular polarization seen is indeed due to spins. The Hanle field, \(B_h\), was applied by a home-built electromagnet and its direction was normal to the sample plane. The circular polarization of the light as a function of the PL energy and \(B_h\) is shown in Fig. 4.8. An optical bandpass filter (centered at 820 nm and width of 10 nm) was used for these experiments and the data were collected at 17 K. As can be seen, the circular polarization does get suppressed at higher fields as expected form spin physics, verifying successful spin injection. Fig. 4.9 shows a line cut along the magnetic field direction at the wavelength corresponding to maximum circular polarization. Also shown is a fit to the curve, which is slightly modified from the standard Lorentzian. One needs to account for the fact that the spins are not created and collected along the same axis. This may

\(^2\)It should be noted that the relative intensities of the two peaks are dependent on the power of excitation. At higher pump power, the localized states of the impurities (1.48 eV) get saturated and the band edge (1.51 eV) becomes relatively stronger.
Figure 4.7: Photoluminescence (left axis) and circular polarization of light (right axis) as a function of light energy (bottom axis). The peak at 1.514 eV (819 nm) corresponds to the bandedge of GaAs at 15K, while the peak at 1.481 eV (837 nm) is a donor to silicon acceptor transition.

be done by writing a modified form of eqn. 3.11

\[ \mathcal{H}(B_h) \propto \int_0^\infty \exp(-t/\tau_s) \cos\left(\frac{g\mu_B B_h t + \theta}{\hbar}\right) dt, \]

A spin lifetime of 88 ns is extracted from the fitting and matches well with literature for this doping level [?].

4.2.2 Spin-PL measurements on membrane sample using fiber based injection

After successfully observing spin physics in the unetched sample, the next set of experiments was done on the membrane sample described in Sec. 4.1 using the geometry shown in Fig. 4.10. An optical fiber was used to pump circularly polarized light into the sample. This was achieved by carefully inserting the fiber into the back etched
Figure 4.8: The figure shows the dependence of circular polarization of PL on the photon energy and transverse magnetic field $B_h$. A bandpass filter was used to remove pump scatter for these measurements. (Adapted from Y. Jung [50])

hole and gluing it 15 $\mu$m from the back of the membrane. The fiber core is 3 $\mu$m in diameter and allows for an injection spot size of $\sim 7$ $\mu$m (see Fig. 4.11)

Presented in Fig. 4.12 is a comparison of the Hanle curve from the previously discussed unetched sample using free space injection (red) and large pump sizes, with that obtained from the membrane using fiber injection (blue). As can be seen, the membrane with fiber based injection has a much broader Hanle curve. The most likely culprit for this is the rather high power density and will be investigated in further detail in the next subsection.

Also, shown in Fig. 4.13 is a comparison of the Hanle response measured by spectrometer and photodiode (DET110A, Thorlabs). In this case, the light polarization was modulated between the two circular states using the EOM, and a lock-in detec-
Figure 4.9: Figure shows the Hanle response of an unetched sample (circles) and a fitting to extract the $\tau_s$. The extracted lifetime is 88 ns. Please see text for the details of the fitting. (Adapted from Y. Jung [50])

tor was used to demodulate and measure the signal with high accuracy. Typically lock-in time constants in the range of $0.5 \text{s} - 3 \text{s}$ were used for these measurements. Luminescence is not usually detected using a photodiode, since normally a wavelength resolved spectrum is required. However, spectrometer-based detection is not compatible with modulation of the signal, as is needed for experiments involving the cantilever and micromagnet. Furthermore, photodiodes are much less expensive than a spectrometer.

The spectrometer data integrates over photon wavelengths that are in the range of a few angstroms (set by the resolution of the spectrometer grating and slit size), while the photodiode’s response is integrated over the 10 nm width of bandpass filter. However, based on widths of the Hanle responses in Fig. 4.13, this integration does not seem to affect the effective spin lifetime significantly.
4.2.3 Spin-PL measurements in free space confocal geometry

The main impetus for the fiber based injection was to be able to implement optical injection and detection in a cryostat with no optical access, like those more commonly used for scanned probe microscopes. While, it was possible to collect the spin-PL data using fiber based injection, it was not a very stable system. Any small stress or mechanical motion can change the polarization state of light. A more prudent approach was to use a free space injection in an optical cryostat. While this did complicate the design inside the cryostat, it has significant advantages in optics.

A design based on a confocal microscope was chosen going forward. Such a design (Fig. 4.14) shares an objective lens for both the pump and detection, thus naturally aligning them. Typically objective lenses have higher numerical aperture than other lens. This allows for smaller focused spot size of the pump. This is important for the experiments involving the the effects of a local magnetic probe. Such effects will be
Figure 4.11: Figure shows a GaAs membrane with an optical fiber glued behind it. The small white spot is the pump being transmitted by the fiber. The pump spot size on the sample is \( \sim 7 \mu m \).

more pronounced if the size of the micromagnet is comparable to that of the pump beam, since this will result in most of the injected spins experiencing the field from the magnet. Also, as in a conventional confocal microscope, a dichroic mirror was used to reduce the amount of pump reaching the detector. Reducing the pump light reaching the detector is very crucial as it can be 100,000 times stronger than the PL signal. Thus any noise on the pump can significantly affect the SNR. The dichroic mirror filters the 780 nm pump light in the transmission path of the detection optics, while allowing it to reflect with minimal losses. In conjunction with the dichroic filter, a 780 nm notch filter and 820 nm bandpass filter were also used. For these experiments a fixed wavelength solid state diode laser (56RCS-HS from Melles-Griot) was used. The wavelength of the laser was slightly tunable around 780 nm by adjusting its temperature, and the temperature was adjusted to obtain minimum transmission through the notch filter.
Figure 4.12: Comparison of the Hanle responses obtained in a membrane sample with fiber based injection (blue), and unetched sample with free space injection (red). The broad Hanle response with the membrane sample is most likely due to the high power density (resulting from a small injection size). The inset shows zoomed in view of the data close to zero field.

I also moved to the photodiode-based detection mentioned previously, since the photon energy resolution afforded by the spectrometer is not required while studying local magnetic probe effects. In these measurements, the pump was modulated between two linear states by a square wave input to the EOM driver. The dichroic mirror reflects linear states more accurately than circular states. The linearly polarized light is converted to the circular states by the quarter waveplate (QWP), which also converts the circularly polarized light in the PL into linear states. A linear polarizer then selects one of these circular states (which has been converted to a linearly polarized light by the QWP), and its intensity is detected by the photodiode.

To establish the reason for broad Hanle curves observed with the membrane sample, a more careful study of the Hanle response was made as a function of the incident pump power density and radius. The confocal geometry using the objective lens al-
Figure 4.13: A comparison of Hanle response using two different detectors—spectrometer (left axis, blue circles) and photodiode (right axis, red circles).

This allows for better imaging capabilities, thus allowing for better estimates of the pump radius, and thus power density also. The pump size/radius was modified by placing two additional lens between the EOM and the dichroic mirror. By moving these lens relative to each other the pump beam was controllably decollimated at the input of the objective. The focus spot size is directly proportional to the collimation of the beam. In this manner the pump size could be varied while keeping the detection optics in focus at all times.

Hanle curves for two pump diameters are presented in Fig. 4.15. This data was measured at 10K. It should be noted that each curve has been multiplied and offset individually to enhance the visibility. As an example, the curves at smallest power densities (in each panel) have been multiplied by 200 and 500 times for panel a) and b) respectively.

The Hanle halfwidths, $B_{1/2}$, can be obtained for each of the curves in Fig. 4.15. This is shown in panel a) of Fig. 4.16. Two trends can be noticed from this data: 1)
Figure 4.14: Optical set-up for measuring PL using confocal geometry. The pump and the detection share an objective lens, thus ensuring a better alignment between the two.

Hanle curves broaden as the power density increases, 2) Hanle curves also broaden as the pump size decreases. The former can be attributed to competition between spin relaxation and carrier recombination dynamics as explained in Sec. 2.3 and eqn. 2.12c. The Hanle halfwidth is inversely proportional to the effective spin lifetime and should thus increase with the power density, which is directly proportional to the generation rate. It should be noted that the Hanle halfwidths can be well fit by a line as a function of power density. The intercept of this line should give the intrinsic spin lifetime of the spins. As can be seen from the plot, the intercept of 2.8 G corresponds to 95 ns for the spin lifetime, which is consistent with the first measurements using the spectrometer and with values reported in literature.

The second observation can be understood in the following manner. The recombination region of the charge carriers, which determines the effective detection area for PL, is close to the injection/excitation region. As the pump size decreases, so does
the effective detection region. However, spin diffusion still exists, and thus longer lived spins will diffuse away from the injection/detection region. Thus, PL essentially collects the shorter lived spins, resulting in a broader Hanle response.

Shown in Fig. 4.17 is a comparison of the Hanle response from different regions of the samples. While the Hanle curve widths do vary a little in different regions of the sample, the effect of the pump intensity is much more significant, and any strain due to the fabrication process does not seem to affect the spin lifetime significantly.

4.2.4 Local field effects on spin-PL

Having established the ability to measure spin-PL in the confocal geometry, I now turn my attention to measuring the effects of local magnetic fields on the spin-PL signal. All the data in the previous subsection and this subsection are collected using the cryostat described in Sec. 4.1.3. Fig. 4.18, shows two optical micrographs: one of a membrane with the cantilever behind it, but visible (along with the micromagnet) in transmitted light; another of the pump beam from the front aligned to the micromagnet. Also seen in the images is the red light from the fiber used for interferometry and the faint outline of the optical fiber.

Fig. 4.19 shows the effect of the magnetic tip on the Hanle response of the spin-PL signal from the GaAs membrane. The moment of the magnetic tip was measured by cantilever magnetometry to be $1.35 \times 10^{-10} \text{ J/T}$. The red dotted data is the Hanle response with the tip far away (> 100 $\mu$m away), while the blue dotted curve was measured while the micromagnet was touching the sample. Please note that the effective dipole of the micromagnet sits at its center, and thus is $\sim 3 \mu$m away from the sample. A broadening of the Hanle response can be seen when the tip is close to the sample. Such a broadening was predicted from the numerical analysis in Sec. 3.5.

$^3$Normally an invisible 1550 nm laser is used for interferometry. However, the red light is used for assisting with the optical alignment process.
Figure 4.15: Hanle response in unetched membranes for various pump power densities. Panel a) shows the data for large pump size (≈ 30 µm; Panel b) shows the data for small pump size (≈ 10 µm). The data was taken at 10 K. It should be noted that each curve has been multiplied and offset individually to enhance the visibility.
Figure 4.16: a) Hanle halfwidth extracted from Fig. 4.15 as a function of the power density for small and large pump sizes. The inset shows a zoomed-in version for the large pump size. b) A comparison at similar power densities showing a broadening of the Hanle response due to diffusion.
Figure 4.17: Comparison of Hanle response from three different regions of the sample—unetched part (red), etched membrane (blue), and in a region close to where the membrane had broken (green). The figure shows that the Hanle width is set by factors other than the strain in the sample.

Figure 4.18: Optical micrograph of membrane sample with the cantilever as seen from behind the membrane in transmitted light (red image). The black dot is the micromagnetic particle. The second micrograph shows the pump beam aligned to the micromagnet.
However, the results are not understood well. At the closest distances, fields from the micromagnet are expected to be of the order of kilogauss and should have resulted in greater broadening of the Hanle. The motion of the micromagnet in direct response to the applied magnetic field complicates the analysis. The cantilever can twist due to the magnetic field and its distance and orientation relative to the spins can change. Analyzing that, especially in the presence of vibrational noise, is non-trivial.

Another way to examine the effects of local fields might be to image the spin polarization using the CCD camera. Fig. 4.20 demonstrates the ability to measure spin-PL using a CCD. The images are taken with the micromagnet aligned to the pump beam and touching the membrane. The figure shows false color CCD images of the intensity of PL spot. It shows two images taken when the pump was left circularly polarized (LCP1, LCP2) and one when it was right circularly polarized (RCP1). The other two panels show the differences LCP1 − LCP2 and LCP1 − RCP1. The difference of the two left circularly polarized images is essential noise, while the LCP1 − RCP1
Figure 4.20: The top row shows two CCD images taken with LCP light, while the third is with RCP light. The bottom rows shows the differences LCP1 − LCP2 and LCP1 − RCP1. The images are taken with the micromagnet aligned to the pump beam and touching the membrane.

shows a pattern. Any background pump light should not show spatial dependence of it circular polarization. The pattern seen in this difference image is reminiscent of the precession of spins seen in panel (a) of Fig. 3.14. Spins precess around the local magnetic field vector from the dipole moment, and can result in the image seen here.

4.3 Magnetic force detection of spin-injection

I will now move on to the effect of these spins on the micromagnet and the cantilever. The dipolar coupling and the resulting force have been discussed in Sec. 3.7.

Fig. 4.21 and Fig. 4.22 together show the closest indication of the first force detection of spin injection in GaAs (or any material). This data was taken with a special SmCo coated Si cantilever ($k = 1-5$ N/m), obtained from Veeco Instruments at 17 K. Fig. 4.21a shows the cantilever spectrum when the the circularly polarized pump (780 nm) was being modulated at the cantilever resonance frequency determined from self excitations. The self excitations for this set of data were achieved using piezo
disc and a self-excitation circuit. The various curves correspond to various magnetic fields. Fig. 4.21b shows a similar set of data, except that the cantilever resonance frequency was determined by driving the cantilever into self excitation by simply turning up the power of the interferometer laser. It provides a less noisy excitation than the piezo disc, and allows for a better measurement of the resonant frequency. It should be noted that it is the DC power of the laser being adjusted. The exact nature of this excitation of the cantilever is not understood. Such laser effects have been seen in bilayered cantilevers and are attributed to thermal effects. However, it was not reproducible from one cool down to the next, or even at various times during a cooldown. Such a coupling was never seen with the Si$_3$N$_4$ cantilevers.

Fig. 4.22 presents the data for the peak values at various fields from two cantilever spectrum figures. The peak values have been converted into a displacement using the calibration obtained from interferometry data (9.27 Å/mV). The cantilever amplitude clearly decreases at higher magnetic fields, as expected from precession physics. The solid line is a Lorentzian fit, but is only meant to serve as a guide to the eye, since it is extremely difficult to get an exact model for fitting$^4$. The magnitude of the force can be computed from $F = k z_0 / Q$, where $z_0$ is the cantilever amplitude and $Q$ is the quality factor of the cantilever. This suggests a force of 5-20 N depending on the exact spring constant.

For comparison, the spin-PL signal with the tip far far away from the sample is also presented (red curve). As can be seen the Hanle response of the force signal is much broader. Such behavior has also been seen in simulations (Sec. 3.7). This could be due to the fact that the force signal is mostly sensitive to the nearby spins that strongly interact with the tip. These spins feel a strong field parallel to the injected direction that results in a broadening of the force signal’s Hanle response. No local

$^4$Unfortunately, this cantilever was a sample, and I do not have enough information regarding the magnetic coating on the cantilever.
Figure 4.21: Cantilever power spectral density for various values of $B_h$. Data were measured while the cantilever was 200 nm away from the sample and the pump was aligned to the cantilever tip. The pump was being modulated at the cantilever’s resonant frequency for a given $B_h$. Panel a) shows the data when the resonant frequency was determined by self exciting with piezo, while for panel b) the resonant frequency was determined by the laser drive (please see text for more information).
Figure 4.22: The blue curve (left axis) is a plot of the peak positions at various fields from the data in Fig. 4.21, converted to cantilever amplitude using calibration from interferometry. The solid line is a Lorentzian fit, but is only meant as a guide to the eye. Also shown for comparison is the spin-PL signal when the cantilever is far away (red curve, right axis). The inset shows an optical micrograph taken during the measurement. It shows the cantilever (dark rectangular shape) tip aligned to the pump beam (brighter white spot).

Field effects could be seen in the PL signal, while measuring the cantilever was close. This may be due because the force signal is mostly sensitive to the nearby spins that strongly interact with the tip, where as the PL signal is global. If the pump size is larger than the region where the tip-field is significant, then a measurable change in spin-PL may not be seen, since most of the spins are not perturbed by the probe.
Chapter 5

CONCLUSIONS AND FUTURE OUTLOOK

5.1 Widely applicable numerical tool for spintronics

This dissertation addressed issues related to understanding the effects of inhomogeneous environments on the behavior of spins injected into semiconductors. While my main motivation was to understand the effects of spatially varying magnetic fields of a magnetic force detector, I and my collaborators have developed a numerical tool that can address a variety of spatial and even temporal variations in a spintronic system. I believe that this can be a very useful tool in guiding experiments and in understanding the performance of devices. I have also analyzed the effect of various field components on the spin precession of spin-polarized populations, and the analysis provides a way for interpreting and understanding my numerical results.

Along with my collaborators, I have proposed a new technique for imaging spin properties of spatially inhomogeneous samples that achieves applicability to a wide variety of materials by relying on proven spin polarization detection techniques such as electrical and optical detection. The localized information is obtained by detecting the local modification to the local spin density by the confined magnetic field of a magnetic dipole scanned over the sample. To this end, we have developed a method for simulating spin density in a medium with spatial or temporal inhomogeneities of
the magnetic field, lifetime, or diffusion constant, regardless of injection or detection
technique. As in MRI, this technique has a spatial resolution inversely proportional
to the gradient of the magnetic field down to length scales comparable to the spin
diffusion length. Below this, resolution improves more slowly with increasing gradient.

My numerical analysis of spin transport emphasizes the importance of experimental
characterization of microscopic inhomogeneity, and of models that incorporate
these phenomena. In particular, inhomogeneities can confound the interpretation of
spin polarization and lifetime measurements, as in Hanle curves. Microscopic imaging
of spin properties of real-world devices will be essential for improving their spin
fidelity and functionality.

5.1.1 Future work for simulations

While the numerical results so far have been very helpful in understanding the basic
behavior of spins in inhomogeneous environments, the tool is far from having reached
its limits. One of the limits of Euler’s method is its instability in the presence of
high fields or strong gradients. The algorithm can give diverging results. This can
be avoided by using more sophisticated algorithms such as Runge-Kutta method.
My collaborators and I have already implemented the Runge-Kutta algorithm. The
ability to model high field behavior is important for extending the numerical tool
to scientifically interesting cases where spin-orbit and hyperfine interactions can be
strong.

We are looking to extend the simulations to 3-dimensional samples, which will be
important for better estimates of the forces that can result on a cantilever. The 3rd
dimension is important since the magnetic field gradient from the tip need not be
constant on the scale of the thickness of the sample.

We are also looking to incorporate electric field effects, especially spatially vary-
ing electric fields, such as those might exist in interesting materials like graphene. Spatially varying electric fields might also be a way to incorporate disorder potential in the spin-transport equation.

5.2 Experiments on interaction of a micromagnet with injected spin populations

On the experimental side after establishing successful spin injection, I was able to measure the broadening of Hanle curve in the proximity of a magnetic tip, as predicted by the simulations. More work needs to be done to improve the reproducibility of the results. I was also able to measure the force from the optically injected spin population in n-GaAs. These forces are extremely small (femto-newtons). This was predicted from the force calculations. A clear distinction between the Hanle response of the spin-PL and force measurements was seen. This is explained by the fact that only the subset of the over all spins, that experiences high parallel field from the tip, contributes to the force signal.

5.2.1 Future work for experiments

The signal needs to be optimized to improve the signal-to-noise ratio and allow for better modeling. More careful studies are required to rule out any spurious effects that might confound the force measurements.

Optimizing the signal will require the guidance of simulations to explore the parameter space of pump size, tip moment and tip-sample separation to find the conditions that give the maximum force. Reducing the spurious requires the implementation of sophisticated measurement techniques that can get rid of backgrounds due to direct magnetic coupling of the magnetic field and the micromagnet. Such techniques, used in the MRFM community, rely on varying the phase of the light polarization
modulation, relative to the cantilever frequency.

The other focus of the experiments is to image spin properties using the proposal for scanned perturbation imaging that I laid out in Sec. 3.6. One such experiment using electrical detection in graphene is already being pursued by Andrew Berger. Another approach would be to use linearly varying fields from commercial electromagnets to study spin properties on larger lengths scales.


