Essays on House Prices and Consumption

Dissertation

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By

In Ho Song, B.A., M.A.

Graduate Program in Economics

The Ohio State University

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Dissertation Committee:
Aubhik Khan, Advisor
Paul Evans
Julia Thomas
Abstract

Micro data suggests that housing and consumption tend to move together and are classified as complements. However, most macroeconomic models rule out the possibility of complementarity between housing and consumption by adopting separable preferences, usually based on a log utility function. Along with complementarity, housing transaction costs are also crucial to the analysis of housing and consumption. Recognizing the importance of both complementarity and transaction costs, I develop a dynamic stochastic general equilibrium (DSGE) model that allows for realtor fees and nonseparable preferences over housing and consumption.

The assumption of complementarity is supported by an estimate of the elasticity of intratemporal substitution (EIS) using the National Income and Product Accounts (NIPA) data over the period 1970 Q1 to 2009 Q1. I estimate the EIS at 0.68; thus, housing and non-housing consumption are complements. This result is strong evidence against models that impose preferences characterized by additive separability.

Using my DSGE model, I investigate the consumption responses of heterogeneous households following changes in both house prices and interest rates. The model predicts that credit constrained households will be substantially more responsive to changes in both house prices and interest rates than unconstrained households. I confirm these predictions of my model using household data from the Consumer Expenditure Surveys.
I also investigate whether monetary policy responding to house prices improves the stability of inflation and output. Efficient frontier lines show that responding to house prices lowers the volatility of both inflation and output when the central bank gives sufficient weight to output stability. This result is consistent with a vector autoregression (VAR) analysis examining the interaction of the federal funds rate, house prices, inflation and output. Furthermore, using an extension of the Rudebusch and Svensson (1998) model, I find that there exists a house price effect in monetary policy. These results indicate that, to the extent that house prices affected output, monetary policy responded during the 1970 through 2007 period.
This work is dedicated to my wife, Haeran Suh. Her enthusiasm and support have been essential throughout my graduate studies.
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Vita

2000  ...............  B.A. German Literature and Linguistics, Korea University, Seoul, South Korea
2002  ...............  (Temporary Rest: M.A.) Finance, Korea University, Seoul, South Korea
2004  ...............  M.A. Statistics, Columbia University
2006  ...............  M.A. Economics, The Ohio State University
2006 to present       Graduate Teaching Assistant, Department of Economics, The Ohio State University

Fields of Study

Major Field: Economics
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Chapter 1: INTRODUCTION

A house not only serves as an important asset, but it also is a large component of overall consumption expenditure. In recent years, researchers have begun to study the role of house prices in explaining movements in macroeconomic series. Consistent with this focus, aggregate data show the comovement between house prices, residential investment and Gross Domestic Product (GDP).

In particular, a house has several essential characteristics compared to other durable goods: (1) the total value of houses is as large as 1.4 times GDP; (2) housing and consumption are complements, while durable goods and consumption are substitutes; (3) a house depreciates very slowly;\(^1\) (4) to the extent to which housing is collateralized, the conventional law of demand is subordinated through the collateral effect.\(^2\)

1.1 House prices, Consumption, and Monetary Policy

Chapter 2, “House Prices and Consumption”, studies the consumption responses of two groups of households following changes in both house prices and interest rates.

\(^1\)Davis and Heathcote (2003) show the effect of slow depreciation in housing on housing production. Specifically they address the depreciation rate of only 1.6 percent for housing and 21.4 percent for durable goods.

\(^2\)Typically housing is highly leveraged.
I show that the common assumption, that household period utility is separable in housing and consumption, can be consistent with the observed comovement between these two series only in the absence of housing transaction costs. When these costs are introduced into dynamic stochastic general equilibrium (DSGE) models characterized by separable preferences, consumption no longer increases after a rise in house prices. However, it is well known that transaction costs are an important ingredient in house sales. I address this issue by developing a model that allows for non-separable preferences in housing and consumption alongside housing transactions costs. The results of my model closely match the aggregate data. Furthermore, it predicts that credit constrained households will be substantially more responsive to changes in both house prices and interest rates than unconstrained households. Following a rise in house prices, consumption among constrained households increases by far more than the consumption of unconstrained households. Following a rise in interest rates, constrained households’ consumption falls by more than that of unconstrained households. I trace this differing responsiveness in consumption to the house loan-to-value ratios of credit constrained households. Higher loan-to-value ratios imply larger differences in their elasticity of response relative to unconstrained households. I also find that the differences widen with the degree of complementarity between housing and consumption. These predictions of my model are confirmed by household data from the Consumer Expenditure Surveys.

In Chapter 3, “Complementarity between Housing and Consumption”, using the NIPA data over the period 1970 Q1 to 2009 Q1, I estimate the elasticity of intratemporal substitution (EIS) between housing and consumption. I find that the estimate of the EIS is 0.68; thus, housing and non-housing consumption are complements.
This result is strong evidence against models that impose preferences characterized by additive separability. The finding is also robust to a variety of estimation methods including Dynamic Ordinary Least Squares (DOLS), Canonical Cointegrating Regression (CCR) and Fully Modified OLS (FMOLS). This estimate is also consistent with the previous microeconomic literature. Both Flavin and Nakagawa (2004) and Siegel (2004) find complementarity in microeconomic data: the Panel Study of Income Dynamics (PSID).

In Chapter 4, “Do House Prices Matter for Monetary Policy?” I empirically investigate whether the house price is a significant factor in monetary policy. Then I construct a theoretical framework that explains empirical results. For the empirical work, extending the Rudebusch and Svensson (1998) model (RS model) to include the house price, I compare the original RS model’s versus the extended model’s ability to predict movements in the federal funds rate. I find that the extended RS model better matches the data over the periods of 1970 through 2007. This result suggests that, to the extent that house prices affect aggregate demand, monetary policy responds.

For the theoretical framework, I construct a DSGE model characterized by complementarity between housing and consumption. The purpose of the model is to obtain the efficient frontier lines with respect to the volatility of inflation and output. Then I investigate whether a monetary policy rule responding to house prices lowers aggregate volatility. The result shows that responding to the house price improves the stability of inflation and output as the central bank gives more weight to output stability. Complementarity and collateral play an important role in monetary policy in the model. The effect of the house price on monetary policy is confirmed by a VAR

\[^{3}\text{The additive separability in the representative macroeconomic literature is adopted by Iacoviello (2005).}\]
analysis examining the interaction of the federal funds rate, house prices, inflation and output.

1.2 The House Price Mechanism

Suppose house prices rise. This increases both the values of collateral, for credit-constrained households, and net worth for all households. In particular, the increase in net worth of credit constrained households rises more because of the collateral effect. So, the consumption of credit constrained households is more responsive to house prices than that of unconstrained households. An increase in net worth makes it easier for households to access house related credit to finance housing and consumption. A rise in the collateral value increases borrowing capacity. The increase in both net worth and the collateral value raises housing demand. Overall, as a result, spending on housing and consumption tends to rise together, showing complementarity. The increase in housing demand leads to a further rise in house prices in a circular process. House prices are also amplified through complementarity and loan-to-value ratios. This circular process can be to some degree abated by housing transaction costs. Nonetheless, the fundamental comovement between housing and consumption continues to deliver an amplification effect. In other words, complementarity and collateral make consumption more volatile. To the extent that house prices affect consumption (consumption is a major component of output), monetary policy responds. By design, the monetary policy response to variation in house prices lowers the volatility of both inflation and output.
1.3 Future Work

In ongoing research, I am using a related framework to examine the role of financial shocks in determining household consumption. Since down-payments associated with mortgages are determined by banks, house loan-to-value ratios are, to some extent, exogenous for households. Thus, I develop a model where changes in loan-to-value ratios serve as financial shocks and use it to investigate the effect of such shocks on consumption and house prices. In particular, I study how constrained households react to a financial innovation that lowers loan-to-value ratios.
Chapter 2: HOUSE PRICES AND CONSUMPTION

2.1 Introduction

In this chapter, I develop a dynamic stochastic general equilibrium (DSGE) model that incorporates two key components: housing transaction costs and nonseparable preferences over housing and consumption. Households differ in that unconstrained households are not affected by their house collateral value, while constrained households are affected by the collateral value of their houses. Constrained households are assumed to be relatively impatient compared to unconstrained households. The purpose of this chapter is to investigate the consumption responses of each group of households following changes in both the house price and the interest rate. There are two essential elements in this study: complementarity and housing transaction costs.

Complementarity between housing and consumption exists when the elasticity of \textit{intrag}temporal substitution (EIS) is less than unity. In this paper, the EIS is estimated at 0.59 through minimum distance estimation. This finding of complementarity is consistent with previous finding in National Income and Product Accounts (NIAP) data (Song (2007)), PSID data (Flavin and Nakagawa (2004), Siegel (2004), Li et al. (2009)), CEX data (Stokey (2007))\textsuperscript{4}, and data from the Housing Allowance Demand

\textsuperscript{4}Using the NIPA data from 1970 to 2009, Song (2007) finds evidence at complementarity. Flavin and Nakagawa (2004) apply complementarity to asset pricing implications. They find that the
Experiment (Hanushek and Quigley (1980)). Hence, introducing complementarity under nonseparable preferences over housing and consumption is justified based on the data.

Along with complementarity, proportional housing transaction costs are also crucial in the analysis of housing and consumption. Eberly (1994) and Lam (1989) show the importance of transaction costs as a determinant of durable goods consumption. Furthermore, Grossman and Laroque (1990) argue that transaction costs significantly affect the number of house sales. They show that a transaction cost that is 5 percent of the value of the existing house implies an average time between purchases of 20 to 30 years. It follows that models without transaction costs may not capture an essential aspect influencing movements in the data, and their predicted price movements may be misleading.

Most of the existing model-based macroeconomic literature rules out the possibility of complementarity between housing and non-housing consumption by adopting separable preferences, which are usually based on a log utility function as in Aoki et al. (2004a) and Iacoviello (2005). Elsewhere, Piazzesi et al. (2007) find evidence suggesting that housing and consumption could be substitutes. They do not directly consider housing, but instead consider the elasticity of substitution between durable goods and nondurable goods.\(^5\) They adopt a parameter value ranging in the interval \([1.04, 1.43]\) with 95 percent confidence, as in Ogaki and Reinhart (1998). This EIS is less than one, arguing housing and consumption are not separable in preferences. In fact, they estimate the EIS to be 0.13. Li et al. (2009) estimate the EIS at 0.33 through the Method of Simulated Moments (MSM) using PSID data. Stokey (2007) also estimates the EIS at 0.23 and 0.45 using CEX data.

\(^5\)Piazzesi et al. (2007) estimate the EIS using the NIPA data over the sample period 1947 to 2001. But they ignore the EIS estimate less than one by using long sample periods (1936 - 2001) of NIPA data.
estimate of that elasticity of substitution, however, is misleading about the direct elasticity between consumption and *housing services*.

This paper is related to the seminal contribution of Iacoviello (2005). His analysis of housing assumes separable preferences between consumption and housing, and abstracts from housing transaction costs. When housing transaction costs are introduced with separable preferences into the Iacoviello (2005) model, consumption no longer increases following a rise in house prices. This is inconsistent with the empirical evidence showing a positive comovement between house prices and consumption.

Recognizing the importance of both complementarity and transaction costs, I address this issue by developing a DSGE model that allows for realtor fees and nonseparable preferences over housing and consumption. In my model, transaction costs, complementarity and house loan-to-value ratios drive different consumption responses across credit-constrained and unconstrained households. Complementarity between housing services and non-housing consumption makes the model consistent with the comovement seen in the data. The transmission mechanism of house prices is as follows. Suppose house prices rise. This increases both the values of collateral, for credit-constrained households, and net worth. As a result, spending on consumption and housing rises. The increase in housing demand leads to a further rise in house prices. Transaction costs abate this circular process somewhat. Nonetheless, the fundamental comovement between housing and consumption continues to deliver an amplification effect.

Song (2007) finds a significant estimate of the housing versus non-housing consumption elasticity to be 0.68. This finding of complementarity is robust to a variety of estimation methods including Dynamic OLS (DOLS), Canonical Co-integrating Regression (CCR) and Fully Modified OLS (FMOLS).
I find that constrained households are substantially more responsive to changes in both house prices and interest rates than unconstrained households. A rise in house prices increases the consumption of constrained households by far more than the consumption of unconstrained households. Furthermore, following a rise in interest rates, the consumption of constrained households falls by more than that of unconstrained households. This difference in the responsiveness of consumption is driven by the house loan-to-value ratio characterizing credit-constrained households. Higher loan-to-value ratios imply larger differences in the consumption response across constrained and unconstrained households.

Stronger complementarity also widens the gap between the consumption responses across the two groups of households. With stronger complementarity, constrained households reduce their consumption even further in response to an interest rate rise. Higher house loan-to-value further magnifies the effect of changes in interest rates, so that the real economy becomes more volatile with respect to house prices and consumption. I confirm these predictions of my model using household data from the Consumer Expenditure Surveys.

The remainder of this paper is as follows. In section 2.2, I provide a set of empirical regularities regarding house prices and the demand for consumption goods and housing. There, I document the positive correlation between house prices and consumption that I seek to address. Next, in section 2.3, I construct a DSGE model allowing for both housing transaction costs and nonseparable preferences over housing and non-housing consumption. In section 2.4, I calibrate my model using long-run average values and shares of macroeconomic series. Next, I estimate the structural parameter governing the consumption-housing elasticity using a minimum distance
method that minimizes the difference between impulse responses from the model versus the data. Results are presented in section 2.5 and section 2.6. There, I show that my model generates time series consistent with those in the data. I also evaluate my model’s comparative statics predictions regarding house loan-to-value ratios and housing transaction costs. I show that, following an increase in the interest rate, higher loan-to-values lead to more volatile responses in both house prices and consumption. Section 2.7 concludes.

2.2 Stylized Facts

In 2007, the value of real estate in households amounted to over $20 trillion, which is 1.4 times greater than nominal annual gross domestic product (GDP). The mortgage related market, totaling $8.6 trillion in 2007, has grown aggressively to become a dominant component of financial markets since 1997. New issuance of mortgage related securities amounted to over $2.1 trillion, which is 272 percent of outstanding Treasuries and 180 percent of the corporate bond market.7

Throughout the post-war period, US house prices, residential investment and GDP have comoved. Figure 2.1 shows the comovement of house prices, consumption and residential investment. At the onset of each recession, all variables tend to move downward together. Residential investment is very volatile and severely affected during a recession. In 2005, the contribution of residential construction to GDP was 6.3 percent; by 2009, it had fallen to 2.4 percent.8

7These observations are drawn from the Securities Industry and Financial Markets Association (SIFMA) data, which is available at: www.sifma.org/uploadedFiles/Research/Statistics/SIFMA_U.SBondMarketOutstanding.pdf
In Figure 2.1, I examine the direct relationship between house prices and consumption using a scatter-plot of de-trended personal consumption expenditure (PCE) and de-trended house prices (CMHPI). The data is from the Bureau of Economic Analysis (BEA) and Freddie Mac, and covers the 1970 to 2009 period. I use an HP-Filter to de-trend the data. The correlation between house prices and consumption is 0.42. As house prices increase (decrease), households tend to increase (decrease) consumption. Given the positive correlation between house prices and both consumption and residential investment (housing demand), housing and consumption appear to move as complements.
Houses are different from other durable goods in several important respects. First, changes in housing stocks incur substantial transaction costs proportional to house values. Second, the average housing depreciation rate (1.6 percent) is very small compared to the depreciation rates of other durable goods (averaging 21 percent). Third, with a required down payment, houses are highly leveraged. Fourth, house values are three times as big as all other durable good stocks. Fifth, houses perform a dual role; they are both an investment asset and a shield against fluctuations in stock prices. In the section below, I develop a model that captures these special qualities unique to housing.

Couch (2004) examines the correlation between house prices and stock returns and finds that it is relatively low.
2.3 The Model

The economy is populated by unconstrained households, constrained households, entrepreneurs, retailers and the central bank. Households are heterogeneous in that unconstrained households are unaffected by their collateral, provide loans to constrained households, receive lump sum profits from the ownership of retail firms and accumulate realtor fees. Constrained households are affected by house collateral, receive loans from unconstrained households and are relatively impatient in comparison to unconstrained households. Since proportional transaction costs come from realtor fees, constrained households and entrepreneurs have to pay transaction costs to unconstrained households (realtors), whenever there are housing transactions. Entrepreneurs produce intermediate goods using labor, houses and capital as inputs. Retailers, who buy intermediate goods from entrepreneurs and sell final consumption goods to households, are monopolistically competitive and have infrequent opportunities to adjust prices. The central bank takes the nominal interest rate as a policy instrument. Monetary policy is assumed to follow a Taylor rule responding to past inflation and past output.

The model in this paper follows the same structure as Iacoviello (2005). However, it differs in two crucial ways. First, the household utility function is not separable in housing and consumption. Introducing nonseparable preferences is a critical extension to the analysis of housing in macroeconomics, since complementarity in preferences is indicated by both micro and macro data. Second, introducing housing transaction costs leads to important changes given the nonseparable utility function.\textsuperscript{10}

\textsuperscript{10}Transaction costs reduce the effect of house prices on consumption. Moreover transaction costs are important determinants of durable good consumption (Eberly (1994)) and (Ogaki and Reinhart (1998)).
In this paper, housing transaction costs proportional to the sale price of houses are introduced into the budget constraints of households and entrepreneurs. Housing transaction costs capture two important facts. First, new house purchases heavily depend on the sale of existing housing. These purchases are associated with realtor fees. Second, realtor fees as a main component of housing transaction costs reduce the effect of changes in house prices on consumption. As a result, transaction costs can dampen movements in house prices. In fact, introducing transaction costs moves the model closer to the data, as will be shown in section 2.5 and 2.6.

2.3.1 Unconstrained Households

Unconstrained households receive utility from consumption and housing services derived from the quantity of housing stock they hold. They also provide labor for production. Their preference for consumption and housing services is represented by a time separable constant elasticity of substitution (CES) utility function. Unconstrained households maximize the following objective function:

$$\max_{\{b_{1,t}, C_{1,t}, H_{1,t}, N_{1,t}\}_{t=0}^\infty} \left\{ E_0 \sum_{t=0}^\infty \beta_1 t \left( \frac{1}{1-\varsigma} \left[ \left( C_{1,t}^{\frac{\varsigma-1}{\epsilon}} + j_t(H_{1,t})^{\frac{\varsigma}{\epsilon}} \right)^{\frac{\epsilon}{\varsigma}} \right]^{1-\varsigma} - \frac{N_{1,t}^{m_t}}{\eta_1} \right) \right\}, \quad (2.1)$$

where $E_0$ denotes the expectation operator. $C_{1,t}$ denotes non-housing consumption. $H_{1,t}$ denotes the stock of housing. The discount factor $\beta_1$ for unconstrained households is assumed to be greater than that of constrained households, or $0 < \beta_2 < \beta_1 < 1$. The parameter $\epsilon$ is the elasticity of intratemporal substitution between housing

---

11Housing transaction costs include time costs, moving costs, tax, and realtor fees. I use realtor fees as a representative of housing transaction costs.

12Throughout the paper, the subscripts 1, 2, and e refer to unconstrained households, constrained households, and entrepreneurs, respectively.

13Housing services are assumed to be proportional to the stock of housing.
and (non-housing) consumption. As long as $\varepsilon$ is less than one, housing and consumption are complements. The parameter $\zeta$ can be interpreted as the households’ coefficient of relative risk aversion. $N_{1,t}$ denotes labor hours.\textsuperscript{14} I also assume that the housing demand shock, $j_t$, follows the stochastic process:

$$\ln j_t = (1 - \rho_j) \ln \bar{j} + \rho_j \ln j_{t-1} + \varepsilon_{j,t},$$

(2.2)

where $\bar{j} > 0$, $\rho_j \in (-1, 1)$ measures the persistence of the shock, and $\varepsilon_{j,t}$ is independent and identically distributed $i.i.d. \sim N (0; \sigma_j^2)$ with mean zero and variance $\sigma_j^2$.

Each period, unconstrained households choose a level of consumption, houses and hours of work, given the period budget constraint:

$$P_t C_{1,t} + (H_{1t} - H_{1t-1}) Q_t + R_{t-1} B_{1,t-1} + \varphi_1 Q_t H_{1t-1} \leq W_{1,t} N_{1,t} + B_{1,t} + F_t + S_t.$$  

(2.3)

Households begin with a stock of housing, which has the nominal market value, $H_{1t-1} Q_t$. They also receive lump sum profits, $F_t$, from the ownership of retail firms, and nominal realtor fees, $S_t$, from constrained households and entrepreneurs. $S_t$ follows

$$S_t = \varphi_e Q_t H_{et-1} + \varphi_1 Q_t H_{1t-1} + \varphi_2 Q_t H_{2t-1},$$  

(2.4)

where $\varphi_1$ is zero because unconstrained households are realtors, while $\varphi_e$ and $\varphi_2$ denote realtor fees, which are proportional to the sale value of houses.\textsuperscript{15} Unconstrained households lend in nominal terms, setting $B_{1,t} \leq 0$, and they receive nominal repayment $| R_{t-1} B_{1,t-1} |$ from loans issued in the previous period, where

\textsuperscript{14}Households allocate their time between labor hours, $N_{1,t}$ and leisure, $L_{1,t}$. Their endowment of time is one, hence $N_{1,t} + L_{1,t} = 1$.

\textsuperscript{15}I assume that households have housing transactions at every period, so that realtors receive fees on the entire stocks on $H_{et-1}$ and $H_{2t-1}$. However, the real housing transaction costs are above just realtor fees due to moving costs and time costs. Hence the realtor fees on $\varphi_e$ and $\varphi_2$ capture all these housing transaction costs.
$R_{t-1}$ is the nominal interest rate. It is assumed that houses do not depreciate. Using a numeraire $P_t$, we can obtain the real budget constraint from the nominal budget constraint above:

$$C_{1,t} + (H_{1,t} - H_{1,t-1}) q_t + \frac{R_{t-1}}{\pi_t} b_{1,t-1} + \varphi_1 q_t H_{1,t-1} \leq w_{1,t} N_{1,t} + b_{1,t} + f_t + s_t, \quad (2.5)$$

where $w_{1,t}$ is the real wage households earn from their labor supply, $q_t$ is the real house price, and $\pi_t$ denotes inflation, or $\pi_t = \frac{P_t}{P_{t-1}}$. Since the nonseparable utility function is associated with transaction costs, the curvature parameter, $\varsigma$, can be greater than, equal to or less than the inverse of the marginal rate of intertemporal elasticity of substitution. I define the curvature of the utility function as $\varsigma = \frac{1}{\sigma}$. When $\varepsilon$ is equal to $\sigma$, the utility function becomes separable. When the limit is taken as $\varepsilon$ goes to one, the utility function becomes the Cobb-Douglas specification. The household optimization problem is to choose $b_{1,t}, C_{1,t}, H_{1,t}$, and $N_{1,t}$ to maximize Equation (2.1), subject to constraints Equation (2.2) and Equation (2.5).

From the unconstrained household problem, the following two optimality conditions are derived:

$$1 = E_t \left( \frac{U_{C_{1,t+1}}}{U_{C_{1,t}}} \frac{\beta_1 R_t}{\pi_{t+1}} \right), \quad (2.6)$$

where $U_{C_{1,t}} = \left\{ C_{1,t}^{\frac{\varepsilon-1}{\varepsilon}} + j_t (H_{1,t})^{\frac{\varepsilon-1}{\varepsilon}} \right\}^{\frac{\sigma}{\sigma - 1}} C_{1,t}^{-1}$, and

$$\frac{U_{H_{1,t}}}{U_{C_{1,t}}} = q_t \left( 1 + \varphi_1 \right) - \left( \frac{\pi_{t+1}}{R_t} \right) q_{t+1}, \quad (2.7)$$

where $U_{H_{1,t}} = \left\{ C_{1,t}^{\frac{\varepsilon-1}{\varepsilon}} + j_t (H_{1,t})^{\frac{\varepsilon-1}{\varepsilon}} \right\}^{\frac{\sigma}{\sigma - 1}} (H_{1,t})^{\frac{1}{\varepsilon}} j_t$. Equation (2.6) is the standard Euler equation requiring that the household expected discounted real marginal utility return from saving one unit is equal to the current marginal utility cost of giving up a unit of consumption. Equation (2.7) determines the marginal rate of substitution.
between housing and consumption. These two equations can be combined into

\[
\frac{U_{H_{1,t}}}{U_{C_{1,t}}} = \frac{(H_{1,t})^{\frac{1}{\varepsilon}}}{C_{1,t}^{\frac{1}{\varepsilon}}} = \left(\frac{C_{1,t}}{H_{1,t}}\right)^{\frac{1}{\varepsilon}} j_t. \tag{2.8}
\]

Equation (2.8) gives the optimal ratio of consumption to housing, which is related to house prices, transaction costs, housing demand shocks and the nominal interest rate. The higher the interest rate is, the higher is consumption relative to housing, because the demand for housing is more sensitive to the interest rate. The higher the positive housing demand shock is, the lower is the ratio of consumption to housing, because housing is more valuable.

### 2.3.2 Constrained Households

The optimization problem for constrained households is

\[
\max_{\{b_{2,t}, C_{2,t}, H_{2,t}, N_{2,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} (\beta_2)^t \left\{ \left[ \left( C_{2,t}^{\frac{1}{\varepsilon}} + j_t (H_{2,t})^{\frac{1}{\varepsilon}} \right) \frac{\varepsilon-1}{\varepsilon} \right] \right. - \frac{N_{2,t}^{\eta_2}}{\eta_2}, \tag{2.9}
\]

subject to

\[
C_{2,t} + (H_{2,t} - H_{2,t-1}) q_t + \varphi_2 q_t H_{2,t-1} + \frac{R_{t-1}}{\pi_t} b_{2,t-1} \leq w_{2,t} N_{2,t} + b_{2,t}, \tag{2.10}
\]

where

\[
R_t b_{2t} \leq m_2 E_t (q_{t+1} \pi_{t+1} H_{2t}). \tag{2.11}
\]

For constrained households, housing transactions incur realtor fees, \(\varphi_2 q_t H_{2,t-1}\). The transaction costs are proportional to the value of house sales and are paid to unconstrained households. Since it is assumed that constrained households discount the

\[16\] Equation (2.8) can be rearranged with respect to the ratio of consumption to housing: \( \frac{C_{1,t}}{H_{1,t}} = \frac{q_t}{\left( q_{t+1} \pi_{t+1} H_{2t} \right)^{\frac{1}{\varepsilon}}} \).
future more heavily than unconstrained households, Equation (2.11), which represents the collateral constraint, binds. As in Kiyotaki and Moore (1997), when debtors default on obligations, creditors repossess houses by paying \((1 - m_2)\beta q_{t+1} \pi_{t+1} H_{2,t}\). Hence, \((1 - m_2)\) can be interpreted as a down-payment ratio. The loan-to-value ratio is denoted as \(m_2\), where \(0 \leq m_2 \leq 1\). Liquidation, however, takes time and bears costs. The collateral constraint implies that mortgages cannot exceed the house value.

I assume that the housing demand shock, \(j_t\), for constrained households is the same as that for unconstrained households described above.

The first order conditions imply the following Euler equation and housing price equation:

\[
U_{C_{2,t}} = R_t \lambda_{m_{2,t}} + U_{C_{2,t+1}} \frac{R_t}{\pi_{t+1}} \beta_2, \quad (2.12)
\]

\[
q_t = \frac{U_{H_{2,t}}}{(1 + \varphi_2) U_{C_{2,t}}} + \frac{\beta_2 (1 - \delta_{h2})}{(1 + \varphi_2)} \left( \frac{U_{C_{2,t+1}}}{U_{C_{2,t}}} \right) q_{t+1} + \frac{\lambda_{m_{2,t}}}{U_{C_{2,t}}} \frac{m_{2t} \pi_{t+1} \pi_{t+1}}{(1 + \varphi_2)}, \quad (2.13)
\]

where \(U_{C_{2,t}} = \left\{ C_{2,t}^{\frac{\varepsilon - 1}{\varepsilon}} + j_t (H_{2,t})^{\frac{\varepsilon - 1}{\varepsilon}} \right\} \frac{1 - \sigma_2}{1 - \varepsilon} C_{2,t}^{-\frac{1}{\varepsilon}}\), and

\[U_{H_{2,t}} = \left\{ C_{2,t}^{\frac{\varepsilon - 1}{\varepsilon}} + j_t (H_{2,t})^{\frac{\varepsilon - 1}{\varepsilon}} \right\} \frac{1 - \sigma_2}{1 - \varepsilon} (H_{2,t})^{\frac{1}{\varepsilon}} j_t.\]

\(\lambda_{m_{2,t}}\) is the Lagrange multiplier associated with the borrowing constraint. House prices depend on \(\varepsilon, \sigma\), and housing demand preferences. Composition risks, which are ratio fluctuations in the share of housing to consumption affect house prices. Consumption risks are also associated with house prices. The realtor fee rate, \(\varphi_2\), affects house prices negatively. When \(\varepsilon\) rises relative to \(\sigma\), households are more willing to substitute housing services and nonhousing consumption within a period than they are to substitute overall consumption between the periods. When \(\varepsilon\) is equal to \(\sigma\), house prices are fully determined by composition risk and consumption risk. House prices in the steady state satisfy

\[
q = \frac{\bar{j}}{(1 + \varphi_2) - \beta_2 - (\beta_1 - \beta_2) m_2} \left( \frac{C_2}{H_2} \right)^{\frac{1}{\varepsilon}}. \quad (2.14)
\]

18
The house price depends on the housing demand shock, transaction costs, loan-to-values, and the ratio of consumption to house. The degree of intratemporal substitutability, $\varepsilon$, is also an important element in determining asset prices. House prices increase when the house loan-to-value ratio, $m_2$, increases and when the realtor fee $\varphi_2$ decreases, ceteris paribus. When transaction costs are equal to zero, house prices are determined by the discount factors of lenders and borrowers, the ratio of nonhousing consumption to housing for constrained households, alongside $\varepsilon$. From the budget constraint, net worth is used to finance the difference of new debt and the next unit of housing. Selling one’s existing house affects net worth, which is used to purchase a new house. The difference between the price of a new house and net worth is the households’s new debt. Hence, houses for constrained households satisfy,

$$H_{2,t} = \frac{1}{E_t} \left(\frac{q_t - \frac{m_2 q_{t+1} \pi_{t+1}}{R_t}}{\pi_t}\right) \left(q_t H_{2,t-1} - \frac{R_t b_{2,t-1}}{\pi_t}\right). \quad (2.15)$$

This equation is consistent with the result of Kiyotaki and Moore (1997). As the loan-to-value, $m_2$, rises, house prices increase. This occurs because a higher loan-to-value results in higher housing demand. As house prices rise from both date $t$ and $t + 1$, the current net worth, $q_t H_{2,t-1} - \frac{R_t b_{2,t-1}}{\pi_t}$, and the collateral value, $\frac{q_{t+1} \pi_{t+1}}{R_t}$, increase, which induces an increase in housing demand. The extent to which houses can be used as collateral determines the extent to which the conventional law of demand is overturned. In fact, an increase in house prices induces a rise in housing demand through the collateral effect. For the extreme case of no collateral value, or zero loan-to-value, there is a trade-off relationship between house prices and house

---

17In this paper, the elasticity of intratemporal substitution, $\varepsilon$, is estimated at 0.592, implying complementarity.
quantity demanded. As long as the loan-to-value is positive, however, house prices are positively related to house quantity demanded.

From Equation (2.15), we see there is a transmission mechanism that connects house prices to macroeconomic fluctuations. Suppose house prices rise. Overall, a rise in house prices increases borrowing capacity and net worth. Housing and consumption are boosted because of heightened collateral values and net worth. In turn, higher housing demand further increases house prices. The change in housing net worth is associated with a high leverage effect, measured in the loan-to-value, which makes net worth much higher relative to a required down payment. Indeed, leverage plays an amplifying role in net worth. The converse holds when there is a decline in house prices. As the leverage ratio rises, it reduces net worth. Housing and consumption can shrink because of tighter borrowing constraints, as a fall in collateral values and net worth further depresses housing and consumption. Therefore, changes in house prices may amplify fluctuations in macroeconomic variables.

2.3.3 Entrepreneurs

Entrepreneurs derive utility only from consumption. They produce intermediate goods according to the following constant return to scale Cobb-Douglas production function with labor, housing and capital as inputs:

\[ Y_t = Z_t \left( K_{t-1}^{v} H_{t-1}^{1-v} \right)^{\mu} \left( N_{1,t}^{\alpha} N_{2,t}^{(1-\alpha)} \right)^{(1-\mu)}, \]

(2.16)

where \( Y_t \) denotes output and \( Z_t \) represents productivity in period \( t \). \( K_{t-1} \) and \( H_{t-1} \) denote capital and housing in period \( t-I \), respectively. The shock to \( Z_t \) follows the

\[^{18}\text{This is consistent with Kiyotaki and Moore (1997)'s observation that "The usual notion that a higher land price, } q_t, \text{ reduces the farmers demand is more than offset by the facts that (i) they can borrow more when } q_t + 1 \text{ is higher, and (ii) their net worth increases as } q_t \text{ rises."} \]
stochastic process:

\[
\ln Z_t = (1 - \rho_z) \ln Z + \rho_z \ln Z_{t-1} + \varepsilon_{Z,t},
\]

where \(Z > 0\), \(\rho_Z \in (-1, 1)\) measures the persistence of the shock and \(\varepsilon_{Z,t}\) is a white noise process with mean zero and variance \(\sigma^2_Z\). Capital accumulation follows

\[
K_t = (1 - \delta) K_{t-1} + I_{Kt} - \frac{\xi (K_t - K_{t-1})^2}{2 K_{t-1}},
\]

where \(I_{Kt}\) denotes investment in capital, and adjustments to the capital stock incur quadratic costs parameterized by \(\xi\). Capital depreciates at the rate \(\delta\). Housing accumulation follows

\[
H_{et} = H_{et-1} + I_{Het} - \varphi_e H_{et-1},
\]

where \(I_{Het}\) denotes investment in houses. Housing transactions are associated with transaction costs like realtor fees, \(\varphi_e q_t H_{et-1}\), which are paid to unconstrained households.

The utility maximization problem for entrepreneurs may be written as follows:

\[
\max_{\{C_{et}, b_{et}, I_{et}, K_t, H_{et}, N_1, N_2, q_t\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \gamma^t \ln C_{et},
\]

subject to

\[
\frac{Y_t}{X_t} + b_{et} \geq C_{et} + \frac{R_{t-1}}{\pi_t} b_{et-1} + w_{1,t} N_{1,t} + w_{2,t} N_{2,t} + I_{Kt} + I_{Het} + \xi_{Kt} + \xi_{Het}, \quad (2.19)
\]

where \(I_{Kt} = K_t - (1 - \delta) K_{t-1}\), \(\xi_{Kt} = \frac{\psi_K}{\delta} \left( \frac{I_{Kt}}{K_{t-1}} - \delta \right)^2 K_{t-1}\), \(I_{Het} = \{H_{et} - H_{et-1}\} q_t\), \(\xi_{Het} = \varphi_e q_t H_{et-1}\), and

\[
R_t b_{et} \leq m_e E_t (q_{t+1} \pi_{t+1} H_{et}). \quad (2.20)
\]

Equation (2.20) represents the collateral constraint facing entrepreneurs, with the constant parameter \(m_e\) reflecting their loan-to-value ratio.
2.3.4 Retailers

Retailers in this model face precisely the same problems as in Iacoviello (2005) and Bernanke et al. (1999). They produce a continuum of differentiated final goods indexed along the unit interval. They supply their goods in monopolistically competitive markets, and their profits are paid to unconstrained households. The retailers’ supply curve implies a Phillips curve, determined by an optimal pricing equation. Retail prices are sticky, and the probability of any retailer being able to adjust its nominal price is \( 1 - \theta \). Retailers purchase intermediate goods from entrepreneurs at price \( P_I^t \) and sell final goods to households at price, \( P_t(g) \). Final goods are

\[
Y_t = \left( \int_0^1 Y_t(g)^{(-1)/\xi} dg \right)^{\frac{1}{\xi-1}} \quad \text{with} \quad \xi > 1.
\]

The solution to the firms’ cost minimization problem gives a demand function for intermediate goods and a price index, \( P_t = \left( \int_0^1 P_I^t(g)^{1-\xi} dg \right)^{\frac{1}{1-\xi}} \). Profit maximization by each retail firm implies that the optimal price of retail goods, \( P_t^*(g) \), satisfies

\[
\sum_{i=0}^{\infty} \theta^i E_t \left( \beta \frac{C_{1,t}}{C_{1,t+1}} \left( \frac{P_t^*(g)}{P_{t+1}^*} \frac{X_t}{X_{t+1}} \right) Y_{t+i}(g) \right) = 0,
\]

where \( X_t \) is the markup. \( P_t^* \) makes discounted marginal revenue be equal to expected discounted marginal cost. The aggregate price index is

\[
P_t = \left( \theta P_t^{\xi} + (1 - \theta) P_t^{* (1-\xi)} \right)^{\frac{1}{1-\xi}}.
\]

From the optimization of retailers, the log-linearized aggregate supply curve can be derived as

\[
\hat{\pi}_t = \beta \hat{\pi}_{t+1} - k \hat{X}_t + \hat{\epsilon}_{u,t},
\]

where \( k = \frac{(1-\theta)(1-\beta\theta)}{\theta} \).

2.3.5 The Central Bank

The central bank is assumed to follow a Taylor rule taking the nominal interest rate as the instrument for monetary policy:

\[
R_t = (R_{t-1})^{\gamma_R} \left( \frac{Y_{t-1}}{Y} \right)^{\frac{\gamma_R}{\gamma}} e_{R,t} \quad (2.21)
\]
2.3.6 Equilibrium

Given initial conditions $H_{e,0}$, $H_{1,0}$, $H_{2,0}$, $b_{e,0}$, $b_{1,0}$ and $b_{2,0}$, the dynamic stochastic equilibrium is a sequence of allocations, \( \{Y_t, C_{e,t}, C_{1,t}, C_{2,t}, H_{e,t}, H_{1,t}, H_{2,t}\}^{\infty}_{t=0} \) and a sequence of prices, \( \{w_{1t}, w_{2t}, R_t, q_t, P_t, P^*_t, \lambda_t\}^{\infty}_{t=0} \), at which all agents satisfy their budget constraints, borrowing constraints and first order conditions, and all market clear in each date. For the markets to clear, the bond market for $b_t$, goods market for $Y_t$ and the house market for $H_{e,t}$, $H_{1,t}$ and $H_{2,t}$ should meet the following conditions:

(a) $N_{1t} = N_{2t}$  
(b) $H_{e,t} + H_{1,t} + H_{2,t} = 1$,  
(c) $C_{e,t} + C_{1,t} + C_{2,t} + I_t = Y_t$,  
(d) $I_t = I_{Kt} + I_{H_{e}t} + I_{H_{1}t} + I_{H_{2}t}$,  
(e) $s_t = \varphi_e q_t H_{e,t-1} + \varphi_1 q_t H_{1,t-1} + \varphi_2 q_t H_{2,t-1}$,  
(f) $b_t + b_{1t} + b_{2t} = 0$

Wages are different across households and based on their productivity. In the steady state, unconstrained households receive wage bills of $w_1 N_1 = \alpha (1 - \mu - v) Y$, while constrained households receive a wage bill of $w_2 N_2 = (1 - \alpha) (1 - \mu - v) Y$.

2.3.7 Solution Method

The solution of the dynamic model involves non-linear equations and identity equations. The steady state values of the state variables are recovered from a non-linear system of equations. This step uses the Broyden method (Miranda and Fackler (2002)) for solving systems of nonlinear equations. After obtaining steady state values, the model is solved as a local (log-linear) approximation around steady state. All derivatives of the first order conditions are computed numerically using the Jacobian command in Matlab. Finally, the Uhlig toolkit (Uhlig (1999)) is used to solve the linear system of stochastic difference equations.
2.4 Calibration and Estimation

2.4.1 Calibration

The model is calibrated to imply that, in steady state, both unconstrained households and constrained households work one-third of their available time. The housing preference parameter, \( j \), is set to 0.21 to match 1.4 times GDP. Following Luengo-Prado and Sorensen (2008), and Li et al. (2009), and Stokey (2007), the rate of transaction costs for constrained households is 5 percent. This is consistent with the data from the Department of Justice in 2009. For entrepreneurs, transaction costs are 3 percent in the benchmark calibration.\(^{19}\) Unconstrained households, as realtors, pay no transaction costs, as mentioned above.

To facilitate comparison to previous results obtained in the model of Iacoviello (2005), I set the values of all common parameters to match that earlier study. Thus, the discount factors for unconstrained households, constrained households and entrepreneurs, the share of housing to consumption, the parameters of production across capital and housing, the probability of the price adjustment and the depreciation rate for housing are taken from Iacoviello (2005). The resulting parameter set is listed in Table 2.1.

\(^{19}\)Home builders usually pay less realtor fees than residential households. They do pay mostly buyer side fees. In a sensitivity analysis below, I confirm that alternative entrepreneur transactions fees do not change model results in any substantive way.
### Table 2.1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Data to Match</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta, \gamma, \beta_2$</td>
<td>0.99, 0.98, 0.97</td>
<td>subjective discount rate</td>
</tr>
<tr>
<td>$j$</td>
<td>0.21</td>
<td>share of housing to consumption</td>
</tr>
<tr>
<td>$\varphi_1, \varphi_e, \varphi_2$</td>
<td>0.0, 0.03, 0.05</td>
<td>housing transaction costs</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1.01</td>
<td>elasticity of labor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.03</td>
<td>depreciation of capital</td>
</tr>
<tr>
<td>$\mu, \nu$</td>
<td>0.3, 0.03</td>
<td>share of capital and housing</td>
</tr>
<tr>
<td>$X$</td>
<td>1.05</td>
<td>markup</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.75</td>
<td>probability of the fixed price</td>
</tr>
</tbody>
</table>

### 2.4.2 Estimation

(a) Monetary Policy

For monetary policy, I assume that the nominal interest rate responds to past inflation and past output through the following Taylor rule, which is a log-linearization of Equation (2.21):

$$
\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1 - \alpha_R) \left( (1 + \alpha_\pi) \hat{\pi}_{t-1} + \alpha_Y \hat{Y}_{t-1} \right) + \hat{\epsilon}_{R,t}.
$$

I estimate the parameters of the Taylor rule using the OLS method and report the results in Table 2.2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{t-1}$</td>
<td>0.84</td>
<td>0.035</td>
<td>23.63</td>
</tr>
<tr>
<td>$\hat{Y}_{t-1}$</td>
<td>0.011</td>
<td>0.008</td>
<td>1.35</td>
</tr>
<tr>
<td>$\hat{\pi}_{t-1}$</td>
<td>0.264</td>
<td>0.06</td>
<td>4.19</td>
</tr>
<tr>
<td>Constant</td>
<td>0.024</td>
<td>0.045</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 2.2: Taylor Rule: Estimation using OLS
Monetary policy makers respond to past inflation and output through a lagged interest rate. The coefficients are estimated: \( \hat{R}_t = 0.84 \hat{R}_{t-1} + 0.26 \hat{\pi}_{t-1} + 0.011 \hat{y}_{t-1} \).\(^{20}\) Hence Equation (2.20) can be written as

\[
\hat{R}_t = 0.84 \hat{R}_{t-1} + 0.16 \left( 1.5 \hat{\pi}_{t-1} + 0.069 \hat{y}_{t-1} \right). \tag{2.23}
\]

(b) Other Structural Parameters

The crucial parameter for the model is the elasticity of substitution of housing for consumption. Here, I discuss two estimates of this parameter. The first involves a cointegration method. The second uses minimum distance estimation between model and data impulse responses.

Using NIPA data, I use a variety of cointegrating methods such as Dynamic Ordinary Least Square (DOLS), Canonical Cointegrating Regression (CCR), and Fully Modified Ordinary Least Squares (FMOLS). The results show that the estimate of the EIS is 0.68, 0.74, and 0.74, respectively. The sample period is important because different sample periods provide different significance of statistical analysis. The sample period of the NIPA data is from 1970 through 2009, which is different from the samples from 1936 to 2001 as in Piazzesi et al. (2007). The relative price is the ratio of the price index of housing services to the price index of nondurable consumption. I use cointegration approach to parameter estimation. Piazzesi et al. (2007) also use NIPA data to estimate the parameter of the elasticity of substitution using cointegration method. The data in their paper is used in two different categories: long sample periods (1936-2001) and post-war sample periods (1947-2001). The results are different based on the the beginning period of 1936 and 1947. They ignore the

\(^{20}\)Following Equation (2), the coefficients are transformed into \( (1 - \alpha_R) (1 + \alpha_\pi) = 0.26 \rightarrow \alpha_\pi = \frac{0.26}{(1-\alpha_R)} - 1, (1 - \alpha_R) \alpha_Y = 0.011 \rightarrow \alpha_Y = \frac{0.011}{(1-\alpha_R)}. \)
result from post-war sample periods. However, the periods from 1970 through 2009 as in Song (2009) significantly provide cointegration trends. This finding is consistent with the results of Flavin and Nakagawa (2004) who apply GMM to the PSID and estimate the EIS at 0.13.

For robustness, instead of using cointegrating method through which I identified complementarity, I broadly follow the methodology of Iacoviello (2005) in that I undertake a minimum distance estimation using macro data from 1970 Q1 to 2008 Q4 including the federal funds rate (fedfunds), inflation (GDP deflator), house prices (CMHPI), and real GDP (GDPC1). However, this estimation differs from Iacoviello (2005) in two substantive ways. First, I use a different time period. Second, I add two more structural parameters: 1) the elasticity of \( \varepsilon \) intratemporal substitution, and 2) risk aversion, \( \varsigma \).

As seen in Table 2.3, the EIS, \( \varepsilon \), is estimated at 0.592 with a standard error of 0.053, thus providing strong support of preferences over housing and consumption exhibiting nonseparability and complementarity. I estimate the empirical impulse responses using the VAR method. The Choleski ordering is the federal funds rate, inflation, house prices and the output gap. The distance function is \( f(b) = IRF_M(b) - IRF_D \), where \( IRF_M(b) \) represents the impulse responses obtained from the model, \( IRF_D \) represents the empirical VAR responses, and \( b \) is the vector of parameters \( (\varepsilon, \sigma, \sigma_u, \sigma_j, \sigma_a, r_u, r_j, r_a, a, m_1, m_2) \). These parameters solve the problem of minimizing \( J_t(b) = \min_b \left[ f(b)^T \Phi f(b) \right] \), where \( \Phi = W\Omega^{-1} \), and \( \Omega^{-1} \) is the inverse matrix of the sample variance of the IRFs.
### Table 2.3: Estimation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>STD.e</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon) = elasticity of substitution</td>
<td>0.592</td>
<td>0.053</td>
</tr>
<tr>
<td>(\varsigma) = curvature</td>
<td>2.317</td>
<td>0.357</td>
</tr>
<tr>
<td>(a) = share of wage</td>
<td>0.553</td>
<td>0.046</td>
</tr>
<tr>
<td>(m_e = \text{loan-to-value}_e)</td>
<td>0.412</td>
<td>0.066</td>
</tr>
<tr>
<td>(m_2 = \text{loan-to-value}_2)</td>
<td>0.861</td>
<td>0.02</td>
</tr>
<tr>
<td>(\phi_\pi = \text{autoregressive}_\pi)</td>
<td>0.629</td>
<td>0.043</td>
</tr>
<tr>
<td>(\phi_j = \text{autoregressive}_j)</td>
<td>0.885</td>
<td>0.017</td>
</tr>
<tr>
<td>(\phi_a = \text{autoregressive}_a)</td>
<td>0.177</td>
<td>0.1</td>
</tr>
<tr>
<td>(\sigma_A = \text{stdev}_a)</td>
<td>9.195</td>
<td>3.5</td>
</tr>
<tr>
<td>(\sigma_j = \text{stdev}_j)</td>
<td>0.021</td>
<td>1.64</td>
</tr>
<tr>
<td>(\sigma_R = \text{stdev}_R)</td>
<td>0.24</td>
<td>0.009</td>
</tr>
<tr>
<td>(\sigma_\pi = \text{stdev}_\pi)</td>
<td>0.154</td>
<td>0.024</td>
</tr>
</tbody>
</table>

#### 2.5 Results

##### 2.5.1 The Issue of Separable Preferences with Housing Transaction Costs

In Figure 2.3, I examine the effect of housing transaction costs on aggregate consumption following a positive house price shock of 1 percent under separable preferences. A persistent change in house prices is generated by a common, exogenous shock to \(j\) that affects all households. In this first example, I set \(\varepsilon = 1\) to eliminate complementarity between housing and consumption. Thus utility becomes separable in housing and non-housing consumption. Panel (A) shows results when there are no transaction costs. In contrast, Panel (B) introduces transaction costs. The triangle, squared, and circled lines are the impulse responses of aggregate consumption to the increase in house prices. The house loan-to-value ratios of those three lines for
entrepreneurs and constrained households are set to (0.1 percent, 0.1 percent), (45 percent, 30 percent) and (89 percent, and 55 percent), respectively.

Figure 2.3: Consumption Responses to a Rise in House Prices When Preferences are separable

Panel (A) is a replication of Iacoviello (2005), which assumes a log utility function and no housing transaction costs. With loan-to-value ratios of 89 percent for entrepreneurs and 55 percent of constrained households, aggregate consumption rises by 0.24 percent after an 1 percent increase in house prices. In this consumption response, the loan-to-value ratios are critical factors, which make the model successful, so that the response matches the data within the 90 percent confidence intervals. The other loan-to-value ratios do not induce effective comovement between housing
and consumption. Hence, the comovement between house prices and consumption is sensitive to the loan-to-value ratio.

To examine the effects of proportional housing transaction costs on the aggregate consumption response, I compare the circled line in Panel (A) with that in Panel (B), where transaction costs of 5 percent and 3 percent for constrained households and entrepreneurs, respectively, are introduced.21 Critically, once we introduce empirically plausible levels of transaction costs, the positive consumption response to a rise in house prices disappears. Transaction costs dominate in dampening the effects of loan-to-value ratios so that aggregate consumption no longer increases following a rise in house prices. In other words, housing transaction costs nullify the effect of loan-to-value ratios on consumption. Thus, a separable utility function between housing and consumption in this model does not permit the comovement between house prices and consumption when housing transactions costs are introduced.22

2.5.2 Complementarity between Housing and Consumption

Figure 2.4 resolves the problem caused by housing transaction costs. The solution relies on the introduction of nonseparable preferences over housing and consumption. The extent of non-separability of preferences is jointly determined by the elasticity of intratemporal substitution, $\varepsilon$, and curvature, $\varsigma$, which are set at the estimated values from Table 2.3, (0.592 and 2.317, respectively).

21In a sensitivity analysis available on request, I consider an alternative formulation where both constrained households and entrepreneurs face a 6 percent housing transaction cost and find that the same negative result appears.

22Confidence intervals are drawn from the empirical impulse responses, which come from a VAR using data on the federal funds rate, real house prices, real personal consumption expenditures, real GDP and inflation. Inflation is the change in the GDP deflator. All variables are de-trended by bandpass filtering the logarithms of the series. The data, data transformation, and the order
Figure 2.4: Consumption Responses to a rise in House Prices under Complementarity in Preferences

As before, a rise in house prices is generated by a shock to housing demand, \( j \), common across households. The house loan-to-value ratios, for entrepreneurs and constrained households, are set to 41 percent and 86 percent, respectively. Panel (A) shows that complementarity induces a positive aggregate consumption response following a positive house price shock of 1 percent. With housing transaction costs of 5 percent and 3 percent for constrained households and entrepreneurs, respectively, the rise in house prices leads aggregate consumption to rise by 0.31 percent, which falls within the 90 percent confidence intervals of the data.

in the VAR are the same as in Iacoviello (2005). However, the time period differs, covering 1970 Q1 to 2008 Q4.
Panel (B) examines the sensitivity of my result with respect to housing transaction costs, fixing the loan-to-value ratios for entrepreneurs and constrained households to 41 percent and 86 percent, respectively. With a common transaction cost of 6 percent, the aggregate consumption response at the impact date is 0.29 percent. As transaction costs are lowered, the complementarity in preferences carries an ever-larger role in the consumption response. With a common transaction costs near zero (0.001), consumption rises 0.57 percent at the date of the shock, placing the response outside the 90 percent confidence interval. In other words, when there are no transaction costs, the consumption response is counterfactually large.

2.5.3 Consumption Across Households

In Figure 2.5, I investigate how a 1 percent increase in house prices affects the consumption of different households as transaction costs vary. There are two types of households: constrained and unconstrained. Aggregate consumption data on these types of households are taken from Consumer Expenditure Surveys from 1984 Q1 to 2008 Q4. Constrained households are assumed to be young households (below 35 years old) whose housing purchases are typically highly leveraged, as in Flavin and Yamashita (2002). On the other hand, unconstrained households are assumed to be relatively old households (over 35 years old). Throughout the figure, the degree of complementarity in preferences and the loan-to-value ratios for entrepreneurs and constrained households are held at their baseline values, (0.59, 0.41 and 0.86, respectively).
Following a 1 percent rise in house prices, the gains in consumption are not equally distributed across households. In other words, the two types of households react differently to a shock in house prices. A key result of the model is that constrained households are more responsive to a change in house prices than unconstrained households.

![Figure 2.5: Consumption Responses by Household Type](image)

Panel (A) shows the consumption response of unconstrained households. Initially, unconstrained households increase consumption by around 0.2 percent. Moreover, the slope of their consumption response is generally flat. Interestingly, their consumption response does negligibly vary under alternative transaction cost specifications.
The consumption response for constrained households is presented in Panel (B). Overall, constrained households are substantially more responsive to the increase in house prices than unconstrained households. Depending upon the degree of transaction costs, they raise their consumption at impact by 0.5 percent, 0.66 percent, or 0.81 percent in this panel. Thus we see that, unlike unconstrained households, transaction costs are a significant dampening factor in the consumption response of credit-constrained households.

The logic is as follows. Suppose there is a rise in house prices. The marginal propensity of consumption (MPC) out of gains in house prices for constrained households is larger than that for unconstrained households. This difference in the MPC across households leads to a change in overall aggregate consumption. Constrained households are likely to use their increase in housing collateral value to finance an increase in consumption that exceeds the house price increase.

2.5.4 The Effect of the Elasticity of Intertemporal Substitution on Consumption across Households

Figure 2.6 illustrates how a 1 percent increase in house prices affects the consumption of each type of household under alternative assumptions about their elasticity of intertemporal substitution, \( \sigma = 1/\varsigma \). The consumption response to a 1 percent increase in house prices is displayed with respect to \( \varsigma \). Throughout this figure, the EIS is held fixed at 0.59, and transaction costs and loan-to-values for entrepreneurs and constrained households are held at their baseline values (3 percent, 5 percent, 41 percent and 86 percent, respectively). I consider parameter values of 2.3, 6 and 10 for \( \varsigma \) to explore the cases when households’ risk aversion is normal, high and very high.
Panel (A) displays how alternative levels of risk aversion affect the consumption responses of unconstrained households. For these households, the consumption responses change negligibly with changes in $\zeta$. In other words, the degree of risk aversion does not significantly affect the consumption of unconstrained households.

![Figure 2.6: Consumption Sensitivity under Alternative Choices of Risk Aversion](image)

Panel (B) shows how risk aversion affects the consumption responses of constrained households. Generally, higher risk aversion implies a lesser rise in consumption following the positive house price shock. At the extreme case of $\zeta = 10$, the initial rise in the consumption of constrained households is completely overturned. At the benchmark, calibration with risk aversion at 2.3, the consumption response to a 1 percent rise in house prices is 0.5 percent.
Overall, risk aversion is important in the consumption decisions of constrained households. Even a positive house price shock can lead to a fall in consumption when risk aversion is sufficiently high, that is when households are unwilling to substitute consumption inter-temporally. Alternatively, a sufficiently low elasticity of \textit{inter}temporal substitution can offset the positive effect of a rise in house prices.

### 2.6 Implications of Changes in Monetary Policy

This section studies the effect of interest rate shocks on housing and consumption. My first result illustrates how changes in $\varepsilon$, the \textit{intra}temporal elasticity of substitution between housing and consumption, affects the effective degree of risk aversion across households. This leads to different consumption responses in response to an increase in interest rates.

Figure 2.7 shows the cumulative consumption loss across households after a 1 percent increase in interest rates under the benchmark parameter specification of my model. Panel (A) is the aggregate cumulative consumption response to tighter monetary policy. Aggregate consumption falls by 1.92 percent initially. Panel (B) shows that an increase in interest rates reduces consumption for both constrained and unconstrained households. However, the effect is considerably larger for constrained households.
Figure 2.7: Consumption Loss with a Rise in the Interest Rate: Benchmark Model

Figure 2.8 shows the case when the degree of complementarity, $\varepsilon$, is stronger (lower value and bold lines) than in the benchmark model. Aggregate consumption falls further in response to the increase in interest rates under the stronger complementarity. For constrained households, the increase in the interest rate with the stronger complementarity induces them to significantly decrease consumption. Since they are constrained by collateral and an increase in the interest rate reduces the values of their collateral, consumption falls by more with stronger complementarity. A negative shock induces them to be more risk averse, so their consumption drops by 5.38 percent, as compared to 3.14 percent in the benchmark model.
The role of complementarity in the consumption response of unconstrained households works oppositely relative to that with regard to constrained households. Unconstrained households are not significantly sensitive to an increase in interest rates because they are not constrained by housing collateral, and, as lenders, they can initially benefit from a rise in interest income. Stronger complementarity magnifies the effect of interest rates, so that the gap between two consumption responses across heterogeneous households widens.

The dotted lines in Figure 2.8 show the case where complementarity is weaker (higher $\varepsilon$) relative to that in the benchmark parameter set. For constrained households, the precautionary saving motive is reduced as complementarity is weakened,
and the negative interest effect on consumption for constrained households becomes smaller. A higher degree of substitutability makes constrained households effectively less risk averse, so they reduce their consumption by less compared to the benchmark, only 2.64 percent at impact. Unconstrained households, by contrast, are more sensitive to the increase in the interest rate when housing and non-housing consumption are more substitutable. Overall, the disparity in the consumption responses of the two types of households is reduced.

![Graph showing responses to a rise in the interest rate](image)

Figure 2.9: Responses to a Rise in the Interest Rate: Benchmark Model

We now study the effect of loan-to-value ratios on the sensitivity of the economy to monetary policy. Figure 2.9 shows the effects of a 1 percent rise in the interest rate on aggregate consumption, and on house prices, for the benchmark model where...
curvature, $\varsigma$, and complementarity, $\varepsilon$, are 2.3 and 0.592, respectively. Loan-to-values for entrepreneurs and constrained households are set to 42 percent and 86 percent, respectively. In this benchmark case, the rise in the interest rate serves to significantly decrease house prices and consumption, as shown in Panel (A) and Panel (B). The reduction in consumption is larger for constrained households, as described above.

Figure 2.10 revisits the response to tightened monetary policy, this time assuming higher loan-to-values ratios. When loan-to-value ratios are reset to 80 percent and 91 percent for entrepreneurs and constrained households, house prices and aggregate consumption fall further in response to a rise in the interest rate. Importantly, higher
loan-to-value ratios increase the volatility of house prices caused by changes in monetary policy. A 1 percent rise in interest rates now reduces house prices by 1.31 percent at impact, and it ultimately reduces house prices by 4.59 percent. House prices become even more volatile. Furthermore, the increase in the interest rate now leads households to reduce their consumption by more. The initial consumption losses across constrained households and unconstrained households are 4.59 percent and 1.02 percent, respectively. As before, the consumption loss following an increase in interest rates is significantly larger for constrained households. Thus the gap between the two impulse responses widens as loan-to-value ratios increase.

2.7 Conclusion

This chapter has explored the implications of shocks to housing demand and monetary policy in a model where period utility is nonseparable in housing and non-housing consumption, and where housing sales involve transaction costs. When housing transaction costs are introduced into dynamic stochastic general equilibrium models where period utility is additively separable in housing and consumption, we have seen that such models fail to reproduce the comovement between these series observed in the data. By eliminating the log utility assumption and allowing for complementarity in preferences, I have developed a model that can accommodate the realism of housing transaction costs while retaining the comovement between housing and consumption.

My model generates differing elasticities of response across households following aggregate shocks. Following changes in both house prices and interest rates, the consumption response of credit-constrained households is greater than that of unconstrained households. I have traced this differing responsiveness in consumption to
the house loan-to-value ratios characterizing credit-constrained households. Loan-to-value ratios amplify the effect of changes in interest rates on consumption especially for credit-constrained households. I have also found that the differences widen with the degree of complementarity between housing and consumption.

Following a rise in house prices, the net worth of credit-constrained households increases, which boosts their access to credit. This, in turn, allows constrained households to raise their spending on consumption and housing. Moreover the resulting rise in housing demand leads to a further increase in house prices. While transaction costs abate this process somewhat, the fundamental comovement between housing and consumption persists.
2.8 Appendix A: Steady State Details

2.8.1 For Entrepreneurs:

\[ K = \frac{\gamma \mu Y}{(1 - \gamma (1 - \delta_k)) X} = \varsigma_1 Y \quad (2.24) \]

\[ I_{Kt} = K_t - (1 - \delta) K_{t-1} \rightarrow I_K = \delta K = \delta \varsigma_1 Y \quad (2.25) \]

\[ w_{1,t} N_{1,t} = \frac{\alpha (1 - \mu - v) Y_t}{X_t} \quad (2.26) \]

\[ w_{2,t} N_{2,t} = \frac{(1 - \alpha) (1 - \mu - v) Y_t}{X_t} \quad (2.27) \]

\[ q = \frac{\gamma v}{((1 + \varphi_{he}) - \gamma (1 - \delta_{he}) - \chi m_e (\beta - \gamma)) X} \left( \frac{Y}{H_e} \right) = \varsigma_2 \left( \frac{Y}{H_e} \right) \quad (2.28) \]

\[ C_e = \frac{Y}{X} - w_{1,t} N_{1,t} - w_{2,t} N_{2,t} - \delta K - \left( \delta + \varphi_e \right) - \left( 1 - \frac{1}{\beta} \right) \beta m_e \chi \right) \varsigma_2 \quad (2.29) \]

\[ b_e = \frac{\chi m_e (q \pi H_e)}{R} = \beta \chi m_e q \pi H_e = \beta \chi m_e \varsigma_2 \quad (2.30) \]

\[ \lambda_e R = \frac{1}{C_e} (1 - R \gamma) = \frac{1}{C_e} R (\beta - \gamma) \quad (2.31) \]
2.8.2 For Unconstrained Households:

\[ C_1 = (1 - R) b_1 + w_1 N_1 + F + Fee, \text{ where } b_1 = -(b_e + b_2) \]  

(2.32)

\[ C_1 = \begin{array}{c} (1 - \beta) \chi (m_e s_2 + m_2 s_4 C_2) + w_1 N_1 + F + s_2 \varphi_e + s_4 C_2 \varphi_2 \end{array} \]  

(2.33)

\[ q = \frac{j}{1 - \beta_1} \left( \frac{C_1}{H_1} \right)^{\frac{1}{2}} = \varsigma_3 \left( \frac{C_1}{H_1} \right)^{\frac{1}{2}} \]  

(2.34)

\[ R = \frac{1}{\beta_1} \]  

(2.35)

\[ \frac{C_1}{H_1} = (q_t)^{\varepsilon} \left( \frac{1 + \varphi_1}{j_t} - \frac{\pi_{t+1} q_{t+1}}{j_t R_t q_t} \right)^{\varepsilon} \]  

(2.36)

2.8.3 For Credit Constrained Households:

\[ q = \frac{j}{(1 + \varphi_2) - \beta_2 (1 - \delta h_2) - (\beta - \beta_2) m_2 \chi} \left( \frac{C_2}{H_2} \right)^{\frac{1}{2}} = \varsigma_4 \left( \frac{C_2}{H_2} \right)^{\frac{1}{2}} \]  

(2.37)

\[ C_2 = \frac{1}{(1 + (1 - \beta) \chi m_2 \varsigma_1 + \varphi_2 \varsigma_4)} w_2 N_2 \]  

(2.38)

\[ \lambda_\mu = \lambda_2 (\beta - \beta_2) \]  

(2.39)
2.9 Appendix B: First Order Condition Details

For entrepreneurs:

\[ \frac{q_t}{q} = E_t \left[ \frac{C_t}{1 + \varphi_{he}} \left( \frac{\gamma}{C_{t+1}} \left( v \frac{Y_{t+1}}{X_{t+1}H_t} + (1 - \delta_{he}) q_{t+1} \right) + \lambda_{mt} \chi_t m_{et} q_{t+1} \pi_{t+1} \right) \right] \]

(2.40)

For constrained households with respect to house prices:

\[ \frac{q_t}{q} = \left( \frac{U_{H_{2,t}}}{U_{C_{i,t}}} + \beta_2 (1 - \delta_{h2}) \frac{U_{C_{i,t+1}}}{U_{C_{i,t}}} q_{t+1} + \frac{\lambda_{iit}}{U_{C_{i,t}}} m_{2t} \chi_t E_t q_{t+1} \pi_{t+1} \right) / (1 + \varphi_2) q \]

(2.41)

For unconstrained households with respect to house prices:

\[ (1 + \varphi_{h1}) \frac{q_t}{q} = \left( \frac{U_{H_{1,t}}}{U_{C_{1,t}}} + (\beta_1 \psi_t) (1 - \delta_{h1}) \frac{U_{C_{1,t+1}}}{U_{C_{1,t}}} q_{t+1} \right) / q \]

(2.42)

For entrepreneurs’ Euler equation:

\[ \left( 1 + \frac{\psi}{\delta} \left( \frac{I_t}{K_{t-1}} - \delta \right) \right) = \gamma \frac{C_t}{C_{t+1}} E_t \left( \frac{\mu Y_{t+1}}{K_{t}X_{t+1}} + 1 + \frac{\psi}{\delta} \left( \frac{I_{t+1}}{K_t} - \delta \right) \left( \frac{1}{2} \left( \frac{I_{t+1}}{K_t} + \delta \right) + 1 \right) \right) \]

(2.43)

\[ 1 = R_t C_{et} \left( E_t \left( \frac{\gamma}{\pi_{t+1}} \frac{1}{C_{et+1}} \right) + \lambda_{et} \right) \]

(2.44)

For constrained households’ Euler equation

\[ R_t \lambda_{m2t} = U_{C_{iit}} - U_{C_{2,t+1}} \frac{1}{\pi_{t+1}} \beta_2 \]

(2.45)

For unconstrained households’ Euler equation:

\[ 1 = \beta_t E_t \left( \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \frac{R_t}{\pi_{t+1}} \right) \]

(2.46)

For entrepreneurs’ borrowing constraints:

\[ R_t / R = \frac{m_e}{\zeta_t} E_t (q_{t+1} \pi_{t+1} H_t) / b_{et} \]

(2.47)
For credit constrained households’ borrowing constraints:

\[
\frac{R_t}{R} = \frac{m^2}{R} \zeta_t E_t (q_{t+1} \pi_{t+1} H_{et}) / b_{2t}
\]  

(2.48)

For constrained households with respect to labor:

\[
N_{1,t}^\eta = U_{Cit} \alpha (1 - \mu - v) \frac{Y_t}{X_t}
\]  

(2.49)

For flow of funds:

\[
b_{et} = C_{et} + \frac{R_{t-1}}{\pi_t} b_{et-1} - (\mu + v) \frac{Y_t}{X_t} + q_t (H_{et} - (1 - \delta) H_{et-1}) + I_t + \psi_{K,t} + \varphi H_{et} q_t
\]  

(2.50)

\[
C_{2,t} + q_t (H_{2,t} - H_{2,t-1}) + \varphi_2 q_t H_{2t} + \frac{R_{t-1}}{\pi_t} b_{2,t-1} = w_{2,t} L_{2,t} + b_{2,t}
\]  

(2.51)
2.10 Appendix C: Log-linearization Details

For housing market clearing condition:

\[ 0 = H \hat{H}_t + H_1 \hat{H}_{1,t} + H_2 \hat{H}_{2,t} \quad (2.52) \]

For production function:

\[ Y_t = \tilde{Z}_t + v \hat{H}_{t-1} + \mu \hat{K}_{t-1} + \alpha (1 - \mu - v) \hat{N}_{1,t} + (1 - \alpha) (1 - \mu - v) \hat{N}_{2,t} \quad (2.53) \]

For the Philips curve:

\[ \hat{\pi}_t = \beta \pi_{t+1} + k \hat{X}_t + \hat{u}_t \quad (2.54) \]
Chapter 3: COMPLEMENTARITY BETWEEN HOUSING AND CONSUMPTION

3.1 Introduction

The purpose of this chapter is to measure the elasticity of intratemporal substitution (EIS) using a variety of cointegrating methods. I construct a nonseparable utility function over housing services and non-housing consumption. The representative household receives utility from these two series. In the optimization problem, households choose housing and consumption subject to intertemporal budget constraints in order to maximize expected discounted lifetime utility. I derive the first order conditions and log-linearize the Euler equation. The parameter value of the EIS is estimated using a cointegration analysis. In this research, housing services are directly dealt with by both the model and the data from the National Income and Product Accounts (NIPA). Hence, the relative consumption ratio and the relative price ratio are constructed from the NIPA.\textsuperscript{24}

\textsuperscript{23}Housing and housing services are used interchangeably.

\textsuperscript{24}In the other macroeconomic studies, either the Conventional Mortgage House Price Index (CMHPI) or Office of Federal Housing Enterprise Oversight (OFHEO)’s Housing Price Index are sometimes used to a proxy for the price of housing services.
Several branches of the macroeconomic literature have ignored nonseparable complementarity between housing services and consumption in defining preferences. Instead, in most macroeconomic analysis, separability of preferences over housing and consumption is widely assumed through a log utility function. In another branch of the literature, substitutability of preferences (instead of complementarity) over housing and consumption is assumed by setting the parameter value of the EIS between 1.04 and 1.43.\(^{25}\) In fact, these assumptions are not consistent with microeconomic data from the Consumer Expenditure Survey (CEX) and the Panel Study of Income Dynamics (PSID). Complementarity between housing and consumption is found in both CEX data and PSID data.

Consistent with this research, I find complementarity between housing and consumption, using the data from the federal funds rate, Gross Domestic Product (GDP), the GDP deflator, Consumer expenditure and the Conventional Mortgage Housing Price Index (CMHPI) as shown in Chapter 2. Using the minimum distance method, I estimate the EIS between housing and consumption at 0.59, which implies that housing and consumption are complements. Along the same lines, this chapter is aimed at measuring the EIS using macroeconomic data, the NIPA, which provides further evidence for complementarity between housing and consumption. I use a variety

\(^{25}\)Piazzesi et al. (2005) deal with the parameter of the elasticity of substitution between housing and consumption. They study nonseparable preferences and base their calibration on the results of Ogaki et al. (1998). Their data is based on the difference between durable good consumption and non-durable good consumption with housing services. The elasticity of substitution between durable goods and non-durable goods has been estimated to fall between [1.04, 1.43] with 95 percent confidence by Ogaki et al. (1998), but this estimation has nothing to do with housing services. This line of research treats housing services as durable good consumption. Piazzesi, Schneider, Tuzel (2005) used a co-integration method to estimate the parameter of the elasticity of substitution, which they found was not significant statistically.
of cointegrating methods such as Dynamic Ordinary Least Square (DOLS), Canonical Cointegrating Regression (CCR), and Fully Modified Ordinary Least Squares (FMOLS). The results show that the estimate of the EIS is 0.68, 0.74, and 0.74 with the standard error of 0.26, 0.24, and 0.23, respectively. The measurement of the EIS using NIPA data is important in that (1) macroeconomic data is consistent with previous microeconomic results and (2) these estimates provide robustness to the results found in Chapter 2.

The structure of this paper is as follows: In section 3.2, I construct a representative household consumption model allowing for nonseparable preferences over housing and consumption. In section 3.3, I conduct a cointegrating analysis. In section 3.4, a variety of cointegrating method provides the statistical results for the estimate of the EIS.

3.2 The Model

3.2.1 The Model Framework

I assume that preferences over housing and consumption are represented by the Constant Elasticity of Substitution (CES) utility function.\(^{26}\) Households receive utility from housing services\(^{27}\) and consumption at each period. The utility function of the representative household at date \(t\) is assumed to be given by

\[
U(C_t, H_t) = \frac{1}{1 - 1/\sigma} \left\{ C_t^{\frac{\sigma + 1}{\sigma}} + j_t(H_t)^{\frac{\sigma + 1}{\sigma}} \right\}^{\frac{\sigma}{\sigma - 1}},
\]

\(3.1\)

\(^{26}\)The parameters of the CES utility function are eventually represented by a cointegration equation. After constructing the utility function, the cointegrating analysis is applied to the Euler equation derived from the first order condition in the consumer optimization problem.

\(^{27}\)Housing services come from holding real housing stocks.
where \( C_t \) is consumption, and \( \tilde{H}_t \) is housing services.\(^{28}\) Let \( j_t \) be a preference shock to housing. At time 0, the expected discounted utility sum of future period for the representative household is given by

\[
E_0 \sum_{t=0}^{\infty} (\beta)^t U(C_t, H_t)^{1-1/\sigma},
\]

(3.2)

where \( E_0 \) is an expectation operator conditional on the information at time \( t \), and \( \beta < 1 \) is the discount factor.

The household budget constraint is

\[
P_tC_t + q_t H_t = E_t,
\]

(3.3)

where \( E_t \) is total expenditure. Let \( P_t \) denote the purchase price of consumption and \( q_t \) denote the house service price. The household chooses housing and consumption for all \( t \geq 0 \) in order to maximize expected discounted utility in Equation (3.2) subject to the budget constraints in Equation (3.3).

The parameter, \( \varepsilon \), is the parameter of the EIS. As the parameter \( \varepsilon \) goes to infinity, housing and consumption become perfect substitutes. As the parameter \( \varepsilon \) goes to zero, housing and consumption become perfect complements. Since \( \varepsilon \) determines whether housing and consumption become substitutes or complements, estimating the parameter, \( \varepsilon \), is crucial. The parameter, \( \sigma \), denotes the intertemporal elasticity of substitution (IES). The first order conditions with respect to consumption and housing lead to the following equation:

\[
\frac{U_{C_{1,t}}}{U_{H_{1,t}}} = \frac{P_t}{q_t} \rightarrow \frac{1}{j_t} \left( \frac{C_t}{\tilde{H}_t} \right)^{-\frac{1}{2}} = \frac{P_t}{q_t}.
\]

(3.4)

\(^{28}\)Housing services can be assumed to be proportional to real housing stock so that \( \tilde{H}_t = \vartheta \cdot H_t \).
Marginal utility of consumption to housing services is represented by the relative price of consumption to housing services.\textsuperscript{29} Defining the relative price $\bar{P}_t \equiv \frac{q_t}{P_t}$, Equation (3.4) is loglinearized yielding the following equation:

$$\ln \bar{P}_t - \frac{1}{\varepsilon} \ln \left( \frac{C_t}{H_t} \right) = \ln (j_t).$$

(3.5)

For estimation of the EIS, Equation (3.5) can be rearranged:

$$\frac{1}{\varepsilon} \ln \left( \frac{C_t}{H_t} \right) = \ln \bar{P}_t - \ln (j_t) \rightarrow \ln \left( \frac{C_t}{H_t} \right) = \varepsilon \ln \bar{P}_t + \varepsilon \ln (j_t).$$

(3.6)

Equation (3.6) is essential to determine $\varepsilon$ using a cointegrating method. I borrow the approach of Ogaki and Reinhart (1998) for demand and supply. The first order condition provides demand from households. For supply, an endowment processes with quantities, $C_t^*$ and $H_t^*$. I denote that $c_t^* = \log (C_t^*)$ and $h_t^* = \log (H_t^*)$. In equilibrium, $c_t = c_t^* = \log (C_t^*)$ and $h_t = c_t^* = \log (H_t^*)$.\textsuperscript{30} I assume that $C_t$ and $H_t$ are difference stationary so that Equation (3.6) may be stationary.\textsuperscript{31} Equation (3.6) can be also expressed as

$$\bar{P}_t \left[ \frac{C_t}{H_t} \right]^{-\frac{1}{\varepsilon}} \sim \text{stationarity, where } \bar{P}_t = \frac{\text{price index of housing services}}{\text{price index of consumption}}.$$  

(3.7)

### 3.3 The Cointegrating Analysis

The parameter of the EIS, $\varepsilon$, is crucial in this chapter in order to study the long-run relationship between housing services and consumption. The elasticity of

\textsuperscript{29}The house price is such that the household is indifferent to giving up a marginal unit of housing for consumption.

\textsuperscript{30}Consumption follows the assumed trend properties $c_t^*$. The trend properties of consumption are closely related to the technology shock to both consumption and housing in the production economy.

\textsuperscript{31} $\frac{1}{\varepsilon} \left( \frac{C_t}{H_t} \right)^{-\frac{1}{\varepsilon}}$ is stationary so that $\frac{P_t}{q_t}$ may be stationary.
substitution determines whether housing and consumption are either complements or substitutes. In Equation (3.5), $\varepsilon$, $\ln(j_t)$ is assumed to be strictly stationary.\textsuperscript{32} Consumption is also assumed to be non-storable.\textsuperscript{33} The cointegrating analysis is applicable to the integration order of 1, or $I(1)$. I use the Augmented Dickey-Fuller Unit Root Test to check whether both the dependent variable and the explanatory variable are $I(1)$. Table 3.1 shows that the dependent variable is $I(1)$. According to the test, the hypothesis that the relative ratio of real nondurable consumption to housing services has a unit root can not be rejected. The critical value is inside of any significance level of 1 percent, 5 percent, or 10 percent.

Null Hypothesis: $\frac{\log(C)}{\log(H)}$ has a unit root
Exogenous: Constant
Lag Length: 1

<table>
<thead>
<tr>
<th>Test</th>
<th>Significance level</th>
<th>t-Statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller Test</td>
<td>test statistic</td>
<td>-0.98</td>
<td>0.775</td>
</tr>
<tr>
<td>Test critical values:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1% level</td>
<td>-3.472</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5% level</td>
<td>-2.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% level</td>
<td>-2.57</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: Augmented Dickey-Fuller Test (1)

\textsuperscript{32}The estimation of the elasticity of substitution is conducted similarly to that of durable good consumption and non-durable good consumption in Ogaki and Reinhart (1998).

\[ \hat{P}_t \left( \frac{C_{DL}}{C_{NDL}} \right)^{-\frac{1}{\varepsilon}} : \text{stationary}, \quad \text{where } \hat{P}_t = \frac{\text{Price Index of Durable Good Consumption}}{\text{Price Index of Non-durable Good Consumption}} \]

where $C_{DL}$ is the real stock of durable good consumption and $C_{NDL}$ is the real stock of non-durable good consumption.

\textsuperscript{33}Piazzesi et al. focus on relative price based on housing price index instead of housing services price index.
However the null hypothesis that the first difference of the relative ratio of real consumption to housing services has a unit root can be rejected, so that the consumption ratio between consumption and housing, $\frac{\log(C)}{\log(H)}$, is $I(1)$ as in Table 3.2.

Null Hypothesis: $D\left(\frac{\log(C)}{\log(H)}\right)$ has a unit root

<table>
<thead>
<tr>
<th>Test</th>
<th>Significance level</th>
<th>t-Statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller Test</td>
<td>test statistic</td>
<td>-0.99</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 3.2: Augmented Dickey-Fuller Test (2)

By the same token, the relative price of housing services to consumption is also $I(1)$. Table 3.3 shows that the hypothesis that $\frac{\log(q)}{\log(P)}$ has a unit root cannot be rejected. However, Table 3.4 shows that the hypothesis that $D\left(\frac{\log(q)}{\log(P)}\right)$ has a unit root can be rejected in the 99 percent significant level. Therefore, since both $\frac{\log(C)}{\log(H)}$ and $D\left(\frac{\log(q)}{\log(P)}\right)$ are $I(1)$, the cointegrating method can be applicable.

Null Hypothesis: $\frac{\log(q)}{\log(P)}$ has a unit root

<table>
<thead>
<tr>
<th>Test</th>
<th>Significance level</th>
<th>t-Statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller Test</td>
<td>test statistic</td>
<td>-0.258</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Table 3.3: Augmented Dickey-Fuller Test (3)
Null Hypothesis: \( D\left(\frac{\log(q)}{\log(P)}\right) \) has a unit root

Exogenous: Constant

Lag Length: 1

<table>
<thead>
<tr>
<th>Test</th>
<th>Significance level</th>
<th>t-Statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller Test</td>
<td>test statistic</td>
<td>-4.27</td>
<td>0.00</td>
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<td>Test critical values:</td>
<td>1% level</td>
<td>-3.46</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5% level</td>
<td>-2.87</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10% level</td>
<td>-2.57</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: Augmented Dickey-Fuller Test (4)

Furthermore, I need to determine whether a stochastic trend exists in these series. For this, I shows Figure 3.1 and Figure 3.2. They show that the relative consumption ratio and the relative price appear to have stochastic trends. However, deterministic trends do not seem to exist. Without consideration of deterministic trends, cointegration of these two series is tested by the Johansen Cointegration method as in Table 3.5.

The null hypothesis of no cointegration can be rejected. Moreover, the null hypothesis of at most one cointegration between these two series can not be rejected. Hence, \( \ln\left(\frac{C_t}{H_t}\right) \) and \( \ln(\tilde{P}_t) \) are cointegrated.

---

34Figures clearly show stochastic trends. Hence deterministic trends can be ignored.

35This result is sensitive to the sample period. Although the results do not change between 1970 Q1 through 2009 Q1 and 1970 Q2 through 2009 Q1, the statistics do significantly change. In the period of 1970 Q2 through 2009 Q1, the eigenvalue and trace statistic are 0.09 and 18.22 for no cointegration hypothesis with probability of 0.01 and 0.01 and 2.25 for at most one cointegration hypothesis with probability of 0.13.
Figure 3.1: The Ratio of Consumption to Housing Services (1)

Figure 3.2: The Ratio of Consumption to Housing Services (2)
Trend assumption: No deterministic trend  
Series: log(C/H) and log(q/P)  
Lag Length: 6

<table>
<thead>
<tr>
<th>No. of Coint</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0.095</td>
<td>17.07</td>
<td>12.32</td>
<td>0.007</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.007</td>
<td>1.23</td>
<td>4.12</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table 3.5: Johansen Cointegration Test

### 3.3.1 Data

I use NIPA data to examine whether cointegration exists for relative consumption and the relative price. Data for consumption and housing services come from first quarter 1959 to first quarter 2009. I apply the Consumer Price Index (CPI) to obtain real nondurable consumption and housing services. The relative price is the ratio of real housing service price to the real nondurable consumption price. Relative consumption is the ratio of real consumption to real housing services.

I also use NIPA data for the housing service price index. Some economists use the CMHPI, the OFHEO, or the S&P Case-Shiller Index as a proxy for the housing service price index. However, as far as housing services are concerned, using the housing service price index in the NIPA is desirable, since housing services are a main argument in the utility function.

---

36 Real housing services come from the index of housing services adjusted by the CPI. Real nondurable consumption is obtained from non-durable consumption adjusted by the CPI. The real prices of housing services and nondurable good consumption are obtained from NIPA data adjusted by the CPI.
3.3.2 DOLS

Although linear combinations of integrated order of one (I(1)) variables are cointegrated, the coefficients of OLS do not follow a $t$-distribution. Hence, cointegrating analysis such as DOLS, CCR and FMOLS must be used to estimate cointegrated coefficients. First, I use DOLS to estimate the coefficient of the relative price between housing services and consumption. Let $y_t$ and $x_t$ denote $\ln \left( \frac{C_t}{H_t} \right)$ and $\ln \left( \frac{q_t}{P_t} \right)$ respectively.

$$y_t = c' x_t + \varepsilon_t \quad (3.8)$$

A general form of the regression equation following Ogaki\footnote{Chapter 12 in Ogaki (2007)} can be

$$y_t = h'd_t + c' x_t + \varepsilon_t, \quad (3.9)$$

where $d_t$ is a time associated vector function, or $\left( \begin{array}{c} 1 \\ t \end{array} \right)$ and $x_t$ is a vector difference stationary process. Equation (3.8) is stationary when $\varepsilon_t$ is stationary.

Let Equation (3.8) be transformed to a DOLS form.

$$y_t = h'd_t + c' x_t + \gamma (L^{-1}) \Delta x_t + \eta (L) \Delta x_t + \varepsilon_t, t = 1, 2, 3, \cdots, \quad (3.10)$$

where $\gamma (L^{-1}) = \gamma_1 L^{-1} + \cdots + \gamma_p L^{-p}$ and $\eta (L) = \eta_0 + \eta_1 L + \cdots + \eta_q L^q$. Both $\gamma_1 L^{-1} + \cdots + \gamma_p L^{-p}$, and $\eta_0 + \eta_1 L + \cdots + \eta_q L^q$ are $1 \times k$ vectors. Equation (3.9) can be applied to the real relative consumption ratio and the real relative price. Since the variables do not show deterministic trends, $h'd_t$ is not included.
\[
\log \left( \frac{C_t}{H_t} \right) = c' \log \left( \frac{q_t}{P_t} \right) \gamma (L^{-6}) \Delta \log \left( \frac{q_t}{P_t} \right) + \eta (L^{+6}) \Delta \log \left( \frac{q_t}{P_t} \right) + \varepsilon_t, \quad t = 1, 2, 3, \ldots ,
\]

(3.11)

where \( \gamma (L^{-6}) = \gamma_1 L^{-1} + \gamma_2 L^{-2} + \gamma_3 L^{-3} + \gamma_4 L^{-4} + \gamma_5 L^{-5} + \gamma_6 L^{-6} \) and \( \eta (L^{+6}) = \eta_0 + \eta_1 L^1 + \eta_2 L^2 + \eta_3 L^3 + \eta_4 L^4 + \eta_5 L^5 + \eta_6 L^6 \). The key element is to estimate \( c' \), so that the cointegration vector may be \( \begin{pmatrix} 1 \\ -c' \end{pmatrix} \).

### 3.3.3 Canonical Cointegrating Regression (CCR)

I apply the CCR approach to estimate the long run covariance parameter used in the cointegration equation:

\[
y_t = h'd_t + c'x_t + \varepsilon_t
\]

\[
\nabla x_t = v_t,
\]

(3.12)

where \( d_t \) is a deterministic trend term with time trends. Both \( y_t \) and \( x_t \) are difference stationary, and both \( \varepsilon_t \) and \( v_t \) are stationary. Define

\[
w_t = (\varepsilon_t, v_t)'.
\]

(3.13)

Let \( \Phi (i) = E (w_tw'_{t-i}), \sum = \Phi (0), \Gamma = \sum_{i=0}^\infty \Phi (i) \) and \( \Omega = \sum_{i=-\infty}^\infty \Phi (i) \). Now transformations are considered,

\[
y_{*t} = y_t + \pi'_y w_t
\]

(3.14)

\[
x_{*t} = x_t + \pi'_x w_t.
\]

(3.15)

Both \( y_{*t} \) and \( x_{*t} \) are cointegrated because \( w_t \) is stationary with the cointegrating vector \( (1, -c) \).
3.4 Results

3.4.1 Results from DOLS

Table 3.6 presents the result for the estimate of the cointegrating regression as in Equation (3.10). The estimate is $c' = 0.68$ and the standard error is low (0.26).\(^\text{38}\)

![Table 3.6: Dynamic OLS Method (Lead=6, Lag=6)](attachment:table36.png)

In addition, the hypothesis that $c' = 1$ can be rejected. This point estimate is consistent with the results in both microeconomic data and my study in Chapter 2. The cointegration equation is confirmed by the Hansen Parameter Instability Test. Table 3.7 shows that the hypothesis that series are cointegrated can not be rejected. Therefore, I can conclude that housing services and consumption are complements since the EIS, $\varepsilon$, is less than one.

\(^{38}\)Here $c'$, $\varepsilon$, and the EIS are used interchangeably. Piazzesi et al. have a similar result for $\varepsilon$ less than 1, using the data between 1936 and 2001. However their result is not significant. Their finding is sensitive to their sample period. The recent period 2005 through 2008 influences the cointegration results.
Series: \( \log(C/H) \) and \( \log(q/P) \)
Null Hypothesis: Series are cointegrated
Equation deterministics: \( \log(C/H) \) \( \log(q/P) \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 3.7: Cointegration Test - Hansen Parameter Instability

### 3.4.2 Results from CCR

The main purpose is to find the cointegration vector using CCR. If so, as in Table 3.8, the coefficient of \( c \) is estimated to be 0.74 with standard deviation error of 0.24. The result from the CCR approach is consistent with the DOLS approach with low standard error.

```
Dependent Variable: log(C/H)
Sample: 1975Q1 2009Q1
Long run variance estimate

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Price</td>
<td>0.74</td>
<td>0.24</td>
<td>3.11</td>
<td>0.002</td>
</tr>
</tbody>
</table>
```

Table 3.8: Canonical Cointegrating Regression (CCR)

The estimate of the EIS is 0.74, the hypothesis that \( c' = 1 \) can be rejected. Hence housing and consumption are complements.

### 3.4.3 Results from Fully Modified OLS (FMOLS)

Finally, I use FMOLS since it can eliminate the problems caused by the long run correlation between equation and error terms. Table 3.9 shows the results to be
consistent with DOLS and CCR. The coefficient for the relative price is estimated at 0.74 with a standard error of 0.23. By the same token, the low standard error does not lead to the hypothesis that $c' = 1$. The results from all three cointegrating methods lead to the conclusion that housing and consumption are complements.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Price</td>
<td>0.74</td>
<td>0.23</td>
<td>3.13</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Table 3.9: Fully Modified Least Squares (FMOLS)

3.4.4 Results from Generalized Linear Model

Using logged detrended housing services and consumption, I examine whether there exists a comovement between these two series. The data comes from the NIPA and is detrended using the HP filter. Figure 3.3 shows that there exists the comovement between housing services and consumption. The vertical axis represents percentage change from the trend. In the 1970s and 2000s, the comovement is clearly evident. The results from DOLS, CCR, and FMOLS can be seen in the comovement seen in Figure 3.3.
3.5 Conclusion

Measuring the parameter value of the EIS using NIPA data sheds light on the question of whether there exists complementarity between housing and consumption in macroeconomic research.\textsuperscript{39} The estimate found in NIPA data shows complementarity between housing and consumption, and helps to integrate microeconomic evidence with macroeconomic evidence.

Complementarity exists when the parameter value of the EIS is less than one. Using a variety of cointegrating methods (DOLS, CCR and FMOLS), I have found

\textsuperscript{39}But until now, most of the macroeconomic literature has used the assumption that preferences over housing and consumption are separable. Other macroeconomic literature considers substitutability in preferences over housing and consumption. These approaches in macroeconomic research, however, are not consistent with the microeconomic evidence for nonseparable complementarity preferences between housing and consumption.
that the estimate is 0.68, 0.74, or 0.74, respectively. The standard error is sufficiently low that the hypothesis that the estimate is one can be rejected. The results from statistical estimation can be confirmed using data, which shows comovement between housing and consumption.

The finding of complementarity between housing and consumption is helpful in developing DSGE models that more successfully generate a positive comovement between housing and consumption. In particular, the estimate of the EIS between housing and consumption can provide a basis to successfully account for the unique feature that housing and consumption are complements in macroeconomic data.
3.6 Appendix: The Effect of House Prices on Consumption

I examine whether the elasticity of house price is positive or not. The elasticity can be measured by

\[
Elasticity_{Consumption, Housing} = \frac{\% \Delta \text{ in Consumption}}{\% \Delta \text{ in price of Housing Services}}.
\]

Following equation provides the elasticity of demand.

\[
\log(C_t) = \beta_1 + \beta_2 \log(P_t) + \beta_3 \log(q_t) \tag{3.16}
\]

As expected, housing service prices cause an increase in consumption. Table 3.10 shows that the elasticity of consumption to housing price is 0.03.

<table>
<thead>
<tr>
<th>Dependent Variable: log(C)</th>
<th>Sample: 1971Q1 2009Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Coefficient</td>
</tr>
<tr>
<td>log(P):Consumption Price</td>
<td>-1.32</td>
</tr>
<tr>
<td>log(q):Housing Prices</td>
<td>0.03</td>
</tr>
<tr>
<td>Constant</td>
<td>18.22</td>
</tr>
</tbody>
</table>

Table 3.10: Generalized Linear Model
Chapter 4: DO HOUSE PRICES MATTER FOR MONETARY POLICY?

4.1 Introduction

The focus of this chapter is to investigate whether monetary policy responds to changes in house prices in empirical models. I also investigate whether a theoretical model can account for the improvement of monetary policy when it responds to house prices.

For an empirical model, I consider a typical Taylor rule following the Rudebusch and Svensson (1998) model (RS model). In the RS model, there are two elements explaining the interest rate: inflation and output. I extend the RS model by adding house prices to the demand equation. I call this extension the modified RS model. I find that the modified RS model better fits the federal funds rate than the RS model. This suggests that monetary policy responds to house prices. I conclude that to the extent that house prices affect output, monetary policy responds.

For a theoretical framework, I construct a dynamic stochastic general equilibrium (DSGE) model, which embeds the complementarity found in Chapter 3. I investigate whether monetary policy responding to house prices improves the stability of inflation and output. The model allows for housing transaction costs and complementarity.
between housing and consumption. In particular, complementarity and loan-to-value ratios play an important role in monetary policy. Monetary policy is evaluated by the efficient frontier lines with respect to the standard deviation of inflation and output. The result shows that monetary policy which responds to house prices reduces the volatility of inflation and output.

There are arguments among economists that monetary policy should not respond to fluctuations in asset prices (e.g., stock prices) because fundamental factors and non-fundamental factors of prices are hard to distinguish. Iacoviello (2005) evaluates monetary policy with the addition of house prices to a Taylor rule. Then he compares the difference of monetary policy between the typical Taylor rule and an extension of the Taylor rule. He concludes that there are marginal improvements between these two policy rules. His argument seems to be consistent with the previous literature claiming that monetary policy does not need to respond to asset prices. Since responding to house prices does not reduce volatility, the central bank may not have an incentive to consider house prices as an independent force for monetary policy.

However, most of the macroeconomic literature has ignored complementarity between housing and consumption by adopting a separable utility function in a DSGE model. In Chapter 3, I found nonseparable complementarity between these two series using National Income of Product Accounts (NIPA) data. Complementarity and loan-to-value ratios amplify movements in consumption. House prices in the presence of complementarity and loan-to-value ratios make the real economy more volatile so that monetary policy is more effective when it responds to house prices. The results of my model indicate that interest rates responding to house prices provide a better tradeoff frontier across inflation and output. As monetary policy starts to give more
weight to output stability, the interest rate can decrease the volatility of inflation and output. Regardless of controversial arguments about interest rate responses to asset prices, I argue that house prices may be an independent force in monetary policy.

The transmission mechanism with complementarity in preferences is as follows: suppose house prices rise. The higher house prices are, the larger both collateral value and net worth are likely to be. The increase in borrowing capacity allows households to receive more credit. An increase in borrowing tends to increase consumption. As a result, consumption increases with house prices. Housing demand also increases through complementarity and the rise in net worth. A rise in housing demand further increases house prices. Overall this circular process leads to further increases in consumption.

The opposite case occurs when house prices fall. Net worth decreases with the drop in house prices. Borrowing limits are tightened due to the fall in net worth. Credit to constrained households is reduced. Consumption decreases with the fall in loans. Housing demand shrinks due to complementarity and the fall in net worth. A fall in housing demand further decreases house prices. This process continues to decrease consumption.

In section 4.2, I show that the housing market has become the largest financial market participant since 2000. In section 4.3, the VAR analysis provides the stylized facts among the federal funds rate, house prices, inflation and output. In section 4.4, the empirical model for monetary policy is investigated by showing the differences between the RS model and the modified RS model. In section 4.5, for the theoretical framework, I construct a DSGE model employing the complementarity between housing and consumption. In section 4.6, estimation and calibration are introduced.
In 4.7, the results show that monetary policy responding to house prices improves the stability of inflation and output.

4.2 The Housing Market

![Graph showing Treasuries, Housing related Bonds and Corporate Bonds over time.](image)

Figure 4.1: Treasuries, Housing related Bonds and Corporate Bonds

The housing market has become a significant participant in financial markets since 2000. Figure 4.1 shows that the total volume of outstanding mortgage related securities has exceeded both the Treasuries and the corporate debt market. The housing market volume reached about $9.3 trillion in 2007, while the corporate bond market and the Treasury market reached $6 trillion and $5 trillion, respectively.
The real interest rate was relatively low compared to the growth rate of real house price over the period of 2002 through 2006. However, the downturn of house prices over the period of 2008 and 2009 caused the housing market to stop growing and to begin to decline, while the Treasury bond market substantially increased.

4.3 VAR

I investigate interrelations among the interest rate, house prices, inflation and output, using VAR analysis.
In order to detrend these variables, I use the BP filter. The quarterly data for the interest rate, house prices, output, and inflation are the federal funds rate, the Conventional Mortgage Home Price Index (CMHPI), the GDPC1, and the GDP deflator from 1970 Q1 to 2009 Q1. The CMHPI is highly correlated with both the Office Of Federal Housing Enterprise Oversight (OFHEO)’s housing price index and the Case-Shiller Home Price Indices.\textsuperscript{40} The output gap is obtained by logged detrended real GDP. Inflation is from the growth rate of GDP deflator. The Cholesky ordering for the VAR is the interest rate, inflation, the real house price and the output gap.\textsuperscript{41}

The ordering is not important for the main results. The number of lags is two. Figure 4.2 shows that the interest rate increases with house prices in the first column. Along the same lines, the interest rate increases as both inflation and output rise. In the third column, as expected, house prices decrease by responding to an increase in interest rates. In addition, house prices rise with output. In the fourth column, output increases as house prices rise.

Interestingly, in the second column, inflation begins to increase to 0.05 percent in the second quarter, rather than immediately responding to the initial shock to house prices. This delayed response implies that house prices lead inflation by two quarters. I normalize the house price shock to one percent. Figure 4.3 shows that the response of interest rates and inflation to a percent increase in house prices.

These responses are consistent with Figure 4.4 as well. The logged detrended house price and inflation shows the two quarter differences between these two series. Figure 4.4 displays the plot of house prices against inflation (CPI).

\textsuperscript{40}The correlation between the CMHPI and the OFHEO is 99.99 percent.

\textsuperscript{41}Figure 4.9 shows detrended inflation, the output gap and house prices; the interest rate is the federal funds rate.
Figure 4.3: The Impulse Response of the Interest Rate and Inflation to a 1% increase in House prices

Figure 4.4: House prices and Inflation
In the 1970s and the latter half of 2000s, inflation clearly lags by two quarters following a house price shock.\textsuperscript{42} The causality is examined by taking pairwise Granger Causality Tests with the lag of one as shown in Table 4.1.

<table>
<thead>
<tr>
<th>Null Hypothesis:</th>
<th>Obs</th>
<th>F-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>house prices Growth does not Granger Cause Inflation</td>
<td>38</td>
<td>20.02</td>
<td>0.00</td>
</tr>
<tr>
<td>Inflation does not Granger Cause the house price Growth</td>
<td>38</td>
<td>1.35</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 4.1: Pairwise Granger Causality Tests: LAG 1 (1)

I can reject the null hypothesis that house price growth does not cause inflation and instead, I cannot reject the hypothesis that inflation does not cause house price growth. Hence Granger causality shows uni-directioned causality from house prices to inflation. The correlation between house prices and inflation is positive, or 0.371. However, the correlation does not establish any causality between them.

According to an announcement by the Federal Reserve in 2008, interest rate were adjusted following the downturn of house prices. This announcement is consistent with my VAR analysis. While the response of monetary policy to asset prices is controversial among economists, policy makers seem to consider house prices when setting interest rates. Figure 4.5 shows that house prices and interest rates move in the same direction.

\textsuperscript{42}In the data, house prices have seemed to act as an inflation indicator for decades.
The patterns seen in house prices and interest rates are remarkably similar. The interest rate is the nominal federal funds rate, while house prices are the logged detrended CMHPI. The percentage change from the trend in house prices slightly leads inflation. This leading indicator is confirmed by Table 4.2. I check the causality during the period of 1975 through 2008 using Pairwise Granger Causality Tests. In the data, there is clear evidence of an effect of house prices on monetary policy.

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Obs</th>
<th>F-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest does not Granger Cause house prices</td>
<td>34</td>
<td>0.42</td>
<td>0.65</td>
</tr>
<tr>
<td>House prices do not Granger Cause Interest</td>
<td>34</td>
<td>9.23</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 4.2: Pairwise Granger Causality Tests: LAG 1 (2)
4.4 The Rudebusch and Svensson IS Model and an Extension

4.4.1 The Effect of House Prices on Output Fluctuations

If the characteristics of housing are ignored in macroeconomic research, and housing is regarded as one of many durable goods with a typical asset price, then the intrinsic effect of house prices on the real economy may not be clear. The questions arise from the unique characteristics of housing. (1) In the real-world data, do house prices affect monetary policy? (2) If so, can a DSGE model account for this feature as in data?

To empirically investigate whether house prices are a significant factor in monetary policy, I extend the RS model by including real house prices in the Rudebusch and Svensson aggregate IS curve (the modified RS model).\textsuperscript{43} First, I investigate the effect of house prices on output fluctuations in the modified RS model. Then I compare the RS model with the modified RS model by evaluating how closely the results match with the actual data from the federal funds rate. The empirical RS model is a backward looking model in which the IS curve is a function of the interest rate and output. Aggregate demand in the RS model is:

\[ y_{t+1} = \varphi_1 y_t + \varphi_2 y_{t-1} + \varphi_3 (\hat{r}_t - \bar{\pi}_t) + \eta_t. \]  
\[ (4.1) \]

Aggregate supply in the RS Model is:

\[ \pi_{t+1} = \alpha_1 \pi_t + \alpha_2 \pi_{t-1} + \alpha_3 \pi_{t-2} + \alpha_4 \pi_{t-3} + \alpha_5 y_t + \varepsilon_t. \]  
\[ (4.2) \]

Monetary policy is assumed to follow a standard Taylor rule:

\[ R_t = \bar{r} + \pi_t + 0.5(\pi_t - \pi^*_t) + 0.5y_t, \]  
\[ (4.3) \]

\textsuperscript{43}In the RS model, there is no house price
where $\pi_t^* = 2$, $\bar{r}$ is the long-run real interest rate, and $y_t$ is the output gap.

The modified RS model is characterized by Equation (4.4). The output gap in the Taylor rule feeds on house prices through the demand equation.

$$y_{t+1} = \gamma_1 y_t + \gamma_2 y_{t-1} + \gamma_3 (\bar{i}_t - \bar{\pi}_t) + \gamma_4 E_t(q_{h_t}) + \epsilon_t.$$  \hspace{1cm} (4.4)

As shown in equation (4.4), the IS curve is a function of output, inflation, the interest rate, and house prices. The data is de-meaned and annualized.\(^{44}\) I use CMHPI data for the house prices. I take the logarithm of the CMHPI and de-trend it using the HP Filter. I estimate the coefficients of the aggregate demand and the aggregate supply using OLS. The model has the restriction that the lag coefficients of inflation sum up to one. The average level of the real interest rate in equilibrium is restricted to be zero as well. The output gap is applied as the relative rate between real GDP and potential GDP. The federal funds rate comes from the quarterly data from 1970 Q1 to 2009 Q4. The estimated aggregate supply is:

$$\pi_{t+1} = 0.48 \pi_t + 0.14 \pi_{t-1} + 0.16 \pi_{t-2} + 0.21 \pi_{t-3} + 0.07 y_t,$$  \hspace{1cm} (4.5)

with $\text{Adj}R^2=0.81$, $\text{SE} = 0.62$, $\text{DW}=1.93$.

The estimated aggregate demand equation for the RS model is:

$$y_{t+1} = 1.1 y_t - 0.24 y_{t-1} - 0.13 (\bar{i}_t - \bar{\pi}_t),$$  \hspace{1cm} (4.6)

\(^{44}\)All variables are de-meaned and annualized. $\pi_t = 400(\ln p_t - \ln p_{t-1}), \bar{\pi}_t = \frac{1}{T} \sum_{j=0}^{3} \pi_{t-j}, \bar{i}_t = \frac{1}{3} \sum_{j=0}^{3} i_{t-j}, y_t = \frac{100(q_t - q^*_t)}{q_t}$.

\(^{45}\)This model is similar to Ball(1999a) and Svensson (1997). In the Ball-Svensson Model: $y_t = \beta (r_{t-1} - r^*) + \lambda y_{t-1}, \pi_t = \pi_{t-1} + \alpha y_{t-1}$.
with \( \text{Adj} R^2 = 0.78 \), \( SE = 1.5 \), \( DW = 1.92 \).

The aggregate demand equation for the modified RS Model is:

\[
y_{t+1} = 1.06 \underbrace{y_t}_{(0.07)} - 0.25 \underbrace{y_{t-1}}_{(0.07)} - 0.14 \underbrace{(\bar{r}_t - \bar{\pi}_t)}_{(0.07)} + 0.05 \underbrace{q_{ht}}_{(0.02)}, \tag{4.7}
\]

with \( \text{Adj} R^2 = 0.78 \), \( SE = 1.5 \), \( DW = 1.93 \).

In Equation (4.7) it can be seen that a rise in the interest rate causes output to decrease. The coefficient estimate is -0.14. On the other hand, an increase in house prices by 1 percent causes output to rise by 0.05 percent.\(^{46}\)

### 4.4.2 Results from the Monetary Policy Rule

I assume that the policy instrument is the nominal interest rate which follows a Taylor rule. Under the Taylor rule, there are two main arguments: inflation and the output gap. Aggregate demand and aggregate supply are integrated with the equation for the monetary policy rule. I solve this system of equations for the interest rate.\(^{47}\)

Overall, as shown in Figure 4.6, the modified RS model better matches the data of the solid line from 1970 to 2007 compared to the RS model. Although the two models are similar, there is a significant gap, which captures the role of house prices. From the late 1980s, the models follow the data very well. The closeness to data of the modified RS model can be seen overall from 1970 to 2007. In particular, the period of 1970 though 1980 shows a significant difference between the two models. The existence of the house price effect and the closeness to the data implies that

\(^{46}\) Aoki et al. (2004b) also regressed output on lagged output, interest and house prices. His estimated coefficient for house prices, which was derived from UK data, was not significant.

\(^{47}\) There are several Taylor rules. However in this paper I examined the typical Taylor Rule. The results do not change when I consider different Taylor rules.
interest rates react to house prices. Since output responds to house prices in the demand equation, monetary policy responds to the extent that house prices affect output.

![Graph showing the Fed Fund Rate, Rudebusch Svensson Model, and Housing Model from 1970 to 2005.](image)

Figure 4.6: The RS model and the modified RS model

I compare the models by evaluating how closely the volatility, mean and maximum follow the data (the federal funds rate). Table 4.3 shows the value of mean, median, maximum, minimum and the standard deviation of the data. The statistics of the federal funds rate and the output gap are closer to the simulated data from the modified RS model. In other words, all the statistics of the modified RS model better fit the data than the RS Model. The results prove that adding house prices to the model helps our understanding of the actual interest rate movement.
### Table 4.3: Data and Models: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RS model</th>
<th>modified RS model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fed Rate</td>
<td>Output</td>
<td>Fed Rate</td>
</tr>
<tr>
<td>Mean</td>
<td>1.61</td>
<td>0.016</td>
<td>2.63</td>
</tr>
<tr>
<td>Median</td>
<td>1.39</td>
<td>0.055</td>
<td>1.78</td>
</tr>
<tr>
<td>Maximum</td>
<td>4.44</td>
<td>3.92</td>
<td>10.5</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.13</td>
<td>-4.75</td>
<td>-3.2</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>0.86</td>
<td>1.54</td>
<td>2.68</td>
</tr>
</tbody>
</table>

4.5 The Model

I construct a DSGE model with complementarity between housing and consumption. In the model, the unique characteristics of housing are implemented: housing transaction costs, loan-to-value ratios, collateral, slow depreciation, and complementarity with consumption. The purpose of the model is to obtain the efficient monetary policy frontier lines and check whether monetary policy responding to house prices matters. I also investigate whether complementarity is a significant element in monetary policy. The structure of the model in this economy is the same as in Chapter 2.

4.5.1 Unconstrained Households

The utility function for households is a life time discounted CES function. In particular, unconstrained households life time utility is:

\[
\max_{\{b_{1,t}, C_{1,t}, H_{1,t}, N_{1,t}\}_{t=0}^{\infty}} \left\{ E_0 \sum_{t=0}^{\infty} \beta_1^t \frac{1}{1-\varsigma} \left[ \left\{ C_{1,t}^{\frac{\varsigma-1}{\varsigma}} + j_t (H_{1,t})^{\frac{\varsigma-1}{\varsigma}} \right\}^{\frac{1}{1-\varsigma}} - \frac{N_{1,t}^{\eta_1}}{\eta_1} \right] \right\}, \quad (4.8)
\]

where \( E_0 \) denotes the expectation operator.
The budget constraint is:

\[ C_{1,t} + (H_{1t} - H_{1t-1})q_t + \frac{R_{t-1}}{\pi_t} b_{1,t-1} + \varphi_1 q_t H_{1t-1} \leq w_{1,t} N_{1,t} + b_{1,t} + f_t + s_t, \quad (4.9) \]

where \( w_{1,t} \) is the real wage households earn from their labor supply, \( q_t \) is the real house price, and \( \pi_t \) denotes inflation, or \( \pi_t = \frac{P_t}{P_{t-1}}. \)

I assume that the housing demand shock \( j_t \) follows the stochastic process,

\[ \ln j_t = (1 - \rho_A) \ln \bar{j} + \rho_j \ln j_{t-1} + \varepsilon_{j,t}, \quad (4.10) \]

where \( \bar{j} > 0, \rho_j \in (-1, 1) \) measures the persistence of the shock, and \( \varepsilon_{j,t} \) is a white noise process with mean zero and variance \( \sigma_j^2. \)

4.5.2 Constrained Households

The optimization problem for constrained households is

\[
\max_{\{b_{2,t}C_{2,t},H_{2,t},N_{2,t}\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty (\beta_2)^t \frac{1}{1-\zeta} \left[ \left\{ C_{2,t}^{\frac{\zeta}{1-\zeta}} + j_t (H_{2,t})^{\frac{\zeta}{1-\zeta}} \right\}^{1-\zeta} \right] - \frac{N_{2,t}^{m_2}}{\eta_2}, \quad (4.11)
\]

subject to

\[ C_{2,t} + (H_{2t} - H_{2t-1})q_t + \varphi_2 q_t H_{2t-1} + \frac{R_{t-1}}{\pi_t} b_{2,t-1} \leq w_{2,t} N_{2,t} + b_{2,t}, \quad (4.12) \]

where

\[ R_t b_{2t} \leq m_2 E_t (q_{t+1} \pi_{t+1} H_{2t}). \quad (4.13) \]

4.5.3 Entrepreneurs

Entrepreneurs derive utility only from consumption. They produce intermediate goods according to the following constant return to scale Cobb-Douglas production function with labor, housing and capital as inputs,

\[ Y_t = Z_t \left( K_{t-1}^\mu (H_{t-1}^{\frac{1-\nu}{1-\mu}}) \right)^{\frac{\mu}{1-\mu}} \left( N_{1,t}^{\alpha} N_{2,t}^{(1-\alpha)} \right)^{(1-\mu)^\alpha}, \quad (4.14) \]
where $Y_t$ denotes output and $Z_t$ represents productivity in period $t$. $K_{t-1}$ and $H_{t-1}$ denote capital and housing in period $t-1$, respectively. The shock to $Z_t$ follows the stochastic process:

$$\ln Z_t = (1 - \rho_z) \ln \bar{Z} + \rho_z \ln Z_{t-1} + \varepsilon_{Z,t},$$  \hspace{1cm} (4.15)

where $\bar{Z} > 0$, $\rho_z \in (-1, 1)$ measures the persistence of the shock and $\varepsilon_{Z,t}$ is a white noise process with mean zero and variance $\sigma^2_Z$. Capital accumulation follows

$$K_t = (1 - \delta) K_{t-1} + I_{Kt} - \frac{\xi (K_t - K_{t-1})^2}{2 K_{t-1}},$$

where $I_{Kt}$ denotes investment of capital. The parameter $\xi$ determines the costs of capital adjustment. Capital depreciates at the rate $\delta$. Housing accumulation follows

$$H_{et} = H_{et-1} + I_{Het} - \varphi_e H_{et-1},$$

where $I_{Het}$ denotes investment of houses. Housing transactions are associated with costs like realtor fees, $\varphi_e q_t H_{et-1}$, which are paid to unconstrained households. The utility function, budget constraint and borrowing constraint of entrepreneurs lead to the following utility maximization problem:

$$\max_{\{C_{et}, b_{et}, I_{Kt}, H_{et}, N_{1,t}, N_{2,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \gamma^t \ln C_{et},$$ \hspace{1cm} (4.16)

subject to

$$\frac{Y_t}{X_t} + b_{et} \geq C_{et} + \frac{R_{t-1}}{\pi_t} b_{et-1} + w_{1,t} N_{1,t} + w_{2,t} N_{2,t} + I_{Kt} + I_{Het} + \xi_{Kt} + \xi_{Het},$$  \hspace{1cm} (4.17)

where $I_{Kt} = K_t - (1 - \delta) K_{t-1}$, $\xi_{Kt} = \frac{\psi_K}{2b} \left( I_{Kt} K_{t-1} - \delta \right)^2 K_{t-1}$, $I_{Het} = \{ H_{et} - H_{et-1} \} q_t$, $\xi_{Het} = \varphi_e q_t H_{et-1}$, and

$$R_t b_{et} \leq m_e E_t (q_{t+1} \pi_{t+1} H_{et}).$$  \hspace{1cm} (4.18)
There is a proportional housing transaction costs parameter, $\varphi_e$, in the budget constraint. Finally, $m_e$ is the parameter which determines the loan-to-value ratio of entrepreneurs.

### 4.5.4 Retailers

Retailers produce a continuum of differentiated final goods modeled under monopolistic competition. Since retailers have market power, profits are assumed to be paid to unconstrained households. The retailers’ supply curve implies a Phillips curve, determined by an optimal pricing equation. Retail prices are sticky, and the probability of any retailer being able to adjust its nominal prices is $1 - \theta$. Retailers purchase intermediate goods from entrepreneurs at price $P^I_t$ and sell final goods to households at price, $P_t(g)$. Final goods are $Y_t = \left(\int_0^1 Y_t(g)^{(\xi-1)/\xi}dg\right)^{\frac{\xi}{\xi-1}}$ with $\xi > 1$. The solution to the firms’ cost minimization problem gives a demand function for intermediate goods and a price index, $P_t = \left(\int_0^1 P^I_t(g)^{1-\xi}dg\right)^{\frac{1}{1-\xi}}$. Profit maximization by each retail firm implies that the optimal price of retail goods, $P^*_t(g)$, satisfies

$$\sum_{i=0}^{\infty} \theta^i E_t \left( \beta \frac{C_{1,t+i}}{C_{1,t+i}} \left( \frac{P^*_t(g)}{P^*_t} - \frac{X_t}{X_{t+i}} \right) Y_{t+i}(g) \right) = 0,$$

where $X_t$ is the markup. $P^*_t$ makes discounted marginal revenue be equal to expected discounted marginal cost.

The aggregate price index is $P_t = \left(\theta P^*_{t-1} + (1 - \theta) P^*_t\right)^{\frac{1}{1-\xi}}$. From the optimization of retailers, the log-linearized aggregate supply curve can be derived as

$$\tilde{\pi}_t = \beta \tilde{\pi}_{t+1} - k \tilde{X}_t + \tilde{\epsilon}_{u,t},$$

where $k = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. 

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4.5.5 The Central Bank

It is assumed that the central bank takes the nominal interest rate as an instrument for monetary policy, which follows a Taylor rule:

\[ R_t = (R_{t-1})^{\gamma_R} \left( \frac{\pi_t^{1+\gamma_q} Y_t^{\gamma_q} q_t^{\gamma_q}}{Y_{t-1}^{\gamma_q} q_{t-1}^{\gamma_q}} \right)^{1-\gamma_R} \varepsilon_{R,t} \]  

(4.19)

First, I evaluate the two interest rules by setting \( \gamma_q = 0 \) and \( \gamma_q > 0 \). Then I derive the two different efficient frontier lines with respect to the volatility of both inflation and output. The previous literature shows the marginal difference between the two lines.

4.5.6 Equilibrium

The dynamic stochastic equilibrium for given \( H_{e,0}, H_{1,0}, H_{2,0}, b_{e,0}, b_{1,0} \) and \( b_{2,0} \) is a sequence of allocations, \( \{Y_t, C_{e,t}, C_{1,t}, C_{2,t}, H_{e,t}, H_{1,t}, H_{2,t}, b_{e,t}, b_{1,t}, b_{2,t}, N_{1,t}, N_{2,t}\}_{t=0}^\infty \) and a sequence of prices, \( \{w_{1,t}, w_{2,t}, R_t, q_t, P_t, P_t^*, \lambda_t\}_{t=0}^\infty \), at which all agents satisfy their budget constraints, borrowing constraints and first order conditions, and all market clear in each date. For the markets to clear, the bond market for \( b_t \), goods market for \( Y_t \) and the house market for \( H_{e,t}, H_{1,t} \) and \( H_{2,t} \) should meet the following conditions:

(a) \( N_{1t} = N_{2t} \)
(b) \( H_{e,t} + H_{1,t} + H_{2,t} = 1 \)
(c) \( C_{e,t} + C_{1,t} + C_{2,t} + I_t = Y_t \)
(d) \( I_t = I_{Kt} + I_{H_{e,t}} + I_{H_{1,t}} + I_{H_{2,t}} \)
(e) \( s_t = \varphi_e q_t H_{e,t-1} + \varphi_1 q_t H_{1,t-1} + \varphi_2 q_t H_{2,t-1} \)
(f) \( b_t + b_{1t} + b_{2t} = 0 \)

Wages are different across households and based on their productivity. In the steady state, unconstrained households receive wage bills of \( w_1 N_1 = \alpha (1 - \mu - v) Y \), while constrained households receive a wage bill of \( w_2 N_2 = (1 - \alpha)) (1 - \mu - v) Y \).
4.5.7 Solution Method

The solution of the dynamic model involves non-linear equations and identity equations. The steady state values of the state variables are recovered from a non-linear system of equations. This step uses the Broyden method (Miranda and Fackler (2002)) for solving systems of nonlinear equations. After obtaining steady state values, the model is solved as a local (log-linear) approximation around steady state. All derivatives of the first order conditions are computed numerically using the Jacobian command in Matlab. Finally, the Uhlig toolkit (Uhlig (1999)) is used to solve the linear system of stochastic difference equations.

4.6 Calibration and Estimation

4.6.1 Calibration

The discount factors for unconstrained households, constrained households and entrepreneurs, the share of housing to consumption, the parameters of production across capital and housing, the probability of the price adjustment and the depreciation rate for housing are the same values as in Chapter 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Data to Match</th>
</tr>
</thead>
<tbody>
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<td>$\beta, \gamma, \beta_2$</td>
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<td>subjective discount rate</td>
</tr>
<tr>
<td>$j$</td>
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<td>share of housing to consumption</td>
</tr>
<tr>
<td>$\varphi_1, \varphi_e, \varphi_2$</td>
<td>0.0, 0.03, 0.05</td>
<td>housing transaction costs</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1.01</td>
<td>elasticity of labor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.03</td>
<td>depreciation of capital</td>
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<tr>
<td>$\mu, \nu$</td>
<td>0.3, 0.03</td>
<td>share of capital and housing</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>1.05</td>
<td>markup</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.75</td>
<td>probability of the fixed price</td>
</tr>
</tbody>
</table>

Table 4.4: Calibration

84
4.6.2 Estimation of Monetary Policy

I assume that the nominal interest rate responds to past inflation and past output through the following Taylor Rule, which is a log-linearization of Equation (4.19):

\[ \hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R) \left( (1 + \gamma_{\pi}) \hat{\pi}_{t-1} + \gamma_Y \hat{Y}_{t-1} \right) + \hat{\varepsilon}_{R,t}. \] (4.20)

I estimate the parameters of the Taylor rule using the OLS method.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{R}_{t-1} )</td>
<td>0.84</td>
<td>0.035</td>
<td>23.63</td>
</tr>
<tr>
<td>( \hat{Y}_{t-1} )</td>
<td>0.011</td>
<td>0.008</td>
<td>1.35</td>
</tr>
<tr>
<td>( \hat{\pi}_{t-1} )</td>
<td>0.264</td>
<td>0.06</td>
<td>4.19</td>
</tr>
<tr>
<td>Constant</td>
<td>0.024</td>
<td>0.045</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 4.5: Taylor Rule: Estimation using OLS (1)

As an alternative, I consider a Taylor rule where the nominal interest rate responds to past inflation, past output and past house prices in order to consider the effect of the response of monetary policy to house prices. I estimate the parameter using the OLS method after log-linearizing Equation (4.19).

\[ \hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R) \left( (1 + \gamma_{\pi}) \hat{\pi}_{t-1} + \gamma_Y \hat{Y}_{t-1} + \gamma_q \hat{q}_{t-1} \right) + \hat{\varepsilon}_{R,t}. \] (4.21)

Based upon these two monetary policy rules, I compare the effect of house prices on monetary policy. In the second Taylor rule, OLS estimation shows that a one percent increase in house prices causes a 0.02 percent increase in the interest rate.
Table 4.6: Taylor Rule: Estimation using OLS (2)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{R}_{t-1}$</td>
<td>0.88</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>$\hat{Y}_{t-1}$</td>
<td>0.004</td>
<td>0.02</td>
<td>0.82</td>
</tr>
<tr>
<td>$\hat{\pi}_{t-1}$</td>
<td>0.18</td>
<td>0.07</td>
<td>0.01</td>
</tr>
<tr>
<td>$\hat{q}_{t-1}$</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Constant</td>
<td>0.01</td>
<td>0.04</td>
<td>0.78</td>
</tr>
</tbody>
</table>

4.6.3 Estimation of Structural Parameters

The crucial parameter listed below is the elasticity of intratemporal substitution of housing for consumption. Further discussion of these parametric estimates is contained in Chapter 2.

Table 4.7: Estimation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>STD.e</th>
<th>Parameter</th>
<th>Value</th>
<th>STD.e</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$ = elasticity of substitution</td>
<td>0.592</td>
<td>0.053</td>
<td>$\phi_{\pi} = \text{autoregressive}_{\pi}$</td>
<td>0.629</td>
<td>0.043</td>
</tr>
<tr>
<td>$\varsigma$ = curvature</td>
<td>2.317</td>
<td>0.357</td>
<td>$\phi_{j} = \text{autoregressive}_{j}$</td>
<td>0.885</td>
<td>0.017</td>
</tr>
<tr>
<td>$a$ = share of wage</td>
<td>0.553</td>
<td>0.046</td>
<td>$\phi_{a} = \text{autoregressive}_{a}$</td>
<td>0.177</td>
<td>0.1</td>
</tr>
<tr>
<td>$m_e =$ loan – to – value</td>
<td>0.412</td>
<td>0.066</td>
<td>$\sigma_{A} = \text{stdev}_{a}$</td>
<td>9.195</td>
<td>3.5</td>
</tr>
<tr>
<td>$m_2 =$ loan – to – value</td>
<td>0.861</td>
<td>0.02</td>
<td>$\sigma_{j} = \text{stdev}_{j}$</td>
<td>0.021</td>
<td>1.64</td>
</tr>
<tr>
<td>$\sigma_R = \text{stdev}_{R}$</td>
<td>0.24</td>
<td>0.009</td>
<td>$\sigma_{\pi} = \text{stdev}_{\pi}$</td>
<td>0.154</td>
<td>0.024</td>
</tr>
</tbody>
</table>

4.7 Monetary Policy Evaluation

4.7.1 The Loss Function

Monetary policy is evaluated with respect to the volatility of both inflation and output. The loss function can be solved by minimizing a weighted sum of the standard
deviation of inflation and output by choice of $\gamma_\pi$, $\gamma_y$, and $\gamma_q$. The loss function is expressed as:

$$J(\gamma_\pi, \gamma_y, \gamma_q) = \min \left\{ \varpi \text{VAR}(Y) + (1 - \varpi) \text{VAR}(\pi) \right\}. \tag{4.22}$$

Optimal $\gamma^*_\pi$, $\gamma^*_y$, and $\gamma^*_q$ are the solutions to the above loss function subject to the following extension of the Taylor rule which has been augmented with house prices:

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R) \left( (1 + \gamma_\pi) \hat{\pi}_{t-1} + \gamma_y \hat{Y}_{t-1} + \gamma_q \hat{q}_{t-1} \right) + \hat{\epsilon}_{R,t}.$$  

For comparison, I also derive the optimal $\gamma^*_\pi$ and $\gamma^*_y$ when Equation (4.22) is subject to the typical Taylor rule,

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R) \left( (1 + \gamma_\pi) \hat{\pi}_{t-1} + \gamma_y \hat{Y}_{t-1} \right) + \hat{\epsilon}_{R,t}.$$  

Using the solutions to the above problems, I can compare the differences between the two monetary policy rules.

### 4.7.2 Efficient Frontier Lines

Two efficient lines, as shown in Figure 4.7, are obtained by the solutions in the loss function based upon two different Taylor rules. Each Taylor rule consists of the different sets of inflation and output. Equation (4.22) makes clear that when $w$ reaches zero, monetary policy only focuses on inflation stability, while when $w$ reaches one, its emphasis is only on output stability. The weight $\varpi$ changes by 0.05 steps from 0 to 1. Hence twenty monetary policy sets are shown in each Taylor rule. Each set is associated with the each optimal $\gamma^*_\pi$, $\gamma^*_y$, and $\gamma^*_q$ to produce the lowest standard deviation of the weighted sum of inflation and output. Each pair of standard deviations that solve a problem given $w$ is plotted.
The bold blue line represents monetary policy using a typical Taylor rule, while the red dotted line represents the Taylor rule augmented by house prices. If there is a distinction between the two lines, then the lower line represents better monetary policy. Overall, the red dotted line is lower than the blue line implying that responding to house prices improves the stability of both inflation and output. However, the lines are initially hard to distinguish. This implies that when monetary policy only focuses on the inflation stability, a response to house prices has negligible effect in lowering volatility. On the other hand, when monetary policy gives more consideration to output stability, responding to the house price significantly improves the overall volatility of inflation and output.

Figure 4.7: Monetary Policy
This result is substantially different from Iacoviello (2005) in that there is a clear difference between a Taylor rule and a monetary policy rule introducing house prices. The crucial difference comes from housing and consumption being complements. In this sense, complementarity matters for monetary policy.

4.7.3 The Interest Rate responding to House Prices

Figure 4.8 shows the effect of house prices on the interest rate. $\gamma_q$ is no longer zero. The higher is $\gamma_q$, the more the interest rate response is likely to be. As seen in the figure, $\gamma_q$ becomes an important force in determining the interest rate until the weight of the output stability reaches 25 percent (the inflation stability has corresponding weight of 75 percent). As the importance of the output stability becomes larger than 30 percent, the interest rate response to house prices movements decreases substantially.

In addition, when monetary policy only focuses on inflation stability, the response of the interest rate to house prices is almost zero.

Figure 4.11 shows the effect of output on the interest rate. The coefficient, $\gamma_Y$, changes as monetary policy changes the importance of output stability. In particular, $\gamma_Y$ becomes larger because of the greater importance of output stability.

48 In the model, the monetary policy rule smooths the interest rate. The effect of house prices on interest rates can be obtained by multiplying $\gamma_q$ by 0.16 to control for the persistence parameter for the interest rate. This shows that monetary policy makers respond to a 1 percent increase in house prices by raising the nominal rate by 0.06 percent.

49 This result is consistent with the study of Bernanke and Gertler (1995). As seen in the VAR responses in Figure 4.10, there is a clear response of interest rates to house prices.
4.8 Conclusion

VAR analysis provides stylized facts that show that the nominal interest rate increases with house prices. I use an empirical monetary policy model and a micro-founded model to account for this feature of the data. First, comparing the RS model with a modified RS model, I find that house prices affect monetary policy. In the modified RS model, output responds to house prices. The result shows that to the extent to which house prices affect output, monetary policy is likely to respond.

Second, assuming complementarity between housing and consumption as found in Chapter 3, I have constructed a DSGE model in order to investigate whether house prices play an independent role in monetary policy. The result shows that monetary
policy responding to house prices improves the stability of both inflation and output when the central bank values output stability. I derive two efficient frontier lines from two different monetary policy rules. The efficient line responding to house prices reduces the volatility of both inflation and output. Put another way, as monetary policy makers start to give a sufficient weight to output stability, the interest rate role responding to house prices decreases volatility. However, when monetary policy focuses on inflation stability, the volatility of both inflation and output between the two efficient frontier lines corresponding to the two monetary policy rules is hard to distinguish. These results are consistent with the stylized facts derived from VAR analysis. The VAR suggests that monetary policy responds to house prices.
### 4.9 Table

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y(output)</td>
<td>0.1</td>
<td>0.02</td>
<td>0.001</td>
</tr>
<tr>
<td>P(inflation)</td>
<td>0.1</td>
<td>0.05</td>
<td>0.091</td>
</tr>
<tr>
<td>Q(house price)</td>
<td>0.02</td>
<td>0.01</td>
<td>0.054</td>
</tr>
<tr>
<td>C(constant)</td>
<td>0.01</td>
<td>0.03</td>
<td>0.74</td>
</tr>
<tr>
<td>Interest rate(-1)</td>
<td>1.26</td>
<td>0.08</td>
<td>0.000</td>
</tr>
<tr>
<td>Interest rate(-2)</td>
<td>-0.33</td>
<td>0.08</td>
<td>0.000</td>
</tr>
<tr>
<td>Q(-1)</td>
<td>-0.02</td>
<td>0.01</td>
<td>0.175</td>
</tr>
<tr>
<td>Y(-1)</td>
<td>-0.07</td>
<td>0.02</td>
<td>0.001</td>
</tr>
<tr>
<td>P(-1)</td>
<td>0.02</td>
<td>0.05</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Table 4.8: Monetary Policy
Figure 4.9: House Prices: Q, output: Y, Inflation: P, and Interest Rate: R
Figure 4.10: VAR from the DSGE Model
The importance of $Y$ relative to $\pi$

Figure 4.11: Optimal Interest Rate Response (2)
Chapter 5: CONTRIBUTIONS

I solve an issue that exists in previous work. Existing research papers typically assume that utility is separable in housing and consumption. However under this assumption, housing and consumption comove only in the absence of housing transaction costs. When these costs are introduced into DSGE models characterized by separable preferences, aggregate consumption no longer increases after a rise in house prices. I resolve this problem by developing a model that allows for nonseparable preferences in housing and consumption alongside housing transaction costs. The nonseparable utility function is characterized by complementarity between housing and consumption. Empirical estimates yield a level of complementarity, which is sufficiently strong that the model can generate the comovement found in the data in spite of the presence of housing transaction costs.

In addition, I address the importance of complementarity between housing and consumption in monetary policy. While it is controversial to suggest that monetary policy should react to changes in asset prices, I provide evidence that it has responded to changes in house prices. Further, I explain why policymakers might have responded in this manner. Specifically, efficient frontier lines defined between the standard deviation of output and inflation show that, when the nominal interest rate rule includes house prices, the volatility of both inflation and output fall.
My results are important in several ways. First, the model in this thesis is helpful in developing DSGE models that are more successful in reproducing the positive comovement between housing and consumption. Second, the model captures the essential characteristics of housing, such as highly leveraged house purchases, high house values relative to GDP, and a significant amount of realtor fees. Third, I investigate the amplification effect of loan-to-value ratios which increase volatility of house prices and consumption following shocks in interest rates. Lastly, I provide further evidence of complementarity between housing and consumption using NIPA data. Therefore my finding establishes agreement between microeconomic estimates and macroeconomic data.


