Novel Techniques for Enhancing SAR Imaging using Spatially Variant Apodization

THESIS

Presented in Partial Fulfillment of the Requirements for the Degree Master of Science in the Graduate School of The Ohio State University

By

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2011

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Abstract

Conventional radar imaging techniques have long been plagued by problematic sidelobes and poor resolution. Indeed, numerous algorithms exist for combating these two issues. In tackling the first problem the technique spatially variant apodization (SVA) has been utilized for quite some time with tremendously positive results. A more recent enhancement to this technique known as Super-SVA has potential for tackling the second problem. In this thesis, a method is developed for utilizing Super-SVA to enhance down-range and cross-range resolution while minimizing sidelobes without distorting scatterer mainlobes.
Acknowledgments

I would like to thank my graduate advisor and supervisor Dr. Robert Burkholder, whom provided me with a fantastic opportunity within the Electro Science Laboratory at The Ohio State University. I would also like to thank Northrup Grumman for their financial support throughout my graduate degree. Additionally, I would like to thank my mother and father for providing me with the encouragement that I found invaluable throughout my engineering career. Finally, I would also like to thank my friends Joseph Lavalley and Philip Nord for being there when I needed them and providing me with a place to live throughout my graduate studies.
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- Electromagnetics
- SAR Radar Imaging
- Radar Development
- Signal Processing
# Table of Contents

Abstract .................................................................................................................................................. ii
Acknowledgments .................................................................................................................................... iii
Vita .......................................................................................................................................................... iv
List of Figures .......................................................................................................................................... vii
Chapter 1: Introduction ...................................................................................................................... 1
  1.1 Previous Work ................................................................................................................................. 2
  1.2 Problem Definition ......................................................................................................................... 4
Chapter 2: One-Dimensional Spatially Variant Apodization .............................................................. 8
  2.1 Introduction to 1D SVA (Spatially Variant Apodization) ................................................................. 8
  2.2 Derivation of the 1D SVA Windowing Function ............................................................................. 10
  2.3 MATLAB Simulated Results ........................................................................................................ 18
    2.3.1 Simulation 1: Two Scatterers well separated in Down-Range ............................................ 22
    2.3.2 Simulation 2: Two Scatterers Merged in Down-Range ..................................................... 25
  2.4 Experimental Results .................................................................................................................... 27
Chapter 3: Two-Dimensional Spatially Variant Apodization ............................................................ 29
  3.1 Introduction to 2D SVA .................................................................................................................. 29
  3.2 Derivation of the 2D SVA Windowing Function .......................................................................... 30
    3.2.1 Method 1: General 2D SVA Method .................................................................................... 34
    3.2.2 Method 2: Simplified 2D SVA Method ............................................................................... 38
  3.3 Derivation of a Two-Step 2D SVA Procedure ............................................................................. 42
    3.3.1 Method 3: Step 1, 1D SVA in Down-Range ......................................................................... 42
    3.3.2 Method 3: Step 2, 1D SVA in Cross-Range ........................................................................ 45
  3.4 MATLAB Simulated Results ........................................................................................................ 47
  3.5 Experimental Results .................................................................................................................... 55
Chapter 4: One-Dimensional Super Spatially Variant Apodization .................................................. 59
  4.1 Introduction to Super-SVA (SSVA) ................................................................................................. 59
  4.2 Derivation of the 1D Super-SVA Method ...................................................................................... 62
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIGURE 1.1:</td>
<td>INSIDE OF AN ANECOIC CHAMBER [1]</td>
</tr>
<tr>
<td>FIGURE 1.2:</td>
<td>ILLUSTRATED EFFECTS OF SVA &amp; SUPER-SVA</td>
</tr>
<tr>
<td>FIGURE 1.3:</td>
<td>UNMANNED AIRBORNE VEHICULAR RADAR (MILITARY DRONES) [7]</td>
</tr>
<tr>
<td>FIGURE 1.4:</td>
<td>A THROUGH-WALL IMAGING SCENARIO</td>
</tr>
<tr>
<td>FIGURE 1.5:</td>
<td>ILLUSTRATION OF THE SUPER SVA PROCEDURE</td>
</tr>
<tr>
<td>FIGURE 1.6:</td>
<td>THE ILLUSTRATED EFFECTS OF 1D SVA</td>
</tr>
<tr>
<td>FIGURE 1.7:</td>
<td>THE SVA WINDOWING FUNCTION</td>
</tr>
<tr>
<td>FIGURE 1.8:</td>
<td>THE SVA WINDOWING FUNCTION</td>
</tr>
<tr>
<td>FIGURE 1.9:</td>
<td>LOW LEVEL BLOCK DIAGRAM OF THE 1D SVA PROCEDURE</td>
</tr>
<tr>
<td>FIGURE 1.10:</td>
<td>THE HANN &amp; HAMMING WINDOWS</td>
</tr>
<tr>
<td>FIGURE 1.11:</td>
<td>1D SVA CONTRASTED WITH THE HANN &amp; HAMMING WINDOWS</td>
</tr>
<tr>
<td>FIGURE 2.1:</td>
<td>THE ILLUSTRATED EFFECTS OF 2D SVA</td>
</tr>
<tr>
<td>FIGURE 2.2:</td>
<td>THE SIMULATED 2D HANN WINDOWED IMAGE</td>
</tr>
<tr>
<td>FIGURE 2.3:</td>
<td>THE SIMULATED 2D HANN WINDOWED IMAGE</td>
</tr>
<tr>
<td>FIGURE 2.4:</td>
<td>THE SIMULATED 2D HAMMING WINDOWED IMAGE</td>
</tr>
<tr>
<td>FIGURE 2.5:</td>
<td>THE SIMULATED 2D HAMMING WINDOWED IMAGE</td>
</tr>
<tr>
<td>FIGURE 2.6:</td>
<td>THE SIMULATED 2D SVA (METHOD 3) WINDOWED IMAGE</td>
</tr>
<tr>
<td>FIGURE 2.7:</td>
<td>THE SIMULATED 2D SVA (METHOD 2) WINDOWED IMAGE</td>
</tr>
<tr>
<td>FIGURE 2.8:</td>
<td>THE SIMULATED 2D SVA (METHOD 2) WINDOWED IMAGE</td>
</tr>
<tr>
<td>FIGURE 2.9:</td>
<td>ALPHA &amp; BETA COMPUTED FOR 2D SVA</td>
</tr>
<tr>
<td>FIGURE 2.10:</td>
<td>ALPHA &amp; BETA COMPUTED FOR 2D SVA</td>
</tr>
<tr>
<td>FIGURE 2.11:</td>
<td>AN EXPERIMENTALLY OBTAINED INITIAL IMAGE</td>
</tr>
<tr>
<td>FIGURE 2.12:</td>
<td>THE EXPERIMENTAL 2D HANN WINDOWED IMAGE</td>
</tr>
<tr>
<td>FIGURE 2.13:</td>
<td>THE EXPERIMENTAL 2D HAMMING WINDOWED IMAGE</td>
</tr>
<tr>
<td>FIGURE 2.14:</td>
<td>THE EXPERIMENTAL 2D SVA (METHOD 3) WINDOWED IMAGE</td>
</tr>
<tr>
<td>FIGURE 2.15:</td>
<td>THE EXPERIMENTAL 2D SVA (METHOD 2) WINDOWED IMAGE</td>
</tr>
<tr>
<td>FIGURE 2.16:</td>
<td>ILLUSTRATION OF THE INVERSE WINDOWING FUNCTION W(K)</td>
</tr>
<tr>
<td>FIGURE 2.17:</td>
<td>PLOTS OF S(K), F(K), &amp; THE EXPANDED SIGNAL</td>
</tr>
<tr>
<td>FIGURE 2.18:</td>
<td>ILLUSTRATION OF THE EFFECTS OF 1D SUPER SVA</td>
</tr>
<tr>
<td>FIGURE 2.19:</td>
<td>HIGH LEVEL BLOCK DIAGRAM OF THE 1D SUPER SVA PROCEDURE</td>
</tr>
<tr>
<td>FIGURE 2.20:</td>
<td>ILLUSTRATION OF THE SINGULARITIES WITHIN F(K)</td>
</tr>
<tr>
<td>FIGURE 2.21:</td>
<td>SIGNAL CORRESPONDING TO A SINGLE POINT SCATTERER</td>
</tr>
<tr>
<td>FIGURE 2.22:</td>
<td>DOWN-RANGE PROFILE OF A SINGLE POINT SCATTERER</td>
</tr>
<tr>
<td>FIGURE 2.23:</td>
<td>DOWN-RANGE PROFILE OF A 1D SVA WINDOWED POINT SCATTERER</td>
</tr>
<tr>
<td>FIGURE 2.24:</td>
<td>ILLUSTRATION OF THE INVERSE WINDOWING FUNCTION W(K)</td>
</tr>
<tr>
<td>FIGURE 2.25:</td>
<td>PLOTS OF THE SINGULARITIES WITHIN THE EXPANDED SIGNAL</td>
</tr>
<tr>
<td>FIGURE 2.26:</td>
<td>DOWN-RANGE PROFILES FOR 5 ITERATIONS OF 1D SUPER-SVA</td>
</tr>
<tr>
<td>FIGURE 2.27:</td>
<td>1D SUPER-SVA APPLIED TOWARDS MERGED SCATTERERS</td>
</tr>
<tr>
<td>FIGURE 2.28:</td>
<td>1D SUPER-SVA APPLIED TOWARDS WELL SEPARATED SCATTERERS</td>
</tr>
<tr>
<td>FIGURE 2.29:</td>
<td>1D SUPER-SVA APPLIED TOWARDS EXPERIMENTAL DATA</td>
</tr>
</tbody>
</table>

vii
FIGURE 5.1: ILLUSTRATED EFFECTS OF 2D SUPER-SVA .................................................................80
FIGURE 5.2: HIGH LEVEL BLOCK DIAGRAM OF THE 2D SUPER-SVA PROCEDURE ........................81
FIGURE 5.3: THE SUPER SVA BANDWIDTH TRUNCATION PROCEDURE .......................................82
FIGURE 5.4: A SIMULATED INITIAL IMAGE OF A POINT SCATTERER .............................................85
FIGURE 5.5: THE SIMULATED 2D SVA (METHOD 2) WINDOWED IMAGE ........................................85
FIGURE 5.6: THE SIMULATED 2D SUPER-SVA WINDOWED IMAGE (AFTER TRUNCATION) ...............85
FIGURE 5.7: POINT SCATTERER WIDTH & HEIGHT BEFORE & AFTER 2D SUPER-SVA ......................86
FIGURE 5.8: THE EXPANDED SIGNALS FOR 5 2D SUPER-SVA ITERATIONS .....................................87
FIGURE 5.9 THE DOWN-RANGE PROFILES FOR 5 2D SUPER-SVA ITERATIONS ...............................88
FIGURE 5.10: A SIMULATED INITIAL IMAGE OF TWO POINT SCATTERERS MERGED IN DOWN-RANGE ......89
FIGURE 5.11: THE SIMULATED 5 ITERATION 2D SUPER-SVA WINDOWED IMAGE .............................90
FIGURE 5.12: THE SIMULATED INITIAL IMAGE OBTAINED USING REAL DATA FOR THE EXTRAPOLATED BANDWIDTH .........................................................................................................................90
FIGURE 5.13: A SIMULATED INITIAL IMAGE OF TWO POINT SCATTERERS MERGED IN CROSS-RANGE ......91
FIGURE 5.14: THE SIMULATED 5 ITERATION 2D SUPER-SVA WINDOWED IMAGE .............................92
FIGURE 5.15: THE SIMULATED INITIAL IMAGE OBTAINED USING REAL DATA FOR THE EXTRAPOLATED BANDWIDTH .........................................................................................................................92
FIGURE 5.16: THE FIRST 2D SUPER-SVA EXPERIMENTAL CONFIGURATION (ANECHOIC CHAMBER) ..........93
FIGURE 5.17: THE SECOND 2D SUPER-SVA EXPERIMENTAL CONFIGURATION (OCCUPIED ROOM) ........94
FIGURE 5.18: AN EXPERIMENTAL INITIAL IMAGE OF A SCATTERING SCENARIO (ON LEFT) CONTRASTED WITH ITS 2D SVA (METHOD 2) WINDOWED IMAGE (ON RIGHT) ..........................................................95
FIGURE 5.19: THE 2D SVA (METHOD 2) WINDOWED IMAGE (ON LEFT) CONTRASTED WITH ITS SUPER-SVA WINDOWED IMAGE (ON RIGHT, ITERATION 1 OF 5) .................................................................96
FIGURE 5.20: THE 2D SVA (METHOD 2) WINDOWED IMAGE (ON LEFT) CONTRASTED WITH ITS SUPER-SVA WINDOWED IMAGE (ON RIGHT, ITERATION 2 OF 5) .................................................................96
FIGURE 5.21: THE 2D SVA (METHOD 2) WINDOWED IMAGE (ON LEFT) CONTRASTED WITH ITS SUPER-SVA WINDOWED IMAGE (ON RIGHT, ITERATION 3 OF 5) .................................................................97
FIGURE 5.22: THE 2D SVA (METHOD 2) WINDOWED IMAGE (ON LEFT) CONTRASTED WITH ITS SUPER-SVA WINDOWED IMAGE (ON RIGHT, ITERATION 4 OF 5) .................................................................97
FIGURE 5.23: THE 2D SVA (METHOD 2) WINDOWED IMAGE (ON LEFT) CONTRASTED WITH ITS SUPER-SVA WINDOWED IMAGE (ON RIGHT, ITERATION 5 OF 5) .................................................................98
FIGURE 5.24: THE INITIAL IMAGE CORRESPONDING TO REAL DATA FOR THE EXTRAPOLATED BW (ON LEFT) CONTRASTED WITH THE 5 ITERATION SUPER-SVA WINDOWED IMAGE (ON RIGHT) .................................98
FIGURE 5.25: THE SVA WINDOWED IMAGE CORRESPONDING TO REAL DATA FOR THE EXTRAPOLATED BW (ON LEFT) CONTRASTED WITH THE 5 ITERATION SUPER-SVA WINDOWED IMAGE (ON RIGHT) ..........98
FIGURE 5.26: A SECOND EXPERIMENTAL INITIAL IMAGE OF A SCATTERING SCENARIO (ON LEFT) CONTRASTED WITH ITS 2D SVA (METHOD 2) WINDOWED IMAGE (ON RIGHT) .................................................100
FIGURE 5.27: THE EXPERIMENTAL 2D SUPER-SVA WINDOWED IMAGE (ITERATION 1 OF 5) .................100
FIGURE 5.28: THE EXPERIMENTAL 2D SUPER-SVA WINDOWED IMAGE (ITERATION 2 OF 5) .................100
FIGURE 5.29: THE EXPERIMENTAL 2D SUPER-SVA WINDOWED IMAGE (ITERATION 3 OF 5) .................101
FIGURE 5.30: THE EXPERIMENTAL 2D SUPER-SVA WINDOWED IMAGE (ITERATION 4 OF 5) .................101
FIGURE 5.31: THE EXPERIMENTAL 2D SUPER-SVA WINDOWED IMAGE (ITERATION 5 OF 5) .................101
FIGURE 5.32: THE INITIAL IMAGE CORRESPONDING TO REAL DATA FOR THE EXTRAPOLATED BW (ON LEFT) CONTRASTED WITH THE 5 ITERATION SUPER-SVA WINDOWED IMAGE (ON RIGHT) .................................102
FIGURE 5.33: THE SVA WINDOWED IMAGE CORRESPONDING TO REAL DATA FOR THE EXTRAPOLATED BW (ON LEFT) CONTRASTED WITH THE 5 ITERATION SUPER-SVA WINDOWED IMAGE (ON RIGHT) ........102
Chapter 1: Introduction

Developers of radar have long sought methods for improving their images. The desire for image enhancement can stem from the use of limited hardware, environmental limiting conditions, and/or insufficient data.

Two desirable qualities in radar formulated images are high resolution and the absence of sidelobes. The signal bandwidth received by the radar can be artificially expanded to improve this first desirable trait. Carefully windowing this signal’s
bandwidth to stress the importance of particular frequencies over others can aid in improving the second desirable trait. In this thesis methods are developed for applying spatially variant apodization (SVA) and Super-SVA (SSVA); the first of which aids in sidelobe reduction and the second actually enhances resolution by expanding bandwidth. Figure 1.2 below illustrates the effect of both of these image enhancing utilities for 1 dimensional down-range images.

Figure 1.2: Illustrated effects of SVA & Super-SVA

1.1 Previous Work

Early apodization, a term used today to describe a technique for adaptive sidelobe reduction, has traditionally made use of frequency windowing functions. These
windows were often only a function of the frequency alone and were applied across the frequency data received by a radar. The shape corresponding to a windowing function is formulated to compromise between sidelobe reduction and the fidelity of scattering objects. Two common windowing functions, the Hann and Hamming windows, utilize the shape of a cosine and a raised cosine respectively.

SVA (spatially variant apodization) has been utilized for quite some time as an effective algorithm for reducing sidelobes introduced into radar reconstructed images without sacrificing resolution. The algorithm was first introduced by Rodney J. Dallaire and Herbert C Stankwiz in a patent published on Sept. 20th 1994 [2]. They introduced the concept of a spatially optimized windowing function formulated around the spatially static cosine windowing function. In SVA, each position within the spatial domain corresponds to its own scaled cosine window. This cosine window is optimized over each spatial position to ideally reduce sidelobes while preserving information pertaining to scattering objects. Hence, the window itself is adaptively defined at each image point.

In a paper introduced by Allen J. Bric [3], Bric compares the spatially static Hann window with SVA. Bric utilizes a unique form of the SVA method, one that relies upon three spatial values within the initial non-apodized image. The spatially scaled window that he utilizes is a raised cosine function. He first performs the comparison upon an 8 scattering point simulation. He then contrasts these methods again by applying them towards inverse synthetic aperture (ISAR) imaging of a BQM-74E Target Drone. His
results demonstrate that SVA performs far better in reducing the sidelobe artifacts within reconstructed images than a static window.

More recently, a patent published on November 11\textsuperscript{th} 1997 by Herbert C. Stankwitz and Michael R. Kosek [4] introduced a significant enhancement upon the SVA algorithm. They termed their hybrid method Super-SVA. In addition to reducing sidelobes inherited through the use of the inverse Fourier transform, these two researchers had found that SVA can be utilized to improve down-range resolution by iteratively expanding bandwidth.

In another paper introduced by Ping Zhang, Jian Shang, Ruliang Yang in April of 2009 [5], the group proposes a technique for utilizing Super-SVA to enhance 1D and 2D images generated from interferometric SAR (INSAR) data. The paper shows that in imaging a simulated point scatterer from INSAR, Super-SVA can be used to reduce sidelobes and enhance down-range and cross-range resolution. An SVA algorithm, similar to that in the Bric paper, is utilized requiring three spatial values in computing the SVA enhanced image.

1.2 Problem Definition

Super-SVA has been shown to be applicable towards 2D problems; offering the capacity to reduce sidelobes and enhance both down-range and cross-range resolution if sufficient angular data is collected. Expanding the initial bandwidth enhances down-range resolution. If frequency data is collected from a wide range of angles with respect to an object being imaged, improved down-range resolution corresponds to improved
cross-range resolution. But what of problems with limited angular aspects? Fixed aperture radar, slow moving radar, and radar used for imaging very distant objects can all benefit from computationally introduced cross-range resolution improvement. In one such scenario a military drone radar (see Figure 1.3) may be limited in look angles due to hostile territory. Furthermore, in such scenarios communication restrictions may limit the uploading of radar data to high performance computers, so rapid image reconstruction may also be desired. The scenario of interest considered here is the through-wall radar imaging problem, which nearly always suffers from limited angular data [6], as shown in Figure 1.4. The material within the 6 comprising chapters of this thesis attempts to address these problems.

Figure 1.3: Unmanned Airborne vehicular radar (Military Drones) [7]
Figure 1.4: A through-wall imaging scenario

Cross-range resolution is inversely proportional to center frequency [8]. This thesis investigates a method by which Super-SVA is used to expand the initial bandwidth for the purpose of raising the center frequency. Multiple iterations of Super-SVA are utilized to further increase bandwidth and explore the capacity of this extrapolated signal to resolve closely spaced scatterers.

Chapter 2 presents the basic SVA approach for 1D down-range images and several techniques for applying 2D SVA are developed and explored in Chapter 3. The derived 2D SVA techniques compromise between generality (higher generality corresponding to additional sidelobe reduction) and computational performance (image development within a reasonable time frame on a general computer). In Sections 3.2 and 3.3 2D SVA techniques are introduced sequentially ranging from most to least general. Compromising between generality and computational performance Method 2 of 3 is later applied towards 2D Super-SVA to assist in the reconstruction of low
distortion images. Chapter 4 introduces the Super-SVA algorithm for 1D down-range images and Chapter 5 develops the novel method for applying SSVA to real 2D images. Conclusions are discussed in Chapter 6.
Chapter 2 : One-Dimensional Spatially Variant Apodization

2.1 Introduction to 1D SVA (Spatially Variant Apodization)

The production of a coherent radar image begins with the acquisition of phase and magnitude data over a band of frequencies. This data is then transformed into a spatial representation through the use of the inverse Fourier transform. Sidelobes are introduced into the reproduced image through the transformation.

The magnitude signal corresponding to a single point scatterer within the frequency domain takes the form of a rectangular pulse over the received bandwidth. Performing the inverse Fourier transformation upon a rectangular pulse corresponds to a sinc function within the spatial domain. This phenomenon is the origin of sidelobes in radar images.

Various techniques for reducing sidelobes have been utilized; this is known as sidelobe apodization. Many such techniques involve the use of windowing functions that reshape the initially received radar signal. Such windowing functions generally taper the upper and lower frequencies of the signal while stressing the middle frequencies. The various window shapes that have been designed, such as the cosine
and triangle based windows, typically compromise between the reduction of sidelobes and the width of the mainlobe. The reduction of the upper and lower frequencies tends to widen the mainlobe, thereby reducing resolution.

Spatially Variant Apodization, or SVA, introduces a spatially variant windowing function. Every spatial position of the reconstructed down-range profile, image, or hologram utilizes a scaled form of a selected windowing function. A scaling value is selected at each point such that sidelobes are optimally reduced while the mainlobe width is not affected. In 1 dimension, the spatial domain values corresponding to the inverse Fourier transformation upon the radar received signal is known as the down-range profile. In this chapter the application of SVA upon down-range profiles will be detailed. Figure 2.1 below illustrates a down-range profile enhanced by the 1D SVA technique that is to be derived in the next section.

![Figure 2.1: The illustrated effects of 1D SVA](image-url)
2.2 Derivation of the 1D SVA Windowing Function

A six step procedure for applying 1D SVA is illustrated below by Figure 2.2.

Initially, for a simulation or experiment it is necessary to specify a frequency bandwidth. Once broadcasted and then received through scatterer reflections, the acquired signal corresponds to the scatterering range. The inverse Fourier transform may be utilized to obtain the complex down-range spatial values corresponding to this scattering range, from which a down-range profile can be plotted. On application of 1D SVA this down-range profile can be enhanced through the reduction of sidelobes. The derivations that follow will provide the tools necessary to apply this procedure.

Figure 2.2: High level block diagram of the 1D SVA procedure
Beginning with Step 1, application of the 1D procedure first requires the selection of an initial bandwidth. This bandwidth is to be utilized within a simulation or physically broadcasted by a radar. Following with step 2, the simulation’s signal values are then computed, or the scattered signal is physically captured by a radar’s antenna.

Moving on to step 3 the Fourier transform is then applied upon the signal data. Suppose that there exists a set of signal data collected from an antenna comprising both magnitude and phase information. The corresponding down-range profile may be obtained as follows:

Let:

\[ k = \text{Wave-number (rad/m)} \]
\[ x = \text{Spatial Displacement (m)} \]
\[ K_{bw} = \text{Initial Wave-number Bandwidth} \]
\[ S(k) = \text{Complex Wave-number Domain Signal} \]
\[ g(x) = \text{Complex Down-Range Profile} \]

Where:

\[ k_1 \leq k \leq k_N \]
\[ -\infty < x < \infty \]
Then:

\[ g(x) = \frac{1}{K_{bw}} \int_{k_1}^{k_N} S(k) e^{j2\pi x} dk \]

Proceeding to step 4, the magnitude of the initial down-range profile \( g(x) \) can now be plotted as illustrated by the upper plot within Figure 2.1. Sometimes background effects and noise can make it difficult to visualize scatterers within the down-range profile. Developing a plot with respect to a dB scale can greatly assist in isolating scatterers from uninteresting artifacts.

Moving on to step 5 a frequency window optimized for each spatial location is derived. This spatially dependent window serves to diminish sidelobes within this initial image all while preserving mainlobes associated with scatterers in the field. SVA commonly makes use of a spatially scaled cosine function as follows:

Let:

\[ f(x) = \text{Complex SVA Enhanced Down-Range Profile} \]

\[ W(k) = \text{Windowing Function} \]

\( \alpha(x) = \text{Spatial Dependent Value with the Range [0, 0.5]} \]

\( k_c = \text{Center Wave-Number} \)

Where:

\[ f(x) = \frac{1}{K_{bw}} \int_{k_1}^{k_N} W(k)S(k) e^{j2\pi x} dk \]
Illustrated by Figure 2.3 is the SVA windowing function $W(k)$ for various values of $\alpha$. The first plot within the figure (upper left plot) pictorializes the rectangular window. The application of a rectangular window corresponds to the preservation of the original signal values $S(k)$. The second plot (upper right) visualizes the SVA term $\alpha(x) \cos \left( \frac{2\pi [k - k_c]}{K_{bw}} \right)$, which is optimized for each spatial position. This window has the form of a single cycle cosine function with an amplitude that ranges from 0 to 1 (for values of $\alpha$ restricted between $[0,0.5]$). The last plot (on bottom) corresponds to the SVA windowing function $W(k)$. The SVA window is normalized such that $\int_{k_1}^{k_N} W(k) \, dk$ is constant for all values of $\alpha$. This can be understood intuitively because the integration of a single cycled cosine function must be zero.
Figure 2.3: The SVA windowing function

Windowed signals and down-range profiles corresponding to a single point scatterer are illustrated below by Figure 2.4. The signal corresponding to a point scatterer is constant in magnitude. This is why the magnitude of the windowed signal appears to match the windowing function for $\alpha = 0$. As $\alpha$ increases, the mainlobe width increases and sidelobe magnitude decreases. For a value of $\alpha = 0$ the SVA windowing function is equivalent to a rectangular window. This preserves the original single that corresponds to the initial un-windowed down-range profile.
Ideally, mainlobe width would be preserved and sidelobes maximally reduced. This cannot be achieved if alpha is constant over the entire down-range profile. However, if alpha is permitted to vary spatially this can be achieved. Next, an expression will be derived for alpha that accomplishes this.

To apply this window $\alpha(x)$ must be computed for each spatial location. Utilizing the information provided up to this point an open form solution for the SVA enhanced down-range profile can be derived in terms of $\alpha(x)$. Following this, an expression for the spatially variant scalar $\alpha(x)$ can be derived through a least squares minimization of the power in the enhanced down-range profile $f(x)$. In the following, $\alpha(x)$ is constrained to be real.

Figure 2.4: 1D SVA (windowed signals & down-range profiles)
Open Form SVA Solution:

\[ f(x) = \frac{1}{K_{bw}} \int_{k_1}^{k_N} W(k)S(k) e^{jk2\pi x} dk \]

\[ = \frac{1}{K_{bw}} \int_{k_1}^{k_N} \left[ 1 + 2\alpha(x) \cos \left( \frac{2\pi [k - k_c]}{K_{bw}} \right) \right] S(k) e^{jk2\pi x} dk \]

\[ = g(x) + \alpha(x) h(x) \]

Where:

\[ h(x) = \frac{2}{K_{bw}} \int_{k_1}^{k_N} S(k) \cos \left( \frac{2\pi (k - k_c)}{K_{bw}} \right) e^{jk2\pi x} dk \]

Least Squares Minimization of \(|f(x)|^2|:

\[ \frac{d}{d\alpha}|f(x)|^2 = 0 \]

Then:

\[ |f(x)|^2 = |g(x) + \alpha(x) h(x)|^2 \]

\[ = [g^*(x) + \alpha(x) h^*(x)] [g(x) + \alpha(x) h(x)] \]

\[ = |g(x)|^2 + \alpha^2(x) |h(x)|^2 + \alpha(x) g^*(x) h(x) + \alpha(x) g(x) h^*(x) \]

\[ = |g(x)|^2 + \alpha^2(x) |h(x)|^2 + 2\alpha(x) Re[g^*(x) h(x)] \]

Taking the Derivative with Respect to \(\alpha\):

\[ \frac{d}{d\alpha}[f(x)]^2 = 2\alpha(x) |h(x)|^2 + 2 Re[g^*(x) h(x)] = 0 \]
\[ \alpha(x) = \frac{-Re[g^*(x)h(x)]}{|h(x)|^2} \]

It is noted that \( \alpha(x) \) could become singular if \( h(x) = 0 \). However, in conventional SVA we constrain \( \alpha(x) \) to the range \([0,0.5]\). This may be written as:

\[ \alpha(x) = 0, \text{ for } \tilde{\alpha}(x) < 0 \]
\[ \alpha(x) = \tilde{\alpha}(x), \text{ for } 0 \leq \tilde{\alpha}(x) \leq 0.5 \]
\[ \alpha(x) = 0.5, \text{ for } 0.5 < \tilde{\alpha}(x) \]

Where:

\[ \tilde{\alpha}(x) = \frac{-Re[g^*(x)h(x)]}{|h(x)|^2} \]

Finally, arriving at step 6, the 1D SVA enhanced down-range profile \( f(x) \) can now be plotted as illustrated by the lower plot within Figure 2.1. In the beginning of this section a high level block diagram was illustrated outlining the 1D SVA procedure. Figure 2.5 illustrates the block diagram once more with the derived 1D SVA equations provided.
2.3 MATLAB Simulated Results

Utilizing the SVA derivations obtained in Section 2.2, a comparison can be made with two commonly implemented spatially static windows known as the Hann and Hamming Windows. These two windowing functions don’t utilize a spatially optimized scaling value, so $\alpha(x)$ is constant for all values of $x$. The mathematical expressions corresponding to these two windowing functions are as defined below. The Hann window corresponds to a scaled cosine function that just touches zero, whereas the Hamming window corresponds to a raised cosine function. Beneficially, the Hann
window provides faster sidelobe roll off. However, the Hamming window provides superior mainlobe shape retention and is optimized to minimize the maximum/nearest sidelobes (9). The Hann and Hamming windows are illustrated by Figure 2.6 below.

![Figure 2.6: The Hann & Hamming windows](image)

Let:

\[ W_{Hann}(k) = \text{Hann Windowing Function} \]

\[ W_{Hamming}(k) = \text{Hamming Windowing Function} \]

\[ f_{Hann}(x) = \text{Complex Hann Reconstructed Down-Range Profile} \]

\[ f_{Hamming}(x) = \text{Complex Hamming Reconstructed Down-Range Profile} \]

Where:

\[ f_{Hann}(x) = \frac{1}{K_{bw}} \int_{k_1}^{k_N} W_{Hann}(k)S(k)e^{jkx} dk \]
Utilizing these definitions, open form solutions for the Hann and Hamming windowed down-range profiles can be derived.

Solving for the Hann Windowed Down-Range Profile:

\[ f_{\text{Hann}}(x) = \frac{1}{K_{bw}} \int_{k_1}^{k_N} W_{\text{Hann}}(k) S(k) e^{jk2x} \, dk \]

\[ = \frac{1}{K_{bw}} \int_{k_1}^{k_N} \left[ 0.5 + 0.5 \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) \right] S(k) e^{jk2x} \, dk \]

\[ = 0.5g(x) + 0.25h(x) \]

Solving for the Hamming Windowed Down-Range Profile:

\[ f_{\text{Hamming}}(x) = \frac{1}{K_{bw}} \int_{k_1}^{k_N} W_{\text{Hamming}}(k) S(k) e^{jk2x} \, dk \]

\[ = \frac{1}{K_{bw}} \int_{k_1}^{k_N} \left[ 0.54 + 0.46 \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) \right] S(k) e^{jk2x} \, dk \]

\[ = 0.54g(x) + 0.23h(x) \]
Point scatterers are commonly utilized to represent objects encountered by radar signals. The signal corresponding to a unit amplitude 1D point scatterer over an infinite bandwidth has the following expression:

Let:

\[ x_{scat} = \text{Point Scatterer Spatial Position} \]

\[ g(x) = \text{Complex Down-Range Profile} \]

Where:

\[ g(x) = \delta(x - x_{scat}) \]

Point Scattering Signal:

\[ S_{scat}(k) = \int_{-\infty}^{\infty} g(x)e^{-jkx}dx = \int_{-\infty}^{\infty} \delta(x - x_{scat})e^{-jkx}dx = e^{-jkx_{scat}} \]

MATLAB has been used to simulate two point scatterer scenarios. The first simulation corresponds to two point scatters that are well separated with respect to down-range resolution. The second simulation corresponds to two point scatterers that are separated by less than the down-range resolution (and hence appear merged). In both simulations a bandwidth of 200 MHz comprised of 200 frequencies ranging from 1 to 1.2 GHz was utilized. The theoretical down-range resolution in both scenarios is 0.75 meters.
2.3.1 Simulation 1: Two Scatterers well separated in Down-Range

Two point scatterers have been separated by 20 meters; one scatterer positioned at 10 meters and another positioned at 30 meters. The down-range profile corresponding from the initial radar signal, without a window applied, has been defined as $g(x)$. The 1D Hann and Hamming windowing functions have been contrasted with the optimized 1D SVA windowing function. The results of this study are illustrated by Figure 2.7.

![Hann Window](image1)

![Hamming Window](image2)

![SVA Window](image3)

Figure 2.7: 1D SVA contrasted with the Hann & Hamming windows
From the illustration of Figure 2.7 it is apparent that SVA, with its spatially optimized parameter $\alpha(x)$, offers superior sidelobe reduction over these two spatially static windowing functions. Also apparent is that SVA provides sidelobe reduction without the disadvantage of widening the mainlobes corresponding to the point scatterers. Illustrated by Figure 2.8 is the spatially varying parameter $\alpha(x)$ for the entire down-range profile.

Figure 2.8: Alpha computed for a 1D SVA simulation
From Figure 2.8 it can be seen that $\alpha(x)$ is smallest within the scatterer mainlobe regions. The SVA windowing function has been provided again below to show why this might occur. When $\alpha(x)$ is zero or very small the windowing function $W(k)$ equals one. When this occurs no windowing function is effectively applied. As a result, when $\alpha(x)$ is very small or zero the original shape of the down-range profile is preserved. Since $\alpha(x)$ approaches zero near the mainlobes within Figure 2.8, mainlobe shape is preserved.

\begin{equation}
W(k) = 1 + 2\alpha(x) \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right)
\end{equation}

1D SVA Windowing Function:

$0 \leq \alpha(x) \leq 0.5$

$0 \leq W(k) \leq 2$

Alternatively, when $\alpha(x)$ is 0.5 or a relatively large value the SVA windowing function is actively scaling the values of the initial signal. This results in the suppression of sidelobes. Upon examining Figure 2.8 it can be seen that $\alpha(x)$ is relatively large outside the mainlobe regions signifying that the SVA windowing function is actively reducing sidelobe levels.

Taking another survey of the SVA windowing function it can be seen that when $\alpha(x)$ equals 0.25 this corresponds to the Hann window. Similarly, when $\alpha(x)$ equals approximately 0.23 the SVA windowing function corresponds to the Hamming window. The value of $\alpha(x)$, as described in Section 2.2, may take on any real value. However, in
practice, the range of $\alpha(x')$ is constrained between [0,0.5]. This practice has been put into effect for all simulations presented by this thesis.

### 2.3.2 Simulation 2: Two Scatterers Merged in Down-Range

Two point scatterers have been separated by 1 meter; one scatterer positioned at 10 meters and another positioned at 11 meters. The down-range profile corresponding from the initial radar signal, without a window applied, has been defined as $g(x)$. The 1D Hann and Hamming windowing functions were once again contrasted with the optimized 1D SVA windowing function. The results of this study are illustrated by Figure 2.9.
From the illustration of Figure 2.9 it is apparent that SVA again provides superior sidelobe reduction for the case of two point scatterers spaced closely in down-range. Additionally, the width of the merged scattering mainlobe has been retained. The application of SVA did not resolve the scatterers merged in down-range, however this can be achieved through the application of Super-SVA which will be discussed further in Chapter 4.
2.4 Experimental Results

Finally, Figure 2.10 illustrates an experimental test configuration from which real radar data had been acquired. The recovered radar signal was comprised of phase and magnitude values corresponding to 101 frequencies ranging from 900 MHz to 2.3 GHz in steps of 14 MHz. A single scattering trihedral had been positioned approximately 1.9 meters in down-range from the radar (about 6 feet). The derived 1D SVA technique was then applied towards the data obtained and the results are illustrated by Figure 2.11.

Figure 2.10: 1D SVA experimental configuration
Upon observation of Figure 2.11 it can be seen that the application of 1D SVA has maintained the integrity of the scattering trihedral’s down range position. Some sidelobes can be seen to be reduced while others still remain within the down-range profile. The remaining sidelobes are not sidelobes, but can be attributed to background clutter. As will be seen in Chapter 4 the application of 1D Super-SVA further assists in the further suppression of sidelobes while also improving upon down range resolution.
Chapter 3: Two-Dimensional Spatially Variant Apodization

3.1 Introduction to 2D SVA

SVA can be and has been utilized for enhancing 2D SAR images. A SAR based aperture may consist of multiple antenna elements in a fixed aperture, or the aperture may be synthetic and comprised of multiple measurement locations of a single antenna element, or the SAR aperture may consist of a combination of these two approaches. In all cases frequency, phase, and magnitude data is collected from multiple spatial positions. Reconstruction of the image once again requires the application of the inverse Fourier transformation upon the measured data allowing for the introduction of sidelobes in both down-range and cross-range.

As in the 1D case, sidelobes can be reduced through the application of a specified windowing function over the frequency domain data. SARs, being comprised of a collection of measurement locations, acquire more than one set of frequency data. A windowing function can be applied across frequency and antenna location.

2D SVA utilizes a spatially varying window that is optimized for each spatial coordinate. Every spatial position of the reconstructed SAR image utilizes a scaled form
of the selected windowing function. A scaling value is selected such that sidelobes are optimally reduced while maintaining scatterer width/shape integrity. Figure 3.1 below illustrates an image of two point scatterers enhanced by the 2D SVA technique that is to be derived in the next section.

![Initial Image vs SVA Image](image)

**Figure 3.1:** Illustrated effects of 2D SVA

### 3.2 Derivation of the 2D SVA Windowing Function

A six step procedure for applying 2D SVA is illustrated below by Figure 3.2.

Initially, for a simulation or experiment it is necessary to specify a frequency bandwidth. Once broadcasted and then received through scatterer reflections the acquired signal corresponds to the scatterering field. Typically, when producing images as opposed to down-range profiles, multiple signals are collected from different locations in 2D or 3D
space, denoted in the following by the index \( m \). The inverse Fourier transform may be utilized to obtain the initial complex spatial values for each antenna location, from which an image corresponding to this scattering field can be generated by a coherent sum. On application of 2D SVA this spatial image can be enhanced through the reduction of sidelobes. The derivations that follow will provide the tools necessary to apply this 2D procedure.

**2D SVA Procedure**

1. Specify an Initial Bandwidth
2. Compute or Capture The Corresponding Signals \( S(m,k) \)
3. Perform Inv-Fourier Transform to Obtain The Initial Image values \( g(x,y) \)
4. Plot the Initial Image \( \text{abs}(g(x,y)) \)
5. Plot the 2D SVA Enhanced Image \( \text{abs}(f(x,y)) \)
6. Apply the optimized Frequency & element windows \( W(k) & W(m) \)

![Figure 3.2: High level block diagram of the 2D SVA procedure](image)

Beginning with Step 1, application of the 2D procedure first requires the selection of an initial bandwidth. This bandwidth is to be utilized within a simulation or physically broadcasted by one or more antennas from multiple locations. Following
with step 2, the simulation’s signal values are then computed, or the scattered signal(s) are physically received by a radar.

Moving on to step 3 the inverse Fourier transform is then applied upon the signals received. Suppose that there exists a set of signal data collected from more than one antenna comprising both magnitude and phase information. The corresponding back projection spatial image may be obtained as follows:

Let:

\[ k = \text{Wave-number (rad/m)} \]
\[ m = \text{The Antenna Element} \]
\[ x, y = \text{Spatial Displacements (m)} \]
\[ K_{bw} = \text{Wave-number Bandwidth} \]
\[ M = \text{Number of SAR antenna elements} \]
\[ S(m, k) = \text{Wave-number Domain Signal for each Antenna} \]
\[ g(m, x, y) = \text{Complex Spatial Domain Image for each Antenna} \]
\[ g(x, y) = \text{Complex Spatial Domain Image for the SAR Aperture} \]
\[ \vec{r}^1(x, y) = \text{Vector Mapping of } x, y \text{ in meters} \]
\[ \vec{r}^2[m] = \text{Vector Antenna Position in meters} \]

Where:

\[ k_1 \leq k \leq k_N \]
\[ 1 \leq m \leq M \]
Then:

\[
g(m, x, y) = \frac{1}{K_{bw}} \int_{k_1}^{k_N} S(m, k) e^{i k_2 |\mathbf{r}_2[m]| - i \mathbf{r}_1(x,y)} \, dk
\]

\[
g(x, y) = \frac{1}{M} \sum_{m=1}^{M} g(m, x, y)
\]

Proceeding to step 4, the initial image \( g(x, y) \) can now be plotted as illustrated by the left plot within Figure 3.1. The values obtained for \( g(x, y) \) can be complex. One approach to plotting complex spatial values is to utilize their absolute magnitude. Furthermore, large dynamic range can make it difficult to visualize scatterers within the image. Developing a plot with respect to a dB scale can greatly assist in isolating scatterers from uninteresting artifacts such as sidelobes.

Moving on to step 5, 2D SVA may be achieved through the application of two independent spatially variant windowing functions. Down-range sidelobe reduction may be obtained through the use of a spatially variant window over the data in the frequency domain. Cross-range sidelobe reduction may be obtained through the use of a spatially variant window over the antenna elements (by array theory analogy). In the following, a linear array of \( M \) equally spaced elements is assumed.
3.2.1 Method 1: General 2D SVA Method

Let:

\[ f(m, x, y) = \text{Complex Spatial Domain SVA Enhanced Image for each Antenna} \]

\[ f(x, y) = \text{Complex Spatial Domain 2D SVA Enhanced Image for the SAR Aperture} \]

\[ W(m, k) = \text{Signal Data Windowing Function} \]

\[ W[m] = \text{Antenna Element Windowing Function} \]

\[ \alpha(m, x, y) = \text{Spatial/Element Dependent Value with the Range } [0, 0.5] \]

\[ \beta(x, y) = \text{Spatial Dependent Value with the Range } [0, 0.5] \]

\[ k_c = \text{Center Wave-Number} \]

Where:

\[ k_c = \frac{1}{2} (k_N + k_1) \]

\[ W(m, k) = 1 + 2\alpha(m, x, y) \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) \]

\[ W[m] = 1 + 2\beta(x, y) \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) \]

\[ f(m, x, y) = \frac{1}{K_{bw}} \int_{k_1}^{k_N} W(m, k)S(m, k)e^{jk2|f2[m] - f1(x, y)|}dk \]

\[ f(x, y) = \frac{1}{M} \sum_{m=1}^{M} W[m]f(m, x, y) \]

Utilizing this information an open form solution for the 2D SVA enhanced image can be derived in terms of \( \alpha(m, x, y) \) and \( \beta(x, y) \). Following this, expressions for the
spatially variant scalars $\alpha(m, x, y)$ and $\beta(x, y)$ can be derived through a least squares minimization of the power in the enhanced image $|f(x, y)|^2$. In the following, both $\alpha(m, x, y)$ and $\beta(x, y)$ are continuous functions in terms of $x$ and $y$ whose optimized computed values may lie outside the range of $[0, 0.5]$. Again, in practice we constrain them to this range.

Open Form 2D SVA Solution:

$$f(x, y) = \frac{1}{M} \sum_{m=1}^{M} \frac{W[m]}{K_{bw}} \int_{k_1}^{k_N} W(m, k)S(m, k)e^{jk2[\bar{r}^2[m]-\bar{r}T(x,y)]} dk$$

$$= \frac{1}{M} \sum_{m=1}^{M} \frac{W[m]}{K_{bw}} \int_{k_1}^{k_N} \left[ 1 + 2\alpha(m, x, y) \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) \right] S(m, k)e^{jk2[\bar{r}^2[m]-\bar{r}T(x,y)]} dk$$

$$= \frac{1}{M} \sum_{m=1}^{M} W[m] \left[ g(m, x, y) + 2\alpha(m, x, y) \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k)e^{jk2[\bar{r}^2[m]-\bar{r}T(x,y)]} dk \right]$$

$$= \frac{1}{M} \sum_{m=1}^{M} \left[ 1 + 2\beta(x, y) \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) \right] g(m, x, y) + 2\alpha(m, x, y) \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k)e^{jk2[\bar{r}^2[m]-\bar{r}T(x,y)]} dk$$
\[
\begin{align*}
&= \frac{1}{M} \sum_{m=1}^{M} g(m, x, y) \\
&\quad + \frac{2}{MK_{bw}} \sum_{m=1}^{M} \alpha(m, x, y) \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{jk2[\bar{r}\bar{z}[m] - \bar{r}1(x,y)]]} \, dk \\
&\quad + \frac{2\beta(x, y)}{M} \sum_{m=1}^{M} \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) g(m, x, y) \\
&\quad + \frac{4\beta(x, y)}{MK_{bw}} \sum_{m=1}^{M} \alpha(m, x, y) \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{jk2[\bar{r}\bar{z}[m] - \bar{r}1(x,y)]]} \, dk
\end{align*}
\]

Let:
\[
\begin{align*}
\hspace{1cm} h_1(m, x, y) &= \frac{2}{K_{bw}} \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{jk2[\bar{r}\bar{z}[m] - \bar{r}1(x,y)]]} \, dk \\
\hspace{1cm} h_2(m, x, y) &= 2 \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) g(m, x, y) \\
\hspace{1cm} h_3(m, x, y) &= \frac{4}{K_{bw}} \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{jk2[\bar{r}\bar{z}[m] - \bar{r}1(x,y)]]} \, dk
\end{align*}
\]

Further Let:
\[
\hspace{1cm} h_2(x, y) = \frac{1}{M} \sum_{m=1}^{M} h_2(m, x, y)
\]

Then:
\[
\begin{align*}
f(m, x, y) &= g(m, x, y) + \alpha(m, x, y)h_1(m, x, y) + \beta(x, y)h_2(m, x, y) \\
&\quad + \alpha(m, x, y)\beta(x, y)h_3(m, x, y)
\end{align*}
\]
\[ f(x, y) = g(x, y) + \frac{1}{M} \sum_{m=1}^{M} \alpha(m, x, y)h_1(m, x, y) + \beta(x, y)h_2(x, y) + \frac{\beta(x, y)}{M} \sum_{m=1}^{M} \alpha(m, x, y)h_3(m, x, y) \]

Least Squares Minimization of \( f(x, y) \):

\[
\frac{d}{d\alpha} \frac{d}{d\beta} |f(x, y)|^2 = 0
\]

Where:

\[
|f(x, y)|^2 = \left| g(x, y) + \frac{1}{M} \sum_{m=1}^{M} \alpha(m, x, y)h_1(m, x, y) + \beta(x, y)h_2(x, y) + \frac{\beta(x, y)}{M} \sum_{m=1}^{M} \alpha(m, x, y)h_3(m, x, y) \right|^2
\]

Both \( \alpha \) and \( \beta \) may be solved for through the use of a numerical search method. This can be achieved through minimizing \( |f(x, y)|^2 \) with respect to both alpha and beta simultaneously. Unfortunately, these 2D SVA solutions are very computationally intensive. Beta must be solved for numerically for each 2D spatial location, but the values of alpha must also be solved for each antenna element. To improve performance \( \alpha \) can be simplified to depend upon only \( x \) and \( y \). This requires that alpha and beta be solved for numerically only for each 2D spatial location.
3.2.2 Method 2: Simplified 2D SVA Method

Let:

\[ \alpha(x, y) = \text{Spatial Dependent Value with the Range } [0, 0.5] \]

\[ \beta(x, y) = \text{Spatial Dependent Value with the Range } [0, 0.5] \]

\[ W(k) = 1 + 2\alpha(x, y) \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) \]

\[ W[m] = 1 + 2\beta(x, y) \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) \]

Then:

\[ f(x, y) = \frac{1}{M} \sum_{m=1}^{M} \frac{W[m]}{K_{bw}} \int_{k_1}^{k_N} W(k) S(m, k) e^{jk2[\overline{r}_2[m] - \overline{r}_1(x,y)]} dk \]

\[ = \frac{1}{M} \sum_{m=1}^{M} g(m, x, y) \]

\[ + \frac{2\alpha(x, y)}{MK_{bw}} \sum_{m=1}^{M} \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{jk2[\overline{r}_2[m] - \overline{r}_1(x,y)]} dk \]

\[ + \frac{2\beta(x, y)}{M} \sum_{m=1}^{M} \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) g(m, x, y) \]

\[ + \frac{4\alpha(x, y)\beta(x, y)}{MK_{bw}} \sum_{m=1}^{M} \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{jk2[\overline{r}_2[m] - \overline{r}_1(x,y)]} dk \]

Let:

\[ h_1(x, y) = \frac{2}{MK_{bw}} \sum_{m=1}^{M} \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{jk2[\overline{r}_2[m] - \overline{r}_1(x,y)]} dk \]
Then:

\[ f(x, y) = g(x, y) + \alpha(x, y)h_1(x, y) + \beta(x, y)h_2(x, y) + \alpha(x, y)\beta(x, y)h_3(x, y) \]

In this simplified solution for \( f(x, y) \), the functions \( h_1(x, y) \) and \( h_3(x, y) \) have been separated from \( \alpha \). Utilizing the least squares method, non-linear expressions for both \( \alpha \) and \( \beta \) may be obtained. Through these expressions the function \( f(x, y) \) may be solved for numerically and independent from \( m \).

**Least Squares Minimization of \( f(x, y) \):**

\[
|f(x, y)|^2 = |g(x, y) + \alpha(x, y)h_1(x, y) + \beta(x, y)h_2(x, y) + \alpha(x, y)\beta(x, y)h_3(x, y)|^2 \\
= |g + \alpha h_1 + \beta h_2 + \alpha \beta h_3|^2 \\
= \langle g + \alpha h_1 + \beta h_2 + \alpha \beta h_3 | g + \alpha h_1 + \beta h_2 + \alpha \beta h_3 \rangle \\
= (|g|^2 + \alpha^2|h_1|^2 + \beta^2|h_2|^2 + \alpha^2 \beta^2|h_3|^2) + \alpha(g^*h_1 + gh_1^*) + \beta(g^*h_2 + gh_2^*) \\
+ \alpha \beta(g^*h_3 + gh_3^*) + \alpha^2 \beta^2(h_1^*h_3 + h_1 h_3^*) + \alpha \beta^2(h_2^*h_3 + h_2 h_3^*) \\
= (|g|^2 + \alpha^2|h_1|^2 + \beta^2|h_2|^2 + \alpha^2 \beta^2|h_3|^2) + 2\alpha Re(g^*h_1) + 2\beta Re(g^*h_2) \\
+ 2\alpha \beta Re(g^*h_3) + 2\alpha^2 \beta Re(h_1^*h_3) + 2\alpha \beta^2 Re(h_2^*h_3)
Taking the Derivative with Respect to $\alpha$:

$$\frac{\partial}{\partial \alpha} |f(x,y)|^2 = 0$$

$$= 2\alpha|h_1|^2 + 2\alpha^2|h_3|^2 + 2Re(g^*h_1) + 2\beta Re(g^*h_3) + 4\alpha\beta Re(h_1^*h_3) + 2\beta^2 Re(h_2^*h_3)$$

$$\alpha = -\frac{2Re(g^*h_1) + 2\beta Re(g^*h_3) + 2\beta^2 Re(h_2^*h_3)}{2|h_1|^2 + 2\beta^2|h_3|^2 + 4\beta Re(h_1^*h_3)}$$

Taking the Derivative with Respect to $\beta$:

$$\frac{\partial}{\partial \beta} |f(x,y)|^2 = 0$$

$$= 2\beta|h_2|^2 + 2\alpha^2|h_3|^2 + 2Re(g^*h_2) + 2\alpha Re(g^*h_3) + 2\alpha^2 Re(h_1^*h_3) + 4\alpha\beta Re(h_2^*h_3)$$

$$\beta = -\frac{2Re(g^*h_2) + 2\alpha Re(g^*h_3) + 2\alpha^2 Re(h_1^*h_3)}{2|h_2|^2 + 2\alpha^2|h_3|^2 + 4\alpha Re(h_2^*h_3)}$$

Where:

$$\alpha = -\frac{2Re(g^*h_1) + 2\beta Re(g^*h_3) + 2\beta^2 Re(h_2^*h_3)}{2|h_1|^2 + 2\beta^2|h_3|^2 + 4\beta Re(h_1^*h_3)}$$

$$\beta = -\frac{2Re(g^*h_2) + 2\alpha Re(g^*h_3) + 2\alpha^2 Re(h_1^*h_3)}{2|h_2|^2 + 2\alpha^2|h_3|^2 + 4\alpha Re(h_2^*h_3)}$$

$\alpha$ and $\beta$ may be solved using the above coupled non-linear equations, and then constraining $\alpha$ and $\beta$ to the range $[0,0.5]$. However, in practice it is easier to simply search over the space $0 \leq \alpha \leq 0.5$ and $0 \leq \beta \leq 0.5$ for the minimum of $|f(x,y)|^2$.

Finally arriving at step 6, the 2D SVA enhanced image $f(x,y)$ can now be plotted as illustrated by the right plot within Figure 3.1. In the beginning of this section a high
level block diagram was illustrated outlining the 2D SVA procedure. Figure 3.3 illustrates the block diagram once more with the derived 2D SVA equations provided.

\[
W(k) = 1 + 2\alpha(x,y) \cos \left( \frac{2\pi[k - k_0]}{K_{bw}} \right)
\]

\[
W[m] = 1 + 2\beta(x,y) \cos \left( \frac{2\pi[m - 1]}{M - 1} \right)
\]

*Where:*

\[
f(x,y) \text{ is 2D SVA technique dependent}
\]

\[
g(m, x, y) = \frac{1}{K_{bw} \cdot k_0} \int_{K_1}^{K_N} S(m, k) e^{j2\pi[f(m) - f(x,y)]} dk
\]

\[
g(x,y) = \frac{1}{M} \sum_{m=1}^{M} g(m, x, y)
\]

**Figure 3.3: Low level block diagram of the 2D SVA procedure**

Fortunately, a faster technique can be developed to perform 2D SVA. The 2D SVA solution can be decomposed into two steps; the first step involving the application of 1D SVA in down-range, and following this with the application of 1D SVA in cross-range. As will be explored throughout the remainder of this chapter, this can be accomplished and with the added benefit of higher computational performance.
However, a loss in generality makes this approach less powerful with respect to sidelobe reduction.

3.3 Derivation of a Two-Step 2D SVA Procedure

Alternatively, the 2D SVA method can be simplified through a two step procedure. First, a spatially variant window is applied over the signal data corresponding to each antenna. Next, a spatially variant window is applied over the aperture. This is equivalent to first applying 1D SVA in down-range, and then following this with the application of 1D SVA in cross-range. This technique is less general than that previously explored. The initial values computed for alpha predetermine the values computed for beta. The general 2D SVA solution accounts for alpha and beta simultaneously. Step one of the simplified procedure shall be derived first.

3.3.1 Method 3: Step 1, 1D SVA in Down-Range

Suppose that there exists a set of signal data collected from more than one antenna comprising both magnitude and phase information. The corresponding spatial image may be obtained as follows:

Let:

\[ k = \text{Wave-number (rad/m)} \]
\[ m = \text{The Antenna Element} \]
\[ x, y = \text{Spatial Displacements (m)} \]
\[ K_{bw} = \text{Wave-number Bandwidth} \]
\[ M = \text{Number of SAR antenna elements} \]
\[ S(m, k) = \text{Wave-number Domain Signal for each Antenna} \]
\[ g(m, x, y) = \text{Complex Spatial Domain Image for each Antenna} \]
\[ \vec{r}1(x, y) = \text{Vector Mapping of } x, y \text{ in meters} \]
\[ \vec{r}2[m] = \text{Vector Aperture Position in meters} \]

Where:
\[ k_1 \leq k \leq k_N \]
\[ 1 \leq m \leq M \]
\[ -\infty < x < \infty \]
\[ -\infty < y < \infty \]

Then:
\[ g(m, x, y) = \frac{1}{K_{bw}} \int_{k_1}^{k_N} S(m, k) e^{jk|\vec{r}2[m|-\vec{r}1(x,y)|} dk \]

A 1D spatially variant window is then applied upon the signal data corresponding to each antenna element. This provides an image optimized in down-range, with reduced sidelobes and highlighted mainlobes, for each antenna element.

Let:
\[ \alpha(m, x, y) = \text{Spatial/Element Dependent Value with the Range } [0, 0.5] \]
\[ W(m, k) = \text{Signal Data Windowing Function} \]

\[ f_{dr}(m, x, y) = \text{Complex Spatial Domain Images 1D SVA Windowed in Down-Range} \]

\[ f_{dr}(x, y) = \text{Complex Spatial Domain Aperture Image 1D SVA Windowed in Down-Range} \]

Where:

\[ W(k) = 1 + 2\alpha(m, x, y) \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) \]

\[ f_{dr}(m, x, y) = \frac{1}{K_{bw}} \int_{k_1}^{k_N} W(m, k) S(m, k) e^{j k_2 [\rho_2[m] - \Phi_T(x, y)]} dk \]

\[ f_{dr}(x, y) = \frac{1}{M} \sum_{m=1}^{M} f_{dr}(m, x, y) \]

Open Form SVA Solution:

\[ f_{dr}(m, x, y) = \frac{1}{K_{bw}} \int_{k_1}^{k_N} W(m, k) S(m, k) e^{j k_2 [\rho_2[m] - \Phi_T(x, y)]} dk \]

\[ = \frac{1}{K_{bw}} \int_{k_1}^{k_N} \left[ 1 + 2\alpha(m, x, y) \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) \right] S(m, k) e^{j k_2 [\rho_2[m] - \Phi_T(x, y)]} dk \]

\[ = g(m, x, y) + \frac{2\alpha(m, x, y)}{K_{bw}} \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{j k_2 [\rho_2[m] - \Phi_T(x, y)]} dk \]

Let:

\[ h_{dr}(m, x, y) = \frac{2}{K_{bw}} \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{j k_2 [\rho_2[m] - \Phi_T(x, y)]} dk \]
Least Squares Minimization of \( f_{dr}(m, x, y) \):

\[
|f_{dr}(m, x, y)|^2 = |g(m, x, y) + \alpha(m, x, y)h_{dr}(m, x, y)|^2 = 0
\]

\[
= [g^*(m, x, y) + \alpha(m, x, y)h^*_{dr}(m, x, y)][g(m, x, y) + \alpha(m, x, y)h_{dr}(m, x, y)]
\]

\[
= |g(m, x, y)|^2 + \alpha(m, x, y)^2|h_{dr}(m, x, y)|^2 + \alpha(m, x, y)g^*(m, x, y)h^*_{dr}(m, x, y)
\]

\[
+ \alpha(m, x, y)g(m, x, y)h^*_{dr}(m, x, y)
\]

\[
= |g(m, x, y)|^2 + \alpha(m, x, y)^2|h_{dr}(m, x, y)|^2 + 2\alpha(m, x, y)\text{Re}[g^*(m, x, y)h_{dr}(m, x, y)]
\]

Taking the Derivative with Respect to \( \alpha \):

\[
\frac{d}{d\alpha}|f_{dr}(m, x, y)|^2 = 0
\]

\[
= 2\alpha(m, x, y)|h_{dr}(m, x, y)|^2 + 2\text{Re}[g^*(m, x, y)h_{dr}(m, x, y)]
\]

\[
\alpha(m, x, y) = \frac{-\text{Re}[g^*(m, x, y)h_{dr}(m, x, y)]}{|h_{dr}(m, x, y)|^2}
\]

3.3.2 Method 3: Step 2, 1D SVA in Cross-Range

A 1D spatially variant window is then applied over the aperture. This provides an image enhanced both in down-range and cross-range. However, as previously propounded, this final computed image is not necessarily optimal.

Let:

\( \beta(x, y) = \text{Spatial Dependent Value with the Range [0, 0.5]} \)

\( W[m] = \text{Aperture Windowing Function} \)

\( f(x, y) = \text{Complex Spatial Domain Image SVA Enhanced in Down-Range & Cross-Range} \)
Where:

\[ W[m] = 1 + 2\beta(x, y) \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) \]

\[ f(x, y) = \frac{1}{M} \sum_{m=1}^{M} W[m] f_{dr}(m, x, y) \]

Open Form SVA Solution (Down-Range & Cross-Range):

\[ f(x, y) = \frac{1}{M} \sum_{m=1}^{M} W[m] f_{dr}(m, x, y) \]

\[ = \frac{1}{M} \sum_{m=1}^{M} \left[ 1 + 2\beta(x, y) \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) \right] f_{dr}(m, x, y) \]

\[ = \frac{1}{M} \sum_{m=1}^{M} f_{dr}(m, x, y) + \frac{2\beta(x, y)}{M} \sum_{m=1}^{M} \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) f_{dr}(m, x, y) \]

\[ = f_{dr}(x, y) + \frac{2\beta(x, y)}{M} \sum_{m=1}^{M} \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) f_{dr}(m, x, y) \]

Let:

\[ h_1(x, y) = \frac{2}{M} \sum_{m=1}^{M} \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) f_{dr}(m, x, y) \]

Least Squares Minimization of \( f(x, y) \):

\[ |f(x, y)|^2 = |f_{dr}(x, y) + \beta(x, y) h_1(x, y)|^2 = 0 \]

\[ = |f_{dr}^*(x, y) + \beta(x, y) h_1^*(x, y)||f_{dr}(x, y) + \beta(x, y) h_1(x, y)| \]

\[ = |f_{dr}(x, y)|^2 + \beta(x, y)^2 |h_1(x, y)|^2 + \beta(x, y) f_{dr}^*(x, y) h_1(x, y) \]

\[ + \beta(x, y) f_{dr}(x, y) h_1^*(x, y) \]
\[ f_{dr}(x, y) = |f_{dr}(x, y)|^2 + \beta(x, y)^2|h_1(x, y)|^2 + 2\beta(x, y)Re[f_{dr}^*(x, y)h_1(x, y)] \]

Taking the Derivative with Respect to \( \beta \):

\[
\frac{d}{d\beta} [f_{cr}(x, y)]^2 = 2\beta(x, y)|h_1(x, y)|^2 + 2Re[f_{dr}^*(x, y)h_1(x, y)] = 0
\]

\[ \beta(x, y) = \frac{-Re[f_{dr}^*(x, y)h_1(x, y)]}{|h_1(x, y)|^2} \]

### 3.4 MATLAB Simulated Results

Utilizing the simplified 2D SVA methods, derived in Sections 2 and 3 of this chapter, a comparison can be made with the two commonly implemented spatially static windows known as the Hann and Hamming Windows. These two windowing functions do not utilize a spatial/element optimized scaling value; \( \alpha \) and \( \beta \) are constant for all values of \( x, y \), and \( m \). These windowing functions are mathematically defined as shown below.

Let:

\[ W_{\text{Hann}}(k)W_{\text{Hann}}[m] = 2D \text{ Hann Windowing Function} \]

\[ W_{\text{Hamming}}(k)W_{\text{Hamming}}[m] = 2D \text{ Hamming Windowing Function} \]

\[ f_{\text{Hann}}(x, y) = \text{Complex Hann Reconstructed Spatial Domain Image} \]

\[ f_{\text{Hamming}}(x, y) = \text{Complex Hamming Reconstructed Spatial Domain Image} \]
Where:

\[ W_{Hann}(k) = 0.5 + 0.5 \cos \left( \frac{2\pi [k - k_c]}{K_{bw}} \right) \]

\[ W_{Hamming}(k) = 0.54 + 0.46 \cos \left( \frac{2\pi [k - k_c]}{K_{bw}} \right) \]

\[ W_{Hann}[m] = 0.5 + 0.5 \cos \left( \frac{2\pi [m - 1]}{M - 1} \right) \]

\[ W_{Hamming}[m] = 0.54 + 0.46 \cos \left( \frac{2\pi [m - 1]}{M - 1} \right) \]

\[
\begin{align*}
    f_{Hann}(x, y) &= \frac{1}{K_{bw}M} \sum_{m=1}^{M} W_{Hann}[m] \int_{k_1}^{k_N} W_{Hann}(k)S(m, k)e^{ik2[\tau_2[m] - \tau_1(x, y)]} \, dk \\
    f_{Hamming}(x, y) &= \frac{1}{K_{bw}M} \sum_{m=1}^{M} W_{Hamming}[m] \int_{k_1}^{k_N} W_{Hamming}(k)S(m, k)e^{ik2[\tau_2[m] - \tau_1(x, y)]} \, dk
\end{align*}
\]

The Hann and Hamming windows, \( W_{Hann}(k) \) and \( W_{Hamming}(k) \), are equivalent to those utilized in Section 2.3 and were illustrated by Figure 2.6. Similarly, the windows \( W_{Hann}[m] \) and \( W_{Hamming}[m] \) conform to the same shape, but for discrete as opposed to continuous values. Each signal, or antenna element by radar analogy, receives its own weighted value. Utilizing these definitions open form solutions for the 2D Hann and Hamming windowed images can be derived.

Solving for the 2D Hann Windowed Image:

\[
\begin{align*}
    f_{Hann}(x, y) &= \frac{1}{K_{bw}M} \sum_{m=1}^{M} W_{Hann}[m] \int_{k_1}^{k_N} W_{Hann}(k)S(m, k)e^{ik2[\tau_2[m] - \tau_1(x, y)]} \, dk
\end{align*}
\]
\[
\frac{1}{K_{bw}M} \sum_{m=1}^{M} W_{Hann}[m] \int_{k_1}^{k_N} \left[ 0.5 + 0.5 \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) \right] S(m, k) e^{j2\pi[m-k] - \tilde{\gamma}(x,y)} dk
\]

\[
= 0.5 \sum_{m=1}^{M} W_{Hann}[m] g(m, x, y)
\]

\[
+ \frac{0.5}{K_{bw}M} \sum_{m=1}^{M} W_{Hann}[m] \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{j2\pi[m-k] - \tilde{\gamma}(x,y)} dk
\]

\[
= 0.5 \sum_{m=1}^{M} \left[ 0.5 + 0.5 \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) \right] g(m, x, y)
\]

\[
+ \frac{0.5}{K_{bw}M} \sum_{m=1}^{M} \left[ 0.5 \right]
\]

\[
+ 0.5 \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{j2\pi[m-k] - \tilde{\gamma}(x,y)} dk
\]

\[
= 0.25 g(x, y)
\]

\[
+ \frac{0.25}{M} \sum_{m=1}^{M} \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) g(m, x, y)
\]

\[
+ \frac{0.25}{K_{bw}M} \sum_{m=1}^{M} \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{j2\pi[m-k] - \tilde{\gamma}(x,y)} dk
\]

\[
+ \frac{0.25}{K_{bw}M} \sum_{m=1}^{M} \cos \left( \frac{2\pi[m - 1]}{M - 1} \right) \int_{k_1}^{k_N} \cos \left( \frac{2\pi[k - k_c]}{K_{bw}} \right) S(m, k) e^{j2\pi[m-k] - \tilde{\gamma}(x,y)} dk
\]

Solving for the 2D Hamming Windowed Image:

\[
f_{Hamming}(x, y)
\]

\[
= \frac{1}{K_{bw}M} \sum_{m=1}^{M} W_{Hamming}[m] \int_{k_1}^{k_N} W_{Hamming}(k) S(m, k) e^{j2\pi[m-k] - \tilde{\gamma}(x,y)} dk
\]
\[
= \frac{1}{K_{bw} M} \sum_{m=1}^{M} W_{\text{Hamming}}[m] \int_{k_1}^{k_N} \left[ 0.54 + 0.46 \cos \left( \frac{2\pi [k - k_c]}{K_{bw}} \right) \right] S(m, k) e^{ik_2 \overline{r}[2][m] - \overline{r}(x,y)} \, dk
\]

\[
= \frac{0.54}{M} \sum_{m=1}^{M} W_{\text{Hamming}}[m] g(m, x, y)
\]

\[
+ \frac{0.46}{K_{bw} M} \sum_{m=1}^{M} W_{\text{Hamming}}[m] \int_{k_1}^{k_N} \cos \left( \frac{2\pi [k - k_c]}{K_{bw}} \right) S(m, k) e^{ik_2 \overline{r}[2][m] - \overline{r}(x,y)} \, dk
\]

\[
= \frac{0.54}{M} \sum_{m=1}^{M} \left[ 0.54 + 0.46 \cos \left( \frac{2\pi [m-1]}{M-1} \right) \right] g(m, x, y)
\]

\[
+ \frac{0.46}{K_{bw} M} \sum_{m=1}^{M} \left[ 0.54 + 0.46 \cos \left( \frac{2\pi [m-1]}{M-1} \right) \right] \int_{k_1}^{k_N} \cos \left( \frac{2\pi [k - k_c]}{K_{bw}} \right) S(m, k) e^{ik_2 \overline{r}[2][m] - \overline{r}(x,y)} \, dk
\]

\[
= 0.2916 g(x,y)
\]

\[
+ \frac{0.2484}{M} \sum_{m=1}^{M} \cos \left( \frac{2\pi [m-1]}{M-1} \right) g(m, x, y)
\]

\[
+ \frac{0.2484}{K_{bw} M} \sum_{m=1}^{M} \int_{k_1}^{k_N} \cos \left( \frac{2\pi [k - k_c]}{K_{bw}} \right) S(m, k) e^{ik_2 \overline{r}[2][m] - \overline{r}(x,y)} \, dk
\]

\[
+ \frac{0.2116}{K_{bw} M} \sum_{m=1}^{M} \cos \left( \frac{2\pi [m-1]}{M-1} \right) \int_{k_1}^{k_N} \cos \left( \frac{2\pi [k - k_c]}{K_{bw}} \right) S(m, k) e^{ik_2 \overline{r}[2][m] - \overline{r}(x,y)} \, dk
\]

Point scatterers are commonly utilized to represent objects encountered by radar signals. The signal corresponding to a 2D point scatterer has the following expression:
Let:

\[ \vec{r}_{\text{scat}} = \text{Point Scatterer Spatial Position} \]

\[ g(x, y) = \text{Complex Spatial Domain Image for the Aperture} \]

Where:

\[ g(x, y) = \delta(\vec{r}\,\vec{T} - \vec{r}_{\text{scat}}) \]

Point Scattering Signal:

\[ S_{\text{scat}}(m, k) = \int_{-\infty}^{\infty} g(x, y) e^{j k z |\vec{r}_2[m] - \vec{r}_1(x, y)|} \, dr_1 \]

\[ = \int_{-\infty}^{\infty} \delta(r_1 - r_{\text{scat}}) e^{j k z |\vec{r}_2[m] - \vec{r}_1(x, y)|} \, dr_1 \]

\[ = e^{-j k z |\vec{r}_{\text{scat}} - \vec{r}_2[m]|} \]

MATLAB has been used to simulate two point scatterers in 2D space. A spatial image corresponding to a scattering field is known as the 2D reconstruction. Figure 3.4 illustrates the reconstructed image corresponding to the simulated point scatterers. These scatterers are separated by a distance of 1.5 feet and are located 9 feet from a simulated SAR aperture. This provides a narrow margin with respect to the theoretical cross-range resolution, approximately 1.3 feet, for the simulation configuration.

For this reconstruction, 37 antenna elements spaced 3 inches apart were used. Each antenna element is aligned along the x-axis and the center of the aperture has been set at 6 feet. Antenna element spacing is less than one-half the minimum
wavelength broadcasted, and total aperture width is approximately 9 feet.

Furthermore, the SAR broadcasted signal consisted of 70 test frequencies ranging from 500 MHz to 1 GHz in steps of 7.14 MHz.

![Figure 3.4: A simulated initial image](image)

The both the 2D Hann and Hamming windowing functions have been contrasted with the generalized 2D SVA windowing function. The results corresponding to this study are illustrated by Figure 3.5 through Figure 3.8.

![Figure 3.5: The simulated 2D Hann windowed image](image)
From the illustrations of Figure 3.5 through Figure 3.8 it is apparent that 2D SVA, with its spatial/element optimized parameters $\alpha$ and $\beta$, offers superior sidelobe
reduction over these two spatially static windowing functions. Also apparent is that SVA provides sidelobe reduction without adversely widening the mainlobe corresponding to the point scatterers. It can be seen from both the Hann and Hamming windowed reconstructions that the two isolated scatterers have somewhat melded together as a result of mainlobe widening.

From Figure 3.7 and Figure 3.8 it is apparent that the more general numerical approach to 2D SVA provides superior sidelobe reduction. In contrast, within Figure 3.8, sidelobe reduction occurs more rapidly as radial distance is traversed from the point scatterers. However, this superior result requires additional computation. The values of \( \alpha \) and \( \beta \) may be computed directly through the use of the simplified two step 2D SVA approach. Illustrated by Figure 3.9 are the numerically computed spatially varying parameters \( \alpha(x, y) \) and \( \beta(x, y) \). In implementing the generalized method, 25 iterations of 2D bisection were used to compute the values of \( \alpha(x, y) \) and \( \beta(x, y) \). Increasing the number of numerical iterations improves sidelobe reduction.

![Figure 3.9: Alpha & beta computed for 2D SVA (Method 2)](image-url)
From the illustration of Figure 3.9 it can be seen that $\alpha(x, y)$ and $\beta(x, y)$ are minimized near the mainlobes. This effectively serves to preserve mainlobe magnitude and shape.

Apparent in the plot of $\alpha(x, y)$ are blue spiraling arms extending from the point scatterers. These low valued arms indicate the failure of the spatially variant parameter $\alpha(x, y)$ to effectively reduce cross-range sidelobes. However, the high valued regions within this plot, indicated by the darkening shades of the color red, indicate that down-range sidelobes are effectively suppressed.

Apparent in the plot of $\beta(x, y)$ is a widening blue fan extending from the center of the aperture. This low valued fan indicates the failure of the spatially variant parameter $\beta(x, y)$ to effectively reduce down-range sidelobes. However, the high valued regions within this plot indicate that cross-range sidelobes are effectively suppressed.

Implemented together, alpha and beta can be used to reduce both down-range and cross-range sidelobes respectively. The values of alpha and beta, as defined throughout this thesis, may take on any finite value, but in practice their ranges are constrained between $[0,0.5]$. This practice has been put into effect for all simulations presented in this thesis.

3.5 Experimental Results

Real SAR data was collected from a preconfigured scattering environment. The scattering environment comprised a metal trihedral and sphere within an anechoic
chamber. Figure 3.10 illustrates the horn radar utilized along with the environmental and test parameters. Figure 3.11 through Figure 3.15 illustrate the initial, Hann, Hamming, and 2D SVA reconstructed images respectively.

Figure 3.10: 2D SVA experimental configuration

Figure 3.11: An experimentally obtained initial image
Figure 3.12: The experimental 2D Hann windowed image

Figure 3.13: The experimental 2D Hamming windowed image

Figure 3.14: The experimental 2D SVA (Method 3) windowed image
Upon reviewing the initial image in contrast with the windowed images illustrated above it can be seen that 2D SVA clearly provides superior sidelobe reduction. Both the Hann and Hamming windowed reconstructions exhibit scatter enlargement, while the 2D SVA windowed reconstructions preserve scatterer fidelity. Upon close examination it can be seen the more general numerical 2D SVA approach has provided superior sidelobe reduction to that of the simplified 2 step approach.
Chapter 4: One-Dimensional Super Spatially Variant Apodization

4.1 Introduction to Super-SVA (SSVA)

Super spatially variant apodization (Super-SVA) relies upon traditional SVA for its non-linear transformation imposed upon the spatial domain data. This non-linear operation results in a modified signal that is no longer band limited permitting the Super-SVA technique for bandwidth expansion.

The signal magnitude of a point scatterer has the form of a rectangular pulse within the frequency domain. The width of this pulse corresponds to the bandwidth used. Applying the inverse Fourier transform upon a rectangular pulse corresponds to a sinc function within the spatial domain. While a sinc function is comprised of a particular bandwidth of values within the frequency domain, a truncated sinc mainlobe is not band limited. The application of SVA upon a down-range profile is roughly equivalent to representing point scatterers, within the spatial domain, by their corresponding sinc mainlobes.

Following the application of 1D SVA, the enhanced down-range profile is then mapped back into the wave-number domain via the Fourier Transform (see Figure 4.1).
Signal values outside the initial bandwidth are obtained by applying the transform over a larger bandwidth. Since the extended bandwidth corresponds to the SVA enhanced down-range profile, not the initial down-range profile, an inverse windowing function is required to restore the signal that would-have-been received over the expanded bandwidth. The importance of this inverse windowing function is made clear by its derivation in Section 4.2. Figure 4.2 below contrasts the initial signal corresponding to a single point scatterer ($S(k)$ in blue) with the SVA windowed signal ($F(k)$ in green). The application of the inverse windowing function upon $F(k)$ provides $S_{SV,A}(k)$ which closely approximates the initial signal.

![Figure 4.1: Illustrated procedure for obtaining $F(k)$](image-url)
Upon obtaining the extrapolated initial signal it can be used in the reconstruction of a higher resolution down-range profile, as shown in Figure 4.3. In the case of radar this is tantamount to artificially widening the broadcasted bandwidth to enhance down-range resolution. This correspondence is mathematically expressed by the widely known equation below:

Let:

\[ \Delta R = \text{Down-Range Resolution in meters} \]
\[ c = \text{Speed of Light in a Vacuum } 3\times 10^8 \text{ m/s} \]
\[ F_{\text{BW}} = \text{Frequency Bandwidth in Hz} \]

Then:

\[ \Delta R = \frac{c}{2F_{\text{BW}}} = \frac{\pi}{K_{\text{BW}}} \]
4.2 Derivation of the 1D Super-SVA Method

An iterative step procedure for applying 1D Super-SVA is illustrated below by Figure 4.4. The first two high-level blocks can be substituted by the 1D SVA procedure illustrated by Figure 2.2 and derived in Section 2.2. The vertical stack of high-level blocks represents the additional steps required by 1D Super-SVA. Upon each iteration of this stack the initial bandwidth is artificially expanded. Any number of iterations may be performed; however, with each iteration processing time increases and signal extrapolation relies more heavily on data from previous extrapolations. Each block
within the stack will be explored in this section. The derivations that follow will provide the tools necessary to apply the 1D Super-SVA procedure.

![1D Super-SVA Algorithm](image)

**Figure 4.4:** High level block diagram of the 1D Super-SVA procedure

Starting with the first block in the stack, suppose that there exists a reconstructed down-range profile corresponding to an antenna. Further suppose that SVA has been applied to this down-range profile and that its corresponding complex spatial values have been retained. The Fourier transformation then may be used to obtain the frequency domain values corresponding to this SVA windowed down-range profile.
Let:

\[ k = \text{Wave-number (rad/m)} \]
\[ x = \text{Spatial Displacement (m)} \]
\[ P = \text{The Number of Spatial Positions} \]

\[ f(x) = \text{Complex Spatial Domain SVA Windowed Down-Range Profile} \]

\[ F(k) = \text{The Wave-number Domain Fourier Transformation of } f(x) \]

Then:

\[ F(k) = K_{bw} \int_{x_1}^{x_p} f(x)e^{-j2\pi kx}dx \]

Where:

\[ k_1 \leq k \leq k_{N'} \text{, The Expanded Bandwidth} \]
\[ x_1 \leq x \leq x_p \text{,} \]

Moving on to the second block in the stack the 1D Super-SVA window is derived.

To derive this window consider the signal corresponding to a point scatterer for a particular bandwidth.

Let:

\[ x_{scat} = \text{Point Scatterer Spatial Position} \]

\[ S_{scat}(k) = \text{Complex Wave-number Domain Signal Corresponding to a point Scatterer} \]
Where:

\[ k_1 \leq k \leq k_N \]

\[ S_{\text{scat}}(k) = e^{-jk_2 x_{\text{scat}}} \]

\[ g(x) = \frac{1}{K_{bw}} \int_{k_1}^{k_N} S_{\text{scat}}(k) \, e^{jk_2 x} \, dk \]

By applying substitution the spatial domain down-range profile corresponding to a generalized point scatterer can be derived in closed form.

Let:

\[ k_1 \leq k \leq k_N \]

\[ k' = k - k_c = k - \frac{1}{2}(k_N + k_1) \]

\[ k_1 - k_c \leq k' \leq k_N - k_c \]

\[ k = k' + k_c = k' + \frac{1}{2}(k_N + k_1) \]

\[ \frac{dk'}{dk} = 1, \text{ and } dk' = dk \]

By Substitution:

\[ g(x) = \frac{e^{jk_c 2x}}{K_{bw}} \int_{-K_{bw}/2}^{K_{bw}/2} S_{\text{scat}}(k' + k_c) \, e^{jk_2 x} \, dk' \]

\[ = \frac{e^{jk_c 2x}}{K_{bw}} \int_{-K_{bw}/2}^{K_{bw}/2} e^{-j(k' + k_c)2x_{\text{scat}}} \, e^{jk_2 x} \, dk' \]

\[ = \frac{e^{jk_c 2(x-x_{\text{scat}})}}{K_{bw}} \int_{-K_{bw}/2}^{K_{bw}/2} e^{jk'_2(x-x_{\text{scat}})} \, dk' \]
By approximating the SVA windowed down-range profile and further applying substitution an open form solution for the windowing function may be derived.

Let:

\[ S_{SSVA}(k) = \text{The Approximation to } S(k) \text{ over the Super-SVA Expanded Bandwidth} \]

\[ W_{SSVA}(k) = \text{The Windowing Function used to recover } S_{SSVA}(k) \text{ from } F(k) \]

Then for the Case of a Single Point Scatterer:

\[ g(x) = \text{sinc}(K_{bw}[x - x_{scat}])e^{jkc^2(x-x_{scat})} \]

\[ f(x) \approx \text{sinc}(K_{bw}[x - x_{scat}])e^{jkc^2(x-x_{scat})}, x_{scat} - \frac{\pi}{K_{bw}} \leq x \leq x_{scat} + \frac{\pi}{K_{bw}} \]

\[ f(x) \approx 0, \text{ elsewhere} \]

\[ F(k) = K_{bw} \int_{x_1}^{x_p} f(x)e^{-j2\pi k x} \, dx = S_{SSVA}(k)W_{SSVA}(k) \]
Solving for $W_{SSV_A}(k)$:

$$W_{SSV_A}(k) = \frac{F(k)}{S_{SSV_A}(k)} = \frac{K_{bw} \int_{-\infty}^{\infty} f(x)e^{-j2\pi k x} \, dx}{K_{bw} \int_{-\infty}^{\infty} g(x)e^{-j2\pi k x} \, dx}$$

$$\approx \frac{\int_{x_{\text{scat}} - \pi/K_{bw}}^{x_{\text{scat}} + \pi/K_{bw}} \text{sinc}(K_{bw}[x - x_{\text{scat}}])e^{jk_{c}2(x-x_{\text{scat}})}e^{-j2\pi k x} \, dx}{e^{-jk_{c}2\pi x_{\text{scat}}}}$$

$$= e^{-jk_{c}2\pi x_{\text{scat}}} \int_{x_{\text{scat}} - \pi/K_{bw}}^{x_{\text{scat}} + \pi/K_{bw}} \text{sinc}(K_{bw}[x - x_{\text{scat}}])e^{j2\pi k_{c}x}e^{-j2\pi k x} \, dx$$

Let:

$$x = x' + x_{\text{scat}}$$

$$x' = x - x_{\text{scat}}$$

$$x_{\text{scat}} - \frac{\pi}{K_{bw}} \leq x \leq x_{\text{scat}} + \frac{\pi}{K_{bw}}$$

$$-\frac{\pi}{K_{bw}} \leq x' \leq \frac{\pi}{K_{bw}}$$

$$\frac{dx'}{dx} = 1, \text{ and } \, dk' = dk$$

By Substitution:

$$W_{SSV_A}(k) = \frac{e^{-jk_{c}2\pi x_{\text{scat}}} \int_{x_{\text{scat}} - \pi/K_{bw}}^{x_{\text{scat}} + \pi/K_{bw}} \text{sinc}(K_{bw}[x - x_{\text{scat}}])e^{j2\pi k_{c}x}e^{-j2\pi k x} \, dx}{e^{-jk_{c}2\pi x_{\text{scat}}}}$$

$$= e^{-jk_{c}2\pi x_{\text{scat}}} \int_{-\pi/K_{bw}}^{\pi/K_{bw}} \text{sinc}(K_{bw}x')e^{j2(x'+x_{\text{scat}})k}e^{-j2(x'+x_{\text{scat}})k} \, dx'$$

$$= \frac{e^{-j2\pi x_{\text{scat}}k} \int_{-\pi/K_{bw}}^{\pi/K_{bw}} \text{sinc}(K_{bw}x')e^{j2x'k}e^{-j2x'k} \, dx'}{e^{-jk_{c}2\pi x_{\text{scat}}}}$$
\[ \int_{-\pi/K_{bw}}^{\pi/K_{bw}} \text{sinc}(K_{bw}x')e^{j2x'k_c}e^{-j2x'k}dx' \]

Approximate Open Form Solution for \( W_{SSVA}(k) \):

\[ W_{SSVA}(k) \approx \int_{-\pi/K_{bw}}^{\pi/K_{bw}} \text{sinc}(K_{bw}x)e^{-j2x(k-k_c)}dx \]

With the windowing function derived the expanded initial signal \( S_{SSVA}(k) \) can be computed from \( F(k) \) as shown below. \( F(k) \) will have singularities outside the bounds of \( k \) indicated below, as shown in Figure 4.5. To prevent singularities, the bandwidth is expanded by no more than \( \sqrt{2}K_{bw} \) per iteration of Super-SVA. This accommodates the doubling of the initial bandwidth following the application of two 1D Super-SVA iterations. Throughout this thesis the bounds on \( k \) defined below are utilized for each iteration of 1D Super-SVA.

The Extrapolated Initial Signal:

\[ S_{SSVA}(k) = \frac{F(k)}{W_{SSVA}(k)} \]

Where:

\[ k_c - \sqrt{2} \frac{K_{bw}}{2} < k < k_c + \sqrt{2} \frac{K_{bw}}{2} \]
Proceeding to the third block within the stack, the initial signal over the initial bandwidth is reinserted into the extrapolation. By utilizing the original signal a more accurate reconstructed down-range profile can be computed. This initial signal is reinserted for each iteration of 1D Super-SVA.

Completing the stack with the last high level block, the inverse Fourier transform is applied to the extrapolated initial signal (with the initial signal substituted into $S_{SSVA}(k)$) and 1D SVA is once again applied. This new down-range profile corresponds to a wider bandwidth and hence inherits a higher down-range resolution. Furthermore, the reapplication of 1D SVA reduces the sidelobes of this resolved down-range profile. Following the application of 1D SVA the 1D Super-SVA method may be reapplied to achieve even greater resolution. With the chosen boundaries on $k$, the bandwidth, and hence the resolution, is doubled for every two iterations of 1D Super-SVA.
4.3 MATLAB Simulated Results

MATLAB has been used to perform two simulations involving 1D Super-SVA. The first simulation involves resolving a single point scatterer through 3 iterations of 1D Super-SVA. The second simulation involves two narrowly spaced point scatterers. These two scatterers are spaced such that they are closer together than the theoretical down-range resolution. For both simulations an initial bandwidth of 1 to 1.2 GHz is utilized. The bandwidth comprises a total of 200 evenly distributed test frequencies.

4.3.1 Simulation 1: A Single Point Scatterer

A single point scatterer has been simulated at a distance of 60 meters. Figure 4.6 illustrates the initial bandwidth in the wave-number domain. Figure 4.7 and Figure 4.8 illustrate the initial and 1D SVA windowed down-range profiles.

![Figure 4.6: Signal corresponding to a single point scatterer](image1)

![Figure 4.7: Down-range profile of a single point scatterer](image2)
Performing the Fourier transformation upon the SVA windowed down-range profile provides its corresponding wave-number domain signal $F(k)$ (see Figure 4.5). The inverse of the windowing function $W_{SVA}(k)$ is illustrated by Figure 4.9. The recovered signal $S_{SVA}(k)$ is contrasted with the initial signal $S(k)$ via Figure 4.10.
Performing the Fourier transformation upon the extrapolated signal yields a new down-range profile with a higher down-range resolution. Figure 4.11 illustrates the comparison between the initial down-range profile, $g(x)$, with the first 3 iterations of applied 1D Super-SVA.
**4.3.2 Simulation 2: Narrowly spaced Point Scatterers**

Two point scatters have been narrowly spaced in a MATLAB simulation. These scatterers were placed at distances of 10 and 10.7 meters from a simulated radar. Computing the theoretical down-range resolution for the 200 MHz bandwidth utilized corresponds to a value of 0.75 meters. Figure 4.12 illustrates the down-range profiles for these point scatterers for 4 consecutive iterations of 1D Super-SVA.
Figure 4.12: 1D Super-SVA applied towards merged scatterers

It can be seen from the plots of Figure 4.12 that the two narrowly spaced point scatterers are eventually resolved. This is not apparent until the second iteration of 1D Super-SVA. Subsequent iterations further resolve the two merged point scatterers. Every two iterations of 1D Super-SVA provides a 100% theoretical improvement in down-range resolution. Hence, Super-SVA is able to super-resolve to a certain extent. However, the scatterers within the final reconstruction do not appear as well resolved as those in Figure 4.13. Figure 4.13 shows the down-range profile reconstructed using
un-extrapolated signal data for a bandwidth of 800 MHz. Furthermore, height appears to be distributed between the two resolved point scatterers within Figure 4.12. Figure 4.14 corresponds to two point scatterers separated by more than the theoretical down-range resolution. It can be seen that these well separated scatterers maintain their heights following several iterations. Despite these issues, 1D Super-SVA has provided a reasonable approximation.

Figure 4.13: 1D Super-SVA contrasted with extrapolated data
Figure 4.14: 1D Super-SVA applied towards well separated scatterers

4.4 Experimental Results

Illustrated by Figure 4.15 is the application of 1D Super-SVA upon the real radar data utilized within Chapter 2. It can be seen throughout the four Super-SVA iterations that physical scattering features are sharpened and sidelobes are attenuated. The integrity of the trihedral’s position is also maintained while the resolution in down-range is iteratively enhanced. Despite the multiple iterations of Super-SVA some residual sidelobes still remain as a result of clutter effects.
Figure 4.15: 1D Super-SVA applied towards experimental data
5.1 Introduction to 2D Super-SVA

As was the case for SVA, Super-SVA may also be applied toward 2D applications. More than one approach can be taken to accomplish 2D Super-SVA, but the variation implemented in this study relies upon bandwidth extrapolation and truncation. This approach is advantageous in that it relies upon the techniques developed in Chapters 2 through 4.

The other approach is to apply Super-SVA in cross-range as well as downrange. However, Super-SVA relies on the Fourier – inverse Fourier transform relationship between the frequency and the down-range domains. The relationship is not as simple in cross-range, making it difficult to transform back to the frequency/aperture domain from the imaging scene.

Bandwidth extrapolation can be utilized to improve upon down-range and cross-range image resolution. The correspondence between down-range resolution and bandwidth was elaborated upon in Section 4.1; these two quantities are indirectly proportional. Similarly, Cross-range resolution is indirectly proportional to center frequency; this relationship is provided below [8]. By expanding the bandwidth to include higher frequencies the center frequency can be increased. This approach has
been taken in the study presented here to improve cross-range resolution (Figure 5.1 shows a typical result).

Let:

\[ \delta_{cr} = \text{Cross-Range Resolution in meters} \]

\[ R = \text{Range in meters} \]

\[ \lambda_c = \text{Center Frequency Wavelength in meters} \]

\[ D = \text{Aperture Width in meters} \]

\[ f_c = \text{Center Frequency in Hz} \]

\[ k_c = \text{Center Wave-number in radians/meter} \]

Then:

\[ \delta_{cr} \approx \frac{R \lambda_c}{D} = \frac{R c}{D f_c} = \frac{2\pi R}{D k_c} \]
5.2 Derivation of the 2D Super-SVA Method

The high-level block diagram portrayed by Figure 5.2 illustrates the 2D Super SVA technique. The variation of 2D Super-SVA utilized here builds upon the techniques developed in Chapters 2 through 4. This approach to 2D Super-SVA is straightforward and does not require additional mathematical derivation.
The first two steps can be substituted for the block diagram provided for the 1D Super SVA method illustrated by Figure 4.4. The only difference here is that 1D Super SVA is applied to more than one set of signal data. For a SAR based radar comprised of several measurement locations 1D Super SVA is applied upon each captured signal individually.

Step 3, bandwidth truncation, involves truncating the lower portion of the expanded bandwidth to raise center frequency. The increase in center frequency is indirectly proportional to the new cross-range resolution. The truncation point can be judiciously selected, but the approach taken within this thesis is to truncate the extrapolated signal below the lowest frequency within the initial signal. A less conservative approach may be taken to obtain greater improvements in cross-range resolution. However, further truncation puts a heavier value upon the accuracy of the extrapolated signal during image reconstruction (see Figure 5.3).
Step 4 involves two applications of 2D SVA. The SVA windows are applied in the frequency domain and across antenna location as in Method 2 of Chapter 3. Throughout the remainder of this thesis 2D Super-SVA is performed utilizing 2D SVA Method 2; this holds true for the simulations and experiments that follow. The final step, step 6, is plotting this result.

Despite being built upon the techniques developed thus far there are additional problems to consider. By expanding the initial bandwidth to include higher frequencies grating lobes may be introduced. By carefully considering SAR element spacing, prior to artificial bandwidth expansion, this problem can be circumvented. Antenna elements that are spaced too far apart allow for the introduction of grating lobes, which distort the reconstructed image. Grating lobes occur as periodic images of the SAR mainbeam. This phenomenon occurs when measurement spacing is greater than half the wavelength of the highest signal frequency.
Let:

\[ d = \text{Measurement Spacing in meters} \]
\[ \lambda_{\text{max}} = \text{Wavelength at maximum Frequency in meters} \]

Then:

\[ d \leq 0.5\lambda_{\text{max}}, \text{Avoidance of Grating Lobes} \]

Furthermore, the artificially expanded bandwidth is not perfect. Recovery of \( S_{SSVA}(k) \) from the windowing function and \( F(k) \) results in some distortion. This distortion takes the form of inaccurate signal shape outside the initial bandwidth and discontinuities within the signal data. Shape distortion outside the initial bandwidth is apparent in Figure 4.10. What’s more, additional iterations of 1D Super-SVA invariably worsens these distortions.

Cross-range resolution enhancement is accomplished through truncating the lower portion of the expanded signal \( S_{SSVA}(k) \) bandwidth in order to raise the center frequency. The unreliability of the artificially expanded portion of the bandwidth makes this difficult. Truncating too much of the original signal data and/or utilizing too much of the expanded signal data may result in distortion within the reconstructed 2D image. Successful application of 2D Super-SVA could mean running multiple simulations utilizing different truncation points to arrive at the most uniform and undistorted image.
5.3 MATLAB Simulated Results

5.3.1 Simulation 1: Single Point Scatterer

A feasible scattering configuration, with respect to available laboratory equipment, has been simulated by MATLAB. The goal of this simulation was to observe if reasonable down-range and cross-range resolution improvement could be obtained through the application of the derived 2D Super-SVA method. Throughout the simulation the 2D SVA method, Method 2 of Chapter 3, was utilized and the parameters $\alpha(x, y)$ and $\beta(x, y)$ were solved for numerically by way of 2D bisection.

The simulation corresponds to a single point scatterer positioned 9 feet from the center of a 37 element SAR (comprising an aperture of 9 feet). Prior to the application of 2D Super-SVA the initial signal comprised 70 test frequencies ranging from 500 MHz to 1 GHz. This frequency configuration accounts for an initial bandwidth of 500 MHz and center frequency of 750 MHz.

Five iterations of Super-SVA are applied to increase bandwidth from 500 MHz to approximately 2.8 GHz. The new expanded bandwidth, following Super-SVA, ranges from negative 664 MHz to 2.16 GHz. Next, the extrapolated signal is truncated below 500 MHz to provide a new bandwidth of 1.66 GHz and center frequency of 1.33 GHz. This procedure corresponds to theoretical down-range and cross-range resolution improvements of approximately 332% and 178% respectively. Figure 5.4 through Figure 5.6 illustrate the results of the simulated point scatterer experiment.
It is clear from the initial and 2D SVA enhanced initial images that both down-range and cross-range sidelobes are significantly reduced. The 2D Super-SVA windowed
image, Figure 5.6, clearly indicates that both down-range and cross-range resolution with respect to the point scatterer has been enhanced. Lastly, the introduction of negative frequencies via Super-SVA bandwidth expansion does not appear to adversely affect the final image. Figure 5.7 below illustrates that the theoretically calculated improvements in resolution closely agree with simulation.

![Figure 5.7: Point scatterer width & height before & after 2D Super-SVA](image)

Based upon the values between the initial and 2D Super-SVA enhanced images the simulated down-range resolution improvement is approximately 315% while the simulated cross-range resolution improvement is approximately 181%. These values closely agree with their respective theoretical values of 332% and 178%.

Examining a single antenna element’s frequency and spatial domain characteristics provides insight into the 2D Super-SVA solution. Figure 5.8 illustrates the expanded wave-number domain signal for all 5 Super-SVA iterations corresponding to the middle antenna element. From this illustration the accuracy of the expanded signal can be examined. The final expanded signal appears to deviate by no more than
approximately 2% of what would be the actual signal. Ideally, the magnitude corresponding to the point scatterer in this plot should be a horizontal line.

Illustrated by Figure 5.9 are the successive 1D down-range profiles corresponding to the middle antenna element for each of the 5 Super-SVA iterations. With each successive iteration the bandwidth is artificially extended by $\sqrt{2}$ providing a final theoretical improvement in down-range resolution of approximately 570%. The width of the scattering mainlobe can be seen to sharpen with each iteration of Super-SVA. Upon completion of the 5 iterations of Super-SVA the bandwidth is then truncated for frequencies less than 500 MHz.
5.3.2 Simulation 2: Dual Point Scatterers

Resolution enhancement offered by 2D Super-SVA was also examined for two closely spaced scatterers. Scatterers were separated by less than the theoretical down-range and cross-range resolutions. 2D Super-SVA was then utilized to determine if the merged scatterers could be resolved.
Figure 5.10 illustrates two point scatterers separated by a distance of 0.8 feet down-range of a 37 element SAR (comprising an aperture width of 9 feet). The two scatterers were respectively positioned 8.2 and 9 feet down-range from the SAR’s center.

![Image 1](image1.png)  
![Image 2](image2.png)

**Figure 5.10: A simulated initial image of two point scatterers merged in down-range**

The initial bandwidth was 500 MHz, comprised of 70 test frequencies ranging from 500 MHz to 1 GHz, corresponding to a theoretical down-range resolution of approximately 1 foot. Since the point scatterers were separated by less than the theoretical down-range resolution they appear merged within Figure 5.10. The bandwidth was then extended to approximately 1.66 GHz following 5 iterations of 2D Super-SVA and bandwidth truncation for frequencies below 500 MHz. This corresponded to a new theoretical down-range resolution of approximately 0.3 feet. As a result of the higher down-range resolution the two point scatterers appear separated within Figure 5.11. Figure 5.12 illustrates the image reconstructed had the entire signal been obtained without extrapolation.
Next, Figure 5.13 illustrates two point scatterers separated by a distance of 1.1 feet in cross-range. Again, the two point scatterers were positioned down-range from a 37 element SAR (comprising an aperture width of 9 feet). Both scatterers were positioned 9 feet down-range of the SAR’s center.
Figure 5.13: A simulated initial image of two point scatterers merged in cross-range

The same bandwidth was utilized providing a center frequency of 750 MHz and a theoretical cross-range resolution of approximately 1.3 feet. Since the point scatterers were separated by less than theoretical cross-range resolution they appear merged within Figure 5.13. Following the application of five 2D Super-SVA iterations the bandwidth was again expanded and then truncated for frequencies below 500 MHz. This approximately corresponded to a new bandwidth and center frequency of 1.66 GHz and 1.33 GHz respectively. As a result the theoretical cross-range resolution was improved to approximately 0.7 feet. Unfortunately, this time the scatterers still appear merged in Figure 5.14. Figure 5.15 illustrates the image reconstructed had the entire signal been obtained without extrapolation.
Figure 5.14: The simulated 5 iteration 2D Super-SVA windowed image

Figure 5.15: The simulated initial image obtained using real data for the extrapolated bandwidth

5.3.3 Interpretation of the 2D Super-SVA Simulation Results

Following the single point scatterer simulation it appears that 2D Super-SVA offers an effective means of shrinking scatterer width. Furthermore, following the two point scatterer simulations it was found that 2D Super-SVA can resolve closely spaced merged scatterers in down-range. Unfortunately, this was not also the case for scatterers spaced closely in cross-range. While the point scatterers were both narrower
in down-range and cross-range they were not resolved in cross-range and so they still appeared as merged.

5.4 Experimental Results

Two experiments are performed in this section. In the first experiment 2D Super SVA is applied towards the test configuration shown by Figure 5.16. This test was performed within an anechoic chamber. In the second experiment the testing environment is moved outside of the anechoic chamber into a room occupied by furniture and other items (see Figure 5.17).

Figure 5.16: The first 2D Super-SVA experimental configuration (anechoic chamber)
Steps were taken to prevent the introduction of grating-lobes following bandwidth extrapolation. Radar measurements were taken uniformly every 3 inches to prevent grating-lobes for frequencies below approximately 2 GHz.

An initial bandwidth of 500 MHz, ranging from 500 MHz to 1 GHz, was extrapolated through 5 iterations of 2D Super SVA. 70 test frequencies were utilized in steps of 7.1875 MHz. Following signal extrapolation and truncation the final bandwidth was 1.66 GHz, ranging from 500 MHz to 2.16 GHz. Furthermore, the initial center frequency was increased from 750 MHz to 1.330 GHz.

The aperture width was 9 feet and was positioned approximately 9 feet from the experimental metal scatterers. The specified initial bandwidth and SAR configuration...
provided initial down-range and cross-range resolutions of approximately 1.0 and 1.3 feet respectively. Following the application of 5 iterations of Super-SVA and bandwidth truncation the final theoretical down-range and cross-range resolutions were improved to approximately 0.3 and 0.7 feet respectively. The new theoretical resolutions, following Super-SVA, correspond to improvements of approximately 332% and 178% respectively.

5.4.1 Experiment 1: Scatterers within an Anechoic Chamber

Figure 5.18 illustrates the initial image before and after the application of 2D SVA (Method 2). Figure 5.19 through Figure 5.23 correspond to the first 5 iterations of 2D Super SVA. Lastly, Figure 5.24 and Figure 5.25 correspond to the initial and 2D SVA windowed images reconstructed using real signal data between 500 MHz and 2.16 GHz.

Figure 5.18: An experimental initial image of a scattering scenario (on left) contrasted with its 2D SVA (Method 2) windowed image (on right)
Figure 5.19: The 2D SVA (Method 2) windowed image (on left) contrasted with its Super-SVA windowed image (on right, iteration 1 of 5)

Figure 5.20: The 2D SVA (Method 2) windowed image (on left) contrasted with its Super-SVA windowed image (on right, iteration 2 of 5)
Figure 5.21: The 2D SVA (Method 2) windowed image (on left) contrasted with its Super-SVA windowed image (on right, iteration 3 of 5)

Figure 5.22: The 2D SVA (Method 2) windowed image (on left) contrasted with its Super-SVA windowed image (on right, iteration 4 of 5)
Figure 5.23: The 2D SVA (Method 2) windowed image (on left) contrasted with its Super-SVA windowed image (on right, iteration 5 of 5)

Figure 5.24: The initial image corresponding to real data for the extrapolated BW (on left) contrasted with the 5 iteration Super-SVA windowed image (on right)

Figure 5.25: The SVA windowed image corresponding to real data for the extrapolated BW (on left) contrasted with the 5 iteration Super-SVA windowed image (on right)
Following 5 consecutive iterations, 2D Super-SVA has demonstrated the ability to enhance image sharpness and resolution. Cross-range improvement is much more modest per iteration. Truncating from a frequency higher than 500 MHz would provide additional improvement. However, doing so places a greater weight upon the reliability of the extrapolated signal. With each iteration the accuracy of the extrapolated data becomes increasingly important.

Figure 5.24 and Figure 5.25 indicate that 2D Super-SVA has provided a reasonable extrapolation. Some minor down-range and cross-range distortions exist in the final extrapolated image, which can be reduced through the use of additional real data. Furthermore, the final extrapolation provides an upper frequency of approximately 2.16 GHz. However, due to spacing grating-lobes start to become a problem for frequencies somewhat less than 2 GHz.

5.4.2 Experiment 2: Through-Wall Images

Figure 5.26 illustrates the initial image before and after the application of 2D SVA. Figure 5.27 through Figure 5.31 correspond to the first 5 iterations of 2D Super SVA. Lastly, Figure 5.32 and Figure 5.33 correspond to the initial and 2D SVA windowed images reconstructed using real signal data between 500 MHz and 2.16 GHz.
Figure 5.26: A second experimental initial image of a scattering scenario (on left) contrasted with its 2D SVA (Method 2) windowed image (on right)

Figure 5.27: The experimental 2D Super-SVA windowed image (iteration 1 of 5)

Figure 5.28: The experimental 2D Super-SVA windowed image (iteration 2 of 5)
Figure 5.29: The experimental 2D Super-SVA windowed image (iteration 3 of 5)

Figure 5.30: The experimental 2D Super-SVA windowed image (iteration 4 of 5)

Figure 5.31: The experimental 2D Super-SVA windowed image (iteration 5 of 5)
Once again, 2D Super-SVA has demonstrated the ability to enhance image sharpness and resolution. Other objects within the room are seen to be resolved right along with the metal sphere and trihedral. The environment of the room has not prevented the visual identification of these scatterers. Clutter still exists within the final Super-SVA image and some degree of distortion is introduced. Lastly, Figure 5.32 and Figure 5.33 indicate that 2D Super-SVA has provided a reasonable extrapolation.
Chapter 6: Conclusions

An experimental 2D Super-SVA method was developed and implemented algorithmically within MATLAB. Simulations and experiments were performed and indeed the method demonstrated that its application can provide clearer and sharper radar images. Scatterer sidelobes were significantly reduced without adversely widening the scatterers themselves. Furthermore, point scatterers were sharpened both in down-range and cross-range. Closely spaced scatterers merged in down-range were resolved, but those merged in cross-range were not. Despite this, the experimental method still provides a means for sharpening scatterers in 2 dimensions. This has been shown for the MATLAB simulations undertaken in this thesis and for two experimental scenarios. One scenario corresponded to an anechoic chamber environment and a second to through-wall data.

Chapter 2 walked through the development of a 1D SVA method that accommodated continuous independent values for spatial position. Chapter 4 expanded upon this method and developed a method for 1D Super-SVA. MATLAB was used to simulate point scattering problems to theoretical verify both 1D methods. Experiments were then performed to verify these methods through the use of real radar data.
Next, the 2D counterparts corresponding to these methods were developed. Chapter 3 expanded upon the 1D SVA method through the introduction of a second spatially variant window. This second window was applied across a SAR aperture to reduce cross-range sidelobes. Two open form non-linear solutions were derived for 2D SVA in Section 3.2. In section 3.3 a simplified open form solution was derived for 2D SVA. The simplified solution was linear in form and provided superior computational performance, but resulted in the loss of some generality.

Chapter 4 introduced 1D Super-SVA and provided its derivation. Directly following this Chapter 5 walked through the development of the experimental 2D Super-SVA method. It was also verified through MATLAB simulation and experimentation. This method relies upon the 1D Super-SVA method to extrapolate each signal individually. Of course the 1D Super-SVA method was built upon the 1D SVA method. After bandwidth extrapolation, bandwidth truncation was performed to improve cross-range resolution. The method developed for 2D SVA was then used to reduce the sidelobes within the 2D Super SVA resulting image.

Bandwidth truncation was not required for down-range resolution enhancement in 2D. The entire extrapolated signal could be utilized, this included the lower frequencies during extrapolation. However, enhancement in cross-range resolution required that the bandwidth be truncated. Discarding too much of the initial signal places a heavier weight on the extrapolated data during reconstruction. It was shown that the extrapolated signal did not perfectly match the physically captured signal. Instead of capturing additional physical data, techniques for reducing extrapolation
error could aid in this problem. This would allow for additional sidelobe reduction and further resolution enhancement.

1 dimensional down-range profiles are resolved through the application of 1D Super-SVA. The 2D Super SVA method explored here expanded the bandwidth of each signal individually by application of the developed 1D Super SVA method (measurement location by SAR analogy). While this approach readily lends itself to parallel processing, it would be of interest to alternatively reconstruct the entire 2D image following each 1D Super SVA iteration.
Bibliography


