EVALUATION OF EXPANDERS FOR USE IN A SOLAR-POWERED RANKINE CYCLE HEAT ENGINE

A Thesis

Presented in Partial Fulfillment of the Requirements for the Degree Master of Science in the Graduate School of the Ohio State University

By

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ABSTRACT

The objective of this research is physical evaluation of a heat engine concept for converting solar energy or waste heat to electric power. This type of heat engine could be useful for distributed power applications delivering power at the rate of three to seven kilowatts. The concept calls for a Rankine cycle using R123 for the working fluid. It uses a positive-displacement expander coupled to an electric generator for electric output. In the laboratory, the energy input comes from steam.

The evaluation includes characterizing the behaviors of available positive-displacement expanders for use in the heat engine: scroll and gerotor. Laboratory tests determine the isentropic efficiency and energy conversion efficiency of the expanders at different operating conditions. The system performance is determined by combining the measured expander performance with a thermodynamic model of a complete system.

A parametric study shows the relationships between operating conditions and system performance. This information can be used to select the appropriate heat exchangers and liquid pump to build a heat engine. Both the scroll and gerotor expanders are suitable for use in a low-temperature heat engine. The predicted performance for a system operating between 350°F and 70°F using one of these rotary positive-displacement expanders is 5kW power output with 14% thermal efficiency.
Dedicated to my wife
ACKNOWLEDGMENTS

I thank Rich Christensen, my adviser, for connecting me with the research project that led to this thesis and advising me to "make it work".

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I also want to thank James Mathias and Jiming Cao, who conducted all of the testing with me.
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CHAPTER 1

INTRODUCTION

1.1 Purpose

The objective of this research is physical evaluation of a heat engine concept for converting solar energy or low-grade heat to electric power. This type of heat engine could be useful for distributed power applications delivering power at the rate of three to seven kilowatts. The critical component of the heat engine is the expander used to convert fluid energy into mechanical energy.

Low-grade energy is abundant, but seldom used. Most renewable energy sources provide low-grade energy. These include solar energy, geothermal energy, and low-grade fuels like biomass. Industry is also a source of low-grade energy. Approximately half of the heat generated in industry is rejected as waste heat from powerplant condensers and other industrial processes [13]. An efficient method for converting this low-grade energy could help satisfy growing power needs and reduce fuel consumption and pollution.

Solar energy is the most abundant of the low-grade energy sources. Photovoltaic cells are capable of converting solar radiation directly into useable electricity. Current technology is capable of converting, at best, 20% of incident radiant power on the
collector to electrical power [6]. A major drawback of photovoltaic systems is the need for a separate backup system for when there is insufficient solar radiation.

A heat engine capable of converting solar energy into electrical power has some advantages over photovoltaic systems for some applications. It could be configured to use a backup heat source when there is insufficient solar radiation. The heat source could even be a low-grade combustion fuel. An entire backup power system is not necessary, only a backup heat source.

Some applications may be better suited for power from a low-temperature solar-thermal heat engine than from a photovoltaic collector. If the energy will be used to pump water – a common need in the locations with the most sunshine – the heat engine could deliver mechanical power rather than electrical power to the pump. This would eliminate some of the inefficiency associated with the conversion from electrical power to mechanical power. If the energy is used for cooling – also a common need in sunny places – the heat engine could be coupled with an absorption chiller. The mechanical power created by the heat engine could be used to drive the pump of the chiller and the heat rejected by the engine could be used as the heat source for the chiller. This could be more efficient than using a photovoltaic system to power an electric refrigeration unit.

1.2 Research Goals

Most electrical power is produced at powerplants that burn fuel to make heat that runs a heat engine. The heat engine uses steam as the working fluid in a Rankine cycle. At temperatures below 700°F, the steam Rankine cycle is not economically feasible [13].
A low temperature heat engine cannot use a steam Rankine cycle. The organic Rankine cycle – named for the organic working fluid used instead of steam – has been identified as the best option to make use of low-grade heat sources [5].

The objective of this research is physical evaluation of a heat engine concept for converting solar energy or waste heat to electric power. Steam powerplant technology does not transfer to an organic Rankine cycle heat engine. The possible working fluids behave differently than steam. These fluids also have undesirable qualities when compared with benign, abundant water. The low power density of the energy sources prohibits large power output and limits the scale of the engine and its required components.

In theory, a heat engine operating on an organic Rankine cycle could convert low-temperature heat to useful work with 10%-20% thermal efficiency. Presently, no low-temperature solar-thermal heat engine has been able to deliver power close to this theoretical efficiency.

This heat engine concept uses solar collectors for heat input at 350°F and a rotary positive-displacement expander, either a scroll or a gerotor, for conversion from fluid to mechanical energy. The shaft of the expander is coupled to an electric generator for electric output. Figure 1 shows a simple diagram of a Rankine heat engine.
Figure 1: Rankine cycle heat engine

The scroll expander is a refrigeration compressor connected to operate in the reverse flow direction, expanding rather than compressing the fluid. The compressor's induction motor acts as a generator when the shaft rotates faster than the synchronous speed.
The gerotor expander is a positive displacement pump constructed with a larger output pocket volume than input pocket volume. This allows the fluid to expand as it passes through. Several configurations are possible with the external shaft of the gerotor expander. It can provide direct mechanical power or drive an electric generator.

The focus of this research is characterizing the expander performance. This research also requires identifying the proper working fluid, heat exchangers, and liquid pump to build a prototype heat engine. The system performance is determined in laboratory tests measuring fluid state points throughout the cycle. The measurement of system performance includes power output and overall efficiency from heat input to electric output. A parametric study will show the relationships between thermodynamic states and system performance. The research also considers the feasibility of a heat engine that can deliver 3-7kW electrical power and the associated efficiency. This research is not concerned with the solar collector design.

This research will increase the volume of knowledge about using low-grade heat sources for useful power. The performance characteristics for these rotary positive-displacement expanders will be useful in future heat-engine design. Regardless of this concept's viability, the conclusions from this research will provide direction for future research.
CHAPTER 2

THEORY AND LITERATURE REVIEW

2.1 Rankine Cycle

Although the finest details of the theory and equipment of the steam Rankine cycle have been researched and documented, relatively little work has been done on organic Rankine cycles. It is worth consideration since it has been identified as the most promising thermodynamic cycle for extracting useful energy from low-grade energy sources [4]. Although the physical implementation is different, the thermodynamic principles are the same for all Rankine cycles regardless of working fluid.

Four reversible processes describe the ideal Rankine cycle [8]. The cycle is shown on a pressure-enthalpy diagram for water in Figure 1. Low pressure saturated liquid enters the pump at point 1. The liquid is isentropically compressed to high pressure by mechanical work put into the system between points 1 and 2. The subcooled liquid enters the evaporator at point 2. Heat is added at constant pressure from point 2 to 3. The fluid changes phase from liquid to vapor. Superheated vapor enters the turbine or expander at point 3. Work is removed from the system at the expander as the fluid expands isentropically between points 3 and 4. Saturated vapor enters the condenser at
point 4. The fluid rejects heat at constant pressure as it condenses from point 4 to 1. The useful work out of the system is greater than the work put into the system.

The ideal amount of work into the system at the pump is equal to the change in enthalpy between points 1 and 2. The pump work from low to high pressure can also be calculated as the product of the specific volume of the fluid at point 1 and the difference in pressure between points 1 and 2:

$$ w_{pump} = h_2 - h_1 = v(P_2 - P_1) \tag{1} $$

The ideal amount of heat added to the fluid in the evaporator is equal to the change in enthalpy between points 2 and 3:

$$ q_{in} = h_3 - h_2 \tag{2} $$

The ideal amount of work out of the system at the expander is equal to the change in enthalpy between points 3 and 4:

$$ w_{expander} = h_3 - h_4 \tag{3} $$

The ideal amount of heat rejected from the fluid in the condenser is equal to the change in enthalpy between points 3 and 4:

$$ q_{out} = h_4 - h_1 \tag{4} $$

The net work from the cycle is the difference between the expander work and the pump work. It can also be calculated from the difference between the heat in and the heat out:

$$ w_{net} = w_{expander} - w_{pump} = q_{in} - q_{out} \tag{5} $$

7
Figure 2: Ideal Rankine cycle for steam
The ability of the system to convert heat into useful work can be described by the thermal efficiency. The thermal efficiency of the cycle is the ratio of the net work to the heat in:

\[
\eta_{\text{in}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}}
\]  \hfill (6)

An actual Rankine cycle has a few differences from an ideal Rankine cycle. The actual cycle has pressure drop from flow resistance, irreversible expansion, irreversible compression. Point 1 must be subcooled liquid, not saturated liquid, to avoid cavitation in the pump. This means the condenser pressure must be greater than the saturation pressure at the lowest temperature. There is pressure drop in the heat exchangers. This makes the pressure difference across the pump greater than the pressure difference across the expander. The greater pressure difference requires more work from the pump. There are irreversibilities in the pump and the expander, so the compression and expansion processes are not isentropic. The work required by the pump is greater and the work out of the expander is less than for the ideal cycle. The departure from ideal expansion in the expander can be expressed as the isentropic efficiency, the ratio of the actual work to the work for isentropic expansion. The isentropic efficiency of the pump is just the inverse:

\[
\eta_{\text{expander}} = \frac{w_{\text{actual}}}{w_{\text{isentropic}}} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}
\]  \hfill (7)

\[
\eta_{\text{pump}} = \frac{w_{\text{isentropic}}}{w_{\text{actual}}} = \frac{h_{2s} - h_1}{h_{2a} - h_1}
\]  \hfill (8)
An actual Rankine cycle is shown in figure 2 along with an ideal Rankine cycle between the same high and low temperatures. The differences are exaggerated for clarity.

The maximum possible thermal efficiency of a cycle between two temperatures is the Carnot efficiency. Carnot cycle efficiency is a function of only the high and low absolute temperatures:

$$\eta_{\text{Carnot}} = 1 - \frac{T_{\text{low}}}{T_{\text{high}}}$$  \hspace{1cm} (9)

The thermal efficiency of a Rankine cycle between two temperatures is less than the thermal efficiency of a Carnot cycle between the same temperatures. A Carnot cycle has constant \textit{temperature} heat addition and rejection, while an ideal Rankine cycle has constant \textit{pressure} heat addition and rejection. When the fluid is changing phase, however, both pressure and temperature are constant. The thermal efficiency of a Rankine cycle is, therefore, a function of the high and low temperatures and the properties of working fluid. The selection of the working fluid determines how well the Rankine cycle approximates the Carnot cycle.
Figure 3: Actual Rankine cycle for steam
2.2 Working Fluid

One of the challenges of building a low temperature Rankine cycle heat engine is finding the proper working fluid. Many researchers have tried to identify the best working fluids for an organic Rankine cycle. No fluid meets all the criteria for a good working fluid, but some come close.

Dr. Wali [30] identified the following criteria for the selection of a working fluid for a solar powered Rankine cycle: Safety, Efficiency, Thermal stability at operating conditions, Pressure drop, and Mean heat transfer coefficient. Dr. Wali selected R-113 as the best working fluid for this cycle, but an international treaty has already led to its phase out [2]. These criteria are incomplete. Several fluids that meet the above criteria may contribute to ozone depletion and global warming. It is appropriate to add environmental safety to the list.

Dr. Hung also described criteria for selecting an appropriate working fluid for an organic Rankine cycle. The saturated vapor curve is the most crucial characteristic of such a working fluid [13]. It affects the fluid applicability, cycle efficiency, and the equipment used in the heat engine. An isentropic fluid has nearly constant entropy of saturated vapor for all temperatures. This means that saturated vapor entering the expander remains saturated vapor throughout an isentropic expansion process. The evaporator temperature can be very close to the high temperature in the cycle. This approaches the constant temperature heat addition of a Carnot cycle. The heat engine also needs no superheater after the evaporator for an isentropic working fluid. Since the
fluid is saturated vapor as it leaves the expander, there is no need for a regenerator to ‘Carnotize the cycle’ [18]. R-123 and R-134a were identified as ozone-safe isentropic fluids.

Dr. Lee evaluated the theoretical performance of several potential working fluids in an ideal organic Rankine cycle. The evaluation also considered the heat transfer capability of the working fluid at evaporation and condensation. R-123 was again identified as one of the best ozone-safe working fluids for an organic Rankine cycle [15].

Previous research at Ohio State University analyzed the theoretical thermodynamic performance of nearly 150 possible working fluids for a low temperature Rankine cycle. It simply compared the latent heat of vaporization at the lower of 300°F or 300psia with the work of isentropic expansion from saturated vapor at that condition. The results showed again that R123 has the best thermodynamic performance of the available ozone-safe fluids [10].

R-123 is not a perfect working fluid. The Ozone Depletion Potential (ODP) is small, but not zero. It will eventually be phased out, along with other HCFCs, by the year 2030 [2]. It has a stronger interaction with most plastics and elastomers than other common refrigerants [7], [11]. System components would need to be carefully selected for material compatibility. R-123 is also more toxic than other refrigerants. The recommended 8-hour average exposure limit is 50ppm for R-123 versus 1000ppm for R-134a [19], [18].

Another fluid that could possibly be used in a low-temperature Rankine cycle not considered in the above studies is HFC-43-10, a cleaning solvent with thermodynamic
properties similar to R-113 [17]. Its Ozone Depletion Potential is zero. HFC-43-10 is compatible with more plastics and elastomers than R-123, but the system components still must be carefully selected. R-43-10 is also less toxic than R-123. The 8-hour exposure limit is 200ppm [28]. The saturated vapor curve is not isentropic like the saturated vapor curve of R-123, so the thermal efficiency of an HFC-43-10 Rankine cycle is probably less than that of an R-123 Rankine cycle.

Some inorganic fluids have also been considered. Ammonia and Butane have unique advantages and disadvantages. Ammonia has the highest thermal conductivity by an order of magnitude, but low thermal efficiency. It also has the most severe affect on people of the fluids considered. Butane has good conductivity and fair thermal efficiency, but it is flammable.

Table 1 compares thermodynamic performance, liquid thermal conductivity at atmospheric pressure, ODP, recommended exposure limits and the slope of the saturated vapor curve for the most promising working fluids for an organic Rankine cycle. The thermodynamic cycle was modeled using Engineering Equation Solver; a software package with built in thermodynamic properties for many common fluids [9]. It does not contain property data for HCF-43-10 since it is not used in refrigeration systems. That cycle was manually calculated using data supplied by the manufacturer [28]. The temperature limits are 70°F and 350°F with 20°F pinch in the heat exchangers to bring the temperature limits in the cycle to 90°F and 330°F. All four state points in the cycle are at least 10°F away from saturated conditions. The high pressure is limited to 400psia. For comparison, the Carnot efficiency between 90°F and 330°F is 29%.
This comparison revealed that R-141b actually has a better thermal performance in both thermal efficiency and thermal conductivity than R-123 for these conditions. Its potential affect on stratospheric ozone, however, is greater than that of R-123. R-141b is scheduled for phase out by the year 2004 [29]. The two zero ozone-depleting refrigerants, R-134a and HFC-43-10, have thermal efficiencies lower than R-123. R-123 and HFC-43-10 are the best fluids for this application, but neither is perfect.

Based on the criteria of thermal efficiency, heat transfer capability and environmental and human safety, R-123 has been selected as the working fluid for these tests. The second choice would be HFC-43-10 for increased environmental safety at the price of efficiency.
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<th>Safety</th>
<th>Saturated Vapor Curve</th>
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<td></td>
<td>$\eta_\pi$ (%)</td>
<td>$k$ (Btu/hr-ft$^{-\circ}$R)</td>
<td>ODP</td>
<td>TEL (ppm)</td>
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<tr>
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<td>5</td>
<td>0.393</td>
<td>0</td>
<td>-</td>
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<td>20</td>
<td>0.042</td>
<td>0.8</td>
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<td>0.061</td>
<td>0</td>
<td>800</td>
<td>Isentropic</td>
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</tbody>
</table>

Table 1: Comparison of potential working fluids for organic Rankine cycle. [18], [19], [22], [25], [28]
2.3 Heat Engine

The heat engine is the physical implementation of thermodynamic theory. It puts heat energy into the working fluid and extracts useful work from it. The proper physical components are vital to the success of any heat engine concept.

Some solar-powered steam powerplants, operating on a steam Rankine cycle, are used for commercial power generation. One example is operated by Lutz International in the Mojave Desert [6]. Large fields of tracking parabolic solar collectors heat oil to over 700°F, which then produces steam. The system uses conventional steam turbines like any other multi-megawatt powerplant. The solar-to-electricity conversion efficiency is 14% on average, with a peak efficiency of 22%. The maximum power output is 80MW.

Attempts at low-temperature, small-scale solar heat engines have not been as successful. Several researchers have attempted small organic Rankine cycle heat engines. The heat engines have generally used heat sources with temperatures lower than 200°F so the maximum possible thermal efficiencies have been low. Inefficiencies in the system components, especially the expander, which will be discussed later, further decreased the overall efficiencies of these heat engines. This heat engine should have a better thermal efficiency with the greater temperature difference and a more efficient expander.

Al-Haddad, et al. developed a heat engine for pumping water using a 120°F heat source and R-11 and air as working fluids [1]. The engine has no moving parts, just fluids pushing against fluids. The unique thermodynamic cycle is a vapor cycle but not a
Rankine cycle. The power output was less than 1 Watt and thermal efficiency was less than 1%. It demonstrated that even a small temperature difference could be used to extract useful work with a crude heat engine.

Spindler, et al. analyzed another heat engine used for pumping water [26]. This machine uses R-113 in an organic Rankine cycle operating between 170°F and 70°F. The mechanical design had the following goals: small, inexpensive, simple, and robust. The engine has a piston expander driving piston pumps for the liquid R-113 and water. The pumped water removed heat from the working fluid in the condenser. Measured thermal efficiency for this heat engine was also less than 1%. The order of magnitude difference between the Carnot efficiency and the system efficiency was attributed to low expander and pump efficiencies.

Nguyen, et al. describes the development of a Rankine cycle engine inside a thermosyphon [21]. Like a heat pipe, fluid evaporates at one end of a cylinder and condenses at the other end. A thermosyphon uses gravity rather than a wick to return the liquid from the condenser to the evaporator. This heat engine design forces the vapor through a turbine on its way to the condenser. The study looked at two simple turbines for use in such a system. Using low-pressure steam for the working fluid and high and low temperatures of 145°F and 90°F, the thermal efficiency was again less than 1%. Losses were attributed to rotational losses in the turbine and the inefficiencies in various components.

Lewis evaluated a heat engine for pumping water operating on a pseudo-Rankine cycle [17]. The cycle operated between 150°F and 80°F with R-113 for the working
fluid. The expander in this system uses a rolling diaphragm piston. The rolling diaphragm effectively seals the piston without excessive friction. The heat engine demonstrated 1-2% thermal efficiency. Flow losses (valves and orifices) and mixing losses (high pressure fluid filling low pressure volume) were identified as the most likely causes of poor efficiency. Those are the problems associated with reciprocating piston expanders in heat engines.

Oomori, et al. used a Rankine cycle heat engine for heat recovery from an automobile engine [23]. R-123 replaced the engine coolant in a 4-cylinder spark ignition engine and acted as the working fluid in the Rankine cycle. The cooling water jacket of the engine acted as the evaporator and a replacement for the radiator acted as the condenser. A scroll expander was used to extracted work from the fluid. Operating between 220°F and 80°F the thermal efficiency was 3.2%. This system suffered from excessive pressure drop in the unmodified cooling water jacket. The system also could not sustain a large pressure ratio at high flow rates. The scroll expander worked much better than the expanders used in other organic Rankine cycle heat engines.

2.4 Expander

2.4.1 Expander Theory

The expander may be the most important piece of equipment in the heat engine system. The thermal efficiency is a strong function of expander efficiency. Consider the relative magnitudes of the heat input, expander work and pump work in the Rankine cycle. The ability of the expanding device to convert fluid energy into work is critical.
The previously mentioned heat engines each used a different device for the expander. Clearly the best one was the rotary positive-displacement scroll expander. The discrete, reciprocating movement of the piston expander contributes to flow and mixing losses. Simple turbines showed other mechanical problems. Badr, et al. [4] and Hinsenkamp, et al. [12] determined that rotary positive-displacement expanders have distinct advantages over turbines and reciprocating-piston expanders for small, low-temperature heat engines. They have the potential to combine the best characteristics of pistons and turbines in one machine.

There does not need to be so much guesswork finding the proper expansion device. In his book, Kreider [14] presents a method for selecting the proper expander for different heat engines. The efficiency of an expander is related to the isentropic expander work, volumetric flow rate, shaft speed and expander size. Each type of expander (piston, rotary positive-displacement, radial turbine, axial turbine) has a different combination of these parameters at its optimum operating condition. For example, at low shaft speeds and high isentropic work, a piston expander has the highest efficiency. At high shaft speeds, a turbine would have the highest efficiency. The similarity parameters used in this method are the specific speed \( N_S \) and the specific diameter \( D_S \). Efficiencies of different expander types are shown on an \( N_S-D_S \) map in Figure 3.

\[
N_S = N Q^{1/2} / H_{ad}^{3/4} \tag{10}
\]

\[
D_S = D H_{ad}^{1/4} / Q^{1/2} \tag{11}
\]

\( N \) is the shaft speed in rpm. \( D \) is the diameter in feet. \( Q \) is the inlet flow rate in \( \text{ft}^3/\text{sec} \). \( H_{ad} \) is the adiabatic enthalpy drop across the expander in \( \text{ft-lbf/lbm} \).
For a small organic Rankine cycle heat engine, these parameters can be calculated to determine what type of expander to use and to estimate the expander efficiency. \( N \) is 3600 rpm for an AC induction generator. \( D \) is a few inches: 0.25 ft. Expander work is calculated in the cycle modeling: about 20Btu/lbm. For 7kW output, the flow rate of the working fluid would be about 0.3lbm/sec. The specific volume at the expander inlet is about 0.1\( \text{ft}^3/\text{lbm} \), so \( Q \) is 0.03\( \text{ft}^3/\text{sec} \). Converting units for expander work makes \( H_{ad} \) about 15,000 ft-lbf/lbm.

\[
N_S = \left( \frac{3600}{0.03} \right)^{1/2} / (15000)^{3/4} = 0.5 
\]

\[
D_S = \left( \frac{0.25}{15000} \right)^{1/4} / (0.03)^{1/2} = 16 
\]

The map shows that piston expanders or rotary piston expanders can operate efficiently in this range.
Figure 4: Efficiency map of different expander types [14].
Morishita, et al. looked at applying the high efficiency of a scroll compressor to an expander application [20]. The scroll compressor has low rubbing speed compared to a reciprocating piston. This allows it to have a tight mechanical seal without high friction losses. The continuous, unidirectional flow of a scroll expander leads to smooth, steady torque, unlike a reciprocating piston.

2.4.2 Scroll Expander

One expander under consideration is this study is a commercial refrigeration scroll compressor. As a compressor the scroll has low friction and tight seals. The compressor can become an expander simply by reversing the flow direction. The built in electric motor can act as an electric generator. It has already been carefully designed for use with refrigerants and has been in production for years. A heat engine based on this expander would benefit from using “off the shelf” equipment, resulting in lower system cost than for a heat engine with a unique new turbine design. Figure 5 shows a simple illustration of the mechanism of a scroll expander.

A scroll expander has two basic parts: a fixed scroll and an orbiting scroll [27]. The scrolls are conjugates. The surfaces contact at several points to create discrete pockets and the two scrolls remain in contact. The contact points move and the pockets increase in size as the orbiting scroll moves relative to the fixed scroll. Figure 6 illustrates the motion of the scrolls and the expansion of one pocket.
Figure 5: Scroll expander
Figure 6: Scroll expander - expansion process

The orbiting scroll stays oriented the same way relative to the stationary scroll as it rubs along the surface. The top row shows the pocket filling as it is exposed to high-pressure fluid at the center. The second row shows the closed pocket expanding. The third row shows the pocket opening and releasing fluid to the low-pressure side. The process is one continuous rotary motion, unlike a piston.
2.4.3 Gerotor Expander

The other expander under consideration is this study is a custom gerotor pump. Like a scroll compressor, a gerotor pump is a positive displacement mechanism with both good volumetric efficiency because of tight mechanical seals and low friction because of the low relative speed of the rubbing parts [24]. Figure 7 shows a simple illustration of the pumping mechanism for a gerotor expander.

The pumping mechanism is simply an inner rotor gear and an outer stator gear, located eccentric to each other. The rotor has one less tooth than the stator. The contact points between the two gears seal off a series of pockets. As the shaft turns, each pocket increases in volume through half of each rotation and decreases in volume through the other half of rotation. The inlet port is located to start filling the pocket at its smallest volume, and close it off at the proper size relative to the largest volume. The exit port is located to start venting the pocket at its largest volume and stop at its smallest volume. The following figure illustrates every 45 degrees of one rotation for one pocket of a five-pocket gerotor.
Figure 7: Gerotor expander
Figure 8: Gerotor expander – expansion process

The top row shows the pocket increasing in size; the bottom row shows the pocket decreasing in size. In the first two frames the pocket is exposed to the inlet port. The third through fifth frames show the closed expansion process. The pocket is exposed to the exit port while it decreases in size.

The gerotor has low friction for mechanical seals. The rotor advances one tooth relative to the stator for each shaft rotation. Rubbing speed at the seal between pockets is a fraction of that for a reciprocating piston. A five-tooth stator rotates at one fifth the shaft speed, relative to the rotor, so a 3600rpm shaft speed translates to a 720rpm relative speed. These properties of the gerotor should lead to high expander efficiency and low mechanical losses.
CHAPTER 3

EXPERIMENTAL METHOD

3.1 System Components

The purposes of this experiment are characterizing the performance of the scroll expander and sizing the other components to match it in a prototype system. The temperature and pressure limits seen by the expander on test stand should not be restricted by inefficiencies of the other system components for the preliminary testing. In the heat engine research mentioned previously the performance of the individual components could not be evaluated because the entire heat engine was new and unknown. In the heat engine tested by Oomori, et al. mentioned previously, for instance, the small pressure ratio at high scroll speeds could have been a limitation of the scroll expander. It could have also been the effect of large pressure drop in the evaporator at high flow rates or the nature of the liquid pump – lower head at higher flow rates. The liquid pump and the heat exchangers should therefore be over capable in a system used to characterize the performance of a new expander.

Figure 9 illustrates the components and state points of an ideal R-123 cycle between 330°F and 90°F. The system has four main components: expander, liquid pump, evaporator, and condenser. This section describes the components used in the test stand.
Figure 9: Ideal Rankine cycle for R-123
3.1.1 Expander

One expander used in this experiment is a Copeland scroll compressor. Expander volume ratios of 3.4 and 2.7 were included in the experiment for comparison. The scroll size is fixed, so the 3.4 volume ratio is achieved with a smaller inlet pocket size and the same exit pocket size as the 2.7 volume ratio scroll. It is important to see the relationship of the inlet volume and volume ratio on expander efficiency. Table 2 lists the sizes of the two scrolls.

<table>
<thead>
<tr>
<th>Scroll Type</th>
<th>Inlet Volume (in^3)</th>
<th>Exit Volume (in^3)</th>
<th>Volume Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>B</td>
<td>1.26</td>
<td>3.4</td>
<td>2.7</td>
</tr>
<tr>
<td>C</td>
<td>1.26</td>
<td>3.4</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Table 2: Scroll sizes

The other type of expander used in this experiment is a prototype gerotor expander. The gerotor expander volume ratios were larger than the scroll expanders were. The inlet and exit volumes can also scale together by increasing the depth of the gerotor. Table 3 lists the gerotor sizes.
<table>
<thead>
<tr>
<th>Gerotor Type</th>
<th>Inlet Volume (in^3)</th>
<th>Exit Volume (in^3)</th>
<th>Volume Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.39</td>
<td>3.85</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>0.77</td>
<td>7.7</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>0.96</td>
<td>3.85</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 3: Gerotor sizes

Other unknowns of the expander performance are the effects of flow rate and pressure on expander efficiency. At higher flow rates less work is required to produce the same power. There could be a tradeoff of expander efficiency for power output and an optimum point for power generation. The limited expansion ratio of the scroll could also mean that efficiency decreases beyond some pressure ratio.

The shaft power is converted to electric power by an A/C induction motor. With a 60 Hz AC excitation voltage applied to the 3-phase motor, it will draw current at speeds below 3600 RPM and produce current at speeds above 3600 RPM. Power measurement will be discussed later.

Proper flow rate at the previously presented conditions of R-123 can be estimated from the size of the expander inlet volume and the fluid density at point 3 of the cycle. The mass flow rate is the product of the density and the volume flow rate. The volume flow rate in a positive-displacement pump is the chamber volume times the cycle rate, usually the shaft speed.
\[ \dot{m} = \rho_3 \dot{V}_3 = \frac{V_{inlet} N}{v_3} \]  

(14)

The estimated mass flow rate for the expander can be used to select the other system components. The estimated flow rates for scroll types A and B are 22lbm/min and 28lbm/min respectively. The estimated flow rates for gerotor types A, B and C are 8lbm/min, 16lbm/min, and 20lbm/min respectively.

3.1.2 Liquid Pump

As discussed in section 2.1, the actual Rankine cycle differs from the ideal Rankine cycle because of pressure drop in the heat exchangers and inefficiency of the expander and pump. This results in a larger pressure difference across the pump than across the expander. The ideal pump output is 28lbm/min at 340psid. The pump for the test stand should be capable of at even more to compensate for pressure drop in the heat exchangers.

Modular Products sells a pump capable of 30lbm/min at 350psid. It requires a 3hP motor, but pump efficiency is not critical for expander evaluation. A perfectly matching, efficient pump can be selected for the system when the expander performance is understood.

The flow could be varied to see the effect on expander performance two ways: control the pump speed or route some flow around the expander. A motor controller could vary the frequency of the current sent to the pump. The controller would produce undesirable electromagnetic noise that could disturb the data acquisition. The other option, a fluid circuit from the high side to the low side of the pump could short-circuit
the excess flow. A valve in the bypass would control how much of the flow at the pump got sent to the expander. The fluid bypass was selected as the simplest method for controlling flow at this stage of the research.

3.1.3 Evaporator

The evaporator for the experiment uses steam from the OSU powerplant on the hot side and R-123 on the cold side. The requirements of the evaporator are determined using the estimate of mass flow rate calculated in equation (14) and the change in enthalpy between points 2 and 3 of the ideal Rankine cycle. The estimated heat load is 30kw. The required flow is 30lpm/min of R123 with less than 1psid pressure drop.

Two different evaporators were used in these experiments. The first is a brazed-plate heat exchanger. The second is a smaller fluted-tube-in-tube heat exchanger. The brazed-plate heat exchanger was replaced with the fluted-tube-in-tube heat exchanger to discourage lubricating oil from collecting in the evaporator.

3.1.4 Condenser

The requirements of the condenser are determined using the estimate of mass flow rate calculated in equation (14) and the change in enthalpy between points 4 and 1 of the ideal Rankine cycle. Heat load of the condenser in an actual Rankine cycle is greater than in an ideal Rankine cycle. The estimated heat load of the condenser is also 30kw. The required flow is again 30lpm/min of R123 with less than 1psid pressure drop.

A brazed-plate heat exchanger was used for the condenser.
3.1.5 Oil pump

The expanders require mineral oil for lubrication and sealing. The scroll expander requires an oil pump to collect oil from the expander housing and feed it back into the fluid before it enters the expander. A metering pump was selected for the low flow rate and high differential pressure.

The gerotor design does not collect oil, so the oil must be added to the working fluid. Five percent of the mass of the working fluid was mineral oil added to the system at charging.

3.1.6 Filter

Mechanical filters are used in the system to protect the liquid pump and the expander. The tight clearances of the positive-displacement machines cannot tolerate foreign objects. The filter on the low-pressure side, just before the pump, has a 0.03in screen size. The one on the high-pressure side, just before the evaporator, has a 0.002in screen size.

3.1.7 Working fluid

The system is charged with about 20lbm of R-123. The closed container is connected to the system near the condenser via a hose and a closed valve. The system is evacuated with a vacuum pump, and then the valve is opened to allow the R-123 into the system. The amount of refrigerant in the system is measured by weighing the container before and after charging. The liquid pump and cooling water in the condenser can be used to keep the pressure low until enough fluid is in the system.
3.2 Instrumentation

The cycle state points and power output are required to describe expander performance. The instrumentation is therefore another important part of the system. Figure 4 illustrates the necessary measurements. There are four temperatures and four pressures of interest in the fluid cycle plus four temperatures for the steam and cooling water inlet and outlet temperatures. Other measured quantities are fluid flow rate and expander power.

3.2.1 Cycle state points

The four state points of the cycle can be described with a temperature and a pressure at each point. T-type thermocouples are used for temperature measurement at each point in the cycle. Absolute pressure transducers are located at the same four points. The mass flow of the fluid is measured using an ABB coriolis mass flow meter. It is located at point 2 of the cycle – high pressure, low temperature liquid – for the best accuracy.

3.2.2 Evaporator

The inlet and outlet temperatures of the steam for the evaporator are measured using T-type thermocouples.
Figure 10: Instrumentation diagram.
3.2.3 Condenser

The inlet and outlet temperatures of the cooling water for the condenser are measured using T-type thermocouples. A rotameter measures cooling water flow rate to the condenser. These pieces of information can be used to verify heat out of the cycle at the condenser.

\[ \dot{q} = \dot{m}C_p\Delta T_{\text{water}} \]  

(15)

3.2.4 Power

The electric power from the expander is measured using two different methods. One method uses a power meter. The three current lines are connected through the power meter that automatically measures voltage, current, and phase of all three lines. It then sends a voltage proportional to the power to the data acquisition system. The other method requires measuring the voltage, current, and phase of all three lines manually. Figure 5 illustrates the wiring for this method.

The power factor of the current is related to the three measured currents in each leg. The current through the capacitor is 90° out of phase with the real current. This fact is used to calculate the real current as shown in the following equation.

\[ I_{\text{real}} = \sqrt{\left(I_{\text{motor}}\right)^2 - \left(\frac{\left(I_{\text{total}}\right)^2 - \left(I_{\text{motor}}\right)^2 - \left(I_{\text{capacitor}}\right)^2}{2I_{\text{capacitor}}}\right)^2} \]  

(16)
3.2.5 Shaft speed

Shaft speed of the gerotor expander was measured using an optical tachometer. A piece of reflective tape on the shaft allowed the tachometer to count cycles and output a voltage proportional to the speed.

The scroll expander has no external shaft. The relationship between power and shaft speed is linear near the synchronous speed. The manufacturer provided an equation to calculate the shaft speed from the power output, $P_{elec}$, in kW.

$$N = 22.986 \times P_{elec} + 3600$$  \hspace{1cm} (17)

![Diagram of power measurement with capacitors.](image)

*Figure 11: Power measurement with capacitors.*
3.3 Data Collection

The instrumentation was connected to a data acquisition system consisting of analog to digital converters, a laptop and data acquisition software. The Omega A/D converter has dedicated boards for each type of input signal. A WinBook laptop computer connects via serial port to the A/D converter. The laptop runs LabTech to display and record the measurements.

The system is intended to measure the steady-state performance of the heat engine. The measurements are displayed on the screen the entire time it is on. Data starts to be recorded to a file at the press of a button. The system samples each instrument every four seconds. The average of at least one minute of data constitutes one measurement point. Measurements that do not go through the data acquisition system (cooling water flow, shaft speed, power output) must be recorded in the lab book with a reference to the matching data file.

3.4 Calibration

All components of the instrumentation were calibrated for steady-state measurements. Two-point calibration was used for the thermocouples at freezing and boiling water. The slope-intercept equation of the linear calibration was incorporated into the LabTech setup to correct the data as it was collected. Four-point calibration was used for the pressure transducers over the full measurement range of the transducers with a dead weight tester. A linear fit of the points was used to enter a slope-intercept equation into the LabTech setup.
The mass flow meter uses the coriolis acceleration to measure the mass flow rate. The fluid reacts to vibration of the specially designed pipe with a vibration perpendicular to the input vibration. The coriolis mass flow meter simultaneously measures density and temperature. It uses that information to continuously compensate for changes in the fluid being measured. The flow meter can measure 0-250 lbm/min ±0.25% of full scale. That implies that measurements are within 1 lbm/min of the actual flow rate. All measured flow rates were at the low end of the range. At 20 lbm/min the accuracy would be ±5%. Two validation tests showed that the measured value at 10 lbm/min was 0.2 lbm/min low: only ±2%.

The test setup was improved to increase the accuracy of the power measurements. Simultaneous power measurements using both methods showed that the capacitor method reads slightly lower than the power meter: 0.1 kW at 1.6 kW. This can be attributed to truncation error in the current measurement. The solid-state power meter was used for the remaining tests.

3.5 Test Plan

The test plan consists of steps for turning on the steam, starting the heat engine, stopping the heat engine, and turning off the steam. The steps must be done in order for the safety of the people and equipment.
3.5.1 Turning on the steam

- Open the first bypass valve to equalize pressure across the first main valve on the steam station.
- Repeat for the second and third pairs of valves.
- Close the bypass valves.
- Open the valve in the steam lab to send steam to the heat engine, but leave the valve closed to the evaporator.
- Set the pressure regulator for a reasonable starting temperature. (About 30psi)

3.5.2 Starting the heat engine

- Turn on data acquisition system to display measurements.
- Make sure all steam valves are open except the valve to the evaporator.
- Make sure all valves in the fluid circuit are open except for the valve to the expander.
- Turn on cooling water to the condenser. (About 3gpm)
- Partially open valve to allow steam through the evaporator.
- Turn on the liquid pump.
- Turn on the oil-metering pump. (Scroll system)

3.5.3 Stopping the heat engine

- Close the steam valve to the boiler.
- When the pressure in the evaporator is less than 100psi, simultaneously close the valve to the expander and turn off the electricity to the generator.
• Turn off the liquid pump

• Turn off the oil-metering pump. (Scroll system)

3.5.4 Turning off the steam

• Close the valve in the steam lab.

• Close the three main valves at the steam station.

3.5.1 Adjusting operating conditions

A few tools can adjust the cycle during operation. More steam flow and higher steam temperature will increase both the temperature and pressure at the expander inlet. Increasing the pump bypass will increase the expander inlet temperature and decrease the mass flow rate and pressure difference. Increasing the cooling water flow will reduce the expander exit pressure. Removing refrigerant from the system will reduce all pressures.
CHAPTER 4

DATA AND RESULTS

4.1 Experimental Results

4.1.1 Scroll Results

Three different scroll configurations were tested. The first scroll, A, had a 3.4 volume ratio and the other two scrolls, B and C, each had a 2.7 volume ratio. One 2.7 volume ratio expander, scroll C, had a modified inlet for reduced pressure loss. See Table 2.

Scroll A was tested at three operating conditions. The maximum temperature was 270F and the maximum pressure was 218psia. The expander produced the maximum power at those conditions. The maximum power output was 1.35 kW.

Scroll B was tested at fourteen different operating conditions. Maximum temperatures ranged from 262F to 329F and maximum pressures ranged from 171psia to 263psia. Maximum measured power was 1.38 kW at 293F and 246psia.

More data was acquired using scroll C: fifteen different operating conditions. It was tested with high temperatures ranging from 259F to 311F and pressures up to 296psia. Maximum measured power was just under 3kW at 310F and 296psia.
4.1.2 Gerotor Results

Three different gerotor expanders were tested. The first gerotor, A, had an expansion ratio of 10. Gerotor B had twice the inlet volume of gerotor A and the same expansion ratio. The last gerotor, C had an expansion ratio of four with an inlet volume close to that of gerotor B. See Table 3.

The first test used gerotor A. The system operated between 183F and 90F with a maximum pressure of 60psia. Power was measured using the DC generator. The maximum measured power was 280W.

The second test used gerotor B, this time with an AC motor with 1800rpm synchronous speed for conversion to electric power. The system operated between 294F and 110F with a maximum pressure of 273psia. The condenser pressure of 33psia prevented full utilization of the 10:1 expansion ratio. Maximum measured power was 480W.

The third test used gerotor C. Maximum measured power was 2070W. The fluid power, determined from the measured temperatures and pressures, does not agree with the measured electrical power for this test. Other methods verify the heat input and power output. An energy balance of the condenser indicates that the measured heat out is accurate. The motor speed matches the manufacturer’s correlation for 2kW electric power. The difference between the measured steam inlet temperature and the measured expander inlet temperature is about 30°F for these tests. On all other tests, the difference was less than 10°F. The expander inlet temperatures were adjusted to 10°F less than the steam inlet temperature.
<table>
<thead>
<tr>
<th>Test #</th>
<th>Expander</th>
<th>Mass flow</th>
<th>$T_3$</th>
<th>$P_3$</th>
<th>$T_4$</th>
<th>$P_4$</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>lbm/min</td>
<td>°F</td>
<td>psia</td>
<td>°F</td>
<td>psia</td>
<td>kW</td>
</tr>
<tr>
<td>1</td>
<td>Scroll A</td>
<td>6.2</td>
<td>245.7</td>
<td>168.8</td>
<td>148.1</td>
<td>22.1</td>
<td>0.82</td>
</tr>
<tr>
<td>2</td>
<td>Scroll A</td>
<td>7.8</td>
<td>259.9</td>
<td>195.7</td>
<td>160</td>
<td>22.4</td>
<td>1.2</td>
</tr>
<tr>
<td>3</td>
<td>Scroll A</td>
<td>8.7</td>
<td>270.2</td>
<td>217.9</td>
<td>163.6</td>
<td>23.1</td>
<td>1.35</td>
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<tr>
<td>4</td>
<td>Scroll B</td>
<td>7.3</td>
<td>277.2</td>
<td>178.6</td>
<td>185.3</td>
<td>20.4</td>
<td>0.69</td>
</tr>
<tr>
<td>5</td>
<td>Scroll B</td>
<td>7.2</td>
<td>277.2</td>
<td>178.2</td>
<td>185.7</td>
<td>20.4</td>
<td>0.69</td>
</tr>
<tr>
<td>6</td>
<td>Scroll B</td>
<td>7.3</td>
<td>277.4</td>
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<td>Scroll B</td>
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<td>178.1</td>
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<td>8</td>
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<td>200.6</td>
<td>200.1</td>
<td>21.6</td>
<td>0.7</td>
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<td>226</td>
<td>222.2</td>
<td>24.1</td>
<td>0.79</td>
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<td>11</td>
<td>Scroll B</td>
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<td>261.7</td>
<td>178.7</td>
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<td>174.8</td>
<td>179.8</td>
<td>27.4</td>
<td>0.84</td>
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<tr>
<td>13</td>
<td>Scroll B</td>
<td>7.3</td>
<td>263.2</td>
<td>170.9</td>
<td>176.1</td>
<td>21.9</td>
<td>0.82</td>
</tr>
<tr>
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<td>10.9</td>
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<td>242.2</td>
<td>178</td>
<td>31.3</td>
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<tr>
<td>15</td>
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<td>9.7</td>
<td>272.1</td>
<td>217.5</td>
<td>173.3</td>
<td>29.1</td>
<td>1.16</td>
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<tr>
<td>16</td>
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<td>300.9</td>
<td>263.3</td>
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<td>267.9</td>
<td>210.2</td>
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<p>| Table 4: Test conditions for Scroll A and Scroll B |</p>
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<tr>
<th>Test #</th>
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<th>$T_3$</th>
<th>$P_3$</th>
<th>$T_4$</th>
<th>$P_4$</th>
<th>Power</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>lbm/min</td>
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Table 6: Test conditions for Gerotors  
<sup>a</sup>Corrrected temperature based on steam temperature
4.2 Data Reduction

The cycle state points used in the data analysis are time averages of the measured values. A data point is a time average over at least one minute of steady-state readings. The original plan was to only record data when the heat engine had been at steady state for 30 minutes, however that greatly increases the time needed to run a test. Later in the study, data for the entire operation of the heat engine was recorded. Steady-state conditions were then identified within the aggregate data.

Engineering Equation Solver (EES) software has built in thermodynamic properties for several fluids, including R-123. That software is used to analyze the data. The four pairs of temperatures and pressures are input into EES to calculate the enthalpy, entropy, and density associated with each state point. These values are used to calculate the isentropic efficiency.

Some other parameters help describe the operating conditions more than just the state points. EES can also calculate the pressure difference, pressure ratio, fluid volume ratio and amount of superheating at the expander inlet. The fluid volume ratio is compared to the expander volume ratio to calculate a new parameter, the expansion-matching ratio, $EM_{ratio}$.

$$EM_{ratio} = \frac{V_{ratio, fluid}}{V_{ratio, exp}} \quad (18)$$
The shaft speeds and inlet volumes are used to see how well the volume flow rate matches the volume rate of the expander inlet pocket to calculate the volume-matching ratio, $\hat{V}M_{\text{ratio}}$.

$$\hat{V}M_{\text{ratio}} = \frac{v \cdot \dot{m}}{V_{\text{inlet}} \cdot N} \tag{19}$$

These two matching ratios reveal limitations of positive displacement expanders. The expansion ratio is fixed, so the expander can only extract work from the fluid on that expansion ratio. In addition, the discrete pockets of the expander and synchronous speed of the induction generator determine the volume flow rate through the expander. If the matching ratios are not 1, than some of the fluid energy is lost outside the expander pocket.

### 4.3 Expander Performance

#### 4.3.1 Isentropic Efficiency

The isentropic efficiency is calculated by the ratio of the change in enthalpy during the actual expansion process to that of an isentropic expansion. (See equation (7)) The change in entropy for isentropic expansion is measured from the actual state point at the expander inlet to an isentropic state point at the same final pressure as the expander exit.
4.3.2 Power Conversion Efficiency

The isentropic efficiency tells about the fluid side of the expander machine while the power conversion efficiency tells about the mechanical side. An expander can allow isentropic expansion of the fluid, but still not efficiently convert the fluid power to useful power. Power conversion efficiency is the ratio of the electrical power out of the expander to the fluid power out of the expander.

\[
\eta_{\text{conversion}} = \frac{\text{Power}_{\text{electric}}}{\text{Power}_{\text{fluid}}} \tag{20}
\]

Power conversion efficiency can improve by reducing mechanical losses and heat transfer in the expander. Mechanical losses decrease the electric power. Heat transfer could increase the measured fluid power.

The following three tables present the performance of the expander for each test with a summary of other parameters describing the operating conditions.
<table>
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<tr>
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<th>$\text{Power}_{\text{electric}}$</th>
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<th>$\eta_{\text{conversion}}$</th>
<th>$\Delta P$</th>
<th>$P_{\text{ratio}}$</th>
<th>$V_{\text{ratio,fluid}}$</th>
<th>$\Delta T_{3,\text{superheat}}$</th>
<th>$E_{\text{Mratio}}$</th>
<th>$V_{\text{Mratio}}$</th>
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Table 7: Expander performance for scroll A and scroll B
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Table 8: Expander performance for scroll C
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<th>$P_{\text{ratio}}$</th>
<th>$V_{\text{ratio,fluid}}$</th>
<th>$\Delta T_{3,\text{superheat}}$</th>
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Table 9: Expander performance for gerotors
4.4 System Performance

4.4.1 Efficiency

The system thermal efficiency (see equation 6) can be calculated in terms of the actual system or a system with properly sized heat exchangers and liquid pump. The pressure drops through the test system would be unacceptable for production. A commercial system would be packaged with shorter lines and fewer adapters than the test system. The simple fluid bypass was an inefficient method to control the flow rate. Sometimes the pump used more power than the expander produced. The actual isentropic efficiency for the pump was very low. The isentropic efficiency for a properly sized pump operating at the right speed could reasonably be about 85 percent.

These total system performance calculations estimate 5psid in each heat exchanger and use an estimate of pump work for a properly sized pump. The low temperature is replaced with the temperature for 10°F subcooled liquid at the pump inlet pressure.

4.4.2 Net Power

The projections for net power also use the estimate of pump power for a properly sized pump. The net power is just the power from the expander minus the power to the liquid pump.
4.4.3 Comparison with Theoretical Performance

The ideal performance of a system operating at these conditions is calculated using the same program as for the cycle efficiency modeling with different working fluids in Table 1. The ideal cycle uses the same high temperature and low pressure as measured in testing. The high pressure is adjusted to make the fluid 10°F superheated at the expander inlet. There is no pressure drop through the heat exchangers. The expansion process is isentropic and all work is converted to electricity. This represents the best possible thermal efficiency for a Rankine cycle at the given conditions.

The ideal power output is estimated by assuming 3600rpm shaft speed and perfect volume matching: the volume flow rate of the fluid into the expander is equal to the volume rate of the expander inlet pocket. With the induction generators used for most of the testing, the shaft speed was within 1% of 3600rpm.

The following tables compare system performance with the actual expander to an ideal Rankine cycle. The left side shows the measured flow rates, inlet temperatures and pressures, and exit temperatures and pressures along with the projected net powers and thermal efficiencies for complete systems at those conditions. The right side shows the same values for an ideal cycle.
| Test # | Expander | Actual | | | | Ideal | | | |
|-------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|       |          | Flow   | $T_3$  | $P_3$  | $T_4$  | $P_4$  | $\eta_{th}$ | Flow   | $T_3$  | $P_3$  | $T_4$  | $P_4$  | $\eta_{th}$ |
|       |          | lbm/min|  °F    | psia   |  °F    | psia   | kW      | lbm/min|  °F    | psia   |  °F    | psia   | kW      |
| 1     | Scroll A | 6.2    | 245.7  | 168.8  | 148.1  | 22     | 0.79    | 0.1    | 1.76   | 0.2    |        |        |        |        |        |
| 2     | Scroll A | 7.8    | 259.9  | 195.7  | 160    | 22     | 1.15    | 0.1    | 2.26   | 0.2    |        |        |        |        |        |
| 3     | Scroll A | 8.7    | 270.2  | 217.9  | 163.6  | 23     | 1.29    | 0.1    | 2.65   | 0.2    |        |        |        |        |        |
| 4     | Scroll B | 7.3    | 277.2  | 178.6  | 185.3  | 20     | 0.65    | 0.1    | 4.46   | 0.2    |        |        |        |        |        |
| 5     | Scroll B | 7.2    | 277.2  | 178.2  | 185.7  | 20     | 0.65    | 0.1    | 4.46   | 0.2    |        |        |        |        |        |
| 6     | Scroll B | 7.3    | 277.4  | 178.3  | 185.7  | 20     | 0.65    | 0.1    | 4.49   | 0.2    |        |        |        |        |        |
| 7     | Scroll B | 7.3    | 277.4  | 178.1  | 185.5  | 20     | 0.65    | 0.1    | 4.5    | 0.2    |        |        |        |        |        |
| 8     | Scroll B | 7.3    | 277.9  | 178.1  | 185.3  | 20     | 0.64    | 0.1    | 4.52   | 0.2    |        |        |        |        |        |
| 9     | Scroll B | 8.2    | 301    | 200.6  | 200.1  | 22     | 0.64    | 0.1    | 6.38   | 0.2    |        |        |        |        |        |
| 10    | Scroll B | 9.2    | 329.1  | 226    | 222.2  | 24     | 0.72    | 0.1    | 9.45   | 0.2    |        |        |        |        |        |
| 11    | Scroll B | 7.8    | 261.7  | 178.7  | 185.6  | 36     | 0.44    | 0.1    | 2.56   | 0.1    |        |        |        |        |        |
| 12    | Scroll B | 7.6    | 262    | 174.8  | 179.8  | 27     | 0.8     | 0.1    | 2.99   | 0.2    |        |        |        |        |        |
| 13    | Scroll B | 7.3    | 263.2  | 170.9  | 176.1  | 22     | 0.78    | 0.1    | 3.42   | 0.2    |        |        |        |        |        |
| 14    | Scroll B | 10.9   | 280.3  | 242.2  | 178    | 31     | 1.07    | 0.1    | 3.84   | 0.2    |        |        |        |        |        |
| 15    | Scroll B | 9.7    | 272.1  | 217.5  | 173.3  | 29     | 1.09    | 0.1    | 3.46   | 0.2    |        |        |        |        |        |
| 16    | Scroll B | 11.6   | 300.9  | 263.3  | 200.7  | 48     | 1.28    | 0.1    | 4.32   | 0.1    |        |        |        |        |        |
| 17    | Scroll B | 9.2    | 267.9  | 210.2  | 173.3  | 31     | 1.23    | 0.1    | 3.09   | 0.2    |        |        |        |        |        |

Table 10: Comparison to ideal cycle for scroll A and scroll B
| Test # | Expander | Actual | | | Ideal | | | | | | Flow | T<sub>3</sub> | P<sub>3</sub> | T<sub>4</sub> | P<sub>4</sub> | Power<sub>net</sub> | η<sub>th</sub> | Flow | T<sub>3</sub> | P<sub>3</sub> | T<sub>4</sub> | P<sub>4</sub> | Power<sub>net</sub> | η<sub>th</sub> |
|--------|----------|--------|--------|--------|----------------|--------|--------|--------|--------|--------|--------|----------------|--------|--------|--------|--------|----------------|--------|
|        |          | lbm/min | °F     | psia  | °F     | psia  | kW    |        | lbm/min | °F     | psia  | °F     | psia  | kW    |        |
| 18     | Scroll C | 12.1   | 259    | 182.7 | 177    | 34    | 1.42  | 0.09   | 11.8    | 259    | 175.7 | 159.3 | 34    | 2.5   | 0.14 |
| 19     | Scroll C | 18.8   | 293.4  | 245.7 | 205.7  | 40.6  | 2.12  | 0.09   | 17.8    | 293.4  | 253.7 | 173.6 | 40.6  | 4.18  | 0.15 |
| 20     | Scroll C | 12.7   | 248.7  | 171.8 | 184.6  | 56.3  | 0.98  | 0.08   | 10.4    | 248.7  | 156.3 | 184.7 | 56.3  | 1.36  | 0.09 |
| 21     | Scroll C | 14.6   | 267.1  | 200.5 | 200.6  | 65.5  | 0.83  | 0.08   | 13      | 267.1  | 192.1 | 196.4 | 65.5  | 1.78  | 0.1  |
| 22     | Scroll C | 16     | 290    | 226.4 | 221.3  | 73    | 1.44  | 0.08   | 17.1    | 290    | 245   | 205.9 | 73    | 2.6   | 0.11 |
| 23     | Scroll C | 18.9   | 310.6  | 259.6 | 239.6  | 85    | 1.61  | 0.08   | 21.9    | 310.6  | 301.4 | 216.8 | 85    | 3.41  | 0.11 |
| 24     | Scroll C | 15.5   | 262.4  | 184.4 | 176.9  | 32.6  | 1.25  | 0.1    | 12.3    | 262.4  | 182.4 | 157.7 | 32.6  | 2.73  | 0.14 |
| 25     | Scroll C | 18.6   | 278    | 217.6 | 194.5  | 35.8  | 1.59  | 0.08   | 14.8    | 278    | 216.1 | 165   | 35.8  | 3.43  | 0.15 |
| 26     | Scroll C | 21.1   | 297    | 247.8 | 210.1  | 38.7  | 2.01  | 0.08   | 18.6    | 297    | 263.2 | 171.5 | 38.7  | 4.56  | 0.15 |
| 27     | Scroll C | 22.4   | 291.4  | 244.7 | 201    | 33.5  | 2.23  | 0.08   | 17.4    | 291.4  | 248.6 | 163.3 | 33.5  | 4.48  | 0.16 |
| 28     | Scroll C | 21.9   | 293.8  | 246.8 | 203.7  | 34.6  | 2.25  | 0.08   | 17.9    | 293.8  | 254.7 | 165.3 | 34.6  | 4.59  | 0.16 |
| 29     | Scroll C | 24.5   | 295.9  | 264.3 | 200.4  | 37.1  | 2.37  | 0.09   | 18.3    | 295.9  | 260.3 | 169.1 | 37.1  | 4.58  | 0.16 |
| 30     | Scroll C | 25.1   | 302.3  | 272   | 209.3  | 35.9  | 2.48  | 0.08   | 19.8    | 302.3  | 277.6 | 168   | 35.9  | 5.18  | 0.16 |
| 31     | Scroll C | 26.7   | 304.5  | 282.3 | 210.6  | 37.6  | 2.54  | 0.08   | 20.3    | 304.5  | 283.8 | 170.6 | 37.6  | 5.25  | 0.16 |
| 32     | Scroll C | 28.3   | 309.5  | 296   | 212.6  | 36.2  | 2.68  | 0.08   | 21.6    | 309.5  | 298.2 | 169   | 36.2  | 5.81  | 0.17 |

**Table 11:** Comparison to ideal cycle for scroll C
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<th>P3 (psia)</th>
<th>T4 (°F)</th>
<th>P4 (psia)</th>
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Table 12: Comparison to ideal cycle for gerotors
4.5 Parametric Study

4.5.1 Data Correlation

It would be useful if the results of these tests could be used to predict expander performance for any operating conditions. The goal is to improve expander efficiency, conversion efficiency, net power and thermal efficiency. It is important to know what affects these performance characteristics when designing a heat engine with one of these expanders.

Correlating the data shows what are the most important factors affecting expander efficiency, conversion efficiency, net power and thermal efficiency. Statistical correlation calculates if values in one set of numbers are related to values in another set. The correlation coefficient for two sets of numbers is the covariance of the two sets divided by the product of their standard deviations as shown in equation 19.

\[ \rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \cdot \sigma_Y} \]  

where

\[ \text{cov}(X,Y) = \frac{1}{n} \sum (x_i - \mu_x)(y_i - \mu_y) \]  

and

\[ \sigma_X^2 = \frac{1}{n} \sum (x_i - \mu_x)^2 \]  

and

\[ \sigma_Y^2 = \frac{1}{n} \sum (y_i - \mu_y)^2 \]  

If the correlation coefficient is one, the two sets are directly related. If the correlation coefficient is negative one, the two sets are also related: large values of one
correlate with small values of the other. If the correlation coefficient is zero, the two sets are completely independent. The following table shows the correlation coefficients for the performance characteristics with the operating condition parameters.

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<th>$\eta_{\text{expander}}$</th>
<th>$\eta_{\text{conversion}}$</th>
<th>$\text{Power}_{\text{net}}$</th>
<th>$\eta_{\text{th}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{m}$</td>
<td>-0.32581</td>
<td>0.225907</td>
<td>0.836199</td>
<td>-0.44534</td>
</tr>
<tr>
<td>$T_3$</td>
<td>-0.22525</td>
<td>0.179777</td>
<td>0.565874</td>
<td>0.101847</td>
</tr>
<tr>
<td>$P_3$</td>
<td>-0.06219</td>
<td>0.196347</td>
<td>0.502258</td>
<td>0.438866</td>
</tr>
<tr>
<td>$P_4$</td>
<td>0.422775</td>
<td>0.206411</td>
<td>0.357016</td>
<td>-0.20798</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>-0.17806</td>
<td>0.151402</td>
<td>0.432858</td>
<td>0.516801</td>
</tr>
<tr>
<td>$P_{\text{ratio}}$</td>
<td>-0.27307</td>
<td>-0.02767</td>
<td>-0.05211</td>
<td>0.642716</td>
</tr>
<tr>
<td>$\Delta T_{3,\text{Superheat}}$</td>
<td>-0.24737</td>
<td>-0.05437</td>
<td>0.100796</td>
<td>-0.54078</td>
</tr>
<tr>
<td>$EM_{\text{ratio}}$</td>
<td>-0.28139</td>
<td>0.273951</td>
<td>0.209696</td>
<td>0.515744</td>
</tr>
<tr>
<td>$VM_{\text{ratio}}$</td>
<td>-0.23482</td>
<td>-0.12774</td>
<td>0.262268</td>
<td>-0.57308</td>
</tr>
</tbody>
</table>

Table 13: Correlation coefficients from test data of all expanders

The first thing to notice is that there is no silver bullet to improve all performance characteristics at once. For each characteristic, a few parameters correlate and some do not. For instance, the net power is nearly directly related to the mass flow rate and independent of the pressure ratio. The expansion-matching ratio correlates best with the energy conversion efficiency, but at only 27%. It is most likely a property of the expander that is only slightly affected by other factors.
4.5.2 Regression Analysis

Linear regression can describe the performance characteristics as a linear function of the operating condition parameters. The top three parameters correlating with each will have the most influence so only these are used for the regression. Table 11 lists the coefficients, the $R^2$ values and the standard error for each. $R^2$ is the coefficient of determination or the square of the correlation coefficient. The standard error measures the amount of variation due to other factors.

<table>
<thead>
<tr>
<th></th>
<th>$\eta_{\text{expander}}$</th>
<th>$\eta_{\text{conversion}}$</th>
<th>$\text{Power}_{\text{net}}$</th>
<th>$\eta_{\text{th}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6514</td>
<td>0.2829</td>
<td>0.1825</td>
<td>0.0724</td>
</tr>
<tr>
<td>$m$</td>
<td>-0.0085</td>
<td>0.0036</td>
<td>0.0727</td>
<td>-</td>
</tr>
<tr>
<td>$T_3$</td>
<td>-</td>
<td>-</td>
<td>-0.0027</td>
<td>-</td>
</tr>
<tr>
<td>$P_3$</td>
<td>-</td>
<td>-</td>
<td>0.0037</td>
<td>-</td>
</tr>
<tr>
<td>$P_4$</td>
<td>0.0046</td>
<td>0.0035</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$P_{\text{ratio}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0040</td>
</tr>
<tr>
<td>$T_{3,\text{Superheat}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0002</td>
</tr>
<tr>
<td>$EM_{\text{ratio}}$</td>
<td>-0.0118</td>
<td>0.0654</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$VM_{\text{ratio}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0019</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.5536</td>
<td>0.2109</td>
<td>0.7557</td>
<td>0.5362</td>
</tr>
<tr>
<td>$S_{y-x}$</td>
<td>0.0652</td>
<td>0.1549</td>
<td>0.3520</td>
<td>0.0127</td>
</tr>
</tbody>
</table>

Table 14: Linear regression coefficients
4.6 Error Analysis

Measurement errors propagate through the calculations and affect the results. The EES software can take the error associated with each measurement and calculate how much the final numbers may vary. The measurements and uncertainty are listed along with the performance characteristics and uncertainty in Table 12. The uncertainty propagation for the performance characteristics shows that the uncertainty associated with the regression analysis is about the same as the uncertainty of the original calculation. The errors caused by excluding factors from the estimate are no larger than the errors caused by original measurement system.

<table>
<thead>
<tr>
<th>Experimental Measurement</th>
<th>Uncertainty</th>
<th>Performance Characteristic</th>
<th>Calculation Uncertainty</th>
<th>Linear Regression Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{m}$</td>
<td>1lbm/min</td>
<td>$\eta_{\text{expander}}$</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>$T_3, T_4$</td>
<td>2F</td>
<td>$\eta_{\text{conversion}}$</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>$P_3, P_4$</td>
<td>1psia</td>
<td>$\text{Power}_{\text{net}}$</td>
<td>0.1kW</td>
<td>0.1kW</td>
</tr>
<tr>
<td>$\text{Power}_{\text{electric}}$</td>
<td>0.1kW</td>
<td>$\eta_{\text{th}}$</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 15: Uncertainty propagation
CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Optimum Operating Conditions

Based on the data from the expanders tested in this study, the operating conditions can be adjusted to increase the system performance. One of the goals of the heat engine concept is to provide 3 to 7 kilowatts of electricity. It should operate between 350°F and 70°F. The analysis shows that net power output is a strong function of mass flow rate and expander inlet temperature and pressure. The ideal inlet condition is high-pressure, saturated vapor. The required flow rate can be calculated with the function.

\[
Power_{net} = 0.1825 + 0.0727 \dot{m} - 0.0027 T_i + 0.0037 P_i \quad (25)
\]

\[
5kW = 0.1825 + 0.0727 \dot{m} - 0.0027(330°F) + 0.0037(363psia) \quad (26)
\]

\[
\dot{m} = 60 \text{ lbm/min} \quad (27)
\]

Thermal efficiency is proportional to the pressure ratio, so the exit pressure should be as low as possible. It is inversely proportional to the amount of superheating and the volume-matching ratio. The thermal efficiency of the cycle can be estimated.
\[ \eta_{th} = 0.0724 + 0.004(P_{ratio}) - 0.0019(\dot{V}M_{ratio}) - 0.0002(T_{s,superheat}) \]  
(28)

\[ \eta_{th} = 0.0724 + 0.004(363\text{psia}/21\text{psia}) - 0.0019(1) - 0.0002(10^\circ F) \]  
(29)

\[ \eta_{th} = 14\% \]  
(30)

Fourteen-percent thermal efficiency is an order of magnitude greater than any of the organic Rankine heat engines mentioned previously.

These values are extrapolated using linear fits to experimental data. The resulting estimate gives a better idea of the proper sizes for the other system components than the model of an ideal cycle originally used. The estimates assume an efficient pump capable of the flow and pressure difference, not the one used in testing. Also note that none of the expanders tested are capable of utilizing the entire 20:1 pressure ratio of the cycle in this estimate.

5.2 Matching components

The liquid pump that is required for this system should be able to deliver 60lbfm R123 at 342psid with 85% isentropic efficiency. Such a pump may not currently exist.

A heat engine with 14% thermal efficiency would require 35kW from the solar collectors for 5kW of power. The evaporator and condenser can be specified based on the 35kW heat load and the operating pressures.

5.3 Recommendations

The induction generators used in this study may have limited the expanders by forcing the shaft speed. Further study with these machines should consider other
generators and see how the volume rate of the expander affects performance. There are published speed/flow efficiency maps for most turbomachines. That same information would be valuable for designing a system using one of these new expanders.

Both the scroll and gerotor expanders are suitable for use in a low-temperature heat engine. The next step is to scale up the system components to build a 5kw system. A control strategy to keep the expander inlet state just slightly superheated would be helpful to compensate for varying heat input.
REFERENCES


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