Robust Heart Rate Variability Analysis using Gaussian Process Regression

THESIS

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Abstract

Heart rate variability (HRV) is a non-invasive way of measuring autonomic nervous system dynamics as influenced by one’s emotional state. By studying beat to beat variations, the dynamic process regarding homeostasis can be studied. Quantification of HRV in frequency domain has traditionally been done in two ways: firstly, interpolating the unevenly sampled inter-beat intervals to a fixed number of samples followed by applying Fast Fourier Transform and secondly by using Lomb’s periodogram.

In this thesis, a different approach is presented. Gaussian Process Regression (GPR) is used in order to interpolate the inter-beat intervals (training data set) to get a mean and variance bound, thus quantifying the time domain tachogram. This tachogram is then used as input to a FFT in order to characterize the frequency domain of HRV. The quantification of HRV using Gaussian Process Regression and Lomb’s Periodogram is compared in lossy data conditions and in the presence of motion artifacts. The results show that GPR based FFT technique is less susceptible to outliers than lomb’s periodogram. The effect of outliers and techniques for their removal are discussed for GPR based processing.
Dedication

Dedicated to my parents Heena and Sagar Shah.
Acknowledgment

This thesis arose in part out of two year of development that has been done since I joined IPS lab. During this time, I have worked with a great number of people who have contribution in assorted ways to the research and the making of the thesis deserved special mention. It is a pleasure to convey my gratitude to them all in my humble acknowledgment. Firstly, I would like to convey my gratitude to Dr. Emre Ertin for his guidance from the very early stage of my Masters as well as his constant motivation and intuitive ideas which have helped me constantly learn new things from him.

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CHAPTER 1
INTRODUCTION

Heart rate variability refers to the beat by beat variation in the heart rate. Technological advancements, have helped in quantification and studies of heart rate variability. Advances in measuring instruments include the galvanometer, the kymograph, the inkwriting polygraph and finally modern digital signal processing systems. Digital signal processing has given us the power of post processing, algorithm development, accurate quantification techniques and real time statistical inference. With the power of embedded technology and signal processing, devices are becoming smaller in size and more powerful.

AutoSense [1] is one such technology, which is used as is a guiding system that motivates the problem studied in this thesis. AutoSense has been developed for accurate monitoring of physiological signals in a completely unobtrusive manner. Wireless technology and the growing semiconductor industry have miniaturized systems leading to social and personal acceptance of wearability of devices such as AutoSense. Unobtrusive monitoring of a subject opens doors to a variety of different situations where monitoring can be done.

Over the years, physiological monitoring has been restricted to laboratories with large wall instruments and with wired systems that are bulky, expensive, and have proprietary technology which cannot be changed. With the development of a systems such as AutoSense, now there is an inexpensive device with a long lifetime having a wireless connection which does not cause social embarrassment.
The AutoSense module uses wireless transmission of data packets. This leads to problems such as lost packets if the channel is lossy. In the field, subject compliance to a set of rules regarding position and placement of the device may be compromised. Where some of the data collected by the sensors does not reach the mobile device for processing. One way to reduce this loss is by providing a distributed algorithm as explained in the chapters below. This reduces the number of packets transmitted over the radio and therefore the radio transmission time. The lower the radio transmission time the longer the life time of the device. A second challenge is posed by motion artifacts which, similar to the lossy channel problem, cannot be controlled in the field when the device is worn by the subjects. Motion artifacts are a major source of false positives and negatives. For accurate detection of inter-beat intervals and quantification of time domain and frequency domain features, motion artifacts and lossy networks need to be dealt with.

The computation of heart rate variability features require the interpolation of inter beat intervals because it is an unevenly sampled data set. Over the years people have used different techniques to help find the frequency domain features such as cubic spline interpolation of the inter beat intervals or the use of Lomb’s periodogram.

In this thesis, the Gaussian Process Regression model is proposed for obtaining a time domain tachogram. This tachogram is then expressed in the frequency domain using a conventional Fourier Transform to get the frequency domain features of the heart rate. Comparisons are drawn between Lomb’s periodogram and Gaussian process regression based analysis in order to quantify the ability to handle lossy data and motion artifacts.

1.1 Contribution

The main contributions of the thesis are as follows.
• Proposing a new distribute framework for HRV analysis on low power sensors (section 3.3)
• Gaussian process regression for quantification of heart rate variability (section 4)
• Outlier rejection method that incorporates probabilistic framework setup by Gaussian process regression (section 4.2.1)
• Empirical comparison with Lomb’s periodogram for heart rate variability analysis (section 5.1)
2.1 Physiology of the human heart

The human heart [34] is controlled by a series of electrical discharges from a specific localized nodes within the cardiac muscle. These discharges propagate through the cardiac muscle and stimulate contractions in a coordinated manner in order to pump deoxygenated blood to the lungs for oxygenation and back into the vascular system. The physical action of the heart is therefore induced by a local periodic electrical stimulation. As a result of the electrical stimulation a change in potential of the order of 1mV can be measured during the cardiac cycle between two surface electrodes attached to the patient’s upper torso. This signal is known as the electrocardiogram (ECG). The ECG detector works mostly by detecting and amplifying the tiny electrical changes on the skin that are caused when the heart muscle depolarizes during each heart beat.
The electrical signal starts in the sinoartial node (SA node) which is the pacemaker of the human heart. The electrical vector moves from the SA node towards the ventricles. The polarization and depolarization of the different heart muscles creates the P-Q-R-S-T wave as shown in figure 2.1.

2.2 Heart Rate Variability

Heart rate variability is a non-invasive way of measuring autonomic nervous system dynamics as influenced by one’s emotional state [33]. It is indicative of neurocardiac fitness and overall health. Thus, by studying beat to beat variations, the dynamic process regarding homeostasis can be studied. Heart rate (HR) and Heart rate variability (HRV) are inversely correlated. That is, stimuli that increase HR often depress the variability of the HR in the short term. Conversely, activities that cause a drop in the average HR can lead to an increase in short term HRV. Although the strength of this correlation can change over time and from individual to individual, it is useful to consider the cardiovascular system from a static perspective in order to gain an insight into the relationship between the cardiovascular parameters.
Our body continuously responds to external and internal changes. Responses such as muscle tension, sweating and breathing rate continuously change. These changes are fundamental for maintaining homeostasis. A stress response similarly leads to many physiological changes, including increased heart rate, increased breathing rate, muscle tension and sweating. Some of these changes are quick to respond while some are slower. Thus the body tries to balance between catabolic (Body is breaking down tissue) and anabolic (Body is building or repairing tissue) states to maintain equilibrium. The Autonomic Nervous System (ANS) [6] helps to send signal from the brain to the target organs. The ANS consists of two parts: the parasympathetic nervous system (PNS) and the sympathetic nervous system (SNS). The SNS response is generally catabolic and leads to responses that require spending of energy; on the other hand the PNS response is anabolic which conserves energy.

Heart rate variability was greatly ignored before Appel et al. [9] (1989) put the question forward. The presence of beat to beat variations was widely known but because of lack of digital resources it was studied in lesser detail. Appel et al. put forward the question - ‘Are the fluctuations in HR noise or music?’ Since then several statistical methods have been applied to the HR signal and observations have been made regarding the meaning of the variability. These studies raised questions regarding how the HR was responsive to changes in behavior. In situations such as running, the heart beats faster but also heart rate does increase while a person is seated in an arm chair and presented psychological stimulus, such as frightening pictures, arithmetic tasks, loud noise etc. Thus both the PNS and the SNS can affect HR. In conclusion, physiological signals can inform as about the status of ANS and it’s adaptation to different situations.

The main steps of HRV analysis are as follows

- Detection of heart beats from ECG
- Computation of time domain tachogram
2.2.1 Heart Beat Detection

In this subsection, we review a popular ECG beat detection algorithm, Pan and Tompkins algorithm [10], that serves as a starting point to the distributed HRV analysis framework proposed in Section 3.3.

In order to first attenuate noise, the signal is passed through a digital band pass filter. The desirable pass band for this filter is between 5-15 Hz. After filtering, the signal is differentiated to provide the QRS-complex slope information. The differentiation stage gets out the information regarding the QRS complex. The frequency response of this derivative is nearly linear between dc and 30 Hz. Thus, it approximates an ideal derivative over this range. These steps are shown in figure 2.2.

![Figure 2.2. Raw, Bandpassed and differentiated ECG signal](image)

Next, as shown in figure 2.3, is squaring of the differentiated signal which is followed by a moving window integration implemented as a low pass filter. The result is a non-negative signal with nonlinear amplification of the output of the derivative emphasizing the
higher frequencies (i.e., predominantly the ECG frequencies). The squaring process actually intensifies the differentiated signal which reduces the false negatives that are caused by the T waves. The final low pass filter smooths the signal giving us both information of the slope as well as the width of the QRS complex.

![Squared and Low passed ECG signal](image)

Figure 2.3. Squared and Low passed ECG signal

The thresholds are automatically adjusted relative to the noise. Low thresholds are possible because of the improvement of the signal-to-noise ratio by the bandpass filter. The algorithm does have two phases: learning phase and detection phase. The thresholding part of the algorithm uses a dual threshold as described below. The dual threshold helps to reduce false negatives and find missed beats.

In figure 2.4, we can clearly distinguish the initial learning phase of the algorithm. This phase helps the algorithm to set up the initial conditions required for the dual thresholding algorithm to work efficiently. After the initial phase any change in the power of the detector can be adjusted to in real time. This unique feature helps to adjust not only to any physiology of heart beat but also to sudden changes due to any change in signal power.
The set of thresholds initially applied to the low pass filtered waveform is computed using the following equations [10]:

\[ SPK = 0.125 \times PEAK + 0.875 \times SPK \] \hspace{1cm} (2.1)

where \( PEAK \) is the signal peak and,

\[ NPK = 0.125 \times PEAK + 0.875 \times NPK \] \hspace{1cm} (2.2)

where \( PEAK \) is the noise peak. The thresholds are given by

\[ THRESHOLDI = NPK + 0.25 \times (SPK - NPK) \] \hspace{1cm} (2.3)

\[ THRESHOLDII = 0.5 \times THRESHOLDI \] \hspace{1cm} (2.4)
where,

PEAK is the overall peak,

SPK is the running estimate of the signal peak,

NPK is the running estimate of the noise peak,

THRESHOLD I is the first threshold applied, and

THRESHOLD II is the second threshold applied.

If the algorithm does not detect a beat for 166 percent of the average R-R interval it searches back to quantify if it has missed any of the peaks by using the second threshold which is half of the first threshold. Two R-R interval averages are maintained. One is the average of the eight most-recent beats. The other is the average of the eight most-recent beats having RR intervals that fall within certain limits. The reason for maintaining these two separate averages is to be able to adapt to quickly changing or irregular heart rates. The first average is the mean of the eight most-recent sequential RR intervals regardless of their values.

\[
RRAVERAGE_1 = 0.125 \times (RR_{n-7} + RR_{n-6} + \ldots + RR_n) \tag{2.5}
\]

where \(RR_n\) is the most-recent RR interval.

The second average is based on selected beats.

\[
RRAVERAGE_2 = 0.125(RR'_{n-7} + RR'_{n-6} + \ldots + RR'_n) \tag{2.6}
\]
where \( RR'_n \) is the most recent RR interval that fell between the acceptable low and high RR-interval limits. The RR-interval limits are

\[
RR \text{ LOW LIMIT} = 92\% \text{ RRAVERAGE2} \tag{2.7}
\]

\[
RR \text{ HIGH LIMIT} = 116\% \text{ RRAVERAGE2} \tag{2.8}
\]

\[
RR \text{ MISSED LIMIT} = 166\% \text{ RRAVERAGE2} \tag{2.9}
\]

The search back techniques reduces buffer length required to store a processed signal. Also, on detection of a QRS complex there is a 200 ms second period where no detections are considered as valid detections. This is useful to avoid false negatives that are caused by the T-wave.

The algorithm’s adaptive thresholds reduces false negatives and also allows adaptation to various different ECG morphologies. The algorithm was modified in order to be run on the Autosense system as described later in chapter 3 of the thesis. The distributed computing [5] of the algorithm helps in reducing radio time. It is very important to evaluate a QRS detector algorithm using a standard arrhythmia database. Over recent years, advances in hardware technology have made possible the acquisition of large databases of multi-channel ECG’s such as the Harvard and Massachusetts Institute of Technology Division of Health Science and Technology’s MIT-BIH arrhythmia database [32]. This comprises hundreds of two-channel ECGs recorded from patients who suffer from various known heart conditions, as well as examples of healthy ECGs. These records have been annotated by clinicians and thus can be used to develop diagnostic software. Tools, available from MITs web site, enable the
programmer or researcher to call libraries that read and compare the clinician-annotated files (for each patient) with any test algorithm. The database and libraries of comparative tests conform to the relevant American National Standards Institute (ANSI) guidelines developed by the Association for the Advancement of Medical Instrumentation (AAMI). The MIT-BIH database is usually quoted as the ECG data source when results of detection algorithms are presented in the literature, and all the associated analysis libraries are in ANSI C. The MIT tool set includes a waveform browser which loads and displays the original file together with an annotation file (either the standard attribute file scored by clinicians or user generated). This therefore makes it an ideal database on which to test and evaluate any new algorithm development.

2.3 Time domain features of inter-beat intervals

Once we have computed the R-R intervals based on the ECG wave, we can use the intervals to find various time domain and frequency domain features. In time domain, some of the measures that are measured are [8]:

- SDNN (ms) - standard deviation of inter beat intervals (IBI’s) usually over 24 hours
- SDANN (ms) Standard deviation of the averages of NN intervals in all 5-minute segments of the entire (24-hour) recording.
- RMSSD (ms) The square root of the mean of the sum of the squares of differences between adjacent NN intervals
- SDSD (ms) Standard deviation of differences between adjacent NN intervals.
- NN50 (count) Number of pairs of adjacent NN intervals differing by more than 50 ms in the entire recording; three variants are possible: counting all such NN intervals pairs, counting only pairs in which the first interval is longer, and counting only pairs in which the second interval is longer.
• pNN50 Percentage of adjacent NN differing by more than 50ms over an entire 24-hour ECG recording.

Over the years many of these statistics have been computed and results have been shown with varying amount of success in correlating these features to stress and alcohol abuse [23].

2.4 Frequency domain features of inter-beat intervals

First the detected beats at times \( t_k \) are transformed to time series data of the form \( x(t_k) \) where \( x(t_k) = t_k - t_{k-1} \). Then the Fourier transform [35] of this unevenly sampled time series is obtained to compute Energy in various frequency bands. Long time frequency analysis is done over 24 hour signals.

- The slowest responses are captured in the ULF-ultra low frequency power(0.0001-0.0003Hz)
- The VLF-very low frequency responses(0.003-0.04Hz)
- The LF-low frequency responses(0.04-0.15Hz)
- The HF-high frequency(0.15-0.5Hz)

The motivation for splitting the spectrum into these frequency bands lies in the belief that the distinct biological regulatory mechanisms that contribute to HRV act at frequencies that are confined within these bands. Fluctuations below 0.04Hz in the VLF and ULF bands are thought to be due to long-term regulatory mechanisms such as the thermoregulatory system. Serrador et al. [11] demonstrated that the ULF band appears to be dominated by contributions from physical activity and that HRV in this band tends to increase during exercise.

Two bands have been suggested for short time frequency analysis.

- The LF-low frequency responses(0.04-0.15Hz)
- The HF-high frequency (0.15-0.5Hz)

The ratio of LF to HF is used as an indication of sympatho-vagal balance. The rationale behind this index is that, compared to HF, LF is more influenced by the SNS.

Spectral analysis involves decomposing a signal into a sum of sine waves of different amplitudes and frequencies. The power spectrum presents the squared amplitude of the sine waves as a function of frequency. Fourier analysis needs a fixed sampling rate. Since the RR tachogram is a representation of the beat-to-beat variability of each systole in the cardiac cycle with both axes representing the time between beats, it is inherently a discrete, uneven time series (otherwise there would be no variability in the heart rate).

However, almost all of the published power spectral density estimation techniques described in the relevant literature require evenly sampled data. Pre-processing of the RR tachogram with re-sampling techniques (such as linear or cubic spline re-sampling) is usually the means of producing an evenly sampled time series. Re-sampling introduces an implicit assumption about the form of the underlying variation in the RR tachogram; for example, cubic spline techniques assume that the variation between beats can be modelled accurately by a cubic polynomial. Thus, to apply Fourier analysis the R-R intervals need to be interpolated to a fixed number of data points and then traditional Fourier methods can be used for computation of the spectrum. The second method involves the use of Lomb’s periodogram [12].

In 1976, Lomb described a method of deriving the power spectral density of an unevenly sampled signal. Methods for PSD estimation based directly on irregularly sampled time series have been used, though not in HRV analysis. Methods such as the Lomb’s periodogram entirely avoid the problems associated with resampling and sample replacement. In 1989, Press and Rybicki [13] published a fast algorithm for obtaining an arbitrarily accurate approximation to the Lomb’s periodogram. In Lomb’s method the data are weighed on a
Lomb and Fourier transform spectra are derived using $O(N \log N)$ algorithms, thus computationally there may be little reason to choose one over another. Lomb’s periodogram is supposed to produce robust PSD estimates in presence of noise and ectopy. In the sections below, Gaussian Process Regression is used in order to estimate the PSD. Also, empirical analysis is done to analyze the effect of noise on frequency domain analysis using both Lomb’s periodogram and Gaussian Process Regression.
2.5 Gaussian Process Regression

In statistics, regression analysis includes the techniques of analyzing the relationship between dependent variables based on independent variables. The history of linear regression dates back to 1875 when Sir Francis Galton [36] applied the technique to the inherited characteristics of sweet peas. The first rigorous treatment of regression and correlation was published in 1896 by Pearson in the Philosophical Transactions of the Royal Society of London. This laid the foundation for linear regression theory. This model though could only analyze the relationship between the input and the output by a linear function, this approximations gave poor predictions.

The connection between the linear model and the Gaussian process regression model comes from projecting the inputs into a higher dimensional feature space where we may use the linear model. The concept of Gaussian process regression was named after Carl Friedrich Gauss because of it being based on the Gaussian distribution. A Gaussian process is thus a generalization of the Gaussian probability distribution.

**Definition**: A Gaussian process is a collection of random variables, any finite number of which have a joint Gaussian distribution [7]

Thus, a Gaussian process (GP) can be specified by giving the second order characteristics: mean function and covariance function. Let us define the mean function as $m(x)$ and the covariance function as $k(x, x')$

$$m(x) = \mathbb{E}[f(x)], \quad (2.11)$$
\[ k(x, x') = \mathbb{E}[(f(x) - m(x))(f(x') - m(x'))] \] (2.12)

and the GP is written as

\[ f(x) \sim \text{GP}(m(x), k(x, x')) \] (2.13)

A GP automatically implies the consistency property which simply means that if the GP specifies \((y_1, y_2) \sim \mathcal{N}(\mu, \Sigma)\) then it must also specify \(y_1 \sim \mathcal{N}(\mu_1, \Sigma_{11})\) where \(\Sigma_{11}\) is a relevant sub-matrix of \(\Sigma\). This basically would mean that examination of a larger set of variables does not change the distribution of the smaller set. Let us consider a simple Bayesian linear regression model \(f(x) = \phi(x)^T w\) with prior \(w \sim \mathcal{N}(0, \Sigma_p)\). Thus we have the mean and covariance to be

\[ \mathbb{E}[f(x)] = \phi(x)^T \mathbb{E}[w] = 0, \] (2.14)

\[ \mathbb{E}[f(x)f(x')] = \phi(x)^T \mathbb{E}[ww^T] \phi(x') = \phi(x)^T \Sigma_p \phi(x') \] (2.15)

Thus \(f(x)\) and \(f(x')\) are jointly Gaussian distributed with mean and covariance as given in the equations above. The choice of different covariance functions allows us to take into consideration different aspects of the signal. In this particular case, the choice made was the squared exponential covariance function, given as follows:
\[ k(x, x') = \sigma_f^2 \exp\left[\frac{-(x - x')^2}{2l^2}\right] + \sigma_n^2 \delta(x, x') \] (2.16)

where \( \sigma_f^2 \) gives us the maximum allowable signal covariance. Thus when \( x = x' \) then \( k(x, x') \) approaches its maximum which is how the Gaussian process appears to be a smooth function as its neighbors are alike.

The parameter \( l \) effects the length of the dependence. This parameter determines if \( x \) is far away from \( x' \) for \( k(x, x') \approx 0 \). Further, parameter \( \sigma_f^2 \) helps to decide the covariance of the noise, and \( \delta(x, x') \) is the Kronecker delta function.

![Effect of Hyperparameter on GPR](image)

Figure 2.6. GPR using different hyperparameters

In the figure 2.6 sample data generated was used to generated from a GP with hyperparameters \((l, \sigma_f, \sigma_n) = (2, 1.27, 0.3)\) for the first subplot. Using Gaussian process prediction we obtain a 95 percent confidence region for the underlying function. Subplots 2 and 3 show
the Gaussian process predictions on the same data set using different hyper-parameters 
(0.5, 1.27, 0.3) and (0.1, 1.27, 0.3) respectively.

In subplot 3 of figure 2.6, we notice that the error variance is larger for the input 
values that are distant from the training data. When we have the length scale very large 
such as in subplot 1 the regressed mean does not pass near any training point. Thus, there 
is a need of closely studying the hyper-parameters in order to get the right regression curve. 

It can be shown that the squared exponential covariance function corresponds to a 
Bayesian linear regression model with an infinite number of basis functions ?? . We can also 
obtain the covariance function from a linear combination of an infinite number of Gaussian- 
shaped basis functions.

Given \( n \) observations \( y \) at test points \( x \) our objective is to predict \( y_* \) at a set of 
prediction points \( x_* \). The GP can be represented as a sample from a multivariate Gaussian 
distribution, as

\[
\begin{bmatrix}
y \\
y_*
\end{bmatrix} \sim \mathcal{N}
\begin{pmatrix}
0, \\
K & K^T \\
K_* & K_{**}
\end{pmatrix}
\] 

(2.17)

The three matrices in the covariance matrix are given by:

\[
K = 
\begin{bmatrix}
k(x_1, x_1) & k(x_1, x_2) & \ldots & k(x_1, x_n) \\
k(x_2, x_1) & k(x_2, x_2) & \ldots & k(x_2, x_n) \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
k(x_n, x_1) & k(x_n, x_2) & \ldots & k(x_n, x_n)
\end{bmatrix}
\] 

(2.18)
\[ K_s = \begin{bmatrix} k(x_s, x_1) & k(x_s, x_2) & \ldots & k(x_s, x_n) \end{bmatrix}, K_{ss} = k(x_s, x_s) \] (2.19)

Thus for \( n \) training points and \( n_s \) test points, \( K(x, x_s) \) is a \( n \times n_s \) matrix. To get the posterior distribution over the function we need to restrict the joint prior distribution to contain only those functions which agree with the observed data points. Thus we need to condition the joint Gaussian prior distribution on the observation.

\[
y_s | y \sim \mathcal{N}(K_sK^{-1}y, K_{ss} - K_sK^{-1}K_s^T) \quad (2.20)
\]

Thus giving us our best estimate \( y_s \) as the mean of this distribution:

\[
\bar{y}_s = K_sK^{-1}y, \quad (2.21)
\]

and the variance in our estimate as

\[
var(y_s) = K_{ss} - K_sK^{-1}K_s^T \quad (2.22)
\]

The above expressions can be written in a further simplified form. Consider the mean prediction as a linear combination of observations \( y \). Thus, we look at the equation as a linear combination of \( n \) kernel functions, each one centered on a training point. Thus,
\[ \bar{y}_* = \sum_{i=1}^{n} \alpha_i k(x_i, x_*) \] (2.23)

where \( \alpha = K^{-1} y \).

The variance of a Gaussian process is the difference between two terms: the first term \( K(x_*, x_*) \) is the prior covariance from which we subtract the information the observations or test points give us about the function. One particular implementation of Gaussian process regression is by using Cholesky decomposition, instead of directly inverting the matrix. This is faster and numerically more stable. In Matlab notations, the use of the following equations is recommended [16]:

\[ L = \text{cholesky}(K) \] (2.24)

\[ \alpha = L^T (L \backslash y) \] (2.25)

\[ \bar{y}_* = k_*^T \alpha \] (2.26)

\[ v = L \backslash k_* \] (2.27)

\[ V[y_*] = k(x_*, x_*) - v^T v \] (2.28)
The reliability of our Gaussian process is dependent on how well we select the covariance function. Thus choices of $l$, $\sigma_f$, and $\sigma_n$ are vital. Thus, it is useful to introduce the marginal likelihood $p(\theta|x,y)$ at this point of the explanation. Marginal likelihood is the integral of the likelihood times its prior.

$$p(\theta|X) = \int p(\theta|y,X)p(y|X)df$$

(2.29)

where $\theta = (l, \sigma_f, \sigma_n)$

To maximize the a posteriori estimate of $\theta$, $p(\theta|x,y)$ has to be at its greatest. Thus assuming we have little prior knowledge about what $\theta$ should be, we need to maximize the log $p(y|x,\theta)$ which is given by

$$\log p(y|x,\theta) = -\frac{1}{2} y^T K^{-1} y - \frac{1}{2} \log |K| - \frac{n}{2} \log 2\pi$$

(2.30)

If the above method of cholesky decomposition is used then the log marginal likelihood can be calculated as

$$\log p(y|X) = -\frac{1}{2} y^T \alpha - \sum_i \log L_{ii} - \frac{n}{2} \log 2\pi$$

(2.31)
CHAPTER 3

REVIEW OF LOW POWER WIRELESS SYSTEM

In section 3.1-3.2 we review the system level design for low power wireless systems such as AutoSense and provide results of traditional HRV analysis of the data collected from autosense lab and field studies to motivate the HRV analysis method proposed in this thesis. In section 3.3 we introduce a distributed HRV analysis Framework applicable to low power wireless devices.

Low power wireless sensors provide a new and unique way to measure physiological signals in the natural environment. Subjects can have their vital signs measured continuously in real time. Real time analysis and inferencing can lead to timely medical treatment which may be crucial for patients. The advent of such devices also provides a method to measure alcohol and drug abuse in subjects. Studies regarding stress can be conducted and important correlations can be drawn between physiological signals and stress. The effect of atmospheric conditions on the body and its reaction to these changes can be studied. Based on these reactions steps can be taken in order to artificially change surrounding condition in order to make the subject more comfortable.
AutoSense [2] is one such low power wireless system. In this chapter we review the system level design for low power wireless systems such as AutoSense. The AutoSense project is part of the Genes Environment and Health Initiative (GEI) at the National Institutes of Health (NIH). Within GEI, Autosense is one of five cooperative agreement programs under the Exposure Biology Program. The AutoSense project is supported by the National Institute on Drug Abuse (NIDA).

The project involves researchers from Computer Science, Electrical Engineering, Behavioral Science, Physiology, and Biochemistry, spread across the Carnegie Mellon University, The Ohio State University, the University of Memphis, the University of Minnesota, and the University of Pittsburgh citeAuto3.

Commercially available wearable sensors for collecting physiological measurements from the field have one or more of the following shortcomings. They have only a few sensors, are cumbersome to wear on a daily basis without causing social embarrassment, have a short lifetime, or are expensive. For example, LifeShirt [31] from Vivometrics is a vest-like device that can monitor ECG, respiration, skin temperature, and physical activity. However, this wired system is bulky, expensive, and uses proprietary wireless technology not compatible with mobile phones. In contrast, an increasingly popular wireless health monitor from AliveTech [30] provides only ECG and 3 axis accelerometer signals using a Bluetooth radio (compatible with mobile phones), and has a lifetime of less than 24 hours. Lorincz et al. [14] describe SHIMMER a wireless sensor platform for supporting long-term field studies that is fine-tuned for motion analysis. The design of the platform and the associated techniques for managing energy and radio bandwidth are matched to the motion and activity monitoring task. While physiological sensors can be added to the platform the resulting system won’t cover all the modalities required for stress inference. The power management
techniques designed for accelerometer sensors will be suboptimal for the physiological sensor modalities.

Figure 3.1. AutoSense Mote

AutoSense is an unobtrusively wearable wireless sensor system for continuous assessment of personal exposures to addictive substances and psychosocial stress as experienced by human participants in their natural environments. The AutoSense system distinguishes itself by its wearability, real time multi-modal stress index calculations and long lifetime achieved through low-power sensor design and distributed signal processing for reducing data bandwidth. The hardware design integrates TI- MSP430, bioamplifiers, power management and Nordic Semiconductor nRF24AP2 transceiver in a small form factor. Autosense was used in a scientific behavioral study, where 35 participants wore the system in a clinical setting where they were subjected to three lab stressors.

It was also worn by 23 participants in their natural environment for one full day, where they took a college exam, but otherwise carried out their daily routine. The sensory measurements responded well to the stressors both in the lab and in the field. Finally, Autosense was reported to be unobtrusively wearable by the participants and was not found to cause any social embarrassment in their natural environment [1].
AutoSense uses two (2.5 square-inch) circuit boards: Chest band Node ECG, Temp, Accel, GSR; Chest band, with an optional RIP daughter board and Bridge Node: Bluetooth Bridge for connection to the phone. Each board is integrated with bioamplifiers, TI MSP430 and nRF24AP2 transceiver from Nordic semiconductor, and includes a 750mAh battery with built in USB charging circuit. The ECG and RIP channels are sampled at 128 Hz and 64 Hz which are down sampled to half rate for transmission after digital filtering.

All other sensors are sampled at 32 Hz, but all could be sampled more frequently if needed. All sensor boards include digital sensor/radio power switches for duty cycling of sensors and radio in software. Sensor measurements (or features derived from them on the mote) can be transmitted wirelessly to a laptop or PC by plugging a mote programmed with TOSBase. Measurements are displayed on the screen using a modified version of the Oscilloscope program and the measurements are logged. A visual interface also allows the study coordinator to provide labels (such as starting and ending times of stress sessions) that can be used during the analysis.

To facilitate wireless connection between mote class sensors and a mobile device, two approaches have been used: first, the mote class devices can be made Bluetooth enabled. This is arguably the most elegant approach, but, Bluetooth radio can drain the battery that
is shared with the sensors. Our team adopted another approach, namely adding a bridge node in between the wearable sensors and the mobile device. Adding a Bluetooth gateway node frees up the constraints of the sensor network designer and the mobile device application developer, yet enables the two communities to combine forces in developing human centric mobile sensing applications.

A bridge node can help pair up any mobile phone and wireless sensor mote combination without introducing strong coupling requirements. In our design, we connect the BlueTooth serial module (BlueRadio-SC40A) to the UART1 lines of the MSP430. Our team at the Ohio State University used the UART1 lines on the MSP430 as it is not shared with any other peripherals and does not need any resource arbitration. We use a queuing mechanism on the bridge node to buffer the data from the sensors before sending it out on the Bluetooth link, to minimize packet loss. By using a moderate buffer size of 12 we are able to keep the packet loss rate to 2% on the phone. We have also included a digital switch on the bridge to enable duty cycling of the Bluetooth radio.

The two lead electrocardiogram (ECG) is used for measurement of electricity of the heart. The system can use alternatively standard disposable adhesive gel electrodes and e-fabric electrodes embedded into the chest band itself. Electrocardiography (ECG) sensing is the primary method for assessing cardiovascular activity in subjects. The SA node produces electrical impulses to stimulate heart muscles to control the pumping action of the heart. ECG measures this electrical activity through electrodes applied to the skin. In particular, electrodes placed on different sides of the heart measure activity at different parts of the heart. Two lead ECG informs about the overall rhythm of the heart providing precise timing information of the heart beat intervals. A multiple lead ECG sensor measures potentials across different vectors across the heart and can indicate problems or weakness in particular parts of the heart muscle.
In AutoSense we are interested in the timing of the heart beats and therefore use the standard two-lead configuration measuring potential from left to right. The typical potential across the heart is of small magnitude on the order of 1 mV which should be amplified a factor of 100 before the sampling circuit. The electric field interference from power lines and other electrical equipment can cause signals of similar magnitude unless a differential amplifiers with high common-mode rejection is employed.

A second problem with two-lead systems is baseline wander since the third grounding leg electrode is missing. In addition, electrode-skin impedances can vary between the two electrodes leading to source impedance unbalance which produces differential mode voltage that will be amplified by the differential amplifier. Therefore baseline needs to be extracted and subtracted from the main signal to provide the maximum dynamic range of the ADC.

This baseline correction circuit provides a way to integrate galvanic skin response sensing into the same electrode through applying a fixed known differential between the electrodes. Finally, appropriate analog filtering must be employed to attenuate high frequency noise sources and to prevent aliasing in the analog-to-digital sampling stage.

Heart Rate Variability (HRV) analysis provides information on the autonomous nervous system activity, with links to disease conditions and mental and emotional states. HRV analysis requires precise determination of beat locations. Providing the minimum rate of 64 Hz on ECG requires 12.8 packets/sec making it the most significant source of power consumption due to frequent radio transmissions. Providing the preferred sampling rate for HRV at 512Hz would reduce the battery life drastically and preclude the possibility of continuous streaming of ECG signal with high sampling rate to a mobile computing platform for analysis.
3.1 Lab Data Analysis

AutoSense was used by behavioral scientists in National Institute of Health sponsored studies of stress, both in the lab and in the field [1]. To compute the group average first the time series of each subject $j$ is normalized by computing the $z$-score $z_j(t) = (x_j(t) - \mu_j) / \sigma_j$ using the subject's standard deviation $\sigma_j$ and mean $\mu_j$. Then, the $z$-scores are averaged for each time step using $z(t) = Mean[z_j(t)]$ and the average $z$-score is rescaled back using the group standard deviation and mean as $x(t) = z(t)\sigma + \mu$.

In another lab study, a 40 min stress protocol consisting of two sessions of public speaking (4 minutes each), mental arithmetic in standing position (4 min), mental arithmetic sitting position (4 min), and cold pressor test (90 seconds) with 5 min rest periods between each task was performed. Before and after the stress protocol, there was a 30 min rest and recovery periods were administered. Public speaking was to simulate social, evaluative challenge. Mental arithmetic problem was to simulate cognitive challenge. Cold pressor test was to simulate acute physical challenge. Finally, this was followed by a period of rest.

As can be seen from the lab study, there are four significant peaks related to the heart rate. Each of these four peaks relate to the application of stressors - Public Speaking, mental arithmetic standing, mental arithmetic sitting and the cold pressor test. The period before and after the application of the stressors gives the baseline for the study. Here to we can identify sections when the four stressors were applied. A significant change is observed at the time when the final stressor- Cold pressor test id applied. The RSA and LF plot behaves as expected. On the application of stress the homeostasis makes the low frequency section of the heart rate reduce and the high frequency section of the heart rate increase. The important aspect to be noted in the figure shown above is that when we process the data form the lab study, it can be seen that with minimum packet loss the computations for the LF and RSA are very responsive to stress.
3.2 Field Data Analysis

The field days are unstructured except the field study dates are chosen to include a college exam period, when the subjects are expected to be under stress. Although more than 35 subjects have undergone the lab study and 23 subjects have worn AutoSense Auto4 for two full days in the field, we present here analysis of data collected from 24 subjects in the lab and from 8 subjects in the field.

In the field study, physiological measurements from all sensors were wirelessly transmitted to an Android G1 smart phone via the bridge. Over 30 features were computed from the physiological measurements to make four inferences: whether the subject is stressed,
whether the subject is speaking (from respiration measurements) to help contextualize the inference of stress, changes in posture (from accelerometer) to again help contextualize the inference of stress, and intensity of physical activity that was used to suspend the inference of stress, since under intense physical activity, it is difficult to filter out the changes induced by stress.

On average, over 13 hours of data was collected per day from each participant. Participants reported that the Respiration band was comfortable to wear in the field and did not cause any social embarrassment. After experienced heavy packet loss in the field which was nearly 30%. After wearing the devices for 12 hours, some felt an itching, but said it was not to the level of discomfort. An exit survey of the participants found out that 46% of men and 54% of women found wearing the band enjoyable. In future, the respiration band can be padded to eliminate the itching.

The field study was used to evaluate the feasibility of the chest band sensor suite for data collection and processing in a natural setting. Also, it was used to assess participant compliance with, and acceptance of, the Autosense data collection protocol over a two day period. Finally, it was used to assess the accuracy/concordance of Autosense-derived alcohol and stress indexes against self-reported alcohol intake and perceived stress. For the field study, the data we show a 3 hour period before, during and after a scheduled exam. Increase in heart rate and LF/HF ratio is observable preceding the exam period. Similar effects after the exam is likely to be associated with physical activity.

Unlike the lab study, the field study were not monitored and that resulted in more packet loss. This evidently affects the LF and RSA computations. The mobile terminal computes Lomb’s periodogram at every one minute time interval in order to estimate the LF and RSA section. The comparison of the lab and field study motivates the need to have a more robust method to estimate the frequency domain features.
3.3 Distributed system for robust HRV computation

Low power wireless systems can be set up in order to perform distributed processing. Data processing on the low power wireless device reduces radio on time and improves the life time of the device considerably. Computationally inexpensive algorithms may be implemented in order to compute features in real time in order to reduce the number of packets. Complex algorithms can be reduced to a certain computational level for there execution.
Figure 3.5. Distributed HRV computation within limited number of clock cycles. Low power systems have a number of limitations such as lack of floating point arithmetic, limited memory and speed.

AutoSense [3] [4] provides a distributed computation method of HRV spectra between the AutoSense Mote and a mobile computing platform that increases the beat detection resolution while reducing the required communication drastically. The algorithm has three main components: 1) Local computation of potential beat locations with associated confidence measures at the heart rate monitor 2) Smoothing and interpolation using Gaussian Process Regression for labeling of True and False Positives and 3) Spectrum calculation by Gaussian process regression at a mobile terminal. From figure 3.5, it can be concluded that by distribution of the peak detection algorithm among the mobile computation device and the
chest band sensor node. The chest band sensor node processes the ECG signals to extract beat-to-beat interval information.

![Graph showing peaks transmitted vs Actual QRS detections](image)

Figure 3.6. Peaks transmitted vs Actual QRS detections

The estimated positions of the beats are calculated in real time using Pan and Tompkins algorithm running on the mote. Limited memory and limited clock cycles forces the use of infinite impulse response (IIR) filters. IIR filters are computationally less intensive for the same cut-off frequency when compared to finite impulse response (FIR) filters. The search back algorithm in order to search for missing beats is also not implemented because of the lack of memory. In order to reduce the number of false negatives the thresholds are lowered. This increases the number of false positive detected. The mobile terminal then needs to decide whether the detections should be considered as a true beat or a false positive. Even with these limitations the distributed algorithm improves the life time of the AutoSense mote considerably. For a nominal heart beat frequency of 1 beat/sec, the beat locations require 1.32 packets per minute assuming 10% false positive rate. This provides 116:1 reduction in communication bandwidth over the 2.56 packets/sec rate of the streaming mode.
Potential R-wave locations are computed locally at the ECG Mote using a modified version of the QRS detection algorithm proposed by Pan-Tompkins. In the mobile terminal an outlier removal algorithm based on peak-to-peak interval normalized by mean and standard deviation over each minute is used to reject too long or too short intervals.

While computing the inter beat interval validation strategies are used to deal with lost packets on the wireless channel. We adopt an outlier detection algorithm based on the maximum norm residual test. A preselector algorithm is used in order to select false positives. These preselected false positives are then processed in order to detect true outliers which are removed from the dataset. The R-R intervals which now remain can be used also to calculate time domain HRV features. Time domain parameters are mean, standard deviation and higher moments of RR interval measurements. The frequency domain parameters are computed on the mobile terminal.

Therefore, we run a modified lightweight Pan and Tompkin’s algorithm on sensor node in order to detect potential inter beat intervals. Then the potential inter-beat intervals are transmitted to a mobile terminal on which we use GP regression to interpolated and identify false positive and negatives. Finally, we compute probabilistic frequency domain HRV features with confidence intervals using sample path averages.
CHAPTER 4
GAUSSIAN PROCESS REGRESSION APPLIED TO HEART RATE VARIABILITY ANALYSIS

4.1 Gaussian Process Regression applied to Inter-Beat Intervals

Traditionally spline interpolation is used on inter beat intervals in order to perform Fourier analysis [24]. This provides uniformly sampled inter beat interval information on which standard Fourier analysis can be performed. Here we employ GPR to provide alternative sample paths which are consistent with the non uniformly sampled inter beat data. GPR method has two distinct advantages over previously suggested techniques. First, it provides confidence intervals on the computed frequency domain features which can serve as a data quality measure that can be incorporated into higher layer inference algorithms for classifying stress. Second, it provides a principled albeit a computationally intensive method of identifying outliers based on expected natural variation in HR variability.

Selection of the covariance function is a crucial ingredient in setting up the Gaussian process predictor, as it holds the assumptions about function we wish to interpolate. It is clear that the notion of similarity between data points is highly crucial in order that the test points (regression curve) should be informative of the adjoining training points (actual data set). In order the regression curve to capture the information contained on the data set the first step is to choose a specific form of covariance function. The Squared Exponential (SE) Covariance function was used in the discussion below.
\[ k(x, x') = \sigma_f^2 \exp\left[-\frac{(x - x')^2}{2l^2}\right] + \sigma_n^2 \delta(x, x') \quad (4.1) \]

The SE covariance function is categorized under stationary covariance functions. It uses three parameters namely - signal noise variance, signal variance and characteristic length scale. The signal noise variance would result from the inaccurate detection of the QRS complex in the ECG waveform.

In embedded module while working with limited hardware and duty cycles Infinite Impulse Response (IIR) filters are preferred. An empirical approach was used in order to quantify the exact delay that the set of filters create in the detection algorithm. Therefore, while using the algorithm on the AutoSense mote, the covariance matrix parameters this empirical calculation of the delay helps us to quantify the signal noise variance. Thus the noise parameter needs to be less than 1.

We then maximise the marginal likelihood ratio with respect to \( \sigma_n, \sigma_f \) and \( l \). A clean set of inter beat intervals computed from 10 minute long ECG data was chosen in order to model the covariance matrix. A section of this is shown in figure 4.1

Traditionally HRV analysis is done on data that is at least 5 minute long. Thus, the above data set was chosen to be of 10 minute duration. The data chosen was for a period of stress collected during the lab study. The signal standard deviation parameter and noise standard deviation were varied to maximize the marginal likelihood equation.

In order to maximize the marginal likelihood function, the toolbox provided by Rasmussen and Williams [29] was used on the clean data set as shown above. The length parameter was then set at 73 maximizes the Marginal likelihood function for a noise standard deviation of 0.5 (0.000134 sec) and signal standard deviation of 5 samples (0.00911 sec).
Using the new parameters of signal std deviation of 5, length dependence parameter of 73 and noise std of 0.5, we regress the 10 minute dataset as shown in the figure 4.2.

This regressed path can be now used to perform standard Fast Fourier transform to get the frequency domain characteristics of the inter beat intervals. Another important property of the Gaussian regressed data set is that we can use the statistics of the covariance function to generate infinite number of test data paths that shall have the same statistical characteristics to the actual data points.

The data paths are generated by spatially coloring white noise to match a signal covariance matrix estimated from real subject data. After passing the noise paths through the covariance matrix for coloring the spectrum we add back the mean path in order to get a new data path that has statistically the same characteristics as all the other generated paths.

\[ y_\star \vert y \sim \mathcal{N}(K_\star K^{-1}y, K_{\star \star} - K_\star K^{-1}K_\star^T) \]  

(4.2)
As seen in the above equation the covariance is given by

\[
\text{covar}(y_*) = K_{**} - K_* K^{-1} K_*^T
\]

(4.3)

The computational complexity of the equation above increase with the size of the test data. This is mainly due to the inverse of the matrix K. Suppose that a vector has covariance matrix Kxx. Since this matrix is Hermitian symmetric and positive semidefinite, by spectral factorization, we can diagonalize or factor the matrix in the following way.

\[
K_{xx} = E \Lambda E^T
\]

(4.4)
where $E$ is the orthogonal matrix of eigenvectors and $\Lambda$ is the diagonal matrix of eigenvalues.

We can write the 1st and 2nd moment properties of this random vector with mean and covariance matrix $Kxx$ using the following transformation of a white vector $w$ of unit variance:

$$x = H \ast w + \mu$$

(4.5)

where,

$$H = E\Lambda^{\frac{1}{2}}$$

(4.6)

Thus, the output of this transformation has expectation of

$$\mathbb{E}\{x\} = H \mathbb{E}\{w\} + \mu = \mu$$

(4.7)

and covariance matrix of

$$\mathbb{E}\{(x - \mu)(x - \mu)^T\} = H\mathbb{E}\{ww^T\}H^T = HH^T = E\Lambda^{\frac{1}{2}}\Lambda^{\frac{1}{2}}E^T = Kxx$$

(4.8)

Eigenvalue decomposition of the matrix helps reducing computational complexity of the above equations. Thus, passing a random noise vector through matrix $H$ colors the random vector in order to give us the vector which has the same covariance matrix as the covariance matrix found from the real data points.
Once we have computed a single regressed path for the data points we can now proceed to get infinite number of such data paths. Ten such paths are shown in the figure 4.3.

### 4.1.1 Frequency Domain Analysis using Fast Fourier Transform

The motivating factor here is that we can have a frequency domain response of the data set with a variance bound around it which helps us quantify the error in the frequency domain analysis when we compute statistics regarding respiratory sinus arrhythmia (RSA) and low frequency (LF) power of the heart rate. After Gaussian process regression analysis the dataset is uniformly sampled in time. Spectral analysis can simply be performed using conventional Fourier analysis.

As seen in the figure 4.4, once we have the randomly generated data paths we can perform separate FFT analysis on each in order to get a similar confidence bound in the frequency domain calculations.

The new sampling frequency is given be the equation:

\[
\text{New Sampling Frequency} = \frac{\text{Number of points used in regression}}{(\text{Time length of data in min} \times 60)} \text{ Hz}
\]
The same data set was passed through Lomb’s periodogram analysis.

<table>
<thead>
<tr>
<th></th>
<th>GPR based FFT</th>
<th>Lomb Periodogram</th>
</tr>
</thead>
<tbody>
<tr>
<td>LF</td>
<td>645.76 +/- 4.57</td>
<td>653.26</td>
</tr>
<tr>
<td>RSA</td>
<td>77.41 +/- 1.109</td>
<td>78.39</td>
</tr>
</tbody>
</table>
4.2 Outlier Removal

Artifacts in the electrocardiography lead to spurious quantification of R-R intervals. This problem has been well documented in the literature [19]. In case of AutoSense, a variety of different conditions lead to outliers.

- **Lossy Channel** - AutoSense may suffer because of the wireless channel being lossy. Dropping packets in the streaming mode can lead to loss of an entire raw ECG wave. This can be reduced by feature detection on the Mote. The feature mode reduces the problems by reducing the packet rate and also gives a chance to implement a retransmission scheme.

- **Motion Artifacts** - AutoSense, when used as a field device is highly susceptible to motion artifacts. Though the QRS detection algorithm in software and bias removal circuit in hardware, which was discussed in chapter 3, reduce these artifacts by using adaptive techniques but there may be situations when false positives do occur. Removal of the back search algorithm and lowering of the adaptive threshold increases false positives.

Thus, lossy channel may lead to false negatives while motion artifacts may lead to false positives or negatives. This make it necessary that good outlier removal algorithms is put in place in order to improve the HRV analysis. Large data sets make artifact processing time consuming and the resulting temptation is to use short cuts or bypass the artifact processing. In the discussion below the impacts of these outliers are discussed with respect to the various frequency analysis techniques. Because the system is wireless body effect on the antenna can lead to high packet loss. Quantification of HRV requires the use of clean outlier free data; further RSA and LF computations based on Lomb’s periodogram suffer greatly with lossy data as shown in chapter 5. Lomb’s periodogram suffers from an important limitation. It is not invariant to time translation the outlier removal used to reject
false inter beat intervals should be taken into account. With the distributed HRV analysis setup on the mobile computing device and the chest band sensory device the prospects of highly sophisticated algorithms are possible.

4.2.1 Outlier Removal using Gaussian Process Regression

Outlier rejection method used in Gaussian process regression is done in a recursive manner. We first use Grubb’s test for outliers to find the suspected beats. The suspected beats detection algorithm is based on the maximum norm residual test (also known as the ubbs test). The Grubbs test statistic is defined as:

$$G = \max_{i=1,...,N} \frac{|Y_i - Y|}{S}$$  \hspace{1cm} (4.9)

with Y and S denoting the sample mean and standard deviation. This is the two sided version of the test.

Grubbs’ test is based on the assumption of normality. Grubbs’ test detects one outlier at a time. Multiple iterations change the probabilities of detection, and the test should not be used for sample sizes of six or less since it frequently tags most of the points as outliers. The suspected beat is eliminated from the set of inter-beat intervals and Gaussian process regression is performed. Once we have the regressed curve, with the beat being suspected as an outlier removed, the suspected outlier beat is reinserted to check it falls in the confidence bound. If it does fall in the 95% confidence bound, the beat is judged to be a true beat. If not in the confidence bound the beat is expunged from the dataset and the test is iterated until
all the suspected outliers are not processed. Following is the illustration of the algorithm given with the help of a flow chart.

This technique of outlier removal is very slow because of the repetitive iterations to perform Gaussian process regression and not effective for real time outlier detection. The ad-hoc method of suspected outlier selection can be improved in order to reduce false suspected positives. Though in order to reduce false negatives and to remove all the outliers, Grubb’s test provides a useful method for suspected outlier selection. An illustration of this technique is shown in the empirical data section 5.2.
CHAPTER 5
EMPIRICAL DATA ANALYSIS

In this section we inject packet losses in Autosense lab data series to assess RSA and LF computations for Lomb’s periodogram and Gaussian Process Regression based FFT analysis.

5.1 Empirical comparison of GPR based FFT and Lomb’s periodogram for heart rate variability analysis

![Clean Data for Analysis](image)

Figure 5.1. Clean Data for Analysis
The data has no packet losses as well as no ectopic beats. Gaussian Process regression was performed using the Squared Exponential covariance matrix. The hyperparameters used were $\theta = (l = 7.3, \sigma_n = 0.5, \sigma_f = 5)$. Figure 5.2 shows the result of the regression process.

Figure 5.3 shows the frequency domain analysis using GPR based technique as well as using Lomb's periodogram.

Figure 5.2. GPR on the clean data

Figure 5.3. Frequency analysis of the clean data

As seen the clean data set has a similar low pass section for both GPR based FFT and Lomb’s periodogram while higher frequencies are reduced in the Gaussian Process Regression based Fourier Transform model due to the regression process.
Next, randomly 5%, 10% and 15% of data loss was simulated in order to see the effect on the frequency domain computations on Lomb’s periodogram and GPR based FFT.

Figure 5.4. 5 percent data loss

Figure 5.5. 10 percent data loss
Figure 5.6. 15 percent data loss

Table 5.1. Frequency Domain Comparison using GPR and Lomb’s Periodogram

<table>
<thead>
<tr>
<th>Data Loss</th>
<th>GPR(LF)</th>
<th>GPR(RSA)</th>
<th>Lomb’s(LF)</th>
<th>Lomb’s(RSA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>645.76 +/- 4.57</td>
<td>77.41 +/- 1.09</td>
<td>653.26</td>
<td>78.39</td>
</tr>
<tr>
<td>5%</td>
<td>651.04 +/- 6.07</td>
<td>87.10 +/- 1.91</td>
<td>638.88</td>
<td>90.62</td>
</tr>
<tr>
<td>10%</td>
<td>657.91 +/- 7.60</td>
<td>94.32 +/- 2.26</td>
<td>598.49</td>
<td>94.58</td>
</tr>
<tr>
<td>15%</td>
<td>623.69 +/- 12.25</td>
<td>101.15 +/- 3.9262</td>
<td>591.72</td>
<td>97.03</td>
</tr>
</tbody>
</table>

Table 5.2. Frequency Domain LF/RSA ratio using GPR and Lomb’s Periodogram

<table>
<thead>
<tr>
<th>Data Loss</th>
<th>GPR(Ratio)</th>
<th>Lomb’s(Ratio)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>8.33</td>
<td>8.33</td>
</tr>
<tr>
<td>5%</td>
<td>7.47</td>
<td>7.05</td>
</tr>
<tr>
<td>10%</td>
<td>6.97</td>
<td>6.32</td>
</tr>
<tr>
<td>15%</td>
<td>6.16</td>
<td>6.09</td>
</tr>
</tbody>
</table>

The HRV analysis of the data in the absence of data losses serves as a golden standard using which we can assess the robustness of GPR and Lomb’s periodogram to data losses.
First, we observe that the confidence intervals on the LF and RSA measures grow with the data loss rate as expected. Therefore these confidence intervals may serve as a good measure of data fidelity. Second, the LF component estimate of HRV is stable for GPR method in contrast the rapidly deteriorating estimate provide by Lomb. Third, both methods produce RSA features that are biased upward.

5.2 Outlier Recognition and Elimination using Gaussian Process Regression

As discussed in section 4.2.1, clean set of inter beat intervals are introduces with 1.5% of outliers. The outlier removal algorithm tested on this data. Grubb’s test is used for suspected outlier detection. In figure 5.7 we see the clean data, the added outliers and the suspected outliers detected by Grubb’s test.

![Figure 5.7. Suspected Outliers using Grubb’s test](image)

In order to detect the outliers, each and every suspected outlier is expunged and Gaussian process regression is performed. Once we have the regression, the suspected outlier is re-introduced and if it lies outside the 95% confidence bound it is declared as a outlier. If not the suspected outlier is an actual inter beat interval. The table 5.3 below shows that after introducing 10 outliers, the Grubb’s test for suspected outliers detected 24 candidates.
After repetitive iterations 11 out of the suspected 24 candidates were deemed to be actual outliers. Thus all the added 10 outliers were detected and one of the inter beat interval was removed as it was considered an outlier. This limited empirical experiment suggests that the proposed GPR based outlier rejection can serve as an effective outlier rejection tool to determine the false positive and negatives from the lightweight beat detection algorithm running in the low power sensor node.

<table>
<thead>
<tr>
<th>Outliers added</th>
<th>Suspected outliers</th>
<th>Outliers removed</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>24</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 5.3. Outlier Removal
CHAPTER 6
FUTURE WORK

In the following section we discuss potential venues for future work. First, a closed form solution for the continuous time equation can be found in order to get the continuous time Fourier transform of the tachogram. Second, a direct FFT approach for the GPR can be formed such that the covariance and mean are directly formed for the frequency domain. Thirdly, the algorithm can be simplified in order to integrate it into the mobile device. This will facilitate real time processing of RSA and LF which may reliable then be used for stress monitoring. The confidence bound can be used in order to judge the reliability of the computations. The mobile computing device can use this information to decide which estimated values are to be used for stress estimation. For outlier removal, ad-hoc procedures that give lesser suspected outliers while detecting all the actual outliers may be found. This reduces computation time and increase the feasibility to use the algorithm on a mobile computing device at real time.

AutoSense provides a comprehensive suite of ultra-low power sensors that can be worn unobtrusively in the natural environment of subjects to enable collection of physiological measurements associated with stress response. Additional capability can be integrated into AutoSense to make it suitable for wider adoption. First, the respiration information can be jointly processed with the heart rate variability data to account for the RSA component of heart rate variability to study the sympathetic/parasympathetic balance in detail.
Second, wearability can be further improved by integrating all sensors onto a single board and designing body-conforming packaging. Third, additional unobtrusive sensors could be integrated to monitor addictive behaviors such as smoking and drinking so their interaction with stress can be jointly investigated in the natural environment.
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