With rising demand for energy and depletion of non-renewable sources, new avenues of energy production are being explored. Wind energy is one popular alternative to meet the energy needs of the future. Apart from visual impact and dependence on strong wind conditions for good performance efficiency, aerodynamic noise from wind turbines serves to be a major obstacle for their societal acceptance. This study tries to understand the propagation of wind turbine noise to the far-field, particularly in the lower frequencies using a technique based on inverse acoustic methods, the Acoustic Intensity Based Method (AIBM). At first, far-field noise radiation from NACA-2409 and NREL S809 airfoil is predicted using AIBM coupled with 2-D Computational Fluid Dynamics (CFD) simulations. Far-field noise radiation predictions for the airfoils are shown to be accurate. Aeroscousic characteristics of NREL Phase VI Horizontal Axis Wind Turbine rotor are then analyzed using AIBM. The flow field in 3-D is obtained by solving the Unsteady Reynolds Averaged Navier-Stokes Equations (U-RANS). The CFD results of the flow field are validated against experimental data and they show good agreement. Thickness noise, created by displacement of a mass of air by the rotating blades and loading noise, created by the rotor interacting with local flow deficiencies, are identified as the two causes of low frequency noise based on the calculated directivity patterns. The ultimate objective is to evaluate
the capability of a hybrid approach involving flow field data and AIBM to predict far-field noise for practical applications such as wind turbines.
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CHAPTER 1

INTRODUCTION

1.1 Background

An increasing need for energy coupled with depletion of non-renewable resources that we have depended on so far has opened the doors to exploring new alternatives to meet energy requirements. Wind energy is one such alternative. The U.S. Department of Energy and the American Wind Energy Association have set a goal of generating 20% of the total electric power from wind energy by 2030 [15]. Two issues that stand in the way of wind turbine technology becoming popular are (i) drop-off in wind power with wind speed and (ii) societal rejection of wind turbines in developed areas due to acoustic pollution. The present work addresses the second issue.

A wind turbine is a device that is used to extract kinetic energy from a stream of wind through wind-blade interaction. A wind turbine in operation produces noise which may be a cause of annoyance for people who live close to the turbines. The perceived noise at a given observer location depends on the turbine construction, its operation and many situational factors such as distance between the turbine and the living area, natural barriers to the propagation of noise such as hills and trees, wind turbine components such as airfoil shape, tower, blade tip etc. [20]. A lot of
these influences have been studied by the National Renewable Energy Laboratory (NREL) resulting in extensive experimentation and publications [13]. With recent technological advancements, it is possible to design turbine blades that have very good aerodynamic and aeroacoustic characteristics. To carry out this multi-objective design problem, the conflicting design objectives, i.e., maximum aerodynamic performance and minimum noise production, need to be addressed simultaneously. In such cases, there is no single optimal solution for which the optimization scheme might search. Instead, a set of compromised solutions, known as the Pareto-optimal solutions is produced [6].

Experimental study on wind turbines have been followed by comprehensive computational analyses. Computational study helps us understand the physics of the problem a lot better as a lot of detailed analyses can be done with the obtained data set. The ideal, frictionless efficiency of a wind turbine was first predicted using an actuator disc model wherein the rotor was approximated by a permeable disc. The original model was developed by Rankine and Froude based on 1D momentum theory [5]. This technique was then combined with blade-element analysis by Glauert resulting in the classical blade-element momentum technique [10]. However, in both these techniques, the geometry of the blades and the viscous effects are not resolved. Several computational models have validated experimental findings [7]. Modifications to existing designs, such as adding winglets to the blade tip etc. have been proposed based on computational studies on wind turbine blades [9]. Computational models have also made major contributions to our understanding of the driving mechanisms behind aerodynamic noise that gets propagated to the far field. Such information can be utilized to modify the design of the blades so that they are quieter. The capability
of the commercially available computational fluid dynamics (CFD) software, ANSYS
FLUENT, for wind turbine applications has been amply demonstrated [8]. FLUENT
contains broad physical modeling capabilities that are needed to model flow, turbu-
ulence, heat transfer and reactions. In this study, FLUENT is used to predict the
sources of noise. This requires the solution of the full 3D Navier-Stokes equations.
The acoustic calculations, which are based on the Euler equation, are then carried out
based on the data obtained from the CFD simulations using the Acoustic Intensity
Based Method (AIBM). Details of this method are presented in the following chapter.

In summary, a comprehensive numerical study has been carried out for the NREL
Phase VI horizontal axis wind turbine (HAWT) rotor and 2D airfoil sections. Two
different airfoils, NACA-2409 and NREL’s S809 airfoil used in the Phase VI HAWT
rotor, have been considered in this study. The aerodynamic performance of the
turbine rotor and airfoils, and the far-field acoustic radiation in the lower frequencies
have been calculated and analyzed. The effectiveness of a hybrid approach involving
CFD simulations to generate the flow field and AIBM to predict far-field noise has
been demonstrated for practical problems. An introduction to aero-acoustics and the
mathematical formulation of AIBM are discussed in the next chapter. Discussions on
fluid flow and aero-acoustic analyses carried of the 2D airfoil sections are described
in Chapter 3. CFD modeling and simulation around NREL Phase VI horizontal axis
wind turbine rotor and its far-field aero-acoustic predictions are discussed in Chapter
4 followed by important conclusions in Chapter 5.
CHAPTER 2

AEROACOUSTICS AND THE ACOUSTIC INTENSITY BASED METHOD (AIBM)

In this chapter, the basic concepts of aeroacoustics, the theory behind wind turbine noise and the Acoustic Intensity Based Method (AIBM) are discussed. The derivation of the governing equations that AIBM is based on is presented in detail in the later sections of this chapter. Details regarding the implementation of AIBM are also discussed.

2.1 Aeroacoustics

Aeroacoustics is a branch of science that deals with the propagation of acoustic waves or noise generated by the interaction of a turbulent fluid flow or aerodynamic forces with solid surfaces. Pressure fluctuations occur in an unsteady flow in order to balance out the fluctuations in momentum. These fluctuations propagate outward from their source [11]. Aerodynamic sound is largely concerned with pressure fluctuations that occur far from the source, where the amplitude of motion is small and the effects of compressibility are important. Acoustic waves are high velocity waves with amplitudes several orders of magnitude smaller than the flow structures, making them less dissipative. One of the major breakthroughs in the field of aeroacoustics was the
concept of acoustic analogy that was proposed by Lighthill [14]. The conservation
equations of mass, momentum and energy describe the entire flow that comprises the
base flow with small perturbations superimposed on the base flow. Lighthill’s acoustic
analogy is obtained from the basic conservation equations after some modifications.
The modifications include the subtraction of the base or mean from the total compo-
nent of the flow variable and upon rewriting the governing equations in terms of the
perturbation quantities, the acoustic wave equation is recovered with a source term
that includes non-linear effects from the flow viz. the fluctuating Reynolds stresses.
This equation thus formed the bridge between acoustic wave propagation and fluid
flow, explaining that the viscous, non-linear effects in the flow act as a source of noise.
Subsequent theories in this field are based on an approach that is similar to Lighthill’s
acoustic analogy.

Wind turbine noise can be divided into two components, mechanical noise and
aerodynamic noise. Mechanical noise originates from the different machinery com-
ponents, such as the gearbox and the generator. This noise is transmitted along the
structure of the turbine and radiated from the surfaces. Aerodynamic noise is created
by the interaction of turbulence with the surface of the blades. Turbulence can be
contained in the incoming flow or generated by the viscous flow around the turbine
blades. The reduction of mechanical noise is relatively easy as there are engineering
methods already available. Aerodynamic noise presents a problem as the cause and
propagation of the noise is itself not well understood.

Flow-induced noise can be attributed to a variety of causes. Steady thickness
noise caused by the rotation of the blades, unsteady loading noise caused by the
interaction of the rotor with local flow deficiencies, inflow turbulence and airfoil self
noise are identified as the major sources of flow induced noise associated with wind turbines. Low frequency noise is caused by the periodic change in the angle of attack of the wind interacting with the blades due to blade rotation. Radiation of low frequency noise can be influenced greatly by the supporting tower. This noise is directly related to the blade passing frequency. Another source of aerodynamic noise is inflow-turbulence noise. The interaction of natural turbulence with the blades of the rotor causes noise radiation that has a broadband nature. The next major source of noise is termed as airfoil self-noise. This comprises of the following: trailing-edge noise, laminar boundary layer vortex shedding noise, tip noise, separated/stalled flow noise, blunt trailing edge noise and noise due to blade surface imperfections [20].

Trailing edge noise is caused by the interaction of boundary layer turbulence with the trailing edge of the airfoil. This noise has a broadband character and is the major source of high frequency noise. Beneath the boundary layer, turbulence induces fluctuations in pressure field which gets propagated to the far-field. The intensity of sound generated by the trailing edge-turbulent boundary layer interaction increases with the Mach number. At low mach numbers, this is an inefficient source of noise. Laminar boundary layer noise arises from non-linearities in the laminar boundary layer over the blade surface. Tonal noise is produced when the non-linear instabilities interact with the surface of the blade. A resonant interaction of the trailing edge can occur with the boundary layer that may be reinforced by an acoustic field acting as a feedback loop. This is a significant source of noise when the rotor operates at very low speeds that establish laminar flow until almost the tip of the blade. One way to minimise this noise is to trip the boundary layer. Tip noise is generated as a result of the interaction of tip turbulence with blade tip surface. This has been found to have
a broadband nature. Tip vortices are formed due to cross flow at the tip arising from the pressure differences between the pressure side and suction side. Brooks, Pope and Marcoloni [4] reported that tip noise added 1-2 dB to the high frequency range in the spectra. Stall separation noise is produced as a result of the interaction of excess turbulence with the surface of the blade. Stall occurs when large separation occurs at high angles of attack. The separation results in excess turbulence that interacts with the blade surface to result in noise production. Blunt trailing edge noise is caused by shedding vortices at the trailing edge of the blade. This component of noise has a tonal nature. Von Karman type vortex streets are produced in the case of a blunt trailing edge. Noise is caused by the alternating vortices in the near wake region of the blade. These tend to produce higher fluctuations in the pressure field. When the ratio of the trailing edge thickness to the boundary layer thickness is increased beyond a particular value, the noise band-width decreases and when the ratio becomes large enough, a dipole like characteristic is observed. This noise can be reduced by sharpening the edge.

A wide range of frequencies are present in the noise spectra of a typical wind turbine. The spectrum ranges from very low frequencies (5-10 Hz) up to higher frequencies of the order of tens of Kilohertz. The former is caused by the rotation of the blades and their interaction with the tower or the wake. This end of the frequency spectra is related to the blade passing frequency. Inflow turbulence results in broadband noise while the other mechanisms of noise production such as trailing edge noise, tip noise, laminar boundary layer noise and blunt trailing edge separation noise occur at higher frequencies [2]. High frequency noise radiation has been quite extensively studied in the past. Highly computationally expensive Large-Eddy simulations
of wind turbines have shed light on the highest of frequencies in the noise spectra associated with wind turbines. But the limits on resource requirement made it feasible to run the simulation for a very short duration of the operation of the turbine rotor, around $20^\circ$ of blade rotation only [3]. Information about lower frequency noise cannot be obtained from such simulations because of prohibitively high requirement of resources. This is due to the fact that lower frequencies can be resolved only when the simulation is run long enough. At least 2 to 3 revolutions of the blades would have to be simulated in order to obtain useful information about the nature of lower frequencies. This number is dependent on the flow and operating conditions. Low frequency noise, on the other hand, was not seen as a source of concern by some researchers in the past. Recent studies have however shown that low frequency noise, even if they cannot be heard, can influence the physiological functioning of the human body, causing headache and nausea [19].

These are the various sources of aerodynamic noise for a wind turbine. A discussion of the Acoustic Intensity Based Method is now presented.

### 2.2 Acoustic Intensity Based Method

With the governing agencies around the world setting strict noise emission standards for airports and other types of industries, there is an increasing need to accurately assess the far-field acoustic implications of different applications such as jet engines, wind turbines, etc. Since acoustic waves have small amplitude and travel at high speeds, they are non-dispersive and hence, traditional methods used in computational fluid dynamics cannot be used for aeroacoustic calculations because they become computationally very expensive. This is because high accuracy is required
in almost the entire domain over which the acoustic equations are solved and the
domain is typically very large since the acoustic waves need to propagated to all the
way to the far-field. This would mean that the computational mesh used cannot be
coarsened much towards the outer bounds of the domain as it would compromise the
accuracy of the solution.

The Acoustic Intensity Based Method (AIBM) is a far-field sound radiation prediction method. Integral methods for far-field noise prediction such as Kirchhoff or the
Ffowcs-Williams and Hawkings require contour integration over a closed surface for
each observer location in the far-field. This makes the technique very ineffective for
large 2D and for most 3D problems. Inverse acoustic methods work based on acoustic
pressure information available on a surface. Inverse acoustic methods do not require
the characteristics of the noise sources to be known \textit{a priori} like in integral methods.
Instead, acoustic measurements in the radiated field are used for characterization of
the unsteady noise sources present in the flow. Based on the Helmholtz equation, the
acoustic fields are computed more efficiently. One drawback is that these methods also
require input specification on a closed surface. AIBM is an inverse acoustic method
that can be used to compute the far-field sound radiation from open surfaces. It is
based on the Helmholtz Equation Least Squares method [22]. One another variable
that is specified on the input surface along with the acoustic pressure is the co-located
normal derivative of the acoustic pressure on the surface. The additional information
is required to establish uniqueness of the predicted solution [23]. From the flow field
data, the acoustic pressure and its co-located normal derivative are obtained on an
input surface that can be either open or closed and fed into AIBM, which then pro-
vides us with the acoustic pressure prediction on a chosen far-field observer surface.
The process is depicted in (Figure 2.1). The AIBM assumes that sound propagation is governed by the modified Helmholtz equation on and outside a control surface that encloses all the nonlinear effects that act as noise sources.

![Figure 2.1: Methodology to obtain far-field noise using AIBM.](image)

### 2.2.1 Mathematical formulation

#### 2.2.1.1 Linearized Euler Equations

The starting point for AIBM is the acoustic wave equation. This equation is obtained by linearizing the governing equations of fluid flow. For the propagation of acoustic waves, diffusive properties of the medium such as viscosity and thermal conductivity have little to no effect and are neglected. The acoustic wave propagation is hence governed by the Euler equations as opposed to the Navier-Stokes equations. Using tensor notation, the unsteady mass conservation equation over a control volume without sources or sinks for a compressible fluid is written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0 \quad (2.1)$$

Eq. (2.1) can be expanded as

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u_i}{\partial x_i} + u_i \frac{\partial \rho}{\partial x_i} = 0 \quad (2.2)$$
Note that Einstein’s summation convention is adopted when the numerator and denominator of the partial derivatives carry the same index. The conservation of momentum over a control volume for the $i^{th}$ velocity component is written as

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i}$$  \hspace{1cm} (2.3)$$

This can be expanded as

$$\rho \frac{\partial u_i}{\partial t} + u_i \frac{\partial \rho}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} + u_i \rho u_j \frac{\partial \rho}{\partial x_j} + \frac{\partial p}{\partial x_i} = 0$$ \hspace{1cm} (2.4)$$

Upon combining 2$^{nd}$ and 4$^{th}$ terms in (2.4) and using (2.1), (2.4) can be simplified to give

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} + \frac{\partial p}{\partial x_i} = 0$$ \hspace{1cm} (2.5)$$

Next, the conservation of energy over a control volume yields

$$\frac{\partial (\rho U)}{\partial t} + \frac{\partial (\rho U u_i)}{\partial x_i} = -p \frac{\partial u_i}{\partial x_i}$$ \hspace{1cm} (2.6)$$

For an ideal gas,

$$U = C_v T$$ \hspace{1cm} (2.7)$$

$$p = \rho R T \implies T = \frac{1}{R} \frac{p}{\rho}$$ \hspace{1cm} (2.8)$$

Noting that $\gamma=C_p/C_v$ and $R=C_p-C_v$, we get

$$U = \frac{C_v p}{R \rho}$$

$$= \left[ \frac{1}{\gamma - 1} \right] \frac{p}{\rho}$$ \hspace{1cm} (2.9)$$

Substituting (2.9) into (2.6), the energy equation upon simplifying is obtained as

$$\frac{\partial p}{\partial t} + u_i \frac{\partial p}{\partial x_i} + \gamma p \frac{\partial u_i}{\partial x_i}$$ \hspace{1cm} (2.10)$$
At this point, a description of variables is provided. \( \rho \) is the density of the fluid, \( p \) is the pressure, \( u_i \) is the \( i^{th} \) component of velocity where \( i \) ranges from 1 to 2 in 2D space and from 1 to 3 in 3D space and \( U \) is the internal energy. \( R \) is the characteristic gas constant, \( C_p \) and \( C_v \) are specific heat at constant pressure and specific heat at constant volume respectively and \( \gamma \) is the ratio of the specific heats, the adiabatic index. The governing equations are now linearized by introducing the following variables that correspond to acoustic perturbations of the respective variables. The flow variables are assumed to be represented as the sum of a time invariant mean, represented by \( \bar{()}' \) and time dependent fluctuations, represented by \( ()' \).

\[
p = \bar{p} + p', \quad \rho = \bar{\rho} + \rho', \quad u_i = \bar{u}_i + u_i'
\] (2.11)

Substituting (2.11) into the mass conservation equation (2.2),

\[
\frac{\partial \bar{p}}{\partial t} + \frac{\partial \rho}{\partial t} + \frac{\partial \bar{\rho} \bar{u}_i}{\partial x_i} + \frac{\partial \rho' \bar{u}_i}{\partial x_i} + \frac{\partial \rho' u_i'}{\partial x_i} = 0
\] (2.12)

In a quiescent fluid, perturbations do not exist and hence for the mean flow,

\[
\frac{\partial \bar{p}}{\partial t} + \frac{\partial \bar{\rho} \bar{u}_i}{\partial x_i} = 0
\] (2.13)

By neglecting the product of perturbations and expanding out the remaining terms in (2.12), we obtain

\[
\frac{\partial \rho'}{\partial t} + \bar{\rho} \frac{\partial u_i'}{\partial x_i} + u_i' \frac{\partial \bar{\rho}}{\partial x_i} + \rho' \frac{\partial \bar{u}_i}{\partial x_i} + \bar{u}_i \frac{\partial \rho'}{\partial x_i} = 0
\] (2.14)

Next, the linear Euler equations are obtained by substituting (2.11) into the momentum conservation equation (2.5). The following equation is obtained for the mean flow,

\[
\bar{\rho} \frac{\partial \bar{u}_i}{\partial t} + \bar{\rho} \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{p}}{\partial x_i} = 0
\] (2.15)
Subtracting (2.15) from (2.5), the following linearized momentum equation is obtained

\[ \bar{\rho} \frac{\partial u_i'}{\partial t} + \bar{\rho} \bar{u}_j \frac{\partial u_i'}{\partial x_j} + \bar{\rho} u_j' \frac{\partial \bar{u}_i}{\partial x_j} + \rho' \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial p'}{\partial x_i} = 0 \]  \tag{2.16}

For uniform fluid flow, the derivatives of the mean quantities can be neglected and hence (2.14) and (2.16) become

\[ \frac{\partial \rho'}{\partial t} + \bar{\rho} \frac{\partial u_i'}{\partial x_i} + \bar{u}_i \frac{\partial \rho'}{\partial x_i} = 0 \]  \tag{2.17}

\[ \bar{\rho} \frac{\partial u_i'}{\partial t} + \bar{\rho} u_j \frac{\partial u_i'}{\partial x_j} + \frac{\partial p'}{\partial x_i} = 0 \]  \tag{2.18}

### 2.2.1.2 Acoustic Wave Equation

The acoustic wave equation is obtained from (2.17) and (2.18). First, the time derivative of (2.17) is calculated as

\[ \frac{\partial^2 \rho'}{\partial t^2} + \bar{\rho} \frac{\partial^2 u_i'}{\partial t \partial x_i} + \bar{u}_i \frac{\partial^2 \rho'}{\partial t \partial x_i} = 0 \]  \tag{2.19}

Next, the divergence of (2.18) is obtained,

\[ \bar{\rho} \frac{\partial^2 u_i'}{\partial x_i \partial t} + \bar{\rho} u_j \frac{\partial^2 u_i'}{\partial x_i \partial x_j} + \frac{\partial^2 p'}{\partial x_i^2} = 0 \]  \tag{2.20}

Writing the 2nd term of (2.20) as \( \bar{u}_j \frac{\partial}{\partial x_j} \left( \bar{\rho} \frac{\partial u_i'}{\partial x_i} \right) \) and substituting for the term within brackets from (2.17), the following equation is obtained

\[ \bar{\rho} \frac{\partial^2 u_i'}{\partial x_i \partial t} + \bar{u}_i \frac{\partial}{\partial x_i} \left( - \frac{\partial \rho'}{\partial t} - \bar{u}_j \frac{\partial \rho'}{\partial x_j} \right) = - \frac{\partial^2 p'}{\partial x_i^2} \]  \tag{2.21}

To be consistent with the indices in (2.17), the indices \( j \) and \( i \) in the second term of (2.21) are interchanged without any loss of generality to result in

\[ \bar{\rho} \frac{\partial^2 u_i'}{\partial x_i \partial t} + \bar{u}_i \frac{\partial}{\partial x_i} \left( - \frac{\partial \rho'}{\partial t} - \bar{u}_j \frac{\partial \rho'}{\partial x_j} \right) = - \frac{\partial^2 p'}{\partial x_i^2} \]  \tag{2.22}
Eqn. (2.22) is now subtracted from (2.19) to get
\[
\frac{\partial^2 \rho'}{\partial t^2} + 2 \bar{u}_i \frac{\partial^2 \rho'}{\partial t \partial x_i} + \bar{u}_i \bar{u}_j \frac{\partial^2 \rho'}{\partial x_i \partial x_j} = \frac{\partial^2 p'}{\partial x_i^2} \tag{2.23}
\]

It can be seen that (2.23) contains both pressure and density perturbations. Pressure is a function of density, \(\rho\) and entropy, \(s\) and may be written as
\[
p = p(\rho, s) \implies dp = \left(\frac{\partial p}{\partial \rho}\right)_s d\rho + \left(\frac{\partial p}{\partial s}\right)_\rho ds \tag{2.24}
\]

For an isentropic process, (2.24) reduces to
\[
dp = \left(\frac{\partial p}{\partial \rho}\right)_s d\rho = c_0^2 d\rho \tag{2.25}
\]

From (2.25), a relation between \(p'\) and \(\rho'\) is obtained using the speed of sound, \(c_0\) as
\[
p' = c_0^2 \rho' \tag{2.26}
\]

Using (2.26) and (2.23), the acoustic wave equation in terms of the pressure perturbations with a homogeneous mean flow is obtained,
\[
\frac{\partial^2 p'}{\partial t^2} + 2 \bar{u}_i \frac{\partial^2 p'}{\partial t \partial x_i} + \bar{u}_i \bar{u}_j \frac{\partial^2 p'}{\partial x_i \partial x_j} = c_0^2 \frac{\partial^2 p'}{\partial x_i^2} \tag{2.27}
\]

When the mean flow is absent, the classical wave equation is obtained,
\[
\frac{\partial^2 p'}{\partial t^2} = c_0^2 \frac{\partial^2 p'}{\partial x_i^2} \tag{2.28}
\]

Eqs. (2.27) and (2.28) may be obtained in terms of the density perturbations using (2.26).

### 2.2.1.3 Acoustic Helmholtz Equation

The propagation of acoustic waves in the time domain is governed by the acoustic wave equation (2.27) or (2.28) depending on the flow conditions. This approach is
suitable for the study of broadband noise wherein there is no single frequency that is
dominant in the power spectrum. When distinct frequency components are present
in the flow, it is more meaningful to study the propagation of acoustic waves in the
frequency domain. In the frequency domain, acoustic wave propagation is governed
by the Helmholtz equation. The Helmholtz equation is obtained by Fourier trans-
formation of the acoustic wave equation. The following derivation is based on the
method proposed by Yu et al. [24]. The mean flow is assumed to be in the $x_1$ direction
only, implying that $\bar{u}_2 = 0$ and $\bar{u}_3 = 0$. The wave equation (2.27) becomes

$$\frac{\partial^2 p'}{\partial t^2} + 2\bar{u}_1 \frac{\partial^2 p'}{\partial t \partial x_1} + \bar{u}_1^2 \frac{\partial^2 p'}{\partial x_1^2} = i_{0}^2 \frac{\partial^2 p'}{\partial x_1^2} \quad (2.29)$$

For the transformation to frequency domain, the acoustic wave is assumed to be
harmonic, making it possible to split the wave into a harmonic component represen-
ted by $e^{i\omega t}$ and a complex amplitude represented by $P$. Thus, at the point $x_i$, the
transformation is given by

$$p'(x_i, t) = P(x_i, \omega) \cdot e^{i\omega t} \quad (2.30)$$

Substituting (2.30) into (2.29) and differentiating with respect to time,

$$-\omega^2 P + 2\bar{u}_1 i\omega \frac{\partial P}{\partial x_1} + \bar{u}_1^2 \frac{\partial^2 P}{\partial x_1^2} = c_{0}^2 \frac{\partial^2 P}{\partial x_1^2} \quad (2.31)$$

Upon dividing (2.31) by $c_{0}^2$ and defining the wave number, $k = \frac{\omega}{c_{0}}$ and the Mach
number, $M_a = \frac{\bar{u}_1}{c_{0}}$, the following equation is obtained:

$$-k^2 P + 2M_a ik \frac{\partial P}{\partial x_1} + M_a^2 \frac{\partial^2 P}{\partial x_1^2} - \frac{\partial^2 P}{\partial x_1^2} = 0 \quad (2.32)$$

This equation (2.32) however does not resemble the standard Helmholtz equation
due to the additional terms resulting from the influence of mean flow. If $M_a = 0$, (2.32) reduces to the standard Helmholtz equation. To convert (2.32) into the
standard Helmholtz equation, an additional transformation is performed. It is a coordinate transformation using the Prandtl-Gläuer factor \( \sqrt{1 - M_a^2} \) operating on the \( x_1 \) coordinate, resulting in a new coordinate space,

\[
\tilde{x}_i = \begin{pmatrix} \frac{x_1}{\sqrt{1 - M_a^2}} \\ x_2 \\ x_3 \end{pmatrix}
\]

The derivatives are evaluated using the chain rule,

\[
\frac{\partial P}{\partial x_1} = \frac{\partial P}{\partial \tilde{x}_1} \cdot \frac{1}{\sqrt{1 - M_a^2}}
\]

\[
\frac{\partial^2 P}{\partial x_1^2} = \frac{\partial^2 P}{\partial \tilde{x}_1^2} \cdot \frac{1}{1 - M_a^2}
\]

Using (2.34), (2.32) can be modified as

\[-k^2 P + \frac{2M_ik}{\sqrt{1 - M_a^2}} \frac{\partial P}{\partial \tilde{x}_1} - \frac{\partial^2 P}{\partial \tilde{x}_1^2} = 0 \]  

The only term that now remains to be treated to obtain the standard Helmholtz equation is the second term resulting from the mean flow influence. To eliminate this term, the mean flow influence is separated from the pressure as shown:

\[ P(\tilde{x}_1) = \hat{P}(\tilde{x}_1) \cdot e^{\zeta \tilde{x}_1} \text{ where } \zeta = \frac{M_ik}{\sqrt{1 - M_a^2}} \]

The derivates are now modified as

\[
\frac{\partial P}{\partial \tilde{x}_1} = \left( \frac{\partial \hat{P}}{\partial \tilde{x}_1} + \zeta \hat{P} \right) e^{\zeta \tilde{x}_1}
\]

\[
\frac{\partial^2 P}{\partial \tilde{x}_1^2} = \left( \frac{\partial^2 \hat{P}}{\partial \tilde{x}_1^2} + 2\zeta \frac{\partial \hat{P}}{\partial \tilde{x}_1} + \zeta^2 \hat{P} \right) e^{\zeta \tilde{x}_1}
\]

By substituting the new derivatives obtained from (2.37) into (2.35), the Helmholtz equation in its standard form is obtained:

\[ -\left( \frac{k^2}{1 - M_a^2} \right) \hat{P} - \frac{\partial^2 \hat{P}}{\partial \tilde{x}_1^2} = 0 \text{ or } \frac{\partial^2 \hat{P}}{\partial \tilde{x}_1^2} + \left( \frac{k^2}{1 - M_a^2} \right) \hat{P} = 0 \]

This is the equation governing the propagation of acoustic waves in the frequency domain. Sound radiation is predicted by AIBM based on this governing equation which
is valid for any observer location in the far-field. The input to AIBM is the acoustic pressure on an input surface that is outside the minimum sphere which encloses all the sources of aerodynamic noise. The input consists of the acoustic pressure and its co-located normal derivative resolved into frequency domain representation on the input surface. This surface can be open if normal pressure gradient information is available on it because this additional information is used to establish the uniqueness of the solution based on the unique continuity theory of elliptic equations. If, on the other hand, the normal gradient of acoustic pressure is not available, the input surface has to be closed. A schematic is shown in (Figure 2.2). Thus, if \( \nu \) is the outward normal on an open input surface defined by \( \Gamma_1 \in \Gamma \), sound radiation is predicted by AIBM by solving the equation

\[
\frac{\partial^2 \tilde{P}}{\partial \tilde{x}_i^2} + \frac{k^2}{1 - M_a^2} \tilde{P} = 0 \tag{2.39}
\]

subject to the boundary conditions,

\[
\tilde{P}|_{\Gamma_1} = g_1, \quad \frac{\partial \tilde{P}}{\partial \nu}|_{\Gamma_1} = g_2 \tag{2.40}
\]

The solution to the Helmholtz equation in 3D is based on a spherical coordinate system. For a constant mean flow in the \( x_3 \) direction, the solution is given by

\[
P\left(\tilde{r}, \tilde{\theta}, \tilde{\phi}\right) = e^{i \frac{kM_a}{1 - M_a^2} \tilde{r}} \cos \theta \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left( a_{nm} \cos m\tilde{\phi} + b_{nm} \sin m\tilde{\phi} \right) L_n^m(\cos \tilde{\theta}) G_n \left( \frac{k}{1 - M_a^2} \tilde{r} \right) \tag{2.41}
\]

where \( L_n^m \) is the associated Legendre polynomial of degree \( n \) and order \( m \) for \( m = 0, 1, 2, \ldots, n \) and \( G_n \) is the generalized Hankel function or spherical Hankel function. Note that the solution given in (2.41) is in terms of \( P \) and not in terms of \( \tilde{P} \). This is
obtained after a reverse transformation given by the exponential term in the beginning of (2.41). The coordinate transformation involved in obtaining this solution is

\[
\tilde{x}_i = \begin{pmatrix} x_1 \sqrt{1 - M_a^2} \\ x_2 \sqrt{1 - M_a^2} \\ x_3 \end{pmatrix} = \begin{pmatrix} \tilde{r} \sin \tilde{\theta} \cos \tilde{\phi} \\ \tilde{r} \sin \tilde{\theta} \sin \tilde{\phi} \\ \tilde{r} \cos \tilde{\theta} \end{pmatrix}
\]

(2.42)

where \( \tilde{r} = \sqrt{\tilde{x}_1^2 + \tilde{x}_2^2 + \tilde{x}_3^2} \). Singularities are revealed in the above formulation when the normal derivative is computed because normal derivative of pressure is required by AIBM. To avoid this, an improved technique was proposed [23]. This involves a transformation into Cartesian coordinates which results in the elimination of the singularity terms after their multiplication with suitable trigonometric functions. Using Euler’s formula, the trigonometric functions are written in terms of Cartesian coordinates,

\[
\begin{align*}
\cos m\tilde{\phi} &= \frac{1}{2} \left( e^{im\tilde{\phi}} + e^{-im\tilde{\phi}} \right) = \frac{1}{2\tilde{r}^m \sin m\tilde{\phi}} \left[ (\bar{x} + i\bar{y})^m + (\bar{x} - i\bar{y})^m \right] \\
\sin m\tilde{\phi} &= \frac{1}{2i} \left( e^{im\tilde{\phi}} - e^{-im\tilde{\phi}} \right) = \frac{1}{2i\tilde{r}^m \sin m\tilde{\phi}} \left[ (\bar{x} + i\bar{y})^m - (\bar{x} - i\bar{y})^m \right]
\end{align*}
\]

(2.43)

Upon substitution of (2.43) into (2.41), the improved solution is obtained,

\[
P(x_1, x_2, x_3) = e^{\frac{ikMa}{\beta^2} \tilde{x}_3} \sum_{n=0}^{\infty} \sum_{m=0}^{n} (a_{nm} \xi_m + b_{nm} \eta_m) Q_n^m (\tilde{r}_3) G_n \left( \frac{k\tilde{r}}{\beta^2} \right) / (2\tilde{r}^m)
\]

(2.44)
\[
\beta = \sqrt{1 - M_a^2}, \quad \xi_m = (\bar{x} + i\bar{y})^m + (\bar{x} - i\bar{y})^m \quad \text{and} \quad \eta_m = (\bar{x} + i\bar{y})^m - (\bar{x} - i\bar{y})^m
\] (2.45)

where, \(Q_n^m = L_n^m / \sin^m \tilde{\theta}\). The gradient of pressure \(P\) along the direction of the normal vector to the surface, \(\mathbf{n}\) is now calculated as

\[
\frac{\partial P}{\partial \mathbf{n}}(x_1, x_2, x_3) = \frac{\partial P}{\partial \bar{x}_1} \beta n_{x_1} + \frac{\partial P}{\partial \bar{x}_2} \beta n_{x_2} + \frac{\partial P}{\partial \bar{x}_3} n_{x_3}
\] (2.46)

This formulation now enables us to compute the sound radiation for one single frequency. In reality, more than one frequency component may be dominant in the flow. In such cases, the total sound radiation is obtained by a linear combination of the individual frequency solutions.

### 2.2.1.4 Numerical Implementation

The numerical solution to the acoustic Helmholtz equation is obtained by approximating the infinite summation in (2.44) by a finite summation.

\[
P_N(x_1, x_2, x_3) = e^{ikM_a \bar{x}_3} \sum_{n=0}^{N} \sum_{m=0}^{+} (a_{nm} \xi_m + b_{nm} \eta_m) \frac{Q_n^m(\bar{x}_3)}{(2\bar{r})^m/(2\bar{r}^m)}
\] (2.47)

where \(0 \leq N \leq N_0\). The upper limit of \(N\), i.e. \(N_0\), is chosen to be large enough so as to accurately reconstruct the pressure on the input surface but not so high that the system of equations becomes unstable or computational effort becomes prohibitive. Numerical instability due to a large \(N\) can result in very poor prediction because the coefficients are determined based on the least squares technique. The numerical procedure is initiated from a small value for \(N\) and gradually incremented up to \(N_0\) in a loop. The optimum value for \(N\) is chosen for the far-field prediction based on the minimum overall error between the input and reconstructed solution on the input surface [25]. Referring to (2.40), the data at a given point \(x_i\) on the input surface
may be written using (2.47) as

\[ g_1(x_i) = e^{ik\lambda x_3} \sum_{n=0}^{N} \sum_{m=0}^{n} (a_{nm}\xi_m + b_{nm}\eta_m) Q_n^{m}(\frac{x_3}{r}) G_n^{m}(\frac{k\sqrt{\beta}}{\beta^2}) / (2\sqrt{r}) \] (2.48)

The normal derivative of the pressure at that point may be calculated as

\[ g_2(x_i) = \frac{\partial}{\partial n} \left( e^{ik\lambda x_3} \sum_{n=0}^{N} \sum_{m=0}^{n} (a_{nm}\xi_m + b_{nm}\eta_m) Q_n^{m}(\frac{x_3}{r}) G_n^{m}(\frac{k\sqrt{\beta}}{\beta^2}) / (2\sqrt{r}) \right) \] (2.49)

An over determined system of equations can be formed from (2.48) and (2.49). The left hand side of the above two equations can be assembled into a vector \( B \) that corresponds to the surface measurements of pressure and the co-located normal derivative. The coefficients that need to be calculated, namely \( a_{nm} \) and \( b_{nm} \) are condensed into a vector \( X \). The other constants that remain in the summation comprising the Legendre polynomials and Hankel functions are arranged into a matrix \( A \). For \( M \) points on the input surface, the matrix equation becomes

\[ B = A \cdot X \] (2.50)

with,

\[ B = [g_1(1,1), \ldots, g_1(1,M), g_2(1,1), \ldots, g_2(2,M)]^T_{(2M \times 1)} \]

\[ X = [a_0, \ldots, a_N, b_1, \ldots, b_N]^T_{(2N+1 \times 1)} \]

\[ A = \begin{bmatrix} Y & Z \\ Y' & Z' \end{bmatrix}_{(2M \times (2N+1))} \]

where \( Y, Z \) correspond to the pressure equation for \( g_1 \) and \( Y', Z' \) correspond to the equation for the normal derivative of pressure, \( g_2 \). The coefficient vector \( X \) is estimated by AIBM using the linear least squares technique that minimises the standard \( \| \cdot \|_2 \)-norm as

\[ \| AX - B \|_2 = minimum \] (2.52)
The coefficients are estimated for the current value of $N$ by singular value decomposition. A set of reconstructed solutions is obtained on the input surface for the chosen values of $N$ based on the estimated coefficients. The optimum value of $N$ that is used for the far-field prediction is that value $N$ for which the total error between the input pressure and the reconstructed pressure on the input surface is minimum. Once the optimum $N$ is established, the acoustic pressure at any far-field location can be ascertained very quickly as the analytical solution is already available (2.44). The accuracy of the reconstruction and prediction is also dependent on the accuracy with which the Legendre polynomials and Hankel functions are evaluated at each point and each order. They are computed based on recurrence relations and some of those details are included in the appendices.
CHAPTER 3

NOISE RADIATION FROM 2D AIRFOIL SECTIONS

A detailed discussion of the aeroacoustic equations and AIBM was presented in the previous chapter. In the current chapter, a hybrid method that combines computational fluid dynamics (CFD) and AIBM for a 2D problem is presented in order to validate the AIBM for physical problems. This problem concerns the flow past a 2D airfoil section at a moderately high angle of attack. Two airfoils, NACA-2409 and NREL S809, of chord 1\( \text{m} \) are selected for this study. The latter airfoil is used in the NREL Phase VI Horizontal Axis Wind Turbine (HAWT) rotor, which is analysed in the following chapter. In order to obtain the acoustic pressure on an input surface, the flow field has to be generated first. The flow field around the airfoil is obtained using CFD simulations in this study. The acoustic pressure and its co-located normal derivative are then extracted on an input surface for AIBM. This information is utilized for far-field acoustic prediction. A description of the mesh, the CFD method and AIBM far-field predictions is presented in the following sections of this chapter. A schematic of the problem is shown in (Figure 3.1)
3.1 Flow Field Generation

The flow field was obtained in the near-field region using CFD simulations on a structured mesh. For standard CFD simulations, it is usual practice to have a refined mesh close to solid surfaces to sufficiently resolve the flow structures and gradually coarsen the mesh close to the far-field. Far from solid surfaces, the mesh usually tends to be very coarse. This is good enough for CFD because the main region of interest is in the region close to the surfaces present in the flow. For acoustics, the considerations are slightly different. As explained previously, acoustic waves are less dispersive. They have small amplitudes and travel very fast. The mesh would have to be fine enough to capture the acoustic waves sufficiently accurately. Hence, compared to a CFD simulation meant to just visualize the flow structures, the mesh for aeroacoustics
purposes would have to be sufficiently finer farther away from surfaces present in the flow.

For this study, keeping the points discussed above in mind, the mesh was refined close to the surface of the airfoil to resolve the wall $y^+$ within the viscous sub-layer. The domain extends up to 12.5 chord lengths in the transverse direction and 20 chords in the stream-wise direction from the trailing edge. A pressure based compressible flow solver with $k – \omega$ turbulence model was used. The outer edges of the domain were modeled as pressure far-field with a free stream Mach number of 0.2. The angle of attack was set to 20°. A high angle of attack was chosen in order to gain an insight into the performance of the two airfoils when the flow is separated from the surface. It is important that the flow remains attached to the surface of a wind turbine blade at all locations so that its performance is not compromised by flow separation. At 20°, the flow is expected to be separated and consequently give rise to more aerodynamic noise due to separation. A comparison of performance between NACA-2409 and S809 airfoil at this angle of attack would be helpful to evaluate the design of S809 and its applicability for use on wind turbines. The computational grid, shown in (Figure 3.2), consists of 121,630 cells for NACA-2409 airfoil and 63,140 for NREL S809 airfoil. They were created in Gambit. To validate the CFD results, the pressure coefficient, $C_P$, was compared against data available in the literature. Computational results from Yu et. al. [12] for NACA-2409 and experimental data from [18] for S809 were used for validation. There is very good agreement between the $C_P$ values predicted by Fluent and the reported data. These are plotted in (Figure 3.3). A plot of instantaneous velocity contours with superimposed streamlines is shown in (Figure 3.4). It is worth mentioning that the flow separates at close to 30% chord.
length from the leading edge of S809 airfoil while separation is observed almost right off the leading edge for NACA-2409 airfoil.

### 3.2 Spectrum Analysis and Acoustic predictions

For the calculation of far-field noise, the acoustic pressure is required. Acoustic pressure or perturbation pressure may be defined as the time-wise fluctuations in gauge pressure about a time-invariant mean value. The time variation of the perturbation pressure (mean subtracted) and its corresponding frequency spectrum at a point right behind the trailing edge of the airfoil are shown in (Figure 3.5) and (Figure 3.6) respectively. The fluctuations are large because the point is right in the wake of the airfoil where the pressure fluctuations are high. The most dominant frequency present in the fluctuations was found to be around 56Hz for the S809 airfoil and around 41Hz for NACA-2409 airfoil. With the flow field at hand, the acoustic radiation signature of the airfoil was predicted using AIBM. A circular input surface that enclosed all noise sources was defined at a radius of 11m from the leading edge. Perturbation pressure and its normal derivative were obtained on this surface. The
Figure 3.3: $C_P$ comparison for the airfoils.
Location of the input surface is chosen for good agreement between predicted pressure and input pressure from the CFD simulation results. Beyond a radius of 11m, the prediction matched well with the CFD solution and hence, the input surface was defined at a radius of 11m. The reconstructed solution is plotted against the perturbation pressure obtained from the CFD simulation in (Figure 3.7). The overall agreement between the reconstructed pressure and the input is seen to be very good. CFD data
Figure 3.6: Perturbation pressure vs Frequency.

was used to validate the reconstructed solution from AIBM due to the non availability of acoustic data for the airfoils in the literature in the regime of operation. The acous-

tic pressure for the two airfoils were predicted at a distance of 50m (Figure 3.8). The observer location is defined as a circular surface at a radius of 50m from the leading edge of the airfoil. Separation results in stronger interaction between the turbulent eddies and the trailing edge, producing stronger fluctuations in the pressure field.

Figure 3.7: Comparison of input and reconstructed solution on input surface.
The pressure fluctuations are seen to be higher for NACA-2409 than for S809 due to extensive separation of flow over the former airfoil. Higher sound pressure levels for the NACA-2409 airfoil, shown in (Figure 3.9), can be attributed to this fact.

For both the airfoils, the execution time for AIBM was in seconds. This reinforces the fact that AIBM is a very fast method for far-field acoustic predictions since the prediction is based on an analytical solution. The major portion of the execution time is spent in determining the optimum value of $N$ in (2.47), the number of coefficients to be included in the series summation. Once this is determined, acoustic pressure at the observer location is predicted using the analytical solution using the coefficients calculated based on $N_{\text{optimum}}$. For both the airfoils, $N_{\text{optimum}}$ was determined to be 8, which resulted in a total of 17 coefficients that had to be included in the truncated series for the best accuracy in acoustic pressure prediction. This study has proved the effectiveness of AIBM for studying far-field acoustic radiation for practical problem.
Figure 3.9: Sound pressure level (SPL) prediction on observer surface.

The method is now extended to a 3D problem involving the NREL Phase VI HAWT rotor.
CHAPTER 4

ACOUSTIC ANALYSIS OF NREL PHASE VI WIND TURBINE ROTOR

In the previous chapter, the applicability of AIBM to practical problems was established. The AIBM is now extended to a 3D problem to predict the far-field noise for a wind turbine rotor. Some of the sources of noise from an operating wind turbine were discussed in Chapter 2. As in the previous exercise, a hybrid approach involving CFD simulations and AIBM prediction is adopted. The generation of the flow field around the wind turbine rotor using CFD and the evaluation of noise radiation from the turbine are discussed in detail in the following sections of this chapter.

4.1 Flow Field Generation around Wind Turbine Rotor

As in the previous case, the flow field around the wind turbine rotor was obtained using CFD. The turbine rotor model was generated using data from the basic machine parameters [13], [7]. The blades are modeled using the NREL S809 airfoil with varying chord and twist along the span. The model consists of 2 blades, one in the 12 o’clock position and the other one in the 6 o’clock position centered about the origin. The blade tip is 5.532m from the origin. The computational domain extends up to 2.6 radii in the radial direction, 2 radii upstream and 4 radii downstream of the rotor in axial
direction. An unstructured mesh was created around these blades using FLUENT’s preprocessor, GAMBIT. The mesh is significantly more complicated than for the airfoils because of the complex geometry. The complexity of the geometry warranted an unstructured mesh in order to control the size of the cells in the domain more effectively. The mesh consists of 8.69 million cells in 3 volumes. A boundary layer mesh was attached to the surface of the blades to resolve the boundary layer. A sizing function was defined while the mesh was created to ensure that the size of the cells increased gradually from the surface of the blades to the far-field. Unstructured mesh provides us with the flexibility to cluster cells in a region of interest and this advantage was utilized. The blades are positioned at the origin, the span of the blades being aligned with the Y axis. The mean flow is in the +Z direction with the blades rotating about the origin in the XY plane. (Figure 4.1) shows the arrangement of the blades and the mesh. A small volume was defined close to the surface of the blades to ensure good distribution of cells is in the vicinity of the blades. A second intermediate volume was defined outside this volume to capture the wake geometry behind the rotor. The clustering of cells drops as the distance from the blades increases. An outer volume was created to define the far-field. The cells close to the edge of this volume are the coarsest. The multiple reference frame (MRF) capability in FLUENT was utilized for the simulations. This implies that the force terms corresponding to centrifugal and Coriolis effects are included in the governing equations in those volumes that are set to rotate at a specified angular velocity. The inner volumes were set to rotate with the blades at an angular velocity of 7.54 rad/s. The free stream wind speed for this case was set to 7 m/s at 0° yaw. The time step for the simulations was set at $8.4 \times 10^{-05}$ seconds which is much smaller than the constant for dynamic stall defined
as \((\text{chord}/R\Omega)\). The minimum dynamic stall constant was found to be around 0.01 for these simulations. Realizable \(k-\epsilon\) turbulence model with enhanced wall functions was used in the simulations. Enhanced wall functions were used because trial runs showed that the wall \(y^+\) could not be brought to realistic values using the basic \(k-\epsilon\) turbulence model with enhanced wall functions. In order to improve the solution on the blades, the realizable \(k-\epsilon\) turbulence model with enhanced wall functions was used. With the use of enhanced wall functions, the value of wall \(y^+\) need not be limited to very small numbers, but at the same time, should not be excessively high. A typical range of 50 to 150 is preferred. With the use of enhanced wall functions, the average value on the blade was found to be around 110. Better turbulence models would be used in the future once the effective acoustic predictions for the wind turbine rotor are demonstrated. As we had pointed out, the major focus is on obtaining the far-field acoustics of the turbine blade and our aim is to have a good CFD model if not the best. Once favorable results are obtained, efforts to have a model with higher accuracy would be in place. The major focus of this study is in the low end of the frequency spectrum. The cause of noise at the lower end is due to the rotation of the blades and is directly linked to the blade passing frequency. To analyse low frequency noise, U-RANS models would be the best choice since this is computationally least expensive compared to other techniques that are available for modeling turbulent flows such as LES, Direct Numerical Simulations(DNS) or Detached Eddy Simulations(DES). Moreover, for the chosen operating condition, there is no separation of flow from the blades, making predictions from the U-RANS model even more reliable. The unsteady simulation was run up to five and a half revolutions of the blades on not less than 32 processors on the Ohio Supercomputer Center’s Glenn cluster that uses
AMD opteron multi-core technology with an average processing speed of 2.5GHz per processor. The coefficient of pressure \( (C_P) \) comparisons served as the main metric to assess the accuracy of the results obtained from our CFD model. The classical definition of \( C_P \) uses dynamic pressure based on the free stream velocity to normalize the gauge pressure at the wall. \( C_P \) for this problem was computed as

\[
C_P = \frac{\text{gauge pressure}}{\frac{1}{2} \rho_{\infty} (V_{\infty}^2 + (r\omega)^2)}
\]  

(4.1)

In (Figure 4.2), time averaged \( C_P \) values from one full revolution of the rotor with

Figure 4.1: Computational domain and unstructured mesh.
Figure 4.2: $C_P$ comparison at 4 span-wise locations.
Figure 4.3: Instantaneous velocity contours at 4 span-wise locations.
data stored for around every 14° of blade revolution are compared against experimental data. Variations in $C_P$ among the different spanwise locations can be observed in (Figure 4.2). The twist in the blade and rotational effects result in a change in the relative angle of attack along the blade span thus changing the pressure distribution along the blade span. This is observed in both the experimental data and numerical computation. The agreement between the simulation and the experiment is good especially at the lower stations while it seems to drop as we move higher up the blade. The reasons for this may be due to the chosen turbulence model or mesh quality. The $k - \epsilon$ model is prone to over-predict the turbulent quantities and it might very well be the case here as we see consistent over prediction of $C_P$ at all the locations.

Instantaneous relative velocity contours at the respective locations are presented in (Figure 4.3). It may be inferred from (Figure 4.3) that the flow is attached to the blade at all the locations. This is especially important to verify because U-RANS predictions are good so long as there is no flow separation from solid surfaces. The blade thrust coefficient was computed from the simulations as,

$$C_T = \frac{T}{\frac{1}{2} \rho_{\infty} (\pi R^2) V_{\infty}^2}$$

and its variation with time is shown in (Figure 4.4). It was reported in [16] that $C_T$ converged to a value of around 0.52. The value of $C_T$ predicted by FLUENT is between 0.532 and 0.533, as shown by (Figure 4.4) is only slightly higher than the value reported in [16]. Iso-surface of $Q$-criterion colored by velocity magnitude is shown in (Figure 4.5). The wake behind the turbine rotor is clearly visible in this figure. Wake expansion is also visible in (Figure 4.5).
Figure 4.4: $C_T$ vs Time

Figure 4.5: Wind turbine wake
4.2 Aeroacoustic Analysis

The flow field was obtained using CFD and the validation of the results was described in the previous section. In this section, frequency analysis and aeroacoustic characteristics are discussed. The time trace for the variation of acoustic pressure at a point very close to the blade tip is plotted in (Figure 4.6). It is shown that the fluctuations in pressure have become periodic. The corresponding spectra for the acoustic pressure in magnitude and decibel scale are shown in (Figure 4.7). The reference pressure was chosen to be $20\mu Pa$. The dominant peak is seen to be in the low end of the spectrum around the blade passing frequency. Up to two harmonics can be observed in the spectrum. Higher up in the spectrum, lots of energy containing components are observed. Noise level contributed by different frequency components is observed to decrease as we move from the low frequency end of the spectrum to
Figure 4.7: Perturbation pressure vs Frequency.

the high frequency end. For the spectrum shown in (Figure 4.7), it may be concluded that low frequency components contribute towards wind turbine noise. Low frequency noise from wind turbines is a subject that has not been researched extensively. Low frequency noise is produced by two factors. One is the displacement of air as the blade passes through. The fluctuations arise from the air trying to move back to the original position after the blade has passed through. This is called thickness noise and it has the nature of a monopole source. Monopole radiation does not have a directional preference and the acoustic waves generated by a monopole are omnidirectional. The second factor that causes low frequency noise is unsteady loading that is caused by the rotor blade interacting with local flow deficiencies which results in low frequency
noise. This is called loading noise and it acts like a dipole source. Dipole radiation has a directional preference and acoustic waves generated by a dipole have a bi-directional nature. The axial velocity profile is shown in (Figure 4.9). It can be seen that there is a velocity deficit only in the region behind the blades. There is no deficit in the space between the blades and this is because the nacelle was not included in the model. This gives rise to higher velocities in the region between the blades which will not be present in a real wind turbine flow field. The wavelength associated with the low frequency waves is high compared to the dimensions of the blade making them acoustically compact [21]. Compact acoustic sources result in low frequency noise that has a dipole character. The interaction between flow deficiencies and the rotor results in unsteady loading which gives rise to sound radiation that has a dipole character. The frequency spectrum from the simulation shows a dominant peak at 5.5Hz that is more than the blade passing frequency for this case, which is 2.4Hz. We can hence conclude that a combination of thickness and loading noise responsible for the spectrum shown in (Figure 4.7). The other peaks that can be seen in the rest of the low frequency range up to around 90Hz may be attributed to the fluctuations in blade loading. To compute far-field noise radiation, a cylindrical input surface was defined at a radius of 8.8m from the centre of the rotor such that the axis of the input surface aligned with the vertical axis. The input surface has 100 points in the circumferential direction and 16 points in the axial direction so that the even the shortest wavelength included in the analysis was amply resolved. It has been observed from previous experience that at least 16 points are needed per wavelength of the wave corresponding to the frequency component being analyzed. A schematic of the input surface is shown in (Figure 4.8). The observer location
was defined as a cylindrical surface in the X-Z plane at a radius of 40m from the centre of rotation. The acoustic pressure was obtained in the frequency domain using Fast Fourier Transform (FFT). Unlike in the 2D aeroacoustic analysis, a broader range of frequencies is seen to make significant contributions to the spectrum that cannot be neglected. As a result, many frequency components were included in AIBM.
calculations to predict the far-field noise with increased accuracy. Since AIBM is a
frequency domain technique, the code was executed for a range of frequencies from
2Hz up to 125Hz and the far-field overall sound pressure level (OASPL) was computed
based on the contributions from the different frequency components. Comparison
of the real and imaginary components of the acoustic input and the reconstructed
pressure on the input surface are shown in (Figure 4.10) and (Figure 4.11) for the
dominant frequency component of 5.5Hz. It may be noted that the reconstruction
is very accurate on the input surface. The acoustic pressure was then predicted
on the observer surface using a limited set of frequencies from the overall spectrum
at the input surface. To gain a better insight into the directivity characteristics of
the frequency components, OASPL calculations were made for a subset of the total
frequencies that were analyzed. The directivity of OASPL computed from just the
lowest four frequency components ranging from 2Hz to 11.5Hz (RANGE1) are shown
in (Figure 4.12). The OASPL directivity computed for a frequency range of 77Hz to
125Hz (RANGE2) is shown in (Figure 4.13). It can be seen that the lowest frequencies
tend to have a monopole-like radiation pattern while the higher frequencies have a
dipole-like radiation pattern. This confirms that the radiation of thickness noise
shows a monopole behavior resulting from the displacement of mass and loading
noise shows a dipole-like radiation caused by fluctuating forces on the blades [20].
The predicted overall sound pressure level (OASPL) at the observer location based on
all the computed frequency components is shown in (Figure 4.14). It may be observed
that the spatial distribution of sound level at the observer location appears resemble
sound radiation from a dipole source. Low frequency noise radiation patterns recorded
for a large-scale HAWT by Shepherd and Hubbard [17] are shown in (Figure 4.15).
The predicted directivity pattern roughly matches the pattern reported in [17].

Figure 4.10: Comparison of real part of acoustic pressure

Figure 4.11: Comparison of imaginary part of acoustic pressure

The reason for the discrepancy could be caused by the absence of the tower in the computational model. Since amplification of low frequency noise is, to a good extent, caused by the rotor tower interaction, the next step would be to include the tower and study its implications on the acoustic field.
Figure 4.12: Prediction for RANGE1 set of frequencies

Figure 4.13: Prediction for RANGE2 set of frequencies
Figure 4.14: Prediction of sound level for all frequencies

Figure 4.15: Reported sound pressure level plot for a large scale wind turbine [17]
The Acoustic Intensity Based Method was successfully extended to real-time problems and the far-field acoustic radiation pattern for NACA-2409, NREL S809 airfoil and NREL Phase VI HAWT rotor were predicted. For the 2D and 3D problems studied, a hybrid approach involving CFD and AIBM was adopted to predict far-field noise. The 2D acoustic analysis problem helped validate AIBM for noise prediction for stalled airfoils. Useful insights into the performance characteristics of NACA-2409 and NREL S809 airfoils subject to the same flow conditions and the consequent acoustic implications were obtained from the 2D flow/acoustics analysis. The prediction for the NREL Phase VI HAWT rotor was based on CFD-URANS simulations of the rotor operation with a free stream velocity of $7m/s$ at $0^\circ$ yaw. The simulations predicted no flow separation along the span of the rotor blade. The simulation results were validated against experimental measurements. The acoustic spectrum was calculated for the low frequency components to understand their generation and propagation. Thickness noise and loading noise were identified to be the two sources of low frequency noise. The thickness noise was found to have a monopole-like radiation pattern and the loading noise was found to have a directional dependence, giving it a dipole-like character. The next step would be to include the wind turbine
tower and nacelle in the analysis to get an insight into the influence of rotor tower interaction on the far-field acoustics. More realistic operating conditions and better turbulence models would be applied in the next stage to improve the predictions of the CFD model and consequently, of AIBM. AIBM is a suitable choice for a hybrid far-field acoustic prediction tool as it has been shown to work efficiently with 2D and 3D data sets through this study.
APPENDIX A

ASSOCIATED LEGENDRE POLYNOMIAL

The derivation of the recurrence relation for the associated Legendre polynomial is based on [1] and is shown below. For $1 \leq x \leq 1$, the associated Legendre polynomials are defined in terms of ordinary Legendre polynomials by

$$L_m^m(x) = (-1)^m (1 - x^2)^{m/2} \frac{d^m}{dx^m} L_n(x) \quad (A.1)$$

where

$$L_n(x) = L_n^0(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n \quad (A.2)$$

Introducing the stable recurrence for $n$,

$$(n - m)L_n^m = x(2n - 1)L_{n-1}^m - (n + m - 1)L_{n-2}^m \quad (A.3)$$

The starting value is defined by the close-form expression

$$L_m^m = (-1)^m (2m - 1)!!(1 - x^2)^{m/2} \quad (A.4)$$

The notation $n!!$ denotes the product of all odd integers less than or equal to $n$.

Using (A.3) with $n = m + 1$ ans setting $L_{n-1}^m = 0$, we obtain

$$L_m^{m+1} = x(2m + 1)L_m^m \quad (A.5)$$
Eqs. (A.4) and (A.5) provide the two starting values required for (A.3) for general $n$.

The derivative of $L^m_n$ with $m < n$ can be derived from its definition

$$\frac{\partial L^m_n}{\partial x} = \frac{-mx}{1-x^2}L^m_n - \frac{1}{\sqrt{1-x^2}}L^{m+1}_n$$  \hspace{1cm} (A.6)

For $m = n$, the derivative is

$$\frac{\partial L^m_n}{\partial x} = \frac{-mx}{1-x^2}L^m_n$$  \hspace{1cm} (A.7)

Let

$$Q^m_n(x) = \frac{L^m_n}{1-x^{2m/2}} = (-1)^m \frac{d^m}{dx^m}L_n(x)$$  \hspace{1cm} (A.8)

This results in the recurrence relation for $Q^m_n(x)$ as

$$(n-m)Q^m_n = x(2n-1)Q^m_{n-1} - (n + m - 1)Q^m_{n-2}$$  \hspace{1cm} (A.9)

$$Q^m_m = (-1)^m(2m - 1)!!$$  \hspace{1cm} (A.10)

$$Q^m_{m+1} = (-1)^m(2m + 1)!!x$$  \hspace{1cm} (A.11)

The derivative for $Q^m_n(x)$ with $m < n$ is

$$\frac{\partial Q^m_n}{\partial x} = -Q^m_{n+1}$$  \hspace{1cm} (A.12)

and when $m = n$,

$$\frac{\partial Q^m_n}{\partial x} = 0$$  \hspace{1cm} (A.13)
APPENDIX B

SPHERICAL HANKEL FUNCTION

The $0$–th order and first order spherical Hankel functions based on [1] are

$$G_0(r) = \frac{1}{r} \sqrt{\frac{2}{\pi}} \exp \left[ -i \left( r - \frac{\pi}{2} \right) \right]$$  (B.1)

$$G_1(r) = \frac{1}{r} \sqrt{\frac{2}{\pi}} \exp \left[ -i (r - \pi) \right] \left( 1 - \frac{i}{r} \right)$$  (B.2)

Employing the recurrence formulation, $n$–th order can be obtained based on the $0$–th and first order formulations.

$$G_n(r) = \frac{2n - 1}{r} G_{n-1} - G_{n-2}$$  (B.3)

The corresponding derivative formulations for $0$–th order is just simply derived from $G_0(r)$ and the higher orders are obtained by recurrence relations.

$$G'_0(r) = -G_0 \left( \frac{1}{2} + i \right)$$  (B.4)

$$G'_n(r) = -\frac{n+1}{r} G_n + G_{n-1}$$  (B.5)
APPENDIX C

COMPARISON OF INPUT AND RECONSTRUCTED SOLUTION ON THE INPUT SURFACE FOR SOME OTHER FREQUENCIES

Figure C.1: $f = 2.86Hz$. 
Figure C.2: $f = 8.59\, Hz$.

Figure C.3: $f = 11.45\, Hz$.

Figure C.4: $f = 14.31\, Hz$. 
Figure C.5: $f = 17.17\, Hz$. 

(a) Real part

(b) Imaginary part
BIBLIOGRAPHY


