MIDDLE SCHOOL DEAF STUDENTS’ PROBLEM-SOLVING BEHAVIORS
AND STRATEGY USE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy in the Graduate
School of The Ohio State University

By
ChongMin Lee, B.S., M.A.

Graduate Program in Education
THE OHIO STATE UNIVERSITY

2010

Dissertation Committee:
Dr. Douglas T. Owens, Advisor
Dr. Scot Danforth
Dr. Brenda Brueggemann
ABSTRACT

The purpose of this research is to describe and understand the ways in which deaf middle school students understood and solved compare word problems, and to examine their overall strategy use in learning mathematics. The participants in the study were deaf middle school students, attending a residential state school for the deaf. Most of them used sign language as their primary language and who had different communication modes and learning styles. This study used grounded theory to shape its methodological framework. Data were collected using four methods. 1) A think-aloud technique was utilized in order to explore the complex cognitive processes of solving word problems. 2) Interviews were conducted using a structured questionnaire, to determine deaf students’ use of self-regulation learning strategy. 3) A computation test was administered to assess the students’ computation abilities, which are necessarily related to word problem solving. 4) Student background surveys were administered to examine their primary language environments at home.

The think-aloud protocols were video recorded, then transcribed and analyzed using a constant comparative method. Four problem-solving behaviors were found in this study. First, deaf students have more difficulty understanding inconsistent language (IL) problems than consistent language (CL) problems. That is, they committed reversal errors on IL problems more frequently than on CL problems. These reversal errors resulted from their lack of syntax knowledge, lack of ability to make inferences from the problem.
details, lack of fractional knowledge, and use of key word strategy. Second, the students tended to use key word strategy, which is identified as a direct-translation approach (DTA), as they read and solved word problems, regardless of the problem type. Unlike hearing students in previous studies, most deaf students used a meaning-based approach (MBA) with easier problems and a DTA approach with complex problems. Third, the students’ insufficient fraction knowledge contributed to their difficulties with word problems. Fourth, students’ problem-solving behaviors varied depending not only on the complexity of problems and their prior knowledge about specific ideas, but also on their language mode, communication styles, and the amount of relevant knowledge they possessed. Finally, with regard to strategy use, the students tend to rely on asking their teachers for help when they encountered difficulties with mathematics, rather than trying to solve the problems themselves using a cognitive or metacognitive strategy. These participants should therefore be given opportunities to learn the cognitive and metacognitive strategies necessary to become independent learners, and to enhance their academic achievement in reading and mathematics.

Overall, this study contributes to understanding why deaf middle school students have difficulty with word problems. The major finding is that the deaf students in this study do not have many opportunities to experience a variety of problem structures in their classes. Accordingly, in order to enhance deaf students’ word problem-solving ability, these findings suggest that teachers should understand and address the students’ characteristics, and provide more challenging problems with a variety method of representing the problems.
Dedicated to my hearing father, mother, and three hearing sisters

with all my love and respect
ACKNOWLEDGMENTS

I am Korean and deaf, and the word “deaf” has a special meaning in my life. Honestly, I was constantly confused about who I was during my whole life, and it was not easy for me to accept my identity as a deaf person until I travelled to the U.S.A to pursue a Ph.D. degree. It helped for me to begin communicating through sign language because I have lived in a hearing world where I needed to communicate mostly through spoken language with hearing people. It may be possible for me to learn spoken English, but I don’t think it is worthwhile for me to do so because I can communicate with hearing individuals in American Sign Language with the assistance of sign language interpreters. At the end of this journey, I have learned that being deaf is neither a good thing nor bad thing but, to me, it is just my life. I have found my identity here as a deaf person.

I started this journey with two goals: to break down individuals’ prejudice toward people with disabilities, especially deaf people, and to share the experiences with my deaf students and friends who use sign language, but learned written language without sounds. Through my experiences, I wanted to understand why they have problems with written language, and I believe I have found nearly all the answers to my questions.

I would like to take this opportunity to thank all the deaf students who participated in the dissertation and all their parents and teachers who helped me and encouraged my study. Without their dedication, I would not have been able to complete...
this PhD journey.

I wish extend my deepest gratitude to my advisor, Dr. Douglas T. Owens, who has taught me to walk with wisdom on this journey, for his patience, expertise, and encouragement of this project. You know this journey was really difficult for me. I thank you for all the support, assistance, and kindness you granted me during this journey. Honestly, I can’t imagine completing this journey with your all support. It has indeed been an honor.

I couldn’t have realized this goal without Dr. Scot Danforth whose inclusive education and disability studies in education provided me not only an opportunity to explore this study from my own perspective as a deaf person, but also encouraged me to grow as a qualitative researcher and an independent and critical thinker. I still remember two books you introduced me to when I took an independent study with you titled “No Pity: People with Disabilities Forging a New Civil Rights Movement” and “Nothing About Us Without Us: Disability Oppression and Empowerment.” With what I learned from these books, I am certain that I can enter a big hearing world with more confidence. I greatly appreciate all your support, encouragement, and help through this journey. Your thoughts and ideas you brought to the projects are sincerely appreciated. I am very fortunate to have such a good mentor in this journey. I am looking forward to working with you on future projects.

I also would like to thank Dr. Brenda Brueggemann for agreeing to be on my committee and for her expertise, support, and encouragement in literacy and deaf culture. Her course on deaf culture and deaf people helped me find my identity as a deaf person.

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This really was important to me for my research and my life as a whole.

I am grateful for the contributions of Dr. Peter V. Paul, not only for your financial support, but also contributing to my theoretical and professional growth in this field of deaf education. Without your knowledge and support, I would not have had this opportunity. I would like to also extend thanks to Dr. Laurie Katz for your support and advice for the first year of my PhD program. I will remember your kindness and advice you granted me through this journey.

Thank you to my deaf friends in deaf churches and hearing churches and the deaf community and the hearing community in both Korea and U.S.A. Without your support and prayers, I would not have finished this journey. I am thankful to my former advisor in Korea, Dr. GwangBok Lee in Mathematics and Dr. YoungYook Kim in Special Education, for their support and guidance. I want to also thank a former visiting scholar at OSU, Dr. ChangWook Kang, for his support, advice, and God’s words he gave me while writing this dissertation. Special thanks go to Dr. Heo, for all the encouragement he has given me during my Ph.D. studies. I want to thank my OSU colleagues – JeeYoun Yang, Tia Jones, Claudia Kinder, and Linda Ross,- who made me feel at home and supported my endeavors. I especially want to thank my dear friend, Emily LeGros. You helped introduce me to the Columbus deaf community. I am grateful for your friendship and support through this process. Your words of encouragement, advice, and humor helped me through some difficult times on this journey. You, your fiancé Tom Fermier, and your entire family have given me a “home-away-from-home” and I will cherish the memories we have made.
Last, and most importantly, I thank my parents for their unconditional love, endless sacrifice, support, prayers, and faith in me, and allowing me to be as ambitious as I wanted to be. I am proud to be your daughter. If someone asks me how best to raise and instruct deaf children, I will tell them to meet my parents, who are the best ones to be able to answer your question. I would like to share my deep gratitude for the love, patience, and support of my three hearing sisters, YeJin, MyungJin, SunMin and their soul mates. Without your support and teaching, I would not have been possible to complete this degree and make my dreams come true. To my niece, JiHo and nephew JiMin, I want you to know how much your expressions of love meant to me.

I am ever grateful to God, who gave me the talent to pursue and complete this journey. God, I love You as much as You do me!
VITA

April 16, 1974 Born - Seoul, Korea

1997 B.S. Mathematics
Dankook University

1998-2002 Researcher and Editor,
Korean Associate of the Deaf

2003 M.A., Special Education,
DanKook University, Seoul, Korea

2003-2005 A Lecturer, Dankook University and Cheonan University

2005-2010 Graduate Teaching Assistant, The Ohio State University
Columbus, Ohio

Publications


Lee, C.M., Kim, Y.W. (2002). Working Memory of Deaf Signers,

Fields of Study
Major Field Education
Minor Field Deaf Education
Research Methodologies
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CHAPTER 1

INTRODUCTION

This study addresses the limited research on the process of understanding and solving word problems, and the affiliated cognitive strategies, among d/Deaf and hard of hearing middle school students. Although a few studies on word problem solving have involved deaf elementary school and college students, none have focused on middle school students. In addition, no previous studies have explored d/Deaf and hard of hearing students’ specific problem-solving behaviors and strategy use.

Despite the importance of understanding word problem strategies, it is surprising that most studies have paid attention to students’ mathematical problem-solving performance (i.e., their arithmetic operation skills) rather than to their process of understanding the problems. Additionally, these studies have analyzed performance in terms of reading comprehension or language development, especially in terms of vocabulary or syntax, rather than in terms of mathematical knowledge or domain-specific knowledge (or metacognitive skills).

Overall, mathematics instructors and researchers tend to think that “language is at the heart of [d/Deaf students’] difficulties” (Barham & Bishop, 1991, p.180). Marschark, Lang, and Albertini (2002) argued that d/Deaf or hard of hearing students do not
“strategically apply their knowledge spontaneously” (p. 125) because they lack metacognitive skills (e.g., when, why, and how to apply these strategies) and mathematical knowledge. In addition, recent studies showed that deaf students who used sign language performed differently on problem-solving tasks related to English than hearing students did (Ansell & Pagliaro, 2006; Kelly, 2008), suggesting that the specific characteristics of deaf students (in this case, language mode) influence problem-solving performance.

Second, there is debate regarding the similarities and differences in language development and mathematics problem solving between deaf or hard of hearing students and hearing students. A study by Kelly, Lang, Mousley, and Davis (2003) has revisited Lewis and Mayer’s (1987) Consistency Hypothesis, which was developed to investigate the process of understanding word problems. The study supported the consistency effect, indicating that d/Deaf and hard of hearing college students are more likely to miscomprehend inconsistent language problems, and to commit more reversal errors on inconsistent word problems than on consistent word problems. Deaf college students likewise exhibited different responses to reversal errors and goal-monitoring errors, compared to hearing college students, in the Lewis and Mayer’s (1987) study. Although Kelly et al (2003) supported the effect of consistency in terms of error products, they argued that deaf college students in this study committed 35% more reversal errors on IL problems, as compared to the 6.9% of reversal errors committed by hearing college students in Lewis and Mayer’s (1987) study. They concluded that deaf college students performed differently from hearing college students on word problem tasks in terms of
the types of errors.

However, Kelly and colleagues (2003) do not provide empirical evidence for why deaf students made more reversal errors on IL problems than hearing students. Therefore it is of interest to explore the notion of the Qualitative Similarity Hypothesis (QSH) suggested by Paul and Lee (2010), in order to examine the similarities and differences in problem-solving behaviors and strategy use between d/Deaf and hard of hearing students and hearing students.

**Purpose of the Study**

The purpose of this study is to understand and describe the problem-solving behaviors and strategy use in deaf or hard of hearing middle school students, utilizing a Think-Aloud protocol and interviews. More specifically, the study explores 1) deaf middle school students’ problem-reading and problem-solving behaviors while reading, understanding, and solving *compare* word problems; 2) how the participants’ patterns of problem-solving behaviors are related to their characteristics, such as language fluency or mode and relevant knowledge; and 3) the use of self-regulated learning (SRL) strategies employed by deaf middle school students while learning mathematics. The ultimate goal of this study is to examine why deaf students have difficulty understanding and solving word problems.

**Research Questions**

The following research questions will guide the design of the study and the
analysis of the data.

1. What kind of problem-solving behaviors are used by deaf or hard of hearing middle school students while reading and solving *compare* word problems?

1.1 What are these students’ word problem-solving success rates?

1.2 What types of errors do these students commit while solving compare word problems?

1.3 How is specific mathematics knowledge associated with the patterns of problem-solving approaches?

1.4 What types of problem-solving approaches do these students use while reading and solving compare word problems?

1.5 How is language mode of these students associated with problem-reading approaches?

2. What SRL strategies do deaf or hard of hearing middle school students choose as they learn mathematics?

**Rationale for the Study**

Recent trends in mathematics education have shifted the discussion from the traditional instruction of symbolic computation procedures and memorization to examining how students understand mathematical concepts (Battista, 2001; English & Halfford, 1995). The reformation movement has been influenced by cognitive psychology, particularly the model of constructivism. Traditionally, mathematics education has heavily emphasized the simple acquisition of a set of isolated concepts and procedural
skills. Within the context of such education, students have limited opportunities to solve problems that may involve various configurations that go beyond basic computations.

Current theories in mathematics education focus on the integrated attainment of conceptual understanding, mathematical reasoning and thinking, and more effective use of problem-solving strategies, by emphasizing the balance between skills and processes (De Corte, Verschaffel, & Eynde, 2000; English & Halfford, 1995; for d/Deaf and hard of hearing students, see Pagliaro, 2006). Scholars view mathematics education as an active and constructive process that requires learners to take control of their own learning while constructing mathematical knowledge and solving problems (De Corte et al., 2000; Pape, 2003, 2004).

Problem solving is an important aspect of mathematics education, because it facilitates the development of a deep understanding of mathematical ideas as well as the application of sub-skills and conceptual knowledge (Schroeder & Lester, 1989). The National Council of Teachers of Mathematics states that “problem solving is not only a goal of learning mathematics but also a major means of doing so…. By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations” (NCTM 2000, p. 52).

With regard to the mathematics reform movement and deafness, the critical need for change in mathematics education has been addressed by the National Action Plan for Mathematics Education Reform for the Deaf (NAPMERD; see Dietz, 1995), supporting the NCTM standards and emphasizing the importance of problem solving (Pagliaro, 1998, 2006; Pagliaro & Ansell, 2002). It has been well documented that deaf or hard of hearing
students still lag behind with mathematics compared to their hearing classmates, even though their performance in mathematics is better than in reading comprehension. Some researchers have argued that poor performance in mathematics by d/Deaf or hard of hearing students is due to the fact that these students do not have sufficient opportunities to engage in challenging problem-solving activities. Like some general education instructors, teachers of d/Deaf or hard of hearing students often focus more on practice or drill exercises that do not require much higher-level thinking skills, but provide limited instruction in problem solving (NAPMERD; Dietz, 1995; Pagliaro, 1998, 2006). Therefore, one major thrust of the current reform movement is to encourage all students, including d/Deaf and hard of hearing students, to think critically and strategically to improve their achievement in mathematics. Within this perspective, word problems can be an effective tool to promote the integration of information and the application of knowledge to other contexts (Pape, 2004). Despite the importance of word problem solving in mathematics curriculum, very few studies on word problem exist in this field. Therefore, this study explores the process of understanding and solving word problems from cognitive perspectives, focused on cognitive processes in mathematics problem solving and a text processing model.

**Theoretical Framework**

The theoretical framework of this study is grounded in constructivism, especially focused on the cognitive perspective in problem solving and a text processing model of reading. Here, constructivism is used to illustrate the way the study is shaped through a
deaf lens. Cognitive perspectives are used to illustrate the major cognitive process in word problem solving, as discussed by Mayer (1992, 2002). The text processing model of reading is integrated into the process of problem representation, in order to understand better the construction of a coherent mental representation of the problem. Therefore, this section briefly discusses the three theoretical frameworks that guide and dominate this study.

**Constructivism**

This study is grounded in constructivist approaches to learning mathematics and language, which guide both its theoretical framework and methodological framework. Constructivism emphasizes that truth is relative and reality can only be known in a subjective way (Tobin & Tippins, 1993). Knowledge is thus constructed by learners’ meanings, based on experiences in social contexts and through social interactions (von Glasersfeld, 1995). The function of cognition is “adaptive and serves the organization of the experiential world, not the discovery of ontological reality” (von Glasersfeld, 1989, p. 1). That is, learners develop their cognitive abilities and build their knowledge through experiences. With regard to language and communication, von Glasersfeld (1989) argued that “language users, therefore, build up their meanings on the basis of their individual experience, and the meanings remain subjective, no matter how much they become modified and homogenized through the subject’s interactions with other language users” (p.3). This theoretical framework takes into account the difficulties of language development and learning mathematics for deaf students who do not have full access to
communication at home and who exhibited delayed experiences in language and learning.

The reform movement in mathematics education was mostly influenced by a constructivist view of learning. Within mathematics education, constructivists have focused on how students learn mathematics effectively, asserting that mathematical ideas are actively constructed by learners as they engage in a variety of meaningful problem-solving activities with real-world applications (Battista, 2001; National Research Council, 1999). However, traditional mathematics instruction has emphasized computation procedures through memorizing what the teachers taught. However, in this way students may be not able to learn mathematics effectively, and they may be not able to develop better transfer skills that apply existing knowledge to new situations (Battista, 2001). The constructivists’ perspective allows students to make sense of mathematics, that is, to learn with understanding-- the central goal in the reform of mathematics education (Battista, 2001).

Cognitive Processes in Mathematical Problem Solving

According to Mayer (1992, 2002), mathematics problem-solving consists of four major cognitive processes: problem translating, problem integrating, solution planning, and solution executing. Mayer argued that these four processes are not mutually exclusive or self-contained; rather, they are often intertwined with each other. Mayer further categorizes these four processes according to function, distinguishing between problem representation and problem solution, a distinction which emphasizes the importance of problem representation (English & Halford, 1995; Mayer,1992, 2002).
In problem representation, individuals first read the given word problem and then construct a mental representation of the problem or of the situation that they are attempting to solve. This process includes translation and integration, and is dependent on linguistic, semantic, and schematic knowledge. By contrast, problem solution involves planning/executing a solution strategy and carrying out it to reach a final answer (Mayer, 1992, 2002). To solve a problem, in this step, strategic knowledge and procedural knowledge are needed. A great number of studies have demonstrated that the process of representing word problems plays a critical role in problem-solving success, rather than just the process of solving word problems (Lewis & Mayer, 1987; Nathan, Kintsch, & Young, 1992). Although Mayer’s work (1992, 2002) on mathematical problem solving has contributed to our understanding about the importance of both representation and solution in word problem-solving success, there is little or no research on the process of understanding and solving word problems in d/Deaf and hard of hearing students. The present study assumes that d/Deaf or hard of hearing students have more difficulty with processes of understanding (i.e., linguistic and schematic knowledge) than of solving problems (i.e., mathematical and procedural knowledge). Specifically, deaf middle school students who use sign language as their primary language are more likely to have difficulty representing and understanding word problems.

**Compare Word Problems.** A substantial body of research has demonstrated that *compare* word problems with relational terms (e.g., *more/less than*) are the most difficult types of mathematics problems (e.g., *combine/change/compare*) for students of various levels, including deaf students, because these problems involve linguistic and
mathematical complexity (Cummins, Kintsch, Reusser, & Weimer, 1988; Mayer, 1992; Pape, 2003). Difficulties with compare problems are attributed to a lack of specific mathematical conceptual and procedural knowledge (Pape, 2004), misrepresentation (Cummins et al., 1988; English, 1997; Lewis & Mayer, 1987; Reed, 1999), inadequate metacognitive awareness or strategies (Montague, 1989, Montague & Bos, 1986a, 1986b for students with learning disabilities), and difficulties in reading comprehension (Kelly, Lang, Mousley, & Davis, 2003 for deaf students; Muth, 1984 for hearing students).

Although the reasons for difficulty with word problems vary, there is a consensus that most students have difficulty in problem representations, rather than in executing procedures (Cummins et al., 1988; Lewis & Mayer, 1987; Mayer & Hegarty, 1996; Pape, 2003, 2004). These results are consistent with the fact that many students tend to make representation errors rather than computation errors (Cummins et al., 1988; Mayer & Hegarty, 1996; Pape, 2003 & 2004; De Corte et al, 2000; Versaffel, 1994). It is obvious that mental representation and reasoning about the situation described in the problem play a crucial role in problem solving success. Despite the importance of this research, until recently there has been surprisingly little work on the process of understanding word problems by deaf or hard of hearing students.

**The Consistency Hypothesis.** Lewis and Mayer (1987) developed a model of the comprehension process for *compare* word problems in order to understand why students have difficulty with word problems with relational terms. They called it the *Consistency Hypothesis*, and distinguish between two forms of language problems: consistent language problems (CL) and inconsistent language problems (IL). A consistent language
problem (CL) is one that includes a relational term (e.g., *more than*) that is consistent
with the required arithmetic operation (e.g., addition). For example: *Joe has 3 marbles. Tom has 5 more marbles than Joe. How many marbles does Tom have?* (Lewis & Mayer, 1987, p. 364). On the other hand, in an inconsistent language (IL) problem the relational
term (e.g., *more than*) conflicts with the required arithmetic operation (e.g., subtraction).
For example: *Joe has 8 marbles. He has 5 more marbles than Tom. How many marbles
does Tom have?* (Lewis & Mayer, 1987, p. 364).

Lewis and Mayer (1987) suggested that students have a preferred form for the
order in which information should be presented (in this case, the CL problem form). The
preferred form is developed through experience with these problems. When the form of
the relational statement is not consistent with their schemata, students are likely to
rearrange the relational terms in IL problems mentally until the terms match their
preferred form (in this case, a consistent relational statement). In the transformation
process, problem solvers tend to commit reversal errors on IL problems (i.e., use of an
opposite operation) that result from a failed rearrangement into a preferred format (i.e., an
established set of schemata), which leads to miscomprehension and misrepresentation.
Additionally, Lewis and Mayer (1987) argued that with more complicated problems, the
rearrangement of information in inconsistent language increases the demands on working
memory during the transformation stage (Lewis & Mayer, 1987; Pape, 2003; Verschaffel
et al., 1992; Verschaffel, 1994).

Supportive research for the Consistency Hypothesis has been documented with
hearing students of different ages (Lewis & Mayer, 1987; Pape, 2003; van Schoot,
Arkema, Horsley, & Lieshout, 2009; Verschaffel, 1994; see Kelly, Lang, Mousley, & Davis, 2003 for D/Deaf and hard of hearing students). These studies have examined by using a variety of measurement techniques such as observations, eye-fixations (Hygerty, Mayer, & Green, 1992; van Schoot et al., 2009; Verschaffel et al., 1992), think-aloud protocols (or retelling), and recall data (Pape, 2003, 2004; Verschaffel, 1994). These results indicate that students made more errors on IL problems than on CL problems, and they predominantly committed reversal errors on IL problems compared to CL problems. In addition, they took longer to solve, integrate, and plan solutions for the IL problems than for the CL problems. Verschaffel et al. (1992) argued that these longer solution times are due to longer fixation times on the relational sentences of the IL problems.

Additionally, several studies have shown differences in problem-solving behaviors between accurate problem solvers and inaccurate problem solvers (Hegarty et al., 1992, Hegarty, Mayer, & Monk, 1995; Pape, 2003, 2004; Schoenfeld, 1985, 1987). Schoenfeld (1985, 1987) suggested that accurate problem solvers tend to detect structural features, which focus on understanding the situation described in the word problem, whereas inaccurate problem solvers focus on only the surface features, such as the use of key words and numbers without understanding. In another study, Hegarty, Mayer, and Monk (1995) reported that accurate problem solvers construct a mental model of the situation described by the text before seeking a solution. This has been labeled the problem-model strategy or meaning-based approach. On the other hand, inaccurate problem solvers look at key words and numbers, a process labeled the direct translation approach (English & Halford, 1995; Mayer, 1998; Pape & Wang, 2003; Verschaffel et al,
A Text Processing Model of Reading

The text processing model of reading was influenced by constructivist studies related to reading comprehension, and argues that meaning is not in the reader’s head alone but takes place through interaction among text, reader, and contexts. Several studies have demonstrated that a coherent mental representation plays a critical role in word problem solving (Greeno & Kintsch, 1985; Lewis & Mayer, 1987; Nathan, Kintsch, & Young, 1992). To understand how deaf students understand these problems, a text processing mode of reading is integrated into the problem representation suggested by Mayer’s (1992, 2002) cognitive process. A text processing model of reading takes account of the process of understanding word problems in terms of both text and knowledge processing, which is grounded in Goldman and Rakestraw (2000).

According to the text processing model of reading, understanding includes the construction of a coherent mental representation of the text (Goldman & Rakestraw; 2000; Kintsch & Greeno, 1985). To understand texts, readers must rely both on text-driven processing and knowledge-driven processing, dealing with both textual comprehension and prior knowledge (Goldman & Rakestraw, 2000). In this model, text-driven processing refers to the use of specific words and segments in the text as a basis for the construction of mental representations (Goldman, & Rakestraw, 2000). By contrast, knowledge-driven processing refers to the important role of prior knowledge in the process of a mental representation of what readers read: they depend heavily on prior
knowledge about specific ideas or topics to build a mental representation of what they read when they have high content knowledge of the topics (Goldman & Rakestraw, 2000). As illustrated by Goldman and Rakestraw (2000), to construct a coherent mental representation of the problem, readers must interpret information in the problems based on both text-driven processing and knowledge-driven processing (Goldman & Rakestraw, 2000).

However, students’ processing of text can vary depending on the complexity of texts, their background knowledge, and their experiences (Goldman & Rakestraw, 2000). That is, in a situation with high content knowledge, students may rely on structural aspects of the text because they use their prior knowledge to build coherent mental representations of what they read. Processing may be more knowledge-driven. In contrast, students with low content knowledge may rely on the cues in the text to understand meanings of problems. In particular, the model emphasizes the importance of prior knowledge in textual comprehension to help readers build a coherent representation of the information contained in the text. Readers’ use of prior knowledge and problem structures, that is, mutually determine their processing of what they read. Kintsch and Greeno (1985) argued that a model of text processing can predict the difficulty of problems students encounter based on these processing differences, by exploring the process of understanding texts. This model provides a plausible explanation of why students have difficulty with word problems and how the complexity of the different kinds of problems influences text processing and procedures (Kintsch & Greeno, 1985).
Self-Regulated Learning Strategy

Research on self-regulated academic learning began in the mid-1980’s by researchers’ efforts to encourage a focus on how students learn mathematics and become “masters of their own learning processes” (Zimmerman, 2001). Much research in mathematics education has indicated that strategic behaviors play a critical role in achieving problem-solving success (English, 1997; Pape, 2004; Pape & Wang, 2003; Schoenfeld, 1992; Verschaffel et al., 2000). In particular, self-regulated strategies are highly related to academic achievement in mathematics (Pape & Wang, 2003; Purdie, Hattie, & Douglas, 1996; Zimmerman & Martinez-Pons, 1986, 1990). NCTM (2000) emphasized the importance of self-regulated behaviors in problem solving. Students, it noted, need to “apply and adapt a variety of appropriate strategies to solve problems” and to “monitor and reflect on the process of mathematical thinking” (p. 52). The standards of the NCTM seem to be consistent with the goal of self-regulated learning strategies.

When applying this concept to academic learning, self-regulated learning (SRL) refers to students’ use of various cognitive, metacognitive, and motivational skills to control and regulate their learning processes (Garcia & Pintrich, 1994; Pintrich, 2000; Zimmerman, 2000). Self-regulated learners are able to analyze tasks, set appropriate goals to achieve tasks, and monitor their progress during problem-solving activities (Pintrich, 1999; Zimmerman, 2000). With regard to problem-solving strategy behaviors, research indicates that metacognitive knowledge and monitoring facilitates learning and improves mathematical problem solving (Schoenfeld, 1987). For example, poor readers are less likely to use monitoring and evaluation skills than are good readers, who
consciously use a variety of strategies as they solve problems. In particular, young or struggling students may misunderstand the story with respect to reading, or similarly misunderstand word problems, because they rarely monitor their knowledge (Wagoner, 1983).

**Perspectives on Deafness**

To understand how deaf and hard of hearing students perform mathematics problems, we first need to understand how their communication methods influence the way they construct English texts. In general, there have been two major perspectives on deafness in deaf education: the pathological perspective and the cultural perspective. Both perspectives have affected how research on deafness has been conducted. From the pathological perspective, deafness can be explained as a function loss of hearing. Within this perspective, hearing loss is a deficit that requires some kind of remediation. Scholars who support this perspective believe that spoken language can be the most beneficial communication method for deaf individuals, because it procures educational success and assimilation into hearing society. The perspective thus often holds that the development of spoken language through speech and lip reading is the most effective means of cognitive development.

On the other hand, from the cultural perspective deafness is a more complex phenomenon: deaf people consider themselves as a distinct cultural and language minority group, using American Sign Language (ASL) as their first language, one whose syntax structures differ from English. The first letter of the term “Deaf” is always
represented by a capital “D” in this perspective, indicating their cultural identity as a minority language group. Accordingly, deafness can be viewed as a positive difference that should not be perceived as a loss of hearing. Rather, deafness relates positively to one who relies on visual input for getting information rather than auditory input. Proponents of the cultural approach believe that deaf children are not deficient, but rather visual learners who must use their vision to learn or communicate. This perspective provides some insight into the socio-cultural context and deaf perspective, instead of identifying deaf people as deficient in hearing or lacking in English understanding. This study is based on a difference model on deafness that views deaf students as members of language minority groups. However, in this study I use the term “deaf or hard of hearing students” or “d/Deaf or hard of hearing students” to refer to the participants, because deaf individuals are determined their own identities and I did not examine those identities in the course of the study.

Additionally, this study was influenced by researchers arguing for the viability of cognition and learning among deaf individuals, as well as the qualitative differences in mathematics and science learning between hearing students and deaf students (Ansell & Pagliaro, 2006; Marschark, 2008; Strassman, 1997). Marschark and Hauser (2008) pointed out that

what most [deaf or hard of hearing] students have in common is their diversity: they tend to come to the classroom with experiences that vary more widely than their hearing peers and, partly as a consequences of those experiences, they have developed different problem-solving and learning strategies (p. 7).

They further claimed that potential differences between hearing students and deaf
students or among deaf or hard of hearing students may result from “the possibility of interactions among individual characteristics, content, and settings” (p. 7).

**Mathematics and Deaf or Hard of Hearing Students**

Among these studies there is debate regarding the reasons for low mathematics academic achievement or difficulties in learning mathematics by deaf populations, but there is agreement that deaf students lag behind in mathematics achievement when compared to their hearing counterparts. Until recently, most studies related to problem solving have investigated the mathematics problem-solving performance of deaf or hard of hearing students in terms of English reading comprehension (Frostad & Ahlberg, 1999; Kelly, Lang, Mousley, & Davis, 2003; Sereno Pau, 1995). These studies seem to emphasize the importance of English reading comprehension in learning mathematics, indicating that there is a high correlation between deaf students’ low reading comprehension and their mathematical problem-solving performance (e.g., Kelly et al., 2003; Kidd et al, 1993; Kelly & Mousley, 2001; Mousley & Kelly, 1998; Sereno Pau, 1995; Wood, Wood, Griffith, & Howarth, 1986). These earlier studies seemed to be influenced by a deficit model, which focused on deaf individuals’ inability to understand English and to learn mathematics. Additionally, these researchers seemed to support the Qualitative Similarity Hypothesis suggested by Paul (2002, 2009), arguing that deaf students learn reading or mathematics, regardless of the specific subject, in the same ways that hearing students learn.

In contrast, more recent studies have indicated that the difficulty of word problem
solving among deaf students is due to a combination of linguistic, cognitive, and experiential factors (Ansell & Pagliaro, 2006; Hyde, Zevenbergen, & Power, 2003; Kelly et al., 2003). In particular, some studies indicated that deaf or hard of hearing students may differently perform word problems and some cognitive tasks related to English from their hearing peers, because of their different language modes or relevant knowledge necessary to problem solving (Ansell & Pagliaro, 2006; Kelly, 2008; Kelly et al., 2003; Marschark, 2008). These studies base a different model, arguing that the different characteristics of deaf students may influence their problem solving. Therefore, it is hypothesized that their language modes may influence their word problem solving. However, other factors also may influence their word problem-solving performance, above and beyond language modes. These factors can include cognitive skills, a lack of mathematics knowledge, insufficient strategies, monitoring skills, and beliefs and confidence (Kelly, 2008; Kelly et al., 2003; Marschark, 2008; Pagliaro & Ansell, 2002).

Therefore, this study explores why deaf middle school students have difficulties with word problem solving by examining the ways in which they read, understand, and solve word problems.

**Definition of Terms**

*Compare Problems.* Problems involving the static comparison of two quantities, usually containing a relational statement or term (*more, less or fewer*). For example, Joe has 2 marbles. Tom has 6 more marbles than Joe. How many marbles does Tom have?

*Consistency Hypothesis.* This hypothesis explains why inconsistent language
problems are more difficult than consistent language problems. It is assumed that problem solvers have a preference for a particular order in which problem information is presented to them.

Consistent Language Problems. Problems in which a key relational term (e.g., more than) is consistent with a required arithmetic operation (e.g., addition) (Lewis & Mayer, 1987). For example, Mary runs about 6 miles per week. Sandy runs 3 times as many miles per week as Mary. How far does Sandy run in a week?

Inconsistent Language Problems. Problems in which the relational term (e.g., less than) conflicts with a required arithmetic operation (addition) (Lewis & Mayer, 1987). For example, Joe runs 6 miles a week. He runs 1/3 as many miles a week as Ken does. How many miles does Ken run in a week?

One-step Compare Problems. In general, compare problems have three sentences such as an assignment sentence, a relational sentence, and a question sentence, involving either one or two computational steps. In one-step problems, students are asked to find a value for the unknown quantity for the question sentence. For example: “At ARCO gas sells for $1.13 per gallon (e.g., assignment sentence). Gas at Chevron is 5 cents less than gas at ARCO (e.g., relational sentence). How much does 1 gallon of gas cost at Chevron (e.g., question sentence)?” This problem requires only one computational step to solve.

Two-step Compare Problems: The third sentence of a compare word problem asks for a multiple of the value for the unknown quantity. For example: “At ARCO gas sells for $1.13 per gallon. Gas at Chevron is 5 cents less than gas at ARCO. How much do 5 gallons of gas cost at Chevron?” For the third sentence, this problem requires two
computational steps to solve. For example, students must multiply the quantity five times to reach an answer after determining the cost of 1 gallon of gas at Chevron.

*Unmarked Terms.* Specific “positive” terms such as *more* and *good* are more semantically salient than that of their marked opposites-namely, negative terms such as *less* and *bad.* Semantic memory representation of unmarked term is easier than that of the negative or marked term. Thus, unmarked terms are easier to understand and process in memory (Clark, 1977).

*Marked Terms.* Marked terms are stored in more complex form in memory, making them harder to understand than unmarked terms (Clark, 1977).

*Direction Translation Approach.* A “compute first and think later” approach that focuses on numbers or key relational terms of word problems without context. Most unsuccessful problem-solvers who use this approach fail to represent the problem accurately (Mayer et al., 1995).

*Problem-Model Approach or Meaning-Based Approach.* Refers to a mental model of the situation described by the problem. Problem-solvers who use this approach focus on understanding the relations among the variables in the problem (Mayer et al., 1995).

Chapter 1 presents the problem, research questions, rationale and theoretical framework. Chapter 2 will review the literature relevant to mathematics learning, problem solving in mathematics and mathematics learning by deaf learners.
CHAPTER 2

REVIEW OF RELEVANT LITERATURE

This chapter discusses mathematics reform and cognitive process in mathematics problem solving in more detail and reviews the literature most relevant to this study and is divided into three major sections. The first section involves an overview of mathematics reform movement, Mayer’s analysis of mathematical problem solving, and the text processing model of reading. The second section describes literature related to mathematical problem solving and self-regulated learning strategy. Finally the last section briefly describes the unique characteristics of deaf students, in order to explore the way in which they interact with the structure of text; the section also includes a review of relevant literature concerning mathematical problem-solving and deafness.

Mathematics Education Reform

The mathematics reform movement and research on self-regulated academic learning began in the mid-1980’s by a lot of researchers’ efforts to encourage a focus on how students learn mathematics and become “masters of their own learning processes” (p. 1, Zimmerman, 2001). Traditionally, mathematics education has placed more emphasis on fixed symbolic procedures by memorization and imitation than sense making, thinking,
and reasoning (Battista, 2001). Recently, however, the current reform movement in school mathematics has called for changes in teaching practices that move toward the constructivist teaching paradigm (Battista, 2001; Pape & Smith, 2002).

The reform movement in mathematics education was mostly influenced by a constructivist view of learning. Constructivists have focused on how students learn mathematics effectively, assuming that mathematical ideas are actively constructed by learners as they engage in a variety of meaningful problem-solving activities with real-world applications (Battista, 2001; National Research Council, 1999). However, traditional mathematics instruction has emphasized computation procedures through memorizing and what the teachers taught. In this way, students may be not able to learn mathematics effectively and they may be not able to develop better transfer skills that apply existing knowledge to new situations (Battista, 2001). The constructivists’ perspective allows students to make sense of mathematics, that is, to learn with understanding mathematics is the central goal in the reform of mathematics education (Battista, 2001).

Reflecting the constructivists’ view of mathematics learning, these reform movement efforts have continued with the recent publication of the National Council of Teacher of Mathematics (NCTM, 2000), Principles and Standards for School Mathematics (PSSM) that recommended new goals that include emphasis on conceptual understanding and communication of reasoning through problem solving and inquiry (Pape & Smith, 2002). The standards document called for teachers to move from traditional mathematics instruction that emphasizes memorization and imitation toward a
student-centered mathematics classroom that emphasizes problem solving and exploration. In reform classrooms, students can develop a deep conceptual understanding of particular ideas through engagement in mathematical thinking, reasoning, and problems solving as well as a belief about problem solving (Battista, 2001; Pape & Smith, 2002).

Within the context of reform mathematics, as has been mentioned above, problem solving is an important aspect of learning mathematics that facilitates the development of mathematical thinking and a conceptual understanding of specific mathematical ideas. Within mathematical problem solving, self-regulation is important because cognitive processing requires knowing when and how to apply different knowledge (i.e., metacognitive knowledge and control) as well as different types of knowledge and strategy (Pape & Smith, 2002; Schoenfield, 1985). Additionally, Schoenfield (1985) argued that complex cognitive processes of mathematical problem-solving behaviors should be explained in terms of cognition, metacognition, and motivation.

Therefore, the present study assumes that d/Deaf or hard of hearing students have difficulty with both the processes of understanding (i.e., linguistic, semantic, and schematic knowledge) and with the solving of problems (i.e., strategic and procedural knowledge) and their problem solving behaviors may be influenced by their motivation.

**Cognitive Processes in Mathematics Problem Solving**

Problem solving is a complex cognitive process that requires the application of a variety of different knowledge, cognitive skills, strategies, and methods for different
solutions, especially when no single solution method is obvious (Mayer & Wittrock, 2006). Additionally, problem solving is a process that involves the representation and manipulation of knowledge in the problem solver’s cognitive system (Mayer & Wittrock, 2006). As mentioned in the previous chapter, problem-solving processes consist of two major cognitive components: a) problem representation and b) problem solution (Mayer, 1992, 2002). In problem representation students must first read and understand the problem and should build a mental representation of the problem to successfully solve the problem before they attempted to solve it. This process is further divided into both translation and integration, and three types of knowledge are needed to account for these two processes.

On the other hand, in problem solution the problem solver devises and carries out a plan for solution based on his or her understanding. This process involves both planning and execution and depending on both strategic and procedural knowledge (Mayer, 2002; Mayer & Wittrock, 2006). Table 2.1 provides a visual representation of the types of cognitive processes and types of knowledge involved in problem solving.

As shown in Table 2.1, the two major problem-solving processes are divided into four sub-components: translating, integrating, planning/monitoring, and executing. It is important to note that each cognitive process is composed of specific activities that require distinctive cognitive skills and specific knowledge that a problem solver can bring to problem solving. However, all the components of these cognitive processes are interwoven, and require integration of all types of knowledge for successful problem solving. The four components are discussed individually below.
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Table 2.1. Types of Cognitive Processes and Knowledge Employed in Problem Solving (Mayer & Wittrock, 2006)

First, in problem representation, translating occurs when problem solvers translate each statement of the problem into an internal mental representation. In the translation process, students have to understand the words and sentences of the problem to construct a coherent problem situation—a mental representation (Mayer, 1992, 2002). This process is dependent on linguistic and semantic knowledge. Specifically, linguistic knowledge refers to knowledge of English (e.g., grammar or mathematics vocabulary), while semantic knowledge (or factual knowledge) involves the knowledge of facts or knowledge about the world, such as time and units of measure (Mayer, 2002).

In the integrating process, problem solvers bring together certain knowledge related to the situation being described in the problem. To do so, they need to put the statements of the problems together into a coherent structure in order to build an accurate mathematical representation of it (Mayer, 1992, 2002, Mayer & Wittrock, 2006). In the process, schematic knowledge used by students influence their problem solving success. Thus,
students’ difficulties with word problems can result from the lack of appropriate schemas rather than poor arithmetic or logical skills (Mayer & Wittrock, 2006). Furthermore, this mental representation can be created by an internal or external representation. An internal representation can involve cognitive schemata that are developed through a learner’s experience and abstractions of mathematical ideas (Pape & Tchoshanov, 2001). On the other hand, an external representation can be created through the manipulation of concrete objects (e.g., graphically displayed algebraic equations, diagrams, and tables) (Pape & Tchoshanov, 2001).

The final components of the problem-solving processes, which involve the actual problem solution, are typically classified as the third and fourth distinct stages: planning and executing. There are two kinds of knowledge concerning the solution of a problem: strategic knowledge and procedural knowledge (Mayer, 1992, 2002; Mayer & Wittrock, 2006). In the first phase of problem solution, planning occurs when problem solvers devise a method for solving the problem. The process of planning involves monitoring, which evaluates whether the solution method is appropriate or not. Additionally, the process requires strategic knowledge, entailing how to develop and monitor for a solution plan. In the process of forming a solution plan, the strategies chosen by problem solvers can be influenced by their attitudes and beliefs (Mayer & Wittrock, 2006). Finally, execution occurs when problem solvers carry out the solution plan. This process depends on procedural knowledge that refers to knowledge of a formal language or symbolic representations, consisting of rules or procedures (Mayer, 1992, 2002; Mayer & Wittrock, 2006).
As described above, Mayer’s (1992, 2002) cognitive process of problem-solving seems to emphasize a reciprocal relationship between problem representation and problem solution. Mathematical proficiency likewise depends on four different cognitive processes that require a variety of knowledge of the problem solver, including linguistic, factual, schematic, strategic, and procedural knowledge. To become productive mathematical problem solvers, these four processes and specific knowledge are needed. However, Mayer’s work does not explain the process of understanding and representing problems, steps that play a critical role in successful problem solving. To understand the complex process of problem representation better, the text processing model of reading is incorporated into the problem representation model suggested by Mayer (1992, 2002). In the next section later, the chapter describes the general features of the comprehension and representation processes, in terms of the interaction of text processing and knowledge processing after discussing literature relevant to mathematical problem solving.

**Mathematical Problem Solving Research and Hearing Students**

**Compare Word Problems**

The difficulties of *compare* word problems have been well documented in the research involving educational psychology and mathematics education (Lewis & Mayer, 1987; Pape, 2003, 2004; Riley & Greeno, 1988; Verschaffel et al, 1992). A number of studies have demonstrated that compare word problems with relational terms (*more than/less than*) are the most difficult among all types of math problems (*change, combine, and compare problems*) for students of various levels (Riley & Greeno, 1988) because of
differences in values, assumptions, and the complexity of problem structures. It is beyond the scope of this study to provide a comprehensive review of all three types of word problems, so the focus here will be solely on compare word problems. As mentioned above, compare word problems consist of different structures in terms of computational steps, problem type, arithmetical operations, and relational terms. It is important to note the components of these word problem structures, to improve our understanding of the complex psychological aspects of problem solving.

*Compare* word problems are structurally composed of three sentences: assignments, relations, and questions (Hegarty, Myaer, & Monk, 1992). For example, *Joe runs 6 miles a week. Ken runs 3 times as many miles as Joe does. How many miles does Ken run in a week?* (Lewis & Mayer, 1987, p. 364). As mentioned in the example, the first (assignment) sentence introduces a numerical value for the variable of a known quantity. The second sentence presents the relationship between the variable of known quantity and another variable of unknown quantity. Finally, the third question asks for the value of the unknown quantity (Lewis & Mayer, 1987; Pape, 2003). In addition, word problems have two computation steps of constructing problems: one-step computation problems and two-step computation problems. In the last question, the word problem differentiates a one-step problem from a two-step problem: in the former, the student is asked to find the value of the unknown variable (e.g. how many miles Ken runs in a week), while the latter asks students for a multiple of that variable (e.g. how many miles Ken runs in four weeks). Some studies have indicated that two-step word problems are relatively more difficult for students than one-step word problems because they demand a
higher cognitive load when they solve two step word problems (Lewis & Mayer, 1987; Pape, 2003).

Additionally, compare word problems are composed of two types of problems: consistent language problems (CL) and inconsistent language problems (IL). To explain why students inaccurately represent IL problems more frequently than CL problems, Lewis and Mayer (1987) formulated the Consistency Hypothesis, which assumes that problem solvers have a preferred form in the order of information presentation.

According to Lewis and Mayer (1987), a CL problem specifies that the unknown variable is the grammatical subject of the second sentence, and the relational term (e.g., more than) is consistent with the required arithmetic operation (e.g., addition). For example: “Joe has 8 marbles. Tom has 5 more marbles than Joe. How many marbles does Tom have?” (p. 364). In this example, the unknown quantity of the second sentence is the number of Tom’s marbles, Tom is the subject of the second sentence, and the relational term (e.g., more than) provides the quantity of Tom’s marbles in terms of the number of Joe’s marbles. In addition, the relational term (in this case, “more than”) is consistent with the arithmetic operation (i.e., addition) (Lewis & Mayer, 1987; Pape, 2003).

On the other hand, in an inconsistent language (IL) problem, “the unknown set is the object of the second sentence, and the relational term (e.g., more than) conflicts with the necessary arithmetic operation (subtraction)” (p. 363). For example, “Joe has 8 marbles. He has 5 marbles less than Tom. How many marbles does Tom have?” (p.364). In this example, the unknown quantity is the number of Tom’s marbles. Tom is the object
of the second sentence, and the relational term (e.g., less than) is not consistent with the required arithmetic operation “addition.” Thus, IL problems are more difficult to solve for most students, but these difficulties vary depending on the complexity of IL problem.

Research concerning word problem solving has been examined from a variety of perspectives to understand the difficulties students encountered with word problems: text processing (Cummins, Kintsch, Reusser, & Weimer, 1988; Kintsch, 1994; Kintsch & Greeno, 1985), information processing (Mayer, 1992; Silver, 1987; Van Der Schoot, Arkema, Horsley, & Van Lieshout, 2009), reading comprehension (e.g., Goldin, 1992; Muth, 1984, 1988), and the interaction of reading and mathematical skills (Greeno & Kintsch, 1985; Pape, 2004).

Although these studies take different perspectives on problem-solving, there is a consensus that the problem representation (i.e., comprehending it) plays a critical role in problem solving success. In fact, the success or failure of problem solving depends on the mental representation of the problem (English, 1997; Nathan, Kintsch, & Young, 1992). Additionally, Verschaffel et al. (2000) state the importance of the mental model, supporting the suspension of the sense-making hypothesis: “many errors in mathematical problem solving can be attributed to students failure to engage in the process of mental model building—that is, a failure to use their prior knowledge about problem situations to make sense of a current problem” (p.79).

Difficulty with word problems is often attributed to comprehension failures involving the complexity of problem structures and mathematical knowledge. Mental representation and reasoning about the situation described by a problem statement
likewise play a crucial role in solving the problems and, in turn, in enhancing students’ understanding (English, 1997; Verschaffel et al., 2000). Therefore, to be successful problem solvers, students must understand and represent the problem based on a coherent mental representation that leads to the use of appropriate solution strategies, before making any attempt to solve the problem.

**The Consistency Hypothesis**

Several studies have demonstrated that students make more errors on IL problems than on CL problems, and that they more frequently commit reversal errors on IL problems than on CL problems (Hegarty, Mayer, & Green, 1992; Lewis & Mayer, 1987; Pape, 2003; Stern, 1993; Verschaffel, De Corte, & Pauwels, 1992; Verschaffel, 1994). One reason for this, Lewis and Mayer (1987) argued, is that a “preferred format corresponds to the order of information presentation in consistent language problems but is in conflicts with the order of information presentation in the inconsistent language problems” (p. 364). Additionally, they claimed that “comprehension errors will be more likely to occur when the structure of the presented information does not correspond to the problem solver’s preferred format” (p. 365). Lewis and Mayer (1987) argued that the preferred from is developed through experiences (Pape, 2003).

More recently, several studies have extended the consistency hypothesis by Lewis and Mayer (1987) and investigated students of various ages, including those in elementary school (Verschaffel et al., 1992; Verschaffel, 1994), middle school (Pape, 2003), and college (Kelly et al., 2003 for deaf students; Lewis & Mayer, 1987;
Verschaffel et al., 1992). These studies used different techniques such as think-aloud and recall (Pape, 2003), eye-fixation (Verschaffel et al., 1992), and re-telling (Verschaffel, 1994) in order to investigate the processes of understanding and solving word problems. The focus of these studies was on investigating why IL problems are more difficult and why reversal errors occurred in IL problems. Furthermore, all the studies regarding the consistency hypothesis have examined using quantitative research methods and statistical analysis to test their respective hypotheses. However, only one study, that conducted by Verschaffel (1994) using the retelling technique, included some case analysis on the CL and IL inversions.

Verschaffel (1994) analyzed 40 fifth graders’ verbal protocols for one-step word problems. Results indicated that elementary students performed better on one-step CL problems than on one-step IL problems. The common errors for IL problems were reversal errors, indicating that IL problems were frequently retold as CL problem structures and not retold as IL problems, but CL problems were not retold as IL problem structures.

While several studies supported the consistency hypothesis, Verschaffel, De Corte, and Pauwels (1992) found mixed results with it. Their study tested the consistency hypothesis using eye movement procedures in order to examine the process of understanding word problems of elementary and college students. Results indicated that the consistency hypothesis was not supported by college students when one-step problems were presented. These one-step problems are not difficult for college students to solve. By contrast, third graders made more reversal errors on one-step IL problems.
and took longer to solve IL problems when the same word problems were presented. With more complex two-step problems, college students committed reversal errors on IL problems and spent more time on IL problems. Verschaffel and his colleagues (1992) argued that the consistency effect was more commonly revealed in complex word problems with greater cognitive demands.

More recently, Pape’s (2003) study found mixed results concerning the effect of the consistency hypothesis, in an investigation of the difficulties of representing IL problems. In Pape’s (2003) study, participants were ninety-eight middle school students who were at the 7th and 8th grade. The study examined middle school students’ problem-solving behaviors as they read and solved word problems using think-aloud procedures and recalled data. The word problem task included problems with a fraction-of-a-number relational terms (e.g. three-quarters as many as) in order to investigate the difficulty of word problem-solving. Findings indicated that the consistency hypothesis was supported by middle school students in terms of success rate, reversal errors, and recall reversal data, indicating that they committed more errors and reversal errors on IL than on CL problems. Fewer recalled IL problems than CL problems. In addition, IL problems were frequently recalled as CL problem structures following reversal errors. Pape’s (2003) study supported Verschaffel’s (1994) findings on retelling techniques. However, with regard to the consistent effect, Pape (2003) argued that middle school students did not take longer to solve IL problems than CL problems. Pape (2003) concluded that for middle school students, fraction-of-a number relational terms have strongly influenced on their problem-solving success.
Lexically Marked Relational Terms

Lewis and Mayer (1987) investigated college students’ preferences for unmarked versus marked relational terms, in order to examine the difficulties of understanding and representing IL word problems. The researchers hypothesized that the probability of a reversal would be more likely when the relational terms in addition or multiplication IL problems were linguistically marked (e.g., less than, 1/n as many as) than when the relational terms in subtraction or division IL problems were unmarked (e.g., more than, n as many as). The results indicated that college students tend to prefer the use of unmarked keywords (e.g., more than). That is, they made more reversal errors on IL addition (e.g., less than) and multiplication problems (e.g., 1/n as many as) than on IL subtraction (e.g., more than) and division problems (e.g., n times as many as) (Lewis & Mayer, 1987). Lewis and Mayer (1987) argued that marked terms in IL addition and multiplication problems (e.g., less, shorter, younger, and 1/n as many) affect problem solvers more predominantly than unmarked terms in IL subtraction and division problems (e.g., more, longer, older, and n times as many). To account for this preference, Clark (1969) explained the difficulties in a variety of reasoning comparison sentences in terms of the principle of lexically marked terms. According to Clark (1969), the meanings of unmarked terms such as “good” and “more” are stored in memory in a more accessible form than the meanings of their marked terms like “bad” and “less,” which are harder to retrieve from memory because they are stored in a more complex way. Thus, students are less resistant to retrieving unmarked terms from memory because unmarked terms are used more frequently than marked terms.
An interactive effect between the consistency effect and lexically marked terms was found in several studies (Lewis & Mayer, 1987; Verschaffel et al., 1992), indicating that IL addition and multiplication problems with marked terms result in more reversal errors than IL subtraction and division problems with unmarked terms. Pape’s (2003) results are mixed, arguing that the fraction-of-a-number structure of the relational terms had a stronger influence on the reversal errors for middle school students than problems with lexically marked terms. That is, middle school students solved IL subtraction and division problems more correctly than IL addition and multiplication problems. Reversal errors on IL subtraction and division problems were greater than IL addition and multiplication problems. However, students solved fewer CL division problems than IL subtraction problems.

Problem-Solving Approaches

A great number of studies regarding problem solving have investigated the differences in problem-solving behaviors between accurate problem solvers and inaccurate problem solvers (Hegary, Mayer, & Monk, 1995; Pape, 2004). Research on expert-novice differences has indicated that experts are more likely to focus on a qualitative understanding of the problem, while novices are more likely to rely on the quantitative reasoning of a word problem, computing a numerical answer (Hegarty et al., 1995) before using qualitative reasoning. Schoenfeld (1987) demonstrated the differences in problem-solving patterns using verbal protocols in terms of how novice and expert problems solvers interact with word problems. Expert analyze the problem by attempting
many different approaches, use various cognitive processes, such as planning, implementing, and verifying, develop a mental representation for the problem, and monitor their performance more often. In contrast, novice problem solvers use ineffectual solution plans without monitoring their progress and tend to use a single strategy. In another study concerning problem-solving behaviors, Hegarty, Mayer, and Monk (1996) investigated the difference in problem-solving approaches between accurate and inaccurate problem solvers, to understand better college students’ difficulties with word problems. As in the studies referenced above, they found that inaccurate problem solvers used the direct translation strategy during problem solving, while accurate problem solvers used the problem model strategy. In the direct translation strategy, problem solvers attempt to select numbers and some key words from the problem and prepare to perform arithmetic operations based on them. This approach emphasizes computation and quantitative reasoning, so it has been referred to as “compute first and think later.”

However, this approach does not construct a qualitative representation of the situation described in the problem (Mayer et al., 1996). Most students who use this approach fail to solve some problems, because they do not represent the situation in their mind and thus fail to make sense of the problem.

In contrast, accurate problem solvers using the problem model strategy try to understand the general situations described in the problem and then devise a plan based on the resulting mental model. This strategy involves understanding the relations among the variables in the problem (Mayer et al., 1996). In other words, the problem model approach requires not simply reading and understanding, but explicitly reasoning through
each of the situations described in the problem statement. This approach plays a crucial role in problem solving success, because students solve problems based on the construction of mental models of the problem situation, in order to engage in sense making (English, 1997).

Another study of the differences between successful and unsuccessful problem solvers was conducted by Hegarty and his colleague (1995). They investigated problem-solving behaviors of 38 undergraduate students using eye movement as they interacted with the word problems. The participants were divided into two groups: a successful group and an unsuccessful group. The problems included two-step consistent and inconsistent language addition and subtraction problems. Unsuccessful problem solvers reread the problem statements more frequently and focused on the numbers and relational terms more often than did the successful problem solvers.

More recently, Pape (2004) interpreted problem-solving behaviors in terms of the interaction of reading comprehension and mathematical skills. Results indicated that middle school students who participated in the study used more direct translation on a majority of the problems rather than the problem solution approach, though they used a problem situation approach more often on the less difficult problems. Pape (1998) reported that unsuccessful problem solvers were not aware of the differences between CL problems and IL problems and did not monitor their understanding of these problems. In contrast, successful problem solvers spent more time than unsuccessful problem solvers in reading problems, developing a mental representation, solving, and remembering the relationship between two variables in IL problems.
Accurate problem solvers construct a problem model while they comprehend the word problems, while unsuccessful problem solvers tend to use a direct-translation strategy for encoding word problems. Novice problem solvers often have difficulty in detecting structural similarities between problems that have different surface features, because they tend to focus on salient surface features rather than structural features. More accurate problem solvers tend to use problem model strategies (English, 1997; Goldman & Rakestraw; 2000), while less successful solvers are more likely to use a direct translation approach. In complex structure problems, however, this direct translation model can lead to the wrong answer (in this case, IL problems): Jonassen (2003) has argued that the “direct translation strategy results in a lack of conceptual understanding and the inability to transfer any problem-solving skills that are developed” (p.267).

**Self-Regulated Learning Strategies**

Several researchers reported that learning different strategies enables students to become more effective and independent learners of mathematics (Pintrich, 2000; Zimmerman, 2000). As a result, self-regulated learning strategy (SRL) has recently received a significant amount of attention from researchers in educational psychology. Self-regulation refers to “the degree that individuals are metacognitively, motivationally, and behaviorally active participants in their own learning process” (Zimmerman, 1986, p.3). Self-regulation is not only the ability of students to monitor and control their thoughts, learning, and behaviors, but also the ability to motivate their learning intrinsically toward a goal (Paris & Wgnograd, 1990; Pintrich, 1999; Schunk &
Zimmerman, 1994). That is, self-regulated learning can be considered as a broader concept that includes motivation and behavior as well as cognition.

Learning strategies include cognitive strategy, metacognitive strategy, and motivation. Cognitive strategies include rehearsing, memorizing, elaborating, organizing, transformation, and seeking information. Metacognitive strategies include planning, monitoring, checking, and regulating strategies (Pintrich, 1999: Weinstein & Mayer, 1986). Motivation includes three general types of motivational beliefs: self-efficacy beliefs, task value beliefs, and goal orientations. Pintrich (1999) argued that students should be aware of their own behavior, motivation, and cognition by reflecting on these three aspects of learning in order to become successful learners.

Much research in educational psychology has paid attention to the improvement of student learning, assuming that personal responsibility for and control over students’ own acquisition of knowledge and skills can improve their learning (Pintrich, 2000; Zimmerman, 2000). Researchers have attempted to understand how students become adept and independent learners by comparing successful learners and unsuccessful learners in terms of self-regulated learning (Pape & Wang, 2003; Pintrich, Smith, Garcia, & McKeachie, 1991; Zimmerman & Martinez-Pons 1990).

Pintrich, Smith, Garcia, and McKeachie (1991) developed the Motivated Strategies for Learning Questionnaire (MSLQ), which is composed of 81 items, to measure college students’ motivation and different learning strategies relating to a particular course. Students’ cognitive strategy use and self-regulation were highly correlated with higher levels of achievement on all performance measures (Pintrich & De
Zimmerman and Martinez-Pons (1990) likewise investigated the self-regulated learning strategies of 180 students who were at the 5th, 8th, and 11th grade levels. Results indicated that self-regulated learning strategies are a powerful predictor of successful task performance and academic achievement.

Previous empirical research on self-regulated learning strategy and academic achievement has targeted the college population rather than the middle school population. Very few studies concerning self-regulated learning strategies exist for middle school students in various subjects (Pape & Wang, 2003; Pintrich & De Groot, 1990; Yetkin, 2006). Furthermore, all these studies investigated students’ self-regulated learning strategy using different instruments. To my knowledge, no one has measured middle school students using the same instruments.

Pape and Wang (2003) extended Zimmerman and Martinez-Pons’ (1986, 1988, 1990) studies by examining self-reported strategic behaviors in 40 sixth- and 40 seventh-graders. Their structured interview protocol was adapted from Zimmerman and Martinez-Pons (1986, 1988, 1990). They investigated the relative frequencies of these categories among middle school students in terms of two contexts: reading-related contexts and mathematics problem-solving contexts. The researchers identified twelve categories of strategic behavior by middle school students as they performed tasks on word problems. Results indicated that high achieving students reported a greater variety of strategies and categories of strategies (i.e., self-evaluation, organizing and transforming, and goal setting and monitoring) than did their low achieving counterparts. The researchers concluded that the use of different strategies and categories of strategies is related to the
use of a meaningful approach that leads to problem-solving success.

Yetkin (2006) explored self-regulated learning strategy use by four 6th graders, using a questionnaire that was adapted from various previous research studies (Pintrich & DeGroot, 1990; Pintrich et al., 1993; Wolters, 1999, 2004). The questionnaire scales are comprised of 33 items to measure 6th graders’ self-regulated learning strategies in mathematics learning contexts. Results indicated that each student who participated in the study engaged in task activities differently, and interacted with classroom contexts in different ways.

Despite the importance of effective measurement of learning strategies, it is surprising that there has been so little empirical research focused on motivational beliefs and the use of self-regulated learning strategies for middle school deaf or hard of hearing students. The next section provides an overview of current research on mathematical problem-solving and deaf students.

**Mathematical Problem Solving and Deaf Students**

During the past decade, research in deaf education has produced a shift in attention from language development and reading comprehension to mathematics and science education to improve low achievement in those content areas for deaf or hard of hearing students. Low achievement by deaf or hard of hearing students in mathematics has been well documented in research (Mithell, Qi, & Traxler, 2007; Nunes & Moreno, 1998; Traxler, 2000), which has indicated that they lag behind their hearing counterparts, even though they perform better in mathematics than in English (Mithell et al., 2007).
The overall mathematics level of most deaf or hard of hearing students upon graduation from high school is as low as sixth grade (Michtell et al., 2007; Traxler, 2000). This achievement gap between deaf and hard of hearing students and their hearing peers is more pronounced in problem-solving abilities (e.g., word problems) than in computation skills, since problem-solving tasks are mostly associated with their reading comprehension ability in English and require higher-level thinking skills (Kelly, Lang, Mousley, & Davis, 2003; Traxler, 2000).

Despite the differences in grades and the methodologies employed, research projects have produced a number of consistent findings there on deaf students’ difficulty with word problems, and there seems to be a consensus that the difficulties deaf students have with mathematical problems may be attributed to combination of delayed development in linguistic, cognitive, and experiential factors (Davis & Kelly, 2003; Hyde, Zevenbergen, & Power, 2003; Kelly et al., 2003; Kelly & Mousley, 2001; Serrano Pau, 1995). Pagliaro and Ansell (2002) argued that poor mathematics performance of deaf or hard of hearing students may result from students’ formal education that they received from their teachers, specifically the following attributes:

1) Lessons are heavily focused on computation and practices.

2) Presentation of word problems is frequently mismatched between deaf students’ language mode and their teachers’ sign fluency.

3) The quality of problem-solving instruction is often inferior because teachers do not have a mathematics certification.

However, some studies suggest that these difficulties can be reduced through direct
instruction. Nunes and Moreno (2002) investigated the effectiveness of an intervention program to improve 23 deaf students’ numeracy. They concluded the intervention program promoted higher achievement in numeracy. In another study conducted by Krizer and Pagliaro (2003), deaf students’ problem-solving ability in discrete mathematics improved through a variety of classroom activities. Therefore, it should be possible to improve mathematics achievement through instructional intervention by analyzing students’ cognitive processes in mathematical problem-solving.

A few studies reported on the difficulty of mathematical linguistic forms for deaf or hard of hearing students (Kidd, Madsen & Lamb, 1993; Rudner, 1978). The language structures include: “conditionals (if, when), comparatives (greater than, the most), negatives (not, without), inferential (should, could, because, since), low-information pronouns (it, something), and lengthy passages” (Rudner, 1978, p.33). In particular, McAnally, Rose, and Quigley (1994) argued that these English words are not exposed frequently enough in the natural language environment (e.g., home and school) of deaf or hard of hearing students. This may be due to the different linguistic nature between spoken language and sign language. For example, some of these English words do not have a corresponding sign language word. The word “since” is a word with multiple meanings, so the meaning of “since” depends on the context. In particular, the comparative phrases “greater than, the most, more than, less than” are visually and spatially represented in ASL, not expressed in words.

In another study, Kidd, Madsen, and Lamb (1993) reported that five major categories of challenging words often used in the vocabulary of mathematics: multiple
meanings (interest, table, chart), technical vocabulary (annual rate, quarterly rate, reconcile, overdrawn), varied forms (yearly, year, basic and base), abbreviations and special symbols (K/h, m/h, w/(meaning with), 5²), and mathematical special emphasis (maximum, minimum, and total). Hyde, Zevenbergen, and Power (2003) suggested that deaf students’ difficulties may be due to the variability of English language structure, such as complexity of syntactic expression and passive forms of expression.

With regard to word problems, there have been several studies in this field on mathematical problem-solving, particularly word problem-solving among deaf or hard of hearing students. Findings from research in this area provide evidence of difficulty in word problem solving due to reading comprehension (Kelly & Gaustad, 2006; Serrano Pau, 1995), address the quality of instruction related to word problem solving (Kelly et al., 2003; Pagliaro & Ansell, 2002) and evaluate the use of sign language (Pagliaro & Anesell, 2002).

Some studies indicate that deaf students’ reading comprehension ability affects their mathematical problem-solving performance (Kelly & Gaustad, 2006; Serrano Pau, 1995). Serrano Pau (1995) presented three different types of word problems (e.g., change, combine, and compare) to 12 deaf or hard of hearing students, whose ages ranged from 8 to 12 years of age. Findings showed that reading comprehension level is directly related to deaf students’ word problem-solving performance. However, this study did not provide enough information about the subjects’ characteristics and research methods, such as data collection and data analysis, which could influence research results and data interpretation. Similarly, Kelly and Gaustad (2006) reported a correlation between deaf
college students’ mathematics performance and reading ability and knowledge of English morphology. This study focused on the structural aspects of text processing.

There are very few studies of strategy use by deaf or hard of hearing students. Hyde, Zevenbergen, and Power (2003) examined problem-solving strategies used by deaf or hard of hearing students as they solved word problems. Results indicated that deaf students seemed to rely on key words strategies, and did not truly understand the problems. Mousley and Kelly (1998) argued that a lack of metacognitive skills is one of the most common inhibitors to success experienced by deaf students.

**Issues Related to Qualitative Similarity Hypothesis and Deaf Students**

There has been much debate in the field about whether deaf or hard of hearing students learn language and reading using the same processing or methods as hearing students (Marscharck, 2008; Marschark, Lang, & Albertini, 2002; Paul, 1998, 2009). The current controversy regarding the English development of deaf or hard of hearing students is influenced by the theoretical framework of researchers and results in a series of dichotomies: objectivism versus. subjectivism, positivism versus. constructivism, deficit model versus. difference (or cultural) model, process versus. product, and the structure of the text versus. the cognitive structure of individuals -- or the interaction between both.

Given the breadth of research on issues in deaf education, this section is limited to a brief discussion of two major perspectives regarding language development and English development in deaf or hard of hearing students. In this field, the dominant view on this
controversial topic is that English literacy development for deaf or hard of hearing students, regardless of whether it’s their first language or second language, is similar to that of hearing students. Deaf students proceed through the same patterns and make similar errors as hearing students on some English tasks, though they lag developmentally behind in English compared to hearing students.

Paul (2009) formalized this idea as the Qualitative Similarity Hypothesis (QSH). The main principle of the hypothesis is that deaf individuals may eventually reach an appropriate proficiency level someday that matches their hearing counterparts, yet in order to reach that level, all individuals must acquire certain fundamentals such as phonics, phonemic awareness, fluency, vocabulary, and text comprehension in English. As a result, a great number of researchers who advocate this view focus on the role of phonological decoding, which is a lower-level skill used as a means to improve low reading achievement in deaf students, arguing that good deaf readers use a phonological code that predicts the success of further reading comprehension (Cornad, 1979; Paul, 1998, 2009). Furthermore, within this view, these researchers investigated English reading ability by comparing deaf students with hearing students in reading tasks. There is an ongoing tendency to conceive students’ reading difficulties as inherent to the learner and blaming them on deaf students’ inability to read.

Paul (2009) seemed to emphasize the importance of textual or disciplinary structure in English, stating that “it might be that development in any discipline such as English language, English literacy, or mathematics is similar for all children and adolescents because of the difficulty levels associated with the inherent framework of and
degrees of conceptual understanding within the specific discipline” (p. 457). In addition, Paul (2009) was influenced by Phillips and Soltis’s (2004) argument, which emphasizes a distinction between disciplinary structures and cognitive structures of individual students.

However, some relevant studies on educational psychology argue that this distinction is philosophically and practically difficult (Greeno, 1985; Roller, 1990), indicating that reading or problem solving functions as an interaction among the reader, text structure, and context of the reading situation; that is, there is an interactive relationship between disciplinary structures and cognitive structures (Goldman & Rakestraw, 2000; Kendeou & Broek, 2007). As a result, different students interact with the text in different ways, producing different learning outcomes, depending on the characteristics of the reader and their reading behavior (Fox, 2009; van den Broek, Young, Tzeng, & Linderholm, 1999).

Furthermore, researchers who support this view have overlooked not only the world knowledge (including content, domain, and/or background knowledge) that students bring to texts or problem-solving, but also an interaction between the structure of text and a variety of knowledge such as the cognitive skills, strategy knowledge, and metacognitive knowledge of deaf individuals. Several studies in cognitive psychology have demonstrated that success in reading or problem solving depends mostly on students’ cognitive skills, relevant knowledge, and motivation, as well as knowledge of the structure of text (Fox, 2009; van den Broek, Young, Tzeng, & Linderholm, 1999; Mayer, 2002; Pintrich, 2000; Roller, 1990; Zimmerman, 2000). Research within the QSH area did not consider how the characteristics of deaf learners and their primary language
mode interact with textual structures that produce different outcomes.

In contrast to the dominant view, recently emerging research focuses on cognition and deaf students indicated that deaf students generally learn and perform differently from their hearing counterparts on English comprehension, mathematical problem-solving tasks, and memory tasks. The major premise of this argument lies in the influence of the primary language mode (sign language vs. spoken language) and early language exposure that results in different experience and knowledge, causing strategies of deaf or hard of hearing students to differ in some way from those of hearing students. Such differences are more likely to influence learning of a given subject matter (Kelly, 2008; Marschark, 2002; Marschark, Lang, & Albertini, 2002).

Marschark (2008) argues that

Differences in the environments and experiences of deaf children and hearing children might lead to different approaches to learning, to knowledge organized in different ways, and to different levels of skill in various domains. Ignoring this possibility not only denies the reality of growing up deaf in a largely hearing world, but jeopardizes academic and future vocational opportunities for deaf children (p.464).

Marschark’s remarks illustrate that deaf learners can have experiences and interactions in very different ways because of their different language modes and different language environments. Within this view, most research studies have explored deaf students’ memory and problem solving to investigate cognitive differences between deaf and hearing learners or among deaf learners (Kelly, 2008; Marschark, 2008). The studies indicated that deaf students exhibited differences in performance on number concepts, sequencing, and conservation tasks.
Results of QSH research with respect to mathematics indicate that deaf students’ problem-solving ability was qualitatively similar to that of hearing students, but was delayed compared to their hearing counterparts. A study by Kelly and colleagues (2003) concerning Lewis and Mayer’s (1987) consistency hypothesis was conducted with deaf college students. Although this study partially supported the effect of the consistency hypothesis, indicating that deaf students more frequently committed reversal errors on IL problems than on CL problems, they argued that reversal errors on IL problems occurred more often compared to hearing students in the Lewis and Mayer (1987) study and that deaf students made some errors that were not made by hearing students. In another study, Hyde, Zevenbergen, and Power (2003) concluded that performance on combine, change, and compare problems was similar for both deaf and hearing students, yet deaf students were delayed in English language development, which influenced their problem-solving success. There are also mixed results among these studies, indicating that deaf students performed differently from their hearing peers when presented with specific tasks related to English or visual spatial ability (Kelly, 2008).

It is difficult to draw firm conclusions on the QSH from a few studies because there are some errors and response patterns that were not exhibited by hearing students. These studies did not explore individual differences in problem solving by deaf students who have different communication modes. Most researchers in this field do not discuss the interaction between the characteristics of deaf students and the structures of tasks, although there is a consensus that unique and diverse characteristics of deaf students influence their learning and language development. As previously discussed, recently
emerging studies even demonstrated that characteristics of hearing students and their prior knowledge influenced their problem solving success and reading comprehension.

**Issues Related to Language Mode**

It is important to understand that deaf or hard of hearing students form a widely heterogeneous group, with different background knowledge in language and learning, and different learning strategies from their hearing counterparts. In addition, they are diverse in their degrees of hearing loss, family backgrounds, language modes, and language fluency. Recent studies demonstrated that this diversity influences their learning, problem solving, and reading comprehension in different ways. In this section, the characteristics of deaf students will be discussed in terms of their primary languages at home.

**Deaf Family versus Hearing Family**

Lane (1984), and Padden and Humphries (1988) suggested that being (culturally) Deaf is determined by the degree of identification with and participation in the deaf community, and by the use of a sign language, but is not decided by the degree of audiologically hearing loss. Approximately 95% of deaf children are born to hearing families who have no experience with how deaf individual learn and live, while only about 5% of deaf children are born to deaf parents (Hauser, Lukomski, & Hillman, 2008). In particular, some studies reported that there are differences in learning and language development between deaf children of deaf parents and deaf children of hearing parents: deaf children of deaf parents outperformed deaf children of hearing parents in ASL and English literacy.
These differences result from early language exposure and language environment. For example, some deaf children who have hearing families do not communicate with their parents in their parents’ primary spoken language at home. Likewise, the home language of deaf students who have hearing parents may be a spoken language that may not be the students’ first language, because these students are not able to access such language directly as their first language if their residual hearing is low or if they have insufficient language exposure (or input) during their early life (Standley, 2005). In the case of hearing parents, language delays in deaf children may be due to inadequate linguistic input or delayed language exposure, which influence cognitive functioning and further learning. Given inconsistent and relatively insufficient language input, some deaf students who have hearing parents can lag behind their hearing peers when they arrive at the public schools. Of course, it is true that few hearing parents of deaf students communicate effectively with their children in sign language or English (Hauser, Lukomski, & Hillman, 2008). These deaf students are at greater risk for delay in the development of either spoken or sign language, and are more likely to be delayed in learning other subject matter (particularly in public education) because of a lack of incidental learning experiences at home (Mayberry, 1993; Newport, 1990). In addition, these deaf students use different communication modes and styles.

On the other hand, deaf children with deaf parents who were fluent in ASL are not necessarily delayed in language development, and have exhibited the same patterns of development as hearing students do (Mayberry, 1993) because they were raised in a rich language use environment without any communication barriers between parent and child.
They can therefore bring and apply their background knowledge to new learning.

**Chapter Summary**

This chapter discussed the four components of cognitive processes in mathematics problem solving and what kind of knowledge in each cognitive process is needed to solve. A text processing model of reading is incorporated into problem representation in order to better understand the process of representing word problems. The text processing model provided a plausible explanation about the difficulties in processing depending on the complexity of problems (Goldman & Rakestraw, 2000; Kintsch & Greeno, 1985). This model emphasized a reciprocal relationship between text-driven processing and knowledge-driven processing. Furthermore, the section discussed that the use of self-regulated learning strategy is related to academic achievement.

With regard to mathematics and deaf students, this chapter discussed some issues related to mathematics low achievements and difficulties of word problem solving. There is an agreement that deaf students have difficulties with mathematics problem solving and lag behind their hearing peers in problem solving. These difficulties resulted from delayed development in experience and language and a lack of metacognitive and prior knowledge (Marscharck, 2008).

Despite the significance of these studies, little is empirically known about how deaf students solve word problems and what kind of problem-solving behaviors used by deaf students as they read and solved word problems and how the characteristics of deaf students interact with the structures of problems, especially in deaf groups who use sign
language as their primary language. With the lack of research related to problems and strategy use in the field of deaf students, the need for research is great. Chapter 3 describes the method used in this study to investigate and describe problem solving behaviors and strategy use in deaf middle school students in order to explore cognitive processes.
CHAPTER 3

METHODOLOGY

The primary goal of this qualitative study was to explore and describe the problem-solving behaviors utilized by deaf or hard of hearing middle-school students, as they read, understood, and solved mathematical word problems. Furthermore, this study examined how the participants’ problem-solving behaviors were influenced by their characteristics, such as language fluency or mode and prior knowledge about specific ideas. To achieve these goals, think-aloud protocols were used to explore complex cognitive process. By doing so, the study sought to describe and understand why deaf middle school students have difficulty in understanding and solving word problems.

Another goal of this study was to examine the use of self-regulated learning (SRL) strategies employed by the mathematics students. To accomplish this goal, data were collected from multiple resources: video recordings of a think-aloud protocol, interviews using a structured questionnaire, written responses on the think-aloud protocol and a computation test, and document analysis of participants’ demographic information and standardized academic achievement scores in mathematics and reading. This chapter outlines the methods and procedures utilized in this study, and is divided into seven sections: Methodological Framework, Research Design, Research Site and Participants,
The Researcher, Instruments, Procedure, and Data Analysis. The following research questions guided the research methods and frame analysis process.

Research Questions

1. What kind of problem-solving behaviors are used by deaf or hard of hearing middle school students while reading and solving compare word problems?
   1.1 What are these students’ word problem-solving success rates?
   1.2. What types of errors do these students commit while solving compare word problems?
   1.3. How is specific mathematics knowledge associated with the patterns of problem-solving approaches?
   1.4. What types of problem-solving approaches do these students use while reading and solving compare word problems?
   1.5. How is language mode of these students associated with problem-reading approaches?

2. What SRL strategies do deaf or hard of hearing middle school students choose as they learn mathematics?

In this chapter, I describe how the methodology framework was applied to the nature and purpose of the research and data as necessary to respond to the research questions. In the research design section, I describe rational of the research method chosen and how the methodology framework is related to my research purpose and
research design, especially based on relevant literature in this field.

Methodology Framework: Grounded Theory

Qualitative researchers believe that all inquiry is value-laden and has multiple realities. Such research attempts to produce descriptive analysis that focuses on deep and interpretive understandings of the phenomena, under context-specific settings and using a variety of data sources (Glensken, 2006). Patton (2001) claimed that qualitative research is defined as a “real world setting the researcher does not attempt to manipulate the phenomenon of interest” (p. 39). Instead, qualitative researchers seek to illuminate and understand phenomena in natural settings.

As one of many approaches to qualitative inquiry, grounded theory refers to theory that is developed inductively from the data. Glaser and Strauss (1967) asserted a need for “a general methodology for developing theory that is grounded in data systematically gathered and analyzed” (p. 273). Accordingly, grounded theory is generated from data observation, as opposed to theory derived deductively from logical assumptions and existing studies. Strauss & Corbin (1998) argued that grounded theory does not start with “a preconceived theory in mind” (p.12); rather, it begins with “allow[ing] the theory to emerge from the data” (p.12). Furthermore, researchers have adopted grounded theory where little is already known in the literature (Goulding, 1999): the main premise of grounded theory is “to bridge the gap between theoretically ‘uninformed’ empirical research and empirically ‘uninformed’ theory, by grounding theory in data” (Goulding 1999, p. 6).
In math education, there is agreement with regard to deaf students’ learning mathematics that “language is at the heart of their difficulties” (Barham & Bishop, 1991, p.180). In my perspective as an English as a second language learner and a deaf researcher, I believe that other factors influence learning mathematics, and that language is just one source of difficulty. Thus, this study started with a crucial question: what factors influenced deaf students’ problem-solving success or failure? Most previous studies on deaf students’ learning have focused on their English language development or reading comprehension, while topics related to mathematics learning have received relatively superficial attention. Furthermore, none have explored mathematics word problem-solving behaviors and strategy use in deaf or hard of hearing middle school students.

Because of this paucity of previous research, grounded theory is an appropriate methodology for studying the process of understanding and solving word problems among deaf or hard of hearing students. I develop the process of understanding and solving word problems in middle school students from the data, in order to provide teachers with better insights about the cognitive process in deaf learners’ word problem solving. Teachers can thereby better understand their students’ cognitive processes and can develop pedagogical practices that can help their students improve their mathematical problem solving. Therefore, this study examines how deaf or hard of hearing middle school students read, understand, and solve word problems, and how they employ SRL strategies to learn mathematics. In order to understand individual differences in such behaviors and strategy use, the study needed to develop a theory generated from data.
Considering the purpose and research questions in this study, grounded theory was appropriate to develop theory from the data.

**Research Design**

A think-aloud protocol and interviews were used to explore the students’ thought processes, and to obtain direct accounts of their problem-solving behaviors and strategy use. Ericsson and Simon (1993) suggested that think-aloud protocols are an effective tool not only for identifying and measuring children’s thought processes, but also for studying individual differences in problem-solving performance. Additionally, Pressley and Afflerbach (1995) pointed out that think-aloud protocol can be helpful for examining the mental representation students construct as they interact with text structures. Therefore, the use of think-aloud protocols for this study is appropriate to elicit data about participants’ thought processes while they understand and solve the word problems. This purpose of this study was to examine not only how different backgrounds influence participants’ problem-solving success and behaviors, but also their types of errors and problem-solving approaches. The participants all use sign language as their primary communication mode. The think-aloud protocols were thus used to explore complex cognitive processes in word problem solving.

**The Research Site and Participants**

This study was conducted during the 2009-2010 academic year at a deaf residential school in the Midwestern United States. This school serves deaf students from
preschool to high school (K-12). Most of deaf or hard of hearing students in this school have severe to profound hearing loss, as compared to public school programs for deaf or hard of hearing students across the Midwestern of the states. According to the school’s website, they follow a “bilingual-bicultural” philosophy of education and communication philosophy; a mixture of American Sign Language (ASL) and signing in English word order is used for instruction and communication among deaf students and their teachers. English in-print is also a critical component used for instruction and communication. In this school, signed system “choice” (i.e. ASL versus Signed English) seems to largely depend on students’ proficiency in ASL and English and his or her comfort level with one language or the other. The school emphasizes full communication access through fluency in both ASL and English and, respects deaf individuals’ different backgrounds and their communication styles. This school provides support in both auditory and speech training, and students’ communication needs are addressed on an individual basis through the IEP process.

**Participants.** This study employed purposive sampling, a qualitative sampling focusing on a relatively small number of participants that is developed in order to provide in-depth understanding of the selected participants (Patton, 2002). In the current study, purposive sampling was chosen for two reasons. The first reason has to do with the fact that, clinically, not all hearing loss is considered to be identical. Clinically, hearing loss is defined as severe when an individual’s Pure-Tone Average (PTA) in his or her better ear ranges from 71 to 90 dB across the speech frequencies of 500, 1000, and 2000 Hz. Loss greater than 90 dB across the same spectrum of frequencies is categorized as being
profound (Paul, 2008). Because of previous research suggesting that, when compared to mild or moderate hearing loss, these profound levels of loss are more likely to indicate difficulty developing language and, consequently, contribute to difficulty learning school subjects such as reading, mathematics, and science (Paul, 2008). This study was limited to students with severe or profound hearing loss in order to understand the way in which students in this group read, understand, and solve word problems.

Secondly, since deafness is a low incident disability, it is unlikely that many students who meet the criteria find the same school district in Ohio. As such, a residential school was chosen. Most students in this school have profound or severe hearing loss and used sign language as their first language, this school best fits with the purpose of this study.

Four criteria for inclusion in the study were:

1. Students be within the range of academic ability such that they could reasonably be expected to solve at least some of the problems presented

2. Students present with a severe to profound hearing loss.

3. Students do not possess any additional disabilities, except for corrected vision.

4. Students had to consent to participate in the study, and provide parental or guardian consent.

In addition to the purposive criteria used to find and build a subject pool, this study monitors results in terms of other basic pieces of demographic information. Additionally, demographic information such as age, gender, grade, and degree of hearing
loss and information about a primary communication mode at home and school, and the hearing status of their parents and siblings was obtained from students through interviews.

With regard to hearing losses, all were identified with profound or severe hearing losses in their better ear (hearing loss left, $M=104.70$ dB, hearing loss right, $M=98.15$ dB), except for one student who uses a cochlear implant (student 7) who had a moderate hearing losses in both ears. Participants did not have any additional disabilities, except for corrected vision. They all communicated in ASL or a form of signed English.

Three $6^{th}$ graders who initially participated in the study were later excluded from the data analysis. After participation in the structured questionnaire on strategy use and a computation test, it was noted that their reading comprehension and mathematical problem-solving abilities were lower than expected. Their backgrounds and prior knowledge about specific concepts, as well as fluency in both written English and sign language was significantly delayed when compared to other participants in this study.

As a result of the exclusion of these students, students outside the initial pool of middle school aged students were evaluated to determine if they would meet selection criteria. Thus, one fourth-grade student and six ninth-grade students were selected because her academic level was such that it was reasonable to assume she would be able to solve the given word problems; the ninth graders’ achievement scores and background information was in line with the purposive criteria of the study.

Therefore, for this study, a total of 13 students from a residential school were chosen based on the criteria addressed above. The thirteen deaf students, ranging in age from 10.2 to 16.6 years, from four different grade levels: fourth grade (1), seventh grade
(5), eighth grade (1), and ninth grade (6) were included for the data analysis. Of the 13 students, 7 were male and 6 were female. The characteristics of these participants varied depending on their language fluency, language mode, and family background, their academic achievement in mathematics and reading. Table 3.1 provides demographic information of participants.
<table>
<thead>
<tr>
<th>Number</th>
<th>Grade</th>
<th>Gender</th>
<th>Age</th>
<th>PTA* (dB)</th>
<th>Primary/secondary communication modality at home</th>
<th>Parental Hearing status</th>
<th>Sibling hearing status</th>
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</thead>
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<tr>
<td>1</td>
<td>7th</td>
<td>Male</td>
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<td>L: 120</td>
<td>Signed English</td>
<td>Deaf</td>
<td>Hearing</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R: 120</td>
<td></td>
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<td>2</td>
<td>7th</td>
<td>Male</td>
<td>14.8</td>
<td>L: 87</td>
<td>Few signs</td>
<td>Hearing</td>
<td>hearing</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R: 87</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7th</td>
<td>Male</td>
<td>14.3</td>
<td>L: 105</td>
<td>Singed English</td>
<td>Hard of hearing / Hearing</td>
<td>hearing</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R: 107</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7th</td>
<td>Male</td>
<td>13.8</td>
<td>L: 103</td>
<td>spoken</td>
<td>Hearing</td>
<td>hearing</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R: 70</td>
<td></td>
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<tr>
<td>5</td>
<td>7th</td>
<td>Male</td>
<td>14.6</td>
<td>L: 107</td>
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<td></td>
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<td>R: 107</td>
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<tr>
<td>6</td>
<td>8th</td>
<td>Male</td>
<td>13.7</td>
<td>L: 108</td>
<td>Signed English /spoken</td>
<td>Hearing</td>
<td>Hearing</td>
</tr>
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<td></td>
<td></td>
<td>R: 88</td>
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</tr>
<tr>
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<td>4th</td>
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<td>10.2</td>
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<td>Hearing</td>
</tr>
<tr>
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<td></td>
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<td></td>
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<td>few signs /written language</td>
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<td>Hearing</td>
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<tr>
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<td>9th</td>
<td>Female</td>
<td>16.4</td>
<td>L: 82</td>
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<td>Hearing</td>
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<td>5.11</td>
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<td>few signs</td>
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<td>hearing</td>
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<tr>
<td>11</td>
<td>9th</td>
<td>Female</td>
<td>15.6</td>
<td>L: 93</td>
<td>Signed English</td>
<td>Hearing</td>
<td>hearing</td>
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<td></td>
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<tr>
<td>12</td>
<td>9th</td>
<td>Female</td>
<td>16.6</td>
<td>L: 105</td>
<td>ASL</td>
<td>Deaf</td>
<td>Deaf</td>
</tr>
<tr>
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<td></td>
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<tr>
<td>13</td>
<td>9th</td>
<td>Male</td>
<td>16.6</td>
<td>L: 103</td>
<td>Signed English</td>
<td>Hearing</td>
<td>Hearing</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td>R: 77</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

*.Pure Tone Audiograms (PTA)

Table 3.1. Demographic Information

Additionally, academic achievement was summarized based on standardized achievement scores in the areas of mathematics and reading, as shown in Table 3.2. The researcher obtained participants’ standardized achievement scores in mathematics and reading/language arts from the Mathematics Testing Coordinator and Reading Testing
Coordinator of this school. Table 3.2 presents participants’ standardized achievement scores in mathematics and reading and computation scores.

<table>
<thead>
<tr>
<th>Number</th>
<th>Grade</th>
<th>Standardized Score</th>
<th>Computation Score</th>
</tr>
</thead>
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<tr>
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<td></td>
<td>Math</td>
<td>Reading</td>
</tr>
<tr>
<td>1</td>
<td>7th</td>
<td>8.4</td>
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</tr>
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<td>7th</td>
<td>7.2</td>
<td>4.0</td>
</tr>
<tr>
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<td>7th</td>
<td>4.2</td>
<td>3.0</td>
</tr>
<tr>
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<td>7th</td>
<td>4.1</td>
<td>2.5</td>
</tr>
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<td>7.5</td>
<td>5.0</td>
</tr>
<tr>
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<td>8th</td>
<td>5.9</td>
<td>5.5</td>
</tr>
<tr>
<td>7</td>
<td>4th</td>
<td>5.1</td>
<td>4.5</td>
</tr>
<tr>
<td>8</td>
<td>9th</td>
<td>7.1</td>
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</tr>
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<td>9th</td>
<td>5.7</td>
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<td>6.7</td>
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<td>9th</td>
<td>5.9</td>
<td>8</td>
</tr>
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<td>9th</td>
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<td>8.5</td>
</tr>
<tr>
<td>13</td>
<td>9th</td>
<td>8.4</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 3.2. Each Participant’s Standardized Scores in Mathematics and Reading and Computation Scores

**Diversity of Deaf Participants.** It is important to understand the characteristics of deaf participants in this study because their certain characteristics influence problem-solving behaviors (Marschark, 2008). Participants in this study have a variety of backgrounds in language and learning outcomes. They came from either deaf parents or hearing parents. Furthermore, there are differences between deaf students with deaf parents and deaf students with hearing parents in the communication skills or language
mode, and the amount of relevant knowledge as mentioned in the literature. Even though all participants communicated using sign language, the degree of their ASL fluency or communication skills varied depending on their home language environments and quality of their parents’ interaction at home. I will discuss the different characteristics of all deaf participants in the degree of hearing loss, primary language mode at home, standardized academic achievement test, and computation skills.

In this study, only three deaf students (student 1, 5, and 12) were from deaf families who were fluent in ASL and immersed in deaf community; these three students’ signs were entirely different from the signs of the remaining participants who were from hearing families who used signed English or knew few signs. In addition, two students (student 6 and 7) had hearing parents who fluently and effectively communicated with their parents in sign language and written language.

In contrast, 8 of the remaining participants came from hearing families who knew few signs or communicated by using spoken language. There are differences between deaf students who had a shared language with their parents and deaf students who did not have such language in word problem solving. In the chapter 4 and 5, how characteristics of deaf students are related to their problem-solving success or problem-solving approaches will be discussed in more detail. I now describe each student giving gender, age, hearing loss, family hearing and communication status, standardized academic achievement scores in reading and mathematics, and other relevant information.

**Student 1** was a 13-year old male in -7th grade male- with profound, bilateral hearing loss; He was fluent in ASL, but tended to communicate using signed English
when he communicated with classmates or teachers. His family members, with the exception of one older sister, are deaf, and are all fluent in ASL and members of the deaf community. He indicated that his communication mode is largely influenced by his sister, who hears and attends the family’s local public high school, mentioning that he mostly communicates with her at home when he has problems with mathematics or reading. His reading comprehension score was at the 5.5 grade level and mathematics score was the 8.4 grade level. His computation score was 26 (Max: 28) and fraction score was 14 (Max: 16). He failed to solve only one fraction division problem on the computation test.

**Student 2** was a 7th grade male with a bilaterally severe hearing loss. His parents were both hearing and used some signs to communicate with him; he also had a deaf sister. He was a quiet boy and seemed to have difficulty expressing what he thought when he was asked to think “sign” while he read and solved word problems. Thus, I had to ask him to draw or write something he was thinking on the paper. He seemed to be delayed in both expressive language and receptive language. His signs were simple when compared to most other participants in the study. His reading level was at fourth grade level, while his mathematics level was at grade level. Although his mathematics level is at grade level, he solved only one fraction problem on the computation test. He appeared to have a lack of understanding of fraction concepts and procedural knowledge. His computation score was 12 (Max: 28) and fraction score was 2 (Max:16).

**Student 3** was a fourteen-year-old male in 7th grade, and he had severe-to-profound hearing loss in both ears. His father was hard of hearing and can sign fluently and his mother is a sign language interpreter. Thus, his parents communicated with him
using sign language at home. His hearing brothers knew only few signs. Although he had a hard of hearing father who signed fluently, his signs were different from student 1 and student 5, who had deaf parents that were fluent in ASL. He was not fully fluent in ASL. He read at the 3th grade level and performed mathematics at the 4.2 grade level. His computation scores and fractions scores with the computation test were 18 (Max: 28) and 5 (Max: 16), respectively. He had problems with fractions, caused by lack of conceptual and procedural knowledge.

**Student 4** was a 7th grade male with a bilateral profound hearing loss in both. He came from hearing parents and had 3 hearing brothers who do not know sign language, and they communicated with him using spoken language. He was delayed in both written language and sign language. His reading level was at the 2.5 grade level that is the lowest level among participants and mathematics level is at the 4.1 grade level. He is delayed in language, English written language, and mathematics. His total computation score is 15 (Max: 28) and fraction score is 3 (Max: 16).

**Student 5** was a fourteen -year-old Deaf male at 7th grade level. He had a severe-to-profound bilateral hearing loss in both ears. He came from a deaf family who was immersed in ASL and deaf community. He was grown up in a rich language environment where his deaf parents and deaf sister fluently communicated in ASL at home. His parents attended school for the deaf and his sister was graduated from the same school 1 year ago. He was very fluent in ASL and communicated using ASL with his teachers and classmates. The quality of his think-aloud protocols in communication skills were different from one of other participants in this study. He brought sufficient background
knowledge to the problems when he read and solved problems and applied them to the problems when he explained his understanding of problems. He is reading at a 5.0 grade equivalent level and is achieving a 7.5 grade equivalent level in mathematics. He is the only one who correctly solved all problems in a computation test (including fraction problems) for this study (maximum:28/28). He had very fluent arithmetic operation skills.

**Student 6** was a thirteen year-old male and wore hearing aids in both ears with moderate-to-severe hearing loss in both ears. Both his parents are fully hearing but his father can sign, although he is not fluent in sign language. He mentioned that he very frequently communicated with his father at home and if he does not understand some words while reading the book, and then he often asks for a help to his father. He can sign fluently and speak in English. He transferred from a total communication classroom to the school 1 year ago. He achieved 5.5 grade level in reading and 5.9 grade level in mathematics, his computation score was 17 (Max: 28) and fraction score was 5 (Max: 16) for fraction test scores within computation test.

**Student 7** was a female in the 4\(^{th}\) grade, the only student in elementary school and had cochlear implants in an ear. She had cochlear implants at twenty months of age and intensive auditory, speech, and language therapy until she moved to this school. She was very fluent in sign language. She came from hearing family but her mother was a sign language interpreter. Her teacher stated that she had grade-appropriate reading comprehension skills and mathematics skills. She is reading at the 4.5 grade level and her mathematics level is at the 5.1 grade level. Her computation score was 15 (Max: 28) and fraction score was 3 (Max: 16). She has struggled with problems associated with fraction
knowledge in conceptions and procedural knowledge.

**Student 8** was a fifteen-year-old female and at 9th grade level. She had a severe-to-profound hearing loss in both ears and did not have hearing aids. She was fluent in sign language and a good expressive language for this test. She is the only deaf girl in her family and came from a hearing family who can sign some words but was not fluent in sign language. She communicated with his parents and siblings using some sign words and written language. Student 8 said that she does not often communicate with her parents because her parents were not fluent in sign language. She read at the 8.5 grade level and her mathematics is at the 7.1 grade level. Her computation score was 25 (max:28) and fraction score within computation scores were 12 (Max: 16).

**Student 9** was a sixteen-year-old female who has a severe-to-profound loss in both ears. Her father is hard of hearing and does not know sign language but her mother can sign some words in English word order and was not fluent in ASL. She was reading at the 7th grade level and performing at the 5.7 grade level in mathematics. For computation score, she got 16 points (Max: 28) and 4 points (Max:16) for a fraction score within the computation test.

**Student 10** was a fifteen-year-old female who has been profoundly deaf in both ears. She came from a hearing family who can sign some words but were not fluent in either ASL or signed English. She seemed not to have a good expressive language. She read at the 5th grade level and performed at the 6.7 grade level for the mathematics. Her computation scores were 21 (Max: 28) and fraction scores within computation were 9 (Max: 16).
**Student 11** was at the 9th grade level and had profound hearing loss in both ears. Both of her parents are hearing who are fluent in signed English. Her parents were fluent in signed English. Her father is a teacher of deaf children and mother is a sign language interpreter. She was fluent in signed-English and had a very good expressive language in sign language. She read at the 8th grade level and performed mathematics at the 5.9 grade level. For computation test, she got a total of 18 points (Max: 28) and 8 points (Max:16) for fraction scores within computation test. It appears that she lags behind in mathematics when compared to her reading comprehension ability.

**Student 12** was a sixteen-year-old female at 9th grade level with severe-to-profound bilateral hearing loss. She came from deaf family whose both parents were deaf and three older brothers and one older sister were deaf who were active in deaf community and immersed in ASL. Her brothers attended the state school for the deaf in the past. Although she had a deaf big family, she received a speech and hearing training in the school. She had grade-appropriate reading comprehension skills reading at the 8.5 grade level, yet she relatively lagged behind in mathematics, indicating that her achievement level in mathematics was at the 5.7 grade level. Her computation score was 18 (Max: 28) and fraction score was 5 (Max: 16).

**Student 13** was a 9th grade male who had a severe to profound hearing loss bilaterally. His parents were both hearing but his mother learned sign language to communicate with her son but her signs were signed English based on English word orders. He communicated with his classmates in ASL because he was a resident in the dormitory. He read at 7th grade level and performed at 8.4 grade level in mathematics. His
math grade level is higher than his reading grade level. He had 21 scores (Max: 28 points) for computation test and 10 points for fraction scores within computation test.

**The Researcher**

As a deaf PhD student and researcher, I have a different background from many of my peers. I learned English as a second language, and learned sign language relatively late in life. These different experiences have allowed me to develop this research project from a deaf lens, especially focused on a different model of deaf students and their primary language.

As a deaf student, I have been put in mainstreamed classrooms, where all students and teachers were hearing and communication access was very limited for me, because there was no support system for deaf students while I was in school. After graduating from a university, I felt I needed to make deaf friends who can share my experiences as an individual with hearing impairment. Thus, I decided to learn Korean Sign Language and participate in social events in the Deaf community. However, unfortunately, there were miscommunications between deaf friends who used sing language as their first language and I who used signed Korean. Through these experiences, I learned that the language processing of deaf students may differ from that of hearing students, because of the different natures of spoken and sign language. These experiences allowed me to pursue meaningful Ph.D program in U.S.A.

As a deaf researcher of graduate studies, I have volunteered at the same school for the Deaf for 6 months in 2006 and have been a university supervisor in the M.Ed Deaf
Education program since 2005. As a deaf doctoral student and international student, the researcher has a lengthy history with this population. I lived in a dormitory at a residential school for six months between 2006 and 2007 to be fluent in American Sign Language (ASL), which is totally different from Korean Sign Language, just as a spoken language English is different from Korean. The residential school was the best place that I can learn both ASL and deaf culture at the same time because many deaf staff who consider themselves as a member of deaf culture worked at dormitory of the school.

During that time, as a mathematics tutor, the researcher taught mathematics to elementary and middle school deaf students at one of the afterschool programs. As an international doctoral student, these experiences allowed me to feel comfortable communicating with deaf signers and deaf students who use ASL.

Additionally, during the 2006-2007, I had an opportunity to observe a fifth-grade classroom at the school one day per week in order to establish a rapport with deaf students, to observe their communication modes and learning styles, and to learn how they study mathematics and language arts in their classroom, and curriculum. The students from this classroom are currently in middle school. These experiences allowed the researcher to become accustomed to the curriculum, activities in these classrooms, their classroom environments as well as the American educational system.

While observing the classes, I participated in class activities as much as possible, by sitting with students around circular tables during the group activities and assisting students upon their request. As a result, I could find consistent problem-solving behaviors among the group. When presented mathematical problem-solving tasks, the students in
the classroom simply attempted to find a right answer and did not show any steps for the problem solving on their work sheets. When I found their errors, the students were asked “do you think your answer is right? If so, why?” and then they had to work the problem again to demonstrate that their answer is right. Through the reflective process, students could correct their errors themselves without someone’s help. These experiences led me to begin to investigate three issues: How do deaf students develop or construct conceptual knowledge in mathematical problem solving? What is their procedural knowledge about particular problems, and what is metacognitive knowledge possessed by them when they solve mathematical problems? I expect to find the answers to these questions through this study.

Before conducting interviews and while getting parents’ permission, I observed a middle school mathematics classroom at the school for the Deaf for about 2 weeks to explore how students in this classroom interact with the mathematics teacher and with each other. These observations allowed me to interact with the students at a comfortably familiar level and to develop and refine appropriate interview questions and methodology. I conducted all interviews of students. Additionally, during interviewing, I asked follow-up questions students to make sure if they understood given word problems and why they encountered difficulties with the problem-solving for think-aloud protocols. During interviews for self-regulated learning strategy use, I asked students follow-up questions if they marked negatively on scales. For example, why did you negatively mark on the item you just read? During the think-aloud protocols, I just asked students “what are you doing now?”. “Please sign what you were thinking” when they were silent.
Instruments

The data for this study included three different instruments: 1) think-aloud protocols (i.e., video recordings), 2) students’ written responses (i.e., word problem-solving on think-aloud protocols, a computation test, SRL strategy questionnaires, and confidence level before and after a think-aloud protocol), and 3) formal academic achievement test scores in mathematics and reading, and student records (i.e., brief demographic information) from the school. The confidence levels on word problems administered before and after think-aloud protocols were not analyzed in this study.

Self Regulated Learning Strategy Questionnaire

A questionnaire on learning strategies used in this study was excerpted by Yetkin (2006). Yetkin (2006) incorporated the questionnaire from several studies (Pintrich & DeGroot, 1990; Pintrich et al., 1993; Wolters, 1999, 2004). This questionnaire consisted of 33 items that is designed to assess mathematics learning strategies by hearing 6th graders in her study. There are four strategy subscales: Cognitive, Metacognitive strategies, effort regulation, and help seeking strategies (Table, 3.3).

For example, one of the items from this scale is “When I study for math, I read my notes, my homework, and the textbook over and over.” Students were asked to rate ranging from 1 (not at all true) to 5 (very true). All items were scored on a 5-point Likert-type scale in order to facilitate middle school deaf or hard of hearing students’ responses. Table 3.3 presents four sub-strategies of SRL learning strategy questionnaire: See Appendix A for the SRL Strategy Questions.
Table 3.3. Four Components of SRL Strategy Questionnaire.

<table>
<thead>
<tr>
<th>SRL Strategy Questionnaire</th>
<th>Sub-Strategies</th>
<th>Items</th>
<th>The Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cognitive Strategies</td>
<td>1, 2, 3, 10, 11, 12, 18, 25, 26, &amp; 32</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Metacognitive and self-regulation</td>
<td>5, 6, 7, 13, 14, 15, 19, 20, 21, 22, 27, 28, 29, &amp; 33</td>
<td></td>
<td>14</td>
</tr>
<tr>
<td>Effort Regulation</td>
<td>4, 8, 16, 23, &amp; 30</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Help Seeking</td>
<td>9, 17, 24, &amp; 31</td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

Think-Aloud Protocol

The Think-Aloud Protocol for this study was adapted from Pape’s (1998) study and is related to its purpose of understanding problem-solving behaviors. The twelve target word problems were identical to the word problem test used for hearing middle school students in Pape’s (1998) study, with the exception that the researcher changed the store names to names familiar in the community of this district. For example, the word “BP” or “Kroger” replaced with the word “Arco” or “Pathmark.”

The Think-Aloud Protocol consisted of 16 compare word problems including 4 filler problems that are included to avoid stereotyped responses and to provide variation in the types of problems. The four filler problems included word problems with multiplication and division, yet are not compare word problems containing relational terms. They were adapted into easier problems to minimize the effect influencing target problem-solving success or failure.
Target problems are 12 compare word problems (Problem 1, 3, 4, 6, 7, 8, 9, 11, 12, 13, 15, and 16) that are composed of combination of four factors: (1) the type of problems (consistent language vs. inconsistent language problems, (2) the number of computational steps (one-step or two-step), (3) different mathematical arithmetic operations (addition, subtraction, multiplication, and division), and (4) four relational terms (more than, less than, times as many as, and 1/n as many as). Additionally, twelve target word problems are comprised of an equal number of consistent and inconsistent language problems, including eight two-step problems, which is composed of problems requiring addition, subtraction, multiplication, and division operation. Four one-step CL and IL problems included only multiplication and division ones.

Table 3.4 provides the different types of compare word problems and Table 3.5 presents twelve target problems consisting of combination of four different components of problems. The Think-Aloud Protocol is shown in Appendix B.

<table>
<thead>
<tr>
<th>Language Consistency</th>
<th>One-step</th>
<th>Two-step</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Multiplication</td>
<td>Division</td>
</tr>
<tr>
<td>CL</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>IL</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table. 3.4. Components of Word Problems
<table>
<thead>
<tr>
<th>#</th>
<th>Problem Types</th>
<th>Computation Steps</th>
<th>Relational Terms</th>
<th>Arithmetic Operation</th>
<th>Fraction-a-Number-of Relational Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CL</td>
<td>2-step</td>
<td>3 times as many as</td>
<td>Multiplication</td>
<td>Absence</td>
</tr>
<tr>
<td>3</td>
<td>CL</td>
<td>2-step</td>
<td>5 cents more than</td>
<td>Addition</td>
<td>Absence</td>
</tr>
<tr>
<td>4</td>
<td>IL</td>
<td>1-step</td>
<td>5 times as many as</td>
<td>Division</td>
<td>Absence</td>
</tr>
<tr>
<td>6</td>
<td>IL</td>
<td>2-step</td>
<td>20 cents more than</td>
<td>Subtraction</td>
<td>Absence</td>
</tr>
<tr>
<td>7</td>
<td>IL</td>
<td>2-step</td>
<td>2 times as many as</td>
<td>Division</td>
<td>Absence</td>
</tr>
<tr>
<td>8</td>
<td>CL</td>
<td>1-step</td>
<td>3 times as many as</td>
<td>Multiplication</td>
<td>Absence</td>
</tr>
<tr>
<td>9</td>
<td>CL</td>
<td>2-step</td>
<td>5 cents less than</td>
<td>Subtraction</td>
<td>Absence</td>
</tr>
<tr>
<td>11</td>
<td>IL</td>
<td>2-step</td>
<td>1/3 as many as</td>
<td>Multiplication</td>
<td>Presence</td>
</tr>
<tr>
<td>12</td>
<td>IL</td>
<td>1-step</td>
<td>1/3 as many as</td>
<td>Multiplication</td>
<td>Presence</td>
</tr>
<tr>
<td>13</td>
<td>CL</td>
<td>2-step</td>
<td>1/5 as many as</td>
<td>Division</td>
<td>Presence</td>
</tr>
<tr>
<td>15</td>
<td>CL</td>
<td>1-step</td>
<td>1/3 as many as</td>
<td>Division</td>
<td>Presence</td>
</tr>
<tr>
<td>16</td>
<td>IL</td>
<td>2-step</td>
<td>15 cents less than</td>
<td>Addition</td>
<td>Absence</td>
</tr>
</tbody>
</table>

Table 3.5 Twelve Target Problems Consisting of Combination of Four Different Structures

**Computation Test**

The computation test was to measure participants’ computation ability (i.e., addition, subtraction, multiplication, division, and fraction) in relation to mathematical concepts and operation necessary to solve the word problems. The computation test was adapted from the model developed by Pape (1998). Two division problems were added to the original test to measure students’ division computation ability. For example, one of
these two problems was similar to the following example $27 \div \frac{1}{3}$. Two items per problem-type, except for the question 4 which consists of 4 items, result in a total 14 arithmetic problems. Of the 14 problems, 8 problems were fraction problems. The computation test is shown in Appendix C.

Student Background Survey

Students’ demographic information were collected to examine the influence of language mode on problem-solving performances: gender, date of birth, grade, parents’ hearing status, siblings’ hearing status, primary communication mode at home and school. The student background survey is attached in Appendix D.

Procedures

The data were collected from April to June 2010 to explore and describe deaf middle school students’ problem-solving behaviors and strategy use from 4 different classrooms in a residential school for the Deaf. As mentioned in the section on participants, these students were recruited from four different grade levels and classrooms to conduct interviews using a think-aloud protocol and a structured questionnaire about a self-regulated learning strategy in a privacy room within the library of the school.

Student Interview Session about SRL

Before I conducted interviews with students, I observed their mathematics
classrooms for 10 days to investigate their communicative skills and learning styles. I decided to conduct interview using the SRL learning strategy questionnaire at first because I wondered what they think about mathematics and what their beliefs about mathematics are. I met with each participant individually in a privacy room within the library of the school to conduct interviews for this test. All interviews took place during mathematics classroom or study hall time. Each participant was asked to complete the SRL Strategy Questionnaire. After reading each item, the student was asked if he or she understood it and then was asked to rate one scale ranging from 1 (not at all true) to 5 (very true). If the participant did not understand the item in the questionnaire, I gave her or him specific examples to help his/her understanding of each item. If she or he refers to inability to understand words, I explained the meanings of the words to them. These items on scales consisted of abstraction questions. Thus some of the items were difficult for some students who were at 7 and 8th grade level to understand but were not difficult for 9th graders. The first session took approximately 20-30 minutes, depending on students’ reading comprehension abilities. The focus of this study is on student response about the use of learning strategy as they learn mathematics.

Think-aloud Protocol Session

The Think-Aloud Protocols were presented to each participant individually in a privacy room within the library. The purpose of the think-aloud protocols was to explore participants’ cognitive processes in word problem solving. Participants were informed that they were not able to discuss the contents and procedure of the word problems with
other participants. The entire session of the think-aloud protocols was video-taped for analysis. The students were instructed that they were unable to ask any questions to the researcher during problem solving because the focus of the test was on examining their knowledge in both reading comprehension and mathematics. To ensure deaf or hard of hearing students understand the directions for this study, all students were presented two practice problems before presenting twelve target word problems.

Practice problems are as follows:

1. Five children share 12 markers equally. How many markers does each child get?

2. Emily bought 2 gallons of milk. Each gallon cost $2.79. He paid with a $10 bill. How much change did he receive?

During the think-aloud problem-solving session, students were asked to sign whatever came to mind as they attempted to understand, think, and solve each compare problem. If the students were silent, the researcher reminded the students to think aloud as often as necessary by asking “What are you doing now?” After reading each problem, the students were asked to retell the problem questions to the researcher to examine how they understood the problems before attempting to solve the problem. Follow-up questions could include “what does this problem mean?” or “can you explain this problem to me? This allowed the researcher to check for their understanding and misunderstanding of word problems that might exist within participant problem-solving behaviors or monitoring strategy. After explaining the problem, the students can proceed to the solving steps.
If a student did not understand the problem and finish the problem without actively seeking to solve the problem, the student was asked to explain why he or she encountered difficulty with the problem. This session took approximately 30-50 minutes for each participant, depending on his or her reading ability and mathematical ability. Video-tapes were utilized to record interviews that were transcribed for data analysis by the researcher.

**Computation Test Session**

The participants were asked to complete computation problems that consisted of addition, subtraction, multiplication, division, decimal, and a fraction of a number. They were asked to show all solution steps for all problems. The focus of this test was on investing their conceptual and procedural knowledge of multiplication, division, and fraction problems. The use of a calculator was not permitted for this study. The test took about 20-25 minutes, depending on their computation ability.

Finally, the researcher collected brief demographic information such as gender, birth date, their parents and siblings’ hearing status, and communication mode at home and school. The Background Information Form may be found in Appendix D

**Time Line of Data Collection**

The data for this study were collected from April to June after approval of IRB. Interviews were conducted in May after getting permission from participants and their parents and concluded at the first week of June. All interviews were individually
conducted at the privacy room with the library of the school.

**Data Analysis**

The data were collected through multiple sources: 1) the think-aloud protocol (video-recordings), 2) two written responses from each student on the think-aloud protocol and the computation test, 3) a questionnaire about learning strategy, and 4) information on students’ backgrounds and records.

**Procedures for Data Coding the Video Recordings and Written Responses**

There were four sets of data used in this study: video recordings, participants’ written responses to think-aloud protocols, participants’ written responses to computation tests, and participants’ responses to questionnaires regarding strategy use. The think-aloud protocols were reviewed using open coding techniques, as suggested by Strauss and Corbin (1998), based on line by line analysis in order to identify key words and concepts. In the process of open coding, each student’s problem-solving behaviors that emerged from the data were categorized, and each problem-solving behavior was related to each research question. Comparisons within and among participants were conducted to examine the similarities and differences in problem-solving behaviors based on problem-solving success, the patterns of errors, retelling protocols, the number of re-readings, problem-solving approaches, and problem-solving reading comprehension behaviors.

In addition to these behaviors, twelve target word problems were analyzed according to four elements of mathematics problems: problem types, computation steps,
arithmetic operations, and relational terms. The extent of problem-solving success with regard to these elements of problem structure was investigated. The four phases for data coding are discussed below in more detail.

**Phase 1: Analysis of Existing Coding Schema**

Before analyzing the data, I reviewed previous studies’ results from the research literature. The results achieved by previous researchers were analyzed for two reasons: to investigate students’ problem-solving behaviors, including studies with deaf students, and to determine similarities and differences in problem-solving behaviors between hearing students and deaf or hard of hearing students. Thus, two major relevant streams of literature on problem-reading and problem-solving were used for the data analysis, to develop the categories for problem-reading and problem-solving behaviors. First, problem-solving behaviors from previous studies included those from Lewis and Mayer (1987); Pape (2003, 2004); Kelly, Lang, Mousley, and Davis (2003) (for deaf students’ problem-solving performance); and Verscaffel (1998). Secondly, during data analysis, problem-reading behaviors for data analysis were drawn from a text processing model of reading (Goldman & Rakestraw, 2000; Greeno & Kintsch, 1985; Hegarty, Mayer, & Monk, 1998) and the relevant studies on language and deaf students (Kelly, 2008; Marschark, 2008).

Initially, I started data coding and analysis with certain preconceived ideas, which were influenced by literature regarding the consistency hypothesis. My perceptions might be influenced by previous relevant studies (Goulding, 1999). Thus, to avoid undue bias, I
had to review relevant literature on reading comprehension and language and deaf students, to understand deaf students’ characteristics and their effects on problem solving, in order to understand similarities and differences in problem-solving behaviors between deaf students and hearing students.

I created theoretical notes based on the previous studies on reading comprehension and word problem solving, so I could start data analysis with theoretical notes. Goulding (1999) stressed the importance of existing theory in terms of data collection and analysis, stating that “Glaser (1978) discussed the role of existing theory and its importance in sensitizing the researcher to the conceptual significance of emerging concepts and categories. Knowledge and theory are inextricably interlinked. This is vital, for without this grounding in extant knowledge, pattern recognition would be limited to the obvious and the superficial, depriving the analyst of the conceptual leverage from which to develop theory (Glaser, 1978)” (p. 6-7).

However, in the second coding analysis, I did not start with preconceived concepts and did not attempt to bring about any hypothesis during data analysis. Glaser and Strauss (1967) argued that researchers who apply grounded theory for their studies should not have pre-conceived ideas when data collection and analysis occurred.

**Phase 2: Open Coding: Analysis of the Transcript Line by Line**

This process began with open coding. Video recordings of a think-aloud protocol were transcribed in this phase, line by line. Analysis began with identification of the key problem-solving behaviors which connect the participant’s description to the experience,
based on the data for this study (success, types of errors, problem-solving approaches, and the number of re-readings). This process was not easy, and was very time- and-labor consuming because I had to translate students’ videotaped sign language into English when transcribing, which involved repeating their think-aloud protocols over and over. This translation is meaning-based.

In the open coding phase, a variety of new problem-solving behaviors displayed by participants emerged from the data: retelling inversions, the number of re-readings, monitoring strategy, the change of language mode, the use of ASL spatial movement, problem-solving behaviors, and some error types (e.g., multiple errors and problem left blank) that did not address in the previous studies.

**Phase 3: Analysis Using the Constant Comparative Method**

Six major problem-solving behaviors were identified in the previous phase. Each problem-solving behavior was divided into a set of sub-categories, which were developed using the constant comparative method (Glaser & Strauss, 1967). The central feature of grounded theory is the constant comparison and analysis of data. Some sub-categories that emerged from the current data did not match existing problem-solving behaviors. Each participant’s response with regard to each problem-solving behavior was compared with others’ responses, by problem type and type of errors. In this phase, seven types of errors were sub-categorized and seven retelling reversals were sub-categorized.

Participants’ computation test scores were also analyzed in this phase, to determine whether students’ difficulties with fraction operations were due to a lack of
understanding fraction concepts or misapplication of fraction problem-solving procedures, and whether the errors made by students were linguistic errors or fraction errors. Figure 3.1 provides definitions of the six problem-solving behaviors present in the data.

1. Problem-solving success was recorded when the answer and all solving steps used for problem solving were correct.

2. Types of errors: incorrect answers were classified into seven sub-categories.
   a. A linguistic error was coded when the participant did not understand a problem, when a computation step indicated in one of the text propositions was incorrectly performed, or when an erroneous arithmetic operation was used due to a lack of understanding of the problem.
   b. A reversal error was coded when the opposite arithmetic operation was performed to solve the problem, such as when a participant subtracted when the correct operation was addition.
   c. A computation error was coded when students made an arithmetic mistake.
   d. A fraction error was coded when students indicated an inability to compute a fraction.
   e. A goal-monitoring error was coded when a computation step of the two procedural steps was omitted.
   f. Multiple error was coded when two or three errors were found in solving the same problem.
   g. Problem left blank was coded when a student read a problem and then indicated and inability to understand and solve the problem and did not attempt to solve the problem.

Fig.3.1. Definitions of the Data Coding
3. The patterns of the problem-solving approach
   a. *Direct translation approach* was recorded when students referred to the numbers and key words without context or transforming the given information, did not address the given information with its context and relationships, solved problems automatically, habitually reread problems without monitoring and context, or failed to address the situation being described in the context of reading.
   b. *Meaning based approach* was recorded when students referred to variables’ relationships and content while formulating a representation of the problem, used the context of the problem using variable names and relational terms to explain elements of the problem, explained with context despite making reversal errors, or explained the situation described in the problem using ASL.

4. Retelling reversal errors were coded when students made reversal errors in the process of explaining their understanding of the word problem after first reading the given word problem.

5. The number of re-readings included each time students re-read the problem to deepen their understanding of the problem following their initial reading.

6. Problem-reading comprehension behaviors were summarized based on reading comprehension behaviors identified as participants read the problems.

Phase 4: Comparison of Results to Previous Studies

Variables were divided into six categories: problem-solving success, seven types
of errors, retelling reversal errors, the number of re-readings, two major problem-solving approaches, and problem-reading comprehension behaviors. The results of this study, specifically types of errors and problem-solving approaches, were compared with results from previous studies and are reported in Chapter 5.

**Scoring Procedures for Written Responses: SRL Strategy Use and Computation Test**

Students’ written responses to the interview questionnaire about SRL strategy was scored and analyzed using descriptive statistical analysis. A computation test was scored. Both scoring procedures are discussed below in more detail.

**SRL learning strategy score:** As described in the previous chapter, learning strategies consisted of four sub-categories, including cognitive strategy, meta-cognitive strategy, help-seeking, and self-efforts. Participants rated themselves using a questionnaire based on a 5-point Likert-type scale, with scores ranging from 1 (not at all true) to 5 (very true). Scores for each strategy were computed by taking the mean response scores of the items relating to the strategy. In addition, some questions (9, 16, 20, 30, and 33) of the scales were negatively worded. Thus, scores for these questions were reversed by subtracting them from 6. An example of one of these questions appears below:

*Question 9: Even when I have a lot of trouble learning math, I don’t ask for help.*

Most students rated 1 for not at all true on question 9, thus this item becomes 5 by subtracting the original score 1 from 6. A score of 2 would then be evaluated as 4, while a 3 would remain a 3.
**Computation Test Scores:** The computation test was composed of 14 total problems that required addition, subtraction, multiplication, and division, and fraction operational skills. The test contained 8 problems that required the understanding of fraction concepts. Each problem scored 2 points if both the solution process steps and answer were correct. If solution steps were omitted but the answer was correct, the problem was scored 1 point. Fraction problems were also in addition to and separate from the basic operations problems. These scores were computed to investigate students’ abilities to solve fraction problem
CHAPTER 4

FINDINGS

The purpose of this study was to explore the way in which deaf middle school students understood and solved word problems, and to examine their strategy use in learning mathematics. A think-aloud protocol was utilized in order to explore the complex cognitive processes of mathematics problem solving; additionally, interviews using a structured questionnaire were conducted to determine strategy use in the participant group. The ultimate goal of the study was to understand why students have difficulties with word problems and what kinds of factors influence their problem-solving success or failures. To achieve this goal, the study was divided into four processes of analyses on the following lists.

The first analysis examined the problem-solving success rate by middle school students. The focus of analyzing success rates was on comparing the number of problems solved correctly by students according to the four elements of problem structures: 1) problem types (consistent language (CL) problems versus. inconsistent language (IL) problems), 2) computation steps (one-step versus. two-step problems), 3) arithmetic operations (addition, subtraction, multiplication, and division), and 4) relational terms (more than, less than, times as many as, and 1/n as many as).

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The second analysis described the patterns of errors committed by deaf middle school students, with a particular focus on reversal errors. The reversal errors were examined during solutions and retellings. The goal of this analysis was to understand: a) why deaf middle school students had more difficulties with IL problems, b) when and why reversal errors occurred, and c) how these errors were related to students’ knowledge of English language and mathematics.

The third analysis examined mathematics knowledge possessed by students in relation to what was required in the word problems. In this section, I investigated four fraction problem-solving performance on the think-aloud protocol and eight fraction problems on the computation test. The performances in both tasks were compared to determine whether the errors on the think-aloud protocol were fraction errors or other errors. The goal of analyzing fraction knowledge was to investigate conceptual and procedural knowledge of fractions possessed by the participating students.

The fourth analysis highlighted problem-solving approaches used by students as they read, understood, and solved from text comprehension and problem-solving models. In this section, I describe problem-reading and problem-solving behaviors used by students as they read and solved the word problems from the text processing model and problem-solving model suggested by Mayer (2008). The focus of this analysis was on exploring how students’ knowledge in reading and mathematics influenced their problem-solving success and problem-solving approaches. Finally, I briefly describe specific and unique problem-reading behaviors of deaf students that emerged from the data.

To achieve these goals, video recordings of the think-aloud protocol were
transcribed and examined utilizing a systematic procedure of data analysis introduced by Strauss and Corbin (1998). Constant comparative methods were used to explore problem-solving behaviors and issues emerging from the data, as well as to investigate the similarities and differences in problem-solving behaviors among participants.

Finally, this chapter presents the study’s findings in five sections: problem-solving success, error-type including retelling protocols, fractional knowledge, problem-solving approaches, and strategy use.

**Problem-Solving Success**

Twelve target problems consisted of four elements of the problem structures: problem types (consistent language problem vs inconsistent language problems), computation steps (one-step vs two-step problems), arithmetic operations (addition, subtraction, multiplication, and division), relational terms (more than, less than, times as many as, and n/1 as many as). Before investigating students’ problem-solving success rate, I analyzed the elements of twelve target word problems: 1) to explore the relative difficulty of completing word problems in participants and 2) to examine how students interacted with these problem structures.

To achieve these goals, problem-solving success was investigated by the number of problems solved correctly by students according to the four elements of problems. Finally, some case analyses were conducted to examine how students interact with these word problems. As mentioned in the previous chapter, the target problems included four structural components.
1) **Problem Type:** six CL problems and six IL problems

2) **Computation Steps:** four one-step problems and eight two-step problems

3) **Arithmetic Operations:** two addition, two subtraction, four multiplication, or four division operation problems

4) **Relational Terms:** more, less, times as many as (multiples for non-fractional problems), and 1/n as many as (fractional problems).

Results related to problem-solving success by the four elements of word problem structures are discussed below in more detail.

The thirteen deaf middle school students successfully solved a range of one to seven of the 12 target word problems (i.e., 6 CL and 6 IL problems). As shown in Table 4.1, each student solved all of the CL problems, with a range of one to six answered correctly; only 6 students (Student 1, 3, 5, 6, 8, 12) solved one or two of the 6 IL problems; the remaining seven students (student 2, 4, 7, 9, 10, 11, & 13) did not solve any IL problems. Interestingly, two students (student 1 & 5) who used ASL as their first language solved all six CL problems correctly and only one IL problem correctly. Student 6, who used English-based phonology, solved two IL problems and his problem-reading and- solving behaviors were different from those of other students. The students’ problem-reading and solving behaviors will be discussed in more detail in the last section of this chapter. Table 4.1 presents the number of problems correctly solved for CL and IL problems.
The number of CL success | The number of IL success | Total success
---|---|---
Student1 | 6 | 1 | 7
Student2 | 2 | 0 | 2
Student3 | 5 | 1 | 6
Student4 | 1 | 0 | 1
Student5 | 6 | 1 | 7
Student6 | 5 | 2 | 7
Student7 | 3 | 0 | 3
Student8 | 2 | 1 | 3
Student9 | 3 | 0 | 3
Student10 | 4 | 0 | 4
Student11 | 4 | 0 | 4
Student12 | 4 | 1 | 5
Student13 | 5 | 0 | 5

Table 4.1. Each Participating Student’s Problem-Solving Success

1) **Problem Type.** The thirteen students solved a total of 151 word *compare* problems for six CL and six IL problems (12 problems x 13 students - 5 blank = 151 problem solutions). As shown in Table 4.2, participants correctly solved 50 CL problems ($M = 3.85$, $SD=1.57$)) and 7 IL problems ($M= 0.54$, $SD=.66$). Students predominantly solved CL problems more accurately than IL problems as hearing students in the previous
studies (Lewis & Mayer, 1987; Pape, 2003; Versaffel, 1994; Versaffel et al, 1992). The reasons of the difficulties in relation to IL problem-solving will be discussed in the section of types of errors. Table 4.2 presents the number (percentage) of problem-solving success for CL and IL problems.

<table>
<thead>
<tr>
<th>Problem-Solving Success</th>
<th>CL (6 problems)</th>
<th>IL (6 problems)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50 (87.7%)</td>
<td>7 (12.3%)</td>
</tr>
</tbody>
</table>

Table 4.2 The Number (Percentage) of Problem-Solving Success for CL and IL problems

2) Computation Steps. Since all four addition and subtraction problems were two-step problems and there are no addition and subtraction one-step problems, problem-solving success rate of one-step multiplication and division problems was compared only to that of two-step multiplication and division problems. The results indicated a lower problem-solving success rate for two-step multiplication and division problems (N=16, M=1.23, SD=.73) than the equivalent one-step multiplication and division problems (N=24, M=1.85, SD=.80). Table 4.3 presents the percentage of problem-solving success for one-step and two-step word problems.
Table 4.3 The Number (Percentage) of Problem-Solving Success for Computation Steps

As shown in Table 4.3, some students are more likely to have difficulty with two-step word problems than one-step word problems. The reasons of difficulties are related to the types of problems (CL vs. IL problems) and mathematical knowledge (in this case, fractional knowledge). As shown in Table 4.4, one-step CL (M=1.54) and IL problems (M=.31) were solved more correctly than two-step CL (M=1.23) and IL problems (M=.0). Interestingly, no one solved two-step IL problems.

Table 4.4. The Number of CL and IL Problem-Solving Success by Computation Steps

As shown in Table 4.5, fewer two-step word problems with n/1 relational terms (M=0.38, SD=.51) were correctly solved than one-step word problems with n/1 relational terms.
relational terms ($M= 0.77$, $SD=0.60$). Therefore, the success of two-step problem-solving seemed to be influenced by students’ fractional knowledge and problem structure knowledge. Pape (2003) argued that students have difficulty with making inferences of two-step word problems because of the cognitive demands of understanding the problems.

<table>
<thead>
<tr>
<th>computation step problem types</th>
<th>One-step</th>
<th>Two-step</th>
</tr>
</thead>
<tbody>
<tr>
<td>problem-solving success</td>
<td>fraction</td>
<td>non-fraction</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>Total</td>
<td>24</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 4.5. The Number of Fraction and Non-Fraction Problem-Solving Success by Computation Steps.

Additionally, case analysis was conducted to understand why these students tend to have more difficulty in solving two-step word problems than one-step word problems. In this study, students tend to omit the last question due to the complexity of linguistics or mathematics knowledge. They took longer to read two-step problems. The following two examples illustrate the most common errors committed by students while solving two-step problems.

**Example 1**

CL Problem 13: Kroger sells 50 pounds of potatoes a day. Meijer sells $1/5$ as many potatoes as Kroger does in a day. How many pounds of potatoes does Meijer sell in 4 days?
**Student 9:** Kroger sells 50 pounds of potatoes but Meijer sells 1/5 less potatoes … Kroger sells more…The answer is 10… Meijer sells 10 pounds of potatoes.

In this case, the student appeared to ignore the final problem question in the attempt of solving problems, focused on the initial fact provided about Meijer and then immediately solved.

**Example 2**

CL Problem 9: At Speedway, gas sells for $1.13 per gallon. Gas at BP is 5 cents less per gallon than gas at Speedway. How much does 5 gallons of gas cost at BP?

**Student 8:** At speedway gas sells for 1.13 per gallon. Gas at BP is 5 cents cheaper per gallon than... It means subtraction… 5 subtract from 1.13… still thinking about something… How much does 5 gallons of gas cost at BP”?…

**Researcher:** What are you doing?

**Student 8:** 5 cents less and 5 gallons… It means 10 (5 cents add to 5 gallons) subtract from 1.13 because … 5 cents less…5 gallons...

**Researcher:** Why did you add here?

**Student 8:** I don’t know what they mean, just guessed

**Researcher:** Can you tell me what you didn’t understand?

**Student 8:** yes, I do not understand these words.

Student 8 underlined the words “ $ 1.13 per gallon” in the first statement, “5 cents less than per gallon” in the second statement, and “5 gallons” in the third statement. She was thinking and looked at the problem again and thought so I had to ask her to remind

**Researcher:** What are you doing now?

**Student 8:** The answer is 1.13 minus 0.05=1.08

**Researcher:** Why did you subtract 0.05 from 1.13?

**Student 8:** Because here says 5 cents less, less means subtraction

This problem is a two-step problem that requires two-step computations. To solve it,
students must understand the different relationships of quantities (i.e., $1.13 per gallon, 5 cents less per gallon, and 5 gallons of gas) and sets in the problem ($1.13 per gallon at Speedway and 5 cents less per gallon at BP than at Speedway). Student 8 seemed not to understand, and ultimately ignored, the final problem solution because she seemed to confuse the units of gas and the units of money, causing a problem with “5 cents less than” and “5 gallons.” Finally, she did subtract 5 cents to get 1.08 as her answer. She obviously got confused with the last question of the problem, I assume because she did not understand the problem, and had difficulty integrating and interpreting information from the second and last questions. Finally, she attempted to solve it focusing only on numbers, and obtained an incorrect answer. This may be because she was focusing on only numbers and key words without completely understanding the problem as she read it and attempted to solve the problem. Since unlike one-step problems, two-step word problems require one more computational step for the last question, students can have more difficulty in obtaining the right answers (Carpenter, Corbitt, Kepner, Lindquist, & Reyes, 1980). For some participants in this study, two-step word problems seemed to require more cognitive demands than one-step word problems due to the complexity of problem structures and mathematical knowledge (Pape, 2003). Fig 4.1 presents a sample of two-step word problem solved incorrectly by student 9. A sample of her written response for this problem.
3) Problem-Solving Success for the Four Arithmetic Operations. The twelve target problems consisted of two addition, two subtraction, four multiplication, and four division problems. To examine whether there is an influence of arithmetic operations skills, the problem-solving success rate for four arithmetic operations was investigated. As shown in Table 4.6, the students were more successful at solving addition (15.8%) than subtraction (14.0%) and more successful at multiplication problems (43.9%) than division problems (26.3%).

<table>
<thead>
<tr>
<th>Problem-solving Success</th>
<th>Addition (2 problems)</th>
<th>Subtraction (2 problems)</th>
<th>Multiplication (4 problems)</th>
<th>Division (4 problems)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9 (15.8%)</td>
<td>8 (14.0%)</td>
<td>25 (43.9%)</td>
<td>15 (26.3%)</td>
</tr>
</tbody>
</table>

Table 4.6 The Number (Percentage) of Problem-Solving Success for Four Arithmetic Operations
4) Problem-Solving Success for the Four Relational Terms. The twelve word problems were divided into the use of four relational terms: *more, less, times as many as,* and *1/n as many as.* The results indicated that participants solved fewer problems with the “less” relational term correctly (M= .38, SD= .51), than problems with the “more” relational term (M= .92, SD= .76), as illustrated in Table 4.7. Similarly, problems with the “times as many as” relational term that did not require the understanding of the fraction knowledge were more frequently solved correctly than problems with the “1/n as many as” relational term that required the understanding of the fraction knowledge. It appears that fractional knowledge possessed by students influenced deaf middle school students’ problem-solving success. The difficulties that students encountered when solving fractional problems will be discussed in the section of the fraction errors in more detail.

<table>
<thead>
<tr>
<th>Relational Terms</th>
<th>More (2 problems)</th>
<th>Less (2 problems)</th>
<th>As many as (4 problems)</th>
<th>Fractional As many as (4 problems)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem-Solving Success</td>
<td>12 (21.1%)</td>
<td>5 (8.8%)</td>
<td>25 (43.9%)</td>
<td>15 (26.3%)</td>
</tr>
</tbody>
</table>

Table 4.7. The Number (Percent) of Problem-Solving Success for Four Relational Terms

In summary, further analysis indicated that deaf middle school students performed relatively better with CL problems than IL problems, 1-step problems than 2-step problems, multiplication problems than division problems, problems with “more” relational terms than “less” relational terms, and problems with non-fraction-a-number of
relation terms than fraction-a-number-of the relational terms. The types of errors made by
the students in solving each problem were evaluated to understand why the students had
more difficulty with certain problem types than others. In next section, the types of errors
will be discussed in more detail.

Types of Errors

In this section, types of errors were investigated. Transcribed notes of video
recordings of think-aloud protocols, written responses on the think-aloud protocol, and
written responses on computation tests were analyzed and integrated into categories. The
goal here was to understand and describe what types of errors were committed by
students and why students had difficulties in solving these word problems. Seven types
of errors were identified in this study: reversal errors, multiple errors, linguistic errors,
fraction errors, computation errors, goal monitoring errors, and problems left blank.
Descriptive analysis and case analysis were conducted.

Amongst the 156 completed written solutions, including 5 problems left blank,
the 13 participants made a total of 99 errors on CL and IL problems. A variety of error
types were present in the data. The most common error was reversal (41.4%), followed
by multiple error combinations (21.2%), linguistic errors (16.2%), fraction errors (13.1%),
problems left blank (5.1%), computation errors (2.0%), and goal monitoring errors
(1.0%), respectively (see Table 4.8).

Furthermore, the students also showed clear differences in types of errors
produced between CL problems and IL problems. In CL problems, the three most
common errors were multiple errors (n=8, 28.6%), linguistic errors (n=7, 25%), and fraction errors (n=6, 21.4%). In contrast, the most common errors on IL problems were reversal errors (n=40, 56.3%), multiple errors (n=13, 18.3%), and linguistic errors (n=9, 12.7%). The frequency and prevalence of each error for both CL and IL problems are presented in Table 4.8.

<table>
<thead>
<tr>
<th>Error Type</th>
<th>CL (6 problems)</th>
<th>IL (6 problems)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linguistic Error</td>
<td>7 (25%)</td>
<td>9 (12.7%)</td>
<td>16 (16.2%)</td>
</tr>
<tr>
<td>Reversal Error</td>
<td>1 (3.6%)</td>
<td>40 (56.3%)</td>
<td>41 (41.4%)</td>
</tr>
<tr>
<td>Computation Error</td>
<td>2 (7.1%)</td>
<td>0</td>
<td>2 (2.0%)</td>
</tr>
<tr>
<td>Fraction Error</td>
<td>6 (21.4%)</td>
<td>7 (9.9%)</td>
<td>13 (13.1%)</td>
</tr>
<tr>
<td>Blank</td>
<td>3 (10.7%)</td>
<td>2 (2.8%)</td>
<td>5 (5.1%)</td>
</tr>
<tr>
<td>Goal monitoring Error</td>
<td>1 (3.6%)</td>
<td>0</td>
<td>1 (1.0%)</td>
</tr>
<tr>
<td>Multiple Error</td>
<td>8 (28.6%)</td>
<td>13 (18.3%)</td>
<td>21 (21.2%)</td>
</tr>
<tr>
<td>No Errors</td>
<td>50 (87.7%)</td>
<td>7 (12.3%)</td>
<td>57</td>
</tr>
<tr>
<td>Total</td>
<td>28 (28.3%)</td>
<td>71 (71.7%)</td>
<td>99</td>
</tr>
</tbody>
</table>

Table 4.8 The Number (Percentage) of Error Types for CL and IL Problems Committed by Students

Reversal errors were examined in both solving and retelling steps to determine
whether a student understood the difference between CL and IL problem structures. Reversal errors were characterized by multiplying or adding the numbers instead of dividing or subtracting them, or vice versa, during solving. As shown in Table 4.8, thirteen students predominantly made errors on IL problems (n=71; 71.7%) and committed more reversal errors on IL problems than on CL problems: Forty IL problem responses (56.3%) resulted in reversal errors. In comparison, only one CL problem response (3.6%) yielded a reversal error. The results indicated that students committed more reversal errors on IL problems (M=3.08, SD=1.50) than on CL problems (M=.08, SD=.27), as shown in Table 4.8. Figure 4.2 presents a sample of the reversal error committed by student 1 during solving. Student 1 solved the IL problem using key word strategy, focused on relational term “more than” in the second problem that led an incorrect answer.

![Image of a sample reversal error committed by student 1 on IL problem during solving.](image)

Figure 4.2. A Sample of Reversal Error Committed by Student 1 on IL Problem during Solving.
Second, reversal errors during retelling were analyzed to investigate the representation process for CL and IL problems after reading the problem and before choosing an arithmetic operation. Students were asked to retell or explain the word problems after their initial reading. When the students did not completely retell the statements made in the problem, additional questions were provided to prompt the students to recall problem details.

Retelling protocols were scored as correct if students’ retellings correctly recounted problem details, if students’ retellings had the same structures as the original problems, or if both the subject and relational terms were inverted correctly. Retelling protocols were scored as incorrect if students inverted the subject and object of the problem statement or inverted the meaning of relational terms used in the problem during retelling. The incidence of retelling a CL problem as an IL problem, and vice versa, was also analyzed. If students immediately attempted to solve the problems without retelling the statements or if students did not understand a given word problem because of their lack of understanding the problem, then it was categorized as no-retelling. Table 4.9 provides an example of CL and IL problems and Table 4.10 presents examples of correct and incorrect retellings for IL subtraction problems.
Table 4. Example of the Relational Statements of CL and IL Problem

| Statements | S1: Mary runs about 6 miles per week. | S1: Joe runs 6 miles a week. |
| S2: Sandy runs 3 times as many miles per week as Mary. | S2: He runs 1/3 as many miles a week as Ken does. |
| S3: How far does Sandy run in a week? | S3: How many miles does Ken run in a week? |

Table 4.9. An Example of the Relational Statements of CL and IL Problem

Problem 6: At Meijer, a pound of sugar costs 89 cents. That is 20 cents more per pound than at Walmart. How much does 5 pounds of sugar cost at Walmart?

<table>
<thead>
<tr>
<th>Correct Retelling</th>
<th>Incorrect Retelling</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Meijer, sugar costs 89 cents. At Walmart a pound of sugar is 20 cents cheaper than at Meijer.</td>
<td>At Meijer, a pound of sugar costs 89 cents. Walmart is 20 cents more per pound than Meijer.</td>
</tr>
<tr>
<td>At Meijer, sugar costs 89 cents. Walmart is 20 cents less per pound than at Meijer.</td>
<td>At Meijer, a pound of sugar costs 89 cents. Walmart is 20 cents more expensive than Meijer.</td>
</tr>
</tbody>
</table>

Table 4.10. Examples of Correct and Incorrect Retellings of an IL Problem

As shown in Table 4.11, in the process of retelling, 47 CL problems were retold correctly, while 9 IL problems were retold correctly. Students retold 33 IL problems as CL problems (n=33) and 2 CL problems as IL problems (n=2). Participants failed to retell 18 CL problems and 17 IL problems by only focusing on numbers and relational terms without referring to context during retelling.
Table 4.1. The Number of Reversal Errors Committed by Students during Retelling

<table>
<thead>
<tr>
<th>Retelling</th>
<th>Correct</th>
<th>Incorrect</th>
<th>No-Retelling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Reversal</td>
<td>Reversal</td>
<td>No Reversal</td>
</tr>
<tr>
<td>CL problems</td>
<td>47</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>IL problems</td>
<td>9</td>
<td>0</td>
<td>17</td>
</tr>
</tbody>
</table>

Case Analysis of Retelling

The following examples demonstrate how students understood and misunderstood CL or IL problems. Seven types of retelling results were identified across all problems in retelling:

CL problems:
1) Correct retelling/incorrect solution
2) Correct retelling/correct solution
3) Incorrect retelling/correct solution

IL problems:
4) Correct retelling/incorrect solution
5) Incorrect retelling/incorrect solution
6) Correct retelling/correct solution

The seventh result was no-retelling of either CL or IL problems. Below, the seven cases of problem retelling will be discussed in more detail using example case analysis in order to understand and describe individual differences in participants.
Retelling of CL Problems

Case 1: Correct Retelling/Incorrect Solution

Problem 9: At Speedway, gas sells for $1.13 per gallon. Gas at BP is 5 cents less per gallon than gas at Speedway. How much does 5 gallons of gas cost at BP?

Student 13: At Speedway, gas sells for $1.13 per gallon. Gas at Speedway is 5 cents more than gas at BP. How much does 5 gallons of gas cost at BP? The answer is $1.18

Although the student retold correctly by inverting relational term “less than” into “more than” and the subject into the object in the second statement, this problem was solved incorrectly during solving. The student seemed to focus solely on relational term and price difference during solving, which attributed to the incorrect answer.

Case 2: Correct Retelling/Correct Solution

Problem 1: The local farm stand sells about 15 watermelons each day during the summer. The Supermarket sells 3 times as many as the farm stand a day. How many watermelons does the Supermarket sell in 5 days?

Student 12: The farm sells 15 watermelons each day. Another food store sells 3 times more than the local farm. Researcher: Can you tell me how you found your answer?
Student 12: 3 times means multiplication, so 15 multiplied by 3 and then multiply it by 5... the answer is 225.

Case 3: Incorrect Retelling/Correct Solution

A student made a reversal error by retelling a CL problem in the IL structure, but solved correctly.

Problem 9: At Speedway, gas sells for 1.13 per gallon. Gas at BP is 5 cents more than at
Speedway. How much does 5 gallons of gas cost at BP?

**Student 11:** BP is 5 cents less than gas at Speedway
At Speedway, gas is 5 cents more than at BP.
5 cents added to $1.13 is $1.18 and then multiplied by 5 is $5.90

The student made a reversal error on CL problem by inverting only the relational terms “more than” into “less than” in the second statement that is not necessary required to invert during retelling. Although the student incorrectly retold CL problems, she correctly solved the problem during solving.

**Retelling of IL Problems**

Case 4: Correct Retelling/Incorrect Solution

Problem 7: Kroger sells 120 bottles of water a day. That is 2 times as many bottles as Giant’s sells in a day. How many bottles of water does Giant’s sell in 5 days?

**Student 13:** Giant sells less bottles of water than Kroger. Kroger sells more bottles of water. 120 multiplied by 2 times and then multiplied it by 5. Answer is 1200

Although Student 13 retold correctly, he did not produce a correct answer. As in CL problem Case #1, he tends to use key word strategy, focusing on relational term and numbers as he attempted to solve this problem, incorrectly evaluating the relationship between the quantities at each store.

Case 5: Incorrect Retelling/Incorrect Solution

Fifty cases were observed where students solved IL problems incorrectly. In all 33 cases that were retold and solved incorrectly, reversal errors took place where students inverted the object into the subject in the second statement during retelling but they did not invert
the relational terms. To successfully solve IL problems, both relational terms and the subject-and-object in the second statement are inverted.

Problem 16: At Kroger, a pound of pears cost $1.16. That is 15 cents less per pound than at Meijer. How much does 5 pounds of pears cost at Meijer?

   Student 3: At Kroger a pound of pears cost $1.16. Meijer is 15 cents less per pound than at Kroger. How much does 5 pounds of pears cost at Meijer? The answer is $5.05

   In his attempt to comprehend the question, the student committed a reversal error that attributed to an incorrect answer. The majority of students who committed such errors tend to prefer CL structures as they read given word problems. This may be due to the use of key word strategy as they read word problems.

Case 6: Correct Retelling/Correct Solution

In the nine out of 76 cases that were retold and solved correctly, the retellings inverted IL problems into the structure of CL problems correctly by inverting the object into the subject in the second statement as well as the relational terms. The following two examples present correct retelling and solution of IL problems produced by two students.

Problem 4: Last year, the sixth grade sold 125 raffle tickets each day. That is 5 times as many tickets as the fifth grade sold per day. How many tickets did the fifth grade students sell in a day?

   Student 6: 125 raffle tickets are 5 times more… more... than the fifth grade.
   Researcher: Which grade sold more tickets?
   Student 6: Sixth grade sold 5 times more tickets than 5th grade, so I think 125 divided by 5 and 5th grade sold 25 tickets.

Problem 6: At Meijer, a pound of sugar costs 89 cents. That is 20 cents more per pound...
than at Walmart. How much does 5 pounds of sugar cost at Walmart?

**Student 5:** At Meijer, a pound of sugar costs 89 cents. But Walmart is 20 cents less per pound than at Meijer… 69 multiplied by 5 = $3.45

Case 7: No-Retelling

Some students did not retell the word problems when they had difficulty in understanding the problems. When the students focused exclusively on numbers during retelling or when they solved the problems without retelling at all, the cases were categorized as no-retelling. This occurred for 18CL problems and 17IL problems.

Problem 8: Mary runs about 6 miles per week. Sandy runs 3 times as many miles per week as Mary. How far does Sandy run in a week?

**Student 2:** Six miles multiplied by 3 is 18 miles.

After the initial reading, student 2 immediately attempted to solve the problem without retelling, focusing entirely on the numbers in the problem.

**Pronoun and Reversal Errors**

In IL problems, the pronouns in the second sentence refer to the person or the place named in the first statement. As shown in Table 4.9, the second question in the IL problems begins with a pronoun (in this case, “he”). In order to solve the IL problem, students have to understand what the pronoun refers to in the second question. If students were not able to relate the pronoun correctly to information from the first statement in the problem, the second probability that the participants would make errors including
reversal errors would increase. Thus, the numbers of pronouns not signed or read by students was elicited through transcripts in order to understand whether students’ reversal errors were related to difficulties with pronouns.

First, the number of read or signed for pronouns in the second statement were investigated during the initial reading. In addition, the relationship between reversal errors and reading behaviors (in this case, the number of non-read pronouns) was investigated to determine whether the reading behaviors are related to reversal errors committed by students. Finally, some case analysis was conducted to explore similarities and differences in problem reading behaviors among participants.

In total, 76 IL problems were solved by the thirteen students. Students did not sign or read second statement pronouns in 29 IL problems (38.2%) (see Table 4.1). It seemed that resolution of pronouns is related to reversal errors on IL problems.

<table>
<thead>
<tr>
<th>Signed for Pronouns</th>
<th>Not Signed for Pronouns</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading in IL</td>
<td>47</td>
<td>29</td>
</tr>
<tr>
<td>Problems</td>
<td>(61.8%)</td>
<td>(38.2%)</td>
</tr>
</tbody>
</table>

Table 4.12 The Number (Percentage) of Signed for and Non-Signed for Pronouns during IL Problem Reading

**Further Analysis on Pronouns Reading**

Problem 6: At Meijer, a pound of sugar costs 89 cents. That is 20 cents more per pound than at Walmart. How much does 5 pounds of sugar cost at Walmart?
**Student 1:** At the store, a pound of sugar costs 89 cents. 20 cents more per pound than at Walmart. How much does 5 pounds of sugar cost at Walmart?

Student 1 omitted the pronoun clause “that is” during the initial reading as shown in his reading above. Some students tend to neglect the words “that is” in the second question of the IL problems while they read IL problems. It was not clear why they did not attend to the pronoun clause “that is” or whether they understood what the pronoun phrase meant in IL problems. Some students were asked to explain what the pronoun in the IL problem referred to. Most students indicated that “the pronoun” means the quantity or numbers stated in the second statement.

In the example above, student 1 was asked what the phrase “that is” meant in the problem after omitting it in their reading.

**Researcher:** Can you tell me what the pronoun “that is” means here?
**Student 1:** I think “that” here means at Walmart, a pound of sugar costs 20 cents more.

In this problem solving, student 1 did not appropriately recognize and use context clues and did not draw inferences from the problem statements. When he was asked to retell the second statement of the problem 6, he committed a reversal error by inverting only the subject and object, which resulted in an erroneous answer.

**Student 1:** At Walmart, a pound of sugar costs 20 cents more per pound than at Meijer.

In addition to reversal errors, fraction errors were one of the most common errors committed by participants. In the next section, incidences of when students made fraction errors and why students have difficulties in solving problems associated with fraction
knowledge will be discussed in more detail.

**Fractional Error Analysis**

In the think-aloud protocols, of twelve target problems, four word problems (problems 11, 12, 13, & 15) were fraction problems containing \( \frac{1}{n} \) as many as relational terms. These four problems consisted of 2 CL problems and 2 IL problems that required multiplication and division operation skills (see Table 4.13).

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>Problem Type</th>
<th>Problem Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>IL</td>
<td>Sam’s Grocery sells 180 eggs a day. That is ( \frac{1}{3} ) as many eggs as Mike’s Grocery sells a day. How many eggs does Mike’s Grocery sell in 3 days?</td>
</tr>
<tr>
<td>12</td>
<td>IL</td>
<td>Joe runs 6 miles a week. He runs ( \frac{1}{3} ) as many miles a week as Ken does. How many miles does Ken run in a week?</td>
</tr>
<tr>
<td>13</td>
<td>CL</td>
<td>Kroger sells 50 pounds of potatoes a day. Meijer sells ( \frac{1}{5} ) as many potatoes as Kroger does in a day. How many pounds of potatoes does Meijer sell in 4 days?</td>
</tr>
<tr>
<td>15</td>
<td>CL</td>
<td>Donatos sells 120 regular pizza pies a day. Pizza Hut sells ( \frac{1}{3} ) as many regular pies as Donatos in a day. How many regular pizza pies does Pizza Hut sell in a day?</td>
</tr>
</tbody>
</table>

Table 4.13. Only 4 Total CL and IL Problems Associated with Fractional Knowledge.

I reviewed three resources to analyze fraction knowledge of all participating students: transcript notes, written responses on think-aloud protocols, and written responses on computation tests. Fraction errors were recorded when:
1) Problem solvers referred to an inability to compute fraction problems

2) Problem solvers reported an inability to understand fraction concepts

3) Problem solvers referred to an indisposition toward fractions

Problem-solving success for the four problems was also analyzed. In order to investigate whether fraction knowledge influences problem-solving success, the number of fraction errors committed by students on these CL and IL problems were examined.

Secondly, problem-solving success of the four IL multiplication and CL division problems with fractional relational terms that required opposite mathematical operations were compared with that of four IL multiplication and CL division problems that did not contain fractional relational terms, but still required opposite mathematical operations. Finally, individual differences in problem-solving behaviors among these participants were analyzed based on the type of fraction error.

Overall, of 48 problem solutions (13 students x 4 problems - 4 problems left blank) that contained fraction-of-a-number relational terms, a total of 13 CL problems and 2 IL problems associated with fraction operations were solved correctly. Students made fractional errors on 6 CL problems and 7 IL problems. Although students solve more CL problems with fraction operations correctly than IL problems with fraction operations, students made equal numbers of errors for each problem type. The errors quantified in Table 4.14 represent instances where students made only fraction errors. Problems solved with other errors in addition to fraction errors were categorized as “multiple errors.”
Second, the problem-solving success rates of problems that contained fractional relational terms were compared with those of problems that did not contain such relational terms. All participants solved fewer IL multiplication and CL division problems (N=15, 37.5%) that contained a fraction-of-a-number relational terms than IL division and CL multiplication problems (N=25, 62.5%) that did not require fraction knowledge, as shown in Table 4.15. It appears that students had more difficulties with problems that required fraction operations than problems that did not include such operations. Accordingly, fraction knowledge appears to be a factor that influenced problem-solving success for participants.

Table 4.14. The Number of Fractional Errors Committed by Participants, and Fractional Problem-Solving Success for CL and IL Problems

<table>
<thead>
<tr>
<th>Fractional Errors</th>
<th>Fractional CL</th>
<th>Fractional IL</th>
<th>Problem-Solving Success</th>
<th>Fractional CL</th>
<th>Fractional IL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Problems (% of total)</td>
<td>6 (46.2%)</td>
<td>7 (53.8%)</td>
<td>13 (86.7%)</td>
<td>2 (13.3%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.15. The Number of Fraction vs. Non-Fraction Problems Solved Correctly by Participants
Further Analysis on Fraction Errors

The reasons for difficulties with fraction problems varied among participants. As mentioned above, this study indicated that the majority of students who participated in this study had difficulties in solving problems involving fractional knowledge. Some students showed a lack of conceptual understanding of fractions, whereas other students had a lack of procedural knowledge of fractions. Other students reported their indispositions related to fraction problems in their attempt to solve the problems. In the following examples, I highlight some of students’ common misconceptions of fractions, their lack of procedural knowledge of fractions, and their indispositions related to problems associated with fractional knowledge.

Example 1: Lack of Conceptual Fraction Knowledge:

IL Problem 11: Sam’s Grocery sells 180 eggs a day. That is 1/3 as many eggs as Mike’s Grocery sells a day. How many eggs does Mike’s Grocery sell in 3 days?

Student 9 possessed misconceptions of fractional operations.

Student 9: I think Mike’s store sells less and Sam’s store sells more.
Researcher: Why do you think Mike’s store sells less and Sam’s store sells more?
Student 9: Because 1/3 makes anything smaller…1/3 means the whole divided by 3… 180 divided by 3 is 60
Researcher: Why did you divide here?
Student 9: Because 1/3 means divide by 3.

It was apparent that the student misunderstood the concept of fraction, indicating that fraction makes numbers and quantities smaller. When she solved this problem, she focused only on fraction number and did not attempt to understand the situation described
in the IL problem. In this problem, she did not recognize that a fraction operation can increase a number by inverting it and then multiplying. This study demonstrated that some students had misconceptions about fractions, indicating that a fraction always makes numbers or quantities smaller. These students seemed to misinterpret fraction concepts because it seemed that they were not exposed to a variety of problems that have different types of problem structures associated with fraction operations.

**Example 2: Lack of Procedural Fraction Knowledge**

Problem 11: Sam’s Grocery sells 180 eggs a day. That is $\frac{1}{3}$ as many eggs as Mike’s Grocery sells a day. How many eggs does Mike’s Grocery sell in 3 days?

**Student 12:** Sam’s store sells 180 eggs a day... $\frac{1}{3}$ more... $\frac{1}{3}$ more... Mike’s store sells more... Mike’s store sells more eggs than Sam’s store... $\frac{1}{3}$ more eggs... Sam’s store sells 180 eggs.

When she initially read the IL problem, she reread the second statement several times to get the meaning of the second statement, focusing only on the fraction number, yet she could not solve the problem due to lack of procedural knowledge of fraction. After reading and rereading, the student would pause and think, but not progress in solving the problem. Thus, I had to ask her what she was doing.

**Researcher:** What are you doing now?

**Student 12:** I don’t know how to change $\frac{1}{3}$ into ... maybe $\frac{1}{3}$ equal 1.3... 180 multiply by 1.3 is 234

After solving, I asked her why she multiplied 180 by 1.3 to see if this would make her recall how to compute fractions.

**Researcher:** Can you tell me why you multiplied 180 by 1.3 here?
**Student 12**: I don’t know because I just guessed 1/3 is 1.3, so I just simply changed 1/3 into 1.3.

**Researcher**: Have you learned how to compute fractions before?

**Student 12**: Yeah, but I don’t know about how to compute 1/3. I forgot it.

The following pictures showed her computation steps for IL problem with 1/n relational term on the think-aloud protocol (See Fig.4.3).

11. Sam’s Grocery sells 180 eggs a day. That is 1/3 as many eggs as Mike’s Grocery sells a day. How many eggs does Mike’s Grocery sell in 3 days?

![Image of computation steps](image.png)

Fig.4.3. A Sample of the Fraction Error Committed by Student 6 during Solving

When student 12 was asked to solve this problem, she struggled. In her attempt to solve this problem, she often said that she didn’t know how to compute fractions, as evidenced by both her inability to solve problems using fractional knowledge and by the lack of understanding fraction concepts found in her computation test. Thus, it took longer for her to read and solve the problem than other IL problems that did not include fraction-of-a-number relational terms. In her attempt to solve the problem, it was apparent that her confusion of the procedural knowledge of fractions and decimals would
impede her in solving the problem. Another example presents how a lack of procedural knowledge of fraction influenced a student’s reading behaviors and motivation. Figure 4.4 provides a sample of fraction conceptual and procedural errors made by student 12 on the computation.

Fig. 4.4. A Sample of the Fraction Computation Errors Committed by Student 12 on the Computation Test

**Example 3: Lack of Procedural Fraction Knowledge**

Problem 11: Sam’s Grocery sells 180 eggs a day. That is $\frac{1}{3}$ as many eggs as Mike’s Grocery sells a day. How many eggs does Mike’s Grocery sell in 3 days?

After reading the second sentence, the student stopped for a while, thinking about
something.

**Researcher:** What are you doing right now?

**Student 6:** Food store sells 180 eggs a day... I understand that... That is 1/3 as many eggs as Mike’s Grocery sells a day...

**Researcher:** What are you doing? Can you tell me what you are thinking?

**Student 6:** I am not good at fractions... I am not good at... That is 1/3 as many eggs as Mike’s grocery sells a day...

1/3 means... divide?...I am confused with the second statement.

**Researcher:** Have you learned how to compute fractions?

**Student 6:** I learned fractions before but forgot... I think 180 multiplied by 3 is 540.

Student 6 omitted and ignored the second statement of IL problem because he did not know how to compute fraction operations and solved by multiplying 180 eggs by 3 days.

Fig 4.5 provides his written response to IL problem with 1/n relational term.

![Image](11.%20Sam%27s%20Grocery%20sells%20180%20eggs%20a%20day.%20That%20is%201%2F3%20as%20many%20eggs%20as%20Mike%27s%20Grocery%20sells%20a%20day.%20How%20many%20eggs%20does%20Mike%27s%20Grocery%20sell%20in%203%20days?)

Fig. 4.5. A Sample of the Fraction Error Made by Student 6 on the Think-Aloud Protocol

Student 6 solved only one of four problems with fractional relational terms, even though he understood and solved previous IL problems not requiring fractional knowledge. Part of his confusion could be located in both his concept of fractions and procedural knowledge of fractions. In his attempt to solve this problem, he ignored the second statement and moved on to the third sentence, focusing only on the numbers, and
multiplied 180 eggs by 3 days. Despite having apparently sound reading comprehension abilities, the student struggled with fraction operations, which influenced his ability to read, understand, and solve the problem.

In a computation test, the student solved fewer fraction problems and solved incorrectly (see Fig. 4.6). As shown in Fig. 4.6, he had difficulty with how to computation fractions.

6. Draw a picture to illustrate the following examples:
   a) \(\frac{3}{5} \times \frac{2}{5}\) (three times two-fifths)

   ![Fraction multiplication example]

   \(\frac{1}{4} \times 16\) (one-fourth of sixteen)

   ![Fraction multiplication example]

Fig. 4.6. A Sample of Fraction Computation Error Made by Student 6 on the Computation Test

In addition, his reading behaviors with this problem were different from those for problems that did not contain fractional problems. For example, when he read the second statement, he read this statement using signed English in English-order and did not change into his own words or did not use ASL spatial movement to represent the two places or two people described in the problems. In reading other problems, he used ASL spatial movements to represent the two stores and represented the relations of quantities
and variables in his own words. He often addressed his inability to solve the problems by saying “I am not good at fraction operations.” He understood his difficulties with this problem and what was needed to solve it.

In addition to a lack of conceptual and procedural knowledge of fractions, problem-solving success or failure on fractional problems is related to students’ motivation toward fraction problems. Before or after reading word problems, some students often said “I hate fractions,” “I don’t like fractions or mathematics,” “I am not good at fractions,” or “I don’t know how to compute fraction problems.” The following examples present how students’ motivation toward fractions influenced their problem-solving success or failure on fraction problems.

Example 4: Indisposition Toward Fractions

Problem 13: Kroger sells 50 pounds of potatoes a day. Meijer sells 1/5 as many potatoes as Kroger does in a day. How many pounds of potatoes does Meijer sell in 4 days?

After the initial reading of problem 11, Student # 7 attempted to reread to understand the problem better.

**Student 7:** Kroger sells 50 pounds of potatoes a day and 1/5 potatoes in a day as many Meijer does in a day.

She stopped and thought about something.

**Researcher:** What are you doing right now?

**Student 7:** I hate fractions…

The answer is 250 because Kroger sells 50 pounds of potatoes and in 4 days, 50 multiplied by 4=200…50 pounds…1,2,3,4,5… 50 multiplied by 5=250
I am not confident with fraction operations.

The example shows that the student’s motivation influences their problem-solving success. In the case above, when the student encountered a more difficult problem, she reported she is not confident with solving the problem.

As discussed above, participants’ problem-solving behaviors associated with fractions indicated a lack of conceptual understanding of fractions as well as a lack of procedural understanding of factions. Misconceptions about fractions can interrupt the learning process, and students should experience a variety of fraction problems that require different arithmetic operations to correct their misinterpretations of fractions.

**Lexically Unmarked Versus Marked Relational Terms**

Previous studies indicated that IL addition and multiplication problems may be more difficult to solve than IL subtraction and division problems because marked terms in IL addition and multiplication problems (e.g., less than and 1/n as many as) are more prominent and influence students more than unmarked terms in IL subtraction and division problems (e.g., more than and n times as many) (Lewis & Mayer, 1987; Pape, 2003). To further examine the influence of lexically marked terms, students’ solutions on marked IL addition and multiplication problems were compared with unmarked IL subtraction and division problems with regard to reversal errors. Table 4.16 presents marked IL addition and multiplication problems and unmarked IL subtraction and division problems. Unmarked terms like *good* and *more* that are positive terms are stored in easier form in memory because these terms tend to use more frequently than marked
terms, whereas marked terms such as *bad* and *less* are stored in a more complex way because marked term derives its meaning in terms of the first term (i.e., unmarked term).

<table>
<thead>
<tr>
<th>Problem Type</th>
<th>Marked Status</th>
<th>Arithmetic Operation</th>
<th>Problem Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>IL Problem</td>
<td>Marked</td>
<td>Addition</td>
<td>At Kroger, a pound of pears cost $1.16. That is 15 cents less per pound than at Meijer. How much does 5 pounds of pears cost at Meijer?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Multiplication</td>
<td>Sam’s Grocery sells 180 eggs a day. That is 1/3 as many eggs as Mike’s Grocery sells a day. How many eggs does Mike’s Grocery sell in 3 days?</td>
</tr>
<tr>
<td>IL Problems</td>
<td>Unmarked</td>
<td>Subtraction</td>
<td>At Meijer, a pound of sugar costs 89 cents. That is 20 cents more per pound than at Walmart. How much do 5 pounds of sugar cost at Walmart?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Division</td>
<td>Kroger sells 120 bottles of water a day. That is 2 times as many bottles as Giant’s sells in a day. How many bottles of water does Giant’s sell in 5 days?</td>
</tr>
</tbody>
</table>

Table 4.16 An Example of Unmarked IL Problems and Marked IL Problems

Overall, of 76 IL problem solutions except problems left blank, 5 IL subtraction and division problems with unmarked relational terms were solved correctly, whereas only 2 IL addition and multiplication problems that included marked relational terms were solved correctly.

To examine the influence of lexically marked relational terms, the number of reversal errors between IL addition and multiplication problems and IL subtraction and division problems were investigated. Results indicated that thirteen students committed more reversal errors on 23 IL subtraction and division problem solutions (M=1.77; SD=.83) than on 17 IL addition and multiplication problem solutions (M=1.31; SD=.95).
As shown in Table 4.17, the number of a total of errors between IL marked problems and IL unmarked problems were approximately similar. As mentioned above, there is no influence of lexically marked relational terms on reversal errors that seemed to be related to the problems associated with fraction knowledge. Table 4.17 provides the number of problems correctly solved and incorrectly solved and reversal errors on IL marked and unmarked problems.

<table>
<thead>
<tr>
<th>Problem Types</th>
<th>IL Marked Addition and Multiplication IL Problems</th>
<th>IL Unmarked Subtraction and Division IL Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem-Solving Success</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Reversal Error</td>
<td>17</td>
<td>23</td>
</tr>
<tr>
<td>The Number of Total Errors</td>
<td>35</td>
<td>34</td>
</tr>
</tbody>
</table>

Table 4.17. The Number of IL Marked and Unmarked Problems Solved Correctly and Incorrectly and Reversal Errors on These Problems

Generally, children are more successful on addition problems and less successful on division problems. However, it is interesting that deaf middle school students solved more CL division problems (M=1.0; SD=.82) than IL addition problems (M=0.0; SD=.0). It is surprising that no one solved IL addition problems in this study, as presented in Table 4.18. This finding illustrates that students have more difficulty in solving IL problems with lexically marked terms than CL division problems. However, it is not clear that they could not solve IL addition problems that required opposite arithmetic operation.
“subtraction.” As one of the reasons for the error, it may be argued that the use of key-word strategy to the IL problem may result in an incorrect answer. Furthermore, the result from this study demonstrates that students’ difficulties with word problems are not in the inability to compute problems, but in the inability to understand word problems. Understanding of problems can contribute to problem-solving success but is not sufficient for problem-solving success.

<table>
<thead>
<tr>
<th></th>
<th>CL Division</th>
<th>IL Addition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem-Solving Success</td>
<td>13</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.18 The Number of Problem-Solving Success for CL Division and IL Addition

In the next section, I will discuss how students approached given word problems as they read, understood, and solved word problems from an integrated theory of text processing and mathematics knowledge in more detail.

**Problem-Solving Approach Analysis**

In this section, I examined participants’ problem-solving approaches as they read, comprehended, and retold given word problems, specifically from a text processing of reading. Reading comprehension is impacted by both text factors (i.e., syntax and vocabulary knowledge) and knowledge issues (i.e., prior knowledge). The model of reading is well-suited to analyzing and understanding deaf students’ reading difficulties in
problem processing.

The emphasis of the analysis was on describing the process and behaviors of reading using an interactive model of reading for the participant group. The data revealed that students exhibited many of the problem-solving behaviors identified in the reading model. However, their problem-solving behaviors varied for different reasons, depending on the complexity of problems and the degree of problem-solving difficulty they encountered. These problem-solving behaviors were categorized by the types of understanding and reasoning processes employed by the students as they read the word problems.

Two major problem-solving approaches were elicited through transcripts of the video recordings of the students’ verbalizations. These approaches based on the findings of previous studies and new problem-solving behaviors emerged from the data for this study. In the next session, two major problem-solving behaviors will be discussed in more detail: a Direct Translation Approach and a Meaning-Based Approach (Hegarty, Mayer, & Monk, 1992; Pape, 2004; Verscaffel, 1998).

**Problem-Solving Approaches**

As mentioned in the previous chapter, 7 problem-solving sub-behaviors and 4 problem-solving sub-behaviors were categorized as either a direct translation approach or a meaning-based approach.

Direct translation approach (DTA) was reported when students focused on key words and numbers without understanding of the situation presented in the problem or
when students automatically solved problems without retelling. In contrast, a meaning-based approach (MBA) was recorded when students understood problem situations and when they provided the justification and explanation of the relations among variables, quantities, and relational terms described in the problems. The following are examples of situations recorded as direct translation approach (DTA):

1) Students automatically solved the problems without rereading the problems after the initial reading.
2) Students simply referred to key relational terms and numbers without defining the relations between numbers and variables. For example, students simply said that “more” means addition and “less” means subtraction.
3) Students frequently reread problems without monitoring.
4) Students omitted one of the computation steps for problem solution or failed to understand the units.
5) Students read given problems in English word order without any transformative behaviors, regardless of whether it was the initial reading, rereading, or retelling.
6) Students did not understand the problems or mentioned an IL problem resembled another CL problem and the same method was used to solve both problem types.
7) Students referred to an inability to understand word problems that are associated with fraction problems.

In contrast, a meaning-based approach (MBA) was recorded when:
1) Students explained problem situations described in the problem with correct context.

2) Students showed understanding of the relations of variables, quantity and relational terms.

3) Students provided evidence of transformative behaviors. For example, students effectively translated English into their own language ASL to represent problem situations during the initial reading or retelling.

4) Students provided explanation and justification for each computation step using correct context.

Finally, if students did not attempt to solve or left problems blank, these problem-solving behaviors were categorized as no solving approach. As illustrated above, twelve problem-solving behaviors in participants were categorized and integrated into two major problem-solving approaches. The number of problem-solving approaches for each problem used by each participant was counted.

Of the 151 problems, a direct translation approach (DTA) was used for 134 (88.7%) of problems. In contrast, a meaning-based approach (MBA) was used for 17 (11.3%) of the problems. Most participating students tend to use a DTA problem-solving approach as their predominant problem-solving behavior as they read and solved the problems, regardless of the types of problem. More specifically, students used a MBA approach for 9 CL problems (6.0%) and 8 IL problems (5.3%), whereas they used a DTA approach for 66 CL problems (43.7%) and 68 IL problems (45%). Table 4.19 presents the number of problem-solving approaches used for CL and IL problems.
Table 4.19 The Number (Percentage) of Problem-Solving Approaches for CL and IL Problems Used by Participants

<table>
<thead>
<tr>
<th>Problem-Solving Approach</th>
<th>CL Problems</th>
<th></th>
<th>IL Problems</th>
<th></th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MBA</td>
<td>DTA</td>
<td>MBA</td>
<td>DTA</td>
<td>MBA</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>66</td>
<td>8</td>
<td>68</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>(6.0%)</td>
<td>(43.7%)</td>
<td>(5.3%)</td>
<td>(45%)</td>
<td>(11.3%)</td>
</tr>
</tbody>
</table>

When I observed video recordings of the think-aloud protocol for students’ problem-solving behaviors for each problem, their problem-solving behaviors for each problem were diverse. Most students did change their problem-solving behaviors depending on the degree of the complexity of problems or relevant knowledge they possessed. For example, some students used a bottom-up processing strategy that relies heavily on the linguistic features of the problems when given more complex word problems such as problems associated with the fraction of a number of relational terms or IL problems. In contrast, more accurate problem solvers used a top-down processing strategy that depends on their prior knowledge or domain knowledge with easier problems. In this study, middle school students seemed to have more difficulties with problems associated with fraction knowledge because of the lack of fraction knowledge. Thus, problem-solving approaches used by the participants as they solved four fractional CL and IL problems were investigated to determine whether fraction knowledge influenced their problem-solving approaches.

Of 52 fractional problems, 48 fraction CL and IL problems were solved by the students (problems 11, 12, 13, and 15). Results indicated that students used more DTA
problem-solving approaches (n=46, 95.6%) on fractional problems than on non-fractional problems (n=44, 84.6%). In contrast, participants used more MBA approaches on non-fractional problems (n=8, 15.4%) than on fractional problems (n=2, 4.4%). There are some differences in problem-solving behaviors in these participants. It is interesting to note that participants tended to use MBA problem-solving approaches with easier problems but DTA problem-solving approaches with more difficult problems. However, the result is not consistent with previous studies involving hearing students (Hegarty, Mayer, & Monk, 1992; Pape, 2004; Verschaffel, 1994). It appears that, regardless of the types of problems, students tend to use DTA approach for problem-solving as they read and solved given word problems. Table 4.20 presents the number of two major problem-solving approaches used by students for CL and IL problems. I will discuss this issue in Chapter 5 in more detail.

<table>
<thead>
<tr>
<th>Problem-Solving Approaches</th>
<th>Fractional Problems</th>
<th>Non-Fractional Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MBA</td>
<td>DTA</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>(4.4%)</td>
<td>(95.6%)</td>
</tr>
</tbody>
</table>

Table 4.20. The Number (Percentage) of Problem-Solving Approaches for Fractional Problems versus Non-Fractional Problems

Students’ problem-solving approaches varied depending on their knowledge of fraction concepts, reading comprehension, or inference-making ability. Student # 5 was the only student who used a MBA approach on 4 problems (for 3 CL problems and for 1
IL problem), 2 students (student # 6 & 12) used a MBA approach for 3 problems, and the remainder of the students who used a MBA approach did so for 1-2 problems. Student # 6 was the only student who used English-based phonology coding when he read word problems. He can speak in English and is very fluent in sign language; however, he has limited fraction knowledge. His reading and solving behaviors are discussed in more detail later in the chapter (See Table 4.21).

Four students (# 2, 4, 10, and 11) did not use any MBA approaches while solving the word problems. These students lag behind in reading and mathematics achievement scores as well as language fluency compared to the other students who participated in this study. These students seemed to have more difficulties in understanding and solving the word problems because of lacking knowledge of English syntax, background knowledge, and specific mathematical concepts. Furthermore, they seemed to have a lack of metacognitive knowledge and strategic knowledge and did not control any problems over any reading process or solution phase. They tended to focus on key words and numbers without understanding the situation described in the problem. In addition to a lack of knowledge, they did not solve any IL problems and did not display a variety of reading behaviors to produce an understanding of the problems when compared with other students in this study. For example, student #2 did not understand some words in the problems, but solved using a key word strategy, focusing on numbers, which led to correct answers for some CL problems, despite not completely understanding the problems.

These results do not mean that students always used a DTA problem-solving
approach when they had difficulties with these word problems. Student #1 is a seventh grader who was at an 8th grade level in mathematics and at a 5.5 grade level in reading. His mathematics achievement level was higher than grade level, but he lagged behind in reading comprehension. Although he had very good problem-solving skills in mathematics, he did not exhibit his monitoring skills and specific strategy skills as he read and solved the problems. His lower problem-solving success rate is related to his low motivation to complete the tasks for this study as well as a lack of metacognitive skills.
A lack of fractional knowledge primarily contributed to the use of DTA approaches on problems involving fraction operations. Student # 6 was the only one who solved 2 IL problems and used MBA approach for 3 problems. He correctly solved one of four fraction problems. His lack of conceptual and procedural knowledge of fractions influenced his reading behaviors and problem-solving success, despite having good

<table>
<thead>
<tr>
<th></th>
<th>CL Problems</th>
<th></th>
<th>IL Problems</th>
<th></th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MBA</td>
<td>DTA</td>
<td>MBA</td>
<td>DTA</td>
<td></td>
</tr>
<tr>
<td>Student 1</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Student 2</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Student 3</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Student 4</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Student 5</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Student 6</td>
<td>0</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Student 7</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Student 8</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Student 9</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>1</td>
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<tr>
<td>Student 10</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Student 11</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Student 12</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Student 13</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

Table. 4.21. The Number of DTA and MBA Problem-Solving Approaches Used by Each Student.
reading ability and mathematics.

Case analysis was conducted to develop a deep understanding of problem-solving behaviors among these participants. The following five examples illustrate DTA problem-solving approaches used by the students as they read and solved the word problems.

Example 1:

Problem 8: Mary runs about 6 miles per week. Sandy runs 3 times as many miles per week as Mary. How far does Sandy run in a week?

After student #2 read the problem, he said, “I don’t understand ‘per’.” After he solved the problem, I asked how he found an answer.

**Researcher:** Can you tell me what you did here? Where did those numbers come from?

**Student 2:** 6 miles multiplied by 3 is 18 miles.

**Researcher:** Why did you multiply by 3 here?

**Student 2:** 3 times means multiplication.

In his attempt to solve it, the student did not understand the word “per” in the problem and did not exhibit any reading behaviors to construct the meaning of the word or the problem he encountered. He focused only on key words and numbers as he solved problems, which lead to a correct answer for this CL problem.

In another example of using a DTA approach, Student #8 used it to solve IL problem 11, which required fraction knowledge. To understand and solve this problem, a student must possess fraction knowledge in conceptions and procedures for this problem, regardless of their reading comprehension ability.

Example 2:

Problem 11: Sam’s Grocery sells 180 eggs a day. That is 1/3 as many eggs as Mike’s
Grocery sells a day. How many eggs does Mike’s Grocery sell in 3 days?

After reading IL problem 11, the student and I had the following exchange:

**Student 8:** Sam’s grocery sells 180 eggs and Mike’s grocery sells less than Sam’s grocery.
**Researcher:** Can you tell me why do you think that Mike’s grocery sells less than Sam’s grocery?
**Student 8**: Because 1/3 makes something smaller and I think Mike’s grocery sells less eggs.

He attempted to solve it and got an incorrect answer:

**Student 8:** The answer is 120.
**Researcher:** Can you tell me how you found your answer?
**Student 8:** 180 multiplied by 1/3 is 60 and 60 subtracted from 180 is 120, so Mike’s grocery sells 120 eggs.
**Researcher:** Why did you subtract from 180?
**Student 8:** Because I think Mike’s grocery sells less, so I subtracted here.

During reading and solving, Student #8 focused only on the fraction 1/3 in this problem. This student did not understand the second statement, so she could not comprehend the meaning of the problem question, which led to an incorrect answer.

Considering her English reading comprehension ability, it was clear that this question was not difficult for her to solve from that aspect, but her struggle with fraction knowledge prevented her from arriving at a correct answer.

While observing participants’ problem-solving behaviors from video recordings, what I found interesting was most deaf middle school students in this study tended to solve fraction problems by focusing only on the fraction number and did not use other information correctly, such as relational terms or context. As a result, students made more errors on fraction problems than problems that did not require the understanding of
fraction concepts. A lack of fraction knowledge interfered with the students’ ability to understand and solve the problem. Although most deaf students used a DTA approach as they read and solved the word problems, this does not mean that they have difficulties with all of the word problems. As discussed in the previous case analysis, some students automatically solved the problem without transformative behaviors. These problems were not difficult for them to read, understand, and solve because the students had been exposed to these problems in their classrooms, so they were familiar with the types of problems. For example, most students automatically solved problem 1 without re-reading or monitoring, or hesitation, as displayed below.

Example 3:

Problem 1: The local farm stand sells about 15 watermelons each day during the summer. The supermarket sells 3 times as many as the farm stand a day. How many watermelons does the supermarket sell in 5 days?

**Student:** 15 multiplied by 3 times and then 5 days. The answer is 225.

**Researcher:** Can you explain how you found your answer?

**Student:** The first sentence says 15 watermelons each day. The store sells 3 times, meaning 15 multiplied by 3… How many sold in 5 days? So 15 multiplied by 3 is 45 and then multiplied by 5.

**Researcher:** Why did you multiply by 3 here?

**Student:** Because 3 times means multiplication.

The student’s think-aloud protocol for this problem revealed that he focused only on relational terms and numbers during solving, though he found the correct answer. It is not clear whether he understood some CL problems and all IL problems or not. He was above his grade level in mathematics (8th grade level), though below grade level in reading,
which was 5\textsuperscript{th} grade level. His problem-solving behaviors for the rest of the word problems are consistent with the example above. He automatically and immediately solved most problems without attempting to reread after the initial reading.

In another example of using a DTA approach, the student did not recognize the difference in problem structure between a CL problem (problem 8) and IL problem (problem 12). Like other participants, the student seemed to lack knowledge of word problem structures.

Example 4:

Problem 12: Joe runs 6 miles a week. He runs 1/3 as many miles a week as Ken does.

How many miles does Ken run in a week?

\textbf{Student 3}: This problem is the same as problem 8 that I solved on the previous page. So 6 miles multiplied by 3 is 18. The answer is 18.
\textbf{Researcher}: Can you explain how you found the answer?
\textbf{Student 3}: 6 miles multiplied by 3 is 18. Because Joe runs 6 miles, he runs 3 times more miles than Ken.
\textbf{Researcher}: Why did you multiply here?
\textbf{Student 3}: Because it is the same problem I solved on the previous page.

Although his answer is correct, the student thinks that this problem was identical in structure to problem 8. Thus, he used the same method to solve this problem as he used for problem 8. After the initial reading, he did not attempt to reread to understand the problem situation. It was apparent that he did not recognize the difference between CL and IL problem structures. The following is an example of CL and IL problem structure differences.

CL Problem 8: Mary runs about 6 miles per week. Sandy runs 3 times as many miles per week as Mary. How far does Sandy run in a week?
IL problem 12: Joe runs 6 miles a week. He runs 1/3 as many miles a week as Ken does. How many miles does Ken run in a week?

Student 3 appeared to read using word clues (i.e. “as many”) and does not use knowledge of structure in processing the word problem, particularly identifying the arithmetic operation needed with respect to the presence of fractions in the problem. It may be because he has problems with English Syntax or inference ability, as well as with fraction operations. As the previous cases have illustrated, most participants misunderstood the relational terms “more than” and “less than” by indicating that “more” simply means addition or multiplication as the operation to use and “less” means subtraction or division. One last example illustrating this is below.

Example 5:

Problem 16: At Kroger a pound of pears cost $1.16. That is 15 cents less per pound than at Meijer. How much does 5 pounds of pears cost at Meijer?

Student 13: At Kroger, a pound of pears cost $1.16. That is 15 cents less, which means at Meijer, the cost is 5 cents subtracted from $1.16 = $1.01. The cost is $1.01. The last question says multiply $1.10 by 5, which means $1.10 plus, $1.01 plus, $1.01 plus, and plus $1.01…multiplied by 5 equals $5.05. How much does 5 pounds of pears cost at Meijer?

Researcher: Why do think that at Meijer the cost is $1.01?

Student 13: Because less than here means…less…always means…subtraction!

Although student 13 used ASL grammatical space to represent the problem as he read, he immediately attempted to solve the problem without any retelling or monitoring, using key word strategy while he read. In addition, the student had a misconception of the relational term “less than.” Like other participants, he thought the relational term “less” always meant “subtraction.” What was interesting here is that no one correctly solved
the IL problem with relational term “less than.” When students solved the problem, all students simply focused on the relational term “less” and said “less” means “subtraction,” so they used the opposite operation needed for the problem solution.

Deaf middle school students who participated in this study tended to use key word strategy as they read word problems. Students who directly translated the key words in the problem into a set of computation committed more errors on these problems. To successfully solve the word problems, students should build a coherent mental representation of what they read and solved. Nine participants used MBA problem-solving behaviors on 17 out of 151 problems. In particular, students 5, 6, and 12 solved 3 or 4 problems using MBA approach. Students 5 and 12 were from deaf families who were fluent in ASL and supported Deaf culture, whereas student 6 had a hearing family who communicated in spoken English. He can speak English and is fluent in Signed English. These three students’ think-aloud protocols were different from other participants’ in this study. They seemed to have enough background knowledge about the world that can learn through communication with family that helped them to successfully solve word problems. The following three examples present MBA problem-solving approaches used by these three students as they solved word problems.

Example 1:

IL Problem 6: At Meijer, a pound of sugar costs 89 cents. That is 20 cents more per pound than at Walmart. How much does 5 pounds of sugar cost at Walmart?

Student 5’s solution:

**Student 5:** The answer is 20 cents subtracted from 89 cents and then multiplied by
5, which is $3.45.

**Researcher:** Can you tell me how you found the answer?

**Student 5:** At Meijer, a pound of sugar costs 89 cents. At Walmart…less per pound than at Meijer. 89 cents…oh…it is addition…I am wrong.

He focused on the relational term “more” while rereading the problem.

**Student 5:** Oh…see…I think Meijer costs 89 cents and Walmart is cheaper than Meijer.

**Researcher:** Why did you correct your answer?

**Student 5:** I thought it was addition but it says Walmart is cheaper.

Student 5 seemed to understand the problem situations described in the IL problem. He retold using his own language, ASL, and explained relations between variables and quantities using ASL grammatical space to represent the places of two stores and variables. However, in the process of explaining his solution steps, he seemed to become confused with the problem. His confusion may stem from his concept of problem structures that were different than the CL problem structure. He corrected his confusion by rereading and monitoring his solution steps. This problem is the only IL problem that he correctly solved.

**Example 2:**

**IL problem 4:** Last year, the sixth grade sold 125 raffle tickets each day. That is 5 times as many tickets as the fifth grade sold per day. How many tickets did the fifth grade students sell in a day?

**Student 6** solved using a MBA problem-solving approach:

**Student 6:** The six grades sold 125 raffle tickets.

After reading the first statement, he read the second statement several times.
**Researcher:** What are you doing now?

**Student 6:** I understand that...125 raffle tickets... is 5 times more as many tickets as the fifth grade. How many tickets did the fifth grade students sell in a day?

He silently read the second statement again to better understand the meaning of the problem.

**Student 6:** I think it is division...125 tickets divided by 5 is 25 tickets, so the fifth grade sold 25 tickets.

**Researcher:** Can you tell me how you found the answer?

**Student 6:** See here...because 6th grade sells 125 tickets and 6th grade sells 5 times more than 5th grade, so it is divided by 5.

In his problem-solving attempt, student 6 had to reread the problem several times to understand the problem details. It took longer for him to read, think, and solve this problem than other problems he solved. During reading or solving, I often had to gain his attention by questioning him because he read and thought silently. He said that this type of problem made him confused, but he understood the problem details described in the problem. In particular, his reading behaviors were different from those of other participants. Unlike other participants who used sign-based phonology, he was the only one who used English-based phonology as he read the problems. When he attempted to solve IL problems, he often got confused and had to reread several times, frequently monitoring his solution steps. It was not clear that his confusion with IL problems was because of his use of his English-based phonology or because of his knowledge of English syntax and problem-solving strategy.

Example 3:

**Problem 6:** At Meijer, a pound of sugar costs 89 cents. That is 20 cents more per pound than at Walmart. How much does 5 pounds of sugar cost at Walmart?
While student 12 read this problem, she used signed English.

**Student 12:** I don’t understand this problem.
At Meijer, sugar costs 89 cents. That is 20 cents more than Walmart…That means Meijer is more expensive than Walmart. How much does 5 pounds of sugar cost at Walmart?

During reading, she used signed English, but during retelling, she used ASL grammatical space. After reading this problem again, she looked at the problem and stated:

**Student 12:** I don’t understand it… I understand the first statement but this says…I don’t know…[sigh]

She reread the problem silently.

**Student 12:** I think 20 subtracted from 89 cents is 69 cents.
**Researcher:** What are you doing now?
**Student 12:** 89 cents… this says…Meijer is 20 cents more expensive than Walmart…This means 20 cents subtracted from 89 cents is 69 cents. The last sentence is saying” how much does 5 pounds of sugar cost at Walmart?”

She sighed and reread the problem again silently.

**Student 12:** 69 cents divided by 5?
Oh I see … 69 cents multiplied by 5.

This IL problem was the only one that student 12 solved correctly. To grasp the concept of this problem, she had to read it several times and monitor her solution steps. It took her longer to read, think, and solve the IL problem. When she used reading comprehension strategies, such as rereading and monitoring, to help her understand the meaning of problems, her accuracy increased as she progressed through her solution steps. However, it was not clear why she could not solve other IL problems that had the same semantic structures and why she did not use her reading comprehension strategies for other IL problems.
In summary, students altered their problem-solving behaviors depending on the degree of difficulty of a problem or their knowledge of problem structures, arithmetic, and fractions. DTA problem-solving approaches were the predominant approaches used. Students who primarily used a MBA approach reread more difficult problems several times and took longer to read, think about, and solve them. In contrast, students who used a DTA approach for most problems utilized a MBA approach with easier problems but utilized a DTA approach with more difficult problems. It appears that these problem solving behaviors are related to students’ reading comprehension abilities.

**Problem-Reading Behaviors**

Observing the participants as they read and solved problems through video recordings of think-aloud protocols revealed five major types of problem-reading behaviors, which can be classified in terms of language or communication mode, the use of sign-based phonology, family backgrounds (deaf family vs. hearing family), and vocabulary issues. I will briefly describe the specific problem-solving behaviors based on their problem-solving approaches.

1. **Language or Communication Mode.** Though the students differed in language modes, communication skills, and the amount of relevant knowledge, they exhibited some common transformative behaviors for reading problems. With regard to language modes, many studies indicated that since ASL is a bona fide language, there is no one-to-one match between English and ASL words. In addition, syntax and grammar in ASL completely differ from English. ASL has spatial grammar, syntax, and semantics.
all its own. In this study, students who were fluent in ASL tended to use top-down processing strategy during problem reading. By contrast, students who did not share a language with their parents, used signed English, and did not have sufficient background knowledge tended to use bottom-up processing strategy while they read word problems.

Furthermore, students’ problem-reading behaviors were influenced by the degree of problem difficulty. When they read word problems, the communication modes (in this case, the form of sign language or system) they used changed for different reasons. For example, when they encountered complex problems, they switched their preferred communication modes into a signed system which signed every word in English word order; that is, they read these word problems literally (i.e., every word in English) using signed systems, yet when given more difficult problems they did not translate English words into their own words or their own language during retelling. Although they could sign all the words in English, they did not necessarily understand the text they read. In addition, if students found unknown words in problem reading, they tended to fingerspell them, although the words in English can express a sign word in sign language. However, students translated English into their own language, either ASL or their own words, during initial reading and retelling when they completely understood the problems.

2. Vocabulary Issues Related to Relational Terms. Much research has demonstrated that deaf or hard of hearing students have less vocabulary knowledge in English compared to their hearing counterparts (Paul, 1996, 2003). For struggling students or second language learners, the knowledge of vocabulary plays an important role in reading comprehension (Paul, 1996; Stahl, 1995)
In this study, for some students who solved fewer problems, a lack of vocabulary knowledge was a major source of difficulty. However, most students in this study had adequate word recognition skills and vocabulary knowledge about the word problems. Instead, they seemed to have more difficulty in integrating information in the problems and in making inferences from them. Furthermore, some words were inconsistent in their signs, such as “per,” “each,” “grocery,” “raffle,” “gallon,” “pears,” and “as many as.”

Interestingly, signs for “as many as” varied radically among students: most students read the phrase in English word order (“as many as” or “as many”, or “many”), while other students signed for the word “as many as” into “more than.” It is assumed that the phrase “as many as” is rarely presented to deaf students in the classroom.

3. Phonology Coding Issues. The third reason for deaf students’ reading problems is related to the form of sign language used by the participants. All students in this study communicated using sign language and were fluent in sign language, although the degree of ASL fluency varied depending on their primary language at home. Student 6 used English-based phonology codes when he read the word problems, unlike the students who used signed or visual code in their reading behaviors. This student was the only one who correctly solved two of the six IL problems, though he seemed to get confused with the IL problems when he first read them. He took much longer to read, understand, and solve IL problems than the other students who participated in this study. These reading behaviors were similar to hearing students’ reading behaviors in other previous studies (Lewis & Mayer, 1987; Pape, 2003). In contrast, other students who used ASL or signed coding did not seem confused with IL problems.
4. **Number of Re-Readings.** According to the reading model, when students have difficulty with specific problems, they tend to reread the problems several times to construct the meaning of the problems. The number of re-readings for CL and IL problems was investigated to determine if students reread one type more often. The number of re-readings was counted only once, regardless of the number of times students re-read. Table 4.22 presents the number of re-readings for CL and IL problems.

<table>
<thead>
<tr>
<th>Number of Re-Readings</th>
<th>CL Problems</th>
<th>IL Problems</th>
<th>Total Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21</td>
<td>37</td>
<td>58</td>
</tr>
<tr>
<td>(36.2%)</td>
<td>(63.8%)</td>
<td>(100%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.22. The Number (Percentage) of Re-Readings for CL and IL Problems

As shown in Table 4.22, students reread fewer CL problems (n=21, 36.2%) than IL problems (n=37, 63.8%). It appears that IL problems were more difficult for students than CL problems.

5. **Deaf Families Versus Hearing Families.** As mentioned in the previous chapter, participants for this study came from two different types of families in terms of language modes: deaf families which used ASL as their primary language at home, and hearing families which used some signs or spoken language at home. To examine the differences in problem solving performance and academic achievement between students who had a shared language and students who did not, problem-solving success and academic achievement were compared. Group 1 (n=5, students #1, 5, 6, 7 and 12) shared a
common language with their parents, either ASL or English. In contrast, Group 2 (n=8, students #2, 3, 4, 8, 9, 10, 11, and 13) did not, though their parents knew a few sign words and communicated with their children at a surface level. As presented in Table 4.23, participants who had a shared language with their parents, either sign language or English (Group 1), showed better performance in mathematics and reading than participants who did not share a language with their parents (Group 2).

With regard to overall problem-solving success, as presented in Table 4.23, participants in Group 1 solved more CL problems and IL problems (M=6.5) correctly than participants in Group 2 (M=3.5). As a result, this study confirmed that deaf students who have a shared language with their parents performed better on word problems than deaf students who do not share a language with their parents.

<table>
<thead>
<tr>
<th>Problem-Solving Success</th>
<th>Shared Language Group</th>
<th>Not Shared Language Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL</td>
<td>5.25</td>
<td>3.25</td>
</tr>
<tr>
<td>IL</td>
<td>1.25</td>
<td>.25</td>
</tr>
</tbody>
</table>

Table 4.23. The Mean of Problem-Solving Success of the Two Groups

This analysis did not aim to examine which communication mode is the best for deaf students’ academic achievement or improvement, so did not compare problem-solving success and academic achievement scores between both groups. It is clear that there are many factors that influence the academic achievements of deaf students other than language environment at home, such as family involvement in their child’s education.
Thus, this analysis is intended to report the importance of communication between deaf children and their parents in a common language, which does have significant effects on the children’s reading development and academic achievement.

SRL Strategy Analysis

To answer the research question, What SRL strategies do deaf or hard of hearing middle school students chose as they learn mathematics? The questionnaire about learning strategy was administered to deaf middle school students. For this purpose, descriptive statistics were employed. Mean scores and Standard deviation scores for learning strategy scales were examined for all participating students. Furthermore, the correlation analysis was conducted to determine if there were the relationships between each scale within learning strategy variables as well as the relationships between the subscales of the learning strategy and other variables: problem-solving rates, computation scores, and these same variables related to student performances, as obtained by mathematics and reading achievement scores and reports scores. Pearson correlation analysis was conducted to investigate whether each sub-category is related to the others. Means, standard deviations, and correlations for the variables in the study at the sub-scale level are presented in Table 4.24 and 4.25. Table 4.24 provides participants’ individual scores for each subscale in the learning strategy.

The overall mean scores for the self-regulation strategies were for cognitive, metacognitive, help-seeking, and efforts regulation subscales were 3.60 (SD=.51),
The help-seeking in learning strategy subscale had the highest mean score when compared to the other subscales. In particular, their mean scores were 3.60 for cognitive strategy and 3.61 for metacognitive strategy. In this study, students reported that they know what they should do and ask for help to the teacher when they encountered difficulties with mathematics problems.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Student1</td>
<td>3.80</td>
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<tr>
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<td>3.60</td>
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<td>Student3</td>
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<td>3.00</td>
<td>3.80</td>
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<td>4.00</td>
<td>4.00</td>
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<td>4.00</td>
</tr>
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<td>3.00</td>
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<td>3.40</td>
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<td>4.60</td>
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<tr>
<td>Total</td>
<td>3.60</td>
<td>3.61</td>
<td>4.08</td>
<td>3.82</td>
</tr>
</tbody>
</table>

Table 4.24. The Mean and Standard Deviation of Participants’ Individual Scores for Each Subscale

As shown in Table 4.25, the cognitive strategy was highly correlated with the meta-cognitive strategy ($r=.77$) and help-seeking ($r=.63$) but the effort regulation strategy
was not associated with any of the other three variables (r=.26). That is, a correlation between cognitive and meta-cognitive was strong.

<table>
<thead>
<tr>
<th></th>
<th>Cognitive</th>
<th>Metacognitive</th>
<th>Effort</th>
<th>Help-Seeking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cognitive</td>
<td>.774**</td>
<td>.257</td>
<td>.628*</td>
<td></td>
</tr>
<tr>
<td>Metacognitive</td>
<td></td>
<td>.379</td>
<td>.654*</td>
<td></td>
</tr>
<tr>
<td>Effort</td>
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<td>.045</td>
<td></td>
</tr>
<tr>
<td>Help-seeking</td>
<td></td>
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</tr>
</tbody>
</table>

*Correlation is significant at the 0.05 level (2-tailed).
**Correlation is significant at the 0.01 level (2-tailed)

Table 4.25. Correlation Among the Variables in the SRL Strategy Scales

**Chapter Summary**

In this study, middle school deaf students who used sign language as their primary language had difficulty in comprehending word problems rather than in solving them. Although they struggled with problems associated with fractional knowledge, most students had relatively good arithmetic operation skills. Furthermore, this study indicated that students had more difficulty with IL problems vs. CL problems, two-step problems vs. one-step problems, division problems vs. multiplication problems, and problems associated with fractional knowledge vs. non-fraction problems. In particular, most students had difficulty in understanding fraction concepts, except for Students #1 and #5. Most students rarely solved the IL problems and predominantly committed reversal errors on IL problems. These reversal errors are related to their knowledge of syntax in English and prior knowledge of mathematics fractions, as well as their inference-making ability.
and tendency to employ a key word strategy as they read word problems.

With regard to problem-solving approaches, all students predominantly used a DTA problem-solving approach. They tended to focus on problem elements without context, such as key words and numbers, and did not fully comprehend the problem statements with respect to the problem details. The use of key-word strategy can aid students in solving CL problems, yet often led to wrong answers for IL problems. The minority of students who used an MBA approach tended to construct a coherent mental model of the situation presented in the word problems, and tended to have more accurate understanding of word problems, leading to more success at solving them. Although the students predominantly used a DTA problem-solving approach, their problem-solving approaches varied and changed depending on the complexity of the problem, prior knowledge, and motivation.

In terms of problem-solving reading behaviors, deaf students who had a shared language with their parents (either English or ASL) exhibited better performance solving word problems than deaf students who did not share a language with their parents. It is interesting that only one student (#6) used English-based phonology as he read word problems, and that his problem reading behaviors were different from those of students who used sign-based coding. Although this study did not provide empirical evidence for cognitive processing regarding phonology-based or sign-based coding of the word problem, it is assumed that there may be differences in cognitive processes between deaf students and hearing students because of different language modes (sign language vs. spoken language).
CHAPTER 5

CONCLUSION

The purpose of this study was to explore and describe deaf middle school students’ problem-solving behaviors as they read, understood, and solved word problems in a residential school. This study examined the process of representing word problems for middle school deaf students, all of whom were at four different grade levels and used sign language as their primary language but had different language modes and fluency. To achieve the study’s goal, a think-aloud protocol, a powerful tool to explore students’ thought processes in problem solving and reading comprehension, was used. Some case analysis was conducted to investigate individual differences in problem solving among the participants in order to explore how problem-solving behaviors were influenced by knowledge of problem structures and mathematics knowledge. The goal of the study sought to identify the difficulties that deaf students encountered while they read and solved *compare* word problems. In doing so, however, the study did not aim to imply that deaf students are in problem-reading or-solving deficits. Rather, the intention here was to describe and understand the way in which deaf middle school students understood and solved given word problems, especially in terms of the process of representing problems. The ultimate goal of this study was to provide instructors and researchers with new
insights about deaf or hard of hearing learners in word problem solving, as a means of better understanding their cognitive processes.

To achieve these goals, this study approached students’ problem-solving behaviors using the integration theories of text processing model of reading and a cognitive process in mathematical problem solving; these theories guided the research questions and discussions in this chapter as well as produce comprehensive and multi-faceted explanations about students’ problem-solving behaviors. Additionally, the goal of this study was to reflect deaf students’ learning abilities in a more positive way, from my own perspective as a deaf researcher and deaf second language learner, which differs in significant ways from hearing researchers’ perspectives on deafness and deaf learners. As Moores (2010) notes, “deaf education has been controlled by hearing educators who harbor a deficit or deficiency model of deafness and who are insensitive to the needs and learning styles of deaf learners” (p. 447). In opposition to the deficiency model, this study described what deaf individuals were able to do and why they had difficulty with word problem solving, and especially focused on what they will be able to accomplish if more effective instruction methods are provided for them in the future.

Therefore, this study used integrated theoretical perspectives to interpret deaf middle school students’ process of understanding and solving word problems. More specifically, it first identified five major problem-solving behaviors in word problem-solving: problem-solving success rates, types of errors including retelling reversal errors, fraction knowledge, problem-solving approaches, and problem-reading behaviors. Within this context, this study explored how participants’ problem-reading behaviors were
influenced by the structures of the problems, prior knowledge in relevant to word
problem solving, and language mode. Second, this study examined the self-regulated
learning strategies chosen by deaf middle school students as they learn mathematics in
their classrooms. Finally, the ultimate goal of the study was to provide evidence why
middle school students encountered difficulties in understanding and solving
mathematical word problems.

**Conclusions and Discussion**

In this chapter, I will discuss five major findings related to the extant research and
time: 1) problem success rate and the patterns of errors, 2) problem-solving approaches
and problem-reading behaviors; 3) fractional knowledge, 4) language mode, and 5) SRL
strategy use. Based on the findings, issues related to the qualitative similarity hypothesis
(QSH) will be discussed. The goal of the discussion is to better understand problem-
solving behaviors of deaf middle school students who used sign language as their primary
language by examining the similarities and differences in problem-solving behaviors
between deaf students and hearing students in the previous studies or among deaf
students. To that end, I will discuss the limitations of the study and present implications
and recommendations for further study at the end of this chapter.

**Problem-Solving Success and Types of Errors**

Thirteen deaf middle school students in this study performed better on consistent
language (CL) problems than on inconsistent language (IL) problems; on one-step
problems than on two-step problems, on addition problems than on subtraction problems, on multiplication problems than on division problems, on the relational term “more than” than on “less than,” and on non-fraction problems than on fraction problems. In particular, students seemed to have more difficulty with IL problems and two-step word problems than other types of problems.

With regard to patterns of errors, deaf middle school students generated seven different types of errors: linguistic error, reversal error, computation error, fraction error, goal monitoring error, multiple errors, and problems left blank. The common errors for CL problems were multiple errors (28.6%), linguistic error (25%), and fractional error (21.4%), whereas the largest error for IL problem was reversal error (56.3%).

As mentioned above, deaf middle school students made errors more frequently on IL problems than on CL problems and predominantly committed reversal errors on IL problems rather than on CL problems, as hearing students did in the previous studies (Lewis & Mayer, 1987; Pape, 2003; Verschaffel, 1994).

In addition, the findings show that there are differences in the types of errors generated by deaf middle school students for CL and IL problems when compared to patterns of types made by hearing students in the previous studies (Lewis & Mayer, 1987; Pape, 2003). Similar results were found by Kelly and his colleagues (2003) for deaf college students. These differences between hearing students and deaf students in word problem solving will be discussed in the section of the issues related to QSH later. However, Kelly and colleagues (2003) did not provide evidence about the students’ processes of understanding and representing word problems, especially, why reversal
errors on IL problems more frequently occurred than on CL problems among deaf college students.

To understand the process of representing IL problems, the current study investigated why and when reversal errors occurred as deaf middle school students read and solved given word problems using a retelling protocol, which is an accurate window into students’ cognitive process in representing word problems (Pape, 2004).

Findings show that reversal errors are directly related to their syntactic knowledge in English and the abilities to make inferences, indicating that some deaf middle school students ignored the pronouns in the second statement of IL problems. IL problem structures are more complex and require the ability to infer meanings from the problems and integrate information across them and make inferences associated with syntax ability and prior knowledge (Cain & Oakhill, 1999; Greeno & Kintsch, 1985).

To better understand reversal errors on IL problems, this study examined whether there is an effect of lexically marked relational terms. Unlike previous studies (e.g., Lewis & Mayer, 1987; Pape, 2003; Verschaffel et al., 1992), the effect of lexically marked relational terms was not supported by this study. As discussed in the previous chapters, IL addition and multiplication problems include the marked relational term “less” and “1/n as many,” whereas IL subtraction and division problem include the unmarked term “more” and “n as many,” respectively, and both require opposite arithmetic operations. According to Lewis and Mayer (1987), reversal errors would increase when relational terms are marked because students are more likely to resist inverting them during transformation. Lewis and Mayer’s (1987) arguments were backed
by several empirical studies, indicating that hearing students at various ages made more reversal errors on IL addition and multiplication (i.e., with the marked relational terms “less” and “1/n as many”) than IL subtraction and division problems (i.e., with the unmarked relational terms “more” and “n as many”). That is, IL addition and multiplication problems may be difficult for hearing students to solve because marked terms in IL addition and multiplication problems (e.g., “less” and “1/n as many”) are more prominent and influence students more than unmarked terms in IL subtraction and division problems (e.g., “more” and “n times as many”) (Lewis & Mayer, 1987; Pape, 2003; Sterns, 1992; Verschaffel et al., 1992). For this reason, Pape (2003) stated that “the problem solver is more reluctant to transform the marked relational term into its unmarked opposite when translating the relational sentence into its CL format” (p. 401).

In a study conducted by Kelly and colleagues (2003), an effect of lexically marked relational terms in deaf college students was observed, indicating that they generated more reversal errors with IL problems that contained marked relational terms (i.e., “less”) than with IL problems that contained unmarked relational terms (i.e., “more”). However, the current study does not support this finding. Unlike the findings of previous studies, reversal errors on IL problems were not influenced by lexically marked relational terms among deaf middle school students. That is, deaf middle school students in this study committed more reversal errors on IL subtraction and division problems with unmarked terms (i.e., “more” or “n as many”) than IL addition and multiplication with marked terms (i.e., “less” or “1/n as many”). It is still unclear why opposite results were found in this study, yet it is hypothesized for deaf middle school students, reversal errors
may result from the use of key word strategy, a lack of fraction knowledge, or the nature of visual language that was not influenced by such relational terms.

Additionally, there are several possible explanations for reversal errors, based on previous studies and findings of current study: 1) an established schema, 2) the inference ability in English, and 3) the use of the key word strategy. First, as discussed in Chapter 2, Lewis and Mayer (1987) argued that most students have a preferred form for the order of presenting information (in this case, CL problem structure) in word problems, so if the order of information in word problems conflicts with their schema, they tend to transfer IL problems into CL problems to match their preferred format (Lewis & Mayer, 1987; Verschaffel et al, 1992, Verschaffel 1994; Pape, 2003, 2004). In transformation, reversal errors may occur because transforming IL problems to CL problems may require heavy demands on the students’ working memory and thus result in errors (Lewis & Mayer, 1987; Pape 2004; Verschaffel, 1994). This may be true for deaf middle school students, even though the issue related to working memory is not empirically supported by this study, because several studies indicated that the preferred form or schema is constructed (or perhaps developed) through experiences with a CL problem structure (Lewis & Mayer, 1987; Pape, 2003; Stern, 1992). Therefore, for deaf middle school students, difficulty with IL word problems may be a result of not having experience with these problem structures in their classroom (Kelly et al, 2003; Marschark, 2002; Hyde, Zevenbergen, & Power, 2003).

Second, reversal errors seemed to be related to an inability to draw inference from the text as well as a lack of syntax knowledge in English. Although this study did not
assess deaf middle school students’ fluency and syntax in written English, it assumes that they are more likely to have specific difficulties in problems requiring inference-making ability because of a lack of syntactic knowledge in written English (Paul, 2002; Wood, 1984). Indeed, a great number of studies have demonstrated that deaf students have difficulties in their acquisition of semantic knowledge and syntax in English (Paul, 1998, 2009) and in making inferences related to English tasks (Kelly, 2008).

As mentioned above, with regard to difficulty with syntax in this group, findings show that some deaf middle school students in this study indicated that they did not attend properly to pronouns of IL problems during the initial reading or retelling. This difficulty with pronoun resolutions may result in either reversal errors or more erroneous answers. In a study investigating students’ ability to understand pronouns in the sentence, Oakhill, Yuill, and Parkin (1986) suggested that less skilled readers have more difficulty than skilled readers with using pronouns as cues for the problem’s intended referents, which require additional processing demand. Thus, less skilled readers often tend to fail to understand and integrate the information from the text, as well as to use syntactical or semantic clues in it appropriately. Such poor inference skills can be attributed to students’ reading comprehension difficulties (Cain & Oakhill, 1999).

Finally, reversal errors are also related to problem-solving approaches the students used as they read and solved word problems (Hegarty, Mayer, & Monk, 1995; Pape, 2003; Verschaeffel, 1994). In this study, most deaf middle school students used a key word strategy, called the Direct Translation Approach (DTA), and mostly focused only on relational words and numbers without completely understanding the situations described
in the problems. Some researchers have argued that reversal errors have partly been caused by the use of the DTA problem-solving approach, although the use of DTA can lead to the correct answer to a CL problem (Verschaffel et al., 1992; Verschaffel 1994). Problem-solving approaches used by students during reading and solving will be discussed below in more detail.

**Problem-Solving Approaches**

This study investigated problem-solving approaches to understand the ways in which such students understood and interpreted word problems, from the perspective of a theory of text processing. This section discusses problem-solving approaches that emerged from the data. The problem-solving approaches were categorized based on the previous studies (Hegarty et al., 1992; Pape, 2004; Verschaffel et al., 1992): the Direct Translation Approach (DTA) and the Meaning-Based Approach (MBA). Findings of this study indicate that all the deaf middle school students used the DTA problem-solving approach as their predominant problem-solving behavior: only 11.3% of their approaches used the MBA approach. However, their problem-solving behaviors are not consistent for all problems. More specifically, problem-solving behaviors varied and were altered according to: 1) the complexity of problem structures (CL vs. IL problems), 2) the complexity of computation steps (one-step vs. two step problems), 3) prior knowledge about fraction concepts, 4) strategic knowledge, and 5) motivation. These five factors influenced language modes (i.e., signed English or ASL) participants chose while they read and understand word problems.
Furthermore, it is important to point out that there are differences in problem-solving behaviors between students who used the MBA approach and students who used the DTA approach according to the complexity of problems. For example, few students who used the MBA approach to some problems not only understand and solve problems differently from students who used the DTA approach for most problems, but have prior knowledge about fractions and better learning strategies.

As deaf middle school students read and solved word problems, they tend to focus on key words and numbers without understanding the problem situations, regardless of type of problems. Thus, they misinterpreted the relational terms “more” and “less” in word problems as indicating the mathematical operations “addition” and “subtraction.” The use of key word strategy can lead to a correct answer for a CL problem even though students do not completely understand the problem. However, in the IL problems, as mentioned in the previous chapter, the relational term “more” is not consistent with the mathematical operation “addition,” and the meaning of the relational terms “more than” or “less than” depends on contexts described in the problems. These misinterpretations of relational terms in IL problems may result in reversal errors or other erroneous answers.

A possibility with use of the DTA problem-solving approach is related to the fact that most deaf students were not able to recognize the structural differences between CL problems and IL problems. For example, one student mentioned that an IL problem included a CL problem that he solved earlier in the study (in this case, CL problem 8 & IL problem 12), and he utilized the same methods and procedures to solve the IL problem
12. It appears that most deaf students who used DTA approaches in the study read and solved given word problems using structural clues in the context, without a deeper understanding of the problems they read. Although most deaf students solved CL and IL problems using a key word strategy, those who used the DTA approach did not necessarily have difficulty understanding given word problems. For example, during the initial reading a student (Student 1) immediately and automatically attempted to solve the word problems without hesitating because he was familiar with the problems so these problems were easy for him so can activate relevant prior knowledge to understand and solve them. That is, he tends to interpret problems based on his prior knowledge about mathematics. However, it remains unclear whether his reading behaviors were modified for IL problems, as he did not exhibit any transforming behaviors for IL problems. His difficulty with word problems seemed to be associated with a lack of transfer skills or a lack of problem structure knowledge. It may be assumed that most deaf students were rarely exposed to these types of IL problems in their reading or mathematics classrooms.

Comparatively fewer deaf students exhibited the use of the MBA problem-solving approach, for fewer problems. They understood expressing a relation between two variables in the problems and planned their solution based on the construction of a coherent mental representation. Likewise, they used a variety of transforming behaviors such as rereading several times, remedying their misinterpretations, checking on their prior knowledge, and monitoring their understanding (Hegarty et al., 1995; Pape, 2004; Verschaffel, 1994).
**Fractional Knowledge**

In this study, the difficulties of word problems are related to fractional knowledge possessed by students. Deaf middle school students solved fewer problems associated with fraction than non-fraction problems. That is, most students had difficulty understanding and solving word problems associated with fractional knowledge, except for student 1 and 5. These students had a lack of the procedural knowledge of fractional operations and an understanding of underlying conceptions that influenced problem-solving success.

For example, student 6, who had a good reading ability and problem-solving ability, did not solve 3 out of 4 problems associated with fraction knowledge. His problem-solving behaviors were changed when he encountered fraction problems. Despite his high reading comprehension, he relied on structural clues in these problems and failed to understand these problems, even though he attempted to use his metacognitive knowledge and monitor his comprehension by rereading several times and by saying “I am not good at fractions.” Due to his lack of fraction knowledge, his understanding of problems has broken down. His difficulty with word problems is placed in prior knowledge about fraction concepts. Goldman and Rakestraw (2000) argued that “in low context knowledge situations, processing may be more text driven, with readers relying on cues in the text to organize and relate the information and achieve the intended meanings” (p. 313). Goldman and Rakestraw’s (2000) remarks illustrate why Student 6 had trouble with word problems that required fractional knowledge. As a result, this study shows that deaf middle school students tend to depend more on structural aspects of the
problem (i.e., use of clues in the context) when they encountered complex or unfamiliar problems. In an attempt to explain text processing, Goldman and Rakestraw (2000) stated that “in situations of high content knowledge, readers will be less reliant on structural aspects of the text than in low content knowledge situations because they can draw on preexisting information to create accurate and coherent mental representations” (p. 313). Goldman and Rakestraw’s (2000) argument seemed to emphasize the importance of prior knowledge students brought to the problems in the process of understanding the text. When considering different problem-solving behaviors, it is evident that individual differences in relevant prior knowledge influence students’ problem-solving approaches and success.

**Language Mode**

Additionally, there are other differences in problem-solving behaviors between deaf students (either with deaf parents or with hearing parents) who were fully immersed in either ASL or spoken English as their first language and deaf students with hearing parents who do not have a shared language. For example, deaf students who have deaf parents and used ASL as their first language tend to use a top-down processing strategy, heavily relying on their prior or background knowledge to understand word problems as they read them, whereas the latter group tends to use a bottom-up processing strategy, focusing on structural clues in the problems. Another finding from this study indicates that students who had a shared language with their parents better performed word problems than students who failed to access communication with their parents at home.
In such student groups, there may be more delay in language and learning. In addition, some students changed their language mode when they encountered complex problems. For example, students literally read complex problems in English order, whereas with more familiar problems, they changed English problems into their own words or ASL.

In conclusion, as noted earlier in this chapter, this study found that middle school deaf students’ problem-solving behaviors varied depending on the complexity of problem structures and prior knowledge students possessed. There are many factors that influenced problem-solving success, such as mathematics knowledge, primary language mode, different experiences in learning, different learning strategies, knowledge of textual structures, and misinterpretation of relational terms. Students’ problem-solving success or failure seemed to be influenced by an interaction between the complexity of problem structures and their prior knowledge. Therefore, it is important for researchers or instructors to understand that deaf students’ problem-solving behaviors vary across problem structures, the amount of relevant knowledge, primary language mode, and setting. It should be possible for teachers to predict variability in performance within deaf students to provide efficient instruction based on deaf students’ understanding of problem solving.

**Self-Regulated Learning Strategy**

To date, learning strategy use by deaf students has not been explored in math education. This study attempted to investigate these students’ learning strategies using a structured questionnaire consisting of four subscales: cognitive, metacognitive, help-
seeking, and efforts regulation strategy. The findings indicate that deaf middle school students scored the highest on the help-seeking strategy and scored lower on the cognitive and metacognitive strategies. Students tend to rely on the help-seeking strategy when engaged in difficult tasks rather than to attempting to undertake the tasks alone; in contrast, they tend to use less metacognitive or cognitive strategy. Several studies have demonstrated that deaf students have relatively inefficient cognitive and metacognitive strategies (Marschark, 2007, 2008), and this study is consistent with that findings. One reason for this, Marschark (2007) has argued, is that “perhaps because of the ways we teach them, deaf students also may demonstrate instrumental dependence in their reading strategies, looking to teachers and peers for explanations of text rather than attempting to determine figure out the meaning themselves” (p. 176). To encourage deaf students to become self-motivated independent learners, teachers should provide them the opportunity to learn from and resolve their own difficulties or tasks. It is important to note that if effective metacognitive skills and monitoring skills are provided in future instruction, deaf students can become active rather than passive learners.

Issues Related to the Qualitative Similarity Hypothesis

This study explored the notion of the Qualitative Similarity Hypothesis (QSH) to investigate the similarities and differences in problem-solving behaviors between deaf students and hearing students. The results of the current study are partially consistent with those of previous studies by Hegarty, Mayer, and Monk (1995), Lewis and Mayer (1987), Pape (2004), Verschaffel et al (1992), and Verschaffel (1994). Thus, it may be
argued that deaf students’ problem-solving performance on word problem tasks was partially similar to that of hearing students in terms of the production of errors they committed; deaf students showed similar patterns of errors (e.g., committed more reversal errors on IL problems) as they solved word problems compared to hearing students in the previous studies. However, it remains unclear (and difficult to say) if deaf students performed word problem-solving tasks in the same (or a similar) way as hearing students did. Since hearing students are not included as a comparison group, this study does not provide empirical evidence about that issue. Although deaf students exhibited similar patterns of errors when compared to hearing students in the previous, there are some errors (multiple errors/linguistics errors) and problem-solving approaches generated only by deaf students but not by hearing students. In addition, deaf students made more errors on IL problems and committed more frequently reversal errors on IL problems than hearing students in previous studies. This is likely to be due to the primary language mode and difference experience in learning, as Marschark (2008) has mentioned. These errors made only by deaf middle school students were found in Kelly and colleagues (2003)’s study. It is argued that deaf students have a greater variability in their performance on word problem solving because of their different experiential and linguistic factors (Marschark, 2008).

There are other differences in problem-solving approaches between hearing students in the previous studies and deaf students. In the previous studies, hearing students used a MBA problem-solving approach with more difficult problems and used a DTA problem-solving approach with easier problems, indicating that accurate problem
solvers took longer to read and understand more complex problems than less complex problems. In this study, similar results were found: fewer deaf students who correctly solved 1-2 IL problems exhibited similar transformative and strategic behaviors when confronted with complex IL problems, as hearing students did. For example, these students took longer to read and solve the IL problem and frequently had to reread to integrate information from the problems. However, opposite results were also found, indicating that a DTA problem-solving approach was predominantly used by deaf middle school students when they encountered complex problems, but they tended to use an MBA approach with less complex problems.

Although the same tasks were used for both studies, it is clear that the structure of knowledge that participants brought to word problems may be obviously different between hearing students and deaf students, or even among the deaf students in this study. Goldman and Rakestraw (2000) argued that the structure of knowledge that students bring to a text can influence the construction of meaning from text or information. With regard to individual differences in reading, van den Broek, Young, Tzeng, and Linderholm (1999) stated that

Individuals differ in their attentional capacities, knowledge, and comprehension processes. As a consequence, the same text may be processed, interpreted, and remembered very differently by different individuals. Indeed, even within the same individual, comprehension processes may differ from one reading situation to the next, for example, as a result of different reading goals, motivation, and fatigue (p. 90).

With regard to the differences between hearing students and deaf students, Marschark (2002) claimed that “most results suggest that the two groups simply vary in
their approaches to cognitive tasks, are influenced by the primary mode of communication, and differ in their amounts of relevant knowledge” (p. 466).

Van den Broek, Young, Tzeng, and Linderholm (1999) and Marschark (2002)’s remarks illustrate deaf individuals may interact with structures of problems in different way that differently influence their learning outcomes: the variety among deaf students is greater than one among hearing students because of language mode and language environments.

In conclusion, it can be asserted that the QSH is partially supported by this study in terms of patterns of errors committed by deaf students, that is, in terms of product-focused perspective. However, it can be also asserted that the process of understanding and solving word problems of deaf middle school students was different from one of hearing students in terms of process-focused perspective. Furthermore, deaf middle school students who participated in this study are a heterogeneous group with unique backgrounds in language, learning, and experiences, even though they used sign language as their primary language mode. Therefore, these different experiences and language mode may have influenced different interpretations of the same text among both deaf students and hearing students.

**Educational Implications for Future Instruction**

This study examined problem-solving behaviors as deaf middle school students read, understood, thought about, and solved word problems using think-aloud techniques, in order to explore their cognitive process in problem solving. The current study was
searching for an explanation why deaf middle school students have difficulties with word problems. The research findings presented and discussed here indicate that deaf middle school children’s difficulties with word problems are related both to their lack of knowledge in English and mathematics, and to experiences that may result from their different primary language mode and early language exposure and language environments. This study discussed how deaf students do not have experiences with a variety of problem types. The lack of such experience contributed to the difficulties with word problems, rather than a real difference in abilities. If efficient and explicit instruction were provided to deaf students in the future, their difficulties with these types of word problems could be mitigated.

Some researchers and instructors in the field had blamed such difficulties on deaf students’ inability to read, rather than attempting to understand their characteristics in terms of language and learning (Marschark, 2008). Stahl, Kuhn, and Pickle (1999) pointed out that instructors or researchers do not “label to excuse our failures to teach by blaming the students for their failure” (p. 269). Rather, they suggested that “we should accept that some children are harder to teach, and we need to work harder to reach those children” (2006, p. 390). Therefore, teachers should understand and support deaf students’ individual differences in problem-solving activities and learning. Instructors must help struggling deaf students to learn how to solve word problems successfully by providing a variety of problem types, and have high expectations for the problem-solving achievement of their deaf students.

This study does not intend to blame teachers of deaf or hard of hearing students,
but rather proposes a reconceptualization of teacher education within deaf education as a productive way to improve how teachers are prepared for teaching deaf students in content areas as well as English language. Accordingly, I make several recommendations for future instructions.

1. Teachers should understand the characteristics and needs of deaf learners in different language and learning modes before providing explicit instruction about word problems, to meet their level of language and learning appropriately.

2. Teachers should provide challenging problems with a variety of types of problem structures, and teach word problems based on a situational model to avoid misinterpretation of various conceptual aspects (in this case, the relational terms “more” and “less,” and fraction concepts) of problem structures.

3. Teachers should encourage students to learn how to distinguish among different types of problem structures and represent the problems in their own words (Mayer, 2007).

4. Students should receive instruction and practice in developing metacognitive awareness and learning how to use strategies flexibly and efficiently.

Limitations of the Study

There are five major limitations of the current study.

1. This study attempted to investigate the interaction of participants’ knowledge structures and text structures using think-aloud techniques, to understand better students’ cognitive process in problem solving. This method may be not
appropriate for some students who were not familiar with talking about their thinking, due to their limited language or communication abilities, although it is a powerful tool for gaining information about students’ cognitive processes (Ericsson and Simon, 1993).

2. This study included only deaf students who used sign language as their first language in residential school classrooms. Because of time constraints and the difficulty of communication access, this study does not include students who have different communication modes in oral or total communication classrooms.

3. This study does not provide enough explanation for why deaf middle school students mainly committed reversal errors on IL problems. Several studies indicated that reversal errors are related to working memory or memory issues (Mayer & Lewis, 1987; Pape, 2003, 2004; Verschaffel, 1992, 1994).

4. This study did not explore students’ strategy use during word problem solving, but rather investigated their strategy use by utilizing a structured questionnaire to measure their mathematics learning strategy.

5. This study did not include hearing students as a comparison group. It would be inappropriate to compare similarities and differences between deaf students and hearing students, based on the relevant literature.

**Recommendations for Future Study**

This study explored and described the processes of understanding and solving word problems among deaf middle school students, as they read, understood, and solved
certain word problems. Overall, the goal was to examine why deaf middle school students have difficulty with word problems. However, there are two main questions that we were not able to answer in this study: how do deaf middle school students process IL problems, and what kind of metacognitive strategies did they use as they read and solved these problems? This section therefore provides important recommendations for future research.

The first recommendation for future study is to extend this study to students of a variety of ages, in order to gain better perspective about similarities and differences in mathematical problem-solving behaviors between hearing and d/Deaf or hard of hearing students, or among d/Deaf or hard of hearing students. This study included only deaf students who used only sign language in a residential education program. Thus, we need to know more about how deaf students who use different communication modes in other types of educational programs understand and solve word problems, and how their word problem-solving success is influenced by language communication modes and learning environments.

The second suggestion is to explore working memory in terms of word problem performance, in order to determine whether there are similarities and differences in the processing of English word problems between hearing and d/Deaf or hard of hearing students. As mentioned in the previous chapters, the problem-solving behaviors of a deaf student in the study who used English based phonology are different from those of deaf students who used visual or sign based coding when they read word problems. Some studies have suggested that working memory is related to the development of vocabulary
and syntax in English. It would be beneficial to develop curriculum to reflect the special instructional needs of deaf students, as well as to understand why deaf or hard of hearing students have difficulty with word problems.

A third avenue for future study could be to study the process of representing word problems among deaf or hard of hearing students using a variety of visual tools, such as written or graphic representations and blocking, because some students are not familiar with expressing their thoughts using think-aloud protocols. It would be important to have a more precise picture of the cognitive processes in such student groups using a variety of research methods and procedures. Finally, further research is needed to explore the metacognitive strategies deaf students use while they read and solve word problems. Some studies have suggested that deaf students have insufficient metacognitive strategies, which also affects their problem solving success (Kelly, 2008; Marschark, 2008).
List of References


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Appendix A

Self-Regulation Learning Strategy Questionnaire

This questionnaire is designed to help us learn about the strategies you are using in your mathematics class to help you with your work. This learning strategies section includes 31 items regarding your use of different learning strategies. This is not a TEST and there is no right or wrong answers to this questionnaire. We would like to know more about what you think, feel, and do when you learn or study mathematics. Please answer as accurately and honestly as possible.

Thanks for your participant!

Chongmin Lee & Douglas T. Owens

The Ohio State University
Directions: The following questions ask you about your learning strategies and study skills for mathematics class. Please circle the numbers that best describes how you study in mathematics class as accurately as possible. If you do not understand the words of sentences or questions, you can ask me about them and I will explain the meanings of the words or meanings of questions. If necessary, I will give you specific examples to help your understanding.

<table>
<thead>
<tr>
<th>#</th>
<th>Items</th>
<th>Not at all true</th>
<th>Rarely true</th>
<th>Somewhat true</th>
<th>Mostly true</th>
<th>Very true</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>When I study for math, I read my notes, my homework, and the textbook over and over</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>When doing work for math, I try to relate what I’m learning to what I already know.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>When I study for math, I go through my class notes and the text book and try to find the most important ideas</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>In math, I always put a lot of effort into doing my work</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>Before starting a math assignment, I try to figure out the best way to do it</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>When I’m working on math, I stop once in a while and go over what I have been doing</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>I try to change the way I study for math to fit the type of material I am trying to learn</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>I work hard to do well in math class even if I don’t like what we’re doing</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>Even when I have a lot of trouble learning math, I don’t ask for help.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
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<tr>
<td>10</td>
<td>When I study for math, I copy my notes over to help me remember the material</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>11</td>
<td>I try to make all the different ideas fit together and make sense when I study for math</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>When I study for math, I go over my class notes and the textbook and make an outline of important concepts or equations</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>#</td>
<td>Items</td>
<td>Not at all true</td>
<td>Rarely true</td>
<td>Somewhat true</td>
<td>Mostly true</td>
<td>Very true</td>
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</tr>
<tr>
<td>13</td>
<td>Before I begin to study for math, I think about things I will need to do to learn</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>14</td>
<td>In math, I keep track of how much I understand the work, not just if I am getting the right answers</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>15</td>
<td>If what I am working on for math is difficult to understand, I change the way I learn the material</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>16</td>
<td>When work is hard, I either give up or only study the easy parts</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>17</td>
<td>I ask my teacher to clarify math concepts I don’t understand well</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
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<tr>
<td>18</td>
<td>To learn the material for my math class, I practice saying the important material until I know it</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>19</td>
<td>I make up my own math problems to help me understand the important concepts</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>In math, I start my assignments without really planning how I will get it done</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>21</td>
<td>For math assignments, I double check my work to make sure I am doing it right.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>22</td>
<td>When I come across difficulty doing a math problem, I go back and try to figure out where I went wrong.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>23</td>
<td>Even when studying math is dull and uninteresting, I keep working until I finish.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>24</td>
<td>When I don’t understand mathematics, I ask one of my classmates for help.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>25</td>
<td>When I study for math, I memorize key equations or facts.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>#</td>
<td>Items</td>
<td>Not at all true</td>
<td>Rarely true</td>
<td>Somewhat true</td>
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<tr>
<td>26</td>
<td>When I study for math, I put important ideas into my own words.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>27</td>
<td>Before I begin to solve a math problem, I think through it and decide what I need to get done</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>28</td>
<td>When I study for math, I ask myself questions to make sure I understand the material I have been studying</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>29</td>
<td>I try to adapt how I do my math assignments to fit with what the teacher wants or expects</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>30</td>
<td>I don’t try very hard to finish my math assignments</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>31</td>
<td>I know who to ask for help in mathematics assignments</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>32</td>
<td>When I do math homework, I try to remember what the teacher said in class so I can answer the questions correctly.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>33</td>
<td>During class time I often find that I think of other things and don’t really listen to what is being said</td>
<td>1</td>
<td>2</td>
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</tbody>
</table>
Appendix B

Think-Aloud Protocols

Directions:

This purpose of the test is to help me understand what students think about while reading and solving math word problems. This is not a test and is not related in any way to your grade in mathematics class. I will give you two forms for word problem solving. A form is for measurement of your problem-solving ability and another form is for measurement of your confidence level in solving the word problems.

I want to know what comes to mind when reading, thinking, and solving the word problems. You will be asked to think aloud (or think sign) using your preferred communication mode as you read and solve the problems.

You will be asked:

1. SAY or SIGN whatever comes to mind as you attempt to read and solve the word problems. If you are silent, I will remind you to think aloud (or think sign) as often as necessary by asking “What are you doing now?”

2. After reading each problem, you will be asked to recall the problem sentence out loud by being asked “What is this problem asking?” before attempting to solve the problem.

3. After recalling the problem, you will proceed to the solving steps. After solving each problem, you will be asked to describe or explain how you solved these problems. If you cannot solve the problem, I will ask why you feel you cannot
solve the problem

4. Finally, you will be asked to rate your confidence in your own ability to solve the word problem on a scale of 1 to 5 after solving each word problem on a separate paper and then move on to the next problem.

If you were not talking or signing out loud, I will remind you to think aloud or think sign everything that comes to you. Keep in mind you cannot ask any questions to me during word problem solving. If you cannot understand the sentence, just explain to me why you cannot understand that.

I will give you two practice problems before 16 word problems are presented so that you can feel comfortable the think-aloud or think-sign process and you will practice the problems with me.

Here are two “practice problems.”

1. Five children share 12 markers equally. How many markers does each child get?

2. Emily bought 2 gallons of milk. Each gallon cost $2.79. He paid with a $10 bill. How much change did he receive?
1. The local farm stand sells about 15 watermelons each day during the summer. The Supermarket sells 3 times as many as the farm stand a day. How many watermelons does the supermarket sell in 5 days?

2. Emily went to the store 15 times last month. She buys 5 oranges each time he goes to the store. How many oranges did Emily buy last month?

3. At BP, gas sells for $1.13 per gallon. Gas at Speedway is 5 cents less per gallon than gas at BP. How much does 5 gallons of gas cost at Speedway?

4. The school is planning a field trip. There are 540 students and 60 seats on each school bus. How many buses are needed to take the trip?

5. Jane has enough money to buy four books per week. How many weeks until Jane can buy 24 books?

6. At Meijer a pound of sugar costs 89 cents. That is 20 cents more per pound than at Walmart. How much do 5 pounds of sugar cost at Walmart?

7. Kroger sells 120 bottles of water a day. That is 2 times as many bottles as Giant’s sells in a day. How many bottles of water does Giant’s sell in 5 days?

8. Mary runs about 6 miles per week. Sandy runs 3 times as many miles per week as Mary. How far does Sandy run in a week?
9. At BP, gas sells for $1.13 per gallon. Gas at Speedway is 5 cents more per gallon than gas at BP. How much does 5 gallons of gas cost at Speedway?

10. Tom has 4 boxes of apples. Each box holds 8 apples. How many apples does Tom have?

11. Jane has enough money to buy four books per week. How many weeks until Jane can buy 24 books?

12. Sam’s Grocery sells 180 eggs a day. That is 1/3 as many eggs as Mike’s Grocery sells a day. How many eggs does Mike’s Grocery sells in 3 days?

13. Joe runs 6 miles a week. He runs 1/3 as many miles a week as Ken does. How many miles does Ken run in a week?

14. Meijer sells 50 pounds of potatoes a day. Kroger sells 1/5 as many potatoes as Meijer does in a day. How many pounds of potatoes does Kroger sell in 4 days?

15. Danny’s pizzeria sells 120 regular pizza pies a day. Angela’s Pizzeria sells 1/3 as many regular pies as Danny’s in a day. How many regular pizza pies does Angela’s Pizzeria sell in a day?

16. At Kroger a pound of pears cost $1.16. That is 15 cents less per pound than at Walmart. How much does 5 pounds of pears cost at Walmart?
Appendix C

Computation Test

Participation code:

Mathematics Computations: please answer each of the following problems. Show all work of how you solved each problem.

1a. How many feet are in 3 miles? (1 mile=5,280 feet.)

1b. How many feet are in one-third (\(\frac{1}{3}\)) of a mile?

2. Write each fraction in lowest terms or simplest form:
   \[ a) \frac{16}{24} \quad b) \frac{25}{125} \]

3. Compute:
   \[ a) \frac{1}{5} \times 60 = \quad b) \frac{1}{4} \times 32 = \]
   \[ C) 48 \div 2 \quad d) 57 \div \frac{1}{3} \]
4. Compute:
   a) 1.76
   b) 2.97
   − .18
   ÷ .08

5. Compute:
   a) 3.47
   b) 4.57
   × 5
   × 6

6. Draw a picture to illustrate the following examples:
   a) \(3 \times \frac{2}{5}\) (three times two-fifths)
   b) \(\frac{1}{4} \times 16\) (one-fourth of sixteen)
Appendix D

Student Background Survey

Student’s initials: ___________ Participant Code: ___________

1. Gender a. Male b. Female

2. Date of Birth:

3. Grade:

4. Parents’ hearing status
   a. Both Deaf   b. Both hearing   c. Mother: Deaf   Father: Hearing
   d. Father: Deaf, Mother: Hearing   e. 

5. Siblings’ hearing status (the number of Siblings)
   a. hearing   b. deaf   c. hearing and deaf (   )

6. Primary communication mode at home
   a. sign language   b. spoken language

7. Primary communication mode at school
   a. sign language   b. spoken language

From school student record:

8. Hearing Level: L: _____ R:

9. Standardized test score Data:
   Date of Administration:
   Mathematics :
   Reading:__________