SENSORLESS VECTOR CONTROL AND FIELD WEAKENING OPERATION OF PERMANENT MAGNET SYNCHRONOUS MACHINES

DISSERTATION

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By

Yuan Zhang, M. S.

Graduate Program in Electrical and Computer Engineering

The Ohio State University

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Dissertation Committee:

Dr. Longya Xu, Advisor

Dr. Vadim I. Utkin

Dr. Donald G. Kasten

Dr. Jin Wang
ABSTRACT

For the last couple of decades, Permanent Magnet Synchronous Machines (PMSMs) have attracted great attention in various industry applications such as hybrid electric vehicle systems, direct drive wind power generation applications, direct drive washing machines and servo systems etc. due to their high power density, wide range of constant power operation and good torque-speed response. This work focuses on developing and implementing simple yet reliable sensorless vector control algorithms for PM machines for the full speed range including zero speed.

Speed or position sensors used to obtain the rotor position information for high performance vector control of PM machines add more complexity to the hardware system, have poor immunity to electromagnetic noise and reduce the reliability of the system. Two sensorless algorithms are investigated in this work: 1) High frequency carrier signal injection based sensorless algorithm for zero and low speed operation; 2) Voltage model based sensorless algorithm for the middle and high speed operation. Initial rotor position is identified using the magnetic saturation effect.

Difference in the $d$- and $q$-axis inductances results in additional reluctance torque in Interior Permanent Magnet (IPM) synchronous machines, which makes them very suitable for operation with a constant output power over a very wide speed range theoretically. It is known that the field weakening control is required to achieve the
constant power operation. Literature review indicates that stable control of IPM machines in the deep field weakening operation region still remains an issue to achieve the largest possible speed range. A single-current-regulator algorithm which gives a fixed command for the $q$-axis voltage and only controls the $d$-axis current is investigated in this work. The criteria of selecting the optimal $q$-axis voltage in terms of variable speeds and load conditions are proposed to achieve better operating efficiency and robustness of the system in the deep field weakening operation region.

The sensorless vector control algorithms and the single-current-regulator algorithm with optimal $q$-axis voltage control are tested on a 50 kW IPM machine system across the full range of speed and load to substantiate the theoretical work.
Dedicated to people that I love.
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VITA

Jul. 7th, 1980 ......................... Born - Nanjing, Jiangsu, China.

Jul. 2002 .............................. B.S. Nanjing University of Aeronautics and Astronautics, Nanjing, China.

Apr. 2005 .............................. M.S. Nanjing University of Aeronautics and Astronautics, Nanjing, China.

Sept. 2005 - Apr. 2010 ................. Graduate Research and Teaching Associate, The Ohio State University, Columbus, OH.

Apr. 2010 - present .................... Engineer, The Ford Motor Company, Dearborn, MI.

PUBLICATIONS

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FIELDS OF STUDY

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CHAPTER 1

Introduction

Since the first power semiconductor was born more than 50 years ago, the rapid progress in the power semiconductor technology during the past one to two decades has shed light on industry applications of AC electric machines. Super fast microprocessors and modern control theories have made it possible to control an AC electric machine as convenient as a DC machine. AC electric machines have been replacing, in more industry applications, the DC electric machines with brushes, which are maintenance intensive and impossible for high speed applications.

Among various types of AC electric machines, Permanent Magnet (PM) synchronous machines are one of the most competitive AC machines that receive tremendous amount of attention from industry and research organizations. For many industry applications, PM machines beat induction machines or wound synchronous machines due to their high efficiency and high power density [1].

PM machines can be divided into two categories due to different rotor structures demonstrated in Figure 1.1. Interior PM machines have the permanent magnets buried under the surface of the rotor, which causes the difference between the $d$- and $q$-axis inductances and also the reluctance torque. Surface PM machines have the permanent magnets mounted on the surface of the rotor. While this structure is
characterized with equal $d$- and $q$-axis inductances. A synchronous machine, typically having windings wound on the rotor to provide the rotor flux, is also shown for comparison since all these three types of synchronous machines can be modeled with very similar differential equations though they have different rotor structures.

Both the interior PM machines and the wound synchronous machines have saliency on the rotors due to the difference between the $d$- and $q$-axis inductances resulting in reluctance torque. The characteristics of inductances and electromagnetic torque for the above three types of synchronous machines are summarized and compared in Table 1.1.

Table 1.1: Comparisons of three types of synchronous machines

<table>
<thead>
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<th>SPM machine</th>
<th>Synchronous machine</th>
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<td>$L_d$ and $L_q$</td>
<td>$L_d &lt; L_q$</td>
<td>$L_d = L_q$</td>
<td>$L_d &gt; L_q$</td>
</tr>
<tr>
<td>Reluctance torque</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
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1.1 Industrial Applications of PM Synchronous Machines

The most famous industry application of PM machines currently would be the Hybrid Electric Vehicle (HEV). Toyota Motor Company introduced the first hybrid electric vehicle Prius into the automobile market in 2002, which uses two Interior Permanent Magnet (IPM) synchronous machines, one for drive purpose and the other for generating and regenerating purpose, to work with the gasoline engine together to achieve much higher operation efficiency for the vehicles [2]. The power ratings of the IPM synchronous machines increase from about 30 kW for the first generation used on Prius to up to around 100 kW for the IPM machines used on Ford Fusion hybrid cars.

Electro-mechanical energy conversion having the highest converting efficiency compared to all the other forms of energy conversions [3] makes the application of AC electric machines very competitive in the renewable energy conversion area. PM machines, of course, have also been successfully applied into the wind power generation area. Typically, PM generators are used to drive the wind turbine directly without any gearbox coupling them. The PM generator converts the mechanical power from the wind turbine to AC power first then to DC power through a rectifier. The DC power then can be converted to AC power of variable magnitude and frequency. The ratings of PM machines used on the commercial wind turbine products generally vary between tens of kilowatts to a few megawatts.

The most successful application of PM machines in house appliances is the direct drive washing machines, which typically use disk surface PM synchronous machines to drive the washing chamber directly without any belt or chain connection. Commercial washing machine products generally employ surface PM synchronous machines rated
around a few kilowatts, which could save up to 30 percent of energy compared to those traditional belt-driven washing machines.

Engineers in aviation industry have been searching solutions for “more-electric” aircrafts for a long time. The aviation industry has also dipped its toe into the water of application of PM synchronous machine drives [4]. The state-of-the-art big aircraft Boeing 787 dreamliner applies two PM machines as backup generators. PM machines are also used to drive flight control surfaces instead of using hydraulic systems which are vulnerable in combats, require intensive maintenance, and contain miles of complicated pipes and tubing.

Other industry applications of PM machines can be found in servo systems, air compressors, various types of pumps, which will not be listed comprehensively in this work.

1.2 Motivations of This Work

Typically, a modern AC electric machine system consists of the following major components as shown in Figure 1.2: an AC electric machine, a controller, a power converter to drive the machine, current and voltage transducers, and position or speed sensors. The current, voltage and rotor position information is obtained by sensors and fed back to the controller. The controller executes certain control algorithms and output the control signals which are then amplified by the power converter to drive the electric machine ultimately.

Modern AC electric machine control generally applies the vector control and requires the information of rotor position to accurately control the flux and the torque
Figure 1.2: General configuration of modern AC machine drive system

of the machine. Hall-effect sensors, encoders and resolvers are common sensors widely used in AC electric machine systems to obtain the rotor position information.

The position or speed sensors mentioned above are extra hardware mounted on the shaft of electric machines. They increase the system cost and require maintenance periodically. Those sensors can also be subject to the EMI issue caused by the power electronic circuits switching at a frequency varying from several kilo Hz to tens of kilo Hz. Failure in position sensors can cause failure of the whole system, which reduces the system reliability.

One of the motivations in this work is to eliminate position or speed sensors to increase the reliability, to reduce the cost, and to simplify the hardware system of the AC electric drive and generating systems. Position sensorless technology is to employ the existing information of currents and voltages of electric machines and obtain the rotor position information with various estimators or observers. Much effort was expended on investigating various sensorless vector control algorithms for PM machines from theory to implementation in this work.
PM machines, especially interior PM machines, have very good capability of providing constant power in a very wide speed range. Lately, due to advanced electric machine design methods, an IPM machine system with a much improved Constant Power Speed Ratio (CPSR) has been demonstrated. In the demonstrated system, a CPSR as high as 9.5 has been achieved [21]. With substantial progress and achievement in machine design techniques and control theories, IPM machines have become one of the most favorable choices and have been used for even wider and higher power applications.

Literature review indicates that stable control of IPM machines in the deep field weakening operation region still remains an issue to achieve the largest possible speed range. Traditional vector control algorithms for IPM machines apply two independent current regulators, one for the $d$-axis current regulation and the other $q$-axis current regulation. Good performance can be achieved with well regulated currents in the constant torque region and slightly beyond. However, in the deep field weakening operation region, the effects of terminal voltage limitation become serious. Saturation of the two current regulators often occurs when the command voltage is larger than the maximum voltage available. Consequently, the two current regulators conflict with each other, leading to nuisance instability of current and torque of IPM machines. As a result, the theoretical speed range of IPM machines is not attainable.

Therefore, another motivation in this work is to investigate reliable control algorithms for the field weakening operation of PM machines so the designed theoretical speed range can be achieved.
1.3 Literature Review

1.3.1 Sensorless Vector Control of PM Machines

Frequently used sensorless vector control algorithms for PM machines can be categorized by the following:

- Flux estimation by the voltage model;
- Model Reference Adaptive Schemes (MRAS);
- Sliding mode observer;
- Extended Kalman filter;
- Flux estimation using other advanced control theories like neural network, fuzzy logic;
- Saliency detection based on high frequency carrier signal injection.

Flux estimation by the voltage model uses the voltage and current information to estimate the back EMF produced by the permanent magnets first, further to derive the rotor flux linkage \[5,6\]. The current information typically is obtained from the current transducers. The line-to-line or phase voltage can be either from the voltage transducers or calculated with the switching functions and the DC bus voltage information.

Model Reference Adaptive Schemes (MRAS) based sensorless algorithms have also been attractive to many researchers and applications. In general, a MRAS observer consists of three major blocks: the reference model which does not contain the variable to be estimated, the adjustable model which contains the variable to be estimated and
the adaptive mechanism. Typically, two variables estimated from the reference model and the adjustable model are compared to each other and the error between the two variables is used to drive the adaptive mechanism to calculate the rotating speed of the machine. In [7,8], the error between two estimators of the back EMF of the PM machine is used to drive an adaptive mechanism to calculate the machine speed: a state observer model based on the voltage equations in the stationery reference frame and a magnetic flux observer model using the estimated rotor speed. However, this method fails when the back EMF is very low when the speed goes below a certain point. Besides, MRAS based sensorless algorithms heavily rely on the accuracy of the reference model which contains the machine parameters. Therefore, MRAS based sensorless algorithms also suffer from inaccurate knowledge of machine parameters.

The sliding mode observer is another very attractive method to identify the rotor position due to its low sensitivity to machine parameter change and good immunity to system disturbances [9-11]. The sliding mode observer observes the stator currents and compares the estimated currents to the measured stator currents. The sliding mode function is selected to be the estimated back EMF and the sliding mode surface is then chosen to be the difference between the estimated and the measured stator currents. When the trajectory of estimation error reaches the sliding surface, the estimated stator currents converge to the measured stator currents. Hence, the estimated back EMF can be obtained by applying a low pass filter to the switching function. The drawback of the sensorless algorithm based on the sliding mode observer is that the back EMF estimation is not accurate at low speed or even fails at zero speed resulting in inaccurate rotor position estimation.
Sensorless algorithm based on extended Kalman filter is an optimal estimator in the least-square sense for estimating the machine speed using the measured voltages and currents [12, 13]. The extended Kalman filter based sensorless algorithm requires very powerful microprocessors to conduct the complicated calculation. The machine model used is typically in the stationery reference and strongly depends on the machine parameters. Also, the initial rotor position is not available using the extended Kalman filter. Due to these facts, the sensorless algorithm based on extended Kalman filter technique is not really favored by many researchers.

The sensorless algorithms discussed above depend on the back EMF information of the rotor. Therefore, they do not work at very low speed or even fail at zero speed. However, the method of injection of high frequency carrier signal is effective for rotor position estimation at zero and very low speed since this method is based on detecting the rotor saliency. For machines like interior PM machines, the rotor saliency is always there regardless of the machine speed.

The method of injecting a rotating high frequency carrier signal was proposed to detect the saliency of the rotor [14, 15]. The rotating high frequency carrier signal is modulated by the rotor saliency and the resulting high frequency current contains the rotor position information, which is obtained by certain signal processing methods.

The method of injecting an oscillating high frequency carrier signal is proposed to estimate the rotor position for interior PM machines [16,17]. The direction of the injected oscillating high frequency carrier voltage vector is along the estimated $d$-axis of the machine. If the estimated $d$-axis is perfectly aligned with the actual $d$-axis, the applied voltage vector will not excite a $q$-axis current component. However, if the estimated $d$-axis is off the actual $d$-axis of the rotor, the resulting $q$-axis current
component will be used as a measurement of the estimation error and sent to a controller to adjust the estimated rotor position until the error, or the \( q \)-axis current component, converges to zero.

### 1.3.2 Field Weakening Operation of PM Machines

The suitability of interior PM machines for constant power operation over a wide speed range has been investigated in [18, 19, 20]. With advanced machine design techniques, an interior PM machine with a constant power speed ratio as high as 9:1 is achieved in [21]. However, literature review indicates that stable control of PM machines in the deep field weakening operation region still remains an issue to achieve the largest possible speed range [22-26].

Most of field weakening operation algorithms apply two current regulators, one for the \( d \)-axis current control and the other for the \( q \)-axis control. Major effort in expanding the operation speed by various methods is to keep two current regulators from getting saturated.

A comprehensive in-depth discussion of the field weakening control for an interior PM machine is presented in [22]. A feedforward control is introduced to decouple the control of the \( d \)- and \( q \)-axis currents. A voltage command compensation algorithm is also presented for the field weakening operation of IPM machines and experimental results on a 2:1 constant power speed ratio operation are reported.

A method calculating the demagnetizing current command \( i_d \) is proposed to try to expand the operation speed of a surface PM machine[23]. The demagnetizing current command is proportional to the \( q \)-axis current command and a 1.5:1 constant power speed ratio operation is achieved on a surface PM machine.
A method to calculate the compensation for the $d$- and $q$-axis current commands is proposed in terms of the changing rate of torque and speed for an interior PM machine, which helps to improve the dynamic performance of the machine [25]. The operation speed is expanded to 5 times the base speed using this method.

A single-current-regulator algorithm was applied on a surface PM machine to achieve a 4:1 constant power speed ratio operation [26]. The $q$-axis voltage maintains at a constant value in the field weakening operation of the surface PM machine instead of using a current regulator. The $d$-axis is adjusted actively. However, higher current is needed to deliver the same amount of torque or some operation points are not achievable at all if one fixed $q$-axis voltage is used for all speeds and torque conditions.

1.4 Chapter Review

Chapter 2 presents the reference frame transformation theory in AC electric machines and the mathematical modeling of PM machines. Then basic control algorithms of PM machines are reviewed including the $V/f$ open loop control, the current vector control using PWM and the six step method. Operating characteristics and constraints of PM machines are analyzed in the $i_d-i_q$ plane.

Chapter 3 presents two sensorless vector control algorithms for interior PM machines: sensorless algorithm based on the high frequency carrier signal injection and the voltage model based sensorless algorithm. Experimental results using both methods are applied on a 50 kW interior PM machine. The high frequency injection method is employed below 5 Hz and the voltage model based algorithm for speeds higher than 5 Hz (electrical frequency). The system makes a transition from the high
frequency injection method to the voltage model based sensorless algorithm using a weight function smoothly. The effects that the rotor position estimation error has on the torque accuracy are discussed.

Chapter 4 presents the single-current-regulator algorithm with optimal $q$-axis voltage control for the deep field weakening operation of interior PM machines and the voltage phase angle control algorithm for the deep field weakening operation of surface PM machines. Experimental results on a 50 kW interior PM machine and a 500 W surface PM machine are presented.

Chapter 5 summarizes the research work in this dissertation and provides potential future research topics.
CHAPTER 2

Mathematical Modeling and Control Basics of PM Synchronous Machines

The theory of reference frame transformation and the mathematical model of PM machines are presented in this chapter. Basic control methods and operating characteristics of PM machines are reviewed and discussed.

2.1 Theory of Reference Frame Transformation

Generally, the self-inductances and mutual inductances of the three-phase windings are functions of the rotor position for three-phase AC electric machines. Therefore, the voltage and flux linkage equations represented by the three-phase variables for AC electric machine typically contain time-variant functions. It is time consuming to solve these time-variant differential equations. In order to find an easier way to solve these equations, several types of reference frame transformations are widely applied in the mathematical modeling of AC rotating electric machines.
2.1.1 *ABC Stationary Reference Frame to αβ0 Stationary Reference Frame*

In Figure 2.1, \([f_a, f_b, f_c]^T\) represents a three-phase variable, denoted as \(f_{abc}\) in the following contents, which can be a three-phase voltage, current, magnetic flux linkage or self- and mutual inductance of stator windings in the stationery reference frame.

![Diagram of three-phase stationery reference frame, αβ0 stationery reference frame and dq0 rotating reference frame](image)

Figure 2.1: Three-phase stationery reference frame, αβ0 stationery reference frame and dq0 rotating reference frame
The synthesis vector of $f_a$, $f_b$ and $f_c$ can be decomposed into two components perpendicular to each other in the stationery $\alpha\beta0$ reference frame, where the $\alpha$ axis is aligned with vector $f_a$ and the $\beta$ axis is leading the $\alpha$ axis by 90 degrees. For an unbalanced three-phase system, the zero sequence component exists. The transformation from the three-phase stationery reference frame to the $\alpha\beta0$ stationery reference frame is expressed as,

$$f_{\alpha\beta0} = T_{abc \rightarrow \alpha\beta0} \cdot f_{abc}$$  \hspace{1cm} (2.1)

where the transformation matrix $T_{abc \rightarrow \alpha\beta0}$ is written as,

$$T_{abc \rightarrow \alpha\beta0} = \frac{2}{3} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} \sqrt{3} & 0 & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$  \hspace{1cm} (2.2)

Eq. (2.1) is known as the Clarke Transformation. Reversely, a vector can be converted from the $\alpha\beta0$ stationery reference frame to the three-phase stationery reference frame by the following equation,

$$f_{abc} = T_{\alpha\beta0 \rightarrow abc} \cdot f_{\alpha\beta0}$$  \hspace{1cm} (2.3)

where the transformation matrix $T_{\alpha\beta0 \rightarrow abc}$ is the inverse matrix of $T_{abc \rightarrow \alpha\beta0},$

$$T_{\alpha\beta \rightarrow abc} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} \sqrt{3} & 1 & 1 \\ -\frac{1}{2} & -\frac{1}{2} \sqrt{3} & 1 \end{bmatrix}$$  \hspace{1cm} (2.4)

It is necessary to point out that the three-phase voltage, current, flux linkage and inductance in an AC rotating machine still remain rotor position dependent and time varying in the $\alpha\beta0$ reference frame. It is not more convenient to solve machine
equations in the $\alpha \beta 0$ reference frame than in the three-phase stationery reference frame.

The $dq0$ rotating reference frame is then introduced to transfer the sinusoidally changing variables in the stationery reference frame into variables independent of the rotor position of the electric machine.

### 2.1.2 $\alpha \beta 0$ Stationary Reference Frame to $dq0$ Rotating Reference Frame

The $dq0$ reference frame is rotating in the space at a certain speed. In Figure 2.1, the angle between $f_\alpha$ and the rotating $d$-axis is $\theta$ and the $q$-axis is leading the $d$-axis by 90 degrees. For an unbalanced three-phase system, zero sequence component also exists. A vector in the $\alpha \beta 0$ stationery reference frame is expressed as in Eq. (2.5) seen in the $dq0$ synchronous rotating reference frame,

$$f_{dq0} = T_{\alpha \beta 0 \rightarrow dq0} \cdot f_{\alpha \beta 0}$$  \hspace{1cm} (2.5)

where the transformation matrix $T_{\alpha \beta 0 \rightarrow dq0}$ is written as,

$$T_{\alpha \beta 0 \rightarrow dq0} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (2.6)

Similarly, the transformation from the $dq0$ rotating reference frame to the $\alpha \beta 0$ stationery reference frame is expressed as,

$$f_{\alpha \beta 0} = T_{dq0 \rightarrow \alpha \beta 0} \cdot f_{dq0}$$  \hspace{1cm} (2.7)

where the transformation matrix $T_{dq0 \rightarrow \alpha \beta 0}$ is the inverse of $T_{\alpha \beta 0 \rightarrow dq0}$.

$$T_{dq0 \rightarrow \alpha \beta 0} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (2.8)
Eqs. (2.5) and (2.7) are two very important transformations in the AC rotating electric machine theory known as the Park transformation and inverse Park transformation, respectively. For a rotating vector with the same speed of the $dq0$ rotating reference frame, the trajectories of the vector on the $d$- and $q$-axis appear as DC values.

It is important to point out that the transformation between either two reference frames mentioned above is not unique. All transformations discussed above are based on the electric power remaining the same before and after the transformation. They are universal for AC three-phase electric machines such as PM machines, induction machines, doubly-fed induction machines and so on.

### 2.1.3 Mathematical Modeling of PM Machines in Stationary Reference Frame

The three-phase voltage of a PM machine in the stationery reference frame can be expressed by Eq. (2.9)

\[
\begin{bmatrix}
V_{as} \\
V_{bs} \\
V_{cs}
\end{bmatrix}
= R_s
\begin{bmatrix}
i_{as} \\
i_{bs} \\
i_{cs}
\end{bmatrix}
+ \begin{bmatrix}
p\lambda_{as} \\
p\lambda_{bs} \\
p\lambda_{cs}
\end{bmatrix}
\]

where $V_{as}$, $V_{bs}$, and $V_{cs}$ are the three-phase voltage; $i_{as}$, $i_{bs}$, and $i_{cs}$ the three-phase current; $R_s$ the resistance of phase winding; $\lambda_{as}$, $\lambda_{bs}$, and $\lambda_{cs}$ the three-phase stator winding flux linkage; $p$ the differential operator, $d/dt$. The three-phase stator winding flux linkage can be expressed as,

\[
\begin{bmatrix}
\lambda_{as} \\
\lambda_{bs} \\
\lambda_{cs}
\end{bmatrix}
= L_{abc}
\begin{bmatrix}
i_{as} \\
i_{bs} \\
i_{cs}
\end{bmatrix}
+ \begin{bmatrix}
\cos \theta_e \\
\cos (\theta_e - \frac{2}{3}\pi) \\
\cos (\theta_e + \frac{2}{3}\pi)
\end{bmatrix}
\lambda_m
\]

\[
\text{(2.10)}
\]
where $\lambda_m$ is the flux linkage generated by the permanent magnets and $\theta_r$ is the electrical angle between the axis of the permanent magnet and the axis of Phase A winding, which is shown as in Figure 2.2.

![Figure 2.2: Definition of rotor position](image)

The inductance matrix $L_{abc}$ can be written as

$$L_{abc} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}$$

(2.11)

where $L_{xx}$ represents the self-inductance of the phase winding and $L_{xy}$ the mutual inductance between two phase windings. Both the self-inductances and the mutual inductances are functions of the rotor position, which are expressed as Eqs. (2.12) to
\[
L_{aa} = L_{0s} - L_{ms} \cos 2\theta_r \tag{2.12}
\]
\[
L_{bb} = L_{0s} - L_{ms} \cos 2(\theta_r - \frac{2}{3}\pi) \tag{2.13}
\]
\[
L_{cc} = L_{0s} - L_{ms} \cos 2(\theta_r + \frac{2}{3}\pi) \tag{2.14}
\]
\[
L_{ab} = L_{ba} = -L_{0s}/2 - L_{ms} \cos 2(\theta_r - \frac{1}{3}\pi) \tag{2.15}
\]
\[
L_{ac} = L_{ca} = -L_{0s}/2 - L_{ms} \cos 2(\theta_r + \frac{1}{3}\pi) \tag{2.16}
\]
\[
L_{bc} = L_{cb} = -L_{0s}/2 - L_{ms} \cos 2(\theta_r + \pi) \tag{2.17}
\]

$L_{0s}$ in the above inductance equations represents the magnetizing inductance of windings regardless of the rotor position and $L_{ms}$ the magnetizing inductance associated with the rotor position. $L_{0s}$ is positive for various types of synchronous machines.

For surface PM machines with uniform air gap, $L_{ms}$ equals zero. With the saturation effects neglected, both the self-inductances of phase windings and the mutual-inductances between either two windings are constant regardless of the rotor position change.

For synchronous machines without permanent magnets on the rotor, $L_{ms}$ is positive. For interior PM machines, $L_{ms}$ is negative, which is consistent with the fact that the self-inductance of the phase winding is the minimum when the permanent magnet is aligned with the axis of the phase winding resulting in smaller $d$-axis inductance compared to the $q$-axis inductance.

Applying reference frame transformation from the three-phase stationary reference frame to the $\alpha\beta0$ reference frame, the voltage equations of a PM machine Eq. (2.9)
will be rewritten as,

\[
\begin{bmatrix}
V_{as} \\
V_{bs} \\
V_{cs}
\end{bmatrix} = R_s \begin{bmatrix}
i_{as} \\
i_{bs} \\
i_{cs}
\end{bmatrix} + \begin{bmatrix}
p\lambda_{as} \\
p\lambda_{bs} \\
p\lambda_{cs}
\end{bmatrix}
\]

(2.18)

\[
\begin{bmatrix}
V_{as} \\
V_{bs} \\
V_{0s}
\end{bmatrix} = R_s \begin{bmatrix}
i_{as} \\
i_{bs} \\
i_{0s}
\end{bmatrix} + \begin{bmatrix}
p\lambda_{as} \\
p\lambda_{bs} \\
p\lambda_{0s}
\end{bmatrix}
\]

(2.19)

where the flux linkages in \(\alpha\beta\) reference frame are written as,

\[
\begin{bmatrix}
\lambda_{as} \\
\lambda_{bs} \\
\lambda_{0s}
\end{bmatrix} = T_{abc\rightarrow\alpha\beta0} \begin{bmatrix}
\lambda_{as} \\
\lambda_{bs} \\
\lambda_{cs}
\end{bmatrix}
\]

(2.20)

Substituting (2.10) into (2.20), we will get

\[
\begin{bmatrix}
\lambda_{as} \\
\lambda_{bs} \\
\lambda_{0s}
\end{bmatrix} = T_{abc\rightarrow\alpha\beta0} \begin{bmatrix}
l_{as} \\
l_{bs} \\
l_{cs}
\end{bmatrix}
\]

(2.21)

An identical matrix \(I = T_{abc\rightarrow\alpha\beta0}^{-1} T_{abc\rightarrow\alpha\beta0}\) is inserted into Eq. (2.21)

\[
\begin{bmatrix}
\lambda_{as} \\
\lambda_{bs} \\
\lambda_{0s}
\end{bmatrix} = T_{abc\rightarrow\alpha\beta0} L_{abc,s} T_{abc\rightarrow\alpha\beta0}^{-1} T_{abc\rightarrow\alpha\beta0} \begin{bmatrix}
i_{as} \\
i_{bs} \\
i_{cs}
\end{bmatrix}
\]

(2.22)

\[
\begin{bmatrix}
\cos \theta_r \\
\cos (\theta_r - \frac{2\pi}{3}) \\
\cos (\theta_r + \frac{2\pi}{3})
\end{bmatrix}
\]

\[
\begin{bmatrix}
l_{as} \\
l_{bs} \\
l_{0s}
\end{bmatrix}
\]

(2.23)

\[
\begin{bmatrix}
\lambda_{as} \\
\lambda_{bs} \\
\lambda_{0s}
\end{bmatrix} = L_{\alpha\beta0,s} \begin{bmatrix}
i_{as} \\
i_{bs} \\
i_{0s}
\end{bmatrix} + T_{abc\rightarrow\alpha\beta0} \begin{bmatrix}
\cos \theta_r \\
\cos (\theta_r - \frac{2\pi}{3}) \\
\cos (\theta_r + \frac{2\pi}{3})
\end{bmatrix}
\]

(2.24)

Substituting \(T_{abc\rightarrow\alpha\beta0}\) and its inverse matrix into Eq. (2.24), we will get the flux

linkage equations in the \(\alpha\beta0\) reference frame,

\[
\begin{bmatrix}
\lambda_{as} \\
\lambda_{bs} \\
\lambda_{0s}
\end{bmatrix} = L_{\alpha\beta0,s} \begin{bmatrix}
i_{as} \\
i_{bs} \\
i_{0s}
\end{bmatrix} + \begin{bmatrix}
\cos \theta_r \\
\sin \theta_r \\
0
\end{bmatrix} \lambda_m
\]

(2.25)
where the inductance matrix in the $\alpha\beta$ reference frame is expressed as,

$$ L_{\alpha\beta, s} = T_{abc \rightarrow \alpha\beta} L_{abc, s} T_{abc \rightarrow \alpha\beta}^{-1} $$  \hspace{1cm} (2.26)

$$ = \begin{bmatrix}
\frac{3}{2}(L_0 - L_{ms} \cos 2\theta_r) & \frac{3}{2}L_{ms} \sin 2\theta_r & 0 \\
\frac{3}{2}L_{ms} \sin 2\theta_r & \frac{3}{2}(L_0 + L_{ms} \cos 2\theta_r) & 0 \\
0 & 0 & 0 \\
\end{bmatrix} $$  \hspace{1cm} (2.27)

In Eq. (2.19), the zero sequence component of the current, $i_0$, is expressed as,

$$ i_0 = \frac{1}{3}(i_{as} + i_{bs} + i_{cs}) $$  \hspace{1cm} (2.28)

In a PM machine with a wye-connection or delta connection, the summation of the three-phase current should be equal to zero.

$$ i_{as} + i_{bs} + i_{cs} = 0 $$  \hspace{1cm} (2.29)

Therefore, the zero sequence component of the current is equal to zero regardless of whether the three-phase current is balanced or not. Consequently, the zero sequence voltage in Eq. (2.19) also equals zero. Hence, the machine model in the $\alpha\beta$ reference frame can be simplified as expressed in Eqs. (2.30) - (2.31),

$$ \begin{bmatrix} V_{\alpha s} \\ V_{\beta s} \end{bmatrix} = R_s \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix} + \begin{bmatrix} \frac{d\lambda_{\alpha s}}{dt} \\ \frac{d\lambda_{\beta s}}{dt} \end{bmatrix} $$  \hspace{1cm} (2.30)

$$ \begin{bmatrix} \lambda_{\alpha s} \\ \lambda_{\beta s} \end{bmatrix} = \begin{bmatrix}
\frac{3}{2}(L_0 - L_{ms} \cos 2\theta_r) & \frac{3}{2}L_{ms} \sin 2\theta_r \\
\frac{3}{2}L_{ms} \sin 2\theta_r & \frac{3}{2}(L_0 + L_{ms} \cos 2\theta_r) \\
\end{bmatrix} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix} + \lambda_m \begin{bmatrix} \cos \theta_r \\ \sin \theta_r \end{bmatrix} $$  \hspace{1cm} (2.31)

Though the equations representing a rotating electric machine in the $\alpha\beta$ stationery reference frame still remain time-varying and rotor position dependent and solving these equations is not less complicated than solving those in the three-phase stationery reference frame, only two phase currents still appear in the machine model, which means only two current transducers are necessary for the current feedback control.
2.1.4 Mathematical Model in Rotating Reference Frame

It has been shown that all zero sequence components of the voltage, current and flux linkage are zero in the \( \alpha \beta 0 \) stationery reference for a wye-connection or delta-connection PM machine. Hence, the zero sequence components of these variables are also zero in the \( dq \) rotating synchronous rotating reference frame. Therefore, the transformation from the \( \alpha \beta 0 \) to the \( dq0 \) reference frame can then be simplified as,

\[
f_{dq} = T_{\alpha \beta \rightarrow dq}(\theta_r) f_{\alpha \beta}
\]

where the transformation matrix \( T_{\alpha \beta \rightarrow dq} \) is written as,

\[
T_{\alpha \beta \rightarrow dq}(\theta_r) = \begin{bmatrix}
\cos \theta_r & \sin \theta_r \\
-\sin \theta_r & \cos \theta_r
\end{bmatrix}
\]

Applying the \( dq \) transformation to the voltage equation in the \( \alpha \beta \) reference frame, we will get,

\[
T_{\alpha \beta \rightarrow dq}(\theta_r) \begin{bmatrix} V_{\alpha s} \\ V_{\beta s} \end{bmatrix} = R_s T_{\alpha \beta \rightarrow dq}(\theta_r) \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix} + T_{\alpha \beta \rightarrow dq}(\theta_r) \begin{bmatrix} \frac{d\lambda_{\alpha s}}{dt} \\ \frac{d\lambda_{\beta s}}{dt} \end{bmatrix}
\]

\[
\begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} = R_s \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \frac{d}{dt} T_{\alpha \beta \rightarrow dq}(\theta_r) \begin{bmatrix} \lambda_{\alpha s} \\ \lambda_{\beta s} \end{bmatrix} - \frac{d}{dt} T_{\alpha \beta \rightarrow dq}(\theta_r) T_{\alpha \beta \rightarrow dq}(\theta_r) \begin{bmatrix} \lambda_{\alpha s} \\ \lambda_{\beta s} \end{bmatrix}
\]

\[
= R_s \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \frac{d}{dt} T_{\alpha \beta \rightarrow dq}(\theta_r) \begin{bmatrix} \lambda_{\alpha s} \\ \lambda_{\beta s} \end{bmatrix} - \frac{d}{dt} T_{\alpha \beta \rightarrow dq}(\theta_r) d\theta_r dt T_{\alpha \beta \rightarrow dq}(\theta_r) T_{\alpha \beta \rightarrow dq}(\theta_r) \begin{bmatrix} \lambda_{\alpha s} \\ \lambda_{\beta s} \end{bmatrix}
\]

\[
= R_s \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{ds} \\ \lambda_{qs} \end{bmatrix} - \omega_r \begin{bmatrix} 0 \\ -1 \end{bmatrix} \begin{bmatrix} \lambda_{ds} \\ \lambda_{qs} \end{bmatrix}
\]

\[
= R_s \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{ds} \\ \lambda_{qs} \end{bmatrix} - \omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{ds} \\ \lambda_{qs} \end{bmatrix}
\]

\[
= \begin{bmatrix} \lambda_{ds} \\ \lambda_{qs} \end{bmatrix}
\]
The flux linkages in the $dq$ reference frame in Eq. (2.38) are obtained by applying the $dq$ transformation to the flux linkages in the $\alpha\beta$ reference frame,

\[
\begin{bmatrix}
\lambda_{ds} \\
\lambda_{qs}
\end{bmatrix} = T_{\alpha\beta\rightarrow dq} \begin{bmatrix}
\lambda_{\alpha s} \\
\lambda_{\beta s}
\end{bmatrix}
\]

(2.39)

\[
= T_{\alpha\beta\rightarrow dq} \begin{bmatrix}
L_{\alpha \beta} \begin{bmatrix}
i_{\alpha s} \\
i_{\beta s}
\end{bmatrix} + \lambda_m \begin{bmatrix}
\cos \theta_r \\
\sin \theta_r
\end{bmatrix}
\end{bmatrix}
\]

(2.40)

\[
= T_{\alpha\beta\rightarrow dq} L_{\alpha \beta} T^{-1}_{\alpha\beta\rightarrow dq} T_{\alpha\beta\rightarrow dq} \begin{bmatrix}
i_{\alpha s} \\
i_{\beta s}
\end{bmatrix} + T_{\alpha\beta\rightarrow dq} \begin{bmatrix}
\cos \theta_r \\
\sin \theta_r
\end{bmatrix} \lambda_m
\]

(2.41)

By substituting the $dq$ transformation matrix Eq. (2.33) into Eq. (2.41), we will get,

\[
\begin{bmatrix}
\lambda_{ds} \\
\lambda_{qs}
\end{bmatrix} = L_{dq} \begin{bmatrix}
i_{ds} \\
i_{qs}
\end{bmatrix} + \begin{bmatrix}
1 \\
0
\end{bmatrix} \lambda_m
\]

(2.42)

where

\[
L_{dq} = T_{\alpha\beta\rightarrow dq} L_{\alpha \beta} T^{-1}_{\alpha\beta\rightarrow dq}
\]

(2.43)

\[
= \begin{bmatrix}
\frac{3}{2} (L_{0s} - L_{ms}) & 0 \\
0 & \frac{3}{2} (L_{0s} + L_{ms})
\end{bmatrix}
\]

(2.44)

$\frac{3}{2} (L_{0s} - L_{ms})$ and $\frac{3}{2} (L_{0s} + L_{ms})$ in the above equation are regarded as the $d$- and $q$-axis inductances, respectively. $L_{ms}$ equals zero for surface PM machines. Therefore, the $d$- and $q$-axis inductances are equal to each other.

The voltage and flux linkage equations of PM machines in the $dq$ rotating transformation reference frame are summarized as following,

\[
\begin{bmatrix}
V_{ds} \\
V_{qs}
\end{bmatrix} = R_s \begin{bmatrix}
i_{ds} \\
i_{qs}
\end{bmatrix} - \omega_r \begin{bmatrix}
0 & 1 \\
-1 & 0
\end{bmatrix} \begin{bmatrix}
\lambda_{ds} \\
\lambda_{qs}
\end{bmatrix} + \begin{bmatrix}
\frac{d\lambda_{ds}}{dt} \\
\frac{d\lambda_{qs}}{dt}
\end{bmatrix}
\]

(2.45)

\[
\begin{bmatrix}
\lambda_{ds} \\
\lambda_{qs}
\end{bmatrix} = \begin{bmatrix}
L_{ds} & 0 \\
0 & L_{qs}
\end{bmatrix} \begin{bmatrix}
i_{ds} \\
i_{qs}
\end{bmatrix} + \begin{bmatrix}
1 \\
0
\end{bmatrix} \lambda_m
\]

(2.46)
2.1.5 Torque Expression of PM Machines

In this section, two approaches are used to derive the electromagnetic torque equation for PM machines: the basic definition of force and torque and the law of conservation of energy. The results obtained by the two different approaches are consistent with each other.

A. Using Definition of Force and Torque

Figure 2.3 shows the typical $\lambda - i$ curve in an electromagnetic system with the magnetic saturation effects included. The magnetic energy stored in the magnetic field, $W_{mag}$, is defined as the integral of the electric current with respect to the magnetic flux linkage.

![Figure 2.3: Energy and co-energy in electromagnetic system](image.png)
\[ W_{\text{mag}} = \int_0^\lambda i \, d\lambda \]  \hspace{1cm} (2.47)

The magnetic co-energy, \( W_{\text{mag}}^{\text{co}} \), is defined as the integral of the magnetic flux linkage with respect to the electric current,

\[ W_{\text{mag}}^{\text{co}} = \int_0^i \lambda \, di \]  \hspace{1cm} (2.48)

For an object in linear motion, the net force on the object is the partial derivative of electromagnetic co-energy with respect to the displacement.

\[ F_e = F_{\text{mech}} = \frac{\partial W_{\text{mag}}^{\text{co}}(i, x)}{\partial x} \]  \hspace{1cm} (2.49)

For a rotating object in the electromagnetic field, the electromagnetic torque produced is defined as the partial derivative of electromagnetic co-energy with respect to the mechanical rotating angle.

\[ T_e = \frac{\partial W_{\text{mag}}^{\text{co}}(i, \theta)}{\partial \theta} \]  \hspace{1cm} (2.50)

In Figure 2.3, the total magnetic energy is the area enclosed by \( obc \) and the total magnetic co-energy is the area enclosed by \( oac \). Eq. (2.47) and (2.48) can be applied to both the linear and non-linear electromagnetic systems. Most real world electromagnetic systems are non-linear systems and the magnetic energy is not equal to the magnetic co-energy for these systems. However, if the saturation effects are neglected in an electric machine, the magnetic energy and magnetic co-energy would be equal to each other.

\[ W_{\text{mag}} = W_{\text{mag}}^{\text{co}} = \frac{1}{2} i\lambda \]  \hspace{1cm} (2.51)
For a three-phase PM machine, the torque equation then can be expressed as

\[ T_e = \frac{\partial W_{\text{mag}}^{\text{co}}}{\partial \theta_m} \]

\[ = P \frac{\partial W_{\text{mag}}^{\text{co}}}{2 \partial \theta_r} \]  

(2.52)  

(2.53)

where \( \theta_m \) is the mechanical rotor position, \( \theta_r \) the electrical rotor position and \( P \) the number of poles. The magnetic co-energy in a three-phase PM machine is written as

\[ W_{\text{mag}}^{\text{co}} = \frac{1}{2} (i_{\text{as}} \lambda_{\text{as}} + i_{\text{bs}} \lambda_{\text{bs}} + i_{\text{cs}} \lambda_{\text{cs}}) \]

\[ = \frac{1}{2} i_{\text{abc,s}}^T \lambda_{\text{abc,s}} \]  

(2.54)  

(2.55)

where \( i_{\text{abc,s}} \) is a vector composed of the currents in the three-phase windings,

\[ i_{\text{abc,s}} = \begin{bmatrix} i_{\text{as}} \\ i_{\text{bs}} \\ i_{\text{cs}} \end{bmatrix} \]  

(2.56)

and \( \lambda_{\text{abc,s}} \) a vector composed of the magnetic flux linkages of the three-phase windings.

\[ \lambda_{\text{abc,s}} = \begin{bmatrix} \lambda_{\text{as}} \\ \lambda_{\text{bs}} \\ \lambda_{\text{cs}} \end{bmatrix} \]  

(2.57)

Substituting Eqs. (2.10) and (2.11) into Eq. (2.55), we will get,

\[ W_{\text{mag}}^{\text{co}} = \frac{1}{2} i_{\text{abc,s}}^T \begin{bmatrix} L_{\text{abc}}(\theta_r) i_{\text{abc,s}} + \begin{bmatrix} \cos \theta_r \\ \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \lambda_m \end{bmatrix} \]  

(2.58)

Substituting Eq. (2.58) into Eq. (2.53), we will get,

\[ T_e = \frac{1}{2} P \frac{\partial L_{\text{abc}}(\theta_r)}{2 \partial \theta_r} i_{\text{abc,s}} + \frac{1}{2} P \frac{\partial L_{\text{abc}}(\theta_r)}{2 \partial \theta_r} \begin{bmatrix} \cos \theta_r \\ \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \lambda_m \]  

(2.59)

By inserting the identity matrix \( T_{abc\rightarrow dq} T_{abc\rightarrow dq}^{-1} = T_{abc\rightarrow dq}^{-1} T_{abc\rightarrow dq} = I \) into Eq. (2.59), we will get

\[ T_e = \frac{1}{2} P \frac{\partial L_{\text{abc}}(\theta_r)}{2 \partial \theta_r} i_{\text{abc,s}} T_{abc\rightarrow dq}^{-1} T_{abc\rightarrow dq} \frac{\partial L_{\text{abc}}(\theta_r)}{\partial \theta_r} T_{abc\rightarrow dq}^{-1} T_{abc\rightarrow dq} i_{\text{abc,s}} \]  

(2.60)

\[ + \frac{1}{2} P \frac{\partial L_{\text{abc}}(\theta_r)}{2 \partial \theta_r} i_{\text{abc,s}} T_{abc\rightarrow dq}^{-1} T_{abc\rightarrow dq} \frac{\partial}{\partial \theta_r} \begin{bmatrix} \cos \theta_r \\ \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \lambda_m \]  

(2.61)
where $\mathbf{T}_{abc\rightarrow dq}$ is the transformation matrix from the $abc$ stationery reference frame to the $dq$ rotating reference frame.

$$
\mathbf{T}_{abc\rightarrow dq} = \frac{2}{3} \begin{pmatrix}
\cos \theta_r & \cos \left( \theta_r - \frac{2\pi}{3} \right) & \cos \left( \theta_r + \frac{2\pi}{3} \right) \\
-\sin \theta_r & -\sin \left( \theta_r - \frac{2\pi}{3} \right) & -\sin \left( \theta_r + \frac{2\pi}{3} \right)
\end{pmatrix}
$$

(2.62)

$$
\mathbf{T}_{abc\rightarrow dq}^{-1} = \begin{pmatrix}
\cos \theta_r & -\sin \theta_r \\
\cos \left( \theta_r - \frac{2\pi}{3} \right) & -\sin \left( \theta_r - \frac{2\pi}{3} \right) \\
\cos \left( \theta_r + \frac{2\pi}{3} \right) & -\sin \left( \theta_r + \frac{2\pi}{3} \right)
\end{pmatrix}
$$

(2.63)

In Eq. (2.61),

$$
i_{abc,s}^T \mathbf{T}_{abc\rightarrow dq}^{-1} = \begin{bmatrix} i_{ds} & i_{qs} \end{bmatrix}
$$

(2.64)

$$
\mathbf{T}_{abc\rightarrow dq} i_{abc,s} = \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}
$$

(2.65)

$$
\mathbf{T} \frac{\partial \mathbf{L}_{abc}(\theta_r)}{\partial \theta_r} \mathbf{T}^{-1} = \begin{bmatrix} 0 & -3L_{ms} \\ -3L_{ms} & 0 \end{bmatrix}
$$

(2.66)

$$
\mathbf{T}_{abc\rightarrow dq} \frac{\partial}{\partial \theta_r} \begin{pmatrix}
\cos \theta_r \\
\cos \left( \theta_r - \frac{2\pi}{3} \right) \\
\cos \left( \theta_r + \frac{2\pi}{3} \right)
\end{pmatrix} \lambda_m = \begin{bmatrix} 0 \\ 3\lambda_m \end{bmatrix}
$$

(2.67)

By substituting Eqs. (2.64)-(2.67) into Eq. (2.61), we will get the electromagnetic torque equation of a PM machine in the $dq$ synchronous rotating reference frame,

$$
T_e = \frac{3}{2} P \left[ \lambda_m i_{qs} + (L_d - L_q)i_{ds}i_{qs} \right]
$$

(2.68)

In the above torque equation, the second term $(L_d - L_q)i_{ds}i_{qs}$ is called the reluctance torque. The reluctance torque exists in interior PM machines and synchronous machines due to the inequality between the $d$- and $q$-axis inductances caused by the saliency structure of the rotor. For surface PM machines, no reluctance torque exists because the $d$- and $q$-axis inductances are identical caused by the uniform air gap structure.
B. Using Law of Conservation of Energy

In this section, the electromagnetic torque equation is derived using the law of conservation of energy. The input electrical power of an electric machine can be expressed as,

\[ P_e = \frac{3}{2} (V_{ds}i_{ds} + V_{qs}i_{qs}) \]  

(2.69)

Substituting Eq. (2.45) into Eq. (2.69), we will get,

\[ P_{in} = \frac{3}{2} R (i_{ds}^2 + i_{qs}^2) + \frac{3}{2} (i_{ds} \frac{d\lambda_{ds}}{dt} + i_{qs} \frac{d\lambda_{qs}}{dt}) + \frac{3}{2} \omega_e [\lambda_m i_{qs} + (L_{ds} - L_{qs})i_{ds}i_{qs}] \]  

(2.70)

In the above equation, the first item represents the copper loss of the stator windings and the second item the changing rate of the electromagnetic energy. The third part denotes the electromagnetic energy passed from the stator to the rotor through the air gap.

\[ P_e = \frac{3}{2} \omega_e [\lambda_m i_{qs} + (L_{ds} - L_{qs})i_{ds}i_{qs}] \]  

(2.71)

According to the law of conservation of energy, the electromagnetic torque can be expressed as,

\[ T_e = \frac{P_e}{\omega_m} \]  

(2.72)

where \( \omega_m \) is the mechanical angular velocity of the rotor. Substituting Eq. (2.71) into Eq. (2.72), we will get the electromagnetic torque equation which is completely consistent with the result by using the original definition of electromagnetic torque presented in the previous part.
2.2 Review of Control Algorithms for PM machines

This section reviews a few control techniques very widely used in PM machine applications and the operating characteristics of PM machines.

2.2.1 V/f Open Loop Control

Figure 2.4 shows the control block diagram of the simplest open loop control algorithm for PM machines, the constant volts per hertz V/f method which is also applicable to some other types of AC electric machines such as induction machines and wound rotor synchronous machines.

It is known that the back EMF in the windings is proportional to the rotating speed of the rotor if the magnetic flux linkage produced by the rotor, λ, is constant, which indicates that the voltage applied to the machine windings is approximately proportional to the rotating speed of the rotor.

\[ V \approx E = 2\pi f \lambda \]  

Therefore, the open loop V/f control generates a control voltage based on the desired machine speed and a preset ratio of V/f, which is approximately \(2\pi \lambda\).

![Figure 2.4: Control block diagram of constant V/f control](image)
An open loop angle $\theta$ can be generated by integrating the electrical angular velocity of the rotor. The reference frame transformation matrix $T(\theta)$ in Figure 2.4 provides the transformation from the synchronous rotating reference frame to the stationary reference frame. The three-phase reference stator voltage can be obtained by Eqs. (2.74) to (2.76). PWM algorithms such as the Sin-$\Delta$ or the space vector PWM method may be used to generate the PWM switching signals $S_a$, $S_b$ and $S_c$ for the power inverter to drive the PMSM machine.

\[
V_a = V \cos \theta \quad (2.74)
\]
\[
V_b = V \cos (\theta - \frac{2}{3}\pi) \quad (2.75)
\]
\[
V_c = V \cos (\theta + \frac{2}{3}\pi) \quad (2.76)
\]

The open loop angle $\theta$ used in the $V/f$ control does not necessarily represent the real rotor position. Actually, we can apply arbitrary initial value for the open loop position. The angle between the applied voltage vector and the rotor flux linkage is automatically adjusted according to the load condition.

Selecting an appropriate $V/f$ curve plays a very important role in starting and accelerating the machine smoothly. Figure 2.5 demonstrates a typical voltage curve versus the machine speed. According to Eq. (2.73), the dashed line is the theoretical voltage to operate a machine under $V/f$ mode. However, a boosted voltage is always required to start the PM machine from stalled condition due to mechanical friction and power switch dead-time effects. Therefore, the actual control voltage is shown as the solid curve. Once the synchronization between the stator flux linkage and the rotor flux linkage is established, the control voltage would be proportional to the frequency of the rotor. When the speed of the rotor reaches the base speed $\omega_{\text{base}}$,
the voltage also reaches the rated level, \( V_{\text{rated}} \). When the speed further exceeds the base speed, \( \omega_{\text{base}} \), the voltage maintains at the rated value \( V_{\text{rated}} \) and the machine will enter the field weakening operation automatically.

![Figure 2.5: V/f generator function](image)

The \( V/f \) control is a low cost control method since no transducer is needed for detecting the current, voltage or the rotor speed and position information. It can offer satisfactory performance for the steady state operation. However, one \( V/f \) curve is only applicable to one specified electric machine with the load torque changing in a narrow range. For an increasing load, the start voltage \( V_0 \) increases and the base speed \( \omega_{\text{base}} \) for the command voltage to reach the rated value decreases. Generally, a \( V/f \) control curve comes from the data obtained by the trial and error testing. It is always time-consuming to get many \( V/f \) control curves which are suitable for different torque levels.

For applications which require fast transient responses, the \( V/f \) method cannot provide satisfactory performance. In fact, the \( V/f \) method often fails under fast
torque transient situations and heavy load conditions because of loss of synchronization between the stator and rotor flux linkages.

It is important to point out that the $V/f$ control is applicable to most types of three-phase AC electric machines such as induction machines, PM machines, synchronous machines and so on.

### 2.2.2 Current Vector Control

Feedback control is typically necessary in PM machine drive systems to achieve good transient performance in torque and speed. Figure 2.6 shows the control block diagram of a PMSM drive system using current and speed feedback control. For PMSM servo applications, close loop control is also needed for the rotor position control which is shown in Figure 2.7. In Figure 2.6 and 2.7, the most commonly used controller for the speed control and position control can be a PI controller.

For the current controller, current vector control is widely applied in various PMSM industry applications. As mentioned in the previous section, the three-phase current in the stator windings forms a rotating current vector which can be decomposed into two perpendicular axes, $d$- and $q$-axis. Generally speaking, the $d$-axis is aligned with the axis of the north pole of the permanent magnets and the $q$-axis is leading the $d$-axis by 90 deg. The flux linkage of the rotor can be easily controlled by controlling the $d$-axis component of the current vector and the torque controlled by the $q$-axis component of the current vector.

### 2.2.3 Six Step Control

It is known that the maximum line-to-line fundamental voltage which can be obtained by using the space vector PWM method is $V_{DC}$ if linear modulation is
Figure 2.6: Control block diagram using current and speed feedback control

Figure 2.7: Control block diagram of PMSM servo system
assumed. At low speed operation, the back EMF of the machine is relatively low and the DC bus can provide sufficient voltage to produce the required torque. At high operating speed, the back EMF of the machine is high and the torque capability is constrained by the limited bus voltage. Industry applications of PM machines such as hybrid electric vehicles and direct drive washing machines also favor the six step control to obtain higher fundamental voltage to increase the torque capability for high speed operation. The advantages of using the six step control at high speeds includes:

- Dramatically reduced switching losses of power converters. Using PWM methods, the switching frequency of power inverters generally varies between several kHz to tens of kHz. For the six step control, each power switch in the power inverter only switches on and off once and the switching frequency reduces to the fundamental frequency of the electric machine, which is generally several hundred Hz. Therefore, the switching losses of power switches can be dramatically reduced.

- Reduced copper loss. Using the six step control, a higher maximum available fundamental voltage can be achieved, which is $\frac{4}{\pi}V_{dc}$ compared to the maximum available fundamental voltage using the space vector PWM in the linear modulation region, $V_{DC}$. With increased fundamental voltage, the machine current and the conduction losses to achieve the same amount of torque at the same speed will be reduced.

- Simplified control structure. No current controller is applied using the six step control algorithm. The magnitude of the voltage vector applied to the machine
always maintains at the maximum level. The only control variable is the angle between the permanent magnet and the voltage vector.

However, some major disadvantages also come along using the six step control.

- Larger torque ripple and increased iron loss caused by harmonics. Without the current control, the six step control introduces many harmonics into the current, which will cause bigger torque ripple and higher iron losses compared to the well regulated sinusoidal current.

- The transition between the PWM operation mode and the six step operation mode needs very accurate synchronization of the applied voltage vector or the drive system could go unstable easily because of losing synchronization.

2.3 Operation Constraints in $i_d-i_q$ Plane

This section will focus on methods commonly applied in the analysis for PM machines and operation constraints and characteristics of PM machines.

A basic and effective way to analyze the operation of a PM machine is in the $i_d-i_q$ plane. It is necessary to review the operation limitations imposed by current and voltage before other advanced PM machine control algorithms are introduced.

2.3.1 Current Operation Constraint

Due to the thermal limitations of the PM machine itself and the power converter, the operating current should always be constrained under the maximum allowable level, $I_{max}$. In the $i_d-i_q$ plane, this operation constraint is expressed in Eq. (2.77),

\[ i_{ds}^2 + i_{qs}^2 \leq I_{max}^2 \] (2.77)
Obviously, Eq. (2.77) defines a current limit circle in the \( i_d-i_q \) plane as shown in Figure 2.8 and 2.9 regardless the rotor structure of the PM machine. Because of the thermal constraint, the current vector should not exceed the current limit circle.

### 2.3.2 Voltage Operation Constraint

Another important operation constraint is imposed by the limited DC bus voltage. Assume that the maximum available phase voltage of a PM machine is \( V_{\text{max}} \). The magnitude of the voltage vector should not exceed the maximum available phase voltage \( V_{\text{max}} \) either, which is represented in Eq. (2.78).

\[
V_{ds}^2 + V_{qs}^2 \leq V_{\text{max}}^2 \tag{2.78}
\]

It is more straightforward to translate the above voltage imposed operation limit into the \( i_d-i_q \) plane to examine the effects of the voltage limit on the operation of a PM machine. The \( d- \) and \( q- \) axis voltage equations can be simplified to Eqs. (2.79) and (2.80) if the resistance of the machine windings is small and the voltage drop across the resistance can be neglected compared to the back EMF.

\[
V_{ds} = -\omega_e L_{qs} i_{qs} \tag{2.79}
\]

\[
V_{qs} = \omega_e (\lambda_m + L_{ds} i_{ds}) \tag{2.80}
\]

Substituting Eqs. (2.79) and (2.80) into Eq. (2.78), we will get,

\[
(L_{ds} i_{ds} + \lambda_m)^2 + (L_{qs} i_{qs})^2 \leq \frac{V_{\text{max}}^2}{\omega_e^2} \tag{2.81}
\]

For a certain speed \( \omega_e \), Eq. (2.81) depicts in the \( i_d-i_q \) plane a circle for surface PM machines or an ellipse for interior PM machines due to the difference between the \( d- \) and \( q- \) axis inductances. The center of the circle or the ellipse is located at \( (-\frac{\lambda_m}{L_{ds}}, 0) \).
Figure 2.8 and 2.9 show the voltage imposed limit together with the current limit circles for surface PM and interior PM machines, respectively. It can be observed from Eq. (2.81) that the size of the voltage imposed limit circles or ellipses is speed dependent. With increased speed $\omega_e$, the size of the circles or ellipses decreases. Two speeds are demonstrated in Figure 2.8 and 2.9, where $\omega_1$ is smaller than $\omega_2$.

The possible operation region of a PM machine at a certain speed is constrained by both the thermal imposed current limit circle and the voltage imposed limit circle or ellipse. Take the IPM machine case in Figure 2.9 for example. At speed $\omega_1$, the operation region of an interior PM machine is the area enclosed by $OA_1B_1C$ for the motoring mode and $OA_2B_2C$ for the generating mode. With increasing speed, the possible operation speed of the interior PM machine decreases rapidly. At speed $\omega_2$, the operation region shrinks to the area inside the voltage imposed ellipse and the thermal imposed current limit circle is no longer a major constraint.

If the operating point of an interior PM machine at the speed of $\omega_e$ is located inside the voltage imposed limit ellipse corresponding to this speed $\omega_e$, it indicates that the phase voltage is less than the maximum available voltage and the DC bus voltage is not fully used. The DC bus voltage is fully used at this speed if the operation point is located right on the voltage imposed limit ellipse.

### 2.4 Maximum Torque Per Ampere Operation (MTPA) of IPM Machines

The torque equation of a PM machine is written as,

$$T_e = \frac{3P}{2} \left( \lambda_m i_{qs} + (L_{ds} - L_{qs}) i_{ds} i_{qs} \right)$$

(2.82)
Figure 2.8: Operation constraints of surface PM machines in $i_d$-$i_q$ plane
Figure 2.9: Operation constraints of interior PM machines in $i_d$-$i_q$ plane
For a surface PM machine, \(d\)- and \(q\)-axis inductances are equal to each other and no reluctance torque exists. Therefore, the \(d\)-axis current does not contribute to the electromagnetic torque. While for an interior PM machine, due to the difference between the \(d\)- and \(q\)-axis inductances, there exists a Maximum Torque Per Ampere (MTPA) curve along which the machine can achieve the maximum torque operation compared with other current vectors with the same magnitude while a different angle.

A current vector in the \(i_d\)-\(i_q\) plane can be expressed as,

\[i_d^2 + i_q^2 = i_s^2\]  \((2.83)\)

For an interior PM machine \((L_d < L_q)\), a negative \(d\)-axis current is always applied to produce positive reluctance torque.

\[i_{ds} = -\sqrt{i_s^2 - i_{qs}^2}\]  \((2.84)\)

By substituting Eq. (2.84) into Eq. (2.82), the torque equation can be rewritten as,

\[T_e = \frac{3}{2} P \left(\lambda_m i_{qs} - (L_{ds} - L_{qs}) i_{qs} \sqrt{i_s^2 - i_{qs}^2}\right)\]  \((2.85)\)

In the above equation, \(i_s\) is regarded as a constant and \(i_{qs}\) the variable. The derivative of \(T_e\) with respect to the variable \(i_{qs}\) is expressed as,

\[\frac{dT_e}{di_{qs}} = \frac{3}{2} P \left(\lambda_m + (L_{ds} - L_{qs}) \sqrt{i_s^2 - i_{qs}^2} - \frac{(L_{ds} - L_{qs}) i_{qs}^2}{\sqrt{i_s^2 - i_{qs}^2}}\right)\]  \((2.86)\)

Substituting Eq. (2.84) into (2.86), we can get,

\[\frac{dT_e}{di_q} = \frac{3}{2} P \left(\lambda_m + (L_{ds} - L_{qs}) i_{ds} - \frac{(L_{ds} - L_{qs}) i_{qs}^2}{i_{ds}}\right)\]  \((2.87)\)

Let \(\frac{dT_e}{di_{qs}} = 0\), we can find the extreme of \(T_e\).

\[i_{qs} = \pm \sqrt{i_{ds}^2 + \frac{\lambda_m}{L_{ds} - L_{qs}} i_{ds}}\]  \((2.88)\)
The positive and negative signs in Eq. (2.88) represent the motoring and generating operation modes, respectively. The interior PM machine operates in the motoring mode in the second quadrant and the generating mode in the third quadrant. Figure 2.10 demonstrates the Maximum Torque Per Ampere curve together with the constant torque curves in the $i_d$-$i_q$ plane for an interior PM machine.

Figure 2.10: Current limit circle, Maximum Torque Per Ampere (MTPA) curve and constant torque curves for interior PM machines
2.5 Translation Between Torque-speed Map and \( i_d-i_q \) Plane

Figure 2.11 shows the torque-speed capability curve for an interior PM machine in both the motoring and generating modes. Some important operation points are labeled on the torque-speed curve. A comprehensive study is done to associate the operation points on the torque-speed curve with their corresponding positions in the \( i_d-i_q \) plane. The most important benefits for this analysis are to easily examine the possible operation region for certain speeds with both the current and voltage constraints and to search the optimal operation point for a certain speed and load condition. The comparison study between the torque-speed map and the \( i_d-i_q \) plane provides a very crucial guide to machine design as well.

Figure 2.12 maps the above operation points on the torque-speed curve into the \( i_d-i_q \) plane. At Point O, both the speed and torque are zero. It corresponds to the origin in the \( i_d-i_q \) plane since both the \( d \)- and \( q \)-axis currents are zero and they do not produce any torque.

At Point A1a, the speed is zero and the torque is the positive maximum. Typically, A1a should be designed to be close to the intersection point of the current limit circle and the MTPA trajectory in the \( i_d-i_q \) plane.

Point A1 on the torque-speed curve typically represents the operation point where both the phase current and the phase voltage reach the maximum values. The speed corresponding to Point A1 is usually called the base speed \( \omega_b \). The voltage imposed limit ellipse at the base speed \( \omega_b \) also passes through A1, which means that the phase voltage reaches the maximum value.

The operation region where the speed is below the base speed \( \omega_b \) is the so-called constant torque operation. In this region, the maximum torque is always achievable.
Figure 2.11: General torque speed curve of IPM machines
Figure 2.12: Translation of operation points on torque-speed map into \( i_d-i_q \) plane
The current limit circle is the only operation constraint since the speed is below the base speed and the DC bus voltage is always sufficient to generated the desired \(d\)-and \(q\)-axis current. Any point in the area defined by \(OA_{1a}A_1O_1\) in the torque speed map has a corresponding point along the Maximum Torque Per Ampere curve \(OA_1\) in the \(i_d-i_q\) plane.

Above the base speed \(\omega_b\), the phase voltage remains constant and the voltage imposed limit ellipse needs to be taken as an operation constraint. Take the \(\omega_1\)-ellipse for example. At \(\omega_1\), the operation point can never be outside the ellipse. It can be observed that the torque decreases as the constant torque curve moves toward the \(i_d\) axis. Now it is not difficult to conclude that the maximum torque at \(\omega_1\) is produced at the intersection of the current limit circle and the \(\omega_1\) voltage imposed limit ellipse. Therefore, Point B1 should be the intersection of the current limit circle and the \(\omega_1\) ellipse. The torque at this point is about 0.7 p.u. If the speed remains at \(\omega_1\) and the desired torque decreases, the operation point will move from Point \(B_1\) toward the \(i_d\) axis along the \(\omega_1\) ellipse. When the torque decreases to 0.5 p.u., the 0.5-p.u.-constant torque current at \(\omega_1\) and the MTPA trajectory meet at a common point, \(B_{1a}\). Points B1 and B1a have the same speed on the torque-speed map. They stay on the same voltage imposed limit ellipse when mapped into the \(i_d-i_q\) plane.

After Point \(B_{1a}\), the operation point can switch to move along the MTPA trajectory toward the origin when the desired torque decreases further. Moving along the MTPA trajectory can keep the phase current being the minimum value compared to other operation points. Finally, the operation point moves to the origin when the torque becomes zero. Thus, Point \(O_2\) is the origin on the \(i_d-i_q\) plane.
When the speed exceeds $\omega_1$, the voltage imposed limit ellipse shrinks further. There must be an ellipse that only has one common point with the MTPA trajectory. This ellipse is also plotted in the $i_d-i_q$ plane. Assume that the speed corresponding to this ellipse is $\omega_2$. The operation point C1 having the largest torque at $\omega_2$ is the intersection of the current limit circle and the $\omega_2$-voltage-imposed limit ellipse. When the speed remains constant and the desired torque decreases, the operation points moves from $C_1$ toward the $d$-axis along the ellipse. Finally, the operation point reaches the $d$-axis when the torque becomes zero.

Point $D_1$ on the torque-speed curve is the intersection of the current limit circle, $\omega_3$-voltage-imposed limit ellipse and the 0.3-p.u.-constant-torque curve. When the speed is higher, the operation point having a maximum torque will be at the point where the ellipse and the constant torque curve have a common tangent line. As the speed increases, the maximum torque decreases. Finally, Point $O_{inf}$ moves to the center of all the voltage imposed limit ellipses. This point only represents an ideal operation situation where the speed can go to infinite.

Points $A_{2a}$, $A_2$, $B_2$, $B_{2a}$, $C_2$ and $D_2$ in the generating operation mode are symmetrical to Points $A_{1a}$, $A_1$, $B_1$, $B_{1a}$, $C_1$ and $D_1$ long the $i_d$-axis in the $i_d-i_q$ plane.

2.6 Summary

In this chapter, the theory of reference frame transformation is reviewed and the mathematical model of PM machines are derived. The electromagnetic torque equation is obtained using two approaches: definition of torque and the law of conservation.
of energy. Basic control methods for PM machines including the $V/f$ open loop control, the current vector control and the six step control are discussed. Important operating constraints and characteristics are inspected in the $i_d$-$i_q$ plane.
CHAPTER 3

Sensorless Vector Control of PM Synchronous Machines

This chapter presents two sensorless vector control algorithms for interior PM machines: the method based on the high frequency carrier signal injection and the voltage model based sensorless algorithm. Experimental results using both methods are applied on a 50 kW interior PM machine. The high frequency injection method is employed below 5 Hz and the voltage model based algorithm for speeds higher than 5 Hz. The system makes a transition from the high frequency injection method to the voltage model based sensorless algorithm using a weight function smoothly. The effects that the rotor position estimation error has on the torque accuracy is presented at the end of this chapter.

3.1 Sensorless Vector Control Based on High Frequency Injection Method

3.1.1 High Frequency Injection Method Fundamental

Interior PM machines have saliencies on the rotor and the inductances of machine windings are varying with the rotor position as shown in Figure 3.1. When a symmetrical three-phase high frequency voltage is injected into the machine windings, the
resulting high frequency current will be modulated by the inductance of the windings and will not be a symmetrical three-phase current.

For example, Figure 3.2 and 3.3 show the three-phase current with a symmetrical three-phase high frequency carrier voltage injected. Figure 3.2 shows the case with the rotor stationery and permanent magnet perpendicular to the axis of the Phase A winding. It is obvious that the three-phase current is not symmetric. With the rotor at this position, Phase A winding has the maximum inductance. The inductances of Phase B and C windings are the same and smaller than that of the Phase A winding. Therefore, the current in Phase A winding is smaller than those in Phase B and C windings. While Figure 3.3 shows the case when the rotor is rotating. The three-phase current is then a superposition of the fundamental component and the high frequency component.

![Figure 3.1: Inductances of phase windings of interior PM machine changing with rotor position](image)

The high frequency current is modulated by the saliency of the rotor and the magnitude of the current is directly associated with the rotor position. Therefore, the
Figure 3.2: Three-phase current with high frequency voltage excitation with stationary rotor

Figure 3.3: Three-phase current with high frequency voltage excitation with rotor rotating
core idea of the rotor position estimation using the high frequency injection method in this work is to apply certain demodulation process to the three-phase current and obtain the rotor position information.

### 3.1.2 Mathematical Modeling

Both the rotating and pulsating high frequency carrier signals can be used for the rotor position estimation. This work focuses on the method using the rotating high frequency carrier signal injection.

In the stationary reference frame, the injected rotating high frequency carrier voltage can be expressed as in Eq. (3.1),

\[
\begin{align*}
\mathbf{u}_c^s &= \begin{pmatrix} u_{ca} \\ u_{cb} \end{pmatrix} = \begin{pmatrix} u_c \cos \omega_c t \\ u_c \sin \omega_c t \end{pmatrix} = |u_c| e^{j\omega_c t}
\end{align*}
\]

where \(|u_c|\) is the magnitude of the high frequency voltage vector, \(\omega_c\) the electrical angular velocity of the injected high frequency voltage.

In the rotating reference frame synchronous with the rotor, the above high frequency voltage vector can be expressed as Eq. (3.2),

\[
\begin{align*}
\mathbf{u}_c^{dq} &= \begin{pmatrix} u_{cd} \\ u_{cq} \end{pmatrix} = |u_c| e^{j\omega_c t} \cdot e^{-j\omega_r t} = |u_c| e^{j(\omega_c - \omega_r) t}
\end{align*}
\]

where \(\omega_r\) is the electrical angular velocity of the rotor.

The high frequency voltages and high frequency currents in the rotor reference frame are governed by Eq. (3.3),

\[
\mathbf{u}_c^{dq} = \mathbf{L}_{dq} \frac{d\mathbf{i}_s^{dq}}{dt}
\]

where \(\mathbf{L}_{dq}\) is the inductance matrix in the synchronous rotating reference frame.

\[
\mathbf{L}_{dq} = \begin{pmatrix} L_{ds} & 0 \\ 0 & L_{qs} \end{pmatrix}
\]
By substituting Eq. (3.4) into Eq. (3.3), the currents can be written as,

\[
\frac{di_q^d}{dt} = L_{dq}^{-1}|u_c|e^{j(\omega_c - \omega_r)t} \quad (3.5)
\]

\[
i_s^{dq} = \frac{-j|u_c|}{\omega_c L_{ds} L_{qs}} \left[ \frac{1}{2} (L_{qs} + L_{ds}) e^{j(\omega_c - \omega_r)t} + \frac{1}{2} (L_{qs} - L_{ds}) e^{-j(\omega_c - \omega_r)t} \right] \quad (3.6)
\]

Since the currents measured by the current transducers are in the stationary reference frame, the currents in the synchronous rotating reference frame are then translated to the stationary frame,

\[
i_s^* = i_s^{dq} e^{j\omega_r t} = \frac{-j|u_c|}{\omega_c L_{ds} L_{qs}} \left[ \frac{1}{2} (L_{qs} + L_{ds}) e^{j\omega_r t} + \frac{1}{2} (L_{qs} - L_{ds}) e^{-j(\omega_c - 2\omega_r)t} \right] = i_p + i_n \quad (3.7)
\]

where \(i_p\) and \(i_n\) are the positive and negative sequence currents, respectively. It can be seen that only the negative sequence current contains the rotor position information.

To obtain the rotor position, the currents should be demodulated. The demodulation procedure is summarized as in Figure 3.4. The measured current is a superposition of the fundamental current and the high frequency current.

\[
i_{\alpha\beta} = i_s^* + i_1^s \quad (3.8)
\]

where the fundamental current in the stationary reference frame \(i_1^s\) can be expressed as,

\[
i_1^s = I_1 e^{j(\omega_r t + \phi)} \quad (3.9)
\]

where \(I_1\) the magnitude of the fundamental component of the current and \(\omega_r\) the angular frequency of the current.

The first step of demodulation is to translate the currents in the stationary reference frame into the rotating reference frame which is synchronous with the high
frequency voltages.

\[ i_{s}^{\alpha \beta} = i_{s}^{\alpha \beta}e^{-j\omega_{c}t} = (i_{s}^{\alpha} + i_{1}^{\beta})e^{-j\omega_{c}t} \]  
\[ = \frac{-j|u_{c}|}{\omega_{c}L_{ds}L_{qs}} \left[ \frac{1}{2}(L_{qs} + L_{ds}) + \frac{1}{2}(L_{qs} - L_{ds})e^{-j(2\omega_{c} - 2\omega_{r})t} \right] e^{-j(2\omega_{c} - 2\omega_{r})t} + I_{1}e^{j(\omega_{r}t - \omega_{c}t + \phi)} \]  
(3.11)

It can be observed that after the reference frame transformation, the fundamental component of the current is converted to a high frequency component in the new reference frame. The positive sequence component of the high frequency current becomes a DC component. The negative sequence current which contains the rotor position information is still an AC component. The DC component can be removed by a high pass filter.

\[ i_{s}^{\alpha \beta 2} = \frac{-j|u_{c}|}{2\omega_{c}L_{ds}L_{qs}}(L_{qs} - L_{ds})e^{-j(2\omega_{c} - 2\omega_{r})t} + I_{1}e^{j(\omega_{r}t - \omega_{c}t + \phi)} \]  
(3.12)

Then the above current \( i_{s}^{\alpha \beta 2} \) is translated into the rotating reference frame which is rotating in the opposite direction to the injected high frequency voltage vector.

\[ i_{s}^{\alpha \beta 3} = i_{s}^{\alpha \beta 2}e^{j2\omega_{c}t} \]  
\[ = \frac{-j|u_{c}|}{2\omega_{c}L_{ds}L_{qs}}(L_{qs} - L_{ds})e^{-j(2\omega_{c} - 2\omega_{r})t}e^{j2\omega_{c}t} + I_{1}e^{j(\omega_{r}t - \omega_{c}t + \phi)}e^{j2\omega_{c}t} \]  
(3.14)

\[ = \frac{-j|u_{c}|}{2\omega_{c}L_{ds}L_{qs}}(L_{qs} - L_{ds})e^{j2\omega_{c}t} + I_{1}e^{j(\omega_{r}t + \omega_{c}t + \phi)} \]  
(3.15)

In Eq. (3.15), the first item contains the rotor position information. The second term is a high frequency signal compared to the first term. Therefore, the first item containing the rotor position information can be separated out by applying a low pass filter to \( i_{s}^{\alpha \beta 3} \).

\[ i_{s}^{\alpha \beta \text{dem}} = \frac{-j|u_{c}|}{2\omega_{c}L_{ds}L_{qs}}(L_{qs} - L_{ds})e^{j2\omega_{c}t} \]  
(3.16)
Figure 3.4: Control block diagram and demodulation process using high frequency injection based sensorless algorithm
Here, a sixth-order butterworth low pass filter is designed in this work to guarantee the low frequency signal can be clearly separated from the high frequency signal in Eq. (3.15). The cut-off frequency of the low pass filter is decided by the trial and error testing and selected to be 8 Hz. The design detail of the sixth-order low pass filter is discussed in Appendix B.

Therefore, the rotor position can be obtained by Eq. (3.17).

\[
2\theta_r = \tan^{-1}\left(\frac{i_{\beta, \text{dem}}}{i_{\alpha, \text{dem}}}ight)
\]  

(3.17)

The high frequency injection method is effective in finding the saliency of the rotor. However, it is necessary to tell whether the saliency is the north pole or the south pole to know the exact rotor position.

### 3.1.3 Initial Pole Position Identification

The B-H curve shown in Figure 3.5 is used to demonstrate how magnet saturation effect is applied to identify the north pole of the permanent magnet. Assume that Point \( A_0 \) is the original operation point. From Point \( A_0 \) to \( A_2 \), the magnetic field strength \( H \) (or current \( i \)) decreases by \( \Delta i_2 \) when the magnetic flux density decreases by \( \Delta B \). While from Point \( A_0 \) to \( A_1 \), the magnetic field strength \( H \) (or current \( i \)) increases by \( \Delta i_1 \) for the flux density to increase by \( \Delta B \). Because of the magnetic saturation effect, it takes a larger change in the magnetic field strength (or current) to generate the same amount of change in the flux density.

Figure 3.6 illustrates the voltage and current waveforms using magnetic saturation effect to identify the north pole of the permanent magnet. First a voltage vector is applied along the \( d \)-axis of the rotor. The current produced is governed by the following equation which does not include any back EMF item because this voltage
only generates a current vector along the $d$-axis which does not produce any torque to move the rotor. The magnetic flux generated by the current is in the same direction as the flux of the permanent magnet and the inductance of the winding decreases because of the magnetic saturation effect.

$$V_d = L \frac{di_d}{dt} \quad (3.18)$$

When a voltage vector is applied opposite to the $d$-axis, the flux produced by the current is opposite to the flux of the permanent magnet and the inductance of the winding will not decrease. Due to the magnetic saturation effects, the current produced by the positive voltage vector will be larger than that generated by the negative voltage vector.
A 50 kW interior PM machine is used to test and verify the initial rotor position identification algorithm. Some important parameters and ratings of the machine are listed in Appendix A. Figure 3.7 shows some experimental results. The high frequency injection method is applied first to find the axis of the saliency on the rotor. A voltage vector lasting 3.75 ms with a magnitude of 35 V is applied to one end of the saliency and the $d$-axis current rises up to 12.5 A. For the opposite voltage vector lasting the same period of time with the same magnitude, the $d$-axis current increases up to 9 A. Therefore, the end with larger current is identified to be the north pole.

The magnetic characteristics and inductances of different machines could be quite different. Therefore, the trial-and-error method is typically used to figure out the appropriate values for the magnitude and the time duration of the applied voltage.
Figure 3.7: Experimental results of initial rotor position identification
vector. The most essential criteria for selecting the magnitude and the time duration of the voltage vector is to make sure the difference between the two currents is obvious enough to tell the north pole. If the magnitude or the time duration are too small, the accuracy of the method can be affected by the signal noise. A very small voltage vector applied for a long time is not favored because it is always important and good to finish the initial rotor position in as short time as possible for industry applications.

The time between the two opposite voltage pulses is selected so that the current will decay to zero before the second voltage pulse is applied. The whole procedure of the north pole identification shown in Figure 3.7 is completed within 50 ms and will not even be noticed.

Figure 3.8 shows experimental results of the bi-directional operation of the interior PM machine under no load condition using the sensorless algorithm based on the high frequency injection method. The top ones show the estimated rotor position and the actual rotor position given by the incremental encoder. The middle one shows the error between the estimated and the actual rotor positions. The estimation error is within 8 electrical degrees. The bottom waveforms show the currents in Phase A and B windings. Figure 3.9 shows the details of the phase currents which is a superposition of the fundamental component and the high frequency component.

Typically, inspecting the rotor position information can help tell if a machine is running smoothly. It can be noticed from Figure 3.8 that the machine is not running very smoothly for very lower speed, which is indicated by the bumps shown in the rotor position waveforms. This is caused by the cogging torque of the prototype interior PM machine used for the test. Therefore, the high frequency injection method is very effective in indicating the torque ripple.
Figure 3.8: Experimental results of bi-directional operation using high frequency injection based sensorless algorithm
3.2 Voltage Model Based Sensorless Algorithm

As stated before, the back EMF of a PM machine is proportional to the speed of the rotor and is also leading the magnetic flux linkage produced by the permanent magnets by 90 degrees. This section presents the rotor position estimation technique based on the voltage model.

3.2.1 Mathematical Modeling

Figure 3.10 shows the topology of a general purpose three-phase power inverter. Assume the duty cycles for the top three switches are $D_1$, $D_3$ and $D_5$. 

Figure 3.9: Phase currents of bi-directional operation using high frequency injection based sensorless algorithm
Assuming the voltage at the negative DC bus zero, the voltages at Point A, B and C can be expressed as,

\[ V_a = V_{DC} \cdot D_1 \]  
\[ V_b = V_{DC} \cdot D_2 \]  
\[ V_c = V_{DC} \cdot D_3 \]  

The three phase voltages are expressed as,

\[
\begin{bmatrix}
V_{an} \\
V_{bn} \\
V_{cn}
\end{bmatrix} =
\begin{bmatrix}
\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\
-\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\
\frac{1}{3} & -\frac{1}{3} & \frac{2}{3}
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
\]  

Therefore, the phase voltages can be calculated from the information of the DC bus voltage and the switching functions \( D_1, D_3 \) and \( D_5 \). Knowing the phase voltages in the three-phase stationery reference frame, voltages in the \( \alpha\beta \) stationery reference frame can be obtained by applying the reference frame transformation from the \( abc \) reference frame to the \( \alpha\beta \) reference frame.
The voltage equations of a PM machine in the $\alpha\beta$ stationary reference frame can be written as

$$V_{\alpha s}(t) = R_s i_{\alpha s}(t) + \frac{d\lambda_{\alpha s}(t)}{dt}$$  \hspace{1cm} (3.23)$$

$$V_{\beta s}(t) = R_s i_{\beta s}(t) + \frac{d\lambda_{\beta s}(t)}{dt}$$  \hspace{1cm} (3.24)$$

In the above voltage equations, $i_{\alpha s}$ and $i_{\beta s}$ are measurable variables. $V_{\alpha s}$ and $V_{\beta s}$ can be calculated from the DC bus voltage and the switching functions as mentioned before. Therefore, the stator flux linkage in the $\alpha\beta$ reference frame can be obtained by Eqs. (3.25) and (3.26),

$$\lambda_{\alpha s}(t) = \int_0^t [V_{\alpha s}(t) - R_s i_{\alpha s}(t)] \, dt$$  \hspace{1cm} (3.25)$$

$$\lambda_{\beta s}(t) = \int_0^t [V_{\beta s}(t) - R_s i_{\beta s}(t)] \, dt$$  \hspace{1cm} (3.26)$$

The integral calculation in Eqs. (3.25) and (3.26) introduces DC offsets in the final results. Therefore, low pass filters are applied to remove the DC components to get the actual AC stator flux linkage.

On the other hand, the stator flux linkages can be express as a function of the stator currents and the flux linkage of the permanent magnets.

$$\begin{bmatrix}
\lambda_{\alpha s}(t) \\
\lambda_{\beta s}(t)
\end{bmatrix} = \frac{3}{2} \begin{bmatrix}
L_{0s} + 2L_{2s}\cos 2\theta_r & L_{2s}\sin 2\theta_r \\
L_{2s}\sin 2\theta_r & L_{0s} - 2L_{2s}\cos 2\theta_r
\end{bmatrix} \begin{bmatrix}
i_{\alpha s}(t) \\
i_{\beta s}(t)
\end{bmatrix} + \lambda_m \begin{bmatrix}
\cos \theta_r \\
\sin \theta_r
\end{bmatrix}$$  \hspace{1cm} (3.27)$$

Knowing the information of the stator flux linkages and the stator currents, the flux linkage of the permanent magnets can be estimated by Eq. (3.30).

$$\begin{bmatrix}
\cos \theta_r \\
\sin \theta_r
\end{bmatrix} = \begin{bmatrix}
\lambda_{\alpha s}(t) \\
\lambda_{\beta s}(t)
\end{bmatrix} - \frac{3}{2} \begin{bmatrix}
L_{0s} + 2L_{2s}\cos 2\theta_r & L_{2s}\sin 2\theta_r \\
L_{2s}\sin 2\theta_r & L_{0s} - 2L_{2s}\cos 2\theta_r
\end{bmatrix} \begin{bmatrix}
i_{\alpha s}(t) \\
i_{\beta s}(t)
\end{bmatrix}$$  \hspace{1cm} (3.28)$$
Hence, the estimated rotor position $\theta_r$ can be acquired by Eq. (3.29).

$$\theta_r = \tan^{-1} \frac{\lambda_m \sin \theta_r}{\lambda_m \cos \theta_r} \quad (3.29)$$

In Eq. (3.30), the rotor position at the present moment is required to calculate the flux linkage of the permanent magnets. However, only the rotor position at the previous moment is known. Therefore, in experimental implementations, compensation is applied to obtain the current rotor position.

$$\lambda_m \begin{bmatrix} \cos \theta_r[k] \\ \sin \theta_r[k] \end{bmatrix} = \begin{bmatrix} \lambda_{\alpha_s}[k] \\ \lambda_{\beta_s}[k] \end{bmatrix} - \frac{3}{2} \begin{bmatrix} L_{0_s} + L_{2_s} \cos 2\dot{\theta}_r[k] \\ L_{2_s} \sin 2\dot{\theta}_r[k] \end{bmatrix} \begin{bmatrix} L_{0_s} - L_{2_s} \cos 2\dot{\theta}_r[k] \\ L_{0_s} + L_{2_s} \cos 2\dot{\theta}_r[k] \end{bmatrix} \begin{bmatrix} i_{\alpha_s}[k] \\ i_{\beta_s}[k] \end{bmatrix} \quad (3.30)$$

where

$$\dot{\theta}_r[k] = \theta_r[k - 1] + \omega_e \cdot T_s \quad (3.31)$$

where $\omega_e$ is the electrical angular velocity of the rotor and $T_s$ the sampling cycle.

### 3.2.2 Experimental Results

The voltage model based sensorless algorithm is implemented on the 50 kW interior PM machine. Figure 3.11 shows the phase current and the rotor position estimation error when the machine is running at 1000 rpm with a torque step change from 50 Nm to 20 Nm. It can be seen that the jump in the rotor position estimation error during the torque step change is about 5 electrical degrees. Though there is a transient in the estimated rotor position, the controller still works in a very stable manor.

Figure 3.12 shows the actual and estimated rotor positions when the interior PM machine is running at 6000 rpm. Compensation is given to make sure that the
Figure 3.11: Experimental results of torque step change using voltage model based sensorless algorithm
Figure 3.12: Experimental results: rotor position estimation at 6000 rpm
estimated rotor position is lagging the actual rotor position by a few electrical degrees. The reason will be explained in the next section.

The high frequency injection based sensorless algorithm and the voltage model based sensorless algorithm are combined to start the interior PM machine from standstill. Figure 3.13 shows the waveforms of the interior PM machine starting from stationery without any hesitation. The upper waveforms are the estimated rotor positions. The green one is the estimated rotor position by using the voltage model based sensorless algorithm. The blue one is the estimated rotor position by using the high frequency injection method. The black one is the rotor position actually used in the close loop control.

The rotor is stationery from 0 to 1.34s. During this period of time, the initial rotor position is obtained by the high frequency injection method and used for the close loop control. Therefore, the rotor position used for the close loop control (black) is overlapped with the estimated rotor position using the high frequency injection method (blue). The rotor position estimated using the voltage model based sensorless algorithm is not valid during this period. At 1.34 s, a command is given to the motor to start and accelerate. It is can be seen from the waveform of the rotor position that the machine starts immediately without any hesitation. Between 1.34 s to 1.62 s, the machine speed is lower than 5 Hz (75 rpm), the estimated rotor position obtained from the high frequency injection method is always used for the close loop control. At 1.62 s, the machine speed reaches 5 Hz, the rotor position used for the close loop control switches from the high frequency injection method to the voltage model based sensorless algorithm smoothly.
Figure 3.13: Experimental results: machine start up from standstill using sensorless algorithms
Figure 3.14 shows the detail of the rotor position during the transition from using the high frequency sensorless algorithm to the voltage model based sensorless algorithm. Both sensorless algorithms are running during the acceleration of the motor. However, the estimated rotor position obtained from the high frequency injection method is used for the close loop control before the transition. During the transition, the rotor position used for the close loop control is obtained by a weight function applied to the two estimated rotor positions.

\[ \theta_r = (1 - K)\theta_{hfi} + K\theta_{vm} \]  
\[ K = 100(t - t_0) \]

where \( \theta_r \) is the rotor position used for the close loop control, \( \theta_{hfi} \) the estimated rotor position using the high frequency injection method, \( \theta_{vm} \) the estimated rotor position using the voltage model based sensorless algorithm, \( K \) a function of time and \( t_0 \) the time when the transition starts. Therefore, the rotor position switches from the high frequency injection method to the voltage model based method gradually. After the transition is finished, the estimated rotor position by using the voltage model based sensorless algorithm is used for the close loop control and the high frequency injection method stops running. In Figure 3.14, the transition procedure takes around 10 ms.

### 3.3 Study on Effects of Rotor Position Estimation Error on Torque Delivery Accuracy

In Figure 3.15, \( \alpha \beta \) represents the stationary reference frame, \( \hat{d} \) and \( \hat{q} \) the estimated reference frame, \( d \) the actual axis of the permanent magnets. The actual rotor position is \( \theta \). Assume the system reaches the steady state and the error between the estimated rotor position and the actual rotor position is \( \Delta \theta \). The counter-clockwise direction
Figure 3.14: Transition from high frequency injection based sensorless algorithm to voltage model based algorithm

Figure 3.15: Error of estimated rotor position
is regarded to the positive direction. Therefore, the angle that the estimated rotor position is leading the actual rotor position is defined as positive. The actual \( d \)- and \( q \)-axis currents can be expressed by the \( d \)- and \( q \)-axis currents in the estimated reference frame,

\[
\begin{bmatrix}
  i_{ds} \\
i_{qs}
\end{bmatrix} =
\begin{bmatrix}
  \cos \Delta \theta & -\sin \Delta \theta \\
  \sin \Delta \theta & \cos \Delta \theta
\end{bmatrix}
\begin{bmatrix}
  \hat{i}_{ds} \\
  \hat{i}_{qs}
\end{bmatrix}
\] (3.34)

The actual electromagnetic torque can be expressed by the \( d \)- and \( q \)-axis currents in the estimated rotating reference frame,

\[
T_e = \frac{3P}{2} \lambda_m \hat{i}_{ds} \sin \Delta \theta + \hat{i}_{qs} \cos \Delta \theta
\] (3.35)

\[
+ (L_d - L_q)(\hat{i}_{ds} \cos \Delta \theta - \hat{i}_{qs} \sin \Delta \theta)(\hat{i}_{ds} \sin \Delta \theta + \hat{i}_{qs} \cos \Delta \theta)
\] (3.36)

The 50 kW interior PM machine is used for the torque accuracy analysis and the machine parameters can be found in Appendix A. Here the torque accuracy is defined as the difference between the actual torque and the command torque divided by the rated torque,

\[
T_{err} = \frac{T_{act} - T_{cmd}}{T_{rated}} \times 100\% 
\] (3.37)

Figure 3.16 plots the torque accuracy with the estimated \( d \)-axis leading the actual \( d \)-axis by 5 electrical degrees. The torque accuracy is negative for this situation and becomes worse with increasing speed and torque. In this case, it is very likely for an interior PM machine to switch from the motoring mode to the generating mode at high speed operation when a small torque is commanded.

Figure 3.17 presents an opposite situation with the estimated \( d \)-axis lagging the actual \( d \)-axis by 5 electrical degrees. The torque accuracy also grows worse with increasing speed and torque. However, the torque accuracy is always positive when the
machine is operating at high speeds, which indicates that the machine will not switch between the motoring mode and generating mode even if zero torque is commanded.

It is strongly not favored for the machine to switch from the motoring to the generating mode at high speed operation since the power circuits might be damaged if the power generated by the interior PM machine cannot be dissipated. Therefore, a compensation is added to the estimated rotor position to maintain the estimated rotor position lagging the actual rotor position by a few electrical degrees in the actual implementation.

3.4 Summary

This chapter presents the principles of two sensorless vector control algorithms: the method based on the high frequency carrier voltage injection and the voltage
Figure 3.17: Torque accuracy with negative estimation error of rotor position

model based sensorless algorithm. Experimental results using both methods are applied on a 50 kW interior PM machine. The effects that the rotor position estimation error has on the torque accuracy is discussed at the end of this chapter.
CHAPTER 4

Deep Field Weakening Operation of PM machines

Control issues for the deep field weakening operation of PM machines, particularly for the interior PM machines, are discussed in this chapter. Two solutions, the single-current-regulator algorithm and the voltage phase angle control, are presented for the deep field weakening operation of interior and surface PM machines, respectively.

4.1 Control Issues of Deep Field Weakening Operation of PM Machines

One critical control issue for the field weakening operation of the PM machine is due to the limited DC bus voltage. Figure 4.1 shows the traditional current vector control algorithm for a PM machine by using two current regulators. In the field weakening operation region, the voltage commands $v_d^*$ and $v_q^*$ may exceed the maximum available voltage, which will cause saturation of the two current regulators. The saturated regulators will conflict with each other and then result in unacceptable performance of current, torque and speed. The operation point of the IPM machine may switch from the motoring mode to the generating mode or vice versa.

Another critical issue for the deep field weakening operation can be illustrated in Figure 4.2 where the 50 kW interior PM machine is used for demonstration and
the machine parameters can be found in Table A.1. Several important operational limit curves and the Maximum Torque Per Ampere (MTPA) curve are simultaneously shown in the same $i_d$-$i_q$ plane.

As already proved in Chapter 2, the voltage imposed limit ellipse shrinks with increasing speed for an interior PM machine in the $i_d$-$i_q$ plane. Three voltage imposed limit ellipses at different operation speeds are shown in Figure 4.2. The possible operation region of an interior PM machine is limited by both the current limit circle and the voltage imposed limit ellipse. It can be observed from Figure 4.2 that the operation area for 2.4 times the base speed is less than half of the operation area for 1.2 times the base speed.

With increasing speed, the size of the voltage imposed limit ellipse reduces rapidly resulting in smaller operation regions. Stable operation in the deep field weakening region requires accurately controlled current. The very small operational region implies another critical issue for the high speed or deep field weakening operation of PM machines.
Figure 4.2: Voltage imposed limit ellipses causing control issues for deep field weakening operation of interior PM machines
4.2 Single-current-regulator Algorithm

4.2.1 Basic Principle of Single-current-regulator Algorithm

The voltage equations of a PMSM in the synchronous rotating reference frame are written as,

\[ V_{ds} = R_s i_{ds} - \omega_c L_q i_{qs} + L_d \frac{di_{ds}}{dt} \]  \hspace{1cm} (4.1)

\[ V_{qs} = R_s i_{qs} + \omega_c (\lambda_m + L_d i_{ds}) + L_q \frac{di_{qs}}{dt} \]  \hspace{1cm} (4.2)

The above voltage equations can be rewritten as,

\[
\begin{bmatrix}
\dot{i}_{ds} \\
\dot{i}_{qs}
\end{bmatrix} = \begin{bmatrix}
-R_s & -\omega_c L_q \\
\omega_c L_d & -R_s
\end{bmatrix} \begin{bmatrix}
i_{ds} \\
i_{qs}
\end{bmatrix} + \begin{bmatrix}
V_{ds} \\
V_{qs} - \omega_c \lambda_m
\end{bmatrix}  \hspace{1cm} (4.3)
\]

We choose \( i_{ds} \) and \( i_{qs} \) to be the state variables in the system.

\[ x_1 = i_{ds} \hspace{1cm} (4.4) \]

\[ x_2 = i_{qs} \hspace{1cm} (4.5) \]

Meanwhile the control variables are defined as,

\[ u_1 = V_{ds} \hspace{1cm} (4.6) \]

\[ u_2 = V_{qs} - \omega_c \lambda_m \hspace{1cm} (4.7) \]

Therefore, the state space equations of a PM synchronous machine can be written as,

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = A \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
u_1 \\
u_2
\end{bmatrix} \hspace{1cm} (4.8)
\]

where the state matrix \( A \) is expressed as,

\[ A = \begin{bmatrix}
R_s & -\omega_c L_q \\
\omega_c L_d & -R_s
\end{bmatrix} \begin{bmatrix}
\frac{L_d}{L_q} \\
\frac{L_q}{L_q}
\end{bmatrix} \hspace{1cm} (4.9)
\]
where $\omega_e$ is the electrical angular velocity of the rotor. It is assumed that the electrical transient occurs much faster compared to the mechanical transient. Therefore, the electrical angular velocity of the rotor $\omega_e$ is assumed to be a constant during the electrical transient. Therefore, the state matrix $A$ is regarded as a constant matrix in the following analysis.

When the control variables $u_1$ and $u_2$ are not bounded, the state variables $x_1$ and $x_2$ can be controlled independently. In the constant torque region, the machine speed is relatively low and the back EMF is smaller compared to the maximum available terminal voltage which can be provided by the power inverter. In other words, there is no limitation for the control variables $u_1$ and $u_2$. Therefore, currents, $i_{ds}$ and $i_{qs}$, can be controlled independently in the constant torque operation region.

With increasing speed, the back EMF produced in the stator windings increases as well. When the speed reaches the base speed, the winding terminal voltage approaches the maximum available voltage, which is constraint by the limited DC link voltage. When the machine is operating beyond the base speed or in the constant power operation region, both of the control voltages $u_1$ and $u_2$ in Eq. (4.6) and (4.7) are bounded by the maximum available voltage, $V_{max}$. Therefore, the two state variables, $x_1$ and $x_2$, may not be controlled independently any more. In other words, the $d$- and $q$-axis currents are strongly coupled with each other and cannot be controlled independently in the constant power region because the terminal voltage is always bounded by the maximum available DC bus voltage.

Tremendous effort has been put into trying to decouple the control between the $d$- and $q$-axis currents, which is actually against the nature of the electric machine equations. Instead of applying decoupling control to the $d$- and $q$-axis currents, the
The single-current-regulator algorithm tries to implement the coupling relationship between the \(d\)- and \(q\)-axis currents.

Let \(u_2\) be a constant \(C\) in Eq. (4.7). Eq. (4.8) can be rewritten as,

\[
\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + u_1 \tag{4.10}
\]

\[
\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + C \tag{4.11}
\]

In Eq. (4.11), it is not difficult to observe that the state variable \(x_2\) follows the equation naturally if \(x_1\) is controlled strictly to follow a command. If active control is applied to the state variable \(x_2\), the applied control might fight with the nature of the above two equations.

Applying the above algorithm into the PM machine control in the constant power operation region, the \(q\)-axis voltage is kept to be a constant and active control is only applied to the \(d\)-axis current. The \(q\)-axis current will follow Eq. (4.11) automatically. Since active control is applied to the \(d\)-axis current, the required \(d\)-axis voltage can be generated by the \(d\)-axis current controller to satisfy Eq. (4.10).

Figure 4.3 demonstrates the control block diagram of the single-current-regulator algorithm. The speed controller generates the torque command, which is converted into the \(d\)-axis current command by multiplying a negative constant \(K\) simply because the demagnetizing current is always opposite to the flux linkage generated by the permanent magnets. The \(d\)-axis current command contains both the demagnetizing current information and the torque information. The output of the \(d\)-axis current controller is regarded as the \(d\)-axis voltage command. No current control is applied for the \(q\)-axis current. Instead, a fixed value is given to the \(q\)-axis voltage command. The \(q\)-axis current will follow Eq. (4.11) automatically for variable speeds and load conditions. By using the single-current-regulator algorithm, no conflict will occur.
between the two current regulators anymore, resulting in high system stability even in the deep field weakening operation region.

![Single-current-regulator algorithm](image)

Figure 4.3: Single-current-regulator algorithm

### 4.2.2 Optimal Q-axis Voltage Control

In order to achieve better operating efficiency and to fully use the DC bus voltage, an optimal q-axis voltage command is much preferred for variable speeds and load conditions. Figure 4.4 shows three different values for the q-axis voltage command, where $V_{FWC1} < V_{FWC2} < V_{FWC3}$.

The solid curve indicates a constant torque curve and the torque is assumed to be $T_e$. The dash-dot curve is the voltage imposed limit ellipse at the speed of $\omega_e$. The operation point corresponding to $V_{FWC1}$, Point A, is located within the voltage imposed ellipse in the $i_d$-$i_q$ plane, which indicates that the terminal phase voltage is still lower than the maximum allowable voltage and the DC bus voltage is not fully used. Operating at Point A can satisfy the required speed and torque but at the expense of higher current and copper losses.

If $V_{FWC3}$ is chosen to be the q-axis voltage, the operation point has to be Point C if the torque $T_e$ is required. However, Point C is outside the voltage imposed ellipse.
Figure 4.4: Optimal $q$-axis voltage control for single-current-regulator algorithm at the speed of $\omega_e$, which means that the desired speed or torque is not achievable using $V_{FWC3}$.

Therefore, $V_{FWC1}$ is a better choice than $V_{FWC3}$. Compared to $V_{FWC1}$, $V_{FWC2}$ is even better because not only the DC bus voltage is fully used but also the current and conduction losses to achieve the same amount of desired torque are smaller. Therefore, the optimal $q$-axis voltage for a certain speed of $\omega_e$ and the desired torque of $T_e$ is the $q$-axis voltage at the intersection of the voltage imposed ellipse of $\omega_e$ and the constant torque curve of $T_e$.

Figure 4.5 shows the flowchart to calculate the optimal $q$-axis voltage for variable speed and torque. For a certain operating speed and desired torque, the corresponding operating point is first found in the $i_d$-$i_q$ map. The next step is to substitute the $i_d$
and $i_q$ values into the voltage equation under steady state to find out the optimal $q$-axis voltage. Searching the optimal $q$-axis voltage is conducted off-line and a two-dimensional look-up table is used to store the values for the optimal $q$-axis voltage in terms of variable speed and torque.

\[ V_{FWC} = R_s i_{qs} + \omega_e (\lambda_m + L_d i_{ds}) \]

Figure 4.5: Calculating optimal $q$-axis voltage offline

Figure 4.6 shows the optimal $q$-axis voltage in terms of the electromagnetic torque and the mechanical speed for the 50 kW interior PM machine.

It is can be observed from Figure 4.6 that the optimal $q$-axis voltage does not change much for light load operation in a wide speed range. Using a fixed command for the $q$-axis voltage may not affect the machine performance a lot for no load and light load operation. However, under heavy load operation conditions, the optimal $q$-axis voltage change dramatically with the change in speed. If the $q$-axis voltage is fixed for all speeds under heavy load conditions, the machine either cannot achieve
the desired torque or will be able to achieve the command torque with much higher conduction losses.

4.2.3 Computer Simulation and Analysis

Computer simulation is conducted using the 50 kW interior PM machine. The parameters and ratings of the machine can are found in Table A.1. In computer simulations, the DC bus voltage is reduced so that the field weakening operation starts from 300 rpm instead of 850 rpm.

Figure 4.7 shows that the IPM machine accelerates from 300 rpm to 2100 rpm which is 7 times the base speed, maintains at 2100 rpm for 10 s and decelerates again with a constant load of 40 Nm.
First, the $q$-axis voltage command $v_q$ is always maintained at a constant value for the entire process of acceleration and deceleration in the field weakening operation region. Figure 4.8 shows the waveforms of the phase current and the phase voltage.

Secondly, the optimal $q$-axis voltage control adaptive to variable speeds and load conditions discussed in the previous part is applied to run the machine over the same speed profile and load condition as in the first simulation. The simulation results are provided in Figure 4.9.

In Figure 4.8, the phase voltage does not reach the maximum level when the machine is accelerating and decelerating, which means that the DC bus voltage is not fully used. In Figure 4.9, the phase voltage always maintains at the maximum level for all speeds, which means that the DC bus voltage is always fully utilized. Since the DC bus voltage is not fully utilized for the fixed $q$-axis voltage control, as
Figure 4.8: Simulation results: phase current and phase voltage when interior PM machine accelerating to 7 times the base speed using single-current-regulator algorithm with fixed $q$-axis voltage control.

As expected, a larger phase current is needed to run the machine at the same speed and load condition.

### 4.2.4 Experimental Results

The adaptive single-current-regulator algorithm is implemented on a 50 kW interior PM machine. The detail of hardware system setup is described in Appendix A. The system control block diagram is shown in Figure 4.10. Sensorless algorithms presented in the previous chapter are used to obtain the rotor position used for the close loop vector control.

Figure 4.11 shows the experimental results when the IPM motor accelerates from 300 rpm to 2000 rpm. The top waveform is the load torque and the bottom one is
Figure 4.9: Simulation results: phase current and phase voltage when interior PM machine accelerating to 7 times the base speed using single-current-regulator algorithm with optimal $q$-axis voltage control
Figure 4.10: System block diagram using single-current-regulator with optimal $q$-axis voltage control
the machine speed. At steady state, the IPM motor outputs a mechanical power of 49.5 kW at 2000 rpm, which is about 2.4 times the base speed. Figure 4.12 shows the DC bus voltage and the three-phase current. The detail of the three-phase current is shown in Figure 4.13.

Figure 4.14 to 4.16 show the experimental results when the IPM motor accelerates from about 300 rpm to 6000 rpm which is 7 times the base speed. At 6000 rpm, the mechanical output power of the IPM machine is about 28 kW.

![Graph](image)

**Figure 4.11:** Experimental results: speed and torque of interior PM machine accelerating to 2000 rpm with full load

In the deep field weakening operation region, the single-current-regulator algorithm demonstrates its superiority over the conventional two-current-regulator algorithm. By using the single-current-regulator algorithm, the conflict between the two
Figure 4.12: Experimental results: DC bus voltage and phase current of interior PM machine accelerating to 2000 rpm with full load

Figure 4.13: Experimental results: three-phase current of interior PM machine accelerating to 2000 rpm with full load
Figure 4.14: System Block Diagram

Figure 4.15: Experimental results: DC bus voltage and three-phase current when IPM motor accelerating to 6000 rpm with 50% load
saturated current regulators no longer exists. Repeated tests show that the IPM machine can run in a stable manner even when the operation region is reduced to a very small ellipse in the $i_d$-$i_q$ plane.

### 4.3 Voltage Phase Angle Control

This section presents the voltage phase angle control algorithm for the deep field weakening operation of PM machines. In the deep field weakening operation region, it is the angle between the voltage vector and the axis of the permanent magnets that substantially affects the torque delivery of the machine at certain speed rather than the magnitude of the applied voltage vector since we have noticed that the terminal voltage of a PM machine almost reaches the maximum available value and does not change much under the field weakening operation.
The power angle, $\gamma$, of a PM machine is defined as the angle between the voltage vector applied to the machine and the back EMF which is always leading the axis of the permanent magnet by 90 electrical degrees.

The power of a PM machine can be expressed by the power angle as in Eq. (4.12) [41].

$$P = \frac{V_s E \sin \gamma}{X}$$

(4.12)

where $X$ is the reactance of the machine windings. Therefore, the electromagnetic torque can also be expressed by the power angle,

$$T_e = \frac{V_s E \sin \gamma}{\omega_m X}$$

(4.13)

Assuming $V_s$, $E$, $X$ and $\omega_m$ are constant in the above equation, the electromagnetic torque is directly associated with the power angle, $\gamma$. Therefore, the voltage phase angle control applies a voltage vector with a constant magnitude and the angle of the voltage vector is generated by the torque command information. Figure 4.17 shows the control block diagram of the voltage phase angle algorithm.

When the speed is below the base speed, the traditional two current regulators are applied, one for the $d$-axis current regulation and the other for the $q$-axis current regulation. When the speed is approaching the base speed, the system switches to the voltage vector control algorithm, where the error between the command speed and the actual speed is regarded as the input of the phase angle controller. The output of the phase angle controller $\gamma$ contains both the torque information and the field weakening information. The commands of $d$- and $q$-axis voltages can be obtained from the equations enclosed in the dash-line box in Figure 4.17, where $V_m$ is the maximum available phase voltage. By using the voltage vector control, the
magnitude of the phase voltage always maintains at the maximum available level and the direction of the voltage vector will be adjusted automatically according to the command speed and load conditions.

Compared to the traditional current vector control, the direct voltage vector control algorithm only applies one phase angle controller. Hence, the conflict between the two current regulators is completely eliminated. The direct voltage vector control algorithm also makes full use of the DC bus voltage, which helps to achieve the maximum possible torque capability in the field weakening operation region.

Experiments have been carried out on a surface PM machine applied on direct drive washing machines. Some of the important ratings and parameters of the PM machine under test are listed in Appendix A at the end of this work. Some experimental results are shown in Figure 4.18 and 4.19. When the speed is below the base speed 250 rpm, two current regulators are applied for the current vector control. In Figure 4.18, the PM motor accelerates from 250 rpm to 1200 rpm, which is about 5 times
the base speed. The first blue line represents the magnitude of the phase voltage, which always remains at the maximum level throughout the whole field weakening operation region. The third green waveform is the Phase A current from 250 rpm to 1200 rpm and the detailed phase current at 1200 rpm at steady state is shown in Figure 4.19.

![Graph showing experimental results](image)

Figure 4.18: Experimental results: magnitude of phase voltage, speed and phase current of surface PM machine accelerating to 5 times the base speed

### 4.4 Summary

This chapter starts with a discussion of the control issues of the deep field weakening operation of PM machines. A single-current-regulator algorithm with optimal $q$-axis control voltage control is proposed for the deep field weakening operation of interior PM machines. Computer simulations and experimental results on a 50 kW
Figure 4.19: Experimental results: current detail of surface PM machine accelerating to 5 times the base speed

interior PM machine are shown including a 7:1 constant power speed ratio operation. The phase angle control algorithm is applied to the deep field weakening operation of a surface PM machine. The phase angle control applies a voltage vector with a fixed magnitude and the angle of the voltage vector is regulated in terms of variable speed and torque.
CHAPTER 5

Conclusions and Future Work

5.1 Summary and Conclusions

The major motivations and goals of this work are to develop simple yet reliable sensorless vector control algorithms for PM machines over a wide range of speed from zero speed to the top speed. This work gives a thorough introduction to topics including the mathematical modeling, operation capabilities and constraints, sensorless vector control algorithms and deep field weakening control for PM machines.

The major conclusions drawn from the research work in this dissertation include:

- The limited DC bus voltage is the major operation constraint and imposes a difficult control issue for the deep field weakening operation of PM machines. Command voltage exceeding the maximum available voltage results in saturation of two current regulators which then conflict with each other. Deep field weakening operation also requires a very accurate control of the current vector within a small area, which causes another difficult control issue due to limited switching frequency of the power switches.

- The $d$- and $q$-axis currents are strongly coupled in the field weakening operation region. This work proves that trying to control the $d$- and $q$-axis currents
independently could violate the machine equations. Therefore, the coupling relationship between the \( d \)- and \( q \)-axis currents is implemented in the deep field weakening control instead of trying to apply decouple control for the \( d \)- and \( q \)-axis currents. The single-current-regulator algorithm applies constant \( q \)-axis voltage to the machine and the \( d \)-axis current is controlled actively. Once the \( d \)-axis current is well regulated, the \( q \)-axis current will be generated automatically in terms of variable speed and load.

- Using a fixed value for the \( q \)-axis voltage command in the deep field weakening operation could sacrifice the operation efficiency or torque capability to gain the robustness, which is not an optimal control for variable speed and load. Therefore, criteria for optimal \( q \)-axis voltage for the single-current-regulator algorithm is proposed and implemented on a 50 kW interior PM machine.

- The open loop rotor position estimation based on the voltage model is simple to be implemented. To acquire accurate estimation of the rotor position, the machine parameters should be well-known. In this work, numerous testing was done to identify the machine parameters before the implementation of the voltage model based sensorless algorithm. The high frequency carrier signal based sensorless algorithm is less dependent on the machine parameters. It is a very effective approach to estimate the rotor position at zero and low speed for interior PM machines while many other sensorless algorithms fail. However, under heavy load conditions, the saliency effect of the rotor becomes more vague due to the magnetic saturation effect, which will cause inaccuracy in rotor position estimation.
• It is proved that it is not favored to have the estimated rotor position leading the actual rotor position, which could cause the machine to switch between the motoring and generating modes during high speed operation when a small torque is commanded. On the other hand, making the estimated rotor position lag the actual rotor position by adding some compensation is safer for the deep field weakening operation of PM machines at the cost of losing a small amount of torque capability.

5.2 Future Work

• Improve the dynamic performance of the PM machines using sensorless algorithms.

It is of great interest to do more investigations on the sensorless algorithms applied in this work for various transient operating conditions like the torque step change and the fast speed sweep change. It is definitely more challenging to achieve satisfying performance during the transient operation using sensorless vector control than using traditional position sensors. Using sensorless algorithms, the rotor position estimation and the controller are coupled and always interact with each other. Inaccuracy in rotor position estimation will make the controller to apply inaccurate control further. Inaccurate control will result in the estimation further off the right track, which then cause the whole system diverge and go unstable finally. Therefore, a lot of effort should be put on investigating the robustness and fast response of the rotor position estimation algorithms during fast transients.
• Investigation on other sensorless vector control algorithms.

A brief review was given to many sensorless vector control algorithms in Chapter 1. Among them, sensorless algorithm using pulsating high frequency carrier voltage injection with close loop correction and the close loop sliding mode observer are absolutely competitive solutions for sensorless vector control of PM machines. The sliding mode observer method does not work at zero or very low speed. Therefore, combining the high frequency injection method and the sliding mode observer can be investigated further.

• Improve the dynamic performance of PM machines using the single-current-regulator algorithm.

The voltage command typically will exceed the maximum available voltage during fast transients in the field weakening operation. Large amount of test has proven that the single-current-regulator algorithm is very robust in the deep field weakening operation region under steady state and slow transient conditions. However, extreme fast transient operation conditions like torque step change and very fast speed change could possibly make the $d$-axis current controller saturated. Therefore, further theoretical study and experiments should be conducted to test the single-current-regulator algorithm under fast transient working conditions.

• Torque ripple compensation.

A very interesting topic on interior PM machine control is minimizing the torque ripple from the control point of view. Although various advanced machine design techniques can be applied to minimize or cancel part of the torque ripple
of an interior PM machine. It is impossible to remove all harmonic torque ripples from the machine design side. Therefore, minimizing the torque ripple from the machine control point of view becomes absolutely very important for some applications of interior PM machines which require very smooth torque production such as hybrid electric vehicles.
APPENDIX A

Experimental Setup

A.1 50 kW Interior PM Machine Test Setup

Some important parameters and ratings of the 50 kW interior PM machine are listed in Table A.1.

Table A.1: Important parameters and ratings of IPM machine under test

<table>
<thead>
<tr>
<th>Parameters and ratings</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power $P_{\text{rated}}$ (kW)</td>
<td>50</td>
</tr>
<tr>
<td>Rated line-to-line voltage $V_{\text{rated}}$ (V rms)</td>
<td>480</td>
</tr>
<tr>
<td>Rated phase current $I_{\text{rated}}$ (A rms)</td>
<td>120</td>
</tr>
<tr>
<td>Characteristic current $I_{ch}$ (A rms)</td>
<td>76</td>
</tr>
<tr>
<td>Base speed (rpm) $\omega_b$</td>
<td>850</td>
</tr>
<tr>
<td>Top speed (rpm) $\omega_{\text{max}}$</td>
<td>8000</td>
</tr>
<tr>
<td>Number of poles</td>
<td>8</td>
</tr>
<tr>
<td>D-axis inductance $L_{ds}$ (mH)</td>
<td>10</td>
</tr>
<tr>
<td>Q-axis inductance $L_{qs}$ (mH)</td>
<td>22</td>
</tr>
</tbody>
</table>

The configure of the interior PM machine system test setup is shown in Figure A.1. The controller used is dSPACE DS1103 which is also used for the data acquisition.

The dPSACE interface circuit boards include three major functions:
• signal conditioning for the outputs of the voltage and current transducers before they are sent to the A/D ports of the dSAPCE controller board;

• converts the electrical switching signals from the dSPACE control board to light signals which are then sent to the gate drive board by fiber optics to drive the inverter;

• routes the outputs of the incremental encoder to the dSPACE controller board.

Figure A.1: System configuration of 50 kW Interior PM machine test setup

The 480 V three-phase voltage is fed to a 100 hp power converter through a three-phase isolation transformer. The power converter consists of a rectifier and an inverter. The switching signals are sent to the inverter from fiber optics. A DC chopper circuit is connected to the DC link of the inverter for dumping the energy to
the resistor bank in case the machine loses control at high speed and the high back EMF produced by the permanent magnets could potentially damage the bus of the inverter.

One end of the Device Under Test (DUT) interior PM machine is coupled to a 460 kW AC induction machine used as the load by a gearbox with a ratio of 2.52:1. The induction machine is driven by an Allen-Bradley drive. It has an incremental encoder and a slip ring mounted on the other end of the PM machine shaft. Some thermal couples installed on the rotor to detect the magnet temperature are wired out through the slip ring. A picture of the real system is shown in Figure A.2.

Figure A.2: 50 kW interior PM machine test system
A.2 500 W Disk Surface PM Machine Setup

Some important machine parameters and ratings of the 500 W surface PM machine are listed as following. The stack length of the surface PM machine is much shorter compared to the diameter of the machine, which makes the machine look like a disk. That is why it is also called a disk motor. A picture of the test system is shown as in Figure A.3. The surface PM machine is installed on a Magtrol hysteresis dynamometer.

Table A.2: Parameters and ratings of 500 W surface PM machine user on washer

<table>
<thead>
<tr>
<th>Parameters and ratings</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power $P_{\text{rated}}$ (W)</td>
<td>500</td>
</tr>
<tr>
<td>Rated DC link voltage $V_{DC:\text{rated}}$ (V)</td>
<td>300</td>
</tr>
<tr>
<td>Rated phase current $I_{\text{rated}}$ (A rms)</td>
<td>6</td>
</tr>
<tr>
<td>Base speed (rpm) $\omega_b$</td>
<td>250</td>
</tr>
<tr>
<td>Top speed (rpm) $\omega_{\text{max}}$</td>
<td>1250</td>
</tr>
<tr>
<td>Number of poles</td>
<td>24</td>
</tr>
<tr>
<td>Resistance of phase winding (Ohms)</td>
<td>4.3</td>
</tr>
<tr>
<td>Self-inductance of phase winding $L_s$ (mH)</td>
<td>40</td>
</tr>
<tr>
<td>Flux linkage of permanent magnet $\lambda_m$ (V-s)</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Figure A.3: 500 W surface PM machine test system
Filter Design for Sensorless Vector Control Algorithm Based on High Frequency Carrier Signal Injection

B.1 Second-order High Pass Filter

The transfer function of a second-order high pass filter in the s-domain is expressed as,

\[ H(s) = \frac{(\frac{s}{\omega_c})^2}{(\frac{s}{\omega_c})^2 + d \frac{s}{\omega_c} + 1} \]  \hspace{1cm} (B.1)

where \( \omega_c \) is the cut-off frequency and \( d \) the damping factor. The second-order high pass filter is applied to separate the information of the carrier current component which has a frequency of 200 Hz from the fundamental current component which is typically lower than 5 Hz.

The cut-off frequency and the damping factor for the high pass filter are from the trial and error testing. In this application, the cut-off frequency is chosen to be 20 Hz and the damping factor is selected to be 1.414.

The discrete form of the above second-order high pass filter is written as,

\[ H(z) = \frac{\alpha(1 - 2z^{-1} + z^{-2})}{0.5 - \gamma z^{-1} + \beta z^{-2}} \]  \hspace{1cm} (B.2)

The difference equation for the above equation is expressed as,

\[ y(n) = 2\{\alpha[x(n) - 2x(n - 1) + x(n - 2)] + \gamma y(n - 1) - \beta y(n - 2)\} \]  \hspace{1cm} (B.3)
where the coefficients $\alpha$, $\beta$ and $\gamma$ in the above equation are defined as,

$$\beta = \frac{1}{2} \left(1 - 0.5 \xi \sin \theta_c\right)$$

(B.4)

$$\gamma = (0.5 + \beta) \cos \theta_c$$

(B.5)

$$\alpha = \frac{0.5 + \beta + \gamma}{4}$$

(B.6)

where the digital domain normalized frequency $\theta_c$ equals $2\pi f_c/f_s$. $f_s$ is the sampling frequency of the controller.

### B.2 Sixth-order Butterworth Low Pass Filter

The transfer function of a sixth-order low pass filter in the frequency domain is expressed as,

$$H(s) = \frac{1}{\left(\frac{s}{\omega_c}\right)^2 + d_1 \frac{s}{\omega_c} + 1} \cdot \frac{1}{\left(\frac{s}{\omega_c}\right)^2 + d_2 \frac{s}{\omega_c} + 1} \cdot \frac{1}{\left(\frac{s}{\omega_c}\right)^2 + d_3 \frac{s}{\omega_c} + 1}$$

(B.7)

where $\omega_c$ is the cutoff frequency and $d_k$ is the $k^{th}$ damping factor.

The discrete form of the above second-order high pass filter is written as,

$$H(z) = \frac{\alpha_1 (1 + 2z^{-1} + z^{-2}) \cdot \alpha_2 (1 + 2z^{-1} + z^{-2}) \cdot \alpha_3 (1 + 2z^{-1} + z^{-2})}{0.5 - \gamma_1 z^{-1} + \beta_1 z^{-2}} \cdot \frac{\alpha_2 (1 + 2z^{-1} + z^{-2})}{0.5 - \gamma_2 z^{-1} + \beta_2 z^{-2}} \cdot \frac{\alpha_3 (1 + 2z^{-1} + z^{-2})}{0.5 - \gamma_3 z^{-1} + \beta_3 z^{-2}}$$

(B.8)
where the coefficients in the above equation are defined as,

\[
\beta_k = \frac{1 - 0.5d_k \sin\theta_c}{2(1 + 0.5d_k \sin\theta_c)} \quad \text{(B.9)}
\]

\[
\gamma_k = (0.5 + \beta_k) \cos\theta_c \quad \text{(B.10)}
\]

\[
\alpha_k = \frac{(0.5 + \beta_k - \gamma_k)}{4} \quad \text{(B.11)}
\]

where the digital domain normalized frequency \( \theta_c \) equals \( 2\pi f_c/f_s \). The damping factors of the sixth-order low pass filter are given by Eq. (B.12),

\[
d_k = 2 \sin \frac{(2k - 1)\pi}{2N} \quad \text{(B.12)}
\]

\[
y_1(n) = 2\alpha_1[x(n) + 2x(n - 1) + x(n - 2)] + \gamma_1y_1(n - 1) - \beta_1y_1(n - 2) \quad \text{(B.13)}
\]

\[
x_2(n) = y_1(n) \quad \text{(B.14)}
\]

\[
y_2(n) = 2\alpha_2[x_2(n) + 2x_2(n - 1) + x_2(n - 2)] + \gamma_2y_2(n - 1) - \beta_2y_2(n - 2) \quad \text{(B.15)}
\]

\[
x_3(n) = y_2(n) \quad \text{(B.16)}
\]

\[
y(n) = 2\alpha_3[x_3(n) + 2x_3(n - 1) + x_3(n - 2)] + \gamma_3y(n - 1) - \beta_3y(n - 2) \quad \text{(B.17)}
\]
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